AGGREGATE PRODUCTION PLANNING MODELS
INCORPORATING DYNAMIC PRODUCTIVITY

By

BEHROKH KHOSHNEVIS

Bachelor of Science
Arya-Mehr University of Technology
Tehran, Iran
1974

Master of Science
Oklahoma State University
Stillwater, Oklahoma
1975

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Thesis Approved:

Philip M. Wolfe
Thesis Adviser

M. P. Terrell

Joe H. Mize

Hamed K. Eldin

Donald W. Grace

Norman N. Durham
Dean of the Graduate College

PREFACE

This research incorporates the effects of the dynamic productivity phenomena present in most industrial situations into the aggregate planning problem. The research originates the introduction of the effect of disruptions in productivity improvement, progress and retrogression to this production and workforce planning area. Aggregate production planning of both long cycle and short cycle product situations are considered and models peculiar to each case are developed and analyzed. The new models are shown to have significant economic impact in the majority of situations.

The general solution methodology utilized in this research was developed by W. H. Taubert [65]. Chapter IV of this research presents a summary of this methodology and contains a number of quotes from his work. The analysis of the manpower disruption effects presented in earlier parts of Chapter V draws heavily from the original efforts of E. B. Cochran [19]. With his permission, quotes from his work are used in this chapter.

I wish to express my most sincere thanks to my uncle, Mr. Ali Khoshnevis, for his unending devotion and faith which have enriched and strengthened me throughout my life.

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CHAPTER I

INTRODUCTION

General

The decisions regarding aggregate production planning play a major role in today's systematic view of the operations planning and control functions. These decisions are of primary importance to many manufacturing concerns, because in the face of a predictable, fluctuating demand pattern, production management is always confronted with broad basic question such as:

— To what extent should inventories be used to absorb the fluctuations in demand throughout the planning horizon?

— How much of the demand fluctuations should be absorbed through varying the size of the workforce?

— How much of the demand fluctuations should be absorbed through changing the production rates by resorting to the alternative ways of workforce utilization (assignment of overtime or undertime)?

— To what extent and when is subcontracting justified?

— To what extent and when should a portion of demand not be met?

In most instances it is true that the utilization of any one of the above strategies to the fullest extent is not as effective as resorting to a balance among them. Each of these strategies implies a set of costs. In general, the following types of costs may be involved:

- Inventory carrying costs
- Costs related to the workforce level
- Costs of changes in the workforce level
- Basic production costs related to the level of production
- Production change costs which arise from changing the current rate of production
- Subcontracting costs
- Costs of out-of-stock or shortages

The objective in aggregate production planning is to develop a least-cost combination of strategies which copes with the predicted demands over some planning horizon. The essence of the outcome of this planning technique is a sequence of the optimum workforce levels and production rates (independent decision variables) throughout the given planning horizon. Since this technique is not concerned with the detailed item requirement, but rather deals in terms of aggregated demand and productive capacity, it has been called aggregate planning or scheduling as well as production planning, programming or smoothing.

Statement of the Problem

The aggregate planning problem has received a great deal of attention over the last two decades. Models and decision rules have been developed for many special cases, and a variety of solution techniques have been suggested. However, all of these models, except two, utilize a constant productivity factor; that is, the expected rate of output capability per employee is unchanging over time.

It is known that the productivity rates in many organizations change with additional manufacturing experience. Empirical studies have demonstrated that an increase in productivity can be systematically related to the cumulative output of the firm. This phenomenon can be quantifiably represented as an improvement curve, or manufacturing progress function.

Even though learning curve analysis and aggregate planning have been largely treated as separate areas of research, the two are inherently interrelated. This is true because improvement curve analysis addresses itself to the productivity factor (the measure of output per unit workforce), which in turn is a major determinant of the shape of the response surface of the objective functions in almost all aggregate planning models. For example, the results of a detailed sensitivity analysis [67] performed on the most famous aggregate planning model with actual data [37] have indicated that the cost function in this model is most sensitive to the variations in the level of the productivity factor. Table I presents a summary result of this analysis. Notice should be made of the dramatically higher amount of loss in the total utility resulting from a 1% change in the level of the productivity factor (C_4), as compared to the losses resulting from the same magnitude of change in the level of the other coefficients in the cost function. This analysis implies the importance of careful considerations in estimation of the level of the productivity factor in the aggregate planning models. According to the empirical studies, the improvement curve analysis provides the best approach for such a critical estimation. The advantages of the joint consideration of aggregate planning and improvement curve analysis are further highlighted by Ebert [24]:

> Three of the purported uses of learning curve analysis are: (1) cash flow analysis, (2) assistance in product pricing

TABLE I

SUMMARY RESULTS OF SENSITIVITY ANALYSIS
ON THE PAINT FACTORY COST MODEL

Change in Coefficient	Related Cost Segment	Loss in Utility
$\Delta C_1 = .01C_1$	Regular Payroll	.4495
$\Delta C_2 = .01C_2$	Hiring and Layoff	.0167
$\Delta C_3 = .01C_3$	Alternative Workforce Utilization	.0072
$\Delta C_4 = .01C_4$	Alternative Workforce Utilization	5.8758
$\Delta C_6 = .01C_6$	Alternative Workforce Utilization	.3070
$\Delta C_{12} = .01$	Alternative Workforce Utilization	1.1070
$\Delta C_7 = .01C_7$	Inventory	.0110
$\Delta C_8 = .01C_8$	Inventory	.8448
$\Delta C_9 = .01$	Inventory	2.4956

Source: From Van De Panne, C. and Bose, P., "Sensitivity of Cost Coefficient Estimates: The Case of Linear Decision Rules for Employment and Production," Management Science, Vol. 9, 1962.

> decisions, and (3) manpower planning. Learning curve analysis recognizes the existence of systematic productivity changes over the life of a product. Such analysis, however, typically ignores scheduling costs that results from changing workforce size, workforce utilization, and inventory fluctuations. The purpose of aggregate planning, on the other hand, is to develop a time-phased program for meeting anticipated demand while incurring minimum overall cost of operation. Clearly, elements of aggregate planning problems are directly related to the three uses of learning curve analysis. First, many of the cost elements in aggregate planning formulations involve each cash outlays and hence should be part of cash flow analysis. Second, aggregate planning formulations reflect operating costs which in addition to other costs should consider not only the learning phenomenon, but also the operating costs associated with alternative strategies of employing and utilizing a variable workforce. (page 172)

The idea of combining learning curve analysis and aggregate planning has been suggested by Greene [33], Niland [52] and Taubert [65], although the methods for doing so have not been presented. Models combining changing productivity situations with aggregate planning have been reported in the literature; however, all have certain limitations and unrealistic assumptions. Given the importance of the problem, it is surprising that only two models have been reported which incorporate the changing productivity considerations. The lack of incorporation of the effect of disruption in productivity improvement, resulting from workforce level changes, and the lack of proper recognition and separate treatment of aggregate production planning of long cycle and short cycle products are two major drawbacks of the existing models. (A more detailed explanation of these models and their limitations is presented in Chapter III).

The importance of a joint consideration of aggregate planning and improvement curve analysis, the advantages that result form this consideration, and the insufficiencies of the exisiting models suggest further exploration of the problem and construction of more reliable models.

Research Objectives

The objective of this research is to develop and evaluate aggregate production planning models incorporating the changing productivity considerations for both long cycle and short cycle product situations. Proper incorporation of the disruption effects resulting from manpower transactions, progress and retrogression effects, and adaptation of suitable solution methodologies are embodied in this objective.

Summary of Results

The objectives of this research have been met and the new models developed in the research are evaluated using a traditional cost model, and where applicable, actual data. The evaluation results of these models have indicated their significant economic impact in most situations. The major conclusions are:

1. The relative performances of the new models over the existing constant productivity and changing productivity models reach their highest levels when the firm passes the transitional start-up period and reaches the steady production state. It has been shown that in the steady production states, these relative performances can be as high as 30%. This magnitude will still be higher for larger production sequences.
2. The relative performance of the existing changing productivity model (applicable to the long cycle product situations) over the constant productivity models becomes insignificant in the steady production states. This model performs slightly better than the constant productivity model in the short start-up

period. However, even in this period, the new model developed for the long cycle product situations still performs better.

3. The impact of the new models is subject to the nature of various operational restrictions imposed on the planning problem. The tighter the restrictions on various production smoothing strategies (except the workforce level fluctuations) the higher the impact of the new models.
4. The impact of the new models is highly related to the levels of the cost coefficients in the objective function of the aggregate planning problem. For example, a potential cost saving of 89% is shown for modified levels of two cost coefficients in a model tested on the actual data.
5. The new models have higher impacts for sharper slopes of the applicable cost reduction curves.
6. The model developed for the short cycle product situations provides information for construction of more realistic aggregate planning cost models. This model allows the incorporation of variable payroll cost and variable overtime length.

Contributions

This research has made several contributions. One major contribution is the definition of the basic assumptions and elements essential to the development of aggregate planning models with dynamic productivity. Another major contribution is the development of models based on those assumptions. Other contributions include:

1. The general solution methodology applied in this research is a heuristic method called the Search Decision Rule. The

computer subprograms developed for long cycle and short cylce product situations are not peculiar to any specific search technique. Also, the routine developed for the long cycle product case can be utilized for all existing aggregate planning cost models. Utilization of the routine developed for the short cycle product case may require minor modifications in the structure of these functions. Generally, these programs can serve as standard routines to convert any current constant productivity aggregate planning model (which applies the Search Decision Rule for solution) to a model which incorporates the effect of the dynamic productivity phenomenon.

2. The methodology used for improving the computational efficiency of the optimizations performed in this research can be generalized for heuristic optimization of all complex functions, provided that approximations of these functions with simplier functions is possible.
3. Development of the analysis of compounded disruption effect is a contribution of this research to the general area of the improvement curve analysis. Application of the new analysis is not limited to the aggregate planning problem. This analysis is useful in a variety of production and workforce planning and scheduling problems.
4. The methodology developed for quantifying the relative performances of the new models developed in this research can be used as standard evaluation methods for future dynamic productivity models.

CHAPTER II

BACKGROUND

Introduction

The present research concerns both the aggregate planning problem and improvement curve analysis. Since these two subjects have been treated as separate areas of research, the background of each area will be independinetly reviewed, in brief, in this chapter. A detailed exploration of the existing models which merge the aggregate planning problem and changing productivity considerations will be presented in the next chapter.

Background of Aggregate Planning

The methodology of aggregate planning was first developed as part of the great post-World War II management science movement. Since then work has continued at an accelerated pace. This work has been motivated, in part, by the tremendous economic consequences of aggregated decisions and by the current development and improvement of research methodologies in the management science field. The initial thrust of this work was in the use of mathematical optimizing techniques, such as differential calculus and linear programming, to solve simplified aggregate planning cost models. Solving the models yielded a set of decisions, or decision rules, which produced mathematically optimum results with respect to the cost model.

More recently, perhaps following a newer wave of management science emphasis, new proposals for solving the aggregate planning problem have taken the form of decision rules which are based on heuristic problem-solving approaches and computer search methods. The objective of these newer methodologies is to enable the model builder to introduce greater realism into his model. This added realism should, hopefully, more than compensate for the fact the heuristic and computer search techniques do not guarantee mathematically optimum decision rules. Advocates of heuristic and search decision rule approaches argue that since the decisions produced by a model can be no better than the model itself, it follows that greater realism should produce better overall results. All three approaches have one thing in common, they address one of the most important problems in industry today. In this section the background of the studies relative to the above three areas will be presented briefly.

Mathematically Optimal Decision Rules

Bowman [12] for the first time proposed the use of the distribution model of linear programming for aggregate planning in 1956. The structure of the model is simple, and it focuses on the objective of assigning units of productive capacity in such a way that combined production plus storage costs are minimized while sales demands within the constraints of available capacity are satisfied. The greatest drawbacks of the distribution model are that the cost of changes in production levels are not accounted for and there is no penalty for back order or lost sales. The limitations and assumptions of the distribution model have caused investigators to continue their search for more effective models.

The simplex method of linear programming was proposed later as a framework for the aggregate planning problems. Its main advantage over the distribution model was that production level change costs as well as shortage costs could be included. McGarrah [47] developed a basic simplex model of aggregate planning for "one period" in which change and inventory cost functions were segmented into two to four linear functions which met the linearity requirements. The disadvantage of this model is that it looks ahead only one period (single stage). This disadvantage is so severe as to eliminate this method from serious consideration as a general approach for aggregate planning, unless one can assume that the constant sales continue into the future for a reasonable planning horizon. Otherwise, the model could suggest changes in workforce levels which might be negated in the subsequent period by a solution to the model requiring exactly the opposite action. The planning horizon time is of critical importance. Furthermore, the model does not express the results of the solution in a collection of decision variables that one really wants to know about; that is, number of employees hired or fired and how much overtime to schedule. Rather, one must work backwards from a new proposed production rate in order to determine how to implement the new rate; with workforce, overtime, or both.

Simplex models which expand the horizon time have also been developed by McGarrah and by Hanssmann and Hess [34]. The McGarrah model involves minimizing change costs plus inventory holding costs with change cost defined as two linear functions, one for production increases and one for decreases. This model expands the size of the simplex matrix considerably, but the major disadvantages are still that the model does not approach real life situations well enough and does not deal directly with the managerial decision variables of size of workforce and production rate.

The Hassmann-Hess simplex formulation isolates workforce and production rate as independent variables while regular payroll, hiring, layoff, overtime, inventory, and shortage costs are considered as dependent variables during the given planning horizon time. This model has been widely applied in industrial aggregate planning situations.

The interest in the area of aggregate planning reached a peak with the publication of "Planning Production, Inventories, and Workforce" by Holt, Modigliani, Muth and Simon [37] in 1960. The orientation of this book was based on an intensive research study conducted by the authors of an empirical situation. Their formulation of the problem was based on the assumption that the costs involved in aggregate scheduling could be represented by linear or quadratic functions which when combined gave a quadratic cost model. The resulting cost model was then minimized by differentiation with respect to the decision variables, production rate and workforce level. This operation produced a set of linear equations which could be solved for the value of the two decision variables. The net result was a set of two linear decision rules (the model is therefore referred as Linear Decision Rule Model—LDR) which related the present state of the system and the forecasted sales for an infinite time horizon to give the minimum cost values for the production rate and workforce level for the next time period. The major advantages of this formulation were its ability to give an analytical optimum with respect to the cost function on the basis of a sales forecast which needs to be unbiased, and the ease of solution to the resulting two linear equations that could be processed in a matter of minutes with a desk calculator. The model was first tested in a paint factory. The results of this analysis have been referenced by many studies in the field ever since.

The LDR model specified four cost components. For any particular period t, the sum of these component cost functions represents a function to be minimized. However, each monthly decision has cost effects which extend into the planning horizon. The result is a cost criterion function which adds these component costs for each month and in turn sums these monthly costs over the planning horizon. The problem then is to minimize monthly cost over N periods. This simple model is mathematically presented as:

$$\text{Min } C_N = \sum_{t=I}^{N} C_t$$

and,

$$\begin{aligned} C_t = {} & [(C_1 W_t) && \text{Regular Payroll Costs} \\ & + C_2(W_t - W_{t-1})^2 && \text{Hiring and Layoff Costs} \\ & + C_3(P_t - C_4 W_t)^2 + C_5 P_t - C_6 W_t && \text{Overtime Costs} \\ & + C_7(I_t - C_8 - C_9 S_t)^2] && \text{Inventory Connected Costs} \end{aligned}$$

Subject to restraints,

$$I_{t-1} + P_t - S_t = I_t \quad t = 1, 2, \ldots., N$$

In the above formulation, W_t, P_t, I_t and S_t represent workforce level, production rate, inventory level, and demand for period t, respectively. Numerical values for the coefficients C_1, ... C_9, are statistically estimated from accounting data. C_4 is the constant productivity rate (units of output per man-month).

Holt, et al., supported their idea of fitting quadratic curves to the cost function by stating that since the optimality of decision rules depends on the accuracy with which the mathematical cost function approximates the true structure of costs, it is desired to know how close the approximation really needs to be. It turns out that fairly large errors in estimating and in approximating the cost relations with quadratic functions lead to small differences in the decisions. Differences in the decisions lead to even smaller differences in the costs incurred when the rules are applied. Thus, only reasonable accuracy in estimating and approximating the cost relationships is sufficient.*

In commenting on the comparison between the LDR and the Hanssmann and Hess models it is believed that one could just about flip a coin. In reality the various component cost functions found in practice are probably neither all linear nor all quadratic, but rather a mixture of various forms. What is needed is a methodolody which is free of mathematical forms in constructing models for specific company situations. Heuristic and computer search methods seem to be promising in this regard.

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*Results of the present research and the sensitivity analysis of the LDR cost coefficients [67] have made the significance of this statement highly questionable!

An extension of the LDR model has been developed by Sypkens [63] which considers plant capacity as a decision variable in addition to the workforce and production rate. This model is expected to perform best if it were used with the kind of production systems in which capacity can be divided into identical units.

In a recent paper Schwarz and Johnson [59] state a hypothesis claiming that the incremental benefit of aggregate planning (all aggregate planning models) over the improved inventory management alone may be quite small. They base their claim on the results of a reported application of the LDR (the paint factory case). The authors discover that in this particular case, most of the LDR cost savings have been due to the reduction in the total inventory costs.*

Gaalman [29] has recently presented an interesting method for aggregating on multi-item version of the HMMS model. He has applied the necessary conditions to reduce the multi-item model to a one item model. The aggregation technique makes use of the structural properties of the inventory-production part of the model, and can be performed regardless of the structure of the workforce-total production part. The author shows that dissagregation of the optimal decisions of the aggregate model gives the optimal decisions of the multi-item model.

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*The author strongly disagrees with Schwarz and Johnson's claim. The particular cost structure of the selected model is surprisingly biased in favor of this claim; therefore, their stated hypothesis can not be generalized for all cases, and specifically for <u>all</u> aggregate planning models. These authors' discovery of the nature of the LDR cost savings is not original: reference [26], which was published years before the publication of the article in question, clearly analyzed the problem (item 3, page 122). However, those conclusions and generalizations were not justified by the author of the latter article!

Chang and Jones [18] generalized the LDR model to yield both aggregate and disaggregate planning in a multiproduct environment and extended their work to handle the situation in which production cannot be started and completed in the same period.

Several other solution methods that provide optimal results have been developed and applied with different degrees of success. The exhaustive enumeration of all feasible solutions is one of these methods. The approach is practically feasible only when a finite number of decision variables exist, that is, when the planning horizon is comparatively short. (See Chapter IV.)

Bellman [6] has proposed dynamic programming formulations of the aggregate planning situation which conceptually are interesting. As in many dynamic programming formulations, what is conceptually interesting is not often computationally feasible. The major restriction in this case is the number of possible production states available at each stage (period). Where a limited number of production levels are a realistic assumption, the application of dynamic programming may become feasible. The advantage of the dynamic programming approach over other optimizing techniques in this area is that it is independent of cost structure.

Goodman [31] has presented a goal programming approach to solving nonlinear aggregate planning models. He applies his technique to the Holt quadratic model and concludes that the effectiveness of such an approach is highly dependent upon the degree of non-linearity which the GP model must approximate. The results suggest that for relatively low degree models goal programming may provide an efficient solution approach, while for higher degree models the approach may be inappropriate.

The optimizing procedures for aggregate planning problems with stochastic demand are currently receiving attention. Kleindorfer and Kunreuther [41] recently published a paper relating to this case. They have developed a methodology for showing how forecast horizons for stochastic planning problems relate to the planning procedures. To illustrate the approach they have chosen a relatively straight forward production problem in which the firm can meet a fluctuating demand pattern through a combination of overtime and inventory-related options. Their conclusion indicates how this methodology can be utilized for specifying stochastic horizons for more general aggregate planning decisions.

Heuristic Decision Rules

The LDR and its extensions continue to provide a harsh standard for comparing the effectiveness of new approaches to the problem, simply because the LDR methods provide known optimum solutions to specific test situations. The difficulty with mathematical methods is the requirement that cost functionsbe expressed with either quadratic or linear relationships, thus limiting the realism which can be incorporated in the model. The new heuristic methods, as well as computer search methods to be discussed later, are more free of the constraints of mathematical forms. Thus, a tradeoff must be made between the desirability of obtaining a known optimum solution to a relatively simplified model versus obtaining a near optimum solution to a richer, more realistic model.

Bowman [13] has proposed a new and different approach to many managerial problems and has used the aggregate planning problem as a sample for study and demonstration of his proposed approach to managerial decision making. His approach establishes the 'form' of decision rules for aggregate planning through rigorous analysis; however, it develops the 'coefficients' for the decision rules through statistical analisis of management's own past performance (decisions). This is in constrast to the LDR, in which both the form and the coefficients are determined by mathematical analysis. Bowman determines the coefficients by regression analysis on management's past actual decisons. Bowman's theory is based on the assumption that management is actually sensitive to the same criteria used in analytical models and that management behavior tends to be highly variable rather than off center. In terms of Bowman's theory then, management's performance using the decision rules can be improved considerably simply by applying the rules more consistently; since, in terms of the usual dish shaped criterion function, variability in applying decision rules is much more costly than being slightly off center from optimum decisions, but consistent in those decisions.

Gordon [32] developed Management Coefficient models for the Chain Brewing Company. He also developed an LDR model for the brewery, for the purpose of comparison. The procedure used simulated the behavior of the production system under each of the alternate sets of decision rules by generating production and workforce decisions over a 52-week period. He concluded that the Management Coefficient Model had a total cost performance advantage somewhere between the LDR and actual. However, one serious drawback of the MCM method is the required subjective selection of the form of the rule. It can very easily be selected incorrectly. Obviously, in this case the use of the rule would lead to less than ideal results.

Vergin [68] argues that in many cases the current state of the art does not allow the analytical solution of a mathematical model that is representative of the prototype situation. Therefore, he claims that the best approach is to model the actual cost functions accurately in the form of a computer program so that functions more complex than those allowed in such approaches as linear programming and the LDR can be included. As in any simulation the approach is to systematically vary the variables (e.g., the workforce sizes and production rates) until a reasonable (and hopefully near optimal) solution is obtained. Tests performed by Vergin have shown that substantial benefits may result from the use of simulation approaches.

Jones [40], in his "Parametric Production Planning," postulates the existence of two linear feed-back rules: one for the workforce, the second for the production rate. Each rule contains two parameters. For a likely sequence of forecasts and sales the rules are applied with a particular set of the four parameters, thus generating a series of workforce levels and production rates. The relevant costs are evaluated using the actual cost structure of the firm under consideration. Using a suitable search technique the best set of parameters is determined. Again, the results with this simulation approach are quite encouraging. With Jones' method, there are no limitations in mathematical form of the cost functions; rather they are simply the best estimates of the cost functions that can be constructed. The selected parameters are incorporated into the two decision rules to make the rules specific for a given firm. Thus, while the decision rules are not optimum in the sense of a mathematically provable optimum, the procedure introduces aggregate production plans involving cost which can not be easily improved.

Search Decision Rules

One of the most recent approaches to the aggregate planning problem has been through optimum-seeking computer search methods. Taubert developed the basic search decision rule methodology in 1967 using the paint company data with the LDR optimum solutions as test functions. The results obtained with the paint company data have since been validated by other authors. Extensions in other environments with much greater complexity have been developed by Buffa and Taubert [64] and others. It has been proven that search decision rules can actually produce realistic decisions in situations so complex that no other known mathematical programming techniques could be used including linear, nonlinear, and dynamic programming.

The Search Decision Rule (SDR) approach does not guarantee global optimality, but it does offer a new way of breaking through the restrictive barrier imposed by the analytic model, the optimal solution methods discussed before. The SDR approach proposes building the most realistic cost or profit model possible and expressing it in the form of a computer subroutine which has the ability to compute the cost associated with any given set of decision variable values. Mathematically, the subroutine defines a multidimensional cost response surface with a dimensionality determined by the number of decision variables and the number of the time periods included in the planning horizon. In short, the cost model forms a multistage decision system model in which each state represents the cost structure of the operation at the point in time when decisions are made, such as monthly, quarterly, etc. A computerized search routine is then used to systematically search the response surface of the cost model for the point (combination of decisons) producing the lowest total cost over the planning horizon. A mathematically optimum solution is not guaranteed, but the solutions found by the model cannot easily be improved.

Background of the Improvement Curves

The Improvement Curve is a graphical or analytical representation of the anticipated reduction in input resources as a production process is repeated. The reduction in cost or the increase in the rate of production is achieved in part by the improvement and performance of direct labor. Other improvements come from management and supporting staff organizations. The airframe industry was first to use the predictive value of improvement curves. Empirical evidence supporting the learning phenomena soon found acceptance in a cross section of manufacturing industries. Today improvement curves are widely used as an integral part of production planning and control, as well as a means of controlling the learning rate of individual operators.

A review of improvement curve applications by Balloff [5] indicates that the power function formulation can be used to determine the productivity increases which accompany the introduction of new products in a variety of labor intensive assembly situations, including the manufacture of airframes, electronic and electro-mechanical components, and machine tools, In addition, the model has also found applicability in instances of both products and process startup in the machine-intensive manufacture of steel, glass, paper, and electrical products.

The literature so far has concentrated mainly on a somewhat standardized power function usually referred to as a linear curve. This formulation was first introduced by Wright [71] in 1936. Since then, it has managed to survive in essentially its original form, even though it has many basic weaknesses. However, a number of alternative functions have been proposed. Most of these were intended for specific applications and, therefore, have not affected the popularity of the power function to any degree. For instance, Cochran [19] proposed an S-type function which was based on the assumption of a gradual startup. An S-type function has the shape of a cumulative normal distribution function for the startup curve and the shape of an operating characteristic function for the learning curve. Guilbert (French) proposed a complicated multiparameter function with several restrictive assumptions.

More recent works hold somewhat greater promise. Among these, DeJong [23] proposed a version of the power function which generates two components, a fixed component which is set equal to the irreducible portion of the task and a variable component which is subject to learning. Levy [46] has presented a new type of firm learning function and shows it to be useful in explaining how firms adapt to new processes and in isolating the variables that may influence the firm's rate of learning. Levy's learning function reaches a plateau and does not continue to decrease or increase as does the power function. Asher [3] reported on a variety of different approaches, most of which were proposed during and immediately following World War II. The main drawback of these proposed functions is the difficulty associated with parameter estimation.

However, the proposed alternatives have not been able to dislodge the power function which, apparently, is still the most common one in use at present time. Pegels [55] offered an alternative exponential function and demonstrated that it provides a better fit to several sets of empirical data than the traditional algebraic power function. Bevis et al. [7] related the learning curve to an exponential law commonly found in physical systems, which characterized the rise time and final value of output rate.

However, as each proponent of an improvement curve model claims relative superiority of a particular model over those of earlier researchers, and as each also gives his examples of specific industrial situations in which his model performed better, it is recommended that any new user of improvement curves make some test runs when selecting an appropriate model for his own situation. Shultz and Conway [21] conclude that improvement curves predict a firm's progress function with tolerable amounts of error better than any other device known to the authors. Thus efforts devoted to determining a proper model and a proper estimation of parameters will yield a greater return.

Another important consideration in the theory of improvement curves is the effect of disruption. Disruption constitutes a definite cost. Hoffman [36], using a displacement of the origin (beginning cumulative production for a number of repetitions), developed the improvement curve for a repeat lot and suggested that the amount of displacement is a function of the amount of learning retained from previous lots. Cochran [19] presents a comprehensive study of the effects of various disruptions on the production of long cycle products.

Carlson, et al. [17] described disruption by a negative delay function comparable to the delay observed in electrical condensers. They assumed that an individual's memory is the equivalent of storing electrical charges in the brain, thus the delay analogy appears reasonable. Obviously, the rate and amount of delay would depend on "how much" has been learned or where the task is interrupted in the process of learning. They showed performance expected as a function of chronological time or equivalent units. This can also be expressed in terms of the number of units completed and equivalent units which could have been completed after an interruption. The amount of forgetting and the corresponding level of performance is thus showed as a function of both the performance at the time the process was interrrupted (or total amount learned) and the length of the interruption. The authors also showed that if the work performed during the interruption was of a similar nature, then the rate of forgetting was reduced. Their model is named the LFL model which represents the Learn-Forget-Learn Phenomena.

Learning curves are applicable to many aspects of production planning and control today. They can be used to predict the cost per unit of production, establish selling price, quantity discounts, and forecast capital needs for budget planning. Learning curves influence delivery schedules, measurement of shop efficiency, setting of labor standards, evaluation of employee training programs and improvement of wage incentive schemes. Finally, the learning curve concept can be introduced to the aggregate production planning area to handle the changing productivity cases which exist in most real situations.

Since the current research makes use of the somewhat standardized linear improvement curves, a brief presentation of the analysis of such curves will follow.

Analysis of the Linear Improvement Curves

Empirical studies have demonstrated that incremental improvement in productivity decreases as the quantity produced increases (Figure 1.a). This relationship is known as an improvement curve. Improvement curves, when plotted on a log-log graph paper, result in a straight line (Figure 1.b). This straight line is easily expressed by a simple algebraic equation.

Letting C(n) represent the cost in manhours of a given cumulative unit n, then the improvement curve can be written as:

$$C(n) = f.n^{-b} \quad \text{where } f = C(1) \tag{2.1}$$

or as:

$$\log C(n) = \log C(1) - b.\log n \tag{2.2}$$

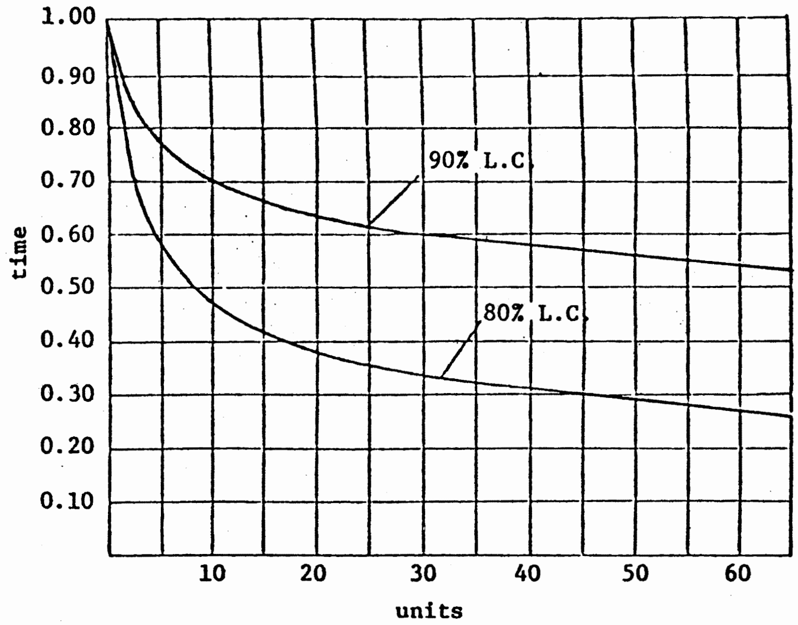
C(1) is the cost of unit one, known as the theoretical base unit cost. The exponent "b" is a measure of the slope of the linear cost reduction line.

Equation (2.1) has some important characteristics. Given any two cost curves $C_1(n)$, $C_2(n)$ which have the same value for b, then

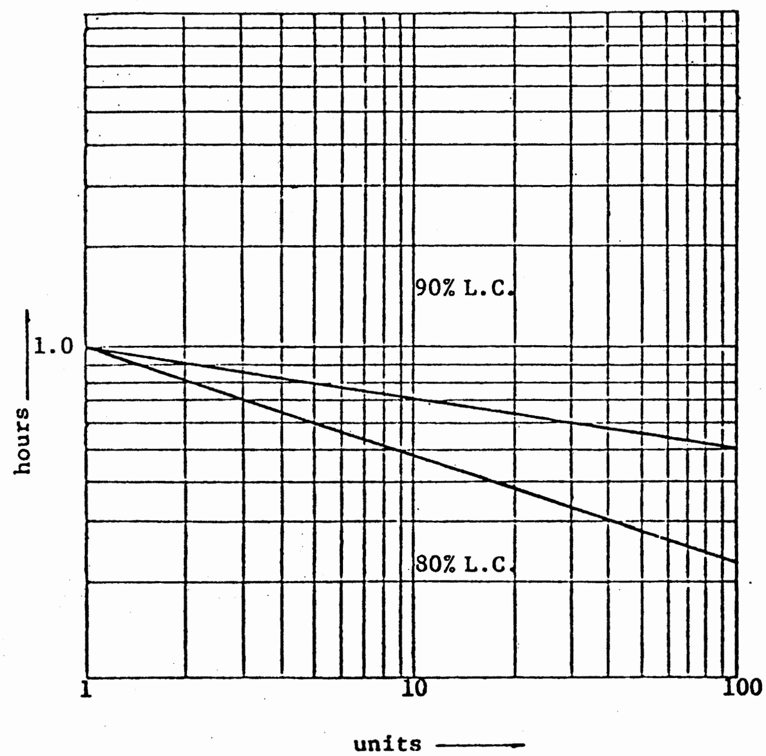
$$C_1(n) = f_1 n^{-b} \quad \text{and} \quad C_2(n) = f_2 n^{-b}$$

where f_1 does not equal f_2. Now these two expressions can be related as follows:

$$\frac{C_1(n)}{C_2(n)} = \frac{f_1 n^{-b}}{f_2 n^{-b}} = \frac{f_1}{f_2} \tag{2.3}$$



a.) Learning Curve Data Plotted on Arithmetic Scale Graph Paper



b.) Learning Curve Data Plotted on Log-Log Scales

Figure 1. Linear Learning Curves

This result indicates that the ratio of unit costs between the two curves is a constant (f_1/f_2), no matter what value n takes. On a logarithmic scale, a constant ratio means a constant distance between the two lines on the graph. Hence the two lines must be parallel to one another; they have the same "slopes."

There are some other characteristics of Equation (2.1) that are of major importance. Primary among these is the ratio of unit costs for any two units m and n:

$$\frac{C(m)}{C(n)} = \frac{C(1)m^{-b}}{C(1)n^{-b}} = \left[\frac{m}{n}\right]^{-b} \tag{2.4}$$

Taking M = 2n, the above equation would be written as:

$$\frac{C(2n)}{C(n)} = 2^{-b} = S \tag{2.5}$$

This is conventionally used to define the slope (S) of an improvement curve. The slope is defined as the ratio of the cost of units in a doubled quantity relationship. Therefore, a 90% improvement curve applies in situations where the manufacture of cumulative unit 2n of output requires only 90% of the manpower that was needed to produce cumulative unit n.

Given the slope of an improvement curve, one can use Equation (2.5) to compute the exponent b:

$$\text{Log } S = -b \log 2$$

or,

$$b = -\log S/\log 2 \tag{2.6}$$

The quantity b is referred to as the measure of slope throughout this research.

Often it is intended to not only determine the cost of a unit but that of a range of units. This total cost is usually referred to as the "block" cost of units. To approximate the block cost over a range of cumulative output, from n_1th to n_2th unit, one can simply integrate Equation (2.1):

$$\sum_{n_1}^{n_2} C(n) \simeq \int_{n_1-0.5}^{n_2+0.5} C(n) = \int_{n_1-0.5}^{n_2+0.5} f.n^{-b}$$

$$= \frac{f}{1-b} [(n_2 + 0.5)^{1-b} - (n_1 - 0.5)^{1-b}] \qquad (2.7)$$

Equation (2.7) can be simplified when n_1 and n_2 are large by simply ignoring the 0.5 terms.

CHAPTER III

AGGREGATE PLANNING MODELS INCORPORATING PRODUCTIVITY—AN OVERVIEW

Introduction

All aggregate planning models discussed in the previous chapter utilize a constant productivity factor. In this chapter an overview of the state of the art in combining aggregate planning models with changing productivity considerations will be presented.

As mentioned earlier, only two models combining changing productivity considerations with aggregate planning have been reported in the literature; however, both have certain limitations and unrealistic assumptions. A brief explanation of these models and their major limitations and drawbacks follows.

Orrbeck Model

The first aggregate planning model which incorporated the effect of worker productivity was developed by Orrbeck et al. in 1968 [53]. This model is an extension of the Hanssmann-Hess model [34] which presents a linear programming formulation of the aggregate planning problem. The cost elements considered in the Hanssmann-Hess model are regular payroll costs, overtime pay, cost of hiring and firing workers, and storage and shortage costs. The sum of these costs accounts for the total relevant cost in any period. The problem is then one of choosing production and employment patterns in order to minimize the sum of the total relevant costs over the planning horizon. The regular payroll costs in any period t are assumed to be proportional to the number of workers employed in that period. The cost of overtime is found after first establishing an upper limit on the production that can take place during regular time. Any production in excess of this amount must be done on overtime. To establish the upper limit of regular time production, the model assumes that each employee can produce exactly the same constant amount in a period. Including hiring, firing, inventory and shortage costs, the aggregate planning problem is then to determine P_t and W_t (t = 1, , , , N) in order to minimize

$$\overset{*}{C} = \sum_{t=1}^{N} [C_r W_t + C_o (KP_t - W_t)^+ + C_h (W_t - W_{t-1})^+$$

$$+ C_f (W_t - W_{t-1})^- + C_I I_t^+ + C_s I_t^-]$$

subject to

$$P_t \geq 0,\ W_t \geq 0,\ I_t = I_{t-1} + P_t - D_t,\ t=1, \ldots, N$$

where

n = number of periods in the planning horizon

W_t = workforce level in period t

C_r = regular payroll cost per employee

C_o = overtime payroll cost per employee

$1/K$ = number of unit of output per employee per period

---

*The function a^+ is defined as a if $a > 0$ and 0 if $a \leq 0$. Its counterpart a^- is 0 if $a > 0$ and $-a$ if $a \leq 0$.

C_h = hiring cost per employee

C_f = firing cost per employee

C_I = inventory cost per period per unit

C_s = shortage cost per unit

I_t = inventory level in period t

By using the proper transformations the problem can be converted into linear form and be solved by standard linear programming methods.

As previously stated the Hanssmann-Hess model assumes a constant productivity rate for employees. In their extended model, Orrbeck, etal. drop this assumption and add the assumption that workers are assumed to have increasing productivity rates. To accomplish this they assume that all employees fall into one of e experience classes, where class e represents the most experienced class of workers. Certain productivity rates are attributed to certain experience classes.

The essence of the extended model is the assumption that the number of workers in an experience class will be the number of workers in the next most experienced class in the preceeding period, minus the number of workers released from the group. Exceptions are the first and last groups. The first will consist of newly hired workers, and the most experienced class will consist of employees in this group in the previous period plus those promoted into the class by the passage of time.

Furthermore, this model assumes that the least experienced workers are fired first, if workers are to be released. Should the number of workers released in a period exceed the number of employees in the first class of the previous period, some workers from the second experience class would have to be released. Also, requirements governing the assignment of overtime are added. One requirement is that unduly large amounts of overtime not be assigned to any class of employees. Another requirement is that if overtime is used, workers will be called upon in order of seniority. Thus the most experienced workers will work overtime first subject to the limit of their capacity. If overtime work still remains, the next most experienced class will be called upon.

As a result of the above assumptions, a set of new constraints are added to the original Hanssman-Hess model and necessary transformation to convert the problem into linear programming format are provided. The formulated model prior to transformation has the following structure:

$$\text{Min. } C = \sum_{t=1}^{T} \left\{ \sum_{i=1}^{e} N_t{}^i c^i + C_h N_t{}^1 + C_f N_t{}^f + a \sum_{i=1}^{e} \frac{c^i}{p^i} O_t{}^i + \frac{1}{2} C_I (I_t + I_{t-1}) \right\}$$

Subject to the following constraints

$$I_t = I_{t-1} + X_t - S_t$$

$$O_t = [X_t - \sum_{i=1}^{e} p^i N_t{}^i]^+$$

$$R_t{}^i = [O_t - \sum_{j=i+1}^{e} (\ell-1) p^j N_t{}^j]^+ \qquad i=1,\ 2,\ \ldots,\ e-1$$

$$O_t{}^i = R_t{}^i - R_t{}^{i-1} \qquad i=1,\ 2,\ \ldots,\ e$$

$$X_t \leq \sum_{i=1}^{e} \ell p^i N_t{}^2$$

$$N_t{}^i = \left[N_{t-1}^{i-1} - \left(\sum_{j=1}^{i=2} N_{t-1}^{2} - N_t{}^f \right)^- \right]^+ \qquad i=2,\ 3,\ \ldots,\ e-1$$

$$N_t^e = \left[N_{t-1}^e + N_{t-1}^{e-1} - \left(\sum_{j=1}^{e-2} N_{t-1}^j - N_t^f\right)^-\right]^+$$

$N_t^i \geq 0$, $O_t^i \geq 0$, $N_t^f \geq 0$, $X_t \geq 0$, $I_t \geq 0$, $i=1, 2, \ldots, e$ and $t=1, 2, \ldots, T$ for all constraints.

where,

T = number of periods in the planning horizon

e = maximum number of experience classes

p^i = productivity level of the ith class

O_t = total amount of overtime in period t

O_t^i = amount of overtime work assigned to class i in period t

X_t = production in period t

S_t = demand in period t

N_t^i = number of men in class i in period t

N_t^f = number of men fired in period t

c^i = regular payroll cost per employee in class i

ℓ = a constant such that maximum production by class i during overtime equals $\ell \cdot p^i$

a = a constant such that $a \cdot c^i$ equals overtime payment per employee in class i

The remainder of variables are as defined in the Hanssmann-Hess original model.

Through numerical calculations, Orrbeck et al. have demonstrated that when the difference in productivity between old and new workers is considerable, as would be the case in a skilled-labor-intensive industry, the extended model represents a substantial improvement over the original Hanssmann-Hess model.

Although the above model considers a variety of relevant assumptions, it has two major drawbacks. First, this model assumes that the productivity rate of each experience class is related to the time span during which the experience classes are involved in the firm's activities. That is, the productivity rates are only related to the passage of time. However, as mentioned earlier, empirical studies demonstrate that an increase in productivity can be systematically related to the cumulative output of the firm. This cumulative output is not necessarily directly proportional to elasped time. Orrbeck et al.'s assumption could be relevant if employees are utilized only on regular time. In such a case the output per employee could be assumed proportional to the number of production periods. In reality, however, utilization of overtime and undertime is frequently experienced by the firms. Due to these alternative ways of workforce utilization, groups of employees starting at identical productivity levels may have different productivity rates after one or more production periods. The lack of proper consideration of this phenomenon may be the major drawback of the Orrbeck et al.'s model.

A second drawback of this model is its computational limitation in the majority of empirical situations. This limitation is due to the large number of variables and constraints encountered in the linear program formulation. For example, for a 12 month period (usually considered in aggregate plans) and 6 experience classes (higher numbers may be assumed by most firms), 288 variables (excluding slack and artificial variables) and 168 constraints will be required in the model (after necessary transformations). Therefore, this model seems to be computationally unattractive.

As will be seen later, the assumptions regarding experience classes considered by Orrbeck et al are too unrealistic to be considered for the firms producing long cycle products. Some of these assumptions are only relevant for the case of short cycle products. Furthermore, the effects of progress and retrogression, which will be explained later, are not incorporated in the Orrbeck et al model.

Ebert Model

The second and the most recent model which merges productivity considerations and aggregate planning was developed by Ebert in 1976 [25]. The advantage of this model over the earlier one is due to the direct use of the learning curve analysis in aggregate planning. Ebert's model can also be applied using more complex cost functions.

The cost structure for production planning in each time period (t) used by Ebert to illustrate his proposed method is:

$$T_c = \sum_{t=1}^{N} T_{c_t}$$

where

$$\begin{aligned}
T_{c_t} &= C_1 W_t && \text{Direct Labor} \\
&+ C_2(P_t - C_{4t}W_t)/C_{4t} + C_3[(P_t - C_{4t}W_t)/C_{4t}]^2 && \text{Overtime} \\
&\text{if } P_t > C_{4t}W_t && \\
&+ C_5 W_t + C_6 C_1 W_t && \text{Variable Labor Overhead} \\
&+ C_7(W_t - W_{t-1}) \text{ if } W_t > W_{t-1} && \text{Hiring} \\
& && \text{or} \\
&+ C_8(W_{t-1} - W_t) \text{ if } W_t > W_{t-1} && \text{Firing}
\end{aligned}$$

$$+ C_9(I_t + I_t^x) \text{ if } I_t > I_t^x$$
$$+ C_{10}(I_t - I_t^x) \text{ if } I_t < I_t^x$$

Inventory Carrying
or
Inventory Stortage

where W_t = director workforce size, P_t = production quanitity, I_t = ending inventory, C_{4t} = average production per worker (same as 1/K in Orrbeck, et al. model), and I_t^x = desired ending inventory.

Evidently, other forms of cost functions could have been used to demonstrate this model. Ebert does not assume a constant value for C_4, rather he assumes that it changes as a function of the cumulative output of the manufacturing facility. The learning curve is usually expressed in terms of man-month per unit output, the inverse of C_4. For proposed levels of output across several future time periods, cumulative output will increase and average productivity will vary from period to period. The expected productivity for each of these time periods can be obtained from the manufacturing progress function (learning curve) and subsequently used as C_4.

To determine the expected productivity for a range of proposed output in a future time period, the general form of manufacturing progress function considered by Ebert is:

$$Y_i = K\, i^{-b} \tag{3.1}$$

where Y_i = man-month required to produce the ith cumulative unit of output, K = man-months required to produce the first unit of output (initial productivity), b = the absolute value of slope of the progress function and i varies continuously.

The average productivity over a range of cumulative output (from Ath to Bth units) proposed for a future month is obtained by first integrating (3.1) to obtain (3.2).

$$f(i) = \int_A^B Ki^{-b}di = K[B^{(1.0-b)} - A^{(1.0-b)}]/(1.0-b) \tag{3.2}$$

Then (3.2) is divided by B-A, thus,

$$Y'_{A,B} = K[B^{(1.0-b)} - A^{(1.0-b)}]/[(1.0-b)(B-A)] \tag{3.3}$$

The value of C_4 is then given by $C_4 = 1.0/Y'_{A,B}$. Thus, the cost of any proposed production plan can be approximated, once the parameters in (3.3) are specified.

A search routine is utilized to determine the solution to the above model. This search model consists of three major sub-components: a main program, an evaluation routine, and an exploratory subroutine. The manufacturing progress function is incorporated in the evaluation routine. For any proposed change in P_t (made by the main program or by the exploratory subroutine), the productivity factor C_4 in the objective function of the evlauation routine changes on the manufacturing progress function. The expected productivitiy (C_4) for the modified range of cumulative production output is used to evaluate the cost for each new plan. The main program in this model makes major changes in the decision vector values based on favorable change indicated by the exploratory routine. The exploratory routine modifies the existing decision vectors by small increments.

Ebert shows the potential significance of his model by generating a series of aggregate plans for various learning rates. These plans are then proposed to be used to develop manpower schedules, for cashflow analysis, and for making product pricing decisions.

The Ebert's model has one major drawback: the model does not incorporate the effect "learning" properly, and under rare situations it can only take into account the effect of "progress" on the productivity. The term "improvement" is usually applied to the general relationship between unit cost reduction and the cumulative number of units produced. The term "learning" is applied strictly to that portion of cost reduction which occurs without major method or design changes, and the term "progress" to the effect of those changes.

To notice the limitations of this model, one can consider the case of short cycle products, for example, in which different experience classes with different productivity rates can be recognized. A close look at Ebert's model in this case highlights the fact that the model treats every member of the workforce level in every period as if he were hired at the beginning of the first period. The productivity rate of a new worker is assumed to be the same as the productivity rate of the most experienced one. This is due to the fact that in this model the basis for determination of the productivity rate of a given employee in a given period is the cumulative product units produced by the workforces, without consideration to when an employee was hired.

This model could be almost valid only in a situation where the workforce level at every period of the planning horizon comprises only those employees (or a proportion of those) who have been hired at the beginning of the first period. That is, where the workforce level is monotonically non-increasing. This situation, of course, is not very likely to occur, since the decision rules in aggregate planning usually indicate fluctuating levels of workforce for the purpose of coping with the fluctuating demand throughout the planning horizon. Furthermore, even under this rare situation, and for the case of production of long cycle products where a crew of men is usually assigned on a job, the reduction in the size of workforce generates significant disruption in the improvement pattern of productivity. This disruption is not incorporated in Ebert's model.

To illustrate the impact of the above point, an extreme case where the workforce level is monotonically increasing could be imagined. Figure 2 portrays such a case for a six month planning horizon. Notice that the workforce in each period is comprised of different classes of employees with different experience levels. The rectangulars in each column represent these classes. The rectangulars with lower numbers represent the classes of employees with higher experience levels and therefore with higher productivity rates.

In the above case the model would assume similar productivity rates for all classes of employees in each period. For example, in the 6th period all lower five classes are treated the same as the upper class which has the highest productivity; the productivity rate of the most experienced class (rectangular No. 1) is applied even for the newest class of employees (rectangular No. 6). However, it is evident that the workforce level in period No. 6 is comprised of 6 classes of employees with different experience levels and productivity rates.

The above discussion concludes that Ebert's model is not a true representation of the production system in situations where the total

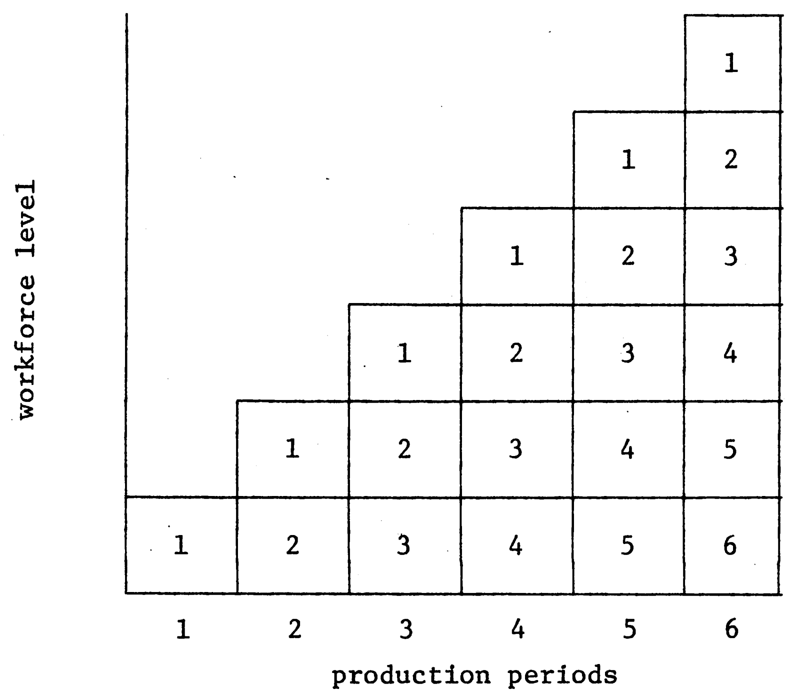


Figure 2. Experience Classes in a Monotonically Increasing Workforce Level Situation

productivity improvement pattern is not solely due to the progress effect (almost all situations with the exception of highly automated processes). The search routine developed by Ebert to obtain solutions to his model is the one developed by Hooke and Jeeves [38]. Since development of this routine, a number of other routines have been developed which are more efficient in handling complex functions which may even be subject to a set of constraints. Application of such routines should have a greater advantage since aggregate decisions are usually subject to constraints such as: limited storage space, limited assignment of overtime, limitations on the rate of hiring and firing manpower, and other restrictions. These constraints and the consideration of the production as function of workforce skills incorporated in an aggregate planning model could enhance the applicability of the model and reduce implementation and operational problems.

Considering the importance of precise determination of the productivity factor in the cost function of the aggregate planning problem, the insufficiencies of the models discussed in this chapter seem to be critical.

More reliable models based on more realistic assumptions for both cases of long cycle and short cycle products have been developed in this research. They will be presented in Chapter V and VI, respectively, following the discussion of the general solution approach.

CHAPTER IV

SOLUTION METHODOLOGY

Introduction

The two models developed in this research utilize the same solution methodology. Since the structures of these models are oriented toward the concepts of the applied solution methodology and these concepts are frequently referred to during the course of model description, it is appropriate to present a detailed description of the selected solution methodology prior to the presentation of the models.

The purpose of this research is to introduce more realism into mathematical models of aggregate planning. As a general rule, small incremental improvements in model realism require exponential increase in the mathematical complexity, and the more complicated and realistic the model, the more critical the problem of choosing a promising solution technique. Therefore, selection of an appropriate solution technique is of special importance for the current research.

There are numerous optimization techniques that can be used to solve mathematical models. Some are strictly analytical in nature: differential calculus, Lagrangian multipliers, linear programming and dynamic programming. Others are quasi-analytical, such as the gradient following techniques, and still others are strictly heuristic in nature. Both the quasi-analytical and heuristic techniques offer the user the hope of finding a global optimum, but not the guarantee of finding one.

At this time no single optimization technique can be used to solve all mathematical models. This means that optimization is still an art involving a careful match between technique and model. This match must be made skillfully, with constant concern for the basic fact that a solution to a model can be no better than the model itself. Consequently, the model builder faces the dilemma that the more complicated he makes the model, the lower the probability of finding the global optimum. In the past this problem was so serious that the model builder had to restrict himself to simple models that could be solved by analytic techniques. Today the computer has made possible many new-quasi-analytical and heuristic search techniques. These techniques have increased significantly the probability of finding the global optimum of a complex model and have placed before the model builder a very powerful set of mathematical tools.

The incorporation of dynamic productiviy consideration into the aggregate planning problem, as will be seen later, introduces a considerable amount of complexity which almost eleminates the possibility of utilizing an analytic solution technique for these models. For example, the dynamic nature of the cost functions assumed by these models results in heterogeneous decision systems (systems with stage dependent structures) that necessitate the utilization of heuristic solution techniques. Although these techniques do not guarantee the global optimum, better overall results are achieved by developing highly realistic models that are near optimum in preference to the globally optimum models that are unrealistic. After all, it is the real-world situation we wish to optimize rather than a model.

Because of the above fundamental reasons, the Search Decision Rule is the solution methodology chosen for the models developed in this research. A detailed description of this methodology follows.

The Search Decision Rule (SDR) Methodology

The heart of the SDR approach lies in a synthesis of computer optimization methods and multistage decision theory. In essence the approach proposes building the most realistic cost model possible and expressing it in the form of a computer subroutine which has the ability to compute the cost associated with any given set of values for decision variables. Mathematically, the subroutine defines a multidimensional cost response surface with a dimensionality determined by the number of decision variables and the number of time periods included in the planning horizon. In short, the cost model forms a multistage decision system model where each stage represents the cost structure of the operation at the point in time when decisions are made, such as monthly, quarterly, etc. A computerized search routine is then used to systematically search the response surface of the cost model for the point (combination of decisions) producing the lowest total cost over the planning horizon. A mathematically optimum solution is not guaranteed, but the method finds solutions which can not be easily improved.

Multistage Model Development

The basic building block of the SDR approach is the one stage decision model. This model represents the cost structure of the firm at some particular point in time when decisions are to be made. This is usually monthly. The one stage models are then joined together to form a multistage model which represents the operation of the firm over the planning horizon. Figure 3 portrays the one-stage model construction process and identifies the state and decision vector inputs, the stage returns and the state vector output.

