A NUMERICAL SIMULATION OF BOREHOLE TEMPERATURE DISTRIBUTIONS WHILE DRILLING

By

RICHARD E. BENNETT

Bachelor of Science

Oklahoma State University

Stillwater, Oklahoma

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Thesis Approved:

Roger J. Schaengold
Thesis Adviser

W. Gomer

N. Durham
Dean of the Graduate College

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

A	Coefficient Matrix of Equation (3-41) Modified by Substitution of (3-52).
AA	Coefficient Matrix of Equation (3-40).
a,b	Constants Linearly Relating T_f (i,1) to T_A(i).
A_A	Area Normal to Flow in the Annulus, sq. ft.
A_D	Area Normal to Flow in the Drill Pipe, sq. ft.
AZ	Defined in Equation (3-40).
$B_{i,j}$	Elements of a Matrix after Gaussian Elimination Has Been Applied.
c	Constant Relating a New Step Size to the Old Step Size.
C	Right Hand Side Vector of Equation (3-41) Modified by Substitution of (3-52).
CC	Right Hand Side Vector of Equation (3-40).
C_i'	Elements of a Vector after Gaussian Elimination Has Been Applied.
C_p	Specific Heat of the Circulating Fluid, Btu/lb.-^oF.
C_{pf}	Formation Specific Heat, Btu/lb.-^oF.
D	Characteristic Diameter, ft.
DC	Vector Generated During Gaussian Elimination of A-Matrix Used to Update the Vector C.
DD	Vector Generated During Gaussian Elimination of AA-Matrix Used to Update the Vector CC.

DZ	"New" Grid Increment Used when Simulating a Well Being Drilled, ft.
F	Unknown Vector of Equation (3-54).
FF	Unknown Vector of Equation (3-53).
G	Mass Velocity of Drilling Fluid, lb/hr-sq. ft.
H	Defined in Equation (3-51).
h_f	Borehole Wall Heat Transfer Coefficient, Btu/hr-sq. ft.-^{o}F.
h_i	Heat Transfer Coefficient at the Inside Wall of the Drill Pipe, Btu/hr-sq. ft.-^{o}F.
h_o	Heat Transfer Coefficient at Outside Wall of Drill Pipe, Btu/hr-sq. ft.-^{o}F.
i	Denotes Grid Point Location Along Z Axis.
j	Denotes Grid Point Location Along r Axis.
K	Number of Grid Points in r Direction.
k	Thermal Conductivity of Circulating Fluid, Btu/hr-ft-^{o}F.
$k_{drill\ pipe}$	Thermal Conductivity of Drill Pipe Metal, Btu/hr-ft-^{o}F.
k_f	Formation Thermal Conductivity, Btu/hr-ft-^{o}F.
L	Total Depth Considered, ft.
M	Number of Grid Points in Z Direction.
N	Denotes Time Step
N_{st}	Stanton Number
Q_p	Heat Flow Between Drill Pipe and Annulus, Btu/hr.
R', RR'	Residuals Defined in Equation (4-1) and (4-2).
r_B	Radius of Borehole Wall, ft.
r_D	Radius of Drill Pipe, ft.

r_h	Hydraulic Radius Equal to Area Normal to Flow Dived by the Perimeter, ft.
r_{ID}	Inside Radius of Drill Pipe, ft.
r_{OD}	Outside Radius of Drill Pipe, ft.
r_∞	Distance from the Wellbore Where no Temperature Disturbance is Felt Throughout the Total Time of Fluid Circulation, ft.
R,Q,S	Defined in Equations (3-32), (3-33), and (3-34).
W,V,E_j,G_j,GE_j	Defined in Equations (3-35), (3-36), (3-37), (3-38), and (3-39).
T_1,T_2	Defined in Figure 2.
t	Time, hr.
T_A	Temperature of Fluid in the Annulus, ^oF.
T_D	Temperature of Fluid in the Drill Pipe, ^oF.
T_f	Formation Temperature, ^oF.
T_{DO}	Fluid Inlet Temperature into Drill Pipe, ^oF.
t_1	Time at Which Drilling of the Increment DZ Begins, hr.
T_∞	Geothermal Formation Temperature, ^oF.
U	Over-all Heat Transfer Coefficient Between Drill Pipe and Annulus, Btu/hr-ft-^oF.
V_A	Velocity of Circulating Fluid in the Annulus, ft/hr.
V_D	Velocity of Circulating Fluid in the Drill Pipe, ft/hr.
X	First Element of Right Hand Side Vector in a System of Equations Similar to the Matrix Equation (3-42).
y(x)	Arbitrary Function.
Z,r	Cylindrical Coordinates
ZZ,ZA	Defined in Equation (3-40).

$\Delta r(j)$	Radial Step Size Increment Between Grid Point j and j-1, ft.
Δt	Time Step Increment, hr.
ΔZ	Grid Increment in Depth, ft.
μ	Viscosity of Drilling Fluid, lb/hr-ft.
(μ_w/μ_B)	Ratio of Fluid Viscosity Evaluated at the Wall Temperature Divided by the Fluid Viscosity Evaluated at the Bulk Stream Temperature.
ρ	Density of Circulating Fluid, lb/ft^3.
ρ_f	Formation Density, lb/ft^3.

CHAPTER I

INTRODUCTION

Origin and Objective of this Study

During recent studies at Oklahoma State University on regional geothermal gradients, the necessity became apparent of the need of further study of temperature disturbances caused by drilling fluid circulation. Undisturbed subsurface temperature data are not readily available in many areas; however, the maximum temperatures recorded while logging wells soon after they have been drilled to total depth are available for most areas. These temperatures usually differ from the true geothermal temperatures due to the heat transfer that has taken place while circulating the drilling fluids.

More accurate knowledge of subsurface temperatures is of interest to the oil industry, particularly in recent years because of higher temperatures associated with deeper drilling and the drilling of geothermal wells. Subsurface temperature information is useful when considering potential oil and gas exploration areas, prediction and control of downhole mud properties, lost circulation, cementing material, casing design, log analysis, optimum location of geothermal wells and in reservoir engineering applications. Consequently, a better understanding of the factors that affect temperatures while the well is being drilled, and in the thermal stabilization period thereafter, would improve drilling operations, and aid in the search for energy sources within the earth.

Mathamatical equations describing wellbore heat transfer were found in the literature; however, a direct solution of these equations was not possible, and no approximate, practical solution for engineering analysis has ever been given. With the availability of larger, faster digital computers, a numerical solution of the problem is now feasible. The finite-difference method lends itself to a broad application in which differing boundary conditions and heterogeneous systems can be considered. This method has been successfully used in similar heat transfer problems and a great variety of other problems involving physical phenomena. With a little ingenuity and proper application of numerical techniques a practical solution, that is both accurate and efficient, can be obtained for a specific problem.

The objective of this study is to develop a practical numerical method capable of modeling accurately and efficiently the non-steady temperature distributions created by fluid circulation in a drilling well. The method must be practical in the sense that computer simulation time is short enough to allow an economical engineering analysis of the problem.

Previous Developments

The following literature survey covers the major developments, pertinent to this study, that have been made in wellbore heat transmission.

The general theory describing heat transfer in physical situations, such as those found in the wellbore, has been long established and is largely accepted. Most of the work that has been done in the area of wellbore heat transmission, therefore, has been directed towards the solution of system(s) of equation(s) describing physical phenomena that

have been derived from the basic concepts of heat transfer. The earlier work in this area was directed towards the determination of temperature profiles caused by fluid injection into an existing well. In 1959, Moss and White (9) obtained solutions describing the temperature behavior of water injected into a well. A quasi-steady state was assumed, thus lessening the importance of time on the temperature behavior. In addition, Moss and White found it necessary to discretize the spatial coordinate, depth.

A more rigorous approach to the problem of estimating temperature profiles as a result of hot-water injection was taken by Squier et al. (15). The solutions of Moss and White were slightly improved upon by solving analytical expressions that were continuous in the independent variables time and depth. However, the solutions of Moss and White compared favorable with those of Squier et al., thus adding to the validity of the quasi-steady state assumption for the long term injection problem.

Ramey (10) investigated the temperature disturbance caused by either hot or cold fluid injection into a well. He developed a more general approach to solving the wellbore heat transmission problem by considering total-energy and mechanical-energy equations simultaneously in order to yield both temperature and pressure distributions. However, in order to solve these equations, he found it necessary to make many simplifying assumptions, thus reducing his solutions to those of approximate total energy equations.

Edwardson et al. (4), in an effort towards better interpretation of electric logs, developed a method of calculating formation temperature disturbances caused by mud circulation. This method was based on

solutions of the radial diffusivity equation in which all initial conditions were approximated using techniques of superposition.

In 1966 Schoeppel and Gilarranz (13) showed that maximum recorded temperatures taken while logging could be used to analyze regional geothermal gradient trends. Formation temperatures were known to have been disturbed during drilling prior to logging; and, due to this, they were previously assumed to be unrepresentative of the steady-state formation temperature. However, by calculating formation temperature build-up using the results of earlier workers and a solution to the radial diffusivity equation, Schoeppel and Gilarranz showed that the temperatures taken at the time of logging were sufficiently close to the true geothermal temperatures to merit their use. This was further demonstrated by correlating known geological data in Oklahoma with regional geothermal gradients, which were computed using log temperature data only.

The most recent work in which heat transfer due to circulation of mud in the wellbore has been considered is that of Raymond (11) who, in 1969, presented a general technique for calculating the temperature distribution in a circulating drilling fluid. Three coupled partial differential equations were solved numerically for the drill pipe and annulus temperatures as functions of time and depth, and the surrounding formation temperatures as functions of depth, time and radial distance from the wellbore. The partial differential equations were well known heat-flow equations that have been presented by Bird, Stewart, and Lightfoot (1).

Raymond's numerical solution technique is of interest to this study for comparative reasons. Therefore, an outline of his solution

method is presented here:

1. Using the specified circulating conditions, physical dimensions of the system and fluid properties, heat transfer coefficients were calculated from the Seider-Tate correlation (14).

2. Using the initial condition of geothermal temperature at the initial time, the drill pipe and annular temperature profiles were calculated for the first time step by solving the wellbore heat flow equations by explicit finite differences. The formation temperature used in the first step of the calculation was the geothermal temperature at that particular depth.

3. From the calculated annular profile, the flux of heat into the annulus from the formation was calculated.

4. Based on the flux found, the formation temperatures were calculated using a solution to the radial diffusivity equation presented by van Everdingen and Hurst (5).

5. Using the values of formation temperatures calculated, the annular temperature profiles, the flux and the formation temperatures were recalculated. This iterative procedure was continued until the flux or formation temperature did not change significantly with iteration number.

6. At the next time step, the previous values of heat flux between the annulus and formation were used to predict the new formation temperatures by the principle of superposition. The drill-pipe and annular temperature profiles were calculated as before, and iterations were carried out until the calculations converged.

7. The above process was continued until the calculations were completed for a total time of calculation specified by the particular problem.

Although Raymond completely described the temperature distribution in a circulating drilling fluid, the iterative explicit solution technique appeared to be undesirable for use in solving a larger class of problems. One such problem is the determination of the temperature distributions in the drilling fluid and surrounding formation throughout an entire drilling period. This problem arises from the need for more complete information concerning formation temperature parameter sensitivity.

Statement of the Problem

Given a well in which there is drilling fluid circulation, the basic problem is to determine the dynamic, thermal response of the fluids in the wellbore and the surrounding formation. A problem of heat flow may be posed in several ways; however, one of the principal considerations is to determine the type, or types, of heat transfer that will be considered in the problem. Differential equations dynamically describing the particular type(s) of heat flow are then formulated, along with initial and boundary conditions applicable to the particular problem being studied. In all facets of the problem development, simplifying assumptions are made only if they are necessary in order to solve the problem.

Physical System

The physical situation studied is represented in Figure 1. Fluid enters the drill pipe at the surface and passes down the drill pipe. The fluid exits the drill pipe through the bit and enters the annulus at the bottom. The fluid then passes up the annulus and exits the annulus at the surface. There is heat exchange between the fluid in

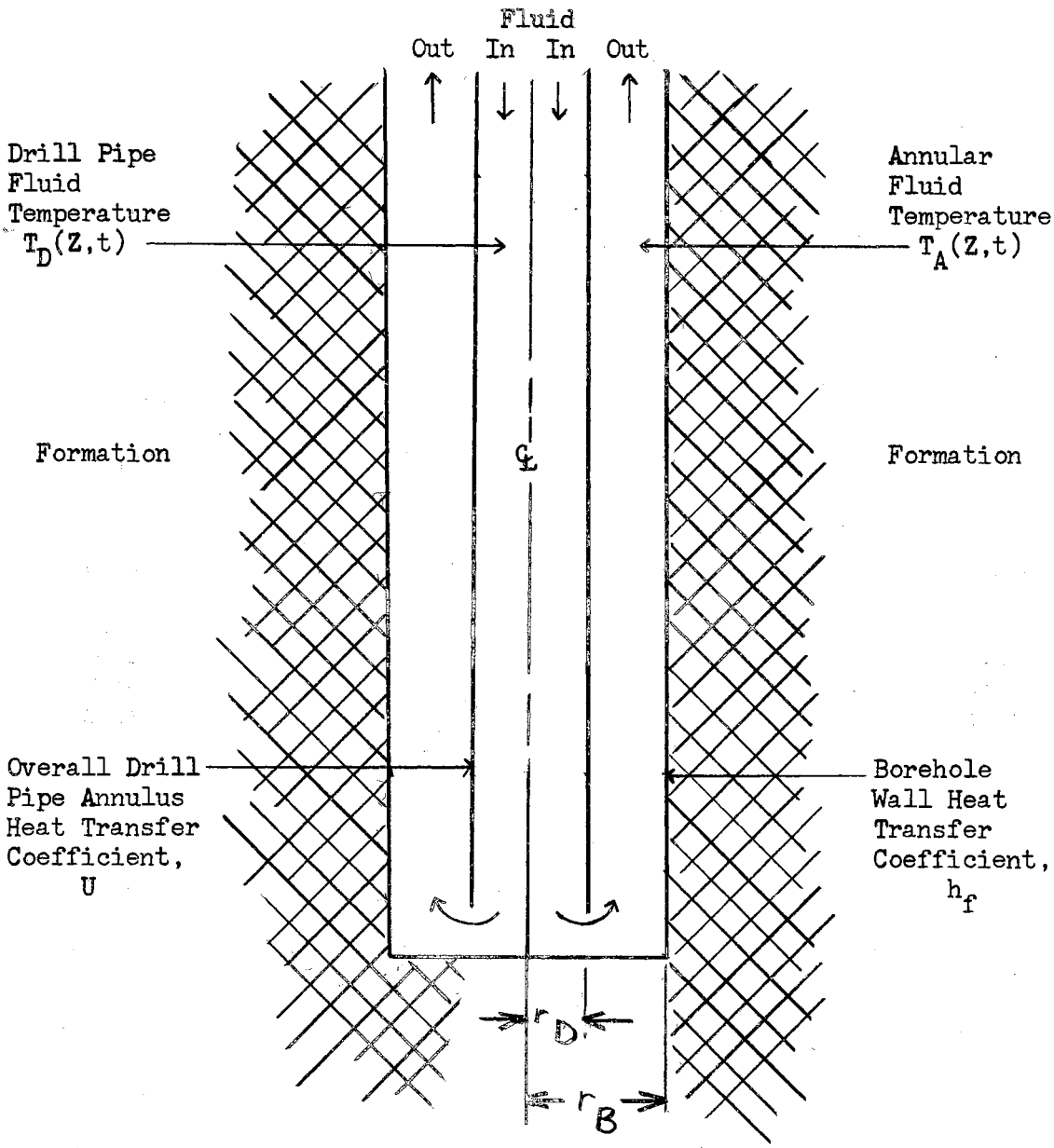

Figure 1. Schematic of Circulating Fluid System

the drill pipe and that in the annulus, and between the annulus fluid and surrounding formation.

Mathematical Statement of the Problem

A. Differential Equations

It was desired to solve the problem in general, with as few limiting assumptions as possible. The differential equations, therefore, selected as most suitable are:

$$A_D \rho V_D C_p \frac{\partial T_D(Z,t)}{\partial Z} + 2\pi r_D U\left[T_D(Z,t) - T_A(Z,t)\right] = -\rho A_D C_p \frac{\partial T_D(Z,t)}{\partial t} \qquad (1\text{-}1)$$

$$A_A \rho V_A C_p \frac{\partial T_A(Z,t)}{\partial Z} + 2\pi r_D U\left[T_D(Z,t) - T_A(Z,t)\right] + 2\pi r_B h_f \left[T_f(r_B, Z, t) - T_A(Z,t)\right] = \rho A_A C_p \frac{\partial T_A(Z,t)}{\partial t} \qquad (1\text{-}2)$$

$$\frac{\partial T_f(r_B, Z, t)}{\partial t} = \frac{k_f}{\rho_f C_{pf}} \frac{1}{r} \frac{\partial}{\partial r}\left[r \frac{\partial T_f(r_B, Z, t)}{\partial r}\right] \qquad (1\text{-}3)$$

The Z and r are cylindrical coordinates, t represents time, T_D, T_A, and T_f, are the drillpipe, annulus, and formation temperatures, respectively, U is the over-all heat transfer coefficient between the drill pipe and annulus, h_f is the borehole wall transfer coefficient, k_f is the formation thermal conductivity, ρ_f is formation density, C_{pf} is

formation specific heat, C_p is the specific heat of the drilling fluid, V_D is the velocity of the fluid in the drill pipe, V_A is the velocity of the fluid in the annulus, A_D and A_A are cross-sectional areas of the drill pipe and annulus, respectively, and r_D and r_B are the radii of the drill pipe and borehole wall, respectively. These equations assume that the temperature of the fluid as it passes down the drill pipe is determined by the rate of heat convection down the drill pipe, the rate of heat exchange between the drill pipe and the annulus, and time. As fluid flows up the annulus, its temperature is determined by the rate of heat convection up the annulus, the rate of heat exchange between the annulus and the drill pipe, the rate of heat exchange between the formation adjacent to the annulus and the fluid in the annulus, and time.

These equations were used by Raymond (11). A simplified form of the equations was used by Squier et al. (15) to determine hot water injection profiles. Many investigators, of which Edwardson et al. (4), Ramey (10), and Schoeppel and Gilarranz (13) are some, have used the radial diffusivity equation (equation (1-3)) to describe heat conduction in the formation adjacent to the wellbore. Bird, Stewart, and Lightfoot (1) and Dusinberre (3) have presented derivations of similar equations.

B. Assumptions

The derivation of equations (1-1), (1-2), and (1-3) will not be presented here; however, the assumptions that have been made in their development are necessary for an understanding of the problem.

The following assumptions have been made:

1. The effect of heat generated by bit action is negligible.

2. There is no fluid flow into or from the formation.

3. There is no fluid flow in the annulus other than that of the circulating drilling fluid.

4. Heat transfer by radiant energy is negligible.

5. Heat transfer by conduction in the circulating fluid is negligible.

6. Heat transfer in the formation is by conduction in the horizontal direction only.

Other assumptions will be given later when the fluid parameters, heat transfer coefficients, and initial and boundary conditions are considered.

Boundary Conditions

The dimensions and cylindrical coordinates of the system are shown by Figure 2.

