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A SYSTEM OF INVENTORY CONTROL FOR
RETAIL FARM EQUIPMENT DEALERS

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## PREFACE

This study is concerned with the inventory control policies being used, at the present, for the parts inventory of retail farm implement dealers. The main objective is to minimize the costs of carrying inventory and to balance these costs against the costs of being without stock. The ABC classification of inventory was used to divide the inventory into categories. A random sample of approximately 50 items was chosen from the total of " A " items and analyzed to determine carrying and ordering costs for both the present in use and the proposed systems.

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## INTRODUCTION AND SETTING OF THE PROBLEM

The most important asset for many firms is inventory, and the skill with which inventory is managed greatly affects net income. This is particularly true of the retail farm equipment business.

Even though inventory management is crucial to profitable operations, Arrow, Karlin, and Scarf (11) say that economic theory has said little about inventories, and this is generally conceded in the inventory literature. However, economic theory has addressed the motives for holding cash, and although these motives are not universally accepted, the general consensus is that the three motives are: the transaction, precautionary, and speculative motives. Arrow, Karlin, and Scarf (11) suggest that these three motives can be applied to inventory.

The application of these motives to inventory is also pointed out by Dopuch and Birnberg (45) as follows:

1. The transaction motive is related to the need to hold inventory to meet production and sales requirements whenever production cannot, economically or feasibly, respond to demand in an instantaneous fashion;
2. A firm is motivated to hold precautionary amounts of inventory when the costs associated with not having the inventory on hand are greater than the costs of holding the additiond inventory;
3. A firm, in anticipation of future input or output price changes, may decide to hold moreor less inventory, speculating on the expected increase or decrease in future prices.

There are primarily two types of uncertainty that activate the transaction and precautionary motives for holding inventory. They are: (1) the uncertainty of demand, and (2) the uncertainty of lead time (time necessary to receive or manufacture the needed items). These two uncertainties make the decision of the level of inventory difficult to determine for a retail dealer who must purchase inventory.

Before decisions can be made regarding the inventory level, the objectives of the firm must be established. The determination of the inventory level is complicated by trade-offs among the alternative goals of a firm. For example, the sales manager's objective is to make the largest possible number of sales, and so that all orders can be filled promptly, he may want a large inventory. However, the objective of the sales manager could conflict with that of the controller of having a minimum of the firm's capital invested in inventory (54, p. 50). If there is no conflict in objectives, then management can solve each problem separately. Whenever there is conflict between the different objectives of a firm, attaining goal congruency is very difficult as there is no common measure of value for these different objectives. As Miller and Starr (84, p. 48) point out, "when objectives are dependent, the optimization of one can result in a lower degree of attainment for at least some others." This condition is known as suboptimization."

One approach in overcoming the conflict in objectives is that proposed by the "quantitative school" of management and requires the
measurement of the values of alternative strategies. This quantitative approach lends itself very well to inventory management and therefore a proliferation of models have been espoused for inventory control. However, before any of theṣe various models can be implemented, it must be possible to quantify costs. Buffa and Taubert (27) point out that our ability to quantify and develop models of most managerial problems depends on the behavior of costs, and in order to apply these models, we must be able to obtain these cost data.

Most of the authors of the different models assume the necessary costs are available and therefore do not discuss how the inputs to the model are to be obtained (95, p. 139) (45, p. 266). Other authors discuss the relevant costs but offer no suggestions about the measurement of these costs. The end result is that there very little in the literature on measurement of costs necessary to implement the model selected. All of the models minimize total inventory costs by balancing conflicting costs. These costs can be controlled by timing the placement of the order and varying the quantity ordered. Therefore, as most authors suggest, the inventory problem can be solved by answering two questions: (1) when to order, and (2) how much to order. Brown (24) defines an inventory control system as a coordinated set of rules for answering these questions routinely and for calling attention to the nonroutine situations that the rules do not cover.

Initially, the firm must establish an inventory policy based primarily on the service it wishes to provide for its customers. The problem then becomes one of minimizing total inventory costs within the objectives of the firm. Minimizing total inventory costs can be
accomplished by balancing the costs associated with carrying inventory against the penalties (costs) of being without stock.

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Objectives of the Study
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The primary objectives of this study are: (1) to investigate the present repair parts inventory control policies being used by the selected retail farm equipment firms and determine their awareness of the inventory management techniques available, and to determine how adequately they are managing their repair parts inventory; (2) to point out weaknesses in present policies which may be improved through the use of inventory management techniques and quantitative models and determine whether the new techniques are applicable to the problems of the farm machinery industry; and (3) to state conclusions and make recommendations for the application of the new techniques to the industry if such application is found to be both useful and practical.

Some specific objectives are as follows:

1. Discover if repair parts sales by the selected firms are seasonal as they have been in the past, or if they have changed due to the dramatic changes that have taken place in the industry.
2. Study the relevant costs needed in making inventory policy decisions. The three classes of costs to be considered are procurement costs, carrying costs, and out-of-stock costs.
3. Compare retail parts sales and inventory policies of the firms participating in the study.
4. Determine if the firm attempts to minimize total inventory costs.

The first chapter of the study includes the introduction, a discussion of the problem, the objectives, the organization and the limitations of the study. The next two chapters contain a literature review of previous work done in the field of inventory control and is divided as follows: Chapter II examines inventory costs and Chapter III presents the different inventory control models and their applications. The research methodology is presented in Chapter IV and includes a discussion of the types of analyses performed on the data collected.

Results of the analyses performed on the data and an interpretation of the results is presented in Chapter V. This is followed by the final chapter which summarizes the results of the study, presents the conclusions drawn and makes recommendations on the most suitable inventory policies for adequate control of the inventory of a retail farm equipment business.
*

Research Design and Methodology

This study contains an extensive review of the literature, which is divided into two chapters for the following reasons:

1. to provide an adequate background of information on specific techniques and inventory control models that have been proposed; and
2. to discover the techniques for identifying and measuring the inputs (costs) needed to implement the models.

Inventory control is a very fertile area for quantitative researchers, and therefore, many models have been proposed. On the
other hand, the measurement of the cost inputs for these models has, more or less, been assumed as given, thus many authors have neglected to include what these costs are, or how they should be measured.

In accomplishing the first objective of the study (determining if parts sales are seasonal in nature) retail parts sales figures were collected for the participating firms for five years on a monthly basis. These sixty data items were then plotted to determine if sales of repair parts are, in fact, seasonal. The anticipated results were that sales are seasonal, and upon making this determination, a seasonal index was computed using the ratio-to-moving average method to eliminate seasonal fluctuations.* The elimination of seasonal fluctuations allows the data to be analyzed for trend, cyclical, and irregular patterns without seasonal interference.

The inventory policy selected to control inventory often depends upon the characteristics of the inventory items. This difference in characteristics brought about the well-known ABC classification of inventory. Numerous studies of the inventories of different business firms have found that the same general relationships exist between usage value in dollars and the number of items in inventory. Plossl and Wight (93) describe the following groupings as typical:
"A" items: High value -- those relatively few items whose value accounts for $75-80 \%$ of the total value of the inventory. These will usually be from $15-20 \%$ of the items.
"B" items: Medium value -- a larger number in the middle of the list, usually about $30-40 \%$ of the items, whose total value accounts for about $15 \%$ of the total.
*For a discussion of this technique, see Robert D. Mason, Statistical Techniques in Business and Economics (Homewood, I11, 1970), pp. 473-482.
"C" items: Low value -- bulk of the items, usually about $40-50 \%$, whose total inventory value is almost negligible, accounting for only $5-10 \%$ of the value (p. 57).

The repair parts inventories of the selected firms in the study were subjected to an $A B C$ analysis within the above suggested ranges. From this analysis, inventory control policies can be suggested. An examination was made of those costs necessary to implement inventory control models. There are three classes of costs necessary to make an inventory control model operational. Ballou (12) defines these three classes as: procurement costs, carrying costs, and out-ofstock costs. These three costs were studied for each participating firm.

Finally, total inventory costs were compared under the present system being used by the firm and those costs that would have been incurred if a different form of the Economic Order Quantity (EOQ) model had been in use. Since the model selected uses expected future events and the data collected is actual historical data, it was necessary to assume that current sales would be the same as those for last year.

To keep the amount of data at manageable proportions, on1y the " A " inventory items were used for this comparison. However, as previously mentioned, the " $A$ " inventory items account for approximately 90 percent of sales dollars.

Selection of the variation of the Economic Order Quantity model depends upon the cost constraints imposed in obtaining the necessary information to implement the model and how the model fits the farm machinery industry, particularly the selected firms.

The three firms were selected for the study from three states served by the Kansas City, Missouri branch of the John Deere Company, a
division of Deere and Company and the company cooperating for this study. The three firms were selected from different geographical locations and serviced by three different parts depots. The firms are of different sizes in dollar parts sales in order to encompass a variety of dealers. Care was taken to select firms which have no capital 1imitations that might affect the level of inventory.

## Limitations of the Study

This study was limited to the repair parts inventories of the retail farm equipment business. The farm equipment industry was chosen for the study because the researcher had considerable experience in this industry, and also had access to Deere and Company inventory records. New farm equipment inventories were excluded from the analysis for two reasons: (1) for the past few years new farm equipment has been in short supply; and (2) because the new equipment inventory of a retail farm equipment business is subject to many variations in demand (due to such factors as; weather, crop market prices, etc.) and must be individually analyzed. Therefore, it is not adaptable to routine quantitative models.

The study was limited to only three retail farm equipment firms for the following reasons: (1) each dealership's repair parts inventory consists of approximately 15,000 items, thus making the gathering of more data from more firms too time consuming; and (2) the cost for gathering and processing more data is beyond the resource capabilities of this writer.

Another limitation of the study was the assumptions of the model chosen to process the data. This limitation was minimized by the
selection of the EOQ model. The model was as sophisticated as possible; that is, it contains as few assumptions as is feasible within the cost constraints imposed.

The final limitation was the conclusions that can be drawn from the analyses of the data collected. Because of the number of firms to be studied (three), general conclusions about the retail farm equipment business as a whole are impossible. However, this study is a beginning and other studies in this area in the future could lead to generalizations about the control of the repair parts inventory of retail farm equipment dealers.

## CHAPTER II

## INVENTORY COSTS AND THEIR MEASUREMENT

## Introduction

There are three types, or classes, of costs associated with inventories. These costs are: (1) ordering or procurement costs, (2) carrying or holding costs, and (3) out-of-stock costs. A11 of these costs are important to the inventory decision process and must be considered in the inventory model.

The first cost, ordering or procurement costs, includes all costs that increase because an order is placed to replenish the inventories. The second cost, the holding or carrying costs, are those costs that result from storing or holding the goods on hand until they are sold to the customer. These costs include: (1) space, (2) taxes, (3) insurance, (4) material handling, (5) physical inventory count, (6) shrinkage, and (7) cost of capital.

The third cost associated with inventories is the out-of-stock cost. Out-of-stock costs can be divided into two categories: (1) cost of lost sales, and (2) backorder costs. The determination of which situation the firm faces depends upon consumer reactions. If the customer can and does go elsewhere to purchase the product, then the firm must consider the cost of lost profit and the cost of customer goodwill. On the other hand, if the customer is willing to wait for his order to
be filled, then costs of additional processing, transportation, and handling must be considered. It is also possible to incur loss of customer goodwill even though the customer is willing to wait.

These three costs will be examined extensively in this chapter. Firms should carefully decide which costs are appropriate in their particular situation.

Ordering Costs

## General Description of Ordering Costs

and Their Behavior

A general definition of ordering costs is: all costs incurred due to placing an order for items to carry in inventory or eliminated by the decision not to order the item. The scope of these costs starts with the signal to order, processing the purchase order, receiving and inspecting the goods, placing them in stock, updating the inventory cards, and continues through the payment to the supplier. Thus, order costs can be shown as follows:

$$
\text { Total Ordering Cost }=S \frac{D}{Q}
$$

where $D$ is the aggregate demand for the period and $Q$ represents the quantity ordered each time an order is placed. Since D divided by Q gives the number of orders placed during the period, then $S$ must represent the cost of placing each individual order. If examination of a cost reveals that it varies with the quantity ordered, this cost should be included in the purchase cost. However, if the cost varies with the frequency of ordering, then these costs are included in ordering costs.

It is possible to show mathematically that order cost must be independent of the quantity ordered. This is shown by adding a second order cost term $S_{2}$ which represents a portion of ordering cost that varies with the quantity ordered. Total order cost would then be shown as follows:

Total Order Cost $=\frac{D}{Q} S+\frac{D}{Q} S_{2} Q$.
This hypothetical order cost term $\frac{D}{Q} S_{2} Q$ simplifies to $D_{2}$ which is a constant. This illustrates that the order cost parameter should include only those costs that do not change with order quantity.

The behavior of ordering costs is shown in Figure l. Because order costs are independent of the size of the order, the unit cost of placing an order decreases at a decreasing rate as the order quantity increases.


Figure 1. Variation of Annual Ordering Cost with Order Quantity

Or, to state this in another way, for a given annual demand fewer orders are needed if a larger quantity is ordered each time an order is placed.

Approaches to Determining Ordering Costs

The primary approach for determining the ordering costs that appear the most in the literature is to study the present accounting records to determine which costs should be included as ordering costs. These costs are then totalled and divided by the number of orders processed during the period, thereby arriving at the cost per order.

The literature suggests two variations of computing this cost. The first technique came from a special study of the National Association of Accountants (NAA) on the subject of inventory management and is presented in Table I.

TABLE I
COMPUTATION OF ORDER COST -- ILLUSTRATION 1

|  |  |
| :--- | ---: |
| Cost of Receiving Department | $\$ 35,000$ |
| Cost of Planning Department | 15,000 |
| Cost of Purchasing Department | $\frac{30,000}{}$ |
| Total Annual Cost | $\$ 80,000$ |
| Number of Purchase Orders Placed Per Year | 7,500 |
| COST PER PURCHASE ORDER | $\$ 10.67$ |

Source: (88; p. 16)

The second technique is a procedure by Levin and Kirkpatrick (73) to determine the aggregate costs which a firm would have at two different levels of activity for processing orders and is shown in Table II.

TABLE II
COMPUTATION OF ORDER COST -- ILLUSTRATION 2

|  | At 3,000 Orders/Year |  |  | At 5,000 Orders/Year |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Annual <br> Salary | Number Required | Annual Cost | Number Required |  |
| Purchasing Department |  |  |  |  |  |
| Chief | \$12,000 | 1 | \$12,000 | 1 \$ | \$ 12,000 |
| Buyers | 7,000 | 3 | 21,000 | 5 | 35,000 |
| Assistant Buyers | 5,000 | 2 | 10,000 | 3 | 15,000 |
| Follow-up Men | 4,000 | 1 | 4,000 | 2 | 8,000 |
| Clerks | 3,000 | 3 | 9,000 | 4 | 12,000 |
| Typists | 2,800 | 2 | 5,600 | 3 | 8,400 |
| Supplies | --- | --- | 1,500 | --- | 2,500 |
| Receiving Clerks | 4,000 | 2 | 8,000 | 3 | 12,000 |
| Receiving Supplies |  | --- | 300 | --- | 500 |
| Accounts Payable |  |  |  |  |  |
| Clerks | 4,200 | 3 | 12,600 | 4 | 16,800 |
| Accounting Supplies | --- | - | 450 | --- | 750 |
| TOTAL EXPENSES |  |  | \$84,450 |  | \$122,950 |

Source: (73, p. 115)

By computing the difference in aggregate costs ( $\$ 122,950.00-\$ 84,500.00$ equals $\$ 38,500.00$ ) and then dividing by the change in the two levels of ordering activity ( $\$ 5,000.00-\$ 3,000.00-\$ 2,000.00),(\$ 38,500 \div$ $\$ 2,000.00$ equals $\$ 19.25$ ) to arrive at what these authors say is the incremental cost per order.

Pritchard and Eagle (96) arrive at an approach similar to Levin and is shown in Table III below.

TABLE III
COMPUTATION OF ORDER COST -- ILLUSTRATION 3

| Position | Orders <br> Placed <br> Per <br> Year | Current Work |  | Reduced Work |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Number | Cost | Number | Cost |
| Chief Buyer \$18,000 |  |  | \$ 18,000 |  | \$ 18,000 |
| Secretary 4,200 |  | 1 | 4,200 | 1 | 4,200 |
| Machine |  |  |  |  |  |
| Operator 2,000 |  | 1 | 2,000 | 1 | 2,000 |
| Buyer 14,000 | 300 | 3 | 42,000 | 2 | 56,000 |
| Ass't. Buyer 10,000 | 600 | 7 | 70,000 | 5 | 50,000 |
| Typist 2,900 |  | 2 | 5,800 | 1 | 2,900 |
| Supplies |  |  | 2,400 |  | 1,600 |
| Utilities |  |  | 600 |  | 600 |
| TOTALS |  |  | \$144, 800 |  | \$107, 300 |
| Orders placed |  |  |  |  |  |
| Per Year |  | 5,100 |  | 3,400 |  |
| Average Order Cost |  | \$28.39 |  | \$31.56 |  |
| Marginal Order Cost |  |  |  | \$22.06 |  |
| Items in Inventory |  | 1,700 |  | 1,133 |  |

Source: (96, p. 81)

By computing the difference in aggregate costs ( $\$ 144,800-\$ 107,300=$ $\$ 37,500$ ) and then dividing by the change in the two levels of ordering activity (5,100-3,400 = 1,700), ( $\$ 37,500 \div 1,700=\$ 22.06$ ) to arrive at what these two authors say is the marginal cost per order.

Most of the suggested approaches that appear in the literature are similar to those above, in that they attempt to compute a cost per order from the available historical accounting records. Several prominent weaknesses should be apparent in the use of this approach. With the first approach there is no separation in the fixed and variable elements of these costs in the accounting records and no attempt is made to distinguish between the two. For example, some of the replenishment costs may be common costs for different activities and these costs may not change if one of these activities is discontinued. The purchasing department of a firm is seldom established to purchase inventory items only and even if a stock of inventory is discontinued costs of this department may not be reduced. Therefore, the inclusion of total costs of the department as inventory replenishment costs are suspect. A second shortcoming is the use of historical cost as the relevant cost concept. Inventory control models use the sacrifice of resources for a particular purpose and the cost concept that should be used is the opportunity cost in the specific situation. This failure to use the relevant cost (opportunity cost) could lead to significant errors in establishing the actual cost needed for the model.

A further weakness of the examples appearing in the literature is that it is not clear what makes up the items of costs. Examination of Tables II and III reveals the costs of physical facilyities were excluded. In the NAA illustration, Table I, it is not clear whether facilities are included because only aggregate figures are shown. The NAA also included an item not included in the other examples, costs of the planning department. However, it is not at all clear what planning costs the NAA has included. Only inventory planning costs would be
the relevant costs, but this is not specified. All of the examples in the literature are vague and failure of the authors to include any discussion of how these costs were determined causes the reader to be unable to decide if his situation is of a similar nature. This makes it impossible to discover anything of great value from the information and only serves as a jumping off point in trying to determine what costs should be included.

Probably a much greater weakness, as far as the necessary information for the model, is the fact that the initial purchase and subsequent purchase will usually differ considerably as far as costs are concerned. When initial orders are placed, it may be necessary to contact various suppliers to obtain the best possible price. This is quite different from the amount of costs necessary to make a simple reorder. It is the additional order costs for subsequent orders that is the relevant cost needed for the inventory control model. The last two examples of inventory carrying costs (Tables II and III) may alleviate this problem to some extent.

Some other weaknesses should also be considered. For example, the behavior of costs is not considered as they relate to the individual cost components. The best example of this is that receiving costs may vary with the quantity ordered and not with the number of orders processed.

In summary, these approaches as suggested in the literature are deficient for three major reasons: first, the presence of common costs, which cannot be satisfactorily separated; second, the possibility that some order costs may vary with the quantities ordered rather than the number of replenishment orders placed throughout the year; and third,
the possibility of significant differences in the cost between initial and subsequent orders.

Grouped Classifications of Types of

## Procurement Costs

Ronald Ballou (12) has defined four general areas of costs that are associated with the procurement process as follows:

1. the cost of processing the order through the accounting and purchasing departments;
2. the cost of transmitting the order to the supplier, usually by mail or by some electronic means;
3. the cost of transporting the order when transportation charges are not included in the purchased goods; and
4. the cost of any materials for handling or processing of the order at the receiving dock (p.281).

Ballou's classification is representative of other authors on inventory control. Any procurement costs associated with inventory would fall into one of these four general areas and this classification is beneficial in identifying all costs associated with the procurement process.

Processing the Order -- Purchasing
and Accounting

Costs of the purchasing department necessary to process an order begin with the order signal. This component of the ordering costs includes the incremental costs incurred by the inventory control system in signaling that an order should be placed and the quantity that should be ordered. This cost is normally small in relation to other order costs and varies with the type of system used by the firm. If the firm uses a fixed order quantity system, irrespective of the technique used, the signal to place an order is given by the person involved or may be
signaled automatically. If a fixed order point system is used, a reorder signal is given by the arrival of the reorder date. Regardless of the system used, the cost of the order signal activity is relevant to the model because the cost varies with the frequency of ordering. After it is determined that an order needs to be placed, the purchasing department must place the order and this usually involves a number of possible resources.

Figure 2 illustrates some steps involved in placing an order.


Source: (112, p. 40)
Figure 2. The Steps Involved in Placing an Order

## The Activity of Receiving, Inspecting,

and Placing in Stock

The receiving of goods at the firm's receiving dock is the next step in the replenishment process. This activity consists of inspecting the merchandise received to determine that the quantities received are correct and in good condition. If there are no discrepencies in the amount ordered and the amount received and there is no damage to the merchandise, then each item is placed in its proper inventory location. These various steps need to be examined to determine the resources used to perform each activity so that it will be possible to decide which costs are related to the inventory model.

Lee and Dobler (71) have grouped the basic activities performed in the receiving area into four steps, which are useful in determining which resources are typically used.

1. Unloading and examining the order. The number of containers unloaded from the carrier's vehicle is compared to the carrier's manifest to determine that the complete shipment has been delivered. The containers are examined for external damage and the receipt form of the shipper is signed.
2. Unpacking and inspecting the material. When the goods are packed, a packing slip itemizing and describing the contents is included in the shipping container. The material received is compared with the packing slip and, if available, against a copy of the firm's purchase order to verify that the correct items have been shipped by the supplier. The quantities are verified in the same manner. The condition of the material is inspected to determine if any damage was
incurred during shipment. If further inspection procedures are required to provide assurance that the material conforms to the buyer's specifications, an additional inspection procedure may be performed by the firm's internal inspection department.
3. Completion of the receiving report. The receiving report may be a separate document or it may be prepared as a by-product of the purchase order. This form is completed by entering what has been received. If a copy of the original purchase order is available, any items on the order that remain open may be indicated. After the quantity and condition of the goods received has been verified, the receiving report is completed and a copy is sent to the accounting department.
4. Delivery of the goods. The goods next are transferred from the receiving department to their proper destination. In the case of inventory materials, the goods commonly are delivered to the storeroom. Either the receiving department or a separate internal transportation department may be responsible for this delivery. Upon delivery of the material to its proper destination, the receiving function is relieved of further responsibility for the goods.

