# THE STATE MATRIX METHOD FOR 

 THE SYNTHESIS OF DIGITALLOGIC SYSTEMS

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## LOGIC SYSTEMS

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## CHAPTER I

## INTRODUCTION


#### Abstract

The technology of switching circuit theory, although relatively young, has found great application and utility in modern design. Most of the theory has been developed for application in electrical engineering since electronics has dominated the field of computation and logic for the last dew decades.

Recent years have seen a rebirth of the use of a fluid medium to perform the logic and computation in sequential machines. The newly emerging field of fluid technology termed "fluidics" is one major reason for this rebirth of fluid logic. Since fluid power is often used as the muscle of machines, it is convenient also to use the fluid itself for the required computation in order to avoid the electrical to fluid interfaces.

To realize maximum utilization of fluid logic devices, it is necessary to develop a technology of switching circuits applicable to fluid circuits. The theory should consider the unique properties of fluid devices not only in the implementation of circuits, but also in the synthesis procedure itself. The synthesis procedure presented in this


thesis does take advantage of the unique properties of devices in order to produce simple fluid circuits containing minimal hardware.

## Background

Modern switching theory had its origin in 1938 when C. E. Shannon (9), of M.I.T., applied the laws of Boolean algebra to the representation of electrical switches. ${ }^{1}$ Although this was a great advancement for combinational switching circuits, there was no formal procedure for the synthesis of sequential switching circuits until 1954 when D. A. Huffman (3) and E. F. Moore (8) independently developed the synthesis technique which is used today. This technique has gained such widespread use and application that today it is taught at every major university and is even referred to as the "classical method". The synthesis procedure presented in this thesis relies upon much of the notation of the classical method. The reader not familiar with this method, should refer to a book on classical switching theory (2), (5), (7), (8).
E. C. Fitch (2), of Oklahoma State University, was one of the first authors to apply the methods of sequential switching circuit theory to hydraulics. However, his work did not take into account any special properties of hydraulic valves except in the implementation of logic circuits.

[^0]Later work at Oklahoma State University by J. H. Cole (1) did consider the properties of devices in the synthesis procedure. Dr. Cole used the properties of the passive memory devices to produce extremely simple circuits for the feedback sequential type problem. This work has been a major advancement for the field even though its scope of application is limited.
G. E. Maroney (6) extended Cole's tabular method to include the random input type circuit. This method was fundamentally the same as Cole's except that the random input possibility necessarily complicated the execution of the method. This technique also utilized the passive memory effect to reduce hardware.

Development of the State Matrix Method

The state matrix synthesis procedure evolved from the assumption that the outputs are related to the inputs and the past state of the system. This relationship can be written in matrix form as:

$$
[\mathrm{Z}]=[\mathrm{M}][\mathrm{X}]
$$

Here, the outputs are contained in the $[Z]$ vector, the $[X]$ vector contains the inputs, and the matrix $[M]$ contains output and memory information. This binary matrix changes with time to yield different outputs representing the different states of a sequence.


#### Abstract

Early experiments with this type of synthesis were restricted to the feedback sequential type problems because of their simplicity. A close examination of the resulting equations revealed that they were essentially identical to those obtained from Cole's method. This was very encouraging since Cole's method was known to produce valid expressions. The matrix arrangement of this method also gave insight to many of the hidden subtleties of Cole's method. Once the rules for the synthesis of feedback sequential circuits using state matrices were defined, the method was extended to handle the random input problems. The main difference between the state matrix methods for random input and feedback sequential problems was the input vector used. The feedback sequential input vector contained only the changed input, whereas the random input vector contained the total input state.

The random input form of the state matrix synthesis procedure has since received more attention since it is the more general procedure. This form will also handle the feedback sequential problems in some respects better than the original state matrix method. Hereafter, the random input form of this method will be referred to simply as the "state matrix method", and the method using the changed input vector will be referred to as the "feedback sequential state matrix method."


Scope and Results of Study

Although the state matrix synthesis procedure is the most important item in this thesis, many other original topics have arisen from this study. The major accomplishments of this study are:
(1) The development of the feedback sequential state matrix synthesis procedure. (Chapter II)
(2) The development of the state matrix synthesis procedure for random input circuits.
(Chapter III)
(3) A digital computer program to perform the state matrix synthesis procedure. (Chapter V)
(4) The development of a simulation technique to check the logical implications of digital equations. (Chapter IV)
(5) A digital computer program to perform the digital equation simulation and to formulate the implied primitive flow table. (Chapter V)
(6) The definition of a standard format for the primitive flow table. (Chapter IV)

The state matrix synthesis procedures have the following distinguishing features:
(1) The basic concepts of circuit synthesis are much easier to grasp than those of other methods.
(2) The execution of the procedure is straightforward with few or no exceptions to
established rules.
(3) The resulting digital equations have few of the usual logical complications.
(4) The procedure takes advantage of device properties to produce circuits with fast response and minimal hardware.
(5) There is virtually no limitation upon the size or length of the problems which can be handled.

The simulation method presented here provides a check upon the digital equations resulting from a synthesis procedure. Each possible input change is systematically inspected for its effect upon circuit equations and the resulting transitions are recorded in a primitive flow table. This flow table may then be compared to the original flow table which should contain identical information.

In comparing the simulated flow table to its original primitive flow table, it is convenient, if not necessary, to establish a standard flow table format. For this reason, a method similar to the simulation method is used to define the canonical flow table format.

The computer programs included in Appendix B perform the mechanics of synthesis or simulation rapidly and accurately. These programs encompass all of the defined rules and methods for the analysis of digital logic systems and can be utilized to good advantage in design work.

Although feedback sequential circuits are comparatively simple, they have found a large field of application in modern automation. Consequently, the synthesis of such circuits is of major importance to industrial designers.

Feedback sequential circuits are characterized by their use of a signal indicating the completion of one event to initiate the next event in a prescribed sequence. Feedback sequential circuits are automatic and, once started, require no further attention to sustain sequential action.

## Formal Matrix Representation

In sequential circuits, each element is associated with one corresponding output from the logic circuit. In a hydraulic circuit this element is typically a hydraulic cylinder and the output is the fluid flow which actuates the cylinder. Since there is usually more than one element in a sequential machine, it is convenient to let $Z_{1}$ represent the output which extends cylinder one and $\bar{Z}_{1}$ represent the
retract output for the cylinder. ${ }^{1}$ The signal $X_{2}$ is used as an input to the logic circuit indicating the full extension of cylinder two, and the signal $\bar{X}_{z}$ appears when cylinder two is fully retracted. Figure 1 illustrates a physical realization of these variables. The reader who is unfamiliar with hydraulic circuit notation should refer to the literature.


Figure 1. Hydraulic Circuit Illustrating

Using this notation, a sequence involving two cylinders
${ }^{1}$ This notation is somewhat unfortunate since $Z_{1}$ is used in this chapter to specify only the change of cylinder one, not its continuous state. Also, $Z_{1}$ and $Z_{1}$ are not perfect complements since the specification of one does not imply the other. A more appropriate notation would be $\Delta Z_{1}$, etc.; however, the $Z, \bar{Z}$ notation is used here for simplicity. A similar statement is true for the inputs $X$ and $\bar{X}$ used in this chapter only.
can be written as $Z_{1}, Z_{2}, \bar{Z}_{1}, \bar{Z}_{2}$. This implies that cylinder one extends, then cylinder two extends, cylinder one retracts, cylinder two retracts, and then the entire sequence is repeated indefinitely. Each event is initiated by the completion of the proceeding event.

The synthesis of a circuit to execute this sequence proceeds from the assumption that the required outputs from the logic circuit are related to the inputs by the matrix equation given below.

$$
\left[\begin{array}{c}
z_{1}  \tag{1}\\
\bar{Z}_{1} \\
\vdots \\
\vdots \\
z_{n} \\
\bar{Z}_{n}
\end{array}\right]=\left[\begin{array}{cccc}
m_{1 I} & m_{12} & \cdots & m_{1 n} \\
\cdot & & & \cdot \\
\cdot & & & \cdot \\
\cdot & m_{1 j} & & \cdot \\
\cdot & & & \cdot \\
\cdot & & & \cdot \\
m_{n I} & \cdots & \cdots & \cdots \\
m_{n n}
\end{array}\right]\left[\begin{array}{c}
x_{1} \\
\bar{x}_{1} \\
\vdots \\
x_{n} \\
\bar{x}_{n}
\end{array}\right]
$$

Recall from the rules of matrix multiplication that when multiplying the matrix $[M]$ by the $[X]$ vector, every entry in the $j^{\text {th }}$ column of $[M]$ is multiplied by the element in the $j^{\text {th }}$ row of $[X]$. Thus, each column in $[M]$ is associated only with the corresponding input element of $[x]$.

For the sequence under consideration, the first event is the extension of cylinder one which results from the previous retraction of cylinder two. Thus, the state number 1 is entered in the matrix in the row of the $Z_{1}$ output and the column associated with the $\bar{X}_{2}$ input (column four). See Table I.

The next event, the extension of cylinder two, is initiated by the full extension of cylinder one. Hence, the state number 2 is entered in the $\mathrm{Z}_{2}$ row and the $\mathrm{X}_{1}$ column. Similarly, state 3 is in the $\bar{Z}_{1}$ row and the $X_{2}$ column. The sequence is completed by state 4 in the $\bar{Z}_{2}$ row, $\bar{X}_{\mathrm{I}} \quad$ column.

TABLE I
THE DEVELOPING STATE MATRIX RELATION FOR THE SEQUENCE $\mathrm{Z}_{1}, \mathrm{Z}_{2}, \overline{\mathrm{Z}}_{1}, \overline{\mathrm{Z}}_{2}$
$\left.\left[\begin{array}{l}z_{1} \\ \bar{z}_{1} \\ z_{2} \\ \bar{z}_{2}\end{array}\right] \sim \begin{array}{|l|l|l|l|}\hline & & & 1 \\ \hline & & 3 & \\ \hline 2 & & & \\ \hline & 4 & & \\ \hline\end{array}\right]\left[\begin{array}{l}x_{1} \\ \bar{x}_{1} \\ x_{2} \\ \bar{x}_{2}\end{array}\right]$

After all state numbers are entered into Table $I$, the state matrix must be inspected to ensure that each state is unique and does not represent any contradictions. For this extremely simple problem, this is true and further attention is not required. Table $I$ may now be written matrix form by placing a logical "1" for each state and a "O" elsewhere.

$$
\left[\begin{array}{l}
z_{1}  \tag{2a}\\
\bar{z}_{1} \\
z_{2} \\
\bar{z}_{2}
\end{array}\right]=\left[\begin{array}{llll}
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{array}\right]\left[\begin{array}{c}
x_{1} \\
\bar{x}_{1} \\
x_{2} \\
\bar{x}_{2}
\end{array}\right]
$$

The matrix in Table $I$ is termed the state matrix since it only shows the states of the sequence. The matrix in Equation (2a) is termed the output matrix because Equation (2a) is merely a set of digital output equations in matrix notation. Writing Equations (2a) in longhand, one has:

$$
\begin{align*}
& \mathrm{z}_{1}=\overline{\mathrm{X}}_{2} \\
& \overline{\mathrm{Z}}_{1}=\mathrm{X}_{2}  \tag{2b}\\
& \mathrm{Z}_{2}=\mathrm{X}_{1} \\
& \overline{\mathrm{Z}}_{2}=\overline{\mathrm{X}}_{1}
\end{align*}
$$

Note that the variables used in digital equations are Boolean or binary logic variables.

Since this introductory problem is simple and requires no memory, one could almost predict the results without the use of any formal synthesis procedure. However, further problems in this chapter illustrate the general case.

## Persistent States

The problem of persistent states are prevalent in almost every feedback sequential circuit. Persistent states result when signals remain on long enough to form a
contradiction. The exact cause and remedy for this can best be illustrated by an example.

Consider as example 2 the sequence $Z_{1}, \bar{Z}_{1}, Z_{2}, \bar{Z}_{2}$. The state numbers are entered into Table II in exactly the same fashion as the previous example. That is, state 1 is in the $\mathrm{Z}_{\mathrm{I}}$ row, $\overline{\mathrm{X}}_{2}$ column. State 2 is in the $\overline{\mathrm{Z}}_{1}$ row, $\mathrm{X}_{1}$ column. The remaining state numbers are entered similarily and the resulting state matrix is shown in Table II.

TABLE II
THE STATE MATRIX RELATION FOR THE SEQUENCE $\mathrm{Z}_{1}, \overline{\mathrm{Z}}_{1}, \mathrm{Z}_{2}, \overline{\mathrm{Z}}_{2}$


If this state matrix were now converted into the output matrix by placing a "1" for the states and a "O" elsewhere, the following equations would result:

$$
\begin{align*}
& \mathrm{Z}_{\mathrm{I}}=\overline{\mathrm{X}}_{2} \\
& \overline{\mathrm{Z}}_{1}=\mathrm{X}_{1} \tag{3}
\end{align*}
$$

Reference to these equations and the state sequence in Table II reveals that cylinder one would be extended by $\bar{X}_{2}$ and subsequently retracted by $X_{1}$. However, at the time of retraction the extent signal $\overline{\mathrm{X}}_{2}$ would still be on, because cylinder two has not been changed since its retraction. Hence, there is a contradiction because cylinder one is trying to extend and retract simultaneously. The signal which remains on creating a contradiction is called a persistent state. In this case, the persistent state is the signal $\overline{\mathrm{X}}_{2}$ from state 1 . This problem arises because only the changed input and the changed output are used in the state matrix relation. An event is specified only by the variables that change, not by the present state of all variables.

This condition can be alleviated by entering a shut-off memory element at the persisting state and its complement at the contradiction. The memory element should be in the "set" position prior to the persistent state and should be in the "reset" position either prior to or on the contradicting state. The complemented memory signal is not used in state signal formulation; it is only used as a reminder when it should be off or in the "reset" position.

For the problem under consideration, the persistent state 1 contradicts state 2 . Consequently, state 1 must be modified with a shut-off memory element, say $W_{12}$. This element can then be used to shut-off the persistent signal thereby avoiding a contradiction. Further examination of

Table 2 reveals that state 3 is persisting at state 4. Hence, the memory element $W_{34}$ is assigned to state 3. The state matrix for example 2.1 may now be written in output equation form as:

$$
\left[\begin{array}{l}
z_{1}  \tag{4}\\
\bar{z}_{1} \\
z_{2} \\
\bar{z}_{2}
\end{array}\right]=\left[\begin{array}{cccc}
0 & 0 & 0 & W_{12} \\
1 & 0 & 0 & 0 \\
0 & W_{34} & 0 & 0 \\
0 & 0 & 1 & 0
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
\bar{x}_{1} \\
x_{2} \\
\bar{x}_{2}
\end{array}\right]
$$

Equations (4) give all of the required output equations to sustain the desired sequential action only if the shutoff memory elements are switched at the proper times. $W_{12}$ must be in the "set" position in order to formulate the state signal 1 ; therefore, it may be set prior to its state. In this case, $W_{12}$ is set. by the state signal 4 which is $X_{2}$. $W_{12}$ must be reset either prior to, or by, state 2 . Since the previous state is the persisting state, its signal may not be used to reset itself. Therefore, the contradicting state must be used to shut-off or reset the memory element. Thus, the switching conditions for $W_{12}$ may be shown as follows:


The notation adopted for subscripting the $W$ elements is quite fortunate since the subscripts of $W_{12}$ (read $W$ one, two) give both the persisting and the contradicting states,
respectively. The switching conditions may then be stated by simply observing the subscripts. For example, the memory element $W_{34}$ is set prior to the persistent state 3 and is reset by the contradicting state 4. Thus, the complete logic specifications for example 2.1 are:

Output Equations:

$$
\begin{align*}
& \mathrm{z}_{1}=\overline{\mathrm{x}}_{2} \mathrm{~W}_{12} \\
& \overline{\mathrm{Z}}_{1}=\mathrm{X}_{1}  \tag{5}\\
& \mathrm{Z}_{2}=\overline{\mathrm{X}}_{1} \mathrm{~W}_{34} \\
& \overline{\mathrm{Z}}_{2}=\mathrm{X}_{2}
\end{align*}
$$

Switching Conditions:


Before going any further into synthesis procedures, it might be helpful to demonstrate the circuit implementation for this problem. If the circuit shown in Figure 2 is not selfexplanatory, the reader is advised to consult a text on fluid circuits. Refer to Figure 1 for the circuit implied by the boxes representing the cylinders.

Persistent states always occur when two events involving one cylinder are consecutive; however, the same problem arises anytime there is a possibility for a contradiction. This problem may best be illustrated by an example. Consider for example 2.2 the three cylinder sequence $Z_{1}, Z_{2}$,


Figure 2. Synthesized Hydraulic Circuit for $Z_{1}, \bar{Z}_{1}, Z_{2}, \bar{Z}_{2}$
$Z_{3}, \bar{Z}_{3}, \bar{Z}_{2}, \bar{Z}_{1}$. Following through the sequence, it is found that $Z_{2}$ is caused by $X_{1}$ in event two. Later, in event five, $\overline{\mathbf{Z}}_{z}$ is required. However, since cylinder one is not retracted between events two and five, the signal $\mathrm{X}_{1}$ from event two is still on. Thus, state 2 is a persisting state contradicting event 5. A shut-off memory, $W_{25}$, is required to modify state 2. Since states 2 and 5 are not consecutive, the shut-off memory element $W_{2 s}$ can be reset just prior to the contradiction, state 5, rather than by the contradiction itself. This is usually more desirable; however, the particular circuit hardware might dictate otherwise.

