SELECTIVE FACTOR ROTATION:

A ROTATIONAL PROCEDURE

YIELDING HIERARCHIAL

FACTOR PATTERNS

By

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CHAPTER I

FACTOR ANALYSIS: AN INTRODUCTION

The Historical Perspective

The birth of factor analysis is generally accredited to Charles Spearman. However, as is the case with most of the principles and procedures found in psychology, factor analysis is not the result of one man's work, but slowly emerged from the work of Francis Galton and Karl Pearson to achieve its initial theoretical application in the work of Spearman. It was Galton's keen interest to uncover the principles of the inheritance of manifest characteristics which, in turn, led to his borrowing from the work of the French mathematician, Quetelet, who is responsible for the earlier mathematical efforts in correlation (Burt, 1962). Also, from his work in inheritance, Galton became familiar with scatterplots and what he termed the principle of "regression toward mediocrity." In 1866, he formalized that notion into the "index of co-relation" (Galton, 1866). Pearson, fascinated by Galton's attempt to mathematize biological and psychological principles, took this index and developed it into what is known today as the product-moment correlation coefficient.

Since the correlation coefficient, $\underline{r}$, plays such an important part in psychological research, it deserves a closer look. The Pearson $\underline{r}$ measures linear dependence or the amount of the ratio of the covariance to the geometric mean of the variances. Mathematically, it is the first mixed central moment of two random variables divided by the product of their respective standard deviations. Hence, the equation with which the correlation coefficient is found is

$$r_{xy} = \frac{COV_{xy}}{s_x s_y} \tag{1-1}$$

where COV_{xy} is the covariance of variables x and y, and s_x, s_y represent the standard deviation of x and y respectively. The Pearson r can range from +1.00 to -1.00, where 0.00 indicates no relationship at all. The direction of the relationship is determined by the sign of the coefficient. If the sign is positive, then the variables are directly proportional; and if the sign is negative, then the variables are inversely related. However, it is the square of the Pearson r that concerns the factor analyst most. One may interpret r-squared multiplied by a hundred as the percent of variance in one variable that can be accounted for by the variance of the other variable. For example, assume $r_{xy} = 0.70$. Then, there is 49 per cent of the variance in x that can be attributed to the variance in y.

As the use of the correlation coefficient gained popularity as a measure of the relationship between variables, there arose a need for a structural theory to account for these relationships (Mulaik, 1972, p. 2). Two answers were proposed. Pearson (1901) contended there should be a method of closest fit of lines and planes to the points in space. An important variation of Pearson's work is that of component analysis (Hotelling, 1933). The purpose of component analysis is to define the basic dimensions of the data.

The second major solution of this early era was by Spearman (1904), who proposed the first common factor approach to factor analysis. He postulated that a variable could be broken down into general and specific parts. In the case of more than one variable, each variable should depend on the general factor, but not necessarily with the same amount of dependency; and the specific factor peculiar to each variable. Mathematically, this can be represented by the equation

$$X_i = a_i G + U_i \qquad (1\text{-}2)$$

where X_i is the _i_th variable (e.g. a test score of intelligence), a_i is the weight indicating how much of the general factor, G, can be found in X_i, and U_i is the unique portion of X_i which is uncorrelated with G. Hence, each specific factor is uncorrelated with the general factor and the other specific factors.

Through additional research, Spearman found that this is only the case if the correlations between all possible groups of four variables are such that

$$r_{12}r_{34} - r_{13}r_{24} = 0. \qquad (1\text{-}3)$$

This is known as the tetrad difference criterion (Wolfle, 1940). If all tetrad differences in a set of variables equal zero, then one may split these variables into a general factor and specific factors. Thus, there are three important ideas emerging out of Spearman's work. First, one finds that the variation of each variable can be explained by exactly two factors, one general and one specific. Next, before the correlation matrix can be factored, it must be sifted to eliminate variables not meeting the tetrad criterion. Finally, Spearman approaches the problem of factor analysis as one of geometric linearity.

As evidence accumulated, many psychologists contemporary with Spearman became dissatisfied with the simple two factor theory. According to Harman (1967, p. 7) the movement to group factors in the late 1920's and early 1930's constitutes the early modern period of factor analysis. Although there was an extensive revision of theory during this period, the basic methods of extracting the factors remained the same. In group factor patterns, the general factor remains unchanged and explains as much of the correlations as possible. The residual correlations, then, are explained by the postulation of group factors.

Holzinger's bifactor technique is a representative of this era. For him, a test score is dependent on a general factor, one group factor, and a specific factor. One of the most important concepts emerging from his theory is the general factor accounts for the most variance and group factors relatively less, depending on whether they are major or minor group factors (Wolfle, 1940). For Holzinger, the geometrical approach is planar. However, as with Spearman, all factors are mutually orthogonal (i.e. uncorrelated). There have been several variations on this theme (Burt, 1949; Kelley, 1935; Tryon, 1958). The basic model is somewhat different from Spearman's and is reminiscent of the equation for multiple correlation. It can be represented mathematically as

$$X_i = a_iG + b_{i1}G_1 + b_{i2}G_2 + c_{i1}H_1 + c_{i2}H_2 + \ldots. + U_i \quad (1\text{-}4)$$

where X_i is the *i*th variable, a_i, b_{i1}, b_{12}, c_{i1}, c_{i2}, ..., are the weights, G is the general factor, G_1 and G_2 are the major group factors, H_1, H_2, ..., are the minor group factors, and U_i is the portion of X_i that is unique from all the other variables.

A break from this traditionally British way of thinking is found in L. L. Thurstone. He agreed that group factors were an important concept, but disregarded the notion of a general factor (Thurstone, 1935). This approach has become known as multiple factor analysis. His major concern was to reduce the data to the minimum number of common factors necessary to reproduce the original data. Simple structure (Thurstone, 1935) became a very useful and popular approach to factor patterning in the United States. Probably, this is due to the fact that simple structure can be objectively defined, which makes it readily applicable to computers. Factor analysis, under Thurstone, became nothing more than the mathematical formulation of a reductionistic model. This could possibly explain why the usefulness of factor analysis in the 1950's and 1960's has been such a hotly debated topic.

Emerging from the work of Thurstone and the British factor analysts, there are two rather diverse views concerning the nature of intelligence. Vernon (1951, p. 30) uses the traditionally British group factor method as the basis for his hierarchial structure of intelligence, while Thurstone (1938), using the notion of simple structure, says intelligence can be divided into primary mental abilities. There have been considerable amounts of research generated in an attempt to resolve the issue. However, as yet, there is no formidable evidence one way or the other and the resolution of which is correct remains purely a person preference.

It is interesting that Harman (1967, p. 9) considers the next period of factor analysis the late modern era, within which factor analysis grew both in theory and application, while Mulaik (1972, p. 9) considers it the blind era of factor analysis. During this period, both agree that factor analysis was widely applied to research in many areas, but they disagree as to whether factor analysis provided meaningful explanations for the relationships found among the variables. For example, Mischel (1968) points out that the trait theorists in personality research have not produced factors that have been universally accepted as explanatory concepts of human behavior.

Presently, the major theoretical considerations have centered around the use of factor analysis as a method for hypothesis testing. The problem was first encountered by Mosier (1939) when he presented a mathematical rationale for oblique rotation to a target matrix. He referred to the target matrix as a reference structure. His approximate solution for rotating a factor matrix to the best least squares fit to the target reference structure has been subsumed under the heading of procrustean transformations. Horst (1956) and Hurley and Cattell (1962) have strongly suggested this method as a way of obtaining approximate solutions. Harman (1967, p. 251) refers to this procedure as a general method giving the matrix of transformation between any two solutions the same common factor space. There have also been exact methods given. These are obtained by adhering to the restrictions of orthogonal rotation (Cliff, 1966; Shonemann, 1966). Most recently, Joreskog (1970) has provided mathematical rationale and computer programs for simultaneous solutions of several populations to a hypothetical factor pattern matrix. These new approaches, it has been suggested, will make factor analysis a more useful tool in the development of structual theories (Mulaik, 1972, pp. 11, 294).

The Theoretical Perspective

The methods of factor analysis were developed primarily for identifying the principal dimensions of mentality, but as the methods became widely recognized, other applications were found. As has been stated, the factor problem is to account for the observed correlations among all variables in terms of the smallest number of factors with the smallest possible residual error. Hence, factor analysis does not add anything to the original data, but is a method of simplification.