The above identification should be used only as a guideline and each individual situation should be studied in determining which resource applications are relevant for a particular firm.

The costs of a purchasing department may be determined by an examination of the budget used to plan and control the purchase department's expenditures. Lee and Dobler present a typical operating budget for a purchasing department and is shown in Table IV.

TABLE IV

PURCHASING DEPARTMENT OPERATING BUDGET FOR FIRMS IN DURABLE GOODS AND NONDURABLE GOODS INDUSTRIES

| Budget Item | Per Cent of Budget, <br> Durable Goods <br> Producers | Per Cent of Budget, <br> Nondurable Goods <br> Producers |
| :--- | :---: | ---: |
| Salaries and Wages | 79.3 | 78.0 |
| Travel | 3.2 | 1.9 |
| Telephone and Telegraph | 6.0 | 3.6 |
| Printing and Stationery | 2,7 | 1.4 |
| Empolyee Benefits | 2.4 | 4.6 |
| Space Rental | 1.5 | 5.0 |
| Dues and Subscriptions | .3 | .1 |
| Rental Equipment | .2 | .1 |
| Maintenance and Repairs | .2 | .5 |
| Interviewing | .1 | .1 |
| Insurance and Taxes | .1 | .1 |
| Depreciation | .3 | .4 |
| Legal Fees | .1 | -1 |
| Utilities | .1 | .1 |
| Contributions | .1 | .1 |
| Miscellaneous | 3.4 | 4.0 |

(Figures are averages, expressed as a \% of the total operating budget.).
Source: (71, p. 445)

The figures in Table IV are averages only of a sample of companies studied by the National Association of Purchasing Agents and should be useful as an indicator of the relative importance of each element of cost associated with the purchasing activity. The aggregate cost of salaries and wages, for example, constitute over 80 percent of purchase department costs.

Böer (19) points out that costs of the purchasing activity are easy to identify, but are more difficult to measure when determining
the amount of cost relevant for inventory decisions. As with other replenishment costs, the costs for this activity may be either entirely fixed, entirely variable, or a combination of the two, semivariable. For some firms, the purchasing operation could be performed by salaried personnel who have no other duties and, therefore, the cost would be fixed and irrelevant as far as inventory ordering decisions are concerned. On the other hand, if these employees have other duties or are paid overtime for the purchasing activity, then the costs would be semivariable. It is also possible for these costs to be entirely variable. For example, the merchandise could be stored in a public warehouse and charges may be based on a per unit handled basis. The accounting department must process the receipt of the inventory items and record the liabilities and payments to creditors. In accomplishing the first procedure, the usual process is to compare a copy of the purchase order with the receiving report and with the invoice furnished by the supplier. The quantities received, price of each item, price extensions, and credit terms must agree with the firm's purchase order. The liability is then recorded and payment made according to the terms of the purchase.

The next step is to record the purchases in the inventory records. The recording of this information is made from the receiving report and is done by the accounting department or, alternatively, by the parts department, depending upon the particular procedure used by the purchasing firm. If the purchasing firm uses a perpetual inventory system, which requires a constant up-dating of inventory records, then the purchasing costs will vary with the number of orders placed.

Cost Behavior and Measurement of
Ordering Costs

The costs relevant in making inventory decisions are those costs that will change because a decision is made to add to the inventory. Ordering costs, therefore, are those costs which will change when an order is placed.

Many of the costs we have discussed remain fixed, such as, personnel for purchasing, accounting, and the parts departments. These costs usually remain fixed over the relevant range, but it is possible that at some point personnel must be added in one or more of the departments making the costs semifixed as shown in Figure 3 . Figure 3 also shows


Source: (20)
Figure 3. Order Costs Behavior in Relation to the Number of Orders P1aced
that most companies have costs that are semivariable in behavior and vary directly the the number of orders placed. Total costs are then determined by adding semifixed and semivariable costs together.

As the number of orders placed reaches a certain point, costs take a significant jump. Germain and Elinor Boer (20) point out the following in regard to semifixed costs. If the accountant ignores semifixed costs, variable costs may be so low that the firm decides to buy in very small quantities, which results in a large number of purchase orders. The larger number of purchase orders may cause the firm to móve up to a higher level on the semifixed cost curve and may cause total costs to increase more than if a larger per order cost had been used originally to compute the number of orders.

Two techniques are proposed in the literature for solving the dilemma of cost behavior. The first proposed technique is to assume that all of the costs association with this activity are semivariable and use least squares regression to fit a line to the cost function. The second technique proposed is to describe the resources utilized by the use of a cost step-function (55).

## Carrying or Holding Costs

Another variable needed in applying the inventory model chosen is the cost of holding inventory. A firm chooses to incur this holding cost for either one, or both, of two reasons: (1) because profits will be increased by having the goods available to meet future demand; or (2) because profits will be increased from a price change in the future. The holding of inventory in anticipation of a price increase, however, will not be included in this study because of its minor
consideration in the literature. Also, many firms establish policies to prevent the holding of inventory for this speculative purpose. However, it may be that this goal will be given more consideration in the future because of the consistent inflation over the last several years.

The inventory holding process may be accomplished in a wide variety of ways, but the two primary methods are: (1) the use of independent warehouse facilities, or (2) the use of facilities owned, rented, or leased by the firm. In the case of public or independent warehouses, the holding costs are primarily determined by the contract agreement and are usually measured by the out-of-pocket expenses incurred. Normally, most merchandising firms own their warehouse facilities and this should be the case for the retail farm equipment firm. Therefore, this study will consider the multiple costs that are incurred by the firm which uses its own facilities for holding inventory. The specific costs of holding inventory will be examined, and the different concepts suggested by the various authors for measuring the various costs will be studied. The concepts for measuring these costs will be analyzed to particular inventory models selected for inventory control.

## General Description of Carrying Costs

 and Their BehaviorHolding, or carrying, costs are those costs that a firm incurs because of the acquisition of goods that are to be used, or needed, for sale at a later date. This has brought about the general definition of holding costs as consisting of all costs that could have been avoided if the firm did not maintain an inventory. These costs may be incurred either externally, as in case of the payment to others, or internally,
by using the resources of the firm. Internal costs are more elusive as far as measurement is concerned and consist of things such as: loss from theft, shrinkage, obsolescence and the implicit cost of funds invested in inventory.

Included in external costs are insurance, taxes, and expenditures to maintain storage facilities (either payment of rental charges or out-of-pocket expenses incurred on facilities owned by the firm). Both internal and external costs are usually incurred by a firm, but the internal costs are the major component of the total holding costs.


Figure 4. Variation of Annual Carrying Cost with Order Quantity

Both current and permanent inputs of resources are associated with the holding process. The current inputs of resources are normally acquired externally and include items such as: cost of insurance, property taxes, and maintaining of storage facilities. Permanent resource inputs include costs such as: physical facilities and personnel. Total holding costs consist of costs incurred either internally or externally and the use of current or permanent resources in the activity necessary in the carrying of inventory.

The behavior of carrying costs is illustrated in Figure 4. As the quantity in stock increases, carrying costs, which are usually expressed as a percentage of the unit purchase cost, will necessarily increase.

Approaches in the Determination of Inventory
Holding Costs in the Literature

There are several suggestions in the literature as to what costs should be included in determining the total inventory carrying cost. While most authors agree that the relevant costs to be considered are incremental costs and opportunity costs, they do not agree upon which incremental costs and opportunity costs to include (64) (93)(95). However, the differences in the approaches to the problem seems to be determined by the inability to measure some costs rather than differences in theory.

The literature suggests three different approaches for determining the total inventory carrying cost which should be discussed (117). The first approach is the most expedient in computing inventory carrying costs because it considers only the cost of capital. The rationale
for this approach is that other costs of holding inventory such as: insurance, taxes, and storage are generally found to be fixed over the relevant range and that interest, or opportunity costs, on the money invested in inventory is the only cost that is directly variable with the quantity held in inventory (26). The weakness of this approach is very evident; it excludes all other costs from consideration.

A second approach in determining inventory holding costs uses the relationship between the existing inventory level and the existing costs. In this approach, aggregate carrying costs are accumulated and related to the present inventory level on a percentage basis. The percentage determined in this manner is then presumed to be the cost of carrying inventory. An example of this procedure is presented in Table V.

The problem with this approach should be obvious; all costs are assumed to be variable with the change in inventory level. One would have very little difficulty in showing that this is an invalid assumption. Many costs are, in fact, fixed and do not change as the level of inventory changes.

A third method appearing in the literature is the use of historical book cost as the cost to be accumulated in costing for the resources used in the inventory carrying activity. The greatest advantage of this method is that book cost is readily available. However, this method will not provide the opportunity costs necessary for the inputs to the model in some instances. An example of this method is shown in Table VI.

TABLE V

INVENTORY CARRYING COST

| Average Inventory | $\begin{aligned} & \text { 1st Year } \\ & \$ 100,000 \\ & \hline \end{aligned}$ |  | 2nd Year$\$ 125,000$ |  | $\begin{aligned} & \text { 3rd Year } \\ & \$ 110,000 \\ & \hline \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Taxes | \$ | 3,000 | \$ | 3,400 | \$ | 3,220 |
| Insurance |  | 1,400 |  | 1,500 |  | 1,400 |
| Obsolescence Losses |  | 2,500 |  | 2,000 |  | 1,500 |
| Depreciation |  | 500 |  | 800 |  | 400 |
| Total Cost | \$ | 7,400 | \$ | 7,700 | \$ | 6,500 |

Assuming the opportunity cost
(or interest) on the money invested
in inventory to be $10 \%$, the carry-
ing cost of the inventory would be:
$\$ 21,600$ (total cost) $+.10=.16$
$\$ 335,000$ (total value of inventory)

In other words, the average cost of carrying \$100 worth of inventory for one year is $\$ 16$.

Source: (50, p. 136)

TABLE VI

INVENTORY CARRYING COSTS USING HISTORICAL
BOOK COST

|  | Per Cent |
| :--- | :---: |
| Storage Facilities | 0.25 |
| Insurance | .25 |
| Taxes | .50 |
| Transportation | .50 |
| Handling and Distribution | 2.50 |
| Depreciation | 5.00 |
| Interest | 6.00 |
| Obsolescence | $\underline{10.00}$ |

Source: (2, p. 4-58)

# Analysis of Individual Components of Inventory <br> Carrying Costs and Their Measurement 

Space

One of the major components of the inventory carrying cost is the physical facilities in which the inventory is stored. These facilities can vary considerably because of the differences in the types of inventories carried by the many different types of firms. These facilities include: the building, bins or shelves, and in some instances, special climate control equipment. The facilities may be rented or leased by the firm, or they may be owned by the firm, or a combination of both. If the storage space is rented and the amount of rental is determined by the square feet used, then the space cost is a function of the bulk of each particular inventory item. Therefore, the bulk of an item may be the inventory characteristic that will allow a determination of the warehousing costs for many types of inventories. For some inventory items, it may be necessary to express the space cost as a percentage of the purchase cost of the item. This is true for inventories that the bulk of the items cannot be used as an appropriate measure of the storage facilities cost.

A more difficult problem arises when the storage facilities are owned by the firm. The original decision, to carry an inventory to meet consumer demand, necessitated that storage space be provided to carry the basic level of inventory. Böer and Böer (20, p. 8) point out, ". . . certain warehousing costs will remain constant regardless of the level of inventory, and other costs will vary with the level of inventory on hand." Usually, the relevant cost for inventory management


#### Abstract

decisions is that the amount of costs for storage space will vary if the level of inventory is increased. If the storage space is used for other income producing purposes, or could be rented to someone else, then the value of the space to other activities, or the amount of rent foregone would be the appropriate cost concept (opportunity cost). If there is not an alternative use for the space by the firm and if the building cannot be rented or sold, then the opportunity cost is zero, as far as inventory management decisions are concerned.

Several approaches for introducing space cost into the inventory control model have been proposed, but the selection of the most appropriate concept depends upon the particular circumstances of the firm.


## Taxes

Many states, counties, and municipalities levy a property tax on inventory and in some cases, it is a significant amount. The form of this property tax varies with the different governmental units. Some taxes are levied on the average inventory on hand during the year, while others compute the tax at one particular point in time, usually the end of the year. Most authors agree that if the tax is based on the average inventory throughout the year, then taxes should be a component of the inventory control model. However, some suggest that if the tax is levied at one point in time, then it is not relevant in inventory management decisions because it does not vary with the level of inventory. Others, such as Wagner, suggest that if taxes are of such magnitude that the firm will drastically reduce inventory at tax date to avoid this cost, then a dynamic programming model should be
used as an alternative which considers the fact that the taxes occur at a given time (115).

## Cost of Capital

Another primary component of inventory carrying cost is the cost of funds invested in inventory. This cost is easy to determine if the firm borrows the money necessary to purchase inventory. In this situation, the cost is simply the interest cost on the borrowed funds. However, it should be made clear that the accountant should consider any compensating balance that might be required by the institution making the loan and also, adjust the interest rate if the interest on the loan is collected in advance or if the loan is to be repaid in installments.

In summary, what the accountant must use in computing the cost is the effective rate of interest. Unfortunately, this is rarely the case. If the investment in inventory is made with funds generated internally, then it is necessary to impute a cost of capital.

It is widely recognized in the literature that a cost of capital should be imputed as noted by Bierman and Smidt (16):

An investment in plant assets will invariably lead to funds being tied up in working capital. This will include the cash necessary to meet payro11 and other bills, funds invested in raw material, work-in-process and finished goods inventory, and receivables from customers. The size of these items will depend on the exact nature of the capital investment, but all the above-mentioned fund requirements will usually accompany an investment in long-lived assets (p. 132).

However, how this imputed cost should be determined is subject to considerable disagreement. The term, cost of capital, has resulted in several interpretations. Perhaps the most accepted definition is,
"the rate of return required on a proposed investment so that the price of the firm's shares does not fall" (94, p. 144). This implies that a condition of uncertainty exists and that this risk should be considered in conjunction with return in capital budgeting. It is suggested by Porterfield (94) that "the market rate of discount (of a firm) reflects the relative uncertainty of the stream of future dividends and that changes in the rate reflect changes in this relative uncertainty" (p. 122). In spite of the difficulty in making the cost of capital operational, there is a definite consensus that it is essential to quantify the concept in order to make sound decisions.

If the cost of capital is considered to be the internal rate of return on an investment, it can then be described as "the discount rate that equates the initial cash inflow from a funds source with the present value of the subsequent cash flows association with that source" (94, p. 59). On the other hand, if the cost of capital is viewed as the implicit cost of an opportunity for the investment of funds, then it can be defined as "the rate of return on other investments available to the firm in addition to that currently being considered" (94, p. 61). Therefore, opportunity cost is the relevant cost concept in determining the cost of capital for making inventory decisions, where the capital is internally generated. It might be noted here that a situation might exist that, even if the funds are borrowed, opportunity cost may be the relevant cost. If the firm has limited access to funds, then the borrowing of funds to purchase inventory may eliminate the possibility of borrowing funds for alternative investments having a greater return than the effective interest rate. The return
on the opportunity foregone by the purchasing of additional inventory would then be the relevant cost.

Opportunity cost cannot be measured directly; therefore, a surrogate must be used. Archer and D'Ambrosio (10, p. 203) point out that "the opportunity cost of capital (its implicit interest cost) is the expected return from other investments of comparable risk." The opportunity cost of money invested in holding inventory should be the expected rate of return on investments of the same or equal risk.

One view, advocated by some authors, is that the cost of capital invested in carrying inventories should be the interest rate that prevails in the market or from banks for the particular firm involved. The rationale for this approach seems to be the different risks for the various types of assets held by the firm. For example, the risk of holding cash may be very minimal; such as the risk of erosion of the purchasing power of the dollar, while the risk of a patent on an invested product may be very risky. It may be possible that inventory has a lower risk, for some firms, than the average risk on the aggregate assets of the firm. If this is the case, then the rationale of this approach may be considered sound. Under this view, the appropriate opportunity cost would be the rate of return foregone on an investment of similar risk.

A second viewpoint is that the cost of capital is the opportunity cost that represents the over-all cost of capital to the firm. The traditional view is that the total outlay needed for an investment project is only relevant in the aggregate and not individual parts of the investment project. The reasoning for this is that in computing the rate of return, or net present value, aggregate outlay for the
project in question is the relevant concept. Individual, or particular investment projects are not matched with specific sources of funds.

This over-all approach is contended by different views as to why it should be used. Modigliani and Miller (86) argue that a firm's cost of capital along the supply function is independent of the mix of equity and debt sources of capital. However, another view is that the debt and equity mix does affect the cost of capital and therefore, depending upon this mix, cost of capital to the firm will vary, but an optimum mix exists and at this point cost of capital to the firm is at a minimum.

Capital budgeting theory implies that if funds are returned by an investment project they will earn at least a minimum return equal to the cost of capital to the firm. Therefore, inventories have been equated with investment projects in that they represent a relatively permanent investment with risk comparable to buildings, plant, or equipment. Magee (78) points out:

Much inventory carried in business is as much a part of the permanent investment as the machinery and buildings. The inventory must be maintained to make operations possible as long as the business is a going concern . . . How much more riskless is this than other fixed manufacturing assets (p. 46)?

The risk to equity holders may be increased by the use of debt and at some point, debt may not be increased until the equity base is increased; therefore, the using of debt has an opportunity cost of the over-all cost of capital to the firm.

Bock (18) another proponent of this theory, states:
The marginal cost should not be used; the right figure is the long-run average cost of the entire capital structure. Such an evaluation will result in assigning considerably higher capital costs to this element of the inventory analysis (p. 61).

Terborgh (109) conducted a survey in which he asked what minimum rate of return the respondent firms required from capital investment projects. The percentage of firms that required a minimum rate of 20 percent, or more, are presented in Table VII below.

TABLE VII

PERCENT OF COMPANIES WITH A REQUIRED RATE OF RETURN ON INVESTMENT OVER 20 PERCENT

|  | (a) | (b) |
| :--- | :---: | :---: |
| A11 Manufacturing | 59 | 52 |
| A11 Business | 52 | 46 |

Source: (109, p. 273)

Hupp (92) found what to him was astonishing from his empirical study:

One of the most startling findings of the informal poll we made throws light on the whole question of inventory management; fewer than 10 percent of the companies we talked to make use of the concept of opportunity costs in their inventory decisions . . . More than nine out of ten of the companies think of the value of money as being what they have to pay for it rather than as the goal they set up for the return they want to earn on their investment (pp. 9-10).

Hupp's findings were startling to him because of his belief that the aggregate cost of capital, rather than a risk-adjusted rate, should be the relevant concept. This should not be too surprising since many authors advocate that firms should use the interest rate on debt as a
surrogate for the opportunity cost of capital for assets in this risk class.

The conclusion by Magee (78) noted previously (that inventories represent a permanent investment such as: machinery and buildings, and that therefore, the risk is approximately the same) is subject to question. While it is true that a firm must maintain a certain amount of inventory, this inventory is constantly turning over; making the risk considerably less than that of fixed assets.

Of the many viewpoints, only two have been discussed here and they represent the two extremes. The lowest extreme was noted by Böer (20); that by having money invested in inventory, the firm foregoes the interest it could earn on some type of an interest-bearing instrument; to the highest as indicated from studies as cited above by Terborgh (109) and Lanziletti (69).

Determination of the cost of capital may lie somewhere in between these two extremes; the decision will have to be made by management along with what the opportunity cost should be after giving careful study to the particular situation of their firm.

## Insurance

Inventories are usually covered by insurance; although management may elect to run the risk of a loss by not having this coverage. The insurance policy may have a face amount based on the average inventory level of the past period. This face amount would not be subject to change unless the inventory level changed substantially. With no planned change in inventory level, the insurance expense will be a
fixed cost. If a significant change in inventory level is planned for the subsequent period, the insurance cost becomes a semifixed expense.

The insurance premium may change as the level of inventory changes. If this is the case, the cost of insurance becomes relevant for inventory policy making decisions. If, as mentioned above, the insurance expense is a semifixed cost, the method of dealing with this situation will be discussed in Chapter III.

## Obsolescence or Spoilage

With a few exceptions, every inventory has a cost of spoilage or obsolescence. Spoilage is not very difficult to measure, in most situations, because it is merely the loss of the cost of replacing the inventory item; for example, perishable produce. However, some types of inventory lose only a part of their value through spoilage; for example, day-old bread. This cost is also fairly easy to compute. It is simply the loss of revenue from selling the day-old bread at a lower price.

Obsolescence is more difficult to discern. When is an item of inventory obsolete and what is the cost of that obsolescence? Heskett, Glaskowsky, and Ivie (64) suggest the following:

Most firms identify obsolete merchandise, or perhaps more accurately "dead stock," in terms of the frequency of orders for such items. For example, an item for which no order has been received at a distribution center for perhaps 90 days is considered, in some companies, to be dead and a candidate for reallocation to other distribution centers that are shipping it (p. 352).

Even if some period of time is established for determining the cut-off point of obsolescence, does this mean that the item has lost all of its value or only partially and if so, how much? Just because
the item has not been sold in 90 days does not mean it will not be sold in the future at full list price. Additional or above normal profit might even result from a price increase over the longer period of time the item of inventory remains in stock.

Hadley and Whitin (60) state that obsolescence costs always occur at a fixed point in time, but this date of obsolescence often cannot be predicted with any certainty in advance. The truth in this statement must depend upon the definition of obsolescence. If, obsolescence is defined as a total loss in value (except for a salvage value), then it may be said to occur at a particular point in time. However, there may be a gradual decline in value, or at least in incremental steps. The second part of the statement by Hadley and Whitin is true, except in rare cases. For example, if a firm comes out with a new product which replaces one presently on the market, it then becomes possible to determine, with some degree of accuracy, the point of obsolescence for the old item. Hadley and Whitin also call attention to the argument that a firm should make a charge against each of the items in inventory in proportion to the length of time the item is in inventory and in this manner a fund is set up to allow for obsolescence. This method may lead to less than optimal decisions and if obsolescence is a major problem, the firm should incorporate this cost into the decision model. How this can be accomplished will be discussed in Chapter III, which investigates most of the different inventory decision models and their ramifications.

## Physical Inventory Count

Periodically, a firm should count the items in inventory for one
or more reasons, depending upon their particular situation. First, even if the firm maintains perpetual inventory records, a count is made to determine the accuracy of those records. If the records are judged as excellent, this counting may be done on a sample basis and at other times than at the end of the accounting period. This procedure may significantly reduce costs by allowing this work to be done during slack periods.

Second, the firm may use a periodic inventory system and this will usually demand a physical inventory count at the end of the period. For some firms, this may cause the suspension of operations for as long as a week, while the inventory is counted. Finally, if the firm must issue audited financial statements, generally accepted auditing standards require an inventory count.

The main resource used in taking a physical inventory count is personnel. A large number of man-hours may be required to count, price, extend, and total. And, in the case of perpetual records, the results must be posted to the inventory records. The circumstances under which a count is made should be examined carefully because the determination of the costs involved depends upon whether an incremental cost is present or if the cost is fixed. Of course, the costs would not exist if the firm did not maintain an inventory of goods, but once the decision has been made to have an inventory, only the incremental cost is relevant for the inventory decision model. If additional personnel are employed, or if overtime is paid to regular employees, to take the inventory count, these costs are easily discernable. A careful analysis of the situation is required if the firm halts operations
while the inventory is taken. Loss of sales may result from the shutdown and the opportunity cost of lost profits would be a relevant cost.

