THE DEVELOPMENT OF FINITE ELEMENT MODELING TECHNIQUES OF WEBS AND THE ANALYSIS OF WEB WRINKLE FORMATION

By

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OF WEB WRINKLE FORMATION

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NOMENCLATURE

A	element area
A	amplitude of wrinkles (Chapter IV)
E	Young's modulus
G	shear modulus
ρ	length of wave
P, Q	constants defined by Equations (2.19a) and (2.19b)
T	initial uniform tension force
t	thickness of material
S	initial uniform shear force
x, y	rectangular coordinates
u, v	displacements in x and y directions
W	width of the web
WR	number of wrinkles
ε_x, ε_y	direct strains in rectangular-coordinate system
γ_{xy}	shear strain in rectangular-coordinate system
ε_1, ε_2	principal strains
ν	Poisson's ratio for material
λ	function determining strain in direction perpendicular to wrinkles, "variable Poisson's ratio"
σ_x, σ_y	direct stresses in rectangular-coordinate system
σ_{xy}	shear stress in rectangular-coordinate system
σ_1, σ_2	principal stresses

CHAPTER I

INTRODUCTION

The problem of wrinkles being formed in webs is a major one in web handling. When attempts to remove the formed wrinkles are not successful, wrinkled material will be wound that most likely becomes waste.

In a situation where one edge of the web advances in position over the other edge, shear stress is produced in addition to the already existing tensile stress. Diagonal wrinkles generally form pointing in the direction of high tension (the direction of one principal stress, shear stress being zero in this direction). Shear stresses always appear in perpendicular and opposite pairs. It is obvious that the same effect could be produced by displacing the trailing edge of a sheet in the transverse direction, while the leading edge is fixed along a cross-machine direction line. This often occurs when shifting an unwinding roll laterally at a speed too high for the web to move over the next carrying roll. Wrinkles can be introduced in the web due to tensile loads alone. High tension causes excessive lateral contraction, thus allowing wrinkles to form. In this case the wrinkles are parallel to the longitudinal direction of the web.

This type of wrinkling is classic in thin webs in aircraft structures. It is historically nomenclated as semi-tension field web wrinkling. The boundary conditions, however, are very different. In aircraft semi-monocoque structures, each thin web is given support on four sides such that fixed boundary conditions can be assumed. The guided web is supported on two sides only. This has a different effect on the shear and tension field stress distributions.

At present, no real attempt has been made to solve this problem. Some empirical methods have been successful in partly eliminating the problem. Techniques such as introducing grooves in the roller occasionally remove some of the incoming wrinkles. However, no one has been able to present a satisfactory explanation as to why this works. Others profess that grooved rollers can introduce more wrinkles than they eliminate. Thus the development of a general method to predict the onset of wrinkling and eliminate it is very much in order.

1.1 Objectives

The objectives of this study are:

1. To develop a finite element computer code to analyze models of the web. The program must be able to predict wrinkling in the web and completely analyze the states of strain and stress on each element.

2. To investigate the effect of material properties (modulus of elasticity and Poisson's ratio), various loading conditions, and web geometry on the behavior of the web and the amount of lateral contraction.

3. To derive equations to predict the overall lateral contraction of the web for known input quantities such as average web tension and material properties.

4. To model the web using NASTRAN and observe its buckling modes, visualize the number of wrinkles formed, and calculate an average wrinkle amplitude.

1.2 Literature Survey

As mentioned previously, there has not been any conclusive work done on this problem. Most work of this nature has been done on thin-plate and membrane elements found in flight vehicle applications.

Studies of stability and failure of thin-plate elements have been conducted by Rivello [1] and Timoshenko [2], among others. The small deflection elastic buckling theory for perfectly flat plates was investigated. These studies were aimed at predicting the critical loads, since stress distribution and stiffness of the structure are affected by buckling. Results for buckling of plates with various loading and boundary conditions have been presented by Flugge [3].

There is a large amount of literature on the subject of membrane structures due to their vast applications on space vehicles and lightweight structures. Hideki, Okamura, and Kawaguchi [4] have presented a practical method of shape finding and nonlinear analysis of membrane structures considering the wrinkling effect. Partly wrinkled membranes have been the subject of investigation conducted by Miller et al. [5], Hedgepeth et al. [6], and Mikulas [7]. In Reference [6], a theory is derived to predict the stresses and deformations of stretched-membrane structural components for loads under which part of the membrane wrinkles. This theory studies average displacements of the wrinkled material. Geometric features of wrinkling such as lateral overcontraction were incorporated into a Hookean material model by appropriately increasing the local effective value of Poisson's ratio in wrinkled regions.

In later work, Miller et al. [5] presented a generalization and numerical implementation of his previous work on partly wrinkled membranes. Slack or unstressed regions have been included in the analysis by setting the local modulus of elasticity equal to zero in such regions. This generalized work was limited to flat membranes with in-plane loading.

From this review of the literature, it became evident that theory developed for partly wrinkled membranes can be applied on web wrinkling as well. A membrane by definition has no bending stiffness and can carry no compressive stress. This is the case in webs as well, since most web materials are able to resist little or no compression at all. In this study, a method developed by Miller et al. [5] is implemented in a computer code to predict the onset of wrinkling in the web model due to loading in the transverse and lateral directions.

1.3 Organization

Chapter II consists of the theory behind this analysis. The theory developed by Miller et al. [5] and the basic theory of elasticity are presented. Chapter III discusses the computer code developed for the implementation of the constitutive relationships derived in Chapter II. The modeling of the web by this code and the structural analysis computer code NASTRAN are also presented in the same chapter. Chapter IV presents the results of this analysis. Data tables and plots of the different parameters are provided for better understanding. Chapter V presents conclusions formulated during the study and a summary. Recommendations for further investigation of the subject are also included in Chapter V.

CHAPTER II

DEVELOPMENT OF THE FINITE ELEMENT EQUATIONS

In this chapter general forms of the equations employed are presented. A brief explanation is given on the meaning of each relationship. Familiarity of the reader with basic theory of elasticity and finite element methods is assumed.

The subject of how the constitutive relations [D] matrix is modified to fit both slack and taut web behaviors is discussed in section 2.2. The theory on partially wrinkled membranes, developed by Stein and Hedgepeth [6], is discussed in section 2.3. The principles governing the study and the resultant modification of matrix [D] are discussed. Finally, summarized in this section are the constitutive relationships employed to assist this study.

2.1 Strain, Stress, and Displacement Equations

In a two-dimensional field, each displacement component at each point is a function of the two coordinate directions, that is,

$$u = f\ (x,y) \tag{2.1a}$$

and

$$v = g\ (x,y) \tag{2.1b}$$

The objective of every analytical and finite element analysis is to determine the equations corresponding to $f(x,y)$ and $g(x,y)$. The finite element approximations for these functions are continuous, piecewise, smooth equations defined over the individual elements. The general form displacement equations in two dimensions are:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = [N] \; \{U^{(e)}\} \tag{2.2}$$

where $\{U^{(e)}\}$ is a column vector containing the element nodal displacements. The matrix [N] contains the element shape functions. A general triangular element defining the individual components of the $\{U^{(e)}\}$ vector is shown in Figure 1.

The strain vector $\{\varepsilon\}$ is defined as

$$\{\varepsilon\} = \{\varepsilon_x \; \varepsilon_y \; \gamma_{xy}\}^T \tag{2.3a}$$

Similarly, a stress vector is defined as

$$\{\sigma\} = \{\sigma_x \; \sigma_y \; \sigma_{xy}\}^T \tag{2.3b}$$

The strain components in $\{\varepsilon\}$ and the displacements are related. The set of the strain-displacement equations consists of

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{2.4}$$

A general matrix [B] is defined to relate the nodal displacements to the strain vector:

$$\{\varepsilon\} = [B] \; \{U^{(e)}\} \tag{2.5}$$

The matrix [B] is called the gradient matrix. The first row of [B] is obtained by differentiating the displacement equation for u with respect to x, that is, $\partial u/\partial x$. The second row contains $\partial v/\partial y$, and so on. The stress

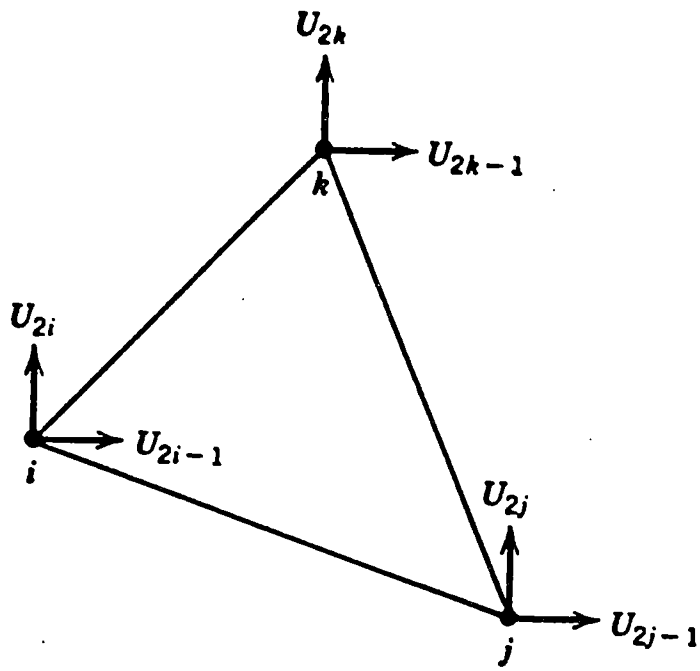

Figure 1. Nodal Displacements for a Triangular Elasticity Element

and elastic strain components are related by a set of coefficients known as the generalized Hooke's law. This law can be written as

$$\{\sigma\} = [D]\ \{\varepsilon\} \tag{2.6}$$

where [D] is an equivalent elasticity matrix.

The element stiffness matrix and the element force vector are the element's contribution to the system of equations that result when the potential energy is minimized. Strain energy considerations define the element stiffness matrix as

$$[K^{(e)}] = [B]^T\ [D]\ [B] \tag{2.7}$$

The element nodal displacement vector is obtained using the equality

$$\{U^{(e)}\} = [K^{(e)}]^{-1}\ \{F\} \tag{2.8}$$

where {F} is the applied forces vector. Equation (2.5) can be solved to yield the strain vector $\{\varepsilon\}$. Knowing $\{\varepsilon\}$ enables the solution of Equation (2.6) for the stress vector $\{\sigma\}$.

2.2 Matrix [D] for Plane Stress-Isotropic Material

The matrix [D] in Equation (2.6) can be explicitly stated for any material. For the plane stress state in an isotropic material by definition:

$$\varepsilon_x = \frac{1}{E}\ (\sigma_x - \nu\sigma_y) \tag{2.9a}$$

$$\varepsilon_y = \frac{1}{E}\ (\sigma_y - \nu\sigma_x) \tag{2.9b}$$

$$\gamma_{xy} = \frac{2}{E}\ (1+\nu)\ \tau_{xy} \tag{2.9c}$$

Solving the above for the stresses yields the elastic matrix [D] as

$$[D] = \frac{E}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1 - \nu)}{2} \end{bmatrix} \tag{2.10}$$

This [D] matrix applies to elements within the elastic region when characteristics of taut behavior are exhibited. That is, when both principal strains ε_1 and ε_2 are positive or when ε_1 is positive and the observed ε_2 can be predicted accurately by multiplying ε_1 times Poisson's ratio. It is assumed that ε_1 is always greater than or equal to ε_2.

Similarly, when ε_1 is less than zero, compressive loads are present. However, by definition no compressive stresses can be carried by a membrane. Hence, it is said that the membrane has assumed slack behavior. The constitutive relations are therefore set equal to zero on elements for which ε_1 is negative. Thus,

$$[D] = 0 \tag{2.11}$$

for slack or unstressed elements.

2.3 Elasticity Matrix for Partly Wrinkled Membranes

This analysis of partly wrinkled membranes is based on the observation that when wrinkles develop within a membrane parallel to, for example, the x-direction, the associated overall contraction in the y-direction exceeds that predicted by the Poisson's ratio effect. This feature of wrinkling may be incorporated into a Hookean material model by appropriately increasing the local effective value of Poisson's ratio in wrinkled regions. This value may be determined by imposing a locally uniaxial stress state in a wrinkled region. The net result is that effective Hookean material properties become dependent on the local state of strain.

It is convenient to look first at the principal stresses. If both principal stresses are positive, the membrane is in tension and thus will not wrinkle. If both principal stresses are zero, the membrane is unloaded and thus will not wrinkle. In a wrinkled membrane, one principal stress must be zero and the other nonzero. The nonzero principal stress may be assumed to act along the wrinkle. This may be justified as follows. Assume σ_1 is the nonzero principal stress. In a wrinkled membrane in tension, tide strips in the longitudinal direction may be observed parallel to the wrinkles. Those strips are in the direction of maximum stress. Since σ_2 is zero, it follows that σ_1 acts parallel to the wrinkles.

Corresponding to the nonzero principal stress σ_1, the principal strain ε_1 parallel to the wrinkle at each point would be expected to be

$$\varepsilon_1 = \frac{\sigma_1}{E} \tag{2.12}$$

Because of the "overcontraction" behavior of a wrinkled membrane in the direction normal to the wrinkles, a "variable Poisson's ratio" λ is defined so that

$$\varepsilon_2 = -\lambda \frac{\sigma_1}{E} \tag{2.13}$$

The quantity λ allows an average measure to be made of the ε_2 strain that would otherwise be either indeterminate or dependent on detailed large deflection analysis. At points where a wrinkled region borders on an unwrinkled region, λ must equal Poisson's ratio for the material.

From the relationships between the strains in the rectangular-coordinate directions and the stresses,

$$\varepsilon_x = \frac{1}{E} (\sigma_x - \lambda\sigma_y) \tag{2.14a}$$

$$\varepsilon_y = \frac{1}{E} (\sigma_y - \lambda\sigma_x) \tag{2.14b}$$

$$\gamma_{xy} = 2 \ (1 + \lambda) \ \tau_{xy}/E \tag{2.14c}$$

The preceding equations, through the quantity λ, define the average strains in rectangular coordinates. The "variable Poisson's ratio" λ can be written as

$$\lambda = -\frac{\varepsilon_2}{\varepsilon_1} \tag{2.15}$$

Consideration of the geometry of Mohr's circle for strains yields

$$\varepsilon_1 = R + D \tag{2.16a}$$

$$\varepsilon_2 = -R + D \tag{2.16b}$$

where $R = 1/2 \ (\varepsilon_1 - \varepsilon_2)$ is the radius of Mohr's circle for strains and $D = 1/2 \ (\varepsilon_x - \varepsilon_y)$ is the distance from the origin of the coordinate system to the center of the circle. Hence, Equation (2.15) may be written in an alternative form:

$$\lambda = \frac{(\varepsilon_1 - \varepsilon_2) - (\varepsilon_x + \varepsilon_y)}{(\varepsilon_x + \varepsilon_y) + (\varepsilon_1 - \varepsilon_2)} \tag{2.17}$$

The expression for λ in Equation (2.17) may be substituted into Equation (2.14). Separation of variables is then performed to transform the three resulting equations into the form of Equation (2.6).

The resulting plane stress formulation for initially flat membranes with in-plane loading is identical to the linear elastic case described by Zienkiewicz [8]. The equivalent elasticity matrix [D] in Equation (2.6) takes the following form:

$$[D] = \frac{E}{4}\begin{bmatrix} 2(1+P) & 0 & Q \\ 0 & 2(1-P) & Q \\ Q & Q & 1 \end{bmatrix} \tag{2.18}$$

for partly wrinkled membranes, where

$$P = (\varepsilon_x - \varepsilon_y)/(\varepsilon_1 - \varepsilon_2) \tag{2.19a}$$

$$Q = \gamma_{xy}/(\varepsilon_1 - \varepsilon_2) \tag{2.19b}$$

Matrix [D] of Equation (2.18) satisfies all of the conditions imposed due to the presence of wrinkles. Thus when [D] is substituted into Equation (2.6), either σ_1 or σ_2 becomes zero. For a zero σ_1, σ_2 has a nonzero value and vice versa.

2.4 Conclusions

In this chapter the constitutive relationships employed in this analysis were described. The elasticity matrix [D] was defined for all three allowable behaviors (taut, wrinkled, and slack). A useful algorithm for choosing the [D] matrix may be expressed as

$$[D] = [D_S]; \qquad \varepsilon_1 < 0 \tag{2.20a}$$

$$= [D_W]; \qquad 0 < \varepsilon_1 \text{ and } \nu\varepsilon_1 \leqq -\varepsilon_2 \tag{2.20b}$$

$$= [D_T]; \qquad \text{Otherwise} \tag{2.20c}$$

where the [D] matrices are defined as

Slack behavior:

$$[D_S] = [0] \tag{2.21a}$$

Taut behavior:

$$[D_T] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1-\nu)}{2} \end{bmatrix} \tag{2.21b}$$

Wrinkled behavior:

$$[D_W] = \frac{E}{4}\begin{bmatrix} 2(1+P) & 0 & Q \\ 0 & 2(1-P) & Q \\ Q & Q & 1 \end{bmatrix} \tag{2.21c}$$

where $P = (\varepsilon_x - \varepsilon_y)/(\varepsilon_1 - \varepsilon_2)$ and $Q = \gamma_{xy}/(\varepsilon_1 - \varepsilon_2)$. This algorithm is employed in a computer code described in Chapter III for the analysis of wrinkling in webs.

CHAPTER III

ANALYTICAL STUDY

The purpose of this study is to examine the contribution of variables such as modulus of elasticity, Poisson's ratio, thickness, tension, and shear loading in the formation of wrinkles in webs. How the overall lateral contraction of the web is affected by these parameters is examined. An average wrinkle amplitude is calculated for an estimated number of wrinkles in a given length-to-width ratio of the web.

For the purpose of applying the constitutive relationships developed in Chapter II, a finite element computer code was developed. A good understanding of how this code implements the constitutive relationships to the finite element model is very important. A description of the logic behind the FORTRAN program STRESS.FOR (Appendix) is presented in this chapter. A description of how the web was modeled as well as the approach taken to the solution of this problem is also presented.

3.1 Description of the Computer Code STRESS.FOR

The computer code STRESS.FOR was written around an already existing code under the same name (see Reference [9]). The program is used to completely analyze every element in the finite element model. Its final version performs tasks such as searching the model to identify wrinkled elements, compute element stresses and strains, and calculate nodal displacements.

It becomes apparent through descriptions of the main program and its subroutines how this code uses the constitutive relationships in Equations (2.19) and (2.20) to perform its functions. This code was written to handle triangular elements. Only two-dimensional, plane stress elasticity problems can be analyzed. Possibilities such as body forces, thermal changes, or composite material construction are beyond the scope of this study and were not included in this code.

The elements in the models under study can assume three different behaviors: slack, taut, and wrinkled. The matrix [D] in Equation (2.6) is dependent on these different behaviors. In Chapter II, three different [D] matrices have been defined, one for each case. Equation (2.19) is employed to define the state of each element and assign the appropriate [D] matrix to it.

Any time there is a transition of elements from one behavior to another, the computer code analyzes its effect on the neighboring elements. In Chapter II, the element stiffness matrix is defined as

$$[K^{(e)}] = [B]^T \; [D] \; [B] \; t \; A \tag{3.1}$$

where [B] is the gradient matrix, and [B] and [D] consist entirely of constant terms. Since the element stiffness matrix [K] is used for the calculation of the nodal displacements, according to Equation (2.8), any changes in [D] yield different displacements. The altered displacements affect the strain calculations, since

$$\{\varepsilon^{(e)}\} = [B] \; \{U^{(e)}\} \tag{3.2}$$

where $\{U^{(e)}\}$ is the element displacement vector. The element strains are used in the calculation of the principal strains. These calculations are therefore affected by any change in $\{\varepsilon^{(e)}\}$. The principal strain values are used in the constitutive relationships to decide the type of the [D] matrix. The computer code performs a global search of the model to reassign [D] matrices where needed. This procedure is followed until the behavior of all elements has been determined for the same loading condition.

During execution, the calculated strains for the last two passes for each wrinkled element are stored in two different arrays. After all elements have been examined and all necessary changes in [D] are performed, the two arrays are checked for closeness. For this purpose a convergence criterion has been preassigned. Upon satisfaction of this criterion, a load step is added to the previous load and all steps are repeated.

