OUTPUT NOISE SPECTRUM WHEN AN IDEAL LIMITER-DISCRIMINATOR DEMODULATES FM

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1968

Submitted to the Faculty of the Graduate College
of the Oklahoma State University
in partial fulfillment of the requirements
for the Degree of
MASTER OF SCIENCE
May, 1969

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PREFACE

The bulk of all frequency modulation theory was developed by 1940. However, a frequency modulation theory was presented to me that indicated that a noise component had been neglected in the accepted analysis. I then committed myself to a thesis that would explore this contention.

Experimental investigation coupled with subsequent re-examination of the mathematical analysis proved that the earlier work was not in error. Although the original premise of this investigator was disproved, the resulting thesis contains a unified treatment of the spectral analysis of a function that is the product of two or more time functions, and the resultant when this function is passed through a differentiator.

I wish to acknowledge my indebtedness to Dr. Bennett L. Basore of Oklahoma State University for his guidance and help when the theoretical obstacles seemed insurmountable. I wish to thank Van Schallenberg of Oklahoma State University for many valuable and illuminating conversations regarding the practical aspects of communication equipment as well as his making available personal equipment that greatly expedited progress many times.

TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER I

INTRODUCTION

The specific topic to be examined by this thesis is the noise spectrum at the output of the limiter-discriminator of a frequency modulation receiver under large signal-to-noise ratio conditions. At the time experimental work began on this thesis, the author believed the output noise spectrum to be composed of independent contributions from the in-phase as well as the quadrature component of the input noise phasor. However, subsequent mathematical and experimental analysis proved the initial assumption invalid. The output noise was found to be an exact differential form of only the quadrature component of the input noise. Therefore, special emphasis is placed on the result obtained when a function of the form $z(t) \cdot \sin(\omega t)$ is passed through a differentiator.

The concept of frequency modulation was present even prior to 1920. But it was in 1922 that Carson (1) published an analysis which showed the need for greater bandwidths with frequency modulation than with amplitude modulation. At that time all emphasis was placed on finding methods by which bandwidth requirements could be diminished. Consequently, FM as a possibly useful system was discarded. In 1936 Armstrong (2) demonstrated the phenomenon of noise suppression with wide band frequency modulation. This was made possible by the use of very high frequency carriers which allowed him many times the bandwidth needed for AM. In 1937 Carson and Fry (3) used an approach involving variable frequency circuit theory to arrive at an analysis of FM. They applied themselves to the question of noise discrimination with and without amplitude limitation. They concluded that frequency modulation in combination with severe amplitude limiting of the received wave results in substantial suppression of the output noise. The work of the 1940's and 50's has been given unified treatment in the text by Schwartz (4).

The experimental equipment used by the author consisted of Army-Navy radio receiver R-237B/VR, bridge oscillator, spectrum analyzer and oscilloscope. The input signal was inserted into the converter stage of the receiver. The IF noise spectrum was examined at the grid input to the first limiter stage. The mean carrier power was determined at the input to the discriminator. The output noise spectrum was measured at the output of the discriminator.

The experimental results verified that the output noise spectrum of the discriminator was indeed ω^2 times the IF noise spectrum shifted in center frequency from 455 KHz to 0Hz. Specifically it was shown that the output noise power at zero frequency was actually zero under the experimental restriction of high carrier to noise ratio. It was further demonstrated that the presence of a modulating signal did not affect the output noise statistics.

CHAPTER II

MATHEMATICAL ANALYSIS

A balanced ideal FM detector has been defined as one which provides $\frac{d\Phi}{dt}$ as its output, where Φ is the instantaneous phase of the combined signal and noise phasor. The FM signal is represented by $A \cos[\omega_o t + b\int s(t)dt]$ where ω_o is the unmodulated carrier frequency, s(t) is the modulating signal and b is a constant of proportionality. Therefore, it is noted that in the absence of noise, $\frac{d\Phi}{dt}$ reproduces b s(t) as it desired. $\frac{d\Phi}{dt}$ is now to be examined when noise as well as modulated carrier is present.