## Material Handling

Warehouse costs for the handling of material may arise for a number of reasons. At the time a requisition is received for the withdrawl of an item from stock, a handling cost is incurred. However, this cost is a part of the usage function and is not needed for inventory ordering decisions. In other cases, the cost of handling depends on the aggregate amount of inventories held; i.e., items requiring an area of height may result in improper stacking or crowding. This could cause additional handing costs because it may be necessary to move boxes that are stacked in order to reach other cartons or crates. This extra handling of material costs would be eliminated if material is received in the amounts needed for immediate use in production.

The appropriate costing method for the handling of the materials component in regard to the storage activity is also dependent on the availability of the resources utilized. If extra handing of inventory goods is necessary, then it is probable that either extra resources or the extraordinary utilization of existing resources will be required. The best method for costing the labor input may be the standard cost of a unit of employee services and the best method for costing the use of materials handling equipment may be the standard cost of a unit of machine services. If the need for extra handing occurs only infrequently, then, since the normal duties will be performed anyway, the utilization of resources for this incidental purpose may not necessarily cause a firm to incur any additional costs.

## Out-of-Stock Costs

The third and final cost parameter to be considered is the cost associated with goods being out-of-stock. A cost is incurred by a firm when it is unable to satisfy or fill customer demand for goods from inventory. This cost can take two primary forms: (1) the costs incurred by having to backorder the item and delivering to the customer at a later date, or (2) the cost of a lost sale. For most firms, unless a monopoly exists, a combination of the two types of situations will exist. One form usually will be very dominant and can be readily identified. For example, in the retail sales of repair parts for farm machinery, the costs are primarily backorder costs. The reason for this is that the customer cannot obtain the needed part from another source without traveling a long distance to another dealer. However, lost sales do occur at harvest times when the customer is in urgent need of repair parts immediately. Again, as in many other costs we have examined, it is relatively easy to identify which costs a particular firm may have, but the measuring of these costs is extremely difficult.

Heskett, Glaskowsky, and Ivie (64) have divided stock-out costs into two categories which they call "hard" and "soft." "Hard" stockout costs are costs that are most easy to measure and include: duplicate or special order costs, telephone or other communications (which is often at the company's expense) and transportation costs, which are either extra transportation costs or at a higher rate. "Soft" costs are costs which are nearly impossible ţo measure, such as: lost profit, customẻr goodwill and lost selling time.

## Approaches in Determining Out-of-

Stock Costs

The ideal approach in determining stock-out costs is to establish a safety or buffer stock of inventory such that the cost of carrying the buffer stock balances the expected cost of stock-out, thereby minimizing total costs. The nearly impossible task of measuring stock-out costs has generally led to approaches that disregard the measurement of this cost. Many managers make the decision not to allow any shortage to occur, which is making the assumption that the cost of being out-ofstock is infinite (87). Almost this extreme was found by Starr and Miller (103) when an imputed cost of $\$ 25,000$ for existing policies was quite common.

Another approach suggested by Biegel (15) is, "Safety stock levels should be set to provide economic protection against being out-of-stock. Thus, it is necessary to know the desired level of customer service" (p. 97).

However, "economic protection" is a very vague term, and Biegel makes no effort to measure this penalty cost. A cost of some sort is implied in deciding what level of customer service to provide, but as Naddor (87) points out, there may not be any stated numerical value.

Decisions affecting inventory are made all the time by managers . . . it is evident that in making the decision the managers are actually placing a value on the unit cost of shortage. They may not be able to state numerically what this value is but their decisions imply that such a value exists (p. 38).

Looking at it in another way, the decision maker must decide that
it is not important that we are able to measure stock-out costs.

Pritchard and Eagle (96) have indicated this attitude by the following statement:

We feel that shortage costs should be viewed not as a tangible, measurable, immutable characteristic of an inventory but as a powerful device to adjust individual item reorder points to the realities of the environment in which a manager must operate (p. 263).

These approaches are not really what one could call approaches in determining out-of-stock costs, but rather, they ignore any measurement of out-of-stock costs. Although, this may be a method of handing the problem, the absence of a quantitative measurement makes it impossible to evaluate the process with any basic meaning.

From what has been examined so far, it appears the more common attitude of most authors is that the amount of reserve stock to be maintained can be computed by, "assuming that the cost of backorder has been determined" (68, p. 81, 82).

Oral, Salvador, Reisman and Dean (90) take exception to the general rule of ignoring the problem of determining stock-out costs. Empirical testing of their approach found it to be valid for a particular situation, but it might not fit other situations, especially if customer goodwill, which they ignored, is an import factor.

Qualitative Factors and a Service Level

The determination of the amount of safety stock to carry is usually made by most firms using factors other than cost minimization. The service level to be provided by a firm may be established using qualitative factors, such as marketing objectives, their distribution system, and how they choose to measure or define a stock-out. The firm needs to examine these three major qualitative factors and determine
if there are others that might influence their decision on the amount of safety stock to maintain. The three major factors will be examined below.

Marketing Objectives. When establishing the service level a firm wishes to provide (setting the amount of safety stock) the firm should give consideration to its marketing objectives. Profit maximization (cost minimization) may be the overall major objective of the firm, but, at least in the short-run, other objectives might override the long-run primary objective. Some other basic objectives in the marketing area might be to establish a particular share of the market for the product, become the industry leader, or at the other extreme, survival of the firm.

Profit maximization provides a basic starting point in analyzing the objectives of the firm because profits are necessary for the firm to survive and for its capacity to grow. In establishing the safety stock, or level of service, the firm (in the case of balancing carrying costs against the out-of-stock costs) is either explicitly or implicitly (in the establishing of some service level) implying that a cost of being out-of-stock does exist.

Marketing and economic literature both point out that objectives of the firm vary during the life of the company depending upon the stage of development of the firm. The firm's growth may follow the sales curve of a product line or of a single product. This growth, or sales curve, is usually divided into three phases: establishment, expansion and stabilization.

The amount of customer service a firm wishes to provide will vary significantly from one stage to another. For example, a strategy to enter a market in the establishment phase would not be the same as in the expansion stage, where the firm may be in a highly competive situation. This would surely be different than the last stage where sales are declining. Selling and marketing is stressed during the growth period of a product; therefore, a high level of customer service is desired during this growth stage and the early stage of maturity. The customer service level will normally be reduced during the last phase when demand for the product diminishes. The firm should be careful not to cut back too much of their service level during this stage; particularly, if they are a multi-product producer. Sales of other products could be affected by customer dissatisfaction with service on the declining product.

Regardless of the stage in which a firm may be operating, other factors may influence the service level the firm wishes to provide. Variability in either demand or lead time can be a significant factor, which will affect the cost of carrying a safety stock. The higher the variability in either lead time or demand will require a larger reserve stock to maintain a specific customer service level. Looking at this from another viewpoint, the safety stock inventory will have a slower turn-over, and the cost of carrying this inventory will be greater than the cost of carrying stock required to meet expected average demand. Management should examine the variability of demand and decide to maintain a lower service level for items with a high degree of variability because of the higher carrying cost.

A second factor, which may affect the level of service that a firm decides to provide, is the competition in the particular market. If competition is particularly acute in a specific field, the firm may choose to have a very high service level, using availability of the product as a tool against competition.

A final factor may be the type of product itself. For example, products have long been categorized as either convenience, shopping, or specialty items. Convenience items are characterized by substitution. If some particular item is not available, a similar item may be substituted, or picked up at another store.

As the term implies, shopping items are those for which a customer will shop around. Substitution is possible in this category because the customer will not pick up just any like item. Specialty items are those on which a customer is willing to wait for delivery, because they either desire or must have that particular item. Inasmuch as a higher level of service will need to be provided for convenience items than shopping items, and the level of service for specialty items can be significantly lower; the firm should study how their items fit into these categories. Again, care should be taken not to set the level too low on specialty items because the consumer could rebel. The customer may not be able to change or substitute in the short-run, but he may change brands if the reliability of service is unsatisfactory.

In summary, the marketing situation, competition, and the type of product have a significant influence on the service level a firm should provide. The reliability of service can strongly influence sales, especially where the buyer can substitute another product. As service becomes a more significant factor, the reliability of service becomes
even more important. Magee (77) points out the importance of the reliability of the service level to a firm as follows:

In a recent case, a manufacturer of industrial equipment decided to increase inventories to improve service. Orders for additional production were issued to bring inventories into a new, higher relation to demand. He quickly found, however, that as material became more readily on hand, orders grew; the more he had, the more he could sell. The new inventory demand relationship was finally reached at a much higher demand level (p. 90).

From the above comments, it becomes clear that the level of service is very important as a sales tool and that management should make every effort to determine what the customer wants, or expects, from a service level standpoint.

Physical Distribution System. The ability of a firm to provide a level of service to its customers may be affected by the physical distribution system employed to market its product. In designing this system, the firm should make two decisions: (1) how to move the product to the destination designated by the customer, and (2) how much inventory to maintain at the point from which the product originates (117). A number of methods are available to move the goods to the customer, ranging from customer pickup, shipment by rail or truck to some form of special delivery, such as by express or by air. The second of the two decisions was discussed under "Determining Out-ofStock Costs." However, the problem of moving the product to the customer was considered as being separate and apart from the service level to be maintained, when in reality they are actually interrelated.

The interrelationship of the level of service and the physical distribution system is very prevalent in the literature. In being discussed by the various authors, however, it is assumed that the firm
will carry enough inventory to meet average expected demand, and the problem becomes one of the: amount of safety stock to maintain. The interrelationship of the distribution system to the amount of safety stock should be apparent. If the firm has near-by sources of supply and the goods can be inexpensively and quickly moved from the near-by location, the necessary safety stock can be greatly reduced. The location of warehouses and the means of transportation has a significant effect on the amount of reserve stock, and therefore, the level of customer service.

Unfortunately, the physical distribution system has been looked upon as fixed, and the decision of safety stock has been made from the standpoint that the stocking points are given. As Magee (78, p. 88) points out, ". . . most companies, in fact, prefer to establish a policy of availability as a basis for the location of stock points and a policy of reliability of shelf service to govern inventory investment."

The separation of the physical distribution system and the decision on the amount of reserve stock to maintain results in suboptimization. The rationale for separating these decisions has been that different steps are taken for the two decisions (78). However, this would seem to be little reason to justify the separation of these decisions when they are so interrelated.

How Stock-out is Measured. The final factor, that will have an effect on the amount of safety stock to carry, is how the firm selects a measurement of the stock-out. To measure the stock-out should be a simple procedure (when an item is depleted, you are out-of-stock), but
such is not the case. Many ways have been presented to measure stockout, such as the length of time out-of-stock, the fraction of orders shipped complete, the maximum number of units short, time-weighted units short, and the occurrence measure, just to mention a few. Each of these different measures requires a different model in setting the amount of safety stock. This makes the task of measuring stock-out more difficult than determining either the ordering period or economic order quantity. Pritchard and Eagle (96) have indicated a possible reason for this problem:

This abundance of objectives for setting reserve stock may stem from the fact that the mathematical study of safety stock is more recent than the study of order quantities. As computation of safety stock receives more study, alternatives which now seem reasonable and worth pursuing may be cast out as impractical or atypical (p. 37).

Since management will need to determine which measure they are going to select, several measures will be discussed in order to aid in making this decision.

One way to measure a shortage is to express the out-of-stock situation as an occurrence. That is, as soon as a single shortage occurs, this is the measure of that shortage, and neither the number of units nor the length of time that the out-of-stock situation persists is considered. However, this measure is only appropriate under one condition; the cost of the shortage must be proportional to the occurrence of the stock-out and have nothing to do with the number of units short.

Both the costs of necessary actions to fill backorders and the intangible costs of customer goodwill are given consideration under the "occurrence method." The difficulty in using this method is computing the cost of goodwill. One approach to measuring goodwill is to
estimate the average number of customers inconvenienced by ordered goods being placed on backorder by the supplier. Then, even if only a few customers are affected, what is the dollar amount associated with each customer? Richard Ziegler (117) suggests that a single goodwill cost component may not be meaningful if the firm has a wide variety of customers. Some authors have suggested that it is impractical to measure goodwill, since it is almost impossible to do so. And, of course, there are those writers who persist that goodwill should be measured.

The maximum number of units short is another out-of-stock measurement and is based upon the expected maximum number of stock-out units over the stock-out period. This measurement is very similar to the "occurrence" method described above. Both methods determine a flat penalty cost for the stock-out. The primary difference between the occurrence method and the units short method is that the maximum number of units short method measures the cost associated with the maximum number of units backordered. The costs necessary to obtain the replenishment order is combined with the cost of goodwill in determining the total, or aggregate, backorder cost. Goodwill, as noted previously, must be estimated.

The maximum number of units short method may be particularly appropriate when demand occurs at a constant rate and lead-time is known. This situation is most prevalent when the firm is a manufacturing firm and its demand is from a single customer. Stock-out can be more readily measured using this method because the penalty for being out-of-stock is idle resources and customer goodwill may not be a consideration.
"Time out-of-stock" may be the appropriate measurement when the out-of-stock cost is related to the period of time during which the stock-out continues. This form of measurement is particularly useful, when the lead-time is known and cost of stock-out is proportional to lead-time. This measurement is similar to the "occurrence" method, and the requirements for its use are similar to those discussed for the "occurrence" method.

The "time-weighted units short" measurement assumes that the cost of having an item placed on backorder is a function of both the length of time out-of-stock and the number of units short. Two costs are included in this method. First, the cost per unit of processing the necessary additional order and the special handling required to receive it into stock; and second, a cost for goodwill, based on the time the shortage lasts. The time-weighted units short measurement is the most realistic of the costs incurred and their behavior. Unfortunately, this measurement includes a cost for goodwill, which as previously pointed out, is impossible for most firms to quantify. Under this approach, goodwill is a non-linear function of time. As the customer waits for the product, the cost increases at an exponential rate, and at some point the customer will permanently be lost to another company. Because of the inability to find a surrogate for the cost of goodwill, this approach is considered to be impractical, notwithstanding its theoretical appeal.

Many authors recommend that a particular service level be established because of the difficulty of measuring stock-out cost directly. This particular approach is implemented by maintaining a certain level of safety stock to provide a specified percentage of probability of
being able to supply demand. For example, a safety stock can be selected so that the firm will be able to have the probability of supplying demand 95 percent of the time.

This approach imputes a stockout cost because of the assumption that the marginal cost of carrying the safety stock is equal to the marginal cost of avoiding a stock-out. Magee (78, p. 126) states, "the choice by management of any desired inventory of service-risk level implies a cost attributed intuitively or indirectly to service failure." Starr and Miller (103) suggest that the imputed stock-out cost should be computed for a variety of service levels, thus allowing management to select the level they deem to be appropriate for their firm. A service level could be selected using both the imputed cost and judgement of the circumstances the firm encounters in the market.

The use of the service level approach is predominant in the literature. Plossl and Wight (93, p. 120) present an excellent discussion on the service level approach and suggest it "is a fairly simple one to handle statistically and one that is easy to relate to real business situations." The statistical model for setting a service level will be discussed in Chapter III on inventory models.

Determination of Out-of-Stock Costs --
The Backorder Case

If the situation a firm faces is one of backorder costs, the management of these costs is not only possible, but can be accomplished by maintaining a reasonably high service level. Three variations of backorder situations and the different circumstances as to how they may be handled have been discussed in the literature. They are as follows:

1. The first situation that may be a tẏpical position of some firms is that they may be able to take corrective action immediately and avoid a stock-out situation, or at least, be able to take such prompt action in filling the customer ${ }^{\wedge}$ s order that the customer is not aware that a shortage existed. This option is available to a firm if its inventory situation is such that another item can be substituted, or the item can be obtained quickly from another location or obtained from another supplier. Under these circumstances, the customer may be aware of the shortage, but the delay in filling his order is so short that he is not inconvenienced. In this situation, the costs incurred are the extra costs over and above those normally incurred to fill the order and can be measured. Buchan and Koenigsberg (26, p. 289) point out similar backorder cost characteristies of a stock-out situation: "Shortages can be met by priority shipments; the penalty cost is then the difference between the cost of priority shipments and cost of routine deliveries."
2. The second situation is when the stock-out results in a delay in filling the customer's order. The difference between this condition and the one discussed in number one above is that the customer's order is not filled immediately, and the customer is notified as to when he can expect delivery. Costs are not only incurred in backordering the item and special shipping charges, but the firm may incur loss of customer goodwill. This situation, if repeated too often or continues for too long a time, could eventually lead to the loss of the customer to another firm. This case is typically the one described in the inventory literature and consists of both the tangible costs of processing a backorder and the intangible costs of loss of customer goodwill, or the customer.
3. The third choice a firm has is to simply carry the stock-out item on backorder for the customer until the inventory is replenished through the normal ordering cycle. The only tangible costs incurred, if this alternative is chosen, are the costs of maintaining the backorder records. However, the intangible costs of loss of customer goodwill and loss of the customer may be very significant. In describing a typical backorder situation, Buchan and Koenigsberg (26, p. 289) make the following observation: "Shortages can sometimes be satisfied by backorders which are delivered to a customer when available; here the penalty cost is a loss of goodwill and may be reflected in the future buying habits of the customer."

In examining the above situations a firm might face or the plan it chooses to follow, the penalty costs for being out-of-stock varies considerably. In case number one, the costs are only the extra costs incurred in filling the backorder. In the second case, both the cost of filling the backorder and the intangible cost of loss of customer goodwill and the loss of customer will be incurred. In the last choice, very small costs are involved in maintaining backorder records, while the loss of customer goodwill may be very significant.

Although it is quite possible for a firm to face all three situations (if a firm has very diverse inventories) it is reasonable to assume that a uniform policy should be established for inventory shortages and should be determined by the goals of the firm, which in all instances may not be monetary goals.
ordering or procurement costs, (2) carrying or holding costs, and (3) out-of-stock costs.

Ordering costs start with the signal to order and continues with the processing of the order, receiving and inspecting the goods, placing the goods in stock, up-dating the inventory records, and finally, the payment to the supplier. There are several approaches that can be used to determine the order cost, but most are deficient due to the accounting records used to obtain the necessary costs. The only relevant costs are those that increase or decrease because of a decision to place an order.

Costs resulting from the storing or holding the goods on hand until sold to a customer are the holding or carrying costs. These costs include: (1) space, (2) taxes, (3) insurance, (4) material handling, (5) physical inventory count, (6) shrinkage, and (7) cost of capital.

Finally, out-of-stock costs are those incurred by the firm when they are unable to fill customer demand for goods. These costs have two primary forms: (1) the cost of having to backorder the item and delivering it to the customer at a later date, or (2) the cost of a lost sale.

This chapter has attempted to present the approaches in the literature of determining which costs are relevant to inventory decisions and how these costs should be measured. These approaches should be used as a guide for finding relevant costs, continually keeping in mind that only incremental or opportunity costs are relevant for the inventory decision process.

Inventory policy decisions are intended to minimize total costs. There are two basic questions involved when making an inventory policy decision: (1) when, or how often, to order, and (2) what quantity should be ordered. Even though these basic questions seem simple enough, a large number of people have given considerable time and effort in developing inventory models to answer these two questions, and thereby, solve the inventory decision problem. Exactly what is a policy, and what is its function? A policy is a statement describing in general terms a course of action. It has two functions: (1) to serve as a base to formulate operating procedures; (2) to serve as a general guide when making decisions about unusual problems that do not fall within clear-cut procedural boundaries (71).

## Possible Inventory Policies

Inventory policies may range from the very simplest type, such as looking at the inventory and making a judgement as to the type of policy needed, to a very complex, computerized inventory control system. Regardless of the method used to establish the inventory policy, this decision must be made before the selection of an inventory model can be made.

Influences on the Choice of an Inventory Policy

## The Influence of an Inventory System's

Cost on the Choice of a Policy

An inventory policy is either a fixed order interval policy (P) or a fixed order quantity policy (Q) or some combination of the two; the most common is referred to by Buffa (27) as the "optimal replenishment" policy, also called the $S, s$ and the min-max policy. Regardless of which policy is selected, the costs associated with each will vary a great deal. This variance is caused by the difference in the information needed to operate them. In the case of the fixed order interval policy, the basic information needs are: the periodic review date, the level of inventory on-hand, and the desired replenishment level. Under this method, the quantity on hand is determined by a physical count on the periodic review date. At this time, an order is placed for the number of the item needed to bring the inventory level up to the desired level. An alternative would be a perpetual inventory record to keep account of the quantity on hand, instead of a periodic physical count.

In opposition to the fixed order interval policy, the fixed order quantity model requires knowledge of what the level of inventory should be when the item is to be reordered and the quantity that needs to be reordered. When the stock on hand falls to this predetermined order level, an order is placed for a fixed quantity of that item. A perpetual inventory information system is normally associated with this method, especially if a computerized information system is used. This allows an immediate determination of when the reorder level has been
reached. If perpetual records must be maintained manually, the cost may be prohibitive. An alternative in this case would be some variation of what is known as the "two-bin" system. This method either physically or identifiably segregates the particular item into two separate parts. As the name suggests, this can be accomplished by having two separate storage bins. In some cases it can be accomplished either with separate containers in the same bin or a painted line within the bin thatmarks the reorder level. The first section of the item is used in supplying demand until the reorder point is reached, and the second section is used to satisfy demand until the new order is received.

Another possible cost difference associated with the fixed order interval and the fixed order quantity policies arises from the methods used in meeting the variations in demand and the variations in leadtime. In using the fixed order quantity policy, increases or decreases in demand are countered by changing the period between orders. Fluctuations in demand result in an increase or decrease in the number of orders. In using the fixed order policy, variations in demand are countered by an adjustment of the order size. Higher demand calls for a higher order quantity, and lower demand is met by reducing the order size. The placing of orders at regular intervals may have the effect of having too large a number of orders for smaller quantities under the fixed order interval policy, while the quantity is determined in advance using the fixed order size policy. Under the fixed order interval plan, the inventory level has a tendency to be close to its maximum level, but under the fixed quantity order plan, the level is
lower because an order is not placed until the inventory level has fallen to the reorder point.

Costs may be affected by the various constraints imposed by the firm's suppliers. The firm may minimize costs by using the fixed order interval policy because orders may be accumulated for many items that are ordered from the same supplier. This policy may be necessary to take advantage of the prepayment of freight charges by the supplier. Another constraint may be imposed by the suppliers if they have fixed shipping dates. The fixed interval policy may also permit more exact scheduling of work by the purchasing, receiving, and warehousing departments. The use of the fixed order quantity policy may be necessitated because of quantity order size restrictions by the supplier and by savings in freight charges made possible by shipping larger quantities of goods, that is, if the buyer is required to pay the freight charges.

The availability of information and the costs involved in obtaining this information has abig influence on the decision of which policy to select. Also, the constraints imposed by the supplier, mentioned above, have a determination on the costs incurred, and therefore, the selection of a policy. In retrospect, particular combinations of these constraints may lead to a combination of the fixed order interval and fixed order quantity policies, such as the previously mentioned "op.timal replenishment" policy. In conclusion, costs of information and the availability of that information, in conjunction with the costs asso ciated to the particular policy, will have a very strong influence on the selection of a policy.