In every case a maximum load in the longitudinal direction and a maximum in the lateral direction are defined. The computer code is made to iterate on the longitudinal load initially, until maximum load has been reached. It then proceeds to iterate on the lateral load with the longitudinal load still present. The program stops executing when the maximum lateral load has been reached.

Initially, the general model is formed using triangular elements. Before the program can be executed, a file with the designated name DATA.DAT must be created. This file contains the node numbers and their location in the model with respect to a preassigned (x-y) coordinate system. The information above is followed by the element numbers and the node numbers associated with the particular element. These node numbers must be presented in a counterclockwise fashion, with the starting node selected at random (see Figure 1).

The file TEMP.DAT is created by the computer code. It temporarily stores all calculated element stresses and strains for each step before moving to next. If, for any reason other than the one defined by the user the program terminates execution, the intermediate results can be retrieved from that file. The file OUT.DAT is created by the computer code also. This file stores the final solution to the problem. The solutions to the longitudinal load applied alone are always included.

The code enables the user to define a load step for the output of the results when load in the transverse direction is applied. For example, when the user defines a print load step of 5 lbs, the computer will output the intermediate results every time the magnitude of the lateral load applied is a multiple of 5.

For the purpose of making this code "user friendly," check points designed to detect common errors have been added throughout the program. Subroutine CORRECT prints on the screen all user input up to the point it is called. The user is given the opportunity to correct any incorrect input.

In cases where symmetry prevails, it is often unnecessary to output the calculated stresses and strains for all the elements. The user is cautioned to identify the smallest element number for which output is saved. All elements with smaller identification numbers are excluded from the output.

The general model is defined as soon as the file DATA.DAT is read. The code is then able to calculate the bandwidth for the particular model. The bandwidth is used to dimension and initialize the storage vector {A}. The global stiffness matrix [K] and the force vector {F} are stored in {A}. The calculated nodal displacements are stored in {A} also. Vector storage eliminates the need to change the dimensions of the stiffness matrix each time a different model is examined.

Initially, for start-up purposes, taut behavior is assumed and $[D_t]$ is used for the assembly of the initial global stiffness matrix. Since at this stage the material is unloaded, elastic, and Hooke's law applies, this assumption is justifiable.

Subroutine ELSTMX performs the assembly of the element stiffness matrix. This subroutine evaluates the element stiffness matrix, $[K^{(e)}]$, for a linear triangular element using the relationship in Equation (3.1). The subroutine also evaluates the gradient matrix [B] in a loop which calculates the stress component in each element. A small loop after the subroutine call assembles the global stiffness matrix which is stored in the vector {A}.

Subroutine MODIFY enables the user to apply concentrated loads to the structure. It also allows nodes to be restrained in one or both degrees of freedom. This subroutine was written to facilitate the loading pattern on the models in this study. Forces can be applied in both X and Y directions. However, forces in the X-direction may only be of the same magnitude (uniform). The same restriction holds true for the forces applied in the Y direction. MODIFY allows the definition of maximum forces and their direction of application. Load increments for iteration are input in this subroutine. Boundary conditions are defined within this subroutine. Any degree of freedom of any node can be set to zero to achieve a fixed condition.

Subroutine DCMPBD decomposes the global stiffness matrix [K] into an upper triangular form using the method of Gaussian elimination. This subroutine assumes [K] to be symmetric and only those elements within the bandwidth and on or above the main diagonal are stored. The programming logic is not easy to follow since the coefficients of [K] are stored in a vector rather than a two-dimensional array.

The subroutine SLVBD decomposes the global force vector, {F}, and solves the system of equations using back substitution. The solution to this system yields the nodal displacements.

Once the nodal displacements are known, the strain vector for each element is obtained according to Equation (3.2). Similarly, Equation (2.6) is employed to yield the stress vector for each element. The principal stresses and strains are calculated using the relationships

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} + \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2} \tag{3.3a}$$

$$\varepsilon_1, \varepsilon_2 = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{1}{2}\left[(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2\right]^{1/2} \quad + \left[\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \gamma_{xy}^2\right]^{1/2} \tag{3.3b}$$

The angle ϕ between the principal axis and the x axis is

$$\tan 2\phi = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} \tag{3.4}$$

The calculated principal strains are used to define the matrix $[D_W]$ (wrinkled behavior).

At this point the computer code performs all the checks mentioned earlier in this section. Subroutines are called as needed and all intermediate results are stored in the file TEMP.DAT.

Subroutine PRINTR writes the final results in the output file OUT.DAT in a formated manner. The results are read from the temporary storage file TEMP.DAT.

An element can assume three different behaviors (taut, wrinkled, and slack). One [D] matrix for each case has been defined in Chapter II.

Subroutine CHECK assigns the appropriate [D] matrix to each element. Any time there is a transition in the behavior of an element (taut to wrinkled, etc.), the element stiffness matrix changes and so does the global stiffness matrix [K]. Subroutine MODIFI reapplies the restrictions placed on the degrees of freedom during execution.

Subroutine LOAD increases the magnitude of the load in the force vector by a load step each time the program has finished computing the results for the current loading condition.

3.2 Finite Element Modeling of the Web

A portion of the web is modeled to simulate its behavior between rollers. The different models used in this study consist of linear triangular elements (Figure 2). This is due in part to the fact that the computer code employed is limited to triangular elements. However, linear triangular elements are preferred over bilinear rectangular elements because they can assume any orientation where the sides of the rectangular elements must remain parallel to the x-y coordinate system.

Two carrying rolls are aligned so that the web path is accurately tangent to both surfaces, i.e., the web between the two rolls could be replaced by a plane touching both ends of both rolls. For the purpose of achieving good traction, the angles of approach and departure for the web differ greatly (Figure 3). Large wrap angles allow for large contact area between the web and the roller. Surface elements of the web material meeting surface elements of a carrying roller remain in a one-to-one correspondence with each other for a finite and measurable distance of travel. Lack of traction is relative motion between adjacent points, which may be supported by film shear. When tension changes going over a

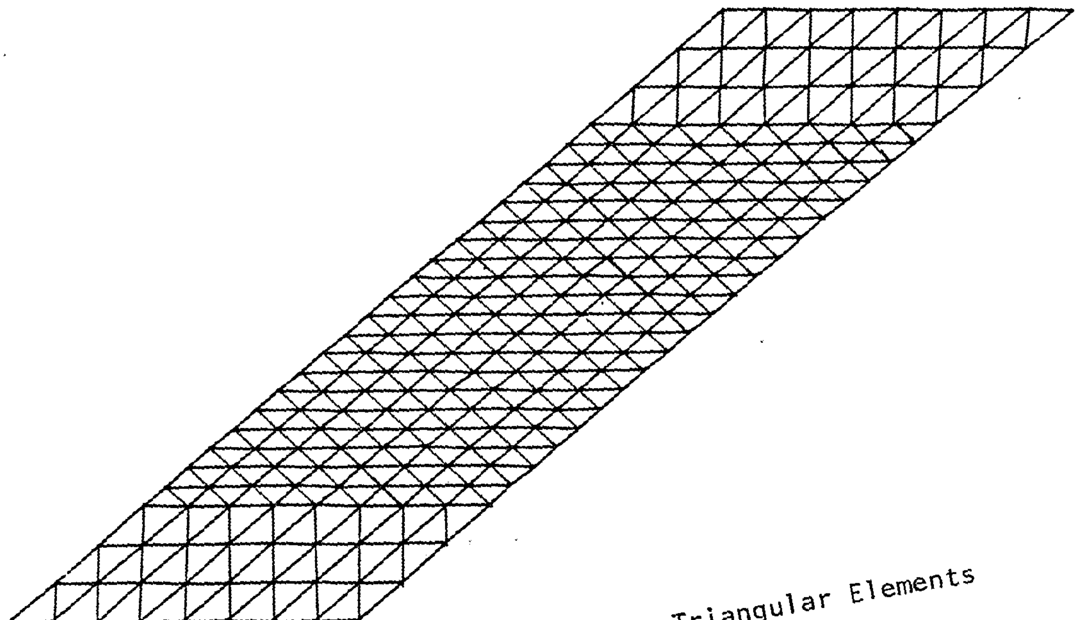

Figure 2. Web Model Using Triangular Elements

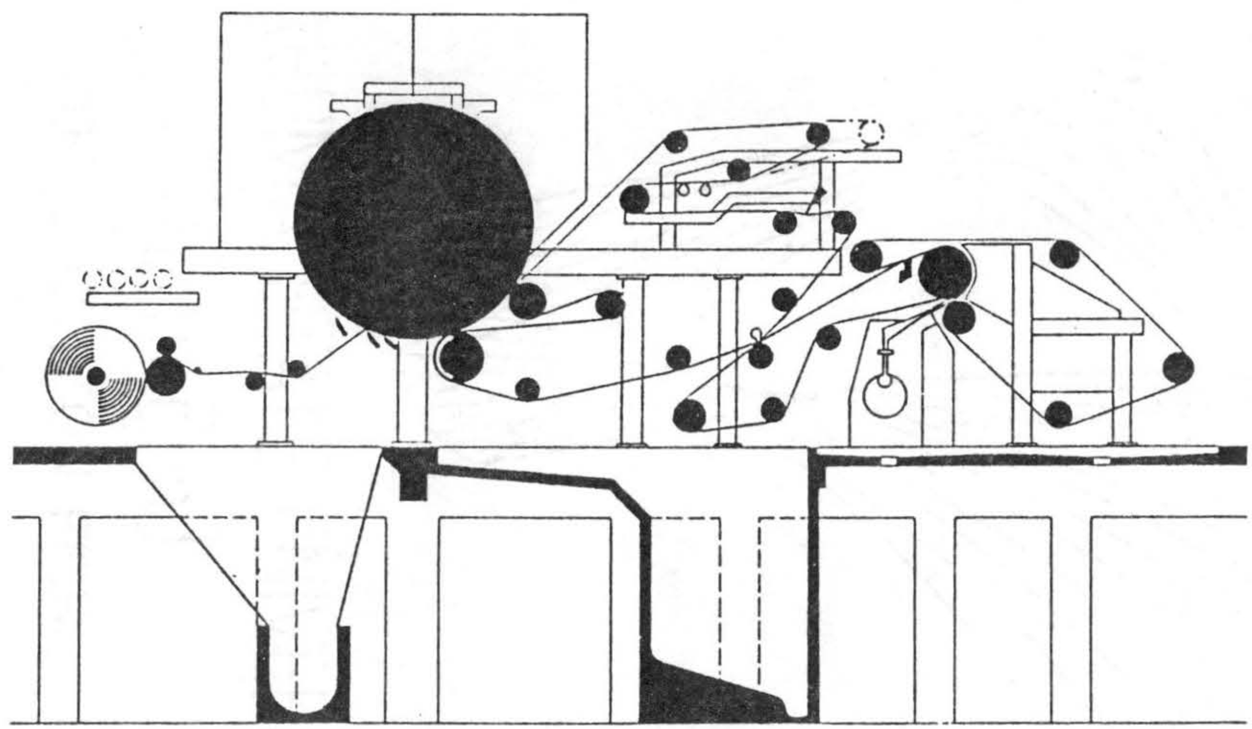

Figure 3. Typical Array of Rollers in a Web Winding System

roll, there may be a zone of traction followed by a short zone not in traction, where either the tension becomes too high or the normal force becomes too low to maintain traction. If ideal traction is achieved between the incoming web and carrying roll, there is no relative displacement between any two adjacent points in the web in both the longitudinal and lateral directions. Therefore, in modeling the leading end of the web, fixed end conditions may be assumed. On the opposite end, just before the web leaves the roller surface, it is still in a one-to-one correspondence with the carrying roll in the lateral direction. No relative displacement occurs in the lateral direction between any two adjacent points in the web. However, relative displacement does exist in the direction of the web travel. This is due to the tensile force induced in the web by the combined effect of traction and speed differential between the carrying roll and its preceding roll. Therefore, the computer model of the web is allowed to translate in the longitudinal direction at the trailing end.

The boundary conditions of the web model are summarized as follows. The leading end in the direction of travel is held completely fixed, while the trailing opposite end is allowed to only translate in the longitudinal direction. The remaining two ends, parallel to the direction of motion, are free of constraints.

3.3 NASTRAN Model Details

In the last phase of this study, buckling analysis is performed on the web model. For this purpose the finite element code known as NASTRAN is employed. NASTRAN is an acronym for NASA Structural Analysis. The objective is to use NASTRAN's buckling analysis to predict the number of wrinkles for a given length-to-width ratio. NASTRAN's plotting capabilities are employed to obtain plots of the different modes. The calculated out-of-plane deformations define the formed wrinkles which can be visualized on the plots. The number of wrinkles counted on the plots for a given length-to-width ratio is checked against the number of stress reversals counted along the lateral web direction. If the number of wrinkles in a web and the overall lateral contraction for a given loading condition are known, an average wrinkle amplitude may be calculated.

The first step is to discretize the web model in terms of geometric grid points located in a Cartesian coordinate system. These grid points were aligned to form rectangular elements when connected. These points can have from zero to six degrees of freedom in a particular coordinate system.

The second step included choosing the elements of which the model would be composed. The quadrilateral element CQUAD2 was selected. This element, with both in-plane and bending stiffness, assumes a solid homogeneous cross section. Transverse shear flexibility is also included within CQUAD2. The NASTRAN model of the web is shown in Figure 4. The quadrilateral elements are intended for use when the surfaces are flat and the geometry near rectangular. Membrane elements could not be employed as there is no ability to model out-of-plane deflections incurred during wrinkling. The property card MAT1 defines the structural properties of the quadrilateral element. This data card includes such entities as Young's modulus, Poisson's ratio or the shear modulus, and density. The boundary conditions in the NASTRAN models remain the same as the previous ones. The data card GRID is used to indicate the location

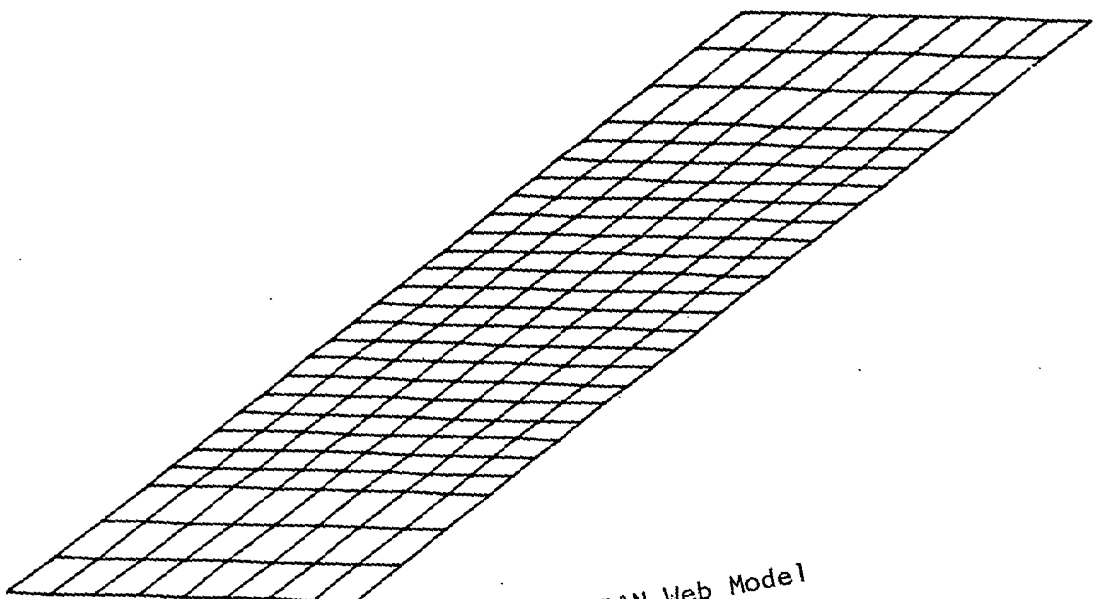

Figure 4. NASTRAN Web Model

of each grid point on the model. It also serves to restrain any degree of freedom on a particular node.

3.4 Summary

In this chapter a full description of the computer code STRESS.FOR was given. It becomes obvious through that description how the constitutive relationships are employed to predict web behaviors. In section 3.2, the final form of the computer model of the web was derived through careful consideration of the actual system. In section 3.3, the NASTRAN model was presented. The elements used to model the web were described and reasons were given to justify their choice. Chapter IV in this study presents the results of this analysis. A number of plots and data tables are presented to verify the derived relationships among the various parameters.

CHAPTER IV

ANALYTICAL RESULTS

As stated in the introduction, the objectives of this study are to investigate the effect of material properties (Poisson's ratio, modulus of elasticity), web geometry (thickness, length-to-width ratio), and various loading conditions on the formation of wrinkles in webs. Equations are derived to predict the overall lateral contraction of the web, based on these observations.

4.1 Results of Web Variables Variation

Element density in the model is important for numerical accuracy. The selected general model is 32 inches wide and 128 inches long. In order to achieve a relatively dense model, the nodes were placed four inches apart in the lateral direction, yielding a total of nine nodes along the width of the web. Similarly, nodes were placed four inches apart in the longitudinal direction. However, close to the leading and trailing ends of the web, the density of the elements drops to half the original density (see Figure 2). Simple static analysis runs are made to examine the numerical accuracy of the calculations for the given model density.

Initially, reference values are assigned to the variables under investigation and a general solution is obtained. The effect of each parameter on the overall lateral contraction of the web is examined in the following manner. The variable under investigation is varied over a specified domain while the remainder of the variables are held constant at their reference values. A region is defined on the web, in an area where wrinkles are most likely to form. In each run, for each isolated variable, the computed principal strain epsilon two for a wrinkled element in that region is recorded. Hence for each parameter value (datum) there exists a corresponding functional value epsilon two. The accumulated data are curve fitted to yield how the average principal strain epsilon two of an element varies with respect to the variable under investigation. The same procedure is followed for all variables to be examined.

Two sets of equations are obtained relating each variable to epsilon two. One set consists of the equations derived when the web is loaded in tension only. The second set of equations shows the relationship between the variables and epsilon two when both tensile and shear loading are applied to the web. Each set of equations is collapsed separately to yield two general equations that relate all variables to the principal strain epsilon two. The combined effect of all parameters involved may then be determined through the use of these equations. The domains over which the examined variables were allowed to vary were determined by the properties of the webs most often encountered in the industry. The reference values assigned to each variable are 10 lbs/in. tension loading, 3.75 lbs/in. shear loading, 0.001 in. thickness, 0.3 Poisson's ratio, 500,000 psi modulus of elasticity, 32 in. width, and 128 in. length model dimensions. The load in the transverse direction is applied midway through the model. Therefore, the effective length is reduced to half its original in this case. Thus, in the case of the general model when shear loading is present, the length of width ratio is two.

Poisson's ratio is allowed to vary over a domain of 0.2 to 0.4. The recorded values for epsilon two for the simple tension loading and the combined tension and shear loading are listed in Table I. Linear regression (general form y = b0 + b1x) curve fitting is employed to reveal the linear relationship between epsilon two and Poisson's ratio in both loading cases (relative error <0.03%). The resulting equations are:

$$\varepsilon_2(\nu) = -1.959968*10^{-6} - 0.0200364\,\nu \qquad (4.1a)$$

when tension is the only applied load, and

$$\varepsilon_2(\nu) = -0.0024081 - 0.0238358\,\nu \qquad (4.1b)$$

when both tension and shear are present. Equation (4.1a) may be approximated by

$$\varepsilon_2(\nu) = -0.0200364\,\nu \qquad (4.2)$$

with no significant error (less than 0.02%). Plots of the relationships in Equations (4.2) and (4.1b) are shown in Figures 5 and 6, respectively.

