STUDY OF BASE PLATES FOR AXIALLY LOADED COLUMNS

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NOMENCLATURE

a	radius
$\{a\}$	derivative of stress invariants
$[B]$	strain matrix
C_1, C_2, C_3	constants dependent on failure criteria
$[D]$	material property matrix in elastic range
$[D_b]$	property matrix of a bond-link element
$[D_{ep}]$	elasto-plastic material property matrix
$[D_p]$	plastic material property matrix
$[D_1]$, $[D_2]$	partitioned property matrices
E	modulus of elasticity
f'_c	ultimate compressive strength of concrete
F	yield surface
$\{F\}$	force vector
$\{\Delta F\}$	load increment vector
$\|\|F_i\|\|$	norm to applied load vector
G	shear modulus
H	parameter equal to the slope of uniaxial stress equivalent plastic strain curve
J_1, J_2, J_3	stress invariants
k	yield stress in pure shear
k_1, k_2	constants
K_h	stiffness coefficient of a bond-link element in horizontal direction

K_v	stiffness coefficient of a bond-link element in the vertical direction
$[K]$	stiffness matrix
$\{N\}$	normal to yield surface
N_i	shape function for node i
$[N]$	matrix of shape function
$[N,_r]$	shape function derivative with respect to r
$[N,_z]$	shape function derivative with respect to z
$\{p\}$	unbalanced force vector
$\{q\}$	vector of force intensity
Q	plastic potential
r, r_1	factors for correcting stress increments
r, z	cylindrical coordinates
$\{R\}$	residual load vector
$\|\|R\|\|$	norm to residual force vector
S_x, S_y, S_z	deviatoric stresses
t	convergence factor
u	displacement in r or x direction
$\{\hat{u}\}$	displacement field
$\{U\}$	vector of nodal displacements
$\{U_i\}$	ith generalized displacement increment
v	displacement in y or z direction
x, y, z	rectangular coordinates
β	ratio of biaxial strength to uniaxial strength of concrete
$\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$	shear strain in rectangular coordinates
γ_{xz}^{p}	plastic incremental shear strain in cylindrical coordinates
ε	uniaxial strain

ε_p	plastic strain
$\bar{\varepsilon}_p$	equivalent uniaxial plastic strain
$\varepsilon_x, \varepsilon_y, \varepsilon_z$	strains in the rectangular coordinates
$\varepsilon_r, \varepsilon_z, \varepsilon_\theta$	strains in the cylindrical coordinates
$\{\varepsilon\}$	strain vector
$\{\Delta\varepsilon\}$	incremental strain vector
$d\varepsilon_r^p, d\varepsilon_z^p, d\varepsilon_\theta^p$	plastic incremental strain in cylindrical coordinates
ε_{yd}	yield strain
Δ_r, Δ_z	relative displacement in horizontal and vertical directions between a bond-link element node
κ	hardening parameter
ν	Poisson's ratio
ξ, η	local natural coordinates
Π	total potential energy
σ	uniaxial stress
$\bar{\sigma}$	invariant for failure criteria
σ_m	octahedral normal stress or invariant for failure criteria
$\sigma_x, \sigma_y, \sigma_z$	stresses in rectangular coordinates
$\sigma_r, \sigma_z, \sigma_\theta$	stresses in cylindrical coordinates
$\sigma_1, \sigma_2, \sigma_3$	principal stresses
σ_{yd}	yield stress
$\{\sigma\}$	stress vector
$\{\sigma_{ex}\}$	excessive stress vector
τ	shear stress
τ_{oct}	octahedral shear stress
τ_{rz}	shear stress in cylindrical coordinates

τ_{xy}, τ_{xz}, τ_{yz}	shear stresses in rectangular coordinates
ϕ	invariant for failure criteria

CHAPTER I

INTRODUCTION

1.1 General

The design procedure recommended by the American Institute of Steel Construction (1) assumes that the bearing pressure under a base plate is of uniform intensity. A permissible bearing stress related to the ratio of the loaded area to the surface area of concrete limits the minimum dimensions of the base plate. The column load is assumed to be uniformly distributed over an effective area which is approximately equal to the depth times the width of the column section. The plate thickness is determined by considering the portions of the plates which extend beyond the effective column area to act as cantilevers; the plate thickness is chosen to limit the flexural stresses at specified critical sections.

The American Institute of Steel Construction (AISC) specifies that the basic allowable bearing stress for concrete is 35 percent of the compressive strength of concrete (1) (2). The American Concrete Institute specifies that the basic bearing stress is 30 percent of concrete strength (3). Both specifications permit the basic bearing strength to be increases as much as 100 percent where only a portion of the concrete surface is subjected to bearing.

In the event of modest column loads, it is possible to calculate a small plate area which has no overhang beyond the critical section. In such a circumstance the designer has no specified procedure to establish a suitable plate thickness. The assumption of a uniform distribution of stresses between a base plate and a concrete footing is not true for relatively flexible plates. The factor of safety which results from the use of the method is unknown. These problems demonstrate that the current design procedure is not rational.

A knowledge of the actual behavior of the base plate system in both the elastic and inelastic stages is of fundamental importance for design. The safety of most structures can be correctly assessed if their ultimate load carrying capacity can be predicted analytically. With the recent development of numerical methods in general, and of the finite element method in particular, solutions of complex structural systems are now possible. The application of this displacement method results in a system of linear simultaneous equations which can be solved on digital computers. Nonlinear problems can be solved either by iterations or as a sequence of consecutive linear problems.

1.2 Scope

The objective of this study is to develop a reliable procedure for analyzing circular base plate systems through the entire elastic and inelastic ranges of loading. The proposed procedure can be used to predict the ultimate load carrying capacity and the behavior of the base plate system throughout the load history.

A mathematical model is formulated for the base plate system that reflects the behavior of the steel base plate and the plain concrete footing in the actual system. The analytical study is limited to small deflections and to short time behavior of an axisymmetric base plate system under monotonically increasing static loads. The main emphasis is placed on the behavior in the inelastic range. An incremental-iterative procedure is used for solving the nonlinear problem. After each load increment the forces and deformations are computed. The adequacy of the proposed procedure is illustrated by comparing analytical solutions for some base plate systems with experimental results. A limited parametric study is formed to investigate the major variables which influence the behavior of the base plate system.

CHAPTER II

HISTORICAL AND LITERATURE REVIEW

2.1 Bearing Capacity of Concrete

The problem of applying large loads to limited areas of concrete is one frequently encountered in structural design. Base plates for steel columns resting on concrete footings, anchor plates in post-tensioned concrete structures, and bridge bearings over piers are a few examples of the numerous bearing problems.

Previous investigations (4) through (13) of the bearing capacity of concrete are discussed in detail in Reference (14). In these studies investigators concentrated their attention on a few principal variables influencing the bearing strength of concrete loaded through rigid plates.

The main conclusions of these investigations were as follows:

1. Bearing strength increases continuously for an increase in the ratio of the footing area to the loaded area; for a large ratio any benefit of a further increase is small.

2. Bearing strength is dependent on the depth of the concrete footing.

3. The higher the compressive strength of concrete the lower is the ratio of the bearing strength to the compressive strength of concrete.

4. A footing supported on a compressible bed will have a decreasing strength.

5. Lateral reinforcement in a footing increases the ultimate bearing strength of concrete.

6. Friction on the base of footings does not influence the bearing strength of concrete.

Hawkins (15) investigated the bearing strength of concrete loaded thorugh flexible plates; the effects of the thickness and yield strength of the plate, the strength of the concrete, and the ratio of the loaded area to total area of the plate were considered. Test results showed that the ultimate bearing capacity for flexible plates increased linearly with the plate thickness, whereas for semi-flexible plates the rate of increase of the ultimate bearing capacity was continuous until a maximum value is reached corresponding to the capacity of rigid plates. Hawkins observed that the first indication of impending collapse was the formation of short vertical cracks on the sides of the specimens, and for flexible plates there was an almost solid core surrounded by a cone of crushed concrete. He developed expressions to predict the ultimate bearing load for flexible and semi-flexible plates based on the yield line theory. He also developed an expression for determining the thickness of rigid plates. Figure 1 compares the experimental and the computed strengths of test series B and D. The curve represents the proposed theoretical results. It is linear over the range for which the plate is flexible. At the end of the semi-flexible range, the ultimate load increases significantly for an increase in the plate thickness until a limiting value is reached which represents the strength of rigid plates. Experimental results showed that the bearing capacity for flexible plates increased in a direct proportion to the concrete strength raised to 0.7 power and the square root of the yield stress of the bearing plate.

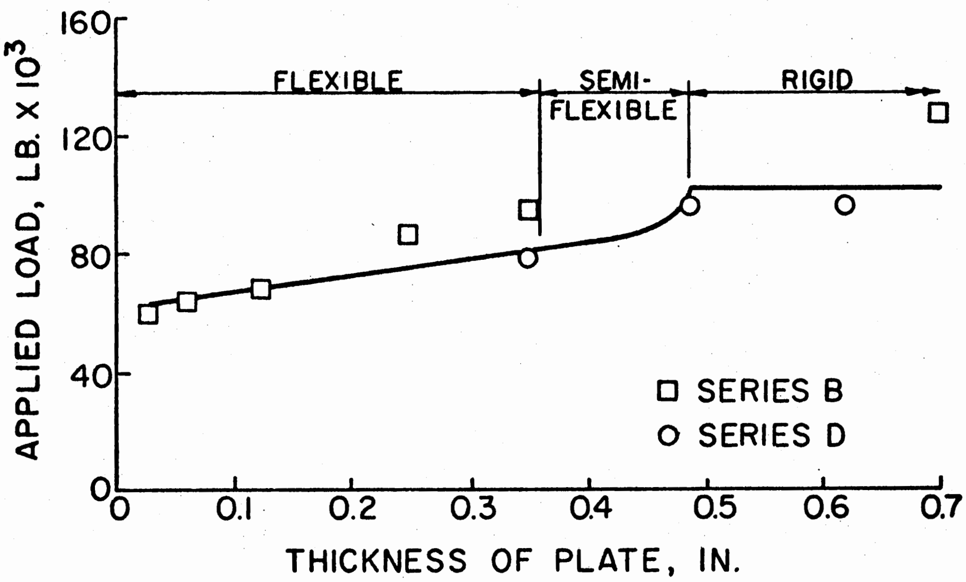


Figure 1. Comparison of Measured and Computed Strengths for Series B and D (From Reference (15))

2.2 Stress-Strain Response and Strength of Concrete

In many experimental investigations concrete has been subjected to complex states of stress. Most of the available information about the behavior of concrete under load has been obtained from static uniaxial compression or tensile tests. Popovics (16) reviewed the stress-strain relationship for concrete subject to uniaxial loading. He discussed the available empirical formulas for describing the stress-strain behavior of concrete.

Concrete specimens subject to any state of stress can support up to 60 percent of the ultimate load before any major internal structural changes occur. Under low states of stress most deformations are linear and recoverable. According to Griffith's theory (17) at the end of the linear range the presence of micro-cracks and stress concentrations result in the slow propagation of micro-cracks of unstable length. Under steady sustained load the slow crack propagation will continue as the excess strain energy is dissipated in the formation of new surface until a stage is reached where the stressed material is in equilibrium with the external loading system. The system will become unstable if the load is increased and severe cracks will start to propagate. Newman (18) defined the load stage at which more severe cracking begins as "discontinuity."

Investigators (19) through (23) have attempted to analyze structural and interparticle models of concrete to explain the mechanism and modes of failure. Although the failure of concrete has been the subject of much research, there is still no universal theory of failure for concrete. Several investigators (24) through (35) attempted to develop theories of failure for concrete under complex states of stress. These studies are summarized and discussed in Reference (14).

Bresler and Pister (24) suggested a criterion relating the octahedral shear stress to the octahedral normal stress at failure in the form

$$\tau_{oct} = f(\sigma_m) \tag{2.1}$$

where

τ_{oct} = octahedral shear stress; and

σ_m = octahedral normal stress.

The octahedral stresses have been widely used for expressing the failure of concrete subjected to multiaxial stresses. Kupfer et al. (31) obtained a failure envelope based on their extensive tests on concrete under a biaxial state of stresses which can be expressed in octahedral shear stresses.

Mills and Zimmerman (36) observed two distinct types of failure in their study of the compressive strength of concrete under multiaxial loading conditions. There was a difference in the compressive strength of concrete between the type I test where $\sigma_1 > \sigma_2 = \sigma_3$ and the type II test where $\sigma_1 = \sigma_2 > \sigma_3$. They developed a criterion for failure of concrete based on the results of their tests in which they used octahedral stresses. The two types of failure may be a result of the difference in testing procedure and loading sequence as discussed by Pandit (37). The test results were presented in the form

$$\sigma_1 = f'_c + k_1\sigma_2 + k_2\sigma_3 \tag{2.2}$$

where

$\sigma_1, \sigma_2, \sigma_3$ = principal stresses;

f'_c = uniaxial compressive strength of concrete; and

k_1, k_2 = constants.

Mills and Zimmerman pointed out that they do not propose this form for multiaxial compressive strength of concrete because of the limited test data available.

Kostovos and Newman (38) recently identified distinct levels of change in the behavior of concrete when it is subjected to multiaxial states of stress. The onset of localized cracking occurs primarily as a result of breakdown of the concrete matrix. After this stage is reached the concrete exhibits distinctly inelastic properties but can still behave in a stable manner. The onset of continuous cracking occurs mainly as a result of fractures within the matrix, after which the material disrupts in an unstable manner when cracks continue to propagate. They suggested that stresses and strains at the onset of stable fracture propagation form an envelope which may serve as a basis for a lower bound failure criterion. For the onset of unstable fracture propagation, the envelope may be used as a basis for the upper bound failure criterion.

2.3 Nonlinear Finite Element Analysis of Structures

The nonlinear analysis of structures was one of the most intractable problems prior to the widespread use of high speed digital computers. There are three categories of nonlinearity: geometric nonlinearity, which arises from nonlinear terms in the kinematic equations; material nonlinearity, which arises from nonlineartieis in the constitutive equations; and combined geometric and material nonlinearity.

Progress in the area of inelastic analysis was accelerated by the simultaneous development of the direct stiffness method by Turner et al. (39) and the principal of the initial strain method developed by Mendelson and Manson (40). Wilson (41) successfully applied a matrix method to the analysis of materially nonlinear framed structures. Gallagher et al. (42) adapted the method of initial strain to the finite element analysis by calculating an initial force vector. Goldberg and Richard (43) extended the applicability of the finite element method to nonlinear problems. Wilson (44) applied an incremental load procedure to the analysis of non-linear structures.

Subsequent application of the initial strain method in the area of plane solids were made by Percy et al. (45), Argyris et al. (46), and Jensen et al. (47). The method evaluates the change in the plastic strain to re-evaluate the stress distribution. The same stiffness matrix is utilized throughout the iteration to reduce the computational time.

The Tangent Modulus Method was developed for the analysis of elastic-plastic problems. The method makes use of the linearity of the incre-mented stress-strain laws to assemble a new element stiffness at each stage. The equations for the tangent modulus were developed by Pope (48), Swedlow and Yang (49), and Marcal and King (50).

Felippa (51) investigated the application of refined displacement finite elements to the analysis of linear and nonlinear problems in struc-tural mechanics. Yamada et al. (52) obtained an explicit expression of the incremental stress and strain matrix for Prandtl-Reuss equations. Akyzy and Merwin (53) investigated plane strain indentation for cylind-rical identers, and Lee and Kobayashi (54) studied plane strain and axi-symmetric flat punch identation into specimens of finite dimensions using the finite element method.

Marcal (55) found many similarities between the initial strain method and the tangent modulus method. He concluded that the constant strain approach does not converge in the case of elastic-perfectly plastic material problems because of the large plastic strains which occur in these cases.

The initial stress method was developed by Zienkiewicz et al. (56). This method appeared to be suitable for general plastic behavior because it relies on the fact that a unique stress exists for an increment of strain. The total incremental stress-strain relation are used to correct the total value of stress at the end of each increment and the matrix of elastic constants is retained unchanged during the loading history.

The research conducted in the area of nonlinear finite element analysis has continued since these earlier investigations. In general, recent investigations have only refined the initial nonlinear techniques of analysis. Oden (57) has presented a comprehensive review of the nonlinear structural analysis techniques. The principal methods of solution for geometrically nonlinear problems are discussed thoroughly in his paper.

The first application of the finite element method to concrete structures was carried out by Rashid (58) who analyzed prestressed concrete vessels as axisymmetric solids. He used several elements to model the composite structure. Rashid (59) later modified the procedure to include cracks in the concrete and the effects of plastic deformations in the steel. Nago and Scordelis (60) used an elastic linear two-dimensional analysis to determine principal stresses in reinforced concrete beams with predefined crack patterns. Nilson (61) introduced nonlinear material properties and a nonlinear bond-slip relationship into his analysis and used an incremental loading technique.

Corum and Kirshnamuthy (62) investigated a series of models of prestressed reactor vessels using a three-dimensional finite element program developed by Cornell et al. (63). The structure was modeled by using tetrahedral concrete elements, uniaxial bars, and triangular membrane steel elements. The results from the three-dimensional model were improved compared to those from the two-dimensional analysis, but the computer time was significantly increased.

In the early nonlinear analysis of concrete structures, the cracking was accounted for by stopping the solution when an element was cracked; then a new cracked structure had to be redefined before resuming the solution.

Franklin (64) advanced the capability of the analytical methods by developing a nonlinear finite element program which accounted for cracking within the finite elements and redistributed the stresses into the system. It was possible to analyze the structural system in one continuous computer program. Incremental loading with iterations within each increment was used to account for the cracking and the nonlinear properties of the material. Reinforced concrete frames with or without infilled shear panels were analyzed using layered frame type elements, quadrilateral plane stress elements and link elements. Cervenka (65) analyzed shear-wall panels and compared the analytical results with those of his experimental studies.

Studies of reinforced concrete slabs using the finite element method have been presented by Jofriet and McNiece (66) and by Bell and Elms (67). Cracking in plate bending elements was considered by changing the bending stiffness of the cracked elements. Scanlon (68) has developed a method of incorporating both cracking and the dependent effects of creep and shrinkage in slabs. He used layered rectangular slab elements which can be cracked progressively layer by layer, and assumed that cracks propagate only parallel to and perpendicular to the orthogonal reinforcements. Hand et al. (69) used a layered finite element to analyze reinforced concrete slabs and shells. Lin (70) investigated the behavior of reinforced concrete slabs and shells in the nonlinear range of loading.

Lassker (71) studied the nonlinear behavior of reinforced concrete beams using the initial strain method and simulated the inelastic behavior by quasi-anisotropic finite elements. Salem (72) analyzed reinforced concrete box culverts under high embankments and planar frames using the initial stress method. He considered bond-slip action between steel reinforcement and concrete. In the last decade several studies have adapted the finite element technique to the investigation of the behavior of concrete structures. A comprehensive list of references on the subject may be found in a state-of-the-art paper by Scordelis (73).

Phillips and Zienkiewicz (74) recently analyzed reinforced concrete structures using the finite element technique. Tensile cracking, comprehensive strength of concrete, and yield of steel reinforcement were studied. They used isoparametric elements and special elements to simulate reinforcement. The bond-slip between steel and concrete was not considered. Incremental, nonlinear finite element programs were developed which used both variable and constant stiffness methods of solutions. Several realistic concrete structures were analyzed and their solutions were compared with experimental results.

CHAPTER III

FINITE ELEMENT IDEALIZATION

3.1 General

Most analytical investigations of structures have been on isolated structural elements. More recently, it has been recognized that attention needs to be focused on integrated structural systems. The success of analytical solutions depends on the selection of realistic idealizations of both the structural system and the behavior of materials. The recent advances in digital computers and numerical methods, such as the finite element method, provide accurate solutions for many complex problems.

In the finite element analysis of a continuum, the continuous body is represented by an assemblage of discrete elements connected at various nodal points to make up a discretized model of the body. Simple displacement functions can be chosen to approximate the variation of the actual displacement field over each discrete element. A variational principle of mechanics is usually employed to obtain a set of equilibrium equations for each element. Then the equilibrium equations for the entire system are obtained and modified for the given displacement boundary conditions. The overall behavior of the continuum is represented by a set of linear algebraic equations. The process of connecting the elements to form the discretized model is a topological one and is independent of the physical nature of the problem and its linearity or nonlinearity.

In order to achieve a realistic modeling of the base plate system by the finite element method, it is necessary to examine the behavior of the steel plate, the response of the concrete footing and bond-link elements connecting the plate and the footing. One major difficulty in attempting the analysis of concrete structures is the continuous change in the topology which results from cracking of concrete under increasing load. Concrete is a nonhomogeneous material and it is difficult to idealize its actual behavior. Constitutive relationships and failure criteria of concrete under combined stress states are still incomplete.

The difficulties encountered in the analysis of concrete structures and the determination of the material constants were eliminated through the proper idealization of the structure and the material properties. The choice of the material and the structural idealization is governed by the structural system, the required accuracy of results, and method of solution employed.

In the finite element analysis circular base plates and cylindrical footings loaded thorugh circular columns are considered. In the axisymmetric system the vertical and radial components of displacement in any plane section of the body along its axis of symmetry define completely the state of strain and stress. Furthermore, the stresses and strains do not vary in the tangential direction. Thus, from a mathematical point of view, the axisymmetric system is two-dimensional in nature.

3.2 Material Behavior

3.2.1 Concrete

The tensile strength of concrete was found experimentally to be about 10 percent of the compressive strength. Concrete behaves in a brittle fashion under various tensile stress states. Test results of Kupfer et al. (31) on the strength of concrete under biaxial stresses are shown in Figure 2. These results show that the tensile strength of concrete is not strongly affected by the presence of tensile stresses in the other direction. Also, the tensile strength of concrete in one direction is not greatly affected if compressive stress is present in the other direction. Therefore, the maximum principal stress was used as a criterion for cracking of concrete in this study. It is assumed that tensile cracks occur normal to the direction of the principal stress. After a crack has formed in the element, for all subsequent loadings tensile stresses cannot be transmitted across the crack, whereas the compressive stresses of concrete remain unchanged in the direction parallel to the crack. However, the material is capable of transmitting shear stresses parallel to the crack. Shear stresses can be carried across the crack by mechanical interlock. It was assumed that the shear carrying capacity and the shear modulus of the cracked material is 50 percent of values for uncracked concrete. On further loading, if tensile stresses exist parallel to the crack, the maximum principal stress is used as a criterion for cracking in that direction.

The strength of concrete under multiaxial compressive stress is higher than that under uniaxial stress as a result of the compressive confinement which slows the propagation of microcracks. Kupfer et al. (31) reported that Poisson's ratio remains constant up to 75 percent of the ultimate load. In this study Poisson's ratio is assumed to be constant. The uniaxial stress-strain curve for concrete was idealized by an approximate linear piecewise curve as shown in Figure 3.

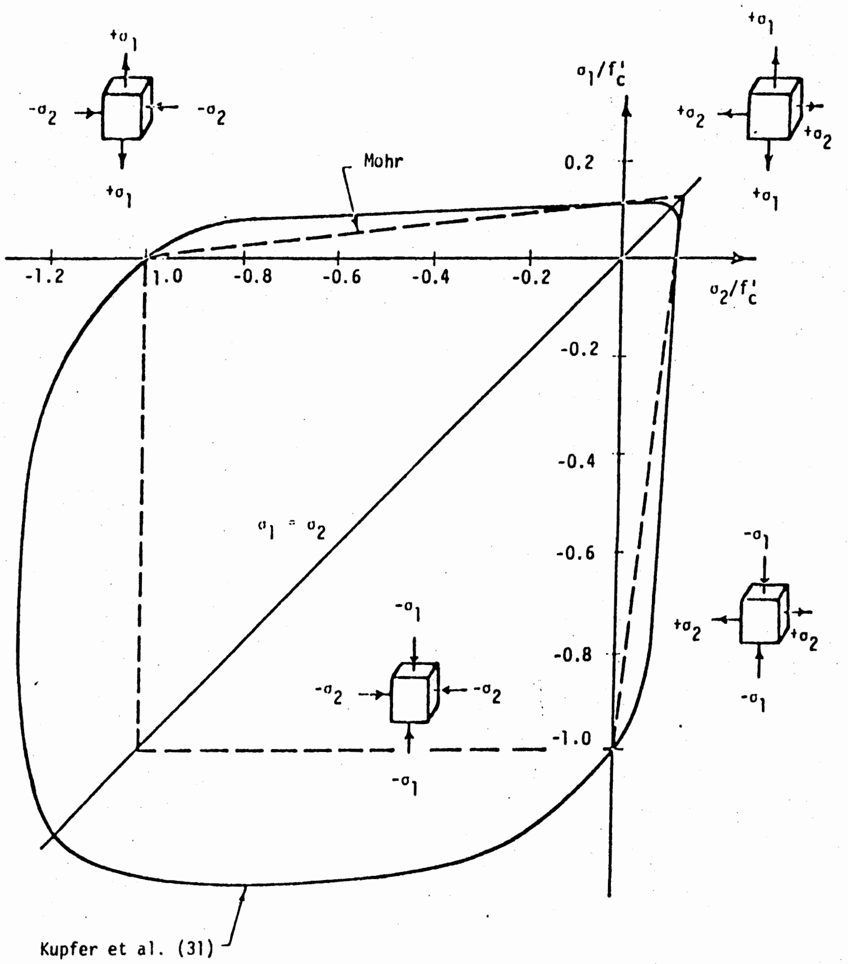


Figure 2. Biaxial Strength of Concrete

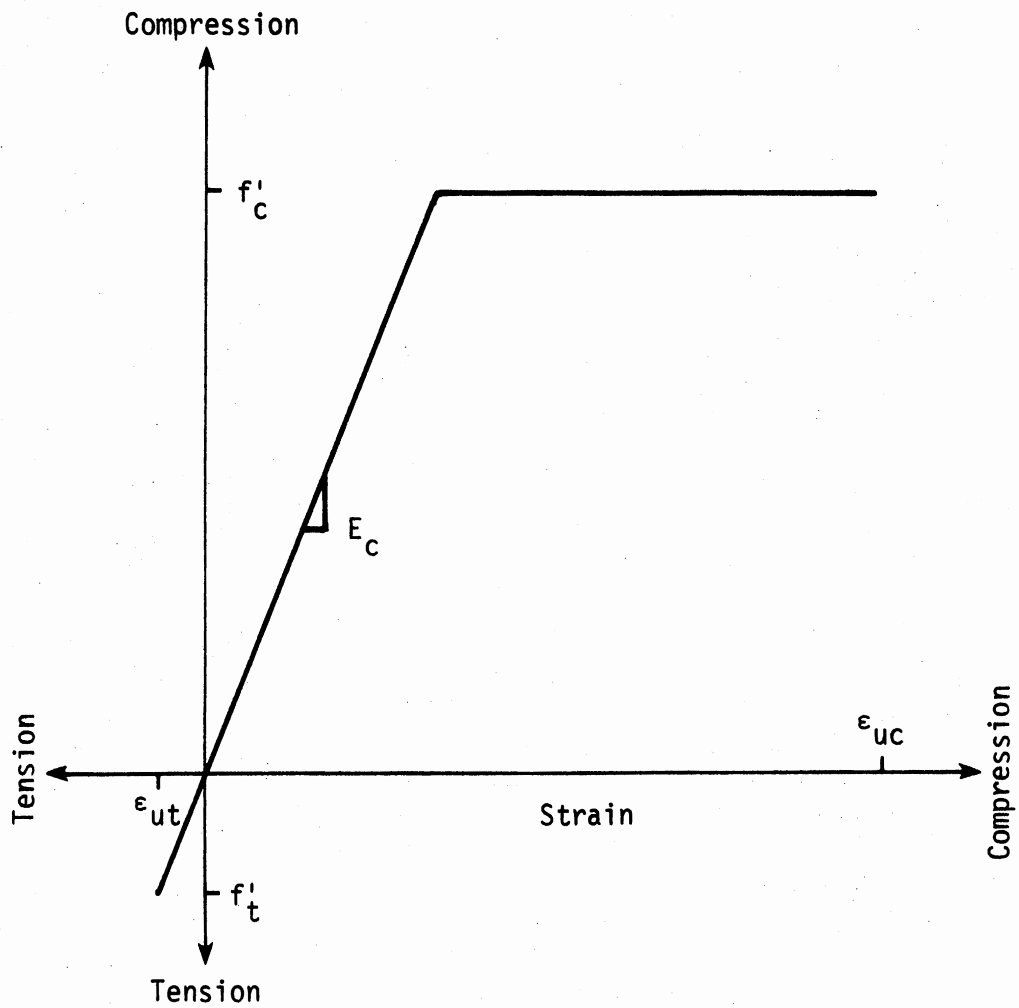


Figure 3. Idealized Stress-Strain Curve for Concrete

Mikkola and Schonrich (75) obtained a close agreement with experimental results of Kupfer et al. (31) by using octahedral stresses. The octahedral stresses can be expressed as follows:

$$\tau_{oct} + \sqrt{2}\,\frac{\beta - 1}{2\beta - 1}\,\sigma_m - \frac{\sqrt{2}}{3}\,\frac{\beta}{2\beta - 1}\,f'_c = 0 \qquad (3.1)$$

where

τ_{oct} = octahedral shear stress;

σ_m = octahedral normal stress;

f'_c = uniaxial compressive strength of concrete; and

β = ratio of biaxial compressive strength of concrete to uniaxial compressive strength in which $\sigma_1/f'_c = \sigma_2/f'_c$.

The above expression is used as a yield criterion for concrete to indicate the boundary between linear and nonlinear behavior in the compression region. The yield criterion is conservative and has been used in this study for elements under multiaxial compressive stresses. Concrete will crush if the equivalent plastic strain, $\bar{\varepsilon}_p$, exceeds the ultimate compressive strain, where the incremental equivalent plastic strain is given by

$$d\bar{\varepsilon}_p = [\frac{1}{3}\,(2d\varepsilon_r^{p^2} + 2d\varepsilon_z^{p^2} + 2d\varepsilon_\theta^{p^2} + d\gamma_{rz}^{p^2})]^{1/2} \qquad (3.2)$$

where

$d\varepsilon_r^p$, $d\varepsilon_z^p$, $d\varepsilon_\theta^p$ = plastic incremental strain in cylindrical coordinates; and

$d\gamma_{rz}^p$ = plastic incremental shear strain in cylindrical coordinates.