There are three other persistent states in this sequence. The reader is encouraged to develop the state matrix for this sequence and verify the memory assignment and switching conditions represented by Equations (6). The output matrix for the sequence $Z_{1}, Z_{2}, Z_{3}, \bar{Z}_{3}, \bar{Z}_{2}, \bar{Z}_{1}$ is:

$$
\left[\begin{array}{l}
z_{1}  \tag{6}\\
\bar{z}_{1} \\
z_{2} \\
\bar{z}_{2} \\
z_{3} \\
\bar{Z}_{3}
\end{array}\right]=\left[\begin{array}{cccccc}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & W_{61} & 0 & 0 \\
W_{25} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & W_{52} \\
0 & 0 & W_{34} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{array}\right]\left[\begin{array}{c}
x_{1} \\
x_{1} \\
x_{2} \\
\bar{x}_{2} \\
x_{3} \\
\bar{x}_{3}
\end{array}\right]
$$

where the switching conditions are:

|  | Set | Reset |
| :---: | :---: | :---: |
| $W_{25}$ | $\bar{X}_{1}$ |  |
| $W_{34}$ | $X_{3}$ |  |
| $\mathrm{X}_{1}$ | $W_{25}$ | $\mathrm{X}_{3}$ |
| $W_{61}$ | $\bar{X}_{3}$ | $W_{52}$ |
| $W_{52}$ | $\bar{X}_{1}$ |  |
| $\mathrm{X}_{3}$ | $\overline{\mathrm{X}}_{1}$ |  |

When determining persistent states, it is convenient to partition the state matrix according to outputs. The two rows for $Z_{1}$ and $\bar{Z}_{1}$ represent the output partition one, etc. The two columns for $\mathrm{X}_{2}$ and $\overline{\mathrm{X}}_{2}$ are input partition two, etc. With the matrix partitioned in this manner, a systematic method for determining persisting states can be defined. This method requires the individual investigation of each output partition. Starting with the first entry in an output partition, each state is checked by investigating the next entry in the output partition. This next entry is always in the complementary half of the output partition. These two states are always contradictory if they are consecutive and are not within a diagonal partition. A diagonal partition is the four entry square formed by the intersection of an output partition and its corresponding input partition. This square will always be on the diagonal of the matrix. Two consecutive entries in a diagonal partition are not contradictory since the first event turns itself off by the next entry. For the same reason, the event in the output partition following an entry in its diagonal partition is not contradictory. States not covered
by the above rule must be examined by applying the following general rule. If the next entry in the output partition is not consecutive and is not within a diagonal partition, then the complementary event of the state immediately preceding the first entry in the output partition must occur before the next entry in the output partition. In other words, the signal that initiated the first entry in the output partition must be negated or turned-off prior to the next entry in the output partition, otherwise the first entry will be a persisting state. The application of these rules is discussed in detail for the example given in the Procedure Summary .

## Memory Assignment

In most sequences, an element is cycled more than once, thus causing an input signal to appear more than once during the sequence. Often, this input signal will initiate a different event each time it appears. In order to determine which event is called for when that input appears, memory of previous events is required.

Consider for example 2.3 the sequence $Z_{1}, Z_{2}, \bar{Z}_{1}, \bar{Z}_{2}$, $Z_{1}, \bar{Z}_{1}$. The state matrix shown in Table III is constructed by entering the state numbers as previously discussed. A close examination of this sequence reveals that state 5 is a persistent state. The element $W_{56}$ is assigned to state 5 to prevent the contradiction at state 6. This element is then
entered into the output matrix for state 5. This is the only persistent state in this sequence.

TABLE III
THE STATE MATRIX RELATION FOR
$\mathrm{Z}_{1}, \mathrm{Z}_{3}, \overline{\mathrm{Z}}_{1}, \overline{\mathrm{Z}}_{2}, \mathrm{Z}_{1}, \overline{\mathrm{Z}}_{1}$
$\left.\left[\begin{array}{l}z_{1} \\ \bar{z}_{1} \\ z_{2} \\ \bar{z}_{2}\end{array}\right] \sim \begin{array}{|c|c|c|c|}\hline & 1 & & 5 \\ \hline 6 & & 3 & \\ \hline 2 & & & \\ \hline & 4 & & \\ \hline\end{array}\right] \quad\left[\begin{array}{l}x_{1} \\ \bar{x}_{1} \\ x_{2} \\ x_{2}\end{array}\right]$

Columns one and two of Table III contain more than one stable state per column. The states 6 and 2 in column one indicate that there are two separate outputs initiated by the input $X_{1}$. One time the input signal $X_{1}$ initiates the output $Z_{3}$; the next time $X_{1}$ appears, the output $\bar{Z}_{I}$ is desired. In order to distinguish between these states, a memory element is assigned to one of these states and its complement is assigned to the other. For instance, the memory element $Y_{26}$ is assigned to state 2 and $\bar{Y}_{26}$ is assigned state 6. In accordance with the $W$ elements, the $Y$ elements are subscripted to denote their associated states. The element $Y_{2 s}$ is used to distinguish between states 2 and 6, and is set prior to state 2 and is reset before 6. A
similar condition exists between states 1 and 4. The memory element $\mathrm{Y}_{14}$ is used to make each of these states unique.

The output matrix is constructed by entering all of the Y elements to distinguish between common input states. The W elements are entered at their persisting states and a "1" is entered for any stable state which does not require memory. A "O" is entered elsewhere. The resulting output matrix for example 2.3 is given by Equation (7).

$$
\left[\begin{array}{c}
\mathrm{Z}_{1}  \tag{.7}\\
\overline{\mathrm{Z}}_{1} \\
\mathrm{Z}_{2} \\
\overline{\mathrm{Z}}_{2}
\end{array}\right]=\left[\begin{array}{cccc}
0 & \mathrm{Y}_{14} & 0 & \mathrm{~W}_{56} \\
\overline{\mathrm{Y}}_{26} & 0 & 1 & 0 \\
\mathrm{Y}_{26} & 0 & 0 & 0 \\
0 & \overline{\mathrm{Y}}_{14} & 0 & 0
\end{array}\right]\left[\begin{array}{l}
\mathrm{X}_{1} \\
\overline{\mathrm{X}}_{1} \\
\mathrm{X}_{2} \\
\overline{\mathrm{X}}_{2}
\end{array}\right]
$$

Written out, these equations are:

$$
\begin{aligned}
& \mathrm{Z}_{1}=\overline{\mathrm{X}}_{1} \mathrm{Y}_{14}+\overline{\mathrm{X}}_{2} \mathrm{~W}_{56} \\
& \overline{\mathrm{Z}}_{1}=\mathrm{X}_{1} \overline{\mathrm{Y}}_{26}+\mathrm{X}_{2} \\
& \mathrm{Z}_{2}=\mathrm{X}_{1} \mathrm{Y}_{26} \\
& \overline{\mathrm{Z}}_{2}=\overline{\mathrm{X}}_{1} \overline{\mathrm{Y}}_{14}
\end{aligned}
$$

The switching conditions are:

|  | Set | Reset |
| :---: | :---: | :---: |
| $\mathrm{Y}_{26}$ | $\overline{\mathrm{X}}_{1} \quad \mathrm{Y}_{14}$ | $\overline{\mathrm{X}}_{2} \mathrm{~W}_{56}$ |
| $\mathrm{Y}_{14}$ | $\mathrm{X}_{1} \quad \overline{\mathrm{Y}}_{26}$ | $\mathrm{X}_{2}$ |
| $W_{56}$ | $\begin{array}{ll}\overline{\mathrm{X}}_{1} & \overline{\mathrm{Y}}_{14}\end{array}$ | $\mathrm{X}_{1} \overline{\mathrm{Y}}_{26}$ |

Figure 3 is a hydraulic implementation of the logic circuit for this sequence. The passive memory effect is


Figure 3. Hydraulic Implementation for $Z_{1}, Z_{2}, \bar{Z}_{1}, \bar{Z}_{2}, Z_{1}, \bar{Z}_{1}$
utilized in this circuit to reduce circuit complexity and hardware. At this point, the reader should refer to Appendix A for a complete discussion of the passive memory effect, assignment, and implementation for hydraulic and fluidic circuits.

## Counting Sequences

Counting sequences are characterized by their repetitious cycling of outputs. For example, a $2,2,1$ counter cycles (i.e., extends, retracts) the first element twice, the second twice, and the third once and then repeats the sequence. Counting sequences are handled in exactly the same manner as any other automatic circuit; however, their uniqueness deserves special mention.

In synthesizing this circuit, the usual formal notation is dropped and the simplified approach is introduced. The first simplification is the omission of the output and input vectors. Instead of writing a formal state matrix relation, the rows and columns of the state matrix are labeled corresponding to their associated vectors. With this simplified approach, the state numbers representing the sequence are entered into the matrix as usual. The required memory elements are then assigned adjacent to their state number eliminating the need for rewriting the state matrix into the output matrix form. The output equations are written directly from the completed state matrix.

The first step in synthesizing the equations for
example 2.4 is to enter the state numbers representing the sequence into the state matrix as shown in Table IV. This sequence is written as $\mathrm{Z}_{1}, \overline{\mathrm{Z}}_{1}, \mathrm{Z}_{1}, \overline{\mathrm{Z}}_{1}, \mathrm{Z}_{3}, \overline{\mathrm{Z}}_{2}, \mathrm{Z}_{2}, \overline{\mathrm{Z}}_{2}$, $Z_{3}, \bar{Z}_{3}$.

TABLE IV
THE STATE MATRIX FOR A $2,2,1$ COUNTER

|  | $\mathrm{X}_{1}$ | $\overline{\mathrm{X}}_{1}$ | $\mathrm{X}_{2}$ | $\overline{\mathrm{x}}_{2}$ | $\mathrm{X}_{3}$ | $\bar{X}_{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Z}_{1}$ |  | $3 \mathrm{Y}_{35}$ |  |  |  | $1 \mathrm{~W}_{12}$ |
| $\overline{\mathrm{Z}}_{1}$ | $\begin{aligned} & 2 Y_{24} \\ & 4 \bar{Y}_{24} \end{aligned}$ |  |  |  |  |  |
| $\mathrm{Z}_{2}$ |  | $5 \mathrm{Y}_{35} \mathrm{~W}_{56}$ |  | $7 \mathrm{Y}_{\text {79 }}$ |  |  |
| $\overline{\mathrm{Z}}_{2}$ |  |  | $\begin{array}{\|ll\|} \hline 6 & \mathrm{Y}_{68} \\ 8 & \overline{\mathrm{Y}}_{68} \\ \hline \end{array}$ |  |  |  |
| $\mathrm{Z}_{3}$ |  |  |  | $9 \mathrm{Y}_{79} \mathrm{~W}_{\text {rio }}$ |  |  |
| $\bar{Z}_{3}$ |  |  |  |  | 10 |  |

The next step is the determination of the existence of any persistent states. Applying the rules from page 18 to the matrix under consideration, it is found that states 1 and 2 are contradictory since they are consecutive entries within the same output partition and different input partitions. States 2 and 3 and 3 and 4 are both within diagonal partitions and, thus, are not contradictory. Since state 4
is in a diagonal partition, there is no contradiction between states 4 and 1. Entries like 5 and 6 in the second partition and 9 and 10 in the third partition are contradictory. By similar application of these rules, it can be seen that these are the only three contradictions in this sequence. The shut-off memories ( $W$ elements) are now entered into Table IV adjacent to their corresponding persistent states (e.g., $W_{12}$ at $1, W_{56}$ at 5, and $W_{910}$ at 9).

The next step of the procedure is the assignment of input memory elements (Y elements). Here, the rule is simple: whenever there is more than one state in a column, a secondary memory state must be assigned to make each state in the column unique. In Table IV there are four such columns requiring memory. The memory elements $Y_{24}, Y_{35}, Y_{68}$, Yyg are assigned to their corresponding states in accordance to Appendix A.

The last step in the procedure is the specification of the output equations and switching conditions. The output equations are written directly from the state matrix in the same manner as initially discussed. The switching conditions are determined directly from the element subscripts. The complete logical specifications for example 2.4 are given below:

Output equations:

$$
\begin{align*}
& \mathrm{Z}_{1}=\overline{\mathrm{X}}_{1} \mathrm{Y}_{35}+\overline{\mathrm{X}}_{3} \mathrm{~W}_{12} \\
& \overline{\mathrm{Z}}_{1}=\mathrm{X}_{1} \mathrm{Y}_{24}+\mathrm{X}_{1} \overline{\mathrm{Y}}_{24}=\mathrm{X}_{1} \\
& \mathrm{Z}_{2}=\overline{\mathrm{X}}_{1} \overline{\mathrm{Y}}_{35} \mathrm{~W}_{56}+\overline{\mathrm{X}}_{2} \mathrm{Y}_{79}  \tag{8}\\
& \overline{\mathrm{Z}}_{2}=\mathrm{X}_{2} \mathrm{Y}_{68}+\mathrm{X}_{2} \overline{\mathrm{Y}}_{68}=\mathrm{X}_{2} \\
& \mathrm{Z}_{3}=\overline{\mathrm{X}}_{2} \overline{\mathrm{Y}}_{79} \mathrm{~W}_{910} \\
& \overline{\mathrm{Z}}_{3}=\mathrm{X}_{3}
\end{align*}
$$

Switching conditions:

|  | Set | Reset |
| :---: | :---: | :---: |
| $\mathrm{Y}_{24}$ | $\overline{\mathrm{X}}_{3} \quad \mathrm{~W}_{1}{ }_{2}$ | $\overline{\mathrm{X}}_{1} \quad \mathrm{Y}_{35}$ |
| $\mathrm{Y}_{35}$ | $\begin{array}{ll}\mathrm{X}_{1} & \mathrm{Y}_{24}\end{array}$ | $\mathrm{X}_{1} \quad \overline{\mathrm{Y}}_{24}$ |
| $\mathrm{Y}_{68}$ | $\begin{array}{llll}\mathrm{X}_{1} & \overline{\mathrm{Y}}_{35} & \mathrm{~W}_{56}\end{array}$ | $\overline{\mathrm{X}}_{2} \quad \mathrm{Y}_{7}{ }_{9}$ |
| $\mathrm{Y}_{79}$ | $\mathrm{X}_{2} \quad \mathrm{Y}_{68}$ | $\mathrm{X}_{2} \overline{\mathrm{Y}}_{68}$ |
| $\mathrm{Y}_{1} 2$ | $\mathrm{X}_{3}$ | $\mathrm{X}_{1} \quad \mathrm{Y}_{24}$ |
| $\mathrm{W}_{56}$ | $\begin{array}{ll}\mathrm{X}_{1} & \overline{\mathrm{Y}}_{24}\end{array}$ | $\mathrm{X}_{2} \quad \mathrm{Y}_{68}$ |
| $\mathrm{W}_{910}$ | $\mathrm{X}_{3} \quad \overline{\mathrm{Y}}_{68}$ | $\mathrm{X}_{3}$ |

Notice that the equations for $\bar{Z}_{1}$ and $\bar{Z}_{2}$ both reduce, thereby eliminating a memory element. This does not imply that these memory elements are not required. These two signals (states 2 and 4) must be unique since they are used to switch other memories to prepare the proper transition paths.

Procedure Summary

The procedure for the synthesis of feedback sequential digital control circuits is summarized by the following
four steps:

1. Enter State Numbers - Write down the specified sequence and number each event in the sequence. Starting with the first event, sequentially enter the state numbers into the state matrix in the row corresponding to the desired output and the column corresponding to the previous event.
2. Correct Persistent States - Whenever a state signal remains on to form an extend-retract contradiction, the persistent state signal must be modified by a $W$ memory element.
3. Assign Memory States - Whenever there is more than one state in any column of the state matrix, memory states are required to make each of these states unique.
4. Determine Output and Switching Conditions The digital output equations are obtained from the state matrix by replacing each state number by a logical "1" and all blank entries in the matrix by "O" and then multiplying the matrix. The switching conditions are determined from the memory subscripts.

The following example encompasses all of the defined rules for the synthesis of feedback sequential logic circuits and is worked in detail as a final illustration of this synthesis procedure. The entire problem is presented
on page 31 and the procedure is discussed in detail below.
First of all, the sequence is specified and written with state numbers below it, as shown on page 31. This sequence is then entered into the state matrix by placing the state numbers in the row of the desired output and the column of the present input. For example, the state number 1 is entered in the $Z_{1}$ row and the $\bar{X}_{3}$ column since the first event, $Z_{1}$, is initiated by the previous event which is the retraction of cylinder three. The next event is the retraction of cylinder one; accordingly, state 2 is located in the $\bar{Z}_{I}$ row and $X_{1}$ column. The remainder of the sequence is entered into the state matrix in the same fashion.

The next step of the procedure requires the investigation of each output partition for the possibility of persistent states. The first partition is investigated by starting with state 1 . The next entry in this partition is state 2. Since this is a consecutive entry not within a diagonal partition, states 1 and 2 are contradictory and must be corrected by modifying the persistent state (state 1) with the memory element $W_{1 a} . W_{1 a}$ is entered in the matrix adjacent to state number 1. The next entry in this partition is state 3. This entry, as well as the next, is within a diagonal partition and is not contradictory. The next entry in partition one after state 4 is state 7 . Since state 4 is within a diagonal partition, its initiating signal is negated prior to the next entry (state 7). States 7 and 9 form a contradiction since event 6 has not been
negated before state 9. Accordingly, the memory element $W_{7}$. is entered by state 7. The final entry in partition one is state 1 . Since event 8 is not negated before state $1, W_{9}$ is placed beside state 9 to correct this contradiction.

The next partition has only two states (5 and 8). It can be seen that these states do not form a contradiction since event 4 is negated by event 7. Similarly, state 8 is not persisting at state 5 .

The possible contradiction in partition three (6 and 10) is eliminated since event 5 is negated by event 8. Thus, the signal causing state 6 is turned off before state 10. The state prior to state 10 (state 9) is negated before state 6 eliminating this possible contradiction.

Now that all persistent states have been corrected, the next step in the procedure is the assignment of any required memory states. Column one of the state matrix contains three states (2, 4, and 8). Each of these states must be made unique by modifying the states with the proper memory state. This is done by placing $Y_{28} Y_{24}$ at state $2, Y_{28} \bar{Y}_{24}$ at state 4 and $\overline{\mathrm{Y}}_{28}$ at state 8 . (Notice the double subscript notation.) Column two also contains three states, 3, 5, and 10 , and the memory elements $Y_{35}$ and $Y_{310}$ are assigned accordingly. There are no other columns requiring memory.

