A GENERALIZATION OF SOME WELFARE MEASURES IN A MULTI-MARKET FRAMEWORK

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CHAPTER I

INTRODUCTION

The Problem

In recent research considerable attention has focused on the desirability of farm policies from the standpoint of producer and consumer welfare. The tool of analysis has centered upon classical welfare measurements of producer and consumer surplus. Several different approaches for evaluating these surpluses are found in the literature. The first is a partial equilibrium approach offered by Mishan (1968), which showed that the area above a competitive supply curve conditioned by a set of fixed inputs measures returns or quasi rents to fixed production factors when all variable input supplies are perfectly elastic. It can also be demonstrated that consumer surplus in an input market measures quasi rents to producers who use that input. In contrast to this partial equilibrium approach, Anderson (1974) examines welfare from a general equilibrium approach where all other prices in the economy are allowed to vary. More recently, Just and Hueth (1979) examine welfare measures arising from a price distortion in a competitive single-factor single-product vertical sector of the economy. They demonstrate that when a market price within the sector is forcibly altered, total change in sector welfare is given by the producer and consumer surplus change measured from the general equilibrium supply and demand functions of the altered market level.

As a consequence of these results many questions have been raised regarding the relationship of surpluses when horizontal as well as vertical markets exist. Given that multi-product multi-factor firms represent a common situation in the economy, the interpretation of welfare measures in this context is certainly relevant. Indeed, it was suggested by Harberger (1971) that possibilities may exist for measuring the distribution of welfare when markets are horizontally and vertically related. However, within the literature one finds little guidance as to how to proceed and interpret surpluses derived from horizontal and vertical markets. This study is an attempt to resolve this issue. That is, the relationship of surpluses is examined when multi-product and multi-factor conditions occur in a vertical market framework.

Objectives

The general objective of this study is to examine the relationship of surpluses for the case of a sector comprised of a number of interdependent competitive industries, with each industry producing multiple outputs which are sold to other industries or to final consumers, and using a set of fixed inputs and multiple variable inputs purchased from other related industries or from the initial resource suppliers. In this context, the actions of any industry may affect all prices and quantities in the economy. Specifically, the objectives are to:

1. Investigate the interpretation of welfare measures for both horizontally and vertically related markets.

2. Investigate the interpretation of welfare measures derived

from alternative industry supply and demand specifications.

3. Examine the empirical implications of using the theoretical results developed in objectives 1 and 2.

Organization of Remainder of Thesis

In Chapter II, a brief historical sketch of consumer and producer surplus is offered. In Chapter III, Mishan's results are examined when supplies and demands are perfectly elastic. In this chapter, it is demonstrated that producer surplus measures only quasi rents when variable input supplies are perfectly elastic and consumer surplus of an input market measures quasi rents when demands are perfectly elastic. Chapter III also considers total sector welfare in a vertical market framework. In this case it is demonstrated that when supplies and demands are of a general equilibrium nature total welfare of the sector can be found by summing producer and consumer surplus at any industry level. Then, in Chapter IV, the generalization of horizontal and vertical market sector welfare is examined in a general equilibrium framework. Then, in Chapter V welfare measures are examined under alternative supply and demand specifications. Chapter VI examines the empirical implications of using producer and consumer surplus in applied problems. Finally, in Chapter VII the conclusions are presented.

CHAPTER II

THE CONCEPT OF WELFARE MEASUREMENT

The term economic surplus encompasses surpluses which accrue to buyers (consumer surplus) and surpluses which accrue to sellers (producer surplus). In this section, these concepts and their applicability in applied welfare economics are reviewed.

Consumer Surplus

Jules Dupuit (1844) is attributed with the invention of consumers surplus. Dupuit defined consumer surplus as the difference between the sacrifice which the purchase would be willing to make in order to obtain a good and the purchase price he has to pay in exchange. Dupuit claimed that this surplus can be measured by the triangle-like area below the demand curve and above the price line. Marshall (1930) popularized the concept in his Principles and qualified Dupuit's definition with the requirement that the marginal utility of money must be constant. After Dupuit and Marshall, Hicks (1940) redefined the concept of consumer surplus using an ordinal indifference curve following the introduction of the commodity at a particular price. Hicks defined four measures of the change in a consumer's welfare that results from a price change. Using Hicksian terminology, the four measures are:

1. compensating variation - the amount of compensation, paid or received, that will leave the consumer in his initial welfare

position following the change in price if he is free to buy any quantity at the new price.

2. compensating surplus - the amount of compensation, paid or received, that will leave the consumer in his initial welfare position following the change in price if he is constrained to buy at the new price the quantity he would have bought at that price in the absence of compensation.

3. equivalent variation - the amount of compensation, paid or received, that will leave the consumer in his subsequent welfare position in the absence of the price change if he is free to buy any quantity of the commodity at the old price.

4. equivalent surplus - the amount of compensation, paid or received, that will leave the consumer in his subsequent welfare position in the absence of the price change if he is constrained to buy at the old price the quantity he would have bought at that price in the absence of compensation.

These four welfare measures for the case of a price decrease are depicted in Figure 1. To illustrate these measures, assumed the consumer has income of the amount of OI_0. The initial price for the good Y is given by the slope P_0. If the price falls to the slope indicated by P_1, the compensating variation is given by I_0I_1. Compensating surplus is BD, equivalent variation is I_0I_2 and equivalent surplus is AC.

Hicks (1956) also attempted to clarify the conditions in which his four measures coincided with the Marshallian result. An important contribution regarding this issue was the development of the Hicksian compensated demand curve. While the ordinary curve, from which the

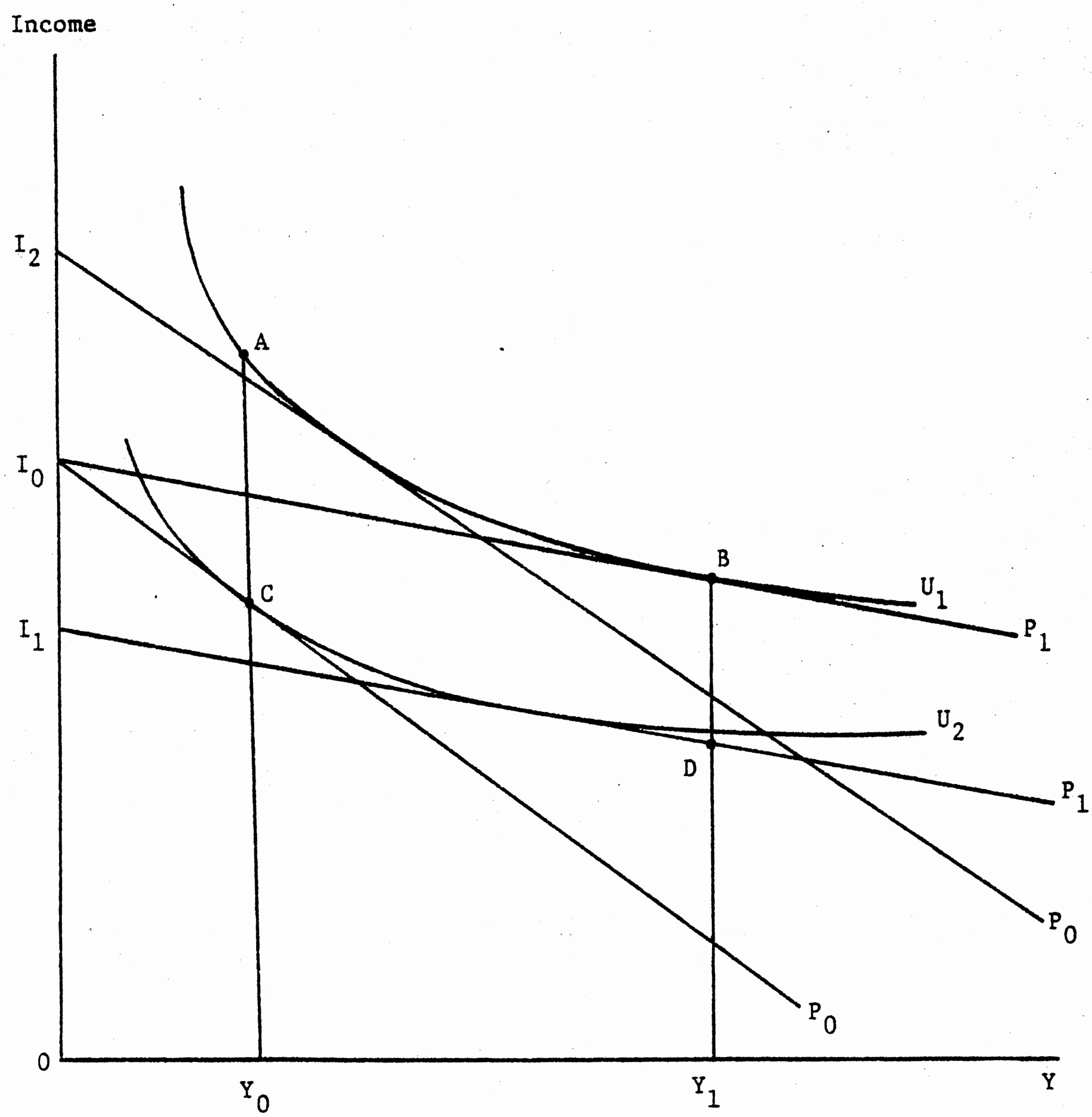

Figure 1. Alternative Welfare Measures for a Consumer Given a Price Decrease.

Marshallian consumer surplus is measured, indicates the quantity that a utility maximizing consumer with a given income level will demand at each price, the compensated demand curve reveals the quantity a consumer will demand at each price, provided that his income is adjusted so that he remains on his initial indifference curve. Hence, an ordinary demand curve reflects a substitution and income effect, whereas a Hicksian demand curve reflects only a substitution effect. As a result of these demand considerations, Hicks noted that all four Hicksian measurements and the Dupuit-Marshallian triangle coincide if the income effect is zero. This was, of course, a great practical implication for applied welfare economists. All that was necessary for consumer surplus to be a valid welfare measurement was the income effect to be small. In fact, Hicks stated that:

> what in the light of this approach, we have been trying to do is to establish, more precisely than Marshall thought necessary, the conditions needed for the Marshall measure (i.e., the relevant area below the ordinary demand curve) to be a good measure. And, so considered, the result of our inquiry is very simple. In order that the Marshall measure of consumer's surplus should be a good measure, one thing alone is needful--that the income effect should be small (p. 177).

Later, the Marshallian consumer surplus began to be viewed with skepticism. As Samuelson (1976) revealed in his Foundations of Economic Analysis in a more general utility framework, whether or not, and to what extent changes represent improvements are dependent upon income and distribution effects. Samuelson indicated that not only are the relative marginal utilities of the affected individuals important, but also the relative weighting attached by society to different individuals should be considered. Subsequent work in welfare theory has reflected this stance.

As a result of Samuelson's criticisms another approach has become popular recently, as evidenced by Willig (1973), Richter (1974) and Bergson (1975). This new approach does not claim to measure social welfare but simply adopts a value judgment that changes should be made or not be made, depending on whether the gainers can bribe the losers to change (the Kaldor-Hicks criterion) or whether the losers can bribe the gainers to forego the change (the Scitovsky criterion). In the Kaldor-Hicks criterion, the appropriate quantitative measure of effects on each individual or group of individuals is the Hicksian compensating variation. With the Scitovsky criterion the appropriate measure is the Hicksian equivalent variation. Willig has shown that these measures have great empirical applicability in a variety of approaches. Willig demonstrated that consumers surplus as measured by an ordinary demand curve is a reasonable approximation of the Hicksian compensating and equivalent variations.

Producer Surplus

The concept of producer surplus was introduced by Marshall (1930). Marshall related the concept of consumer surplus to producers by indicating that a seller as well as a buyer may receive some sort of surplus from a transaction. Marshall indicated that when an individual makes a sale he generally receives something which has a greater direct or indirect utility to him than the item he gives up. Marshall defined producer surplus as the excess of the gross receipts which a producer gets for any of his commodities over their prime cost and used the area above the product supply curve and below the price line as a measure of this surplus.

Currie et al. (1971) have indicated that Marshall's use of the term producer's surplus rather than quasi rents is unfortunate since both relate to the same phenomena. In fact, Mishan (1968) has argued that the term producer's surplus is misleading and should be struck from the economist's vocabulary in favor of the more general concept of quasi rents. Mishan came to this conclusion by noting that producer surplus is symmetrical to quasi rents when factor supplies are perfectly elastic, but overestimates quasi rents when factor supplies are not perfectly elastic. However, Hueth, Just and Schmitz (1980) have demonstrated that when variable input supplies are perfectly elastic, the change in producer surplus or Mishan's equivalent measure of quasi rents is an exact measure of both the producer's compensating and equivalent variations. Furthermore, Just and Hueth (1979) have shown that when variable input supplies are not perfectly elastic, producer surplus measures more than the equivalent and compensating variations of income to producers in the market of interest. But rather, producer surplus measures the initial resource suppliers surplus plus all quasi rents in all industries involved in transforming the initial resource into its present form at the market of interest. In order to examine the reasons why Mishan and Just and Hueth came to these conclusions, it is convenient to expand the analysis into a multi-market framework. By doing so, the relationships between producer and consumer welfare in related markets can also be examined.