What does factor analysis do? There have been many different approaches to this answer. Harman (1967, p. 2) says "it is a tool in the empirical sciences. The major objective of such a tool is to provide an explanation for the underlying behavior of the data." Similar to the middle of the road approach of Harman is that of Mulaik (1972, p. 3). In a more mathematically pure approach, Horst (1965, p. 17) states "the primary concern of factor analysis is to investigate a table of measures to determine whether the table may be simplified in some way." Thurstone (1935), on the other extreme, feels that factor analysis not only discovers the underlying order of the data, but says that one may identify their nature.

Keeping this differentiation in mind, one may infer that factors may represent three theoretical levels. Most simply, it may be a formal concept expressing the mathematical relationships found in the raw data (Horst, 1965, p. 16). When taking this approach, one refrains from making aggressive assertions about the nature of a factor. Secondly, a factor may be a theoretical construct. This is the most common theoretical perspective and it represents the belief that factors define the causal network underlying the observed patterns. This easily can be seen in the application of factor analysis to the study of intelligence (Spearman, 1904; Thurstone, 1935; Vernon, 1951). The final level at which a factor can be viewed is as an empirical concept. An example of this is found in the factor analytic personality theorists, where typologies play an extremely important part in the understanding of personality. In this case, factors categorize the concommitent relationships. Therefore, factor analysis can become an exploratory device for uncovering these basic concepts. These three views, however, do not represent the only definitional approaches to factors, but are points on a continuum which goes from a purely mathematical bleaching to the colorful identification of factors as representatives of reality.

The Mathematical Perspective

In general, there are two mathematical approaches to factor analysis: (1) common factor analysis and (2) component analysis. The basic and most traditional is common factor analysis. There have been two basic refinements of this technique. Guttman (1953) developed image analysis in order to remove the problem of communality. He felt he could accomplish this by operationally defining "commonness." The other refinement was by Rao (1955) when he extended factor analysis to the realm of statistical tests of significance. His procedure has become known as canonical factor analysis. The essence of common factor analysis is the division of the variables into their common and unique parts, while component analysis (Hotelling, 1933) does not make this distinction. Of course, there are other mathematical models found in factor analysis, but these two are the classical techniques and the majority of research involves their usage (Gorsuch, 1970).

Common Factor Analysis

This technique was first developed by Spearman (1927) for his two factor theory and later extended to multiple factors by Thurstone (1935). This is similar to a partial correlation approach in that one divides the variance of the variable into two distinct parts. The first is that variable of X_j which is common to or related to the variance of the other variables in the study. This part is technically known as the communality of X_j. The other part, as expected, is the uniqueness of X_j or the variance of X_j not common to the other variables under study. Schematically, this is shown by Figure 1.

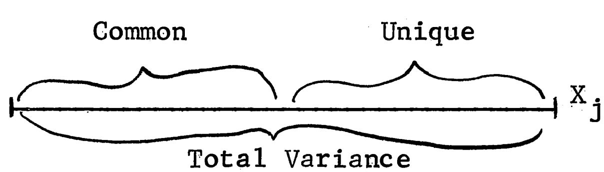

Figure 1. The Common and Unique Parts of X_j

With respect to the above definitions, the following notation can be developed. (In subsequent chapters, the following technique will be used to introduce important symbols and their definitions. In addition to notation, equations which are necessary to derivations will also be introduced using this technique.)

1.1 Notation Z_j the standardization of variable X_j

h_j^2 the communality of Z_j

H_{mm}^2 a diagonal matrix of communalities, where h_{ii}^2 is the communality of variable Z_i with respect to the other (m - 1) variables

u_j^2 the unique part of variable Z_j

U_{mm}^2 a diagonal matrix of uniquenesses, where u_{ii}^2 is the uniqueness of variable Z_i with respect to the other (m - 1) variables

Using the preceeding notation, it is rather simple to mathematically express the variance of Z_j as

$$u_j^2 + h_j^2 = 1.00 \tag{1-5}$$

With the use of matrix notation, the case of <u>m</u> variables can be expressed as 1-6, where I_{mm} is an identity matrix.

$$U_{mm}^2 + H_{mm}^2 = I_{mm} \tag{1-6}$$

The common factor model is similar to equation 1-4, but there is no distinction made between general, major group or minor group factors. In general, common factor analysis defines the hypothetical unknown factors related to the common variance components of the variables. The common factor equations are

$$\begin{aligned}
X_1 &= a_{11}S_1 + a_{12}S_2 + \cdots + a_{1p}S_p + a_{1u}S_{1u} \\
X_2 &= a_{21}S_1 + a_{22}S_2 + \cdots + a_{2p}S_p + a_{2u}S_{2u} \\
&\vdots \\
X_m &= a_{m1}S_1 + a_{m2}S_2 + \cdots + a_{mp}S_p + a_{mu}S_{mu}
\end{aligned} \tag{1-7}$$

where S_p are the p number of common factors, S_{mu} is the unique factor for each variable, and the coefficients a_{mp} and a_{mu} are the scalars indicating the weighting of each variable on the common factors and unique factor respectively. It is these coefficients that emerge as the factor loadings when factor analyzing a data matrix using common factor analysis.

Matrix notation can be readily applied to the techniques of factor analysis, and using this notation can express the basic equations in a much simpler form. For example, the equations of 1-7 can be expressed as the following using a set of standardized variables, Z_i, rather than the raw score form, X_i, where there are m variables and n cases.

1.2 Notation F_{mp} a matrix of loadings, a, for each variable on each common factor

S_{np} a matrix of factor scores for each case on each common factor

$S^{\#}_{nm}$ a matrix of factor scores for each case on the unique factor

$$Z_{nm} = S_{np} F'_{mp} + S^{\#}_{nm} U_{mm} \qquad (1\text{-}8)$$

The object of common factor analysis is, then, to determine $S_{np}F'_{mp}$. It is at this point that one of the most basic derivations of factor analysis needs to be presented.

1.3 Notation R_{mm} the intercorrelation matrix

Φ_{mm} the null matrix, where all elements are zero

1.4 Derivation of the Basic Equation of Factor Analysis

The starting point for this derivation is equation 1-8. Taking the minor product of both sides and simplifying algebraically, the following equations emerge.

$$Z'_{nm}Z_{nm} = (S_{np}F'_{mp} + S^{\#}_{nm}U_{mm})'\ (S_{np}F'_{mp} + S^{\#}_{nm}U_{mm}) \quad (1\text{-}9)$$

$$= F_{mp}S'_{np}S_{np}F'_{mp} + U_{mm}S^{\#'}_{nm}S_{np}F'_{mp} + F_{mp}S'_{np}S^{\#}_{nm}U_{mm}$$

$$+ U_{mm}S^{\#'}_{nm}S^{\#}_{nm}U_{mm} \quad (1\text{-}10)$$

Since the common and unique factors are uncorrelated, one may express that mathematically in the following manner.

$$S^{\#'}_{nm}S_{np} = S'_{np}S^{\#}_{nm} = \Phi_{mp} \quad (1\text{-}11)$$

By substituting 1-11, 1-10 can be simplified to arrive at 1-12.

$$Z'_{nm}Z_{nm} = F_{mp}S'_{np}S_{np}F'_{mp} + U_{mm}S^{\#'}_{nm}S^{\#}_{nm}U_{mm} \quad (1\text{-}12)$$

Now, if one makes the assumption that S_{np} and $S^{\#}_{nm}$ have been standardized and the factor loadings are orthogonal, then the following holds.

$$S'_{np}S_{np} = nI_{pp} \text{ and } S^{\#'}_{nm}S^{\#}_{nm} = nI_{mm} \quad (1\text{-}13)$$

In making this substitution and simplifying, one obtains the final solution. It should be pointed out that in simplifying (n^{-1}) is multiplied through as a constant.

$$(n^{-1})\ Z'_{nm}Z_{nm} = F_{mp}F'_{mp} + U^2_{mm} \quad (1\text{-}14)$$

One final simplification can be made. Given 1-15 and removing U^2_{mm} from both sides, one obtains the most widely accepted version of the basic equation.

$$R_{mm} = (n^{-1})Z'_{nm}Z_{nm} \quad (1\text{-}15)$$

$$R_{mm} = F_{mp}F'_{mp} + U^2_{mm} \quad (1\text{-}16)$$

$$R_{mm} = U^2_{mm} = F_{mp}F'_{mp} \quad (1\text{-}17)$$

It is this final equation that numerous authors refer to as the basic equation of factor analysis (Mulaik, 1972, p. 100; Thurstone, 1947; Harman, 1969, p. 28). However, there are two fundamental indeterminacies found in this derivation. First of all, an infinite number of matrices F_{mp} can be found which will reproduce

$R_{mm} - U^2_{mm}$ (Guilford and Hoepfner, 1969). Guilford and Hoepfner (1969) have examined this problem and emphasize that in an unexplored area of research, one has trouble determining which solution represents the genuine psychological variables. The other problem, which is peculiar to the common factor solution, is that one must identify the unique parts of the variables before solving for F_{mp}. This is a problem, in that, the unique portions cannot be known until the common factors are defined. What is usually done is to estimate the uniqueness. This raises the question as to whether differences in these unique estimates have any effect on the factor loadings. Thurstone (1947, p. 285) presents factor results for three different sets of estimates. The factor structures differ significantly. Hence, there is a circular effect. It is this problem that is known in many texts as the problem of communality, and as yet there has been no suitable solution reached.

