69-8595

FIEDLER, Leigh Allan, 1930-
A COMPARISON OF ACHIEVEMENT RESULTING FROM LEARNING MATHEMATICAL CONCEPTS BY COMPUTER PROGRAMMING VERSUS CLASS ASSIGNMENT APPROACH.

The University of Oklahoma, Ph.D., 1969
Education, theory and practice

University Microfilms, Inc., Ann Arbor, Michigan

THE UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

A COMPARISON OF ACHIEVEMENT RESULTING FROM LEARNING MATHEMATICAL CONCEPTS BY COMPUTER PROGRAMMING VERSUS CLASS ASSIGNMENT APPROACH

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

DOCTOR OF PHILOSOPHY

BY

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Norman, Oklahoma

1969

A COMPARISON OF ACHIEVEMENT RESULTING FROM LEARNING MATHEMATICAL CONCEPTS BY COMPUTER PROGRAMMING VERSUS CLASS ASSIGNMENT APPROACH

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ACKNOWLEDGEMENT

The research reported in this dissertation would not have been possible without the involvement and support of many persons. The author expresses deep gratitude especially to his major advisor, Dr. John W. Renner, for careful direction and encouragement during the investigation and preparation of the report. The author also wishes to thank the Advisory Committee members, Dr. Herbert Hengst, Dr. Harold Huneke, and Professor Eunice Lewis for the interest that they have shown in the project and for planning an interesting and appropriate program of advanced graduate study.

The administration of the trial runs of the evaluative instruments developed for the study was supervised by Dr. William Glynn, Western Illinois University, Macomb, Illinois, Dr. Harry Nelson, Augustana College, Rock Island, Illinois, Mr. Donald Fuller, Northern Illinois University, DeKalb, Illinois and Mr. James Brunner, Black Hawk College, Moline, Illinois. To these colleagues, the author expresses sincere appreciation.

Finally, the author expresses profound gratitude to his wife, Andrée, for her inspiration, determination, and perseverance during the years of graduate study and research.

He also thanks her for her diligent effort in the initial typing of the dissertation.

The work reported in this dissertation was in partial fulfillment of the requirements for the degree of Doctor of Philosophy at the University of Oklahoma, Norman, Oklahoma.

TABLE OF CONTENTS

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

A COMPARISON OF ACHIEVEMENT RESULTING FROM LEARNING MATHEMATICAL CONCEPTS BY COMPUTER PROGRAMMING VERSUS CLASS ASSIGNMENT APPROACH

CHAPTER I

INTRODUCTION

Computers and Higher Education

A recent report of the President's Science Advisory Committee states "We believe that it is in the national interest to have adequate computing for educational use in all our institutions of higher education by 1971-72. We believe that this can be achieved, but we believe that it can be done only with the government assistance."[1] The report recommends that the federal government underwrite three-fourths of a $400-million-a-year program to give every college student in the nation access to a computer by 1971.[2] The report also declares "By sometime in the 1970's it is doubtful that more than a few percent of the students will graduate without having made some use of computers."[3]

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[1]Report of the President's Science Advisory Committee, Computers in Higher Education (Washington: U.S. Government Printing Office, 1967), p. 3.

[2]*Ibid.*, p. 4.

[3]*Ibid.*, p. 12.

The use of computers in educational facilities has increased greatly in the past decade and undoubtedly will continue to do so. Yet there is a considerable lag between the use of computers in business, industry and government versus use in education. Thus if the deficit in educational computing continues, the students of higher education in the 1970's will be poorly prepared for the world of the 1980's and 1990's. In 1967 there were 35,000 computers in operation in the United States, by 1975 the Science Advisory Committee report projects that approximately 80,000 computers will be in operation.[4] Since these machines can be used in many ways, considerable thought should be given to utilizing them in the most effective manner possible.

Digital computers can be useful in college work in at least three ways, by:

1. students in conjunction with courses,
2. students in vocational programs,
3. school administration for data processing.

This investigation is concerned with the first of these, specifically with respect to mathematics courses.

Statement of the Problem

This investigation treats the following problem: Does a student gain a greater understanding of a mathematical concept by programming it for a digital computer than when he

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[4] *Ibid.*, p. 59.

studies that concept in the usual course structure? This question is motivated by many claims which have been made in favor of using the computer to teach mathematical concepts. Forsythe in The American Mathematical Monthly states "Whereas we think we know something when we learn it, and are convinced we know it when we can teach it, the fact is we don't *really* know it until we can code it for an automatic computer."[5] Walter Hoffman in an article in The Mathematics Teacher declares, "In addition to the obvious motivational advantage, students who write computer programs acquire a better understanding of the mathematical concepts involved."[6] In Needed Research in Mathematical Education, Professor Kemeny asserts "Then the right way to do the algorithm in practice is to program it for a computer. Thus the computer is being used in such a way as to force the student to explain the given algorithm to a computer. If a student succeeds in this, he will have a depth of understanding of the problem which will be much greater than anything he has previously experienced."[7] George Handelman, Chairman of the Mathematics Department at

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[5]George E. Forsythe, "The Role of Numerical Analysis in an Undergraduate Program," The American Mathematical Monthly, 66 (October, 1959), p. 656.

[6]Walter Hoffman et al., "Computers for School Mathematics," The Mathematics Teacher, 58 (May 1965), p. 395.

[7]John G. Kemeny, "The Role of Computers and Their Applications in the Teaching of Mathematics," Needed Research in Mathematical Education, ed. Howard A. Fehr, (New York: Teachers College Press, Columbia University, 1966). p. 10.

Renssalaer Polytechnic Institute expressed himself about the introduction of computer programming into the numerical analysis courses as follows: "It's so fantastic it's impossible to say how much better the course is. The amount of practice the students get is up by one to two orders of magnitude."[8] Similar claims can be found in many current journal articles and books.[9,10,11,12]

This investigation studies the notion of learning mathematics through computer programming as opposed to the usual classroom procedures. Specifically, the following question was investigated. Does a student develop deeper understanding of selected mathematical topics by programming problems involving those topics for a digital computer than by doing the usual homework assignments? The topics which were investigated were selected from Analytic Geometry and Calculus and two distinct groups of students were used.

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[8]Report of the President's Science Advisory Committee, op. cit., p. 63.

[9]Alexandra Forsythe, "Mathematics and Computing in High School: A Betrothal," The Mathematics Teacher, 57 (January, 1964), pp. 2-7.

[10]Darrel G. Littlefield, "Computer Programming for High Schools and Junior Colleges," The Mathematics Teacher, 54 (April, 1961), p. 223.

[11]Elliot Pierson, "Junior High Mathematics and the Computer," The Mathematics Teacher, 56 (May, 1963), p. 298.

[12]Raymond Sweet, "High Speed Computer Programming in the Junior High School," The Mathematics Teacher, 56 (November, 1963), p. 535.

The two groups consisted of the students enrolled in Analytic Geometry and Calculus I at Black Hawk College, Moline, Illinois for the Spring semester of the 1967-68 academic year.

Statement of Hypotheses

Throughout this report the two groups of students mentioned above will be designated as follows:

Group A - Traditional Class Assignment Group.

Group B - Computer Programming Group.

Group A was taught in the usual course structure. That is, there was lecture and discussion during the class sessions and the students were given homework assignments to be done with pencil and paper outside the class. This process will be called Treatment 1. Group A, then, received Treatment 1.

Group B was also taught in the lecture-discussion structure. The students in this group wrote computer programs outside the class to solve problems assigned as homework. The programs were run on the IBM 1401 computer by Data Processing staff operators at the College. This process will be called Treatment 2. Group B, then, received Treatment 2.

The specific problem of the study is to determine if there is a measurable difference in learning each topic selected for the investigation between the students who were given Treatment 1 and the students who were given Treatment 2. Before each of the concepts selected for the investigation was introduced in class lecture and discussion, a fifty-minute test designed to measure the student's knowledge of that concept was given as a pretest. At the conclusion of the class sessions concerned with the topic, the same test was given as a criterion test (post test) to measure the student's achievement during the time period devoted to the concept.

The possibility was considered that a student with greater prior knowledge of the concept might perform differently on the criterion test than a student with less prior knowledge. On the basis of pretests designed to measure knowledge of each concept selected for the investigation, each group of students was divided into two groups (levels). Thus an attempt was made to see if separate levels of prior knowledge would contribute to a difference in learning from the two treatments.

The possibility was also considered that the treatments might produce different results depending upon the amount of prior knowledge that a student had of a given concept. That is, there was a possibility of interaction between the treatments and levels used in the experiment. Thus an attempt was made to see if the two treatments would contribute to a difference in learning in separate levels of prior knowledge of the topic.

To examine the three situations just mentioned, three hypotheses were investigated for each topic used in the study. Stated in null form, these hypotheses were:

<u>Hypothesis 1</u> - Treatments

After adjustment for the scores on the pretest, there is no difference in the results on the criterion test between the two treatments.

Hypothesis 2 - Levels

There is no difference in achievement as measured by the criterion test between the two levels.

Hypothesis 3 - Interaction

There is no interaction between treatments and levels--the treatments produce similar results at both levels.

The Need for the Study

The claims cited in the previous section, that students learn mathematical concepts better by computer programming than in the traditional manner, and other similar statements, have been made most enthusiastically, but these claims have not been subjected to research. That research is needed in this area is strongly suggested by Professor Kemeny.[13] No research, however, has been reported in any of the journals, pamphlets and reference volumes cited in the bibliography of this report. The bibliography includes *Dissertation Abstracts* from 1955 to the present, government reports and journals reporting educational research. A recent issue of *The Mathematics Teacher* proposed a new department of the journal called "Computers in Mathematics Instruction" and issued an invitation to teachers who are experimenting in this area to

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[13] Kemeny, *op. cit.*

contribute articles.[14] There is a considerable lack of information on activities which are being conducted in the use of computers for mathematics education. Also in the not too distant future, especially if the recommendations of the President's Science Advisory Committee are implemented, computers will be available to nearly all college students and provision should be made for their optimum use. For the foregoing reason, the next few paragraphs consider the computer as a pedagogical tool.

For the sake of clarity, the distinction between this investigation and Computer-Assisted Instruction (CAI) must be pointed out. Much work has been done on the latter by Suppes,[15] Bundy[16] and others. In CAI the course material is written (programmed) in a "programmed learning" format. The material in the programmed learning format is then coded (programmed) for a digital computer. (There is a very unfortunate dual use of the word "program" in programmed learning and in computer programming which frequently causes confusion.) The student then works through the course material seated at a computer console or a remote terminal input/output device and puts his responses into the computer which,

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[14]"An Invitation to Contribute to Computers in Mathematics Instruction," The Mathematics Teacher, 61 (February, 1968), p. 147.

[15]Patrick Suppes, "Tomorrow's Education," Education Age 2 (January-February 1966), pp. 4-11.

[16]Robert F. Bundy, "Computer-Assisted Instruction: Now and For the Future," Audiovisual Instruction, (April 1967), pp. 344-348.

in turn, tells the student immediately if he is correct or incorrect and gives him the next question (or directs him to it). The student does not need to know a computer language and the questions are posed for him by the computer.

In this investigation the students who used the computer as a tool analyzed the mathematical problems and wrote programs in a mathematics-oriented computer language which allowed the computer to solve the problems. Hence, the essential distinction between the two approaches is that in CAI the computer directs the student whereas in this investigation the student directed the computer. The investigator feels that the approach used in this study requires greater skill and understanding than CAI.

One reason for studying the computer as a pedagogical tool is suggested by Professor Kemeny in Random Essays on Mathematics, Education and Computers. He states:

> "The advent of computing machines provides an additional opportunity for encouraging mathematical talent. Programming for a high-speed computer requires the type of systematic thought, ingenuity, and logical precision that is excellent training for the mathematically talented student. This is particularly true in his late high school and beginning college years."[17]

Additional support for using the computer in mathematics instruction is given by the CUPM (Committee on the Undergraduate Program in Mathematics) report to the Mathematical Association of America. The report stated:

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[17]John G. Kemeny, Random Essays on Mathematics, Education and Computers, (Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1964), p. 57.

"The prevalence of the high-speed automatic computer affects the teaching of mathematics in a very general way. Many mathematically trained students will work closely with computers, and even those who do not should be taught to appreciate the type of algorithmic approach that enables a problem to be handled by a machine. This point of view should therefore be presented, along with the more classical differential equations, linear algebra, etc.

"If there is a computer on campus, or if one is otherwise accessible, it is likely that elementary programming instruction will be available to students early in their academic careers. This, in turn, makes it possible to take advantage of the computer throughout the mathematics program, and material should be presented to make use of this opportunity."[18]

Other evidence that future mathematicians need to be aware of the capabilities of digital computers is cited in Employment Outlook for Mathematics and Related Fields:

"The demand for mathematicians in research and development is closely associated with the use of high speed electronic computers. These computers have made it possible to solve a wide variety of complex problems in engineering, and natural and social science research, and also have opened broad new fields for mathematics in business management."[19]

Thus computers, while already playing an important tool in business and industry, will be playing a larger role in education.

In summary, the computer is said to be a valuable pedagogical tool because:

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[18]A General Curriculum in Mathematics for Colleges (Berkeley, California: Committee on the Undergraduate Program in Mathematics, 1965), p. 14.

[19]United States Department of Labor. Employment Outlook for Mathematics and Related Fields: Mathematicians, Statisticians, Actuaries (Washington, D.C.: U.S. Government Printing Office, 1966), p. 3.

1. the student must be able to state his problem in a very precise manner to program it for machine solution.

2. the student is forced to <u>rethink</u> his problem carefully; if the problem <u>is not</u> stated in a precise manner, the computer will reject the problem or give absurd results.

3. many students taking mathematics today will be directly involved with computers in one way or another in their future.

4. the student of today will live in a world where some understanding of computers would be desirable.

This study investigates a procedure for merging into mathematics education the teaching of the skill to formulate mathematical problems for machine solution. There is a reluctance and inertia on the part of mathematics teachers to accept and attempt innovations in the teaching of mathematics. This is probably true because the teacher is usually most comfortable with the method he is currently (and perhaps, has been) using. In describing the plans for this investigation to his colleagues, the investigator received comments ranging from "It can't be done!" to "Of course, this is the natural way to do it!" Both types of comments provided incentive to continue.

The establishment of guidelines which will influence how computer programming should be introduced into the curriculum is important at the present moment; later may be too late. The next few years will be characterized by an increased accessibility of computers for educational uses.

An elective course entitled "Computer Programming and Related Mathematics" has been offered at Black Hawk College since the Spring semester of 1963. Enrollments have been small (approximately 10-17 students per semester) and many students have stated that they desire to take the course but their schedules are filled with required courses. Hence if programming would supplement and reinforce already existing courses, this seems to be the desirable way to incorporate the computer into classroom teaching. If a course such as Computer-Oriented Mathematics is not required or programming is not introduced into mathematics courses which are already required, the CUPM recommendation that the advantages of the computer be utilized throughout the mathematics program could not be realized due to the fact that only a minority of the students would have the necessary training. Thus, research concerned with the most feasible way to incorporate computer training into the mathematics curriculum is essential.

Review of the Literature

During the summer of 1967 the investigator carried out an intensive literature search for information concerning research conducted on the problem of teaching mathematics via computer programming. Several articles describing experiences in teaching computer languages to students from the junior high level through the freshman and sophomore years of college were noted. However, as mentioned in the previous section of this report, no formal research has been reported on this problem.

Kemeny reports that each year at Dartmouth College eighty percent of the freshman class is trained in the rudiments of computer programming. The plan is incorporated into second semester freshman mathematics, which is calculus for the physical science students and finite mathematics for other students.[20] Professor Kemeny states "I am of the opinion that no other academic program yields as high a dividend, per time invested, as the freshman computer program."[21]

Hoffman states that beside the motivational advantages, students writing computer programs understand better the mathematical concepts involved.[22] However, no evidence is given to support this statement. The article suggests many topics from secondary school mathematics as candidates for student-written computer programs.

Littlefield describes his experiences in teaching a course in computer programming to secondary school and junior college students. He recommends that high schools and junior colleges look to the computer as a natural educational resource.[23]

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[20]Report of the President's Science Advisory Committee, op. cit., p. 76.

[21]Ibid.

[22]Hoffman et al., op. cit., p. 395.

[23]Littlefield, op. cit., p. 223.

Forsythe relates her experiences in teaching a computer language (BAGOL) to high school students on a Burroughs 220 at Stanford University. She states that learning a mathematically oriented computer language makes it easier to put computing into existing mathematics courses and thus the computer and mathematics have a good chance to reinforce each other.[24] She asserts, "It is very important for students of mathematics at all levels to learn to apply their mathematics, and this is harder than one might think."[25] She feels that new mathematics programs, intentionally or not, seem to ignore the applications of the topics presented.

Whitacre describes her experiences in teaching computing on a UNIVAC 80 to twenty-one selected high school sophomores during the summer of 1962. She found that class morale was spirited and that the students were eager to present their clever and sometimes ingenious solutions to the assigned problem.[26] Pierson taught GOTRAN on an IBM 1620 to junior high students during a one week period. He states that the students could master the fundamentals of this language in one or two sessions and that the main objective of the work with the computer was the study of mathematics.[27]

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[24]Forsythe, op. cit., p. 6.

[25]Ibid, p. 7.

[26]Lillian Whitacre, "Computer Programming for High School Sophomores," The Mathematics Teacher, 56 (May, 1963), pp. 340-343.

[27]Pierson, op. cit.

Sweet used a CDC 1604 at the University of Texas with his junior high students. He felt that the students seemed to assimilate the programming language with no more difficulty than learning other junior high subjects.[28]

However, as noted previously, none of the authors made a formal study to discover whether or not the mathematical concepts learned via computer programming were learned better in that manner than in the usual classroom situation. Perhaps one could conclude from these articles that there is a charismatic effect of computers upon young students (and/or teachers!)

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[28] Sweet, *op. cit.*

CHAPTER II

DEVELOPMENT OF EVALUATIVE INSTRUMENTS

Mathematical Topics Selected for the Investigation

The course utilized during the investigation was Analytic Geometry and Calculus I which is taught at Black Hawk College, Moline, Illinois. The textbook used for this course is Calculus with Analytic Geometry, Third Edition by R. E. Johnson and F. L. Kiokemeister.[29] The book was selected in 1964 because it was judged to contain a combination of theory and application of mathematics appropriate for first year college students.

Four mathematical concepts which are used in many branches of mathematics were selected for investigation. The topics selected were: (1) functions, (2) limits and differentiation, (3) iterative techniques, and (4) integration. To distinguish between the phases of the study, the topics will be designated as follows:

Phase I - FUNCTIONS

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[29] R. E. Johnson and F. L. Kiokemeister, Calculus with Analytic Geometry, Third Edition, (Boston: Allyn and Bacon, Inc., 1964).

Phase II - LIMITS AND DIFFERENTIATION

Phase III - ITERATIVE TECHNIQUES

Phase IV - INTEGRATION

Functions, limits and differentiation, and integration play major roles in the study of calculus itself as well as in other branches of analysis. The principle of iteration is of great importance in numerical analysis, functional analysis and the theory of differential equations. The course analytic geometry and calculus gives the student an introduction to iterative methods. Usually only a few relatively simple problems are assigned due to the fact that meaningful problems involve extended calculations and many iterations. By utilizing a computer, the student need not concern himself with the tediousness of lengthy calculations and the possibility of computational errors; he can concentrate on the ideas involved. Such a procedure allows the student to cope with problems that were considered impractical in mathematics courses only a few years ago. In this investigation, the idea of iteration was interpreted to include problems where the use of the positive integers is essential in a given problem.

Development of Evaluative Instruments

For each phase of the study an instrument was designed to test the concept being examined. No standardized tests are available which evaluate each of the topics of the investigation as an isolated concept. For example, the

Co-operative Mathematics Tests of the Educational Testing Service were examined to see if using these tests would be feasible. However, each of these tests is designed to measure student achievement over the entire course and not on isolated concepts.

Thus evaluative instruments which could be employed to test the concepts selected for each phase of the study were developed; i.e., four tests were constructed. The first draft of each instrument consisted of forty-five multiple-choice items with five alternative answers for each test item. The positions of the correct responses were chosen randomly on each test item. The instruments were then submitted to three Black Hawk College mathematics instructors who independently examined each instrument to assure that it contained items concerned only with the single concept being tested by that instrument. Since each of the instructors had previously taught Analytic Geometry and Calculus I, they also checked the instruments' face validity. That is, they assured that the test items represented the content of the course to which the test would be applied. Each instructor worked through the test to check the correctness of the answers provided by the investigator. The three instructors agreed that forty-five items were too many to be completed within a fifty-minute class period, therefore items were revised and eliminated. The final draft of each of the four instruments consists of thirty-five multiple choice items with five alternative answers each. Copies of the four evaluative instruments used in the investigation are included in Appendix B of this report.

Testing the Evaluative Instruments

The next step in the development of the evaluative instruments was to determine if the tests distinguished among students of different achievement levels in the course used for the study. This step was necessary because the instruments were to be used during the teaching experiment, described in the next chapter, to compare the achievement of two treatment groups.

