RAO, Gujar Nagendra Seshappa, 1931-
COMPUTATIONAL METHODS AND ERROR ANALYSIS IN WATER SURFACE PROFILES OF GRADUALLY VARIED FLOW IN OPEN CHANNELS.

The University of Oklahoma, Ph.D., 1969
Engineering, civil

University Microfilms, Inc., Ann Arbor, Michigan

THE UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

COMPUTATIONAL METHODS AND ERROR ANALYSIS IN WATER SURFACE PROFILES OF GRADUALLY VARIED FLOW IN OPEN CHANNELS

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

DOCTOR OF PHILOSOPHY

BY

GUJAR NAGENDRA SESHAPPA RAO

Norman, Oklahoma

1969

COMPUTATIONAL METHODS AND ERROR ANALYSIS

IN WATER SURFACE PROFILES OF GRADUALLY

VARIED FLOW IN OPEN CHANNELS

APPROVED BY

J. F. Harp

Geo. W. Reid

R D Bauman

John E. Brixey

DISSERTATION COMMITTEE

COMPUTATIONAL METHODS AND ERROR ANALYSIS

IN WATER SURFACE PROFILES OF GRADUALLY

VARIED FLOW IN OPEN CHANNELS.

BY

GUJAR NAGENDRA SESHAPPARAO

MAJOR PROFESSOR: Dr. Jimmy Frank Harp

A generalized method to compute any of the twelve different water surface profiles of gradually varied flow in prismatic open channels has been developed in a form suitable for use with high speed digital computers. The differential equation has been transformed to a form suitable for integrating by numerical methods. All the twelve profiles are shown to lead to suitable integral functions and eight different polynomial approximations in the form of power series are fitted to these integral functions for different range of values for the ratio of normal depth to stream depth depending on the convergence of the series. Chebyshev polynomial approximation and Lanczos method of Economization of Power Series are found to have a dramatic effect in summing the infinite power series arising in the case of adverse slopes.

This study also deals with the error analysis and the problem of economization of number of steps required for the desired accuracy in the water surface profile computations by the most popular "Step Methods". Improved procedures have been presented and their accuracy and advantages are discussed in the text.

ACKNOWLEDGEMENTS

Words are scarce to express fully my deep gratitude to my major professor, Dr. J. F. Harp, under whom this research was undertaken. Without his encouragement, guidance, and advice, this work would not have been possible. I am highly grateful to him for his untiring interest in my work and marathon patience in going through the manuscript of this dissertation at various stages with corrections, suggestions, and helpful comments.

I also wish to express my sincere appreciation to other members of my Doctoral Committee, Professor G. W. Reid, Dr. R. D. Bauman, and Dr. J. C. Brixey, for reviewing the reading copy with constructive criticism and suggestions. The financial aid from the State Highway Department of Oklahoma, and the National Science Foundation through the Oklahoma Research Institute, throughout my graduate studies, is gratefully acknowledged.

My thanks are also due to Mary Zimbelman and Keven Amrein who typed this dissertation.

Finally, I wish to express my appreciation to my wife, Vasantha, for her patience, understanding, and encouragement in all my endeavors.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

A . . . . Cross Sectional Area of Flow

α . . . . Kinetic Energy Correction Factor

b . . . . Bottom Width of Channel

C . . . . Chezy Coefficient

E . . . . Specific Energy

g . . . . The Acceleration of Gravity

h_f. . . . . Head Loss

M . . . . Hydraulic Exponent for Critical Flow Computation

N . . . . Hydraulic Exponent for Uniform Flow Computation

n . . . . Manning's Friction Factor

P . . . . Wetted Perimeter

Q . . . . Volumetric Rate of Discharge

R . . . . Hydraulic Mean Radius

S_c. . . . . Critical Bed Slope

S_o. . . . . Longitudinal Bed Slope

S_f. . . . . Friction Slope

T . . . . Top Width of Channel

U . . . . y/y_n

V . . . . Velocity of Water

X . . . . Length of Backwater or Drop-Down Curve

y . . . . Depth of Flow in Channel

y_n. . . . . Normal Depth in Uniform Flow

y_c. . . . . Critical Depth

z . . . . Side Slope

Z . . . . Datum Height

CHAPTER I

INTRODUCTION

General: The hydraulic engineer is often called upon to design canals, flood-channels, and other open-channel works, the consturction of which has proceeded rapidly in the last half-century. The study of hydraulics of flow in open channels is not always subject to analysis in an exact or rigorous manner due to the large number of fundamental variables involved. The results are obtained by simplified assumptions and, therefore, are only approximate, but the limitations are compatible with the desired accuracy of results in practical engineering problems. In order to have a steady uniform flow, a constant area and shape of section must be maintained, and the mean velocity must be the same at all sections. The water depth and the bed slope must be constant. However, if any of these fundamental conditions and characteristics are changed, the flow becomes non-uniform or varied. Usually, in this case the cross-sectional area, the velocity, and the hydraulic slope vary from section to section. The sections are irregular, the bed or the banks may be continually changing. So the balance between the friction loss and the slope is disturbed and the surface line is not parallel to the bottom. On the other hand, the position of the surface curve must be computed for the limiting cases of non-uniform flow. In natural watercourses such a computation is necessary. The classical example of varied flow is that of a backwater curve produced by a dam, weir or other structure impeding the flow. These structures will cause the water to back up the stream thereby increasing the depth. In such cases, the important question of how much the water will be raised at a given distance upstream from the point of obstruction arises. Most of the problems in gradually varied flow are concerned with the determination of the surface profile of flow in both natural and uniform channels where changes of depth are so gradual that the flow may be considered essentially parallel to the stream bed. The derivation of water surface profiles serves two major purposes; one - to determine tail-water rating curves and, two - to trace the backwater curve above a dam or other obstruction into the main stream or tributary where the effects are manifested. In some cases a complete determination of a family of profiles, depending on various conditions of stage and discharge of the stream, is necessary. Such a family of profiles is necessary in (1) determining the economical height of a dam, where the initial elevation is indeterminate (2) tracing the flow profiles in a tributary stream for different stages, and discharges into the main river (3) connecting two reservoirs by canal for changing reservoirs' elevations and variable discharges.

Tail water curves are used in the desing of power plants, pumping plants, and energy dissipators such as stilling basins. In the design of large dams, these curves also furnish useful information for making stability and stress analyses. A primary use of the backwater curve above a dam is for reservoir land acquisitions and easements.

Backwater information is also necessary in the design of bridges, power plants or other hydraulic structures located along or above the reservoir.

It must be noted that the fact that actual curvature of these profiles is very slight, except in the immediate vicinity of the critical depth and also the relative length of the several curves, for a given rate of discharge, will vary with relative depth of flow, since the rate of loss is proportional to the square of the velocity. It will be seen from figure (1) that except C_3 and M_3 profiles, all the other curves are asymptotic to the normal depth line, to the bottom, or to the horizontal. These curves are infinite in extent, mathematically speaking, although for practical purposes it is reasonable to consider that a curve has reached its end when the depth in within 1 to 2 per cent of its asymptotic limit. Longitudinal water surface curves, in general, include not only backwater curves, but also water surface profiles through the hydraulic jump, over weirs, around sharp bends, and through abrupt changes in cross section. These abrupt or sharply curving profiles cannot be solved by backwater computations and therefore are not studied in this effort.

Purpose and Scope of Research: The purpose of this research is to develop a generalized method of computing surface profiles of stready, gradually varied flow in uniform channels, suitable for high speed digital computers whereby results can be obtained more directly than are possible hitherto with the existing methods. These methods require extensive tables, charts, and graphs developed by earlier

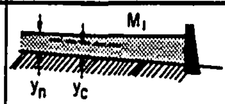
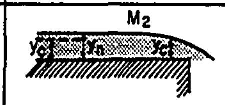
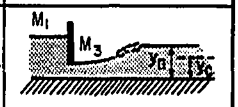
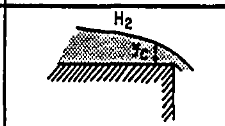
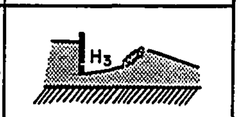
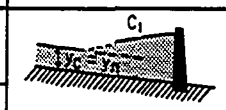
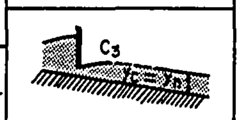
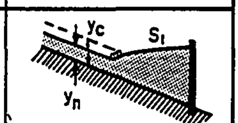
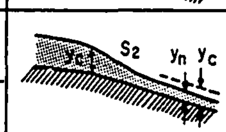
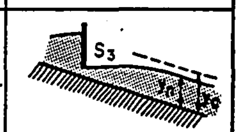
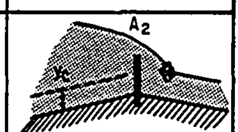
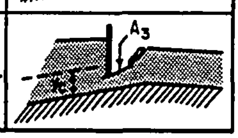
CHANNEL SLOPE	DEPTH RELATIONS	$\frac{dy}{dx}$	TYPE OF PROFILE	SYMBOL	TYPE OF FLOW	CONFIGURATION
MILD $0<S_o<S_c$	$y > y_n > y_c$	+	BACKWATER	M_1	SUBCRITICAL	
	$y_n > y > y_c$	−	DROPDOWN	M_2	SUBCRITICAL	
	$y_n > y_c > y$	+	BACKWATER	M_3	SUPERCRITICAL	
HORIZONTAL $S_o = 0$ $y_n = \infty$	$y > y_c$	−	DROPDOWN	H_2	SUBCRITICAL	
	$y_c > y$	+	BACKWATER	H_3	SUPERCRITICAL	
CRITICAL $S_n = S_c$ $y_n = y_c$	$y > y_c = y_n$	+	BACKWATER	C_1	SUBCRITICAL	
	$y_c = y = y_n$		PARALLEL TO BED	C_2	UNIFORM, CRITICAL	
	$y_c = y_n > y$	+	BACKWATER	C_3	SUPERCRITICAL	
STEEP $S_o>S_c>0$	$y > y_c > y_n$	+	BACKWATER	S_1	SUBCRITICAL	
	$y_c > y > y_n$	−	DROPDOWN	S_2	SUPERCRITICAL	
	$y_c > y_n > y$	+	BACKWATER	S_3	SUPERCRITICAL	
ADVERSE $S_o < 0$ $y_n = \infty$	$y > y_c$	−	DROPDOWN	A_2	SUBCRITICAL	
	$y_c > y$	+	BACKWATER	A_3	SUPERCRITICAL	

Fig. 1. Different Types of Water Surface Profiles

investigators which are no longer of widespread interest in view of the extensive and rapidly growing use of high speed digital computers of this era. So the problem is to devise a new method to develop a generalized computer program, flexible enough to solve any of the twelve profiles encountered in practice.

The second phase of the study deals with the error analyses in the most simple DIRECT STEP method which is most popular but laborious and time consuming. However, when only a few surface curves have to be computed and elevations are needed all along the channel, the Step Method is most indispensable. The ultimate accuracy of the solution will depend upon the number of steps used for the computation. The smaller the depth increments, the more accurate are the results because of the assumptions made regarding energy loss. Which is the most economical way to obtain the required accuracy with minimum effort? This problem has been studied in this work. From the available literature it has been seen that most of the authors have not been enthusiastic in recognizing this problem. Numbers of methods are available in solving the problem in gradually varied flow. Considerable work has been done to devise many labor-saving methods for the convenience of the human operator, who can quickly and calmly look up tables or trace solutions on a graph, but who finds it more lengthy and difficult to do arithmetical calculations. Also he prefers his calculations to be direct and explicit, and not to involve the tedious repetition of trial-and-error processes which are often indispensable in open-channel flow problems.

The high speed computer takes quite a different view of the matter. It can calculate with great speed, and is not deterred by the repetitions required in trial processes. Also the human programmer directing the machine can easily and quickly write the instructions governing these trial processes. The computer can look up tables just as readily as it can perform calculations, but tables and graphs are generally regarded as a nuisance in computer work for two reasons: They take up valuable storage space in the machine, and the programmer must transfer them manually to tape or cards and then load them into the machine.

In using graphs their ordinates would be listed in the machine as tables, and the scanning of a graph is simulated by looking up the table. But still these scanning and interpolation procedures in a computer are quite awkward. It appears therefore that the computer's view of the labor problem almost exactly reverses the view of the human operator. It does not follow, however, that an engineer with access to a computer need scrap all the procedures developed extensively so far in water surface profile computations. For quick spot checks or for small schemes, the engineer will always have a use for methods that can be operated at the desk or in the drawing office, and that will yield solutions quickly. For comprehensive reviews of large schemes, the computer comes into its own but until it becomes cheap and small it will not completely displace the methods which have been developed so far in open channel flow problems.

CHAPTER II

REVIEW OF LITERATURE

Most of the earlier investigations are confined mainly to the backwater and draw-down curves in uniform channels on mild slopes which form the more important classes of surface profiles encountered in practice. The computation of gradually varied flow profiles involves basically the solution of the differential equation of gradually varied flow. The main objective of the computation is to determine the shape of the flow profile. Broadly classified, there are three methods of computation; namely, the graphical-integration method, the direct-integration method, and the Step-Methods.

The theory of varied flow was first postulated in a complete and comprehensive manner by J. M. Belanger in 1828. His method contains the general differential equation for gradually varied flow and method of solutions by successive approximation. The fundamental principles were so well covered that little has since been added to modify the differential equation in its original form. All the subsequent investigations have been mainly confined to the method of solution of the gradually varied flow equation by direct, approximate and graphical integration.

The earlier attempts to obtain an analytical solution by direct integration of the differential equation were restricted to flow in channels of special form. The case of rectangular channel of great-width was treated by Dupuit (1848) and Ruhlmann (1880) both ignoring the effect due to changes of velocity. The same case was treated in complete form by Bresse (1860), considering the effect of friction as well as that of the change of kinetic energy, and Bresse used the Chezy formula for the evaluation of friction. After experimental studies had shown that the Chezy C was not constant, Bresse's function fell into disuse and it is known that the shape of the channel may have an appreciable effect, so that Bresse's curves cannot be used if the greatest accuracy is desired. Bresse's method of integration of the differential equation leads to the result

$$x = \frac{y}{S_0} - y_n \left[\frac{1}{S_0} - \left(\frac{C^2}{g}\right)\right] \varphi$$

where φ is known as Bresse's function, and is equal to

$$\varphi = \int \frac{du}{1-u^3} = \frac{1}{6}\left[\log \frac{u^2+u+1}{(u-1)^2}\right] - \frac{1}{\sqrt{3}} \tan^{-1} \frac{\sqrt{3}}{2u+1} + A,$$

where $u = y/y_n$ and A, is a constant of integration.

The case of broad parabolic channel was treated by Tolkmitt (1898). Many other solutions for a rectangular channel based on a variable coefficient in Chezy equation were due to Masani (1900), Schaffernak-Ehrenberger (1914), Baticle (1921, approximate trapezoid), Husted (1924), Kosney (1928), Schokliseh (1930), and Gunder (1943).

The first attempt to arrive at a solution suitable for any type of cross section was begun by Bakhmeteff (1932) and subsequently presented in a more complete form by Mononobe (1938). Mononobe (15) who assumed both A and P to be monomial functions of y - an assumption that appears to introduce little error for the depth range of the M_1 and the M_2 curves in many types of channel. Bakhmeteff's approach has met with wide spread favor among hydraulic engineers. Finding that for prismatic channels the product A^2C^2R may satisfactorily be represented as a constant power of y over a considerable depth range, Bakhmeteff was able to develop by analytical and graphical means a series of integral tables, use of which greatly simplifies routine computation of surface profiles for positive profiles. Under Bakhmeteff's guidance, Matzke (13), extended this method to the case of adverse slopes, the nature of which requires a different set of tables.

In an attempt to improve Bakhmeteff's method, Mononobe introduced two assumptions for hydraulic exponents. By these assumptions the effects of velocity change and friction head are taken into account integrally without the necessity of dividing the channel length into short reaches. Thus the Mononobe method affords a more direct and accurate computation to these two types of surface profile, Mononobe submitted plots of his integration function, compared his results at length with those of nine earlier investigators, and presented a series of ingenious laboratory tests to substantiate his computations. In order to verify the curves obtained by the methods of previous investigators, Stevens (1936) checked the Step-by-Step method for M_1-profile, a backwater curve with results scarcely discernible from the experimental data.

Although the general principles of gradually varied flow have been developed over a period of many years, it remained to Bakhmeteff (2) to organize these principles for ready engineering use. In the Bakhmeteff

Method, the channel length under consideration is divided into short reaches. The change in critical slope within the small range of the varying depth in each reach is assumed constant and the integration is carried out by short range steps with the aid of a varied flow function. Mononobe Method provides a direct procedure whereby results can be obtained without recourse to successive steps. In applying this method to practical problems, it has been found that some of his assumptions are not satisfactory in many cases, and another drawback of this method perhaps lies in the difficulty of using the accompanying charts, which are not sufficiently accurate for practical purposes. Bakhmeteff's varied flow equation is

$$S_0 \frac{dx}{dy} = \frac{1 - \beta(\frac{y_n}{y})^N}{1 - (\frac{y_n}{y})^N}$$

where

$$\beta = C^2 S_0 b/g \cdot p$$

A complete integration of the differential equation of non-uniform flow has been developed by Woodward and Posey (1943) (23), for channels with horizontal bottom grade, but this integration is not applied to channels with sustaining slopes. Because of the importance of non-uniform flow and the particular difficulties in computing backwater curves in closed conduits, C. T. Keifer and H. H. Chu (1954) (7) have devised new procedures facilitating backwater computations. The methods developed by earlier investigators cannot be applied accurately to closed conduits when flow is near the top. C. J. Keifer and H. H. Chu proposed a method for separating many of the involved factors in the non-uniform flow equation so that it can be integrated numerically with respect to the relative depth of flow, y/D, in which y represents the depth of flow and D is the diameter of the conduit. Later, Lee (1947) (14) and Von Seggern (1950) (22) suggested new assumptions which result in more satisfactory solutions. Von Seggern introduced a new varied flow function in addition to the function used by Bakhmeteff; hence, an additional table for the function is necessary in his method. In Lee's method, however, no new function is required.

Many investigators have suggested means of refining Bakhmeteff's original work. Ven Te Chow (1955) (3) in particular has developed methods which extend and consolidate Bakhmeteff's work while retaining the same form of varied flow function. Ven Te Chow's varied flow equation is

$$dx = \frac{y_n}{S_0}\left[1 - \frac{1}{1-U^N} + \frac{\left(\frac{y_c}{y_n}\right)^M U^{N-M}}{1-U^N} \right] dU$$

where

$$U = y/y_n.$$

He has given extensive tables and charts which are simple and time saving for practical application.

J. M. Lara and K. B. Schroeder (1959) (9) developed two methods in their work with the Bureau of Reclamation for the computation of water surface profiles in natural streams. First method is a trial and error procedure which involves step-by-step computations. The method has a very wide scope of application, particularly applicable to the irregular channel in which the cross section consists of a main channel and separate overbank areas having individual "n" coefficients. Velocity head changes are taken into account by weighting process and corrections can be included for eddy losses within the reach. Reach length representing the flow path between sections, however, are assumed equal for the main channel and overbank areas. The second method is also a trial and error procedure involving step computations. However, it differs from first method in that reach lengths representing the flow path between sections are different for the main channel and overbank and the overbank reach length could be considerably shorter.

A relatively simple and practical method has been proposed for the computation of water surface profiles in natural streams, by P. A. Argyropoulos (1961) (1). It is an advantageous method when several backwater profiles must be determined in the channel. The velocity head corrections have been taken into account and the effect of bend losses, bridge-pier losses, and losses owing to change in shape of the cross section can be included when necessary. The method is based on the assumption that the velocity of flow is not uniformly distributed over the area of any cross section.

J. A. Ligget (10) (1961) developed a different procedure for the general solution for open channel flow profiles. This method is claimed to lend itself either to hand calculation or to calculation on the digital computer. His procedure deals with the derivation of the simplified equation of non-uniform flow after applying certain boundary conditions and solving that equation by Newton-Raphson Method of Successive Approximation. The present classical solutions do not allow for the variations in channel width or cross section. Also, many of the present methods are troublesome near points of critical depth. However, the author himself has admitted that the method is evidently somewhat more laborious than the methods now in use.

The effects of bed slope and roughness on the form of M_1 - type backwater profile in rectangular channel has been studied by H. R. Vallentine (1964) (21). For channels of given width-depth ratios and maximum-to-normal depth ratios, non-dimensional profile forms are seen to depend on the normal-flow Froude numbers. There is little readily available information on the actual proportions of the profiles and the effect on these proportions of variations in flow geometry, roughness, flow rate and bed slope. In this study which is limited to M_1 - curves, the variations of profile with bedslope, roughness and channel width are presented in graphical form. Non-dimensional plots of the profiles show a measure of similarity not readily recognizable from the usual analytical approaches. It appears practical to generalize, at least approximately, regarding the vertical displacement of the water surface and the length of the profile. These measures can be of value as approximate guides, useful for preliminary design purposes, thus eliminating computation when precise values are not required.

In the treatment of irregular channels, it is possible, just as in the case of uniform channels, to replace the time consuming, tedious and trial process by graphical methods based on plotting certain properties of the cross section against the water level, or stage. Leach (1919), Grimm (1928), Steinberg (1939), Escoffier (1946), and Ezra (1954) developed such graphical methods which are useful for determining a family of flow profiles quickly. But the advantages of graphical procedures are offset by thier limited accuracy and applicability.

The first attempt to use computers (5) for calculations of backwater curves was in Canada (1956). One machine did the work of 50 men on the calculations of backwater curves, so vital to the hydraulic juggling of the St. Lawrence River. The figures it produced have been of constant use to Canadian and U.S. engineers all along the 40 mile stretch of river from Barnhart Island to Lake Ontario. The British Digital computer, called "Ferut" was able to produce results in a little over a year from the start of operations to the final report on a job which it was calculated, would have taken 50 man-years to accomplish. The machine was used to analyze all possible variables that might have developed. By manual calculations only about one-eigth of the variables could have been considered.

Another study has been made recently by William F. Pickard (1963) (17) in the application of the computer to longitudinal profiles. It has been shown that the problem of integrating the differential equation that governs the non-uniform flow in open channel can be reduced to the evaluation of three transcendental functions. This later problem was shown to reduce to that of computing certain simple algebraic quantities and calculating various low-order polylogarithm functions. A sub-program has written in Fortran for use on 7090 I.B.M. computer. However, the derivation of approximating polynomials for the several polylogarithm function presents considerable difficulties and it requires extensive tables of polylogarithms and also it demands high degree of mathematical skill.

Hydro: The hydro (12) (1966) is a content-oriented computer language system, developed by the Department of Civil Engineering, Carnegie Institute of Technology, for the solution of hydraulic engineering problems. This system has two principal components: a compiler and a procedure library. When a HYDRO program is run, the compiler translates it and assembles from the library an equivalent ALGOL program which is then executed. Two generalized computer programs in ALGOL language has been developed only for natural rivers, using Standard Step Method. One computes a profile for a single regime -- subcritical or supercritical -- emanating from a single control point. The second computes profiles past many control points, with changes of regime permitted. Both procedures conpute curves for each of the input discharges supplied by the user.

U.S. Army Corps of Engineers, Hydrologic Center, Sacramento (1967) (19), have developed very recently a generalized computer program of backwater computations for Frotran II for the I.B.M. 1620 and G. E. 225 computers, using the "Standard Step Method" for natural rivers. This program will compute the water surface profile for sections of any shape using up to 100 points to describe the cross section and using up to 10 different "n" values. Backwater through bridges may be made by a special routine for low flow control and pressure flow. Correction for bridge deck area and wetted perimeter can be made for a bridge section when using the normal backwater routine. However, a more fundamentally based formulation is required if one is to achieve very much generality and flexibility.

CHAPTER III

DEVELOPMENT OF PROCEDURE FOR DIRECT INTEGRATION

In the methods by direct integration the attempt is made to obtain a direct solution of the varied flow function to dispense with the large numbers of steps required by Step Methods. In order to integrate the equation of gradually varied flow, it is necessary to establish the relationship between the surface width, area, friction slope and depth of flow. For a given rate of flow in a given uniform channel, these variables may be expressed in terms of the depth y i.e., $dx = \varphi(y)dy$ in which $\varphi(y)$ is usually of a form that is most difficult, if not impossible, to integrate by methods of ordinary calculus.

This chapter presents a new method of integrating the equation of gradually varied flow in prismatic channels. The differential equation of gradually varied flow is integrated under given assumptions, resulting in an equation which contains varied flow functions, belonging to the type same as Ven Te Chow's (3) varied flow function. Formulas for hydraulic exponents derived by earlier investigators have been used throughout the computations. But the Voluminous Tables, curves, and graphs developed by earlier investigators are no longer of widespread interest in view of the extensive and rapidly growing use of high speed digital computers of this era and the use of tables, curves and graphs will have to be outdated sooner or later. Therefore, a new procedure has to be developed, suitable for programming to digital computers.

