AN ANALYTICAL INVESTIGATION OF AUDIT DECISION MAKING UNDER UNCERTAINTY

By

JANET ILLEEN KIMBRELL

Bachelor of Science in Education
Southeastern Oklahoma State University
Durant, Oklahoma
1969

Master of Professional Accounting
University of Texas at Arlington
Arlington, Texas
1976

Submitted to the Faculty of the Graduate College
of the Oklahoma State University
in partial fulfillment of the requirements
for the Degree of
DOCTOR OF PHILOSOPHY
December, 1979

AN ANALYTICAL INVESTIGATION OF AUDIT

DECISION MAKING UNDER

UNCERTAINTY

Thesis Approved:

Larry Christensen

Thesis Adviser

William T Anderson

Billy M. Thompson

Alan E. Bayer

John D Hampton

Norman N Durham

Dean of the Graduate College

PREFACE

This study consists of an investigation of the auditor's behavior under conditions of uncertainty, with an explicit consideration of risk aversion on the part of the auditor. The primary objective is to determine the effect of uncertainty and risk aversion on the output decisions of an auditor.

The presentation of this study is a deviation from the regular dissertation format due to the fact that the dissertation was funded by the Peat, Marwick, and Mitchell Foundation and prepared in a form to be presented to the Foundation.

The author wishes to express her appreciation to the members of her dissertation committee: Dr. Lanny G. Chasteen (Chairman), Dr. Wilton T. Anderson, Dr. Alan Baquet, Dr. John Hampton and Dr. Billy Thornton for their assistance.

Gratitude is also expressed to the Peat, Marwick, and Mitchell Foundation for their generous financial support.

A special thanks goes to Richard, Sean and Adam for their patience and understanding, and to Chester and Hazel Kimbrell for their encouragement.

Finally, a special appreciation is extended to Dr. James R. Boatsman for his advice, encouragement, constant support, and most importantly, his confidence.

TABLE OF CONTENTS

LIST OF FIGURES

Introduction

Interest in auditing services[1] is brought about by an aversion to uncertainty--uncertainty concerning the economic condition of a business firm. In order to reduce that uncertainty and protect the public welfare, the Securities Act of 1933 required that an independent accountant certify financial statements contained in registration statements. The Securities Exchange Act of 1934 further required that listed companies file annual reports and that those annual reports be certified by independent accountants.

In this paper a theory will be developed which will explain auditor behavior under uncertainty. In particular, the theory explains how the economically rational auditor makes decisions on the amount of auditing services he will offer in the face of uncertainty. It is important to be able to explain auditor behavior, because if the factors which influence auditor decision making are understood, then these factors can be manipulated to provide "natural" incentives to facilitate the regulation of auditor behavior. Considerable interest has been expressed in regulating auditor behavior, i.e., inducing auditors to make preferred decisions. Two congressional committees have expressed an increased concern regarding this issue. It is therefore important to understand the "natural" incentives which cause an auditor to make a decision, because there is reason to believe that operating on incentives is a more effective regulatory tool than is <u>imposition</u> from

[1]Auditing services as used in this paper applies only to the attest functions--not to internal auditing or any other work performed by public accountants.

outside the profession, such as government regulation.

Theory construction will begin by specifying an auditor decision model. But before the model is formulated, the nature and environment of the auditing profession will be discussed. Given that this is an economic analysis, it is important to understand the auditor's economic environment. The purpose here is to compare the auditing profession with other industries or professions on the basis of economic characteristics. Following this will be an explanation of the axiomatic structure underlying the development of the decision model to be considered. The next section specifies the model under several different economic scenarios. The final section discusses the results of varying these scenarios and the use of these results in inducing preferred auditing decisions.

Nature of the Auditing Profession

The majority of the 54 U. S. jurisdictions have certain regulatory statutes which restrict the licenses to express opinions on financial statements to persons having the title of Certified Public Accountant. Due to the structure of the regulatory statutes when considered together (the Securities Acts and these state laws), independent accountant has become synonymous with an independent Certified Public Accountant.

The auditor is responsible for making an independent verification of the financial statements and issuing an audit report on the findings of that verification. To obtain this verification, the auditor first studies and then makes an evaluation of the internal control system of the client firm. The auditor performs tests of compliance to evaluate the effectiveness of the system. Using the results of the evaluation, the auditor will then decide how much additional audit evidence should be collected. In general, there is an inverse relation between the assessed quality of internal control and the quantity of the requisite additional evidence. This process of collecting additional evidence includes physical observation, confirmation, etc., of the balances shown on the financial statements. Such procedures are referred to as substantive tests.

After reviewing events which have occurred after the balance sheet date, the auditor then assesses all the test results and makes a determination of the type of audit report to be expressed on the financial statements of the firm. The facet of the audit process of interest in this paper is substantive testing--specifically the choice of a level of substantive testing after the evaluation of internal control has been made.

The Securities Act of 1933 also established the auditor as being liable to any purchaser of the securities when the certified financial statements in the registration statement were shown to be misleading or false. If auditors are found to be negligent in the collection of audit evidence, they are liable for this negligence to those who relied upon the financial statements. Subsequent litigation has tended to support that view. It is appropriate, then, to conceive of some amount of audit evidence that is optimal for society which has to be performed before the auditor can be considered to be not negligent and thus devoid of liability. If the auditor shows "reasonable and due care" in the collection of audit evidence, then the liability is reduced. The model used in this paper will contain an explicit characterization of this potential liability or loss to the auditor.

Competitive Aspects of the Auditing Environment

Before looking at the model and methodology to be employed in this paper, it will be useful to consider some aspects of the supply and demand for auditing services.

The initial demand for auditing services is faily regular, since firms are required to have their statement certified on a regular basis. This is like most other services--except medical care or warfare military services. However, after the audit is undertaken and the evaluation of internal control is made, the auditor has to make a decision as to how many units of audit services he will supply (i.e, how many units of time will be allocated to the gathering of audit evidence).

The demand for audit services is associated with some probability of a loss to the firm. Without the audit service, the business firm would not be allowed to operate (or at least be unable to procure funds in the capital markets). With audit services, it is still not certain the business firm will continue to operate. It is possible that the audit may reveal information harmful to the firm's status, or even worse, it is possible the audit may not reveal information which, if discovered at a later time, could be harmful to the firm. Therefore, the need for audit services is risky in itself, making it similar to other commodities such as medical care. Choosing a physician will not necessarily mean a cure or prevention of illness. It is not being argued here that auditing is as risky as medical care, but that it bears some slight resemblence in this aspect.

The auditor is expected to behave differently from other sellers of services or products. Due to the requirement that he be independent, he is viewed to have less of a self-interest in the financial statements of his client. Due to the large potential damage which can come to third party users, the auditor has certain ethical considerations in offering his services--considerations beyond those, for example, of a clothing dealer or a book salesman. Society's expectation is that the auditor's concern for the correct information will override the desire to please the customers. (It could also be argued the auditor's shared liability for the large potential damage would override the desire to please the customers.) The auditor is viewed as a professional who provides expert certification of the financial statements of the client company. It is not apparent that this characteristic is as pronounced as in the medical profession, as evidenced by the amount of charity work which occurs in the medical profession (Mushkin, 1958), but there is less self-interest involved than in many fields. In fields in which self-interest is low, such as the medical profession, it is also claimed that financial considerations are not the main factors in the amount and type of services to be offered (Arrow, 1971). The opposite would hold for fields where self-interest is high. It is contended here that auditing services lie somewhere in the range between these two extremes and definitely not at the same end of the spectrum as the medical care profession. It would be difficult to defend the position that the auditor could avoid the consideration of the potential liability to him.

Product uncertainty is greater in the market for audit services than in that for many markets. The only way a consumer can test the quality of auditing service is through consumption and any results are not immediate since time is usually required to verify quality. There may also be the belief that the auditor, by virtue of being an expert in possession of highly technical skills, possesses information that the client does not have--information as to the manner in which the statements should be presented and the consequences of that presentation. This may or may not be true; but it could be perceived by the client as such.

To reduce this quality uncertainty regarding auditors' services, provision of auditing services is restricted to holders of the CPA certificate, which supposedly ensures their competence to perform the attest function. This licensing restricts entry to the profession, which of course, restricts supply and increases the cost of audit services. The extent to which the CPA certificate does this is unclear. It is contended by some (Friedman and Kuznets, 1945; also Pichler, 1973) that it is substantial. It has been pointed out that in the past, the failure rate on the CPA examination greatly exceeds that for the examinations to qualify doctors, lawyers, and dentists. It has been stated by unofficial AICPA sources that approximately 10% of those who take all four parts pass them on the first attempt (Pichler, 1973). Observations of a few of the state newsletters indicate it may be even lower. It has been pointed out, though, that the eventual pass rate by serious candidates writing the examination two or more times is probably as high as 80% and any restriction is more in the nature of a delay than an outright barrier to entry (Revsine and Juris, 1973).

In the past, the costs of an accounting education have been comparable to other fields requiring only a basic college education.

But the costs are higher than the cost of training for most trades. The rewards, though, for successful auditors are high--higher than for many other areas in business.

There is also evidence that the education requirements for CPA certificates are increasing. Before 1936, the educational requirements for writing the examination in all states was only the completion of high school (Carey, 1970). Since that time, almost all jurisdictions require some formal training above high school and a majority require a baccalaureate degree. The trend has been toward more years of formal education and a reduction in required work experience (Anderson, 1972). In 1969 the Board of Examiners for the American Institute of CPAs adopted a proposal to raise the examination standards by 1975 to a level of competence which requires at least five years of college work (Hendrickson, 1971).

Three states have recently passed laws requiring CPA candidates to have five years of college training and one other state is considering such laws (The Federation of Schools of Accountancy, 1977). There is also a trend towards Professional Schools of Accounting, most emphasizing a five-year program. There are now thirteen such schools in operation and sixteen more are being considered, revised, or discussed (Anderson, 1979). A parallel move in this direction is the accreditation by the AACSB of five-year programs in accounting (see Miller, 1977; Miller and Davidson, 1978; Olson, 1979; Pearson, 1979). The net result is higher education requirements associated with obtaining the CPA certificate. A related trend is also emerging for continuing education for those already holding CPA certificates (Anderson and Dowell, 1978). Raising the education requirements may not necessarily mean a limitation in the supply of auditors, but simply better trained accountants. One university which has a professional school with a five-year program recently published a report that 80% of its graduates who sat for the CPA exam for the first time in 1978 passed all four parts (National Association of State Boards of Accountancy, 1979).

It is argued that the five year accreditation plan for accounting schools would eliminate some sources of training as the number of schools achieving accreditation might be small and thus accreditation could limit supply. Pearson (1979) and Pichler (1973) have argued that standards are being raised because the educational backgrounds have increased; but the percentage passing remains constant. Pichler also points to the high passing rate for other fields such as medicine. His conclusion does not necessarily follow. Arrow (1971) contends the high passing rate for physicians is due to the extremely high standards for entrance to medical school, and thus the qualifying exam for physicians is redundant. Knowing the passing rates for examinations in two different areas says little about the standards in those areas.

It is also erroneous to argue that a constant percentage implies an increase in standards. The technical knowledge required to be an expert in accounting has increased significantly and it seems only reasonable that the amount of knowledge contained in the examination should also increase significantly.

An additional factor which concerns quality is that the attest function can only be performed by a CPA. Less trained personnel, such as accountants without CPA certificates, cannot legally offer these audit services in most states. This limits the range of quality of audit services. In a market where many qualities are offered, there are varying prices. By licensing, many alternative qualities of audit services are eliminated, and also the alternative prices that might occur. This same situation occurs in the medical profession where there are only a few paramedics, etc., performing medical care services (Arrow, 1971).

Pricing practices include fees for service and very little pre-payment. There also appears to be little closed panel practice (contracts binding the client to a particular auditing firm) whereas in many business firms, prepayment and exclusive service contracts are common competitive practices.

Until recently competitive bidding was forbidden by the AICPA code of ethics. The Antitrust Division of the Department of Justice brought suit to enjoin it from limiting competitive bidding and the AICPA then deleted this restriction from its code. But most states still have such competitive bidding sanctions; and these prove to be valid since the Sherman Act does not apply to state law. The extent, then, to which this rule is applied varies from state to state (Causey, 1976).

Advertising was also not previously permitted; but in 1978 the AICPA Rule 502 was amended to allow advertising after a U. S. Supreme Court ruled that restrictions on advertising were unconstitutional (Wood and Ball, 1978). It is too early at this point to determine the effect of this ruling change.