## The Influence of Inventory Item Characteristics

on the Choice of an Inventory Policy

The "ABC" Analysis of Characteristics. The characteristics of the different items carried in an inventory may differ to a large extent. These major differences in characteristics of items carried in most inventories brought about the " $A B C$ " method of classification of inventory that has become so prevelant in the literature. The concept originated with the great nineteenth-century, Italian philosopher, Pareto, who discovered that a very large part of all the material wealth of Italy was in the hands of a very small number of people. Ford Dickie adopted Pareto's concept: and applied it to the classification of inventory. He called his technique the "ABC" method. The "A" items are those items with a high unit cost comprising a small group which represents a majority of total annual usage dollars. The "B" group represents a larger group of medium priced units and represents a much smaller share of the total annual usage dollars. Finally, the "C" group is comprised of the largest number of items and represents a very small share of the total annual usage dollars. Figure 5 depicts what the distribution of most inventories would look like. Of course, the percentage of each classification would differ from one inventory to another. Plossl and Wight (93) describe the following ranges as typical for most inventories:
"A" items: High value -- those relatively few items whose value accounts for $75-80 \%$ of the total value of the inventory. These will usually be from $15-20 \%$ of the items.
" ${ }^{\prime \prime}$ items: Medium value -- a larger number in the middle of the list, usually about $30-40 \%$ of the items, whose total value accounts for about $15 \%$ of the total.
"C" items: Low value -- the bulk of the items, usually about 40-50\%, whose total inventory value is almost negligible, accounting for only $5-10 \%$ of the value ( $p$. 57).


Several other authors have found similar percentages in their research with the "ABC" method.

Obviously, there is a close relationship between the class with which an inventory item is associated and those procedures that the firm adopts for control. For "A" items, the firm may want to maintain a close control by using a perpetual inventory system and a fixed order quantity policy. On the other hand, the firm might prefer to adopt a fixed order interval policy with a periodic physical count for "C" items because of low unit cost or usage. A problem arises with "B" items; the selection of a policy becomes more difficult. For "B"items, either the reorder period policy with a physical count, or a reorder point doctrine, combined with perpetual inventory records might be
appropriate. It may be anticipated that many items will fall between class intervals and the decision of which class should be used will require judgement on the part of the inventory manager. Thomas $R$. Prince (95) states:

It goes without saying that the analyst must exercise considerable judgement in classifying the items in inventory. He may make mistakes in both directions: too much control as well as, too little control. The non-publicized condition of too much control may be of more financial importance to the business firm than the publicized condition of too little control (p.367).

Classification of Inventory Systems

Besides the "ABC" analysis of characteristics, there are many other factors to be considered in deciding upon inventory classification. Among these, in order of their importance, are: (1) consistency of the usage rate, (2) peculiarities in lead-time and procurement, (3) risk of obsolescence, (4) suseptability to theft and (5) the physical size of the item (117, p.118).

Classification in this manner needs to be associated with a study of the various aspects of the different inventory control policies in order to achieve the lowest costs for the total inventory control system. Different procedures will be required for the different classes of items in the inventory and management needs to be aware that the same policies are not appropriate for each and every item in the inventory.

Several methods of classification of inventory systems are presented in the literature, and while all are closely related in concept, there is naturally some disagreement on how they should be classified.

The following is a classification of these systems as suggested by Ploss1 and Wight (93): (1) the two-bin system, (2) visual review, (3) order point system, (4) periodic review system, and (5) materials planning. No classification system has been able to establish a clear cut distinction between systems. A system may have characteristics of more than one classification, however, the systems known at this time will be related to one of the above classifications. Classifications and their characteristics will be explained in the next several paragraphs.

## The Two-Bin System

This system is used primarily for low-cost items where perpetual records are not maintained and customers will not be greatly inconvenienced by a stock-out. A predetermined amount of an item is set aside, either in a separate bin or separated from the main stock in the same bin. This reserve supply is not used until the main stock has been exhausted. At the time the reserve stock is opened, or broken into, a replenishment order is placed immediately.

## Visual Review System

As the title of the system suggests, the level of stock is visually checked periodically and replenishment orders are placed after each review to bring the inventory le'vel up to a predetermined maximum of the amount on hand plus the amount on order. This system is used either for low-cost items or could be used for higher priced items if the periodic review is frequent.

## Order Point System

Under the order point system, when sales brings the quantity on hand down to a predetermined level as shown on perpetual inventory records (the order point) a replenishment order is placed usually for an amount of the economic order quantity. This system is described as having a fixed order quantity and a variable order interval. An economic order quantity model falls within this classification. Both the order point system and economic order quantity models will be discussed in detail later in this chapter because of their predominant use in industry.

## Periodic Review System

Perpetual inventory records are examined periodically, for example, once a week or once a month. An order for replenishment is placed for enough units to bring the amount on hand plus on order up to a predetermined maximum level. This process is comparable to the visual review, except perpetual inventory records are maintained.

## Materials Planning System

This system is used for raw materials for a manufacturing firm where a preplanned program of production enables the manager to order in amounts and on time schedules to meet a planned demand.

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Fixed Order Quantity Models -- Q Models
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Assumptions of the Model

The classical model assumes: (1) that demand for the item is at a
constant rate and is known in advance by the decision maker, and (2) the lead-time, the time between the placement of the order and its receipt into inventory or the time for acquiring an item, is also known.

Although these assumptions are not realistic in a real world situation, the simple model can be developed and realistic factors introduced into the model later. However, the simplest model is usable in many situations to great advantage because of the insensitivity of the model to the various inputs (12). The different inputs should be examined and management should concentrate on the most critical ones.

The simple model is concerned with only two of the three costs previously discussed in Chapter II, carrying and ordering costs. Carrying costs are balanced against only ordering costs. Because of the simplifying assumptions of known demand and lead-time, out-of-stock costs are not relevant. There are no lost sales. These costs are controlled by varying the quantities ordered and the timing of the placement of the order. If a firm's inventory policy is to order in large quantities infrequently, the higher carrying costs may be more than the saving in ordering costs. On the other hand, if a smaller quantity is ordered more frequently, the lower costs of carrying a lower average inventory may be more than offset by the increased ordering costs. What is needed is to find the optimum order quantity and the timing of the placement of the order that will minimize total inventory costs to the firm.

Ballou (12) depicts the extreme inventory policies as shown in Figure 6. Somewhere between the inventory level ( $\mathrm{AIL}_{1}$ ) that results
from ordering in large quantities infrequently and the inventory level ( $\mathrm{AIL}_{2}$ ) that results from ordering frequently in small quantities, is the policy that will minimize the firm's total inventory costs.


Source: (12, p. 280)
Figure 6. Effect of Extreme Inventory Policies on Inventory Level Under Constant Demand and Constant LeadTime Conditions

Economic Order Quantity Mode1

The basic model for solving the multiple-order problem was first
introduced by Ford Harris of the Western Electric Company in 1915. This model has become known as the Economic Order Quantity (EOQ) model and is the basis of most models today. The EOQ model answers the questions of how much to order and when to order. There are four generally recognized approaches for solving the EOQ. These four approaches are discussed below.

The Tabular Approach. This approach is by trial and error. First, a number of different purchase quantities (lot sizes) are selected. Second, the total costs for each lot's size is determined and third, the lot size which minimizes total cost is chosen. Thierauf (111) illustrates this approach as shown in Table VIII below. The limitation of this approach is the great number of alternatives which

TABLE VIII

TABULAR APPROACH TO EOQ

| Orders |  |  | Carrying <br> Charges | Ordering <br> Costs | Total <br> Cost |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Year | Lot | Average <br> Inventory | Size <br> $20 \%$ Per <br> Sear | $\$ 12.50$ <br> Per Order | Per <br> Year |
| 1 | 8,000 | 4,000 | $\$ 800.00$ | $\$ 12.50$ | $\$ 812.50$ |
| 2 | 4,000 | 2,000 | 400.00 | 25.00 | 425.00 |
| 4 | 2,000 | 1,000 | 200.00 | 50.00 | 250.00 |
| 8 | 1,000 | 500 | 100.00 | 100.00 | 200.00 |
| 12 | 667 | 333 | 66.00 | 150.00 | 216.00 |
| 16 | 500 | 250 | 50.00 | 200.00 | 250.00 |
| 32 | 250 | 125 | 25.00 | 400.00 | 425.00 |

Source: (111. p. 188)
must be examined without being sure that the lowest possible cost alternative has been examined.

The Graphic Approach. The most prevalent approach in the literature for solving EOQ is the graphic approach. Figure 7 shows how total costs, at first, decrease to a low point where inventory ordering costs equal inventory carrying costs and then increases as the quantity ordered increases. The graphic approach pictorially presents the EOQ so that it can be readily seen that the quantity (EOQ) should be the quantity where total costs are at a minimum; i.e., carrying costs and ordering costs are equal. Unfortunately, as discovered from the discussion in Chapter II, carrying costs and ordering costs are very difficult to measure and without accurate, specific costs, the plotting of these curves is not reasible.


Figure 7. Economic Ordering Quantity (EOQ) Graph

Algebraic Approach. From the graphic approach it is seen that the most economical EOQ is the point where carrying costs equal ordering costs. The algebraic method uses this concept to solve for the EOQ. Thierauf and Grosse (111) have a very easy-tb-follow explanation of the algebraic approach which is presented below:

$$
\left.\begin{array}{rl}
Q= & \text { economic ordering quantity (EOQ) or optimum number } \\
& \text { of units per order to minimize total cost for the }
\end{array}\right\} \begin{aligned}
& \text { firm; }
\end{aligned}
$$

Total inventory carrying costs are determined in the following manner:
$\frac{Q}{2}$
x
$C \times I \quad=\quad \frac{Q}{2} C I$

Average $\quad$ Cost of carrying
Inventory x one unit of $=$ Total inventory Quantity inventory per year carrying cost

Total ordering costs are derived as follows:
$\frac{D}{Q} \quad x \quad S \quad=\frac{D}{Q} S$

Total inventory carrying costs are then set equal to total ordering costs and solve for Q :

$$
\begin{aligned}
\frac{Q}{2} C I & =\frac{D}{Q} S \\
Q C I & =\frac{2 D S}{Q} \\
Q^{2} C I & =2 D S
\end{aligned}
$$

$$
\begin{align*}
& Q^{2}=\frac{2 \mathrm{DS}}{\mathrm{CI}} \\
& \mathrm{Q}=\sqrt{\frac{2 \mathrm{DS}}{\mathrm{CI}}} \tag{3.1}
\end{align*}
$$

The equating of inventory ordering costs to carrying costs can be used to determine two other very useful inventory control answers: (1) the optimum number of orders per year and (2) the optimum number of days supply per order.

The optimum number of orders per year is determined as shown in Equation 3.2 shown below:
$\mathrm{N}=$ Optimum number of orders per year to minimize total costs to the firm;
$T=$ Total dollars amount of annual usage;
S = Ordinary costs per order;
I = Inventory carrying costs expressed as a percentage of the value of average inventory.

Total inventory carrying costs are computed as follows:
$\begin{array}{llllll}\frac{T}{N} & x & \frac{I}{2} & x & 1 & \frac{T I}{2 N}\end{array}$
Dollar amount x Average inventpry $\mathrm{Inventory}=$ Total per order
Average inventory
under constant

usage $\quad$\begin{tabular}{ll}
Inventory <br>
carrying cost <br>
percentage

$\quad$

Total <br>
Inventory <br>
Carrying <br>
Cost
\end{tabular}

Total ordering cost per year is determined as follows:


Set inventory carrying costs equal to inventory ordering costs and solve for $N$.

$$
\frac{\mathrm{TI}}{2 \mathrm{~N}}=\mathrm{NS}
$$

$$
\begin{align*}
2 \mathrm{~N}^{2} \mathrm{~S} & =\mathrm{TI} \\
\mathrm{~N}^{2} & =\frac{\mathrm{TI}}{2 \mathrm{~S}} \\
\mathrm{~N} & =\sqrt{\frac{\mathrm{TI}}{2 \mathrm{~S}}} \tag{3.2}
\end{align*}
$$

The optimum number of days supply order is determined as shown in equation 3.3 that follows:
$\mathrm{M}=$ Optimum number of days supply per order;
D = Total annual quantity requirement;
$\mathrm{S}=$ Ordering costs per order placed;
I - Inventory carrying costs, expressed as a percentage of the value of average inventory;
$\mathrm{C}=$ Cost value of one unit;
$365=$ Calendar days per year .
Total carrying costs per year is computed as follows:

| $\frac{\mathrm{DC}}{365 / \mathrm{M}}$ | x | $\frac{\mathrm{I}}{2}$ | x | I |  | $\frac{\mathrm{DCT}}{730 \mathrm{M}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dollars per order | x | Average <br> Inventory under Constant usage | x | Inventory <br> Carrying Cost Percentage |  | total <br> Inventory <br> -Carrying <br> Costs |

Total ordering costs per year are determined as follows:
$\frac{365}{M}$
x
$S \quad=\frac{365 \mathrm{~S}}{\mathrm{M}}$

Number of Ordering Cost Total
Orders per $x$ per Order $\quad=$ Ordering
Year Costs

Set ordering costs equal to carrying costs and solve for $M$.
$\frac{\mathrm{DCI}}{730 / \mathrm{M}}=\frac{365 \mathrm{~S}}{\mathrm{M}}$
$\frac{\text { DCIM }}{730}=\frac{365 \mathrm{~S}}{\mathrm{M}}$

$$
\begin{align*}
\mathrm{M}^{2} \mathrm{DCI} & =266,450 \mathrm{~S} \\
\mathrm{M}^{2} & =\frac{266,450 \mathrm{~S}}{\mathrm{DCI}} \\
M & =\sqrt{\frac{266,450 \mathrm{~S}}{\mathrm{DCI}}} \tag{3.3}
\end{align*}
$$

Differential Approach. The best approach for solving for EOQ is the differential approach because this approach does not have the limitations of the other three approaches.

Refer to Figure 7 and note that the slope of the total cost curve is the sum of the slopes of the carrying cost curve and the ordering cost curve. As the cost curves move from the vertical axis, the ordering cost curve has a negative slope, and the inventory carrying cost curve has a positive slope. As the quantity increases on the horizontal axis, a point is reached where the slope of the carrying cost curve and the slope of the ordering cost curve are the same in magnitude but opposite (one negative and one positive) so that when they are added together the slope of the total cost curve is zero. This, of course, is what the EOQ model is trying to determine, the quantity where total costs are at a minimum or when the total cost curve has a zero slope.

Reviewing the equation from the algebraic approach where we set inventory carrying costs equal to inventory ordering costs,

$$
\frac{Q}{2} C I=\frac{D}{Q} S
$$

Then, total costs would be

$$
\mathrm{TC}=\frac{\mathrm{Q}}{2} \mathrm{CI}+\frac{\mathrm{D}}{\mathrm{Q}} \mathrm{~S}
$$

Differentiating this equation will express the slope of the total cost curve.

$$
\frac{d(T C)}{d(Q)}=\frac{C I}{2}-\frac{D S}{Q^{2}}
$$

Then, this first derivative is set equal to zero to determine the optimal point.

$$
\begin{align*}
\frac{\mathrm{CI}}{2}-\frac{\mathrm{DS}}{\mathrm{Q}^{2}} & =0 \\
\mathrm{Q}^{2} \mathrm{CI} & =2 \mathrm{DS} \\
\mathrm{Q}^{2} & =\frac{2 \mathrm{DS}}{\mathrm{CI}} \\
\mathrm{Q} & =\sqrt{\frac{2 \mathrm{DS}}{\mathrm{CI}}} \tag{3.1}
\end{align*}
$$

This is the same equation arrived at by the algebraic method. However it could not be determined under the algebraic method whether this point was a maximum or a minimum. By taking the second derivative, the maximum or minimum of this point can be determined.

$$
\frac{d^{2}(T C)}{d(Q)^{2}}=0+\frac{2 R S}{Q^{3}}>0
$$

The plus sign in the second derivative confirms that we have the minimum total cost point, and also, indicates that this total cost curve is increasing upward. Therefore, the differential approach confirms that total costs to the firm are at a minimum, which cannot be accomplished using the first three methods.

EOQ Model with Intermittent Demand

In its most basic form, the standard economic order quantity formula assumed demand was known, and that sales occured at a uniform rate. The assumption of known demand was relaxed, but sales were still assumed to be fairly continuous, and the EOQ formula was modified to cope with this situation。

However, requirements may be intermittent (sales may come in large lots at irregular periods). When sales come at irregular periods, a
slightly different approach is necessary, but the basic concept of balancing carrying and ordering costs remains the same. If, for example, the demand occurs in five weeks of the year, then five different orders could be placed. Alternatively, the demand for two weeks could be combined into one order resulting in four orders. By trial and error, the various combinations can be examined to determine the lowest total cost. This presents a more complex calculation, but intermittent demand makes it a necessity.

## Complications in Computing EOQ Caused

 by Quantity DiscountsQuantity discount schedules, freight rate savings, and anticipated price increases can create a situation, where it becomes advantageous to order a quantity larger than the EOQ. The simplest approach for determining whether to order the larger amount and receive a quantity discount is the "price comparison approach." To use this approach, a comparison of the material cost saving by purchasing a larger amount is made with the cost of carrying that many more units in stock. The most economical decision then becomes obvious (71). Some quantity discount schedules may be more complicated than others, but they can be incorporated into the EOQ.*

## Reorder Point Models

## Fixed Order Point--Fixed Order Quantity Model

When conditions of uncertainty exist, one of the more basic
*For a more throrough discussion on quantity discount schedules, see Thierauf and Grosse (111, pp. 195-202).
approaches to inventory control is very similar to the economic order quantity model used under conditions of certainty. To use this model, a minimum inventory level is predetermined, and at the point this predetermined level is reached, an economic order quantity will be ordered to replenish the stock inventory.

Order Point Determination. The minimum level of stock carried in the inventory should be an amount sufficient to meet the determined level of demand during the order cycle. However, the order point may need to be adjusted to effectively take care of uncertainty; therefore, future demand must be predictable, subject to some differences between actual and predicted demand (64).

Determination of Safety Stock. In the classical inventory model, demand and lead-time are known. However, in most situations this is considered unrealistic because demand does vary, and in many situations, supply lead-time also may vary.

To carry a stock inventory so large as to eliminate the possibility of being out-of-stock is obviously impractical. Therefore, an optimum level of safety stock should be established. The calculation of a reorder point under uncertainty should be one that continues to increase our reorder point, and thus, our average inventory, until the additional cost of adding one unit to inventory equals the savings that will result from this decision. That is, the optimum safety stock level is where combined carrying costs and stock-out costs are minimized.

Two methods of setting safety stock levels are as follows:

1. Minimizing total costs where stock-out costs can be measured. An example similar to that of Heskett, Glaskowsky and Ivie (64) to more
clearly show this cost minimization approach using a frequency demand distribution for an inventory item is presented in Table IX below.

TABLE IX
AN EXAMPLE OF FREQUENCY DEMAND DISTRIBUTION FOR AN INVENTORY ITEM

| Units <br> Demanded | Number <br> of Days | Units | Frequency | Cumulative <br> Frequency |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 60 | 0 | $50 \%$ | $50 \%$ |
| 1 | 44 | 44 | $37 \%$ | $87 \%$ |
| Tota1 | 16 | 32 | $13 \%$ | $100 \%$ |

An examination of orders during this period would reveal that lead-time has also varied. For example, during the time period of 120 days, four orders were placed. One order was received six days after being placed; one order was received four days after being placed, and the other two orders were received within five days after being placed. This results in a probability of 50 percent that the lead-time will be five days and a probability of 25 percent each for four and six days.

Rules for probability allow us to compute the probability of sales (x) varying from 0 to 12 during lead-time. For example, the probility of encountering a six-day lead-time and six straight days during which demand is 2 units each day is computed as follows:

$$
p(x=12)=.25(.13)(.13)(.13)(.13)(.13)(.13)=.000001206702
$$

The probability of demand being two units in a day is .13 , and being six units in two consecutive days is $.13 \times 13$, etc.

The probability of encountering a six-day lead-time is 25 percent. If all of these probabilities are multiplied, the probability of "x" equaling twelve is one in one million. It now becomes evident that it is not feasible to carry a level of safety stock to be prepared for this eventuality.

To check the occurrence that "x" equals eleven, sales of two units each day for five days and a sale of one unit on the sixth day would be necessary. Since this could happen in six different patterns, the basic probability equation is multiplied by six. There is a 37 percent chance of encountering a sale of one unit on one day and a 13 percent chance that two units would be sold on each of the other five days. Therefore:

$$
p(x=11)=6[.25(.37)(.13)(.13)(.13)(.13)(.13)]=.000020606762
$$

The probability of sales equal to eleven units and lead-time equal to six days is two out of 100,000 . The results of computing all of the probabilities for "x" are shown in Table X.

The underlying principle of the calculation for a reorder point under uncertainty is that units should be added to the reorder point until the additional cost of adding a unit equals the cost of a stockout. Therefore, if the cost of carrying an additional unit, rvQ/S, is set equal to the cost and probability of having a stock-out, $\mathrm{p}(\mathrm{x}>\mathrm{P}$ ) $(\pi)$, the following formula may be used for determining the optimum probability of a given size penalty which can be tolerated in an order situation:

TABLE X

PROBABILITIES FOR X

| x | $\mathrm{p}(\mathrm{x})$ | $\mathrm{P}(\mathrm{x}) *$ |
| :---: | :---: | :--- |
| 12 | .0000 | ---- |
| 11 | .0000 | .0000 |
| 10 | .0002 | .0000 |
| 9 | .0062 | .0002 |
| 8 | .0056 | .0064 |
| 7 | .0200 | .0120 |
| 6 | .0549 | .0320 |
| 5 | .1174 | .0869 |
| 4 | .1933 | .2043 |
| 3 | .2390 | .3976 |
| 2 | .2069 | .6366 |
| 1 | .1214 | .8435 |
| 0 | .0351 | .9649 |
| Total | 1.0000 |  |
| Cumulative probability of more than x. |  |  |

$$
p(x>P)=\frac{r v Q}{\pi S}
$$

where:
$r=$ the annual inventory carrying charge (as a percentage of product cost or value)
$v=$ average cost, or value, per unit of product (in dollars)
$\mathrm{Q}=$ the quantity ordered (in units)
$S=$ the annual demand or usage of the product (in units)
$\pi=$ cost of a stock-out situation (per unit)
$\mathrm{p}=$ probability of an occurrence
$\mathrm{P}=$ reorder point (in units of stock on hand and on order less those promised out)
$\mathrm{x}=$ usage or demand during an order cycle, in units. This computation, of course, is dependent upon the ability of the firm to measure the cost of a stock-out. If the firm faces a backorder situation without a cost of customer goodwill, as discussed in Chapter II, then this procedure can be used. If stock-out cost cannot be measured then the firm should use a service level procedure.

Service Level Safety Stock. Ploss1 and Wight (93) point out that the amount of reserve stock required is a function of five elements: (1) the ability to forecast demand accurately, (2) the length of leadtime, (3) the ability to forecast or control lead-time accurately, (4) the size of the order quantity, and (5) the service level desired (93). Some statistical techniques can be quite helpful in establishing the proper amount of safety stock necessary to provide a certain service level. When it is impossible to measure the out-of-stock costs, these statistical techniques are particularly important. The techniques are based upon the probability of a firm being able to fill the demand.