The appropriate boundary conditions for this problem are:

$$T_D(Z=0,t) = T_{D_o}(t) \tag{1-4}$$

$$T_D(Z=L,t) = T_A(Z=L,t) \tag{1-5}$$

$$T_f(r_\infty, Z, t) = T_\infty(Z) \tag{1-6}$$

$$2\pi r_B h_f \left[T_f(r_B, Z, t) - T_A(Z, t)\right] = 2\pi r_B k_f \left[\frac{\partial T_f(Z,t)}{\partial r}\right]_{r=r_B} \tag{1-7}$$

Here r_∞ is sufficiently far from the wellbore such that its temperature, $T_\infty(Z)$, is undisturbed throughout the total time of drilling fluid circulation. At total depth, Z = L, fluid leaves the drill pipe and enters the annulus, therefore, the annulus and drill pipe temperatures

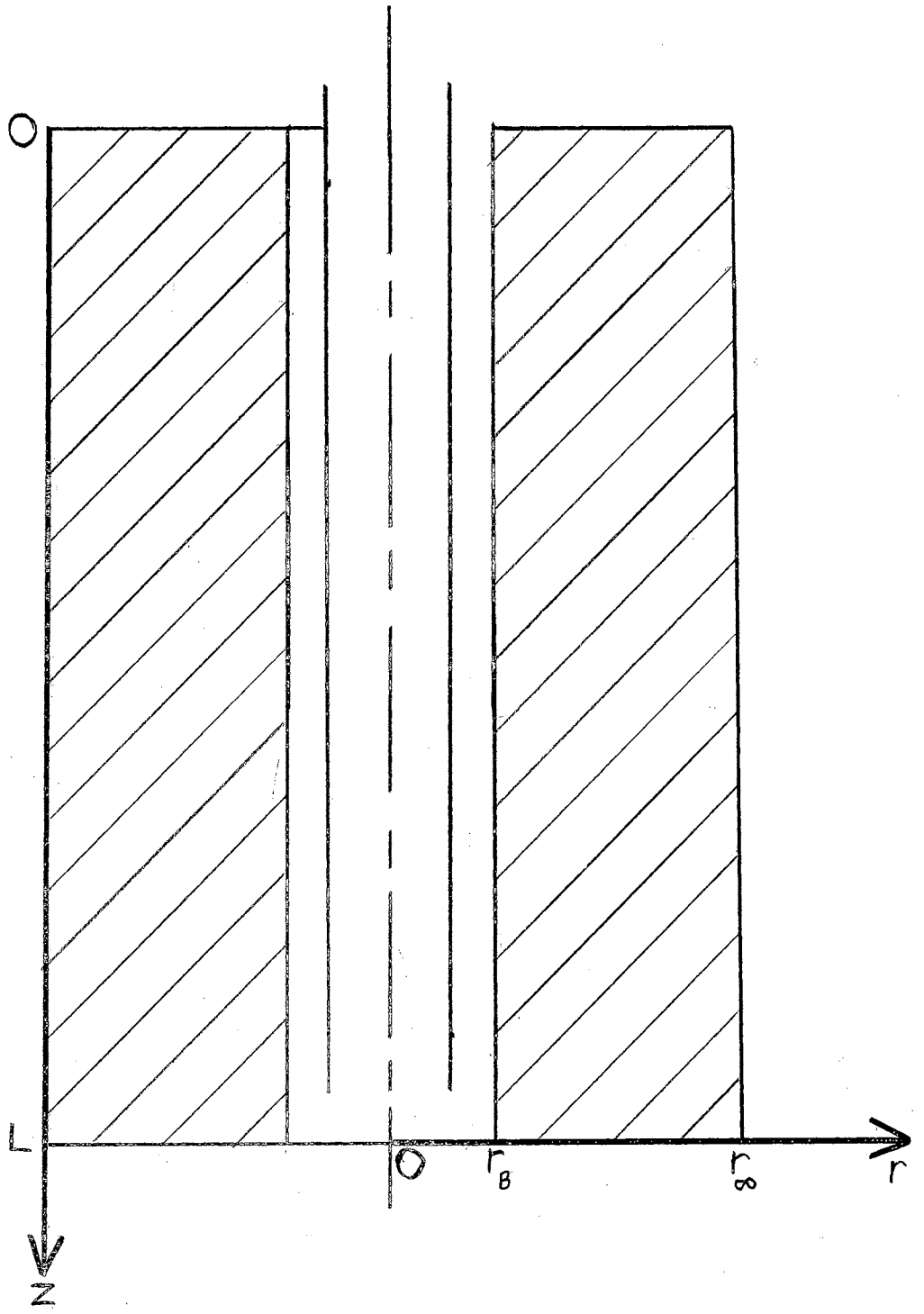

Figure 2. Assumed Coordinate System for Numerical Model

must be the same. The inlet temperature of the drilling fluid coming into the drill pipe from the mud pits must be specified. It is reasonable to specify this temperature as a constant for circulation at a constant depth, and as an increasing function with depth as the well is drilled deeper. Equation (1-7) states that the flux of heat entering the annulus from the formation must equal the heat flux lost by the formation.

Initial Conditions

The initial conditions considered are:

$$T_D(Z,0) = T_A(Z,0) = T_f(r,Z,0) = T_\infty(Z). \tag{1-8}$$

While drilling, the conditions are modified somewhat in that

$$T_D(Z+DZ,t_1) = T_A(Z+DZ,t_1) = T_D(Z,t_1) \tag{1-9}$$

and,

$$T_f(r,Z+DZ,t_1) = T_\infty(Z), \tag{1-10}$$

Here DZ is a discreet increment in depth in which drilling is assumed to take place starting at time, t_1.

CHAPTER II

HEAT TRANSFER COEFFICIENTS

To solve equations (1-1), (1-2), and (1-3) the heat transfer coefficients U and h_f must first be determined. The overall heat transfer coefficient, U, considers the thermal resistances at both the inside tube walls of the drill pipe, and the resistance to heat flow through the drill pipe. This coefficient is defined in the following equation (refer to Figure 3):

$$Q_p = 2\pi r_D U (T_D - T_A) \tag{2-1}$$

also,

$$Q_p = 2\pi R_{ID} h_i (T_D - T_1) \tag{2-2}$$

$$Q_p = \frac{2\pi k_{drill\ pipe}}{\ln\left(\frac{R_{OD}}{R_{ID}}\right)} (T_1 - T_2) \tag{2-3}$$

$$Q_p = 2\pi R_{OD} h_o (T_2 - T_A) \tag{2-4}$$

since,

$$T_D - T_A = T_D - T_1 + T_1 - T_2 + T_2 - T_A \tag{2-5}$$

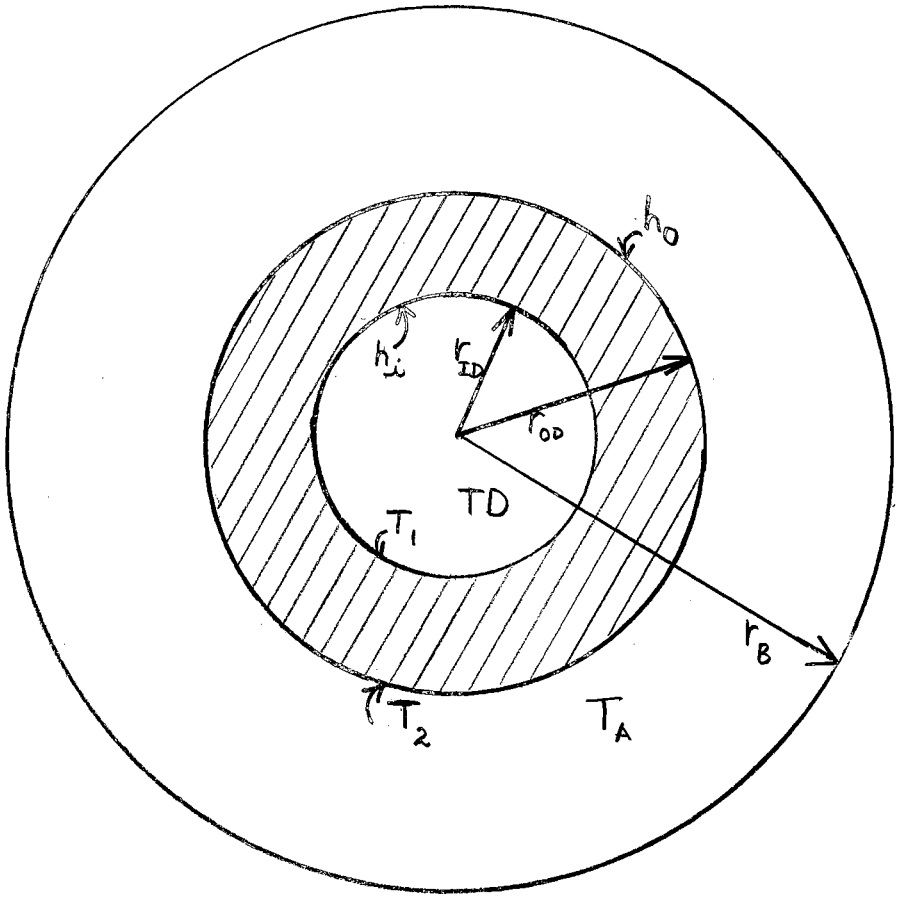

Figure 3. Cross-sectional View of the Wellbore and Associated Heat Transfer Coefficients

$$T_D - T_A = Q_P \left[\frac{1}{2\pi R_{ID} h_i} + \frac{\ln\left(\frac{R_{OD}}{R_{ID}}\right)}{2\pi k_{drill\ pipe}} + \frac{1}{2\pi R_{OD} h_o} \right] \quad (2\text{-}6)$$

$$U = \frac{1}{\frac{R_D}{R_{ID} h_i} + \frac{R_D \ln(R_{ID}/R_{ID})}{k_{drill\ pipe}} + \frac{R_D}{R_{OD} h_o}} \quad (2\text{-}7)$$

The coefficients, h_i, h_o, and h_f can be approximated using correlations presented in the literature (7, 14). These correlations have been developed by using dimensional analysis to form parameter groupings. Emperical data was then used to relate these parameter groups. The parameter groups that were considered significant for this study were the Prandtl number,

$$\frac{C_p \mu}{k} \quad (2\text{-}8)$$

the Reynolds number,

$$DG/\mu \quad (2\text{-}9)$$

and the Stanton number,

$$\frac{h_f}{C_p G} \quad (2\text{-}10)$$

where μ is the viscosity of the drilling fluid, G is the mass velocity of the fluid, k is the thermal conductivity of the drilling fluid, and D is a characteristic diameter equal to $2r_{ID}$ for h_i, and $2(r_B - r_{OD})$ for h_o and h_f (after McAdams (7), page 241).

For this study the fluid properties were considered to be invariant with temperature. This assumption has been made due to lack of sufficient data on the temperature dependency of drilling fluid properties. If data were available it could easily be incorporated into the problem solution.

McAdams (7) presented the following correlation of the parameter groups (2-8), (2-9), and (2-10):

$$\text{for } (L/2r_B) > 60 \text{ and,}$$
$$2100 < (DG/\mu) < 10{,}000$$

$$\frac{h}{C_p G}\left(\frac{C_p \mu}{k}\right)^{2/3}\left(\frac{\mu_w}{\mu_b}\right)^{0.14} = \frac{0.023}{(DG/U)^{.2}} \qquad (2\text{-}11)$$

$$\text{for } (DG/\mu) > 10{,}000$$
$$(L/2r_B) > 60$$

$$\frac{h}{C_p G}\left(\frac{C_p \mu}{k}\right)^{2/3} = \frac{0.023}{(DG/\mu)^{.2}} \qquad (2\text{-}12)$$

The viscosity ratio (μ_w/μ_b) is a ratio of the fluid viscosity evaluated at the wall temperature divided by the fluid viscosity evaluated at the bulk stream temperature. Since the fluid parameters are considered temperature invariant in this study this term reduces to unity, and equation (2-11) becomes equation (2-12). Thus, equation (2-12) may be used to evaluate the heat transfer coefficients h_i, h_o, and h_f.

CHAPTER III

NUMERICAL SOLUTION

Finite-Difference Equations

An approximate solution of equations (1-1), (1-2), and (1-3) may be obtained by replacing each derivative by its finite-difference approximation and solving the resulting set of algebraic equations either explicitly or implicitly. The basic methods are explained adequately in most texts on numerical analysis (e.g., Collatz (2), McCracken and Dorn (8), and Richtmyer (12)); however, a derivation of a finite-difference approximation for the first and second derivative, when the step size is not constant, is necessary here.

If a function y(x) and all its derivatives exist at a point x_o, then y(x) can be expanded in a Taylor's series as:

$$y(x) = y(x_o) + (x - x_o)\,y'(x_o) + \frac{(x - x_o)^2}{2!}\,y''(x_o) + \frac{(x - x_o)^3}{3!}\,y'''(x_o) + \ldots \text{higher-order terms} \tag{3-1}$$

With the notation $y_i = y(x_o + i\Delta x_i)$, equation (3-1) can be written:

$$y_1 = y_o + (\Delta x_1)\,y_o' + \frac{(\Delta x_1)^2}{2}\,y_o'' + \frac{(\Delta x_1)^3}{3!}\,y_o''' + \ldots \tag{3-2}$$

$$y_{-1} = y_0 - (\Delta x_{-1})\, y_0' + \frac{(\Delta x_{-1})^2}{2} y_0'' - \frac{(\Delta x_{-1})^3}{3!} y_0''' + \cdots \qquad (3\text{-}3)$$

Subtracting equation (3-3) from (3-2) and, ignoring terms of order 2 and higher, a finite-difference form for the derivative $\frac{dy}{dx}$ at the point x_o is obtained, i.e.:

$$y_0' = \frac{y_1 - y_{-1}}{\Delta x_1 + \Delta x_{-1}} \qquad (3\text{-}4)$$

Similarly, addition of (3-2) and (3-3) yields a finite-difference form for the second derivative, i.e.:

$$y_0'' = \frac{y_1 - 2y_0 + y_{-1} + (\Delta x_{-1} - \Delta x_1) y_0'}{\frac{1}{2}(\Delta x_1^2 + \Delta x_{-1}^2)} \qquad (3\text{-}5)$$

When $\Delta x_1 = \Delta x_{-1}$, equations (3-4) and (3-5) reduce to the well knwon centered-difference finite-difference forms. These are:

$$y_0' = \frac{y_1 - y_{-1}}{2\Delta x} \qquad (3\text{-}6)$$

$$y_0'' = \frac{y_1 - 2y_0 + y_{-1}}{\Delta x^2} \qquad (3\text{-}7)$$

An explicit method was used by Raymond (11) in his solution. However, in this study an implicit method was used. The implicit method usually requires more numerical calculations for a solution at

at each time step, but the time step size is not limited by convergence criteria common to explicit methods. In addition, when solving coupled equations, an implicit approach can eliminate the necessity of applying iterative techniques. The choice of the implicit method was made for these reasons. However, it was also realized that, if necessary, the method would have to be altered by some corrector technique if the round-off error became significant.

The implicitly formulated finite-difference equations for equations (1-1) and (1-2) (ΔZ = constant) are, respectively:

$$\left\{\frac{A_D \rho V_D C_p}{2\Delta Z}\left[T_D(i+1) - T_D(i-1)\right] + 2\pi r_D U\left[T_D(i) - T_A(i)\right]\right\}^{N+1} =$$

$$\frac{-\rho A_D C_p}{\Delta t}\left[T_D^{N+1}(i) - T_D^{N}(i)\right] \qquad (3\text{-}8)$$

$$\left\{\frac{A_A \rho V_A C_p}{2\Delta Z}\left[T_A(i+1) - T_A(i-1)\right] + 2\pi r_D U\left[T_D(i) - T_A(i)\right] + \right.$$

$$\left. 2\pi r_B h_f\left[T_f(i,1) - T_A(i)\right]\right\}^{N+1} = \frac{\rho A_A C_p}{\Delta t}\left[T_A^{N+1}(i) - T_A^{N}(i)\right] \qquad (3\text{-}9)$$

The subscript i denotes the grid point along the Z-axis, and the subscript N denotes the time step, with N + 1 being the new time at each step.

Equation (1-3), formulated implicitly with a variable radial step size, becomes:

$$\frac{1}{r(j)}\left[\frac{T_f(j+1)-T_f(j-1)}{\Delta r(j)+\Delta r(j+1)}\right]^{N+1}+\frac{2}{\Delta r^2(j)+\Delta r^2(j+1)}\left\{T_f(j+1)\right.$$

$$\left.-2T_f(j)+T_f(j-1)+\left(\Delta r(j)-\Delta r(j+1)\right)\left[\frac{T_f(j+1)-T_f(j-1)}{\Delta r(j)+\Delta r(j+1)}\right]\right\}^{N+1}=$$

$$\frac{\rho_f C_{pf}}{K_f \Delta t}\left[T_f^{N+1}(j)-T_f^{N}(j)\right] \qquad (3\text{-}10)$$

The subscript j denotes the grid point along the r-axis (see Figure 4).

These finite-difference equations are not the only difference equations that could be used. More accurate difference representations may be developed; however, the second order difference representations were considered adequate for this study in view of the added computational work required by higher order methods.

Equations (3-8), (3-9), and (3-10) completely define the temperature history of the physical system. The complete solution of these equations consists of knowledge of,

$$T_D(i),\ i=1,M$$

$$T_A(i),\ i=1,M \qquad (3\text{-}11)$$

$$\left[T_f(i,j),\ j=1,k\right]\ i=1,M$$

for each position in time, N.

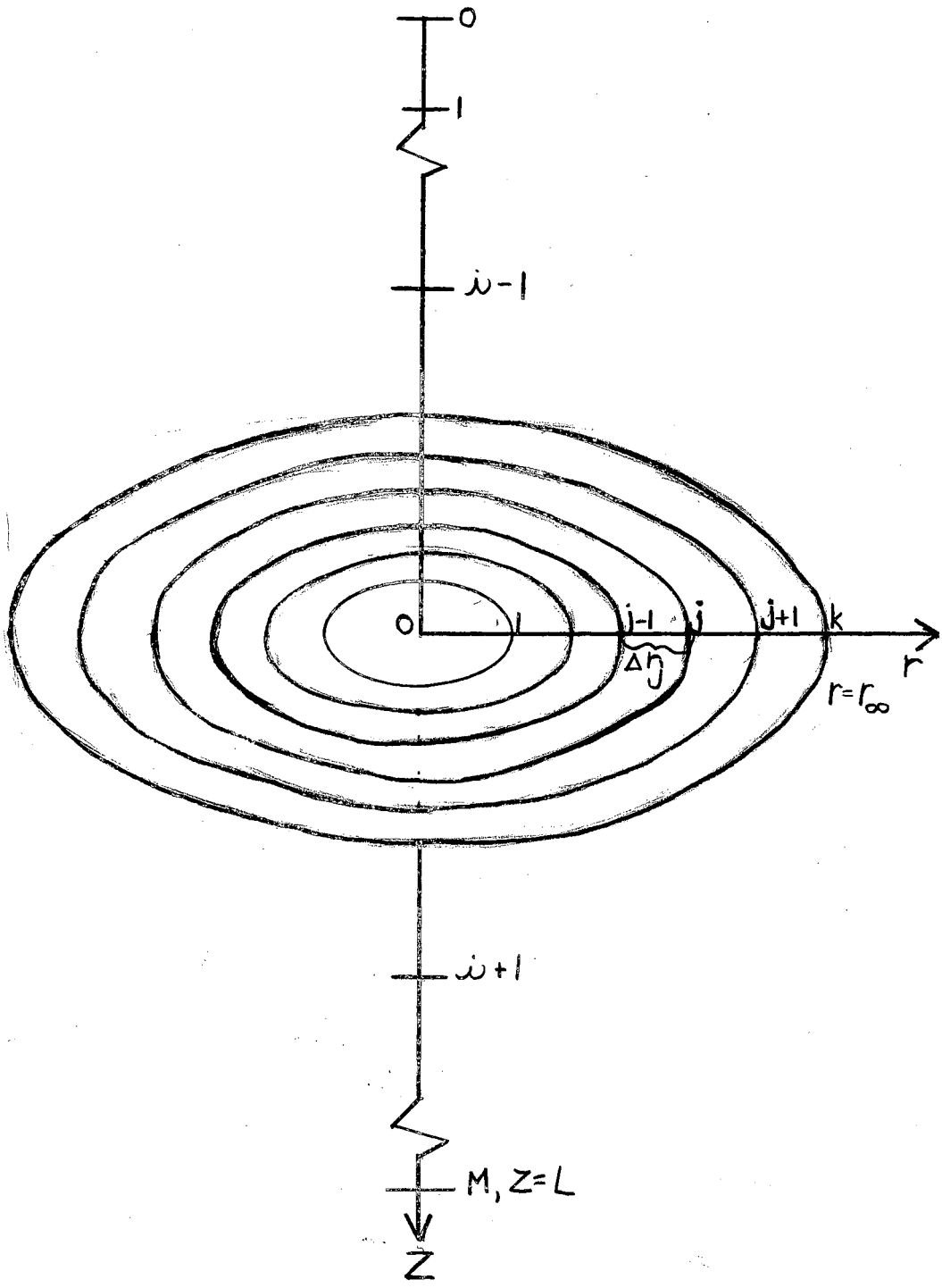

Figure 4. Numbering of Grid Points

These equations are algebraic equations; however, assuming appropriate boundary and initial conditions are specified, these equations are implicit in the sense that none of the temperatures can be determined without simultaneously determining all the temperatures of the solution set, (3-11).

Boundary Conditions

To carry out the solution of equations (3-8), (3-9), and (3-10) a knowledge of either the temperatures or heat fluxes at some of the boundaries must be known. This knowledge comes from the boundary conditions which were specified in Chapter I. These conditions are now applied to equations (3-8), (3-9), and (3-10), and the resulting finite-difference forms are considered.