In summary, the regulatory bodies and courts may be making the auditing profession more competitive, whereas the actions of the auditing profession itself (by raising of standards or educational requirements) may be restricting competition. The upshot of the above is that it cannot be argued that auditing exists in a "pure" competition environment. There are limited restrictions to entry and not extensive price competition. But auditing is also not a true monopoly given the number of small and large firms in the industry. For this reason, the auditor's decision model considered in this paper will initially be examined under pure competition. Then the model will be examined under monopolistic conditions. By considering both extremes (price competition and monopoly) the analysis will facilitate prediction of the impact of changes in the degree of competition which may occur in the future.

Development of the Model and Its Underlying Axioms

Theory construction in auditing itself is in an embryonic state. One approach involves fitting some statistical model to a data set consisting of auditor decisions, e.g., internal control evaluations, obtained in some experimental setting. Examples include Ashton (1974), Warren (1975), and Joyce (1976). The difficulties with such hypothetical data are well known (Slovic and Lichtenstein, 1971). The major difficulty is the development of a general decision model from specific cases.

Another approach is the development of statistical decision rules. Both classical (Elliot and Rogers, 1972; Arkin, 1974; and Roberts, 1974) and Bayesian (Birnberg, 1964; Kraft, 1968; Tracy, 1969; Knoblett, 1970; Corless, 1972; and Smith, 1972) rules have been developed. Such works usually take factors such as the determination of audit risk as exogenous, and thus can only be considered tangential to the development of a theory about auditor behavior.

A third approach is exemplified by works such as Scott (1973, 1975), which view the auditor as a completely altruisitic agent who makes decisions in order to minimize the opportunity loss of some third party user(s)--remaining all the while oblivious to his own utility. The reasonableness of this view has been severely criticized by Magee (1975).

A fourth approach involves viewing the audit decision process as that of a rational economic being, i.e., one who seeks to maximize his own expected utility. Examples of this approach are provided by

Kinney (1975) and Magee (1977). However, these works have assumed that the auditors' utility functions are linear, i.e., that auditors are indifferent to risk. And there is evidence that auditors are indeed risk averse (Newton, 1977).

The proposed research is an expansion of the fourth approach described above. The method will be analytical and the auditor will be viewed as a rational economic agent. Further, risk aversion on the part of the auditor will be dealt with explicity. A particular audit decision--the choice of a level of substantive testing--will be formulated in competely general terms.

Models incorporating risk aversion have received little attention in the auditing literature; however, they have received extensive attention in the economics literature. Before 1959, uncertainty had played a relatively small role in decision models. In 1959, Mills showed that randomness (uncertainty) in demand will cause different results from the classical economics approach where demand is taken as deterministic. Other studies considering demand as a random variable and incorporating risk were Nelson (1961), Oi (1961), Tisdell (1963), Dhrymes (1964), Hymans (1966), Dreze and Gabszewicz (1967), Stigum (1969), Zabel (1970), Baron (1970 and 1971), Sandmo (1971), and Leland (1972). These studies, concerned with competitive and monopolistic firms, showed that output under uncertainty will be something different than output under certainty, and that the risk attitudes of the entrepreneur affect output.

Studies of uncertainty conditions were made for taxation effects (Penner, 1967; Mossin, 1968; and Stiglitz, 1969). These also showed that incorporating risk into the model will produce different consequences. Other studies along the same line considered the effect of uncertainty on savings decisions (Leland, 1968; Sandmo, 1970); on factor services (Walters, 1960); on utilization of capital (Smith, 1969; Meyer, 1975); on insurance decisions (Borch, 1963 and 1966; Mossin, 1968); and illegal activities (Becker, 1968; Allingham and Sandmo, 1972; Block and Heineke, 1973; Ehrlich, 1973; Sjoquist, 1973; Block and Heineke, 1975). The last group of studies have a slight similarity to this study in that there is an explicit penalty involved in both models. The auditor has a liability for not exercising due care and the law-breaker is penalized when convicted of breaking the law.

The model developed and analyzed in this study will use the axiomatic structure employed in many previous economic analyses--the expected utility model of von Neumann and Morgenstern. It is thus assumed that the expected utility model is a reasonable characterization of the actual behavior of auditor preferences for risky outcomes.[2] The expected utility model rests upon the five axioms discussed on the following pages.

---

[2]The following discussion on the expected utility model and the underlying axioms was developed from six sources: von Neumann and Morgenstern (1947), Luce and Raiffa (1957), Markowitz (1959), Arrow (1963), Borch (1968), Horowitz (1970), and Keeney and Raiffa (1976).

Axiom 1

A preference or indifference relation exists between any two items or any two sets of items. X_1 is at least as desirable as X_2 ($X_1 \succsim X_2$) or X_2 is at least as desirable as X_1 ($X_2 \succsim X_1$). In other words, the auditor can express whether he prefers X_1 to X_2 ($X_1 \succ X_2$), X_2 to X_1 ($X_2 \succ X_1$) or is indifferent between X_1 and X_2 ($X_1 \sim X_2$). This axiom is needed, for if an individual could not state his preference for a set of actions, then he would not be led to make a preferred decision.

Axiom 2

Preferences are transitive. If one exhibits a preference for X_1 over X_2, and X_2 over X_3, then X_1 should be preferred to X_3. $X_1 \succsim X_2$ and $X_2 \succsim X_3$ implies $X_1 \succsim X_3$. The X_i could represent different choices of careers or several risky undertakings composed of a set of payoffs x_j, each of which has a probability p_{ij}. Each of the x_j is subject to a transitive preference or indifference relation. This axiom can be written as:

$$X_i = ([p_{i1}]\ x_1;\ [p_{i2}]\ x_2;\ \ldots;\ [p_{ij}]\ x_j;\ \ldots;\ [p_{in}]\ x_n),\ i = 1, \ldots, m$$

If these are numbered such that $x_1 \succsim x_2 \succsim \cdots \succsim x_{j-1} \succsim x_j \succsim x_{j+1} \succsim \cdots \succsim x_n$, it would imply that x_1 is the most preferred payoff and x_n the least preferred.

Suppose there are three alternatives, X_1, X_2, and X_3, and the payoffs (say, net cash flows) are uncertain for each project. However, management makes the following probability assessments:

X_1 = ([.3] 400; [.2] 300; [.2] 200; [.3] 100)

X_2 = ([.2] 400; [.2] 300; [.1] 200; [.5] 100)

X_3 = ([.4] 400; [.2] 300; [.4] 0)

Labeling the x_j, $x_1 = 400$, $x_2 = 300$, ..., $x_5 = 0$, then $x_1 \succsim x_2 \succsim \cdots \succsim x_5$. Also, X_3 could be written as:

X_3 = ([.4] 400; [.2] 300; [0] 200; [0] 100; [.4] 0),

where $p_{31} = .4$, $p_{32} = .2$, $p_{33} = 0$, etc. Thus, the payoffs can be ordered and all projects can be considered to contain each of the payoffs, even if some happen to have a probability (p_{ij}) of zero.

The second axiom implies that individuals are consistent in their preferences. This may be difficult to believe as complexity of some situations may introduce ambiguity, but this analysis assumes that a given individual will be able to order preferences in a manner that is consistent with actual beliefs. This ordering is based upon attitudes toward risk and a prior ordering of payoffs (x_j). This second axiom is important because the individual will usually make the decision that is preferred; and by observing this consistency in preferences, a second party can determine the choice preferred by a decision maker.

Axiom 3

When the projects X_i (i = 1, ..., m) are themselves offered with probability of d_i as a subset of some superproject Y_k,

Y_k = ([d_1] X_1; [d_2] X_2; ... ; [d_i] X_i; ... ; [d_m] X_m), there can be determined a project $X^* = ([p_1]\ x_1; \ldots ; [p_j]\ x_j; \ldots ; [p_n]\ x_n)$ containing only the original payoffs x_j ($j = 1, \ldots, n$) such that one is indifferent between Y_k and X^* ($Y_k \sim X^*$). Also,

$$p_j = \sum_{i=1}^{m} d_i\ p_{ij}.$$

Consider the example used previously with X_1, X_2, and X_3 and these X_i each have probabilities of .2, .2, and .6, respectively. $Y_k = ([.2]\ X_1; [.2]\ X_2; [.6]\ X_3)$. One may compute the probability (p_j) of each payoff. For example, the probability of a 400 payoff (p_1) is given by the probability of getting X_1 times the probability of receiving 400 after choosing X_1 plus the probability of getting X_2 times the probability of receiving 400 after getting X_2, and so forth. Therefore,

$$p_1 = .2\ (.3) + .2\ (.2) + .6\ (.4) = .34$$

$$p_2 = .2\ (.2) + .2\ (.2) + .6\ (.2) = .20$$

$$p_3 = .2\ (.2) + .2\ (.1) + .6\ (0) = .06$$

$$p_4 = .2\ (.3) + .2\ (.5) + .6\ (0) = .16$$

$$p_5 = .2\ (0) + .2\ (0) + .6\ (.4) = .24$$

Note that the $\sum_{i=1}^{m} p_j = 1$, as all possible payoffs are considered. This axiom implies that given a project $X^* = ([.34]\ 400; [.20]\ 300; [.06]\ 200; [.16]\ 100; [.24]\ 0)$, and a superproject Y_k, then one would be indifferent between the two. In essence, the X^* project represents a simplification of Y_k, but does not change the payoffs or their probabilities. The necessity for this axiom is the need to compute and compare payoffs and their probabilities so that they may be ordered in a preference ranking.

Axiom 4

An individual can express preference and indifference between a certain or guaranteed payoff x_j and a risky alternative $\tilde{x}_j$ which involves only two alternatives, x_1 and x_n, where $x_1 \succ x_n$. In addition, probabilities t_j and $1-t_j$ can be found such that (1) $\tilde{x}_j \sim x_j$, where $\tilde{x}_j = ([t_j]\ x_1;\ [1-t_j]\ x_n)$; and (2) probabilities can be found such that $\tilde{x}_j' \succ \tilde{x}_j$, if and only if $t_j' \succ t_j$, where $\tilde{x}_j' = ([t_j']\ x_1;\ [1-t_j']\ x_n)$.

For example, suppose there is a certain payoff x_j = \$300 and a risky payoff $\tilde{x}_j$ which involves x_1 = \$400 with some probability t_j and x_n = \$0 with some probability $(1-t_j)$. Dependent upon the probabilities t_j and $(1-t_j)$, an individual can express preferences or indifferences. For example, he might prefer x_j = \$300 to an $\tilde{x}_j$ where $t_j = .4$. Furthermore, it is possible to ascertain probabilities for which he would be indifferent, in this case, say, $t_j = .75$. It can then be seen that if the probability of receiving x_1 = \$400 is greater than $t_j = .75$, then the risky alternative $\tilde{x}_j$, will be preferred to the certain x_j. When an individual is indifferent between two alternatives, one of which is an amount to be received for certain, that certain amount is called the "certainty equivalent" of the other, risky choice.

If, as in the above example, the certainty equivalent (or the certain amount, x_j) is equal to the expected value of the risky payoff $\tilde{x}_j$, the individual is referred to as being <u>linear</u> in risk. In other words, an individual is "indifferent" to risk if he is indifferent between the gamble (the risky payoff, $\tilde{x}_j$) or receiving the expected value of the gamble with certainty. Consider the cash flow values in the previous example, where x_1 = \$400, x_n = \$0. For the individual linear in risk, the following relations hold:

400 ∿ ([1] 400; [0] 0); 300 ∿ ([.75] 400; [.25] 0); 200 ∿ ([.5] 400; [.5] 0); 100 ∿ ([.25] 400; [.75] 0); 0 ∿ ([0] 400; [1] 0).

A risk-evader (or an individual who is said to be averse to risk) will not accept the risky alternative unless its expected value exceeds the certain payoff. A risk-taker will accept the risky alternative even though its expected value is less than the certain payoff. The risk-evader demands that the expected value be greater than the value of the certainty equivalent (the certain payoff) before he accepts it. The risk-taker is willing to accept a gamble which has an expected value less than its certainty equivalent. For example, a risk-taker may have the following relation: 300 ∿ ([.6] 400; [.4] 0). A possible relation for a risk-evader is 300 ∿ ([.9] 400; [.1] 0).

This axiom does not imply that all risk-evaders or risk-takers need the same probability to accept a gamble; nor does it imply that all individuals will consistently behave as one or the other. Risk attitude depends upon the situation. The axiom only implies that, in a given situation, a given individual will be indifferent between the risky alternative or the certain payoff, for some probabilities, t_j and $1-t_j$. Axiom 4 also states that there will exist a certainty equivalent for every risky choice. The implication of Axiom 4 may be depicted as shown in Figure 1.