Poisson's ratio dictates the prewrinkled behavior of the web model primarily. For small Poisson's ratios, larger loads are required to produce large enough lateral contraction to allow wrinkles to form. Large Poisson's ratios result in the formation of wrinkles at an early stage and small loads. In the extreme case, for small enough Poisson's ratios, wrinkles may not be allowed to form even when large loads are applied because of the resulting small lateral contraction. Even

TABLE I

EVALUATION OF PRINCIPAL STRAIN EPSILON TWO
FOR VARYING POISSON'S RATIO

Poisson's Ratio	Epsilon Two: Shear = 0 lbs/in.	Epsilon Two: Shear = 3.75 lbs/in.
0.20	-0.0040092	-0.0071730
0.25	-0.0050110	-0.0083671
0.30	-0.0060129	-0.0095621
0.35	-0.0070150	-0.0107530
0.40	-0.0080163	-0.0119390

though Poisson's ratio is replaced by the "variable Poisson's ratios" λ when an element assumed wrinkled behavior, as shown in Equations (4.1b) and (4.2), the variation of Poisson's ratio has an effect on wrinkled elements due to the influence on the surrounding taut behavior elements. It may be seen by comparing Figures 5 and 6 that larger lateral contraction results when loads are applied in both the longitudinal and transverse directions.

With Poisson's ratio set equal to its reference value of 0.3, the modulus of elasticity of the material is allowed to vary from 100,000 to 500,000 psi, at an increment of 100,000. The principal strain epsilon two is recorded for both the applied loading conditions. These values are listed in Table II. Examination of the recorded values indicates that epsilon two is inversely proportional to the modulus of elasticity. Therefore, stiffening of the material reduces the amount of lateral contraction.

TABLE II

CALCULATED VALUES OF EPSILON TWO FOR VARIOUS MODULI OF ELASTICITY

Modulus of Elasticity (PSI)	Epsilon Two	
	Shear = 0 Lbs/In.	Shear = 3.75 Lbs/In.
100,000	-0.0300630	-0.0478070
200,000	-0.0150310	-0.0239030
300,000	-0.0100210	-0.0159360
400,000	-0.0075315	-0.0119520
500,000	-0.0060129	-0.0095621

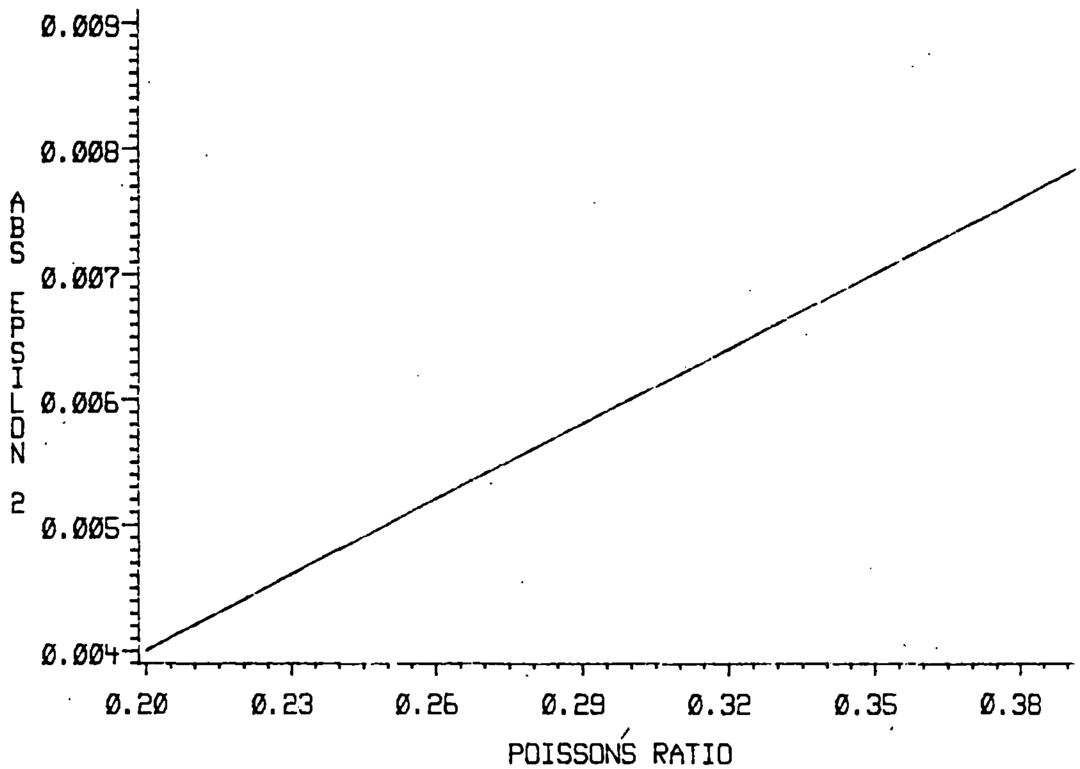

Figure 5. Graph of Poisson's Ratio Effect for Simple Tensile Loading

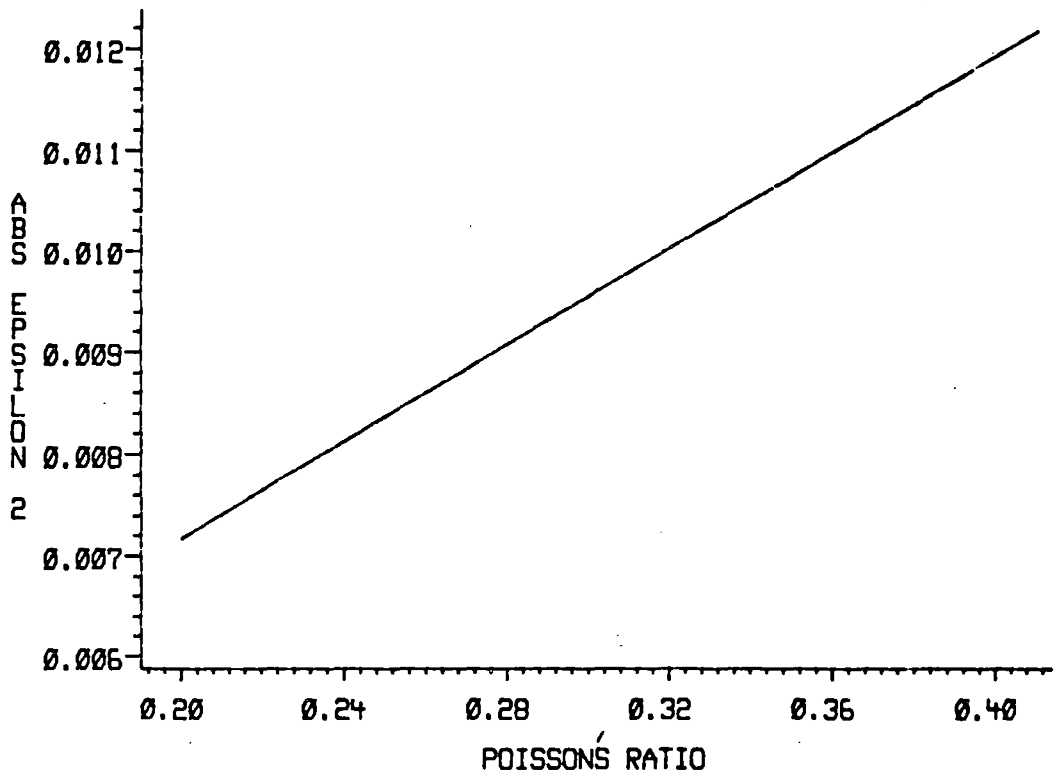

Figure 6. Graph of Poisson's Ratio Effect for Combined Tension and Shear Loading

An inverse X (general form y = (b0x + b1)/x) curve fitting routine is employed to yield a close fit. The resulting equations are:

$$\varepsilon_2(E) = \frac{(-6.749153 * 10^{-6}E - 3005.503)}{E} \tag{4.3a}$$

$$\varepsilon_2(E) = \frac{(-4.762779 \times 10^{-7}E - 4780.63)}{E} \tag{4.3b}$$

Equation (4.3a) is valid when tension is the only applied load, and Equation (4.3b) when both tension and shear forces are applied on the web. Equations (4.3a) and (4.3b) may be approximated by

$$\varepsilon_2(E) = \frac{-3005.503}{E} \tag{4.4a}$$

$$\varepsilon_2(E) = \frac{-4780.63}{E} \tag{4.4b}$$

introducing very small error (less than 0.003%). Plots of Equations (4.4a) and (4.4b) are shown in Figures 7 and 8, respectively. It may be seen that the amount of lateral contraction in the web decreases rapidly for Young's modulus values ranging from 100,000 to 350,000 psi. The rate of decrease in epsilon two becomes smaller beyond the 350,000 psi mark. Even though both plots assume the same shape, the plot in Figure 8 predicts larger lateral contraction due to the presence of shear loading.

In the equations already described and in the ones to follow, it is understood that the constant terms have units associated with them such that the final answer has units of strain (in./in.). Thus in Equation (4.2), for example, the constant term has units of strain (in./in.) since ν being a ratio is unitless.

The thickness of the model becomes a factor in the calculations of the element stiffness matrix as may be seen from Equation (3.1). It may

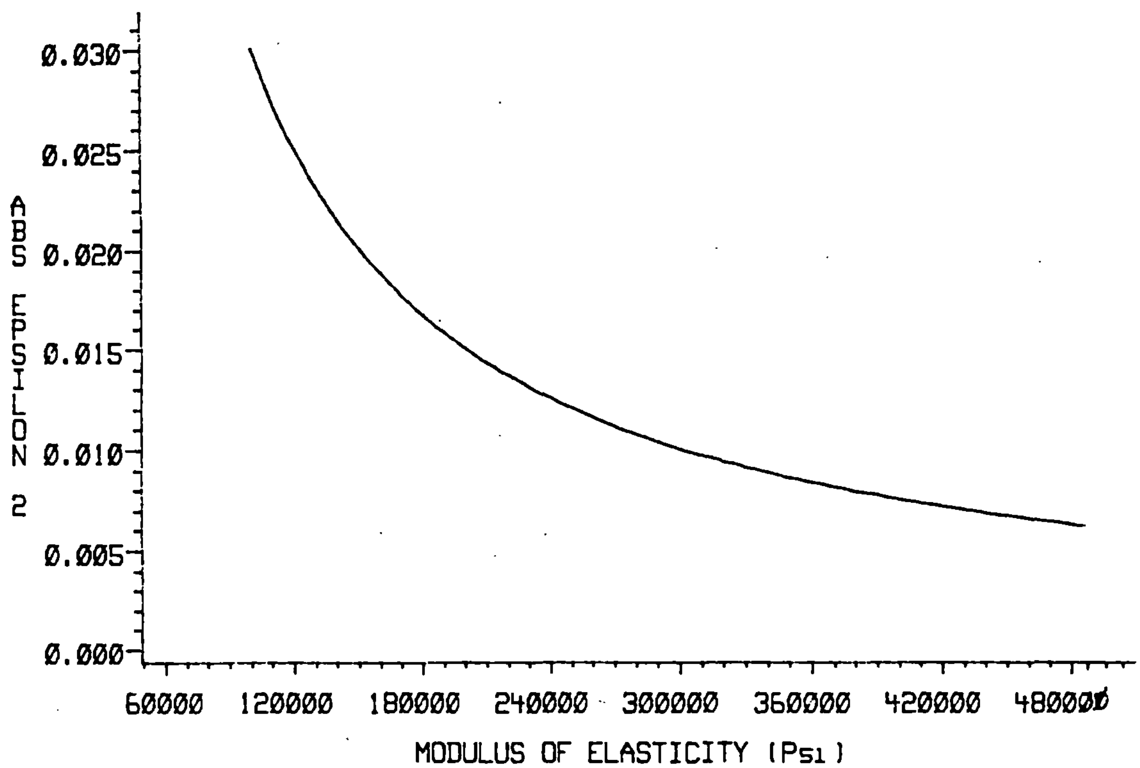

Figure 7. Graph of the Effect of Modulus of Elasticity for Tensile Loading

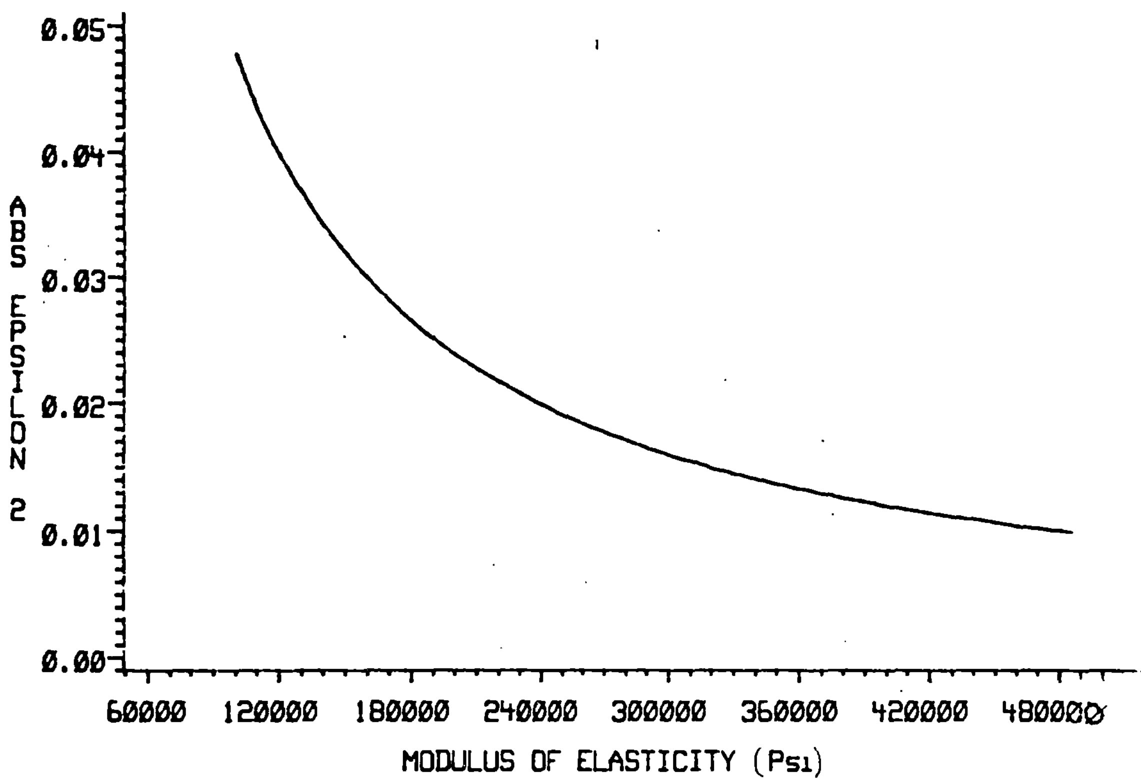

Figure 8. Graph of the Effect of Modulus of Elasticity for Combined Tensile and Shear Forces

be predicted that a decrease in thickness yields larger nodal displacements which in turn yield larger element strains according to Equations (2.5) and (2.8). The thickness of the web is allowed to vary over a range of values from one-tenth of an inch to one ten-thousandth of an inch. The calculated values of the principal strain epsilon two are listed in Table III. It is apparent from the examination of the tabulated values that epsilon two is inversely proportional to the material thickness. Each order of magnitude decrease in thickness results in an order of magnitude increase in epsilon two. This is true for both the tensile loading case and the combined tension and shear loads.

TABLE III

CALCULATED VALUES OF EPSILON TWO FOR VARIOUS MODEL THICKNESSES

Thickness (In.)	Epsilon Two	
	Shear = 0.0 Lbs/In.	Shear = 5 Lbs/In.
0.1000	$-0.60132*10^{-4}$	$-0.11658*10^{-3}$
0.0100	$-0.60130*10^{-3}$	$-0.11656*10^{-2}$
0.0010	$-0.60129*10^{-2}$	$-0.11656*10^{-1}$
0.0001	$-0.60130*10^{-1}$	-0.11657

Curve fitting of the data pairs yields the following equations:

$$\varepsilon_2(t) = \frac{1}{-166300.6\ t} \tag{4.5a}$$

$$\varepsilon_2(t) = \frac{1}{-85777.58\ t} \tag{4.5b}$$

Equation (4.5a) applies when the tension force acts alone and Equation (4.5b) applies when the combination of tension and shear forces is present. The abrupt changes in epsilon two are evident in the plots of Equations (4.5a) and (4.5b) shown in Figures 9 and 10. The reactions of web models to the variation of the material thickness result in the same abrupt changes in epsilon two regardless of how the web is loaded. As in the previous cases, it is obvious that larger lateral contraction results when the model is loaded in both the longitudinal and transverse directions.

The effects of the applied tension load on the lateral contraction of the web is examined. Initially there is no force applied in the lateral direction. The tension force is allowed to vary from 5 lbs/in. to a maximum of 25 lbs/in. For the purpose of better numerical accuracy, the load step is defined at 1 lb/in. in this case. The computer runs are repeated after the force in the transverse direction (shear) is added to the model. The average principal strain epsilon two is recorded for wrinkled elements in the predefined region. The values of epsilon two for both loading conditions are listed in Table IV. The best curve fit for this set of data is linear regression for the simple, tension only loading case, and a second order polynomial in the case of the combination loading in the lateral and transverse directions of the web. The two equations relating the applied tension force to epsilon two are

$$\varepsilon_2(T) = -1.97615*10^{-8} - 6.01292*10^{-4}\ T \tag{4.6a}$$

$$\varepsilon_2(T) = -4.585975*10^{-3} - 1.7423*10^{-4}\ T - 1.4628*10^{-5}\ T^2 \tag{4.6b}$$

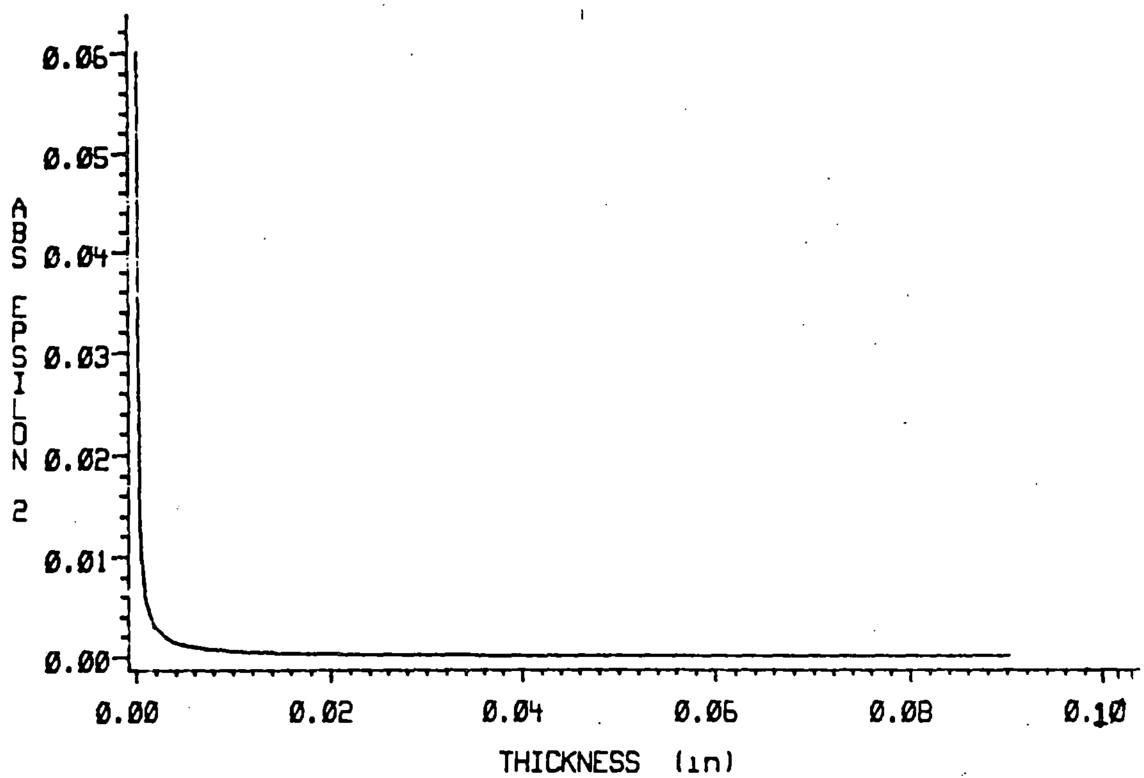

Figure 9. Graph of the Effect of Variable Material Thickness for Simple Tensile Loading

