THE OPTIMIZATION OF GAS WITHDRAWAL RATES FROM SINGLE WELL AND MULTI WELL RESERVOIRS

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Scope and Method of Study: This paper presents a new method for obtaining a steady state solution of an integrated gas system model made up of pipelines in either storage or natural gas fields. By use of this very flexible solution technique, it is possible to solve directly for system parameters when field information is available. Linear programming will be used in this model, a technique which as of late has made inroads into many segments of the petroleum industry. The final model will be developed in steps by first exploring the general equations of gas flow applied to a single well in a single producing field, to the more common multi well reservoir, multi reservoir field where the performance of one well is directly influenced by the performance of the adjoining wells.

Findings and Conclusions: The developed equations modeling the typical gas reservoirs in use today are valid only when the assumptions employed in their derivation are recognized. Application of the equations indicate that an optimal solution is possible through the use of linear programming techniques.

An important implication of these results is that management now has at its disposal an optimization model based upon deriving the maximum dollar profits.

ADVISER'S APPROVAL Kent Mingo

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PREFACE

This report is concerned with the maximization of total gas withdrawals from gas producing fields during producing seasons and requires the identification of optimal withdrawal rates from the active wells. The transient behavior of the wells during the short producing season and the mutual interference of the wells make the physical problem complicated. During the early parts of the producing season, the demand rates are low and it is easy to produce the demand rate from just a few wells. However, later in the season, all wells need to be operated at peak capacity, or at least at optimal capacity in order to meet the demand. To compensate for some of this fluctuation, some of the abandoned oil fields are converted to storage facilities and are filled during the period of low demand and emptied during peak loads. However, the problem of the optimal operation exists whether the field is natural or one converted to storage.

I would like to take this opportunity to express my appreciation for the assistance and guidance given me by: J. Scott Turner, who suggested that I explore this topic and was always available for counsel, and A. G. Comer of the Department of Mechanical and Aerospace Engineering at

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CHAPTER I

INTRODUCTION

In recent years considerable progress has been made in the development and application of mathematical techniques for the solution of certain problems involving economic strategies. Such a problem might involve, for example, the scheduling of shipments of a number of goods from a number of sources to a number of destinations. The object would thus be to schedule the shipments in a manner so as to satisfy the destination requirements and at the same time minimize the transportation costs.[1] The solution to such a problem is not always intuitively obvious. The obvious solution is frequently far from the optimum. For example, if the shipments are to be made from only two sources to four destinations, the optimum solution is readily found. However, if shipments are to be made from fifteen or twenty sources to a hundred or so destinations, even competent and experienced schedulers may spend considerable time in finding merely a reasonable answer. Even at that, the scheduler has no guarantee that his solution is the optimum solution. Moreover, the techniques employed will give no indication as to how far the solution is from the optimum answer. This leads to uncertainty; should the scheduler accept this solution or seek a better one.[2]

Before the advent of the large high speed computers, little more could be done with such problems because of their great size and multiple solutions. A problem involving twenty sources and fifty destinations would require choosing from a very large number of possible combinations, the optimum combination of one thousand variables. The best one could do was to utilize intuition, extrapolation from past experience, and other non-exact approaches. With a high speed computer, however, such problems can be solved providing a reasonable computational procedure (or algorithm) can be utilized.

The oil and gas industry became aware of the great powers of linear programming through the pioneering work of Charnes, Cooper, and Mellon (1952, 1954) and the work of Gifford Symonds (1953).[3] Up until five or ten years ago there were few people in the petroleum industry who had heard of such things as "basic solution" or "convex set." Today, these terms are very familiar. The lag time between the theory and the application of linear programming is due to the fact that educational processes are involved and educational processes are notoriously slow.

As technology improves, the problems become more interwoven and complex. The problems of the oil and gas industry are no exception. They can be logically grouped into categories according to different phases: exploration and land lease; drilling and production; manufacturing; and distribution and marketing.[4] An integrated oil company must first of all carry out exploration activities to determine where exactly petroleum is to be found. The land must then be acquired or leased and a "wildcat" well or exploratory well is drilled. If the well indications are favorable, additional wells are drilled to develop the field and production gets under way.

An oil or gas field may be produced in many different ways. Which is best? The complexity of a modern refinery is staggering. What is the best operating plan? What is meant by "best"? Of course, not all of these problems lend themselves to linear programming, but some of them do. The techniques of linear programming have become quite extensive in many different segments of the oil and gas industry, but very little work has been done to date in extending these methods to the area of underground gas production. Specifically, progress has been made in the area of gasolene blending, complete refinery operations, distribution of products from refinery to bulk plants, distribution from multi-refineries to bulk plants, scheduling of ships and routing of trucks from the bulk plant to the service station.[5] It is true that even the simplest reservoir behavior problem is non linear in both geometry and time, and hence does not lend itself readily to linear programming models. Yet this same objection has been made in the past in regard to almost all of the above categories. This report develops some linear programming models and indicates where they may have some applicability. Since a firm footing is always required for future blocks of knowledge, a short introductory section to the basics of the oil and gas industry will be found in Appendix A. For one with no prior petroleum experience, it would be advisable to read this section before proceeding into the body of the paper. Also included, in Appendix B, are a few notes on the "state of the art" of linear programming in the petroleum industry today. From here then, the models will be developed for the optimal extraction of gas from natural sources or gas reservoirs previously injected.

FOOTNOTES

[1]Cyrus Derman, "A Simple Allocation Problem," *Management Science*, 1959, p. 453.

[2]Samuel B. Richmond, *Operations Research for Management Decisions* (New York, 1968), Chapter 10.

[3]W. W. Garvin, H. W. Crandall, J. B. John, and R. A. Spellman, "Applications of Linear Programming in the Oil Industry," *Management Science*, 1958, p. 407.

[4]*Ibid.*, p. 408.

[5]*Ibid.*, p. 409.

CHAPTER II

A DETAILED APPROACH TO PETROLEUM TECHNOLOGY

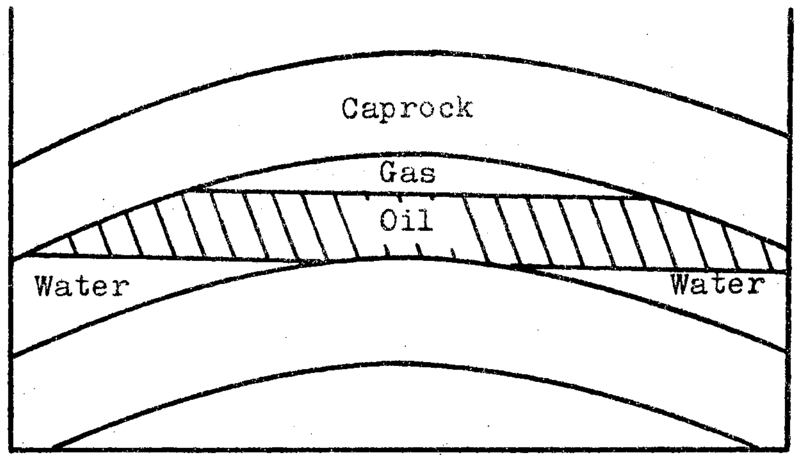
The demand for fuels is not constant due to the variable weather conditions. When gas or LP gas is produced in one locality and consumed in another, cheap transportation is essential. The lowest cost of transporting fluid fuels is normally through pipelines which operate as close to capacity as possible throughout the year. Storage of the fluid fuels at or near the point of utilization is the mechanism of permitting the pipelines to operate at capacity during periods of low consumption and to permit them to accept a constant supply. Consequently, since man has learned the importance of underground storage of fluids and gases, and since petroleum has been stored in underground reservoirs for millions of years by being trapped below a caprock and confined by underlying water, both the optimal removal of gases from natural sources as well as that from underground storage fields needs to be explored.[1]

Natural gas is not the only fluid which may be stored within the natural caverns of the earth. Brine produced in connection with oil production is returned to the earth, sometimes even to other zones. Earthen pits are used after freezing the earth to form an impervious container for liquified natural gas or refrigerated propane. The new ecological emphasis on pollution of streams and the atmosphere will no doubt provide future use of the earth for storage of waste materials.

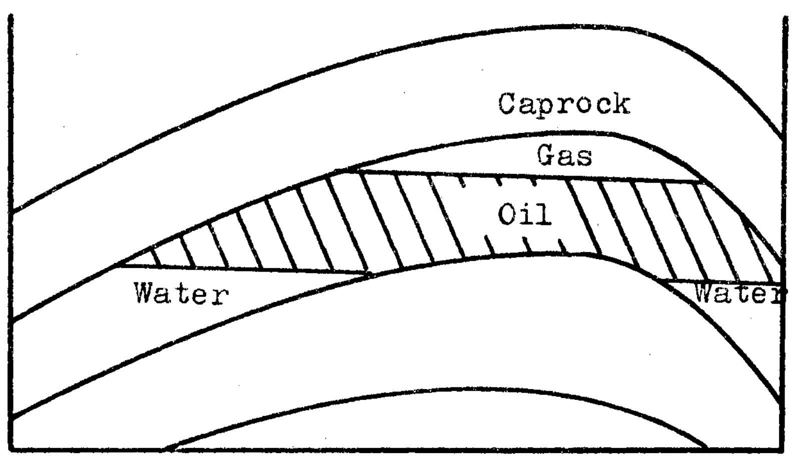
From various fields such as geology, petroleum engineering, water resources, hydrology, and the engineering area of underground storage comes the information needed for the understanding of the nature of the strata near the surface which might become storage zones.

One very often misunderstood concept of the nature of the containers for underground storage is that there exists large mysterious caverns many miles deep and equally as many miles of endless branches. The only natural caverns underground are the relatively rare solution cavities in the carbonate rocks or the man made salt cavities created by solution mining.

Oil and gas are found in a large variety of underground structures and their forms are so numerable that they are beyond the scope of this paper. The most common shape, however, has already been mentioned and is depicted in Figure 1. It is termed the buried hill or anticline. The features of interest are the height of the hill from the highest adjoining valley. This height is called the closure since it represents the size of the container closed to normal horizontal movement. Typical heights are as low as 100 to 300 feet and this distance is relatively small as compared to the lateral distances which can be several miles.[2]



Symmetrical Anticline



Asymmetrical Anticline

Figure 1. Common Anticlines

Figure 1 should not be considered to scale because the vertical scale has far less real distance per unit of the figure than the horizontal scale. The sides of the hill may have a slope of 100 to 200 feet per mile or an angle of 1 to 2 degrees with the horizontal, and usually, one side of the hill is steeper than the other.

The actual nature of the gas reservoir is learned during the initial drilling period. The presence of the hydrocarbons testifies to the presence of a container and the quality of the caprock to hold fluids below it.[3]

The common elements of an underground storage reservoir are depicted in Figure 2, for the many natural hydrocarbon fuels. First, there is a structure under which gas may accumulate. Second, there is a container, a porous bed of rock into and out of which fluids may flow through wells. Third, there is a water filled caprock which prevents the stored fluid from rising vertically due to buoyant forces or from moving laterally to rise elsewhere. Fourth, there is depth or overburden to allow storage to take place under pressure much above atmosphere. Fifth, the water is present to confine the stored fluid from all directions. Below the stored gas, water moves under a pressure gradient to make room for the stored gas while in the caprock it seals the tight rock from penetration by the gas phase. Depth is considered an element of importance since economical consideration requires enough depth to permit sufficient fluid pressures to be used to get satisfactory stored quantities

