AN EVALUATION OF THE FEEDBACK

CONTROL OF A STRIPPING

COLUMN MODEL

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PREFACE

A feedback control system has been added to a lumped parameter model, developed to predict the transient behavior of stripping column after an upset in the feed. The effect of column efficiency upon the operation and control of the column was studied. Inconsistencies in the internal stream compositions were also studied. The results indicate that the model can be adequately controlled by conventional feedback control methods.

I wish to express my sincere thanks to Dr. John H. Erbar for the guidance and advice that he gave me while serving as my research adviser. I would also like to thank Dr. R. N. Maddox for his helpful suggestions.

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CHAPTER I

INTRODUCTION

In recent years computer control in the process indus-
tries has become increasingly important. With the advent
of modern technology, solution of industrial problems ne-
cessitates the reliability and rapid service which the
computer offers.

The distillation column, an integral part of most
processes in the chemical industry, must be accurately
controlled. The product compositions and column perfor-
mance must be maintained at an operating level to insure
the desired results of the process.

A lumped parameter model has been developed which has
been shown to predict with some degree of accuracy the
transient behavior of a stripping column. The model re-
quires only that data which can be obtained from plant
column or computer calculations. The model offers sim-
plicity of operation and can be programmed on a computer
which can be economically installed in a plant. However,
to be suitable for industrial application, the model must
be able to control the conditions of the column.

This research project had three objectives. The
first objective was to explore the controllability of the

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lumped parameter model. A second objective was to apply the control system to a column in which the tray efficiency was variable. By varying the tray efficiency, actual operating conditions of a real-life column could be approximated. The third objective was to resolve inconsistencies in the internal stream compositions between the values predicted by the model and those experimental values taken by Burman (3).

CHAPTER II

REVIEW OF THE LITERATURE

In the last decade many papers have been published in the area of fractionator control. Most of the column and control models involve complex tray-to-tray calculations and matrix manipulations. A description of several of the models will be given in the following paragraphs.

Luyben, et al. (10, 11) did extensive work in the feed-forward control of distillation columns. The transient behavior of a ten-tray pilot-plant column was described by a set of 26 differential equations. Two equations were required for each tray. One equation described the composition-time behavior, while the second showed the liquid rate-time behavior. The remaining equations described the reboiler, reflux drum and sections of the top and feed tray used to preheat cold feed streams to those trays. The 26 equations were then converted by Laplace transforms into the frequency domain. At a particular value of the frequency, 26 algebraic equations resulted with coefficients which were complex numbers. The equations for the top tray or the tenth tray are given by

$$\frac{dL_{10}}{dt} = (1/a)(R - L_{10})$$ (1)

and

$$\overline{\delta}_{10} \frac{dx_{10}}{dt} = (\overline{x_d} - \overline{x_{10}})R - (\overline{y_{10}} - \overline{y_9})V_{10} + \overline{R}\, x_d$$

$$- (\overline{L_{10}} + \overline{V_{10}}\, \overline{m_{S_{10}}})\, x_{10} + \overline{V_9}\, \overline{m_{S_9}}\, \overline{x_9}, \qquad (2)$$

where the term, a, is a constant and barred quantities are the steady-state values. In the frequency domain, these equations become

$$j\omega x_{10} = b_2 R - b_3 V_{10} - b_4 x_{10} + b_5 x_9 \qquad (3)$$

$$j\omega L_{10} - b_6 R - b_6 L_{10}, \qquad (4)$$

where the term, b, is a constant and $j = \sqrt{-1}$. Similar equations for tray 9 can be derived as

$$j\omega x_9 = b_7 L_{10} - b_8 V_9 + b_9 x_{10} - b_{10} x_9 + b_{11} x_8 \qquad (5)$$

$$j\omega L_9 = b_6 L_{10} - b_6 L_{10}. \qquad (6)$$

Substitution of Equations (3) and (4) into (5) yields

$$x_8 = (b_{12} + jb_{13})V_r - (b_{14} + jb_{15})R. \qquad (7)$$

The stepping procedure was continued on down the column to the feed plate. A similar procedure was followed from the bottom plate to the feed plate. Equations for the feed plate are

$$R = (b_{24} + jb_{25})\, F + (b_{26} + jb_{27})x_f \qquad (8)$$

and

$$V_s = (b_{28} + jb_{29})F + (b_{30} + jb_{31})x_f. \tag{9}$$

The feedforward controller (FFC) was made up of four separate transfer functions. In matrix notation, these are

$$FFC = \begin{vmatrix} R/F & R/x_f \\ V_s/F & V_s/x_f \end{vmatrix}. \tag{10}$$

Therefore, if the transfer functions are known, the proper values for the corrective inputs are determined by

$$\begin{vmatrix} R \\ V_s \end{vmatrix} = |FFC| \begin{vmatrix} F \\ x_f \end{vmatrix}. \tag{11}$$

Gerster (5) applied the same mathematical model described by Luyben (10, 11) to a ten-tray stripping column. The binary system studied was composed of benzene and acetone. The column and control system were simulated on an analog computer. The liquid temperature on the seventh tray was the controlled variable. The manipulated variable was the vapor rate to the column. Feedback control, as well as feedforward control, was tested on the column.

Brosilow (2) implemented feedback control on a fifteen tray pilot scale rectifying column. The process model consisted of linearized material balance equations for each tray. These equations were very similar in form to those used by Luyben (10, 11). The control law used in the study is described by the following equations:

$$m(t) = a \qquad \text{if } \beta'x < 0$$
$$= b \qquad \text{if } \beta'x > 0$$

$$\beta'x = \sum_{n=1}^{N} \beta_n \, x_n \qquad (12)$$

$$\beta = PC \qquad (13)$$

$$A'P + PA = -Q, \qquad (14)$$

where a and b are scalar constraints on the control effort, x_n is the deviation of liquid composition on tray n from the steady-state and β,P and C are matrix quantities.

Rose, et al. (19, 20) described the control of a five-plate distillation column. The column was represented by a series of finite difference equations, which were very similar to those of Luyben (10, 11). These equations were in differential form and related the rate of change of the composition as a function of the traffics and holdup on each plate. An example of this equation is given for plate 5 by

$$\delta_5 \, \frac{dx_5}{dt} = V_4 y_4 + L_d y_5 - V_5 y_5 - L_5 x_5 , \qquad (15)$$

where the subscripts refer to the plate number. The terms, V and L, are not constant with time during the transient condition and were represented in transfer notation by

$$V_n = \frac{V_{n-1}}{\tau_1 p + 1}, \qquad (16)$$

where $p = \frac{d}{dt}$, τ is a time constant and n is a generalized

plate number.

A generalized model for a distillation column was presented by Holland (8). The component material balances were similar to those presented by Rose (19, 20) and were represented by a set of integral-difference equations. These equations were converted to a set of algebraic equations and put into matrix notation.

The calculational procedure is initiated by assuming a set of temperatures and traffic streams for each plate. The matrix is then solved for the component flow rates from each plate. After convergence has been achieved, a new set of temperatures and flow rates are predicted for the end of the next time period. Holland used this calculational procedure and developed controller equations for both feedback and feedforward control.

Rosenbrook (21) had a different approach to the control of a column. Rather than calculate transient data, he was concerned only with initial and final steady-state date. He defined a "disturbance function", D_s, and a "variation of product", G. For a binary system, D_s is given by

$$D_s = \Sigma_r \left| \frac{d}{dt} \delta_r x_r \right|, \qquad (18)$$

where the sum is taken over all plates. δ_r is the liquid holdup. If the column were operating at steady-state and the reflux rate were increased by ΔL and if all liquid flows were increased by the same amount, the material

balance around the condenser, the reboiler, and each plate would yield

$$\frac{d}{dt} \delta_{n+1} x_{n+1} = 0 \qquad (19)$$

$$\frac{d}{dt} \delta_0 x_0 = \Delta L(x_1 - x_0) \qquad (20)$$

$$\frac{d}{dt} \delta_r x_r + \Delta L(x_{r+1} - x_r). \qquad (21)$$

The subscripts refer to the plate number. The values of the composition, x, are those in the previous steady-state. On summing, the amount of disturbance in the column after the change in reflux is found to be

$$D_S = \Delta L(x_{n+1} - x_0). \qquad (22)$$

A similar equation can be written for an increase in vapor flow rate, ΔV, as

$$D_S = \Delta V(y_n - x_0). \qquad (23)$$

Optimum control would have the property of making the rate of decrease of D_S a maximum. Therefore, if a disturbance should enter the column through the feed, control would be applied by a subsequent disturbance in the liquid or vapor flow rate which would annul part of the disturbance previously introduced.

Lupfer, et al. (12, 13, 14) have also done extensive work in the area of feedforward control. However, their work has been confined to actual column operation. A

rather complicated control scheme was suggested to optimize column operation. General equations for a debutanizer were developed which stated that the product purities are functions of 16 variables. By applying internal reflux control, three of the variables were removed from consideration. The addition of feed enthalpy control reduced the number of variables to 11. The regulation of the reboiler heat input reduced the number of independent variables to 10. The general equation for the controlled variables are given by

$$B = f_9(C_3, iC_4, nC_4, iC_5, nC_5, F, F_t, F_e, iC_{5d}, nC_{4b})$$

$$R_1 = f_{10}(C_3, iC_4, nC_4, iC_5, nC_5, F, F_t, F_e, iC_{5d}, nC_{4b}),$$

$$(24)$$

where F_t is the feed tray location, F_e is the feed enthalpy, and R_1 is the internal reflux rate. The equations show that the bottoms flow rate and the internal reflux rate required to maintain the desired product purities are a function of feed composition, feed flow rate, feed tray location, feed enthalpy, and product specification. The values of the feed enthalpy and feed tray location were optimized to minimize operating costs. The development of an equation for predictive control of the internal reflux rate involved a regression analysis of data obtained by tray-to-tray calculations. The data were "curve-fitted" to a forty-five term second-order equation. A multi-order time lag was used on the bottom and reflux

flow rates. Feedback control was then used to compensate for the predictive system errors which result because of measurement errors, imperfect equations, and computing equipment errors. A composition analyzer was placed on the product stream, and a controller trimmed the computed bottom product signal to insure specified overhead product purity.

Williams, et al. (23, 24) and Williams in conjunction with Rose (19, 20) concerned himself with determining the optimum sampling point and the range of stability for a given sampling period. The mathematical model, presented by Rose, was used. The maximum sampling period was found to be less than one-half the system time constant. The sampling point was found to vary according to the amount of dead space error.

Other articles have been written which, although they do not discuss a particular mathematical model, contain useful material. Talbot (22) discusses the various methods which are presently being used in fractionator control. The advantages and disadvantages of each control scheme are discussed. Bertrand (1) also discusses the different control schemes for fractionators. Composition, pressure, and inventory controls are illustrated.

Development of the Lumped Parameter
Model (16, 17)

In 1962 Marr (15) suggested a new concept for

predicting the transient behavior of a distillation
column. He suggested that a model should be developed in
which some parameter could be used to describe the degree
of separation that was occurring in a distillation column.
All model development until that time had been directed
toward the complicated plate-to-plate model.

Reynolds (18) developed a model which used the concept
of the separation parameter. He envisioned a distillation
column as being composed of several sections in which
there could be any number of trays. According to the
section concept, as shown in Figure 1, a section of a
distillation column is that part of the column which lies
between the points at which the feed streams enter or
product streams leave the column. According to Reynolds,
the rate at which mass is transferred from the vapor phase
to the liquid phase can be expressed by the equation

$$N_{n,i}^{V} = -J_{n,i}(y^* - y)_{n,i} \, , \tag{25}$$

where $J_{n,i}$ is the parameter which describes the degree of
separation occurring in a section and $(y^* - y)_{n,i}$ is the
driving force for mass transfer in the section.

Although Equation (25) appears to be identical in form
to the equation normally used for mass transfer

$$N = K_{OG}(y^* - y) \, ,$$

there are fundamental differences. The coefficient, K_{OG},
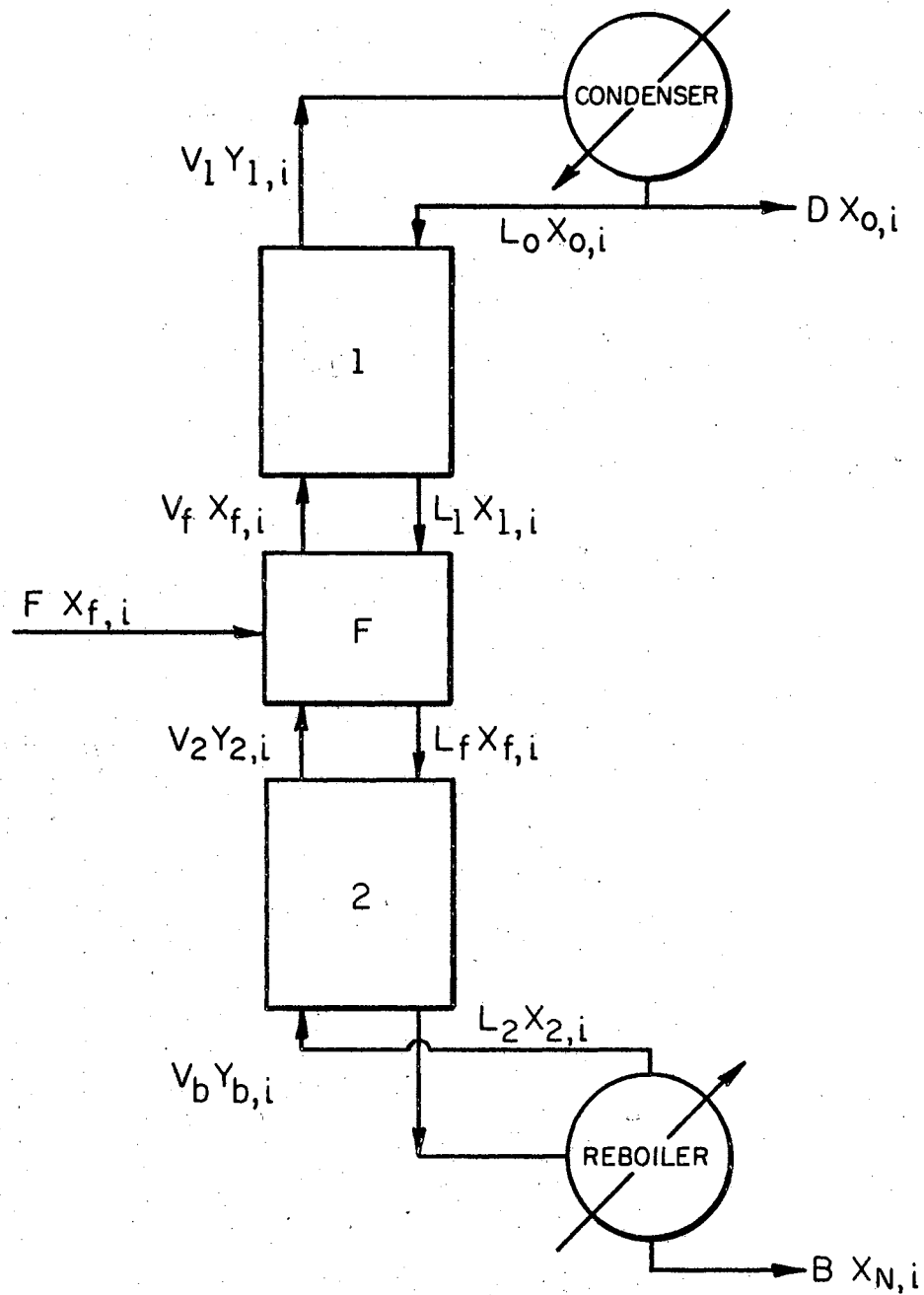is related to the diffusivity of the component being

Figure 1. A Distillation Column According
to the Section Concept

transferred. The coefficient, $J_{n,i}$, however, is not directly related to the physical properties of the component being transferred. $J_{n,i}$ is a parameter that describes the degree of separation that occurs in a section and is an empirically determined factor.

Reynolds developed a set of differential equations to predict the transient behavior of the liquid and vapor streams leaving the section. Two major assumptions were made by Reynolds. The first assumption was that $J_{n,i}$ remains constant for small changes in column conditions. The second was constant molal overflow throughout the section. Reynolds was unable to get good agreement between values predicted by his model and experimental data.

Osborne (16) felt that Reynold's concept was basically sound. Rather than assume constant molal overflow, he extended Reynold's model to relate the net rate at which mass is transferred from the liquid phase to the net rate at which mass is transferred from the vapor phase. His development of the model follows.

From the law of conservation of matter,

$$\text{input} - \text{output} = \text{accumulation} \tag{26}$$

$$\text{input} = V_{n+1}\, y_{n+1,i} \tag{27}$$

$$\text{output} = V_{n+1}\, y_{n+1,i} + \frac{\partial(V_n\, y_{n,i})}{\partial z}\, dz + N_{n,i}^V \tag{28}$$

$$\text{accumulation} = \frac{\partial(\delta_n^V\, y_{n,i})}{\partial t}. \tag{29}$$

The output term is the sum of all of the material leaving the vapor phase, either by flow or mass transfer. The flow rate of a component from the section is equal to the flow rate in $V_{n+1}\, y_{n+1,i}$ plus the increase in the stream flow rate that occurs in the section $\dfrac{\partial(V_n\, y_{n,i})dz}{\partial z}$. The vapor phase is represented by the term $N^V_{n,i}$. The material balance equation becomes

$$\frac{\partial(\delta^V_n\, y_{n,i})}{\partial t} = -\frac{\partial(V_n\, y_{n,i})}{\partial z}\, dz - N^V_{n,i}. \qquad (30)$$

A similar material balance equation

$$\frac{\partial(\delta^L_n\, x_{n,i})}{\partial t} = -\frac{\partial(L_n\, x_{n,i})}{\partial z}\, dz - N^L_{n,i} \qquad (31)$$

can be derived for the liquid stream passing through a section of a distillation column.

If the change in height Δz is small, the term, $\dfrac{\partial(V_n\, y_{n,i})}{\partial z}$ can be approximated by

$$\frac{\partial(V_n\, y_{n,i})}{\partial z} = \frac{V_n\, y_{n,i} - V_{n+1,i})}{\Delta z}. \qquad (32)$$

Since $\delta^V_n\, y_{n,i}$ is now a function of time only, the partial derivative with respect to time can be replaced with a total derivative. Osborne then assumed constant molal holdup and a constant rate of interphase mass transfer in a section. According to Reynolds,

$$N^V_{n,i} = -\, J_{n,i}(y^* - y)_{n,i}. \qquad (33)$$

Equation (30) can then be written in the form

$$\delta V_n \frac{d(y_{n,i})}{dt} = -(V_n\, y_{n,i} - V_{n+1}\, y_{n+1,i})$$
$$+ J_{n,i}(y^* - y)_{n,i}. \qquad (34)$$

If a section is considered to be subdivided into an infinite number of subsections, the driving force for one of these infinitesimal subsections can be represented by the equation

$$(y^* - y)_{k,i} = (Kx)_{k-1,i} - y_{k+1,i}. \qquad (35)$$

By summing the values of the subintervals, the driving force for the section can be approximated by the equation

$$(y^* - y)_{n,i} = (Kx)_{n-1,i} - y_{n+1,i}. \qquad (36)$$

Using Equation (36), Equation (34) can be rewritten in the form

$$\delta V_n \frac{d(y_{n,i})}{dt} = -(V_n\, y_{n,i} - V_{n+1}\, y_{n+1,i})$$
$$+ J_{n,i}[(Kx)_{n-1,i} - y_{n+1,i}]. \qquad (37)$$

A similar derivation for the liquid phase gives the equation

$$\delta L_n \frac{d(x_{n,i})}{dt} = -(L_n\, x_{n,i} - L_{n-1}\, x_{n-1,i})$$
$$- J_{n,i}[(Kx)_{n-1,i} - y_{n+1,i}]. \qquad (38)$$

By assuming that $J_{n,i}$ is constant for small changes

in column conditions, a solution to the transient behavior of the composition of the vapor and liquid streams leaving the section can be obtained by integrating Equations (37) and (38).

At steady-state $\frac{d(y_{n,i})}{dt} = 0$. Therefore, the value of $J_{n,i}$ can be calculated by the equations

$$J_{n,i} = \frac{(V_n\, y_{n,i} - V_{n+1}\, y_{n+1,i})}{(Kx)_{n-1,i} - y_{n+1,i}} \qquad (39)$$

or

$$J_{n,i} = -\frac{(L_n\, x_{n,i} - L_{n-1}\, x_{n-1,i})}{(Kx)_{n-1,i} - y_{n+1,i}} \qquad (40)$$

and with steady-state data.

Introduction to Feedback Control (6, 25)

A feedback control system is defined as one in which the controlled variable is compared with the reference variable, and any difference between the two is used to reduce the difference. The control instrument continuously measures the output variable of the controlled process and compares this output with a pre-established desired value. The instrument then uses any resulting error to compute the required correction to the setting of a basic element of the piece of equipment being controlled.

A control system must fulfill three basic requirements:

1. The controlled system must be stable. The

response to a disturbance must reach a
steady value within a reasonable period
of time. Instability in the system pro-
duces oscillations of the output variable
and could drive the output to some
limiting value.

2. The second requirement, accuracy, insures
that the control system will reduce any
error to some tolerable value.

3. Speed of response, the third requirement,
is essential in maintaining the desired
product specifications.

The three modes of control normally used in control
systems are proportional, integral, and derivative.
Stability, accuracy, and speed of response of the control
system are usually obtained by some combination of the
three modes with appropriate proportionality constants.
Gardner (4) and Hoffman (7) discuss the common combina-
tions of the control modes. A discussion of each control
mode follows:

Proportional control applies a correction to the
process which is directly proportional to the error. The
equation which describes this action is

$$\Delta m = K_1 \ E_r .\tag{41}$$

For any particular column or apparatus being controlled,
there is a small range of values of K_1 which gives optimum

correction. Too small a value of K_1 results in a continuous steady-state error. Too large a value results in instability and overcorrection. An error must be present for a correction to be made; therefore, proportional control can never completely correct an upset in the column.