Component Analysis

Harold Hotelling (1933, 1936) proposes a method which attempts to rid the factor analyst of the problem of communality. It is the analysis of variables into their principal components that removes estimation of communalities by employing the following model.

$$\begin{aligned} X_1 &= a_{11}S_1 + a_{12}S_2 + \cdots + a_{1p}S_p \\ X_2 &= a_{21}S_1 + a_{22}S_2 + \cdots + a_{2p}S_p \\ &\vdots \\ X_m &= a_{m1}S_1 + a_{m2}S_2 + \cdots + a_{mp}S_p \end{aligned} \tag{1-18}$$

where S_p are the first p principal components and the coefficients a_{ij}, i = 1, 2, ..., m and j = 1, 2, ..., p, are the scalars which represent the weighting of the variables on each component. It is important at this point to note that it will require m components to reproduce the correlation matrix of m variables exactly. However, the technique which Hotelling developed extracted the components in such a manner that each new component accounts for the greatest possible portion of the total variance of the variables unaccounted for by preceding components. Thus, when results are presented, the factors (components) reported are those which significantly contribute to the variance.

The extraction of factors or components in decreasing order of variance accountability offers a method which serves to provide a unique solution for equation 1-21. In fact, several methods have been proposed (Hotelling, 1933; Kelley, 1935, p. 145; Thomson, 1934). These methods involve iterative schemes which obtain the roots of the characteristic equation, mathematically speaking. These are known as eigenvalues. In matrix theory, the mathematical usage of eigenvalues and eigenvectors is easily seen. Generally, there is a number, e, and an m-dimensional column vector, q_{m1} such that

$$R_{mm}q_{m1} = (e)\ q_{m1} \qquad (1\text{-}19)$$

Any number, e_1, satisfying this equation is called an eigenvalue of R_{mm} and its corresponding vector q_{m1} is called the eigenvector of R_{mm}. Due to the mathematical complexity of these extraction procedures, they were unpopular techniques in the pre-computer age. Hence, many researchers felt that common factor analysis was a more appropriate technique due to its relative ease of calculation (McCloy, Metheny, and Knott, 1938). However, as Gorsuch (1970) points out, most decisions, concerning the model and methods used in a factor analytic study, depend on the availability of computer facilities and programs.

CHAPTER II

THE ROTATIONAL PROBLEM

There are two possible steps one could use in obtaining the final factor solution: (1) extraction and (2) rotation. Thurstone (1937) points out that it should be the theme of all those who factor analyze, in psychology, to achieve psychologically significant factors, which (1) can be replicated, (2) can be easily interpreted in light of some theoretical position, and (3) can be investigated meaningfully by other techniques. There are several methods in current use which use only extraction to achieve the final solution (Joreskog, 1970; Harman and Jones, 1966). However, some extraction methods lend themselves to rotation. This is the case with principal components (Hotelling, 1933), principal factor (Thomson, 1934), and centroid (Thurstone, 1947, Chapter 4) extraction procedures.

The value of rotation is not found in its mathematical basis, but rotation is beneficial to the researcher in that it allows one to include subject matter considerations. In fact, rotation, mathematically, leads one again into the problem of indeterminateness, for there are an infinite number of rotations which one can make. Thus, it is the extraction technique which defines the minimum dimensions of the data, and the rotational procedure which makes the factors substantively interesting. However, one does not rotate blindly. There are several criteria which must be considered when one uses rotation in factor analysis. It is important, then, to be somewhat cautious when using rotational procedures.

The concept of parsimony is a key factor in determining rotated factor matrices. In factor extraction, parsimony is met by using the smallest number of factors to account for the observed correlations in the variables. However, the meaning of the term, with respect to the rotational problem is neither explicit nor precise. Since the axes can be rotated to an infinite number of positions, a question arises as to which is the most parsimonious description. Ferguson (1954) suggests "some function of the sum of products of the coordinates might serve as a measure of parsimony." If one accepts this view, then the resulting factor pattern is that of simple structure (for a fuller discussion of this concept see the next chapter), which may or may not be of theoretical interest. Hence, it is thought by the author that a more versatile definition of parsimony is needed with respect to the problem of rotation. Therefore, a factor pattern is parsimonious if the factor structure one obtains by rotation is the most expeditious in making the transfer from the data to the theoretical model. In other words, parsimony is viewed with respect to the theoretical model one is considering as well as its traditional aspect of using the simplest solution.

A second major criterion of rotation is factor invariance. In fact, Kaiser (1958) states that the ultimate criteria of a rotational procedure is factor invariance. By invariance, one means the constancy of factors from analysis to analysis. Hence, invariant factors always delineate the same variables regardless of the other variables included in the analysis. If a factor is invariant, under changing samples of tests, there is evidence that the inferences regarding the domain of the factors are correct (Thurstone, 1947, p. 360-361). It is felt that Kaiser is placing undue emphasis on the notion of invariance. Although one may increase the generalizability of the factors, Reyburn and Taylor (1943) show that invariance is a function of subjective identification of factors. Hence, they feel that while invariance will aid in the verification of theoretical formulations, it is not as necessary as parsimony in the final factor solution.

The final criterion used in governing rotational procedures is based on the orthogonality-obliqueness issue. The major distinction between the two approaches is orthogonality guarantees independent factors, whereas oblique rotational processes do not. It must be noted, however, that orthogonal rotation is a subset of oblique rotation. There exists a great controversy over which approach is better. In order to evaluate the positions of both sides, it is necessary to delineate the characteristics of each rotational procedure.

Orthogonal rotation yields factors which have an inner product equal to zero, implying the factors are independent. The factor scores are linearly independent and uncorrelated. Also, the communality of each variable remains invariant through the rotational process. Probably the most important characteristic associated with orthogonal rotation is that the equation

$$F^*_{mp} = Z'_{nm} Z^*_{np} (n^{-1}) \tag{2-1}$$

where F^*_{mp} is the rotated factor matrix, Z_{nm} is the data matrix, and Z^*_{np} is the rotated factor score matrix, is still applicable. Hence, the factor loadings are correlation coefficients of the original data and the factor scores. Fruchter (1952) found that orthogonal solutions resulted in conceptual clarity when contrasted to oblique rotations. Other arguments in favor of orthogonal solutions are simplicity, mathematical independence, and the ease with which orthogonal solutions can be applied to subsequent manipulations.

Oblique rotation, on the other hand, defines the clusters of variables more precisely. However, there is a loss of independence between factors. This is due to the fact that the factors are rotated individually until the best fit is obtained. Harris and Knoell (1948) offer an extensive analysis of the oblique solution. It is evident from their discussion, that in obtaining the additional information from oblique solutions, one obsures the basics of factor analysis, such as communality, factor structure and pattern, and the percent of variance accounted for by each factor. The most convincing argument offered for the use of oblique solutions is by Cattell (1952, p. 122-123). His reasoning is mainly epistemological. Phenomena, according to Cattell, are always interrelated, and the factors must express this reality.

Rummel (1970, p. 388) states that the issue of orthogonality-obliqueness is not a question of either/or. One should try both in an effort to find the best theoretical solution. This author feels that this is not the case. The position of orthogonality seems to be a more potent alternative. As Harris and Knoell (1948) point out, the oblique solution led to theoretical problems with respect to their model. Also, the results of oblique solutions were not as straight forward as the orthogonal results. It appears that by accurately defining each cluster, one is creating more theoretical obscurity than providing information. Hence, in oblique solutions there is a greater problem with interpretability than one encounters with orthogonal solutions. Also, there is not a necessity for higher-order factors in orthogonal solutions since the factors are uncorrelated.

It is proposed, then, that there are three major criteria governing the rotational procedure. The factor patterns should be parsimonious with respect to the theoretical model and in the traditional sense. It is felt that this is the most important criteria to be met. Secondly, the factors should remain orthogonal during the rotational process in order not to cloud their interpretability. The third property one must keep in mind is factor invariance. While it is desirable to haveinvariance, it is not an essential property. Having factor invariance increases the interpretability of the factors, but does not directly influence the factors themselves. Using these as rotational guide to aid in finding the right rotation to best understand the subject matter, it is now appropriate to consider the mathematical aspects of rotation.

