SUBOPTIMAL CONTROL OF INTERCONNECTED POWER SYSTEMS

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CHAPTER I

INTRODUCTION

1.1 Statement of the Problem

Nowadays most electric power generating units are tied together to form a large interconnected power system. The primary motivation for interconnection is to obtain the economic benefits of new large-scale generation and transmission facilities. Advantage is taken to transfer power generation over interconnecting tie-lines from an area of low demand to one of high demand. It also enables utilities to share spinning reserves during emergencies so that spinning reserve requirement in each area is reduced. Thus, overall operating economies and high reliability are achieved.

Interconnections, however, increase the degree of complexity of power system operating problems. These problems arise as a consequence of the complexity of the network topology and of dynamic behavior of the system when subject to disturbances occurring not only internally but also elsewhere in other systems in the interconnections. In order to cope with this problem, control of power system dynamics has become the focus of considerable interest.

Power system control can be divided into two separate problems, namely synchronous machine control and load frequency control. Both of them can be viewed as dynamic control systems in which control is required to damp out system oscillations or swings. Feedback signals of some or all variables are usually used as input to the system controller. The difference between them is that synchronous machine control deals with interactions between one or a few generating units with the rest of the system considered as an infinite bus while load frequency control deals with all of the generating units within the system with equal attention. Since there is a large number of generating units within a system, and the main variables of interest are load demand and energy supply, it is general practice to eliminate electrical dynamics from load frequency control system.

The applications of modern control theory to stabilize power system dynamics for both synchronous machine control and load frequency control problems were proposed in 1970. The technique yielded good damping results. However, if all state variables are not available to be fed back and if the dimension of the system is increased, the computer time and memory for optimal gain calculation will be increased significantly. It is correspondingly important to investigate a new approach to the solution of optimal control of the systems such that it can be applied to large interconnected power systems.

1.2 Literature Review

In this section a review of works in both synchronous machine control and load frequency control are presented. The development of load frequency control is described in the first part. The development of synchronous machine control is described in the second part. Since the conventional control techniques for the load frequency problem is well established and has been used widely for many years, only a general discussion of the technique is presented. More reviews are given to those studies dealing with modern control theory.

1.2.1 Load Frequency Control of Power Systems

The conventional approach to load frequency control of power systems is called tie-line bias control. The comprehensive presentation of the control design can be found in Reference [12]. In this approach the problem is considered as a static one. The design of system control involves steady state quantities. There are three steps to deal with load frequency control. First, total system generation must be matched to total system load. This can be done by speed governor control of the system. The criterion for determining when total demand has been satisfied is an unchanging system frequency. Second, total system generation among the areas is allocated so that each follows its own load chnages and does its share of frequency regulation. This objective is accomplished by net interchange tie-line bias control such that area net interchange is on schedule, i.e., area control error is reduced to zero. Third, each unit should operate at the same incremental cost in order to minimize combined system cost. This is the function of economic dispatch control. Studies of tie-line bias control during dynamic period were carried out by Concordia and Kirchmayer [13] [14].

The control design using optimal control theory was proposed by Elgerd and Fosha [23] [27]. They made use of linear models of the turbine, speed regulator, and power system. Then they derived the optimal feedback gain that minimized the standard integral quadratic objective function via Riccati's equation. The simulation results of system behavior following a disturbance were given. Cavin et al. [11] applied stochastic control theory to the load-frequency control problem. The Kalman filter was used for estimation and the separation theory was used to derive the control law. Miniesy and Bohn [45] considered the demand to be an unknown. Two methods were suggested for demand identification. The first method made use of differential approximation. The second method made use of Luenberger observer. Bohn and Miniesy [7] applied sampled-data control to the problem considered earlier [45]. In that paper an adaptive observer was introduced and its effectiveness was illustrated. Glover and Schweppe [30] proposed a discrete time, linear-plus-deadband, feedback control law. A simulation of system response to a step load change was presented. Calovic [9] considered the control based on a combination of conventional and optimal control design. Results of a digital simulation of the optimal system showed significant improvement of system transients while maintaining the desired steady-state characteristics. His proportional-plus-integral control law extended to multi-area interconnections was presented in Reference [10]. Recently, Kwatny et al. [41] formulated the load frequency control as a tracking problem instead of a regulation one. In their paper the prime mover energy source was recognized as a part of the system dynamic model. The control system included estimation and prediction of loads which were used to regulate power flow and frequencies.

1.2.2 Synchronous Machine Control System

In the early studies of stabilization of synchronous machine dynamics most researchers focused their attention on the so-called excitation control. Ellis et al. [24] made a stability study of the Peace River transmission system and proposed that the stability could be improved by using speed error as an input to the excitation system. Shier and Blythe [55] confirmed that idea by computer simulation and field tests. They demonstrated that a practical stabilizer can be devised using simple electrical devices. Hanson et al. [33] studied the oscillation control by reducing gains on automatic voltage regulators. They carried out a series of tests and found that the system obtained was properly damped. deMello and Concordia [19] reported analytical results concerning excitation control of power system dynamic stability. The gain parameters of the voltage regulator that stabilized the system was derived. The transfer function for speed-derived signals was also studied. Byerly et al. [8] studied the use of electrical power as an auxiliary signal input to the excitation system. The paper included the effects of rotor-iron saturation on generator damping. Schleif et al. [54] showed that damping was improved by supplementing excitation control with a derived function of frequency deviation. Results of the studies were verified in actual field tests.

The application of the modern control theory to power systems was proposed by Yu et al. [58]. They applied the optimal control to minimize an integral quadratic performance index of a power system. All the state variables were assumed to be measurable. The constant feedback gain was obtained by solving the Riccati's equation. Anderson [1] reported a similar approach to a slightly different model. In his paper simplified Park's synchronous machine variables [49] were used. The comparison of the optimal control technique to that of the excitation control was carried out by Yu and Siggers [59]. They found that a well-designed system obtained from an excitation control technique yielded the results which were as good as those obtained from optimal control technique. However, the design procedure of the excitation control technique had to be done in trial-and-error fashion. Davison and Rao [18] considered the problem where not all state variables were available for measurement. They solved this problem by using a gradient method of parameter optimization. Elangovan and Kuppurajulu [21] considered another approach to the limited state variable feedback problem. They reduced the dimension of the original state vector to the one that had only measurable variables. The technique which retained dominant eigenvalues was applied to the problem. Yu and Moussa [60] made a study of multimachine control system. A reduced-order model was used. They found that a controller obtained from the multimachine system design was better than the one that was obtained from a one-machine infinite-bus system design. Moussa and Yu [46] developed a method to determine the weighting matrix Q such that the dominant eigenvalues were shifted to the left in the complex plane as far as the practical controllers permitted. They applied the eignevalue sensitivity analysis technique to the problem. By this method the weighting matrix Q can be determined analytically. Habibullah and Yu [31] presented a method to determine both weighting matrices Q and R. Their controls were found to be able to stabilize the system under a wide range of operations. Elmetwally et al. [25] presented a method of optimal control in which the system controllable parameters were selected so as to correspond to the region of near zero sensitivity. Elmetwally et al. [26] and Newton and Hogg [47] reported the implementation of the optimal controller to real micro-machines. Experimental results showed that the controller worked well under small disturbances. Daniels et al. [17] developed a technique to determine a control which is a linear combination of some selected state variables. They used an unconstrained optimization routine to minimize the performance index with respect to the nonlinear system of differential equations. The synthesized controller was implemented on a micro-machine and the experimental results demonstrated the advantages of the technique. Raina et al. [52] presented a method of optimal control of power systems. Modification to the usual proportional controller was suggested and a good damping response was found under a wide range of operating conditions. Quintana et al. [51] studied an optimal output feedback control design with a compensator. A number of combinations of measurable output variables were used as input to the controller.

1.3 Research Objective

The objective of this research is to develop a suboptimal control technique for interconnected power systems. In the first part, a fixed configuration control whose controllers are a linear transformation of some certain state variables will be formulated. Attempts will be made to subdivide the interconnected system into subsystems. Necessary conditions for optimality as functions of these subsystems will then be derived. Since the dimensions of the subsystem matrices are less than those of the original interconnected system, it is expected that calculations of optimal gain in subsystem equations will require less computer time and memory than using the original equations. In the second part, applications of the results obtained from the first part to interconnected power systems will be studied. Linear system models for both multi-area load frequency control systems and interconnected synchronous machine control systems will be formulated. The optimal control and the suboptimal control gains will be calculated and compared.

CHAPTER II

OPTIMAL LIMITED STATE VARIABLE FEEDBACK CONTROL OF LINEAR STOCHASTIC SYSTEMS

2.1 Introduction

Over the past years considerable contributions were made in the area of optimal limited state variable feedback control systems. Different models were used by different researchers. Levine and Athans [42] reported necessary conditions for a deterministic linear time-invariant control system. The initial condition for the state vector was assumed to be a set of random variables which were uniformly distributed on the surface of the n-dimensional unit sphere. Sims and Melsa [56] worked on a linear stochastic system. A filter which was a linear combination of state variables and control variables was used. The dimension of the filter was prespecified. The control was assumed to be a linear transformation of the filter. They found that performance does not depend on the filter dynamics. McLane [43] considered a system in which the plant noise was dependent on both state and control variables. In his study the measurement noise was not presented. Recently, Mendel [44] provided necessary conditions for a linear time-invariant stochastic system. In that paper an infinite final time for the performance index was considered.

Assumptions were made so that the compensation plant matrix could be optimized.

In this study the interconnected power system will be represented by a linear time-invariant model with or without plant noise. Since the results for the deterministic case was given in Reference [42], in this chapter necessary conditions for limited state variable feedback control of a linear time-invariant stochastic system with perfect measurement will be derived. It will be seen that even though the assumptions and the approach used in this derivation are different from those of Reference [42] the results are very similar. Thus, with a minor change the approach given in this chapter is applicable to both the deterministic case and the stochastic case with perfect measurement.

2.2 Optimization Problem Formulation

Consider a first-order system of linear equations.

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t) + Dw(t) \tag{2.1}$$

$$y(t) = Cx(t) \tag{2.2}$$

where $x(t)$ is a state vector of dimension n; $u(t)$ is a control vector of dimension s; $y(t)$ is an output vector of dimension m; and $w(t)$ is a noise vector of dimension ℓ. A, B, C, and D are constant matrices of compatible dimensions. The noise vector is assumed to be white with zero mean and its covariance is:

$$E\{w(t)w^T(\tau)\} = V\delta(t-\tau) \tag{2.3}$$

where V is a positive definite noise intensity matrix. Mean and covariance of the initial states are:

$$E\{x(t_o)\} = x_o \tag{2.4a}$$

$$E\{x(t_o)x^T(t_o)\} = Q_o \tag{2.4b}$$

The performance index to be minimized is:

$$J_o = E \int_{t_o}^{t_f} \{x^T(t)\; Qx(t) + u^T(t)\; Ru(t)\}dt \tag{2.5}$$

where Q is a positive semi-definite matrix and R is a positive definite matrix. Let the control, u(t), be constrained to be a linear transformation of the output vector, i.e.,

$$u(t) = Ky(t) \tag{2.6}$$

where K is the constant matrix to be determined. From Equations (2.1), (2.2), (2.5), and (2.6) we have

$$\frac{dx(t)}{dt} = (A + BKC)\; x(t) + Dw(t) \tag{2.7}$$

$$J_o = E \int_{t_o}^{t_f} x^T(t)\; [Q + C^TK^TRKC]\; x(t)dt \tag{2.8}$$

From Theorem 1.54 of Reference [39], the Equations (2.7) and (2.8) may be rewritten as:

$$J_o = tr\{P(t_o)Q_o + \int_{t_o}^{t_f} DVD^TP(t)dt\} \tag{2.9}$$

where P(t) satisfies

$$-\frac{dP(t)}{dt} = (A + BKC)^T\; P(t) + P(t)(A + BKC) + Q + C^TK^TRKC \tag{2.10}$$

$$P(t_f) = 0$$

If (A + BKC) is asymptotically stable, P(t) has a steady state value as t approaches infinity. Let P be the steady state value of P(t). Equation (2.10) becomes

$$0 = (A + BKC)^T P + P(A + BKC) + Q + C^T K^T RKC \quad (2.11)$$

If t_f approaches infinity, Equation (2.9) becomes

$$J_1 = \lim_{t_f \to \infty} J_o = \lim_{t_f \to \infty} \{tr[P(t_o)Q_o + (t_f - t_o)DVD^T P]\} \quad (2.12)$$

In order to avoid an infinite number in Equation (2.12), let us define a new performance index:

$$J = \lim_{t_f \to \infty} \frac{J_1}{t_f - t_o}$$

$$J = tr(DVD^T P). \quad (2.13)$$

2.3 Statement of the Problem

Given the plant matrices A, B, C, D, white noise intensity V, weighting matrices Q, R, and the performance index,

$$J = tr(DVD^T P) \quad (2.14)$$

where P is the solution of the equation,

$$(A + BKC)^T P + P(A + BKC) + Q + C^T K^T RCK = 0 \quad (2.15)$$

Find the real constant matrix K which minimizes J assuming that K makes (A + BKC) a stable matrix, i.e., all of its eigenvalues have negative real parts.

2.4 Necessary Conditions for Optimality

The main result is summarized in the following theorem.

Theorem 2.1: Let K be a real matrix. Assuming that (A + BKC) is stable, then, in order for K to be optimal for the problem defined in section 2.3, it is necessary that

$$K = -R^{-1}B^TPZC^T(CZC^T)^{-1} \tag{2.16a}$$

where Z satisfies the equation

$$DVD^T + (A + BKC)Z + Z(A + BKC)^T = 0 \tag{2.16b}$$

and P satisfies the equation

$$(A + BKC)^TP + P(A + BKC) + Q + C^TK^TRKC = 0 \tag{2.16c}$$

Proof:

The necessary conditions for optimality are derived by applying the gradient matrix concept [2] [3] to an augmented function L.

Define:

$$L = tr[DVD^TP] + tr\{[(A + BKC)^TP + P(A + BKC) + Q + C^TK^TRKC]Z^T\} \tag{2.17}$$

where Z is an n x n multiplier matrix.

The conditions for extremum are

$$\frac{\partial L}{\partial K} = 0 \tag{2.18a}$$

$$\frac{\partial L}{\partial P} = 0 \tag{2.18b}$$

$$\frac{\partial L}{\partial Z} = 0 \qquad (2.18c)$$

By some matrix manipulation of Equation (2.18) and using the results of Reference [2], Equation (2.16) is obtained.

2.5 Properties of Matrices at the Extremum Condition

It should be noted that Equation (2.16b) and (2.16c) have the same form as the Lyapunov matrix equation. Properties of this equation are given in the following theorems. The proof of these theorems can be found in References [5] and [28].

<u>Theorem 2.2</u>: Given the Lyapunov matrix equation

$$AX + XB + C = 0 \qquad (2.19)$$

where A, B, and C are $m \times m$, $n \times n$, and $m \times n$ matrices, respectively. Let λ_i, $i = 1, 2, \ldots, m$, and μ_j, $j = 1, 2, \ldots, n$ denote the eigenvalues of A and B, respectively. Then Equation (2.19) has a unique $m \times n$ matrix solution X if and only if for all i, j

$$\lambda_i + \mu_j \neq 0.$$

<u>Theorem 2.3</u>: Given the Lyapunov matrix Equation (2.19). If all the eigenvalues of A and B have negative real parts, then Equation (2.19) has a unique solution given by:

$$X = \int_0^\infty e^{At} C \, e^{Bt} dt. \qquad (2.20)$$

Theorem 2.4. Given the Lyapunov matrix Equation (2.19). If C is nonnegative definite, all eigenvalues of A and B have negative real parts, and

$$A = B^T$$

then X is a constant symmetric nonnegative definite matrix.

From the above theorems, properties of matrices satisfying Equation (2.16) are as follow:

(1) It can be proven by means of Theorem 2.3 that Equation (2.16) has a unique solution, K, P, and Z, that yields a stable system. Furthermore, it follows from Theorem 2.4 that P and Z are nonnegative definite.

(2) The reverse of (1), however, is not true. It has been found that there exists a K which satisfies Equation (2.16) but does not stabilize the system. The corresponding P and Z are not nonnegative definite. This has been a cause of trouble in determining optimum feedback gain of the system.

(3) If C is an identity matrix, the necessary conditions, Equation (2.16), are the same as those of the state feedback control problem.

(4) For the deterministic case the necessary conditions, to solve for K, are obtained by substituting for DVD^T in Equation (2.16b), an identity matrix. The result is the same as the one given in Reference [42]. It should be noted that this does not imply that either D or V is an identity matrix.

(5) If D or V is a null matrix, we get a singular problem. It can be verified by writing the closed-form solution of Equation (2.16b) as

$$Z = \int_{0}^{\infty} \{\exp\,(A + BKC)^{T}t\}\,\{DVD^{T}\}\,\{\exp(A + BKC)t\}dt$$

if $DVD^{T} = 0$, then $Z = 0$.

(6) The solution of Equation (2.16) is, clearly, dependent on the noise intensity V. However, even though V has a significant effect on Z, it has been found that V has a smaller effect on K.

CHAPTER III

SUBOPTIMAL CONTROL OF INTER-CONNECTED SYSTEMS

3.1 Introduction

One problem that is encountered very often in practical controller design of interconnected systems is long computer time and large memory requirements. It arises as a consequence of the large dimensions of the systems. Because an unlimited computer capability is not usually available, the design is generally carried out by using a simplified model. Two methods of model simplification which are usually found are decoupling and deleting of state variables. The decoupled system model is utilized if certain portions of the system are weakly coupled such that it may be possible to break the system into several mutually exclusive low-order subsystems. This model is usually used in systems consisting of many subsystems or components and the effects of interaction between them are negligible. Deletion of state variables can be applied to those variables which have small contribution to system dynamics. But in some cases where computer burden indicates that it is necessary to simplify the model, state variables which are not considered negligible are also eliminated. Here the designer must depend on the physical understanding of the system in selecting which state variable to delete. It must be done very carefully and always with some risk. These two methods of model simplification can reduce the computation to a large extent. However, the quality of system performance is sometimes unsatisfactory and instability may result if the model is simplified improperly.

In this chapter another technique for limited state variable feedback controller design is presented. It is achieved by dividing the interconnected system into several subsystems. The feedback gain matrix is derived by Taylor series expansion of matrices K, P, and Z (see Chapter II) with respect to a coupling parameter. This technique does not require model simplification so that generally a better performance should be obtained. It also makes use of the low-order subsystem to offer less computation. Thus it is expected that the technique is suitable for large scale system controller design without requiring large computer capability.

The method of Taylor series expansion in linear systems was applied earlier by Kokotovic et al. [37] [38] [53] to find an approximate solution to Riccati's equation. In this chapter the method is applied to necessary conditions derived in Chapter II. The results are applicable to both complete and limited state variable feedback control systems with and without plant stochastic noise.

3.2 Interconnected System

Suppose that the system considered in Chapter II consists of subsystems. Each subsystem is described by

$$\frac{dx_i(t)}{dt} = A_i x_i(t) + \varepsilon \sum_{j=1}^{q} A_{ij} x_j(t) + B_i u_i(t)$$

$$+ D_i w_i(t); \qquad i = 1, 2, \ldots, q \qquad (3.1)$$

$$y_i(t) = C_i x_i(t); \qquad i = 1, 2, \ldots, q \qquad (3.2)$$

where $x_i(t)$, $y_i(t)$, $u_i(t)$, $w_i(t)$ are the state, output, control, and noise vectors of the i^{th} subsystem, respectively. $w_i(t)$ is a white noise process with zero mean and

$$E\{w_i(t)\ w_i^T(\tau)\} = V_i \delta(t-\tau); \ i = 1, 2, \ldots, q \qquad (3.3)$$

where V_i is a positive definite noise intensity matrix. Interactions between subsystems are represented by a parameter ε which has a value between 0 and 1. If $\varepsilon = 0$, the interactions are neglected and the interconnected subsystems are decoupled. If $\varepsilon = 1$, Equations (3.1) through (3.3) represent the original interconnected system.

In Chapter II the control is assumed to be a linear combination of the output:

$$u(t) = Ky(t)$$

The problem is to determine the matrix K that minimizes the performance index given in Equation (2.5). With some manipulation and approximations, necessary conditions for optimum K are given in Equation (2.16).

In this chapter K, P, and Z are approximated by a finite term Taylor series expansion about $\varepsilon = 0$, i.e.,

$$K(\varepsilon) \cong \sum_{j=0}^{r-1} \frac{\varepsilon^j}{j!} K^j(0) \tag{3.4a}$$

$$P(\varepsilon) \cong \sum_{j=0}^{r-1} \frac{\varepsilon^j}{j!} P^j(0) \tag{3.4b}$$

$$Z(\varepsilon) \cong \sum_{j=0}^{r-1} \frac{\varepsilon^j}{j!} Z^j(0) \tag{3.4c}$$

where the superscript j on K, P, and Z represent j^{th} partial derivatives of K, P, and Z with respect to ε, respectively, and r is the number of terms in the series. We shall derive necessary conditions for the terms in the series of Equation (3.4).

In order to simplify the problem, we shall work with a system consisting of two coupled subsystem so that

$$A = \begin{bmatrix} A_1 & A_3 \\ A_4 & A_2 \end{bmatrix}$$

$$= \begin{bmatrix} A_1 & \varepsilon A_{12} \\ \varepsilon A_{21} & A_2 \end{bmatrix}$$

where

$$A_{12} = \frac{A_3}{\varepsilon}, \quad A_{21} = \frac{A_4}{\varepsilon}$$

$$B = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}$$

$$C = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}$$

$$D = \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix}$$

$$R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}$$

$$V = \begin{bmatrix} V_1 & 0 \\ 0 & V_2 \end{bmatrix}$$

$$Q = \begin{bmatrix} Q_1 & Q_3 \\ Q_4 & Q_2 \end{bmatrix}$$

$$= \begin{bmatrix} Q_1 & \varepsilon Q_{12} \\ \varepsilon Q_{21} & Q_2 \end{bmatrix}$$

where

$$Q_{12} = \frac{Q_3}{\varepsilon}, \quad Q_{21} = \frac{Q_4}{\varepsilon}$$

It should be noted that the approach used in this study can be extended to a system consisting of an arbitrary number of coupled subsystems. However, one may be faced with a very long expression.

3.2.1 Special-Type Matrices

Since we are going to deal with matrices consisting of four submatrices, some of which have two null submatrices on either the diagonal or the off-diagonal, it is useful to define symbols for some types of those matrices and submatrices.

Let M be any matrix consisting of four submatrices, we shall write

$$M = \begin{bmatrix} M_1 & M_{12} \\ M_{21} & M_2 \end{bmatrix} \tag{3.5a}$$

Let us define two types of matrices:

1. A matrix whose off-diagonal submatrices are null matrices is called an α-type matrix and is written as

$$M_\alpha \triangleq \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \tag{3.5b}$$

2. A matrix whose diagonal submatrices are null matrices is called a β-type matrix and is written as

$$M_\beta \triangleq \begin{bmatrix} 0 & M_{12} \\ M_{21} & 0 \end{bmatrix}$$

From the above definitions it can be verified that:

(i) $M = M_\alpha + M_\beta$ (3.6a)

(ii) If $L = M_\alpha N_\alpha$ (3.6b)

then L is of α-type, where

$$L_1 = M_1 N_1 \quad (3.6c)$$

$$L_2 = M_2 N_2 \quad (3.6d)$$

(iii) If $L = M_\alpha N_\beta$ (3.6e)

then L is of β-type, where

$$L_{12} = M_1 N_{12} \quad (3.6f)$$

$$L_{21} = M_2 N_{21} \quad (3.6g)$$

(iv) If $L = M_\beta N_\alpha$ (3.6h)

then L is of β-type, where

$$L_{12} = M_{12} N_2 \quad (3.6i)$$

$$L_{21} = M_{21} N_1 \quad (3.6j)$$

(v) If $L = M_\beta N_\beta$ (3.6k)

then L is of α-type, where

$$L_1 = M_{12} N_{21} \quad (3.6l)$$

$$L_2 = M_{21} N_{12} \quad (3.6m)$$

We can apply these equations to find submatrix equations from a given matrix equation. For example, let

$$L = M_\alpha N_\beta S_\beta T_\alpha$$

By means of Equation (3.6) we get

L is of α-type

$$L_1 = M_1 N_{12} S_{21} T_1$$

$$L_2 = M_2 N_{21} S_{12} T_2$$

With the definitions in Equation (3.5), A and Q can be written as

$$A = A_\alpha + \varepsilon A_\beta \tag{3.7a}$$

$$Q = Q_\alpha + \varepsilon Q_\beta \tag{3.7b}$$

It also can be seen that the matrices B, C, D, and R are of α-type.

3.3 Necessary Conditions for Optimality in Subsystem Forms

When a system is described by Equations (3.1) through (3.3), the matrices K, P, and Z can be written as Equation (3.4). Necessary conditions for matrices in the series of K, P, and Z are presented in the following theorem. They are written in a general form.

Theorem 3.1: If the matrices K, P, and Z of Equation (2.16) can be presented by the Taylor series, the following equations are necessary conditions for the i^{th} derivative of j^{th} submatrices of the matrices in the series:

$$K_j^i = F(P_j^i,\ Z_j^i) \tag{3.8a}$$

$$(A_m + B_m K_m^o C_m)^T P_j^i + P_j^i (A_n + B_n K_n^o C_n) + G_1 = 0 \quad (3.8b)$$

$$(A_m + B_m K_m^o C_m) Z_j^i + Z_j^i (A_n + B_n K_n^o C_n)^T + G_2 = 0 \quad (3.8c)$$

where $i = 0, 1, 2, 3, \ldots$.

If i is even,

$j = 1, 2$

$m = n = j$.

If i is odd,

$j = 12, 21$

$m = 1$

$n = 2$.

F, G_1, and G_2 are matrix functions of other submatrices. Since submatrices of K, P, and Z of lower derivative than i have already been determined in an earlier step, these submatrices can be considered as constants. By this assumption G_1 and G_2 become constant matrices and F becomes a matrix function of P_j^i and Z_j^i.

The proof of this theorem is presented in the next section. Necessary conditions for the first few terms of the series of K, P, and Z are derived. It can be seen that Equation (3.8) is the generalized form of those equations.

3.4 Derivation of the Results

In the derivation of Theorem 3.1 a certain set of matrix equations is involved. The solution of this set of equations is given in the following lemma.

Lemma 3.1: Given

$$W = C_1XC_2 + C_3YC_4 \tag{3.9a}$$

$$D_1WD_2 + D_2^TW^TD_1^T + D_3X + XD_3^T = 0 \tag{3.9b}$$

$$E_1WE_2 + E_2^TW^TE_1^T + E_3Y + YE_3^T = 0 \tag{3.9c}$$

where W, X, and Y are sxm, nxn, and nxn unknown matrices, respectively; and C_i, i = 1, . . . , 4; D_j and E_j, j = 1, . . . , 3 are constant known matrices of compatible dimension. 0 is an nxn null matrix.

If unique solutions of Equation (3.9) exist, they are

$$W = 0 \tag{3.10a}$$

$$X = 0 \tag{3.10b}$$

$$Y = 0 \tag{3.10c}$$

This result is obtained when one recognizes that Equation (3.9) can be transformed to a set of homogeneous system of equations.

The set of necessary conditions, Equation (2.16), can be rewritten by using Equation (3.7) as

$$K = -R^{-1}B^TPZC^T(CZC^T)^{-1} \tag{3.11a}$$

$$DVD^T + (A_\alpha + \varepsilon A_\beta + BKC)Z + Z(A_\alpha + \varepsilon A_\beta + BKC)^T = 0 \tag{3.11b}$$

$$(A_\alpha + \varepsilon A_\beta + BKC)^TP + P(A_\alpha + \varepsilon A_\beta + BKC) + Q_\alpha + \varepsilon Q_\beta + C^TK^TRKC = 0 \tag{3.11c}$$

Derivations of matrices in the series are presented as follow:

(i) If $\varepsilon = 0$, the two subsystems are completely decoupled. In this case Equation (3.11) may be expressed in submatrix form as follows:

$$K_i^o = -R_i^{-1}B_i^T P_i^o Z_i^o C_i^T \ (C_i Z_i^o C_i^T)^{-1};\ i = 1,\ 2 \qquad (3.12a)$$

$$D_i V_i D_i^T + (A_i + B_i K_i^o C_i) Z_i^o + Z_i^o (A_i + B_i K_i^o C_i)^T$$
$$= 0;\ i = 1,\ 2 \qquad (3.12b)$$

$$(A_i + B_i K_i^o C_i)^T P_i^o + P_i^o (A_i + B_i K_i^o C_i)$$
$$+ Q_i + C_i^T K_i^{oT} R_i K_i^o C_i = 0;\ i = 1,\ 2 \qquad (3.12c)$$

Since K^o, P^o, Z^o are α-type matrices consisting of K_i^o, P_i^o, Z_i^o; $i = 1, 2$, as their diagonal submatrices, the first terms of these unknowns are obtained.