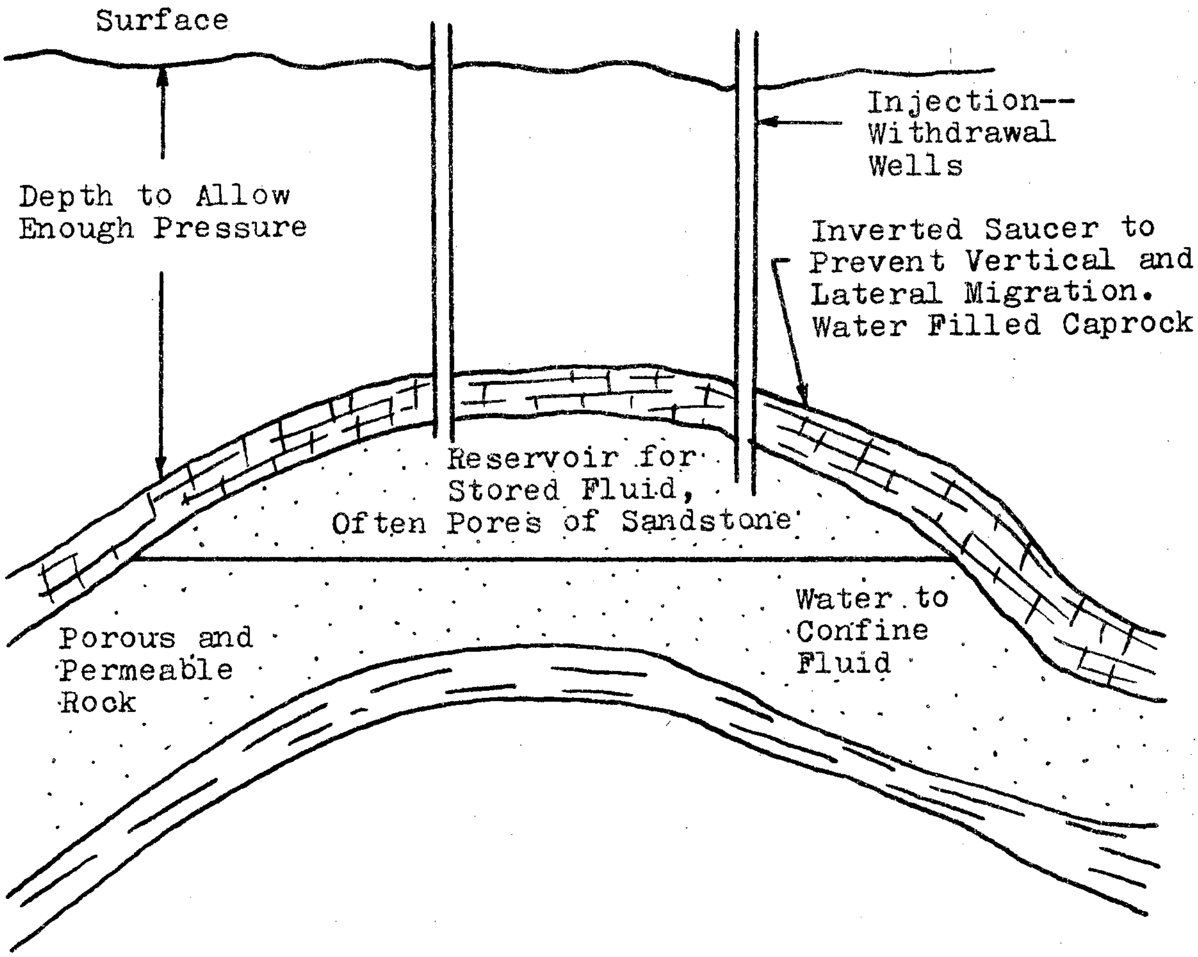


Figure 2. Common Elements of Underground Storage

into a given space and to readily move them into and out of a storage container.[4]

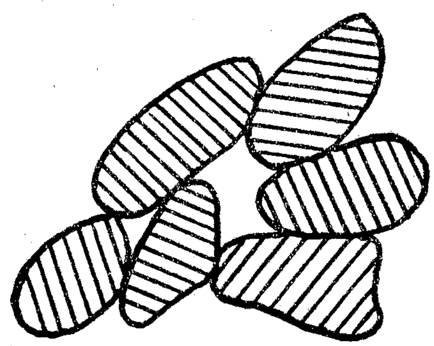
In a free system, oil and gas float to the top of water, and so it is in the underground storage container. A storage reservoir caprock is needed to hold buoyant fluids at a given depth.

The caprock must not only stop vertical movement, but it must be shaped to prevent lateral movement as well. The anticline or inverted saucer type of structure shown in the figure is common. The hill on the underground structure may be nonappearing on the surface of the earth and so is only located through core drilling.

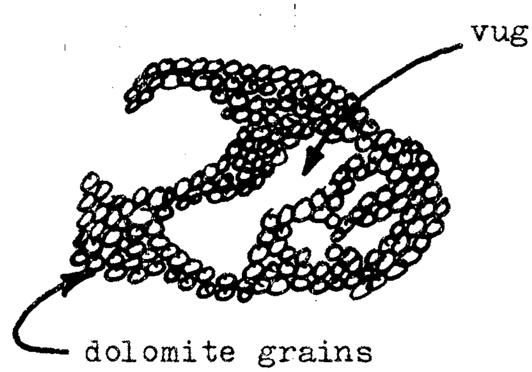
The conception of underground storage containers generally starts with the pore space in the rock, structures for containing the buoyant fluids, characteristics of the caprock to hold gases below them and the quality of the rocks that allow gases to move through them.[5]

All underground strata, whether unconsolidated solids like soils, or rocks like sandstones or limestones have some amount of pore space not occupied by the solid substance. For example, a pile of sand probably consists of 65 percent solids and 35 percent voids filled with air if the sand is dried. A brick may well have a porosity of 15 percent of its volume. The sandstone used for building garden walls may have a porosity of 15-20 percent. Sometimes underground layers are unconsolidated and may have porosities of 25-40 percent.[6]

Therefore, it is the porous rocks which serve as reservoirs for oil and gas deposits. Since the pore space within the rock cannot be viewed directly, the porosity concepts come from indirect measurements. Figure 3 shows the two primary types of pores inside rocks. Granular materials such as sand have pore spaces between the grains with porosities often in the range of 10-35 percent depending upon the range of size of grains in the sand mix. The mixing of several sizes of grains can reduce the porosity significantly. Limestones or dolomites may have solution channels and cavities known as vugs to enlarge the pore space above that found between the grains. Matrix porosity with its fine pores may behave quite differently from the larger solution channels or cavities. When rocks are composed of very fine particles such as shales, often described as compressed and dehydrated clay, they will have very small pores. Their porosities may be from 6-12 percent and they will permit water to flow through them very slowly with a high pressure drop. Other rocks which transmit single phase fluids slowly are low porosity (2-8 percent) limestones, dolomites, and anhydrates. If one takes core specimens of such caprocks and cleans and dries them in the laboratory, they will permit gas to flow through them very slowly, but fast enough so that a continued flow through several acres could accumulate to a large quantity. Therefore, it is not the low permeability of a rock per se which is characteristic of a caprock, but the quality of not



Space Between Sand Grains



Dolomite with Intergranular
Solution Porosity

Figure 3. Primary Types of
Pores

permitting one substance (oil or gas) to displace another substance such as water which characterizes it as the confining layer.[7]

The mental picture then of a caprock is a low permeability--low porosity rock filled with water and gas pressing from below at small menisci in the pores. Each rock has measurable characteristics of the gas pressure required for gas to force the meniscus to move. This threshold pressure measurement was devised to test caprock for gas storage reservoirs. The fact is generally accepted that water within the caprock is the sealing force for confining the stored gas in either a natural source field or a gas reservoir.

Walls of buildings made of a single tier of cement blocks have been observed to pass water in a driving rain. Just as natural rocks have capacity to hold volumes of substances in their pores, fluids can be made to flow through them, Water wells which draw water through porous beds to the well bore depend upon the permeability of the layer to pass water. In contrast to the low permeability needed for caprocks, the rock comprising the storage zone beneath the caprock must permit fluids to flow through it readily. The pressure drop required to cause the flow depends upon the quantity flowing per unit of cross section and, hence, is greatest at the well bore. Not only are the permeabilities of the storage zone needed to predict gas flow, they also are necessary in aquifer storage reservoirs for predicting water movement rates when injected gas is displacing the native water from the formation. Considerable time is required to develop aquifer storage reservoirs and the orderly displacement of the water depends both on the pressure differential and the formation permeability. Because of the high viscosity of water relative to gas, its rate of travel under a given pressure drop is much slower than for gas.

In gas and oil fields, the potential of the wells found in the development indicates the permeability of the rock. High open flow gas wells indicate that the producing formation is permeable and that injection withdrawal rates in storage operations can be high. For aquifer storage fields, core tests and water pumping rates permit predictions to be made of the rock permeability as it is in the earth.[8]

Basically there are two kinds of pressure to consider in underground strata: fluid pressures in the pores of the rocks and the overburden pressure exhibited by the solids.[9] The first of these, fluid pressure, is of primary importance in gas storage while the second, overburden pressures, has much to do with hydraulic fracturing.

A static column of water (with a density of 1 gm/ml or 62.4 pounds/cu ft) has a vertical pressure gradient of .433 pounds per square inch per foot of depth. An open well casing full of water to a depth of 2000 feet will have a bottom hole pressure of 2000 x .433 or 866 psi above the top hole pressure. Since the rocks which comprise the surface of the earth are essentially water filled rocks through which the water exerts pressure gradients, wells completed at various depts will find fluid pressures similar to .6 psi/foot. In some wells, lack of rock permeability may cause water to enter the borehole very slowly and the water pressure may appear to be much less. Also, there are some variations in earth fluid pressures due to dynamic hydraulic situations. The overburden pressures are those which represent the load represented by the rock between a given depth and the surface.

Given that the reservoir has an initial pressure then of Po, which is a combination of fluid and overburden pressure, any disturbance such as a drilled well will cause a pressure drop which varies with time as well as distance from the disturbance. Because this pressure varies throughout the reservoir, any further disturbance, or any other wells drilled, will cause a change in this pressure gradient and consequently will effect a change in the performance of each well.

Consider a circular reservoir of exterior radius r_e with a well radius of r_w at its center. The initial reservoir pressure is Po and is uniform. The well is opened and produced at a constant rate. Figure 4 illustrates the nature of the unsteady state pressure behavior in the reservoir. The pressure transient moves out into the reservoir toward the exterior boundary r_e. The plot of P^2 versus log of the radius is nearly linear near the well bore.

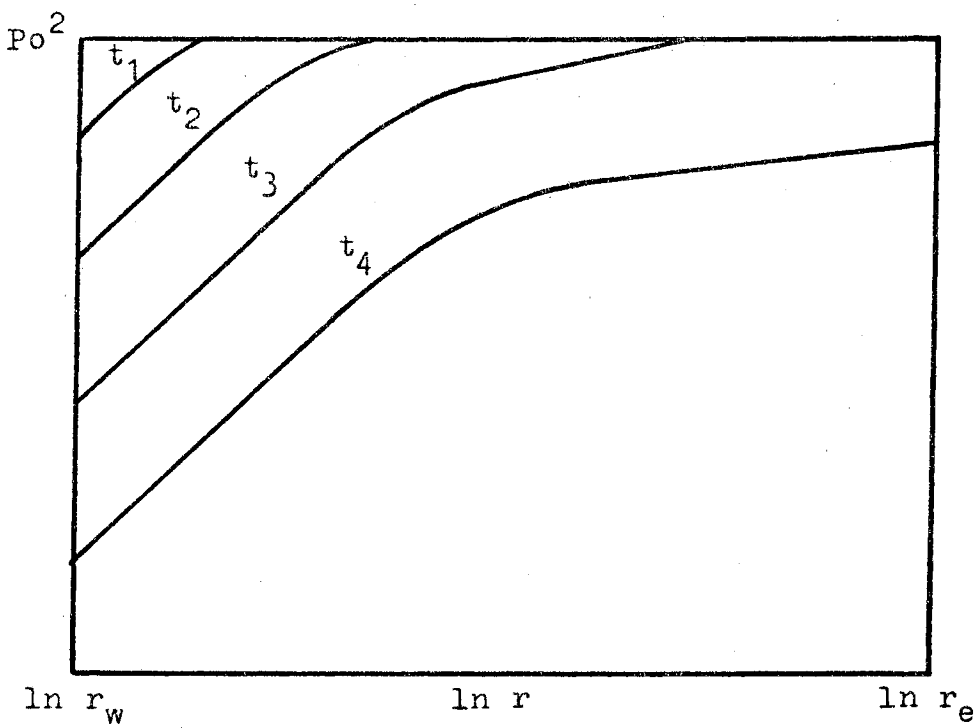


Figure 4. Reservoir Pressure Behavior

Extrapolation of this linear portion to initial pressure Po^2 introduces the concept of a "drainage radius," r_d. The period of time $0 < t < t_3$ is referred to as unsteady state and is a period when pressure varies with time at any given position.[10]

If the exterior boundary is maintained at a constant pressure, Po, then shortly after time t_3 the well will stabilize at a constant flow rate and constant flowing bottom hole pressure. Even though pressure varies with position, it no longer changes with time and a steady state condition prevails. A permeability pinchout at r_e or the pressure of the neighboring wells in a pattern of well spacing $2r_e$ results in a closed or no flow boundary at r_e. In this case, as in the earlier unsteady state position, pressure varies with time throughout the reservoir. However, theory and experience show that the rate of pressure decline is nearly independent of position. That is dP/dt is constant throughout the reservoir and a quasi-steady state condition prevails.

It should be established that around the well bore where most of the pressure drop occurs, much of the gas is flowing through the sand as contrasted to a pressure depletion.

The question of well deliverability involves prediction of well performance under steady state or quasi-steady state conditions. Unfortunately though, well performance data are frequently obtained under unsteady state conditions.