Three of the most common probability distributions of sales are: the normal, Poisson, and negative exponential.* From the appropriate distribution, the amount of safety stock can be determined for the desired service leve1. For example, if sales have a normal distribution, the adding of one standard deviation to the anticipated demand during lead-time will provide an 84 percent service level, and the

[^0]addition of two standard deviations will raise the service level to 98 percent. Stated another way, the firm is able to supply the demand 98 percent of the time, and an out-of-stock situation will occur only 2 percent of the time. The most common service level is between 90 and 95 percent.

If, as pointed out previously, out-of-stock costs are not subject to measurement, these statistical techniques are far superior to the popular "rules-of-thumb," such as the level of safety stock is one month's supply or 10 percent of the demand during lead-time.

Fixed Order Interval Models -- P Models

The fixed-order-interval system, also called the fixed-interval reorder system, or the fixed-reorder cycle system, is a periodic reordering system in which the time interval between orders, such as a week, a month, or a quarter, is fixed, but the size of the order is not, and order quantity varies according to usage since the last review. This type of inventory control system is employed when it is more convenient to examine inventory stocks on a fixed-time cycle. Figure 8 below is an example of how this system works over a period of time for one item.

The fixed-order-interval system is used to reduce the cost of constantly reviewing inventory levels, a major disadvantage of fixed-order-quantity models. Another advantage of the fixed-order-interval model is a cost reduction that can be realized when a great number of items are purchased from a single supplier. Quantity discounts may present considerable savings and transportation charges greatly reduced by ordering one shipment due to scheduling reorders on the same day.


Source: (93, p.127)
Figure 8. Periodic Review System Fixed Order Cycle-Variable Quantity

A major disadvantage is the amount of safety stock needed to protect against a stock-out is usually higher because, under this system, it is possible to have a stock-out, not only during the lead-time, but also before the reorder date is reached.

Another disadvantage of the fixed-reorder-cycle system is the determination of what is most commonly referred to as a maximum value to order, or an "order-up-to-leve1.!" A typical difficulty encountered is instability in reordering habits and inventory levels caused by attempts to outguess the market or usage rates, and to assume that high or low demand indicates an established trend which must be anticipated.

Lack of care in selecting the review period can lead to instabil: ity in inventory balances. This lack of care may also lead to excess requirements of safety stock to preserve customer service。

## Combinations or Variations of Models

## Optional Replenishment Systems Model (S,s Model)

The optional replenishment system ( $\mathrm{S}, \mathrm{s}$ system) is a combination of the fixed-order-point and the fixed-order-interval models. It combines the essential control devices of both models and has a fixed-orderinterval, a fixed order point, and a fixed-order-quantity.

The optional replenishment system establishes a maximum inventory level, designated as "S," which is a total of on-hand and on order. A reorder point "s" is dểtermined; thus, the name $\mathrm{S}, \mathrm{s}$ model. At each review date, if the amount on-hand and on order is below the reorder point, an order is placed for an amount to bring the inventory level up to maximum inventory level "S." The reorder quantity is not fixed, as most authors suggest, but semi-fixed. A lower limit is placed on the order quantity. This lower limit is the difference between "S," the maximum inventory level, and "s," the reorder point level. However, by the time the review date comes around, the order quantity could vary from the lower limit up to "S." If the order point " s " has not been reached on a designated review date, no order is placed until the same process occurs at the next review date.

The combination of both systems maintains close control over inventory levels. However, there is the disadvantage of carrying a
high level of safety stock, in some instances; inasmuch as, it is possible to run out of stock between review periods.

Stochastic Dynamic Continuous Review Model

Wagner (115) has developed a model for inventory control that encompasses probabilistically described demand. He calls this model a stochastic dynamic continuous review inventory model. Discussion of this model will show that although all factors could be previously incorporated into a model, it was not until the coming of the computer age that it became usable. Wagner's earlier derivations will be dispensed with, and only the final formulae, which is used to determine the average cost per unit of time, will be presented.*

Expected average
(1) $\begin{aligned} & \text { cost per unit } \\ & \text { of time }\end{aligned} \equiv E[A C]=\frac{K M}{Q}+c M+h\left(\frac{Q}{2}-M_{\ell}+s\right)+$

$$
\left(\frac{h M_{\ell}}{2 Q}+\frac{M \pi}{Q}\right) \sum_{q_{\ell}>s}\left(q_{\ell}-s\right) p_{\ell}\left(q_{\ell}\right)
$$

where:

| M | $=$ expected number of items demanded per unit of time; |
| :---: | :---: |
| L | $=$ lead-time, the length of the interval between placing and receiving an order; |
| M | $=$ expected number of items demanded during an interval of $L$ units of time; |
| q | = actual demand during an interval between placing and receiving an order; |
| h | $=$ holding cost for each item per unit of time; |
| K | = setup or reorder cost; |
| c | $=$ unit purchasing cost; |
| $\pi$ | = penalty or out-of-stock cost; |
|  | = reorder point; |
| p (q ) | = probability distribution of demand during each lead-time; |
| Q | = order quantity. |

[^1]Partially differentiating E $\bar{A} C]$ with respect to " $Q$," setting the derivative equal to zero, and solving for $Q$, gives the formula (115, p. 815).
(2) optimal $Q=\sqrt{\frac{2 K M}{h}+\left(M_{\ell}+\frac{2 M \pi}{h}\right) \sum_{q_{\ell}>s}\left(q_{\ell}-s\right) p_{\ell}\left(q_{\ell}\right)}$

If the cumulative demand distribution is defined as,

$$
\begin{equation*}
P_{\ell}^{\prime}(\mathrm{y}) \equiv \sum_{\mathrm{q}_{\ell}=0}^{\mathrm{y}} \mathrm{P}_{\ell}\left(\mathrm{q}_{\ell}\right) \tag{3}
\end{equation*}
$$

It can then be shown that the optimal $s \geq 0$ is the smallest integer, such that,

$$
\begin{equation*}
P_{\ell}(s) \geq R \text { (determination of the reorder point) } \tag{4}
\end{equation*}
$$

where the critical ratio is,

$$
\begin{equation*}
R \equiv 1-\frac{\mathrm{hQ}}{\underline{\mathrm{hM}_{\ell}+\mathrm{M} \mathrm{\pi}}} \tag{5}
\end{equation*}
$$

The order quantity "Q" and the reorder point "s" are optimal values if they simultaneously satisfy equations (2) and (4) (115).

As previously stated, the purpose of presenting Wagner's stochastic dynamic continuous review model is to show that sophisticated models and the computer are available to solve any inventory problem.

## Summary

Inventory policies ranging from the simplest type, such as observation of the quantity on hand and merely using judgement on the amount to order, to the most sophisticated computerized models have been examined in this chapter. Also, the influence of the cost of a system on the choice of an inventory policy has been discussed, as well as, the
influence that the characteristics of the inventory items have on the choice of a policy.

Inventory systems are classified into: (1) the two-bin system, (2) visual review, (3) order point, (4) periodic review, and (5) material planning system.

Models to implement the above inventory systems are usually divided into two primary forms: (1) the fixed-order-quantity ( 0 models) and (2) the fixed-order-interval ( P models). The fixed-orderquantity model has become well known as the economic order quantity (EOQ) model. This model balances inventory carrying costs with ordering costs in minimizing total inventory costs to the firm. A fixed quantity (EOQ) is ordered when inventory falls to the reorder point. The fixed-order-interval model places orders at fixed, specified intervals. Under this method, the quantity on hand is determined, and an order is placed for the number of the item needed to bring the inventory level up to the desired level.

Combinations of $P$ and $Q$ models, such as the fixed-order-point --fixed-order-quantity model and the optional replenishment system, combine good characteristics of both models. Finally, the ultimate computerized model was developed by Wagner (115), the stochastic dynamic continuous review model.

## RESEARCH DESIGN AND METHODOLOGY

The objective of this chapter is to explain the methodology and design of the research conducted to provide evidence supporting, or not supporting, the research hypothesis. As outlined in Chapter $I$, one of the specific objectives of the study was to discover if repair parts sales by the selected firms are seasonal, as they have been in the past, or if they have changed due to the dramatic changes that have taken place in the industry.

Further objectives were to examine the relevant costs needed in making inventory policy decisions, to compare the retail parts sales of the firms participating in the study, and to determine if the firm had attempted to minimize total inventory costs.

An explanation of the research design and methodology used in trying to attain the above objectives is presented in the balance of this chapter.

The Research Design

Three retail farm equipment dealers from the states of Oklahoma, Nebraska, and Missouri were selected for the study. They are all John Deere farm implement dealers because of the willingness of Deere and Company and each dealer to participate in the study. The three dealers were selected for the following two reasons: first, each dealer
receives his parts stock from a different parts depot, and second, they are all using the John Deere "D-Parts" system.

The D-Parts system is a computerized inventory control system that maintains records on computer tape by direct communication from the dealer to Deere and Company in Moline, Illinois. This information is sent to Deere and Company at the close of each day's operations, where perpetual inventory records are maintained. At the end of each week, the dealers receive a weekly analysis.

The dealers chosen have different size operations with inventories of $9,861,12,574$, and 12,818 items in stock. Annual dollar sales range from $\$ 236,770$ to $\$ 441,692$. It was felt that a better sample could be tested by choosing dealers from different geographical locations and with different size operations. Potential differences can exist in the items carried in inventory because of the different growing seasons. For this reason, the geographic location is considered important. Both the number of parts carried in inventory and the volume of parts sold can be affected by the size of the dealer's operation. Care was taken to select dealers with no capital limitations which might affect the inventory leve1.

Repair parts sales for the chosen firms were examined to see if the sales of repair parts are seasonal in nature. It was determined that they are seasonal and seasonality was removed in order to examine trend.

An ABC analysis of the data of each dealer for the fiscal year April 1, 1976, through May 31, 1977, was conducted. The two-fold purpose of this analysis was to: (1) separate the data into three categories, so that inventory policies can be recommended, and (2)
statistically sample the data from the "A" group to compare costs under the inventory system now in use and that proposed by the study.

The final design was to compute the total inventory costs for the fiscal year, April 1, 1976, through March 31, 1977, under the inventory system presently being used by each dealer and compare this cost with the system recommended by the study. The hypothesis is that the proposed system will reduce total inventory costs without a significant increase in the number of stock-out occurrences.

Methodology

To attain the objectives previously stated, the methodology used is outlined in the following section.

The first step was to gather and graph the monthly parts sales data, for a five-year period from 1971 through 1975, for each of the three selected dealers, to determine if sales are seasonal. It was anticipated that sales would be seasonal and after making this determination, seasonality was removed by the generally accepted ratio-tomoving average technique (81). After the removal of seasonality, the data was graphed to study trend, cyclical, and irregular patterns. A percentage of increase in sales, for each dealer for each year, comparison was made.

As previously noted, the inventory policy selected often depends upon the characteristics of the inventory items. The well known $A B C$ classification of inventory was useful in dividing inventory into "A;" "B," and "C" categories. As shown by Plossl and Wight (92) 15 to 20 percent of the items represent 75 to 80 percent of the total value of the inventory. These items are classified as "A" items and are
considered high value items. "B" items are medium value items and usually comprise between 30 to 40 percent of the items and 15 percent of the total inventory value. The low value items are "C" items, which 40 to 50 percent of the total items account for only 5 to 10 percent of the inventory value. It was anticipated that the data would fall in these ranges, and from this analysis, inventory control policies were suggested.

After the inventory was divided into "A," "B," and "C" categories, the "A" group was statistically sampled for use in the analysis comparing the total inventory costs of each dealer, using their present inventory control system, with the system proposed by this study.

## Sample Selection Procedure

Using a random number table, a sample of 50 items was chosen from the total population of " $A$ " items for each of the three dealers. The number of "A" items ranged from 1293 to 1894 and accounted for approximately 90 percent of dollar sales. Only the " A " items were sampled because they comprise the largest part of the dollar sales, and therefore, the " $B$ " and " $C$ " items were omitted.

## Test Procedure

The sample items were analyzed to determine the costs incurred for those items under each dealer's present inventory system. Carrying costs were computed by averaging the number of items on hand at the beginning and end of the month. Ordering costs were simply the number of times an order was placed multiplied by the cost per order. Carrying and ordering costs used were 24.5 percent and $\$ .50$ respectively.

These costs were established by a study made by Deere and Company, and while they may be subject to argument, they were used consistently under both systems. Total inventory cost was then computed for the sample by adding carrying and ordering costs.

Under the system proposed by this study, sales were examined for two years for the sample items. The first year's sales were used to predict sales for the second year. From this prediction, a reorder point was computed using predicted sales plus safety stock. The amount on hand was then determined by deducting actual sales in order to determine what would have happened using the proposed system.

The proposed system used a period of six weeks to predict sales for the second year even though lead-time is actually one week. The purpose for this was two-fold: (1) only one-year's data were available (as more becomes available it should be added), and (2) as it was determined that sales are highly seasonal, the seasons starting point varies.

Safety stock was determined by computing the standard deviation of demand from the average of the six-weeks sales. Sales may approximate a normal, Poisson, or negative exponential distribution. Well-Known statistical techniques were used to determine the amount of safety stock to add to the average demand during the lead-time in establishing the reorder point. For example, if the distribution is normal, two standard deviations provide a confidence level of 97.5 percent. Safety stock, under the system now in use ( $D$-Parts system) is set as a percentage (for example, 10 percent) of demand during lead-time. The proposed safety stock technique proved far superior to this popular "rule-ofthumb" technique (93).

An arbitrarily chosen service, or confidence level, of 97.5 percent was used in this study. In other words, when a customer calls for a certain item it can be supplied to him 97.5 percent of the time. Although a 97.5 percent level of confidence was used in this study, this does not mean that a 97.5 percent level of confidence will always be chosen. Each dealer should choose the level of customer service that he feels is satisfactory for his particular circumstances. While it is obviously impractical to maintain a 100 percent service level, because the cost would be prohibitive, good judgment indicates that anything below 80 percent could result in extreme customer dissatisfaction. Therefore, a customer service level somewhere between 80 and 100 percent should be used depending upon the dealer's environment.

## Hypothesis

The difference between the two cost distributions was tested by the Wilcoxon matched pair signed rank test. This is a nonparametric test of significance developed by Frank Wilcoxon which requires that the data be, at least, ordinally scaled and the two samples be related. The Wilcoxon test was chosen because the data are related in that costs were computed using two different methods for the same inventory items. Since costs were computed for each item using the D-Parts system and the proposed system, each inventory item represents a matched pair of data. The Hypothesis tested is stated as follows:

Ho: There is not significant difference between the inventory costs of the D-Parts system and the proposed system.

Since the proposed system produced a significantly lower cost than the $D$-Parts approach for the 50 sampled items, the difference in the
costs for the entire inventory of " $A$ " items was estimated using ratio estimation. A ratio between the two costs was developed by the following formula:

$$
R=\frac{\text { Proposed System Cost }}{\text { D-Parts System Cost }}
$$

The total cost differential was estimated by applying the ratio to the total cost for the D-Parts system. Once this estimate was derived, a confidence interval was computed at the 95 percent level of confidence as follows:

$$
\begin{aligned}
& \text { Estimate } \pm 2 \text { standard error of the estimate when, } \\
& \text { Standard Error of }=N \sqrt{\frac{\sum Y^{2}+R^{2} \Sigma X^{2}-2 R \sum X Y}{n(n-1)}} \sqrt{1-\frac{n}{N}}
\end{aligned}
$$

The 95 percent level was chosen because statistitions typically use this level when evaluating inventory systems.

## Measurement of Stockrout

The occurrence measure of stock-out was used because, in most cases, a backorder situảtion exists and not a lost sales problem. This does not mean that a customer will never go to another John Deere dealer to purchase the part in an emergency.

No distinction among backorders, either on the basis of the number of units short or the length of time the stock-out exists, is made by the occurrence measurement. A single shortage exists as soon as demand for an item exceeds the stock on hand. This measure is considered appropriate when shortage cost is proportional to its occurrence and not to the number of units short. Also, for this measurement to be appropriate, special actions must be taken to promptly replenish the inventory.

The occurrence of stock-out under the D-Parts system of inventory control now in use was compared to the number of stockouts under the inventory control system proposed by this study.

## ANALYSIS AND INTERPRETATION OF THE DATA

This chapter presents the analysis and interpretation of the research conducted according to the design and methodology described in Chapter IV.

In the first part of the chapter, the sales data for each of the three implement dealers selected for the study are examined to determine if sales are seasonal. It was determined that sales are seasonal; seasonality was removed and sales examined for trend, cyclical, and irregular patterns. This was followed by a comparison of growth in dollar volume repair parts sales experienced by each of the three implement dealers.

The ABC analysis performed on the data enabled a sample to be taken from the "A" group. The sample items were then used to make a comparison of inventory costs for each dealer between the system the dealers are presently using and the system proposed by this study.
Analysis of Seasonality

Sales of the farm equipment repair parts departments for each of the three dealers participating in the study were gathered for the years 1971 through 1975 on a monthly basis. To guarantee anonymity, the three dealers were designated as "D," "G," and "Y."


Figure 9. 1971-1975 Retail Parts Sales for Dealer D


Figure 10. 1971-1975 Retail Parts Sales for Dealer G


Figure 11. 1971-1975 Retail Parts Sales for Dealer Y

Figures 9, 10, and 11 are graphs of the dollar sales of the monthly sales for each dealer. All three graphs reveal a seasonal pattern of sales.

First, in Figure 9, company "D" shows sales in January to be normally the lowest of the year, followed by a slight increase in February. March brings the first large increase to one of the two high points of the year. April normally has a moderate increase followed by a significant decline for May. June remains approximately steady with July showing a signigicant decline. A slight increase is shown for August, followed by the most significant rise in September, to the point of highest sales in October. November takes a tremendous plunge followed by another slight decline in December. Thus, it appears that dealer "D" has two seasons that normally reach their peak in April and May for the first peak and in September and October for the second.

Seasonal patterns for dealers "G" and "Y," as shown in Figures 10 and 11 , respectively, are similar to those of dealer "D." There are two high points, or dual seasonality, with some variation between dealers because of their different geographical locations. The first high for dealer "G" occurs in May and June; the second peak is in October. Dealer " Y " has a first high in May and June; while the second is in September.

It is therefore determined that the seasonal pattern of demand for the three dealers used in this study provides some evidence that sales of repair parts for retail farm equipment dealers is seasonal. However, generalizations, that all dealers have seasonal sales patterns, cannot be assumed from the results of the analysis of the three dealers in
this study. The seasonal results for the three dealers should, however, be given careful consideration when formulating an inventory control system.

The Sales Trend for Repair Parts

The purpose in determining the seasonal index is to remove the seasonal effect on sales so that trend may be examined without the seasonal influence. The ratio-to-moving average technique was used in this study to compute the seasonal index. Appendix A contains the . computer printout of seasonal index computations for each of the dealers selected for the study. A comparison of the three dealer's sales after the removal of the seasonal effects is shown in Figure 12 . From this comparison, it is apparent that retail sales of parts show an increasing trend. The average annual rate of growth for the three dealers was 19.6 percent for dealer "G," 22.3 percent for dealer "Y," and 23.7 percent for dealer "D."

## ABC Analysis

Appendix B contains analysis of the sales data for each dealer. All three dealers are very similar in that approximately 16 percent of the items represent approximately 90 percent of the dollar sales; while "B" and "C" items comprise the other 10 percent of dollar sales. The results of this analysis were very different from those suggested in the literature. Typically, 15 to 20 percent of the items encompass 75 to 80 percent of sales. The cut-off point of the " $A$ " items was made near the bottom of this range because a higher percentage would
significantly reduce the " B " and " C " categories. If a higher percentage of the items had been used, the percentage of sales would have exceeded 95 percent.

TABLE XI
COMPARISON OF "A" ITEMS FOR THE
THREE DEALERS

|  | Total <br> Number <br> of Items |  |  |  | Number of <br> in Stock |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Total Parts <br> Sales in $\$$ <br> "A" Items | Items <br> as a of of <br> Total | Sales as <br> a \% of <br> Total Sales |  |  |  |
| Dealer G | $236,769.88$ | 9,861 | 1,293 | 16.9 | 89.4 |
| Dealer Y | $331,203.25$ | 12,574 | 1,519 | 15.2 | 90.1 |
| Dealer D | $441,691.33$ | 12,818 | 1,894 | 17.5 | 88.6 |

TABLE XII
COMPARISON OF INVENTORY COSTS FOR FIFTY SAMPLE ITEMS BETWEEN PRESENT D-PARTS SYSTEM AND SYSTEM PROPOSED BY THIS STUDY

|  | Dealer "D" | Dealer "G" | Dealer " Y " |
| :--- | :---: | :---: | :---: |
| Cost Under System Now <br> in Use | $\$ 1,288.76$ | $\$ 932.47$ | $\$ 1,047.68$ |
| Number of Stock-outs | 42 | 31 | 18 |
|  |  |  |  |
| Cost under Proposed System | $\$ 989.11$ | $\$ 550.98$ | $\$ 683.69$ |
| Number of Stock-outs | 20 | 38 | 65 |
| Difference in \$'s | $\$ 299.65$ | $\$ 381.49$ | $\$ 363.99$ |
| Savings Percentage | 23.3 | 40.9 | 34.7 |



Figure 12. Comparison of the Three Dealers' Sales After Removal of Seasonal Effects

Cost Comparison of the Two Systems

The actual sales, orders, and receipts data were used for a oneyear period in computing the ordering and carrying costs for the system now in use. The proposed system determined the quantities and timing of orders in the following manner.

Sales for the previous year were used in determining the reorder point for each item. To estimate the amount needed during the week of lead-time, sales for a siz-week period of the previous year were averaged. As noted in Chapter IV, the reason for using a six-week period to predict demand was two-fold: (1) the amount of past data was limited to one year, and (2) demand is seasonal, and the reason may vary slightly from one year to another. The next step in determining the reorder point was to add the amount of safety stock to the average sales computed for the six-week period. This constantly changing, or dynamic, reorder point allows the inventory level to reach zero during periods in which little or no sales occur. This is particularly important when sales are highly seasonal, as was determined for the three dealers in this study. The D-Parts system, using a fixed reorder point, sometimes causes inventory items to be carried in stock, for some items at an extremely high inventory level, during periods of slack demand or in which no sales occur. The dynamic reorder point of the proposed system alleviates the problem.

Safety stock was based on a 97.5 percent level which was achieved by adding two standard deviations to the average weekly sales. As pointed out in Chapter IV, this provides a 97.5 percent service level if sales have a normal distribution. The majority of the items
examined in the study had a normal distribution. If a different service level than 97.5 percent is preferred, it can be achieved by adding one standard deviation for a 84.1 percent service level, or by adding three standard deviations, a service level of 99.9 percent could be maintained.

After determining the reorder point, actual sales were deducted from the beginning inventory of each item and when the inventory on hand fell to the reorder point, an order was placed for the economic order quantity computed for that item. One week later (the period of lead-time) the order was received, the quantity recorded and added to the inventory on hand. The computer program used to determine orders and receipts under the proposed system is shown in Appendix C.