Integral control applies a correction to the process which is directly proportional to the time integral of the error. Expressed mathematically,

$$\Delta m = K_2 \int_0^t E_r \, dt. \tag{42}$$

The main advantage of integral control is the elimination of the steady-state error. The integral controller continues to correct until the error is zero. If, however, the error develops a change in sign, the accumulated correction must be integrated out before the proper correction can be made. For example, if there had been a large positive error, the integral of the error would be represented by a positive area under the error curve. If the error should then change sign, the negative area under the error curve would have to equal the positive area before the total area under the error curve could change sign permitting the proper correction to be made. Therefore, integral control tends to be sluggish and can lead to overcorrections and instability.

Derivative control applies a correction to the process which is directly proportional to the time derivative of the error. Expressed mathematically,

$$\Delta m = K_3 \frac{dE_r}{dt}. \tag{43}$$

Derivative control is desirable, because it responds to the rate of change of error and can apply a large correction while the error is forming. However, derivative control cannot be used alone, because it will not respond to a steady-state error.

Combination of the control modes can utilize the good qualities of each and eliminate the undesirable characteristics of each individual mode. Proportional plus integral control is often used to obtain the stability of proportional control and the error elimination of integral control. The addition of derivative control provides the early detection of the error and adds damping to the system, permitting a higher proportional gain.

CHAPTER III

DESCRIPTION OF CONTROL MODEL

Osborne (16, 17) developed a mathematical model which
described the transient behavior of a distillation column.
The model offers a more simple solution than the conven-
tional tray-to-tray model. A single parameter, $J_{n,i}$,
describes the separation occurring in each section of the
column. The model's simplicity makes it possible to use
smaller computers for transient condition calculations.

A feedback control system was developed as an exten-
sion of the lumped parameter model, in order to test the
controllability of a column described by the model. A
stripping column, shown in Figure 2, was selected for the
simulated control study. The composition of the bottoms
product was selected as the controlled variable. The re-
boiler heat duty was adjusted by a controller at each time
interval to maintain the desired bottoms composition.
Simple proportional, integral, or derivative control modes
or combinations of these modes were selected for the simu-
lated controller.

Osborne assumed that all flow rates changed to their
final steady-state values instantaneously after the feed
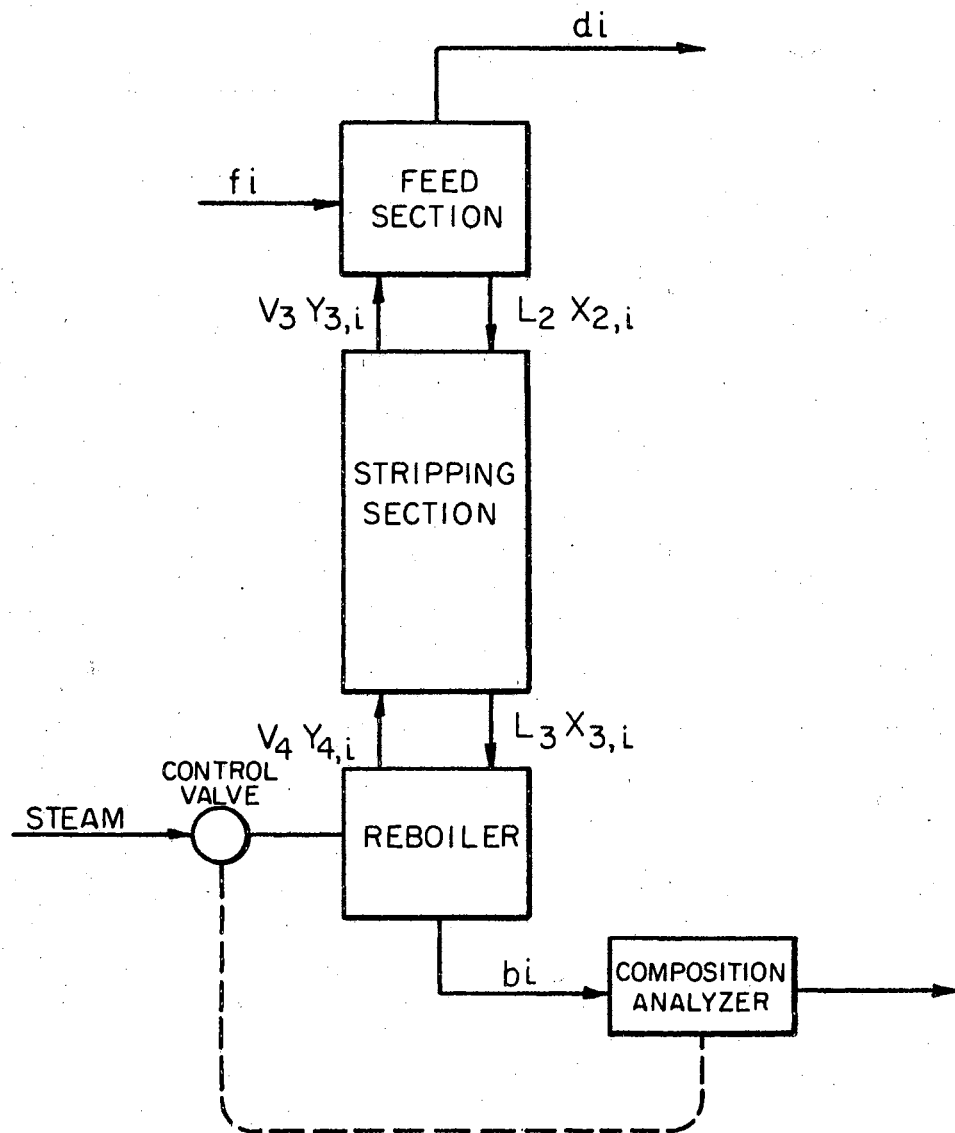upset occurred. Rather than make this assumption, the

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Figure 2. Stripping Column Control Model

flow rates were calculated by heat and material balances at each time interval. An over-all heat and material balance can be written as

$$F = D + B \qquad (44)$$

$$F\, h_f = D\, H_d + B\, h_b - Q_r. \qquad (45)$$

The molar enthalpies, h_f, H_d, and h_b, were assumed to remain constant for small changes in the respective flow rates. This assumption is valid at small time increments, because the temperature and composition of the components change only slightly. Equations (44) and (45) may then be written as

$$\Delta F = \Delta D + \Delta B \qquad (46)$$

$$h_f \Delta F = H_d \Delta D + h_b \Delta B - \Delta Q_r. \qquad (47)$$

Substitution of (46) into (47) yields

$$\Delta D = \Delta F\, \frac{(h_b - h_f) - \Delta Q_r}{(h_b - H_d)} \qquad (48)$$

$$\Delta B = \Delta F - \Delta D. \qquad (49)$$

Similar equations used for the reboiler are

$$\Delta B = \Delta L_3 - \Delta V_4 \qquad (50)$$

$$\Delta Q_r + h_3 \Delta L_3 = h_3 \Delta B + H_4 \Delta V_4 \qquad (51)$$

$$\Delta V_4 = \frac{\Delta B(h_b - h_3) - \Delta Q_r}{(h_3 - H_4)} \qquad (52)$$

$$\Delta L_3 = \Delta B + \Delta V_4. \tag{53}$$

Assuming that the column is at thermal steady-state at all times, the following equations can be written for the stripping section

$$\Delta L_2 + \Delta V_4 = \Delta L_3 + \Delta V_3 \tag{54}$$

$$h_2 \Delta L_2 + H_4 \Delta V_4 = h_3 \Delta L_3 + H_3 \Delta V_3. \tag{55}$$

Simultaneous solution of Equations (54) and (55) yields

$$\Delta V_3 = \frac{h_2(\Delta L_3 - \Delta V_4) + H_4 \Delta V_4 - h_3 \Delta L_3}{(H_3 - h_2)} \tag{56}$$

$$\Delta L_2 = \Delta L_3 + \Delta V_3 - \Delta V_4. \tag{57}$$

The new flow rates are calculated as

$$X = X_{initial} + \Delta X, \tag{58}$$

where X equals the respective flow rates.

The control procedure can now be described. The values of the separation parameter are determined from the steady-state operating conditions. Once the separation parameters have been determined, the feed rate and composition are changed to new values. The reboiler duty is then adjusted by the simulated controller to maintain a constant composition of one of the components in the bottoms product. Heat and material balances are solved to determine the new flow rates.

The new values of the flow rates are then used in the lumped parameter model to determine the new composition of

the bottom product. The reboiler duty is again adjusted. This procedure is continued until the column reaches steady-state at the desired composition level or the computer time allotted for the run is exceeded.

CHAPTER IV

TESTS OF THE CONTROL MODEL

A binary system was selected to test the control model. The hexane-heptane system was selected, because data for this system was readily available. Pertinent data were selected from Osborne's work. Due to the generality of the model, any binary or multicomponent system could reasonably be expected to exhibit similar results to those shown by this system, provided reasonably good thermodynamic data are available and the system is not a complex one.

In order to test the controllability of the model, disturbances in the feed composition and feed flow rate were induced on the model of the stripping column. The three types of feed disturbances considered were step, ramp, and sinusoidal. Combinations of proportional, reset or integral, and derivative control modes were used to maintain the composition of the bottoms product at the desired level. Another test of the model was to vary the column efficiency and observe the action taken by the controller. The tests and results are discussed in the remainder of the chapter.

Proportional Control

The working equation for proportional control, as previously discussed, is

$$\Delta m = K_1 \, E_r. \tag{41}$$

The error is defined by the equation

$$E_r = x_{bd} - x_{bm}. \tag{59}$$

Substitution of the appropriate variables into Equation (41) gives

$$\Delta Q_r = K_1 (x_{bd} - x_{bm}). \tag{60}$$

The measured value of the composition was compared with the specified composition after each time interval. The reboiler duty was then adjusted to its new value.

The value of the proportionality constant was varied to obtain the best control for a step disturbance in composition and flow rate to the feed. Selected results are shown in Figure 3. By observing these results, certain conclusions can be made. With a value of the proportional constant, K_1, equal to 10.0, little correction to the error can be observed. Increasing the value of the constant by an order of magnitude, correction is observed; however, control is still sluggish, and a large amount of off-quality bottoms product would result. When the value of the constant was increased to 1000.0 to obtain an early correction, an overcorrection resulted. Values of the
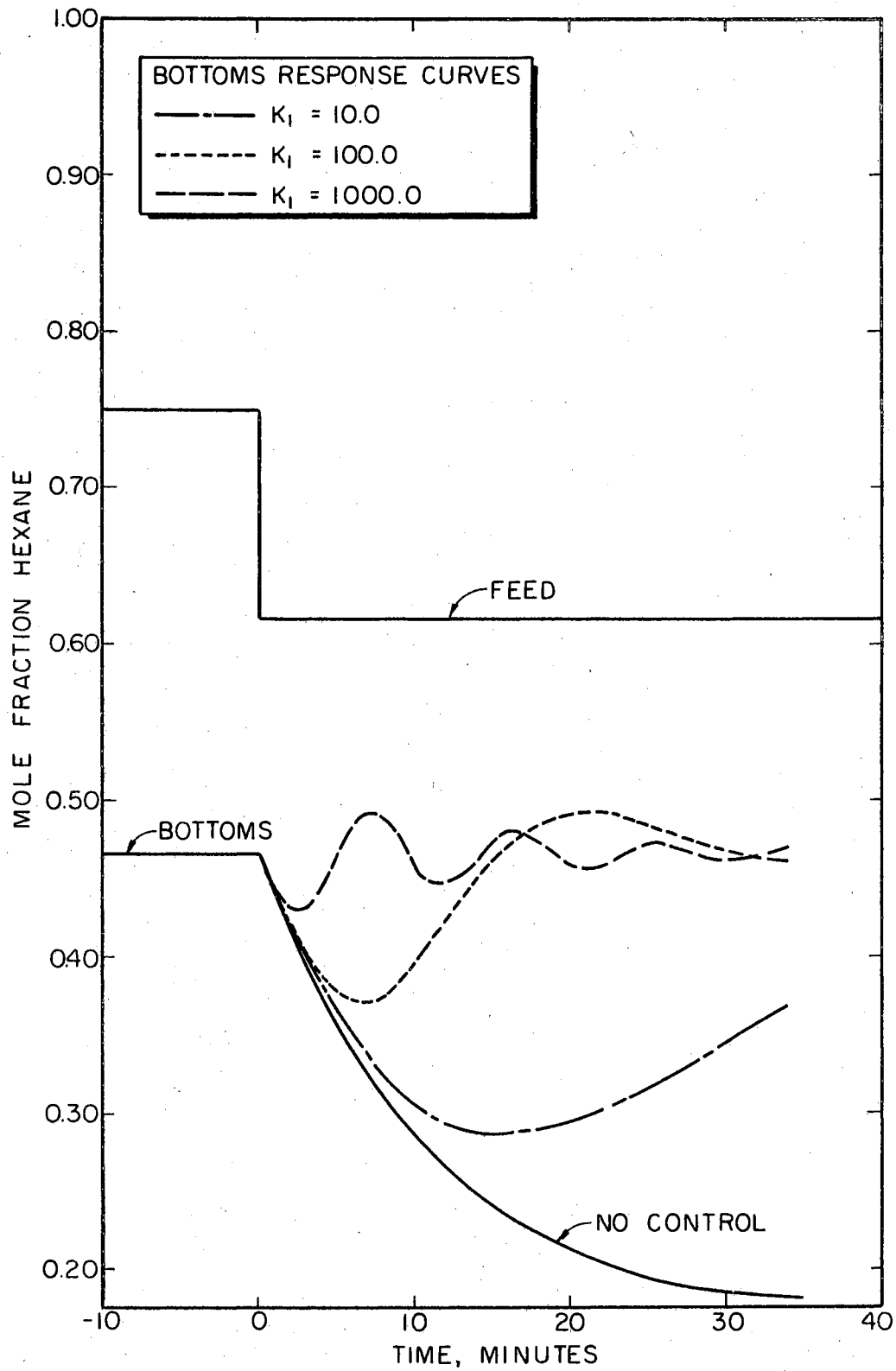
Figure 3. Simple Proportional Control of Bottoms Composition

constant larger than 1000.0 resulted in calculational
instability of the model.

Integral Control

The equation for integral control is given by

$$\Delta m = K_2 \int_0^t E_r \, dt \qquad (42)$$

and

$$\Delta Q_r = K_2 \int_0^{t'} (x_{bd} - x_{bm}) \, dt'. \qquad (61)$$

The trapezoidal rule was used to approximate the
value of the integral. The trapezoidal rule can be
expressed as

$$I = \frac{\Delta t'}{2} (E_{r2} + E_{r1}). \qquad (62)$$

The areas, calculated for each time increment, were summed
to obtain the total area under the error curve at time,
t'. The method is illustrated in Figure 4.

Several runs were made with the integral controller
in which K_2 was varied to obtain the best control for a
step disturbance in composition and flow rate to the feed.
The results are shown in Figure 5. The results reveal
that simple integral control is unacceptable for good con-
trol. A value of the constant, K_2, equal to 1000.0 pro-
vided a reasonable degree of control but caused
instabilities in the calculational procedure of the lumped
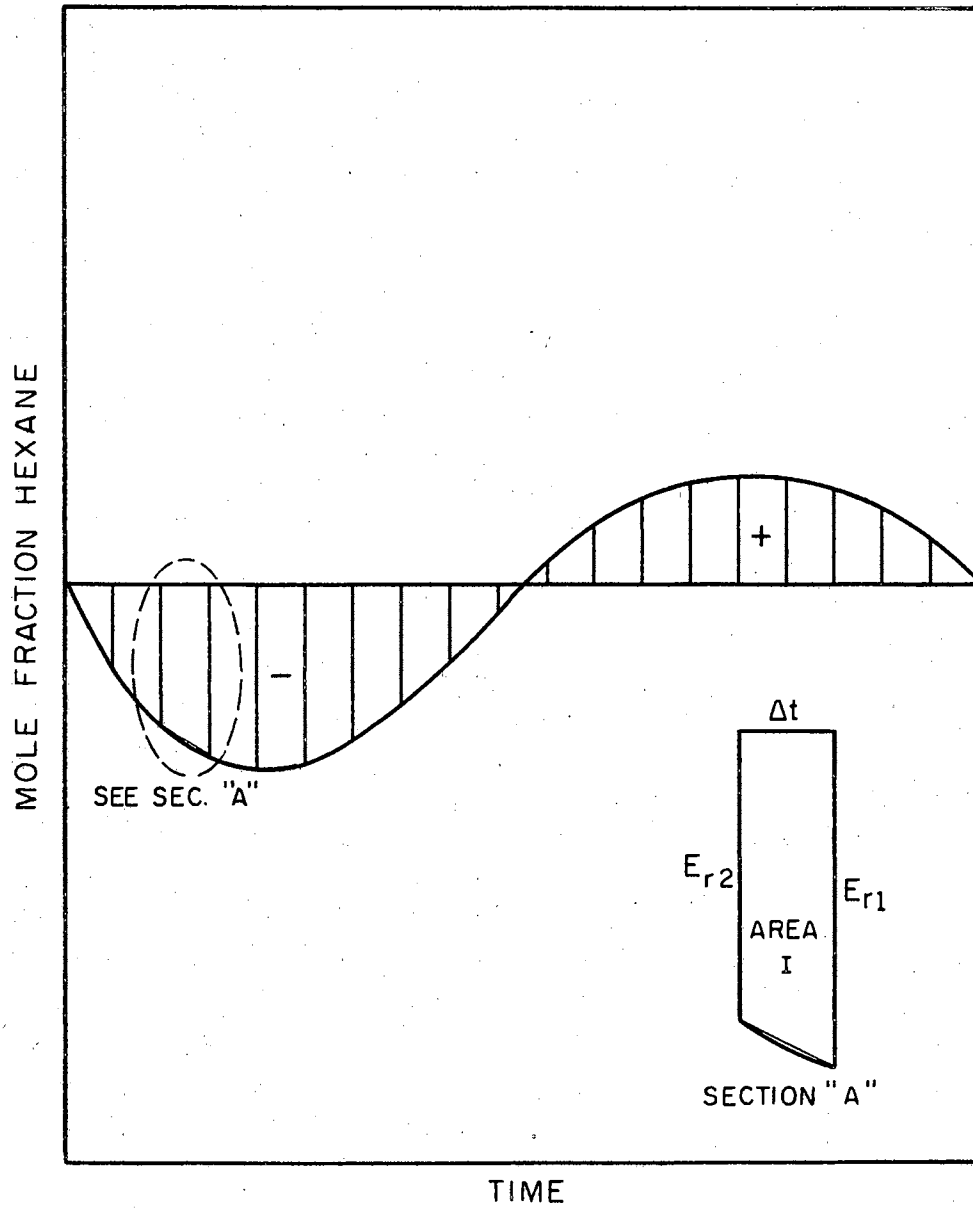parameter model. Smaller values of the constant, K_2,

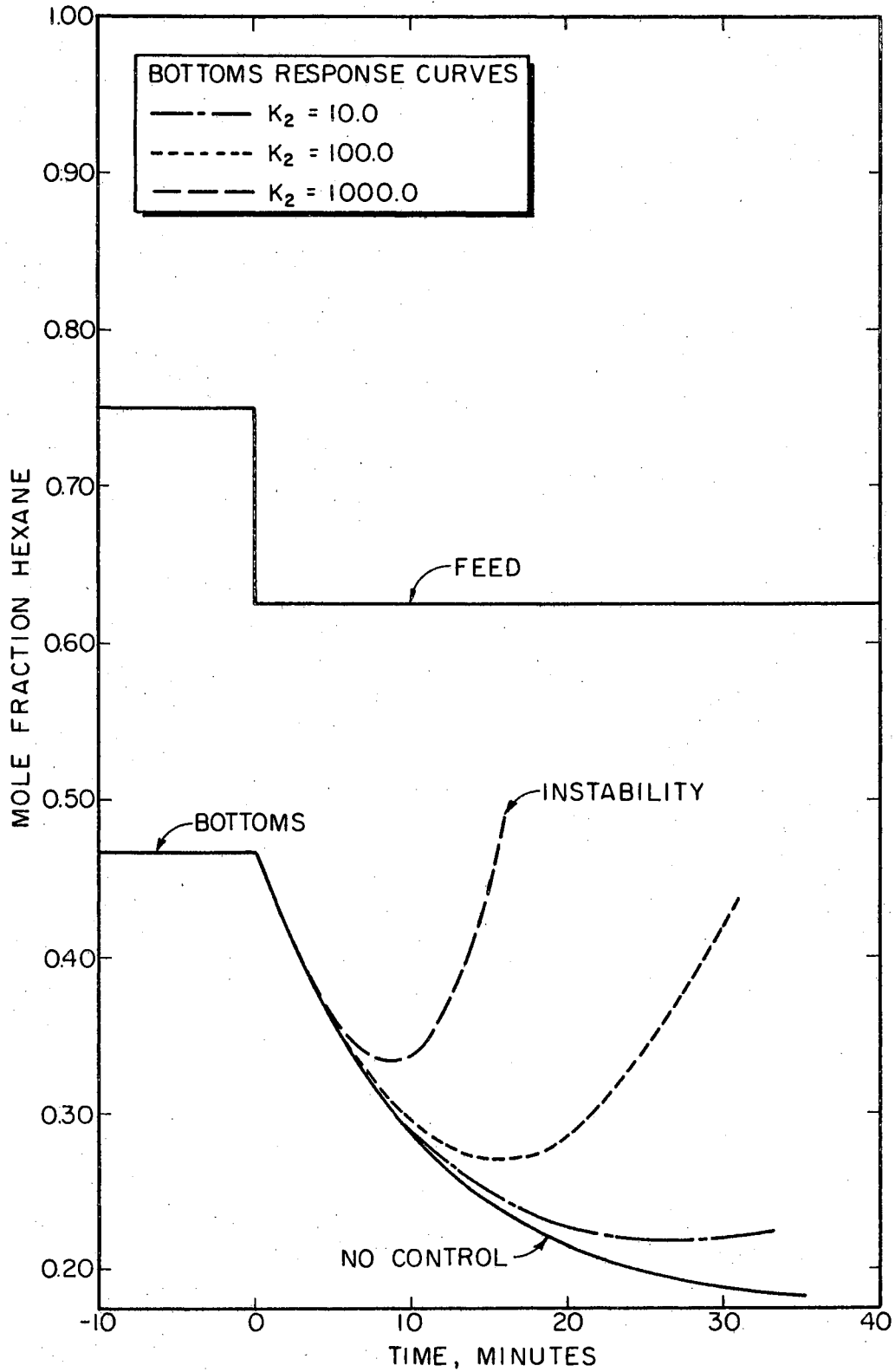Figure 4. Integration of Error Curve by
Trapezoidal Rule

Figure 5. Simple Integral Control of Bottoms Composition

allowed stable operation, but little control. These
results are verified in the theory presented earlier. In
practice, pure integral control tends to be sluggish and
can lead to instability.