Thus the problem is to deduce from these unsteady state data the steady state well performance.[11]

The procedure employed to treat gas fields in interference, i.e., more than one well per reservoir, entails the principle of superposition.[12] Mathematically the superposition theorem states that the linear combination of particular solutions to a linear and homogeneous differential equation is a solution to the differential equation. The superposition theorem is a useful tool for treating systems upon which involved boundary conditions are imposed. The general solution is the summation of the particular solutions obtained by treating one boundary condition at a time. In this particular multi well per reservoir problem, the performance of multiple gas wells in a common acquifer or reservoir can be evaluated from the separate solutions obtained by dealing with one well at a time. In essence, if the pressure change associated with producing each and every well were computed individually (i.e., ignoring the presence of all gas wells but one) for a time instant, t, and at some arbitrary point in the reservoir system, then the total pressure change at this point and time instant is given by the sum of all individual changes. For that matter, the arbitrary point may well be the effective center or any other appropriate point in a gas field and the time instant may represent some assigned future date at which it is desired to predict the behavior of the system.

For illustration, assume two wells A and B, located in a common aquifer. Producing well A is accompanied by a pressure change in A and a relatively smaller pressure change in B. Similarly, producing well B is accompanied by simultaneous pressure changes in B and A.

Let P_{Aa} = pressure change in A due to A's production, and

P_{Ab} = pressure change in A due to B's production.

And define similarly P_{Bb} and P_{Ba}. Then the total pressure change in A:

$$P_A = P_{Aa} + P_{Ab}$$

and the total pressure change in B:

$$P_B = P_{Bb} + P_{Ba} \quad .$$

The pressure change terms depend on the rates of production and the physical characteristics of the system.

The superposition principle demands the same assumptions on the gas field. That is, it assumes homogeniety of matrix rock with regard to permeability, porosity and thickness; homogeniety of the gas; etc. It also assumes that in the case of aquifers, the compressibility coefficient which is defined as the sum of the water and the rock effective compressibilities is assumed constant and independent of the pressure.[13]

Aquifers are merely water bearing zones extending over distances of miles. Water may enter a sandstone at a high elevation and flow downwards toward an outcrop. The flow rates are usually slow in terms of a few feet of motion per year. Many sandstone layers are overlain by impervious shales or other rocks of caprock qualities. When a closed structure with the sandstone capped by shale is found, gas can be injected and stored in the porous sand. This type of storage operation is described as aquifer storage in that gas displaces water in an aquifer. The water just moves away from the gas injection well by compressing the water into the formation as the pressure rises.

Katz and Coats state that the development of aquifers into gas storage reservoirs includes the location of the underground structure and a determination of the quality of caprock.[14] To test the caprock, water is pumped into the porous media to find if a pressure differential across the caprock will cause water movement through it. Once a structure has been located and all signs point to an impermeable caprock, pilot gas injections are made to initiate the gas bubble and further test the caprock. Such gas injection involves gas pressures above the initial aquifer pressure to make the water move. Development of such a gas reservoir ready to serve a storage demand may well take two to four years to initiate and the project may increase in size over a period of ten years or more.

Once the aquifer storage area is developed, it will operate like a normal gas field which has a comparable degree of water drive. Sometimes caprocks permit gas migration upward and gas is collected in shallower strata. Although every operator hopes the caprock for his aquifer storage reservoir will not leak, it is possible to operate successfully when gas leaks through the caprock. Gas collected in upper strata can be allowed to accumulate and be produced in the winter time. Continuous recycling of leaked gas from an upper collection zone back to the storage zone is a mechanism of maintaining operation.

FOOTNOTES

[1]Robert A. Wattenbarger, "Maximization of Seasonal Withdrawals from Gas Storage Reservoirs," Society of Petroleum Engineers of AIME, SPE # 2406, p. 1.

[2]Dorsey Hager, *Fundamentals of the Petroleum Industry* (New York, 1939), Chapter VII.

[3]George Sell, *The Petroleum Industry* (New York, 1963), p. 23ff.

[4]Donald L. Katz and Keith H. Coats, *Underground Storage of Fluids* (Michigan, 1968), p. 2ff.

[5]*Ibid*., p. 4.

[6]*Ibid*., p. 5.

[7]Sell, p. 20.

[8]Paul R. Stewart, "Low Permeability Gas Well Performance at Constant Pressure," Society of Petroleum Engineers of AIME, SPE # 2604, p. 2.

[9]Katz and Coats, p. 10.

[10]*Ibid*., p. 150.

[11]Michael A. Stoner, "Steady State Analysis of Gas Production, Transmission and Distribution Systems," Society of Petroleum Engineers of AIME, SPE # 2554, p. 1.

[12]Mohamed Mortada, "A Practical Method for Treating Oil-field Interference in Water Drive Reservoirs," *Petroleum Transactions, AIME* (1955), p. 218.

[13]*Ibid*.

[14]Katz and Coats, p. 39ff.

CHAPTER III

EQUATIONS OF FLOW--SINGLE WELL RESERVOIRS

Linear Programming Basics

The flow of gas in reservoirs has been treated in literally hundreds of technical papers over the past thirty years. A wealth of field data has been presented along with increasingly complex mathematical treatments.[1] The equations used in this paper have been selected as those pertinent to the steady state condition and then adapted, by various assumptions, to conform to the restrictions involved in linear programming.

The basic problems solved by linear programming are those of maximizing or minimizing some linear objective function subject to one or more linear constraints.[2] In more general terms:

Maximize or Minimize

$$E = \sum_{j=1}^{n} c_m x_j \qquad (1)$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad (2)$$

where there are m such restrictions, and i takes the values from i = 1 for the first restriction to i = m for the m^{th}. Attached to these constraints is the nonnegativity requirement. That is:

$$x_i \geq 0 \qquad \text{for all } i . \qquad (3)$$

This states that none of the variables may be negative.

For the two examples in this paper, the objective function and its constraints will be derived and the solution will be determined algebraically through the use of one of the library programs stored in the Oklahoma State University IBM 360 MOD 50 digital computer. One should bear in mind that the same examples may be solved graphically, providing that the number of variables does not exceed three, with the solution appearing at one of the corners of the convex set.

Gas Properties and Assumptions Used In Models

The properties of natural gas products stored in the earth are generally well known for use in engineering calculations. These properties include densities, viscosities, and effect of expansion on cooling and hydrate forming conditions.

The density of a natural gas is treated by use of the gas law, including the compressibility factor:

$$PV = ZnRT = \frac{ZW}{29\ G}\ RT \qquad (4)$$

where

P = pressure, psia

V = volume, cu. ft.

Z = compressibility factor (dimensionless)

n = pound moles

R = gas constant = 10.73

T = absolute temperature, ^{o}R

W = pounds

G = gas gravity, molecular weight/29.0 .

When Equation (4) is written for a pound of fluid, W = 1 and V becomes the specific volume, cu.ft./lb.

The viscosities of natural gases have been measured and found to be a function of the gas gravity. Charts for the viscosities of various gases as a function of temperature have been prepared and may be found in any handbook of natural gas engineering.[3]

With regard to the flow of gases through porous media, the present state of knowledge is far from being fully developed. The difficulty lies in the non linearity of partial differential equations which describe both real and ideal gas flow. The solutions which are available consist of approximate analytical solutions, graphical solutions, analogue solutions, and numerical solutions.

The earliest attempt to solve this problem involved the method of successions of steady states proposed by Muskat.[4] Approximate analytical solutions were obtained by linearizing the flow equation for ideal gas to yield a diffusivity-type equation. Such solutions, though widely used and easy to apply to engineering problems are of limited value because of idealized assumptions and restrictions imposed upon the flow equation.

Numerical methods using finite difference equations and digital computing techniques have been used extensively for solving both ideal and real gas equations. The most important contribution to the theory of flow of ideal gases through porous media was the conclusion reached by Aronofsky and Jenkins that solutions for the liquid flow case could be used to generate approximate solutions for constant rate production of ideal gases.[5]

The mechanism of fluid flow through porous medium is governed by the physical properties of the matrix, geometry of flow, PVT (Pressure-Volume-Temperature) properties of the fluid and pressure distribution within the flow system. In deriving the majority of the flow equations and establishing the solutions, the following assumptions are made. The medium is homogeneous, the flowing gas is of constant composition and the flow is laminar and isothermal. The equations of flow which will be used in this paper will be those derived by VanEverdingen and Hurst as applied to Darcy's laws of permeability.[6]

Bottom hole pressures in gas wells must take into account the changing density with pressure but can be computed from well head pressures, gas properties, well temperature, and depth.

The flowing pressure or the pressure drop during flow includes the static pressure (bottom hole pressure) at the mean flowing pressure and temperature and the friction loss of the fluid flowing in a pipe.

For a given field with fixed well size, well depth, well temperature, and gas gravity, it is possible to compute the static pressure gradient and the flowing gradients for a series of flow rates. In this way it is not necessary to go through the calculations of either static or flowing pressure gradient each time the value is desired. Figure 5 is such a chart for the Hersher-Mt. Simon reservoir based on well head conditions.[7] Such charts can be prepared based on bottom hole pressures to be used when converting flowing bottom hole pressure to flowing well head pressure.

Generalized Model Equations

The present method of representing permeability was established in 1935 by a scientist named Darcy in whose behalf the units of permeability were credited--darcy and millidarcy. A cube of rock one centimeter on an edge that passes fluid of one centipoise viscosity (water at 68°F) between two faces at a rate of one cubic centimeter per second when the pressure drop is one atmosphere (14.7 psi) is said to have a permeability of a darcy. Since few formations used to store gas have permeabilities this high, the term millidarcy of 1/1000 of a darcy is generally used.

Although there is a tendency of increasing permeability with increased porosity, it is by no means universal. A porous medium can easily have a high porosity but have the

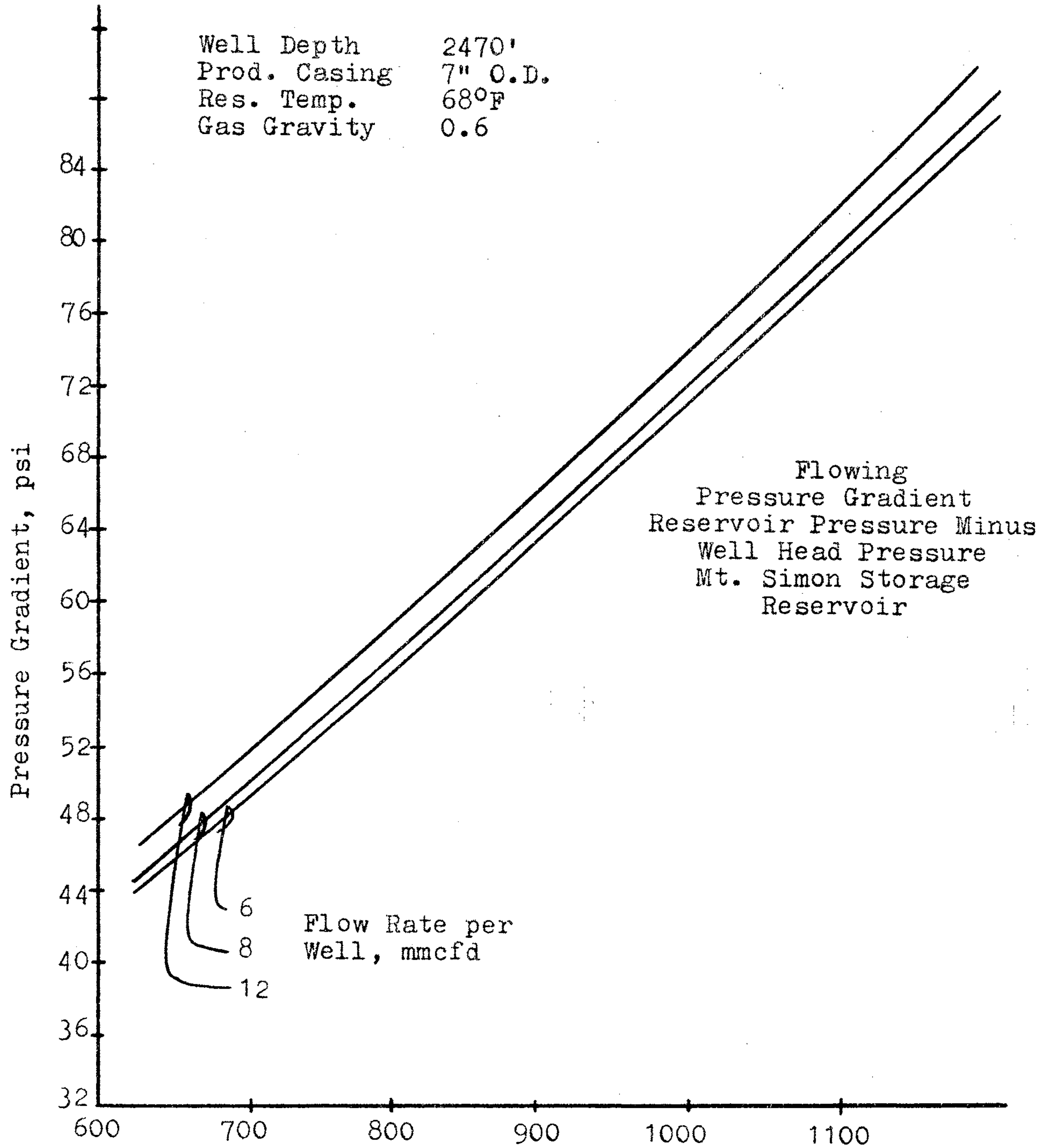


Figure 5. Pressure Gradients for Mt. Simon Reservoir

passage so circuitous or restricted that the permeability is very low. In many formations, the gas must flow into and out of the wells through the matrix rock. In these cases, rock having a permeability for the storage zone as low as one millidarcy has a good chance of being marginal or unacceptable. Rocks with permeability of 100 millidarcy are usually quite acceptable for gas flow while permeabilities above 1000 millidarcy or a darcy are excellent and quite rare.

