SUBSTRUCTURING AND COMPONENT MODES

By

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LIST OF SYMBOLS

A	number of fixed interface modes
A_n	number of fixed interface modes assigned to nth substructure
@	at
$[B_n]$	transformation matrix for nth substructure
$[C]$	damping matrix
C_n	modal damping
D	number of attachment modes
DOF	degrees of freedom
EI_o	flexural rigidity
$\{F_D\}$	damping forces at the floor level
$\{F_I\}$	inertia forces
ft	feet
g	gravitational acceleration
I_n	number of physical coordinates internal to the nth substructure
J_n	number of physical coordinates on the interface of the nth substructure
$[K]$	stiffness matrix
kip	1000 pounds
K_n	modal stiffness
$[K_n]$	stiffness matrix for nth substructure
$[M]$	mass matrix
M_n	modal mass
$[M_n]$	mass matrix for nth substructure

M_1	point mass
mbr	member
N	number of geometric degrees of freedom in the original idealization
No.	number
N-S	north-south
P	point load (static load)
{R}	load vector
{r}	vector of amplitude of the generalized displacements
Sub	substructure
S	number of approximate modes used as generalized coordinates
T	fundamental period of vibration
{V}	amplitude of the modal components
$\{\dot{V}_n\}$	modal velocity
$\{\ddot{V}_n\}$	modal acceleration
W	wide flange section
wt	weight
XA	cross sectional area
XI	moment of inertia
x	horizontal component of translation
{x}	relative displacement of the stories
$\{\dot{x}\}$	relative velocity of the stories
$\{x_g\}$	horizontal support displacement
$\{\ddot{x}_t\}$	total acceleration of the stories
y	vertical component of translation
z	number of substructures
ξ	percent of critical damping
ω	natural frequencies (eigenvalues)

θ	angular rotation
ϕ	generalized coordinate mode shapes
$[\phi]$	transformation matrix consisting of orthogonal modes
$\sum$	summation
⑥	joint number
$\{1\}$	unit column vector
$=$	equal to
$\neq$	not equal to

Subscripts

n	substructure number

Notes

(1)	a dot above a quantity denotes time derivative

CHAPTER I

INTRODUCTION

1.1 Statement of the Problem

1.1.1 General

In the analysis of large complex structures, the number of natural degrees of freedom very often strain the capacity of available computer facilities. This is compounded for structural dynamic problems which are formed as a succession of static solutions corresponding to all times of interest in the response history. It is much more complex and time consuming than a static analysis. However, in the analysis of building frames, experience has shown that satisfactory accuracy can be obtained by using ten percent or less of the number of degrees of freedom used in the static analysis. These degrees of freedom must be carefully selected in order to attain satisfactory representation. Thus, it is essential that methods of obtaining quality reduced order models for these complex structures are made available to the structural engineer.

Two general approaches have been used effectively to reduce the dynamic degrees of freedom. The first is based on the assumption that inertia forces are associated with only certain selected degrees of freedom of the original idealization; the remaining degrees of freedom are not explicitly involved in the dynamic analysis and can be condensed from the dynamic formulation. In the second approach the number of dynamic degrees of freedom is limited by assuming that the displacements of the structure are combined in selected patterns, the amplitudes of which become the generalized coordinates of the dynamic analysis (1). These approaches have been combined in a variety of specific techniques; the essential features used in this study are discussed in the sections that follow.

1.1.2 Substructuring

Analysis of large structural systems by substructuring is accomplished by physically dividing the structure into convenient components, expressing the structural properties of each in terms of the kinematic freedoms on the substructure interfaces only. The reduced substructure models are combined to formulate the total structural problem in terms of the interface freedoms. Results of the solution of this reduced order model are introduced into the individual substructure models yielding the detailed solution for all components.

For a given substructure the reduced order stiffness matrix is obtained by the well-known technique of static condensation (2). The same congruent transformation used in the condensation of the stiffness matrix transforms the inertia properties in a consistent manner.

1.1.3 Generalized Coordinates and Their Formulation

An alternative approach is to combine the kinematic freedoms in patterns corresponding to selected displaced shapes of the total structure. The amplitudes of these shapes serve as generalized coordinates, from which a reduced order model is formed by the Rayleigh-Ritz method

(1). The accuracy which can be achieved depends both on the number and quality of the selected deformation patterns.

A systematic scheme to obtain a quality reduced order model is to select displacement patterns corresponding to the lower modes of free vibration. These shapes constitute independent displacement patterns, the amplitudes of which may serve as generalized coordinates to approximate any form of displacement. The mode shapes thus serve the same purpose as the trigonometric function in a Fourier series, and they are advantageous for the same reason--because of their orthogonality properties and because they describe the displacements efficiently, good approximations can be made with few terms (1).

For large complex structures the solution of the free-vibration problem requires many computations and tremendous computer storage. Most often these problems exceed the capacity of available computers. Thus, it is again essential that some method of reduction in the number of degrees of freedom be employed.

1.1.4 Modal Synthesis

During the past decade there has been developed, within the field of structural dynamics, a technique that may be identified by the term modal synthesis. A large number of papers and reports have been written to introduce methods or variations of methods falling within the general scope of this technique. All of these methods developed around the central idea of combining features of substructuring and generalized coordinates.

The structure is divided into components or substructures, each of which is analyzed as a separate unit for the purpose of constructing a set of modes or displacement shapes that can be used as generalized coordinates to define displacements both in the interior of the component and on its boundaries or interfaces with other components. Stiffness and mass properties of each component related to this set of generalized coordinates are computed together with damping properties, if applicable. Imposition of the requirements of displacement compatibility at the component interfaces leads to a synthesis of all of the component coordinates to form a set of generalized coordinates applicable to the complete structure. At the same time properties of the complete structure are formed from the collected properties of the components. The equations of motion for the complete structure are thus formulated. When these are solved, the pertinent information in the form of displacements, velocities, and accelerations may be carried through the coordinate transformations to give corresponding information related to each separate component.

1.1.5 Substructures and Component Modes

The present method combines the techniques of substructuring and generalized coordinates with special provisions to exploit data accumulated in the course of a developing design. The structure is divided into components. Generalized coordinates of two types are formed. The first type are the deflected shapes of the total structure subjected to selected static loads as predicted by conventional substructure analysis. These are referred to as attachment modes and are augmented with generalized coordinates corresponding to selected responses of the individual substructures with interface freedoms suppressed. These generalized coordinates are called fixed interface modes.

Alternative fixed interface modes may be derived from the free vibration mode shapes of the fixed interface substructures.

1.1.5.1 Coordinate Transformation Matrices. Each deflected shape of the total structure is divided into a component mode for each substructure. These component modes and those obtained from the substructures are arranged into prescribed order to form a transformation matrix for each substructure. These matrices are used to compute the first reduced order of [M], [K], and R (generalized mass, stiffness, and load vector) for each substructure.

1.1.5.2 Computer Model. In general, the component modes do not have the orthogonality properties of true mode shapes. Thus, the off-diagonal terms of the generalized mass and stiffness matrices do not vanish. However, a good choice of component mode shapes will make these off-diagonal terms relatively small. From these generalized mass and stiffness matrices for the substructures, the computer model is formed for the total structure by superposition. The computer model is used to solve static and dynamic problems for the structure. When necessary, a better quality reduced order model may be formed by single subspace iteration (1). This was unnecessary for this study.

The reduced order free vibration problem is solved by standard eigen-value procedure. The eigen-values of the dynamic matrix so obtained represent approximations to the true frequencies of the lower modes of vibration, the accuracy generally being excellent for the lowest modes. When the corresponding eigenvectors are normalized, they represent mode shapes expressed in generalized coordinates. It is of interest to note that these mode shapes are orthogonal with respect to the generalized mass and stiffness matrices. These are transformed to geometric coordinates resulting in approximate mode shapes for the structure. These approximate geometric mode shapes are orthogonal with respect to the mass and stiffness expressed in geometric coordinates. They can therefore be used in the standard mode-superposition dynamic-analysis procedure.

1.1.5.3 Modal Superposition. The approximate geometric mode shapes discussed in the previous section are used to form new transformation matrices for each substructure. These transformation matrices are used to compute new reduced orders of [M], [K], and [C] (generalized mass, stiffness, and damping) for each substructure. These new reduced orders of [M], [K], and [C] are used to form the equation of motion. This equation is numerically integrated. In this study a fourth-order Runge-Kutta technique is used (3). The response of the structure is computed by superposition of the various modal contributions, hence the name modal superposition.

1.1.5.4 Numerical Demonstration. The theory was tested on three different models. They consisted of a simply supported beam model, a multi-story steel frame model, and a twenty-story reinforced concrete frame model. For the simply supported beam model, the lower natural frequencies were computed and compared with well-known classical results. The lower natural frequencies were computed for the steel frame using only deflected shapes as component modes. The frame was then modeled using different combinations of attachment modes and fixed interface modes computed from the free vibration problem for the substructures. The lower frequencies and mode shapes were computed and compared with a conventional solution of the free vibration problem for the total structure. For the third model the lower frequencies and orthogonal modes were computed. The lower modes and frequencies were used to form the equation of motion for the structure. This equation was integrated for earthquake excitation using the referenced fourth-order Runge-Kutta technique. The response of the structure was compared with the solution given in References (1), (4), and (5).

1.2 Purpose and Scope

The primary objective of this study was to develop an accurate and economical technique for dynamic analysis of complex substructures. The proposed technique combines substructuring and mode superposition in terms of component modes used as generalized coordinates.

In each case the structure was divided into substructures and transformation matrices formed for each substructure. These transformation matrices were formed from component modes which were the attachment modes for the total structure and fixed interface modes for the substructures. Reduced orders of [M] and [K] were computed for the substructure and the computer model formed.

From the solution of the free vibration problem for the computer model, the lower frequencies and normal modes were computed. These lower frequencies and mode shapes were used in a standard modal transformation to formulate the equation of motion. The equation of motion was solved using the Reinforced Concrete model. Results obtained were compared with classical or referenced solutions.

The review of literature is presented in Chapter II. Chapter III gives a detailed description of the methodology. The verification of the results of the study are presented in Chapter IV. Chapter V contains the summary, conclusions, and recommendations of the research.

CHAPTER II

LITERATURE REVIEW

In recent years much attention has been given to the dynamic behavior of structural systems. To overcome the many problems and difficulties encountered in the analysis of large complex structures, many substructuring technqiues have been developed. The terms modal synthesis, substructure coupling, component modes, and component modal synthesis are all techniques of solving complex dynamic problems. The basic scheme is to divide the system into parts,or components, to describe physical displacements of the components in terms of reduced generalized coordinates and to combine the reduced component models through the use of interface compatibility. The reduced order properties of the complete structure are used to formulate and solve the equations of motion for displacements, velocities, and accelerations. Carrying these back through a coordinate transformation, the corresponding information for each substructure is determined.

2.1 Earlier Investigations

The earliest concepts of modal synthesis were restricted to structures with statically determinate interfaces and have been overshadowed by later developments. The early works of Serbin (6), Sofrin (7), and Bishop (8) treated the structure as a composite system. A significant development of these methods with solutions by means of electrical analog systems was developed by MacNeal (9). Hunn (10) introduced the idea of combining vibration modes of component parts of an airplane structure in dealing with the problem of dynamic response. These early works did not make use of matrix algebra because digital computers were not readily available at this time. Further, no attempt at dealing with the redundancies at the interface of component parts was undertaken.

One of the first to use matrix algebra was Hurty (11). He was also the first to deal with redundancies in the interconnections (12). The early papers of Hurty (11) through (14) introduced the concept of modal synthesis. The first of these papers appeared in 1960. However, these techniques were not applied in the aerospace industry until the mid 1960's. This was perhaps due to the fact that in the mid 1960's computers were available and computer programs dealing with matrix algebra were in use. Bamford (15) developed a computer program using Hurty's method with some modifications.

Goldman (16) (17) introduced a new technique which employed free interface substructure normal modes. This proved that constraint modes as used in other techniques are not needed. This always leads to an eigen-solution of the single matrix. Since the mid 1960's, modal synthesis has become a popular subject in research and industrial fields.

2.2 Later Developments

In section 2.1 most of the pioneers in modal synthesis were listed. Authors covered in this section did excellent work to improve or modify the work of these early pioneers.

Craig and Bampton (18) made an improvement to Hurty's method (12) (13) by showing that it is not essential to separate the set of constraint modes into rigid-body modes and redundant constraint modes. That is, all modes associated with boundary freedoms are treated alike, which leads to simplicity of computer programming and the possibility of shorter computer time. Similar proof was presented by Bajan, Feng, and Jaszlics (19) (20).

Hou (21) developed a technique of employing free-interface normal modes which is similar to work presented earlier by Goldman (16) (17). However, he used a different technique for generating the system transformation which is less complex than Goldman's approach.