(ii) Taking the derivative of Equation (3.11) with respect to ε and letting $\varepsilon = 0$, the following set of equations is obtained.

$$B^T P^1 Z^o C^T + B^T P^o Z^1 C^T + RK^1 CZ^o C^T + RK^o CZ^1 C^T = 0$$
$$(3.13a)$$

$$(A_\alpha + BK^o C) Z^1 + Z^1 (A_\alpha + BK^o C)^T + (A_\beta + BK^1 C) Z^o$$
$$+ Z^o (A_\beta + BK^1 C)^T = 0 \qquad (3.13b)$$

$$(A_\alpha + BK^o C)^T P^1 + P^1 (A_\alpha + BK^o C) + (A_\beta + BK^1 C)^T P^o$$
$$+ P^o (A_\beta + BK^1 C) + Q_\beta + C^T K^{1T} RK^o C$$
$$+ C^T K^{oT} RK^1 C = 0 \qquad (3.13c)$$

Let,

$$P^1 = P_\alpha^1 + P_\beta^1 \tag{3.14a}$$

$$Z^1 = Z_\alpha^1 + Z_\beta^1 \tag{3.14b}$$

$$K^1 = K_\alpha^1 + K_\beta^1 \tag{3.14c}$$

Substitute Equation (3.14) into Eqatuion (3.13). Since α-type and β-type matrices are independent, two sets of equations are obtained. The first set is:

$$B^T P_\alpha Z^o C^T + B^T P^o Z_\alpha^1 C^T + RK_\alpha^1 C Z^o C^T + RK^o C Z_\alpha^1 C^T = 0 \tag{3.15a}$$

$$(A_\alpha + BK^o C) Z_\alpha^1 + Z_\alpha^1 (A_\alpha + BK^o C)^T + (BK_\alpha^1 C) Z^o + Z^o (BK_\alpha^1 C)^T = 0 \tag{3.15b}$$

$$(A_\alpha + BK^o C)^T P_\alpha^1 + P_\alpha^1 (A_\alpha + BK^o C) + (BK_\alpha^1 C)^T P^o + P^o (BK_\alpha^1 C) + C^T K_\alpha^{1T} RK^o C + C^T K^{oT} RK_\alpha^1 C = 0 \tag{3.15c}$$

The second set of equations is:

$$B^T P_\beta^1 Z^o C^T + B^T P^o Z_\beta^1 C^T + RK_\beta^1 C Z^o C^T + RK^o C Z_\beta^1 C^T = 0 \tag{3.16a}$$

$$(A_\alpha + BK^o C) Z_\beta^1 + Z_\beta^1 (A_\alpha + BK^o C)^T + (A_\beta + BK_\beta^1 C) Z^o + Z^o (A_\beta + BK_\beta^1 C)^T = 0 \tag{3.16b}$$

$$(A_\alpha + BK^oC)^T P^1_\beta + P^1_\beta(A_\alpha + BK^oC) + (A_\beta + BK^1_\beta C)^T P^o$$
$$+ P^o(A_\beta + BK^1_\beta C) + Q_\beta + C^T K^{1T}_\beta R K^o C$$
$$+ C^T K^{oT} R K^1_\beta C = 0 \tag{3.16c}$$

Equation (3.15) has the same form as Equation (3.9). Thus from Lemma 3.1:

$$K^1_\alpha = 0, \quad P^1_\alpha = 0, \quad Z^1_\alpha = 0 \tag{3.17}$$

Equation (3.16) may be written in submatrix form as

$$K^1_{12} = -R_1^{-1}(R_1 K^o_1 C_1 Z^1_{12} C^T_2 + B^T_1 P^o_1 Z^1_{12} C^T_2 + B^T_1 P^1_{12} Z^o_2 C^T_2)$$
$$(C_2 Z^o_2 C^T_2)^{-1} \tag{3.18a}$$

$$(A_1 + B_1 K^o_1 C_1) Z^1_{12} + Z^1_{12}(A_2 + B_2 K^o_2 C_2)^T$$
$$+ (A_{12} + B_1 K^1_{12} C_2) Z^o_2 + Z^o_1 (A_{21} + B_2 K^1_{21} C_1)^T = 0$$
$$\tag{3.18b}$$

$$(A_1 + B_1 K^o_1 C_1)^T P^1_{12} + P^1_{12}(A_2 + B_2 K^o_2 C_2)$$
$$+ (A_{21} + B_2 K_{21} C_1) \; P^o_2 + P^o_1(A_{12} + B_1 K^1_{12} C_2)$$
$$+ Q_{12} + C^T_1 K^{1T}_{21} R_2 K^o_2 C_2 + C^T_1 K^{oT} R_1 K^1_{12} C_2 = 0$$
$$\tag{3.18c}$$

K^1_{12}, P^1_{12}, and Z^1_{12} can be obtained by solving Equation (3.18) simultaneously. The set of equations that is used to solve for K^1_{21}, P^1_{21}, and Z^1_{21} is the same as Equation (3.18) except all of the subscripts must be changed from 1 to 2, from 2 to 1, from 12 to 21, and from 21 to 12.

(iii) Taking the derivative of Equation (3.11) with respect to ε and letting $\varepsilon = 0$, the following set of equations is obtained:

$$B^T P^2 Z^o C^T + 2B^T P^1 Z^1 C^T + B^T P^o Z^2 C^T + RK^2 C Z^o C^T + 2RK^1 C Z^1 C^T + RK^o C Z^2 C^T = 0 \tag{3.19a}$$

$$(A_\alpha + BK^o C)Z^2 + Z^2(A_\alpha + BK^o C)^T + (BK^2 C)Z^o + Z^o(BK^2 C)^T + 2(A_\beta + BK^1 C)Z^1 + 2Z^1(A_\beta + BK^1 C)^T = 0 \tag{3.19b}$$

$$(A_\alpha + BK^o C)^T P^2 + P^2(A_\alpha + BK^o C) + (BK^2 C)^T P^o + P^o(BK^2 C) + 2(A_\beta + BK^1 C)^T P^1 + 2P^1(A_\beta + BK^1 C) + C^T K^{2T} RK^o C + C^T K^{oT} RK^2 C + 2C^T K^{1T} RK^1 C = 0 \tag{3.19c}$$

Equation (3.19) may be rewritten in submatrix form by using the same procedure as before. Separating P^2, Z^2, and K^2 into α-type and β-type matrices, we get two sets of equations. The first set is used to solve for P^2_α, Z^2_α, and K^2_α. The second set is used to solve for P^2_β, Z^2_β, and K^2_β. Applying Lemma 3.1 to the second set of equations we have

$$P^2_\beta = 0, \quad Z^2_\beta = 0, \quad K^2_\beta = 0. \tag{3.20}$$

The submatrix form of the first set of equations is

$$K_1^2 = -R_1^{-1}(B_1^T P_1^2 Z_1^o C_1^T + 2B_1^T P_{12}^1 Z_{21}^1 C^T + B_1^T P_1^o Z_1^2 C_1^T$$
$$+ 2R_1 K_{12}^1 C_2 Z_{21}^1 C_1^T + R_1 K_1^o C_1 Z_1^2 C_1^T)(C_1 Z_1^o C_1^T)^{-1} \quad (3.21a)$$

$$(A_1 + B_1 K_1^o C_1)Z_1^2 + Z_1^2(A_1 + B_1 K_1^o C_1)^T + (B_1 K_1^2 C_1)Z_1^o$$
$$+ Z_1^o(B_1 K_1^2 C_1)^T + 2(A_{12} + B_1 K_{12}^1 C_2)Z_{21}^1$$
$$+ 2Z_{12}^1(A_{12} + B_1 K_{12}^1 C_2)^T = 0 \quad (3.21b)$$

$$(A_1 + B_1 K_1^o C_1)^T P_1^2 + P_1^2(A_1 + B_1 K_1^o C_1) + (B_1 K_1^2 C_1)^T P_1^o$$
$$+ P_1^o(B_1 K_1^2 C_1) + 2(A_{21} + B_2 K_{21}^1 C_1)^T P_{21}^1$$
$$+ P_{21}^1(A_{21} + B_2 K_{21}^1 C_1) + C_1^T K_1^{2T} R_1 K_1^o C_1$$
$$+ C_1^T K_1^{oT} R_1 K_1^2 C_1 + 2C_1^T K_{21}^{1T} R_2 K_{21}^1 C_1 = 0 \quad (3.21c)$$

K_1^2, P_1^2, and Z_1^2 can be obtained by solving Equation (3.21) simultaneously. The set of equations that is used to solve for K_2^2, P_2^2, and Z_2^2 is the same as Equation (3.21) except all of the subscripts must be changed from 1 to 2, from 2 to 1, from 12 to 21, and from 21 to 12.

3.4.1 Summary of the Procedure

The procedure to derive necessary conditions for the terms in the series of K, P, and Z can be summarized as follows:

(a) For K^o, P^o, and Z^o necessary conditions are obtained by decoupling of the system. Then Equation (2.16) is applied directly to each decoupled subsystems.

(b) For K^i, P^i, and Z^i, where i = 1, 2, . . . , the derivation proceeds as follows:

(i) Take the i^{th} derivative of Equation (3.11) with respect to ε and let $\varepsilon = 0$.

(ii) Separate the equations obtained into two sets of equations. The first set is an α-type matrix equation. The second set is a β-type matrix equation.

(iii) Apply Lemma 3.1 to the equations. If i is odd, the α-type matrix equation yields

$$K^i_\alpha = 0, \quad P^i_\alpha = 0, \quad Z^i_\alpha = 0.$$

If i is even, the β-type matrix equation yields

$$K^i_\beta = 0, \quad P^i_\beta = 0, \quad Z^i_\beta = 0.$$

(iv) Write necessary conditions for nonzero elements of K, P, and Z in submatrix form by using Equation (3.6).

Theorem 3.2: Let K^i_a, P^i_a, and Z^i_a, i = 1, 2, . . . , be the i^{th} terms in the series of the optimal matrices satisfying Equation (3.8) when $\varepsilon = \varepsilon_a$. Let K^i_b, P^i_b, and Z^i_b, i = 1, 2, . . . , be the i^{th} terms in the series of the optimal matrices satisfying Equation (3.8) when $\varepsilon = \varepsilon_b$. Then,

$$\varepsilon^i_a K^i_a = \varepsilon^i_b K^i_b$$

$$\varepsilon^i_a P^i_a = \varepsilon^i_b P^i_b$$

$$\varepsilon^i_a Z^i_a = \varepsilon^i_b Z^i_b$$

for all $i = 1, 2, \ldots,$ provided Q_β of Equation (3.7b) is a null matrix.

Proof: This theorem can be proved by using the following procedure for each of the i^{th} derivative sets of K, P, and Z.

(i) Multiply the necessary conditions for K_a^i, P_a^i, and Z_a^i where $\varepsilon = \varepsilon_a$ by $(\varepsilon_a/\varepsilon_b)^i$.

(ii) A_β for $\varepsilon = \varepsilon_b$ is equal to A_β for $\varepsilon = \varepsilon_a$ multiplied by $\varepsilon_a/\varepsilon_b$.

(iii) Compare these equations to necessary conditions for K_b^i, P_b^i, and Z_b^i. Using the fact that the solutions are unique (Theorem 2.3), the above results are obtained.

Corollary: The matrices K, P, and Z, whose series terms are solutions of Equation (3.8), are the same for every finite and nonzero value of ε, provided Q_β of Equation (3.7b) is a null matrix.

Proof: This corollary can be proved by substituting the results of Theorem 3.2 into Equation (3.8). It can be seen that both sets yield the same solution.

3.5 Computational Algorithm

The difficulties with solving a set of equations of the same form as Equation (2.16) or (3.8) have been reported elsewhere [42] [44]. In this section three methods of solving Equation (2.16) or (3.8) are presented. The first method is an iterative method. It is found that conventional fixed-point iteration [16] suggested in Reference [42] yields divergence in nearly all of the numerical problems examined in this study. So another iterative algorithm is considered. In this algorithm an increment of K_j^i for the next iteration is a fraction of the difference between its old value and its new value. The second method makes use of an existing optimization algorithm to find K_j^i such that when substituting into Equation (3.8) its residue is a minimum. The third method uses the optimization algorithm to minimize $tr(DVD^TP)$. Some modification is made so that K stabilizes the system. An additional algorithm to solve the Lyapunov equation is also presented.

3.5.1 Iterative Algorithm

The iterative algorithm to solve Equation (3.8) is as follows:

(1) Make an initial guess for K_j^i.

(2) Substitute K_j^i into Equation (3.8b). Solve for P_j^i.

(3) Substitute K_j^i into Equation (3.8c). Solve for Z_j^i.

(4) Substitute P_j^i and Z_j^i into Equation (3.8a). Solve for $K_{j,new}^i$.

(5) Update the value of K_j^i by using this equation:

$$K_j^i = K_j^i - \alpha(K_j^i - K_{j,new}^i) \qquad (3.22a)$$

where

$$\alpha = \frac{\delta}{|K_j^i - K_{j,new}^i|_{max}} \tag{3.22b}$$

$|K_j^i - K_{j,new}^i|_{max}$ is the maximum element of the absolute value of the residue of K_j^i.

(6) If the maximum residue of K_j^i is not as small as required, go to step (2). Otherwise stop.

Convergence and speed of convergence of this algorithm depends on α. If $\alpha = 1$, it is the same as the fixed-point iteration. It has been found that convergence is achieved only if α is small. But a small value of α results in slow convergence. Thus α should be adaptable to convergence conditions. The choice of α given in Equation (3.22b) has been found to give satisfactory results. However, the value of δ should be adaptable also. The following is a typical example of δ and α.

$\delta = 1$ if the maximum residue of K_j^i is greater than 5.

$\delta = 0.5$ if the maximum residue of K_j^i is less than 5.

$\alpha = \delta$ if the maximum residue of K_j^i is less than 1.

3.5.2 Residue Minimization Algorithm

This algorithm uses a standard unconstrained multivariable optimization subroutine to minimize weighted sum of square of residues of K_j^i. The algorithm is as follows:

(1) Make an initial guess for K_j^i.

(2) The main program calls the optimization subroutine. K_j^i is transferred to the subroutine.

(3) The optimization subroutine calls a secondary subroutine to evaluate the residue of K_j^i.

(4) The secondary subroutine substitutes K_j^i into Equation (3.8b) and (3.8c). It solves for P_j^i and Z_j^i. Then it substitutes P_j^i and Z_j^i into Equation (3.8a) and solves for $K_{j,new}^i$. The last duty of this subroutine is evaluating weighted sum of square of $(K_j^i - K_{j,new}^i)$. After finishing this it returns to the calling subroutine.

(5) The optimization subroutine compares the residues for several values of K_j^i and proceeds to the one that has a minimum residue.

The Powell's optimization algorithm [50] [40] is employed in this study.

3.5.3 DVDP Minimization Algorithm

The solution of Equation (2.16) can be obtained by minimizing Equation (2.13) with respect to Equation (2.11) using a multivariable optimization algorithm [18] [51]. It was pointed out earlier in section 2.5 that K, which does not stabilize the system but satisfies Equation (2.16), can be found. The corresponding P may not be nonnegative definite. This means that the value of K may yield a negative performance index, $tr(DVD^TP)$, which is less than that of the real optimum K. In order to avoid this difficulty, a stability indicator should be included in the performance index.

In this study Powell's optimization algorithm [50] [40] is used. The performance index is modifed as follows:

Let ρ be the maximum real part of eigenvalues of (A+BKC), then

$$J = tr(DVD^TP); \quad \text{if } \rho \leq 0 \tag{3.23a}$$

$$J = tr(DVD^TP) + (\rho \cdot \xi); \quad \text{if } \rho > 0 \tag{3.23b}$$

where ξ is an arbitrarily large positive number.

3.5.4 Kronecker Product Algorithm

The general form of Equation (3.8b) and (3.8c) is

$$AX + XB + C = 0 \tag{3.24}$$

The solution of this equation is obtained by applying the Kronecker Product method [5] [6] to Equation (3.24).

Define:

$$F = A * I_1 + I_2 * B^T \tag{3.25}$$

where * is the Kronecker product operator.

I_1 and I_2 are identity matrices of compatible dimension. It can be proven that [5] [6]

$$Fy = -z \tag{3.26}$$

where y is the vector consisting of all elements of X and z is the vector consisting of all elements of C.

Then Equation (3.26) is solved for y by the Gauss elimination method [16]. The result is obtained by transforming the vector y to the matrix X.

3.5.5 Comparison of the Algorithms

The solution of Equation (2.16) can be found by means of the three algorithm. The iterative algorithm yields the solution within the shortest time. A FORTRAN program for the algorithm is less complicated than that of the other two. However, it inherits certain disadvantages. It has been found that in some numerical problems the algorithm results in a solution, K, which does not stabilize the system. The only way to get the right K when this difficulty happens is by making an initial guess of K which is very close to the right solution. It is impossible to make such a guess in a practical problem. Another disadvantage of the iterative algorithm is that optimal values for α and δ have to be chosen by trial and error. In the DVDP minimization algorithm the problem of getting the wrong K has been solved by using a modified performance index described in section 3.5.3. Since eigenvalues have to be calculated every time the performance index is evaluated, the algorithm requires large amounts of computer processing time. Writing a FORTRAN program for this algorithm is somewhat more complicated than the iterative algorithm, even though standard subprograms for optimization and eigenvalue evaluation are used. In some numerical problems this algorithm cannot find a very accurate solution because the value of the performance does not change for a small change of K. In that case the residue minimization algorithm or the combination of the DVDP minimization algorithm and the iterative algorithm can be used.

The solution for the suboptimal control problem presented in this chapter can be obtained by means of the iterative algorithm or a combination of residue minimization algorithm and DVDP minimization algorithm. The use of the iterative algorithm to solve Equation (3.8) is practically the same as described above. In the second algorithm the DVDP minimization algorithm is used to solve the zeroth-order of Equation (3.8) or (3.12). The algorithm minimizes $D_i V_i D_i^T P_i$ with respect to K_i with Equation (3.12c) as an equality constraint, where i = 1, 2. For the values of the first order and higher of K, P, and Z in Equation (3.8) the residue minimization algorithm is used. It should be noted that after K_1^o and K_2^o, which are solutions to Equation (3.12) and which stabilize the subsystems, are obtained the solution for Equation (3.8) is unique. This can be proven by Theorem 2.3. So there is no need to calculate eigenvalues in the residue minimization algorithm and the computer time required for this algorithm is not too large. In general, the iterative algorithm should be used if it is expected to converge to the right solution, since it is faster and sometimes gives more accurate results. Residue + DVDP minimization algorithm should be used if the iterative algorithm does not work.

3.5.6 Some Symmetrical Properties

By applying Theorem 2.4 to the equations obtained earlier it can be proved that:

(1) Even derivatives of submatrices of Z and P are symmetric. The submatrices are Z_1^o, Z_2^o, P_1^o, P_2^o, Z_1^2, Z_2^2, P_1^2, P_2^2, ...

(2) "12" submatrices of odd derivatives of Z and P are equal to transpose of "21" submatrices of their own matrices, e.g.,

$$Z_{12}^1 = Z_{21}^{1T}$$

$$P_{12}^1 = P_{21}^{1T}$$

These symmetrical properties are very useful. They may be used to simplify the computer program and reduce the computer burden to a large extent.

3.6 Conclusion

It has been discussed that solving necessary condition Equation (2.15) for optimal feedback gain of a large system is not a trivial job. In order to cope with this problem an approach to suboptimal gain calculation is developed in this chapter. The method involves Taylor series expansion of the matrices K, P, and Z with respect to the system coupling parameter, ε. Necessary conditions to be solved for the matrices in the series are derived. With some matrix manipulation, these equations can be presented as functions of submatrices. The general form of the equations is given in Theorem 3.1. The proof of the theorem is carried out by an induction method. In this study the first few terms of matrices in the series of K, P, and Z are derived. A general procedure of derivation is given so that higher derivative terms can be derived if necessary. Computational methods for solving Equation (2.16) or (3.8) are presented in section 3.5. The iterative algorithm offers the fastest speed. Unfortunately, in some problems it converges to wrong solutions or does not converge at all. When this problem arises, the DVDP minimization algorithm, or the residue minimization algorithm, or a combination of both is recommended. These algorithms make use of an existing optimization routine to solve the problem. The listing of programs used in this study is shown in the appendices.

CHAPTER IV

LOAD FREQUENCY CONTROL OF MULTIAREA POWER SYSTEMS

4.1 Introduction

Load frequency control of electric power systems represents the first realization of large scale complex system control. It has made the operation of interconnected systems possible. The objective of load frequency control is to maintain a balance between system's generation and consumption. Today the tie-line bias control is widely applied. A linear combination of net interchange error and frequency deviation, called area control error, is used to control the system generating units. Each area tends to reduce the area control error to zero. When this aim is achieved the system frequency equals the desired value and the interchange schedule is met. The conventional approach to this problem is mainly concerned with steady-state power balance. Little attention has been paid to the optimization of system transients. Recently, some attempts have been made to apply linear optimal control theory to the load frequency problem. The main purpose of those studies is to stabilize power swings which occur when the system is subjected to disturbances.

The approach results in a minimum of weighted sum of power swings (state variable deviations) and control efforts. Normally, it is assumed that all variables are measurable and the feedback gain is calculated by solving the Riccati equation.

In this chapter the load frequency control is modeled as a limited state variable feedback control system. Then the approach given in Chapter III is applied to calculate the suboptimal feedback gain. As an example, the control of a two-area system will be considered. The results of the suboptimal approach will be compared to those of the optimal approach.

4.2 System Modeling

The development of the system model is considered in this section. Turbines and their speed-governing systems are very important components in the load frequency control system, so considerable details are presented in the first part. The relationship between the system power balance and its frequency is given in the second part. Then the models of each component are grouped to form a model of load frequency control system. It is presented in a standard state variable form in the third section.

4.2.1 Speed-Governing System and Turbine Models

Standard modeling of steam turbines and hydroturbines and their speed-governing systems was provided by the IEEE

Task Force [35]. The model descriptions were typical of those in use by utilities and service centers. The basic diagram showing location of speed-governing system and turbine relative to the system is shown in Figure 1. A general model for speed-governing systems is shown in Figure 2. In the model many nonlinearities are neglected except rate limits which may occur for large, rapid speed deviations and position limits which may correspond to wide-open valves or the setting of a load limiter. Rate limiting of servomotor is shown at the input to the integrator representing the servomotor. This model shows the load reference as an initial power P_o. This initial value is combined with the increments due to speed deviation to obtain the valve position, h, subject to the time lag, T_3, introduced by the servomechanism.

Models for different types of steam turbine systems are shown in Figure 3. In these models flows into and out of any steam vessel are related by a simple time constant. The time constants T_{CH}, T_{RH}, and T_{CO} represent delays due to the steam chest and inlet piping, reheaters, and crossover piping, respectively.

A general model for speed-governing system for hydroturbines is shown in Figure 4. Linear characteristics of the distributor valve and gate servomotor, and the dashpot feedback are utilized. Position limits are presented at the output of the system. Nonlinearities in rate limits, permanent droop compensation, etc. are neglected.

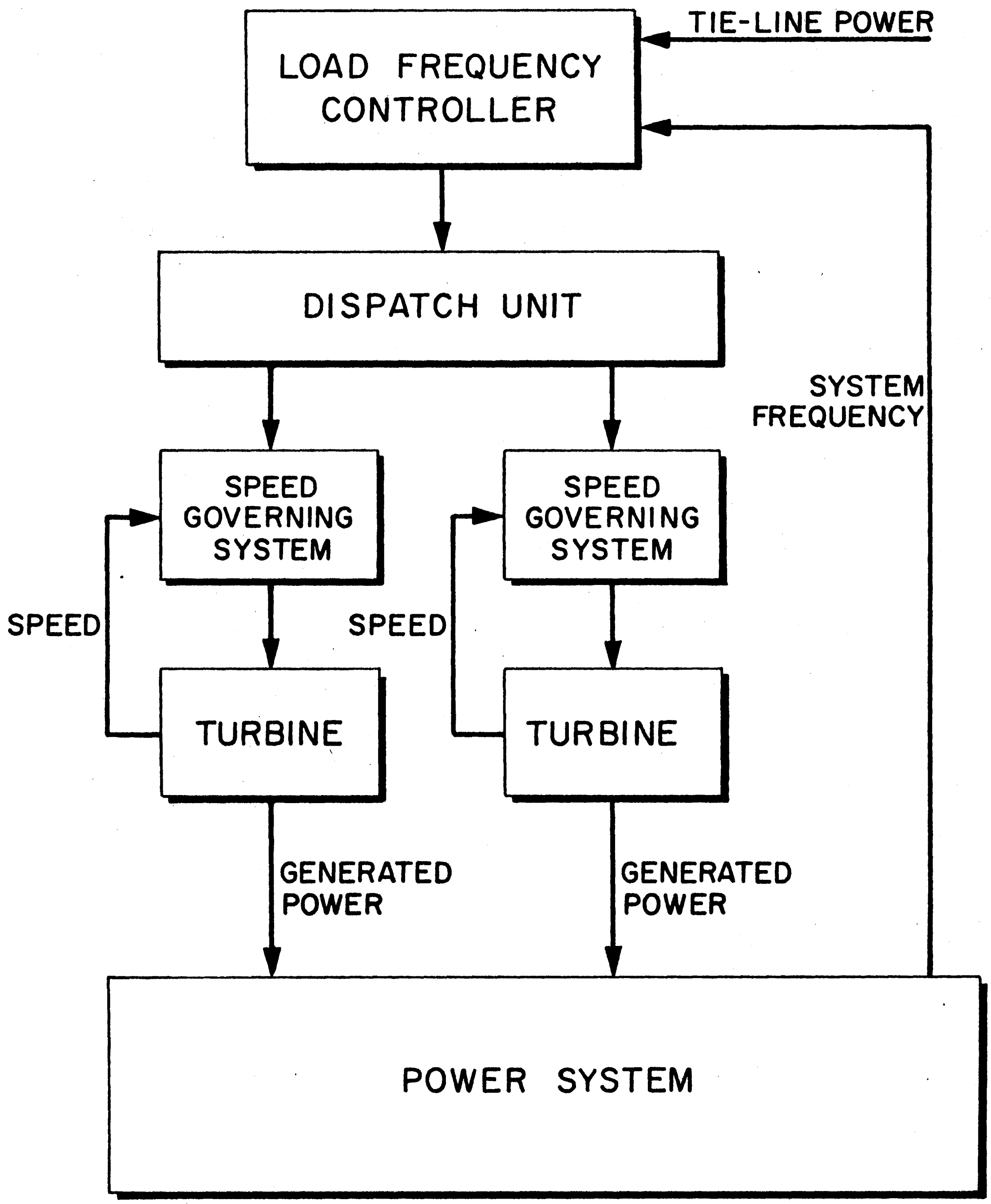


Figure 1. Block Diagram of Single Area Load Frequency Control System

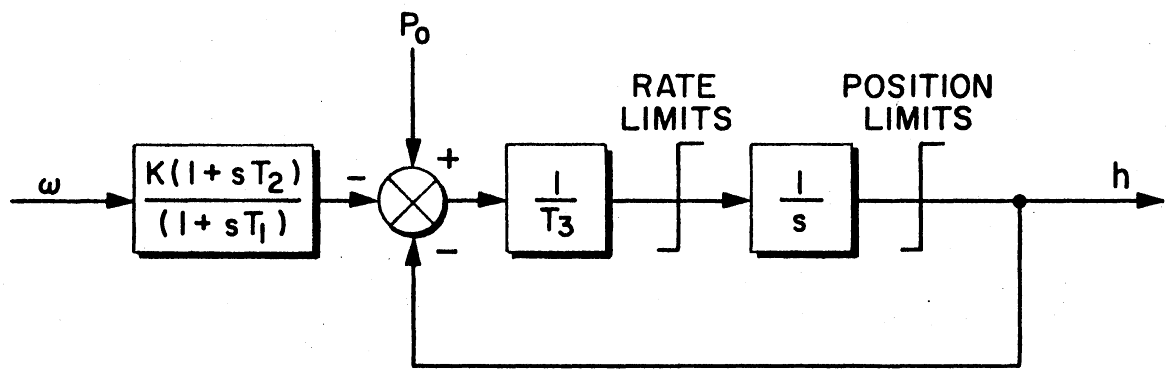
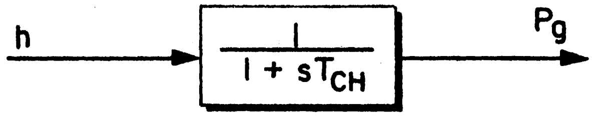
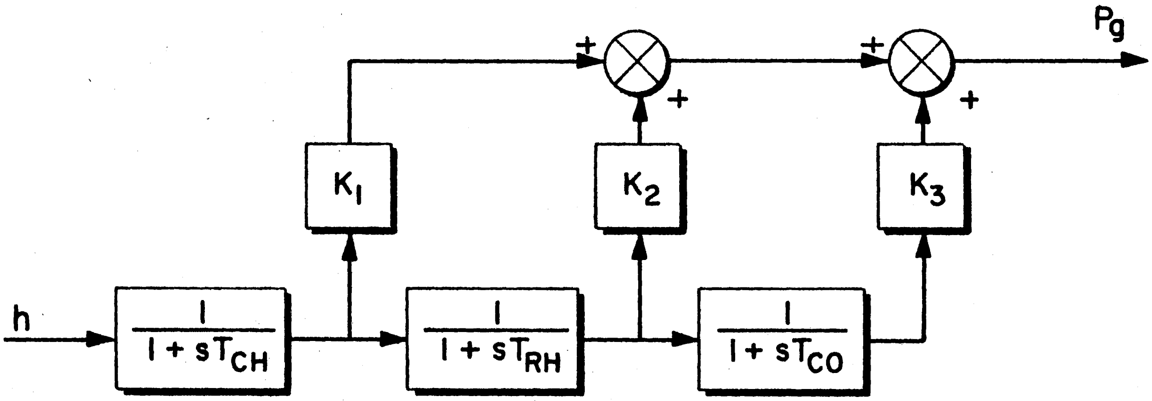