The final step of the procedure is the specification of output and switching conditions. The output equations are obtained by mentally replacing each state number by the
logical "1" and multiplying the matrix by the input vector. The switching conditions for the memory elements are obtained from the element subscripts. For example, $W_{12}$ is set prior to state 1 by state 10 and is reset by state 2 . $W_{\text {rg }}$ is set by state 6 and is reset by state 8 . $Y_{24}$ is set by state 1 and reset by state 3 , etc.

This problem is worked to completion on the following page.

EXAMPLE PROBLEM

| Sequence: | 1 | $\overline{1}$ | 1 | $\overline{1}$ | 2 | 3 | 1 | $\overline{2}$ | $\overline{1}$ | $\overline{3}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| State Nos: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |


|  | $\mathrm{X}_{1}$ | $\overline{\mathrm{X}}_{1}$ | $\mathrm{X}_{2}$ | $\overline{\mathrm{X}}_{2}$ | $\mathrm{X}_{3}$ | $\overline{\mathrm{x}}_{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Z}_{1}$ |  | $3 \mathrm{Y}_{310} \mathrm{Y}_{35}$ |  |  | $7 \mathrm{~W}_{79}$ | $1 \mathrm{~W}_{1,2}$ |
| $\bar{Z}_{1}$ | $\begin{array}{lll} 2 & Y_{28} & Y_{24} \\ 4 & Y_{28} & \bar{Y}_{24} \\ \hline \end{array}$ |  |  | $9 \mathrm{~W}_{91}$ |  |  |
| $\mathrm{Z}_{2}$ |  | $5 \mathrm{Y}_{310} \mathrm{Y}_{35}$ |  |  |  |  |
| $\overline{\mathrm{Z}}_{2}$ | $8 \mathrm{Y}_{28}$ |  |  |  |  |  |
| $\mathrm{Z}_{3}$ |  |  | 6 |  |  |  |
| $\overline{\mathbf{Z}}_{3}$ |  | $10 \mathrm{Y}_{310}$ |  |  |  |  |

Output Equations:

$$
\begin{align*}
& \mathrm{Z}_{1}=\overline{\mathrm{X}}_{1} \mathrm{Y}_{310} \mathrm{Y}_{35}+\mathrm{X}_{3} \mathrm{~W}_{79}+\overline{\mathrm{X}}_{3} \mathrm{~W}_{12} \\
& \overline{\mathrm{Z}}_{1}=\mathrm{X}_{1} \mathrm{Y}_{28} \mathrm{Y}_{24}+\mathrm{X}_{1} \mathrm{Y}_{28} \overline{\mathrm{Y}}_{24}+\overline{\mathrm{X}}_{2} \mathrm{~W}_{91}=\mathrm{X}_{1} \mathrm{Y}_{28}+\overline{\mathrm{X}}_{2} \mathrm{~W}_{91} \\
& \mathrm{Z}_{2}=\overline{\mathrm{X}}_{1} \mathrm{Y}_{310} \overline{\mathrm{Y}}_{35} \\
& \overline{\mathrm{Z}}_{2}=\mathrm{X}_{1} \overline{\mathrm{Y}}_{28}  \tag{9}\\
& \mathrm{Z}_{3}=\mathrm{X}_{2} \\
& \overline{\mathrm{Z}}_{3}=\overline{\mathrm{X}}_{1} \overline{\mathrm{Y}}_{310}
\end{align*}
$$

Switching Conditions:

|  | Set | Reset |
| :---: | :---: | :---: |
| $Y_{24}$ | $\bar{X}_{3} \quad W_{1}{ }_{2}$ | $\bar{X}_{1} \mathrm{Y}_{310} \mathrm{Y}_{35}$ |
| $Y_{38}$ | $\bar{X}_{3} \quad W_{12}$ | $\mathrm{X}_{3} \mathrm{~W}_{79}$ |
| $\mathrm{Y}_{35}$ | $\begin{array}{llll}\mathrm{X}_{1} & \mathrm{Y}_{28} & \mathrm{Y}_{24}\end{array}$ | $\begin{array}{llll}\mathrm{X}_{1} & \mathrm{Y}_{28} & \overline{\mathrm{Y}}_{24}\end{array}$ |
| $Y_{310}$ | $\begin{array}{llll}\mathrm{X}_{1} & \mathrm{Y}_{28} & \mathrm{Y}_{24}\end{array}$ | $\overline{\mathrm{X}}_{3} \quad \mathrm{~W}_{9}$ |
| $\mathrm{W}_{9} \mathrm{I}$ | $\mathrm{X}_{1} \quad \overline{\mathrm{Y}}_{28}$ | $\overline{\mathrm{X}}_{1} \quad \overline{\mathrm{Y}}_{310}$ |
| $\mathrm{W}_{79}$ | $\mathrm{X}_{2}$ | $\mathrm{X}_{1} \overline{\mathrm{Y}}_{28}$ |
| $\mathrm{W}_{1} 2$ | $\overline{\mathrm{X}}_{1} \overline{\mathrm{Y}}_{310}$ | $\begin{array}{llll}\mathrm{X}_{1} & \mathrm{X}_{28} & \mathrm{Y}_{24}\end{array}$ |

The hydraulic implementation for this circuit is shown in Figure 4. In this circuit, the actual switching signals have been replaced by the notation $S_{24}, R_{35}$, etc., where $S_{24}$ denotes the "set" signal for $\mathrm{Y}_{24}$ from the above switching conditions.


Figure 4. Hydraulic Implementation for $1, \overline{1}, 1, \overline{1}, 2,3,1, \overline{2}, \overline{1}, \overline{3}$

CHAPTER III

THE STATE MATRIX SYNTHESIS PROCEDURE
FOR RANDOM INPUT CIRCUITS

Unlike feedback sequential circuits, random input circuits do not anticipate the next input; consequently, every possible input change must be considered. An example of this type of circuit is the secret combination lock in which only one sequence of input changes will result in the proper output (i.e., the opening of the lock). Other sequences might result in different outputs, return to starting position, or many other conceivable situations. In any event, the response to all input change possibilities from any state in the sequence must be specified before a circuit to perform the required logic can be synthesized.

## The Primitive Flow Table

The synthesis of a circuit to perform certain logic sequences must proceed from the word statement of the possible inputs and the desired response to input changes. For every input change, two things must be specified: the resulting output and the desired transition paths from that state. These specifications are most conveniently
represented by the information table termed the Primitive Flow Table.

The primitive flow table contains the complete logic specifications for a problem and is arranged as follows. The columns of the table indicate all of the possible input combinations. These input states are usually labeled above each column according to the Gray code (one variable change between columns). Each row of this table represents the state of the logic system and its corresponding output, $Z$. Numbers with parentheses around them indicate stable states of the circuit and the unparenthesized numbers show the possible transition paths from one stable state to another.

As example 3.1, consider the primitive flow table shown by Table V. This example has two inputs, $X_{1}$ and $X_{2}$, and one output, $Z_{1}$. The table indicates that the logic circuit must provide a path from state (1) to state (2) when the input changes from "00" to "10" as indicated by the transition path numbered 2 in the first row. Also, the circuit must return from (2) to (1) by the path indicated in the second row, first column. Notice that no transition path is shown from input "OO", state (1), to input "11", since this would require two inputs to be changed at exactly the same instant, which is highly improbable.

TABLE V
PRIMITIVE FLOW TABLE FOR EXAMPLE 3.1

| $\mathrm{X}_{1} \mathrm{X}_{2}$ |
| :--- |
| 00 |


| $(10$ | 11 | 01 | $\mathrm{Z}_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | - | 3 | 0 |
| 1 | $(2)$ | 4 | - | 0 |
| - | - | 4 | $(3)$ | 0 |
| 2 | $(4)$ | 3 | 1 |  |

In Table $V$, the output $Z_{I}$ results when both inputs are actuated by either the path from state 2 or 3 . As is the case with this example, the primitive flow table should specify every possible transition path and should form a closed loop in that there is a path back to the origin or any other state. The above example is extremely simple and requires no memory. When the sequences get larger and inputs are cycled, the need for memory arises as is shown in the next example.

Consider for example 3.2 the primary sequence 00,10 , 11, $01,11,10$, which results in the output $Z_{1}$. The sequence $00,01,11$ results in the $Z_{2}$ output. All other possible sequences are considered and the transition paths are shown in the completed primitive flow table, Table VI.

TABLE VI
PRIMITIVE FLOW TABLE FOR EXAMPLE 3.2

| $\mathrm{X}_{1} \mathrm{X}_{2}$ |
| :--- |
| 00 | $\mathbf{1 0}^{2}$

0

|  | 11 | 01 | $Z_{1}$ | $Z_{2}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $(1)$ | 2 | - | 7 | 0 | 0 |
| 1 | $(2)$ | 3 | - | 0 | 0 |
| - | 2 | $(3)$ | 4 | 0 | 0 |
| 1 | - | 5 | $(4)$ | 0 | 0 |
| - | 6 | $(5)$ | 4 | 0 | 0 |
| 1 | $(6)$ | 3 | - | 1 | 0 |
| 1 | - | 8 | $(7)$ | 0 | 0 |
| - | 9 | $(8)$ | 7 | 0 | 1 |
| 1 | $(9)$ | 3 | - | 0 | 0 |

Before synthesizing a circuit to perform the indicated logic of Table VI, it is advantageous, although not completely necessary, to administer two additional steps to the primitive flow table. First of all, the primitive flow table should be checked for the possibility of redundant states. Two stable states are said to be redundant if and only if they have the same input state, the same output state, and the same or equivalent transition paths. For example, the states (2) and (9) in Table VI are redundant since they have the same input (they are in the same column), the same output $\left(\bar{Z}_{1} \bar{Z}_{\mathcal{Z}}\right)$, and the same transition paths (1 and 3). For this reason, the row containing state (9) may be completely removed and all of the transition
paths 9 may be replaced with the path indicator 2 . There are no more redundancies in this table and the resulting flow table is termed the reduced primitive flow table.

Another advantageous operation on this flow table is the transformation to the canonical flow table. This operation is not completely necessary for the purposes of this chapter, so the definition and detailed discussion of it is deferred until Chapter IV. Briefly though, the basic concept is to order the states according to systematic input changes. The canonical flow table for the problem under consideration (which includes the above mentioned reduction) is shown in Table VII.

TABLE VII
CANONICAL FLOW TABLE FOR EXAMPLE 3.2

| $\mathrm{X}_{1} \mathrm{X}_{2}$ |
| :--- |
| 00 | $1^{10}$


| $(1)$ | 2 | - | 3 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $(2)$ | 4 | - | 0 | 0 |
| 1 | - | 5 | $(3)$ | 0 | 0 |
| - | 2 | $(4)$ | 6 | 0 | 0 |
| - | 2 | $(5)$ | 3 | 0 | 1 |
| 1 | - | 7 | $(6)$ | 0 | 0 |
| - | 8 | $(7)$ | 6 | 0 | 0 |
| 1 | $(8)$ | 4 | - | 1 | 0 |

## Formal Matrix Representation

Once a problem has been completely specified and the canonical flow table has been derived, the next step is the synthesis of circuit equations to perform the required logic. This synthesis can be reduced to the determination of a unique matrix $[M]$ satisfying the relation.

$$
[\mathrm{Z}]=[\mathrm{M}][\mathrm{X}]
$$

This is a statement that the outputs $[Z]$ are related to the inputs $[X]$ and previous events. The matrix $[M]$ provides this relationship and contains memory information which defines the present state. The only difference between this matrix relation and the one used for the feedback sequential circuits is the input and output vectors used. In feedback sequential circuit synthesis, the changed input and the changed output vectors are used. For random input circuit synthesis, the input vector contains the total input state (present state of all inputs) and the output vector represents the continuous output state (present state of each output) rather than the change output.

As a first step toward constructing this matrix, the state numbers from the canonical flow table are entered into each of the output partitions. States with an output of $Z_{1}$ are entered in the top half of the $i^{\text {th }}$ output partition and states with the $\bar{Z}_{1}^{\prime}$ output are entered in the bottom half. This determines the rows in which states are entered. To
determine the proper entry column, recall from the rules of matrix multiplication that each column in the matrix is multiplied only by a corresponding row of the input vector [X]. Thus, a column of the matrix represents events associated with only one input state. Hence, state numbers are entered in the proper row of the output partition and in the column associated with that input state.

To illustrate the state matrix synthesis concept, consider example 3.1 as represented by Table V. This primitive flow table is entered into the state matrix by entering the stable state numbers in the row of the individual output and the column of the present input similar to the way it was done in Chapter II. This matrix is given by Table VIII.

TABLE VIII
THE STATE MATRIX RELATION FOR EXAMPLE 3.1


To obtain the output equation, replace every state number by the logical "1" and place a "O" elsewhere. Multiplying the matrix yields the result:

$$
\begin{equation*}
\mathrm{Z}_{1}=\mathrm{X}_{1} \mathrm{X}_{2} \tag{10}
\end{equation*}
$$

The above example illustrates the basic concept of circuit synthesis using state matrices. This problem did not require memory; a more general problem requiring memory is discussed below.

As another example of circuit synthesis, consider
example 3.2 represented by the canonical flow table given in Table VIII. The state numbers are entered into the matrix as described above and the result is termed the state matrix relation. See Table IX.

TABLE IX
THE STATE MATRIX RELATION FOR EXAMPLE 3.2


Memory Assignment

As can be seen from Table IX, the only time the output
$Z_{1}$ appearsis state 8. Since state 8 is associated with the input "10", one would be tempted to state that the output $\mathrm{Z}_{1}$ is equal to $X_{1} \bar{X}_{2}$. However, this is not the case since state 2 also has the input "10" but does not have the output $\mathrm{Z}_{\mathrm{I}}$. Thus, some method to distinguish between states 2 and 8 is required. This is most conveniently done by assigning a memory state at both states. If a memory element was in the "set" position for 2 and in the "reset" position for 8, then these two states would be a unique combination of the input and memory states. This memory element may be represented by placing $Y_{z s}$ adjacent to every 2 in Table IX and its logical complement $\bar{Y}_{28}$ by states 8. This double subscript notation implies that the memory element $Y_{28}$ is used to distinguish between states 2 and 8 and is set prior to 2 and is reset prior to 8.

A similar condition exists in column four. Although states 3 and 6 do not have differing outputs, they still required uniqueness since they have different transition paths and their signals are used to switch different memory elements. Therefore, the memory element $Y_{36}$ is assigned to state 3 and its complement $\bar{Y}_{36}$ is assigned to state 6. States 4, 5, and 7 in column three also require memory to demand their uniqueness. The memory state $Y_{47} Y_{45}$ is assigned to state $4, \mathrm{Y}_{47} \overline{\mathrm{Y}}_{45}$ to state 5 , and $\overline{\mathrm{Y}}_{47}$ to state 7 . Here again, the switching conditions are inferred by the subscripts. At this point, the reader should refer to

Appendix A for further information concerning the passive memory.

The matrix shown in Table $X$ has all of the above memory modifications. Now, each state in this matrix has a unique representation.

TABLE X
THE UNIQUE STATE MATRIX RELATION FOR EXAMPLE 3.2


Output and Switching Conditions

The purpose of any synthesis procedure is to give every state a unique signal representation. This signal (or variations upon this signal) is then used either as an output signal or as a switching signal for other memory elements. The above steps produce a state matrix in which every state is represented uniquely by a certain combination of input
and memory states. The only remaining step is the specification of the output and switching conditions.

The output equations are obtained from the state matrix relation by replacing every state number designation in the state matrix by the logical "1" and by placing a logical "0" elsewhere. Once this substitution has been made, the resulting matrix is termed the output matrix since it now represents a set of digital equations rather than a state matrix relation. These equations can be rewritten in the individual equation form by multiplying the matrix by the input vector.

The final step in the synthesis procedure is the one which insures the proper circuit operation; this is the specification of when each memory element is to be switched to the proper state. These switching conditions are inferred from the element subscripts and the flow table. For example, the memory element $Y_{1 j}$ is set prior to the state "i" and is reset prior to state "j". This information is obtained from the flow table by observing the possible transition paths to states $i$ and $j$. The corresponding previous states are to be used for switching signals.