Gladwell (22) presented a component modal substitution method that is suitable for statically determinate models, which he called the "Branch Mode" analysis. The component modes are determined by allowing the component to vibrate with distortion, while all other components are assumed attached and rigid. Using the same concepts, Benfield and Hruda (23) developed a comprehensive matrix analysis offering alternative procedures for constructing the branch stiffness and mass matrices. A primary advantage of component mode substitution is the ability to select fewer generalized coordinates and still obtain satisfactory results. Their technique included an approximation to the interface loading effect of one component on another and also permits treatment of redundantly interconnected components. Four variations of the basic theme of branch modes are described by Benfield and Hurda.

In 1971, Hart, Hurty, and Collins (24) presented a survey of modal synthesis methods. Each method discussed is based upon the Rayleigh-Ritz energy principle. They found that the methods that adapt themselves easily to the digital computer fall in three categories:

1. Free-Free Modal Synthesis--where composite structural dynamic characteristics are developed from the free-free mode shapes of the components.

2. Component-Mode Synthesis--where composite structural dynamic characteristics are developed using boundary-fixed mode shapes of the components and the boundary displacement functions.

3. Component-Mode Substitution--where free-free model displacement functions are improved by the consideration of interface loading at the attachment points.

2.3 Recent Developments

The work of Bamford, Wada, Garda, and Chisholm (25) outlined the basic steps of a generalized substructure coupling procedure. Emphasis was placed on experience and techniques deemed necessary to obtain accurate solutions. The authors pointed out that one disadvantage was the understanding required by the engineer to properly select the substructures, interface, and displacement functions. Craig and Chang (26) (27) presented a more general procedure with detailed definitions.

Hurty (28) presented a convergence criterion. He developed a method for predicting truncation errors for unused component modes. Morosow and Abbott (29) derived a technique for determining the modal participation factors, or weighing factors, to assess which component modes should be included.

Dowell (30) (31) introduced the use of Lagrange multipliers to enforce constraints in structures represented by component modes. Klein (32) in his thesis presented similar work related to the Lagrange multiplier technique for component mode synthesis.

Klosterman (33) (34) and Klosterman and Lemon (35) described modal synthesis techniques which combined analytical and experimental data for components and also provided for residual flexibility and inertia restraint. Good correlations between calculated and measured frequency response was noted.

In 1972, a Symposium on Substructure Testing and Synthesis was sponsored by NASA Marshall Space Flight Center. Benfield, Bodley, and Morosow (36) presented a paper on modal synthesis methods. They compared the accuracy of several modal synthesis methods based on a two-component truss with redundant interface connections.

Hasselman and Hart (37), using Hurty's component mode synthesis method, considered the effect of random structural properties on system modal properties. They presented a minimization technique for dealing with convergence.

Neubert (38) presented a paper on the development of improved mathematical models of substructures based on two synthesis approaches, the modal synthesis method and the impedance or dynamic stiffness method.

Berman (39) presented a method of modal synthesis which made use of substructure response to forces at the interface coordinates and performed an exact reduction of the substructure to those coordinates, instead of using modes of the components as general coordinates of the system.

MacNeal (40) presented a procedure for employing mixed or hybrid component modes (i.e., modes obtained with some interface coordinates free and other fixed). He also employed statically derived modes to improve the representation. Rubin (41) presented a method of component-modes which employs an incomplete set of free-boundary normal modes of vibration, augmented with an account for "residual effects" (contributions of neglected modes). This method adds residual inertial and dissipative effects to the residual flexibility introduced by MacNeal (40). Chang (42) in his thesis and Craig and Cheng (43) (44) discussed the coupling of sbustructures represented by the Rubin component-modes model (41) and presented several numerical examples.

Most of the methods of modal synthesis deal with the undamped free-vibration problem. However, a few studies of damping in component modal synthesis have been conducted. Klein and Dowell (45) in a continuation of their early works (30) (31) (32) employed the Lagrange Multiplier technique to couple damped substructures. Klosterman (33) (34) and Klosterman and Lemon (35) employed damping and experimentally-determined component modes to determine system frequency response.

Most of the work presented in this section can be classified as modifications or improvements to previous works.

2.4 Summary of Modal Synthesis Methods

In survey and review papers, Craig and Chang (26), Hinze (46), and Craig (47) presented a complete description of the various methods of modal synthesis. These papers contain many numerical examples and compare the accuracy of the different methods.

Modal synthesis is based on the Rayleigh-Ritz procedures. It is a method for formulating and solving dynamic problems when dealing with complex structural systems. Solutions obtained from modal synthesis are approximate in that the motion of the structure is limited to a few modes or displacement functions characterizing the behavior of the total structure. Some methods are found to be more suitable for certain dynamic problems than others. The literature has shown that no single method is generally preferred over another one.

Hurty is considered to be the pioneer in modal synthesis. His work is one of the simplest and has demonstrated excellent results. It has been widely accepted and is the basis of most methods now in use.

The major objective of all the various modal synthesis methods is the economical use of computers. The method presented in this study has the same objective with special provisions for use of data accumulated in the design phase of a structural system.

CHAPTER III

METHOD OF ANALYSIS

The present scheme follows the basic pattern of modal synthesis with special provision to exploit data accumulated in the course of a developing design. Precomputed static mode shapes are directly used in the formulation of substructure component modes. This approach permits the creation of a reduced order dynamic model of a structure by sequential manipulation of substructure data utilizing data computed from service loads.

3.1 Substructure Model

The structure to be analyzed is partitioned into convenient substructures as shown schematically in Figure 1.

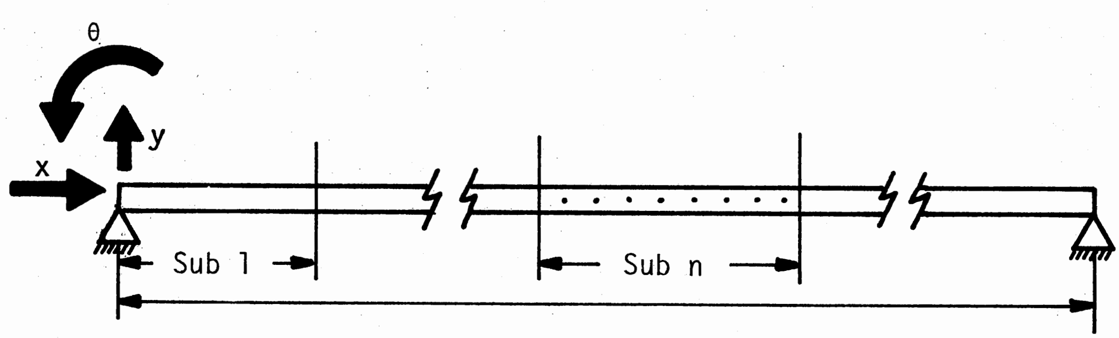


Figure 1. Schematic Substructure Partitions

Physical coordinates are the x,y components of translation and the rotation (θ) at each joint.

3.2 Attachment Modes

In this study attachment modes are the deflected shapes of the total structure computed from service loads using conventional substructure analysis. These attachment modes are used as component modes for the substructures. Figure 2 illustrates the attachment mode concept.

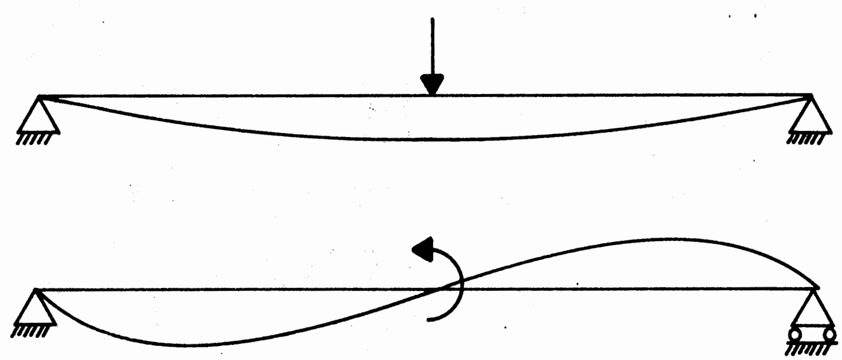


Figure 2. Structure Attachment Modes

3.3 Fixed Interface Modes

Linear combinations of the physical coordinates interior to the nth substructure are chosen as the generalized coordinates for the particular substructure. These are the deformed shapes of the substructure arising from arbitrary loads with the interfaces rigidly constrained. Figure 3 illustrates the geometric interpretation of these coordinates.

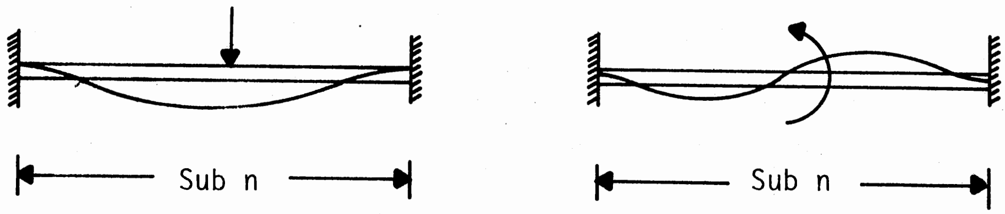


Figure 3. Substructure Fixed Interface Modes

Fixed interface modes may alternatively be obtained by computing the natural modes of vibration for a substructure with interfaces fixed.

3.4 Approximate Modes As Generalized Coordinates

Each attachment mode or fixed interface mode constitutes an approximate mode for the total structure. Those approximate modes are treated as generalized coordinates to form transformation matrices for the substructures. The number of approximate modes used is arbitrary and should be somewhat greater than the number of degrees of freedom retained in the final solution.

3.5 Coordinate Transformation

The chosen generalized coordinates lead to simple coordinate transformations which permit the serial assembly of the generalized stiffness, mass, etc., by substructures.

The matrix relating the generalized coordinates, with the global coordinates belonging to the nth substructure, including its boundaries, is shown schematically partitioned below (Figure 4), where the dimensions

S = total number of generalized coordinates;

A = number of fixed interface modes;

D = number of attachment modes;

A_n = number of fixed interface modes assigned to the nth substructure;

I_n = number of physical coordinates internal to the nth substructure; and

J_n = number of physical coordinates on interfaces of the nth substructure.

The columns of the populated blocks are the deformed shapes of the substructure for the fixed and deformed interface modes, respectively.

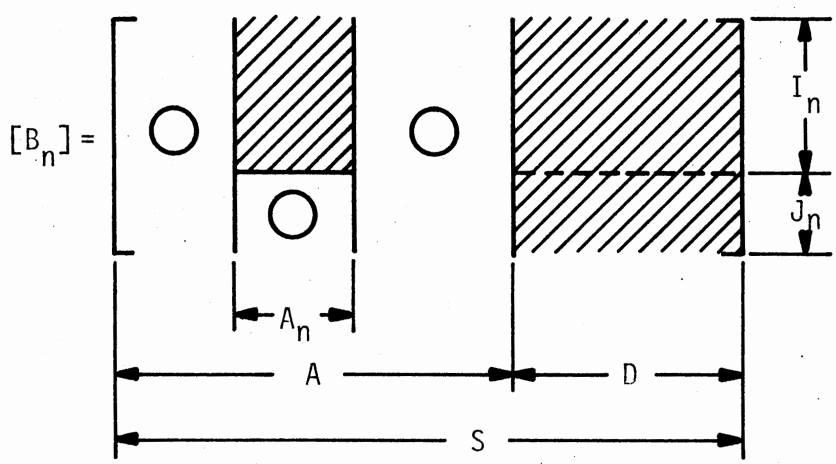


Figure 4. Transformation Matrix

3.6 Model Assembly

The substructure matrices are assembled into the generalized model by the familiar transformations:

$$[K] = \sum_{n=1}^{z} [B_n^t] [K_n] [B_n] \tag{3.1a}$$

$$[M] = \sum_{n=1}^{z} [B_n^t] [M_n] [B_n] \tag{3.1b}$$

$$R = \sum_{n=1}^{z} [B_n^t] \{R_n\} \tag{3.1c}$$

where [K], [M], and {R} are the generalized stiffness, mass, and load matrices, respectively; $[K_n]$, $[M_n]$, and $\{R_n\}$ represent the same quantities for the nth substructure; and z is the number of substructures.

The reduced order static problem is

$$[K] \{r\} = \{R\}$$

while the generalized form of the free vibration problem is

$$[K - \omega^2 M] \{r\} = 0$$

where {r} is the vector of amplitudes of the generalized displacements.

3.7 Example of Coordinate Transformation

To clarify the concept of attachment modes, fixed interface modes, and the physical interpretation of coordinate transformation, the following example is given (Figure 5). The simply supported beam of Figure 5 has 5 joints, 4 bar elements, and 15 degrees of freedom (x_1, y_1, θ_1, x_2, y_2, θ_2, . . . , θ_5). It is divided into two substructures. For this example four generalized coordinates are assigned, two to the total structure and one to each substructure. Two static load conditions are applied to the total structure and two attachment modes computed (Figure 6). Also one static load condition is applied to each substructure and two fixed interface modes computed (Figure 7). The attachment modes are separated into component modes for the substructures. These component modes plus the fixed interface modes for the substructures are arranged in prescribed order to form transformation matrices for the substructures.