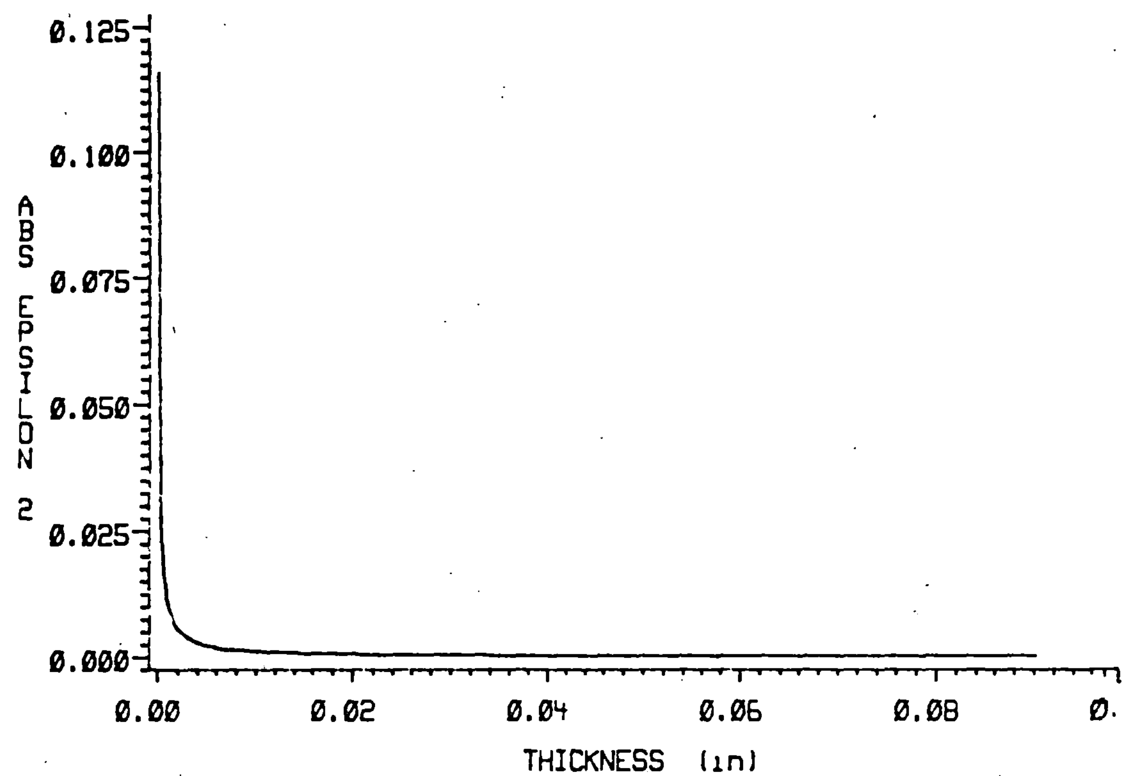

Figure 10. Graph of the Effect of Variable Material Thickness for Combined Tension and Shear Loads

TABLE IV

CALCULATED VALUES OF EPSILON TWO FOR
VARIABLE TENSILE LOADING

Tension (Lbs/In.)	Epsilon Two Shear = 0.0 Lbs/In.	Shear = 2.5 Lbs/In.
5.0	-0.0030065	-0.0058282
7.5	-0.0045097	-0.0066994
10.0	-0.0060129	-0.0078072
12.5	-0.0075162	-0.0090441
25.0	-0.0150320	-0.0134025

Since the constant term in Equation (4.6a) is a very small number, it may be excluded from the equation. Hence

$$\varepsilon_2(t) = -6.01292*10^{-4}\ T \tag{4.7}$$

This approximation introduces negligible error (less than 0.0004%) to the expression. Plots of Equations (4.7) and (4.6b) are shown in Figures 11 and 12, respectively. In the case where there is no load acting in the transverse direction, the relationship is linear (Figure 11). Large applied tension load produces large amounts of lateral contraction. These amounts are sufficient for wrinkles to be formed in the longitudinal direction. Small nonlinearities arise when shear loading is present (Figure 12). However, the overall effect on the lateral contraction of the web, by the variable tension load, remains the same as before (the larger the load the greater the lateral contraction).

In order to further examine the effect the various loading forms have on the lateral contraction of the web, the shear force is allowed to vary while the uniform tension load is held constant at 10 lbs/in. The shear force is induced to the web through a guide roller or a misaligned roll. This load is therefore always smaller than the tension load which is induced to the web by means of driver rollers. The shear load is allowed to vary from 0 lbs/in. to 5 lbs/in. The resultant values of the principal strain epsilon two are listed in Table V. A third order polynomial curve fit is employed to define the relationship between epsilon two and shear for the data pairs in Table V. Curve fittings yield the following expression:

$$\begin{aligned}\varepsilon_2(s) = {} & -6.012328*10^{-3} - 1.075685*10^{-4}\ s \\ & - 2.829943*10^{-4}\ s^2 + 1.575681*10^{-5}\ s^3\end{aligned} \tag{4.8}$$

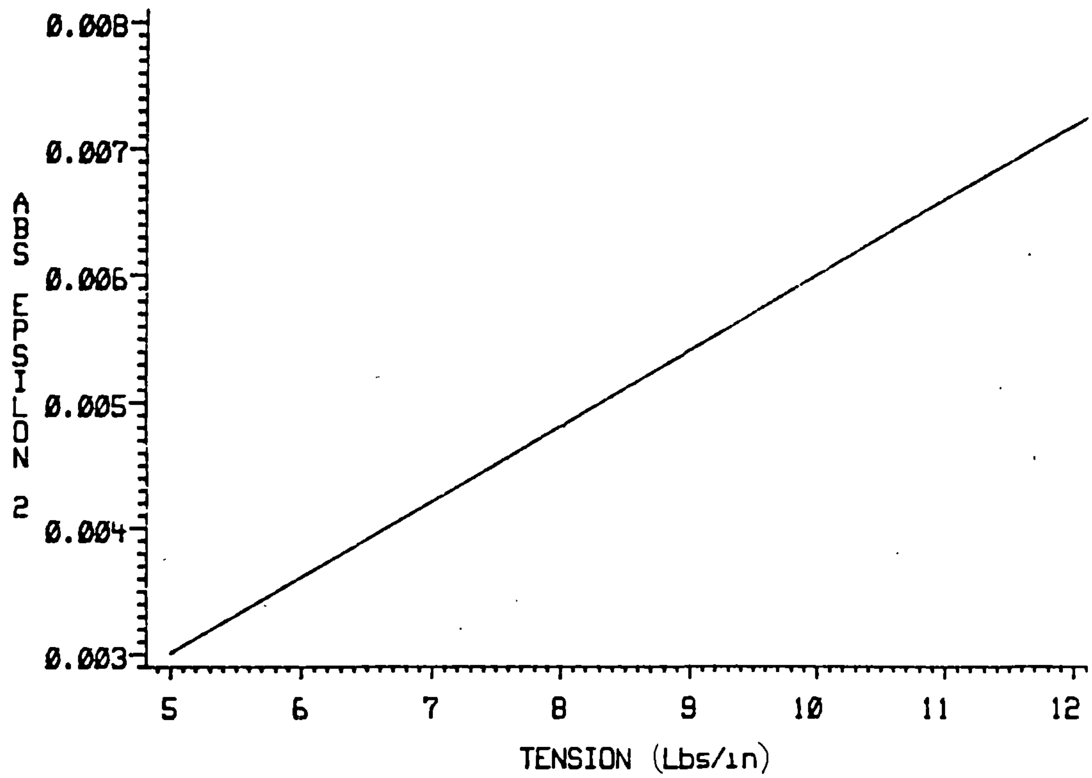

Figure 11. Graph of Equation (3.7) Showing Variations in Epsilon Two Due to Variable Tensile Loading

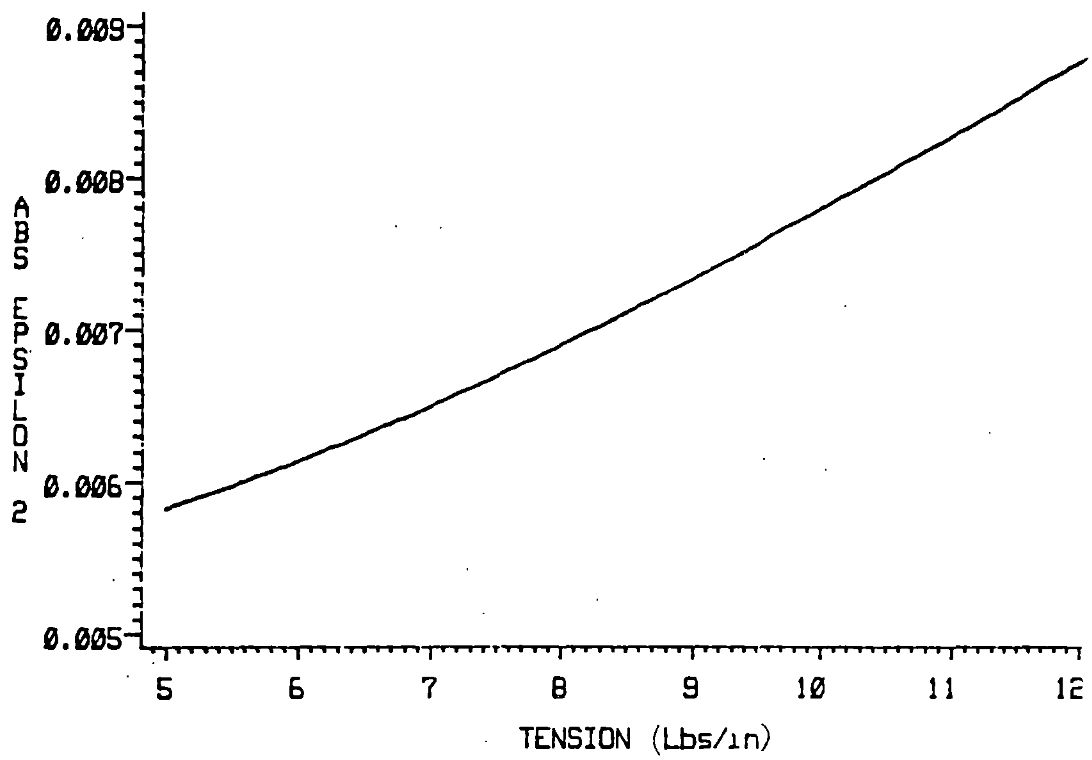

Figure 12. Graph of Variable Tension Force for Combined Model Loading

TABLE V

CALCULATED EPSILON TWO FOR VARIABLE SHEAR FORCE

Shear (Lbs/In.)	Epsilon Two (In./In.)
0.00	-0.0060129
1.25	-0.0065559
2.50	-0.0078072
3.75	-0.0095621
5.00	-0.0116560

A plot of Equation (4.8) is shown in Figure 13. The third order polynomial yields the best fit for the data set (relative error less than 0.004%). The graph assumes the shape of a parabola for values of shear ranging from 0 lbs/in. to 2 lbs/in. For shear force values greater than 2 lbs/in., the relationship approaches that of a linear function.

The dimensions of the web are varied to study the effect on the overall lateral contraction. Various length-to-width ratios are examined to determine the effect of the different dimensions on the principal strain epsilon two. In order to obtain the various length-to-width ratios, either the width of the model is held constant while the length is allowed to vary or the width is varied while the length is held constant. The element density of the model, however, must be held relatively constant during these variations. This is because finite element methods become inaccurate if the model is not adequately dense (nodes placed too far apart). For this reason several new web models are input with nodes placed two inches apart in both x and y directions.

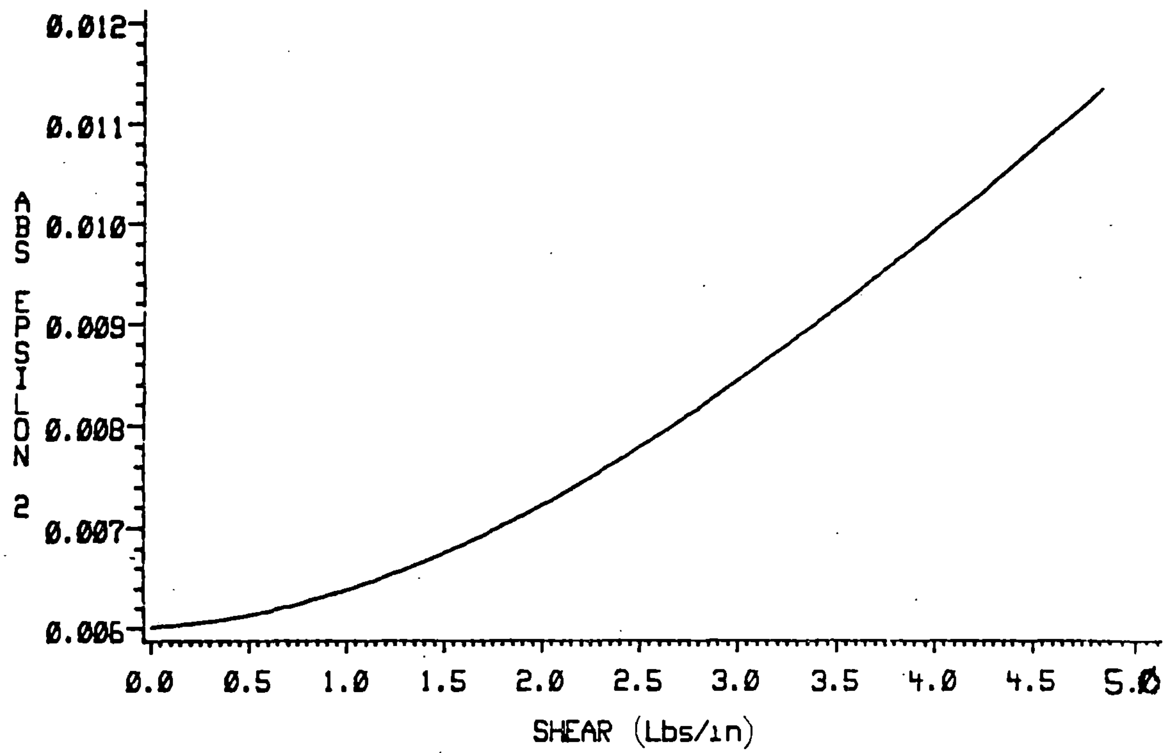

Figure 13. Graph of Variation of Lateral Contraction of Web for Variable Shear Loading

The results of these calculations are recorded for both loading forms. One model is assigned to be the reference model at a length-to-width ratio of two. The predefined region from which the epsilon two value is recorded for a wrinkled element within that region is defined for each separate length-to-width ratio. The ratios of the dimensions of the reference model to the model under investigation are taken to determine the new location of that region. The recorded values of epsilon two are listed in Table VI. Figures 14 and 15 show plots of epsilon two versus various length-to-width ratios.

In the case where tension is the only applied force, the amount of lateral contraction is constant for the various ratios. The amount of lateral contraction is larger in the case of combined loading. Small variations in the epsilon two values are evident in this case. This is due to the nature of the finite element modeling of the web. Better refined models reduce the numerical inaccuracy in large. Relatively large variations had been observed for small ratios (1.0 to 2.0). The use of refined models, however, proved those variations in epsilon two to be erroneous (see Figure 15). The same holds true for length-to-width ratios greater than 3.5. Better refined models eliminate these small variations.

Curve fitting of the data sets in Table VI can only be accomplished through the use of higher order polynomials. This is because of the small variations in epsilon two. However, since the lateral contraction is actually constant, no attempt is made to obtain a relationship.

The contribution of each of the selected variables to the lateral contraction of the web has been examined. Expressions have been derived to relate each individual variable to the principal strain epsilon

TABLE VI

CALCULATIONS OF EPSILON TWO FOR VARIABLE LENGTH-TO-WIDTH RATIOS

L/W Ratio	Epsilon Two	
	Shear = 0.0 Lbs/In.	Shear = 3.75 Lbs/In.
1.000	-0.0061903	-0.0088448
1.250	-0.0061448	-0.0086799
1.375	-0.0061138	-0.0091436
1.500	-0.0061006	-0.0086989
1.625	-0.0060695	-0.0086971
1.750	-0.0060439	-0.0086924
2.000	-0.0060129	-0.0095621
2.286	-0.0060321	-0.0094694
2.667	-0.0060058	-0.0088866
3.167	-0.0059949	-0.0100530
3.500	-0.0060109	-0.0087418
4.000	-0.0060085	-0.0083948
4.250	-0.0059903	-0.0107760
4.500	-0.0059908	-0.0083167

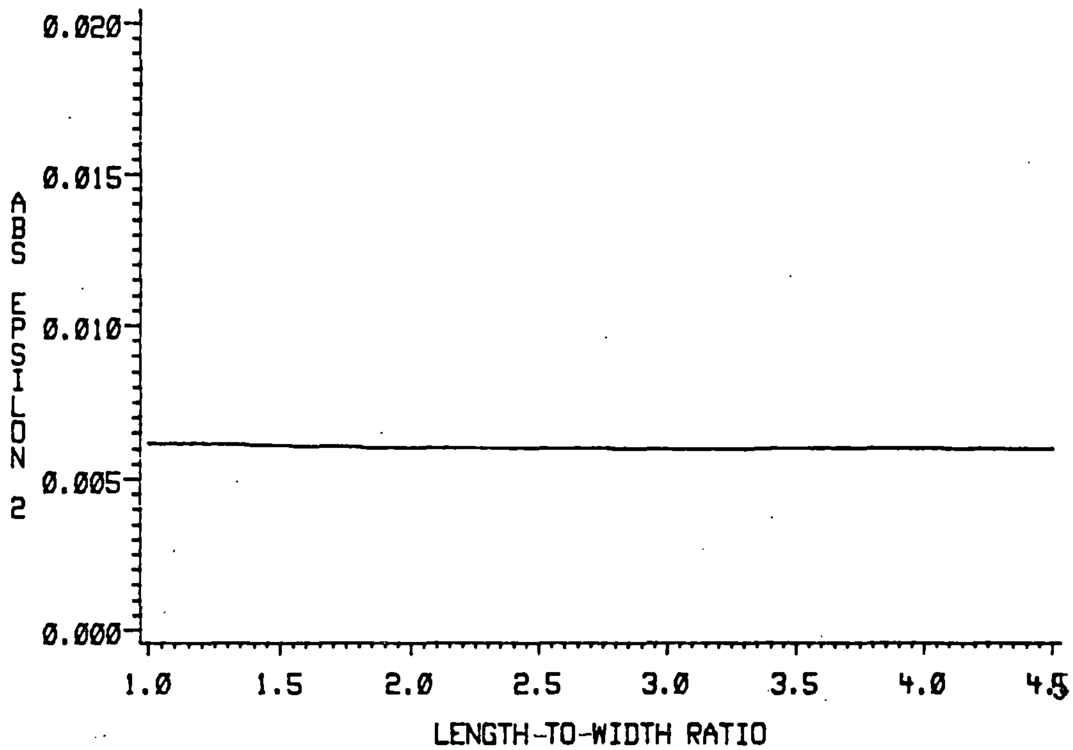

Figure 14. Graph of Length-to-Width Ratio Effect on Epsilon Two for Simple Tension Loading

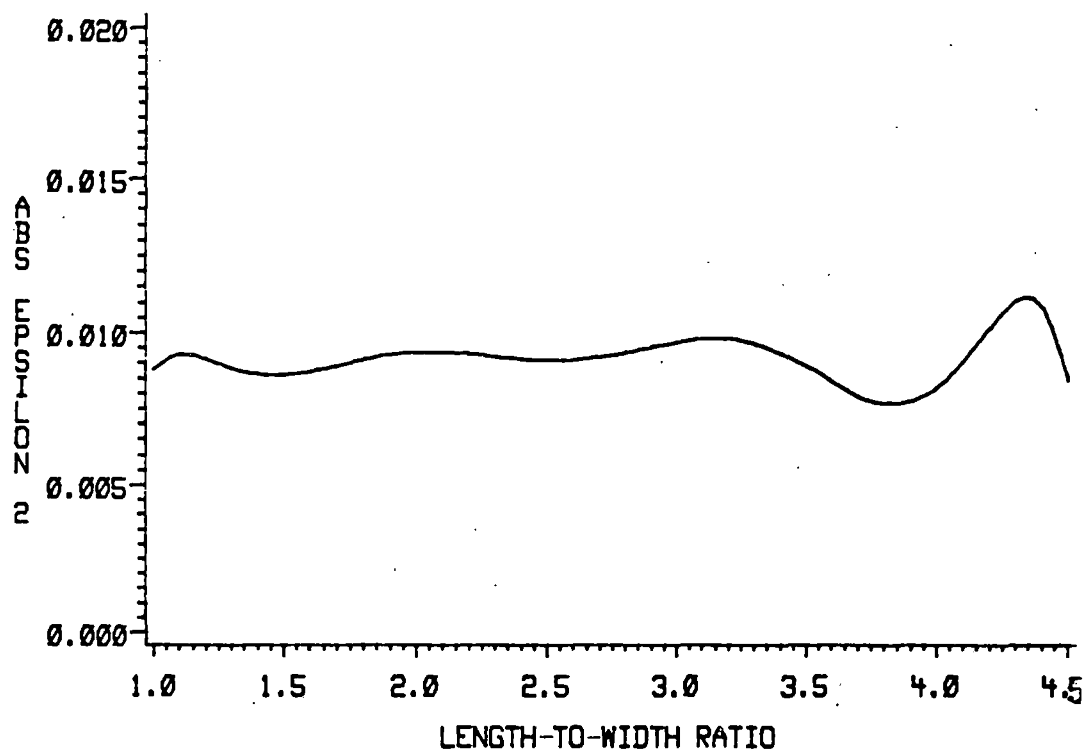

Figure 15. Graph of Variable Length-to-Width Ratio Effect on Epsilon Two for Combined Loading

two. The objective of this study is to derive general expressions to calculate epsilon two. All equations relating each variable to the principal strain epsilon two are combined to yield two equations. The first one calculates epsilon two for the case where there is only tension force applied. The second calculates epsilon two for the combined loading case where both tension and shear forces are applied. The changing dimensions of the web model due to variable length-to-width ratios do not affect the amount of lateral contraction. Therefore, the general expressions do not include length and width terms.

