Note to the Editor

Theoretical and Experimental Bases for More Precise Elastic Properties of Epoxy

Sir:

This note is concerned with determination of more precise elastic properties for epoxy resin. In the discussions of several papers presented at the Third Annual Symposium on High Performance Composites held at Washington University, October 26-27, 1967, considerable discrepancy was noted between the measured shear modulus G of epoxy and the value predicted by the classical isotropic elastic relation:

$$G = E/[2(1+\nu)] \tag{1}$$

where E = Young's modulus, and ν = Poisson's ratio. The purpose here is to derive a more accurate relation than equation (1) by considering the fact that epoxy has different properties in tension and compression and to check its validity with results of carefully controlled experiments. The resulting value can be used in various micromechanics analyses to predict elastic properties of epoxy-matrix composites reinforced with various fibers.

THEORETICAL CONSIDERATIONS

The elastic strain energy per unit volume, U, for an isotropic material subjected to a pure, uniform shear-stress field is given by

$$U = \sigma_{xy}\epsilon_{xy}/2 = \sigma_{xy}^2/2G \tag{2}$$

where σ_{xy} = shear stress and ϵ_{xy} = shear strain.

By means of the Mohr stress circle for a pure shear-stress field, it can be shown easily that the pure shear-stress field can be replaced by a biaxial normal-stress field $(\sigma_{11}, \sigma_{22})$ oriented at 45° to the x, y axes, respectively, such that $-\sigma_{22} = \sigma_{11} = \sigma_{xy}$. The strain energy associated with σ_{11} and σ_{22} is

$$U = (\sigma_{11}\epsilon_{11} + \sigma_{22}\epsilon_{22})/2 \tag{3}$$

where ϵ_{11} and ϵ_{22} are the normal strains in directions 1 and 2. Generalized Hooke's law for an isotropic material with different elastic properties in tension (subscript t) and compression (subscript c) and subject to a tensile principal stress in direction 1 and a compressive principal stress in direction 2 can be expressed as follows:

$$\epsilon_{11} = (\sigma_{11}/E_t) - (\nu_c/E_c)\sigma_{22}, \ \epsilon_{22} = -(\nu_t/E_t)\sigma_{11} + (\sigma_{22}/E_c) \qquad (4)$$

Assuming isothermal conditions, conservation of energy [1] requires that the compliance matrix be symmetric about its main diagonal, i.e.

$$\nu_c/E_c = \nu_t/E_t \qquad (5)$$

Putting equations (4) into equation (3) and then equating the resulting expression for U with that in equation (2) yields the following result:

$$G = [(1+\nu_t)E_t^{-1} + (1+\nu_c)E_c^{-1}]^{-1} \qquad (6)$$

When $E_c \neq E_t$, G predicted by equation (6) can be quite different from that predicted by equation (1). For materials having $E_c = E_t = E$, $\nu_c = \nu_t = \nu$ and equation (6) reduces to equation (1). Equation (6) was also derived in [2] in a different way.

EXPERIMENTAL RESULTS

Experimental determinations of the elastic properties of Epon 828/1031/MNA/BDMA epoxy were conducted on resin castings cured as follows: 2 hr. at 190°F, 1 hr. at 250°F, 1 hr. at 300°F, and 2 hr. at 350°F. Tension and compression tests were made on an Instron machine at a crosshead speed of 0.01 in./min. Axial strain was measured with an extensometer and transverse strain was measured simultaneously with a strain gage. Shear modulus was determined by the solid-rod torsion test from a plot of angle of twist versus torque.

In all cases, the load-strain curve consisted of an initial linearly elastic portion, followed by a smoothly curving function until failure. All moduli were obtained from the initial linear portion, and the results are summarized in Table 1.

Table 1. Elastic Properties of Epon 828/1031/MNA/BDMA

Property	No. of Tests	Mean Value
E_t	9	0.547×10^6 psi
ν_t	5	0.334
E_c	22	0.591×10^6 psi
ν_c	14	0.386
G	13	0.212×10^6 psi

COMPARISON OF THEORETICAL PREDICTIONS WITH EXPERIMENTAL RESULTS

Table 2 presents results of various theoretical predictions.

Table 2. Various Theoretical Predictions

Relation	Predicted Value of G, psi
I. $E_t/[2(1+\nu_t)]$	0.205×10^6
II. $E_c/[2(1+\nu_c)]$	0.213×10^6
III. $[(1+\nu_t)E_t^{-1}+(1+\nu_c)E_c^{-1}]^{-1}$	0.209×10^6
IV. $[(1+2\nu_t)E_t^{-1}+E_c^{-1}]^{-1}$	0.211×10^6

In practice, it is frequently easier to obtain accurate tensile properties than compressive properties. Consequently, relation I would be the more commonly used form of equation (1). The fact that relation II gives better agreement with the experimental value is interesting and merits further investigation. Relation III is identical to equation (6); also, it is noted that

$$G_{\mathrm{III}}^{-1} = 2(G_{\mathrm{I}}^{-1} + G_{\mathrm{II}}^{-1})$$

The aforementioned difficulty in conducting accurate compressive tests suggests that the value of ν_c computed by the reciprocity relation equation (5), might be more accurate than the experimental value. Substituting equation (5) into equation (6) yields relation IV, which results in a value of shear modulus very close to the measured value, 0.212×10^6 psi.

CONCLUSION

It is concluded that relations III and IV, which take into account the difference between elastic properties in tension and compression, give better predictions of the shear modulus of this epoxy than does the commonly used form of the classical relation, equation (1).

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