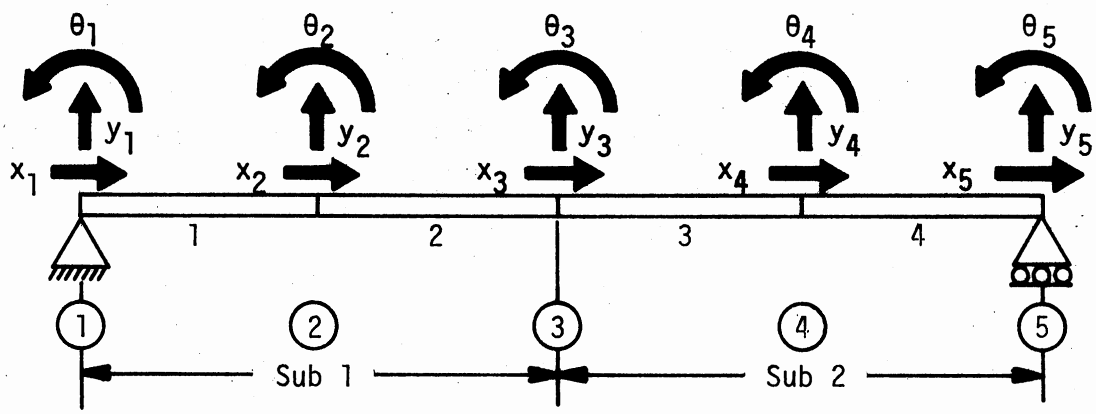
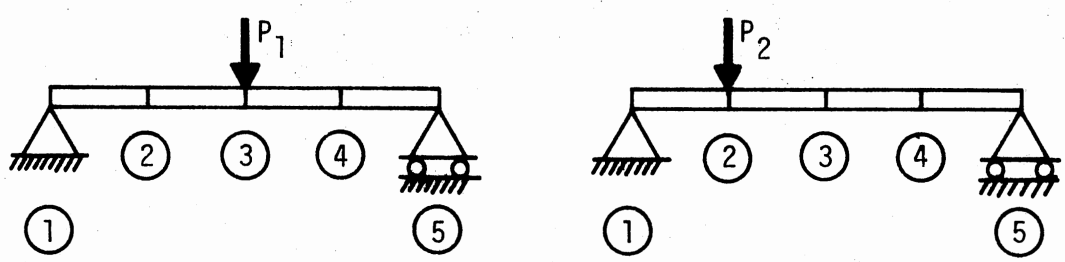


Figure 5. Simply Supported Beam With Degrees of Freedom Shown



Load Condition No. 1

Load Condition No. 2

Figure 6. Load Conditions for Attachment Modes

For simplicity the displacements are represented by the joints and and load condition numbers as subscripts. The first subscript is the joint number and the second subscript is the load condition number.

Attachment Mode No. 1

Component Mode No. 1,1:

$x_{11} = 0$

$y_{11} = 0$

$\theta_{11} \neq 0$

$x_{21} \neq 0$

$y_{21} \neq 0$

$\theta_{21} \neq 0$

$x_{31} \neq 0$

$y_{31} \neq 0$

$\theta_{31} \neq 0$

Component Mode No. 2,1:

$x_{31} \neq 0$

$y_{31} \neq 0$

$\theta_{31} \neq 0$

$x_{41} \neq 0$

$y_{41} \neq 0$

$\theta_{41} \neq 0$

$x_{51} \neq 0$

$y_{51} = 0$

$\theta_{51} \neq 0$

Attachment Mode No. 2

Component Mode No. 1,2:

$x_{12} = 0$

$y_{12} = 0$

$\theta_{12} \neq 0$

$x_{22} \neq 0$

$y_{22} \neq 0$

$\theta_{22} \neq 0$

$x_{32} \neq 0$

$y_{32} \neq 0$

$\theta_{32} \neq 0$

Component Mode No. 2,2:

$x_{32} \neq 0$

$y_{32} \neq 0$

$\theta_{32} \neq 0$

$x_{42} \neq 0$

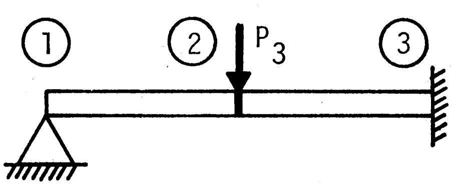
$y_{42} \neq 0$

$\theta_{42} \neq 0$

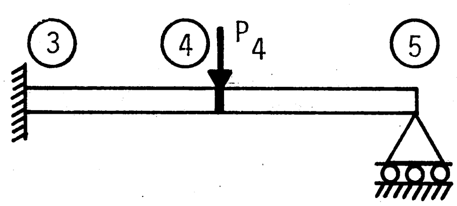
$x_{52} \neq 0$

$y_{52} = 0$

$\theta_{52} \neq 0$



Load Condition No. 3



Load Condition No. 4

Figure 7. Load Conditions for Fixed Interface Modes

Fixed Interface Mode No. 1

$$\left.\begin{array}{l} x_{13} = 0 \\ y_{13} = 0 \\ \theta_{13} \neq 0 \\ x_{23} \neq 0 \\ y_{23} \neq 0 \\ \theta_{23} \neq 0 \\ x_{33} = 0 \\ y_{33} = 0 \\ \theta_{33} = 0 \end{array}\right\} \text{Component Mode No. 1,3}$$

Fixed Interface Mode No. 2

$$\left.\begin{array}{l} x_{34} = 0 \\ y_{34} = 0 \\ \theta_{34} = 0 \\ x_{44} \neq 0 \\ y_{44} \neq 0 \\ \theta_{44} \neq 0 \\ x_{54} \neq 0 \\ y_{54} = 0 \\ \theta_{54} \neq 0 \end{array}\right\} \text{Component Mode No. 2,3}$$

3.7.1 Transformation Matrices

$$[B_1] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \theta_{13} & 0 & \theta_{11} & \theta_{12} \\ x_{23} & 0 & x_{21} & x_{21} \\ y_{23} & 0 & y_{21} & y_{21} \\ \theta_{23} & 0 & \theta_{21} & \theta_{21} \\ 0 & 0 & x_{31} & x_{31} \\ 0 & 0 & y_{31} & y_{31} \\ 0 & 0 & \theta_{31} & \theta_{31} \end{bmatrix} \qquad [B_2] = \begin{bmatrix} 0 & 0 & x_{31} & x_{32} \\ 0 & 0 & y_{31} & y_{32} \\ 0 & 0 & \theta_{31} & \theta_{32} \\ 0 & x_{44} & x_{41} & x_{42} \\ 0 & y_{44} & y_{41} & y_{42} \\ 0 & \theta_{44} & \theta_{41} & \theta_{42} \\ 0 & x_{54} & x_{51} & x_{52} \\ 0 & 0 & 0 & 0 \\ 0 & \theta_{54} & \theta_{51} & \theta_{52} \end{bmatrix}$$

Note that $[B_1]$ is made up of physical coordinates for joints 1, 2, 3, and $[B_2]$ is made up of physical coordinates for joints 3, 4, 5. The interface displacements (x_{31}, x_{32}, y_{31}, y_{32}, θ_{31}, and θ_{32}) appear in both $[B_1]$ and $[B_2]$, thus ensuring compatibility.

3.7.2 Coordinate Transformations

$$[\overline{K}_1] = [B_1^T]\,[K_1]\,[B_1] \qquad [\overline{M}_1] = [B_1^T]\,[M_1]\,[B_1]$$

$$[\overline{K}_2] = [B_2^T]\,[K_2]\,[B_2] \qquad [\overline{M}_2] = [B_2^T]\,[M_2]\,[B_1]$$

$$[K] = [\overline{K}_1] + [\overline{K}_2] \qquad [M] = [\overline{M}_1] + [\overline{M}_2]$$

$$\{\overline{R}_1\} = [B_1^T]\,\{R_1\} \qquad \{\overline{R}_2\} = [B_2^T]\,\{R_2\}$$

$$\{R\} = \{\overline{R}_1\} + \{\overline{R}_2\}$$

3.8 Reduced Order Free Vibration Problem

The reduction scheme in the previous section reduced the structure from N degrees of freedom, as represented by the geometric coordinates, to S degrees of freedom representing the number of generalized coordinates and corresponding approximate modes (where N is the number of geometric degrees of freedom in the original idealization and S is the number of approximate modes used). From the solution of the reduced order free vibration problem, the lower frequencies and mode shapes expressed in generalized coordinates are computed. The frequencies represent approximations to the true frequencies of the lower modes of vibration, the accuracy is generally very good for the lowest modes and very poor in the highest modes. The generalized coordinate mode shapes (ϕ)

represent a square S x S matrix. These mode shapes are orthogonal with respect to the generalized mass and stiffness matrices. That is:

$$[\phi_m]^T\ [M]\ [\phi_n] = 0 \qquad m \neq n \qquad (3.4a)$$

$$[\phi_m]^T\ [K]\ [\phi_n] = 0 \qquad m \neq n \qquad (3.4b)$$

As many approximate modes as desired may be used in the original reduction. In general, it may be advisable to use as many as S approximate modes if it is desired to obtain S/2 vibration mode shapes and frequencies with good accuracy (1).

The set of generalized-coordinate mode shapes are normalized by dividing by some reference coordinate. The product of the approximate mode shapes and the generalized-coordinate mode shapes gives approximate mode shapes in geometric coordinates, which is of dimension N x S. These mode shapes are orthogonal with respect to the mass and stiffness expressed in geometric coordinates. They are used in the standard mode-superposition dynamic-analysis procedure (1).

3.9 Modal Transformation

The mode superposition method is a very effective procedure for the dynamic analysis of any linearly elastic structure to any prescribed dynamic excitation. One of its major advantages is that the same general technique is used to obtain any desired degree of accuracy. A simple approximate solution is provided by considering only the fundamental mode of vibration. Adding more modes increases the accuracy of the solution.

The second reduction is a standard modal superposition by substructures. The lower modes (up to one-half of the approximate modes if desired) are used to form new component modes for the substructures.

These component modes are orthogonal and the new reduced orders of [M] and [K] are true diagonal matrices. The damping matrix is also diagonal when modal damping is used. Summing these new reduced orders of [M], [K], and [C] for the substructures is synonymous to modal superposition for the total structure.

3.10 The Equation of Motion

The equation of motion, when the excitation is applied to a building in the form of horizontal support motion, has the form:

$$[M]\{\ddot{X}_t\} + [C]\{\dot{X}\} + [K]\{X\} = 0 \qquad (3.5)$$

where $\{X_t\}$ denotes total displacement, and $\{X\}$ is relative displacement. The inertia-force term depends on total motion, while the damping and elastic forces depend only on relative motion. The mass of the structure is assumed to be lumped or concentrated at the joints. The inertia forces (F_I) of the structure are given by:

$$\{F_I\} = [M]\ \{\ddot{X}_t\} \qquad (3.6)$$

in which

$[M]$ = diagonal matrix of the structure masses; and

$\{\ddot{X}_t\}$ = total acceleration of the stories.

The damping forces at the floor levels, $\{F_D\}$, may be expressed similarly:

$$\{F_D\} = [C]\ \{\dot{X}\} \qquad (3.7)$$

in which

$[C]$ = damping matrix of the building; and

$\{\dot{X}\}$ = relative velocity of the stories.

The relationship between total and relative displacements is expressed as:

$$\{X_t\} = \{1\}X_g + \{X\} \tag{3.8}$$

where

$\{X_g\}$ = horizontal support displacement; and

$\{1\}$ = unit column vector.

Differentiating Equation (3.8) twice with respect to time and substituting into Equation (3.5) leads to the following equation involving only the relative displacement as the dependent variable:

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = -[M]\{1\}\ddot{X}_g \tag{3.9}$$

where $-[M]\{1\}\ddot{X}_g$ represents the effective force acting in the structure. The structure responds as though dynamic forces were applied at each floor or story level. This force is equal to the product of the story mass and the ground acceleration.

The mode superposition method of analysis is based on a transformation of the relative displacement coordinates of the structure to generalized mode shapes. The orthogonality of the modes permits the definition of modal mass, stiffness, and damping (for damping proportional to mass or stiffness matrices):

$$\{\phi_n\}^T [M]\{\phi_n\} = M_n \tag{3.10a}$$

$$\{\phi\ \}^T [K]\{\phi_n\} = K_n \tag{3.10b}$$

$$\{\phi\ \}^T [C]\{\phi_n\} = C_n \tag{3.10c}$$

where $\{\phi_n\}$ is the nth mode. In this study the damping matrix is proportional to the mass matrix which leads to the following relationship (1):

$$[C] = \begin{bmatrix} \xi_1\omega_1 m_1 & 0 & 0 & . & 0 \\ 0 & \xi_2\omega_2 m_2 & 0 & . & 0 \\ 0 & 0 & \xi_3\omega_3 m_3 & . & 0 \\ . & . & . & . & 0 \\ 0 & 0 & 0 & 0 & \xi_n\omega_n m_n \end{bmatrix} \qquad (3.11)$$

where ξ is the percent of critical damping. The transformation which results in mode superposition is expressed as:

$$\{X\} = [\phi]\{V\} \qquad (3.12)$$

where

$[\phi]$ = transformation matrix made up of the mode shapes; and

$\{V\}$ = amplitude of the modal components.