Equations (4.2), (4.4a), (4.5a), and (4.7) calculate epsilon two for the simple tensile loading case. Each of these equations contains no higher order terms. Tension and Poisson's ratio are directly proportional to epsilon two, unlike thickness and modulus of elasticity which are inversely proportional. Therefore, it may be assumed that a general expression combining all four variables may be written as

$$\varepsilon_2(\nu,\ T,\ t,\ E) = K\,\frac{\nu T}{tE} \tag{4.9}$$

where K is a constant of proportionality. Each one of the Equations (4.2), (4.4a), (4.5a), and (4.7) may be written in a general form:

$$\varepsilon_2(t) = -\frac{1}{a\ t} \tag{4.10a}$$

$$\varepsilon_2(\nu) = -b\ \nu \tag{4.10b}$$

$$\varepsilon_2(E) = -\frac{c}{E} \tag{4.10c}$$

$$\varepsilon_2(T) = -d\ T \tag{4.10d}$$

where a = 166300.6 in.$^{-1}$, b = 0.0200364 in./in., c = 3005.503 lbs/in., and d = 601.292*10^{-6} in./lbs. Equating each one of Equations (4.10) to

Equation (4.9) enables a unique solution for the unknown constants:

$$\frac{1}{a} = -k_1 \frac{\nu T}{E} \tag{4.11a}$$

$$b = -k_2 \frac{T}{tE} \tag{4.11b}$$

$$c = -k_3 \frac{\nu T}{t} \tag{4.11c}$$

$$d = -k_4 \frac{\nu}{tE} \tag{4.11d}$$

where $k_1 = -1.00220124$, $k_2 = -1.00182$, $k_3 = -1.00183433$, and $k_4 =$ -1.00215333. The arithmetic average of the four constants yields the constant K since the expressions involved are linear. The general expression of Equation (4.9) becomes

$$\varepsilon_2(\nu,\ T,\ t,\ E) = -1.00200223 \frac{\nu T}{tE} \tag{4.12}$$

which is a general expression to calculate epsilon two when there is no shear force applied and all parameters are within their limiting values. If a unit area is defined as A = t * unit width, Equation (4.12) may be written in an alternative form

$$\varepsilon_2(\nu,\ E,\ \sigma_L) = -1.00200223 \frac{\nu\ \sigma_L}{E} \tag{4.13}$$

where σ_L is the longitudinal stress in the web. The relative error between the calculated epsilon two and that predicted using Equation (4.12) or (4.13) is less than 0.5 percent, which is acceptable for numerical solutions such as this.

In the case of tension and shear forces acting on the web simultaneously, the amount of lateral contraction may be calculated for each individual variable using Equations (4.1b), (4.4b), (4.5b), (4.6b), and

(4.8). The presence of the shear force introduces nonlinearities and higher order terms in the expression for the tension force (Equation (4.6b)). A general expression combining all variables may be written as

$$\varepsilon_2(\nu,\ t,\ E,\ T,\ S) = C\,\frac{K_\nu\ K_S\ K_T}{K_t\ K_E} \tag{4.14}$$

where K_ν, K_E, K_t, K_T, and K_S are values of epsilon two which may be calculated using Equations (4.1b), (4.4b), (4.5b), (4.6b), and (4.8), respectively. C is a constant of proportionality which acts as a "correction factor." Values of the factor C may be extracted from the graph in Figure 16 or evaluated using the expression

$$C = \frac{P}{(1.226584\ P + 0.3770973)} \tag{4.15}$$

P is the variable product times 100. The variable product is defined as

$$\text{Product} = \frac{K_\nu\ K_S\ K_T}{K_t\ K_E} \tag{4.16}$$

The procedure used to obtain the constant K in Equation (4.9) is employed in this case to obtain C. However, nonlinearities are involved in this case and the data set is curve fitted to yield an expression for C in terms of P (product $*\ 10^2$).

A relatively large error is introduced in the case of curve fitting the data set to obtain an expression for the constant C. Hence the total error amounts to 10 percent approximately. This is due, in part, to the nonlinearities involved and the large number of curve fitting.

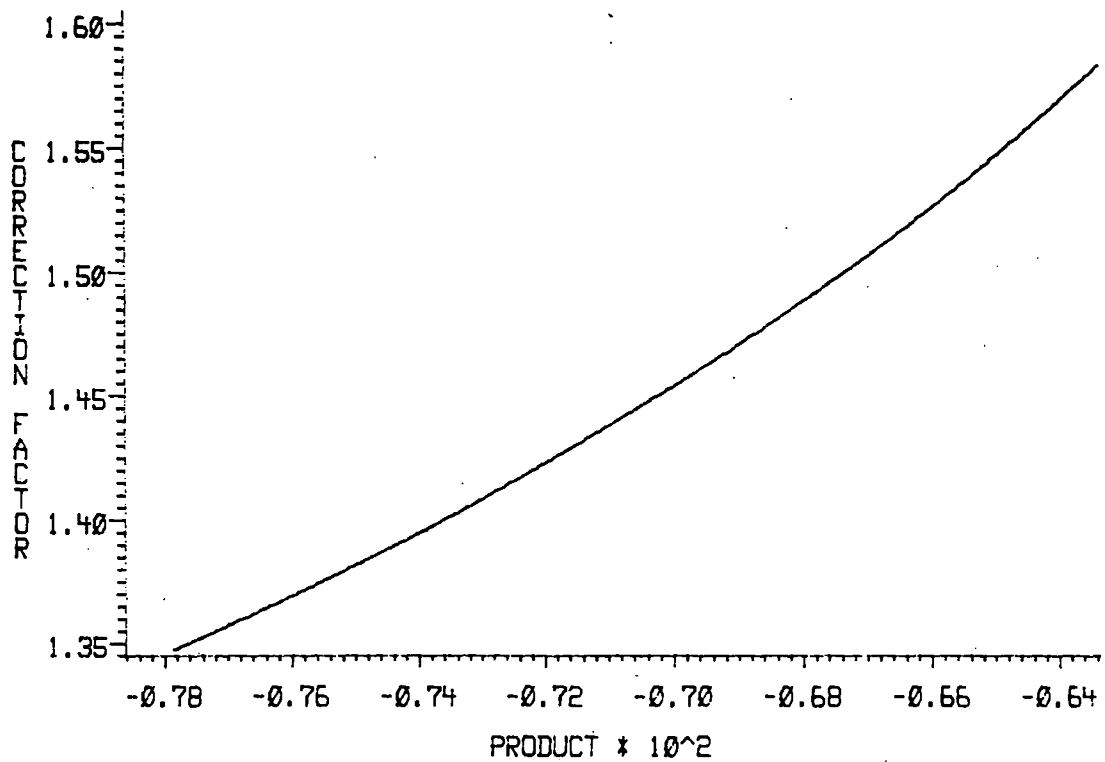

Figure 16. Graph of Correction Factor C
for Equation (4.14)

Equations (4.12), (4.13), and (4.14) calculate the average lateral strain of an element in the model. By definition

$$\varepsilon_2 = \frac{\Delta w}{w} \tag{4.17}$$

Therefore, the total lateral contraction of a web with width w is $\varepsilon_2 * w$.

Figure 17 shows the principal stress directions for each element for a successful run of the general model. The tension load in this run is 4 lbs/in. and the shear force is 2.5 lbs/in. The elements for which directions for both principal stresses are shown have assumed taut behavior. Those with one principal stress direction are wrinkled elements. According to the theory on partly wrinkled membranes (Chapter II), in a wrinkled element one of the principal stresses is zero while the nonzero principal stress acts parallel to the wrinkle. The formation of wrinkles along the diagonal is evident in Figure 17. No element has assumed slack behavior.

4.2 NASTRAN Buckling Analysis

The NASTRAN version of the web model (Figure 4) is subjected to buckling analysis. The objective is to force the model to buckle in order to determine the number of formed wrinkles. This may be accomplished either by counting the stress reversals in the transverse direction or by using the NASTRAN plotting capabilities to plot the various mode shapes and visualize the formed wrinkles. A relationship is to be derived to predict the number of wrinkles as a function of the various length-to-width ratios.

The quadrilateral element CQUAD2 is employed for its ability to model out-of-plane deflections incurred during wrinkling. The boundary

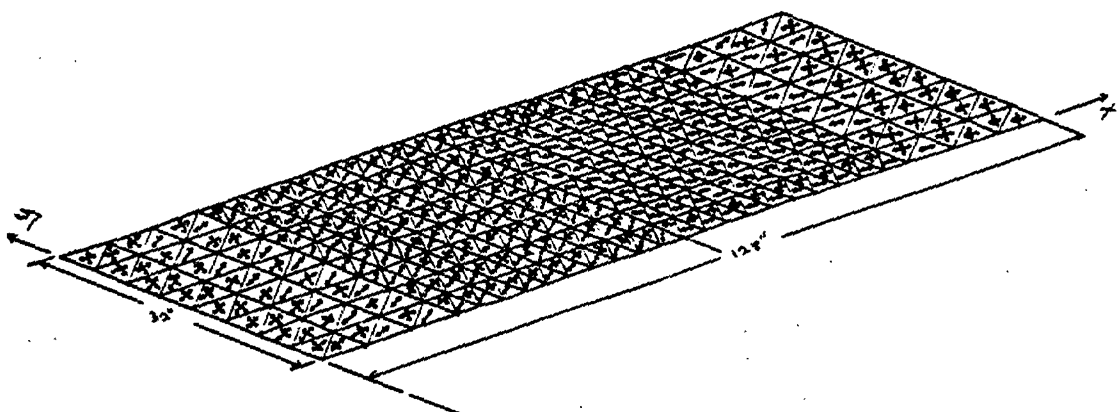

Figure 17. Directions of Principal Stresses for Model Loaded in Both Tension and Compression

conditions in the NASTRAN models remain the same as in the general model consisting of triangular elements. The inverse power method is employed in the analysis for symmetric matrix operations. The observed number of wrinkles for the input length-to-width ratios are listed in Table VII. These data pairs are curve fitted to yield the inverse relationship

$$WR = \frac{(2.3319R + 2.854609)}{R} \tag{4.18}$$

where R denotes the length-to-width ratio. A plot of Equation (4.18) is shown in Figure 18. It may be observed that the number of wrinkles decreases asymptotically to a small number at large ratios (greater than 3.5). Changes in the number of wrinkles formed at large ratios occur slowly and at large intervals (number of wrinkles may drop to $2\frac{1}{2}$ for a ratio of 7.0). Abrupt increases in the number of wrinkles is observed for length-to-width ratios less than 2.0.

TABLE VII

OBSERVED NUMBER OF WRINKLES FOR VARIOUS LENGTH-TO-WIDTH RATIOS

Length-to-Width Ratio	Number of Wrinkles
1.0	5.0
1.5	4.5
2.0	4.0
2.5	3.5
3.0	3.0
3.5	3.0
4.0	3.0
4.5	3.0
5.0	3.0

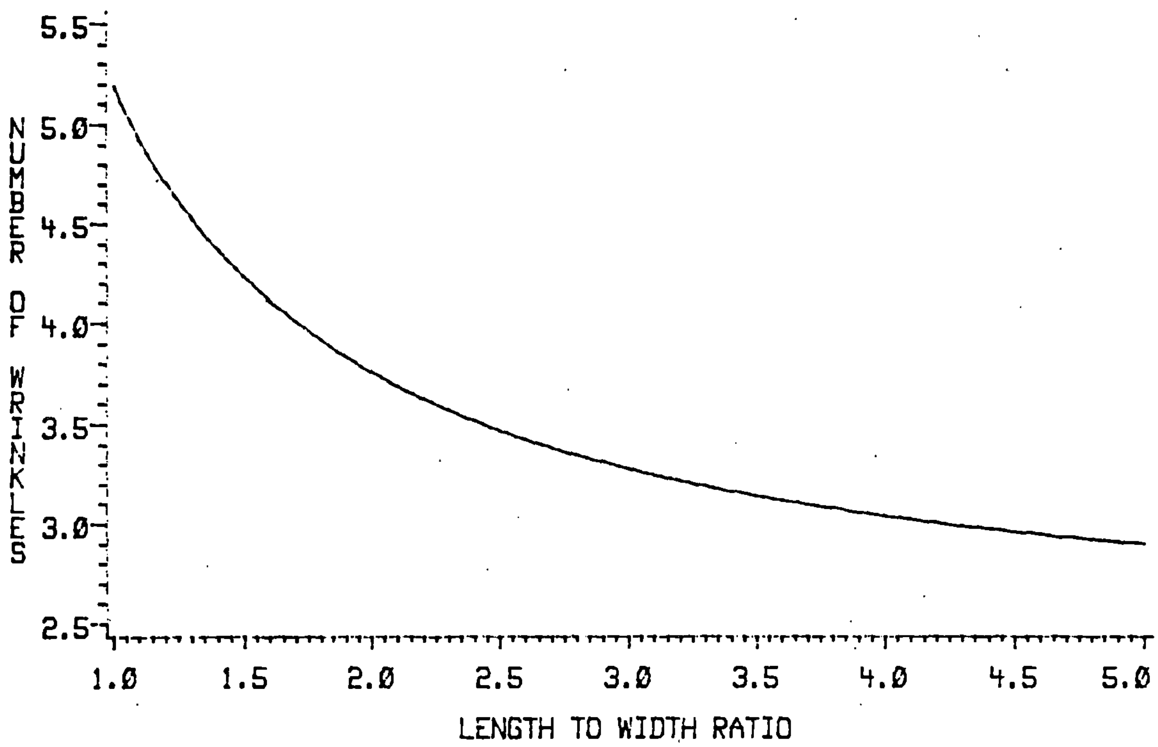

Figure 18. Graph of Number of Wrinkles as a Function of Various Length-to-Width Ratios

Figures 19, 20, and 21 show the results of the buckling analysis. The number of wrinkles formed becomes apparent for the various length-to-width ratios.

4.3 Calculation of the Wrinkle Amplitude

The final task in this study is the calculation of the amplitude of the formed wrinkles. Two assumptions are made to assist this task. The wrinkles are assumed to be sinusoidal in form of constant amplitude. These assumptions are justified through observations of real time web processing and examination of the wrinkles formed during the buckling analysis previously mentioned.

The amplitude of the sine wave may be calculated as follows. The wavelength of the sine wave may be obtained using the expression

$$\rho = \frac{W - \Delta W}{WR} \tag{4.19}$$

where W is the width of the web, ΔW is the overall lateral contraction, and WR is the number of wrinkles in the web. The sine wave takes the form

$$y(x) = A \sin \frac{2\pi}{\rho} x \tag{4.20}$$

where A is the amplitude of the formed wrinkles. The total length of the arcs forming the sine wave may be obtained:

$$W = WR * \int_0^{\rho} [1 + y'^2]^{1/2} \, dx \tag{4.21}$$

The integration of this expression is a difficult task. Therefore, Simpson's rule is applied to obtain a solution. The expression of Equation (4.21) may be written as

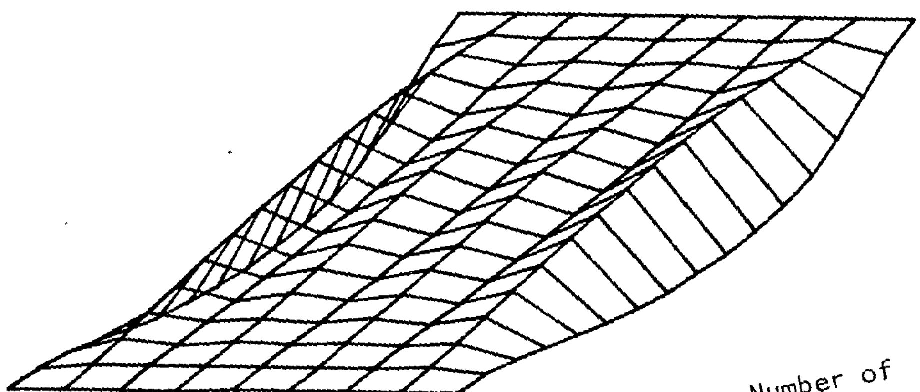

Figure 19. Graph of Mode Shape Showing Number of Wrinkles for Length-to-Width Ratio of 2

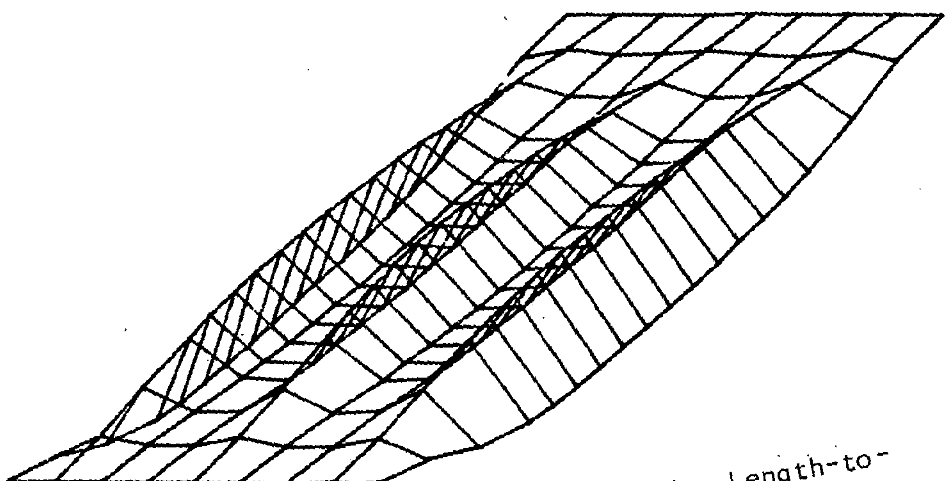

Figure 20. Graph of Mode Shape for Length-to-Width Ratio of 3

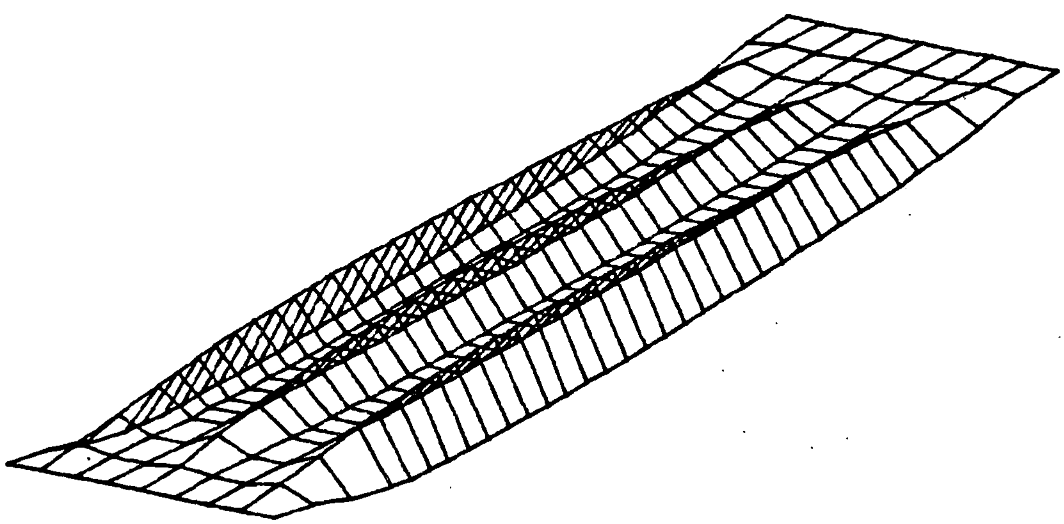

Figure 21. Graph of Mode Shape for Length-to-Width Ratio of 5

$$f(A) = WR * \int_0^{\rho} [1 + y'^2]^{\frac{1}{2}} dx - W \qquad (4.22)$$

The solution of Equation (4.22) is the amplitude of the sine wave. A combination of Simpson's 1/3 and 3/8 rules is employed to numerically integrate the integral part of the expression. A listing of the computer algorithms is presented in the Appendix. The Modified Linear Interpolation scheme is applied to obtain a final solution. For a known number of wrinkles in the web and a known width, a curve may be obtained showing the relationship between the amplitudes and the amount of lateral contraction. One such curve is shown in Figure 22. The width of the web is 32 inches and the number of wrinkles is four. The amplitude of the wrinkles is predicted over a range of contraction from 0.5 to 3.0 inches. For example, for a one-inch contraction of the web, the wrinkle amplitude is predicted to be 0.43 inches.

