Theoretical and Experimental Bases for More Precise Elastic Properties of Epoxy

Sir:

This note is concerned with determination of more precise elastic properties for epoxy resin. In the discussions of several papers presented at the Third Annual Symposium on High Performance Composites held at Washington University, October 26-27, 1967, considerable discrepancy was noted between the measured shear modulus $G$ of epoxy and the value predicted by the classical isotropic elastic relation:

$$G = \frac{E}{2(1 + \nu)}$$

where $E = \text{Young's modulus}$, and $\nu = \text{Poisson's ratio}$. The purpose here is to derive a more accurate relation than equation (1) by considering the fact that epoxy has different properties in tension and compression and to check its validity with results of carefully controlled experiments. The resulting value can be used in various micromechanics analyses to predict elastic properties of epoxy-matrix composites reinforced with various fibers.

**THEORETICAL CONSIDERATIONS**

The elastic strain energy per unit volume, $U$, for an isotropic material subjected to a pure, uniform shear-stress field is given by

$$U = \frac{\sigma_{xy}\epsilon_{xy}}{2} = \frac{\sigma_{xy}^2}{2G}$$

where $\sigma_{xy}$ = shear stress and $\epsilon_{xy}$ = shear strain.

By means of the Mohr stress circle for a pure shear-stress field, it can be shown easily that the pure shear-stress field can be replaced by a biaxial normal-stress field $(\sigma_{11}, \sigma_{22})$ oriented at $45^\circ$ to the $x, y$ axes, respectively, such that $-\sigma_{22} = \sigma_{11} = \sigma_{xy}$. The strain energy associated with $\sigma_{11}$ and $\sigma_{22}$ is

$$U = (\sigma_{11}\epsilon_{11} + \sigma_{22}\epsilon_{22})/2$$

where $\epsilon_{11}$ and $\epsilon_{22}$ are the normal strains in directions 1 and 2. Generalized Hooke's law for an isotropic material with different elastic properties in tension (subscript $t$) and compression (subscript $c$) and subject to a tensile principal stress in direction 1 and a compressive principal stress in direction 2 can be expressed as follows:

\[ J. \text{COMPOSITE MATERIALS, Vol. 2, No. 4 (Oct. 1968), p. 506.} \]
Assuming isothermal conditions, conservation of energy [1] requires that the compliance matrix be symmetric about its main diagonal, i.e.

$$\epsilon_{11} = \left( \frac{\sigma_{11}}{E_t} \right) - \left( \frac{\nu_c}{E_c} \right) \sigma_{22}, \quad \epsilon_{22} = -\left( \frac{\nu_t}{E_t} \right) \sigma_{11} + \left( \frac{\sigma_{22}}{E_c} \right)$$

(4)

Putting equations (4) into equation (3) and then equating the resulting expression for $U$ with that in equation (2) yields the following result:

$$G = \left\{ \left( 1 + \nu_t \right) E_t^{-1} + \left( 1 + \nu_c \right) E_c^{-1} \right\}^{-1}$$

(6)

When $E_c \neq E_t$, $G$ predicted by equation (6) can be quite different from that predicted by equation (1). For materials having $E_c = E_t = E$, $\nu_c = \nu_t = \nu$ and equation (6) reduces to equation (1). Equation (6) was also derived in [2] in a different way.

**EXPERIMENTAL RESULTS**

Experimental determinations of the elastic properties of Epon 828/1031/MNA/BDMA epoxy were conducted on resin castings cured as follows: 2 hr. at 190°F, 1 hr. at 250°F, 1 hr. at 300°F, and 2 hr. at 350°F. Tension and compression tests were made on an Instron machine at a crosshead speed of 0.01 in./min. Axial strain was measured with an extensometer and transverse strain was measured simultaneously with a strain gage. Shear modulus was determined by the solid-rod torsion test from a plot of angle of twist versus torque.

In all cases, the load-strain curve consisted of an initial linearly elastic portion, followed by a smoothly curving function until failure. All moduli were obtained from the initial linear portion, and the results are summarized in Table 1.

<table>
<thead>
<tr>
<th>Property</th>
<th>No. of Tests</th>
<th>Mean Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_t$</td>
<td>9</td>
<td>0.547 x 10⁶ psi</td>
</tr>
<tr>
<td>$\nu_t$</td>
<td>5</td>
<td>0.334</td>
</tr>
<tr>
<td>$E_c$</td>
<td>22</td>
<td>0.591 x 10⁶ psi</td>
</tr>
<tr>
<td>$\nu_c$</td>
<td>14</td>
<td>0.386</td>
</tr>
<tr>
<td>$G$</td>
<td>13</td>
<td>0.212 x 10⁶ psi</td>
</tr>
</tbody>
</table>

**COMPARISON OF THEORETICAL PREDICTIONS WITH EXPERIMENTAL RESULTS**

Table 2 presents results of various theoretical predictions.
Table 2. Various Theoretical Predictions

<table>
<thead>
<tr>
<th>Relation</th>
<th>Predicted Value of G, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. $E_t/[2(1 + \nu_t)]$</td>
<td>$0.205 \times 10^6$</td>
</tr>
<tr>
<td>II. $E_t/[2(1 + \nu_t)]$</td>
<td>$0.213 \times 10^6$</td>
</tr>
<tr>
<td>III. $[(1 + \nu_t)E_t^{-1} + (1 + \nu_c)E_c^{-1}]^{-1}$</td>
<td>$0.209 \times 10^6$</td>
</tr>
<tr>
<td>IV. $[(1 + 2\nu_t)E_t^{-1} + E_c^{-1}]^{-1}$</td>
<td>$0.211 \times 10^6$</td>
</tr>
</tbody>
</table>

In practice, it is frequently easier to obtain accurate tensile properties than compressive properties. Consequently, relation I would be the more commonly used form of equation (1). The fact that relation II gives better agreement with the experimental value is interesting and merits further investigation. Relation III is identical to equation (6); also, it is noted that

$$G_{III}^{-1} = 2(G_{T}^{-1} + G_{II}^{-1})$$

The aforementioned difficulty in conducting accurate compressive tests suggests that the value of $\nu_c$, computed by the reciprocity relation equation (5), might be more accurate than the experimental value. Substituting equation (5) into equation (6) yields relation IV, which results in a value of shear modulus very close to the measured value, $0.212 \times 10^6$ psi.

CONCLUSION

It is concluded that relations III and IV, which take into account the difference between elastic properties in tension and compression, give better predictions of the shear modulus of this epoxy than does the commonly used form of the classical relation, equation (1).

The first author wishes to acknowledge that the experimental work cited was supported by Naval Air Systems Command Contract N00019-67-C-0354.

Richard C. Novak
Aeronutronic Division
Philco-Ford Corporation
Newport Beach, California

Charles W. Bert
School of Aerospace and Mechanical Engineering
University of Oklahoma
Norman, Oklahoma

REFERENCES


(received May 24, 1968)