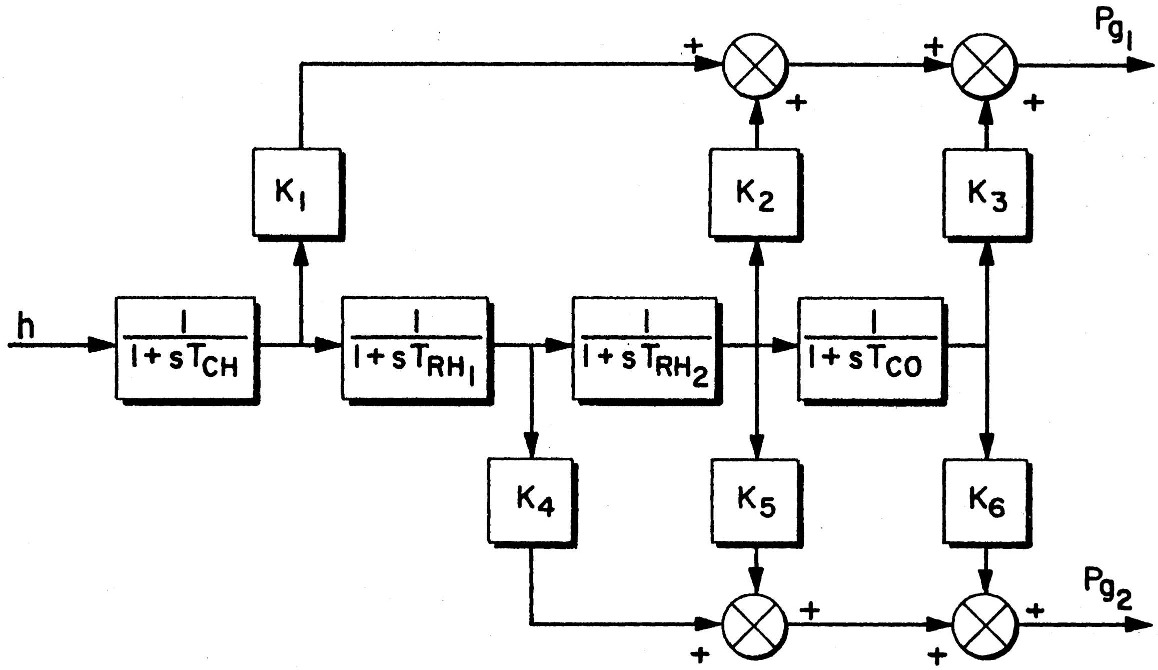
Figure 2. General Model for Speed-Governing System of Steam Turbine Systems



a) NONREHEAT



b) TANDEM COMPOUND, SINGLE REHEAT



c) CROSS COMPOUND, DOUBLE REHEAT

Figure 3. Linear Models for Steam Turbine Systems

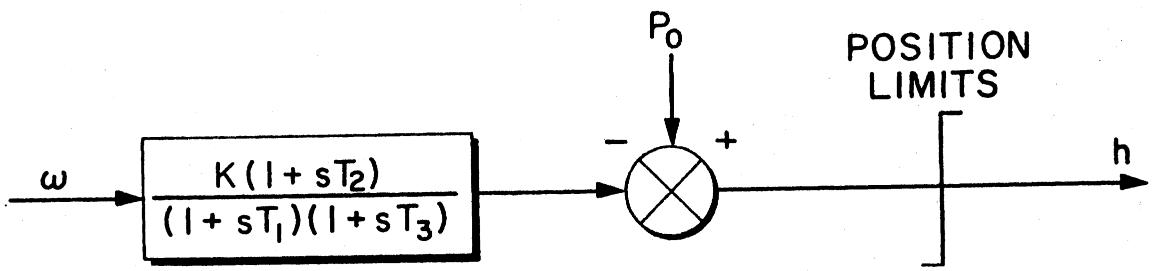


Figure 4. General Model for Speed-Governing Systems of Hydroturbine Systems

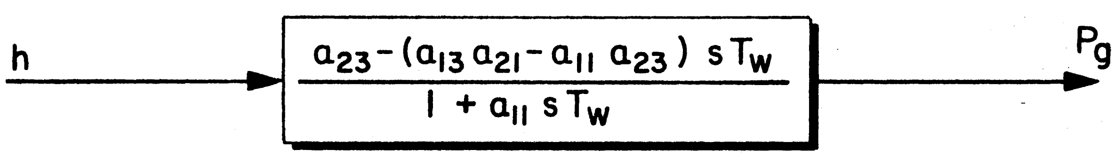


Figure 5. Approximate Linear Model for Hydroturbines

A linear model for hydroturbines which is most often used is shown in Figure 5. The transient characteristics of hydroturbines are determined by dynamics of water flow in the penstock. The time constant T_w is called the water starting time or water time constant. A method for estimating this time constant is given in Appendix II of Reference [35].

4.2.2 Power System Inertia Model

Whenever there is an imbalance in the applied torques of a power generating unit, acceleration takes place. The mechanical torque equilibrium equation can be written as

$$\frac{Jd\omega}{dt} + D\omega = \Delta T \qquad (4.1)$$

where

J = moment of inertia of the moving parts;

D = damping coefficient, including mechanical viscous friction plus electrical damping torque from field coil and damping coil;

ΔT = change of torque from equilibrium state; and

ω = angular velocity.

It is customary to normalize Equation (4.1) using the inertia constant H which is defined to be the kinetic energy at rated speed ω_o divided by the generator MVA base S_b:

$$H = \frac{\frac{1}{2} J\omega^2}{S_b} \qquad (4.2)$$

Linearizing Equation (4.1) around the operating point and making use of Equation (4.2), we get

$$\frac{2H}{f_o}\frac{d\Delta f}{dt} + K_d \Delta f = P \tag{4.3}$$

where

Δ = incremental operator;

f = system frequency;

f_o = rated frequency;

P = power output of the system in per unit; and

$K_d = \dfrac{2\pi\omega_o D}{S_b}$.

Change of power output from equilibrium state, ΔP, of Equation (4.3) takes the form

$$\Delta P = \Delta P_g - \Delta P_d - \Delta P_t \tag{4.4}$$

where

P_g = power generation;

P_d = increment in load demand; and

P_t = increment in tie-line power imported from other areas.

The increment in tie-line power can be represented by

$$\Delta P_t = \sum_i S_i(\Delta\delta_i - \Delta\delta) \tag{4.5}$$

where

δ = angular displacement of the area;

δ_i = angular displacement of the remote area i; and

S_i = synchronizing coefficient between the area and the remote area i.

Thus the load frequency control system is described by the following equation:

$$\frac{d}{dt}\Delta f = \frac{f_o}{2H}(\Delta P_g - \Delta P_d - \Delta P_t - K_d \Delta f) \qquad (4.6a)$$

$$\frac{d}{dt}\Delta P_g = \frac{1}{T_t}(\Delta h - \Delta P_g) \qquad (4.6b)$$

$$\frac{d}{dt}\Delta h = \frac{1}{T_g}(\Delta u - \frac{\Delta f}{R} - \Delta h) \qquad (4.6c)$$

$$\Delta P_t = 2\pi \sum_i S_i \left(\int \Delta f_i dt - \int \Delta f dt\right) \qquad (4.6d)$$

where

T_t = time constant of the turbine;

T_g = time constant of the speed-governing system;

h = valve position;

u = input to speed-governing system; and

R = speed regulation parameter.

4.2.3 Integrated Model for Load Frequency Control Systems

Unlike models designed for transient or dynamic stability studies, the objective is a model to represent the interplay between system load demand and mechanical energy supply. This model must describe the system dynamics with sufficient accuracy and at the same time must be of reasonably small dimension such that its solutions are attainable. In order to achieve this goal the following assumptions must be made because there are a large number of power generating units within an area of a power system. First, the effects of network electrical dynamics can be eliminated from the load frequency problem. Second, all power generating units belonging to an area are similar and they are tied via stiff lines such that they have coherent phase and frequency. Third, variations of variables are small so linearization of system equations around a nominal operating condition is permitted. With these assumptions a power system area for load frequency control can be represented by a single power generating unit.

The block diagram model of a load frequency area is shown in Figure 6. It consists of a turbine, its speed-governing system, and a power-frequency transfer function. The turbine is assumed to be a nonreheat steam type which has only one time constant representing time delay in its steam chest. The speed-governing system is assumed to be a mechanical-hydraulic type with negligible speed relay time constant. So it can be represented by a first order system. The power-frequency transfer function can be derived from Equation (4.6a) where

$$K_p = \frac{1}{K_d} \tag{4.7a}$$

$$T_p = \frac{2H}{f_o K_d} \tag{4.7b}$$

Nonlinearities in every component are neglected since we shall consider system dynamics under small disturbances.

4.3 Control of Two-Area System

In this section we shall consider an interconnected power system consisting of two areas. The state equation for the system can be written as

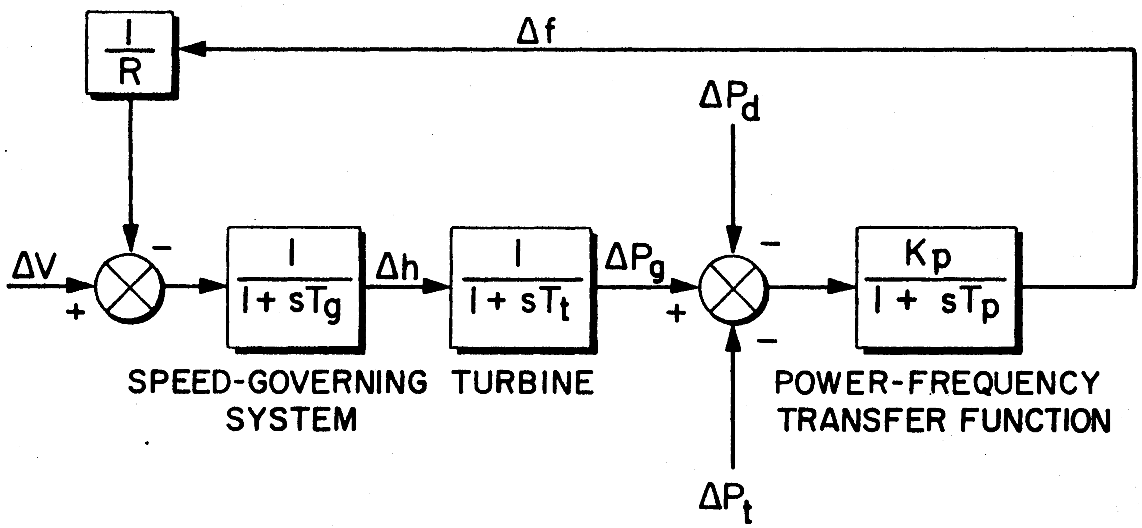


Figure 6. Linear Model for Single-Area Load Frequency Control System

$$\frac{dz}{dt} = Az + Bu + Ev \tag{4.8}$$

where

$$z^T = [\int \Delta f_1 dt,\ \Delta f_1,\ \Delta P_g,\ \Delta h_1,\ \int \Delta f_2 dt,$$

$$\Delta f_2,\ \Delta P_{g2},\ \Delta h_2]$$

$$u^T = [\Delta u_1,\ \Delta u_2]$$

$$v^T = [\Delta P_{d1},\ \Delta P_{d2}]$$

The matrices A, B, and E are shown in Table I. In order that modern optimal control technique can be applied to this problem, Equation (4.8) must be modified to the standard form:

$$\frac{dx}{dt} = Ax + Bu \tag{4.9}$$

Several methods have been suggested such that the system equations can be written as Equation (4.9) [9][27][29]. Since we are interested only in dynamic aspects of the problem, in this study the method in Reference [27] is used. The new vector x is defined by

$$x = z - z_{ss} \tag{4.10a}$$

where z_{ss} is the steady state value of z, and

$$x(o) = -z_{ss} \tag{4.10b}$$

With this modification the matrices A and B of Equations (4.8) and (4.9) are still the same.

4.4 Two Area Control System Example

In the study of the two-area load frequency control system an iterative algorithm is used to calculate the optimal and suboptimal gain matrices. The program is designed

TABLE I

MATRICES A, B, AND E OF THE TWO-AREA LOAD FREQUENCY CONTROL SYSTEM

$$
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{-f_o S_{12}}{2H_1} & \frac{-f_o K_{d1}}{2H_1} & \frac{f_o}{2H_1} & 0 & \frac{f_o S_{12}}{2H_1} & 0 & 0 & 0 \\
0 & 0 & \frac{-1}{T_{t1}} & \frac{1}{T_{t1}} & 0 & 0 & 0 & 0 \\
0 & \frac{-1}{R_1 T_{g1}} & 0 & \frac{-1}{T_{g1}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\frac{-a_{12} f_o S_{12}}{2H_2} & 0 & 0 & 0 & \frac{a_{12} f_o S_{12}}{2H_2} & \frac{-f_o K_{d2}}{2H_2} & \frac{f_o}{2H_2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{-1}{T_{t2}} & \frac{1}{T_{t2}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{-1}{R_2 T_{g2}} & 0 & \frac{-1}{T_{g2}}
\end{bmatrix}
$$

TABLE I (Continued)

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{T_{g1}} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{T_{g2}} \end{bmatrix} \qquad E = \begin{bmatrix} 0 & 0 \\ \frac{-f_o}{2H_1} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{-f_o}{2H_2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

to calculate three terms in the series of the matrices, K, P, and Z. Eigenvalues of the closed-loop system, $A+BK^{o}C$, are evaluated after K^{o}, P^{o}, and Z^{o} are found to make sure that the feedback gain yields a stable system. The program is constructed in such a way that it can be used for subsystems of dimension up to 10. Since the necessary conditions of the optimal gain are similar to those of the first term of the series of K, P, and Z, the first part of the program can be used to calculate the optimal gain. The program is implemented on a 370/158 computer system. A G-level FORTRAN compiler is used.

Numerical data for the system under study are shown in Table II. The matrices D, Q, and R are assumed to be identity matrices. The performance index is an infinite integral of sum of square of all state and control variables. Both deterministic and stochastic cases are considered. In the deterministic case the matrix DVD^{T} is replaced by an identity matrix. The numerical value of V for the stochastic case is given in Table III. The number of output variables in each subsystem varies from 2 to 4. For the load frequency control system, the minimum number of the feedback variables which can stabilize the system is 2. They are the frequency and the phase angle of the area. If the generated power is assumed measurable, the dimension of the output vector of the area is 3. If all of the state variables are measurable, the dimension of the output vector of the area is 4 and this case is equivalent to the optimal linear regulator problem.

TABLE II

DATA OF THE TWO AREA LOAD FREQUENCY CONTROL SYSTEM

Variable	Area No. 1	Area No. 2
T_g	0.08	0.1
T_t	0.3	0.25
R	2.4	2.5
H	5.0	8.0
K_d	0.008	0.01
S_{12}	0.545	0.545
a_{12}	-1	-1
f_o	60	60

TABLE III

STOCHASTIC NOISE INTENSITY MATRIX

.0001	0	0	0	0	0	0	0
0	.0005	0	0	0	0	0	0
0	0	.0003	0	0	0	0	0
0	0	0	.0007	0	0	0	0
0	0	0	0	.0002	0	0	0
0	0	0	0	0	.0009	0	0
0	0	0	0	0	0	.0005	0
0	0	0	0	0	0	0	.0008

The results of the study are presented in Tables VI through XXVII. It is observed that:

1. The optimal feedback gain matrix, K, for the deterministic system and the stochastic system are very close.

2. The coupling coefficient, ε, having the values between 0 and 1, results practically in the same suboptimal feedback gain matrix. The time spent in the calculation, however, is not the same for different ε's. It is found that the value of $\varepsilon = 0.5$ converges faster than other values. Thus this value is used throughout the study.

3. When the generated power of the area is used as an input to the controller in addition to the frequency and the phase angle, the performance of the system is significantly improved. Thus it is beneficial to transmit the generation power signal other than the frequency and phase angle signals of each area to other areas in the pool for the purpose of an automatic power generation control of the interconnected system. The valve position of the turbine, on the other hand, yields only small improvement in the system performance when all other variables are used. Thus this variable need not be used as a control signal because it will increase the cost of telemetering while it does a small contribution to the system stabilization.

4. The performance of the optimal system is better when more variables are used as the controller inputs. This is not surprising since in that case more information is obtained. But the performance of the suboptimal control

system may not follow the rule. For example, the performance index of the suboptimal control system using 8 variables as inputs to the controller is higher than that using 6 variables. This happens because the error when calculating the suboptimal gain for the system with 8 dimensional output vector is more than that with 6 dimensional output vector.

5. Only two terms in the series of the suboptimal gain matrix give results which are considerably close to those of the optimal control system. It is consequently believed that using two terms is enough for suboptimal feedback gain calculation of any two-area load frequency control system design.

6. The suboptimal gain calculation consumes much less computer burden than the optimal gain calculation. In this study the optimal gain calculation which starts from the results of the suboptimal gain calculation spent about four to seven times longer execution time than that of the suboptimal gain calculation which starts from an arbitrary value (0 is used). It is expected that if both methods start from the same initial value, the optimal method will use more than ten times the computer time used by the suboptimal method for the same 8 dimensional system. The memory required for the optimal gain calculation is three to four times more than that of the suboptimal gain calculation. This amount of memory saving is very attractive for those using a computer with limited memory size to calculate the feedback gain for a large system. However, the program architecture is more complex and compilation time is longer for the suboptimal

method. But these are trivial disadvantages compared to the benefits described above.

7. When all the state variables are available for measurement, i.e., C = I, the matrix Z has no effect on the optimal feedback gain. In this case the optimal matrix K is the same for both the deterministic system and the stochastic system.

4.5 Conclusion

In this chapter the suboptimal approach for the feedback gain calculation of linear limited state variable feedback developed in Chapter III is applied to multiarea load frequency control. In section 4.2 models of components within an area of load frequency control system is presented; it consists of a turbine, speed-governing system, and power-frequency transfer function. The integrated model of these components for two-area interconnected system is described in section 4.3. It is presented in a standard form of state equations. The system with and without plant noise are studied. The iterative algorithm is used to calculate the suboptimal and optimal feedback gains. Convergence is obtained in all of the problems considered in this step. The results are presented in Tables VI through XXVII. They show that the suboptimal approach yields results which are close to those of the optimal approach but the suboptimal approach requires much less computer burden. From the study of the load frequency control system it is suggested that frequency, phase angle, and power generation be used as feedback signals to the system controller.

TABLE IV

NOMENCLATURE OF CASE STUDIES

Case Study	Type of Dynamic System	Dimension of Output Vector
D4	Deterministic	4
D6	Deterministic	6
D8	Deterministic	8
S4	Deterministic	4
S6	Deterministic	6
S8	Deterministic	8

TABLE V

NOMENCLATURE OF FEEDBACK GAIN MATRICES

Feedback Gain Matrix	Number of Terms Used	Formula
K_a	1	$K_a = K^o$
K_b	2	$K_b = K^o + \varepsilon K^1$
K_c	3	$K_c = K^o + \varepsilon K^1 + \frac{\varepsilon^2 K^2}{2!}$
K^*	∞	$K^* = K^o + \varepsilon K^1 + \frac{\varepsilon^2 K^2}{2!} + \frac{\varepsilon^3 K^3}{3!} + \ldots$

TABLE VI

FEEDBACK GAIN MATRICES OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY D4

$$K_a = \begin{bmatrix} .1566 & -.0388 & 0 & 0 \\ 0 & 0 & .0812 & -.1141 \end{bmatrix}$$

$$K_b = \begin{bmatrix} .1566 & -.0388 & -.4316 & -.1761 \\ -.3301 & -.0940 & .0812 & -.1141 \end{bmatrix}$$

$$K_c = \begin{bmatrix} .1500 & -.0634 & -.4316 & -.1761 \\ -.3301 & -.0940 & .0311 & -.1674 \end{bmatrix}$$

$$K^* = \begin{bmatrix} .1382 & -.0666 & -.4500 & -.1803 \\ -.3576 & -.1073 & .0270 & -.1679 \end{bmatrix}$$

TABLE VII

COMPUTER BURDEN OF FEEDBACK GAIN MATRIX CALCULATION OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY D4

Feedback Gain Matrix Obtained	Memory (K-Byte)	Number of Iterations#	Execution Time# (Seconds)	Compilation Time (Seconds)
K_a	120	19	8.74	11.65
K_b	132	41	15.17	15.85
K_c	148	64	24.34	20.74
K*	440	15	76.15	9.81

#The initial values used to calculate K_a, K_b, and K_c are zeros. Then K_c is used as the initial value to calculate K*.

TABLE VIII

PERFORMANCE INDICES OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY D4

Feedback Gain Matrix Used	Performance Index
K_a	---
K_b	8.357
K_c	8.306
K*	8.288

TABLE IX

EIGENVALUES OF THE CLOSED-LOOP SYSTEM, (A+BKC), OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY D4

$(A+BK_aC)$	$(A+BK_bC)$	$(A+BK_cC)$	$(A+BK^*C)$
-13.36	-13.37	-13.41	-13.41
-11.00	-10.99	-11.06	-11.06
$-1.46 \pm j2.44$	$-1.09 \pm j2.47$	$-1.05 \pm j2.62$	$-1.04 \pm j2.62$
-1.43	-1.04	-1.02	-1.00
$-0.71 \pm j3.33$	$-0.93 \pm j3.16$	$-0.92 \pm j3.23$	$-0.91 \pm j3.23$
0.22#	-0.48	-0.49	-0.54

#Unstable eigenvalue.

TABLE X

FEEDBACK GAIN MATRICES OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY D6

$$K_a = \begin{bmatrix} .2432 & -.3184 & -1.0483 & 0 & 0 & 0 \\ 0 & 0 & 0 & .0620 & -.4225 & -.7827 \end{bmatrix}$$

$$K_b = \begin{bmatrix} .2432 & -.3184 & -1.0483 & -.6556 & -.1530 & -.1051 \\ -.4777 & -.1057 & -.1020 & .0620 & -.4225 & -.7827 \end{bmatrix}$$

$$K_c = \begin{bmatrix} .1134 & -.3613 & -1.1035 & -.6556 & -.1530 & -.1051 \\ -.4777 & -.1057 & -.1020 & -.0813 & -.4879 & -.8350 \end{bmatrix}$$

$$K^* = \begin{bmatrix} .0916 & -.3670 & -1.1110 & -.7313 & -.1809 & -.1291 \\ -.5433 & -.1261 & -.1314 & -.1085 & -.4941 & -.8417 \end{bmatrix}$$

TABLE XI

COMPUTER BURDEN OF FEEDBACK GAIN CALCULATION OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY D6

Feedback Gain Matrix Obtained	Memory (k-byte)	Number of Iterations[#]	Execution Time[#] (Seconds)	Compilation Time (Seconds)
K_a	120	10	4.19	11.59
K_b	132	20	7.94	15.05
K_c	148	30	12.60	20.93
K^*	440	9	50.84	9.66

[#]The initial values used to calculate K_a, K_b, and K_c are zeros. Then K_c is used as the initial value to calculate K^*

TABLE XII

PERFORMANCE INDICES OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY D6

Feedback Gain Matrix Used	Performance Index
K_a	---
K_b	6.921
K_c	6.762
K*	6.737

TABLE XIII

EIGENVALUES OF THE CLOSED-LOOP SYSTEM, (A+BKC), OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY D6

$(A+BK_aC)$	$(A+BK_bC)$	$(A+BK_cC)$	$(A+BK^*C)$
-8.11	-8.28	-7.77	-7.84
$-5.51 \pm j2.32$	$-5.27 \pm j2.71$	$-5.32 \pm j3.05$	$-5.38 \pm j3.22$
$-3.90 \pm j2.63$	$-2.93 \pm j1.90$	$-3.29 \pm j1.59$	$-2.99 \pm j.144$
$-1.57 \pm j2.70$	$-2.32 \pm j1.96$	$-2.05 \pm j2.24$	$-2.15 \pm j2.20$
0.173#	-0.596	-0.831	-1.04

#Unstable eigenvalue.

TABLE XIV

FEEDBACK GAIN MATRICES OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY D8

$$K_a = \begin{bmatrix} .2951 & -.7653 & -.1407 & -.6581 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .0490 & -.8613 & -.9536 & -.6622 \end{bmatrix}$$

$$K_b = \begin{bmatrix} .2951 & -.7653 & -.1407 & -.6581 & -.9504 & -.2353 & -.1210 & -.0287 \\ -.6778 & -.1551 & -.1412 & -.0229 & .0490 & -.8613 & -.9536 & -.6622 \end{bmatrix}$$

$$K_c = \begin{bmatrix} .0902 & -.8389 & -.2089 & -.6689 & -.9504 & -.2353 & -.1210 & -.0287 \\ -.6778 & -.1551 & -.1412 & -.0229 & -.1766 & -.9649 & -1.0127 & -.6761 \end{bmatrix}$$

$$K^* = \begin{bmatrix} .0573 & -.8486 & -1.4821 & -.6700 & -1.0652 & -.2809 & -.1453 & -.0343 \\ -.7782 & -.1890 & -.1702 & -.0274 & -.2139 & -.9755 & -1.0172 & -.6771 \end{bmatrix}$$

TABLE XV

COMPUTER BURDEN OF FEEDBACK GAIN CALCULATION OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY D8

Feedback Gain Matrix Obtained	Memory (k-byte)	Number of Iterations#	Execution Time# (Seconds)	Compilation Time (Seconds)
K_a	120	18	7.82	11.50
K_b	132	37	15.70	15.79
K_c	148	57	23.93	21.49
K^*	440	18	98.49	9.69

#The initial values used to calculate K_a, K_b, and K_c are zeros. Then K_c is used as the initial value to calculate K^*.

TABLE XVI

PERFORMANCE INDICES OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY D8

Feedback Gain Matrix Used	Performance Index
K_a	---
K_b	7.802
K_c	7.491
K*	6.655

TABLE XVII

EIGENVALUES OF THE CLOSED-LOOP SYSTEM, (A+BKC), OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY D8

$(A+BK_aC)$	$(A+BK_bC)$	$(A+BK_cC)$	$(A+BK^*C)$
-21.18	-21.19	-21.20	-17.84
-14.18	-14.18	-14.18	-14.18
-2.91	-2.37 ± j1.18	-2.64	-3.05
-2.20 ± j1.67	-1.17	-2.00 ± j1.33	-2.50 ± j1.94
-1.12 ± j3.91	-1.21 ± j3.88	-1.18 ± j3.93	-2.03 ± j2.62
0.15#	-0.50	-0.64	-0.95

#Unstable eigenvalue.