4.4 Summary

In this chapter the analytical results of this study were presented. The selection of the "important" variables to influence the amount of lateral contraction and how they were examined was presented. Relationships were derived for epsilon two as a function of each individual variable. Plots for each expression were presented for a specified domain of variable values. Two general equations were derived to predict the overall lateral contraction of the web by means of combining the individual variables into one expression.

NASTRAN was employed to perform buckling analysis on the web models. Plots of the various mode shapes were obtained to show the number

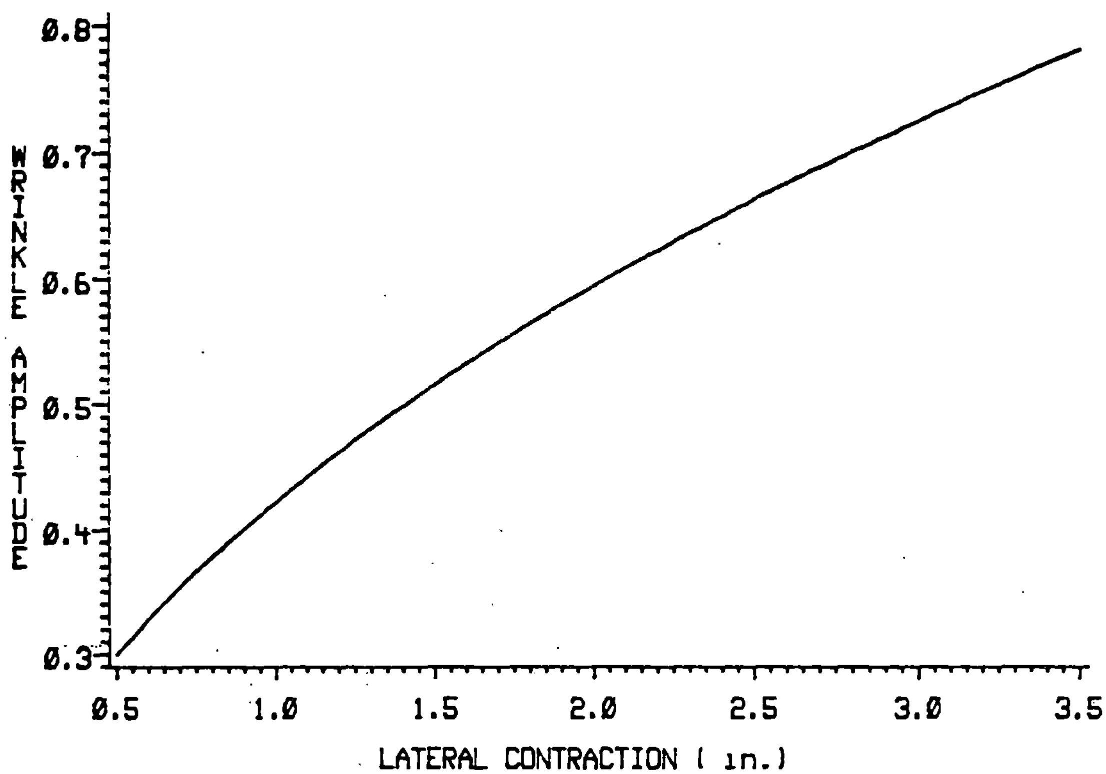

Figure 22. Graph of Wrinkle Amplitude for 32-Inch Web Width and Four Wrinkles

of formed wrinkles. A relationship was obtained to predict the number of wrinkles formed for a given length-to-width ratio.

Finally, an algorithm was developed to predict the amplitude of the wrinkles as a function of the number of wrinkles formed and the overall lateral contraction.

CHAPTER V

SUMMARY AND CONCLUSIONS

5.1 Overview

In this analytical study, finite element methods were employed to study the formation of wrinkles in webs. The effects of several parameters on the amount of lateral contraction of the web was studied, since excessive lateral contraction allows wrinkles to form. The computer code STRESS.FOR was the major instrument of investigation. An element may assume three different behaviors: taut, wrinkled, and slack. The computer code is able to completely analyze the element and most importantly calculate the average lateral strain on it. A "variable Poisson's ratio" λ replaced the classic ratio ν, since it has been determined by Reference [1] that in the wrinkled region ν fails to predict the excessive lateral contraction.

The parameters thought to be of importance and therefore closely examined were the material properties (Poisson's ratio, modulus of elasticity), thickness of the region, various loading conditions (tensile loading, combination of tension and shear forces), and geometry of the web (various length-to-width ratios). The obtained results revealed various relationships between the lateral strain and the examined parameters. In the case where only tensile loading was applied, the lateral strain varied in a linear fashion with the tensile loading and Poisson's ratio. Large tension caused large lateral contraction. For the same loading form, the thickness of the material and Young's modulus were inversely proportional to the lateral strain. When the combined tensile and shear loading form was applied, nonlinearities were introduced to the various relationships. Expressions including higher order terms were obtained. Two different sets of equations were assembled relating the examined parameters to the element lateral contraction, one for each loading form. The equations in each set were combined to form two general expressions. These expressions predict the overall lateral contraction of a web for various parameter values within the domains specified in Chapter IV.

The finite element code NASTRAN was employed to perform buckling analysis on the web model. The capability of this code to calculate out-of-plane displacements allowed the number of wrinkles formed for given length-to-width ratios to be determined. Several models of various ratios were examined. The data set of the recorded number of wrinkles for a given ratio enabled the derivation of an expression relating the two. Thus the number of wrinkles may be predicted analytically for a known length-to-width ratio. This relationship revealed that the number of wrinkles is inversely proportional to the various ratios. The number of formed wrinkles becomes smaller at large length-to-width ratios. The reverse is also true.

Finally, an algorithm was developed for the calculation of the average wrinkle amplitude. The amplitude is dependent on the number of wrinkles and the amount of overall lateral contraction present. The resulting expressions require simple numerical integration and root finding schemes.

5.2 Conclusions Regarding Modeling Characteristics

In this study, the performed analysis of the web assumed a static model. The boundary conditions of the two nonfree ends were as follows. One end was completely fixed in both x and y directions while the opposite end was allowed to translate in the longitudinal direction. These were considered suitable in this study, since it is desirable for the traveling web to ride onto the carrying roller without any relative displacements in the x or y directions. The material of the web was assumed to be of uniform thickness and all thermal effects are neglected. The analysis is restricted to in-plane element interaction.

The physical length of the model was twice its "effective" length. This was implemented to account for the portion of the web traveling toward the guiding roller and enable the application of the tensile loading upstream from it. The model consisted of triangular elements which are orientation independent. Hence no transformation of axes was required. The element density of the model is important for good numerical accuracy. Nodes placed far apart result in inaccurate displacement calculations which in turn affect the overall outcome.

Basic web loading characteristics were incorporated in the model. The tensile loading is induced to the web by the driver roller. The shear forces were applied along the midspan of the model. These forces are the result of the effect of the guiding roller on the web. In the case of a misaligned roller, the forces induced are applied to the web at an angle. Breaking these forces into components in the x and y directions enables the computer code to simulate the effects of the misaligned roll on the web. As is the case with all numerical solutions, an iterative process was employed in the application of the various loads.

The obtained results are accurate and simulate the behavior of the web up to a small distance away from the boundaries (roller surface). The small region before the web comes in contact with the roller surface is a region of high stresses. This is because shear forces can be resisted in that area. The validity of the obtained results in that region was not examined. The study of web behavior over a roller is a problem which cannot be analyzed with the present model. The results of the present study, however, may serve as "boundary conditions" for a study to determine the behavior (failure criteria) of wrinkles as they travel around the roller wrap.

5.3 Recommendations for Future Studies

In future research the boundary conditions should be modified to better simulate the process of the web traveling over the roller surface. The web slides in the lateral direction as it travels over the carrying roller. This may be simulated by defining spring reactions in the lateral direction. The same concept should be applied in both ends with the spring reactions acting opposite to each other in the longitudinal direction.

The computer code STRESS.FOR may be modified easily to accommodate various web behaviors. For example, the "spreading" effect of the curved axis roller may be simulated by defining permanent displacements on the web in the region near and on the roller.

In summary, it is noted that extensive experimental analysis is required to verify all of the derived relationships. Due to the simplistic approach taken in this study, some variables which affect web behavior may have been overlooked.