CHAPTER III

WELFARE MEASURES IN A VERTICAL MARKET SECTOR

In this chapter, the relationship between producer and consumer surplus and quasi rents are examined. Initially all the assumptions which have been commonly attached to welfare measures are made. That is, perfectly elastic variable input supply and product demand curves (i.e., fixed prices at the firm and industry levels) are assumed. This assumption implies that all supply and demand curves will initially be partial equilibrium curves. Furthermore, as previously indicated by Mishan, producer surplus at any market level will be shown to measure profits plus fixed costs and thus, measures quasi rents to the owners of the fixed production factors.

Producer Surplus At An Intermediate Vertical Market Level

For expository convenience, assume that there are K competitive industries in an industry sector which are so ordered that each industry k produces as an output Y_k, using a single variable factor input Y_{k-1}, which is the output produced at the preceding industry level in the sector with fixed prices P_k, k=1, ..., K and a set of fixed inputs. The indirect profit function for the industry or quasi rents is given by,

$$\pi_k^*(P) = P_k\, Y_k^*(P) - P_{k-1} Y_{k-1}^*(P) \tag{1}$$

This industry profit function is determined by substituting the derived profit maximizing levels of output and input for given prices, $Y_k^*(P)$ and $Y_{k-1}^*(P)$ into the direct or primal profit function.1/

In order to demonstrate the relationship of profits or quasi rents and producer surplus in industry k, observe by the envelope theorem that 2/,

$$\frac{\partial \pi_k}{\partial P_k} = Y_k(P) \tag{2}$$

Since $Y_k(P)$ is the supply curve for industry k when prices P_k and P_{k-1} are fixed (i.e., the partial equilibrium supply curve), the change in profits or quasi rents associated with an output price change from P_k^0 to P_k^1 is given by (3)

$$\Delta\pi_k = \int_{P_k^0}^{P_k^1} \frac{\partial \pi_k}{\partial P_k}\, dP_k = \int_{P_k^0}^{P_k^1} Y_k(P)\, dP_k, \tag{3}$$

where $\Delta\pi_k$ denotes the change in quasi rents for industry k. To interpret (3), note that the far right hand term is the change in producer surplus associated with the output price change of P_k^0 to P_k^1. Hence (3) can be rewritten as,

$$\Delta\pi_k = \Delta PS_k, \tag{4}$$

where ΔPS_k denotes the change in producer surplus. Hence, as Mishan pointed out, the change in producer surplus is equal to the change in profits plus fixed costs or quasi rents to the set of fixed production factors when variable input supply is perfectly elastic. This result can be graphically illustrated in Figure 2. Let Y_k be the supply function given by (2). Now, if P_k is altered from P_k^0 to P_k^1 the shaded area represents the change in producer surplus which is equivalent to quasi rents given by (4).

Consumer Surplus At An Intermediate

Vertical Market Level

In order to demonstrate the relationship between quasi rents in industry k and consumer surplus in industry k-1 when supply and demand are perfectly elastic, observe that by the envelope theorem one can also obtain from (1),

$$\frac{\partial \pi_k}{\partial P_{k-1}} = -Y_{k-1}(P), \tag{5}$$

which is the input demand function for industry k. Now, the change in quasi rents for industry k for an input price change of P_{k-1}^0 to P_{k-1}^1 is,

$$\Delta\pi_k = \int_{P_{k-1}^0}^{P_{k-1}^0} \frac{\partial \pi_k}{\partial P_{k-1}} dP_{k-1} = \int_{P_{k-1}^0}^{P_{k-1}^1} -Y_{k-1}(P)\, dP_{k-1} \tag{6}$$

or,

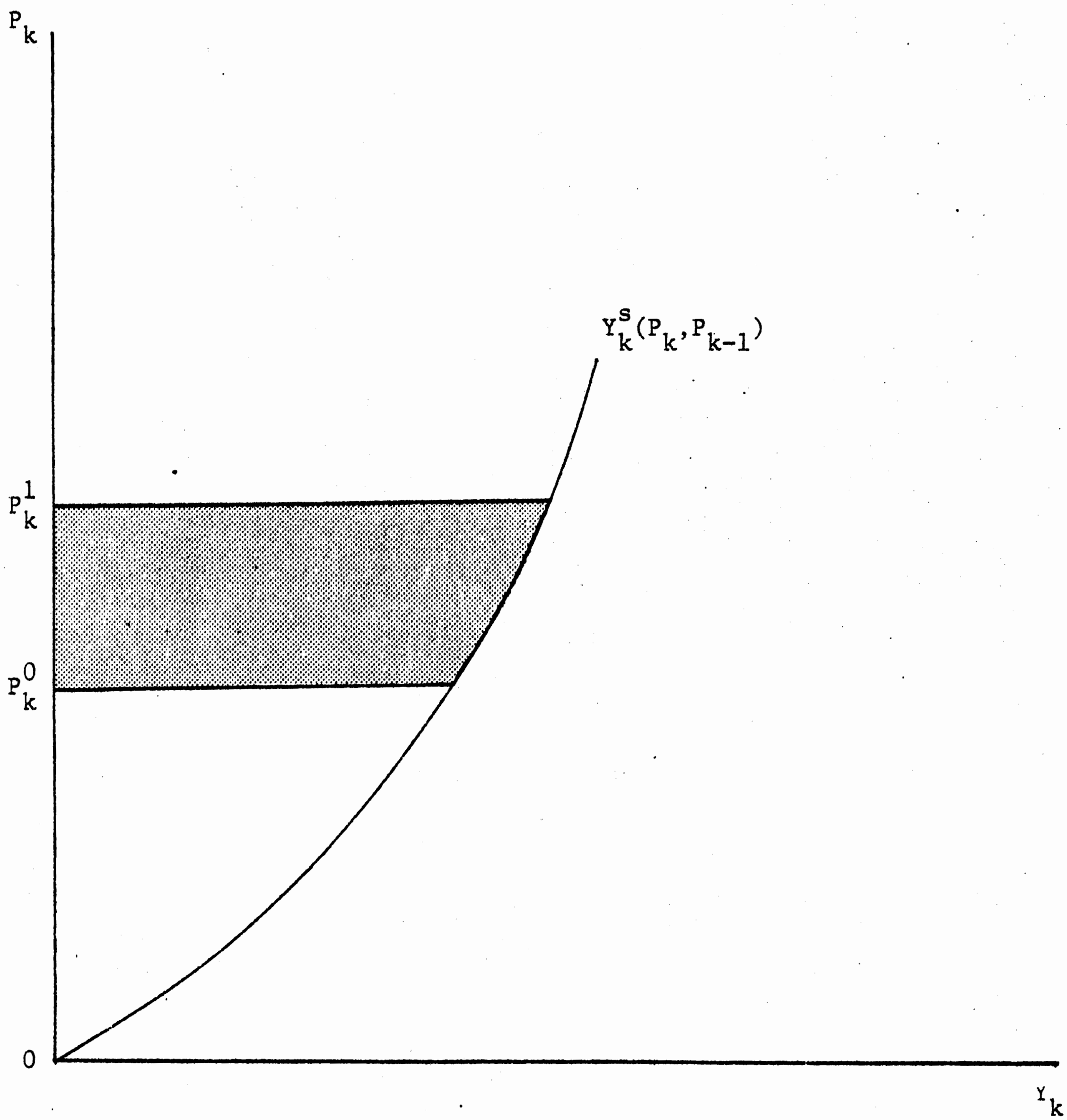

Figure 2. Producer Surplus and Quasi Rents

$$\Delta\pi_k = \Delta CS_{k-1},$$

and the term ΔCS_{k-1} measures precisely the change in area behind the derived demand for Y_{k-1} or the change in consumer surplus in industry k-1. Hence, consumer surplus in the input market k-1 is the same as profits plus fixed costs, which is identical to producer surplus from equation (4) in market k when demand and variable input supply are perfectly elastic. Hence, if input demands are zero when output supplies are zero one can write[3/],

$$PS_k = CS_{k-1} = \pi_k. \tag{7}$$

Producer and Consumer Surplus As a Measure Of Total Vertical Market Sector Welfare

Now suppose that the assumptions of perfectly elastic variable input supply and demand are relaxed, so that total sector welfare of a chain of markets as well as the distribution of welfare throughout the chain can be examined. As before, the assumed objective of each industry is to maximize profits. The indirect profit function for the kth industry is again found in (1); however, now industry prices are assumed to adjust with industry usage.

Suppose that prices in all industries are related through competition at the industry level so that as price P_n is forcibly altered, the entire price vector of the sector changes monotonically following P_n. As pointed out by Mishan (1968), evaluation of the welfare impact of such a distortion in this case requires looking beyond the purchasers and sellers in market n. Consider first the effects on any industry k in the chain where $n < k$. By the envelope theorem, one may find from (1) that,

$$\frac{\partial \pi_k}{\partial P_n} = Y_k(P) \frac{\partial P_k}{\partial P_n} - Y_{k-1}(P) \frac{\partial P_{k-1}}{\partial P_n} \tag{8}$$

Now, integration for a specific price change from P_n^0 to P_n^1 implies that,

$$\Delta\pi_k = \int_{P_n^0}^{P_n^1} \frac{\partial \pi_k}{\partial P_k} dP_n = \int_{P_1^0}^{P_n^1} Y_k(P) \frac{\partial P_k}{\partial P_n} dP_n - \int_{P_n^0}^{P_n^1} Y_{k-1}(P) \frac{\partial P_{k-1}}{\partial P_n} dP_n \tag{9}$$

where as before, $\Delta\pi_k$ denotes the change in quasi rents for industry k. In order to interpret (9), note that the first right-hand term is the change in the area below demand and above price or consumer surplus ΔCS_k for industry k. This occurs since, when $n < k$, integration in (9) is along equilibrium quantities in market k as the supply curve influenced by P_n is shifted. Hence, as the supply curve shifts we are measuring the change in the area below demand. Thus, the first right-hand term of (9) can be rewritten as,

$$\Delta CS_k = - \int_{P_n^0}^{P_n^1} Y_k(P) \frac{\partial P_k}{\partial P_n} dP_n = - \int_{P_k(P_n^0)}^{P_k(P_n^1)} \tag{10}$$

Notice, however, that ΔCS_k is not calculated with respect to the usual partial equilibrium demand curve. But rather, ΔCS_k is determined according to the sector equilibrium demand curve which is equivalent to a general equilibrium demand curve that accounts for adjustments in other industries through the sector as the price P_n is forcibly adjusted. The integration in (9) when $n < k$ can be graphically shown in Figure 3. If P_n is forcibly altered from P_n^0 to P_n^1 then the supply curve in the kth market shifts from $Y_k^s(P_k^0)$ to $Y_k^s(P_k^1)$. Hence, integration for the first right hand term in (9) calculates the change in consumer surplus of the kth market represented by the shaded area in Figure 3.

To interpret the remaining right-hand term in (9), again note that when $n < k$ integration is along equilibrium quantities as variable input supply is being shifted due to the altering of industry price P_n. Hence, the remaining integral measures the change in the area below demand in industry k-1 and above the industry price P_{k-1}, which can be written as,

$$\Delta CS_{k-1} = -\int_{P_n^0}^{P_n^1} Y_{k-1}(P) \frac{\partial P_{k-1}}{\partial P_n} dP_n = -\int_{P_{k-1}(P_n^0)}^{P_{k-1}(P_n^1)} Y_{k-1}(P)\, dP_{k-1}. \qquad (11)$$

As before, the demand curve for industry k-1 is an industry general equilibrium demand which accounts for adjustments by other industries.