Referring to the phasor diagrams of Figure 1 it is readily ascertained that

$$\Phi = D + \tan^{-1} \frac{r \sin(\theta - D)}{A + r \cos(\theta - D)}$$

Taking the derivative with respect to time of both sides of the equation yields:

$$\dot{\Phi} = \dot{D} + \left[\frac{(A + r \cos(\theta - D))^2}{(A + r \cos(\theta - D))^2 + r^2 \sin^2(\theta - D)}\right] \left[\frac{(\dot{r} \sin(\theta - D) + (\dot{\theta} - \dot{D}) r \cos(\theta - D)) - r \sin(\theta - D)(\dot{r} \cos(\theta - D) - (\dot{\theta} - \dot{D}) r \sin(\theta - D))}{(A + r \cos(\theta - D))^2}\right]$$

$$\dot{\Phi} = \dot{D} + \left[\frac{A\dot{r} \sin(\theta - D) + A(\dot{\theta} - \dot{D}) r \cos(\theta - D) + r^2(\dot{\theta} - \dot{D})(\cos^2(\theta - D) + \sin^2(\theta - D))}{A^2 + 2Ar \cos(\theta - D) + r^2(\cos^2(\theta - D) + \sin^2(\theta - D))}\right]$$

$$\dot{\Phi}=\dot{D}+\frac{A\dot{r}\sin(\theta-D)+A(\dot{\theta}-\dot{D})r\cos(\theta-D)+r^2(\dot{\theta}-\dot{D})}{A^2+2Ar\cos(\theta-D)+r^2}$$

Once again referring to Figure 1, it is noted that the resultant phasor

$$G=\Big[[A+r\cos(\theta-D)]^2+r^2\sin^2(\theta-D)\Big]^{\frac{1}{2}}$$

$$G^2=A^2+2Ar\cos(\theta-D)+r^2[\cos^2(\theta-D)+\sin^2(\theta-D)]$$

Therefore,

$$\dot{\Phi}=\dot{D}+\frac{A}{G^2}[\dot{r}\sin(\theta-D)+(\dot{\theta}-\dot{D})r\cos(\theta-D)]+\frac{r^2(\dot{\theta}-\dot{D})}{G^2}$$

At this point in the analysis high carrier-to-noise ratio is assumed. For this assumption $G^2 \gg r^2$ and $A \doteq G$ yielding

$$\dot{\Phi}\doteq\dot{D}+\frac{\dot{r}\sin(\theta-D)+(\dot{\theta}-\dot{D})r\cos(\theta-D)}{G}$$

It was here that the first analysis ran into difficulty. Those original steps are now retraced in order to clarify an important point regarding the autocorrelation function of the sum of two seemingly independent time functions. When given a generalized function $\dot{a}\sin b+\dot{b}\,a\cos b$ where a and b are functions of time and further it is known that a and $\dot{a}$ are independent, one is led to conclude that since sin b and cos b are independent, $\dot{a}\sin b$ and $\dot{b}\,a\cos b$ must be independent. Because a sin b and $\dot{b}\,a\cos b$ are obviously orthogonal, the next conclusion is that the autocorrelation function of the sum of these "two independent functions" is simply the sum of the two autocorrelation functions. And finally it could be concluded that the power density spectrum of the sum was simply the sum of the two individual power density spectra.

The point to be emphasized is that the preceding conclusions do not hold because $\dot{a}$ sin b + $\dot{b}$ a cos b is the total derivative of a sin b.

Returning now to

$$\dot{\Phi} \doteq \dot{D} + \frac{\dot{r}\sin(\theta-D) + (\dot{\theta}-\dot{D})r\cos(\theta-D)}{G}$$

realizing $y = r\sin(\theta-D)$ and, therefore, $\dot{y} = \dot{r}\sin(\theta-D) + (\dot{\theta}-\dot{D})r\cos(\theta-D)$ leads to $\dot{\Phi} \doteq \dot{D} + \frac{\dot{y}}{G}$.

Since the modulating signal D and the noise r(t) are independent, $\dot{D}$ is the modulating signal contribution to $\dot{\Phi}$ and, (ignoring the $\frac{r^2(\dot{\theta}-\dot{D})}{G^2}$ term) then $\frac{\dot{y}}{G}$ is the noise contribution to $\dot{\Phi}$. Since it has been shown that $y(t) = r(t)\sin(\theta-D)$ is the only noise component making a significant contribution to $\dot{\Phi}$ under the assumption of high carrier-to-noise ratio, this time function will be examined now. y(t) is the product of two time functions, r(t) and $\sin(\theta-D)$. Accordingly the power spectrum $S_{yy}(\omega)$ of y(t) is the convolution of the power spectra of r(t) and $\sin(\theta-D)$, i.e., $S_{yy}(\omega) = S_{rr}(\omega) * S_{ss}(\omega)$. Referring to Figure 2, r(t) is idealized as band limited noise with $S_{rr}(\omega)=1$, $|\omega| < B$ and the $\sin\omega_1 t$ has $S_{ss}(\omega) = \pi[\delta(\omega+\omega_1) + \delta(\omega-\omega_1)]$. Upon convolving, (Figure 2c), $[S_{rr}(\omega) + S_{ss}(\omega)]$ is obtained. Since a differentiator is a system whose output is the derivative of the input and its transfer function is $H(j\omega) = j\omega$, it is obvious that $S_{y\dot{y}}(\omega) = -j\omega S_{yy}(\omega)$ and $S_{\dot{y}\dot{y}}(\omega) = \omega^2 S_{yy}(\omega)$. Therefore, Figure 2d, $\omega^2[S_{rr}(\omega) * S_{ss}(\omega)]$ represents $S_{\dot{y}\dot{y}}(\omega)$. In general, it is seen that the power spectrum of the derivative of a continuous time function is always zero at $\omega = 0$.