Darcy's law used in measuring permeability assumes the flow rate is proportional to pressure drop. The equation defining Darcy flow is:[8]

$$q = \frac{1000\ kA}{14.7\mu} \frac{(P_1 - P_2)}{L} \qquad (5)$$

where:

q = flow rate, cu. cm./sec.

k = millidarcy or .001 darcy

A = cross-sectional area to flow, $cm.^2$

μ = fluid viscosity, centipoise

P = pressure, lb. per sq. in. absolute

L = length of core, cm.

Subscript 1 = at entrance to core

Subscript 2 = at exit from core.

At higher flow rates, the flow does not increase as rapidly as the pressure drop increases, and non darcy or turbulent flow is said to occur. The quadratic equation which represents this flow through cores is:

$$-\frac{dP}{dL} = \frac{\mu V}{k} + \beta \rho V^2 \qquad (6)$$

where

$-\frac{dP}{dL}$ = pressure drop per unit length

V = velocity

ρ = fluid density

β = turbulence factor.

This equation with $\beta = 0$ can be integrated for steady state flow to relate flow rate and pressure drop as:

$$Pe^2 - Pw^2 = \frac{1424\ T2\mu q}{kh} \ln \frac{r_e}{r_w}$$

where:

q = production rate, mcf/day

k = permeability, md.

h = reservoir thickness, ft.

Pe = pressure at exterior radius of reservoir, r_e, psia

Pw = well pressure at well radius, r_w, psia

T = reservoir temperature, $^oR = 460 + {^oF}$

μ = gas viscosity, cp

Z = gas compressibility factor, average between Pe and Pw

r_e = exterior reservoir radius, ft., a mean radius approximated in non-circular reservoirs

r_w = well radius, ft.

Van Everdingen and Hurst then took Darcy's equations for flow and related the unsteady state well pressure to production or injection rate for flow of slightly compressible liquids. One of several cases that they considered was the variation of well pressure with time caused by production at a constant rate starting from a shut in condition of uniform pressure. The counterpart to their liquid flow solution for the case of gas flow is:[9]

$$Po - Pw = q\, f(t_D) \qquad (7)$$

where:

Po = initial reservoir pressure, psia

Pw = flowing well pressure at some later time, t, psia

$f(t_D)$ = dimensionless pressure drop (influence function)

t_D = dimensionless time.

The dimensionless time, t_D, describes how the well pressure Pw changes with time when the flow rate changes abruptly from 0 to a constant rate, q. Mathematically, t_D is defined as:

$$t_D = \frac{.00633\ k\bar{P}t(\text{days})}{\mu\ \Phi\ r_w} = \frac{.000264\ k\bar{P}t(\text{hrs})}{\mu\ \Phi\ r_w} \qquad (8)$$

where:

$\bar{P}$ = mean pressure (psia) between Pw and Po

Φ = fractional porosity.

An assumption implicit in Equation (7) is that the drawdown Po - Pw is not large. The use of an average pressure $\bar{P}$ in the definition of dimensionless time is valid only for a small drawdown.

If the dimensionless time happens to be greater than 100 the $f(t_D)$ may be approximated by:[10]

$$f(t_D) = \tfrac{1}{2}(\ln t_D + .809) \, . \tag{9}$$

This approximation is good for an infinite reservoir or for a finite reservoir before the pressure transient reaches the outer boundary. The restriction of $t_D > 100$ for validity of Equation (9) is of little concern in practice since a dimensionless time of 100 generally corresponds to a very small real time.

Equation (7) represents the technology of flow through a porous media--the behavior of underground reservoirs. Physically, the equation describes an underground reservoir which contains gas at an initial pressure Po. At time $t = 0$, gas production commences from a well bore at a constant flow rate, q. This equation thus gives the flowing well pressure Pw at any later time, t. It is assumed also that the function $f(t_D)$ has been precalculated for discrete values of time, t_i. Thus, the equation expresses a linear relationship between Pw and q.

This equation of flow will first be applied as a scheduling problem of optimal gas withdrawal from a combined system of wells wherein there exists only one well per reservoir as depicted in Figure 6.

It will be assumed that each reservoir contains a single ideal gas in a homogenous porous medium under the influence of an infinite water drive. This assures a uniform

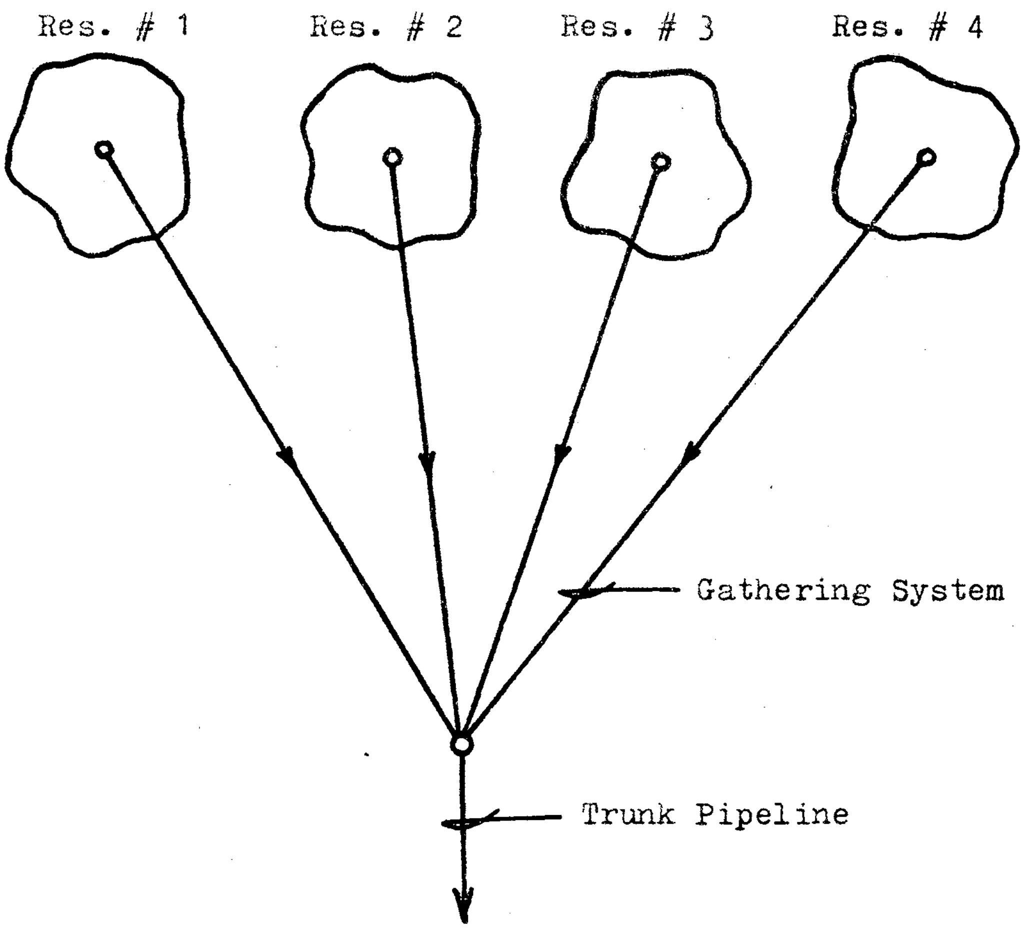


Figure 6. Single Well--Multi Reservoir Schematic

gas with a uniform reservoir pressure.

The well schedule should result in maximum profit where the system is subject to certain restrictions. Equation (7) is superimposed to a general form to permit variations in q for different lumped wells and for a series of time periods representing times of different demands.

$$Pw_j = Po_j - \sum_{k=1}^{i} (q_{k,j} - q_{k-1,j}) \cdot f_j(t_i - t_{k-1}) \qquad (10)$$

where:

$q_{i,j}$ = average production rate of the j^{th} reservoir during the i^{th} time period

Po_j = original pressure of the j^{th} reservoir

t_i = a specified time

Pw_j = well pressure of well in j^{th} reservoir.

This linear program is constructed on the basis that Pw_j at some time, t_i, and $q_{i,j}$ represent the unknown variables Hence, the well pressure can take on any value provided it does not go below some specified value, D_j at time t_i. This then gives the first linear constraint

$$Pw_j \geq D_j \qquad (11)$$

where:

$$j = 1,2,3, \ldots, J .$$

The next set of constraints is derived from the overall material balance of the system which merely states that the cumulative production cannot be greater than the reserves:

$$\sum_{i=1}^{I} q_{i,j} \Delta t_i \leq B_j \qquad (12)$$

where:

$\Delta t_i = t_i - t_{i-1}$

B_j = the removable oil in place for the j^{th} reservoir

$j = 1, 2, 3, \ldots, J.$

Next it is required that the production rate in any time period should not exceed the pipeline capacity, R_i

$$\sum_{j=1}^{J} q_{i,j} \Delta t_i \leq R_i \qquad (13)$$

where:

$\Delta t_i = t_i - t_{i-1}$

R_i = the flow of gas through the trunk pipeline in the i^{th} time period.

In periods of low demand, R_i may be reduced directly and considered to be the actual demand.

The problem now becomes one of decifering from all of the sets of values of the $q_{i,j}$ which satisfy the constraints (11), (12), (13), that particular set for which the profit is the largest. That is, the equation to maximize is:

$$\Psi = \sum_{j=1}^{J} \sum_{i=1}^{I} q_{i,j} d_{i,j} \Delta t_i \qquad (14)$$

where profit is defined to include all economic factors that are involved in producing and selling the gas to a pipeline facility. All of the economic factors pertaining to revenues and expenses are contained in the profitability index, $d_{i,j}$. The profit function Ψ has the dimension of dollars and it represents the total profit over the entire lifetime of the project.