TABLE XVIII

FEEDBACK GAIN MATRICES OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY S4

$$K_a = \begin{bmatrix} .1796 & -.0745 & 0 & 0 \\ 0 & 0 & .1000 & -.1434 \end{bmatrix}$$

$$K_b = \begin{bmatrix} .1796 & -.0745 & -.3496 & -.1328 \\ -.3570 & -.0686 & .1000 & -.1434 \end{bmatrix}$$

$$K_c = \begin{bmatrix} .1363 & -.0860 & -.3496 & -.1328 \\ -.3570 & -.0686 & .0583 & -.1963 \end{bmatrix}$$

$$K^* = \begin{bmatrix} .1372 & -.0945 & -.3806 & -.1439 \\ -.3667 & -.0857 & .0494 & -.1943 \end{bmatrix}$$

TABLE XIX

COMPUTER BURDEN OF FEEDBACK GAIN CALCULATION OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY S4

Feedback Gain Matrix Obtained	Memory (K-byte)	Number of Iterations #	Execution Time # (Seconds)	Compilation Time (Seconds)
K_a	120	20	9.27	10.51
K_b	132	42	17.32	14.32
K_c	144	67	27.31	19.45
K^*	440	19	103.86	9.61

#The initial values used to calculate K_a, K_b, and K_c are zeros. Then K_c is used as the initial value to calculate K^*.

TABLE XX

PERFORMANCE INDICES OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY S4

Feedback Gain Matrix Used	Performance Index
K_a	---
K_b	.002896
K_c	.002861
K*	.002855

TABLE XXI

EIGENVALUES OF THE CLOSED-LOOP SYSTEM, (A+BKC), OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY S4

$(A+BK_aC)$	$(A+BK_bC)$	$(A+BK_cC)$	$(A+BK^*C)$
-13.42	-13.42	-13.44	-13.45
-11.05	-11.04	-11.12	-11.11
$-1.45 \pm j2.55$	$-1.15 \pm j2.54$	$-1.11 \pm j2.67$	$-1.08 \pm j2.67$
-1.32	$-0.93 \pm j3.31$	-0.93	-0.93
$-0.74 \pm j3.43$	-0.92	$-0.89 \pm j3.34$	$-0.90 \pm j3.35$
0.24 #	-0.38	-0.44	-0.47

#Unstable eigenvalue.

TABLE XXII

FEEDBACK GAIN MATRICES OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY S6

$$K_a = \begin{bmatrix} .2361 & -.3355 & -1.0922 & 0 & 0 & 0 \\ 0 & 0 & 0 & .0544 & -.4427 & -.8238 \end{bmatrix}$$

$$K_b = \begin{bmatrix} .2361 & -.3355 & -1.0922 & -.6652 & -.1505 & -.1011 \\ -.4759 & -.1062 & -.1046 & .0544 & -.4427 & -.8238 \end{bmatrix}$$

$$K_c = \begin{bmatrix} .1010 & -.3787 & -1.1420 & -.6652 & -.1505 & -.1011 \\ -.4759 & -.1062 & -.1046 & -.0959 & -.5082 & -.8756 \end{bmatrix}$$

$$K^* = \begin{bmatrix} .0802 & -.3842 & -1.1492 & -.7395 & -.1779 & -.1204 \\ -.5446 & -.1260 & -.1308 & -.1222 & -.5142 & -.8787 \end{bmatrix}$$

TABLE XXIII

COMPUTER BURDEN OF FEEDBACK GAIN CALCULATION OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY S6

Feedback Gain Matrix Obtained	Memory (K-byte)	Number of Iterations#	Execution Time# (Seconds)	Compilation Time (Seconds)
K_a	120	9	3.72	10.63
K_b	132	18	7.25	14.79
K_c	144	29	11.82	19.62
K^*	440	8	44.33	9.59

#The initial values used to calculate K_a, K_b, and K_c are zeros. Then K_c is used as the initial value to calculate K^*.

TABLE XXIV

PERFORMANCE INDICES OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY S6

Feedback Gain Matrix Used	Performance Index
K_a	---
K_b	.002306
K_c	.002258
K^*	.002251

TABLE XXV

EIGENVALUES OF THE CLOSED-LOOP SYSTEM, (A+BKC), OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY S6

$(A+BK_aC)$	$(A+BK_bC)$	$(A+BK_cC)$	$(A+BK^*C)$
-7.64	-7.83	-7.27	-7.33
-5.49 + j2.70	-5.36 ± j3.09	-5.40 ± j3.36	-5.43 ± j3.46
-4.15 ± j2.60	-3.24 ± j1.89	-3.50 ± j1.54	-3.27 ± j1.42
-1.58 ± j2.66	-2.14 ± j1.89	-2.00 ± j2.18	-2.07 ± j2.11
0.16#	-0.60	-0.85	-1.06

#Unstable eigenvalue.

TABLE XXVI

PERFORMANCE INDICES OF THE LOAD FREQUENCY CONTROL SYSTEM, CASE STUDY S8#

Feedback Gain Matrix Used	Performance Index
K_a	---
K_b	.002495
K_c	.002436
K*	.002201

#Other results of Case Study S8 are the same as Case Study D8.

TABLE XXVII

COMPARISONS OF PERFORMANCE INDICES OF THE LOAD FREQUENCY CONTROL SYSTEM

Dimension of Output Vector	Feedback Gain Matrix Used	Performance Index	
		Deterministic System	Stochastic System
4	K_a	---	---
4	K_b	8.357	.002896
4	K_c	8.306	.002861
4	K*	8.288	.002855
6	K_a	---	---
6	K_b	6.921	.002306
6	K_c	6.762	.002258
6	K*	6.737	.002251
8	K_a	---	---
8	K_b	7.802	.002495
8	K_c	7.491	.002436
8	K*	6.655	.002201

CHAPTER V

CONTROL OF INTERCONNECTED SYNCHRONOUS MACHINES

5.1 Introduction

Within the past few years studies have been made to apply optimal control theory to synchronous machine stabilization problems. With the increasing size and complexity of power systems improved techniques are required in order to achieve a better stability limit. The first part of the works reported in the literature is primarily concerned with state feedback strategies [1][31][46][58][59]. The results of the controller design in the real implementation on a micro-machine shows a good dynamic response for a small disturbance [26][47]. One of the main disadvantages of this technique is that all the state variables are not always available for measurement. To overcome this difficulty output feedback control has been considered [18][20][51][52].

The publications described above confine themselves to a model consisting of one machine connected to an infinite bus. However, there are some situations where a multimachine model is preferred. When using this model computational difficulty has been experienced because of the large dimension of the system. Usually optimal control design of the multimachine system is carried out by using a reduced-order model for each machine [60].

Instead of deleting some state variables which result in reduced-order system design or semi-decoupling the system which results in a one-machine-infinite-bus design, in this chapter the suboptimal technique developed in Chapter III is applied to the multimachine control problem. The technqiue requires less calculation than the optimal design given in Chapter II so it is suitable for multimachine design problem whose dimension is, in general, large. However, since it is desirable to compare the results of the suboptimal control to those of the optimal control, the reduced-order models for synchronous machine and its exciter are used. An interconnected network consisting of two machines and an infinite bus is considered in this study.

5.2 Interconnected Synchronous Machine Model

5.2.1 Synchronous Machine Equations

Comprehensive mathematical equations describing the behavior of a synchronous machine, both during steady state and transient state, were derived by Park [49]. Since then there have been numerous publications dealing with mathematical models of the machine. Different forms of the model can be found in different problems. References [15] [22] [32] [36] [48] are examples of papers and books that present the machine's equations. The following are synchronous machine equations in per unit (except for time). The variables consist of magnetic flux, voltage, and current in direct and quadrature axes, and field circuit.

$$\psi_f = x_f i_f - x_{fd} i_d + x_{fkd} i_{kd} \quad (5.1a)$$

$$\psi_d = x_{fd} i_f - x_d i_d + x_{akd} i_{kd} \quad (5.1b)$$

$$\psi_{kd} = x_{fkd} i_f - x_{akd} i_d + x_{kkd} i_{kd} \quad (5.1c)$$

$$\psi_q = -x_q i_q + x_{akq} i_{kq} \quad (5.1d)$$

$$\psi_{kq} = -x_{akq} i_q + x_{kkq} i_{kq} \quad (5.1e)$$

$$v_f = \frac{1}{\omega_o} \frac{d\psi_f}{dt} + r_f i_f \quad (5.1f)$$

$$v_d = \frac{1}{\omega_o} \frac{d\psi_d}{dt} - r_a i_d - \frac{\omega}{\omega_o} \psi_q \quad (5.1g)$$

$$v_q = \frac{1}{\omega_o} \frac{d\psi_q}{dt} - r_a i_q + \frac{\omega}{\omega_o} \psi_d \quad (5.1h)$$

$$0 = \frac{1}{\omega_o} \frac{d\psi_{kd}}{dt} + r_{kd} i_{kd} \quad (5.1i)$$

$$0 = \frac{1}{\omega_o} \frac{d\psi_{kq}}{dt} + r_{kq} i_{kq} \quad (5.1j)$$

$$v_t^2 = v_d^2 + v_q^2 \quad (5.1k)$$

$$T_a = \psi_d i_q - \psi_q i_d \quad (5.1l)$$

$$\frac{d\omega}{dt} = \frac{\omega_o}{2H} [T_i - T_a - K_d \omega] \quad (5.1m)$$

$$\frac{d\delta}{dt} = \omega \quad (5.1n)$$

where subscripts f, d, q, kd, kq stand for field, d-axis armature, q-axis armature, d-axis amortisseur, and q-axis amortisseur windings, respectively; and

r_j = resistance of circuit j;

x_j = reactance of circuit j;

i_j = current in circuit j;

v_j = voltage in circuit j;

ψ_j = flux linkage of circuit j;

ω = angular velocity of rotor;

ω_o = base angular velocity;

v_t = machine terminal voltage;

T_a = air gap electromagnetic torque of synchronous machine;

T_i = prime mover input torque;

H = per unit inertia of the generating unit;

K_d = system damping coefficient; and

δ = phase angle of machine.

In this study a linearized third-order model of a synchronous machine and a first order excitation system is used. The block diagram of the exciter is shown in Figure 7. The third-order model of a synchronous machine is obtained by neglecting effects of amortisseur windings, armature resistance, and time rate of change of magnetic fluxes. The results are four state equations and seven algebraic equations as follow:

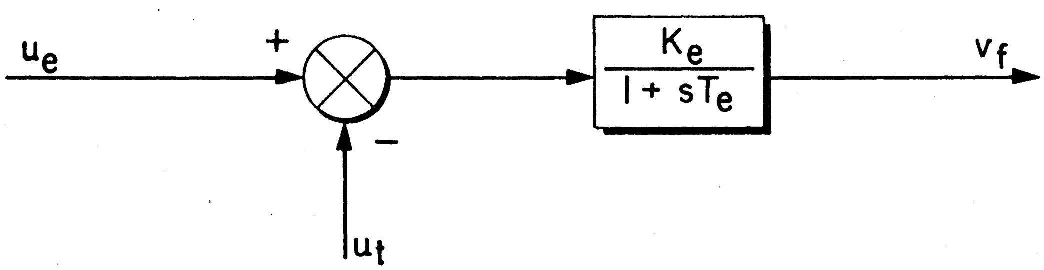


Figure 7. Excitation System

State equations:

$$\frac{d}{dt}\Delta\psi_f = \omega_o \Delta v_f - \omega_o r_f \Delta i_f \tag{5.2a}$$

$$\frac{d}{dt}\Delta\delta = \Delta\omega \tag{5.2b}$$

$$\frac{d}{dt}\Delta\omega = -\frac{\omega_o}{2H}[\Delta T_a + K_d \Delta\omega] \tag{5.2c}$$

$$\frac{d}{dt}\Delta v_f = \frac{1}{T_e}[K_e \Delta u - K_e \Delta v_t - \Delta v_f] \tag{5.2d}$$

Algebraic equations:

$$\Delta\psi_f = x_f \Delta i_f - x_{fd} \Delta i_d \tag{5.3a}$$

$$\Delta\psi_d = x_{fd} \Delta i_f - x_d \Delta i_d \tag{5.3b}$$

$$\Delta\psi_q = -x_q \Delta i_q \tag{5.3c}$$

$$\Delta v_t = \frac{v_d}{v_t}\Delta v_d + \frac{v_q}{v_t}\Delta v_q \tag{5.3d}$$

$$\Delta v_d = -\Delta\psi_q - \frac{\psi_q}{\omega_o}\Delta\omega \tag{5.3e}$$

$$\Delta v_q = \Delta\psi_d + \frac{\psi_d}{\omega_o}\Delta\omega \tag{5.3f}$$

$$\Delta T_a = \psi_d \Delta i_q + i_q \Delta\psi_d - \psi_q \Delta i_d - i_d \Delta\psi_q \tag{5.3g}$$

The state Equation (5.2) may be written as:

$$\frac{dx}{dt} = A_1 x + Bu + Cz \tag{5.4}$$

where

$x^T = [\Delta\delta, \Delta\omega, \Delta v_f, \Delta\psi_f]$;

$z^T = [\Delta i_f, \Delta T_a, \Delta v_t]$;

$u = \Delta u$;

and A_1, B, and C are shown in Table XXVIII.

By manipulation of Equation (5.3a) and (5.3d) through (5.3g) the following equation is obtained:

$$z = EW + GI \tag{5.5}$$

where

$$W^T = [\Delta\psi_f, \Delta\omega]$$
$$I^T = [\Delta i_d, \Delta i_q]$$

and E and G are shown in Table XXVIII.

From Equation (5.3a), (5.3b), (5.3e), and (5.3f) we have

$$V = RW + SI \tag{5.6}$$

where

$$V^T = [\Delta v_d, \Delta v_q]$$

and R and S are shown in Table XXVIII.

5.2.2 Multimachine Equations

In order to make use of equations and symbols presented in section 5.2 and extend it to a two-machine system, we shall modify equations as follow. Suppose an equation for a one-machine system is of the form

$$X = HY \tag{5.7}$$

where

$$X^T = [x_1, x_2]$$
$$Y^T = [y_1, y_2]$$

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

We shall write Equation (5.7) for a two-machine system as

$$X_m = H_m Y_m \tag{5.8}$$

where

$$X_m^T = [x_{1a}, x_{2a}, x_{1b}, x_{2b}]$$

$$Y_m^T = [y_{1a}, y_{2a}, y_{1b}, y_{2b}]$$

$$H_m = \begin{bmatrix} h_{11a} & h_{12a} & 0 & 0 \\ h_{21a} & h_{22a} & 0 & 0 \\ 0 & 0 & h_{11b} & h_{12b} \\ 0 & 0 & h_{21b} & h_{22b} \end{bmatrix}$$

Using this notation Equations (5.4), (5.5) and (5.6) may be written for a multimachine system as

$$\frac{d}{dt} x_m = A_{1m} x_m + B_m u_m + C_m z_m \tag{5.9}$$

$$z_m = E_m W_m + G_m I_m \tag{5.10}$$

$$V_m = R_m W_m + S_m I_m. \tag{5.11}$$

5.2.3 Transmission Network Equations

The transmission network under study is shown in Figure 8. It consists of three buses. Two of them are connected to synchronous machines. The third is an infinite bus. The equation for the network current and voltage is

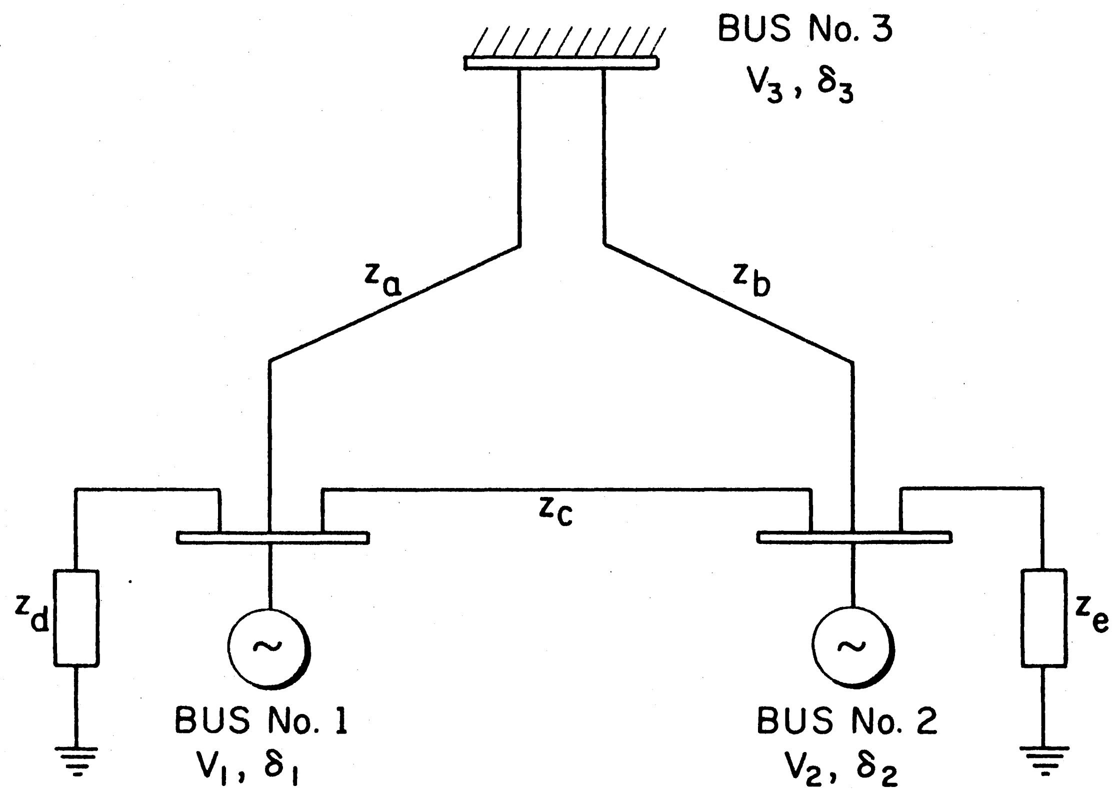


Figure 8. Transmission Network

$$I_n = Y_n V_n \tag{5.12}$$

where

$$I_n^T = [i_{D1}, i_{Q1}, i_{D2}, i_{Q2}, i_{D3}, i_{Q3}]$$

$$V_n^T = [v_{D1}, v_{Q1}, v_{D2}, v_{Q2}, v_{D3}, v_{Q3}]$$

and Y_n is a bus admittance matrix as shown in Table XXVIII.

The relationship between the machine-reference quantities and the network-reference quantities is given by Taylor [57] as

$$V_{mm} = FV_n \tag{5.13}$$

$$I_{mm} = FI_n \tag{5.14}$$

where

$$V_{mm}^T = [v_{d1}, v_{q1}, v_{d2}, v_{q2}, v_{d3}, v_{q3}]$$

$$I_{mm}^T = [i_{d1}, i_{q1}, i_{d2}, i_{q2}, i_{d3}, i_{q3}]$$

and F is a function of δ_1, and δ_2, the displacement between the machine reference and the network reference. It is shown in Table XXVIII.

From Equations (5.14) and (5.12)

$$I_{mm} = FY_n V_n$$

since

$$F^{-1} = F^T$$

$$I_{mm} = FY_n F^T V_{mm} \tag{5.15}$$

Equation (5.15) can be written in linearized form and using the fact that bus No. 3 is infinitely strong, i.e., $\Delta\delta_3 = 0$, we get

$$\Delta I_{mm} = FY_n F^T \Delta V_{mm} + (\frac{\partial F}{\partial \delta_1} Y_n F^T + FY_n \frac{\partial F}{\partial \delta_1}^T) V_{mm} \Delta \delta_1$$

$$+ (\frac{\partial F}{\partial \delta_2} Y_n F^T + FY_n \frac{\partial F}{\partial \delta_2}^T) V_{mm} \Delta \delta_2$$

$$= FY_n F^T \Delta V_{mm} + T \Delta \delta_{mm} \qquad (5.16)$$

where

$$\Delta \delta_{mm}^T = [\Delta \delta_1, \Delta \delta_2]$$

$$T = [(\frac{\partial F}{\partial \delta_1} Y_n F^T + FY_n \frac{\partial F}{\partial \delta_1}^T) V_{mm}, (\frac{\partial F}{\partial \delta_2} Y_n F^T + FY_n \frac{\partial F}{\partial \delta_2}^T) V_{mm}]$$

From definitions, the following equations are obtained:

$$W_m = J_1 x_m \qquad (5.17)$$

$$\Delta \delta_{mm} = J_2 x_m \qquad (5.18)$$

$$I_m = J_3 \Delta I_{mm} \qquad (5.19)$$

$$\Delta V_{mm} = J_4 V_m \qquad (5.20)$$

where J_1, J_2, J_3, and J_4 are shown in Table XXVIII.

By manipulation of Equations (5.9), (5.10), (5.11), (5.17), (5.18), (5.19), and (5.20), the state equation in standard form is obtained:

$$\frac{dx_m}{dt} = \{A_{1m} + C_m E_m J_1 + C_m G_m (I_m - J_3 F \; Y_n F^T J_4 S_m)^{-1}$$

$$\times [J_3 F \; Y_n F^T J_4 R_m J_1 + J_3 T J_2]\} x_m + B_m u_m \qquad (5.21)$$

TABLE XXVIII

CONSTANT MATRICES IN THE SYNCHRONOUS MACHINE CONTROL SYSTEM

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{\omega_o K_d}{2H} & 0 & 0 \\ 0 & 0 & -\frac{1}{T_e} & 0 \\ 0 & 0 & \omega_o & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{K_e}{T_e} \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{\omega_o}{2H} & 0 \\ 0 & 0 & -\frac{K_e}{T_e} \\ -\omega_o r_f & 0 & 0 \end{bmatrix}$$

TABLE XXVIII (Continued)

$$E = \begin{bmatrix} \dfrac{1}{x_f} & 0 \\[2ex] \dfrac{i_q \; x_{fd}}{x_f} & 0 \\[2ex] \dfrac{v_q \; x_{fd}}{v_t \; x_f} & \dfrac{v_q \psi_d - v_d \psi_q}{\omega_o \; v_t} \end{bmatrix}$$

$$G = \begin{bmatrix} \dfrac{x_{fd}}{x_f} & 0 \\[2ex] \dfrac{i_q \; x_{fd}^2}{x_f} - i_q x_d - \psi_q & \psi_d + i_d x_q \\[2ex] \dfrac{v_q}{v_t} \left(\dfrac{x_{fd}^2}{x_f} - x_d \right) & \dfrac{v_d x_q}{v_t} \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & -\dfrac{\psi_q}{\omega_o} \\[2ex] \dfrac{x_{fd}}{x_f} & \dfrac{\psi_d}{\omega_o} \end{bmatrix}$$

$$S = \begin{bmatrix} 0 & x_q \\[2ex] \dfrac{x_{fd}^2}{x_f} - x_d & 0 \end{bmatrix}$$

TABLE XXVIII (Continued)

$$Y_n = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}$$

where

$$y_{ij} = \begin{bmatrix} g_{ij} & -b_{ij} \\ b_{ij} & g_{ij} \end{bmatrix}$$

g_{ij} = real part of admittance y_{ij}; and

b_{ij} = imaginary part of admittance y_{ij}.

$$F = \begin{bmatrix} \cos\delta_1 & \sin\delta_1 & 0 & 0 & 0 & 0 \\ -\sin\delta_1 & \cos\delta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos\delta_2 & \sin\delta_2 & 0 & 0 \\ 0 & 0 & -\sin\delta_2 & \cos\delta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

TABLE XXVIII (Continued)

$$J_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$J_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

5.3 Interconnected System Example

In the study of the interconnected synchronous machine control system, minimization algorithms which are discussed in section 3.5 are used to calculate the optimal gain and the suboptimal feedback gain matrices. The iterative algorithm has been tried but very often it converged to the wrong solution (see section 2.5). The program is designed to calculate the first three terms of the series of the matrices K, P, and Z. The DVDP minimization algorithm is used to find the first terms: K^o, P^o, and Z^o. Eigenvalues of the closed loop system, $A+BK^oC$, are calculated in each function evaluation to insure that the solution yields closed-loop stability. After a solution is obtained the iterative algorithm is applied to refine the result to a more accurate one. For the terms of higher derivative the residue-minimization algorithm is used.

The network under study in this chapter is depicted in Figure 8. The numerical data for the two synchronous machines, network impedances and voltages, are shown in Tables XXIX and XXX. The matrices D, Q, and R are assumed to be identity matrices. The numerical value of the matrix A is shown in Table XXXI. Both deterministic case and stochastic case are considered. In the deterministic case the matrix DVD^T is replaced by an identity matrix. The numerical value of the matrix V for the stochastic case is given in Table III. The number of state variables for the interconnected system is eight. The numbers of feedback variables of six and eight are studied. The system of four

TABLE XXIX

SYNCHRONOUS MACHINES' CONSTANTS

Constants	Machine No. 1	Machine No. 2
r_f	0.0010	0.0016
x_f	1.5	1.47
x_{fd}	0.9	1.33
x_d	1.1	1.20
x_q	0.85	1.07
H	5.0	3.20
K_d	0.003	0.001
K_e	50.0	35.0
T_e	0.1	0.08

TABLE XXX

NETWORK DATA

Variable	Numerical Value
V_1	1.05 p.u.
V_2	1.00 p.u.
V_3	1.00 p.u.
δ_1	5 degree
δ_2	3 degree
δ_3	0 degree
Z_a	0.020 + j0.40 p.u.
Z_b	0.030 + j0.50 p.u.
Z_c	0.015 + j0.25 p.u.
Z_d	2.120 + j0.076 p.u.
Z_e	1.050 + j0.49 p.u.

TABLE XXXI

PLANT MATRIX A OF THE EXAMPLE SYSTEM

0	1.0000	0	0	0	0	0	0
-45.8617	-0.0698	0	-20.1032	8.0283	-0.0126	0	-1.3920
50.2127	-0.3048	-10.0000	-60.2574	-100.4260	-0.5633	0	-216.7320
-0.1796	-0.0008	376.9910	-0.4423	0.1086	0.0004	0	0.1782
0	0	0	0	0	1.0000	0	0
27.5730	0.0444	0	15.8685	-146.9050	-0.3531	0	-181.8650
42.5993	-0.0442	0	-1.5547	-171.7630	-0.9996	-12.5000	-382.4629
0.2338	0.0010	0	0.2780	-1.5496	-0.0059	376.9910	-2.7467

feedback variables was tried but accurate results for the nonlinear matrix equations were very difficult to get, so it is disregarded.

The results of the study of interconnected synchronous machine control systems are presented in Tables XXXII through XLVIII. From these results the following statements can be made:

1. The optimal feedback gain matrix, K, is not sensitive to changes in values of the noise intensity V.

2. When more variables are used as the controller inputs the performance index is better for both optimal system and suboptimal system.

3. Using the first two terms of the series to calculate K yields results which are close to those of the optimal system. However, for a system of higher dimension it is wise to check whether or not more terms are necessary.

4. Like the case of load frequency control, it is found that using three feedback variables for each machine gives results close to those using four feedback variables. So it is suggested that output feedback variables be frequency, phase angle, and field voltage for each generating unit. The field flux linkage does not contribute a significant improvement in the system performance and it is practically unmeasurable. So the variables should not be used as an output feedback variable.

5. The optimal gain matrix calculation requires much more calculation time than the suboptimal one, even if the

former uses the results of the latter as a starting value which is very close to the optimal value. The memory requirement for the optimal gain calculation is about three times more than that for the suboptimal gain calculation.

6. When the minimization algorithm is used, the memory requirement for the optimal gain calculation is about three times more than that of the suboptimal gain. The computation time is also longer for the optimal method, even if the results of the suboptimal gain method are used as a starting value for the optimal value. It should be noted that the optimal gain matrix for this problem is obtained by the iterative method, since an initial value which is near the optimal gain matrix is available. The minimization algorithm is applied to calculate the optimal gain matrix for the sake of comparison only.

5.4 Conclusion

The applications of the suboptimal control technique to interconnected synchronous machine system are studied in this chapter. In section 5.2 a model for interconnected synchronous machines in the standard state variable form is developed. It represents dynamic aspects of the three-bus power system network. One of the buses is assumed to be infinitely strong. Control signals are derived from some of the state variables: frequencies, phase angles, field voltages, and flux linkages of field windings. Optimal and suboptimal feedback gain matrices are calculated and compared. The minimization algorithm is used. The results of the study show that the suboptimal gain calculation is very effective. It requires much less calculation than that of the optimal gain while the former technique results in a small amount of performance degradation.