The demands of the risk-evader lie above the straight line, because a higher probability of winning (a higher expected value) than that of the person linear in risk is required before acceptance of a risky alternative. The opposite will hold for the risk-taker as his demands lie below the straight line. Because it is assumed that the individual will always prefer a higher probability of winning to a lower probability of winning, the curves will be assumed to be monotonically increasing. It is possible for individuals to have a curve which might, at some point, lie above the line and some points below the line; but individuals are usually classified as being either risk-takers or risk-evaders. The risk-evader's curve will be assumed to be strictly concave and the risk-taker's curve will be assumed to be strictly convex. This is done for ease of mathematical treatment, even though there may be some points for which it does not always hold.

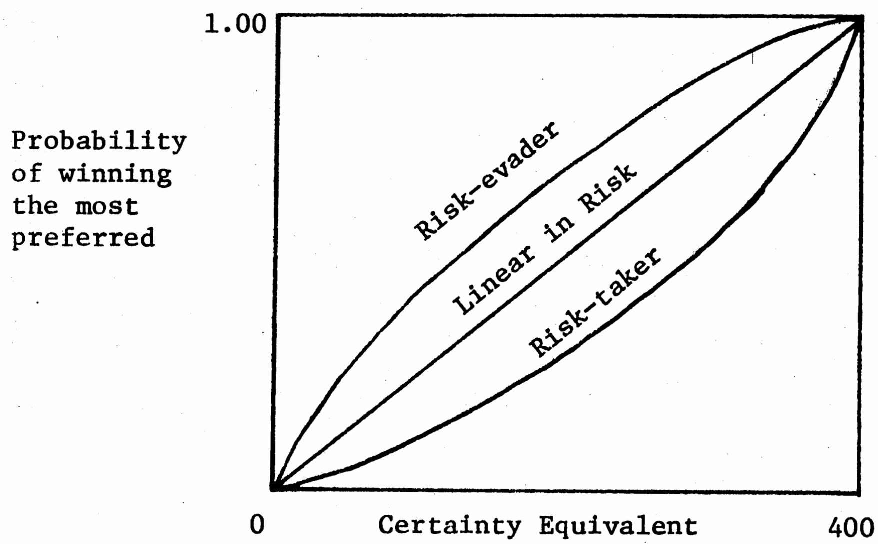

Figure 1. Utility Functions

Axiom 5

If the risky alternative $\tilde{x}_j$ is equally preferred to x_j and replaces the payoff x_j in a project X_i, the new project containing $\tilde{x}_j$ will be equally preferred to the old project containing x_j. In effect if

$\tilde{x}_j \sim x_j$, and $X_i' = ([p_{i1}]\ x_1; \ldots ; [p_{ij}]\ \tilde{x}_j; \ldots ; [p_{in}]\ x_n)$, then $X_i' \sim X_i$. This axiom implies that all of the x_j's could be replaced by their equally preferred risky alternatives. Invoking Axiom 3 then implies that the superproject X_i can then be replaced by the superproject $\tilde{X}_i$, which implies $\tilde{X}_i^* \sim \tilde{X}_i$, where $\tilde{X}_i$ is nothing more than a superproject containing subprojects $\tilde{x}_j = ([t_j]\ x_1; [1-t_j]\ x_n)$. Then a project, $\tilde{X}_i^* \sim \tilde{X}_i$, can be determined where $X_i^* = ([p_i]\ x_1; [1-p_i]\ x_n)$ and from the second axiom, as $X_i^* \sim \tilde{X}_i$ and $\tilde{X}_i \sim X_i$; then $X_i^* \sim X_i$. X_i may be replaced with X_i^* which only involves the most preferred and least preferred alternatives. The X_i^* which has the highest probability of winning will be the more preferred, or $X_i^* \succ X_k^*$ when $p_i > p_k$. (This does not mean that this analysis will tell an individual which decision he should make, but which decision is actually preferred. In particular $p_i > p_k$ only points out the fact X_i is preferred to X_k; $X_i \succ X_k$ not because $p_i > p_k$, but because $X_i \succ X_k$, p_i has to be greater than p_k.)

To illustrate the meaning of this fifth axiom, consider again the example used to explain the second axiom which had choices of X_1, X_2, and and X_3, and specific probabilities (p_{ij}) for each payoff x_j (see p. 16). Furthermore, suppose a particular individual has the following preferences relations:

$400 \sim ([1]\ 400; [0]\ 0)$; $300 \sim ([.90]\ 400; [.10]\ 0)$; $200 \sim ([.80]\ 400; [20]\ 0)$; $100 \sim ([.70]\ 400; [.30]\ 0)$; $0 \sim ([0]\ 400; [1]\ 0)$.

This individual displays the characteristics of risk aversion and for him, $X_1 \sim \tilde{X}_1$, where

$$\tilde{X}_1 = [.3]\ ([1]\ 400; [0]\ 0); [.2]\ ([.90]\ 400; [.10]\ 0); [.2]\ ([.80]\ 400; [.20]\ 0); [.3]\ ([.70]\ 400; [.30]\ 0); [0]\ ([0]\ 400; [1]\ 0).$$

Then using the third axiom, $X_1 \sim X_1^*$, where $X_1^* = ([.85]\ 400; [.15]\ 0)$. Also, $X_2 \sim X_2^* = ([.81]\ 400; [.19]\ 0)$; and $X_3 \sim X_3^* = ([.58]\ 400; [.42]\ 0)$. For this risk averse individual, $p_1 > p_2 > p_3$ and this implies that $X_1 \succ X_2 \succ X_3$. Since $X_1^* \sim X_1$, $X_2^* \sim X_2$, and $X_3^* \sim X_3$, then $X_1 \succ X_2 \succ X_3$. Thus X_1 is the preferred project for this specific individual. This is also the project preferred by the individual that is linear in risk as it is the project with the highest expected value. And it has been shown (von Neumann and Morgenstern, 1947) that individuals linear in risk will choose the project with the highest expected value. But the risk-taker will not necessarily choose X_1. In fact, the risk-taker will choose X_3 in this case, because it offers the highest probability of receiving 400 (and incidentally, the highest probability of receiving 0). If the risk-taker asserted equivalences as did the risk-averse individual above, it could also be shown in this way that he would choose X_3.

Axiom 5 (sometimes referred to as the substitution principle) implies that if two payoffs are equally preferred, it does not matter (1) which is promised, (2) whether one is risky, or (3) if one or both are certain. This says that the risky situation itself does not affect one's preference. For example, suppose an individual can draw from an urn which contains one red and one blue ball and he is indifferent between receiving \$300 with certainty and gambling on \$400 with a probability of .5. Then the individual will also be indifferent when given the following two choices: (1) draw a red ball and receive \$300 or draw a blue ball and receive zero; or (2) draw a red ball and get to accept the gamble or draw a blue ball and get nothing. It is not the risky situation itself which affects one's decision--it is the individual's set of preferences, attitudes toward risk, and the set of probability assessments made by the individual. When any one of these differs among different individuals, then it is possible (and most likely) that their decisions will be different.

An individual who conforms to these axioms will be able to state preferences and by observing his preferences, a utility function $U = u(x_j)$ denoting his preferences can be derived. It has been shown (von Neumann and Morgenstern, 1947) that an individual will choose that alternative which will maximize expected utility. The function $U = u(x_j)$ reflects one's index or ranking for certain and uncertain outcomes. Alternatively, it might be thought of as a rule by which the individual orders preferences.

Axiom 4 implies that $u(x_j)$ increases with increasing x_j which means $u'(x_j) > 0$. The shape of the utility function is not derived from the axiom, but rather from the personal risk attitudes of the individuals.

Assume the simplest linear case where $u(x_j) = x_j$, as shown in Figure 2. Now consider the gamble $\tilde{x} = ([p]\ x;\ [1-p]\ 0)$, where p is a given probability. The expected value of $\tilde{x}$ is px. The individual can also state a certainty equivalent, C, an amount to which he would be indifferent between receiving that amount for certain and the gamble, $\tilde{x}$. Thus, by assumption, the utility of C is equal to the expected utility of its equivalent gamble.

Now the utility of the expected value of the gamble is:

$$u(px) = px.$$

The expected utility of the risky situation $\tilde{x}$ (the gamble) is:

$$E[u(\tilde{x})] = pu(x) + (1-p)\ u(0)$$

$$= pu(x)$$

Since $px = pu(x)$, then $E[u(\tilde{x})] = u(px) = pu(x)$.

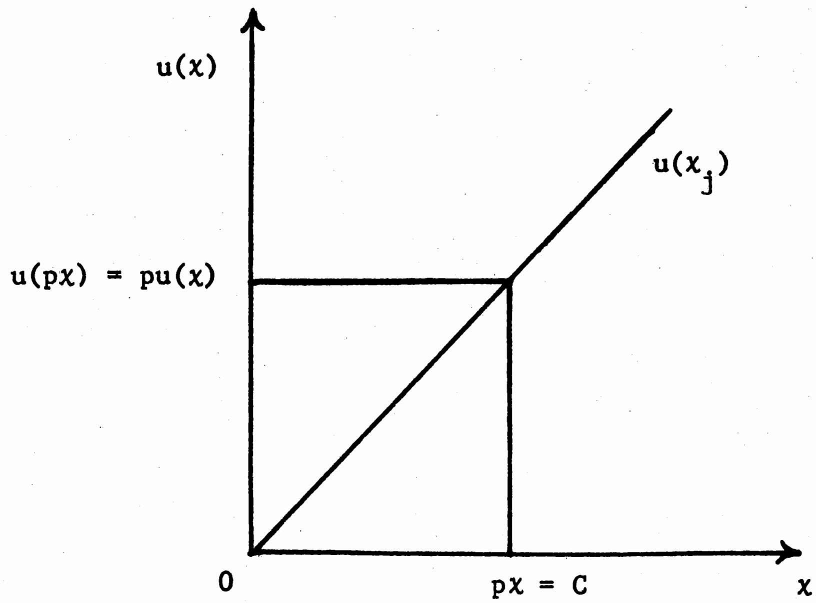

Figure 2. A Risk Linear Utility Function

which states, in words, that the utility of the expected value of the gamble is equal to the expected utility from the gamble. In addition, since, $u(p\chi) = pu(\chi) = p\chi = C$, then the expected value of the gamble is also equal to the certainty equivalent of the gamble.

As noted earlier, an individual will choose the alternative that maximizes expected utility. Thus, when $u(\chi_j)$ is linear, only the expected value of the risky situation need be used in making choices between alternatives.

Now consider the case where $u(\chi_j)$ is concave. This will imply a curve of the form in Figure 3, for which $u'(\chi_j) > 0$ and $u''(\chi_j) < 0$.

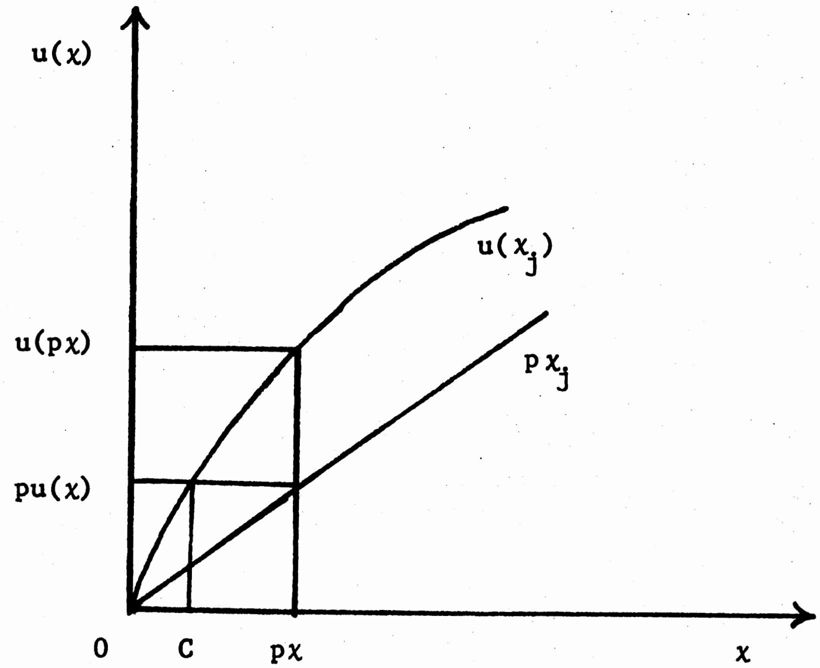

Figure 3. A Risk Averse Utility Function

Consider again the gamble:

$$\tilde{\chi} = ([p]\ \chi;\ [1-p]\ 0)$$

where p is a given probability and pχ is the expected value of $\tilde{\chi}$. There will exist a certainty equivalent C for which the individual will be indifferent between receiving C for certain and $\tilde{\chi}$. Again the utility of C is equal to the expected utility of its equivalent gamble, $\tilde{\chi}$.