Summarizing, the generalized model for the single well per reservoir problem at any time t is represented by the following linear program:

Objective function:

$$\Psi = \sum_{j=1}^{J} \sum_{i=1}^{I} q_{i,j} d_{i,j} \Delta t_i$$

subject to these constraints

$$D_j \leq Po_j - \sum_{k=1}^{i} (q_{k,j} - q_{k-1,j}) \cdot f_j(t_i - t_{k-1})$$

$$B_j \geq \sum_{i=1}^{I} q_{i,j} \Delta t_i$$

$$R_i \geq \sum_{j=1}^{J} q_{i,j} \Delta t_i \quad .$$

A Numerical Example

As an example, consider the following situation. Located in each of two reservoirs is a single well. For each reservoir arbitrary values are assigned for permeability, viscosity, outer radius, initial pressure, etc. It will also be assumed that all of the reservoir parameters will remain constant throughout the time period being considered. The well pressures (in both reservoirs) are not allowed to go below some arbitrary value, such as 1 atmosphere. Thus, there are four constraints on the production rates, $q_{i,j}$ (one for each of two reservoirs for each of two time periods). The set of four inequalities are:

$$Pw_1(t_1) \geq D_1 \quad \text{where} \quad D_1 = 1 \text{ atm.} \tag{15}$$

$$Pw_2(t_1) \geq D_2 \quad \text{where} \quad D_2 = 1 \text{ atm.} \tag{16}$$

$$Pw_1(t_2) \geq D_1 \quad \text{where} \quad D_1 = 1 \text{ atm.} \tag{17}$$

$$Pw_2(t_2) \geq D_2 \quad \text{where} \quad D_2 = 1 \text{ atm.} \tag{18}$$

Since the right hand of Equation (10) can be substituted for the Pw_j of Equation (11):

$$14.7 = 1 \text{ atm} \leq Po_1 - [(q_{1,1} - q_{0,1}) \cdot f_1(t_1 - t_0)] \quad \text{for} \quad (i = 1 \quad j = 1) \tag{19}$$

$$14.7 = 1 \text{ atm} \leq Po_2 - [(q_{1,2} - q_{0,2}) \cdot f_2(t_1 - t_0)] \quad \text{for} \quad (i = 1 \quad j = 2) \tag{20}$$

$$14.7 = 1 \text{ atm} \leq Po_1 - [(q_{1,1} - q_{0,1}) \cdot f_1(t_1 - t_0) + (q_{2,1} - q_{1,1}) \cdot f_1(t_2 - t_1)] \quad \text{for} \quad (i = 2 \quad j = 1) \tag{21}$$

$$14.7 = 1 \text{ atm} \leq Po_2 - [(q_{1,2} - q_{0,2}) \cdot f_2(t_1 - t_0) + (q_{2,2} - q_{1,2}) \cdot f_2(t_2 - t_1)] \quad \text{for} \quad (i = 2 \quad j = 2) \,. \tag{22}$$

These above four inequalities constrain the individual reservoir production rates. It will be remembered that initial flow values are zero as are t_0's. Therefore, the equations of constraint reduce to

$$14.7 \leq Po_1 - [(q_{1,1}) \cdot f_1(t_1)] \tag{23}$$

$$14.7 \leq Po_2 - [(q_{1,2}) \cdot f_2(t_1)] \tag{24}$$

$$14.7 \leq Po_1 - [(q_{1,1}) \cdot f_1(t_1) + (q_{2,1} - q_{1,1}) \cdot f_1(t_2 - t_1)] \tag{25}$$

$$14.7 \leq Po_2 - [(q_{1,2}) \cdot f_2(t_1) + (q_{2,2} - q_{1,2}) \cdot f_2(t_2 - t_1)] \,. \tag{26}$$

Let it be assumed for the example:

Reservoir 1	Reservoir 2
Po_1 = 1000 psia	Po_2 = 1200 psia
k_1 = 20 md.	k_2 = 25 md.
μ_1 = .015 cp.	μ_2 = .017 cp.
Φ_1 = .12 fractional porosity	Φ_2 = .14 fractional porosity
r_{w_1} = .25 ft.	r_{w_2} = .25 ft.

From Van Everdingen and Hurst's equations for dimensionless time t_D, Equation (8), it follows that for time periods of one year where $t_0 = 0$, $t_1 = 1$ yr., $t_2 = 2$ yrs:

For reservoir # 1:

$$t_1 = \frac{.00633\ k\bar{P}t(\text{days})}{\mu\Phi r_w^2}$$

$$= \frac{.00633(20\text{ md})(1000\text{ psia})(365\text{ days})}{(.015\text{ cp})(.12)(.0625\text{ ft}^2)}$$

$$= 41.1 \times 10^7$$

$$t_2 = \frac{.00633(20\text{ md})(1000\text{ psia})(730\text{ days})}{(.015\text{ cp})(.12)(.0625\text{ ft}^2)}$$

$$= 82.2 \times 10^7$$

For reservoir # 2:

$$t_1 = \frac{.00633\ k\bar{P}t(\text{days})}{\mu\Phi r_w^2}$$

$$= \frac{.00633(25\text{ md})(1200\text{ psia})(365\text{ days})}{(0.17\text{ cp})(.14)(.0625\text{ ft}^2)}$$

$$= 46.6 \times 10^7$$

$$t_2 = \frac{.00633(25\ \text{md})(1200\ \text{psia})(730\ \text{days})}{(.017\ \text{cp})(.14)(.0625\ \text{ft}^2)}$$

$$= 93.2 \times 10^7 \quad .$$

Solving for the respective influence functions from Equation (9) since $t_D > 100$:

$$f_1(t_1) = \tfrac{1}{2}(\ln 41.1 \times 10^7 + .809)$$

$$= 10.32$$

$$f_1(t_2) = \tfrac{1}{2}(\ln 46.6 \times 10^7 + .809)$$

$$= 10.38$$

$$f_2(t_1) = \tfrac{1}{2}(\ln 46.6 \times 10^7 + .809)$$

$$= 10.38$$

$$f_1(t_2 - t_1) = f_1(82.2 \times 10^7 - 41.1 \times 10^7)$$

$$= \tfrac{1}{2}(\ln 41.1 \times 10^7 + .809)$$

$$= 10.32$$

$$f_2(t_2 - t_1) = f_2(93.2 \times 10^7 - 46.6 \times 10^7)$$

$$= \tfrac{1}{2}(\ln 46.6 \times 10^7 + .809)$$

$$= 10.38 \quad .$$

Substituting these values into the above four constraint equations, Equations (23), (24), (25), and (26) yield:

$$14.7 \leq 1000 - [q_{1,1}(10.32)] \qquad (27)$$

$$14.7 \leq 1200 - [q_{1,2}(10.38)] \qquad (28)$$

$$14.7 \leq 1000 - [q_{1,1}(10.32) + (q_{2,1} - q_{1,1})(10.32)] \quad (29)$$

$$14.7 \leq 1200 - [q_{1,2}(10.38) + (q_{2,2} - q_{1,2})(10.38)]. \quad (30)$$

Rearranging:

$$14.7 \leq 1000 - 10.32\ q_{1,1} \quad (27a)$$

$$14.7 \leq 1200 - 10.38\ q_{1,2} \quad (28a)$$

$$14.7 \leq 1000 - 10.32\ q_{2,1} \quad (29a)$$

$$14.7 \leq 1200 - 10.38\ q_{2,2}\ . \quad (30a)$$

The material balance constraints from Equation (12) yield:

$$q_{1,1}(t_1 - t_0) + q_{2,1}(t_2 - t_1) \leq B_1$$

$$q_{1,2}(t_1 - t_0) + q_{2,2}(t_2 - t_1) \leq B_2\ .$$

Assume for this example that:

$$B_1 = 60 \times 10^6\ ft^3$$

$$B_2 = 70 \times 10^6\ ft^3$$

and since $t_0 = 0$, substitution gives:

$$q_{1,1}(41.1 \times 10^7) + q_{2,1}(41.1 \times 10^7) \leq 60 \times 10^6 \quad (31)$$

$$q_{1,2}(46.6 \times 10^7) + q_{2,2}(46.6 \times 10^7) \leq 70 \times 10^6\ . \quad (32)$$

The final constraints are for the production rates. From Equation (13):

$$q_{1,1}(t_1 - t_0) + q_{1,2}(t_1 - t_0) \leq R_1$$

$$q_{2,1}(t_2 - t_1) + q_{2,2}(t_2 - t_1) \leq R_2\ .$$

Assume:

$$R_1 = 100{,}000 \text{ ft}^3/\text{day}$$
$$R_2 = 200{,}000 \text{ ft}^3/\text{day} .$$

Substitution gives:

$$q_{1,1}(41.1 \times 10^7) + q_{1,2}(46.6 \times 10^7) \leq 100{,}000 \qquad (33)$$
$$q_{2,1}(41.1 \times 10^7) + q_{2,2}(46.6 \times 10^7) \leq 200{,}000 . \qquad (34)$$

In sum, the number of constraints altogether are IJ + I + J, or in this case 2(2) + 2 + 2 = 8 constraint equations. They are in order of derivation

$$14.7 \leq 1000 - 10.32\, q_{1,1} \qquad (27a)$$
$$14.7 \leq 1200 - 10.38\, q_{1,2} \qquad (28a)$$
$$14.7 \leq 1000 - 10.32\, q_{2,1} \qquad (29a)$$
$$14.7 \leq 1200 - 10.38\, q_{2,2} \qquad (30a)$$

$$60 \times 10^6 \geq 41.1 \times 10^7\, q_{1,1} + 41.1 \times 10^7\, q_{2,1} \qquad (31)$$
$$70 \times 10^6 \geq 46.6 \times 10^7\, q_{1,2} + 46.6 \times 10^7\, q_{2,2} \qquad (32)$$
$$100{,}000 \geq 41.1 \times 10^7\, q_{1,1} + 56.6 \times 10^7\, q_{1,2} \qquad (33)$$
$$200{,}000 \geq 41.1 \times 10^7\, q_{2,1} + 46.6 \times 10^7\, q_{2,2} . \qquad (34)$$

The problem now is to find from all the sets of values of $q_{i,j}$ which satisfy the above constraints that particular set for which the profit is the largest. That is, one must maximize Equation (14):

$$\Psi = q_{1,1}d_{1,1}(t_1 - t_0) + q_{2,1}d_{2,1}(t_2 - t_1)$$
$$+ q_{1,2}d_{1,2}(t_1 - t_0)$$
$$+ q_{2,2}d_{2,2}(t_2 - t_1) .$$

The following table represents the assumed profit per ft^3 of gas from the two sources over the two time periods. In any practical application of this work, a careful economic study would have to be made in order to estimate unit profits for time periods in the future.

	Reservoir	
Time	1	2
1	.00010	.00013
2	.00017	.00018

$d_{i,j} = \$/ft^3$

= potential profit

Substituting these values gives the profit equation

$$\Psi = q_{1,1}(.00010)(41.1 \times 10^7) + q_{2,1}(.00013)(41.1 \times 10^7) + q_{1,2}(.00017)(46.6 \times 10^7) + q_{2,2}(.00018)(46.6 \times 10^7)$$

$$\Psi = 4.1 \times 10^4 \; q_{1,1} + 5.33 \times 10^4 \; q_{2,1} + 7.91 \times 10^4 \; q_{1,2} + 8.39 \times 10^4 \; q_{2,2} \qquad (35)$$

or the equation to be maximized subject to the constraints (27a)-(34).

The algebraic solution of this model will contain the production rate for each of the two wells for each time period. More numerous reservoirs would not involve any new general equations, however, the constraint equations would become more complicated, as well as the objective function. The actual numerical solution obtained from the IBM 360 MOD 50 computer and an analysis of the results appears in Appendix C.

FOOTNOTES

[1]A. F. Van Everdingen, "The Application of the Laplace Transformation to Flow Problems in Reservoirs," Petroleum Transaction, AIME (1949), p. 305.

[2]Richmond, p. 317.

[3]C. George Segeler, Marion D. Ringler, and Evelyn M. Kafka, Gas Engineers' Handbook (New York, 1965).

[4]R. Al-Hussainy, H. J. Ramey, and P. B. Crawford, "The Flow of Real Gases Through Porous Media," Journal of Petroleum Technology (1966), p. 264.

[5]J. S. Aronofsky and R. Jenkins, "A Simplified Analysis of Unsteady Radial Gas Flow," Petroleum Transactions, AIME (1954), p. 149.

[6]Van Everdingen, p. 306.

[7]Katz and Coats, p. 148.

[8]Ibid., p. 58.

[9]Ibid., p. 152.

[10]Ibid., p. 153.

CHAPTER IV

EQUATIONS OF FLOW--MULTI WELL RESERVOIRS

Generalized Model Equations

The first model in Chapter III is quite limited even without considering the required assumptions as homogenous gas, etc., for very seldom is only one well used per field. Consequently, the logical extension of the model would be to consider reservoirs that contain two or more wells. Accordingly, one must then re-examine the derived relations between well pressures and flow rate; that is, the influence function.

Figure 7 depicts a reservoir with two wells l and m. Since the pressure decline in each well is influenced by the production in the other well, a generalized influence function will be introduced to describe the interaction. When developed, it will be assumed that the same generalized function will be applicable for any number of wells.