The differential equation of the gradually varied flow in open channel flow can be written as

$$\frac{dy}{dx} = S_o \cdot \frac{1 - ({}^{y_n}/_y)^N}{1 - (y_c/y)^M} \tag{3.1}$$

It is not possible to perform this integration by conventional methods. Equation (3.1) is integrated to give the distance x between y_1 and y_2. This equation represents the slope of the water surface with respect to the bottom of the channel. It can therefore be used to describe the characteristics of avrious flow profiles. The flow profile represents the surface curve of the flow. It will represent a backwater curve if the depth of flow increases in the direction of flow and draw down curve if the depth decreases in the direction of flow. Twelve different types of flow profiles (Figure 1) are possible in practical engineering problems. All these cases are analyzed by the newly proposed technique and compared with the other most common method and errors occurring in each method are discussed. For the proposed analysis, all the twelve profiles can be classified under four groups as following.

I Group: M_1, S_1, S_2, C_1 ----------------------$S_o > 0$, $y/y_n > 1$

II Group: M_2, M_3, S_3, C_3 ----------------------$S_o > 0$, $y/y_n < 1$

III Group: H_2, H_3 ------------------------------$S_o = 0$, $y_n = \infty$

IV Group: A_2, A_3 ------------------------------$S_o < 0$, $y_n < 0$

Positive slopes:

Consider equation (3.1),

$$\text{Let } y/y_n = U, \quad dy = yn\, dU$$

$$dx = (1/S_o) \cdot \frac{1 - (y_c/y)^M}{1 - (y_n/y)^N} \; dy$$

$$= (1/S_o) \cdot \frac{1 - (y_c/y_n)^M \, (y_n/y)^M}{1 - (y_n/y)^N} \; dy$$

$$= (\frac{1}{S_o}) \cdot \frac{1 - (y_c/y_n)^M \, (1/U)^M}{1 - (1/U)^N} \cdot y_n \; dU$$

$$dx = + (y_n/S_o) \cdot \left[\frac{(y_c/y_n)^M \, U^{N-M}}{1 - U^N} - \frac{U^N}{1 - U^N} \right] dU \qquad (3.2)$$

This equation can also be written without any other substitution by dividing each numerator and denominator by U^N as following.

$$dx = + (y_n/S_o) \left[- \frac{(y_c/y_n)^M \, U^{-M}}{1 - U^{-N}} + \frac{1}{1 - U^{-N}} \right] dU \qquad (3.3)$$

The object of writing the equation (3.3) in this form is to fit a converging series to the integral function for a particular range of value for the ratio $y/y_n = U$.

Negative Slopes or Adverse Slopes:

For negative slopes it can be shown that the differential equation takes the form *

$$dx = -(1/S_o) \cdot \frac{1 - (y_c/y_n)^M (y_n/y)^M}{1 + (y_n/y)^N} dy$$

Let $\qquad y/y_n = U, \qquad dy = y_n \; dU$

$$dx = -(1/S_o) \cdot \frac{1 - (y_c/y_n)^M (\frac{1}{U})^M}{1 + (\frac{1}{U})^N} \cdot y_n \; dU$$

$$dx = +(y_n/S_o) \cdot \left[\frac{(y_c/y_n)^M U^{N-M}}{1 + U^N} - \frac{U^N}{1 + U^N} \right] dU \qquad (3.4)$$

This can also be written without any other substitution

$$dx = (y_n/S_o) \cdot \left[\frac{(y_c/y_n)^M U^{-M}}{1 + U^{-N}} - \frac{1}{1 + U^{-N}} \right] dU \qquad (3.5)$$

From the above equations, it can be seen that the terms

A Group: $\dfrac{U^{N-M}}{1-U^N}$, $\dfrac{U^{-M}}{1-U^{-N}}$, $\dfrac{U^{N-M}}{1+U^N}$, $\dfrac{U^{-M}}{1+U^{-N}}$,

B Group: $\dfrac{U^N}{1-U^N}$, $\dfrac{1}{1-U^{-N}}$, $\dfrac{U^N}{1+U^N}$, and $\dfrac{1}{1+U^{-N}}$ present

considerable difficulties in integration for different values of M and

---

*Chow, V. T., Hydraulics of Open Channel Flow. McGraw Hill Book Co. Inc., 1955.

N since direct integration of these terms is impossible. The procedures only to integrate the first four terms in Group A are required since the last four terms fortunately are the special cases of the terms of Group A, when M is equated to zero in writing the program for the computer.

There are limiting conditions to the surface profiles. For example as y approaches y_c, the denominator of Equation (3.1) approaches zero. Thus dy/dx becomes infinite and the curve crosses the critical depth line perpendicular to it. Hence surface profiles in the vicinity of $y = y_c$ are only approximate. Similarly when y approaches y_n, the numerator approaches zero. Thus the curve approaches the normal depth y_n asymptotically. Finally, as y approaches zero, the surface profile approaches the channel bed perpendicularly which is impossible under the assumption concerning the gradually varied flow.

Thus all the intervals mentioned above fall in the domain of singular integrals. A variety of procedures exist for dealing with singular integrals, whether for singular integrals or for infinite range of integration. In such cases, ignoring the singularity may be successful. Under certain circumstances it is enough to use more and more arguments y_i until a satisfactory result is obtained. Series expansions of all or part of the integral, followed by term by term integration, is a popular procedure provided convergence is adequately fast.

As already pointed out earlier, most of the curves are infinite in extent, mathematically speaking, although for practical purposes it is reasonable to consider that a curve has reached it's end when the depth is within 1 to 2 per cent of it's asymptotic limit. Thus in the present analysis singular points in the integral functions are avoided.

Positive Slopes

Drop-Down Curves:

Now consider the integral $\displaystyle\int \frac{U^{N-M}}{1 - U^N}\, dU$, where $y/y_n = U$.

When $U < 1$, profiles of the II group, M_2, M_3, S_3, and C_3 -- drop-down curves can be computed. Expanding the function in power series

$$\frac{U^{N-M}}{1 - U^N} = \left[U^{N-M} \quad 1 + U^N + U^{2N} + U^{3N} + \ldots . + \ldots . \right]$$

$$= U^{N-M} + U^{2N-M} + U^{3N-M} + \ldots \ldots \ldots \ldots$$

$$\int \frac{U^{N-M}}{1 - U^N}\, dU = \frac{U^{N-M+1}}{N-M+1} + \frac{U^{2N-M+1}}{2N-M+1} + \frac{U^{3N-M+1}}{3N-M+1} + \ldots .$$

$$= U^{-M+1} \left[\frac{U^N}{N-M+1} + \frac{U^{2N}}{2N-M+1} + \frac{U^{3N}}{3N-M+1} + \ldots \right] \tag{3.6}$$

$$= U^{-M+1} \quad (S_1) \tag{3.7}$$

Where S_1 is a hypergeometric series of equation (3.6) and it converges for all values of $U<1$, since the practical range of values for N being $2{\cdot}2 \leqslant N \leqslant 5{\cdot}5$ and for M being $3 \leqslant M \leqslant 5$, as given by V.T. Chow (4). Similarly, it can be shown that the integral

$$\int \frac{U^N}{1 - U^N}\, dU = U \cdot \left[\frac{U^N}{N + 1} + \frac{U^{2N}}{2N + 1} + \frac{U^{3N}}{3N + 1} + \ldots . \right] \tag{3.8}$$

$$= U \cdot S_5 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \tag{3.9}$$

Where S_5 is the hypergeometric series in equation (3.8)

Finally the equation (3.2) for the drop-down curve takes the form

$$dx = \frac{yn}{s_o} \cdot \left[\left\{ \frac{y_c}{y_n} \right\}^M \cdot U^{-M+1} \cdot S_1 - U \cdot S_5 \right], \quad U < 1 \quad \ldots\ldots \quad (3.10)$$

Backwater curves:

Now consider the integral $\int \frac{U^{-M}}{1-U^{-N}} dU$, where $y/yn = U$.

When $U > 1$, profiles of the I group, M_1, S_1, S_2, and C_1 -- backwater curves can be computed. Expanding the function in Power Series.

$$\frac{U^{-M}}{1-U^{-N}} = U^{-M} \left[1 + U^{-N} + U^{-2N} + U^{-3N} + \ldots\ldots \right]$$

$$= U^{-M} + U^{-N-M} + U^{-2N-M} + U^{-3N-M} + \ldots\ldots$$

$$\int \frac{U^{-M}}{1-U^{-N}} dU = \frac{U^{-M+1}}{-M+1} + \frac{U^{-N-M+1}}{-N-M+1} + \frac{U^{-2N-M+1}}{-2N-M+1} + \ldots$$

$$= \frac{U^{-M+1}}{-M+1} - U^{-M+1} \cdot \left[\frac{U^{-N}}{N+M-1} + \frac{U^{-2N}}{2N+M-1} + \frac{U^{-3N}}{3N+M-1} + \ldots \right] \quad (3.11)$$

$$= -U^{-M+1} \left[S_2 - \frac{1}{-M+1} \right] \quad \ldots\ldots\ldots \quad (3.12)$$

Where S_2 is the hypergeometric series of equation (3.11) and it converges for all values of $U > 1$, and for the practical range of values of N and M as stated earlier.

Similarly, it can be shown that the integral

$$\int \frac{1}{1-U^{-N}} dU = U - U \cdot \left[\frac{U^{-N}}{N-1} + \frac{U^{-2N}}{2N-1} + \frac{U^{-3N}}{3N-1} + \ldots\ldots \right] \quad (3.13)$$

$$= -U\left[S_6 - 1\right] \qquad (3.14)$$

Where S_6 is the hypergeometric series in equation (3.13).

Finally the equation (3.3) for the backwater curve takes the form

$$dx = \frac{y_n}{s_o}\left[\left\{\frac{y_c}{y_n}\right\}^M \cdot U^{-M+1} \cdot \left(S_2 + \frac{1}{M-1}\right) - U \cdot (S_6 - 1)\right], \; U > 1 \qquad (3.15)$$

Adverse Slopes

Drop-down curves:

Next consider the integral $\int \frac{U^{N-M}}{1 + U^N} dU$. When $U < 1$ profiles of IV group A_2 -- drop-down curves can be computed. Expanding this term in Power Series

$$\frac{U^{N-M}}{1+U^N} = U^{N-M}\left[1 - U^N + U^{2N} - U^{3N} + U^{4N} - \ldots + \ldots\right]$$

$$= U^{N-M} - U^{2N-M} + U^{3N-M} - U^{4N-M} + \ldots - \ldots$$

$$\int \frac{U^{N-M}}{1+U^N} dU = \frac{U^{N-M+1}}{N-M+1} - \frac{U^{2N-M+1}}{2N-M+1} + \frac{U^{3N-M+1}}{3N-M+1} - \ldots + \ldots$$

$$= U^{-M+1}\left[\frac{U^N}{N-M+1} + \frac{U^{2N}}{2N-M+1} + \frac{U^{3N}}{3N-M+1} - \ldots + \ldots\right] \qquad (3.16)$$

$$= U^{-M+1} \cdot S_3 \qquad (3.17)$$

Where S_3 is again an hypergeometric series of equation (3.16) with alternative signs. For all values of $U < 1$, this oscillating series converges. Similarly it can be shown that the integral

$$\int \frac{U^N\, dU}{1+U^N} = U\left[\frac{U^N}{N+1} - \frac{U^{2N}}{2N+1} + \frac{U^{3N}}{3N+1} - \ldots\ldots + \ldots\right] \quad (3.18)$$

$$= U \cdot S_7 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.19)$$

Where S_7 is the hypergeometric series of equation (3.18).

Therefore, the equation (3.4) takes the form

$$dx = \frac{y_n}{s_o}\left[(y_c/y_n)^M \cdot U^{-M+1} \cdot S_3 - U \cdot S_7\right], \quad U < 1 \quad (3.20)$$

Similarly, the equation (3.5) can be written as follows.

$$dx = \frac{y_n}{s_o}\left[\left\{\frac{y_c}{y_n}\right\}^M \cdot U^{-M+1}\left(S_4 + \frac{1}{M-1}\right) - U \cdot (S_8 - 1)\right], \quad U > 1 \quad (3.21)$$

Where $S_4 = \dfrac{U^{-N}}{N+M-1} - \dfrac{U^{-2N}}{2N+M-1} + \dfrac{U^{-3N}}{3N+M-1} - \ldots\ldots + \ldots$

and $S_8 = \dfrac{U^{-N}}{N-1} - \dfrac{U^{-2N}}{2N-1} + \dfrac{U^{-3N}}{3N-1} - \ldots\ldots + \ldots$

From equation (3.21), for $U > 1$, profile of the IV group, A_3 -- backwater curve can be computed. For all values of $U > 1$, the oscillating hypergeometric series S_4 and S_8 converge. Now two questions; whether the above derived series converge fast enough and whether they are useful for practical computational purposes, remain unanswered. These questions will be discussed in detail in the next chapter.

Horizontal Channels

Computations of profiles in III group for horizontal channel do not present any difficulty. The differential equation for this case can be shown (4)

$$\frac{dy}{dx} = S_c \left[\frac{p^{M-N}}{1-p^M} \right], \text{ where } p = y/y_c$$

Integrating and solving for x,

$$x = (y_c/S_c) \left[\frac{p^{N-M+1}}{N-M+1} - \frac{p^{N+1}}{N+1} \right] \quad \ldots\ldots\ldots\ldots (3.22)$$

CHAPTER IV

PRACTICAL EVALUATION OF FUNCTIONS DEVELOPED IN WATER SURFACE PROFILE COMPUTATIONS

This chapter deals with the study of actual methods of numerical computation with the subject of function evaluation. The functions can be expressed in terms of polynomials which arise quite naturally from the truncation of infinite power series. A polynomial approximation of degree n can be obtained by simply truncating the infinite series, and the series must converge for all values of U. The number of terms which must be retained to guarantee an accuracy ϵ will, however, clearly depend upon the value of U. In order to determine the number of terms, U should be restricted to a certain known interval. Thus if U is on the interval $0 \leq U \leq 1$, then themaximum error occurs at the end point $U = 1$. Indeed some series converge so slowly that the amount of work required to evaluate the approximate polynomial becomes prohibitively large. Even more important, the addition of a large number of terms will eventually lead to a serious loss of accuracy, owing to round-off error accumulation. It is seen in the previous chapter that all the integral functions give rise to eight different infinite power series (See Table 1). But it is not possible to sum an infinite number of terms. Therefore, some sort of approximation becomes a necessity. Now the most

TABLE I

DIFFERENT TYPES OF POWER SERIES ARISING IN INTEGRAL FUNCTIONS

$$\left[\frac{U^{N}}{N-M+1} + \frac{U^{2N}}{2N-M+1} + \frac{U^{3N}}{3N-M+1} + \ldots + \ldots \right] = S_1$$

$$\left[\frac{U^{-N}}{N+M-1} + \frac{U^{-2N}}{2N+M-1} + \frac{U^{-3N}}{3N+M-1} + \ldots + \ldots \right] = S_2$$

$$\left[\frac{U^{N}}{N-M+1} - \frac{U^{2N}}{2N-M+1} + \frac{U^{3N}}{3N-M+1} - \ldots + \ldots \right] = S_3$$

$$\left[\frac{U^{-N}}{N+M-1} - \frac{U^{-2N}}{2N+M-1} + \frac{U^{-3N}}{3N+M-1} - \ldots + \ldots \right] = S_4$$

$$\left[\frac{U^{N}}{N+1} + \frac{U^{2N}}{2N+1} + \frac{U^{3N}}{3N+1} + \ldots + \ldots \right] = S_5$$

$$\left[\frac{U^{-N}}{N-1} + \frac{U^{-2N}}{2N-1} + \frac{U^{-3N}}{3N-1} + \ldots + \ldots \right] = S_6$$

$$\left[\frac{U^{N}}{N+1} - \frac{U^{2N}}{2N+1} + \frac{U^{3N}}{3N+1} - \ldots + \ldots \right] = S_7$$

TABLE I (cont'd)

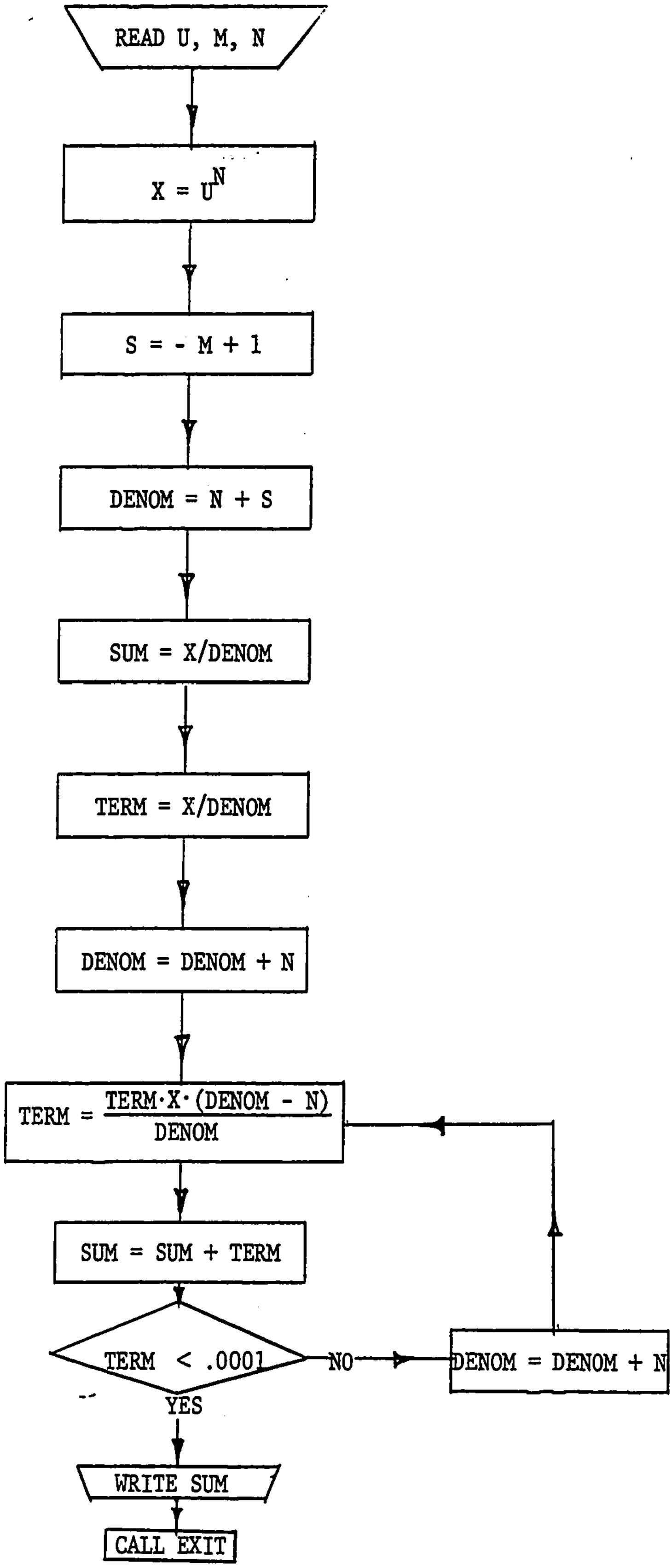
$$\left[\frac{U^{-N}}{N-1} - \frac{U^{-2N}}{2N-1} + \frac{U^{-3N}}{3N-1} - \ . \ . \ . \ + \ . \ . \ . \right] \qquad = S_8$$

important question is what constitutes a good approximation. This depends on the rapidity of convergence of the series. The hypergeometric series developed in the previous chapter converges very fast for all values of U in the range $1.1 < U < \infty$ and in the range $1.01 < U < 1.1$, the rate of convergence is very slow, the range $1 < U < 1.01$ being of no practical importance. Actual computations with numerical values of practical range have been done on a digital computer to know the nature of convergence in direct summation of the series. The program and the flow chart for the summation of this series are shown in Page 32. The series S_6 and S_8 have been tested by evaluating term by term with the values, U = 1.01, N = 2.2 and M = 0.0, which are the minimum values possible as stated earlier and also give rise to slowest possible converging series. Similarly, the series S_1 and S_3 with numerical values, U = 0.99, N = 2.2, M = 5, the series S_5 and S_7 with values U = 1.01, N = 2.2, and M = 3.0, have been tested for convergence. As expected, they are rather slow in this range, (see Table II). However, the convergence is guaranteed. At modern computing speeds, especially with electronic machines like the I.B.M. 360, the slow convergence may not be a good reason to rule out a computing algorithm. The accuracy expected in open channel flow problems is generally of the order of 0.001, therefore the question of slow convergence does not appear to be of great concern. For the series S_1, S_2, S_5, and S_6 any other method of approximation other than a direct summation up to N terms with the tolerable truncation error, would involve too much additional work and is not advisable. However, for the other series, S_3, S_4, S_7, and S_8, good approximations are available, and they will be discussed later.

TABLE II. EVALUATION OF SERIES SHOWING THE NUMBER OF TERMS REQUIRED FOR THE GIVEN ACCURACY

Series	U	N	M	Accuracy	Value	No. of Terms Required
S_1	0.99	2.2	5.0	.001	3.373	81
				.0001	3.4001	156
S_2	1.01	2.2	3.0	.001	1.313	76
				.0001	1.3469	156
S_3	0.99	2.2	5.0	.001	-2.691	80
				.0001	-2.6918	156
S_4	1.01	2.2	3.0	.001	.1457	80
				.0001	.1452	156
S_5	0.99	2.2	0.0	.001	1.461	76
				.0001	1.494	156
S_6	1.01	2.2	0.0	.001	2.215	78
				.0001	2.2469	156
S_7	0.99	2.2	0.0	.001	.199	80
				.0001	.1993	156
S_8	1.01	2.2	0.0	.001	..633	80
				.0001	.6337	156

FLOW CHART TO COMPUTE S_1 OR S_5 SERIES



FORTRAN IV SUB-PROGRAM FOR COMPUTING S_1 OR S_5

(IBM 1130)

```
//bJOBbT

//bFOR

*IOCS (CARD, 1132 PRINTER)

*EXTENDED PRECISION

*ONE WORD INTEGERS

*LIST SOURCE PROGRAM

C SUBPROGRAM FOR COMPUTATIONS OF SERIES S1 OR S5.

  READ (2,99) U, HM, HN

99 FORMAT (3F 10.4)

X = U* * HN

S = - HM + 1.

DENOM = HN + S

SUM = X/DENOM

TERM = X/DENOM

DENOM = DENOM + HN

5  TERM = TERM * X * (DENOM - HN)/ DENOM

SUM = SUM + TERM

IF (ABS(TERM) - .0001) 16, 16, 12

12 DENOM = DENOM + HN

   GO TO 5

16 WRITE (3,100) SUM

100 FORMAT (E 15.8)

CALL EXIT

END
```

Many techniques exist for finding approximating polynomials which reduce the amount of work and guarantee a specified accuracy. Perhaps the best known of these is Chebyshev economization. The truncated series have the property that the maximum error occurs at the end points of the interval of interest. The objective is to find a polynomial approximation to a given function which (1) is of lower degree than the truncated power series, (2) spreads the error evenly over the whole interval, and (3) provides the same required accuracy. Chebyshev polynomials are only useful for a particular range of value, $-1 \leq U \leq 1$. Any polynomial of degree n is uniquely expressible as a linear combination of the Chebyshev polynomials. The first ten of the Chebyshev polynomials are listed in Column A of Table VII in the Appendix A.

Another simply but very useful tool in Chebyshev approach for the present analysis is what is known as "Economization of Power Series" mainly due to Lanczos (8). If the truncated series approximates a given function

$$f(U) = a_0 + a_1 U + a_2 U^2 + \text{-----} + a_n U^n \tag{4.1}$$

in the interval $0 \leq U \leq 1$, the error tends to be large at the ends of the interval and small in the middle. Using Table VII given in Appendix A, the truncated series is converted to an expansion in Chebyshev polynomials.

$$f(U) = b_0 + b_1 T_1(U) + b_2 T_2(U) + \text{----} + b_n T_n(U) \tag{4.2}$$

This is an expansion in orthogonal polynomials. In fact, the expansion of a function into Chebyshev polynomials is mere reinterpretation of the expansion of an even function into a Fourier cosine series. This fundamental relation, which translates the outstanding properties of the Fourier Series into the realm of power expansion, is the most important property of Chebyshev polynomials. For many functions the expansion in Chebyshev polynomials converges more rapidly than the expansion in any other set of orthogonal polynomials. Thus it is expected to find b_k of Equation (4.2) becoming small very rapidly as compared with A_k of Equation (4.1). The Chebyshev expansion may be converted back to a polynomial using Table VII (Appendix A). In general, it is expected that if a power series consisting of many terms, is converted to a Chebyshev expansion, then a much lower order polynomial approximation can be obtained by dropping many of the later Chebyshev terms without greatly increasing the error over that of the error due to originally taking a finite number of terms in the power series. For many functions, especially those with slowly converging power series, the telescoping effect can be quite dramatic. Hastings (6), has shown that some polynomial derived from the Chebyshev approach, containing only five or six terms approximate certain functions like log $(1 + U)$ to an accuracy of .0000015 for all U on the interval $0 \leq U \leq 1$. The Taylor Series expansion for some of those functions converge so slowly that many hundreds of terms would be required for the same accuracy. The procedure for economization of power series can be summarized as follows.