As was previously mentioned, $D = b\int s(t)dt$ so $\dot{D} = b\,s(t)$ and for the assumption of a sinusoidal modulating signal, $b\,s(t) = \Delta\omega\cos\omega_m t$ where $\Delta\omega$ is the frequency deviation. It is then seen that the mean-squared signal output is $S_o = \frac{(\Delta\omega)^2}{2}$.

Returning now to the noise contribution, since $A \doteq \dot{G}$, as a further simplification $\frac{\dot{y}}{G}$ is replaced by $\frac{\dot{y}}{A}$.

Assuming a flat band-limited IF noise spectrum of constant magnitude η_o (Figure 2e), the discriminator output noise spectrum can be calculated easily.

$$S_{yy}(f) = \eta_o \qquad f_o - B < f < f_o + B$$

$$S_{\frac{yy}{AA}}(f) = \frac{\eta_o}{A^2} \qquad f_o - B < f < f_o + B$$

$$S_{\frac{\dot{y}\dot{y}}{AA}}(f) = \frac{\eta_o \omega^2}{A^2} \qquad -B < f < B$$

This spectrum is represented in Figure 2f.

Since the total IF noise is $2\eta_o B = N$ and the mean-squared carried voltage is $\frac{A^2}{2} = S_c$, substitution yields,

$$S_{\frac{\dot{y}\dot{y}}{AA}}(f) = \frac{\pi^2}{B} f^2 \left(\frac{N}{S_c}\right), \quad -B < f < B$$

The total output noise is simply the integration of $S_{\frac{\dot{y}\dot{y}}{AA}}(f)$ from $-f_m$ to $+f_m$.

$$N_o = \int_{-f_m}^{f_m} \frac{\pi^2}{B} \frac{N}{S_c} f^2 df = \frac{\pi^2}{B}\left(\frac{N}{S_c}\right)\frac{f^3}{3}\Big|_{-f_m}^{f_m} = \frac{2f_m^3\pi^2}{3B}\left(\frac{N}{S_c}\right)$$

So:

$$\frac{S_o}{N_o} = \frac{(\Delta\omega)^2/2}{\frac{2f_m^3\pi^2}{3B}\left(\frac{N}{S_c}\right)}$$

The modulation index $= \beta = \frac{\Delta\omega}{\omega m} = \frac{\Delta\omega}{2\pi f_m}$. Substituting,

$$\frac{S_o}{N_o} = \frac{3\beta^2\left(\frac{S_c}{N}\right) B}{f_m}$$

Also, $N_c = N \frac{f_m}{B}$, where N_c is the equivalent noise in the modulation bandwidth. Therefore,

$$\frac{S_o}{N_o} = 3\beta^2 \left(\frac{S_c}{N_c}\right)$$

This result derived for FM under conditions of high carrier-to-noise ratio, and with modulation present, goes beyond most treatments of FM, but confirms the results published prior to 1940.

CHAPTER III

QUANTITATIVE EXPERIMENTAL RESULTS

The reader is referred to Chapter IV for a detailed discussion of the experimental set-up of the test equipment.

Referring to Figure 3b, $S_c = \frac{A^2}{2}$, the mean-squared carrier voltage, is seen to be -8dB with 60dB of attenuation yielding a net value of 52dB. Previously it was stated that the output noise power spectrum is $\frac{2\omega^2}{A^2}$ times the input noise power spectrum for frequencies between $\pm$ B. This theoretical conclusion will now be tested for first 5 KHz, then 10 KHz and finally 2 KHz.