TABLE XXXII

FEEDBACK GAIN MATRICES OF THE SYNCHRONOUS MACHINE CONTROL SYSTEM, CASE STUDY D8

$$K_a = \begin{bmatrix} -.0620 & 1.0078 & -1.5472 & -.9653 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .2477 & 1.0026 & -1.5902 & -.9398 \end{bmatrix}$$

$$K_b = \begin{bmatrix} -.0620 & 1.0078 & -1.5472 & -.9653 & .3546 & -.0797 & .1289 & .4414 \\ -.0726 & .0883 & .1138 & .0634 & .2477 & 1.0026 & -1.5902 & -.9398 \end{bmatrix}$$

$$K_c = \begin{bmatrix} -.0689 & 1.0044 & -1.5473 & -.9730 & .3546 & -.0797 & .1289 & .4414 \\ -.0726 & .0883 & .1138 & .0634 & .2468 & 1.0011 & -1.6183 & -1.0016 \end{bmatrix}$$

$$K^* = \begin{bmatrix} -.0689 & 1.0029 & -1.5463 & 0.9721 & .3478 & -.0839 & .1291 & .4420 \\ -.0676 & .0908 & .1130 & .0636 & .2473 & .9993 & -1.6160 & -.9983 \end{bmatrix}$$

TABLE XXXIII

COMPUTER BURDEN OF FEEDBACK GAIN CALCULATION OF THE SYNCHRONOUS MACHINE CONTROL SYSTEM, CASE STUDY D8

Feedback Gain Matrix Obtained	Memory (K-byte)	Number of Function Evaluated#	Execution Time# (Seconds)	Compilation Time (Seconds)
K_a	136	328	60	16.62
K_b	148	431	89	21.27
K_c	164	512	114	26.65
K*	440	>100	>300	10.04

#Approximate value.

TABLE XXXIV

PERFORMANCE INDICES OF THE SYNCHRONOUS MACHINE CONTROL SYSTEM, CASE STUDY D8

Feedback Gain Matrix Used	Performance Index
K_a	4.01368
K_b	3.96699
K_c	3.96593
K^*	3.96592

TABLE XXXV

EIGENVALUES OF THE CLOSED-LOOP SYSTEM, (A+BKC), OF THE SYNCHRONOUS MACHINE CONTROL SYSTEM, CASE STUDY D8

$(A+BK_aC)$	$(A+BK_bC)$	$(A+BK_cC)$	$(A+BK^*C)$
$-367 \pm j200$	$-385 \pm j194$	$-386 \pm j199$	$-386 \pm j199$
$-295 \pm j374$	$-283 \pm j378$	$-292 \pm j386$	$-292 \pm j386$
-149.184	-136.787	-129.496	-129.580
-16.519	-17.430	-17.150	-17.139
-2.877	-2.927	-2.961	-2.963
-1.313	-1.261	-1.310	-1.311

TABLE XXXVI

FEEDBACK GAIN MATRICES OF THE SYNCHRONOUS MACHINE CONTROL SYSTEM, CASE STUDY D6

$$K_a = \begin{bmatrix} -.1319 & .0988 & -.5279 & 0 & 0 & 0 \\ 0 & 0 & 0 & .2840 & .4555 & -.9877 \end{bmatrix}$$

$$K_b = \begin{bmatrix} -.1319 & .0988 & -.5279 & .1535 & .2067 & -.5031 \\ -.0643 & .0764 & .2831 & .2840 & .4555 & -.9877 \end{bmatrix}$$

$$K_c = \begin{bmatrix} -.1218 & .1308 & -.4812 & .1535 & .2067 & -.5031 \\ -.0643 & .0764 & .2831 & .3113 & .4544 & -1.0365 \end{bmatrix}$$

$$K^* = \begin{bmatrix} -.1223 & .1298 & -.4973 & .1564 & .1990 & -.5250 \\ -.0700 & .0740 & .2355 & .3106 & .4552 & -1.0187 \end{bmatrix}$$

TABLE XXXVII

COMPUTER BURDEN OF FEEDBACK GAIN CALCULATION OF THE SYNCHRONOUS MACHINE CONTROL SYSTEM, CASE STUDY D6

Feedback Gain Matrix Obtained	Memory (K-byte)	Number of Function Evaluated[#]	Execution Time[#] (Seconds)	Compilation Time (Seconds)
K_a	136	179	34	16.95
K_b	148	436	104	20.87
K_c	164	534	134	26.09
K^*	440	>100	>300	10.12

[#]Approximate value.

TABLE XXXVIII

PERFORMANCE INDICES OF THE SYNCHRONOUS MACHINE CONTROL SYSTEM, CASE STUDY D6

Feedback Gain Matrix Used	Performance Index
K_a	4.55949
K_b	4.18517
K_c	4.01546
K^*	4.00669

TABLE XXXIX

EIGENVALUES OF THE CLOSED-LOOP SYSTEM, (A+BKC), OF THE SYNCHRONOUS MACHINE CONTROL SYSTEM, CASE STUDY D6

$(A+BK_aC)$	$(A+BK_bC)$	$(A+BK_cC)$	$(A+BK^*C)$
-235	$-171 \pm j335$	$-176 \pm j327$	$-175 \pm j323$
$-165 \pm j285$	$-145 \pm j49$	$-131 \pm j51$	$-129 \pm j51$
$-71 \pm j99$	-77.54	-81.73	-89.39
-9.74	$-65 \pm j2.1$	-18.91	-18.73
-3.98	-1.25	-2.94	-2.91
-1.08		-1.31	-1.30

TABLE XL

FEEDBACK GAIN MATRICES OF THE SYNCHRONOUS MACHINE CONTROL SYSTEM, CASE STUDY S8

K_a =	-.0620	1.0078	-1.5472	-.9653	0	0	0	0
	0	0	0	0	.2477	1.0026	-1.5902	-.9398
K_b =	-.0620	1.0078	-1.5472	-.9653	.3541	-.0812	.1305	.4434
	-.0725	.0879	.1143	.0637	.2477	1.0026	-1.5902	-.9398
K_c =	-.0688	1.0038	-1.5469	-.9726	.3541	-.0812	.1305	.4434
	-.0725	.0879	.1143	.0637	.2463	.9993	-1.6164	-.9992
K^* =	-.0689	1.0031	-1.5464	-.9721	.3478	-.0839	.1291	.4420
	-.0676	.0908	.1130	.0636	.2473	.9993	-1.6160	-.9983

TABLE XLI

COMPUTER BURDEN OF FEEDBACK GAIN CALCULATION OF THE SYNCHRONOUS MACHINE CONTROL SYSTEM, CASE STUDY S8

Feedback Gain Matrix Obtained	Memory (K-byte)	Number of Function Evaluated#	Execution Time# (Seconds)	Compilation Time (Seconds)
K_a	136	279	54.39	17.52
K_b	148	376	81.87	21.02
K_c	164	452	103.41	25.29
K*	440	>100	>300	11.35

#Approximate value.

TABLE XLII

PERFORMANCE INDICES OF THE SYNCHRONOUS MACHINE CONTROL SYSTEM, CASE STUDY S8

Feedback Gain Matrix Used	Performance Index
K_a	.0005780
K_b	.0005720
K_c	.0005718
K^*	.0005718

TABLE XLIII

EIGENVALUES OF THE CLOSED-LOOP SYSTEM, (A+BKC), OF THE SYNCHRONOUS MACHINE CONTROL SYSTEM, CASE STUDY S8

$(A+BK_aC)$	$(A+BK_bC)$	$(A+BK_cC)$	$(A+BK^*C)$
$-368 \pm j200$	$-385 \pm j194$	$-386 \pm j199$	$-386 \pm j199$
$-295 \pm j374$	$-283 \pm j377$	$-292 \pm j386$	$-292 \pm j386$
-149.18	-136.80	-129.41	-129.58
-16.52	-17.43	-17.15	-17.15
-2.88	-2.93	-2.96	-2.96
-1.31	-1.26	-1.31	-1.31

TABLE XLIV

FEEDBACK GAIN MATRICES OF THE SYNCHRONOUS MACHINE CONTROL SYSTEM, CASE STUDY S6

$$K_a = \begin{bmatrix} -.1156 & .1057 & -.4508 & 0 & 0 & 0 \\ 0 & 0 & 0 & .2918 & .4613 & -.9555 \end{bmatrix}$$

$$K_b = \begin{bmatrix} -.1156 & .1057 & -.4508 & .1672 & .2183 & -.4784 \\ -.0859 & .0660 & .3986 & .2918 & .4613 & -.9555 \end{bmatrix}$$

$$K_c = \begin{bmatrix} -.1133 & .1352 & -.3820 & .1672 & .2183 & -.4784 \\ -.0859 & .0660 & .3986 & .3440 & .4778 & -1.0740 \end{bmatrix}$$

$$K^* = \begin{bmatrix} -.1149 & .1337 & -.4017 & .1746 & .2135 & -.5088 \\ -.0945 & .0633 & .3352 & .3379 & .4745 & -1.0371 \end{bmatrix}$$

TABLE XLV

COMPUTER BURDEN OF FEEDBACK GAIN CALCULATION OF THE SYNCHRONOUS MACHINE CONTROL SYSTEM, CASE STUDY S6

Feedback Gain Matrix Obtained	Memory (K-byte)	Number of Function Evaluated#	Execution Time# (Seconds)	Compilation Time (Seconds)
K_a	136	182	35	16.50
K_b	148	387	91	20.84
K_c	164	491	122	26.46
K^*	440	>100	>300	10.95

#Approximate value.

TABLE XLVI

PERFORMANCE INDICES OF THE SYNCHRONOUS MACHINE CONTROL SYSTEM, CASE STUDY S6

Feedback Gain Matrix Used	Performance Index
K_a	.0007028
K_b	.0006082
K_c	.0005931
K^*	.0005913

TABLE XLVII

EIGENVALUES OF THE CLOSED-LOOP SYSTEM, (A+BKC), OF THE SYNCHRONOUS MACHINE CONTROL SYSTEM, CASE STUDY S6

$(A+BK_aC)$	$(A+BK_bC)$	$(A+BK_cC)$	$(A+BK^*C)$
-209.92	-155 ± j360	-167 ± j339	-167 ± j338
-158 ± j292	-134 ± j61	-107 ± j56	-109 ± j66
-63 ± j112	-73.15	-113.57	-104.24
-10.64	-10.68	-20.95	-20.58
-3.79	-5.10	-2.83	-2.84
-1.09	-1.24	-1.31	-1.30

TABLE XLVIII

COMPARISONS OF PERFORMANCE INDICES OF THE SYNCHRONOUS MACHINE CONTROL SYSTEM

Dimension of Output Vector	Feedback Gain Matrix Used	Performance Index	
		Deterministic System	Stochastic System
6	K_a	4.55959	.0007028
6	K_b	4.18517	.0006082
6	K_c	4.01546	.0005931
6	K^*	4.00669	.0005913
8	K_a	4.01368	.0005780
8	K_b	3.96699	.0005720
8	K_c	3.96593	.0005718
8	K^*	3.96592	.0005718

CHAPTER VI

SUMMARY AND CONCLUSIONS

6.1 Summary

The purpose of this research was to find a control design technique which does not require extensive off-line computation. Such a technique is useful for the development of interconnected power system control and other large interconnected control schemes. The control is required to be a linear transformation of only some state variables which are measurable. The performance index is an integral quadratic type with infinite final time. In order to satisfy these conditions the feedback gain matrix of the control must be the solution of a set of nonlinear matrix equations, called necessary conditions, and it must stabilize the closed-loop system. Usually a considerable effort is needed to solve the necessary conditions. If the system is a large one, the problem involved with calculation of the optimal feedback gain is not trivial. A solution to this problem is obtained by applying the technique developed in this research.

In this study the system is assumed to consist of two subsystems. The interactions between the subsystems are functions of a coupling coefficient. When the coefficient is zero the interactions are neglected and the two subsystems are independent. This makes the calculations using the subsystem matrices possible. With the use of the coupling coefficient the optimal feedback gain matrix can be approximated by a finite term Taylor series expansion. The terms in the series of the suboptimal feedback gain matrix can be calculated from sets of equations which are functions of these subsystem matrices. If the number of terms is selected properly the suboptimal approach potentially offers large reductions in computational requirements while introducing only a small amount of performance degradation.

It is shown that the sets of equations used to solve for the terms in the series for the suboptimal feedback gain has a similar structure to necessary conditions of the optimal feedback gain. Furthermore, the even derivative terms of the series are of α-type and the odd derivative terms of the series are of β-type. It is also proved that the nonzero coupling coefficient has no effect on the feedback gain matrix if the weighting matrix Q is of α-type.

Three numerical methods to solve the sets of the matrix equations are developed. They are the iterative algorithm, the DVDP minimization algorithm, and the residue minimization algorithm. The iterative algorithm requires less computer processing time but convergence to the right solution is not guaranteed. The DVDP algorithm usually converges to the right solution but it requires a longer execution time and the result may not be very accurate. Furthermore, the performance index to be minimized must be well defined. If it is not, for example in the case of necessary conditions for derivative terms of the series of the feedback gain matrix, the residue minimization algorithm can be applied. This algorithm requires more processing time but a more accurate result can be obtained.

The applications of the techniques to interconnected power systems are studied. Dynamic models for the load frequency control system and the synchronous machine control system are developed. The optimal and suboptimal feedback gain matrices are calculated and compared. The results show that the suboptimal technique results in a closed-loop control system whose performance is almost the same as the performance of the optimal system but it requires much less computer burden.

6.2 Conclusions

The suboptimal control design technique presented herein makes use of decoupling of the interconnected system into smaller subsystems. By this method the difficulties in solving the nonlinear set of necessary conditions as well as the processing time and the rapid access memory requirements for large scale systems have been greatly reduced. Even though the technique is suitable for interconnected power systems, it may be applied to any large-scale dynamic systems using a relatively small capability computer. The choice of the coupling coefficient may be selected from a physical parameter but it can be introduced as a computational tool. The dimensions of the two subsystems need not be equal even though equal dimensions are used in this research.

Since the coupling coefficient of the interconnected system can be chosen quite arbitrarily, it is felt that the technique presented can be applied to a single large system or a system consisting of more than two subsystems. This is accomplished by dividing the system into two parts. The validity of the suboptimal technique to such a system offers a topic for further investigation.

Three terms of the series of the suboptimal feedback gain were used in this study and satisfactory results were obtained for the eighth-order power system considered. However, a criterion to judge the number of terms of the series required for a satisfactory performance of the closed-loop system when the optimal performance is unknown is still open for further research. One suggested method is by observing the performance improvement when one more term is added to the series. If the performance index is decreased only a very small amount, this should be the indication that enough terms have been used.

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APPENDIX A

PROGRAM TO EVALUATE THE MATRICES IN THE SERIES OF THE SUBOPTIMAL FEEDBACK GAIN MATRIX USING THE ITERATIVE ALGORITHM

```
C/////////////////////////////////////////////////////////////////////
C
C     THIS PROGRAM FINDS THE SUBOPTIMAL FEEDBACK GAIN MATRIX
C     ITERATION ALGORITHM IS USED
C
C/////////////////////////////////////////////////////////////////////
      INTEGER S1,S2,T1,T2
      REAL K011(10,10),K012(10,10),K021(10,10),K022(10,10)
      REAL K112(10,10),K121(10,10)
      REAL K211(10,10),K222(10,10)
      DIMENSION A1(10,10),A2(10,10),B1(10,10),B2(10,10),C1(10,10),
     *   C2(10,10),D1(10,10),D2(10,10),Q1(10,10),Q2(10,10),R1(10,10),
     *   R2(10,10),V1(10,10),V2(10,10),A12(10,10),A21(10,10),Q12(10,10),
     *   Q21(10,10),PO11(10,10),PO12(10,10),PO21(10,10),PO22(10,10),
     *   ZO11(10,10),ZO12(10,10),ZO21(10,10),ZO22(10,10),
     *   P211(10,10),P222(10,10),Z211(10,10),Z222(10,10) ,
     *   GTO11(10,10),GTO22(10,10),GT121(10,10),GT112(10,10) ,
     *   RESP1(10,10),RESP2(10,10),RESZ1(10,10),RESZ2(10,10),
     *   RINV1(10,10),RINV2(10,10)
      DIMENSION GO11(10,10),GO22(10,10),G112(10,10),G121(10,10),
     *          GG211(10,10),GGT211(10,10),GG222(10,10),GGT222(10,10),
     *          Z112(10,10),Z121(10,10),P112(10,10),P121(10,10),
     *          BT1(10,10),BT2(10,10),CT1(10,10),CT2(10,10),
     *          X1(10,10),X2(10,10),Y1(10,10),Y2(10,10),
     *          RESK1(10,10),RESK2(10,10),CZC1(10,10),CZC2(10,10)
      COMMON/AA/N1,M1,S1,T1,N2,M2,S2,T2
      COMMON/BB/A1,A2,A12,A21,B1,B2,C1,C2,D1,D2,Q1,Q2,Q12,Q21,
     *   R1,R2,V1,V2,K011,K022,K012,K021
      COMMON/IS/ISUB
      COMMON/DIFF1/K112,K121
      COMMON/DIFF2/K211,K222
      COMMON/INV/ICHECK
C
      LIMIT1=30
      LIMIT2=30
      EPSIL=.5
      ICHECK=0
      EPS1=1.E-5
      EPS2=1.E-5
      EPS3=.001
      NN=4
C
C
      DO 1 I=1,4
      DO 1 J=1,8
      JJ=J-4
      IF(J.LE.4) READ(5,2) A1(I,J)
      IF(J.GT.4) READ(5,2) A12(I,JJ)
1     CONTINUE
2     FORMAT(20X,E20.6)
      DO 3 I=1,4
      DO 3 J=1,8
      JJ=J-4
      IF(J.LE.4) READ(5,2) A21(I,J)
      IF(J.GT.4) READ(5,2) A2(I,JJ)
3     CONTINUE
C
C     WRITE INPUT
C
      WRITE(6,20)
20    FORMAT('1SUBSYSTEM NUMBER 1',///,
     *   3X,'I',4X,'J',8X,'A1',10X,'B1',10X,'C1',10X,'D1',10X,'Q1',10X,
     *   'R1',10X,'V1',10X,'K1')
      DO 101 I=1,N1
      WRITE(6,10)
10    FORMAT(1H0)
      DO 101 J=1,N1
101   WRITE(6,30) I,J,A1(I,J),B1(I,J),C1(I,J),D1(I,J),Q1(I,J),R1(I,J),
     *   V1(I,J),K011(I,J)
30    FORMAT(1H ,I3,I5,2X,8(2X,F10.4))
      WRITE(6,21)
21    FORMAT(/////,' SUBSYSTEM NUMBER2',///,
     *   3X,'I',4X,'J',8X,'A2',10X,'B2',10X,'C2',10X,'D2',10X,'Q2',10X,
     *   'R2',10X,'V2',10X,'K2')
      DO 102 I=1,N2
      WRITE(6,10)
      DO 102 J=1,N2
102   WRITE(6,30) I,J,A2(I,J),B2(I,J),C2(I,J),D2(I,J),Q2(I,J),
     *   R2(I,J),V2(I,J),K022(I,J)
      WRITE(6,22)
22    FORMAT(/////,' COUPLING MATRICES',///,
     *   3X,'I',4X,'J',8X,'A12',9X,'A21',9X,'Q12',9X,'Q21')
      DO 103 I=1,N1
      WRITE(6,10)
      DO 103 J=1,N1
103   WRITE(6,40) I,J,A12(I,J),A21(I,J),Q12(I,J),Q21(I,J)
40    FORMAT(1H ,I3,I5,2X,4(2X,F10.4))
C
C
      CALL TRANSP(B1,N1,S1,BT1)
      CALL TRANSP(C1,M1,N1,CT1)
      CALL INVERT(R1,S1,RINV1)
      CALL TRANSP(D1,N1,T1,X1)
      CALL MULT(V1,X1,T1,T1,N1,X2)
      CALL MULT(D1,X2,N1,T1,N1,Y1)
C     NOTE  Y1=D1*V1*D1
      CALL TRANSP(B2,N2,S2,BT2)
      CALL TRANSP(C2,M2,N2,CT2)
      CALL INVERT(R2,S2,RINV2)
      CALL TRANSP(D2,N2,T2,X1)
      CALL MULT(V2,X1,T2,T2,N2,X2)
      CALL MULT(D2,X2,N2,T2,N2,Y2)
C     NOTE  Y2=D2*V2*D2
C
C     DECOUPLED FEEDBACK GAIN
C
      ISUB=1
      CALL ZERO(A1,B1,C1,D1,V1,Q1,R1,Y1,BT1,CT1,RINV1,N1,M1,S1,T1,
     *   K011,PO11,ZO11,GO11,GTO11)
150   ISUB=2
      CALL ZERO(A2,B2,C2,D2,V2,Q2,R2,Y2,BT2,CT2,RINV2,N2,M2,S2,T2,
     *   K022,PO22,ZO22,GO22,GTO22)
C
C     FIRST DERIVATIVE OF FEEDBACK GAIN
C
200   MMM=0
      DO 210 I=1,N1
      DO 210 J=1,N2
210   A12(I,J)=A12(I,J)/EPSIL
```

```
      DO 211 I=1,N2
      DO 211 J=1,N1
211   A21(I,J)=A21(I,J)/EPSIL
C
C     FIND CZC1,CZC2
C
      CALL MULT(C1,Z011,M1,N1,N1,X1)
      CALL MULT(X1,CT1,M1,N1,M1,X2)
      CALL INVERT(X2,M1,CZC1)
C
      CALL MULT(C2,Z022,M2,N2,N2,X1)
      CALL MULT(X1,CT2,M2,N2,M2,X2)
      CALL INVERT(X2,M2,CZC2)
C
C     ITERATION LOOP
C
250   MMM=MMM+1
      CALL ZZ1(K112,K121,A12,A21,B1,B2,C1,C2,G011,GT022,Z011,Z022,
     *   N1,M1,S1,N2,M2,S2,Z112,RESZ1,G112,GT121,ITERZ1)
      CALL ZZ1(K121,K112,A21,A12,B2,B1,C2,C1,G022,GT011,Z022,Z011,
     *   N2,M2,S2,N1,M1,S1,Z121,RESZ2,G121,GT112,ITERZ2)
C
      CALL PP1(GT121,G112,P011,P022,GT011,G022,Q12,K011,K022,K121,K112,
     *   R1,R2,CT1,C2,N1,M1,S1,N2,M2,S2,P112,RESP1,ITERP1)
      CALL PP1(GT112,G121,P022,P011,GT022,G011,Q21,K022,K011,K112,K121,
     *   R2,R1,CT2,C1,N2,M2,S2,N1,M1,S1,P121,RESP2,ITERP2)
C
      CALL KK1(R1,K011,C1,CT2,C2,Z112,Z022,BT1,P011,P112,N1,M1,S1,
     *   N2,M2,S2,Y1,CZC2,RINV1)
      CALL KK1(R2,K022,C2,CT1,C1,Z121,Z011,BT2,P022,P121,N2,M2,S2,
     *   N1,M1,S1,Y2,CZC1,RINV2)
C
C     FIND RESIDUE OF K1
C
      BIGR=0.
      DO 220 I=1,S1
      DO 220 J=1,M2
      RESK1(I,J)=K112(I,J)+Y1(I,J)
      RKK=ABS(RESK1(I,J))
220   BIGR=AMAX1(BIGR,RKK)
      DO 225 I=1,S2
      DO 225 J=1,M1
      RESK2(I,J)=K121(I,J)+Y2(I,J)
      RKK=ABS(RESK2(I,J))
225   BIGR=AMAX1(BIGR,RKK)
C
C     WRITE OUTPUT
C
      WRITE(6,230) MMM,BIGR,ITERZ1,ITERZ2,ITERP1,ITERP2
230   FORMAT('1ITERATION NUMBER',I5,//,
     *   ' MAXIMUM RESIDUE OF K1',E20.6,//,
     *   ' NUMBER OF ITERATION USED:',10X,'ITERZ112 =',I4,10X,
     *   'ITERZ121 =',I4,10X,'ITERP112 =',I4,10X,'ITERP121 =',I4,//,
     *   ' FIRST DERIVATIVE OF FEEDBACK GAIN',///,
     *                  3X,'I',4X,'J',9X,'Z112',
     *   9X,'RESZ1',10X,'Z121',9X,'RESZ2',10X,'P112',9X,'RESP1',9X,1X,
     *   'P121',9X,'RESP2')
      DO 240 I=1,NN
      WRITE(6,10)
      DO 240 J=1,NN
240   WRITE(6,245) I,J,Z112(I,J),RESZ1(I,J),Z121(I,J),RESZ2(I,J),
     *    P112(I,J),RESP1(I,J),P121(I,J),RESP2(I,J)
245   FORMAT(1X,I3,I5,2X,8(2X,E12.5))
      WRITE(6,260)
260   FORMAT(1H1,/,3X,'I',4X,'J',9X,'G011',9X,'GT022',10X,'G112',
     *    10X,'G121', 9X,'K112(N),',4X,'-K112(N+1)',6X,'K121(N)',4X,
     *    '-K121(N+1)')
      DO 265 I=1,NN
      WRITE(6,10)
      DO 265 J=1,NN
265   WRITE(6,245) I,J,G011(I,J),GT022(I,J),G112(I,J),G121(I,J),
     *    K112(I,J),Y1(I,J),K121(I,J),Y2(I,J)
C
C     TEST FOR TERMINATION
C
      IF(MMM.GE.LIMIT1) GO TO 300
      IF(BIGR.LT.EPS1) GO TO 300
C
C     UPDATE K1
C
      ALFA=1.
      IF(BIGR.LT.1.) ALFA=.5
      IF(BIGR.LT.1.) BIGR=1.
      DO 270 I=1,S1
      DO 270 J=1,M2
270   K112(I,J)=K112(I,J)-ALFA*RESK1(I,J)/BIGR
      DO 275 I=1,S2
      DO 275 J=1,M1
275   K121(I,J)=K121(I,J)-ALFA*RESK2(I,J)/BIGR
      GO TO 250
C
C     SECOND DERIVATIVE OF FEEDBACK GAIN
C
C
300   MMM=0
C
C     ITERATION LOOP
C
310   MMM=MMM+1
      CALL MULT(K211,C1,S1,M1,N1,X1)
      CALL MULT(B1,X1,N1,S1,N1,GG211)
      CALL TRANSP(GG211,N1,N1,GGT211)
      CALL MULT(K222,C2,S2,M2,N2,X1)
      CALL MULT(B2,X1,N2,S2,N2,GG222)
      CALL TRANSP(GG222,N2,N2,GGT222)
C
C
      CALL ZZ2(K211,B1,C1,Z011,Z121,Z112,G112,GT112,G011,GT011,
     *   N1,M1,S1,N2,M2,S2,GG211,GGT211,Z211,RESZ1,ITERZ1)
      CALL ZZ2(K222,B2,C2,Z022,Z112,Z121,G121,GT121,G022,GT022,
     *   N2,M2,S2,N1,M1,S1,GG222,GGT222,Z222,RESZ2,ITERZ2)
C
C
      CALL PP2(GGT211,GG211,P011,P121,P112,R1,R2,G121,GT121,GT011,
     *   G011,C1,CT1,K011,K121,K211,N1,M1,S1,N2,M2,S2,P211,RESP1,ITERP1)
      CALL PP2(GGT222,GG222,P022,P112,P121,R2,R1,G112,GT112,GT022,
     *   G022,      C2,CT2,K022,K112,K222,N2,M2,S2,N1,M1,S1,P222,
     *   RESP2,ITERP2)
C
C
```

```
      CALL KK2(R1,RINV1,BT1,C1,C2,CT1,P211,P112,P011,Z011,Z121,Z211,
     *   K112,K011,CZC1,N1,M1,S1,N2,M2,S2,Y1)
      CALL KK2(R2,RINV2,BT2,C2,C1,CT2,P222,P121,P022,Z022,Z112,Z222,
     *   K121,K022,CZC2,N2,M2,S2,N1,M1,S1,Y2)
C
C     FIND RESIDUE OF K211, K222
C
      BIGR=0.
      DO 320 I=1,S1
      DO 320 J=1,M1
      RESK1(I,J)=K211(I,J)+Y1(I,J)
      RKK=ABS(RESK1(I,J))
320   BIGR=AMAX1(BIGR,RKK)
      DO 325 I=1,S2
      DO 325 J=1,M2
      RESK2(I,J)=K222(I,J)+Y2(I,J)
      RKK=ABS(RESK2(I,J))
325   BIGR=AMAX1(BIGR,RKK)
C
C
      WRITE(6,330) MMM,BIGR,ITERZ1,ITERZ2,ITERP1,ITERP2
330   FORMAT('1ITERATION NUMBER',I5,//,
     *   ' MAXIMUM RESIDUE OF K2',E20.6,//,
     *   ' NUMBER OF ITERATION USED:',10X,'ITERZ211 =',I4,10X,
     *   'ITERZ222 =',I4,10X,'ITERP211 =',I4,10X,'ITERP222 =',I4,//,
     *   ' SECOND DERIVATIVE OF FEEDBACK GAIN',///,
     *   3X,'I',4X,'J',9X,'Z211',9X,'RESZ1',10X,'Z222',9X,'RESZ2',
     *   10X,'P211',9X,'RESP1',9X,1X,'P222',9X,'RESP2')
      DO 340 I=1,NN
      WRITE(6,10)
      DO 340 J=1,NN
340   WRITE(6,245) I,J,Z211(I,J),RESZ1(I,J),Z222(I,J),RESZ2(I,J),
     *   P211(I,J),RESP1(I,J),P222(I,J),RESP2(I,J)
      WRITE(6,360)
360   FORMAT(1H1,/,3X,'I',4X,'J',9X,'GG211',8X,'GG222',8X,
     *   'K211(N)',6X,'-K211(N+1)',5X,'RESK1',9X,'K222(N)',6X,
     *   '-K222(N+1)',5X,'RESK2')
      DO 365 I=1,NN
      WRITE(6,10)
      DO 365 J=1,NN
365   WRITE(6,245) I,J,GG211(I,J),GG222(I,J),K211(I,J),Y1(I,J),
     *   RESK1(I,J),K222(I,J),Y2(I,J),RESK2(I,J)
C
C     TEST FOR TERMINATION
C
      IF(MMM.GE.LIMIT2) GO TO 400
      IF(BIGR.LT.EPS2) GO TO 400
C
C     UPDATE K2
C
      IF(BIGR.LT.1.) ALFA=.5
      IF(BIGR.LT.1.) BIGR=1.
      DO 370 I=1,S1
      DO 370 J=1,M1
370   K211(I,J)=K211(I,J)-ALFA*RESK1(I,J)/BIGR
      DO 375 I=1,S2
      DO 375 J=1,M2
375   K222(I,J)=K222(I,J)-ALFA*RESK2(I,J)/BIGR
      GO TO 310
400   STOP
      END
      SUBROUTINE ZERO(A,B,C,D,V,Q,R,DVD,BT,CT,RINV,N,M,S,T,K,P,Z,ABKC,
     *   ABKCT)
C************************************************************************
C     THIS SUBROUTINE FINDS DECOUPLED FEEDBACK GAIN
C************************************************************************
      INTEGER S,T
      REAL K(10,10)
      COMPLEX ZZZ,W
      DIMENSION W(10),ZZZ(10,10),WK(120)
      DIMENSION ABKCT(10,10)
      DIMENSION BT(10,10),CT(10,10),RINV(10,10),A(10,10),B(10,10),
     *   C(10,10),D(10,10),V(10,10),Q(10,10),R(10,10),P(10,10),Z(10,10),
     *   DVD(10,10),X1(10,10),X2(10,10),X3(10,10),X4(10,10),ABKC(10,10),
     *   RESP(10,10),RESK(10,10),RESZ(10,10)
      COMMON/DIFFO/ LIMITO,EPSO,IPRTO
      COMMON/SL/H,EPS
      COMMON/IS/ISUB
C
      MMM=0
100   MMM=MMM+1
C
C     FIND Z
C
      CALL MULT(K,C,S,M,N,X1)
      CALL MULT(B,X1,N,S,N,X2)
      CALL ADD(A,X2,N,N,ABKC)
      CALL TRANSP(ABKC,N,N,X3)
      CALL TRANSP(ABKC,N,N,ABKCT)
      CALL SOLN(X3,ABKC,DVD,N,N,Z,RESZ,MZ)
C
C     FIND P
C
      CALL MULT(R,X1,S,S,N,X2)
      CALL TRANSP(X1,S,N,X4)
      CALL MULT(X4,X2,N,S,N,X1)
      CALL ADD(Q,X1,N,N,X2)
      CALL SOLN(ABKC,X3,X2,N,N,P,RESP,MP)
C
C     FIND K
C
      CALL MULT(Z,CT,N,N,M,X1)
      CALL MULT(C,X1,M,N,M,X2)
      CALL INVERT(X2,M,X3)
      CALL MULT(X1,X3,N,M,M,X4)
      CALL MULT(P,X4,N,N,M,X1)
      CALL MULT(BT,X1,S,N,M,X2)
      CALL MULT(RINV,X2,S,S,M,X4)
C     NOTE  X4=RINV*BT*P*Z*CT*((C*Z*CT)**-1)
C
C     FIND RESIDUE OF K
C
      BIGR=0.
      DO 1 I=1,S
      DO 1 J=1,M
      RESK(I,J)=K(I,J)+X4(I,J)
      RR=ABS(RESK(I,J))
1     BIGR=AMAX1(BIGR,RR)
C
C
```

```
      WRITE(6,10) MMM,MP,MZ,BIGR,ISUB
10    FORMAT('1OUTPUT OF SUBROUTINE ZERO',///,
     *    ' ITERATION NUMBER',I5,///,
     *    ' NUMBER OF ITERATION USED TO FIND P',I5,///,
     *    ' NUMBER OF ITERATION USED TO FIND Z',I5,///,
     *    ' MAXIMUM RESIDUE OF K',E15.6,///,
     *    ' DECOUPLED FEEDBACK GAIN OF SUBSYSTEM NUMBER',I5,///,
     *    1H0,2X,'I',4X,'J',7X,'Z(I,J)',7X,'RESZ(I,J)',8X,'P(I,J)',
     *    7X,'RESP(I,J)',7X,'KZ(I,J)',9X,'K(I,J)',7X,'RESK(I,J)')
      DO 3 I=1,N
      WRITE(6,20)
20    FORMAT(1H0)
      DO 3 J=1,N
3     WRITE(6,30) I,J,Z(I,J),RESZ(I,J),P(I,J),RESP(I,J),X4(I,J),
     *    K(I,J),RESK(I,J)
30    FORMAT(1H ,I3,I5,2X,7(2X,E13.6))
C
C     FIND THE PERFORMANCE INDEX
C
      CALL MULT(DVD,P,N,N,N,X1)
      PFX=0.
      DO 300 I=1,N
300   PFX=PFX+X1(I,I)
      WRITE(6,500) PFX
500   FORMAT(//,10(1H*),5X,'PERFORMANCE INDEX =',E14.6)
C
C     TEST FOR TERMINATION
C
      IF(MMM.GE.LIMITO) GO TO 400
      IF(BIGR.LT.EPSO) GO TO 400
C
C     UPDATE K
C
200   ALFA=1.
      IF(BIGR.LT.1.) ALFA=.5
      IF(BIGR.LT.1.) BIGR=1.
      DO 4 I=1,S
      DO 4 J=1,M
4     K(I,J)=K(I,J)-ALFA*RESK(I,J)/BIGR
      GO TO 100
400   IA=10
      IZ=10
      IJOB=2
      CALL EIGRF(ABKC,N,IA,IJOB,W,ZZZ,IZ,WK,IER)
      WRITE(6,1600) WK(1),IER,(I,W(I),I=1,N)
1600   FORMAT('1WK(1)=',E15.6,//,' IER=',I10,//
     *    8(' W(',I2,')=',2E15.6,//))
      RETURN
      END
$ENDLIST
```