Step 1. Expand $f(U)$ into a Taylor Series valid on interval $[0,1]$. Truncate this series to obtain a polynomial

$$P_n(U) = a_0 + a_1U + a_2U^2 + \ . \ . \ . + a_nU^n$$

which approximates $f(U)$ to within error ε for all U in $[0,1]$.

Step 2. Expand $P_n(U)$ into a Chebyshev Series

$$P_n(U) = b_0 + b_1\ T_1(U) + \ . \ . \ . + b_n\ T_n(U),$$

making use of the Table VII (Appendix A).

Step 3. Retain the first k term in this series,

$$S_k(U) = b_0 + b_1\ T_1(U) + \ . \ . \ . + b_k\ T_k(U)$$

choosing k so that the maximum error given by

$$f(U) - S_k(U) \leq \varepsilon + b_{k+1} + \ . \ . \ . + b_n$$

is acceptable.

Step 4. Replace T_J ($J = 0, 1, \ . \ . \ ., k$) by its polynomial form using Table VIII (Appendix A), and rearrange to obtain the economized polynomial approximation of degree k in standard form,

$$f(U) \approx b_0 + b_1U + b_2U^2 + \ . \ . \ . + b_kU^k$$

Now consider the S_3, S_4, S_7, and S_8 of IV Group which arise in the case of adverse slopes. Since these are oscillating series, it is possible to sum the terms by less direct methods. Chebyshev polynomial approximation has been found to have quite a dramatic effect on the evaluation of these series. Letting $U^N = x$ and $-M+1 = S$, and $0 < x < 1$

$$S_3 = \frac{x}{N+S} - \frac{x^2}{2N+S} + \frac{x^3}{3N+S} - \frac{x^4}{4N+S} + \ . \ . \ . \ - \ . \ . \ . + \frac{x^9}{9N+S} - \ . \ . \ .$$

using the Table given in Appendix A, this truncated series is converted to an expansion in Chebyshev polynomials.

$$S_3 = \frac{(T_0+T_1)}{2(N+S)} - \frac{(3T_0+4T_1+T_2)}{8(2N+S)} + \frac{(10T_0+15T_1+6T_2+T_3)}{32(3N+S)} - \ . \ . \ .$$

The Chebyshev expansion may be converted back to a polynomial, using Table VIII, Appendix A. The detail procedures for finding Chebyshev polynomial coefficients are found elsewhere.* After using Lanczos' method of Economization of Power Series, it can be shown that the following polynomial of only 8 terms can be obtained with the same accuracy that can be achieved by directly summing many hundreds of terms of the original series.

$$S_3 \approx \frac{.9999964239x}{(N+S)} - \frac{.9997482476x^2}{(2N+S)} + \frac{.9953970774x^3}{(3N+S)} - \frac{.9629352336x^4}{(4N+S)} + \frac{.8382703555x^5}{(5N+S)} - \frac{.5719763382x^6}{(6N+S)} + \frac{.2526194559x^7}{(7N+S)} - \frac{.0516283536x^8}{(8N+S)}, \text{ where } S = -M + 1, \quad (4.3)$$

Exactly the same Chebyshev polynomial coefficients hold good for other series S_4, S_7, and S_8 of this group. It is amazing to see that the evaluation of only 8 terms will give accuracy far superior to direct summation of many hundreds of terms of the original series. Therefore this polynomial has been uniquely used in the general computer program for computations of water surface profiles with adverse slopes.

---

* M. A. Snyder. Chebyshev Methods in Numerical Approximation. Prentice-Hall Inc. Englewood Cliffs, N.J., 1966.

*C. Hastings. Approximations for Digital Computers. A research study by the Rand Corporation. Princeton University Press, Princeton, N.J., 1966.

TABLE III. COMPARISON BETWEEN DIRECT SUMMATION AND CHEBYSHEV POLYNOMIAL APPROXIMATION

					No. of Terms Required	
SERIES	U	N	M	Value	Chebyshev Polynomials	Direct Summation
S_3	.99	2.2	5.0	-2.6914408	8	180
S_4	1.01	2.2	3.0	.14516469	8	180
S_7	.99	2.2	0.0	.19929927	8	172
S_8	1.01	2.2	0.0	.63365053	8	172

CHAPTER V

COMPUTATIONS OF CRITICAL DEPTH AND NORMAL DEPTH IN TRAPEZOIDAL CHANNELS

Computations of Critical Depth: Critical flow is an important problem in the hydraulics of open channels. The problem of computing the critical depth is encountered frequently, so that it is advantageous for hand computations, to have dimensionless curves and relative tables as computational aids. The direct integration method of solving a non-uniform flow equation requires the determination of both the critical depth and the normal depth for each particular discharge. For trapezoidal cross sections, this is rather difficult. When the discharge in a trapezoidal channel is given, the Equation (5.1) for critical depth would yeild a sixth degree equation, the solution of which would involve a cumbersome trial and error procedure. To overcome this difficulty, various graphical methods have been suggested by a number of investigators, N. N. Pavlovskij,[1] S. Kolupaila,[2] N. Rajarathnam and A. Thiru-[3]

---

[1] N. N. Pavlovskij, in Kratkij Gridrauliceskij Spravocnik Tech-gosizdat. Brief Consulting Book of Hydraulics. Leningrad, 1940 (Russian).

[2] S. Kolupaila, Universal Diagram Gives Critical Depth in Trapezoidal Channels. Civil Engr. Vol. 12, Dec. 1950.

[3] N. Rajarathnam and A. Thiruvengadam, Critical Depth in Open Channels. Journal of India Institute of Engineers. Vol. 41., No. 8, Part I. April 1961.

vengadam, and R. M. Advani[4]. Critical depth may be defined as the depth at which a certain discharge Q flows in a channel of a given cross section with a minimum content of specific energy. It can be shown that for critical flow, there exists the relationship

$$\alpha Q^2/g = A_c^{\,3}/T \tag{5.1}$$

For a trapezoidal channel,

$$\alpha Q^2/g = \left[(b+zy_c)y_c\right]^3/(b+2zy_c)$$

This reduces to a sixth degree equation

$$z^3y_c^{\,6} + 3bz^2y_c^{\,5} + 3b^2zy_c^{\,4} + b^3y_c^{\,3} - 2C_ozy_c - bC_o = 0 \tag{5.2}$$

where $C_o = \alpha Q^2/g$ = constant for the given discharge. It can be seen that a direct solution for critical depth y_c is not easy where large numbers of computations are to be performed for various discharges in different trapezoidal channels. The first successful attempt in this direction was made by S. Koulupaila, who introduced a diagram that is actually dimensionless, and is based on a three parameter equation

$$x = b/y_c \qquad \alpha Q^2/gb^5 = N = (x+z)^3/x^5(x+2z), \tag{5.3}$$

which is claimed to be universal in application. Curves can be plotted as N versus x for different values of z. Knowing N and z from the given conditions of flow, x can be obtained from the appropriate curve; hence y_c is calculated.

Equation (5.2) is modified further, to yield a more useful two

[4]R. M. Advani, Critical Depth in Trapezoidal Channels. A.S.C.E. Proceedings. J. of Hydraulics Division. Vol. 88. No Hy3, May 1962.

parameter equation by later investigators.

Rajarathnam and Thiruvengadam

$$\frac{Qz^{3/2}}{(\frac{b}{2})^{5/2}} = 4 \cdot \left[\frac{\left\{(y_c{}^2/(b/2z)^2) + (2y_c/(b/2z))\right\}^{3/2}}{\left\{1 + y_c/(b/2z)\right\}^{1/2}} \right] \tag{5.4}$$

Advani

$$\frac{\alpha Q^2}{gb^5} = N_1 = \frac{\left((b/ny_c) + 1\right)^3}{(\frac{b}{ny_c})^5 . ((\frac{b}{ny_c}) + 2)} \tag{5.5}$$

Dimensionless curves have been prepared using the above equations to calculate y_c.

Proposed procedure: It appears that all of the above methods are of negligible value for digital computers, and the problem of finding the critical depth seems to be ridiculously simple on computers, provided the critical depth Equation (5.1) is reduced to suitable form. It can be done as follows:

$$Q^2/g = A^3/T = (b+zy_c)^3 y_c{}^3 \Big/ (b+2zy_c)$$

with

$$C_o = Q^2/g = \text{constant}$$

$$y_c{}^3 = C_o \; (b+2zy_c)/(b+zy_c)^3$$

$$y_c = C_o{}^{1/3} \cdot (b+2zy_c)^{1/3}/(b+zy_c) \tag{5.6}$$

$$= f(y_c) \tag{5.7}$$

The procedure to solve for y_c involves

(1) Finding an approximate root

(2) Refining the approximation to the prescribed degree of

accuracy.

The first approximation, or initial guess is known from physical considerations. Refining the approximation is done by the method of successive approximation or iterative technique.

Let $(y_c)_1 = f\ (y_c)_0$

Next approximation $(y_c)_2 = f\ (y_c)_1$

- - - - - - - - - - - -

N^{th} approximation $(y_c)_N = f\ (y_c)_{N-1}$

It is seen from Table IV that $(y_c)_N$ converges to a solution of Equation (5.7) very fast as N increases for different values of variables in a practical range. The initial approximation is governed by a physical consideration based on optimum critical velocity that may occur in common open channel flow problems (See computer program).

Computation of Normal Depth: Computation of normal depth in a trapezoidal channel presents more difficulty than that of finding critical depth. For uniform flow in trapezoidal channel

$$Q = (b+zy_n)y_n \cdot \frac{1.486}{n} \cdot \left\{(b+zy_n)y_n \Big/ (b+2\,y_n\sqrt{1+z^2})\right\}^{2/3} \cdot S_o^{1/2} \qquad (5.8)$$

This equation reduces to a 10th degree equation,

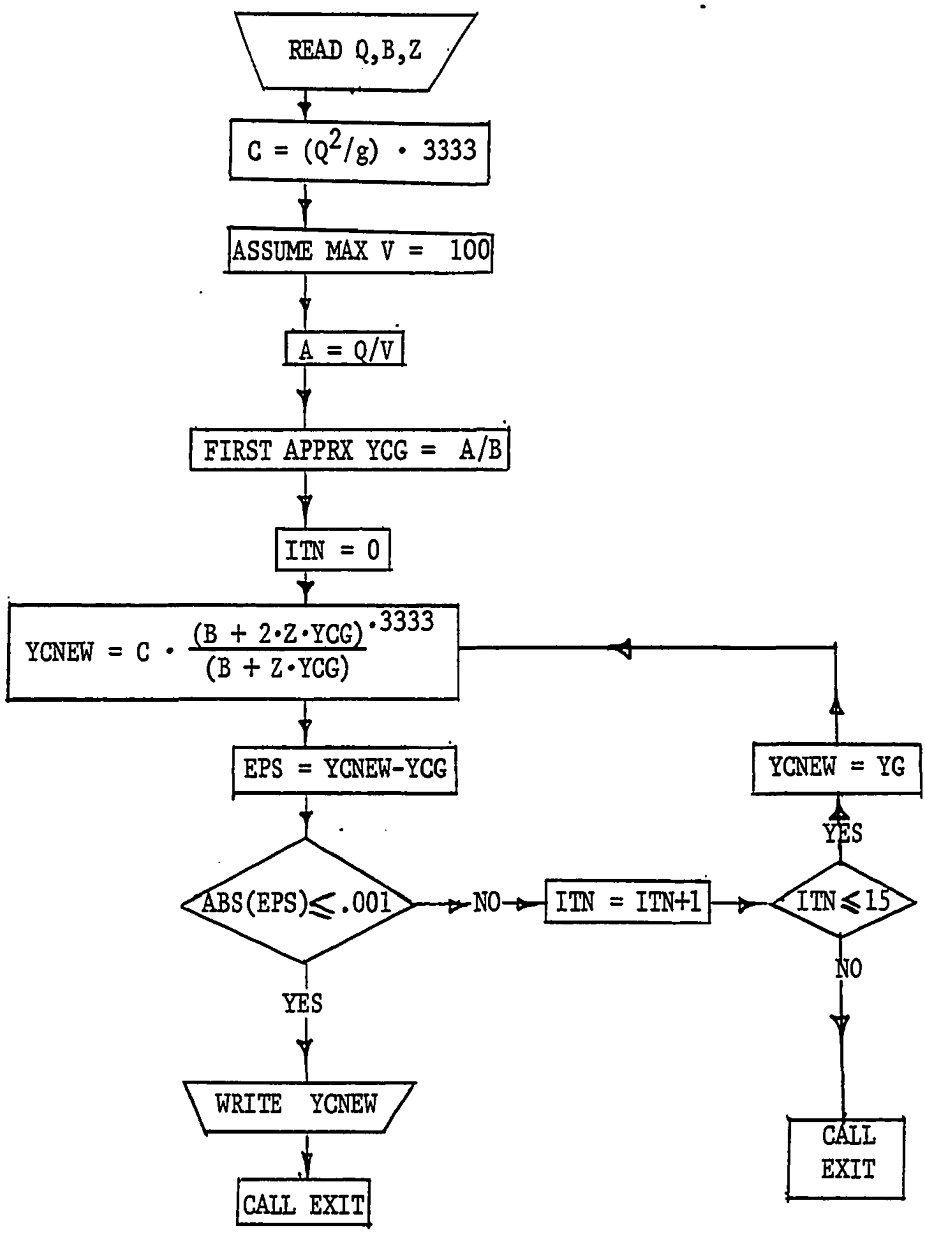
$$z^5y_n^{10} + 5bz^4y_n^9 + 10b^2z^3y_n^8 + 10b^3z^2y_n^7 + 5zb^4y_n^6$$

$$+ b^5y_n^5 - c_2c_1^2y_n^2 - 2c_1c_2by_n - c_2b^2 = 0 \qquad (5.9)$$

where

$c_1 = 2\sqrt{1+z^2}$ and $c_2 = Q\cdot\left(n/1.486\right)\sqrt{S_0}$

FLOW CHART FOR COMPUTING CRITICAL DEPTH



FORTRAN IV SUB PROGRAM FOR CRITICAL DEPTH

(IBM 1130)

```
//bJOBbT

//bFOR

*IOCS(CARD,1132 PRINTER)

*EXTENDED  PRECISION

*ONE WORD INTEGERS

C SUB PROGRAM FOR CRITICAL DEPTH INTRAPL CHANNELS

C Q = DISCHARGE CFS, B = BOTTOM WIDTH, Z = SIDE SLOPE

  READ (2,99) Q, B, Z

99 FORMAT (3F10.4)

  C = (Q**.2/32) ** .3333

C ASSUME MAXIMUM VELOCITY V = 100FT PER SEC

  A = Q
      V

C YCG = FIRST APPROXIMATION FOR CRITICAL DEPTH

C YCNEW = CRITICAL DEPTH

  YCG = A/B

  ITN = 0

5 YCNEW = C* (B + 2.* Z * YCG) ** .3333/(B + Z* YCG)

  EPS = YC - YCG

  IF (ABS (EPS) - .001) 42, 42, 43

43 ITN = ITN + 1

  IF (ITN - 15) 16, 16, 12

16 WRITE (3,100) YCNEW

100 FORMAT ( F 15.7 )
```

```
YCG = YNEW
  GO TO  5
12 CALL EXIT
42 WRITE (3,100) YCNEW
  CALL EXIT
  END
```

TABLE IV. COMPUTATIONS OF CRITICAL DEPTH BY ITERATIVE METHOD

Q	b	z	y_c	i = No of iterations
15	4.0	.25	.747	3
47	3.0	.5	1.773	5
160	8.0	.75	2.157	4
240	20.0	.5	1.624	3
300	12.0	1.73	2.381	5
400	20.0	3.0	2.075	5
450	10.0	2.0	3.192	7
500	18.0	1.0	2.733	4
600	12.0	2.0	3.487	6
800	15.0	1.5	3.884	6
900	20.0	1.5	3.613	5
1000	20.0	1.5	3.852	4
2000	15.0	2.0	6.211	8
3000	20.0	2.5	6.70	8
4000	50.0	1.5	5.505	5
6220	100.0	1.0	4.851	4
10000	50.0	2.5	9.153	6
50000	300.0	2.5	9.263	4
100000	500.0	4.0	10.437	4
150000	500.0	4.0	13.555	4

the direct solution for normal depth y_n for the given discharge from this equation is formidable, though not impossible. At present, several methods are available, trial and error approach, graphical methods and the use of hydraulic tables being more popular. But again, none of them are suitable for digital computers. Exactly similar method of iterative technique as that used for finding critical depth is suggested for finding normal depth. The procedure is as follows.

From equation of uniform flow (5.8)

$$c_2 = \left[(b+zy_n)^{5/3}/(b+c_1y_n)^{2/3}\right] \cdot y_n^{5/3}$$

where

$$c_2 = Q \cdot \left(n/1.486\right) \cdot S_o^{1/2} \text{ , and } c_1 = 2\sqrt{1+z^2}$$

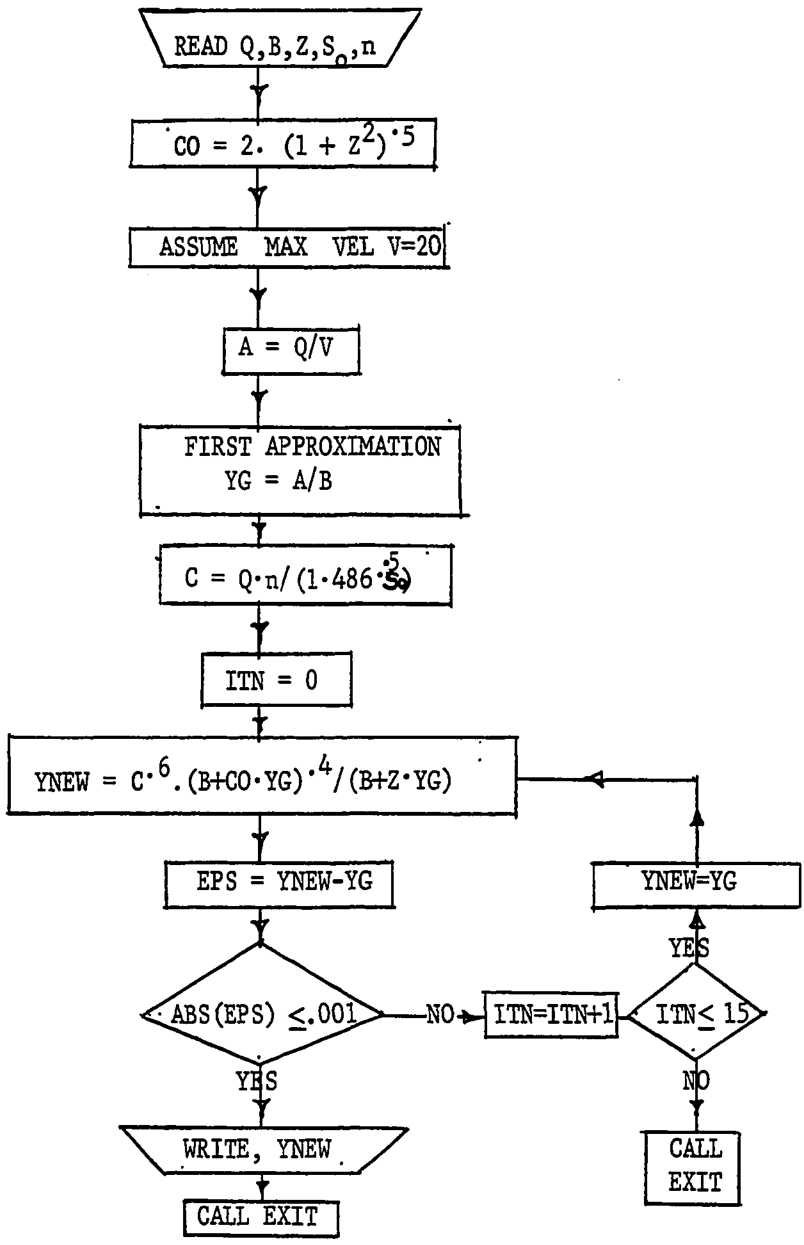
$$y_n = c_2^{3/5}(b+c_1y_n)^{2/5}/(b+zy_n)$$

$$= c_2^{0.6}(b+c_1y_n)^{0.4}/(b+zy_n) \qquad (5.10)$$

$$y_n = f(y_n) \qquad (5.11)$$

With a practically reasonable first approximation for y_n, it is possible to solve Equation (5.11) with the required degree of accuracy by the iterative technique. (See Table V.) The above mentioned procedures for the evaluation of critical depth and normal depth are also simple for hand computations. A uniform velocity of 20 ft/sec and a critical velocity of 100 ft/sec have been found reasonable assumptions which give rise to minimum values of y_n and y_c as first approximation and then

FLOW CHART FOR COMPUTING NORMAL DEPTH



FORTRAN SUB PROGRAM FOR NORMAL DEPTH

(IBM 1130)

```
//bJOBbT

//bFOR

* IOCS (CARD,1132 PRINTER)

* EXTENDED PRECISION

* ONE WORD INTEGERS

C SUB PROGRAM FOR NORMAL DEPTH IN TRAPESOIDAL CHANNELS

C Q = DISCHARGE.CFS, B = BOTTOM WIDTH, Z = SIDE SLOPE

C S = BED SLOPE, AN = MANNING'S FRICTION COEFFT.

  READ (2,99) Q,B, Z, So, AN

99 FORMAT (5F 10.4)

  CO = 2. * (1. + Z * * 2.) * * .5

C ASSUME MAXIMUM VELOCITY V = 20 FT PER SEC

  A = Q/V

C YG = FIRST APPROXIMATION FOR NORMAL DEPTH

  YG = A/B

  C = Q * AN/ (1.486* (So* * .5))

  ITN = 0

C YNEW = NORMAL DEPTH

5 YNEW (( C ** .6) * (B + CO * YG) * * .4) / (B + Z * YG)

  EPS = YNEW - YG

  IF (ABS ( EPS ) - .001) 42, 42, 43

43 ITN = ITN + 1

  IF (ITN - 15) 16, 16, 12
```

```
16  WRITE (3,100) YNEW
100 FORMAT ( F 15.8)
  YG = YNEW
  GO TO 5
12 CALL EXIT
42 WRITE (3,100) YNEW
   CALL EXIT
   END
```

TABLE V. COMPUTATIONS OF NORMAL DEPTH BY ITERATIVE METHOD

Q cfs-	b-ft	z	S_o	n	y_n	i No. of iterations
15	4.0	.25	.004	.016	.838	4
47	3.0	.5	.005	.025	2.372	3
160	8.0	.75	.0005	.017	4.057	4
240	20.0	.5	.0002	.015	3.818	4
300	12.0	1.73	.002	.015	2.633	4
400	20.0	3.0	.00085	.015	2.888	5
450	10.0	2.0	.0016	.015	4.727	7
500	18.0	1.0	.0025	.025	2.468	3
600	12.0	2.0	.003	.012	4.809	6
800	15.0	1.5	.0003	.03	9.773	7
900	20.0	1.5	.00015	.015	7.831	6
1000	20.0	1.5	.0001	.025	11.933	7
2000	15.0	2.0	.001	.04	12.066	9
3000	20.0	2.5	.001	.025	10.294	8
4000	50.0	1.5	.0001	.012	11.541	5
6220	100.0	1.0	.0001	.022	15.108	3
10000	50.0	2.5	.005	.04	11.756	5
50000	300.0	2.5	.0005	.045	24.803	5
100000	500.0	4.0	.001	.045	22.505	5
150000	500.0	4.0	.0002	.012	21.105	4

converge to a real value in either case. Even lesser values of first approximation should also work since the uniform velocity never exceeds 10-12 ft/sec in manmade channels or in natural streams. Newton-Raphson Method (26) can also be applied to the Equations (5.6) and (5.10) but the only disadvantage is that the evaluation of the first derivative of these equations which are essential in this method, is rather cumbersome. However, in most of the cases, Newton-Raphson Method may be superior to iterative method which has been used here fro a critical depth and normal depth computations.

CHAPTER VI

STEP METHODS AND ERROR ANALYSIS

The Step Method is perhaps more widely used than the others in computing surface profiles of flow in uniform channels. It is less mathematically involved than the method by direct integration which requires different sets of tables for varied flow functions of the existing methods. For hand computations, the use of tables is not very satisfactory in computing distance between the sections close to each other since a slight error due to interpolation and round off may introduce a wide discrepancy in results. The Step Method on the other hand, increases in precision as sections become closer and when only a few surface curves have to be computed and elevations are needed all along the channel, Step Method will be most convenient.