Mathematically, rotation is a relatively simple process.

2.1 Notation T_{pp} an orthonormal rotation matrix (i.e. $I_{pp} = T'_{pp}T_{pp}$)

F^*_{mp} rotated factor loading matrix

Z^*_{np} rotated factor score matrix

Using the above notation, one can rotate the factor loading matrix, F_{mp}, and the factor score matrix, Z_{np}, respectively, using equations 2-2 and 2-3.

$$F^*_{mp} = F_{mp} T_{pp} \tag{2-2}$$

$$Z^*_{np} = Z_{np} T_{pp} \tag{2-3}$$

There are two general methods for obtaining T_{pp}. The graphical procedure is subjective and somewhat laborious. Fruchter and Novak (1958) point out that, in addition to being laborious, the accuracy of the method is questionable when compared to present computer methods. The other method of rotation is the analytic approach. Analytic rotations are attempts to specify mathematically how the rotation is to be performed. This will remove the infinite rotational possibilities and make the solution determinate. Fruchter and Novak (1958) found that the analytic method is the most objective and requires the least amount of judgmental decisions.

CHAPTER III

THURSTONE'S SIMPLE STRUCTURE

The Theoretical Position

In an effort to make the rotational process as mathematically complete as possible, Thurstone defined simple structure as the most parsimonious factor solution (Thurstone, 1935, p. 150-151). Simple structure is an attempt to maximize each factor colinearly with a cluster of variables. The initial criteria for determining simple structure is as follows (Thurstone, 1935, p. 156):

1) Each row should have at least one zero.
2) Each column should have at least r zeros (where r is the number of factors).
3) For every pair of columns, there should be at least r variables whose entries vanish in one column but not the other.

There have been many individuals who have made numerous proposals for procedures to attain the goal of simple structure. Since this was the first attempt to objectively define a factor structure, it became very popular in America.

In 1947, Thurstone added two more criteria to make sure the factors were distinct and overdetermined by the data (Thurstone, 1947, Chapter 14). At present, the five criterion for simple structure are (Thurstone, 1947, p. 335):

1) Each row of the factor matrix should have at least one zero.

2) For each column of the factor matrix, there should be a distinct set of r linearly independent tests whose factor loadings are zero.
3) For each pair of columns, there should be several variables whose entries vanish in one column but not the other.
4) For every pair of columns, a large proportion of variables should have zero entries in both columns. This applies only to factor problems with four or more factors.
5) For every pair of columns, there should preferably be only a small number of variables with nonvanishing entries in both columns.

Schematically, pure simple structure can be seen in the following manner.

TABLE I

THE SIMPLE STRUCTURE PATTERN

Unrotated Factors			Simple Structure Rotation		
I_0	I_1	I_2	I_0	I_1	I_2
X	X		X		
X	X		X		
X		X	X		
	X	X		X	
X	X			X	
X	X	X		X	
X		X			X
	X	X			X
X		X			X

Note: X represents a factor loading other than zero.

Using the five criterion, one has a good assurance that the factors are defining distinct clusters of variables. There are several properties which are inherent in simple structure. As seen in Table I, each variable is loaded on one factor. Hence, simple structure minimizes the number of factors necessary to account for a single variable. Similarly, the number of variables loaded on each factor is minimized. This removes a general factor or any bipolar factors, leaving only group factors. It is from this that Thurstone lays the foundations of his primary mental abilities approach to intelligence (Thurstone, 1938, pp. 71-73).

Two other general characteristics usually appear when one employs the simple structure criteria. The majority of extraction processes that lead to rotation (see page 16) extract the factors by maximizing the variance each factor represents. Hence, the magnitude of variance associated with each factor decreases as the factors are extracted. (This is especially true when using the principal components or the centroid procedure.) However, after applying the simple structure criteria to rotation the variance associated with each factor has approximately the same magnitude. The other property one usually finds is that the factors define clusters of interelated variables. Again, this provides a strong argument for Thurstone in favor of primary mental abilities (Thurstone, 1938, pp. 89-91). This is one of the main reasons why simple structure is so frequently used. The factors lend themselves more easily to invariance, since the clusters are interrelated and distinct from the other clusters. Thus, the addition of new variables in the study will not affect the correlations of those already highly correlated.

Analytic Rotations to Simple Structure

The popularity of Thurstone's simple structure increased with the development of analytic procedures to rotate factor matrices into simple structure. Applying these techniques to computers (e.g. Kaiser, 1959), has increased its popularity even more (Gorsuch, 1970). At present, there are two major analytic orthogonal procedures enjoying extensive use. They are the quartimax and the verimax. The varimax has greater popularity (Gorsuch, 1970), because it is not as biased as the quartimax with respect to the weighting of the first factor.

Quartimax

Several researchers independently developed the quartimax rationale (Carroll, 1953; Ferguson, 1954; Neuhaus and Wrigley, 1954). Although the approaches are mathematically equivalent, they arrive at their solutions emphasizing different theoretical aspects. Ferguson (1954) approaches the problem by attempting to objectify the notion of parsimony (see page 18). On the other hand, Neuhaus and Wrigley (1954) and Carroll (1953) place their emphasis on finding a mathematical formulation of Thurstone's five criteria (1947, p. 335). Carroll's aim centers around criteria three, four and five, while Neuhaus and Wrigley attended the concentration of test variance which is related to criteria one and two. Since all three are mathematically equivalent, it is sufficient to describe one in detail. Neuhaus and Wrigley are chosen to serve this purpose, mainly because the term "quartimax" is introduced in their article.

The aim of quartimax, according to Neuhaus and Wrigley, is to find an orthogonal transformation which maximizes the variance of the factor contributions. The variance of the factor contribution is given by the square of the factor loading, since the factor loading may be viewed as a correlation.

3.1 Notation F_{mp} an unrotated factor matrix

(f_{ij}) an element of F_{mp} $(i=1, 2, \ldots, m; j=1, 2, \ldots p)$

G_{mp} a rotated factor matrix with the variance of the squared elements maximized

(g_{ij}) an element of G_{mp}

T_{pp} an orthonormal rotation matrix

Recalling equation 2-2, the form of rotation is $F_{mp}T_{pp}=G_{mp}$. The problem then is to find T_{pp} so that G_{mp} is actually the maximum variance of the squared elements. Neuhaus and Wrigley give the variance of all the (f_{ij}^2) as

$$\sigma^2 = (\Sigma\Sigma\ f_{ij}^4 - (\Sigma\ (\Sigma\ f_{ij}^2))^2/mp)/mp. \qquad (3\text{-}1)$$

This may also be written as

$$\sigma^2 = (mp)^{-1}\Sigma\Sigma\ f_{ij}^4 - (m^2p^2)^{-1}\ (\Sigma\ (\Sigma\ g_{ij}^2)^2. \qquad (3\text{-}2)$$

The (g_{ij}^2) may be substituted since, for any row, i, the variance must remain the same due to the conditions of orthogonality. From this, they conclude to find G_{mp}, one has to find T_{pp} which maximizes the fourth powers of the elements in the rotated matrix. Hence, the name quartimax is chosen.

Due to the mathematical complexity of determining the angle of rotation to simultaneously rotate the entire factor space, a two by two transformation matrix is set up and the entire factor speace is rotated one plane at a time. The procedure takes two factors at a time and maximizes the squared factor loadings for those factors. The process continues until all $((p-1)p)/2$ possible combinations have been rotated. The transformation can be written

$$F_{mp}T_{12}T_{13}\cdots T_{ij}\cdots T_{p(p-1)} = G_{mp}. \qquad (3\text{-}3)$$

A two by two transformation matrix is given by the form

$$\begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix}.$$

Neuhaus and Wrigley, then, give the following formula for determining Θ, which will maximize the squared factor loadings.

$$\tan 4\Theta = \frac{2\sum(2f_{ki}f_{kj})(f_{ki}^2 - f_{kj}^2)}{\sum((f_{ki}^2 - f_{kj}^2)^2 - (2f_{ki}f_{kj})^2)} \qquad (3\text{-}4)$$

In the above equation, i and j are the two factor being rotated. One will then obtain a different Θ for each pair of factors.

Varimax

The varimax can be distinguished from the quartimax in that it tries to simplify columns rather than rows. Kaiser (1958) develops two versions of the varimax: (1) "raw" and (2) "normalized." Since Kaiser (1958, p. 193) states "there is a more fundamental rationale to establish the normal varimax as a mathematical definition of rotation." Hence the term "varimax" from here forward will refer to the "normal" varimax criteria. The normal varimax is given by Kaiser as

$$V = m\sum\sum(f_{ij}^2/h_j^2)^2 - \sum(\sum f_{ij}^2/h_j^2)^2 \qquad (3\text{-}5)$$

In this equation h_j^2 is the communality of the jth test.