Likewise, the utility of the expected value of the gamble is:

$$u(p\chi) = p\chi.$$

The expected utility of the risky situation $\tilde{\chi}$ (the gamble) is:

$$E[u(\tilde{\chi})] = pu(\chi) + (1-p)\ u(0) = pu(\chi).$$

Since $u(x_j)$ is strictly concave, then,

$$u(px) > pu(x),$$

which, is exactly the definition for risk averse utility functions (Keeney and Raiffa, 1976).

This implies that for the risk averse individual, the utility of the expected value of the gamble is greater than the expected utility of the gamble.

Also, since

$$u(C) = E[u(\tilde{x})] = pu(x)$$

then

$$u(px) > u(C)$$

and $u(x_j)$ being concave,

$$px > C.$$

Thus, for the risk averse case, the expected value of a gamble is also greater than the certainty equivalent of that gamble.

The convex case, where $u'(x) > 0$ and $u''(x) > 0$, is portrayed in Figure 4.

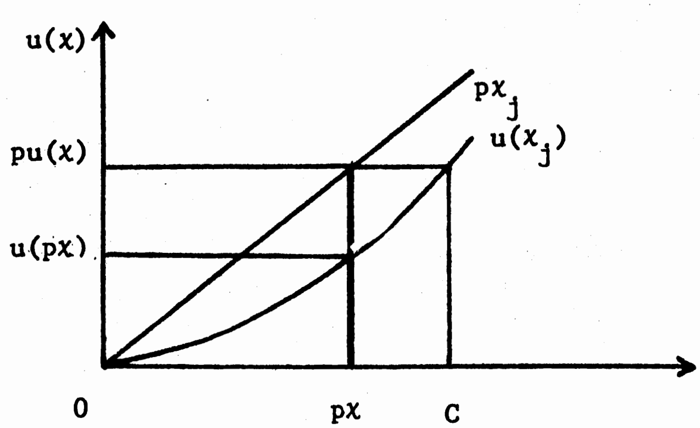

Figure 4. A Risk-Loving Utility Function

Assume the same risky gamble, $\tilde{x}$, as in the two previous cases, with expected value px, and a certainty equivalence C, where

$$u(C) = E[u(\tilde{x})],$$

$$u(px) = px,$$

and

$$E[u(\tilde{x})] = pu(x).$$

As $u(x_j)$ is strictly convex, then

$$u(px) < p[u(x)]$$

which is the definition for a risk loving utility function. Then,

$$u(px) < u(C)$$

implying

$$px < C.$$

In words, for the risk loving individual, the utility of the expected value of the gamble is less than the expected utility of the gamble, <u>and</u> the expected value of the gamble is less than the certainty equivalent of the gamble.

This is not to say that all individuals are strictly concave or strictly convex, as there is some evidence that individuals have utility functions which are convex in one interval and concave in another interval (Friedman and Savage, 1948). It has also been shown that it is unlikely for individuals to be risk indifferent (Borch, 1963). But in this analysis, it is assumed that consideration is made of only the relevant portion of that curve depicted by an individual's utility function and can regard the individual as being total risk indifferent or totally risk averse within that portion.

With this basic understanding of the expected utility hypothesis, the model for the auditor can be developed. This analysis will first consider the auditor in a competitive environment as if he were indifferent to risk and then averse to risk. The impact of varying these conditions on the auditor's decision to produce units of audit service will be assessed. The analysis will then consider the auditor with a linear utility function versus a concave utility function in a monopolistic setting. Additional analysis will consider the effect of the ways penalty is assessed upon the auditor.

Model I--An Auditor's Model Under Pure Competition

Under the conditions of pure competition the auditor has no control over the price of the units of credit services to be offered. The auditor may incur, in addition to the deterministic costs to be incurred in the production of the units of audit service, a penalty or additional loss because of his failure to offer enough audit units. Therefore, an auditor's profit function (π) will be of the form $\pi = PQ - C_1(Q) - C_2(Q)$, where Q denotes demand for units of audit services. Demand is the amount of services that society (or courts of law) will deem as being a sufficient amount for this particular audit. This can be thought of as an operationalization of the "due audit care" standard. If the auditor does not meet this amount, i.e., exercise due audit care, then there will be some penalty assessed. The amount of Q demanded by society is unknown at the time the auditor must make a decision. Let $Q = D(P, v)$ where v is a random variable distributed by g(v) with zero mean and P is the price for one unit of audit service.[2] For simplicity, the unit will refer to a unit of time, which assumes that time reflects audit quality. While few would argue that time is synonomous with audit quality, the two terms certainly exhibit strong positive association. In any event, under pure competition, P is given, Q itself becomes a random variable and for simplicity, the probability density function of Q will be represented by f(Q) (instead of D(P,v)).

---

[2] See footnote 3 on page 43 for an additional explanation of g(v).

The direct cost of producing Q is $C_1 = c_1(Q)$. Also, $C_2 = c_2(Q)$ is the penalty costs associated with the failure of the auditor to produce what may ultimately be determined to be an adequate audit. Q^* is the amount the audit firm decides to produce for a particular audit. If the amount supplied by the firm, (Q^*), is less than the amount demanded by society (Q), there will be a penalty. If $Q \leq Q^*$, then no penalty will be assessed. This analysis will make the assumption that there will be a penalty assessed for all $Q > Q^*$. In Model I, assume this penalty C_2 to be a function of Q, the amount that should have been audited. There is no way of knowing if all "incorrect" audits will be protested, but it can be argued that, in general, over a period of time any negligence will be discovered.

The auditor's utility function is denoted as $U = u(\pi)$ where U is assumed to be a monotonically increasing function. The auditor's profit function will consist of two parts:

$$\pi_1 = PQ - c_1(Q) \quad \text{for } Q \leq Q^*, \text{ and}$$

$$\pi_2 = PQ - c_1(Q) - c_2(Q) \quad \text{for } Q \geq Q^*.$$

Given the axiomatic structure described earlier, the auditor's decision can be characterized in terms of maximizing expected utility:

$$E[u(\pi)] = \int_0^{Q^*} u[PQ^* - c_1(Q^*)]\, f(Q)dQ + \int_{Q^*}^{\infty} u[PQ^* - c_1(Q^*) - c_2(Q)]f(Q)dQ. \tag{1}$$

For this expression to be at a maximum, two conditions must hold:

$$\frac{dE[u(\pi)]}{dQ^*} = 0,$$

and

$$\frac{d^2E[u(\pi)]}{dQ^*} < 0.$$

The satisfaction of the second order condition is shown in the Appendix. The first derivative of the expected utility is:

$$\frac{dE[u(\pi)]}{dQ^*} = \int_0^{Q^*} u'(P - \frac{dc_1}{dQ^*}) f(Q)dQ + (1)[PQ^* - c_1(Q^*)] f(Q^*) -$$

$$(0)\ u[PQ^* - c_1(Q^*)] f(Q^*) + \int_{Q^*}^{\infty} u'[P - \frac{dc_1}{dQ^*} - \frac{dc_2}{dQ^*}]$$

$$f(Q)dQ - (1)\ u[PQ^* - c_1(Q^*) - c_2(Q^*)] f(Q^*), \tag{2}$$

where $u' = \frac{du}{d\pi}$.

The last term collapses to $u[PQ^* - c_1(Q^*)] f(Q^*)$ as this is the expression evaluated where $Q = Q^*$ and here $c_2 = 0$. Thus the expression becomes:

$$\frac{dE(u)}{dQ^*} = \int_0^{Q^*} u'(P - \frac{dc_1}{dQ^*}) f(Q)dQ + \int_{Q^*} u'(P - \frac{dc_1}{dQ^*} - \frac{dc_2}{dQ^*}) f(Q)dQ. \tag{3}$$

$$\frac{dE(u)}{dQ^*} = \int_0^{\infty} u'(P - \frac{dc_1}{dQ^*}) f(Q)dQ - \int_{Q*}^{\infty} u'(P - \frac{dc_1}{dQ^*}) f(Q)dQ +$$

$$\int_{Q*}^{\infty} u'(P - \frac{dc_1}{dQ^*} - \frac{dc_2}{dQ^*}) f(Q)dQ. \tag{4}$$

$$\frac{dE(u)}{dQ^*} = \int_0^{\infty} u'(P - \frac{dc_1}{dQ^*}) f(Q)dQ + \int_{Q*}^{\infty} u'(- \frac{dc_2}{dQ^*}) f(Q)dQ. \tag{5}$$

As P and $\frac{dc_1}{dQ^*}$ are constants, then:

$$\frac{dE(u)}{dQ^*} = (P - \frac{dc_1}{dQ^*}) \int_0^{\infty} u'\ f(Q)dQ - \int_{Q*}^{\infty} u'(\frac{dc_2}{dQ^*}) f(Q)dQ. \tag{6}$$

Setting (6) equal to zero,

$$(P - \frac{dc_1}{dQ^*}) \int_0^{\infty} u' \; f(Q)dQ = \int_{Q^*}^{\infty} u' \; (\frac{dc_2}{dQ^*}) \; f(Q)dQ \tag{7}$$

$$P = \frac{dc_1}{dQ^*} + \frac{\int_{Q^*}^{\infty} u' \; (\frac{dc_2}{dQ^*}) \; f(Q)dQ}{\int_0^{\infty} u' \; f(Q)dQ} \tag{8}$$

This is the optimal output condition under uncertainty and perfect competition. Given that c_2 is a decreasing function of Q^*, $\frac{dc_2}{dQ^*} < 0$, the second expression on the right side of (8) will always be negative. This implies that marginal costs of production will always be greater than price, which, in turn, implies that output increases as uncertainty is introduced into the model. To understand why output increases under uncertainty, consider the model under certainty. Here the auditor acts as if expected demand ($\bar{Q}$) is the certain demand ($\bar{Q} = \int_0^{\infty} Qf(Q)dQ$), so the firm will produce at $\bar{Q}$ under certainty. Given that price is fixed, then:

$$P = \frac{dc_1}{d\bar{Q}} + \frac{\int_{Q^*}^{\infty} u' \frac{dc_2}{d\bar{Q}} \; f(Q)dq}{\int_0^{\infty} u' \; f(Q)dQ} \tag{9}$$

for the certainty output and

$$P = \frac{dc_1}{dQ^*} + \frac{\int_{Q^*}^{\infty} u' \; \frac{dc_2}{dQ^*} \; f(Q)dq}{\int_0^{\infty} u' \; f(Q)dQ} \tag{10}$$

for the uncertainty output. Then:

$$\frac{dc_1}{d\bar{Q}} + \frac{\int_{Q^*}^{\infty} u' \; \frac{dc_2}{d\bar{Q}} \; f(Q)dQ}{\int_0^{\infty} u' \; f(Q)dQ} = \frac{dc_1}{dQ^*} + \frac{\int_{Q^*}^{\infty} u' \; \frac{dc_2}{dQ^*} \; f(Q)dQ}{\int_0^{\infty} u' \; f(Q)dQ} \; . \tag{11}$$

But $\bar{Q} = Q$ implies $\dfrac{dc_2}{d\bar{Q}} = 0$, then:

$$\frac{dc_1}{d\bar{Q}} = \frac{dc_1}{dQ^*} + \frac{\int_{Q^*}^{\infty} u' \frac{dc_2}{dQ^*} f(Q)dQ}{\int_0^{\infty} u' f(Q)dQ} \tag{12}$$

and $\dfrac{dc_1}{d\bar{Q}} < \dfrac{dc_1}{dQ^*}$ as $\dfrac{dc_2}{dQ^*} < 0$.

If marginal costs for production are rising, then the output under uncertainty is greater than the output under certainty and then $\bar{Q} \leq Q^*$. If marginal costs are falling, then $\bar{Q} \geq Q^*$, but the auditor will produce only as long as MR = P $\geq$ AVC or MC $\geq$ AVC which is where marginal cost is rising. The firm would not operate if marginal costs are falling, and $\bar{Q} = Q^* = 0$. This is not a case to be considered for the rational auditor.

Introducing uncertainty into the auditor's model, then, will cause an increase in the number of units to be offered. In effect, a larger output is produced to lessen the probability of demand being greater than production. This has the effect of reducing the probability of a penalty. The penalty will increase <u>total</u> marginal costs; and to avoid that increase in total marginal costs, output is increased to cause a decrease in the expected penalty costs.

Looking again at the expression of maximization from (10), it can be rearranged to give it a more meaningful interpretation:

$$P \int_0^{\infty} u' f(Q)dq = \frac{dc_1}{dQ^*} \int_0^{\infty} u' f(Q)dQ + \int_{Q^*}^{\infty} u' \frac{dc_2}{dQ^*} f(Q)dQ, \tag{13}$$

and $\int_0^{\infty} u' f(Q)dQ$ is the expected marginal utility.