Direct Step Method In the direct Step Method, solution involves integration of the varied flow equation by steps, beginning with known conditions at the control section, the size of the steps determining the accuracy of the results. Energy equation for the non-uniform flow between Section 1 and 2 (See Figure 2)

$$V_1^2/2g + y_1 + z_1 = V_2^2/2g + y_2 + z_2 + h_f \tag{6.1}$$

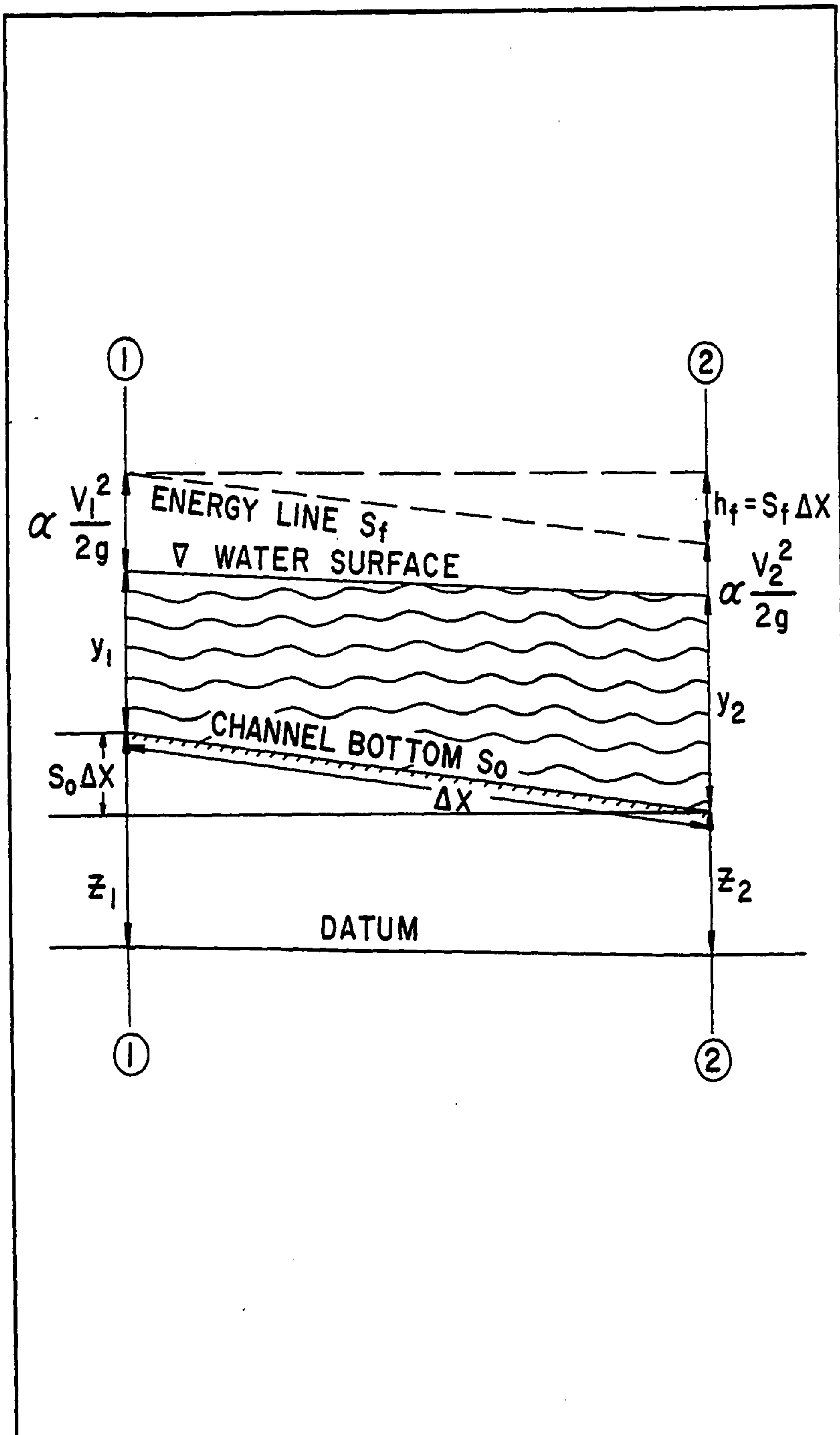


Fig. 2 Channel Reach for the Direct Step Method

$$(V_2^2/2g + y_2) - (V_1^2/2g + y_1) = (Z_1 - Z_2) - h_f$$

$$E_2 - E_1 = S_o \ \Delta x - \bar{S}_f \ \Delta x$$

$$\Delta x = \Delta E/(S_o - \bar{S}_f) \qquad (6.2)$$

where $\bar{S}_f = (S_{f\,1} + S_{f\,2})/\ 2$, average friction slope between any two reaches. In the equation (6.2) the magnitude of Δx is determined for the corresponding difference in depth Δy . Evidently the slope of the energy line then represents a mean over the distance Δx. So the assumption in the direct Step Method is that the slope of the energy grade line for this distance Δx is equal to the average of the slope of the energy grade lines, corresponding to uniform flow at the two sections $\bar{S}_f = (S_{f\,1} + Sf_{\,2})/\ 2$, where S_{f1} and S_{f2} are found from Manning's equation. This change in depth from the control section to the limit which the curve approaches is first divided into an appropriate number of increments, thereby establishing a series of vertical sections along the curve for which the depths of flow are known quantities. The locations of these sections is yet unknown. This has been explored by the method of error analysis in this present study. For the given discharge it is possible to compute for each sections, the cross section area, the velocity, the velocity head and the specific energy. Then ΔE represents the change in specific energy and the quantity $\bar{Sf}$, the mean friction slope may be computed for values of y_1 and y_2, and evaluation of Δx is at once possible. This simple process is applicable to any type of surface curve and natural water courses as well as artificial channels. It is evident, however, that the ultimate accuracy of the solution will depend upon the factors 1) the extent to which assumptions leading to equation (6.2) are justified. 2) the number of steps adopted for the integration; the smaller the depth increments, the more accurate are the results because of the assumptions made regarding energy loss. The depth increments should be smaller as the profile approaches uniform depth. If the length of the varied flow profile between sections where the depths y_o and y_k is to be calculated, accuracy whould be better if the depth increments, for example, $(y_o - y_k)/100$ were used instead of $(y_o - y_k)/10$; So the calculation effort would be many times greater than 10 intervals were used. This becomes a very big problem, especially for hand computations when curves of considerable length say, 20000 or 40000 feet have to be computed. Which is the most economical way to obtain maximum accuracy with minimumeffort? Table (VI) shows some numerical examples which give some idea about the magnitude of error and it's behavior. This study deals with the problem of economization of number of intervals in surface profile computations.

Direct Step Method to compute the length of profile is a particular numerical method used to integrate the differential equation (6.2). Figures (3 & 4) represent the graphical representation of the problem of integrating the equation approximately. This is based on the assumption that friction slope line is a straight line and S_f is calculated as the arithmetic mean of the two slopes at points 1 and 2. Therefore, the error arises in each step due to this wrong assumption and finally it

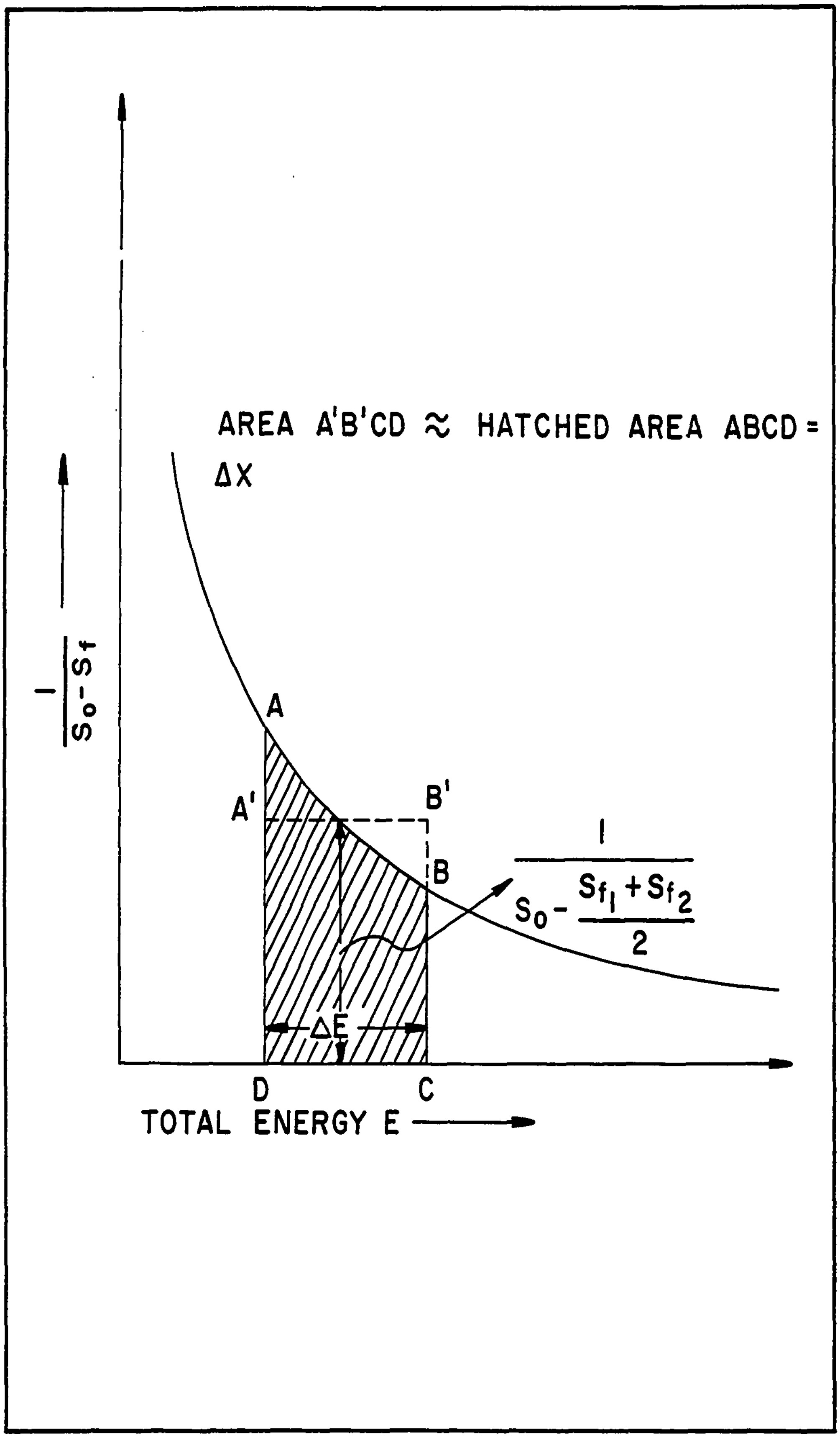


Fig. 3 Graphical Representation of the Equation $\Delta x = \frac{\Delta E}{S_o - \bar{S}_f}$ for Backwater Curve, (Existing Method)

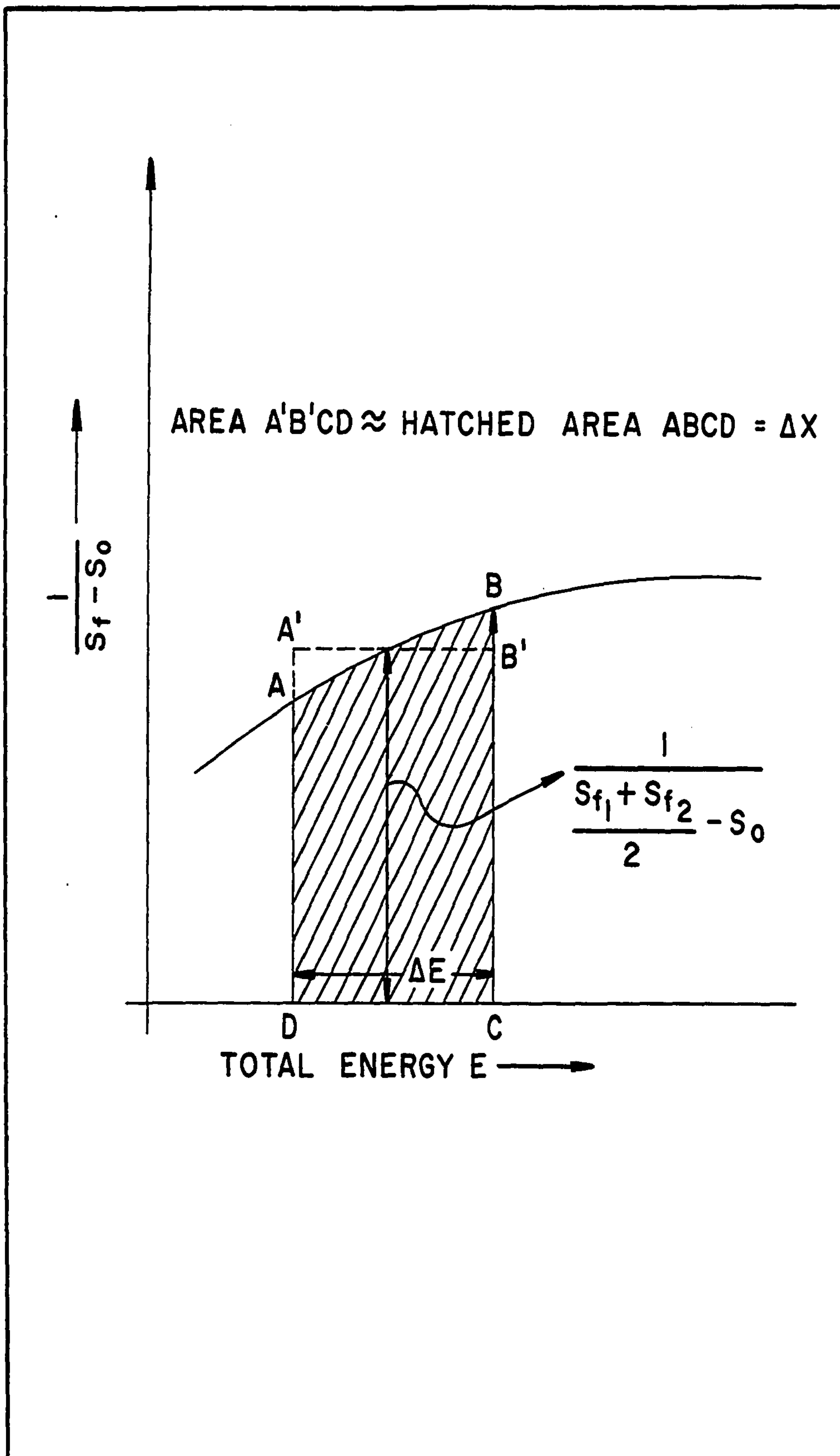


Fig. 4 Graphical Representation of the Equation $\Delta x = \dfrac{\Delta E}{\bar{S}_f - S_o}$ for Drop Down Curve. (Existing Method)

accumulates to a considerable extent.

Error Analysis: It can be seen that the area under the curve A B C D may be replaced by an equivalent rectangular area $A^1B^1C\ D$, the height of which being $1/\ (S_o - \bar{S}_f)$ in the Figure (3) , and $1/\ (\bar{S}_f - S_o)$ in Figure (4). The distance along the channel between the two sections is therefore

$$\Delta x = \int dE/\ (S_o - \bar{S}_f) = \text{Area ABCD} = \Delta E/\ (S_o - \bar{S}_f) \quad \text{(Fig. 3)}$$

$$\Delta x = \int dE/\ (\bar{S}_f - S_o) = \text{Area ABCD} = \Delta E/\ (\bar{S}_f - S_o) \quad \text{(Fig. 4)}$$

in which $\bar{S}_f$ represents the arithmetic mean of friction slope. If the value of S_f between two given depths of flow is known, the equation (6.2) may be used to find the distance along the channel between these depths. It has been said earlier that the mean friction slope however, cannot be determined conveniently, and some assumptions are made in it's evaluation. It is necessary to divide the entire channel into several reaches, each of such length that the mean friction slope may be assumed equal to the arithmetic mean of the friction slopes at the beginning and end of reach. Under this assumption, equation (6.2) becomes

$$\Delta x = \Delta E.\ 1/\ [\ S_o - (S_{f1} + S_{f2})/2] = \text{rectangle } A^1B^1C\ D \quad \text{(Fig. 3)}$$

$$\Delta x = \Delta E.\ 1/\ [(S_{f1} + S_{f2})/2 - S_o] = \text{rectangle } A^1B^1C\ D \quad \text{(Fig. 4)}$$

But in the present study, the area ABCD is taken as

$$\Delta x = \frac{\Delta E_1 + \Delta E_2}{2} \cdot \frac{1}{3} \cdot \left[\frac{1}{S_o - Sf_1} + \frac{4}{S_o - Sf_2} + \frac{1}{S_o - Sf_3} \right] \tag{6.3}$$

(Fig. 5)

$$\Delta x = \frac{\Delta E_1 + \Delta E_2}{2} \cdot \frac{1}{3} \cdot \left[\frac{1}{Sf_1 - S_o} + \frac{4}{Sf_2 - S_o} + \frac{1}{Sf_3 - S_o} \right] \tag{6.4}$$

(Fig. 6)

This is evidently the famous Simpson's rule of Numerical Integration, if $\Delta E_1 = \Delta E_2$. By introducing this concept, the existing method is suitably modified and computations by both the methods are done and the results are compared (Table VI). The advantage of this modification is to explore the possibility of error analysis as it becomes formidable in the former Average Friction Slope Method. Thus, it can be seen that although this method is known as Arithmetic Direct Step Method in literature this can lead to one of the most widely known and used techniques in Numerical Integration, Simpson's Parabolic Formula, and this is certainly not more complex than that being used at present.

Estimation of Error Bound in Direct Step Method

The integral of a function $f(y)$ may be obtained by integrating it's Taylor Series expansion term by term and approximate formulas, together with the corresponding truncated errors may then be determined from the integrated series. The truncated error committed in Simpson's rule can be shown (16) equal to

$$R_n \leq [(b' - a')^5 / 180\ (2N)^4] \cdot M_4, a' < y < b' \tag{6.5}$$

where, $2N$ = number of intervals and M_4 is the maximum modulus of the fourth derivative of the integrand over the interval $[a', b']$. It can

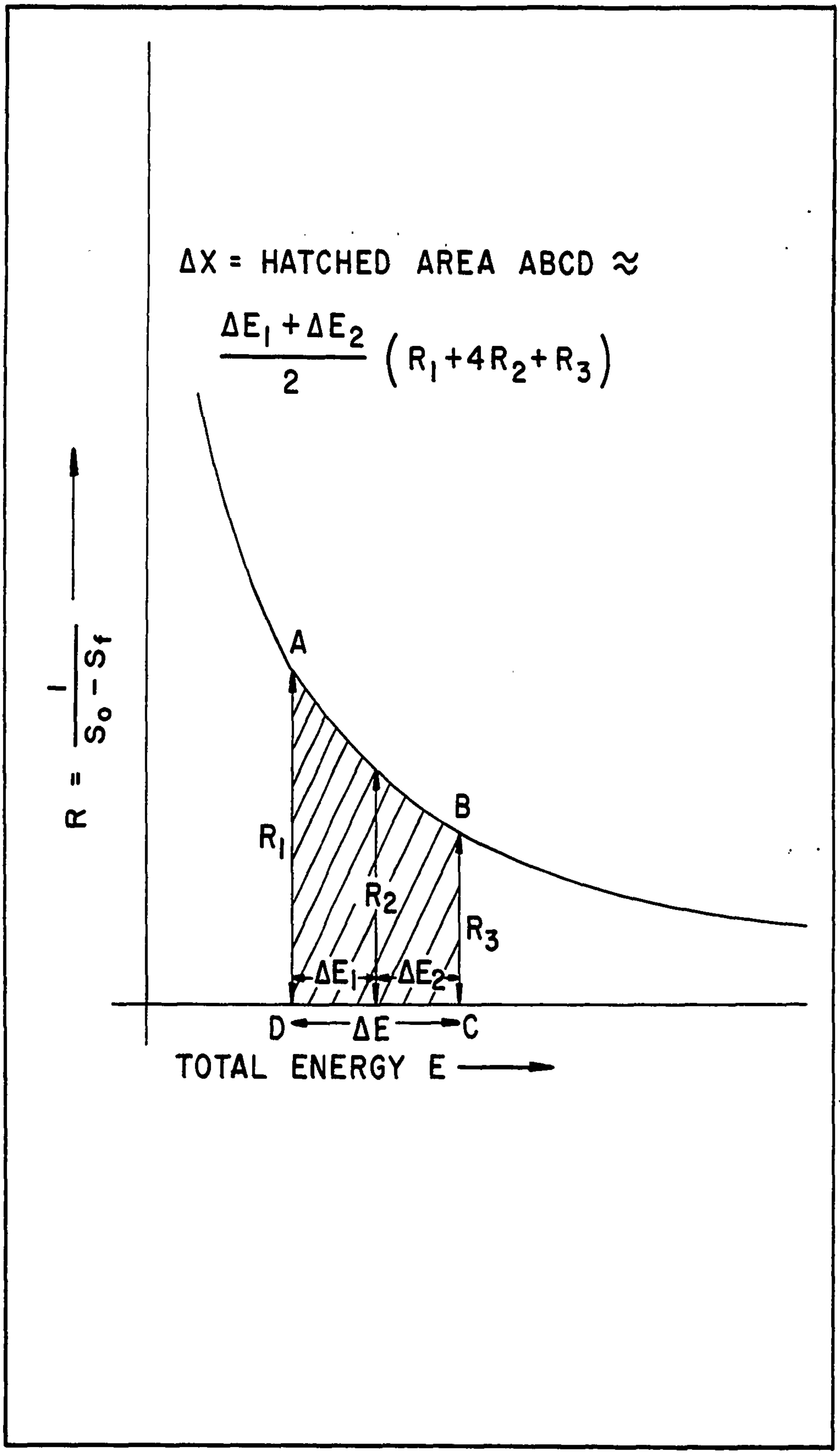


Fig. 5 Graphical Representation of the Equation $\Delta x = \frac{\Delta E}{S_o - S_f}$ for Backwater Curve, (Proposed Method)

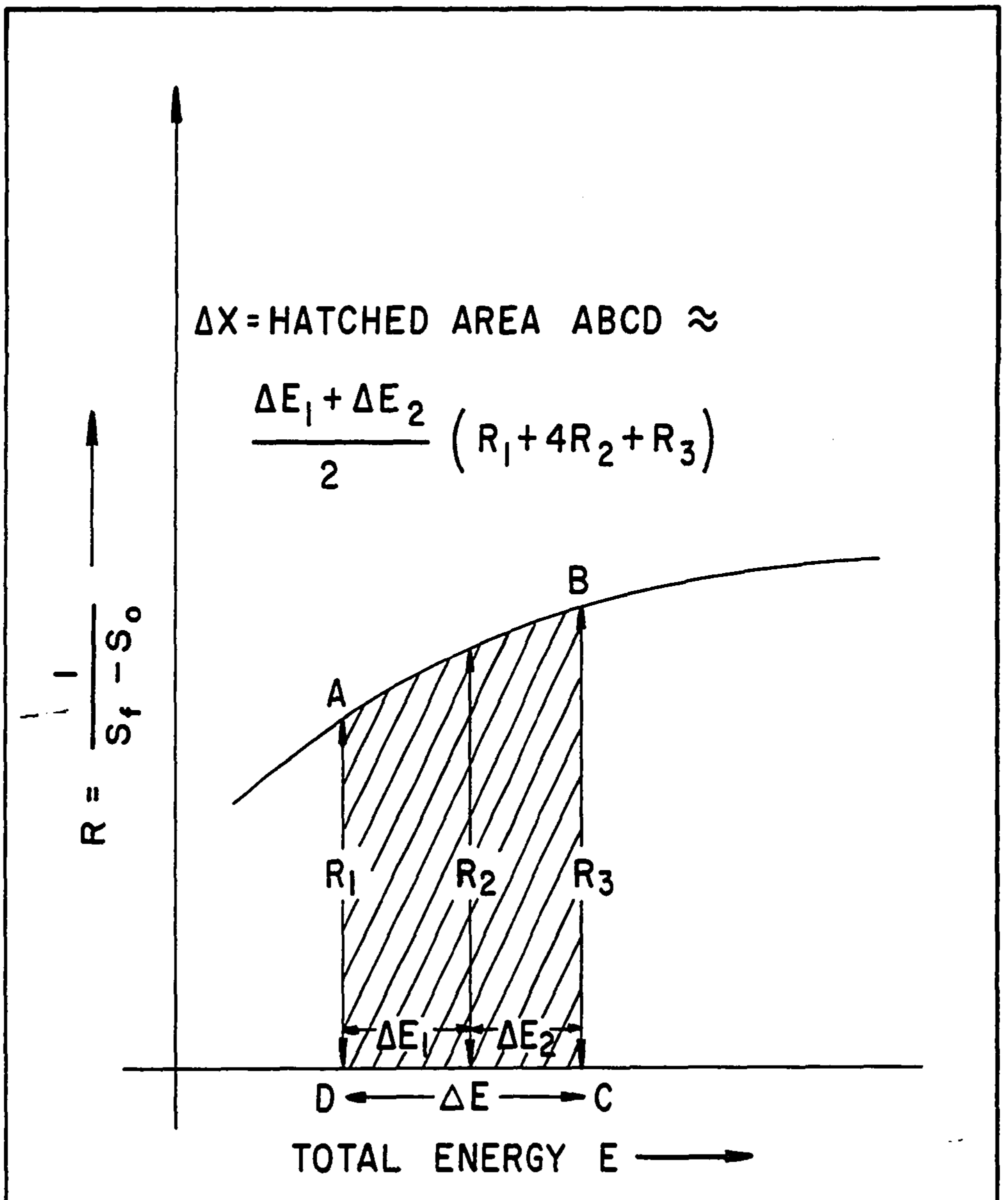


Fig. 6 Graphical Representation of the Equation $\Delta x = \frac{\Delta E}{S_f - S_o}$ for Drop Down Curve. (Proposed Method)

be seen that the remainder term of the Parabolic Formula decreases as $1/N^4$; i.e. Simpson's rule converges significantly fast, and the computational technique is not complex. Unfortunately, the equation (6.5), is not helpful for computational purposes, since the maximal value of the fourth order derivative of the integrand appears in this equation. Expressing the equation (6.2) in terms of y,

$$E = y + Q^2 / [(b + z\ y)\ y]^2 .\ 2g$$

$$S_o - S_f = S_o - (n^2\ Q^2 / 2.22) \cdot \frac{(b + 2 \cdot \sqrt{1 + z^2} \cdot y)^{4/3}}{(b + zy)^{10/3} \cdot y^{10/3}}$$

it can be seen that f^{iv} (y) is too much complicated and is not worth evaluation practically. Therefore, this difficulty can be overcome if the relations showing the dependence between derivatives and differences of one and the same order, are used.

$$* \Delta^m y_k = h^m f^m (\xi) \tag{6.6}$$

where, $h = y_n + 1 - y_k$

ξ = a point between $y_n + 1$ and y_k

From (6.5) and (6.6) $* \left| R_n \right| \leq (b' - a') \max \left| \Delta^4 y \right| / 180$

To carry the transformation further, the differences can be written in terms of the ordinates.