Writing the single well influence function for the l^{th} well in the j^{th} reservoir:

$$Po_j - Pw_{i,j,l}(t_i) = q_{i,j,l}f_{j,l}(t_i) \qquad (36)$$

where

Po_j = the original pressure in the j^{th} reservoir

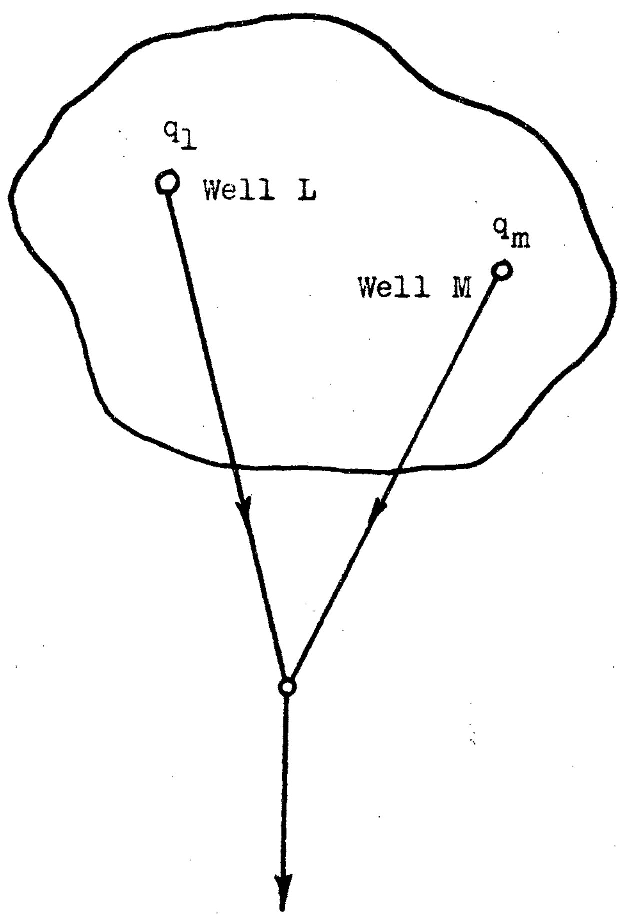


Figure 7. Single Reservoir--
Multi Well Schematic

$Pw_{i,j,l}$ = the well pressure in the i^{th} time period, the j^{th} reservoir and the l^{th} well

$q_{i,j,l}$ = a constant flow rate in the j^{th} reservoir and the l^{th} well

$f_{j,l}(t_i)$ = the influence function.

The generalized influence function will then be of the form:

$$Po_j - Pw_{i,j,l}(t_i) = q_{i,j,m}g_{j,l,m}(t_i) \qquad (37)$$

or, rearranging:

$$\frac{Po_j - Pw_{i,j,l}(t_i)}{q_{i,j,m}} = g_{j,l,m}(t_i)$$

where

$q_{i,j,m}$ = the constant flow rate at well m

$g_{j,l,m}(t_i)$ = the general influence function. For the i^{th} time period and the j^{th} reservoir, it expresses the partial pressure in the l^{th} well due to a constant flow rate in well m.

This might be understood more easily if the equation were written:

$$Pw_{i,j,l}(t_i) = Po_j - q_{i,j,m}g_{j,l,m}(t_i) \ . \qquad (38)$$

And then re-written for the effects of the flow from the well l on well m:

$$Pw_{i,j,m}(t_i) = Po_j - q_{i,j,l}g_{j,m,l}(t_i) \qquad (39)$$

where:

$Pw_{i,j,m}(t_i)$ = well pressure in the i^{th} time period, the

j^{th} reservoir and the m^{th} well

$q_{i,j,l}$ = constant flow rate at well l

$g_{j,m,l}(t_i)$ = the general influence function for the i^{th} time period, the j^{th} reservoir; it expresses the partial pressure in the m^{th} well due to a constant flow rate at the l^{th} well.

Equations (38) and (39) will be combined through superposition as described in Chapter II to yield Equation (40). This equation represents the well pressure in all wells, all reservoirs, all time periods, and for any arbitrary flow rates. A typical example is depicted in Figure 8.

The equation representing the multi well reservoir is:

$$Pw_{i,j,l} = Po_j - \sum_{m=1}^{M_j} \sum_{k=1}^{i} [(q_{k,j,m} - q_{k-1,j,m}) g_{j,l,m}(t_i - t_{k-1})] \; . \tag{40}$$

Figure 8 shows several reservoirs, $j = 1,2,3, \ldots, J$ and within each reservoir, several wells, $l = 1,2,3, \ldots, M$ are producing gas. It will be assumed that each reservoir has its own pipeline gathering system which feeds into its own trunk pipeline. The trunk pipelines merge into a major transportation link which transports the gas to a refinery or final destination.

The dimensionless time, t_D, will again be defined at t_i or the dimensionless time t_i. From the first model:

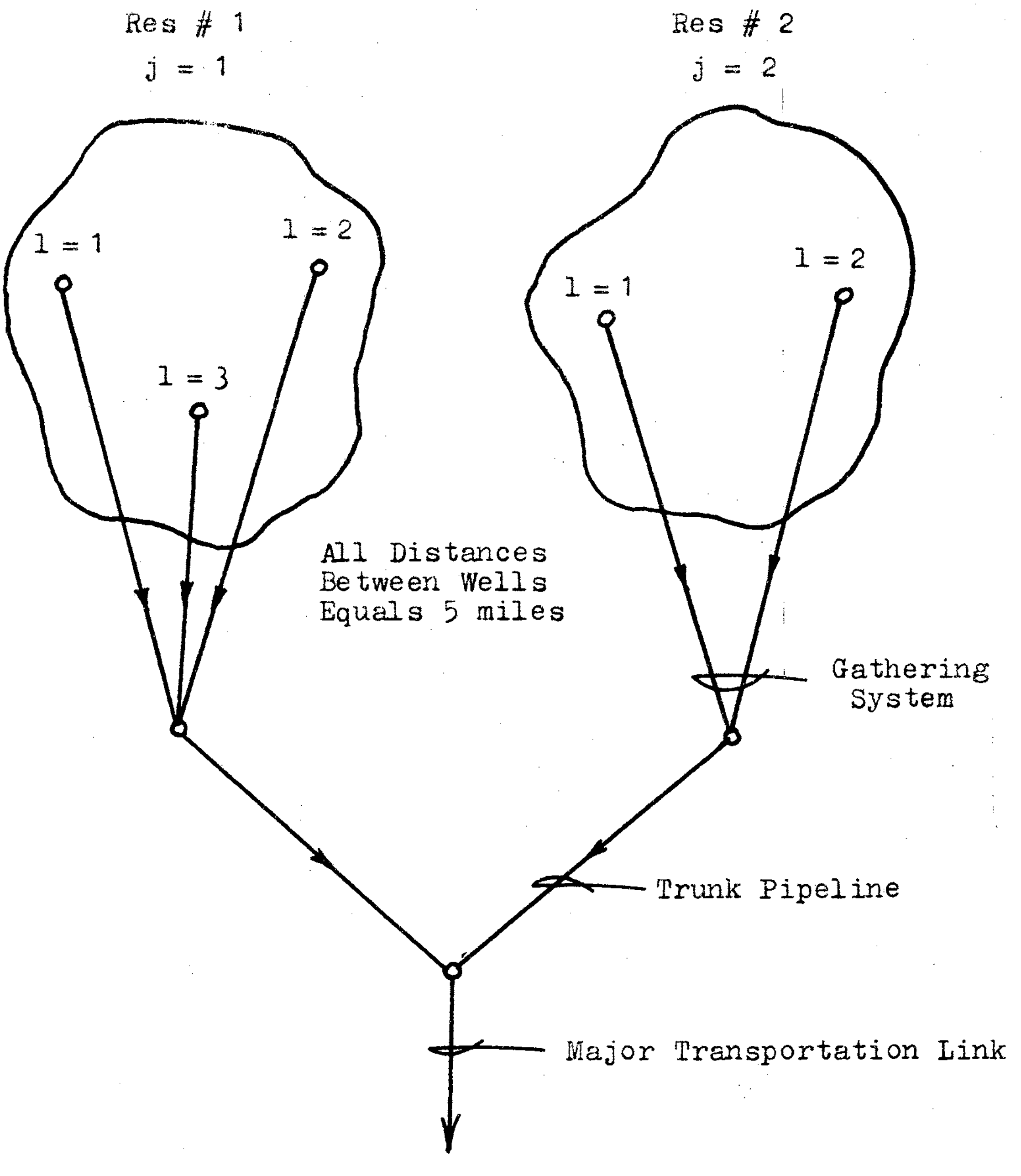


Figure 8. Multi Well--Multi Reservoir Schematic

$$t_D = \frac{.00633\ k\bar{P}t(\text{days})}{\mu\Phi r_w} = \frac{.000264\ k\bar{P}t(\text{hrs})}{\mu\Phi r_w} \tag{41}$$

where:

$\bar{P}$ = mean pressure (psia) between Pw and Po

Φ = fraction porosity

μ = gas viscosity, cp.

k = permeability, md.

r_w = well radius, ft.

t = time (hrs or days).

Again, it will be assumed that the drawdown pressure, Po - Pw is not large. J. S. Aronofsky and A. S. Lee have determined that the influence function for interference may be approximated by:[1]

$$g_{j,l,m}(t_i) = \tfrac{1}{2}[\ln \frac{4t}{r^2_{l,m}} - .57722] \tag{42}$$

for $$\frac{4t}{r^2_{l,m}} \geq 2000$$

where:

$r_{l,m}$ = the well spacing between two wells defined as the ratio:

$$\frac{\text{distance between the two well centers}}{\text{well drawdown radius}}.$$

This approximation is good for an infinite reservoir with constant pressure Po.

The constraints to the model are much similar to those of the single well per reservoir model. More specifically: Equation (43) requires that the well pressure must always be larger than a stated constant.

$$Pw_{i,j,l} \geq D_j \quad \text{where} \quad i = 1,2, \ldots, I \quad j = 1,2, \ldots, J \quad l = 1,2, \ldots, M . \qquad (43)$$

Equation (44) requires that the cummulative production from all wells in a reservoir over the entire time period cannot exceed the recoverable reserves for that reservoir.

$$\sum_{l=1}^{M_j} \sum_{i=1}^{I} [q_{i,j,l} \Delta t_i] \leq B_j \quad \text{where} \quad j = 1,2, \ldots, J . \qquad (44)$$

Equation (45) states that all the production from one reservoir should not exceed in any one time period, the capacity of that particular trunk pipeline:

$$\sum_{l=1}^{M_j} [q_{i,j,l} \Delta t_i] \leq R_{i,j} \quad \text{where} \quad i = 1,2, \ldots, I \quad j = 1,2, \ldots, J . \qquad (45)$$

Equation (46) requires that the flow from all the trunk pipelines must not be greater than the capacity of the major transportation link. This equation also acts to cover varying demands. All that is required is to change W_i for different demands.

$$\sum_{j=1}^{J} \sum_{l=1}^{M_j} [q_{i,j,l} \Delta t_i] \leq W_i \quad \text{where} \quad i = 1,2, \ldots, I . \qquad (46)$$

Finally, Equation (47) is the functional that must be maximized:

$$\Psi = \sum_{l=1}^{M_j} \sum_{j=1}^{J} \sum_{i=1}^{I} [(q_{i,j,l}\Delta t_i)d_{i,j,l}] \quad . \tag{47}$$

This model now enables one to consider multi well models that permit interactions between wells. The disadvantage is that the new linear program contains more inequalities than previously, specifically (I·J·M+I·J+I+J) restraints. However, in many cases, this model will be still of tractable size.

Again, this model will only be valid for wells that have been drilled; that is, a completely developed field.

In summary, the model will be to maximize Equation (47) subject to the constraint Equations (43), (44), (45), and (46).

A Numerical Example

To demonstrate the versatility of this model, an example problem will now be solved based on the developed field depicted in Figure 8. For simplicity, only one time period will be considered with its own demand. Likewise, as shown, only two reservoirs will be used with three wells in the first and two in the second.