Substituting (10) and (11) into (9) gives the difference equation,

$$\Delta\pi_k = \Delta CS_{k-1} - \Delta CS_k \qquad k = n+1, \ldots, K,$$

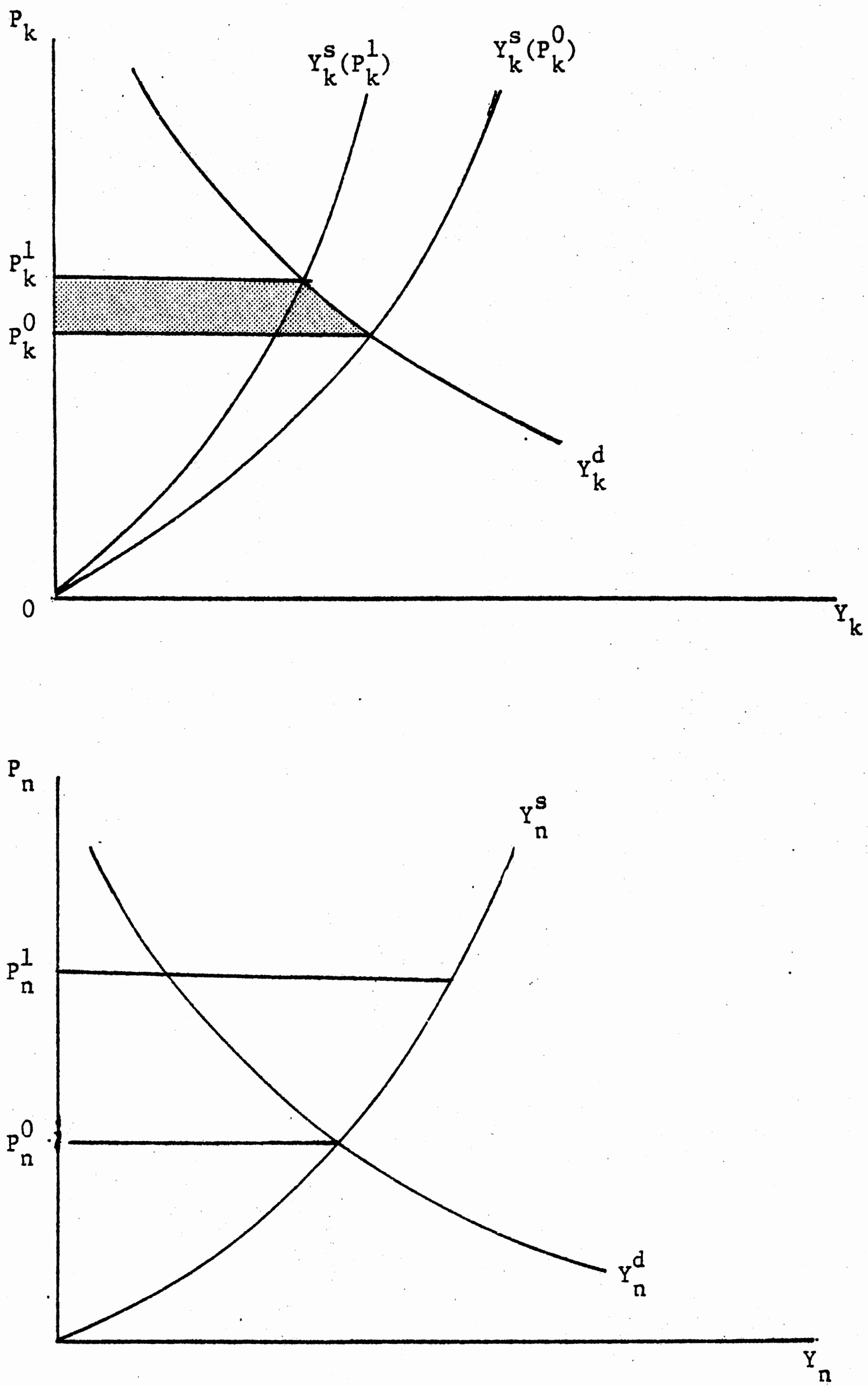

Figure 3. Representation of the Change in Consumer Surplus Through Altering Industry Price when $n < k$.

which upon solving yields the following,

$$\Delta CS_n = \sum_{k=n+1}^{K} \Delta\pi_k + \Delta CS_k, \qquad (12)$$

where ΔCS_K represents the change in final consumer surplus for the final product at the end of the market chain. Thus, at any altered market level n in a vertical sector of industries related by supplies and demands which are not perfectly elastic, the consumer surplus measure is equal to final product consumer surplus ΔCS_K plus the change in quasi rents on fixed factors in all forward industries involved in transforming the commodity produced in industry n into its final form. Since industry K is the final product, the validity of the consumer surplus measure in this industry is clear following Willig (1973). That is, if the final product demand curve is a Marshallian demand then Willig's results can be used to determine the closeness of approximation to the proper Hicksian welfare concept. However, if the demand is a Hicksian demand curve, then the measure ΔCS_K holds the proper welfare significance without approximation.

To show the relationship between producer surplus and quasi rents in related markets, let industry price P_n be altered for the case where $n \geq k$. In this case, when considering industry k, demands are being shifted rather than supplies. Hence, integration in (9) is along equilibrium quantities supplies as demand is shifted. This implies that the first right-hand term of (9) is equivalent to,

$$\Delta PS_k = \int_{P_n^0}^{P_n^1} Y_k(P) \frac{\partial P_k}{\partial P_n} dP_n = \int_{P_k(P_n^0)}^{P_k(P_n^1)} Y_k(P)\, dP_k, \tag{13}$$

where ΔPS_k is producer surplus for the kth industry. Similarly, integration of the remaining right-hand term yields,

$$\Delta PS_{k-1} = \int_{P_n^0}^{P_n^1} Y_{k-1}(P) \frac{\partial P_{k-1}}{\partial P_n} dP_n = \int_{P_{k-1}(P_n^0)}^{P_{k-1}(P_n^1)} Y_{k-1}(P)\, dP_{k-1} \tag{14}$$

Substituting (13) and (14) into (9) obtains the difference equation,

$$\Delta\pi_k = \Delta PS_k - \Delta PS_{k-1} \qquad k = 1, \ldots, n,$$

and upon solving reveals that,

$$\Delta PS_n = \Delta PS_0 + \sum_{k=1}^{n} \Delta\pi_k, \tag{15}$$

where ΔPS_0 is the initial resource suppliers surplus. Thus, at any market level n in a vertical chain which are related by supplies and demands which are not perfectly elastic, producer surplus at the nth level measures the initial resource suppliers surplus plus all quasi rents involved in transforming the initial resource into its present nth form.

Summing the surplus results from (12) and (15) in industry n obtains,

$$\Delta CS_n + \Delta PS_n = \Delta PS_0 + \Delta CS_K + \sum_{k=1}^{K} \Delta\pi_k. \qquad (16)$$

Hence, where market 0 is an initial resource market and market K is a final product market, it is found that the sum of producer and consumer surplus at any intermediate market level n measures total sector welfare when supplies and demands are not perfectly elastic.

Questions relating to the distribution of welfare to a particular market can be studied by determining either producer and consumer surplus at each industry level and then, applying (12) for consumer surplus and (15) for producer surplus.

FOOTNOTES

[1]The super subscript * will be dropped for notational convenience.

[2]For a formal proof of the envelope theorem see Silberberg 1978, p. 168

[3]This assumes the constant of integration is the same for all supplies and input demands.

CHAPTER IV

WELFARE MEASURES IN A VERTICAL-HORIZONTAL MARKET SECTOR

The previous analysis has demonstrated that when measuring surpluses in a vertical industry sector composed of a single product and single variable factor, total welfare can be accounted by summing producer and consumer surplus at any industry level in the vertical sector. However, in many types of analyses in agriculture and non-agriculture industries, more than one variable input is used and more than one product is produced within a vertical market sector. Hence, in these situations it is important to know the relationships of surpluses when multi-product, multi-factor conditions occur. That is, how can total as well as the distribution of welfare be accounted when a vertical industry structure is composed of many horizontal markets at each level in a vertical market sector? In the succeeding analysis, the previous results are generalized to include multi-product and multi-factor markets within a chain of vertical industries.

Consumer Surplus At an Intermediate Market Level

For notational convenience, assume there are k competitive industries within a vertical market sector, which are ordered so that, each industry k produces m outputs facing output prices $P_{k,m}$, m=1,..., M, and uses as variable inputs the products $Y_{k-1,m}$, with input prices $P_{k-1,m}$, m=1,..., M_{k-1}, which are produced at the preceeding industry level. This vertical and horizontal market sector is depicted in Figure 4. Suppose also, that supplies and demands between the vertical industries are related through an industry implicit production function. Maximization of profit subject to this implicit production will give an indirect profit function for the kth vertical industry,

$$\pi_k = \sum_{m=1}^{M_k} P_{k,m} Y_{k,m}(P) - \sum_{m=1}^{M_{k-1}} P_{k-1,m} Y_{k-1,m}(P). \tag{17}$$

where profit maximizing levels of outputs and inputs at given prices are denoted respectively by $Y_{k,m}(P)$, m=1,..., M_k, and $Y_{k-1,m}(P)$, m=1,..., M_{k-1} and where P is a matrix of sector prices.

Now suppose that prices in all industries are related through competition at the industry level so that, as price $P_{n,1}$ is forcibly altered, all industry prices change monotonically following $P_{n,1}$. Consider first the effects when $n < k$. Employing the envelope theorem on (17) results in,

$$\frac{\partial \pi_k}{\partial P_{n,1}} = \sum_{m=1}^{M_k} Y_{k,m}(P) \frac{\partial P_{k,m}}{\partial P_{n,1}} - \sum_{m=1}^{M_{k-1}} Y_{k-1,m}(P) \frac{\partial P_{k-1,m}}{\partial P_{n,1}} \tag{18}$$

As before, integration for a specific price change from $P^0_{n,1}$ to $P^1_{n,1}$ implies,

Final	K	1, 2, 3, . . . M_K
	.	. . . .
	.	. . . .
Vertical	.	. . . .
Markets	3	1, 2, 3, . . . M_3
	2	1, 2, 3, . . . M_2
	1	1, 2, 3, . . . M_1
Initial	0	1, 2, 3, . . . M_0

Figure 4. Illustration of Horizontal and Vertical Market Sector

$$\Delta\pi_k = \int_{P^0_{n,1}}^{P^1_{n,1}} \frac{\partial\pi_k}{\partial P_{n,1}} dP_{n,1} = \sum_{m=1}^{M_k} \int_{P^0_{n,1}}^{P^1_{n,1}} Y_{k-1,m}(P) \frac{\partial P_{k,m}}{\partial P_{n,1}} dP_{n,1}$$

$$- \sum_{m=1}^{M_{k-1}} \int_{P^0_{n,1}}^{P^1_{n,1}} Y_{k-1,m}(P) \frac{\partial P_{k-1,m}}{\partial P_{n,1}} dP_{n,1}, \tag{19}$$

where $\Delta\pi_k$ represents the change in quasi rents for vertical industry k. In order to interpret (19), recall that the first set of terms on the right-hand side measure changes in the areas behind the general equilibrium demands for the commodities $Y_{k,m}$, $m=1,\ldots,M_k$ at the kth vertical industry. This is clear since when $n < k$ integration is along equilibrium quantities in industry k as supplies influenced by $P_{n,1}$ are shifted. Hence, the first set of integrations can be written as,

$$\sum_{m=1}^{M_k} \Delta CS_{k,m} = - \sum_{m=1}^{M_k} \int_{P^0_{n,1}}^{P^1_{n,1}} Y_{k,m}(P) \frac{\partial P_{k,m}}{\partial P_{n,1}} dP_{n,1}$$

$$= - \sum_{m=1}^{M_k} \int_{P^0_{k,m}(P^0_{n,1})}^{P_{k,m}(P^1_{n,1})} Y_{k,m}(P)\, dP_{k,m}. \tag{20}$$

To interpret the remaining set of right-hand terms in (19) when $n < k$, note that integration is along equilibrium quantities in industry k-1 as variable input supplies influenced by $P_{n,1}$ are altered. Hence, the remaining integrations measure changes in the areas below the demands and above market prices in industry k-1,

$$\sum_{m=1}^{M_{k-1}} \Delta CS_{k-1,m} = - \sum_{m=1}^{M_{k-1}} \int_{P^0_{n,1}}^{P^1_{n,1}} Y_{k-1,m}(P) \frac{P_{k-1,m}}{P_{n,1}} dP_{n,1}$$

$$= - \sum_{m=1}^{M_{k-1}} \int_{P_{k-1,m}(P^0_{n,1})}^{P_{k-1,m}(P^1_{n,1})} Y_{k-1,m}(P)\ dP_{k-1,m}. \tag{21}$$

Substituting (20) and (21) into (19) implies,

$$\Delta\pi_k = \sum_{m=1}^{M_{k-1}} \Delta CS_{k-1,m} - \sum_{m=1}^{M_k} \Delta CS_{k,m} \qquad k = n + 1, \ldots, K, \tag{22}$$

which reveals upon solving the difference equation for $\Delta CS_{n,m}$, that

$$\sum_{m=1}^{M_n} \Delta CS_{n,m} = \sum_{k=n+1}^{K} \Delta\pi_k + \sum_{m=1}^{M} \Delta CS_{K,m} \tag{23}$$

where as before $\Delta CS_{K,m}$ represents the changes in final consumer surpluses of the last M_K industry products. Thus, the sum of consumer surpluses in industry n associated with an alteration of one of the prices $P_{n,1}$ in industry n, measures the sum of final consumer surpluses plus all industry rents involved in transforming the commodities traded at industry n into their final consumption form.

The welfare significance of $\Delta\pi_k$ is the same as in Mishan (1968), only in this case, $\Delta\pi_k$ measures the rents associated with multi-product and multi-factor production. That is, $\Delta\pi_k$ is the measure of quasi rents to all of the $Y_{k,m}$ products. The welfare significance of $\Delta CS_{K,m}$ unlike the single production and factor case, is more complicated since more than one price is changed at the final consumption level. In this case, if the final demands are calculated according to Marshallian demands, one must know the path of prices to determine the closeness of approximation to the proper Hicksian concept. However, if final demands are calculated as Hicksian demands, then the welfare measurements of $\Delta CS_{K,m}$ hold the proper Hicksian welfare significance without approximation. This occurs since Hicksian demands are path independent of prices, which is well understood following Silberberg (1972).