In order to perform a small standardization of the evaluative instruments, mathematics students who had recently completed the first course in analytic geometry and calculus were sought to take the tests. During the 1967-68 academic year there was only one section of the second semester of Analytic Geometry and Calculus being taught at Black Hawk College. That was not a sufficient population to use to study the instruments. In addition to the Black Hawk sample, the mathematics departments of Western Illinois University (Macomb), Northern Illinois University (De Kalb) and Augustana College (Rock Island) were invited and agreed to take part in the standardization process. This selection of schools provided a cross section of types of institutions of higher education; i.e., a junior college, a four-year liberal arts college, and two state universities. Copies of the tests were mimeographed and sent to these schools. The cooperating instructors gave the tests early in the Spring semester of 1968.

The students who took the test recorded their answers with electrographic pencils on IBM mark-sense answer cards. The students also recorded the grade that they received on the first course in analytic geometry and calculus. The cover letter giving the instruction for administering the test is contained in Appendix A of this report. A reproduction of the type of answer card which was provided is also shown in Appendix A.

The raw scores, obtained by the students who were used in the trial of the evaluative instruments, are listed in Appendix C of this report. The scores are classified by the course grades received in the first course in analytic geometry and calculus. For each instrument, the following hypothesis, stated in null form, was tested.

H_0: After being classified by course grade, there is no difference between the means of the scores obtained by the students who received the various grades.

For each instrument, the analysis of variance model,[30] which yields the F-statistic, was used to test the hypothesis at the 0.01[31] level of significance. Applying

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[30]Bernard Ostle, <u>Statistics in Research</u>, (Ames, Iowa: The Iowa State University Press, 1963), pp. 134-135.

[31]Some authors write the 0.01 level of significance as the 1% level of significance.

the 0.01 significance level means that the probability of rejecting a true hypothesis is less than 0.01. A tabular representation, which describes the calculations involved in calculating the F-statistic for analysis of variance, is shown in Appendix D of this report. If the calculated F value is larger than the value obtained from an F-distribution table,[32] the calculated F-value is significant at the 0.01 level and the null hypothesis will be rejected.

Table 1 summarizes the results of the trial for Test I - Functions. A total of ninety-six students from Western Illinois University, Northern Illinois University, Augustana College, and Black Hawk College participated in this trial. The F-value obtained from the analysis of Test I was 4.99 which is significant at the 0.01 level.

TABLE 1

ANALYSIS OF VARIANCE TABLE
FOR TEST I - FUNCTIONS

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F-ratio	$F_{.01}$
Mean	1	60652	60652.00		
Among groups	3	197	65.67	4.99	4.01
Within groups	92	1210	13.15		
Total	96	62059			

[32]Ostle, *op. cit.*, pp. 529-543.

Table 2 summarizes the results of the trial for Test II - Limits and Differentiation. A total of seventy-five students from Western Illinois University, Northern Illinois University, and Black Hawk College participated in this trial. The F-value obtained from the analysis of Test II was 4.61 which is significant at the 0.01 level.

TABLE 2

ANALYSIS OF VARIANCE TABLE
FOR TEST II - LIMITS AND DIFFERENTIATION

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F-ratio	$F_{.01}$
Mean	1	18534	18534.00		
Among groups	3	457	152.33	4.61	4.08
Within groups	71	2345	33.03		
Total	75	21336			

The results of the trial of Test III - Iteration are shown in Table 3. A total of fifty-five students from Western Illinois University and Black Hawk College were used as subjects in this trial. Students from Augustana College also took Test III, but the Chairman of the Mathematics Department stated that iterative techniques were not presented in the first course in analytic geometry and calculus at Augustana. For the foregoing reason, the Augustana students' scores for Test III were deleted. The F-value obtained from the analysis of Test III was 8.11 which is significant at the 0.01 level.

TABLE 3

ANALYSIS OF VARIANCE TABLE
FOR TEST III - ITERATION

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F-ratio	$F_{.01}$
Mean	1	14080	14080.00		
Among groups	3	205	68.33	8.11	4.19
Within groups	51	430	8.43		
Total	55	14715			

The results of the trial of Test IV - Integration are shown in Table 4. A total of seventy-two students from Northern Illinois University, Augustana College, and Black Hawk College participated in this trial. The F-value obtained from the analysis of Test IV was 8.07 which is significant at the 0.01 level.

Since F-values for each of the four tests were significant at the 0.01 level, the null hypothesis was rejected in each case and it was inferred that, at the 0.01 level of significance, there is a difference in the means of the examination scores as classified by course grade. This indicates that the evaluative instruments distinguish among students of different achievement levels in the first course in analytic geometry and calculus.

TABLE 4

ANALYSIS OF VARIANCE TABLE
FOR TEST IV - INTEGRATION

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F-ratio	$F_{.01}$
Mean	1	19372	19372.00		
Among groups	3	465	155.00	8.07	4.09
Within groups	68	1306			
Total	72	21143			

Further information concerning the evaluative instruments was obtained by calculating the correlation coefficient between the students grades in the first semester of analytic geometry and calculus and test scores. The results of these calculations are presented in Appendix E of this report.

Thus, the results of the trial of the evaluative instruments indicated that the instruments distinguished among the students of different achievement levels in the course used for the study. Also the examination of the instruments by three Black Hawk College mathematics instructors indicated that the test items reflected the course content. On the basis of the foregoing information, the instruments were judged to be valid for measuring the relative achievement of two treatment groups to be used during the teaching experiment. The teaching experiment performed for this study is described in the next chapter.

CHAPTER III

THE EXPERIMENT

Experimental Design

The analysis of the experimental data employed a 2 x 2 treatments by levels analysis of covariance design. The analysis of covariance technique combines the concepts of analysis of variance and regression to supply a more discriminating analysis than would be afforded by either of the two component parts. Another very important feature of the analysis of covariance statistical model is that it adjusts the scores on the criterion test (post test) according to the associated values on the pretest. After discussing randomized groups and matched-group designs, Mouly in The Science of Educational Research states:

> "The more sophisticated and adequate method of handling the situation is to rely on statistical equation of the groups through analysis of covariance. This technique is a procedure which permits statistical adjustments to be made in the dependent variable in order to compensate for any lack of equivalence between the groups in the independent variables."[33]

Borg points out that "because much educational research must be done on groups that are already in existence,

[33]George S. Mouly, The Science of Educational Research, (New York: American Book Company, 1963), p. 344.

covariance analysis is an extremely valuable tool for use by research workers."[34]

In this report, the pretest score will be designated as the prediction variable, X, and the post test score will be designated as the criterion variable, Y. The pretest scores were used to divide each of the two groups into high and low levels with respect to prior knowledge of the concept being tested. The design model for each phase of the investigation has the configuration shown in Table 5.

TABLE 5

EXPERIMENTAL DESIGN

	TREATMENTS	
LEVELS	I. TRADITIONAL	II. COMPUTER
1. High	H T	H C
2. Low	L T	L C

The hypotheses, in null form, tested in each phase of the investigation were:

1. After adjustment for scores on the pretest, there is no difference in the results on the criterion test between the two treatments.

2. There is no difference in achievement as measured by the criterion test between the two levels used in the

[34]Walter R. Borg, Educational Research, (New York: David McKay Company, Inc., 1963), p. 144.

experiment.

3. There is no interaction between the treatments and levels.

The 0.10 level of significance was used for the analysis. To justify the selection of this level of significance, one must consider the two types of error possible in the inference drawn concerning the hypothesis tested.

If the null hypothesis (H_O) is actually true and the statistical analysis indicates that the investigator reject H_O, then an incorrect decision has been made. This kind of error is called the type I error.[35] The level of significance is a measure of type I error. Using the 0.10 significance level means that the null hypothesis will be rejected 10 per cent of the time when it is actually true.

A type I error in an educational situation could imply that the procedure or system which is being investigated may be changed when the change would not improve the procedure or system. Assume that a type I error is made in an experiment with ninth grade algebra where a proposed new teaching method is being compared with the method in current use. Two algebra classes were selected for the investigation and one class was taught using the current method. The null hypothesis is that there is no difference in the achievement in the two classes as a result of using the two different

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[35] Ostle, op. cit., pp. 107-108.

methods. If a type I error is made, the investigator will conclude that there is a difference in achievement when, in fact, there is no difference. Thus a change in method may be made when the change will not produce an improvement. Students will achieve the same as they did previous to the change in method and the only harm done is the annoyance to the teachers of learning the new method. The students, however, will be neither helped nor hindered by the change in method.

On the other hand, if the null hypothesis (H_o) is false and the statistical analysis indicates that H_o be accepted, then an incorrect decision has also been made. This kind of error is called the <u>type II error</u>.[36] The size of the type II error will increase if the size of the type I error is decreased. In this study, the 0.10 level of significance has been selected to reduce the size of the type II error. It is highly undesirable that the study conclude that there is a difference between the two treatments when, in fact, there is no difference. Mathematics teachers who retain or revise their methods based on such false conclusions would be doing a disservice to their students.

Reconsider the ninth grade algebra teaching experiment described previously. If a type II error is made, the investigator will conclude that there is no difference in achievement when, in fact, there is a difference. Thus no

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[36] <u>Ibid</u>.

change in method will be made when a change could possibly produce better results. A chance for improvement could be overlooked and future students would be deprived of the opportunity of benefitting from the better method and, hence from superior achievement. Since the welfare of the student is of prime importance in an educational procedure, size of the type II error should be small in an educational experiment. Table 6 summarizes the discussion on the two types of error.

TABLE 6

TWO TYPES OF ERROR

	H_o true	H_o false
Reject H_o	Type I error (Students not hindered)	Correct decision
Accept H_o	Correct decision	Type II error (Students hindered)

H_o - null hypothesis

Population and Sampling

Due to schedule conflicts and other factors inherent in a junior college educational program, assigning students to each course section on an individual basis was not possible. Since the course has prerequisites, which are strictly adhered to, the two groups had approximately the same mathematical background. However, the statistical model, analysis of covariance, which was used to analyze the results of the experiment has the useful property that it adjusts for initial differences between the groups. Borg states in his book Educational Research that "this technique [Analysis of Covariance] overcomes many of the difficulties of both matching and random assignment."[37]

The students used as subjects for the investigation were enrolled in Mathematics 124 - Analytic Geometry and Calculus I, a four-credit course, at Black Hawk College, Moline, Illinois, during the Spring semester of the 1967-68 academic year. The class met four hours per week - Monday through Thursday.

At the beginning of the semester Group A consisted of 25 students. Twenty-three of the students in this group were males and two were females. Due to attrition during the semester, at the close of the course there were 22 students in this group. Twenty of the students who completed the course were males and two were females.

At the beginning of the semester Group B consisted of 24 students. Nineteen of the students in this group were males and five were females. At the close of the semester there were 19 students in this group. Fifteen of the students who completed the course were males and four were females.

All, except seven, students used as subjects in the investigation had taken college algebra and trigonometry, the

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[37]Borg, op. cit., p. 304.

normal prerequisite, the previous semester. Seven students, four in Group A and three in Group B, were repeating Analytic Geometry and Calculus I after failing the course during the Fall semester. All of the subjects, except two, were college freshmen in their second semester of college work. Two students were classified as sophomores.

The students were pre-engineering, physical science, and mathematics majors. The freshman year, especially in a junior college, however, is an exploratory experience for many students. The college freshman is trying to find where his strengths, weaknesses, and interests lie. In general, the students used as subjects in this investigation could be characterized as typical of those students who enroll in the first course in analytic geometry and calculus.

General Procedures

The first day of class both groups were told that they were to participate in an experiment involving learning calculus through programming a digital computer. They were told that one of the two groups would be using the computer during the entire semester, but, at this time, they were not told which group. The investigator taught both groups.

During the first eight class sessions of the semester, the two groups were taught the FORTRAN IV computer language. Two weeks (eight sessions) of instruction would allow the students to learn the basic ideas of FORTRAN and yet would not use so much time that some of the material normally presented in the course would have to be omitted. FORTRAN is a computer language similar to algebra (which was already familiar to the students) and was designed specifically for problems of a mathematical nature.

During these first two weeks of the semester, the students in both groups wrote simple programs to solve elementary problems in order to gain skill and confidence. These programs were tested on the IBM 1401 computer at Black Hawk College. After these eight sessions, the two treatments were randomly assigned, Treatment 1 to Group A and Treatment 2 to Group B, and the classes began the usual course work of Analytic Geometry and Calculus I. Group A was not allowed to submit programs after the preliminary sessions.

As each of the four selected mathematical concepts was encountered during the course, the phase of the study associated with that concept was undertaken.[38] Before the concept was introduced in class lecture and discussion, the fifty-minute test associated with that phase of the study was given as a pretest. The scores on the pretest were listed in descending order and on the basis of the test results each group was divided into a high level and a low level.

As the material was covered in class, Group A did their homework assignments in the usual way with pencil and paper and Group B submitted programs as homework. Exemplary

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[38] _Supra_, pp. 16-17.

problems assigned for solution are shown in Appendix I of this report. The programs were processed on the computer by Data Processing Staff operators. The conclusion of each phase of the investigation consisted of a fifty-minute criterion test which was the same as the pretest.

Throughout the investigation the students in Group B were told to make their programs as general as possible and to use problems assigned by the instructor as data to test their programs. All of the material of Analytic Geometry and Calculus I is not amenable to computer programming. Thus in each phase of the study the students in Group B were sometimes assigned problems involving mathematical proofs and simple manipulative-type problems to be done with pencil and paper. The emphasis of the assignments, however, was on the computer programs assigned as homework.

The students keypunched their own programs. The programs were submitted by placing them in an "in" box in the computer room. After the programs were run on the computer, they were placed in an "out" box in the computer room. The student then picked up his program and, when he was satisfied that his solution was correct, he presented the program listing and the computer output to the course instructor, who verified and recorded the results and returned the listing and output to the student.

Specific Procedures

Phase I of the investigation dealt with the mathematical idea of function. Six class sessions were concerned specifically with this concept. During Phase I of the study Group A consisted of 25 students and Group B consisted of 24 students. One of the members of Group A was absent the day the pretest was given and was omitted from this phase of the study. Thus there were 48 subjects involved in this phase of the investigation. The results of the pretests and criterion tests for this phase are shown in Table 7. The mean of the pretest scores is higher on this phase than on subsequent phases of the investigation because the concept of function is also used in courses which are prerequisites for analytic geometry and calculus.

Phase II of the investigation was the study of limits and differentiation. Sixteen class sessions were devoted to this phase of the study. During Phase II there were twenty-two subjects in Group A and twenty subjects in Group B. One student in Group A took the pretest late and hence was eliminated from this phase. Another student in Group A was randomly selected and deleted to equalize the treatments by levels cells. Thus there were 40 subjects included in this phase of the investigation. The pretest and criterion test scores for this phase of the investigation are displayed in Table 8.

Phase III of the investigation was the study of iterative techniques. Five lectures were devoted to iterative methods. During Phase III of the investigation there were twenty-one students in Group A and twenty students in

TABLE 7

TEACHING EXPERIMENT - PHASE I

Pretest (X) and Post Test (Y) Scores

	Treatment 1 X	Treatment 1 Y	Treatment 2 X	Treatment 2 Y	
Level 1	27	32	28	30	
	26	32	27	33	
	26	30	25	28	
	25	29	25	31	
	24	29	24	27	$\bar{X}_{1.}=23.4$
	24	31	23	31	
	23	31	23	30	$\bar{Y}_{1.}=29.5$
	23	29	22	33	
	22	28	22	26	
	21	28	22	28	
	19	31	21	27	
	19	28	20	25	
	$\bar{X}_{11}=23.3$	$\bar{Y}_{11}=29.8$	$\bar{X}_{12}=23.5$	$\bar{Y}_{12}=29.1$	
Level 2	19	25	20	28	
	18	28	20	27	
	17	29	19	24	
	17	25	18	20	
	16	25	18	30	$\bar{X}_{2.}=15.9$
	16	24	18	22	
	15	26	18	21	$\bar{Y}_{2.}=29.5$
	14	24	16	21	
	14	22	15	22	
	11	29	15	23	
	11	13	15	24	
	9	19	12	25	
	$\bar{X}_{21}=14.8$	$\bar{Y}_{21}=24.1$	$\bar{X}_{22}=17.0$	$\bar{Y}_{22}=23.9$	
	$\bar{X}_{.1}=19.5$		$\bar{X}_{.2}=20.3$		$\bar{X}_{..}=19.8$
	$\bar{Y}_{.1}=26.9$		$\bar{Y}_{.2}=26.5$		$\bar{Y}_{..}=26.7$

TABLE 8

TEACHING EXPERIMENT - PHASE II

Pretest (X) and Post Test (Y) Scores

	Treatment 1		Treatment 2		
	X	Y	X	Y	
	16	27	13	13	
	12	22	9	11	
	12	16	8	17	
	11	18	8	13	$\bar{X}_{1.}= 9.0$
Level 1	9	20	8	19	
	9	12	8	22	$\bar{Y}_{1.}=18.2$
	8	22	7	17	
	7	13	7	17	
	7	19	7	21	
	7	18	7	26	
	$\bar{X}_{11}= 9.9$	$\bar{Y}_{11}=18.7$	$\bar{X}_{12}= 8.2$	$\bar{Y}_{12}=17.6$	
	7	18	7	13	
	7	19	6	22	
	7	16	6	16	$\bar{X}_{2.}= 4.7$
Level 2	7	20	6	20	
	7	7	6	17	$\bar{Y}_{1.}=15.8$
	5	13	5	19	
	5	19	4	11	
	2	17	2	11	
	1	17	1	13	
	$\bar{X}_{21}= 5.2$	$\bar{Y}_{21}=15.7$	$\bar{X}_{22}= 4.2$	$\bar{Y}_{22}=15.9$	
	$\bar{X}_{.1}= 7.5$		$\bar{X}_{.2}= 6.2$		$\bar{X}_{..}= 6.8$
	$\bar{Y}_{.1}=17.2$		$\bar{Y}_{.2}=16.8$		$\bar{Y}_{..}=17.0$

Group B. One student from Group A was randomly selected and deleted to equalize the number of students in each cell. Thus there were 40 subjects included in Phase III. The pretest and criterion test scores for this phase are presented in Table 9.

Phase IV of the investigation was the study of integration. Fourteen class sessions were devoted to this concept. During Phase IV there were twenty students in Group A and nineteen students in Grcup B. One student from Group A took the pretest late and was deleted. One additional student was randomly selected from each group and omitted to equalize the number of subjects in each cell. Thus there were 36 students included in this phase of the study. The pretest and criterion test scores for Phase IV are shown in Table 10.

Statistical Treatment of Data

The computation for the analysis of covariance and the regression equations in this study were performed by an IBM 1401 computer. The programs were written and tested by the investigator and the program which computes the analysis of covariance is listed in Appendix F. The output of this program is an analysis of covariance table. The program which computes linear regression equations is listed in Appendix G. The output of this program is a regression equation for each treatment by level cell and a total regression

TABLE 9

TEACHING EXPERIMENT - PHASE III

Pretest (X) and Post Test (Y) Scores

	Treatment 1		Treatment 2		
	X	Y	X	Y	
Level 1	19	23	20	20	
	18	25	17	23	
	18	22	17	27	
	16	24	16	18	$\bar{X}_{1\cdot}=15.8$
	15	25	16	30	
	15	22	15	29	$\bar{Y}_{1\cdot}=23.2$
	15	20	14	25	
	15	20	14	17	
	14	20	14	25	
	14	23	13	26	
	$\bar{X}_{11}=15.9$	$\bar{Y}_{11}=22.4$	$\bar{X}_{12}=15.6$	$\bar{Y}_{12}=24.0$	
Level 2	13	18	12	26	
	13	24	12	23	
	13	15	12	22	
	13	16	10	23	$\bar{X}_{2\cdot}=10.4$
	13	26	9	17	
	11	25	9	18	$\bar{Y}_{2\cdot}=20.5$
	10	18	9	20	
	10	18	8	21	
	9	15	7	25	
	9	21	6	19	
	$\bar{X}_{21}=11.4$	$\bar{Y}_{21}=19.6$	$\bar{X}_{22}=9.4$	$\bar{Y}_{22}=21.4$	
	$\bar{X}_{\cdot 1}=13.7$		$\bar{X}_{\cdot 2}=12.5$		$\bar{X}_{\cdot\cdot}=13.1$
	$\bar{Y}_{\cdot 1}=21.0$		$\bar{Y}_{\cdot 2}=22.7$		$\bar{Y}_{\cdot\cdot}=21.9$

TABLE 10

TEACHING EXPERIMENT - PHASE IV

Pretest (X) and Post Test (Y) Scores

	Treatment 1		Treatment 2		
	X	Y	X	Y	
Level 1	17	21	16	17	
	16	25	12	15	
	14	30	11	15	
	13	24	11	27	$\bar{X}_{1\cdot}$=10.8
	11	14	11	21	
	9	19	8	28	$\bar{Y}_{1\cdot}$=21.0
	8	12	8	22	
	8	22	8	20	
	7	23	7	23	
	$\bar{X}_{11}$=11.4	$\bar{Y}_{11}$=21.1	$\bar{X}_{12}$=10.2	$\bar{Y}_{12}$=20.9	
Level 2	7	12	6	20	
	7	17	6	16	
	7	17	6	21	
	7	21	5	15	$\bar{X}_{2\cdot}$= 5.3
	7	21	5	23	
	6	21	5	26	$\bar{Y}_{2\cdot}$=19.2
	5	19	3	24	
	5	13	3	24	
	4	16	1	21	
	$\bar{X}_{21}$=8.1	$\bar{Y}_{21}$=17.4	$\bar{X}_{22}$=4.4	$\bar{Y}_{22}$=20.9	
	$\bar{X}_{\cdot 1}$= 8.8		$\bar{X}_{\cdot 2}$= 7.3		$\bar{X}_{\cdot\cdot}$= 8.1
	$\bar{Y}_{\cdot 1}$=19.3		$\bar{Y}_{\cdot 2}$=20.9		$\bar{Y}_{\cdot\cdot}$=20.1

equation. Both programs are written to accept the same input data.