As a specific example, the output and switching conditions for the problem given in Table $X$ are as follows.. The output matrix equation is:


Since the outputs $Z_{1}$ and $\bar{Z}_{1}$ are perfect complements, only the equations for $Z_{1}$ and $Z_{2}$ need to be specified. These are:

$$
\begin{align*}
& \mathrm{Z}_{1}=\mathrm{X}_{1} \overline{\mathrm{X}}_{2} \quad \overline{\mathrm{Y}}_{28} \\
& \mathrm{Z}_{2}=\mathrm{X}_{1} \mathrm{X}_{2} \quad \mathrm{Y}_{47} \overline{\mathrm{Y}}_{45} \tag{11b}
\end{align*}
$$

The switching conditions as determined from the subscripts and the flow table (Table VII) are:

$$
\begin{aligned}
\mathrm{Y}_{28}: \quad \text { Set } & =\text { States } 1+4+5 \\
& =\overline{\mathrm{X}}_{1} \overline{\mathrm{X}}_{2}+\mathrm{X}_{1} \mathrm{X}_{2} \mathrm{Y}_{47} \mathrm{Y}_{45}+\mathrm{X}_{1} \mathrm{X}_{2} \mathrm{Y}_{47} \overline{\mathrm{Y}}_{45} \\
& =\overline{\mathrm{X}}_{1} \overline{\mathrm{X}}_{2}+\mathrm{X}_{1} \mathrm{X}_{2} \mathrm{Y}_{47} \\
\text { Reset } & =\text { State } 7 \\
& =\mathrm{X}_{1} \mathrm{X}_{2} \overline{\mathrm{Y}}_{47} \\
\mathrm{Y}_{47}: \quad \text { Set } & =\text { States } 2+8 \\
& =\mathrm{X}_{1} \overline{\mathrm{X}}_{2} \mathrm{Y}_{28}+\mathrm{X}_{1} \overline{\mathrm{X}}_{2} \overline{\mathrm{Y}}_{28} \\
& =\mathrm{X}_{1} \overline{\mathrm{X}}_{2} \\
\text { Reset } & =\text { State } 76 \\
& =\overline{\mathrm{X}}_{1} \mathrm{X}_{2} \overline{\mathrm{Y}}_{36}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Y}_{45}: \text { Set } & =\text { States } 2+8 \\
& =\mathrm{X}_{1} \overline{\mathrm{X}}_{2} \\
\text { Reset } & =\text { State } 3 \\
& =\overline{\mathrm{X}}_{1} \mathrm{X}_{2} \mathrm{Y}_{36} \\
\mathrm{Y}_{36}: \quad \text { Set } & =\text { States } 1+5 \\
& =\overline{\mathrm{X}}_{1} \overline{\mathrm{X}}_{2}+\mathrm{X}_{1} \mathrm{X}_{2} \mathrm{Y}_{47} \overline{\mathrm{Y}}_{45} \\
\text { Reset } & =\text { States } 4+7 \\
& =\mathrm{X}_{1} \mathrm{X}_{2} \mathrm{Y}_{47} \mathrm{Y}_{45}+\mathrm{X}_{1} \mathrm{X}_{2} \overline{\mathrm{Y}}_{47}
\end{aligned}
$$

In more compact notation, the switching conditions are:


Procedure Summary

The state matrix synthesis procedure consists of the following four steps:

1. Develop Primitive Flow Table - From the word statement of the problem, construct a primitive flow table showing all possible input changes, all possible transitions, and the corresponding outputs. If desired, this flow table may then be transformed into the canonical flow table.
2. Form State Matrix - Enter the stable state numbers into the state matrix. Each state
number appears in every output partition under the proper column.
3. Assign Memory States - Whenever there is more than one stable state number in a column, make each state unique by assigning the appropriate memory state.
4. Determine Output and Switching Conditions The output equations are obtained by replacing each state number by "1" and placing a "O" elsewhere and then multiplying the matrix. The output complement need not be specified. The switching conditions are determined from the element subscripts and previous events shown in the flow table.

As a final example of the state matrix synthesis procedure, example 3.3 is worked to completion on page 51 , and each step is explained in detail below. The reader may refer to Appendix $C$ for further example problems and their solutions.

Before working the final example, some of the formality of the method can be dropped and the shorthand notation introduced. First of all, the formal matrix representation is omitted and the rows and columns of the matrix itself are merely labeled according to their outputs and inputs. Next, the intermediate step of writing the output matrix is eliminated by mentally multiplying the matrix rather than rewriting it. As a matter of fact, the matrix representation
itself can be eliminated by working directly with the primitive flow table once the reader is familiar with the technique. However, this step is not presented here.

Consider for example 3.3 a secret combination lock in which there is only one proper sequence of output actuations which will open the lock (output $Z_{1}$ ). Any deviation from this sequence sounds an alarm (output $Z_{2}$ ). The correct sequence is $X_{1}, X_{2}, \bar{X}_{2}, X_{2}, \bar{X}_{1}$; where $X$ means actuate and hold, $\overline{\mathrm{X}}$ means release. Even though a mistake sounds the alarm, there should be a path provided back to the origin. This primitive flow table is shown on page 51 and is not transformed into the canonical form.

Once the primitive flow table is developed, the next step is the formation of the corresponding state matrix. This is done by entering each state number in the column of the input state and the rows of the individual outputs. For the first output, all state numbers except state 6 are entered in the lower half of output partition one, since all of them have the $\bar{Z}_{1}$ output. State 6 is then entered into the $Z_{1}$ row of the state matrix. Next, states 1 through 6 are entered in partition two in the $\bar{Z}_{2}$ row and states 7 through 10 are entered in the $\mathrm{Z}_{2}$ row. These two row partitions comprise the state matrix for this example.

The next step is the determination of memory requirements. To do this, each column, representing one combination of the inputs, is treated separately. Reference to the state matrix reveals that every column has multiple states
and requires memory to make each state unique. Column one has two states, 1 and 8 , requiring one memory element, $Y_{1}$. $Y_{1}$ e is thus entered beside every 1 in the matrix, and its complement $\bar{Y}_{18}$ is entered adjacent to states 8. Similarily, column two contains three states, 2, 4, and 9. Each of these states is made unique by assigning two memory elements, $Y_{29}$ and $Y_{24}$, in accordance with Appendix A. Column three has three states and column four has two. Memory elements are assigned to these states in the same manner as above.

After the state matrix is formed and the memory requirements are entered adjacent to their respective states, the output equations are obtained by mentally replacing the state numbers with "1's" and then multiplying the matrix by the input vector. The output complements do not have to be specified. The output $Z_{1}$ appears at state 6 only. The $Z_{2}$ output appears at states 8, 9, 10, and 7.

The final step is the specification of the switching conditions; this step ensures proper circuit operation. If the double subscript notation is used to denote memory elements, the switching conditions are stated from knowledge of the subscripts and the flow table. The subscripts indicate when an element should be in the "set" or "reset" position and the flow table shows the possible transitions to these states. For example, $Y_{18}$ is set by any state immediately preceding state 1 and is reset by states preceding state 8. From the flow table, it can be seen that the only transition
path to only transition path to state 1 is from state 6. There are transition paths to state 8 from states 2, 4, 7, and 9. Thus, $Y_{18}$ is set by state 6 and is reset by state 2 , 4, 7, or 9. The element $\mathrm{Y}_{\text {2 }}$ is set by state 1 or 8 and is reset by state 5 or 10 . The remaining switching conditions are determined in the same fashion and the complete table or switching conditions is given below.

This problem is shown on the next page and the logic circuit schematic is shown in Figure 5.

TABLE XI

THE PRIMITIVE FLOW TABLE FOR EXAMPLE 3.3

| $\mathrm{X}_{1} \mathrm{x}_{2}$ |
| :--- |
| 00 |


| $(1)$ | 10 | 11 | 01 | $\mathrm{Z}_{1}$ | $\mathrm{Z}_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | $(2)$ | 3 | 7 | 0 | 0 |
|  | 4 | $(3)$ | 7 | 0 | 0 |
| 8 | $(4)$ | 5 |  | 0 | 0 |
|  | 9 | $(5)$ | 6 | 0 | 0 |
| 1 |  | 10 | $(6)$ | 1 | 0 |
| 8 |  | 10 | $(7)$ | 0 | 1 |
| $(8)$ | 2 |  | 7 | 0 | 1 |
| 8 | $(9)$ | 10 |  | 0 | 1 |
|  | 9 | $(10)$ | 7 | 0 | 1 |

The State Matrix:

| $\mathrm{Z}_{1}$ | 00 | 10 | 11 | 01 |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $6 \mathrm{Y}_{67}$ |
| $\overline{\mathrm{Z}}_{1}$ | $\begin{aligned} & 1 \mathrm{Y}_{18} \\ & 8 \overline{\mathrm{Y}}_{18} \end{aligned}$ | $\begin{array}{lll} 2 & Y_{29} & Y_{24} \\ 4 & Y_{29} & \bar{Y}_{24} \\ 9 & \bar{Y}_{29} & \end{array}$ | $\begin{array}{rll} 3 & \mathrm{Y}_{310} & \mathrm{Y}_{35} \\ 5 & \mathrm{Y}_{310} & \overline{\mathrm{Y}}_{35} \\ 10 & \overline{\mathrm{Y}}_{310} & \end{array}$ | $7 \mathrm{Y}_{6} 7$ |
| $\mathrm{Z}_{\text {a }}$ | $8 \bar{Y}_{18}$ | $9 \mathrm{X}_{29}$ | $10 \overline{\mathrm{Y}}_{310}$ | $7 \bar{Y}_{67}$ |
| $\overline{\mathbf{Z}}_{2}$ | $1 \mathrm{Y}_{18}$ | $\begin{array}{lll} 2 & Y_{29} & Y_{24} \\ 4 & Y_{29} & \bar{Y}_{24} \end{array}$ | $\begin{array}{lll} 3 & Y_{310} & Y_{35} \\ 5 & Y_{310} & \bar{Y}_{35} \end{array}$ | $6 \mathrm{Y}_{67}$ |

Output Equations:

$$
\begin{align*}
& \mathrm{Z}_{1}=\overline{\mathrm{X}}_{1} \mathrm{X}_{2} \mathrm{Y}_{67} \\
& \mathrm{Z}_{2}=\overline{\mathrm{X}}_{1} \overline{\mathrm{X}}_{2} \overline{\mathrm{Y}}_{18}+\mathrm{X}_{1} \overline{\mathrm{X}}_{2} \overline{\mathrm{Y}}_{29}+\mathrm{X}_{1} \mathrm{X}_{2} \overline{\mathrm{Y}}_{310}+\overline{\mathrm{X}}_{1} \mathrm{X}_{2} \overline{\mathrm{Y}}_{67} \tag{12}
\end{align*}
$$

Switching Conditions:

|  | Set | Reset |
| :---: | :---: | :---: |
| $\mathrm{Y}_{18}$ | $\overline{\mathrm{X}}_{1} \mathrm{X}_{2} \quad \mathrm{Y}_{67}$ | $\mathrm{X}_{1} \overline{\mathrm{X}}_{2}+\overline{\mathrm{X}}_{1} \mathrm{X}_{2} \overline{\mathrm{Y}}_{6}{ }_{7}$ |
| $\mathrm{Y}_{29}$ | $\overline{\mathrm{X}}_{1} \overline{\mathrm{X}}_{2}$ | $\mathrm{X}_{1} \mathrm{X}_{2} \mathrm{Y}_{310} \overline{\mathrm{Y}}_{35}+\mathrm{X}_{1} \mathrm{X}_{2} \overline{\mathrm{Y}}_{310}$ |
| $\mathrm{Y}_{24}$ | $\overline{\mathrm{X}}_{1} \overline{\mathrm{X}}_{2}$ | $\mathrm{X}_{1} \mathrm{X}_{2} \mathrm{Y}_{310} \mathrm{Y}_{35}$ |
| $\mathrm{Y}_{310}$ | $\mathrm{X}_{1} \mathrm{X}_{2} \quad \mathrm{Y}_{29} \mathrm{Y}_{\mathrm{Z}_{4}}$ | $\overline{\mathrm{X}}_{1} \mathrm{X}_{2}+\mathrm{X}_{1} \overline{\mathrm{X}}_{2} \overline{\mathrm{Y}}_{29}$ |
| $\mathrm{Y}_{35}$ | $\mathrm{X}_{1} \overline{\mathrm{X}}_{2} \quad \mathrm{Y}_{29} \mathrm{Y}_{24}$ | $\mathrm{X}_{1} \overline{\mathrm{X}}_{2} \quad \mathrm{Y}_{29} \overline{\mathrm{Y}}_{24}$ |
| $\mathrm{Y}_{67}$ | $\mathrm{X}_{1} \mathrm{X}_{2} \quad \mathrm{Y}_{310} \overline{\mathrm{Y}}_{3 \text { б }}$ | $\bar{X}_{1} \bar{X}_{2}+\mathrm{X}_{1} \mathrm{X}_{2} \mathrm{Y}_{310} \mathrm{Y}_{35}+\mathrm{X}_{1} \mathrm{X}_{2} \overline{\mathrm{Y}}_{310}$ |



Figure 5. Logic Circuit for Example 3.3

## CHAPTER IV

## DIGITAL EQUATION SIMULATION AND THE CANONICAL FLOW TABLE

All synthesis procedures will produce valid equations for the representation of the specified logic when the procedure is executed correctly. However, some methods are not easily understood or require personal preference in certain steps. Often, intuitively designed circuits do not function properly or for some reason the circuit action needs to be analyzed. To do this, the implied equations of the circuit can be written.

Whether for verification or analysis, it is often necessary to check the system equations. For this reason, a systematic digital equation simulation method has been developed. This method involves the systematic excitation of the inputs to the equations to produce a primitive flow table. This simulated flow table representing the equations may then be compared to the desired circuit action to ascertain if the equations represent the required logic.

Once the simulated flow table is obtained, the task of comparing this table to the original flow table may be larger than the original task of verifying the equations if the state numbers do not coincide. For this reason, it is
advantageous, if not mandatory, to define a standard format for flow tables. The canonical flow table defined in this chapter satisfies this requirement.

## Digital Equation Simulation

The simulation technique presented here offers a systematic method for checking equations and in no way assumes prior knowledge of system response. The basic idea is to change one input from some base state and then observe the resulting output and memory states. If these output and memory states are different from any previously determined, then a new state is defined. If they are the same as some other state, then this new state is redundant and is replaced by its equivalent state. By extending this procedure, there finally results a closed flow table. The flow chart shown in Figure 6 illustrates the complete simulation method.

The method may best be explained by an example. Table XII illustrates the step-by-step development of the simulation discussed below. Consider the logic represented by the following equations as derived by the classical method:

$$
\begin{align*}
& \mathrm{Z}_{1}=\mathrm{X}_{1} \overline{\mathrm{X}}_{2} \overline{\mathrm{Y}}_{1} \mathrm{Y}_{2} \\
& \mathrm{Z}_{2}=\overline{\mathrm{X}}_{1} \mathrm{X}_{2} \mathrm{Y}_{1} \mathrm{Y}_{2}  \tag{13}\\
& \mathrm{Y}_{1}=\mathrm{S}_{1}+\mathrm{Y}_{1} \overline{\mathrm{R}}_{1} \\
& \mathrm{Y}_{2}=\mathrm{S}_{2}+\mathrm{Y}_{2} \overline{\mathrm{R}}_{2}
\end{align*}
$$



Figure 6. Flow Table for Simulation Method

Where the switching conditions are given by:

$$
\begin{align*}
& \mathrm{S}_{1}=\mathrm{X}_{1} \mathrm{X}_{2}+\mathrm{X}_{2} \mathrm{Y}_{2} \\
& \mathrm{R}_{1}=\overline{\mathrm{X}}_{1} \overline{\mathrm{X}}_{2}+\overline{\mathrm{X}}_{2} \mathrm{Y}_{2}  \tag{14}\\
& \mathrm{~S}_{2}=\mathrm{X}_{1} \overline{\mathrm{X}}_{2} \mathrm{Y}_{1}+\overline{\mathrm{X}}_{1} \mathrm{X}_{2} \mathrm{Y}_{1} \\
& \mathrm{R}_{2}=\mathrm{X}_{1} \mathrm{X}_{2} \mathrm{Y}_{1}+\overline{\mathrm{X}}_{1} \overline{\mathrm{X}}_{2} \overline{\mathrm{Y}}_{1}
\end{align*}
$$

The simulation is started by the initial assumption of a memory state and an input state. For convenience, assume that all memories are in the reset position, $\left(\overline{\mathrm{Y}}_{1} \overline{\mathrm{Y}}_{2}\right)$, and that all inputs are off, ( 00 ). This state is termed the temporary base and is entered into a flow table by placing a (1) in the first row under the input column "00". The corresponding output and memory states are also indicated for this row. Starting with this state, (1), as a base, each input is excited individually to determine the system response. First, the input $X_{1}$ is excited. This defines a new state, (2), in the "10" column of Table XII (a). The transition path to stable state (2) is indicated by the unparenthesized 2 in row one. Reference to the equations reveal that the corresponding output and memory states do not change. Next, input two is changed from the base, resulting in the new state (3) in the "O1" column. Again, the output and memory states remain the same. This completes the investigation from base (1) and the resulting response is indicated by Table XII (a).

The next step is to return to the oldest new state and repeat the procedure with this state as the base.

TABLE XII

## STEP-BY-STEP DEVELOPMENT OF DIGITAL EQUATION SIMULATION

| $\mathrm{X}_{1} \mathrm{X}_{2}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 00 | 10 | 11 | 01 | $\mathrm{Z}_{1}$ | $\mathrm{Z}_{2}$ | $\mathrm{Y}_{1}$ | $\mathrm{Y}_{2}$ |
| (1) | 2 | - | 3 | 0 | 0 | 0 | 0 |
|  | (2) |  |  | 0 | 0 | 0 | 0 |
|  |  |  | (3) | 0 | 0 | 0 | 0 |


| $\mathrm{X}_{1} \mathrm{X}_{2}$ |
| :---: |
| 00 |


| $(10$ | 11 | 01 | $\mathrm{Z}_{1}$ | $\mathrm{Z}_{2}$ | $\mathrm{Y}_{1}$ | $\mathrm{Y}_{2}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $(2)$ | 4 | - | 0 | 0 | 0 | 0 |
|  |  |  | $(3)$ | 0 | 0 | 0 | 0 |
|  |  | $(4)$ |  | 0 | 0 | 0 | 0 |

(b) Investigation of Base (2)

| $\mathrm{X}_{1} \mathrm{X}_{2}$ |
| :--- |
| 00 | $1_{10}$


| $(1)$ | 2 | - | 3 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $(2)$ | 4 | - | 0 | 0 | 0 | 0 |
| 1 | - | 4 | $(3)$ | 0 | 0 | 0 | 0 |
|  |  | $(4)$ |  | 0 | 0 | 1 | 0 |

(c) Investigation of Base (3)

TABLE XII (Continued)

| $\mathrm{X}_{1} \mathrm{X}_{2}$ |
| :--- |
| 00 |


| $\mathbf{1 0}$ | 10 | 11 | 01 | $\mathrm{Z}_{1}$ | $\mathrm{Z}_{2}$ | $\mathrm{Y}_{1}$ | $\mathrm{Y}_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1})$ | 2 | - | 3 | 0 | 0 | 0 | 0 |
| 1 | $(2)$ | 4 | - | 0 | 0 | 0 | 0 |
| 1 | - | 4 | $(3)$ | 0 | 0 | 0 | 0 |
| - | 6 | $(4)$ | 5 | 0 | 0 | 1 | 0 |
|  |  |  | $(5)$ | 0 | 1 | 1 | 1 |
|  | $(6)$ |  |  | 1 | 0 | 0 | 1 |

(d) Investigation of Base (4)

|  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 00 | 10 | 11 | 01 | $\mathrm{Z}_{1}$ | $\mathrm{Z}_{2}$ | $\mathrm{Y}_{1}$ | $\mathrm{Y}_{2}$ |
| ( 1 ) | 2 | - | 3 | 0 | 0 | 0 | 0 |
| 1 | (2) | 4 | - | 0 | 0 | 0 | 0 |
| 1 | - | 4 | (3) | 0 | 0 | 0 | 0 |
| - | 6 | (4) | 5 | 0 | 0 | 1 | 0 |
| 1 | - | 4 | (5) | 0 | 1 | 1 | 1 |
| 1 | (6) | 4 | - | 1 | 0 | 0 | 1 |
| (e) | Investigations of Bases (5) and (6) and the Final Simulated Flow Table |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

At this point, the oldest new state is (2). With "10" as a new base, changing the first input defines a transition to the "OO" column. The reader is encouraged to check both the output and switching equations to verify that the resulting output and memory states for this possible transition remain the same. The new state defined in column one is redundant since it is equivalent to (1). Hence, a transition path from (2) to (1) is indicated by a 1 entered in column one. Next, the second input is changed from the base. This defines a new state, (4), in the "11" column and the input "11" sets $Y_{1}$. This completes the investigation of (2). The result is shown in Figure XII (b).