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APPENDIX

```
C*********************************************
C ----------PROGRAM  STRESS.FOR --------------
C*********************************************
C
C      The computer program STRESS is used to
C  analyze two-dimensional, plane stress
C  elasticity problems.The program includes
C  the possibility of wrinkles being formed
C  within the model due to loading.The
C  program uses only the three node triangular
C  element.
C
C*********************************************
       COMMON/ELMATX/ESM(6,6),X(3),Y(3),D(3,3)
     + ,DT(3,3),DW(3,3),IELR
       COMMON/GRAD/B(3,6),AR2
       COMMON/MTL/EM,PR,TH,NE,NN
       COMMON/TLE/TITLE(20)
       COMMON/VARS/TMAX,TDF,SMAX,DSF,NUM,NUM1
       COMMON/TEMPS/IAN(400),AF(400),
     +  IASN(400),ASF(400)
       COMMON/AV/A(35000),JGF,JGSM,NP,NBW,
     +  IPC(400),PC1(400),NUM2
       COMMON/AA/ARB(500),ARC(500),
     +  DTEMP(500,9),KEL
       DIMENSION NEL(500,3),XC(400),YC(400)
       DIMENSION NS(6),U(6),STRA(3),STRE(6)
     +  ,ICK(500)
       DIMENSION AEC(500),AEB(500)
        CHARACTER DN,D1
C
C*********************************************
C    DEFINITION OF THE INPUT VARIABLES
C*********************************************
C
C  TITLE AND PARAMETERS
C    TITLE - A DESCRIPTIVE STATEMENT OF THE
C            PROBLEM
C    NN    - NUMBER OF NODES
C    NE    - NUMBER OF ELEMENTS
C
C  MATERIAL PROPERTIES AND THICKNESS
C    EM    - MODULUS OF ELASTICITY
C    PR    - POISSON'S RATIO
C    TH    - THICKNESS OF THE REGION
C
C  NODAL COORDINATES
C    XC(I) - X COORDINATES OF THE NODES IN
C             NUMERICAL SEQUENCE
C    YC(I) - Y COORDINATES OF THE NODES IN
C             NUMERICAL SEQUENCE
C
C  ELEMENT DATA
```

```
C    N         - ELEMENT NUMBER
C    NEL(N,I) - NUMERICAL VALUE OF NODE I
C    NEL(N,J) - NUMERICAL VALUE OF NODE J
C    NEL(N,K) - NUMERICAL VALUE OF NODE K
C
C************************************************
C------ INPUT SECTION OF THE PROGRAM -----------
C************************************************
C
      OPEN(10,FILE='DATA.DAT',STATUS='UNKNOWN')
      OPEN(9,FILE='TEMP.DAT',STATUS='UNKNOWN')
      OPEN(15,FILE='OUT.DAT',STATUS='UNKNOWN')
C
C   INPUT OF THE TITLE CARD AND PARAMETERS
C
      WRITE(6,*) 'ENTER PROJECT TITLE '
      READ(5,3) TITLE
3     FORMAT(20A4)
      WRITE(6,*) 'ENTER NUMBER OF ELEMENTS ?'
      READ(5,*) NE
      WRITE(6,*) 'ENTER NUMBER OF NODES ?'
      READ(5,*) NN
C
C  COMPARISON CHECK OF NN AND NE WITH THE
C   VALUES USED IN THE DIMENSION STATEMENTS
C
      ISTOP = 0
C**** CHECK NUMBER OF NODES ****
      IF (NN.LE.400) GO TO 6
      WRITE(6,10)
10    FORMAT(10X,'NUMBER OF NODES EXCEEDS 400'/
     $ 10X,26HCHECK DIMENSION STATEMENTS/
     $ 10X,20HEXECUTION TERMINATED)
      ISTOP = 1
C
C   CHECK NUMBER OF ELEMENTS
C
6     IF (NE.LE.500) GO TO 1
      WRITE(6,2)
2     FORMAT(10X,'NUMBER OF ELEMENTS EXCEEDS'
     $ ' 500'/10X,'CHECK DIMENSION STATEMENTS'/
     $ 20HEXECUTION TERMINATED)
      ISTOP = 1
1     IF (ISTOP.EQ.1) STOP
C
C   INPUT OF THE MATERIAL PROPERTIES
C
      WRITE(6,*) 'ENTER YOUNGS MODULUS ?'
      READ(5,*) EM
      WRITE(6,*) 'ENTER POISSONS RATIO ?'
      READ(5,*) PR
      WRITE(6,*)'ENTER THICKNESS OF REGION ?'
      READ(5,*) TH
```

```
C --- CORRECT ANY INCORRECT INPUT ---
C
      CALL CORRECT
C
      NP = 2*NN
C
      WRITE(6,*)'ENTER PRINT STEP FOR SHEAR ?'
      READ(5,*) ISTEP
C
C ELEMENT NUMBERS SMALLER THAN THE ONE ENTERED
C     WILL BE EXCLUDED FOR THE OUTPUT
C
      WRITE(6,*)'ENTER LOWEST ELEMENT NUMBER'
      WRITE(6,*)'TO BE INCLUDED IN THE OUTPUT'
      READ(5,*) KEL
C
C  INPUT NODAL COORDINATES FROM FILE DATA.DAT
C
      READ(10,*) (XC(I),YC(I),I=1,NN)
C
C     OUTPUT OF TITLE AND DATA HEADINGS
C
      WRITE(15,4) TITLE,NN,NE
4     FORMAT(1H1////10X,20A4//13X,'NN ='I6/
     $13X,5HNE  =I6)
      WRITE(15,16) EM,PR,TH
16    FORMAT(//10X,'PARAMETER VALUES'
     $/13X,4HEM =,E15.5/13X,4HPR =,
     $ E15.5/13X,4HTH =,E15.5)
C
C  INPUT AND ECHO PRINT OF ELEMENT DATA
C CHECK TO SEE IF THE ELEMENTS ARE IN SEQUENCE
C
      NID = 0
      DO 9 KK = 1,NE
       READ(10,*) N,(NEL(N,I),I=1,3)
       IF ((N-1).NE.NID) WRITE(6,17) N
17    FORMAT(/10X,7HELEMENT,I4,' NOT IN SEQUENCE')
       NID = N
9     CONTINUE
       CLOSE(10)
C
C*********************************************
C     ANALYSIS OF THE NODE NUMBERS
C*********************************************
C
C   INITIALIZATION OF A CHECK VECTOR
C
      DO 20 I = 1,NN
20     ICK(I) = 0
C
C CHECK TO SEE IF ANY NODE NUMBER EXCEEDS NP
C
```

```
      DO 25 I = 1,NE
       DO 30 J = 1,3
        K = NEL(I,J)
        ICK(K) = 1
30     IF (K.GT.NN) WRITE(6,35) J,I,NN
35      FORMAT(/10X,4HNODE,I4,11H OF ELEMENT
     $ ,I4,13H EXCEEDS NN =,I4)
25    CONTINUE
C
C  CHECK TO SEE IF ALL NODE NUMBERS
C     THROUGH NN ARE INCLUDED
C
      DO 40 I = 1,NN
40     IF (ICK(I).EQ.0) WRITE(6,45) I
45    FORMAT(/10X,4HNODE,I4,' DOES NOT EXIST')
C**********************************************
C  CREATION AND INITIALIZATION OF
C    THE STORAGE VECTOR {A}
C**********************************************
C   CALCULATION OF THE BAND WIDTH
C
      IEL = 0
      INBW = 0
      NBW = 0
      DO 50 KK = 1,NE
       DO 55 I = 1,3
55     NS(I) = NEL(KK,I)
      DO 50 I = 1,2
       IJ = I + 1
      DO 50 J = IJ,3
       NB = IABS(NS(I)-NS(J))
       IF (NB.EQ.0) WRITE(6,60) KK
60     FORMAT(/10X,7HELEMENT,I3,' HAS TWO'
     $ ' NODES WITH THE SAME NODE NUMBER')
      IF (NB.LE.NBW) GO TO 50
       INBW = KK
       NBW = NB
50    CONTINUE
       NBW = (NBW+1)*2
       WRITE(6,65) NBW,INBW
65     FORMAT(/10X,12HBANDWIDTH IS,I4,
     $ 11H IN ELEMENT,I4)
C
C  INITIALIZATION OF THE COLUMN VECTOR {A}
C
       JGF = NP
       JGSM = JGF + NP
       JEND = JGSM + NP * NBW
       IF (JEND.GT.35000) GO TO 70
       JL = JEND - JGF
      DO 75 I = 1,JEND
75     A(I) = 0.0
       GO TO 85
```

```
70      WRITE(6,80) JEND
80      FORMAT(10X,'DIMENSION OF {A} VECTOR'
     $   ' HAS BEEN EXCEEDED'/
     $10X,'DIMENSION MUST BE EQUAL TO',E10.0,/
     $          10X,'EXECUTION TERMINATED')
       STOP
C
C*********************************************
C     GENERATION OF THE SYSTEM EQUATIONS
C*********************************************
C
85     DO 86 I = 1,NN
        ASF(I) = 0.0
        IASN(I) = 0.0
        AF(I) = 0.0
        IAN(I) = 0.0
        PC1(I) = 0.0
        IPC(I) = 0
86     CONTINUE
C
C  INITIALIZATION OF [Dw] WRINKLED BEHAVIOR
C     MATRIX
C
       DO 205 I = 1,3
        DO 205 J = 1,3
205     DW(I,J) = 0.0
C
C  GENERATION OF (DT) TAUT BEHAVIOR MATRIX
C
        R = EM /(1.0-PR**2)
        DT(1,1) = R
        DT(2,2) = DT(1,1)
        DT(3,3) = R*(1.0-PR)/2.0
        DT(1,2) = PR*R
        DT(2,1) = DT(1,2)
        DT(1,3) = 0.0
        DT(3,1) = 0.0
        DT(2,3) = 0.0
        DT(3,2) = 0.0
C
        IFLAG2 = 0
        IFLAG = 0
C
C   SET  [DT] EQUAL TO [D]
C
       DO 135 I = 1,3
        DO 135 J = 1,3
135     D(I,J) = DT(I,J)
C
       DO 140 I = 1,NE
        AEC(I) = 0.0
        AEB(I) = 0.0
        ARB(I) = 0
```

```
140    ARC(I) = 0
C
C START GENERATING THE ELEMENT MATRICES
C
145    IFLAG1 = 0
        IFLG = 0
        IELR = 0
        KK = 1
C*********************************************
C  GENERATION OF THE NODAL DEGREES OF FREEDOM
C    RETRIEVAL OF THE NODAL COORDINATES
C*********************************************
C
C     SUBROUTINE CHECK ASSIGNS THE
C      APPROPRIATE [D] MATRIX TO EACH ELEMENT
C
150    CALL CHECK(KK)
C
       DO 155 I = 1,3
        J = NEL (KK,I)
        NS(2*I-1) = J*2-1
        NS(2*I) = J*2
        X(I) = XC(J)
155     Y(I) = YC(J)
C
C     CALCULATION OF ELEMENT MATRICES
C
        CALL ELSTMX(KK)
C
C   DIRECT STIFFNESS PROCEDURE
C
      DO 160 I = 1,6
       II = NS(I)
        DO 160 J = 1,6
         JJ = NS(J) + 1-II
         IF (JJ.LE.0) GO TO 160
         J1=JGSM+(JJ-1)*NP+II-(JJ-1)*(JJ-2)/2
         A(J1) = A(J1) + ESM (I,J)
160   CONTINUE
       KK = KK + 1
       IF (KK.LE.NE) GO TO 150
C*********************************************
C  CALCULATION OF THE ELEMENT STRESS AND
C   STRAIN COMPONENTS AND THE PRINCIPAL
C    STRESS VALUES
C*********************************************
C
       IF (IFLAG2.EQ.0) CALL MODIFY
        IF (IFLAG2.EQ.1) CALL MODIF1
        CALL DCMPBD
        CALL SLVBD
C
        IELR = 1
```

```
         DO 165 KK = 1,NE
C
C  GENERATION OF THE NODAL DEGREES OF FREEDOM
C   RETRIEVAL OF THE NODAL COORDINATES
C
        DO 170 I = 1,3
         J = NEL(KK,I)
         NS(2*I-1) = 2*J-1
         NS(2*I) = 2*J
         X(I) = XC(J)
170      Y(I) = YC(J)
C
C RETRIEVAL OF THE ELEMENT NODAL DISPLACEMENTS
C
        DO 175 I = 1,6,2
         NS1 = NS(I)
         NS2 = NS(I+1)
         U(I) = A(NS1)
175      U(I+1) = A(NS2)
C
C CALCULATION OF THE STRAIN VECTOR
C    STRAIN = [B] [U]
C
        CALL ELSTMX(KK)
         DO 180 I =1,3
          STRA(I) = 0.0
           DO 180 K=1,6
180       STRA(I) = STRA(I)+B(I,K)*U(K)/AR2
C
C CALCULATION OF THE STRESS VECTOR
C       STRESS = [D] [STRAIN]
C
        CALL CHECK(KK)
        DO 185 I=1,3
         STRE(I) = 0.0
         DO 185 K = 1,3
185      STRE(I) = STRE(I)+D(I,K)*(STRA(K))
C
C  CALCULATION OF THE PRINCIPAL STRESSES
C
       AA = (STRE(1) +STRE(2))/2.0
       AD=((STRE(1)-STRE(2))/2.0)**2+STRE(3)**2
       AB = SQRT(AD)
       S1 = AA + AB
       S2 = AA - AB
       W1 = STRA(1) + STRA(2)
       W2 = (STRA(1) - STRA(2))**2 + STRA(3)**2
       E1 = (W1 + SQRT(W2)) / 2.0
       E2 = (W1 - SQRT(W2)) / 2.0
       IFLAG2 = 1
       IFLAG = 1
       TM = AB
       IF (ABS(STRA(1)-STRA(2)).LT.0.0001) GO TO 190
```

```
      AC = ATAN2(STRA(3),STRA(1)-STRA(2))
      THM = ((180.0/3.14159265)*AC)/2.0
      GO TO 195
190   THM = 90.0
C
195    ARB(KK) = ARC(KK)
       AEB(KK) = AEC(KK)
C
       AEC(KK) = E1
C
       IF (ARC(KK).EQ.2.0) THEN
        S1 = EM * E1
        S2 = 0.0
       END IF
C
       ARC(KK) = 0
       IF (E1.LT.0) ARC(KK) = 1
       IF (E1.GT.0.AND.PR*E1.LT.-E2) ARC(KK)=2
C
C  DEFINE [Dw] WRINKLED BEHAVIOR MATRIX
C
       IF (ARC(KK).NE.2.0) GO TO 215
C
       PP = (STRA(1) - STRA(2)) / ( E1-E2 )
       QQ = STRA(3) / ( E1-E2 )
C
C
       DW(1,1) = EM * (1+PP)/ 2.0
       DW(2,2) = EM * (1-PP)/ 2.0
       DW(3,3) = EM / 4.0
       DW(1,3) = QQ * EM / 4.0
       DW(2,3) = DW(1,3)
       DW(3,1) = DW(1,3)
       DW(3,2) = DW(2,3)
C
       M = 1
       I = 1
       J = 1
210    DTEMP(KK,M) = DW(I,J)
       J = J + 1
       M = M + 1
       IF (J.GT.3) THEN
        I = I + 1
        J = 1
       ENDIF
      IF (M.GT.9) GO TO 215
      GO TO 210
215   WRITE(9,*) KK,STRA(1),STRE(1),S1,STRA(2)
     $,STRE(2),S2,STRA(3),STRE(3),TM,E1,E2,THM
C
165   CONTINUE
C
C  CHECK FOR SIMILAR PREVIOUS @ CURRENT
```

```
C      [D] MATRICES
C
       DO 220 KK = 1,NE
        IF (ARC(KK).NE.2.0) GO TO 220
       IF(ABS(ARB(KK)-ARC(KK)).EQ.0.) GO TO 220
        IFLAG1 = 1
220     CONTINUE
C
C    ADJUST AND RETURN IF THE ABOVE IS FALSE
C
        IF (IFLAG1.EQ.1) THEN
         REWIND(9)
         CALL LOAD
         GO TO 145
        ENDIF
C
C    CONVERGE ON PRINCIPAL STRAIN E1
C     WHEN WRINKLES ARE PRESENT
C
        DO 225 KK = 1,NE
         IF (ARC(KK).NE.2.0) GO TO 225
       IF(ABS(AEB(KK)-AEC(KK)).LE.0.0001) THEN
          GO TO 225
       END IF
        IFLG = 1
225     CONTINUE
C
C   ADJUST AND RETURN IF NO CONVERGENCE
C
        IF (IFLG.EQ.1) THEN
          REWIND(9)
          CALL LOAD
          GO TO 145
        ENDIF
C
C   IF TMAX IS REACHED @ SHEAR IS ZERO
C      THEN PRINT RESULTS
C
       IF (ABS(AF(2)).GE.ABS(TMAX)
      $   .AND.ASF(2).EQ.0.0) THEN
        REWIND(9)
        CALL PRINTR(ASF(2),TMAX,NE)
       ENDIF
C
       IF (ASF(2).EQ.0.0) GO TO 228
C
C   PRINT INTERMEDIATE RESULTS FOR EVERY
C    ISTEP INCREASE IN SHEAR LOADING
C
       IF (INT(ASF(2))/ISTEP*ISTEP
      $.EQ.INT(ASF(2))) THEN
        REWIND(9)
        CALL PRINTR(ABS(ASF(2)),TMAX,NE)
```

```
      ENDIF
C
228   IF(ABS(AF(2)).GE.ABS(TMAX)) GO TO 230
C
C
      DO 235 I = 1,NUM
       IB = IAN(I)
       BV = AF(I) + TDF
      IF(I.EQ.1.OR.I.EQ.NUM) THEN
        BV = (AF(I)*2 + TDF)/2.0
      END IF
       AF(I) = BV
235    CONTINUE
       CALL LOAD
       REWIND(9)
       GO TO 145
C
230   IF(ABS(ASF(2)).GE.ABS(SMAX)) STOP
C
      DO 240 I = 1,NUM1
       IB = IASN(I)
       BV = ASF(I) + DSF
       IF (I.EQ.1.OR.I.EQ.NUM1) THEN
        BV = (ASF(I)*2 + DSF) / 2.0
       END IF
        ASF(I) = BV
240   CONTINUE
       CALL LOAD
       REWIND(9)
       GO TO 145
      END

C*********************************************
      SUBROUTINE PRINTR(ASF,AF,NE)
C*********************************************
      COMMON/AA/ARB(500),ARC(500),
     $ DTEMP(500,9),KEL
      COMMON/AV/A(35000),JGF,JGSM,NP,NBW,
     $ IPC(400),PC1(400),NUM2
      COMMON/TLE/TITLE(20)
C
       WRITE(15,320) TITLE
320    FORMAT(1H1///10X,20A4)
C
       WRITE(15,300) AF,ASF
300    FORMAT(//7X,14HTENSION LOAD =,E15.6,
     $  5X,14H SHEAR LOAD = ,E15.6/)
C
       WRITE(15,301)
301    FORMAT(//10X,'NODAL DISPLACEMENT'
     $ ' VALUES'/)
       NN = NP/2
       DO 303 I = 1,NN
```

```
         WRITE(15,302) I,A(I*2-1),A(I*2)
302      FORMAT(11X,'NODE',I4,3X,
     $   ' X=',E15.6,3X,' Y=',E15.6)
303     CONTINUE
C
315    READ(9,*) I,SSTRA1,SSTRE1,SS1,SSTRA2,
     $SSTRE2,SS2,SSTRA3,SSTRE3,TTM,EE1,EE2,TTHM
C
       IF (I.LT.KEL)THEN
        I = I + 1
        GO TO 315
       END IF
C
        WRITE(15,305) I
305     FORMAT(/10X,7HELEMENT,I4)
C
        IF (ARC(I).EQ.0.0) THEN
         WRITE(15,306)
306      FORMAT(10X,'TAUT BEHAVIOR [D] MATRIX'/)
        ENDIF
C
       IF (ARC(I).EQ.1.0) THEN
        WRITE(15,307)
307    FORMAT(10X,'SLACK BEHAVIOR [D] MATRIX'/)
       ENDIF
C
       IF (ARC(I).EQ.2.0) THEN
        WRITE(15,308)
308    FORMAT(10X,'WRINKLED BEHAVIOR'
     $' [D] MATRIX'/)
       ENDIF
C
       WRITE(15,310) SSTRA1,SSTRE1,SS1,SSTRA2,
     $SSTRE2,SS2,SSTRA3,SSTRE3,TTM,EE1,EE2,TTHM
310    FORMAT(15X,5HEXX =,E12.5,5X,5HSXX =,
     $ E12.5,5X,5HS1  =,E12.5/15X,5HEYY =,
     $ E12.5,5X,5HSYY =,E12.5,5X,5HS2  =,
     $E12.5/15X,5HGXY =,E12.5,5X,5HTXY =,E12.5,
     $ 4X,6HTMAX =,E12.5/15X,5HE1 = ,E12.5,5X,
     $ 5HE2 = ,E12.5,5X,7HANGLE =,F6.2,4H DEG/)
       I = I + 1
       IF (I.LE.NE) GO TO 315
C
       RETURN
       END

C*********************************************
        SUBROUTINE ELSTMX(KK)
C*********************************************
        COMMON/MTL/EM,PR,TH,NE,NN
        COMMON/GRAD/B(3,6),AR2
        COMMON/ELMATX/ESM(6,6),X(3),Y(3),D(3,3)
     $       ,DT(3,3),DW(3,3),IELR
```

```
         DIMENSION C(6,3)
C
C    GENERATION OF THE B MATRIX
C
       DO 400 I=1,3
        DO 400 J=1,6
400       B(I,J) = 0.0
          B(1,1) = Y(2)-Y(3)
          B(1,3) = Y(3) - Y(1)
          B(1,5) = Y(1) - Y(2)
          B(2,2) = X(3) - X(2)
          B(2,4) = X(1) - X(3)
          B(2,6) = X(2) - X(1)
          B(3,1) = B(2,2)
          B(3,2) = B(1,1)
          B(3,3) = B(2,4)
          B(3,4) = B(1,3)
          B(3,5) = B(2,6)
          B(3,6) = B(1,5)
          AR2 = X(2)*Y(3)+X(3)*Y(1)+X(1)*Y(2)-
     $     X(2)*Y(1)-X(3)*Y(2)-X(1)*Y(3)
         IF (IELR.EQ.1) RETURN
C
C MATRIX MULTIPLICATION TO OBTAIN C = [BT] [D]
C
         DO 405 I = 1,6
          DO 405 J = 1,3
          C(I,J) = 0.0
           DO 405 K = 1,3
405        C(I,J) = C(I,J) + B(K,I) * D(K,J)
C
C  MATRIX MULTIPLICATION TO OBTAIN ESM
C  ESM = [BT] [D] [B] = [C] [B]
C
         DO 410 I = 1,6
          DO 410 J = 1,6
          SUM = 0.0
          DO 415 K = 1,3
415       SUM = SUM + C(I,K) * B(K,J)
          ESM(I,J) = SUM*TH/(2.0*AR2)
410       CONTINUE
          RETURN
          END

C***********************************************
       SUBROUTINE DCMPBD
C***********************************************
       COMMON/AV/A(35000),JGF,JGSM,NP,NBW,
     $  IPC(400),PC1(400),NUM2
C
C  DECOMPOSITION OF A BANDED MATRIX INTO AN
C   UPPER TRIANGULAR FORM USING GAUSSIAN
C    ELIMINATION
```

```
C
      NP1 = NP - 1
      DO 500 I = 1,NP1
       MJ = I + NBW - 1
       IF (MJ.GT.NP) MJ = NP
       NJ = I+1
       MK = NBW
       IF ((NP-I+1).LT.NBW) MK = NP -I +1
       ND = 0
      DO 505 J =NJ,MJ
       MK = MK - 1
       ND = ND + 1
       NL = ND + 1
      DO 505 K = 1,MK
       NK = ND + K
       JK = JGSM+(K-1)*NP+J-(K-1)*(K-2)/2
       INL= JGSM+(NL-1)*NP+I-(NL-1)*(NL-2)/2
       INK= JGSM+(NK-1)*NP+I-(NK-1)*(NK-2)/2
       II = JGSM + I
505   A(JK)=A(JK)-A(INL)*A(INK)/A(II)
500   CONTINUE
      RETURN
      END

C**********************************************
      SUBROUTINE SLVBD
C**********************************************
      COMMON/AV/A(35000),JGF,JGSM,NP,NBW,
     $ IPC(400),PC1(400),NUM2
       NP1 = NP - 1
C
C  DECOMPOSITION OF THE GLOBAL FORCE VECTOR
C
      DO 550 I = 1,NP1
       MJ =I + NBW - 1
       IF (MJ.GT.NP) MJ = NP
       NJ = I + 1