3.2.2 Steel

The behavior of steel base plates is idealized as an elastic-perfectly plastic material as shown in Figure 4, where σ_{yd}, ε_{yd} are the yield stress and yield strain, respectively. The material properties can be determined directly from the uniaxial stress-strain of the material. The von Mises yield criterion, widely used for steel, was adopted in this study. The yield surface can be expressed as follows:

$$F = [\frac{1}{2}(\sigma_x - \sigma_y)^2 + \frac{1}{2}(\sigma_y - \sigma_z)^2 + \frac{1}{2}(\sigma_z - \sigma_x)^2 + 3(\tau_{yz}^2 + \tau_{zx}^2 + \tau_{xy}^2)]^{1/2} - \sigma_{yd} \quad (3.3)$$

where

F = yield surface;

σ_x, σ_y, σ_z = normal stress component;

τ_{xy}, τ_{yz}, τ_{xz} = shear stress component; and

σ_{yd} = uniaxial stress at yield.

3.3 Elemental Stiffness Matrices

3.3.1 General

Cost and accuracy are the major factors to be considered in the finite element analysis of structures. The use of higher order elements or a fine mesh is restricted to cases in which higher accuracy is a necessity. This will result in increasing the complexity of an element and increasing the computational time. The linear rectangular element with four integrating points was used in this study to idealize steel base plates and concrete footings. A bond-link element was used to simulate the bond-slip phenomenon between the concrete footing and the steel plate.

A number of alternative methods are available for the formulation of elemental stiffness matrices. The variational approach based on the principal of minimum potential energy is adopted here. A comprehensive review

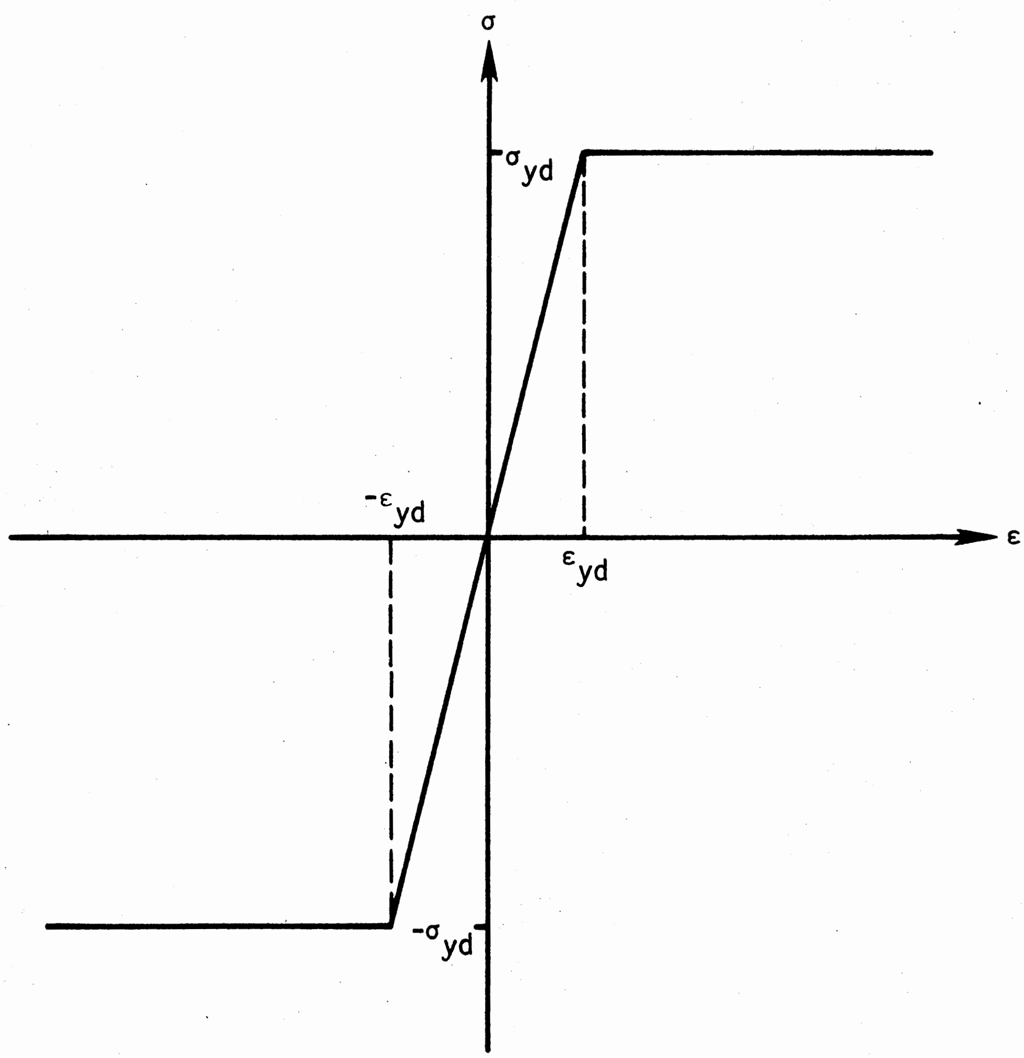


Figure 4. Idealized Stress-Strain Curve for Steel

of the theory and application of the methods is given in textbooks (76) (77). The formulation procedure can be summarized as follows: the basic step in determining the properties of the element is to define the displacement field $\{\hat{u}\}$ in terms of the nodal displacements $\{u\}$ by a set of equations given as

$$\{\hat{u}\} = [N]\{u\} \tag{3.4}$$

where [N] is the matrix of shape function. The strain-displacement relations in the element can be expressed as follows:

$$\{\varepsilon\} = [B]\{u\} \tag{3.5}$$

where $\{\varepsilon\}$ is the strain vector, and [B] is the strain matrix. The stresses may be determined from a constitutive relationship in the form

$$\{\sigma\} = [D]\{\varepsilon\} \tag{3.6}$$

where $\{\sigma\}$ is the stress vector, and [D] is the material property matrix. For distributed forces the potential energy can be expressed as

$$PE = \int_s \{U\}^T\{q\}\, ds \tag{3.7}$$

where $\{q\}$ is the vector of force intensity. The strain energy in the elements is the integral of internal work

$$SE = \int_v d\{\varepsilon\}^T\{\sigma\}\, dv \tag{3.8}$$

The total potential energy Π of the element is the sum of its strain energy and potential energy. Thus:

$$\Pi = \int_v d\{\varepsilon\}^T\{\sigma\}\, dv - \int_s \{U\}^T\{q\}\, ds$$

or

$$\Pi = \int_v \{U\}^T[B]^T[D][B]\{U\}\, dv - \int_s \{U\}^T\{N\}^T\{q\}\, ds \tag{3.9}$$

Application of the principle of minimum potential energy in order to ensure equilibrium will result in the desired element stiffness matrix [K] and the nodal force vector [f].

$$[K] = \int_V [B]^T[D][B]\, dv \tag{3.10}$$

$$[f] = \int_S \{N\}^T\{q\}\, ds \tag{3.11}$$

The force displacement relation for the overall structure can be obtained by the summation of element stiffnesses after modifying for boundary conditions as follows:

$$\{F\} = [K_S]\{U_S\} \tag{3.12}$$

where

$\{F\}$ = generalized nodal forces;

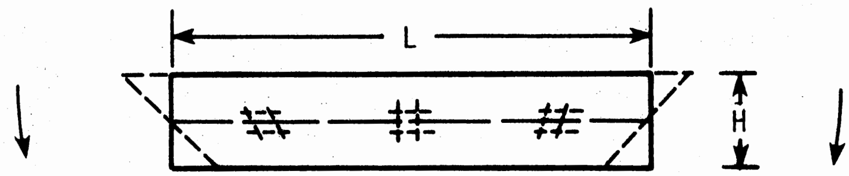
$[K_S]$ = stiffness matrix of structure; and

$\{U_S\}$ = generalized nodal displacements.

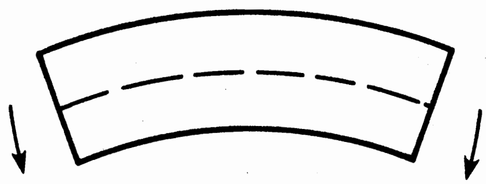
The unknown displacements can be obtained by solving the above equation. Thus the nodal displacements, strains, and stresses can be computed for each element from Equations (3.4), (3.5), and (3.6), respectively.

3.3.2 Linear Rectangular Element

The linear edge displacement rectangular element produces displacements due to direct stresses. However, when it is used in problems in which the bending behavior is important, such as the bending of the base plate, a very fine mesh is needed to avoid the effect of parasitic shear as shown in Figure 5. Zienkiewicz and Too (78) used a reduced integration technique to count for parasitic shear in plate and shell elements. Wilson et al. (79) added two quadratic shape functions to the basic shape function to eliminate parasitic shear from the behavior of the lower order



(a) Constraint Mode



(b) True Node

Figure 5. Parasitic Shear Stresses Induced in a Linear Element Under Bending Mode (78)

elements. The only drawback in this procedure is that displacements along the common edges are not solely dependent on the displacements of the terminal nodes. The details of the derivation of the element stiffness matrix can be summarized as follows: the rectangular element with its natural coordinates are shown in Figure 6. The natural coordinates can be related to the global cartesian coordinates through the shape functions N_i (ξ,η). The coordinate transformation can be written in the form

$$r(\xi,\eta) = \sum_{i=1}^{4} N_i \bar{r}_i$$

$$z(\xi,\eta) = \sum_{i=1}^{4} N_i \bar{z}_i \qquad (3.13)$$

where $\bar{r}_i$ and $\bar{z}_i$ are the global coordinates at the nodal points. The shape functions can be expanded in the form

$$N_1 = \frac{1}{4}(1 - \xi)(1 - \eta)$$

$$N_2 = \frac{1}{4}(1 + \xi)(1 - \eta)$$

$$N_3 = \frac{1}{4}(1 + \xi)(1 + \eta)$$

$$N_4 = \frac{1}{4}(1 - \xi)(1 + \eta) \qquad (3.14)$$

The displacement field within the element is approximated by the shape functions as follows:

$$\hat{u}(\xi,\eta) = [N]\{u\}$$

$$\hat{v}(\xi,\eta) = [N]\{v\} \qquad (3.15)$$

where

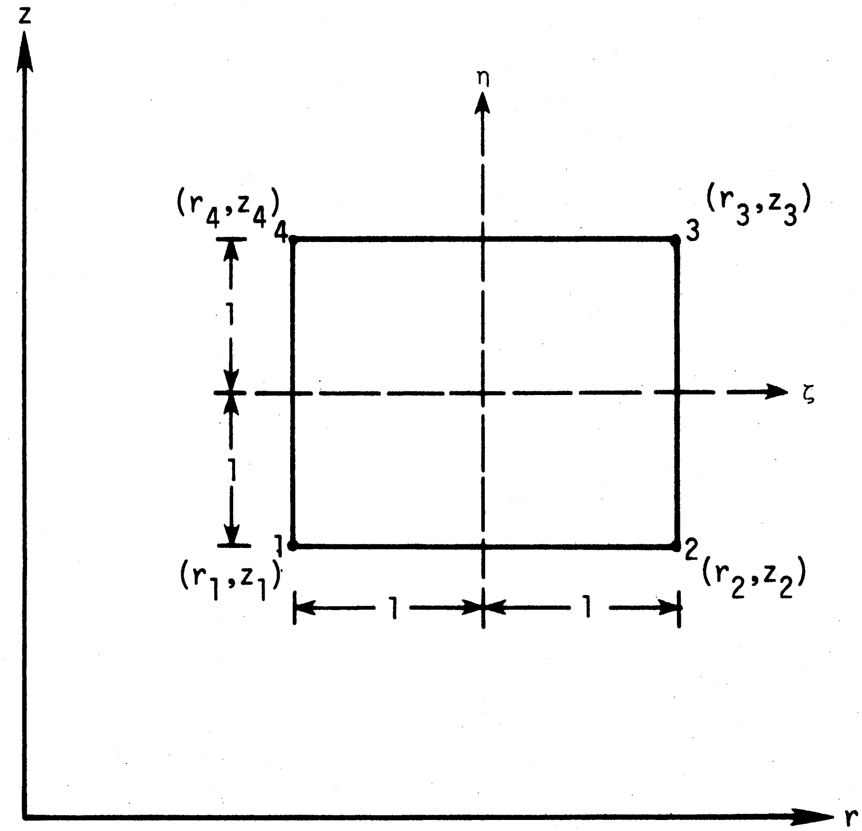


Figure 6. Linear Rectangular Element

[N] = matrix of shape function;

{u} = column vector of nodal displacement in the r direction; and

{v} = column vector of nodal displacement in the z direction.

For axisymmetric problems the strain displacement relations are

$$\varepsilon_r = \frac{\partial u}{\partial r} = [N,_r]\{u\}$$

$$\varepsilon_z = \frac{\partial v}{\partial z} = [N,_z]\{v\}$$

$$\varepsilon_\theta = \frac{u}{r} = \frac{1}{r}[N]\{u\}$$

$$\gamma_{rz} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = [N,_z]\{u\} + [N,_r]\{v\}$$

where

$[N,_r]$ = shape function derivative with respect to r; and

$[N,_z]$ = shape function derivative with respect to z.

In matrix form,

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \gamma_{rz} \end{Bmatrix} = \begin{bmatrix} [N,_r] & [0] \\ [0] & [N,_z] \\ [\frac{N}{r}] & [0] \\ [N,_z] & [N,_r] \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} \qquad (3.16)$$

which can be expressed as

$$\{\varepsilon\} = [B]\{u\}$$

where

$\{\varepsilon\}$ = column vector of strain;

[B] = strain matrix; and

{u} = vector of nodal displacements.

The stress-strain relation can be written as

$$\{\sigma\} = [D]\{\varepsilon\}$$

where $\{\sigma\}$ is the stress vector and $[D]$ is the material property matrix.

Using the principal of minimum potential energy the equilibrium equations can be expressed as

$$\{F\} = [K]\{U\}$$

where

$\{F\}$ = force vector;

$[K]$ = element stiffness matrix; and

$\{U\}$ = displacement vector.

A four point, numerical integration technique based on the Gauss quadrant rule is employed to obtain Equation (3.10).

The elemental stiffness matrix for bending elements can be obtained by using a simple technique in which

$$[K] = \int_V [B]^T[D_1][B]\, dv + \int_V [B]^T[D_2][B]\, dv \qquad (3.17)$$

where

$[D_1]$ = partitioned material property matrix containing no shear modulus; and

$[D_2]$ = partitioned material property matrix containing shear modulus terms only.

The first part is integrated about the four integration points based on the Gauss quadrant rule. The second part is integrated only about the center of the element.

3.3.3 Stiffness Matrix of Bond-Link Element

The bond-link element is represented in the r,z plane and is shown in Figure 7. To compute the stiffness matrix, the stiffness coefficient K_h and K_v are assumed in the r,z directions, respectively. The stress-strain relation in the matrix notation can be expressed in the form

$$\{\sigma\} = [D_b]$$

$$\begin{Bmatrix} \sigma_r \\ \sigma_z \end{Bmatrix} = \begin{bmatrix} K_h & 0 \\ 0 & K_v \end{bmatrix} \begin{Bmatrix} \Delta_r \\ \Delta_z \end{Bmatrix} \tag{3.18}$$

where Δ_r and Δ_z are the relative displacements between the adjacent steel and concrete nodes. The strain displacement can be verified as

$$\Delta_r = \bar{U}_3 - \bar{U}_1$$

$$\Delta_z = \bar{U}_4 - \bar{U}_2 \tag{3.19}$$

The total deformations in terms of the nodal displacement can be written as

$$\{U_s\} = [B]\{\bar{U}\}$$

$$\begin{Bmatrix} U_{sh} \\ U_{sv} \end{Bmatrix} = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{Bmatrix} \tag{3.20}$$

The stiffness matrix can be evaluated from the relation

$$[K] = [B]^T[D][B] \tag{3.21}$$

Lassker (71) and Salem (72) have found in their studies of reinforced concrete structures that this type of bond mechanism simulated the interaction between steel and concrete quite accurately. In this study the bond-link elements are assumed to separate if there are tensile forces

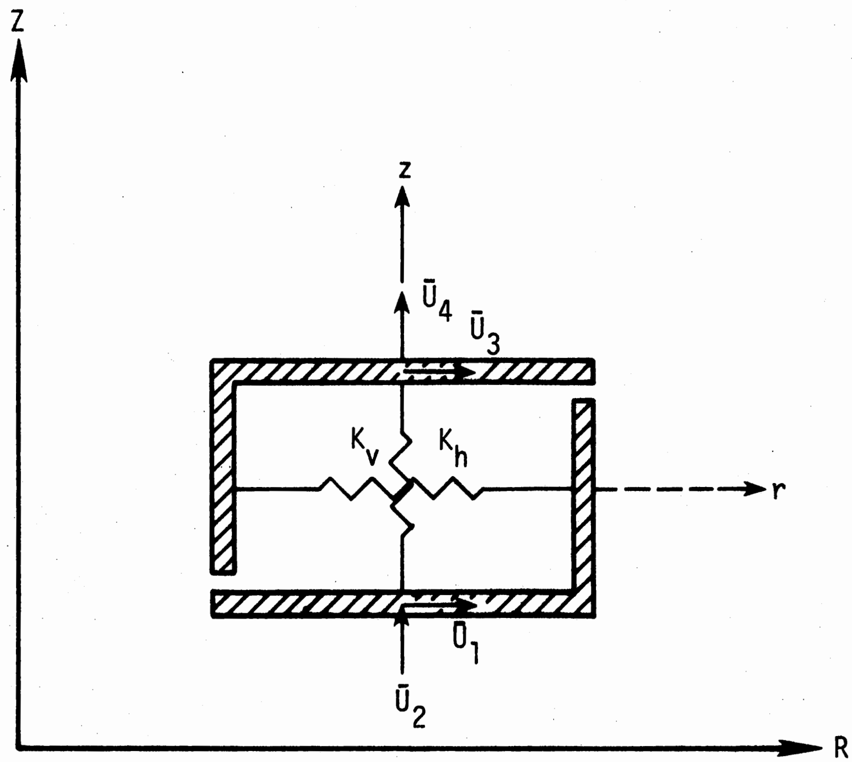


Figure 7. Bond-Link Element

present in the elements, and to slip if the horizontal force component is greater than three times the vertical force component.

CHAPTER IV

METHOD OF SOLUTION

4.1 General

One of the most important applications of the finite element method is in the analysis of nonlinear structures. Nonlinearities occur in three different forms. The first is material nonlinearity which results from nonlinear constitutive laws. The second is geometric nonlinearity which is encountered when a structure experiences large deformations. The third is the combined geometric and material nonlinearity. Only the material nonlinearity is considered in this study because of the relatively small deformations in the base plate system.

The solution of material nonlinear problems using the finite element method will result in a set of nonlinear simultaneous equations which may be written in the form

$$[K]_n \{U\} = \{F\} \tag{4.1}$$

The nonlinearity occurs in the stiffness matrix $[K]_n$ which is a function of the nonlinear material properties. The coefficients of the [D] matrix are strains evaluated according to the stress-strain relationship of the constitutive laws. The basic variational approach for obtaining the element stiffness and load matrices for nonlinear problems is the same as presented in section 3.3. The basic techniques used in the solution of nonlinear problems are the incremental, the iterative, and the incremental-iterative procedures. The techniques for the solution of nonlinear problems are discussed in detail in the textbook by Desai and Able (77).

The main advantage of the incremental procedure is its general applicability and the ability to obtain a load-deformation history. On the other hand, the iteration method is easier to use and convergence is achieved faster than in the incremental procedure. The principal disadvantages of the iterative procedure are that the deformations and the stresses can be determined only for the total load and there is no assurance that the method will converge to the exact solution. Because of these limitations, a mixed incremental-iterative procedure which combines the advantages of both the incremental and iterative procedures is widely used. This method tends to minimize the disadvantage of the other procedures. The additional computation effort is justified by the higher accuracy and a more complete description of the load-deformation of the problem.

4.2 Nonlinear Solution

An incremental-iterative procedure for solving the nonlinear problem, based on the initial stress method developed by Zienkiewicz and co-authors (56) (80), is used in this study. The nonlinearity results from the nonlinear form of the constitutive relations of concrete and steel. Relatively small load increments are applied to the structure to predict the actual path of the load-deformation as closely as possible. Assuming a linear strain-displacement relationship, one can obtain a nonlinear solution by iterating until the constitutive laws and equilibrium are satisfied.

The incremental-iterative method is illustrated in Figure 8. The method can be written for the ith increment as follows.

$$[K_i] \quad \{U_i^j\} = \{R^{j-1}\} \tag{4.2}$$

where

$[K_i]$ = incremental stiffness matrix for load step i;

$\{U_i^j\}$ = incremental displacement vector for load step i and iteration j; and

$\{R^{j-1}\}$ = residual load vector computed from previous iteration j-1.

The residual load vector is caused by the excessive stresses (σ_{ex}) that the element can no longer sustain at the current strain level because of cracking and yielding of the materials. As the result of cracking of concrete, the tangential stiffness matrix $[K_i]$ is computed at the beginning of each load step and used to analyze the structure during the iterations for that load increment. The iterative procedure is terminated when convergence is achieved. The choice of the criterion for convergence may be based on the degree of approximation desired, acceptable accuracy, and financial feasibility. It is cumbersome to check and compute the residuals or displacements for each degree of freedom. In this study the norm of the applied load vector {F} and the norm of the residual load vector {R} are computed during the iterative solution. Convergence is assumed to occur when the norm of the residuals to the norm of the applied load is less than a preselected convergence factor which can be written as

$$\frac{||F_i||}{||R_i||} \leq t \tag{4.3}$$

$||F_i||$ and $||R_i||$ are norm to the applied loads and the residuals, and t is a convergence factor of about 0.01 or 0.02.

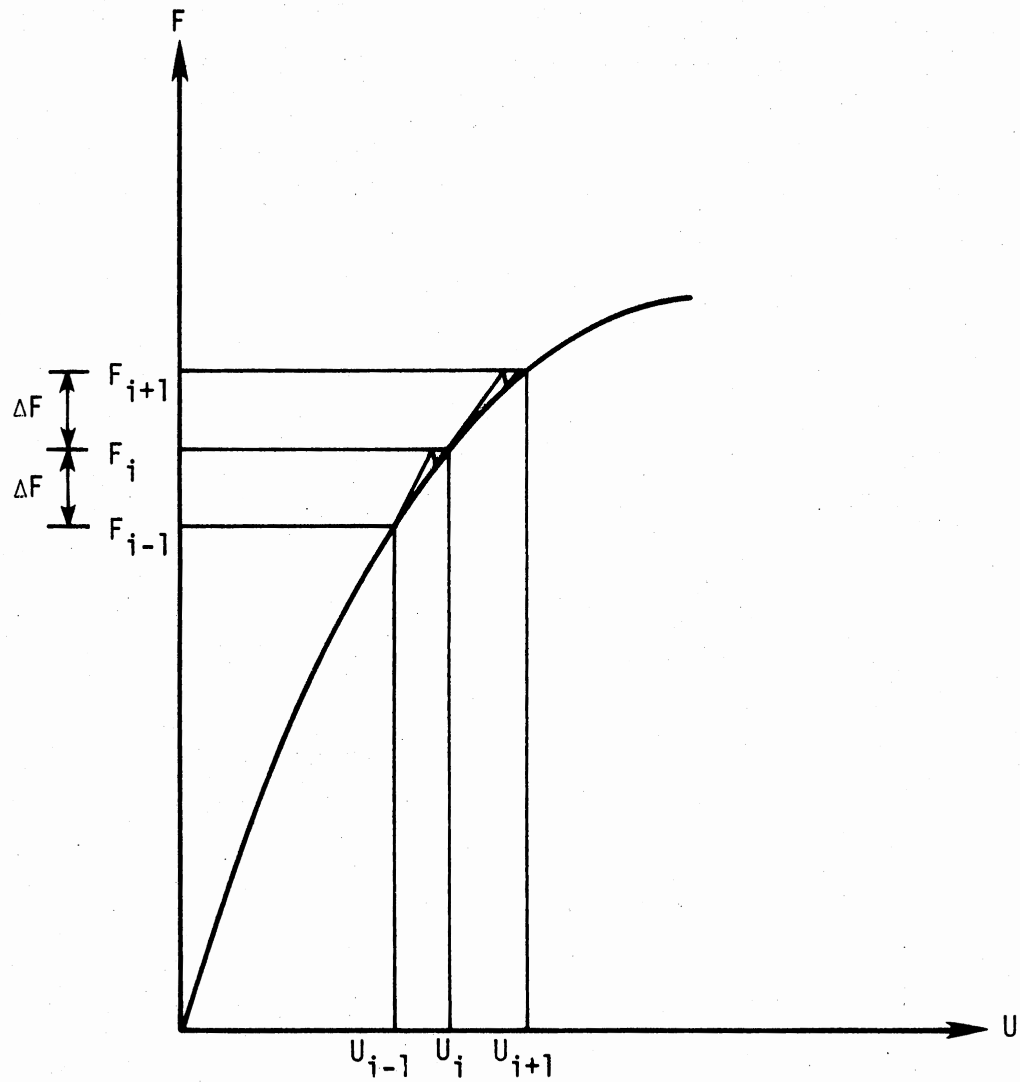


Figure 8. Incremental-Iterative Procedure

The combination of both the incremental and iterative technique, in which the stiffness matrix is updated at the beginning of the load step, is believed to be the most economical of all available procedures of solving nonlinear material problems (73).

The over-relaxation method proposed by Nayak and Zienkiewicz (81) to improve convergence has not been used in this study. This technique caused divergence of the solution of concrete structures (72) (73).

4.3 Constitutive Laws of Incremental Plasticity

A law defining the limit of elastic behavior under any possible combination of stress is known as a criterion of yielding. Mathematically it is expressed by a surface in the stress space. The general form of the yield surface is in the form

$$F(\{\sigma\}, \{\varepsilon_p\}, \kappa) = 0 \tag{4.4}$$

where $\{\sigma\}$ contains the relevant stress components, $\{\varepsilon_p\}$ is the accumulated plastic strain, and κ is the hardening parameter which describes the modification in the yield surface during the plastic flow. In this equation $F < 0$ indicates an elastic state, $F = 0$ denotes a plastic state.

The flow rule relates the plastic strain increments $d\{\varepsilon_p\}$ to stresses and their increments. The plastic potential Q to which the normality rule is applicable can be expressed as

$$Q(\sigma, \varepsilon_p, \kappa_o) = 0 \tag{4.5}$$

If the plastic potential is identical with the yield surface $F \equiv Q$, the flow rule is referred to as the associated flow rule. Then the normality rule can be expressed as

$$d\{\varepsilon_p\} = d\lambda \frac{dF}{d\{\sigma\}} = d\lambda\{N\} \tag{4.6}$$

where dλ is a non-negative constant to be determined, and {N} is the normal to the yield surface.

The incremental strain is separable into elastic and plastic portions during an infinitesimal increment of stress

$$d\{\varepsilon\} = d\{\varepsilon_e\} + d\{\varepsilon_p\} \tag{4.7}$$

The total differential of the yield function is

$$dF = \{\frac{dF}{d\{\sigma\}}\}^T d\{\sigma\} + \{\frac{dF}{d\{\varepsilon_p\}}\}^T d\{\varepsilon_p\} + \frac{dF}{d\kappa} d\kappa = 0 \tag{4.8}$$

Equation (4.8) can be written as

$$dF = \{N\}^T d\{\sigma\} - H\, d\lambda = 0 \tag{4.9}$$

where

$$H = -\frac{1}{d\lambda} [\{\frac{dF}{d\{\varepsilon_p\}}\}^T d\{\varepsilon_p\} + \frac{dF}{d\kappa} d\kappa]$$

Substituting Equation (4.6) into Equation (4.7) gives

$$d\{\varepsilon\} = [D]^{-1} d\{\sigma\} + d\lambda\{N\} \tag{4.10}$$

Pre-multiplying Equation (4.10) by $\{N\}^T[D]$ and eliminating d{σ}, one obtains

$$d\lambda = \frac{\{N\}^T[D]\, d\{\varepsilon\}}{H + \{N\}^T[D]\{N\}} \tag{4.11}$$

Substituting dλ in Equation (4.10) and rearranging the terms, one obtains

$$d\{\sigma\} = ([D] - [D_p])\, d\{\varepsilon\} \tag{4.12}$$

in which

$$[D_p] = \frac{[D]\{N\}\{N\}^T[D]}{H + \{N\}^T[D]\{N\}}$$

Rewriting Equation (4.12)

$$d\{\sigma\} = [D_{ep}]\ d\{\varepsilon\} \tag{4.13}$$

where $[D_{ep}]$ is the elasto-plastic material property matrix which is symmetric and positive definite.

The isotropic hardening rule assumes a uniform expansion of the initial yield surface. The hardening parameter κ depends on the plastic strain hardening and the uniaxial yield stress.

Two definitions have been proposed for κ (82). One definition states that κ is a function of the plastic work only; hence, it is dependent of the strain path such that

$$d\kappa = \{\sigma\}^T \cdot d\{\varepsilon_p\} \tag{4.14}$$

The other definition is based on the assumption that the work hardening parameter κ is a function of the equivalent plastic strain

$$d\kappa = d\bar{\varepsilon}_p \tag{4.15}$$

The two definitions of κ lead to identical results for von Mises yield criterion. The work hardening is more general and is used by Nayak and Zienkiewicz (80). They showed that for isotropic hardening the parameter H in Equation (4.9) is the slope of the uniaxial stress-equivalent plastic strain curve.