Consider first 5 KHz. The input noise power at (455 $\pm$ 5) KHz is approximately -25dB with 5dB of attenuation or a net of -20dB. (Figure 3a). Therefore, the output noise power spectral density at 5 KHz should be

$$S_{\dot{y}\dot{y}}(f) \Big|_{f=5000} = \left[-20 + 10 \log_{10}(3.14\text{x}10^4)^2 - 52\right] \text{dB} = 18\text{dB}$$

Reference to Figure 3c shows -32dB with 50dB attenuation. So,

$$S_{\dot{y}\dot{y}}(f) \Big|_{f=5000} = (-32 + 50)\text{dB} = 18\text{dB}$$

Next at (455 $\pm$ 10) KHz the input noise spectral density is again -25dB with 5dB attenuation for a net of -20dB.

$$S_{\dot{y}\dot{y}}(f) \Big|_{f=10,000} = \left[-20 + 10 \log_{10}(6.28\text{x}10^4)^2 - 52\right] \text{dB} = 24\text{dB}$$

Referring to Figure 3c,

$$S_{\dot{y}\dot{y}}(f)\Big|_{f=10,000} = (-26 + 50)\text{dB} = 24\text{dB}$$

However, at (455 $\pm$ 2) KHz where the output noise spectral density should be $[-20 + 10\log_{10}(12.56 \times 10^3)^2 - 52]$dB or 10dB reference to Figure 3e shows -38dB with 50dB attenuation or a net of 12dB. The error here at 2000 Hz is attributed to difficulty in determining exact zero vertical position for spectrum analyzer trace and nonlinearity of scale at small readings.

For zero frequency the output noise spectral density by theory must be zero. Reference to Figure 3f verifies that the noise spectrum density is indeed zero for zero frequency.

It is easily ascertained from Figure 3c that the bandwidth is approximately 26 KHz.

CHAPTER IV

DESCRIPTION OF TEST EQUIPMENT

The FM receiver used in the experiment was an Army-Navy receiver. Its model number is R-237B/VR. Its schematic is labeled Figure 4. (The squelch circuit was disconnected throughout the experiment.) Early experimental work indicated that the output noise spectrum was not zero for zero frequency when modulation was present. After a re-examination of the mathematical analysis revealed that the output noise spectrum should have been zero at zero frequency determining why experimental results did not bear this out was of paramount interest. The solution was found in the initial alignment of the receiver for a flat response across the passband under modulation. The noise was at saturation level and, consequently, as a result of apparent alignment, the signal power was decreased by an approximate power of ten at either end of the passband versus the center frequency. The $\frac{r^2(\dot{\theta}-\dot{D})}{G^2}$ term was now a factor near center frequency. To correct the problem the alignment was re-established while the tube of the first IF amplifier was removed to greatly decrease the noise present.

The GR 1330A bridge oscillator was used as a signal source. While the GR 1330A is supposedly an amplitude modulated instrument, it was determined in the laboratory that it was actually frequency modulated in the frequency range of interest.

A Tektronix oscilloscope type 585 with type G plug-in was used. The

Singer model FM5 panalyzer was used to determine all power spectra. The test equipment interconnection diagram is illustrated in Figure 4.

Referring to Figure 5, the input signal from the bridge oscillator was inserted into the first converter stage. The IF noise spectrum generated in the receiver front end was fed to the spectrum analyzer from the grid, pin 4, of the first limiter. The mean carrier power was determined at the plate, pin 3, of V9, the discriminator. The output noise spectrum was measured at pin 4 of the discriminator.

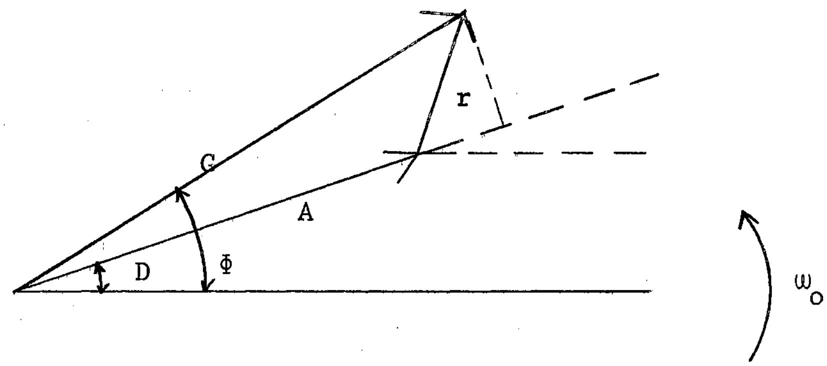
CHAPTER V

CONCLUSIONS

Given that $f(t) = x(t) + y(t)$, x and y linearly independent and orthogonal are necessary but not sufficient conditions for the power density spectrum $S_{ff}(\omega)$ to equal the sum of the power density spectra $S_{xx}(\omega) + S_{yy}(\omega)$. It was shown in Chapter II that a third condition is that $x(t) + y(t)$ cannot represent a total derivative.