Derivative Control

The working equation for the derivative controller is
written as

$$\Delta m = K_3 \frac{dE_r}{dt'} \tag{43}$$

or

$$\Delta Q_r = K_3 \frac{E_r|_{t'=t'} - E_r|_{t'=t'_o}}{t' - t'_o}. \tag{63}$$

The feed disturbance was assumed to occur at $t'_o = 0$.
Because the system was assumed to be at the desired steady-
state before the upset, the initial error was assumed to
be zero. Equation (63) reduces to

$$\Delta Q_r = \frac{K_3 \; E_r|_{t'=t'}}{t'}, \tag{64}$$

where t' equals the time elapsed since the column upset.

The results of the simple derivative controller are
presented in Figure 6 for the control of the column which
underwent a step disturbance in feed composition and flow
rate. Small values of the constant K_3 provided rather
slow control. With K_3 equal to 500.0, the system came
close to a condition of being critically damped. When the
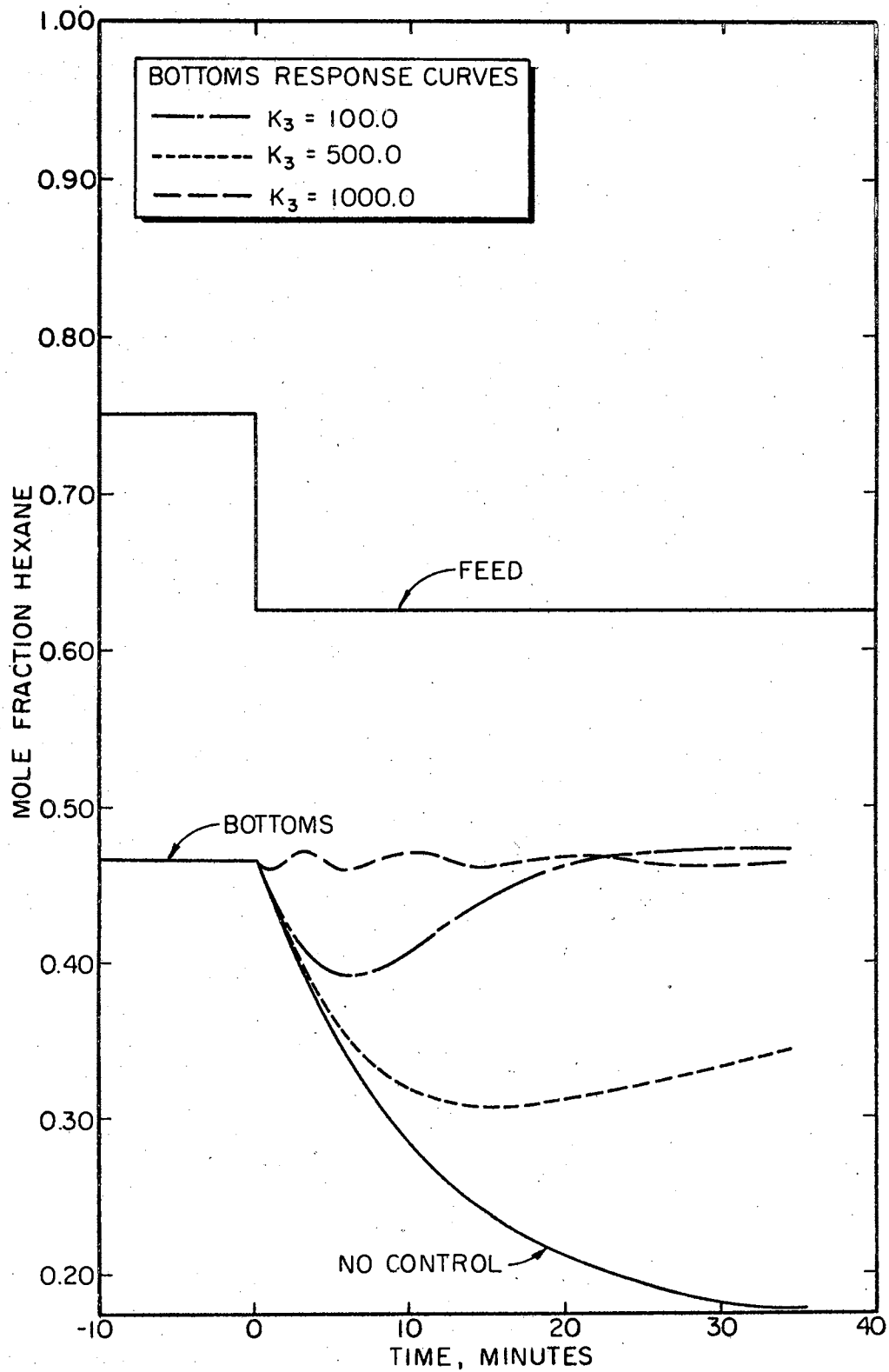value of the constant was increased by two orders of

Figure 6. Simple Derivative Control of Bottoms
Composition

magnitude, good control was achieved, despite oscillation about the desired value.

The derivative controller functioned well and provided better control than the pure proportional or pure integral controller. The derivative controller detected the error more quickly than the other modes and applied a large correction at small values of time, t'. However, at larger values of time, t', the amount of control diminished. In practice, derivative control is not used alone. Because of its mathematical nature, derivative control will not respond to a steady-state error. This is shown when K_3 is equal to 500.0. Although the composition curve levels out at a steady value, this value is above the desired product composition. For this reason, derivative control must be used in conjunction with the other control modes.

Proportional and Integral Control

The proportional and integral control modes can be combined to give

$$\Delta Q_r = K_1 E_r + K_2 \int_0^{t'} E_r \, dt'. \tag{65}$$

This combination of control modes is frequently used in industry. Because of its relative importance, this control scheme was tested on three types of feed disturbances.

The first type of disturbance considered was the step disturbance. The feed composition and flow rate were stepped by the same amount as the case of the pure control

modes discussed earlier. The values of the constants K_1
and K_2 were varied to obtain optimum control. Selected
results for the step disturbance are presented in Figure 7.

The control curves reveal that the main control effort
results from the proportional controller. The effect of
integral control appears to be negligible. The importance
of the integral controller, however, lies in its ability
to eliminate all the error.

A ramp disturbance in the feed composition and flow
rate was the second type of disturbance considered in the
tests. The slope of the ramp function was varied while
the constants K_1 and K_2 remained at a fixed value. The
values of K_1 and K_2 were selected as those which gave good
results during the step disturbance.

In Figure 8 the control curve is similar to the curve
obtained during a step change. The slope of the ramp
function was decreased in Figure 9, and the control was
much better. A further decrease in the slope allowed
almost perfect control, as shown in Figure 10. The results
are much as one should expect. A steep slope of the ramp
function closely approximates a step change. As the slope
of the ramp decreases, the severity of the disturbance
also decreases and can be approximated by a series of
small step changes.

The third type of disturbance studied was the
sinusoidal variation of the feed composition and flow
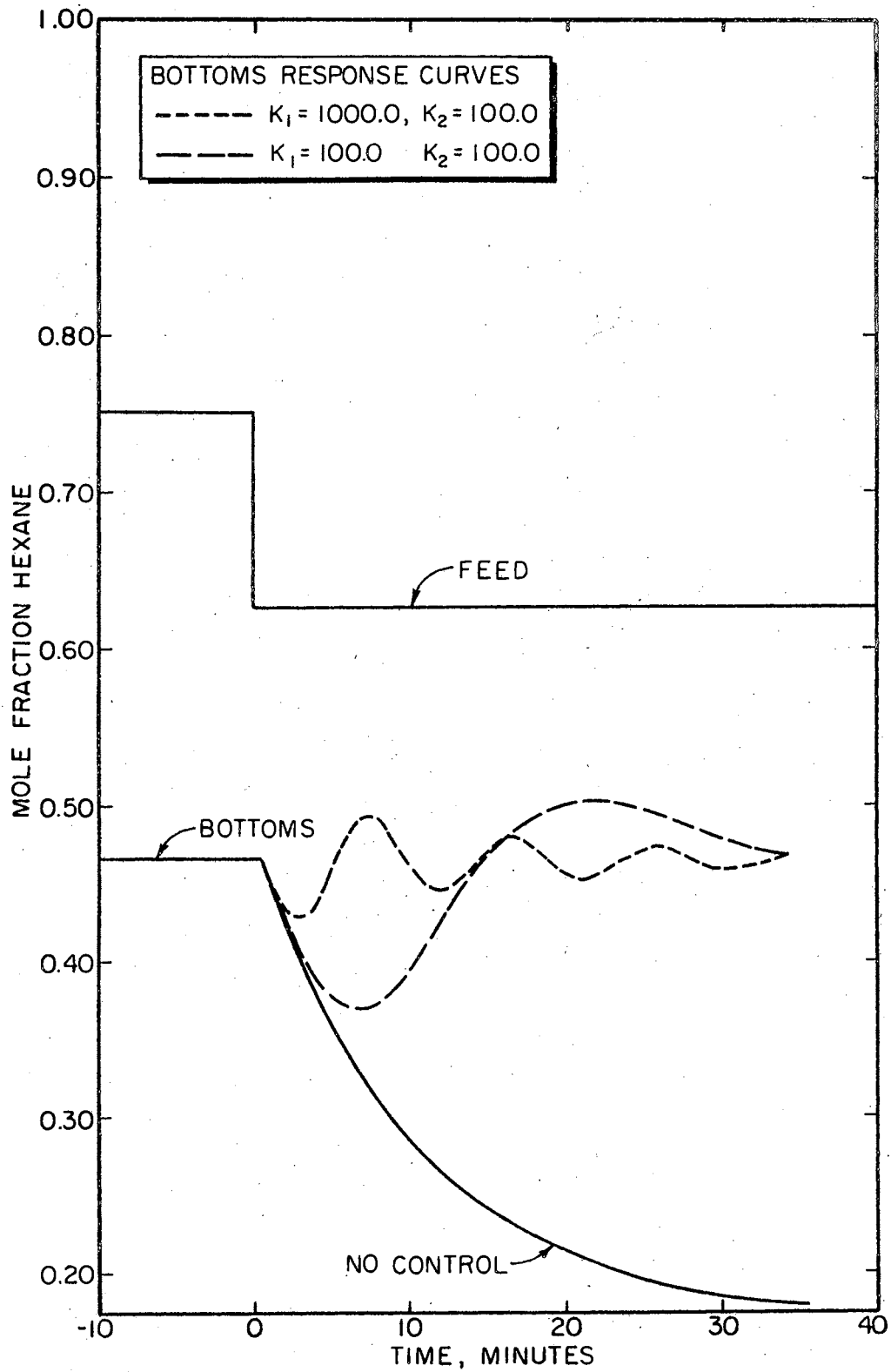rate. The frequency of the sine wave was varied with the

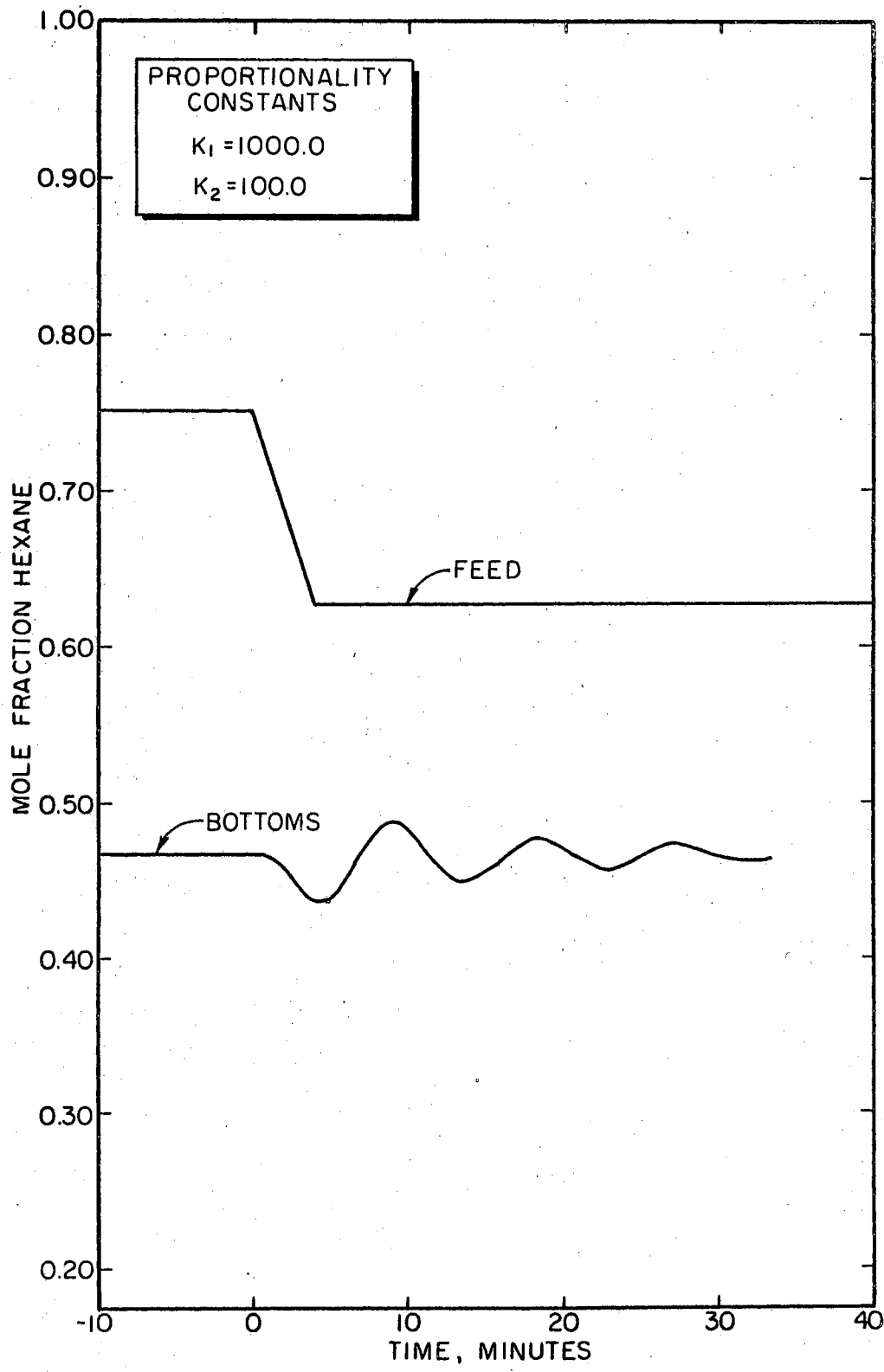Figure 7. Proportional and Integral Control
With Step Disturbance

Figure 8. Proportional and Integral Control With
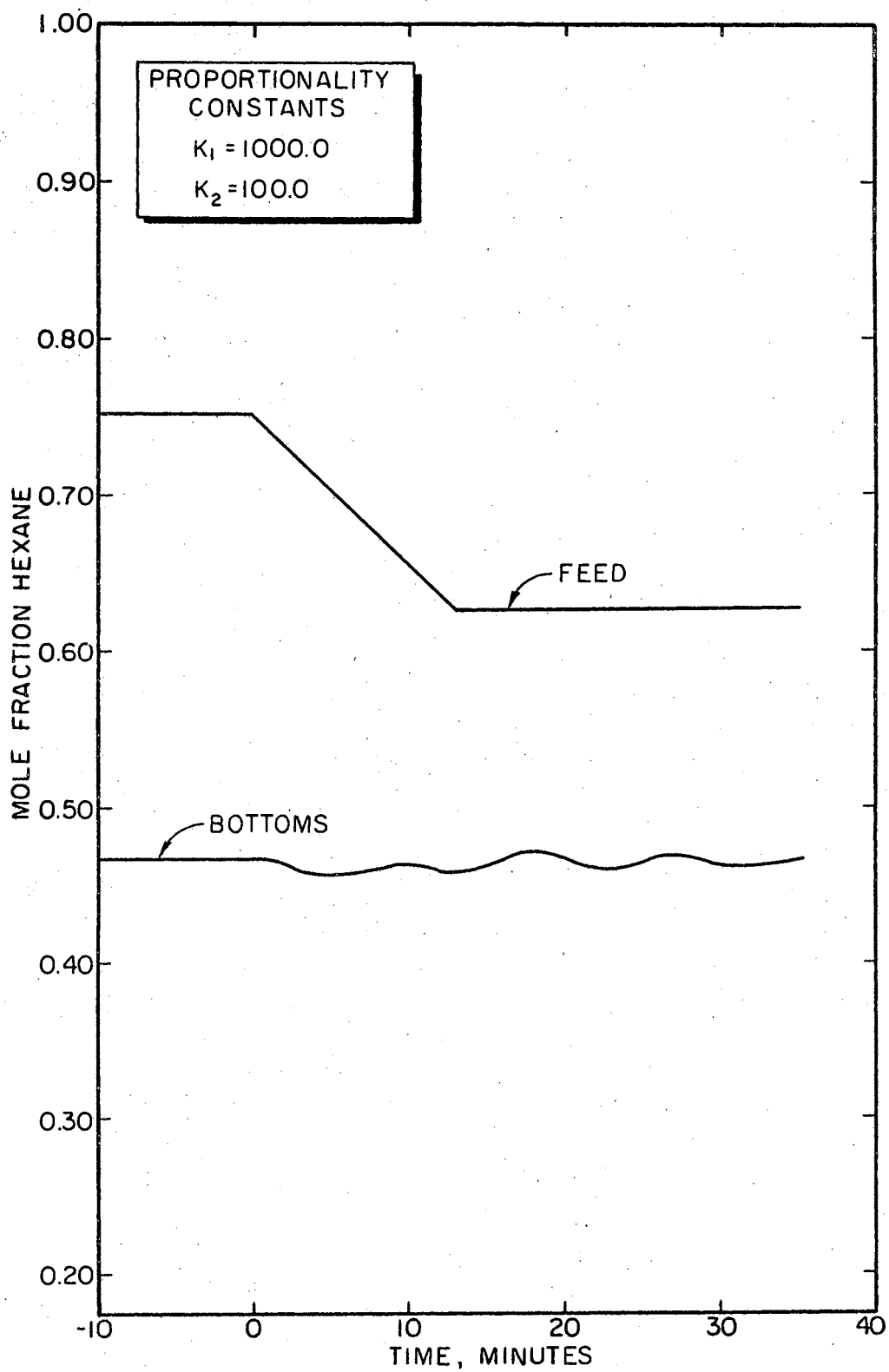Ramp Disturbance of Steep Slope

Figure 9.   Proportional and Integral Control With
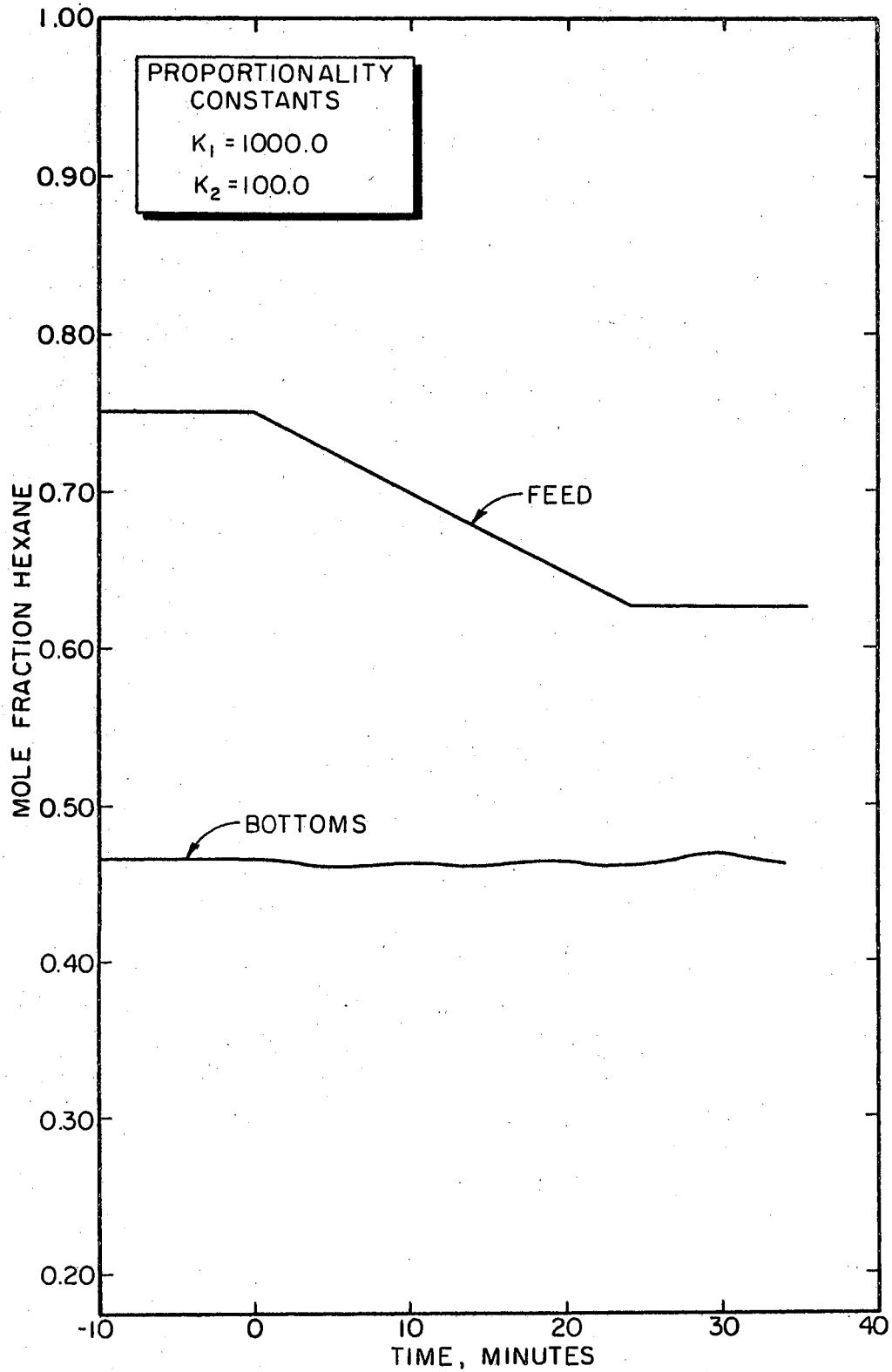Ramp Disturbance of Moderate Slope

Figure 10. Proportional and Integral Control
With Ramp Disturbance of
Gradual Slope

constants K_1 and K_2 set at the same values used to control the ramp disturbance. The amplitude of the sine function was set at one-half the magnitude of the step change.