The expected marginal utility when $Q \geq Q^*$ is $\int_{Q^*}^{\infty} u' \, f(Q)dQ$. Thus, (13) is an optimal condition which states that the expected marginal utility of price equals the expected marginal utility of marginal costs.

For the individual that is linear in risk preferences (i.e., indifferent to risk), u' is a constant, $\int_0^{\infty} f(Q)dQ = 1$, and

$$P = \frac{dc_1}{dQ^*} + \frac{dc_2}{dQ^*} \int_{Q^*}^{\infty} f(Q)dQ \tag{14}$$

or price is equal to expected marginal costs.

The expression $\int_{Q^*}^{\infty} f(Q)dQ$ is the probability that $Q \geq Q^*$ and thus that a penalty will occur. The last term in (14) then is the expected marginal cost of a penalty. The individual will produce where price is equal to the marginal production costs, reduced by some expected marginal penalty costs. The auditor produces more to allow for this reduction.

For the individual who displays risk aversion, the expression remains as in (8). If it can be shown that the maximization output for the risk linear individual is less than the maximization output for the risk averse individual, then this implies that the auditor would audit more as he becomes more risk averse.

Since P is fixed, and letting Q_L^* represent output in the linear case, Q_A^* represent output in the risk averse case, then:

$$\frac{dc_1}{dQ_L^*} + \int_{Q^*}^{\infty} \frac{dc_2}{dQ_L^*} f(Q)dQ = \frac{dc_1}{dQ_A^*} + \frac{\int_{Q^*}^{\infty} u'\left(\frac{dc_2}{dQ_A^*}\right) f(Q)dQ}{\int_0^{\infty} u' \, f(Q)dQ} \, . \tag{15}$$

Now consider only the expressions

$$\int_{Q^*}^{\infty} f(Q)dQ \quad \text{and} \quad \frac{\int_{Q^*}^{\infty} u'\ f(Q)dQ}{\int_{0}^{\infty} u'\ f(Q)dQ}$$

to determine their relationship. It is of interest whether

$$\int_{Q^*}^{\infty} f(Q)dQ \gtreqless \frac{\int_{Q^*}^{\infty} u'\ f(Q)dQ}{\int_{0}^{\infty} u'\ f(Q)dQ} . \tag{16}$$

The symbol "?" will be used until we have determined the sign of (16).

$$\int_{Q^*}^{\infty} f(Q)dQ \ ?\ \frac{\int_{Q^*}^{\infty} u'\ f(Q)dQ}{\int_{0}^{\infty} u'\ f(Q)dQ} . \tag{17}$$

$$\int_{Q^*}^{\infty} f(Q)dQ \int_{0}^{\infty} u'\ f(Q)dQ \ ?\ \int_{Q^*}^{\infty} u'\ f(Q)dQ. \tag{18}$$

$$[1 - \int_{0}^{Q^*} f(Q)dQ] \int_{0}^{\infty} u'\ f(Q)dQ \ ?\ \int_{Q^*}^{\infty} u'\ f(Q)dQ. \tag{19}$$

$$\int_{0}^{\infty} u'\ f(Q)dQ - \int_{0}^{Q^*} f(Q)dQ \int_{0}^{\infty} u'\ f(Q)dQ \ ?\ \int_{0}^{\infty} u'\ f(Q)dQ - \int_{0}^{Q^*} u'\ f(Q)dQ. \tag{20}$$

$$-\int_{0}^{Q^*} f(Q)dQ \int_{0}^{\infty} u'\ f(Q)dQ \ ?\ -\int_{0}^{Q^*} u'\ f(Q)dQ. \tag{21}$$

Note that for $Q \leq Q^*$, profits will not exceed the π_1 level (where $\pi_1 = PQ^* - c_1(Q^*)$ as defined on page 28), $\frac{du}{d\pi}$ is a constant function at $\frac{du(Q^*)}{d\pi(Q^*)} = \hat{u}'$. Therefore:

$$-\int_{0}^{Q^*} f(Q)dQ \int_{0}^{\infty} u'\ f(Q)dQ \ ?\ -\hat{u}' \int_{0}^{Q^*} f(Q)dQ. \tag{22}$$

$$-\int_{0}^{\infty} u'\ f(Q)dQ \ ?\ -\hat{u}'. \tag{23}$$

$$-\hat{u}' \int_0^{Q^*} f(Q)dQ + \int_{Q^*}^{\infty} u' \, f(Q)dQ \ ? \ - \hat{u}'. \tag{24}$$

$$-\int_{Q^*}^{\infty} u' \, f(Q)dQ \ ? \ - \hat{u}' \left[1 - \int_0^{Q^*} f(Q)dQ\right]. \tag{25}$$

$$-\int_{Q^*}^{\infty} u' \, f(Q)dQ \ ? \ - \hat{u}' \left[\int_{Q^*}^{\infty} f(Q)dQ\right]. \tag{26}$$

Given that $\frac{d(u')}{dQ} = u''(\frac{d\pi}{dQ}) = u''\ (-\frac{dc_2}{dQ})$, and $u'' < 0$, $\frac{dc_2}{dQ} > 0$, then $u''\ (-\frac{dc_2}{dQ}) > 0$. This implies u' is an increasing function of Q. Therefore, $\hat{u}'$ is never greater than any u' on the left and $u' > \hat{u}'$ and $-u' < -\hat{u}'$, making the questioned sign "?" a "<". Therefore,

$$\int_{Q^*}^{\infty} f(Q)dQ < \frac{\int_{Q^*}^{\infty} u' \, f(Q)dQ}{\int_0^{\infty} u' \, f(Q)dQ} . \tag{27}$$

In addition, we know both expressions are less than one, but greater than zero. Now, reconsider the equality in (15).

$$\frac{dc_1}{dQ_L^*} + \int_{Q^*}^{\infty} \frac{dc_2}{dQ_L^*} f(Q)dQ = \frac{dc_1}{dQ_A^*} + \frac{\int_{Q^*}^{\infty} u'(\frac{dc_2}{dQ_A^*}) \ f(Q)dQ}{\int_0^{\infty} u' \, f(Q)dQ} \tag{15}$$

For this equality to hold, there are three cases. It is known from the assumptions about costs that $\frac{dc_1}{dQ^*} > 0$, $\frac{d^2c_1}{dQ^{*2}} > 0$, $\frac{dc_2}{dQ^*} < 0$, and $\frac{d^2c_2}{dQ^{*2}} > 0$.

Case I. Suppose the second term on the left side of the equality is greater than the second term on the right side of (15). This implies $\frac{dc_1}{dQ_L^*} < \frac{dc_1}{dQ_A^*}$ which in turn implies $Q_L^* < Q_A^*$. If $Q_L^* < Q_A^*$, then $C_{2_L} > C_{2_A}$ which means $\frac{dc_2}{dQ_L^*} < \frac{dc_2}{dQ_A^*}$. This is feasible.

Case II. The second term on the left is less than the second term on the right which implies $\frac{dc_1}{dQ^*_L} > \frac{dc_1}{dQ^*_A}$. Then $Q^*_L > Q^*_A$, $C_{2_L} < C_{2_A}$ and $\frac{dc_2}{dQ^*_L} > \frac{dc_2}{dQ^*_A}$. But this cannot be mathematically possible, so this case does not hold.

Case III. If the two second terms on each side of the equality of (15) are equal, then $Q^*_L = Q^*_A$ and $\frac{dc_2}{dQ^*_L} = \frac{dc_2}{dQ^*_A}$, which is also not mathematically possible knowing that

$$\int_{Q^*}^{\infty} f(Q)dQ < \frac{\int_{Q^*}^{\infty} u' f(Q)dQ}{\int_{0}^{\infty} u' f(Q)dQ}$$

unless $Q^*_L = Q^*_A = 0$, which is not a practical consideration for the auditor.

The first case, where $Q^*_L < Q^*_A$ is the only possible one, which means that a risk averse individual will audit more than a risk indifferent individual.

Model II--An Auditor's Model Under Pure Competition

Perhaps a more realistic case is where the penalty is some function of the number of units audited (Q^*) and the number of units that should have been audited (Q). In particular, the penalty is a function of the difference between Q and Q^*. It would be appropriate to argue that as the amount demanded becomes closer to the amount produced, then the costs of a penalty should be less.

The other terms will be defined as before, except now the penalty is represented by $C_2 = c_2\,(Q - Q^*)$. The profit will consist of the following two parts:

$$\pi_1 = PQ^* - c_1\,(Q^*) \text{ for } Q \leq Q^*,$$

$$\pi_2 = PQ^* - c_1\,(Q^*) - c_2\,(Q - Q^*) \text{ for } Q \geq Q^*.$$

If the auditor overaudits, then the profit is simply π_1; no penalty is assessed and the only costs are those deterministic ones associated with Q^*. If the auditor underaudits, as in π_2, there is an additional cost which is dependent upon the deficiency $(Q - Q^*)$, i.e., how much the auditor failed to audit.

In this case, the auditor will wish to maximize:

$$E[u(\pi)] = \int_0^{Q^*} u\,[PQ^* - c_1\,(Q^*)]\,f(Q)dQ + \int_{Q^*}^{\infty} u\,[PQ^* - c_1\,(Q^*) - c_2\,(Q - Q^*)]\,f(Q)dQ. \tag{28}$$

Then,

$$\frac{dE[u(\pi)]}{dQ^*} = \int_0^{Q^*} u'(P - \frac{dc_1}{dQ^*})\,f(Q)dQ + (1)\{u[PQ^* - c_1(Q^*)]\}\,f(Q^*) - 0 + \int_{Q^*}^{\infty} u'[P - \frac{dc_1}{dQ^*} - \frac{dc_2}{d(Q - Q^*)}(-1)]\,f(Q)dQ - (1)\{u[PQ^* - c_1\,(Q^*) - c_2(Q - Q^*)]\}\,f(Q)dQ. \tag{29}$$

$$\frac{dE(u)}{dQ^*} = \int_0^{Q^*} u'(P - \frac{dc_1}{dQ^*}) f(Q)dQ + \int_{Q^*}^{\infty} u'[P - \frac{dc_1}{dQ^*} + \frac{dc_2}{d(Q - Q^*)}] f(Q)dQ. \tag{30}$$

$$\frac{dE(u)}{dQ^*} = \int_0^{\infty} u'(P - \frac{dc_1}{dQ^*}) f(Q)dQ - \int_{Q^*}^{\infty} u'(P - \frac{dc_1}{dQ^*}) f(Q)dQ + \int_{Q^*}^{\infty} u'[P - \frac{dc_1}{dQ^*} + \frac{dc_2}{d(Q - Q^*)}] f(Q)dQ. \tag{31}$$

$$\frac{dE(u)}{dQ^*} = \int_0^{\infty} u'(P - \frac{dc_1}{dQ^*}) f(Q)dQ + \int_{Q^*}^{\infty} u'[\frac{dc_2}{d(Q - Q^*)}] f(Q)dQ. \tag{32}$$

$$(P - \frac{dc_1}{dQ^*}) \int_0^{\infty} u'f(Q)dQ + \int_{Q^*}^{\infty} u'[\frac{dc_2}{d(Q - Q^*)}] f(Q)dQ = 0. \tag{33}$$

$$(P - \frac{dc_1}{dQ^*}) \int_0^{\infty} u'f(Q)dQ = - \int_{Q^*}^{\infty} u'[\frac{dc_2}{d(Q - Q^*)}] f(Q)dQ. \tag{34}$$

$$P = \frac{dc_1}{dQ^*} - \frac{\int_{Q^*}^{\infty} u'[\frac{dc_2}{d(Q - Q^*)}] f(Q)dQ}{\int_0^{\infty} u'f(Q)dQ}, \tag{35}$$

or

$$P \int_0^{\infty} u'f(Q)dQ = \frac{dc_1}{dQ^*} \int_0^{\infty} u'f(Q)dQ - \int_{Q^*}^{\infty} u'[\frac{dc_2}{d(Q - Q^*)}] f(Q)dQ. \tag{36}$$

This model has similar results to Model I. For linear risk preferences, u' is a constant and (36) becomes

$$P = \frac{dc_1}{dQ^*} - \int_{Q^*}^{\infty} \frac{dc_2}{d(Q - Q^*)} f(Q)dQ. \tag{37}$$

Since $\frac{dc_2}{dQ^*} < 0$, by assumption, $\frac{dc_2}{dQ^*} = \frac{dc_2}{d(Q - Q^*)} (-1)$, then $\frac{dc_2}{d(Q - Q^*)} > 0$; and the last term on the right will be negative. The auditor indifferent to risk would produce where price is equal to marginal production costs less the expected marginal costs of the penalties, or where P = expected MC.