Any output of a differentiator that is the total derivative of a particular time function or product of time functions must have a power spectrum that is zero at zero frequency.

With the restriction of high carrier-to-noise ratio, it is experimentally concluded through prolonged observation that the noise spectrum at the output of the discriminator, in the vicinity of zero frequency, is not affected by the presence of a modulating signal.



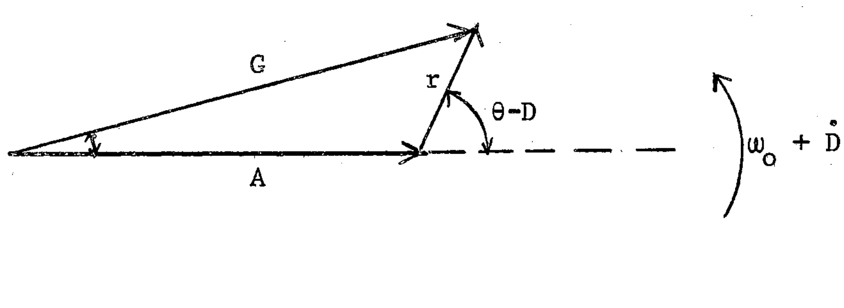


Figure 1. Phasor Diagrams

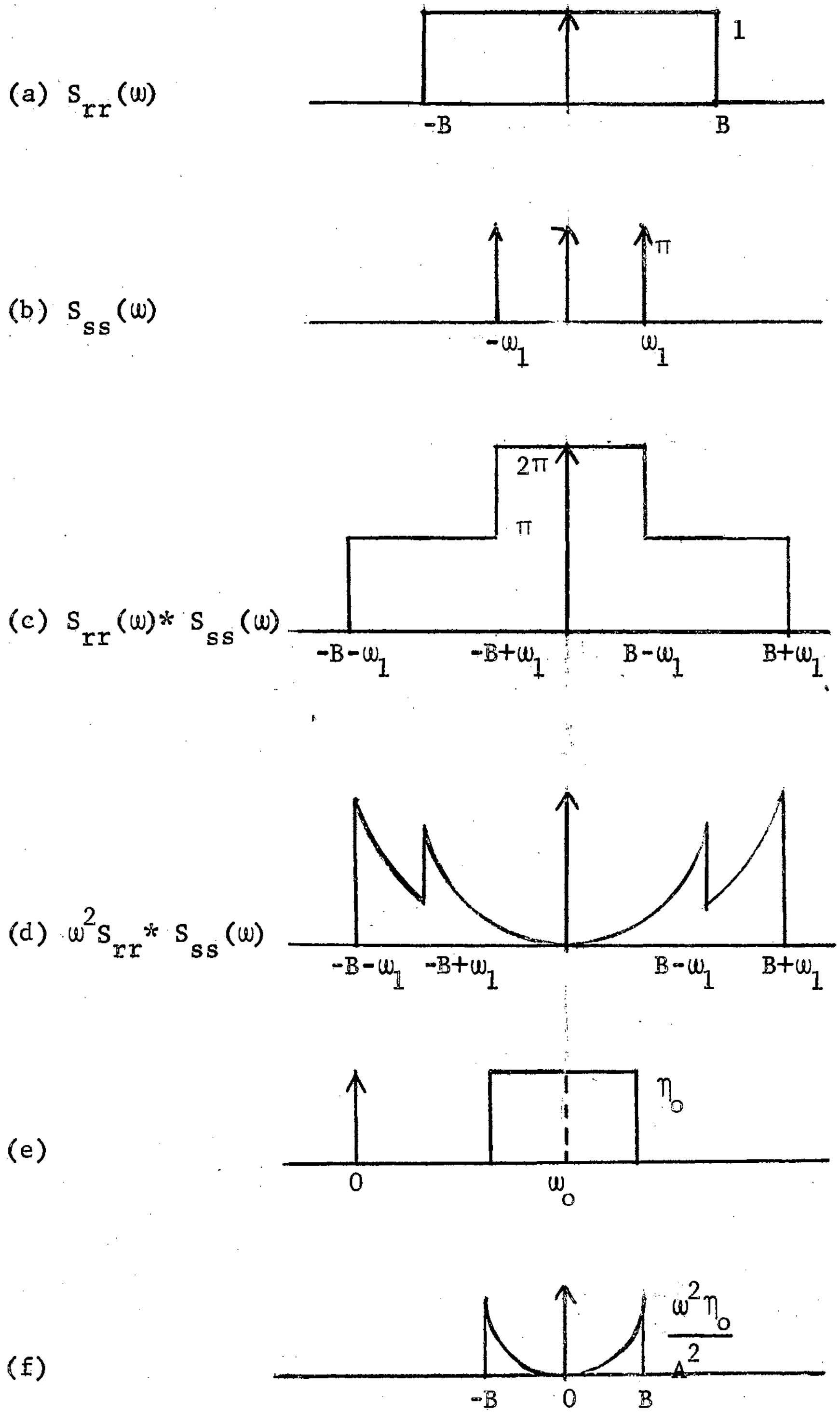
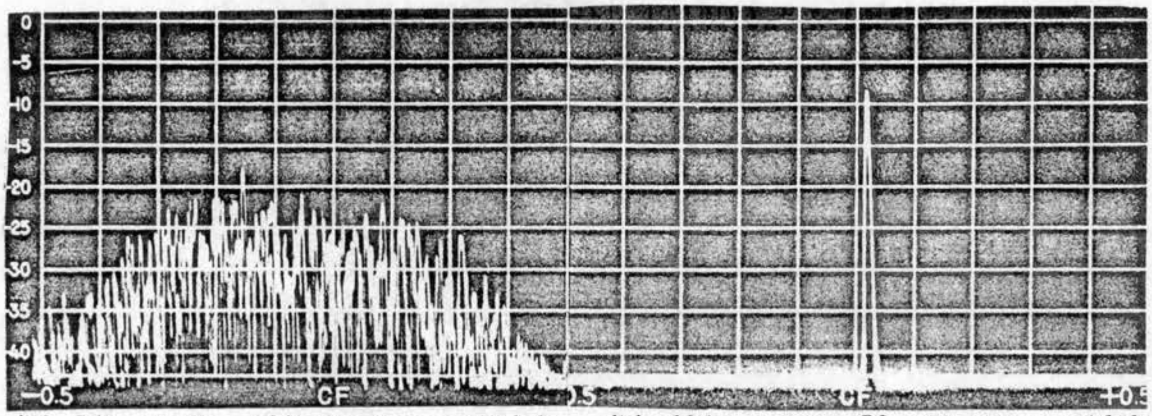
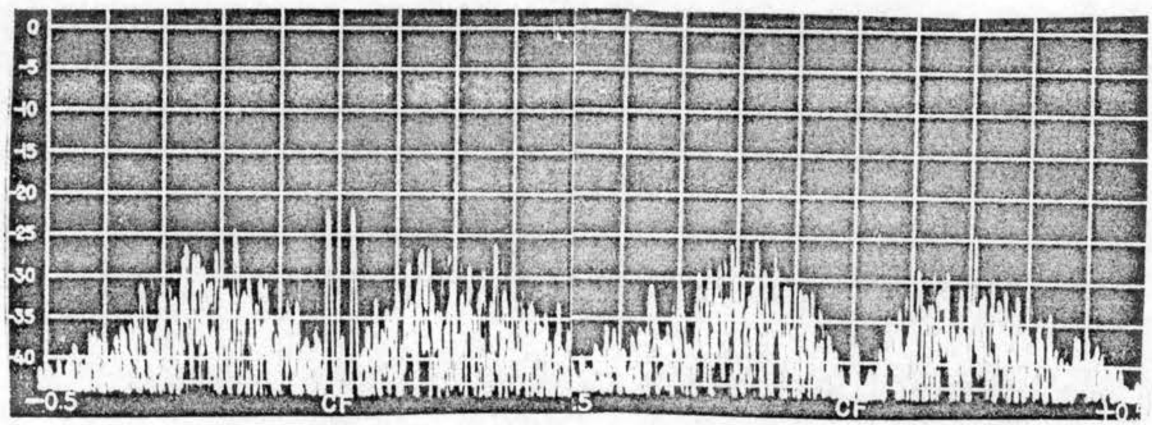