At sufficiently high or low frequencies, the desired bottoms composition was maintained quite well. At intermediate values of frequency, the amplitude of the oscillation about the desired bottoms composition increased. At high frequencies, the oscillation of the feed composition closely approximates a steady-state value. At low frequencies the severity of the disturbance decreases in much the same manner described for the ramp function. The results are presented in Figures 11, 12, and 13.

An intermediate frequency was selected and the constants K_1 and K_2 were varied. Typical results are shown in Appendix C.

Integral, Proportional, and Derivative Control

Although the combination of integral and proportional modes provided satisfactory control, the success of the pure derivative control prompted its inclusion in the control scheme. The working equation for the combination of the three control modes becomes

$$\Delta Q_r = K_1 E_r + K_2 \int_0^{t'} E_r \, dt' + K_3 \frac{dE_r}{dt'}. \qquad (66)$$

The control scheme was tested on the step disturbance only. The results obtained with the proportional-integral
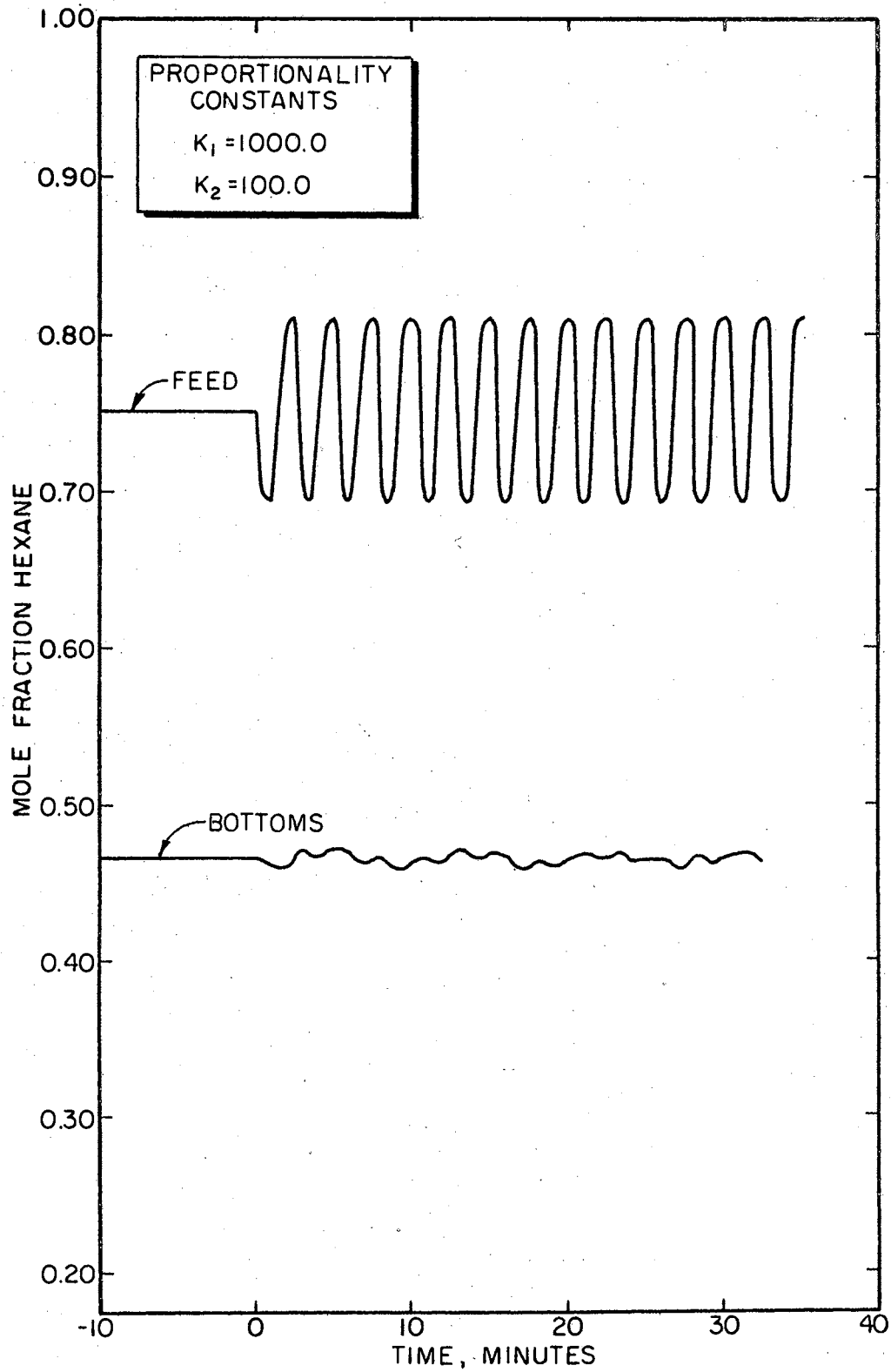
Figure 11.   Proportional and Integral Control
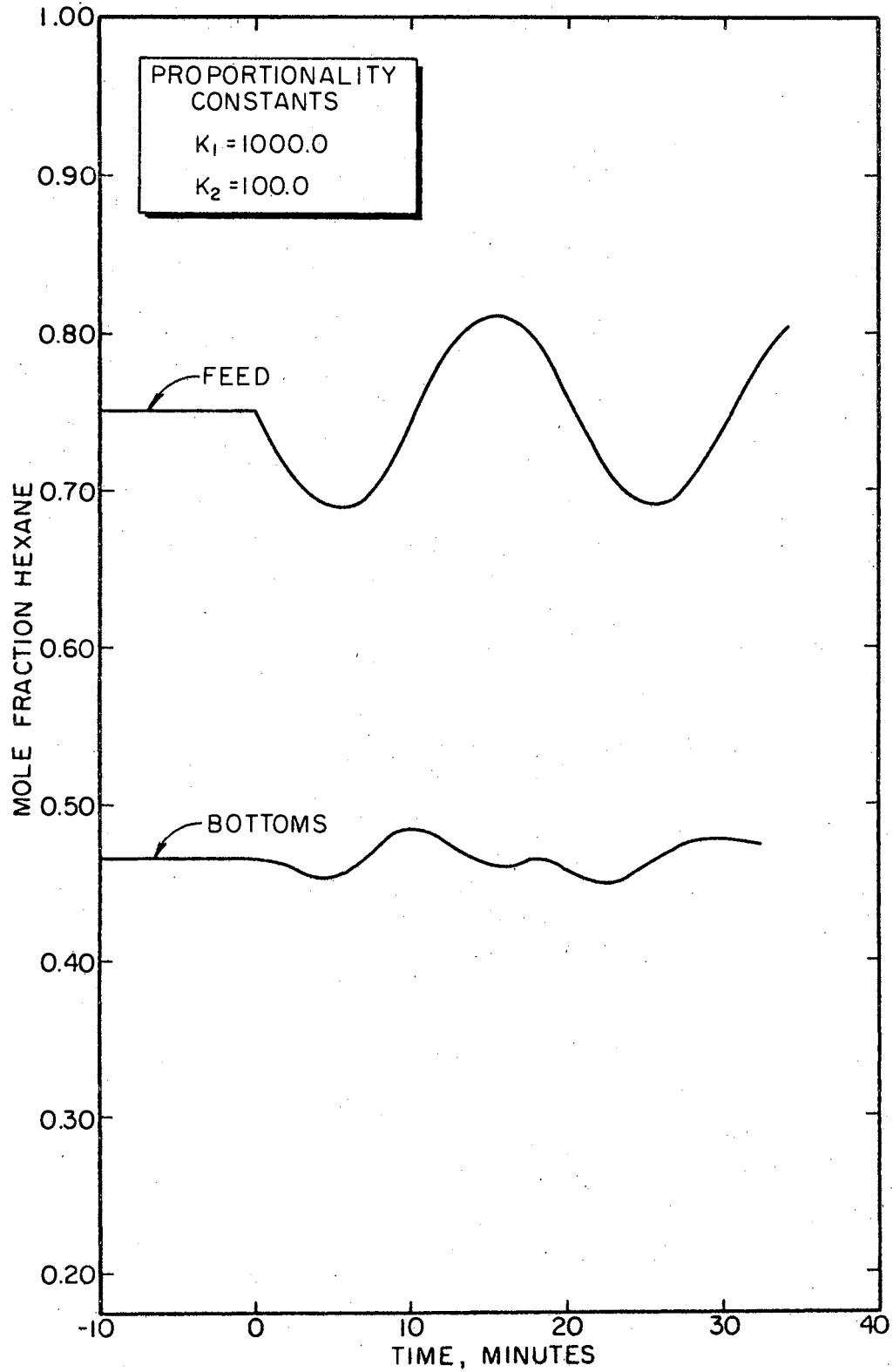With High Frequency Sinusoidal
Disturbance

Figure 12.   Proportional and Integral Control
With Moderate Frequency
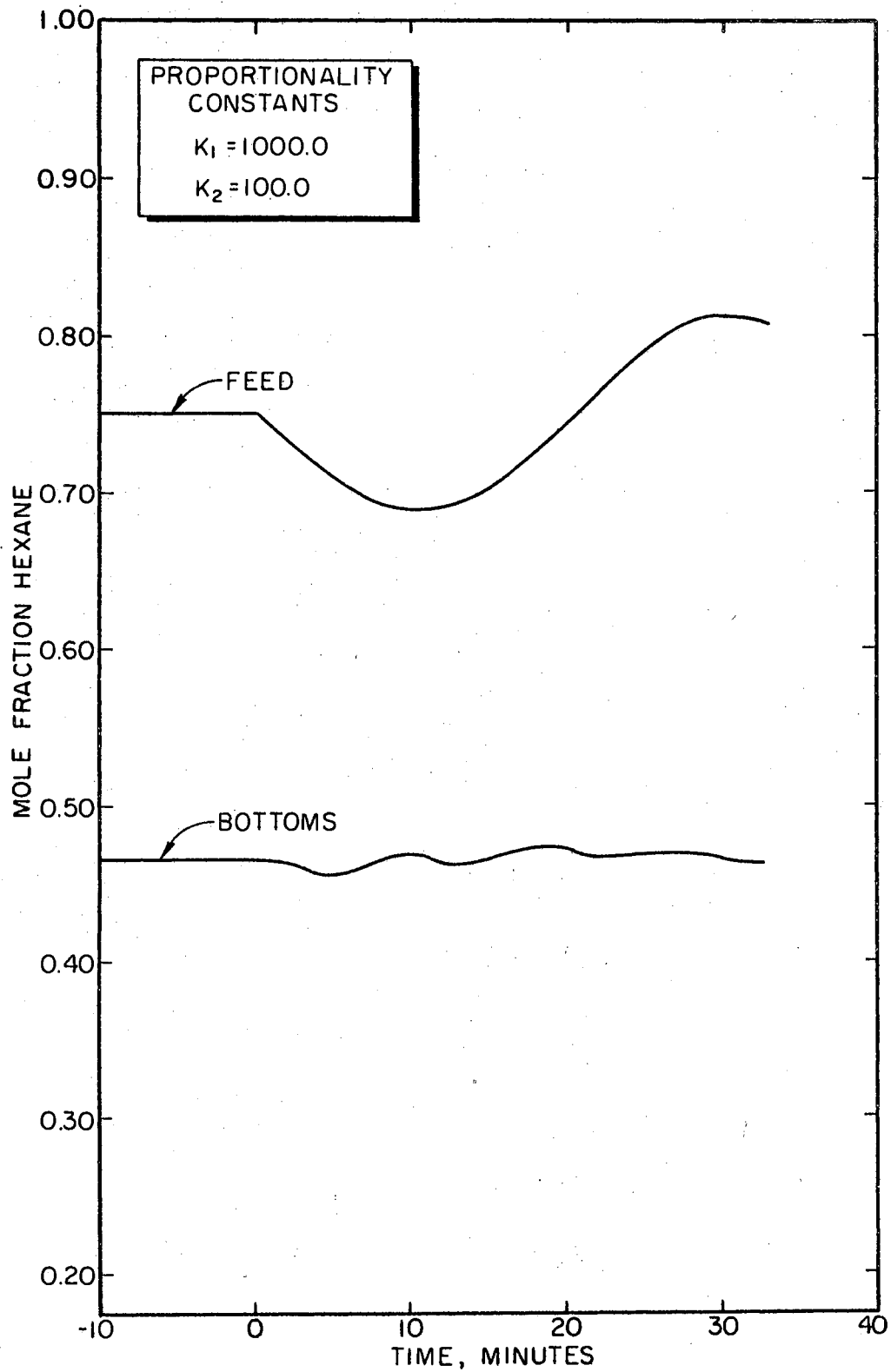Sinusoidal Disturbance

Figure 13. Proportional and Integral Control
With Low Frequency Sinusoidal
Disturbance

controller have shown that good control of the step disturbance also provides good control of the ramp and sinusoidal disturbances. The best control was achieved with large values of K_3 and moderate values of K_1 and K_2. Typical results are presented in Figure 14 and Appendix C.

Variation of the Separation Parameter

Osborn (16, 17), when developing the lumped parameter model, made an assumption that the value of the separation parameter remained constant. The efficiency of the plates in a "real" column will vary as a function of vapor and liquid loadings (9). A typical curve is presented in Figure 15 with efficiency as a function of the vapor flow rate. The Murphree equation for efficiency is

$$E = \frac{y_n - y_{n+1}}{y^* - y_{n+1}}, \tag{67}$$

where the subscript, n, is the tray number. This equation is very similar in form to Equation (39) derived for the separation parameter, $J_{n,i}$. Since there appears to be a relationship between tray efficiency and the separation parameter, the separation parameter in a "real" column could be assumed to be a function of the liquid and vapor traffics. A quadratic equation was selected as the form of the function. The maximum efficiency was assumed to occur at the initial steady-state operation of the column. The separation parameter was then decreased as a function
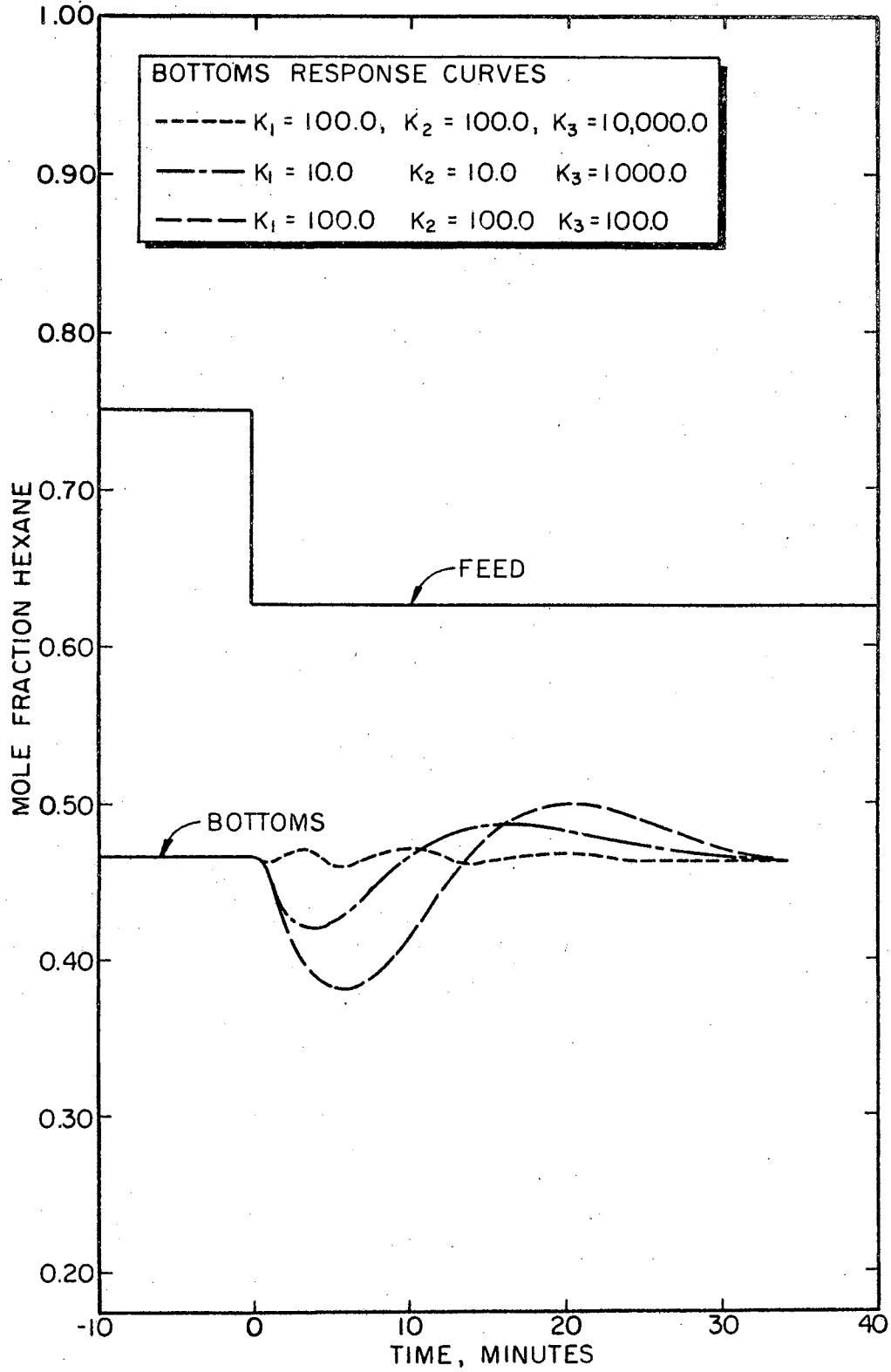
Figure 14. Proportional, Integral, and
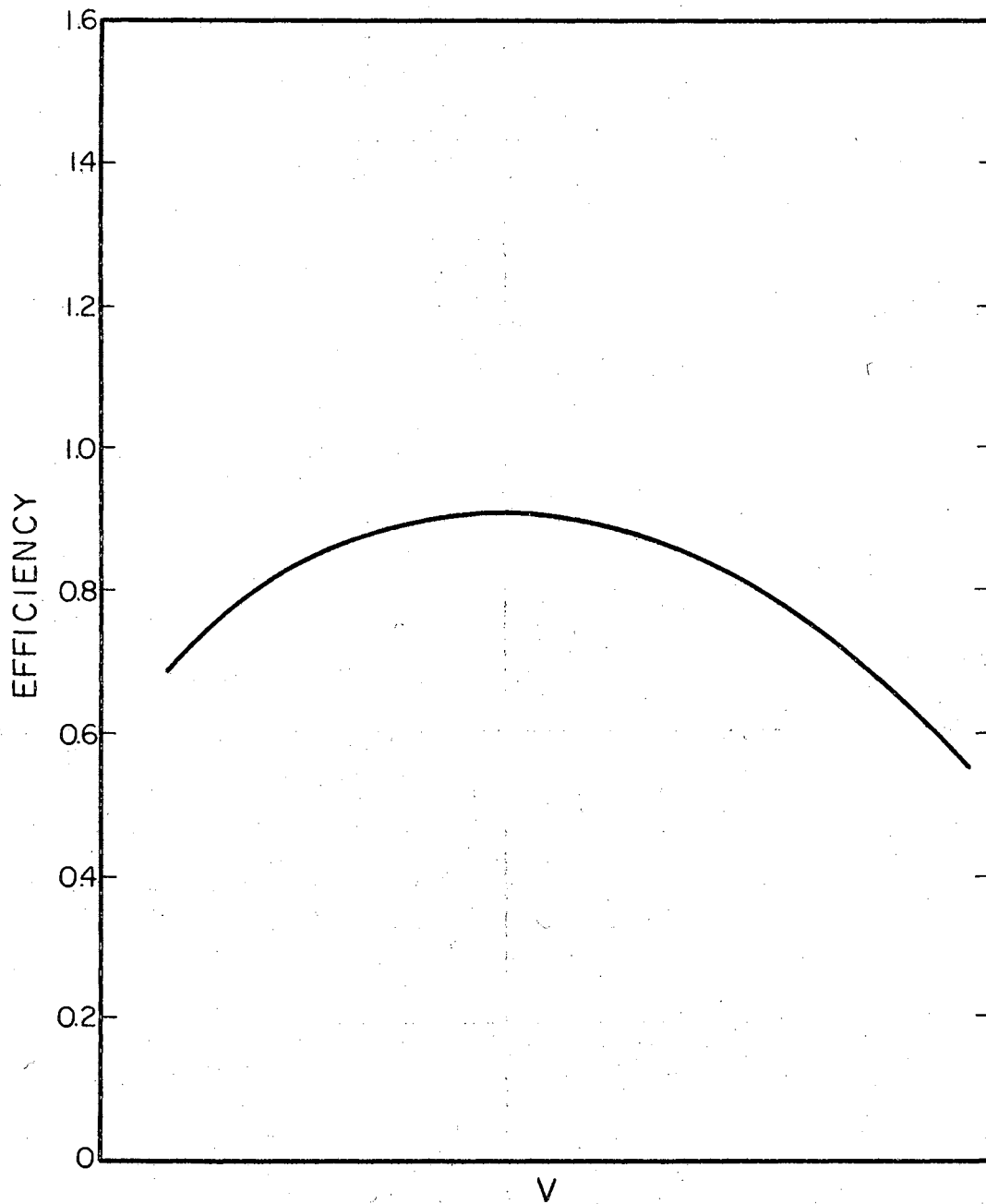   Derivative Control With
            Step Disturbance

Figure 15.   A Typical Variation of Tray Efficiency
With Vapor Flow Rate

of the vapor boilup rate after the feed upset. Typical
variations of $J_{n,i}$ with vapor flow rate are presented in
Figure 16.