<u>at i = 0</u>

Equation (3-8) becomes:

$$\left\{\frac{A_D \rho V_D C_p}{2\Delta Z}\left[T_D(2)-T_{D_0}\right]+2\pi r_D U\left[T_D(1)-T_A(1)\right]\right\}^{N+1} =$$

$$-\frac{\rho A_D C_p}{\Delta t}\left[T_D^{N+1}(i)-T_D^{N}(i)\right] \qquad (3\text{-}12)$$

The annulus temperature at this boundary is not specified. In order to maintain the same number of unknowns as equations, a forwards difference representation of the derivative at this point is written. Equation (3-9), therefore, becomes:

$$\left\{\frac{A_A \rho V_A C_\rho}{\Delta Z}\left[T_A(2)-T_A(1)\right]+2\pi r_D U\left[T_D(1)-T_A(1)\right]+\right.$$

$$\left.2\pi r_B h_f\left[T_f(1,1)-T_A(1)\right]\right\}^{N+1}=\frac{\rho A_A C_\rho}{\Delta t}\left[T_A^{N+1}(1)-T_A^{N}(1)\right] \qquad (3\text{-}13)$$

at i = M

The drill pipe temperature at this boundary is not specified. Therefore, a backwards difference representation of the derivative is written. Equation (3-8) becomes:

$$\left\{\frac{A_D \rho V_D C_\rho}{\Delta Z}\left[T_D(M)-T_D(M-1)\right]+2\pi r_D U\left[T_D(M)-T_A(M)\right]\right\}^{N+1}=$$

$$-\frac{\rho A_D C_\rho}{\Delta t}\left[T_D^{N+1}(M)-T_D^{N}(M)\right] \qquad (3\text{-}14)$$

Here, the annulus temperature is given in terms of the drill pipe temperature. Equation (3-9) is replaced by the following equation:

$$\left\{T_A(M)=T_D(M)\right\}^{N+1} \qquad (3\text{-}15)$$

j = 1, all i

At j = 1, r = r_B and the heat flux leaving the formation must equal that going into the annulus. Therefore, a boundary condition for equation (3-10) may be obtained in terms of the annulus temperature at that particular depth, i. Replacing equation (1-7) by its central-difference, finite difference form gives:

$$\left\{2\pi r_B h_f \left[T_f(i,1) - T_A(i)\right] = \frac{2\pi r_B k_f}{2\Delta r(1)} \left[T_f(i,2) - T_f(i,0)\right]\right\}^{N+1} \qquad (3\text{-}16)$$

Equation (3-10) written at j = 1 is:

$$\left\{\frac{1}{2r\Delta r(1)} \left[T_f(i,2) - T_f(i,0)\right] + \frac{1}{\Delta r^2(1)} \left[T_f(i,2) - 2T_f(i,1) + T_f(i,0)\right]\right\}^{N+1} = \frac{\rho_f C_{pf}}{k_f \Delta t} \left[T_f(i,1)^{N+1} - T_f(i,1)^N\right] \qquad (3\text{-}17)$$

The ficticious temperature, $T_f(i,0)$, is determined from equation (3-16),

$$\left\{T_f(i,0) = T_f(i,2) - \frac{2\Delta r(1) h_f}{k_f} \left[T_f(i,1) - T_A(i)\right]\right\}^{N+1} \qquad (3\text{-}18)$$

and equation (3-17) is written:

$$\left\{\left[\frac{1}{\Delta r^2(1)} - \frac{1}{2r\Delta r(1)}\right]\left[T_f(i,2) - \frac{2\Delta r(1) h_f}{k_f}\left[T_f(i,1) - T_A(i)\right]\right] + \frac{1}{2r\Delta r(1)}\left[T_f(i,2)\right] + \frac{1}{\left[\Delta r(1)\right]^2}\left[T_f(i,2) - 2T_f(i,1)\right]\right\}^{N+1} =$$

$$\frac{\rho_f C_{pf}}{k_f \Delta t}\left[T_f(i,1)^{N+1} - T_f(i,1)^N\right] \qquad (3\text{-}19)$$

j = k, all i

At this boundary the temperature is a constant equal to the geothermal temperature at depth, i. Equation (3-10) becomes:

$$\frac{1}{r(k)}\left[\frac{T_\infty - T_f(k-1)}{\Delta r(k)+\Delta r(k+1)}\right]^{N+1} + \frac{2}{[\Delta r(k)]^2+[\Delta r(k+1)]^2}\left\{T_\infty - 2T_f(k) + T_f(k-1) + [\Delta r(k) - \Delta r(k+1)]\left[\frac{T_\infty - T_f(k-1)}{\Delta r(k)+\Delta r(k+1)}\right]\right\}^{N+1} =$$

$$\frac{\rho_f C_{pf}}{K_f \Delta t}\left[T_f^{N+1}(k) - T_f^{N}(k)\right] \qquad (3\text{-}20)$$

Initial Conditions

Values of $T_D(i)$, $T_A(i)$, and $T_f(i,j)$ at time N must be known in order to obtain a solution at time N + 1. In order to begin the solution of equations (3-8), (3-9), and (3-10) initial values of the variables must be specified. These values are the initial conditions given in Chapter I. In terms of the grid system these conditions are:

for all i, N = 0

$$T_D(i) = T_\infty(i) \qquad (3\text{-}21)$$

$$T_A(i) = T_\infty(i) \qquad (3\text{-}22)$$

for all i, all k, N = 0

$$T_f(i,k) = T_\infty(i) \qquad (3\text{-}23)$$

For simulation of a well that is being drilled, the initial conditions that are appropriate for each new interval, DZ, of depth considered are:

<u>for the grid points, i', contained in DZ</u>

$$T_D(i') = T_D(M) \tag{3-24}$$

$$T_A(i') = T_A(M) \tag{3-25}$$

$$T_f(i',k) = T_\infty(i') \tag{3-26}$$

Spatial Step Size Change

Equations (3-8) and (3-9) require that the spatial variable, Z, be discretized into equal increments of size, ΔZ. For reasons of accuracy and practicality it is often desirable to determine temperature distributions from data corresponding to depths that are unequally spaced along the Z axis. A step size may be multiplied by an integer, and the computational procedure continued, with little difficulty (the boundaries of Z are assumed to stay the same). However, when the step size is multiplied by anything other than an integer, an approximation for some of the dependent variables must be made.

Consider the portion of the Z axis shown in Figure 4. A step size change has been made, starting at point i. In order to calculate the derivative at point i the temperature at the ficticious step (see Figure 5) must be approximated from knowledge of nearby temperatures. A second order approximation of $T_{ficticious}$ is justified since the derivatives of equations (3-8) and (3-9) are calculated considering second

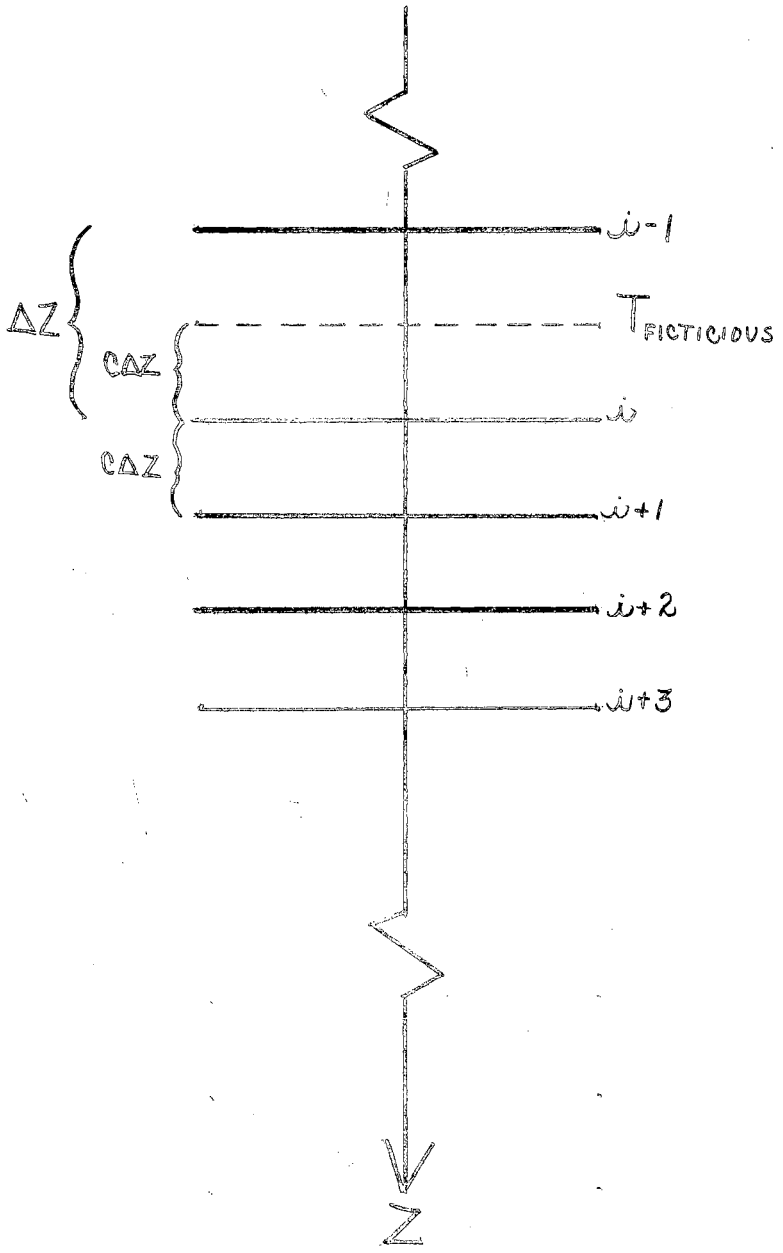

Figure 5. Grid Nomenclature for a Step Size Change

order effects. However, for simplicity, a first order approximation is used. This requires that the temperature at the ficticious grid point be:

$$T_{FICTICIOUS} = c\,T_{i+1} + (1-c)\,T_i \qquad (3\text{-}27)$$

Therefore, the central difference approximation for the derivative at point i is:

$$\frac{\partial T_i}{\partial Z} = \frac{T_{i+1} - [c\,T_{i+1} + (1-c)\,T_i]}{2c\,\Delta Z} \qquad (3\text{-}28)$$

Simultaneous Solution of the Equations

Equations (3-8), (3-9), and (3-10) with their associated end conditions (equations (3-12), (3-13), (3-14), (3-15), (3-19), and (3-20) constitute a set of 2M(1+k) algebraic equations with 2M(1+k) unknowns. In order to accurately simulate the temperature distributions, caused by mud circulation in a deep well, it was found necessary to use values of M = 48 and K = 30. This is a set of 2976 equations. Since these equations were formulated implicitely, they must be solved simultaneously in order to determine their solutions. In addition, the equations must be solved again at each time step (around 75 times for a deep well).

This problem cannot be solved practically if the usual solution techniques are used (i.e., matrix inversion, standard Gauss elimination, Gauss-Seidel iterative method, etc.). Rather than simplify the problem by introducing further approximations, or use time consuming iterative techniques, an efficient method of solving this problem was developed by modifying some of the usual methods of matrix manipulation so as to take

advantage of the particular structure of the problem. This method is developed in the following pages.

Equations (3-8), (3-9), and (3-10) can be written:

$$\left\{\frac{V_D \Delta t}{2\Delta Z}\left[T_D(i+1)-T_D(i-1)\right]+\left[\frac{2\pi r_o U \Delta t}{\rho A_D C_p}+1\right]\left[T_D(i)\right]\right.$$

$$\left.-\frac{2\pi r_o U \Delta t}{\rho A_D C_p}\left[T_A(i)\right]\right\}^{N+1} = T_D^N(i) \qquad (3\text{-}29)$$

$$\frac{V_A \Delta t}{2\Delta Z}\left[T_A(i-1)-T_A(i+1)\right]+\left[\frac{2\pi r_B U \Delta t}{\rho A_A C_p}+1+\frac{2\pi r_B h_f \Delta t}{\rho A_A C_p}\right] T_A(i)$$

$$\left.-\frac{2\pi r_o U \Delta t}{\rho A_A C_p}\left[T_D(i)\right]-\frac{2\pi r_B h_f \Delta t}{\rho A_A C_p}\left[T_f(1,i)\right]\right\}^{N+1} = T_A^N(i) \qquad (3\text{-}30)$$

$$-\frac{k_f \Delta t}{\rho_f C_{pf}}\left\{\frac{1}{r(j)\left[\Delta r(j)+\Delta r(j+1)\right]}+\frac{2}{\Delta r^2(j)+\Delta r^2(j+1)}\left[1+\frac{\Delta r(j)-\Delta r(j+1)}{\Delta r(j)+\Delta r(j+1)}\right]\right\}$$

$$T_f^{N+1}(j+1)+\left[\frac{4 k_f \Delta t}{\rho_f C_{pf}\left[\Delta r^2(j)+\Delta r^2(j+1)\right]}+1\right] T_f^{N+1}(j)-\frac{k_f \Delta t}{\rho_f C_{pf}}$$

$$\left\{-\frac{1}{r(j)\left[\Delta r(j)+\Delta r(j+1)\right]}+\frac{2}{\Delta r^2(j)+\Delta r^2(j+1)}\left[1-\frac{\Delta r(j)-\Delta r(j+1)}{\Delta r(j)+\Delta r(j+1)}\right]\right\}$$

$$T_f^{N+1}(j-1) = T^N(j) \qquad (3\text{-}31)$$

The following parameter groups are defined to be constant in order that this development may be followed more clearly. The method of solution; however, does not require that these parameters be invariant. They may vary as a function of any one, or all of the independent variables.

$$R = \frac{V_D \Delta t}{2\Delta Z} \tag{3-32}$$

$$Q = \frac{2\pi r_D U \Delta t}{\rho A_D C_p} \tag{3-33}$$

$$S = \frac{V_A \Delta t}{2\Delta Z} \tag{3-34}$$

$$W = \frac{2\pi r_D U \Delta t}{\rho A_A C_p} \tag{3-35}$$

$$V = \frac{2\pi r_B h_f \Delta t}{\rho A_A C_p} \tag{3-36}$$

The following parameter groups vary as a function of r only.

$$G_j = \frac{k_f \Delta t}{\rho_f C_{pf}} \left\{ \frac{1}{r(j)[\Delta r(j) + \Delta r(j+1)]} + \frac{2}{\Delta r^2(j) + \Delta r^2(j+1)} \left[1 + \frac{\Delta r(j) - \Delta r(j+1)}{\Delta r(j) + \Delta r(j+1)} \right] \right\} \tag{3-37}$$

$$E_j = \frac{k_f \Delta t}{\rho_f C_{pf}} \left\{ \frac{-1}{r(j)\left[\Delta r(j) + \Delta r(j+1)\right]} + \frac{2}{\Delta r^2(j) + \Delta r^2(j+1)} \left[1 - \frac{\Delta r(j) - \Delta r(j+1)}{\Delta r(j) + \Delta r(j+1)} \right] \right\} \tag{3-38}$$

$$GE_j = \frac{4 k_f \Delta t}{\rho_f C_{pf} \left[\Delta r^2(j) + \Delta r^2(j+1)\right]} + 1 \tag{3-39}$$

Substituting equations (3-37), (3-38), and (3-39) into equations (3-31), (3-19), and (3-20) and writing in matrix form gives:

$$\begin{bmatrix} AZ & -ZA & & & & & \\ & \ddots & \ddots & & & & \\ & & G_j & GE_j & E_j & & \\ & & & \ddots & \ddots & \ddots & \\ & & & & G_K & GE_K & \end{bmatrix}^{N+1} \begin{bmatrix} T_f(i,1) \\ T_f(i,2) \\ \vdots \\ T_f(i,j-1) \\ T_f(i,j) \\ T_f(i,j+1) \\ \vdots \\ T_f(i,k-1) \\ T_f(i,k) \end{bmatrix} + \begin{bmatrix} ZZT_A(i) \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}^{N+1} = \begin{bmatrix} T_f(i,1) \\ T_f(i,2) \\ \vdots \\ T_f(i,j-1) \\ T_f(i,j) \\ T_f(i,j+1) \\ \vdots \\ T_f(i,k-1) \\ -E_K T_\infty(i) + T_f(i,k) \end{bmatrix}^{N} \tag{3-40}$$

$$AZ = \left\{\left[\frac{1}{\Delta r^2(1)} - \frac{1}{2r\Delta r(1)}\right]\left[\frac{2\Delta r(1)h_f}{k_f}\right] + \frac{2}{\Delta r^2(1)}\right\}\frac{k_f \Delta t}{\rho_f C_{pf}} + 1$$

$$ZA = -\left[\frac{2}{\Delta r^2(1)}\right]\left[\frac{k_f \Delta t}{\rho_f C_{pf}}\right]$$

$$ZZ = -\left[\frac{1}{\Delta r^2(1)} - \frac{1}{2r\Delta r(1)}\right]\left[\frac{2\Delta r(1)h_f}{k_f}\right]\left[\frac{k_f \Delta t}{\rho_f C_{pf}}\right]$$

Substituting equations (3-32) through (3-36) into equations (3-29), (3-30), and end conditions (3-12) through (3-15) and writing in matrix form gives:

$$\begin{bmatrix}
Q+1 & R & 0 & -Q & 0 & & & & & & & & & & \\
-R & Q+1 & R & 0 & -Q & 0 & & & & & & & & & \\
0 & R & Q+1 & 0 & 0 & -Q & R & & & & & & & & \\
-W & 0 & 0 & W+V & -2S & 0 & 0 & 0 & & & & & & & \\
0 & -W & 0 & S & W+V+1 & -S & 0 & 0 & 0 & & & & & & \\
 & 0 & -W & 0 & S & W+V+1 & 0 & 0 & 0 & S & & & & & \\
 & & -R & 0 & 0 & 0 & Q+1 & R & 0 & -Q & 0 & & & & \\
 & & & 0 & 0 & 0 & -R & Q+1 & 0 & 0 & -Q & R & & & \\
 & & & & 0 & 0 & -R & Q+1 & 0 & 0 & -Q & R & & & \\
 & & & & & S & -W & 0 & 0 & W+V+1 & -S & 0 & 0 & 0 & \\
 & & & & & & 0 & -W & 0 & S & W+V+1 & -S & 0 & 0 & 0 \\
 & & & & & & & 0 & -W & 0 & S & W+V+1 & 0 & 0 & -S \\
 & & & & & & & & -R & 0 & 0 & 0 & Q+1 & R & 0-Q \\
 & & & & & & & & & 0 & 0 & 0 & -R & Q+1 & R 0-Q \\
 & & & & & & & & & & 0 & 0 & -R & Q+1 & 0 0-Q R \\
 & & & & & & & & & & & -2R & Q+2R & 0 & 0-Q \\
 & & & & & & & & & & & 0 & -W & 0 & S W+V+1-S 0 \\
 & & & & & & & & & & & & 0 & -W & 0 S W+V+1-S \\
 & & & & & & & & & & & & & 0-1 & 0 0 1
\end{bmatrix}
\begin{matrix} N+1 \\ \begin{bmatrix} T_D(1) \\ T_D(2) \\ T_D(3) \\ T_A(1) \\ T_A(2) \\ T_A(3) \\ \vdots \\ T_D(i-1) \\ T_D(i) \\ T_D(i+1) \\ T_A(i-1) \\ T_A(i) \\ T_A(i+1) \\ \vdots \\ T_D(M-2) \\ T_D(M-1) \\ T_D(M) \\ T_A(M-2) \\ T_A(M-1) \\ T_A(M) \end{bmatrix} \end{matrix}
+
\begin{matrix} N+1 \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ -VT_f(1,1) \\ -VT_f(2,1) \\ -VT_f(3,1) \\ \vdots \\ 0 \\ 0 \\ 0 \\ -VT_f(i-1,1) \\ -VT_f(i,1) \\ -VT_f(i+1,1) \\ \vdots \\ 0 \\ 0 \\ 0 \\ -VT_f(M-2,1) \\ -VT_f(M-1,1) \\ 0 \end{bmatrix} \end{matrix}
=
\begin{matrix} N \\ \begin{bmatrix} T_D(1)+RT_{D0} \\ T_D(2) \\ T_D(3) \\ T_A(1) \\ T_A(2) \\ T_A(3) \\ \vdots \\ T_D(i-1) \\ T_D(i) \\ T_D(i+1) \\ T_A(i-1) \\ T_A(i) \\ T_A(i+1) \\ \vdots \\ T_D(M-2) \\ T_D(M-1) \\ T_D(M) \\ T_A(M-2) \\ T_A(M-1) \\ 0 \end{bmatrix} \end{matrix}$$

(3-41)

Neither equation (3-40) or (3-41) can be solved as they are written, each contain too many unknowns. However, the two equations combined contain sufficient information to obtain a complete solution. An obvious, yet unique, approach to this problem is to determine $T_f(i,1)$ as a function of $T_A(i)$, substitute this relation into equation (3-41), and solve the resulting set of 2M equations and 2M unknowns.

Equation (3-40) can be solved most efficiently by applying a recursion relationship given by Richtmyer (12). Upon examination, it was found that this recursion relationship was nothing more than standard Gauss elimination applied to a tridiagonal matrix. Therefore, the method of Gauss elimination was used to restructure the coefficient matrix of equation (3-40) into an upper bi-diagonal matrix. New values of the constants in the right hand side vector were formed by multiplying this vector by a working vector, DD(j), formed in the elimination process. Upon examination it was found that a new algorithm could be developed in which the temperature $T_f(i,1)$ is written as a linear function of $T_A(i)$. To illustrate the development of the algorithm consider the following system of four equations that are to be solved simultaneously.

$$\begin{bmatrix} A_{11} & A_{12} & 0 & 0 \\ A_{21} & A_{22} & A_{23} & 0 \\ 0 & A_{32} & A_{33} & A_{34} \\ 0 & 0 & A_{43} & A_{44} \end{bmatrix} \begin{Bmatrix} FF_1 \\ FF_2 \\ FF_3 \\ FF_4 \end{Bmatrix} = \begin{Bmatrix} X \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix} \qquad (3\text{-}42)$$

Where the A's and C's are specified constants, the FF's are unknown temperatures, and X is an unknown.