The students recorded their answers on the pretests and post tests on the answer cards described in Chapter II of this report. The answer cards were processed by the Automatic Test Scoring System (ATSS) provided by the Black Hawk College Data Center.

The "teacher" variable was controlled in the study since the investigator taught both groups. The same lectures were given to both groups; only the homework assignments and discussion of homework were different. Other variables which were controlled are:

1. the number of class sessions spent discussing each mathematical concept used in the study were the same for both groups.

2. the initial differences between the groups were adjusted by using analysis of covariance.

3. both groups were introduced to the FORTRAN IV computer language.

4. the length of time allowed to complete the pretests and criterion tests was the same for both groups.

5. both groups of students used the same textbook.

6. both groups of students were of approximately the same mathematical background.

Due to the nature of the study, some variables were difficult to control. These were:

1. the amount of time that each student studied each concept outside of class.

2. discussion between the members of the two treatment groups outside of class.

The assumption must be made that these variables were random and hence did not influence the results of the experiment.

CHAPTER IV

RESULTS AND DISCUSSION

Preliminary Discussion

A summary of pretest and criterion test scores by treatment and level for Phase I, II, III and IV of the investigation were listed in Tables 5, 6, 7 and 8 respectively.

In Figure 1 the total regression line of Y and X has been plotted for Phase I. The mean values of X and Y and the regression lines for each cell have also been plotted on the graph. The notation of Table 1 has been used to label the regression lines for each cell. The mean value points for the high-computer (HC), high traditional (HT) and low-traditional (LT) cells lie above the total regression line. This indicates that the observed mean value of the criterion test scores was higher than the predicted mean value in the HC, HT and LT cells. The mean value point of the low-computer (LC) cell lies below the total regression line. This indicates that the observed mean value of the criterion test scores was lower than the predicted mean value in the LC cell. Analysis of covariance will be employed to decide if these differences are statistically significant.

FIGURE 1

REGRESSION LINES - PHASE I

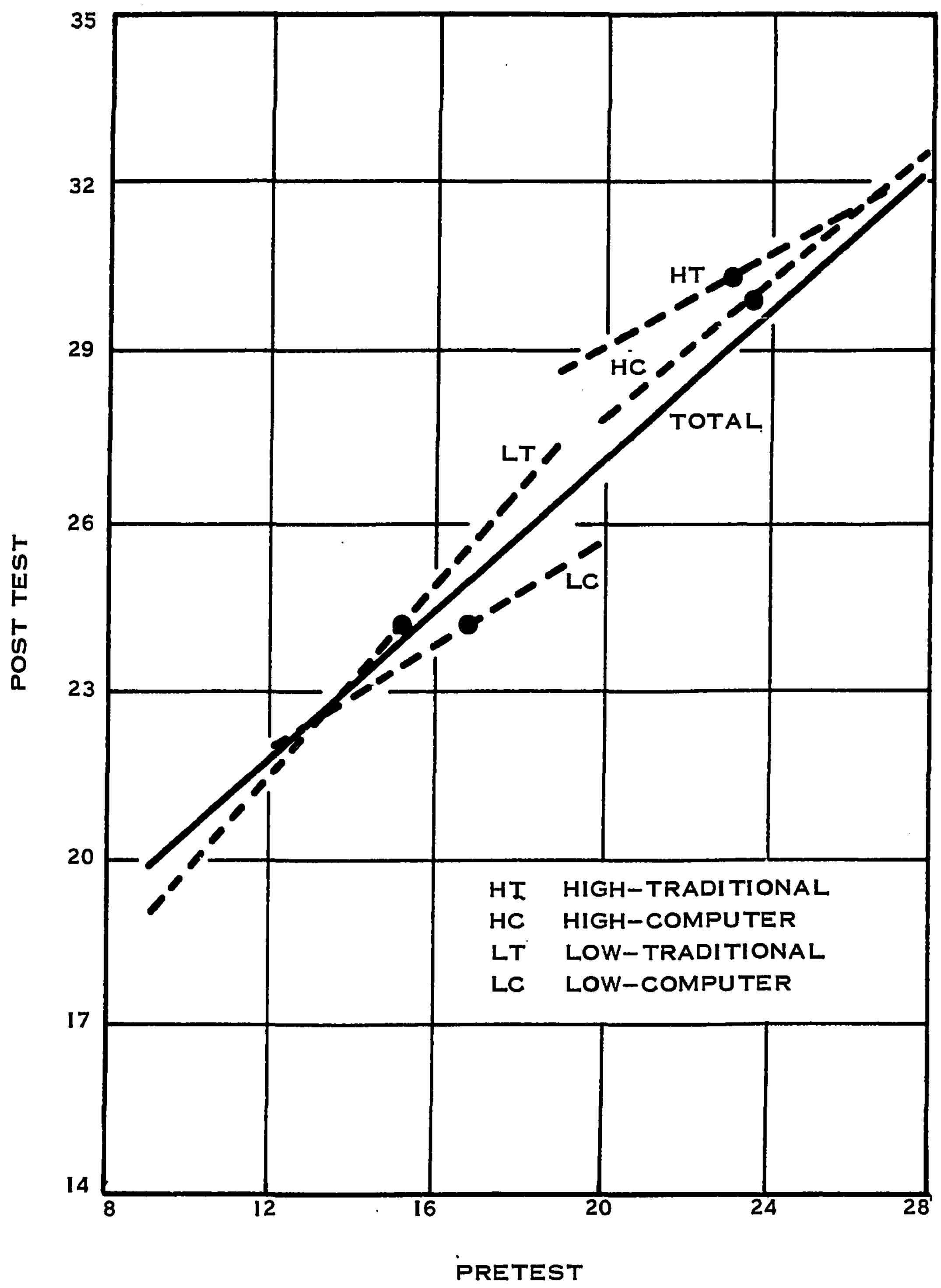


Figure 2 is similar to Figure 1 except that Figure 2 applies to Phase II. In this case the points associated with the means of LC, LT and HC are below the total regression line while the mean value point of HT is above the total regression line. However, all of these points do not appear to deviate very much from the total regression line. In Figure 2 the HC cell regression line has a negative slope. This indicates that subjects having higher pretest scores in this cell did not gain as much in achievement as subjects having lower pretest scores. This phenomenon was also observed in the graphs of the regression lines for Phase IV. Comments concerning this occurrence will be made in the General Discussion section of this chapter.

Figure 3 illustrates the graphs of the regression lines for Phase III. In Figure 3 the LC and HC mean value points are below the total regression line. This indicates that at both levels the observed means of the group which received Treatment 2 (the computer group) were higher than the predicted means, whereas the observed means of the traditional group were lower than the predicted means at both levels. However, the analysis of covariance model must be applied before a decision can be made as to whether the differences are significant.

Figure 4 shows the regression lines for Phase IV. In this case the mean value points for the LC, HC and HT cells are above the total regression line and the mean value point

FIGURE 2

REGRESSION LINES - PHASE II

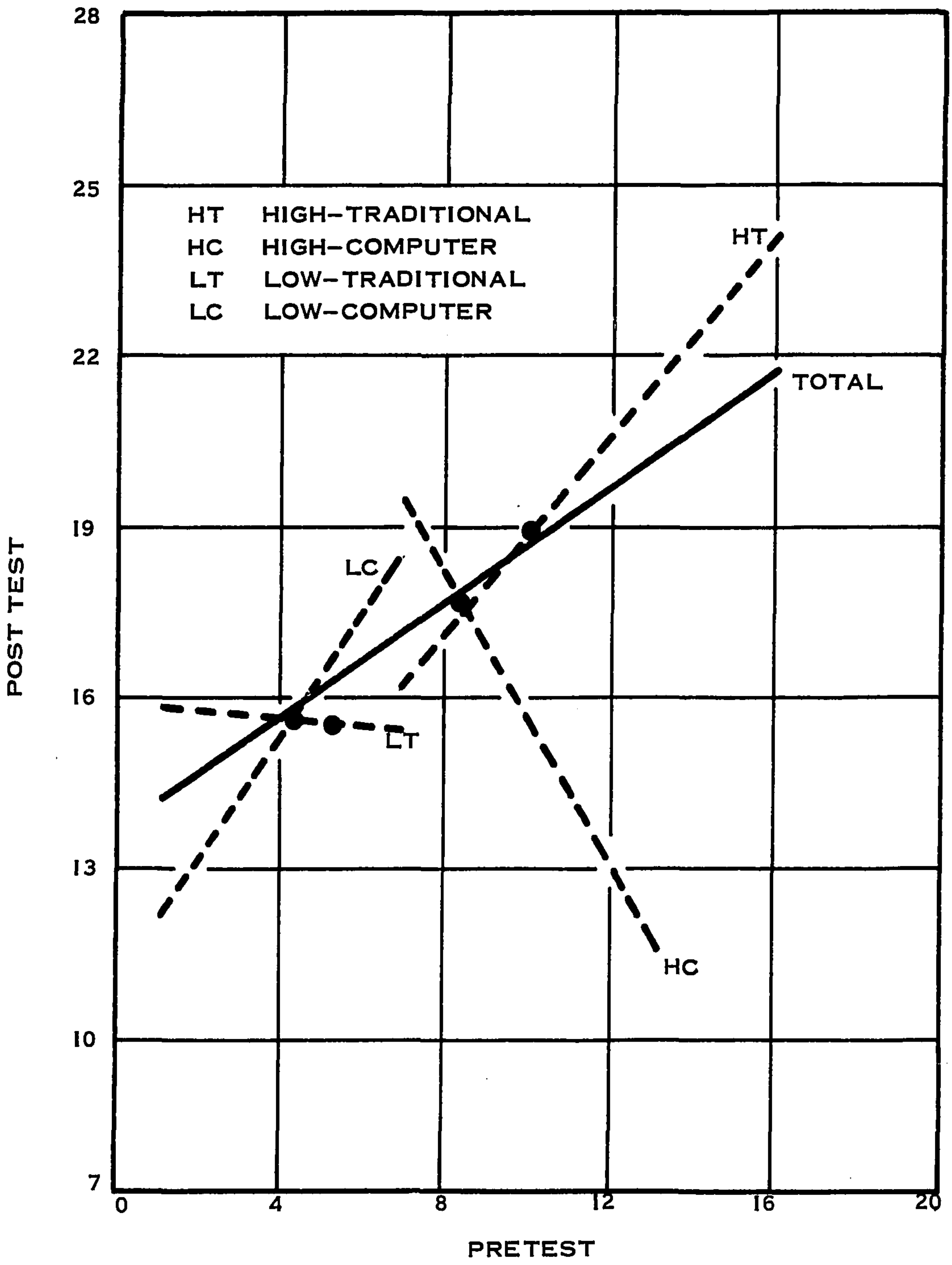


FIGURE 3

REGRESSION LINES - PHASE III

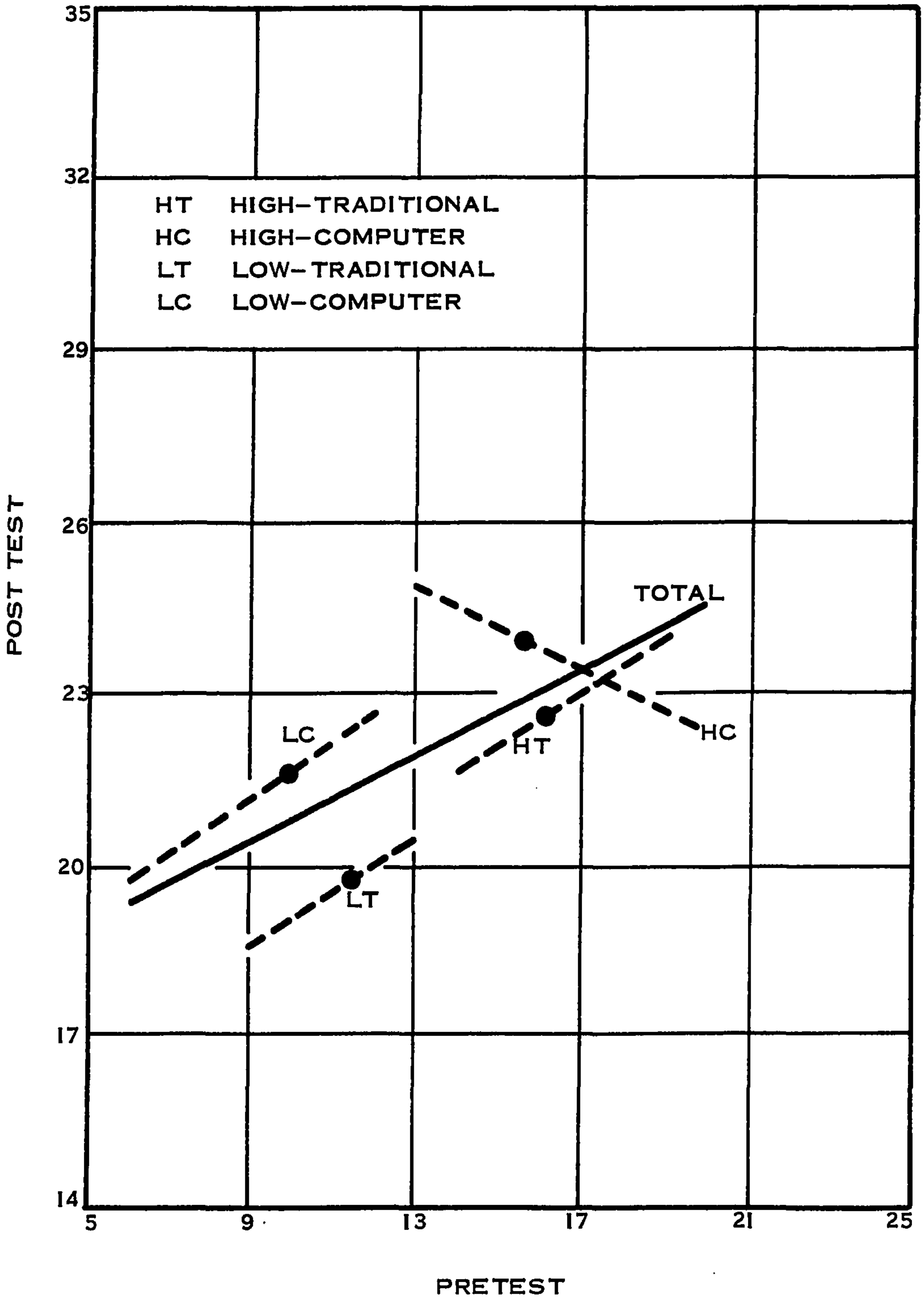
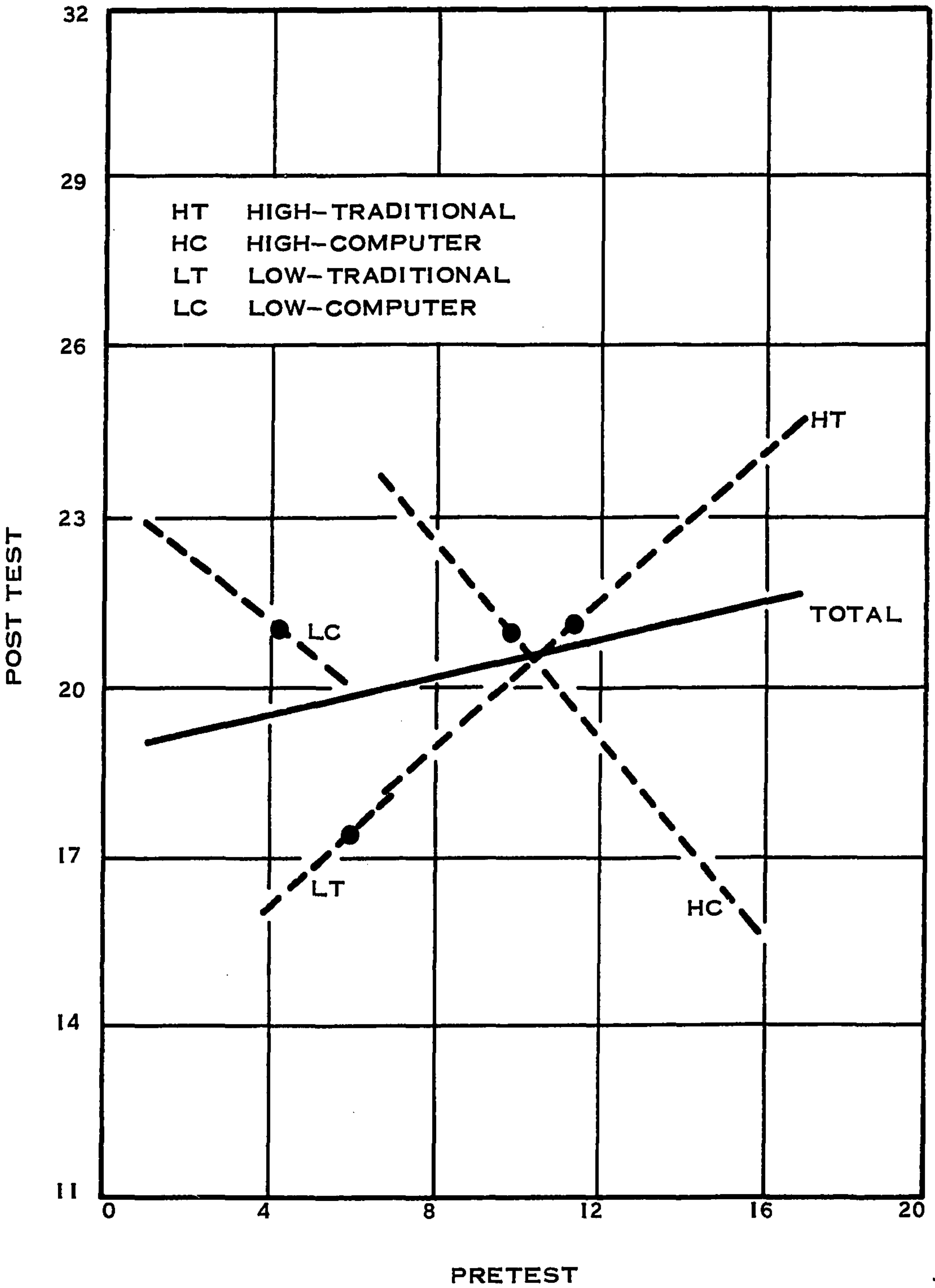


FIGURE 4

REGRESSION LINES - PHASE IV



for the LT cell is below the total regression line. As in Phase II, the regression lines for the LC and HC cells have negative slopes.

Tests of Hypotheses

The hypotheses, in null form, which were tested by the investigation are:

Hypothesis 1. There is no difference between the adjusted means of the two treatment groups indicated by the criterion test.

Hypothesis 2. There is no difference between the adjusted means of the two levels indicated by the criterion test.

Hypothesis 3. There is no interaction between treatments and levels on the adjusted means.

An explanation of the equations used to obtain the numerical entries in the analysis of covariance tables is presented in Appendix H of this report. The computer program which was written to perform the necessary calculations is listed in Appendix F.

Application of the analysis of covariance model yields the F-statistic.[39] The significance of the obtained F-ratio can be determined by consulting a standard table of F-ratios.[40] To enter the F-table, the degrees of freedom for the numerator and denominator of the F-ratio must be known.

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[39]Ostle, *op. cit.*, p. 453.

[40]*Ibid.*, pp. 529-541.

For each phase of the study, the degrees of freedom for the denominator depends upon the number of students who participated in a given phase. As was noted in Chapter III, some students withdrew from the course during the semester. For the foregoing reason, the degrees of freedom of the denominator of the F-ratio was not the same for all four phases of the investigation.

Tests of Hypotheses for Phase I - Functions

The analysis of covariance table for Phase I of the investigation is shown in Table 11. To reject the null hypothesis at the 0.10 level of significance for 1 and 43 degrees of freedom the F-ratio must exceed 2.83. Since the F-ratio obtained for the two treatments (1.84) is less than 2.83, Hypothesis 1 was not rejected. Thus for Phase I of the study the investigator concluded that there is no difference in achievement between the groups as a result of the two treatments.

The F-ratio obtained for the two levels used in the experiment (0.93) is not significant at the 0.10 level. Hypothesis 2 was not rejected and the investigator concluded that there is no difference in achievement between the two levels. The F-ratio obtained for the treatments by levels interaction (0.09) is not significant at the 0.10 level. Hypothesis 3 is not rejected and the investigator concluded that there is no interaction between treatments and levels.