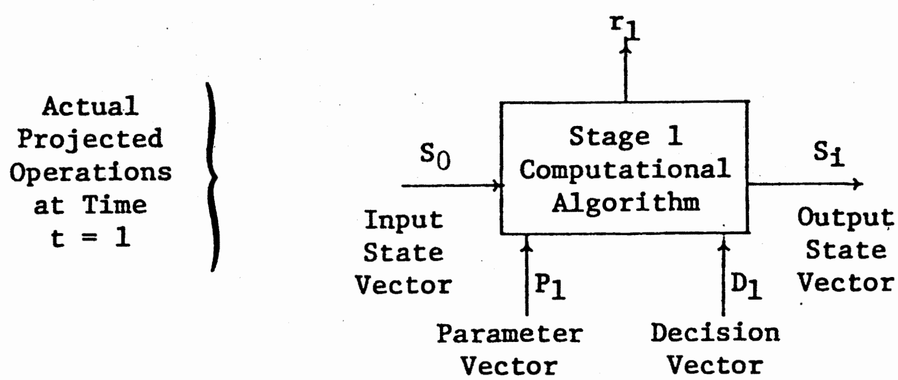


Figure 3. One Stage Decision Model

The terms illustrated in Figure 3 and used in the SDR approach are defined as:

Stage: Any real or abstract entity in which transformation takes place. In the context of the operations planning problem, a stage represents the point in time when decisions are made concerning the operation of the system. At each stage a decision (D) creates a return (r) and places the system in a new state (S).

Input State Vecotr S : A j component vector S = $(S_{01}, S_{02}, \ldots, S_{0j})$ which transmits information to stage 1 and serves to describe the state of the system at the beginning of the stage 1 transformation.

Output State Vector S_1: A j component vector $S_1 = (S_{11}, S_{12}, \ldots, S_{1j})$ which transmits information to stage 2 and serves to describe the state of the system at the end of the stage 1 transformation. The transition function is given by $S_1 = T_1(S_0, D_1, P_1)$.

Parameter Vector P_1: An i component vector $P_1 = (p_{11}, p_{12}, \ldots p_{1i})$ containing those factors that affect r_1 and S_1 and must be specified to define the problem.

Decision Vector D_1: A k component vector $D_1 = (d_{11}, d_{12}, \ldots d_{1k})$ which controls the operation of stage 1, given S_0 and P_1.

Stage Return r_1: A scalar used to measure the utility of the stage as a single valued function of the input state, parameter and decision vector $r_1 = f_1(S_0, D_1, P_1)$.

A one stage decision system model is constructed for each month in the N month planning horizon and then the models are joined together to form a serial mutlistage decision system. A serial mutlistage system is termed homogeneous if the individual stages are identical, and termed heterogeneous if they are not. A heterogenous system is produced when the coefficients of the cost function change from one stage to the other. The models developed in this research are of heterogeneous type, because the productivity rates are assumed to be changing from one period to another. The structure of the cost function in these models is dynamic in nature.

A simple multistage system is illustrated in Figure 4. The operations planning model in its simplest form consists of optimizing the total expected N stage return of the multistage model shown in this figure. This optimization is performed over the decision vectors $(D_1, D_2, \ldots, D_N)$, given the values specified by the initial system state vector S_0, and subject to possible constraints on both the decision and state variables. The decision made at each stage should be optimal with respect to the entire N stage system rather than optimal with respect to a particular stage.

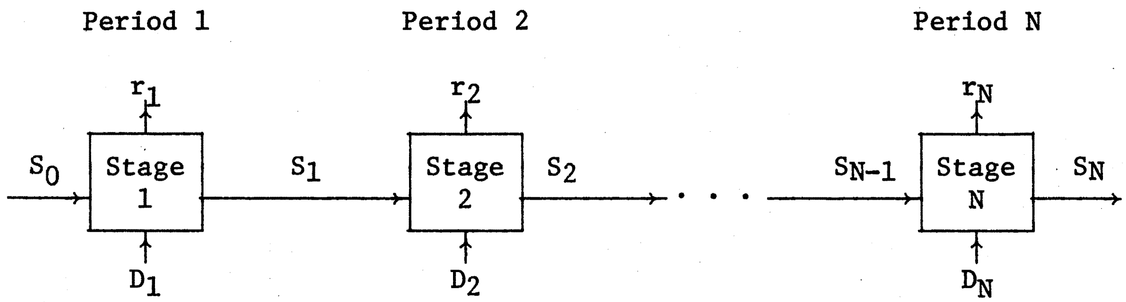


Figure 4. Multistage SDR Decision System

The SDR technique for optimizing the return from the multistage decision model is shown in Figure 5. It should be noted that the individual components of each decision vector for each month are considered as separate independent variables of the total multistage system. The computer optimization routine attempts to optimize all stages simultaneously; therefore, it must deal with a response surface with dimensionality determined by the product of the number of decisions per stage (K) times the number of stages (N) in the planning horizon. To do this the search routine measures its progress with reference to the total return (R) produced by the multistage system.

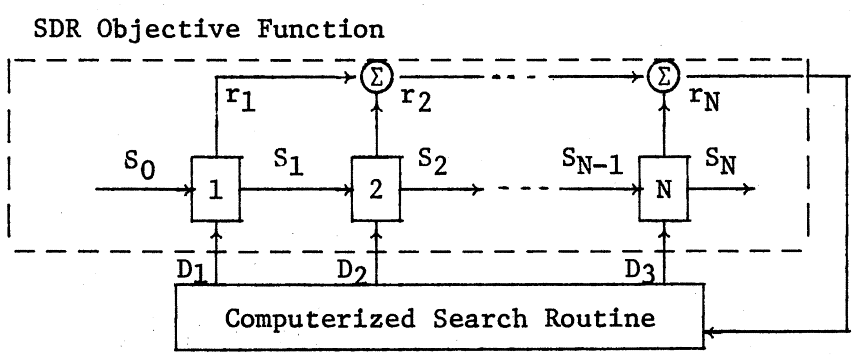


Figure 5. SDR Method for Solving a Multistage decision System

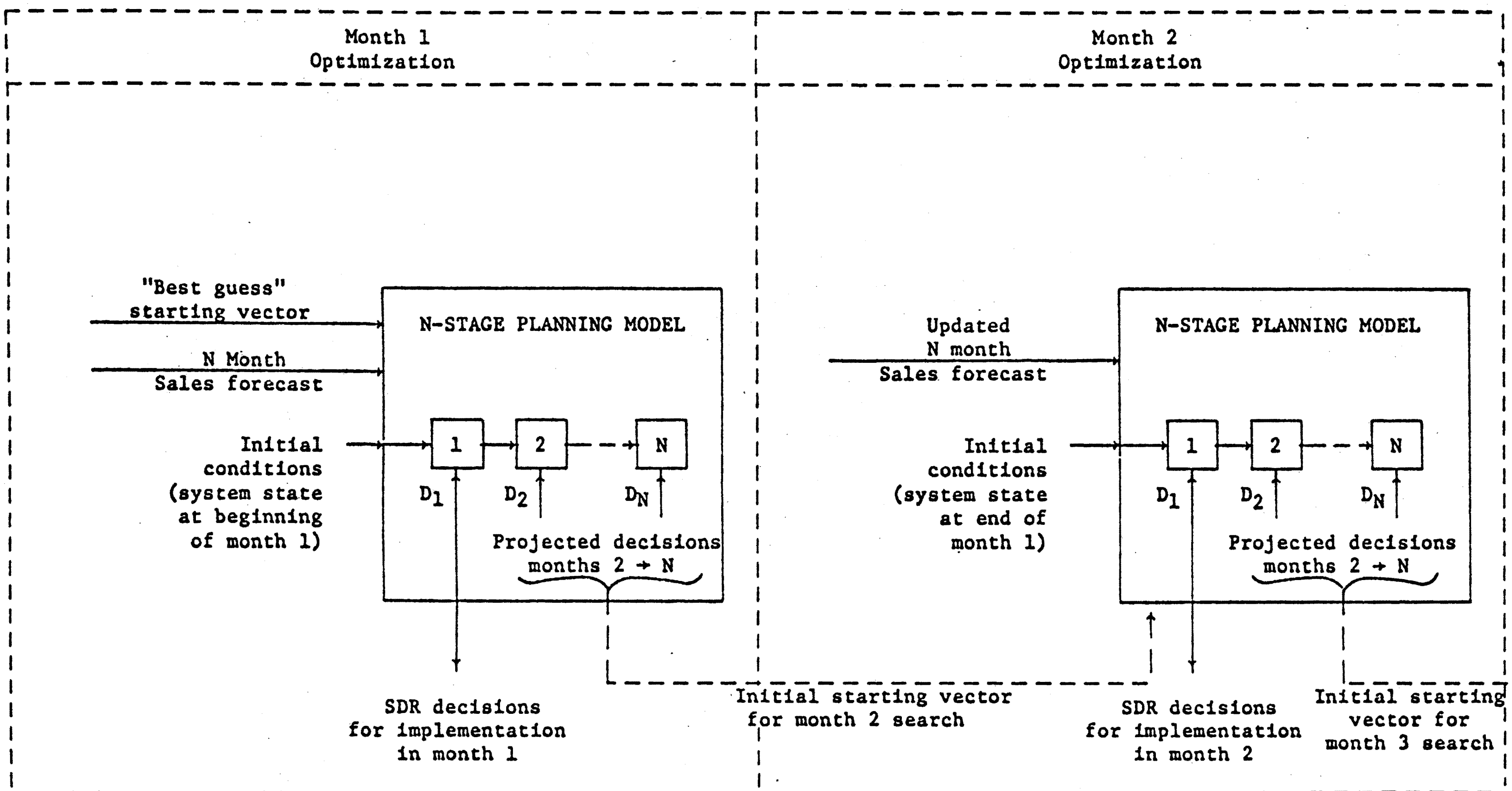
The computer search solution to the SDR model provides decisions for each month of the N month planning horizon. Normally, one is interested only in implementing the month 1 decisions contained in vector D_1. Decision vectors D_2, D_3, ..., D_N provide a planning purpose forecast of possible actions, but they are based on a successively shorter planning horizon. Therefore, when it is time to make decisions for the next month, the model is updated with a new sales forecast, initial conditions, etc., and optimized again.

The total return (R) need not be additive as shown in Figure 5 . It might consist of a weighted sum, such as present value discounting techniques; or it might consist of a complex formulation based on utility concepts. The SDR approach provides great flexibility in this respect.

The information flow in a typical SDR monthly cycle is shown in Figure 6. Following the month 1 optimization of the model by the search routine, the decisions contained in D_1 are reviewed and implemented by management. The projected decisions for the remaining months in the planning horizon (D_2, D_3, ..., D_N) are used to form the SDR starting vector for the month 2 search (performed at the beginning of the second month). In this way the search routine does not have to start all over again from a randomly selected starting point in N-dimensional space. Use of the SDR computed starting vector sharply reduces the search time and thereby reduces the cost of computing the decisions for month 2 and all subsequent decisions.

SDR Programming System

The complete SDR programming system for homogeneous models consists of a main program and two subroutines containing the search



Source: From Tauber, W. H., "The Search Decision Rule Approach to Operations Planning," Unpublished Ph.D. Dissertation, UCLA, 1968

Figure 6. Information Flow in a Typical SDR Monthly Updating Cycle

routine and the cost model. The operating sequence of the system is shown in Figure 7. The main program initializes all variables and reads in the sales forecasts, the starting decision vectors, the initial state vector and all model parameters. The main program then calls the search routine which systematically explores the response surface of the cost model until either the limit on the number of cost function evaluations is reached or a better point cannot be found.

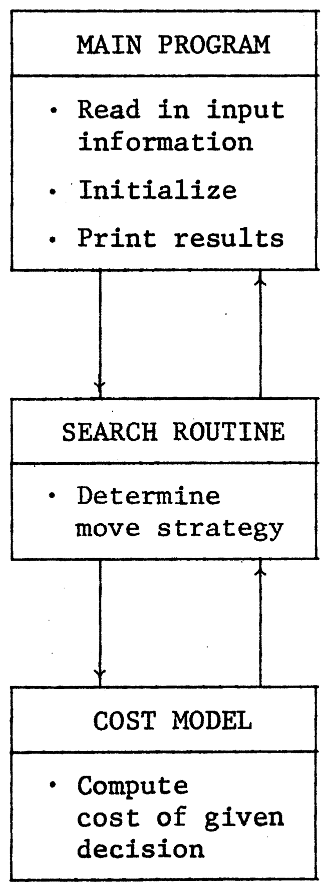


Figure 7. SDR Programming System

The search routine continuously varies the decision vector components in an attempt to minimize the total cost of operation over the entire planning horizon. At the conclusion of the search, control is returned to the main program for printing out the final decision vector and other information relating to the operation of the cost model. Typical computer times for a complete SDR search ranges from three seconds to two minutes on most medium-size computers, depending upon the complexity of the cost model and the number of dimensions.

For heterogeneous models, the program should be supported by a subroutine which systematically constructs the cost model at each stage. As will be seen later, such routines are developed in this research to properly re-structure the cost model at every stage with respect to the levels of state and decision vectors in the preceeding stages. Since these routines include a considerable number of computations and are called for every proposed change in the level of decision vectors, the computer time required for the search process is relatively higher for these models.

SDR Advantages and Disadvantages

The advantages and disadvantages of using the SDR approach as opposed to the traditional analytic model (optimal solution approach) are summerized below:

SDR Advantages

1. Permits realistic modeling free from restrictive assumptions, such as closed form mathematical expressions, linear/quadratic cost functions, etc.

2. Permits a variation in mathematical structure from stage to stage (heterogeneous stages) so that anticipated system changes, such as productivity improvement, wage increases, etc., can be considered.

3. Provides the operating manager with a set of current and projected decisions.

4. Permits optimized disaggregate decision making.

5. Lends itself to evolutionary cost model development and provides solutions at desired points in the iterative process.

6. Facilitates sensitivity analysis and provides sensitivity data while the search routine is converging on a solution.

7. Easily handles cash flow discounting, nonlinear utility functions, multiple objectives, and complex constraints.

8. Offers the potential of solving many otherwise impossible operations planning problems.

SDR Disadvantages

1. Optimization using computer search routines is an art and it is currently impossible to state which search routine will give the best performance on a particular objective function.

2. Decisions made by this methodology may not represent the absolute global optimum.

3. Response surface dimensionality appears to limit the efficiency.

Search Routine Operation

The heart of the SDR approach is the computerized search routine. A large number of search routines have been developed during the recent years and vary in design from traditional gradiant approaches to rather heuristic programs. Regardless of the particular design, all search routines may be classified by the way they answer two questions. Assuming that the routine has selected a particular point on the response surface by specifying the decision vector, then the two key questions are:

1. What is the next direction of movement?

2. How far should the movement be in the given direction?

The direction of movement may be along the gradient, along the deflected gradient, along each of the coordinate axis, or in a randomly selected direction. Once the direction has been determined; one step, several steps, or a one-dimensional line search may be made. In quantitative terms, the questions are answered by the following iterative equation:

$$D_{i+1} = D_i + \lambda_i P_i,$$

where D_i is an n-dimensional decision vector with components (d_1, d_2, ..., d_n) representing the trial point for the ith trial or iteration, λ_i is a positive constant, and P_i is an n-dimensional direction vector evaluated at the ith iteration. The vector P_i answers the first question by specifying the direction to be taken in moving away from point D_i and the magnitude of $\lambda_i P_i$ answers the second question by specifying a step is to be taken in that direction.

Figure 8 is a flow chart illustrating the major elements of a comprehensive search routine. The routine starts by selecting the initial starting vector for the search (box 1).

Boxes 2, 3, 4, 5 and 7 constitute what is frequently called the search code, or search algorithem. Collectively, they determine the location of the next point, evaluate the response surface at that point, determine the best direction of movement, and at the same time, monitor the progress to see if any action need by taken to speed up convergence.

Box 6 contains appropriate logic to check if the search code has moved the search outside of the feasible region; if so, it computes

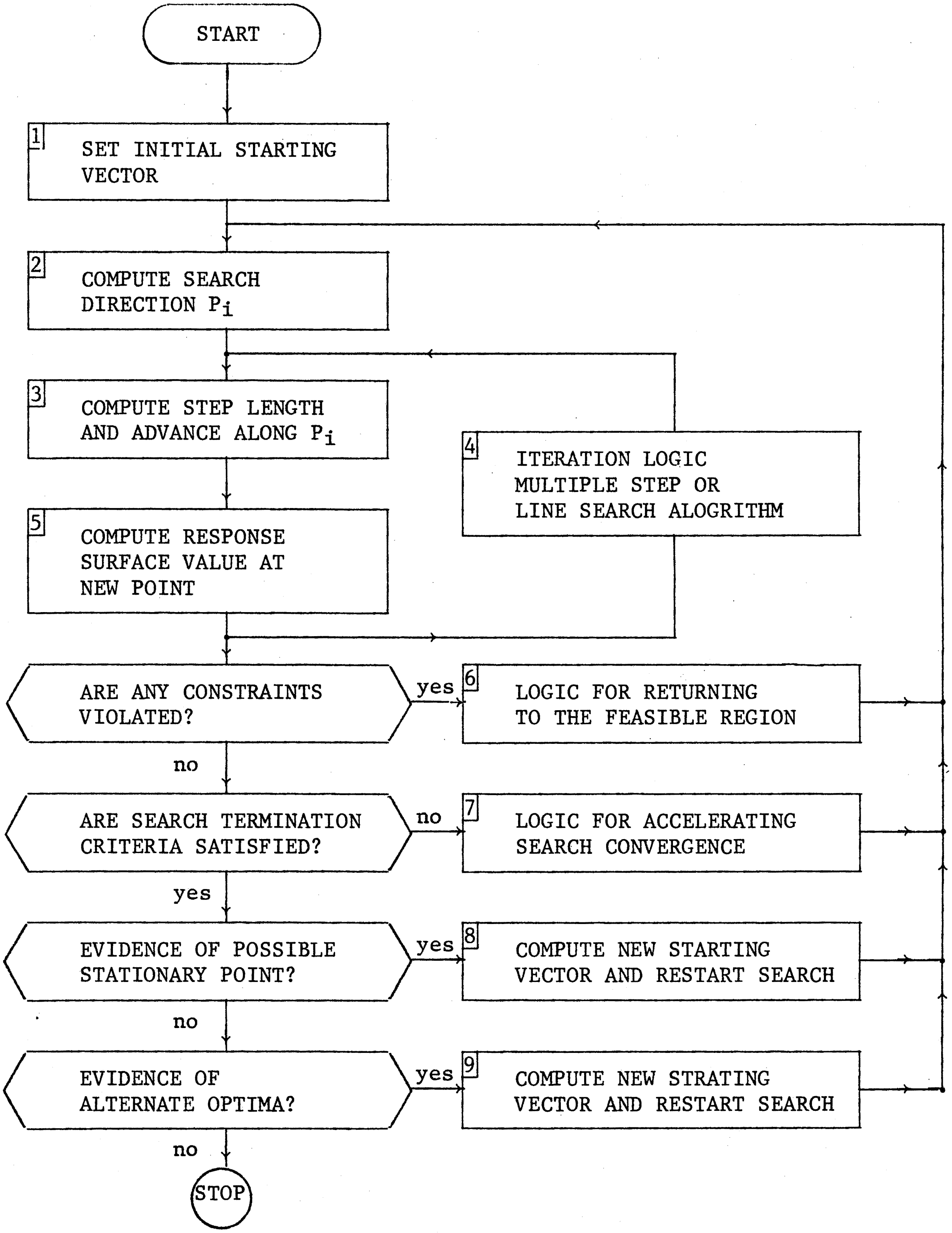


Figure 8. Major Elements of a Comprehensive Search Routine

the necessary step size and direction to bring the search back. There are a large number of sophisticated techniques available for use, but most require the solution of bounding problems that are almost as complicated as the objective function itself. As a result, many users prefer to transform the problem to one without constraints. This transformation is done by adding penalty functions to the original objective function and then optimizing the model as if it were unconstrained.

Box 8 represents the logic used to determine if the routine has become stuck on a relatively flat portion of the response surface. Typical tests for this condition include either random or symetric spot checks at various trial points in the neighborhood of the suspected stationary point. If no improvement is noted, the routine moves to the final test for alternate optima. If a better point is found, the search is restarted using the location of the new point as the starting vector.

Box 9 conducts a test for the alternative optima by restarting the search from different locations on the response surface. If they all converge to the same point, there is increased probability that the global optimum has been found.

Selection of a Search Code

A considerable number of search routines have been developed during the last two decades which utilize different search codes. As it was stated earlier, it is currently impossible to make a priori prediction concerning the performance of a search code on a particular response surface. Often no amount of argument can ever determine before hand if a particular method will work more efficiently then another, or even work at all. The performance of a search code depends on the particular type of system to be optimized and how well the program itself has been written.

Taubert [65] has conducted a comprehensive experiment to test the performance of four promising search codes applied to the SDR methodology. These codes are: conjugate gradient [28], variable metric [22,27], pattern [38,69] and simplex [11,51].

Conjugate gradient and variable metric codes consider the response surface in terms of a quadratic Taylor's series approximation model and base their move strategy on this representation. If the response surface is quadratic, the routines are guaranteed to locate the optimum point on an n-dimensional response surface in n-steps, assuming rounding errors are not significant. If the response surface is not quadratic, the routines use a local quadratic approximation of the surface to generate promising search directions. The resulting procedure is then iterative rather then n-step with the rate of convergence determined by the routines' response to the local quadratic approximation from one iteration to the next.

The conjugate gradient search code uses the method of conjugate gradients as the main vehicle for solving the quadratic series model of the response surface. The solution is used to produce new search direction vectors.

The variable metric code approaches the problem by approximating the matrix of second derivatives, or Hessian matrix, using information about the way in which the first derivatives vary from one iteration to another. Both conjugate gradient and variable metric methods require the numerical computation of first partial derivatives.

The pattern and simplex search codes are stricIty heuristic in nature and do not view the response surface in terms of a specific analytic model. Consequently, they do no require the numerical estimation of derivatives. In a series of performance tests on a 60-dimensional SDR model, Taubert concluded that the pattern search code performed better than the other codes. Faster convergence was the basic criterion for this comparison.

In the course of his work, Taubert developed a new code which is essentially a modified version of the pattern search code. The new code is termed the adaptive pattern search. The modification was made to the code developed by Weisman, Wood and Rivlin [69]. This code features a system for independently controlling the step size of each variable as well as some rather sophisticated search termination logic. These features considerably enhance search efficiency and performance over the original Hooke and Jeeves version.

The operation of the Weisman et al. version of the pattern search code was carefully studied by Taubert in connection with the performance tests on a SDR model. Taubert found that the performance of the pattern search code could be improved by a factor of two, and sometimes more, by systematically changing the pattern growth multiplier. The multiplier is used to control the length of the pattern move as defined by the equation:

$$T(I) = P(I) + G \cdot [C(I) - P(I)]$$

T is the new temporary base point computed by the pattern move equation, P is the old base point prior to the present pattern move and exploratory search sequence, C is the search location at the end of the exploratory search, G is a constant called the pattern growth multiplier and (I) represents the ith component of each vector. It can be seen that increasing the value of G causes the search to make larger pattern moves, while smaller values reduce the distance covered by the move.

Weisman et al. used a fixed value of G = 2.0 in their code, thereby, permitting a simplification of the general equation to:

$$T(I) = 2C(I) - P(I)$$

The pattern move is basically an acceleration device to move the search rapidly along relativley straight ridges in n-dimensional space. Consequently, it is hypothesized that if the value of G could be varied based on the progress made by the search code, then it might help the acceleration move adapt to the local terrain and thereby improve the overall efficiency of the search code. In other words, the multiplier would be varied if the search appeared to be slowing down or stuck at some point.

The adaptive control logic is designed around a measurement of the rate of convergance made by the search as it moves toward the minimum. If the rate is high, no change in G is made; if it is low, a small change is made; and, if it is very low, a large change is made. Determination of what is a high, low and very low rate depends on the particular response surface under study. The values used in this work are based on empirically determined values from investigations performed on the multistage SDR test models. A FORTRAN listing of the complete adaptive pattern search code with documentation, and the glossary of the variables in this program are included in the Appendix.

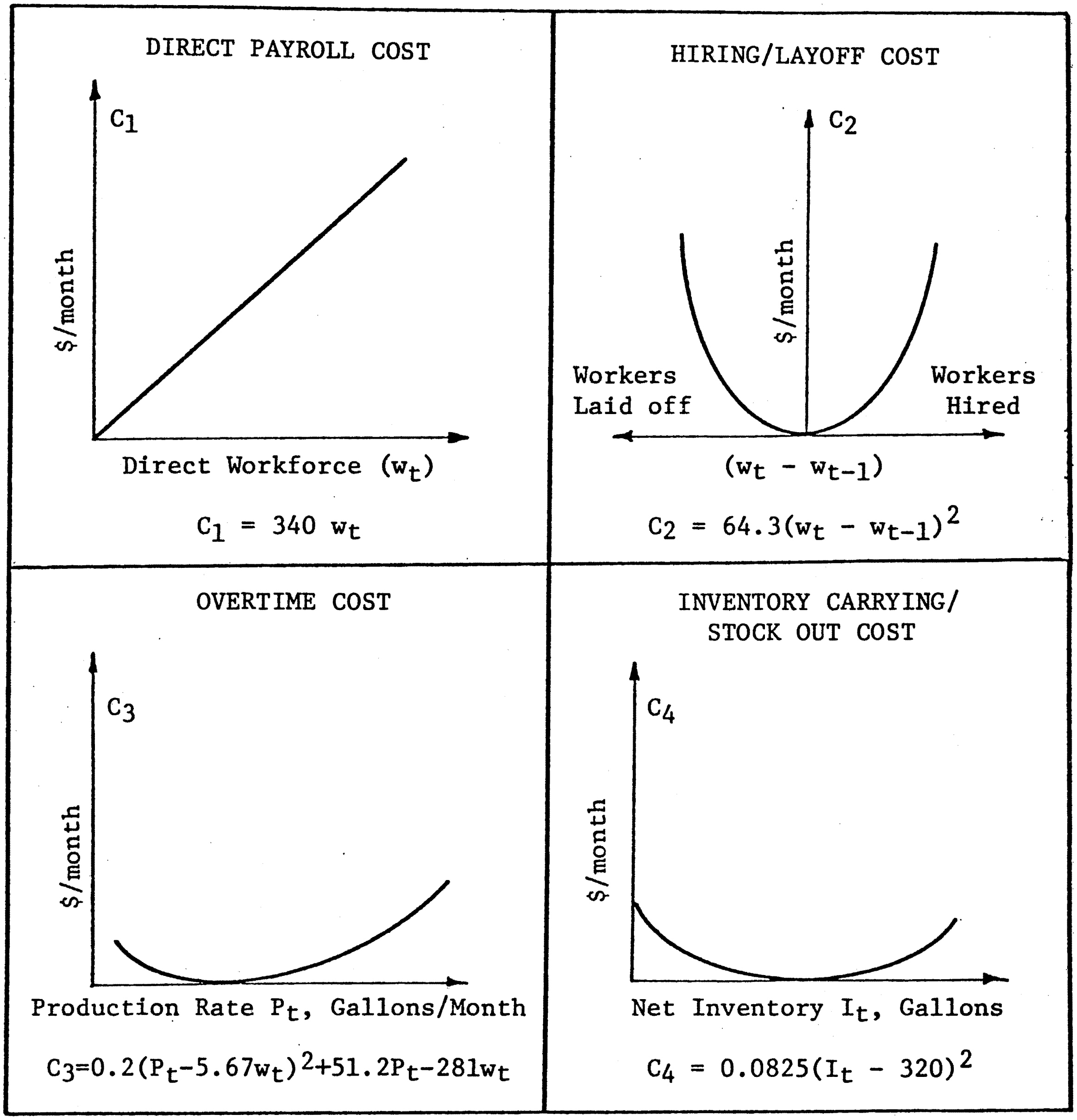
Application of SDR to the Aggregate Planning Problem

Since the current research utilizes the original HMMS cost structure as the basis for analysis of the research results, it is appropriate to demonstrate the application of the SDR methodology to optimization of this particular cost model. The HMMS model application using the quadratic paint factory cost function is probably the most widely studied in the aggregate planning field. This model has achieved the status of the standard of comparison for other aggregate planning techniques. Most of the challenging techniques have been unable to equal the LDR's performance of the paint factory model.

Figure 9 summarizes the four basic cost equations that makeup the paint factory cost model. In this model, the regular payroll cost for a period is a linear function of the workforce level in that period.

The hiring and layoff costs are not associated with the size of the workforce, but rather with changes in its size. This relationship is approximated by a quadratic function.

The overtime cost function is not based on the size of the workforce or on the production rate, but rather on the production rate given the workforce. The overtime rate may be 50% of regular pay for week days and 100% of regular pay for weekends. Hence a quadratic approximation is assumed to be more accurate than a linear approximation. It should be noticed that any idle time incurred is included in the direct payroll cost function in this model. The factor 5.67 in the overtime cost function is the average productivity factor (units



Source: From Holt, C. C., Modigliani, F., Muth, J. F., and Simon, H. A., Planning Production, Inventories and Workforce, Prentice-Hall, 1960.

Figure 9. Cost Relationships of the Paint Factory Cost Model

of output per man-month) which is assumed to be constant in this model. As production, P_t, exceeds the level 5.67 W_t (a level set by the size of the workforce), overtime costs are incurred.

Inventory and shortage costs depend on the inventory on hand and on the orders that could not be filled. As the inventory varies from some optimum level, the inventory holding and shortage cost also vary. This relationship is also approximated with a quadratic function. The net inventory, I_t, in period t is the amount of inventory or shortages at the end of period t. The sum of the above four basic costs gives the total monthly cost, and the sum of the total monthly cost for each month in the planning horizon gives the total cost to be minimized.

The paint factory model can be formulated as a multistage SDR model. The main elements of such a model are defined as:

Decision Vector: $D_t = (d_{t1}, d_{t2})$

d_{t1} = workforce level for month t, or W_t

d_{t2} = production rate for month t, or P_t

State Vector: $S_t = (S_{t1}, S_{t2})$

S_{t1} = ending workforce level for month t, or W_t

S_{t2} = ending inventory for month t, or I_t

where $S_{t2} = S_{t-1,2} + d_{t2} - P_{t1}$

Parameter Vector: $P_t = (P_{t1}, P_{t2}, \ldots, P_{t8})$

P_{t1} = sales forecast for month t

P_{t2} through P_{t8} = cost coefficients for the stage return equation for month t

Stage Return: $r_t = f_t(S_{t-1}, D_t, P_t)$ = total cost/period t

$$r_t = P_{t2}d_{t1} + P_{t3}(d_{t1} - S_{t-1,1})^2 + P_{t4}(d_{t2} - P_{t5}d_{ti})^2$$
$$+ P_{t5}d_{t2} - P_{t6}d_{t1} + P_{t7}(S_{t-1,2} + d_{t2} - P_{t1} - P_{t8})^2$$

Total Return:

$$R = \sum_{t=1}^{N} r_t$$

Since the cost structure of the paint factory model does not change with time, the stage return equation would have a simplified form, including the constant parameters:

$$\begin{aligned} r_t = {} & 340\ d_{t1} && \text{(Regular Payroll)} \\ & + 64.3\ (d_{t1} - S_{t-1,1})^2 && \text{(Hiring and Firing)} \\ & + 0.2\ (d_{t2} - 5.67\ d_{t1})^2 + 51.2\ d_{t2} - 281\ d_{t1} && \text{(Overtime)} \\ & + .0825\ (S_{t-1,2} + d_{t2} - P_{t1} - 320)^2 && \text{(Inventory)} \end{aligned}$$

It can be seen from the composition of the decision vector $D_t = (d_{t1}, d_{t2})$ that each month included in the forecast horizon requires the addition of one complete stage, or two additional independent variables (dimensions) to the multistage model. One variable is for the workforce level and the other is for the production rate. Therefore, an N period planning horizon would require an optimization of a 20-dimensional function.

Assuming an initial inventory level of 263 units and initial workforce level of 81 men, the SDR solution to the paint factory model for a 10 month planning horizon is demonstrated in Table II.

The results generated by the SDR show that this methodology has the ability to virtually duplicate the results of the mathematically

TABLE II

RESULTS OF OPTIMIZATION OF THE PAINT FACTORY COST MODEL

A. DECISIONS AND PROJECTIONS

Month	Demand	Work Force	Production	Inventory	Average Productivity
1	430	77.7	470.5	303.5	5.67
2	447	74.3	444.1	300.6	5.67
3	440	70.9	417.1	277.7	5.67
4	316	67.7	381.7	343.4	5.67
5	397	65.1	376.2	322.5	5.67
6	375	62.7	363.8	311.4	5.67
7	292	60.7	348.9	368.3	5.67
8	458	59.0	359.4	269.7	5.67
9	400	57.4	329.3	199.0	5.67
10	350	56.1	272.2	121.2	5.67

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	26,406.07	715.19	2,447.78	22.45	29,591.40
2	25,247.18	747.03	1,978.09	31.04	28.003.34
3	24,105.72	724.73	1,476.17	147.88	26,454.50
4	23,029.24	644.56	511.47	45.05	24,230.33
5	22,122.18	457.64	986.83	0.53	23,567.18
6	21,327.44	351.32	1,015.87	6.13	22,700.76
7	20,639.67	263.11	810.97	192.46	21,906.21
8	20,068.63	181.38	1,936.16	208.91	22,395.08
9	19,509.80	173.70	740.77	1,207.64	21,631.91
10	19,080.42	102.55	-1,408.60	3,259.04	21,033.41
					241,514.22

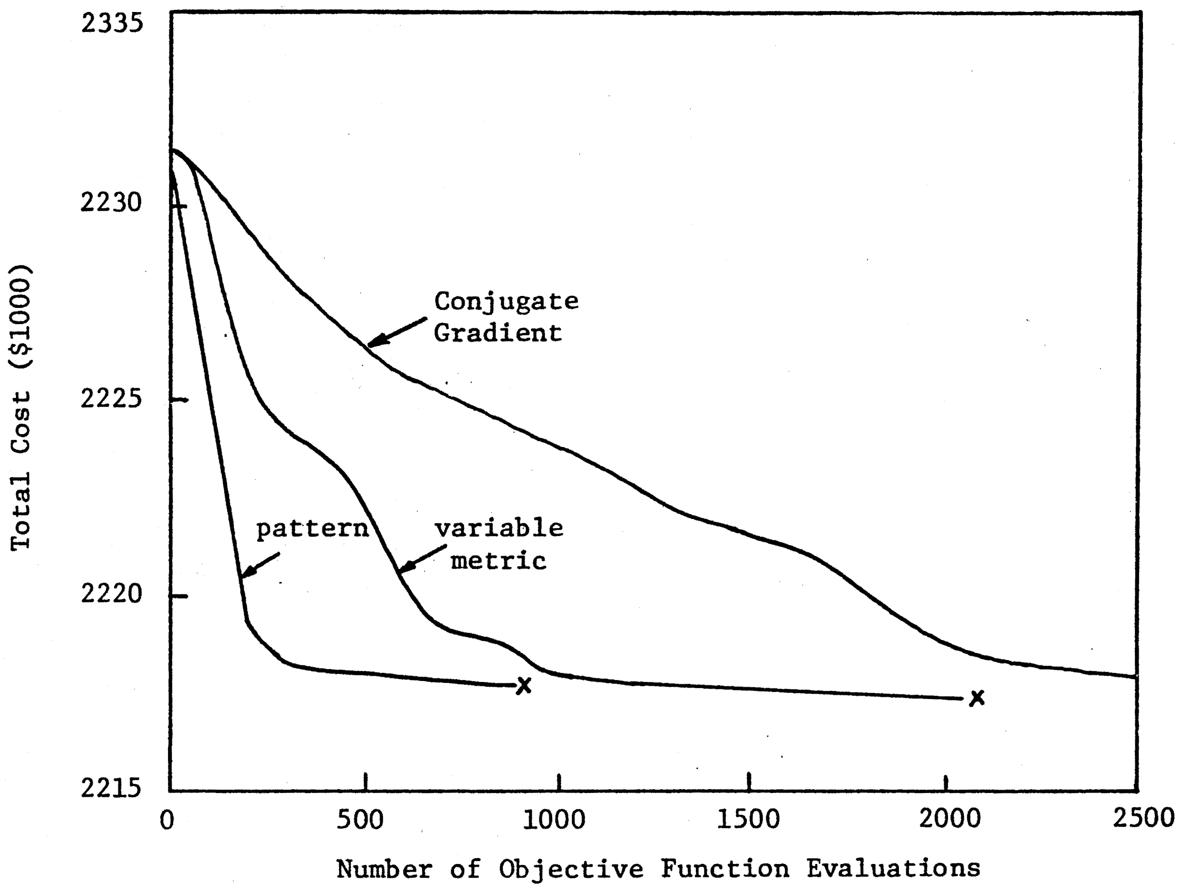
optimum rule (LDR). The total costs indicated by the SDR deviate within 0.1% from the ones generated by the LDR.

Once again, it should be noticed that the structure of the paint factory cost model is static. By means of the SDR approach it is possible to eleminate this restriction, as will be seen in the later chapters.

The aggregate decisions are usually subject to a variety of constraints: ending inventories are limited to some predetermined amount; assigned overtime is limited by the size of workforce; the normal workforce size is limited; there are limitations on the rate of hiring and firing new employees; etc.

The SDR methodology can incorporate the above constraints into the cost model and thereby generate solutions with less operational and implementation difficulties. The cost models optimized by SDR are not restricted to linear and quadratic forms; therefore, more realistic and general cost models can be developed and optimized using this methodology. The efficiency of the SDR methodology is highly dependent upon the type of the search code utilized. As mentioned before, the pattern search code has demonstrated the best performance in this regard.

Figure 10 shows the results of a typical test run in conjuntion with performances of three search codes, namely conjugate gradient, variable metric and the pattern search, as have been used for optimization of the paint factory model. It should be noted that in every case pattern search exhibits the fastest convergence.



Source: From Taubert, W. H., "The Search Decision Rule Approach to Operations Planning," Unpublished Ph.D. Dissertation, UCLA, 1968.

Figure 10. Results of a Typical Test Run to Select the Best Search Code for the Paint Factory SDR Application

CHAPTER V

MODEL I. LONG CYCLE PRODUCTS

Introduction

This research distinguishes between the long cycle products and the short cycle products. The output rate of firms producing long cycle products cannot be directly related to the productivity rates of the individual workers, as is the case for short cycle products. Long cycle operations are inherently a team activity. Long cycle work is long cycle because it is complex and requires a large amount of manpower. Therefore, it must be broken down into separate tasks in order to permit enough labor effort to be expended to meet schedules. In many areas this requires establishment of teams of men on the shop floor, each performing his specific task on the same basic unit. The unit cost (timewise) usually requires operating at low unit quantities of inventory, so that when a man or group falters, the effect on those around is felt quickly. The need to coordinate their efforts closely ties the workers together as a team.

There are several consequences of this team relationship. For example, the pace of each work center tends to be limited by the slower members. At the same time, those members may work better than they might have alone. In such a case, the improvements found by one member may help the others. The rhythm of the production process is generally more difficult for a long cycle work center to acquire. This may have some real effects on the pattern of cost reduction. The constant flow of minor method changes also upsets this rhythm to a greater degree. This prevents the operation from gaining the full benefit of those changes.

The changes in the manpower level can also impose significant changes on the unit cost of long cycle products. Aggregate production plans usually indicate a fluctuating workforce level schedule for the purpose of optimally coping with the fluctuating demand levels. The purpose of this chapter is to perform a study of the disruption effect on the productivity improvement pattern as a result of manpower level changes as these changes relate to the aggregate production planning problem.

Analysis of the Disruption Effect of Manpower Changes

As stated in the previous section, the unit cost of the long cycle product cannot be directly attributed to the productivity rates of individual workers. That is, the productivity rates of different workers or groups of workers cannot be separately considered and treated. This is due to the nature of team activities. In such activities, whenever an increase or decrease occurs in the level of the workforce, the productivity of the whole team is usually affected. As an example, the effect of a simple manpower increase could be considered. Some possible results of this change are:

- Some tasks are continued by those already performing them.
- Some tasks are assigned to new men who have no experience in performeing them.
- Some tasks are reassigned from men already performing them to the new men. Presumably the supervisor establishes the

new distribution of tasks with reference to individual capabilities.

— Some tasks are removed from men already performing them and reassigned to others of the original team (men with some experience) to secure better balance of tasks or other advantages.

It is conceivable that a major crew expansion could result in such a reshuffling of tasks that everyone receives a new reassignment. This would of course have a serious effect on productivity and is thus usually avoided.

Estimating the cost effect of manpower changes in long cycle manufacturing is of special importance because of the substantial cost premiums generated by the constant addition of untrained people and loss of trained ones. This not only reflects personnel turnover, but also the unique needs of long cycle manufacturing to meet production acceleration demands, implement frequent process improvement, and adjust manpower to match the relentless reduction of unit costs and the corresponding rise in the rate of output.

The method of estimating the effect of design change can be used as the basis for quantifying the effect of manpower changes on productivity. This is a reasonable approach because manpower changes have characteristics very similar to those of design changes. In both cases the work is new to the operator, the penalty of the change is larger for events occuring further in the production sequence, and it shrinks rapidly as production continues. This is true because, for instance, at the early stages of production, in which the employees are unexperienced and have low productivites, introduction of design change or addition of new (unexperienced) manpower do not introduce significant drops (disruptions) in the overall productivity level. In the later stages of production, the result is opposite.

Since new work is not added by a manpower change, it could be expected to resemble a task "turnover" most closely. By definition, when a task is deleted of the same size—timewise—as that which is added, the change is called a task turnover. However, a manpower change is less severe than a design change since supervision, tooling, support personnel, and other crew members are left unaltered.

Measuring the cost effect of manpower changes requires four steps:

1. Define the type of change which will be covered;
2. Measure the effect of each type of change on crew assignments to develop an "index of new manpower;"
3. Translate the index into an equivalent task turnover ratio;
4. Use the ratio and the methods based on the study of the cost effect of design changes to estimate the cost of the manpower changes.

In order to conduct the above analysis it is first necessary to analyze the cost effect of design changes and then interpret the concept of the task turnover in terms of manpower changes. Consequently, the following section is devoted to the analysis of the cost effect of design changes.

Cost Effect of Design Changes

Cost reduction arising from design changes is better described as "progress" rather than as "learning." However, such changes can increase as well as reduce unit time costs, and often generate irregular and confusing cost patterns.

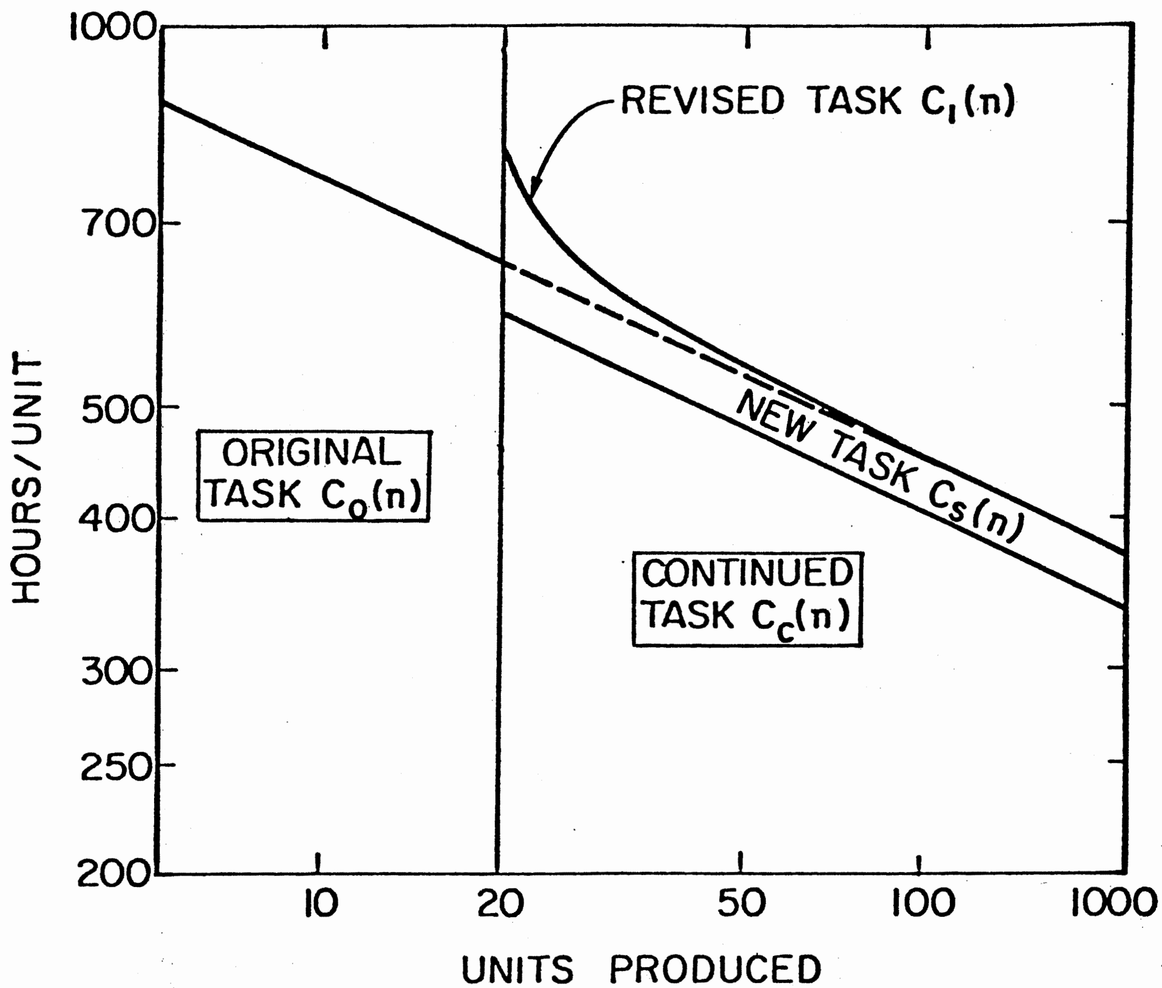
Any change generates a substantial initial labor cost penalty even if the ultimate effect is cost reduction, termed "progress."

This fact is logically interpreted as a "learning" phenomenon, since change introduces new work into a work center performing an existing task. The learning curve analysis provides a method of estimating the cost effect of task changes.

Of the many possible applications of learning curve cost analysis, long cycle products have the greatest need for this application since learning continues throughout the life of a product. The use of learning curve methods to predict changes in cost is based on the linear learning curve. The discussion here will therefore focus on the computation of change costs for linear curves.

Figure 11 illustrates a typical change on a long cycle product, occuring at unit 20. The high penalty paid on the first few units following the change with a rapid recovery of the original base cost level is illustrated in this figure. Such a cost pattern will naturally cause a temporary slow-down in the rate of output, unless extra men are assigned. The logic of this situatuion is quite conceivable, but it first requires the definition of what is meant by "a change " which involves the following elements:

1. The original task being performed by the work center. This is designated by $C_o(n)$ — The original log linear curve.
2. The portion $C_r(n)$ which is <u>removed</u> from the original task $C_o(n)$.
3. The portion $C_c(n)$ which continues to be performed after the change is incorporated. But since this equals the <u>difference</u> between the original task and that removed, this portion can also be written as $C_c(n) = C_o(n) - C_r(n)$
4. The new task $C_s(n)$ which replaces the deleted portion $C_r(n)$ of the original task. Normally it will continue at the same slope as before.
5. The revised <u>total task</u> after the changes have been made, is designated by $C_1(n) = C_c(n) + C_s(n)$.



Source: From Cochran, E. B., Planning Production Costs Using the Improvement Curve, Chandler Publishing Co., 1968.

Figure 11. Unit Time Cost Effect of a Typical Change for Long Cycle Product

In order to demonstrate the application of the above definitions to a typical case the following situation can be considered: Consider an 85 percent slope for an original task $C_o(n)$, such that the unit cost of unit 500 is 300 hours ($C_o(500) = 300$). The base cost can be found using the basic log linear curve formula:

$$C(n) = f \cdot n^{-b}$$

or,

$$b \doteq -\log(.85)/\log(2) = .2344$$

thus

$$f_o = (300)(500)^{(.2344)} = 1{,}287 \text{ hours}$$

Now suppose that of this task some 10 percent is deleted after unit 20. This makes the continued task 90 percent of the original one, or:

$$C_c(500) = (.9)(300) = 270 \text{ hours}$$

and the theoretical base unit cost for this task is:

$$f_c = (270)(500)^{(.2344)} = 1{,}158 \text{ hours}$$

For simiplicity, let us take the <u>new</u> task $C_s(t)$ to be exactly the same size as the task removed $C_r(n)$. This will also permit the measurement of the cost of making a change when the final task $C_1(n)$ is the same size as the original task $C_o(n)$. This will illustrate a significant cost penalty in such a case.

Being the same type of work as $C_o(n)$, the new task will also be on an 85 percent slope with a base unit cost equal to 10 percent of the original task (the size of the new task is 10 percent of the original one). Therefore, f_s = 128.7 hours.

Now the unit cost calculations for the revised task as the result of this disruption can be performed using the above data. Since the change was made after completion of unit 20, the first unit of the revised design is unit 21. The cost of this unit originally would have been:

$$C_o(21) = 1{,}287 \cdot (21)^{(-.2344)} = 630 \text{ hours.}$$

The revised cost $C_1(21)$ can be found by using definition (5) above. This cost can be computed in two steps, corresponding to the two elements of the revised task itself:

1. The first element, $C_c(21)$, represents the continued portion of the task which continues to be performed. This equals:

$$C_c(21) = f_c \cdot (21)^{(-.2344)} = (1{,}158)(.490) = 567 \text{ hours}$$

2. The second element, $C_s(21)$, represents the new portion of the task, and equals:

$$C_s(21) = f_s \cdot (1)^{-.2344} = (128.7)(1)^{(-.2344)} = 129 \text{ hours}$$

Therefore, the combined cost will be:

$$C_1(21) = C_c(21) + C_s(21) = 567 + 129 = 696 \text{ hours}$$

and this value compares with the original level as follows:

$$C_1(21) - C_o(21) = 696 - 630 = 66 \text{ hours (10.5 percent)}$$

The 66 hours difference is approximately 10.5 percent of the original level.

The subtantial cost premium of 10.5 percent is noticed, even though a task was added of the same size as that removed.

Similarly, the cost of a subsequent unit—say, unit 30—can be computed with some interesting further conclusions:

$$C_o(30) = (1{,}287)(30)^{(-.2344)} = 580$$

$$C_c(30) = (1{,}158)(30)^{(-.2344)} = 522$$

$$C_s(30) = (129)(30-20)^{(-.2344)} = 75$$

$$C_1(30) = 531 + 75 = 597$$

The unit cost difference between the revised task and the original task is $C_1(30) - C_o(30) = 17$ hours. This indicates a cost premium of only 3 percent.

The cost premium drops from 66 hours to 17 hours when only ten additional units are produced. The more units produced after a change, the closer is the revised cost level to the original one; and finally, after some number of units, the disruption effect will damp and the revised curve will approach the original curve. As Figure 11 indicates, a sharp slope immediately after the disruption occurs. It helps to understand this if one recalls that the slope of the learning curve plotted on the log/log scale reflects the <u>ratio</u> of cost reduction between succeeding units.

Tabulation of the costs which contribute to the revised curve in this example shows exactly how the gradually reducing slope occurs:

N	$C_o(n)$	$C_c(n)$	$C_s(n)$	$C_1(n)$
21	630	567	129	696
22	623	561	110	671
23	617	555	99	654

For the basic cost line $C_o(n)$, the cost for unit 22 drops only 7 hours or 1 percent below that for unit 21. But for the new cost line $C_1(n)$, there is a reduction of 25 hours or almost 4 percent. This large change creates the sharper slope shown in Figure 11. Such a change occurs because of the large reduction in the new portion of $C_1(n)$, namely $C_s(n)$. The drop from unit 21 to unit 22 amounts to 19 hours, or 15 percent, because those costs represent only the first and second units of the new work produced. It should be noticed that $C_c(n)$, the continued portion, has the same ratio of cost reduction (slope) as does $C_o(n)$ itself. Going from unit 22 to 23, $C_o(n)$ drops 6 hours and 1 percent while $C_1(n)$ drops 17 hours and 3 percent. The reason is the same, but now the cost of the new work $C_s(n)$ drops less rapidly so that $C^1(n)$ does also. The slope of $C_1(n)$ is therefore not so sharp between units 22 and 23 as between 21 and 22. And so the leveling trend continues as more units are produced.

Since this example assumes that a task is deleted of the same size as that which is added (10 percent of the original task), the change is called a "task turnover" of 10 percent. Usually a design change involves adding a task of different size than that deleted. Consequently, when the difference is larger there can be a substantial shift in the entire level of the curve. However, this latter case is not of major concern of this research, since the cost effect of the manpower change, which at this point is the subject of the research concern, generally corresponds to the task turnover.

General Formula For Unit and Block Change Costs

Further definitions can simplify the formulation of the change costs. Let us define:

n_o = The unit produced <u>before</u> changing the task.

r = The ratio which the task removed bears to the basic cost curve, or $r = C_r(n)/C_o(n)$.

s = The ratio which the new task bears to the basic cost curve, or $s = C_s(n)/C_o(n-n_o)$.

It should be noted that $C_s(n)$, the new task, must be related to $C_o(n-n_o)$ rather than $C_o(n)$ because each value reflects n_o fewer performances of the task. Therefore, $C_s(n) = sC_o(n-n_o)$.

Now since $C_1(n) = C_c(n) + C_s(n)$, and $C_s(n) = C_o(n) - C_r(n)$, then:

$$C_1(n) = C_o(n) - C_r(n) + C_s(n)$$

or

$$C_1(n) = C_o(n) - rC_o(n) + sC_o(n-n_o)$$

and with minor rearrangements:

$$C_1(n) = (1-r)C_o(n) + sC_o(n-n_o)$$

It should be noted that except for $C_o(n)$, the original cost curve, all other cost functions have been eleminated. Hence all calculations can be made using this curve. In the event of a "task turnvoer," r and s are equal in the above expression. Therefore, in this case the formula becomes:

$$C_1(n) = (1-r)C_o(n) + rC_o(n-n_o)$$

The unit cost difference, the difference between the revised task and the original task for the n^{th} unit is:

$$UCD(n) = C_1(n) - C_o(n) = (1-r)C_o(n) + sC_o(n-n_o) - C_o(n)$$

or,

$$\mathrm{UDR}(n) = s\frac{(n-n_o)^{-b}}{(n)^{1-b}} - r = s(1-\frac{n_o}{n})^{-b} - r$$

This equation indicates that as n becomes larger, n_o/n becomes smaller; a steady decline occurs in the UDR as more units are produced after a change. Eventually, the value of UDR approaches s - r. This means that a difference between r and s eventually creates a corresponding permanent shift up or down in the cost level. However, in the event of task turnover, this difference is zero meaning, that the two curves will eventually merge to a unique one (the original curve).