It can also be shown for Model II that the risk averse individual would audit more than the risk indifferent individual. Using the same approach as in Model I, first compare

$$\int_{Q^*}^{\infty} f(Q)dQ \quad \text{to} \quad \frac{\int_{Q^*}^{\infty} u'f(Q)dQ}{\int_{0}^{\infty} u'f(Q)dQ}$$

to determine their relationship. As before, "?" will be used until the direction of the sign has been ascertained. Then

$$\int_{Q^*}^{\infty} f(Q)dQ \;\; ? \;\; \frac{\int_{Q^*}^{\infty} u'f(Q)dQ}{\int_{0}^{\infty} u'f(Q)dQ} \,. \tag{38}$$

This becomes

$$-\int_{Q^*}^{\infty} u'f(Q)dQ \;\; ? \; -\hat{u}'[\int_{Q^*}^{\infty} f(Q)dQ] \,, \tag{39}$$

where $\hat{u}'$ is a constant function of Q and represents the u' of $\int_{0}^{Q^*} u'f(Q)dQ$. Now, $\frac{d(u')}{dQ} = u''(\frac{d\pi}{dQ}) = u''[\frac{-dc_2}{d(Q - Q^*)}(1)]$; $u'' < 0$ and $\frac{dc_2}{d(Q - Q^*)} < 0$, therefore u' is an increasing function of Q. Thus, $\hat{u}'$ is never greater than any u' on the left side of (39). As $u' \geqslant \hat{u}'$, and $-u' < -\hat{u}'$, the questioned sign becomes <, the same as in the case of Model I.

$$\int_{Q^*}^{\infty} f(Q)dQ < \frac{\int_{Q^*}^{\infty} u'f(Q)dQ}{\int_{0}^{\infty} u'f(Q)dQ} \,. \tag{40}$$

Given than p is fixed, the risk averse optimality condition of (35) and the risk linear condition of (37) will be equal. If Q^*_L is used to represent the risk linear output and Q^*_A to represent the risk averse output, then:

$$\frac{dc_1}{dQ^*_L} - \int_{Q^*}^{\infty} \frac{dc_2}{d(Q - Q^*_L)} f(Q)dQ = \frac{dc_1}{dQ^*_A} - \frac{\int_{Q^*}^{\infty} u' \frac{dc_2}{d(Q - Q^*_A)} f(Q)dQ}{\int_0^{\infty} u' f(Q)dQ} . \quad (41)$$

There are three cases which may occur.

Case I. The second term on the left of the equality sign is greater than the second term on the right. This makes $\frac{dc_1}{dQ^*_L} > \frac{dc_1}{dQ^*_A}$ which implies $Q^*_L > Q^*_A$ making $C_{2_L} < C_{2_A}$ and thus $\frac{dc_2}{d(Q - Q^*_L)} < \frac{dc_2}{d(Q - Q^*_A)}$. But this is not mathematically possible as it cannot hold that $\frac{dc_2}{d(Q - Q^*_L)} < \frac{dc_2}{d(Q - Q^*_A)}$,

$$\int_{Q^*}^{\infty} f(Q)dQ < \frac{\int_{Q^*}^{\infty} u' f(Q)dQ}{\int_0^{\infty} u' f(Q)dQ}$$ and have

$$\int_{Q^*}^{\infty} \frac{dc_2}{d(Q - Q^*_L)} f(Q)dQ > \frac{\int_{Q^*}^{\infty} u' \frac{dc_2}{d(Q - Q^*_A)} f(Q)dQ}{\int_0^{\infty} u' f(Q)dQ} .$$

Case II. Suppose the second term on the left of the equality sign in (41) is less than the second term on the right. Then $\frac{dc_1}{dQ^*_L} < \frac{dc_1}{dQ^*_A}$, which implies $Q^*_L < Q^*_A$, $C_{2_L} > C_{2_A}$ and $\frac{dc_2}{d(Q - Q^*_L)} > \frac{dc_2}{d(Q - Q^*_A)}$. This particular situation is feasible.

Case III. This is where $Q^*_A = Q^*_L = 0$, which is not a point at which the firm wishes to operate.

Therefore, the risk linear individual will audit less than the risk averse individual when the penalty is perceived to be a function of the difference between Q and Q*. As the auditor becomes more averse to risk, output will be increased to avoid the penalty.

The Effects of a Change in the Auditor's Environment

It will be helpful to examine Models I and II, compare their results and determine if a change in the way penalties are assessed will cause a change in the auditor's output. This comparison can be used to assess the effect of a change in the auditor's legal environment.

Consider just the linear models, denoting these as Q^*_I and Q^*_{II}.

$$P = \frac{dc_1}{dQ^*_I} + \frac{dc_2}{dQ^*_I} \int_{Q^*}^{\infty} f(Q)dQ \ , \tag{42}$$

and

$$P = \frac{dc_1}{dQ^*_{II}} + \int_{Q^*}^{\infty} - \frac{dc_2}{d(Q - Q^*_{II})} f(Q)dQ \ . \tag{43}$$

Let $h(Q) = \dfrac{dc_2}{dQ^*_I}$ and $n(Q) = - \dfrac{dc_2}{d(Q - Q^*_{II})}$. It is known that if $\dfrac{d[h(Q) - n(Q)]}{dQ} > 0$, then $h(Q) > n(Q)$.

$$\frac{d\left[\dfrac{dc_2}{dQ^*_I} - \dfrac{-dc_2}{d(Q - Q^*_{II})}\right]}{dQ} = 0 + \frac{d^2c_2}{d(Q - Q^*_{II})^2} (1), \tag{44}$$

and $\dfrac{d^2c_2}{d(Q - Q^*_{II})} > 0$, so $\dfrac{dc_2}{dQ^*_I} > - \dfrac{dc_2}{d(Q - Q^*_{II})}$.

Therefore:

$$\frac{dc_2}{dQ^*_I} \int_{Q^*}^{\infty} f(Q)dQ > \int_{Q^*}^{\infty} - \frac{dc_2}{d(Q - Q^*_{II})} f(Q)dQ \ . \tag{45}$$

If (45) is true, then $Q^*_I < Q^*_{II}$.

Using the same method, these results can be found for the risk averse situation for both models. It can be seen, then, that a change (a perceived change) in the risk structure will cause a change in output. In this particular analysis, it implies a greater output under Model II than under Model I. This could be explained as the auditor, perceiving a reward for how much is audited, will audit more under Model II. In effect, there is a "reward" for increasing output. Whereas in Model I, the penalty is a function of Q and the amount audited will not affect the penalty unless production reaches Q or surpasses it.

Interestingly, Section II(e) of the Securities Act of 1933 would seem to impose penalties in a fashion more similar to Model I. That is, plaintiffs may recover their losses. And these losses would, in turn, seem to be more a function of the scale of the audit, Q, than the amount of auditor negligence, Q - Q*. It appears that penalties are based upon how much the auditor should have audited, without consideration of what was actually produced. If the intent of the Act is, in part, to guarantee collection of a sufficient amount of audit evidence, and if what is deemed a sufficient amount of evidence is an unknown until after the audit has been completed, then the intent of the Act is more likely to be served under an alternative (Model II) legal structure.

Model III--An Auditor's Model Under Monopolistic Conditions

Consider Model I under different economic conditions, in particular, consider a scenario where the firm has some control over price. Demand at a price of P is given by $Q = D(P,v)$ where v is a random variable, distributed by $g(v)$ with a zero mean.[3] The cost functions are defined as before with $C_1 = c_1(Q)$ being the determinstic production cost and $C_2 = c_2(Q)$ is the penalty costs. Q^* is the production by the auditor. The maximization will be at the values of P and Q^* such that $\frac{\partial E(u)}{\partial P} = 0$, $\frac{\partial E(u)}{\partial Q^*} = 0$, and certain second order conditions are satisfied.[4]

The profit function is of the same form as before, and the auditor will wish to maximize:

$$E[u(\pi)] = \int_0^{Q^*} u[Q^*P - c_1(Q^*)]\, f(Q)dQ + \int_{Q^*}^{\infty} u[Q^*P - c_1(Q^*) - c_2(Q)]\, f(Q)dQ. \quad (46)$$

---

[3] $\bar{v} = E[v] = \int_{-\infty}^{\infty} vg(v)dv = 0$ and v is distributed independently of P, so $Q = D(P,v) = D(P) + v$. $D(P) + v > 0$, so that $Q \geq 0$ to avoid being buyers instead of sellers. Then, for any price P^* expected demand $\bar{Q} = E[Q/P^*] = E[D(P^*) + v] = E[D(P^*)] + E[v] = D(P^*)$. The assumption of independence is necessary to perform some of the integration. The function $g(v) \equiv g([Q - D(P)) \equiv f(Q)$ as before for simplification.

[4] The second order conditions are that

$$\frac{\partial^2 E[u(\pi)]}{\partial Q^{*2}} < 0 \quad \text{and} \quad \frac{\partial^2 E[u(\pi)]}{\partial P^2} < 0.$$

See the Appendix for the satisfaction of these conditions.

First, look at the condition concerning Q*:

$$\frac{\partial E[u(\pi)]}{\partial Q^*} = \int_0^{Q^*} u'(P - \frac{dc_1}{dQ^*})\ f(Q)dQ + \int_{Q^*}^{\infty} u'(P - \frac{dc_1}{dQ^*} - \frac{dc_2}{dQ^*})\ f(Q)dQ. \tag{47}$$

Setting this equal to zero, then

$$P = \frac{dc_1}{dQ^*} + \frac{\int_{Q^*}^{\infty} u' \frac{dc_2}{dQ^*} f(Q)dQ}{\int_0^{\infty} u' f(Q)dQ}, \tag{48}$$

or

$$P \int_0^{\infty} u' f(Q)dQ = \frac{dc_1}{dQ^*} \int_0^{\infty} u' f(Q)dQ + \int_{Q^*}^{\infty} u' \frac{dc_2}{dQ^*} f(Q)dQ. \tag{49}$$

In the risk linear case, this becomes

$$P = \frac{dc_1}{dQ^*} + \int_{Q^*}^{\infty} \frac{dc_2}{dQ^*} f(Q)dQ. \tag{50}$$

The other first order condition which has to be satisfied is,

$$\frac{\partial E[u(\pi)]}{\partial P} = \int_0^{Q^*} u'(Q^* + \frac{dQ^*}{dP} P)\ f(Q)dQ + \int_{Q^*}^{\infty} u'(Q^* + \frac{dQ^*}{dP} P)\ f(Q)dQ, \tag{51}$$

which becomes

$$(Q^* + \frac{dQ^*}{dP} P) \int_0^{\infty} u'\ f(Q)dQ = 0, \tag{52}$$

and further reduces to

$$Q^* + \frac{dQ^*}{dP} \cdot P = 0\ . \tag{53}$$

This term is the change in revenues for a unit change in price. This simply suggests where the auditor should set price. Therefore

$$P^* = -\ Q^* \ / \ \frac{dQ^*}{dP} \tag{54}$$

and $\frac{dQ^*}{dP} < 0$, by assumption, which insures that $P^* > 0$.

Looking at (53), which is $\frac{dR}{dP}$, then

$$\frac{dR}{dP} = Q^* + \frac{dQ^*}{dP} P, \tag{55}$$

and by substituting the expression for P from the other first order condition,

$$\frac{dR}{dP} = Q^* + \frac{dQ^*}{dP} \left(\frac{dc_1}{dQ^*} + \frac{\int_{Q^*}^{\infty} u' \frac{dc_2}{dQ^*} f(Q)dQ}{\int_0^{\infty} u' f(Q)dQ}\right). \tag{56}$$

Dividing through by $\frac{dQ^*}{dP}$, then

$$\frac{dR}{dP} / \frac{dQ^*}{dP} = Q^* / \frac{dQ^*}{dP} + \frac{dc_1}{dQ^*} + \frac{\int_{Q^*}^{\infty} u' \frac{dc_2}{dQ^*} f(Q)dQ}{\int_0^{\infty} u' f(Q)dQ} . \tag{57}$$

Since $MR = \frac{dR}{dQ^*} = (\frac{dR}{dP})(\frac{dP}{dQ^*}) = (\frac{dR}{dP})/(\frac{dQ^*}{dP})$, then (57) becomes

$$MR = Q^* / \frac{dQ^*}{dP} + \frac{dc_1}{dQ^*} + \frac{\int_{Q^*}^{\infty} u' \frac{dc_2}{dQ^*} f(Q)dQ}{\int_0^{\infty} u' f(Q)dQ} , \tag{58}$$

or

$$MR \int_0^{\infty} u' f(Q)dQ = Q^* / \frac{dQ^*}{dP} \int_0^{\infty} u' f(Q)dQ + \frac{dc_1}{dQ^*} \int_0^{\infty} u' f(Q)dQ + \int_{Q^*}^{\infty} u' \frac{dc_2}{dQ^*} f(Q)dQ. \tag{59}$$

For the linear case, this would reduce to

$$MR = Q^* \,/\, \frac{dQ^*}{dP} + \frac{dc_1}{dQ^*} + \int_{Q^*}^{\infty} \frac{dc_2}{dQ^*} f(Q)dQ, \tag{60}$$

or

$$MR = MC + Q^* \,/\, \frac{dQ^*}{dP} \; . \tag{61}$$

Here marginal revenue is something less than expected marginal costs as $\frac{dQ^*}{dP} < 0$, by assumption. (A similar interpretation holds for the risk aversion case of (59), except that marginal utility enters into the model.)