---

*S. B. Norkin., Elements of Computational Mathematics. (Trans. from Russian). MacMillan Company, New York. (1965)

$$|\Delta^4 y| = (f_{i-2} - 4f_{i-1} + 6f_i - 4f_{i+1} + f_{i+2})$$

Finally, $|R_n| \leq [(b' - a')/180] \max (f_{i-2} - 4f_{i-1} + 6f_i - 4f_{i+1}$

$$+ f_{i+2}) \qquad (6.7)$$

Actually 8 typical profiles (Table VI) with a different set of variables from discharge 400 cfs to 150,000, are analyzed and the error in each method, the present as well as the proposed, is computed. Curves are drawn (Figs. 7 to 14) with error V/S number of divisions. It can be seen from the curves, that errors decrease monotonically in a well behaved manner in either case, and the proposed procedure proves better in the range of desired accuracy. Error Bounds are also computed analytically by the equation (6.7), and the results are compared (Table VI). Since the error predominates in the last few steps as the profile approaches the normal depth, the equation has to be applied to this range to find the maximum modulus of the fourth order differences. From this error analyses, it is found out that it is possible to determine the number of steps required for the desired accuracy from equation (6.7) with relatively few computations before actually going through the most tedious computations, using the number of steps in random fashion without having any idea of it's accuracy whatsoever.

Standard Step Method

In natural watercourses, or artificial channels with frequent changes of section and grade, Standard Step Method becomes more convenient than the integration methods, described before. The detail procedure can be found elsewhere (4). The underlying theory for the computation of water surface elevations is the principle of the conservation

TABLE VI COMPARISON OF ACCURACY AND NUMBER OF STEPS REQUIRED IN THE EXISTING AND PROPOSED STEP METHODS

Profile

1. $Q = 400.0$ $W = 20.0$ $S_o = 0001.$ $Z = 0$ $n = .017$ $y_n = 8.34$ $y_c = 2.96$ $y_1 = 6.0$ $y_2 = 8.20$

EXISTING METHOD			PROPOSED METHOD			
No. of Steps	Length $L_{.985}$	Absolute Error	No. of Steps	Length $L_{.985}$	Error Ft	Error Bound Predicted from Eq. 6.7
4	60405	12825	4	86395	13165	
8	68200	5030	8	76408	3178	
11	70178	3052	16	73771	541	
22	72321	809	22	73350	120	
44	72987	243	44	73249	19	
88	73167	63	88	73230	0	$\leq$ 27 ft
176	73230	18				

(Note: $L_{.985}$ = Length of profile up to a section where $y/y_n = .985$)

TABLE VI (CON'T) COMPARISON OF ACCURACY AND NUMBER OF STEPS REQUIRED IN THE EXISTING AND PROPOSED STEP METHODS

Profile

2. $Q = 750$ $W = 10$ $L\ 1.01$ $S_o = .0004$ $Z = 1$ $n = .0215$ $y_n = 9.5$ $L\ 1.01$ $y_c = 4.75$ $y_1 = 15.0$ $y_2 = 9.6$

EXISTING METHOD			PROPOSED METHOD			
No. of Steps	Length L 1.01	Absolute Error	No of Steps	Length L 1.01	Error Ft	Error Bound Predicted from Eq. 6.7
4	27619	2861	8	34236	2757	
8	28747	1733	16	31479	940	
27	30114	366	32	30729	250	
54	30377	103	54	30539	40	
108	30444	35	108	30499	20	
216	30479	10	162	30479	0	$\leq$ 68

Profile 3 $Q = 800$ $W = 15$ $S_o = .0003$ $Z = 1$ $n = .03$ $y_n = 10.8$ $y_c = 4.05$ $y_1 = 4.08$ $y_2 = 10.065$

EXISTING METHOD			PROPOSED METHOD			
No. of Steps	Length $L_{.932}$	Absolute Error	No. of Steps	Length L.932	Absolute Error	Error Bound Predicted from Eq. 6.7
9	9498	548	18	10041	5	
18	9898	148	36	10043	3	
45	10021	25	90	10045	1	
90	10039	7	180	10046	0	0

Profile 4 $Q = 1000$ $W = 20$ $S_o = .0001$ $Z = 1.5$ $n = .025$ $y_n = 11.85$ $y_c = 3.85$ $y_1 = 3.85$ $y_2 = 11.65$

No. of Steps	Length $L_{.98}$	Absolute Error	No. of Steps	Length $L_{.98}$	Absolute Error	Error Bound Predicted from Eq. 6.7
4	36680	24892	4	89201	19391	
10	53318	8254	8	69810	7215	
20	58775	2797	20	62595	884	
40	60733	839	40	61711	119	
80	61358	114	80	61592	20	
200	61572	0	120	61572	0	$\leq$ 9.45

Profile 5 $Q = 4000$ $W = 50$ $S_o = .001$ $Z = 0$ $n = .025$ $y_n = 10.95$ $y_c = 5.84$ $y_1 = 20.0$ $y_2 = 11.2$

EXISTING METHOD			PROPOSED METHOD			
No. of Steps	Length $L_{1.02}$	Absolute Error	No. of Steps	Length $L_{1.02}$	Absolute Error	Error Bound Predicted from Eq. 6.7
4	17379	1196	4	23837	5253	
8	17852	723	8	20215	1631	
11	18061	514	16	18981	397	
22	18377	198	32	18674	90	
44	18516	59	44	18610	26	
88	18562	22	88	18584	0	$\leq$ 36
176	18575	9				

Profile 6 $Q = 6220$ $W = 100$ $S_o = .0004$ $Z = 1$ $n = .022$ $y_n = 10.0$ $y_c = 4.9$ $y_1 = 25.0$ $y_2 = 10.2$

	$L_{1.02}$			$L_{1.02}$		
4	55253	2195	8	66570	9122	
10	55499	1949	20	59221	1773	
20	56398	1050	40	57813	365	
40	57024	424	80	57490	42	
74	57284	164	148	57448	0	$\leq$ 148
148	57398	50				

Profile
7 $Q = 500000$ $W = 300$ $Z = 2.5$ $S_o = .0005$ $n = .045$ $y_n = 24.8$ $y_c = 9.263$ $y_1 = 50.01$ $y_2 = 25.01$

EXISTING METHOD			PROPOSED METHOD			
No. of Steps	Length $L_{1.01}$	Absolute Error	No. of Steps	Length $L_{1.01}$	Absolute Error	Error Bound Predicted from Eq. 6.7
5	88999	11097	10	123983	23887	
10	92229	7767	20	107796	7700	
25	96658	3438	50	101286	1190	
50	98720	1376	100	100303	207	
100	99661	435	200	100131	35	
			400	100096	0	< 123

Profile
8 $Q = 150000$ $W = 500$ $Z = 4.0$ $S_o = .0002$ $n = .012$ $y_n = 21.1$ $y_c = 13.55$ $y_1 = 41.3$ $y_2 = 21.3$

	$L_{1.01}$			$L_{1.01}$		
5	164763	16359	10	217283	36155	
10	169728	11394	20	192146	11018	
20	175030	6092	40	183806	2678	
40	178612	2510	80	181639	517	
80	180321	801	160	181230	108	
160	180872	250	320	181122	0	< 273