APPENDIX B

PROGRAM TO EVALUATE THE MATRICES IN THE SERIES OF THE SUBOPTIMAL FEEDBACK GAIN MATRIX USING THE MINI- MIZATION ALGORITHM

```
C//////////////////////////////////////////////////////////////////////
C
C     THIS PROGRAM FINDS THE SUBOPTIMAL FEEDBACK GAIN MATRIX
C     THE COMBINATION OF ALGORITHMS IS USED
C
C//////////////////////////////////////////////////////////////////////
      INTEGER S1,S2,T1,T2
      DIMENSION A1(10,10),A2(10,10),B1(10,10),B2(10,10),C1(10,10),
     *   C2(10,10),D1(10,10),D2(10,10),Q1(10,10),Q2(10,10),R1(10,10),
     *   R2(10,10),V1(10,10),V2(10,10),A12(10,10),A21(10,10),Q12(10,10),
     *   Q21(10,10),P011(10,10),P012(10,10),P021(10,10),P022(10,10),
     *   Z011(10,10),Z012(10,10),Z021(10,10),Z022(10,10),
     *   RESP1(10,10),RESP2(10,10),RESZ1(10,10),RESZ2(10,10)
     *   P211(10,10),P222(10,10),Z211(10,10),Z222(10,10)
     *   GT011(10,10),GT022(10,10),GT121(10,10),GT112(10,10)
     *   RINV1(10,10),RINV2(10,10),X(100),E(100),W(130)
      DIMENSION CZC1(10,10),CZC2(10,10),RESK1(10,10),RESK2(10,10),
     *           BT1(10,10),BT2(10,10),CT1(10,10),CT2(10,10),
     *           Z112(10,10),Z121(10,10),P112(10,10),P121(10,10),
     *           G011(10,10),G022(10,10),G112(10,10),G121(10,10),
     *           GG211(10,10),GGT211(10,10),GG222(10,10),GGT222(10,10),
     *           X1(10,10),X2(10,10),Y1(10,10),Y2(10,10)
      REAL K011(10,10),K012(10,10),K021(10,10),K022(10,10)
      REAL K112(10,10),K121(10,10)
      REAL K211(10,10),K222(10,10)
C
C
      COMMON/AA/ N1,M1,S1,T1,N2,M2,S2,T2
      COMMON/BB/ A1,A2,A12,A21,B1,B2,C1,C2,D1,D2,Q1,Q2,Q12,Q21,
     *   R1,R2,V1,V2
      COMMON/CC/ BT1,BT2,CT1,CT2,RINV1,RINV2,CZC1,CZC2
      COMMON/DD/ Z011,Z022,P011,P022,G011,G022,GT011,GT022
      COMMON/EE/ Z112,Z121,P112,P121,G112,G121,GT112,GT121
      COMMON/FF/ Z211,Z222,P211,P222,GG211,GG222,GGT211,GGT222
      COMMON/DD1/ K011,K022
      COMMON/EE1/ K112,K121
      COMMON/FF1/ K211,K222
      COMMON/IS/ISUB
      COMMON/OPT/ E,ESCALE,MAXIT,IPRINT,NI,NO
      COMMON/INV/ICHECK
      COMMON/CALC/ IDER
C
C
      EPSIL=.5
      ICHECK=0
      EPS1=.001
      EPS2=.001
      NN=4
      IDER=0
C
C
      DO 1 I=1,4
      DO 1 J=1,8
      JJ=J-4
      IF(J.LE.4) READ(5,2) A1(I,J)
      IF(J.GT.4) READ(5,2) A12(I,JJ)
1     CONTINUE
2     FORMAT(20X,E20.6)
      DO 3 I=1,4
      DO 3 J=1,8
      JJ=J-4
      IF(J.LE.4) READ(5,2) A21(I,J)
      IF(J.GT.4) READ(5,2) A2(I,JJ)
3     CONTINUE
C
C     WRITE INPUT
C
      WRITE(6,20)
20    FORMAT('1SUBSYSTEM NUMBER 1',///,
     *   3X,'I',4X,'J',8X,'A1',10X,'B1',10X,'C1',10X,'D1',10X,'Q1',10X,
     *   'R1',10X,'V1',10X,'K1')
      DO 101 I=1,N1
      WRITE(6,10)
10    FORMAT(1H0)
      DO 101 J=1,N1
101   WRITE(6,30) I,J,A1(I,J),B1(I,J),C1(I,J),D1(I,J),Q1(I,J),R1(I,J),
     *   V1(I,J),K011(I,J)
30    FORMAT(1H ,I3,I5,2X,8(2X,F10.4))
      WRITE(6,21)
21    FORMAT(/////,' SUBSYSTEM NUMBER2',///,
     *   3X,'I',4X,'J',8X,'A2',10X,'B2',10X,'C2',10X,'D2',10X,'Q2',10X,
     *   'R2',10X,'V2',10X,'K2')
      DO 102 I=1,N2
      WRITE(6,10)
      DO 102 J=1,N2
102   WRITE(6,30) I,J,A2(I,J),B2(I,J),C2(I,J),D2(I,J),Q2(I,J),
     *   R2(I,J),V2(I,J),K022(I,J)
      WRITE(6,22)
22    FORMAT(/////,' COUPLING MATRICES',///,
     *   3X,'I',4X,'J',8X,'A12',9X,'A21',9X,'Q12',9X,'Q21')
      DO 103 I=1,N1
      WRITE(6,10)
      DO 103 J=1,N1
103   WRITE(6,40) I,J,A12(I,J),A21(I,J),Q12(I,J),Q21(I,J)
40    FORMAT(1H ,I3,I5,2X,4(2X,F10.4))
C
C
      CALL TRANSP(B1,N1,S1,BT1)
      CALL TRANSP(C1,M1,N1,CT1)
      CALL INVERT(R1,S1,RINV1)
      CALL TRANSP(D1,N1,T1,X1)
      CALL MULT(V1,X1,T1,T1,N1,X2)
      CALL MULT(D1,X2,N1,T1,N1,Y1)
C     NOTE  Y1=D1*V1*D1
      CALL TRANSP(B2,N2,S2,BT2)
      CALL TRANSP(C2,M2,N2,CT2)
      CALL INVERT(R2,S2,RINV2)
      CALL TRANSP(D2,N2,T2,X1)
      CALL MULT(V2,X1,T2,T2,N2,X2)
      CALL MULT(D2,X2,N2,T2,N2,Y2)
C     NOTE  Y2=D2*V2*D2
C
C     DECOUPLED FEEDBACK GAIN
C
      ISUB=1
      CALL ZERO(A1,B1,C1,D1,V1,Q1,R1,Y1,BT1,CT1,RINV1,N1,M1,S1,T1,
     *   K011,P011,Z011,G011,GT011)
      CALLZERO9(A1,B1,C1,D1,V1,Q1,R1,Y1,BT1,CT1,RINV1,N1,M1,S1,T1,
     *   K011,P011,Z011,G011,GT011)
```

```
150   ISUB=2
      CALL ZERO(A2,B2,C2,D2,V2,Q2,R2,Y2,BT2,CT2,RINV2,N2,M2,S2,T2,
     *   K022,P022,Z022,G022,GT022)
      CALLZERO9(A2,B2,C2,D2,V2,Q2,R2,Y2,BT2,CT2,RINV2,N2,M2,S2,T2,
     *   K022,P022,Z022,G022,GT022)
C
C     FIRST DERIVATIVE OF FEEDBACK GAIN
C
      IDER=1
      DO 210 I=1,N1
      DO 210 J=1,N2
210   A12(I,J)=A12(I,J)/EPSIL
      DO 211 I=1,N2
      DO 211 J=1,N1
211   A21(I,J)=A21(I,J)/EPSIL
C
C
      CALL MULT(C1,Z011,M1,N1,N1,X1)
      CALL MULT(X1,CT1,M1,N1,M1,X2)
      CALL INVERT(X2,M1,CZC1)
      CALL MULT(C2,Z022,M2,N2,N2,X1)
      CALL MULT(X1,CT2,M2,N2,M2,X2)
      CALL INVERT(X2,M2,CZC2)
C
C
      NNN=(M1*S1)+(M2*S2)
      NW=NNN*(NNN+3)
      DO 261 I=1,S1
      DO 261 J=1,M2
      II=(I-1)*M2+J
261   X(II)=K112(I,J)
      DO 262 I=1,S2
      DO 262 J=1,M1
      II=(S1*M2)+(I-1)*M1+J
262   X(II)=K121(I,J)
      CALL BOTM(X,E,NNN,EF,ESCALE,IPRINT,MAXIT,W,NI,NO,NW)
C
C     SECOND DERIVATIVE OF FEEDBACK GAIN
C
      IDER=2
      DO 361 I=1,S1
      DO 361 J=1,M2
      II=(I-1)*M2+J
361   X(II)=K211(I,J)
      DO 362 I=1,S2
      DO 362 J=1,M1
      II=(S1*M2)+(I-1)*M1+J
362   X(II)=K222(I,J)
      CALL BOTM(X,E,NNN,EF,ESCALE,IPRINT,MAXIT,W,NI,NO,NW)
      STOP
      END
      SUBROUTINE CALCFX(NNN,X,F)
C#####################################################################
C     THIS SUBROUTINE CALLS OTHER SUBROUTINE
C     TO EVALUATE THE PERFORMANCE INDEX
C#####################################################################
      DIMENSION X(100)
      COMMON/CALC/ IDER
      IF(IDER.EQ.0) CALL ZERO55(NNN,X,F)
      IF(IDER.EQ.1) CALL FIRST(NNN,X,F)
      IF(IDER.EQ.2) CALL SECOND(NNN,X,F)
      RETURN
      END
      SUBROUTINE ZERO(A,B,C,D,V,Q,R,DVD,BT,CT,RINV,N,M,S,T,K,P,Z,
     *   ABKC,ABKCT)
C#####################################################################
C     THIS SUBROUTINE CALCULATES DECOUPLED FEEDBACK GAIN BY BOTM ROUTINE
C#####################################################################
      INTEGER S,T,SS
      REAL K(10,10)
      DIMENSION A(10,10),B(10,10),C(10,10),D(10,10),V(10,10),Q(10,10),
     *   R(10,10),DVD(10,10),BT(10,10),CT(10,10),RINV(10,10),P(10,10),
     *   Z(10,10),ABKC(10,10),ABKCT(10,10),X(100),W(130),E(100)
      DIMENSION AA(10,10),BB(10,10),CC(10,10),PP(10,10),QQ(10,10),
     *   RR(10,10),DDVDD(10,10),AACC(10,10),AACCT(10,10)
C
      COMMON/CAL/ AA,BB,CC,PP,QQ,RR,DDVDD,AACC,AACCT,MM,SS,NN
      COMMON/BOT/ E,ESCALE,MAXIT,IPRINT,NI,NO
C
      DO 1 I=1,S
      DO 1 J=1,M
      II=(I-1)*M+J
 1    X(II)=K(I,J)
C
      CALL EQUAL(A,N,N,AA)
      CALL EQUAL(B,N,S,BB)
      CALL EQUAL(C,M,N,CC)
      CALL EQUAL(Q,N,N,QQ)
      CALL EQUAL(R,S,S,RR)
      CALL EQUAL(DVD,N,N,DDVDD)
C
      MM=M
      SS=S
      NN=N
      NNN=M*S
      NW=NNN*(NNN+3)
      CALL BOTM(X,E,NNN,EF,ESCALE,IPRINT,MAXIT,W,NI,NO,NW)
      DO 2 I=1,S
      DO 2 J=1,M
      II=(I-1)*M+J
2     K(I,J)=X(II)
      CALL EQUAL(AACC,N,N,ABKC)
      CALL EQUAL(AACCT,N,N,ABKCT)
      RETURN
      END
      SUBROUTINE ZERO55(NNN,X,F)
C#####################################################################
C     THIS SUBROUTINE CALCULATES OBJECTIVE FUNCTIONS
C#####################################################################
      INTEGER S,T
      COMPLEX Z,W,ZN
      REAL K(10,10)
      DIMENSION A(10,10),B(10,10),C(10,10),P(10,10),Q(10,10),R(10,10),
     *   DVD(10,10),X1(10,10),X2(10,10),X3(10,10),X4(10,10)
      DIMENSION W(10),Z(10,10),WK(120),RW(10)
      DIMENSION ABKC(10,10),ABKCT(10,10)
      DIMENSION RESP(10,10)
      DIMENSION X(100)
      COMMON/CAL/ A,B,C,P,Q,R,DVD,ABKC,ABKCT,M,S,N
C
```

```
C
      DO 1 I=1,S
      DO 1 J=1,M
      II=(I-1)*M+J
1     K(I,J)=X(II)
C
      CALL MULT(K,C,S,M,N,X1)
      CALL TRANSP(X1,S,N,X2)
      CALL MULT(R,X1,S,S,N,X3)
      CALL MULT(X2,X3,N,S,N,X4)
      CALL ADD(Q,X4,N,N,X3)
C
      CALL MULT(B,X1,N,S,N,X2)
      CALL ADD(A,X2,N,N,ABKC)
      CALL TRANSP(ABKC,N,N,ABKCT)
C
      CALL SOLN(ABKC,ABKCT,X3,N,N,P,RESP,MP)
      CALL MULT(DVD,P,N,N,N,X1)
      F=0.0
      DO 2 I=1,N
2     F=F+X1(I,I)
C
C     FIND EIGENVALUES
C
      IA=10
      IZ=10
      IJOB=2
      CALL EIGRF(ABKC,N,IA,IJOB,W,Z,IZ,WK,IER)
C
      EIGMAX=0.
      DO 50 I=1,N
      RW(I)=REAL(W(I))
50    IF(RW(I).GT.EIGMAX) EIGMAX=RW(I)
      F=F+(EIGMAX*10.E20)
100   RETURN
      END
      SUBROUTINE ZERO9(A,B,C,D,V,Q,R,DVD,BT,CT,RINV,N,M,S,T,K,P,Z,ABKC,
     *   ABKCT)
C#######################################################################
C     THIS SUBROUTINE FINDS DECOUPLED FEEDBACK GAIN BY ITERATION METHOD
C#######################################################################
      INTEGER S,T
      REAL K(10,10)
      COMPLEX ZZZ,W
      DIMENSION BT(10,10),CT(10,10),RINV(10,10),A(10,10),B(10,10),
     *   C(10,10),D(10,10),V(10,10),Q(10,10),R(10,10),P(10,10),Z(10,10),
     *   DVD(10,10),X1(10,10),X2(10,10),X3(10,10),X4(10,10),ABKC(10,10),
     *   RESP(10,10),RESK(10,10),RESZ(10,10)
      DIMENSION ABKCT(10,10)
      DIMENSION W(10),ZZZ(10,10),WK(120)
      COMMON/DIFFO/ LIMITO,EPSO,IPRTO
      COMMON/SL/H,EPS
      COMMON/IS/ISUB
C
      MMM=0
100   MMM=MMM+1
C
C     FIND Z
C
      CALL MULT(K,C,S,M,N,X1)
      CALL MULT(B,X1,N,S,N,X2)
      CALL ADD(A,X2,N,N,ABKC)
      CALL TRANSP(ABKC,N,N,X3)
      CALL TRANSP(ABKC,N,N,ABKCT)
      CALL SOLN(X3,ABKC,DVD,N,N,Z,RESZ,MZ)
C
C     FIND P
C
      CALL MULT(R,X1,S,S,N,X2)
      CALL TRANSP(X1,S,N,X4)
      CALL MULT(X4,X2,N,S,N,X1)
      CALL ADD(Q,X1,N,N,X2)
      CALL SOLN(ABKC,X3,X2,N,N,P,RESP,MP)
C
C     FIND K
C
      CALL MULT(Z,CT,N,N,M,X1)
      CALL MULT(C,X1,M,N,M,X2)
      CALL INVERT(X2,M,X3)
      CALL MULT(X1,X3,N,M,M,X4)
      CALL MULT(P,X4,N,N,M,X1)
      CALL MULT(BT,X1,S,N,M,X2)
      CALL MULT(RINV,X2,S,S,M,X4)
C     NOTE  X4=RINV*BT*P*Z*CT*((C*Z*CT)**-1)
C
C     FIND RESIDUE OF K
C
      BIGR=0.
      DO 1 I=1,S
      DO 1 J=1,M
      RESK(I,J)=K(I,J)+X4(I,J)
      RR=ABS(RESK(I,J))
1     BIGR=AMAX1(BIGR,RR)
C
C     WRITE OUTPUT IF IPRT=1
C
      IF(IPRTO.NE.1) GO TO 200
      WRITE(6,10) MMM,MP,MZ,BIGR,ISUB
10    FORMAT('1OUTPUT OF SUBROUTINE ZERO',///,
     *   ' ITERATION NUMBER',I5,///,
     *   ' NUMBER OF ITERATION USED TO FIND P',I5,///,
     *   ' NUMBER OF ITERATION USED TO FIND Z',I5,///,
     *   ' MAXIMUM RESIDUE OF K',E15.6,///,
     *   ' DECOUPLED FEEDBACK GAIN OF SUBSYSTEM NUMBER',I5,///,
     *   1H0,2X,'I',4X,'J',7X,'Z(I,J)',7X,'RESZ(I,J)',8X,'P(I,J)',
     *   7X,'RESP(I,J)',7X,'KZ(I,J)',9X,'K(I,J)',7X,'RESK(I,J)')
      DO 3 I=1,N
      WRITE(6,20)
20    FORMAT(1H0)
      DO 3 J=1,N
3     WRITE(6,30) I,J,Z(I,J),RESZ(I,J),P(I,J),RESP(I,J),X4(I,J),
     *   K(I,J),RESK(I,J)
30    FORMAT(1H ,I3,I5,2X,7(2X,E13.6))
C
C     FIND THE PERFORMANCE INDEX
C
      CALL MULT(DVD,P,N,N,N,X1)
      PFX=0.
      DO 300 I=1,N
300   PFX=PFX+X1(I,I)
```

```
      WRITE(6,500) PFX
500   FORMAT(//,10(1H*),5X,'PERFORMANCE INDEX =',E14.6)
C
C     TEST FOR TERMINATION
C
      IF(MMM.GE.LIMITO) GO TO 400
      IF(BIGR.LT.EPSO) GO TO 400
C
C     UPDATE K
C
200   ALFA=1.
C
      IF(BIGR.LT.1.) ALFA=.05
      IF(BIGR.LT.1.) BIGR=1.
      DO 4 I=1,S
      DO 4 J=1,M
4     K(I,J)=K(I,J)-ALFA*RESK(I,J)/BIGR
      GO TO 100
400   IA=10
      IZ=10
      IJOB=2
      CALL EIGRF(ABKC,N,IA,IJOB,W,ZZZ,IZ,WK,IER)
      WRITE(6,1600) WK(1),IER,(I,W(I),I=1,N)
1600   FORMAT('1WK(1)=',E15.6,//,' IER=',I10,//
     *   8(' W(',I2,')=',2E15.6,//))
      RETURN
      END
      SUBROUTINE FIRST(NNN,X,F)
C#######################################################################
C     THIS SUBROUTINE FINDS THE FIRST DERIVATIVE TERM
C#######################################################################
      INTEGER S1,S2,T1,T2
      REAL K011(10,10),K022(10,10),K112(10,10),K121(10,10)
      DIMENSIONX(100),A1(10,10),A2(10,10),A12(10,10),A21(10,10),
     *   B1(10,10),B2(10,10),C1(10,10),C2(10,10),D1(10,10),D2(10,10),
     *   Q1(10,10),Q2(10,10),R1(10,10),R2(10,10),V1(10,10),V2(10,10),
     *   BT1(10,10),BT2(10,10),CT1(10,10),CT2(10,10),
     *   RINV1(10,10),RINV2(10,10),CZC1(10,10),CZC2(10,10),Z011(10,10),
     *   Z022(10,10),P011(10,10),P022(10,10),G011(10,10),G022(10,10),
     *   GT011(10,10),GT022(10,10),Z112(10,10),Z121(10,10),P112(10,10),
     *   P121(10,10),G112(10,10),G121(10,10),GT112(10,10),GT121(10,10),
     *   WEIGHT(10,10),Y1(10,10),Y2(10,10)
      DIMENSION RESZ1(10,10),RESZ2(10,10),RESP1(10,10),RESP2(10,10)
      DIMENSION Q12(10,10),Q21(10,10)
      COMMON/AA/ N1,M1,S1,T1,N2,M2,S2,T2
      COMMON/BB/ A1,A2,A12,A21,B1,B2,C1,C2,D1,D2,Q1,Q2,Q12,Q21,
     *   R1,R2,V1,V2
      COMMON/CC/ BT1,BT2,CT1,CT2,RINV1,RINV2,CZC1,CZC2
      COMMON/DD/ Z011,Z022,P011,P022,G011,G022,GT011,GT022
      COMMON/EE/ Z112,Z121,P112,P121,G112,G121,GT112,GT121
      COMMON/DD1/ K011,K022
      COMMON/EE1/ K112,K121
      COMMON/WW/ WEIGHT
      DO 10 I=1,S1
      DO 10 J=1,M2
      II=(I-1)*M2+J
10    K112(I,J)=X(II)
      DO 20 I=1,S2
      DO 20 J=1,M1
      II=(S1*M2)+(I-1)*M1+J
20    K121(I,J)=X(II)
C
C
      CALL ZZ1(K112,K121,A12,A21,B1,B2,C1,C2,G011,GT022,Z011,Z022,
     *   N1,M1,S1,N2,M2,S2,Z112,RESZ1,G112,GT121,ITERZ1)
      CALL ZZ1(K121,K112,A21,A12,B2,B1,C2,C1,G022,GT011,Z022,Z011,
     *   N2,M2,S2,N1,M1,S1,Z121,RESZ2,G121,GT112,ITERZ2)
      CALL PP1(GT121,G112,P011,P022,GT011,G022,Q12,K011,K022,K121,K112,
     *   R1,R2,CT1,C2,N1,M1,S1,N2,M2,S2,P112,RESP1,ITERP1)
      CALL PP1(GT112,G121,P022,P011,GT022,G011,Q21,K022,K011,K112,K121,
     *   R2,R1,CT2,C1,N2,M2,S2,N1,M1,S1,P121,RESP2,ITEPP2)
      CALL KK1(R1,K011,C1,CT2,C2,Z112,Z022,BT1,P011,P112,N1,M1,S1,
     *   N2,M2,S2,Y1,CZC2,RINV1)
      CALL KK1(R2,K022,C2,CT1,C1,Z121,Z011,BT2,P022,P121,N2,M2,S2,
     *   N1,M1,S1,Y2,CZC1,RINV2)
      F1=0.
      F2=0.
      DO 220 I=1,S1
      DO 220 J=1,M2
      FFF=K112(I,J)+Y1(I,J)
      F1=F1+FFF*FFF*WEIGHT(I,J)
220   CONTINUE
      DO 230 I=1,S2
      DO 230 J=1,M1
      FFF=K121(I,J)+Y2(I,J)
      F2=F2+FFF*FFF*WEIGHT(I,J)
230   CONTINUE
      F=F1+F2
      RETURN
      END
      SUBROUTINE SECOND(NNN,X,F)
C#######################################################################
C     THIS SUBROUTINE FINDS THE SECOND DERIVATIVE TERM
C#######################################################################
      INTEGER S1,S2,T1,T2
      REAL K011(10,10),K022(10,10),K112(10,10),K121(10,10)
      REAL K211(10,10),K222(10,10)
      DIMENSIONX(100),A1(10,10),A2(10,10),A12(10,10),A21(10,10),
     *   B1(10,10),B2(10,10),C1(10,10),C2(10,10),D1(10,10),D2(10,10),
     *   Q1(10,10),Q2(10,10),R1(10,10),R2(10,10),V1(10,10),V2(10,10),
     *   BT1(10,10),BT2(10,10),CT1(10,10),CT2(10,10),
     *   RINV1(10,10),RINV2(10,10),CZC1(10,10),CZC2(10,10),Z011(10,10),
     *   Z022(10,10),P011(10,10),P022(10,10),G011(10,10),G022(10,10),
     *   GT011(10,10),GT022(10,10),Z112(10,10),Z121(10,10),P112(10,10),
     *   P121(10,10),G112(10,10),G121(10,10),GT112(10,10),GT121(10,10),
     *   WEIGHT(10,10),Y1(10,10),Y2(10,10)
      DIMENSION RESZ1(10,10),RESZ2(10,10),RESP1(10,10),RESP2(10,10)
      DIMENSION Q12(10,10),Q21(10,10)
      DIMENSION Z211(10,10),Z222(10,10),P211(10,10),P222(10,10),
     *   GG211(10,10),GG222(10,10),GGT211(10,10),GGT222(10,10),X1(10,10)
      COMMON/AA/ N1,M1,S1,T1,N2,M2,S2,T2
      COMMON/BB/ A1,A2,A12,A21,B1,B2,C1,C2,D1,D2,Q1,Q2,Q12,Q21,
     *   R1,R2,V1,V2
      COMMON/CC/ BT1,BT2,CT1,CT2,RINV1,RINV2,CZC1,CZC2
      COMMON/DD/ Z011,Z022,P011,P022,G011,G022,GT011,GT022
      COMMON/EE/ Z112,Z121,P112,P121,G112,G121,GT112,GT121
      COMMON/FF/ Z211,Z222,P211,P222,GG211,GG222,GGT211,GGT222
      COMMON/DD1/ K011,K022
      COMMON/EE1/ K112,K121
      COMMON/FF1/ K211,K222
```

```
      COMMON/WW/ WEIGHT
C
      DO 10 I=1,S1
      DO 10 J=1,M2
      II=(I-1)*M2+J
10    K211(I,J)=X(II)
      DO 20 I=1,S2
      DO 20 J=1,M1
      II=(S1*M2)+(I-1)*M1+J
20    K222(I,J)=X(II)
      CALL MULT(K211,C1,S1,M1,N1,X1)
      CALL MULT(B1,X1,N1,S1,N1,GG211)
      CALL TRANSP(GG211,N1,N1,GGT211)
      CALL MULT(K222,C2,S2,M2,N2,X1)
      CALL MULT(B2,X1,N2,S2,N2,GG222)
      CALL TRANSP(GG222,N2,N2,GGT222)
C
C
      CALL ZZ2(K211,B1,C1,Z011,Z121,Z112,G112,GT112,G011,GT011,
     *   N1,M1,S1,N2,M2,S2,GG211,GGT211,Z211,RESZ1,ITERZ1)
      CALL ZZ2(K222,B2,C2,Z022,Z112,Z121,G121,GT121,G022,GT022,
     *   N2,M2,S2,N1,M1,S1,GG222,GGT222,Z222,RESZ2,ITERZ2)
      CALL PP2(GGT211,GG211,P011,P121,P112,R1,R2,G121,GT121,GT011,
     *   G011,C1,CT1,K011,K121,K211,N1,M1,S1,N2,M2,S2,P211,RESP1,ITERP1)
      CALL PP2(GGT222,GG222,P022,P112,P121,R2,R1,G112,GT112,GT022,
     *   G022,     C2,CT2,K022,K112,K222,N2,M2,S2,N1,M1,S1,P222,
     *   RESP2,ITERP2)
      CALL KK2(R1,RINV1,BT1,C1,C2,CT1,P211,P112,P011,Z011,Z121,Z211,
     *   K112,K011,CZC1,N1,M1,S1,N2,M2,S2,Y1)
      CALL KK2(R2,RINV2,BT2,C2,C1,CT2,P222,P121,P022,Z022,Z112,Z222,
     *   K121,K022,CZC2,N2,M2,S2,N1,M1,S1,Y2)
C
C
      F1=0.
      F2=0.
      DO 220 I=1,S1
      DO 220 J=1,M2
      FFF=K211(I,J)+Y1(I,J)
      F1=F1+FFF*FFF*WEIGHT(I,J)
220   CONTINUE
      DO 230 I=1,S2
      DO 230 J=1,M1
      FFF=K222(I,J)+Y2(I,J)
      F2=F2+FFF*FFF*WEIGHT(I,J)
230   CONTINUE
      F=F1+F2
      RETURN
      END
$ENDLIST
```