Producer Surplus At An Intermediate Market Level

In order to examine the relationships of producer surpluses when multi-product and multi-factor conditions occur, consider the effects of a similar alteration of price $P_{n,1}$, when $n \geq k$. In this case, demands rather than supplies in industry k are affected so that integration of (19) is along equilibrium quantities supplies as demands are being shifted. Thus, the first set of integrations for industry k can be written as,

$$\sum_{m=1}^{M_k} \Delta PS_{k,m} = \sum_{m=1}^{M_k} \int_{P^0_{n,1}}^{P^1_{n,1}} Y_{k,m}(P) \frac{\partial P_{k,m}}{\partial P_{n,1}} dP_{n,1},$$

$$= \sum_{m=1}^{M_k} \int_{P_{k,m}(P^0_{n,1})}^{P_{k,m}(P^1_{n,1})} Y_{k,m}(P)\, dP_{k,m}, \tag{24}$$

where $\Delta PS_{k,m}$ represents producer surpluses at the kth industry level, for the commodities $m=1,\ldots,M_k$. Furthermore, the remaining set of integrations in (19) when $n \geq k$ measure producer surpluses in industry k-1, since integrations are along equilibrium quantities of input supplies in industry k-1 as input demands are shifted due to an alteration of P_n. Hence,

$$\sum_{m=1}^{M_{k-1}} \Delta PS_{k-1,m} = \sum_{m=1}^{M_{k-1}} \int_{P^0_{n,1}}^{P^1_{n,1}} Y_{k-1,m}(P) \frac{\partial P_{k-1,m}}{\partial P_{n,1}} dP_{n,1},$$

$$= \sum_{m=1}^{M_{k-1}} \int_{P_{k-1,m}(P^0_{n,1})}^{P_{k-1,m}(P^1_{n,1})} Y_{k-1,m}(P)\, dP_{k-1,m}. \tag{25}$$

Substituting (24) and (25) into (19) gives the difference equation,

$$\Delta\pi_k = \sum_{m=1}^{M_k} \Delta PS_{k,m} - \sum_{m=1}^{M_{k-1}} \Delta PS_{k-1,m} \qquad k = 1,\ldots,n. \tag{26}$$

Solving (26) obtains,

$$\sum_{m=1}^{M_n} \Delta PS_{n,m} = \sum_{k=1}^{n} \Delta\pi_k + \sum_{m=1}^{M_0} \Delta PS_{0,m} \tag{27}$$

where $\Delta PS_{0,m}$ represents the change in the initial resource suppliers surpluses of the m=1,...,M_0 initial factors. Thus, summing the changes in producer surplus triangles in industry n associated with a price change $P_{n,1}$ measures the sum of the initial resource supplier surpluses plus all industry rents involved in transforming the initial resources into their present form at industry n.

Total Welfare Change

Summing the consumer and producer surpluses at the nth industry level obtains,

$$\sum_{m=1}^{M_n} \Delta CS_{n,m} + \sum_{m=1}^{M_n} \Delta PS_{n,m} = \sum_{m=1}^{M_K} \Delta CS_{K,m} + \sum_{m=1}^{M_0} \Delta PS_{0,m} + \sum_{k=1}^{K} \Delta\pi_k .$$

It is tempting to argue that for a price distortion all one has to do is sum the producer and consumer surpluses at the nth altered level to obtain the total welfare effect. However a closer examination reveals that the producer and consumer surpluses of the commodities $Y_{n,2},\ldots,Y_{n,M_n}$ are measured along the same path of integration. This occurs because both supplies and demands shift for the commodities $Y_{n,m}$ m=2,...,M_n. Furthermore, since $\Delta CS_{n,m}$ m=2,...,M_n are derived from the π_{n+1} industry we have,

$$\sum_{m=2}^{M_n} \Delta CS_{n,m} = - \sum_{m=2}^{M_n} \int_{P_{n,m}(P^0_{n,1})}^{P_{n,m}(P^1_{n,1})} Y_{n,m}\, dP_{n,m}$$

and from the π_n industry the producer surpluses $\Delta PS_{n,m}$ $m=2,\ldots,M_n$,

$$\sum_{m=2}^{M} \Delta PS_{n,m} = \sum_{m=2}^{M_n} \int_{P_{n,m}(P^0_{n,1})}^{P_{n,m}(P^1_{n,1})} Y_{n,m}\, dP_{n,m} .$$

Hence we find that,

$$\sum_{m=2}^{M_n} \Delta CS_{n,m} + \sum_{m=2}^{M_n} \Delta PS_{n,m} = 0 .$$

Thus one can write,

$$\Delta CS_{n,1} + \Delta PS_{n,1} = \sum_{m=1}^{M_K} \Delta CS_{k,m} + \sum_{m=1}^{M_0} \Delta PS_{0,m} + \sum_{k=1}^{K} \Delta\pi_k . \tag{28}$$

Hence, where industry 0 is an initial resource industry and industry K is a final consumption industry, the sum of producer and consumer surplus of the altered commodity $Y_{n,1}$ measures the change in total sector welfare. Notice this result is a generalization of Chapter III results (single-product and factor industries). In both cases the relevant total welfare measure of a price distortion is to sum the producer and consumer surplus of the distorted commodity.

In summary, the results so far have emphasized the welfare measures for: (a) partial equilibrium condition; and (b) general equilibrium conditions. The results have demonstrated that the change in total sector welfare is found from the general equilibrium changes, while under partial equilibrium analysis only the change in welfare to the directly affected parties is forthcoming. However, in applied research there exists many possible theoretical supply and demand specifications (ranging from partial equilibrium at one extreme to general equilibrium at the other extreme) depending upon the assumptions the research makes regarding adjustments by industry prices. Hence, the issue is raised regarding whether one can determine what welfare results are being measured under alternative supply and demand specifications. The answer to this issue is addressed in the following chapter.

CHAPTER V

WELFARE MEASURES UNDER ALTERNATIVE INDUSTRY SUPPLY AND DEMAND SPECIFICATIONS

The results in Chapter IV have shown that when supplies and demands are of a general equilibrium nature, total sector welfare is found by summing producer and consumer surpluses of all markets at the industry of interest. An important empirical question is under what type of supply and demand specifications are other welfare measures forthcoming? This question arises because in many policy problems some industry prices may be omitted because of lack of data on these industries or simply because the policy may not affect the price of a particular industry. Thus, the following analysis examines situations where some industry prices are indeed constant and do not depend upon industry usage, and where some prices are held constant but depend upon industry usage.

Welfare Measures Arising From Fixed Horizontal Prices

Consider the effects when industry price $P_{k,1}$ is forcibly altered from its equilibrium value. Suppose the indirect profit maximizing objective function for the k+1 industry is given by,

$$\pi_{k+1} = \sum_{m=1}^{M_{k+1}} P_{k+1,m} Y_{k+1,m} - \sum_{m=1}^{M_k} P_{k,m} Y_{k,m}. \tag{29}$$

Suppose also, that the industry prices for $P_{k,m}$, $m=2,\ldots,M_k$ are perfectly elastic (do not depend upon industry usage) while all other prices in the sector are dependent upon industry usage and monotonically change due to an alteration of $P_{k,1}$. This supposition implies that the demand function for $Y_{k,1}$ and the supply functions for $Y_{k+1,m}$ will be of the form,

$$Y_{k,1} = Y^d_{k,1}\ (P_{k,1},\ldots,P_{k,m}).$$

$$Y_{k+1,m} = Y^s_{k+1,m}\ (P_{k+1,m},P_{k,2},\ldots,P_{k,m})$$

Note that since the prices $P_{k,m}$ $m=2,\ldots,M_k$ are perfectly elastic at the industry level (fixed and do not depend upon industry usage) then the above functional forms can be thought of as general equilibrium functions. However, if the prices $P_{k,m}$ $m=2,\ldots,M_k$ do adjust as industry usage changes then the above functional forms are neither a partial equilibrium or general equilibrium form since the former would include all $P_{k-1,m}$, $m=1,\ldots,M_{k-1}$ prices while the latter would include only the price $P_{k,1}$.

Now since the horizontal prices at the kth level are perfectly elastic the envelope theorem on (29) yields,

$$\frac{\partial \pi_{k+1}}{\partial P_{k,1}} = \sum_{m=1}^{M_{k+1}} Y_{k+1,m}\ \frac{\partial P_{k+1,m}}{\partial P_{k,1}} - Y_{k,1} \tag{30}$$

Integrating (30) for a specific price change from $P^0_{k,1}$ to $P^1_{k,1}$ gives,

$$\int_{P_{k,1}^0}^{P_{k,1}^1} \frac{\partial \pi_{k+1}}{\partial P_{k,1}} dP_{k,1} = \sum_{m=1}^{M_{k+1}} \int_{P_{k,1}^0}^{P_{k,1}^1} Y_{k+1,m} \frac{\partial P_{k+1,m}}{\partial P_{k,1}} dP_{k,1} - \int_{P_{k,1}^0}^{P_{k,1}^1} Y_{k,1}\, dP_{k,1},$$

$$\Delta\pi_{k+1} = \sum_{m=1}^{M_{k+1}} \int_{P_{k+1,m}(P_{k,1}^0)}^{P_{k+1,m}(P_{k,1}^1)} Y_{k+1,m}\, dP_{k+1,m} - \int_{P_{k,1}^0}^{P_{k,1}^1} Y_{k,1}\, dP_{k,1},$$

$$\Delta\pi_{k+1} = - \sum_{m=1}^{M_{k+1}} \Delta CS_{k+1,m} + \Delta CS_{k,1} \tag{31}$$

Notice that (23) for n=k+1 can be rewritten as,

$$\sum_{m=1}^{M_{k+1}} \Delta CS_{k+1,m} = \sum_{j=k+2}^{K} \Delta\pi_j + \sum_{m=1}^{M_K} \Delta CS_{K,m}. \tag{32}$$

Substituting (32) into (31) and solving for $\Delta CS_{k,1}$ yields,

$$\Delta CS_{k,1} = \sum_{j=k+1}^{K} \Delta\pi_j + \sum_{m=1}^{M_K} \Delta CS_{K,m}. \tag{33}$$

Thus, we find that the change in consumer surplus of the first commodity at the kth level measures all forward quasi rents plus final consumer surpluses of the finished goods. This result occurs because the prices $P_{k,m}$ $m=2,\ldots,M_k$ are held constant (i.e., their changes in consumer surplus for $Y_{k,m}$, $m=2,\ldots,M_k$ are zero).

Now consider the effects of an alteration of $P_{k,1}$ on the kth industry. The indirect profit function is,

$$\pi_k = \sum_{m=1}^{M_k} P_{k,m}Y_{k,m} - \sum_{m=1}^{M_{k-1}} P_{k-1,m}Y_{k-1,m}. \tag{34}$$

Since the prices $P_{k,m}$, $m=2,\ldots,M_k$ do not depend upon industry usage then the supply function for $Y_{k,1}$ and the demand functions for $Y_{k=1,m}$ will be of the form,

$$Y_{k,1} = Y^s_{k,1} \ (P_{k,1},\ldots,P_{k,m})$$

$$Y_{k-1,m} = Y^d_{k-1,m} \ (P_{k-1,m},P_{k,2},\ldots,P_{k,m}).$$

Employing the envelope theorem on (34) one obtains,

$$\frac{\partial\pi_k}{\partial P_{k,1}} = Y_{k,1} - \sum_{m=1}^{M_{k-1}} Y_{k-1,m} \frac{\partial P_{k,m}}{\partial P_{k,1}} . \tag{35}$$

Integrating (35) along the supply function for $Y_{k,1}$ and shifting demands for $Y_{k-1,m}$, $m=1,\ldots,M_k$ implies that areas behind supply and above price are being measured. Therefore one can obtain,

$$\int_{P^0_{k,1}}^{P^1_{k,1}} \frac{\partial\pi_k}{\partial P_{k,1}} dP_{k,1} = \int_{P^0_{k,1}}^{P^1_{k,1}} Y_{k,1} \, dP_{k,1} - \sum_{m=1}^{M_{k-1}} \int_{P^0_{k,1}}^{P^1_{k,1}} Y_{k-1,m} \frac{\partial P_{k,m}}{\partial P_{k,1}} dP_{k,1}$$

$$\Delta\pi_k = \int_{P^0_{k,1}}^{P^1_{k,1}} Y_{k,1}\, dP_{k,1} - \sum_{m=1}^{M_{k-1}} \int_{P_{k,m}(P^0_{k,1})}^{P_{k,m}(P^1_{k,1})} Y_{k-1,m}\, dP_{k-1,m}$$

$$\Delta\pi_k = \Delta PS_{k,1} - \sum_{m=1}^{M_{k-1}} \Delta PS_{k-1,m}\,. \tag{36}$$

Note that when n=k-1, equation (27) can be written as,

$$\sum_{m=1}^{M_{k-1}} \Delta PS_{k-1,m} = \sum_{j=1}^{k-1} \Delta\pi_j \quad \sum_{m=1}^{M_0} \Delta PS_{0,m}. \tag{37}$$

Substituting (37) into (36) and solving for $\Delta PS_{k,1}$ obtains,

$$\Delta PS_{k,1} = \sum_{j=1}^{k} \Delta\pi_j + \sum_{m=1}^{M_0} \Delta PS_{0,m}. \tag{38}$$

Hence, when industry price adjustments $P_{k,m}$ $m=2,\ldots,M_k$ are assumed to be unaffected by the change in P_n, the producer surplus measure $\Delta PS_{k,1}$ measures all backward quasi rents plus the initial producer surpluses of the beginning raw resources.