Several tests were conducted with various lower limits
on the separation parameter. The only effect noted in the
system was a displacement of the liquid and vapor rates.
As the value of the separation parameter decreased, the
boilup vapor rate increased to compensate for the decrease
in the amount of separation occurring in the column. How-
ever, the limiting vapor velocity within the fractionator
was not reached. Subsequent work in this area would be to
describe what would happen if flooding occurred. The con-
trol of the bottoms composition was not seriously affected
as a result of the decrease in efficiency. The control
curve output was quite similar to those curves which were
obtained with a constant separation parameter, as shown in
Figure 17.

Tests to Resolve an Inconsistency in the
Internal Vapor Composition

Burman (3) described an inconsistency in the internal
stream traffics predicted by the mathematical model and
data measured in a twelve-inch diameter column. The model
consistently predicted a vapor composition leaving the
reboiler higher in the more volatile component than was
actually observed in the experimental column. His results
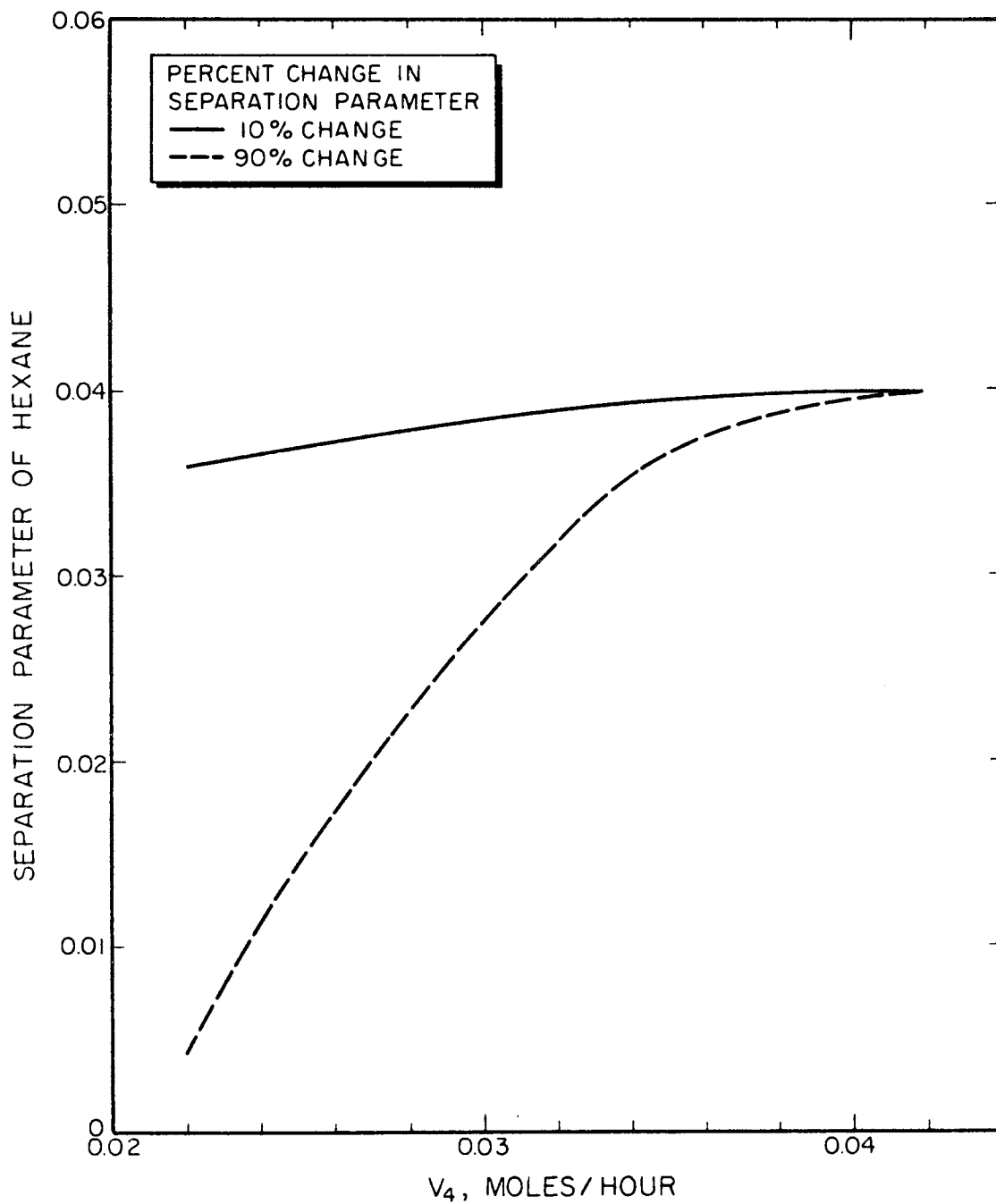are shown in Figure 18.

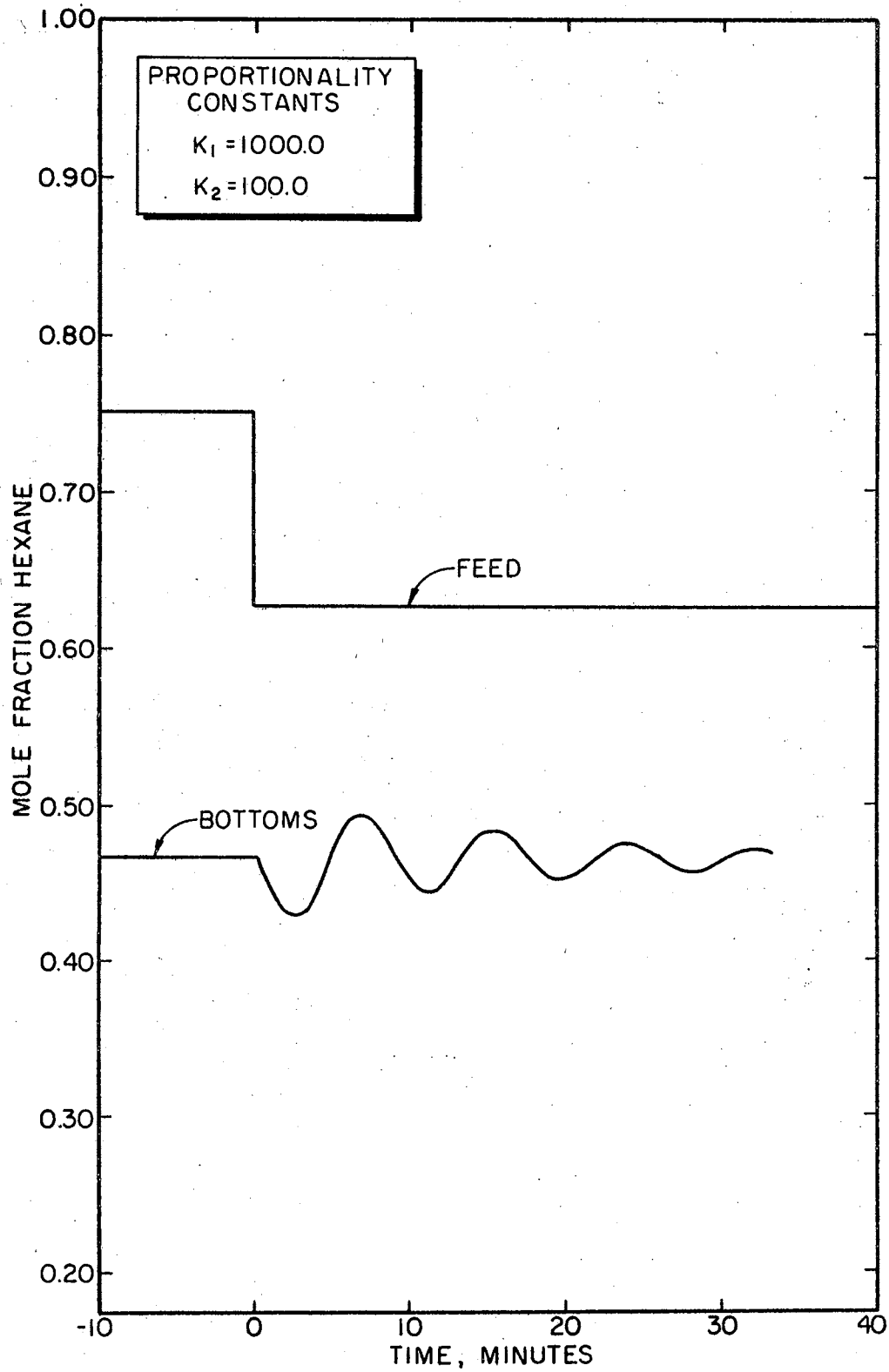Figure 16. Variation of Separation Parameter With Vapor Boilup

Figure 17. Integral and Proportional Control With 90% Change in Separation Parameter
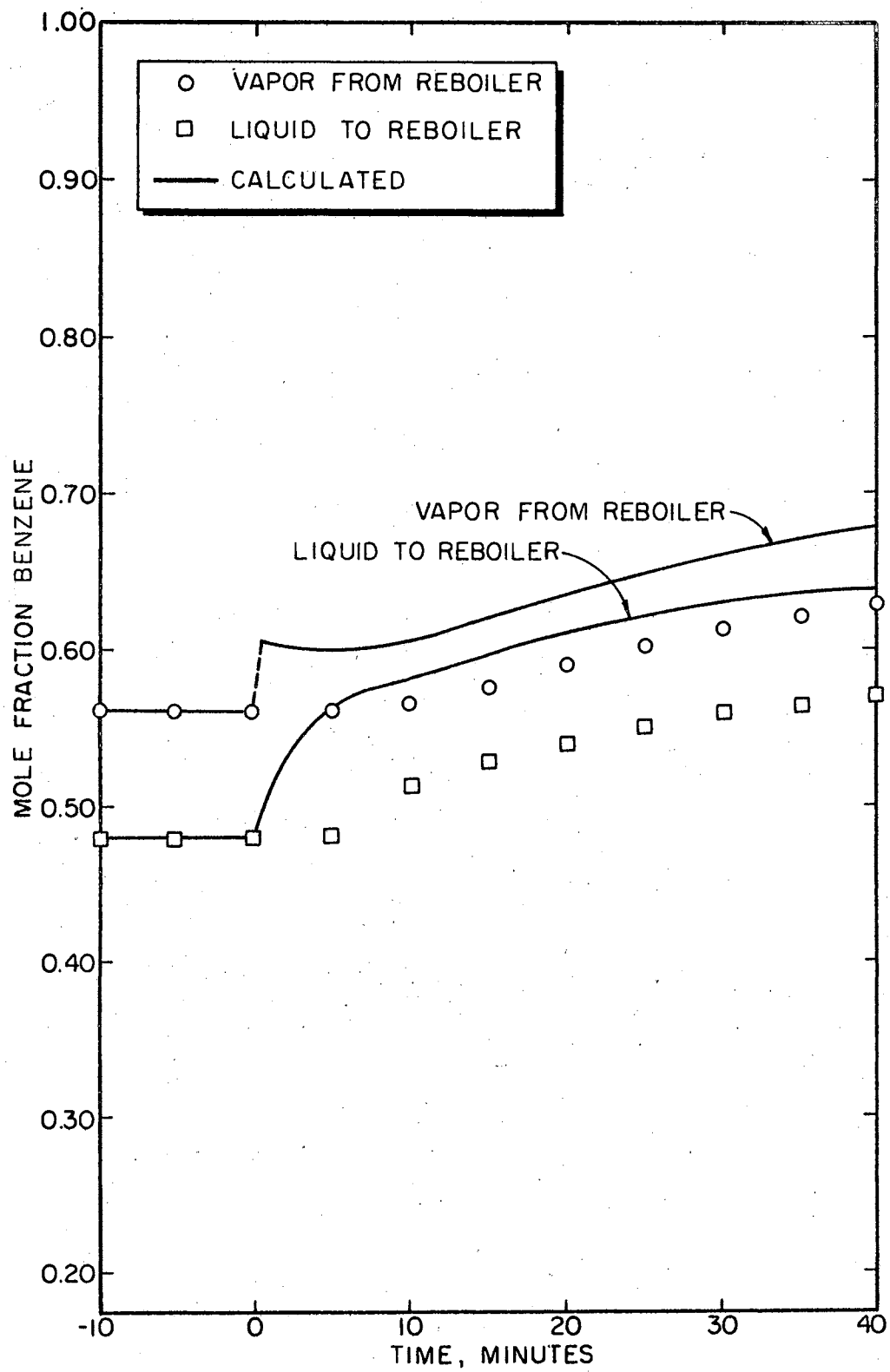
Figure 18. Experimental and Calculated Data
of Burman

Osborne's model, which Burman used in his research,
assumes that the reboiler is an equilibrium stage. By
taking the experimental compositions of the vapor leaving
the reboiler and the bottoms product, one can calculate a
set of "K values". Differences were found when comparing
these "K values" with thermodynamic K values. The sets of
K values are plotted in Figure 19. The difference had a
maximum value near the time of the feed disturbance and
decreased as the column approached the final steady-state
condition. This leads to the conclusion that the reboiler
is not operating as a theoretical stage during transient
operation.

An efficiency term which forced the thermodynamic K
values to equal those calculated from experimental data
was placed in the reboiler calculations in the model. The
bottom composition of the model was controlled to follow
the compositions measured on the experimental column. A
combination of proportional, integral, and derivative
modes was used to control the reboiler heat input. The
agreement with the experimental data improved; however,
the difference was not completely resolved.

When an upset occurs in the feed section of a column,
a time lag occurs before the effect is noted in the bottoms
product stream. By adjusting the reboiler holdup term or
the column holdup term in the column model, the time lag
in the experimental column can be more closely approximated.
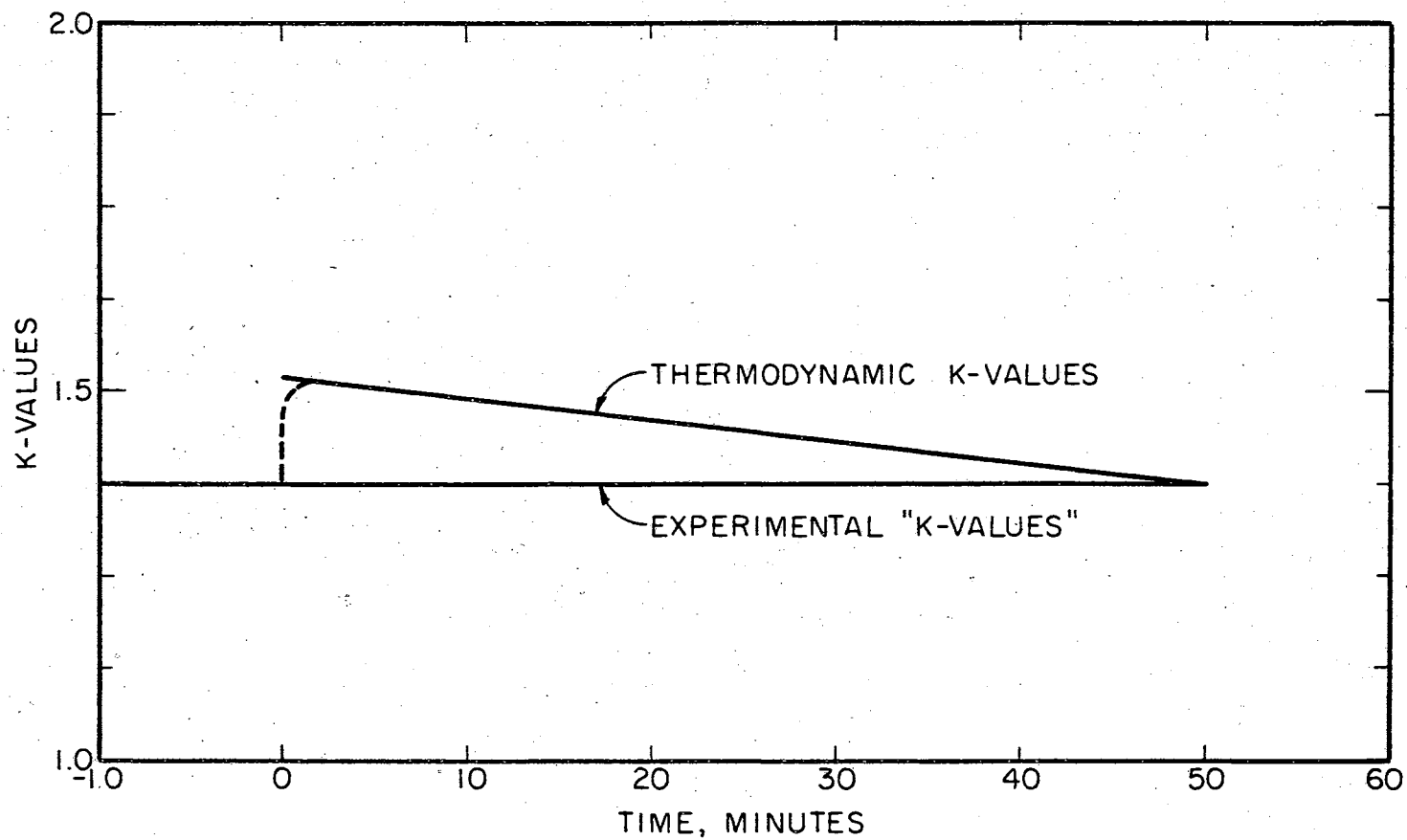Doubling the reboiler holdup in the calculations gave the

Figure 19. Comparison Between Experimental and Thermodynamic K Values

best results. The vapor boilup concentration was more closely approximated. However, there was still a discrepancy between the calculated and experimental values of the concentration of the liquid stream to the reboiler.

A third possible factor which could explain the inconsistency is the external and internal material balances. Burman assumed that there was constant molal overflow during the initial and final steady-state. However, Osborne's model does not make this assumption. Therefore, the initial steady-state values of the internal stream flow rates were varied to observe the effect upon the internal composition. The inconsistency in the liquid stream composition was decreased; however, the value of the separation parameter, which is calculated from the initial steady-state data, became negative when the best results were achieved. A negative separation parameter of the more volatile component violates the principles of distillation, because there must be some movement of the more volatile component up the column. However, it is felt that accurate measurement of the steady-state internal flow rates will give better correlation than was exhibited by assuming constant molal overflow.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The objectives of this research project were three-
fold. The first objective was to apply feedback control
to the lumped parameter model. The second objective was
to vary the column efficiency to observe the results upon
the model and the control of the model. The third objec-
tive was to attempt to resolve inconsistencies in the
internal vapor composition from the reboiler.

Step, ramp, and sinusoidal disturbances were induced
in the feed rate and composition. The bottoms composition
was controlled by adjusting the reboiler heat duty.
Combinations of the proportional, integral, and derivative
modes were used to achieve the desired control.

The results indicate that the lumped parameter model
can be readily controlled within a feedback control loop.
The control curves appear to be consistent with normal
control modes and exhibit characteristics of the control
of a first order differential equation. The derivative
controller appears to provide the bulk of the control.
However, the combination of the three modes should be used
to allow for the possibility of a steady-state error.

A decrease in the column efficiency resulted in an increase of the vapor boilup rate. The control function of the model was not seriously affected by the variable separation parameter.

The inconsistency in the internal vapor composition of the model and Burman's data can be partially resolved by placing an efficiency factor on the reboiler, by considering a time lag between the feed upset and the corresponding change in the bottoms composition and by measuring accurately the steady-state internal flow rates.

Recommendations

The dynamic model and control scheme should be extended to describe the operation and control of a complete column.

A combination of feedforward and feedback controllers should be developed. This control scheme would provide much faster and more reliable control of the column products.

A separation parameter should be determined for the reboiler. This would probably eliminate the error in the vapor boilup composition.

To obtain an actual working control system, a small computer should be tied into an experimental distillation column. Putting the theory into practice would more thoroughly test the practicality of the control model.

The effect of a variable tray efficiency upon the control function should also be tested experimentally.

NOMENCLATURE

Major Symbols

English Letters

B	=	bottoms product flow rate, moles/hour.
D	=	distillate flow rate, moles/hour.
D_s	=	disturbance function.
E	=	efficiency.
E_r	=	error in the controlled variable.
F	=	feed flow rate, moles/hour.
F_e	=	feed enthalpy, BTU/lb.-mole.
F_t	=	feed tray location.
h	=	enthalpy of liquid, BTU/lb.-mole.
H	=	enthalpy of vapor, BTU/lb.-mole.
J	=	parameter which describes the degree of separation occurring in a column section, moles/hour-mole fraction.
K	=	vapor liquid equilibrium coefficient.
K_1	=	constant for proportional control.
K_2	=	constant for integral control.
K_3	=	constant for derivative control.
L	=	liquid flow rate, moles/hour.
m	=	manipulated variable which is adjusted to correct error in controlled variable.

m_s = slope of equilibrium curve.

$m(t)$ = deviation of reflux flow rate about the steady-state.

N = net rate of mass transfer between phases, moles/hour.

Q_r = reboiler heat duty, BTU/hour.

R = reflux flow rate, moles/hour.

t = time, hour.

t' = time, minutes.

V = vapor flow rate, moles/hour.

x = liquid composition, mole fraction.

y = vapor composition, mole fraction.

z = height of the column section, ft.

Greek Letters

δ = holdup in a section or on a tray, moles.

τ = time constant, hour.

ω = frequency, cycles/hour.