Substituting Equation (3.12) into Equation (3.9) leads to

$$[M][\phi]\{\ddot{V}\} + [C][\phi]\{\dot{V}\} + [K][\phi]\ \{V\} = -[M]\{1\}\ddot{X}_g \qquad (3.13)$$

Premultiplying by the transpose of the transformation matrix, the equation of motion becomes

$$[\phi^T][M][\phi]\{\ddot{V}\} + [\phi^T][C][\phi]\{\dot{V}\} + [\phi^T][K][\phi]\{V\} = -[\phi^T][M]\{1\}\ddot{X}_g \qquad (3.14)$$

or

$$M_n\{\ddot{V}_n\} + C_n\{\dot{V}_n\} + K_n\{V_N\} = -[\phi^T][M]\{1\}\ddot{X}_g \qquad (3.15)$$

3.11 Solution of Equation of Motion

For this study a computer program that established the various matrices and necessary transformations was developed. A subprogram based on a fourth-order Runge-Kutta technique (3) was added to this program. Thus a step-by-step integration of the equation of motion is carried out.

CHAPTER IV

NUMERICAL APPLICATIONS

To demonstrate the effectiveness of the present scheme for solving structural dynamic problems, three different models were selected and analyzed. The theory was first applied to a simply supported beam and the lower natural frequencies computed. All approximate modes used for this model consisted of deflected shapes computed from static loads.

The second model consisted of a thirteen-story steel frame. The lower natural frequencies were computed using only approximate modes from deflected shapes due to static loads. The frame was then analyzed for the lower frequencies and modes using a combination of fixed interface natural free vibration modes for the substructures and deflected shapes for the total structure under static loads.

The third model consisted of a twenty-story reinforced concrete frame. The lower frequencies and modes were computed using fixed interface natural modes for the substructures and static deflected shapes for the connected model. The lower frequencies and modes were used in a modal transformation to formulate the equation of motion. The response envelopes of displacements and moments for the girders and interior columns due to earthquake excitation were computed and compared with those given in References (1), (4), and (5).

4.1 Simply Supported Beam Model

4.1.1 Beam Properties

The beam is model with 10 uniform bar elements and 11 joints shown in Figure 8. Masses of (1.294) slugs are lumped at the interior joints. The beam was substructured as shown in Figure 9. Each substructure has 6 joints and 5 bar elements.

4.1.2 Approximate Modes

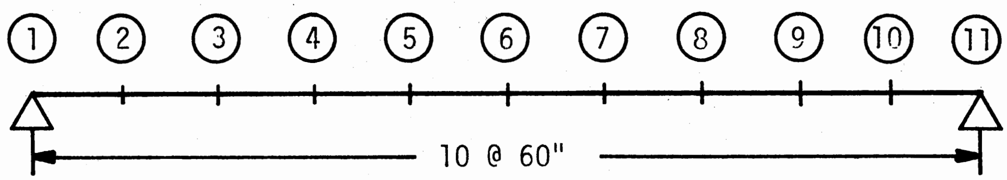
The connected model and substructures were loaded as shown in Figure 10 and static mode shapes were computed for each load condition. These load conditions were chosen arbitrarily.

The beam model was analyzed using seven approximate modes as generalized coordinates (static modes computed from all load conditions except 5, 6, and 10). The model was then analyzed using all 10 approximate modes as generalized coordinates.

4.1.3 Numerical Results for Beam Model

To evaluate the results obtained from the reduced order free vibration problem, the natural frequencies for the total structure were computed in the conventional manner. These are the baseline solution for the model with no reduction in degrees of freedom. Table I shows the lowest six of those natural frequencies.

Table II shows the natural frequencies predicted by seven generalized coordinates corresponding to the first three attachment modes augmented with the first four fixed interface modes of Figure 10. Reasonable accuracy is shown for the lowest three modes.



Beam Properties

Constant section

11 Joints

10 Members

Moment of inertia 1500 $in.^4$

○ Joint Number

33 Degrees of freedom (DOF)

3 Degrees of freedom at each joint

Figure 8. Simply Supported Beam Model

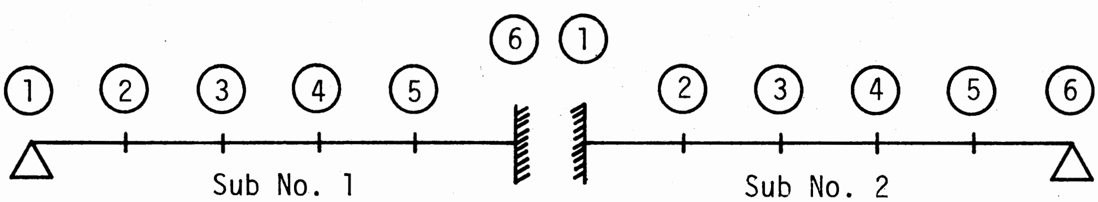
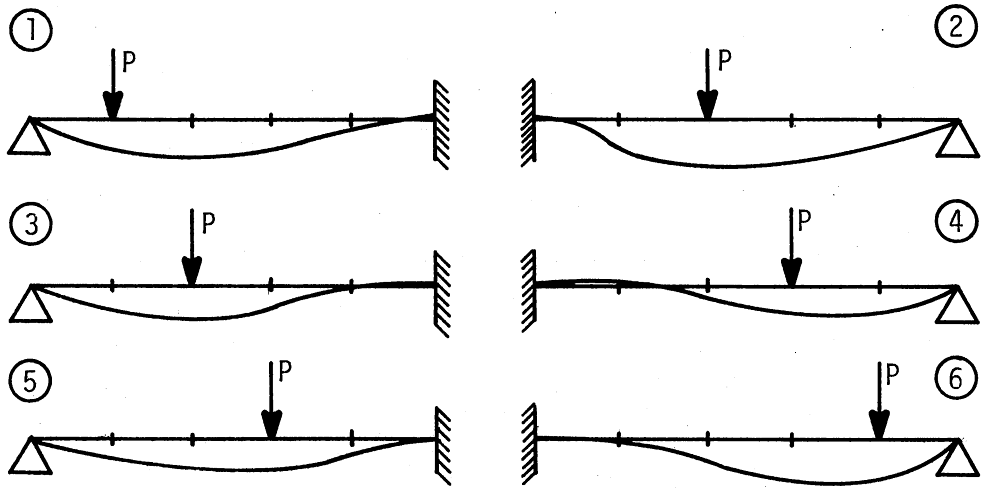
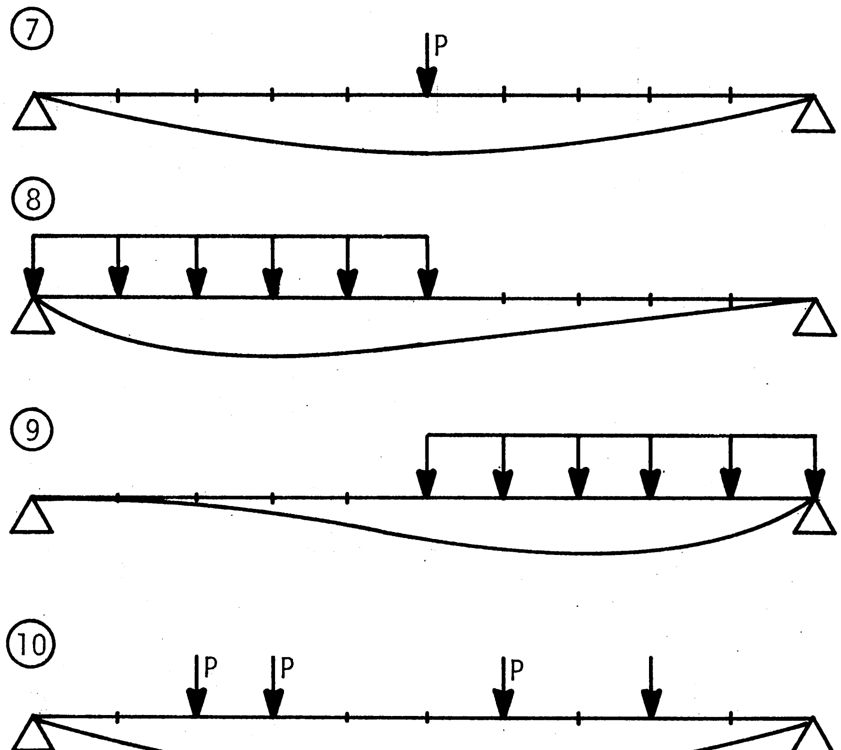


Figure 9. Substructures



(a) Fixed Interface Modes



(b) Attachment Modes

Figure 10. Static Mode Shapes

TABLE I

SIMPLE BEAM FREQUENCIES--PHYSICAL MODEL

Mode	1	2	3	4	5	6
Frequency (Hz)	2.01	8.56	18.11	32.15	50.00	71.18

TABLE II

SIMPLE BEAM FREQUENCIES--GENERALIZED MODEL
OF ORDER SEVEN

Mode	1	2	3	4	5	6
Frequency (Hz)	2.01	8.56	21.13	61.19	100.00	154.30

The approximate reduced order mode is refined by including all static modes shown in Figure 10, giving a ten-degree-of-freedom approximation. Excellent correlation is shown in the first six modes as shown in Table III.

TABLE III

SIMPLE BEAM FREQUENCIES--GENERALIZED MODEL
OF ORDER TEN

Mode	1	2	3	4	5	6
Frequency (Hz)	2.01	8.05	18.11	32.15	50.00	71.21

4.2 Steel Frame Model

4.2.1 Frame Dimensions and Properties

For the second model a thirteen-story rigid jointed planar steel frame was selected. The frame has 56 joints and 91 bar elements (Figure 11).

Using the portal method for wind loads and computed gravity loads, members were selected in the conventional way and are shown in Figure 12. The properties of the members are given in Table IV.

Masses were lumped at the joints with the following numerical values:

M_1 = 12.94 lb-in./sec^2 at top girder exterior column joints

M_2 = 28.40 lb-in./sec^2 at top girder interior column joints

M_3 = 32.00 lb-in./sec^2 at all other exterior column joints

M_4 = 71.20 lb-in./sec^2 at all other interior column joints.

The lower natural frequencies of this frame were required. The structure has global kinematic degrees of freedom corresponding to two translations and one rotation at each joint.

4.2.2 Approximate Modes

The connected model and substructures were loaded as shown in Figures 13 and 14. Static mode shapes were computed for the eight load conditions.

The frame was analyzed first using only the four attachment modes as generalized coordinates. It was then analyzed using a combination of attachment modes and the fixed interface static modes computed from the load conditions shown in Figure 14.

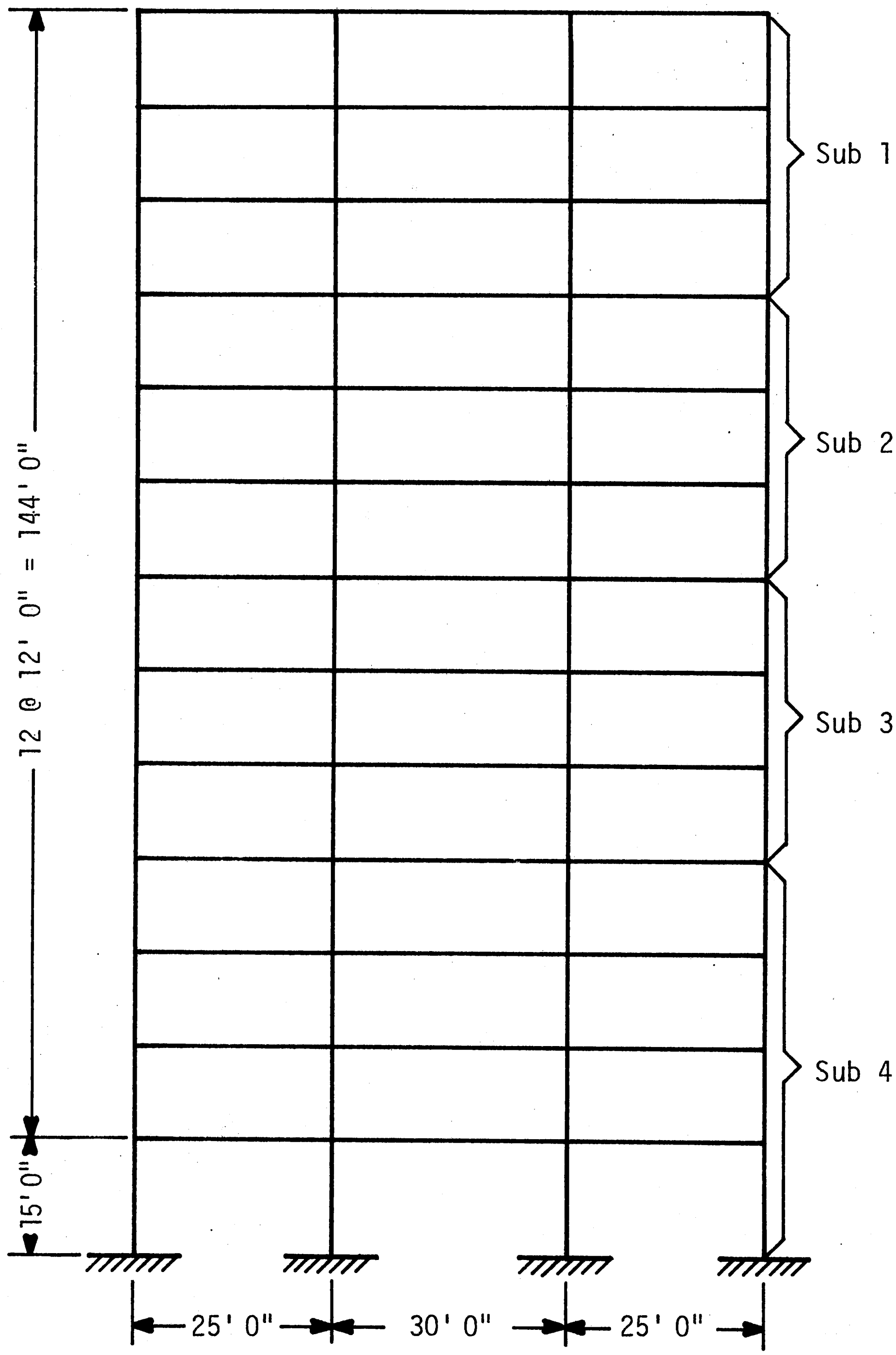


Figure 11. Building Frame--Substructure Model With 56 Joints and 168 Degrees of Freedom