TABLE 11

ANALYSIS OF COVARIANCE - PHASE I
FUNCTIONS

Source of Variation	Degrees of Freedom	Criterion (Y) Sum of Squares	Sum of Products (X·Y)	Pretest (X) Sum of Squares
Treatments	1	2.52	-6.88	18.75
Levels	1	357.52	491.25	675.00
Treatments x Levels	1	1.02	3.50	12.00
Within Cells	44	430.42	166.25	307.50
Total	47	791.48	654.13	1013.25
Treatments + Within Cells	45	432.94	159.38	326.25
Levels + Within Cells	45	787.94	657.50	982.50
Treatments x Levels + Within Cells	45	431.44	169.75	319.50

TABLE 11 - Continued

Source of Variation	Adjusted Degrees of Freedom	Adjusted Criterion Sum of Squares	Adjusted Mean Square	F-ratio	$F_{0.10}$
Treatments	1	14.55	14.55	1.84	2.83
Levels	1	7.40	7.40	0.93	2.83
Treatments x Levels	1	0.72	0.72	0.09	2.83
Within Cells	43	340.53	7.92		
Treatments + Within Cells	44	355.08			
Levels + Within Cells	44	347.93			
Treatments x Levels + Within Cells	44	341.25			

Tests of Hypotheses for Phase II -
Limits and Differentiation

The analysis of covariance table for Phase II of the study is displayed in Table 12. At 1 and 35 degrees of freedom the F-ratio needed to reject the null hypothesis at the 0.10 level of significance must be larger than 2.86. The F-ratio obtained for the two treatments (0.10) is not significant at the 0.10 level. Hypothesis 1 was not rejected and the investigator concluded that there is no difference in achievement between the two treatment groups for Phase II.

The F-ratio obtained for the two levels (0.30) is not significant at the 0.10 level. Thus Hypothesis 2 was not rejected and the investigator concluded that there is no difference in achievement between the two levels. The F-ratio obtained for the treatments X levels interaction (0.12) is not significant at the 0.10 level. Hence Hypothesis 3 was not rejected and the investigator concluded that there is no interaction between treatments and levels.

Tests of Hypotheses for Phase III -
Iteration

The analysis of covariance table for Phase III is presented in Table 13. At 1 and 35 degrees of freedom the observed F-ratio must be greater than 2.86 to reject the null hypothesis at the 0.10 level of significance. The F-ratio obtained for the two treatments (2.81) is not significant at the 0.10 level. Thus Hypothesis 1 was not rejected and the

TABLE 12

ANALYSIS OF COVARIANCE - PHASE II
LIMITS AND DIFFERENTIATION

Source of Variation	Degrees of Freedom	Criterion (Y) Sum of Squares	Sum of Products (X·Y)	Pretest (X) Sum of Squares
Treatments	1	2.03	4.73	11.03
Levels	1	55.23	95.18	164.03
Treatments x Levels	1	4.23	3.58	3.03
Within Cells	36	653.50	61.50	186.90
Total	39	714.98	164.98	364.98
Treatments + Within Cells	37	655.53	66.23	197.93
Levels + Within Cells	37	708.73	156.68	350.93
Treatments x Levels + Within Cells	37	657.73	65.08	189.93

TABLE 12 - (Continued)

Source of Variation	Adjusted Degrees of Freedom	Adjusted Criterion Sum of Squares	Adjusted Mean Square	F-ratio	$F_{0.10}$
Treatments	1	0.10	0.10	0.01	2.86
Levels	1	5.51	5.51	0.30	2.86
Treatments x Levels	1	2.16	2.16	0.12	2.86
Within Cells	35	633.26	18.09		
Treatments + Within Cells	36	633.37			
Levels + Within Cells	36	638.78			
Treatments x Levels + Within Cells	36	635.43			

TABLE 13

ANALYSIS OF COVARIANCE - PHASE III ITERATION

Source of Variation	Degrees of Freedom	Criterion (Y) Sum of Squares	Sum of Products (X·Y)	Pretest (X) Sum of Squares
Treatments	1	28.90	-19.55	13.23
Levels	1	72.90	144.45	286.23
Treatments x Levels	1	0.10	-0.85	7.23
Within Cells	36	445.20	31.40	136.10
Total	39	547.10	155.45	442.78
Treatments + Within Cells	37	474.10	11.85	149.33
Levels + Within Cells	37	518.10	175.85	422.33
Treatments x Levels + Within Cells	37	445.30	30.55	143.33

TABLE 13 - Continued

Source of Variation	Adjusted Degrees of Freedom	Adjusted Criterion Sum of Squares	Adjusted Mean Square	F-ratio	$F_{0.10}$
Treatments	1	35.20	35.20	2.81	2.86
Levels	1	6.92	6.92	0.55	2.86
Treatments x Levels	1	0.83	0.83	0.07	2.86
Within Cells	35	437.96	12.51		
Treatments + Within Cells	36	473.16			
Levels + Within Cells	36	444.88			
Treatments x Levels + Within Cells	36	438.79			

investigator concluded that there is no difference between the two treatments in Phase III.

The F-ratio obtained for the two levels (0.55) is not significant at the 0.10 level. Hypothesis 2 was not rejected and the investigator concluded that there is no difference between the two levels. The F-ratio obtained for the treatments by levels interaction (0.55) is not significant at the 0.10 level. Hypothesis 3 was not rejected and the investigator concluded that there is no interaction between treatments and levels.

Tests of Hypotheses for Phase IV - Integration

The analysis of covariance table for Phase IV is shown in Table 14. At 1 and 31 degrees of freedom the F-ratio must exceed 2.88 in order to be significant at the 0.10 level. The F-ratio obtained for the two treatments (1.16) is not significant at the 0.10 level. Hypothesis 1 was not rejected and the investigator concluded that there is no difference in achievement as a result of the treatments in Phase IV.

The F-ratio obtained for the two levels (0.50) is not significant at the 0.10 level. Thus Hypothesis 2 was not rejected and the investigator concluded that there is no difference between achievement on the two levels. The F-ratio obtained for the treatments by levels interaction (1.53)

TABLE 14

ANALYSIS OF COVARIANCE - PHASE IV
INTEGRATION

Source of Variation	Degrees of Freedom	Criterion (Y) Sum of Squares	Sum of Products (X·Y)	Pretest (X) Sum of Squares
Treatments	1	23.36	-20.94	18.78
Levels	1	30.25	91.67	277.78
Treatments x Levels	1	30.25	-3.67	0.44
Within Cells	32	616.89	8.78	208.89
Total	35	700.75	75.83	505.89
Treatments + Within Cells	33	640.25	-12.17	227.67
Levels + Within Cells	33	647.14	100.44	486.67
Treatments x Levels + Within Cells	33	647.14	5.11	209.33

TABLE 14 - Continued

Source of Variation	Adjusted Degrees of Freedom	Adjusted Criterion Sum of Squares	Adjusted Mean Square	F-ratio	$F_{0.10}$
Treatments	1	23.08	23.08	1.16	2.88
Levels	1	9.89	9.89	0.50	2.88
Treatments x Levels	1	30.49	30.49	1.53	2.88
Within Cells	31	616.52	19.89		
Treatments + Within Cells	32	639.60			
Levels + Within Cells	32	626.41			
Treatments x Levels + Within Cells	32	647.01			

is not significant at the 0.10 level. Hypothesis 3 was not rejected and the investigator concluded that there is no interaction between treatments and levels for Phase IV.

General Discussion

Failure to reject Hypothesis 1 for all phases of the investigation suggests that there is no evidence that there is a difference in the two treatments; i.e., the students learned the selected mathematical concepts just as well by computer programming as they did by solving problems in the usual homework structure. Figure 3 indicated a possible difference in treatments in Phase III and, indeed, the F-ratio for treatments on this phase was the highest F-ratio obtained in the study. Perhaps the explanation for the "relatively high" F-ratio is that iterative techniques are more amenable to computer methods than the other concepts. Problems involving iterative methods solved by traditional methods involve tedious arithmetic operations, especially after the first or second iteration. The student tends to become confused by the numerical operations and forgets the process. This is one reason why teachers and textbook authors sometimes omit problems of this type in a traditional course. By using computer methods a student can concentrate more on the process and less upon the computations involved in a given problem.

Failure to reject Hypothesis 2 in all phases indicates that each level performed relatively the same on the pretest as on the post test. A few individual students performed relatively better on the post test than the pretest, but as a group the low level on the pretest remained low on the post test and the high level remained high. This indicates that, in general, the amount of mathematical knowledge gained during the course was a factor of the amount of mathematical knowledge that a student had prior to beginning the course.

Failure to reject Hypothesis 3 for all phases of the study indicates that there is no interaction between treatments and levels. The treatments produced similar results at both levels. The largest F-ratio for interaction (1.53) was obtained in Phase IV. In this case the criterion means were the same for both levels in the computer programming group (see Table 8). Studying the concept of integration via computer programming appeared to "equalize" these two cells. However, the F-ratio is not significant at the 0.10 level and the null hypothesis for interaction was rejected for Phase IV.

The influence of the computer upon the motivation of the student must not be overlooked. The idea that computers do motivate mathematics students is suggested by the following statement from Computer Oriented Mathematics:

"The spectacular development of computing machines and their impact on contemporary society has interested many teachers who have worked to make their courses of study consistent with the modern world. Moreover, teachers are discovering that the computer can serve as an excellent tool in the motivation of the study of mathematics."[41]

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[41] Computer Oriented Mathematics (Washington: The National Council of Teachers of Mathematics, Inc., 1963), p. 6.

In order to consider the effect of the computer upon motivation, the investigator made some observations which are thought to be important about the attitudes of the computer programming group during the course of the experiment. These observations are not a part of the formal study, but reflect impressions that were obtained by working with the students.

Many of the students in the computer programming group seemed to enjoy working with the computer and programming their homework problems. One student was taking a physics course and was granted permission to write programs to solve problems for that course. After all four phases of the study were completed, the students in the computer group were given a brief questionnaire. Of the nineteen students who completed the course, ten "felt" that they learned mathematical concepts better by programming than in the usual way and five did not. Four students responded with a "partial yes" in favor of programming. Seventeen students declared that learning FORTRAN programming for its own sake was a worthwhile effort, whereas two students stated that the effort was not worthwhile.

At first, strict attention to detail necessary to write workable programs and the precision and objectivity with which the computer treated their programs was difficult for the students to comprehend. Most of the students overcame their initial reactions, but for several students writing computer programs was an arduous experience. For these students the idea of getting back a highly objective criticism of their errors and going over the problem again and again until it was absolutely correct was foreign to their temperaments. In their former educational experiences they had submitted problems and received partial credit for partially correct solutions and then continued to the next lesson. A few students stopped submitting programs since they could not accept the idea that their program was incorrect every time. Frequently a student would blame the computer for making a mistake on his program.

Thus the observation of the investigator is that most students were positively motivated by the computer while a few were negatively motivated. There was no noticeable difference in ability between these two categories of students. Some students who the investigator judged to be of low mathematical ability would try very hard and experienced success in their programming efforts; whereas others with apparently higher ability would "give up". Perhaps there are personality traits which influence the way that computer programming affects motivation.

The investigator suggests that the statement in Computer Oriented Mathematics and similar claims are highly subjective judgements. Often the teacher educating students in computer programming is working with volunteers or students who are already interested in computers. Also the teacher is not likely to keep accurate records of what the student does on the computer and hence may recall only the enthusiasm of the students (perhaps a reflection of his own enthusiasm) and remember only the accomplishments of those students who performed exceptionally well.

The students who participated in this study were not volunteers nor were they selected because of high interest in computers or ability in mathematics. They represented the usual set of students enrolled in Analytic Geometry and Calculus I at Black Hawk College. There is no reason to believe that Black Hawk College students are significantly different from college students in other institutions of higher education.

CHAPTER V

SUMMARY, CONCLUSIONS, RECOMMENDATIONS

Restatement of the Problem

The purpose of this investigation was to determine whether students develop a deeper understanding of mathematical concepts by programming problems involving those concepts for a digital computer than by doing the usual homework assignments. The understanding of the concepts achieved by the students who participated in the study was measured by separate examinations which were shown to be valid by using students who were not involved in the study and who had satisfactorily completed the course wherein the selected mathematical concepts were taught.

Two groups of students (denoted as Group A and Group B) were used for the study. The groups consisted of all students enrolled in Analytic Geometry and Calculus I during the Spring semester of the 1967-68 academic year at Black Hawk College, Moline, Illinois. Four mathematical concepts were selected for the investigation: (1) functions, (2) limits and differentiation, (3) iteration, and (4) integration. The study consisted of four phases; each phase was concerned with one of the concepts selected for the study.

Evaluative Instruments

Four evaluative instruments were constructed to test the student's understanding of the four concepts selected for the study. Each instrument consisted of thirty-five multiple-choice items with five alternative answers per test item. Each instrument tested one of the concepts selected for the investigation.

The four instruments were examined by three college mathematics instructors to assure that each test dealt only with the concept being examined by that instrument and that the test items reflected the content of the first course in analytic geometry and calculus. In order to perform a small standardization of the instruments, students from four colleges and universities who recently completed the first course in analytic geometry and calculus took the tests before they were applied to the teaching experiment.

The first test concerns the concept of functions and includes items on polynomial, rational, algebraic, and composite functions. The second test is on limits and differentiation and contains problems involving limit theorems and their applications, derivatives, maxima and minima, and the first and second derivative tests. The third test, on iteration, includes items on Newton's method, the fixed point method, the rule of false position, and basic recurrence relations. The fourth instrument is concerned with integration and contains items on basic integration formulas, Riemann sums, the trapezoidal rule, and Simpson's rule. All four tests are shown in Appendix B of this dissertation.

Experimental Design

A 2 x 2 treatments by levels design was used in the investigation. Treatment 1 consisted of solving problems assigned as homework with pencil and paper. Treatment 2 consisted of writing computer programs in FORTRAN IV which would allow a digital computer to solve problems assigned as homework. Group A received Treatment 1 and Group B received Treatment 2.

Since it was not possible to assign students randomly to the two groups, analysis of covariance was utilized to evaluate the results. Tests of significance were made at the 0.10 level. The evaluative instruments which were constructed for each concept selected for the study were utilized to measure the students' understanding of the concepts.

For each phase of the study each group was divided into a high and a low level on the basis of the evaluative instrument for that phase of the study applied as a pretest. Thus it was possible not only to determine if the treatments produced different results, but also to determine if students with greater prior knowledge of the given concept performed differently than students with less prior knowledge.

Procedures

During the first eight class sessions both groups were taught the FORTRAN IV computer language. Both groups of students wrote programs to solve simple problems in order to gain skill and confidence. The programs were run on an IBM 1401 computer by the Data Processing Staff. At the end of the first eight class meetings Group A was assigned Treatment 1 and Group B was assigned Treatment 2.

As each of the concepts selected for the study were encountered during the course, the phase of the investigation associated with that concept was undertaken.[42] Prior to lectures on the concept, a pretest was given and each treatment group was divided into a high level and a low level. During the portion of the semester devoted to the given concept, Group A did their homework assignments the usual way (with pencil and paper) and Group B submitted computer programs to solve problems assigned as homework. Each phase of the study proceeded in the same fashion and concluded with the post test given as a criterion test to measure achievement on the concept under investigation.

Findings

The hypotheses of the investigation were tested by interpreting the results obtained from the analysis of covariance model. A 0.10 level of significance was used. As a result of these tests the investigator found:

1. In each phase of the study, there was no statistically significant difference in achievement between the

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[42] Supra, pp. 16-17.

treatment groups; i.e., as measured by the criterion tests, the group which received Treatment 1 did not perform significantly different than the group which received Treatment 2.

2. In each phase of the study, there was no statistically significant difference in achievement between the two levels; i.e., as measured by the criterion tests, the group of students with a lesser amount of prior knowledge of the selected concepts did not gain significantly more in understanding than the group of students with greater prior knowledge.

3. In each phase of the study, there was no statistically significant interaction between treatments and levels; i.e., as measured by the criterion tests, the two treatments produced similar results at the high and low levels of prior knowledge of the selected concepts.

Conclusions

The population used as subjects for the study consisted of all students at Black Hawk College enrolled in Analytic Geometry and Calculus I for the Spring semester of the 1967-68 academic year. Conclusions drawn from the study are applicable to that population and based upon the evaluative instruments used in the investigation. Generalizations to other situations must be drawn with care.

For the mathematical concepts, evaluative tests, and specific population used, the following conclusions were possible:

1. There is no apparent difference in achievement between students who learn mathematical concepts by computer programming and those who learn the concepts in the usual homework structure.

2. There is no difference in achievement between the two levels used in each phase of the study. Students with more previous knowledge of a given concept have a greater knowledge after the concept has been studied than students who have a lesser previous knowledge of that concept.

3. There is no apparent interaction between the treatments and levels used in the study. The treatments produce similar results at both levels.

Recommendations for Mathematics Education

Since both treatments used in the investigation produced similar results, a mathematics department may select either method to teach the mathematics concepts involved in the experiment to students. If a computer is available, computer programming could be used to supplement regular course work. Departments which continue with traditional methods will be as effective as departments which utilize computer programming in teaching the mathematical concepts involved in this study. If the results of this investigation are generalized, the use of the computer in mathematics courses must be justified by arguments concerned with its impact on modern society rather than the claim that students learn better by computer programming than by traditional methods.

This study was concerned primarily with the freshman-sophomore calculus sequence. In making the following recommendations for mathematics education, the investigator has considered the advice of the CUPM Committee of the Mathematical Association of America and the President's Science Advisory Committee cited in Chapter I of this dissertation as well as the results of his own study.

The results of this investigation indicated that the students used in the experiment learned mathematical concepts just as well by computer programming as they did by solving problems in the usual homework structure. There are, however, benefits to the student to be gained by knowledge of computer programming in addition to learning mathematical concepts. In subsequent courses, students of mathematics, statistics, engineering, and science will be expected to make use of the computer to solve assigned problems. College and university instructors in advanced courses assign problems for computer solution and assume that the student has learned computer programming in a previous course. Sometimes, during a class session, an instructor illustrates his lecture with a computer program which solves a particular problem and expects the students to comprehend the program. Students who later do research for advanced degrees probably will need the services of a computer to analyze data obtained from their experiments.

Many graduate students have had projects delayed because they lack experience in computer programming. The present demand for mathematically trained personnel is directly associated with the growth of computer utilization in business, government, and industry.[43] A person with an otherwise adequate background in mathematics frequently does not obtain a desired position because he has not had computer training. The advantages of knowing how to program a digital computer mentioned above are in evidence now, but they will be even more in evidence in the future, as the influence and numbers of computers in academic and professional life increase. Mathematics departments need to realize that they are not necessarily teaching mathematical concepts better by providing computer training, but are meeting an additional need of contemporary mathematics students.

Thus, since computers are available in institutions of higher education, mathematics departments should use them to provide all mathematics students with the advantages to be gained by knowing how to program a digital computer. Many mathematics departments offer a separate introductory course in programming, but the number of students who take this course is small compared to the total number of students who take mathematics courses. Aside from a few exceptional cases, the first course in analytic geometry and calculus is required of all mathematics, engineering, and physical science students.

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[43]United States Department of Labor, op. cit., p. 8.

Hence this course is the most feasible one in which to teach computer programming since the majority of science and mathematics students will be reached.

The investigator recommends that the freshman-sophomore calculus sequence be extended to four semesters. During the first six weeks of the first semester intensive effort should be given to teach the student a mathematically oriented computer language. During this period, the student should develop proficiency in coding the selected language. Problems to be solved on the computer should come from algebra, trigonometry and geometry. This would force the student to synthesize his previous mathematical understandings. In this way the CUPM recommendation that computer work should be introduced as early as possible into college mathematical training will be met. The student would then be prepared to take advantage of the computer throughout his mathematics education.

After the initial training in computer coding, the student would begin the calculus sequence. As opportunities arise during the course of study, problems should be assigned for computer solution. These problems should be designed to demonstrate the role of computing in contemporary mathematics. By the end of the fourth semester the student would have a working knowledge of the calculus through differential and difference equations as well as computer programming and basic ideas of numerical analysis.

There are three difficulties to be overcome in order to implement the recommendation outlined above. First, teachers must be trained in computer programming. A few teachers already have had considerable training, many have had some training, but most teachers have not had any training. Courses in computer programming should be required of every prospective secondary school and college mathematics teacher. Also in-service programs and institutes should be expanded.

A second problem to be solved is that of appropriate textbooks. Textbooks including topics and problems for computer solution are beginning to appear on the market.[44] An instructor can however incorporate work with computers into any textbook he is currently using. For example, numerical integration and Newton's method are excellent topics for computer problems.

A third difficulty to be overcome is curriculum reform, The problem here is to find the proper balance between how much theory and how much application of mathematics should be presented. A fourth semester in the calculus sequence seems to be desirable if both theory and applications of mathematics are to be presented sufficiently to show their interdependence.

Implications for Further Research

As is frequently the case in educational research, an investigation of one problem suggests other problems and

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[44]Donald Greenspan, Introduction to Calculus, New York: Harper and Row, Publishers, 1968).

approaches. Some research projects suggested by this study are the following:

1. The academic level at which computer programming should be introduced should be investigated. Perhaps the most feasible level to introduce a mathematically oriented computer language is the eighth or ninth grade when algebra is introduced.

2. An experiment to teach mathematical concepts by programming to students who already know a computer language should be conducted. Perhaps learning a computer language and mathematical concepts via that language, as was done in this research, is too difficult to be accomplished during the same semester.

3. An experiment similar to the one described in this report should be conducted using courses other than the first course in analytic geometry and calculus. Perhaps the unified course in college algebra and trigonometry could be used.