The block (cumulative cost for a range of units) change cost formula can be written on the basis of the same analysis. Using the same symbols and terms as before, the block cost may be expressed as the sum of two separate tasks, namely the one continued after the change is made and the new task. The simplest way to express this mathematically is to sum the costs of all units produced for the block in question. Therefore, for all units produced from unit $n_1 (n_1 \geq n_o)$ and through n_2 the formula will be:

$$\sum_{n_1}^{n_2} C_1(n) = \sum_{n_1}^{n_2} [(1-r)C_o(n) + sC_o(n-n_o)]$$

or

$$\sum_{n_1}^{n_2} C_1(n) = (1-r) \sum_{n_1}^{n_2} C_o(n) + s \sum_{n_1}^{n_2} C_o(n-n_o)$$

or

$$\sum_{n_1}^{n_2} C_1(n) = \frac{f(1-r)}{1-b} \cdot [(n_2+.5)^{(1-b)} - (n_1-.5)^{(1-b)}]$$

$$+ \frac{f \cdot s}{1-b} \cdot [(n_2-n_o+.5)^{(1-b)} - (n_1-n_o-.5)^{(1-b)}]$$

Notice that like the unit cost, the block cost of the design change can be calculated only with reference to the original cost curve. Figure 12 is a schematic representation of the unit cost computations.

Effect of Manpower Changes

As mentioned earlier, the analysis of the effect of design changes (given in the previous sections) is the basis for quantifying the effect of manpower changes. On the basis of this analysis, the discussions regarding the four steps in quantifying the effect of manpower changes are presented as follows.

Defining the Type of Change

There are three basic types of manpower changes: addditions, deductions, and the change which includes the first two at the same time. Additions and deduction to the workforce may be made to an existing production line or shift, or they may accompany changes in the number of lines or shifts working on a given product, as when output is raised or lowered to meet schedule requirements.

With regard to design changes there are four distinct tasks: the original one, the task removed, the task added, and the task continued. In a manpower change the entire old task continues, but several different manpower assignments are involved. As an example,

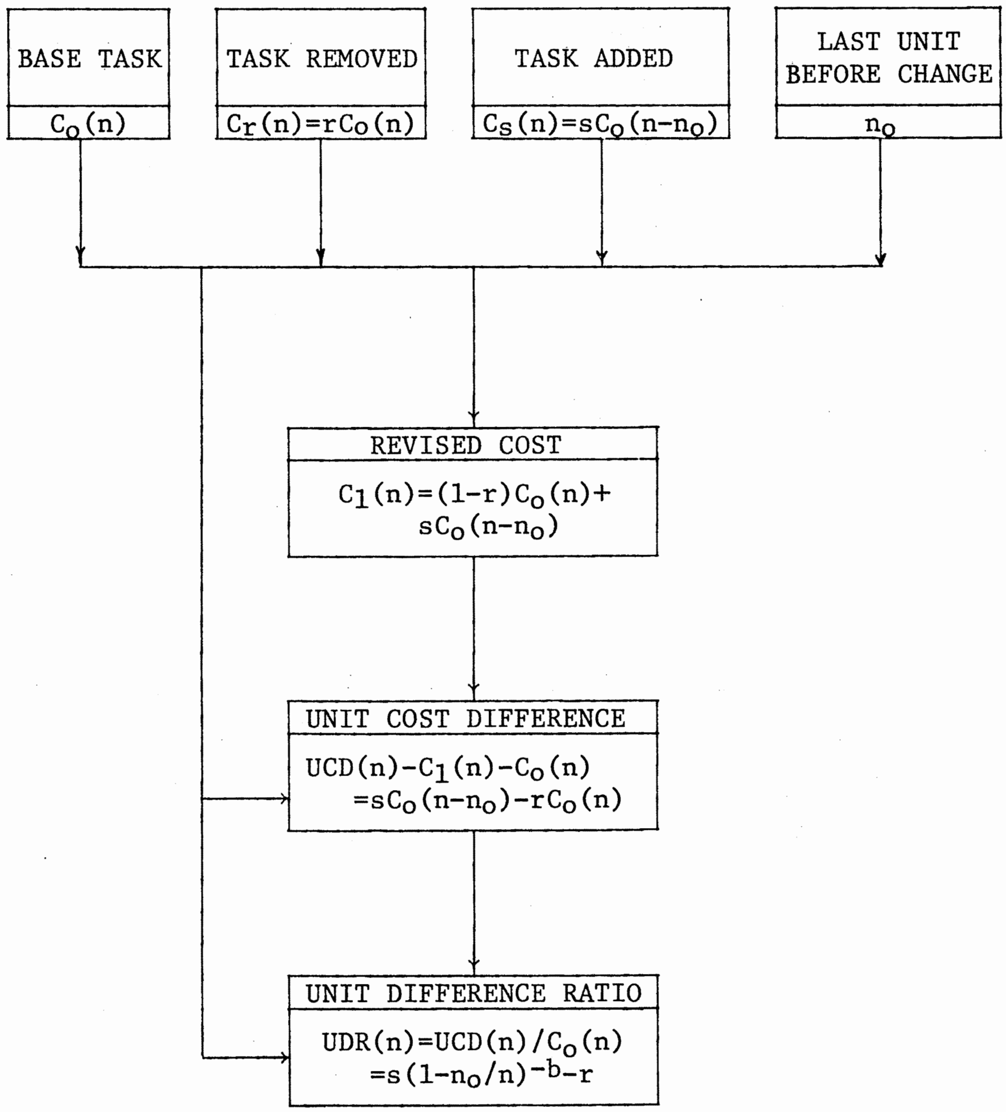


Figure 12. Unit Cost Computations for Long Cycle Products

the case for a simple manpower increase mentioned earlier in this chapter may be considered.

Aggregate plans only indicate the quantity of the workforce level and the direction of changes (increase or decrease) in this level at every period in the planning horizon. No detailed schedule of the third type of manpower change is provided by aggregate plans; this study will, therefore, consider only the changes as a result of manpower addition and deduction, based on the theory available in the literature of learning curve and possible contribution provided later in this chapter.

Measuring the Change

Once the quantity and the type of change is known, the proportion of new men can be translated into a numerical measure of tasks new to the revised crew. Designating the number of people before and after the change by P_1 and P_2 respectively, a "new man ratio" for manpower addition can be defined as:

$$t_a = (P_2 - P_1)/P_2$$

If, for example, manpower is increased by half, t_a is always 33 percent, as shown below:

$$t_a = (1.5P_1 - P_1)/1.5P_1 = .33 = 33\%$$

In such a case t_a measures the minimum proportion of the crew which now performs tasks new to those men.

A similar approach gives a "new man ratio" for a crew decrease:

$$t_d = (P_1 - P_2)/P_1$$

In general the value of t (t_a or t_d) depends only on the proportion of personnel change; the actual number is not important for the current purpose. Therefore, for any event the new man ratio will be:

$$t = |P_1 - P_2|/\mathrm{Max}(P_1,P_2)$$

Translating Manpower Changes into Cost

The index of new manpower cannot be interpreted simply as a task turnover (r) of the same magnitude, for the cost of manpower change would be over stated. Manpower change is less severe than design change, since supervision, support personnel, and other crew members are left unaltered. Therefore, some means of deluting the index is necessary. Furthermore, there is an obvious difference between the cost effect of a manpower decrease and those of an increase. The decrease involves only reassignment of crew members already in the work center, who may be considered already familiar with the task. It is also a common experience that minor manpower changes (such as normal turnover, bumping, etc.) do not affect cost considerably. This may include the flow of minor changes constantly occuring in plant operations, whose effects are already part of the cost base. It may also mean that small changes can be absorbed by a trained workforce with some extra effort by both direct and indirect personnel. In any event, it appears that the new manpower effect must exceed a certain "threshold" level before its cost effects need by taken into account.

Therefore, the index of manpower requires some modifications before it can be used as a "task turnover." Letting't' denote the task turnover whose cost will represent that of the new manpower ratio, t_a, the following expression will suitably translate into t':

$$t' = K_1(t_a - K_2)^+ \qquad K_1, K_2 < 1$$

and for further deluting the effect of manpower decrease, the following expression can be applied:

$$t' = K_1(K_3 t_d - K_2) + \qquad K_3 < 1$$

The values of K_1, K_2 and K_3 are subject to the type of plant conditions and no single set constants can be expected to apply to every plant or department. However, values of 0.5 for K_1, 0.05 for K_2 and .65 for K_3 have empirically been found to give a reasonable result [19].

Applying the Translated Index to Calculate the Effect

At this point the translated index t' is interpreted as a task turnover of size r = t' and the general formula developed for the design change case is used to calculate the effect of the manpower transaction on the unit or the block cost for the desirable units.

The Effect of Compounded Disruptions

A continuous change in the level of workforce indicated by the optimum levels of the decision variables of aggregate plans is always expectable. A possible result of such a situation is the occurrence of sequential compounded disruptions.

A significant manpower transaction at the beginning of a period may occur, while the effect of disruption(s) in the previous periods has not yet been discovered. Under this condition the cost effect of the new disruption is not directly a function of the original curve, but a function of the previously revised curve, simply because the current disruption is not imposed on the original curve, but the revised one.

To a manpower transaction occuring at the beginning of a period, the compounded disruption effect is applied only if there is a significant deviation between the original curve and the revised curve carried over this period. In other words, this effect is to be considered only where the unit cost difference ratio at the beginning of the period in question exceeds a certain minimum level below which the assumption of the identity of the original and the revised curves could be justified.

The possible occurrence of the compounded disruption effect is not limited to aggregate plans only. It's presence should also be expected in most situations involving cost improvement. Since the learning curve literature does not include the analysis of such an effect, the following section presents an approach to quantifying the effect of compounded disruption.

General Formula for Cost Effect of Compounded Disruptions

The approach taken in this section for computation of unit cost is very close to the one used before for the single disruption case. For every manpower change an index of new manpower is calculated and translated into an equivalent task turnover ratio. This ratio is then used to estimate the cost of the manpower change by the methods developed to estimate the cost effect of design changes.

Since manpower transactions generate task turnovers (the case where $r = s$), for the purpose of simplicity only the cost effect of changes as a result of turnover will be considered here.

The elements involved in this new situation are:

$C_o(n)$ - The very original task being performed by the work center - the original linear curve.

r_i - The equivalent task turnover of the ith manpower transaction.

n_i - The unit produced before the occurence of the (i+1)th manpower transaction.

Figure 13 illustrates the simplest case where two sequential disruptions have taken place. n_o is the unit number after which the first manpower transaction occurs and n_1 is the respective unit number in the second transaction. According to the former analysis, the revised task, as the result of the first transaction, could be written in terms of the original task $C_o(n)$, n_o, and r_1:

$$C_1(n) = (1-r_1)C_o(n) + r_1C_o(n-n_o) \qquad n_o < n \leq n_1$$

Now since the second disruption is imposed on this revised task, $C_1(n)$, and not on the original one, $C_o(n)$, the same general conclusion could hold true about the nature of the second revised task, if in the above formula $C_o(n)$ is replaced by $C_1(n)$ in the above equation. Therefore, the equation yielding the cost of the units produced after the cumulative unit n_1 could be written as:

$$C_2(n) = (1-r_2)C_1(n) + r_2C_o(n-n_1) \qquad n > n_1$$

It should be noticed that the second term in the right hand side of the equation is still directly related to the original task and not the revised one. This is true because this part of the equation reflects that portion of the task totally new to the crew (or that portion of men unfamiliar to the task). However, the cost of the current unit n is not referred to on the original curve, simply because this segment of the total cost considers the portion which considers n_1 fewer performances than n.

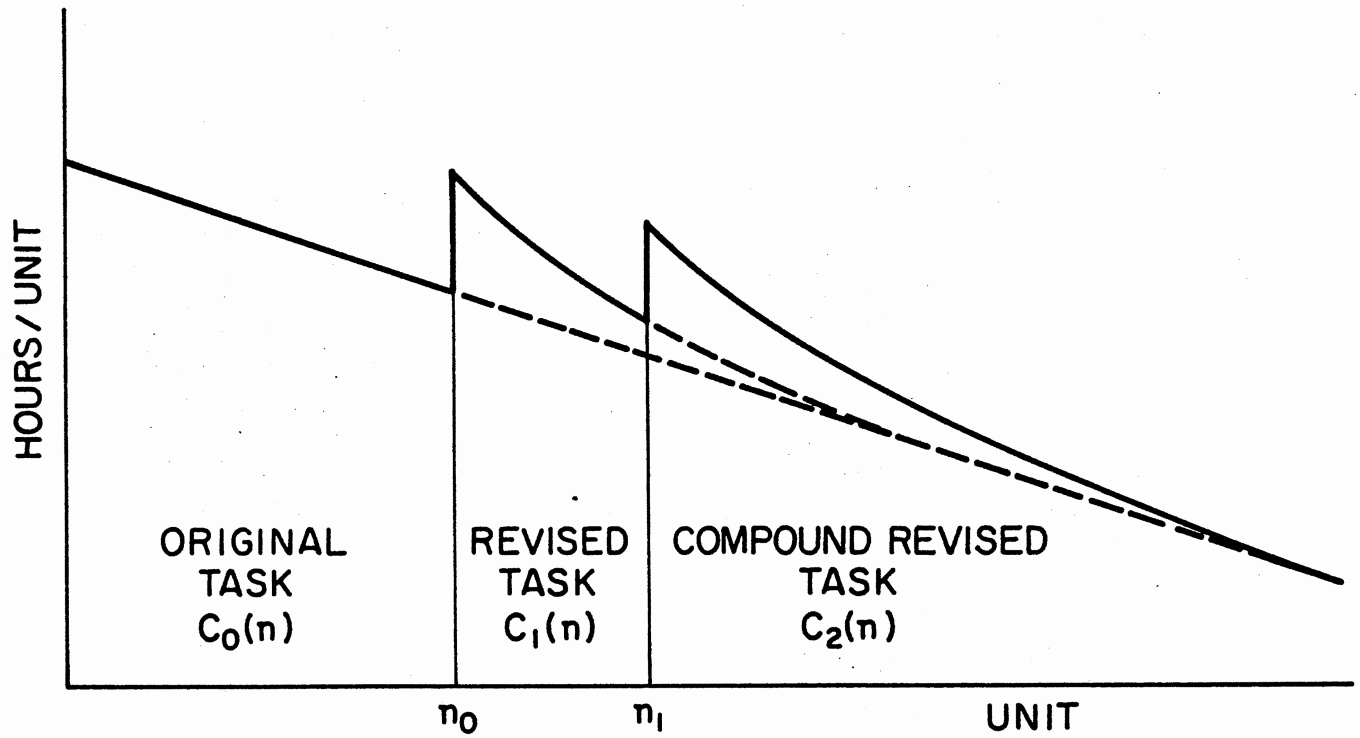


Figure 13. Compounded Disruption Effect Resulting from Two consecutive Disruptions

The compound revised task, $C_2(n)$, can still be written in terms of the original curve by substituting the corresponding value of $C_1(n)$ in the above equation:

$$C_2(n) = (1-r_2)[(1-r_1)C_o(n) + r_1C_o(n-n_o)] + r_2C_o(n-n_1)$$

$$n > n_1$$

To demonstrate the application and the significance of the above consideration, a typical case could be considered: Consider the example given earlier in this chapter, and assume that a second manpower transaction generating a turnover of 10 percent occurs after completion of unit 24 (the first disruption took place after unit 20 with the same turnover ratio). The values of parameters in this example are:

$b = .2344$

$f = 1{,}287$ hours

$n_o = 20$

$n_1 = 24$

$r_1 = .1$

$r_2 = .1$

The cost of any unit produced after unit 24 can be found using the formula above. Let us consider unit 27:

$$C_2(27) = (1287)\{(1-.1)[(1-.1)(27)^{-.2344} + (.1)(27-20)^{-.2344}] + (.1)(27-24)^{-.2344}\} = 654 \text{ hours}$$

The unit cost for unit 27, not incorporating the compounded disruption as if the second disruption is assumed to be imposed on the original task is calculated as:

$$C_2^1(27) = (1287)[(1-.1)\cdot(27)^{-.2344} + (.1)(27-24)^{-.2344}]$$
$$= 634 \text{ hours}$$

If only the first disruption were in effect, the cost for unit 27 would have been:

$$C_1(27) = (1287)[(1-.1)(27)^{-.2344} + (.1)(27-20)^{-.2344}]$$
$$= 616 \text{ hours}$$

Finally, the cost for unit 27 on the original curve is:

$$C_o(27) = (1278)(27)^{-.2344} = 584 \text{ hours.}$$

Figure 14 is a schematic representation of this example.

The formula developed for the double disruption case could be generalized on a recursive basis to incorporate the effect of compounded disruptions of any number. For example, consider the case which incorporates five consecutive disruptions, each occuring when the effect of the previous one is still significant (as shown in Figure 15). The unit costs for every relative cumulative unit range are as follows:

$$C_o(n) \qquad n \leq n_o$$

$$C_1(n) = (1-r_1)C_o(n) + r_1C_o(n-n_o) \qquad n_o < n \leq n_1$$

$$C_2(n) = (1-r_2)[(1-r_1)C_o(n) + r_1C_o(n-n_o)] + r_2C_o(n-n_1) \qquad n_1 < n \leq n_2$$

$$C_3(n) = (1-r_3)[(1-r_2)[(1-r_1)C_o(n) + r_1C_o(n-n_o)] + r_2C_o(n-n_1)] + r_3C_o(n-n_2) \qquad n_2 < n \leq n_3$$

$$C_4(n) = (1-r_4)[(1-r_3)[(1-r_2)[(1-r_1)C_o(n) + r_1C_o(n-n_o)] + r_2C_o(n-n_1)] + r_3C_o(n-n_2)] + r_4C_o(n-n_3) \qquad n_3 < n \leq n_4$$

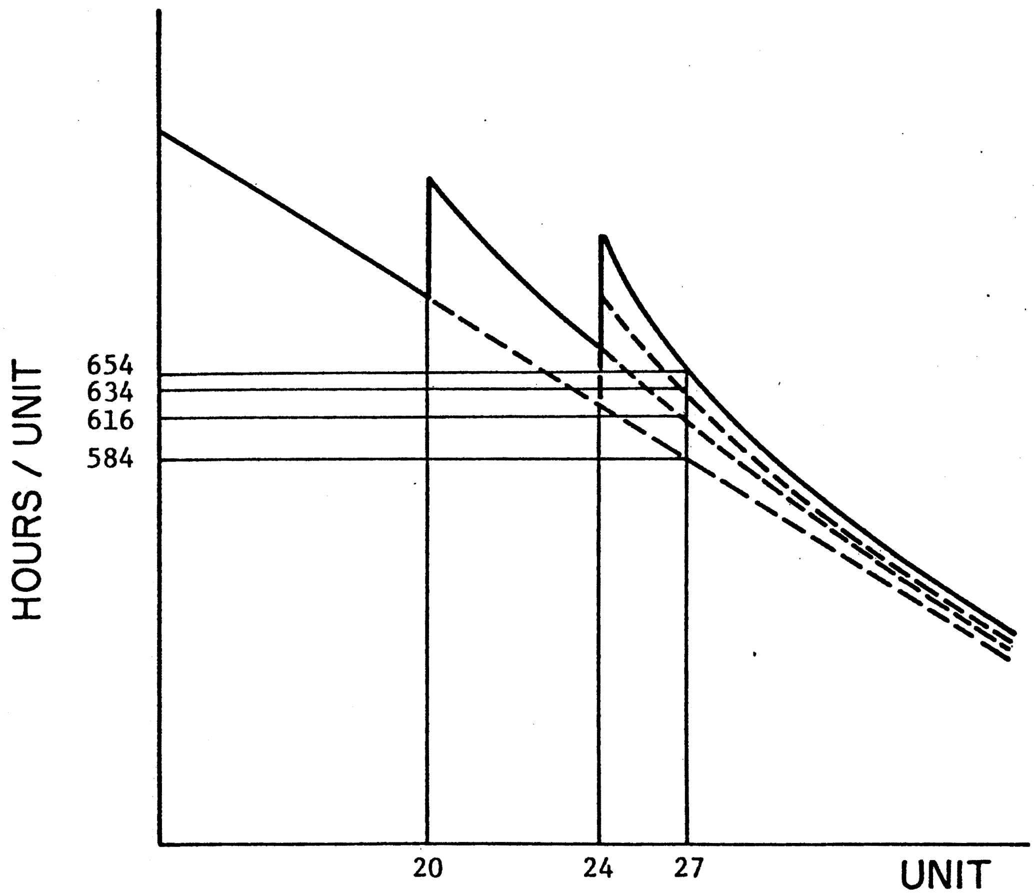


Figure 14. Graphical Illustrations of the Example of a Double Disruption Case

$$C_5(n) = (1-r_5)[(1-r_4)[(1-r_3)[(1-r_2)[(1-r_1)C_o(n) + r_1C_o(n-n_o)] + r_2C_o(n-n_1)] + r_3C_o(n-n_2)] + r_4C_o(n-n_3)] + r_5C_o(n-n_4) \qquad n > n_4$$

It should be noticed that all revised curves are interpreted in terms of the original curve. The original curve reflects the cost of any unit according to the basic formula:

$$C_o(n) = f \cdot n^{-b}$$

Therefore, given the slope of the applicable learning curve and the cost of the first unit, cost calculations can be performed for any desired unit.

The block cost the compounded disruption case can be found for any range of the cumulative units by integrating the applicable revised unit costs over the desired range. For example, for the range (A-B) shown in Figure 15 , where $n_o < A < n_1$, and $n_1 < B < n_2$, the block cost will be:

$$\sum_{A}^{n_1} C_1(n) + \sum_{n_1+1}^{B} C_2(n) = (1-r_1) \sum_{A}^{n_1} C_o(n) + r_1 \sum_{A}^{n_1} C(n-n_o)$$

$$+ (1-r_2)[(1-r_1) \sum_{n_1+1}^{B} C_o(n) \cdot + r_1 \sum_{n_1+1}^{B} C_o(n-n_o)]$$

$$+ r_2 \sum_{n_1+1}^{B} C_o(n-n_1)$$

At this point the effect of disruption in productivity rate , as a result of occurences of manpower transactions,has been analyzed sufficiently. This provides the basic tools for incorporation of dynamic productivity consideration into the aggregate planning problem.

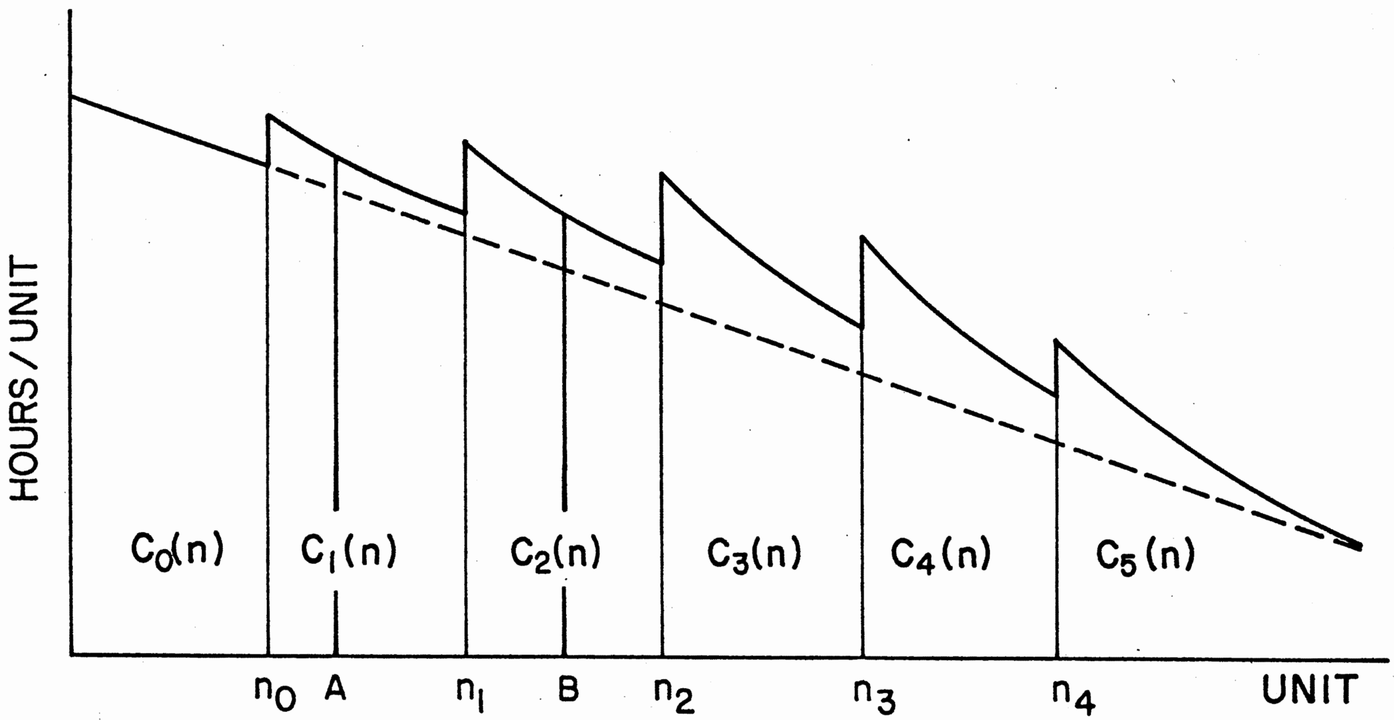


Figure 15. A Typical Multiple Compounded Disruption Situation

The succeeding sections of this chapter will present the methodology developed for the joint consideration of the disruption effect and the aggregate planning problem.

Aggregate Planning and Disruption Effect

Proper determination of the average productivity rate applicable to each period in the planning horizon of an aggregate plan, necessitates the following input elements:

1. Specifications of the learning curve applicable to the firm, $C_o(n)$
2. Cumulative number of units produced by the firm until the beginning of the planning horizon, cum_o, and the initial workforce level, W_o
3. Production rate scheduled for each period in the planning horizon, P_t, $P_t = 1, 2, \ldots, N$
4. Workforce level scheduled for each period in the planning horizon, W_t, $W_t = 1, 2, \ldots, N$
5. Values of the parameters to be used in translation of manpower index into task turnover ratio, K_1, K_2 and K_3
6. A chosen minimum unit cost difference ratio level above which the carried-over disruption effect is considered significant, E.

Task turnover ratios at the beginning of each period can be determined using initial workforce level and elements (4) and (5) above.

When the effect of compounded disruption is ignored, the original improvement curve and the possible task turnovers at the beginning of each period, along with cumulative number of units produced before and after the period provide the necessary information for determining the block cost for the period in question. This block cost is then divided by the number of units produced during the peried, P_t, resulting in the appropriate average productivity applicable to that period.

In order to incorporate the effect of compound disruption, the cumulative number of units at the end of a period, during which a disruption has been in effect, could be used to determine the unit cost difference ratio at the end of the period. This ratio, if checked against the selected minimum unit cost difference ratio (element number 6 above), indicates if there is a significant carry over and thus consideration of compounded disruption in the succeeding period. If this is the case, calculation of the block cost for a period may involve consideration of a sequence of events that took place during the previous periods. Therefore, in this situation, the calculation of average productivity for a period is not supposed to be performed independently from the previous periods. The proper approach in this case would involve sequential calculations, starting with the first period and ending with the last one.

Upon determination of the applicable cost curve for a period in the above sequential process, a trace back to the preceding periods must be done to search for the number of compounded disruptions, and the units at which each disruption has taken place. Then a recursive computation of unit costs is performed, starting with period corresponding to the first disruption in the sequence and ending with the current period.

An example of the application of the foregoing discussion clarifies the approach. Figure 16 illustrates a planning horizon containing 12 periods. In this hypothetical case, all possible combinations of disruption occurences are incorporated.

Some useful definitions are:

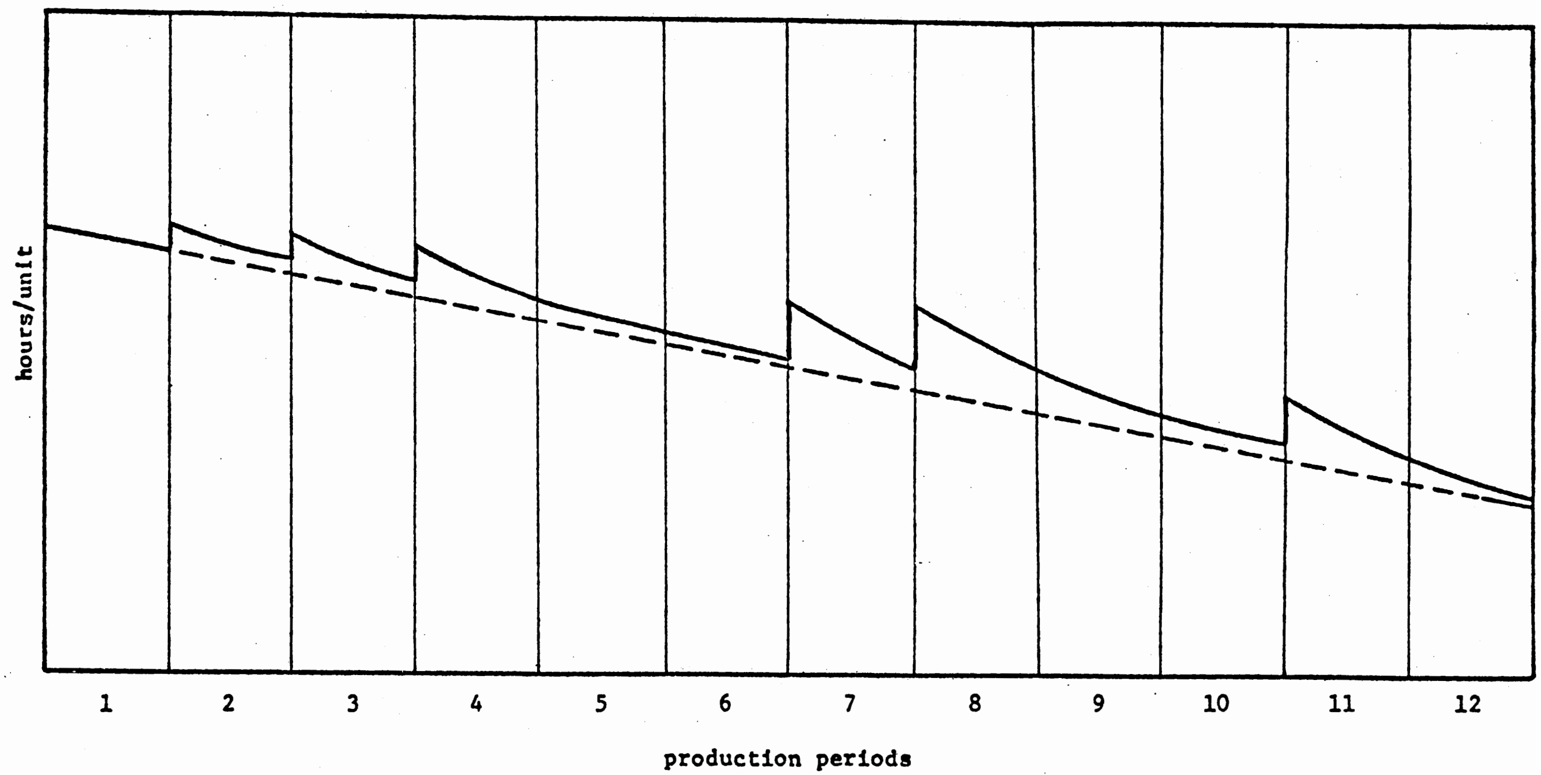


Figure 16. Unit Cost Curve Over 12 Month Period for the Hypothetical Case (disruptions are caused by manpower level changes occuring at the beginning of related periods.)

cum_i - Cumulative number of units produced by the end of period i

r_i - Task turnover ratio resulted from manpower transaction at the beginning of period i

$INDEX_i$ - 1 if the disruption effect at the beginning of period i is significant $r_i \geq K_2$

0, otherwise

$ICHK_i$ - 1 if the unit cost difference ratio at the end of period i is large enough to indicate a carried-over effect, $UDR(cum_i) \geq \varepsilon$

0, otherwise

Using the original unit cost curve, equations developed for single and compounded disruption cost effects, and the approach discussed in this section, the unit cost curves applicable to those conditions depicted in Figure 16 are listed in Table III.

Some of the unit cost functions in this table have not been expanded in terms of the original curve. For period 4, for example, the expanded cost function is:

$$C_4(n) = (1-r_4)[(1-r_3)C_o(n) + r_3 \cdot C_o(n-cum_2+1)] + r_4 \cdot C_o(n-cum_3+1)$$

These equations are used in the recursive computations that are performed to obtain the unit cost where the compound disruption is in effect.

The values of $ICHK_i$ are determined by simply substituting cum_i in place of n in the respective unit cost function. The block cost for period i is found by integrating the cost function applicable to the period from $cum_{i-1} + 1$ to cum_i. This block cost is then divided by P_i to obtain the average productivity applicable to the period. Values of P_i, W_i and average productivities in every period along with

TABLE III

UNIT COST FUNCTIONS FOR THE EXAMPLE DEPICTED IN FIGURE 16

Period	INDEX	ICHK	Unit Cost Curve	Applicable Unit Range
1	0	0	$C_1(n) = c_o(n)$	$cum_o < n \le cum_1$
2	1	1	$C_2(n) = (1-r_2)C_o(n)+r_2 \cdot C_o(n-cum_1+1)$	$cum_1 < n \le cum_2$
3	1	1	$C_3(n) = (1-r_3)C_2(n)+r_3 \cdot C_o(n-cum_2+1)$	$cum_2 < n \le cum_3$
4	1	1	$C_4(n) = (1-r_4)C_3(n)+r_4 \cdot C_o(n-cum_3+1)$	$cum_3 < n \le cum_6$
5	0	1	$C_5(n) = C_4(n)$	$cum_3 < n \le cum_6$
6	0	1	$C_6(n) = C_4(n)$	$cum_3 < n \le cum_6$
7	1	1	$C_7(n) = (1-r_7)C_4(n)+r_7 \cdot C_o(n-cum_6+1)$	$cum_6 < n \le cum_7$
8	1	1	$C_8(n) = (1-r_8)C_7(n)+r_8 \cdot C_o(n-cum_7+1)$	$cum_7 < n \le cum_{10}$
9	0	1	$C_9(n) = C_8(n)$	$cum_7 < n \le cum_{10}$
10	0	1	$C_{10}(n) = C_8(n)$	$cum_7 < n \le cum_{10}$
11	1	1	$C_{11}(n) = (1-r_{11})C_8\ n)+r_{11} \cdot C_o(n-cum_{11}+1)$	$cum_{10} < n \le cum_{12}$
12	0	1	$C_{12}(n) = C_{11}(n)$	$cum_{10} < n \le cum_{12}$

other parameters in the cost function are used to obtain the total cost of this specific aggregate plan.

Optimization Methodology

In the previous section the methodology for evaluating the performance of a 'specific' aggregate plan under dynamic productivity condition was presented. A specific aggregate plan was recognized by the specific levels of production and workforce at each period of the planning horizon. The index of performance of such a plan is the total cost level resulting from the plan.

Production and workforce levels are the independent, or controllable, decision variables in this operations planning problem. These decision variables can be adjusted within specified boundaries so as to produce the best possible index of system performance (the minimum level of the total operating cost). Therefore, application of a search technique would involve systematic adjustments of the levels of decision variables and exploration of the response surface. The optimum solutions are those which correspond to the minimum response.

The structure of the cost function in the constant productivity models is fixed and independent of the levels of decision variables. However, in a model which incorporates dynamic productivity, this structure is totally dynamic in nature because the productivity coefficients are not constant throughout the planning horizon. These coefficients are related to the levels of decision variables; therefore, they form a new structure for the objective function every time an adjustment in these levels is made. It is conceivable that in such a dynamic system, sensing the performance of a specific set of values for decision variables would necessitate the determination of the respective structure of the objective function first and then evaluation of the response against these levels for decision variables.

The optimization methodology proposed for the current model is based upon the above considerations, and is composed of three interacting major units.

The first unit is the pattern search routine, called PATS. As demonstrated in the diagram in Figure 17 , this routine generates a sequence of levels for decision variables, workforce and production, in every period in the planning horizon.

The second unit is a routine, called PROTCV which receives the information generated by PATS and utilizes the approach discussed in the previous section. This routine generates unit cost curves applicable to each period on the basis of the input information about the magnitudes and patterns of ups and downs in the levels of workforce and production throughout the planning horizon. Based on the unit cost curves, this routine then determines the appropriate average productivity levels applicable to the respective periods.

The third routine, called FCT1, receives the information generated by PROCTIV and forms the updated structure of the objective function. This routine evaluates the response of the newly structured cost function against the corresponding levels of decision variables, the levels originally generated by the PATS routine.

The response evaluated by FCT1 routine is fed to the PATS routine. The magnitude of this response provides the search routine with the necessary information to make appropriate adjustments in the level of decision variables. The newly adjusted levels are fed to the PROCTV

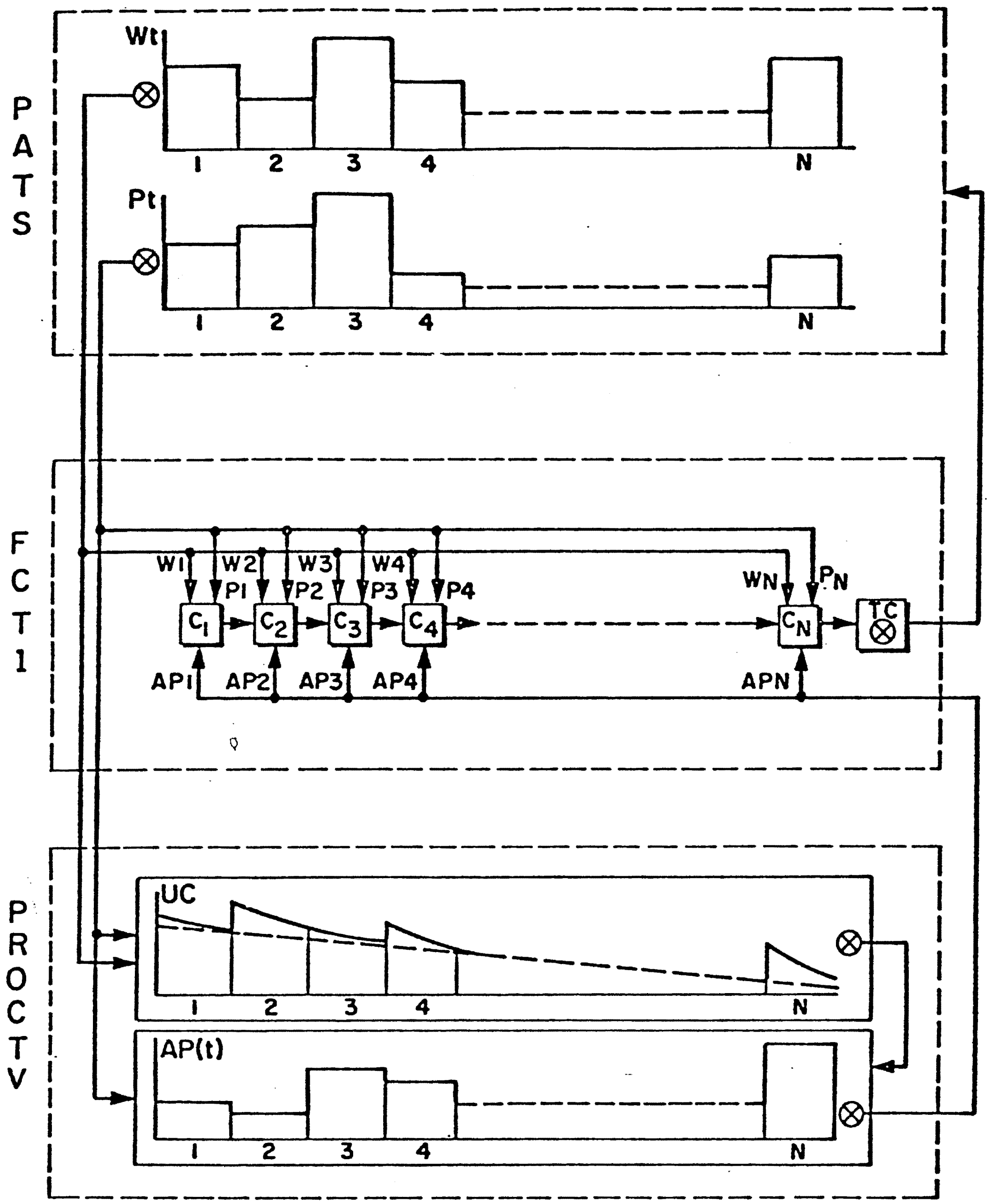


Figure 17. Information Flow Among the Three Major Routines Involved in Optimization of Model I

routine and the whole process is continued until the optimum solution is found.

A considerable increase in the amount of computation time occurs when incorporating the dynamic productivity considerations into the SDR solution technique, since for every proposed change in the level of decision variables made by PATS a call is made to the PROCTV routine. For example, using a 10-period planning horizon, the optimization of a cost function similar to the paint factory model would, on the average, involve 3000 evaluations of the objective function. For each evaluation the PROCTV routine performs a sequence of computations similar to those presented in Table II. A considerable portion of these computations can be avoided and, therefore, a considerable time saving could be achieved, if the procedures described below are used.

One of the major factors affecting the number of required evaluations in all search routines is the starting solution vector. The closer this initial solution is to the optimum, the fewer evaluations required to reach the optimum. Therefore, a procedure which would approximate the optimum solution can always provide a better starting point than a starting point chosen intuitively. For the dynamic productivity model, this procedure could be the one which optimizes a somewhat equivalent average productivity model. Since the cost structure in this model is fixed, the optimization process would involve a considerably small computation process, and therefore a considerably smaller computation time. The dynamic productivity model could then utilize this optimal solution as a starting point and proceed toward its own optimum. The efficiency of the solution technique can be improved further, if an intermediate model, which incorporates the improvement curve but not the disruption effect (Ebert model), is utilized. In this setting, a starting solution vector would be chosen for optimizing the constant productivity model. The optimum solution to this model is then utilized as the starting point for the intermediate model, and the solution to the intermediate model can serve as a starting solution to the final complex model. In this approach, each model represents the real situation more realistically than its preceding one.

Besides saving computation time, the above procedure also provides information regarding the comparative performances of the three models under different situations.

Further improvement in the computation efficiency is still possible by improving the efficiency of the algorithm utilized in the PROCTV routine. A great portion of the computations can be avoided in this routine by observing the fact that average productivites do not have to be computed for all periods every time the level of a decision variable is adjusted. It should be noticed that the average productivity at a given period could only be related to the levels of decision variables in the preceding periods that is, a change in the level of workforce, or production at a given period can only affect the average productivities in the succeeding periods. For example, in a 20-period model, a change in the workforce level at period 18, W_{18}, can only affect the average productivities in periods 18, 19 and 20. The average productivites for the first 17 periods would remain unaffected. Therefore, a check can be made at the beginning of the PROCTV routine to determine the first period for which a decision variable has changed. The computation steps should then be carried on only for the current and the succeeding periods.

The above provisions are all incorporated into the optimization procedure used in this study. A more detailed explanation of this procedure follows.

Procedure

Figure 18 illustrates a block diagram of the optimization procedure. Glossary of code definitions, flow charts and the documented program listing for this procedure are provided in Appendixes A, D and E, respectively.

The main program reads the model parameters and initializes the search routine. The computation of the overall average productivity for the equivalent constant productivity model is performed in this routine. This average productivity is determined by applying the original improvement curve and computing the block cost for units ranging from cum_o (the cumulative production by the beginning of the first period) to $cum_o + \sum_{i=1}^{N} D_i$ (D_i being the demand level for period i).

The PATS routine is then called and the constant productivity model is optimized and the results are printed. The optimum solution of this model is then fed into PROCTV through a call to FCT1. This evaluates the performance of the constant productivity model in situations where the dynamic productivity condition is present. The result of this evaluation is then printed.

A similar set of steps is then followed for the optimization and evaluation of the optimum solution to the Ebert model, the starting solution vector for this optimization being the optimum solution to the first model. The optimum solution to the second model is then utilized as the starting solution for the optimization of the dynamic productivity

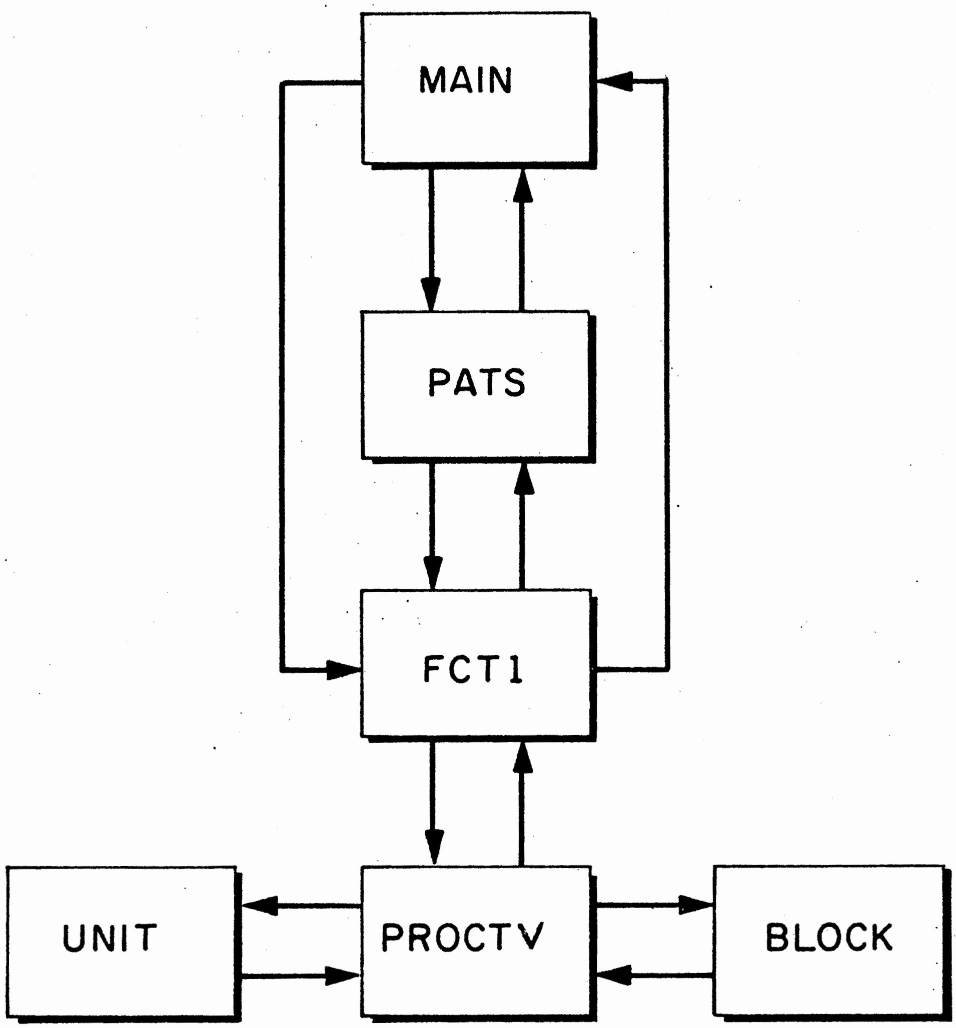


Figure 18. Block Diagram of the Optimization Procedure for Model I

model. This model is optimized and the results are printed. In summary, the outputs of this routine are: the optimum plan and corresponding cost level for the constant productivity model; the cost of implementing this plan in a changing productivity situation; the optimum plan and corresponding cost levels for the Ebert model; the cost of implementing this plan in a changing productivity situation; and the optimum plan for the dynamic productivity model and the associated cost levels. Appendixes C and F should provide a complete description of the pattern search subroutine calles PATS.

Subroutine FCT1 increments the number of function calculations, receives the levels of the decision variables from PATS and directs these levels to PROCTV to receive back the applicable productivity levels. The structure of the cost function is updated, different cost elements are calcualted for every period and the total cost is reported to PATS from this routine. FCT1 also computes and incorporates the penalty of violating the model constraints.

Subroutine PROCTV starts with detecting the effective period: the first period at which the value of a decision variable does not correspond to its previous value. Recognition of types of manpower transactions, computation of manpower ratios, translation of these ratios into the equivalent task turnovers, and setting of the disruption index for each period are then performed. Next, unit and block cost computations for periods with no disruption and single disruption are performed. The carried-over disruption index, ICHK(I), is also set for all periods. The recursive computations for compound disruption effects are performed where applicable. Finally, the values for average productivity levels applicable to each period are computed and reported back to the FCT1 routine.

PROCTV interacts with two function routines UNIT and BLOCK, which compute unit cost for every proposed cumulative unit and block cost for every proposed range of cumulative units, respectively. Computations performed by these routines are based on the original improvement curve, $C_o(n)$, for the given unit number as requested by PROCTV.

The above procedure performs the month one optimization. To perform the next month optimization (at the beginning of the second month, and for a revised forecast) the effect of possible disruption resulting from implementing the month one decisions and possibly continuing through month two is to be considered. The value of the unit cost difference ratio at the end of month one provides the information regarding this effect. Therefore, for the beginning of the second month (in the month two optimization), this value can be added to the turnover ratio resulting from the possible disruption at this period. The optimization procedure is then the same as before for all periods in the planning horizon.

Remarks

Based on the foregoing analysis of disruption effect on the productivity factor in the aggregate planning, Figure 19 demonstrates a hypothetical situation in which the periodical average man-month requirements per unit product (the inverse of average productivities) considered by the constant productivity models, the Ebert model and the dynamic productivity model (new model) are contrasted.

This figure shows that while the Ebert model assumes a continuously increasing average productivity, the new model considers a fluctuating average productivity level. The figure also demonstrates the fact that

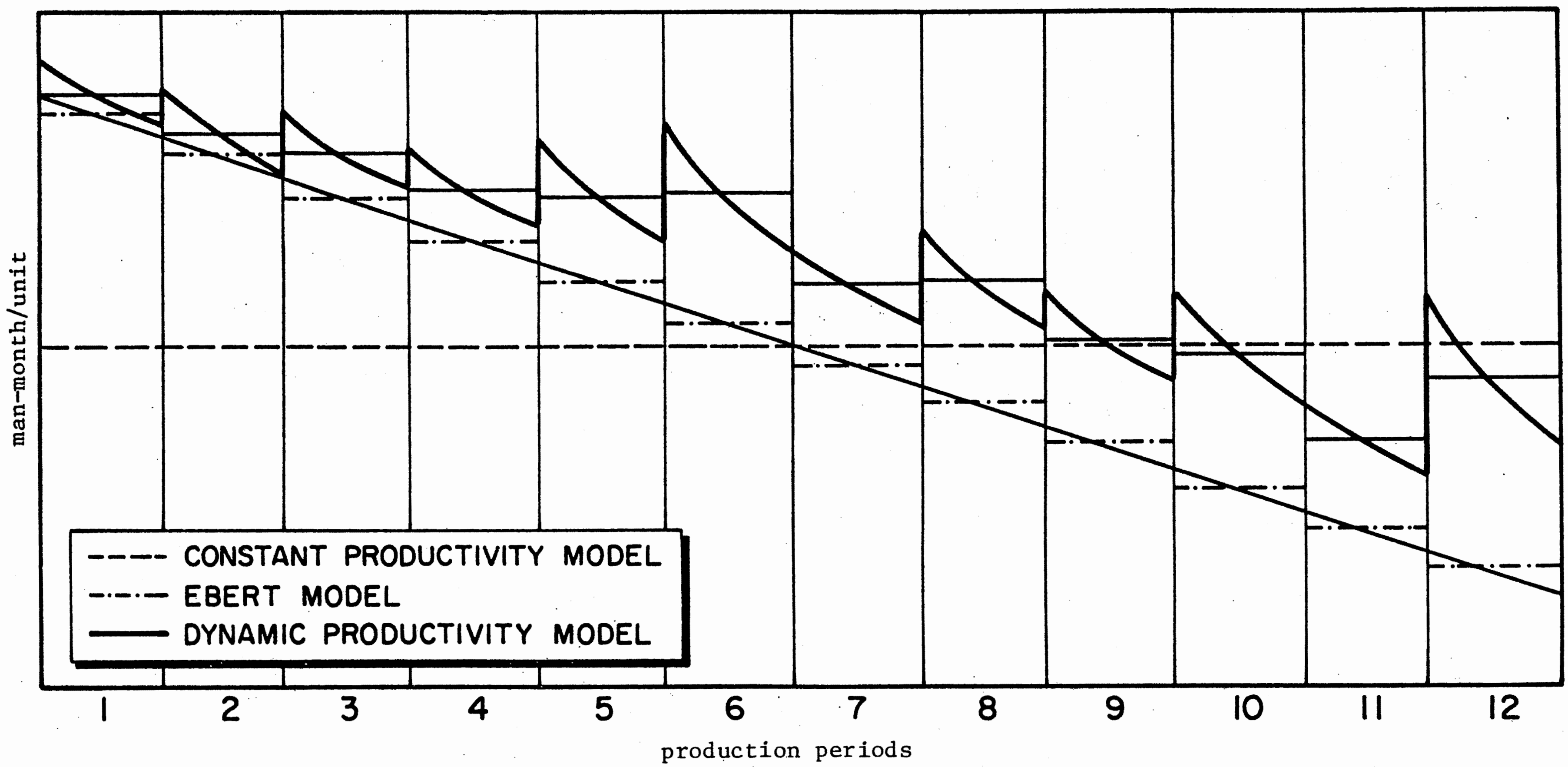


Figure 19. Comparison of the Patterns of the Man-month Requirements per Product Unit (inverse of average productivity) in Constant Productivity, Ebert, and Dynamic Productivity Models for a Hypothetical Situation

disruptions occuring further in the production sequence impose more significant effects on the average productivity level; the difference between the average productivity levels considered by the Ebert model and the new model becomes larger in the further periods. As will be seen in Chapter VII, the analysis of the results numerically support the relevance of the ideas depicted in this figure.

CHAPTER VI

MODEL II. SHORT CYCLE PRODUCTS

Introduction

Most mass production and make-to-stock type of operations can be classified as the short cycle operations. Unlike the case of long cycle products, in the short cycle operations the unit output rate of the firm can be rather directly attributed to the productivity rates of the individual workers.

Short cycle operations are usually uncomplex and are composed of few tasks, all being performable by an individual employee, or by a small number of employees sequentially performing specific tasks. Even though processing of a short cycle product unit may require utilization of more than one employee, the short cycle operation is not a team activity of the type found in long cycle operations. This is due to the independency persisting among the work segments performed by different employees.

There are several consequences of this special nature of the short cycle operations. For example, the primary step for coordination is the choosing of employees with similar productivities when arranging a crew for a production line. The pace of the production line is always limited by the slower members in the sequence, and because of the independency of work, any improvement found by some members does not help the others. Short cycle activities usually involve a number of small production lines each utilizing a homogeneous class of employees. When a need arises for production rate changes through changes in the workforce level, a number of production lines are either dropped or added. A decrease or increase in the workforce level is not usually accompanied by the reassignment of manpower; therefore, disruptions in the average productivity level do not persist as a result of such transactions. Any drop in the overall average productivity level of the firm is due to the addition of slower production lines utilizing newer employees.

It is only in the above organizational settings, that the assumptions regarding experience classes considered by Orrbeck et al. are justifiable. In considering plans for long cycle operations it is conceivable that these assumptions are far from being realistic.