Summing producer and consumer surplus measures for market one in industry k, one finds,

$$\Delta CS_{k,1} + \Delta PS_{k,1} = \sum_{m=1}^{M_K} \Delta CS_{K,m} + \sum_{m=1}^{M_0} \Delta PS_{0,m} + \sum_{k=1}^{K} \Delta\pi_k. \tag{39}$$

Hence, where markets m=1,...,M are initial resource markets and markets m=1,...,M are final product markets, and that by specifying supply and demand of the first market at the kth level to include all horizontal market prices of the kth level, it is found that the sum of changes in producer and consumer surplus of the altered commodity measure the change in total sector welfare. Note that the result of (39) is the same as (28). Hence it is not necessary for one to assume perfect elastic supplies and demands in other related markets to obtain the total welfare change of the sector.

Other Supply and Demand Specifications

From the results so far it is also possible to examine what is being measured under other supply and demand specifications depending upon what assumptions one makes regarding price adjustments. In this section we examine what is being measured under alternative supply and demand specifications. However, unlike the previous section we will make no assumptions regarding adjustments by other prices. For example, the kth vertical level is composed of M_k=2 products and M_{k-1}=2 inputs. The kth industry indirect objective function is,

$$\pi_k = P_{k,1}Y_{k,1} + P_{k,2}Y_{k,2} - P_{k-1,1}Y_{k-1,1} - P_{k-1,2}Y_{k-1,2} \tag{40}$$

One approach in the literature to measure π_k has been to estimate product supplies for $Y_{k,1}$ and $Y_{k,2}$ of the form,

$$Y_{k,i} = Y^S_{k,i}(P_{k,1}, P_{k,2}, P_{k-1,1}, P_{k-1,2}) \qquad i = 1,2. \tag{41}$$

These supplies are partial equilibrium in nature since they contain all product and input prices. Hence, by definition (41) is derived by holding prices constant even though they may depend upon industry adjustment. Now, suppose that one measures producer surplus of $Y_{k,1}$ and $Y_{k,2}$ of (41) and sums them to get π_k. Is this the correct procedure to obtain quasi rents for industry k when supplies are of a partial equilibrium nature? Employing the envelope theorem on (40) obtains,

$$\frac{\partial \pi_k}{\partial P_{k,i}} = Y_{k,i} \qquad i = 1,2 \tag{42}$$

where $Y_{k,i}$ are the supply functions given in (41). Integrating (42) with a product price change from $P^0_{k,i}$=0 to $P^1_{k,i}$ gives,

$$\pi_k = \int_0^{P^1_{k,i}} \frac{\partial \pi_k}{\partial P_{k,i}} dP_{k,i} = \int_0^{P^1_{k,i}} Y_{k,i} dP_{k,i} = PS_{k,i} \qquad i = 1,2. \tag{43}$$

Hence (43) implies that if the constant of integration is the same then

$$\pi_k = PS_{k,1} = PS_{k,2}. \tag{44}$$

Thus, summing the producer surplus measures does not give π_k, but gives $n\pi_k$ where in this case n=2. Furthermore, $PS_{k,i} \neq \pi_i$, where π_i is the proportion of quasi rents to the ith product, but rather $PS_{ki} = \sum_i^2 \pi_i = \pi_k$. Hence, without knowing the proportion of industry variable factor cost that goes into the production of $Y_{k,1}$ or $Y_{k,2}$ it is not possible to examine just the rent to $Y_{k,1}$ or $Y_{k,2}$. A similar conclusion holds for consumer surplus measures derived from partial equilibrium demands.

So far, we have only considered the relationships between surpluses and three supply and demand specifications. The first was a general equilibrium supply and demand in which the quantity supplied or demanded was just a function of its own price. This case implicitly implies that all related industry price adjustments are monotonically made. The second case was a completely partial equilibrium result. In this situation the supply and demand equations were functions of all immediate related prices. In this case producer surplus is a measure of rent to all products contained in the industry objective function and consumer surplus is a measure of rent to all products in the next forward industry. Finally, supply and demand was specified to include all horizontal market prices. However, other supply and demand specifications do exist. Hence, is it possible to tell what is being measured no matter how supplies and demands are specified? For example, suppose the objective function is given by (40), and one estimates a supply function for $Y_{k,1}$ which only includes market prices $P_{k,1}$ and $P_{k-1,1}$. In this case market price adjustments due to an alteration of $P_{n,1}$ is made for $P_{k,1}$, $P_{k,2}$ and $P_{k-1,2}$. Hence, if the price $P_{n,1}$ is forcibly altered where $n \geq k$ one obtains from the envelope theorem,

$$\frac{\partial \pi_k}{\partial P_{n,1}} = Y_{k,1} \frac{\partial P_{k,1}}{\partial P_{n,1}} - Y_{k,2} \frac{\partial P_{k,2}}{\partial P_{n,1}} - Y_{k-1,1} \frac{\partial P_{k-1,1}}{\partial P_{n,1}} - Y_{k-2,2} \frac{\partial P_{k-1,2}}{\partial P_{n,1}} \quad (45)$$

Integrating (45) gives,

$$\Delta\pi_k = \Delta PS_{k,1} - \Delta PS_{k,2} - \Delta PS_{k-1,2},$$

or

$$\Delta PS_{k,1} = \Delta\pi_k + \Delta PS_{k,2} + \Delta PS_{k-1,2}. \tag{46}$$

Hence, for the market price $P_{k-1,1}$ which was held constant, (i.e., its price was not allowed to adjust as a result of altering P_n) the producer surplus measure for $Y_{k-1,1}$ does not show up in the producer surplus measure for $P_{k,1}$. A similar result also holds for prices which are held constant in demands. Table I gives five alternative industry supply and demand specifications and their appropriate welfare measures for the industry objective function given in (40).

Notice that if the industry objective function was different than that shown in (40), the welfare measures would also be different than those obtained in Table I. For example, if the objective function was given by,

$$\pi_k = P_{k,1}Y_{k,1} + P_{k,2}Y_{k,2} - P_{k-1,1}Y_{k-1,1}, \tag{47}$$

then the producer surplus measure for the first supply equation given in Table I would be,

$$\Delta PS_{k,1} = \Delta\pi_k + \Delta PS_{k-1,1} - \Delta PS_{k,2}.$$

Hence, knowing the industry objective function is as important as the specification of the supply and demand functions if one wants to know what welfare results are being measured.

TABLE I

WELFARE MEASURES FOR ALTERNATIVE MEASURES AND DEMAND SPECIFICATIONS

Specification	Welfare Measure
	Supply
$Y^s_{k,1}(P_{k,1})$	$\Delta PS_{k,1} = \Delta\pi_k + \Delta PS_{k-1,1} + \Delta PS_{k-1,2} - \Delta PS_{k,2}$
$Y^s_{k,1}(P_{k,1}, P_{k,2}, P_{k-1,1}, P_{k-1,2})$	$\Delta PS_{k,1} = \Delta\pi_k$
$Y^s_{k,1}(P_{k,1}, P_{k,2})$	$\Delta PS_{k,1} = \Delta\pi_k + \Delta PS_{k-1,1} + \Delta PS_{k-1,2}$
$Y^s_{k,1}(P_{k,1}, P_{k-1,1}, P_{k-1,2})$	$\Delta PS_{k,1} = \Delta\pi_k + \Delta PS_{k,2}$
$Y^s_{k,1}(P_{k,1}, P_{k,2}, P_{k-1,1})$	$\Delta PS_{k,1} = \Delta\pi_k + \Delta PS_{k,2} + \Delta PS_{k-1,1}$
	Demand
$Y^d_{k-1,1}(P_{k-1,1})$	$\Delta CS_{k-1,1} = \Delta\pi_k + \Delta CS_{k,1} + \Delta CS_{k,2} - \Delta CS_{k-1,2}$
$Y^d_{k-1,1}(P_{k,1}, P_{k,2}, P_{k-1,1}, P_{k-1,2})$	$\Delta CS_{k-1,1} = \Delta\pi_k$
$Y^d_{k-1,1}(P_{k-1,1}, P_{k-1,2})$	$\Delta CS_{k-1,1} = \Delta\pi_k + \Delta CS_{k,1} + \Delta CS_{k,2}$
$Y^d_{k-1,1}(P_{k-1,1}, P_{k,1}, P_{k,2})$	$\Delta CS_{k-1,1} = \Delta\pi_k - \Delta CS_{k-1,2}$
$Y^d_{k-1,1}(P_{k-1,1}, P_{k-1,2}, P_{k,1})$	$\Delta CS_{k-1,1} = \Delta\pi_k + \Delta CS_{k,2}$

CHAPTER VI

EMPIRICAL IMPLICATIONS AND AN APPLIED AGRICULTURAL EXAMPLE

In the preceding analysis it was demonstrated that alternative welfare measures arise depending upon the assumptions regarding industry prices and the industry objective function. In this chapter the empirical implications of these results are examined as they relate to applied econometric or linear programming welfare studies. Furthermore, an applied agricultural example is presented to demonstrate the ease in which welfare measures can be calculated from linear supply and demand specifications.

Empirical Implications

The previous results in Chapter IV imply that when all welfare measures are taken along general equilibrium functions (i.e., all quantities and prices in the economy are allowed to monotonically adjust) equation (28) provides a convenient way to evaluate the total change in welfare. For example, consider a large scale econometric model giving a representation of an economy (or of a sector if this sector is facing fixed prices from other sectors of the economy). If the general equilibrium supply and demand curves are linear[1/], then the producer surplus calculations, for the kth industry for a policy change from $P^0_{k,m}$ to $P^1_{k,m}$ $m=1,\ldots,M_k$ is,

$$\sum_{m=1}^{M_k} \Delta PS_{k,m} = \tfrac{1}{2} [P^1_{k,m} - P^0_{k,m}][Y^s_{k,m}(P^0_{k,m}) + Y^s_{k,m}(P^1_{k,m})]. \tag{48}$$

Similarly, the consumer surplus calculations can be represented by,

$$\sum_{m=1}^{M_k} \Delta CS_{k,m} = -\tfrac{1}{2} [P^1_{k,m} - P^0_{k,m}][Y^d_{k,m}(P^0_{k,m}) + Y^d_{k,m}(P^1_{k,m})] \tag{49}$$

Then, by summing (48) and (49) one can obtain the change in total welfare for the economy. Thus, the only information required to evaluate the change in welfare in the economy is the set of general equilibrium prices and quantities in the distorted industry before and after the policy change. These results can usually be estimated fairly easily from econometric models or linear programming. In this context, there is no need to have measurement in other industries of the economy as long as the objective of the researcher is to evaluate the total welfare impact. Furthermore, these results appear to have important implications for empirical welfare analysis since they provide a simple and practical approach to studying welfare in an economy comprised of horizontally and vertically related markets.

If one is interested in the distribution of the welfare change, then there is a need to disaggregate the total welfare effect into impacts on individual industries. In a general equilibrium framework, this amounts to subtracting consumer surpluses using equation (22) or producer surpluses using equation (26).

Notice also, that the supply equations $Y^s_{k,m}$ in (48) and the demand equations $Y^d_{k,m}$ in (49) do not necessarily have to be general equilibrium in nature. That is, since one is only interested in the initial and final vectors of prices and quantities these can be found from partial equilibrium supply and demand functions or any other alternative specifications.[2/]

An Applied Agricultural Example

Results in this chapter have demonstrated the simplicity of examining applied welfare changes in an economy. In the following analysis, estimated supply and demand equations of the corn and soybean industries in the agricultural sector are used to illustrate how the results of this study can be used in applied welfare analysis.

Consider the following industry indirect objective function for corn and soybeans,

$$\pi = A_1 P_c + A_2 P_s + A_3 \frac{P_c^2}{r} + A_4 \frac{P_s^2}{r} + A_5 \frac{P_c P_s}{r} + A_6 P_c T + A_7 P_s T \qquad (50)$$

where P_c ≡ price of corn for U.S. in \$/bu.,

P_s ≡ price of soybeans for U.S. in \$/bu.,

r ≡ price index for variable production items,

T ≡ time.

From the envelope theorem one can obtain the following partial equilibrium supply and demand equations derivable from $\pi(P_c, P_s, r)$,

$$\frac{\partial \pi}{\partial P_c} = Y_c^s = A_1 + 2A_3 \frac{P_c}{r} + A_5 \frac{P_s}{r} + A_6 T, \qquad (51)$$

$$\frac{\partial \pi}{\partial P_s} = Y_s^s = A_2 + 2A_4 \frac{P_s}{r} + A_5 \frac{P_c}{r} + A_7 T, \qquad (52)$$

$$\frac{\partial \pi}{\partial r} = - X^d = A_3 \frac{P_c^2}{r^2} + A_4 \frac{P_s^2}{r^2} + A_5 \frac{P_c P_s}{r^2} \; . \qquad (53)$$

where Y_c ≡ production of corn for U.S. in m.bu.,

Y_s ≡ production of soybeans for U.S. in m.bu.,

X ≡ quantity index of variable inputs used to produce Y_c and Y_s.

One may note that the above system of equations satisfy the homogeneity condition for partial equilibrium functions.