Applying Gauss elimination gives:

$$\begin{bmatrix} B_{11} & B_{12} & 0 & 0 \\ 0 & B_{22} & B_{23} & 0 \\ 0 & 0 & B_{33} & B_{34} \\ 0 & 0 & 0 & B_{44} \end{bmatrix} \begin{Bmatrix} FF_1 \\ FF_2 \\ FF_3 \\ FF_4 \end{Bmatrix} = \begin{Bmatrix} X \\ C_2' \\ C_3' \\ C_4' \end{Bmatrix} \qquad (3\text{-}43)$$

The C′'s are formed by application of the working vector; they are:

$$C_2' = -DD(1)\,X + C_2$$

$$C_3' = -DD(2)\,C_2' + C_3$$

$$C_4' = -DD(3)\,C_3' + C_4$$

or,

$$C_3' = C_3 - DD(2)\,C_2 + DD(1)\,DD(2)\,X$$

$$C_4' = C_4 - DD(3)\,C_3 + DD(2)\,DD(3)\,C_2 \qquad (3\text{-}44)$$
$$- DD(1)\,DD(2)\,DD(3)\,X$$

and,

$$DD(i) = A_{i+1,i} / A_{ii} \qquad (3\text{-}45)$$

Performing back substitution,

$$FF_4 = \frac{C_4'}{B_{44}}$$

$$FF_3 = \frac{C_3'}{B_{33}} - \frac{B_{34} C_4'}{B_{33} B_{44}}$$

$$FF_2 = \frac{C_2'}{B_{22}} - \frac{B_{23} C_{23}'}{B_{22} B_{33}} + \frac{B_{23} B_{34} C_4'}{B_{22} B_{33} B_{44}} . \qquad (3\text{-}46)$$

and,

$$FF_1 = \frac{X}{B_{11}} - \frac{B_{12} C_2'}{B_{11} B_{22}} + \frac{B_{12} B_{23} C_3'}{B_{11} B_{22} B_{33}} - \frac{B_{12} B_{23} B_{34} C_4'}{B_{11} B_{22} B_{33} B_{44}} . \qquad (3\text{-}47)$$

Substituting equations (3-44) into the above expression gives:

$$FF_1 = \left[\frac{1}{B_{11}} + \frac{B_{12} DD(1)}{B_{11} B_{22}} + \frac{B_{12} B_{23} DD(1) DD(2)}{B_{11} B_{22} B_{23}} \right.$$

$$\left. + \frac{B_{12} B_{23} B_{34}}{B_{11} B_{22} B_{33} B_{44}} DD(1) DD(2) DD(3) \right] X +$$

$$\left\{ - \frac{B_{12} C_2}{B_{11} B_{22}} + \frac{B_{12} B_{23}}{B_{11} B_{22} B_{33}} \left[C_3 - DD(2) C_2 \right] - \frac{B_{12} B_{23} B_{34}}{B_{11} B_{22} B_{33} B_{44}} \right.$$

$$\left. \left[C_4 - DD(3) C_3 + DD(2) DD(3) C_2 \right] \right\} \qquad (3\text{-}48)$$

The coefficient of X, as well as the other terms of equation (3-48) are all known constants. The relationship has the form,

$$FF_1 = a X + b \tag{3-49}$$

where a and b are constants.

A new algorithm can be developed by generalizing equation (3-49) to a set of K equations. The constants a and b then have the following form:

$$a = \frac{1}{B_{11}} \left\{ 1 + \sum_{i=2}^{K} \prod_{j=2}^{i} \frac{B_{j-1,j} DD_{j-1}}{B_{jj}} \right\} \tag{3-50}$$

$$b = -\frac{B_{12} C_2}{B_{11} B_{22}} + \sum_{i=3}^{K} - \left\{ (-1)^i \frac{B_{12}}{B_{11} B_{22}} \prod_{j=3}^{i} \frac{B_{j-1,j}}{B_{jj}} \right.$$

$$\left. \left[C_j - DD_{j-1} C_{j-1} - DD_{j-1} H_{j-1} \right] \right\} \tag{3-51}$$

$$H_2 = 0$$

$$H_i = -DD_{i-1} C_{i-1} - DD_{i-1} H_{i-1}$$

This algorithm enables the solution for $T_f^{N+1}(i,1)$ (refer to equation (3-40)) to be written as a function of $T_A^{N+1}(i)$, i.e.:

$$T_f^{N+1}(i,1) = a(i) \left[T_f^{N}(i,1) - ZZ T_A^{N+1}(i) \right] + b(i) \tag{3-52}$$

The constants a(i) and b(i) are determined using equations (3-50) and (3-51). This requires that Gauss elimination be applied to the coefficient matrix of equation (3-40), and the working vector, that is generated, be stored.

When equation (3-52) is substituted for $T_f^{N+1}(i,1)$ in equation (3-41), the unknown, $T_f^{N+1}(i,1)$, is eliminated, enabling a solution to be obtained at time step N + 1 for $T_A(i)$ and $T_D(i)$. Knowing $T_A^{N+1}(i)$, equation (3-40) can then be solved for $T_f^{N+1}(i,j)$.

The coefficient matrix of equation (3-41) is structured in four upper and lower sidebands. This structure becomes more significant as M increases. That is, the number of non-zero elements in the coefficient matrix increases by a factor of 9, rather than by a factor of 2M, as M is increased. Considering this, a form of "Gaussian elimination with back substitution" was selected as the most suitable method for solving this equation.

The Gaussian elimination method is simple and easily applied; however, its use normally results in considerable error due to round-off when it is applied to large matrices. In this case the scope of the method can be extended by applying it to only the non-zero elements contained in the main diagonal, and upper and lower sidebands of the coefficient matrix. By doing this the round-off error, and computer time used, is reduced to an acceptable limit.

For convenience, consider equation (3-40) written in the form:

$$[AA]\{FF\} = \{CC\} \tag{3-53}$$

Similarly, equation (3-41), modified by substitution of (3-52), can be written:

$$[A]\{F\} = \{C\} \tag{3-54}$$

Using this notation the overall computational procedure is:

1. Gauss elimination is applied to the AA matrix, and the working vector generated, DD(j), is stored.

2. The constants a(i) and b(i) are computed.

3. Gauss elimination is applied to the non-zero elements of the A matrix, and the working vector generated, DC(i), is stored.

4. The working vector, DC(i), is applied to C(i).

5. Values at time step N + 1 for T_D (i) and T_A (i) are obtained by back substitution.

6. Knowing $T_A^{N+1}(i)$, $T_f^{N+1}(i,j)$ is obtained by back substitution.

Computer Programming

The method of solving equations (3-40) and (3-41) was programmed in Fortran-IV for an IBM 360 Model 50 computer. For a problem of this size efficient programming was important. A high speed main core of 256 K was available for operation and storage. Considering this, the number of calculations made at each time step were kept to a minimum by storing appropriate information from previous time steps. An example of this is shown in Figure 6. Here, a flow chart of the solution of equations (3-40) and (3-41) is shown when their coefficient matrices do not vary with time. In this case Gaussian elimination was applied only once on the coefficient matrix of each equation. The working vectors, generated by the elimination scheme, contained sufficient information to update the equations at each time step.

The temperatures generated by a solution similar to that shown in Figure 6 model the temperature distributions in and around a well of constant depth that is circulating a drilling fluid. With modifications,

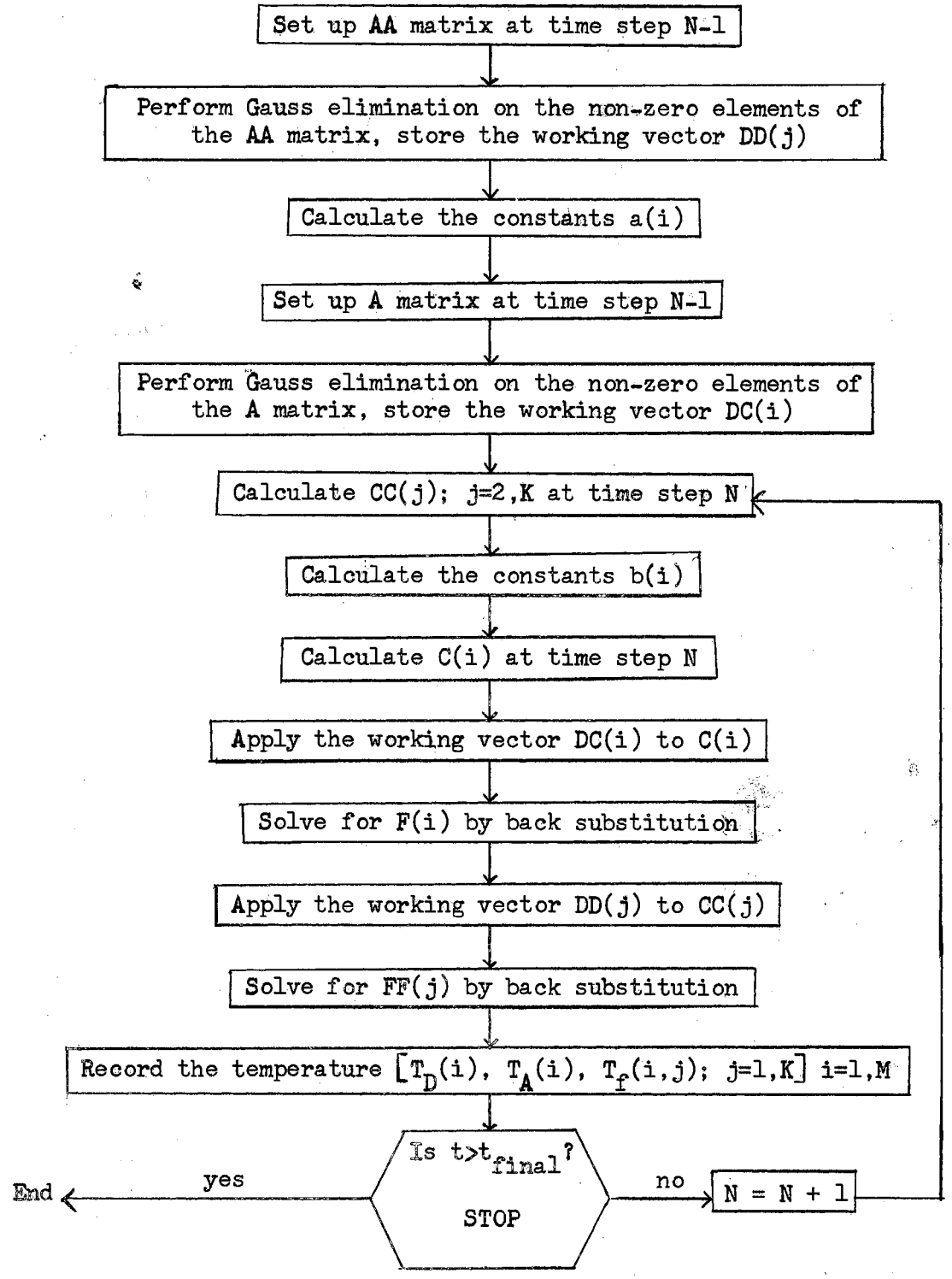

Figure 6. Flow Chart for Repeated Solution of Equations (3-40) and (3-41)

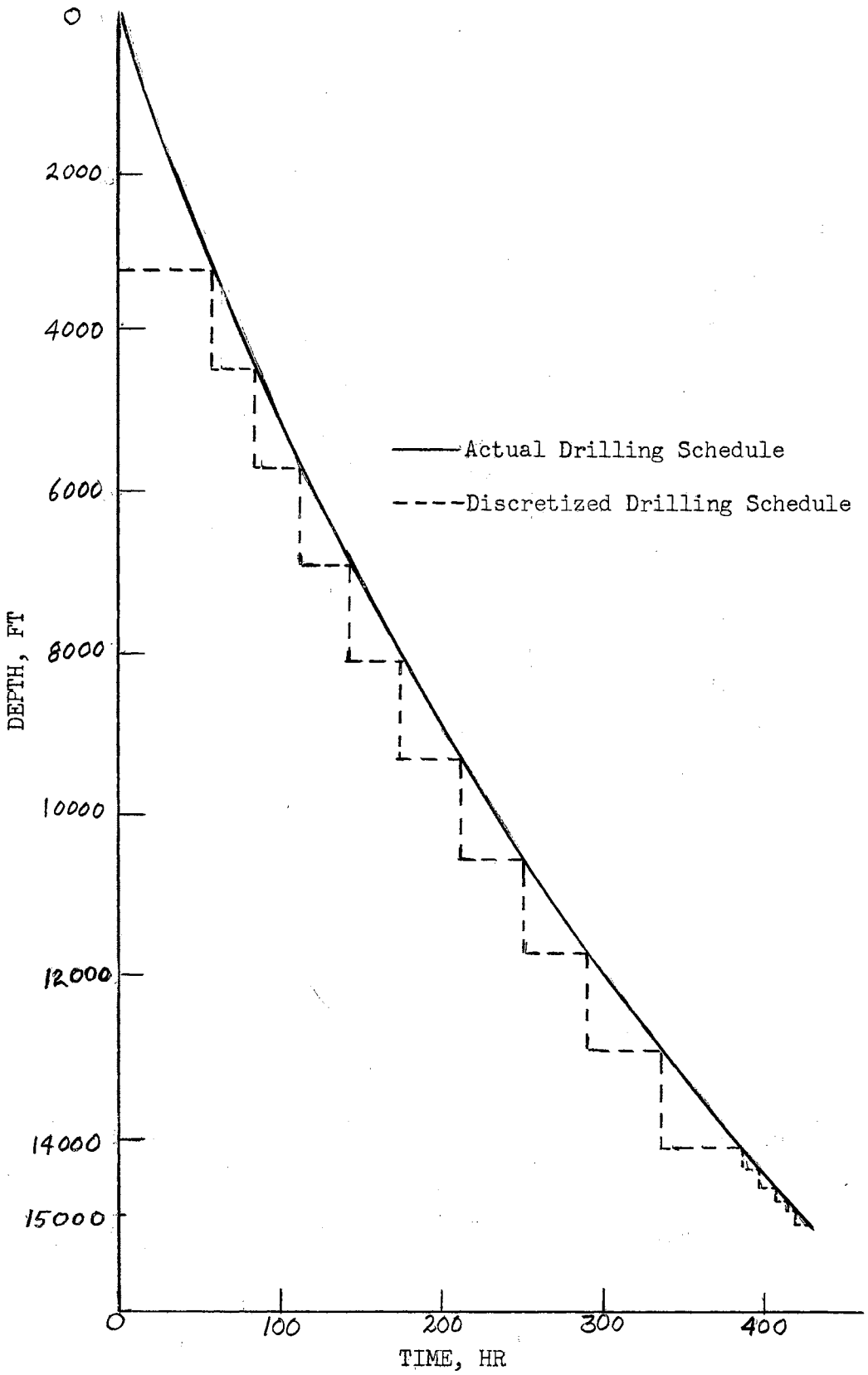

Figure 7. Example of a Discretized Drilling Schedule

temperature distributions developed while drilling were modeled. A drilling schedule was discretized (see Figure 7), and the mud inlet temperature, T_{Do}, was approximated as a function of depth (equation (3-55)).

$$T_{Do} = \frac{20{,}000 + DEPTH}{250} \qquad 0 < DEPTH < 10{,}000$$

$$T_{Do} = \frac{70{,}000 + DEPTH}{667} \qquad 10{,}000 < DEPTH < 25{,}000 \tag{3-55}$$

The basic method shown in Figure 6 was applied within each discreet interval of the drilling schedule. The initial conditions specified by equations (3-24), (3-25), and (3-26) were used at the start of drilling within each interval. Near total depth the coefficient matrix of equation (3-41) was modified (see equation (3-28)) by a reduction in the step size, ΔZ. This insured that the temperatures calculated as those of total depth would truly represent that depth.

CHAPTER IV

NUMERICAL RESULTS

Error Analysis

Equations (1-1), (1-2), and (1-3) with sufficient initial and boundary conditions define a unique solution. Any solution deviating from this solution is in error. The magnitude of this error is a major concern in using any finite-difference method. Unfortunately, there is no known exact solution of equations (1-1), (1-2), and (1-3). This necessitates an indirect analysis of the error. A rigorous, analytical, analysis was not attempted, but a sufficient study was made to show that the error associated with the numerical method is acceptable.

There are at least two types of error associated with a finite difference method. The truncation error (inherent in the finite difference approximations), and the round-off error (due to using finite arithmetic in the calculations). In addition, the numerical method may have convergence criteria which must be met in order to obtain a valid solution.

Truncation Error

The truncation error was investigated by determining the sensitivity of the solution to changes in a step size. Figures 8 through 17 and Tables I, II, III, and IV show the results of this investigation.

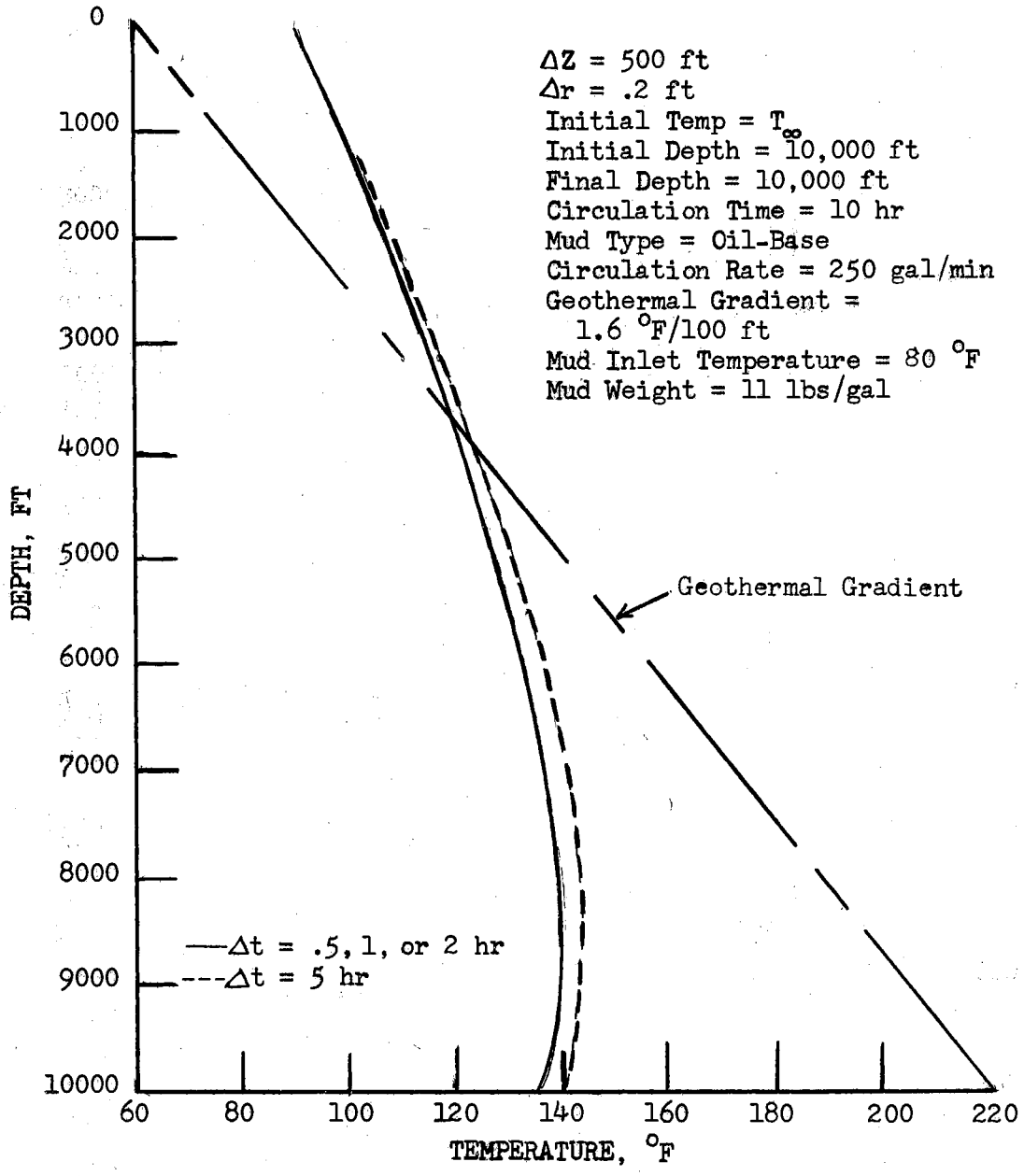

Figure 8. Depth vs Circulating Annulus Temperature as a Function of Time Step Size