APPENDIX C

PROGRAM TO EVALUATE THE PLANT MATRIX "A"

OF THE SYNCHRONOUS MACHINE

CONTROL SYSTEM

```
C/////////////////////////////////////////////////////////////////////
C
C     THIS PROGRAM FINDS THE MATRIX A OF THE SYNCHRONOUS MACHINE SYSTEM
C
C/////////////////////////////////////////////////////////////////////
      REAL J1(10,10),J2(10,10),J3(10,10),J4(10,10),IMAT(4,4),
     *   KD1,KD2,KE1,KE2,ID1,ID2,IQ1,IQ2,I1,I2,KA1,KA2,KB1,KB2
     *   ,IM(10),IN(10)
      DIMENSION A1(10,10),A(10,10),B(10,10),C(10,10),E(10,10),G(10,10),
     *   R(10,10),S(10,10),DF1(10,10),DF2(10,10),DFT1(10,10),DFT2(10,10)
     *   ,T(10,10),TT1(10),TT2(10),X1(10,10),X2(10,10),X3(10,10),
     *   F(10,10),FT(10,10),Y(10,10),VM(10),VN(10),FYF(10,10),
     *   X4(10,10),X5(10,10)
      DATA Y/.823,-6.492,-.239,3.986,-.125,2.49,4*0.,
     *   6.492,.823,-3.986,-.239,-2.49,-.125,4*0.,
     *   -.239,3.986,1.142,-6.339,-.12,1.99,4*0.,
     *   -3.986,-.239,6.339,1.142,-1.99,-.12,4*0.,
     *   -.125,2.49,-.12,1.99,.245,-4.48,4*0.,
     *   -2.49,-.125,-1.99,-.12,4.48,.245,44*0./
C
C
      DATA KA1,KA2,KB1,KB2/4*.5/
      DATA V1,V2,TETA1,TETA2/1.05,1.,.087266,.05236/
      DATA H1,KD1,KE1,TE1,TM1/5.,.003,15.,.1,.55/
      DATA H2,KD2,KE2,TE2,TM2/3.2,.001,10.,.08,.35/
      DATA VT1,VT2,VT3/1.05,1.,1./
      DATA J1,J2,J3,J4,IMAT/400*0.,16*0./
      DATA A1,A,B,C,E,G,R/700*0./
      DATA S,DF1,DF2,DFT1,DFT2/500*0./
      DATA XF1,XFD1,XD1,XQ1,RF1/1.5,.9,1.1,.85,.001/
      DATA XF2,XFD2,XD2,XQ2,RF2/1.47,1.33,1.2,1.07,.0016/
      DATA L1,L2,L3,L4,L5,L6,L10/1,2,3,4,5,6,8/
      DATA WO/376.991/
      DATA M,K/6,1/
      DATA F/100*0./
C
C
      VN(1)=V1*COS(TETA1)
      VN(2)=V1*SIN(TETA1)
      VN(3)=V2*COS(TETA2)
      VN(4)=V2*SIN(TETA2)
      VN(5)=1.
      VN(6)=0.
      CALL MULT(Y,VN,M,M,K,IN)
      BETA1=ATAN2(IN(2),IN(1))
      BETA2=ATAN2(IN(4),IN(3))
      Z=(IN(1)**2)+(IN(2)**2)
      I1=SQRT(Z)
      Z=(IN(3)**2)+(IN(4)**2)
      I2=SQRT(Z)
      Z1=(V1*COS(TETA1))-(XQ1*I1*SIN(BETA1))
      Z1=-Z1
      Z2=(V1*SIN(TETA1))+(XQ1*I1*COS(BETA1))
      DEL1=ATAN2(Z1,Z2)
      Z1=(V2*COS(TETA2))-(XQ2*I2*SIN(BETA2))
      Z1=-Z1
      Z2=(V2*SIN(TETA2))+(XQ2*I2*COS(BETA2))
      DEL2=ATAN2(Z1,Z2)
      F(1,1)=COS(DEL1)
      F(1,2)=SIN(DEL1)
      F(2,1)=-F(1,2)
      F(2,2)=F(1,1)
      F(3,3)=COS(DEL2)
      F(3,4)=SIN(DEL2)
      F(4,3)=-F(3,4)
      F(4,4)=F(3,3)
      F(5,5)=1.
      F(6,6)=1.
      CALL TRANSP(F,M,M,FT)
      CALL MULT(F,Y,M,M,M,X1)
      CALL MULT(X1,FT,M,M,M,FYF)
      CALL MULT(F,VN,M,M,K,VM)
      CALL MULT(FYF,VM,M,M,K,IM)
      WRITE(6,15)
15    FORMAT(1H1,10X,'I',9X,'VN',18X,'IN',18X,'VM',18X,'IM')
      DO 10 I=1,M
      WRITE(6,90)
90    FORMAT(1H0)
10    WRITE(6,20) I,VN(I),IN(I),VM(I),IM(I)
20    FORMAT(I10,4E20.6)
      VD1=VM(1)
      VQ1=VM(2)
      VD2=VM(3)
      VQ2=VM(4)
      ID1=IM(1)
      IQ1=IM(2)
      ID2=IM(3)
      IQ2=IM(4)
      PSID1=VQ1
      PSIQ1=-VD1
      PSID2=VQ2
      PSIQ2=-VD2
      WRITE (6,25) ID1,IQ1,ID2,IQ2
25    FORMAT(/////,' ID1=', E13.6,/,/,' IQ1=',E13.6,//,' ID2=',E13.6,//,
     *   ' IQ2=',E13.6)
      WRITE(6,26) VD1,VQ1,VD2,VQ2
26    FORMAT(/////,' VD1=',E13.6//,' VQ1=',E13.6,//,' VD2=',E13.6,//
     *   ' VQ2=',E13.6)
C
C
      A1(1,2)=1.
      A1(2,2)=-WO*KD1/(2.*H1)
      A1(3,3)=-1./TE1
      A1(4,3)=WO
      A1(5,6)=1.
      A1(6,6)=-WO*KD2/(2.*H2)
      A1(7,7)=-1./TE2
      A1(8,7)=WO
      B(1,3)=KE1/TE1
      B(2,7)=KE2/TE2
      C(2,2)=-WO/(2.*H1)
      C(3,3)=-KE1/TE1
      C(4,1)=-WO*RF1
      C(6,5)=-WO/(2.*H2)
      C(7,6)=-KE2/TE2
      C(8,4)=-WO*RF2
      E(1,1)=1./XF1
      E(2,1)=ID1*XFD1/XF1
      E(3,1)=VQ1*XFD1/(VT1*XF1)
```

```
      E(3,2)=((VQ1*PSID1)-(VD1*PSIQ1))/(VT1*WO)
      E(4,3)=1./XF2
      E(5,3)=IQ2*XFD2/XF2
      E(6,3)=(VQ2*XFD2)/(VT2*XF2)
      E(6,4)=((VQ2*PSID2)-(VD2*PSIQ2))/(VT2*WO)
      G(1,1)=XFD1/XF1
      G(2,1)=(IQ1*XFD1*XFD1/XF1)-(IQ1*XD1)-PSIQ1
      G(2,2)=PSID1+(ID1*XQ1)
      G(3,1)=((XFD1*XFD1/XF1)-XD1)*VQ1/VT1
      G(3,2)=VD1*XQ1/VT1
      G(4,3)=XFD2/XF2
      G(5,3)=(IQ2*XFD2*XFD2/XF2)-(IQ2*XD2)-PSIQ2
      G(5,4)=PSID2+(ID2*XQ2)
      G(6,3)=((XFD2*XFD2/XF2)-XD2)*VQ2/VT2
      G(6,4)=VD2*XQ2/VT2
      R(1,2)=-PSIQ1/WO
      R(2,1)=XFD1/XF1
      R(2,2)=PSID1/WO
      R(3,4)=-PSIQ2/WO
      R(4,3)=XFD2/XF2
      R(4,4)=PSID2/WO
      S(1,2)=XQ1
      S(2,1)=(XFD1*XFD1/XF1)-XD1
      S(3,4)=XQ2
      S(4,3)=(XFD2*XFD2/XF2)-XD2
C
C
      DF1(1,1)=-SIN(DEL1)
      DF1(1,2)= COS(DEL1)
      DF1(2,1)=-COS(DEL1)
      DF1(2,2)=-SIN(DEL1)
      DFT1(1,1)=-SIN(DEL1)
      DFT1(1,2)=-COS(DEL1)
      DFT1(2,1)=COS(DEL1)
      DFT1(2,2)=-SIN(DEL1)
C
C
      DF2(3,3)=-SIN(DEL2)
      DF2(3,4)=COS(DEL2)
      DF2(4,3)=-COS(DEL2)
      DF2(4,4)=-SIN(DEL2)
      DFT2(3,3)=-SIN(DEL2)
      DFT2(3,4)=-COS(DEL2)
      DFT2(4,3)= COS(DEL2)
      DFT2(4,4)=-SIN(DEL2)
C
C
      J1(1,4)=1.
      J1(2,2)=1.
      J1(3,8)=1.
      J1(4,6)=1.
      J2(1,1)=1.
      J2(2,5)=1.
      J3(1,1)=1.
      J3(2,2)=1.
      J3(3,3)=1.
      J3(4,4)=1.
      J4(1,1)=1.
      J4(2,2)=1.
      J4(3,3)=1.
      J4(4,4)=1.
C
C     IMAT=IDENTITY MATRIX
C
      IMAT(1,1)=1.
      IMAT(2,2)=1.
      IMAT(3,3)=1.
      IMAT(4,4)=1.
C
C     FORM MATRIX T
C
      CALL MULT(F,Y,L6,L6,L6,X1)
      CALL MULT(Y,FT,L6,L6,L6,X2)
C
      CALL MULT(DF1,X2,L6,L6,L6,X3)
      CALL MULT(X1,DFT1,L6,L6,L6,X4)
      CALL ADD(X3,X4,L6,L6,X5)
      CALL MULT(X5,VM,L6,L6,L1,TT1)
C
      CALL MULT(DF2,X2,L6,L6,L6,X3)
      CALL MULT(X1,DFT2,L6,L6,L6,X4)
      CALL ADD(X3,X4,L6,L6,X5)
      CALL MULT(X5,VM,L6,L6,L1,TT2)
C
      DO 100 I=1,L6
      T(I,1)=TT1(I)
100   T(I,2)=TT2(I)
C
C     CALCULATE JFYFJRJ+JTJ
C
      CALL MULT(J3,FYF,L4,L6,L6,X1)
      CALL MULT(X1,J4,L4,L6,L4,X2)
      CALL MULT(X2,R,L4,L4,L4,X3)
      CALL MULT(X3,J1,L4,L4,L10,X4)
C
      CALL MULT(J3,T,L4,L6,L2,X1)
      CALL MULT(X1,J2,L4,L2,L10,X3)
      CALL ADD(X4,X3,L4,L10,X5)
C
C     CALCULATE INV(I-JFYFJS)
C
      CALL MULT(J3,FYF,L4,L6,L6,X1)
      CALL MULT(X1,J4,L4,L6,L4,X2)
      CALL MULT(X2,S,L4,L4,L4,X3)
      DO 200 I=1,L4
      DO 200 J=1,L4
200   X4(I,J)=IMAT(I,J)-X3(I,J)
      CALL INVERT(X4,L4,X1)
C
      CALL MULT(X1,X5,L4,L4,L10,X2)
      CALL MULT(G,X2,L6,L4,L10,X3)
      CALL MULT(C,X3,L10,L6,L10,X5)
C
C     CALCULATE CEJ
C
      CALL MULT(C,E,L10,L6,L4,X1)
      CALL MULT(X1,J1,L10,L4,L10,X4)
C
C
C
```

```
      DO 250 I=1,L10
      DO 250 J=1,L10
250   A(I,J)=A1(I,J)+X4(I,J)+X5(I,J)
C
C     WRITE OUTPUT
C
      WRITE(6,300)
300   FORMAT(1H1,5X,'I',4X,'J',9X,'A1',13X,'B',14X,'C',14X,'Y',13X,'FYF'
     *   ,13X,'F',14X,'A')
      DO 310 I=1,L10
      WRITE(6,90)
      DO 310 J=1,L10
310   WRITE(6,320) I,J,A1(I,J),B(I,J),C(I,J),Y(I,J),FYF(I,J),F(I,J),
     *   A(I,J)
320   FORMAT(I6,I5,2X,7(2X,E13.6))
      WRITE(6,400)
400   FORMAT(1H1,5X,'I',4X'J',9X,'E',14X,'G',14X,'R',14X,'S',14X,'T')
      DO 410 I=1,L6
      WRITE(6,90)
      DO 410 J=1,L6
410   WRITE(6,420) I,J,E(I,J),G(I,J),R(I,J),S(I,J),T(I,J)
420   FORMAT(I6,I5,2X,5(2X,E13.6))
C
C     PUNCH OUTPUT
C
      DO 520 IJ=1,3
      DO 500 I=1,L10
      DO 500 J=1,L10
500   WRITE(7,510) I,J,A(I,J),B(I,J)
510   FORMAT(2I10,2E20.6)
520   CONTINUE
      STOP
      END
$ENDLIST
```

APPENDIX D

SUBROUTINES USED IN COMMON BY

OTHER PROGRAMS

```
C///////////////////////////////////////////////////////////////////////
C
C     SUBROUTINES
C
C///////////////////////////////////////////////////////////////////////
      SUBROUTINE ZZ1(K112,K121,A12,A21,B1,B2,C1,C2,G011,GT022,Z011,Z022,
     *   N1,M1,S1,N2,M2,S2,Z112,RESZ,G112,GT121,ITERZ)
C#######################################################################
C     THIS SUBROUTINE CALCULATES Z112,G112,GT121 (AND Z121,GT112,G121)
C#######################################################################
      INTEGER S1,S2
      REAL K112(10,10),K121(10,10)
      DIMENSION A12(10,10),A21(10,10),B1(10,10),B2(10,10),C1(10,10),
     *   C2(10,10),G011(10,10),GT022(10,10),Z112(10,10),RESZ(10,10),
     *   G112(10,10),GT121(10,10),X1(10,10),X2(10,10),X3(10,10),
     *   Z011(10,10),Z022(10,10)
C
      CALL MULT(K112,C2,S1,M2,N2,X1)
      CALL MULT(B1,X1,N1,S1,N2,X2)
      CALL ADD(A12,X2,N1,N2,G112)
      CALL MULT(K121,C1,S2,M1,N1,X1)
      CALL MULT(B2,X1,N2,S2,N1,X2)
      CALL ADD(A21,X2,N2,N1,X1)
      CALL TRANSP(X1,N2,N1,GT121)
C
      CALL MULT(G112,Z022,N1,N2,N2,X1)
      CALL MULT(Z011,GT121,N1,N1,N2,X2)
      CALL ADD(X1,X2,N1,N2,X3)
      CALL SOLN(GT022,G011,X3,N2,N1,Z112,RESZ,ITERZ)
      RETURN
      END
      SUBROUTINE PP1(GT121,G112,P011,P022,GT011,G022,Q12,K011,K022,K121,
     *   K112,R1,R2,CT1,C2,N1,M1,S1,N2,M2,S2,P112,RESP,ITERP)
C#######################################################################
C     THIS SUBROUTINE CALCULATES P112 ( AND P121 )
C#######################################################################
      INTEGER S1,S2
      REAL K121(10,10),K112(10,10),K011(10,10),K022(10,10)
      DIMENSION GT121(10,10),G112(10,10),P011(10,10),P022(10,10),
     *   GT011(10,10),G022(10,10),Q12(10,10),R1(10,10),R2(10,10),
     *   CT1(10,10),C2(10,10),P112(10,10),RESP(10,10),X1(10,10),
     *   X2(10,10),X3(10,10),Y1(10,10),Y2(10,10)
      DIMENSION Q21(10,10)
      CALL MULT(GT121,P022,N1,N2,N2,X1)
      CALL MULT(P011,G112,N1,N1,N2,X2)
      CALL ADD(X1,X2,N1,N2,Y1)
C
      CALL MULT(K022,C2,S2,M2,N2,X1)
      CALL MULT(R2,X1,S2,S2,N2,X2)
      CALL TRANSP(K121,S2,M1,X1)
      CALL MULT(X1,X2,M1,S2,N2,X3)
      CALL MULT(CT1,X3,N1,M1,N2,Y2)
C
      CALL MULT(K112,C2,S1,M2,N2,X1)
      CALL MULT(R1,X1,S1,S1,N2,X2)
      CALL TRANSP(K011,S1,M1,X1)
      CALL MULT(X1,X2,M1,S1,N2,X3)
      CALL MULT(CT1,X3,N1,M1,N2,X1)
C
      CALL ADD(Y1,Y2,N1,N2,X2)
      CALL ADD(X2,X1,N1,N2,X3)
      CALL ADD(X3,Q12,N1,N2,Y1)
      CALL SOLN(G022,GT011,Y1,N2,N1,P112,RESP,ITERP)
      RETURN
      END
      SUBROUTINE KK1(R1,K011,C1,CT2,C2,Z112,Z022,BT1,P011,P112,N1,
     $M1,S1,N2,M2,S2,K112,CZC2,RINV1)
C#######################################################################
C     THIS SUBROUTINE CALCULATES K112 ( AND K121)
C#######################################################################
      INTEGER S1,S2
      REAL K011(10,10),K112(10,10)
      DIMENSION RINV1(10,10)
      DIMENSION CZC2(10,10)
      DIMENSIONR1(10,10),C1(10,10),C2(10,10),CT2(10,10),Z112(10,10),
     $Z022(10,10),BT1(10,10),P011(10,10),P112(10,10),
     $X1(10,10),X2(10,10),Y1(10,10),Y2(10,10)
C
      CALL MULT(R1,K011,S1,S1,M1,X1)
      CALL MULT(X1,C1,S1,M1,N1,X2)
      CALL MULT(X2,Z112,S1,N1,N2,X1)
      CALL MULT(X1,CT2,S1,N2,M2,Y1)
C
      CALL MULT(BT1,P011,S1,N1,N1,X1)
      CALL MULT(X1,Z112,S1,N1,N2,X2)
      CALL MULT(X2,CT2,S1,N2,M2,X1)
      CALL ADD(Y1,X1,S1,M2,Y2)
C
      CALL MULT(BT1,P112,S1,N1,N2,X1)
      CALL MULT(X1,Z022,S1,N2,N2,X2)
      CALL MULT(X2,CT2,S1,N2,M2,X1)
      CALL ADD(Y2,X1,S1,M2,Y1)
C
      CALL MULT(Y1,CZC2,S1,M2,M2,Y2)
      CALL MULT(RINV1,Y2,S1,S1,M2,K112)
      RETURN
      END
      SUBROUTINE ZZ2(K211,B1,C1,Z011,Z121,Z112,G112,GT112,G011,GT011,
     *   N1,M1,S1,N2,M2,S2,GG211,GGT211,Z211,RESZ,ITER)
C#######################################################################
C     THIS SUBROUTINE CALCULATES Z211 ( AND Z222)
C#######################################################################
      INTEGER S1 ,S2
      REAL K211(10,10)
      DIMENSION G011(10,10),GT011(10,10)
      DIMENSION B1(10,10),C1(10,10),Z011(10,10),G112(10,10),GT112(10,10),
     *   ,GG211(10,10),GGT211(10,10),Z211(10,10),Z121(10,10),RESZ(10,10)
     *   ,Z112(10,10),X1(10,10),X2(10,10),X3(10,10),Y1(10,10),Y2(10,10)
C
      CALL MULT(GG211,Z011,N1,N1,N1,X1)
      CALL MULT(Z011,GGT211,N1,N1,N1,X2)
      CALL ADD(X1,X2,N1,N1,Y1)
      CALL MULT(G112,Z121,N1,N2,N1,X1)
      CALL MULT(Z112,GT112,N1,N2,N1,X2)
      CALL ADD(X1,X2,N1,N1,Y2)
      ALFA =2.
      CALL MULT11(Y2,ALFA,N1,N1,X1)
      CALL ADD(Y1,X1,N1,N1,Y2)
      CALL SOLN(GT011,G011,Y2,N1,N1,Z211,RESZ,ITER)
```

```
      RETURN
      END
      SUBROUTINE PP2(GGT211,GG211,PO11,P121,P112,R1,R2,G121,GT121,GTO11,
     *   GO11,C1,CT1,KO11,K121,K211,N1,M1,S1,N2,M2,S2,P211,RESP,ITER)
C#######################################################################
C     THIS SUBROUTINE CALCULATES P211 ( AND P222 )
C#######################################################################
      INTEGER S1,S2
      REAL KO11(10,10),K121(10,10),K211(10,10)
      DIMENSION GG211(10,10)
      DIMENSION GGT211(10,10)              ,PO11(10,10),P121(10,10),
     *   P112(10,10),G121(10,10),GT121(10,10),GTO11(10,10),GO11(10,10),
     *                 C1(10,10),CT1(10,10),P211(10,10),R1(10,10),R2(10,10
     *   ),X1(10,10),X2(10,10),X3(10,10),X4(10,10),Y1(10,10),Y2(10,10),
     *   Y3(10,10),RESP(10,10)
C
      CALL MULT(GGT211,PO11,N1,N1,N1,X1)
      CALL MULT(PO11,GG211,N1,N1,N1,X2)
      CALL ADD(X1,X2,N1,N1,Y1)
      CALL MULT(GT121,P121,N1,N2,N1,X1)
      CALL MULT(P112,G121,N1,N2,N1,X2)
      CALL ADD(X1,X2,N1,N1,Y2)
      ALFA=2.
      CALL MULT11(Y2,ALFA,N1,N1,X1)
      CALL ADD(Y1,X1,N1,N1,Y2)
C
      CALL MULT(KO11,C1,S1,M1,N1,Y1)
      CALL MULT(R1,Y1,S1,S1,N1,X2)
      CALL MULT(K211,C1,S1,M1,N1,Y3)
      CALL TRANSP(Y3,S1,N1,X1)
      CALL MULT(X1,X2,N1,S1,N1,X4)
      CALL ADD(X4,Y2,N1,N1,X1)
C
      CALL MULT(R1,Y3,S1,S1,N1,X2)
      CALL TRANSP(Y1,S1,N1,X3)
      CALL MULT(X3,X2,N1,S1,N1,X4)
      CALL ADD(X1,X4,N1,N1,Y1)
C
      CALL MULT(K121,C1,S2,M1,N1,X1)
      CALL TRANSP(X1,S2,N1,X2)
      CALL MULT(R2,X1,S2,S2,N1,X3)
      CALL MULT(X2,X3,N1,S2,N1,X4)
      CALL MULT11(X4,ALFA,N1,N1,Y2)
      CALL ADD(Y1,Y2,N1,N1,Y3)
C
      CALL SOLN(GO11,GTO11,Y3,N1,N1,P211,RESP,ITER)
      RETURN
      END
      SUBROUTINE KK2(R1,RINV1,BT1,C1,C2,CT1,P211,P112,PO11,ZO11,Z121,
     *   Z211,K112,KO11,CZC1,N1,M1,S1,N2,M2,S2,K211)
C#######################################################################
C     THIS SUBROUTINE CALCULATES -K211 ( AND -K222 )
C#######################################################################
      INTEGER S1,S2
      REAL K211(10,10),K112(10,10),KO11(10,10)
      DIMENSION R1(10,10),RINV1(10,10),BT1(10,10),C1(10,10),CT1(10,10),
     *   C2(10,10),P211(10,10),P112(10,10),PO11(10,10),ZO11(10,10),
     *   Z121(10,10),Z211(10,10),CZC1(10,10),X1(10,10),X2(10,10),
     *   Y1(10,10),Y2(10,10)
      DIMENSION Y3(10,10)
C
      CALL MULT(BT1,P211,S1,N1,N1,X1)
      CALL MULT(X1,ZO11,S1,N1,N1,X2)
      CALL MULT(X2,CT1,S1,N1,M1,Y1)
C
      CALL MULT(BT1,P112,S1,N1,N2,X1)
      CALL MULT(X1,Z121,S1,N2,N1,X2)
      CALL MULT(X2,CT1,S1,N1,M1,X1)
      ALFA=2.
      CALL MULT11(X1,ALFA,S1,M1,X2)
      CALL ADD(Y1,X2,S1,M1,Y2)
C
      CALL MULT(BT1,PO11,S1,N1,N1,X1)
      CALL MULT(Z211,CT1,N1,N1,M1,Y1)
      CALL MULT(X1,Y1,S1,N1,M1,X2)
      CALL ADD(Y2,X2,S1,M1,Y3)
C
      CALL MULT(R1,K112,S1,S1,M2,X1)
      CALL MULT(X1,C2,S1,M2,N2,X2)
      CALL MULT(X2,Z121,S1,N2,N1,X1)
      CALL MULT(X1,CT1,S1,N1,M1,X2)
      CALL MULT11(X2,ALFA,S1,M1,X1)
      CALL ADD(Y3,X1,S1,M1,Y2)
C
      CALL MULT(R1,KO11,S1,S1,M1,X1)
      CALL MULT(X1,C1,S1,M1,N1,X2)
      CALL MULT(X2,Y1,S1,N1,M1,X1)
      CALL ADD(X1,Y2,S1,M1,Y1)
C
      CALL MULT(Y1,CZC1,S1,M1,M1,Y2)
      CALL MULT(RINV1,Y2,S1,S1,M1,K211)
      RETURN
      END
      SUBROUTINE SOLN(B,A,C,M,N,P,RES,IER)
C#######################################################################
C     THIS SUBROUTINE FINDS THE SOLUTION OF :  PB+AP+C=0
C     A(M,M),B(N,N),C(M,N),P(M,N)
C#######################################################################
C
      DIMENSION A(10,10),B(10,10),C(10,10),P(10,10),RES(10,10),
     *   X3(10,10),BT(10,10),CC(100),PP(100)
      DIMENSION X1(25,25),X2(25,25),AB(25,26),AABB(625)
C     THE ABOVE STATEMENT CAN BE USED FOR N=5 OR LESS
C     FOR N=10 OR LESS, THE FOLLOWING STATEMENT MUST BE USED INSTEAD
C     DIMENSION X1(100,100),X2(100,100),AB(100,101),AABB(10000)
      DOUBLE PRECISION AABB,CC
      MM=M*N
      DO 25 I=1,MM
      DO 25 J=1,MM
      X1(I,J)=0.
      X2(I,J)=0.
25    CONTINUE
C
C     KRONECKER PRODUCT  X1=A*I
C
      DO 10 I=1,M
      DO 10 J=1,M
      DO 10 K=1,N
      II=(I-1)*N+K
      JJ=(J-1)*N+K
```