Since (51) and (52) provide estimates of all the parameters in (53) the system of equations to be estimated can be reduced to,

$$Y_c^s = A_1 + \bar{A}_3 \frac{P_c}{r} + A_5 \frac{P_s}{r} + A_6 T + e_1, \qquad (54)$$

$$Y_s^s = A_2 + \bar{A}_4 \frac{P_s}{r} + A_5 \frac{P_c}{r} + A_7 T + e_2 \qquad (55)$$

where $\bar{A}_3 \equiv 2 \cdot A_3$,

$\bar{A}_4 \equiv 2 \cdot A_4$.

Data used to estimate (54) and (55) were for the years 1949-1977 (Agricultural Statistics 1957, 1963 and 1978).

Since corn and soybeans are considered competing crops in production, the error terms in (54) and (55) may be correlated. Hence, a gain in the efficiency of parameter estimates may be achieved by jointly estimating the set of equations as a multivariate system (Zellner 1962).

Economic theory requires $\pi(P_c, P_s, r)$ to be positive definite for a maximum. A sufficient condition for $\pi(P_c,P_s,r)$ to be positive definite is that,

$$\bar{A}_3 > 0, \bar{A}_4 > 0, \bar{A}_3\bar{A}_4 > A_5^2.$$

Furthermore, the symmetry condition,

$$\frac{\partial Y_c^s}{\partial P_s} = \frac{\partial Y_s^s}{\partial P_c} = A_5,$$

was imposed. This restriction amounts to forcing A_5 in (54) to be equal to A_5 in (55).

Estimates of this system are presented in Table II. The R^2 for the system is .914. This is the R^2 that corresponds to the approximate F test on all non-intercept parameters in the system. The F value for imposing the symmetry condition was 2.39 with probability of being exceeded of .12. Furthermore one may note that the sufficient condition for $\pi(P_c,P_s,r)$ to be maximum is met since $\bar{A}_3 > 0$, $\bar{A}_4 > 0$ and $\bar{A}_3\bar{A}_4 > A_5^2$.

In order to demonstrate applied results the demand equations for corn and soybeans remain to be estimated. These equations, for convenience, were specified as general equilibrium functions. Hence, the equations are functions of their own price and time. The demand specifications are,

$$Y_c^d = b_o + b_1P_c + b_2T + e_1, \tag{56}$$

$$Y_s^d = C_o + C_1P_s + C_2T + e_2, \tag{57}$$

TABLE II

PARTIAL EQUILIBRIUM SUPPLY ESTIMATES FOR
U.S. CORN AND SOYBEAN INDUSTRY

	Parameter	Parameter Estimate	Standard Error
Corn	A_1	-7,024.927	1,353.725
	$\bar{A}_3$	116,868.554	43,735.184
	A_5	-39,244.714	21,657.051
Soybeans	A_2	-2,388.746	402.265
	$\bar{A}_4$	29,211.220	12,905.830
	A_5	-39,244.714	21,657.051
	A_7	46.448	5.531

where $Y_c^d \equiv$ quantity demanded excluding exports for U.S. corn production in m.bu.,

$Y_s^d \equiv$ quantity demanded excluding exports for U.S. soybean production in m.bu.

The ordinary least squares estimates for these parameters for the time period 1949-1977 are depicted in Table III.

Using the estimated supply and demand equations one can now examine the welfare changes due to an alteration of one or more of the parameters in the system. For example, suppose that a particular policy is to result in increasing the export demand by 10 percent for corn and soybeans in 1979. What are the welfare changes to producers of corn and soybeans? Furthermore, assume that this effect will not alter the prices paid for variable input items r, and carryover levels of supply are constant. Setting supply equal to domestic demand plus export demand for corn and soybeans the system can be rewritten as,

$$Y_c^s = Y_c^d + E_c,$$

$$A_1 + \bar{A}_3 \frac{P_c}{r} + A_5 \frac{P_s}{r} + A_6 T = b_o + b_1 P_c + b_2 T + E_c, \tag{58}$$

$$Y_s^s = Y_s^d + E_s,$$

$$A_2 + \bar{A}_4 \frac{P_s}{r} + A_5 \frac{P_c}{r} + A_7 T = C_o + C_1 P_s + C_2 T + E_s, \tag{59}$$

where $E_c \equiv$ exports of corn for U.S. in m.bu.,

TABLE III

GENERAL EQUILIBRIUM DEMAND ESTIMATES FOR U.S. CORN AND SOYBEAN INDUSTRY

	Parameter	Parameter Estimate	Standard Error	R^2
Corn	b_o	-1,199.084	400.174	.85
	b_1	-204.434	111.748	
	b_2	79.092	7.008	
Soybeans	c_o	-1,164.895	76.469	.96
	c_1	-1.917	0.888	
	c_2	27.342	1.465	

$E_s \equiv$ exports of soybeans for U.S. in m.bu.

Setting all variables except prices and quantities at their 1978 values and solving (58) and (59) for general equilibrium prices and quantities gives the initial conditions.

$$P_c^o = 2.30 \qquad Y_c^o = 6{,}104.12$$

$$P_s^o = 5.28 \qquad Y_s^o = 1{,}494.38$$

$$r^o = 208 \qquad X^o = 5.54 \tag{60}$$

Substituting the above values into the indirect profit function (50) or into the direct profit function yields the initial general equilibrium profit value,

$$\pi^o = P_c^o Y_c^o + P_s^o Y_s^o - r^o X^o$$

$$= 20{,}794.7 \tag{61}$$

Now suppose that E_c^o and E_s^o increase by 10 percent to E_c^1 and E_s^1. Resolving (58) and (59) for the new general equilibrium conditions yields,

$$P_c^1 = 2.77 \qquad Y_c^1 = 6{,}176.23$$

$$P_s^1 = 6.30 \qquad Y_s^1 = 1{,}548.83$$

$$r^1 = 208 \qquad X^1 = 7.94 \tag{62}$$

Again substituting these values into the direct profit function gives,

$$\pi^1 = P_c^1 Y_c^1 + P_s^1 Y_s^1 - r^1 X^1,$$

$$= 25{,}239.6. \tag{63}$$

Subtracting (63) from (61) gives the change in welfare to corn and soybean producers.

$$\Delta\pi = \pi^1 - \pi^o$$

$$= 4{,}444.9 \tag{64}$$

Let us now examine the surplus changes under the assumption that r is held constant. Given the objective function in (50) one would expect from the results in Chapter IV that,

$$\Delta\pi = \Delta PS_c + \Delta PS_s \tag{65}$$

where ΔPS_c ≡ the change in producer surplus for corn in m.\$.,

ΔPS_s ≡ the change in producer surplus for soybeans in m.\$.

Furthermore, since the partial equilibrium supplies and the general equilibrium demands are linear in P_c and P_s the general equilibrium supplies for corn and soybeans will be linear. Hence, one can use (48) to determine the producer surplus measures for corn and soybeans. Thus, one may write,

$$\Delta PS_c = \tfrac{1}{2} [P_c^1 - P_c^o][Y_c^s(P_c^o) + Y_c^s(P_c^1)], \tag{66}$$

$$\Delta PS_s = \tfrac{1}{2} [P_s^1 - P_s^o][Y_s^s(P_s^o) + Y_s^s(P_s^1)]. \tag{67}$$

Substituting the initial general equilibrium values in (60) and the final general equilibrium values in (62) into (66) and (67) respectively gives,

$$\Delta PS_c = 1552.84,$$

$$\Delta PS_s = 2892.08.$$

Now summing the producer surpluses one finds,

$$\Delta\pi = \Delta PS_c + \Delta PS_s,$$

$$= 4444.9. \qquad (68)$$

Comparing (68) with (64) we find that the two measures are equivalent. Hence, holding input prices constant and letting the price of corn and soybeans adjust to external forces the summation of producer surpluses gives the change in industry rent.

FOOTNOTES

[1]If the general equilibrium functions are non-linear, then equations (48) and (49) provide only an approximated welfare measure. These approximated welfare measures will differ from the true welfare measures by the area difference between the non-linear function and a linear line between the initial and final vectors of prices and quantities.

[2]The exclusion of supplies and demands from being general equilibrium does not prevent the researcher from solving the set of equations for general equilibrium prices and quantities.

CHAPTER VII

SUMMARY AND CONCLUSIONS

This study has investigated welfare measures in an economy constituted of vertically related multi-product multi-factor industries, where a particular industry is subject to an outside distortion. In this chapter the major findings and their use in applied welfare research are re-examined.

Total Sector Welfare

This study has demonstrated that the change in total sector welfare can be found by several alternative means. If supplies and demands are general equilibrium, then total welfare changes can be obtained from summing the producer and consumer surpluses of the horizontal commodities at any vertical level. However, if the general equilibrium supply and demand functions are linear then one can estimate any other theoretical specification besides the general equilibrium prices and quantities. These general equilibrium points will in turn give a precise estimate of the change in total welfare. This particular procedure provides a practical way of evaluating the change in welfare when general equilibrium supply and demand estimates are poor. Furthermore, given the extent of multi-product multi-factor firms and vertical market chains these results can be used in linear programming or econometric simulation analysis.

The Distribution of Welfare

If one is interested in the distribution of the total welfare change, then there is a need to disaggregate the total welfare effect into impacts on individual industries. In a general equilibrium framework this amounts to finding either producer or consumer surpluses at each vertical level and then taking first differences. In a partial equilibrium framework, ordinary supply or demand curves can also be used. In this case, producer surplus is a measure of rent to all multi-products and consumer surplus is a measure of forward rent to forward multi-products (providing of course, that it is not the consumer surplus of the final products). One should also note that the producer surplus measure defined by the partial equilibrium supply measures total rent to all related products rather than just the rent to the product of interest.

Missing data may hinder many practical applications of applied welfare analysis. In these situations one may not be able to estimate the welfare effects directly. However, one can estimate supplies and demands of related industries and still obtain the welfare measure of the industry of interest. For example, suppose that quantity information is not available for the commodities $Y_{k,1}$ and $Y_{k,2}$ which are the products produced. Furthermore, suppose one wants to know the welfare effect on these commodities due to a price change in the input market $Y_{k-1,1}$ for which prices and quantity information is available. In this situation if one estimates a partial equilibrium input demand function,

$$Y_{k-1,1} = Y^{d}_{k-1,1}(P_{k,1}, P_{k,2}, P_{k-1,1}),$$

then the consumer surplus measure of this function is a measure of the rent to the products $Y_{k,1}$ and $Y_{k,2}$. Hence, being able to use data from related markets allows the analyst to overcome data problems and in turn widens the applicability of his tools. In fact, as demonstrated in Chapter V in multi-product multi-factor industries, the economist may have many different ways of measuring the change in quasi-rent in a given industry. This provides some flexibility in welfare analysis when the objective is to investigate welfare distribution.

The results of this study seem to be of direct applicability when vertical market multi-product chains exist. For example, in the petroleum, minerals, fisheries and agriculture sectors of the economy the results of this paper can be used to examine the distribution as well as the total welfare impact of some policy distortion. In the appendixes two examples of the results of this paper are examined. The first in Appendix A is a single-product single-factor case and then in Appendix B a single-product, multi-factor case is examined.

REFERENCES

Anderson, J. E. 1974. "A Note on Welfare Surpluses and Gains From Trade in General Equilibrium." Amer. Econ. Rev., 64:758-62.

Bergson, A. 1975. "A Note on Consumer's Surplus." J. Econ. Literature, 13:38-44.

Currie, J. et al. 1971. "The Concept of Economic Surplus and Its Use in Economic Analysis." The Econ. Journal, 81:741-99.

Dupuit, J. 1844. "On the Measurement of the Utility of Public Works." Annales Des Ponts Et. Chaussees, Second Series, 8. Translation reprinted in D. Mumby, 1968 Transport.

Harberger, A. C. 1971. "Three Basic Postulates for Applied Welfare Economics: An Interpretive Essay." J. Econ. Literature, 9:785-97.

Hicks, J. R. 1940. "The Rehabilitation of Consumer's Surplus." Rev. Econ. Stud., 8:108-16.

Hicks, J. R. 1956. A Revision of Demand Theory. London: Oxford University Press.

Hueth, D. L., Richard E. Just and Andrew Schmitz. 1980. Applied Welfare Economics. Englewood Cliffs, N.J.: Prentics-Hall, Inc. forthcoming.

Just, Richard E. and D. L. Hueth. 1979. "Welfare Measures in a Multimarket Framework." Amer. Econ. Rev., 69:947-54.

Marshall, Alfred. 1930. Principles of Economics. 8th edition. London: Macmillan and Co.

Mishan, E. J. 1968. "What is Producer's Surplus?" Amer. Econ. Rev., 58:1269-82.

Richter, D. K. 1974. Games Pythagoreams Play. Discussion Pap. 743, Dept. of Econ., Univeristy of Rochester.

Samuelson, P. A. 1976. Foundations of Economic Analysis. 8th printing. New York: Atheum and Co.

Silberberg, Eugene. 1972. "Duality and the Many Consumer's Surpluses." Amer. Econ. Rev., 62:942-52.

Silberberg, Eugene. 1978. The Structure of Economics: A Mathematical Analysis. New York: McGraw-Hill Book Co.

U.S. Department of Agriculture. 1957. Agricultural Statistics.

U.S. Department of Agriculture. 1963. Agricultural Statistics.

U.S. Department of Agriculture. 1978. Agricultural Statistics.