Subscripts

b = bottoms.

bd = desired bottoms composition.

bm = measured bottoms composition.

d = distillate.

f = feed.

k = subsection number.

I = internal.

i   = component number.

n   = section number.

o   = initial condition.

r   = rectifying section.

s   = stripping section.

Superscripts

*   = equilibrium value.

Groups

$\dfrac{d}{dt}$ = total derivative with respect to time.

$\dfrac{\partial}{\partial z}$ = partial derivative with respect to distance.

$\dfrac{\partial}{\partial t}$ = partial derivative with respect to time.

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APPENDIX A

FUNCTIONS OF THE FEED DISTURBANCE

FUNCTIONS OF THE FEED DISTURBANCE

Three different types of feed disturbances were used in the study. The disturbances were represented by step, ramp, and sinusoidal functions.

A step change in the feed flow rate and composition can be represented by the equations

$$\Delta F = constant \tag{68}$$

and

$$\Delta x_f = constant. \tag{69}$$

The values of the final steady-state feed flow rate and composition were substituted for the initial values at the time of the upset.

The equations for the ramp functions are given by

$$F = F_0 + a_1 t \tag{70}$$

$$x_f = x_{f_0} + a_2 t, \tag{71}$$

where a_1 and a_2 are arbitrarily determined constants which determine the slopes of the respective ramps. The values of the constants were varied to change the slope or severity of the disturbance. The values of a_1 were 1.05×10^{-3}, 2.1×10^{-3}, 4.2×10^{-3}, 8.4×10^{-3}, and

16.8 x 10^{-3} moles/(hour)^2. The values of a_2 were 0.15,
0.31, 0.62, 1.24, and 2.48 hour^{-1}.

The sinusoidal functions are represented by the
equations

$$F = A_1 \sin\omega t \qquad (72)$$

$$x_f = A_2 \sin\omega t. \qquad (73)$$

The amplitudes of the sine functions were set at one-half
the value of the respective step changes. The frequency
was arbitrarily varied to obtain a range of good operation
or control. The frequency, ω, had values of 1.5, 3, 6,
12, and 24 cycles/hour.

APPENDIX B

INCORPORATION OF CONTROL AND HEAT BALANCE

EQUATIONS INTO OSBORNE'S

COMPUTER PROGRAM

INCORPORATION OF CONTROL AND HEAT BALANCE

EQUATIONS INTO OSBORNE'S

COMPUTER PROGRAM

The calculation of the transient column conditions
was made by a computer program written by Osborne (16, 17).
However, some changes were made in the original program to
incorporate a feedback control scheme.

The first major change involved the calculations of
the flow rates after the upset. Osborne's program assumed
that the flow rates changed to their final steady-state
values at the time of the upset. These steady-state values
were read into the computer on data cards. In the revised
program, the flow rates were calculated through the use of
heat and material balances.

A second change was the addition of a subroutine to
the program. This subroutine contained the heat and mate-
rial balance equations and the controller equations. The
error in bottoms composition is given by

$$E_r = x_{bd} - x_{bm}. \tag{59}$$

The controller corrects the error by the equation

$$\Delta Q_r = K_1 E_r + K_2 \int_0^{t'} E_r \, dt' + K_3 \frac{dE_r}{dt'}. \tag{66}$$

The trapezoidal rule was the method of integration. The differentiation was made by assuming for small intervals that

$$\frac{dE_r}{dt} = \frac{E_r|_{t=t}}{t}. \tag{74}$$

The new value of Q_r is then used in the heat and material balance equations to calculate the new flow rates. The disturbance functions were also included in the subroutine. The subroutine was called by the main program at the desired time intervals. For this study, the time interval was set at one-half minute.