The next base is (3) and investigations from this base reveal that both input changes describe redundant information. The first input change transfers to (4) and the second change transfers to (1). See Table XII (c).

The first input change from the next base, (4), sets $Y_{2}$, subsequently giving the output $Z_{2}$. Since this new state is not redundant, the state number (5) is assigned in the "01" column. Changing the second input from base (4) sets $Y_{2}$. $\bar{X}_{2} Y_{z}$ resets $Y_{1}$ which results in the $Z_{1}$ output. Again, this new state is not redundant and the state number (6) is assigned to this transition. See Table XII (d).

The first input change from state (5) resets $\mathrm{Y}_{2}$ and produces no output. This is equivalent to state (4) so no new state number is assigned. The second input change from (5) resets $Y_{1}$ and then $Y_{2}$, and has no output. This
defines a transition path back to state (1).
The final state to be investigated is state (6). It can be shown that both input changes describe transitions to previously defined states. Since there are no new states to be investigated, this completes the simulation; the final simulated flow table is shown in Table XII (e). The equations examined above were derived from the classical method. In the classical method, each memory state is assigned to a complete row. In the state matrix method, the memory elements are associated with input columns individually, not the complete row. Consequently, when simulating the state matrix equations, the particular sub-memory state associated with a column, not the total memory state, is all that needs to be considered during investigations. With this in mind, it is convenient to place the designation of the memory state beside the state number in the flow table rather than beside the complete row.

## Canonical Flow Table

Considering the previously mentioned need for the canonical flow table and the simulation method discussed above, it seems reasonable to define the canonical flow table in a manner analogous to the simulated flow table. The process used here is the systematic ordering of the rows of a primitive flow table in accordance with the specified response to input changes. Starting with the origin
or first stable state as a base, the state resulting from the first input change is placed in the second row. The state resulting from the second input change is placed in the third row, etc. Upon the completion of the investigation of this base, the oldest new state is then used as a base and the entire process is repeated until all rows have been reordered. The state numbers are then resequenced.

The process is best illustrated by an example. Consider the primitive flow table used in Chapter III, Table VI. The redundant state is eliminated and the reduced primitive flow table is shown in Table XIII (a).

Starting with state (1) as a base, the first input change indicates a transition path to state (2). Since state (2) is already in row two, no reordering is necessary. The second input change indicates a transition path to (7). Hence, the row containing state (7) is placed third as shown in Table XIII (b). This completes the investigation from (1).

The first input change from (2) indicates a path back to a previously ordered state, (1), requiring no reordering. The second input change indicates a path to (3). Since it happens that (3) is already in the next row, no reordering is required. See Table XIII (c).

The next base is (7). This state has transitions to states (8) and (1), respectively. Thus, state (8) is moved to the fourth row and the transition to (1) is already ordered. See Table XIII (d).

TABLE XIII
THE DEVELOPMENT OF THE CANONICAL
FLOW TABLE

| $\mathrm{X}_{1} \mathrm{X}_{2}$ |
| :---: |
| 00 |


| $(1)$ | 10 | 11 | 01 | $\mathrm{Z}_{1}$ | $\mathrm{Z}_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $(2)$ | - | 7 | 0 | 0 |
| - | 2 | $(3)$ | 4 | 0 | 0 |
| 1 | - | 5 | $(4)$ | 0 | 0 |
| - | 6 | $(5)$ | 4 | 0 | 0 |
| 1 | $(6)$ | 3 | - | 1 | 0 |
| 1 | - | 8 | $(7)$ | 0 | 0 |
| - | 2 | $(8)$ | 7 | 0 | 1 |

(a) Original Primitive Flow Table

| $X_{1} X_{2}$ |
| :---: |
| 00 |


|  | 10 | 11 | 01 | $Z_{7}$ | $Z_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $(1)$ | 2 | - | 7 | 0 | 0 |
| 1 | $(2)$ | 3 | - | 0 | 0 |
| 1 | - | 8 | $(7)$ | 0 | 0 |
| - | 2 | $(3)$ | 4 | 0 | 0 |
| 1 | - | 5 | $(4)$ | 0 | 0 |
| - | 6 | $(5)$ | 4 | 0 | 0 |
| 1 | $(6)$ | 3 | - | 1 | 0 |
| - | 2 | $(8)$ | 7 | 0 | 1 |

(b) Initial Investigation From (1)

TABLE XIII (Continued)


| $\mathrm{X}_{1} \mathrm{X}_{2}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 00 | 10 | 11 | 01 | $\mathrm{Z}_{1}$ | $\mathrm{Z}_{2}$ |
| (1) | 2 | - | 7 | 0 | 0 |
| 1 | (2) | 3 | - | 0 | 0 |
| 1 | - | 8 | (7) | 0 | 0 |
| - | 2 | (3) | 4 | 0 | 0 |
| - | 2 | (8) | 7 | 0 | 1 |
| 1 | - | 5 | (4) | 0 | 0 |
| - | 6 | (5) | 4 | 0 | 0 |
| 1 | (6) | 3 | - | 1 | 0 |
| (d) Investigation of State (7) |  |  |  |  |  |

TABLE XIII (Continued)

| $\mathrm{X}_{1} \mathrm{X}_{2}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 00 | 10 | 11 | 01 | $\mathrm{Z}_{1}$ | $\mathrm{Z}_{2}$ |
| (1) | 2 | - | 7 | 0 | 0 |
| 1 | (2) | 3 | - | 0 | 0 |
| 1 | - | 8 | ( 7 ) | 0 | 0 |
| - | 2 | ( 3 ) | 4 | 0 | 0 |
| - | 2 | (8) | 7 | 0 | 1 |
| 1 | - | 5 | (4) | 0 | 0 |
| - | 6 | (5) | 4 | 0 | 0 |
| 1 | (6) | 3 | - | 1 | 0 |


| $X_{1} X_{2}$ |
| :---: |
| 00 |


| $(1)$ | 11 | 01 | $Z_{7}$ | $Z_{2}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | - | 3 | 0 | 0 |
| 1 | $(2)$ | 4 | - | 0 | 0 |
| 1 | - | 5 | $(3)$ | 0 | 0 |
| - | 2 | $(4)$ | 6 | 0 | 0 |
| - | 2 | $(5)$ | 3 | 0 | 1 |
| 1 | - | 7 | $(6)$ | 0 | 0 |
| - | 8 | $(7)$ | 6 | 0 | 0 |
| 1 | $(8)$ | 4 | - | 1 | 0 |

(f) The Completed Canonical Flow Table With Resequenced State Numbers

The reader is encouraged to investigate states (3), (8), (4), and (5) to verify that the remaining states are already in the proper order. Once the rows are in the proper order, the state numbers are then resequenced so that each stable state number corresponds to its row number. The completed canonical flow table is shown in Table XIII (f).

One further point which has not been decided at the time of this writing is the definition of an origin for the primitive flow table. The origin is usually thought of as being the state with the inputs off and having the desired sequence or logic developed from this point. However, a more meaningful definition of the origin should consider the topology of the transitions as being more important than the number of inputs or outputs that are on or off. This definition should be comprehensive enough so that an origin can be uniquely determined for any primitive flow table.

Since an origin is not defined in this chapter, the canonical flow table used here is not unique. The rows are in the proper order, but the origin or first row in the canonical flow table will be the first row given in the primitive flow table. This depends upon the designer's personal preference and will, in general, not be unique. However, for all of the cases investigated by the author, the simulated flow table has resulted with the same origin as the canonical primitive flow table, thereby presenting no problem.

## CHAPTER V

## DIGITAL COMPUTER PROGRAMS

The logic systems program is designed to perform either the synthesis or simulation of digital control systems. In order to perform system synthesis, the user needs only to supply the primitive flow table describing the desired logic; the computer program will then perform the necessary steps to obtain the digital equations by the state matrix synthesis procedure given in Chapter III. These equations may then be implemented to obtain a circuit containing the information represented by the primitive flow table.

With this capability, the designer does not need to know a formal synthesis procedure; he only needs to know how to write a primitive flow table, call the program, and then implement the resulting equations.

The simulation program offers a powerful tool for the analysis of digital systems. This program generates the primitive flow table implied by a set of digital equations by the method described in Chapter IV. The simulation program may be used either to confirm the validity of equations or to analyze the logical implications of existing circuits. This can be advantageous when working with intuitively designed circuits.

The FORTRAN IV source deck listed in Appendix B has been running on the WATFOR terminal of OSU's IBM $360 / 50$ computing facility. A time-share version of the program is also available to allow users with remote teletype terminals to have access to the program from any phone line. A user's guide for the time share program will be made available under a separate cover.

Since the programs are rather lengthy and the listings given in Appendix $B$ contain many of the details of the programs, only the philosophy of the programs is presented in the rest of this chapter. Appendix $C$ shows both the calling information and the computer solutions to many example problems. For further details of the use of this program, see the write-up in Appendix $B$ and the example solutions in Appendix $C$.

## Synthesis Program LOGSYN

Subroutine LOGSYN is the executive subroutine for the synthesis of digital systems. The flow chart showing the relation of subroutines is given by Figure 7. Subroutine LOGSYN reads in the input data concerning the primitive flow table and then uses subroutine PRINT to print the original primitive flow table. This primitive flow table is then examined by subroutine EQUIV to reduce any redundant information which might be contained in the flow table. If two states are found to be redundant, one is eliminated and an indication of this reduction is printed out below the


Figure 7. Flow Diagram for Logic Systems Program
original primitive flow table. This reduced primitive flow table is then put into canonical form by subroutine CANON. In this routine the rows of the primitive flow table are reordered and resequenced as described in Chapter IV. The resulting canonical flow table is then printed by subroutine PRINT.

Subroutine OUTPUT performs most of the steps required for system synthesis. In this routine, the memory requirements for each column are determined and subroutine ASSIGN is used to provide the passive memory assignment code to distinguish between stable states. After each state is made unique by the proper memory assignment, the state signals are printed. This gives the input and memory combination which describes each stable state. Next, the switching conditions required for proper circuit action are printed. The switching conditions are presented by giving the state numbers at which a switch occurs. Finally, the output equations are given by printing the states at which the individual outputs appear. This completes the synthesis procedure and the program then returns to the main calling program to exit.

Simulation Program LOGSIM

Subroutine LOGSIM is the executive program for systems simulation. As can be seen by Figure 7 , this routine reads the data cards containing basic information concerning the system to be simulated. Subroutine LOGSIM then sets up a
loop similar to the one shown in. Figure 6 of Chapter IV. This routine changes an input according to a Gray code. The Gray code is supplied by subroutine ASSIGN. The corresponding system response is determined by subroutine DIGEQN. Subroutine DIGEQN is a subroutine supplied by the user containing the switching and output equations. The input change and the corresponding response determines a new state. This state is then checked for redundancy by subroutine EQUIV. If the new state is not equivalent to a previously defined state, a state number is assigned to this state.

This process is continued until all states have been investigated and no new information is being generated. At this point, the simulation is completed and subroutine PRINT is then used to print the simulated primitive flow table.

Appendix $C$ contains many examples of problems solved with both the synthesis and simulation programs. The reader is referred to the appendices for further information concerning the usage and input for these computer programs.

## CHAPTER VI

## SUMMARY AND CONCLUSIONS

## Summary

The major effort of this thesis has been concentrated upon the development of new techniques for the synthesis and analysis of digital logic systems. The synthesis procedures are based upon the assumption that the outputs are related to the inputs. This relation can be represented by the vector matrix equation

$$
\begin{equation*}
[\mathrm{Z}]=[\mathrm{M}][\mathrm{X}] \tag{15}
\end{equation*}
$$

Since the input vector $[X]$ and the desired output vector $[Z]$ are known, the synthesis reduces to the determination of the binary matrix $[M]$. The entries in this matrix give the relationship between the inputs and the outputs and contain memory information of previous states.

The synthesis proceeds from entering the state numbers from a statement of the desired logic or sequence into the $\operatorname{matrix}[M]$. The memory requirements are then determined and entered into the matrix, producing the set of output equations in matrix form. Specification of the switching
conditions for the memory elements completes the synthesis procedure.

The simulation technique presented here is quite helpful either to verify digital equations or to analyze existing circuits. This technique can also be used to totally redesign existing circuits by first writing the equations for the circuit, obtaining the simulated flow table, and then synthesizing the state matrix equations from this flow table. The canonical flow table is also an aid for analysis and comparison.

The digital computer programs developed to perform either systems synthesis or simulation offer a great design tool to the designer who is unfamiliar with switching circuit theory. These programs perform the steps necessary to synthesize or simulate digital systems as described in Chapters III and IV. With these programs, the designer only needs to be able to write a primitive flow table and to implement equations.

## Comparison to Other Techniques

To fully evaluate the merits of this synthesis technique, a general comparison to existing techniques should be made. This technique is compared to the classical method and those methods suggested by Cole (1) and Maroney (6) on the basis of the following areas:

1. Simplicity of the Synthesis Procedure - The execution of the state matrix synthesis
procedure is much less complicated than the classical method since the merging operation, operational flow table, Karnaugh maps, etc., are eliminated. The total concepts of circuit synthesis are much easier to grasp, partially due to the use of the familiar matrix notation. In comparing to the tabular methods of Cole and Maroney, one can only compare on the basis of procedure simplicity since these methods produce essentially the same equations as the techniques presented here. The philosophy of circuit implementation is also the same. Thus, any comment made about the state matrix equations or circuits is equally applicable to those of the tabular methods.

Cole's tabular technique for the synthesis of feedback sequential circuits handles persistent states in a more straightforward manner than does the matrix method. However, the search procedure for persistent states in the matrix is more mechanical. It is felt that the synthesis concepts using the matrix notation are easier to grasp than the tabular method; but this is a matter of personal preference. Maroney's tabular method handles random input problems in a tabular technique similar
to Cole's method. The random input possibility requires multiple transition paths from states. The transitions from each state are very hard to follow in the tabular form; whereas, the primitive flow table provides a graphic display of transition paths. This causes a slight problem for involved sequences since the designer must keep much of this information in his head rather than on paper. Also, redundant states are harder to sense from the tabular technique than from the primitive flow table. Again, it is felt that the matrix synthesis concepts are easier to grasp.
2. Simplicity of Circuit Implementation Procedure The state matrix and tabular synthesis procedures offer a specific step-by-step procedure for circuit implementation; whereas, the classical method does not lend itself to any set procedure.
3. Circuit Complexity - The number of elements required to implement a circuit is generally a good indication of the circuit complexity. Although the state matrix equations usually require more memory elements, the use of the passive memory effect reduces the total number of elements to about the same or less than that required by the classical method. However, this is not a very rigid basis for comparison
since the classical method offers such a flexibility in writing equations from the Karnaugh maps. Each designer might derive different equations from the classical method depending upon his own personal preference. Thus, to compare on this basis, the equations from the classical Karnaugh maps must be rewritten until a combination with minimum hardware is determined. This is then compared to the state matrix method.
4. Other Circuit Considerations - The state matrix synthesis procedure offers circuit features that are not available from the classical method. Among these are the elimination of switching hazards, cycles, and other logical complications. Another very important feature is the prepared flow path concept. In this procedure, each memory is switched prior to any input change, thus preparing all possible paths from that state. Notice that in the classical method the input change causes the switching of a memory to give the next state. The prepared flow path feature produces circuits in which the only delays are the delays caused by forming the input combination and any transmission time delay. Thus, circuit response time is at a minimum.


#### Abstract

Another important feature stemming from the prepared flow path concept is that the passive memory elements used in this synthesis procedure are never switched when they are under power as they are in the classical method. Switching under power causes undesirable transient pulses in the circuit. This is avoided by switching the element before the passive signal appears.


Suggestions for Further Study

As is true with any study, there are many areas providing interesting further study. Among these are:

1. A Synthesis Procedure Considering Some Combination of the Total Input and Changed Inputs - The synthesis procedure for feedback sequential circuits presented in Chapter II considers only the changed input whereas the procedure for random input circuits (Chapter III) considers only the total input state. Both of these approachs have their own distinguishing merit; however, it is felt that some combination of the two concepts will consistently produce circuits having more of the desirable features of both methods.