The objective of this chapter is to develop an aggregate production planning model suitable for the firms manufacturing short cycle products with special attention devoted to the proper incorporation of the dynamic productivity phenomenon. The new model incorporates some basic assumptions of the Orrbeck et al. model; however, the general approach in the model development and the solution methodology utilized are totally different and new in nature. Direct application of improvement curve analysis with detailed reference to learning, progress and retrogression effects form the essence of the new model. A number of interesting and significant facts about the nature of the objective functions in most aggregate planning models are highlighted as the outcome of this part of research. A detailed explanation of this research will follow in suceeding sections of this chapter.

Analysis of the Effect of Manpower Changes

As it was mentioned previously, in the case of short cycle products, disruption in productivity as a result of task reassignment is not usually experienced. Even for major crew expansion or reduction, reshuffling of tasks such that some experienced workers wind up with brand-new reassignments is not expected to occur; however, lack of such a disruption effect does not mean that manpower transactions do not generate significant effects on the overall average productivity level of a short cycle manufacturing firm.

Implementation of the decisions generated by the aggregate plans may require constant addition of untrained people and loss of trained ones for almost every type of manufacturing firm. These transactions, coupled with fluctuating production output requirement per period result in fluctuating average productivity levels over the periods of the planning horizon. For the short cycle case, the addition of production lines with slower paces, or the shutting down of faster lines as a result of additions or reductions in the level of workforce are the natural causes for such fluctuations.

Fluctuations in the level of workforce result in constant generation of new experience classes, or they vary the number of such classes. However, workforce fluctuations are not the only factors affecting the number and the sizes of the experience classes in different periods.

According to the learning curve analysis, an employee's productivity is a function of the cumulative units produced by that employee, and (as will be discussed later) this productivity is also a function of the "progress" attained by the firm. On the other hand, the options of overtime and idle time, which are frequently utilized by firms and indicated by aggregate plans, can generate situations where groups of employees starting at identical productivity levels may end up producing different cumulative product units after one or more periods. Therefore, the alternative ways of workforce utilization affect the conditions of experience classes in different production periods.

One basic distiction between the new model and the Orrbeck et al. model arises from the above point; that is, the new model recognizes the alternative ways of workforce utilization as a factor affecting the conditions (number of classes, size,and productivity) of the experience classes. As will be shown later, other significant distinctions between the two models do exist. The foregoing considerations structure the assumptions of the new model.

Assumptions of the New Model

The new model adapts the assumptions regarding the effect of workforce fluctuations through hiring and layoff, considered by Orrbeck et al. The assumptions governing the assignment of overtime are also adapted from this model. The basis for recognition of the experience classes in the new model is the productivity rates of the members of each class based upon the cumulative number of product units experienced by those members. Other factors affecting productivity rates are the ones associated with the progress effect and the retrogression effect.

It should be noted that the new assumption differs in nature from the one adapted by Orrbeck et al. who relate the productivity rates to the passage of time. They relate the productivity rate of each experience class to the time span during which the experience class is involved in the firm's activity. Such relationship may exist only if workers are utilized on regular time, a situation which is indeed unrealistic. Based upon the new assumptions presented above, each element of the new model and the logical relationships among the elements are analyzed in the following sections.

Workforce Fluctuations

Fluctuation in the workforce level is an element which affects the conditions of experience classes according to the assumption employed by Orrbeck et al. That is, the least experienced class is the one which consists of the newly hired workers. If workers are to be fired, the least experienced workers are fired first. Should the number of people fired in a period exceed the number of employees in the least experienced class of the previous period, some workers from the next least experienced class would have to be laid off.

Not including the effect of all other relevant factors (which will be explained in the later sections), the effect of workforce fluctuations on a typical situation is demonstrated in Figure 20. In this hypothetical case, a planning horizon consisting of six periods is considered. Originally, the first period includes three experience classes. In Figure 20.b rectangulars with lower numbers indicate experience classes with higher productivities. A typical fluctuation pattern in the level of workforce, as shown in Figure 20.a affects the conditions of experience classes as shown in Figure 20.b.

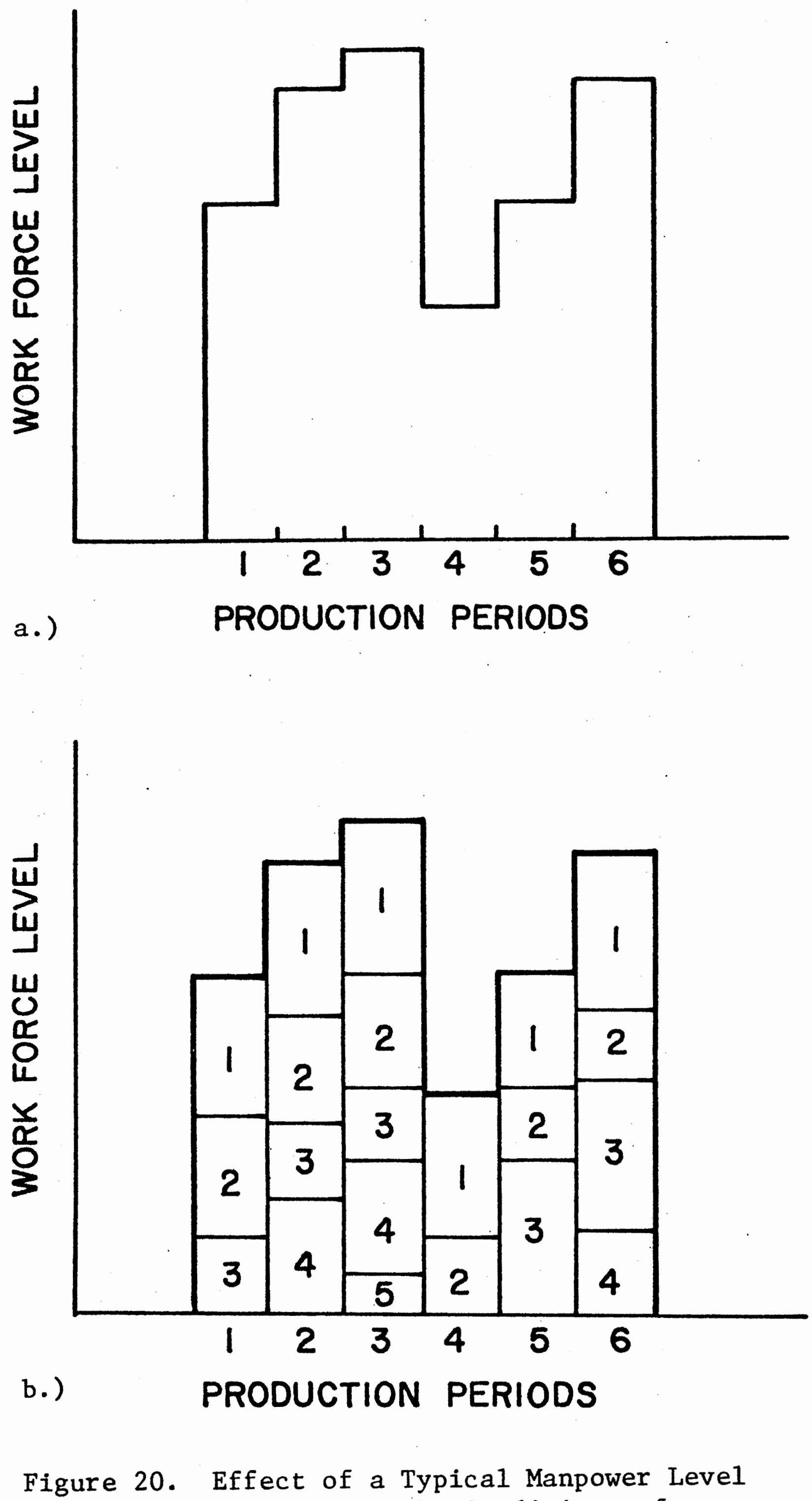


Figure 20. Effect of a Typical Manpower Level Change on the Conditions of Experience Classes

Regular Time

The new model assumes that regular time work is assigned to workers in order of their seniority. Thus, the most experienced workers will work on regular time first subject to the limit of their capacity. If regular time work still remains, the next most experienced class will be utilizied; and similarly, the remaining experience classes will be utilized until all regular time work is assigned.

There are two possible outcomes of the above situation. First, if the amount of regular time work is smaller than the production capability of all experienced classes, some lower experienced workers will remain idle. (This is a realistic situation that Orrbeck et al. do not incorporate in their model.) Second, if the amount of regular time work exceeds the total production capability of all experience classes, the excess production must be performed on overtime. Should this excess amount exceed the production capability of all experience classes on overtime, subcontracting might be utilized. If a particular situation does not include the possibility of utilization of subcontracting, then the model will indicate the infeasibility of the planned production rate in the appropriate period. It should be noted that Orrbeck et al. do not include considerations regarding the subcontracting option. Their model does not incorporate the constraint relative to the production capability on overtime. They first establish an upper limit on the production that can take place on regular time, and then they assume any production in excess of this amount must be done on overtime.

Based upon the assumptions of the new model, the analysis of regular time work and its effect on the condition of experience classes would require the following input information for each period:

— Production rate scheduled for the period

— Workforce level scheduled for the period

— Number of experience classes, size and measure of productivity of each experience class at the beginning of the period

Given the current scheduled workforce level and the workforce level in the preceeding period, the condition of the experience classes in the current period can be revised by applying the approach discussed in the previous section.

To remain consistent with the improvement curve theory, the cumulative output unit per employee in a given class can be chosen as the "measure of productivity" of that experience class. Given the total production rate scheduled for the given period, and also given the number of experience classes, size and productivity measure of each class, the total number of men to be assigned on regular time work can be determined. This is possible by first finding the production capability of each experience class during regular time, and then comparing the total of the production capabilities with the total scheduled production rate for the period.

The production capability of an experience class can be found using a revised form of the block cost expression discussed in the analysis of linear learning curves. According to this expression, the total man-months required to produce units n_1 through n_2 is given by:

$$\sum_{n_1}^{n_2} C(n) = \frac{f}{1 - b} \left[(n_2 + .5)^{(1-b)} - (n_1 - .5)^{(1-b)} \right] \qquad (6.1)$$

For simplicity, ignoring the approximation improvement factor .5 in the above equation, the formula will reduce to:

$$\sum_{n_1}^{n_2} C(n) = \frac{f}{1 - b} [n_2^{(1-b)} - n_1^{(1-b)}] \tag{6.2}$$

Now, given an experience class i with w_i members, each member having produced n_1 units by the beginning of a period, it is desired to find the production capability of this class throughout the period (month). The production capability of the experience class can be directly derived from the above expression by noticing the fact that the expression indicates the total man-months required to produce units n_1 though n_2. Therfore, knowing the left-hand side value (total man-months available), one man-month, and also knowing n_1, the value of n_2 can be found:

$$1 = \frac{f}{1 - b} [n_2^{(1-b)} - n_1^{(1-b)}] \tag{6.3}$$

then

$$n_2 = [\frac{1 - b}{f} + n_1^{(1-b)}]^{(\frac{1}{1-b})} \tag{6.4}$$

where n_2 is the cumulative number of units produced by the end of the period (month) by one member (one man-month). Therefore, the regular time production capability of each member throughout the period is:

$$n' = n_2 - n_1 = [\frac{1 - b}{f} + n_1^{(1-b)}]^{(\frac{1}{1-b})} - n_1 \tag{6.5}$$

Correspondingly, the total production capability of all members in experience class i on regular time in the given period is:

$$P_{reg}(i) = W_i \cdot \left\{ [\frac{1 - b}{f} + n_1^{(1-b)}]^{(\frac{1}{1-b})} - n_1 \right\} . \tag{6.6}$$

To determine the total number of men to be assigned on regular time work based on the order of seniority, the production capability of the first experience class (the highest experienced class) is compared against the scheduled production rate P_t for the period in question. If the production capability of this class is smaller than P_t, the excess amount is compared against the production capability of the second experience class. The comparison process continues until one of three outcomes occurs.

First, if the excess production amount left for the last experience class (with the lowest productivity) exceeds the production capability of this class, the excess amount must be done on overtime. If the total production capability in overtime is smaller than this amount, the excess must be retained through subcontracting (if possible). This case will be explained in more detail in the next section.

Second, if the comparison process of the excess production amount against the production capability of sequentially considered experience classes continues until the production capability of an experience class equals (within an acceptable tolerance) the excess production amount, the current class and all higher classes are assigned on regular time work. The productivity measure of the members in these classes is increased by their respective production capabilities during the given period. This updates their status for the beginning of the next period. In this case all succeeding experience classes (the ones with lower productivities than the current one) will remain idle. The detailed treatment of productivity status of the members of such classes will be explained later in a related section.

Figure 21 illustrates the latter situation. In this figure a workforce level W_t is comprised of five different experience classes. The total production scheduled for the period P_t is compared against production capability of experience class 1, $P_{reg}(1)$. The remainder

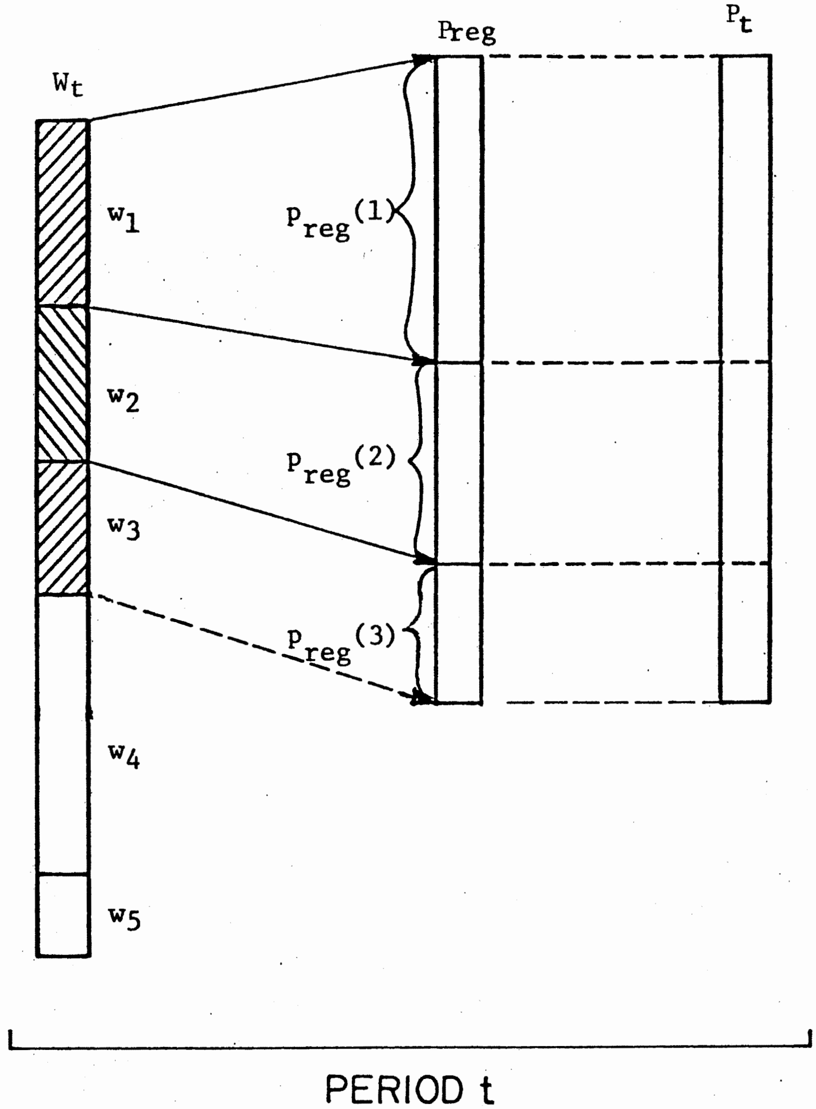


Figure 21. Effect of a Typical Regular Time Work Assignment on the Condition of Experience Classes

of production is compared against $P_{reg}(2)$ and the comparison is continued until the last remainder of P_t is found to be equal to the production capability of the third experience class. In this case the shaded portion of w_t indicates the workforce assigned on regular time. The unshaded portion shows the classes which are left idle.

The third possible outcome of the comparison process is the situation where the procedure continues until the production capability of an experience class exceeds the remaining production amount. In such case, the ratio of the excess amount over the production capability of the experience class in question can indicate the portion of the class which will remain idle (this is true because all members in a class have similar productivity rates). All other remaining classes with lower productivities will also remain idle. It should be noted that in this situation, an experience class is broken into two parts. One which is assigned on regular time work, and the other which remains idle. As a result of such a transaction, the two segments would represent two different productivity rates at the beginning of the next period; one part gains productivity through producing more units during regular time work, the other part loses some productivity due to the retrogression effect. Therefore, productivity measure of each of the working experience classes is increased by the number produced by each member during the period and the status of these classes is updated. The status of each of the idle classes is also updated, using the analysis of retrogression effect (to be discussed later).

Figure 22 illustrates the above situation. In this hypothetical case, the comparison process has continued until it is found that the third experience class is capable of producing more than actually

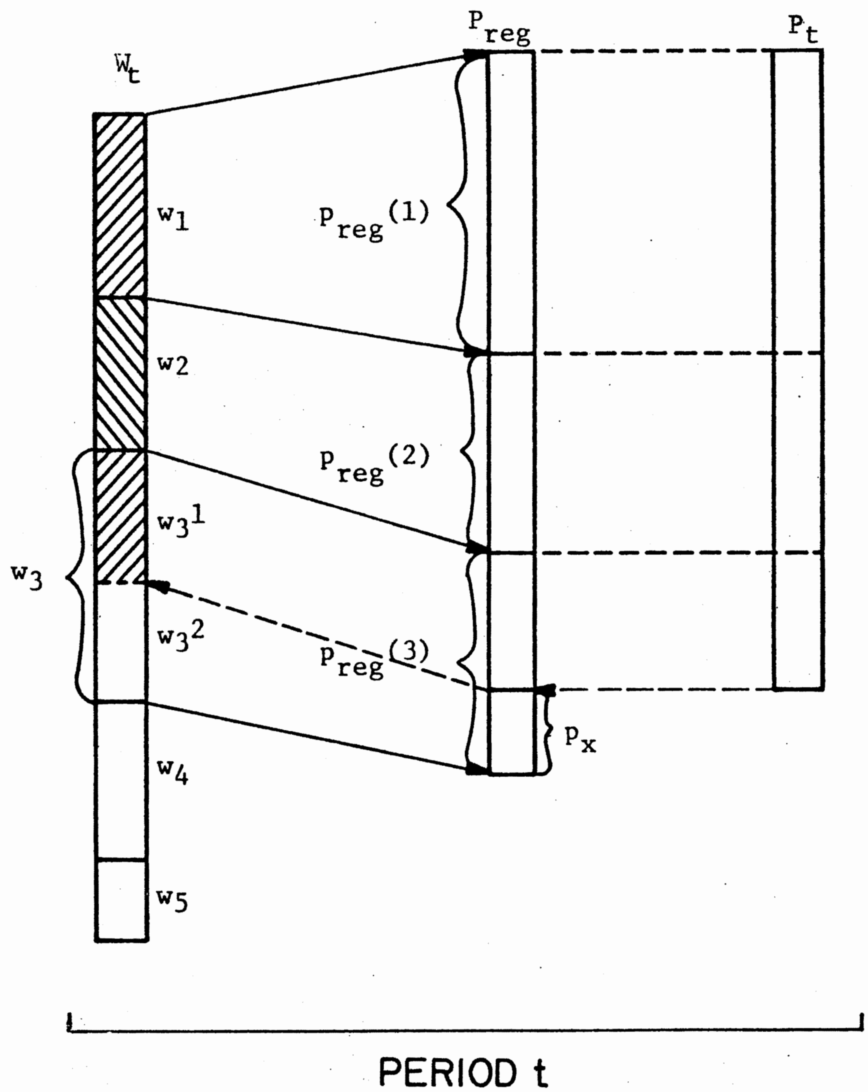


Figure 22. Effect of a Typical Regular Time Work Assignment On the Conditions of Experience Classes, Creating a Partitioned Class

remains to be produced. The ratio of the excess amount P_x, over $P_{reg}(3)$ quantifies the partition of this class into two new classes: the working class comprising w_3^1 members, and the idle class comprising w_3^2 members. This case shows that number, size and productivity level of each experience class can be affected by the regular time assignment of work.

At this point, it must be mentioned that an alternative assumption regarding the latter case may be relevant in some situations: When the production capability of a class exceeds the production amount scheduled for the class, the work may be equally distributed among the members of the class, thereby allowing undertime work for all members in the class. This assumption would considerably simplify the analysis; for in this case, the class is not partitioned and the number of classes and the size of each class would remain unchanged. The productivity measure of each member receiving undertime work would increase by his share of work. This assumption is not incorporated in the present model.

Overtime

Overtime is utilized whenever the production level scheduled for a period exceeds the total production capability of all employees on regular time. The analysis of the overtime effect is very similar to the one applied to the regular time. This analysis is based upon the assumption of Orrbeck et al. regarding the assignment of overtime. That is, when overtime is used, workers will be called upon in order of seniority. Thus, the most experienced workers will work overtime first subject to the limit of their capacity. If overtime work still remains, the next most experienced class will be called upon; and similarly for the remaining experience classes until all overtime work is assigned.

Since the analysis of overtime work is conducted after the inclusion of the effects of workforce fluctuations and the regular time utilization, the only information required for this part of the analysis for each period is:

- Remaining production after the assignment of regular time work
- Number of experience classes, size and the cumulative number of units experienced by the members in each class
- The length of overtime as a percentage of the length of regular time

Given the remaining production amount, and also given the number of experience classes, size and productivity measure of each class, the total number of men to be assigned on overtime work can be determined by finding the production capability of each experience class during overtime, and comparing the result with the remaining production amount.

The production capability of an experience class can be determined by applying the same revised form of the block cost expression which was applied in regular time analysis. The only difference between the two cases is that the overtime duration is usually smaller than the duration of regular time; therefore, if one man can represent one man-month worth of work during regular time, this man can only provide a proportion of a man-month work during overtime. If overtime is assured to be as long as α percent of regular time in a period, then Equation (6.3) can be written as:

$$\alpha \cdot 1 = \frac{f}{1 - b} [m_2^{(1-b)} - m_1^{(1-b)}] \tag{6.7}$$

In this expression, m_1 is the cumulative number of units produced by the employee by the beginning of the period, and m_2 is the expected cumulative unit that will be produced by him during overtime by the end of the period.

In a given period, an employee is assigned on overtime only if he is also assigned on regular time. This indicated that there is an interactive effect between productivity improvements during regular time and overtime. Overtime work is performed during the same day that regular time has been performed. The experience gained during regular time would effect the productivity during overtime. Precise incorporation of this interactive effect requires the breaking of each period into the number of days contained in each period. However, such division would result in a large amount of increased computational time, while the gained precision in the short length of one period (compared to the length of the planning horizon) may not deviate too much from an approximated quantity. This approximation can be done by improving the productivity measure in Equation (6.7). To do this the value m_1 in this equation is set equal to the total of: (1) the cumulative units produced by the beginning of the period, and (2) one-half (average of the total regular time production at the beginning and at the end of the period) of what is produced during regular time:

$$m_1 = n_1 + \frac{n'}{2} \tag{6.8}$$

The variable, n', is the total units produced by a member during regular time in the same period, and is computed using Equation (6.5).

Using Equation (6.7), the overtime production capability of each member with productivity measure of m_1 (at the beginning of the period) throughout the period is:

$$m' = m_2 - m_1 = [\alpha(\frac{1-b}{f}) + m_1^{(1-b)}]^{(\frac{1}{1-b})} - m_1 \qquad (6.9)$$

The total production capability of the ith experience class consisting of w_i members, each with productivity measure of m_1 is, therefore:

$$P_{over}(i) = w_i \left\{ [\alpha(\frac{1-b}{f}) + m_1^{(1-b)}]^{(\frac{1}{1-b})} - m_1 \right\} \qquad (6.10)$$

To determine the total number of men to be assigned on overtime work in a given period (t), first the total amount of overtime work is computed by deducting the total regular time capabilities of all classes from the production rate scheduled for the period. Denoting this remaining production amount by RP_t, then $RP_t = P_t - \sum_i P_{reg}(i)$. Based on the order of seniority, the overtime production capability of the first experience class is compared with RP_t. If the overtime production capability of the first class is smaller than RP_t, then the excess amount is compared against the overtime production capability of the second class. The comparison process continues until one of three possible outcomes occurs.

First, if the excess production amount left for the last experience class exceeds the production capability of this class, then the excess amount must be retained through subcontracting. If, for a specific situation, the subcontracting option is not available, then in the above case an infeasible schedule must be reported. This is true because, in this situation, the scheduled production for the period in question cannot be met even if all employees are fully utilized (both on regular time and overtime).

Second, if the comparison process of the excess production amount against the overtime production capability of the sequentially taken experience classes continues until the production capability of an experience class equals (within an acceptable tolerance) the excess production amount, the current class and all preceeding classes are assigned on overtime work. The productivity measures of the members in these classes are increased by their respective production capabilities during the given period. This updates the status of each class for the beginning of the next period. In this case, all succeeding experience classes are only assigned on regular time and, since this assignment is done prior to consideration of overtime, the status of these classes will remain unchanged.

The third possible outcome of the comparison process is the situation where the production capability of a class in sequence exceeds the remaining production amount. In such case (like the similar case in regular time considerations) the ratio of the excess amount over the production capability of the experience class involved can indicate the portion of the class which is not assigned on overtime. Therefore, an increase in the number of experience classes will result as a consequence of such a situation. The two newly generated classes will differ in the relative productivity measures at the beginning of the next period. Due to the assignment of overtime, the first class will gain higher productivity as compared to the second one, which only performs regular time work.

Figure 23 illustrates the above situation. In this hypothetical case the comparison process has continued until it is found that the third experience class is capable of producing on overtime more than what remains to be produced. The ratio of the excess amount RP_x, over $P_{over}(3)$ is used to partition this class into two new classes: the

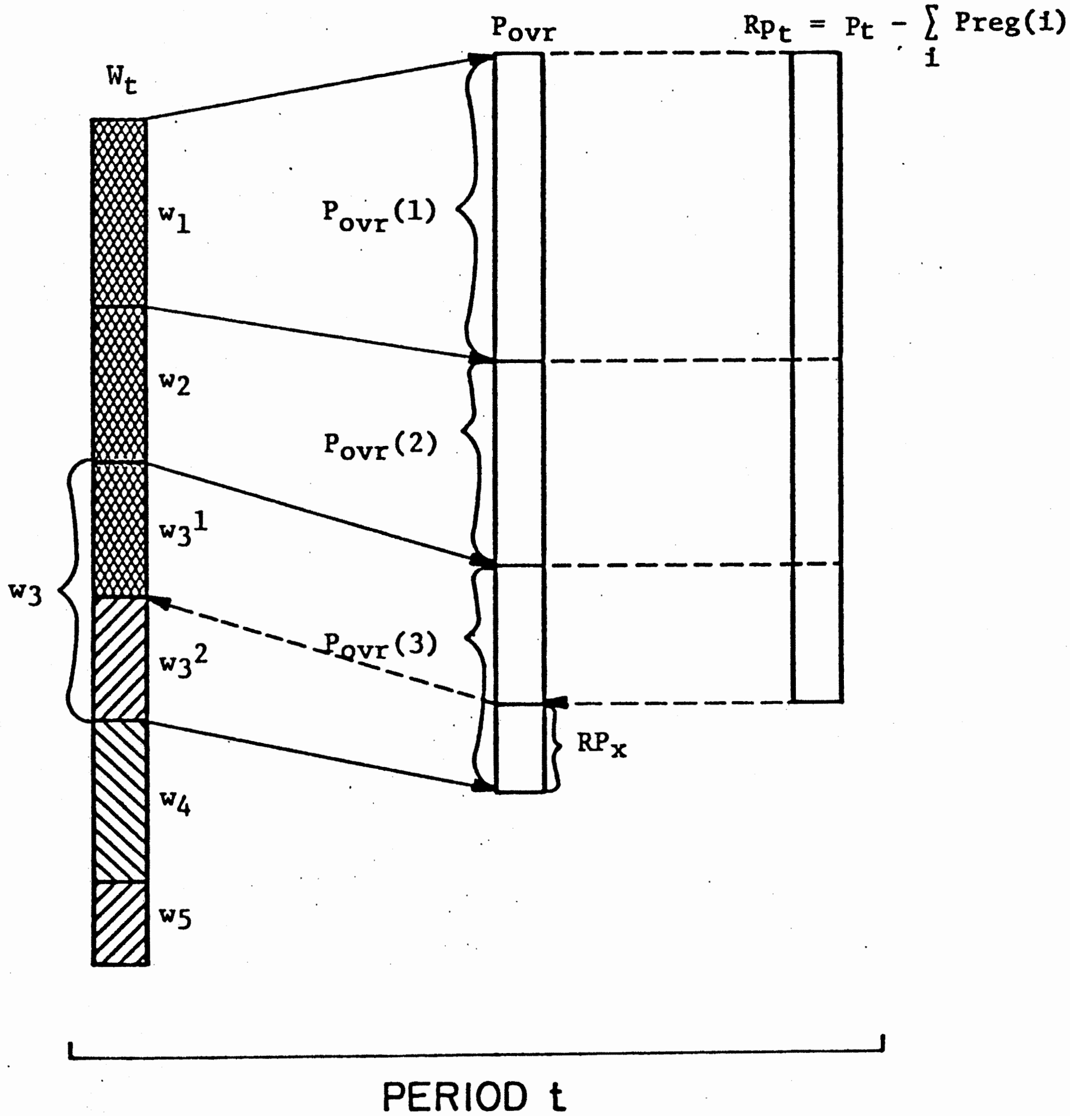


Figure 23. Effect of a Typical Overtime Work Assignment on the Conditions of Experience Classes, Creating a Partitioned Class

upper part consisting of w_3^1 members assigned on overtime, and the lower part consisting of w_3^2 members assigned only on regular time. The double shaded areas in Figure 23 represents those classes assigned both on regular time and overtime.

It should be noticed that the assignment of overtime is another factor affecting number, size and productivity of experience classes involved. In addition, the analysis applied in this section directly determines the total number of workers assigned on overtime for any given period. This number, if incorporated into the total cost function as a variable, can improve the degree of realism of the function; thereby eliminating the possiblity of generating unrealistic results such as the 'negative overtime' indicated for some periods by the HMMS model in the paint company example.

Idle Time

As discussed earlier in the chapter, if in any production period the total regular time production capability of workers exceeds the production rate scheduled for the period, some workers would remain idle. Devotion of special consideration to this phenomenon seem to be necessary because of at least two reasons: first, the idle time payment in some firms is estimated separately and is different from the regular time and the overtime payments. Therefore, inclusion as a variable the number of idle workers in each production period in the cost function of the aggregate planning model would improve the degree of realism of the function. Secondly, the assignment of idle time generates a decay in the productivity rate of the workers involved. This is due to the forgetting effect experienced during idle time. In the literature of learning curve analysis this effect is usually referred to as retrogression or interruption effect. It was noted that the analysis of regular time also revealed the number of idle workers per period where idle time applied. This number can be directly incorporated into the total cost function with the proper cost coefficient.

To analyze the effect of idle time on the productivity rate of idle workers in the aggregate planning problem the analysis of the retrogression effects is considered. According to this analysis [19] [17], the forgetting or interruption phenomenon can be described by a negative decay function comparable to the decay observed in electrical losses in condensers. If it is assumed that an individual's memory is the equivalent of storing electrical charges in the brain, then the decay analogy appears reasonable. In general, the amount of retrogression is a function of the quantity produced by the time that interruption occurs, and the length of the interruption period.

Forgetting patterns show rapid initial decrease in performance followed by a gradual leveling off as a function of the interruption interval. Also the rate and the amount of forgetting decreases as an increased number of units are completed before an interruption occurs. Therefore, the forgetting pattern is very similar to a learning pattern with negative slope. This slope is known as the forgetting slope.

Interruptions, which can take place at any point, and the number of units produced (productivity measure) at this point together with an assumed forgetting slope yield a model for forgetting. At the point of resumption, the performance or unit time can be used to determine the restart point on the original improvement curve. Figure

24 shows the performance versus elapsed weeks for an interrupted operation with a learning slope of 87% and a forgetting slope of 80%.

The expected unit times for an interrupted operation can be determined in a Learn-Forget-Learn (LFL) model by first considering the original learning curve formula for the log linear model. The expected unit cost for cumulative unit X is given as

$$T(X) = T1 \cdot X^{-L}$$

where T1 is the base unit time, L is the measure of learning slope, and X goes from unit 1 to Q1. The latter term being the unit at which the interruption began.

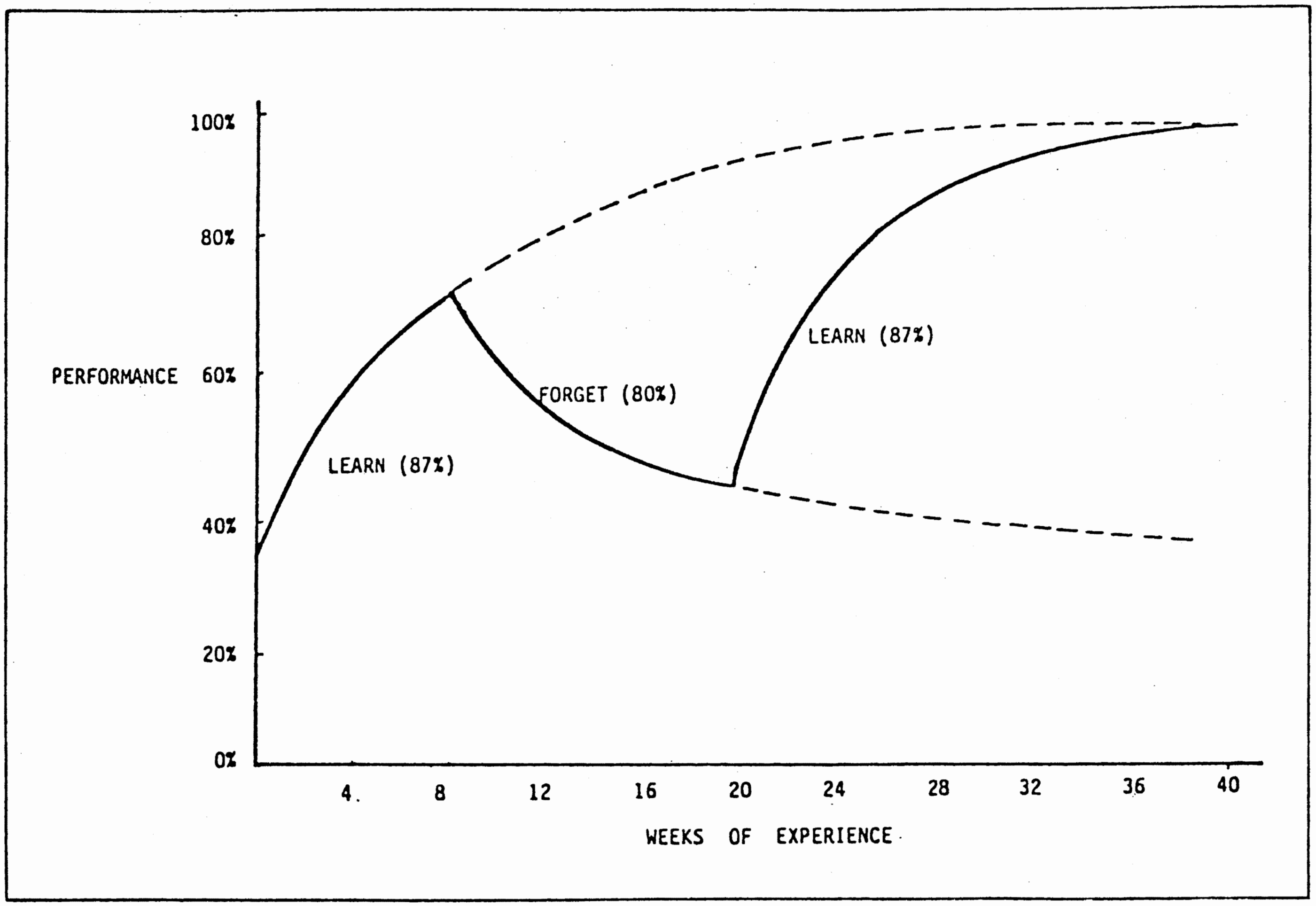
At this point a similar model for forgetting is used to estimate the degradation during the interruption. The equivalent intercept (see Figure 25) can be computed from U1 and Q1, the unit time and quantity completed when the interruption occurred, and an assumed forgetting slope. For example, if a forgetting slope of 80 percent was used, this would mean that ultimately only 80 percent of what was learned would be retained. Using the following formula:

$$T2 = U1 \cdot Q1^{-F}$$

where F is the forgetting slope, the intercept for the forgetting curve can be found. The equivalent unit time at a point where learning is to resume would be found from:

$$U2 = T(X) = T2 \cdot X^{F}$$

This point is the equivalent unit time and quantity at which performance has degraded over the duration of the interruption. An interval expressed in time periods can easily by converted to equivalent



Source: From Carlson, J. C. and A. J. Rowe, "How Much Does Forgetting Cost?", Industrial Engineering, Sept., 1976.

Figure 24. Performance versus elapsed weeks for an interrupted operation

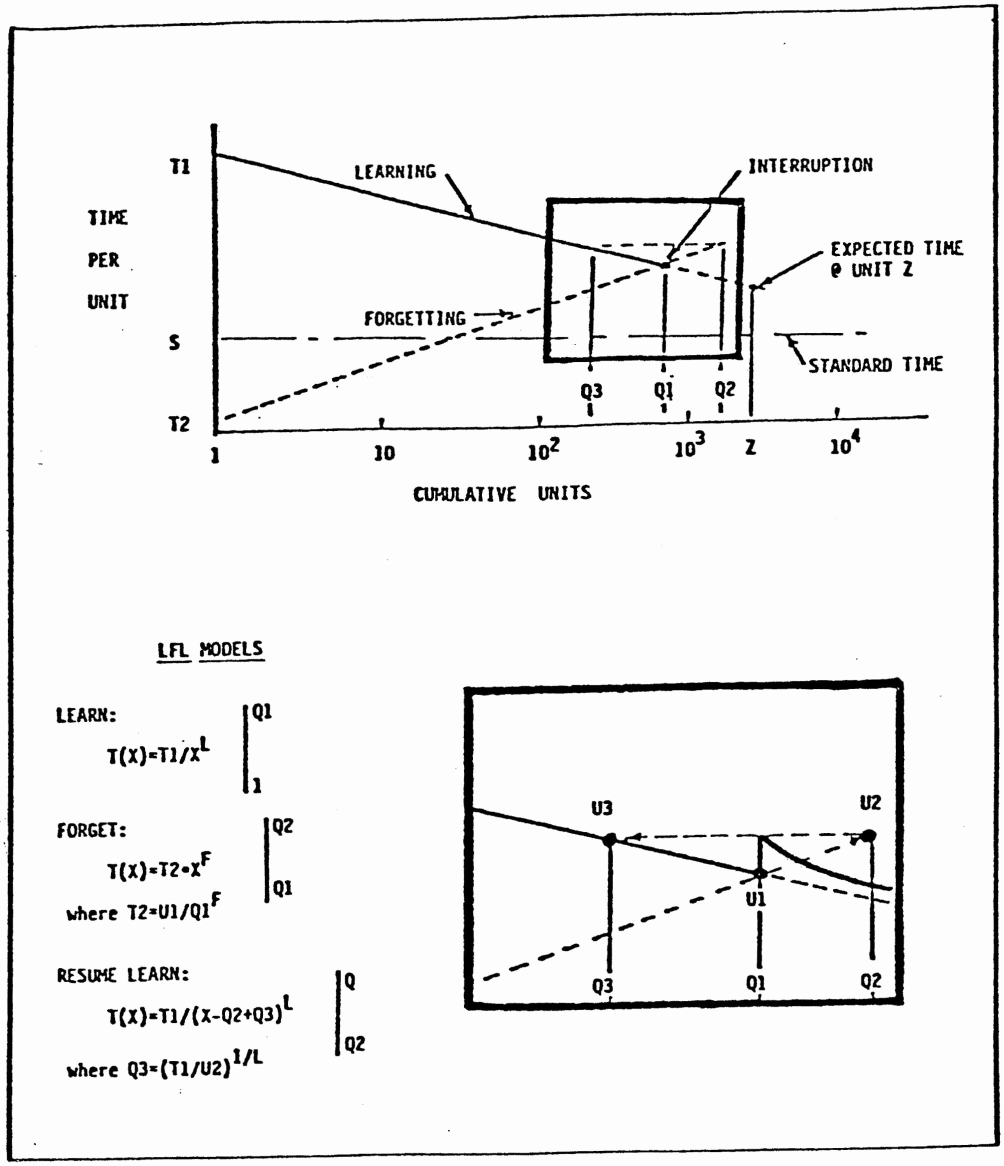
units in a manner similar to that employed for the learning portion of the LFL curve.

Following the resumption of work, the unit times for the learning curve could be expected to follow the original curve but displaced back up the improvement curve to the point equalling the expected re-start unit time. The equivalent start quantity Q3 is found from:

$$U3 = U2 = T1/Q3^{L}$$

Figure 25 shows the expected time as superimposed back over the original learning curve and the scallop shaped curve resulting when these unit times are plotted against the cumulative quantity to date.

The application of the concepts described above to the analysis of the effect of idle time on productivity of workers in the current model happens to be straight forward and requires fewer number of computations than the above methodology. This is due to the fact that in the current model, the status of the productivity measure of each experience class is updated periodically. Therefore, if idle time is experienced, the interval of interruption considered is always one period. For a given period the productivity status of an idle class at the beginning of the following period, can be found by degrading the productivity measure of this class (at the beginning of the current period) over an interrruption length of one period, using the applicable forgetting slope. If the current class is to remain idle in a number of subsequent periods, the degrading process is then imposed sequentially on the updated (degraded) productivity measures throughout the total interruption interval, each time considering an interruption length of one period. If at any period work is to be resumed, the updated status of



Source: From Carlson, G. and A. J. Rowe, "How Much Does Forgetting cost?", Industrial Engineering, September, 1976.

Figure 25. Expected Unit Times for an Interrupted Learning Experience

each experience class would indicate the applicable starting productivity.

To determine the quantity by which the productivity measure of an idle employee degrades throughout one period, let us first consider Equation (6.5) given in the analysis of regular time. This equation actually represents the quantity by which the productivity measure upgrades as a result of learning gained over one period. If in this equation, the forgetting slope, F, is substituted in place of the learning slope, b, this equation can be used to represent the quantity by which the productivity measure degrades as a result of an interruption occuring after production of cumulative unit n_1 and lasting for as long as one period. Therefore, given a productivity measure of n_1 at the beginning of the period, and given a forgetting slope exponent of F, the productivity of an idle employee degrades by:

$$\Delta n = \left[\frac{1-F}{f} + n_1^{(1-F)}\right]^{\left(\frac{1}{1-F}\right)} - n_1 \qquad (6.11)$$

The updated productivity measure at the end of the period (beginning of the next period) is:

$$n'' = n_1 - \Delta n = 2n_1 - \left[\frac{1-F}{f} + n_1^{(1-F)}\right]^{\left(\frac{1}{1-F}\right)} \qquad (6.12)$$

The above formula can be directly used to generate a new productivity status for the idle employees at any applicable period. Since the length of the interruption interval is constant and equal to one period, the only information required is the productivity status when idle time starts and the slope of the forgetting curve. The forgetting slope may be different for different tasks and may depend upon the complexity of the task.

As a final phenomenon affecting productivity in aggregate planning of short cycle products, the analysis of the progress effect will be discussed in the next section.

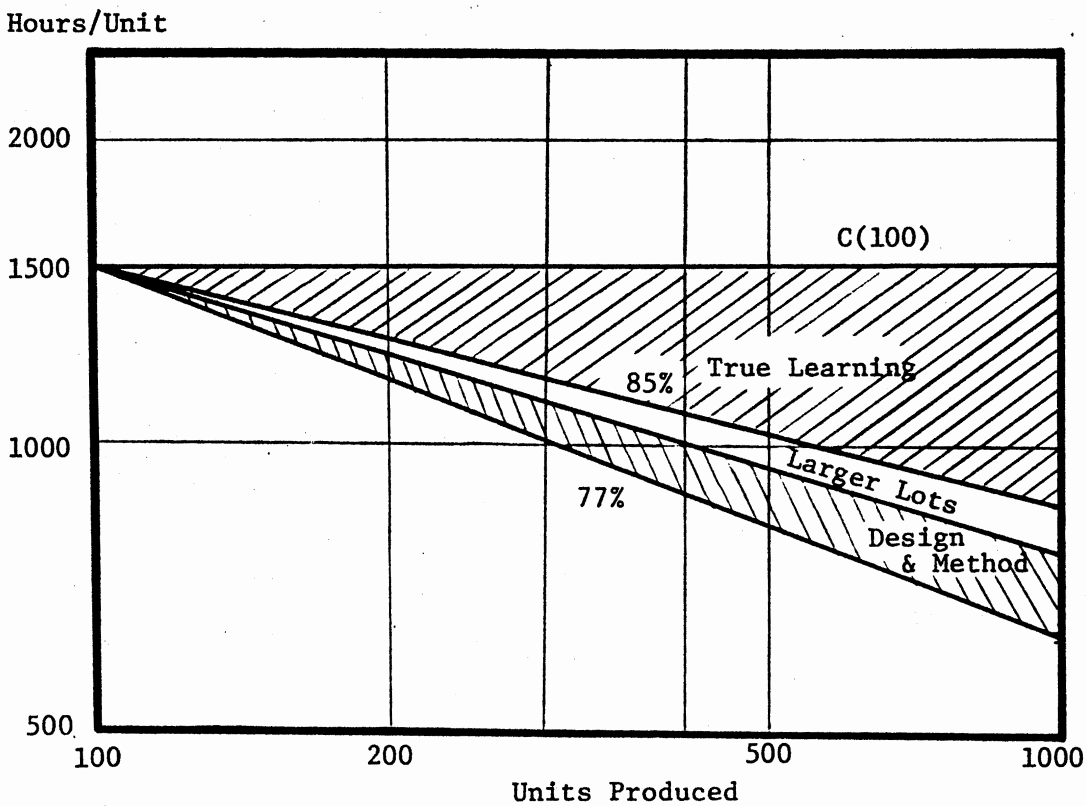
Progress Effect

The unit cost reduction curves used for the long cycle case were termed "improvement curves" and the ones used up to this point for the short cycle case were termed learning curves. Improvement curves incorporate the effect of learning together with the progress effect.

Although both learning and progress effects are present in production of short cycle products, these effects are treated separately for this case. This is due to the need for properly incorporating the productivity measure of the newly hired workers in different time periods.

To clarify the above point, first the distinction between learning and progress should be noticed: the term "improvement" is usually applied to the general relationship between unit cost reduction and the cumulative number of units produced. The term "learning" is applied strictly to that portion of cost reduction which occurs without major method or design changes, and the term "progress" to the effect of those changes.

The total productivity improvement is not solely due to either learning or to progress, Figure 26 presents several kinds of manhour cost reduction that can occur from cumulative units 100 to 1,000. It should be noted that the learning curve portion is by no means, the entire reason for the cost reduction. New methods, design changes and larger lot sizes also contribute significantly to the substantial cost



Source: From Cochran, E. B., Planning Production Costs Using the Improvement Curve, Chandler Publishing Co., 1968.

Figure 26. Learning vs Progress

reduction which occurs between these two units.

Cochran [19] shows that the progress slope can be calculated and that the slope of improvement pattern equals the product of the slopes of learning and progress patterns. Therefore, given the slopes of learning and progress, the portion of cost reduction due to each effect can be determined for any unit.

The progress effect is a function of the cumulative unit output of the firm to date. Therefore, given the cumulative output by period, and the slope of the progress function, the productivity measures for each experience class found by the foregoing analysis can be revised to incorporate the progress effect.

As mentioned earlier, the separate consideration of the progress effect in this analysis is due to the need of developing appropriate productivity measures for the new workers hired in different periods of the planning horizon. It is clear that due to the presence of the progress effect, unexperienced workers hired in different production periods would have different starting productivities. Therefore, the original production capablity of these groups cannot be directly based on the same base unit cost (the cost of original unit produced by the firm). Although all new workers begin at the same learning level, they do not necessarily begin at the same progress level attained by the firm. The progress effect on the cost of the original units produced by them is a function of the cumulative units produced by the firm at the time they are hired. Therefore, the productivity measures of the new employees can be determined by first referring to the learning curve base unit cost and then revising this measure to incorporate the effect of the appropriate progress status.

If the cumulative units produced by period t is denoted by cum_t and the slope of the progress pattern by P, then according to the basic formula of linear cost curve;

$$\beta_t = 1 \cdot (cum_t)^{-P} \tag{6.13}$$

β_t is the cost of unit cum_t, assuming a base unit cost of 1, and only progress being in effect. Therefore, β_t is a factor that can be multiplied by any base unit cost, K, the product is the cost of unit cum_t, assuming base cost of K. The result of such multiplication is, of course, smaller than K. The above factor can also be used to revise the production capability of new workers determined only on the basis of learning effect on the original unit.

In summary, the production capabilities of different experience classes, including the newly hired workers which are first found by solely considering the learning effect, can be revised by incorporating the progress effect. This revision takes place at the time the unrevised measures are formed and prior to the inclusion of these measures as input information for determination of regular time, overtime, etc. Therefore, both learning and progress are in effect when decisions regarding utilization of workforce are analyzed.

Summary of the Analysis

Application of the foregoing analysis to the aggregate planning of short cycle products can be summerized as follows. Given a sequence of fluctuating workforce and production levels throughout the planning horizon, starting with the first period, the workforce level in this period is compared with the initial level of workforce. The initial number of classes and possibly the size of an experience class are adjusted depending upon the type of manpower change. The production rate scheduled for the first period is then compared against the production capabilities of the experience classes. Both effects of learning and progress are incorporated when determining the production capabilities. As a result of this comparision, members are assigned on regular time work, overtime work, or remain idle. The conditions of the experience classes and the productivity measure of each are updated according to their level of contribution to the production, taking into account the decay in the productivity rate of the idle workers. This process sets a new status for experience classes at the beginning of the next period. The workforce on regular payroll, on overtime payroll, idle workforce, and the subcontracting level are the results of the analysis for this period.

After completion of the above computations for the first period, the second period is then treated similarly. The procedure continues for the succeeding periods in sequence until all periods have been considered. The output of each period is then used in evaluating the objective function. The response level of the objective function indicates the total operating cost of the proposed production and workforce schedule throughout the planning horizon.

It should be noted that in this model there is no need to find the average productivities for each period. The average productivity in aggregate planning models is used to evaluate the amount of overtime, undertime, or subcontracting required. These quantities are computed on the current model and are available for evaluation of the objective function. Of course, the objective function in this model does not include the average productivity factor.

Optimization Methodology

The methodology for optimization of Model II is somewhat similar to the one utilized for Model I. In the previous chapter a summary of the methodology for evaluating the performance of an aggregate plan under dynamic productivity consideration was presented. An aggregate plan is represented by its specific levels of production and workforce throughout the planning horizon.

Again, in the new model, production and workforce levels are considered as the independent or controllable variables. These decision variables can be adjusted within specified boundaries so as to produce the best possbile index of system performance. The application of search techniques would involve systematic adjustment of these variables and exploration of the response surface of the objective function. The optimum solutions are those which correspond to the minimum response.

Note that unlike the cost functions of the constant productivity models which only incorporate the independent variables, the cost function in the new model also incorporates a number of dependent variables. These variables could be: workforce levels on regular time, on overtime, idle workforce, amount of subcontracting and the penalty factors related to the production constraint violation. The levels of the above variables are dependent upon the levels of the independent variables and are calculated through the application of the foregoing analysis for the case of short cycle products.

Based upon the above considerations, the optimization methodology proposed for Model II is composed of three major interacting units.

The interactions between these routines are demonstrated in the diagram in Figure 27. The first unit is the pattern search routine, PATS. This routine systematically generates a sequence of levels for the independent decision variables, workforce and production rates for each period of the planning horizon.

The second unit, called CLASS, is a routine which utilizes the analysis developed for the case of short cycle products. The routine receives the levels of independent decision variables generated by PATS and then generates the respective levels for the dependent variables for all periods, based on dynamic productivity considerations.

Input to the third routine, called FCT1, is the information generated by CLASS together with the information generated by PATS. Incorporating all independent and dependent variables into the cost function, this routine evaluates the response of the proposed plan.

The response evaluated by FCT1 is received by the PATS routine. The magnitude of this response provides the search routine with necessary information to make appropriate adjustments in the level of the independent variables. The newly adjusted levels are fed to the CLASS routine and the whole process is continued until the criteria for termination is met and the optimal solution is found.

In the above methodology, every proposed change made by PATS in the level of decision variables necessitates re-evaluation of the dependent variables through the steps of the analysis of dynamic productivity discussed earlier in this chapter. This results in a considerable increase in the computation time involved in the SDR solution methodology. An improvement in the starting solution through the optimization of a somewhat equivalent constant productivity model

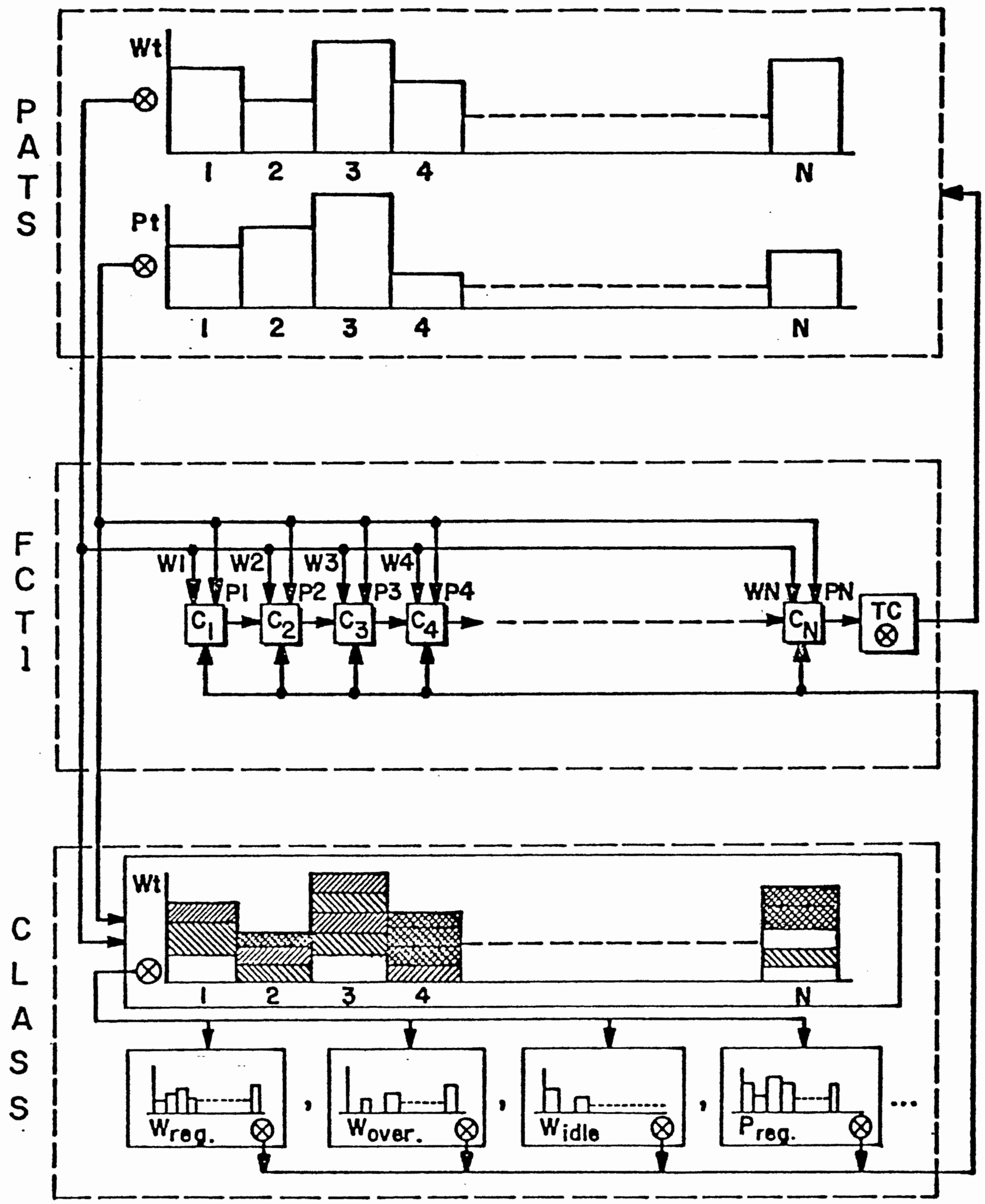


Figure 27. Information Flow Among the Three Major Routines Involved in Optimization of Model II

prior to the optimization of the new model is found to increase the efficiency of the technique.