4. A teaching experiment should be conducted using topics from analytic geometry and calculus other than the ones selected for this study. Perhaps there are other mathematical concepts more tractable to computer programming.

5. The study described in this report should be repeated at a later time. Computer technology is advancing rapidly and the same study repeated five years hence with a simpler programming language and better computers might produce different results than those found in this study.

6. Personality factors affecting success in computer programming should be investigated. Perhaps certain personality types should not be subjected to computer programming. That is, programming may hinder learning in students with certain personality characteristics. There also is the possibility that if such students were identified, they could be helped by individual attention.

Operational Problems

Several problems were encountered during the course of the experiment which may have affected the results. Perhaps the following list will be of assistance to an investigator planning future research in this area. These problems were:

1. Only eight class sessions were allowed to present the FORTRAN IV computer language. Several students remarked that a previous course in programming would be desirable. Perhaps students more fluent in the FORTRAN language would have performed differently.

2. More class periods were absorbed by administering tests than the instructor usually allows. Pre- and post-testing accounted for eight class sessions. Usually the investigator, as an instructor, gives four tests per semester in Analytic Geometry and Calculus I. The extra four sessions for testing plus the eight sessions used to teach FORTRAN made it necessary to move through the course material at a more

rapid pace than usual. This accelerated pace may have affected one of the groups more than the other.

3. The "turn around time" of one day between submitting programs and obtaining results may have been too long. Students having difficulty correcting programs tended to develop a backlog of incomplete programs. This delay may have discouraged some students. The optimum situation would be for the students' programs to be run immediately after being submitted to enable corrections to be made while the problem is still fresh on their minds. More equipment and advances in technology would make this feasible.

4. Occasionally not enough keypunch machines were available and students had to wait in line to keypunch their programs. Some students used this reason for submitting programs late. The availability of more keypunch machines would alleviate this problem.

The first decade of the "computer revolution" is drawing to a close and the second decade is beginning. The advances made during the last ten years have been phenomenal and one can only speculate about what the "state of the art" will be at the conclusion of the second decade. The growth has occurred so quickly that educational research has lagged far behind the developments. Many research projects, similar to the one described in this dissertation, must be conducted to give direction to the educational uses of computers so that the full potential of these devices can be realized.

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APPENDICES

APPENDIX A

INSTRUCTIONS FOR ADMINISTERING

TESTS I, II, III, AND IV.

Dear Colleague,

Thank you very much for your willingness to administer this test to your class. You are performing a service to mathematics education as well as a great favor to me. I hope that some time I will be able to thank you again on a personal basis.

The primary purpose for administering the test is to do a small "standardization" so that the test can be used in a later study. Each test consists of thirty-five multiple choice items and has five alternative answers for each item. The test is designed to be completed in a fifty minute period or less. It is important that the following instructions to the students be given precisely.

<u>Instructions to the Student</u>

"1. With ballpoint pen, to the left of the words FRONT SIDE on the answer card write your name and the title and number of the mathematics course you are now in. To the right of the words FRONT SIDE write the title and number of the math course that you completed previously as a prerequisite to the course you are now in and the name of your university or college. Also write the number of the test you are taking (I, II, III or IV).

EXAMPLE:

K. F. Gauss Math 142	FRONT SIDE	Math 132 Augustana College Test III

2. On the back side of the card, with the black electronic pencil given to you, in number 50 shade the oval corresponding to the grade you received in your previous math course according to the following scheme:

For A, shade 5
For B, shade 4
For C, shade 3
For D, shade 2
For F (or E), shade 1

(Now turn the card over to the front side again.)

3. As you take the test use the black electronic pencil to shade in the oval corresponding to the number of what you think is the best answer to the test item. Multiple responses will be scored as incorrect.

4. Do not linger a long time on any one item. Your score is the number of correct answers. The test is designed to be completed in one class period.

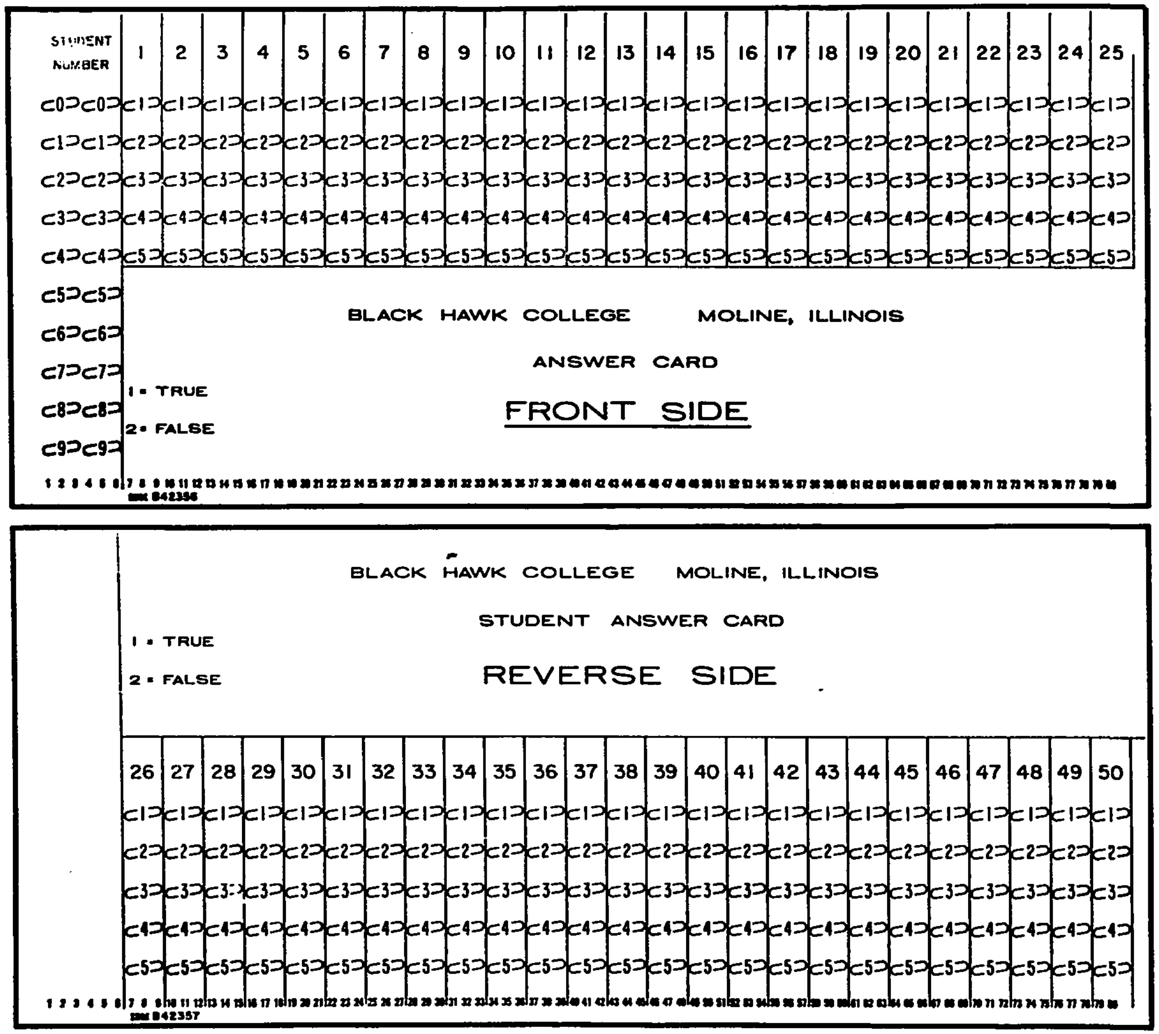
5. Are there any questions? Thank you very much for your cooperation. You may begin the test."

If you wish to know the scores your students received on the test, send me a note with your name, school and test number. I will be glad to send you the results. Please do not give your students any advance tutoring or specific lectures concerning test content or test items as this will invalidate the results. Please return the answer cards and pencils as soon as possible, hopefully before April 1st. If returning the tests is not convenient, you may keep them, but please keep them confidential.

Sincerely,

Leigh A. Fiedler, Head
Mathematics Department
Black Hawk College

ANSWER CARD FOR EVALUATIVE TESTS



APPENDIX B

EVALUATIVE TESTS

Test I

1. If f is a function whose value at x is given by $f(x) = x^2 - 3x + 1$, then $f(1) = (?)$.

   1. 1   2. 5   3. -1   4. 2   5. 10

---

2. Use the function f defined above to find $f(-1)$.

   1. -1   2. 7   3. 10   4. 5   5. 2

---

3. If g is a function whose value at x is given by $g(x) = \frac{x-1}{x+1}$, $x \neq -1$, then $g(1+h) = (?)$.

   1. $\frac{h-1}{h+1}$ 2. $\frac{h}{h+2}$ 3. h   4. 0   5. $1+h^2$

---

4. Use the function g defined above to find $\frac{g(4) - g(2)}{2}$.

   1. 2   2. $\frac{4}{5}$ 3. $\frac{1}{6}$ 4. $\frac{2}{15}$ 5. $\frac{2}{3}$

---

5. If F is the function defined by $F(x) = \sqrt{x}$, $x>0$, $h>0$, then $\frac{F(x+h) - F(x)}{h} = (?)$.

5. 1. $\sqrt{x+h}$ 2. $\dfrac{1}{\sqrt{x+h} + \sqrt{x}}$ 3. $\sqrt{x+h} - \sqrt{x}$ 4. $\sqrt{x}+\sqrt{h}$ 5. 1

---

6. Which of the following equations define polynomial functions in x ?

I. $f(x) = x^{-3}+4x^{-2}+1$ II. $f(x) = 5x$

III. $f(x) = 6x^3 - 4x^5 + 3x -1.$ IV. $f(x) = 3x^{5/2} + 6x -1.$

1. I and IV 2. I and III 3. III and IV

4. II and III 5. all of them

---

7. Which of the following equations define rational functions in y ?

I. $g(y) = 4y^{3/2}+5y^{1/2}-9$ II. $g(y) = \dfrac{3y^{3/4}+6y}{6y^2-1}$

III. $g(y) = \dfrac{8y^2 - 6y+1}{9y^3 + 6y^2-5y+1}$ IV. $g(y) = \dfrac{5y^3 - 1}{6y^2}$

1. IV only 2. I and III 3. II and IV

4. III and IV 5. All of them

---

8. Which of the following equations define algebraic functions in z ?

I. $h(z) = 6z^{3/2}-5z^2 -z\cdot\cos 50$

II. $h(z) = 7z^5-6z^{2/3}+9z^{-12}+4e^z$

III. $h(z) = 5\cdot\cos z+6\cdot\tan z+9e^z$

1. III only 2. I and III 3. II only

4. I only 5. I and II

---

9. If $f(x) = x^2-1$ and $g(x) = 3x+1$ write the expression which defines the function $f+g$.

1. x^2+3x 2. x^2-3x+2 3. $3x^2-2x+1$

4. $3x^3+x^2-3x+1$ 5. $\frac{x^2-1}{3x+1}$

---

10. Use $f(x)$ and $g(x)$ as defined above to write the expression which defines the function fg.

1. x^2+3x 2. x^2-3x+2 3. $3x^3+x^2-3x-1$

4. $3x^2-2x+1$ 5. $\frac{x^2-1}{3x+1}$

---

11. If $g(x) = 4^x$, then $g(t+2) = (?)$.

1. 4^t+16 2. $16g(t)$ 3. $4g(t)$ 4. $4g(x)$ 5. $16g(x)$

---

12. If $f(x) = 11x$, then $f(x+y) = (?)$.

1. $11xy$ 2. $f(x)$ 3. $f(y)$ 4. 11 5. $f(x)+f(y)$

In problems 13, 14, 15 and 16, let $f(x) = x+1$, $g(x) = x-2$, and $F(x) = x^2+x$.

---

13. $f(g(6)) = (?)$.

1. 3 2. 6 3. 5 4. -1 5. 4

---

14. $f \circ F(5) = (?)$.

1. 6 2. 31 3. 180 4. 42 5. 30

---

15. $F \circ f(5) = (?)$.

1. 42 2. 30 3. 180 4. 6 5. 31

---

16. $f(g(x)) = (?)$.

1. x^2-x-2 2. $g(f(x))$ 3. x^2-3 4. $x+3$ 5. $2x+3$

---

17. If the function f is the set $f = \{(x, f(x)) \mid f(x) = \sqrt{25-x^2}\}$, then $6f(0)+f(-3)-f(4) = (?)$.

1. -12 2. 7 3. 180 4. 42 5. 31

---

18. What subset of real numbers is the domain of the function defined in problem 17?

1. $\{x \mid -5 \leq x \leq 5\}$ 2. All real numbers
3. $\{x \mid -5 < x < 5\}$ 4. $\{x \mid x > 5\}$ 5. $\{x \mid x \neq 0\}$

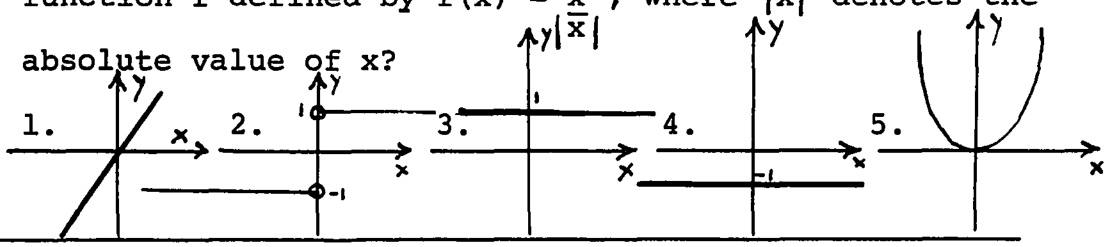
---

19. What is the range of the function defined in problem 17?

1. All real numbers 2. $\{y \mid 0 \leq y \leq 5\}$
3. $\{y \mid y \neq 0\}$ 4. $\{y \mid y > 0\}$ 5. $\{y \mid y = \pm 5\}$

---

20. Which of the following could be the graph of the function f defined by $f(x) = \frac{x}{|x|}$, where $|x|$ denotes the absolute value of x?



---

21. What is the domain of the function g defined by $g(x) = \frac{|1+x|-1}{x}$?

1. $\{x \mid x = 1\}$ 2. $\{x \mid x > 0\}$ 3. $\{x \mid x > -1\}$ 4. $\{x \mid x \neq 0\}$
5. $\{x \mid x = \pm 1\}$

---

22. Which of the following equations define quadratic functions in x?

I. $y = x^4+3x^3-6x+4x-1$ II. $y = 6x-5x^2+1$

III. $y = x^2$ IV. $y = 9x^4+6x^2-8$

1. I only 2. II only 3. IV only
4. III and IV 5. II and III

---

23. Which of the following equations do <u>not</u> define a function f where $y = f(x)$?

I. $x^2+y^2 = 25$ II. $y^3 = x^2$

III. $6x^2-8x+1+9y^2-6y+12 = 0$ IV. $y = 4x^3-6x^2-9x+1$

1. I and III 2. IV only 3. I, II and III
4. I only 5. III only

---

24. If f is the function defined by the equation $f(x,y) = x^2-3xy+4y^2$, then $f(-1,3) = (?)$.

1. 46 2. (1,7) 3. 22 4. 28 5. 2

---

25. Using the function f defined in problem 24, find $f(x+1, y-2)$.

1. $x^2-3xy+4y^2+1$ 2. $2x^2-5x+6y+8y^2+7$
3. $x^2-3xy+4y^2+8x-19y+23$ 4. $4x^2-6x+y$ 5. x^2+y^2

---

26. Given $f(x) = \frac{x-3}{3x-1}$ find $f(f(x))$.

1. x 2. $\frac{x-3}{3x-1}$ 3. $4x^2+3x+1$ 4. $2x^2-3$ 5. $6x^2$

---

27. Given $f(x,y) = x^2 - xy - y^2$, find $f(-x, 6y)$.

1. $-f(x,y)$ 2. $f(-x,y)$ 3. $f(x,y)$ 4. $f(x,-y)$

5. -1

---

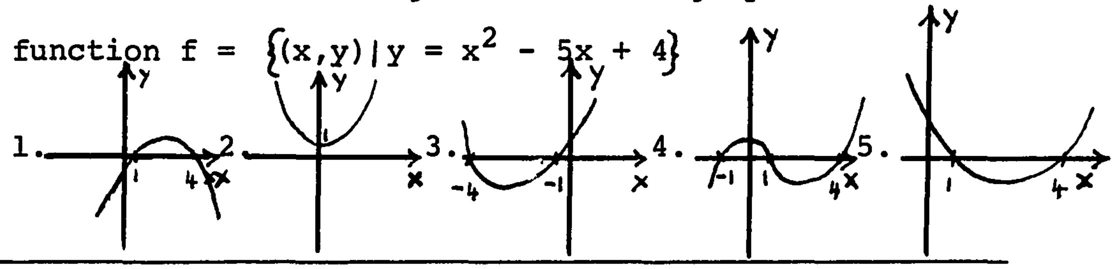
28. If $f(x) = x^2 - x + 1$, $h \neq 0$, find $\dfrac{f(x+h) - f(x)}{h}$.

1. $x^2+2xh+h^2$ 2. $\dfrac{x^2-2xh}{h}$ 3. -1 4. $2x+h-1$

5. $2x$

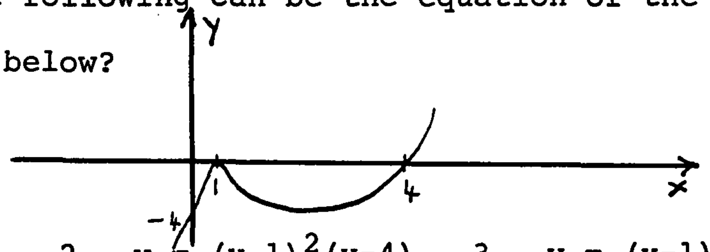
---

29. Which of the following could be the graph of the function $f = \{(x,y) \mid y = x^2 - 5x + 4\}$



---

30. Which of the following can be the equation of the graph shown below?



1. $y = x^3-4$ 2. $y = (x-1)^2(x-4)$ 3. $y = (x-1)(x-4)$

4. $y = (x-1)^3(x-4)$ 5. $y = (x-1)^3$

---

31. If $f(x) = \log_e x$ and $g(x) = e^x$ where e is the base of natural logarithms, then $f(g(x)) = (?)$.

1. e 2. e^x 3. $\log_e x$ 4. $e \log_e x$ 5. x

---

32. If x and y are real numbers, what is the domain of the function defined by $y = \dfrac{x}{\sqrt{9-x2}}$?

32. 1. $\{x \mid -3<x<3\}$ 2. $\{x \mid x\neq 0\}$ 3. $\{0\}$

4. All x 5. $\{x \mid -3\leq x\leq 3\}$

---

33. The straight line defined by the parametric equations

$x = 2t+1$
$y = -3t+2$ intersects the xy plane at the point (?).
$z = t-4$

1. (1,2,-4) 2. (9,-10,0) 3. (2,-3,1)

4. (0,0,0) 5. (-10,9,0)

---

34. If $f(x) = 4x+3$ and $g(x) = x^2-2$, then $f(g(x)) =$ (?).

1. $-2x+1$ 2. $3x^2-7$ 3. $5x+6$

4. $4x^3-6$ 5. $4x^2-5$

---

35. If $F(x) = x^n$ for some positive integer n, find the expression which defines the function $F\circ(F\circ F)$.

1. $3x^{3n}$ 2. x^{n^3} 3. x^{3n} 4. $n\cdot x^{n-3}$ 5. x^3

Test 2

1. Find $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$.

   1. 1/2   2. 1   3. 0   4. $\sqrt{2}$ 5. 2

---

2. If $g(x) = \sqrt{25-x^2}$, then $\lim_{x\to 4} \frac{g(x)-g(4)}{x-4} = (?)$.

   1. 0   2. -3/4   3. 5   4. 1   5. -4/3

---

3. Find $\lim_{x\to a} \frac{x-a}{x^3-a^3}$.

   1. 1   2. $\frac{1}{3a^2}$ 3. $\frac{1}{2a^3}$ 4. 0   5. a^3+3a^2+3a+1

---

4. Find $\lim_{x\to 0^-} \frac{|x|}{x}$.

   1. 1   2. 0   3. ∞ 4. 2   5. -1

---

5. Find $\lim_{t\to \infty} \frac{at^2+bt+c}{dt^2+et+f}$, where a,b,c,d,e,f are constant.

   1. c/f   2. a/d   3. 1   4. ∞ 5. b/e

---

6. Find $\lim_{t\to 4} \frac{x-4}{x^2-x-12}$.

   1. 1   2. 1/7   3. 0   4. 3/8   5. ∞

---

7. Let the function f be defined by $f(x) = \begin{cases} 5 \text{ if } x \geq 3 \\ 2 \text{ if } x < 3 \end{cases}$,

then $\lim_{x \to 3} f(x) = (?)$.