Willig, R. 1973. Consumer's Surplus: A Rigorous Cookbook. Stanford University: Stanford, California: Institute for Mathematical Studies in the Social Sciences, Tech. Rept. No. 98.

Zellner, A. 1962. "An Efficient Method of Estimating Seemingly Unrelated Regressions on Tests for Aggregation Bias." Jour. Amer. Stat. Assoc., 57:348-368.

APPENDIXES

APPENDIX A

AN EXAMPLE OF WELFARE MEASURES IN A SINGLE PRODUCT AND FACTOR SECTOR

Consider a single product-single variable factor sector of the economy. Suppose at the kth level the following conditions pertain:

$Y_k = Y_{k-1}^{1/2}$ is the industry production function and its inverse $Y_{k-1} = Y_k^2$ exists. (A.1)

$P_k = Y_k^{-1/2}$ is the general equilibrium demand equation for Y_k. (A.2)

$P_{k-1} = 2Y_{k-1}$ is the general equilibrium factor supply equation for Y_{k-1}. (A.3)

Industry quasi rents are given by,

$$\pi_k = P_k Y_k - P_{k-1} Y_{k-1}. \qquad (A.4)$$

The first order condition for obtaining the industry derived partial equilibrium input demand function for Y_{k-1} is,

$$\frac{\partial \pi_k}{\partial Y_{k-1}} = \frac{P_k Y_{k-1}^{-1/2}}{2} - P_{k-1} = 0.$$

Solving for Y_{k-1} obtains the partial equilibrium input demand function,

$$Y_{k-1} = Y_{k-1}^d \ (P_k, P_{k-1}) = \frac{P_k^2}{4P_{k-1}^2} . \qquad (A.5)$$

Similarly, the first order conditions for the industry partial equilibrium product supply function for Y_k is,

$$\frac{\partial \pi_k}{\partial Y_k} = P_k - 2P_{k-1} Y_{k-1} = 0.$$

Solving for Y_{k-1} obtains the partial equilibrium supply function,

$$Y_k = Y_k^s\ (P_k, P_{k-1}) = Y_k = \frac{P_k}{2P_{k-1}}\ . \tag{A.6}$$

Notice, however that in order to find the equilibrium prices and quantities, one must obtain general equilibrium functions. Therefore, substituting the production function into (A.2) gives,

$$P_k = Y_k^{-\frac{1}{2}} = (Y_{k-1}^{\frac{1}{2}})^{-\frac{1}{2}} = Y_{k-1}^{-\frac{1}{4}}, \tag{A.7}$$

which is the profit maximizing induced relationship between P_k and Y_{k-1}. Substituting for P_k in (A.5) from (A.7) obtains the general equilibrium input demand function,

$$Y_{k-1} = \frac{(Y_{k-1}^{-\frac{1}{4}})^2}{4P_{k-1}^{\ 2}}\ ,$$

$$Y_{k-1}^{3/2} = \frac{1}{4P_{k-1}^{\ 2}}\ ,$$

$$Y_{k-1} = Y_{k-1}^d(P_{k-1}) = \frac{1}{4^{2/3}P_{k-1}^{\ 4/3}}\ . \tag{A.8}$$

Substituting the inverse industry production function into (A.3) for Y_{k-1} obtains,

$$P_{k-1} = 2Y_{k-1} = 2Y_k^2. \tag{A.9}$$

Now substituting for P_{k-1} from (A.9) into (A.6) yields the general equilibrium supply function,

$$Y_k = \frac{P_k}{2(2Y_k^2)} ,$$

$$Y_k^3 = \frac{P_k}{4} ,$$

$$Y_k = Y_k^s(P_k) = \frac{P_k^{1/3}}{4^{1/3}} . \qquad \text{(A.10)}$$

Setting (A.3) equal to (A.8) and (A.2) equal to (A.10) obtains the resulting general equilibrium prices and quantities for the k and k-1 markets.

$$\bar{P}_{k-1} = .9057 \qquad \bar{Y}_{k-1} = .4528$$

$$\bar{P}_k = 1.2190 \qquad \bar{Y}_k = .6729 \qquad \text{(A.11)}$$

Given the equilibrium values we are now able to determine what is being measured under the alternative supply and demand specifications. Consider first the producer surplus measure for the partial equilibrium supply curve $Y_k = Y_k^s\ (P_k, P_{k-1})$,

$$\Delta PS_k(P_k, P_{k-1}) = \int_0^{\bar{P}_k} Y_k^s(P_k, P_{k-1})\ dP_k ,$$

$$= \int_0^{\bar{P}_k} \frac{P_k}{2P_{k-1}} \, dP_k,$$

$$= \frac{P_k^2}{4P_{k-1}} \Big|_0^{1.2190} \, ,$$

$$= \frac{(1.219)^2}{4(.9057)} \, ,$$

$$= .4101 \qquad \text{(A.12)}$$

Note also that industry rents are found from (A.13),

$$\Delta\pi_k = P_k Y_k - P_{k-1} Y_{k-1},$$

$$= (1.219)(.6729) - (.9057)(.4528),$$

$$= .4101 \qquad \text{(A.13)}$$

Hence, from (A.12) and (A.13) one finds that under partial equilibrium supply functions,

$$\Delta PS_k = \Delta\pi_k . \qquad \text{(A.14)}$$

Now consider what is being measured from the general equilibrium supply curve given in (A.10).

$$\Delta PS_k(P_k) = \int_0^{\bar{P}_k} Y_k^s(P_k) \, dP_k,$$

$$= \int_0^{\bar{P}_k} \frac{P_k^{1/3}}{4^{1/3}} \, dP_k \, ,$$

$$= \frac{P_k^{4/3}}{4^{1/3}(4/3)} \Big|_0^{1.219} = .6152 \qquad (A.15)$$

Hence, one finds that $\Delta PS_k(P_k) \neq \Delta\pi_k$ as in (A.15). However, from (A.3) the producer surplus for the k-1 input is,

$$\Delta PS_{k-1}(P_{k-1}) = \int_0^{\bar{P}_{k-1}} Y^s_{k-1}(P_{k-1})\, dP_{k-1},$$

$$= \int_0^{\bar{P}_{k-1}} \frac{P_{k-1}}{2}\, dP_{k-1} \; \underline{1/},$$

$$= \frac{P_{k-1}^2}{4} \Big|_0^{.9057},$$

$$= .2051 \qquad (A.16)$$

Summing (A.13) and the result obtained in (A.16) gives,

$$\Delta PS_k(P_k) = \Delta\pi_k + \Delta PS_{k-1}(P_{k-1}),$$

$$= .4101 + .2051,$$

$$= .6152 \qquad (A.17)$$

Thus, when the industry price adjustment has been made for the input price, the producer surplus measure for the general equilibrium supply curve $Y_k(P_k)$ measures rent to the kth industry plus the producer surplus measure for the input.

Turning to the consumer surplus measures on the demand side one finds first that consumer surplus for the partial equilibrium function in (A.5) is,

$$\Delta CS_{k-1}(P_k,P_{k-1}) = \int_{P_{k-1}}^{0} Y^d_{k-1}(P_k,P_{k-1})\, dP_{k-1} ,$$

$$= \int_{P_{k-1}}^{0} \frac{P_k^2}{4P_{k-1}^2}\, dP_{k-1} ,$$

$$= - \frac{P_k^2}{4(-1)P_{k-1}} \Big|_{.9051}^{0} ,$$

$$= .4101 \qquad \text{(A.18)}$$

From (A.18), (A.13), and (A.12) one finds that with partial equilibrium supply and input demand, the resulting surplus measures imply,

$$\Delta PS_k = \Delta\pi_k = \Delta CS_{k-1}. \qquad \text{(A.19)}$$

Now the consumer surplus measure for the general equilibrium demand in (A.8) is,

$$\Delta CS_{k-1}(P_{k-1}) = - \int_{P_{k-1}}^{0} \frac{1}{4^{2/3}P_{k-1}^{4/3}}\, dP_{k-1} ,$$

$$= - \frac{1}{4^{2/3}(-1/3)P_{k-1}^{1/3}} \Big|_{.9057}^{0} ,$$

$$= 1.2305 \qquad \text{(A.20)}$$

However, from (A.2) the general equilibrium consumer surplus measure for the product Y_k is,

$$\Delta CS_k(P_k) = \int_{P_k}^{0} \frac{1}{P_k^2} \, dP_k ,$$

$$= \frac{1}{P_k} \Big|_{1.219}^{0} ,$$

$$= .8203 \qquad \text{(A.21)}$$

From (A.21), (A.20) and (A.13) one finds that with general equilibrium demands that,

$$\Delta CS_{k-1}(P_{k-1}) = \Delta CS_k(P_k) + \Delta \pi_k . \qquad \text{(A.22)}$$

Note also that total sector welfare can be found through summing producer and consumer surplus at the kth or k-1 levels. Hence,

$$\Delta PS_k(P_k) + \Delta CS_k(P_k) = .6152 + .8203 = 1.435 ,$$

$$\Delta PS_{k-1}(P_{k-1}) + \Delta CS_{k-1}(P_{k-1}) = .2051 + 1.2305 = 1.435 \qquad \text{(A.23)}$$

Thus, from (A.23) when surpluses are determined from general equilibrium functions, the total change in sector welfare is found at any market level.

FOOTNOTES

[1]Note from (A.3) $Y_{k-1} = \frac{P_{k-1}}{2}$.

APPENDIX B

AN EXAMPLE OF WELFARE MEASURES IN
A SINGLE PRODUCT AND MULTI-FACTOR SECTOR

Consider a vertical-horizontal market sector of the economy where at the k level $M_k=1$ and at the k-1 level $M_{k-1}=2$. Furthermore, assume the following industry conditions exist,

$Y_k = 2Y_{k-1,1}^{\frac{1}{2}} \; Y_{k-1,2}^{\frac{1}{4}}$ is the industry production function. (B.1)

$Y_{k-1,i} = Y_{k-1,i}^{s}(P_{k-1,i}) = \frac{P_{k-1,i}}{2}$ for i = 1,2 are the general equilibrium supply curves for the inputs. (B.2)

$Y_k = Y_k^d(P_k) = \frac{1}{P_k^2}$ is the general equilibrium demand function for the product Y_k. (B.3)

Industry quasi rent at the kth level in this sector is given by,

$$\pi_k = P_k Y_k - P_{k-1,1}\, Y_{k-1,1} - P_{k-1,2}\, Y_{k-1,2}. \qquad \text{(B.4)}$$

Now substituting (B.1) for Y_k in (B.4) then differentiating with respect to $Y_{k-1,i}$ i=1,2 gives the first order conditions for the industry.

$$\frac{\partial \pi_k}{\partial Y_{k-1,2}} = P_k\, Y_{k-1,1}^{-\frac{1}{2}}\, Y_{k-1,2}^{\frac{1}{4}} - P_{k-1,1} = 0 \qquad \text{(B.5)}$$

$$\frac{\partial \pi_k}{\partial Y_{k-1,2}} = 2^{-1}\, P_k\, Y_{k-1,1}^{\frac{1}{2}}\, Y_{k-1,2}^{-3/4} - P_{k-1,2} = 0 \qquad \text{(B.6)}$$

Solving (B.5) and (B.6) gives the industry expansion path,

$$\frac{2Y_{k-1,2}}{Y_{k-1,1}} = \frac{P_{k-1,1}}{P_{k-1,2}}\,. \qquad \text{(B.7)}$$

Now substituting (B.2) for $P_{k-1,i}$ i=1,2 in (B.7) gives,

$$Y_{k-1,1} = Y_{k-1,2} 2^{1/2} .$$

In order to obtain the general equilibrium demand functions for $Y_{k-1,1}$ and $Y_{k-1,2}$ substitute the inverse of (B.3) for P_k in (B.5) then substitute (B.1) for Y_k which gives,

$$P_k\, Y_{k-1,1}^{-1/2}\, Y_{k-1,2}^{1/4} = P_{k-1,1},$$

$$Y_k^{-1/2}\, Y_{k-1,1}^{-1/2}\, Y_{k-1,2}^{1/4} = P_{k-1,1},$$

$$2^{-1/2}\, Y_{k-1,1}^{-1/4}\, Y_{k-2}^{-1/8}\, Y_{k-1,1}^{-1/2}\, Y_{k-1,2}^{1/4} = P_{k-1,1},$$

$$2^{-1/2}\, Y_{k-1,1}^{-3/4}\, Y_{k-1,2}^{1/8} = P_{k-1,1}.$$

Now substituting the inverse of (B.8) for $Y_{k-1,2}$ yields the general equilibrium demand function for $Y_{k-1,1}$,

$$2^{-1/2}\, Y_{k-1,1}^{-3/4}\, Y_{k-1,1}^{1/8}\, 2^{-1/16} = P_{k-1,1},$$

$$2^{-9/16}\, Y_{k-1,1}^{-5/8} = P_{k-1,1}.$$

$$Y_{k-1,1} = Y_{k-1,1}^{d}\,(P_{k-1,1}) = P_{k-1,1}^{-8/5}\, 2^{-9/10} \qquad \text{(B.9)}$$