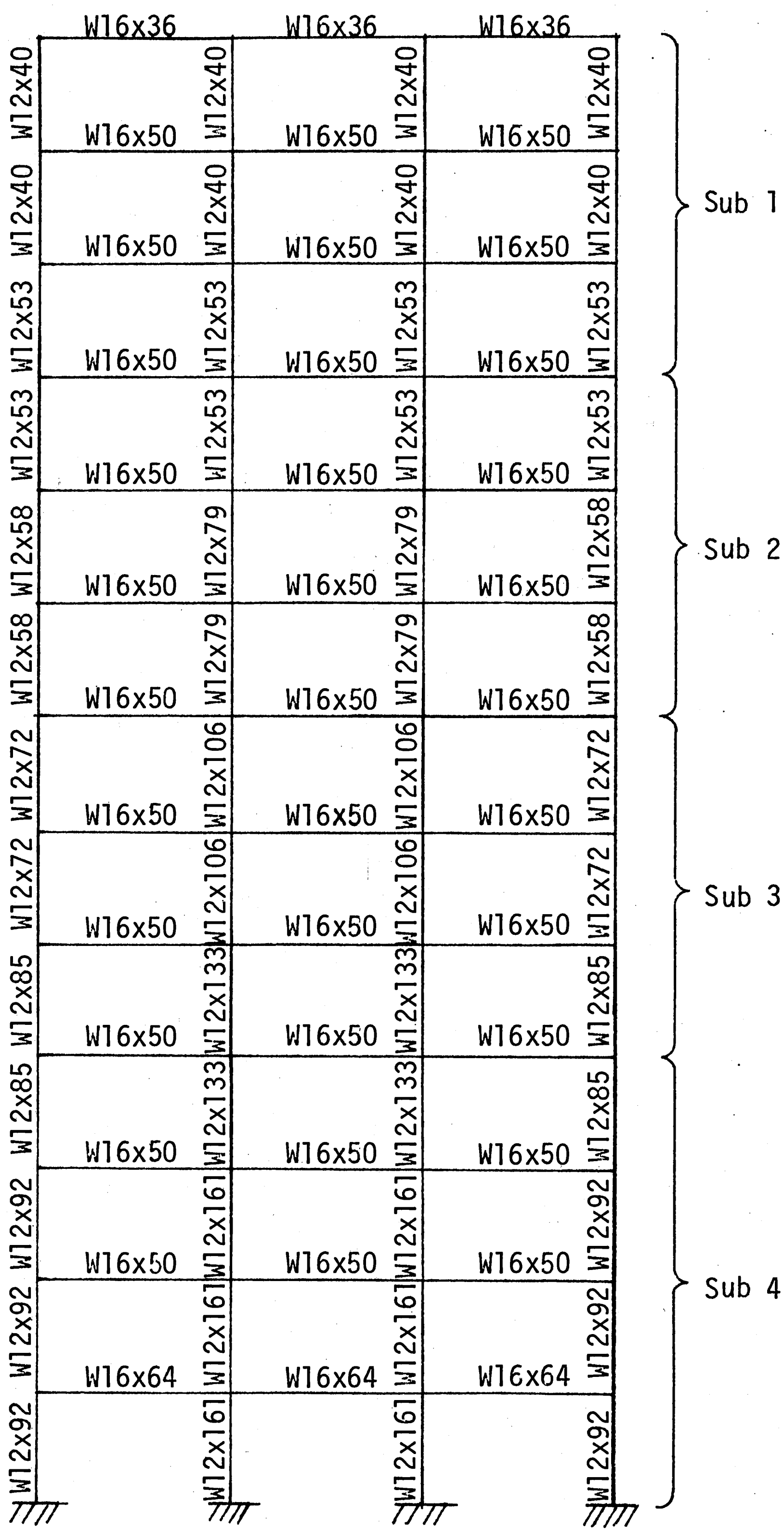
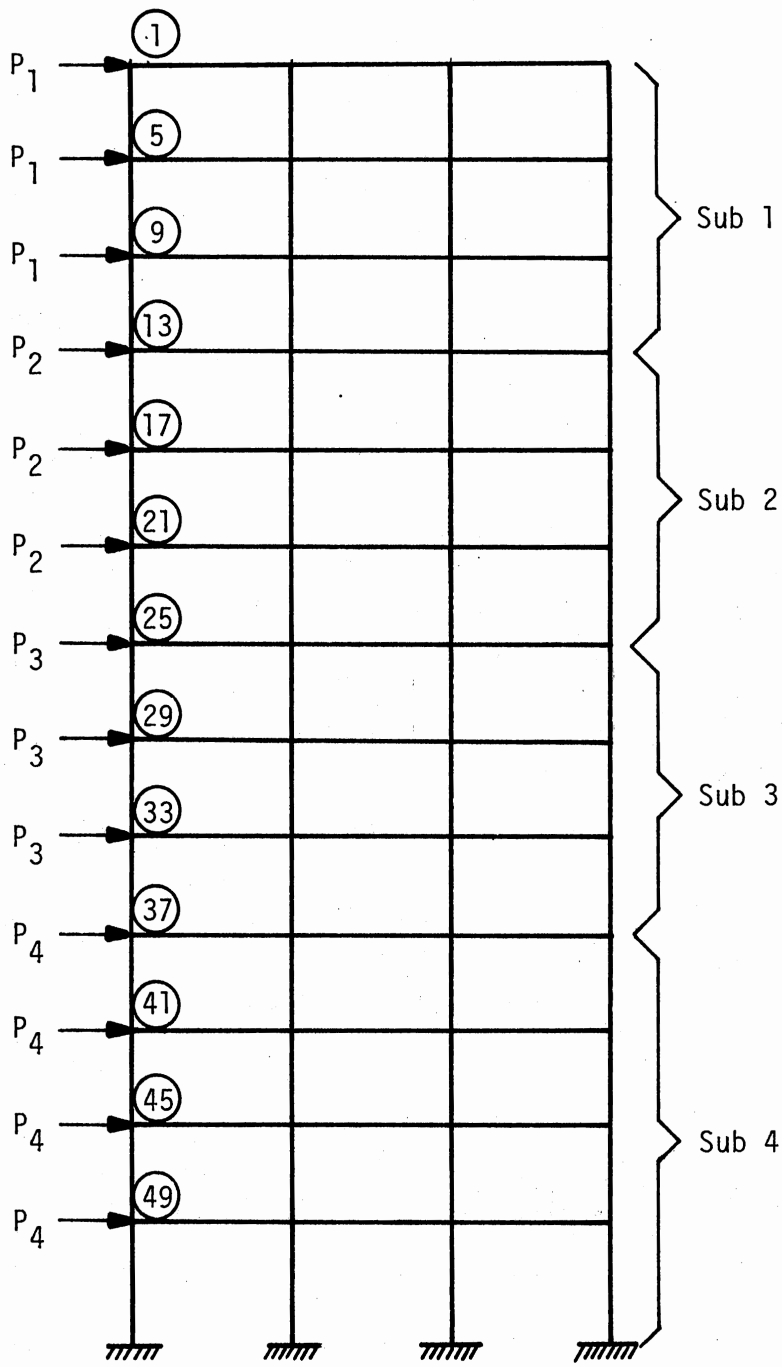


Figure 12. Building Frame--Member Schedule

TABLE IV

MEMBER PROPERTIES

Member	XI	XA	Weight
W12x40	310.10	11.77	40.00
W12x53	426.10	15.59	53.00
W12x58	476.10	17.06	58.00
W12x72	597.40	21.16	72.00
W12x79	663.00	23.22	79.00
W12x85	723.30	24.98	85.00
W12x92	788.20	27.06	92.00
W12x106	930.70	31.19	106.00
W12x133	1071.70	39.11	133.00
W12x161	1541.80	47.38	161.00
W16x36	446.30	10.59	36.00
W16x50	655.40	14.70	50.00
W16x64	833.80	18.80	64.00



P_1's are Load Condition No. 1, P_2's are Load Condition No. 2, etc.

Figure 13. Building Frame--Load Conditions for Attachment Modes

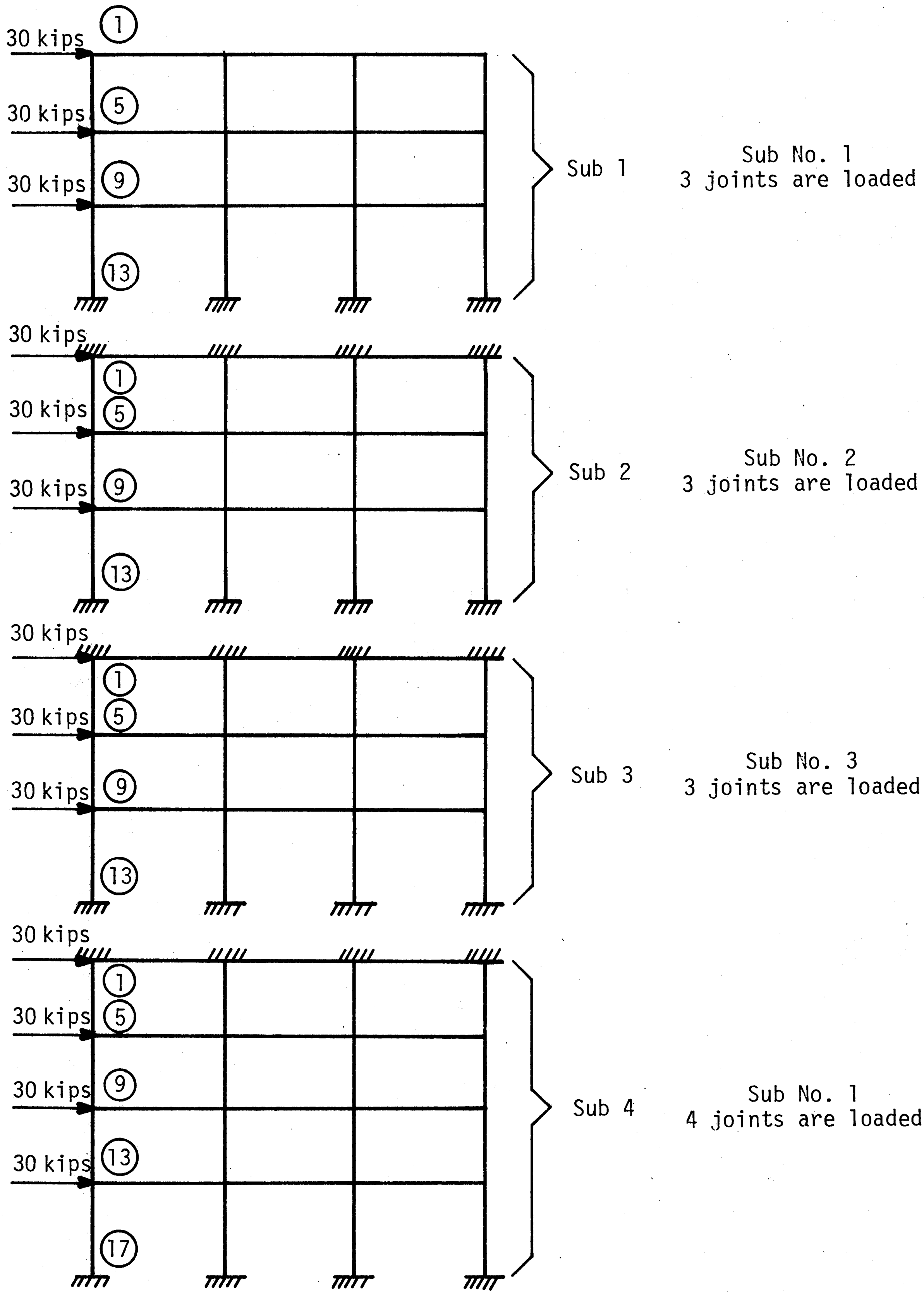


Figure 14. Load Conditions for Fixed Interface Modes

Additional fixed interface modes for the substructures were obtained by solving the free vibration problem for the substructures with fixed interfaces. The first few normal modes are used as generalized coordinates for each substructure.