The failure criteria can be expressed in terms of the three invariant quantities (σ_m, $\bar{\sigma}$, ϕ) as suggested by Nayak and Zienkiewicz (80) as

$$\sigma_m = \frac{J_1}{3} = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z)$$

$$\bar{\sigma} = \sqrt{J_2} = [\frac{1}{2}(S_x^2 + S_y^2 + S_z^2) + \tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2]^{1/2}$$

$$\phi = \frac{1}{3}\sin^{-1}[\frac{-3\sqrt{3}}{2}\frac{J_3}{3}] \qquad -\frac{\pi}{6} < \phi < \frac{\pi}{6} \qquad (4.16)$$

where

$$J_1 = \sigma_x + \sigma_y + \sigma_z$$

$$J_2 = \frac{1}{6}[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)^{1/2}$$

$$J_3 = S_x S_y S_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - S_x\tau_{yz}^2 - S_y\tau_{zx}^2 - S_z\tau_{xy}^2$$

$$S_x = \sigma_x - \sigma_m$$

$$S_y = \sigma_y - \sigma_m$$

$$S_z = \sigma_z - \sigma_m$$

The normal to the yield surface {N} can be obtained in the form

$$\{N\} = \frac{\partial F}{\partial\{\sigma\}} = \frac{\partial F}{\partial\sigma_m}\frac{\partial\sigma_m}{\partial\{\sigma\}} + \frac{\partial F}{\partial\bar{\sigma}}\frac{\partial\bar{\sigma}}{\partial\{\sigma\}} + \frac{\partial F}{\partial\phi}\frac{\partial\phi}{\partial\{\sigma\}}$$

$$\{N\} = C_1\{a_1\} + C_2\{a_2\} + C_3\{a_3\} \qquad (4.17)$$

where

F = yield surface

$$\{a_1\} = \frac{\partial\sigma_m}{\partial\{\sigma\}}$$

$$\{a_2\} = \frac{\partial\bar{\sigma}}{\partial\{\sigma\}}$$

$$\{a_3\} = \frac{\partial\phi}{\partial\{\sigma\}}$$

for the case in which the stresses are σ_x, σ_y, σ_z and τ_{xy}. The derivatives of the stress invariant are

$$\{a_1\}^T = \frac{1}{3} \quad [1, 1, 1, 0]$$

$$\{a_2\}^T = \frac{1}{2\bar{\sigma}} [S_x, S_y, S_z, 2\tau_{xy}]$$

$$\{a_3\}^T = -\frac{\sqrt{3}}{2\cos 3\phi} \left[\frac{1}{\bar{\sigma}^3} \frac{\partial J_3}{\partial\{\bar{\sigma}\}} - \frac{3J_3}{\bar{\sigma}^4} \frac{\partial\bar{\sigma}}{\partial\{\phi\}}\right]$$

where

$$\frac{\partial J_3}{\partial\{\sigma\}} = \begin{Bmatrix} S_y S_z - \tau_{yz}^2 \\ S_x S_z - \tau_{xz}^2 \\ S_x S_y - \tau_{xy}^2 \\ 2(\tau_{yz}\tau_{xz} - S_z\tau_{xy}) \end{Bmatrix} + \frac{1}{3} \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{Bmatrix} \bar{\sigma}^2 \tag{4.18}$$

The values of C_1, C_2, and C_3 are constants dependent on the failure criteria. For von Mises yield criterion the constants are:

$$C_1 = 0, \; C_2 = \sqrt{3}, \; C_3 = 0 \tag{4.19}$$

The constants for the octahedral shear yield criterion given in Equation (3.3) are

$$C_1 = \frac{\beta - 1}{2\beta - 1}, \; C_2 = \frac{1}{\sqrt{3}}, \; C_3 = 0 \,. \tag{4.20}$$

4.4 Evaluation of Excessive Stresses

In the analysis of problems with nonlinear stress-strain laws, the stiffness matrix is constructed by using material properties established during the previous iteration. After solving for displacements, one can obtain the strain increment $d\{\varepsilon\}$ and the stress increment $d\{\sigma\}$. The total stress $\{\sigma_e\}$ is evaluated by adding the stress increment to previous stresses. As a result of the material nonlinearity, the stresses $\{\sigma_e\}$ are different from the true stresses $\{\sigma\}$. Thus excessive stresses can be evaluated as

$$\{\sigma_{ex}\} = \{\sigma_e\} - \{\sigma\} \tag{4.21}$$

During the transition from elastic to plastic conditions, the intermediate stress value at which the yield begins must be determined. Let F_o be the yield function corresponding to stresses $\{\sigma_o\}$ and F_1 be the yield function corresponding to stresses $\{\sigma_o + \Delta\sigma_e\}$

$$F(\sigma_o) = F_o < 0 \tag{4.22}$$

$$F(\sigma_o + \Delta\sigma_e) = F_1 > 0 \tag{4.23}$$

If plasticity is encountered in the increment, a factor r must be determined, such that

$$F(\sigma_o + r\Delta\sigma_e) = 0 \tag{4.24}$$

By linear interpolation an approximate value r can be found as

$$r_1 = \frac{-F_o}{F_1 - F_o}$$

Nayak and Zienkiewicz (80) obtained a better estimate of r by evaluating the instantaneous position of the yield surface, F_2, where

$$F_2 = -\{N\}^T \cdot \Delta\{\sigma\} \cdot \Delta r_1 \tag{4.25}$$

and an improved value of r can be given by

$$r = r_1 - \frac{F_2}{\{N\}^T \ \Delta\{\sigma\}} \tag{4.26}$$

The value of r is used in the correction of the incremental plastic strains and stresses. The strain increment $\Delta\{\varepsilon\}$ may be separated into two parts: an elastic strain increment $r\Delta\{\varepsilon\}$ which corresponds to a stress point on the yield surface; and a plastic strain increment $(1 - r)$ $\Delta\{\varepsilon\}$ in the same manner the stress increment can be separated.

The excessive stresses can be calculated as

$$\{\sigma_{ex}\} = \int_{r\Delta\{\varepsilon\}}^{\Delta\{\varepsilon\}} [D_p]\, d\{\varepsilon\} \tag{4.27}$$

where $[D_p]$ is the plastic material property matrix. The equation can be written in an approximate form as

$$\{\sigma_{ex}\} = (1 - r)\, [D_p]\, \Delta\{\varepsilon\} \tag{4.28}$$

The error resulting from such an approximation is very small and no correction is necessary in the case in which the departure from the yield surface is small.

The excessive stresses are converted into unbalanced nodal forces as

$$\{P\} = \int_v [B]^T \{\sigma_{ex}\}\, dv \tag{4.29}$$

where $\{P\}$ is the unbalanced forces resulting from the excessive stresses $\{\sigma_{ex}\}$.

4.5 Outline of Computational Steps

The computational procedure can be summarized as follows:

1. Divide the total load vector $\{F\}$ to be applied into suitable small increments $\{\Delta F\}$.

2. Apply a load increment $\{\Delta F\}$ to the structure. Store $\{\Delta F\}$ in the residual load vector $\{R\}$.

3. Assemble the stiffness matrix $[K]$ using geometry and elastic data. Analyze the structure using the load vector $\{F\}$. The Gaussian

elimination procedure is used to solve the linear simultaneous equations. Solve for incremental displacements $\{\Delta U\}$. Update the displacements $\{U\}$.

4. Using the incremental displacement $\{\Delta U\}$, compute incremental strains $\{\Delta\varepsilon\}$ and update the strains $\{\varepsilon\}$.

5. Determine incremental stresses $\{\Delta\sigma\}$ using the incremental strains $\{\Delta\varepsilon\}$ and the current material properties. Add the incremental stresses to the previous stresses to obtain the stress vector $\{\sigma'\}$.

6. Check the stresses against possible transition criteria (criteria for cracking, crushing, and yielding). If none of the transition criteria is achieved, proceed to step 10.

7. Calculate material property matrix [D] based on the current strains and stresses. Determine the stress vector $\{\sigma\}$ at which the element can sustain at this strain level.

8. Obtain the excessive element stresses $\{\sigma_{ex}\}$ by subtracting the stresses $\{\sigma\}$ from $\{\sigma'\}$.

9. Convert the excessive stresses $\{\sigma_{ex}\}$ into unbalanced nodal forces $\{p\}$ for the element.

10. Add the unbalanced nodal forces to the global unbalanced nodal loads $\{R\}$. Calculate a new elemental stiffness matrix.

11. Check all elements repeating steps 4 through 10.

12. Use convergence criteria mentioned in section 4.2 to determine if convergence is achieved. If convergence has not been achieved, perform a new iteration cycle starting from step 3.

13. If convergence has been achieved, apply a new load step starting from step 2.

A flow chart for the computational procedure is summarized and presented in Figure 9.

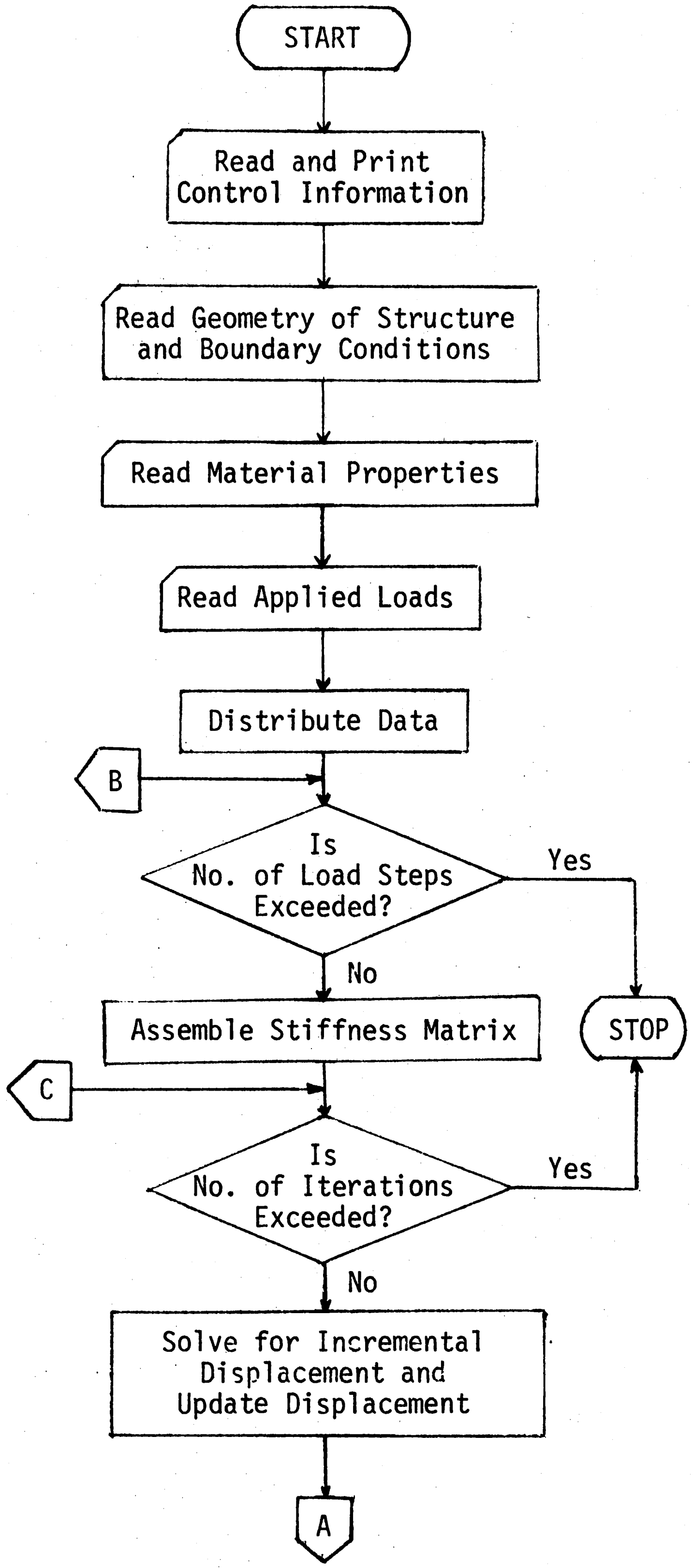


Figure 9. Flow Chart for Computational Steps

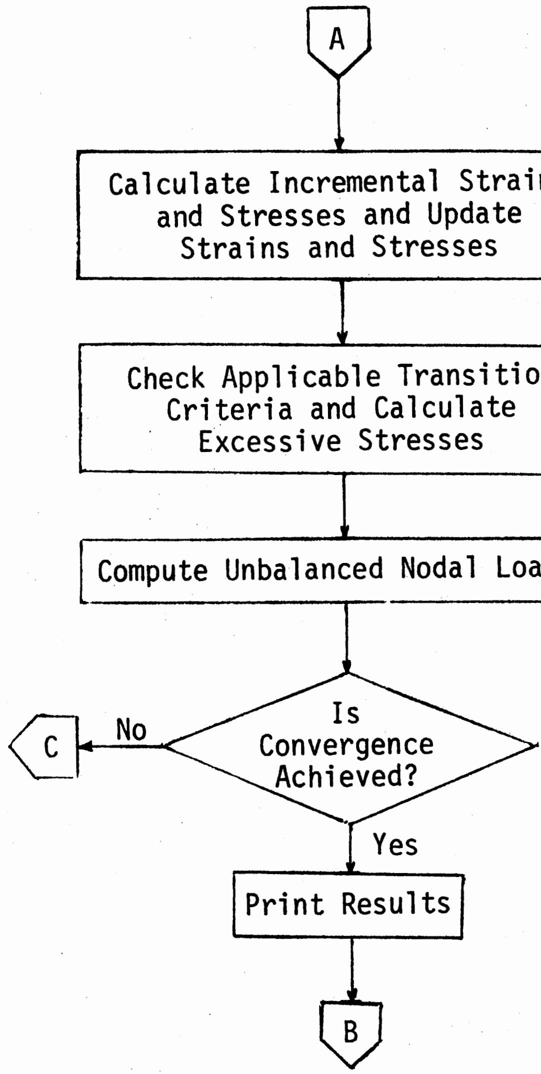


Figure 9. (Continued)

CHAPTER V

ANALYTICAL RESULTS

5.1 General

In order to illustrate the solution capability of the program and to verify the accuracy of the method used in this study, several problems have been solved. Analytical results are compared with experimental results which are presented in Appendix A, and with results obtained by conventional closed form solutions.

In this chapter several problems are described and discussed. Data input and output information is presented in Appendix B and a computer listing for the program is presented in Appendix C. All computations were carried out on the IBM 370/185 computer.

5.2 Example Solutions

5.2.1 Simply Supported Circular Plate

A simply supported circular plate was solved to verify the accuracy of the integration technique used in this study for bending elements in the elastic range of loading. A plate with a radius of 2.61 in. and a thickness of 0.26 in. was selected. The plate was loaded with a concentrated load of 62.83 lb at the center. The plate material had a modulus of elasticity of 10.5×10^6 psi and a Poisson's ratio of 0.33. The exact deflection at the center of the plate is 1.221×10^{-3} in. The plate was idealized by 28 rectangular elements. The central deflection for the bending elements was 1.208 x 10^{-3} in. with a difference of 1 percent. However, the central deflection for the rectangular elements without bending mode was 1.075 x 10^{-3} in. with a difference of 11 percent. This problem demonstrates the adequacy of the integration technique used in this study for bending elements.

5.2.2 Thick Wall Cylinder

A classic case was studied to test the plasticity routine of the computer program. Hodge and White (83) studied an infinitely long thick wall cylinder under internal pressure for the case of elastic-perfectly plastic material using the von Mises yield criterion. The cylinder had an inner radius, a, and an outer radius, 2a. The internal pressure, p, was increased from 0.7 p/k to 1.4 p/k in eight increments, where k is the yield stress in pure shear. The solution was obtained for ten rectantular elements.

The results of the finite element analysis in this study are compared to the solution obtained by Hodge and White in Figures 10 and 11. Figure 10 shows the pressure-external displacement of the cylinder presented in a nondimensionalized form, where G is the shear modulus. Figure 11 shows the distribution of radial, hoop, and axial stresses after the elasto-plastic boundary had propagated to 1.5a. The results of the finite element analysis show excellent agreement with the close form solution of Hodge and White.

5.3 Circular Base Plate System

Eight circular base plate specimens with axisymmetric loading,

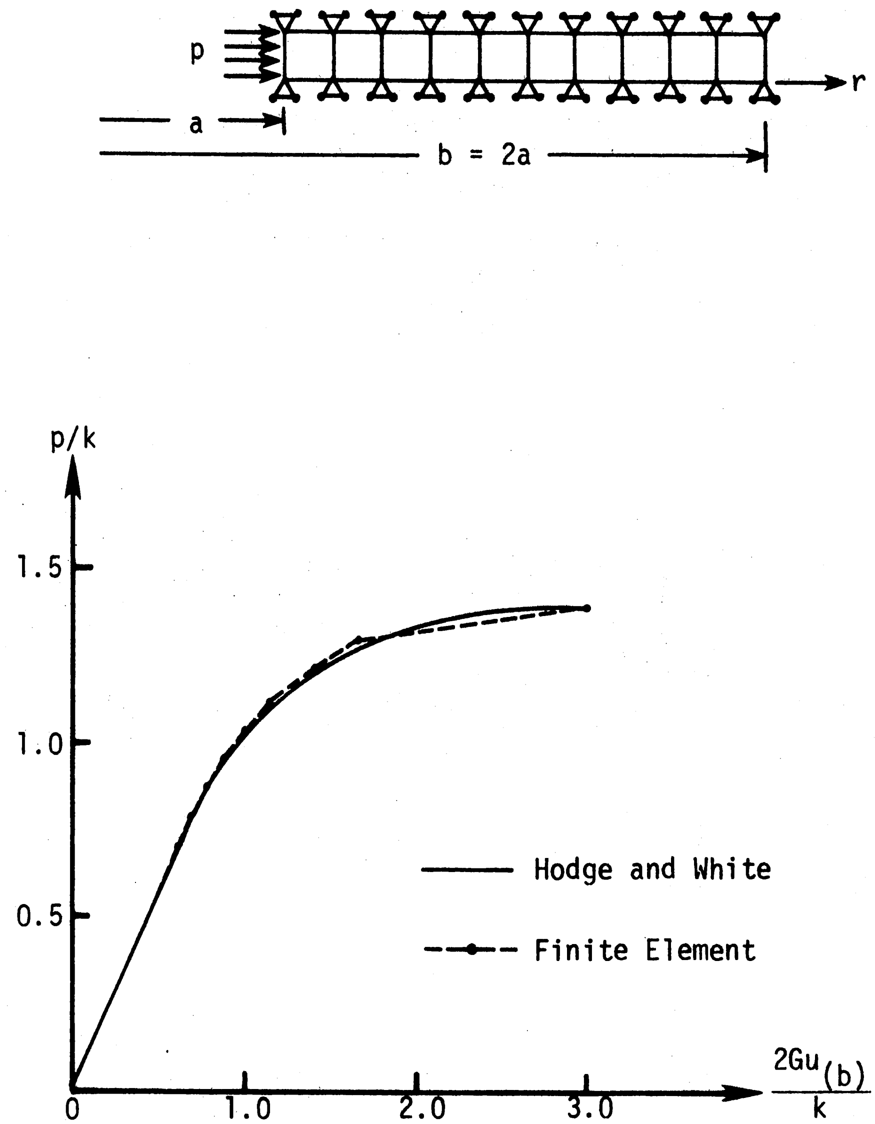


Figure 10. Thick Cylinder Pressure-Displacement Curve

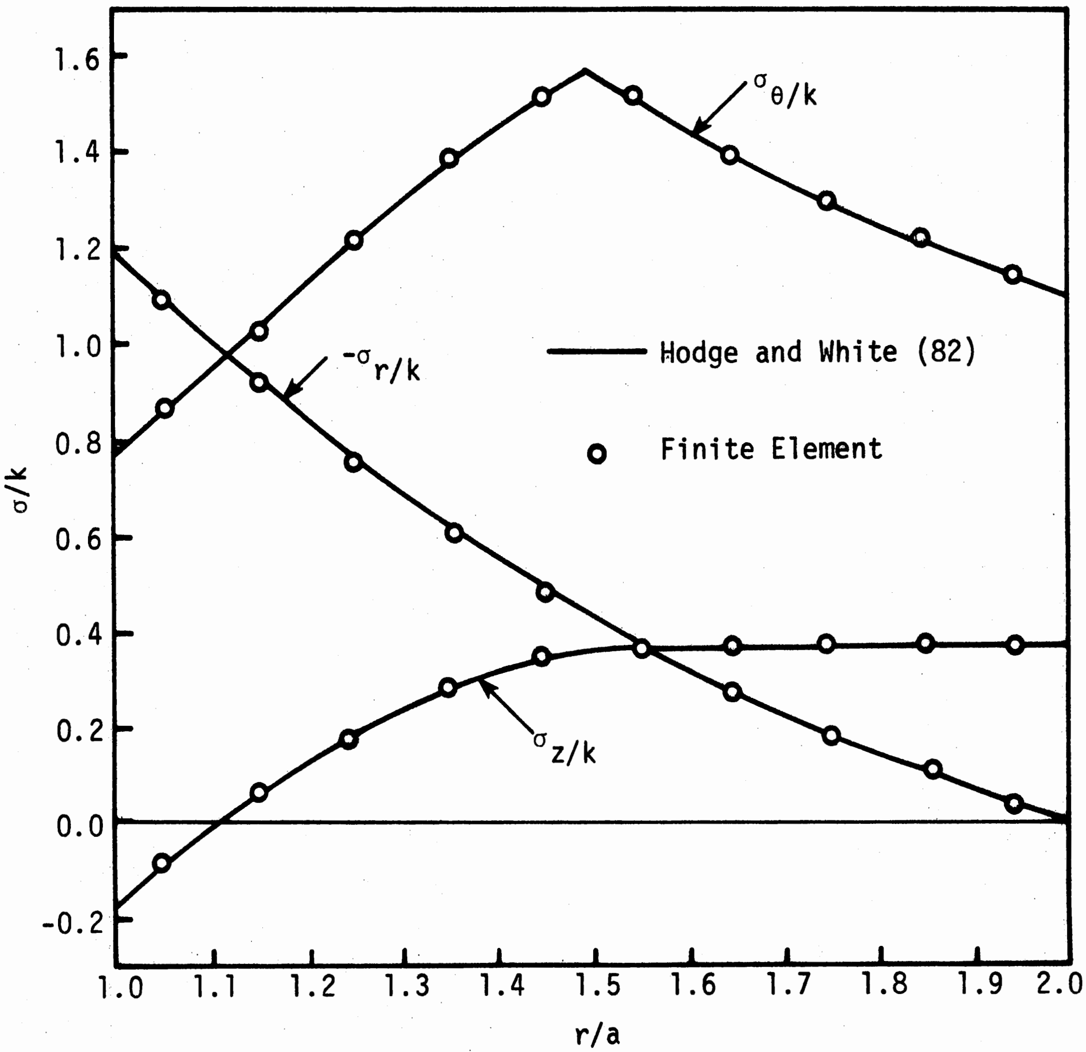


Figure 11. The Distribution of Radial, Hoop, and Axial Stresses When the Radius of the Plastic Zone is 1.5a

boundary, and geometric conditions were studied both experimentally and analytically. The various aspects of the experimental program are presented in detail in Appendix A. The concrete footing portion of the base plate system was idealized by 400 rectangular elements having a height of 1/2 in. and a width of 1/4 in., as shown in Figure 12. The circular steel base plate was connected to the concrete footing by bond-link elements. The stiffness of the bond-link elements was chosen to suppress the relative displacements between the nodes at the bottom surface of the base plate and the nodes at the top surface of the concrete footing. Large numerical values were used for the stiffness of the bond-link elements. The stiffness of the bond-link elements must be chosen with caution because small or extremely large stiffness values may result in erroneous solutions. In this study a combination of constant and variable stiffnesses was used to improve the convergence of the solution.

Analytical load-deformation curves for the eight base plate systems are presented and compared with experimental results in Figures 13 through 20. The vertical displacement of the center of the plates relative to the edge of the plate and the vertical displacement of concrete at the outer edge of the plate relative to the edge of the plate are presented. The predicted ultimate load carrying capacity of the base plate system is in close agreement with experimental results. The load deformation curves of the proposed analytical model compare well with experimental results of this study. The use of a finer mesh, or a more complex material, or smaller load increments might result in better agreement with experimental data. However, the inherent uncertainties associated with the characteristics of the individual experimental specimens may result in some differences when comparisons of this type are made.