Similar substitutions into (B.6) yields the general equilibrium demand function for $Y_{k-1,2}$,

$$Y_{k-1,2} = Y^{d}_{k-1,2}(P_{k-1,2}) = P^{-8/5}_{k-1,2}\ 2^{-11/5}. \qquad \text{(B.10)}$$

Now setting (B.9) equal to (B.2) for i=1 then (B.10) equal to (B.2) for i=2 gives the equilibrium prices and quantities for the factors,

$$\bar{P}_{k-1,1} = 1.027 \qquad \bar{P}_{k-1,2} = .7262$$

$$\bar{Y}_{k-1,1} = .5135 \qquad \bar{Y}_{k-1,2} = .3631$$

In order to obtain the general equilibrium supply function for Y_k substitute the inverse of (B.8) into (B.1) which gives $Y_{k-1,1}$ as a function of Y_k.

$$Y_k = 2Y^{\frac{1}{2}}_{k-1,1}\ 2^{-1/8}\ Y^{\frac{1}{4}}_{k-1},$$

$$Y_{k-1,1} = 2^{-7/6}\ Y_k^{4/3} \qquad \text{(B.11)}$$

Substituting (B.8) into (B.1) yields $Y_{k-1,2}$ as a function of Y_k.

$$Y_{k-1,2} = 2^{-5/3}\ Y_k^{4/3} \qquad \text{(B.12)}$$

Now substituting (B.11) and (B.12) into (B.4) and differentiating with respect to Y_k gives the following industry first order condition,

$$\frac{\quad k}{Y_k} = P_k - P_{k-1,1}\ \frac{4}{3}\ 2^{-7/6}\ Y_k^{1/3} - P_{k-1,2}\ \frac{4}{3}\ 2^{-5/3}\ Y_k^{1/3} = 0. \qquad \text{(B.13)}$$

Since industry prices depend upon industry usage, substitute (B.11) into the inverse of (B.2) for i=1 then (B.12) into the inverse of (b.2) for i=2. This obtains the industry input prices as functions of Y_k,

$$P_{k-1,1} = 2^{-1/6}\, Y_k^{4/3} \qquad \text{(B.14)}$$

$$P_{k-1,2} = 2^{-2/3}\, Y_k^{4/3} \qquad \text{(B.15)}$$

Now substituting (B.14) and (B.15) into (B.11) gives the general equilibrium supply function for Y_k.

$$P_k = 2^{-1/6}\, Y_k^{4/3}\, \frac{4}{3}\, 2^{-7/6}\, Y_k^{1/3} + 2^{-2/3}\, \frac{4}{3}\, 2^{-5/3}\, Y_k^{1/3}\,,$$

$$P_k = \frac{2^{2/3}}{3}\, Y_k^{5/3} + \frac{2^{-1/3}}{3}\, Y_k^{5/3}\,,$$

$$P_k = \frac{(2^{2/3} + 2^{-1/3})}{3}\, Y_k^{5/3}\,,$$

$$Y_k = Y_k^s\,(P_k) = 1.14869\, P_k^{3/5}. \qquad \text{(B.16)}$$

Setting (B.16) equal to (B.3) gives the following equilibrium conditions for Y_k.

$$\bar{P}_k = .9480$$

$$\bar{Y}_k = 1.1125$$

Now, in order to show welfare measures from alternative supply and demand specifications let's consider the demand function for $Y_{k-1,1}$.

In the partial equilibrium case where all prices are assumed constant, the demand function is found by substituting (B.7) into (B.5) which gives,

$$Y_{k-1,1} = Y^{d}_{k-1,1}(P_k, P_{k-1,1}, P_{k-1,2}) = \frac{P_k^4}{2P^3_{k-1,1}P_{k-1,2}}. \tag{B.17}$$

However, if one allows for the industry price adjustment of $P_{k-1,2}$ the demand function is found by substituting the inverse of (B.8) into (B.5) which gives,

$$Y_{k-1,1} = Y^{d}_{k-1,1}(P_k, P_{k-1,1}) = \frac{P_k^4}{2^{\frac{1}{2}}P^4_{k-1,1}} . \tag{B.18}$$

One can also specify the demand of $Y_{k-1,1}$ as a function of just the input prices. In this case the product price adjustment for P_k is accounted for. This specification is found by substituting (B.2) into (B.1) into (B.5) then substituting (B.7) into (B.5) which will give,

$$Y_{k-1,1} = Y^{d}_{k-1,1}(P_{k-1,1}, P_{k-1,2}) = \frac{1}{2P^{7/5}_{k-1,1}P^{1/5}_{k-1,2}}. \tag{B.19}$$

We now have four alternative demand specifications: the general equilibrium case given by (B.9), the partical equilibrium case given by (B.17) and two in-between cases given by (B.18) and (B.19). In order to show the various welfare relationships, consider the general equilibrium case first. In this case, consumer surplus of the factor markets yields,

$$\Delta CS_{k-1,1}(P_{k-1,1}) = \int_{P_{k-1,1}}^{0} P_{k-1,1}^{-8/5}\, 2^{-9/10}\, dP_{k-1,1},$$

$$= - P_{k-1,1}^{-3/5}\, 2^{-9/10}(-3/5)^{-1}\Big|_{\bar{P}_{k-1,1}}^{0},$$

$$= .8789, \tag{B.20}$$

$$\Delta CS_{k-1,2}(P_{k-1,2}) = \int_{P_{k-1,2}}^{0} P_{k-1,2}^{-8/5}\, 2^{-11/5}\, dP_{k-1,2}$$

$$= - P_{k-1,2}^{-3/5}\, 2^{-11/5}(-3/5)^{-1}\Big|_{\bar{P}_{k-1,2}}^{0}$$

$$= .4394 \tag{B.21}$$

And from (B.4) industry rent is,

$$\pi_k = \bar{P}_k\, \bar{Y}_k - \bar{P}_{k-1,1}\, \bar{Y}_{k-1,1} - \bar{P}_{k-1,2}\, \bar{Y}_{k-1,2},$$

$$= .2636 \tag{B.22}$$

And from (B.3) consumer surplus of the product Y_k is,

$$\Delta CS_k\ (P_k) = \int_{P_k}^{0} P_k^{-2}\, dP_k,$$

$$= -P_k^{-1}\Big|_{P_k}^{0},$$

$$= 1.0547 \tag{B.23}$$

Now from the envelope theorem one finds that (B.24) should hold for the general equilibrium demand function in (B.9).

$$\Delta CS_{k-1,1} = \Delta\pi_k + \Delta CS_k - \Delta CS_{k-1,2},$$

$$= .2636 + 1.0547 - .4394$$

$$= .8789 \qquad \text{(B.24)}$$

Hence, for the general equilibrium case the consumer surplus of the input $Y_{k-1,1}$ is equal to the forward rent of the kth industry plus consumer surplus of the product Y_k less consumer surplus of the input $Y_{k-1,2}$. Note also, that (B.24) is a difference equation which can be solved such that the summation of consumer surpluses at the k+1 level measures all forward rents plus all final consumers surpluses of the finished products.

Now, from the partial equilibrium demand function consumer surplus is given by,

$$\Delta CS_{k-1,1}(P_k, P_{k-1,1}, P_{k-1,2}) = \int_{P_{k-1,1}}^{0} \frac{P_k^4}{2P_{k-1,1}^3 P_{k-1,2}} \, dP_{k-1,1},$$

$$= - \frac{P_k^4}{-2^2 P_{k-1,1}^2 P_{k-1,2}} \Big|_{P_{k-1,1}}^{0},$$

$$= .2636. \qquad \text{(B.25)}$$

The result in (B.25) is what would be expected from partial equilibrium analysis. That is, consumer surplus of the input $Y_{k-1,1}$ under a partial equilibrium demand should be equal to the forward industry rent. Comparing (B.25) to (B.22) one finds that this holds.

The two in-between cases remain to be examined; (B.18) and (B.19). In (B.18) the input price $P_{k-1,2}$ is omitted which implies that its effect on $Y_{k-1,1}$ has been accounted for. From the analysis in Chapter IV one would expect (B.26) to hold,

$$\Delta CS_{k-1,1}(P_k,P_{k-1,1}) = \Delta\pi_k - \Delta CS_{k-1,2}(P_k,P_{k-1,2}) \qquad \text{(B.26)}$$

Now since P_k is not allowed to adjust, while the input prices do, the demand for $Y_{k-1,1}$ is given by (B.18) and the demand for $Y_{k-1,2}$ is found by substituting (B.8) into (B.6) and solving which yields,

$$T_{k-1,2} = Y^d_{k-1,2}(P_{k-1},P_k) = 2^{-3}\, P_k^4\, P_{k-1,2}^{-4}. \qquad \text{(B.27)}$$

Integrating (B.18) and (B.27) gives the following consumer surplus measures,

$$\Delta CS_{k-1,1}(P_k,P_{k-1,1}) = -\int_{P_{k-1,1}}^{0} 2^{-\frac{1}{2}}\, P_k^4\, P_{k-1,1}^{-4}\, dP_{k-1,1},$$

$$= -2^{-\frac{1}{2}}\, P_k^4\, P_{k-1,1}^{-3}\, (-3)^{-1}\Big|_{P_{k-1,1}}^{0},$$

$$= .1758, \qquad \text{(B.28)}$$

$$\Delta CS_{k-1,2}(P_k,P_{k-1,2}) = -\int_{P_{k-1,2}}^{0} 2^{-3} P_k^4\, P_{k-1,2}^{-4}\, dP_{k-1,2},$$

$$= - 2^{-3} P_k^4 P_{k-1,2}^{-3}(-3)^{-1} \Big|_{P_{k-1,2}}^{0} ,$$

$$= .0878 \qquad (B.29)$$

Substituting (B.29) and (B.22) into (B.26) one finds,

$$\Delta CS_{k-1,1}(P_k, P_{k-1,1}) = \Delta\pi_k - \Delta CS_{k-1,2}(P_k, P_{k-1,2}),$$

$$= .2636 - .0878$$

$$= .1758$$

Comparing the above result with (B.28) one finds that (B.26) holds.

The last demand specification to be examined is (B.19). In this case the input prices are not allowed to adjust while the output price is allowed to adjust. This particular example would imply,

$$\Delta CS_{k-1,1}(P_{k-1,1}, P_{k-1,2}) = \Delta\pi_k + \Delta CS_k. \qquad (B.30)$$

Integrating (B.19) yields,

$$\Delta CS_{k-1,1}(P_{k-1,1}, P_{k-1,2}) = - \int_{P_{k-1,1}}^{0} 2^{-1} P_{k-1,1}^{-7/5} P_{k-1,2}^{-1/5} dP_{k-1,1}, \qquad (B.31)$$

$$= 2^{-1} P_{k-1,1}^{-2/5} P_{k-1,2}^{-1/5} (-2/5)^{-1} \Big|_{P_{k-1,1}}^{0} ,$$

$$= 1.3183$$

Substituting (B.22) and (B.23) into (B.30) obtains,

$$\Delta CS_{k-1,1}(P_{k-1,1}P_{k-1,2}) = \Delta\pi_k + \Delta CS_k,$$

$$= .2636 + 1.0547,$$

$$= 1.3183 \qquad (B.32)$$

Comparing (B.31) and (B.32) one finds that (B.30) holds.

From the above it has been demonstrated that under alternative demand specifications alternative welfare measures are forthcoming. It can also be demonstrated that the sum of welfare measures in the k industry are equivalent to the sum of welfare measures in the k-1 industry. From (B.2) the general equilibrium producer surplus measures for $Y_{k-1,1}$ and $Y_{k-1,2}$ are,

$$\Delta PS_{k-1,1}(P_{k-1,1}) = \int_0^{P_{k-1,1}} \frac{P_{k-1,1}}{2}\, dP_{k-1,1},$$

$$= \frac{P^2_{k-1,1}}{4} \Big|_0^{1.027},$$

$$= .2636 \qquad (B.33)$$

$$\Delta PS_{k-1,2}(P_{k-1,1}) = \int_0^{P_{k-1,2}} \frac{P_{k-1,2}}{2}\, dP_{k-1,2},$$

$$= \frac{P^2_{k-1,2}}{4} \Big|_0^{.7262},$$

$$= .1318 \qquad (B.34)$$

Furthermore, the general equilibrium producer surplus measure for the product Y_k is found by integrating (B.16),

$$\Delta PS_k(P_k) = \int_0^{.9480} 1.1486\ P_k^{3/5}\ dP_k\ ,$$

$$= .71793\ P_k^{8/5} \Big|_0^{.9480}\ ,$$

$$= .6591 \qquad \text{(B.35)}$$

Adding the general equilibrium surplus results for the kth industry (B.23) and (B.32) yields,

$$\Delta CS_k(P_k) + \Delta PS_k(P_k) = 1.0547 + .6591 = 1.7138 \qquad \text{(B.36)}$$

Similarly, adding (B.20) and (B.21) yields,

$$\Delta CS_{k-1,1}(P_{k-1,1}) + \Delta CS_{k-1,2}(P_{k-1,2}) + \Delta PS_{k-1,1}(P_{k-1,1}) + \Delta PS_{k-1,2}(P_{k-1,2})$$

$$= .8789 + .4394 + .2636 + .1318 = 1.7138 \qquad \text{(B.37)}$$

Comparing (B.36) with (B.37) one finds that total welfare is given at the k or k-1 industry levels.

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