APPENDIX C

SIMULATED COLUMN RESPONSE CURVES
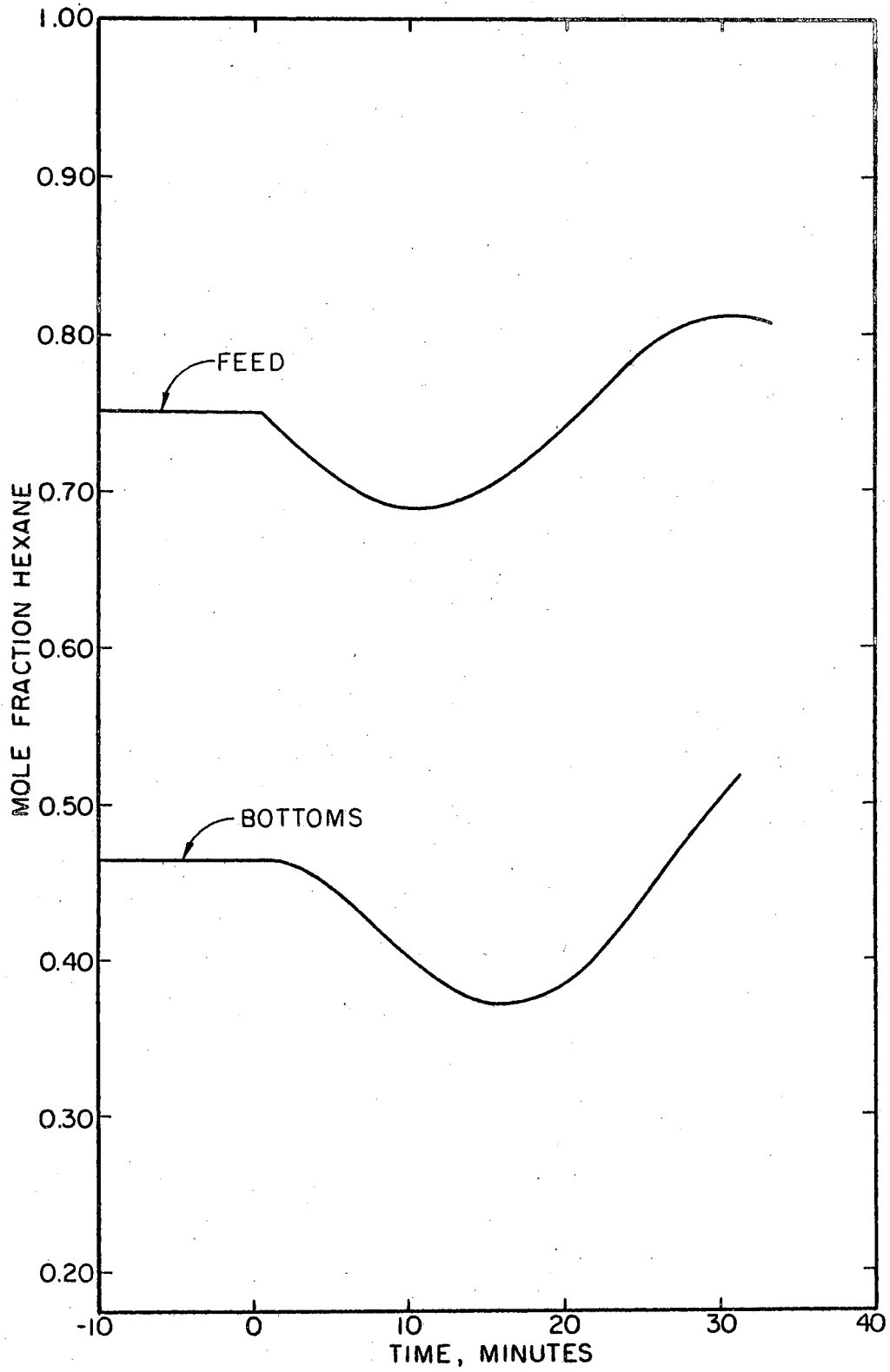
Figure 20. Low Frequency Sinusoidal Disturbance
With No Control
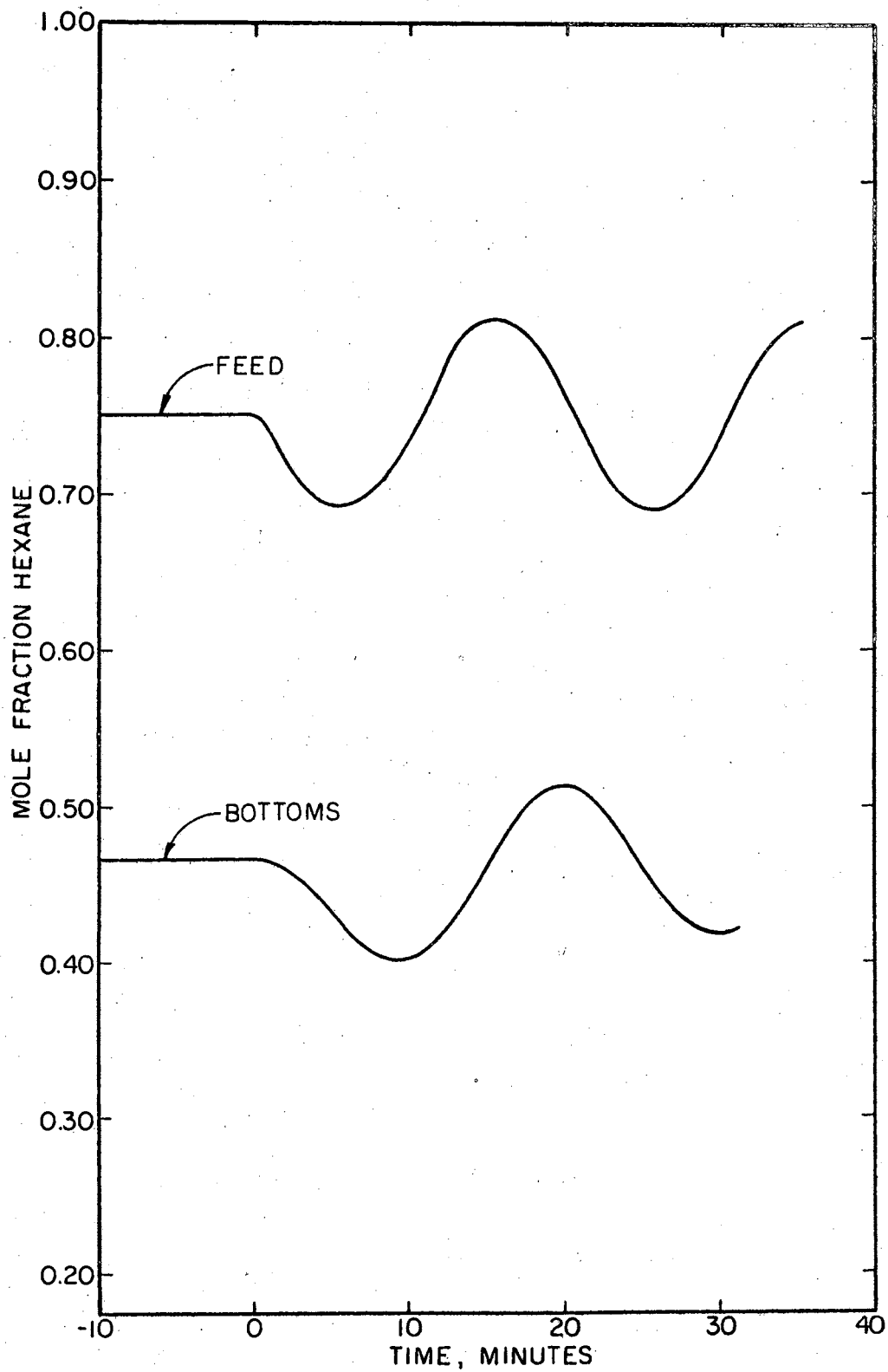
Figure 21. Moderate Frequency Sinusoidal Disturbance
With No Control
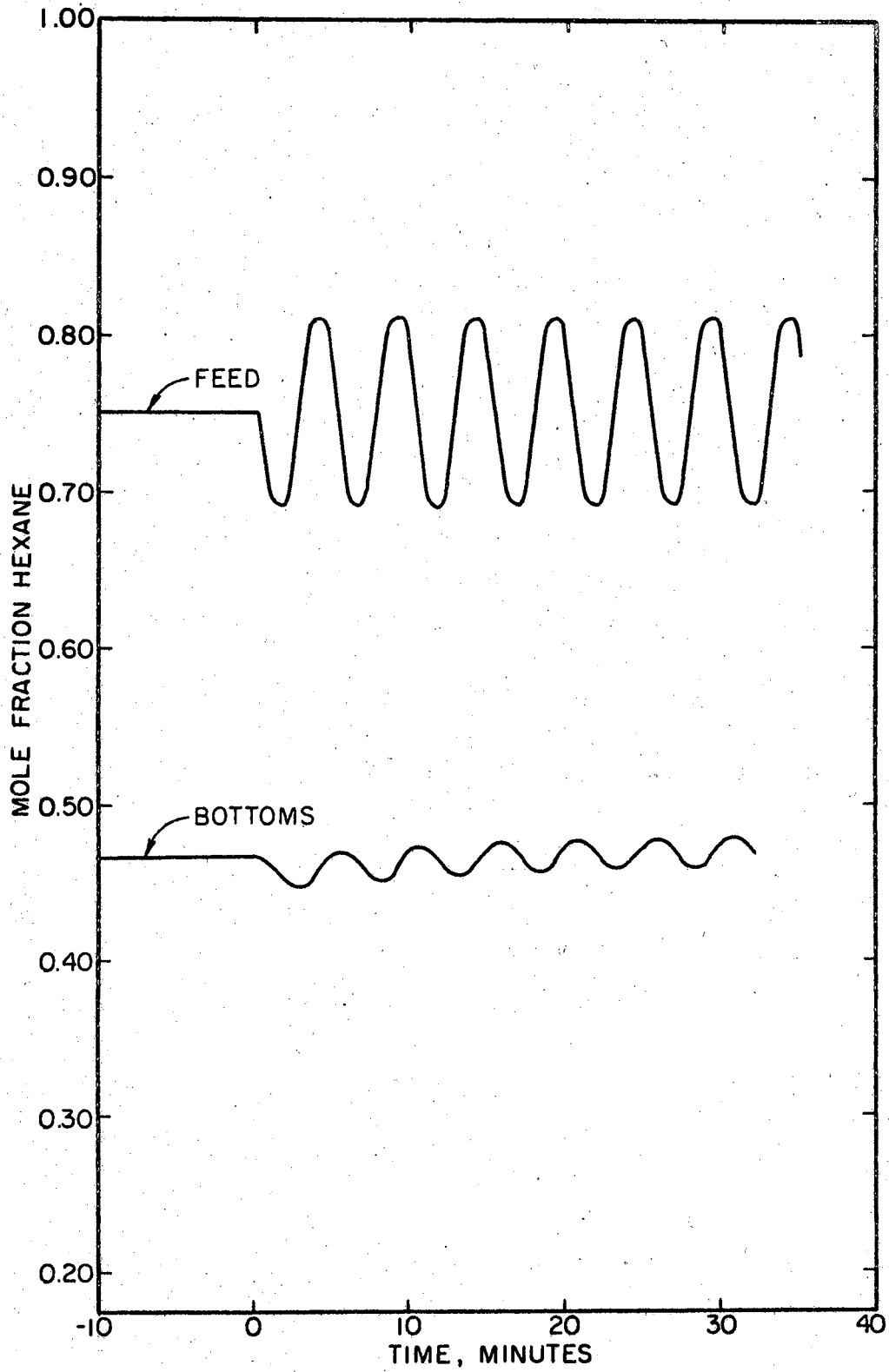
Figure 22. High Frequency Sinusoidal Disturbance
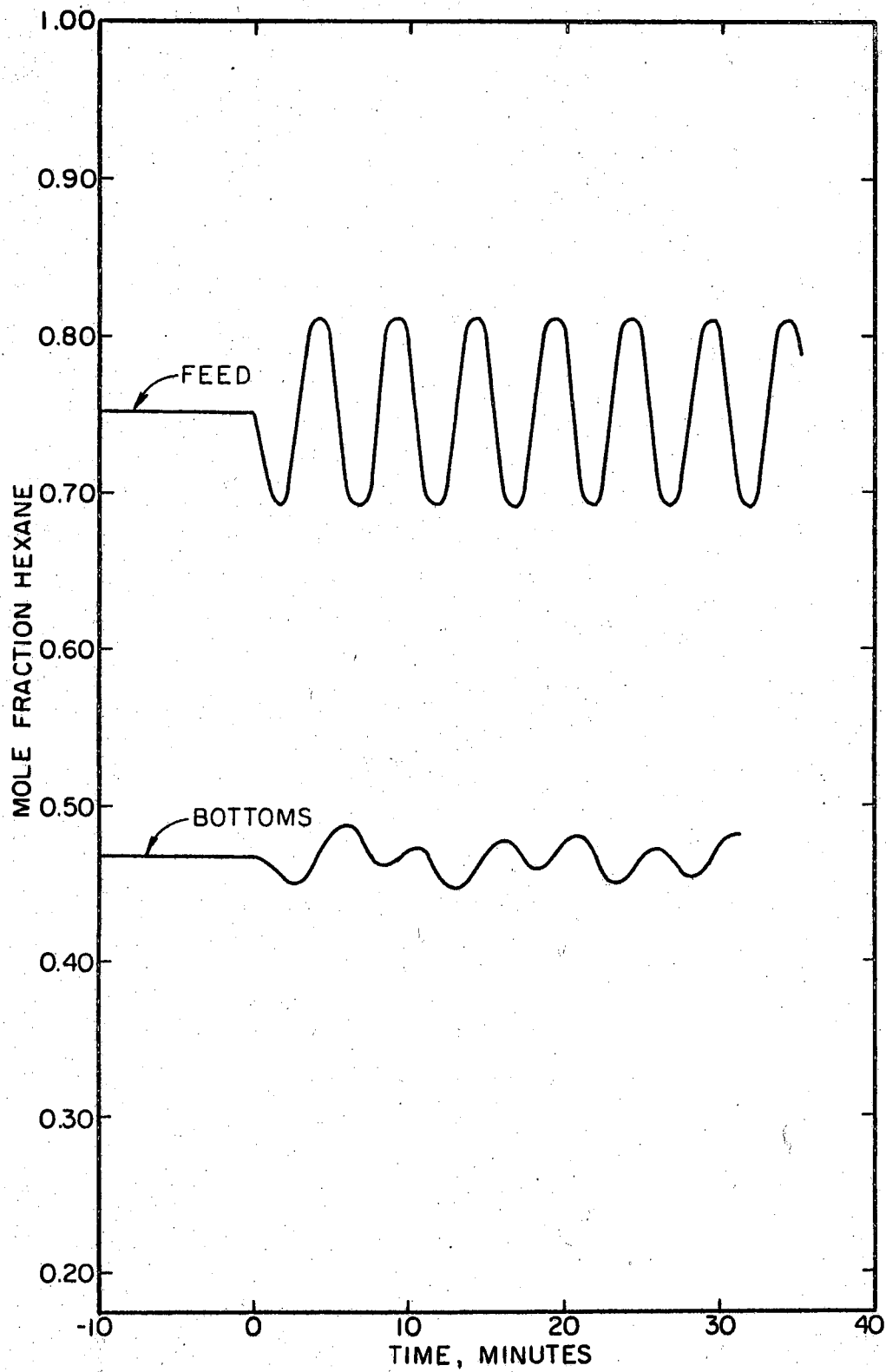With No Control
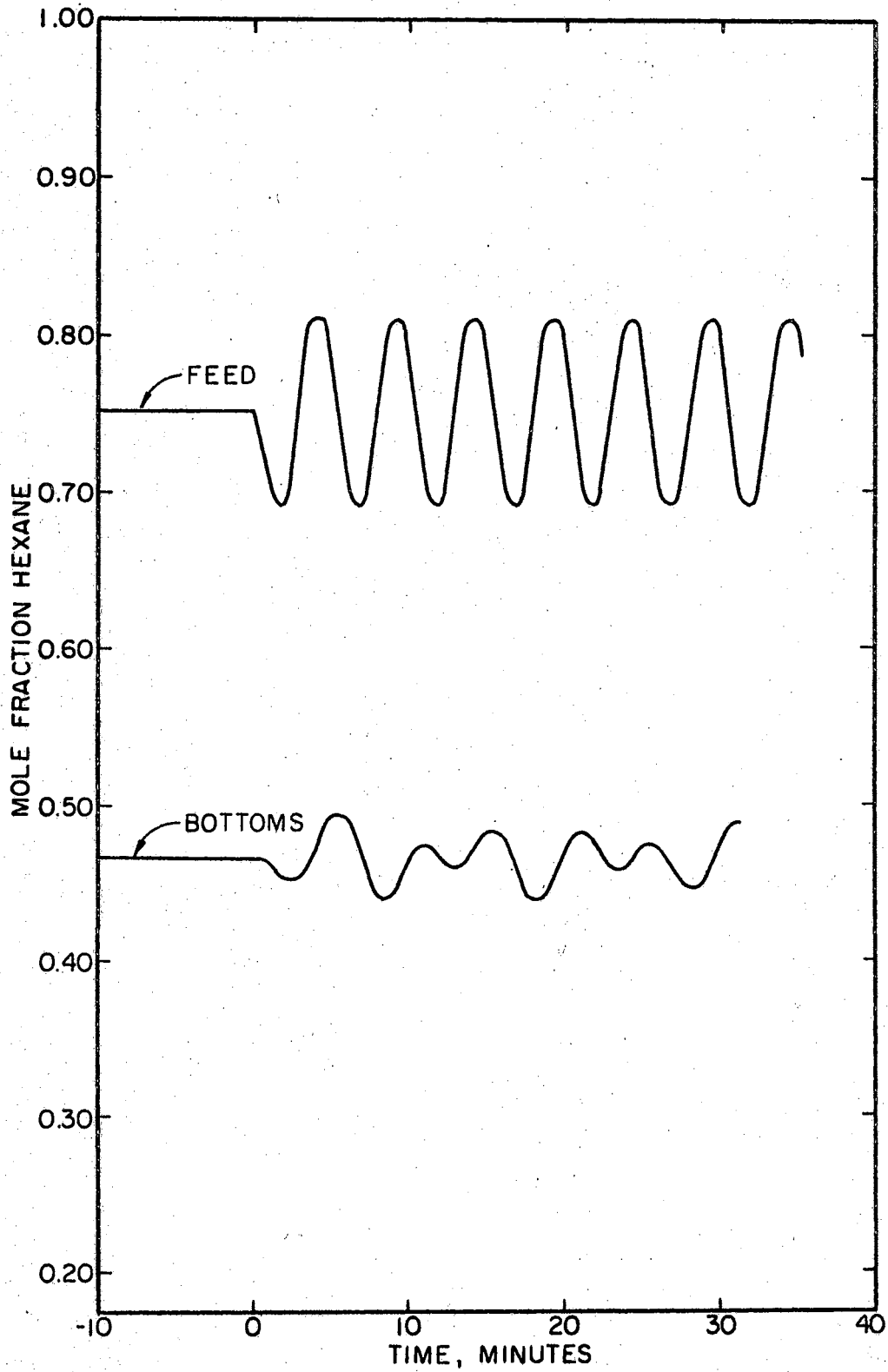
Figure 23. Integral and Proportional Control
With K_1 = 1000.0 and K_2 = 1000.0
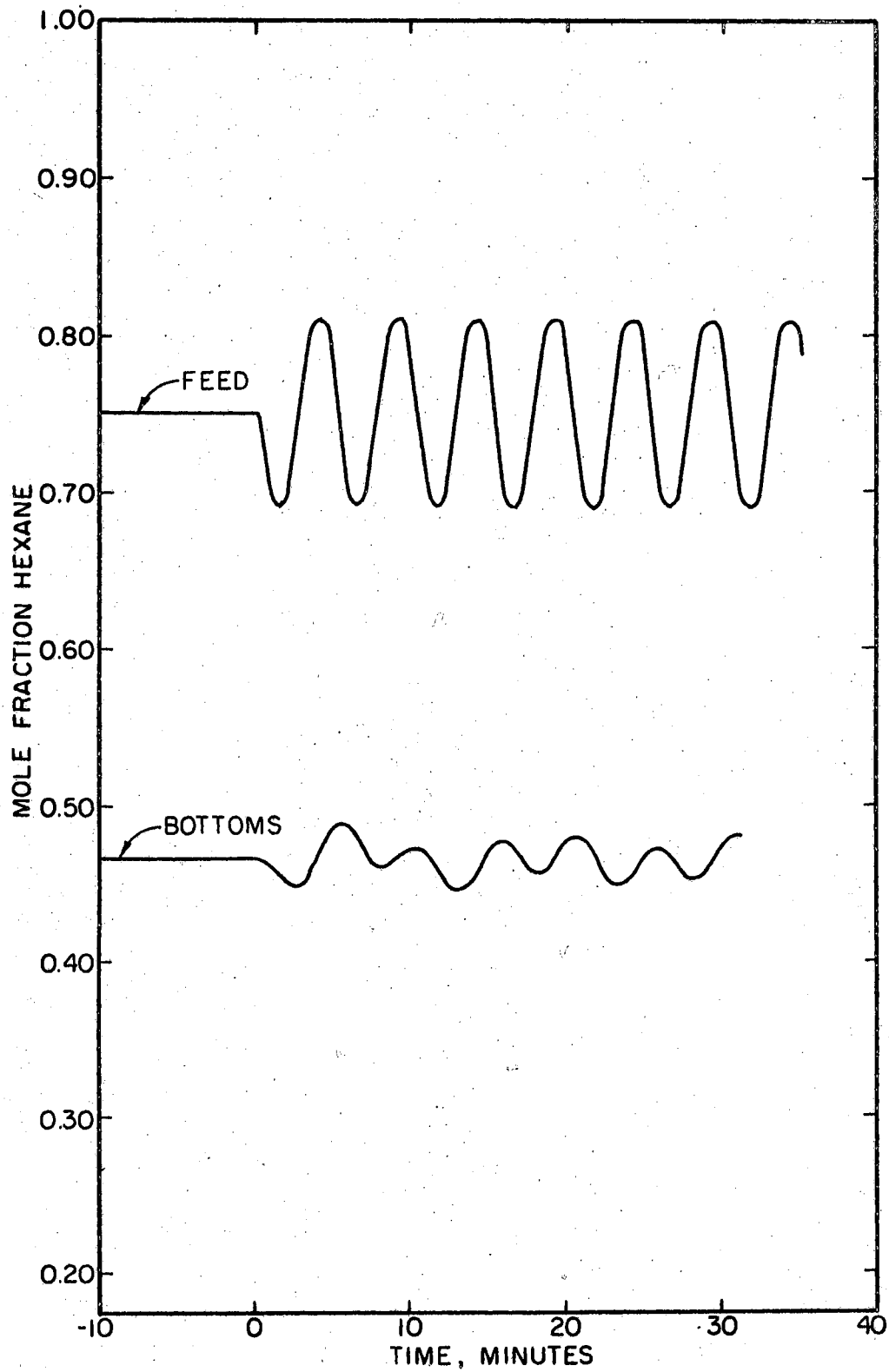
Figure 24.   Integral and Proportional Control With
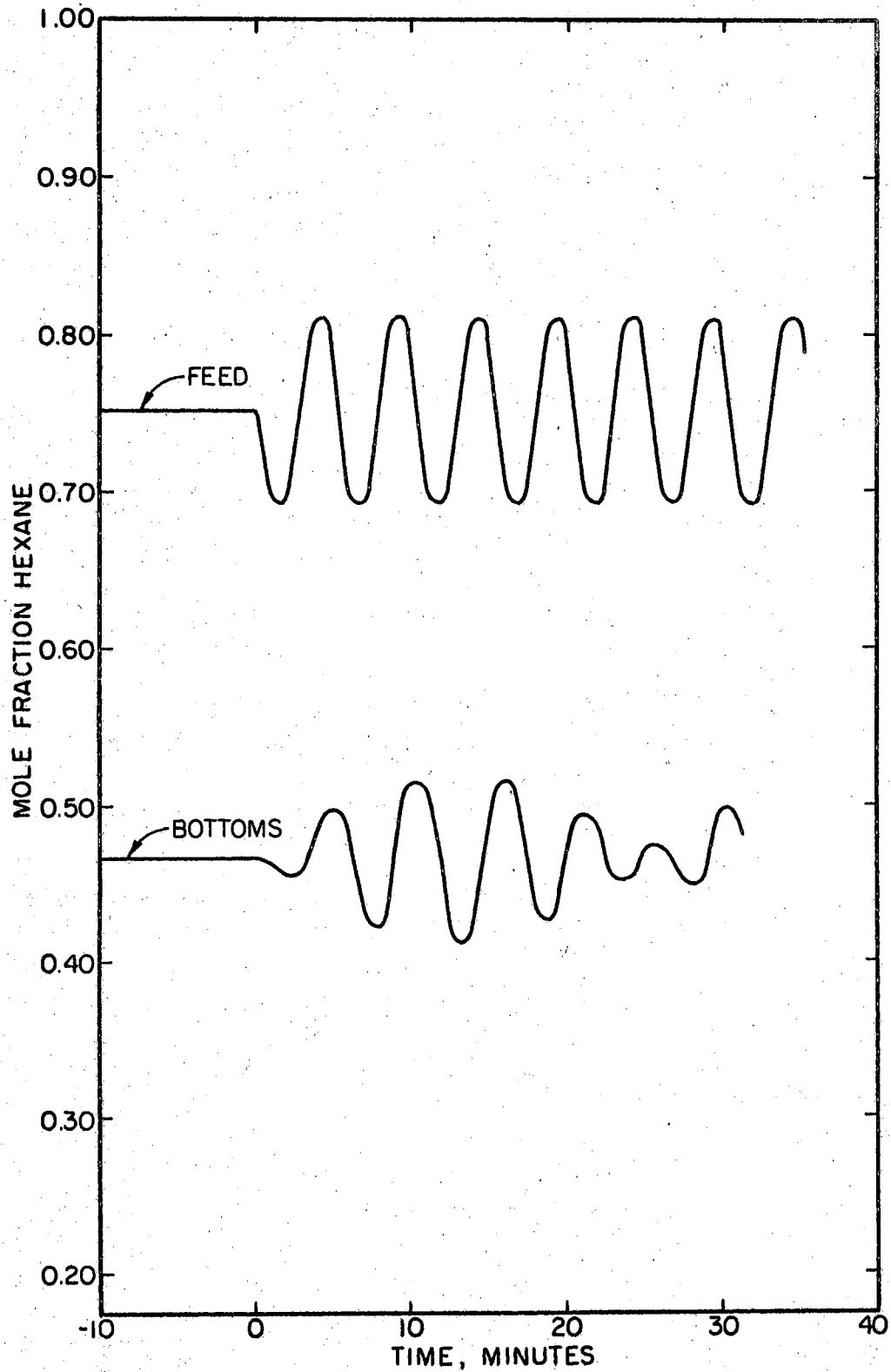$K_1 = 500.0$ and $K_2 = 50.0$
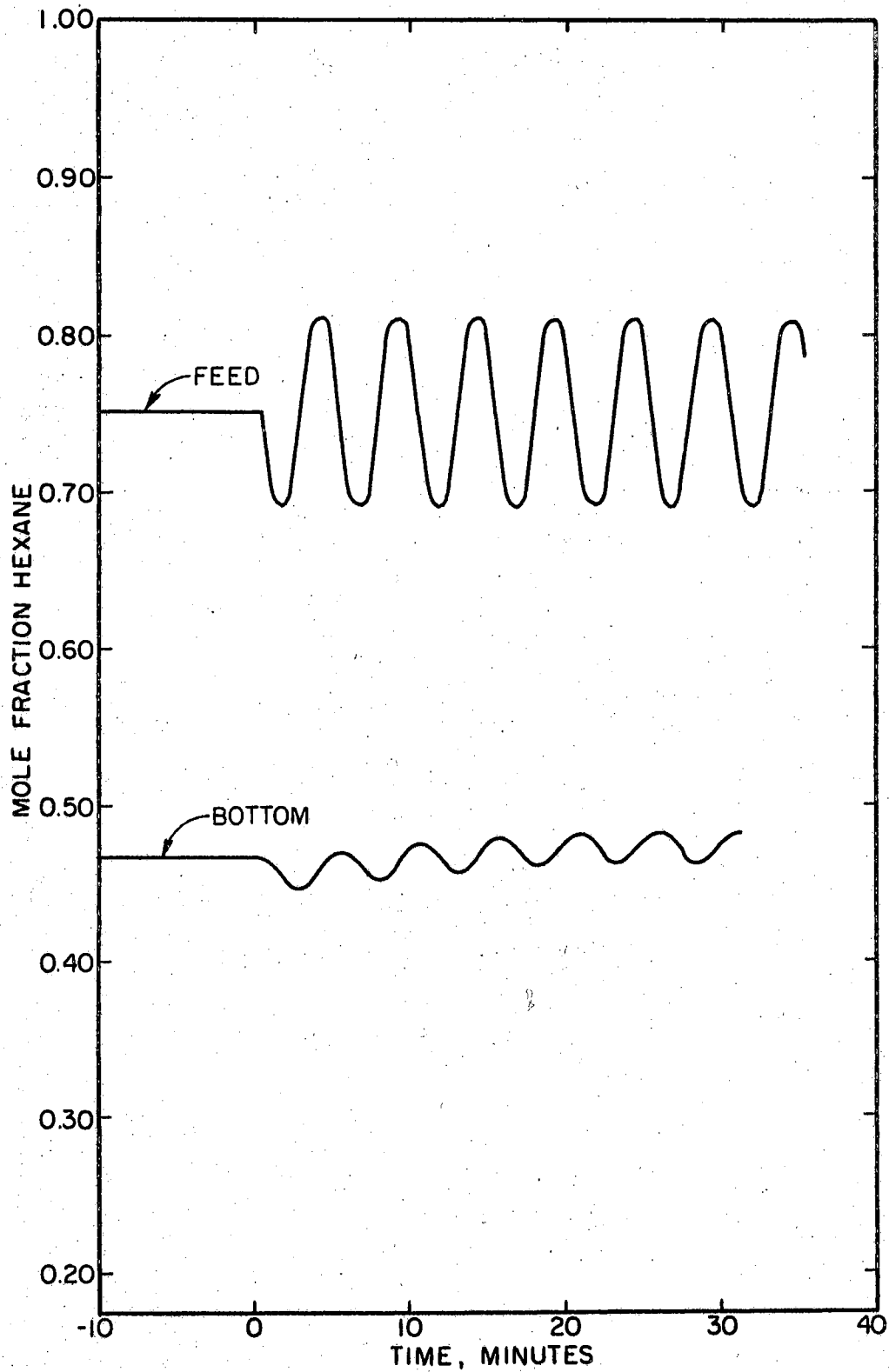
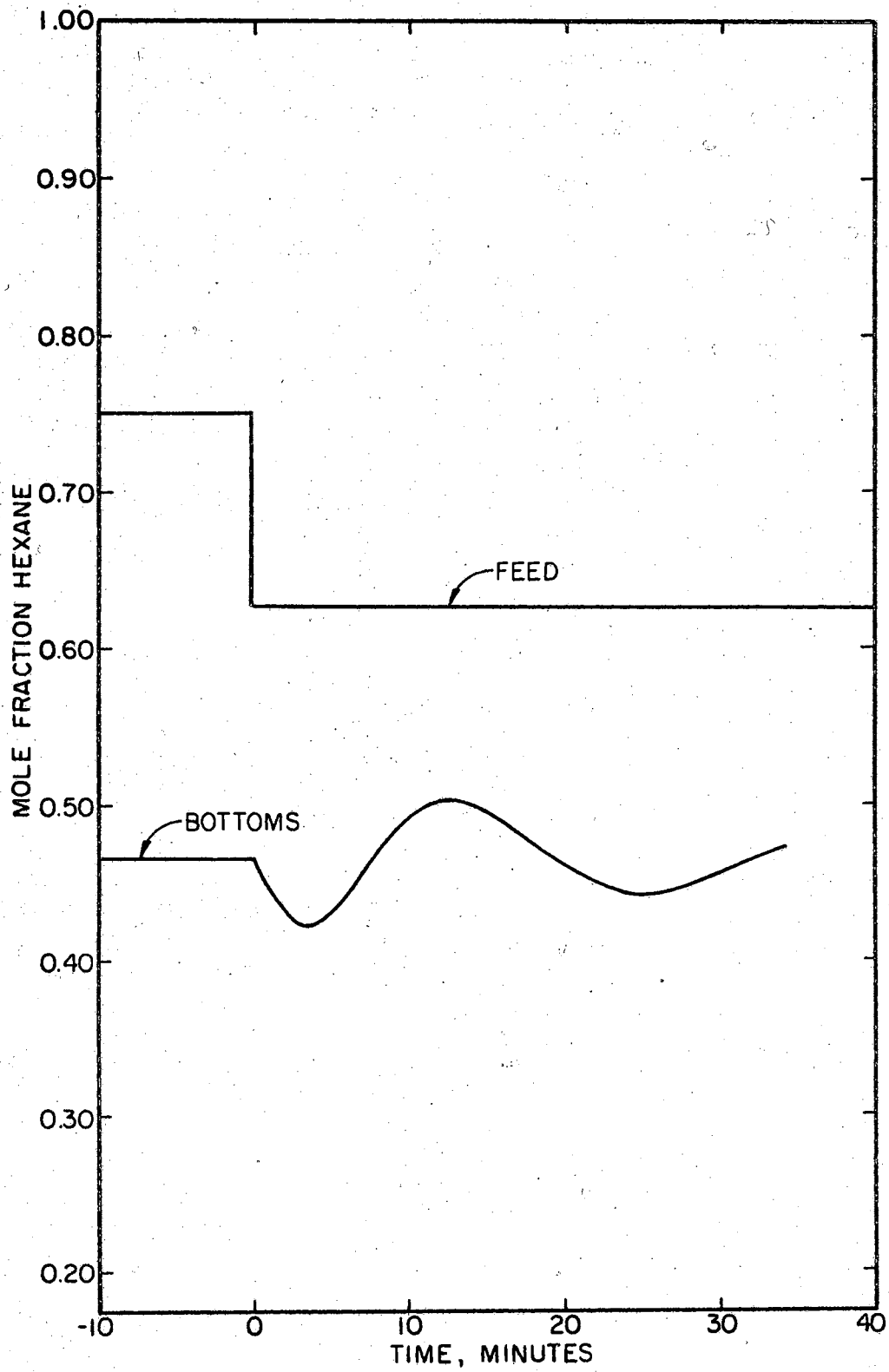Figure 25. Integral and Proportional Control With
$K_1 = 10.0$ and $K_2 = 100.0$
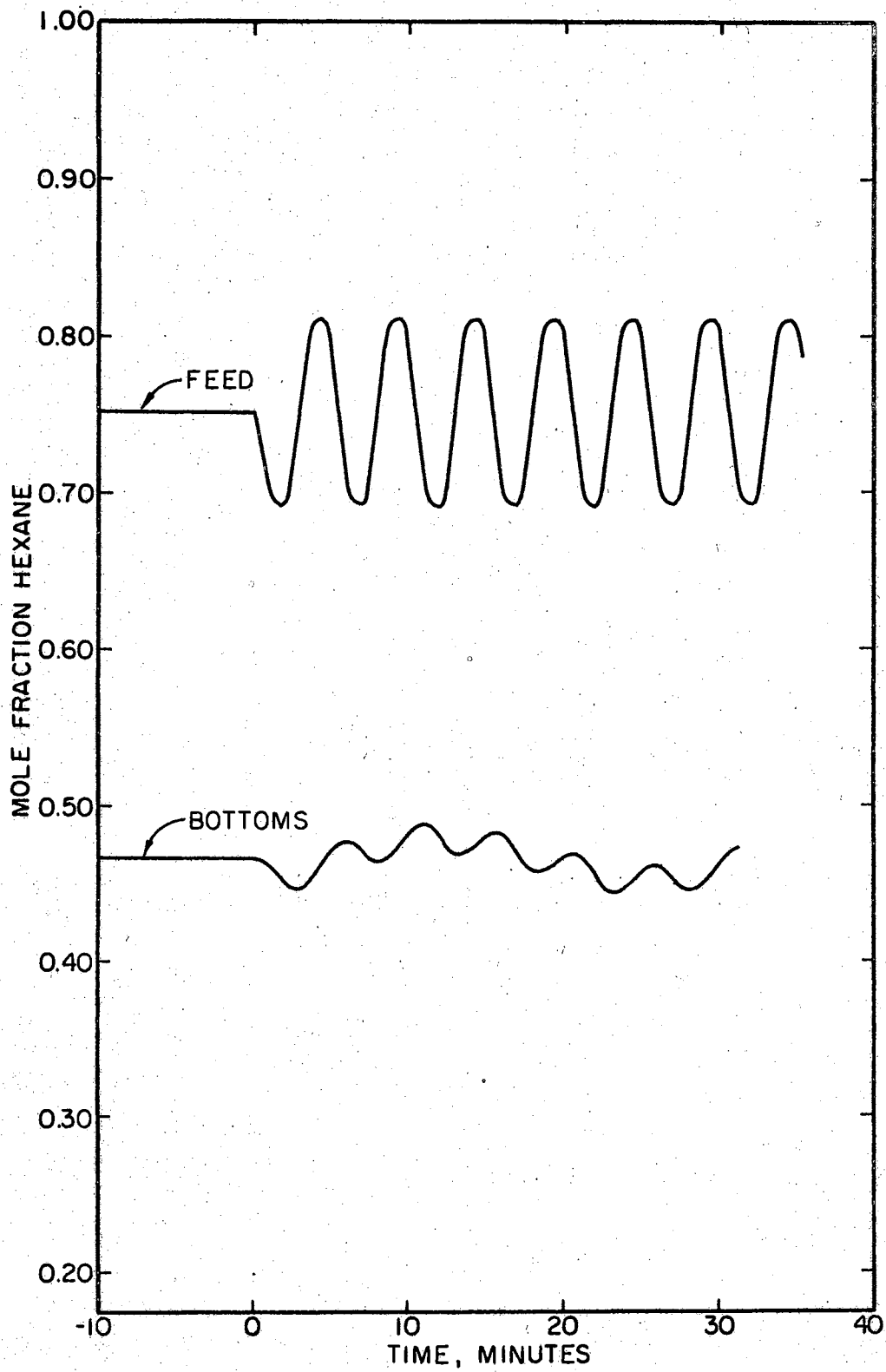
Figure 26. Integral and Proportional Control With
$K_1 = 100.0$ and $K_2 = 1000.0$

Figure 27.   Integral and Proportional Control With
K_1 = 500.0 and K_2 = 100.0

Figure 28. Integral and Proportional Control With $K_1 = 2000.0$ and $K_2 = 100.0$