1. 5 2. 2 3. 7/2 4. Does not exist 5. f(3)

---

8. If $f(x) = 2x^3 - 3x + 1$, evaluate $f'(x)$ at $x = 2$.

1. 11 2. 18 3. 21 4. 9 5. 13

---

9. At what point on the curve $y = 3x^2 + 2x + 1$ is its slope 8?

1. (-5/3,6) 2. (1,6) 3. (-5/16,-69/16)
4. (1,9) 5. (4/3,9)

---

10. What is the equation of the line through (0,1) parallel to the line $3x + 5y = 7$?

1. $3x+5y = 3$ 2. $3x-5y = -5$ 3. $3x+5y = 5$
4. $5x-3y = 7$ 5. $5x-3y = -3$

---

11. If $y = f(x) = \sqrt{x^2+x+1}$, then $f'(x) = (?)$.

1. $(2x+1)\sqrt{x^2+x+1}$ 2. $\frac{2x+1}{2\sqrt{x^2+x+1}}$ 3. $\frac{(2x+1)\sqrt{x^2+x+1}}{2}$

4. $\frac{1}{2\sqrt{x^2+x+1}}$ 5. $\frac{2x+1}{\sqrt{x^2+x+1}}$

---

12. -What is the slope of the line perpendicular to the curve $y = x^2 + 2x$ at the point (2,8)?

12. 1. $-1/4$ 2. $1/8$ 3. $1/6$ 4. $-1/6$ 5. $1/4$

---

13. Evaluate $\lim_{x \to \infty} \frac{2x^2-5x+2}{x^2+5x+6}$.

1. $-1/12$ 2. 2 3. $1/3$ 4. 1 5. Does not exist

---

14. If the line $y = 3x + k$ is tangent to the parabola $y = x^2 - x + 9$, then $k =$ (?).

1. -1 2. 1 3. 3 4. 9 5. 5

---

15. What are all values of x for which the function f defined by $f(x) = x^2 - 4x + 3$ is strictly increasing?

1. $x>2$ 2. $x<2$ 3. $1<x<3$ 4. $x>1$ 5. $x<1, x>3$

---

16. If $y = x^6$, then Δy, the increase in y as x increases from 2 to 2.1, is approximately (?).

1. 10^{-6} 2. 0.5 3. 0.6 4. 3.0 5. 19.2

---

17. If $f(x) = x^{20}$ and if $f^{(n)}(x)$ denotes the n^{th} derivative of f at x what is the smallest n for which $f^{(n)}(x)$ is a constant?

1. 20 2. 11 3. 19 4. 21 5. 22

---

18. What is the slope of the curve $y = x^3 - 3x^2 - 9x + 20$ at its inflection point?

1. -7 2. 0 3. 1 4. -12 5. 9

---

19. If $f(x) = (x-a)^3\phi(x)$ where $\phi(x)$ is differentiable, then $f'(a) = (?)$.

1. 0 2. $\phi'(a)$ 3. $-a^3\phi'(a)$ 4. $3a^2\phi(a)-a^3\phi'(a)$
5. $3(\phi'(a))^2$

---

20. If $3y^3x = 8$, find $\frac{dy}{dx}$.

1. $\frac{8}{3y^3}$ 2. $\frac{1}{9x^2}$ 3. $3y^3$ 4. $\frac{-y}{3x}$ 5. $\frac{-3x}{y}$

---

21. What is the minimum value of the function defined by $y = x^2 - 5x + 4$ on the interval $-1 \leq x \leq 2$?

1. -2 2. $-5/4$ 3. 0 4. 1 5. 2

---

22. Evaluate $\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$ when $x = 2$.

1. 0 2. $1/2$ 3. $\frac{1}{2\sqrt{2}}$ 4. $\frac{2}{\sqrt{2}}$ 5. ∞

---

23. A particle moves on the x- axis so that its distance from the origin at time t is given by $x = t^2 - 8t + 7$. For what value of t is its velocity zero?

1. 7 2. 2 3. 4 4. 1 5. 3

---

24. For what value of c, $1 < c < 4$, will the tangent line to the graph of $f(x) = \sqrt{x}$ be parallel to the secant through the points on the graph where $x = 1$ and $x = 4$?

24. 1. 3 2. 2 3. 3/2 4. 9/4 5. 5/2

---

25. Find the slope of the graph of $y = x^2$ at the point (3,9).

1. 9 2. 2 3. 0 4. 3 5. 6

---

26. If $y = 0.125^{-2/3}$, then dy/dx = (?).

1. -64/3 2. 0 3. -8/3 4. 4 5. 64/3

---

27. Find the derivative of $x\sqrt{3x-5}$ with respect to x.

1. $1/2\sqrt{3x-5}$ 2. $\dfrac{3}{\sqrt{3x-5}}$ 3. $\dfrac{6x-5}{\sqrt{3x-5}}$

4. $\dfrac{7x-10}{2\sqrt{3x-5}}$ 5. $\dfrac{9x-10}{2\sqrt{3x-5}}$

---

28. What are all the values of x for which the function f defined by $f(x) = x^2 - 3x + 4$ is strictly decreasing?

1. $-3/2<x<3/2$ 2. $x<3/2$ 3. $-1<x<4$

4. $x<-1,\ x>4$ 5. $x>3/2$

---

29. If $y = v^{4/3}$ and $v = x^3 - 1$, then dy/dx = (?).

1. $\dfrac{1}{4x^2(x^3-1)^{1/3}}$ 2. $4x^2(x^3-1)^{1/3}$ 3. $\frac{4}{9}\left[\dfrac{(x^3-1)^{1/3}}{x^2}\right]$

4. $\frac{4}{3}(x^3-1)^{1/3}+3x^2$ 5. $\frac{4}{3}(x-1)+3x^2$

---

30. The graph of $y = ax^2 + bx + c$ is concave upward for all x whenever (?).

1. $a>0$ 2. $a<0$ 3. $a>b/2$ 4. $a<b/2$ 5. $a\neq b$

---

31. Find the slope of the curve $x^2y = 4$ at the point (2,1).

1. -2 2. -1 3. -1/2 4. 7/4 5. 1

---

32. If $y = x^{5/4}$, then Δy, the increase in y as x increases from 16 to 16.1, is approximately (?).

1. 1/40 2. 2/25 3. 1/4 4. 1/8 5. 1/2

---

33. If $y = 1/x$, then d^ny/dx^n = (?).

1. $(-1)^n\frac{1}{x^n}$ 2. $\frac{(-1)^n n!}{x^{n+1}}$ 3. $(-1)^n\frac{1}{x^{n+1}}$

4. $(-1)^{n+1}\frac{1}{x^{n+1}}$ 5. $\frac{(-1)^n n!}{x^n}$

---

34. If f and g are differentiable on $a\leq x\leq b$, and if $f(a) = g(a)$ and $f(b) = g(b)$, then there exist two numbers ξ and μ in $a<x<b$ such that (?).

1. $f(\xi) = g(\mu)$ 2. $f''(\xi) = g''(\mu)$ 3. $f'(\xi) = g'(\mu)$

4. $f'(\xi)g'(\mu) = 1$ 5. $f(\xi)g(\mu) = 1$

---

35. The curve given by $y = 1 - x^2 - x^3$ has its maximum slope at $x =$ (?).

1. $-1/3$ 2. $-2/3$ 3. 0 4. $2/3$ 5. 1

Test 3

1. The expanded form of $\sum_{k=1}^{4} \frac{(-1)^k}{2^k}$ is (?).

   1. $0+1+2+3$ 2. $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}$ 3. $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$
   4. $0-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}$ 5. $-\left(\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}\right)$

---

2. The expanded form of $\sum_{k=1}^{4} \frac{k}{1+k}$ is (?).

   1. $1+2+3+4$ 2. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$ 3. $0+\frac{1}{2}+\frac{2}{3}+\frac{3}{4}$
   4. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}$ 5. $\infty+(\propto+1)+(\propto+2)+(\propto+3)$

---

3. Find a if $\sum_{j=1}^{5} aj = 14$.

   1. 14/15 2. 1 3. Impossible to solve for a
   4. 14 5. 4/5

---

4. If the first term of an arithmetic progression is -4, the last term is 4, and the common difference is 0.05, then the number of terms in the progression is (?).

   1. 52 2. 6 3. 17 4. 205 5. 161

---

5. Write in expanded form: $\sum_{j=2}^{5} (j^2+1)$.

5. 1. $4+9+16+25$ 2. $2+5+10+17$ 3. $5+10+17+26$

4. $5+9+13+17$ 5. $5+10+17$

---

6. Find the value of $\frac{(12!)\cdot(8!)}{16!}$.

1. 6 2. 1/6 3. 2/3 4. 12/13 5. 1/24

---

7. Simplify $\frac{n!}{(n-1)!}$.

1. $n!$ 2. $n-1$ 3. $\frac{n}{n-1}$ 4. n^2-1 5. n

---

8. Write the seventh term of $(a^3-b)^9$.

1. $84a^9b^6$ 2. $-36a^6b^7$ 3. $-126a^{12}b^5$

4. a^9b^6 5. $36a^6b^7$

In each of the following problems assume $n = 0,1,2 \ldots$

---

9. If $a_{n+1} = \frac{1-a_n}{1+a_n}$ and $a_0 = x$, then $a_2 = (?)$.

1. $\frac{1-x}{1+x}$ 2. -1 3. x 4. 0 5. $\frac{1}{x}$

---

10. If a_{n+1} is as defined above, then $a_{11} = (?)$.

1. -1 2. x 3. 0 4. $\frac{1-x}{1+x}$ 5. $\frac{1}{x}$

---

11. If $a_{n+1} = F(a_n)$ and $a_0 = x$ where $F(x) = \sqrt{1+x}$, then $a_4 = (?)$.

11. 1. $\sqrt{5}$ 2. $\sqrt{1+x}$ 3. $\sqrt{1+\sqrt{1+x}}$

4. $\sqrt{1+\sqrt{1+\sqrt{1+x}}}$ 5. $\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+x}}}}$

---

12. If $\lambda_{n+1} = \sqrt{2+\lambda_n}$ and $\lambda_0 = \sqrt{x}$, find $\lim\limits_{x\to 4} \frac{\lambda_r - 2}{x-4}$ for r = 1.

1. 1/16 2. Does not exist 3. λ_1 4. 0 5. 1

---

13. If $f(x) = x^n$ where n is a positive integer, then $f^{[k]}(x)$ = (?) where $k<n$ Alternatively, $D^{[k]} f(x)$ = (?).

1. $(n-k-1)x^{n-k}$ 2. $\frac{n!}{(n-k)!}x^{n-k}$ 3. 0

4. nx^{n-1} 5. $n!$

---

14. Newton's method for approximating the real roots of an equation $F(x) = 0$ is derived using the idea of (?), provided Newton's method is applicable.

1. Similar triangles 2. Tangent lines

3. Linear interpolation 4. Arcs of circles

5. Parabolas

---

15. In Newton's method, two successive approximations to the real roots of $F(x) = 0$ are related by (?).

1. $x_{n+1} = x_n - \frac{F'(x_n)}{F(x_n)}$ 2. $x_{n+1} = x_n + \frac{F(x_n)}{F'(x_n)}$

15.

3. $x_{n+1} = x_n - \frac{F'(x_n)}{F(x_n)}$ 4. $x_{n+1} = F(x_n) - \frac{x_n}{F'(x_n)}$

5. $x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$

---

16. What value of x should be used to minimize the function $f(x) = (x-a_1)^2 + (x-a_2)^2 + . . . + (x-a_n)^2$?

1. $\sum_{i=1}^{n} a_i$ 2. $1/n$ 3. 0

4. $\frac{\sum_{i=1}^{n} a_i^2}{n}$ 5. $\frac{\sum_{i=1}^{n} a_i}{n}$

---

17. If $a_0 = \sqrt{2}$ and $a_{n+1} = \sqrt{2a_n}$, then a_2 = (?).

1. $\sqrt{2\sqrt{2\sqrt{2}}}$ 2. $\sqrt{2+\sqrt{2+\sqrt{2}}}$ 3. 8

4. $\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}$ 5. $\sqrt{2\sqrt{2}}$

---

18. A possible iteration scheme to find a root of the equation $3x - \sqrt{1 + \sin x} = 0$ is given by (?).

1. $x_{n+1} = 3\sqrt{1+\sin x_n}$ 2. $x_{n+1} = \frac{1}{3}\sqrt{1+\sin x_n}$

3. $x_{n+1} = \frac{\cos x_n}{6\sqrt{1+\sin x_n}}$ 4. $x_{n+1} = 1-\cos^2 x_n$

5. $x_{n+1} = 9(1-\sin^2 x_n)$

---

19. Let $f(x) = 5-x^2$. To find a zero of the graph of the function f by Newton's method which iterative formula could be used?

1. $x_{n+1} = 5-x_n^2$ 2. $x = \sqrt{5}$ 3. $x_{n+1} = \frac{x_n}{5}$

4. $x_{n+1} = \frac{x_n^2+5}{2x_n}$ 5. $x_{n+1} = \frac{1}{2x_n}$

---

20. The role of x_0 in an iterative scheme could be described as (?).

1. The final answer
2. A first approximation
3. $x = 0$
4. An infinite sequence
5. The only possibility

---

21. The method of "false position" (or regula falsi) can be derived by using which of the following ideas?

1. Arc of circles
2. Similar triangles
3. Parabolas
4. Tangent lines
5. Rolle's theorem

---

22. One way to find a root of the equation $x^2 + 5.4x - 12.3 = 0$ would be to use the iterative equation (?).

1. $x = \frac{-5.4 \pm \sqrt{(5.4)^2+4(12.3)}}{2}$ 2. $x_{n+1} = \frac{12.3}{x_n+5.4}$

22. 3. $x_{n+1} = \frac{5.4}{12.3-x_n}$ 4. $x_{n+1} = 12.3-5.4x_n$

5. There is no possible scheme

---

23. Using Newton's method to find a root of a quadratic equation will fail if the roots are (?).

1. Real 2. Zero 3. Equal 4. Negative

5. Complex

---

24. If successive iterates get closer and closer to some value, the iterative scheme is said to (?).

1. Fail 2. Converge 3. Oscillate 4. Move

5. Diverge

---

25. A possible iterative plan to solve y = 2x, y = 100 + x simultaneously would be (?).

1. $x = 100$ 2. $x_{n+1} = \frac{2x_n}{5}$ 3. $x_{n+1} = 50x_n$

4. $x_{n+1} = 2x_n-100$ 5. $x_{n+1} = \frac{8}{x_n{}^2}$

---

26. A possible iterative scheme to solve y = x, y = f(x) simultaneously could be (?).

1. $x_{n+1}=\frac{1}{f(x_n)}$ 2. $x_{n+1}=f(x_n)$ 3. $f(x_{n+1})=x_n$

4. $x_{n+1}=1-f(x_n)$ 5. $x_n=\frac{f(x_n)}{x_n}$

---

27. To find the 5^{th} root of 100 by Newton's method we could use (?).

1. $x_{n+1} = \dfrac{4x_n^5+100}{5x_n^4}$ 2. $x = \sqrt[5]{100}$

3. $x_{n+1} = \dfrac{x_n^5-100}{5x_n^4}$ 4. $x_{n+1} = x_n - x_{n-1}$

5. $x_{n+1} = \dfrac{100}{x_n^2}$

---

28. Using the iteration $x_{n+1} = x_n^2 + \frac{1}{2}(\frac{8}{x_n} - x_n)$ to solve $x = x^2 + \frac{1}{2}(\frac{8}{x} - x)$ is essentially the same problem as searching for the intersection of the graphs of (?).

1. $y = x^2$, $y = x$ 2. $y = 2x^2+8x_n-1$
$y = x$

3. $y = x^2+\frac{1}{2}(\frac{8}{x} - x)$
$y = x$ 4. $y = x^2-1$
$y = x^3+4$

5. $y = 0$
$y = x^2+\frac{1}{2}(\frac{8}{x} - x)$

---

29. To find the reciprocal of 5 using Newton's method, which of the following equations could be used?

1. $x = \frac{1}{5}$ 2. $x_{n+1} = x_n(2-5x_n)$ 3. $x_{n+1} = \dfrac{x_n^2}{5x_n-1}$

29\. 4. $x_{n+1}=\frac{1}{5x_n}$ 5. $x_{n+1}=\frac{3}{4x_n^2}$

30\. The equation $\frac{1}{1+t} = 1-t+t^2-t^3+t^4- \ . \ . \ .$ is true for (?).

1. All values of t
2. $t=-1$
3. $t<-1,\ t>1$
4. $t=1$
5. $-1<t<1$

31\. Write the first four terms of $\sum_{k=0}^{\infty} \frac{1}{k!}$.

1. $0+1+\frac{1}{2}+\frac{1}{6}$
2. $\infty+1+\frac{1}{2}+\frac{1}{6}$
3. $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$
4. $1+1+\frac{1}{2}+\frac{1}{6}$
5. $1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}$

32\. One of the major problems of iterative techniques is deciding (?).

1. Whether to use them
2. Upon convergence
3. How to use them
4. If oscillation exists
5. How many terms to use

33\. Another problem that must be considered in using an iterative technique is that of choosing (?).

1. A value of n
2. An initial value
3. The proper reference
4. A computer to do the work
5. A friend to talk to about it

---

34. Newton's method will not find (?).

1. Real zeros
2. Negative zeros
3. Positive zeros
4. Complex zeros
5. Multiple zeros

---

35. Newton's method will fail if (?).

1. Too much calculation is necessary
2. The function is discontinuous
3. The initial value is not close
4. The slope of the tangent line becomes zero at some iterative point
5. The initial value is large.

Test 4

1. $\int_1^2 x^2\,dx = (?)$.

1. 2 2. 7/2 3. 7/3 4. 7 5. 3

---

2. If $f'(x) = x^3 + 4x$ and $f(0) = 4$, then $f(x) = (?)$.

1. $3x^2+4$ 2. x^3+4x+4 3. $\frac{1}{4}x^4+2x^2+4$
4. x^4+4x^2+4 5. $\frac{1}{4}x^4+2x^2-4$

---

3. What is the area of the region bounded by the curve $y = x^3$, the x-axis, and the lines $x = 1$ and $x = 2$?

1. 7/3 2. 3 3. 7/2 4. 15/4 5. 4

---

4. $\int \sqrt{x-4}\,dx = (?)$.

1. $2/3\ x^{3/2} - 2x + c$ 2. $1/2\ (x-4)^{-1/2} + c$
3. $1/2\ (x-4)^{3/2} + c$ 4. $3/2\ (x-4)^{3/2} + c$
5. $\frac{2(x-4)^{3/2}}{3} + c$

---

5. If the maximum value of f is 2 and if $f'(x) = 2 - 2x$, then $f(x)$ is given by (?).

1. $-x^2+2x+1$ 2. $-x^2+2x+2$ 3. $-x^2+2x$
4. $-x^2+2x-1$ 5. $-x^2+2x-2$

---

6. $\int_1^3 x^{-2}\,dx = (?)$.

6. 1. $\frac{-26}{27}$ 2. $\frac{-2}{3}$ 3. $\frac{2}{3}$ 4. $\frac{52}{27}$ 5. $\frac{26}{9}$

---

7. What are all the (real) values of n for which the integration formula $\int u^n \, du = \frac{u^{n+1}}{n+1} + c$ is valid?

1. $n \neq 0$ 2. $n > -1$ 3. $n > 0$ 4. $n \neq -1$ 5. All n

---

8. $\int x \sqrt{2x^2 - 1} \, dx = (?)$.

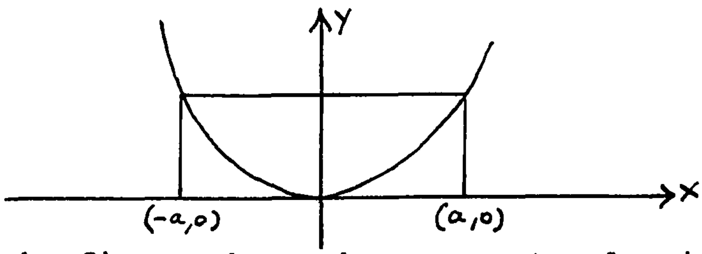
1. $1/6 \ (2x^2-1)^{3/2}+c$ 2. $1/4 \ (2x^2-1)^{3/2}+c$
3. $2/3 \ (2x^2-1)^{3/2}+c$ 4. $\frac{2x^2}{\sqrt{2x^2+1}}$
5. $1/3 \ (2x^2-1)^{3/2}+c$

---

9. If the slope of a curve at (x,y) is 3/2 x and if the curve passes through (4,-11), which of the following is its equation?

1. $y = 3/4 \ x^2$ 2. $y = 3/2 \ x^2 - 25$ 3. $y = 3/2 \ x^2$
4. $y = 3/4 \ x^2 - 23$ 5. $y = 3x^2 - 47$

---

10. 

The figure above shows a rectangle with two of its vertices at (a,0) and (-a,0) and the other two on the parabola $y = x^2$. What fraction of the area of the rectangle lies below the parabola?

10. 1. 1/6 2. 2/5 3. 1/4 4. 1/2 5. 1/3

---

11. What is the area of the region bounded by the line

$y = \frac{x}{2}$ and the parabola $y^2 = x$?

1. 2 2. 4/3 3. 16/3 4. 20/3 5. 32/3

---

12. If $G(t) = \int_2^t f(x)\,dx$, then $G'(t) = (?)$.

1. $f'(t)-f'(2)$ 2. $f'(t)$ 3. $f(t)-f(2)$

4. $f(t)$ 5. $G(t)-G(2)$

---

13. If $dx = 3t^2\,dt$ and $x = 3$ when $t = 1$, what is the value of x when $t = 2$?

1. 6 2. 8 3. 10 4. 12 5. 24

---

14. $\int_1^8 \frac{dx}{x^{1/3}} = (?)$.