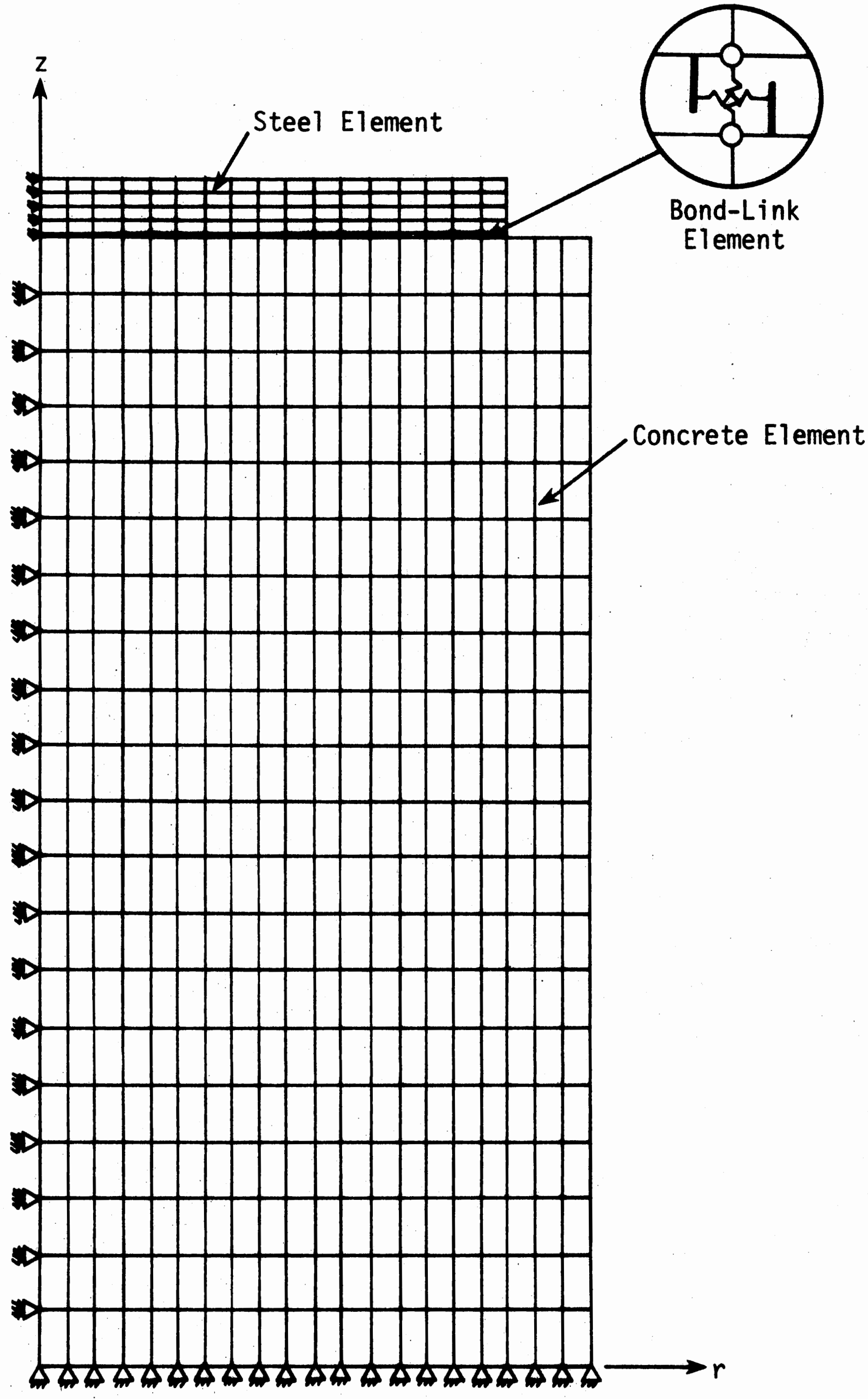


Figure 12. Finite Element Idealization of Circular Base Plate System

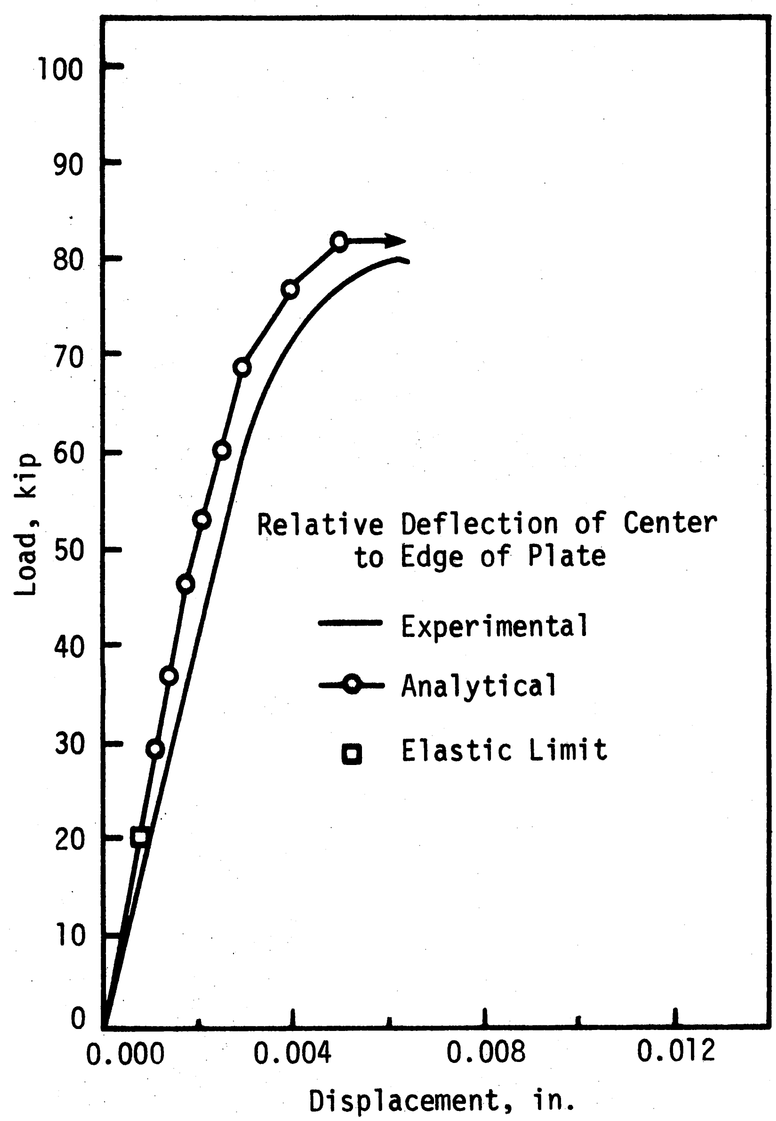


Figure 13. Load-Deformation Curve, Base Plate C1

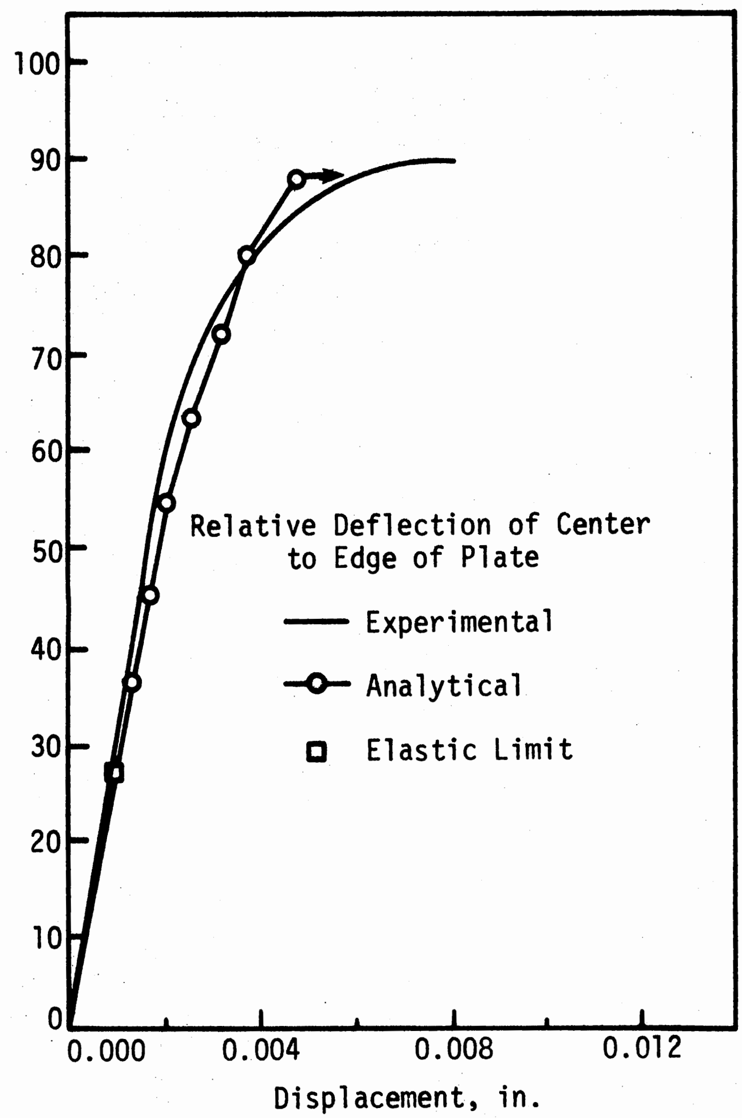


Figure 14. Load-Deformation Curve, Base Plate C2

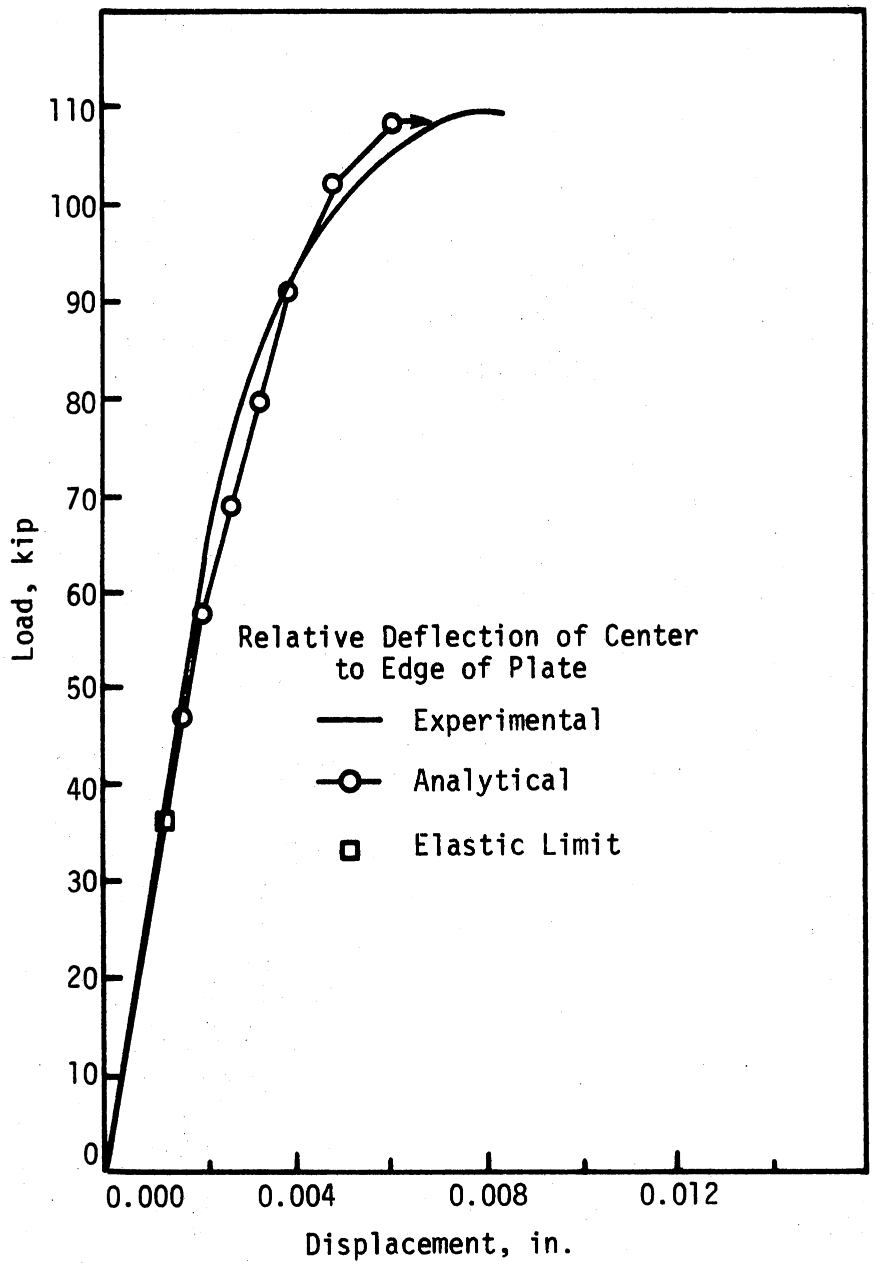


Figure 15. Load-Deformation Curve, Base Plate C3

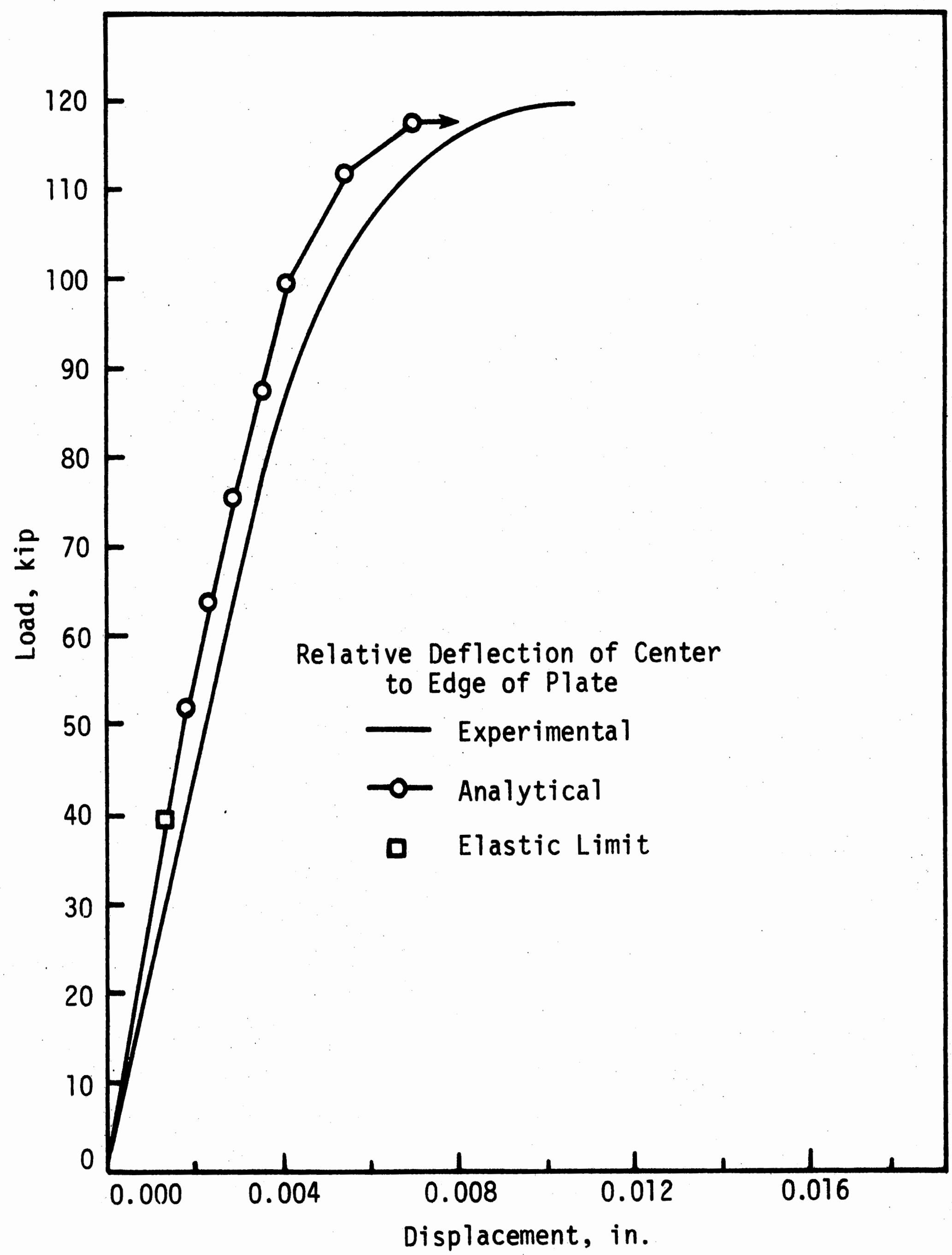


Figure 16. Load-Deformation Curve, Base Plate C4

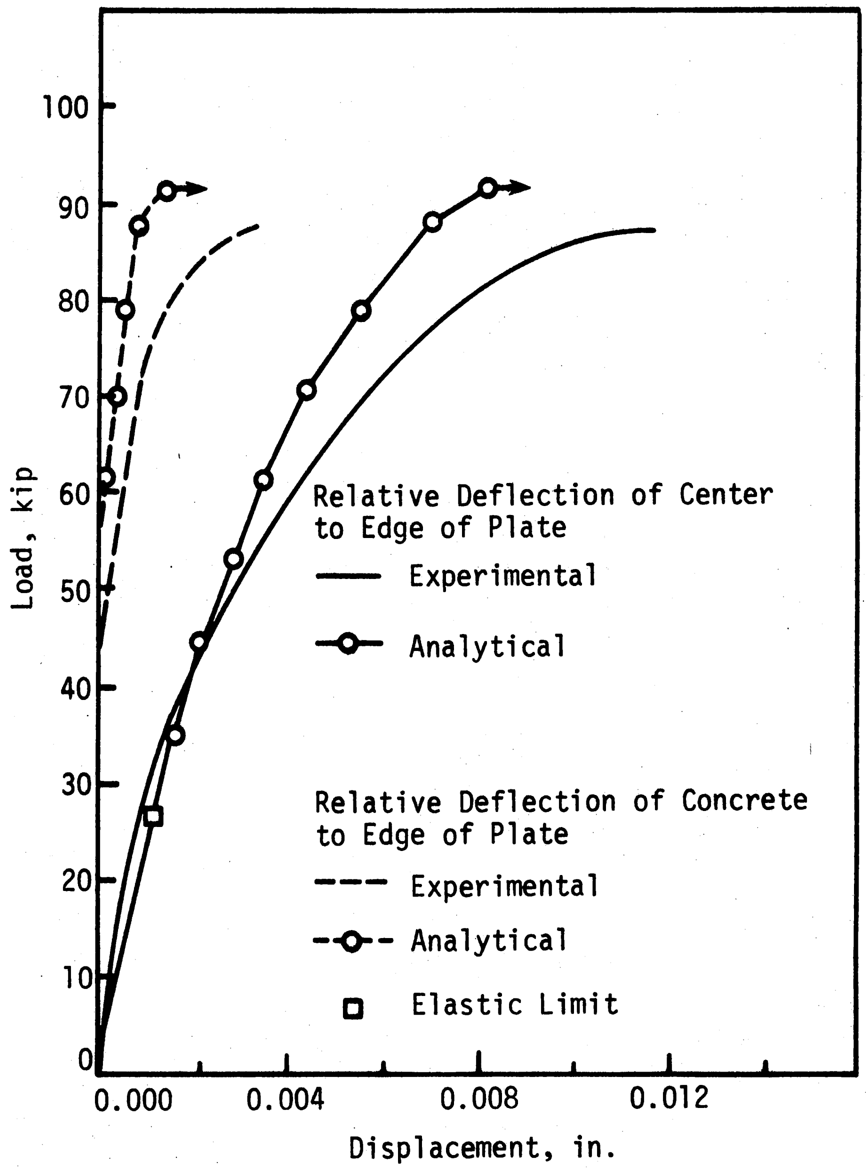


Figure 17. Load-Deformation Curve, Base Plate C5

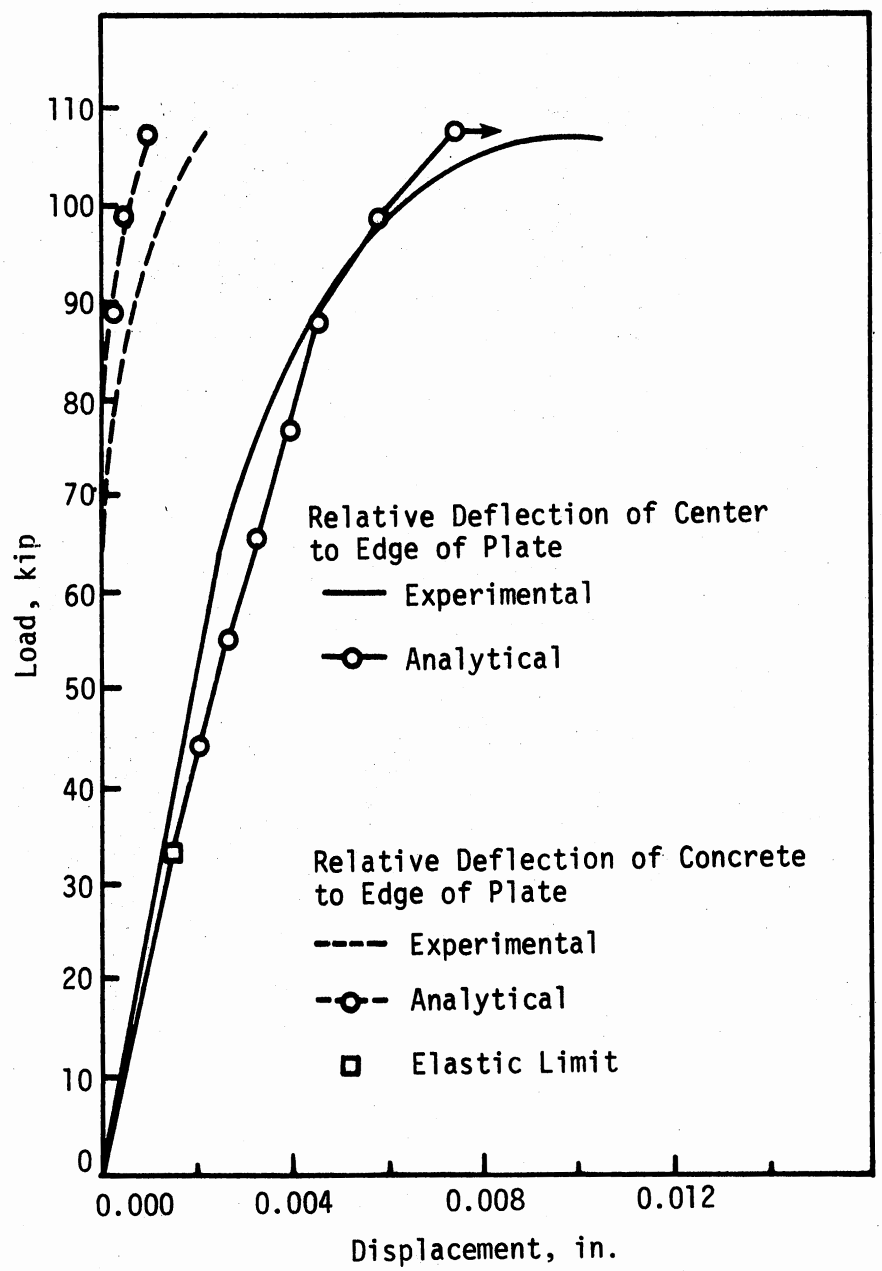


Figure 18. Load-Deformation Curve, Base Plate C6

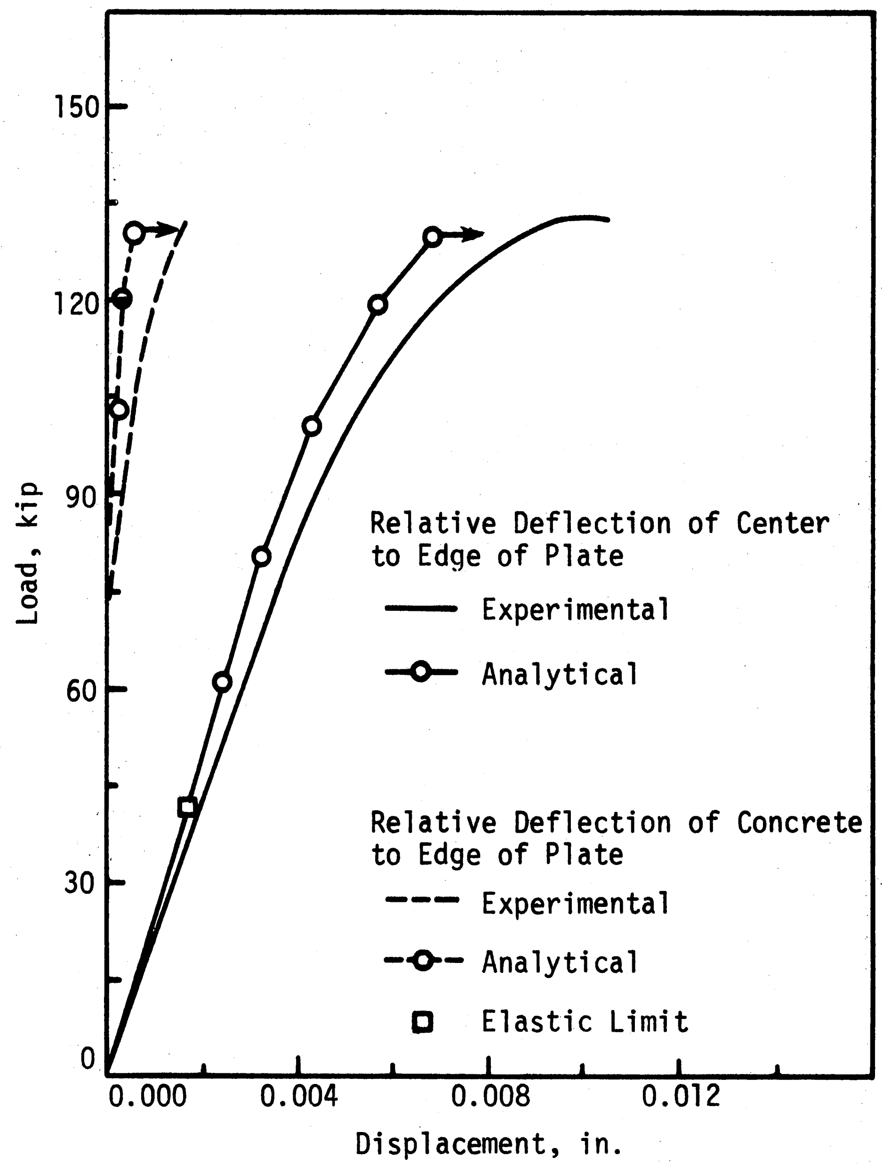


Figure 19. Load-Deformation Curve, Base Plate C7

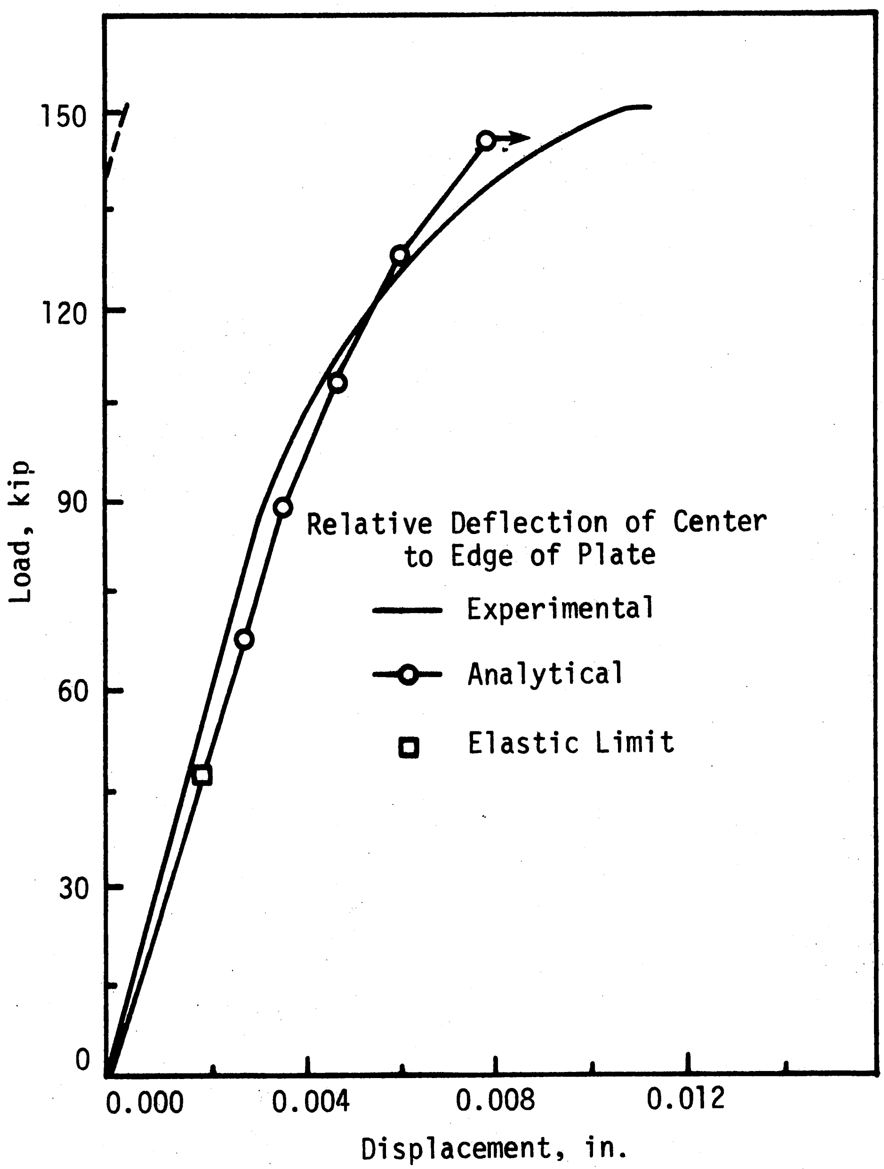


Figure 20. Load-Deformation Curve, Base Plate C8

5.4 Variables Influencing a Circular Base Plate System

Existing design methods express the bearing strength of concrete as a function of the compressive strength of concrete. Increased bearing strength is permitted when the load is applied to only a portion of the surface of the footing. In design, a base plate thickness is chosen to limit the bending stresses in the plate. Therefore, the compressive strength of concrete, yield strength of the steel, and the fraction of the concrete surface under load are the major variables influencing the design and possibly the behavior of the base plate system.

A concrete footing with a diameter of 8.0 in. and a height of 4.0 in., and a base plate having a diameter of 5.0 in. and a thickness of 0.5 in. were selected as a reference to study these variables. This footing was idealized by 64 elements, each having a height of 1/2 in. and a width of 1/2 in. A compressive strength of 3000 psi was used for concrete footing and a yield strength of 36,000 psi was used for the steel. The base plate system was loaded in a manner which corresponded to a loading which would occur with a pipe column having a nominal diameter of 3.0 in. The ultimate load carrying capacity of this system was 71 kips.

The influence on the ultimate capacity of the footing system resulting from variations in the compressive strength of the concrete footing, yield strength of the steel plate, plate thickness and plate diameter on the base plate system are presented in Figures 21 through 24. The increase in the ultimate load carrying capacity of the base plate system resulting from an augmentation in the yield strength of the plate was more significant than the increase in capacity resulting from an augmentation in the compressive strength of concrete. The load carrying capacity

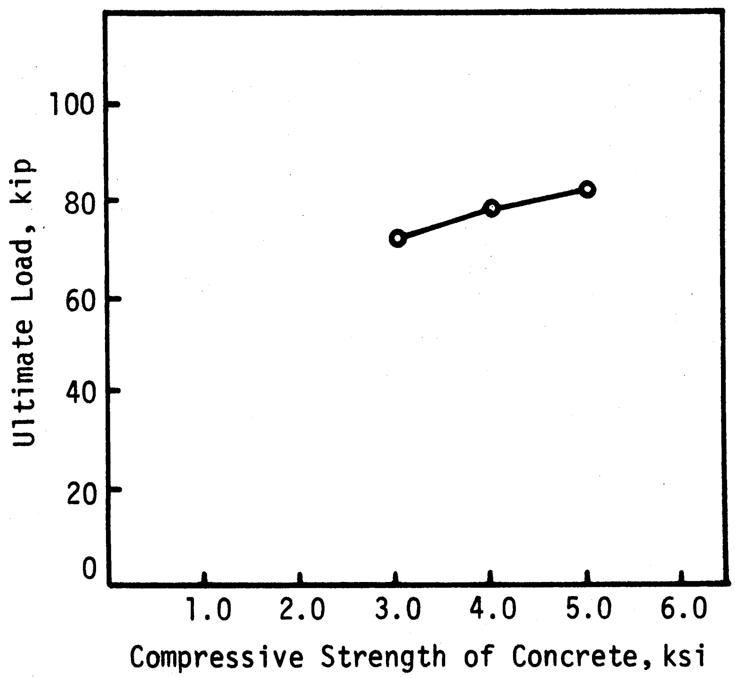


Figure 21. Influence of Compressive Strength of Concrete on Ultimate Load

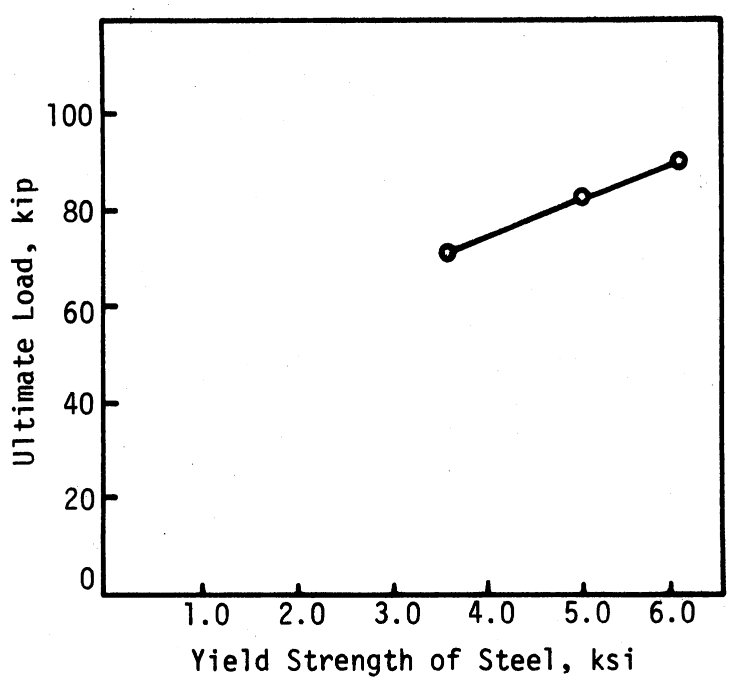


Figure 22. Influence of Yield Strength of Steel on Ultimate Load

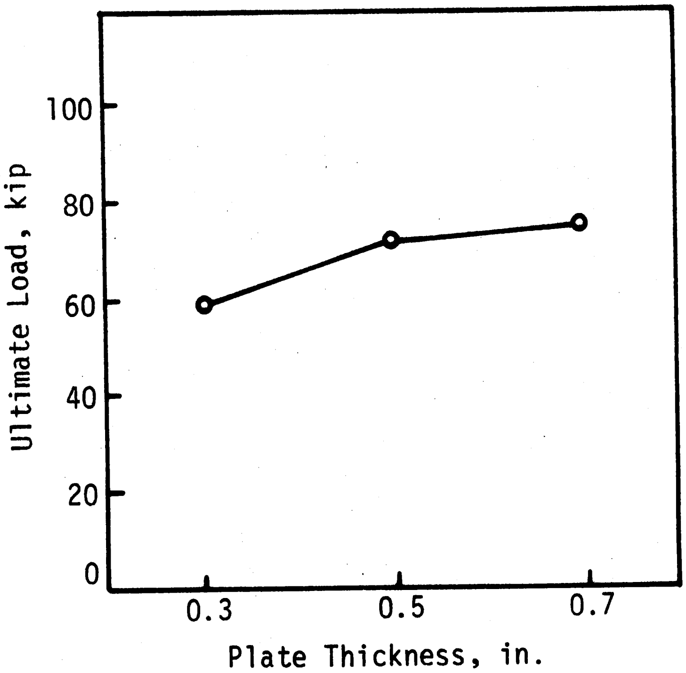


Figure 23. Influence of Plate Thickness on Ultimate Load

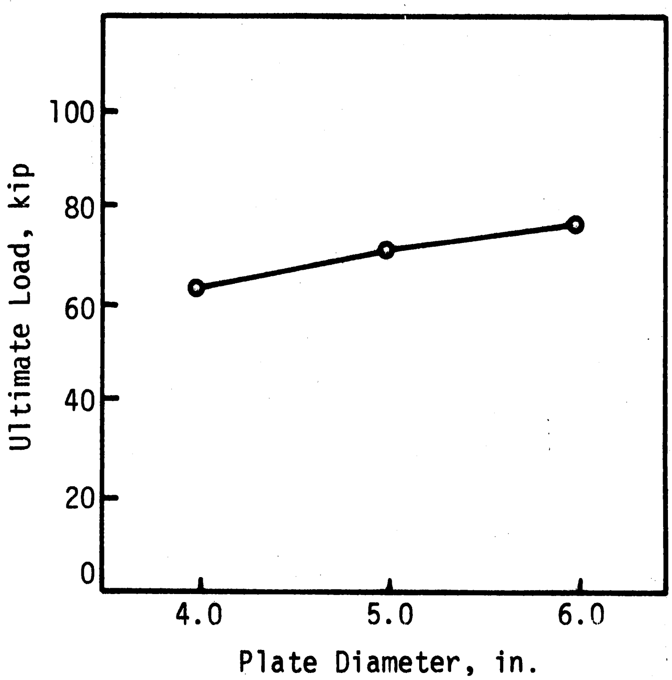


Figure 24. Influence of Plate Diameter on Ultimate Load

of base plates increased with an increase in the plate thickness until a limiting value was reached which represents the thickness of a rigid plate. The capacity of the base plate system also increased as a result of increasing the plate diameter.