APPENDIXES

```
      K=(J-1)*N+I
20    XINV(I,J)=Y(K)
C
C     CHECK THE ACCURACY OF THE RESULT
C
      IF(ICHECK.NE.1) GO TO 30
      WRITE(6,17)
17    FORMAT(////,' THE RESULT OF MATRIX INVERSION',//,
     *  4X,'I',4X,'J',13X,'X',17X,'XINV',16X,'X*XINV')
      CALL MULT(X,XINV,N,N,N,XX)
      DO 15 I=1,N
      WRITE(6,18)
18    FORMAT(1H0)
      DO 15 J=1,N
15    WRITE(6,16) I,J,X(I,J),XINV(I,J),XX(I,J)
16    FORMAT(2I5,3E20.6)
30    RETURN
      END
      BLOCK DATA
C**********************************************************************
C     THIS SUBPROGRAM INITIALIZE VARIABLES
C**********************************************************************
      INTEGER S1,T1,S2,T2
      REAL K011(10,10),K022(10,10)
      REAL K112(10,10),K121(10,10)
      REAL K211(10,10),K222(10,10)
      DIMENSION A1(10,10),A2(10,10),B1(10,10),B2(10,10),C1(10,10),
     *  C2(10,10),D1(10,10),D2(10,10),Q1(10,10),Q2(10,10),R1(10,10),
     *  R2(10,10),V1(10,10),V2(10,10),
     *  Q21(10,10),Q12(10,10),A12(10,10),A21(10,10)
      DIMENSION E(20)
      DIMENSION WEIGHT(10,10)
      COMMON/AA/N1,M1,S1,T1,N2,M2,S2,T2
      COMMON/BB/ A1,A2,A12,A21,B1,B2,C1,C2,D1,D2,Q1,Q2,Q12,Q21,
     *  R1,R2,V1,V2
      COMMON/DD1/ K011,K022
      COMMON/EE1/ K112,K121
      COMMON/FF1/ K211,K222
      COMMON/GG1/ K312,K321
      COMMON/WW/ WEIGHT
      COMMON/SL/H,EPS
      COMMON/DIFFO/ LIMITO,EPSO,IPRTO
      COMMON/BOT/ E,ESCALE,MAXIT,IPRINT,NI,NO
C
C
      DATA LIMITO/30/
      DATA N1,M1,S1,T1,N2,M2,S2,T2/4,3,1,4,4,3,1,4/
      DATA K011,K022,K121,K112,K211,K222/600*0./
      DATA E/20*.01/
      DATA B1/0.,0.,500.,97*0./
      DATA B2/0.,0.,437.5,97*0./
      DATA MAXIT/20/
      DATA ESCALE/5./
      DATA IPRINT/1/
      DATA NI,NO/5,6/
      DATA WEIGHT/100*1./
      DATA EPSO,IPRTO/1.E-5,1/
      DATA V1/.0001,10*0.,.0005,10*0.,.0003,10*0.,.0007,10*0.,56*0./
      DATA V2/.0002,10*0.,.0009,10*0.,.0005,10*0.,.0008,10*0.,56*0./
      DATA Q1/1.,10*0.,1.,10*0.,1.,10*0.,1.,10*0.,1.,10*0.,45*0./
      DATA Q2/1.,10*0.,1.,10*0.,1.,10*0.,1.,10*0.,1.,10*0.,45*0./
      DATA Q12,Q21/200*0./
      DATA C1/1.,10*0.,1.,10*0.,1.,10*0.,1.,10*0.,1.,10*0.,45*0./
      DATA C2/1.,10*0.,1.,10*0.,1.,10*0.,1.,10*0.,1.,10*0.,45*0./
      DATA D1/1.,10*0.,1.,10*0.,1.,10*0.,1.,10*0.,1.,55*0./
      DATA D2/1.,10*0.,1.,10*0.,1.,10*0.,1.,10*0.,1.,55*0./
      DATA R1/1.,10*0.,1.,10*0.,1.,10*0.,1.,10*0.,1.,10*0.,45*0./
      DATA R2/1.,10*0.,1.,10*0.,1.,10*0.,1.,10*0.,1.,10*0.,45*0./
      END
C
C     ..............................................................
C
C        SUBROUTINE MINV
C
C        PURPOSE
C           INVERT A MATRIX
C
C        USAGE
C           CALL MINV(A,N,D,L,M)
C
C        DESCRIPTION OF PARAMETERS
C           A - INPUT MATRIX, DESTROYED IN COMPUTATION AND REPLACED BY
C               RESULTANT INVERSE.
C           N - ORDER OF MATRIX A
C           D - RESULTANT DETERMINANT
C           L - WORK VECTOR OF LENGTH N
C           M - WORK VECTOR OF LENGTH N
C
C        REMARKS
C           MATRIX A MUST BE A GENERAL MATRIX
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           THE STANDARD GAUSS-JORDAN METHOD IS USED. THE DETERMINANT
C           IS ALSO CALCULATED. A DETERMINANT OF ZERO INDICATES THAT
C           THE MATRIX IS SINGULAR.
C
C     ..............................................................
C
      SUBROUTINE MINV(A,N,D,L,M)
      DIMENSION A(1),L(1),M(1)
      DOUBLE PRECISION A,D,BIGA,HOLD,DABS
C
C     ..............................................................
C
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION
C        STATEMENT WHICH FOLLOWS.
C
C     DOUBLE PRECISION A,D,BIGA,HOLD
C
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
C        ROUTINE.
C
C        THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO
C        CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS.  ABS IN STATEMENT
C        10 MUST BE CHANGED TO DABS.
```

```
C
C        ..................................................................
C
C        SEARCH FOR LARGEST ELEMENT
C
      D=1.0
      NK=-N
      DO 80 K=1,N
      NK=NK+N
      L(K)=K
      M(K)=K
      KK=NK+K
      BIGA=A(KK)
      DO 20 J=K,N
      IZ=N*(J-1)
      DO 20 I=K,N
      IJ=IZ+I
   10 IF(DABS(BIGA)-DABS(A(IJ))) 15,20,20
   15 BIGA=A(IJ)
      L(K)=I
      M(K)=J
   20 CONTINUE
C
C        INTERCHANGE ROWS
C
      J=L(K)
      IF(J-K) 35,35,25
   25 KI=K-N
      DO 30 I=1,N
      KI=KI+N
      HOLD=-A(KI)
      JI=KI-K+J
      A(KI)=A(JI)
   30 A(JI) =HOLD
C
C        INTERCHANGE COLUMNS
C
   35 I=M(K)
      IF(I-K) 45,45,38
   38 JP=N*(I-1)
      DO 40 J=1,N
      JK=NK+J
      JI=JP+J
      HOLD=-A(JK)
      A(JK)=A(JI)
   40 A(JI) =HOLD
C
C        DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS
C
   45 IF(BIGA) 48,46,48
   46 D=0.0
      RETURN
   48 DO 55 I=1,N
      IF(I-K) 50,55,50
   50 IK=NK+I
      A(IK)=A(IK)/(-BIGA)
   55 CONTINUE
C
C        REDUCE MATRIX
C
      DO 65 I=1,N
      IK=NK+I
      HOLD=A(IK)
      IJ=I-N
      DO 65 J=1,N
      IJ=IJ+N
      IF(I-K) 60,65,60
   60 IF(J-K) 62,65,62
   62 KJ=IJ-I+K
      A(IJ)=HOLD*A(KJ)+A(IJ)
   65 CONTINUE
C
C        DIVIDE ROW BY PIVOT
C
      KJ=K-N
      DO 75 J=1,N
      KJ=KJ+N
      IF(J-K) 70,75,70
   70 A(KJ)=A(KJ)/BIGA
   75 CONTINUE
C
C        PRODUCT OF PIVOTS
C
      D=D*BIGA
C
C        REPLACE PIVOT BY RECIPROCAL
C
      A(KK)=1.0/BIGA
   80 CONTINUE
C
C        FINAL ROW AND COLUMN INTERCHANGE
C
      K=N
  100 K=(K-1)
      IF(K) 150,150,105
  105 I=L(K)
      IF(I-K) 120,120,108
  108 JQ=N*(K-1)
      JR=N*(I-1)
      DO 110 J=1,N
      JK=JQ+J
      HOLD=A(JK)
      JI=JR+J
      A(JK)=-A(JI)
  110 A(JI) =HOLD
  120 J=M(K)
      IF(J-K) 100,100,125
  125 KI=K-N
      DO 130 I=1,N
      KI=KI+N
      HOLD=A(KI)
      JI=KI-K+J
      A(KI)=-A(JI)
  130 A(JI) =HOLD
      GO TO 100
  150 RETURN
      END
C        SUBROUTINE DGELG
C
C        ..................................................................
```

```
C
C
C        PURPOSE
C           TO SOLVE A GENERAL SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS.
C
C        USAGE
C           CALL DGELG(R,A,M,N,EPS,IER)
C
C        DESCRIPTION OF PARAMETERS
C           R      - DOUBLE PRECISION M BY N RIGHT HAND SIDE MATRIX
C                    (DESTROYED). ON RETURN R CONTAINS THE SOLUTIONS
C                    OF THE EQUATIONS.
C           A      - DOUBLE PRECISION M BY M COEFFICIENT MATRIX
C                    (DESTROYED).
C           M      - THE NUMBER OF EQUATIONS IN THE SYSTEM.
C           N      - THE NUMBER OF RIGHT HAND SIDE VECTORS.
C           EPS    - SINGLE PRECISION INPUT CONSTANT WHICH IS USED AS
C                    RELATIVE TOLERANCE FOR TEST ON LOSS OF
C                    SIGNIFICANCE.
C           IER    - RESULTING ERROR PARAMETER CODED AS FOLLOWS
C                    IER=0  - NO ERROR,
C                    IER=-1 - NO RESULT BECAUSE OF M LESS THAN 1 OR
C                             PIVOT ELEMENT AT ANY ELIMINATION STEP
C                             EQUAL TO 0,
C                    IER=K  - WARNING DUE TO POSSIBLE LOSS OF SIGNIFI-
C                             CANCE INDICATED AT ELIMINATION STEP K+1,
C                             WHERE PIVOT ELEMENT WAS LESS THAN OR
C                             EQUAL TO THE INTERNAL TOLERANCE EPS TIMES
C                             ABSOLUTELY GREATEST ELEMENT OF MATRIX A.
C
C        REMARKS
C           INPUT MATRICES R AND A ARE ASSUMED TO BE STORED COLUMNWISE
C           IN M*N RESP. M*M SUCCESSIVE STORAGE LOCATIONS. ON RETURN
C           SOLUTION MATRIX R IS STORED COLUMNWISE TOO.
C           THE PROCEDURE GIVES RESULTS IF THE NUMBER OF EQUATIONS M IS
C           GREATER THAN 0 AND PIVOT ELEMENTS AT ALL ELIMINATION STEPS
C           ARE DIFFERENT FROM 0. HOWEVER WARNING IER=K - IF GIVEN -
C           INDICATES POSSIBLE LOSS OF SIGNIFICANCE. IN CASE OF A WELL
C           SCALED MATRIX A AND APPROPRIATE TOLERANCE EPS, IER=K MAY BE
C           INTERPRETED THAT MATRIX A HAS THE RANK K. NO WARNING IS
C           GIVEN IN CASE M=1.
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           SOLUTION IS DONE BY MEANS OF GAUSS-ELIMINATION WITH
C           COMPLETE PIVOTING.
C
C     ..................................................................
C
      SUBROUTINE DGELG(R,A,M,N,EPS,IER)
C
C
      DIMENSION A(1),R(1)
      DOUBLE PRECISION R,A,PIV,TB,TOL,PIVI
      DOUBLE PRECISION DABS
      IF(M)23,23,1
C
C     SEARCH FOR GREATEST ELEMENT IN MATRIX A
    1 IER=0
      PIV=0.D0
      MM=M*M
      NM=N*M
      DO 3 L=1,MM
      TB=DABS(A(L))
      IF(TB-PIV)3,3,2
    2 PIV=TB
      I=L
    3 CONTINUE
      TOL=EPS*PIV
C     A(I) IS PIVOT ELEMENT. PIV CONTAINS THE ABSOLUTE VALUE OF A(I).
C
C
C     START ELIMINATION LOOP
      LST=1
      DO 17 K=1,M
C
C     TEST ON SINGULARITY
      IF(PIV)23,23,4
    4 IF(IER)7,5,7
    5 IF(PIV-TOL)6,6,7
    6 IER=K-1
    7 PIVI=1.D0/A(I)
      J=(I-1)/M
      I=I-J*M-K
      J=J+1-K
C     I+K IS ROW-INDEX, J+K COLUMN-INDEX OF PIVOT ELEMENT
C
C     PIVOT ROW REDUCTION AND ROW INTERCHANGE IN RIGHT HAND SIDE R
      DO 8 L=K,NM,M
      LL=L+I
      TB=PIVI*R(LL)
      R(LL)=R(L)
    8 R(L)=TB
C
C     IS ELIMINATION TERMINATED
      IF(K-M)9,18,18
C
C     COLUMN INTERCHANGE IN MATRIX A
    9 LEND=LST+M-K
      IF(J)12,12,10
   10 II=J*M
      DO 11 L=LST,LEND
      TB=A(L)
      LL=L+II
      A(L)=A(LL)
   11 A(LL)=TB
C
C     ROW INTERCHANGE AND PIVOT ROW REDUCTION IN MATRIX A
   12 DO 13 L=LST,MM,M
      LL=L+I
      TB=PIVI*A(LL)
      A(LL)=A(L)
   13 A(L)=TB
C
C     SAVE COLUMN INTERCHANGE INFORMATION
      A(LST)=J
C
C     ELEMENT REDUCTION AND NEXT PIVOT SEARCH
```

```
      PIV=0.D0
      LST=LST+1
      J=0
      DO 16 II=LST,LEND
      PIVI=-A(II)
      IST=II+M
      J=J+1
      DO 15 L=IST,MM,M
      LL=L-J
      A(L)=A(L)+PIVI*A(LL)
      TB=DABS(A(L))
      IF(TB-PIV)15,15,14
   14 PIV=TB
      I=L
   15 CONTINUE
      DO 16 L=K,NM,M
      LL=L+J
   16 R(LL)=R(LL)+PIVI*R(L)
   17 LST=LST+M
C     END OF ELIMINATION LOOP
C
C
C     BACK SUBSTITUTION AND BACK INTERCHANGE
   18 IF(M-1)23,22,19
   19 IST=MM+M
      LST=M+1
      DO 21 I=2,M
      II=LST-I
      IST=IST-LST
      L=IST-M
      L=A(L)+.5D0
      DO 21 J=II,NM,M
      TB=R(J)
      LL=J
      DO 20 K=IST,MM,M
      LL=LL+1
   20 TB=TB-A(K)*R(LL)
      K=J+L
      R(J)=R(K)
   21 R(K)=TB
   22 RETURN
C
C
C     ERROR RETURN
   23 IER=-1
      RETURN
      END
      SUBROUTINE BOTM (X,E,N,EF,ESCALE,IPRINT,MAXIT,W,NI,NO,NW)
C.....................................................................
      DIMENSION X(N), W(NW), E(N)
C
      NUMBER=0
      WRITE (NO,001)
  001 FORMAT (1H1,10X,32HPOWELL-BOTM OPTIMIZATION ROUTINE )
      WRITE (NO,002) N, MAXIT, ESCALE, (I, X(I), I=1,N), (J, E(J), J=1,
     1 N)
  002 FORMAT (//,2X,10HPARAMETERS,//,2X,4HN = ,I2,4X,8HMAXIT = ,I4,4X,
     1 9HESCALE = ,F5.2,//,2X,15HINITIAL GUESSES,//,2(2X,2HX(,I2,4H) = ,
     2 1PE16.8),//,2X,31HACCURACY REQUIRED FOR VARIABLES ,//,2(2X,2HE(,
     3 I2,4H) = ,E16.8) )
C
      DDMAG=0.1*ESCALE
      SCER=0.05/ESCALE
      JJ=N*(N+1)
      JJJ=JJ+N
      K=N+1
      NFCC=1
      IND=1
      INN=1
      DO 4 I=1,N
      W(I)=ESCALE
      DO 4 J=1,N
      W(K)=0.
      IF(I-J)4,3,4
    3 W(K)=ABS (E(I))
    4 K=K+1
      ITERC=1
      ISGRAD=2
      CALL CALCFX(N,X,F)
      FKEEP=2.*ABS (F)
    5 ITONE=1
      FP=F
      SUM=0.
      IXP=JJ
      DO 6 I=1,N
      IXP=IXP+1
 6    W(IXP)=X(I)
      IDIRN=N+1
      ILINE=1
    7 DMAX=W(ILINE)
      DACC=DMAX*SCER
      DMAG=AMIN1 (DDMAG,0.1*DMAX)
      DMAG=AMAX1(DMAG,20.*DACC)
      DDMAX=10.*DMAG
      GO TO (70,70,71),ITONE
C
   70 DL=0.
      D=DMAG
      FPREV=F
      IS=5
      FA=FPREV
      DA=DL
    8 DD=D-DL
      DL=D
   58 K=IDIRN
      DO 9 I=1,N
      X(I)=X(I)+DD*W(K)
 9    K=K+1
      CALL CALCFX(N,X,F)
      NFCC=NFCC+1
      GO TO (10,11,12,13,14,96),IS
   14 IF(F-FA)15,16,24
C
   16 IF (ABS (D)-DMAX) 17,17,18
   17 D=D+D
      GO TO 8
   18 WRITE (NO,019)
   19 FORMAT(5X,38HMAXIMUM CHANGE DOES NOT ALTER FUNCTION)
      GO TO 20
C
```

```
15 FB=F
   DB=D
   GO TO 21
24 FB=FA
   DB=DA
   FA=F
   DA=D
21 GO TO (83,23),ISGRAD
23 D=DB+DB-DA
   IS=1
   GO TO 8
83 D=0.5*(DA+DB-(FA-FB)/(DA-DB))
   IS=4
   IF((DA-D)*(D-DB))25,8,8
25 IS=1
   IF(ABS (D-DB)-DDMAX)8,8,26
26 D=DB+SIGN (DDMAX,DB-DA)
   IS=1
   DDMAX=DDMAX+DDMAX
   DDMAG=DDMAG+DDMAG
   IF (DDMAG.GE.1.0E+60)  DDMAG = 1.0E+60
   IF(DDMAX-DMAX)8,8,27
27 DDMAX=DMAX
   GO TO 8
13 IF(F-FA)28,23,23
28 FC=FB
   DC=DB
29 FB=F
   DB=D
   GO TO 30
12 IF(F-FB)28,28,31
31 FA=F
   DA=D
   GO TO 30
11 IF(F-FB)32,10,10
32 FA=FB
   DA=DB
   GO TO 29
71 DL=1.
   DDMAX=5.
   FA=FP
   DA=-1.
   FB=FHOLD
   DB=0.
   D=1.
10 FC=F
   DC=D
30 A=(DB-DC)*(FA-FC)
   B=(DC-DA)*(FB-FC)
   IF((A+B)*(DA-DC))33,33,34
33 FA=FB
   DA=DB
   FB=FC
   DB=DC
   GO TO 26
34 D=0.5*(A*(DB+DC)+B*(DA+DC))/(A+B)
   DI=DB
   FI=FB
   IF(FB-FC)44,44,43
43 DI=DC
   FI=FC
44 GO TO (86,86,85),ITONE
85 ITONE=2
   GO TO 45
86 IF (ABS (D-DI)-DACC) 41,41,93
93 IF (ABS (D-DI)-0.03*ABS (D)) 41,41,45
45 IF ((DA-DC)*(DC-D)) 47,46,46
46 FA=FB
   DA=DB
   FB=FC
   DB=DC
   GO TO 25
47 IS=2
   IF ((DB-D)*(D-DC)) 48,8,8
48 IS=3
   GO TO 8
41 F=FI
   D=DI-DL
   DD=SQRT ((DC-DB)*(DC-DA)*(DA-DB)/(A+B))
   DO 49 I=1,N
   X(I)=X(I)+D*W(IDIRN)
   W(IDIRN)=DD*W(IDIRN)
49    IDIRN=IDIRN+1
   W(ILINE)=W(ILINE)/DD
   ILINE=ILINE+1
   IF(IPRINT-1)51,50,51
C
50 WRITE(NO,52)ITERC,NFCC,F,(X(I),I=1,10)
52 FORMAT(/10H ITERATION,I5,I15,16H FUNCTION VALUES,10X,3HF =,E15.8/
  +5(E16.8,2X))
   NUMBER=NUMBER+1
   IF(NUMBEK.GT.N) NUMBER=1
   WRITE(6,1000)  NUMBER
1000  FORMAT(5X,'SUB-ITERATION NUMBER',I5,/,100(1H*),//)
   GO TO(51,53),IPRINT
51 GO TO (55,38),ITONE
55 IF (FPREV-F-SUM) 94,95,95
95 SUM=FPREV-F
   JIL=ILINE
94 IF (IDIRN-JJ) 7,7,84
84 GO TO (92,72),IND
92 FHOLD=F
   IS=6
   IXP=JJ
   DO 59 I=1,N
   IXP=IXP+1
59    W(IXP)=X(I)-W(IXP)
   DD=1.
   GO TO 58
96 GO TO (112,87),IND
112 IF(FP-F) 37,37,91
91 D=2.*(FP+F-2.*FHOLD)/(FP-F)**2
   IF (D*(FP-FHOLD-SUM)**2-SUM) 87,37,37
87 J=JIL*N+1
   IF (J-JJ) 60,60,61
60 DO 62 I=J,JJ
   K=I-N
62    W(K)=W(I)
   DO 97 I=JIL,N
97    W(I-1)=W(I)
```

```
   61 IDIRN=IDIRN-N
      ITONE=3
      K=IDIRN
      IXP=JJ
      AAA=0.
      DO 67 I=1,N
      IXP=IXP+1
      W(K)=W(IXP)
      IF (AAA-ABS (W(K)/E(I))) 66,67,67
   66 AAA=ABS (W(K)/E(I))
   67 K=K+1
      DDMAG=1.
      W(N)=ESCALE/AAA
      ILINE=N
      GO TO 7
   37 IXP=JJ
      AAA=0.
      F=FHOLD
      DO 99 I=1,N
      IXP=IXP+1
      X(I)=X(I)-W(IXP)
      IF(AAA*ABS (E(I))-ABS (W(IXP))) 98,99,99
 98   AAA=ABS (W(IXP)/E(I))
   99 CONTINUE
      GO TO 72
   38 AAA=AAA*(1.+DI)
      GO TO (72,106),IND
   72 IF(IPRINT-2)53,50,50
   53 GO TO (109,88),IND
  109 IF (AAA - 0.1) 20,20,76
C
 76   IF(F-FP)35,78,78
   78 WRITE (NO,80)
   80 FORMAT(5X,31HACCURACY LIMITED BY ERRORS IN F)
      GO TO 20
C
   88 IND=1
   35 DDMAG=0.4*SQRT(ABS(FP-F))
      IF (DDMAG.GE.1.0E+60)  DDMAG = 1.0E+60
      ISGRAD=1
C
  108 ITERC=ITERC+1
      IF(ITERC-MAXIT)5,5,81
   81 WRITE (NO,82) MAXIT
 82   FORMAT(I5,29H ITERATIONS COMPLETED BY BOTM)
      IF(F-FKEEP)20,20,110
  110 F=FKEEP
      DO 111 I=1,N
      JJJ=JJJ+1
 111  X(I)=W(JJJ)
      GO TO 20
C
 106  IF(AAA-0.1) 20,20,107
C
 20   EF=F
      RETURN
C
 107  INN=1
      GO TO 35
C
```

```
      END
$ENDLIST
```

VITA

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Doctor of Philosophy

Thesis: SUBOPTIMAL CONTROL OF INTERCONNECTED POWER SYSTEMS

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