Further improvement in the computation efficiency is made possible by improving the efficiency of the algorithm utilized in the CLASS routine. As it was the case for Model I, the levels of the dependent variables at every period are only related to the levels of the independent variables throughout the preceding periods. Therefore, once a change is made in the level of production or workforce in a given period, the re-evaluation of the dependent variables is not necessary for the preceding periods. Thus, a check can be made at the beginning of the CLASS routine to determine the first period for which a decision variable has changed. The computation steps should then be carried on only for the current and the succeeding periods. These provisions have been incorporated into the optimization procedure applied in this research. An explanation of this procedure follows.

Optimization Procedure

Figure 28 illustrates a block diagram of the optimization procedure (also refer to Appendix B and E for variable definiations and semi-descriptive flow charts, respectively). The main program reads the model parameters and initalizes the search routine. The computation of the overall average productivity for the equivalent constant productivity model is then performed based on the applicable improvement curve (which incorporates both learning and progress effects). The slope of this curve is the product of the learning and the progress slopes. The unit numbers considered in this computation range from the initial cumulative units produced by the firm to the cumulative units produced by the end

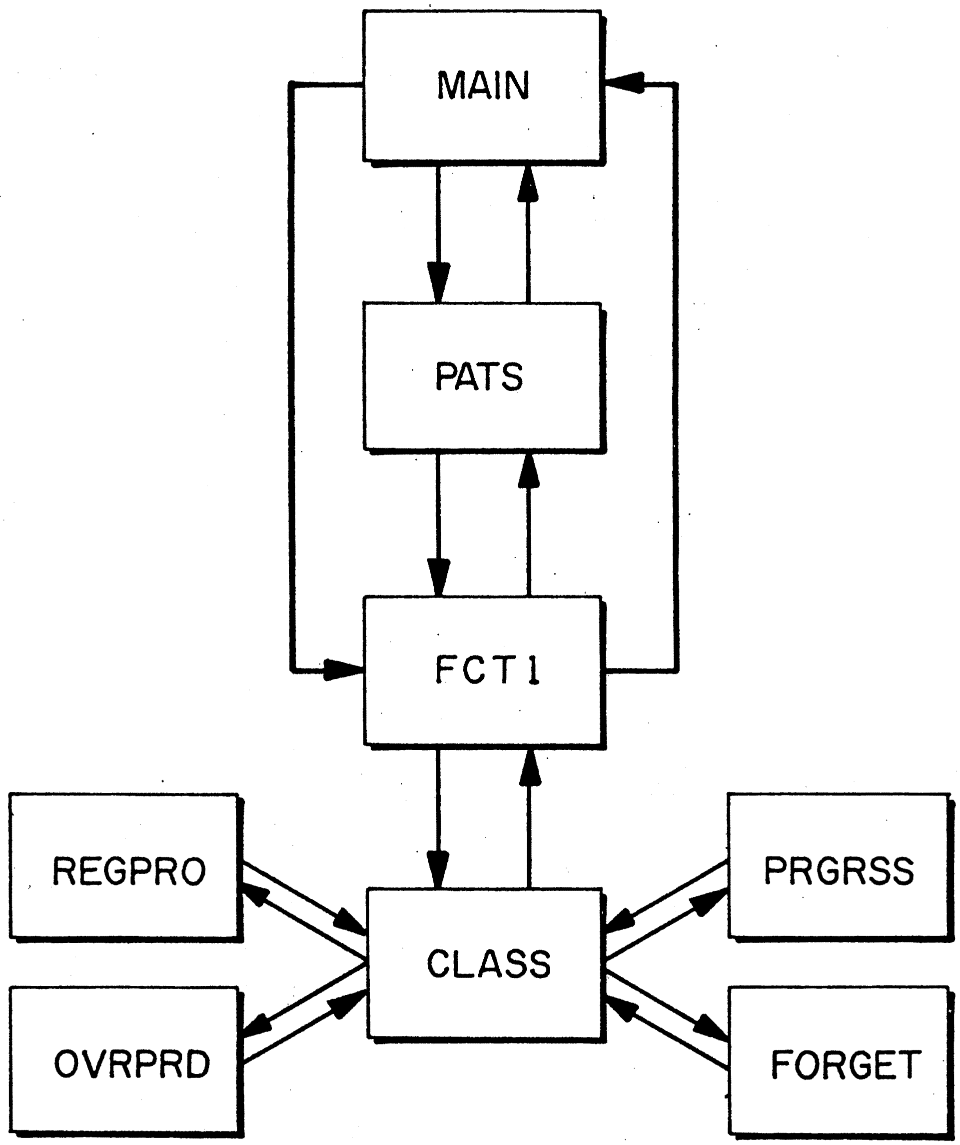


Figure 28. Block Diagram of the Optimization Procedure for Model II

of the planning horizon. The total number of units produced during the planning horizon is assumed to be equal to the total demand during the horizon.

The PATS routine is then called and the constant productivity model is optimized. The results are then printed and input to CLASS through a call to FCT1. This evaluates the performance of the constant productivity model in situations where the dynamic productivity conditions are present. The main routine then prints the results of this evaluation.

The optimum solution found for the constant productivity model is used as the starting solution for the optimization process of the dynamic productivity model. This model is optimized (while the CLASS routine is in effect) and the results are printed.

Subroutine FCT1 increments the number of function evaluations, receives the levels of the decision variables from PATS and directs these levels to the cost function of the constant productivity model, when this model is being optimized. The FCT1 routine directs the information received from PATS to the CLASS routine and receives back the appropriate levels of the dependent variables. These levels, together with the levels of the independent variables received from PATS are then directed to a new (but equivalent) cost function with the same cost coefficients. This new cost function does not include the productivity factors; however, the function includes the dependent variables. The appropriate levels of these variables are received from CLASS.

Subroutine CLASS begins by detecting the period in which the first change is made by PATS in the former levels of the decision variables. This subroutine then incorporates the effect of manpower change (through hiring and firing) into the condition of the experience classes. The new conditions are then recorded in temporary locations.

The analysis of regular time and its effect on the experience classes is then performed in this routine. At this step, the level of the workforce on regular time and the level of the idle workforce (when applicable) are computed for each period. After incorporating the effect of regular time work, a permanent status for each class is then determined.

If the regular time work in any period does not satisfy the total production scheduled for that period, the excess amount is then transferred to another part of the routine which incorporates the effect of the overtime assignment on the conditions of the experience classes. At this step, the levels of the overtime workforce and subcontracting or the penalty factors of exceeding the maximum production capability for an infeasible plan are generated (if subcontracting is not allowed). After the incorporation of the effect of the overtime assignment on the status of the classes, the permanent status for all classes in the applicable periods is established.

Subroutine CLASS interfaces with four other routines: REGPRO, which reports regular time production capability of members for any given productivity measure OVRPRO, which reports overtime production capability of members for any given productivity measure; PRGRSS, which reports the applicable progress factor for any given cumulative units produced by the firm; and FORGET, which reports the amount of decay in productivity measure as a result of idle time, for any given productivity measure. The Model II program listing is provided in Appendix F.

Remarks

The analysis and the optimization methodology developed in this chapter have resulted in an integrated model which represents the workforce body in every period of the planning horizon as a non-homogeneous entity comprised of different classes of employees with their relevant productivity rates. These productivity rates are determined on the basis of the current improvement curve theories. The new model has a number of advantages over the comparable model developed by Orrbeck et al. because it incorporates more realistic assumptions while applying the improvement curve analysis. Considering the solution approach and the computational difficulties, there is also a considerable difference between the two models.

Note that although the new model breaks the workforce body in each period into a number of experience classes, the dimension of the problem (number of independent variables) is still unchanged (twice the number of periods). That is, the independent variables used in the search technique are still the "total workforce" level and the production rate for each period. Consequently, the new model has a considerable advantage over the Orrbeck et al. model. In this latter model each class in each period is represented by a new variable. These variables are incorporated into the objective function and the constraints, thereby resulting in a large model which (considering the available L.P. packages) is computationally unattractive as compared with the performance of the new model. Optimization of the new model on an IBM 370 system for a 10 period planning horizon (20 dimensions) requires a computing time of a little over one minute. As experience shows, the optimization of an equivalent model with structure proposed by Orrbeck et al. would result in a large L.P. model (approximately 200 x 100) which requires much more execution time and fast memory on the same system (approximately 15 minutes).

The new model also has some advantages over all existing aggregate planning models. For example, the length of overtime in this model does not have to be considered as a constant fraction of the length of the regular time for all periods throughout the planning horizon. This quantity may be considered as a variable, thereby introducing a new dimension to the optimization process. That is, three independent variables can be assumed for every period. The optimization process would then include the periodical length of overtime as a variable which may assume different values within the prespecified boundaries. In this case, the dimension of the problem will be three times larger than the number of periods in the planning horizon.

Another general advantage of the new model is its capability of producing additional information about the dependent variables in the system. Workforce level on regular time, overtime and idle workforce in each period along with production level related to each alternative way of workforce utilization are some of these variables which can help the analyst construct more realistic cost functions. In this case, some approximations (such as quadratic approximation of overtime cost) would be unnecessary. Also, note that the new model provides the size and experience level of each class in each period. Therefore, if workers are to receive different wage incentives relative to their productivities, an even more realistic cost function can be constructed and used in the new model.

CHAPTER VII

ANALYSIS OF RESULTS

Introduction

The objective of this chapter is to quantitatively evaluate the impact of the new models developed in this research. The cost structure selected for this purpose is the one developed for the original HMMS paint factory problem which has frequently been utilized as a test criterion by many researchers.

The basic problem encountered in this part of the research originates from the fact that the existing actual data lack information regarding learning productivity, since they are used to evaluate constant productivity models. This may be the reason that Ebert and Orrbeck et al. have used hypothetical data to demonstrate the performance of their models.

The above problem has been overcome in the current research by considering the fact that the constant productivity factor used in the existing constant productivity models is actually the "average" productivity rate determined on the basis of available accounting data. Actually, it is possible to generate several learning curves that yield a given average productivity over a range of cumulative output (from the beginning through the end of the planning horizon). This can be done by proper selection of sets of base unit cost, improvement slope, and initial cumulative output quantity. For the paint factory problem such improvement curves can be developed to provide a basis for rating the relative performances of the new models. The evaluation algorithm developed for this purpose is as follows:

Step 1. On the basis of the given overall average (constant) productivity and a range of cumulative output, generate an applicable improvement curve.

Step 2. Optimize the constant productivity model. (Only the solution is important at this step, the associated cost is not.)

Step 3. Using the dynamic productivity model, compute the total cost associated with the solution found in Step 2 (no optimization is performed at this step).

Step 4. Applying the improvement curve developed in Step 1, optimize the dynamic productivity model.

Step 5. Compare the two costs found in Steps 3 and 4 (evaluate the two possibly different sets of solutions; the solution found in Step 3 is optimal only under constant productivity assumption, and not necessarily optimal for the dynamic model).

Step 6. Repeat the above steps for different values of the model parameters.

Basically, the above algorithm evaluates the outcome of implementing the solutions found by applying constant productivity models versus the dynamic productivity models in situations where a constant productivity assumption does not hold.

Notice that the Ebert model can be substituted in Step 2 of the above algorithm. The outcome of the evaluation in this case would be the impact of disruptions generated by manpower transactions which are not incorporated into the Ebert model.

Evaluation of Model I

As mentioned before, the actual data used in the paint factory problem is chosen as the basis for testing the performance of the new models. It was seen in Chapter IV, that the overall average productivity per employee in this problem is 5.67 units (gallons) per man per month. Although this case does not ideally represent a long cycle product situation (5.67 is rather large output quantity per man-month), this case still seems to be closer to the long cycle product situation rather than to a short cycle one (where the output per man-month is a lot higher than 5.67). Thus, the paint factory data can suitably provide a basis for evaluation purposes regarding Model I.

Both the constant productivity and the Ebert models are compared with the new model in this part of the analysis. The Ebert model is used to show the impact of disruptions caused by the manpower transactions.

Substituting the dynamic productivity factor as a variable in the original cost structure of the paint factory model (shown in Chapter IV), the new cost structure would have the following form:

$$T_c = \sum_{t=1}^{N} \{[340\ W_t] + [64.3(W_t - W_{t-1})^2]$$
$$+ [0.20(P_t - AP_tW_t)^2 + 51.2\ P_t - 281\ W_t]$$
$$+ [0.0825(I_t - 320)^2]\}$$

s.t.

$$I_t = I_{t-1} + P_t - D_t \qquad t = 1, 2, \ldots, N$$

where AP_t represents the average productivity per employee in period t. The initial workforce and inventory levels in this model are 81 men and 283 units, respectively. A planning horizon containing 10 periods is assumed in this analysis, thereby incorporating 20 independent (W_t and P_t) and 10 dependent (AP_t) variables into the cost function. The demand levels (D_t) for 10 consecutive months are given in the second columns of the upper parts of the tables in the following pages. Workforce levels and production rates are restricted to vary in the intervals of 0 to 150 men and 0 to 1000 units, respectively.

Using the algorithm presented earlier in this chapter (Step 1) and applying an improvement slope of 70% over the 10 month planning horizon, a base unit cost of 16.55 man-months per unit is estimated. This estimation is based on the assumption that the firm has produced 5,000 units by the beginning of the planning horizon (cum_o = 5000). This improvement curve is used in the Ebert and in the dynamic productivity models. The minimum threshold level, TRSHL, for a significant manpower turnover and the minimum unit cost difference ratio,EPSY for a significant compounded disruption are .05 and 10%, respectively (see Appendix A).

Table IV shows the results of optimization of the average productivity model with original constant productivity level of 5.67 (on the basis of which the improvement curve parameters are computed). This table contains the same information contained in Table 2 of Chapter IV. The results of optimization of the Ebert model are presented in Table V. Notice that the monthly average productivites in this model increases continuously. The magnitude of increments in the average productivity in a month is proportional to the production rate in that month. Transactions occuring in the workforce level do not affect these magnitudes. It should be noted, that although the average of the monthly average productivity levels in this model is very close to the overall average productivity level in the equivalent constant productivity model, it is not necessarily equal to this level. This is due to the fact that the overall average productivity level is computed on the basis of the

TABLE IV

RESULTS OF OPTIMIZATION OF THE CONSTANT PRODUCTIVITY MODEL CUMØ = 5,000

A. DECISIONS AND PROJECTIONS

Month	Demand	Work Force	Production	Inventory	Average Productivity
1	430	77.7	470.5	303.5	5.67
2	447	74.3	444.1	300.6	5.67
3	440	70.9	417.1	277.7	5.67
4	316	67.7	381.7	343.4	5.67
5	397	65.1	376.2	322.5	5.67
6	375	62.7	363.8	311.4	5.67
7	292	60.7	348.9	368.3	5.67
8	458	59.0	359.4	269.7	5.67
9	400	57.4	329.3	199.0	5.67
10	350	56.1	272.2	121.2	5.67

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	26,406.07	715.19	2,447.78	22.45	29,591.40
2	25,247.18	747.03	1,978.09	31.04	28.003.34
3	24,105.72	724.73	1,476.17	147.88	26,454.50
4	23,029.24	644.56	511.47	45.05	24,230.33
5	22,122.18	457.64	986.83	0.53	23,567.18
6	21,327.44	351.32	1,015.87	6.13	22,700.76
7	20,639.67	263.11	810.97	192.46	21,906.21
8	20,068.63	181.38	1,936.16	208.91	22,395.08
9	19,509.80	173.70	740.77	1,207.64	21,631.91
10	19,080.42	102.55	-1,408.60	3,259.04	21,033.41
					241,514.22

TABLE V

RESULTS OF OPTIMIZATION OF THE
EBERT MODEL CUMØ = 5,000

A. DECISIONS AND PROJECTIONS

Month	Demand	Work Force	Production	Inventory	Average Productivity
1	430	78.4	454.9	287.9	4.947
2	447	75.2	439.9	280.8	5.161
3	440	71.6	418.0	258.8	5.358
4	316	68.0	384.5	327.3	5.537
5	397	64.6	379.4	309.7	5.702
6	375	61.5	267.0	301.6	5.859
7	292	58.8	352.0	361.6	6.006
8	458	56.5	363.5	267.1	6.150
9	400	54.4	226.1	203.2	6.287
10	350	52.9	285.3	138.5	6.407

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	26,651.61	438.99	2,167.94	84.83	29,343.38
2	25,568.29	652.79	1,925.85	126.79	28,273.71
3	24,358.81	813.67	1,501.16	209.27	26,982.92
4	23,113.53	863.94	598.97	4.39	24,579.83
5	21,974.00	721.01	1,287.77	8.77	23,991.56
6	20,924.94	612.15	1,502.55	27.80	23,067.43
7	19,991.34	484.81	1,499.05	142.89	22,118.09
8	19,221.41	329.72	2,776.50	230.54	22,558.18
9	18,505.50	285.09	1,921.73	1,124.68	21,837.00
10	17,976.34	149.92	316.52	2,717.46	21,170.24
					243,922.34

monthly production rates. Evidently, the total production in the planning horizon could differ from the total demands in the horizon. This is due to the existance of the initial and ending inventory levels which are allowed in these models. Of course, the overall average productivity cannot be computed on the basis of the total production, because the monthly production rates are decision variables which are not known prior to the optimization process. However, the monthly demands are assumed to be known prior to determination of these variables; therefore, they are used to estimate the overall average productivity.

As mentioned earlier in this chapter, the cost analysis of the optimization results of the constant productivity and the Ebert models is not relevant for the current purpose; however, the projected decision variables are: the actual cost of implementing these decisions in situations where disruption effects exist is, of course, larger than what is indicated by the models. Tables VI and VII shown the results of implementing these decisions in such a situation. To compute the entries in these tables, the projected decisions by the constant productivity and the Ebert models are used to evaluate the objective function of the dynamic productivity model. The actual average productivities (versus the projected ones) are computed using the PROCTV routine. No optimization is performed at this stage.

The entries in the columns under INDEX indicate the occurance of significant disruption as a result of manpower transactions. If such disruption occurs at the beginning of a given period, the value of INDEX for that period is one; otherwise, it is zero. The entries in the columns under ICHK for a given period indicate the existance of a significant disruption effect carried over from previous period(s). If such a case occurs, the value of ICHK is one; otherwise, it is zero. If both INDEX and ICHK are one in a period, this indicates the occurrence of a compounded disruption.

Notice that implementation of the constant productivity model (Table VI) does not incorporate any disruption. However, implementation of the Ebert model incorporates one significant disruption at the beginning of the fourth period (Table VII). This disruption is caused by a manpower reduction of size 3.6 which is the largest change throughout the planning horizon. Also notice the effect of this change in the productivity level for this period; while the Ebert model indicates a productivity level of 5.537 in this period (Table V), the actual productivity is 5.205 (as indicated by the new model) the difference being due to the disruption occurring at this period.

Since the average productivity level is constant in the constant productivity model, the projected workforce level is relatively smoother in this model. However, since the average productivites are continuously increasing in the Ebert model, the projected manpower level is continuously decreasing in this model. This is why the implementation of the constant productivity model does not incorporate any disruption, but implementation of the Ebert model does.

The cost analysis given in Tables VI and VII are important and can be compared with the cost of implementing the dynamic productivity model. The results of optimization and implementation of the dynamic productivity model are given in Table VIII. Notice that this model projects decisions which indicate the lowest total cost of implementation. Note that the Ebert model performs better than the constant productivity model. This is simply because of the better estimation of the actual

TABLE VI

RESULTS OF IMPLEMENTATION OF THE CONSTANT PRODUCTIVITY MODEL CUMØ = 5,000

A. DECISIONS AND PROJECTIONS

Mo.	Demand	Work Force	Prod.	Inventory	Act Avg Prod.	INDEX	ICHK
1	430	77.7	470.5	303.5	4.951	0	0
2	447	74.3	444.1	300.6	5.169	0	0
3	440	70.9	417.1	277.7	5.367	0	0
4	316	67.7	381.7	343.4	5.544	0	0
5	397	65.1	376.2	322.5	5.708	0	0
6	375	62.7	363.8	311.4	5.863	0	0
7	292	60.7	348.9	368.3	6.010	0	0
8	458	59.0	359.4	269.7	6.152	0	0
9	400	57.4	329.3	199.0	6.287	0	0
10	350	56.1	272.2	121.2	6.403	0	0

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventroy	Total
1	26,406.07	715.19	3,745.86	22.45	30,889.57
2	25,247.18	747.03	2,598.28	31.04	28,623.53
3	24,105.72	724.73	1,698.15	147.88	26,676.48
4	23,029.24	644.56	518.00	45.05	24,236.85
5	22,122.18	457.64	980.90	0.53	23,561.26
6	21,327.44	351.32	1,005.61	6.13	22,690.50
7	20,639.67	263.11	847.06	192.46	21,952.29
8	20,068.63	181.38	1,816.86	208.91	22,275.78
9	19,509.80	173.70	935.04	1,207.64	21,826.18
10	18,080.42	102.55	-314.21	3,259.04	22,127.81
					244,860.25

TABLE VII

RESULTS OF IMPLEMENTATION OF THE EBERT MODEL CUMØ = 5,000

A. DECISIONS AND PROJECTIONS

Mo.	Demand	Work Force	Prod.	Inventory	Act Avg Prod.	INDEX	ICHK
1	430	78.4	454.9	287.9	4.947	0	0
2	447	75.2	439.9	280.8	5.161	0	0
3	440	71.6	418.0	258.8	5.358	0	0
4	316	68.0	384.5	327.3	5.205	1	0
5	397	64.6	379.4	309.7	5.702	0	0
6	375	61.5	367.0	301.6	5.859	0	0
7	292	58.8	352.0	361.6	6.006	0	0
8	458	56.5	363.5	267.1	6.150	0	0
9	400	54.4	366.1	203.2	6.287	0	0
10	350	52.9	285.3	138.5	6.407	0	0

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	26,651.61	438.99	2,167.94	84.83	29,343.38
2	25,568.29	652.79	1,925.85	126.79	28,273.71
3	24,358.81	813.67	1,501.16	309.27	26,982.92
4	23,112.53	863.94	774.21	4.39	24,755.07
5	21,974.00	721.01	1,287.77	8.77	23,991.56
6	20,924.94	612.15	1,502.55	27.80	23,067.43
7	19,991.34	484.81	1,499.05	142.89	22,118.09
8	19,221.41	329.72	2,776.50	230.54	22,558.18
9	18,505.50	285.08	1,921.73	1,124.68	21,837.00
10	17,986.34	149.92	316.52	2,717.46	21,170.24
					244,097.57

TABLE VIII

RESULTS OF OPTIMIZATION (AND IMPLEMENTATION) OF THE DYNAMIC PRODUCTIVITY MODEL CUMØ = 5,000

A. DECISIONS AND PROJECTIONS

Mo.	Demand	Work Force	Prod.	Inventory	Avg Prod.	INDEX	ICHK
1	430	78.4	454.7	287.7	4.947	0	0
2	447	75.2	439.6	280.3	5.160	0	0
3	440	71.7	417.8	258.1	5.258	0	0
4	316	68.1	384.6	326.7	5.536	0	0
5	397	64.9	379.6	309.3	5.701	0	0
6	375	61.9	367.7	302.0	5.859	0	0
7	292	59.2	353.6	363.6	6.007	0	0
8	458	57.1	366.9	272.5	6.151	0	0
9	400	55.1	342.7	215.2	6.291	0	0
10	350	52.3	257.0	122.2	5.860	1	0

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	26,658.12	432.58	2,142.42	86.04	29,319.17
2	25,582.62	643.39	1,893.74	129.72	28,249.47
3	24,382.97	800.51	k,463.13	315.91	26,962.52
4	23,164.05	826.43	556.22	3.69	24,550.38
5	22,049.91	690.44	1,233.81	9.39	23,983.55
6	21,031.91	576.43	1,499.49	26.64	23,084.48
7	20,137.32	445.15	1,462.30	157.03	22,201.80
8	18,412.49	292.23	2,789.73	186.05	22,680.50
9	18,741.61	250.35	2,060.29	905.84	21,958.09
10	17,784.72	509.31	-1,050.31	3,227.67	20,471.39
					243,461.35

average productivity levels performed by the former model.

As this part of analysis indicates, there is no major cost advantage of one model over the other (.57% better performance for the dynamic productivity model over the constant productivity model and .26% over the Ebert model). This is due to the particular level of cum_o (selected on purpose to show the effect).

To clarify the above point, note that improvement curves approach a horizontal line as the cumulative unit number increases; therefore, average productivities indicated by the Ebert model become closer to the overall average productivity computed for a range of large cumulative output quantities. Thus, for these ranges the Ebert model would approximately duplicate the results of the constant productivity model, resulting in a low relative performance. The best relative performance of the Ebert model over the constant productivity model is expected in the early stages of production (low values of cum_o). On the other hand, the performance of the dynamic productivity model indicates that the disruption effects are insignificant in the early stages of production. This is expectable since in the early stages productivities of the old and new workers do not differ much. A manpower addition, for example, does not degrade the overall productivity, significantly. However in the larger production sequences, due to the larger difference between the productivity levels of the old and the new employees, an increase in the manpower level would have a greater impact on the overall average productivity. Therefore, in the early stages of production the performance of the dynamic productivity model is close to the performance of the Ebert model (both models performing better than the constant productivity model). As the production sequence becomes larger, the relative performances of the Ebert and the dynamic productivity model over the constant productivity model become smaller. This decreasing pattern in the relative performances continues until the effect of manpower disruptions starts to become significant (due to the larger production sequences). From this point on, the relative performance of the dynamic productivity model over the other two models follows an increasing pattern.

Tables IX through XIX show the results of implementing the three models for initial cumulative units of 0, 10,000, 25,000, and 50,000 (the cost analysis of the optimization results of the constant productivity and the Ebert models are not included in these tables, because as mentioned before, these analyses are not relevant for the current purpose). In these tables notice that:

1. At cum_o=0 the dynamic productivity model performs exactly similar to the Ebert model (Table X).

2. The deviations of the average productivities projected by the Ebert model from the overall average productivity, projected by the constant productivity model, become smaller as cum_o increases.

3. The relative performance (on the basis of the total cost) of Ebert model over the constant productivity model decays as cum_o increases.

4. More significant disruptions occur as cum_o increases.

5. The deviations of the average productivities projected by the dynamic productivity model from the average productivities projected by the other two models increase significantly as cum_o increases.

6. The relative performance of the dynamic productivity model over the other two models improves as cum_o increases.

Figures 29 through 32 demonstrate those conditions mentioned in (1), (2) and (5) above. Table XX contains a summary analysis of the relative performances. Figure 33 depicts the pattern of the changes in

TABLE IX

RESULTS OF IMPLEMENTATION OF THE CONSTANT PRODUCTIVITY MODEL CUMØ = 0

A. DECISIONS AND PROJECTIONS

Mo.	Demand	Work Force	Prod.	Inventory	Proj. Avg Prod.	Act Avg Prod.	INDEX	ICHK
1	430	86.4	441.8	274.8	2.094	0.717	1	0
2	447	90.6	432.0	259.7	2.094	1.680	0	0
3	440	93.6	413.8	233.5	2.094	2.190	0	0
4	316	96.3	382.6	300.1	2.094	2.579	0	0
5	397	97.8	377.8	281.0	2.094	2.904	0	0
6	375	98.6	363.9	269.9	2.094	3.190	0	0
7	292	99.1	345.0	322.9	2.094	3.442	0	0
8	458	99.7	348.1	213.0	2.094	3.673	0	0
9	400	99.3	302.7	115.7	2.094	3.878	0	0
10	350	99.0	218.4	-15.9	2.094	4.025	0	0

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	29,377.96	1,878.99	11,945.46	168.92	43,371.34
2	30,808.03	1,137.54	8,396.70	299.49	40,641.76
3	31,837.34	589.32	4,352.74	617.00	37,396.40
4	32,727.39	440.64	-900.99	32.58	32,299.62
5	33,259.46	157.47	-2,155.23	125.72	31,387.41
6	33,522.06	38.36	-4,111.43	207.21	29,656.19
7	33,692.18	16.10	-6,395.21	0.70	27,313.77
8	33,886.21	20.94	-6,295.07	944.21	28,556.29
9	33,770.78	7.41	-10,614.60	3,441.85	26,605.44
10	33,648.67	8.29	-16,603.82	9,307.10	26,360.24
					328,804.92

TABLE X

RESULTS OF OPTIMIZATION (AND IMPLEMENTATION) OF THE EBERT AND THE DYNAMIC PRODUCTIVITY MODELS CUMØ = 0

A. DECISIONS AND PROJECTIONS

Mo.	Demand	Work Force	Prod.	Inventory	Avg Prod.	INDEX	ICHK
1	430	84.1	416.7	349.7	0.698	0	0
2	447	86.9	426.8	229.5	1.642	0	0
3	440	88.7	412.0	201.5	2.156	0	0
4	316	89.5	383.9	269.4	2.551	0	0
5	397	89.5	381.9	254.3	2.880	0	0
6	375	88.9	371.5	250.8	3.173	0	0
7	292	87.8	357.9	316.7	3.434	0	0
8	458	86.6	370.0	228.7	3.676	0	0
9	400	85.2	344.9	173.6	3.900	0	0
10	350	84.2	297.4	120.9	4.092	0	0

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	28,599.20	624.04	23,333.75	407.61	53,964.60
2	28,550.70	503.58	13,567.30	675.82	44,297.41
3	30,164.71	209.70	5,907.27	1,158.31	37,439.99
4	30,431.46	39.58	-656.85	211.41	30,025.60
5	30,434.02	0.00	-2,521.15	356.35	28,269.23
6	30,212.87	27.21	-4,342.49	394.90	26,292.49
7	29,847.56	74.23	-5,708.97	0.91	24,213.73
8	29,432.74	94.71	-4,842.77	687.68	25,373.37
9	28,971.41	118.38	-6,255.48	1,769.00	24,603.31
10	28,615.26	70.56	-7,981.78	3,268.91	23,972.94
					317,452.67

TABLE XI

RESULTS OF IMPLEMENTATION OF THE CONSTANT PRODUCTIVITY MODEL CUMØ = 10,000

A. DECISIONS AND PROJECTIONS

Mo.	Demand	Work Force	Prod.	Inventory	Proj. Avg Prod.	Act Avg Prod.	INDEX	ICHK
1	430	71.9	502.9	335.9	7.670	6.038	1	1
2	447	64.3	451.7	340.6	7.670	7.172	1	1
3	440	58.3	413.6	314.3	7.670	6.026	1	1
4	316	53.4	374.2	372.5	7.670	5.977	1	1
5	397	49.9	368.8	344.3	7.670	6.064	1	1
6	375	47.2	357.9	327.2	7.670	6.168	1	1
7	292	45.0	344.2	379.4	7.670	6.755	0	1
8	458	43.3	355.6	277.0	7.670	7.019	0	1
9	400	41.5	324.0	201.0	7.670	7.222	0	1
10	350	40.0	263.5	114.5	7.670	7.375	0	1

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	24,432.25	5,372.10	6,510.09	20.91	36,335.36
2	21,871.78	3,646.65	5,650.17	35.16	31,203.76
3	19,805.27	2,375.35	5,593.02	2.72	27,776.36
4	18,163.59	1,499.10	4,751.26	227.24	24.641.19
5	16,965.79	798.04	5,740.33	48.79	23,552.94
6	16,026.58	480.27	5,969.54	4.31	22,490.70
7	15,294.90	305.97	5,304.64	291.01	21,196.53
8	14,716.01	186.40	6,580.97	152.62	21,636.00
9	14,101.31	210.17	5,053.78	1,168.64	20,533.91
10	13,594.87	142.67	2,451.68	3,485.48	19,674.70
					249,041.43

TABLE XII

RESULTS OF IMPLEMENTATION OF THE EBERT MODEL CUMØ = 10,000

A. DECISIONS AND PROJECTIONS

Mo.	Demand	Work Force	Prod.	Inventory	Proj. Avg Prod.	Act Avg Prod.	INDEX	ICHK
1	430	73.0	487.2	320.2	6.994	6.129	1	1
2	447	66.0	449.4	322.6	7.157	6.237	1	1
3	440	59.9	416.4	299.0	7.304	6.041	1	1
4	316	54.8	378.2	361.2	7.437	5.984	1	1
5	397	50.8	372.3	336.5	7.561	6.023	1	1
6	375	47.6	360.5	322.1	7.680	6.083	1	1
7	292	45.0	346.2	376.3	7.793	6.157	1	1
8	458	42.9	357.8	276.1	7.904	6.761	0	1
9	400	40.8	327.4	203.4	8.010	7.026	0	1
10	350	39.1	270.0	123.4	8.102	7.211	0	1

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	24,822.75	4,106.87	4,742.03	0.00	33,671.66
2	22,435.07	3,171.08	4,754.88	0.54	30,361.57
3	20,376.17	2,357.87	5,035.90	36.32	27,806.27
4	18,637.76	1,680.98	4,464.22	140.21	24,923.16
5	17,287.19	1,014.57	5,647.94	22.57	23,972.28
6	16,189.43	670.30	6,083.38	0.35	22,943.47
7	15,289.45	450.53	6,050.41	261.08	22,051.47
8	14,580.37	279.67	7,190.29	159.28	22,209.61
9	13,869.36	281.20	5,630.80	1,121.04	20,902.39
10	13,309.08	174.61	2,852.99	3,188.83	19,525.51
					248,367.38

TABLE XIII

RESULTS OF OPTIMIZATION (AND IMPLEMENTATION) OF THE DYNAMIC PRODUCTIVITY MODEL CUMØ = 10,000

A. DECISIONS AND PROJECTIONS

Mo.	Demand	Work Force	Prod.	Inventory	Avg Prod.	INDEX	ICHX
1	430	74.8	482.1	315.1	6.301	1	0
2	447	69.1	452.8	320.8	6.414	1	0
3	440	63.8	423.3	304.1	6.494	1	1
4	316	58.6	368.2	356.3	6.246	1	1
5	397	54.1	350.8	310.1	6.147	1	1
6	375	51.4	365.0	300.1	6.863	0	1
7	292	48.9	353.2	361.3	7.137	0	1
8	458	46.8	363.9	267.2	7.343	0	1
9	400	44.6	333.8	201.0	7.514	0	1
10	350	43.0	278.8	129.9	7.650	0	1

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	25,433.55	2,468.06	3,684.13	2.01	31,487.75
2	23,504.36	2,070.15	3,773.86	0.06	29,348.43
3	21,690.38	1,830.30	3,760.75	20.87	27,302.29
4	19,927.43	1,728.74	2,383.15	108.67	24,147.99
5	18,400.41	1,297.01	2,817.83	8.13	22,523.38
6	17,480.71	470.49	4,270.56	32.74	22,254.50
7	16,628.63	403.85	4,346.54	140.87	21,519.88
8	15,901.07	294.44	5,573.33	229.89	21,998.73
9	15,173.72	294.26	4,551.68	1,167.55	21,187.21
10	14,613.89	174.32	2,697.87	2,982.21	20,468.30
					242,338.47

TABLE XIV

RESULTS OF IMPLEMENTATION OF THE CONSTANT PRODUCTIVITY MODEL CUMØ = 25,000

A. DECISIONS AND PROJECTIONS

Mo.	Demand	Work Force	Prod.	Inventory	Proj. Avg Prod.	Act Avg Prod.	INDEX	ICHK
1	430	62.2	571.8	404.8	11.575	7.376	1	1
2	447	49.1	455.6	413.4	11.575	7.276	1	1
3	440	40.6	395.3	268.6	11.575	6.792	1	1
4	316	35.2	352.7	405.4	11.575	6.670	1	1
5	397	32.2	353.3	361.6	11.575	6.984	1	1
6	375	30.3	349.0	335.6	11.575	7.325	1	1
7	292	29.0	340.1	383.7	11.575	8.288	0	1
8	458	27.9	354.4	280.0	11.575	8.731	0	1
9	400	26.3	218.4	198.4	11.575	7.921	1	1
10	350	24.5	248.0	96.4	11.575	7.474	1	1

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	21,135.29	22,816.69	14,371.92	592.78	58,916.68
2	16,695.46	10,964.42	11,272.21	719.12	39,651.20
3	13,792.03	4,688.93	11,707.75	195.12	30.383.84
4	11,953.65	1,879.86	10,976.39	601.25	25,411.15
5	10,935.04	577.13	12,359.42	142.96	24,014.54
6	10,305.21	220.65	12,575.73	20.11	23,121.69
7	9,853.95	113.26	11,262.39	334.56	21,564.17
8	9,502.32	68.78	12,726.20	131.68	22,428.98
9	8,928.80	182.96	11,355.86	1,219.70	21,687.32
10	8,335.07	196.08	6,647.61	4,124.52	19,303.28
					286,482.85

TABLE XV

RESULTS OF IMPLEMENTATION OF THE
EBERT MODEL CUMØ = 25,000

A. DECISIONS AND PROJECTIONS

Mo.	Demand	Work Force	Prod.	Inventory	Proj. Avg Prod.	Act Avg Prod.	INDEX	ICHK
1	430	62.9	561.1	394.1	11.134	7.450	1	1
2	447	50.1	456.3	403.5	11.248	7.454	1	1
3	440	41.5	398.7	362.2	11.344	6.846	1	1
4	316	35.8	355.8	402.0	11.428	6.679	1	1
5	397	32.6	355.1	360.1	11.506	6.933	1	1
6	375	30.5	349.9	335.0	11.583	7.222	1	1
7	292	28.9	340.5	282.6	11.658	7.460	1	1
8	458	27.7	354.8	280.4	11.734	8.349	0	1
9	400	25.9	319.0	199.4	11.806	7.619	1	1
10	350	24.1	249.4	98.8	11.867	7.225	1	1

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	21,386.38	21,062.74	12,767.28	453.46	55,669.85
2	17,027.49	10,568.30	10,669.46	574.76	38,840.00
3	14,094.58	4,774.87	11,401.18	146.79	30,420.42
4	12,183.20	2,038.50	10,859.85	554.21	26,635.76
5	11,073.90	684.46	12,376.01	132.68	24,267.04
6	10,359.17	284.14	12,728.63	18.65	23,390.59
7	9,837.07	151.62	12,413.87	333.23	22,735.79
8	9,431.35	91.56	13,407.38	129.61	23,059.90
9	8,817.68	209.47	11,997.40	1,199.85	22,224.40
10	8,199.44	212.60	7,118.29	4,038.33	19,568.66
					285,812.41

TABLE XVI

RESULTS OF IMPLEMENTATION OF THE DYNAMIC PRODUCTIVITY MODEL CUMØ = 25,000

A. DECISIONS AND PROJECTIONS

Mo.	Demand	Work Force	Prod.	Inventory	Avg Prod.	INDEX	ICHX
1	430	69.1	518.2	351.2	8.321	1	1
2	447	59.7	460.0	364.2	8.401	1	1
3	440	53.2	404.3	328.5	7.970	1	1
4	316	48.1	357.1	369.6	7.711	1	1
5	397	44.8	361.1	333.7	7.857	1	1
6	375	41.8	339.3	297.9	7.785	1	1
7	292	39.7	356.5	362.5	8.876	0	1
8	458	37.9	374.8	279.3	9.323	0	1
9	400	36.0	351.5	230.8	9.644	0	1
10	350	32.6	207.7	88.5	7.534	1	1

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	23,506.68	9,048.55	7,757.18	80.44	40.392.85
2	20,304.08	5,705.02	7,116.98	161.01	33,287.09
3	18,073.53	2,767.45	5,837.42	5.92	26,684.31
4	16,358.77	1,635.52	4,804.37	203.10	23,001.76
5	15,228.95	710.03	5,916.83	15.44	21,871.24
6	14,206.79	581.15	5,667.29	40.18	20,495.41
7	13,497.54	280.59	7,105.29	148.74	21,029.15
8	12,800.25	197.77	8,617.19	136.88	21,852.09
9	12,255.48	231.24	7,872.21	656.56	21,015.49
10	11,099.36	743.46	1,752.91	4,422.16	18,017.89
					247,647.29

TABLE XVII

RESULTS OF IMPLEMENTATION OF THE CONSTANT PRODUCTIVITY MODEL CUMØ = 50,000

A. DECISIONS AND PROJECTIONS

Mo.	Demand	Work Force	Prod.	Inventory	Proj. Avg Prod.	Act Avg Prod.	INDEX	ICHK
1	430	53.3	646.0	479.0	16.070	7.839	1	1
2	447	36.7	444.0	475.9	16.070	7.436	1	1
3	440	27.8	365.0	401.0	16.070	6.586	1	1
4	316	23.4	327.7	412.7	16.070	6.695	1	1
5	397	21.8	341.1	356.8	16.070	7.654	1	1
6	375	21.2	346.4	328.2	16.070	9.091	0	1
7	292	20.8	342.9	379.1	16.070	9.801	0	1
8	458	20.4	360.6	281.7	16.070	10.343	0	1
9	400	18.7	318.6	200.4	16.070	8.763	1	1
10	350	16.7	236.4	86.4	16.070	7.529	1	1

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	18,134.38	49,207.09	28,471.61	2,085.05	97,898.13
2	12,472.98	17,827.95	18,282.65	2,006.42	50,589.99
3	9,441.21	5,112.64	17,518.79	540.74	32,613.38
4	7,945.93	1,243.65	16,079.77	708.87	25,978.22
5	7,419.36	154.23	17,389.80	111.57	25,074.95
6	7,211.87	23.95	16,496.06	5.57	23,737.44
7	7,065.68	11.89	15,596.20	288.55	22,962.31
8	6,923.17	11.30	17,239.01	120.88	24,294.36
9	6,369.73	170.37	15,821.66	1,180.94	23,542.69
10	5,673.64	269.52	9,866.69	4,488.84	20,298.69
					346,990.17

TABLE XVIII

RESULTS OF IMPLEMENTATION OF THE
EBERT MODEL CUMØ = 50,000

A. DECISIONS AND PROJECTIONS

Mo.	Demand	Work Force	Prod.	Inventory	Proj. Avg Prod.	Act Avg Prod.	INDEX	ICHK
1	430	53.6	641.0	474.0	15.866	7.859	1	1
2	447	37.0	445.3	472.3	15.954	7.467	1	1
3	440	28.0	366.9	399.2	16.020	6.597	1	1
4	316	23.4	328.9	412.1	16.076	6.671	1	1
5	397	21.8	341.5	356.6	16.129	7.585	1	1
6	375.0	21.1	346.6	328.2	16.184	9.051	0	1
7	292	20.6	343.0	379.2	16.239	9.765	0	1
8	458	20.1	360.9	282.1	16.295	10.310	0	1
9	400	18.5	318.8	200.9	16.348	8.676	1	1
10	350	16.4	236.2	87.1	16.392	7.427	1	1

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	18,219.73	48,318.99	27,427.99	1,956.42	95,922.24
2	12,566.46	17.776.77	18,147.06	1,913.28	50,403.58
3	9,505.02	5,213.20	17,588.35	517.31	32,823.88
4	7,970.94	1,309.04	16,198.96	699.06	26,178.00
5	7,409.00	175.64	17,577.30	110.49	25,272.43
6	7,172.44	31.13	16,662.57	5.52	23,871.65
7	7,003.37	15.90	15,799.25	289.04	23,107.56
8	6,843.89	14.15	17,522.07	118.77	24,498.88
9	6,273.96	180.67	16,176.41	1,170.78	23,801.83
10	5,563.59	280.69	10,130.12	4,474.05	20,448.45
					346,328.50

TABLE XIX

RESULTS OF OPTIMIZATION (AND IMPLEMENTATION) OF THE DYNAMIC PRODUCTIVITY MODEL CUMØ = 50,000

A. DECISIONS AND PROJECTIONS

Mo.	Demand	Work Force	Prod.	Inventory	Avg Prod.	INDEX	ICHK
1	430	64.5	536.0	369.0	9.490	1	1
2	447	52.8	463.7	385.7	9.671	1	1
3	440	45.6	401.3	346.9	8.986	1	1
4	316	40.1	345.8	376.8	8.428	1	1
5	397	35.7	323.4	303.2	8.103	1	1
6	375	33.9	364.5	292.7	10.114	0	1
7	292	32.2	360.9	361.7	10.914	0	1
8	458	30.6	377.6	281.3	11.475	0	1
9	400	29.1	356.0	237.3	11.907	0	1
10	350	25.9	181.1	68.4	8.179	1	1

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	21,940.93	17,437.56	10,476.74	197.94	50,053.17
2	17,965.68	8,789.83	9,340.49	355,74	36,451.74
3	15,506.75	3,363.14	7,743.33	59.82	26,673.04
4	13,622.44	1,974.96	6,461.92	265.88	22,325.20
5	12,128.83	1,240.88	6,772.39	23.25	20,165.34
6	11,523.24	203.99	9,234.70	61.32	21,023.25
7	10,947.14	184.61	9,450.86	143.31	20,725.91
8	10,417.97	155.75	10,859.05	123.61	21,556.39
9	9,897.29	150.80	10,067.02	563.84	20,678.95
10	8,789.39	682.74	2,191.35	5,222.24	16,885.71
					256,538.71

TABLE XX

PERFORMANCE COMPARISONS

cum_o	Total Cost			Relative Performance		
	C	E	D	E/C	D/C	D/E
0	328,804	317,452	317,452	3.5%	3.5%	0.0%
5,000	244,860	244,097	243,461	.31%	.57%	.26%
10,000	249,041	248,367	242,338	.27%	2.6%	2.33%
25,000	286.452	285,812	247,647	.23%	13.5%	13.27%
50,000	346,990	346,328	256,538	.19%	26%	25.81%

C = Constant Productivity Model
E = Ebert Model
D = Dynamic Productivity Model

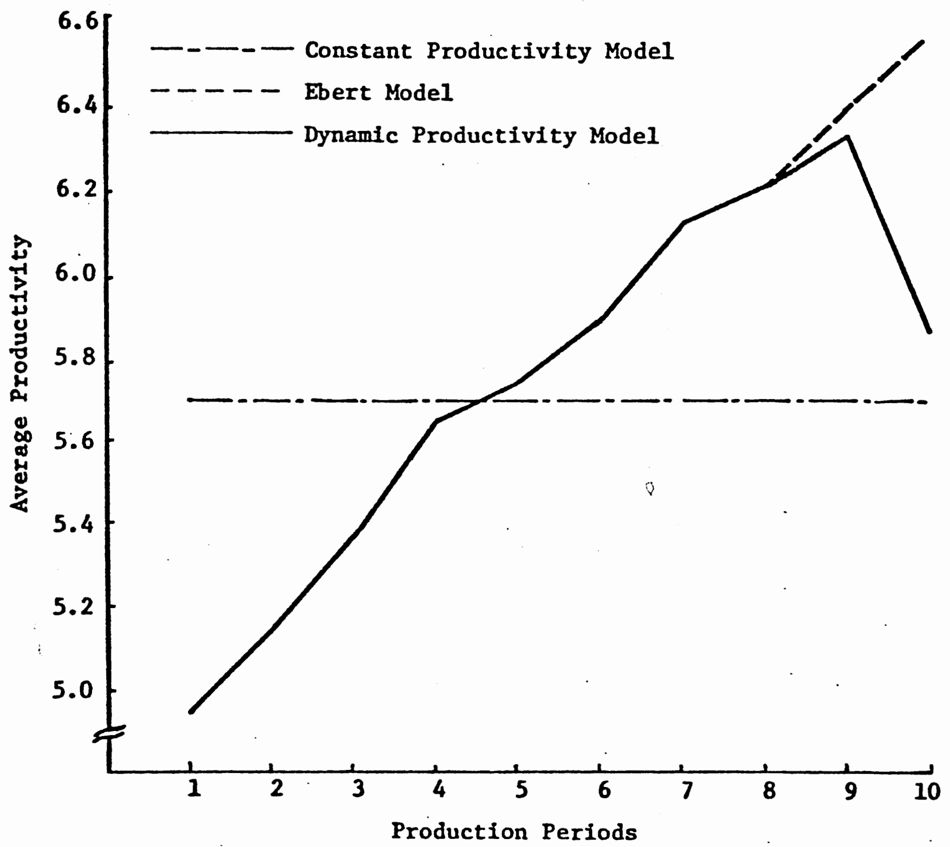


Figure 29. Average Productivity Patterns Over 10 Month Plannig Horizon (cum_o=5000)

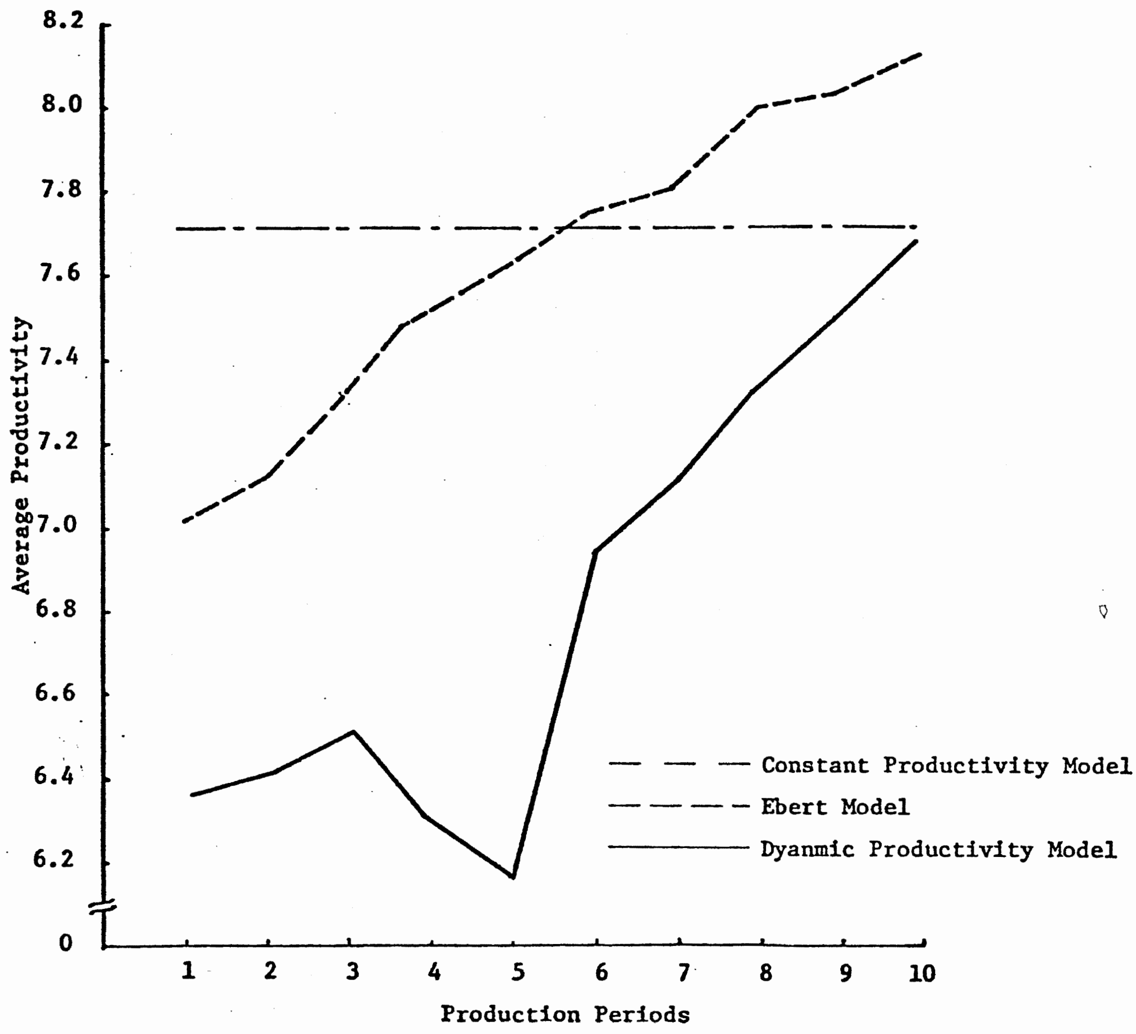


Figure 30. Average Productivity Patterns Over 10 Month Planning Horizon (cum_6=10,000)

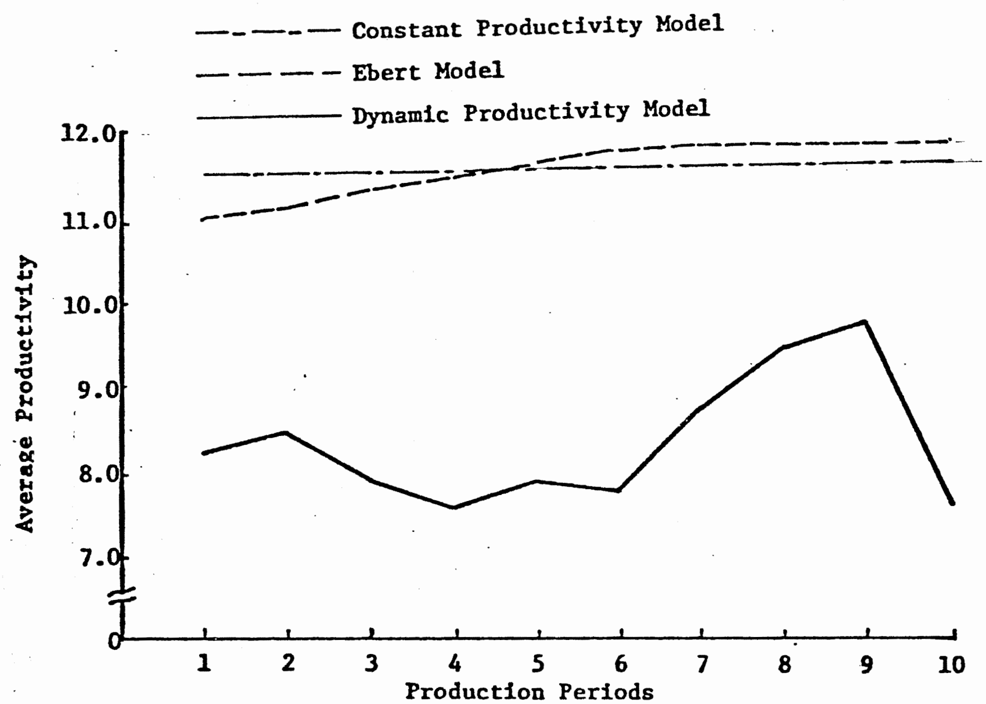


Figure 31. Average Productivity Patterns Over 10 Month Planning Horizon (cum_o=25,000)

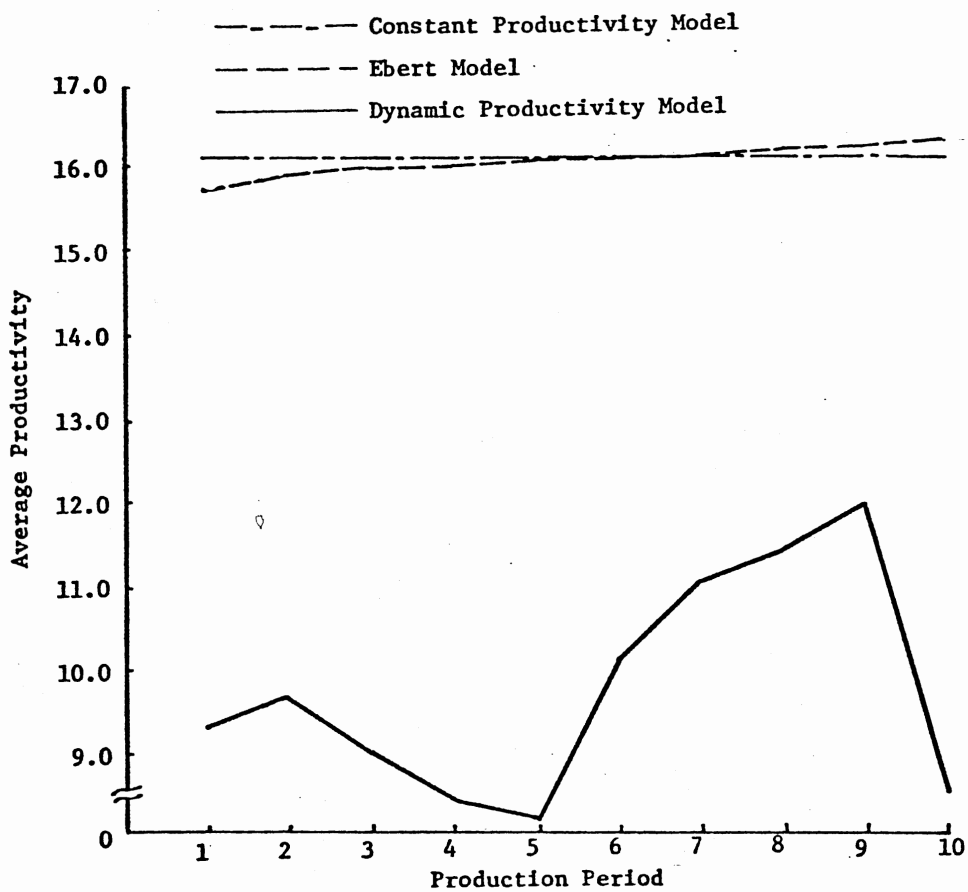


Figure 32. Average Productivity Patterns Over 10 Month Planning Horizon (cum_o=50,000)

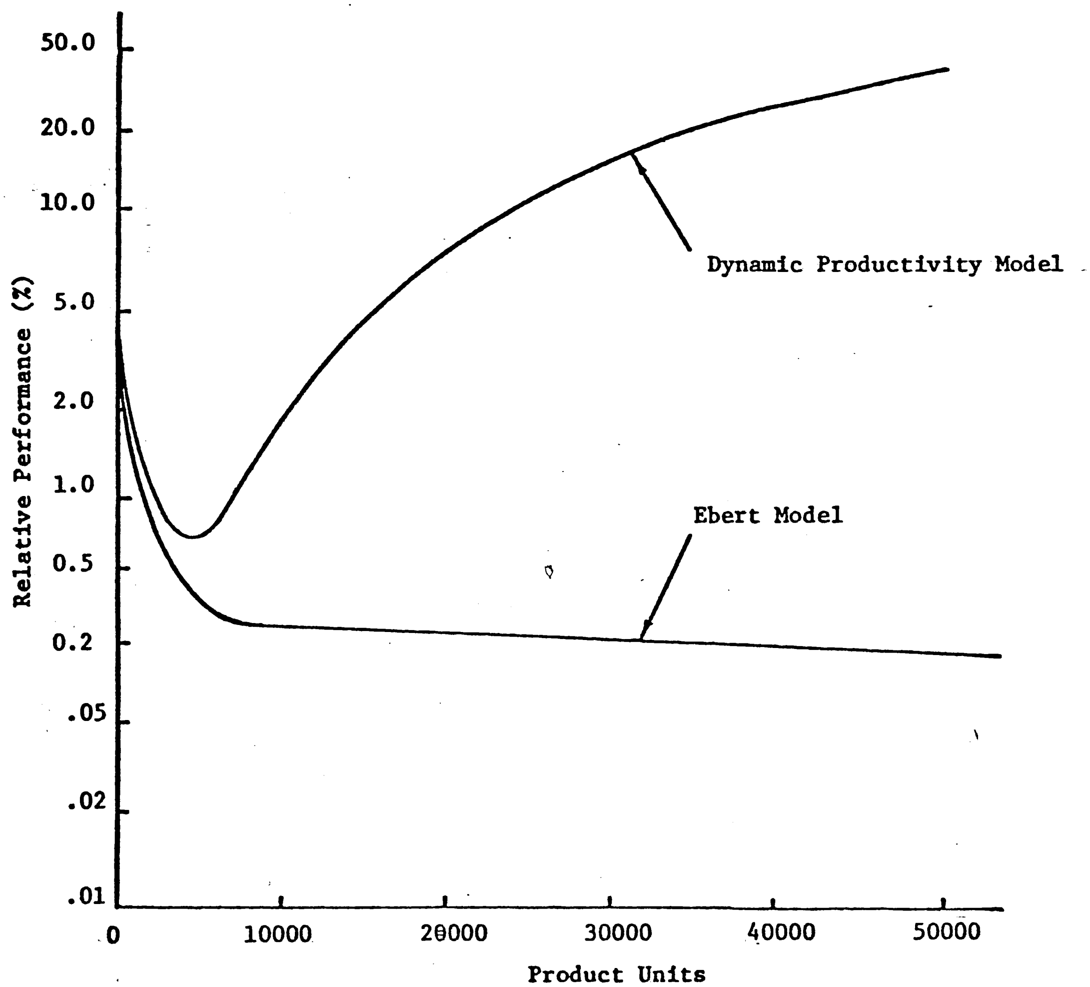


Figure 33. Patterns of relative Performances of the Ebert and the Dynamic Productivity Models Over the Constant Productivity Model

the relative performances of the Ebert and the dynamic productivity models over the constant productivity model as a function of changes in cum_o. Notice in this figure that the dynamic productivity and the Ebert models perform slightly better than the constant productivity model only during the early stages of production. At cum_o=5000 the relative performance of the dynamic productivity model starts to increase sharply, while the relative performance of the Ebert model continues to decline.