These limited parametric studies illustrate the manner in which the program can be used to investigate the variables which influence the capacity of the base plate system. By further experimental and analytical work, improvements in the present design procedures may be realized.

CHAPTER VI

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

6.1 Summary

A model for circular base plates based on the finite element method was developed. The principal characteristics affecting the behavior of the base plate system were incorporated in the model. Steel was idealized as an elastic-perfectly plastic material. Concrete was assumed to be an isotropic homogeneous material when loaded in the elastic range and was assumed to be an elastic-perfectly plastic material in the compression zone. Cracking and crushing of concrete were considered also in the nonlinear range of loading. Steel base plates were connected to concrete footings through bond-link elements in such a way that a slip and separation between the two materials was permitted. Linear rectangular elements were used in this study to represent the steel base plate and the concrete footing. An integration technique was used in this study for elements in which the bending behavior was important.

Several numerical examples were solved to demonstrate the validity of the proposed model. The analytical solutions compared favorably with experimental results. The proposed model appears to be adequate for predicting the ultimate load for circular base plate systems. A limited number of solutions was obtained to study the effects of certain variables influencing the ultimate load carrying capacity of the base plate system.

6.2 Conclusions and Recommendations

The proposed analytical model is capable of analyzing and predicting the ultimate load carrying capacity for circular base plate systems. The effects of the material nonlinearities, cracking, crushing, and yielding were included in the analysis and the material model was adequate to analyze the base plate systems.

The compressive strength of concrete, yield strength of steel, relative area of base plate, and thickness of plates are the major variables influencing the behavior of the base plate system. Experimental results in this study indicated that circular base plate systems exhibited a lower bearing strength than square base plates which were loaded through wide flange sections (14).

The base plate model can be extended to include reinforcement in the concrete footing to better simulate actual design conditions. Also, different boundary conditions can be included in the model to allow for interaction between the structure and the surrounding soil. The results obtained suggested that more general problems, such as a three-dimensional base plate system which includes plates loaded by wide flange columns, can be solved by the method used in this study.

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APPENDIX A

EXPERIMENTAL INVESTIGATION

A.1 Specimens

The tests involved eight specimens. Each specimen consisted of a circular base plate and a cylindrical mortar footing having a diameter of 10 in. and a height of 10 in. The plates were either 6.5 in. or 8.5 in. in diameter. Details of the specimens are given in Table I. The specimens were loaded through a loading apparatus as shown in Figure 25.

TABLE I

OUTLINE OF BASE PLATE SPECIMENS

Speci-men	Base Plate Size		Compressive Strength of Concrete (psi)	Age at Test (days)
	Diameter (in.)	Thickness (in.)		
C1	6.5	0.250	3140	38
C2	6.5	0.375	3140	38
C3	6.5	0.500	3140	39
C4	6.5	0.625	3140	39
C5	8.5	0.375	3140	40
C6	8.5	0.500	3140	40
C7	8.5	0.625	3140	41
C8	8.5	0.750	3140	41

A.2 Materials

The mortar in the investigations contained type I portland cement and river sand meeting relevant ASTM specifications. The mix proportions

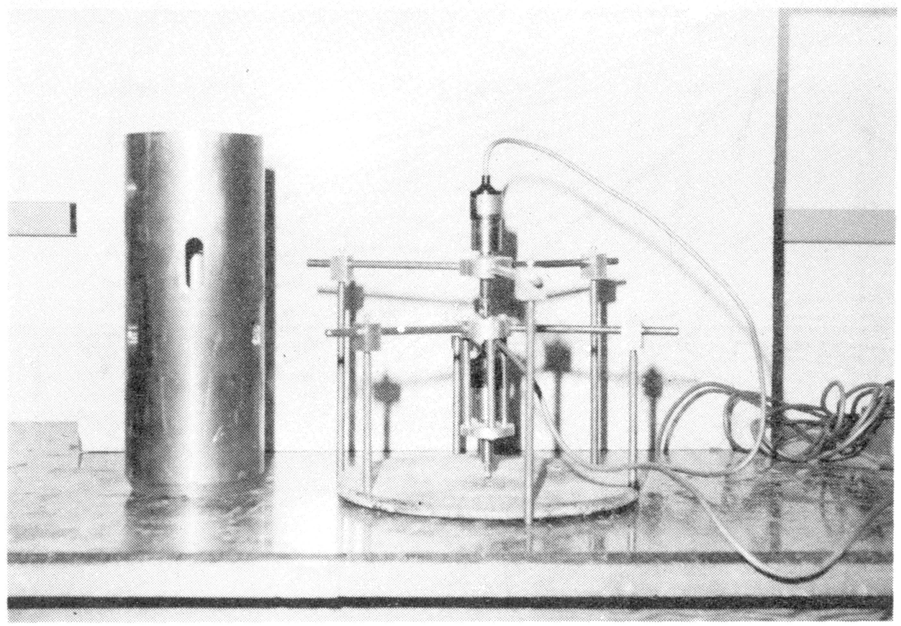

Figure 25. Loading Apparatus for Circular Plates

by weight of cement and sand were 1.00:5.00; the water-cement ratio was 0.7. The nominal strength for the mix was 3000 psi. The mortar was mixed in the laboratory and the compressive strength of concrete was 3140 psi and the tensile splitting strength was 250 psi. A typical stress-strain curve for concrete used in this study is shown in Figure 26. The circular plates were annealed to relieve the effect of the residual stresses which resulted from flame cutting of the plates. Coupons were cut from the plate material and tested in tension; the results of the tests are given in Table II. A typical stress-strain curve for the steel used in this study is shown in Figure 27.

TABLE II

TENSILE PROPERTIES OF PLATES

Plate Thickness (in.)	Strength	
	Yield (psi)	Ultimate (psi)
0.250	39,000	60,500
0.375	30,100	47,400
0.500	28,400	50,600
0.625	36,000	61,600
0.750	34,600	58,200

A.3 Experimental Procedure

Cylindrical footings and control cylinders were cast in disposable cardboard molds. Mortar was consolidated with the use of an electric

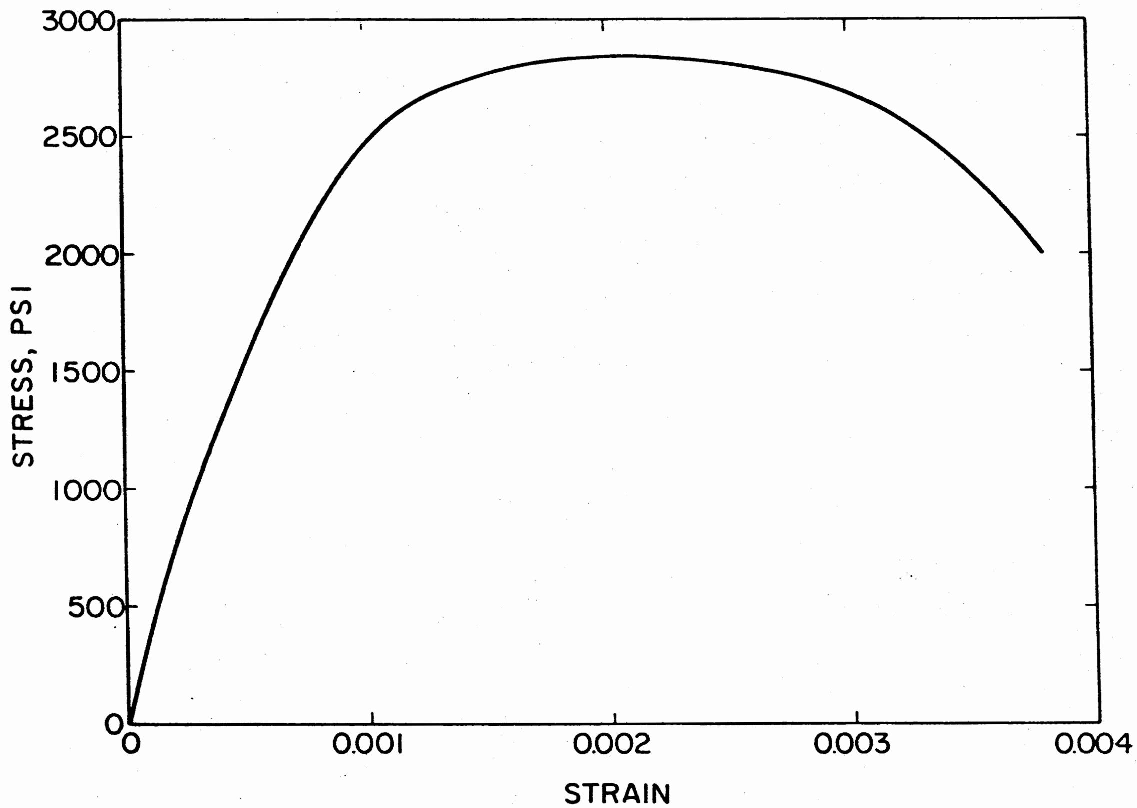

Figure 26. Typical Stress-Strain Curve for Concrete

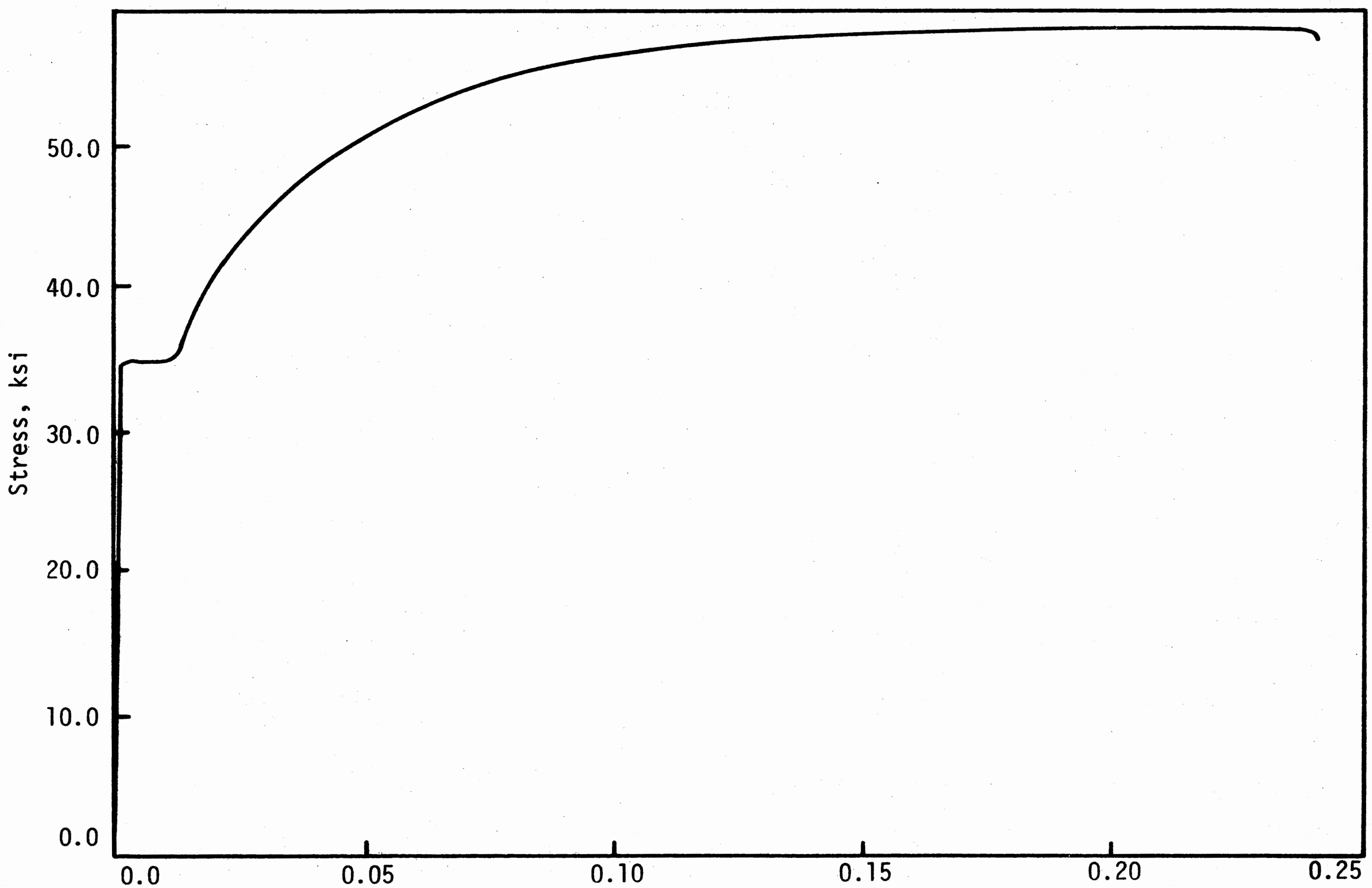

Figure 27. Typical Stress-Strain Curve for Steel

internal vibrator. The cylindrical footings and control cylinders were removed from the forms after one day and cured in a moist room until the time of the test.

Vertical displacement at the center of the plate and of the concrete footing at the outer edge of the plate was measured relative to the edge of the plate by two DCDT displacement transducers as shown in Figure 28. Load displacement curves were continuously plotted during the test by two X-Y recorders. The load was applied concentrically to the plates through a loading apparatus. The loading apparatus had a 3.5 in. outer diameter at the loading surface. Portland cement grout was used to position the plates on the top of the footings and high strength gypsum cement grout was used between the bottom of the footing and the one-inch thick shoe plate. Four dial gages were placed horizontally at 2.0 in. intervals to detect the horizontal motion of the footing as shown in Figure 28.

A.4 Experimental Results

Circular base plates were loaded up to failure and the load was applied in increments of 10 kips. Vertical cracks formed at the surface of cylindrical footings and propagated, thus indicating splitting due to tensile stresses normal to the radial direction. Failure in all cases occurred by formation of a solid core under the base plate followed by splitting and radial cracks. A typical specimen at failure is shown in Figure 29. The load carrying capacity of the base plate system dropped after the ultimate load was reached. Edges of base plates having a diameter of 6.5 in. did not deflect upward and separate from the concrete footing even at higher loads, although edges of base plates having a diameter of 8.5 in. deflected upward and separated from the concrete

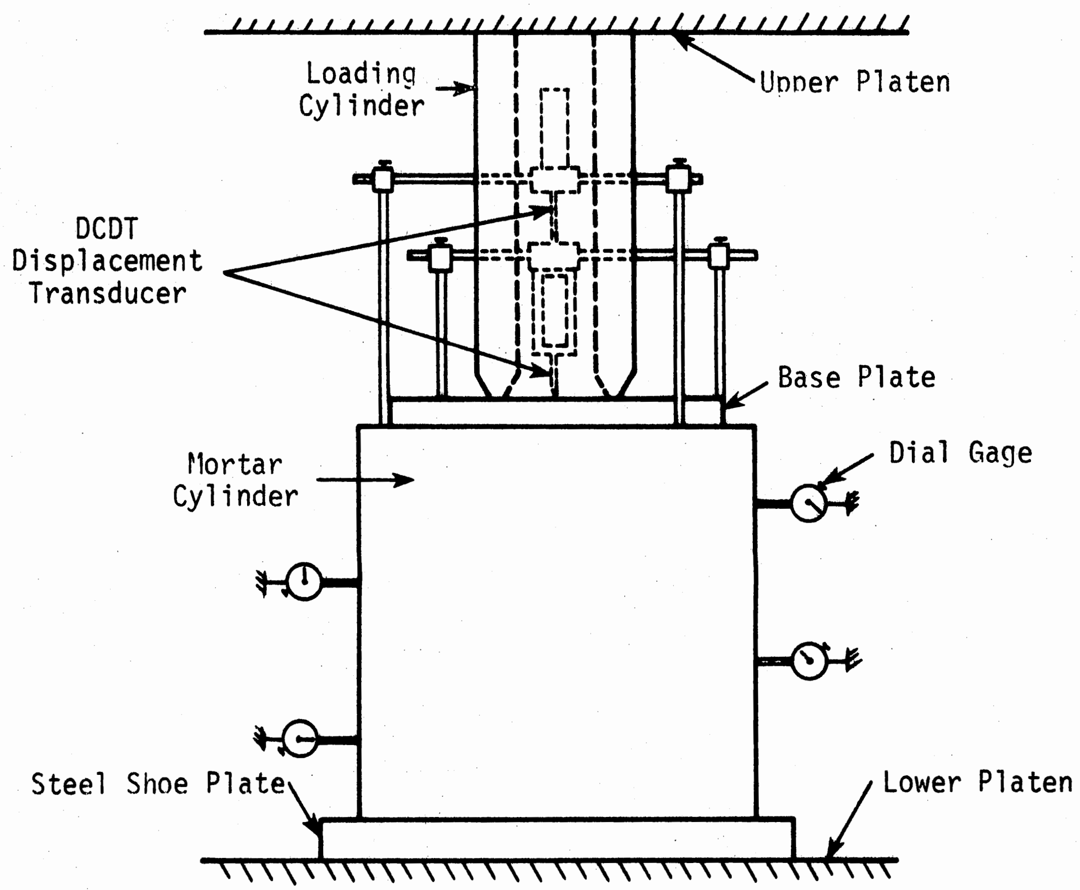

Figure 28. Loading Arrangement for Circular Plates

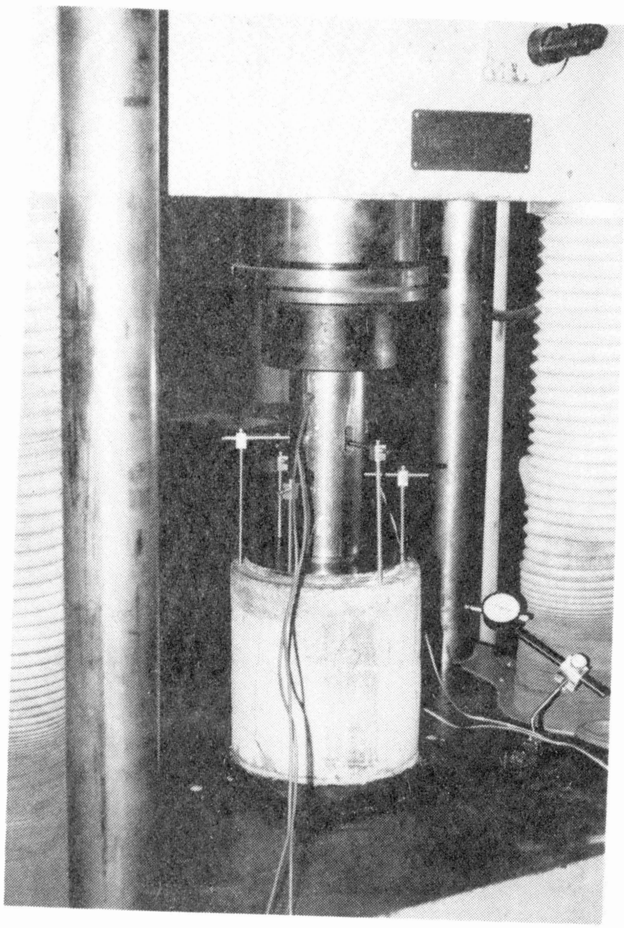

Figure 29. Circular Base Plate System at Failure

footing. The deflections of the center of the base plates and of the concrete footing were measured relative to the edges of the base plate. The ratio of footing area to the plate area, the ultimate bearing stress and the dimensionless relationship between the ultimate bearing pressure and the concrete compressive strength are summarized in Table III.

TABLE III

RESULTS OF CIRCULAR BASE PLATE SPECIMENS

Specimen	Ultimate Load (kips)	R	Ultimate Bearing Stress, qu (psi)	$\frac{qu}{f'_c}$
C1	80	2.36	2,410	0.768
C2	90	2.36	2,710	0.864
C3	108	2.36	3,250	1.036
C4	120	2.36	3,610	1.152
C5	88	1.38	1,550	0.494
C6	110	1.38	1,940	0.617
C7	136	1.38	2,390	0.763
C8	150	1.38	2,640	0.842

APPENDIX B

INPUT/OUTPUT INFORMATION

B.1 INPUT INFORMATION

IDENTIFICATION OF PROBLEM (2 alphanumeric cards per problem)

20 A4
1 — 80

20 A4
1 — 80

CONTROL DATA

NUMNP	NUMEL	NMPRT	NB
I5	I5	I5	I5
5–10	15–20	25–30	35–40

NUMNP = total number of nodal points

NUMEL = total number of elements

NMPRT = number of parts (maximum 10)

NB = number of bond-link elements (maximum 50)

MESH GEOMETRY

NPT	R1	Z1	R2	Z2	NR	NZ	NNS	NES	END
I2	E10.3	E10.3	E10.3	E10.3	I5	I5	I5	I5	A3
3–5	10–20	20–30	30–40	40–50	50–55	55–60	60–65	65–70	76–78

NPT = number of a part

R1 = r coordinate of first node in a part

Z1 = coordinate of first node in a part

R2 = r coordinate of last node in a part

Z2 = z coordinate of last node in a part

NR = number of elements in r direction

NZ = number of elements in z direction

NNS = number of first node in a part

NES = number of first element in a part

All units must be consistent in all input data.

MATERIAL PROPERTIES

NPRO(1)	NPRO(2)	NPRO(3)	NPRO(4)
I5	I5	I5	I5
5 – 10	15 – 20	25 – 30	35 – 40

FROM NODE	TO NODE	NSTEP	E	ν	FT	FC	ALFA	BETA
I5	I5	I5	E10.3	E10.3	E10.3	E10.3	E10.3	E10.3
5 – 10	10 – 15	15 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80

STRESS	STRAIN	EQUIVALENT PLASTIC STRAIN
E10.3	E10.3	E10.3
1 – 10	10 – 20	20 – 30

NPRO(1) = material type (concrete = 1, steel = 2)

NPRO(2) = (no bending mode = 1, bending mode = 2)

NPRO(3) = number of points in a stress-strain curve in tension

NPRO(4) = number of points in a stress-strain curve in compression

E = modulus of elasticity

ν = Poisson's ratio

FT = ultimate uniaxial stress in tension or yield stress in tension

FC = ultimate uniaxial stress in compression or yield stress in compression

ALFA = f'_t/f'_c

BETA = $\sigma_1/f'_c = \sigma_2/f'_c$

BOUNDARY CONDITIONS

FROM NODE	TO NODE	NSTEP	U	V	END
I5	I5	I5	I1	I1	A3
5–10	10–15	15–20	24	25	56–58

APPLIED NODAL LOADS

FROM NODE	TO NODE	NSTEP	FR	FZ	DFR	DFZ	END
I5	I5	I5	E10.3	E10.3	E10.3	E10.3	A3
5–10	10–15	15–20	25–35	35–45	45–55	55–65	71–73

FR = initial load in r direction

FZ = initial load in z direction

DFR = increment of load in r direction

DFZ = increment of load in z direction

<u>ITERATION DATA</u>

NSTEPT	NIT	NRUN	IFRQ	IPRINT	TNC
I5	I5	I5	I5	I5	E10.3
5 – 10	15 – 20	25 – 30	35 – 40	45 – 50	50 – 60

NSTEPT = maximum number of load steps

NIT = maximum number of iterations per step

NRUN = run identification number

IFRQ = frequency of updating stiffness matrix

IPRINT = frequency of printing output data

TNC = convergence factor

B.2 Output Information

A complete list of input data is printed. Calculated results are printed according to an option specified by user. The computed forces and deformations can be printed at each load increment or at any specified number of increments.

For each load increment radial and vertical displacements are calculated at each node. Also, radial, hoop, vertical, and shear stresses and the principal stresses are calculated for each element. Messages are printed for elements when cracks or yield occurs on those elements.