In developing the computer program for varimax, some special notation was developed which aided in finding the angle Θ of rotation for each pair of factors.

3.2 Notation

U	$f_{ki}^2 - f_{ki}^2$
V	$2f_{ki}f_{kj}$
A	$\sum U$
B	$\sum V$
C	$\sum (U^2 - V^2)$
D	$\sum 2UV$

The desired angle of rotation is given by

$$\tan 4\theta = \frac{D - 2AB/m}{C - (A^2 - B^2)/m} . \qquad (3\text{-}6)$$

As in the case of the quartimax, a different θ is computed for each pair of factors. To show differences between the varimax and the quartimax, equation 3-4 can be written in the notation used for equation 3-6.

$$\tan 4\theta = \frac{D}{C} . \qquad (3\text{-}7)$$

It is readily evident that the quartimax criteria is much less complicated than the varimax. Tenopyr and Michael (1963) have shown that when a general factor is expected the quartimax does not remove enough variance and the varimax removes too much. However, Hakstein and Boyd (1972) point out the quartimax is not as accurate as the varimax when compared to solutions obtained from visual rotations. It is the majority opinion that varimax is the best analytic procedure to approximate simple structure.

Criticisms of Simple Structure

There are three main criticisms of simple structure. The most important is that it does not provide a likely theoretical model for several aspects of human nature. For example, as the British expound, the structure of intelligence has more of a hierarchial structure. In fact, the evidence is heavily in favor of a hierarchial structure of intelligence. The second criticism is an outcropping of the first. Reyburn and Raath (1949) point out that there is considerable misuse of simple structure in current research. The final criticism is made by Reyburn and Raath drawing on the evidence given by Reyburn and Taylor (1943). They criticize its invariance property due to the heavy reliance on the identity of factors to support a factor's invariance. Hence, there is considerable subjectivity in interpreting a factor as invariant.

In conclusion, it is recommended that simple structure should be used in two instances. The most obvious case is that in which the theoretical model suggests simple structure as the factor pattern. The other case is as an alternate solution when reporting results. By doing this, one offers the reader a choice and according to the reader's theoretical position. This author recommends that the automatic use of simple structure be discontinued and more selective use of methods available be considered.

CHAPTER IV

HIERARCHIAL STRUCTURE

Unlike simple structure, hierarchial structure is the product of a theoretical school of thought, rather than mathematical assumptions or factor loading patterns. In other words, the theory came before the factor analyses; as opposed to simple structure, where theory comes after simple structure is imposed on the factors. Therefore, it is easier to discuss hierarchial structure in terms of some aspect of psychological research. The area of intellectual abilities has a long history of using hierarchial structure as its theoretical model. It is this area, then, that will serve as the example for developing the discussion of hierarchial structure.

The Structure of the Intellect

In the late nineteenth century, there were two major views concerning the structure of intellectual abilities. They were monism, which stated the mind was a single entity which was indivisible, and plurism, which had as its main tenet the division of the mind into special faculties. However, in 1912, McDougall (1912, pp. 71-121) proposed a compromise to the problem of the mind by combining the monistic and pluristic doctrines. He accomplished this by setting up a hierarchy of levels. This approach has become one of the favorite of British factor analysts.

Burt (1949) uses McDougall's approach as a theoretical explanation for the results of several factor analytic studies. He concludes that there can be no doubt as to the existence of group factors, and their arrangement in some sort of hierarchial schema. This is in contrast with Spearman's two factor hierarchy (1942) which does not postulate group factors, only a general factor and specific ability factors. The following figures should further clarify this distinction.

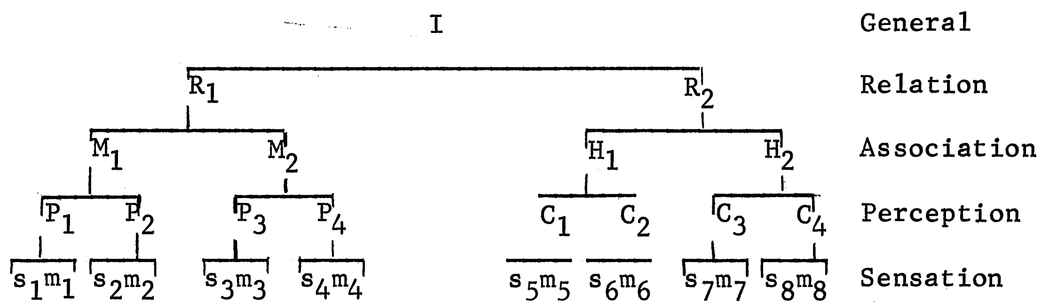

Figure 2. Burt's Hierarchy. (For a fuller discussion, see Burt (1909)).

Several differences can be found between Burt's hierarchy (figure 2) and Spearman's two factor hierarchy (Figure 3). It may first be noted that Spearman's model is linear, while Burt's is branched. Thus, Spearman's approach is continuously graded and Burt's is subdivided into levels. This implies that the group factors of Burt are differentially weighted with respect to the amount of influence they have on the process of intellectual functioning. For example, R_1 would have more influence or account for more variation in a group of tests than would P_3. A final distinction that has already been noted is Spearman's lack of group factors. For him, there is no tendency for specific abilities to cluster together as in Burt's approach. At present, it seems that the majority of evidence points to group factors. It remains up to the individual theorist as to which approach he takes. However, most factor analysts hold to either the hierarchial approach to intelligence of Burt (or some variation thereof) or to Thurstone's primary mental abilities.

G General

S_1 S_2 S_3 ... S_n Specific

Figure 3. Spearman's Hierarchy. (For a fuller discussion, see Spearman (1942)).

The more recent work of Vernon (1956) and Jaynes (1972) add more convincing evidence for a hierarchial structure of intellectual functioning. In general, the evidence now points to a general factor which might best be described as reasoning ability. It should be noted that this general factor is not yet a universal factor. However, given that its generality is limited to western culture, at present, it is a very stable factor. Major group factors are also evident. These seem to be associated with reasoning or cognitive abilities that are greatly influenced by verbal or mechanical skills. These factors are not as stable in that they are influenced by the experience of the individual. Finally, there are minor group factors that appear to reflect sensory or motor capacities.

Jaynes (1972) and Jaynes and Weiner (1973) offer an improved conception of hierarchial structure. They, like Thurstone, use a definition of factor patterns to define his theoretical structure. There are two major conditions used to define the pattern of factor loadings. The first is positive manifold. This implies that the factor loadings of importance will be positive unless there exists "true negativity." The other requirement is in the form of restrictions on the factor loadings. A variate may load on only two factors and one of those factors must be the general factor. Some of the variables are allowed to load only on the general factor, but no variates are allowed to load on two group factors. This creates an arrangement that can be characterized in Table II.

TABLE II

A COMPARISON OF SIMPLE STRUCTURE AND JAYNES' HIERARCHIAL STRUCTURE SOLUTIONS

Simple Structure Pattern			Jaynes' Hierarchial Pattern		
I_0	I_1	I_2	G	I_0	I_1
X			X		
X			X		
X			X		
	X		X	X	
	X		X	X	
	X		X	X	
		X	X		X
		X	X		X
		X	X		X

Note: X represents a factor loading other than zero.

Procedures for Hierarchial Structure

At present, there are two general procedures used to obtain a hierarchial factor structure. Using the notion of oblique factors, Schmid and Leiman (1957) propose a method which transforms the oblique factors into a larger number of orthogonal factors. This procedure is known as higher-order factors. The other procedure was introduced by Burt (1950) which is a group factor solution. Burt's procedure involves the grouping of variables according to their sign pattern in the centroid solution. Jaynes and Weiner (1973) use a variation of this method to obtain factor patterns with hierarchial structure. This procedure is based on the development of an extraction from a hypothesized factor pattern matrix.

Higher order factors are the results of factor analysis of the matrices of correlations of the oblique factors. The data and the factors could possibly be laid out in a hierarchy of orders, with each order representing a factor analysis of the preceeding oblique factor solution. Actually, the higher order factors serve the same function as simple structure by systematically clustering the oblique factors, which already represent clusters of variables. Rummel (1970, p. 425) feels that higher order factors define the basic dimensions of the data, while the higher order factors show the functional relationships among the various clusters of variables.

It is felt by the present author that higher order factors have several disadvantages which makes their use questionable. The major problem with this type of analysis is the final factor solution is a matrix which is not of full rank. Thus, some of the factors play no part in defining the factor space. This produces a violation of parsimony in the traditional sense and implies that some of the factors represent redundant information which may produce interpretation complications. A second disadvantage of the use of higher order factors is that one must use oblique factors. As has already been noted, Harris and Knoell (1948) point out that when using oblique factors, one encounters problems of interpretability.