First, from Equation (43), the following constraints are enumerated:

$$\left.\begin{array}{l} \left.\begin{array}{l} Pw_{1,1,1} \geq D_1 \\ Pw_{1,1,2} \geq D_1 \\ Pw_{1,1,3} \geq D_1 \end{array}\right\rangle j = 1 \\ \left.\begin{array}{l} Pw_{1,2,1} \geq D_2 \\ Pw_{1,2,2} \geq D_2 \end{array}\right\rangle j = 2 \end{array}\right\rangle t = 1$$

As before, these inequalities can be incorporated into Equation (40) as follows

For j = 1:

$$D_1 \leq Po_1 - \{[(q_{1,1,1} - q_{0,1,1})g_{1,1,1}(t_1 - t_0)] + [(q_{1,1,2} - q_{0,1,2})g_{1,1,2}(t_1 - t_0)] + [(q_{1,1,3} - q_{0,1,3})g_{1,1,3}(t_1 - t_0)]\}$$

which reduces to: (since $q_{o,j,l}$ and $t_0 = 0$)

$$D_1 \leq Po_1 - \{[(q_{1,1,1})g_{1,1,1}(t_1)] + [(q_{1,1,2})g_{1,1,2}(t_1)] + [(q_{1,1,3})g_{1,1,3}(t_1)]\} \quad (48)$$

$$D_1 \leq Po_1 - \{[(q_{1,1,1} - q_{0,1,1})g_{1,2,1}(t_1 - t_0)] + [(q_{1,1,2} - q_{0,1,2})g_{1,2,2}(t_1 - t_0)] + [(q_{1,1,3} - q_{0,1,3})g_{1,2,3}(t_1 - t_0)]\}$$

which reduces to

$$D_1 \leq Po_1 - \{[(q_{1,1,1})g_{1,2,1}(t_1)] + [(q_{1,1,2})g_{1,2,2}(t_1)] + [(q_{1,1,3})g_{1,2,3}(t_1)]\} \quad (49)$$

$$D_1 \leq Po_1 - \{[(q_{1,1,1} - q_{0,1,1})g_{1,3,1}(t_1 - t_0)] + [(q_{1,1,2} - q_{0,1,2})g_{1,3,2}(t_1 - t_0)] + [(q_{1,1,3} - q_{0,1,3})g_{1,3,3}(t_1 - t_0)]\}$$

which reduces to:

$$D_1 \leq Po_1 - \{[(q_{1,1,1})g_{1,3,1}(t_1)] + [(q_{1,1,2})g_{1,3,2}(t_1)] + [(q_{1,1,3})g_{1,3,3}(t_1)] \quad (50)$$

For j = 2:

$$D_2 \leq Po_2 - \{[(q_{1,2,1} - q_{0,2,1})g_{2,1,1}(t_1 - t_0)] + [(q_{1,2,2} - q_{0,2,2})g_{2,1,2}(t_1 - t_0)]\}$$

which reduces to

$$D_2 \leq Po_2 - \{[(q_{1,2,1})g_{2,1,1}(t_1] + [(q_{1,2,2})g_{2,1,2}(t_1)]\} \quad (51)$$

$$D_2 \leq Po_2 - \{[(q_{1,2,1} - q_{0,2,1})g_{2,2,1}(t_1 - t_0)] + [(q_{1,2,2} - q_{0,2,2})g_{2,2,2}(t_1 - t_0)]\}$$

which reduces to

$$D_2 \leq Po_2 - \{[(q_{1,2,1})g_{2,2,1}(t_1)] + [(q_{1,2,2})g_{2,2,2}(t_1)]\} \quad (52)$$

Evaluating the dimensionless time for each reservoir using the following measured data and Equation (41)

<u>Reservoir 1</u>	<u>Reservoir 2</u>
Po_1 = 1000 psia	Po_2 = 1200 psia
k_1 = 20 md	k_2 = 25 md
μ_1 = .015 cp	μ_2 = .017 cp
Φ_1 = .12 fractional porosity	Φ_2 = .14 fractional porosity
r_{w_1} = .25 ft	r_{w_2} = .25 ft

For reservoir 1

$$t_1 = \frac{.00633(20 \text{ md})(1000 \text{ psi})(365 \text{ days})}{(.015 \text{ cp})(.12)(.0675 \text{ ft}^2)}$$

$$= 41.1 \times 10^7$$

For reservoir 2

$$t_1 = \frac{.00633(25\text{ md})(1200\text{ psi})(365\text{ days})}{(.015\text{ cp})(.12)(.0675\text{ ft}^2)}$$

$$= 46.6 \times 10^7$$

Assume that the well spacing is given as that in Figure 8. From Equation (42):

Distance = 5 miles = 26,400 ft

Well drawdown radius = 1055 ft

Reservoir 1	Reservoir 2
$r_{1,1} = 0$	$r_{1,1} = 0$
$r_{1,2} = \frac{26,400}{1055} = 250$	$r_{1,2} = 250$
$r_{1,3} = 250$	$r_{2,1} = 250$
$r_{2,1} = 250$	$r_{2,2} = 0$
$r_{2,2} = 0$	
$r_{2,3} = 250$	
$r_{3,1} = 250$	
$r_{3,2} = 250$	
$r_{3,3} = 0$	

Consequently, the respective influence functions will be:

$$g_{1,1,1} = g_{1,2,2} = g_{1,3,3} = g_{2,1,1} = g_{2,2,2} = 0$$

$$g_{1,1,2} = g_{1,1,3} = g_{1,2,1} = g_{1,2,3} = g_{1,3,1} = g_{1,3,2}$$

$$= \tfrac{1}{2}[\ln \frac{4(41.1 \times 10^7)}{(250)^2} - .57722]$$

$$= \tfrac{1}{2}[10.17732 - .57722]$$

$$= 4.80005$$

$$g_{2,1,2} = g_{2,2,1}$$

$$= \tfrac{1}{2}[\ln \frac{4(46.6 \times 10^7)}{(250)^2} - .57722]$$

$$= \tfrac{1}{2}[10.30226 - .57722]$$

$$= 4.86252$$

Let it be assumed that $D_1 = D_2 \geq 1$ atm = 14.7 psi. Substituting these values into Equations (48), (49), (50), (51), and (52) yields

$$14.7 \leq 1000 - \{[(q_{1,1,2})4.80] + [(q_{1,1,3})4.80]\}$$

or

$$14.7 \leq 1000 - [4.80\ q_{1,1,2} + 4.80\ q_{1,1,3}] \ . \qquad (48a)$$

Similarly:

$$14.7 \leq 1000 - [4.80\ q_{1,1,1} + 4.80\ q_{1,1,3}] \qquad (49a)$$

$$14.7 \leq 1000 - [4.80\ q_{1,1,1} + 4.80\ q_{1,1,2}] \qquad (50a)$$

$$14.7 \leq 1200 - [4.86\ q_{1,2,2}] \qquad (51a)$$

$$14.7 \leq 1200 - [4.86\ q_{1,2,1}] \ . \qquad (52a)$$

The next constraint from Equation (44) assuming that $B_1 = 60 \times 10^8\ ft^3$ and $B_2 = 70 \times 10^8\ ft^3$ yields

$$q_{1,1,1}(t_1) + q_{1,1,2}(t_1) + q_{1,1,3}(t_1 \leq B_1 \quad (53)$$

$$q_{1,2,1}(t_1) + q_{1,2,2}(t_1) \leq B_2 \quad . \quad (54)$$

Substitution gives

$$41.1 \times 10^7 \; q_{1,1,1} + 41.1 \times 10^7 \; q_{1,1,2}$$

$$+ \; 41.1 \times 10^7 \; q_{1,1,3} \leq 60 \times 10^8 \quad (53a)$$

$$46.6 \times 10^7 \; q_{1,2,1} + 46.6 \times 10^7 \; q_{1,2,2} \leq 70 \times 10^8 . \quad (54a)$$

From Equation (45) assuming that

$R_{1,1}$ = 500,000 ft^3/day and $R_{1,2}$ = 300,000 ft^3/day

the next constraints are:

$$q_{1,1,1}t_1 + q_{1,1,2}t_1 + q_{1,1,3}t_1 \leq R_{1,1} \quad (55)$$

$$q_{1,2,1}t_1 + q_{1,2,2}t_1 \leq R_{1,2} \quad . \quad (56)$$

Substitution yields:

$$41.1 \times 10^7 \; q_{1,1,1} + 41.1 \times 10^7 \; q_{1,1,2}$$

$$+ \; 41.1 \times 10^7 \; q_{1,1,3} \leq 500{,}000 \quad (55a)$$

$$46.6 \times 10^7 \; q_{1,2,1} + 46.6 \times 10^7 \; q_{1,2,2} \leq 300{,}000 \; . \quad (56a)$$

One should note that Equations (55a) and (56a) resemble Equations (53a) and (54a). Such would not be the case if more than one time period were considered.

The final constraint comes from Equation (46). That is:

$$q_{1,1,1}t_1 + q_{1,1,2}t_1 + q_{1,1,3}t_1 + q_{1,2,1}t_1 + q_{1,2,2}t_1 \leq W_1 \ . \quad (57)$$

For this example assume that $W_1 = 600{,}000 \ ft^3/day$. Substitution gives:

$$41.1 \times 10^7 \ q_{1,1,1} + 41.1 \times 10^7 \ q_{1,1,2} + 41.1 \times 10^7 \ q_{1,1,3} + 46.6 \times 10^7 \ q_{1,2,1} + 46.6 \times 10^7 \ q_{1,2,2} \leq 600{,}000 \ . \quad (57a)$$

The functional to be maximized comes from Equation (47)

$$\Psi = q_{1,1,1}t_1 d_{1,1,1} + q_{1,1,2}t_1 d_{1,1,2} + q_{1,1,3}t_1 d_{1,1,3} + q_{1,2,1}t_1 d_{1,2,1} + q_{1,2,2}t_1 d_{1,2,2} \ .$$

The following table represents the profit in $\$/ft^3$. Such information would again come from an economic study and would probably vary over different time periods. It must be remembered that this model represents only one time period and as a result, the profit table will only consider that time period.

		Reservoir 1	Reservoir 2
	1	.00010	.00013
Wells	2	.00012	.00010
	3	.00011	

$d_{i,j} = \$/ft^3$

Substituting these values for profit into the profit function, yields the functional to be maximized:

$$\Psi = q_{1,1,1}(41.1 \times 10^7)(.00010) + q_{1,1,2}(41.1 \times 10^7)(.00012)$$
$$+ q_{1,1,3}(41.1 \times 10^7)(.00011) + q_{1,2,1}(46.6 \times 10^7)(.00013)$$
$$+ q_{1,2,2}(46.6 \times 10^7)(.00010) \qquad (58)$$

which reduces to

$$\Psi = 41.1 \times 10^3\ q_{1,1,1} + 49.3 \times 10^3\ q_{1,1,2} + 45.2 \times 10^3\ q_{1,1,3}$$
$$+ 60.6 \times 10^3\ q_{1,2,1} + 46.6 \times 10^3\ q_{1,2,2}\ . \qquad (58a)$$

Reviewing, the linear programming model for the multi well reservoir problem consists of:

$$\Psi_{(max)} = 41.1 \times 10^3\ q_{1,1,1} + 49.3 \times 10^3\ q_{1,1,2} + 45.2 \times 10^3\ q_{1,1,3}$$
$$+ 60.6 \times 10^3\ q_{1,2,1} + 46.6 \times 10^3\ q_{1,2,2} \qquad (58a)$$

$$1000 - [4.80\ q_{1,1,2} + 4.80\ q_{1,1,3}] \geq 14.7 \qquad (48a)$$

$$1000 - [4.80\ q_{1,1,1} + 4.80\ q_{1,1,3}] \geq 14.7 \qquad (49a)$$

$$1000 - [4.80\ q_{1,1,1} + 4.80\ q_{1,1,2}] \geq 14.7 \qquad (50a)$$

$$1200 - [4.86\ q_{1,2,2}] \geq 14.7 \qquad (51a)$$

$$1200 - [4.86\ q_{1,2,1}] \geq 14.7 \qquad (52a)$$

$$41.1 \times 10^7\ q_{1,1,1} + 41.1 \times 10^7\ q_{1,1,2}$$
$$+ 41.1 \times 10^7\ q_{1,1,3} \leq 60 \times 10^8 \qquad (53a)$$

$$46.6 \times 10^7 \; q_{1,2,1} + 46.6 \times 10^7 \; q_{1,2,2} \leq 70 \times 10^8 \qquad (54a)$$

$$41.1 \times 10^7 \; q_{1,1,1} + 41.1 \times 10^7 \; q_{1,1,2} + 41.1 \times 10^7 \; q_{1,1,3} \leq 500{,}000 \qquad (55a)$$

$$46.6 \times 10^7 \; q_{1,2,1} + 46.6 \times 10^7 \; q_{1,2,2} \leq 300{,}000 \qquad (56a)$$

$$41.1 \times 10^7 \; q_{1,1,1} + 41.1 \times 10^7 \; q_{1,1,2} + 41.1 \times 10^7 \; q_{1,1,3} + 46.6 \times 10^7 \; q_{1,2,1} + 46.6 \times 10^7 \; q_{1,2,2} \leq 600{,}000 \qquad (57a)$$

One should recognize that the answer will contain the flow rates for five wells for only one time period. For a computer solution of this multi well reservoir example, the reader is referred to Appendix C where the numerical results are given and analyzed.

FOOTNOTES

[1] J. S. Aronofsky and A. S. Lee, "A Linear Programming Model for Scheduling Crude Oil Production," Journal of Petroleum Technology (1958), p. 53.

[2] Hager, Chapter II.

[3] Sell, Chapters I, II, III.

CHAPTER V

CONCLUSION

This paper has attempted to discuss one particular petroleum industry problem, optimization of gas withdrawal rates from single well reservoirs and multi well reservoirs, and to indicate how linear programming can be used to solve it. There can be no doubt that linear programming has made a place for itself in the petroleum industry, particularly in the manufacturing phase. It is beginning to be appreciated by management as an important help in making complicated decisions. It should be pointed out that the successful application of linear programming to practical problems has been made possible by the recent advent of large, high speed computers and by the existence of an efficient linear programming code. If digital computers were nonexistent, the answers would be many years too late.