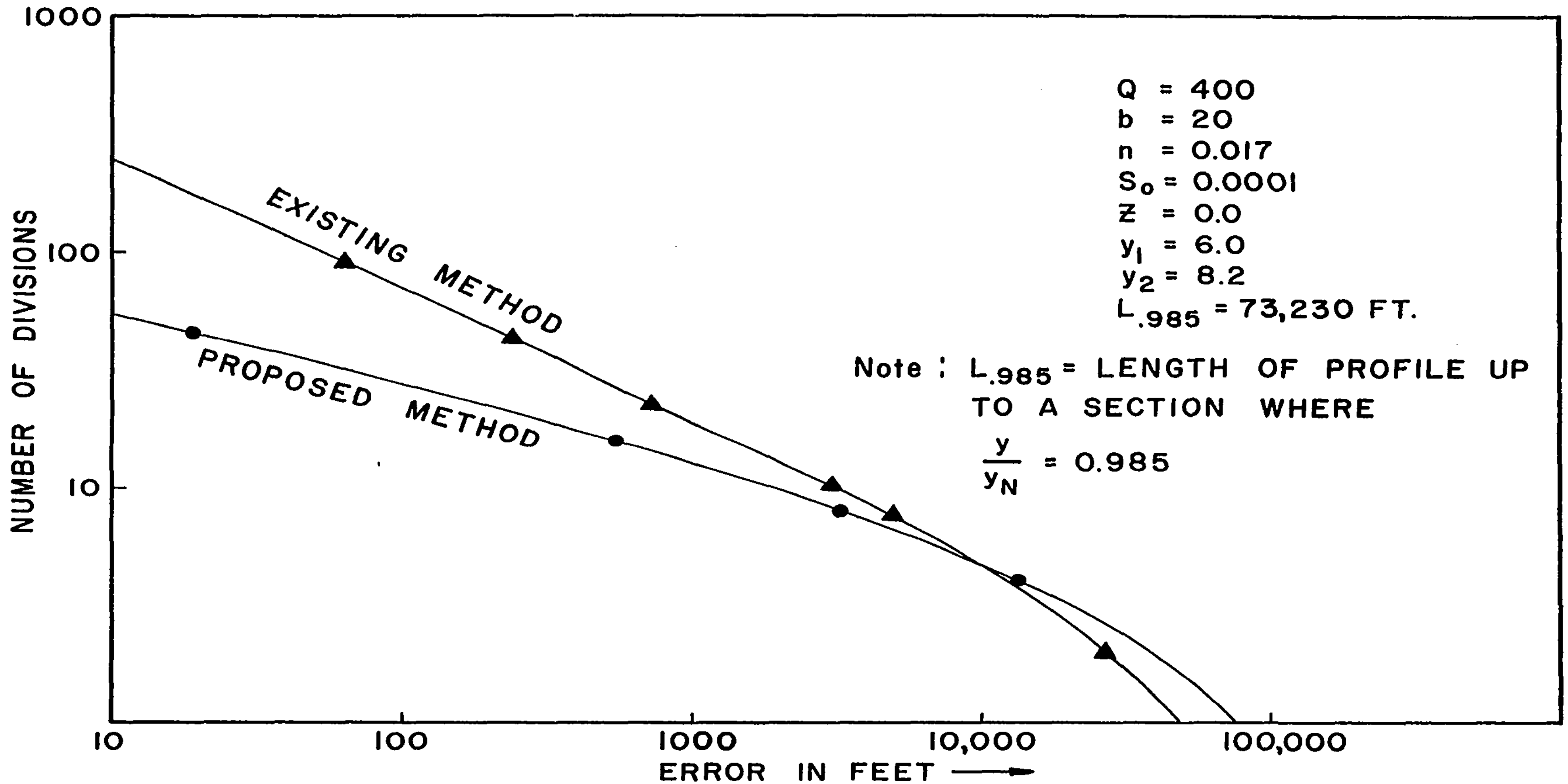


Fig. 7. Profile 1. Drop-Down Curve

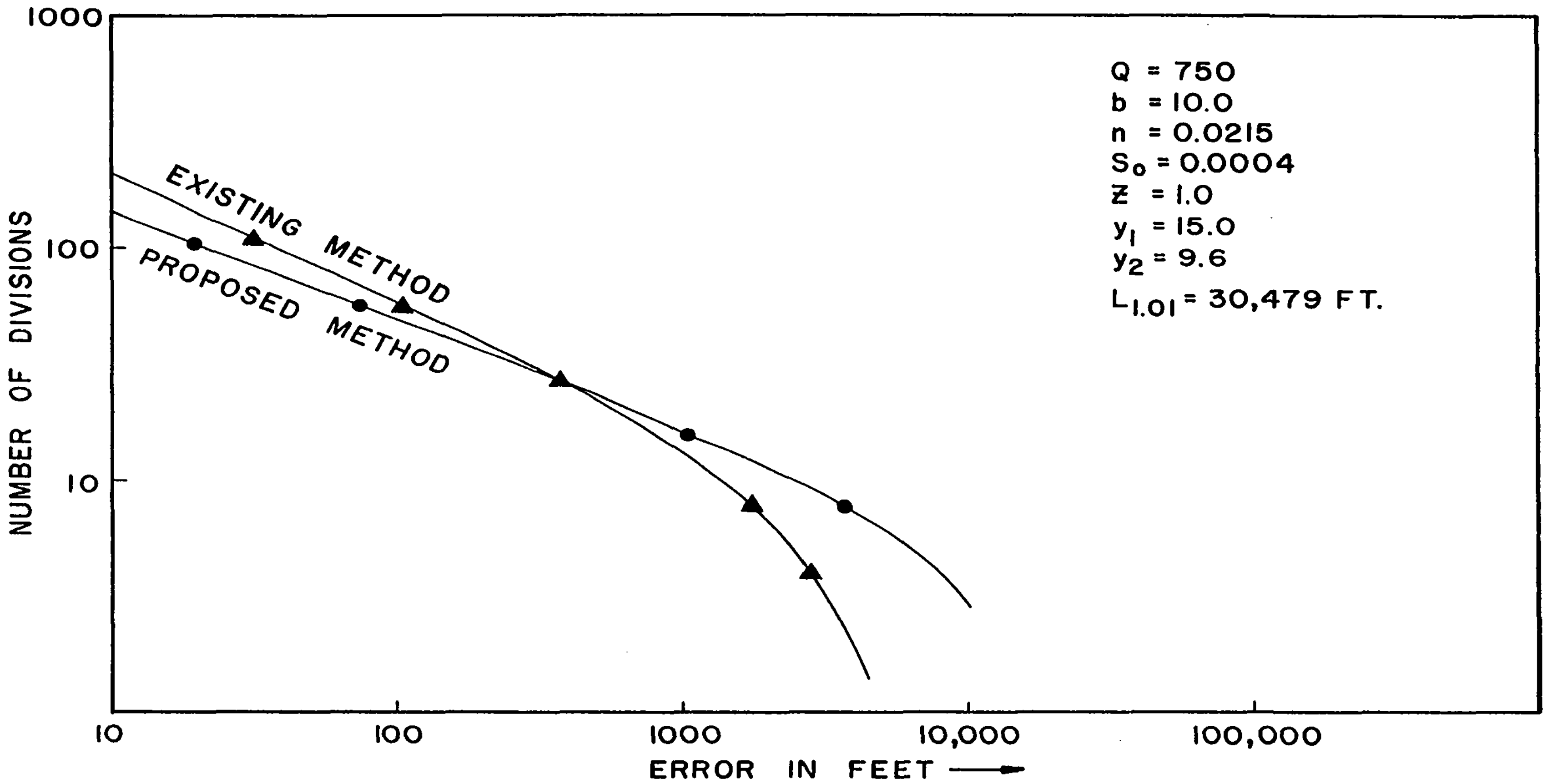


Fig. 8. Profile 2. Backwater Curve

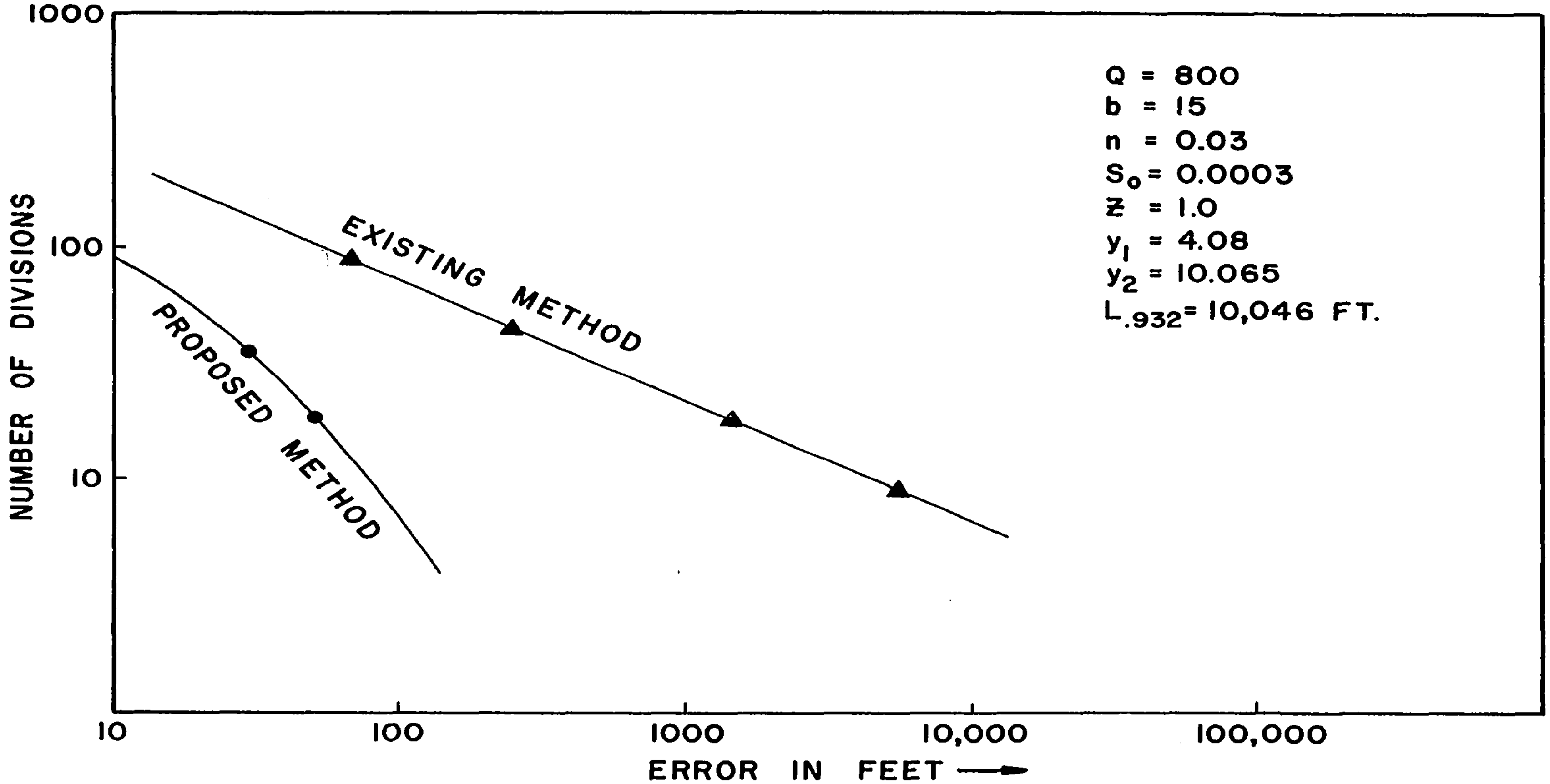


Fig. 9. Profile 3. Drop-Down Curve

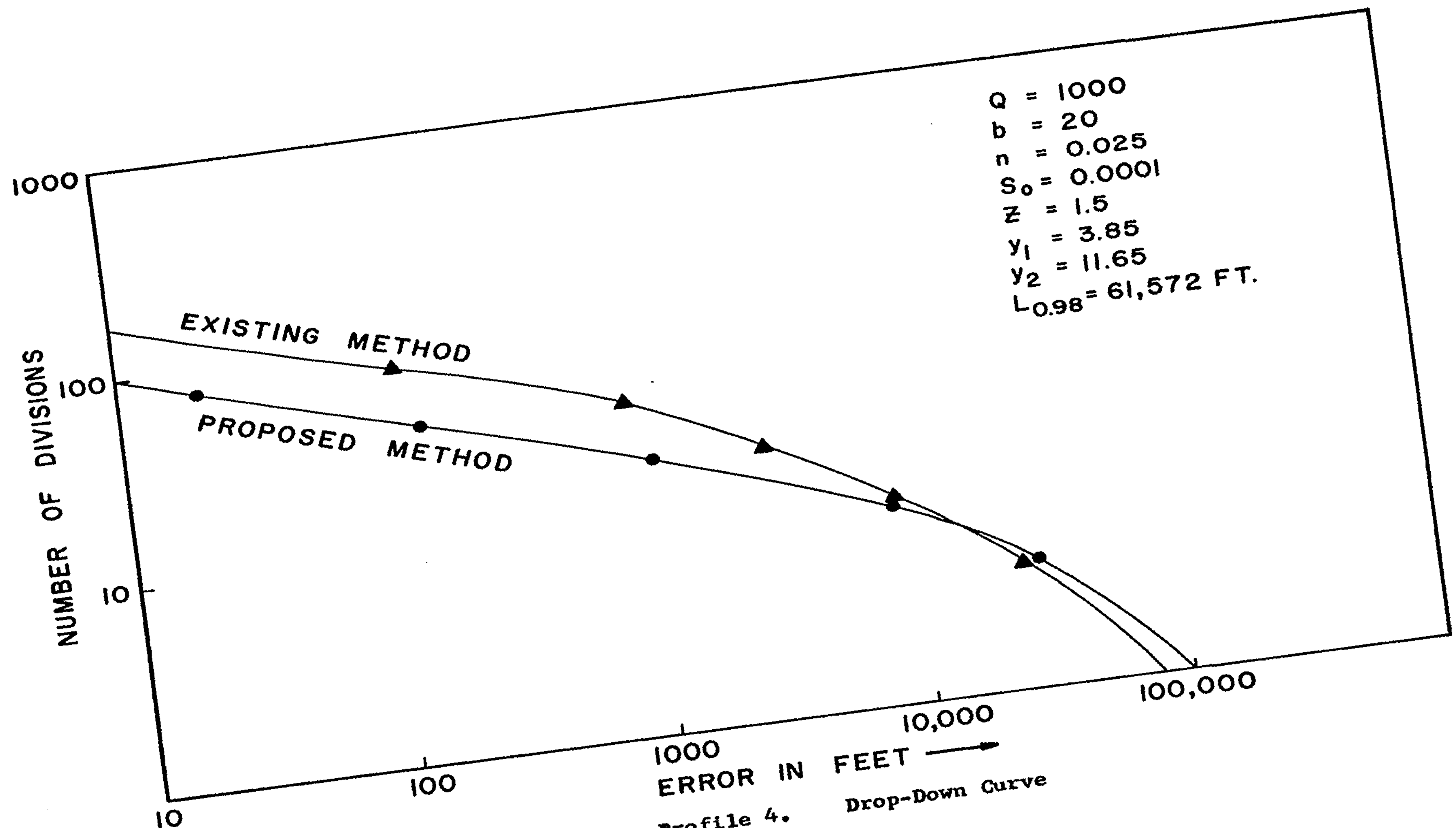


Fig. 10. Profile 4. Drop-Down Curve

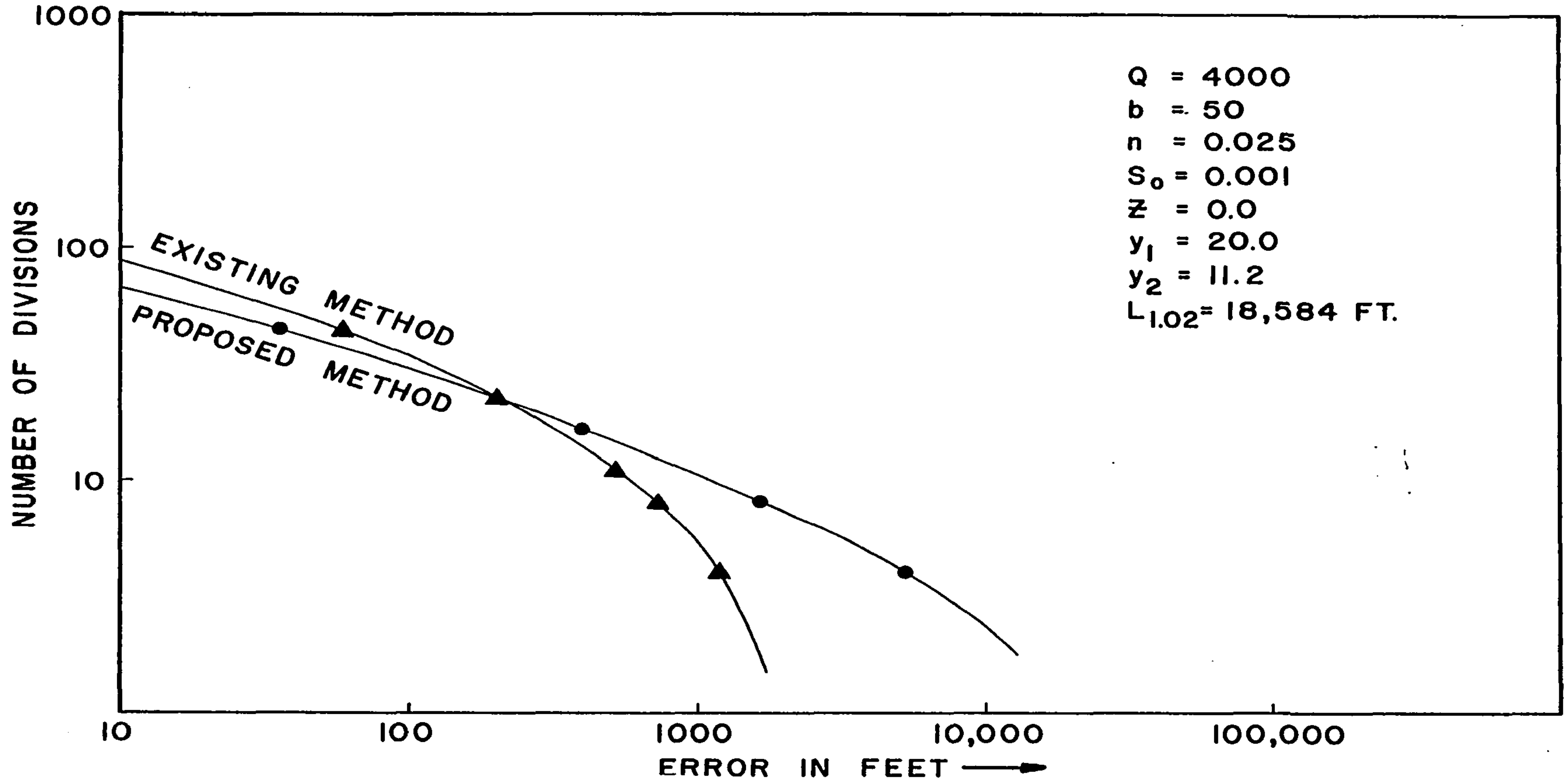


Fig. 11. Profile 5. Backwater Curve

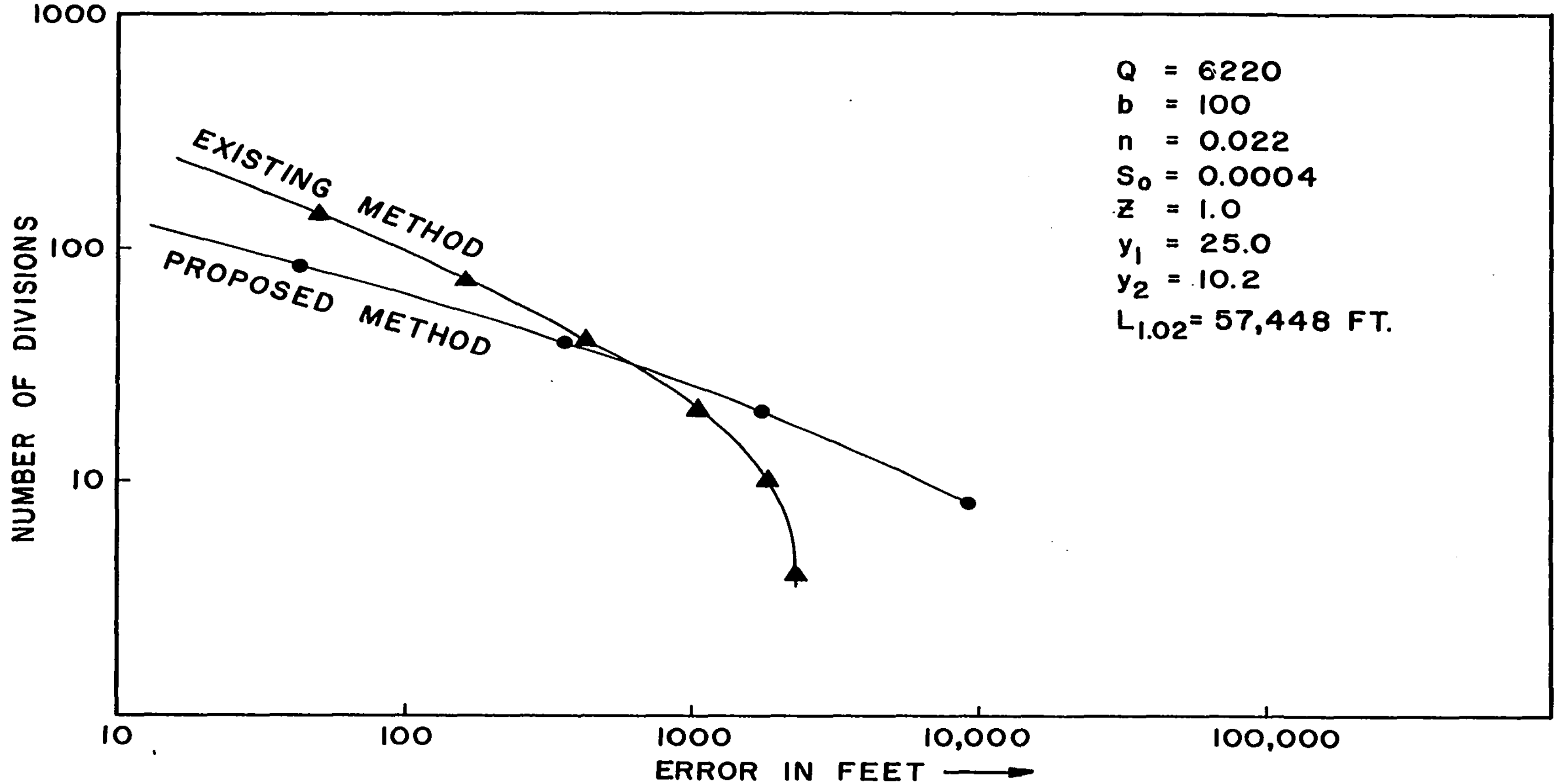


Fig. 12. Profile 6. Backwater Curve

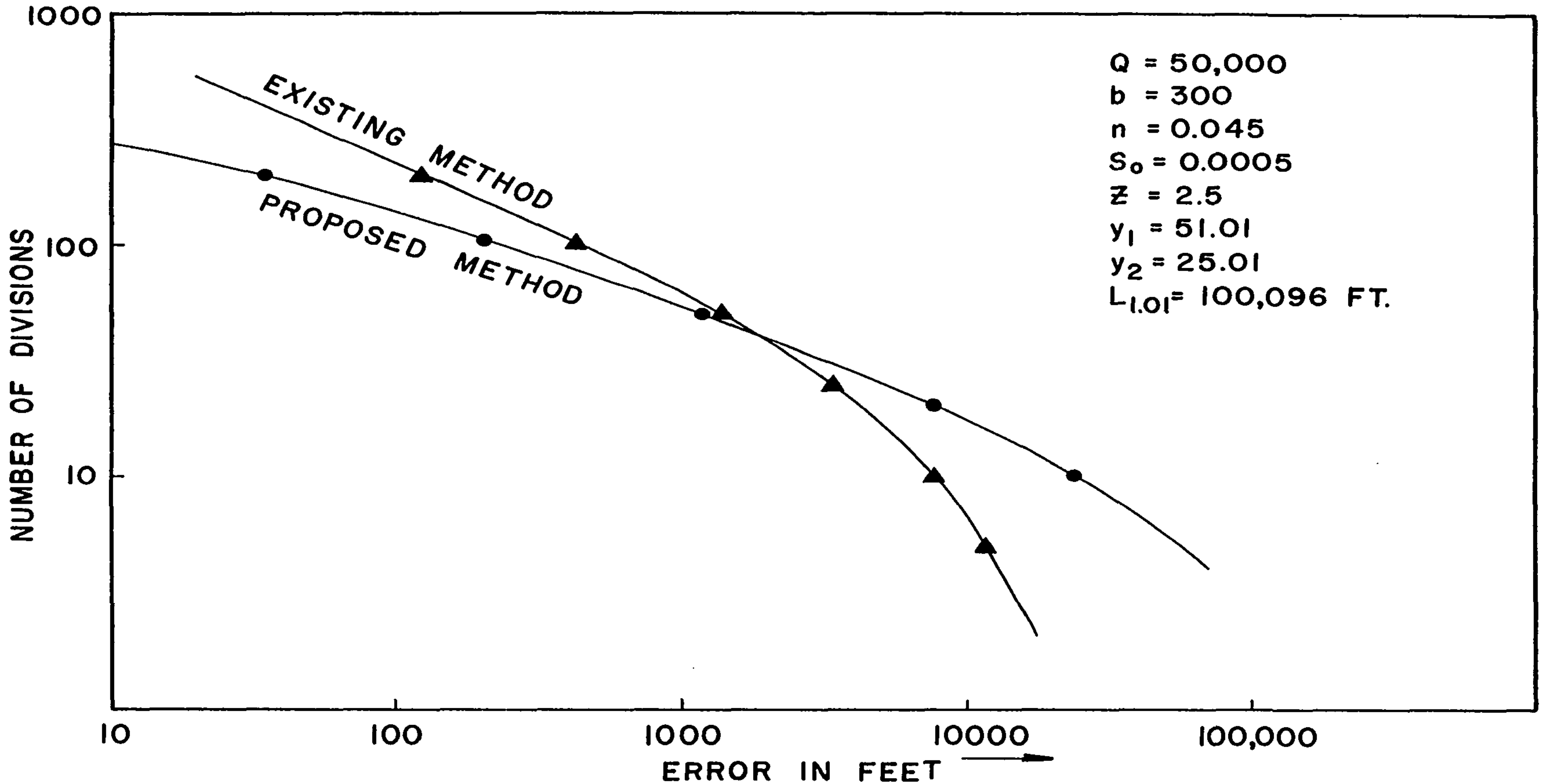


Fig. 13. Profile 7. Backwater Curve

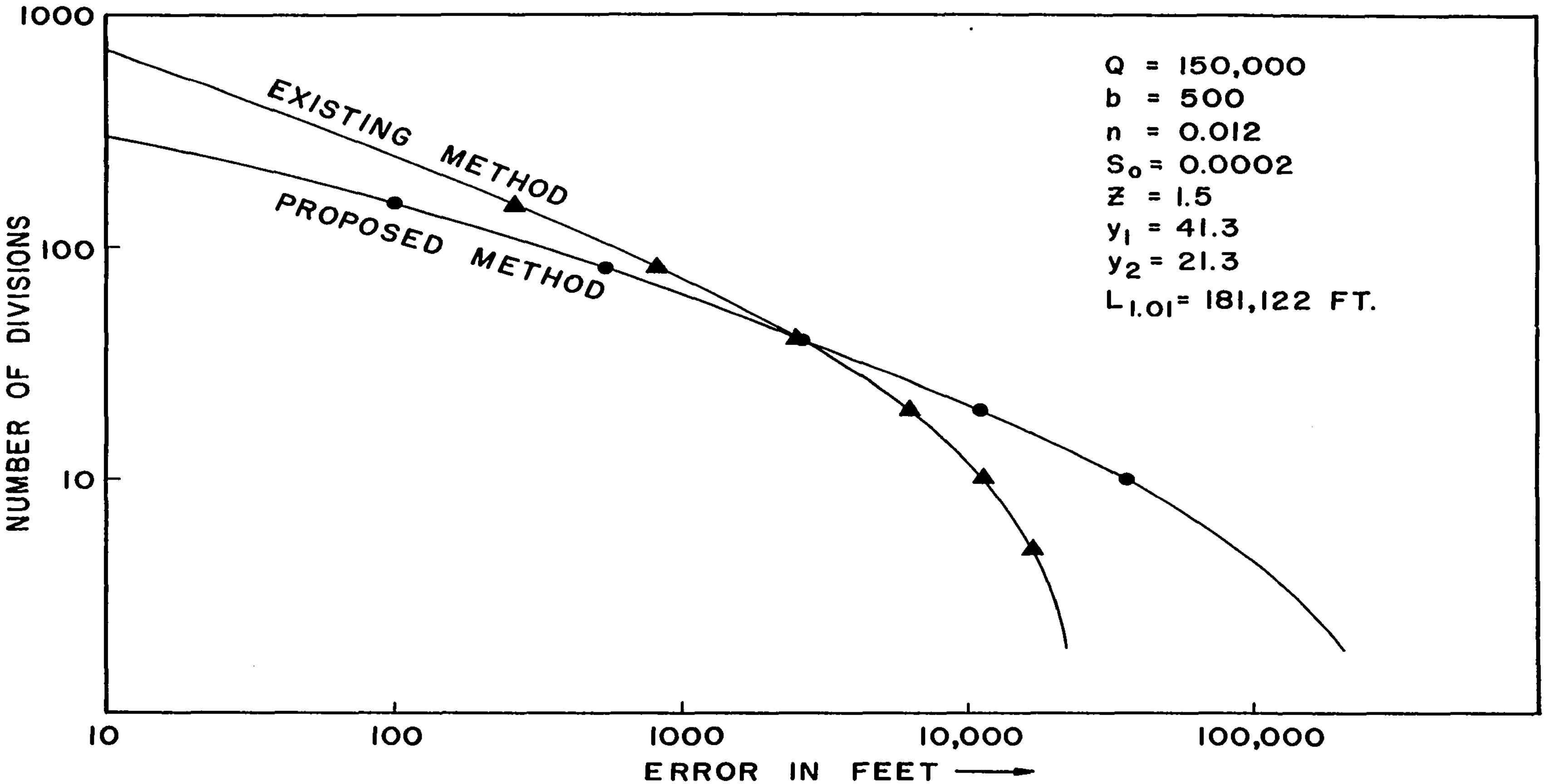


Fig. 14. Profile 8. Backwater Curve

of energy between two cross-sections on a stream and is expressed by the same equation (6.1) and illustrated in Fig. (2). The stretch to be studied is divided into short reaches. Trial-and-error computations are made for each reach, based upon the data for the reach and the result of the computations for the preceding reach. This necessitates carrying the computations step by step from one end of the stretch to the other, generally from downstream to upstream. Selection of Reach: The reaches need to be short enough to reduce to within permissible limits the error in approximating the true water surface slope through the reach by the average of the surface slopes at each end. However, definite criteria for determining the reach length have not been established, but the distance depends primarily on the depth and slope of the stream channel, and on the accuracy of the estimate of the water surface elevation at the initial section. Some empirical methods are being used at present to determine the reach length (19).

Proposed Theory of Criteria for the Determination of Reach Length.

It has already been mentioned that the error arises in the computation of average friction slope which is calculated as the mean of the two slopes at sections 1 and 2. This is based on the assumption that the friction slope line is a straight line. But obviously it is a curve and not a straight line. If the reach lengths are not small enough, the error committed in each step goes on accumulating the finally grossly misleading results may be obtained.

Consider the two adjacent values of the argument, denoted by y_k and y_{k+1} (Fig. 15) , between which lies the given value of y $(y_k < y < y_{k+1})$. Let $(Sf)_k = f\ (y_k)$ and $(Sf)_{k+1} = f\ (y_{k+1})$ be

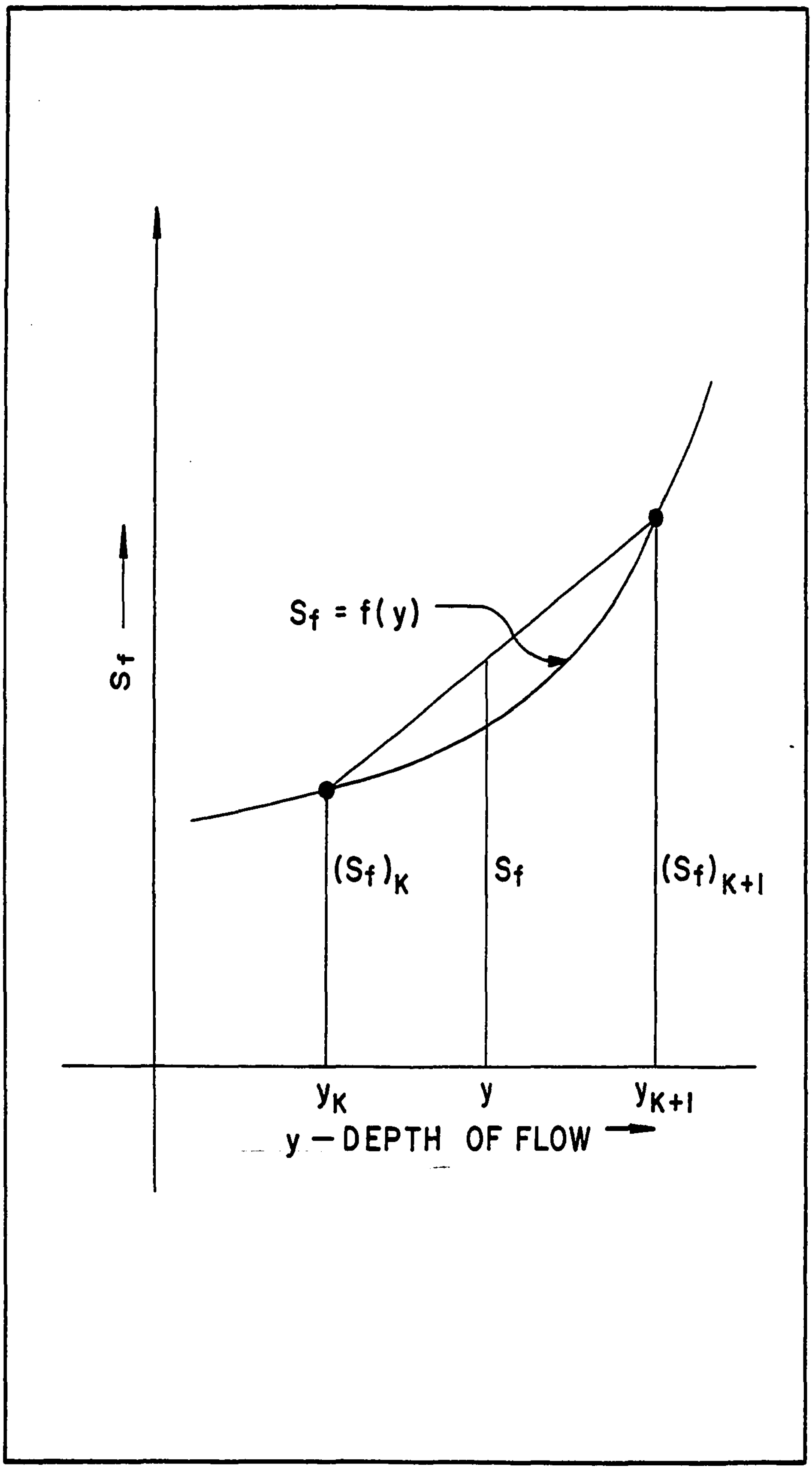


Fig. 15. Graphical Interpretation of Error in the Average Friction Slope Method

the corresponding values of the function. If the function is replaced by a linear function i.e., the arc of the graph of the function is replaced by the chord spanning it, the error arising therefrom can be estimated.

The equation of the line passing through the points (y_k, Sf_k), $(y_k + 1, Sf_k + 1)$ has the form

$$(Sf - Sf_k)/(Sf_{k+1} - Sf_k) = (y - y_k)/(y_{k+1} - y_k)$$

but

$$Sf_{k+1} - Sf_k = \Delta Sf_k$$

$$y_{k+1} - y_k = h$$

$$Sf \approx Sf_k + \Delta Sf_k \quad \frac{y - y_k}{h} \tag{6.8}$$

Now let the difference between the accurate value of the function f (y) and in approximate value as determined by the formula (6.8), be denoted by φ (y)

$$\varphi(y) = f(y) - \left[Sf_k + \Delta Sf_k \quad \frac{y - y_k}{h}\right]$$

Differentiating φ (y)

$$\varphi^{i}(y) = f^{i}(y) - \frac{\Delta Sf_k}{h}$$

$$\varphi^{ii}(y) = f^{ii}(y)$$

and also $\varphi(y_k) = \varphi(y_{k+1}) = 0$

Since the second derivative of the function f (y) is continuous over the range being considered, it satisfies the inequality

$$|f^{ii}(y)| \leq M_2 \tag{6.9}$$

where M_2 is the maximum modulus of the second derivative of the integrand over the interval (y_k , y_{k+1}). Let at some point 'a' in the interval, the function φ (y) attains it's maximum modulus for this interval. Now expanding the function as a Taylor Series in powers of (y - a),

$$\varphi (y) = \varphi (a) + \varphi^{i} (a) (y - a) + (\varphi^{ii} (\xi)/2) (y - a)^2 + \ldots\ldots$$

$$= \varphi (a) + (\varphi^{ii} (\xi)/2) (\bar{y} - a)^2 \qquad (6.10)$$

since φ^{i} (a) = 0, owing to the choice of the point a. Here ξ is some point lying between y and a. Finally, let that point which of the pair of points y_k and y_{k+1}, is the closer to a , be called $\bar{y}$. Then $\varphi (\bar{y}) = 0$ and it follows from (6.10) that $\varphi (a) = (-\varphi^{ii}(\xi)/2) (\bar{y} - a)^2$ Since $|y - a| \leq h/2$ (y is that end of interval y_k, y_{k+1} which is closer to a) then because of (6.9)

$$|\varphi (a)| \leq \frac{\varphi^{ii} (\xi)}{2} \quad \frac{h^2}{4} \leq \frac{M_2 h^2}{8}$$

since $|\varphi (y)| \leq |\varphi (a)|$ on the interval (y_k , y_{k+1}) ,

$$|\varphi (y)| = f (y) - [Sf_k + \Delta Sf_k / h \ (y - y_k)] | \leq M_2 h^2/8$$

for any y from the interval (y_k , y_{k+1}). (6.11)

This is the required estimate for the error of taking the average of friction slopes at two sections based on the erroneous assumption of the friction slope line bring a straight line.

Unfortunately, it can be seen that evaluation of $M_2 = \text{Max } f^{ii}$ (y) (see equation 6.15) is formidable and is not worth evaluation practically. So again, this difficulty can be overcome if the relations showing the dependence between derivatives and differences of one and the same order are used. Thus it can be shown (11)

$$f_i^{ii} = (1/h^2)\cdot(f_{i+1} - 2f_i + f_{i-1}) - (h^2/12)f_i^{iv}$$

$$f_i^{iv} = (1/h^4)\cdot(f_{i-2} - 4f_{i-1} + 6f_i - 4f_{i+1} + f_{i+2})$$

$$f_i^{ii} = (1/12\,h^2)[-f_{i-2} + 16f_{i-1} - 30f_i + 16 + f_{i+1} - f_{i+2}] \qquad (6.12)$$

Neglecting higher order derivatives, a more approximate equation will be

$$f_i^{ii} = (1/h^2)\cdot[f_{i+1} - 2f_i + f_{i-1}] \qquad (6.13)$$

Finally, from (6.11) and (6.13)

$$\varphi\,(y) \leq (1/8)\cdot(f_{i+1} - 2f_i + f_{i-1}) \qquad (6.14)$$

Therefore, it follows that the criterion to determine the length of reaches will be $2f_i \approx f_{i+1} + f_{i-1}$ to avoid errors. This can be satisfied by increasing the number of steps.