Burt's multiple group method (1950) is more advantageous than the solution of Schmid and Leiman, but still is not the best possible solution. Although one can obtain orthogonal factors by using Burt's method, there is still a limitation in that one cannot maximize the amount of variance accounted for by the factors extracted. One of the newer techniques available, Joreskog's maximum likelihood procedure (1970), meets with this same problem. Although these techniques are more advantageous than higher order factors, there still exists a lack of mathematical completeness due to that limitation.

CHAPTER V

SELECTIVE FACTOR ROTATION

At present, there exists a need for rotational procedures which approximate hierarchial structure. As was demonstrated in the last chapter, the present methods are inadequate for their respective reasons. In order to fulfill this need, a rotational procedure is presented that allows not only approximation to hierarchial structure, but to any structure depending upon the researcher's theoretical model. No longer need the factor analyst base his interpretations solely on the outcomes of a rotation which produces simple structure. He can rotate the factors in accord with his theoretical perspective.

This method is based on two important aspects of the present analytic rotational procedures. Recalling equation 3-3, one notes that the entire factor space is not rotated simultaneously, but two factors are rotated at a time while holding the other factors involved in the space constant. This pairwise method of rotation does not affect the factor's orthogonality with another factor. Also, the factors which are being rotated are not affected. Therefore, it seems reasonable to conclude that one can stop the rotational process at any point leaving all factors independent. For example, in a five factor space, one might rotate all pairwise combinations of the first three factors and leave the last two in their original position. This will produce an orthogonal factor space and communality for each variate will remain unchanged.

The second point which acts as a foundation for selective factor rotation is that the amount of variance which is shifted from one factor to another or spread across factors during the varimax procedure influences how well simple structure is achieved. The less variance that is moved the closer simple structure is approximated. In a Monte Carlo study of this phenomena, Younger (1973) has produced fairly substantial evidence for its support. A hypothetical three factor solution was set up. The amount of variance each factor represented was varied, and it was found that when a great deal of variance was shifted about, the number of high loadings decreased and moderate to low loadings increased. In other words, simple structure was more definitive in terms of high factor loadings when the variance was evenly distributed between the three factors. Taking the above features into account, one can now produce a method of rotation which can be coupled by a principal components extraction to produce a hierarchial factor structure.

Selective factor rotation is a very simple process. The factor analyst rotates only those factors which he feels should be rotated. This is very similar to what one does in graphical rotation. By selectively rotating factors and using analytic procedures which are now available, all one does is impose simple structure on only a portion of the factor solution instead of the entire solution. Therefore, by breaking the initial extraction up into groups of factors, according to one's theoretical model, numerous factor patterns emerge. Some examples of possible patterns are given in

Table III. It should be noted that pattern C is not of full rank. Therefore, when one is using selective factor rotation to hold a general factor constant, pattern D is a matrix of full rank. It will be shown in the next chapter that pattern D is the actual result of such a rotation.

TABLE III

SELECTIVE FACTOR ROTATION PATTERNS

Pattern A					Pattern B					Pattern C				Pattern D			
I_0	I_1	I_2	I_3	I_4	I_0	I_1	I_2	I_3	I_4	I_0	I_1	I_2	I_3	I_0	I_1	I_2	I_3
X		X			X			X		X	X			X			
X		X			X			X		X	X			X	X		
X		X			X			X		X	X			X	X		
X			X			X		X		X		X		X		X	
	X		X			X			X	X		X		X		X	
	X		X			X			X	X		X		X		X	
	X			X			X		X	X			X	X			X
	X			X			X		X	X			X	X			X
	X			X			X		X	X			X	X			X

Note: X represents a factor loading other than zero.

Pattern A can be obtained by rotating factors I_0 and I_1 and holding all others constant, and then rotating the remaining factors, while holding the first two constant. The second pattern, B, simply reverses this process. More important is the distinction between patterns C and D. Pattern C is the type one would expect from a solution obtained by higher order factors. As already noted, pattern C is not of full rank. In order to have full rank some variables must load only on the general factor. To obtain this type of pattern, one holds the first factor constant while rotating the remaining factors to simple structure. A more detailed account for rotation to a hierarchial structure can be developed.

The starting point for hierarchial structure, when using selective factor rotation, is Hotelling's principal components method. The amount of variance accounted for by the factor space is maximized. Thus, the first factor always accounts for the most variance, and sometimes there is a substantial difference between the variance of the first factor and the rest of the factors in the solution. Recalling that the less variance disturbed, the closer to simple structure one gets. Therefore it is unwise to rotate the first factor of a principal component solution. This first unrotated factor of the principal components solution acts as the general factor in the hierarchial structure. Now that the general factor is found, one finds the major and minor group factors by selectively rotating.

The major and minor group factors are rotated depending on the theoretical model one is using. As is the case of pattern D, the remaining three factors are rotated together. This is not the only hierarchial solution one can obtain. Table IV illustrates a pattern resembling Burt's hierarchy. In this solution there were two separate rotations. Factors two and five were rotated together and factors three, four, six and seven were rotated together.

TABLE IV

A BRANCHED HIERARCHY

G	R_1	H_1	H_2	R_2	M_1	M_2
X						
X						
X						
X	X					
X	X					
X	X					
X	X	X				
X	X	X				
X	X	X				
X	X		X			
X	X		X			
X	X		X			
X				X		
X				X		
X				X		
X				X	X	
X				X	X	
X				X	X	
X				X		X
X				X		X
X				X		X

When reporting results which have been selectively rotated, the following system of notation should be used. It is assumed, unless otherwise stated, that the analytic method used is the varimax procedure. Selective factor rotation can be abbreviated by SFR. The various groups of factors rotated will appear as subscripts. These groups will be separated by commas. The factors in the groups will be indicated by parentheses. Table IV can be specified by SFR$_{(2,5),(3-4,6-7)}$, while pattern D in Table III is written SFR$_{(2-4)}$. If another analytic procedure, other than the varimax is employed, it should be stated in a footnote.

Combining this rotational procedure with the principal components extraction procedure, the factor analyst no longer has to rely on approximate methods to obtain hierarchial structure. The researcher can use the principal components extraction to ensure that the maximum amount of variance is extracted and then rotate that extraction to fit the theoretical model by breaking it up into groups possessing simple structure. It may also be suggested that breaks in the eigenvalues could indicate the groupings. For example, in Table IV, G is the first principal component and R_1 and R_2 are the second and third components. Thus, as the importance of the factors decreases, the systematic variance they account for also decreases due to the nature of the principal components extraction. Hence, approximations to hierarchial structure can now be obtained as readily as approximations to simple structure have been obtained previously.

CHAPTER VI

THE APPLICATION OF SELECTIVE FACTOR ROTATION TO RESEARCH

For any method to be accepted as a research technique, it must be shown to be useful in an actual research situation. In order to show that selective factor rotation is a useful technique, two studies will be presented. One is a replication of an earlier investigation and the other is an investigation which is exploratory in nature. The replication is taken from the Holzinger-Swineford data (Harman, 1967, p. 124). The second study centers around personality variables and is designed so that the interrelationships of several of the more popular personality inventories are developed.

Intellectual Ability

There presently exists a controversy as to the structure of the intellect. In order to shed some light, selective factor rotation was applied to the classic intellectual ability data collected by Holzinger and Swineford (Harman, 1967, p. 124). The analysis of the twenty-four variables provided evidence for the hierarchial approach to intelligence. A comparison is made to the simple structure solution (Harman, 1967, p. 311) and Jaynes' (1972) multiple group solution. The present solution was obtained from a principal components extraction with a $SFR_{(2-5)}$ rotation applied to the extraction matrix.

TABLE V

CENTROID-VARIMAX SOLUTION FOR INTELLECTUAL ABILITY TESTS

TESTS	FACTORS M_1	M_2	M_3	M_4	Systematic Variance For Variates
Series completion	57	35			54
Numerical puzzles	42			38	41
Problem reasoning	41	36	36		42
Arithmetic problems		34	34	44	47
Deduction	47				42
Number-figure	32		54		46
Figure-word			59		25
Sentence completion		82			75
General information		75			67
Paragraph comprehension		75			67
Word meaning		80			75
Word classification	38	54			52
Paper Form Board	54				32
Cubes	43				21
Flags	54				34
Visual perception	67				53
Word recognition			50		33
Number recognition			50		29
Object-number			64		47
Figure recognition	41		43		37
Counting dots				69	54
Addition				70	57
Code			36	60	52
Straight-curved capitals	41			59	55
Systematic Variance For Factors	3.08	3.50	2.36	2.44	11.38

From Harman (1967, p. 311). The decimal points have been omitted except at the bottom.