APPENDIX C

COMPUTER PROGRAM LISTING

```
C     *****************************************************************
C     *                                                               *
C     *    PROGRAM NFEA                                               *
C     *    NONLINEAR FINITE ELEMENT ANALYSIS OF AXISYMMETRIC STRUCTURES *
C     *                                                               *
C     *    LANGUAGE USED          :  FORTRAN IV                       *
C     *    DIGITAL COMPUTER       :  IBM 370-185                      *
C     *    PROGRAMMER             :  HUSSAM M. GHANEM                 *
C     *                                                               *
C     * THIS PROGRAM SOLVES AXISYMMETRIC STRUCTURES BY USING THE FINITE *
C     * ELEMENT METHOD. AN INCREMENTAL-ITERATIVE PROCEDURE IS USED IN THE *
C     * SOLUTION OF OF NONLINEAR PROBLEMS.                            *
C     *                                                               *
C     *****************************************************************
C
C---->DRIVER FOR PROGRAM NFEA
      IMPLICIT REAL * 8 ( A-H, O-Z )
C---->DIMENSIONS OF 'A' AND 'B' DEPEND ON VALUE OF MAXMBD - SEE BELOW
      DIMENSION A(124,62),B(124)
C---->DIMENSIONS OF FOLLOWING ARRAYS DEPEND ON VALUE OF NPMAX - BELOW
     0DIMENSION X(570), Y(570), FX(570), FY(570), NPCODE(570), SFX(570),
     1          US(570), VS(570), ISTAR(570) , FORCE(1140),SFORCE(1140),
     2          DISPL(1140), DELTA(1140),CFORCE(1140), XFORCE(1140),
     3          SFY(570)
C---->DIMENSIONS OF FOLLOWING ARRAYS DEPEND ON VALUE OF NELMAX - BELOW
      DIMENSION NPC(510,4), E(510), PR(510), MTYP(510), NELPT(510)
      DIMENSION NBC(30,2), SKH(30), SKV(30)
C---->MAXIMUM MESH DATA
C          NUMBER OF NODES
                NPMAX = 570
C          NUMBER OF ELEMENTS
                NELMAX = 510
C          MAXIMUM DIFFERENCE IN ELEMENT NODE NUMBERS
                MAXMBD = 30
C          MAXIMUM STORAGE SIZES
                MWIDE = 2 * MAXMBD + 2
                MLONG = 2 * MWIDE
                NBMAX=30
                NPMAX2 = NPMAX * 2
C---->DIMENSIONS OF 'A' AND 'B' MUST BE A(MLONG, MWIDE), B(MLONG)
     0CALL FINEL2( A, B, X, Y, NPC, E, PR, FX, FY, NPCODE, US, VS ,
     1             NPMAX, NELMAX, MAXMBD, MWIDE, MLONG, ISTAR ,NBMAX ,
     2              NBC, SKH, SKV, NPMAX2, FORCE, SFORCE, DISPL, DELTA,
     3              CFORCE, XFORCE, MTYP, NELPT, SFX, SFY )
      STOP
      END
```

```
      SUBROUTINE FINEL2( A, B, X, Y, NPC, E, PR, FX, FY, NPCODE, US, VS,
     1                   NPMAX, NELMAX, MAXMBD, MWIDE, MLONG, ISTAR,NBMAX,
     2               NBC, SKH, SKV, NPMAX2, FORCE, SFORCE, DISPL, DELTA,
     3                   CFORCE, XFORCE, MTYP, NELPT, SFX, SFY )
C---->MAIN PROGRAM FOR PROGRAM NFEA   - FINITE ELEMENT ANALYSIS OF
C          AXISYMMETRICN NONLINEAR ELASTICITY PROBLEMS
      IMPLICIT REAL * 8 ( A-H, O-Z )
   10 FORMAT(10D12.3)
 2000OFORMAT (     1H1, /////,
     1             51H     FINITE ELEMENT ANALYSIS OF AXISYMMETRIC SOLIDS,
     2             26H PROBLEMS - PROGRAM NFEA      ,//, 5X, 20A4,
     3             ///                             , 5X, 20A4 )
 2010 FORMAT(//,' ***** PROGRAM PROCEEDED ALL COMMANDS ***** ',//)
 2020OFORMAT (     1H1, /////,
     1,10X,' THIS LOAD STEP NUMBER ..................NSTEP= ',I3,//,
     2 10X,' NORM TO THE LOAD VECTOR ...............ELNORM= ',D12.3,/)
 2030 FORMAT(//,2X,' **** ITERATION PROCESS  DOES NOT CONVERGE ',/
     1,2X,'  ITERATION NO. ',I3,' EXCUTION WILL BE TERMINATED ',//)
 2040 FORMAT(//,10X,' START ITERATION NUMBER ..........NI= ',I3,/ )
 2050 FORMAT(//,10X,' NORM OF THE RESIDUAL LOAD VECTOR ....ELFORCE=',
     1 D12.3,/)
 2060 FORMAT(//,10X,' TOTAL NUMBER OF ITERATIONS IN THE LOAD STEP ',I3,
     1 '    .....NI= ',I3,/)
 2070OFORMAT (///45H      MAXIMUM BANDWIDTH EXCEEDED. TO CORRECT: ,
     1   /          45H            1. RENUMBER NODES TO REDUCE MAXIMUM ,
     2   /          30H               BAND PARAMETER TO, I5,
     3   /          47H      OR,  2. CHANGE VALUE OF 'MAXMBD' IN DRIVER
     4   /          48H               AND REDIMENSION ARRAYS 'A' AND 'B'.  )
 2080 FORMAT(//,10X,'*** ELASTIC SOLUTION IS COMPLETED *** HMG ',//)
 2100 FORMAT(//,10X,'*** NONLINEAR SOLUTION ACCOMPLISHED *** HMG ',//)
C---->COMMON STATEMENTS
      COMMON / CONT / ID1(20), ID2(20)
      COMMON / MESH / X1, Y1, X2, Y2, NX, NY, NPT
     0COMMON / MATL / EN(10), PRN(10), ELFA(10) , INI2(10), INL2(10),
     1                NSTEP2(10), UNIAX(10,3,10), FYT(10), FYC(10),
     2                NPRO(4,10), ELBET(10), NMPRT
     0COMMON / DISP / USPN(10), VSPN(10), INI3(10), INL3(10),
     1                NSTEP3(10), KODEXN(10), KODEYN(10), JSP
      COMMON / LOAD / FUN(10), FVN( 10), INI4(10), INL4(10),NSTEP4(10),
     1                SFUN(10), SFVN(10), JL
      COMMON / BOND / INI5(30), INL5(30), BFD(30), NB
      COMMON / ITER / NSTEPT,NIT,NPUN,IFRQ,IPRINT
C---->DIMENSION
     0DIMENSION X(NPMAX), Y(NPMAX), NPC(NELMAX,4), NPCODE(NPMAX),
     1          FX(NPMAX), FY(NPMAX), US(NPMAX), VS(NPMAX), E(NELMAX),
     2          PR(NELMAX), A(MLONG,MWIDE), B(MLONG)   ,NBC(NBMAX,2),
     3          ISTAR(NPMAX) ,SKH(NBMAX), SKV(NBMAX) , FORCE (NPMAX2) ,
     4          DISPL(NPMAX2), DELTA(NPMAX2),SFORCE(NPMAX2),
     5          CFORCE(NPMAX2), XFORCE(NPMAX2), MTYP(NELMAX), SFX(NPMAX)
     6,         SFY(NPMAX), NELPT(NELMAX)
```

```
      DATA ZERO / 0.0000 /
   90 CALL INECHO ( X, Y, NPC, NPMAX, NELMAX, NUMNP, NUMEL, KEY,
     1                SKH, SKV, NBMAX, TNC )
     0CALL DIST ( X, Y, E, PR,NBC,US, VS, NPCODE, FX, FY, NPMAX, NELMAX,
     1            NBMAX, NUMNP, NUMEL, NPC, MBAND, IMBD, ISTAR, SKH,
     2            SKV, NELPT, SFX, SFY, MTYP )
                  NUMNP2 = 2 * NUMNP
C---->SET UP TOTAL STIFFNESS MATRIX
C---->TEST BAND WIDTH EXCEEDED
            IF ( MBAND .LE. MAXMBD ) GO TO 94
                  PRINT 2000, ( ID1(I), I = 1, 20 ), ( ID2(I), I = 1, 20 )
                  PRINT 2070, MAXMBD
                  PRINT 2000, ( ID1(I), I = 1, 20 ), ( ID2(I), I = 1, 20 )
      STOP
   94             MBAND = 2 * ( MBAND + 1 )
            IF ( NRUN .EQ. 0 ) GO TO 300
      CALL    FVEC    ( FX,FY,NUMNP,NPMAX,NPMAX2,FORCE,SFORCE,SFX,SFY )
      CALL STIFF ( X, Y, E, PR, NPMAX, NELMAX, NUMNP, NUMEL, NPC, NELPT,
     1              ELFA, ELBET, FYT, FYC, NPRG )
                  NI=0
                  NSTEP = 1
                  IUPST = 1
                  ICOUNT = 1
            DO 95 I=1,NUMNP2
                  XFORCE(I) = FORCE(I)
                  CFORCE(I) = FORCE(I)
                  DISPL(I) = ZERO
   95       CONTINUE
            GO TO 130
  100       CONTINUE
                  NI=0
                  IUPST = 1
                 NSTEP=NSTEP+1
            IF ( NSTEP.LE.NSTEPT ) GO TO 120
                  NSTEP=NSTEP-1
  110 PRINT 2010
            GO TO 300
  120       DO 130 I=1,NUMNP2
                  FORCE(I) = SFORCE(I)
                  CFORCE(I) = XFORCE(I) + SFORCE(I)
                  XFORCE(I) = CFORCE(I)
  130       CONTINUE
      CALL NORM ( NUMNP2,CFORCE, ELNORM )
      PRINT 2020, NSTEP,ELNORM
  200       CONTINUE
                  NI = NI + 1
                  IUPDAT = 0
                  INVRT=0
            IF ( NI.LE.NIT ) GO TO 210
      PRINT 2030, NIT
                  NI=NI-1
            IF ( NSTEP .GE. NSTEPT ) GO TO 280
            GO TO 300
  210 PRINT 2040, NI
            IF ( NI .NE. IUPST ) GO TO 230
  220 CALL ASMBL ( US, VS, NPCODE, NPMAX, NUMNP, NUMEL, MBAND, MLONG,
     1             MWIDE,A, NBC, NUMBLK, NB, SKH, SKV, NBMAX, E, NELMAX)
                  INVRT=1
                  IUPST = IUPST + IFRQ
  230 CALL FORBLK ( B, MWIDE, MLONG, NPCODE, NPMAX, NUMNP, FORCE, NPMAX2
     1             , US, VS )
C---->SOLVE FOR NODAL DISPLACEMENTS
      CALL BANSOL ( B, A, MBAND, NUMBLK, MWIDE, MLONG )
C---->CALCULATE DISPLACEMNT
                  K=0
                  MSTRT=MWIDE+1
                  MSTOP=MLONG
            DO 240 N=1,NUMBLK
            DO 235 M=MSTRT,MSTOP,2
                  K = K + 2
                  DELTA(K-1) = A(M,N)
                  DELTA(K) = A(M+1,N)
            IF ( K .GE. NUMNP2 ) GO TO 250
  235       CONTINUE
  240       CONTINUE
  250       CONTINUE
C---->UPDATE DISPLACEMENTS
            DO 260 I=1,NUMNP2
                  DISPL(I)=DISPL(I)+DELTA(I)
                  FORCE(I) = ZERO
  260       CONTINUE
C---->ADJUST LOAD THAT STRUCTURE CAN RESIST FROM CURRENT IMPOSED LOADS
      CALL    PLASCL     ( NUMEL, NUMNP, NPMAX, NELMAX, NPMAX2, DELTA,
     1                     DISPL, NBC, NB, SKV, SKH, UNIAX, NPRO,MTYP,
     2                BFD, NBMAX, INVRT, NELPT, NSTEPT, FORCE, IUPDAT )
            DO 275 N=1,NUMNP
                  KEY = NPCODE(N) + 1
            GO TO ( 275, 270, 265, 265, 275 ) , KEY
  265             M = 2*N -1
                  FORCE(M) = US(N)
            IF ( KEY .NE. 4 ) GO TO 275
  270             M = 2*N
                  FORCE(M) = VS(N)
  275       CONTINUE
      CALL NORM ( NUMNP2,FORCE,ELFORC  )
      PRINT 2050, ELFORC
            IF ( NSTEPT.EQ.0 ) GO TO 295
            IF ( ELFORC .LE. TNC*ELNORM ) GO TO 280
            GO TO 200
  280       CONTINUE
```

```
      PRINT 2060, NSTEP,NI
         IF ( NSTEP .NE. ICOUNT ) GO TO 290
285         ICOUNT = ICOUNT+IPRINT
      CALL    PRNT    ( DISPL, NUMEL, NUMNP, NPMAX, NPMAX2, NELMAX, X, Y)
290      CONTINUE
         IF ( NSTEP.EQ.NSTEPT ) GO TO  300
         GO TO 100
295      IF ( NSTEPT.EQ.0 ) WRITE(6,2080)
         IF ( NRUN.EQ.100 ) WRITE(6,2010)
      STOP
300      CONTINUE
      PRINT 2010
      PRINT 2100
      STOP
      END
```

```
      SUBROUTINE INECHO ( X, Y, NPC, NPMAX, NELMAX, NUMNP, NUMEL, KEY,
     1                    SKH, SKV, NBMAX, TNC )
C---->READ AND ECHO INPUT DATA FOR NFEA
      IMPLICIT REAL * 8 ( A-H,O-Z )
      COMMON / CONT / IO1(20), IO2(20)
      COMMON / MESH / X1, Y1, X2, Y2, NX, NY, NPT
     XCOMMON / MATL / EN(10), PEN(10), FLFA(10) , INI2(10), INL2(10),
     1                NSTEP2(10), UNIAX(10,3,10), FYT(10), FYC(10),
     2                NPRO(4,10), FLBET(10), NMPRT
     XCOMMON / DISP / USPN(10), VSPN(10), INI3(10), INL3(10),
     1                NSTEP3(10), KODEXN(10), KODEYN(10), JSP
      COMMON / LOAD / FUN(10), FVN(10), INI4(10), INL4(10),NSTEP4(10),
     1                SFUN(10), SFVN(10), JL
      COMMON / BOND / INI5(30), INL5(30), BFD(30), NB
      COMMON / ITER / NSTEPT,NIT,NFUN,IFRQ,IPRINT
      DIMENSION X(NPMAX), Y(NPMAX), NPC(NELMAX,4), SKH(NBMAX),SKV(NBMAX)
      DATA IEND / 3HEND /
C---->INPUT FORMATS
 1000 FORMAT ( 20A4 )
 1010 FORMAT ( 4 ( 5X, I5 ) )
 1020 FORMAT (  4X,I1,5X, 4E10.3,4I5, 5X, A3 )
 1030 FORMAT ( 4 ( 5X, I5 ) )
 1040 FORMAT ( 5X, 3I5 , 6E10.3 )
 1050 FORMAT ( 3E10.3 )
 1060 FORMAT ( 5X, 3I5, 3X, 2I1, 5X, 2E10.3, 5X, A3 )
 1070 FORMAT ( 5X, 3I5, 5X, 4E10.3, 5X, A3 )
 1080 FORMAT ( 2 ( 5X, I5 ), 5X, 3E10.3 )
 1090 FORMAT ( 5 ( 5X, I5 ) ,  E10.3 )
C---->OUTPUT FORMATS
 2000OFORMAT (     1H1     ,
     1  ///       51H     FINITE ELEMENT ANALYSIS OF AXISYMMETRIC SOLIDS,
     2             26H PROBLEMS - PROGRAM NFEA       ,
     3  ///, 5X, 20A4  ,///, 5X , 20A4 )
 2010 FORMAT (///41H      TABLE 1 - MESH GEOMETRY CONTROL DATA     )
 2020OFORMAT (///29H      RECTANGULAR MESH   PART ,I1,
     1   //       30H           CORNER COORIDINATES
     2   /        20H                   R1 = , 1PD10.3 , 5X, 5HZ1 = , D10.3,
     3   /        20H                   R2 = , D10.3, 5X, 5HZ2 = , D10.3,
     4   /        43H           NUMBER ELEMENTS IN R-DIRECTION = , I5,
     5   /        43H           NUMBER ELEMENTS IN Z-DIRECTION = , I5,
     6   /        29H           NUMBERING NODES IN ,      11HR-DIRECTION )
 2030 FORMAT (///34H      TABLE 2 - MATERIAL PROPERTIES ,/)
 2040 FORMAT( //,
     1          6X,' MATERIAL TYPE (CONCRETE=1,STEEL=2).....NPRO(1)=',I5,/,
     1          6X,' BENDING    MODE(1)......................NPRO(2)=',I5,/,
     2         5X,' NO. OF POINTS IN STRESS-STRAIN CURVE (TENSION) =',I5,/,
     3         5X,' NO. OF POINTS IN STRESS-STRAIN CURVE(COMPRES.) =',I5,/)
 2050OFORMAT (// 50H       FROM   TO  STEP      MODULUS OF      POISSON'S ,
     1 '        YIELD OR ULTIMATE STRESS      ALFHA               BETA ',/,
     2            49H        EL.   EL.           ELASTICITY         RATIO  ,
```

```
     3 '          TENSION      COMPRESSION',/)
 2060 FORMAT (  5X, 3I5, 5X, 1PD12.3, 3X, D12.3 , 3X, 1PD12.3 , 2X ,
     1 1PD12.3 , 2X , D12.3 , 2X, D12.3 )
 2070 FORMAT(//, 5X,' STRESS-STRAIN CURVE OF MATERIAL ',//,
     11X,  '      POINT        STRESS        STRAIN        EQ. STRAIN ',/ )
 2080 FORMAT ( 6X, I2, 6X, 1PD12.3, 2X, 1PD12.3, 2X, 1PD12.3  )
 2090 FORMAT (// 40H      TABLE 3 - SPECIFIED DISPLACEMENTS    )
 2100 FORMAT (// 40H      FROM   TO  STEP  CODE
     1  /         30H      NODE  NODE         UV       /  )
 2110 FORMAT ( 5X, 3I5, 4X, 2I1  )
 2120 FORMAT (///34H      TABLE 4 - APPLIED NODAL LOADS       )
 2130 FORMAT (// 50H      FROM   TO  STEP              FR                 FZ ,
     1 '               SFR                SFZ ',/,
     2               16H      NODE  NODE    / )
 2140 FORMAT ( 5X, 3I5,10X, 1PD12.3, 3X, 1PD12.3, 3X,1PD12.3,3X,1PD12.3)
 2150 FORMAT(/// '      TABLE 5 - BOND LINKS ')
 2160 FORMAT(//'       FROM   TO          KH                KV          BOND ',
     1       /,'       NODE  NODE                                   STRENGTH',//)
 2170 FORMAT (      5X, 2I5, 3X, 1PD12.3, 1X, 1PD12.3, 1X, 1PD12.3 )
 2190 FORMAT(///,'       ITERATION INFORMATION ',//)
 2200 FORMAT(6X,' NUMBER OF LOAD STEPS.....NSTEPT=',I2,/,
     1       6X,' MAX. NO. OF ITERATIONS.......NIT=',I2,/,
     2       6X,' RUN IDENTIFICATION..........NRUN=',I2,/,
     3       6X,' TOLERANCE REQUIRED...........TNC=',D10.3,/ )
C---->READ AND ECHO RUN ID
      READ 1000, ( ID1(I), I = 1, 20 )
      READ 1000, (ID2(I), I = 1, 20 )
      PRINT 2000, ( ID1(I), I = 1, 20 ),( ID2(I), I= 1, 20 )
C---->READ AND ECHO CONTROL DATA
C---->READ AND ECHO MESH GEOMETRY CONTROL DATA
      PRINT 2010
      READ 1010,NUMNPS,NUMELS,NMPRT,NB
  100 READ 1020 ,NPT ,X1,Y1 ,X2 ,Y2 ,NX ,NY ,NNS ,NES ,IENDN
      CALL RECGEN(NPMAX,NELMAX,  X,Y,NPC,NUMNP,NUMEL,NX,NY,NNS,NES,X1,Y1
     1            ,X2,Y2)
      PRINT 2020, NPT ,X1 ,Y1 ,X2 ,Y2 ,NX ,NY
          IF ( IENDN.EQ. IEND ) GO TO 110
          GO TO 100
  110     IF ( NUMNP .GT. NPMAX .OR. NUMEL .GT. NELMAX) GO TO 920
C---->READ AND ECHO MATERIAL PROPERTIES
      PRINT 2030
          DO 130 NPT=1,NMPRT
      READ 1030, ( NPRO(I,NPT) , I=1,4 )
      PRINT 2040,( NPRO(I,NPT) , I=1,4 )
      PRINT 2050
      READ 1040, INI2(NPT), INL2(NPT), NSTEP2(NPT), EN(NPT), PRN(NPT),
     1           FYT(NPT),FYC(NPT),ELFA(NPT),ELBET(NPT)
      PRINT 2060,INI2(NPT), INL2(NPT), NSTEP2(NPT), EN(NPT), PRN(NPT),
     1           FYT(NPT),FYC(NPT),ELFA(NPT),ELBET(NPT)
              NPONT=NPRC(3,NPT)
              NPONC=NPRC(4,NPT)
              NPOINT=NPONT+NPONC
      PRINT 2070
          DO 120 I=1,NPOINT
      READ 1050, (UNIAX(I,J,NPT),J=1,3)
      PRINT 2080, I, (UNIAX(I,J,NPT),J=1,3)
  120     CONTINUE
  130     CONTINUE
C---->READ AND ECHO BOUNDARY CONDITIONS
      PRINT 2090
               JSP = 1
  140 READ 1060, INI3(JSP), INL3(JSP), NSTEP3(JSP), KODEXN(JSP),
     1         KODEYN(JSP), USPN(JSP), VSPN(JSP), IENDN
          IF ( IENDN .EQ. IEND ) GO TO 150
              JSP = JSP + 1
          GO TO 140
  150 PRINT 2100
     CPRINT 2110, ( INI3(I), INL3(I), NSTEP3(I), KODEXN(I), KODEYN(I),
     1                                    I = 1, JSP )
C---->READ AND ECHO APPLIED NODAL LOADS
      PRINT 2120
              JL = 1
  160 READ 1070, INI4(JL), INL4(JL), NSTEP4(JL),FUN(JL), FVN(JL),
     1           SFUN(JL), SFVN(JL), IENDN
          IF ( IENDN .EQ. IEND ) GO TO 170
              JL = JL + 1
          GO TO 160
 170  PRINT  2130
      PRINT 2140,( INI4(I), INL4(I), NSTEP4(I), FUN(I), FVN(I), SFUN(I),
     1               SFVN(I), I=1,JL )
C---->READ AND ECHO BOND LINKS
      PRINT 2150
      PRINT 2160
          IF ( NB .NE. 0 ) GO TO 180
              JB = 1
          GO TO 185
  180     DO 190 JB = 1, NB
  185 READ 1080,  INI5(JB), INL5(JB), SKH(JB), SKV(JB), BFD(JB)
      PRINT 2170, INI5(JB), INL5(JB), SKH(JB), SKV(JB), BFD(JB)
  190     CONTINUE
C---->READ AND ECHO ITERATION INFORMATION
      PRINT 2190
      READ 1090,  NSTEPT, NIT, NRUN, IFRQ, IPRINT, TNC
      PRINT 2200, NSTEPT, NIT, NRUN, TNC
C---->END INPUT
      RETURN
  920 PRINT 9020
 9020 FORMAT (///42H      NUMBER OF ELEMENTS OR NUMBER OF NODES
     1           35H SPECIFIED EXCEEDS DIMENSION SIZE. ,
     2  /        20H      TO CORRECT:
```

```
3   /        30H          1. REDUCE MESH SIZE.
4   /        42H     OR,  2. REDIMENSION ARRAYS IN DRIVER.    )
 STOP
 END
```

```
      SUBROUTINE DIST ( X, Y, E, PR,NBC,US, VS, NPCODE, FX, FY, NPMAX,
     1            NELMAX, NBMAX, NUMNP, NUMEL, NPC, MBAND, IMBD, ISTAR ,
     2                 SKH, SKV, NELPT, SFX, SFY, MTYP )
C---->INITIALIZE AND DISTRIBUTE DATA FORM TABLES 2, 3, 4,5
      IMPLICIT REAL * 8 ( A-H, O-Z )
      COMMON / CONT / ID1(20), ID2(20)
      COMMON / MESH / X1, Y1, X2, Y2, NX, NY, NPT
      COMMON / MATL / EN(10), PRN(10), ELFA(10) , INI2(10), INL2(10),
     1                NSTEP2(10), UNIAX(10,3,10), FYT(10), FYC(10),
     2                NPRO(4,10), ELBET(10), NMPRT
     OCOMMON / DISP / USPN(10), VSPN(10), INI3(10), INL3(10),
     1                NSTEP3(10), KODEXN(10), KODEYN(10), JSP
      COMMON / LOAD / FUN(10), FVN(10), INI4(10), INL4(10),NSTEP4(10),
     1                SFUN(10), SFVN(10), JL
      COMMON / BOND / INI5(30), INL5(30), BFD(30), NB
      COMMON / ITER / NSTEPT,NIT,NFUN,IFRO,IPRINT
     CDIMENSION X(NPMAX), Y(NPMAX), E(NELMAX), PR(NELMAX), NBC(NBMAX,2),
     1    US(NPMAX), VS(NPMAX), FX(NPMAX), FY(NPMAX), NPC(NELMAX,4),
     2    NPCODE(NPMAX), ISTAR(NPMAX), MTYP (NELMAX), NELPT(NELMAX),
     3    SFX(NPMAX), SFY(NPMAX)
      DATA ZERO, ONE / 0.0000, 1.0000 /
      DATA IBLANK / 1H / , IAST / 1H* /
C---->INITIALIZE
   80     DO 100 I = 1, NUMEL
                E(I) = ZERO
                PR(I) = ZERO
                MTYP(I) = 0
                NELPT(I) = 0
  100     CONTINUE
          DO 110 I = 1, NUMNP
                US(I) = ZERO
                VS(I) = ZERO
                NPCODE(I) = 0
                FX(I) = ZERO
                FY(I) = ZERO
                SFX(I) = ZERO
                SFY(I) = ZERO
  110     CONTINUE
C---->DISTRIBUTE TABLE 2 - MATERIAL PROPERTIES
          DO 130 J = 1, NMPRT
                ISTRT = INI2(J)
                ISTOP = INL2(J)
                NSTEP = NSTEP2(J)
          IF ( ISTOP .GT. ISTRT ) GO TO 114
                ISTOP = ISTRT
                NSTEP = 1
  114     DO 120 I = ISTRT, ISTOP, NSTEP
                E(I) = E(I) + EN(J)
                PR(I) = PR(I) +PRN(J)
                MTYP(I) = NPRO(1,J)+MTYP(I)
```

```
              NELPT(I) = J
  120     CONTINUE
  130     CONTINUE
C---->DISTRIBUTE TABLE 3-BOUNDARY CONDITIONS
          DO 150 J = 1, JSP
              ISTRT = INI3(J)
              ISTOP = INL3(J)
              NSTEP = NSTEP3(J)
          IF ( ISTOP .GT. ISTRT ) GO TO 134
              ISTOP = ISTRT
              NSTEP = 1
  134         IKODE = 2 * KODEXN(J) + KODEYN(J)
          IF ( IKODE .GT. 3 ) IKODE = 4
          DO 140 I = ISTRT, ISTOP, NSTEP
              US(I) = USPN(J)
              VS(I) = VSPN(J)
              NPCODE(I) = IKODE
  140     CONTINUE
  150     CONTINUE
C---->TEST ALL NODES RESTRAINED
          DO 151 J = 1, NUMNP
              ISTAR(J) = IAST
  151     CONTINUE
          DO 153 N = 1, NUMEL
          IF ( E(N) .LE. ZERO )  GO TO 153
          DO 152 J = 1, 4
              NP = NPC(N,J)
              ISTAR(NP) = IBLANK
  152     CONTINUE
  153     CONTINUE
          DO 154 J = 1, NUMNP
          IF ( ISTAR(J) .EQ. IBLANK ) GO TO 154
              NPCODE(J) = 4
              US(J) = ONE
              VS(J) = ONE
  154     CONTINUE
C---->DISTRIBUTE TABLE 4 - APPLIED NODAL LOADS
          DO 170 J = 1, JL
              ISTRT = INI4(J)
              ISTOP = INL4(J)
              NSTEP = NSTEP4(J)
          IF ( ISTOP .GT. ISTRT ) GO TO 155
              ISTOP = ISTRT
              NSTEP = 1
  155     DO 160 I = ISTRT, ISTOP, NSTEP
              FX(I) = FX(I) + FUN(J)
              FY(I) = FY(I) + FVN(J)
              SFX(I) = SFX(I) + SFUN(J)
              SFY(I) = SFY(I) + SFVN(J)
  160     CONTINUE
  170     CONTINUE
C---->DISTRIBUTE TABLE 5 - BOND LINK
          IF ( NB .EQ. 0 ) GO TO 210
          DO 250 N = 1,NB
              NBC(N,1) = INI5(N)
              NBC(N,2) = INL5(N)
  250     CONTINUE
          IF ( NB .GT. NBMAX ) GO TO 270
C---->DETERMINE BAND WIDTH
  210         MBAND = 0
          DO 240 I = 1, NUMEL
          DO 230 J = 1, 3
              L = J + 1
          DO 220 K = L, 4
              IDIFF = IABS ( NPC(I,J) - NPC(I,K) )
          IF ( IDIFF .LE. MBAND ) GO TO 220
              MBAND = IDIFF
              IMBD = I
  220     CONTINUE
  230     CONTINUE
  240     CONTINUE
      PRINT 2020
      PRINT 2040 , NUMNP, NUMEL, MBAND, IMBD
      RETURN
 2010FORMAT (     1H1, /////,
    1             51H     FINITE ELEMENT ANALYSIS OF AXISYMMETRIC SOLIDS,
    2             26H PROBLEMS - PROGRAM NFEA       ,//, 5X, 20A4,
    3            ///                          , 5X, 20A4 )
 2020 FORMAT (// 26H          RECTANGULAR MESH     )
 2040FORMAT (    35H          NUMBER OF NODAL POINTS = , I5 ,
    1   /        35H          NUMBER OF ELEMENTS     = , I5 ,
    2   /        35H          MAXIMUM BAND PARAMETER = , I5 ,
    3   /        35H          ELEMENT WITH MAX. BAND = , I5  )
  270 PRINT 2000
 2000 FORMAT ('  NUMBER OF BOND LINKS EXCEEDES DIMENSION ')
      STOP
      END
```

```
      SUBROUTINE FVEC ( FX,FY,NUMNP,NPMAX,NPMAX2,FOFCE,SFORCE,SFX,SFY )
C---->SET UP FORCE VECTOR
      IMPLICIT REAL * 8 ( A-H, O-Z )
      DIMENSION FORCE( NPMAX2 ), FX( NPMAX), FY( NPMAX)
      DIMENSION SFORCE( NPMAX2 ), SFX( NPMAX ), SFY( NPMAX )
      DATA ZERO /0.0000/
              NN= 2 * NUMNP
          DO 100 I = 1, NN
              FORCE(I) = ZERO
              SFORCE(I) = ZERO
  100     CONTINUE
          DO 200 N = 1, NUMNP
              K = 2 *N - 1
              FORCE ( K ) =   FORCE ( K ) + FX ( N )
              FORCE ( K+1 ) = FORCE ( K+1 ) + FY ( N )
              SFORCE(K) = SFORCE(K) + SFX(N)
              SFORCE(K+1) = SFORCE(K+1) + SFY(N)
  200     CONTINUE
      RETURN
      END
```

```
      SUBROUTINE STIFF ( X, Y, E, PR, NPMAX, NELMAX, NUMNP, NUMEL, NPC,
     1                  NELPT, ELFA, ELBET, FPT, FPC, NPRO )
C---->FORM ELEMENTAL STIFFNESS MATRICIES
      IMPLICIT REAL * 8 ( A-H, O-Z )
      DIMENSION X(NPMAX), Y(NPMAX), E(NELMAX), PR(NELMAX), XX(4), YY(4),
     1          C(4,4), SOD(8,8), LB(4), DD(4,8), NPC(NELMAX,4),
     2          ELFA(10), ELBET(10), NELPT(NELMAX), SIG(4), EPS(4),
     3          GY(2), FPT(10), FPC(10), BD(4,8), NPRO(4,10), CC(4,4)
      DATA ZERO, P5, ONE, TWO / 0.0000, 0.5000, 1.0000, 2.0000 /
              IERR = 0
      REWIND 2
      REWIND 8
          DO 400 N = 1, NUMEL
          DO 100 I = 1, 4
              SIG(I) = ZERO
              EPS(I) = ZERO
          DO 100 J = 1, 4
              C(I,J) = ZERO
  100     CONTINUE
              NPT = NELPT(N)
              MODL = NPRO(2,NPT)
C---->PLANE STRESS PROBLEM
              COMM = E(N) /(( ONE +PR(N))*(ONE-TWO*PR(N)))
              C(1,1) = COMM *(ONE - PR(N))
              C(2,2) = COMM *(ONE-PR(N))
              C(3,3) = COMM *(ONE-PR(N))
              C(1,2) = PR(N) * COMM
              C(1,3) = PR(N) * COMM
              C(2,3) = PR(N) * COMM
              C(2,1) = C(1,2)
              C(3,1) = C(1,3)
              C(3,2) = C(2,3)
              C(4,4)= COMM * (ONE - TWO *PR(N) )
C---->SET UP NODAL COORDINATES
             DO 200 I = 1, 4
              LB(I) = NPC(N,I)
              XX(I) = X(LB(I))
              YY(I) = Y(LB(I))
  200     CONTINUE
              AA = P5 * ( XX(3) - XX(1) )
              BB = P5 * ( YY(3) - YY(1) )
              XO = P5 * ( XX(3) + XX(1) )
              YO = P5 * ( YY(3) + YY(1) )
              RO = P5 * ( XX(3) + XX(1) )
      CALL FORMB ( AA, BB, XO, YO, C, BD, MODL )
      CALL AXREC ( AA, BB, C, RO, SOD, MODL )
      WRITE (2) (LB(I),I=1,4), ((SOD(I,J),I=1,8),J=1,8)
              ALFA = ELFA(NPT)
              BETA = ELBET(NPT)
              GY(1) = FPT(NPT)
```

```
            GY(2) = FPC(NPT)
            F0 = -10.0000
            F1 = 1.0010
            ANGLE = ZERO
            H = 1.0010
            GX = ZERO
            SX = ZERO
            MI = 0
            C22 = ZERO
            C44 = ZERO
            SUP = ZERO
            MIO = 0
        DO 300 I =1,4
        DO 300 J =1,4
            CC(I,J) = C(I,J)
300     CONTINUE
      WRITE (8)   (SIG(I),EPS(I),I=1,4),GY(1),GY(2),H,ALFA,BETA,F0,F1,
     1             SUP,GX,SX,C22,C44,MI,MIO,XO,YO,AA,BB,RO,((C(I,J),
     2             I=1,4),J=1,4),((BD(I,J),J=1,8),I=1,4),((CC(I,J),J=1,4
     3             ),I=1,4) ,ANGLE
400     CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ASMBL ( US, VS, NPCODE, NPMAX, NUMNP, NUMEL, MBAND,
     1                    MLONG, MWIDE, A, NBC, NUMBLK, NB, SKH, SKV,
     2                    NBMAX, F, NELMAX )
C---->ASSEMBLE STIFFNESS MATRIX
      IMPLICIT REAL * 8 ( A-H, O-Z )
      DIMENSION NPCODE(NPMAX),NBC(NBMAX,2), A(MLONG,MWIDE), SKH(NBMAX),
     1          SKV(NBMAX), LB(4), SQD(8,8), LM(4), SS(4,4), US(NPMAX),
     2          VS(NPMAX), E(NELMAX), EB(30)
      DATA ZERO,ONE /0.0000, 1.0000 /
            NSTOP=0
            NUMBLK=0
      REWIND 1
            NW = MWIDE / 2
        DO 50 M=1,NUMEL
            E(M) = DABS(E(M))
  50    CONTINUE
        DO 75 M=1,NB
            EB(M) = ONE
  75    CONTINUE
        DO 100 I = 1, MLONG
        DO 100 J = 1, MWIDE
            A(I,J) = ZERO
 100    CONTINUE
C---->FORM STIFFNESS MATRIX BY BLOCKS
 200        NUMBLK = NUMBLK + 1
            NSTART = NSTOP + 1
            NSTOP = NSTART + NW - 1
        IF ( NSTOP .GT. NUMNP ) NSTOP = NUMNP
            KSHIFT = 2 * NSTART - 2
      REWIND 2
        DO 310 N = 1, NUMEL
      READ (2) (LB(I),I=1,4), ((SQD(I,J),I=1,8),J=1,8)
        IF ( E(N) .LE. ZERO ) GO TO 310
        DO 120 I=1,4
        IF ( LB(I) .GE. NSTART .AND. LB(I) .LE. NSTOP ) GO TO 130
 120    CONTINUE
        GO TO 310
 130    CONTINUE
            E(N) = -E(N)
        DO 280 I = 1, 4
            LM(I) = 2 * (LB(I)-1)
 280    CONTINUE
C---->ADD TO TOTAL STIFFNESS MATRIX
        DO 300 I = 1, 4
        DO 300 K = 1, 2
            II = LM(I) + K - KSHIFT
            KK = 2 * ( I - 1 ) + K
        DO 290 J = 1, 4
        DO 290 L = 1, 2
            JJ = LM(J) + L - II + 1 - KSHIFT
```

```
      IF ( JJ .LE. 0 ) GO TO 290
           LL = 2 * ( J - 1 ) + L
           A(II,JJ) = A(II,JJ) + SQD(KK,LL)
  290 CONTINUE
  300 CONTINUE
  310 CONTINUE
C---->ADD SPRINGS TO STIFFNESS MATRIX
      DO 320 N = NSTART, NSTOP
           K = 2 * N - 1 - KSHIFT
      IF ( NPCODE(N) .NE. 4 ) GO TO 320
  315      A(K,1) = A(K,1) + US(N)
           A(K+1,1) = A(K+1,1) + VS(N)
  320 CONTINUE
C---->SET UP BOND LINK STIFFNESS
      IF ( NB .EQ. 0 ) GO TO 510
      DO 510 N = 1 , NB
      IF ( EB(N) .LE. ZERO ) GO TO 510
      DO 420 I = 1 , 2
      IF ( NBC(N,I) .GE.NSTART .AND. NBC(N,I) .LE. NSTOP ) GO TO 430
  420 CONTINUE
      GO TO 510
  430 CONTINUE
           EB(N) = -EB(N)
      DO 440 I = 1, 4
      DO 440 J = 1, 4
  440      SS ( I ,J ) = ZERO
           SS ( 1 ,1 ) = SKH( N )
           SS ( 2 ,2 ) = SKV( N )
           SS ( 3 ,3 ) = SKH( N )
           SS ( 4 ,4 ) = SKV( N )
           SS ( 1 ,3 ) = -SKH( N )
           SS ( 3 ,1 ) = -SKH( N )
           SS ( 2 ,4 ) = -SKV( N )
           SS ( 4 ,2 ) = -SKV( N )
      DO 480 I = 1 , 2
  480      LM( I ) = 2 * ( NBC( N , I ) - 1 )
      DO 500 I = 1 , 2
      DO 500 K = 1 , 2
           II = LM ( I ) + K - KSHIFT
           KK = 2 * ( I - 1 ) + K
      DO 490 J = 1 , 2
      DO 490 L = 1 , 2
           JJ = LM( J ) + L - II + 1 - KSHIFT
      IF ( JJ .LE. 0 ) GO TO 490
           LL = 2 * ( J - 1 ) + L
           A (II,JJ) = A(II,JJ) + SS(KK,LL)
  490 CONTINUE
  500 CONTINUE
  510 CONTINUE
C---->REVISE FCR SPECIFIED DISPLACEMENTS
```

```
           NMAX = NSTOP + MBAND / 2
      IF ( NMAX .GT. NUMNP ) NMAX = NUMNP
      DO 350 N = NSTART, NMAX
     .     KEY = NPCODE(N) + 1
      GO TC ( 350, 340, 330, 330, 350 ), KEY
C---->U - SPECIFIED
  330      M = 2 * N - 1 - KSHIFT
           U = US(N)
   CALL MODFY (   A, MLONG, MWIDE, MBAND, M, U )
      IF ( KEY .NE. 4 ) GO TO 350
C---->V - SPECIFIED
  340      M = 2 * N - KSHIFT
           U = VS(N)
   CALL MODFY (   A, MLONG, MWIDE, MBAND, M, U )
  350 CONTINUE
C---->WRITE BLOCK OF EQUATIONS ON TAPE AND SHIFT UP LOWER BLOCK
  375 WRITE (1) ((A(N,M),M=1,MBAND),N=1,MWIDE )
      DO 390 N = 1, MWIDE
           K = N + MWIDE
      DO 390 M = 1, MWIDE
           A(N,M) = A(K,M)
           A(K,M) = ZERO
  390 CONTINUE
C---->CHECK FOR LAST BLOCK
      IF ( NSTOP .LT. NUMNP ) GC TO 200
      DO 400 N=1,NB
           EB(N) = CABS(EB(N))
  400 CONTINUE
   RETURN
   END
```

```
      SUBROUTINE MODFY (    A, MLONG, MWIDE, MBAND, M, U )
            IMPLICIT REAL *8 (A-H,O-Z)
C----->MODIFY STIFFNESS MATRIX
      DIMENSION A(MLONG,MWIDE)
      DATA ZERO, ONE / 0.0D00, 1.0D00 /
C----->START
            DO 100 I = 2, MBAND
                  K = M - I + 1
            IF ( K .LE. 0 ) GO TO 100
                  A(K,I) = ZERO
  100             K = M + I - 1
                 IF ( MLONG .LT. K ) GO TO 110
                  A(M,I) = ZERO
  110       CONTINUE
                  A(M,1) = ONE
       RETURN
       END
```

```
      SUBROUTINE FORBLK ( B, MWIDE, MLONG, NPCODE, NPMAX, NUMNP, FORCE,
     1                      NPMAX2, US, VS )
      IMPLICIT REAL * 8 ( A-H, O-Z )
C----->SET  FORCE VECTOR IN BLOCKS
      DIMENSION B(MLONG), FORCE(NPMAX2), US(NPMAX), VS(NPMAX),
     1          NPCODE(NPMAX)
      DATA ZERO / 0.0D00 /
C----->START
                NUMBL=0
                NB=MWIDE/2
                NSTOP=0
      REWIND 3
            DO 100 I=1,MLONG
                B(I)=ZERO
  100        CONTINUE
  200           NUMBL=NUMBL+1
                NSTART = NSTOP+1
                NSTOP = NSTART+NB-1
            IF ( NSTOP.GT. NUMNP  ) NSTOP=NUMNP
                KSHIFT=2*NSTART-2
            DO 300 N=NSTART,NSTOP
                K=2*N-1-KSHIFT
                L=2*N-1
                B(K)=B(K)+FORCE(L)
                B(K+1)=B(K+1)+FORCE(L+1)
  300       CONTINUE
C----->DEVICE FOR BOUNDARY COND.
                NMAX=NSTOP+MBAND/2
            IF ( NMAX.GT. NUMNP ) NMAX=NUMNP
            DO 350 N=NSTART,NMAX
                KEY=NPCODE(N)+1
            GO TO ( 350,340,330,330,350 ) , KEY
  330           M=2*N-1-KSHIFT
                B(M)=US(N)
            IF ( KEY.NE.4 ) GO TO 350
  340           M=2*N-KSHIFT
                B(M)=VS(N)
  350       CONTINUE
      WRITE ( 3 )(B(I),I=1,MWIDE)
            DO 390 N=1,MWIDE
                K=MWIDE+N
                B(N)=B(K)
                B(K)=ZERO
  390       CONTINUE
C----->CHECK LAST BLOCK
            IF(NSTOP.LT.NUMNP) GO TO 200
            GO TO 400
  400 RETURN
      END
```

```
      SUBROUTINE BANSOL ( B, A, MBAND, NUMBLK, MWIDE, MLONG )
C---->SOLVE BANDED EQUATIONS
      IMPLICIT REAL * 8 ( A-H, O-Z )
      DIMENSION B(MLONG), A(MLONG,MWIDE)
      DATA ZERO / 0.0D00 /, ONE / 1.0D00 /
C---->START
              NSTART = MWIDE + 1
      REWIND 1
      REWIND 3
      REWIND 4
              NBLK = 0
      READ ( 1 ) (( A(N,M),M=1,MBAND),N=NSTART,MLONG)
      READ ( 3 ) ( B(N),N=NSTART,MLONG)
  100         NBLK = NBLK + 1
          DO 120 N = 1, MWIDE
              M = MWIDE + N
              B(N) = B(M)
              B(M) = ZERO
          DO 110 L = 1, MBAND
              A(N,L) = A(M,L)
              A(M,L) = ZERO
  110     CONTINUE
  120     CONTINUE
          IF ( NBLK .EQ. NUMBLK ) GO TO 130
      READ ( 1 ) (( A(N,M),M=1,MBAND),N=NSTART,MLONG)
      READ ( 3 ) ( B(N),N=NSTART,MLONG)
C---->REDUCE BLOCK
  130     DO 160 N = 1, MWIDE
          IF ( A(N,1) .EQ. ZERO ) GO TO 160
              TEMP = ONE / A(N,1)
              B(N) = TEMP * B(N)
          DO 150 M = 2, MBAND
          IF ( A(N,M) .EQ. ZERO ) GO TO 150
              C = A(N,M) * TEMP
              I = N + M - 1
              J = 0
          DO 140 K = M, MBAND
              J = J + 1
              A(I,J) = A(I,J) - C * A(N,K)
  140     CONTINUE
              B(I) = B(I) - A(N,M) * B(N)
              A(N,M) = C
  150     CONTINUE
  160     CONTINUE
C---->WRITE REDUCED BLOCK ON TAPE
          IF ( NUMBLK .EQ. NBLK ) GO TO 170
      WRITE (4) ( B(N), ( A(N,M), M = 2, MBAND ), N = 1, MWIDE )
          GO TO 100
C---->BACK SUBSTITUTE FOR DISPLACEMENTS
  170     DO 190 M = 1, MWIDE
              N = MWIDE + 1 - M
          DO 180 K = 2, MBAND
              L = N + K - 1
              B(N) = B(N) - A(N,K) * B(L)
  180     CONTINUE
              NM = N + MWIDE
              B(NM) = B(N)
              A(NM,NBLK) = B(N)
  190     CONTINUE
              NBLK = NBLK - 1
          IF ( NBLK .EQ. 0 ) GO TO 200
      BACKSPACE 4
      READ ( 4 ) ( B(N), ( A(N,M), M = 2, MBANC ), N = 1, MWIDE )
      BACKSPACE 4
          GO TO 170
  200     CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RECGEN(NPMAX,NELMAX,X,Y,NPC,NUMNP,NUMEL,NX,NY,NNS,NES,
     1                  X1,Y1,X2,Y2)
      IMPLICIT REAL *8 (A-H,O-Z)
C---->CALCULATE NODAL COOR. & CONNECTIVITY FOR RECTANGLE MESH
      DIMENSION X(NPMAX),Y(NPMAX),NPC(NELMAX,4)
      DATA ZERO,ONE/0.0D00,1.0D00/
              NUMNP=NNS+NX*NY+NX+NY
              NUMEL=NES+NX*NY-1
              DX =(X2-X1)/NX
              DY =(Y2-Y1)/NY
              NSTEP=NX+1
              IMAX=NY+1
          DO 100 I=1,IMAX
              DELX=ZERO
              XI=I-ONE
              YST=Y1+XI*DY
              JSTRT=NNS+(I-1)*NSTEP
              JSTOP=JSTRT+NX
          DO 110 J=JSTRT,JSTOP
              X(J)=X1+DELX
              Y(J)=YST
              DELX=DELX+DX
  110     CONTINUE
  100     CONTINUE
          DO 120 I=NES,NUMEL
              J=I-NES
              K=J/NX
              NNUM=NNS+J+K
              NPC(I,1)=NNUM
              NPC(I,2)=NNUM+1
              NPC(I,3)=NNUM+NX+2
              NPC(I,4)=NNUM+NX+1
  120     CONTINUE
      RETURN
      END
```

```
      SUBROUTINE AXREC ( AA, BB, D, RO, SE, MODL )
C---->FORM ELEMENT STIFFNESS MATRIX FOR AXISYMMETRIC RECTANGLE ELEMENTS
      IMPLICIT REAL * 8 ( A-H,O-Z )
      DIMENSION SE(8,8),D(4,4),BM(4,8),DB(4,8),S(8,8),DD(4,4),A(4,4)
      DATA ZERO,ONE,THREE /0.0D00,1.0D00,3.0D00/
C---->INITIALIZE
        DO 120 I=1,8
        DO 100 J=1,8
              SE(I,J)=ZERO
              S(I,J)=ZERO
  100     CONTINUE
          DO 110 J=1,4
              BM(J,I)=ZERO
              DB(J,I)=ZERO
  110     CONTINUE
  120     CONTINUE
          DO 122 I=1,4
          DO 122 J=1,4
              DD(I,J)=ZERO
              A(I,J) = ZERO
  122     CONTINUE
          DO 123 I=1,3
          DO 123 J=1,3
              DD(I,J) = D(I,J)
  123     CONTINUE
              A(4,4) = D(4,4)
              R3=ONE/DSQRT(THREE)
C---->FORM STIFFNESS MATRIX BY FOUR POINT NUMERICAL INTEGRATION
          DO 190 I=1,4
              ETA=-R3*((-1)**((I-1)/2))
              XI=ETA*((-1)**(I+1))
              RR=XI*AA +RO
C---->FORM B-MATRIX
      CALL DERIVE(ETA,XI,AA,BB,RR,BM)
C---->FORM PRODUCT D*B
          DO 150 II=1,4
          DO 140 JJ=1,8
              TEMP=ZERO
          DO 130 KK=1,4
          IF ( MODL .EQ. 1 ) GO TO 125
              TEMP=TEMP+DD(II,KK)*BM(KK,JJ)
          GO TO 130
  125         TEMP=TEMP+D(II,KK)*BM(KK,JJ)
  130     CONTINUE
              DB(II,JJ)=TEMP
  140     CONTINUE
  150     CONTINUE
C---->FORM PRODUCT BT*D*B
          DO 180 II=1,8
          DO 170 JJ=1,8
```

```
          TEMP=ZERO
      DO 160 KK=1,4
          TEMP=TEMP+BM(KK,II)*DB(KK,JJ)
160   CONTINUE
          S(II,JJ) =S(II,JJ) +TEMP*AA*BB*RR
170   CONTINUE
180   CONTINUE
190   CONTINUE
      IF ( MODL .EQ. 2 ) GO TO 195
          DO 290 I=1,8
          DO 290 J=1,8
          SE(I,J)=S(I,J)
290   CONTINUE
      GO TO 300
195   CONTINUE
      DO 200 I=1,4
      DO 200 J=1,8
      BM(I,J) = ZERO
200   CONTINUE
          ETA=ZERO
          XI =ZERO
          RR = RO
   CALL DERIVE(ETA,XI,AA,BB,RR,BM)
      DO 230 II=1,4
      DO 220 JJ=1,8
          TEMP=ZERO
      DO 210 KK=1,4
          TEMP=TEMP+A(II,KK)*BM(KK,JJ)
210   CONTINUE
          DB(II,JJ)=TEMP
220   CONTINUE
230   CONTINUE
      DO 280 I=1,8
      DO 270 J=1,8
          TEMP=ZERO
      DO 260 K=1,4
          TEMP=TEMP+BM(K,I)*DB(K,J)
260   CONTINUE
          SE(I,J) = SE(I,J)+TEMP*AA*BB*RR
270   CONTINUE
280   CONTINUE
      DO 300 I=1,8
      DO 300 J=1,8
          SE(I,J) = SE(I,J)+S(I,J)
300   CONTINUE
   RETURN
   END
```

```
      SUBROUTINE DERIVE(ETA,XI,AA,BB,RR,BM)
C---->FORM B-MATRIX FOR LINEAR RECTANGLE
      IMPLICIT REAL * 8 ( A-H,O-Z )
      DIMENSION BM(4,8)
      DATA ONE,FOUR /1.0D00,4.0D00/
C---->INITIALIZE CONSTANTS
              C4=ONE/FOUR
              BM(1,1)=C4*(ETA-ONE)/AA
              BM(2,2)=C4*(XI-ONE)/BB
              BM(3,1)=C4*(ONE-XI)*(ONE-ETA)/(RR)
              BM(4,1)=BM(2,2)
              BM(4,2)=BM(1,1)
              BM(1,3)=C4*(ONE-ETA)/AA
              BM(2,4)=-C4*(ONE+XI)/BB
              BM(3,3)=C4*(ONE-ETA)*(ONE+XI)/(RR)
              BM(4,3)=BM(2,4)
              BM(4,4)=BM(1,3)
              BM(1,5)=C4*(ONE+ETA)/AA
              BM(2,6)=C4*(ONE+XI)/BB
              BM(3,5)=C4*(ONE+XI)*(ONE+ETA)/(RR)
              BM(4,5)=BM(2,6)
              BM(4,6)=BM(1,5)
              BM(1,7)=-C4*(ONE+ETA)/AA
              BM(2,8)=C4*(ONE-XI)/BB
              BM(3,7)=C4*(ONE-XI)*(ONE+ETA)/(RR)
              BM(4,7)=BM(2,8)
              BM(4,8)=BM(1,7)
      RETURN
      END
```

```
      SUBROUTINE FORMB ( AA, BB, XO, YO, C ,BD )
          IMPLICIT REAL *8 (A-H,O-Z)
      DIMENSION BM(4,8),BD(4,8)
      DATA ZERO / 0.0000 /
                RO=XO
                ETA=ZERO
                XI=ZERO
                RR=RO
          DO 180 I=1,4
          DO 180 J=1,8
                BM(I,J)=ZERO
180       CONTINUE
      CALL DERIVE(ETA,XI,AA,BB,RR,BM)
          DO 200 II=1,4
          DO 190 JJ=1,8
                BD(II,JJ)=BM(II,JJ)
190     CONTINUE
200     CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PLASCL ( NUMEL, NUMNP, NPMAX, NELMAX, NPMAX2, DELTA,
     1                    DISPL, NBC, NB, SKV, SKH, UNIAX, NPRO,MTYP,
     2             BFD, NBMAX, INVRT, NELPT, NSTEPT, FORCE, IUPDAT )
      IMPLICIT REAL *8 (A-H,O-Z)
      COMMON / IMPT / SIG(4), EPS(4), GY(2), H, ALFA, BETA, FO, F1, SUP,
     1               GX, SX, C22, C44, MI, MIC,  C(4,4), XO, YO, RO,
     2                     AA, BB, BD(4,8), LB(4), ST(8,8),CC(4,4),ANGLE
      DIMENSION  DSIG(4), DEPS(4), NBC(NBMAX,2), SKH(30), SKV(30),
     1     DELTA(NPMAX2),MTYP(NELMAX), NELPT(NELMAX), VEL(8), BFD(30),
     2          NPRO(4,10), UNIAX(10,3,10), FORCE(NPMAX2),DISPL(NPMAX2)
      DATA ZERO / 0.0000 /
      REWIND 2
      REWIND 8
      REWIND 9
          DO 200 M = 1, NUMEL
      READ (2) (LB(I),I=1,4),(( ST(I,J),J=1,8),I=1,8)
      READ (8) ( SIG(I), EPS(I),I=1,4), GY(1), GY(2), H, ALFA, BETA, FO,
     1           F1, SUP, GX, SX, C22, C44, MI, MIC, XC, YO, AA, BB, RO ,
     2          ((C(I,J),I=1,4),J=1,4),((BD(I,J),J=1,8),I=1,4),((CC(I,J),
     3          J=1,4),I=1,4) ,ANGLE
                MTYPE = MTYP(M)
                NPT = NELPT(M)
          DO 100 I=1,4
                DSIG(I) = ZERO
                DEPS(I) = ZERO
  100     CONTINUE
C---->SET UP NODAL DISPLACEMENT AND COORDINATES
          DO 110 I=1,4
                II = 2*I
                KK = 2*LB(I)
                VEL(II-1) = DELTA(KK-1)
                VEL(II) = DELTA(KK)
  110     CONTINUE
C---->CALCULATE STRAINS
          DO 130 II=1,4
                TEMP = ZERO
          DO 120 JJ=1,8
                TEMP = TEMP+BD(II,JJ)*VEL (JJ)
  120     CONTINUE
                DEPS(II) = TEMP
                EPS(II) = EPS(II) + DEPS(II)
  130     CONTINUE
C---->CALCULATE STRESSES
          DO 150 II=1,4
                TEMP = ZERO
          DO 140 JJ=1,4
                TEMP = TEMP+CC(II,JJ)*DEPS(JJ)
  140     CONTINUE
                DSIG(II) = TEMP
  150     CONTINUE
```

```
C---->START PLASTIC CALCULATIONS
C---->CHECK MATERIAL TYPE
            IF ( MTYPE .EQ. 1 ) GO TO 170
       CALL PLASTL ( FORCE, NPMAX2, UNIAX, NSTEPT, INVRT, NPT, DSIG,
     1                        DEPS, IUPDAT, M, NPRC )
            GO TO 180
  170 CALL PLSCON ( FORCE, NPMAX2, UNIAX, NSTEPT, INVRT, NPT, DSIG,
     1                        DEPS, IUPDAT, M, NPRO )
  180       CONTINUE
       WRITE (9) (   LB(I), I=1,4)    , (( ST(I,J), J=1,8), I=1,8),
     1             (SIG(I), EPS(I),I=1,4), GY(1), GY(2), H, ALFA, BETA, FO,
     2            F1, SUP, GX, SX, C22, C44, MI, MIO, XO, YO, AA, BB, RO ,
     3           ((C(I,J),I=1,4),J=1,4),((BD(I,J),J=1,8),I=1,4),((CC(I,J),
     4           J=1,4),I=1,4) ,ANGLE
  200       CONTINUE
C---->CHECK BOND LINKS
            IF ( NB .EQ. 0 ) GO TO 250
            IF ( INVRT .EQ. 0 ) GO TO 250
            DO 250 I=1,NB
                 II=NBC(I,1)*2
                 JJ=NBC(I,2)*2
                 VDISP =         ( DISPL(JJ) - DISPL(II) )
                 HDISP = CABS ( DISPL(JJ-1) - DISPL(II-1) )
            IF ( VDISP .LT. ZERO    ) GO TO 220
            IF ( VDISPL .LT. BFD(I) ) GO TO 220
                 SKV(I) = ZERO
                 SKH(I) = ZERO
  220       CONTINUE
                 VDISP = CABS ( DISPL(JJ) - DISPL(II) )
                 FSKH = SKH(I)*HDISP
                 FSKV = SKV(I)*VDISP
            IF ( FSKH .GT. 3.0D00*FSKV )  SKH(I) = ZERO
  250       CONTINUE
C---->SHIFT TAPES
       REWIND 2
       REWIND 8
       REWIND 9
            DO 300 N = 1,NUMEL
       READ (9)  (    LB(I), I=1,4)    , (( ST(I,J), J=1,8), I=1,8),
     1             (SIG(I), EPS(I),I=1,4), GY(1), GY(2), H, ALFA, BETA, FO,
     2            F1, SUP, GX, SX, C22, C44, MI, MIO, XO, YO, AA, BB, RO ,
     3           ((C(I,J),I=1,4),J=1,4),((BD(I,J),J=1,8),I=1,4),((CC(I,J),
     4           J=1,4),I=1,4),ANGLE
       WRITE (2)(LB(I),I=1,4),(( ST(I,J),J=1,8),I=1,8)
       WRITE (8) (SIG(I), EPS(I),I=1,4), GY(1), GY(2), H, ALFA, BETA, FO,
     1            F1, SUP, GX, SX, C22, C44, MI, MIO, XO, YO, AA, BB, RO ,
     2           ((C(I,J),I=1,4),J=1,4),((BD(I,J),J=1,8),I=1,4),((CC(I,J),
     3           J=1,4),I=1,4) ,ANGLE
  300       CONTINUE
       RETURN
       END
```

```
       SUBROUTINE PLSCON ( FORCE, NPMAX2, UNIAX, NSTEPT, INVRT, NPT, DGE,
     1                      DS, IUPDAT, M, NPRC )
       IMPLICIT REAL * 8 ( A-H, O-Z )
       COMMON / IMPT / G(4), S(4), GY(2), H, ALFA, BETA, FO, F1, SUP,
     1                 GX, SX, C22, C44, MI, MIC,  C(4,4), XO, YO, RO,
     2                    AA, BB, BE(4,3), LB(4), ST(8,8), CC(4,4) ,ANGLE
       DIMENSION DD(4,8),FORCE(NPMAX2), GO(4), GM(4), GPR(4), DGPL(4),
     1           DR(4), A(4), AB(4), D(4), UNIAX(10,3,10), NPRO(4,10),
     2           DS(4), DGE(4), DSPL(4), P(8)
       DATA  ZERO , PS , ONE , TWO , THREE,  ETE , PTE , RAD
     1 /0.0D00,0.5D00,1.0D00,2.0D00,3.0D00,-1.0D-7,1.0D-7,57.2957795D00/
C---->ELASTIC SOLUTION
                 MODL = NPRO(2,NPT)
            DO 50 I = 1,8
                 P(I) = ZERO
   50       CONTINUE
            IF ( NSTEPT .NE. 0 ) GO TO 70
            DO 60 I=1,4
                 GO(I) = ZERO
                 GM(I) = DGE(I)
   60       CONTINUE
            GO TO 760
C---->CALCULATE TOTAL STRESS
   70       DO 80 J=1,4
                 GO(J) = G(J)
                 GPR(J) = G(J)+DGE(J)
   80       CONTINUE
            IF ( MI ) 200,100,300
C---->START PLASTICITY CALCULATIONS
  100 CALL OCTA ( G, GY(1), ALFA, BETA, FO, A, NTEMP, MI )
       CALL OCTA ( GPR, GY(1), ALFA, BETA, F1, C, ISIGN, MI )
C---->CHECK ELASTIC CASE OR ELASTIC UNLOADING
            IF ( F1 .LT. ETE ) GO TO 125
            GO TO 150
  125       DO 130 I=1,4
                 GM(I) = GPR(I)
  130       CONTINUE
            IF ( GPR(3) .GT. UNIAX(2,1,NPT) ) GO TO 500
            GO TO 760
C---->CHECK UNLOADING FROM ELASTIC TO PLASTIC
  150       IF ( F1 .GT. PTE .AND. FO .LT. ETE ) GO TO 155
            GO TO 175
C---->GET AN ESTIMATE OF R
  155            R1 = -( FO/(F1-FO) )
C---->GET IMPROVED VALUE OF R
            DO 160 I=1,4
                 DGPL(I) = G(I)+R1*DGE(I)
  160       CONTINUE
       CALL OCTA ( DGPL, GY(1), ALFA, BETA, F2, A, NTEMP, MI )
                 DENOM = ZERO
```

```
          DO 170 J=1,4
                DENOM = DENOM+A(J)*DGE(J)
  170     CONTINUE
                R = P1-( F2/DENOM )
          GO TO 190
C---->CHECK CASE OF JUST YIELDING
  175     IF ( DABS ( F1 ) .LT. PTE .AND. FO .LT. ETE ) GO TO 180
          GO TO 185
  180           R = ONE
          GO TO 190
C---->CASE OF LOADING FROM YIELD SURFACE
  185           R = ZERO
  190     IF ( MI .NE. 0 ) GO TO 196
C---->CALCULLATE NEW H FOR CASE OF NEW PLASTICITY
          IF ( ISIGN ) 192,193,194
  192           NST = NPRO(3,NPT)
                H = (UNIAX(NST+3,1,NPT)- UNIAX(NST+2,1,NPT) )/
     1               (UNIAX(NST+3,3,NPT)- UNIAX(NST+2,3,NPT) )
                MI = -1
          GO TO 195
  193 PRINT 1100, M
  194           H = ZERO
                MI = 1
  195     IF ( ISIGN .LT.ZERO ) PRINT 1300,M
C---->SCALE PLASTIC STRESSES AND STRAINS
  196     DO 198 J=1,4
                G(J) = G(J)+R*DGE(J)
                DS(J) = (ONE-R)*DS(J)
                DGE(J) = (ONE-R)*DGE(J)
                GM(J) = G(J)
                DSPL(J) = ZERO
  198     CONTINUE
          IF ( ISIGN ) 205,299,294
C---->CASE OF OLD PLASTICITY
  200     DO 201 J=1,4
                GM(J) = G(J)
                DSPL(J) = ZERO
  201     CONTINUE
                IFLAG = IABS(MI)
          GO TO  (205,275), IFLAG
C---->PLASTICITY CALCULATION IN  C,C  REGION  (MI=-1)
  205 CALL OCTA ( GM, GY, ALFA, BETA, FO, A, ISIGN, MI )
C---->CALCULATE CONSTANT DLAMDA
                BETTA = ZERO
                DLAM=ZERO
C---->CALCULATE D=C*A
          DO 210 I=1,4
                AB(I) = A(I)
  210     CONTINUE
          DO 215 I=1,4
                D(I) = ZERO
                DB(I) = ZERO
          DO 212 J=1,4
                D(I) = D(I)+ C(I,J)*A(J)
                DB(I) = DB(I)+ C(I,J)*AB(J)
  212     CONTINUE
                BETTA = BETTA+A(I)*DB(I)
                DLAM = DLAM+D(I)*DS(I)
  215     CONTINUE
                DLAM = DLAM/(H+BETTA)
          IF ( H+BETTA .GT. ZERO ) GO TO 220
          IF ( BETTA .EQ. ZERO ) GO TO 220
      PRINT  2000, M
  220     IF ( DLAM .LE. ZERO ) GO TO 225
          GO TO 235
C---->CHECK ELASTIC UNLOADING OR ZERO PLASTIC STRAIN
  225     CONTINUE
                DLAM = ZERO
          DO 230 I=1,4
                GM(I) = GPR(I)
  230     CONTINUE
          GO TO 760
  235     DO 240 I=1,4
                GM(I) = GM(I)+DGE(I)
  240     CONTINUE
C---->CALCULATE PLASTICITY INCREMENTS
          DO 245 I=1,4
                DSPL(I) = DLAM*A(I)
                GM(I) = GM(I)-DLAM*DB(I)
  245     CONTINUE
C---->CALCULATE EQUIVELENT UNIAXIAL PLASTIC STRAIN
                DENOM = DSQRT(ONE/THREE*(TWO* DSPL(1)**2+TWO*DSPL(2)**2+
     1                           TWO*DSPL(3)**2+DSPL(4)**2))*ISIGN
                SUP = SUP+DENOM
          IF ( ISIGN ) 255 ,250,250
  250 PRINT 1500, M
C---->COMPRESSION
  255           ICASE = 2
                NST = 3+NPRO(3,NPT)
                NSTP = NPRO(4,NPT)+NPRO(3,NPT)
          DO 260 I=NST,NSTP
          IF ( DABS(UNIAX(I,3,NPT)) .LT. DABS(SUP) ) GO TO 260
                H = (UNIAX(I,1,NPT)-UNIAX(I-1,1,NPT) )/
     1             (UNIAX(I,3,NPT)-UNIAX(I-1,3,NPT) )
                GY(ICASE) = GY(ICASE)+H*DENOM
          GO TO 285
  260     CONTINUE
          IF ( INVRT . EQ . 0 ) GO TO 760
      PRINT 1000, M
          DO 270 I=1,4
```

```
         DO 270 J=1,4
                CC(I,J) = ZERO
  270    CONTINUE
                MI = -2
                MIO = MIO+1
         GO TO 760
C---->CASE OF ELEMENT CRUSHED IN COMPRESSION  (MI=-2)
  275    DO 280 I=1,4
                GM(I) = ZERO
  280    CONTINUE
         GO TO 760
C---->APPLY CORRECTION TO STRESSES
  285 CALL OCTA ( GM, GY, ALFA, BETA, F1, D, ISIGN, MI )
                DENOM = ZERO
         DO 290 I=1,4
                DENOM = DENOM+A(I)**2
  290    CONTINUE
                DENOM = F1/DENOM
         DO 292 I=1 ,4
                TYF = DENOM*AB(I)
                GM(I) = GM(I)-TYF
  292    CONTINUE
         GO TO 760
C---->CASE OF JUST CRACKING
  294           S1 = P5*(GM(1)+GM(2))
                S2 = P5*(GM(1)-GM(2))
                S3 = DSQRT(S2**2+GM(4)**2)
                GX = S1+S3
                G2 = S1-S3
                ANGLE = ZERO
         IF ( MI .EQ. 5 )    GO TO 500
         IF ( GM(4)  .EQ. ZERO .AND. S2 .EQ. ZERO ) GO TO 297
                ANGLE = P5*(DATAN2(GPR(4),S2))
  297           C44 = CC(4,4)
                C22 = CC(2,2)-CC(1,2)**2/CC(1,1)
     PRINT 1200, M,ANGLE
         DO 298 I=1,4
                AB(I) = S(I)-DS(I)
  298    CONTINUE
     CALL TRNS ( AB, DB, ANGLE, 1 )
                SX = DB(1)
                T = S1-S3
                TYF = RAD*ANGLE
         IF ( GPR(3) .GT. UNIAX(2,1,NPT) )  FC=ONE
         GO TO 300
  299 PRINT 1500, M
      STOP
C---->CALCULATE PRINCIPAL STRESSES
  300 CALL TRNS ( S(1), A, ANGLE, 1 )
      CALL TRNS ( GPR, D, ANGLE, 2 )
                C33 = C(3,3)
         IF ( GPR(3) .GT. UNIAX(2,1,NPT) )  FC=ONE
                FIX = ZERO
                IFLAG = MI
         GO TC ( 350,390,445 ), IFLAG
C---->CRACKED IN 1 DIRECTION
  350    CONTINUE
         DO 360 I=1,4
                D(I) = G(I)
  360    CONTINUE
     CALL TRNS ( GM , D, ANGLE, 2 )
         IF ( INVRT .GT. 0 ) GO TO 390
         GO TO 420
C---->FIRST REDUCE IN SHEAR
  390           T= P5*C44
                F1 = ZERO
                C11 = 0.1D-5 * C22
     CALL MATRED ( CC, ANGLE, C11, C22, T )
         IF ( FO .EQ. ONE ) CC(3,3) = 0.1D-5*C(3,3)
         IF ( FO .NE. ONE ) GO TO 400
                CC(1,3) = 0.1D-5*C(1,3)
                CC(3,1) = CC(1,3)
                CC(3,2) = CC(1,3)
                CC(2,3) = CC(1,3)
  400           MIO = MIO+1
         IF ( INVRT .GT. 0 )  MI=2
         IF ( INVRT .GT. 0 )   GO TO 420
C---->CALCULATE STRESS ELEMENT CAN PICK
                D(1) = ZERO
                D(2) = C22*A(2)
         IF ( FO .EQ. ONE )  GO TO 417
                D(3) = GPR (3)
         GO TC 418
  417           D(3) = ZERO
  418           D(4) = T *A(4)
  420    IF ( D(2) .LT. ZERO ) GO TO 430
C---->TENSION IN 2-DIRECTIONS
         IF ( D(2).LT. UNIAX(2,1,NPT ) ) GO TC 460
C---->ELEMENT CRACKED IN  2  DIRECTION
                MI = 3
     PRINT 1600, M
                D(2) = UNIAX(2,1,NPT)
                FIX = ONE
         GO TC 445
C---->COMPRESSION IN 2- DIRECTIONS
  430    IF ( A(2) .GT. UNIAX(NPRO(3,NPT)+NPRC(4,NPT),2,NPT)) GO TO 440
C---->CONCRETE HAS CRUSHED IN 2ND DIRECTION
                MI = 3
     PRINT 1600,M
                D(2) = UNIAX(5,1,NPT)
```