The frame was modeled twice using the four original attachment modes and the first four and the the first eight normal modes for each of the substructures. The reduced order free vibration problem was solved and the lower frequencies and modes determined for the total structure. The first 12 frequencies are shown in Table V.

The frame was then loaded as shown in Figure 15 and eight attachment modes were computed for the total structure. The frame was then modeled using combinations of the eight attachment modes and mode shapes for the substructures computed from the free vibration problem. Results are presented in the section that follows.

4.2.3 Numerical Results for Steel Frame

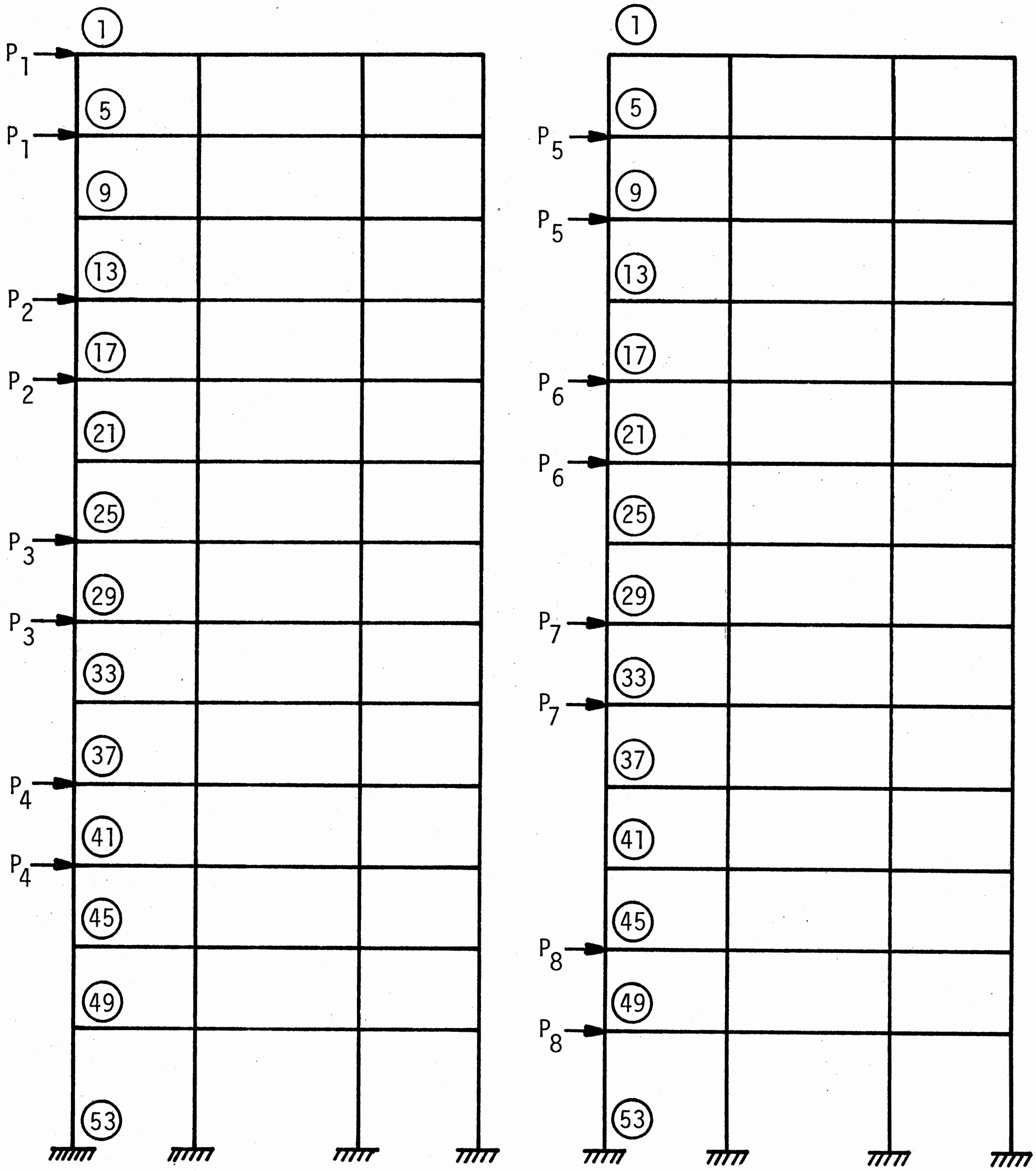
The natural frequencies and modes for the total structure were computed from a conventional method to provide a baseline for assessing the accuracy of the reduced order analysis. The lowest 12 natural frequencies are shown in Table V. The lower frequencies computed from each of the reduced order free vibration problems are also shown in Table V. The number of frequencies above the solid line are one-half of the number of approximate modes used in each idealization. These frequencies show close approximations when the same number of approximate modes are used.

The first 12 modes computed from the conventional method and those computed from the reduced order when 36 component modes were used are plotted in Figure 16.

TABLE V

SUMMARY OF PLANAR FRAME FREQUENCIES

Attachment Modes	4	4	4	4	8	8	8	8	8	
Fixed Interface Static Modes	0	4	0	0	0	0	0	0	0	
Fixed Interface Free Vibration Modes	0	0	4	16	0	4	16	20	28	Conventional Solution of Free Vibration Problem
Mode No.	Frequencies									
1	2.59	2.58	2.59	2.58	2.59	2.59	2.59	2.59	2.58	2.58
2	7.62	7.60	7.62	7.60	7.61	7.61	7.59	7.59	7.59	7.57
3	13.35	13.25	13.30	13.19	13.17	13.16	13.10	13.10	13.07	13.01
4	20.28	19.67	19.69	19.21	19.33	19.25	19.07	19.05	19.01	18.80
5	.	31.08	30.28	27.07	26.31	25.97	25.45	25.38	25.29	25.04
6	.	40.61	39.22	34.60	34.36	33.58	32.89	32.85	32.75	31.86
7	.	59.62	55.80	41.32	43.23	41.91	39.95	39.92	39.88	38.84
8	.	140.19	134.78	49.69	55.38	49.82	45.65	45.60	45.55	45.43
9	.	.	.	53.85	.	74.98	52.75	52.69	52.61	51.57
10	.	.	.	63.89	.	87.22	59.93	59.84	59.67	58.52
11	.	.	.	67.09	.	143.27	67.06	67.06	67.05	66.96
12	.	.	.	79.43	.	271.45	78.54	78.51	78.45	78.09
								. . .	. . .	



P_1's are Load Condition No. 1, P_2's are Load Condition No. 2, etc. All point loads are 30 kips.

Figure 15. Steel Frame--Load Conditions for Additional Attachment Modes

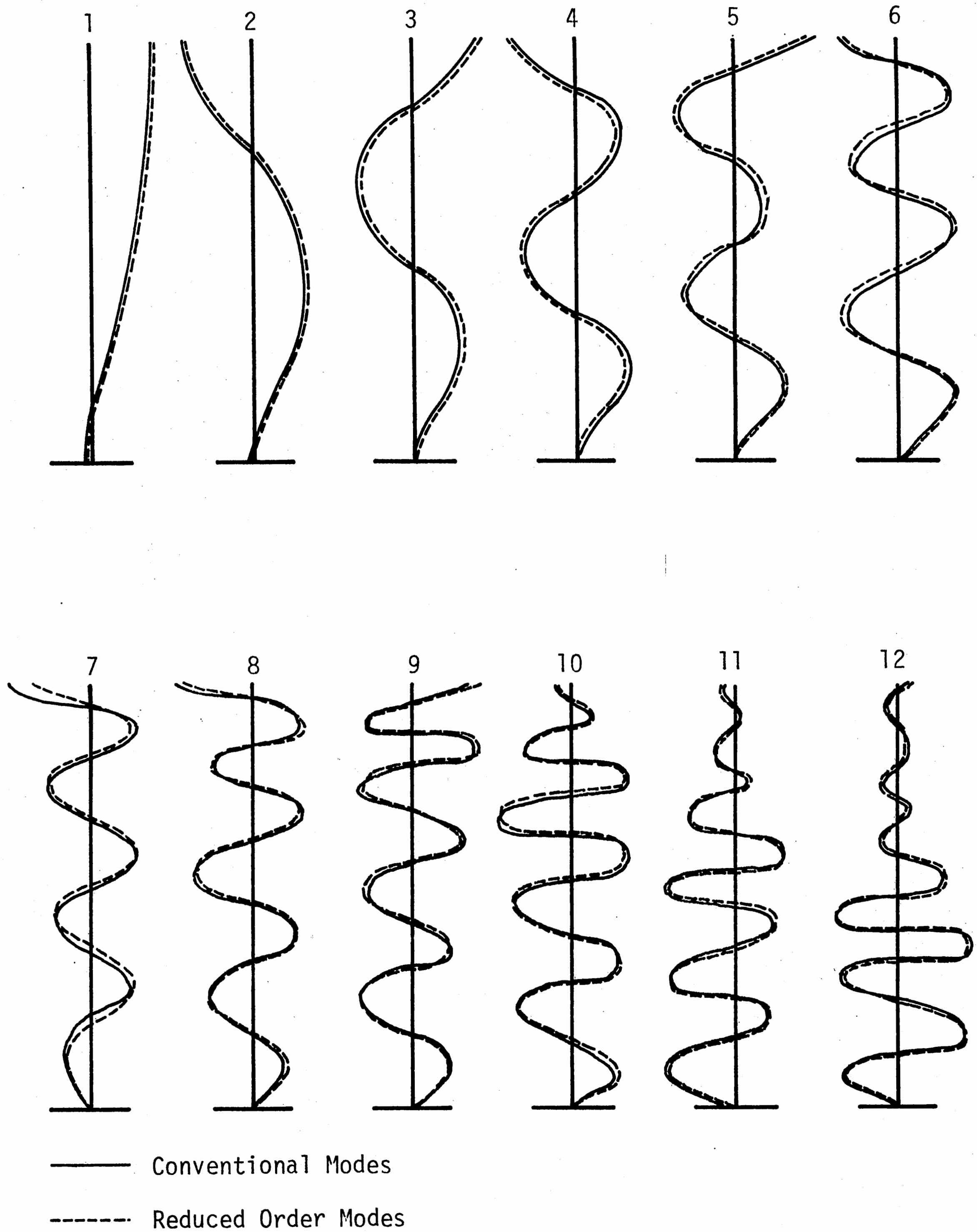


Figure 16. Steel Frame--Summary of Mode Shapes

The lower 12 frequencies and modes computed from the reduced order of 36 show close approximation to the lower natural frequencies and modes computed from the conventional free vibration problem. These lower frequencies and modes may be used in a modal transformation to formulate the equation of motion for the structure.

4.3 Reinforced Concrete Frame Model

For the final model a twenty-story reinforced concrete building frame was selected. The response of a building composed of two parallel sets of these frames subjected to earthquake excitation is given in References (1), (4), and (5). The response was computed using ten percent critical damping in each mode for the first six modes (5). The response was also computed using the present method with ten percent critical damping and six modes. However, there were several differences in the method of modeling.

In the analysis given by the above references axial deformation was neglected for the girders, shear deformation was included, and flexural deformation was considered for all members. The stiffness for the girders was modified to eliminate axial deformation. Several modifications were made to the frame stiffness to eliminate the vertical translation and rotational degrees of freedom, which resulted in a lateral stiffness for the frame. This reduced the number of degrees of freedom for the frame to one per story (5).

In the present method the stiffness matrix for each substructure was established using standard procedures. No modifications were made to the stiffness matrix (all degrees of freedom were considered). Lateral forces computed from the Uniform Building Code (Vol. 1, 1974 edition) were used to compute the displacement of the frame. Using the member properties and dimensions given in the references, the total lateral displacements exceeded those given in the references by approximately 21 percent. These properties were adjusted to provide the same static deflections as those given in the references. The resulting frame stiffness was assumed to be equivalent to the original frames with the stated modifications. (The given EI_0 of 133,500 kip-ft^2 was increased to 158,500 kip-ft^2.)

4.3.1 Model Dimensions and Properties

The geometric arrangement and relative stiffness properties of the original frame as given in References (1), (4), and (5) are shown in Figure 17. The fundamental period of vibration is given as 2.2 seconds.

The structure was divided into four components. Each component has 5 stories, 24 joints, 35 members, and 72 degrees of freedom. The story weight was converted to mass and lumped at the joints according to the area supported by the columns.