1. $\frac{3}{64}$ 2. 2 3. $\frac{8}{3}$ 4. $\frac{9}{2}$ 5. 6

---

15. The slope of the graph of $y = f(x)$ at $(1,-1)$ is 10.

Find the equation of the curve if $\frac{d^2y}{dx^2} = 18x - 8$.

1. $y = 10x-11$ 2. $y = 9x^2-8x-2$ 3. $y = 3x^3-4x^2$

4. $y = 3x^3-4x^2+9x-9$ 5. $y = 9x^2-8x+9$

---

16. If the interval $a \leq x \leq b$ is divided into n subintervals of length $x = \frac{b-a}{n}$, and if n points x_i $(i=1,2 \ . \ . \ .,n)$ are chosen so that there is exactly one of these points inside each of the subintervals, then, for the integrable function f, $\int_a^b f(x)\,dx =$ (?).

1. $\lim\limits_{n \to \infty} \sum\limits_{i=1}^{n} f(x_i)\Delta x$
2. $\lim\limits_{n \to \infty} \sum\limits_{i=1}^{n} f(x_1)$
3. $\lim\limits_{n \to \infty} \sum\limits_{i=1}^{n} f(x_i+\Delta x)$
4. $\lim\limits_{n \to \infty} \sum\limits_{i=1}^{n} [f(x_i+\Delta x)-f(x_i)]$
5. $\lim\limits_{n \to \infty} \sum\limits_{i=1}^{n} [f(x_{i+1})-f(x_i)]\Delta x$

---

17. What is the area of the region bounded by the parabola $y = x^2$ and the line $y = x + 2$?

1. $\frac{9}{2}$ 2. $\frac{3}{2}$ 3. $\frac{21}{2}$ 4. 6 5. $\frac{16}{3}$

---

18. A particle starts from rest at the origin at time $t = 0$ and moves along the x-axis with acceleration $d^2x/dt^2 = t - 2$. The position of the particle at the instant when the acceleration is 0 is at $x =$ (?).

1. $\frac{-16}{3}$ 2. $\frac{-8}{3}$ 3. 2 4. 0 5. -2

---

19. Let $F(x) = \int_0^x (4 - u^2)du$, then at $x = 2$, the graph of $y = F(x)$ has (?).

19. 1. A zero 2. A relative minimum 3. A discontinuity

4. A relative maximum 5. An inflection point

---

20. If Simpson's rule were used to approximate $\int_0^1 \sqrt{1-x^2}\,dx$ with n = 4, which of the following sums would be evaluated?

1. $\frac{1}{12}(1+\sqrt{15}+\sqrt{3}+\sqrt{7})$ 2. $\frac{3}{8}(1+\frac{\sqrt{3}}{2}+2)$

3. $\frac{1}{12}(\sqrt{15}+\sqrt{3}+\sqrt{7})$ 4. $\frac{1}{4}(1+3+5+7)$

5. $\frac{5}{12}(3+7+11+15)$

---

21. Let f be a function continuous for all x such that $a \leq x \leq b$. If no antiderivative of f is known, then it is . . . possible to approximate $\int_a^b f(x)\,dx$ as closely as we desire.

1. Never 2. Usually 3. Sometimes

4. Always 5. Easily

---

22. In using the trapezoidal rule for approximating definite integrals, the given curve is approximated by segments of (?).

1. Straight lines 2. Cubic curves

3. Arcs of circles 4. Parabolas 5. Spirals

---

23. In using Simpson's rule for approximating definite integrals, the given curve is approximated by segments of (?).

23. 1. Parabolas 2. Cubic curves 3. Arcs of circles
4. Straight lines 5. Spirals

---

24. The area bounded by the x-axis, the curve $y = x^3$ and the lines $x = 1$, $x = 5$ is (?).

1. 124 2. 4 3. 156 4. 102 5. 32

---

25. Using Simpson's rule to approximate the area bounded by $y = \frac{1}{1+x^2}$, $x = 1$, $x = 2$, $y = 0$ with $\Delta x = \frac{1}{2}$, which of the following sums will be evaluated?

1. $\frac{3}{8}\left(\frac{1}{2}+\frac{16}{13}+\frac{1}{5}\right)$ 2. $\frac{1}{12}(1+2+3)$ 3. $\frac{5}{9}\left(\frac{3}{2}+\frac{4}{3}+\frac{5}{4}\right)$
4. $\frac{5}{12}\left(\frac{3}{4}+\frac{5}{8}-\frac{7}{12}\right)$ 5. $\frac{1}{6}\left(\frac{1}{2}+\frac{16}{13}+\frac{1}{5}\right)$

---

26. Using the trapezoidal rule to approximate $\int_1^2 x^2\,dx$ with $n = 4$ will necessitate evaluating (?).

1. $\frac{1}{12}\left(1+\frac{25}{8}+\frac{9}{2}+\frac{49}{8}+4\right)$ 2. $\frac{1}{8}\left(1+\frac{25}{8}+\frac{9}{2}+\frac{49}{8}+4\right)$

3. $\frac{3}{8}\left(1+\frac{25}{8}+\frac{9}{2}+\frac{49}{8}+4\right)$ 4. $\frac{5}{9}\left(1+\frac{25}{4}+\frac{2}{9}+4\right)$

5. $\frac{3}{5}(4+5+6+7)$

---

27. Which of the following approximate integration formulas gives an exact answer for polynomials of degree three or less?

27. 1. Trapezoidal rule 2. Simpson's rule

3. Upper sum 4. Riemann sum 5. Chain rule

---

28. Given the function f defined by $f(x) = 1 - x^2$ and the partition $P = \left\{\left[0, \frac{1}{2}\right], \left[\frac{1}{2}, \frac{3}{2}\right], \left[\frac{3}{2}, 2\right]\right\}$, to find the upper sum S of f relative to P, using the maximum value of f in each subinterval, which of the following sums would be evaluated?

1. $1 + \frac{3}{4} - \frac{5}{4}$ 2. $\frac{3}{8} - \frac{5}{4} - \frac{3}{2}$ 3. $\frac{1}{2} + 1 + \frac{1}{2}$

4. $\frac{1}{2} + \frac{3}{4} - \frac{5}{8}$ 5. $\frac{1}{2} + \frac{3}{2} + 2$

---

29. Evaluate $\int_{-1}^{1} |x^3|\, dx$ where $|a|$ denotes the absolute value of a.

1. 0 2. 2 3. 1/2 4. 1 5. -1/2

---

30. Evaluate $\int_{4}^{9} \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx$.

1. 7/6 2. 32/3 3. 5 4. 1 5. 2/3

---

31. Find $\int \frac{\sqrt{1 + \sqrt{x}}}{\sqrt{x}}\, dx$.

1. $\frac{4}{3}(1+\sqrt{x})^{3/2} + c$ 2. Not integrable

31. 3. $\frac{3}{4}(1+\sqrt{x})^{5/2} + c$ 4. $\frac{3}{2}(1+\sqrt{x})^{3/2} + c$

5. $\frac{4}{9}(1+x)^{3/2} + c$

---

32. Find the area of the region bounded by the graphs of $y = x^2$ and $y = 1$.

1. 1/3 2. 8/3 3. 2/3 4. 5/3 5. 4/3

---

33. Find $\int_c^d \sqrt{ax+b}\,dx$ where a, b, c and d are constants.

1. $c - d$ 2. $(ad+b)^{1/2} - (ac+d)^{1/2}$
3. Not integrable 4. $\frac{2}{3a}\left[(ad+b)^{3/2} - (ac+b)^{3/2}\right]$
5. $(ad+b)^{3/2} - (ac+d)^{3/2}$

---

34. Find $\int_0^3 [x]\,dx$ where $[x]$ denotes the greatest integer $\leq x$.

1. Not integrable 2. 0 3. 2 4. 5 5. 3

---

35. The theorem which shows the relationship between the derivative and the integral is called the (?).

1. Fundamental theorem of the integral calculus
2. Mean value theorem
3. Rolle's theorem
4. L'Hopital's rule
5. Cauchy's formula

APPENDIX C

RAW SCORES FOR TRIAL OF TESTS CLASSIFIED BY COURSE GRADE

Test 1

(N = 96)

A	B		C		D
31	19	23	24	23	26
25	33	28	24	23	26
27	29	28	27	25	26
31	28	24	28	19	16
26	28	27	31	23	19
22	25	29	24	24	24
24	32	25	26	20	25
30	25	21	24	25	24
20	21	25	28	23	24
21	28	29	22	24	16
27	20	24	25	24	20
32	17	30	20	27	27
30	25		19	18	24
26			22	29	
25			25	26	
31			24	30	
27			19	29	
28			22	26	
31			25	29	
33					
547		643		926	297

$N_A = 20$

$\overline{X}_A = 27.35$

$N_B = 25$

$\overline{X}_B = 25.72$

$N_C = 38$

$\overline{X}_C = 24.37$

$N_D = 13$

$\overline{X}_D = 22.85$

Test 2

(N = 75)

A	B	C	D
14	15	11	13
9	9	11	5
21	13	13	14
12	11	16	7
23	13	12	9
16	14	8	9
19	15	10	14
25	13	10	8
15	21	11	9
14	17	17	23
26	21	14	11
17	11	13	17
25	14	9	18
20	17	17	
30	21	22	
32	17	15	
	27	18	
	15	21	
	9	9	
		7	
		22	
		28	
		17	
		26	
		13	
		28	
		16	
318	293	411	157
$N_A = 16$	$N_B = 19$	$N_C = 27$	$N_D = 13$
$\overline{X}_A = 19.9$	$\overline{X}_B = 15.4$	$\overline{X}_C = 15.2$	$\overline{X}_D = 12.1$

Test 3

(N = 55)

A	B	C	D
22	17	9	15
24	20	16	13
15	16	14	9
18	16	17	13
17	15	16	16
17	15	14	10
24	19	21	18
16	16	16	13
23	14	21	
	14	13	
	19	6	
	16	9	
	15	12	
	20	14	
	21	14	
		18	
		19	
		8	
		16	
		20	
		17	
		12	
		22	
176	253	344	107
$N_A = 9$	$N_B = 15$	$N_C = 23$	$N_D = 8$
$\overline{X}_A = 19.6$	$\overline{X}_B = 16.9$	$\overline{X}_C = 15.0$	$\overline{X}_D = 13.4$

Test 4

(N = 72)

A	B	C	D
24	17	18	12
20	14	19	11
30	17	24	11
21	14	22	17
17	9	17	11
13	11	22	19
17	13	21	12
16	17	18	8
19	15	21	10
23	21	21	5
13	12	9	
17	23	9	
21	11	17	
26	13	13	
25	15	5	
16	12	18	
21	19	14	
20	15	15	
	21	16	
	17	9	
	19	15	
	18		
	20		
359	363	343	116
$N_A = 18$	$N_B = 23$	$N_C = 21$	$N_D = 10$
$\overline{X}_A = 19.9$	$\overline{X}_B = 15.7$	$\overline{X}_C = 16.3$	$\overline{X}_D = 11.6$

APPENDIX D

TABULAR PRESENTATION OF THE F-TEST FOR EQUALITY OF MEANS OF k GROUPS

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares	F-ratio
Mean	1	M_{yy}	$M = \dfrac{M_{yy}}{1}$	
Among groups	k-1	G_{yy}	$G = \dfrac{G_{yy}}{k-1}$	G/W
Within groups	$\sum_{i=1}^{k}(n_i-1)$	W_{yy}	$W = \dfrac{W_{yy}}{\sum_{i=1}^{k}(n_i-1)}$	
Total	$\sum_{i=1}^{k} n_i$	$\sum Y^2$		

The entries on the preceding table are calculated as follows:

$\sum Y^2$ = sum of the squares of all the test scores

n_i = the number of test scores in the i^{th} group

$$M_{yy} = \frac{T^2}{\sum_{i=1}^{k} N_i}$$

$$G_{yy} = \sum_{i=1}^{k} \frac{G_i^2}{n_i} - M_{yy} \quad \text{and}$$

$$W_{yy} = Y^2 - M_{yy} - G_{yy}.$$

In the previous equati~~ons~~,

$G_i = \sum_{j=1}^{n_i} Y_{ij}$ = total of the scores in the i^{th} group,

$T = \sum_{i=1}^{k} \sum_{j=1}^{n_i} Y_{ij} = \sum_{i=1}^{k} G_i$ = total of all scores, and

$\sum_{i=1}^{k} n_i$ = total number of scores in all the groups combined.

APPENDIX E

RESULTS OF THE TRIAL OF EVALUATIVE INSTRUMENTS

	TEST NAME	Number of Students (N)	Correlation Coefficient[a] (r)	t-value (t)[b]	critical t-value ($t_{.01}$)	significance at 0.01 level
I.	FUNCTION	96	0.37	3.91	2.63	significant
II.	LIMITS AND DIFFEREN-TIATION	75	0.38	3.51	2.64	significant
III.	ITERATION	55	0.48	4.03	2.67	significant
IV.	INTEGRATION	72	0.40	3.61	2.65	significant

[a]The correlation coefficient is between test scores and grades received in the first course in analytic geometry and calculus.

[b] $t = \dfrac{r\sqrt{N-2}}{\sqrt{1-r^2}}$.

CORRELATION COEFFICIENTS BETWEEN EVALUATIVE TEST SCORES

AND COURSE GRADE IN FIRST SEMESTER

ANALYTIC GEOMETRY AND CALCULUS

		SCHOOLS			
	TEST NAME	Augustana College	Black Hawk College	Northern Illinois University	Western Illinois University
I.	FUNCTIONS	0.53	0.61	0.67	0.23
II.	LIMITS AND DIFFEREN-TIATION	a	0.45	0.61	0.57
III.	ITERATION	- 0.05	0.55	a	0.57
IV.	INTEGRATION	0.36	0.54	0.42	a

[a]The school listed above did not participate in the trial of this test.

APPENDIX F

COMPUTER PROGRAM FOR ANALYSIS OF COVARIANCE

```
C     ANALYSIS OF COVARIANCE    L. FIEDLER
     0DIMENSION  DATAX(12,2,2), DATAY(12,2,2),SLX(2,12),
     1TSX(2,12)
      DIMENSION SLY(2,12), XLT(2,2), YLT(2,2),TX(2),TY(2)
      DIMENSION  YL(2),XLX(2),XLY(2),TSY(2,12),XL(2)
      DIMENSION TX(2),TY(2),TITLE(8)
    3 READ(1,58) (TITLE(I), I=1,8)
      WRITE(3,58) (TITLE(I), I=1,8)
      READ(1,55)  NS,NL,NT,MDFTR,MDFL,MDFW,MDFT
      IF(NS-9999) 80,81,80
   81 STOP
   80 CSLT=NS*NL*NT
      CSL=NS*NL
      CST=NS*NT
      CLT=NL*NT
      WRITE(3,59)
      DO 33  I=1,NT
      DO 33 J=1,NL
      READ(1,56) (DATAX(K,I,J), K=1,NS)
   33 WRITE(3,56) (DATAX(K,I,J), K=1,NS)
      WRITE(3,60)
      DO 34  I=1,NT
      DO 34  J=1,NL
      READ(1,56) (DATAY(K,I,J), K=1,NS)
   34 WRITE(3,56) (DATAY(K,I,J), K=1,NS)
      SSX=0.0
      SSY=0.0
      SX=0.0
      SY=0.0
      SXY=0.0
      DO 35  J=1,NT
      DO 35  I=1,NL
      XLT(I,J)=0.0
      YLT(I,J)=0.0
      DO 35  K=1,NS
      WK1=DATAX(K,I,J)
      WK2=DATAY(K,I,J)
      XLT(I,J)=XLT(I,J)+WK1
      YLT(I,J)=YLT(I,J)+WK2
```

```
      SX=SX+WK1
      SY=SY+WK2
      SSX=SSX+WK1*WK1
      SSY=SSY+WK2*WK2
   35 SXY=SXY+WK1*WK2
      DO 36  J=1,NT
      DO 36  K=1,NS
      TSX(J,K)=0.0
      TSY(J,K)=0.0
      DO 36  I=1,NL
      WK1=DATAX(K,I,J)
      WK2=DATAY(K,I,J)
      TSX(J,K)=TSX(J,K)+WK1
   36 TSY(J,K)=TSY(J,K)+WK2
      DO 37  I=1,NL
      DO 37  K=1,NS
      SLX(I,K)=0.0
      SLY(I,K)=0.0
      DO 37  J=1,NT
      WK1=DATAX(K,I,J)
      WK2=DATAY(K,I,J)
      SLX(I,K)=SLX(I,K)+WK1
   37 SLY(I,K)=SLY(I,K)+WK2
      DO 38  J=1,NT
      TY(J)=0.0
      TX(J)=0.0
      DO 38  I=1,NL
      TX(J)=TX(J)+XLT(I,J)
   38 TY(J)=TY(J)+YLT(I,J)
      DO 39  I=1,NL
      XL(I)=0.0
      YL(I)=0.0
      DO 39  J=1,NT
      XL(I)=XL(I)+XLT(I,J)
   39 YL(I)=YL(I)+YLT(I,J)
      SSTY=0.0
      SSTX=0.0
      SPTXY=0.0
      DO 40  I=1,NT
      SSTX=SSTX+TX(I)*TX(I)
      SSTY=SSTY+TY(I)*TY(I)
   40 SPTXY=SPTXY+TX(I)*TY(I)
      CORX=SX*SX/CSLT
      CORY=SY*SY/CSLT
      CORXY=SX*SY/CSLT
      SSTX=SSTX/CSL-CORX
      SSTY=SSTY/CSL-CORY
      SPTXY=SPTXY/CSL-CORXY
      SSLX=0.0
      SSLY=0.0
      SPLXY=0.0
```

```
      DO 41  J=1,NL
      SSLX=SSLX+XL(J)*XL(J)
      SSLY=SSLY+YL(J)*YL(J)
41    SPLXY=SPLXY+XL(J)*YL(J)
      SSLX=SSLX/CST-CORX
      SSLY=SSLY/CST-CORY
      SPLXY=SPLXY/CST-CORXY
      CNS=NS
      WX=0.0
      WY=0.0
      WXY=0.0
      DO 42  I=1,NT
      DO 42  J=1,NL
      WX=WX+XLT(I,J)*XLT(I,J)
      WY=WY+YLT(I,J)*YLT(I,J)
42    WXY=WXY+XLT(I,J)*YLT(I,J)
      WX=SSX-WX/CNS
      WY=SSY-WY/CNS
      WXY=SXY-WXY/CNS
      TSSX=SSX-CORX
      TSSY=SSY-CORY
      TSPXY=SXY-CORXY
      WRITE(3,61)
      WRITE(3,62)
      MDFTL=MDFT-MDFTR-MDFL-MDFW
      SSTLY=TSSY-WY-SSLY-SSTY
      PTLXY=TSPXY-WXY-SPLXY-SPTXY
      SSTLX=TSSX-WX-SSLX-SSTX
      MDFTW=MDFT+MDFW
      MDFLW=MDFL+MDFW
      MDTLW=MDFTL+MDFW
      TPWY=SSLY+WY
      TPWXY=SPLXY+WXY
      TPWX=SSLX+WX
      MAJTW=MDFTR+MDFW
      WPLY=SSTY+WY
      WPLXY=SPTXY+WXY
      WPLX=SSTX+WX
      MAJLW=MDFL+MDFW
      TLPWY=SSTLY+WY
      TLWXY=PTLXY+WXY
      TLPWX=SSTLX+WX
      MATLW=MDFTL+MDFW
      SE=(WXY*WXY)/WX
      AT=SSLY-(TPWXY*TPWXY)/TPWX+SE
      AL=SSTY-(WPLXY*WPLXY)/WPLX+SE
      ATL=SSTLY-(TLWXY*TLWXY)/TLPWX+SE
      AW=WY-SE
      ATPW=AT+AW
      ALPW=AL+AW
      ATLW=ATL+AW
```

```
      MDFAW=MDFW-1
      WK1=MDFAW
      AMSW=AW/WK1
      F1=AT/AMSW
      F2=AL/AMSW
      F3=ATL/AMSW
      WRITE(3,63) MDFTR,SSLY,SPLXY,SSLX,MDFTR,AT,AT,F1
      WRITE(3,64) MDFL,SSTY,SPTXY,SSTX,MDFL,AL,AL,F2
      WRITE(3,65) MDFTL,SSTLY,PTLXY,SSTLX,MDFTL,ATL,ATL,F3
      WRITE(3,66) MDFW,WY,WXY,WX,MDFAW,AW,AMSW
      WRITE(3,67) MDFT,TSSY,TSPXY,TSSX
      M=MAJTW-1
      WRITE(3,68) MAJTW,TPWY,TPWXY,TPWX,M,ATPW
      WRITE(3,69) MAJLW,WPLY,WPLXY,WPLX,M,ALPW
      WRITE(3,70) MATLW,TLPWY,TLWXY,TLPWX,M,ATLW
      GO TO 3
   55 FORMAT(7I5)
   56 FORMAT(16F5.0)
   58 FORMAT(8A10)
   59 FORMAT(8H0X  DATA/)
   60 FORMAT(8H0Y  DATA/)
   61 FORMAT(1H0,10X,28HANALYSIS OF COVARIANCE TABLE/)
   620FORMAT(41H   SOURCE        DF        SSY       SPXY   ,
     143H    SSX    ADJ DF   ADJ SSY    ADJ MS    F  /)
   63 FORMAT(13H TREATMENTS-A,I5,3F10.2,I8,2F10.2,F6.2/)
   64 FORMAT(13H LEVELS-B    ,I5,3F10.2,I8,2F10.2,F6.2/)
   65 FORMAT(13H A X B       ,I5,3F10.2,I8,2F10.2,F6.2/)
   66 FORMAT(13H WITHIN      ,I5,3F10.2,I8,2F10.2/)
   67 FORMAT(13H TOTAL       ,I5,3F10.2,I8,F10.2//)
   68 FORMAT(13H A + WITHIN  ,I5,3F10.2,I8,F10.2/)
   69 FORMAT(13H B + WITHIN  ,I5,3F10.2,I8,F10.2/)
   70 FORMAT(13H AXB + WITHIN,I5,3F10.2,I8,F10.2)
      END
```