The centroid-varimax solution exhibits some clustering, but it does not present a clear definitive picture of simple structure. The moderate loadings (30-39) cloud the issue. This is probably due, as Younger (1973) has pointed out, to the shifting of the systematic variance of the factors. Jaynes' (1972) multiple group solution is much more definitive in its factor pattern. It is this hierarchial structure that is to be replicated by the SFR solution. Jaynes' solution is presented in Table VI and the SFR solution is seen in Table VII. It should be noted that the order of the variables has been changed in the SFR solution to emphasize the pattern obtained.

The most noticeable characteristic of the SFR solution is that a definite hierarchial structure appears. This adds information to the growing evidence in favor of a hierarchial conception of the structure of the intellect. However, when comparing Table VI with Table VII, several major differences appear as well as consistencies between the two solutions. Before dealing with the discrepancies, it is wise to point out how well the Jaynes' solution was replicated.

As is expected, the principal component extraction gives the SFR solution more variance to work with in obtaining the final solution. The general or first factor in both solutions are very similar. It should be noted that the SFR solution produces more variance for the general factor, causing the loadings to be somewhat higher across the board. Another important aspect which needs to be noted is that the variables in all three solutions cluster similarly. This is especially evident in the multiple group and SFR solutions.

TABLE VI

MULTIPLE GROUP SOLUTION FOR INTELLECTUAL ABILITY TESTS

TESTS	FACTORS I.0	I.1	I.2	I.4	I.5	Systematic Variance For Variates
Series completion	71					55
Numerical puzzles	69					51
Problem reasoning	68					50
Arithmetic problems	68					51
Deduction	64					47
Number-figure	62					43
Figure-word	58					36
Sentence completion	48	73				77
General information	50	69				73
Paragraph comprehension	49	69				73
Word meaning	57	65				78
Word classification	51	68				61
Paper Form Board	34		63			53
Cubes	33		57			44
Flags	42		55			50
Visual perception	62		49			64
Word recognition	37			64		57
Number recognition	37			61		51
Object-number	43			55		51
Figure recognition	58			45		60
Counting dots	44				68	67
Addition	43				66	67
Code	52				57	63
Straight-curved capitals	59				52	67
Systematic Variance For Factors	7.50	2.03	1.27	1.41	1.68	13.87

From Jaynes (1972). The decimal points have been omitted except at the bottom.

The most obvious difference between the multiple group solution and the SFR solution is the negative loadings on factor I_1 in the latter solution. This result implies that factors I.2 and I.5 in the multiple group solution are opposite ends of the same continuum. The negative loadings on factor I_1 are from tests which deal with perceptual and spatial abilities, while the positive loadings are from speed type tests which require rapid manipulation and detailed skills. Hence, the continuum might be labeled mental manipulation with one end being in the abstract realm and the other in the concrete. On the other hand, the negativity has not previously been reported. In terms of positive manifold the solution is not as refined as the multiple group solution.

The other major difference existing between the two solutions is that the SFR extracts a new factor, I_3, which is not evident in the multiple group solution. The reason for this factor appearing in the SFR solution might be attributed to the fact that more variance has been extracted. The new factor comes from variables which loaded only on the first factor in the multiple group solution except for Object-number. The tests loading on the new factor measure rote learning ability and deductive processes. One would expect that number-figure would also load on this factor. It loads on the verbal factor, which does not seem entirely congruent. In conclusion, the results of the SFR solution lead one to the following interpretation. The structure of intellectual abilities is hierarchial. In the series of tests, which are the classic twenty-four psychological variables introduced by Holzinger and Swineford, five factors emerge. A general factor, accounting for two thirds of the variation in the factor space, appears as the first factor with four major group factors branching off of it. Factor I_0 is interpreted to be a verbal factor. I_1, which is the bipolar factor, has already been discussed and appropriately labeled cognitive manipulation. It was also noted that one of the poles was abstract and the other was concrete. The third major group factor is centered around tests of recognition and recall. Therefore, I_2 represents memory. The final factor, I_3, is the new factor emerging from the study and as already mentioned is the rote learning factor. It should be noted that if a larger, unselected sample was used, this factor might emerge as a minor factor out of the memory factor.

An Analysis of Personality

Raymond Cattell (1950) and H. J. Eysenck (1953) develop theories of personality based on factor analytic work. Cattell's structure of traits represents a hierarchial conception of personality, while Eysenck's three broad dimensions, extraversion-introversion, neuroticism, and psychoticism, give the appearance of a simple structure approach. Since the work in this area is not as extensive as that of intellectual abilities, there is not a clear cut solution to this difference. It is felt that an analysis of some of the more common personality tests would help resolve this issue.

Method

Eighty seven undergraduates enrolled in a sophomore level psychology course were given a battery of personality tests. The entire battery was given at one sitting and the Ss were divided into groups in order to counter balance the tests to account for any fatigue affects. Age, social class, and nationality were not obtained, but the students were asked to furnish their overall grade point average.

TABLE VII

SFR SOLUTION FOR INTELLECTUAL ABILITY TESTS

TESTS	G	I_0	I_1	I_2	I_3	Systematic Variance For Variates
Series completion	71					57
Numerical puzzles	62					52
Problem reasoning	64					45
Arithmetic problems	67					55
Number-figure	54	41				52
Sentence completion	67	55				77
General information	69	46				70
Paragraph comprehension	69	49				73
Word meaning	69	53				77
Word classification	69	31				59
Paper Form Board	44		-52			61
Cubes	40		-46			49
Flags	51		-34			45
Visual perception	61		-30			60
Counting dots	48		50			67
Addition	47		71			74
Code	57		42			56
Straight-curved capitals	61		35			67
Word recognition	44			57		54
Number recognition	41			61		56
Figure recognition	53			35		59
Object-number	48			34	46	60
Figure-word	47				62	66
Deduction	64				30	54
Systematic Variance For Factors	8.14	1.89	1.80	1.43	1.19	14.45

The decimal points have been omitted except at the bottom.

The test battery consisted of four personality tests which are commonly administered during an intake or preliminary evaluation when and individual is seeking psychological assistance. The four tests are: (1) Fundamental Interpersonal Relations Orientation - Behavior (FIRO-B), (2) Rotter Locus of Control, (3) Multiple Affect Adjective Check List (MAACL), and (4) Mini-Mult (The seventy one question version of the Minnesota Multiphasic Personality Inventory). (See Appendix A for the names of the scales and the abbreviations employed.)

The tests were scored and the twenty one resulting scale scores, along with the reported grade point average, produced a 22 by 22 product moment correlation matrix. A principal components extraction procedure was performed with extraction being halted on the basis of the eigenvalues, the criterion being set at 1.00. Two rotations were applied separately to the factor matrix. The first was the varimax and the second was a $SFR_{(2-5)}$ rotation.

Results

Table VIII contains the principal component solution with all factor loadings being reported. Table IX reports the results of the two rotations. Only loadings having an absolute value greater than 0.30 are reported.

The varimax solution reveals an interesting phenomena. The clusters represent scales of the same test. Therefore, the tests are internally homogeneous while they are externally heterogeneous. This solution also features more positive manifold than the SFR solution. However, the clustering of the tests makes interpretation almost impossible. The SFR solution offers more interpretability. A hierarchial structure with some interesting properties emerges. The first factor looks like a factor measuring within individual stability, while the fourth measures social stability.

TABLE VIII

THE FIRST FIVE PRINCIPAL COMPONENTS OF 22 PERSONALITY VARIABLES

TESTS	FACTORS I	II	III	IV	V	Systematic Variance For Variates
MMPI Sc	90	14	14	-02	01	87
MMPI Hs	79	05	14	01	-08	66
MMPI D	75	-21	26	-03	-04	69
MMPI Pt	74	-32	29	00	-18	78
MMPI Pa	68	-10	35	-17	-03	64
MMPI Hy	51	-31	37	-35	03	63
MMPI Pd	37	-52	46	-27	23	75
MMPI F	77	47	-04	01	12	84
ROTTER IE	70	42	-18	22	-13	79
MMPI L	60	63	-18	-01	08	81
MMPI K	54	68	-09	-10	18	81
MMPI Ma	-05	-55	48	-20	-04	60
MAACL A	38	-69	-09	37	18	82
MAACL D	48	-57	-17	38	27	82
MAACL H	21	-57	-33	52	23	81
FIRO-B Ie	-42	50	39	17	22	65
FIRO-B Aw	-36	17	69	18	01	67
FIRO-B Iw	-22	28	61	34	34	74
FIRO-B Ae	-03	40	59	13	-03	54
FIRO-B Ce	-28	10	21	05	69	63
FIRO-B Cw	24	15	23	66	-27	65
GPA	-15	11	38	23	-48	51
Systematic Variance For Factors	6.15	3.88	2.76	1.61	1.30	15.70

The decimal points have been omitted except at the bottom.