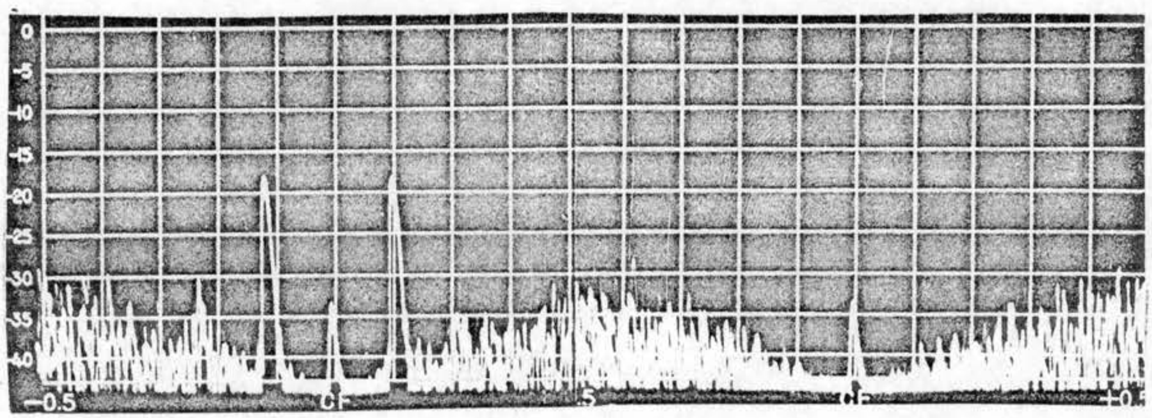
Figure 2. Graphical Convolutions



(a) 5dB Atten. 100 KHz Sweep Width (b) 60dB Atten. 50 KHz Sweep Width



(c) 50dB Atten. 50 KHz Sweep Width (d) 50dB Atten. 50 KHz Sweep Width



(e) 50dB Atten. 10 KHz Sweep Width (f) 50dB Atten. 10 KHz Sweep Width

Figure 3. Noise Spectra

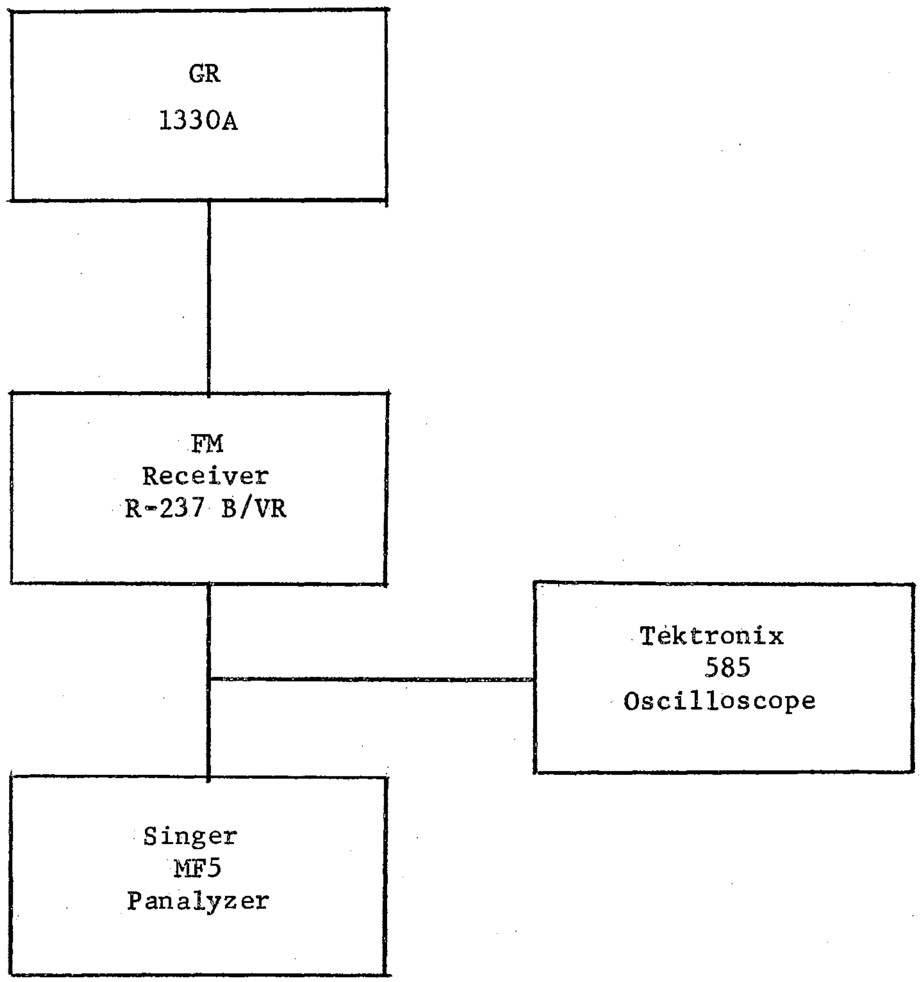