Perhaps one of the most cumbersome and time consuming procedures in open channel flow problems happens to be the water surface profile computations by Standard Step Method because each step requires a crude trial-and-error solution and the number of variables are too many. For a prismatic channel, the energy equation (6.1) can be written for backwater curves in the form as follows

$$y_2 = y_1 + (V_1^2 / 2g) - (V_2^2 / 2g) - S_o\,\Delta x + ((Sf_1 + Sf_2)/2)\,\Delta x$$

$$y_2 = y_1 + Q^2 / [(b + zy_1)\,y_1]^2 2g - Q^2 / [(b + zy_2)y_2]^2 2g - S_o\,\Delta x \;+$$

$$\frac{n^2Q^2}{2.22} \cdot \left[\frac{(b + 2 \cdot \sqrt{1 + z^2} \cdot y_1)^{4/3}}{(b + zy_1)^{10/3}\ y_1^{\ 10/3}} \right] +$$

$$\frac{n^2Q^2}{2.22} \cdot \left[\frac{(b + 2 \cdot \sqrt{1 + z^2}\ y_2)^{4/3}}{(b + zy_2)^{10/3}\ y_2^{\ 10/3}} \right] \qquad (6.15)$$

It can be seen that trial-and-error is the only way to solve this equation, and the same procedure has to be repeated again and again depending on the number of intervals chosen for the total reach of the channel.

Computer Method of Solution. The Computer Program utilizes this equation by starting in the most downstream subreach with a known water surface elevation or depth, a corresponding discharge, and the cross-sectional properties. It there selects for the upstream cross-section of the subreach, a water surface elevation, or depth, which will balance the equation to a selected tolerance limit. This just-chosen upstream water-surface elevation of the next subreach, and the solution progresses upstream, balancing the equation to the tolerance in each subreach.

Equation (6.15) is of the form $y_2 = f\ (y_2)$ and can be solved by the iterative technique described in Chapter V. The initial approximation is possible from physical considerations of the open channel flow. (See Fortran Program page 107). This procedure eliminates assuming values of y_2 in a random fashion, and testing for it's accuracy in each step. The criterion, derived earlier (Equation 6.14), can also be applied to check the accuracy at each step without any considerable extra efforts in computations.

CHAPTER VII

DISCUSSIONS, CONCLUSIONS AND RECOMMENDATIONS

Practically every problem in Engineering Hydraulics involves the prediction by either analytical or experimental methods of one or more characteristics of flow. There are, in brief, three different bases for such prediction. The first, and by far the oldest, is that of "engineering experience" gained in the field by each individual engineer. The second is the laboratory method of studying each specific problem by means of scale models. The third is the process of theoretical analysis which is developing rapidly now-a-days. Hydraulics has occasionally been depreciated as a "Science of Coefficients, Tables, and Charts" because engineers sometimes feel that other branches of engineering have attained a higher state of development than hydraulic engineering which lacks very often, more precise and accurate mathematical analyses. However, the empirical nature of hydraulics is being replaced rapidly by more rational methods of attack. At the same time the increasing availability of digital computers along with the explosive and sophisticated growth of Numerical Analysis have caused great impact on the computational methods in today's hydraulic and hydrologic investigations.

Most of the equations in hydraulics are empirical and the methods of solution involve equally crude trial-and-error procedures and depend on extensive tables and graphs. From this study it is expected that most of these computational methods have to be outdated in this era of digital machinery. This study shows that hydraulic problems encountered in the planning, design, construction and operation of water resources projects, which often require the solution of complex mathematical relationships, the reduction of large volumes of data, the frequent repetition of basic operations, or the evaluation of alternative assumptions and criteria, can be made ideally suited for the use of electronic machines. In order to eliminate the procedures of searching tables, charts, or scanning tabulations of experimental or computed data, it is often more efficient to have the computer calculate the value of a function of particular mathematical relationship each time, however complicated it may be, by employing approximate expressions for the functions. New techniques and procedures in Numerical Analysis are constantly being developed for use with electronic computers which permit the application of these machines in many areas of study hitherto considered impracticable or impossible. The present analysis is an attempt to explore the use of some of the numerical methods available to solve the problems in open channel flow.

An area in which considerable future progress can be made is that of a finding approximate polynomials for the long series derived in the case of positive slopes, instead of direct summation as used here so that some valuable computer time can be saved. Although the programs developed in direct integration method is applicable to all prismatic channels with a few changes to parabolic shape, cannot be applied to circular channels commonly encountered in Sanitary Engineering because the hydraulic exponents M and N vary appreciably when the depth of flow is close to the crown. Even the other programs cannot be claimed that they are suitable for all conditions of open channel flow problems. Further research has to be done to refine some of the programming techniques to achieve more generality and flexibility. In this study, emphasis is given more on the development of Numerical Methods rather than actual programming.

Integrated Civil Engineering System: This system, called ICES, was initiated and being carried out at the M.I.T. civil engineering systems laboratory. This project is a cooperative venture of the government, industry, and university groups interested in the development of a large-scale, computer-based system which integrates advanced information systems and powerful problem solving capabilities. Recently steps were taken to begin development of a hydraulic and hydrologic problem-solving capability for I.C.E.S. Named 'HYDRA,' this subsystem is in the earlier stages of its development. HYDRA presents an opportunity for the research group who can contribute to its initial design and orientation. Therefore, it is hoped that the present work may form some ground work in one of its important phases--water surface profile computations and may conceivably influence the ultimate form or capabilities of the system.

BIBLIOGRAPHY

Specific References

1. Argyopoulos, P. A., Water Surface Profiles in Irregular Natural Streams, A.S.C.E. Proceedings Hydraulics Division, Vol. 87, No. HY 4 July 1961.

2. Bakhmeteff, B. A., Hydraulics of Open Channels, McGraw-Hill Co., N. Y., 1932.

3. Chow, V. T., Integrating the Equation of Gradually Varied Flow. A.S.C.E. Proceedings, Hydraulics Division, Vol. 81, Nov., 1955.

4. Chow, V. T., Open Channel Hydraulics. McGraw-Hill Co., Inc., N. Y., 1959.

5. Engineering News Record. Backwater Curves Computed by Machine. Vol. 156, No. 19. May, 1956.

6. Hastings, C. Approximations to Digital Computers. A Research Study by the Rand Corporation, Princeton University Press, New Jersey, 1966.

7. Keifer, C. J., and Chu, H. H. Backwater Function by Numerical Integration. Transactions, A.S.C.E., Vol. 120, 1955.

8. Lanczos, C. Applied Analysis. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1956.

9. Lara, J. M. and Kenneth, B. S. Two Methods to Compute Water Surface Profiles. A.S.C.E. Proceedings, Hydraulics Division, HY 4, April 1959.

10. Liggett, J. A., General Solution for Open Channel Profiles. A.S.C.E. Proceedings, Hydraulics Division, No. HY 6, Vol. 87, Nov. 1961.

11. McCormick and Salvadori. Numerical Methods in Fortran. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1965.

12. McNally, W. D. Hydro Users Mannual. Carnegie Institute of Technology, Department of Civil Engineering, Dec. 1966.

13. Metzke, A. E. Varied Flow in Open Channels of Adverse Slopes. Transactions, A.S.C.E. Vol. 102., 1937.

14. Ming, Lee. Steady Gradually Varied Flow in Uniform Channels on Mild Slopes. Ph.D. Thesis, University of Illinois. Urbana, 1947.

15. Mononobe, N. Backwater and Drop Down Curves for Uniform Channels. Transactions, A.S.C.E., Vol. 103, 1938.

16. Norkin, S. B. Elements of Computational Mathematics. (Translated from Russian). MacMillan Co., New York, 1965.

17. Pickard, W. F. Solving the Equations of Uniform Flow. A.S.C.E. Proceedings. Hydraulics Division, HY 4, July 1963.

18. Snyder, M. A. Chebyshev Methods in Numerical Approximation. Prentice-Hall Series in Automatic Computation, Englewood Cliffs, N. J., 1966.

19. U.S. Army Corps. of Engineers. Generalized Computer Program, Backwater Any Cross-Section. Hydrologic Engineering Center, Sacramento, June, 1967.

20. U.S. Geological Survey. Computation of Water-Surface Profiles in Open Channels, Surface Water Techniques. Book 1, Chapter 1, 1964.

21. Vallentine, H. R. Characteristics of the Backwater Curve. A.S.C.E. Proceedings, Hydraulics Division, HY 4, July, 1964.

22. Von Saggern, M. E. Integrating the Equation of Non-Uniform Flow. Transactions, A.S.C.E., Vol. 115, 1950.

23. Woodward, S. M., and Posey, C. J. Hydraulics of Steady Flow in Open Channels. John Wiley & Sons, Inc., New York, 1941.

General References

24. Conte, S. D. Elementary Numerical Analysis. McGraw-Hill Co., Inc., 1965.

25. Henderson, F. M. Open Channel Flow. MacMillan Co., New York, 1966.

26. McCracken, D. D., and Dorn, W. S. Numerical Methods and Fortran Programming. John Wiley & Sons, Inc., New York, 1964.

27. Olson, R. M. Engineering Fluid Mechanics. International Textbook Co., Scranton, Pennsylvania, 1961.

28. Rouse, H. Fluid Mechanics for Hydraulic Engineers. Dover Publications, Inc., New York. 1961.

29. Rouse, H. Engineering Hydraulics. John Wiley & Sons, Inc., New York, 1966.

30. Vennard, J. K. Elementary Fluid Mechanics. John Wiley & Sons, Inc., New York, January, 1965.

APPENDIX A

Shifted Chebyshev Polynomials

APPENDIX A

Table VII. The Explicit Formulas for the Various <u>Shifted Chebyshev Polynomials</u> for the Interval $[0,1]$

$T_o\ (X) = 1$ (NOTE: $T_o\ (X)$ enters with halfweight only)

$T_1\ (X) = 2X - 1$

$T_2\ (X) = 8X^2 - 8X + 1$

$T_3\ (X) = 32X^3 - 48X^2 + 18X - 1$

$T_4\ (X) = 128X^4 - 256X^3 - 256X^3 + 160X^2 - 32X + 1$

$T_5\ (X) = 512X^5 - 1280X^4 + 1120X^3 - 400X^2 + 50X - 1$

$T_6\ (X) = 2048X^6 - 6144X^5 + 6912X^4 - 3584X^3 + 840X^2 - 72X + 1$

$T_7\ (X) = 8192X^7 - 28672X^6 + 39424X^5 - 26880X^4 + 9408X^3 - 1568X^2 + 98X - 1$

$$T_8\ (X) = 32768X^8 - 131072X^7 + 212992X^6 - 180224X^5 + 84480X^4 - 21504X^3 + 2688X^2 - 128X + 1$$

$$T_9\ (X) = 131072X^9 - 589824X^8 + 1105920X^7 - 1118208X^6 + 658944X^5 - 228096X^4 + 44352X^3 - 4320X^2 + 162X - 1$$

$$T_{10}\ (X) = 524288X^{10} - 2621440X^9 + 5570560X^8 - 6553600X^7 + 4659200X^6 - 2050048X^5 + 549120X^4 - 84480X^3 + 6600X^2 - 200X + 1$$

$$T_n\ (X) = 2(2X - 1)\ T_{n-1}\ (X) - T_{n-2}\ (X)$$

Table VIII. Relations Defining the Shifted Chebyshev Polynomials and Power Series for the Interval [0, 1]

$1 = T_o$ (NOTE: T_o enters with halfweight only)

$x = 1/2 \quad (T_o + T_1)$

$x^2 = 1/8 \quad (3T_o + 4T_1 + T_2)$

$x^3 = 1/32 \quad (10T_o + 15T_1 + 6T_2 + T_3)$

$x^4 = 1/128 \quad (35T + 56T_1 + 28T_2 + 8T_3 + T_4)$

$x^5 = 1/512 \quad (126T_o + 210T_1 + 120T_2 + 45T_3 + 10T_4 + T_5)$

$x^6 = 1/2048 \quad (462T_o + 792T_1 + 495T_2 + 220T_3 + 66T_4 + 12T_5 + T_6)$

$x^7 = 1/8192 \quad (1716T_o + 3003T_1 + 2002T_2 + 1001T_3 + 364T_4 + 91T_5 + 14T_6 + T_7)$

$x^8 = 1/32768 \quad (6435T_o + 11440T_1 + 8008T_2 + 4368T_3 + 1820T_4 + 560T_5 + 120T_6 + 16T_7 + T_8)$

$x^9 = 1/131072 \quad (24310T_o + 43758T_1 + 31824T_2 + 18564T_3 + 8568T_4 + 3060T_5 + 816T_6 + 153T_7 + 18T_8 + T_9)$

$x^{10} = 1/524288 \quad (92378T_o + 167960T_1 + 125970T_2 + 77520T_3 + 38760T_4 + 15504T_5 + 4845T_6 + 1140T_7 + 190T_8 + 20T_9 + T_{10})$

$$x^n = 2^{1-2n} \quad T_n(X) + \binom{2n}{1} T_{n-1}(X) + \binom{2n}{2} T_{n-2}(X) + \ldots\ldots\ldots\ldots\ldots$$

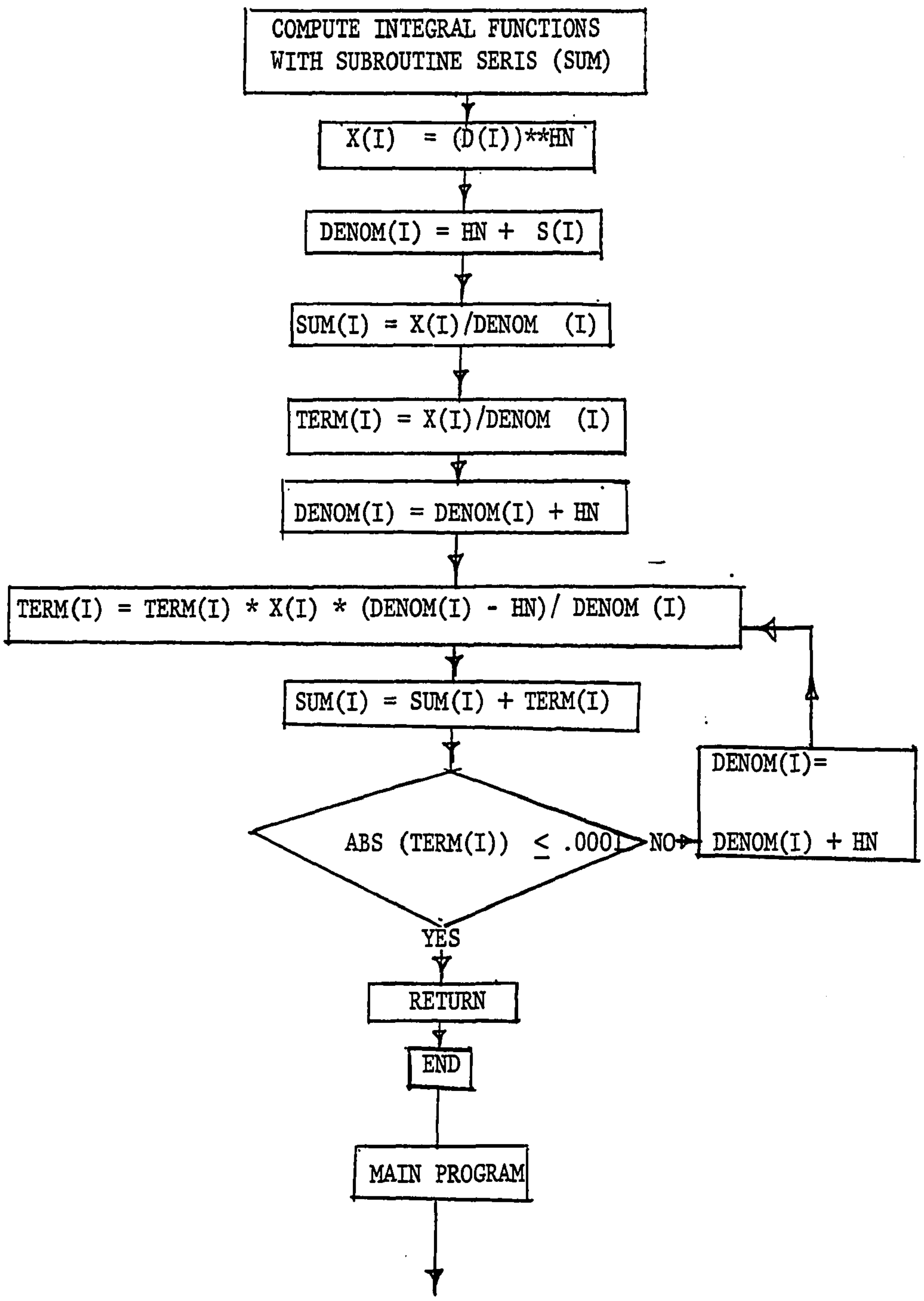
APPENDIX B

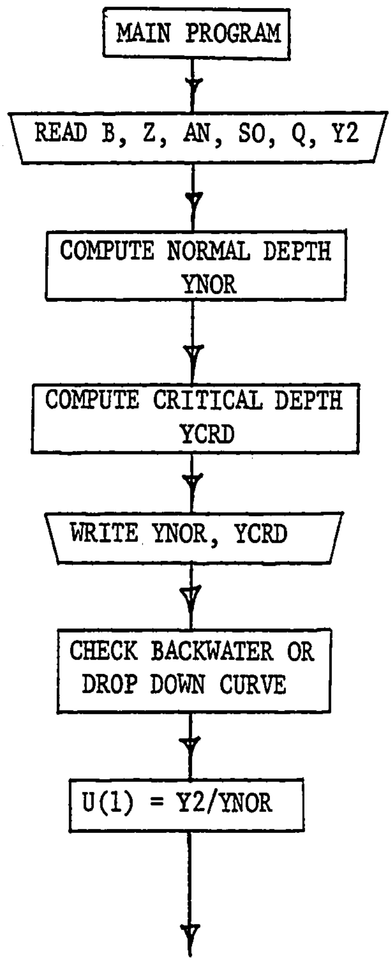
General Fortran IV Program for Water

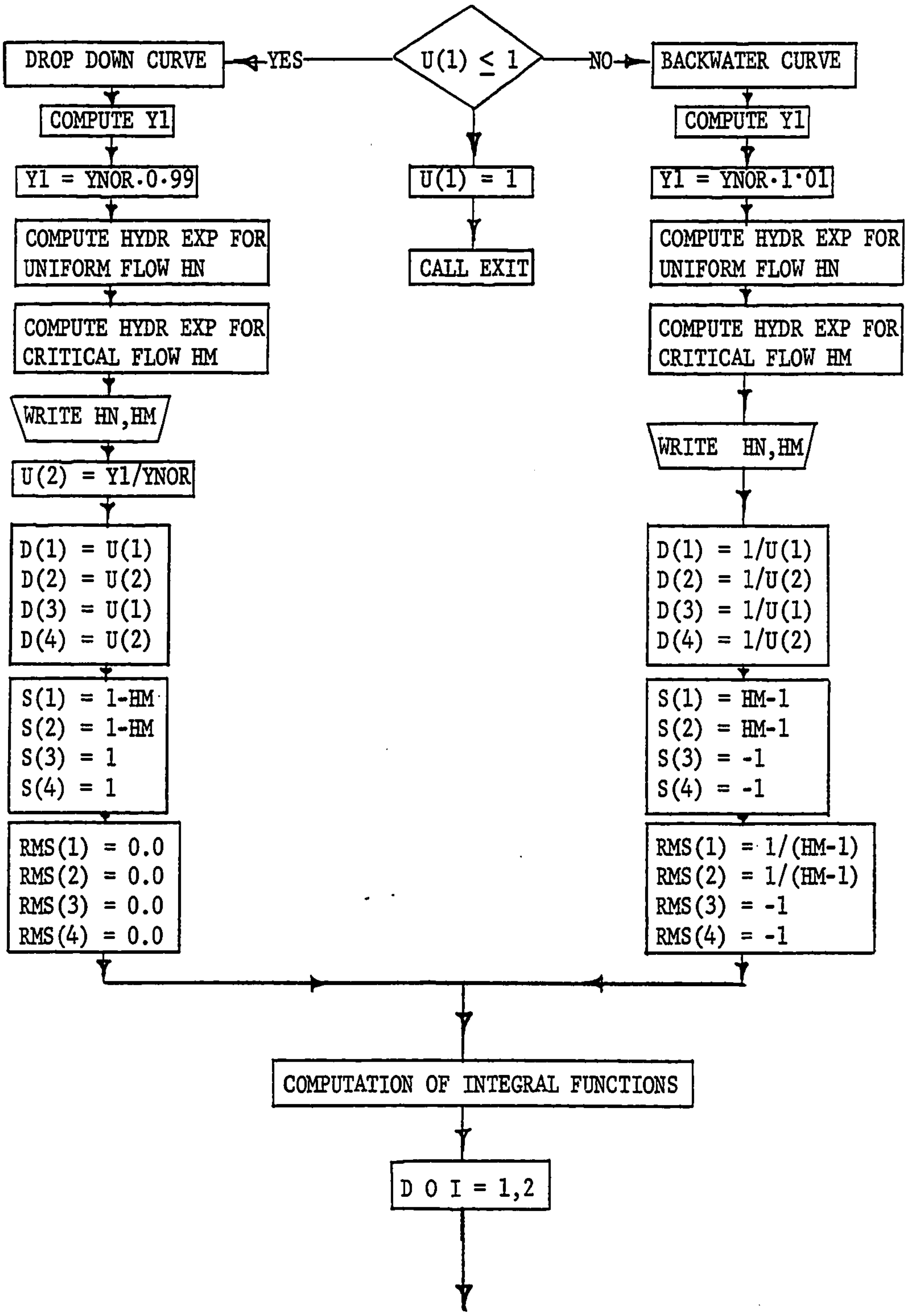
Surface Profile Computations by

Direct Integration Method

FLOW CHART FOR WATER SURFACE PROFILE COMPUTATIONS BY DIRECT INTEGRATION METHOD (POSITIVE BED SLOPE)




MAIN PROGRAM
READ B, Z, AN, SO, Q, Y2
COMPUTE NORMAL DEPTH YNOR
COMPUTE CRITICAL DEPTH YCRD
WRITE YNOR, YCRD
CHECK BACKWATER OR DROP DOWN CURVE
U(1) = Y2/YNOR


U(1) ≤ 1
YES
NO
DROP DOWN CURVE
COMPUTE Y1
Y1 = YNOR·0·99
COMPUTE HYDR EXP FOR UNIFORM FLOW HN
COMPUTE HYDR EXP FOR CRITICAL FLOW HM
WRITE HN,HM
U(2) = Y1/YNOR
D(1) = U(1)
D(2) = U(2)
D(3) = U(1)
D(4) = U(2)
S(1) = 1-HM
S(2) = 1-HM
S(3) = 1
S(4) = 1
RMS(1) = 0.0
RMS(2) = 0.0
RMS(3) = 0.0
RMS(4) = 0.0
U(1) = 1
CALL EXIT
BACKWATER CURVE
COMPUTE Y1
Y1 = YNOR·1·01
COMPUTE HYDR EXP FOR UNIFORM FLOW HN
COMPUTE HYDR EXP FOR CRITICAL FLOW HM
WRITE HN,HM
D(1) = 1/U(1)
D(2) = 1/U(2)
D(3) = 1/U(1)
D(4) = 1/U(2)
S(1) = HM-1
S(2) = HM-1
S(3) = -1
S(4) = -1
RMS(1) = 1/(HM-1)
RMS(2) = 1/(HM-1)
RMS(3) = -1
RMS(4) = -1
COMPUTATION OF INTEGRAL FUNCTIONS
D O I = 1,2

CALL SUBROUTINE SERIS (SUM)

CONTINUE

WRITE SUM(1),SUM(2)

D O I = 3,4

CALL SUBROUTINE SERIS (SUM)

CONTINUE

WRITE SUM(3),SUM(4)

D O I = 1,4

RSUM(I) = SUM(I) + RMS(I)

CONTINUE

$$RUOSM = \left(\frac{YCRD}{YNOR}\right)^{HM} \cdot \frac{1}{(U(2))^{HM-1}} \cdot \left(RSUM(2)\right)$$

$$RUTSM = \left(\frac{YCRD}{YNOR}\right)^{HM} \cdot \frac{1}{(U(1))^{HM-1}} \cdot \left(RSUM(1)\right)$$

RUSMO = U(2) · RSUM(4)

RUSMT = U(1) · RSUM(3)

WATER SURFACE PROFILE LENGTH

SWPL = (YNOR/SO) · (RUSMO - RUSMT - RUOSM + RUTSM)