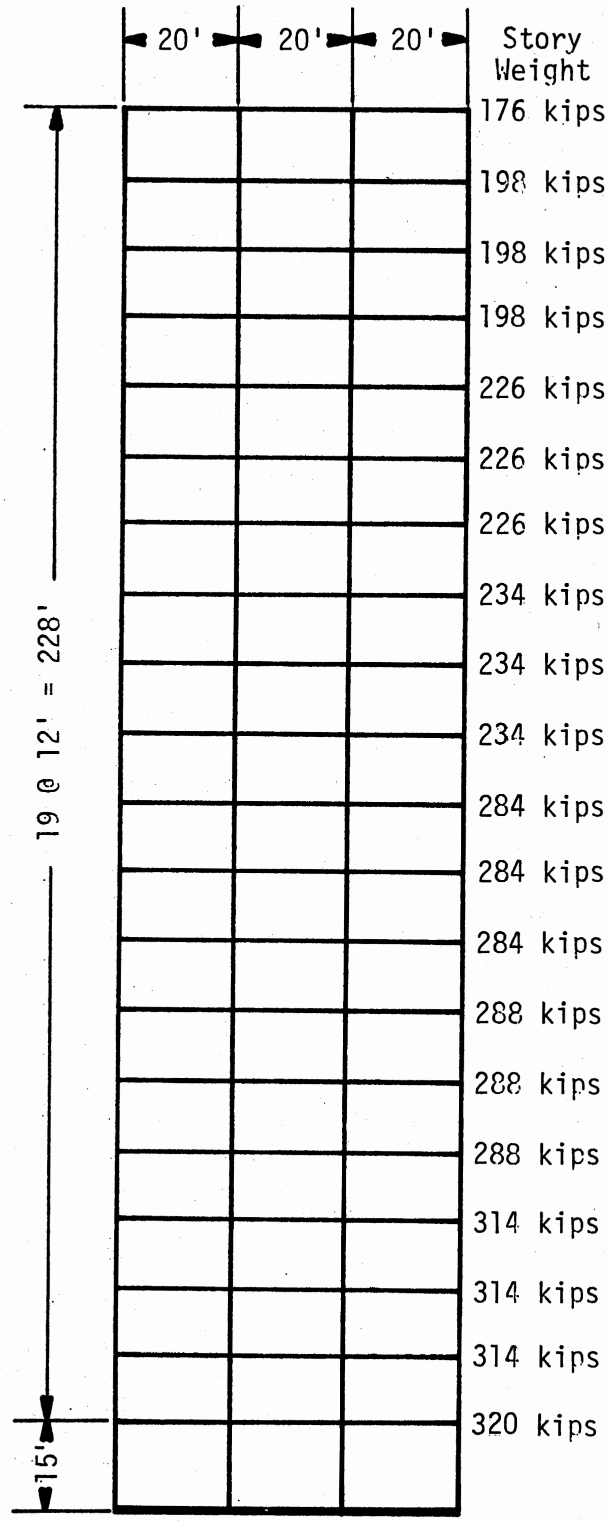
4.3.2 Approximate Modes

The connected model was loaded as shown in Figure 18. Static mode shapes (attachment modes) were computed for these eight load conditions. These static modes were used as component modes for the substructures.

The free vibration problem was solved for each substructure with fixed interfaces. The lower modes are used as component modes for the respective substructures.

EI_o = 133,500 kip-ft^2

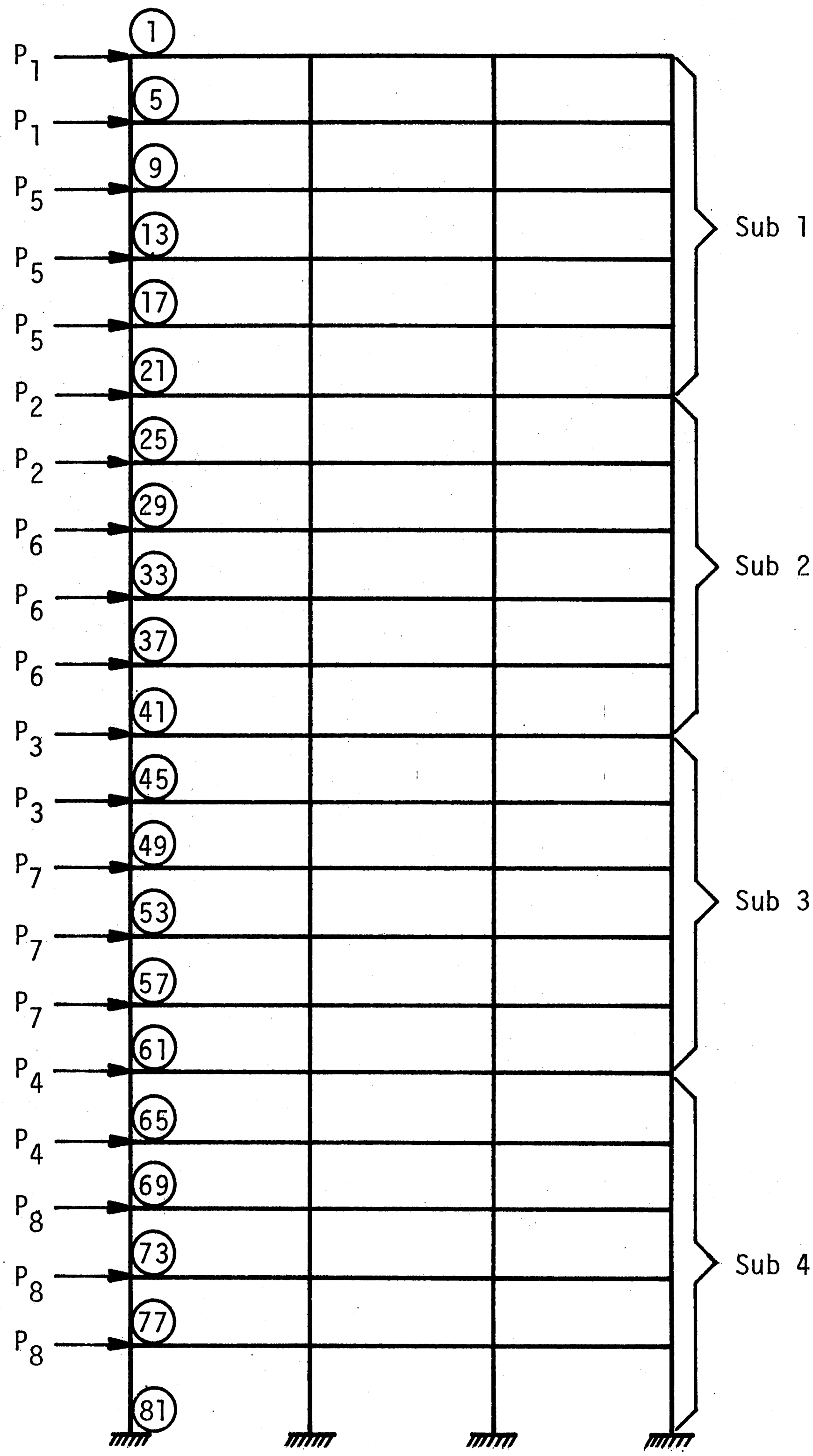
T = 2.2 seconds



Relative Stiffness of Columns and Girders

Ratio $EI:EI_o$		
Columns		Girders
Exterior	Interior	
1.0	2.0	4.0
1.5	3.0	
3.0	6.0	6.0
4.5	9.0	
6.0	12.0	8.0
10.0	20.0	10.0
12.0	24.0	

Figure 17. Concrete Frame Dimensions and Stiffness Properties



P_1's are Load Condition No. 1, P_2's are Load Condition No. 2, etc.

Figure 18. Reinforced Concrete Frame Load Conditions for Attachment Modes

4.3.3 Numerical Results for the Concrete Frame Model

The eight attachment modes and the first seven free vibration modes for each substructure were used to form transformation matrices for each substructure. The reduced orders of [M] and [K] were computed and the reduced order free vibration problem solved. The frame was reduced to 36 degrees of freedom from this transformation.

The fundamental period of vibration was computed to be 2.228 seconds as compared to 2.200 seconds for the original building. This constitutes an error of 1.3 percent for the fundamental period of vibration.

The first 12 lower frequencies are listed in Table VI along with those computed from a conventional solution.

4.3.4 The Equation of Motion

The first six modes computed in the previous section were used to form new transformation matrices for the substructure. From modal superposition new reduced orders of [M], [K], and [C] were computed for each substructure (for this model the damping is porportional to the mass matrix with ten percent critical damping in each mode). The new orders of [M], [K], and [C] were used to form the equation of motion.

The forcing function consisted of the first ten seconds of the N-S component of the El Centro Earthquake of 1940. The first 30 seconds of the digitized ground acceleration was obtained from the Pasadena Institute of Technology. This particular earthquake was selected because of its long duration and intensity. Figure 19 shows the accelerogram for the first ten seconds of the earthquake. The time step for this digitized accelerogram is 0.1 seconds.

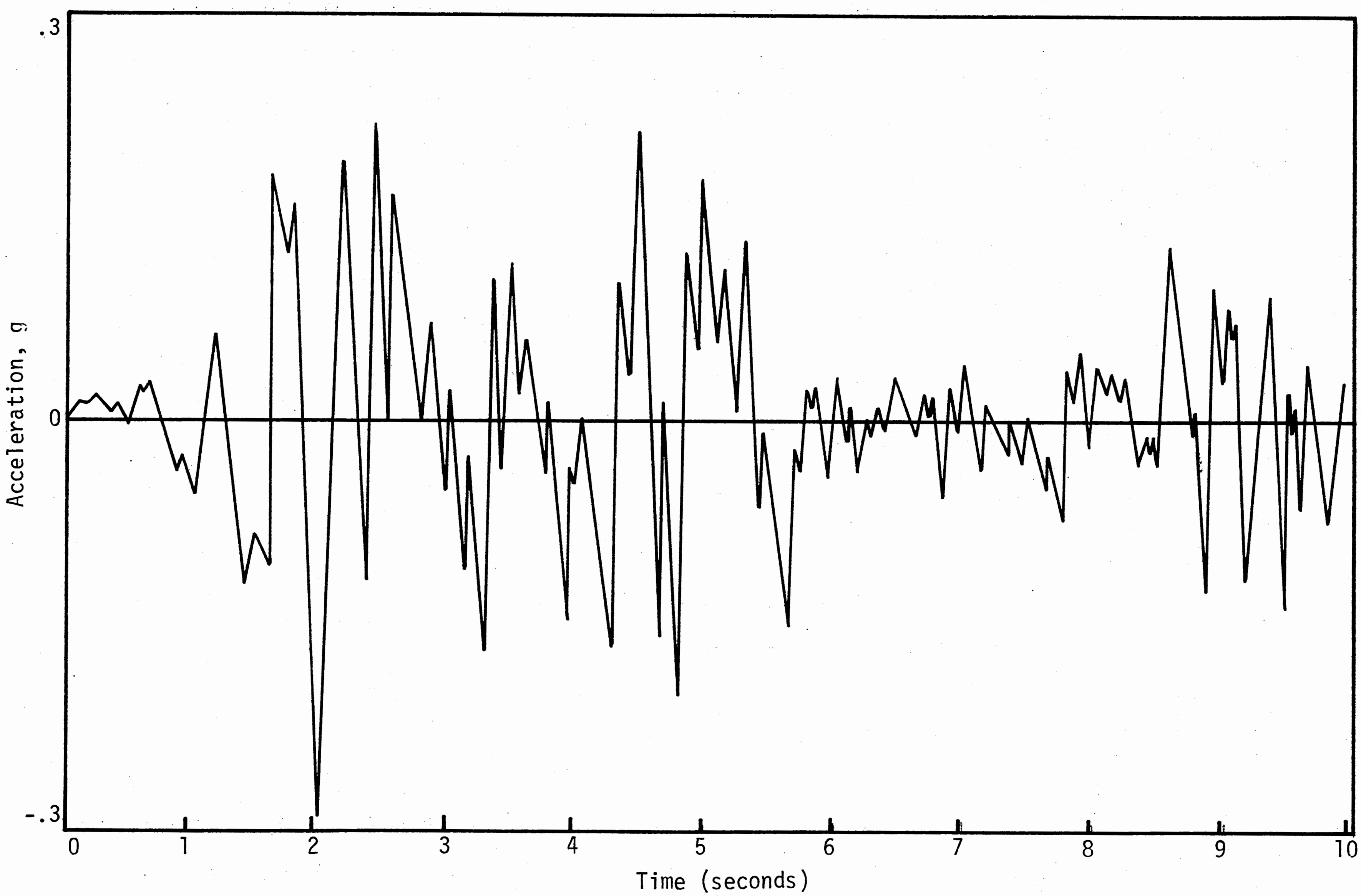


Figure 19. Accelerogram for N-S Component of El Centro Earthquake of 1940

TABLE VI

LOWER FREQUENCIES FOR CONCRETE FRAME MODEL

Mode No.	Reduced Order	Conventional Solution	Percent Variation
1	2.82	2.82	0.00
2	7.29	7.29	0.00
3	12.61	12.60	0.04
4	17.57	17.54	0.19
5	23.17	23.10	0.30
6	29.15	29.05	0.31
7	34.26	34.08	0.53
8	40.74	40.62	0.30
9	46.41	46.21	0.45
10	52.56	51.85	1.40
11	57.83	57.45	0.67
12	63.18	62.64	0.85

The response of the structure was evaluated using 6, 7, 8, 9, and 10 modes. It was also evaluated using the first 15 modes computed in the previous section.