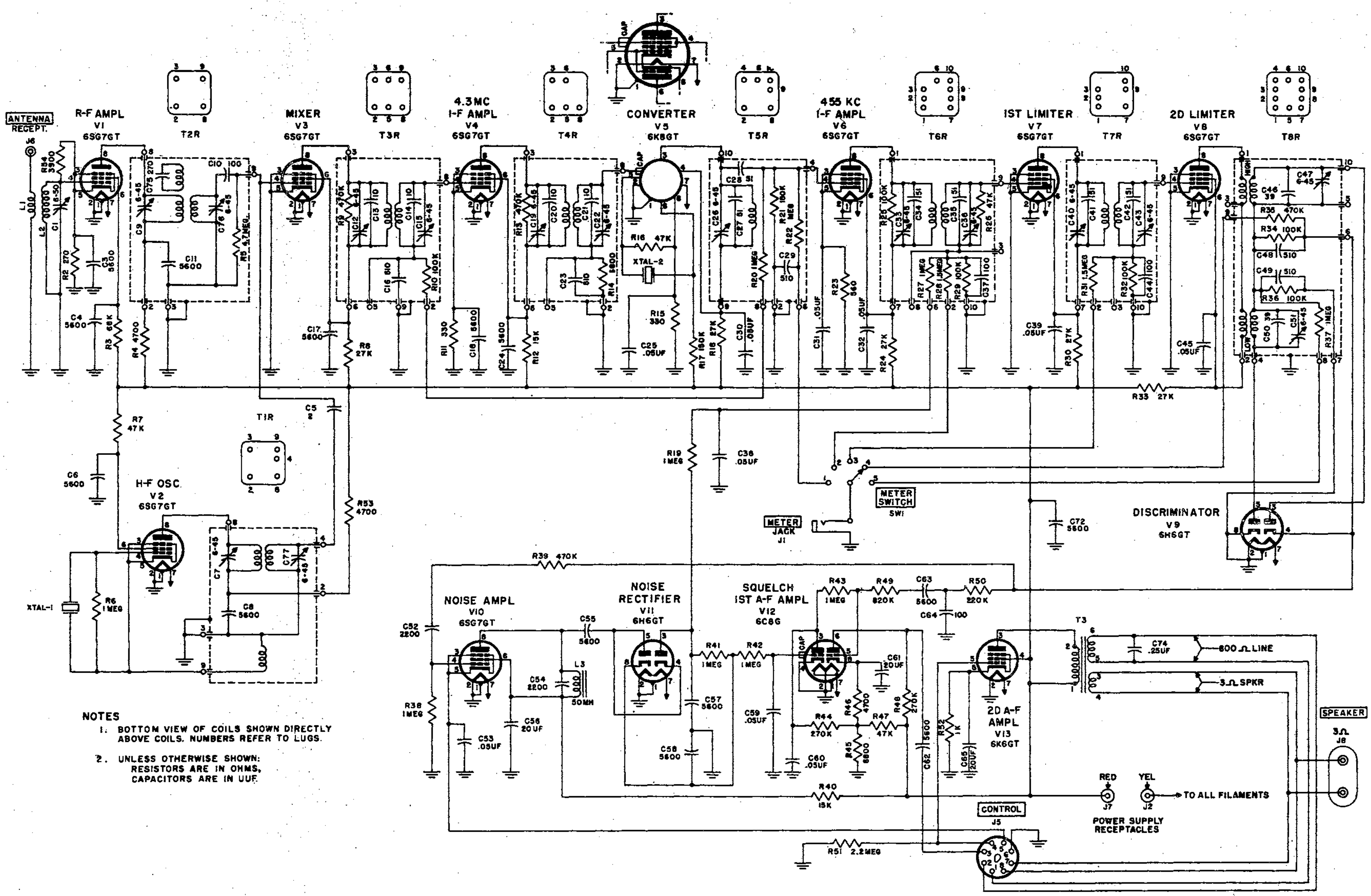


Figure 4. Test Equipment Interconnection Diagram



NOTES

1. BOTTOM VIEW OF COILS SHOWN DIRECTLY ABOVE COILS. NUMBERS REFER TO LUGS.
2. UNLESS OTHERWISE SHOWN: RESISTORS ARE IN OHMS, CAPACITORS ARE IN UUF.

Figure 5. FM Receiver Schematic

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