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            FIX = ONE
        GO TO 445
C---->CHECK ELASTIC OR PLASTIC
  440       NST = NPRC(3,NPT)
        IF ( A(2) .GT. UNIAX(NST+2,2,NPT )) GO TO 460
C---->CONCRETE IS PLASTIC
            A(2) = UNIAX(NST+2,1,NPT)
        GO TO 460
C---->CASE OF ELEMENT CRACKED OR CRUSHED IN 2-DIRECTION
  445   IF ( FO .EQ. ONE ) CC(3,3) = 0.1D-5*C(3,3)
        IF ( INVRT .GT. 0 )  FIX = ZERO
        IF ( FIX .EQ. ONE ) GO TO 460
     CALL MATFED ( CC, ANGLE, 0.1D-5,0.1D-5, F1 )
            MIO = MIO+1
        IF ( INVRT .GT. 0 ) GO TO 460
            D(1) = ZERO
            D(2) = ZERO
        IF ( FO .EQ. ONE ) GO TO 450
            D(3) = GPR(3)
        GO TO 455
  450       D(3) = ZERO
  455       D(4) = ZERO
C---->TRASFORM STRESSES
  460   CONTINUE
     CALL TRNS ( D, GM, -ANGLE, 2 )
        GO TO 760
C
  500   IF ( GPR(3) .LT. UNIAX(2,1,NPT) ) GO TO 760
        DO 550 I=1,4
            GM(I) = GPR(I)
  550   CONTINUE
            GM(3) = UNIAX(2,1,NPT)
            MI = 5
C
  760   IF ( IUPDAT .EQ. 0 ) GO TO 790
     PRINT 10, MI
        GO TO 800
C---->CALCULATE RESIDUAL FORCES
  790   IF ( MI .EQ. 0 ) GO TO 797
        DO 795 J=1,8
        DO 795 I=1,4
            P(J) = P(J)+BE(I,J)*(GPR(I)-GM(I)) *RO
  795   CONTINUE
  797   DO 798 I=1,4
            G(I) = GM(I)
  798   CONTINUE
  800   CONTINUE
        IF ( MIO .EQ. 0 ) GO TO 850
        IF ( INVRT .EQ. 0 ) GO TO 850
     CALL AXREC ( AA, BB, CC, RO, ST, MODL )
            MIO = 0
C---->ADD RESISTED OR RESIDUAL FORCE TO FORCE
  850   DO 860 I=1,4
            II = 2*I
            JJ = LB(I)*2
            FORCE(JJ-1) = FORCE(JJ-1)+P(II-1)
            FORCE(JJ) = FORCE(JJ)+P(II)
  860   CONTINUE
      RETURN
   10 FORMAT(I5)
   20 FORMAT(4D14.5)
 1000 FORMAT ( //,'  THE EQUIVALENT UNIAXIAL PLASTIC STRAIN IN ELEMENT '
     1,I4,' HASEXCEEDED MAX. VALUE',//)
 1100 FORMAT ( //,'  ERROR IN ELEMENT ',I4,//)
 1200 FORMAT(//,'  ELEMENT',I4,'  HAS CRACKED AT ANGLE',D12.4,//)
 1300 FORMAT(/,' NONLINEAR BEHAVIOR OCCURED IN ELEMENT ',I4,/)
 1500 FORMAT ( //,'  ** BRANCHING ERROR OCCURED IN ELEMENT ',I4,//)
 1600 FORMAT ( //,'  ELEMENT',I4,' HAS CRUSHED              ',// )
 2000 FORMAT ( //,'  CRITICAL SOFTENING HAS OCCURED IN ELEMENT',I4, //)
      END
```