In the feedback sequential method, the W elements can often be replaced by "anding" another input signal to the state signal. Rules
for doing this should be investigated.
Another interesting synthesis concept is the use of internal information as an auxiliary input. It seems that as more information is used as input information, the less complicated the resulting circuit. This concept has not yet been pursued.
2. Definition of Origin for Canonical Flow Table The canonical flow table defined in Chapter IV has a unique relationship involving the order of the rows of a primitive flow table. Any two flow tables containing the same information will always result in canonical flow tables having the same row relationships. However, the row appearing first in the table is thus far left to the designer's preference. Although this is usually acceptable, a rigorous definition for the origin or first row of the canonical flow table should be made considering only the topology of the table's transition paths. This would provide a unique format for displaying the information contained in any primitive flow table.
3. Computer Program for Feedback Sequential

Synthesis - Efforts should be made to write a computer program to perform the necessary steps for the synthesis of feedback sequential circuits

```
as presented in Chapter II. The techniques already developed for the present program could be easily adapted to provide a program to accomplish this from a statement of the desired sequence.
```

4. A Logic Synthesis Procedure Considering Proportional As Well As Binary Variables - To date, the synthesis of physical systems using formal logic has been restricted to binary or digital systems. Considering the matrix synthesis philosophy presented in this thesis, it seems natural to extend this technique to include proportional or dynamic variables as well as binary variables. A proportional variable could be entered into the state matrix to modify a state in the same manner as the memory elements are in this thesis. The proportional state modifier would tell not only when to give the output but would also tell how. This "how" could be the proportional signal rather than the binary signal now used.

The author is currently engaged in investigating the possibilities of such a synthesis procedure.

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APPENDIX A

THE PASSIVE MEMORY

## APPENDIX A

## THE PASSIVE MEMORY

This appendix deals with the definition, description, and assignment of passive memory elements.

Definition

Any memory element which does not rely upon an active power source to retain its output state is said to be a passive memory element. In most cases, these devices have a mechanical memory and the logic signal is merely directed through the device according to its mechanical position. The best example of this concept is the four-way, twoposition detent valve shown in Figure 8.


Figure 8. Passive Memory

## Description

This device has many salient features, most important of which is the mechanical memory. Once the device has been switched by either the set or reset signals, the device remains in that position due to the detent hold feature. The signal sent through the device does not necessarily have to be an active signal connected to the supply; this signal may be an input or logic signal which appears only occasionally.

By sending a logic signal through the device, the output XY appears only when the memory element is in the proper position (indicated by $Y$ ) "and" the logic signal $X$ is on. The $X \bar{Y}$ signal appears only when the device is in the "reset" position "and" the signal $X$ is on. This device holds its mechanical position to display memory characteristics and it forms two "and" combinations (X.Y and $X \cdot \bar{Y}$ ); thus, the passive memory device serves the function of three logic elements, memory and two "ands". By utilizing this effect, circuit complexity and hardware can be reduced substantially.

Another advantageous feature of this device is the complementary output. Notice that the device has two outputs, $X Y$ and $X \bar{Y}$; when one is on (pressurized) the other is off (to tank). Thus, the need for the inversion of $Y$ to get its complement $\bar{Y}$ is eliminated.

The pneumatic diaphragm logic device (4) possesses similar mechanical memory characteristics as the valve
described above.
Fluidic passive memory devices without moving mechanical parts do not exist; however, a similar savings in circuit hardware can be made by the use of the two devices shown in Figure 9. The bistable amplifier is an active memory element and its complementary outputs are fed into a passive "and". The passive "and" element has complementary outputs serving the function of two separate "ands" to form $X Y$ and $X \bar{Y}$.


Figure 9. Fluidic Memory Circuit

The latching relay performs the analogous passive memory function in electronic circuits. However, modern
technology has almost phased out the use of relays in compact logic circuits. Even so, the addition of two extra "ands" in an electronic circuit is much less costly than the same for fluid circuits. The usual bistable flip-flop integrated circuit could be built with outputs $X Y$, and $X \bar{Y}$ instead of the usual $Y, \bar{Y}$ where $X$ is some logic signal.

## Assignment

As has been shown above, the passive memory can be used to reduce hardware when distinguishing between two states. The problem of assignment when higher orders of memory are required is discussed next. By using one more passive memory element, the circuit of Figure 8 is modified to form three unique memory states as shown by Figure 10 (a). Four unique states are obtained in Figure 10 (b) by adding one more passive memory element:

As shown by the previous discussion, each time another memory element is added, another unique passive memory state results. In general, $N-1$ passive memory elements describe $N$ unique states. The assignment schematic shown in Figure 11 illustrates the passive memory code. To describe $N$ unique states, omit all memory elements numbered above $\mathrm{N}-1$.

The alternating placement of elements in the assignment code allows the proper balance of fluid power. Higher orders may be obtained in the same alternating pattern.

To illustrate the assignment technique for making each state of an input column unique, consider the three states

(a) Three Unique States, $X \mathrm{Y}_{1} \mathrm{Y}_{2}, \mathrm{X} \mathrm{Y}_{1} \overline{\mathrm{Y}}_{2}$ and $X Y_{1}$

(b) Four Unique States, $X \mathrm{Y}_{1} \mathrm{Y}_{2}, \mathrm{X} \mathrm{Y}_{1} \overline{\mathrm{Y}}_{2}, \mathrm{X} \overline{\mathrm{Y}}_{1} \mathrm{Y}_{3}$, and $X \quad \bar{X}_{1} \bar{Y}_{3}$

Figure 10. Passive Memory Assignment Circuits


Figure 11. Passive Memory Code Schematic

1, 3, and 5. Using the double subscript notation, the memory states are assigned as follows:

$$
\begin{array}{ll}
\text { (1) } & \mathrm{Y}_{15} \mathrm{Y}_{13} \\
\text { (3) } & \mathrm{Y}_{15} \overline{\mathrm{Y}}_{13} \\
\text { (5) } & \overline{\mathrm{Y}}_{15}
\end{array}
$$

The reader is cautioned not to confuse the double subscript notation discussed here and the single subscript notation used in Figure 11. The double subscript notation carries information of the switching conditions. For example, $Y_{15}(r e a d Y$, one, five) is set prior to state 1 , and is reset prior to state 5. As an example of higher order memory state assignment, consider the states 1, 3, 5, 8, 10, and 13. The assignment is as follows:
(1) $\quad Y_{18} \quad Y_{15} \quad Y_{13}$
(3) $\quad Y_{18} Y_{15} \quad \bar{Y}_{13}$
(5) $\quad \mathrm{Y}_{18} \overline{\mathrm{Y}}_{15}$
(8) $\quad \bar{Y}_{18} Y_{813} Y_{810}$
(10) $\quad \bar{Y}_{18} Y_{813} \bar{Y}_{810}$
(13) $\quad \overline{\mathrm{Y}}_{18} \overline{\mathrm{Y}}_{8 \$ 3}$

The reader is encouraged to implement this circuit using Figure 11 as a guideline.

As a final note, it should be pointed out that this synthesis procedure allows every column in the state matrix to be treated independently. In this respect, each input
state (or changed input) may be sent through memory elements as a passive signal.

## APPENDIX B

## LISTING OF COMPUTER PROGRAMS



```
Subroutine logsyn
```



```
    # SUBRDUTINE LOGSYN IS THE EXECUTIVE PROGRAM FOR SYSTEM
    ** SUBRDUTINE LOGSYN IS THE EXECUTIVE PROGRAM FOR SYSTEM *LSPODI22
    ORIGINAL PR:HITIVEFECNH TABLE, CHECKS FOR ANY PFDUNDANT
    INSFORMATIDN IN THE PRIMITIVE FLOH TABLE, REARRANGES THE ROW
    TO FDRM THE CANONICALFLDK TAELE, PRINTS THE CANONICAL F
* TABLE, AND T
    NI = THE NUMBER OF INPUTS
    NO = THE NUMBER OF. OUTPUTS THE PRIMITIVE FLOW TABLE
    NC =THE NUMBER DF COLUMNS IN THE PRIMITIVE FLOH TABLE.
    NC = THE NUMBER OF COLUMNS IN THE PRIMITIVE FLON TABLE. 
    \
            MRIMTIVE FLDH TABLE. STABLE STATES ARE INOICATED 
    SSEIJC,11 = THE NUMBER OFF STABLE STATES IN THE JC-TH COLUMN
    SSM,
#****************************************************************
COMMON /ALL/ N1,NO,NR,NM,NC,IXI4,
COMMON IION/ 1DEN(201
INTEGER S:4S
l FGRMAT(20A4)
3 FORMAT(161411))
4 FORMAT(1614,6I11'LOIC SYNTMESIS',1,27X,'FOR '11,' INPUTS, ',I1,
6 FgRMATIIOX*ORIGINAL PRIMITIVE FLOW TABLE FOR./15X,20A4///)
6 FERMATIIOX'ORIGINAL PRIMITIVE FLOW TABLE FOR'/15X,20A4,
**FORMAT1H1!
lol
lol
    READI5,i` THEARRAY SILES MuSt be Altereo.,
    READ(5,1) IEN NO
    READ(5.2) Ni, NO, NR
    l
    IFINO.GG. SI ERROR=
    M,
    NC =2*NI (1) (1,JC), 1=1,4),JC=1,NC)
    REAO(5,4)(IS(1R,NC),J=1;4),JC=1;NC)
    REAI5,4)ISIIR,ND
    HRITE (6,6) TDEN
    CALL PRINTIO
    synthesis. This program readS the data cards, prints the
    **SPO125
CMMON ALL/ NI,NO,NR,NM,NC,IX(4,16),1Y(36,401,1216,40),S(40,16)
FGRMAT(312)
```



```
    \SP0165
    ** *SP0127
**LSPO128
* *SPO130
* *SPO132
*LSPO133
LSPO135
*LSPO137
    *LSPO138
    * *SPO140
    *LSPO142
* *LSPP145
            LSPO145
            LSPO147
            LSPO149
            LSPO150
            LSPO152
                                LSPO153
            LSPO 154
                        LSPO156
                            lSP0158
                    LSP0160
                    MESPO162
                    $5P164
                            LSPO166
                                LSP0167
                                LSpO169
                                \SPP0170
                                \SPO172
    lSPO173
    CALL PRINTIO:
    LSSP0176
    LSPO178
```




READ 15,4 ) (IMC
On 19 JC=1, NC , $\mathrm{JC}, \mathrm{JN}, \mathrm{JM}=1,191, \mathrm{JC}=1, \mathrm{NCI}$
GI. 1日) WRITE\{6.17) JC, MCIJC. WRITEIG.51 NI, NO, NM, IDEN
CALL ASSIGN(NIS.1)
OD
OD $11 \mathrm{~J}=1, \mathrm{~N}$


GOTO 12
11
12 IR 12 I 1
CALIGEON
NR $=1$
NR $=1$
NRS $=1$

 $\begin{array}{ll}00 & 15 \\ \text { DO } & 15=1,40 \\ \mathrm{~J}=1, \mathrm{NC}\end{array}$ $\mathrm{DS}(1.01$
K
O
O
$15 \mathrm{~S}(\mathrm{I}, \mathrm{J})=0$

$21 \begin{aligned} & 1 \times(1), J C)=16(1, J C) \\ & \times(1)=1 \times(1), ~\end{aligned}$

$\mathrm{x}(1)=\mathrm{NET}(\mathrm{X}(1) \mathrm{I}$
CALL
CRIGEON
NR
NR
NR $=$ NR +1
NRS $=$ NRS +1
DO $2\{=j=1, N C$
DO $22 \quad 11=1, N$
FF(xili) NE. IGIIt.J.J) Go to 23
22

23 CONTINUE 24 IFINR LT. 401 GO TO 25
IFINR ILT;
WRITE 6 , 01
CALL PRINTINMI
25 S(NR.JC) $=$ NRS +1000
S(IR,JC) $=$ NRS
SN
26
26
IY(M,NR) ${ }^{26}$ N(NI
30 DO 30 JEL NO
50
$\mathrm{KS(NR}, J C 1=1$
$\mathrm{X}(1)=\mathrm{NDT}(\mathrm{X}(11)$
CALL EQUIVINMI
DO $60 \quad J R=2, N R$
DO $60 \mathrm{JR=2}=\mathrm{NR}$
DO $60 \mathrm{IC=1,NC}$

kS(JR.iC) $=0$
$\mathrm{IR}=\mathrm{JR}$
$\mathrm{JC}=\mathrm{IC}$


```
M(M)= 1Y(N,1R)
S% Y(M) = IrMM,
CONTINSE
DO 100 I=1,NR
l
MFIIS :LT. O] GO TO 90
0 IF(SIII+N) (EO. IS)
GC TO 100
90 CONTINUE
CONTINUE
110 S(1,j)=-5(1,J)
S(1,J)=-5(1,
CCAL PRNT
RETurn
LSP0319
LSP0320
SPO323
LSPO324
LSPO325
LSPO327
LSPO328
\SP0329
$5P0330
SP0332
LSP0333
LSP0335
```

```
    STATE SHITCHING OR CYCLING.
    * STATE SNITCHING OR CYCLING. SUPPLY THE SHITCHINE EQUATIONS AND
    the OUTPUT EQuATIONS IN THIS SUBROUTINEG the logical
    complement
    x{I] = THE Current state dF the i-Th input
    Y(M) = THE CURRENT STAE OF THE I-TH INPUT MTHEMOR
    ITS(JC)= THE CURRENT STATE OF THE J-TH QUTPUT INPUT STATE FOR THE JC-TH COLUMN
    ITS(JC)= THE TDTAL INPUT STATE FOR THE JC-TH COL
********************************************************************
COMMON SALL/ N1,NO,NR,NH,NC,IX(4,16),IY(36,40),1Z(6,40),S(40,16)
    COMMN /EON/ X(4), Y(36), Z{6), KS
    INTEGER X, Y, Z, S
```




```
        MIgeqnol
        * DIGEQNO2
        *DIGEONO
    THEDIGEONO
    lorer(m)
    lim(t)
    lilll
    ITS(7)={\begin{array}{lll}{x(1)*NOT(x(2))*}&{x(3)*NOT(x)44)}\\{\mathrm{ ITNOT(x(4)}}\end{array})
        iTS( 8)=NOT(x(1)**OT(x(2))*
        ITSI 9) = NOT(X(1))*NOT(X(2))*
        MTS(10)=
        ITS(11) = x(1) *NOT
        ITS.122)=NOT(x(1)
        ITS13)=
        \
--n------
    MS(1) = 1TS(2)#Y(2)*Y(3) +1TS(4)
    MS(1) = ITS(2)#Y(2)*Y(3) + ITS(4)
    MR(1)=ITS(2)*Y(2)*NOT(Y(3))
    MS(2)=ITS(1)+MIS(3)*(
    4S(3) = 1TS(1)*Y{1)
    MR(3)=ITS(1)*NCT(Y{1)] +ITS(3)*Y(4)
    MR{4)=ITS(2)*Y(2)*Y(3) + ITS{4)#Y(5)
    MS(4)=IT{2)*Y(2)*NOTY(3) + +' 
```

SUBROUTINE DIGEQN
 EOUATIONS NECESSARY TO PERFORM SYSTEH SIMULATION. THE USER MAY FIND IT ADVANTAGEDUS TO USE THE TOTAL INPUY STATE ARRAY
ITS JC). INSTEAD OF FORMING THESE STATES IN HIS EQUAIIONS: SWITCHING EQUATIONS ARE REPEATED TO ALLOW FOR ANY INTERNAL


```
        #CH5 Y=1睢
            Y(M)= MEHDRY[Y(M), RS{M),MRIMI)

```

            IF(Y(M).NE. MCH) ICH=1
    15 CONTINAE IF(ICH.EQ. I) GO to 10
c
\#DIGEQNO7
\#DIGEGNOB
*DIGEONI
*OIGEQNI2
*OIGEON13
*OIGEQN15
*DIGEON16
*DIGEN17
* *DIGEQ19
*DIGEON2O
*DIGEN21
*DIGEON23
MIGEQN24
DIGEQN25
MIGE ON28
DIGEQN27
DIGEON28
DIGEQN29
DIEGN3O
DIGEQN31
DIGEQ32
OIGEON34
DIGEQN35
DIGEQN36
OIGEQN37
OIGEON38
D1GEQN39
DIGEQN4O
OIGEQN4O
OIGEON4
DIGEO42
DIGEQN44
DIGEQ44
-DIGEN4,
DIGEN4
c
enter shitching equations-
MR(5) = ITS(3)*NOT(Y(4)

```
\begin{tabular}{|c|c|c|c|}
\hline \multicolumn{4}{|l|}{subroutine equit senses any redundant states in the primitive flow table and replaces these states with their equivalent states. this rdutine is used hith either the synthesis or sinulation prograhs.} \\
\hline \multicolumn{4}{|l|}{\multirow[t]{4}{*}{\begin{tabular}{l}
knm = the number af memory elements to be checked for eouivalence buring system similation. knh \(=0\) for SYSTEM SYNTHESIS. \\
hcise.i) = the number of mehtry elements associated hith. the jc-th column \\
mC(JC.JM+11 = the number designation of the jheth hemory \\
in the jc-th column
\end{tabular}}} \\
\hline & & & \\
\hline & & & \\
\hline & & & \\
\hline
\end{tabular}LSPO 337
LSPO 338
COMRON /EON/ X(4), Y(36), Z(6), KS \((40,26), \mathrm{KC}(16+19)\),
dihension it (16)
integer \(S\)

*IFINR : REMOYED=:
    \(\mathrm{DO} 70 \quad \mathrm{JC}=1, \mathrm{NC}\)
\(\mathrm{NR1}=\mathrm{NR}-1\)
\(\mathrm{DO} 40 \quad 11=1, \mathrm{NR} 1\)
    00 \(4011=1\), NR
    lF(S(II, JC) LE. 1000 ) GO TO 40

    OO \(10 \mathrm{~J}=1\), NO

    IFIIZ(J,II) NE. IZ(J.Iz)! GO To 30
CONTINUE
10 CONTINUE
    IFEKNM .EQ. O) GO TO 12
    \(\mathrm{NMC}=\mathrm{HC}(\mathrm{JC}\),
IF
IFIMC


11 CONTINUE
2 DO \(13, j=1\), NC
    IFIJ.EQ. JC1 60 TO 13
ITIA
\(=\) SII


3 continae
IRC \(=1\)
\(18=\sin 2 . \mathrm{Jcs}-1000\)

    \(1 T(J C)=s(11, \Delta C)\)