The economic order quantity model used was the basic EOQ formula shown below:

$$
\mathrm{EOQ}=\sqrt{\frac{2 \times \mathrm{AD} \times \mathrm{CO}}{\mathrm{CU} \times \mathrm{CC}}}
$$

where: $A D=A n n u a l$ Unit Demand, CO = Cost Per Order (\$.50), CU $=$ Cost Per Unit, CC $=$ Carrying Cost Percentage ( $24.5 \%$ ).

The ordering and carrying costs for 50 items selected from the "A" items were computed for the D-Parts system (the system presently being used by the three dealers selected for this study) and for the system proposed by this study. Annual ordering costs were computed by multiplying the number of orders placed during the year by $\$ .50$.

Annual carrying costs were computed as follows: (1) the beginning inventory as of April 1, 1976, was recorded; (2) the number of items received during the first week were added to the beginning inventory,
and the number of items sold during the same week were subtracted, giving the ending inventory at the end of that week; (3) at the end of each four-week period, the beginning inventory for the period was averaged with the ending inventory of the period; and (4) this average inventory was used to compute the carrying costs.

These annual carrying costs can also be expressed by the formula:

$$
\text { Annual Carrying Costs }=\sum_{i=1}^{\mathrm{n}=12}\left[\left(\frac{\mathrm{BIi}+\mathrm{EIi}}{2}\right)(\mathrm{IC})(.245)(1 / 12)\right]
$$

where:
BIi $=$ Beginning inventory for month i;
ELi = Ending inventory for month i;
IC $=$ Item cost.
The $\$ .50$ per order and 24.5 percent carrying cost charge are those developed by Deere and Company. Since these values are used by both systems, the results were comparable. A comparison for the three dealers for both systems is shown in Table XII.

The savings for dealer "D" is $\$ 299.65$, or 23.3 percent, with a reduction in the number of stock-outs from 42 to 20. Dealer " $G$ " shows a savings of $\$ 381.49$, or 40.9 percent, and an increase in stock-outs from 31 to 38 . Dealer " Y " had a savings of $\$ 363.99$, or 34.7 percent, with an increase in stock-outs from 18 to 65 . These savings were significant at the . 01 level, and therefore, the hypothesis stated below can be rejected at the . 01 level

Ho: There is no significant difference between the inventory costs of the D-Parts system and the proposed system.

Appendix $D$ is a comparison of the total costs, including ordering and carrying cost, for the 50 sampled items for dealer "G" and illustrates the method used to arrive at these cost for all three dealers. These costs savings were achieved with no significant increase in stock-outs. The total number of stock-outs for all three dealers under the D-Parts system was 91 , and 123 for the proposed system. Two dealers had a slight increase, while one dealer had a slight decrease. The number of stock-outs under the proposed system is compatible with the number that should occur with a service level of 97.5 percent. The total number of possible stock-outs is 7,200 (50 items $\times 48$ weeks $\times 3$ dealers). Therefore, the number of stock-outs that should be expected to occur is 180 ( $7,200 \times 2.5 \%$ ). The 123 stock-outs that occurred under the proposed system is below the 180 expected to occur with a 97.5 percent service level. Although these savings are significant, actual cost savings using the proposed system would be greater than that shown in Table XII.

Actual beginning inventory was used to determine costs under both the present system in use (D-Parts system) and the proposed system. Approximately 80 percent of the sample group of 50 items had a beginning inventory that was higher than the beginning inventory would have been had the proposed system been in use the prior year. This beginning inventory was so high in approximately 20 percent of the items that no orders were placed during the year, and costs were the same for both systems. As a result, savings were possible in only 80 percent, or 40 out of the 50 items.

Proposed System vs., the D-Parts System

Using the ratio estimation, the sample costs were used to project costs under the two systems for all "A" items. Once the estimated cost of the proposed system was determined, the reliability of the estimate was shown by establishing a confidence interval.

The actual annual cost for all " A " items under the D-Parts system was not available. Therefore, an estimate of the total cost was computed by projecting the cost for the 50 sampled parts over the entire population of " A " items as follows:

D-Parts Annual Cost $=\left(\frac{\$ \text { Cost for } 50 \text { parts }}{50}\right)($ Total $\#$ "A" item parts). The estimated annual cost for the proposed system was calculated by multiplying the D-Parts annual cost times the ratio of the cost of the sampled parts for the proposed system to the cost of the sampled parts for the D-Parts system as shown below:

Estimated Annual Cost of Proposed System $=$
( $\left.\frac{\$ \text { Cost of Proposed System }}{\$ \text { Cost of D-Parts System }}\right)($ D-Parts Annual Cost).
Once the estimated annual cost of the proposed system was developed, a 95 percent confidence interval was established as follows:

Estimated Annual Cost of Proposed System $\pm$ (Standard Error of the Estimate ) (1.96)
where:

$$
\begin{aligned}
& \text { Standard Error of } \\
& \text { Estimate }
\end{aligned}=N \sqrt{\frac{\sum Y^{2}+R^{2} \Sigma X-2 R \Sigma X Y}{n(n-1)}} \sqrt{1-\frac{n}{N}}
$$

As shown in Table XIII, the costs for dealer " D " under the D-Parts system are estimated to be $\$ 48,818.23$, and under the proposed system $\$ 37,467.99$ using the computed ratio of .7675 . At the 95 percent level of confidence, the $\$ 37,467.99$ cost could be either plus or minus
$\$ 3,099.64$. Therefore, one can be 95 percent confident that the cost of the proposed system would fall between $\$ 34,368.35$ and $\$ 40,567.63$.

TABLE XIII

COST COMPARISON FOR ENTIRE INVENTORY OF "A" ITEMS

| Dealer | No. of Parts | Cost Using <br> D-Parts System | Cost Using <br> Proposed System |
| :---: | :---: | :---: | :---: |
| G | 1,293 | $24,113.67$ | $41,248.77$ |
| Y | 1,519 | $31,828.52$ | $20,771.29$ |
| D | 1,894 | $48,818.23$ | $37,467.99$ |

For dealer "G," the estimated costs of the D-Parts system are $\$ 24,113.67$, and the proposed system's costs would be $\$ 14,248.77$ with a ratio of .5909. Again, using a 95 percent confidence level, the actual cost could range from $\$ 11,847.47$ to $\$ 16,650.07$.

The last dealer, "Y," has an estimated cost of $\$ 31,828.52$ using the D-Parts system and a ratio of .6526 determines costs under the proposed system to be $\$ 20,771.29$. A 95 percent confidence interval ranges from $\$ 16,378.49$ to $\$ 25,164.09$.

These cost savings were achieved by reducing inventories using a dynamic, ever-changing reorder point, which reduced the inventory carrying costs. The seasonality of sales makes it evident that a static reorder point will cause inventory to be carried in stock during
period of low sales, and in many instances, no sales are made for extensive periods of time. The only slightly negative point of the proposed system is the increase in stock-outs, but as shown previously, stock-outs were below the limit consistant with the 97.5 percent level used in the study.

## Summary

Repair parts sales for all three dealers were found to have a seasonal pattern. The ratio-to-moving average method was used to compute a seasonal index to remove the seasonal influence. The removal of seasonality revealed similar sales patterns for all three dealers.

The ABC analysis revealed an unusual situation; a small percentage of the items, 16 percent, accounted for approximately 90 percent of the dollar sales. Typically, 15 to 20 percent of the items represented 75 to 80 percent of dollar sales. This unusual situation allows the dealer to maintain tight control of only a small percentage of the items.

Significant cost savings were found using the proposed system for all three dealers. The difference was found to be significant at the . 01 level for each dealer. Since these costs were based on a sample of 50 parts, ratio estimation was used to project savings over all of the "A" items.

## CHAPTER VI

## SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

## Summary

A study was made of the repair parts inventory system being used by three retail farm machinery dealers, from widely different geographical locations. The study focused on an examination of the inventory system presently in use and compares the total inventory costs of that system with the costs that would have been incurred if the system proposed by this study had been used.

Sales of each of the three dealers were examined for a five-year period to determine if they were seasonal. It was found that sales were seasonal, and therefore, seasonality was removed so that sales could be examined for trend, cyclical, and irregular patterns without seasonal interference。

The next step was to perform an $A B C$ analysis on the data, in an effort to suggest inventory control techniques for " A, " " B, " and " C " classified items. Then, from a random sample of " A " items, a comparison was made between the inventory system in use and the system proposed by the study. The research hypothesis formulated concerned the difference in the ordering and carrying costs of the two systems. Out-of-stock occurrences were also compared for the two systems.

## Conclusions

The analysis of repair part sales for the five-year period, 1971 through 1975, revealed strong seasonal patterns for all three dealers. Although the geographical locations of the three dealers were widely spread, two seasons were evident for all the dealers, but not necessarily occurring simultaneously. The strong seasonal pattern has a significant effect on the inventory control system that should be used. For example, a number of the inventory items examined are sold only during a four-month period. Obviously, care should be exercised so that as few of these items as possible are carried in stock during the other eight months of the year.

After the removal of seasonality, a significant upward trend in sales results (see Figure 12, page 103) with all three dealers showing similar patterns. This points out the need for more emphasis on inventory control, inasmuch as, when sales rise inventories also increase.

The ABC analysis reveals the important information that approximately only 16 percent of the inventory items account for approximately 90 percent of total sales. If, as in the proposed system, the inventory items are divided into "A," "B," and "C" classifications, then the " $A$ " group, or the items representing 90 percent of total sales, can be more tightly controlled, while maintaining very little control on both the " $B$ " and " $C$ " groups. This is significantly different from most inventories as shown by examples of studies in Chapter III, where 15 to 20 percent of the items account for approximately 75 to 80 percent of the value. Under the circumstances found in most inventories discussed in the literature, it was difficult to decide what kind of controls to
use on the " $B$ " items. Because of the unusual results of the $A B C$ analysis, which placed 90 percent of the items into Group "A," this problem may be eliminated and tight control maintained on only the " A " items.

A significant cost savings of the proposed system over the present system being used (D-Parts system) was revealed when a comparison of the two systems was made for all three dealers. The cost savings range from \$9,864.90 for dealer "G," \$11,057.23 for dealer "Y," to a high of $\$ 11,350.24$ for dealer "D." While these costs savings were found to be significant at the . 01 level of confidence, it should be noted that the beginning inventory of most of the items was too large, and that no cost savings were made on approximately 20 percent of the items, because there were no orders or receipts, due to the high amount on hand at the beginning inventory date.

The measurement of stock-out used to compare the two systems should be satisfactory for the purpose for which it was intended, but a more meaningful measure would be to record actual stock-outs; e.g., when a customer calls for the part, the dealer does not have it in stock.

Two primary differences in the two systems are: (1) the reorder point is automatically changed under the proposed system as opposed to a fixed reorder point under the D-Parts system, and (2) a safety stock is established by statistically setting a customer service level under the proposed system as opposed to a percentage being added to the reorder point under the present system.

The above mentioned differences in the two systems is the reason for the significant cost savings when using the proposed system. This savings was accomplished within the confidence level of 97.5 percent.

Under the system proposed by the study, stock-outs occur 2.5 percent of the time, and the number of possible stock-outs would be 2,400 per dealer, or 50 items multiplied by 48 weeks. Therefore, 2.5 percent of 2,400 is 60 stock-outs, or 180 for all three dealers. The actual number of stock-outs for all three dealers was 123 . It should again be mentioned that the stock-out measurement used was not ideal, but should be sufficient to establish that demand is being satisfied approximately 97.5 percent of the time.

## Recommendations

The seasonal analysis revealed strong seasonal patterns, and this information is valuable in pointing out that conventional models of inventory control, in which demand is relatively constant, will not fit this seasonal behavior. The proposed system observes past sales history and by constantly changing the reorder point, maintains inventory levels only during those periods when demand occurs. To establish the reorder point, this study used a six-week period to compute average weekly sales and then added safety stock to that average. A six-week period was used because previous sales data were available for only one year. As more data becomes available, the six-week period could be shortened to four weeks. The reason for having more weeks than the lead-time is because the seasons can vary from year-to-year.

The unusual results of the ABC analysis presents an opportunity to maintain tight inventory control over only 16 percent of the items, and thereby, control those items that account for approximately 90 percent of the sales. Controlling the "B" and "C" items will result in very little savings, inasmuch as together they account for only
approximately 10 percent of sales. The cost of maintaining items on the D-Parts system could be more expensive than beneficial. For example, the cost to record the sale of a paper gasket, with a cost of five cents, is four cents. These items could be controlled by using one of the simplest methods: the two-bin system.

This study added two standard deviations to the average demand for a safety stock that would provide a service level of 97.5 percent. This statistical method of computing safety stock is considered to be superior to that of adding a percentage to average demand and will allow the dealer to provide the service level that he considers appropriate for his particular circumstances.

Since the conclusions reached in this research are based on a limited study of three dealerships, additional research is needed to confirm or refute these findings. In addition, further research is recommended in the selection of the proper economic order quantity model. It is possible that a different economic order quantity model may produce an even greater cost savings for the retail farm equipment dealer.
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APPENDIXES

APPENDIX A

SEASONAL ANALYSES

## EXHIBIT I

SEASONAL ANALYSIS FOR DEALER D

| YEAR | MONTH | DATA | CENTERED MOVING AVERAGE | $\begin{aligned} & \text { SPECIFIC } \\ & \text { SEASONALS } \end{aligned}$ | $\begin{aligned} & \hline \text { SEASONAL } \\ & \text { INDEX } \end{aligned}$ | SEASONALLY ADJUSTED VALUE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1971 | 1 | 9784. |  |  | 47.70 | 20513. |
|  | 2 | 10831. |  |  | 65.53 | 16529. |
|  | 3 | 22275. |  |  | 107.01 | 20816. |
|  | 4 | 31367. |  |  | 121.17 | 25887. |
|  | 5 | 17658. |  |  | 123.07 | 14348. |
|  | 6 | 23518. |  |  | 124.46 | 18896. |
|  | 7 | 18700. | 20998. | 89.06 | 93.75 | 19947. |
|  | 8 | 20755. | 21133. | 98.21 | 95.21 | 21798. |
|  | 9 | 28133. | 21527. | 130.69 | 122.44 | 22976. |
|  | 10 | 28836. | 21648. | 133.44 | 132.63 | 21780. |
|  | 11 | 20654. | 21979. | 93.97 | 87.25 | 23673. |
|  | 12 | 18880 . | 22701. | 83.17 | 79.79 | 23662. |
| 1972 | 1 | 10858. | 22834. | 47.55 | 47.70 | 22765. |
|  | 2 | 13004. | 22817. | 56.99 | 65.53 | 19845. |
|  | 3 | 29540. | 22953. | 128.70 | 107.01 | 27605. |
|  | 4 | 27006. | 23127. | 116.77 | 121.17 | 22288. |
|  | 5 | 29979. | 23198. | 129.23 | 123.07 | 24359. |
|  | 6 | 28516. | 23240 。 | 122.70 | 124.46 | 22911. |
|  | 7 | 16905. | 23372 . | 72.33 | 93.75 | 18033. |
|  | 8 | 22130. | 23698. | 93.38 | 95.21 | 23243. |
|  | 9 | 30013. | 23702. | 126.63 | 122.44 | 24512 . |


|  | 10 | 31201. | 23431. | 153.16 | 132.63 | 23525. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 11 | 20029. | 23825. | 84.07 | 87.25 | 22957. |
|  | 12 | 20513. | 24675. | 83.13 | 79.79 | 25708. |
| 1973 | 1 | 12408. | 25623. | 48.42 | 47.70 | 26015. |
|  | 2 | 19266 | 26491 | 72.73 | 65.53 | 29401. |
|  | 3 | 23380. | 26879. | 86.98 | 107.01 | 21849. |
|  | 4 | 26669. | 27425. | 97.24 | 121.17 | 22010. |
|  | 5 | 39772 ． | 28298. | 140.55 | 123.07 | 32317 。 |
|  | 6 | 39103. | 28886. | 135.37 | 124.46 | 31417 ． |
|  | 7 | 29087. | 29208. | 99.59 | 93.75 | 31027 ． |
|  | 8 | 30772 ． | 29671. | 103.71 | 95.21 | 32319 ． |
|  | 9 | 30683. | 30920 。 | 99.23 | 122.44 | 25059 ． |
|  | 10 | 43643. | 32706 。 | 133.44 | 132.63 | 32907 ． |
|  | 11 | 28538. | 33615 ． | 84.90 | 87.25 | 32710 ． |
|  | 12 | 26109. | 33721. | 77.43 | 79.79 | 32721. |
| 1974 | 1 | 14534. | 34371. | 42.29 | 47.70 | 30472 ． |
|  | 2 | 28252 。 | 35219 。 | 80.22 | 65.53 | 43114. |
|  | 3 | 44384. | 36305. | 122.25 | 107.01 | 41477. |
|  | 4 | 48515. | 37975. | 127.76 | 121.17 | 40040 。 |
|  | 5 | 39738. | 39295. | 101.13 | 123.07 | 32289 ． |
|  | 6 | 41680. | 39927. | 104．39 | 124.46 | 33488 。 |
|  | 7 | 42130. | 40801. | 103.26 | 93.75 | 44940 。 |
|  | 8 | 38067 。 | 41412 。 | 91.92 | 95.21 | 39981. |
|  | 9 | 49459. | 41298. | 119.76 | 122.44 | 40394. |
|  | 10 | 64944. | 41733. | 155.62 | 132.63 | 48968. |
|  | 11 | 38907. | 42914. | 90.66 | 87.25 | 44595. |
|  | 12 | 30924. | 44388 。 | 69.67 | 79.79 | 38756 。 |


| 1975 | 1 | 30678. | 45113. | 63.00 | 47.70 | 64320. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 26791. | 45307 。 | 59.13 | 65.53 | 40885. |
|  | 3 | 43106. | 46312 。 | 93.08 | 107.01 | 40283. |
|  | 4 | 60240。 | 47415 。 | 127.05 | 121.17 | 49716. |
|  | 5 | 56344. | 47582. | 118.41 | 123.07 | 45782. |
|  | 6 | 60460 。 | 47328. | 127.75 | 124.46 | 48577 。 |
|  | 7 | 40736. |  |  | 93.75 | 43453. |
|  | 8 | 44117 。 |  |  | 95.21 | 46335. |
|  | 9 | 67541. |  |  | 122.44 | 55161. |
|  | 10 | 73316. |  |  | 132.63 | 55280 。 |
|  | 11 | 34552 。 |  |  | 87.25 | 39603. |
|  | 12 | 29175. |  |  | 79.79 | 36564 。 |

## EXHIBIT II

SEASONAL ANALYSIS FOR DEALER G

| $\overline{\text { YEAR }}$ | MONTH | DATA | $\begin{aligned} & \text { CENTERED } \\ & \text { MOVING } \\ & \text { AVERAGE } \end{aligned}$ | $\begin{aligned} & \hline \text { SPECIFIC } \\ & \text { SEASONALS } \end{aligned}$ | SEASONAL INDEX | SEASONALLY <br> ADJUSTED VALUE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1971 | 1 | 4782. |  |  | 65.00 | 7347. |
|  | 2 | 7761. |  |  | 61.92 | 12534. |
|  | 3 | 12178. |  |  | 110.73 | 10998. |
|  | 4 | 10415. |  |  | 116.79 | 8917. |
|  | 5 | 10627. |  |  | 132.33 | 8000. |
|  | 6 | 14319. |  |  | 126.93 | 11281. |
|  | 7 | 8736. | 11337. | 77.06 | 78.12 | 11182. |
|  | 8 | 9477 . | 11376. | 83.31 | 88.07 | 10760. |
|  | 9 | 11280. | 11273. | 100.06 | 117.73 | 9582. |
|  | 10 | 16424. | 11462 . | 143.29 | 139.90 | 11740 . |
|  | 11 | 12818. | 11819. | 108.45 | 83.93 | 15272. |
| $\because$ | 12 | 15984. | 11986. | 133.35 | 77.96 | 20504. |
| 1972 | 1 | 7269. | 12007. | 60.54 | 65.09 | 11168. |
|  | 2 | 6199. | 12027. | 51.54 | 61.92 | 10011. |
|  | 3 | 11284. | 12150. | 92.87 | 110.73 | 10191. |
|  | 4 | 15848. | 12149. | 130.44 | 116.79 | 13569. |
|  | 5 | 13763. | 11877. | 115.88 | 132.83 | 10361. |
|  | 6 | 15186. | 11398. | 133.24 | 126.93 | 11964. |
|  | 7 | 8375. | 11057. | 75.74 | 78.12 | 10720. |
|  | 8 | 10319. | 11058. | 93.31 | 88.07 | 11716. |
|  | 9 | 13388. | 11209. | 119.44 | 117.73 | 11372. |


|  | 10 | 14921. | 11221. | 127.36 | 139.90 | 10215. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 11 | 8410. | 11361. | 74.03 | 83.93 | 10020． |
|  | 12 | 8892. | 11686 。 | 76.09 | 77.96 | 11406 。 |
| 1973 | 1 | 6187. | 11952. | 51.77 | 65.09 | 9505. |
|  | 2 | 7314. | 12147 ． | 60.21 | 61.92 | 11812. |
|  | 3 | 13782. | 12326. | 111.81 | 110.73 | 12447 ． |
|  | 4 | 13640. | 12741. | 107.06 | 116．79 | 11679． |
|  | 5 | 19328. | 13256. | 145.80 | 132.83 | 14551. |
|  | 6 | 17424. | 13563. | 128.46 | 126.93 | 13728. |
|  | 7 | 12520. | 13813. | 90.64 | 78.12 | 16026. |
|  | 8 | 10867. | 14137. | 76.87 | 88.07 | 12339. |
|  | 9 | 17129. | 14467 ． | 118.40 | 117.73 | 14550. |
|  | 10 | 20496. | 14931. | 137.27 | 139．90 | 14561 |
|  | 11 | 14583 | 15466 ． | 94.29 | 83.93 | 17375 。 |
|  | 12 | 10087. | 15683. | 64.32 | 77.96 | 12939. |
| 1974 | 1 | 10988. | 15699. | 69.99 | 65.09 | 16881. |
|  | 2 | 10274. | 16061. | 63.97 | 61.92 | 16592. |
|  | 3 | 18761. | 16562 。 | 113.27 | 110.73 | 16944. |
|  | 4 | 19798. | 17155． | 115.41 | 116.79 | 16951． |
|  | 5 | 26011. | 17434. | 149.20 | 132.83 | 19582. |
|  | 6 | 15947. | 17600. | 90.61 | 126.93 | 12564. |
|  | 7 | 14365. | 18043. | 79.61 | 78.12 | 18387 。 |
|  | 8 | 17719。 | 18479 。 | 95.89 | 88.07 | 20118。 |
|  | 9 | 22311. | 18957. | 117.69 | 117.73 | 18952. |
|  | 10 | 29540. | 19469. | 151.73 | 139.90 | 21115. |
|  | 11 | 12236： | 19832． | 61.70 | 83.93 | 14579. |
|  | 12 | 16421： | 20464 | 80.25 | 77.96 | 21064. |


| 1975 | 1 | 15288. | 21240. | 71.98 | 65.09 | 23488. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 16432. | 21549. | 76.25 | 61.92 | 26537. |
|  | 3 | 24081. | 21844. | 110.24 | 110.73 | 21748. |
|  | 4 | 26747 ． | 22511. | 118.82 | 116.79 | 22901。 |
|  | 5 | 27781. | 23039. | 120.58 | 132.83 | 20914. |
|  | 6 | 29336. | 23269. | 126.07 | 126.93 | 23113. |
|  | 7 | 19612. |  |  | 78.12 | 25104. |
|  | 8 | 19894. |  |  | 88.07 | 22588. |
|  | 9 | 27210. |  |  | 117.73 | 23113. |
|  | 10 | 40659. |  |  | 139.90 | 29063. |
|  | 11 | 13769. |  |  | 83.93 | 16405. |
|  | 12 | 20411. |  |  | 77.96 | 26182. |