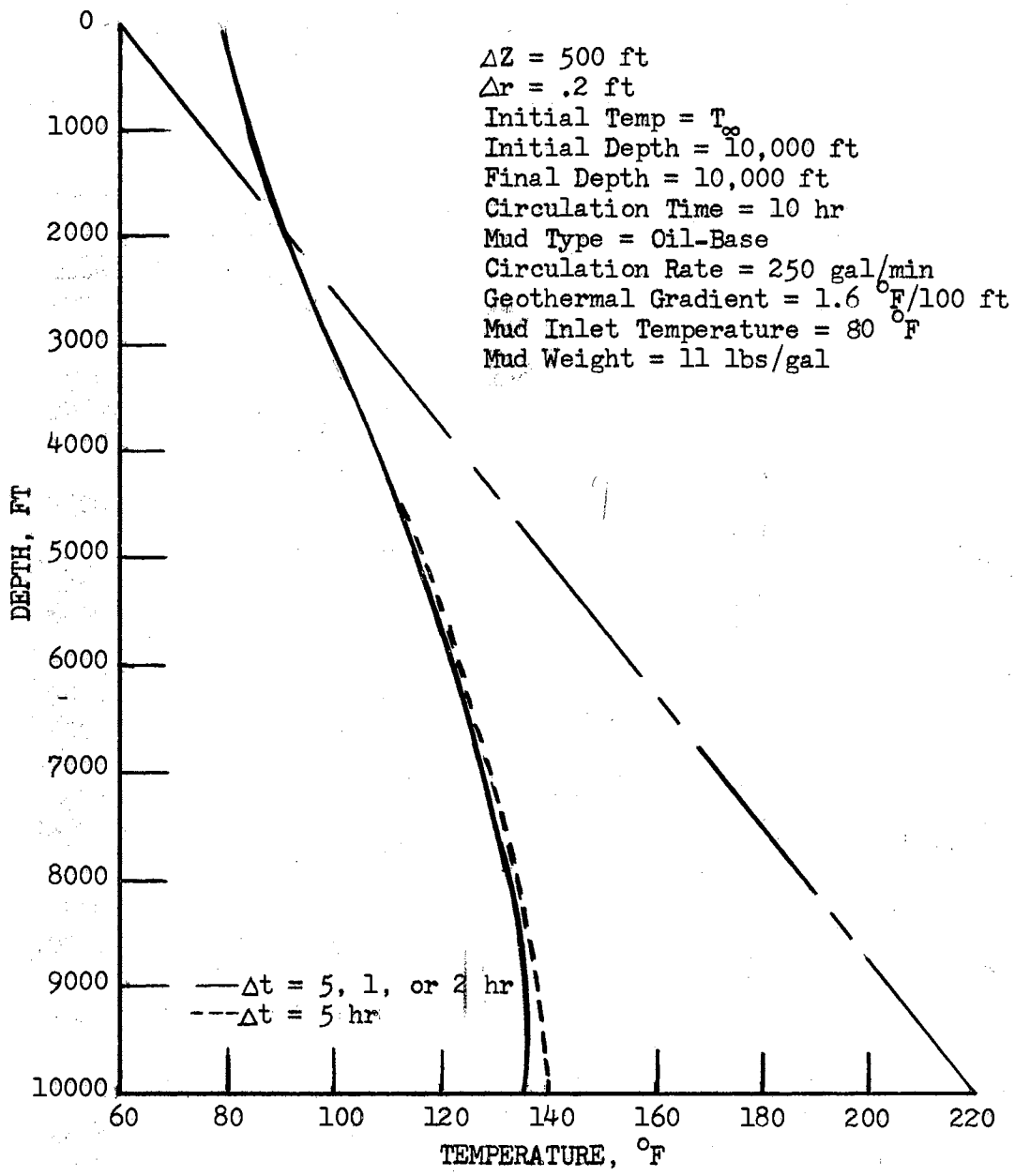

Figure 9. Depth vs Circulating Drill Pipe Fluid Temperature as a Function of Time Step Size

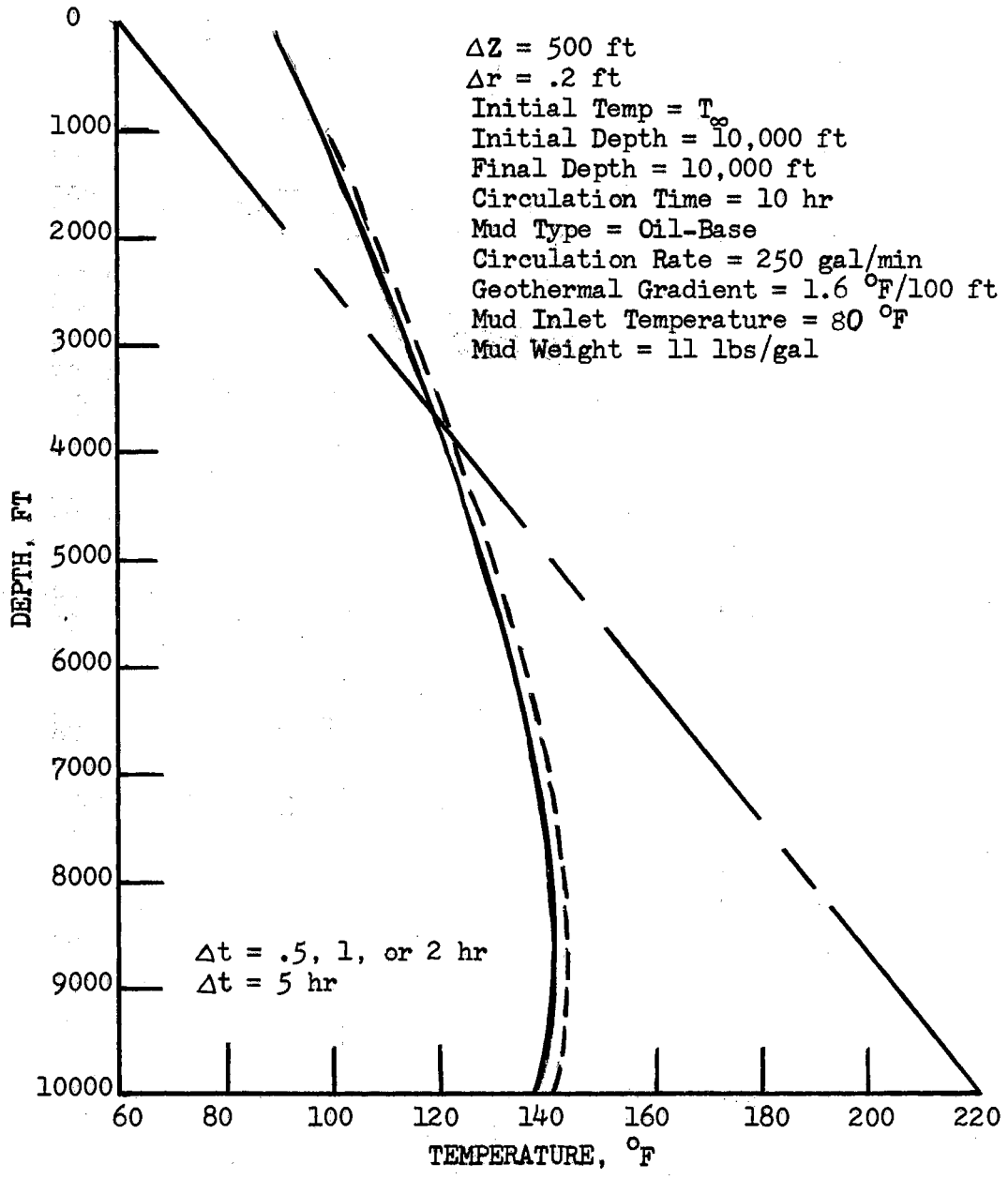

Figure 10. Depth vs Borehole Wall Temperature While Circulating as a Function of Time Step Size

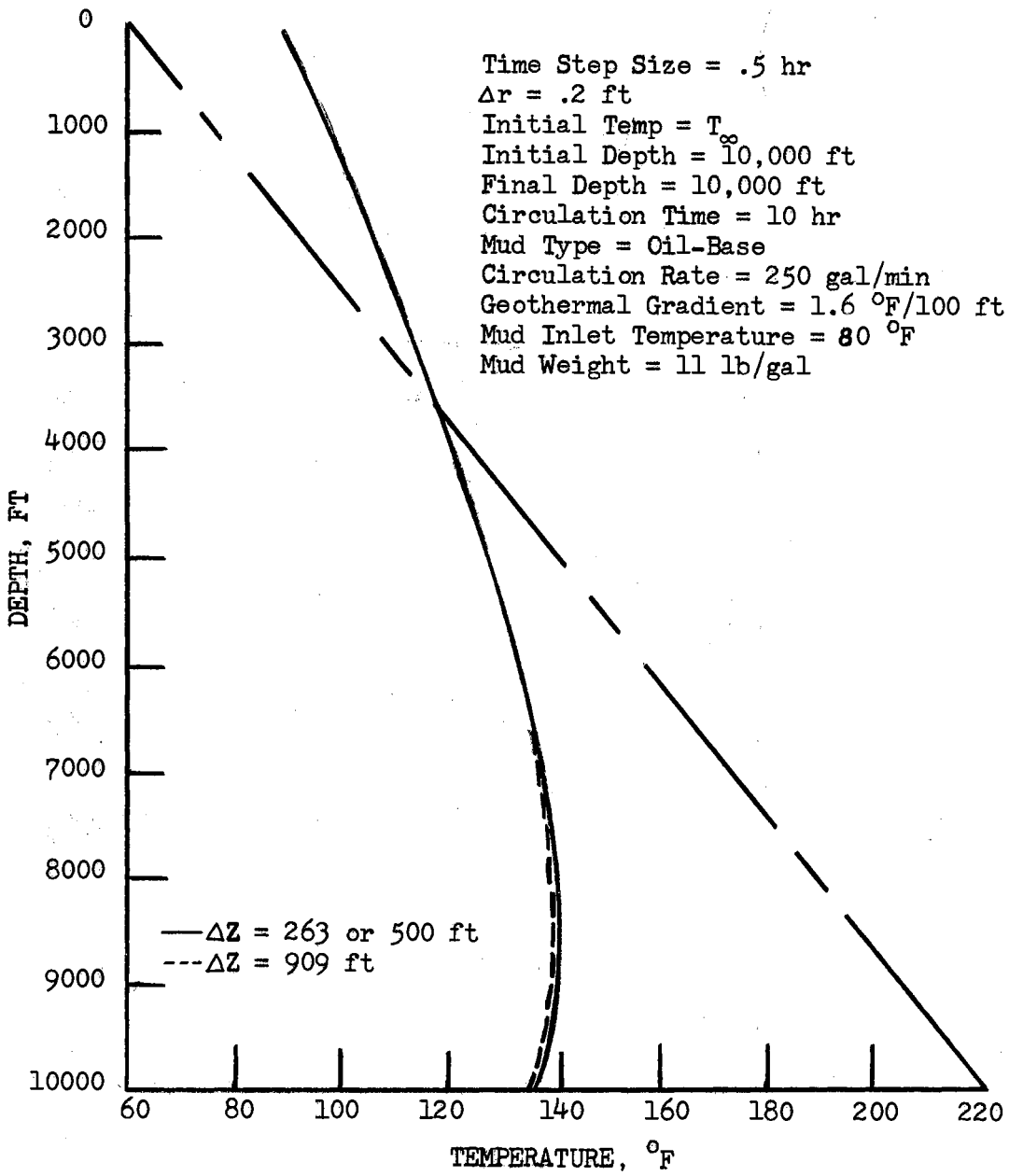

Figure 11. Depth vs Borehole Wall Temperature While Circulating as a Function of the Spatial Step Size ΔZ

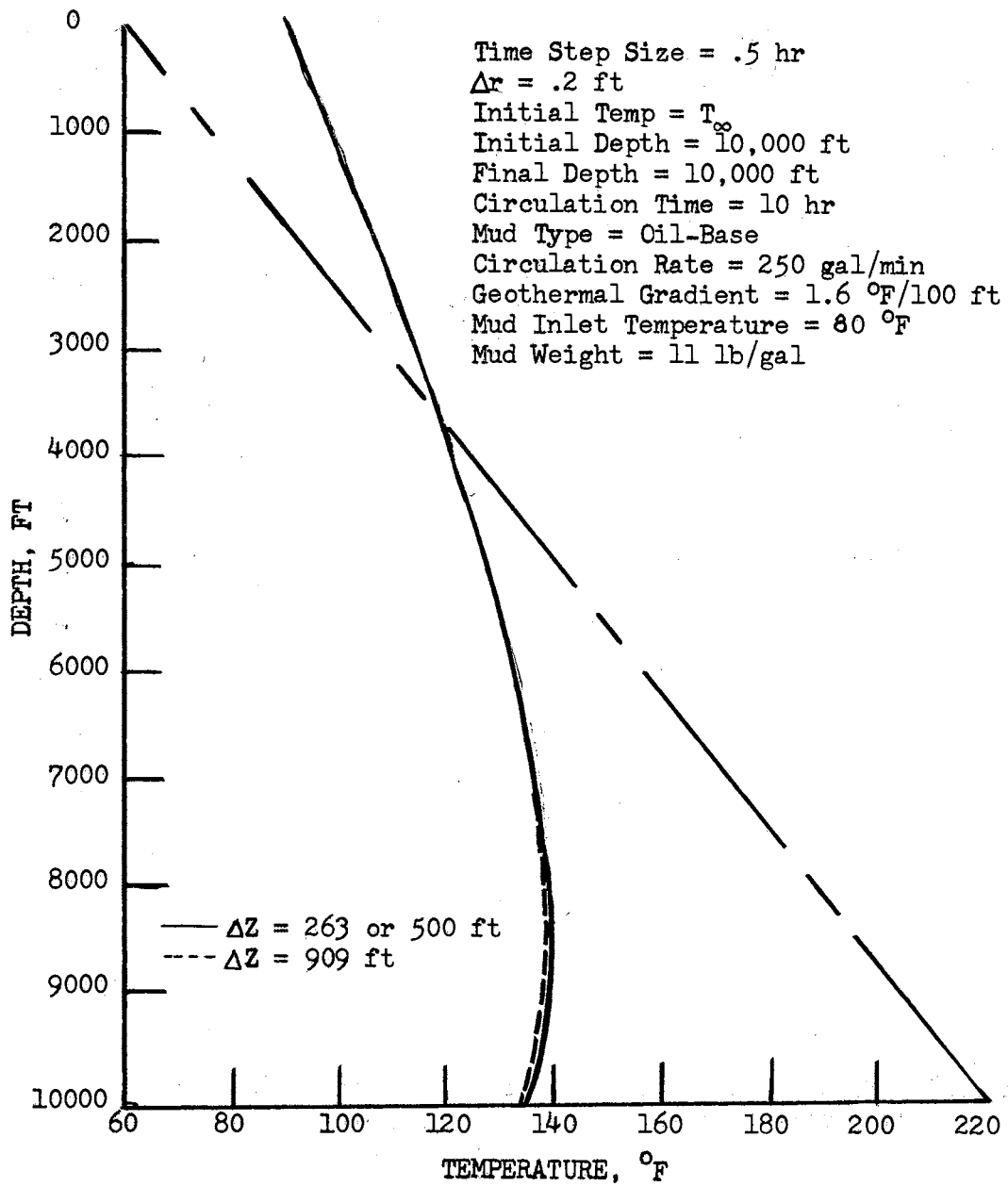

Figure 12. Depth vs Annulus Temperature While Circulating as a Function of the Spatial Step Size ΔZ

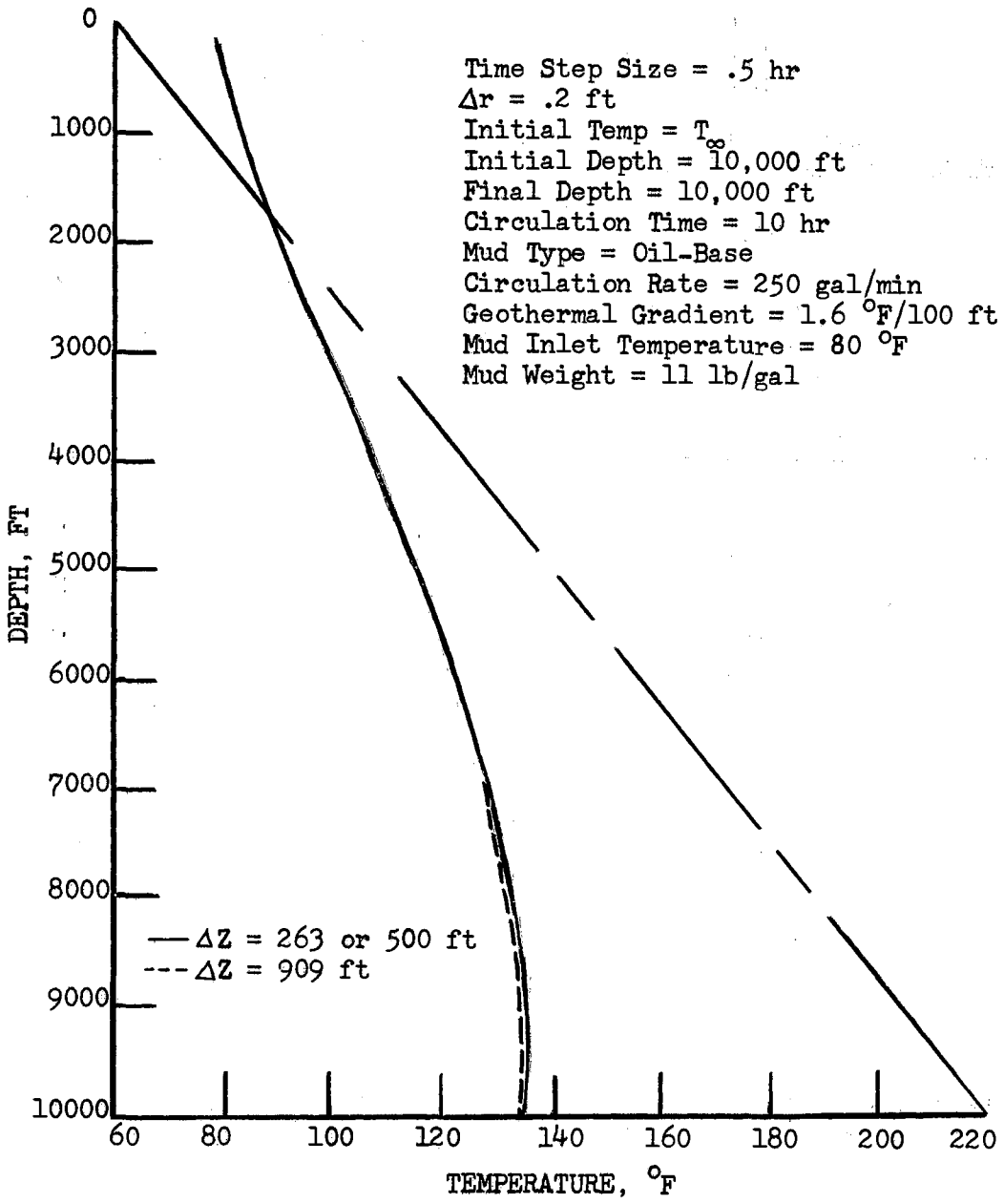

Figure 13. Depth vs Circulating Drill Pipe Temperature as a Function of the Spatial Step Size ΔZ

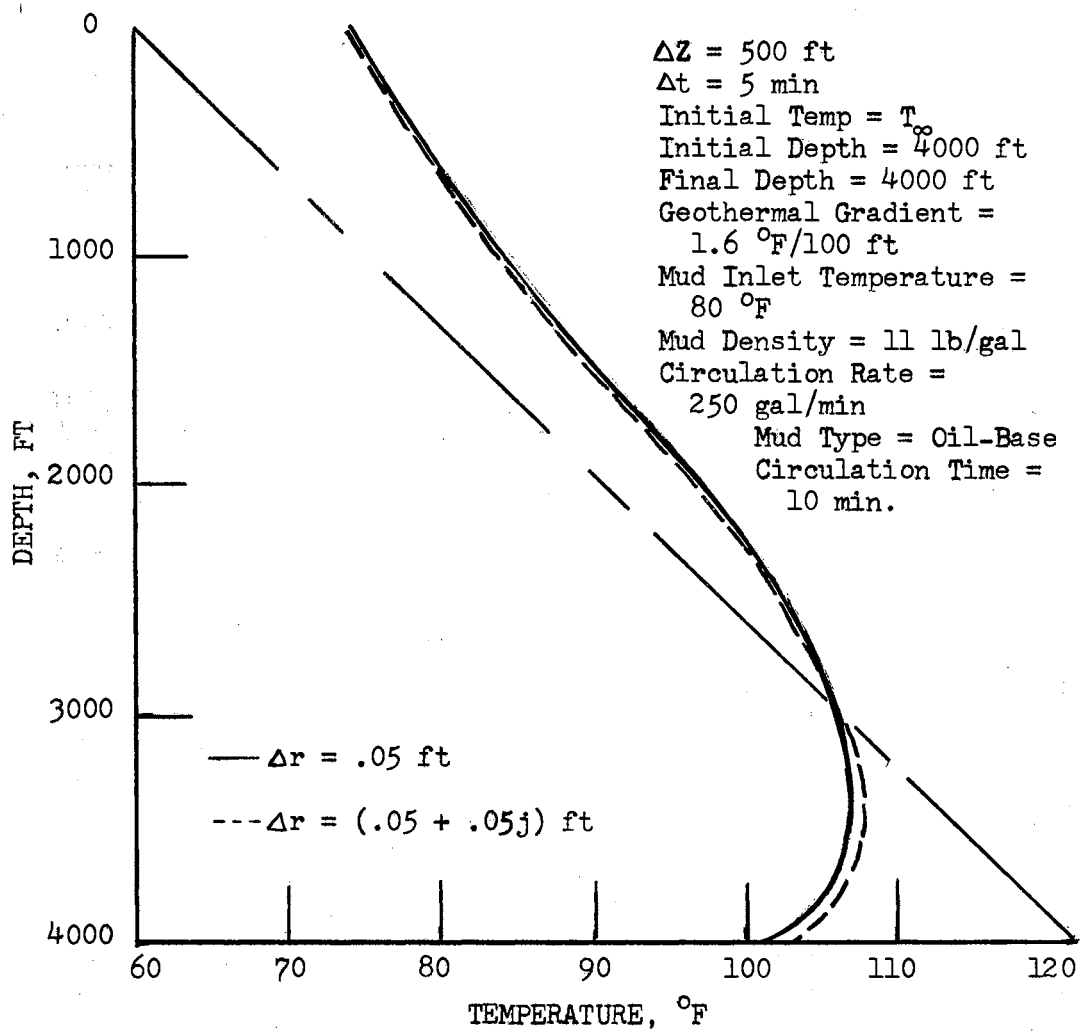

Figure 14. Depth vs Borehole Wall Temperature While Circulating as a Function of the Spatial Step Size Δr