For simplicity, only the risk linear model will be used to compare output under uncertainty with output under certainty. The risk averse case will give the same results, when compared to the certainty case.

From (60) and the second order conditions, $MR(P^*) \leq MC(Q^*)$. The certainty output would be some $Q = \bar{Q}$ to be sold at a price P', and $MR(P') = MC(\bar{Q})$. If $MR(P') = MR(P^*)$ (implying $P' = P^*$), then $MR(P') \leq MC(Q^*)$ and $MR(P') = MC(\bar{Q})$. It follows that $MC(\bar{Q}) \leq MC(Q^*)$ and $\bar{Q} \leq Q^*$, where $P' = P^*$.

If $MR(P') < MR(P^*)$, then $MR(P') < MC(Q^*)$, $MC(\bar{Q}) < MC(Q^*)$ and $\bar{Q} \leq Q^*$, when $P' < P^*$.

When $MR(P') > MR(P^*)$, two possibilities exist: (1) $MR(P') > MC(Q^*)$, implying $MC(\bar{Q}) > MC(Q^*)$ and $\bar{Q} > Q^*$; or (2) $MR(P') < MC(Q^*)$, $MC(\bar{Q}) < MC(Q^*)$ and $\bar{Q} < Q^*$. When $P' > P^*$, it is possible to have $\bar{Q} > Q^*$ or $\bar{Q} < Q^*$, but not $\bar{Q} = Q^*$. The case where $P' > P^*$ and $\bar{Q} < Q^*$ may occur if the average demand under uncertainty was so great that there is a huge penalty, and any reduction in price below P' would lower MR substantially but not influence the penalty by any appreciable amount.

Summarizing, if the optimal price under uncertainty is greater than or equal to the optimal price under certainty, then the optimal output under uncertainty will be greater than that under certainty. If the price under certainty was greater than the price under uncertainty, it is still possible that the output under certainty will be less than the output under uncertainty. But it is not clear this is always true. In general, there are two effects. First, the auditor will raise output in most situations to reduce the possibility of a penalty, and second, a higher price is charged to offset the higher costs which include the penalty.

An additional issue is the comparison of the optimal output for the risk indifferent auditor portrayed by (60) to that of the risk averse auditor in (59).

Knowing that $Q^* / \frac{dP}{dQ^*} = Q^* / \frac{dQ^*}{dP}$, then (59) becomes

$$P_A^* = \frac{dc_1}{dQ_A^*} + \frac{\int_{Q^*}^{\infty} u' \frac{dc_2}{dQ_A^*} f(Q)dQ}{\int_0^{\infty} u' f(Q)dQ}, \qquad (62)$$

where the subscript (A) denotes that this is the P* and Q* for the risk evader.

This appears to be the same optimality condition as in Model I, but P* is not fixed, as is the P in Model I. Using the subscript (L) to denote risk indifference, then (60) is

$$P_L^* = \frac{dc_1}{dQ_L^*} + \int_{Q^*}^{\infty} \frac{dc_2}{dQ_L^*} f(Q)dQ. \qquad (63)$$

Again, several cases may be examined.

Case I. Consider $P_L^* > P_A^*$ which implies $MC_L > MC_A$. This is possible if $\frac{dc_1}{dQ_L^*} > \frac{dc_1}{dQ_A^*}$ and $\frac{dc_2}{dQ_L^*} > \frac{dc_2}{dQ_A^*}$, implying $Q_L^* > Q_A^*$.

Case II. Suppose $P_L^* < P_A^*$ which implies $MC_L < MC_A$. This will occur when $\frac{dc_1}{dQ_L^*} < \frac{dc_1}{dQ_A^*}$ and $\frac{dc_2}{dQ_L^*} < \frac{dc_2}{dQ_A^*}$, which is also possible. Then $Q_L^* < Q_A^*$.

Case III. If $P_L^* = P_A^*$, then $MC_L = MC_A$, $\frac{dc_1}{dQ_L^*} = \frac{dc_1}{dQ_A^*}$ and $\frac{dc_2}{dQ_L^*} = \frac{dc_2}{dQ_A^*}$. But this is not possible in a mathematical sense when

$$\int_{Q^*}^{\infty} f(Q)dQ < \frac{\int_{Q^*}^{\infty} u'f(Q)dQ}{\int_{0}^{\infty} u'f(Q)dQ},$$

which is known from our previous work regarding Model I.

The conclusions that can be made are: (1) as the quantity is increased, price also increases because costs are increasing; and (2) output will change as the auditor becomes more risk averse, but the direction of the change is indeterminable.

It is obvious that the consideration of uncertainty gives results different from the traditional economic approach to output and price determination. It can still be inferred, though, that output will most likely change when uncertainty is introduced and, further, it will change again as the auditor becomes more risk averse.

Even though the results of the neoclassical analysis for monopolies show a smaller output than for pure competition, the results under uncertainty are not clear. The introduction of risk preferences and uncertainty results in ambiguity. Output will increase under pure competition, and it is probable that it will increase under monopoly. Not knowing the magnitude of the increase, however, no comparison of the output between Model I and Model III can be made.

Conclusions

The introduction of uncertainty into the auditor's model will cause marked differences from the results obtained in the neoclassical economic model.

First, the introduction of uncertainty will affect the units of audit service the auditor will perform. In a pure competition environment, where price is fixed, the utility maximizing auditor will produce more when faced with uncertainty. In addition, the more risk averse the auditor is, the more output will increase. This is in contrast to traditional thinking that the maximizing auditor would provide services at a point where marginal cost equals marginal revenue.

Under monopolistic conditions, the results are not as simple. As uncertainty is introduced, it is not obvious that the auditor will always audit more than under certainty. In most instances, the auditor will audit more and also will charge a higher price to compensate for the rise in the costs due to this incremental audit work.

The second insight gained from the model is that the explicit consideration of the penalty in the model caused output to increase. Without the penalty, the output would have been the same as that under certainty, which was less. The penalty itself is an incentive for the auditor to audit more.

A third conclusion from the study is that a change in manner in which penalties are assessed will cause a change in the output of the auditor. This, together with the second result above supplies us with an insight into how auditor behavior might be regulated. Further, this can be applied to subordinates' behavior so that they can make preferred audit output decisions. Using a penalty/reward structure would induce the same results for subordinates also.

The final result from the analysis concerned risk aversion. Given that there is a belief that auditors are risk averse, it is important that the effect of risk aversion on their audit decisions be understood. In a pure competitive situation, the auditor displaying risk aversion will produce more than the auditor that is risk indifferent. Under the economic conditions of monopoly, it can only be determined that the output of a risk averse auditor will be different from that of a risk indifferent auditor. There is also a price effect, which has the same sign as the output effect.

It has thus been shown that uncertainty, risk attitude, the manner in which penalties are assessed, and competitive aspects of auditing are important determinants of the supply of auditing services. Since this is an analytical study, it is premature to speculate on whether these results will actually prove useful in any practical sense. The results have only been shown to follow from a set of assumptions; and whether those assumptions indeed capture the important aspects of the audit environment simply cannot be assessed on a priori grounds. This is an empirical issue and is suggestive of one course for future research.

All theoretical constraints employed in this study have empirical counterparts. Thus the requirement of testability in principal has been achieved. However, it is not entirely clear just how such empirical testing ought best to proceed. The abundance of internal auditing issues suggests laboratory experimentation as a promising candidate.

Additionally, Ng (1978) has developed the rudiments of a theory of the demand for auditing services. A second course for fruitful further research would seem to be an integration of the present study with that of Ng in order to obtain a full-blown equilibrium model of the market for auditing services. And nothing short of an empirically validated full-blown equilibirum model is likely to be very useful in obtaining the degree of understanding upon which sensible regulation can be based.

LITERATURE CITED

Allingham, Michael G. and Agnar Sandmo. "Income Tax Evasion: A Theoretical Analysis." Journal of Public Economics (November, 1972), pp. 323-338.

Anderson, Wilton T. "CPA Requirements of the States." Collegiate News and Views (Spring, 1972), pp. 11-15.

__________. Accounting Departmental Memo, Oklahoma State University, 1979.

Anderson, Wilton T. and D. Dwayne Dowell. "CPA Requirements of the States." Collegiate News and Views (Fall, 1978), pp. 5-10.

Arkin, Herbert. Handbook of Sampling for Auditing and Accounting, 2nd ed. (New York: McGraw-Hill Book Co., 1974).

Arrow, Kenneth J. Social Choice and Individual Values. (New Haven: Yale University Press, 1963.)

__________. Essays in the Theory of Risk-Bearing. (Chicago: Markham Publishing Co., 1971.)

Ashton, Robert H. "An Experimental Study of Internal Control Judgments." The Journal of Accounting Research (Spring, 1974), pp. 143-57.

Baron, David P. "Price Uncertainty, Utility, and Industry Equilibrium in Pure Competition." International Economic Review, Vol. 11 (October, 1970), pp. 463-480.

__________. "Demand Uncertainty in Imperfect Competition." International Economic Review (June, 1971), pp. 196-208.

Becker, Gary S. "Crime and Punishment: An Economic Approach." Journal of Political Economy, Vol. 78 (March/April, 1968), pp. 173-217.

Birnberg, J. G. "Bayesian Statistics: A Review." Journal of Accounting Research (Spring, 1964), pp. 108-16.

Block, M. K. and J. M. Heineke. "The Allocation of Effort Under Uncertainty: The Case of Risk-Averse Behavior." Journal of Political Economy, Vol. 81 (March/April, 1973), pp. 376-385.

__________. "A Labor Theoretic Analysis of the Criminal Choice." The American Economic Review, Vol. 65 (June 1975), pp. 314-324.

Borch, Karl. "Recent Developments in Economic Theory and Their Application to Insurance." ASTIN Bulletin, Vol. 2 (1963), pp. 322-41.

__________. "A Utility Function Derived from a Survival Game." Management Science, Vol. 12 (April, 1966), pp. B289-B295.

__________. The Economics of Uncertainty. (Princeton University Press, 1968.)

Carey, John L. The Rise of the Accounting Profession to Responsibility and Authority, 1937-1969. (New York: AICPA, 1970.)

Causey, Denzil Y., Jr. Duties and Liabilities of the CPA, rev. ed. (Bureau of Business Research, The University of Texas, 1976.)

Corless, J. C. "Assessing Prior Distribution for Applying Bayesian Statistics in Auditing." The Accounting Review (July, 1972), pp. 556-66.

Dhrymes, Phoebus J. "On the Theory of the Monopolistic Multiproduct Firm Under Uncertainty." International Economic Review, Vol. 5 (September, 1964), pp. 239-257.

Dreze, Jacques H. and Jean J. Gabszewicz. "Demand Fluctuations, Capacity Utilization and Prices." Operations Research Verfahren, Vol. 3 (1967), pp. 119-141.

Ehrlich, Isaac. "Participation in Illegitimate Activities: A Theoretical and Empirical Investigation." Journal of Political Economy, Vol. 81 (May/June, 1973), pp. 521-565.

Elliott, R. K. and J. R. Rogers. "Relating Statistical Sampling to Audit Objectives." The Journal of Accounting (July, 1972), pp. 46-55.

The Federation of Schools of Accountancy, Proceedings of the First National Conference of the Federation of Schools of Accountancy, edited by W. Steve Albrecht and K. Fred Skousen. (Institute of Professional Accountancy, 1977.)

Friedman, Milton and Leonard J. Savage. "The Utility Analysis of Choices Involving Risk." Journal of Political Economy, Vol. 16 (August, 1948), pp. 279-304.

Friedman, Milton and Simon Kuznets. Income from Independent Professional Practice. (New York: National Bureau of Economic Research, 1945.)

Hendrickson, H. S. "The Changing Content of the CPA Examination." Journal of Accountancy (July, 1971), pp. 42-49.

Horowitz, Ira. Decision Making and the Theory of the Firm. (New York: Holt, Rinehart and Winston, Inc., 1970.)

Hymans, Saul H. "The Price-Taker: Uncertainty, Utility and the Supply Function." International Economic Review, Vol. 7 (September, 1966), pp. 346-366.