It should be noted that the cumulative unit number is not the only factor affecting the relative performance of the new model. Considering that the impact of this model is due to the incorporation of the significant disruptions resulting from manpower transactions, it becomes clear that any factor affecting the magnitude of fluctuations in the workforce level projected by the aggregate plan is an important factor in this regard. To demonstrate this point, two examples are given as follow:

One alternative strategy to absorb fluctuations in the demand level is the utilization of inventories. If the inventory related costs increase, or storage space becomes a limitation, then less utilization of inventories and more utilization of other strategies including the variations in the manpower level would be justified. A numerical example shows the impact of this point: Assume that in the paint factory model the storage space is limited by the maximum capacity of 150 units. This constraint is incorporated into the objective function of the SDR model. Every time the inventory level exceeds 150, a penalty of 100,000 is added to the total cost function. The results of optimization for the constant productivity and the dynamic productivity models are given in Table XXI and XXII, respectively. The models are optimized with respect to an initial cumulative unit number 50,000. Notice the increased magnitudes of fluctuations in the workforce levels in both models (compare Table XVII with Table XXI and Table XIX with Table XXII). In general, the number of the significant disruptions and the magnitude of each disruption (related to the magnitude of change in the workforce level) have increased. Also note that, although the number of significant disruptions is reduced in the dynamic productivity model, the magnitudes of most disruptions have become larger. The results indicate a better relative performance of new model over the constant productivity model (from 26% to 38%).

As in the second example, consider that changes in the cost parameter can also affect the fluctuations in the workforce level. For instance, a decrease in the cost of hiring and firing or an increase in the overtime cost would result in an increase in the magnitude of workforce fluctuation. To demonstrate this point numerically, the parameter C_2 (related to the hiring and firing cost) is reduced by one half and the parameters C_3 (related to the overtime cost) is increased from .2 to 5 in the original paint factory cost model. The Ebert model and the dynamic productivity model are optimized under the new situation, and the results are presented in Tables XXIII and XXIV. Notice the great differences in performance: a 89% better overall result is obtained by using the new model!

In comparison of Table XXIII with Table XXIV one should note that although the magnitude of manpower changes in the new model is almost as large as the magnitude of changes projected by the Ebert model (and for the first period is even larger), the new model has scheduled the

TABLE XXI

RESULTS OF IMPLEMENTATION OF THE CONSTANT PRODUCTIVITY MODEL (LIMITED INVENTORY)

A. DECISIONS AND PROJECTIONS

Mo.	Demand	Work Force	Prod.	Inventory	Proj. Avg Prod.	Act Avg Prod.	INDEX	ICHK
1	430	46.5	360.1	193.1	16.070	5.800	1	1
2	447	31.7	338.0	84.1	16.070	6.806	1	1
3	440	26.0	338.9	-17.0	16.070	7.026	1	1
4	316	24.7	343.0	10.0	16.070	8.506	1	1
5	397	26.5	532.0	145.0	16.070	8.359	1	1
6	375	23.5	340.0	110.0	16.070	7.785	1	1
7	292	22.8	332.0	150.0	16.070	9.517	0	1
8	458	24.4	456.8	148.8	16.070	8.422	1	1
9	400	23.1	401.2	150.0	16.070	8.899	1	1
10	350	20.8	298.3	98.3	16.070	7.864	1	1

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	15,798.35	76,685.21	7,020.78	1,328.97	200,833.30
2	10,782.69	13,992.97	11,382.26	4,590.56	40,748.46
3	8,839.45	2,100.41	14,926.03	9,369.89	35,235.77
4	8,392.43	111.15	14,164.94	7,928.66	30,597.17
5	9,013.82	214.78	39,058.89	2,526.79	50,814.27
6	7,990.76	582.18	15,737.24	3,638.29	27,948.47
7	7,764.24	28.54	13,211.43	2,384.28	23,388.49
8	8,287.70	152.41	29.193.49	2,417.51	40,051.11
9	7,860.34	101.59	21,684.74	2,384.33	32,030.99
10	7,084.71	334.63	13,029.09	4,056.09	24,504.51
					506,152.56

TABLE XXII

RESULTS OF OPTIMIZATION (AND IMPLEMENTATION) OF THE DYNAMIC PRODUCTIVITY (LIMITED INVENTORY)

A. DECISIONS AND PROJECTIONS

Mo.	Demand	Work Force	Prod.	Inventory	Avg Prod.	INDEX	ICHK
1	430	59.9	260.1	93.1	7.037	1	1
2	447	48.9	438.0	84.1	9.389	1	1
3	440	41.1	338.9	-17.0	8.258	1	1
4	316	36.2	343.0	10.0	8.154	1	1
5	397	34.4	532.0	145.0	10.545	0	1
6	375	32.7	340.0	110.0	11.450	0	1
7	393	31.3	332.0	150.0	11.918	0	1
8	458	32.0	458.0	150.0	12.348	0	1
9	400	30.7	400.0	150.0	12.729	0	1
10	350	28.0	250.9	50.9	9.660	1	1

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	20,351.19	28,745.31	1,689.42	4,248.15	55,034.06
2	16,612.82	7,773.54	8,828.40	4,590.56	37,805.31
3	13,985.76	3,838.76	5,691.96	9,369.89	32,986.37
4	12,300.25	1,580.22	7,856.67	7,928.66	29,665.79
5	11,685.34	210.32	23,333.12	2,526.79	37,755.57
6	11,101.10	189.86	8,462.55	3,638.29	23,391.80
7	10,650.98	112.70	8,537.47	2,384.28	21,685.43
8	10,877.85	28.63	15,251.81	2,384.39	28,542.68
9	10,438.70	107.27	11,869.81	2,384.29	24,800.06
10	9,529.97	459.33	5,047.39	5,975.67	21,012.36
					312,679.46

TABLE XXIII

RESULTS OF IMPLEMENTATION OF THE EBERT MODEL (MODIFIED COST PARAMETERS)

A. DECISIONS AND PROJECTIONS

Mo.	Demand	Work Force	Prod.	Inventory	Proj. Avg Prod.	Act Avg Prod.	INDEX	ICHK
1	430	43.7	685.5	518.5	15.870	6.780	1	1
2	447	26.3	415.2	486.7	15.959	6.352	1	1
3	440	20.9	333.1	379.9	16.019	6.486	1	1
4	316	20.0	321.4	385.3	16.072	8.907	0	1
5	397	21.1	340.9	329.1	16.125	8.215	1	1
6	375	22.0	356.0	310.2	16.180	9.741	0	1
7	292	22.1	359.7	377.9	16.237	10.419	0	1
8	458	21.9	358.4	278.3	16.294	10.924	0	1
9	400	19.0	310.8	189.0	16.347	8.029	1	1
10	350	15.1	245.8	84.8	16.391	6.121	1	1

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	14,844.73	4,5032.74	781,408.92	3,252.05	844,538.45
2	8,939.30	8,744.24	321,814.00	2,292.88	342,790.42
3	7,099.46	945.82	206,629.20	295.61	214,970.08
4	6,806.49	23.98	113,255.31	351.70	120,437.49
5	7,174.83	37,91	151,776.69	6.90	158,996.32
6	7,463.13	23.22	113,148.37	8.00	120,642.73
7	7,518.45	0.86	95,807.01	276.12	103,602.44
8	7,436.25	1.89	83,605.19	143.60	91,186.93
9	6,445.75	274.13	136,278.35	1,414.72	144,412.95
10	5,123.02	488.86	126,331.39	4,562.86	136,396.14
					2,277,973.94

TABLE XXIV

RESULTS OF OPTIMIZATION (AND IMPLEMENTATION) OF THE DYNAMIC PRODUCTIVITY MODEL (MODIFIED COST PARAMETERS)

A. DECISIONS AND PROJECTIONS

Mo.	Demand	Work Force	Prod.	Inventory	Avg Prod.	INDEX	ICHK
1	430	61.6	550.3	383.3	8.916	1	1
2	447	47.3	394.6	330.9	8.357	1	1
3	440	39.5	305.6	196.5	7.663	1	1
4	316	37.6	389.1	269.5	10.369	0	1
5	397	35.7	405.1	277.7	11.399	0	1
6	375	33.9	407.6	310.3	12.059	0	1
7	292	32.2	401.6	419.9	12.537	0	1
8	458	33.0	425.9	387.7	12.921	0	1
9	400	27.4	204.4	192.1	7.475	1	1
10	350	26.0	267.1	109.2	10.300	0	1

B. COST ANALYSIS OF DECISIONS AND PROJECTIONS ($)

Mo.	Payroll	Hiring & Firing	Overtime	Inventory	Total
1	20,956.18	12,111.57	10,858.90	330.63	44,257.29
2	16,074.26	6,659.27	6,919.22	9.78	29,662.53
3	13,446.86	1,928.84	4,564.37	1,258.76	21,198.83
4	12,774.56	126.29	9,363.49	210.09	22,474.43
5	12,136.51	113,75	10,728.13	147.79	23.126.18
6	11,530.67	102.56	11,349.38	7.77	22,990.37
7	10,958.53	91.46	11,535.20	822.99	23,408.18
8	11,219.21	18.99	12,532.65	378.46	24,149.31
9	9,304.65	1,024.20	2,773.95	1,349.54	14,452.34
10	8,849.85	57.79	6,365.43	3,666.66	18,939.74
					244,659.19

right timing for these changes. As the result of the arrangement, the new model incurs lower overtime costs from which the Ebert model has suffered dramatically.

As the results of the analyses show, the effect of the compounded disruption is present in most situations. Most disruption effects are carried over a number of succeeding periods (INDEX and ICHK both take values of one in most periods). In order to evaluate the significance of the compounded disruption effects, EPSY is given a high value (100%). This suppresses the effect of compounded disruption. The problems are optimized for an initial cumulative unit number of 50,000 for the original cost structure. Comparing the new results with the ones presented in Tables XVII to XIX it has been noticed that on the average a 5% reduction in the total cost has resulted in the constant productivity, the Ebert, and the dynamic productivity models. This indicates that the compounded disruption effect, although not considered in the literature of learning of curve, has significant effect on the production and workforce planning problems.

Remarks

The foregoing analyses have indicated that the disruption effects of the manpower transactions on the average productivity significantly affect the performance of the aggregate planning models. This impact becomes more severe as the firm approaches its steady production state. The nature of the cost structure of the firm is also an important factor reflecting the severity of such effects. The compounded disruption phenomenon is important and should be considered in production and workforce planning problems, in general.

Analysis of Model II

As described in detail in Chapter VI, Model II incorporates new assumptions and requires new objective function structures. Consequently, reliable evaluation of the relative performance of this model requires an existing constant productivity model with similar objective function structure to serve as a basis for comparison purposes.

Unfortunately, the original HMMS model and other existing aggregate planning models do not incorporate objective functions with structures equivalent to the one used in the new model. Aside from the assumptions regarding the dynamic productivity phenomenon, the new model differs from most existing models in the way of estimating the levels for dependent variables such as workforce levels utilized on regular time, overtime, idle time, subcontracting level, etc. While the new model computes these level by incorporating proper considerations regarding the production capabilities of the experience classes involved in different periods, the existing models attempt to approximate these levels by fitting a polynominal which would hopefully express the levels for the dependent variables as functions of the independent decision variables. For example, in the HMMS model, the overtime cost is approximated by using a quadratic function of the following form:

$$\text{Overtime cost in period } t = C_3(P_t - C_4W_t)^2 + C_5P_t - C_6W_t$$

which expresses the cost in terms of production rates and workforce levels. As Tables IX and X show, this approximation is not always successful (notice the negative overtime indicated in most periods).

For the analysis purposes, the above segment of the cost function in the HMMS model is changed to a linear function to provide a cost function somewhat equivalent to the one considered by the new model. Also, the following simplifying assumptions are made:

— Idle workers are assumed to receive the same payments as those assigned on the regular time work. Therefore, this variable is not included in the cost function.

— The length of the overtime work is assumed to be 50% of the length of the regular time work.

— The hourly overtime payment is assumed to be twice the amount of regular time payment.

— Subcontracting is assumed to be allowed.

Based on the above assumptions, the overtime portion of the HMMS cost model is converted to the following form which also includes the cost of subcontracting:

$$\text{overtime and subcontracting costs in period } t = 2C_1 \cdot WO_t + C_{10}SB_t$$

where,

$$\left.\begin{aligned} WO_t &= 0 \\ SB_t &= 0 \\ WI_t &= W_t - P_t/C_4 \end{aligned}\right\} \quad \text{if } P_t - C_4W_t \leq 0$$

$$\left.\begin{aligned} WO_t &= (P_t - C_4W_t)/\tfrac{1}{2}C_4 \\ SB_t &= 0 \\ WI_t &= 0 \end{aligned}\right\} \quad \text{if } P_t - C_4W_t > 0 \text{ and } (P_t - C_4W_t) - \tfrac{1}{2}C_4W_t \leq 0$$

$$\left.\begin{aligned} WO_t &= W_t \\ SB_t &= (P_t - C_4W_t) - \tfrac{1}{2}C_4W_t \\ WI_t &= 0 \end{aligned}\right\} \quad \text{if } P_t - C_4W_t > 0 \text{ and } (P_t - C_4W_t) - \tfrac{1}{2}C_4W_t > 4$$

In the above formulation, C_1 is the regular payroll cost per employee; WO_t is the workforce level assigned on overtime work in period t; C_{10} is the cost of subcontracting per product unit; SB_t is the subcontracting level in period t; C_4 is the constant productivity factor (unit output per employee); WI_t is the idle workforce level in period t (not included in the cost function); and W_t and P_t are the workforce level and the production rate scheduled for period t.

The following relationships explain the above formulation:

$C_4 \cdot W_t$ - Production capability of the workforce level during regular time work in period t

$\frac{1}{2}C_4 \cdot W_t$ - Production capability of the workforce level during overtime work in period t

$P_t - C_4 \cdot W_t$ - Production level to be allocated to overtime and possibly subcontracting options in period t

P_t/C_4 - Workforce level required to produce P_t units

Notice that in this formulation, the productivity rate of employees during overtime work is assumed to be one half of their productivities during regular time work. This is because the length of the overtime is assumed to be one half the length of regular time.

An arbitrary set of data which represent a typical short cycle product case is selected. The demand levels assumed in the example problem are listed in the second column of the following tables. The cost parameter C_8 (related to the inventory cost) in the original HMMS model is given a new value of 3200 (due to high demand and production quantities). The cost of subcontracting is assumed to be three dollars per unit. Slopes of 70%, 90% and 80% are assumed for learning, progress and forgetting curves, respectively. A base unit cost of .3 man-month per unit is assumed. The initial inventory level is selected to be 7000 units, and the initial workforce level is 81 men. Initially, a single experience class with productivity measure of 10 units is assumed. The overall average productivity for the constant productivity model is computed over the range of cumulative demand and on the basis of an overall improvement curve with slope of 63% (product of learning and progress slopes). An initial cumulative unit quantity of 1000 units is assumed. The results of optimization and implementation of the constant productivity and the dynamic productivity models are presented in Tables XXV.A and XXV.B respectively. In this particular example, the dynamic productivity model performs almost 30% better than the constant productivity model. Notice the relatively lower workforce levels projected by the constant productivity model. This is due to the fact that this model does not incorporate the effect of disruptions in the overall productivity level. These disruptions are caused by the addition of new employees with low productivity measures. Also, the forgetting effect applied to the idle workers is not considered in the constant productivity model. In general, the constant productivity model assumes relatively higher productivities per employee. Therefore, the workforce level required to meet the production schedule is relatively lower in this model.

Further computer runs for different values of the parameters in the above example have resulted in conclusions similar to most of those indicated by the analyses of Model I. For example, an increase in the level of cum_O is accompanied by an increase in the relative performance of the new model (mainly due to the progress effect). Also, the impact of the new model becomes more significant for sharper slopes of learning, progress and forgetting curves.

It should be noted that in this example the total workforce level in each period is comprised of a number of experience classes each having different productivity levels. However, the detailed information regarding the workforce body is not shown in the table.

TABLE XXV

A. DECISIONS AND PROJECTIONS RESULTING FROM IMPLEMENTATION OF THE CONSTANT PRODUCTIVITY MODEL

Month	Demand	Work Force	Prod	Inven	WORKFORCE Regular Time	WORKFORCE Over-Time	Idle	PRODUCTION Regular Time	PRODUCTION Over-Time	Subcontracting
1	9,000	62	5,292	3,293	62	62	0	2,891	2,252	149
2	13,000	46	12,828	3,121	46	46	0	7,219	4,672	937
3	12,000	33	12,097	3,218	33	6	0	10,876	1,221	0
4	18,000	23	17,982	3,200	23	22	0	11,623	6,359	0
5	14,000	14	14,017	3,217	14	6	0	11,192	2,826	0
6	11,000	8	10,985	3,202	8	3	0	9,141	1,843	0
7	16,000	5	15,993	3,195	5	5	0	7,698	4,120	4,175
8	14,000	5	14,004	3.199	5	5	0	8,438	4,471	1,094
9	15,000	5	15,000	3,199	5	4	0	10,263	4,737	0
10	10,000	3	10,006	3,204	3	1	0	8,798	1,208	0

TABLE XXV (Continued)

B. DECISIONS AND PROJECTIONS OF THE DYNAMIC PRODUCTIVITY MODEL

Month	Demand	Work Force	Prod	Inven	WORKFORCE Regular Time	WORKFORCE Over-Time	WORKFORCE Idle	PRODUCTION Regular Time	PRODUCTION Over-Time	PRODUCTION Subcontracting
1	9,000	69	5,195	3,195	69	55	0	3,200	1,995	0
2	13,000	58	12,987	3,181	58	40	0	8,919	4,067	0
3	12,000	46	12,024	3,205	36	0	9	12,024	0	0
4	18,000	36	17,990	3,195	36	0	0	17,990	0	0
5	14,000	27	14,005	3,200	20	0	6	14,005	0	0
6	11,000	19	11,002	3,202	12	0	7	11,002	0	0
7	16,000	15	15,995	3,198	15	0	0	15,995	0	0
8	14,000	11	14,004	3,201	10	0	0	14,004	0	0
9	15,000	9	14,997	3,199	9	0	0	14,997	0	0
10	10,000	7	10,001	3,200	5	0	1	10,001	0	0

As a simple example demonstrating the performance of Model II, consider a six month planning horizon. Assuming that there are initially two experience classes with sizes of 40 and 30 men and productivity measures of 100 and 50, respectively, the results of computations performed by the CLASS routine are listed in Table 25. The input to this routine are initial status of the experience classes; the values of the parameters for learning, progress, and forgetting curves (in this example, these values are 70%, 90% and 80%, respectively; the base unit cost is assumed to be .3 man-month/unit); and the scheduled values for the workforce levels and the production rates throughout the planning horizon.

Table XXVI shows the effects of manpower fluctuations, production rate fluctuations, regular time work, overtime work, and forgetting on the conditions of the experience classes in each period. For example, notice that in the second period the first experience class in the first period is partitioned into two new classes. This is due to the assignment of overtime work to the upper part of the class. The second experience class in the fifth period is partitioned into two classes in the sixth period as a result of regular time assignment in this period. Notice the effect of forgetting on the productivity measrues of idle workers in the fifth period (the fourth and fifth classes). The productivity measures of these classes are decreased by 18 and 16 units, respectively, by the end of the sixth period. For the purpose of simplicity, inventories are not assumed in this example.

Remarks

As Table XXVI indicates, the new model can provide information

TABLE XXVI

A SAMPLE OF DETAILED OPERATION OF MODEL II

Period	Decisions		Exp. Class	Size	Prod. Measure	Workforce			Production		
	Work Force	Production				Regular Time	Over-time	Idle	Regular Time	Over-time	Subcontac-ting
1	70	12,000	1	40	234	70	70	0	5,203	3,078	3,719
			2	30	147						
2	90	13,000	1	36	473	90	36	0	9,846	3,154	0
			2	4	387						
			3	30	269						
			4	20	4						
3	65	5,000	1	23	843	65	23	0	13,921	3,079	0
			2	13	711						
			3	4	602						
			4	25	448						
4	80	20,000	1	4	1,368	80	4	0	19,320	680	0
			2	19	1,183						
			3	13	1,024						
			4	4	889						
			5	25	696						
			6	15	5						
5	70	15,000	1	4	1,823	36	0	34	15,000	0	0
			2	19	1,607						
			3	13	1,416						
			4	4	871						
			5	25	679						
			6	5	1						
6	60	11,000	1	4	2,363	21	0	39	11,000	0	0
			2	17	2,111						
			3	2	1,605						
			4	13	1,395						
			5	4	853						
			6	20	663						

regarding the size and experience level of each class in each period. This allows the building of more realistic cost models in which the regular time, overtime, and idle time payrolls for each class can be expressed as a function of the productivity level of the class. Also, as mentioned in Chapter VI, the length of overtime does not have to be assumed as a constant in the new model. This length (expressed by OPCNT as a percentage of the regular time length) can be introduced on an independent decision variable in the new model. The optimum periodical levels of this variable can then be determined by the search routine. This would increase the dimension of the problem from two times the number of periods to three time the number of periods in the planning horizon.

It should be noted that the special methodologies used for increasing the computational efficiency of the optimization processes for both models I and II have been very successful. For example, the stagewise improvement of the initial solutions in model I optimization has resulted in a computational time saving over 30% for the early stages of production, where the solution to the Ebert Model is relatively closer to the dynamic productivity model.

CHAPTER VIII

CONCLUSION AND RECOMMENDATIONS

The aggregate production planning problem is one of the most challenging and potentially rewarding problems in industry today. The two-fold economic significance of the problem is by no means minor. At the micro level, the significance to an individual firm can often be measured directly in annual savings by following one policy as opposed to another. On a macro basis, although not often considered explicitly, the variables considered in aggregate planning form major factors in the classic economic indicators. The present research should provide a significant contribution to this area.

The proper incorporation of the dynamic productivity phenomenon present in most empirical situations into the aggregate production planning problem has been the main objective of the research. The research has originated the introduction of workforce level change disruptions, progress and retrogression effects to this production planning area. Aggregate planning of both long cycle and short cycle product cases have been considered and models peculiar to each case have been developed and analyzed. The analyses of these models indicated their significant economic impact in the majority of situations. The relative performances of the new models over the existing ones reach their highest levels when the productive firm passes the transitional start-up production period and reaches the steady production state.

The impact of the models is also subject to the nature of various operational restrictions imposed on the planning problem, and the levels of different cost parameters incorporated into the objective function. The new models have higher impacts for sharper slopes of the applicable cost reduction curves.

The general solution methodology applied in this research is the Search Decision Rule. The computer subprograms PROCTV and CLASS developed for long and short cycle product cases, respectively, are not peculiar to any specific search technique. Also, the routine developed for the long cycle product case can be utilized for all existing aggregate planning cost models. Utilization of the routine developed for the short cycle product case for the existing cost functions may require minor modifications in the structure of these functions. Generally, these programs can serve as standard routines to convert any current constant productivity aggregate planning model, which applies the search decision rule, to incorporate the effect of the dynamic productivity phenomenon. Attachment of the new routines to the existing programs would require minor modifications in the main and the objective function routines of these programs to facilitate the process of transferring the common variables and parameters among the interfacing routines.

The methodology used for improving the computational efficiency of the optimizations performed in this research can be generalized for heuristic optimization of all complex objective functions (provided that approximation of these functions with simpler functions is possible). The essence of this methodology has been the stagewise approximation of the cost function and improvement of the starting solutions.

Development of the analysis of the compounded disruption effect is a significant contribution of this research to the general area of the improvement curve analysis. Application of the analysis of the compounded disruption effect is not limited to the aggregate planning problem. This analysis is useful in a variety of production and workforce planning and scheduling problems.

Finally, the methodology developed for quantifying the relative performances of the new models in this research can be used as evaluation methods for future dynamic productivity models.

The following items are recommended for future research:

1. Extension of Models I and II to multi-department aggregate planning models, in which disruptions as the result of the task changes generated by the inter-departmental transferring of employees affect the productivity rates.

2. Substitution of other existing analytical cost reduction curves in place of the original linear curve models used in this research to provide models suitable for different industrial applications. For example, utilization of the mixed model learning curve [66] will be ideal for the mixed model assembly situations in which more than one model is assembled on the same assembly line; hence, the repetitions of the assembly work are not always the same. Analysis of various disruption effects for these cost reduction curves will be required prior to their incorporation into the dynamic aggregate planning models.

3. Extension of the analysis of Model II for the cost function in which the payroll costs are expressed as functions of productivity rates of employees. Also, incorporation of the overtime length as variable in this model and analysis of the effect of such modifications. These

modifications would require a very small programming effort since the provisions have been made in the current program.

4. Incorporation of the integer restriction on the variables such as workforce level into the SDR methodology.

5. Incorporation of the effects of the dynamic productivity elements considered in this research (cost reduction, task change disruption, manpower level change disruption, compounded disruption and other related elements) into the various production, inventory and work force planning and scheduling areas.

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APPENDIX A

GLOSSARY OF VARIABLES USED IN COMPUTER PROGRAM FOR MODEL-I (EXCLUDING THE SEARCH ROUTINE)

APROD(I)	Average productivity in the ith period computed by PROCT routine
AK1, AK2	Constants used to delute the task turnover in cases of manpower reduction and addition
AK3	Constant used to delute the task turnover in case of manpower reduction
B	Measure (exponent) of slope for improvement curve
BCT(I)	Block cost in the ith period
CAPR	Overall average productivity applied by the constant productivity model
CUM(I)	Cumulative number of units produced by the end of the ith period
CUMØ	Cumulative number of units produced by the beginning of the first period
C1	Cost coefficient for direct payroll cost
C2	Cost coefficient for hiring and firing costs
C3, C5 & C6	Cost coefficients for overtime cost
C7 & C8	Cost coefficients for inventory carrying cost
C9 & C10	Additional cost coefficients (not used in this model)
D(I)	Demand level in the ith period
DINV(I)	Inventory level in the ith period
DPCST(I)	Direct payroll cost in the ith period
EPSY	a minimum unit cost difference level above which the current disruption is assumed to be carried over to the following period
F	Base unit cost (in terms of man-month)
HLCST(I)	Hiring and layoff cost in the ith period
I	A localized index for various loops; generally represents the period number for the consecutive periods.

ICHK(I)	A binary variable used to record the occurance of compound disruption; it is Ø if the unit cost difference at the end of the ith period is less than EPSY; it is 1, otherwise
ICNTRL	A flag used in detecting IEFECT. It is 1 as long as a change in the level of independent variables is not reported; it becomes 2 as soon as a change is detected
IEFECT	The period number at which the first change in the level of an independent variable is made by the search routine. It is used for gaining computational efficiency
INDEX(I)	A binary variable used to record the occurances of significant disruptions. It is 1 if the disruption occuring at the beginning of the ith period is significant; it is Ø, otherwise
IP	An index used to update the level of the cumulative number of units when a change is made in the level of production rate in a period
KFLAG	A flag used to direct the control in PROCTIV and FCT1. If KGLAG = 0, the dynamic productivity model is in effect. If KFLAG = 1, the constant productivity model in in effect. If KFLAG = 2, the Ebert model is in effect.
KPRESS	A variable which when set =1, suppresses a large portion of the detailed output of the search routine (see Appendix C)
KPRINT	A flag controlling the printout in the main routine. It directs the control for printing the results of optimization and evaluation of the three models considered.
LIM	An approximate upper bound on the number of calls of the objective functions (see Appendix C)
N	The number of independent variables in the search, N = 2*NM
NEVAL	Number of function evaluations (see Appendix C)
NI	Computer system input unit code
NM	Number of periods in the planning horizon—length of the planning horizon

NO	Computer system output unit code
OINV	Initial level of inventory (at the beginning of the first period)
OVCST(I)	Overtime cost in the ith period
P(I)	Production rate in the ith period (an independent variable)
PCUM	A localized variable used in computing the cumulative number of units
R(I)	Task turnover ratio considered for the beginning of the ith period
S	Slope of the improvement curve
SN	The response level of the objective function
STCST(I)	Storage (inventory) cost in the ith period
TC(I)	Total operation costs in the ith period
TDND	A localized variable used in computing total demand for the constant productivity model
TMHR	Total man-month requirement (block cost) over the planning horizon. It is used in computing the average productivity level for the constant productivity model
TNMP(I)	Manpower turnover in the ith period (undeluted)
TOTCST	Total cost of operations throughout the planning horizon
TRSHL	The threshold level for significant disruption
UA	The cumulative unit number received by UNIT and BLOCK. If received by BLOCK, UA is the first unit number in the block of units
UB	The cumulative unit number received by BLOCK. It is the last unit number in the block of units
UCDIF(I)	The unit cost difference at the end of the ith period
UC(K)	A localized value used to compute the unit cost of the Kth unit in sequence of units

UCT(I)	Unit cost of the last unit produced in the ith period
UX	A localized variable representing the cumulative unit number at which a 'carried over' disruption takes place
UY	A localized variable representing the cumulative unit number of the first unit produced in a period
UZ	A localized variable representing the cumulative unit number of the last unit produced in a period
W(I)	Workforce level in the ith period (an independent variable
WOLD	A localized variable representing the workforce level in the previous period (=W(I-1))
WØ	Initial workforce level
X(J)	The jth independent variable (see Appendix C)
XMAX(J)	Upper limit on jth independent variable, X(j)
XMIN(j)	Lower limit on jth independent variable, X(j)
ZADJ	The unit cost difference ratio at the end of the first period, used to adjust the task turnover for the beginning of the first period in the next month optimization

APPENDIX B

GLOSSARY OF VARIABLES USED IN COMPUTER PROGRAM

FOR MODEL-II (EXCLUDING THE SEARCH ROUTINE)

B	Measure (exponent) of learning slope
BF	Measure (exponent) of forgetting slope
BP	Measure (exponent) of progress slope
COMØ	Cumulative number of units produced by the beginning of the first period
C1	Cost coefficient for direct payroll cost
C2	Cost coefficient for hiring and firing costs
C3, C4, C5 & C6	Cost coefficients for overtime cost. C4 is the overall average productivity factor for the constant productivity model
C7 & C8	Cost coefficients for inventory carrying cost
D(I)	Demand level in the ith period
DINV(I)	Inventory level in the ith period
DPCST(I)	Direct payroll cost in the ith period
F	Base unit cost (in terms of man-month)
HLCST(I)	Hiring and layoff costs in the ith period
I	A localized index for various loops; generally represents the period number of the consecutive periods
ICNTRL	A flag used in detecting IEFECT. It is 1 as long as a change in the level of independent variables is not reported; it becomes 2 as soon as a change is detected.
IEFECT	The period number at which the first change in the level of our independent variable is made by the search routine. It is used for gaining more computational efficiency
II	Index of the previous period (II = I - 1)
IP	An index used to update the level of the cumulative number of units when a change is made in the level of production rate in a period
KDIRCT	A flag controlling the printout in the main routine. It directs the controls for printing the results of optimization and evaluation of the models considered

KFLAG	A flag used to direct the control in CLASS. If KFLAG = 0, class partition does not apply; otherwise, KFLAG = 1.
KPRESS	A variable which when set =1, suppresses a large portion of the detailed output of the search routine (see Appendix C)
LIM	An approximate upper bound on the number of calls of the objective function (see Appendix C)
N	The number of independent variables in the search, N = 2*NM
NEVAL	Number of function evaluations (see Appendix C)
NI	Computer system input unit code
NM	Number of periods in the planning horizon—the length of the planning horizon
NO	Computer system output unit code
NTEMP	A temporary location for the number of experience classes. NTEMP is updated if a class partition takes place
NXCL(I)	Number of experience classes in the ith period. NXCL takes the updated value of NTEMP
NXCLØ	Initial number of experience classes (at the beginning of the first period)
NXX	A localized variable used to compute NTEMP
OINV	Initial inventory level
OPCNT	The length of overtime as percentage of the length of regular time
OVCST(I)	Overtime cost in the ith period
P(I)	Production rate in the ith period (an independent variable)
PCUM	A localized variable used in computing the cumulative number of units
PCXR(I,J)	Production capability of the jth experience class in the ith period

PNALT(I)	A binary variable recording the violation of the total production capability constraint. When this constraint is violated in the ith period, PNALT(I) is set equal to 1, otherwise it is zero. This variable is multiplied by C10 and the result is incorporated into the objective function
POVR(I)	Total production on overtime in the ith period
PREG(I)	Total production on regular time in the ith period
RP	A localized variable representing the remaining (excess) production level. It is used in computing PREG, POVER, size of partitioned classes, and the subcontracting level
SF	Slope of forgetting
SL	Slope of learning
SN	The response level of the objective function
SP	Slope of progress
SS	Slope of the overall improvement curve (used in the constant productivity model)
STCST(I)	Storage (inventory) cost in the ith period
SUB(I)	Subcontracting level in the ith period
TDND	A localized variable used to compute the total demand for the constant productivity model
TL	A localized variable accumulating numbers of men in the sequentially taken experience classes. It is used to incorporate the effect of manpower reduction
TMHR	Total man-month required (block cost) over the planning horizon. It is used in computing the average productivity level for the constant productivity model
TOTCST	Total operating cost throughout the planning horizon
TWO	A localized variable representing the total number of men assigned on overtime work (in a period)

TWR	A localized variable representing the total number of men assigned on regular time work (in a period)
UF	Level of the productivity measure at which forgetting starts
UP	The applicable cumulative unit number for which the progress effect (UPP) is computed.
UPP	Progress effect factor
UTEMP(J)	A temporary location for productivity measure of the jth class (in a period)
UXCL(I,J)	Productivity measure of the jth experience class in the ith period
UXCLØ(J)	Initial level of productivity measure of the jth experience class
UXX(J)	A localized variable used for storing the productivity measure of the jth experience class
W(I)	Workforce level in the ith period (an independent variable)
WIDL(I)	Total number of idle men in the ith period
WL	A localized variable representing the amount of change in the total level of manpower
WOLD	A localized variable representing the workforce level in the previous period
WOVR(I)	Total number of men assigned on overtime work in the ith period
WREG(I)	Total number of men assigned on regular time work in the ith period
WRDP	The size of upper portion of the class partitioned through assignment of overtime
WRRP	The size of upper portion of the class partitioned through assignment of regular time
WTEMP(J)	A temporary location for storing the number of members in the jth experience class
WXCL(I,J)	The size of the jth experience class in the ith period

WXCLØ(J)	Initial size of the jth experience class
WXX(j)	A localized variable used for storing the size of the jth experience class (in a period)
X(K)	The kth independent variable (see Appendix C)
XMAX(K)	Upper limit on kth independent variable, X(K)
XMIN(K)	Lower limit on kth independent variable, X(K)

APPENDIX C

GLOSSARY OF VARIABLES USED IN

PATTERN SEARCH SUBROUTINE

ALP	Alpha, the factor by which the step size, D(I), grows when a forward move is successful and $L_4 = 2$. (Initialized at 2.0; used in stm 292)
BET	Beta, the multiplicative factor by which step size for an independent variable is reduced if forward and reverse move for that variable fail
D(I)	The current value of the step size for the ith independent variable
DEL	Delta, the multiplier which is used to determine the initial value of D(I), the step size, in accordance with statement 180
DX	A local quantity used to determine whether the lower bound on the step size has been reached. DX is computed and used only between stms 480 and 485, where step size reduction takes place
D1	A quantity used to increment the value of GR in the adaptive logic. D1 is set in stms 802 + 1 and 804; it is used in stm 810
GR	The factor by which the pattern move vector is multiplied to obtain the actual size of the pattern move. (GR is initialized at 2.2 and adjusted upward, usually by increment of .1, in statements 510 through 783.) When GR reaches 3.5 it is reset to 2.2
I	A highly localized variable used as an index in DO loops—see stms 180, 420, 786
ID1	Counter to record passes through stm 802
ID2	Counter to record passes through stm 803
ID3	Counter to record passes through stm 804
ID4	Counter to record passes through stm 801
ITR	A printout control character of little importance; initialized at 1 and left at that value. It is tested in stm 888; if the value is >1 it causes deletion of certain print lines. (In effect, it is not used unless initialized at ITR > 1.)
K	A subscript which defines which independent variable is now being studied—as in X(K), D(K), etc.

KK — A counter for one plus the number of variables studied since the last test for a new base point. When KK reaches N + 1 a test for a new base point is made. (The foregoing sentence applies when the subroutine is in the full exploratory search mode, as signified by $LT \leq 0$—this is the usual mode.) KK is set equal to 1 at stms 180 + 9, 440 + 2, and 784. In the full exploratory search mode ($LT \leq 0$) KK is incremented at 330 and tested at 330 + 1 and 404; in the truncated search mode ($LT \geq 1$) KK is incremented at 778 and tested at 404

KOUNT — Counter of the number of times we enter the adaptive logic preparatory attempting a pattern move. (-has same value as LT7) Incremented at stm 510 + 1

KPRESS — A printout control character which is input on the first data card read by the Main routine. KPRESS = 1 suppresses about 99% of the output generated by Subroutine PATS; KPRESS = 0 allows the full details to be output during PATS.

LA — A Master Monitor of Subroutine Status which tells where to GO TO next. The primary job of LA is to control traffic through the Boundary Check and the objective function subroutine. The following values of LA correspond to the following destinations:

Value of LA	Destination: Stm no.	Destination: Task
1	100	Initialization
2	282	Forward Exploratory Move—Normal
3	463	Reverse Exploratory Move—Normal (Following Fwd Failure)
4	580	Evaluate Base Point Following Pattern Move
5	285	Forward Exploratory Move—Following Pattern Move
6	466	Reverse Exploratory Move—Following Pattern Move (and Fwd Failure)
7	510	Attempt Addaptive Pattern Move
8	500	Terminate Search and Exit From Subroutine

LIM	An approximate upper bound on the number of calls of the objective function during a particular call of subroutine PATS. (LIM is usually set at 3000, but a different value might be appropriate for some applications.) See final remark under NEVAL for identification of location where NEVAL is tested
LSN	Set at 0 in fifth statement of subroutine and kept there—never referred to again except in statement 784 + 1 (See Weisman, Wood, and Rivlin for explanation)
LT	Controls the choice between the standard full exploratory search mode (LT ≤ 0) and the truncated search mode (LT ≥ 1). Under truncated mode the exploratory search is stopped as soon as any move produces an improvement in the last base point. This point is saved as the new base point, and a pattern move is made. The next exploratory search starts with the variable after the one which produced the last success. Under the full exploratory search mode, which is the mode this coding reflects, since LT = 0, an exploratory step is made with all N variables before a pattern move is attempted. LT is set in the fourth statement of the subroutine and does not appear again except in statement 320 and 400.
LT2	Number of times this subroutine has reached status LA = 2*
LT3	Number of times this subroutine has reached status LA = 3*
LT4	Number of times this subroutine has reached status LA = 4
LT5	Number of times this subroutine has reached status LA = 5*
LT6	Number of times this subroutine has reached status LA = 6*
LT7	Number of times this subroutine has reached status LA = 7

*in the event that a proposed forward or reverse move fails the boundary check the appropriate one of the coutners will fail to be incremented

L4	A status variable always equal to 1 or 2: L4 - 1 says we are in the process of making exploratory moves in normal fashion, i.e., we are searching for a pattern. L4 = 2 says we are in the process of making exploratory moves following a pattern move
M1	One plus the number of indpendent variables which have experienced both forward and reverse failures with the minimum step size since the last test for a new base point. The search is terminated when M1 is ≥ N + 1 during the exploratory search mode, i.e., when L4 = 1 and we are attempting to establish a new pattern. Even though M1 may be (redundantly) manipulated while the search is conducting a post-pattern-move exploration (when L4 = 2), it is never used, i.e., tested, except when we are attempting to establish a new pattern, namely when L4 = 1. (In reprogramming, this variable could be reduced by 1, i.e., intialized to 0 and tested for ≥ n, with improved clarity of interpretation; this also applies to M2.) M1 is initialized in stms 180 + 6, 300 + 2, 352 + 1, and 440 + 1; incremented at 490; and tested at 429 + 1
M2	One plus the number of independent variables which have experienced both forward and reverse failures since the last attempt to make a pattern move. The pattern is considered broken and the search is restored to exploratory mode (L4's value changes from 2 to 1) if all variables fail following a single pattern move attempt
N	The number of independent variables, i.e., the dimensionality of the space being searched.
NEVAL	A counter of the number of evulatuation of FCT1, the objective function, which have been made during this particular call of Subroutine PATS. NEVAL is incremented inside Subroutine FCT1; it is not tested every time it is incremented. LIM is an approximate upper bound on NEVAL; the test is made in stm 7, just prior to attempting a pattern move. This is preferable to testing every time FCT1 is called
NEVOLD	Variable used to store the previous value of NEVAL, the "total number of evaluation of the objective function made thus far." NEVOLD is reset at stm 815 + 2 and is used in the computation of V in stm 398

NPF	Counter for the number of successive pattern move (attempts) which are followed by failure of all individual steps that try to adjust the pattern move attempt. (Set at 0 in stms 100 + 5, 300 + 3, 420 + 1; incremented at stms 780 + 1; tested at 353.) See discussion under M2
OLDSN	A variable used to store a certain prior value of SN. It is intialized at stm 180 + 3, reset at stm 815, and used in the computation of V at stm 398 (within the adaptive logic)
OLDV	Variable used to store the previous value of (re-set in stm 815 + 1; used in stm 782 to compare old and new values of V)
P	A highly localized temporary storage variable used only between stms 530 + 3 and 530 + 5, where the pattern move is attempted
Q(I)	A storage matrix for storing the old, i.e., base point, values of the independent variables. These values are initially set equal to the X(I) values and are updated in stm 530 + 4. If an attempted pattern move fails, the old values of X(I) are recovered from Q(I) in stm 420
SC	A variable used to store the value of the objective function at the most recent base point. SC is initialized at stm 180 + 4, reset at stm 530 (just prior to attempting a pattern move), tested at statement 340 to determine whether a new base point has been established, and used to restore SP to the old value in stm 410 if the attempted pattern move fails
SN	The value returned by the objective function, subroutine FCT1; i.e., the cumulation of all costs over the planning horizon, using the current values of X(I) ($1 \leq I \leq N$) as decisions
SNOLD	A redundant variable which is initialized at stm 180 + 2 and never referred to again for any purpose
SP	A variable which (except in the following circumstance) is equal to the minimum value returned by the objective function thus far. The exception occurs when an exploratory search is being conducted immediately after a pattern move. In that situation SP is set (stm 580 + 2) equal to the value of the objective function which reflects

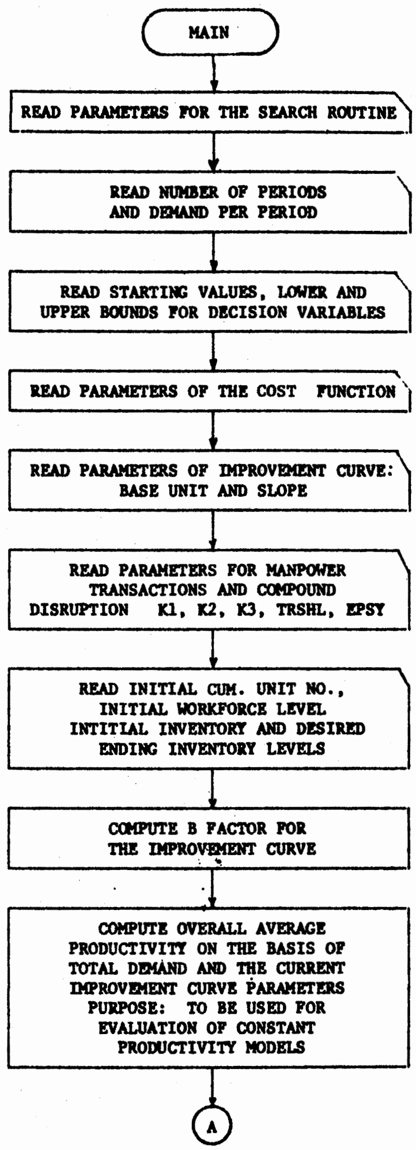
the unadjusted pattern move, and thereafter, as the exploratory search to adjust the pattern move proceeds, SP is updated to reflect the minimum of this value and the best exploratory move to date. After the explorations are completed, this updated value of SP is compared (stm 340) with SC, the value of the objective function at the last base point. If this base point test is passed, the (adjusted) pattern move is declared successful and a new base point is established. If the base point test is not passed, the pattern move attempt is declared unsuccessful, SP is restored to its old value (stm 410), and local explorations are initiated about the old base point in an effort to establish a new pattern

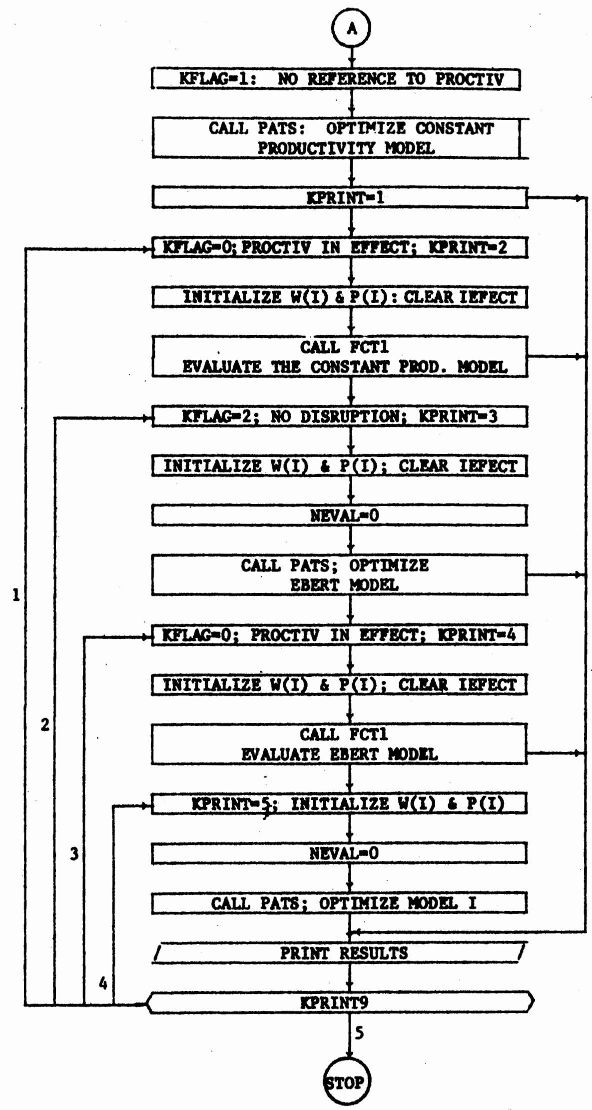
TOL	A quantity used (stm 480 + 2) in obtaining lower bounds on step sized for exploratory moves
V	Percentage improvement in value of objective function per call of objective function. Computed and used in adaptive logic—see stms 398 through 782
X(I)	The independent decision variables (of which ther are N: $1 \le I \le N$). In the Paint Factory application the odd values of the index, I, identify the work force decisions in successive months, while the even values of the index denote production rate decisions for those months. There are N/2 months in the planning horizon
XMAX(I)	The upper bound on the acceptable value of X(I)
XMIN(I)	The lower bound on the acceptable value of X(I)