## EXHIBIT III

## SEASONAL ANALYSIS FOR DEALER Y

| YEAR | MONTH | DATA | CENTERED MOVING AVERAGE | $\begin{aligned} & \hline \text { SPECIFIC } \\ & \text { SEASONALS } \end{aligned}$ | $\begin{aligned} & \text { SEASONAL } \\ & \text { INDEX } \end{aligned}$ | SEASONALLY ADJUSTED VALUE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1971 | 1 | 8907. |  |  | 53.08 | 16780. |
|  | 2 | 7001. |  |  | 73.28 | 9554. |
|  | 3 | 6116. |  |  | 83.28 | 7344. |
|  | 4 | 6574. |  |  | 98.80 | 6654. |
|  | 5 | 12909. |  |  | 148.97 | 8665. |
|  | 6 | 17818. |  |  | 167.17 | 10659. |
|  | 7 | 9633. | 9826. | 98.03 | 100.32 | 9602. |
|  | 8 | 10875. | 9913. | 109.70 | 108.83 | 9993. |
|  | 9 | 14383. | 10381. | 138.56 | 117.26 | 12266. |
|  | 10 | 10940. | 10907. | 100.31 | 107.80 | 10149. |
|  | 11 | 8949. | 11602. | 77.13 | 74.57 | 12001. |
|  | 12 | 4979. | 12244. | 40.67 | 66.66 | 7470. |
| 1972 | 1 | 6571. | 12795. | 51.36 | 53.08 | 12379. |
|  | 2 | 11416. | 13536. | 84.34 | 73.28 | 15579. |
|  | 3 | 12926. | 13845. | 93.36 | 83.28 | 15522. |
|  | 4 | 12385 | 14010. | 88.40 | 98.80 | 12536. |
|  | 5 | 23785. | 14359. | 165.64 | 148.97 | 15966. |
|  | 6 | 22346 . | 14658. | 152.45 | 167.17 | 13367. |
|  | 7 | 18337. | 14844. | 123.53 | 100.32 | 18278. |
|  | 8 | 19953 | 14937. | 133.58 | 108.83 | 18335. |
|  | 9 | 12719. | 15139. | 84.01 | 117.26 | 10847 . |


|  | 10 | 16567. | 15516. | 106.77 | 107.80 | 15369. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 11 | 11702. | 15999. | 73.14 | 74.57 | 15693. |
|  | 12 | 9403. | 17029. | 55.22 | 66.66 | 14106. |
| 1973 | 1 | 6608. | 17953. | 36.81 | 53.08 | 12449 ． |
|  | 2 | 13602. | 18345. | 74.15 | 73.28 | 18562. |
|  | 3 | 15598. | 19477. | 80.08 | 83.28 | 18730 ． |
|  | 4 | 18757. | 20830． | 90.05 | 98．80 | 18985. |
|  | 5 | 29016 | 21765. | 133.32 | 148.97 | 19477. |
|  | 6 | 41836 | 22727. | 184.08 | 167.17 | 25026. |
|  | 7 | 21022. | 23642 。 | 88.92 | 100.32 | 20955. |
|  | 8 | 26660. | 24323. | 109.61 | 108.83 | 24498. |
|  | 9 | 33187. | 24993. | 132.79 | 117.26 | 28303. |
|  | 10 | 28565. | 25859. | 110.47 | 107.80 | 26499. |
|  | 11 | 22145. | 26790. | 82.66 | 74.57 | 29698. |
|  | 12 | 22062. | 27267. | 80.91 | 66.66 | 33097 。 |
| 1974 | 1 | 15892. | 27643. | 57.49 | 53.08 | 29940. |
|  | 2 | 20673 | 28115. | 73.53 | 73.28 | 28211. |
|  | 3 | 24594. | 28030. | 87.74 | 83.28 | 29533. |
|  | 4 | 30546. | 28010 。 | 109.05 | 98.80 | 30918 。 |
|  | 5 | 39587 ． | 28056. | 141.10 | 148.97 | 26573. |
|  | 6 | 42699. | 27943. | 152.81 | 167.17 | 25543. |
|  | 7 | 29178. | 28018. | 104.14 | 100．32 | 29085. |
|  | 8 | 29847. | 27693. | 107.78 | 108．83 | 27426. |
|  | 9 | 27956. | 27007 。 | 103.51 | 117．26 | 23842. |
|  | 10 | 33319. | 26933. | 123.71 | 107.80 | 30909 。 |
|  | 11 | 18487. | 27578. | 67.04 | 74.57 | 24792 。 |
|  | 12 | 23012. | 29086 。 | 79.12 | 66.66 | 34523. |


| 19.75 | 1 | 16747. | 30112. | 55.61 | 53.08 | 31550. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 12004. | 30193. | 39.76 | 73.28 | 16381． |
|  | 3 | 16814. | 30895. | 54.60 | 83.28 | 20190. |
|  | 4 | 36547 ． | 31099. | 117.52 | 98.80 | 36992 ． |
|  | 5 | 49058. | 30831. | 159.12 | 148.97 | 32931. |
|  | 6 | 69432. | 30648 ． | 226.55 | 167.17 | 41535. |
|  | 7 | 27074. |  |  | 100.32 | 26988. |
|  | 8 | 33877. |  |  | 108.83 | 31130 。 |
|  | 9 | 38392. |  |  | 117.26 | 32742 。 |
|  | 10 | 30166. |  |  | 107.80 | 27984. |
|  | 11 | 15203. |  |  | 74.57 | 20388 。 |
|  | 12 | 21904. |  |  | 66.66 | 32860 。 |

## APPENDIX B

ABC ANALYSES

## EXHIBIT I

ABC ANALYSIS FOR DEALER D

| SOURCE OF SUPPLY CODE |  | $\begin{aligned} & \text { DOLLAR } \\ & \text { SALES } \end{aligned}$ | CUMULATIVE DOLLAR <br> SALES | DOLLAR <br> SALES <br> PERCENT | CUMULATIVE DOLLAR <br> PERCENT | $\begin{aligned} & \text { ITEM } \\ & \text { COUNT } \end{aligned}$ | CUMULATIVE ITEM COUNT | ITEM COUNT PERCENT | CUMULATIVE ITEM <br> PERCENT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| OVER \$1000.00 |  | 101, 849.74 | 101, 849.74 | 23.3 | 23.3 | 59 | 59 | . 5 | . 5 |
| \$900.01 TO | \$1000.00 | 12,296.54 | 114,146.28 | 2.8 | 26.1 | 13 | 72 | . 1 | . 6 |
| \$800.01 TO | \$900.00 | 4,246.09 | 118,392.37 | 1.0 | 27.1 | 5 | 77 | . 0 | . 7 |
| \$700.01 T0 | \$800.00 | 9,538.35 | 127,930.72 | 2.2 | 29.2 | 13 | 90 | . 1 | . 8 |
| \$600.01 TO | \$700.00 | 22,669.44 | 150,600.16 | 5.2 | 34.4 | 35 | 125 | . 3 | 1.1 |
| \$500.01 TO | \$600.00 | 16,014,70 | 166,614.86 | 3.7 | 38.1 | 29 | 154 | . 3 | 1.4 |
| \$400.01 TO | \$500.00 | 23,098.94 | 189,713.80 | 5.3 | 43.4 | 52 | 206 | . 5 | 1.9 |
| \$300.01 T0 | \$400.00 | 32,810.51 | 222,524.31 | 7.5 | 50.9 | 96 | 302 | . 9 | 2.8 |
| \$200.01 T0 | \$300.00 | 37,801.79 | 260,326.10 | 8.7 | 59.5 | 157 | 459 | 1.5 | 4.2 |
| \$100.01 T0 | \$200.00 | 65,233.41 | 325,559.51 | 14.9 | 74.5 | 460 | 919 | 4.3 | 8.5 |
| \$90.01 T0 | \$100.00 | 8,211.99 | 333,771.50 | 1.9 | 76.4 | 87 | 1006 | . 8 | 9.3 |
| \$80.01 T0 | \$90.00 | 9,251.35 | 343,022.85 | 2.1 | 78.5 | 109 | 1115 | 1.0 | 10.3 |
| \$70.01 T0 | \$80.00 | 9,443.36 | 352,466.21 | 2.2 | 80.6 | 126 | 1241 | 1.2 | 11.5 |
| \$60.01 T0 | \$70.00 | 10,668.75 | 363,134.96 | 2.4 | 83.1 | 165 | 1406 | 1.5 | 13.0 |
| \$50.01 T0 | \$60.00 | 11,481. 22 | 374,616.18 | 2.6 | 85.7 | 210 | 1616 | 1.9 | 15.0 |
| _\$40.01 T0 | \$50.00 | $12,455.43$ | 387, $071 . \underline{1}$ | $\underline{2} \cdot \underline{9}$ | 88.6 | 278 | 1894 | $\underline{2} \cdot \underline{6}$ | 17.5 |
| \$30.01 T0 | \$40.00 | 12,688.17 | 399,759.78 | 2.9 | 91.5 | 367 | 2261 | 3.4 | 20.9 |

## EXHIBIT I (Continued)

| \$20.01 T0 | \$30.00 | 12,638.57 | 412,398.35 | 2.9 | 94.4 | 511 | 2772 |  | 4.7 | 25.7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \$10.01 T0 | \$20.00 | 13,742.48 | 426,140․ 83 | 3.1 | 97. 5 | 1 | 3733 | B | 8. 9 | 34.6 |
| \$ . 01 TO | \$10.00 | 10,710.33 | 436,851.16 | 2.5 | 100.0 | 2897 | 6630 | C | 26.9 | 61.5 |
| ZERO_SALES |  |  | 51. 16 |  |  |  |  | $38 . \underline{4}$ - 100. $\underline{0}$ - |  |  |
|  | TOTALS | 436,851.16 | 436,851.16 | 100.1 | 100.0 | 10771 | 10771 |  | 99.8 | 100.00 |

ABC ANALYSIS FOR DEALER G

| SOURCE OF SUPPLY CODE |  | $\begin{aligned} & \text { DOLLAR } \\ & \text { SALES } \end{aligned}$ | CUMULATIVE DOLLAR <br> SALES | $\begin{aligned} & \hline \text { DOLIAR } \\ & \text { SALES } \\ & \text { PERCENT } \\ & \hline \end{aligned}$ | CUMULATIVE DOLLAR <br> PERCENT | $\begin{aligned} & \text { ITEM } \\ & \text { COUNT } \end{aligned}$ | CUMULATIVE <br> ITEM COUNT | ITEM COUNT <br> PERCENT | $\begin{aligned} & \text { CUMULATIVE } \\ & \text { ITEM } \\ & \text { PERCENT } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| OVER \$1000.00 |  | 45,605.80 | 45,605.80 | 19.6 | 19.5 | 24 | 24 | . 3 | . 3 |
| \$900.01 TO \$ | \$1000.00 | 5,704.97 | 51,310.77 | 2.4 | 22.0 | 6 | 30 | . 1 | . 3 |
| \$800.01 T0 | \$900.00 | 7,699.27 | 59,010.04 | 3.3 | 25.3 | 9 | 39 | . 1 | . 5 |
| \$700.01 T0 | \$800.00 | 4,492.60 | 63,502.64 | 1.9 | 27.2 | 6 | 45 | . 1 | . 5 |
| \$600.01 TO | \$700.00 | 6,504.70 | 70,007.34 | 2.8 | 30.0 | 10 | 55 | . 1 | - 7 |
| \$500.01 T0 | \$600.00 | 8,138.67 | 78,146.01 | 3.5 | 33.5 | 15 | 70 | . 2 | . 9 |
| \$400.01 TO | \$500.00 | 10,625.95 | 88,771.96 | 4.6 | 38.0 | 24 | 94 | . 3 | 1.2 |
| \$300.01 T0 | \$400.00 | 15,705.76 | 104,477.72 | 6.7 | 44.8 | 45 | 139 | . 6 | 1.8 |
| \$200.01 T0 | \$300.00 | 24,598.03 | 129,075.75 | 10.5 | 55.3 | 102 | 241 A | 1.3 | 3.1 |
| \$100.01 T0 | \$200.00 | 34,364.99 | 163,440.74 | 14.7 | 70.0 | 248 | 489 | 3.3 | 6.4 |
| \$90.01 TO | \$100.00 | 5,809.85 | 169,250.59 | 2.5 | 72.5 | 61 | 550 | . 8 | 7.2 |
| \$80.01 T0 | \$90.00 | 5,722.10 | 174,972.69 | 2.5 | 75.0 | 67 | 617 | . 9 | 8.0 |
| \$70.01 T0 | \$80.00 | 6,444.64 | 181,417.33 | 2.8 | 77.7 | 86 | 703 | 1.1 | 9.2 |
| \$60.01 T0 | \$70.00 | 5,289.19 | 186,706.52 | 2.3 | 80.0 | 82 | 735 | 1.1 | 10.2 |
| \$50.01 T0 | \$60.00 | 7,353.66 | 194,060.18 | 3.2 | 83.2 | 134 | 919 | 1.8 | 12.0 |
| \$40.01 TO | \$50.00 | 6,775.79 | 200,835.97 | 2.9 | 86.1 | 151 | 1070 | 2.0 | 14.0 |
| -\$30.01 T0 | -\$40.00 | -7,749.61 | 208, $58 \underline{5} .58$ | - $3 \cdot 3$ | 89.4 - - | 223 | - -1293 - | 2.9 - | -16.9 - |

## EXHIBIT II (Continued)

| \$20.01 T0 \$30,00 | 8,030.05 | 216,615.63 | 3.4 | 92.3 | 322 | 1615 | B | 4.2 | 21.1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \$10.01 T0_\$20.00 | 9,315.20 | $\underline{225}, \underline{9} 30.83$ | 4.0 | 96. $\underline{8}$ | 642 | $\underline{2257}$ |  | 8.4 | 29.6 |
| \$ . $01 \mathrm{TO} \mathrm{\$ 10.00}$ | 2,270.93 | 233,201.76 | 3.1 | 100.0 | 2006 | 4263 | C | 26.3 | 55.9 |
| ZERO_SALES |  |  | . 0 |  |  | -7625 |  | 44.1 | 100.0 |
| TOTALS | 233,201.76 | 233,201.76 | 100.0 | 100.0 | 7625 | 7625 |  | 100.0 | 100.0 |

## EXHIBIT III

ABC ANALYSIS FOR DEALER Y

| SOURCE OF SUPPLY CODE |  | $\begin{aligned} & \text { DOLLAR } \\ & \text { SALES } \end{aligned}$ | $\begin{aligned} & \text { CUMULATIVE } \\ & \text { DOLLAR } \\ & \text { SALES } \\ & \hline \end{aligned}$ | DOLLAR SALES PERCENT | CUMULATIVE DOLLAR <br> PERCENT | $\begin{aligned} & \text { ITEM } \\ & \text { COUNT } \end{aligned}$ | CUMULATIVE <br> ITEM COUNT | ITEM COUNT <br> PERCENT | $\begin{aligned} & \text { CUMULATIVE } \\ & \text { ITEM } \\ & \text { PERCENT } \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| OVER \$1000. |  | 69, 263.33 | 69,263.33 | 23.2 | 23.2 | 28 | 28 | . 3 | . 2 |
| \$900.01 TO | \$1000.00 | 7,677.12 | 76,940.45 | 2.6 | 25.8 | 8 | 36 | . 1 | . 3 |
| \$800.01 T0 | \$900.00 | 8,315.20 | 85,255.65 | 2.8 | 28.6 | 10 | 46 | . 1 | . 4 |
| \$700.01 T0 | \$800.00 | 9,700.25 | 94,955.90 | 3.3 | 31.8 | 13 | 59 | . 1 | . 5 |
| \$600.01 T0 | \$700.00 | 7,273.65. | 102,229.55 | 2.4 | 34.3 | 11 | 70 | . 1 | . 7 |
| \$500.01 TO | \$600.00 | 15,329.47 | 117,559.02 | 5.1 | 39.4 | 28 | 98 | . 3 | . 9 |
| \$400.01 T0 | \$500.00 | 12,538.55 | 130,097.57 | 4.2 | 43.6 | 28 | 126 | . 3 | 1.2 |
| \$300.01 TO | \$400.00 | 17,810.61 | 147,908.18 | 6.0 | 49.6 | 51 | 177 A | . 5 | 1.7 |
| \$200.01 TO | \$300.00 | 23,637.15 | 171,545.33 | 7.9 | 57.5 | 98 | 275 | 1.0 | 2.7, |
| \$100.01 TO | \$200.00 | 44,699.11 | 216,244.44 | 15.0 | 72.5 | 314 | 589 | 3.1 | 5.8 |
| \$90.01 T0 | \$100.00 | 5,986.83 | 222,231.27 | 2.0 | 74.5 | 63 | 652 | . 6 | 6.5 |
| \$80.01 T0 | \$90.00 | 6,298.42 | 228,529.69 | 2.1 | 76.6 | 74 | 726 | . 7 | 7.2 |
| \$70.01 TO | \$80.00 | 7,930.17 | 236,459.86 | 2.7 | 79.3 | 106 | 832 | 1.1 | 8.3 |
| \$60.01 TO | \$70.00 | 7,338.18 | 243,798.04 | 2.5 | 81.8 | 113 | 945 | 1.1 | 9.4 |
| \$50.01 T0 | \$60.00 | 7,960.15 | 251,758.19 | 2.7 | 84.4 | 145 | 1090 | 1.5 | 10.9 |
| \$40.01 T0 | \$50.00 | 8,426.48 | 260,184.67 | 2.8 | 87.3 | 187 | 1277 | 1.9 | 12.7 |
| _\$30.01 T0 | \$ 40.00 | 8,405.65 | 268, $\underline{\underline{9} 9} . \underline{32}$ | $\underline{2} \cdot \underline{8}$ | 90.1. | 242 | 1519 | 2.4 | -15. 2 |

## EXHIBIT III (Continued)

| \$20.01 T0 | \$30.00 | 9,999.07 | 278.589.39 | 3.4 | 93.4 | 403 | 1922 | B | 4.0 | 19.2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |
| \$ . 01 T0 | \$10.00 | 9,051.11 | 297,985.28 | 3.0 | 100.0 | 2512 | 5153 |  | 25.2 | 51.5 |
| ZERO_SALE |  |  | 297,985 |  |  |  |  | $4 \underline{8} \cdot \underline{4}$ - 100․ 0 |  |  |
|  | TOTALS | 297,985.28 | 297,985.28 | 100.0 | 100.0 | 9987 | 9987 |  | 100.0 | 100.0 |

APPENDIX C

COMPUTER PROGRAM USED TO COMPUTE ORDERS AND
RECEIPTS UNDER PROPOSED SYSTEM

COMPUTER PROGRAM USED TO COMPUTE ORDERS AND
RECEIPTS UNDER PROPOSED SYSTEM

DIMENSION ISALES (52), IORERS (52), IREC (52), TCOST (13), TOTCAR (50), IORD 1ER (52), IRECC (52), ISAL (52), XBAR (52), STAND (52), REORD (52), XINV (52), XI INVIM (52)
$5 \operatorname{READ}(60,10)$ ID1, ICARD,DCOST,INVEN,(ISALES (I) , $I=1,34$ )
10 FORMAT (I2,I1,F6.2,I3,34I2)
$\operatorname{READ}(60,12) \operatorname{ID} 2, \operatorname{ICARD},(\operatorname{ISALES}(I), I=35,55)$
FORMAT (I2, I1, 21I2)
$\operatorname{READ}(60,14)(\operatorname{ISAL}(I), I=1,34)$
14 FORMAT (12X,3412)
$\operatorname{READ}(60,16)(\operatorname{ISAL}(I), I=35,48)$
16 FORMAT (3X,14I2)
ANSALE=0
D0 $1 \mathrm{I}=1,52$
1 ANSALE=ANSALE $+I S A L$ (I)
XDUM=ANSALE/ (DCOST*.245)
EOQ=SQRT (XDUM)
TOTAL=0
IN=1
$\mathrm{K}=2$
$\mathrm{J}=\mathrm{K}+5$
D0 $20 \mathrm{I}=\mathrm{K}, \mathrm{J}$
20 KSALES=KSALES+ISAL(I)
ZSALES=KSALES
XBAR (IN) $=$ ZSALES $/ 6.0$
D0 $30 \mathrm{I}=\mathrm{K}, \mathrm{J}$
TOTAL $=($ TOTAL $+(\operatorname{ISAL}(I)-X B A R(I N)) * * 2)$
TOT=TOTAL/ 6.0
$\operatorname{STAND}(\mathrm{IN})=\mathrm{SQRT}(\mathrm{TOT})$
REORD (IN) $=\operatorname{XBAR}(I N)+S T A N D(I N) * 2$
IN $=1 N+1$
TOTAL=0
$\mathrm{J}=\mathrm{J}+1$
KSALES=KSALES+ISAL (J) -ISAL (K)
K=K+1
ZSALES=KSALES
$\operatorname{XBAR}(I N)=Z S A L E S / 6.0$
DO $40 \mathrm{I}=\mathrm{K}, \mathrm{J}$
TOTAL $=($ TOTAL $+($ ISAL $(\mathrm{I})-X B A R(I N)) * * 2)$
TOT=TOTAL/ 6.0
$\operatorname{STAND}($ IN $)=$ SQRT (TOT)
$\operatorname{REORD}($ IN $)=\operatorname{XBAR}($ IN $)+\operatorname{STAND~(IN)}) * 2$

```
    IF(IN.LT.47) GO TO 35
    D0 500 I=1,52
500
2 CONTINUE
    WRITE(43,31) ID2, ICARD,(IORDER(J), J=1,38)
    FORMAT(I2,I1,38I2)
    ICARD=4
    WRITE(43,41) ID2,ICARD,(IORDER(J),J=39,48)
    41 FORMAT (I2,I1,10I2)
    ICARD=5
    WRITE (43,50) ID2,ICARD,(IRECC (J) ,J=1, 38)
    FORMAT (I2,I1,38I2)
    ICARD=6
    WRITE (43,60) ID2, ICARD, (IRECC (J) ,J=39, 48)
60 FORMAT (I2,I1,10I2)
    IF(ID2.LT.5Q) GO TO 5
99 STOP
END
```