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APPENDIX A

PETROLEUM BASICS*

Although the petroleum industry as it is known today is only a century old and started with the first bore hole drilled in 1859, the crude material was known and put to some use long before the Christian era. In those distant times the main use of the oil which seeped to the surface appears to have been as a waterproofing material or as a mortar in building construction. The "pitch" used by Noah to caulk the ark was probably an inspissated petroleum gathered from the shores of the Dead Sea. Also, the "slime" which upset calculations in the building of the Tower of Babel is referred to in some translations of the Bible as "bitumen" and it is recorded that in building the walls of Babylon use was made of bitumen. As well as bitumen, there are many references in history to the presence of natural gas, a companion to oil, in the earth. The famous Fire Worshippers' Temple at Baku, in the Caucasus, is one example, the temple being erected over a gas seepage. People traveled long distances to worship at the "eternal fires." Likewise,

*The following is primarily a summary of information found in the books by Hager and Sell.[31,32]

at Baba Gurgur, in Iraq and in the Kirkuk oilfield, there still burns the gas of the "fiery furnace" into which Shadrach and Meshach and Abed-nego were cast.

The word "petroleum" is derived from the Latin petra (rock) and oleum (oil) and by the modern definitions of today includes hydrocarbons found in the ground in various forms from the solid bitumen through the normal liquids, to gases. The largest and most important deposits are in the form of liquid crude petroleum, although considerable and increasing quantities of natural gas are being produced.

The actual origin of petroleum is a much debated and postulated subject and as of yet, it cannot be said that the problem has been resolved to the complete satisfaction of all the theorists. However, at the present, it is generally accepted that it is derived primarily from organisms and plant life which have been buried in the earth by the deposition of sediments. Among the theories advanced to account for the transformation of these organic materials to petroleum are the effects of heat, of pressure, of time, or of combinations of these, bacterial action, of low temperature catalysts, or of radio activity. There is also the suggestion that the oil is not formed in the sediments, but is released there, having already been produced in the living organism.

The search for petroleum deposits in the earth no longer relies upon the chance discovery of a surface seepage of oil as an indicator of the possibility of a commercial deposit existing at depth. Today, the geologist no longer has to wildcat his wells where the vehicle carrying the equipment broke down. In his turn, the geophysicist finds that the instruments and techniques available to him are infinitely more precise and reliable than those used by his predecessor. No longer does an impenatrable forest or water present an obstacle to him.

The essential ingredients for the genesis of oil are first, a shallow water area in which the original organic material could have been laid down in sufficient quantity, and secondly, that the dead organisms should have been buried by deposition of sedimentary material. The most common source rocks are believed to be shales and clays, but it is possible that limestones may also be a source rock.

The sedimentary rocks in which the oil is found range in geological age from the pre-Cambrian to the Pleistocene, and the source rocks may have a similar age range, the composition of crude oil varying according to differences in the original substances and to variations in the conditions under which those substances have been transformed. Normally, a source rock will yield only comparatively small amounts of petroleum and some process of concentration is provided by migration of the oil from the source rock to a suitable reservoir rock. This migration occurs in two phases, primary and secondary.

Primary migration, that is, the transfer of hydrocarbons from the source rock to the reservoir rock is most probably due to the compaction of the source rock as it is buried deeper by subsequent sediments. Compaction squeezes fluids, mainly water, out of the rocks, generally upwards and in this process, oil and gas are transferred to the reservoir rock. In the reservoir rock secondary migration takes place. The oil from the source rock will contain varying proportions of dissolved gas. When there is more gas than can be dissolved in the oil, the free gas will rise to the top of the reservoir to form a gas cap.

The two principle requisites of a reservoir rock to make it suitable for the accumulation of petroleum are porosity and permeability. There must be sufficient amounts of pore space to hold a reasonable quantity of fluid and the pores must be interconnected, and of suitable size to make the rock relatively permeable, in order that the fluids or gases can readily flow in it. Most reservoir rocks are the coarse grained sedimentaries such as sandstones, limestones, dolomites, etc.

For oil and gas to accumulate in the reservoir rock, some form of seal is necessary to prevent them from passing out of the reservoir. This seal is normally an impervious caprock overlying the reservoir and of such shape as will prevent the upward escape of oil or gas.

The actual distribution of oil and gas and water in a reservoir rock depends upon their densities, on the physical conditions, and on the details of the rock itself. Thus, in a reservoir rock of uniform properties, the upper zone will be filled with a gas, the middle zone by oil with some gas in solution, and below this will be water. The thickness of the transitions between zones will depend on the physical properties of the fluids and, in those where the gas is completely in solution in the oil, there will be no gas zone. Where no oil is present, the gas will lie directly over the water.

The oil containing layer may be of any thickness, from a few inches to possibly hundreds of feet, and in area may extend over many square miles. In any vertical sequence there may be several oil horizons separated by layers of unproductive beds of varying thickness. Thus an oil field may consist of several horizons from which oil may be produced separately or simultaneously and the petroleums from different horizons may vary considerably in characteristics.

After the deposition of the source and reservoir rocks, the earth's movements caused these beds to become folded or faulted, and in some cases, led to their partial destruction. Thus traps were formed in which oil and gas could be accumulated in addition to those created directly during the course of deposition. One of the most common forms of a trap is that known as the anticline in which the rocks are folded archwise with the limbs dipping away on either side from the crest. A symmetrical anticline has equal dips on both sides. This rarely occurs in nature and more often than not, an anticline is asymmetrical with the limbs dipping at unequal angles.

Still another frequent type of structural trap is a dome, a type of fold in which the beds dip downwards in all directions from the central crest. Where the rocks have been faulted, this movement may result in a reservoir rock on one side of the fault plane being brought against an impervious bed on the other. The trap so formed is called a fault trap.

The preliminary work in locating a possible oil field is to examine the surface evidence which may be available. This may include gas or oil seepages, areal photographs of petroleum bearing anticlines, etc. This is then followed by a detailed geological and geophysical survey in order to select suitable sites for the drilling of test wells. It must be remembered that surface investigations and even geophysical surveys can only give indications of the presence of underground conditions and structures suitable to the accumulation of petroleum in quantity. The drill is the final arbiter concerning the presence or absence of important amounts of petroleum.

APPENDIX B

LINEAR PROGRAMMING--A STATE OF THE ART IN THE PETROLEUM FIELD

As previously mentioned, linear programming has been used quite extensively in the petroleum field considering the actual "scientific" age of both. Upon investigation of any scientific periodical index, one finds countless examples of linear programming applications to not only the petroleum field, but to such fields as agriculture and economics and any other that may be mathematically modeled. In this light then, a few examples will be presented of some linear programming models used as of late in the oil and gas industry.

A paper done by an Oklahoma State University graduate student used linear programming techniques as an approach to the solution of the classical octane economics problem found in the area of gasolene blending. In past years, gasolene blending was comparatively simple. A slide rule and a few response charts were used quite effectively to make numerous grades of gasolene. Often, market conditions will even divert some gasolene component to another product. The combined effect of this is to present the refiner with a problem of how to blend for maximum profit. With today's squeezed profit margins, most refiners have been forced to electronic computer solutions, using such techniques as linear programming.

Three engineers from the Atlantic Richfield Refinery devised a simple but nonetheless instructive example of linear programming applied to petroleum refining. A refinery produces gasolene, furnace oil, and other by products. This same refinery can be supplied with a fairly large number of crude oils, which is usually the case. The available crude oils have different properties and yield different volumes of finished products. Some of these crudes must be refined because of long term, minimum volume commitments, or because of requirements for specialty products. These crudes are considered fixed and yield gasolene and furnace oil volumes. From the remaining crudes and from those crudes which are available in volumes greater than their minimum volume commitment, must be selected those which can supply the required products most economically. These are the incremental crudes. The problem is to determine the minimum incremental cost of furnace oil as a function of incremental furnace oil production, keeping gasolene production and general refinery operations fixed. An actual problem was run with the equations derived by Atlantic. The so called "parametric programming" procedure was used on the IBM 704 LP Code. The results illustrated that the modern refinery is a complicated system with strong interdependence among the activities within it. It also demonstrated the importance of the refiner's experience in correctly isolating portions of the refinery which can be separately considered.[1]

Having considered the refinery and one of the operations within, namely gasolene blending, one might now consider some of the recent work done regarding the flow of the products from the bulk terminal to the service stations. The location and the roads connecting the service stations are considered given. Each service station requires the delivery of so many gallons of gasolene. Different truck types, differing in their capacity and operating characteristics are available for making deliveries. There are a number of each type of truck available for the operation. The problem is to devise a delivery schedule such that the transportation cost is minimized. The Operations Research Group at Atlantic Richfield Company became quite interested in this technique devised and applied it as a means for handling transhipments on a daily basis. A method was developed that is not guaranteed to lead to the optimum solution, but will usually lead to a solution rather close to it.[2]

There can be no doubt that linear programming has made a place for itself in the petroleum industry, particularly in the manufacturing phase. It is beginning to be appreciated by management as an important help in making complicated decisions. One must realize, however, that everything in this world is not linear and that occasionally one will come across constraints which are mathematically not logical.

This is good in a way because if ever a method is devised that solves all problems, life would undoubtedly become rather dull.

FOOTNOTES

[1] Garvin, Crandall, John, and Spellman, pp. 407-430.

[2] *Ibid*.

APPENDIX C

COMPUTER SOLUTIONS OF SINGLE WELL AND MULTI WELL RESERVOIR EXAMPLES

Both sets of example equations were run on the CPS (Conversational Programming System) terminal at Oklahoma State University. This terminal is tied in via phone to the IBM 360 MOD 50 digital computer and contains its own library programs. The specific library program used for the models was LINPRO, a program using the simplex method for solving linear programming problems.

The equations are read into the computer in a tableau, the size of which is limited to an array where $M \leq 13$ and $M + N + G \leq 31$ where M = the number of equations, N = the number of variables, and G = the number of "greater thans" ($\geq$).

The single well, multi reservoir equations were read in and the following solution was obtained.

Let:

$$q_{1,1} = X_1$$
$$q_{1,2} = X_2$$
$$q_{2,1} = X_3$$
$$q_{2,2} = X_4$$

Slack Variables = 5 through 12

Answers:

5	985.3
6	1185.297772
7	985.3
8	60,000,000
9	1185.295545
10	69,700,000
2	21,459.2275
4	42,918.4549

Objective Function Value = $5,298.28

This solution indicates that a volume of 21,459 cu.ft./day in the first time period from the second well plus 42,918 cu.ft./day in the second time period from the second well would result in the maximum value of the objective function of $5,298.28. Evidently, from this solution, the first well would not be operated in either time period.

In the second model, the equations were again read in in tableau. The printed solution consisted of:

Let:

$$q_{1,1,1} = X_1$$
$$q_{1,1,2} = X_2$$
$$q_{1,1,3} = X_3$$
$$q_{1,2,1} = X_4$$
$$q_{1,2,2} = X_5$$

Slack Variables = 6 through 5

Answers:

6	985.296496
7	985.3
8	985.296496
9	1185.3
10	1185.29687
11	.59997 E 10
12	.69997 E 10
13	200,000
4	64,377.682
2	72,992.700

Objective Function Value = $7,499.82

This solution indicates that 64,377 cu.ft./day from the second well, first reservoir plus 72,992 cu.ft./day from the first well, second reservoir will yield the maximum value of $7,499.82 for the objective function. Again, the first and third wells of the first reservoir plus the second well of the second reservoir will not be operated.

To check the model equations further, numerous other examples were run. One specific additional set of equations for the single well, multi reservoir model was:

$$6.98\ X_1 \leq 700.1$$

$$7.63\ X_2 \leq 723.2$$

$$7.25\ X_3 \leq 698.7$$

$$7.02\ X_4 \leq 711.3$$

$$38.3\ X_1 + 39.6\ X_3 \leq 125.0$$

$$39.6\ X_2 + 32.1\ X_4 \leq 103.0$$

$$41.1\ X_1 + 45.1\ X_2 \leq 19.3$$

$$36.9\ X_3 + 42.1\ X_4 \leq 20.4$$

$$\Psi = 40.1\ X_1 + 79.1\ X_2 + 50.2\ X_3 + 78.1\ X_4$$

The solution involved three variables, or in other words, required flow from both wells during one time period. More specifically,

Let:

$$q_{1,1} = X_1$$

$$q_{1,2} = X_2$$

$$q_{2,1} = X_3$$

$$q_{2,2} = X_4$$

Slack Variables = 5 through 12

Answers:

5	700.1
6	719.93483
7	675.81489
8	707.89838
3	315,656.565
10	70.499264
2	42,793.791
4	48,456.057

Objective Function Value = $23,015.36

In other words, in the first time period the second well would supply 42,793 cu.ft./day. In the second time period, a combination of both wells, the first yielding 315,656 cu.ft./day and the second, 48,456 cu.ft./day would provide the maximum value of the objective function of $23,015.36.

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