```

    NR =NR-1. NR+11 GO TO 25
    Na 24 14=12,NR
    20 J=1,NC
    S(14,J)=s(14+1*J)
    IF(KNM.GT. 0) KS(14,J)=KS(14+1,J)
21 124,14) = Iz(J,14+1)
F(KNM OEQ- O) 60 TE 2
DO 22 M=1,\timesNM
22 [Y(M,I4)= IY(M,I4+1
22 CONTINUE
24 CONTINUE
25OD 2G J=1,NC
20 S(NR+1,J) = 0
26 KSNRR+1,
30 CONTNUE
70 CONTNNUE
RETURN

```

```

SUBROUT TNE CANON
LspO420
******************************************************************LSPO421
THE SYNTHESIS PROGRAM AND REORDERS THE ROHS ACCORDING TO *LSPO424

```

```

    numbers are then reSequenced to proouce the ganonical flow
    table.
    COMMON TALL/ NI;ND,NR,NH,NC,IX(4,16),IY(36,40),12(6,40),S{40,16)
COMMON AALL' NI,ND,NR,NH,NC,IX14,16),IY(36,40),12(6,40),S(40,16) LSPO430
INTEGER S, X X ISER IS STAQLE STATE IN THE FIRST ROE
formatiloxithere is no stable state in the first roh.';
DO 10 I=1,NR
10 KS(1,N)=0
IFIS(1,J).-LT. 10008 60 TO 11
IR = 1
GO TO 20
WRITEIb,H
0 DO22 i=1,N1
DO 50 I=1,NI
X(1) =NNT(X(I)
MO 30 N1=1,NC
43.
IFISIIRR,NI EO. O) GO TO 50
DO 26 IRI=IRI,NR
IFIS(IR1,JII-1000 -NE* SITRR,JII) GO TO 26
LR=IR+1
DO 24 JCI=1,NC
S(IR,JCI)= S(IRI.NCI
24 SiI
S{IR1,JC1)=S
L
25. IZ(JINIR) IR =124
% G0 TD 31
26 GONTINUE
31 IF(IR .GE. NR) GO TO 6I
S0 X{11= NOT(X(1))
\0 60 I2=2,NR
OM 60 J2%1,NC
KS(12*J2)=
IRR = I2
MC = 12
GO CONTINUE
$6.100100 \quad I=1$, NR IS = SiIf, it 1000

```


``` \(5(1, J)=-\{i+1000\}\) 6010100
90 CONTINUE
100 CONTINUE
DO \(110 \begin{aligned} & \mathrm{I}=1, N R \\ & 00: 110 \\ & \mathrm{~J}=1, N C\end{aligned}\)
```



```
RETUPM
END
```

LSPO480
LSPO
LS81
LSPO482
LSPO483
LSPP483
LSPO4B4
LSPO
LSP 0484
LSPO485
LSPO4
LSPOR46
LSPO4B7
LSPO487
LSPO488

| LSPP488 |
| :--- |
| LSPO489 |

LSPP490
LSP0491
LSPO491
LSPO492
LSPPO493
LSPO494

```
SUBROUTINE DUTPUT
    Suaroutine output oetermines the memory requirements,
    pRINTS THE STA
    SET(K,M) = THE m-th STATE SIGNAL uSED to SEt the k-th
    RESET(K,M) = MEMORY THETEMENT MOTH STATE SIGNAL USED to RESET THE K-TH
```



```
        *LSP497
        prinis the state signals, the suitching comditions, ano the
        *LSPO498
        *LSP0500
        *LSPO501
        *LLSPO502
        *
        # = HE H-TH PREVIOUS STATE TOL STABLE STATE NOM, *LSPP506
        RESET(K,1) = THE NUMBER OF MRESET" SIGNALS FOR THE K-TH MEMDRY*LSPOSOG
        mc(JC+1) = the nlmber dF memory elements in the je-th columb*Lsposio
    in the resulting print-dut:
        - THE STATE SIGNALS ARE TB BE SUBSTITUTED FOR THE STATE
        _- NUMBERS IN THE SWITCHING AND OUTPUST EQUATIONS
        -- N###IMPLIES THESLOGICAL MAND"
    ***************************************************************
    COMMON IOUT/ SSC(16,21), SSRI40)
    CUMMON IEON/ X(4), Y(1)
DIMENSION SET(36,201, RESET(36,20), PS(20), IZS(6,40), JL(19),
INTEGER S, SSC, SSR,; SET, RESET, NY(5), 10140
    INTEGERS, SNC, SSR, SET;RESET; PS , IOIH, 39*IH+/
DATA IAB,IAN/1H, 1H-%, NY/5*4H* Y/, IO/IH,
FORHAT (15X,1H2,12,4H)
FORMAT115X,1H(12,4H)
```



```
FORMAT(1H+,26x,116{3X,A1,2x1,1)
```



```
FORMAT(10X:SHITCHING CDNDITIONS:',1)
M,
FORMATCI
NNM =0 J=1,NC
20
MOC
NM = NM+ML(J,1)
MFINH EO,
    MO 25 K=1,NM
25 SE
SETEK,1)=0
MMCL
MML=1
HRITE{G*4},
```

30
MC1 $=M C_{1}+M C(J C, 1)$
$30 \mathrm{IF} \mathrm{CLL}-111 \mathrm{IT}, 160,30$
30 CALL ASSIGN(L) 2$)$
$00100 \quad 1=1, L$
$M 1=0$



$\begin{array}{ll}11=0 \\ 00 \\ 90 & \mathrm{~K}=\mathrm{MCL} \\ \mathrm{MC} \\ \mathrm{MC}\end{array}$


$\underset{\operatorname{MPP}(J 101)}{ }=K^{12 \theta}$

NSI $=$ SETiK, IT
NER
$=$ RESETiK,

SE
DE
82.
SE
GO

93 RESETK
$00.64 \times=1, M i 1$ NR1 * M1
84 RESET(K,1)M\&NR1) = PSSN)
85 CONTINUE
85 CONTINUE
YO CONTINUEE
100 JLENTINUE $=11$
100 CONTINUE
$\mathrm{DO}=15.5-\mathrm{KK}=1, \mathrm{~L}$
$\mathrm{J1}=\mathrm{JL}(\mathrm{KK})$
150 GRITE $16,50,151,152,1531, \mathrm{NL}$


 LSTO 155
LSPOS9
LSPO597
 सRITE(6,9MSINOT $4 M, K K 1, M=1, J 1)$
GOTO 155

155 HRIEE(6,9) SINOT(M+KKIOM=10J1)
CONTINUE CONTINUE
WRIIEGG. 16
60 TO 170
160


170 MRITE( 6,96 ,


DO $175 \mathrm{KK1}, \mathrm{NM}$
$\mathrm{HI}=\mathrm{SET}(\mathrm{K}, \mathrm{LI}$



```
    * (a************************************************#****************LSPO532
```




```
    ****************)
    COMON /ALL/NI,NG,NR,NH,NC,IX(4,16),IY(36,40),I2(6,
    INTEGER S IAP, IN3, IXP, IZP/4H ,4HI 1,4,4*3H--,4*IHX,G*1HZ,
    1. FORMAT(1X,4(5X, IN1,3X,6(1X,A1,I11)
    FORMAT{10X,414x,211),3x,6(1X,A1,I11)
    FDRMAT{10X,813x,3111,3x,6:1x,A1,I1);
    FORMAT{12X,41A3)
    FORMAT{2X,41A3)
    8 FORMAT (10X,216,3X,6{2X,111)
    & FORMAT {10x+216,3x,6{2x,11
    9 FORMAT(10X,416,3X,6(2x,I1)
    lo FORMAT (10x,816,3X,5{2X,1i);
    Z ᄃSNAAT(1H+, 50x,1612)
```



```
    45 FORMAT (1H+, 86X,1612)
io FORMAT(1H+,10X,1H{,1X,A4,3t2X,A4),2H
17 FORMAT(1HH,10X,1H1,1X,A4,7(2X,A4),2H {
18 FORMAT(3H+!,A4, A5(2X;A4);2H |)
    INC=1+NC=2+NO
    WRITE(6,1)(IXP(1),I,I=1,NI)
20 WRITE{6,2i(f1\times1,JCi,I=1,NI),JS=1,NC!,IIZP(J),J,J=1,NO)
    WRIE{6,2(tIX WI,JC),I=1,NI
    DO 22 IR=1,NR
    MS(JC)= S(IR,JC)
    MSS(JC)=IAB
    MK(SIR,JC) LLE. 1000)60
MS(SNC)=S(1R
21 CONIINUE
    WRITE6, 8)(MS(JC),NC=1,NC),(IT(J,IR), S=1,NG)
    IF(NNG NE; O) WRITE{6,N12)(I
```



```
    NRIEE(6,6)(1N3(1), I=1,INC)
    OO 27 1R=1,NR
    MO 26 JC=1:NC
    MSS(JC)= IAB
    MSS(JC)=IAB
MSSIJC)=
MRITE(6. 9l(MSIJC1,JC=1,NC),IIZ(J,IR),J=1,NO)
IF{KNM -NE: O} HRITE{6,13)(IY(M,1R), M=1,KNM)
27 WRIIE(6;16)(MSS(JC),JC=1;NC)
    G0 TO 40
SP0689
```



$\begin{array}{ll}00 & 32 \quad \text { R } \mathrm{R}=1 \text {, NR } \\ \text { DO } \\ 31\end{array}$




31 CONTINUE

32 HRITEAG:I7: NASSIJCI, JC=1,NC
3560 TO 40

$0 a 37 \quad \bar{R}=1, N R$
DC $3 S \mathrm{SC=1,NC}$
MS\{JC) $=5: N C$
MSS:JC $=I A B, J$
IFSSIR,JCI $1 E_{0} 1000$ ) 60 TO 36
AS(JC) $=5(I R, J C:-1000$
$M S S(J C)=I A P$
36 MSSNTINGE
GRITE(S,11)(MS(JC),JC=1, RCC), (IIZ(J,IR),J=1,NO)
IFiKNM - NE, O) HRTTE\{ $\left.\sigma_{2} 15\right)$ (IYY $\left.A, 1 R 1, M=1, K N M\right)$
37 HRITETG,183IMSSTJCI,FC=1,NC)
REIURN
40 KRITE $(6,6)(I N 3(H, I=1$, INC)
RE TUR
END



```
    # SUBRGUTINE ASSIGN PRODUCES EITHER THE PASSIVE MEMORY OR
            LSPO725
    gray ASSIGNMENT CODE. THE PASSIVE MEMORY ASSIGNEENT CODE IS
                                *LSP0726
    USEO FOR MEMORY ASSIGNMENT IN THE SYNTHESIS PFOGRAM AND THE 
    the gray code produced gY this susRoutine has the element on
    theg gRay cooe produced gY this Sug.
    NA = OPIION SPECIFYING CODE
    1 = GRAY CODE (INPUT STATES)
    2 = PASSIVE CODEPMEMDATESY ASSIGNMENT:
    *)
    IGII,SI = THE VALUE OF THE I-TH MEMORY IOR INPUTI IN THE
    * J-TH STATE iOR COLUMN).
    *)
    DIMENSIONIACl6), RAC16,
    INTEGER RA (1) RETURN
    NMI = LOG(K;NA)
IFINA.EQ. 2) GO TO 30
    DO 11 I= {,K.
    DO 11 J=1,NM1
    MM=2**NM1-3!
    OO 11
    A1 = 2**J*1AM+0.5
    A2 =2** **(AM-0.5
IFI.GT.AI .OR. I.LE.AZS GO TO Il
1 IG(1)I)}
RETURN
\00 31 =1,19
IG(1,J)= =-1
NRA =1
2 M=1
MO 33 I=1,NRA
IFINRA.GT. 1) IAII: = 1
M M M*2
35 D0 35 J=1,N,2 (1+J*NRA/M)=IA(1+(J-1)*NRA/M) +M/2
IF(M,LT. NRA) 60 to 34
    OO 37- {=1,NRA
IK =0
```