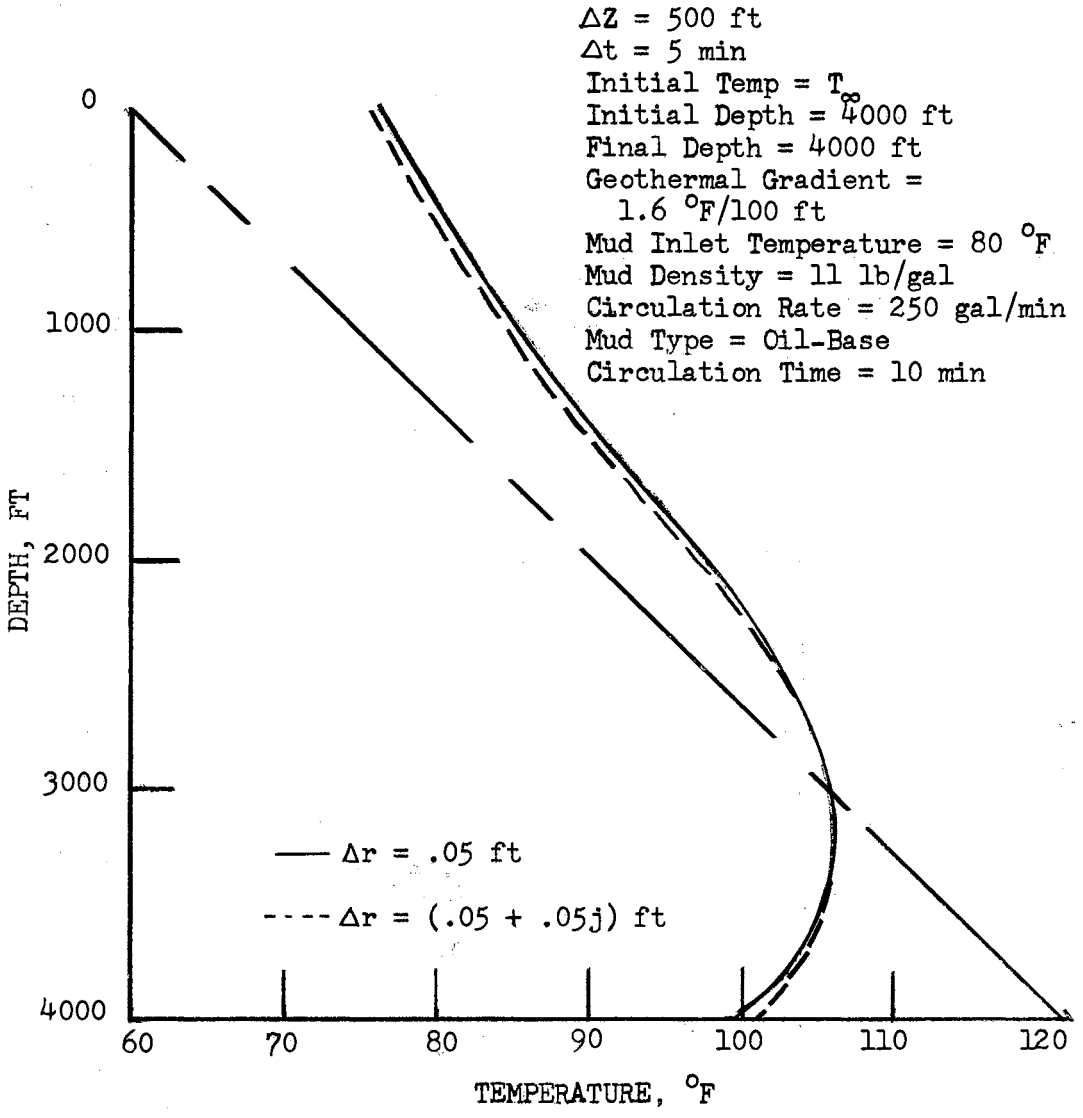

Figure 15. Depth vs Annulus Circulating Fluid Temperature as a Function of the Spatial Step Size Δr

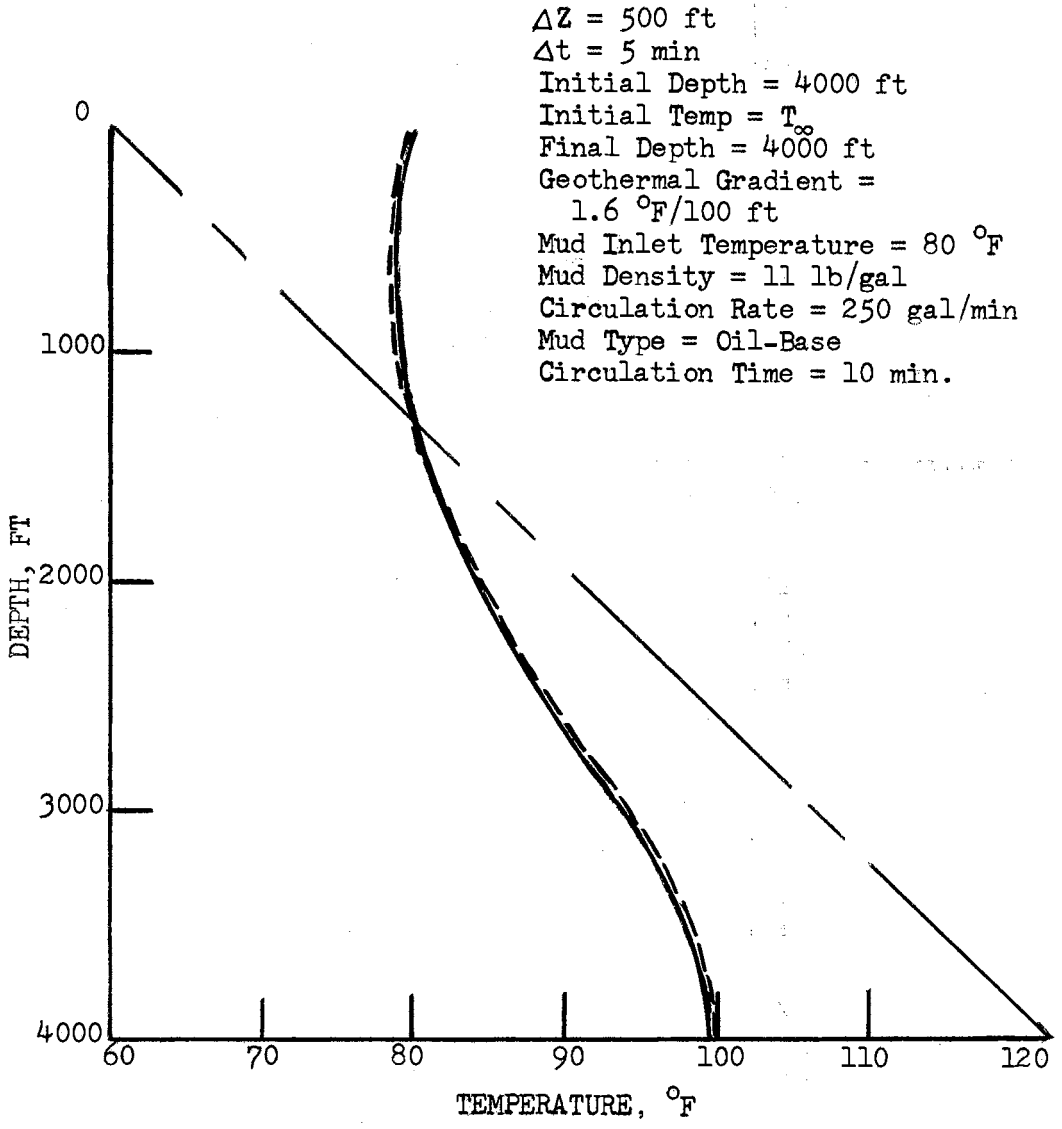

Figure 16. Depth vs Annulus Circulating Fluid Temperature as a Function of the Spatial Step Size Δr

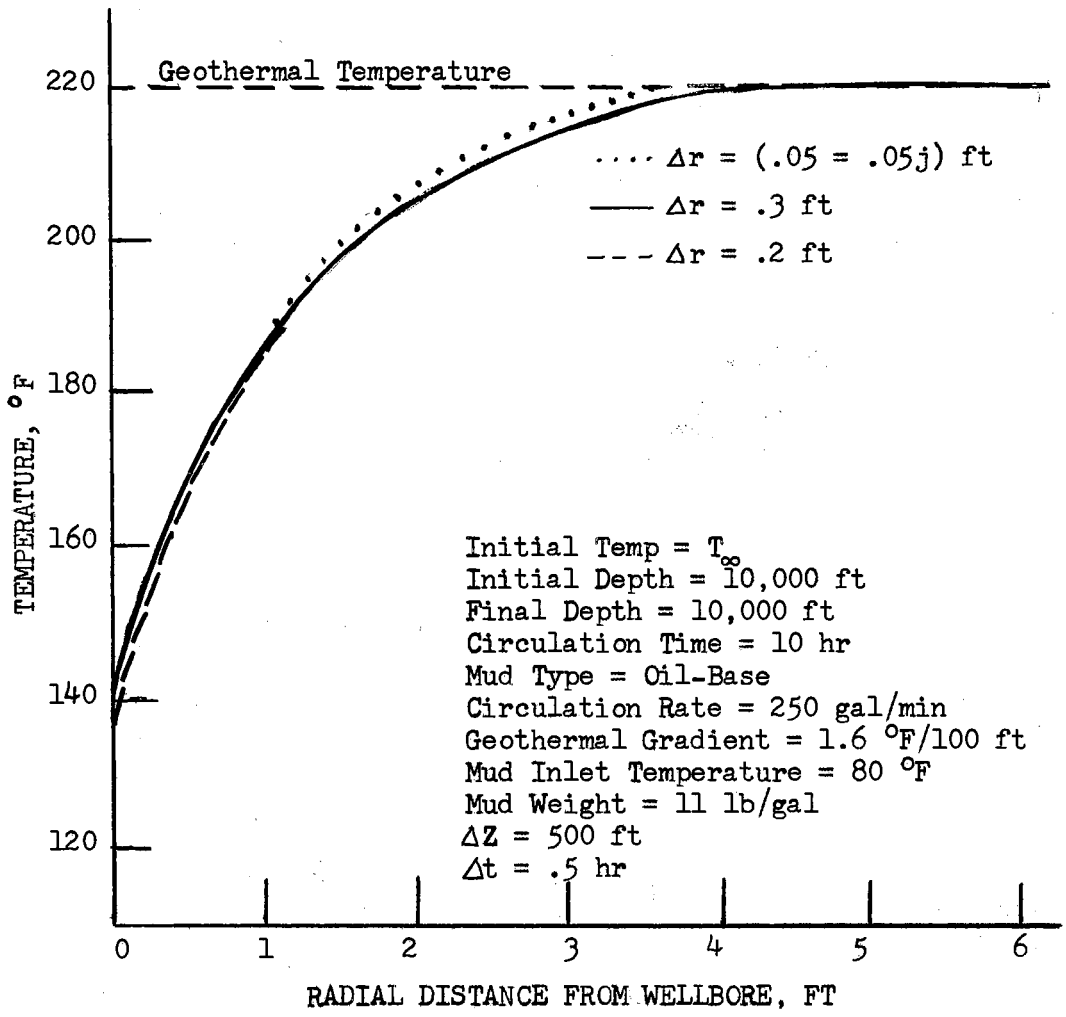

Figure 17. Formation Temperature at Total Depth vs Radial Distance from the Wellbore as a Function of the Radial Step Size

Tables I and II show that the truncation error, due to time step size, becomes less significant as the time since start of circulation increases. This is to be expected since the solutions converge to pseudo-steady state values. Although the error after one and two hours since start of circulation is as great as $3\frac{1}{2}\%$, this would not have to be encountered in any practical application of the method. Any simulation of only 2 hours duration should require intermediate temperature information, and thus use a smaller time step. As a rule of thumb, it was found that, if the total time of simulation was divided into at least 5 time staps, the truncation error was always less than one-half of one per cent.

The truncation error due to the spatial step size, ΔZ, is negligible, as can be seen in Tables III and IV. The temperature solution is practically insensitive to changes in this step size; thus, allowing great freedom in discretizing depth.

The radial step size, $\Delta r(j)$, contributes little to the truncation error when it is divided into very small increments near the wellbore. Figures 14, 15, 16, and 17 show the effect this step size has on the computed temperature distributions.

Round-Off Error

The local round-off error was approximated in all programs that were run by calculating the residuals R′ and RR′ (defined in equations (4-1) and (4-2) at each time step.

$$[A]^N \{F\}^{N+1} - \{C\}^N = \{R'\} \tag{4-1}$$

$$[AA]^N \{FF\}^{N+1} - \{CC\}^N = \{RR'\} \tag{4-2}$$

TABLE I

SENSITIVITY OF DRILL PIPE, OR ANNULUS, TEMPERATURE AT TOTAL DEPTH TO CHANGES IN THE TIME STEP SIZE

Time Since Start of Circulation, Hr.	Δt, Hr.			
	.5	1.0	2.0	5.0
1	170.645	174.050	--	--
2	159.311	161.184	164.964	--
4	148.692	149.679	151.701	--
5	145.494	146.285	--	152.908
10	136.495	136.867	137.631	140.042
20	129.081	129.230	129.538	130.525

Radius of drill pipe = 2.3 in; Wellbore radius = 3.94 in; Mud density = 11 lbs/gal; Circulation rate = 256 gal/min; Formation thermal conductivity = 1.3 Btu/hr-ft-^{o}F; Specific heat of drilling fluid = .94 Btu/lb_m-^{o}F; Specific heat of formation rock = .21 Btu/lb_m-^{o}F; Geothermal gradient = 1.6 F/100 ft; Mud inlet temperature = 80^{o}F; Formation density = 160 lb_m/ft^3; Thermal conductivity of the drilling fluid = .37 Btu/hr-ft-^{o}F; Viscosity of the drilling fluid = 19 cp; Total depth = 10,000 ft; Initial condition for all temperatures = Geothermal temperature; Initial depth at start of circulation = 10,000 ft; Step size (Δr) = .3 ft; Step size (ΔZ) = 500 ft.

TABLE II

SENSITIVITY OF BOREHOLE WALL TEMPERATURE AT TOTAL DEPTH TO CHANGES IN THE TIME STEP SIZE

Time Since Start of Circulation, Hr.	Δt, Hr.			
	.5	1.0	2.0	5.0
1	175.050	179.166	--	--
2	162.673	164.790	169.176	--
4	151.617	152.645	154.798	--
5	148.301	149.120	--	156.270
10	138.942	139.328	140.118	142.627
20	131.187	131.342	131.663	132.691

Parameter values are the same as in Table I.

TABLE III

SENSITIVITY OF BOREHOLE WALL TEMPERATURE AT TOTAL DEPTH TO CHANGES IN THE SPATIAL STEP SIZE ΔZ

Time Since Start of Circulation, Hr.	ΔZ, Ft.		
	263	500	909
1	175.377	175.050	173.691
2	162.859	162.673	161.770
5	148.418	148.301	147.658
10	139.026	138.942	138.421
20	131.245	131.187	130.753

Parameter values are the same as in Table I, except substitute "Time Step Size = 1800 sec" for "Step Size (ΔZ) = 500 ft."

TABLE IV

SENSITIVITY OF DRILL PIPE, OR ANNULUS, TEMPERATURE AT TOTAL DEPTH TO CHANGES IN THE SPATIAL STEP SIZE ΔZ

Time Since Start of Circulation, Hr.	ΔZ, Ft.		
	263	500	909
1	170.978	170.645	169.240
2	159.502	159.311	158.400
5	145.616	145.494	144.838
10	136.582	136.495	135.964
20	129.143	129.081	128.641

Parameter values are the same as in Table I, except substitute "Time Step Size = 1800 sec" for "Step Size (ΔZ) = 500 ft."

The absolute value of R′ ranged from $.3553 \times 10^{-14}$ to $.3638 \times 10^{-12}$, and the absolute value of RR′ from $.7105 \times 10^{-14}$ to $.1137 \times 10^{-12}$. These values certainly indicate that the round-off error is insignificant.

Convergence

The implicit solution method insures the stability of the solution with time; however, there are convergence criteria for the spatial subdivisions. For this type of a problem the criterion for the step size increment, ΔZ, has been shown by Dusinberre (3) to be,

$$\Delta Z \leq \frac{2 r_h}{Nst} \tag{4-3}$$

where r_h is the hydraulic radius, and Nst is the Stanton number. Evaluating this criteria using the fluid properties, and physical dimensions given in Table I results in the limits:

$$\Delta Z_{drillpipe} \leq 2730 \text{ ft.} \tag{4-4}$$

$$\Delta Z_{annulus} \leq 3660 \text{ ft.} \tag{4-5}$$

Fortunately, due to the magnitude of these limits, no practical restrictions are imposed on the problem solution.

Similarly, in a manner suggested by Dusinberre (3), a criterion on the radial step size can be determined. This is:

$$\Delta r \leq 2 r \tag{4-6}$$

This inequality was purposely violated to investigate its validity; however, it proved to be valid since, in all cases, the temperatures generated were completely unrealistic or unbounded. This criterion

imposes no serious limit on the practical applications of the numerical method; however, it must be considered, especially when defining the radial step size increments near the wellbore.

Example Problems

Several example problems have been solved to illustrate to what extent the numerical model may be applied. In Figure 18 a comparison is made of the temperature distributions calculated by Raymond (11) and those calculated by the method developed in this study. The two solutions deviate by less than one per cent.

Figures 19, 20, 21, and 22 show the dynamic response of a circulating fluid system in a 20,000 foot well. The initial condition for these curves was taken as the formation geothermal gradient. The depth was held constant throughout the circulation period.

Figures 23 through 27 show temperature profiles generated while drilling a 14,000 foot well. The drilling schedule used in the simulation of this data is shown in Figure 7. Figure 23 shows that the bottom-hole mud temperature is actually much lower than that used by Edwardson et al. (4) (320 ^{o}F) in his analysis of formation temperature build-up.