The complete response history for the frame was evaluated; however, only the response envelope (i.e., the maximum value achieved by each response quantity at any time during this period of the earthquake) were considered. There was small refinement in the computed responses of the structure when the number of modes were increased from 6 to 10. However, when 15 modes were used instead of 10, no change in the response of the structure was observed. The envelopes of lateral displacements when 6 modes were used are plotted in Figure 20 along with those given in References (1), (4), and (5).

Response envelopes for girder moments and interior column moments were also computed and compared with those given in the References. These plots are shown in Figure 21.

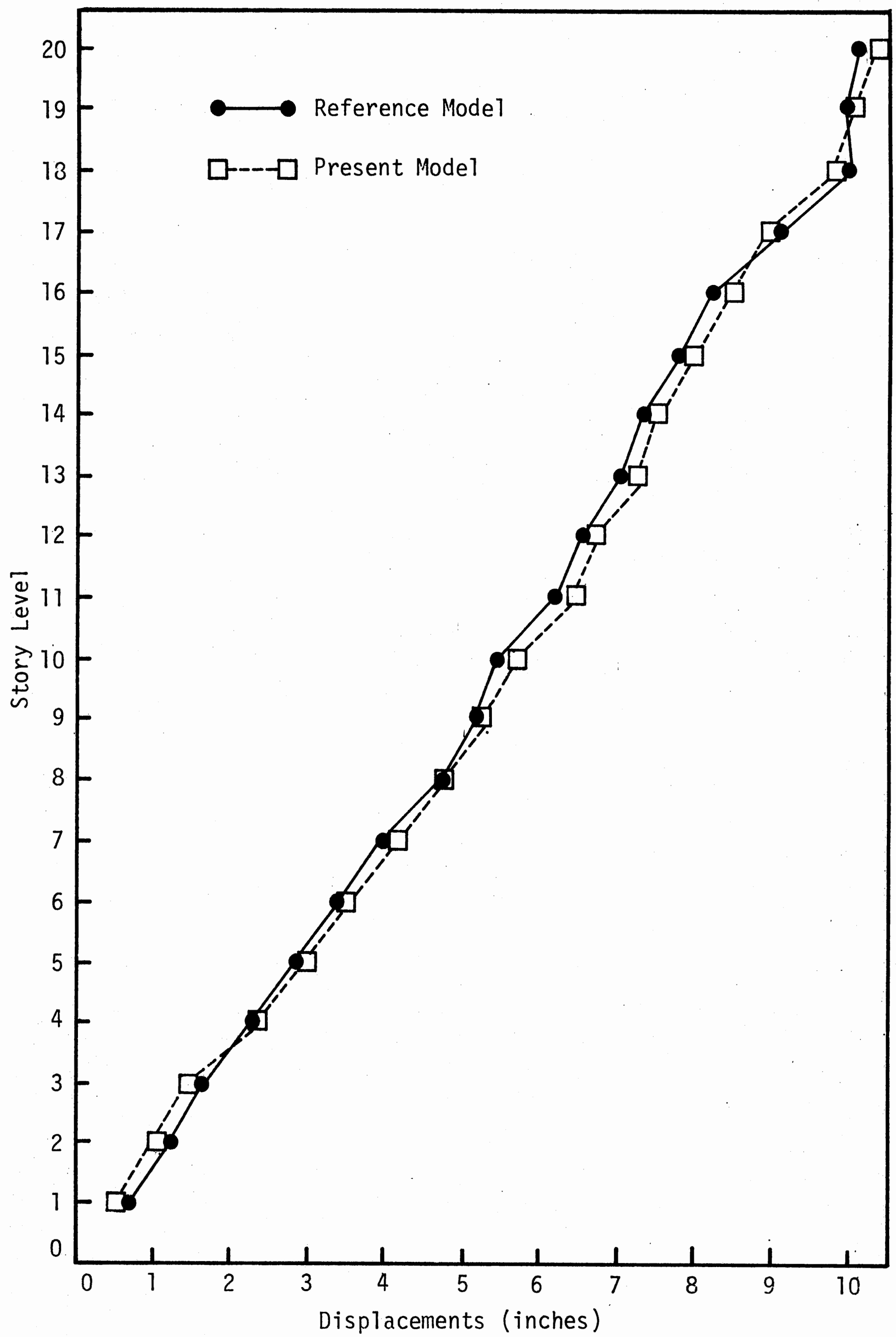


Figure 20. Dynamic Earthquake Response for Reference and Present Method

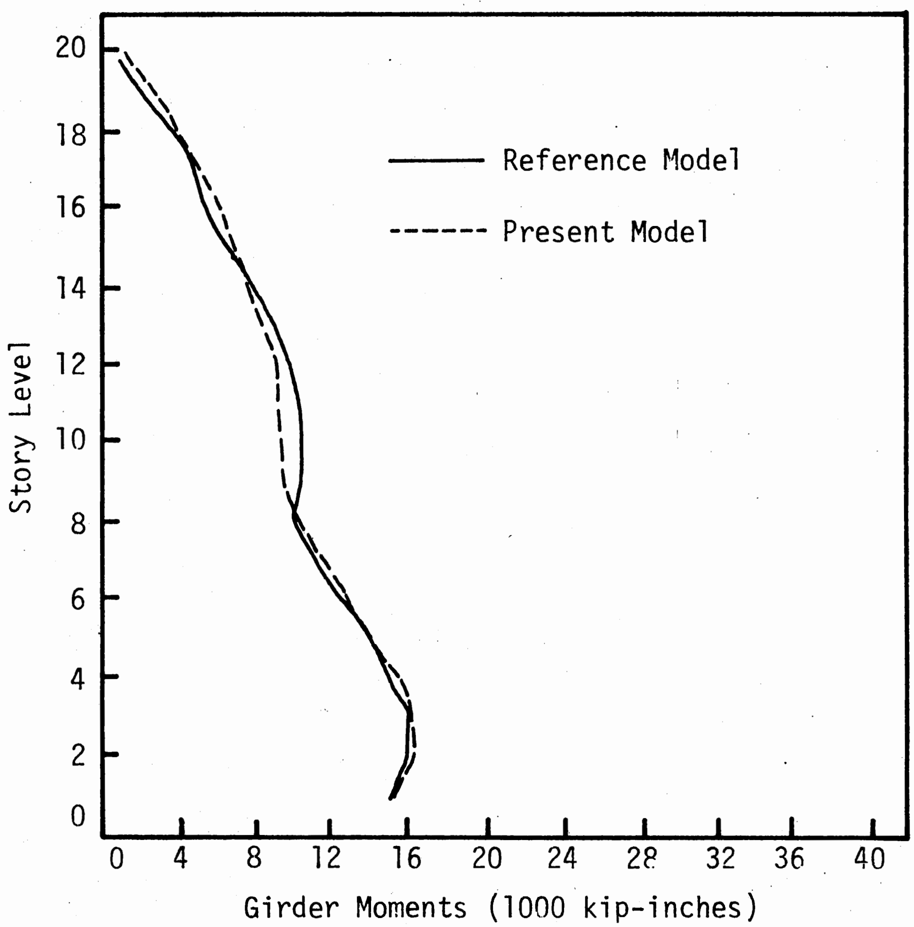


Figure 21. Girder and Interior Column Moments

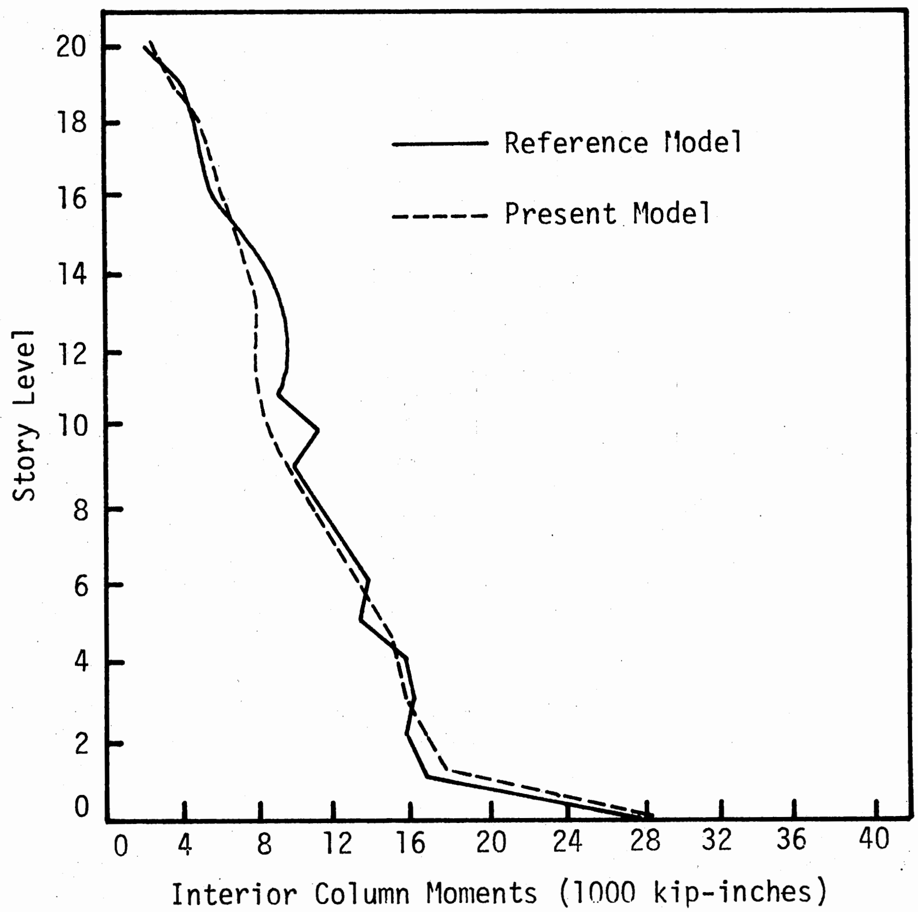


Figure 21. (Continued)

CHAPTER V

SUMMARY AND CONCLUSIONS

5.1 Summary

A simple and economical method for dynamic analysis of large complex models using a combination of substructuring and component modes as generalized coordinates has been presented. It is especially applicable for solving frame-type structures. This method is a variation of the Rayleigh-Ritz procedure, wherein approximate displacement patterns are computed from static deflected shapes. The method parallels existing modal synthesis methods, with the principal novelty found in the formulation and treatment of the interface compatibility problem.

Some mathematical complexity involved in devising compatible modes by other available schemes is avoided in the present work by choosing as coordinates only inherently compatible shapes with the interface displacements either zero or combinations resulting from prescribed loads.

The present technique thus relies on previously computed data (in the design phase of a structure, these deflected shapes should be readily available). Accordingly, the potential economy of the present technique rests on the availability of suitable static displacement data.

Once suitable approximate displacement patterns are identified, a reduced order model in these generalized coordinates is formed by sequential processing of substructure data.

For forced dynamic response problems, the model is further reduced by defining new generalized coordinates corresponding to free vibration modes of the reduced order model. This permits the exploitation of the orthogonality of the vibration modes in subsequent analysis.

Three numerical applications are presented. The first application was to compute natural frequencies and mode shapes of a simply supported beam. The primary motivation for this example was to verify theory and computer code. Excellent correlation to baseline solution was demonstrated.

The second numerical application was the free vibration analysis of a building frame. This was conducted to investigate the accuracy achievable by various combinations of attachment and fixed interface modes.

The final numerical application was the analysis of a standard building frame subjected to earthquake excitation. Numerical integration of the present model gave displacements and stresses comparable to the generally accepted work of Clough et al. loc cit.

5.2 Conclusions

The technique presented in this study is accurate and readily programmable. The use of attachment modes eliminates complex mathematical procedures required by similar techniques.

Results of this study indicate that the number of attachment modes are very significant for the lower frequencies and normal modes. However, the fixed interface modes also have a marked effect on these lower frequencies and normal modes if a considerable number are used. These fixed interface modes are easy to obtain from the free vibration problem, and as many as are necessary may be included.

The number of attachment modes is somewhat arbitrary, but results from this study indicate that about one-fourth of the total number of approximate modes used is a reasonable proportion.

Results from this study reinforce the conclusion that a building designed for Code seismic forces must be expected to suffer overstress when subjected to tremors of the severity investigated. Only elastic stresses are predicted, while in reality plastic action would occur.

5.3 Suggestions for Future Studies

In this study the number of attachment modes used was determined arbitrarily. An extension of this study to determine a scientific method of determining the apportionment of attachment and fixed interface modes is suggested.

It is also recommended that an investigation be made on the feasibility of extending the first step reduction of this technique to non-linear analysis.

A further potential extension is to combine the present technique with the existing method of subspace iteration to exploit the economics of each technique.

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VITA 2

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