Joyce, Edward J. "Expert Judgment in Audit Program Planning." Studies on Human Information Processing in Accounting, Supplement to the Journal of Accounting Research (1976), pp. 29-60.

Keeney, Ralph L. and Howard Raiffa. Decisions with Multiple Objectives: Preferences and Tradeoffs. (New York: John Wiley and Sons, 1976.)

Kinney, William R., Jr. "Decision Theory Aspects of Internal Control System Design/Compliance and Substantive Tests." Studies on Statistical Methodology in Auditing: 1975, Supplement to the Journal of Accounting Research (1975), pp. 14-29.

Knoblett, James Allen. "The Applicability of Bayesian Statistics in Auditing." Decision Science (July-October, 1970), pp. 423-40.

Kraft, William H. "Statistical Sampling for Auditing: A New Look." Journal of Accountancy (August, 1968), pp. 49-56.

Leland, Hayne E. "Saving and Uncertainty: The Precautionary Demand for Saving." Quarterly Journal of Economics, Vol. 82 (August, 1968), pp. 463-473.

__________. "Theory of the Firm Facing Uncertain Demand." The American Economic Review, Vol. 62 (June, 1972), pp. 278-291.

Luce, R. D. and Howard Raiffa. Games and Decision. (John Wiley and Son: New York, 1957.)

Magee, Robert P. "Employment Contracts and Accounting: The Auditor as an Economic Agent." Unpublished working paper, Graduate School of Management, Northwestern University, 1977.

__________. "Discussion of Auditor's Loss Functions Implicit in Consumption-Investment Models." Studies on Statistical Methodology in Auditing: 1975, Supplement to the Journal of Accounting Research (1975), pp. 121-123.

Markowitz, Harry M. Portfolio Selection. (John Wiley and Sons, New York, 1959.)

Meyer, Robert A. "Monopoly Pricing and Capacity Choice Under Uncertainty." The American Economic Review, Vol. 65, No. 3 (June 1975), pp.326-337.

Miller, Herb. "An Update on Accreditation." Speech during Proceedings of the First National Conference of the Federation of Schools of Accountancy. (Institute of Professional Accountancy, 1977.)

Miller, Herb and Sid Davidson. "Accreditation: Two Views." The Journal of Accountancy (March 1978), pp. 56-65.

Mills, Edwin S. "Uncertainty and Price Theory." Quarterly Journal of Economics, Vol. 73 (February 1959), pp. 116-130.

Mossin, Jan. "Taxation and Risk-Taking: An Expected Utility Approach." Economica, Vol. 35 (February, 1968), pp. 74-82.

__________. "Aspects of Rational Insurance Purchasing." Journal of Political Economy, Vol. 76 (July/August, 1968), pp. 553-568.

Mushkin, Selma. "Towards a Definition of Health Economics." Public Health Reports, Vol. 73 (1958), pp. 785-93.

National Association of State Boards of Accountancy. State Board Report. (September, 1979.)

Nelson, Richard R. "Uncertainty, Prediction, and Competitive Equilibrium." Quarterly Journal of Economics, Vol. 75 (February, 1961), pp. 41-62.

Newton, Lauren K. "The Risk Factor in Materiality Decision." The Accounting Review (January, 1977), pp. 97-108.

Ng, David. "An Information Economics Analysis of Financial Reporting and External." The Accounting Review (October, 1978), pp. 910-920.

Oi, Walter Y. "The Desirability of Price Instability Under Perfect Competition." Econometrica, Vol. 29 (January, 1961), pp. 58-64.

Olson, Wallace E. "The Accounting Profession in the 1980s." The Journal of Accountancy (July, 1979), pp. 54-60.

Pearson, David B. "Will Accreditation Improve the Quality of Education." The Journal of Accountancy (April, 1979), pp. 53-58.

Penner, Rudolph G. "Uncertainty and the Short-Run Shifting of the Corporation Tax." Oxford Economic Papers, Vol. 19 (March 1967), pp. 99-110.

Pichler, Joseph A. "An Economic Analysis of Accounting Power." Institutional Issues in Public Accounting, papers and responses from Accounting Colloquium III at the University of Kansas, 1973. (Scholars Book Company, 1974.)

Revsine, Lawrence and Hervey A. Juris. "Accounting Power: An Alternative to the Economic Analysis." Institutional Issues in Public Accounting, papers and responses from Accounting Colloquium III at the University of Kansas, 1973. (Scholars Book Company, 1974.)

Roberts, Donald M. "A Statistical Interpretation of SAP No. 54." Journal of Accountancy (March, 1974), pp. 47-53.

Sandmo, A. "The Effect of Uncertainty on Saving Decisions." Review of Economic Studies, Vol. 37 (July, 1970), pp. 353-360.

__________. "On the Theory of the Competitive Firm Under Price Uncertainty." The American Economic Review, Vol. 61 (March, 1971), pp. 65-73.

Scott, William R. "A Bayesian Approach to Asset Valuation and Audit Size." Journal of Accounting Research (Autumn, 1973), pp. 304-30.

__________. "Auditor's Loss Function Implicit in Consumption-Investment Models." Studies on Statistical Methodology in Auditing: 1975, Supplement to Journal of Accounting Research (1975), pp. 98-117.

Sjoquist, David Lawrence. "Property Crime and Economic Behavior: Some Empirical Results." The American Economic Review, Vol. 63 (June, 1973), pp. 439-446.

Slovic, Paul and Sarah Lichtenstein. "Comparison of Bayesion and Regression Approaches to the Study of Information Processing in Judgment." Organization Behavior and Human Performance (November, 1971), pp. 649-744.

Smith, K. A. "The Relationship of Internal Control Evaluation and Audit Sample Size." The Accounting Review (April, 1972), pp. 260-9.

Smith, Kenneth R. "The Effect of Uncertainty on Monopoly Price, Capital Stock and Utilization of Capital." Journal of Economic Theory, Vol. 1 (1969), pp. 48-59.

Stiglitz, J. E. "The Effects of Income, Wealth, and Capital Gain Taxation and Risk-Taking." Quarterly Journal of Economics, Vol. 83 (May, 1969), pp. 263-283.

Stigum, Bernt P. "Entrepreneurial Choice Over Time Under Conditions of Uncertainty." International Economic Review, Vol. 10, No. 3 (October, 1969), pp. 426-442.

Tisdell, Clem. "Uncertainty, Instability, Expected Profit." Econometrica, Vol. 21 (January-April, 1963), pp. 243-247.

Tracy, J. A. "Bayesian Statistical Methods in Auditing." The Accounting Review (January, 1969), pp. 90-98.

von Neumann, John and Oskar Morgenstern. Theory of Game and Economic Behavior. (Princeton University Press, 1947.)

Walters, A. A. "Marginal Productivity and Probability Distribution of Factor Service." Economic Journal, Vol. 40 (June 1960), pp. 325-330.

Warren, C. S. "Uniformity of Auditing Standards." Journal of Accounting Research (Autumn, 1975), pp. 162-176.

Wood, Thomas D. and Donald A. Ball. "New Rule 502 and Effective Advertising by CPAs." The Journal of Accountancy (June, 1978), pp. 65-70.

Zabel, E. "Monopoly and Uncertainty." Review of Economic Studies, Vol. 37 (April, 1970), pp. 205-219.

APPENDIX

SECOND ORDER CONDITIONS FOR THE AUDITOR'S MODELS

Model I

$$\frac{d^2E(u)}{dQ^*} = \int_0^{Q^*} u'' \left(P - \frac{dc_1}{dQ^*}\right)^2 f(Q)dQ + \int_0^{Q^*} u' \left(- \frac{d^2c_1}{dQ^{*2}}\right) f(Q)dQ +$$

$$\int_{Q^*}^{\infty} u'' \left(P - \frac{dc_1}{dQ^*} - \frac{dc_2}{dQ^*}\right)^2 f(Q)dQ + \int_{Q^*}^{\infty} u' \left(- \frac{d^2c_1}{dQ^{*2}} - \right.$$

$$\left.\frac{d^2c_2}{dQ^{*2}}\right) f(Q)dQ \ . \tag{64}$$

For the risk linear individual, where $u' > 0$ and $u'' = 0$, the above expression will be less than zero. The first and third terms are equal to zero. The second and third terms are negative as $\frac{d^2c_1}{dQ^{*2}} > 0$ and $\frac{d^2c_2}{dQ^{*2}} > 0$.

For the risk evading individual, where $u' > 0$ and $u'' < 0$, the expression in (64) is also negative. All terms are negative, given that $\frac{d^2c_1}{dQ^{*2}} > 0$ and $\frac{d^2c_2}{dQ^{*2}} > 0$ by assumption.

Model II

$$\frac{d^2E(u)}{dQ^{*2}} = \int_0^{Q^*} u'' \left(P - \frac{dc_1}{dQ^*}\right)^2 f(Q)dQ + \int_0^{Q^*} u' \left(- \frac{d^2c_1}{dQ^{*2}}\right) f(Q)dQ +$$

$$\int_{Q^*}^{\infty} u'' \left[P - \frac{dc_1}{dQ^*} + \frac{dc_2}{d(Q - Q^*)}\right]^2 f(Q)dQ +$$

$$\int_{Q^*}^{\infty} u' \left[- \frac{d^2c_1}{dQ^{*2}} - \frac{d^2c_2}{d(Q - Q^*)^2}\right] f(Q)dQ \ . \tag{65}$$

This will be negative for the risk linear auditor as $u' > 0$, $u'' = 0$, $\frac{d^2c_1}{dQ^{*2}} > 0$ and $\frac{d^2c_2}{d(Q - Q^*)^2} > 0$, by assumption. The expression (65) will also be less than zero for risk averse auditor as $u' > 0$, $u'' < 0$, $\frac{d^2c_1}{dQ^{*2}} > 0$ and $\frac{d^2c_2}{d(Q - Q^*)^2} > 0$.

Model III

$$\frac{\partial^2E(u)}{\partial Q^{*2}} = \int_0^{Q^*} u'' \left(P - \frac{dc_1}{dQ^*}\right)^2 f(Q)dQ + \int_0^{Q^*} u' \left(- \frac{d^2c_1}{dQ^*}\right) f(Q)dQ +$$

$$\int_{Q^*}^{\infty} u'' \left(P - \frac{dc_1}{dQ^*} - \frac{dc_2}{dQ^*}\right)^2 f(Q)dQ + \int_{Q^*}^{\infty} u' \left(- \frac{d^2c_1}{dQ^{*2}} - \right.$$

$$\left. \frac{d^2c_2}{dQ^{*2}}\right) f(Q)dQ \ . \tag{66}$$

Given that $u' > 0$, $u'' = 0$, $\frac{d^2c_1}{dQ^{*2}} > 0$, and $\frac{d^2c_2}{dQ^{*2}} > 0$ for the risk linear case, all terms will be zero or less than zero. In the risk averse case, where $u' > 0$ and $u'' < 0$, all terms in (66) will be negative.

$$\frac{\partial^2 E(u)}{\partial P^2} = \left[2 \frac{dQ*}{dP} + \frac{d^2 Q^*}{dP^2} (P) \right] \int_0^\infty u' \; f(Q) dQ +$$

$$[Q^* + \frac{dQ*}{dP} (P)]^2 \int_0^\infty u'' \; f(Q) dQ \; . \qquad (67)$$

The terms $\frac{dQ*}{dP}$ and $\frac{d^2Q*}{dP^2}$ are negative by assumption, and with $u' > 0$, the first term will be negative for both the risk linear and risk averse cases. The second term will be zero for the risk linear individual as $u'' = 0$ for him. The second term will be negative for the risk averse individual as $u'' < 0$.

VITA

Janet Illeen Kimbrell

Candidate for the Degree of

Doctor of Philosophy

Thesis: AN ANALYTICAL INVESTIGATION OF AUDIT DECISION MAKING UNDER UNCERTAINTY

Major Field: Business Administration

Biographical:

Personal Data: Born in Duncan, Oklahoma, April 9, 1945, the daughter of Mr. and Mrs. Chester Kimbrell.

Education: Graduated from Dickson High School, Ardmore, Oklahoma, in May, 1963; received Bachelor of Science in Education degree from Southeastern Oklahoma State University in 1968; received Master of Professional Accounting from University of Texas at Arlington in 1976; completed requirements for the Doctor of Philosophy degree at Oklahoma State University in December, 1979.

Professional Experience: Mathematics instructor for Durant Public Schools, Durant, Oklahoma, in 1968 and 1969; mathematics instructor for Lewisville Public Schools, Lewisville, Texas, in 1969 and 1970; graduate teaching and research assistant at University of Texas at Arlington, 1973-75; instructor and teaching associate at Oklahoma State University, 1975-79.

Professional Organizations: Member of American Accounting Association and American Institute of Certified Public Accountants.