WRITE SWPL

CALL EXIT

END

APPENDIX B

GENERAL FORTRAN IV PROGRAM FOR WATER SURFACE PROFILE COMPUTATIONS WITH <u>POSITIVE BED SLOPE</u> BY DIRECT INTEGRATION METHOD

```
      SUBROUTINE  SERIS(I,SUM,X,TERM,D,DENOM,S,HN)
      DIMENSION SUM(4),X(4),TERM(4),D(4),DENOM(4),S(4)
      X(I)=(D(I)**HN
      DENOM(I)=HN+S(I)
      SUM(I)=X(I)/DENOM(I)
      TERM(I)=X(I)/DENOM(I)
      DENOM(I)=DENOM(I)+HN
 50   TERM(I)=TERM(I)*X(I)*(DENOM(I)-HN)/DENOM(I)
      SUM(I)=SUM(I)+TERM(I)
      IF(ABS(TERM(I))-.0001)51,51,52
 52   DENOM(I)=DENOM(I)+HN
      GO TO 50
 51   RETURN
      END

C     METHOD FOR PRISMATIC CHANNELS WITH POSITIVE SLOPE
C     SYMBOLS  B=BOTTOM WIDTH,Z=SIDE SLOPE,Q=DISCHARGE
C     SO=BED SLOPE,AN=MANNINGS FRICTION COEFFT
C     Y1=DEPTH OF FLOW DOWNSTREAM,Y2=DEPTH UPSTREAM
C     AL=ALPHA=ENERGY COEFFT=1,SWPL=LENGTH OF PROFILE
      DIMENSION SUM(4),X(4),U(2),D(4),S(4),RMS(4),RSUM(4)
      READ(2,97)B,Z,AN,SO,Q
 97   FORMAT(5F10.4)
      READ(2,98)Y2
 98   FORMAT(F10.4)
C     ROUTINE FOR OBTAINING NORMAL DEPTH YNOR
      CO=(1.+Z**2.)**.5
C     FIRST APPROX FOR NORMAL DEPTH, ASSUME VELOCITY=25.0FPS
      V=25.0
      A=Q/V
C     FIRST GUESS FOR NORMAL DEPTH=YNG
      YNG=A/B
      C=Q*AN/(1.486*(SO**.5))
  5   YNOR=((C**.6)*(B+2.*CO*YNG)**.4)/(B+Z*YNG)
      EPS=YNOR-YNG
      IF(ABS(EPS)-.001)16,16,12
 12   YNG=YNOR
      GO TO 5
 16   WRITE(3,100)YNOR
100   FORMAT(1H ,'YNOR= ',F6.3)
C     ROUTINE FOR OBTAINING CRITICAL DEPTH YCRD
C     FIRST APPROXIMATION FOR CRITICAL DEPTH,ASSUME VELOCITY =100.0 FPS
      VC=100.0
      C=(Q**2./32.2)**.3333
```

```
      A=Q/VC
C     FIRST GUESS FOR CRITICAL DEPTH YCG
      YCG=A/B
    6 YCRD=C*(B+2.*Z*YCG)**.3333/(B+Z*YCG)
      EEPS=YCRD-YCG
      IF(ABS(EEPS)-.001)17,17,13
   13 YCG=YCRD
      GO  TO  6
   17 WRITE93,101)YCRD
  101 FORMAT(1H ,'YCRD= ',F6.3)
      U(1)=Y2/YNOR
      IF(U(1)-1.)25,26,27
   27 Y1=YNOR*1.01
      YAV=(Y1+Y2)/2.
      RY=YAV/B
C     OBTAIN HYDRAULIC EXP FOR UNIFORM FLOW,HN  BACK WATER CURVE
      HN1=3.3333*(1.+2.*Z*RY)/(1.+Z*RY)
      HN2=2.6667*(CO*RY)/(1.+2.*CO*RY)
      HN=HN1-HN2
      WRITE(3,102)HN
  102 FORMAT(1H ,'HN= ',F6.3)
C     OBTAIN HYDRAULIC EXP FOR CRITICAL FLOW,HM  BACK WATER CURVE
      HM1=3.*(1.+2.*Z*RY)/(1.+Z*RY)
      HM2=(2.*Z*RY)/(1.+2.*Z*RY)
      HM=HM1-HM2
      WRITE(3,103)HM
  103 FORMAT(1H ,'HM= ',F6.3)
      U(2)=Y1/YNOR
      D(1)=1./U(1)
      D(2)=1./U(2)
      D(3)=1./U(1)
      D(4)=1./U(2)
      S(1)=HM-1.
      S(2)=HM-1.
      S(3)=-1.
      S(4)=-1.
      RMS(1)=1./(HM-1.)
      RMS(2)=1./(HM-1.)
      RMS(3)=-1.
      RMS(4)=-1.
      GO  TO  8
   25 Y1=YNOR*.99
      YAV=(Y1+Y2)/2.
      RY=YAV/B
C     OBTAIN HYDRAULIC EXP FOR UNIFORM FLOW,HN DROP DOWN CURVE
      HN1=3.3333*(1.+2.*Z*RY)/(1.+Z*RY)
      HN2=2.6667*(CO*RY)/(1.+2.*CO*RY)
      HN=HN1-HN2
      WRITE(3,120)HN
  120 FORMAT(1H ,'HN= ',F6.3)
C     OBTAIN HYDRAULIC EXP FOR CRITICAL FLOW,HM DROP DOWN CURVE
```

```
      HM1=3.*(1.+2.*Z*RY)/(1.+Z*RY)
      HM2=(2.*Z*RY)/(1.+2.*Z*RY)
      WRITE(3,121)HM
      HM=HM1-HM2
  121 FORMAT(1H ,'HM= ',F6.3)
      U(2)=Y1/YNOR
      D(1)=U(1)
      D(2)=U(2)
      D(3)=U(1)
      D(4)=U(2)
      S(1)=1.-HM
      S(2)=1.-HM
      S(3)=1.
      S(4)=1.
      RMS(1)=0.0
      RMS(2)=0.0
      RMS(3)=0.0
      RMS(4)=0.0
C     COMPUTATIONS OF INTEGRAL FUNCTIONS START
    8 DO  9  I=1,2
      CALL  SERIS(I,SUM,X,TERM,D,DENOM,S,HN)
    9 CONTINUE
      WRITE(3,30)SUM(1),SUM(2)
   30 FORMAT(1H ,'SUM(1)= ',E15.8,1H ,'SUM(2)= ',E15.8
      DO  10  I=3,4
      CALL  SERIS(I,SUM,X,TERM,D,DENOM,S,HN)
   10 CONTINUE
      WRITE(3,31)SUM(3),SUM(4)
   31 FORMAT(1H ,'SUM(3)= ',E15.8,1H ,'SUM(4)= ',E15.8)
      DO  81  I=1,4
      RSUM(I)=SUM(I)+RMS(I)
   81 CONTINUE
      FTERM=(YCRD/YNOR)**HM
      SS=HM-1.
      RUO=1./(U(2))**SS
      RUT=1./(U(1))**SS
      RUOSM=RUO*RSUM(2)
      RUTSM=RUT*RSUM(1)
      BAA=FTERM*(RUOSM-RUTSM)
      RUSMO=(U(2))*RSUM(4)
      RUSMT=(U(1))*RSUM(3)
      BAB=RUSMO-RUSMT
      SWPL=(YNOR/SO)*(BAB-BAA)
      WRITE(3,106)SWPL
  106 FORMAT(1H ,'SWPL= ',E15.8)
      GO  TO  28
   26 CALL EXIT
   28 CALL EXIT
      END
```

GENERAL FORTRAN IV FOR WATER SURFACE PROFILE COMPUTATIONS
WITH NEGATIVE BED SLOPE BY DIRECT INTEGRATION METHOD

```
      SUBROUTINE  SERIS(I,SUM,X,D,HN,S,E,F,G,H,O,P,T,W)
      DIMENSION SUM(4),X(4),D(4),S(4),E(4),F(4)
      DIMENSION G(4),H(4),O(4),P(4),T(4),W(4)
      X(I)=(D(I)**HN
      E(I)=.9999964239*(X(I)/(HN+S(I))
      F(I)=.9997482476*((X(I))**2.)/(1.*HN+S(I))
      G(I)=.9953970774*((X(I))**3.)/(3.*HN+S(I))
      H(I)=.9629353236*((X(I))**4.)/(4.*HN+S(I))
      O(I)=.8381703515*((X(I))**5.)/(5.*HN+S(I))
      P(I)=.5719763382*((X(I))**6.)/(6.*HN+S(I))
      T(I)=.2526194559*((X(I))**7.)/(7.*HN+S(I))
      W(I)=.0516283536*((X(I))**8.)/(8.*HN+S(I))
      SUM(I)=E(I)-F(I)+G(I)-H(I)+O(I)-P(I)+T(I)-W(I)
      RETURN
      END

C     METHOD FOR PRISMATIC CHANNELS WITH NEGATIVE SLOPE
C     SYMBOLS  B=BOTTOM WIDTH,Z=SIDE SLOPE,Q=DISCHARGE
C     SO=BED SLOPE,AN=MANNINGS FRICTION COEFFT
C     Y1=DEPTH OF FLOW DOWNSTREAM,Y2=DEPTH UPSTREAM
C     AL=ALPHA=ENERGY COEFFT=1,SWPL=LENGTH OF PROFILE
      DIMENSION SUM(4),X(4),U(2),D(4),S(4),RMS(4),RSUM(4)
      READ(2,97)B,Z,AN,SO,Q
   97 FORMAT(5F10.4)
      READ(2,98)Y2
   98 FORMAT(F10.4)
C     ROUTINE FOR OBTAINING NORMAL DEPTH YNOR
      CO=(1.+Z**2.)**.5
C     FIRST APPROX FOR NORMAL DEPTH,ASSUME VELOCITY=25.0FPS
      V=25.0
      A=Q/V
C     FIRST GUESS FOR NORMAL DEPTH=YNG
      YNG=A/B
      C=Q*AN/(1.486*(SO**.5))
    5 YNOR=((C**.6)*(B+2.*CO*YNG)**.4)/(B+Z*YNG)
      EPS=YNOR-YNG
      IF(ABS(EPS)-.001)16,16,12
   12 YNG=YNOR
      GO TO  5
   16 WRITE(3,100)YNOR
  100 FORMAT(1H ,'YNOR= ',F6.3)
C     ROUTINE FOR OBTAINING CRITICAL DEPTH YCRD
C     FIRST APPROXIMATION FOR CRITICAL DEPTH,ASSUME VELOCITY=100.0 FPS
      VC=100.0
      C=(Q**2./32.2)**.3333
```

```
      A=Q/VC
C     FIRST GUESS FOR CRITICAL DEPTH YCG
      YCG=A/B
    6 YCRD=(C*(B+2.*Z*YCG)**.3333/(B+Z*YCG)
      EEPS=YCRD-YCG
   13 IF(ABS(EEPS)-.001)17,17,13
      GO  TO  6
   17 WRITE(3,101)YCRD
  101 FORMAT(1H ,'YCRD= ',F6.3)
      U(1)=Y2/YNOR
      IF(U(1)-1.)25,26,27
   27 Y1=YNOR*1.01
      YAV=(Y1+Y2)/2.
      RY=YAV/B
C     OBTAIN HYDRAULIC EXP FOR UNIFORM FLOW, HN  BACK WATER CURVE
      HN1=3.3333*(1.+2.*Z*RY)/(1.+Z*RY)
      HN2=2.6667*(CO*RY)/(1.+2.*CO*RY)
      HN=HN1-HN2
      WRITE(3,102)HN
  102 FORMAT(1H ,'HN= ',F6.3)
C     OBTAIN HYDRAULIC EXP FOR CRITICAL FLOW, HM  BACK WATER CURVE
      HM1=3.*(1.+2.*Z*RY)/(1.+Z*RY)
      HM2=(2.*Z*RY)/(1.+2.*Z*RY)
      HM=HML-HM2
      WRITE(3,103)HM
  103 FORMAT(1H ,'HM= ',F6.3)
      U(2)=Y1/YNOR
      D(1)=1./U(1)
      D(2)=1./U(2)
      D(3)=1./U(1)
      D(4)=1./U(2)
      S(1)=HM-1.
      S(2)=HM-1.
      S(3)=-1.
      S(4)=-1.
      RMS(1)=1./(HM-1.)
      RMS(2)=1./(HM-1.)
      RMS(3)=-1.
      RMS(4)=-1.
      GO  TO  8
   25 Y1=YNOR*.99
      YAV=(Y1+Y2)/2.
      RY=YAV/B
C     OBTAIN HYDRAULIC EXP FOR UNIFORM FLOW,HN DROP DOWN CURVE
      HN1=3.3333*(1.+2.*Z*RY)/(1.+Z*RY)
      HN2=2.6667*(C)*RY)/(1.+2.*CO*RY)
      HN=HN1-HN2
      WRITE(3,120)HN
  120 FORMAT(1H ,'HN= ',F6.3)
```

```
C     OBTAIN HYDRAULIC EXP FOR CRITICAL FLOW,HM DROP DOWN CURVE
      HM1=3.*(1.+2.*Z*RY)/(1.+Z*RY)
      HM2=(2.*Z*RY)/(1.+2.*Z*RY)
      WRITE(3,121)HM
      HM=HM1-HM2
  121 FORMAT(1H ,'HM ',F6.3)
      U(2)=Y1/YNOR
      D(1)=U(1)
      D(2)=U(2)
      D(3)=U(1)
      D(4)=U(2)
      S(1)=1.-HM
      S(2)=1.-HM
      S(3)=1.
      S(4)=1.
      RMS(1)=0.0
      RMS(2)=0.0
      RMS(3)=0.0
      RMS(4)=0.0
C     COMPUTATIONS OF INTEGRAL FUNCTIONS START
    8 DO    9   I=1,2
      CALL SERIS(I,SUM,X,D,HN,S,E,F,G,H,O,P,T,W)
    9 CONTINUE
      WRITE(3,30)SUM(1),SUM(2)
   30 FORMAT(1H ,'SUM(1)= ',E15.8,1H ,'SUM(2)= ',E15.8)
      DO   10   I=3,4
      CALL SERIS(I,SUM,X,D,HN,S,E,F,G,H,O,P,T,W)
   10 CONTINUE
      WRITE(3,31)SUM(3),SUM(4)
   31 FORMAT(1H ,'SUM(3)= ',E15.8,1H ,'SUM(4)= ',E15.8)
      DO   81  I=1,4
      RSUM(I)=SUM(I)+RMS(I)
   81 CONTINUE
      FTERM=(YCRD/YNOR)**HM
      SS=HM-1.
      RUO=1./(U(2))**SS
      RUT=1./(U(1))**SS
      RUOSM=RUO*RSUM(2)
      RUTSM=RUT*RSUM(1)
      BAA=FTERM*(RUOSM-RUTSM)
      RUSMO=(U(2))*RSUM(4)
      RUSMT=(U(1))*RSUM(3)
      BAB=RUSMO-RUSMT
      SWPL=(YNOR/SO)*(BAB-BAA)
      WRITE(3,106)SWPL
  106 FORMAT(1H ,'SWPL= ',E15.8)
      TO  TO  28
   26 CALL EXIT
   28 CALL EXIT
      END
```

APPENDIX C

WATER SURFACE PROFILE COMPUTATIONS

STEP METHODS WITH ERROR ANALYSIS

APPENDIX C

WATER SURFACE PROFILE COMPUTATIONS DIRECT STEP METHOD WITH ERROR ANALYSIS

```
C      COMPUTE WATER SURFACE PROFILE DIRECT STEP METHOD IN PRISMATIC CHANNELS
C      SYMBOLS  W=BOTTOM WIDTH,Z=SIDE SLOPE,Q=DISCHARGE
C      SO=BED SLOPE,AN=MANNINGS FRICTION COEFFT
C      Y1=DEPTH OF FLOW DOWN STREAM,Y2=DEPTH UPSTREAM
C      ALPHA=ENERGY COEFFT=1,SUMX=LENGTH OF PROFILE
       DIMENSION D(5),A(5),P(5),B(5),V(5),SFR(5),RSFR(5)
       READ(2,95)Y2
   95  FORMAT(F10.4)
       READ(2,97)W,Z,AN,SO,Q
   97  FORMAT(5F10.4)
       READ(2,98)HN,ERR
   98  FORMAT(2F10.4)
C      ROUTINE FOR OBTAINING NORMAL DEPTH YNOR
       CO=(1.+Z**2.)**.5
C      FIRST APPROX FOR NORMAL DEPTH,ASSUME VELOCITY=25.0 FPS
       VEL=25.0
       AREA=Q/VEL
C      FIRST GUESS FOR NORMAL DEPTH YNG
       YNG=AREA/W
       C=Q*AN/(1.486*(SO**.5))
    5  YNOR=((C**.6)*(W+2.*CO*YNG)**.4)/(W+Z*YNG)
       EPS=YNOR-YNG
       IF(ABS(EPS)-.001)16,16,12
   12  YNG=YNOR
       GO TO  5
   16  WRITE(3,100)YNOR
  100  FORMAT(1H ,'YNOR= ',F6.3)
C      DETERMINE  ECONOMICAL  STEP
       U=Y2/YNOR
       IF(U-1.)25,26,27
C      DROP DOWN CURVE
   25  Y1=YNOR*.99
       GO  TO  28
C      BACK  WATER CURVE
   27  Y1=YNOR*1.01
   28  STEP=(Y2-Y1)/HN
   50  YNEW=Y1+STEP*4.0
       D(1)=YNEW
       D(2)=D(1)-STEP
       D(3)=D(2)-STEP
       D(4)=D(3)-STEP
```

```
      D(5)=D(4)-STEP
      DO 6 I=1,5
      A(I)=(W+Z*D(I))*D(I)
      P(I)=W+2.*D(I)*((1.+Z**2.)**.5)
      B(I)=(A(I)/P(I))**1.3333
      V(I)=Q/A(I)
      SFR(I)=((AN**2.)*(V(I))**2.)/(2.22*B(I))
      RSFR(I)=1./(SO-SFR(I))
    6 CONTINUE
      EEPS=RSFR(1)-4.*(RSFR(2)+RSFR(4))+6.*RSFR(3)+RSFR(5)
      TEPS=EEPS*(Y2-Y1)/180.0
      IF(ABS(TEPS)-ERR)18,18,17
   17 STEP=STEP/2.
      GO TO 50
   18 DELY=STEP
      WRITE(3,101)DELY
  101 FORMAT(1H ,'DELY= ',F6.3)
      SUMX=0.0
   10 Y=Y2
      AS=(W+Z*Y)*Y
      PS=W+2.*Y*CO
      RS=AS/PS
      BS=RS**1.3333
      VS=Q/AS
      VSS=VS**2.
      VHS=VSS/64.4
      E=Y+VHS
      SF=(AN**2.)*VSS/(2.22*BS)
      SR=SO-SF
      RSR=1./SR
      DP=Y2-DELY/2.
      YP=DP
      AP=(W+Z*YP)*YP
      PP=W+2.*YP*CO
      RP=AP/PP
      BP=RP**1.3333
      VP=Q/AP
      VSP=VP**2.
      VHP=VSP/64.4
      SFP=(AN**2.)*VSP/(2.22*BP)
      SRP=SO-SFP
      RSRP=1./SRP
      DDP=DP-DELY/2.
      YDP=DDP
      ADP=(W+Z*YDP)*YDP
      PDP=W+2.*YDP*CO
      RDP=ADP/PDP
      BDP=RDP**1.3333
      VDP=Q/ADP
      VSDP=VDP**2.
      VHDP=VSDP/64.4
```

```
      EDP=YDP+VHDP
C     CHANGE IN TOTAL ENERGY
      DLTAE=(E-EDP)/2.
      SFDP=(AN**2.)*VSDP/(2.22*BDP)
      SRDP=SO-SFDP
      RSRDP=1./SRDP
      SOSF=(1./3.)*(RSR+4.*RSRP+RSRDP)
C     SMALL REACH LENGTH
      DLTAX=DLTAE*SOSF
      WRITE(3,102)Y,DLTAE,RSR,RSRP,DLTAX,SUMX
  102 FORMAT(1X,6(E15.8,1X))
      IF(Y2-Y1)13,13,11
   11 Y2=Y2-DELY
      SUMX=SUMX+DLTAX
      GO TO 10
   26 CALL EXIT
   13 CALL EXIT
      END
```

APPENDIX C

WATER SURFACE PROFILE COMPUTATIONS STANDARD STEP METHOD WITH ERROR ANALYSIS

```
C       COMPUTE WATER SURFACE PROFILE STANDARD STEP METHOD
C       METHOD FOR PRISMATIC CHANNELS WITH POSITIVE SLOPE
C       SYMBOLS  W=BOTTOM WIDTH,Z=SIDE SLOPE,Q=DISCHARGE
C       SO=BED SLOPE,AN=MANNINGS FRICTION COEFFT
C       Y1=DEPTH OF FLOW DOWN STREAM,Y2=DEPTH UPSTREAM
C       ALPHA=ENERGY COEFFT=1,DIST=LENGTH OF PROFILE
        DIMENSION  D(3),A(3),P(3),B(3),V(3),SFR(3)
        READ(2,94)DIST
   94   FORMAT(F10.4)
        READ(2,95)Y2
   95   FORMAT(f10.4)
        READ(2,97)W,Z,AN,SO,Q
   97   FORMAT(5F10.4)
        READ(2,98)HN,ERR
   98   FORMAT(2F10.4)
C       ROUTINE FOR OBTAINING NORMAL DEPTH YNOR
        CO=(1.+Z**2.)**.5
C       FIRST APPROX FOR NORMAL DEPTH,ASSUME VELOCITY=25.0 FPS
        VEL=25.0
        AREA=Q/VEL
C       FIRST GUESS FOR NORMAL DEPTH YNG
        YNG=AREA/W
        C=Q*AN/(1.486*(SO**.5))
    5   YNOR=((C**.6)*(W+2.*CO*YNG)**.4)/(W+2*YNG)
        EPS=YNOR-YNG
        IF(ABS(EPS)-.001)16,16,12
   12   YNG=YNOR
        GO  TO  5
   16   WRITE(3,100)YNOR
  100   FORMAT(1H ,'YNOR= ',F6.3)
C       DETERMINE ECONOMICAL  STEP
        U=Y2/YNOR
        IF(U-1.)25,26,27
C       DROP  DOWN  CURVE
   25   Y1=YNOR*99
        GO  TO  28
C       BACK  WATER CURVE
   27   Y1=YNOR*1.01
   28   STEP=(Y2-Y1)/HN
   50   YNEW=Y1+STEP*2.0
        D(1)=YNEW
        D(2)=D(1)-STEP
        D(3)=D(2)-STEP
        DO   6   I=1,3
        A(I)=(W+Z*D(I))*D(I)
```

```
      P(I)=W+2.*D(I)*((1.+Z**2.)**.5)
      B(I)=(A(I)/P(I))**1.3333
      V(I)=Q/A(I)
      SFR(I)=((AN**2.)*(V(I))**2.)/(2.22*B(I))
    6 CONTINUE
      EEPS=SFR(1)-2.*SFR(2)+SFR(3)
      IF(ABS(EEPS)-ERR)18,18,17
   17 STEP=STEP/2.
      GO  TO  50
   18 DELY=STEP
      WRITE(3,101)DELY
  101 FORMAT(1H ,'DELY= ',F6.3)
      DNS=(Y2-Y1)/DELY
      DX=DIST/DNS
      SUMX=0.0
   10 Y=Y2
      ITN=1
      AS=(W+Z*Y)*Y
      PS=W+2.*Y*CO
      RS=AS/PS
      BS=RS**1.3333
      VS=Q/AS
      VSS=VS**2.
      VHS=VSS/64.4
      SF=(AN**2.)*VSS/(2.22*BS)
      SFDX=SF*DX/2.
      SDX=SO*DX
C     FIRST APPROX FOR UPSTREAM DEPTH OF SUBREACH
      GAP=Q/25.0
C     FIRST GUESS FOR UPSTREAM DEPTH OF SUBREACH
      YP=GAP/W
   15 AP=(W+Z*YP)*YP
      VP=Q/AP
      VSP=VP**2.
      VHP=VSP/64.4
      PP=W+2.*CO*YP
      RP=AP/PP
      BP=RP**1.3333
      SFDP=((AN**2.)*VSP)/(2.22*BP)
      SFDPX=SFDP*DX/2.
      AYNEW=Y+VHS+SFDX-SDX-VHP
      SEPS=AYNEW-YP
      IF(ABS(SEPS)-.001)19,19,20
   20 ITN=ITN+1
      IF(ITN-15)44,45,45
   44 YP=AYNEW
      GO  TO  15
   45 CALL EXIT
   19 YTWO=AYNEW
      WRITE(3,105)YTWO,SUMX
```

```
105  FORMAT(1X,2(E15.8,1X))
     IF(SUMX-DIST)21,22,22
 21  SUMX=SUMX+DX
     Y2=YTWO
     GO  TO  10
 26  CALL EXIT
 22  CALL EXIT
     END
```