```
      SUBROUTINE PLASTL ( FORCE, NPMAX2, UNIAX, NSTEPT, INVRT, NPT, DGE,
     1                    DS, IUPDAT, M, NPRO )
      IMPLICIT REAL * 8 ( A-H, O-Z )
      COMMON / INPT / G(4), S(4), GY(2), H, ALFA, BETA, FO, F1, SUP,
     1               GX, SX, C22, C44, MI, MIC,  C(4,4), XO, YO, RO,
     2                  AA, BB, BF(4,8), LB(4), ST(8,8),CC(4,4),ANGLE
      DIMENSION DD(4,8),FORCE(NPMAX2), GO(4), GM(4), GPR(4), DGPL(4),
     1          DB(4), A(4), AB(4), D(4), UNIAX(10,3,10), NPRO(4,10),
     2          DS(4), DGE(4), DSPL(4), P(8), CYM(2)
      DATA ZERO, ONE,TWO,THREE / 0.0D00,1.0D00,2.0D00,3.0D00 /
      DATA TEN, ETE, PTE / 10.0D00, -1.0D-7, 1.0D-7 /
             MMI = 0
             TYF = TEN
             MOCL = NPRO(2,NPT)
         DO 50 I = 1,8
             P(I) = ZERO
   50    CONTINUE
C---->ELASTIC SOLUTION
         IF ( NSTEPT .NE. 0 ) GO TO 200
         DO 100 I=1,4
             GO(I) = ZERO
             GM(I) = DGE(I)
  100    CONTINUE
         IYIELD = 0
         GO TO 760
C---->CALCULATE TOTAL STRESS
  200    DO 300 J=1,4
             GO(J) = G(J)
             GPR(J) = G(J)+DGE(J)
  300    CONTINUE
C---->START PLASTICITY CALCULATIOS
      CALL MISES ( G(1), GY(1), FO, A, NTEMP, MMI )
      CALL MISES ( GPR, GY(1), F1, D, ISIGN, MMI )
C----> CHECK ELASTIC CASE OR UNLOADING
         IF ( F1 .LT. ETE ) GO TO 400
         GO TO 420
  400    DO 410 I=1,4
             GM(I) = GPR(I)
  410    CONTINUE
             IYIELD = 0
         GO TO 760
  420    IF ( F1 .GT. PTE . AND. FO .LT. ETE ) GO TO 430
         GO TO 470
C---->GET AN ESTIMATE OF R
  430        R1 = -( FO/(F1-FO) )
         DO 440 I =1,4
             DGPL(I) = G(I)+R1*DGE(I)
  440    CONTINUE
      CALL MISES ( DGPL, GY(1), F2, A, NTEMP, MMI )
             DENOM = ZERO
         DO 450 J=1,4
             DENOM = DENOM+A(J)*DGE(J)
  450    CONTINUE
             R = R1-(F2/DENOM)
             IFLAG = 0
         IF ( MI .EQ. 0 ) GO TO 460
         GO TO 485
C---->CHECK CASE OF JUST YIELDING
  460        IFLAG = 1
             MI = DABS(F2/TYF)+1
         IF( MI .GT. 5 ) MI =5
         GO TO 485
  470    IF ( DABS(F1) .LE. PTE .AND. FO .LT. ETE ) GO TO 475
         GO TO 500
  475        R = ONE
             IFLAG = 0
         IF ( MI .EQ. 0 ) GO TO 480
         GO TO 485
  480        MI = 1
             IFLAG = 1
C---->CALCULATE NEW H FOR CASE OF NEW PLASTICITY
  485    IF ( IFLAG .NE. 1 ) GO TO 510
         IF ( ISIGN ) 490,500,495
  490        NST = NPRO(3,NPT)
             H = ( UNIAX(NST+3,1,NPT)-UNIAX(NST+2,1,NPT) ) /
     1               ( UNIAX(NST+3,3,NPT)-UNIAX(NST+2,3,NPT) )
         GO TO 510
  495        H = ( UNIAX(3,1,NPT)-UNIAX(2,1,NPT) ) /
     1               ( UNIAX(3,3,NPT)-UNIAX(2,3,NPT) )
         GO TO 510
C---->SCALE ELASTIC STRESSES
  500        R = 0
  510        IYIELD = 1
         DO 520 J=1,4
             G(J) = G(J)+R*DGE(J)
             DS(J) = (ONE-R)*DS(J)/MI
             DGE(J) = (ONE-R)*DGE(J)/MI
             GM(J) = G(J)
             DSPL(J) = ZERO
  520    CONTINUE
             SUPM = SUP
             GYM(1) = GY(1)
             GYM(2) = GY(2)
             ALFAM = ALFA
             HM = H
             NTEMP = MI
         IF ( BETA .EQ. ONE ) PRINT 1500,M
             BETA = ZERO
C---->PLASTICITY CALCULATICNS IN MI STEPS
         DO 750 MM = 1,NTEMP
```

```
      CALL MISES ( GM, GYM, FO, A, ISIGN, MMI )
C---->CALCULATE DLAMDA
               BETTA = ZERO
               DLAM = ZERO
C---->CALCULATE  D=C*A
          DO 530 I=1,4
               AB(I) = A(I)
  530     CONTINUE
          DO 550 I=1,4
               D(I) = ZERO
               DB(I) = ZERO
          DO 540 J=1,4
               D(I) = D(I)+C(I,J)*A(J)
               DB(I) = DB(I)+C(I,J)*AB(J)
  540     CONTINUE
               BETTA = BETTA+A(I)*DB(I)
               DLAM = DLAM+D(I)*DS(I)
  550     CONTINUE
C---->USE HARDENING COEFFICIENT H
               DLAM = DLAM/(H+BETTA)
          IF ( (H+BETTA) .GT. ZERO ) GO TO 560
          IF ( BETTA .EQ. ZERO ) GO TO 560
      PRINT 2000, M
  560     IF ( DLAM .LE. ZERO )  GO TO 570
          GO TO 590
  570          DLAM = ZERO
               IYIELD = 0
          DO 580 I=1,4
               GM(I) = GPR(I)
  580     CONTINUE
          GO TO 760
  590     DO 600 I=1,4
               GM(I) = GM(I) + DGE(I)
  600     CONTINUE
C---->CALCULATE PLASTIC STRAIN & STRESS INCERMENT
          DO 610 I=1 ,4
               DSPL(I) = DLAM*A(I)
               GM(I) = GM(I)-DLAM*DB(I)
  610     CONTINUE
C---->CALCULATE EQVELANT UNIAXIAL PLASTIC STRAIN INCRMPNT
               DENOM = DSQRT(ONE/THREE*(TWO*DSPL(1)**2+TWO*DSPL(2)**2+
     1                    TWO*DSPL(3)**2+DSPL(4)**2))*ISIGN
               SUPM = SUPM+DENOM
C---->LOCATE PRESENT POINT ON STRESS-STRAIN CURVE
          IF ( ISIGN ) 620,630,630
C---->COMPRESSION
  620          ICASE = 2
               NST = 3+ NPRO(3,NPT)
               NSP = NPRO(4,NPT)+NPRO(3,NPT)
          GO TO 640
C---->TENSIO
  630          ICASE = 1
               NST = 3
               NSP = NPRO(3,NPT)
C---->LOCATE CURRENT POINT
  640     DO 650 I=NST,NSP
          IF ( DABS(UNIAX(I,3,NPT)) .LT. DABS(SUPM) ) GO TO 650
               HM = (UNIAX(I,1,NPT)-UNIAX(I-1,1,NPT) )/
     1               (UNIAX(I,3,NPT)-UNIAX(I-1,3,NPT) )
               GYM(ICASE) = GYM(ICASE)+HM*DENOM
          GO TO 700
  650     CONTINUE
      PRINT 1000, M
          DO 660 J=1,4
               GM(J) = ZERO
  660     CONTINUE
               GYM(1) = ZERO
               ALFAM = ZERO
               HM = ZERO
          GO TO 760
C---->APPLY CORRECTION TO STRESS INCREMENT
  700 CALL MISES ( GM, GYM, F1, D, ISIGN, MMI )
               DENOM = ZERO
          DO 720 I=1,4
               DENOM = DENOM+D(I)**2
  720     CONTINUE
               DENOM = F1/DENOM
          DO 730 I=1,4
               GM(I) = GM(I)-DENOM*D(I)
  730     CONTINUE
  750     CONTINUE
C---->CALCULATE  INCRMENTAL FORCE
  760     IF ( IUPDAT .EQ. 0 ) GO TO 780
      PRINT 10, MI
C---->UPDATE VARIABLES
C---->CALCULATE RESIDUAL FORCE
  780     IF ( IYIELD .EQ. 0 ) GO TO 790
          DO 785 J=1,8
          DO 785 I=1,4
               P(J) = P(J)+BE(I,J)*(GPR(I)-GM(I) )*RO
  785     CONTINUE
  790     DO 795 I=1,4
               G(I) = GM(I)
  795     CONTINUE
          IF ( IYIELD .EQ. 0 ) GO TO 800
      CALL AXREC ( AA, BB, CC, RO, ST, MODL )
               SUP = SUPM
               GY(1) = GYM(1)
               GY(2) = GYM(2)
               ALFA = ALFAM
```

```
                H = HM
  800     CONTINUE
C---->ADD RESISTED OR RESIDUAL FORCE
          DO 810 I=1,4
                II = 2*I
                JJ = LB(I)*2
                FORCE(JJ-1) = FORCE(JJ-1)+P(II-1)
                FORCE(JJ) = FORCE(JJ)+P(II)
  810     CONTINUE
      RETURN
   10 FORMAT(I5)
   20 FORMAT(8D14.5)
 1000 FORMAT ( //,'  THE EQUIVELNT UNIAXIAL PLASTIC STRAIN AT ELEMENT',
     1                I4,' EXCEEDS THE MAX. VALUE',/)
 1500 FORMAT(/,' YIELD OCCURED IN ELEMENT ',I4,/)
 2000 FORMAT ( //,'   CRITICAL SOFTENING HAS OCCURED IN ELEMENT',I3,/)
      END
```

```
      SUBROUTINE OCTA ( G, GY, ALFA, BETA, F, A, ISIGN, MI )
      IMPLICIT REAL * 8 ( A-H, O-Z )
C---->EVALUATES THE YIELD FUNCTION AND ITS NORMAL BASED ON
C       THE OCTAHEDRAL SHEAR STRESS CRITERION
      DIMENSION G(4), GY(2), A(4), A1(4), A2(4), S(3)
      DATA ZERO,P5,ONE,TWO,THREE/0.0D00,0.5D00,1.0D00,2.0D00,3.0D00/
                SQ3 = DSQRT(THREE)
C---->PRINCIPAL STRESSES
                S1 = P5*(G(1)+G(2))
                S2 = P5*(G(1)-G(2))
                S3 = DSQRT(S2**2+G(4)**2)
                G1 = S1+S3
                G2 = S1-S3
C---->DEVIATORIC STRESSES
                GM = (G(1)+G(2)+G(3))/THREE
          DO 10 I=1,3
                S(I) = G(I)-GM
   10     CONTINUE
                GB = DSQRT((S(1)**2+S(2)**2+S(3)**2)/TWO+G(4)**2)
                C2 = ONE/SQ3
          IF ( GM .EQ. ZERO .AND. GB .EQ. ZERO ) GB = 1.0D-7
                TEMP = ONE/(TWO*GB)
C---->COMPONENT OF THE NORMAL TO YIELD SURFACE
          DO 20 I=1 ,3
                A1(I) = ONE/THREE
                A2(I) = TEMP*S(I)
   20     CONTINUE
                A1(4) = ZERO
                A2(4) = TEMP*TWO*G(4)
C---->DECIDE IN WHICH REGION POINT EXIST
          IF (MI ) 100,45, 50
   45     IF ( G1  .LT. ZERO .AND. G2 .LT. ZERO ) GO TO 100
          IF ( G1 .LT.1.0D2 .AND. G2 .LT. GY(2) ) GO TO 100
C---->CASE OF ++ OR +-
   50           ISIGN = 1
                TEMP = (ONE-ALFA)/(ONE+ALFA)
                C1 = TEMP
                F = G1 - GY(1)
          GO TO 200
C---->CAES OF --
  100           ISIGN = -1
                TEMP = (BETA-ONE)/(TWO*BETA-ONE)
                C1 = TEMP
                F = C2*GB+TEMP*GM-(BETA*DABS(GY(2)))/(THREE*(TWO*BETA-ONE
     1          ))
C---->EVALUATE THE NORMAL
  200     DO 250 I=1,4
                A(I) = C1*A1(I)+C2*A2(I)
  250     CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MISES ( G, GY, F, A, ISIGN, MI )
      IMPLICIT REAL * 8 ( A-H, O-Z )
C---->EVALUATES THE YIELD FUNCTION AND ITS NORMAL BASED ON
C       THE VON MISES YIELD CRITERION
      DIMENSION G(4), GY(2), A(4), S(3)
      DATA ZERO,P5,ONE,TWO,THREE/0.0D00,0.5D00,1.0D00,2.0D00,3.0D00/
              SQ3 = DSQRT(THREE)
C---->PRINCIPAL STRESSES
              S1 = P5*(G(1)+G(2))
              S2 = P5*(G(1)-G(2))
              S3 = DSQRT(S2**2+G(4)**2)
              G1 = S1+S3
              G2 = S1-S3
C---->DEVIATORIC STRESSES
              GM = (G(1)+G(2)+G(3))/THREE
         DO 10 I=1,3
              S(I) = G(I)-GM
   10    CONTINUE
              GB = DSQRT((S(1)**2+S(2)**2+S(3)**2)/TWO+G(4)**2)
         IF ( GM .EQ. ZERO .AND. GB .EQ. ZERO ) GB = 1.0D-7
              TEMP = ONE/(TWO*GB)
C---->COMPONENT OF THE NORMAL TO YIELD SURFACE
         DO 20 I =1 ,3
              A(I) = SQ3*TEMP*S(I)
   20    CONTINUE
              A(4) = SQ3*TEMP* TWO*G(4)
         IF ( MI ) 40,30,50
   30    IF ( G1 .GT. 1.0D-7 ) GO TO 50
   40         ISIGN = -1
              F = SQ3*GB+GY(2)
      RETURN
   50         ISIGN = 1
              F = SQ3*GB-GY(1)
      RETURN
      END
```

```
      SUBROUTINE TRNS ( B, A, AL, ICON )
      IMPLICIT REAL * 8 ( A-H, O-Z )
C----> ICON1 = 1  TRASFORMS STRAINS
C----> ICON2 = 2  TRASFORMS STRESSES
      DIMENSION A(4), B(4)
      DATA ONE,TWO / 1.0D00,2.0D00 /
              CAL = DCOS(AL)
              SAL = DSIN(AL)
              SAL2 = SAL**2
              CAL2 = CAL**2
              SCAL = SAL*CAL
              C1 = TWO
              C2 = ONE
         IF ( ICON .EQ. 2 ) GO TO 200
              C1 = ONE
              C2 = TWO
  200         A(1) = B(1)*CAL2+B(2)*SAL2+C1*B(4)*SCAL
              A(2) = B(1)*SAL2+B(2)*CAL2-C1*B(4)*SCAL
              A(3) = B(3)
              A(4) = -C2*B(1)*SCAL+C2*B(2)*SCAL+B(4)*(CAL2-SAL2)
      RETURN
      END
```

```
      SUBROUTINE NORM ( NPMAX2, A, VALUE )
C---->CALCULATES NORM OF A VECTOR
      IMPLICIT REAL * 8 ( A-H, O-Z )
      DIMENSION A(NPMAX2)
      DATA ZERO / 0.0D00 /
              VALUE = ZERO
          DO 100 I = 1,NPMAX2
              VALUE = VALUE + A(I)**2
  100     CONTINUE
              VALUE = DSQRT(VALUE)
      RETURN
      END
```

```
      SUBROUTINE MATRED ( C, AL, C11, C22, C44 )
C---->REDUCE THE D-MATRIX
      IMPLICIT REAL * 8 ( A-H, O-Z )
      DIMENSION C(4,4)
      DATA TWO, FOUR / 2.0D00, 4.0D00 /
              SAL = DSIN(AL)
              CAL = DCOS(AL)
              C(1,1) = C11*CAL**4+C22*SAL**4+FOUR*C44*CAL**2*SAL**2
              C(1,2) = (C11+C22)*CAL**2*SAL**2-FOUR*C44*CAL**2*SAL**2
              C(2,1) = C(1,2)
              C(1,4) = -C22*CAL*SAL**3-TWO*C44*CAL**3*SAL+TWO*C44*CAL*
     1                 SAL**3+C11*CAL**3*SAL
              C(4,1) = C(1,4)
              C(2,2) = C11*SAL**4+C22*CAL**4+FOUR*C44*CAL**2*SAL**2
              C(2,4) = -C22*CAL**3*SAL+TWO*C44*CAL**3*SAL-TWO*C44*CAL*
     1                 SAL**3+C11*CAL*SAL**3
              C(4,2) = C(2,4)
              C(4,4) = (C11+C22)*CAL**2*SAL**2+C44*(CAL**2-SAL**2)**2
      RETURN
      END
```

```
      SUBROUTINE PRNT (DISPL, NUMEL, NUMNP, NPMAX, NPMAX2, NELMAX, X, Y)
      IMPLICIT REAL * 8 ( A-H, O-Z )
 2000 FORMAT (     1H1, //// ,
     1             51H     FINITE ELEMENT ANALYSIS OF AXISYMMETRIC SOLIDS,
     2             26H PROBLEMS - PROGRAM NFEA       , //, 5X, 20A4, / ,
     3    /        13H      PROBLEM ,       //,5X , 20A4      )
 2010 FORMAT .// 24H      NODAL DISPLACEMENTS        )
 2020 FORMAT (     1H+, 24X, 13H( CONTINUED ) )
 2030 FORMAT (// 50H         NODE     COORDINATES            DISPLACEMENTS  ,
     1    /        50H                    R       Z             R            Z  /)
 2040 FORMAT ( 5X, I5, 3X, 2F7.2, 5X, 1P2D11.3 )
 2060 FORMAT (// 21H      ELEMENT STRESSES    )
 2070 FORMAT(      1H+, 21X , 13H( CONTINUED ) )
 2080 FORMAT(/'        EL     R       Z      R-STRESS   Z-STRESS    T-STRESS
     1RZ-SHEAR MAX-STRESS MIN-STRESS  MAX-SHEAR ANG1X',/)
 2090 FORMAT(5X,I3,2F7.2,1P7D11.3, 0PF6.2 )
      COMMON / CONT / ID1(20), ID2(20)
      DIMENSION DISPL(NPMAX2), X(NPMAX), Y(NPMAX), SIG(8), GY(2),EPS(4),
     1          C(4,4), BD(4,8)
      DATA ZERO,TWO/ 0.0D00, 2.0D00 /
      REWIND 8
C---->PRINT DISPLACEMENTS
      PRINT 2000, ( ID1(I),I=1,20 ),( ID2(I), I=1,20 )
      PRINT 2010
      PRINT 2030
             KLM = 40
         DO 100 N = 1,NUMNP
             K = 2 * N- 1
      PRINT 2040, N, X(N), Y(N), DISPL(K), DISPL(K+1)
         IF ( N .GE. NUMNP ) GO TO 150
         IF ( N .NE. KLM ) GO TO 100
      PRINT 2000, ( ID1(I), I=1,20 ),( ID2(I),I=1,20 )
      PRINT 2010
      PRINT 2020
      PRINT 2030
             KLM = KLM + 40
  100    CONTINUE
  150    CONTINUE
C---->PRINT STRESSES
             MLINE = 40
      PRINT 2000, ( ID1(I),I=1,20),(ID2(I) ,I=1,20)
      PRINT 2060
      PRINT 2080
         DO 200 M=1,NUMEL
      READ (8) ( SIG(I), EPS(I),I=1,4), GY(1), GY(2), H, ALFA, BETA, FO,
     1       F1, SUP, GX, SX, C22, C44, MI, MIO, XO, YO, AA, BB, RO ,
     2       ((C(I,J),I=1,4),J=1,4),((BD (I,J),J=1,8),I=1,4)
             CC=(SIG(1)+SIG(2))/TWO
             FF=(SIG(1)-SIG(2))/TWO
             CR=DSQRT(FF**2+SIG(4)**2)
             SIG(5)=CC+CR
             SIG(6)=CC-CR
             SIG(7)=CR
             TEMP=CR+(SIG(1)-SIG(2))/TWO
         IF(TEMP.EQ.ZERO) GO TO 170
             SIG(8)=DATAN(-SIG(4)/TEMP)
         GO TO 180
  170        SIG(8)=ZERO
  180    CONTINUE
      PRINT 2090 ,(M,XO ,YO , (SIG(I),I=1,8) )
      PRINT 2090,(M,XO,YO,(EPS(I),I=1,4))
         IF ( M .GE. NUMEL ) GO TO 250
         IF ( M .NE. MLINE ) GO TO 200
      PRINT 2000, ( ID1(I),I=1,20),(ID2(I) ,I=1,20)
      PRINT 2060
      PRINT 2070
      PRINT 2080
             MLINE = MLINE + 40
  200    CONTINUE
  250    CONTINUE
      RETURN
      END
```

VITA

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