APPENDIX D

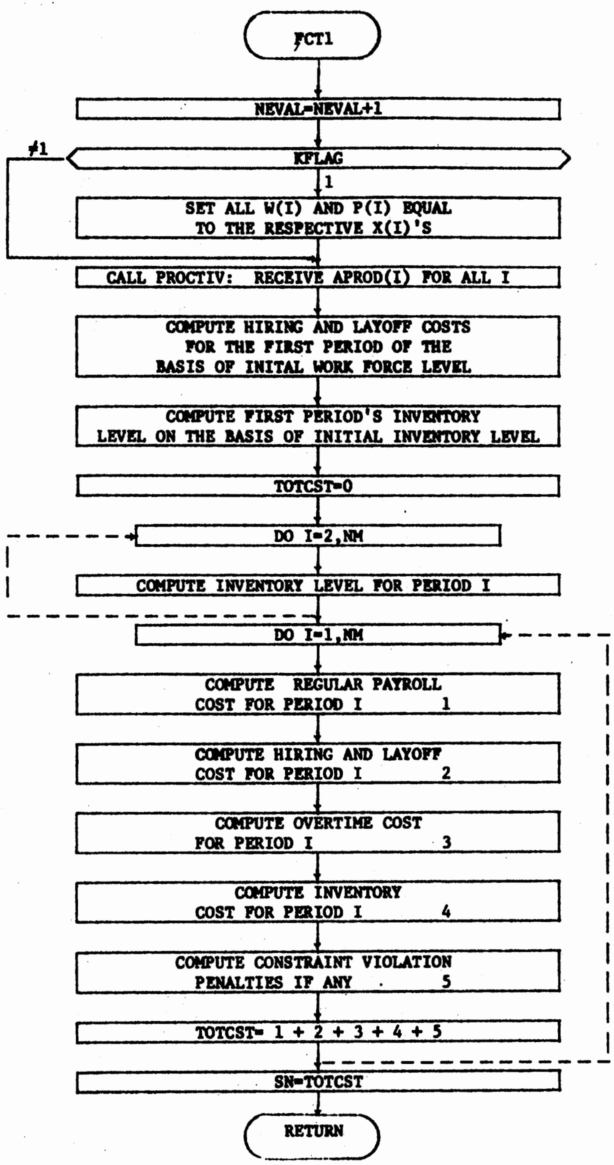
SEMI-DESCRIPTIVE FLOWCHARTS OF

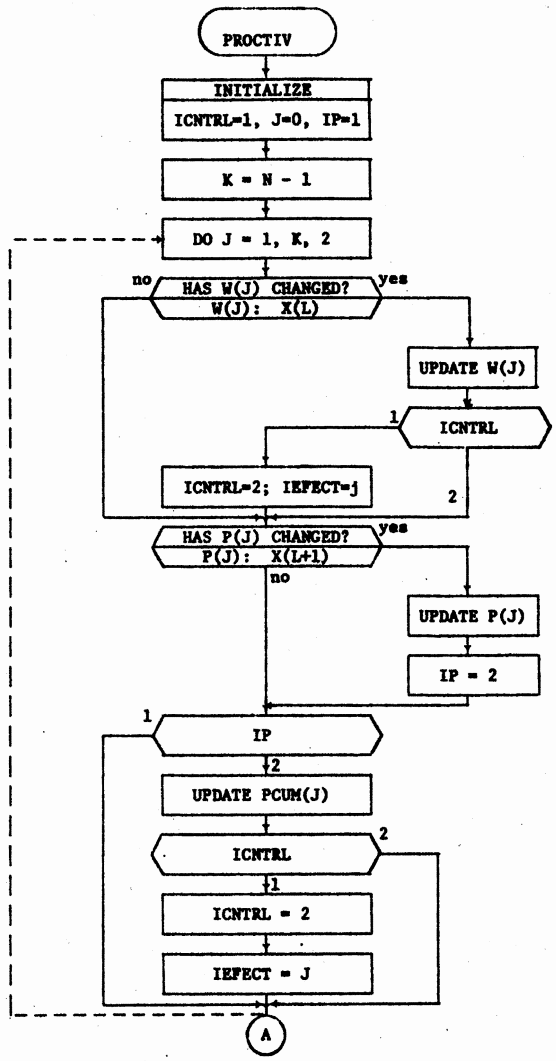
COMPUTER PROGRAM FOR MODEL-I

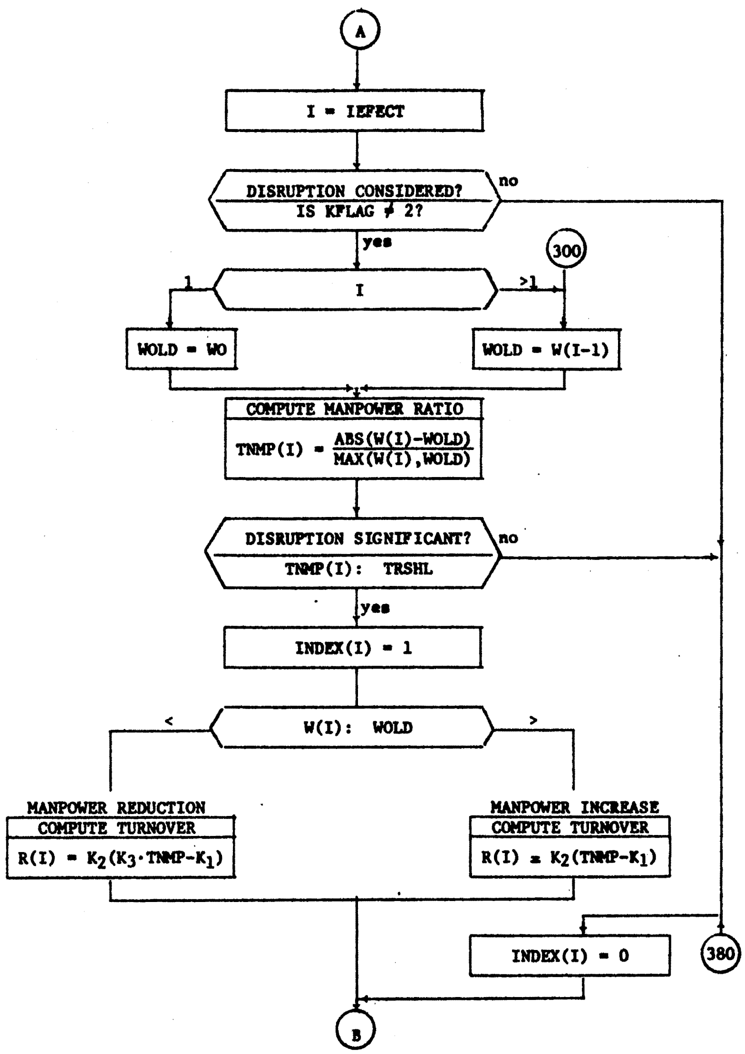

MAIN
READ PARAMETERS FOR THE SEARCH ROUTINE
READ NUMBER OF PERIODS AND DEMAND PER PERIOD
READ STARTING VALUES, LOWER AND UPPER BOUNDS FOR DECISION VARIABLES
READ PARAMETERS OF THE COST FUNCTION
READ PARAMETERS OF IMPROVEMENT CURVE: BASE UNIT AND SLOPE
READ PARAMETERS FOR MANPOWER TRANSACTIONS AND COMPOUND DISRUPTION K1, K2, K3, TRSHL, EPSY
READ INITIAL CUM. UNIT NO., INITIAL WORKFORCE LEVEL INTITIAL INVENTORY AND DESIRED ENDING INVENTORY LEVELS
COMPUTE B FACTOR FOR THE IMPROVEMENT CURVE
COMPUTE OVERALL AVERAGE PRODUCTIVITY ON THE BASIS OF TOTAL DEMAND AND THE CURRENT IMPROVEMENT CURVE PARAMETERS PURPOSE: TO BE USED FOR EVALUATION OF CONSTANT PRODUCTIVITY MODELS
A

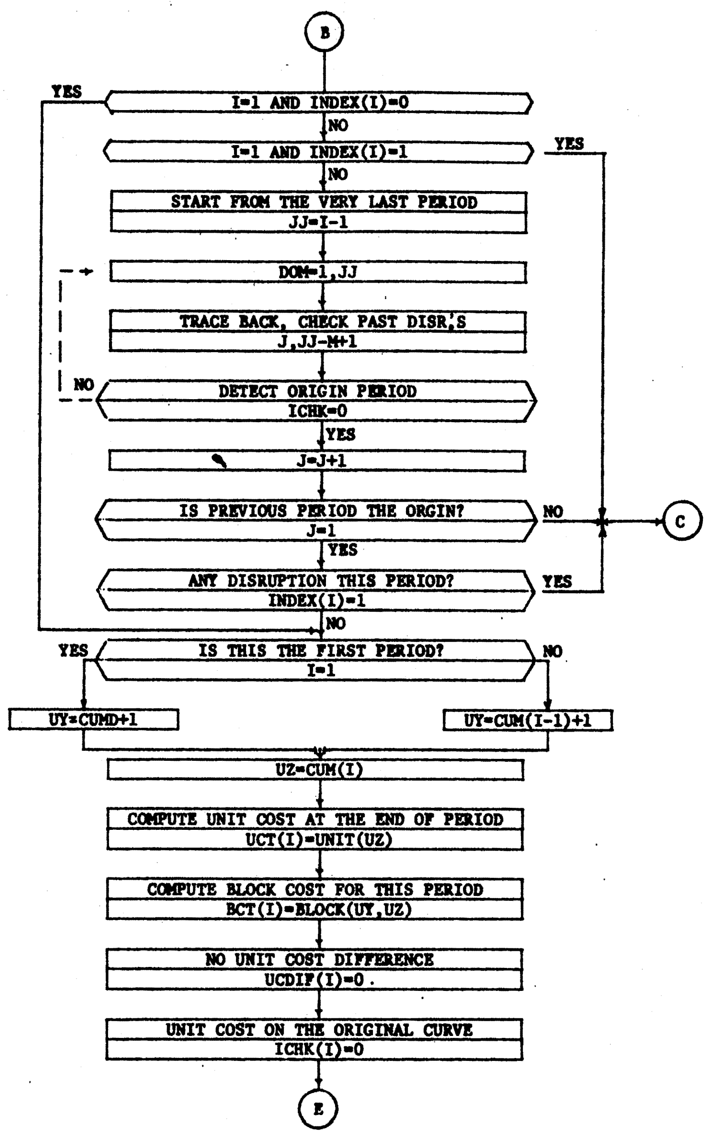


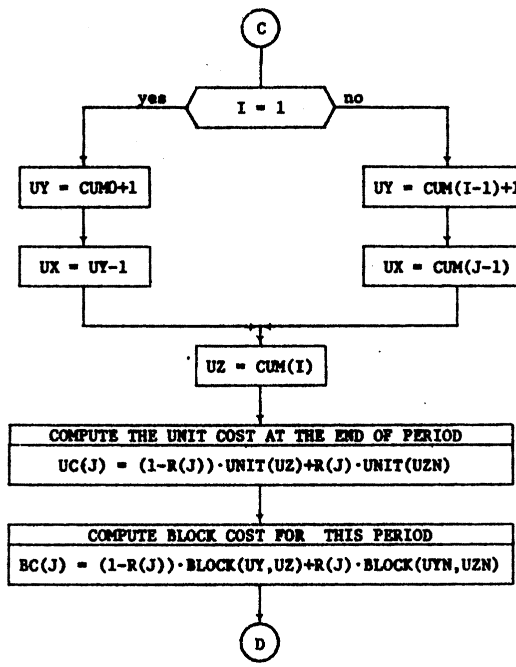
A
KFLAG=1: NO REFERENCE TO PROCTIV
CALL PATS: OPTIMIZE CONSTANT PRODUCTIVITY MODEL
KPRINT=1
KFLAG=0; PROCTIV IN EFFECT; KPRINT=2
INITIALIZE W(I) & P(I): CLEAR IEFECT
CALL FCT1
EVALUATE THE CONSTANT PROD. MODEL
KFLAG=2; NO DISRUPTION; KPRINT=3
INITIALIZE W(I) & P(I); CLEAR IEFECT
NEVAL=0
CALL PATS; OPTIMIZE
EBERT MODEL
KFLAG=0; PROCTIV IN EFFECT; KPRINT=4
INITIALIZE W(I) & P(I); CLEAR IEFECT
CALL FCT1
EVALUATE EBERT MODEL
KPRINT=5; INITIALIZE W(I) & P(I)
NEVAL=0
CALL PATS; OPTIMIZE MODEL I
PRINT RESULTS
KPRINT9
1
2
3
4
5
STOP

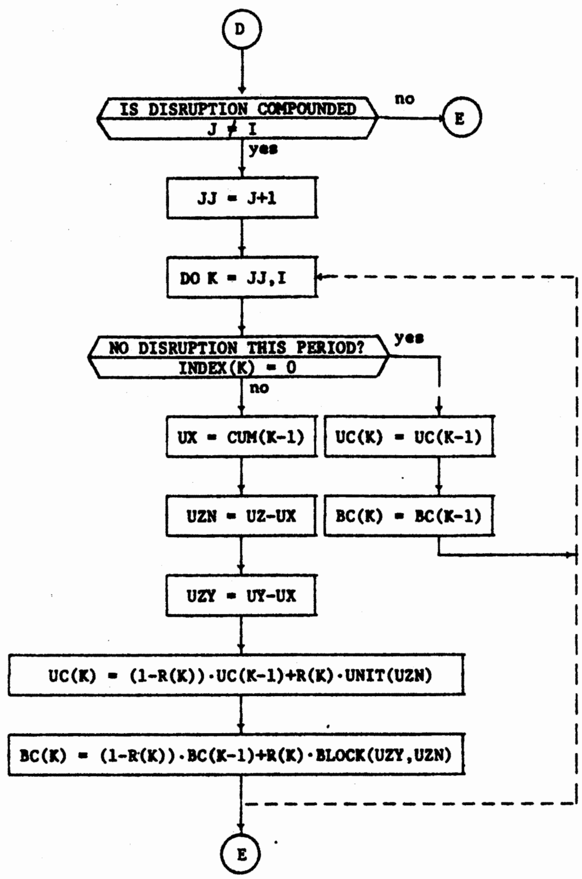

FCT1
NEVAL=NEVAL+1
≠1
KFLAG
1
SET ALL W(I) AND P(I) EQUAL TO THE RESPECTIVE X(I)'S
CALL PROCTIV: RECEIVE APROD(I) FOR ALL I
COMPUTE HIRING AND LAYOFF COSTS FOR THE FIRST PERIOD OF THE BASIS OF INITAL WORK FORCE LEVEL
COMPUTE FIRST PERIOD'S INVENTORY LEVEL ON THE BASIS OF INITIAL INVENTORY LEVEL
TOTCST=0
DO I=2,NM
COMPUTE INVENTORY LEVEL FOR PERIOD I
DO I=1,NM
COMPUTE REGULAR PAYROLL COST FOR PERIOD I 1
COMPUTE HIRING AND LAYOFF COST FOR PERIOD I 2
COMPUTE OVERTIME COST FOR PERIOD I 3
COMPUTE INVENTORY COST FOR PERIOD I 4
COMPUTE CONSTRAINT VIOLATION PENALTIES IF ANY 5
TOTCST= 1 + 2 + 3 + 4 + 5
SN=TOTCST
RETURN

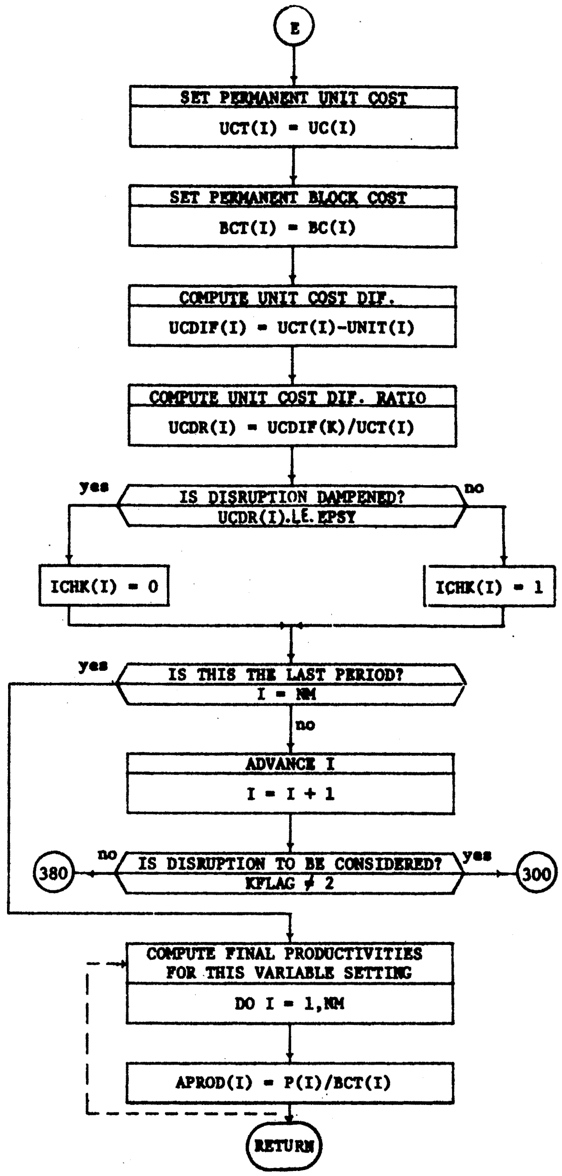

PROCTIV
INITIALIZE
ICNTRL=1, J=0, IP=1
K = N - 1
DO J = 1, K, 2
no
HAS W(J) CHANGED?
W(J): X(L)
yes
UPDATE W(J)
1
ICNTRL
ICNTRL=2; IEFECT=j
2
HAS P(J) CHANGED?
P(J): X(L+1)
yes
no
UPDATE P(J)
IP = 2
1
IP
2
UPDATE PCUM(J)
2
ICNTRL
1
ICNTRL = 2
IEFECT = J
A


A
I = IEFECT
DISRUPTION CONSIDERED?
IS KFLAG ≠ 2?
no
yes
300
I
1
>1
WOLD = WO
WOLD = W(I-1)
COMPUTE MANPOWER RATIO
TNMP(I) = ABS(W(I)-WOLD) / MAX(W(I),WOLD)
DISRUPTION SIGNIFICANT?
TNMP(I): TRSHL
no
yes
INDEX(I) = 1
W(I): WOLD
<
>
MANPOWER REDUCTION
COMPUTE TURNOVER
R(I) = K2(K3·TNMP-K1)
MANPOWER INCREASE
COMPUTE TURNOVER
R(I) = K2(TNMP-K1)
INDEX(I) = 0
380
B


B
I=1 AND INDEX(I)=0
YES
NO
I=1 AND INDEX(I)=1
YES
NO
START FROM THE VERY LAST PERIOD
JJ=I-1
DOM=1,JJ
TRACE BACK, CHECK PAST DISR'S
J,JJ-M+1
NO
DETECT ORIGIN PERIOD
ICHK=0
YES
J=J+1
IS PREVIOUS PERIOD THE ORGIN?
J=1
NO
C
YES
ANY DISRUPTION THIS PERIOD?
INDEX(I)=1
YES
NO
YES
IS THIS THE FIRST PERIOD?
I=1
NO
UY=CUMD+1
UY=CUM(I-1)+1
UZ=CUM(I)
COMPUTE UNIT COST AT THE END OF PERIOD
UCT(I)=UNIT(UZ)
COMPUTE BLOCK COST FOR THIS PERIOD
BCT(I)=BLOCK(UY,UZ)
NO UNIT COST DIFFERENCE
UCDIF(I)=0.
UNIT COST ON THE ORIGINAL CURVE
ICHK(I)=0
E


C
I = 1
yes
no
UY = CUMO+1
UY = CUM(I-1)+1
UX = UY-1
UX = CUM(J-1)
UZ = CUM(I)
COMPUTE THE UNIT COST AT THE END OF PERIOD
UC(J) = (1-R(J))·UNIT(UZ)+R(J)·UNIT(UZN)
COMPUTE BLOCK COST FOR THIS PERIOD
BC(J) = (1-R(J))·BLOCK(UY,UZ)+R(J)·BLOCK(UYN,UZN)
D

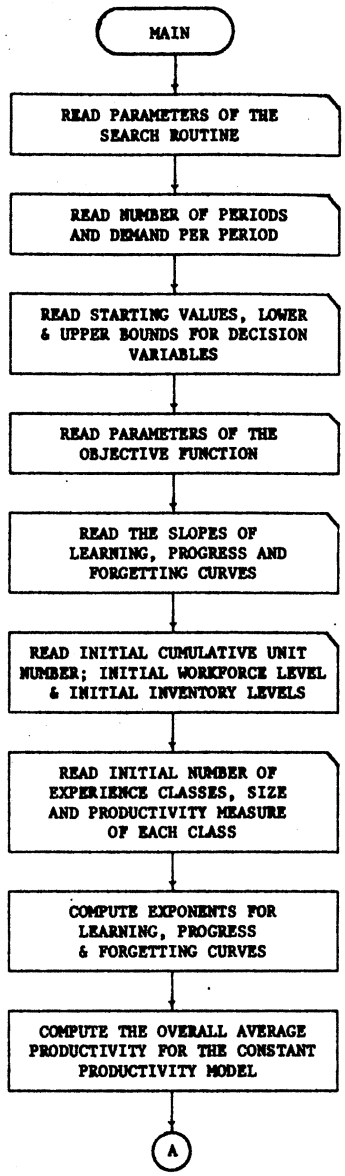

D
IS DISRUPTION COMPOUNDED
J ≠ I
no
E
yes
JJ = J+1
DO K = JJ,I
NO DISRUPTION THIS PERIOD?
INDEX(K) = 0
yes
no
UX = CUM(K-1)
UC(K) = UC(K-1)
UZN = UZ-UX
BC(K) = BC(K-1)
UZY = UY-UX
UC(K) = (1-R(K))·UC(K-1)+R(K)·UNIT(UZN)
BC(K) = (1-R(K))·BC(K-1)+R(K)·BLOCK(UZY,UZN)
E

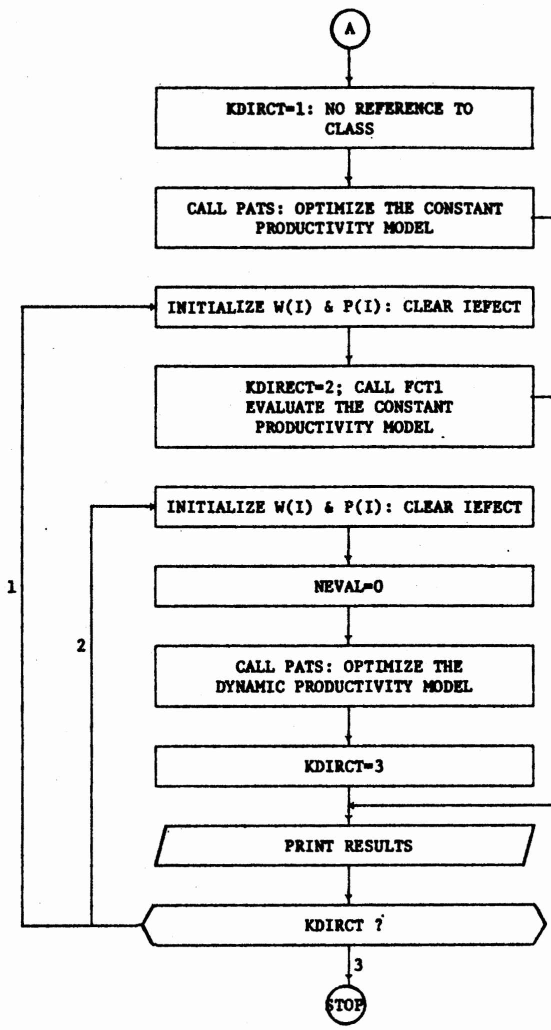

E
SET PERMANENT UNIT COST
UCT(I) = UC(I)
SET PERMANENT BLOCK COST
BCT(I) = BC(I)
COMPUTE UNIT COST DIF.
UCDIF(I) = UCT(I)-UNIT(I)
COMPUTE UNIT COST DIF. RATIO
UCDR(I) = UCDIF(K)/UCT(I)
yes
IS DISRUPTION DAMPENED?
UCDR(I).LE.EPSY
no
ICHK(I) = 0
ICHK(I) = 1
yes
IS THIS THE LAST PERIOD?
I = NM
no
ADVANCE I
I = I + 1
380
no
IS DISRUPTION TO BE CONSIDERED?
KFLAG ≠ 2
yes
300
COMPUTE FINAL PRODUCTIVITIES
FOR THIS VARIABLE SETTING
DO I = 1,NM
APROD(I) = P(I)/BCT(I)
RETURN

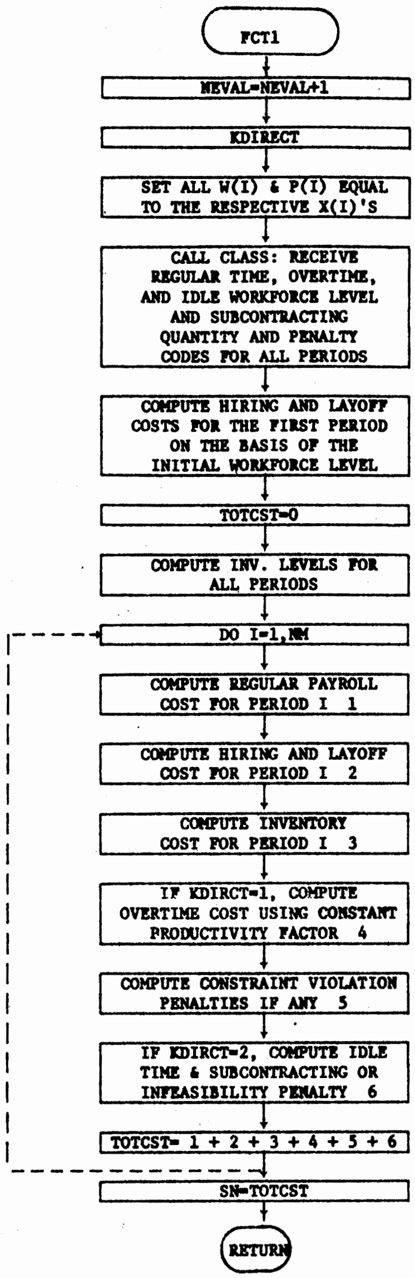
APPENDIX E

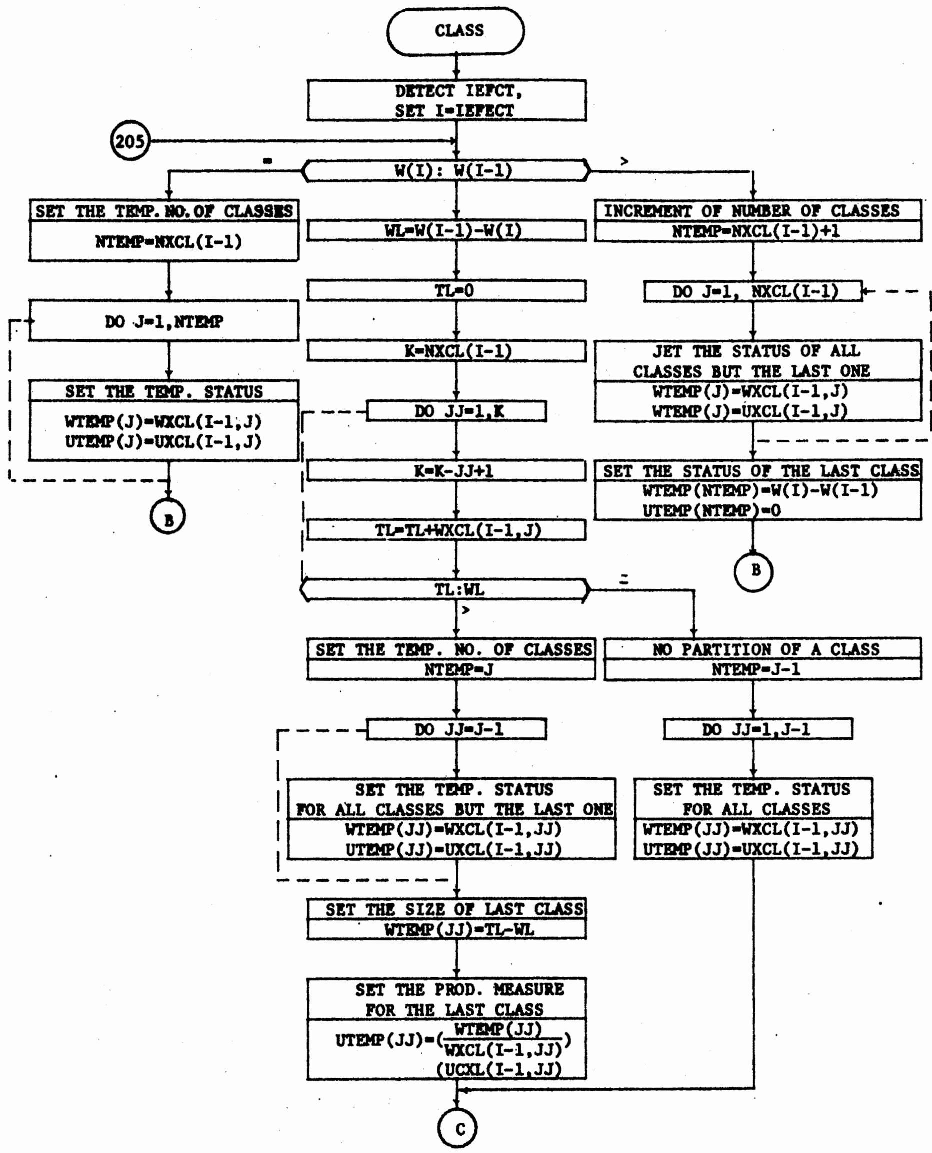
SEMI-DESCRIPTIVE FLOWCHARTS OF

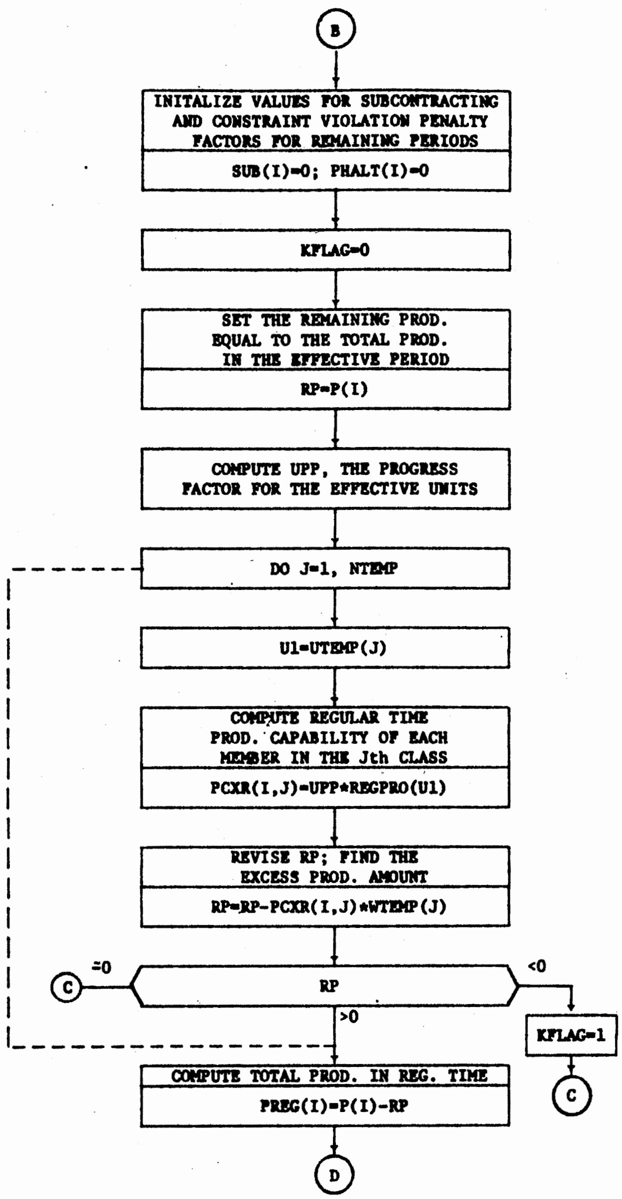
COMPUTER PROGRAM FOR MODEL-II

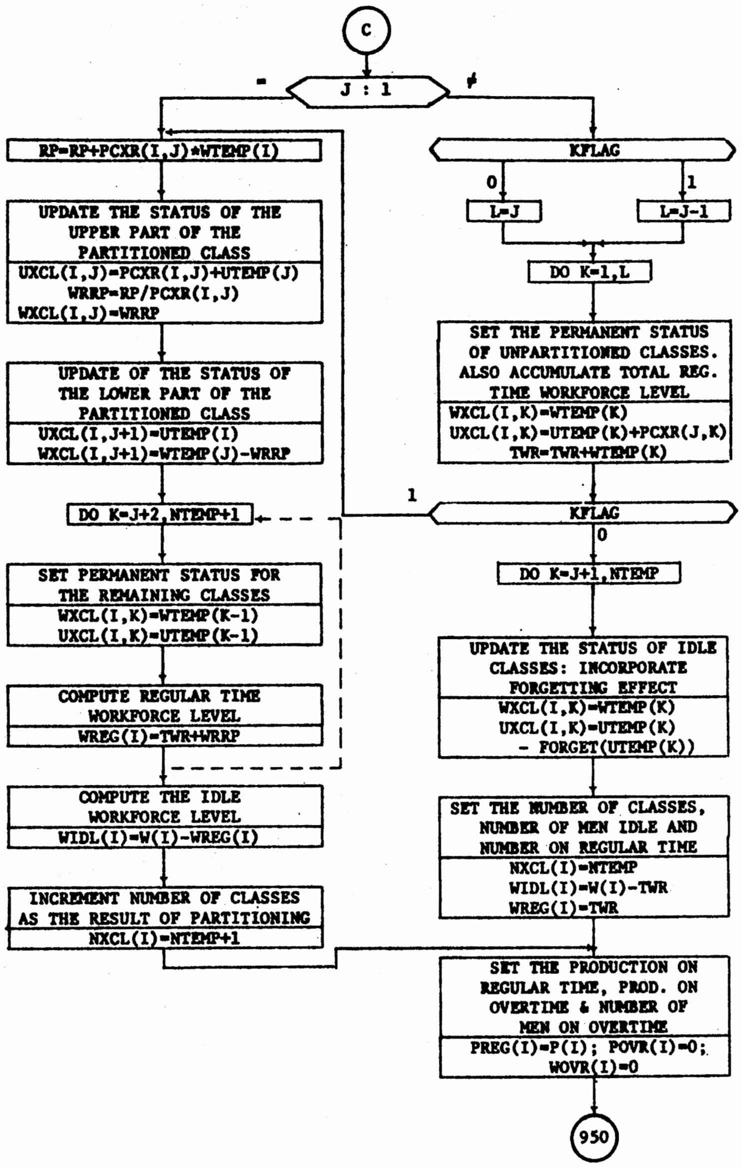

MAIN
READ PARAMETERS OF THE SEARCH ROUTINE
READ NUMBER OF PERIODS AND DEMAND PER PERIOD
READ STARTING VALUES, LOWER & UPPER BOUNDS FOR DECISION VARIABLES
READ PARAMETERS OF THE OBJECTIVE FUNCTION
READ THE SLOPES OF LEARNING, PROGRESS AND FORGETTING CURVES
READ INITIAL CUMULATIVE UNIT NUMBER; INITIAL WORKFORCE LEVEL & INITIAL INVENTORY LEVELS
READ INITIAL NUMBER OF EXPERIENCE CLASSES, SIZE AND PRODUCTIVITY MEASURE OF EACH CLASS
COMPUTE EXPONENTS FOR LEARNING, PROGRESS & FORGETTING CURVES
COMPUTE THE OVERALL AVERAGE PRODUCTIVITY FOR THE CONSTANT PRODUCTIVITY MODEL
A

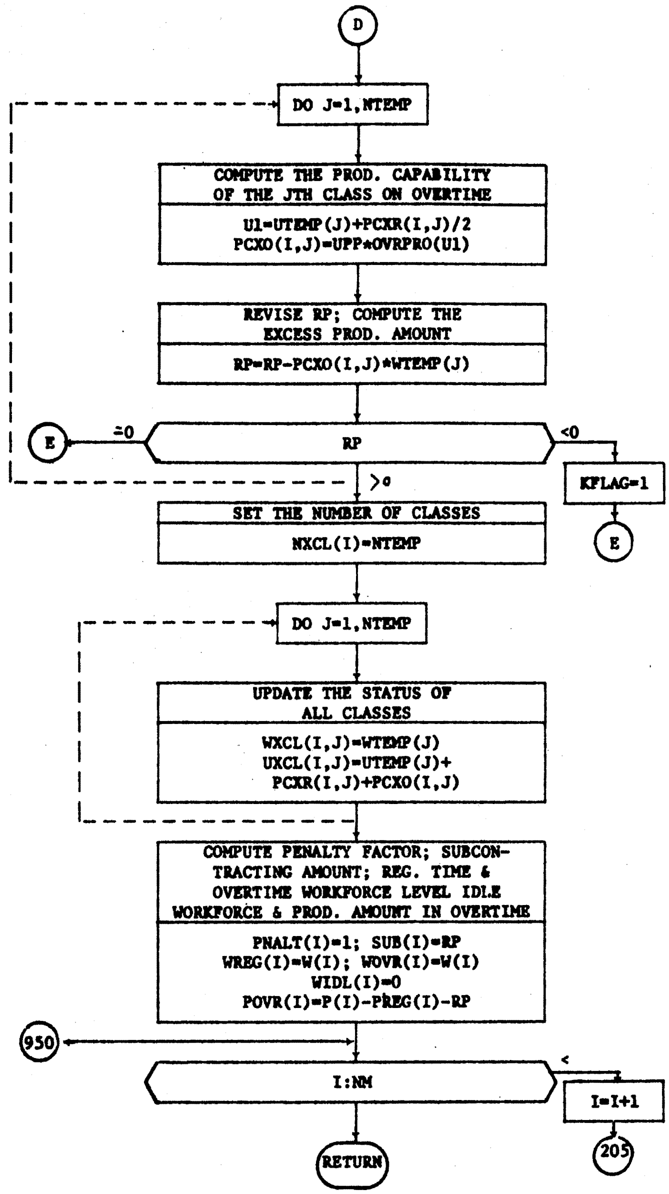

A
KDIRCT=1: NO REFERENCE TO CLASS
CALL PATS: OPTIMIZE THE CONSTANT PRODUCTIVITY MODEL
INITIALIZE W(I) & P(I): CLEAR IEFECT
KDIRECT=2; CALL FCT1 EVALUATE THE CONSTANT PRODUCTIVITY MODEL
INITIALIZE W(I) & P(I): CLEAR IEFECT
NEVAL=0
CALL PATS: OPTIMIZE THE DYNAMIC PRODUCTIVITY MODEL
KDIRCT=3
PRINT RESULTS
KDIRCT ?
1
2
3
STOP

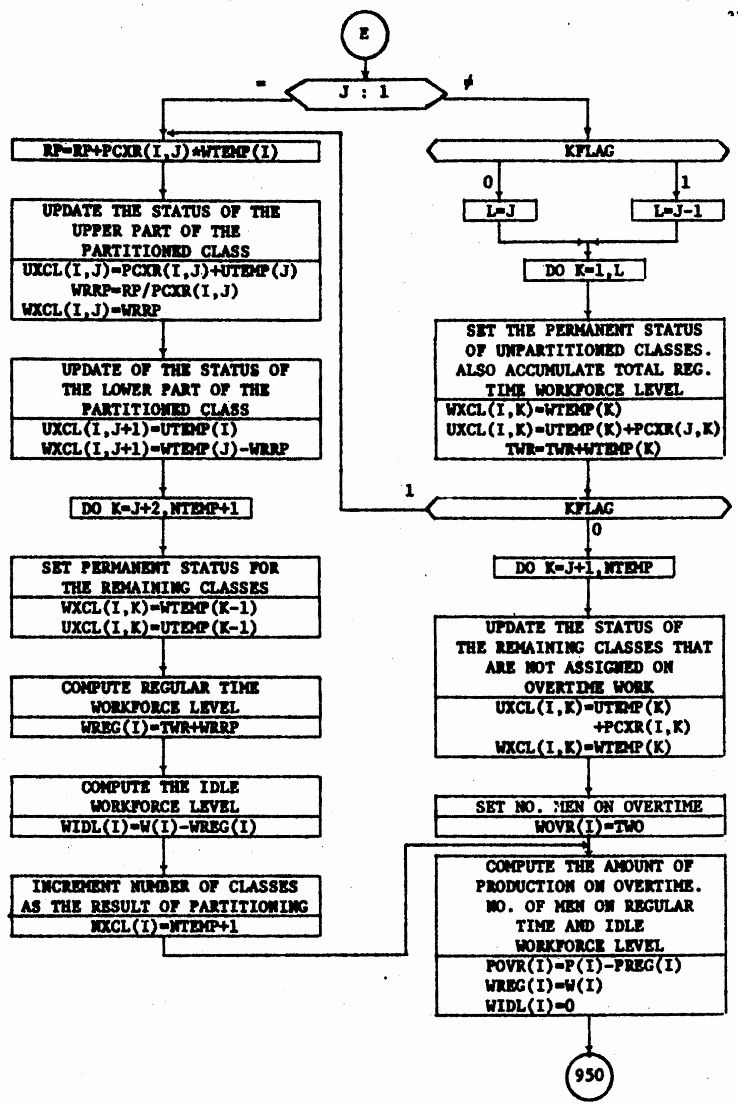

FCT1
NEVAL=NEVAL+1
KDIRECT
SET ALL W(I) & P(I) EQUAL TO THE RESPECTIVE X(I)'S
CALL CLASS: RECEIVE REGULAR TIME, OVERTIME, AND IDLE WORKFORCE LEVEL AND SUBCONTRACTING QUANTITY AND PENALTY CODES FOR ALL PERIODS
COMPUTE HIRING AND LAYOFF COSTS FOR THE FIRST PERIOD ON THE BASIS OF THE INITIAL WORKFORCE LEVEL
TOTCST=0
COMPUTE INV. LEVELS FOR ALL PERIODS
DO I=1,NM
COMPUTE REGULAR PAYROLL COST FOR PERIOD I 1
COMPUTE HIRING AND LAYOFF COST FOR PERIOD I 2
COMPUTE INVENTORY COST FOR PERIOD I 3
IF KDIRCT=1, COMPUTE OVERTIME COST USING CONSTANT PRODUCTIVITY FACTOR 4
COMPUTE CONSTRAINT VIOLATION PENALTIES IF ANY 5
IF KDIRCT=2, COMPUTE IDLE TIME & SUBCONTRACTING OR INFEASIBILITY PENALTY 6
TOTCST= 1 + 2 + 3 + 4 + 5 + 6
SN=TOTCST
RETURN


CLASS
DETECT IEFCT, SET I=IEFECT
205
W(I): W(I-1)
=
>
SET THE TEMP. NO. OF CLASSES
NTEMP=NXCL(I-1)
DO J=1,NTEMP
SET THE TEMP. STATUS
WTEMP(J)=WXCL(I-1,J)
UTEMP(J)=UXCL(I-1,J)
B
WL=W(I-1)-W(I)
TL=0
K=NXCL(I-1)
DO JJ=1,K
K=K-JJ+1
TL=TL+WXCL(I-1,J)
TL:WL
INCREMENT OF NUMBER OF CLASSES
NTEMP=NXCL(I-1)+1
DO J=1, NXCL(I-1)
JET THE STATUS OF ALL CLASSES BUT THE LAST ONE
WTEMP(J)=WXCL(I-1,J)
WTEMP(J)=UXCL(I-1,J)
SET THE STATUS OF THE LAST CLASS
WTEMP(NTEMP)=W(I)-W(I-1)
UTEMP(NTEMP)=0
B
SET THE TEMP. NO. OF CLASSES
NTEMP=J
NO PARTITION OF A CLASS
NTEMP=J-1
DO JJ=J-1
DO JJ=1,J-1
SET THE TEMP. STATUS FOR ALL CLASSES BUT THE LAST ONE
WTEMP(JJ)=WXCL(I-1,JJ)
UTEMP(JJ)=UXCL(I-1,JJ)
SET THE TEMP. STATUS FOR ALL CLASSES
WTEMP(JJ)=WXCL(I-1,JJ)
UTEMP(JJ)=UXCL(I-1,JJ)
SET THE SIZE OF LAST CLASS
WTEMP(JJ)=TL-WL
SET THE PROD. MEASURE FOR THE LAST CLASS
UTEMP(JJ)=(WTEMP(JJ)/WXCL(I-1,JJ))(UCXL(I-1,JJ))
C


B
INITALIZE VALUES FOR SUBCONTRACTING AND CONSTRAINT VIOLATION PENALTY FACTORS FOR REMAINING PERIODS
SUB(I)=0; PHALT(I)=0
KFLAG=0
SET THE REMAINING PROD. EQUAL TO THE TOTAL PROD. IN THE EFFECTIVE PERIOD
RP=P(I)
COMPUTE UPP, THE PROGRESS FACTOR FOR THE EFFECTIVE UNITS
DO J=1, NTEMP
U1=UTEMP(J)
COMPUTE REGULAR TIME PROD. CAPABILITY OF EACH MEMBER IN THE Jth CLASS
PCXR(I,J)=UPP*REGPRO(U1)
REVISE RP; FIND THE EXCESS PROD. AMOUNT
RP=RP-PCXR(I,J)*WTEMP(J)
=0
C
RP
<0
>0
KFLAG=1
C
COMPUTE TOTAL PROD. IN REG. TIME
PREG(I)=P(I)-RP
D


C
J : 1
=
≠
RP=RP+PCXR(I,J)*WTEMP(I)
UPDATE THE STATUS OF THE UPPER PART OF THE PARTITIONED CLASS
UXCL(I,J)=PCXR(I,J)+UTEMP(J)
WRRP=RP/PCXR(I,J)
WXCL(I,J)=WRRP
UPDATE OF THE STATUS OF THE LOWER PART OF THE PARTITIONED CLASS
UXCL(I,J+1)=UTEMP(I)
WXCL(I,J+1)=WTEMP(J)-WRRP
DO K=J+2,NTEMP+1
SET PERMANENT STATUS FOR THE REMAINING CLASSES
WXCL(I,K)=WTEMP(K-1)
UXCL(I,K)=UTEMP(K-1)
COMPUTE REGULAR TIME WORKFORCE LEVEL
WREG(I)=TWR+WRRP
COMPUTE THE IDLE WORKFORCE LEVEL
WIDL(I)=W(I)-WREG(I)
INCREMENT NUMBER OF CLASSES AS THE RESULT OF PARTITIONING
NXCL(I)=NTEMP+1
KFLAG
0
1
L=J
L=J-1
DO K=1,L
SET THE PERMANENT STATUS OF UNPARTITIONED CLASSES. ALSO ACCUMULATE TOTAL REG. TIME WORKFORCE LEVEL
WXCL(I,K)=WTEMP(K)
UXCL(I,K)=UTEMP(K)+PCXR(J,K)
TWR=TWR+WTEMP(K)
1
KFLAG
0
DO K=J+1,NTEMP
UPDATE THE STATUS OF IDLE CLASSES: INCORPORATE FORGETTING EFFECT
WXCL(I,K)=WTEMP(K)
UXCL(I,K)=UTEMP(K)
- FORGET(UTEMP(K))
SET THE NUMBER OF CLASSES, NUMBER OF MEN IDLE AND NUMBER ON REGULAR TIME
NXCL(I)=NTEMP
WIDL(I)=W(I)-TWR
WREG(I)=TWR
SET THE PRODUCTION ON REGULAR TIME, PROD. ON OVERTIME & NUMBER OF MEN ON OVERTIME
PREG(I)=P(I); POVR(I)=0;
WOVR(I)=0
950


D
DO J=1,NTEMP
COMPUTE THE PROD. CAPABILITY OF THE JTH CLASS ON OVERTIME
U1=UTEMP(J)+PCXR(I,J)/2
PCXO(I,J)=UPP*OVRPRO(U1)
REVISE RP; COMPUTE THE EXCESS PROD. AMOUNT
RP=RP-PCXO(I,J)*WTEMP(J)
E
=0
RP
<0
KFLAG=1
E
>0
SET THE NUMBER OF CLASSES
NXCL(I)=NTEMP
DO J=1,NTEMP
UPDATE THE STATUS OF ALL CLASSES
WXCL(I,J)=WTEMP(J)
UXCL(I,J)=UTEMP(J)+
PCXR(I,J)+PCXO(I,J)
COMPUTE PENALTY FACTOR; SUBCONTRACTING AMOUNT; REG. TIME & OVERTIME WORKFORCE LEVEL IDLE WORKFORCE & PROD. AMOUNT IN OVERTIME
PNALT(I)=1; SUB(I)=RP
WREG(I)=W(I); WOVR(I)=W(I)
WIDL(I)=0
POVR(I)=P(I)-PREG(I)-RP
950
I:NM
<
I=I+1
205
RETURN


E
J : 1
=
≠
RP=RP+PCXR(I,J)*WTEMP(I)
UPDATE THE STATUS OF THE UPPER PART OF THE PARTITIONED CLASS
UXCL(I,J)=PCXR(I,J)+UTEMP(J)
WRRP=RP/PCXR(I,J)
WXCL(I,J)=WRRP
UPDATE OF THE STATUS OF THE LOWER PART OF THE PARTITIONED CLASS
UXCL(I,J+1)=UTEMP(I)
WXCL(I,J+1)=WTEMP(J)-WRRP
DO K=J+2,NTEMP+1
SET PERMANENT STATUS FOR THE REMAINING CLASSES
WXCL(I,K)=WTEMP(K-1)
UXCL(I,K)=UTEMP(K-1)
COMPUTE REGULAR TIME WORKFORCE LEVEL
WREG(I)=TWR+WRRP
COMPUTE THE IDLE WORKFORCE LEVEL
WIDL(I)=W(I)-WREG(I)
INCREMENT NUMBER OF CLASSES AS THE RESULT OF PARTITIONING
NXCL(I)=NTEMP+1
KFLAG
0
1
L=J
L=J-1
DO K=1,L
SET THE PERMANENT STATUS OF UNPARTITIONED CLASSES. ALSO ACCUMULATE TOTAL REG. TIME WORKFORCE LEVEL
WXCL(I,K)=WTEMP(K)
UXCL(I,K)=UTEMP(K)+PCXR(J,K)
TWR=TWR+WTEMP(K)
1
KFLAG
0
DO K=J+1,NTEMP
UPDATE THE STATUS OF THE REMAINING CLASSES THAT ARE NOT ASSIGNED ON OVERTIME WORK
UXCL(I,K)=UTEMP(K)
+PCXR(I,K)
WXCL(I,K)=WTEMP(K)
SET NO. MEN ON OVERTIME
WOVR(I)=TWO
COMPUTE THE AMOUNT OF PRODUCTION ON OVERTIME. NO. OF MEN ON REGULAR TIME AND IDLE WORKFORCE LEVEL
POVR(I)=P(I)-PREG(I)
WREG(I)=W(I)
WIDL(I)=0
950