```
RA(I)=JAili}+|
```


$33 \begin{aligned} & 16(M 1,12+1)=1 \mathrm{IG}(\mathrm{M1}, 12)\end{aligned}$
$\operatorname{IGG}(1 \mathrm{~N}, 1 \mathrm{IP})=1$
$\mathrm{G}(\mathrm{IM}, \mathrm{IR}+1)=0$

$\begin{aligned} & 39 \text { CONTINUE } \\ & \text { NR }=\text { NRR }\end{aligned}$
39
NR $=$ NRR
NRA
$4 \begin{gathered}\text { GO. TO } 32 \\ 40 \\ \text { CONTINUE }\end{gathered}$
RETU
ERSO


|  | function notilal | LSPOB21 |
| :---: | :---: | :---: |
| C \#****\#************************************************************SP0822 |  |  |
| c |  | *LSP0823 |
| ${ }_{6}$ | * function not performs the logical complement of the | *LSPCE24 |
| $c$ | * variable ia. | *LSP0825 |
| c |  | *LSP0826 |
| c | *************************************************** | LSP0827 |
|  | IFIA -GE. 11 NOT = | LSP0828 |
|  | IFIIA.EO. O1 NOT = 1 | LSP0829 |
|  | RETURN | LSP0830 |
|  | End | LSP0831 |




APPENDIX C

EXAMPLE COMPUTER SOLUTIONS

DATA CARDS:
0000000011111111112222222222333333333344444444455555555566666666677777777778
2345678901234567890123456789012345678901234567990123456789012345678901234567890
EXAMPLE Col - table xi representing example 3.3
${ }_{00}^{210}{ }^{210} 2^{11} 017$
$\begin{array}{cc}001 & 2 \\ 81002 & 3 \\ 81003 \\ 81004 & 5 \\ ! & 91005 \\ 8 & 10100 \\ 8 & 10100\end{array}$
00
00
00
00
00
00
10
01
01
01
01
01
01

2 Inputs, 2 Dutputs
original primitive flow table for
EXAMPLE C.1.- table xi representing example 3.3

| $\begin{gathered} x_{1} \times 2 \\ 00 \end{gathered}$ | 10 | 11 | 01 | 21 | 12. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ( 11 | 2 | 0 | 7 | 10. | 0 |
| 8 | $(21$ | 3 | 0 | 10 | 0 |
| 0 | 4 | 131 | 7 | - | 0 |
| 8 | 14 | 5 | 0 | - | 0 |
| 0 | 9 | 151 | 6 | 10 | 0 |
| 1 | 0 | 10 | $(6)$ | 1 |  |
| 8 | 0 | 10 | 171 | 10 | 1 |
| 1.81 | 2 | O | 7 | 10 |  |
| 8 | 191 | 10 | 0 | - | 1 |
| 10 | 9 | 1101 | 7 | 10 | 1 |

CANOMTCAI FLOR TABLE FOR
EXAMPLE C-1-TABLE XI REPRESENTING EXAMPLE 3.3

| $\begin{gathered} 1 \times 2 \\ 00 \end{gathered}$ | 10 | 11 | 01 | 2122 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ( 11 | 2 | 0 | 3 | 0 | 0 |
| 14 | $(2)$ | 5 | 0 | 0 | 0 |
| 14 | O | 6 | 131 | 0 | 1 |
| 114 | 2 | 0 | 3 | 0 | 1 |
| 4 | 7 | 151 | 3 | 0 | 0 |
| 10 | B | $(6)$ | 3 | 0 | 1 |
| 14 | ( 71 | 9 | 0 | 0 | 0 |
| 1 4 | ( 8) | 6 | 0 | 0 | 1 |
| 0 | 8 | (9) | 10 | 0 | - |
| 11 | 0 | , | (10) | 1 | - |

(passive memary assignments
state signals:
$\left\{\begin{array}{l}111=00 * Y 1 \\ 141=00 * Y 1\end{array}\right.$
$\left(\begin{array}{l}21 \\ 71 \\ 11 \\ 19\end{array}=10 * Y 2 * Y 3\right.$
$=10 * Y 2 * Y 3$


SWITCHING CONDITIONS:
$\mathrm{Y} 1 \mathrm{SET}=10$
RESET
$=$
$2+3+7+8$
$\times 2 \quad$ SET $=1+4+5$
$\vee 3 \quad$ SET $\quad$ RESET $=1+4$
Y4 $\underset{\substack{\text { SET } \\ \text { RESET }}}{=} \frac{2}{7}+3+3+10$
Y5 SET $=2+8+10$
$\times 6 \quad \underset{\text { RESET }}{\text { SET }}=\mathbf{1}+4+5+6$
Dutput signals:
$21=10$
$12=3+4 * 6+8$
data cards:
000000001111111811222222222333333333334444444445555555555666666666677777777778 12345678901234567890223456789012345678901234567890123456789012345678901234567890
example caz - table o. 4 from fluto logic text by eac. fitch
$\begin{array}{llll}2112 \\ 00^{2} & 11 & 11 & 01 \\ 1001^{2} & & \end{array}$

original primitive floh table for
example c. 2 - TABLE 6.4 fROM fluid logic text by e.c. fitch

| $\begin{gathered} x_{1} \quad x_{2} \\ 00 \end{gathered}$ | 10 | 11 | 01 | 21 |
| :---: | :---: | :---: | :---: | :---: |
| 111 | 2 | 0 | 7 | 0 |
| 8 | $(2)$ | 3 | 0 | 0 |
| 0 | 4 | (3) | 9 | 0 |
| 10 | $(4)$ | 5 | 0 | 0 |
| 0 | 11 | 151 | 6 | 1 |
| 11 | 0 | 12 | $(6)$ | 1.1 |
| 1. | 0 | 0 | 171 | 0 |
| 181 | 2 | 0 | 0 | $\bigcirc$ |
| 0 | - | 3 | 191 | 0 |
| (10) | 4 | 0 | 0 | 0 |
| 0 | (11) | 5 | 0 | 1 |
| 1.0 | 0 | $(12)$ | 6 | 1 | STATE 8 HAS EQUIVALENT TD STATE, I AND HAS BEEN REMOVED.

STATE 12 HAS EQUIVALENT TO STATE 5 AND HAS BEEN REMOVED.
STATE 9 WAS EQUIVALENT TO STATE $\$$ AND HAS BEEN REMOVED.
canonical flow table for
example c. 2 - table 6.4 from fluid logic texy ay E.So fith

(passive memory assignment)

STATE SIGNALS:
$\left(\begin{array}{l}11 \\ (1)=00 * Y 1 \\ 6\end{array}\right)=00 * Y 1$
$\left(\begin{array}{l}2)=10 * Y 2 * Y 3 \\ 15)=10 * Y 2 * Y 3 \\ 19)=10 * Y\end{array}\right.$
$(4)=11 * Y 4$
$(7)=11 * Y 4$
$\left(\begin{array}{l}(3) \\ (8)=01 * Y 5 \\ \text { (1) }\end{array}\right.$
Shitching conottions:
Y $1 \quad \underset{\text { RESET }}{\text { SET }}=2+3+8$
Y2 $\operatorname{SET}_{\text {RESET }}^{=}=1 * 4+6$
$\because 3$ SET $=1$

Y $_{5}^{\text {SET }}=1+4$
output signals
$71=7 * 8+9$
ssob

## CALL STOP END

data cards:
D0000000011111111112222222222333333333344444444445555555555666666686677777777776 12345678901234567890123456789012345678901234567890123456789012345678901234567890
EXAMPLE C. 3 - 4-INPUT. 30 Raw
4
0.630
0000100011000100011011101010001000111011111101110101110110010001


ORIGINAL PRIMITIVE FLOH TABLE FOR
EXAMPLE C.3-4-INPUT, 30 ROW
$\times 1 \times 2 \times 3 \times 4$

| 00 | 1000 | 1100 | 0100 | 0110 | 1110 | 1010 | 0010 | 0011 | 1011 | 1111 | 0111 | 0101 | 1101 | 1001 | 0001 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ( 11 | 2 | 0 | 10 | 0 | 0 | 0 | 13 | 0 | 0 | 0 | 0 | 0 | 0 |  | 14 | 1 | 1 | 0 | - | 0 | 0 | 0 |
| 1 | 121 | 3 | 0 | 0 | 0 | 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 16 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 10 | 2 | $(3)$ | 10 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 17 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 10 | 0 | 3 | 0 | 18 | 14 | 15 | 0 | 0 | 0 | 5 | 0 | 0 | 0 | 0 |  | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 19 | ( 5 ) | 6 | 0 | 17 | 0 | 0 | I | 1 | 1 | 1 | 1 | 0 | 0 |
| 10 | 0 | 0 | 0 | 18 | 0 | 0 | 0 | 7 | 0 |  | 161 | 11 | 0 | 0 | 0 | 1 | 0 | 1 | 1. |  |  | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 13 | 173 | 19 | 0 | 6 | 0 | 0 |  | 8 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 0 | 0 | 0 | 11 | 0 | 16 | ( ${ }^{8}$ |  | 0 | 0 | 0 | 1 | 1 | 0 |
| 1191 | 20 | 0 | 10 | 0 | 0 | 0 | 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 3 | (10) | 18 | 0 | 0 | 0. | 0 | 0 | 0 | 0 | 11 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | (11) | 17 | 0 | 12 | 1 | 0 |  | 0 | 1 |  | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 0 | 0 | 0 | 11 | 0 | 16. | (12) | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 18 | 0 | 15 | (13) | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 0 | 0 | 0 | 11 | 0 | 16. | (14) | 1 | 0 | 0 | 0 | 1 |  | 1 |
| 10 | 2 | 0 | 0 | 0 | 4 | 125 | 13 | 0 | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | c | 0 | 0 |
| 10 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 19 | 0 | 0 | 0 | 17 | (16) | 12 |  | 1 | 0 | 0 | 1 | 0 | 0 |
| 10 | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 0 | 11 | 117 | 16 | 0 | , | 1 | 1 | 0 | 1 | 0 | 0 |
| 10 | 0 | 0 | 10 | 1181 | 4 | 0 | 13 | 0 | 0 | 0 | 6 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 10 | 0 |  | 0 | 0 | 0 | 15 | 0 | 7 | [19] | 5 | 0 | 0 | 0 | 16 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 |
| 121 | (20) | 3 | 0 | 0 | 0 | 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 16 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $1{ }^{1213}$ | 22 | 0 | 10 | 0 | 0 | 0 | 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 14 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 123 | (22) | 3 | 0 | 0 | 0 | 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 16 | 0 | 1 | 1 | 0 | 0 | - | 0 | 0 |
| 1 (23) | 24. | 0 | 10 | 0 | 0 | 0 | 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 14 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 125 | 124) | 3 | 0 | 0 | 0 | 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 16 | 0 | , | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 (25) | 20 | 0 | 10 | 0 | 0 | 0 | 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 26 |  | 0 | 0 | 0 | 0 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 |  | 0 | 0 | 27 | 0 | 0 | 0 | 11 | 0 | 16. | (26) |  | 0 |  | 0 |  | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 28 | (27) | 19 | 0 | 6 | 0 | $\bigcirc$ | 0 | 8 | 1 | 0 | 0 | , | 1 | 0 | 0 |
| 11 | 0 | 0 | 0 | 18 | 0 | 29 | 1281 | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | - | 0 |  | 0 | 0 | 0 |
| 1.0 | 30 | 0 | 0 | 0 | 4 | (29) | 13 | 0 | 19 | $\bigcirc$ | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1301. | 3 | 0 | 0 | 0 | 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 16 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |

STATE 25 WAS EQUIVALENT TO STATE 9 AND HAS BEEN REMOVED.
STATE 14 WAS EqUIVALENT TO STATE 12 AND HAS BEEN REMOVED.

CANONICAL FLOU TABLE FOR
$x_{1} \times 2 \times 3 \times 4$

| 000 | 1000 | 1100 | 0100 | 0110 | 1110 | 1010 | 0010 | 0011 | 1011 | 1111 | 0111 | 0101 | 1101 | 1001 | 0001 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1111 | 2 | 0 | 3 | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | $!$ | 0 | 0 | 0 | 0 | 0 | 0 |
| 1.1 | ( 21 | 6 | 0 | 0 | 0. | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8 | 0 | 1 | 1 | 0 | 0 |  | 0 | 0 |
| 11 | 0 | 6 | 131 | 9 | 0 . | 0 | 0 | 0 | 0 | 0 | 0 | 10 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 9 | 0 | 7 | 14. | 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0. | 1 | 0 | 0 | 1 |  |  | 0 |
| 1.1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11 | 0 | 0 | 0 | 10 | 0 | 8 | ( 5) | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1.0 | 2 | ( 6) | 3 | 0 | 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 13 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 10 | 2 | 0 | 0 | 0 | 12 | 171 | 4 | 0 | 14 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |  | 0 | 0 |
| 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 14 | 0 | 0 | 0 | 13 | (8) | 5 | ; | 1 | 0 | 0 | 1 | 0 | 0 |
| 10 | 0 | 0 | 3 | 19 | 12 | 0 | 4 | 0 | 0 | 0 | 15 | 0 | 0 | 0 | 0 | 1 |  |  | 1 |  |  | 0 |
| 10 | 0 | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 15 | (10) | 13 | 0 | 5 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | (11) | 14 | 0 | 15 | 0 | 0 | 0 | 16 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 6 | 0 | 9 | 1121 | 7 | 0 | 0 | 0 | 17 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |  | 0 |
| 10 | 0 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 17 | 0 | 10 | (13) | 8 | 0 | I |  | 1 | 0 |  | 0 | 0 |
| - 0 | 0 | 0 | 0 | 0 | 0 | 7 | 0 | 11 | (14) | 17 | 0 | 0 | 0 | ${ }^{8}$ | 0 | - 1 | 1 | 0 | 1 | 1 | 0 | 0 |
| - ${ }^{0}$ | 0 | 0 | 0 | 9 | 0 | 0 | 0 | 11 | 0 | 17 | [151 | 10 | 0 | 0 | 0 | 1 | 0 |  |  |  |  | 0 |
| 118 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11 | 0 | 0 | 0 | 10 | 0 | 8 | (16) | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 10 | 0 | 0 | 0 | 0 | - 12 | 0 | 0 | 0 | 14 | 1171 | 15 | 0 | 13 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| (18) | 19. | 0 | 3 | 0 | $\bigcirc$ | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 20 | + | 0 | 0 | 0 | 0 | 0 | 0 |
| 21. | (19) | 6 | 0 | 0 | 0 | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | . | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 118 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 22 | 0 | 0 | 0 | 10 | 0 | 8 | (20) | 1 | 0 |  | 0 | 1 |  | 0 |
| 11211 | 23 | 0 | 3 | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 |  | 0 | 0 | 0 | 0 | - | 0 |
| 10 | 0 |  |  | 0 | 0 | 0 | 24 | (22) | 14 | 0 | 15 | 0 | 0 | 0 | 16 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 125 | (23) |  | 0 | 0 | 0 | 7 | 0 | 0 | 0. | 0 | 0 | 0 | 0 | 8 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | - |
| 1 | 0 | 0 | 0 | 9 | 0 | 26. | (24) | 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| [251 | 27. | 0 | 3 | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 28 | 0 | 0 - | 0 | 12 | (26) | 4 | 0 | 14 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 18 | (27) | 6 | 0 | 0 | 0 | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | (28) | 6 | 0 | 0 | 0 | 7 | 0 | 0 | 0 | - | 0 | 0 | 0 | 8 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |

tPassive memory assignmenti
state signals:
$111=0000 * Y 1 * Y 2$
$118,=0000 * Y \neq Y Y$
$1211=0000 * Y 1 * Y Z$
$125)=0000 * Y 1 * Y 3$

$1231=1000 * Y 4 * Y 5$
$127)=1000 * Y 4 * Y-1$
(28) $=1000$
$31=0100$

```
        (9) = 0110
        (12) = 1110
        (20) =1010* % % 
        (24) =0010* % % %
        (11) =0001* Y Yob
        (14) = 1011
        117)=1111
        1151 =0111
        {10) = 0101
    (13) = 1101
    (8) = 1001
    {55=0001**Y11*Y12
SHITCHING CONDITIONS:
    Y1 SET = 2+3+3+4+5 5+24+28+16+20+27
```



```
    Y S SET = RESET = 23
    Y4 S SET = RESET = 25+26+7+8+18+21
```



```
    YO SET SET =25
    YT SET SET = 18+6+7+8
    Y8 SET = 2 + + + 12+14+19+23+27+28
    Y9. SET SESET=22+7+9+11+18+21+25*26
    Y10 SET = 4+5*14+15+16* 24
```


## CALL STOP END

oata caros:
000000000111111111122222222223333333333444444444655555555566666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
example c.4-8 event counter
EXAMPLL
1116
1001


LOGIC SYNTHESIS
1 INPUTS, 1 OUTPUTS.

ORIGINAL PRIMITIVE FLOW TABLE FOR
EXAMPLE $C-4-8$ EVENT COUNTER


CANOMICAL FLOM TABLE FGR
EXAMPLE C.4-8 EVENT COUNTER

| $\times 1{ }_{0}$ | 1 | 21 |
| :---: | :---: | :---: |
| $1 \cdot 11$ | 2 | 0 |
| $1{ }^{3}$ | 121 | 0 |
| $1(3)$ | 4 | 0 |
| 15 | (4) | 0 |
| 1155 | ${ }^{6}$ | 0 |
| 17 | (6) | 0 |
| 117 | ${ }^{8}$ | 0 |
| 19 | (8) | 0 |
| 1191 | 10 | 0 |
| 111 | 1101 | 0 |
| 1 1113 | 12 | 0 |
| $1{ }^{13}$ | (12) | 0 |
| (13) | 14 | 0 |
| 15 | (14) | 0 |
| (15) | 16 | 0 |
| 1 | 1167 | 1 |

(PASSIVE MEMORY ASSIGNMENT)
state signals:


```
        RESET = 12 * 14
        SET =16
```



```
    Y6 SET SET = 4
    r> SET SES = 12
```



```
    r9 \
    V10 SET SESET = 13; 11 
    N11 SET = = 1
    Y12 SET SESET = 11
    r13 SET SESET = = %
    V14 SET SESET=13
dutput signals:
    21=16
```

DATA CARDS:
00000000011111111122222222223333333333444444444555555555566666666667777777777
example c. 5 - table xil - classical equations
EXAMPL
2.22
022
$\begin{array}{rrrr}0000 & \\ 2 & 1 & 2 \\ 2 & 1 & 2 \\ 2 & 1 & 2 \\ 2 & 1 & 2\end{array}$
MS(1) $=1$ ITS(3) $+X(2) * Y(2)$
MR(1) $=1 T S(1)+X(2) * Y(2)$
$+N O T X(21) * Y(2)$
MS(2) $=I T S(2) * Y(1)+\operatorname{ITS}(4) * Y(1)$
MR(2) $=I T S(3) * Y(1)+\operatorname{ITS}(1) * N O T(Y(1)$
$M R(2)=1 T S(3) * Y(1) * i r s(1)$
$2(1)=1 T S(2) * N O T(Y(1)) * Y(2)$
$2(2)=1 T S(4) * Y(1) * Y(2)$
$(2)=115(4) * \mathrm{Y}(1) * Y(2)$
for 2 inputs, 2 dic simulation 2 memories.

Simulated flow table for
example c.5 - table xil - classical equations

| $x 1$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 02 | 10 | 11 | 01 | 21 | 22 |  |
| 1 | 11 | 2 | 0 | 3 | 0 | 0 |
| 1 | 1 | 21 | 4 | 0 | 0 | 0 |
| 1 | 0 | 4 | 31 | 0 | 0 |  |
| 0 | 6 | 41 | 5 | 0 | 0 |  |
| 1 | 0 | 4 | 51 | 0 | 1 |  |
| 1 | 1 | 6 | 4 | 0 | 1 | 0 |

$0-1000$
-50000
3.308

## CALL STOP LOGSIM STOP SNO

data cards:
12345678901111111112222222222333333333344444444445555555555666666666677777777778
example c. 6 - rotation direction sensor - state matrix equations
$\begin{array}{ll}2 & 14 \\ 2 & 1 \\ 000000 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4\end{array}$

MR(1) $=1$ ITS(2)
$M S(2)=1 T S(3)$
$M R(2)=1 T S(1)$
MS(2) $=1 T S(3)$
$M R(2)=1 T S 11$
$M S(3)=(1 T S(4)$
MR(3) $=$ ITS(2)
MS(4) $=$ ITS(3)
MR(4)


FOR 2 INPUTS, 1 OUTPUTS. 4 mEmories.

Simulated floh table for
example cog - rotation direction sensor - state matrix equations

s. 10 BB
call logsim $\underset{\substack{\text { STOP } \\ \text { ENO }}}{ }$

DATA CarDS:
00000000011111111112222222222333333333344444444445555555555656666666677777777778
12345678901234567890123456799012345676901234567890123456799012345678901234567990
Example c. 2 - table 6.4 from fluid logit by fitch - state matrix equations
EXARLE
2115
1111100
$\begin{array}{lll}1 & 1 & 5 \\ 1 & 1 \\ 2 & 2.3 \\ 1 & 4 \\ 1 & 5\end{array}$
MS(1) $=$ ITS(2)*Y(2)*Y(3) + ITS(4)
MR(1) $=$ ITS(2)*V(2)*NGT(Y(3))
HS(2) $=\operatorname{ITS}(1)+\operatorname{ITS}(3) * Y(1)$
$H R(2)=\operatorname{ITS}(3) * N O T Y(4) 1)$
MS( 2$)=15 S(1) * N(1)$
$M R(3)=15(1) * N O T$
MR( $(3)=\operatorname{ITS}(1) * \operatorname{NOT}(Y(1) 1)+1 T S(3) * Y(4)$
$M S(4)=1 T S(2) * Y(2) * Y(1)+1 T(4) * Y(5)$


MR(5) $=1$ IS $(3) * N O T(Y(4))$
I(1) $=1 T S(2) * N O T(Y(2))+I T S(3) * N O T(Y(4))+I T S(4) * N O T(Y(5))$
for 2 inputs, 1 goutpuis, 5 memories.
simulated flow table for
example c. 2 - table 6.4 from fluid logic by fitch - staje matrix equations

| $\begin{array}{c}x 1 \\ 02\end{array}$ | 10 | 11 | 01 | 21 |
| :---: | :---: | :---: | :---: | :---: |
| 100 | 11 | 2 | 0 | 3 |
| 1 | 1 | 21 | 4 | 0 |
| 1 | 0 |  |  |  |
| 1 | 0 | 4 | 31 | 0 |
| 0 | 5 | 41 | 3 | 0 |
| 1 | 6 | 51 | 7 | 0 |
| 1 | 51 | 5 | 0 | 3 |
| 1 | 0 | 9 | 17 | 8 |
| 1 | 0 | 1 |  |  |
| 1 | 1 | 0 | 7 | 81 |
| 1 | 91 | 7 | 0 | 1 |

[^1]
## VITA

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Master of Science

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Professional Experience: Technical analyst, Nuclear Research Services, Dallas, Texas, 1967; graduate teaching assistant, Southern Methodist University, 1968; graduate research assistant, Oklahoma State University, 1968-69.

Professional Organizations: Member Pi Tau Sigma.


[^0]:    ${ }^{1}$ Numbers in parentheses refer to references in the Selected Bibliography.

[^1]:    