APPENDIX G

COMPUTER PROGRAM FOR LINEAR REGRESSION EQUATIONS

```
C     REGRESSION EQUATIONS FOR TEACHING EXPERIMENT
C     L. FIEDLER
     0DIMENSION  DATAX(12,2,2),DATAY(12,2,2),XLT(2,2),
     1YLT(2,2),XSQLT(2,2),XYLT(2,2),A(2,2),B(2,2),TITLE(8)
   44 READ(1,79)  (TITLE(I), I=1,8)
      WRITE(3,79) (TITLE(I), I=1,8)
      READ(1,80)  NS,NL,NT
      IF(NS-9999) 3,4,3
    4 STOP
    3 DO 33  I=1,NT
      DO 33  J=1,NL
   33 READ(1,81)  (DATAX(K,I,J) , K=1,NS)
      DO 34  I=1,NT
      DO 34  J=1,NL
   34 READ(1,81)  (DATAY(K,I,J), K=1,NS)
      WRITE(3,78)
      SX=0.0
      SY=0.0
      SXY=0.0
      SSX=0.0
      XNS=NS
      TOT=NS*NT*NL
      DO 35  I=1,NT
      WRITE(3,77)
      DO 35  J=1,NL
      XLT(I,J)=0.0
      YLT(I,J)=0.0
      XSQLT(I,J)=0.0
      XYLT(I,J)=0.0
      DO 36  K=1,NS
      WK1=DATAX(K,I,J)
      WK2=DATAY(K,I,J)
      XLT(I,J)=XLT(I,J)+WK1
      YLT(I,J)=YLT(I,J)+WK2
      XSQLT(I,J)=XSQLT(I,J)+WK1*WK1
      XYLT(I,J)=XYLT(I,J)+WK1*WK2
   36 WRITE(3,82)  WK1,WK2,J,I
      DENOM=XNS*XSQLT(I,J)-XLT(I,J)*XLT(I,J)
      A(I,J)=(XNS*XYLT(I,J)-XLT(I,J)*YLT(I,J))/DENOM
```

```
      B(I,J)=(XSQLT(I,J)*YLT(I,J)-XLT(I,J)*XYLT(I,J))/DENOM
      SX=SX+XLT(I,J)
      SY=SY+YLT(I,J)
      SXY=SXY+XYLT(I,J)
      SSX=SSX+XSQLT(I,J)
      AVGX=XLT(I,J)/XNS
      AVGY=YLT(I,J)/XNS
   35 WRITE(3,83)  AVGX,AVGY
      DENOM=TOT*SSX-SX*SX
      ACOEF=(TOT*SXY-SX*SY)/DENOM
      BCON=(SSX*SY-SX*SXY)/DENOM
      WRITE(3,84)
      DO 37  I=1,NT
      DO 37  J=1,NL
   37 WRITE(3,85)  A(I,J), B(I,J), J, I
      WRITE(3,86) ACOEF,BCON
      GO TO 44
   79 FORMAT(8A10)
   80 FORMAT(3I5)
   81 FORMAT(16F5.0)
   78 FORMAT(27H0  X    Y   LEVEL TREATMENT/)
   77 FORMAT(1H0)
   82 FORMAT(F6.1,F5.1,I6,I10)
   83 FORMAT(1H0,2F5.1, 10H  AVERAGES/)
   84 FORMAT(24H0   REGRESSION EQUATIONS/)
   850FORMAT(3H Y=,F10.2,4H X +, F10.2,7H  LEVEL,I4,
     111H  TREATMENT,I4/)
   860FORMAT(4H0 Y=,F10.2,4H X +,F10.2,
     123H  TOTAL REGRESSION LINE)
      END
```

APPENDIX H

EQUATIONS FOR ANALYSIS OF COVARIANCE FOR 2 x 2 TREATMENTS BY LEVELS DESIGN

Source of Variation	Degrees of Freedom	Criterion (Y) Sum of Squares	Sum of Products (X·Y)	Pretest (X) Sum of Squares
Treatments	1	A_{yy}	A_{xy}	A_{xx}
Levels	1	B_{yy}	B_{xy}	B_{xx}
Treatments x Levels	1	$(AB)_{yy}$	$(AB)_{xy}$	$(AB)_{xx}$
Within Cells	n-3	E_{yy}	E_{xy}	E_{xx}
Total	n	S_{yy}	S_{xy}	S_{xx}
Treatments + Within Cells	n-3+1	${}_{T}S_{yy}=A_{yy}+E_{yy}$	${}_{T}S_{xy}=A_{xy}+E_{xy}$	${}_{T}S_{xx}=A_{xx}+E_{xx}$
Levels + Within Cells	n-3+1	${}_{L}S_{yy}=B_{yy}+E_{yy}$	${}_{L}S_{xy}=B_{xy}+E_{xy}$	${}_{L}S_{xx}=B_{xx}+E_{xx}$
Treatments x Levels + Within Cells	n-3+1	${}_{TL}S_{yy}=(AB)_{yy}+E_{yy}$	${}_{TL}S_{xy}=(AB)_{xy}+E_{xy}$	${}_{TL}S_{xx}=(AB)_{xx}+E_{xx}$

APPENDIX H - Continued

Source of Variation	Adjusted Degrees of Freedom	Adjusted Criterion Sum of Squares	Adjusted Mean Square	F-ratio
Treatments	1	$ADJ^{A}yy = A_{yy} - \frac{{}_{T}S^{2}xy}{{}_{T}S_{xx}} + \frac{E^{2}xy}{E_{xx}}$	$T = \frac{ADJ^{A}yy}{1}$	T/W
Levels	1	$ADJ^{B}yy = B_{yy} - \frac{{}_{L}S^{2}_{xy}}{{}_{L}S_{xx}} + \frac{E^{2}xy}{E_{xx}}$	$L = \frac{ADJ^{B}yy}{1}$	L/W
Treatments x Levels	1	$ADJ^{(AB)}yy = (AB)_{yy} - \frac{{}_{TL}S^{2}xy}{{}_{TL}S_{xx}} + \frac{E^{2}xy}{E_{xx}}$	$TL = \frac{ADJ^{(AB)}yy}{1}$	TL/W
Within Cells	n-4	$ADJ^{E}yy = E_{yy} - \frac{E^{2}xy}{E_{xx}}$	$W = \frac{ADJ^{E}yy}{n-4}$	

The entries in the above table are calculated as follows:

$$A_{yy} = \frac{\sum_{j=1}^{a} \left(\sum_{i=1}^{r} \sum_{k=1}^{b} Y_{ijk}\right)^2}{r \cdot b} - \frac{\left(\sum_{i=1}^{r} \sum_{j=1}^{a} \sum_{k=1}^{b} Y_{ijk}\right)^2}{r \cdot a \cdot b}$$

APPENDIX H - Continued

$$A_{xy} = \frac{\sum_{j=1}^{a} \left(\sum_{i=1}^{r} \sum_{k=1}^{b} Y_{ijk}\right)\left(\sum_{i=1}^{r} \sum_{k=1}^{b} X_{ijk}\right)}{r \cdot b} - \frac{\left(\sum_{i=1}^{r} \sum_{j=1}^{a} \sum_{k=1}^{b} Y_{ijk}\right)\left(\sum_{i=1}^{r} \sum_{j=1}^{a} \sum_{k=1}^{b} X_{ijk}\right)}{r \cdot a \cdot b}$$

$$A_{xx} = \frac{\sum_{j=1}^{a} \left(\sum_{i=1}^{r} \sum_{k=1}^{b} X_{ijk}\right)^2}{r \cdot b} - \frac{\left(\sum_{i=1}^{r} \sum_{j=1}^{a} \sum_{k=1}^{b} X_{ijk}\right)^2}{r \cdot a \cdot b}$$

$$B_{yy} = \frac{\sum_{k=1}^{b} \left(\sum_{i=1}^{r} \sum_{j=1}^{a} Y_{ijk}\right)^2}{r \cdot a} - \frac{\left(\sum_{i=1}^{r} \sum_{j=1}^{a} \sum_{k=1}^{b} Y_{ijk}\right)^2}{r \cdot a \cdot b}$$

$$B_{xy} = \frac{\sum_{k=1}^{b} \left(\sum_{i=1}^{r} \sum_{j=1}^{a} X_{ijk}\right)\left(\sum_{i=1}^{r} \sum_{j=1}^{a} Y_{ijk}\right)}{r \cdot a} - \frac{\left(\sum_{i=1}^{r} \sum_{j=1}^{a} \sum_{k=1}^{b} X_{ijk}\right) \cdot \left(\sum_{i=1}^{r} \sum_{j=1}^{a} \sum_{k=1}^{b} Y_{ijk}\right)}{r \cdot a \cdot b}$$

$$B_{xx} = \frac{\sum_{k=1}^{b} \left(\sum_{i=1}^{r} \sum_{j=1}^{a} X_{ijk}\right)^2}{r \cdot a} - \frac{\left(\sum_{i=1}^{r} \sum_{j=1}^{a} \sum_{k=1}^{b} X_{ijk}\right)^2}{r \cdot a \cdot b}$$

$$E_{yy} = \sum_{i=1}^{r} \sum_{j=1}^{a} \sum_{k=1}^{b} Y_{ijk}^2 - \frac{\sum_{i=1}^{r} \left(\sum_{j=1}^{a} \sum_{k=1}^{b} Y_{ijk}\right)^2}{a \cdot b} - \frac{\sum_{j=1}^{a} \sum_{k=1}^{b} \left(\sum_{i=1}^{r} Y_{ijk}\right)^2}{r} +$$

$$\frac{\left(\sum_{i=1}^{r}\sum_{j=1}^{a}\sum_{k=1}^{b} Y_{ijk}\right)^2}{r\cdot a\cdot b}$$

$$E_{xy} = \sum_{i=1}^{r}\sum_{j=1}^{a}\sum_{k=1}^{b} X_{ijk}Y_{ijk} - \frac{\sum_{i=1}^{r}\left(\sum_{j=1}^{a}\sum_{k=1}^{b} X_{ijk}\right)\cdot\left(\sum_{j=1}^{a}\sum_{k=1}^{b} Y_{ijk}\right)}{a\cdot b} -$$

$$\frac{\sum_{j=1}^{a}\sum_{k=1}^{b}\left(\sum_{i=1}^{r} X_{ijk}\right)\left(\sum_{i=1}^{r} Y_{ijk}\right)}{r} + \frac{\left(\sum_{i=1}^{r}\sum_{j=1}^{a}\sum_{k=1}^{b} X_{ijk}\right)\left(\sum_{i=1}^{r}\sum_{j=1}^{a}\sum_{k=1}^{b} Y_{ijk}\right)}{r\cdot a\cdot b}$$

$$E_{xx} = \sum_{i=1}^{r}\sum_{j=1}^{a}\sum_{k=1}^{b} X^2{}_{ijk} - \frac{\sum_{i=1}^{r}\left(\sum_{j=1}^{a}\sum_{k=1}^{b} X_{ijk}\right)^2}{a\cdot b} - \frac{\sum_{j=1}^{a}\sum_{k=1}^{b}\left(\sum_{i=1}^{r} Y_{ijk}\right)^2}{r} +$$

$$\frac{\left(\sum_{i=1}^{r}\sum_{j=1}^{a}\sum_{k=1}^{b} Y_{ijk}\right)^2}{r\cdot a\cdot b}$$

$$S_{yy} = \sum_{i=1}^{r}\sum_{j=1}^{a}\sum_{k=1}^{b} Y^2{}_{ijk} - \frac{\left(\sum_{i=1}^{r}\sum_{j=1}^{a}\sum_{k=1}^{b} Y_{ijk}\right)^2}{r\cdot a\cdot b}$$

$$S_{xy} = \sum_{i=1}^{r} \sum_{j=1}^{a} \sum_{k=1}^{b} X_{ijk}Y_{ijk} - \frac{(\sum_{i=1}^{r} \sum_{j=1}^{a} \sum_{k=1}^{b} X_{ijk}) \cdot (\sum_{i=1}^{r} \sum_{j=1}^{a} \sum_{k=1}^{b} Y_{ijk})}{r \cdot a \cdot b}$$

$$S_{xx} = \sum_{i=1}^{r} \sum_{j=1}^{a} \sum_{k=1}^{b} X_{ijk}^2 - \frac{(\sum_{i=1}^{r} \sum_{j=1}^{a} \sum_{k=1}^{b} X_{ijk})^2}{r \cdot a \cdot b}$$

$$(AB)_{yy} = S_{yy} - A_{yy} - B_{yy}$$

$$(AB)_{xy} = S_{xy} - A_{xy} - B_{xy}$$

$$(AB)_{xx} = S_{xx} - A_{xx} - B_{xx}$$

In the above equations, a = the number of treatments,
b = the number of levels,
n = the total number of students, and
r = the number of students in each treatment by level cell.

APPENDIX I

EXEMPLARY PROBLEMS ASSIGNED AS HOMEWORK

Phase I - Functions

1. a) Computer Problem.--Write a program to evaluate a general polynomial function (say of degree 8 or less) for values x_1, x_2, x_3, . . . x_n where $n \leq 7$.

   Test your program by using $f(x) = x^2 - 3x + 1$ and find $f(0)$, $f(-1)$, and $f(-\sqrt{3})$.

   b) Traditional Problem.--Given a polynomial function f defined by $f(x) = x^2 - 3x + 1$, find $f(0)$, $f(-1)$, and $f(-\sqrt{3})$.

2. a) Computer Problem.--Given $f(x) = x^2 - 1$ and $g(x) = 3x + 1$, compute values of the functions $f+g$, fg, $f \circ g$, $f-g$, f/g, and $g \circ f$ for values of x from -4 to 4 in increments of 0.25.

   b) Traditional Problem.--Given $f(x) = x^2 - 1$ and $g(x) = 3x + 1$, write the equations which define $f+g$, fg, $f \circ g$, $f-g$, f/g, and $g \circ f$.

Phase II - Limits and Differentiation

1. a) Computer Problem.--Write a program which will indicate the following limit:

$$\underset{x \to 0}{\text{limit}} \frac{\sqrt[5]{1+5x^5} - \sqrt[5]{1+x^2}}{\sqrt[5]{1-x^2} - \sqrt[5]{1-x^3}}.$$

b) Traditional Problem.--Find the limit:

$$\underset{x \to 0}{\text{limit}} \frac{\sqrt[5]{1+x^5} - \sqrt[5]{1+x^2}}{\sqrt[5]{1-x^2} - \sqrt[5]{1-x^3}}.$$

2. a) Computer Problem.--Find a $\delta > 0$ such that $|f(x)-L| < \epsilon$ whenever $0 < |x-a| < \delta$ for the function defined by $f(x) = x^2$ where $a = 3$, $L = 9$, and $\epsilon = .3$.

   Write a program to calculate f(x) for twenty different values of x such that $0 < |x-a| < \delta$. Also output $L-\epsilon$ and $L+\epsilon$ as a check on your results.

   b) Traditional Problem.--Given $f(x) = x^2$, $a = 3$, $L = 9$, and $\epsilon = .3$, find a $\delta > 0$ such that $|f(x)-L| < \epsilon$ whenever $0 < |x-a| < \delta$.

Phase III - Iteration

1. a) Computer Problem.--Write a program which will read in an initial value and find a root of a polynomial equation, say of degree 20 or less, by Newton's method.

   Test your program by finding the root of $x^3-5x-3 = 0$ for $2<x<3$. Also find the largest positive zero of $f(x) = x^3-3x^2-8x+10$. Use twenty iterations.

   b) Traditional Problem.--Use Newton's method to find the root of $x^3-5x-3 = 0$ for $2<x<3$ to three significant digits. Also find the largest positive zero of

$f(x) = x^3-3x^2-8x+10$ to three significant digits.

2. a) Computer Problem.--Find a value of x such that $3x = 1 + \cos x$ using a "fixed-point" iterative scheme. Use twenty iterations.

   b) Traditional Problem.--Use a "fixed point" iterative scheme to solve $3x = 1 + \cos x$ to three significant digits.

Phase IV - Integration

1. a) Computer Problem.--Given the function f defined by $f(x) = \frac{1}{x^2}$, find the upper and lower sums with respect to a regular partition of $[1,4]$ into 6 subintervals. Repeat the problem using 60 subintervals. Use the maximum and minimum value of the function in each subinterval for the upper and lower bounds in that subinterval.

   b) Traditional Problem.--Find the upper and lower sums of the function defined by $f(x) = \frac{1}{x^2}$ with respect to a regular partition of $[1,4]$ into 6 subintervals. Use the maximum and minimum value of the function in each subinterval for the upper and lower bounds in that subinterval.

2. a) Computer Problem.--Use Simpson's rule with $n = 6$ to approximate the area of the region bounded by one quarter of the hypocycloid $x^{2/3} + y^{2/3} = 1$. Repeat the program with $n = 40$.

b) Traditional Problem.--Approximate the area of the region bounded by one quarter of the hypocycloid $x^{2/3} + y^{2/3} = 1$ by Simpson's rule with n = 6.

The preceding examples may give the impression that there was a one-to-one correspondence between problems assigned to the computer group and to the traditional group. This was not the case because solving a problem by writing and running an error-free computer program usually takes much more time than solving the same problem by pencil and paper. Also the student, not the computer, does the analysis of the problem. Fewer exercises were assigned to the computer group than the traditional group, but most of the computer programs were written to solve a certain class of problems rather than a specific exercise. Two examples of computer solutions, one simple and one more complex, will be given to illustrate the above remarks.

Problem 1 - Absolute Value

Write a computer program which will evaluate the function f defined by $f(x) = |x|$. Test your program using x = -3, 42, 0, 5.92, 8, -18, and 3.14159.

Solution

```
C     ABSOLUTE VALUE PROGRAM
    7 READ (1,55) X
      IF (X) 3,4,4
    3 FX = -X
      GO TO 5
    4 FX = X
    5 WRITE (3,55) X,FX
      GO TO 7
   55 FORMAT (2F15.4)
      END
```

In this problem the traditional student can write down an answer by rote, but the computer student must apply the definition of absolute value to his program. The FORTRAN language has a built-in absolute value function, but the students were purposely not informed about this fact.

Problem 2 - Derivatives of Polynomials

Write a program to compute the values of a general polynomial f, (say of degree 15 or less), its first derivative, and its second derivative for values of x from a to b in increments of h. Analyze the output table to find and classify the extrema and points of inflection of the graph of f in the interval $[a,b]$.

Test your program by using $f(x) = 4x^5 - 5x^4 - 20^3 + 50x^2 - 40x - 132$ with values of x from -3 to 3 in increments of 0.125. Plot the function from the output table.

Solution

```
C     DERIVATIVES OF POLYNOMIALS
      DIMENSION A(50)
      READ (1,50) XL, XH, XINC
      WRITE (3,60) XL, XH, XINC
      READ (1,51) N, (A(I), I = 1, N)
      WRITE (3,61) N, (A(I), I = 1, N)
      X = XL
   33 FX = 0.0
      IF(X) 11,12,11
   12 FX = A(1)
      FXP = A(2)
      FXPP = 2.0*A(3)
      GO TO 44
   11 DO 3 I = 1, N
    3 FX = FX + A(I)*X**(I-1)
      FXP = 0.0
      DO 4 I = 2, N
      C1 = I - 1
    4 FXP = FXP + C1*A(I)**(I-2)
      FXPP = 0.0
```

```
      DO 5 I = 3, N
      C1 = I - 1
      C2 = I - 2
    5 FXPP = FXPP + C1*C2*A(I)*X**(I-3)
   44 WRITE (3,52)X, FX, FXP, FXPP
      X = X + XINC
      IF (X-XH) 33,33,34
   34 STOP
   50 FORMAT (3F10.4)
   51 FORMAT (I5,(7F10.4))
   52 FORMAT (4F20.4)
   60 FORMAT (1H0,3F10.4//)
   61 FORMAT (I5,7F10.4//)
      END
```

The function selected to test this program has related maximum value, a relative minimum value, and a point of inflection in the interval $[-3,3]$. The student will see both the first and second derivative tests as well as the test for points of inflection illustrated in his program output. Furthermore, he must use the general algorithm for differentiating a polynomial to write his program.

Clearly, there are many ways to write a program which will solve a given problem. Each student wrote his program in his own way. As a matter of fact, it would be unusual if two students wrote the same FORTRAN instructions to solve a particular problem.