APPENDIX D

SAMPLE PER ITEM COST COMPARISON

PER ITEM COST COMPARISON FOR DEALER "G"

| Part No. | B. Inv. | Apr. | May | Jun. | Ju1. | Aug. | Sep. | Oct. | Nov. | Dec. | Jan. | Feb. | Mar. | Total Cost |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AH 78096 (Cost \$8.16 ea.) | 4 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  | 2 | 2 |  |  |  |  | 1 |  | 1 | 1 | 1 |  |
| Actual in Oíders <br> Use System Rēceipts |  |  |  | 1 |  |  |  |  | 1 |  | 1 | 1 1 | 1 | 5.58 |
| Proposed Orders <br> System Receipts | $=$ |  |  | 1 |  |  |  |  | 1 1 | 1 1 | 1 |  | 1 | 5.92 |
| AH 871 B (Cost \$10.40 ea.) | 11 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  | 8 | 4 |  |  |  |  |  |  |  |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  | $\begin{aligned} & 2 \\ & 2 \end{aligned}$ | $\begin{aligned} & 4 \\ & 4 \end{aligned}$ |  |  |  |  |  |  |  |  | 8 | 2 | 20.76 |
| Proposed Orders <br> System Receipts |  | 2 | $\begin{aligned} & 2 \\ & 4 \\ & \hline \end{aligned}$ |  |  |  |  |  |  |  |  |  |  | 10.13 |
| AH 87926 (Cost \$33.33 ea. | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  |  | 1 | 1 |  |  | 4 |  |  |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  |  |  |  | 2 2 |  |  | 5 |  | 1 | 1 |  |  | 34.66 |
| Proposed Orders <br> System Receipts |  |  |  |  |  | 1 | 1 1 | 2 <br> 2 |  |  |  |  |  | 16.97 |
| AK 3997 B (Cost \$9.08 ea.) | 8 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  | 4 | 5 |  | 2 |  | 3 |  |  |  | 2 |  | 5 |  |
| Actual in Orders <br> Use System Receipts |  |  | $\begin{aligned} & 8 \\ & 2 \end{aligned}$ | 6 |  |  |  |  | 3 | 6 | 3 9 | 9 |  | 17.02 |
| Proposed Orders <br> System Receipts |  |  | 2 | 2 |  | 2 |  |  |  | 2 |  | 2 | 2 | 8.19 |


| Part No. | B. Inv. | Apr. | May | Jun. | Ju1. | Aug. | Sep. | Oct. | Nov. | Dec. | Jan. | Feb. | Mar . | Total Cost |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AR 21059 (Cost \$3.39 ea.) | 14 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  | 3 | 5 | 1 | 1 | 2 | 1 | 2 | 3 |  |  | 4 | 16 |  |
| Actual in $\quad$ Orders |  | 2 | $\begin{aligned} & 7 \\ & 7 \end{aligned}$ |  | 7 | 2 | 1 |  | 7 |  | 6 |  |  | 18.97 |
| Use System Receipts |  | 2 | $7$ |  | 7 | 2 |  |  |  | 6 |  |  |  | 18.97 |
| Proposed Orders <br> System Receipts |  |  |  |  |  | 7 |  |  |  |  | 7 | $\begin{aligned} & 7 \\ & 7 \end{aligned}$ | $\begin{array}{r} 7 \\ 14 \end{array}$ | 7.33 |
| System Receipts |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| AR 28589 (Cost \$3.02 ea.) | 5 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  | 2 | 1 |  | 1 |  |  |  |  |  | 1 |  |  |  |
| Actual in Orders |  |  | $5$ |  |  |  |  |  |  |  |  | 1 | 1 | 6.53 |
| Use System Receipts |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Proposed Orders <br> System Receipts |  |  |  |  |  |  |  |  | 3 3 |  |  |  |  | 2.26 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| AR 30155 (Cost \$31.08 ea.) | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  | 1 | 2 | 1 |  |  |  |  | 1 |  |  |  |  |  |
| Actual in $\quad$ Orders |  | 1 | 2 |  |  |  |  |  | 1 |  | 2 | 1 |  | 27.11 |
| Use System Receipts |  | 1 | 2 |  |  |  |  |  | 1 |  | 2 | 1 |  | 27.11 |
| Proposed Orders <br> System Receipts |  |  | 1 | 1 |  |  | 1 | 1 |  |  |  |  |  | 11.65 |
| System Receipts |  |  |  |  |  |  |  | 1 |  |  |  |  |  |  |
| AR 31790 (Cost \$3.15 ea.) | 10 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  | 1 | 1 | 1 | 2 | 1 |  | 2 | 3 | 2 | 2 | 2 | 2 |  |
| Actual in Orders |  | 1 | 1 | 1 | 2 | 1 |  | 2 | 3 | 2 | 2 | 2 | 1 | 16.44 |
| Use System Receipts |  |  | 1 | 2 | 2 | 1 |  | 2 | 2 | 3 | 2 |  | 3 | 16.44 |
| Proposed Orders |  |  |  |  |  |  |  |  | 4 | 4 |  | 4 |  | 5.65 |
| System Receipts |  |  |  |  |  |  |  |  | 4 | 4 |  | 4 |  | 5.65 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| AR36044 (Cost \$14.13 ea.) | 6 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  | 2 |  | 3 | 6 | 1 | 2 | 3 | 3 | 1 | 1 |  |  |
| Actual in Orders |  |  |  | 2 | 2 | 8 |  |  |  | 4 | 2 | 2 | 2 | 22.08 |
| Use System Receipts |  |  | - | 1 | 3 | 6 | 2 |  |  | 4 | 2 | 2 | 2 | 22.08 |
| Proposed Orders |  |  |  |  | 4 | 4 | 2 | 2 | 2 | 2 |  | 2 |  | 14.02 |
| System Receipts |  |  |  |  | 2 | 4 | 2 | 4 |  | 4 |  | 2 |  | 14.02 |



| Part No. | B. Inv. | Apr. | May | Jun. | Ju1. | Aug. | Sep. | Oct. | Nov. | Dec. | Jan. | Feb. | Mar. | Total Cost |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AR 68650 (Cost \$54.75 ea.) | 11 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  | 6 |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  |  |  |  |  |  |  |  |  | 1 |  |  |  | 88.25 |
| Proposed Orders <br> System Receipts |  |  |  |  |  |  |  |  |  |  |  |  |  | 82.72 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| AR 71286 (Cost \$17.73 ea.) | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  |  | 2 | 1 | 1 |  |  |  |  |  |  | 1 |  |
| Actual in Orders <br> Use System Receipts |  |  |  | 2 | 2 4 | 3 | 3 |  |  |  |  |  |  | 18.15 |
| Proposed Orders <br> System Receipts |  |  |  | 1 | 1 | 1 1 |  |  |  |  |  |  | 1 1 | 7.25 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| AR 73712 (Cost \$13.14 ea.) | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  | 3 |  |  | 2 |  |  |  |  |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  |  | 2 | 2 |  |  | 2 | 2 |  |  |  |  |  | 13.94 |
| Proposed Orders <br> System Receipts |  |  | $\begin{aligned} & 2 \\ & 1 \end{aligned}$ | 1 |  | 2 1 | 1 |  |  |  |  |  |  | 5.35 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| AR 20450 (Cost \$21.45 ea.) | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  | 2 | 1 |  |  |  | 1 | 1 |  |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  |  |  |  | 2 1 | 1 | 2 | 2 | 1 1 |  |  |  |  | 16.95 |
| Proposed Orders <br> System Receipts |  |  |  | 1 1 | 1 |  | 1 1 |  |  |  |  |  |  | 12.01 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\frac{\text { AT } 14467 \text { (Cost \$1. } 80 \text { ea.) }}{\text { Actual Sales }}$ | 20 |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 2 | 1 | 2 | 3 | 1 | 2 | 2 | 4 | 2 |  | 1 |  |
| Actual in Orders <br> Use System Receipts |  |  |  |  |  |  |  | 9 |  |  |  |  |  | 7.45 |
| Proposed Orders <br> System Receipts |  |  |  |  |  |  |  |  |  |  |  |  |  | 4.80 |


| Part No. | B. Inv. | Apr. | May | Jun. | Ju1. | Aug. | Sep. | Oct. | Nov | Dec. | Jan. | Feb. | Mar. | Total Cost |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AT 22070 (Cost \$24.45 ea.) | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  | 1 |  |  | 1 | 1 |  | 1 |  |  |  |  | 1 |  |
| Actual in Orders <br> Use System Receipts |  |  |  |  |  | 2 | 2 | 2 |  |  |  |  |  | 12.98 |
| Proposed Orders <br> System Receipts |  |  |  |  | 1 1 | 1 | 1 | $\begin{aligned} & 1 \\ & 1 \end{aligned}$ |  |  |  |  | 1 | 8.24 |
| AZ 1299 (Cost \$24.60 ea.) | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  | 2 | 1 | 1 | 1 |  |  |  |  |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  |  | 2 | 2 | 2 |  |  |  |  |  |  |  |  | 14.81 |
| Proposed Orders <br> System Receipts |  |  | 1 | 1 | 1 | 1 |  |  |  |  |  |  |  | 14.81 |
| B 2354 (Cost \$3.28 ea.) | 9 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  | 2 | 2 | 2 | 2 |  |  |  | 1 |  |  | 2 |  |  |
| Actual in Orders <br> Use System Receipts |  |  | 10 | 10 | 1 |  |  |  | 1 |  |  |  |  | 9.64 |
| Proposed Orders <br> System Receipts |  |  |  |  |  |  |  |  | 3 3 |  |  |  |  | 2.84 |
| E 417 AN (Cost \$4.98 ea.) | 5 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  | 7 | 2 |  |  |  |  |  |  |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  | 2 |  | 2 | 1 |  |  |  |  |  | 1 | $\begin{aligned} & 6 \\ & 4 \end{aligned}$ | 4 | 6.86 |
| Proposed Orders <br> System Receipts |  |  | $\begin{aligned} & 2 \\ & 2 \\ & \hline \end{aligned}$ | $\begin{aligned} & 2 \\ & 2 \\ & \hline \end{aligned}$ |  |  |  |  |  |  |  |  |  | 1.76 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| F 1687 (Cost \$16.79 ea.) | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  | 1 |  |  | 1 |  |  |  | 1 |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  |  | 1 | 1 |  | 1 |  |  |  | 1 | 1 |  |  | 4.91 |
| Proposed Orders <br> System Receipts |  |  | 1 | 1 | 1 |  |  |  |  | 1 |  |  |  | 4.91 |


| Part No. | B. Inv. | Apr. | May | 'Jun. | Ju1. | Aug. | Sep. | Oct. | Nov. | Dec. | Jan. | Feb. | Mar. | Total Cost |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| H 26858 (Cost \$9.53 ea.) | 7 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  |  |  |  | 1 | 1 | 2 | 1 |  |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  |  |  |  |  | 3 3 | 3 | 1 |  |  |  |  |  | 22.51 |
| Proposed Orders <br> System Receipts |  |  |  |  |  |  |  |  | 2 2 |  |  |  |  | 13.63 |
| H 28671. (Cost. \$17.85 ea) | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  |  |  | 1 | 1 |  | 1 | 1. |  | 1. |  |  |  |
| Actual in Orders <br> Use System Receipts |  |  |  |  | 1 | 1 1 |  | 1 | 1 |  | 1 |  |  | 7.24 |
| Proposed Orders <br> System Receipts |  |  | . |  | 1 | 1 |  | 1 | 1 1 |  | 1 |  |  | 7.24 |
| H 80103 (Cost \$16.50 ea.) | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  |  | 1 | 1 |  |  |  |  |  |  |  |  |  | 4.21 |
| Proposed Orders <br> System Receipts |  | 2 1 | 1 |  |  |  |  |  |  |  |  |  |  | 9.25 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| H 80806 (Cost \$6.08 ea.) | 4 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  | 2 |  |  | 3 | 1 | 2 | 2 |  |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  |  | $\begin{aligned} & 2 \\ & 2 \end{aligned}$ |  |  | 6 4 | 5 7 |  |  |  |  |  |  | 12.62 |
| Proposed Orders <br> System Receipts |  |  |  |  | 2 2 |  | 4 4 4 | 2 | 2 2 |  |  |  |  | 8.09 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| H 86259 (Cost \$5.99 ea.) | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  | 1 |  |  |  |  | 2 | 1 |  |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  |  |  |  |  |  |  | 3 2 | 1 |  |  |  |  | 4.36 |
| Proposed Orders <br> System Receipts |  |  |  |  |  |  |  | 1 | 1 1 |  | $\vdots$ |  |  | 3.57 |


| Part No. | B. Inv. | Apr. | May | Jun. | Jul. | Aug. | Sep. | Oct. | Nov. | Dec. | Jan. | Feb. | Mar. | Total Cost |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| JD 1299 (Cost \$24.82 ea.) | 4 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  |  |  |  |  |  |  | 2 |  |  |  | 1 |  |
| Actual in Orders <br> Use System Receipts |  |  |  |  |  |  |  |  |  |  |  |  |  | 20.02 |
| Proposed Orders <br> System Receipts |  |  |  |  |  |  |  |  |  |  |  |  |  | 20.02 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| JD 8545 (Cost \$3.23 ea.) | 7 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  | 2 |  |  | 1 | 5 | 7 | 6 |  | 2 |  |  |  |
| Actual in Orders <br> Use System Receipts |  |  |  | 4 |  | 6 | 1 | 8 | $\begin{aligned} & 4 \\ & 1 \end{aligned}$ | $\begin{aligned} & 4 \\ & 4 \end{aligned}$ | 5 5 |  |  | 8.88 |
| Proposed Orders <br> System Receipts. |  |  |  |  |  |  | $\begin{aligned} & 4 \\ & 4 \end{aligned}$ | $\begin{aligned} & 8 \\ & 8 \end{aligned}$ | $\begin{aligned} & 4 \\ & 4 \end{aligned}$ |  | 4 |  |  | 6.09 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| JD 8913 (Cost \$3.71 ea.) | 12 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  | 4 | 8 | 3 | 2 | 2 |  |  |  | 2 | 2 | 1 |  |  |
| Actual Sales  <br> Actual in Orders <br> Use System Receipts <br> Proposed Orders <br> System Receipts |  | 5 | 5 |  | 5 |  |  |  |  | 5 | 5 | 5 |  | 14.21 |
|  |  |  | 5 5 | 5 |  |  |  |  |  | 5 5 |  |  |  | 6.08 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\frac{\text { P 2020 A (Cost \$14.97 ea.) }}{\text { Actual Sales }}$ | 11 |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | 9 |  |  |  |  |  |  |  | 4 | 9 | 9 |  |
| Actual in Orders <br> Use System Receipts |  |  | 8 | 4 |  |  |  |  |  |  | 4 4 | 10 | $\begin{array}{r}8 \\ 13 \\ \hline 1\end{array}$ | 40.75 |
| Proposed Orders <br> System Receipts |  |  |  |  | 1 |  |  |  |  |  | 1 | 2 1 | 1 2 | 14.27 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| R 15019 (Cost \$11.94 ea.) | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  | 1 |  | 1 |  |  |  |  |  | 1 |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  | 2 |  |  | 1 |  |  |  |  |  |  |  |  | 4.80 |
|  |  | 2 |  |  | 1 |  |  |  |  | 1 |  |  |  | 4.80 |
| Proposed Orders <br> Systems Receipts |  | 2 |  |  | 1 |  |  |  |  | 1 |  |  |  | 5.29 |
|  |  | 2 |  |  | 1 |  |  |  |  | 1 |  |  |  | 5.29 |


| Part No. | B. Inv. | Apr. | May | Jun. | Ju1. | Aug. | Sep. | Oct. | Nov. | Dec. | Jan. | Feb | Mar. | Total Cost |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| R 20246 (Cost \$7.25 ea.) | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  |  | 2 |  |  |  |  |  |  |  |  |  |  |
| Actual in Orders <br> Use-System Receipts |  |  |  | 2 2 |  |  |  |  |  |  |  |  |  | 4.35 |
| Proposed Orders <br> System Receipts |  |  | 1 1 | $\begin{aligned} & 1 \\ & 1 \end{aligned}$ |  |  |  |  |  |  |  |  |  | 5.00 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| R 27018 (Cost \$4.28 ea.) | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  | 2 |  |  |  |  |  |  |  | 2 |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  |  | 2 |  |  | 2 |  |  |  |  |  |  |  | 3.80 |
| Proposed Orders <br> System Receipts |  | $\begin{aligned} & 4 \\ & 2 \\ & \hline \end{aligned}$ | 2 |  |  |  |  |  |  |  |  |  |  | 4.50 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| R 27173 (Cost \$3.58 ea.) | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  | 2 | 1 |  | 1 | 1 |  |  | 2 |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  |  |  |  | 2 2 |  | 4 | 4 |  | 3 3 | 3 3 |  |  | 5.62 |
| Proposed Orders <br> System Receipts |  |  | 3 <br> 3 |  |  |  |  | 3 | 3 |  |  |  |  | 3.30 |
|  |  |  |  |  |  | , |  |  |  |  |  |  |  |  |
| R 31026 (Cost \$20.07 ea.) | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  |  | 2 | 2 |  |  |  | 4 | 2 |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  |  |  | 2 | 2 | 5 7 |  |  | 1 | 3 | 3 5 |  |  | 29.41 |
| Proposed Orders <br> System Receipts |  |  |  | 1 1 | 1 | 1 | 1 | 1 2 | 3 3 | $\begin{aligned} & 2 \\ & 1 \end{aligned}$ | 1 |  |  | 13.11 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| R 36847 (Cost \$2.96 ea.) | 23 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  |  |  | 1 |  |  |  | 2 |  |  |  | 3 |  |
| Actual in Orders <br> Use System Receipts |  | 8 |  |  |  |  |  |  |  | 1 1 |  | : |  | 23.87 |
| Proposed Orders <br> System Receipts |  |  |  |  |  |  |  |  |  |  |  |  |  | 16.56 |


| Part No. | B. Inv. | Apr. | May | Jun. | Jul. | Aug. | Sep. | Oct. | Nov. | Dec. | Jan. | Feb. | Mar. | Total Cost |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| R 45134 (Cost \$44.64 ea.) | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  | 1 |  |  |  |  |  |  | 1 | 1 |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  | $\begin{aligned} & 1 \\ & 1 \\ & \hline \end{aligned}$ |  |  |  |  |  |  | 1 | 1 |  |  |  | 13.35 |
| Proposed Orders <br> System Receipts |  | 2 2 |  |  |  |  |  |  |  | 1 |  | + |  | 19.73 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| R 51392 (Cost \$33.14 ea.) | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  |  |  |  | 1 |  |  |  | 1 |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  |  |  |  |  | 1 |  |  |  | 1 |  |  |  | 9.80 |
| Proposed Orders <br> System Receipts |  |  |  |  |  | 1 |  |  |  | 1 1 |  |  |  | 9.80 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| R 52568. (Cost \$3.06 ea.) | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Astual Sales |  |  |  |  | 1 |  |  |  |  |  |  | 2 |  |  |
| Actual in Orders <br> Use System Receipts |  |  |  |  |  |  |  |  |  |  |  | 3 3 |  | 2.50 |
| Proposed Orders <br> System Receipts |  |  | , |  |  |  |  |  |  |  |  | 2 2 |  | 2.34 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| R 53282 (Cost \$1.10 ea.) | 10 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  | 2 | 3 | 3 | 2 | 2 |  |  |  |  |  |  |  |  |
| Actual in Orders <br> Jse System Receipts |  |  |  | $\begin{aligned} & 11 \\ & 11 \end{aligned}$ |  |  | 11 | 11 |  |  |  |  |  | 5.14 |
| Proposed Orders <br> System Receipts |  |  |  | 9 | 9 |  |  |  |  |  |  |  |  | 2.36 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| R 35303 (Cost \$35.19 ea.) | 4 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  | 1 | 1 | 1 |  |  |  | 2 | 1 |  |  | 5 |  |
| Actual in Orders <br> Use System Receipts |  |  | $\begin{aligned} & 1 \\ & 1 \end{aligned}$ | 3 | 1 3 |  |  | 1 | 2 |  |  |  |  | 41.52 |
| Proposed Orders |  |  |  |  |  |  |  |  | 1 | 1 |  |  |  |  |
| System Receipts |  |  |  |  |  |  |  |  | 15 | 1 |  |  | 3 | 13.28 |


| Part No. | B. Inv. | Apr. | May | Jun. | Ju1. | Aug. | Sep. | Oct. | Nov. | Dec. | Jan. | Feb. | Mar. | Total Cost |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| TY 1856 (Cost \$6.60 ea.) | 7 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  | 1 | 1 | 2 | 1 |  |  | 1 |  |  |  |  | 3 |  |
| Actual in Orders <br> Use System Receipts |  |  | 6 | 5 |  |  |  |  |  |  |  |  |  | 105.60 |
| Proposed Orders <br> System Receipts |  |  |  |  |  |  |  |  |  |  | 1 | 1 | 1 | 45.97 |
| TY 4327 (Cost \$30.77 ea.) | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  | 1 |  |  |  |  |  |  | 1 |  |  | 1 |  |
| Actual in Orders <br> Use System Receipts |  |  | 1 | 1 |  |  |  |  |  | 1 1 |  |  |  | 7.60 |
| Proposed Orders <br> System Receipts |  |  | 1 | 1 |  |  |  |  |  | 1 |  |  | 1 1 | 9.04 |
| T |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  |  |  |  | 6 |  |  | 6 |  |  |  |  |  |
| Actual in Orders |  |  |  |  |  | 6 |  |  |  |  |  |  |  |  |
| Use System Receipts |  |  |  |  |  | 6 |  |  | 6 |  |  |  |  | 30.35 |
| Proposed Orders <br> System Receipts |  |  |  |  |  | 1 |  |  | 1 |  |  |  |  | 12.29 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| YA 579 (Cost \$3.98 ea.) | 4 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  | 8 |  |  |  |  |  |  |  |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  | $\begin{aligned} & 6 \\ & 6 \end{aligned}$ |  |  |  |  |  |  |  |  |  |  | 5 | 4.45 |
| Proposed Orders <br> System Receipts |  | $\begin{aligned} & 4 \\ & 4 \end{aligned}$ | 2 |  |  |  |  |  |  |  |  |  |  | 4.18 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Y 635 AN (Cost \$4.67 ea.) | 13 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  |  | 14 |  |  |  |  |  |  |  |  |  |  |
| Actual in Orders |  |  |  | 3 |  |  |  |  |  |  |  |  | 10 | 7.53 |
| Proposed Orders |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\text { System } \quad \text { Receipts }$ |  |  |  | 8 |  |  |  |  |  |  |  |  |  | 11.11 |


| Part No. | B. Inv. | Apr. | May | Jun. | Ju1. | Aug. | Sep. | Oct. | Nov. | Dec. | Jan. | Feb. | Mar. | Total Cost |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Y 972 N (Cost \$4.12 ea.) | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Actual Sales |  |  | 3 |  |  |  |  |  |  |  |  |  |  |  |
| Actual in Orders <br> Use System Receipts |  |  |  |  |  |  |  |  |  |  | 3 3 |  |  | 1.76 |
| Proposed Orders <br> System Receipts |  |  | 1 | 1 |  |  |  |  |  |  |  |  |  | 1.76 |

VITA<br>Orland S. Lee<br>Candidate for the Degree of<br>Doctor of Philosophy

## Thesis: A SYSTEM OF INVENTORY CONTROL FOR RETAIL FARM EQUIPMENT DEALERS

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[^0]:    *See Ploss1 and Wight (93) for an excellent explanation of the selection criteria in determining which distribution best describes the firm's sales.

[^1]:    *For complete details of how these formulae were derived, see (115, pp. 811-816).