APPENDIX F

FORTRAN PROGRAM LISTINGS

```
C         FORTRAN CODES FOR AGGREGATE PLANNING MODELS
C                WITH DYNAMIC PRODUCTIVITY
C
C                      PROGRAMMED BY
C                      B. KHOSHNEVIS
C
C                       APRIL  1979
C
C
C
C              *****************************
C              *                           *
C              *  M A I N   P R O G R A M  *
C              *                           *
C              *    F O R   M O D E L -I   *
C              *                           *
C              *****************************
C
C
C
C          FORTRAN CODE FOR AGGREGATE PLANNING OF
C                 LONG CYCLE PRODUCTS
C
C
C
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /SHARE/ X(20),N,NM,SN
      COMMON /SRCHP/ XMAX(20),XMIN(20),LIM,KPRESS,NO
      COMMON /VALUE/ NEVAL
      COMMON /PRODY/ UCT(20),BCT(20),ICHK(20),INDEX(20),R(20),
     1CUMO,TRSHL,EPSY,AK1,AK2,AK3,ZADJ
      COMMON /THREE/ W(20),P(20),APROD(20),WO,KFLAG
      COMMON /OBJFN/ D(20),DPCST(20),HLCST(20),OVCST(20),DINV(20),
     1STCST(20),TC(20),OINV,TOTCST,C1,C2,C3,C4,C5,C6,C7,C8,C9,C10
      COMMON /CURVE/ B,F
C
C  SET THE INPUT-OUTPUT UNIT NUMBERS
      IN=5
      NO=6
C  READ IN THE MASTER CONTROL DATA - SEE GLOSSARY FOR DEFINITIONS
      READ(IN,1)KPRESS,LIM,NM
      N=2*NM
C  READ IN THE MONTHLY DEMAND LEVELS
      READ(IN,2) (D(I),I=1,NM)
C  READ IN THE INITIAL, STARTING VALUES FOR THE DECISION VARIABLES
C  PRODUCTION RATES AND WORKFORCE LEVELS
      READ(IN,3) (X(I),I=1,N)
C  READ IN THE UPPER AND LOWER LIMITS FOR DECISION VARIABLES
      DO 10 I=1,N
   10 READ(IN,4) XMAX(I),XMIN(I)
C  READ IN THE VALUES OF THE COST PARAMETERS IN THE OBJECTIVE FUNCTION
      READ(IN,5) C1,C2,C3,C4,C5,C6,C7,C8,C9,C10
C  READ IN THE PARAMETERS OF THE IMPROVEMENT CURVE
      READ(IN,6)BS,F
C  READ IN THE PARAMETERS OF MANPOWER CHANGE DISRUPTIONS AND
C  COMPOUNDED DISRUPTION EFFECTS
      READ(IN,7) AK1,AK2,AK3,TRSHL,EPSY
C  READ IN THE INITIAL CUMULATIVE UNIT NUMBER,INITIAL WORKFORCE
C  AND INITIAL INVENTORY LEVELS
      READ(IN,8) CUMO,WO,OINV
```

```
C  COMPUTE THE EXPONENT VALUE OF THE IMPROVEMENT CURVE
      TWO=2.
      B=-DLOG(S)/DLOG(TWO)
C
      DO 100 I=1,NM
      INDEX(I)=0
  100 ICHK(I)=0
      KFLAG=1
C  COMPUTE THE OVERALL AVERAGE PRODUCTIVITY ON THE BASIS OF THE
C  TOTAL DEMAND, THE INITIAL CUMULATIVE UNIT NUMBER AND THE CURRENT
C  IMPROVEMENT CURVE PARAMETERS--PURPOSE: TO BE USED FOR EVALUATION
C  OF THE CONSTANT PRODUCTIVITY MODEL
C
      TDND=0.
      DO 200 I=1,NM
  200 TDND=TDND+D(I)
      T=TDND+CUMO
      TMHR=BLOCK(CUMO,T)
      CAPR=TDND/TMHR
C
C  SET THE AVERAGE MONTHLY PRODUCTIVITY LEVELS EQUAL TO THE CONSTANT,
C  OVERALL AVERAGE PRODUCTIVITY LEVEL
      DO 300 I=1,NM
  300 APROD(I)=CAPR
C
C  OPTIMIZE THE CONSTANT PRODUCTIVITY MODEL AND PRINT THE RESULTS
      CALL PATS
      KPRINT=1
      GO TO 800
  701 KFLAG=0
      KPRINT=2
C
C  INITIALIZE THE DECISION VARIABLES TO CLEAR 'IEFECT' --THE COPY
C  OF THE ORIGINAL VALUES IS SAVED IN X(.)
      DO 400 I=1,NM
      W(I)=0.
  400 P(I)=0.
C  EVALUATE THE RESULTS OF THE CONSTANT PRODUCTIVITY MODEL BY AN
C  INDIRECT CALL TO 'PROCTV' ,THROUGH 'FCT1' . -PRINT THE RESULTS
      CALL FCT1
      GO TO 800
C
C  SUPPRESS THE EFFECT OF MANPOWER  DISRUPTION,INITIALIZE VARIABLES
C  AND OPTIMIZE THE 'EBERT' MODEL--PRINT THE RESULTS
  702 KFLAG=2
      KPRINT=3
      DO 500 I=1,NM
      W(I)=0.
  500 P(I)=0.
      NEVAL=0
      CALL PATS
      GO TO 800
C
C  EVALUATE THE RESULTS OF OPTIMIZATION OF THE EBERT MODEL--PRINT RESULT
  703 KFLAG=0
      KPRINT=4
      DO 600 I=1,NM
      W(I)=0.
  600 P(I)=0.
      CALL FCT1
      GO TO 800
C
C  PUT THE DISRUPTION BACK TO EFFECT AND OPTIMIZE THE DYNAMIC
C  PRODUCTIVITY MODEL -- PRINT THE RESULTS
  704 KPRINT=5
```

```
      DO 700 I=1,NM
      W(I)=0.
  700 P(I)=0.
      NEVAL=0
      CALL PATS
  800 DO 900 I=1,NM
  900 WRITE(NO,1000) I,D(I),W(I),P(I),DINV(I),APROD(I),INDEX(I),ICHK(I)
C     DO 910 I=1,NM
  910 WRITE(NO,1001) I,DPCST(I),HLCST(I),OVCST(I),STCST(I),TC(I)
      WRITE(NO,1002) TOTCST
C
C  DIRECT THE CONTROL TO THE PROPER STAGE IN THE ABOVE SEQUENCE
      GO TO (701,702,703,704,999),KPRINT
C
    1 FORMAT(3I5)
    2 FORMAT(5X,F10.0)
    3 FORMAT(F10.0)
    4 FORMAT(5X,2F10.0)
    5 FORMAT(10F8.3)
    6 FORMAT(5X,F5.4,F10.3)
    7 FORMAT(5F10.5)
    8 FORMAT(5X,3F10.0)
 1000 FORMAT(1X,I2,2X,4(F5.1,2X),F10.3,2X,2(I2,2X))
 1001 FORMAT(8X,I2,5(F11.2,2X))
 1002 FORMAT(35X,F10.2)
  999 STOP
      END
C
C
C
C           *********************************
C           *                               *
C           * OBJECTIVE FUNCTION SUBROUTINE *
C           *                               *
C           *********************************
C
C
      SUBROUTINE FCT1
C
C
C  THIS ROUTINE EVALUATES THE RESPONSE OF THE COST FUNCTION AGAINST
C  THE VALUES OF THE DECISION VARIABLES AND AVERAGE PRODUCTIVITY LEVELS
C  RECEIVED EITHER FROM THE MAIN PROGRAM OR THE PATTERN SEARCH ROUTINE.
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /SHARE/ X(20),N,NM,SN
      COMMON /VALUE/ NEVAL
      COMMON /THREE/ W(20),P(20),APROD(20),W0,KFLAG
      COMMON /OBJFN/ D(20),DPCST(20),HLCST(20),OVCST(20),DINV(20),
     1STCST(20),TC(20),OINV,TOTCST,C1,C2,C3,C4,C5,C6,C7,C8,C9,C10
C  INCREMENT THE NUMBER OF OBJECTIVE FUNCTION EVALUATIONS
      NEVAL=NEVAL+1
C  THE FOLLOWING STM DIRECTS THE CONTROL TO EVALUATION OF EITHER
C  CONSTANT PRODUCTIVITY; OR EBERT AND DYNAMIC PRODUCTIVITY MODELS.
C  THE VALUES OF THE DECISION VARIABLES ARE FIRST SET FOR EVALUATION
C  OF THE CONSTANT PRODUCTIVITY MODEL
C
      IF(KFLAG.NE.1) GO TO 200
      J=0
      K=N-1
      DO 100 L=1,K,2
      J=J+1
      W(J)=X(L)
      P(J)=X(L+1)
  100 CONTINUE
      GO TO 300
```

```
C  RECEIVE THE MONTHLY LEVELS OF THE AVERAGE PRODUCTIVITIES ASSOCIATED
C  ASSOCIATED WITH THE CURRENT DESICION -- 'PROCTV' PROVIDES THIS
C
  200 CALL PROCTV
C  COMPUTE MONTHLY AND TOTAL COSTS RELATED TO THE CURRENT DESICION
  300 TOTCST=0.
      HLCST(1)=C2*(W(1)-W0)**2
      DINV(1)=P(1)+DINV-D(1)
      DO 400 I=2,NM
  400 DINV(I)=P(I)+DINV(I-1)-D(I)
      DO 500 I=1,NM
      DPCST(I)=C1*W(I)
      IF(I.NE.1) HLCST(I)=C2*(W(I)-W(I-1))**2
      OVCST(I)=C3*(P(I)-APROD(I)*W(I))**2+C5*P(I)-C6*W(I)
      STCST(I)=C7*(DINV(I)-C8)**2
C  INCORPORATE THE PENALTY FOR CONSTRAINT VIOLATION
      IF(DINV(I).GT.900) PNLT=100000.
      PNLT=0.
      TC(I)=DPCST(I)+HLCST(I)+OVCST(I)+STCST(I)+PNLT
      TOTCST=TOTCST+TC(I)
  500 CONTINUE
C
C  DIRECT THE THE RESPONSE LEVEL TO THE CALLING ROUTINE
      SN=TOTCST
      RETURN
      END
C
C
C         ************************
C         *                      *
C         * DYNAMIC PRODUCTIVITY *
C         *      SUBROUTINE      *
C         *                      *
C         ************************
C
C
      SUBROUTINE PROCTV
C
C
C  THIS ROUTINE COMPUTES THE MONTHLY AVERAGE PRODUCTIVITY LEVELS
C  FOR ANY SET OF DECISION VARIABLES RECEIVED FROM THE CALLING ROUTINE.
C  (SEE THE GLOSSARY FOR VARIABLE DEFINITIONS; AND RELATED FLOW-CHARTS)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION CUM(20),TNMP(20),UC(20),BC(20),UCDIF(20),UCDR(20)
      COMMON /SHARE/ X(20),N,NM,SN
      COMMON /PRODY/ UCT(20),BCT(20),ICHK(20),INDEX(20),R(20),
     1CUMO,TRSHL,EPSY,AK1,AK2,AK3,ZADJ
      COMMON /THREE/ W(20),P(20),APROD(20),W0,KFLAG
      COMMON /CURVE/ B,F
C
C  THE FOLLOWING STNS (THRU STM 200+1) DETECT THE EFFECTIVE PERIOD
C  AT WHICH THE FIRST CHANGE IN THE LEVEL OF A DECISION VARIABLE
C  IS MADE BY THE SEARCH ROUTINE
      ICNTRL=1
      J=0
      IP=1
      K=N-1
      DO 200 L=1,K,2
      J=J+1
      IF(W(J)-X(L)) 120,110,120
  120 W(J)=X(L)
      GO TO (130,110),ICNTRL
  130 ICNTRL=2
      IEFCT=J
  110 IF(P(J)-X(L+1)) 140,100,140
```

```
  140 P(J)=X(L+1)
      IP=2
  100 GO TO (200,145),IP
  145 IF(J-1) 160,150,160
  150 PCUM=CUMO
      GO TO 170
  160 PCUM=CUM(J-1)
  170 CUM(J)=PCUM+P(J)
      GO TO (180,200),ICNTRL
  180 ICNTRL=2
      IEFCT=J
  200 CONTINUE
      I=IEFCT
C
C  DIRECT THE CONTROL TO COMPUTATION OF AVERAGE PRODUCTIVITIES FOR
C  EITHER EBERT OR DYNAMIC PRODUCTIVITY MODELS
      IF(KFLAG.EQ.2) GO TO 380
      IF(I-1) 300,250,300
  250 WOLD=WO
      GO TO 350
C  COMPUTE THE UNDELUTED MANPOWER TUNOVER RATIO FOR THE MANPOWER LEVEL
C  IN THE CURRENT PERIOD -- CHECK THE SIGNIFICANCE OF DISRUPTION
C
  300 WOLD=W(I-1)
  350 ADJ=ZADJ
      IF(I.NE.1) ADJ=0.
      TNMP(I)=(DABS(W(I)-WOLD))/DMAX1(W(I),WOLD)
      IF(TNMP(I).LE.TRSHL)GO TO 380
      INDEX(I)=1
C
C  CHECK FOR MANPOWER REDUCTION OR ADDITION -- DELUTE THE TURNOVER RATIO
      IF(W(I).LT.WOLD)GO TO 370
      R(I)=AK1*(TNMP(I)-AK2)+ADJ
      GO TO 400
  370 R(I)=AK1*(AK3*TNMP(I)-AK2)+ADJ
      GO TO 400
  380 INDEX(I)=0
  400 IF(I.EQ.1.AND.INDEX(I).EQ.0) GO TO 600
      IF(I.EQ.1.AND.INDEX(I).EQ.1) GO TO 650
C  TRACE BACK FOR DETECTION OF THE ORIGINATION OF THE COMPOUNDED
C  DISRUPTION
      JJ=I-1
      DO 500 M=1,JJ
      J=JJ-M+1
      IF(ICHK(J)) 500,550,500
  500 CONTINUE
  550 J=J+1
      IF(I-J) 700,590,700
  590 IF(INDEX(I)) 650,600,650
  600 IF(I-1) 620,610,620
  610 UY=CUMO+1.
      GO TO 640
  620 UY=CUM(I-1)+1.
  640 UZ=CUM(I)
C
C  NO DISRUPTION -- COMPUTE THE UNIT COST AT THE END OF THE PERIOD
C  (ON THE ORIGINAL CURVE)
      UCT(I)=UNIT(UZ)
C  COMPUTE THE BLOCK COST OF THE CURRENT PERIOD(ON ORIGINAL CURVE)
      BCT(I)=BLOCK(UY,UZ)
C  SET THE UNIT COST DIFFERENCE AT THE END OF THIS PERIOD EQUAL TO
C  ZERO, BECAUSE NO DISRUPTION HAS OCCURED AND THERE IS NO CONTINUING
C  EFFECT OF THE PREVIOUS ONES(IF ANY)
      UCDR(I)=0.
      ICHK(I)=0
```

```
      GO TO 900
C
C  STMS 650 THRU 730+4  COMPUTE THE SINGLE DISRUPTION EFFECT OCCURING
C  AT THE BEGINNING OF THE CURRENT PERIOD
  650 J=I
  700 IF(I-1) 720,710,720
  710 UY=CUM0+1.
      UX=UY-1.
      GO TO 730
  720 UY=CUM(I-1)+1.
      UX=CUM(J-1)
  730 UZ=CUM(I)
      UZN=UZ-UX
      UYN=UY-UX
      UC(J)=(1.-R(J))*UNIT(UZ)+R(J)*UNIT(UZN)
      BC(J)=(1.-R(J))*BLOCK(UY,UZ)+R(J)*BLOCK(UYN,UZN)
C
C  FOLLOWING STMS (THRU STM 850+4) DETERMINE THE COMPOUNDED DISRUPTION
C  EFFECT FOR THE CURRENT PERIOD THROUGH RECURSIVE COMPUTATIONS.
      IF(I-J) 740,850,740
  740 JJ=J+1
      DO 800 K=JJ,I
      IF(INDEX(K)) 765,755,765
  755 UC(K)=UC(K-1)
      BC(K)=BC(K-1)
      GO TO 800
  765 UX=CUM(K-1)
      UZN=UZ-UX
      UZY=UY-UX
      UC(K)=(1.-R(K))*UC(K-1)+R(K)*UNIT(UZN)
      BC(K)=(1.-R(K))*BC(K-1)+R(K)*BLOCK(UZY,UZN)
  800 CONTINUE
  850 UCT(I)=UC(I)
      BCT(I)=BC(I)
      UCDIF(I)=UCT(I)-UNIT(UZ)
      UCDR(I)=UCDIF(I)/(UCT(I))
C  CHECK TO SEE IF THE DISRUPTION EFFECT CARRIED OVER THE NEXT PERIOD
C  IS SIGNIFICANT
      IF(UCDR(I).LE.EPSY) GO TO 870
      ICHK(I)=1
      GO TO 900
  870 ICHK(I)=0
  900 IF(I.EQ.NM) GO TO 999
C  INCREMENT THE PERIOD NUMBER -- REPEAT THE PROCESS FOR THE NEXT PERIOD
      I=I+1
      IF(KFLAG.EQ.2) GO TO 380
      GO TO 300
C
C  COMPUTE THE FINAL VALUES OF THE MONTHLY AVERAGE PRODUCTIVITY LEVELS-
C  DIRECT THE VALUES TO THE CALLING ROUTINE
  999 DO 1000 I=IEFCT,NM
 1000 APROD(I)=P(I)/(BCT(I))
      ZADJ=UCDR(1)
      RETURN
      END
C
C
C     ****************
      FUNCTION UNIT(UA)
C     ****************
C
C  THIS FUNCTION COMPUTES THE UNIT COST FOR ANY CUMULATIVE UNIT NUMBER
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /CURVE/ B,F
```

```
      IF((UA**B).LT.1) GO TO 1
      UNIT=F/(UA**B)
      RETURN
    1 UNIT=F
      RETURN
      END
C
C
C     ********************
      FUNCTION BLOCK(UA,UB)
C     ********************
C
C  THIS FUNCTION COMPUTES THE UNIT COST FOR ANY RANGE OF CUMULATIVE
C  UNIT NUMBERS , UA THRU UB
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /CURVE/ B,F
      IF(UA-UB) 2,1,2
    1 BLOCK=0
      RETURN
    2 BLOCK=(F/(1.-B))*((UB+.5)**(1.-B)-(UA-.5)**(1.-B))
      RETURN
      END
      SUBROUTINE PATS
```

```
C               ******************************
C               *                            *
C               *  M A I N   P R O G R A M   *
C               *                            *
C               *    F O R   M O D E L-II    *
C               *                            *
C               ******************************
C
C
C
C           FORTRAN CODE FOR AGGREGATE PLANNING OF
C                   SHORT CYCLE PRODUCTS
C
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /SHARE/ X(40),N,NM,SN
      COMMON /SRCHP/ XMAX(20),XMIN(20),LIM,KPRESS,NO
      COMMON /VALUE/ NEVAL
      COMMON /CCLSS/ NXCL(20),WXCL(20,50),WXCLO(10),UXCLO(10),CUMO
     1,NXCLO
      COMMON /THREE/ W(20),P(20),WREG(20),WOVR(20),WIDL(20),
     1PREG(20),POVR(20),SUB(20),PNALT(20),WO
      COMMON /OBJFN/ D(20),DPCST(20),HLCST(20),OVCST(20),DINV(20),
     1STCST(20),OINV,TOTCST,C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,KDIRCT
      COMMON /LEARN/ B,F
      COMMON /PROGS/ BP
      COMMON /FORGT/ BF,F1
      COMMON /OVPCT/ OPCNT
C  SET THE INPUT-OUTPUT UNIT NUMBER
      NI=5
      NO=6
C  READ IN THE MASTER CONTROL DATA -- SEE GLOSSARY FOR DEFINITIONS
      READ(IN,1) KPRESS,LIM,NM
      N=2*NM
C  READ IN THE MONTHLY DEMAND LEVELS
      READ(IN,2) (D(I),I=1,NM)
C  READ IN THE INITIAL, STARTING VALUES FOR THE DECISION VARIABLES
      READ(IN,3) (X(I),I=1,N)
C  READ IN THE UPPER AND THE LOWER LIMITS ON THE DECISION VARIABLES
      DO 10 I=1,N
   10 READ(IN,4) XMAX(I),XMIN(I)
C  READ IN THE VALUES OF THE COST PARAMETERS IN THE OBJECTIVE FUNCTION
      READ(IN,5) C1,C2,C3,C4,C5,C6,C7,C8,C9,C10
C  READ IN THE INITIAL NUMBER OF EXPERIENCE CLASSES,INITIAL CUMULATIVE
C  OUTPUT UNIT NUMBER, AND INITIAL INVENTORY LEVEL
      READ(IN,6) NXCLO,CUMO,OINV
C  READ IN THE SIZE AND PRODUCTIVITY MEASURE OF EACH INITIAL CLASS
      DO 20 I=1,NXCLO
   20 READ(IN,7) WXCLO(I),UXCLO(I)
C  READ IN THE SLOPES OF LEARNING, PROGRESS, AND FORGETTING CURVES
C  ;AND THE BASE UNIT COST
      READ(IN,8) SL,SP,SF,F
      F1=F
C  READ IN THE LENGTH OF OVERTIME WORK AS A PERCENTAGE OF THE REGULAR
C  TIME WORK
      READ(IN,9) OPCNT
C  COMPUTE THE EXPONENT VALUES FOR THE THREE CURVES
      TWO=2.
      B=-DLOG(SL)/DLOG(TWO)
      BP=-DLOG(SP)/DLOG(TWO)
      BF=-DLOG(SF)/DLOG(TWO)
```

```
C  COMPUTE THE TOTAL INITIAL MANPOWER LEVEL
      W0=0.
      DO 100 I=1,NXCLO
  100 W0=W0+WXCLO(I)
C
C  COMPUTE SLOPE AND EXPONENT OF THE OVERALL IMPROVEMENT CURVE, AND ON
C  THE BASIS OF THIS CURVE AND THE TOTAL DEMAND, COMPUTE THE OVERALL
C  AVERAGE(CONSTANT) PRODUCTIVITY LEVEL -- PURPOSE: TO BE USED IN THE
C  EVALUATION OF THE CONSTANT PRODUCTIVITY MODEL
C
      SS=SL*SP
      BC=-DLOG(SS)/DLOG(TWO)
      TDND=0.
      DO 150 I=1,NM
  150 TDND=TDND+D(I)
      T=CUM0+TDND
      TMHR=(F/(1.-BC))*((T+.5)**(1.-BC)-(CUM0-.5)**(1.-BC))
      C4=TDND/TMHR
      KDIRCT=1
      NEVAL=0
C  OPTIMIZE THE CONSTANT PRODUCTIVITY MODEL -- PRINT THE RESULTS
      CALL PATS
      GO TO 450
C  INITIALIZE THE DECISION VARIABLES TO CLEAR IEFCT - A COPY OF THE
C  ORIGINAL VALUES IS SAVED IN X(.)
  250 DO 300 I=1,NM
      W(I)=0
  300 P(I)=0.
      KDIRCT=2
C
C  EVALUATE THE RESULTS OF THE CONSTANT PRODUCTIVITY MODEL BY AN
C  INDIRECT CALL TO 'CLASS', THROUGH 'FCT1'--PRINT THE RESULS
      CALL FCT1
      GO TO 550
C  INITIALIZE THE DECISION VARIABLES AND OPTIMIZE THE DYNAMIC
C  PRODUCTIVITY MODEL  -- PRINT THE RESULTS
  350 DO 400 I=1,NM
      W(I)=0.
  400 P(I)=0.
      NEVAL=0
      CALL PATS
      KDIRCT=3
      GO TO 550
  450 DO 500 I=1,NM
  500 WRITE(NO,1000) I,D(I),W(I),P(I),WOVR(I),WIDL(I),SUB(I)
      GO TO 700
  550 DO 600 I=1,NM
  600 WRITE(NO,1001) I,D(I),W(I),P(I),DINV(I),WREG(I),WOVR(I),
     1WIDL(I),PREG(I),POVR(I),SUB(I)
      DO 650 I=1,NM
      SUCST=C10*SUB(I)
  650 WRITE(NO,1003) I,DPCST(I),HLCST(I),OVCST(I),STCST(I),SUCST
  700 WRITE(NO,1002) TOTCST
C
C  DIRECT THE CONTROL TO THE PROPER STAGE IN THE ABOVE SEQUENCE
      GO TO (250,350,999),KDIRCT
C
    1 FORMAT(3I5)
    2 FORMAT(5X,F10.0)
    3 FORMAT(F10.0)
    4 FORMAT(5X,2F10.0)
    5 FORMAT(10F8.3)
    6 FORMAT(5X,I5,2F10.0)
    7 FORMAT(2F5.0)
    8 FORMAT(5X,3F5.4,F10.3)
```

```
    9 FORMAT(F5.4)
 1000 FORMAT(2X,I2,6F11.1)
 1001 FORMAT(2X,I3,10F8.0)
 1002 FORMAT(15X,F14.2)
 1003 FORMAT(5X,I2,5F10.0)
  999 STOP
      END
C
C
C          **********************************
C          *                                *
C          * OBJECTIVE FUNCTION SUBROUTINE *
C          *                                *
C          **********************************
C
C
      SUBROUTINE FCT1
C
C
C  THIS ROUTINE EVALUATES THE RESPONSE OF THE COST FUNCTION AGAINST
C  THE VALUES OF THE INDEPENDENT DECISION VARIABLES AND THE VALUES
C  OF THE DEPENDENT VARIABLES COMPUTED BY THE 'CLASS' ROUTINE.
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /SHARE/ X(40),N,NM,SN
      COMMON /VALUE/ NEVAL
      COMMON /THREE/ W(20),P(20),WREG(20),WOVR(20),WIDL(20),
     1PREG(20),POVR(20),SUB(20),PNALT(20),WO
      COMMON /OBJFN/ D(20),DPCST(20),HLCST(20),OVCST(20),DINV(20),
     1STCST(20),OINV,TOTCST,C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,KDIRCT
      COMMON /OVPCT/ OPCNT
      NEVAL=NEVAL+1
C  THE FOLLOWING STM DIRECTS THE CONTROL TO EVALUATION OF COST FUNCTION
C  FOR EITHER CONSTANT PRODUCTIVITY, OR DYNAMIC PRODUCTIVITY MODELS.
C  THE VALUES OF DECISION VARIABLES ARE FIRST RESET FOR EVALUATION OF
C  CONSTANT PRODUCTIVITY MODEL.
      IF(KDIRCT.NE.1) GO TO 200
      J=0
      K=N-1
      DO 100 L=1,K,2
      J=J+1
      W(J)=X(L)
      P(J)=X(L+1)
  100 CONTINUE
      GO TO 300
C
C  RECEIVE THE MONTHLY LEVELS OF INDEPENDENT AND DEPENDENT VARIABLES
C  FROM THE CLASS ROUTINE
  200 CALL CLASS
C
C  COMPUTE MONTHLY AND TOTAL COSTS ASSOCIATED WITH THE CURRENT DECISION
C  (SEE GLOSSARY FOR DEFINITIONS)
C
  300 TOTCST=0.
      HLCST(1)=C2*(W(1)-WO)**2
      DINV(1)=P(1)+OINV-D(1)
      DO 400 I=2,NM
  400 DINV(I)=P(I)+DINV(I-1)-D(I)
      DO 500 I=1,NM
      DPCST(I)=C1*W(I)
      IF(I.NE.1) HLCST(I)=C2*(W(I)-W(I-1))**2
      IF(KDIRCT.EQ.1) GO TO 490
      GO TO 495
  490 PER=P(I)-C4*W(I)
      IF(PER) 491,491,492
```

```
  491 WOVR(I)=0.
      SUB(I)=0.
      WIDL(I)=-PER/C4
      GO TO 495
  492 SB=PER-OPCNT*C4*W(I)
      IF(SB) 493,493,494
  493 WOVR(I)=PER/(OPCNT*C4)
      SUB(I)=0.
      GO TO 495
  494 WOVR(I)=W(I)
      SUB(I)=SB
      WIDL(I)=0.
  495 OVCST(I)=C1*WOVR(I)
      STCST(I)=C7*(DINV(I)-C8)**2
      TOTCST=TOTCST+DPCST(I)+HLCST(I)+OVCST(I)+STCST(I)+C10*SUB(I)
     1+C3*(P(I)-C5)**2
  500 CONTINUE
C
C  DIRECT THE RESPONSE LEVEL TO THE CALLING ROUTINE
      SN=TOTCST
      RETURN
      END
C
C
C
C          ************************
C          *                      *
C          * DYNAMIC PRODUCTIVITY *
C          *      SUBROUTINE      *
C          *                      *
C          ************************
C
C
      SUBROUTINE CLASS
C
C
C  THIS ROUTINE COMPUTES THE MONTHLY LEVELS OF THE DEPENDENT DECISION
C  VARIABLES AND THE STATUS OF EXPERIENCE CLASSES FOR ANY SET OF
C  INDEPENDENT DECISION VARIABLES RECEIVED FROM THE CALLING ROUTINE.
C  (SEE  GLOSSARY FOR VARIABLE DEFINITIONS AND THE RELATED FLOWW-CHARTS)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION CUM(20),WXX(50),UXX(50),WTEMP(50),UTEMP(50),
     1PCXR(20,50),PCXO(20,50),UXCL(20,50)
      COMMON /SHARE/ X(40),N,NM,SN
      COMMON /CCLSS/ NXCL(20),WXCL(20,50),WXCLO(10),UXCLO(10),CUMO
     1,NXCLO
      COMMON /THREE/ W(20),P(20),WREG(20),WOVR(20),WIDL(20),
     1PREG(20),POVR(20),SUB(20),PNALT(20),WO
      COMMON /LEARN/ B,F
      COMMON /PROGS/ BP
      COMMON /FORGT/ BF,F1
      COMMON /OVPCT/ OPCNT
      TOL=1.
C
C  THE FOLLOWING STMS (THRU STM 200+1) DETECT THE EFFECTIVE PERIOD
C  AT WHICH THE FIRST CHANGE ON THE LEVEL OF A DECISION VARIABLE IS
C  MADE BY THE SEARCH ROUTINE
      ICNTRL=1
      IP=1
      J=0
      K=2*NM-1
      DO 200 L=1,K,2
      J=J+1
      IF(W(J)-X(L)) 120,110,120
```

```
  120 W(J)=X(L)
      GO TO (130,110),ICNTRL
  130 ICNTRL=2
      IEFCT=J
  110 IF(P(J)-X(L+1)) 140,100,140
  140 P(J)=X(L+1)
      IP=2
  100 GO TO (200,145),IP
  145 IF(J-1) 160,150,160
  150 PCUM=CUMO
      GO TO 170
  160 PCUM=CUM(J-1)
  170 CUM(J)=PCUM+P(J)
      GO TO (180,200),ICNTRL
  180 ICNTRL=2
      IEFCT=J
  200 CONTINUE
      I=IEFCT
C
C  THE FOLLOWING STMS (THRU STM 330+2) COMPUTE THE EFFECT OF MANPOWER
C  CHANGE ON THE SIZE AND THE NUMBER OF EXPERIENCE CLASSES IN THE
C  CURRENT PERIOD. THE RESULTS ARE STORED IN TEMPORARY LOCATIONS AS
C  POSSIBLE CHANGES RESULTING FROM CLASS PARTITIONNING IS NOT KNOWN
C  AT THIS STAGE.
C
  205 II=I-1
      IF(II) 220,210,220
  210 DO 230 J=1,NXCLO
      WXX(J)=WXCLO(J)
  230 UXX(J)=UXCLO(J)
      NXX=NXCLO
      WOLD=WO
      GO TO 240
  220 WOLD=W(II)
      NXX=NXCL(II)
      DO 225 J=1,NXX
      WXX(J)=WXCL(II,J)
  225 UXX(J)=UXCL(II,J)
  240 IF(W(I)-WOLD) 270,260,250
  260 NTEMP=NXX
      DO 265 J=1,NXX
      WTEMP(J)=WXX(J)
  265 UTEMP(J)=UXX(J)
      GO TO 350
  250 NTEMP=NXX+1
      DO 280 J=1,NXX
      WTEMP(J)=WXX(J)
  280 UTEMP(J)=UXX(J)
      WTEMP(NTEMP)=W(I)-WOLD
      UTEMP(NTEMP)=0.
      GO TO 350
  270 WL=WOLD-W(I)
      TL=0.
      DO 290 JJ=1,NXX
      J=NXX-JJ+1
      TL=TL+WXX(J)
      IF(TL-WL) 290,300,310
  290 CONTINUE
  300 NTEMP=J-1
      DO 320 J=1,NTEMP
      WTEMP(J)=WXX(J)
  320 UTEMP(J)=UXX(J)
      GO TO 350
  310 NTEMP=J
      K=J-1
```

```
      DO 330 J=1,K
      WTEMP(J)=WXX(J)
  330 UTEMP(J)=UXX(J)
      WTEMP(NTEMP)=TL-WL
      UTEMP(NTEMP)=UXX(NTEMP)
C
  350 DO 360 L=I,NM
      SUB(L)=0.
  360 PNALT(L)=0.
      KFLAG=0
C
C  THE FOLLOWING STMS (THRU STM 500+2) COMPUTE THE EFFECT OF REGULAR
C  TIME PRODUCTION AND IDLE TIME ON THE CONDITION(NUMBER,SIZES,AND
C  PRODUCTIVITY MEASURES) OF EXPERIENCE CLASSES IN THE CURRENT PERIOD.
C  WHEN THE SCHEDULED PRODUCTION IS SATISFIED THROUGH REGULAR TIME
C  WORK,THE VALUES OF THE DEPENDENT VARIABLES ARE COMPUTED AND THE
C  PERMANENT STATUS FOR CLASSES IS SET.
C
      TWR=0.
      TWO=0.
      RP=P(I)
      UP=CUM(II)+P(I)/2.
      UPP=PRGRSS(UP)
      DO 370 J=1,NTEMP
      U1=UTEMP(J)
      PCXR(I,J)=UPP*REGPRO(U1)
      RP=RP-PCXR(I,J)*WTEMP(J)
      IF(DABS(RP).LE.TOL) GO TO 380
      IF(RP) 390,380,370
  370 CONTINUE
      PREG(I)=P(I)-RP
      GO TO 600
  390 KFLAG=1
      IF(J.NE.1) GO TO 380
  470 RP=RP+PCXR(I,J)*WTEMP(J)
      WRRP=RP/PCXR(I,J)
      UXCL(I,J)=PCXR(I,J)+UTEMP(J)
      WXCL(I,J)=WRRP
      M=J+1
      UXCL(I,M)=UTEMP(J)
      WXCL(I,M)=WTEMP(J)-WRRP
      MM=M+1
      NN=NTEMP+1
      DO 410 K=MM,NN
      L=K-1
      WXCL(I,K)=WTEMP(L)
      UF=UTEMP(L)
  410 UXCL(I,K)=UTEMP(L)-FORGET(UF)
      WREG(I)=TWR+WRRP
      WIDL(I)=W(I)-WREG(I)
      NXCL(I)=NTEMP+1
      GO TO 500
  380 IF(KFLAG) 420,430,420
  420 L=J-1
      GO TO 440
  430 L=J
  440 TWR=0.
      DO 450 K=1,L
      TWR=TWR+WTEMP(K)
      WXCL(I,K)=WTEMP(K)
      UXCL(I,K)=UTEMP(K)+PCXR(I,K)
  450 CONTINUE
      IF(KFLAG) 470,460,470
  460 KK=J+1
      DO 480 K=KK,NTEMP
```

```
      UF=UTEMP(K)
      UXCL(I,K)=UTEMP(K)-FORGET(UF)
  480 WXCL(I,K)=WTEMP(K)
      NXCL(I)=NTEMP
      WIDL(I)=W(I)-TWR
      WREG(I)=TWR
  500 PREG(I)=P(I)
      POVR(I)=0.
      WOVR(I)=0.
      GO TO 950
C
C   THE FOLLOWING STMS (THRU STM 920+6) COMPUTE THE EFFECT OF OVERTIME
C   PRODUCTION ON THE CONDITIONS OF EXPERIENCE CLASSES. THE VALUES  OF
C   THE DEPENDENT VARIABLES ARE COMPUTED AND THE PERMANENT CLASS STATUS
C   IS SET FOR CLASSES IN THE CURRENT PERIOD.
C
  600 DO 570 J=1,NTEMP
      U1=UTEMP(J)+PCXR(I,J)/2
      PCXO(I,J)=UPP*OVRPRO(U1)
      RP=RP-PCXO(I,J)*WTEMP(J)
      IF(DABS(RP).LE.TOL) GO TO 580
      IF(RP) 590,580,570
  570 CONTINUE
      GO TO 900
  590 KFLAG=1
      IF(J.NE.1)GO TO 580
  670 RP=RP+PCXO(I,J)*WTEMP(J)
      WROP=RP/PCXO(I,J)
      UXCL(I,J)=UTEMP(J)+PCXO(I,J)+PCXR(I,J)
      WXCL(I,J)=WROP
      M=J+1
      UXCL(I,M)=UTEMP(J)+PCXR(I,J)
      WXCL(I,M)=WTEMP(J)-WROP
      MM=M+1
      NN=NTEMP+1
      DO 610 K=MM,NN
      L=K-1
      WXCL(I,K)=WTEMP(L)
  610 UXCL(I,K)=UTEMP(L)+PCXR(I,L)
      WOVR(I)=TWO+WROP
      NXCL(I)=NTEMP+1
      GO TO 700
  580 IF(KFLAG) 620,630,620
  620 L=J-1
      GO TO 640
  630 L=J
  640 TWO=0.
      DO 650 K=1,L
      TWO=TWO+WTEMP(K)
      UXCL(I,K)=UTEMP(K)+PCXR(I,K)+PCXO(I,K)
  650 WXCL(I,K)=WTEMP(K)
      IF(KFLAG) 670,660,670
  660 KK=J+1
      DO 680 K=KK,NTEMP
      UXCL(I,K)=UTEMP(K)+PCXR(I,K)
  680 WXCL(I,K)=WTEMP(K)
      NXCL(I)=NTEMP
      WOVR(I)=TWO
  700 POVR(I)=P(I)-PREG(I)
      WREG(I)=W(I)
      WIDL(I)=0.
      GO TO 950
  900 NXCL(I)=NTEMP
      DO 920 J=1,NTEMP
      UXCL(I,J)=UTEMP(J)+PCXR(I,J)+PCXO(I,J)
```

```
  920 WXCL(I,J)=WTEMP(J)
      PNALT(I)=1.
      SUB(I)=RP
      WREG(I)=W(I)
      WOVR(I)=W(I)
      POVR(I)=P(I)-PREG(I)-RP
      WIDL(I)=0.
  950 IF(I-NM) 990,1000,990
C
C  INCREMENT THE PERIOD NUMBER -- REPEAT THE PROCESS FOR THE NEXT PERIOD
  990 I=I+1
      TWR=0.
      TWO=0.
      GO TO 205
 1000 RETURN
      END
C
C
C     *******************
      FUNCTION REGPRO(UA)
C     *******************
C
C  THIS FUNCTION COMPUTES THE PRODUCTION CAPABILITY OF A MEMBER
C  OF AN EXPERIENCE CLASS WITH PRODUCTIVITY MEASURE OF 'UA'
C  DURING ONE PERIOD ON REGULAR TIME WORK.
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /LEARN/ B,F
      REGPRO=((1.-B)/F+UA**(1.-B))**(1./(1.-B))-UA
      RETURN
      END
C
C
C     *******************
      FUNCTION OVRPRO(UA)
C     *******************
C
C  THIS FUNCTION COMPUTES THE PRODUCTION CAPABILITY OF A MEMBER
C  OF AN EXPERIENCE CLASS WITH PRODUCTIVITY MEASURE OF 'UA'
C  DURING ONE PERIOD ON OVERTIME WORK.
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /LEARN/ B,F
      COMMON /OVPCT/ OPCNT
      OVRPRO=(((1.-B)/F)*OPCNT+UA**(1.-B))**(1./(1.-B))-UA
      RETURN
      END
C
C
C     *******************
      FUNCTION PRGRSS(UA)
C     *******************
C
C  THIS FUNCTION COMPUTES THE PROGRESS FACTOR FOR ANY CUMULATIVE
C  OUTPUT UNIT 'UA' .
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /PROGS/ BP
      PRGRSS=UA**BP
      RETURN
      END
C
C
C     *******************
      FUNCTION FORGET(UA)
C     *******************
C
C  THIS FUNCTION COMPUTES THE AMOUNT OF DECAY IN PRODUCTIVITY LEVEL
```

```
C  OVER ONE PERIOD FOR A MEMBER OF A CLASS WITH PRODUCTIVITY MEASURE
C  OF 'UA' AT THE BEGINNING OF THE PERIOD.
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /FORGT/ BF,F1
      FORGET=((1.-BF)/F1+UA**(1.-BF))**(1./(1.-BF))-UA
      RETURN
      END
```

```
C                ****************************
C                *                          *
C                * PATTERN SEARCH SUBROUTINE *
C                *                          *
C                ****************************
C
C
      SUBROUTINE PATS
C
C
C  THIS ROUTINE IS BASED LARGELY ON PRIOR WORK BY TAUBERT WHO,
C  IN TURN, RELIED HEAVILY ON A PRIOR SEARCH ROUTINE SOURCE
C  CODE BY WEISMAN-WOOD-RIVLIN,WHICH WAS ITSELF AN ADAPTATION
C  OF THE ORIGINAL HOOKE-JEEVES PATTERN SEARCH.THE ROUTINE  IS
C  OBTAINED FROM A PART OF THE RESEARCH CONDUCTED BY TOM SIKES
C  (UCLA),RELATIVE TO THE SDR APPLICATION IN THE AGGREGATE
C  PLANNING PROBLEM. THE CODE IS DIVIDED INTO 18 SUBSETS CALLED
C  'BOXES' ,WITH ENTRANCES AND EXITS FOR EACH BOX EXHAUSTIVELY
C  IDENTIFIED. THIS FORMS A COMPLETE BASIS FOR A FLOW CHART, IF
C  THE NEED SHOULD ARISE.
C
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION D(20),Q(20)
      COMMON /SHARE/ X(40),N,NM,SN
      COMMON /SRCHP/ XMAX(20),XMIN(20),LIM,KPRESS,NO
      COMMON /VALUE/ NEVAL
C
C
C <BOX 1> PURPOSE: INITIALIZE DATA. ENTRANCES: FROM START OF THE
C  SUBROUTINE AND FROM BOX 18 VIA STM 100. EXITS: TO 190 IN BOX 5.
C
  100 TOL=1.E-5
      BET=.1
      DEL=.1
      LT=0
      LSN=0
      NPF=0
      ITR=1
      ALP=2.
      GR=2.2
      LA=1
      LT2=0
      LT3=0
      LT4=0
      LT5=0
      LT6=0
      LT7=0
      ID1=0
      ID2=0
      ID3=0
      ID4=0
      KOUNT=0
      V=0.
      OLDV=0.
      NEVOLD=0
      DO 180 I=1,N
```

```
      Q(I)=X(I)
C  NOTE FROM THE NEXT STATEMENT THAT THE INITIAL VALUE OF D(K)
C  DEPENDS ON HOW FAR APART THE UPPER AND LOWER BOUNDRIES ARE.
C
  180 D(I)=DEL*(XMAX(I)-XMIN(I))
      CALL FCT1
      SNOLD=SN
      OLDSN=SN
      SC=SN
      SP=SN
      M1=1
      M2=1
      K=1
      KK=1
      L4=1
      IF(KPRESS.NE.0) GO TO 190
      WRITE(NO,999) DEL
  999 FORMAT('0',20X,'PATTERN SEARCH   DEL=',F6.2)
      WRITE(NO,991)NEVAL,KOUNT,LT2,LT3,LT4,LT5,LT6,LT7,K,KK,M1,M2,NPF,
     1LA,L4,ID1,ID2,ID3,ID4,SN,V,GR,D(1),D(2),D(3)
      GO TO 190
C
C
C <BOX 18> PURPOSE: CALL OBJECTIVE FUNCTION. ENTRANCES: FROM
C  BOXES 17, 7, AND 14. EXITS: TO STMS 100 AND 500, TO 282 AND
C  285 IN BOX 2, TO 463 AND 466 IN BOX 11, AND TO 580 IN BOX 15,
C  TO 510 IN BOX 13. COMMENT: 'LA' CONTROLS THE TRAFFIC THROUGH
C  THIS BOX, SEE GLOSSARY FOR INTERPRETATION OF VALUES OF LA.
C
  270 CALL FCT1
      GO TO (100,282,463,580,285,466,510,500),LA
C
C
C <BOX 2> PURPOSE: EVALUATE FORWARD MOVES. ENTRANCES:FROM BOX 18
C  VIA STMS 282 AND 285. EXITS: TO 360 IN BOX 6 WHEN FORWARD
C  MOVE FAILS, TO 300 IN BOX 3 WHEN FORWARD MOVE SUCCEEDS.
C  COMMENT: THIS BOX IS REACHED WHEN LA=2 OR 5.
C
C  INCREMENT STATE COUNTER
C
  282 LT2=LT2+1
      GO TO 280
  285 LT5=LT5+1
  280 IF(SN-SP)290,360,360
C  TEST FOR SUCCESS OR FAILURE (290, OR 360)
  290 GO TO (300,292),L4
C  IF THE SEARCH IS IN AN EXPLORATORY MODE FOLLOWING AN ATTEMPTED
C  PATTERN MOVE,MULTIPLY THIS SUCCESSFUL FORWARD STEP SIZE BY ALP.
C
  292 D(K)=D(K)*ALP
C
C
C <BOX 3> PURPOSE: RESET SOME VARIABLES. ENTRANCES: FROM BOXES 2
C  AND 11,BOTH VIA STM 300. EXIT: TO 305 IN BOX 4.
C
  300 SP=SN
      M2=1
      M1=1
      MPF=0
C
C
C <BOX 4> PURPOSE: DECIDE WHETHER TO TEST FOR A NEW BASE POINT.
C  ENTRANCES: FROM BOXES  3 AND 12 VIA STM 305. EXITS:TO 340 IN
C  IN BOX 7 WHEN A BASE POINT TEST IS REQUIRED, TO 200 IN BOX 5
C  OTHERWISE.  COMMENT: THIS BOX IS THE ONLY ENTRANCE TO THE BASE
```

```
C  POINT TEST. ALSO NOTE THAT WHEN LT=0,AS IT NORMALLY IS, IT IS
C  ONLY HERE THAT KK IS INCREMENTED AND TESTED.
C
  305 K=K+1
      IF(K-N)320,320,777
  777 K=1
  320 IF(LT)330,330,340
  330 KK=KK+1
      IF(KK-N)200,200,340
C
C
C <BOX 5> PURPOSE: SET SOME INDICES AND INCREMENT THE VALUE OF
C  X(K) IN PREPARATION FOR TESTING A FORWARD MOVE IN THE K TH
C  VARIABLE. ENTRANCES: FROM BOX 1 VIA STM 190, FROM BOXES 4 AND
C  8 VIA STM 200, FROM BOX 10 VIA STM 190, FROM BOX 15 VIA STM 210
C  EXITS: TO 490 IN BOX 12 WHEN VARIABLE IS FIXED AND CANNOT BE
C  PETURBED, TO 230 IN BOX 17 ,OTHERWISE. COMMENT: THE VALUE OF
C  X(K) IS ADJUSTED HERE  TO TRY A FORWARD MOVE.
C
  200 GO TO (190,210),L4
  190 L4=1
      LA=2
      GO TO 220
  210 LA=5
  220 IF(D(K))225,490,225
  225 X(K)=X(K)+D(K)
      GO TO 230
C
C
C <BOX 6> PURPOSE: WHEN FORWARD MOVE FAILES THIS BOX IS REACHED
C  AND THE VALUE OF X(K) IS SET IN PREPARATION FOR ATTEMPTING A
C  A REVERSE MOVE. ENTRANCES: FROM BOXES 2 AND 17 , BOTH VIA STM
C  360. EXIT: TO TO 230 IN BOX 17(BOUNDARY CHECK). COMMENT: STMS
C  360 THROUGH 380 SHOULD BE SET,THEN TESTED AND RESET IF NECESSARY
C  WHEN PROGRAMMING FOR SPEED.
C
  360 GO TO (370,380),L4
  370 LA=3
      GO TO 390
  380 LA=6
  390 X(K)=X(K)-2*D(K)
      GO TO 230
C
C
C <BOX 17> PURPOSE: BOUNDARY CHECK IN PREPARATION FOR POSITIVE
C  OR NEGATIVE MOVE OF X(K). ENTRANCE: FROM BOXES 5 AND  6 VIA
C  STM 230.  EXITS: TO 270 IN BOX 18 WHEN BOUNDARY CHECK IS PASSED
C  ,TO 500 IN CERTAIN EVENTS WHEN FAILED, TO 360 IN BOX 6 WHEN
C  POSITIVE MOVE FAILS THE BOUNDARY CHECK, TO 480 IN BOX 2 WHEN
C  NEGATIVE MOVE FAILS THE BOUNDARY CHECK. COMMENT: 270 IMPLIES
C  BOUNDARY WAS NOT VIOLATED.
C
  230 IF(X(K)-XMAX(K))250,270,260
  250 IF(XMIN(K)-X(K))270,270,260
  260 GO TO (500,360,480,500,360,480,500,500),LA
C
C
C <BOX 7> PURPOSE: TEST TO SEE WHETHER A NEW BASE POINT HAS BEEN
C  ESTABLISHED. ENTRANCE: FROM BOX 4 VIA STM 340. EXITS: TO 400 IN
C  BOX 8 WHEN TEST FOR A NEW BASE POINT FAILS, TO 270 IN BOX 18
C  WHEN NEW BASE POINT IS ESTABLISHED.
C
  340 IF(SP+.0001-SC)350,400,400
C
C  NOTE THAT 350 MEANS THAT A SUCCESSFULLY NEW BASE POINT IS
```

```
C  IDENTIFIED.IF L4=1, THE FIRST PATTERN MOVE IN THIS SERIES
C  IS TO BE ATTEMPTED.
C
   350 GO TO (352,353),L4
   352 LA=7
       M1=1
       GO TO 270
   353 IF(NPF-5)352,400,400
C
C
C <BOX 8> PURPOSE: HAVING FAILED TO ESTABLISH A NEW BASE POINT, THIS
C  BOX IS REACHED TO DECIDE WHERE TO GO NEXT. ENTRANCE: FROM BOX 7
C  VIA STM 400. EXITS: TO 200 IN BOX 5 WHEN KK.LE.N, AFTER BASE POINT
C  TEST; WHEN FAILURE OCCURED AT THE CONCLUSION OF ADJUSTMENTS
C  FOLLOWING AN ATTEMPTED PATTERN MOVE,AND TO 440 IN BOX 10 WHEN IN
C  THE PROCESS OF SEARCHING FOR A PATTERN AND THE VALUE OF M1 IS LESS
C  THAN N+1-TO 500 IN  BOX 16 WHEN NO NEW PATTERN IS FOUND AND SEARCH
C  IS TO BE ESTABLISHED.
C
   400 IF(LT)404,404,778
   778 KK=KK+1
   404 IF(KK-N)200,200,779
   779 GO TO (429,410),L4
   429 L4=2
       IF(M1-N)440,440,500
C
C
C <BOX 9> PURPOSE:RESTORE THE VALUES OF X(I) AFTER UNSUCCESSFUL
C  ATTEMPT TO ESTABLISH NEW BASE POINT. ENTRANCE: FROM BOX 8 VIA STM
C  410. EXIT: TO 440 IN BOX 10.
C
   410 SP=SC
       DO 420 I=1,N
   420 X(I)=Q(I)
       NPF=0
C
C
C <BOX 10> PURPOSE: REST SOME COUNTERS (SEE GLOSSARY FOR DEFINITIONS)
C  ENTRANCE: FROM BOX 8 AND 9, BOTH VIA STM 440. EXIT: TO 190 IN BOX 5.
C
   440 M2=1
       M1=1
       KK=1
       GO TO 190
C
C
C <BOX 11> PURPOSE: EVALUATE A REVERSE MOVE (CORRESPONDING TO BOX 2'S
C  FORWARD MOVE EVALUATION).ENTRANCE: FROM BOX 18 VIA STMS 463,466.
C  EXITS: TO 480 IN BOX 12 WHEN WHEN REVERSE MOVE FAILS,
C  TO 300 IN BOX 3 WHEN REVERSE MOVE SUCCEEDS. COMMENT: NOTE THAT
C  NAS USUAL THE COUNTERS LT3 AND LT6 ARE SET, DEPENDING WHETHER
C  THIS BOX IS REACHED WITH LA=3 OR 6.
C
   463 LT3=L3+1
       GO TO 460
   466 LT6=LT6+1
   460 IF(SN-SP)470,480,480
C
C  SUCCESS! THEREFORE IN THE FUTURE A FORWARD MOVE WILL BE WHAT HAS
C  HERETOFORE BEEN A BACKWARD MOVE - HENCE, STM 470
C
   470 D(K)=-D(K)
       GO TO 300
C
C
```

```
C <BOX 12> PURPOSE: AFTER REVERSE MOVE FAILURE THIS BOX REDUCES STEP
C  SIZE AND ATTEMPTS TO TRY AN EXPLORATORY STEP FOR ANOTHER VARIABLE.
C  ENTRANCES: FROM BOX 5 VIA STM 490, FROM BOXES 17 AND 11 VIA STM 480.
C  EXIT: TO 305 IN BOX 4.
C
  480 X(K)=X(K)+D(K)
      D(K)=D(K)*BET
      DX=DABS(X(K)/D(K)*TOL)
      IF(1-DX)481,482,484
  481 D(K)=D(K)*DX
C
  482 DX=DABS(1.E-30/D(K))
      IF(1-DX)483,490,490
  483 D(K)=D(K)*DX
      GO TO 490
  484 DX=DABS(1.E-30/D(K))
      IF(1-DX)485,490,492
  485 D(K)=D(K)*DX
  490 M1=M1+1
  492 GO TO (495,493),L4
  495 GO TO 309
  493 M2=M2+1
      IF(M2-N)309,309,780
  780 M2=1
      NPF=NPF+1
  309 GO TO 305
C
C
C <BOX 13> PURPOSE: ADAPTIVE LOGIC TO COMPUTE GR, A MULTIPLICATIVE
C  FACTOR THAT GOVERNS THE SIZE OF THE PATTERN MOVE ATTEMPT.
C  ENTRANCE: FROM BOX 18 VIA STM 510. EXIT: TO 888 IN BOX 14.
C  COMMENT: BY OBSERVING THE OUTPUT FROM PATS IN THE HMMS MODEL, IT
C  IS SEEN THAT 99% OF THE TIME GR SIMPLY GROWS BY INCREMENTS OF .1
C  FROM 2.2 TO 3.5 - THEN IS RESET TO 2.2 FOR ANOTHER CYCLE.
C  THE ADAPTIVE PATTERN SEARCH LOGIC SEEMS TO OFFER GREAT OPPORTUNITIES
C  FOR IMPROVEMENT.
C
  510 LT7=LT7+1
      KOUNT=KOUNT+1
      IF(MOD(KOUNT,1))888,398,888
  398 V=(OLDSN-SN)*100/(OLDSN*(NEVAL-NEVOLD))
      IF(.6-V)801,781,781
  781 IF(V-.3)802,782,782
  782 IF(OLDV-V)803,804,804
  801 ID4=ID4+1
      GO TO 815
  802 ID1=ID1+1
      D1=.1
      GO TO 810
  803 ID2=ID2+1
      GO TO 815
  804 ID3=ID3+1
      D1=.05
  810 GR=GR+D1
      IF(GR-3.5)815,783,783
  783 GR=2.2
  815 OLDSN=SN
      OLDV=V
      NEVOLD=NEVAL
C
C
C <BOX 14> PURPOSE: SET X(I) TO MAKE PATTERN MOVE ATTEMPT.
C  ENTRANCE: FROM BOX 13 VIA STM 888. EXITS: TO 270 IN BOX 18(ALMOST
C  ALWAYS) , TO 500 IN BOX 16 WHEN NEVAL VIOLATES LIMIT, CAUSING A
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C  RETURN TO THE MAIN ROUTINE. COMMENT: NOTICE THAT A BOUNDARY CHECK
C  IS CONTAINED IN THIS BOX AND IS SLIGHTLY DIFFERENT FROM THE
C  ONE IN BOX 17 INSOFAR AS THE CONCEQUNCES OF FAILING THE CHECK ARE
C  CONCERNED.
C
  888 IF(MOD(KOUNT,ITR))7,8,7
    8 IF(KPRESS.NE.0) GO TO 7
      WRITE(NO,991)NEVAL,KOUNT,LT2,LT3,LT4,LT5,LT6,LT7,K,KK,M1,M2,NPF,
     1LA,L4,ID1,ID2,ID3,ID4,SN,V,GR,D(1),D(2),D(3)
    7 IF(LIM-NEVAL)500,784,784
  784 KK=1
      IF(LSN)530,530,785
  785 SP=SN
  530 SC=SP
      LA=4
      DO 570 I=1,N
      P=Q(I)
      Q(I)=X(I)
      X(I)=P+GR*(X(I)-P)
      IF(X(I)-XMAX(I))550,570,786
  786 X(I)=XMAX(I)
      GO TO 570
  550 IF(XMIN(I)-X(I))570,570,787
  787 X(I)=XMIN(I)
  570 CONTINUE
      GO TO 270
C
C
C <BOX 15> PURPOSE: SET CERTAIN COUNTERS PRIOR TO ADJUSTMENTS OF THE
C  PATTERN MOVE ATTEMPT. ENTRANCE: FROM BOX 18 VIA STM 580.
C  EXIT: TO 210 IN BOX 5 ALWAYS.
C
  580 LT4=LT4+1
      L4=2
      SP=SN
      GO TO 210
C
C
C <BOX 16> PUPOSE: PRINT OUT FINAL PARAMETER VALUES AND TERMINATE THE
C  SEARCH. ENTRANCE: FROM BOXES 17 AND 18 , FROM BOXES 8 AND 14 , ALL
C  VIA STM 500.  EXIT: RETURN TO THE MAIN ROUTINE.
C
  500 LA=8
  990 WRITE(NO,1111)
 1111 FORMAT(1H0)
      WRITE(NO,991)NEVAL,KOUNT,LT2,LT3,LT4,LT5,LT6,LT7,K,KK,M1,M2,NPF,
     1LA,L4,ID1,ID2,ID3,ID4,SN,V,GR,D(1),D(2),D(3)
  991 FORMAT(1H ,I5,4I4,1X,3I4,4I3,3I2,1X,4I3,F14.4,F8.5,F5.2,3F9.6)
      RETURN
      END
END OF DATA
```

VITA

Behrokh Khoshnevis

Candidate for the Degree of

Doctor of Philosophy

Thesis: AGGREGATE PRODUCTION PLANNING MODELS INCORPORATING DYNAMIC PRODUCTIVITY

Major Field: Industrial Engineering and Management

Biographical:

Personal Data: Born in Sari, Iran, December 31, 1950, the son of Mr. and Mrs. Islam Khoshnevis.

Education: Graduated from Azar High School, Tehran, Iran, in June, 1969; received Bachelor of Science degree in Industrial Engineering from Arya-Mehr University of Technology, Tehran, Iran, in January, 1974; received Master of Science degree in Industrial Engineering and Management from Oklahoma State University in December, 1975; completed requirements for the Doctor of Philosophy degree at Oklahoma State University in December, 1979.

Professional Experience: Instructor of Physics and Technical Graphics in Tehran public high schools, 1970-1973; Assembly Line Supervisor, Mercedes Benz Factory, Tehran, summer, 1972; Graduate Research and Teaching Assistant, Oklahoma State University, 1975-1979; Visiting Assistant Professor, teaching Operations Research, Dynamic Programming, Systems Simulation and Project Management, Oklahoma State University, 1979 to present.

Professional Organizations: American Institute of Industrial Engineers, Operations Research Society of America, The Institute of Management Sciences, Alpha Pi Mu.