Figure 29. Step Disturbance With K_1 = 100.0,
K_2 = 1000.0, and K_3 = 1000.0
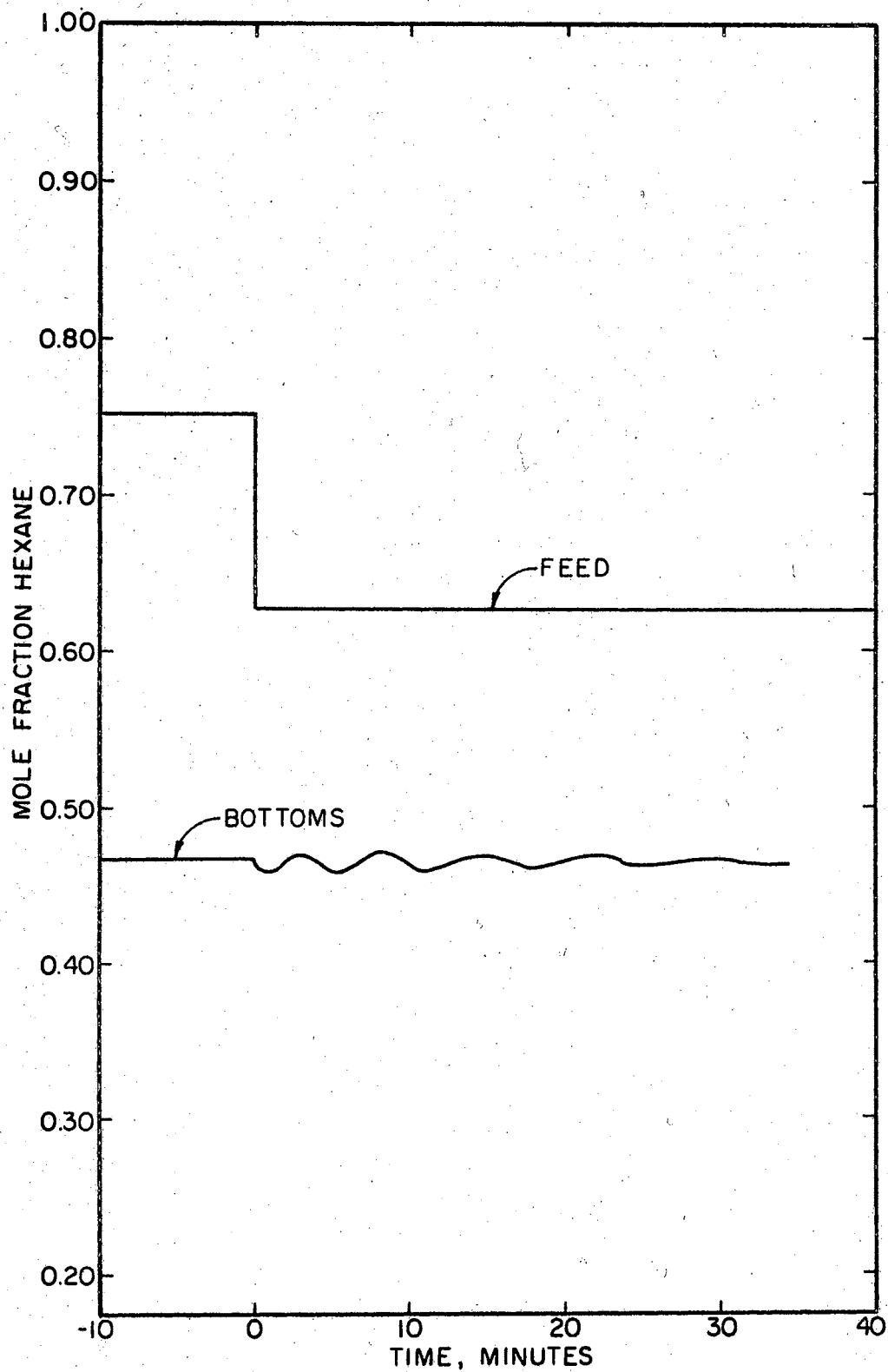
Figure 30. Step Disturbance With K_1 = 1000.0,
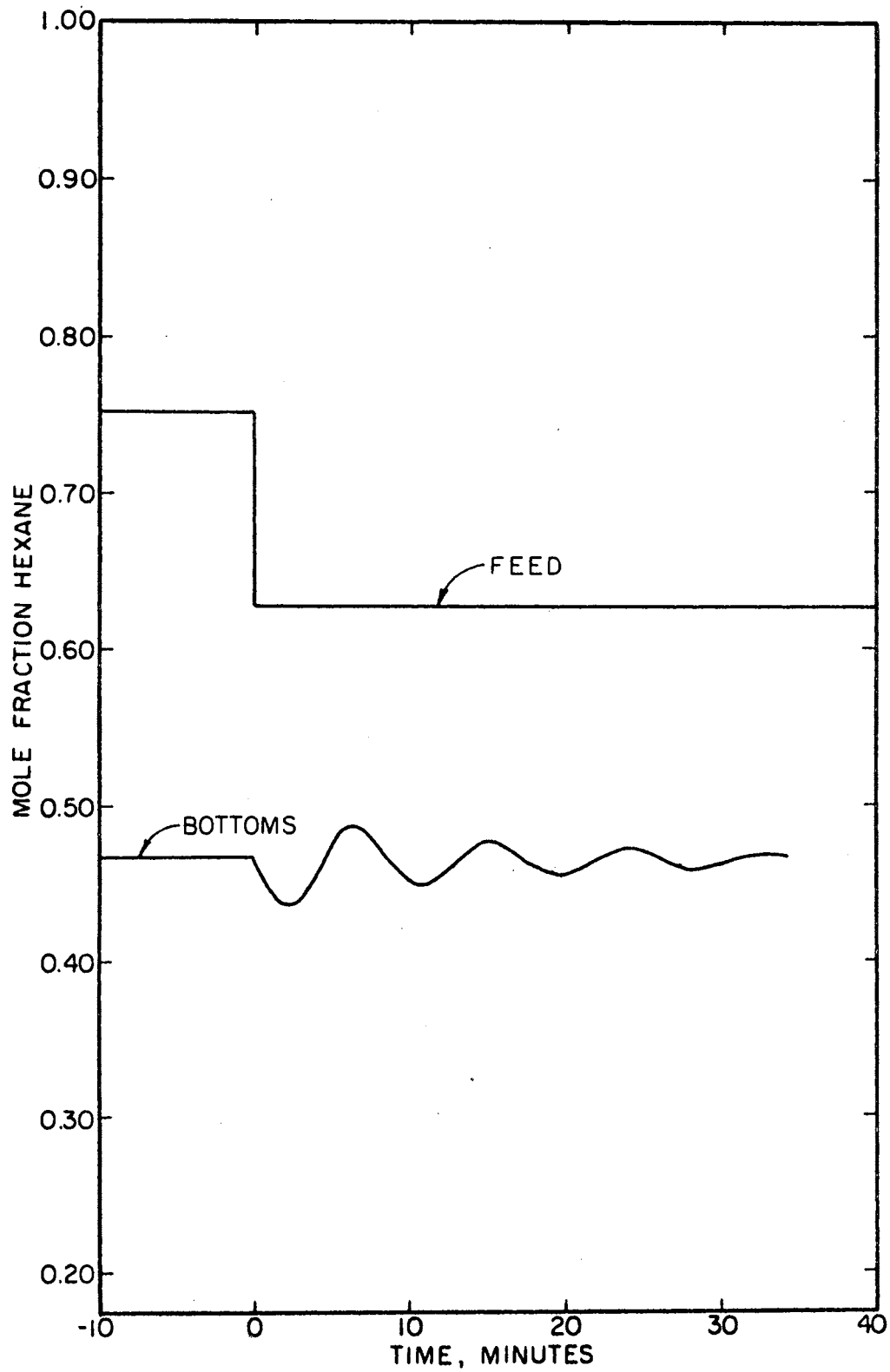K_2 = 100.0, and K_3 = 10000.0

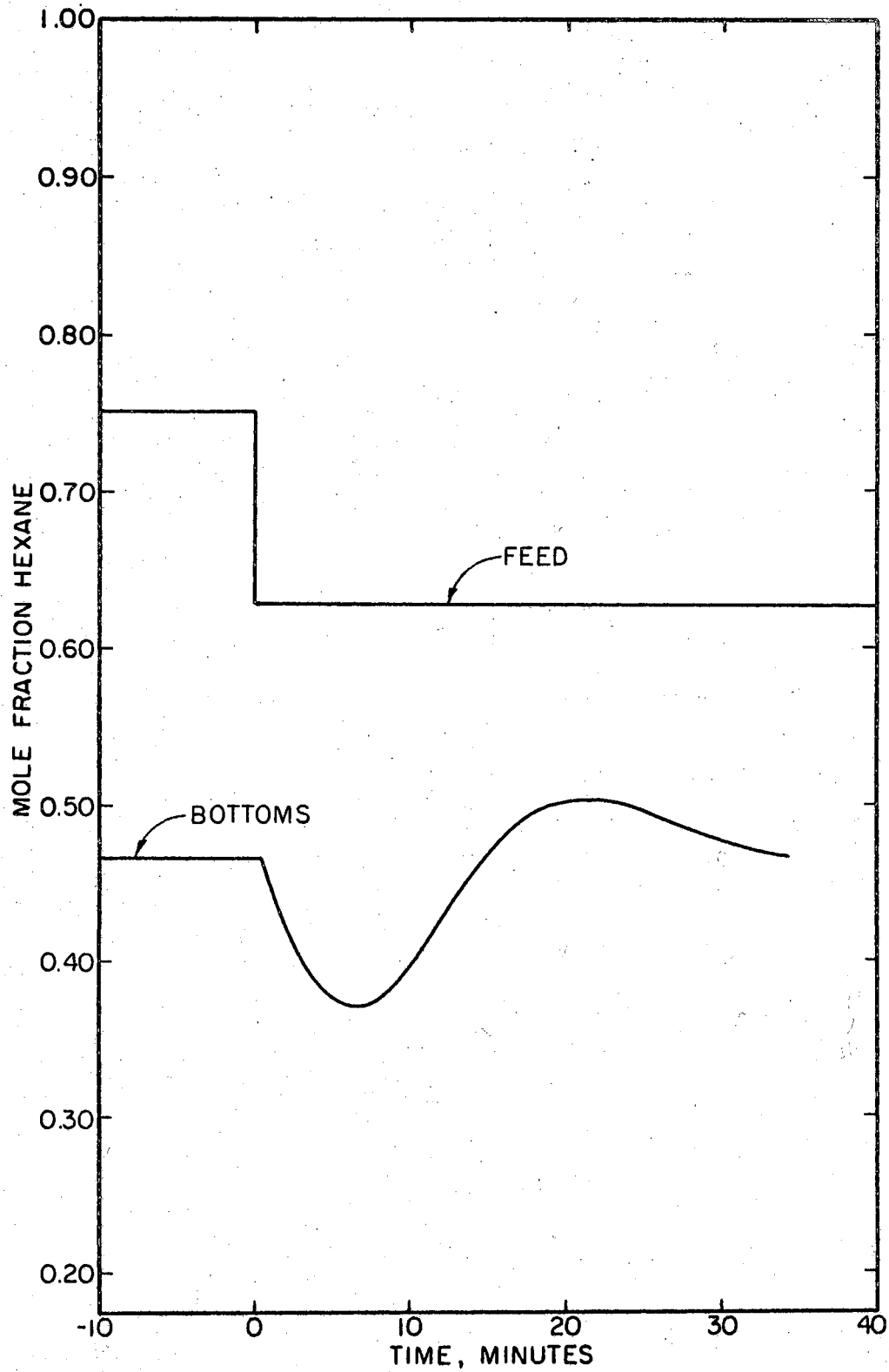Figure 31. Step Disturbance With K_1 = 1000.0,
K_2 = 100.0, and K_3 = 1000.0

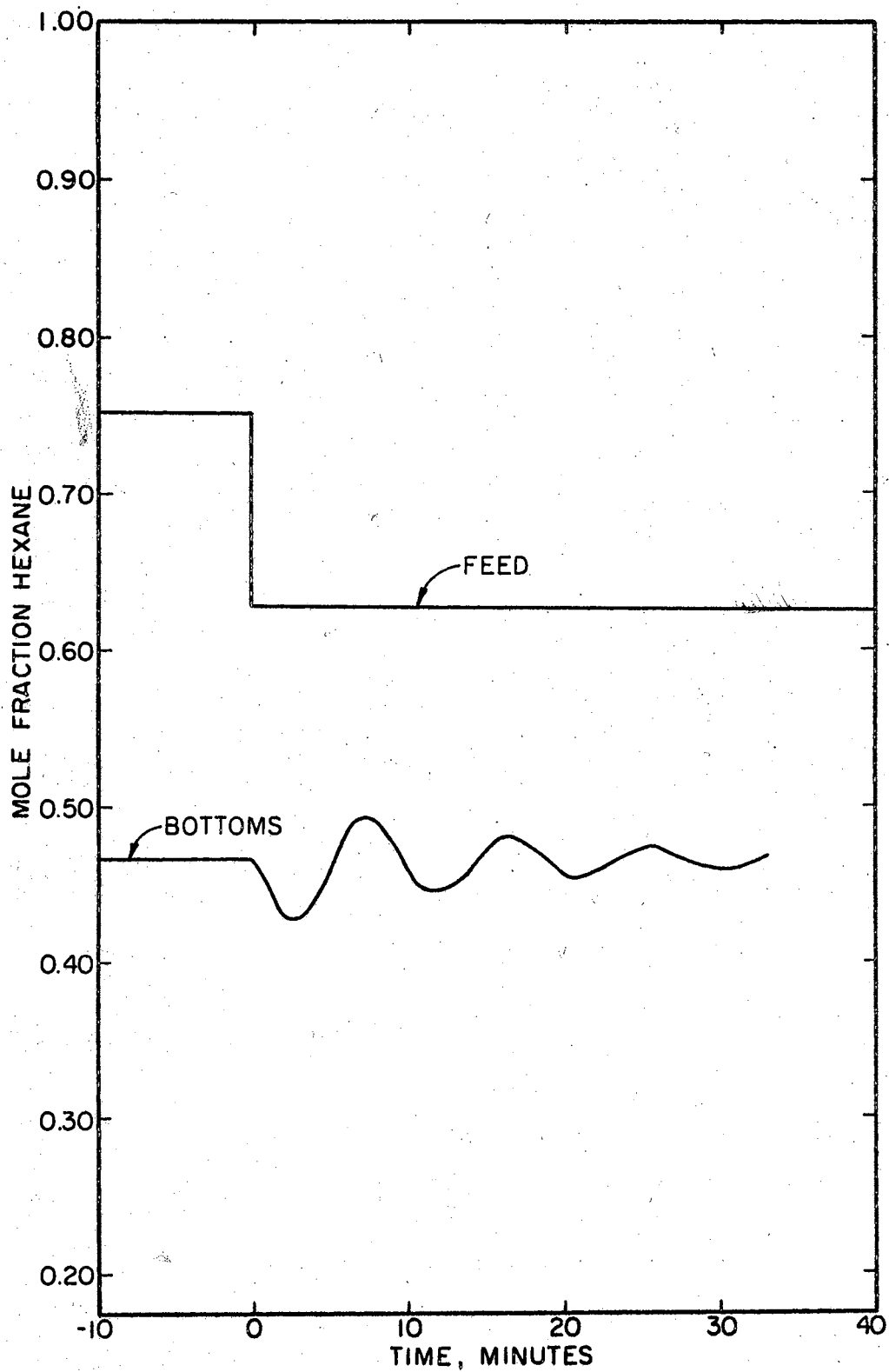Figure 32. Step Disturbance With $K_1 = 100.0$, $K_2 = 100.0$, and $K_3 = 10.0$

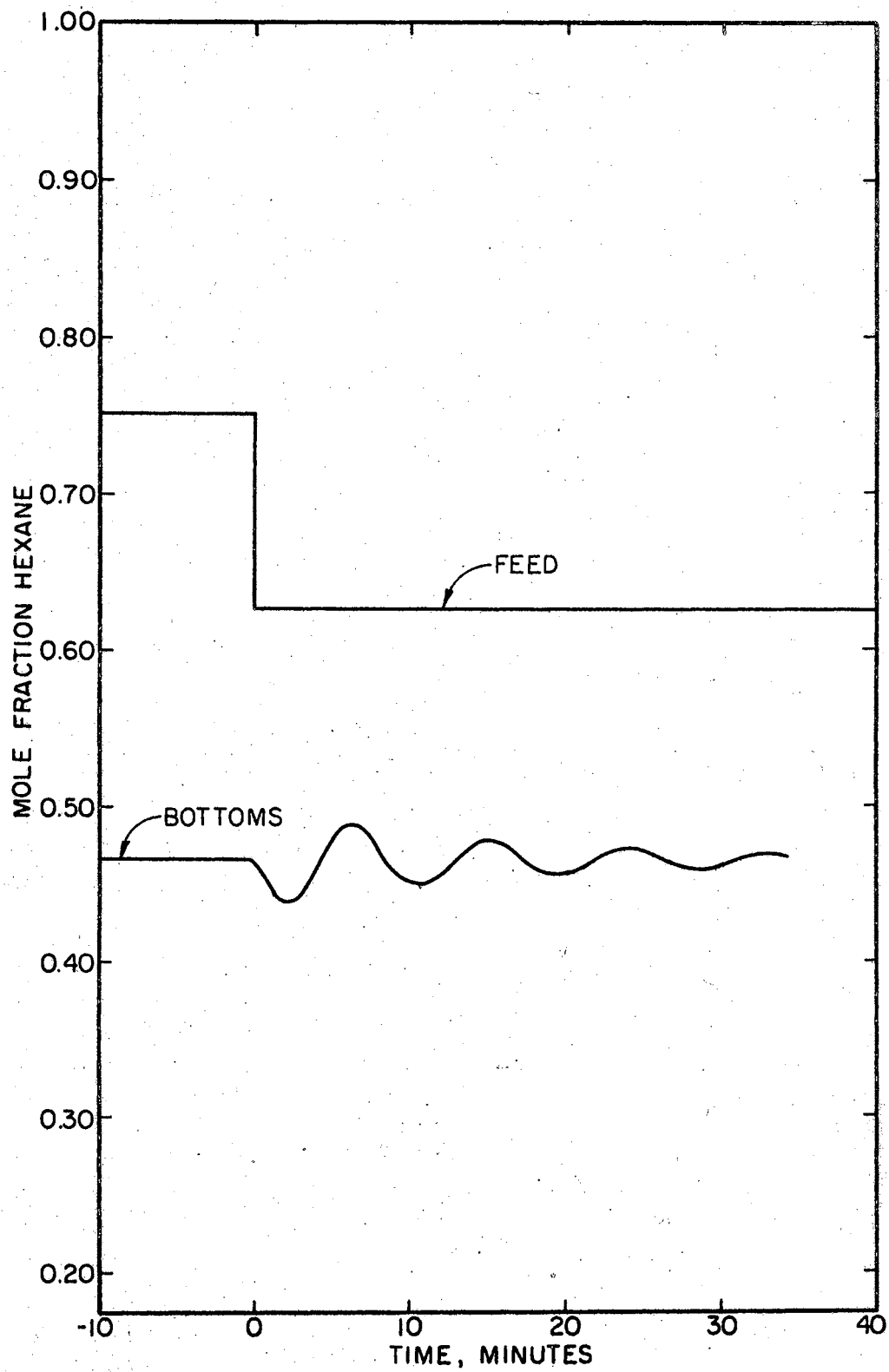Figure 33. Step Disturbance With K_1 = 1000.0, K_2 = 100.0, and K_3 = 100.0

Figure 34. Step Disturbance With K_1 = 1000.0,
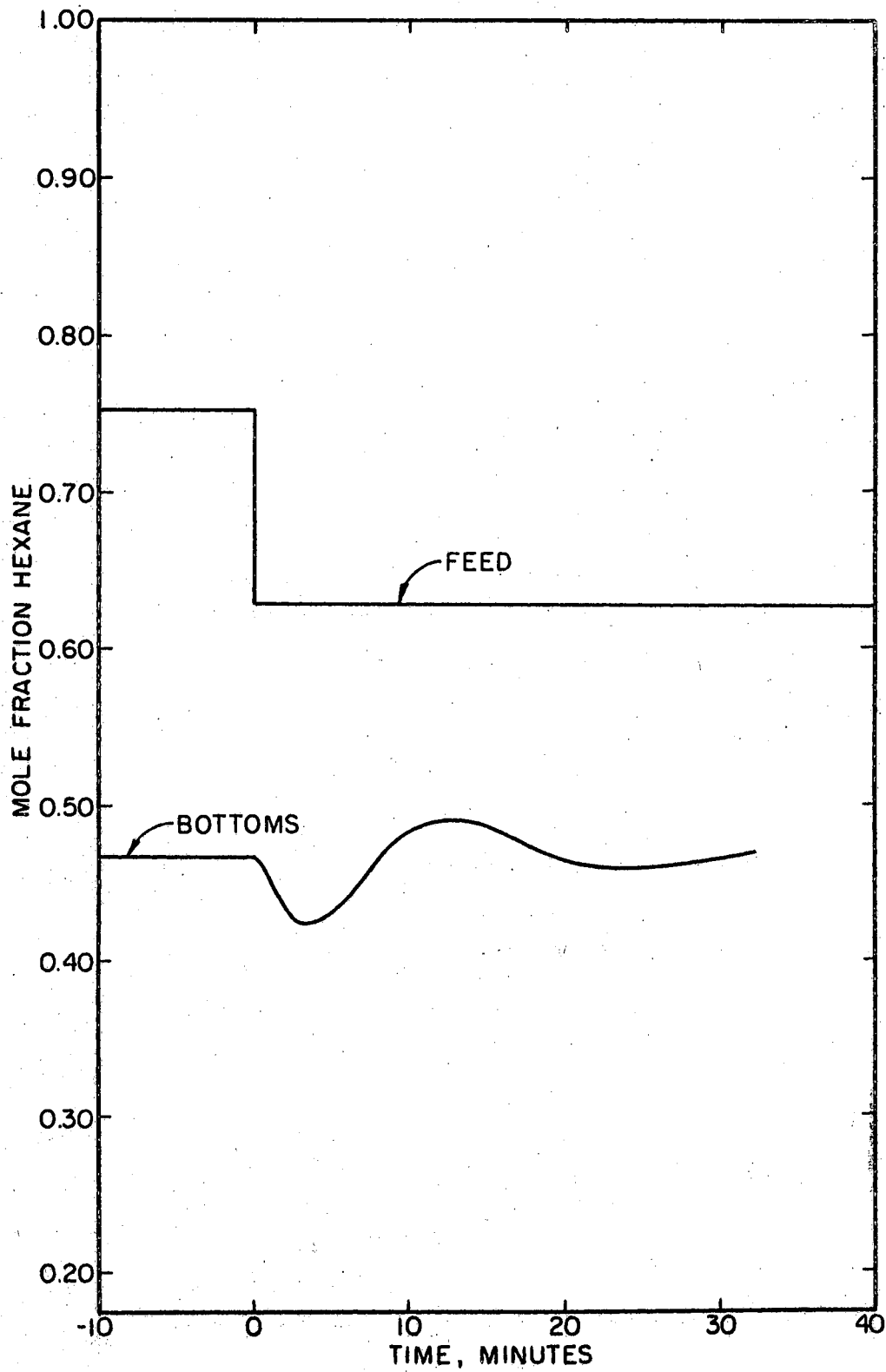K_2 = 1000.0, and K_3 = 1000.0

Figure 35. Step Disturbance With K_1 = 100.0, K_2 = 100.0, and K_3 = 1000.0

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