The computer used for all calculations was an IBM 360 Model 50 with 256 K main core and 2361 K large core storage under OSMFT Release 15/16. The high speed main core with a Fortran-G compiler was used for all calculations. Exact computer times were not available, but there were available rough approximations, normally intended for accounting purposes. Figure 28 shows the computer time as a function of M. The

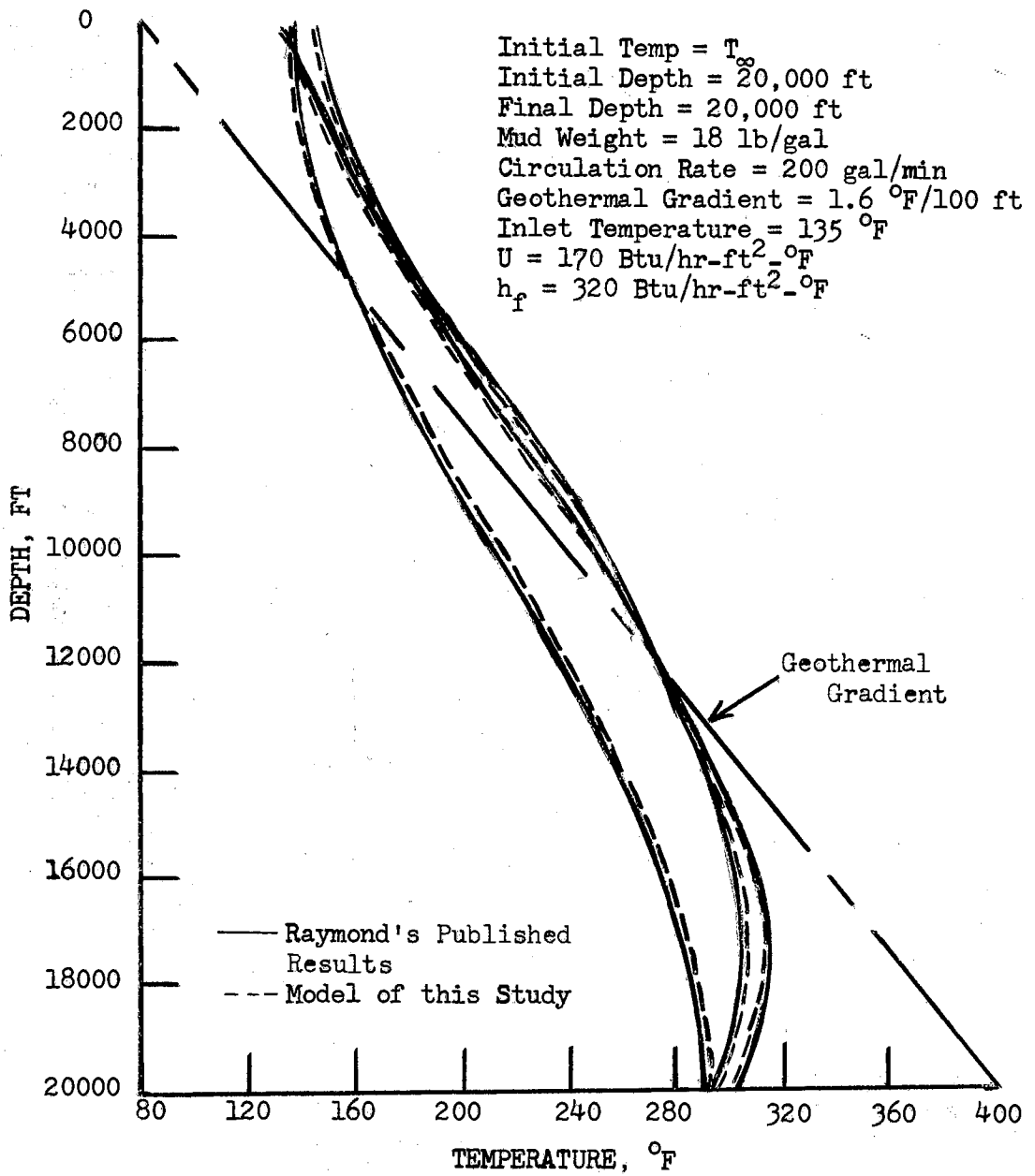

Figure 18. Temperature as a Function of Depth after Two Hours of Circulation at Total Depth for a Simulated Well

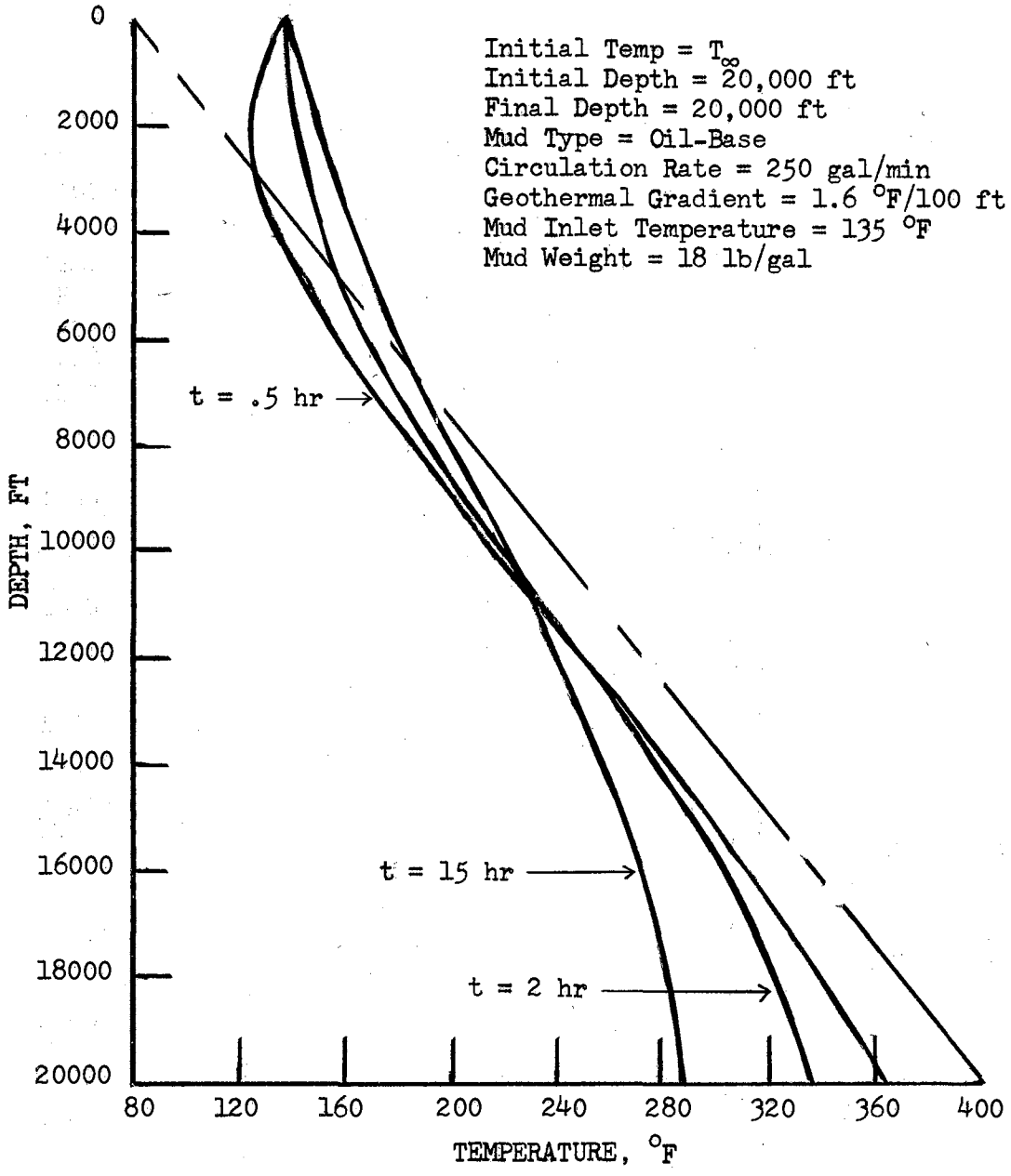

Figure 19. Drill Pipe Fluid Temperature vs Depth as a Function of Circulation Time for a Simulated Well

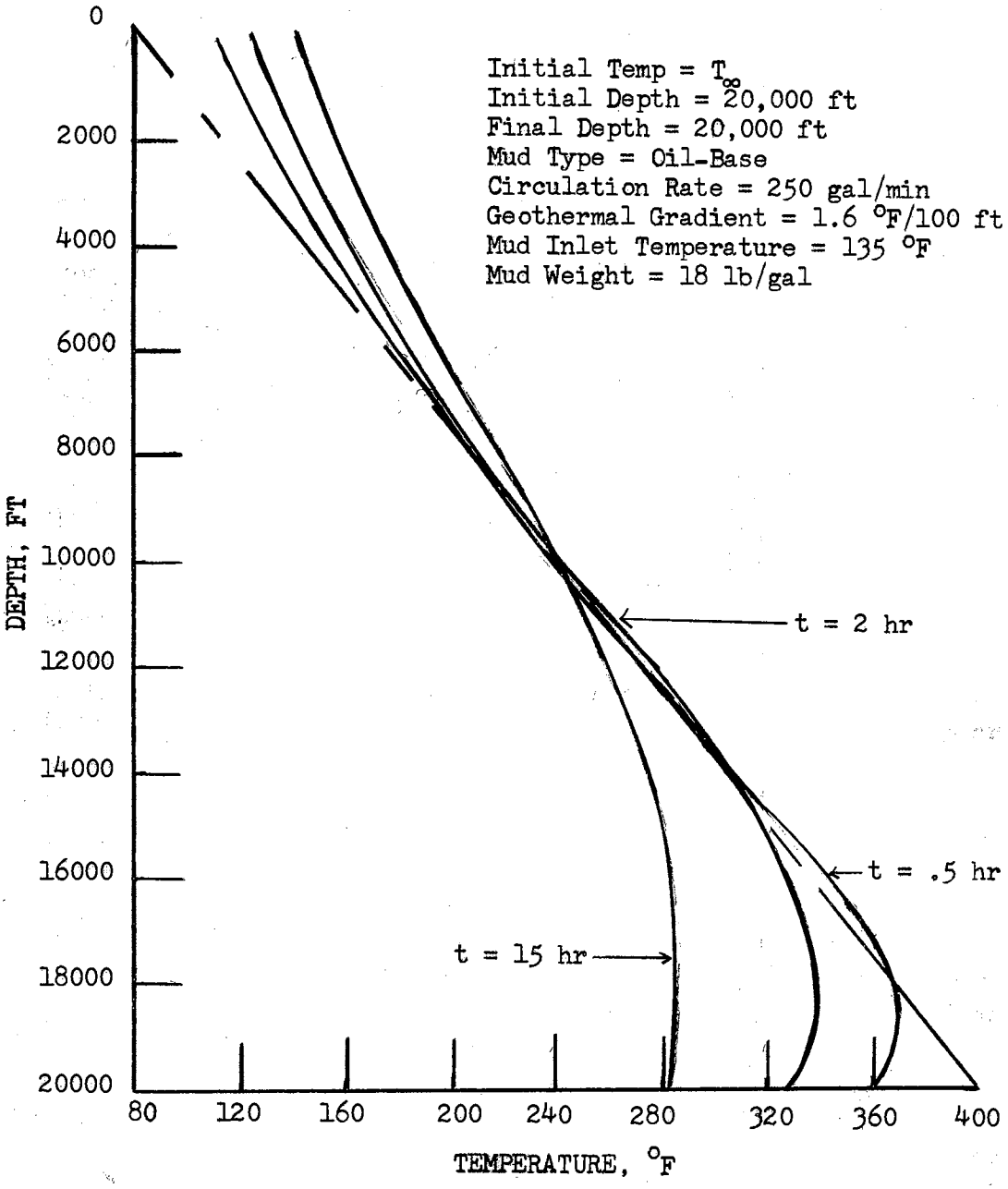

Figure 20. Annulus Fluid Temperature vs Depth as a Function of Circulation Time for a Simulated Well

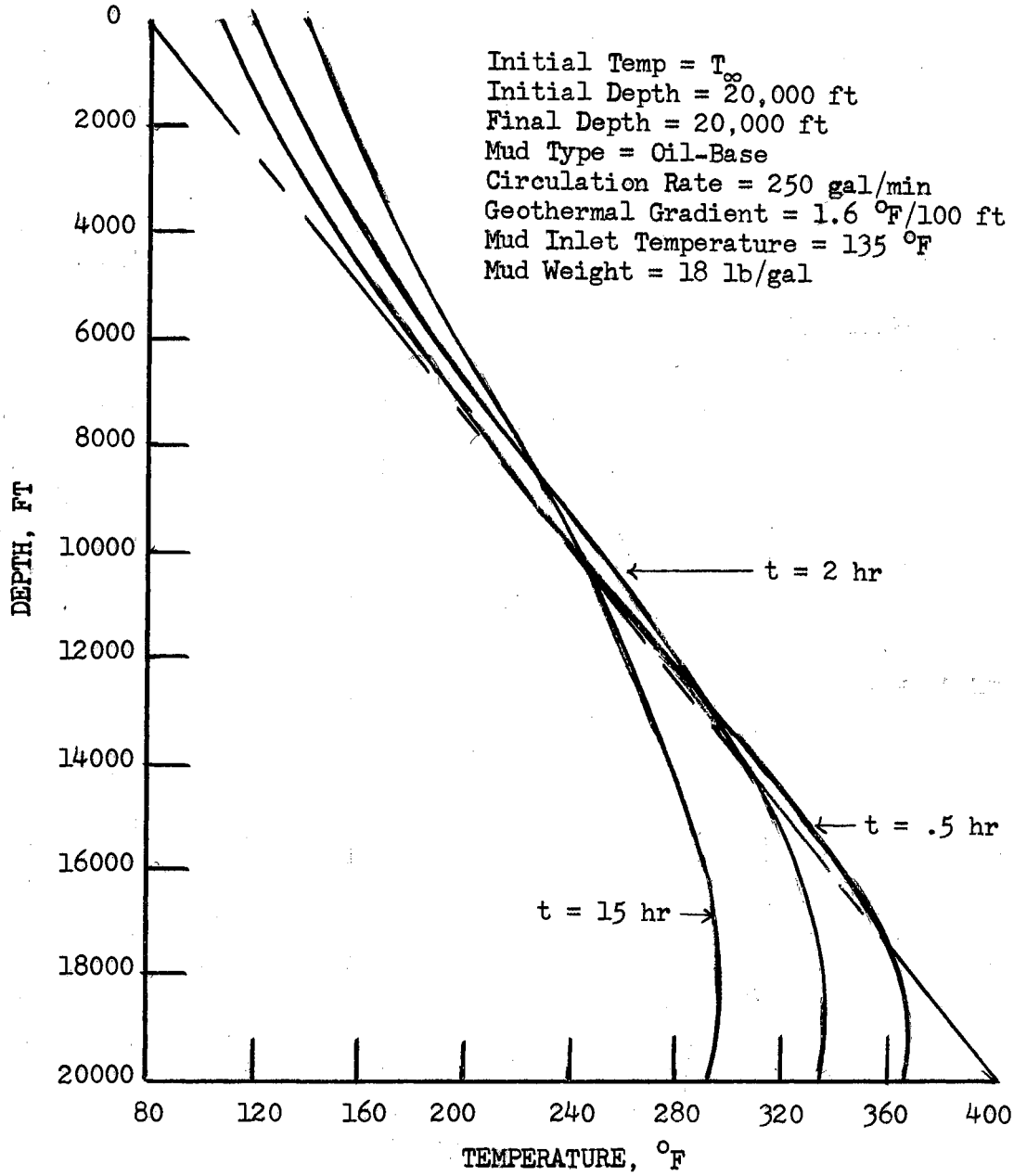

Figure 21. Borehole Wall Temperature vs Depth as a Function of Circulation Time for a Simulated Well

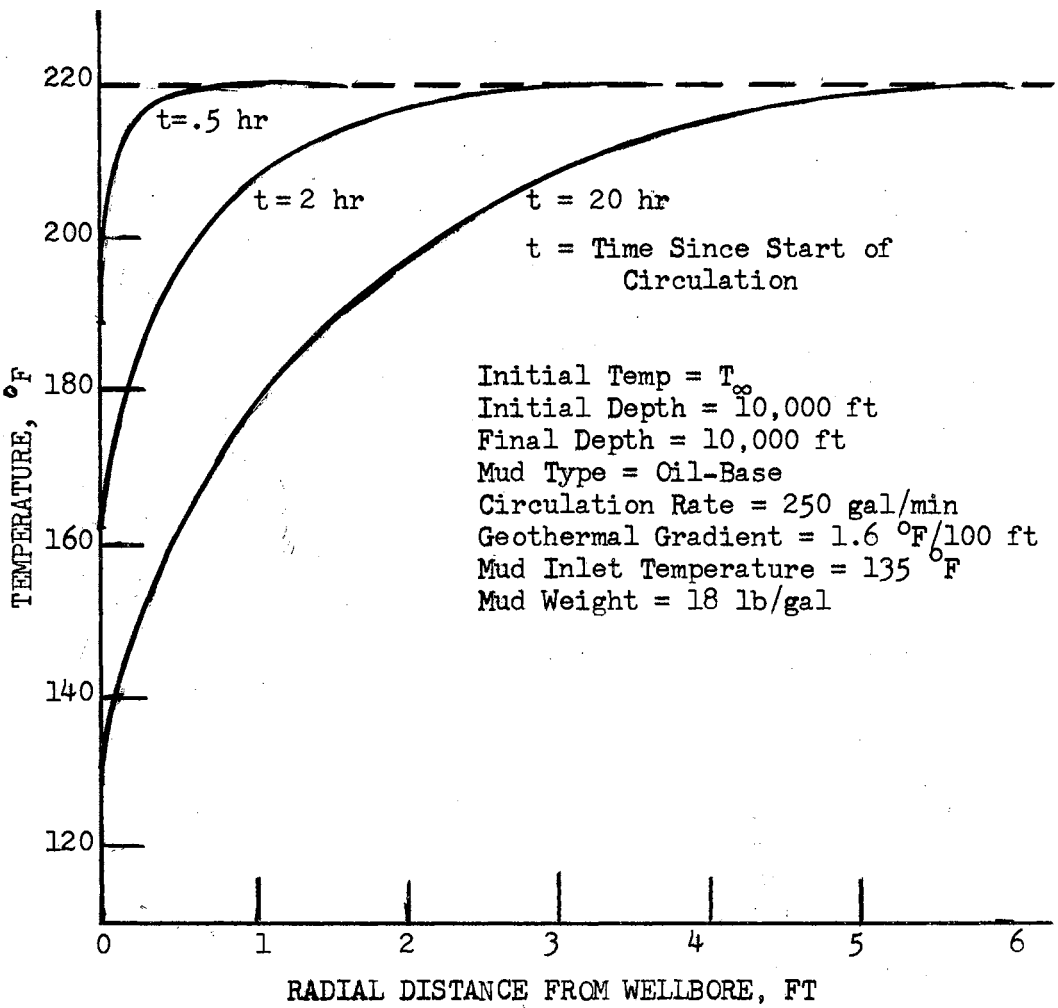

Figure 22. Formation Temperature at Total Depth vs Radial Distance from the Wellbore as a Function of Circulation Time for a Simulated Well

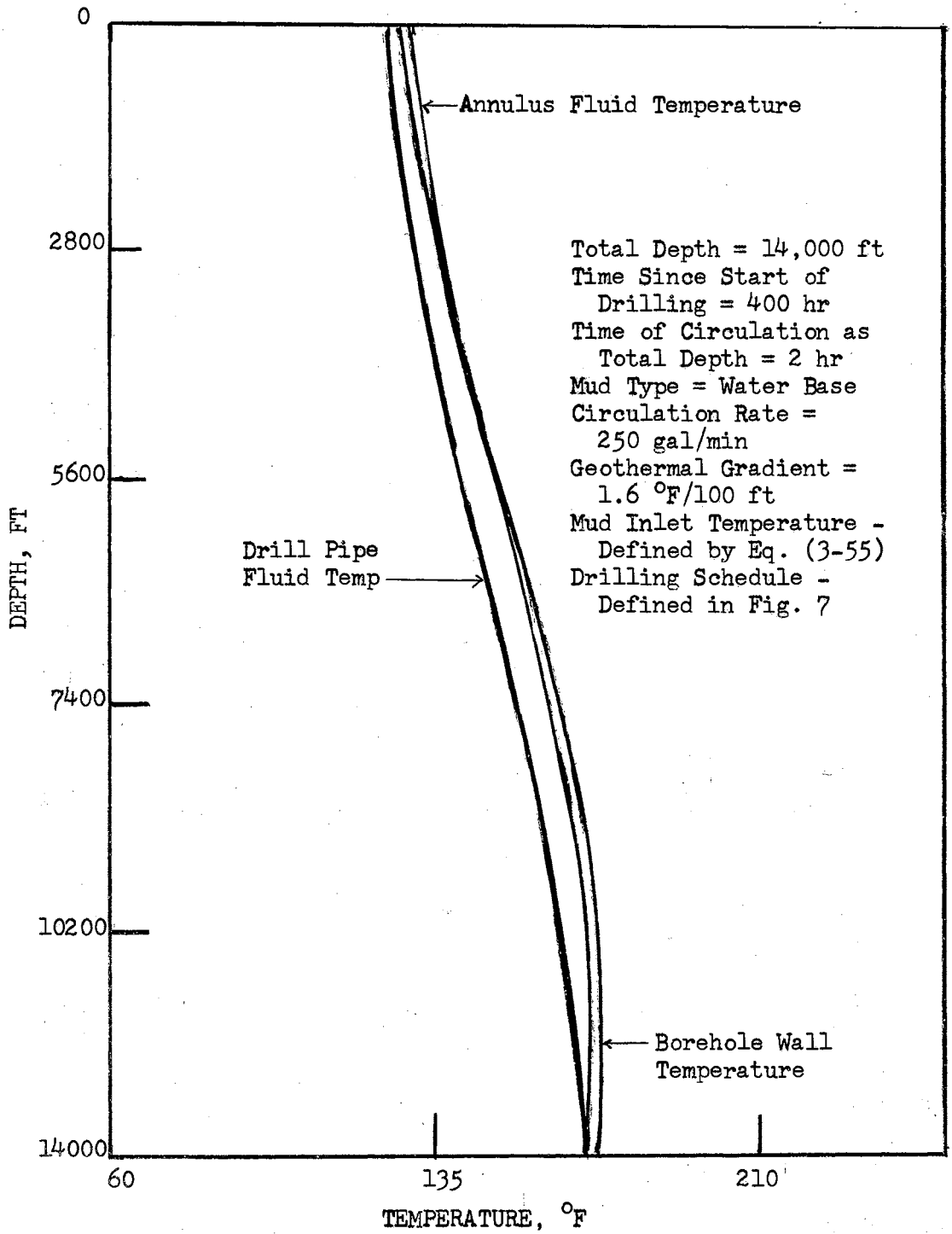

Figure 23. Temperature as a Function of Depth Upon Cessation of Drilling a Simulated Well