       L = 1
      DO 550 J = NJ,MJ
       L=L+1
       IL=JGSM+(L-1)*NP+I-(L-1)*(L-2)/2
550  A(JGF+J)=A(JGF+J)-A(IL)*A(JGF+I)/A(JGSM+I)
C
C  BACKWARD SUBSTITUTION FOR DETERMINATION OF
C    THE NODAL VALUES
C
      A(NP)= A(JGF+NP)/A(JGSM+NP)
      DO 555 K = 1,NP1
       I = NP - K
       MJ = NBW
       IF((I+NBW-1).GT.NP) MJ=NP-I+1
       SUM = 0.0
      DO 560 J = 2,MJ
```

```
        N = I+J-1
        IJ=JGSM+(J-1)*NP+I-(J-1)*(J-2)/2
560     SUM = SUM + A(IJ)*A(N)
555     A(I)=(A(JGF+I)-SUM)/A(JGSM+I)
        RETURN
        END

C**********************************************
       SUBROUTINE CHECK(KK)
C**********************************************
       COMMON/AA/ARB(500),ARC(500),
     $  DTEMP(500,9),KEL
       COMMON/ELMATX/ESM(6,6),X(3),Y(3),D(3,3)
     $   ,DT(3,3),DW(3,3),IELR
C
       IF (IFLAG.NE.1) RETURN
C
       IF (ARC(KK).EQ.0) THEN
        DO 600 I = 1,3
        DO 600 J = 1,3
600      D(I,J) = DT(I,J)
       ENDIF
C
       IF (ARC(KK).EQ.1) THEN
        DO 605 I = 1,3
         DO 605 J = 1,3
605       D(I,J) = 0.0
       ENDIF
C
       IF (ARC(KK).EQ.2) THEN
        M = 1
        I = 1
        J = 1
610     DW(I,J) = DTEMP(KK,M)
        J = J + 1
        M = M + 1
        IF (J.GT.3) THEN
        I = I + 1
        J = 1
        ENDIF
        IF (M.GT.9) GO TO 615
        GO TO 610
615     DO 620 I = 1,3
         DO 620 J = 1,3
620      D(I,J) = DW(I,J)
        ENDIF
        RETURN
        END

C**********************************************
       SUBROUTINE MODIFY
C**********************************************
        COMMON/TEMPS/IAN(400),AF(400),
```

```
     $   IASN(400),ASF(400)
         COMMON/VARS/TMAX,TDF,SMAX,DSF,NUM,NUM1
         COMMON/AV/A(35000),JGF,JGSM,NP,NBW,
     $   IPC(400),PC1(400),NUM2
C
        NUM = 0
        NUM1 = 0
        NUM2 = 0
        SMAX = 0
        DSF = 0
C**********************************************
C  INPUT OF THE NODAL FORCE VALUES
C     NV - NODE NUMBER
C     BV - VALUE OF THE FORCE
C     IB - DEGREE OF FREEDOM OF THE FORCE
C**********************************************
      WRITE(6,90)
90    FORMAT(/3X,'*** STRUCTURE LOADING ***'/)
91    WRITE(6,*)'LOAD STRUCTURE IN X-DIR (Y/N)?'
      READ(5,'(A1)') D1
      IF (D1.EQ.'N') GO TO 109
      IF (D1.EQ.'Y') GO TO 92
      GO TO 91
C
92    WRITE(6,93)
93    FORMAT(/3X,'ENTER DIRECTION OF FORCE
     $ APPLICATION'/
     $  3X,'    XP ---  POSITIVE  X DIRECTION'/
     $  3X,'    XN ---  NEGATIVE  X DIRECTION')
      READ(5,'(A2)') DN
      IF (DN.NE.'XP'.AND.DN.NE.'XN') GO TO 92
C
        IF (DN.EQ.'XP') WRITE(6,94)
        IF (DN.EQ.'XN') WRITE(6,95)
94    FORMAT(/3X,'ENTER NUMBER OF NODES ON WHICH
     $ FORCE'/,3X,'IN THE POSITIVE
     $  X DIRECTION IS TO BE APPLIED ?')
95    FORMAT(/3X,'ENTER NUMBER OF NODES ON WHICH
     $ FORCE'/,3X,'IN THE NEGATIVE
     $  X DIRECTION IS TO BE APPLIED ?')
       READ(5,*) NUM
       WRITE(6,*)'   ENTER NODE NUMBERS ?'
       READ(5,*)(IAN(I),I=1,NUM)
C
98     DO 96 I = 1,NUM
        WRITE(6,97) I,IAN(I)
97      FORMAT(3X,I3,' ---   NODE',I4)
96     CONTINUE
99     WRITE(6,*)'ANY INCORRECT NODES (Y/N) ?'
       READ(5,'(A1)') D1
       IF (D1.EQ.'N') THEN
      WRITE(6,*)'ENTER NUMBER OPPOSITE TO NODE ?'
       READ(5,*) I
```

```
      WRITE(6,*)'ENTER CORRECT NODE NUMBER ?'
      READ(5,*) IAN(I)
      GO TO 98
      END IF
      IF (D1.EQ.'Y') GO TO 100
      GO TO 99
100   DO 101 I = 1,NUM
101    IAN(I) = IAN(I)*2 - 1
102   WRITE(6,*)'ENTER LOAD STEP FOR TENSION FORCE ?'
       READ(5,*) TDF
105    WRITE(6,104) TDF
104    FORMAT(/3X,'IS ',F10.2,' THE CORRECT
     $INCREMENT (Y/N) ?')
      READ(5,'(A1)') D1
      IF (D1.EQ.'Y') THEN
       IF (DN.EQ.'XN') TDF = -TDF
        GO TO 106
      END IF
      IF (D1.EQ.'N') GO TO 102
       GO TO 105
106   WRITE(6,*)'ENTER MAX FORCE IN X-DIR ?'
        READ(5,*) TMAX
        TMAX = ABS(TMAX)
107    WRITE(6,108) TMAX
108   FORMAT(/3X,'IS ',E10.2,' THE CORRECT
     $ MAX FORCE  (Y/N) ?')
       READ(5,'(A1)') D1
       IF (D1.EQ.'Y') GO TO 109
       IF (D1.EQ.'N') GO TO 106
       GO TO 107
C
109   WRITE(6,*)'LOAD STRUCTURE IN Y-DIR (Y/N)?'
       READ(5,'(A1)') D1
        IF (D1.EQ.'N') GO TO 800
        IF (D1.EQ.'Y') GO TO 110
        GO TO 109
C
110     WRITE(6,111)
111    FORMAT(/3X,'ENTER DIRECTION OF FORCE
     $ APPLICATION'/
     $  3X,'   YP ---   POSITIVE Y DIRECTION'/
     $  3X,'   YN ---   NEGATIVE Y DIRECTION')
       READ(5,'(A2)') DN
       IF (DN.NE.'YP'.AND.DN.NE.'YN') GO TO 110
        IF (DN.EQ.'YP') WRITE(6,112)
        IF (DN.EQ.'YN') WRITE(6,113)
112    FORMAT(/3X,'ENTER NUMBER OF NODES ON
     $ WHICH FORCE'/,3X,'IN THE POSITIVE
     $  Y DIRECTION IS TO BE APPLIED ?')
113    FORMAT(/3X,'ENTER NUMBER OF NODES ON
     $ WHICH FORCE'/,3X,'IN THE NEGATIVE
     $  Y DIRECTION IS TO BE APPLIED ?')
        READ(5,*) NUM1
```

```
      WRITE(6,*)'ENTER NODE NUMBERS ?'
      READ(5,*)(IASN(I),I=1,NUM1)
C
117   DO 114 I = 1,NUM1
      WRITE(6,115) I,IASN(I)
115   FORMAT(3X,I3,' ---   NODE',I4)
114   CONTINUE
C
116  WRITE(6,*)'ANY INCORRECT NODES (Y/N)?'
     READ(5,'(A1)') D1
      IF (D1.EQ.'N') THEN
      WRITE(6,*)'ENTER NUMBER OPPOSITE'
      WRITE(6,*)'TO INCORRECT NODE ?'
       READ(5,*) I
      WRITE(6,*)'ENTER CORRECT NODE NUMBER ?'
       READ(5,*) IASN(I)
       GO TO 117
      END IF
      IF (D1.EQ.'Y') GO TO 118
      GO TO 116
C
118   DO 119 I = 1,NUM1
119   IASN(I) = IASN(I)*2
C
120  WRITE(6,*)'ENTER LOAD INCREMENT FOR SHEAR ?'
     READ(5,*) DSF
121   WRITE(6,122) DSF
122   FORMAT(/3X,'IS ',F10.3,' THE CORRECT
     $ INCREMENT (Y/N) ?')
      READ(5,'(A1)') D1
      IF (D1.EQ.'Y') THEN
       IF (DN.EQ.'YN') DSF = -DSF
       GO TO 123
      END IF
      IF (D1.EQ.'N') GO TO 120
      GO TO 121
C
123   WRITE(6,*)'ENTER MAX FORCE IN Y-DIR ?'
       READ(5,*) SMAX
124    WRITE(6,125) SMAX
125    FORMAT(/3X,'IS ',E10.2,' THE CORRECT
     $ MAXIMUM FORCE (Y/N) ?')
      READ(5,'(A1)') D1
      IF (D1.EQ.'Y') GO TO 800
      IF (D1.EQ.'N') GO TO 123
      GO TO 124
C************************************************
C  INPUT OF PRESCRIBED NODAL VALUES AND/OR
C  CONSTRAINED DOF
C    NV - NODE NUMBER
C    BV - THE VALUE OF THE DISPLACEMENT
C    IB - DOF OF THE KNOWN DISPLACEMENT
C************************************************
```

```
800    WRITE(6,801)
801    FORMAT(/3X,'SEQUENCE TO RESTRAIN NODES'/
      $ 3X,'ANY NODES TO BE RESTRAINED  (Y/N) ?')
       READ(5,'(A1)') D1
       IF (D1.EQ.'N') RETURN
        IF (D1.EQ.'Y') GO TO 802
       GO TO 800
C
802    WRITE(6,*)'ENTER NUMBER OF NODES TO BE'
       WRITE(6,*)'RESTRAINED IN THE X-DIRECTION ?'
       READ(5,*) NN1
       IF (NN1.EQ.0) GO TO 807
       WRITE(6,*)'ENTER NODE NUMBERS ?'
       READ(5,*)(IPC(I),I=1,NN1)
C
803     DO 804 I = 1,NN1
         WRITE(6,805) I,IPC(I)
805      FORMAT(3X,I3,' ---   NODE',I4)
804     CONTINUE
806      WRITE(6,*)'ANY INCORRECT NODES (Y/N) ?'
         READ(5,'(A1)') D1
         IF (D1.EQ.'N') THEN
         WRITE(6,*)'ENTER NUMBER OPPOSITE TO'
         WRITE(6,*)'INCORRECT NODE ?'
         READ(5,*) I
          WRITE(6,*)'ENTER CORRECT NODE NUMBER ?'
         READ(5,*) IPC(I)
         GO TO 803
         END IF
         IF (D1.EQ.'Y') GO TO 807
         GO TO 806
C
807      DO 808 I = 1,NN1
808      IPC(I) = IPC(I)*2-1
C
         WRITE(6,*)'ENTER NUMBER OF NODES TO BE'
         WRITE(6,*)'RESTRAINED IN THE Y-DIR ?'
         READ(5,*) NN2
         IF (NN2.EQ.0) THEN
          NUM2 = NN1
          GO TO 815
         END IF
         NUM2 = NN1 + NN2
C
         WRITE(6,*)'   ENTER NODE NUMBERS ?'
         READ(5,*)(IPC(I),I=NN1+1,NUM2)
810      DO 811 I = 1,NN2
         WRITE(6,812) I,IPC(NN1+I)
812      FORMAT(3X,I3,' ---   NODE',I4)
811      CONTINUE
813      WRITE(6,*)'ANY INCORRECT NODES (Y/N) ?'
         READ(5,'(A1)') D1
         IF (D1.EQ.'N') THEN
```

```
      WRITE(6,*)'ENTER NUMBER OPPOSITE TO'
      WRITE(6,*)'INCORRECT NODE ?'
       READ(5,*) I
       I = NN1+I
      WRITE(6,*)'ENTER CORRECT NODE NUMBER ?'
       READ(5,*) IPC(I)
       GO TO 810
       END IF
      IF (D1.EQ.'Y') THEN
      DO 814 I = NN1+1,NUM2
814   IPC(I) = IPC(I)*2
      GO TO 815
      END IF
      GO TO 813
C
C   MODIFICATION OF THE [K] MARTIX DUE TO
C    THE PRESENCE OF RESTRAINTS
C
815   DO 816 I = 1,NUM2
      IB = IPC(I)
      BV = 0
      K = IB -1
      DO 820 J = 2,NBW
      M = IB + J - 1
      IF (M.GT.NP) GO TO 825
      IJ = JGSM + (J-1)*NP+IB-(J-1)*(J-2)/2
      A(JGF+M) = A(JGF+M) - A(IJ)*BV
      A(IJ) = 0.0
825   IF (K.LE.0) GO TO 820
      KJ=JGSM+(J-1)*NP+K-(J-1)*(J-2)/2
      A(JGF+K) = A(JGF+K) - A(KJ)*BV
      A(KJ) = 0.0
      K = K-1
820   CONTINUE
      A(JGF+IB) = A(JGSM+IB)*BV
816   CONTINUE
      RETURN
      END

C**********************************************
      SUBROUTINE MODIF1
C**********************************************
      COMMON/AV/A(35000),JGF,JGSM,NP,NBW,
     $ IPC(400),PC1(400),NUM2
C
      DO 650 I = 1,NUM2
       IB = IPC(I)
       BV = PC1(I)
       K = IB - 1
      DO 655 J = 2,NBW
       M = IB + J - 1
       IF (M.GT.NP) GO TO 660
        IJ = JGSM + (J-1)*NP+IB-(J-1)*(J-2)/2
```

```
          A(JGF+M) = A(JGF+M) - A(IJ)*BV
          A(IJ) = 0.0
660      IF (K.LE.0) GO TO 655
          KJ = JGSM+(J-1)*NP+K-(J-1)*(J-2)/2
          A(JGF+K)=A(JGF+K) - A(KJ)*BV
          A(KJ) = 0.0
          K = K - 1
655      CONTINUE
         A(JGF+IB) = A(JGSM+IB)*BV
650      CONTINUE
         RETURN
         END

C***********************************************
        SUBROUTINE LOAD
C***********************************************
         COMMON/VARS/TMAX,TDF,SMAX,DSF,NUM,NUM1
         COMMON/TEMPS/IAN(400),AF(400),IASN(400)
     $    ,ASF(400)
         COMMON/AV/A(35000),JGF,JGSM,NP,NBW,
     $    IPC(400),PC1(400),NUM2
C
         JEND2 = JGSM + NP * NBW
         DO 700 I = 1,JEND2
700      A(I) = 0.0
C
         DO 702 I = 1,NUM
          IB = IAN(I)
          BV = AF(I)
          A(JGF+IB) = BV
702      CONTINUE
C
         IF (ABS(AF(2)).GE.ABS(TMAX).AND.
     $    ASF(2).EQ.0.0) RETURN
         IF (ABS(AF(2)).GE.ABS(TMAX)) THEN
           IF (SMAX.EQ.0.0) RETURN
           DO 704 I = 1,NUM1
            IB = IASN(I)
            BV = ASF(I)
            A(JGF+IB) = BV
704        CONTINUE
         END IF
         RETURN
         END

C***********************************************
         SUBROUTINE CORRECT
C***********************************************
         COMMON/MTL/EM,PR,TH,NE,NN
C
711      WRITE(6,710) NE,NN,EM,PR,TH
710     FORMAT(/5X,'1 ---  NUMBER OF ELEMENTS
     $      = ',I4,/5X,'2 ---  NUMBER OF NODES
```

```
$          = ',I4,/
$5X,'3 ---  MODULUS OF ELASTICITY = ',E10.4,/
$5X,'4 ---  POISSONS RATIO         = ',F6.4,/
$5X,'5 ---  THICKNESS OF REGION   = ',F10.5,/)
       WRITE(6,*)'TO CHANGE ANY VALUE ENTER NUMBER'
       WRITE(6,*)'OPPOSITE TO SELECTION'
       WRITE(6,*)'TO CONTINUE ENTER THE NUMBER 6'
       WRITE(6,*)'PLEASE ENTER CHOICE ?'
       READ(5,*) CHOICE
C
       IF (CHOICE.EQ.6.0) RETURN
C
       IF (CHOICE.EQ.1.0) THEN
        WRITE(6,*)'ENTER NEW NUMBER OF ELEMENTS ?'
         READ(5,*) NE
         GO TO 711
       END IF
C
       IF (CHOICE.EQ.2.0) THEN
         WRITE(6,*)'ENTER NEW NUMBER OF NODES ?'
         READ(5,*) NN
         GO TO 711
       END IF
C
       IF (CHOICE.EQ.3.0) THEN
        WRITE(6,*)'ENTER NEW YOUNGS MODULUS ?'
        READ(5,*) EM
        GO TO 711
       END IF
C
       IF (CHOICE.EQ.4.0) THEN
        WRITE(6,*)'ENTER NEW POISSONS RATIO ?'
        READ(5,*) PR
        GO TO 711
       END IF
C
       IF (CHOICE.EQ.5.0) THEN
        WRITE(6,*)'ENTER NEW THICKNESS  ?'
        READ(5,*) TH
        GO TO 711
       END IF
C
       GO TO 711
       END
```

```
C*********************************************
C  ----------  PROGRAM  AMPL.FOR  ----------  *
C*********************************************
C
C    THIS PROGRAM CALCULATES WRINKLE AMPLITUDES
C FOR A GIVEN WEB WIDTH AND NUMBER OF WRINKLES
C
       COMMON/FUN/F(10000),X(10000)
       COMMON/PAR/N,H,NUMR,W,DSTEP,OM
       PARAMETER(PI=3.141592654)
       OPEN(9,FILE='DATA.DAT',STATUS='UNKNOWN')
       WRITE(6,*)'ENTER WITH OF THE WEB ?'
       READ(5,*) W
       WRITE(6,*)'GUESS TWO AMPLITUDES ?'
       READ(5,*) G1,G2
      WRITE(6,*)'ENTER MIN @ MAX CONTRACTION ?'
      READ(5,*) DWMIN,DWMAX
      WRITE(6,*)'ENTER CONTRACTION INCREMENT ?'
      READ(5,*) WSTEP
      WRITE(6,*)'ENTER NUMBER OF WRINKLES ?'
      READ(5,*) NUMR
C
C   WAVELENGTH CALCULATION
C
      DW = DWMIN
      WL = (W-DW)/NUMR
      XTOL = 0.001
      FTOL = 0.0001
      NLIM = 50
C
2     OM = 2.0*PI/WL
      H = 0.001
      DSTEP = 0.0
      N = ANINT(WL/H)
      DO 6 I = 1,N
6     F(I) = 0.0
      A1 = G1
      A = A1
      FLAG = 0
5     DO 10 I = 1,N
      X(I) = DSTEP
      YP = OM*A*COS(OM*X(I))
      F(I) = SQRT(1+YP**2)
      DSTEP = H * I
10     CONTINUE
      CALL SIMPS(N,H,RESULT)
      IF (FLAG.NE.0) GO TO 15
      STORE1 = RESULT
      A2 = G2
      A = A2
      DSTEP = 0.0
      FLAG = 1
      GO TO 5
```

```
15     STORE2 = RESULT
       IF (DW .GT. DWMAX) STOP
       F1 = NUMR * STORE1 - W
       F2 = NUMR * STORE2 - W
       XR = 0.0
       I = 0
       CALL MDLNIN(F1,F2,A1,A2,XR,XTOL,FTOL,
      $ NLIM,I)
       WRITE(9,*) DW,XR
        DW = DW + WSTEP
        WL = (W-DW)/NUMR
        GO TO 2
        END
C
       SUBROUTINE MDLNIN (F1,F2,X1, X2, XR,XTOL,
      $ FTOL,NLIM,I)
C
C SUBROUTINE FOR ROOT FINDING BY MODIFIED
C   LINEAR INTERPOLATION
C    PARAMETERS ARE -
C      X1,X2    INITIAL VALUES OF X. F(X) MUST
C                 CHANGE SIGNS AT THESE POINTS
C      XR     RETURNS THE ROOT TO MAIN PROGRAM
C      XTOL,FTOL  TOLERANCE VALUES FOR X AND F(X)
C                 TO TERMINATE ITERATIOS
C      NLIM     LIMIT TO NUMBER OF ITERATIONS
C      I      A SIGNAL OF HOW ROUTINE TERMINATED
C      I = 1     MEETS TOLERANCE FOR X VALUES
C      I = 2     MEETS TOLERANCE FOR F(X)
C      I = -1    NLIM EXCEEDED
C      I = -2 F(X1) NOT OPPOSITE IN SIGN TO F(X2)
C
       COMMON/FUN/F(10000),X(10000)
       COMMON/PAR/N,H,NUMR,W,DSTEP,OM
       LOGICAL PRINT
       PRINT = .TRUE.
       IF (I.NE.0) PRINT = .FALSE.
       IF (F1*F2 .GT. 0) GO TO 50
       FSAVE = F1
       DO 20 J = 1,NLIM
       XR = X2-F2*(X2-X1)/(F2-F1)
       DO 7 I = 1,N
7      F(I) = 0.0
       STP = 0.0
       DO 6 L = 1,N
       X(L) = STP
       TEMP = OM*XR*COS(OM*X(L))
       F(L) = SQRT(1+TEMP**2)
       STP = H*L
6      CONTINUE
       CALL SIMPS(N,H,RESULT)
       FR=RESULT*NUMR-W
       XERR = ABS(X1-X2)/2.0
```

```
      IF (.NOT.PRINT) GO TO 5
      WRITE(6,199) J,XR,FR
199   FORMAT(1H , 13HAT ITERATION ,I4,
     $5H X = ,E12.5,9H, F(X) = ,E12.5)
5     IF (XERR .LE. XTOL) GO TO 60
      IF (ABS(FR) .LE. FTOL) GO TO 70
      IF (FR*F1 .LT. 0 ) GO TO 10
      X1 = XR
      F1 = FR
      IF (FR*FSAVE . GT. 0) F2 = F2/2.0
      FSAVE = FR
      GO TO 20
10    X2 = XR
      F2 = FR
      IF (FR*FSAVE .GT. 0) F1 = F1/2.0
      FSAVE = FR
20    CONTINUE
C  WHEN LOOP IN NORMALLY COMPLETED, NLIM
C    IS EXCEEDED
      I = -1
      WRITE(6,200) NLIM, XR,FR
200   FORMAT(1H0,26HTOLERANCE NOT MET.
     $ AFTER ,I4,15H ITERATIONS X = ,E12.5,
     $ 12H AND F(X) = , E12.5)
      RETURN
C THIS SECTION FOR RETURN WHEN F(X1) AND F(X2)
C   NOT OPPOSITE IN SIGN
50    I = -2
      WRITE(6,201)
201   FORMAT(1H0, 35HFUNCTION HAS SAME SIGN
     $ AT X1 AND X2)
      RETURN
C  THIS SECTION RETURNS AFTER MEETING XTOL
60    I = 1
      WRITE(6,202) J,XR,FR
202   FORMAT(1H0, 19HX TOLERANCE MET IN ,I4,
     $ 18H ITERATIONS. X = ,E12.5,8H F(X) = , E12.5)
      RETURN
C  THIS SECTION RETURNS AFTER MEETING F(X) TOLERANCE
70     I = 2
      WRITE(6,203) J,XR,FR
203   FORMAT(1H0, 19HF TOLERANCE MET IN , I4,
     $18H ITERATIONS. X = ,E12.5, 8H F(X) = ,E12.5)
      RETURN
      END

      SUBROUTINE SIMPS (N,H,RESULT)
C  THIS ROUTINE PERFORMS SIMPSON'S RULE INTEGRATION
C   OF A FUNCTION DEFINED BY A TABLE OF EQUISPACED
C    VALUES
C      PARAMETERS ARE
C        F    ARRAY OF VALUES OF THE FUNCTION
C        N    NUMBER OF POINTS
```

```
C        H    UNIFORM SPACING BETWEEN X VALUES
C
      COMMON/FUN/F(10000),X(10000)
C
C NUMBER OF PANELS = N -1 ; EVEN ?
C
      NPANEL = N -1
      NHALF = NPANEL / 2
      NBEGIN = 1
      RESULT = 0.
      IF ((NPANEL - 2*NHALF) .EQ. 0) GO TO 5
C   NUMBER OF PANELS IS ODD. USE 3/8 RULE ON
C    FIRST THREE PANELS
      RESULT = 3.0*H/8.0*(F(1) + 3.0*F(2) +
     $ 3.0*F(3) + F(4))
      NBEGIN = 4
C
5     RESULT = RESULT + H/3.0*(F(NBEGIN)+
     $ 4.0*F(NBEGIN+1) + F(N))
      NBEGIN = NBEGIN + 2
      IF (NBEGIN .EQ. N) RETURN
C
      NEND = N-2
      DO 10 I = NBEGIN, NEND, 2
10    RESULT = RESULT + H/3.0*(2.*F(I) +
     $ 4.0*F(I+1))
      RETURN
      END
```

VITA

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