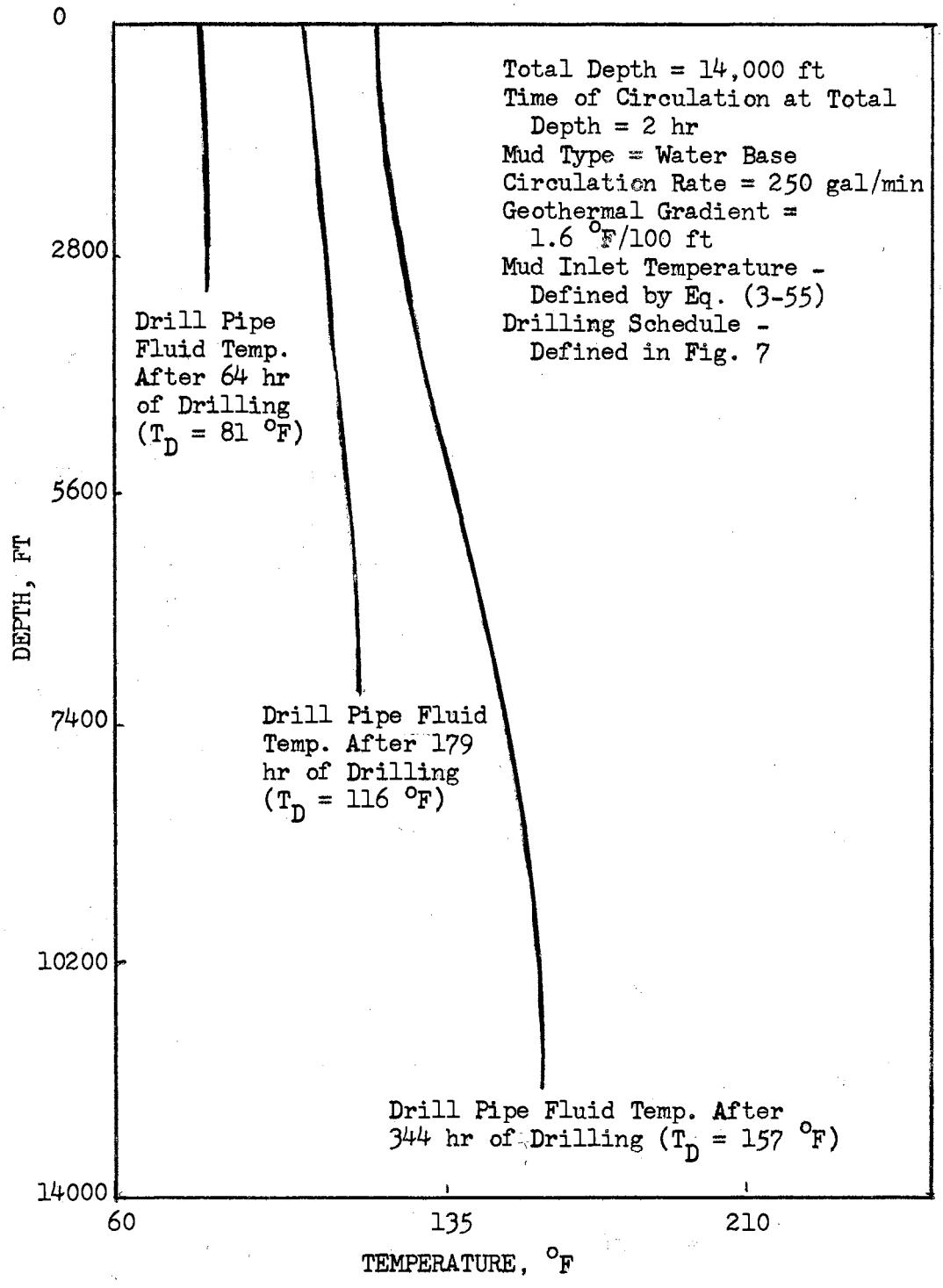

Figure 24. Drill Pipe Circulating Fluid Temperature as a Function of Drilling Depth During Simulation of a Drilling Well

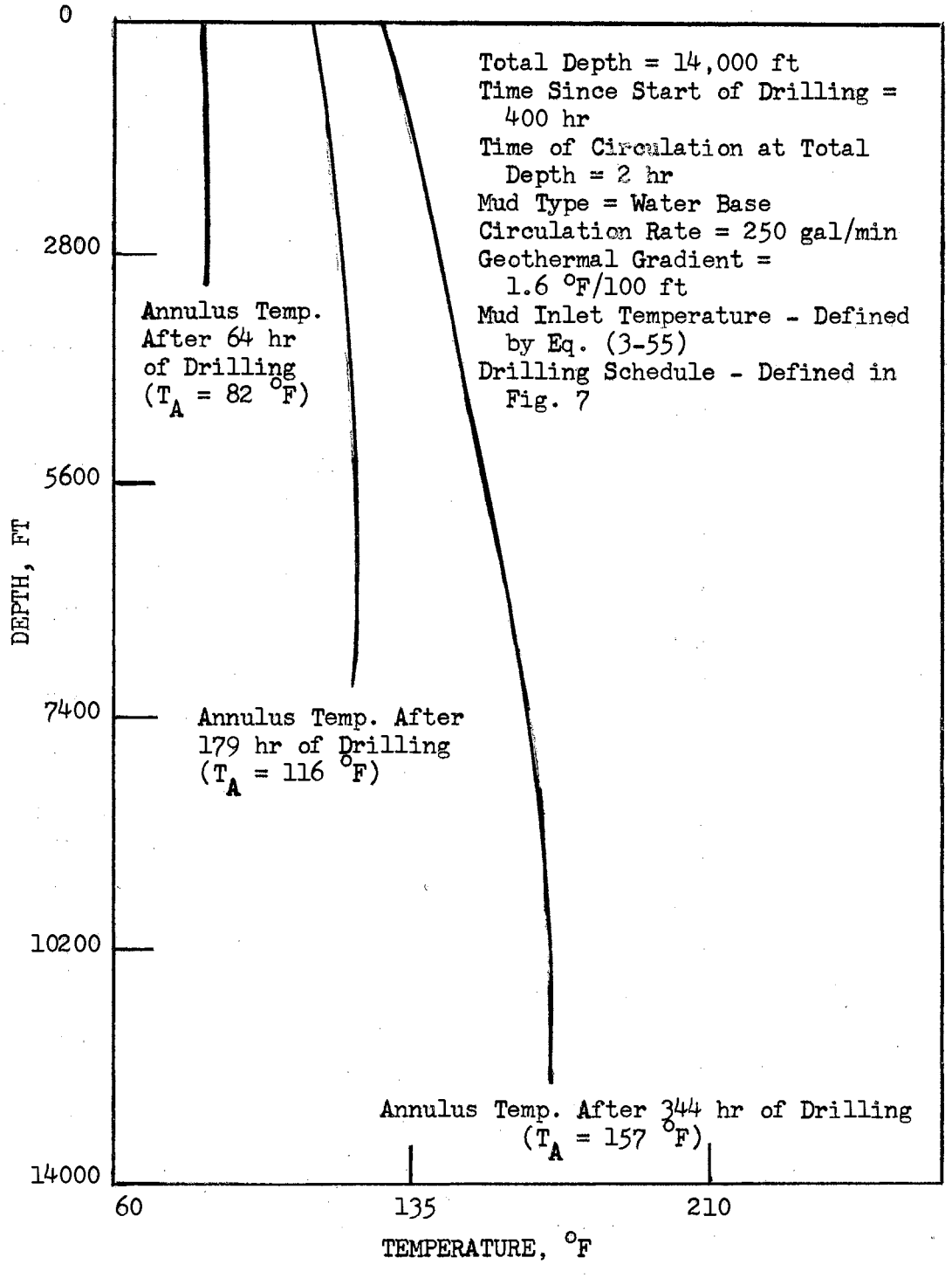

Figure 25. Annulus Circulating Fluid Temperature as a Function of Drilling Depth During Simulation of a Drilling Well

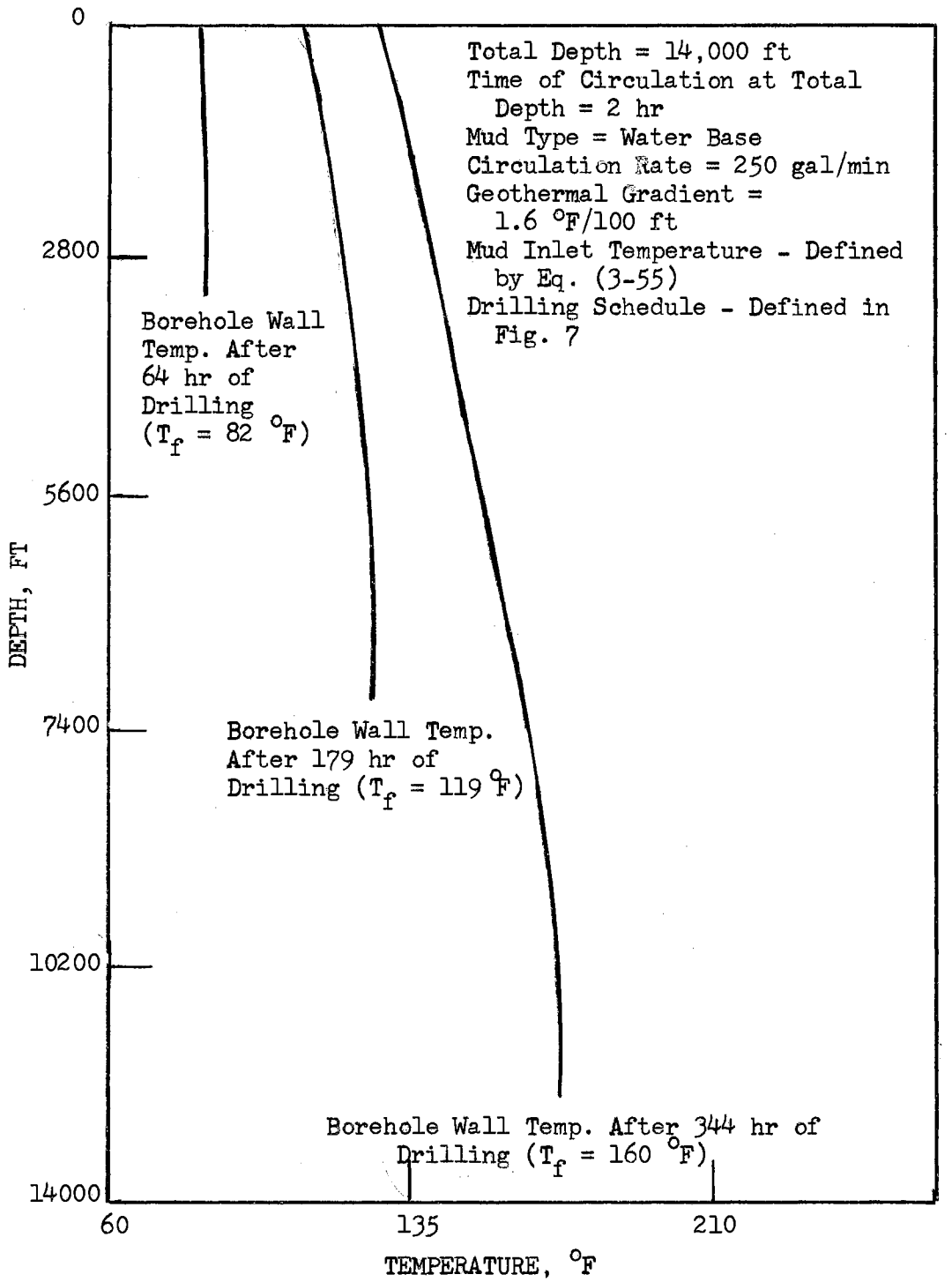

Figure 26. Borehole Wall Temperature as a Function of Drilling Depth During Simulation of a Drilling Well

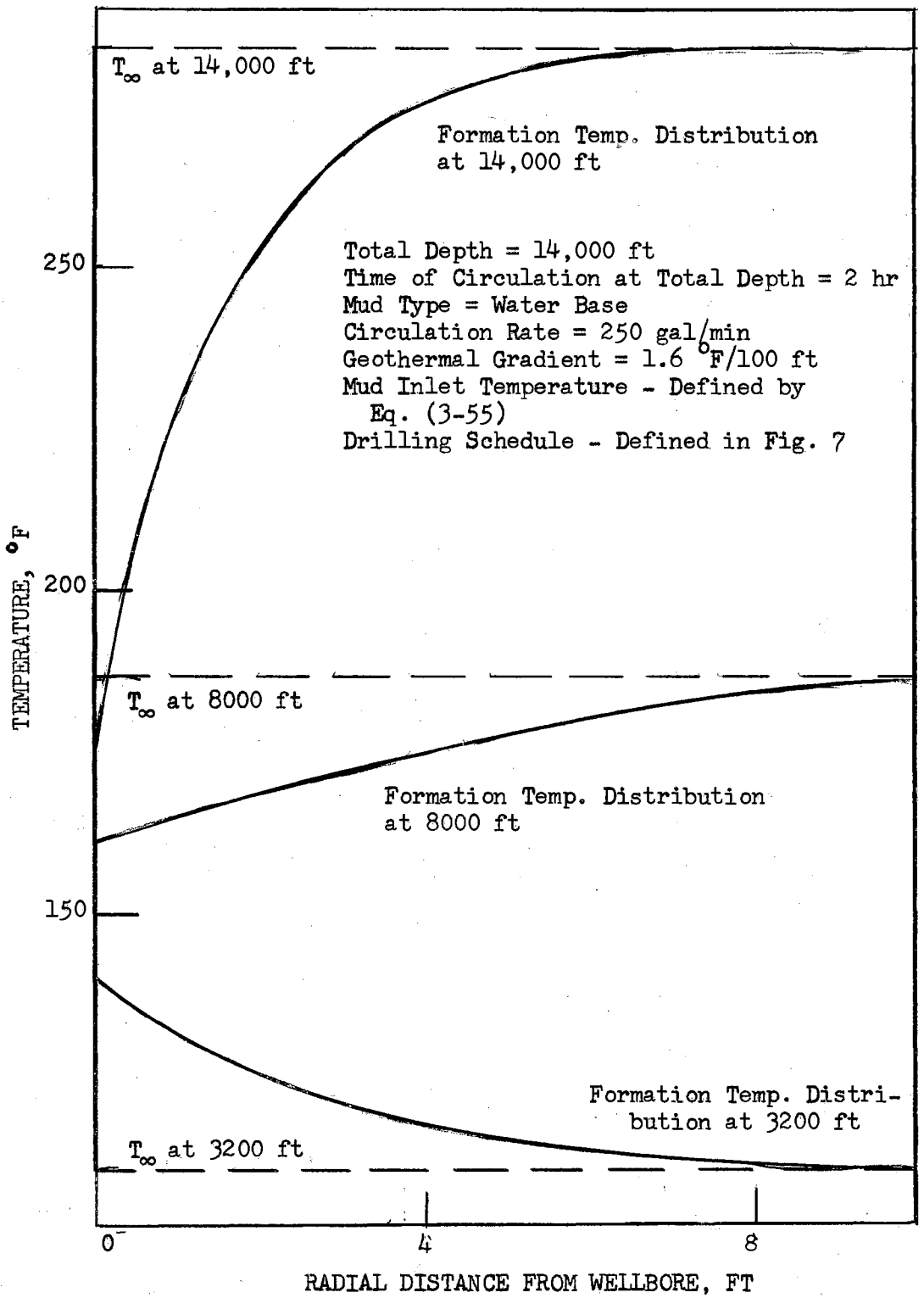

Figure 27. Formation Temperature vs Radial Distance from Wellbore Upon Cessation of Drilling a Simulated Well

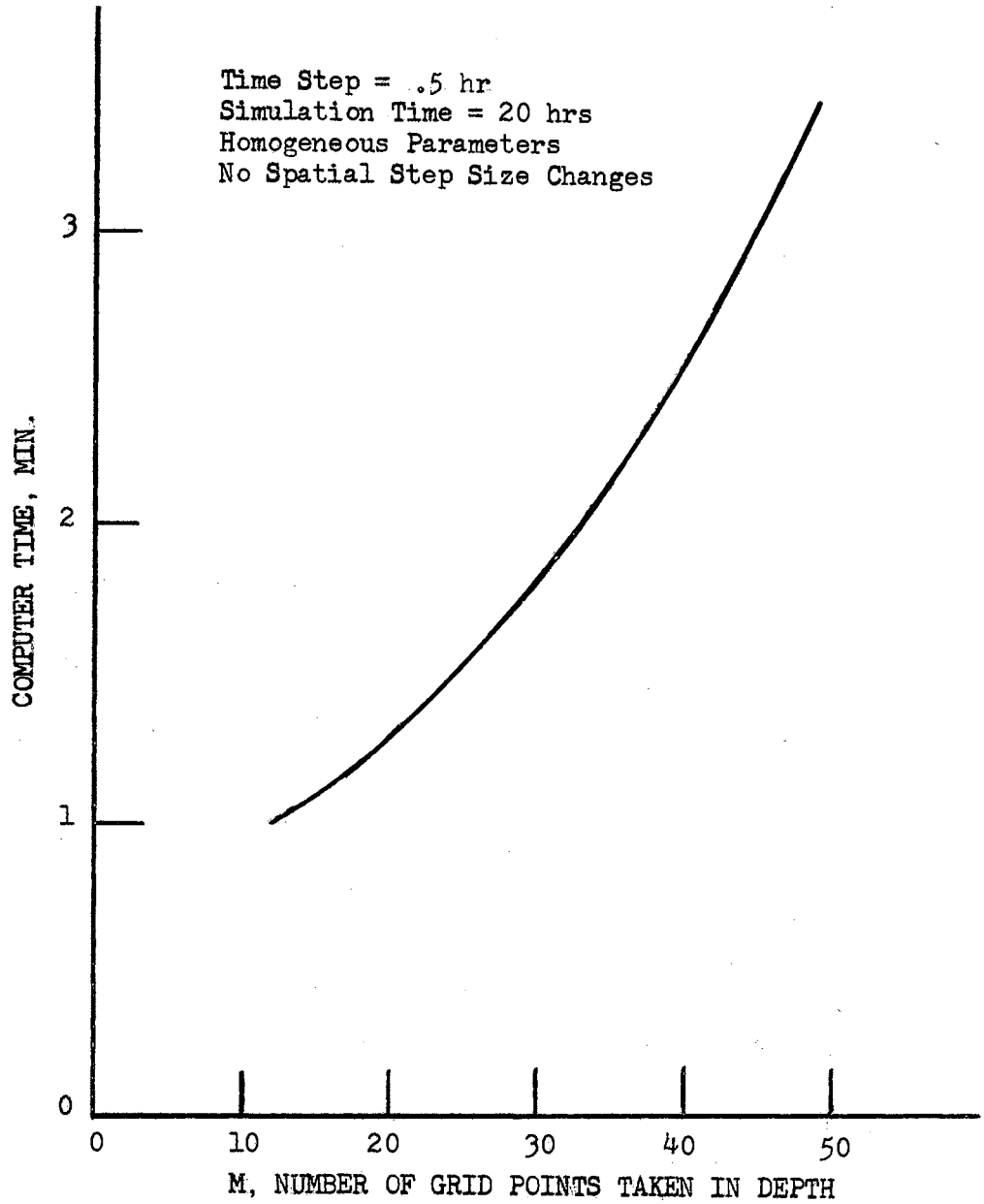

Figure 28. Computer Time Required to Solve Numerical Model as a Function of the Number of Increments Used to Discretize Depth

longest computer time used to simulate a drilling well was 3 minutes 20 seconds. Most simulations were completed within 2 minutes 30 seconds.

A complete parameter study is beyond the scope of this study and will not be presented here. However, the numerical model is structured so that a study may be made of parameters distributed heterogeneously as well as homogeneously. With slight modifications, parameters could also be imputed as functions of time or temperature, or both.

CHAPTER V

SUMMARY AND CONCLUSIONS

A method was developed to numerically model the unsteady temperature distributions in a circulating drilling fluid, and the surrounding formation. The model was based on a fourth order set of partial differential equations describing heat flow by forced convection in the wellbore, and by conduction in the adjacent formation. These equations were formulated into implicit finite-difference approximations resulting in two sets of coupled, algebraic equations. An algorithm, based on Gaussian elimination of a tri-diagonal matrix, was developed in order to couple the two sets of equations. The equation sets were then solved individually by applying Gaussian elimination to only the non-zero elements of the coefficient matrices. In the computer programming, advantage was taken of the problem structure in order to apply the numerical method most efficiently.

A numerical error analysis showed that solution error due to truncation and round-off was not significant. Solution convergence criteria were established, and shown to impose no practical limits on the application of the method. Example problems were given, and a comparison was made between a solution developed by a different numerical method (Raymond (11)), and the one developed in this study.

The following conclusions are made from this study:

1. Computer simulation time for solution of the numerical model

as proposed herein is relatively short; thus, the model provides the only published solution of this problem that is practical for engineering analysis.

2. The numerical method developed herein provides an accurate solution to the problem of predicting the non-steady temperature distributions associated with the drilling of a well.

3. The numerical model is capable of simulating real systems that have heterogeneous, and/or time dependent properties. No other method has ever been published in which this could be done practicaly.

4. For problems of this type, general methods of matrix manipulation cannot be used to generate an efficient solution. In addition, a high degree of accuracy is usually associated with an efficient numerical method due to the dependency of round-off error on the number of calculations made; therefore, a general method may not even be capable of obtaining a valid solution. Considering this one should understand the development and application of a numerical technique before it is blindly used.

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VITA

2

Richard E. Bennett

Candidate for the Degree of

Master of Science

Thesis: A NUMERICAL SIMULATION OF BOREHOLE TEMPERATURE DISTRIBUTIONS WHILE DRILLING

Major Field: Mechanical Engineering

Biographical:

Personal Data: Born in Waynesville, Missouri, July 6, 1946, the son of Mr. and Mrs. Eugene N. Bennett.

Education: Graduated from Putnam City High School, Oklahoma City, Oklahoma, in May 1964; entered Oklahoma State University in September, 1964; received the Bachelor of Science degree from Oklahoma State University in 1969, with a major in Mechanical Engineering; completed requirements for the Master of Science degree at Oklahoma State University in May, 1970.

Professional Experience: Engineering Assistant, Standard Oil Co. of Calif. W.O.I., Summer 1967; Project Engineer, Northern Natural Gas Co., Summer 1968; graduate research assistant, School of Mechanical and Aerospace Engineering, Oklahoma State University, 1969-70.