The systematic variance for variates is not presented in Table IX because the rotations are orthogonal and the variance for variates does not change during rotation. The validity scales load positively on the general intraindividual stability factor, but negatively on the group factors emerging from it. The interindividual stability factor, S_0, has a group factor emerging from it also. Due to the fact that wanted control and GPA load heavily on it, it could be labeled as an academic achievement or dependency factor. Hence, it is not an instability factor, but a factor indicating the individual's orientation to interpersonal relations.

Discussion

The results of this analysis lead one to several interesting conclusions. Although a hierarchy does appear, two separate aspects emerge, each having group factors branching from it. The first hierarchy is found in the first three factors, I_0, I_1, and I_2. I_0 is the general factor of this group and is interpreted as an intra-individual stability factor. This factor closely resembles Eysenck's Neuroticism factor. The group factors emerging from this factor may be interpreted as follows. Since I_1 has the MMP Ma scale loading on it as the highest variable, it can be labeled as an energy factor. If an individual has a high factor score on this factor, there would be a great deal of energy within his system. Factor I_2, since it is made of the scales from the MAACL, could be labeled as a self-image factor. The higher the factor score on this factor would indicate a lower self-image. A distinction is made here between the Mini-Mult and the Multiple Affect Adjective Checklist from the standpoint of their different techniques of measurement. Since, both are measuring similar aspects of personality, the differences causing their separation in the factors is a result of their different approaches to the measurement of those aspects.

TABLE IX

VARIMAX AND SFR ROTATIONAL SOLUTIONS
FOR PERSONALITY VARIABLES

TESTS	VARIMAX FACTORS I	II	III	IV	V	SFR FACTORS I_0	I_1	I_2	S_0	S_1
MMPI Sc	61	68				90				
MMPI Hs	52	55				79				
MMPI D		72				75	32			
MMPI Pt		83				74	42			
MMPI Pa		75				68	35			
MMPI Hy		68				51	57			
MMPI Pd		77				37	73			
MMPI F	85	31				77	-37			
ROTTER IE		30				70	-47			
MMPI L	91					60	-56	-31		
MMPI K	89					54	-51	-40		
MMPI Ma	-61	57					76			
MAACL A		30		83		38		75		
MAACL D				87		48		73		
MAACL H				84				80		
FIRO-B Ie			63	-32		-42			64	
FIRO-B Aw			79			-36			65	
FIRO-B Iw			84						83	
FIRO-B Ae			33		32				63	
FIRO-B Ce			34		-70				44	-57
FIRO-B Cw			65						38	59
GPA			40		54					60
Systematic Variance For Factors	4.81	4.35	2.73	2.69	1.37	6.15	3.06	2.59	2.49	1.41

The decimal points have been omitted except at the bottom.

The interindividual factors deal mainly with social skills and the individual's satisfaction in those relationships. Factor S_0 is the general social stability factor and can be related to Eysenck's Extroversion factor. Factor S_1 branches off to form a dependency factor. The recognition of those two hierarchies is important in the verification of certain personality theories. They seem to provide indirect support for Eysenck's approach. Since the first two factors he found were Neuroticism and Extraversion, these results add to the evidence that those two factors are fundamental in the personality of man. These results indicate that man is a two-sided being. Therefore the fact that inter- and intraindividual factors exist needs to be a fundamental supposition in the development of theories of personality.

CHAPTER VII

SUMMARY

This study investigated the possibilities of developing a rotational procedure to produce hierarchial solutions. A brief background was presented and it was shown that present factor analytic work was stuck in a simple structure mold. It was shown that there are several theorists who hold to a hierarchial theory of intelligence. Therefore, the simple structure is inadequate.

Selective factor rotation was developed because of this inadequacy. It is based on two properties of the present analytic procedures. The plane by plane rotation technique is employed as a logical basis of development and it was noted that shifting great amounts of variance causes more problems in the approximation of simple structure. It was shown that by using the principal components extraction procedure and then rotating the factors according to a theoretical position with selective factor rotation certain hierarchial structures could be obtained.

Finally, the procedure was applied to two different sets of data. A replication of the Hilzinger and Swineford data was presented. It was found that Jaynes' (1972) hierarchial solution was closely approximated, with the exception of two factors. The other study dealt with an investigation of several personality tests and their factor structure. It was found that two hierarchies appear. One represents Eysenk's Neuroticism factor or intraindividual stability and the other represents Eysenck's Extraversion factor or inter-individual stability. The group factor associated with the latter factor is a dependency factor. The group factors associated with the intraindividual factor are energy level and self image. The use of selective factor rotation in a replication and exploration proved to be worthwhile.

The major flaw that is seen in the technique is the increased negativity in the factor loadings after rotation. This was seen in both studies. Therefore, if selective factor rotation is to be fully useful to the researcher, some refinement needs to be made to aid in the production of positive manifold. When this refinement is realized, selective factor rotation will allow the researcher to have the tools necessary to find the most parsimonious solution from a theoretical perspective.

SELECTED BIBLIOGRAPHY

Burt, Cyril. Experimental tests of intelligence. British Journal of Psychology, 1909, 3, 94-177.

Burt, Cyril. The structure of the mind: A review of the results of factor analysis. British Journal of Educational Psychology, 1949, 19, 100-111, 176-199.

Burt, Cyril. Group factor analysis. British Journal of Psychology, Statistical Section, 1950, 3, 40-75.

Burt, Cyril. Francis Galton and his contribution to psychology. British Journal of Statistical Psychology, 1962, 15, 1-49.

Carroll, John. An analytic solution for approximating simple structure in factor analysis. Psychometrika, 1953, 18, 23-38.

Cattell, R. B. Personality: A systematic, theoretical, and factual study. New York: McGraw-Hill, 1950.

Cattell, R. B. Factor analysis: An introduction and manual for the psychologist and social scientist. New York: Harper and Row, 1952.

Cliff, Norman. Orthogonal rotation to congruence. Psychometrika, 1966, 31, 33-42.

Eysenck, H. J. The structure of human personality. New York: Wiley, 1953.

Ferguson, George. The concept of parsimony in factor analysis. Psychometrika, 1954, 19, 281-290.

Fruchter, Benjamin. Orthogonal and oblique solutions to a battery of aptitude, achievement and background variables. Educational and Psychological Measurement, 1952, 12, 20-38.

Fruchter, B. J., & Novak, E. A comparitive study of three methods of rotation. Psychometrika, 1958, 23, 211-221.

Galton, Francis. Hereditary genius, an inquiry into its laws and consequences. London: University of London Press, 1869.

Gorsuch, R. L. A comparison of Biquartimin, Maxplane, Promax and Varimax. Educational and Psychological Measurement, 1970, 30, 861-868.

Guilford, J. P., & Hoepfner, R. Comparisons of varimax rotations with rotations to theoretical targets. Educational and Psychological Measurement, 1969, 29, 3-17.

Guttman, Louis. Image theory for the structure of quantitative variables. Psychometrika, 1953, 18, 277-296.

Hakstein, A. R., & Boyd, W. M. An empirical investigation of some special cases of the general "orthomax" criteria for orthogonal factor transformation. Educational and Psychological Measurement, 1972, 32, 3-14.

Harman, Harry. Modern factor analysis. Chicago: University of Chicago Press, 1967.

Harman, Harry, & Jones, W. H. Factor analysis by minimizing residuals (Minres). Psychometrika, 1966, 31, 351-368.

Harris, C., & Knoell, P. M. The oblique solution in factor analysis. Journal of Experimental Psychology, 1948, 39, 385-403.

Horst, P. A simple method of rotating a centroid factor matrix to a simple structure hypothesis. Journal of Experimental Education, 1956, 24, 251-258.

Horst, P. Factor analysis of data matrices. New York: Holt, Rinehart and Winston, 1965.

Hotelling, H. Analysis of a complex of statistical variables into principal components. Journal of Educational Psychology, 1933, 24, 417-441, 498-520.

Hotelling, H. Simplified calculations of principal components. Psychometrika, 1936, 1, 27-35.

Hurley, J. L., & Cattell, R. The procrustes program producing direct rotation to test an hypothesized factor structure. Behavioral Science, 1962, 7, 258-262.

Jaynes, W. An improved hierarchial conception for factor analysis with applications to intelligence and instructor ratings. Paper presented at the meeting of the Southwestern Psychological Association, Dallas, May, 1972.

Jaynes, W., & Weiner, B. Temperament factors and work preferences. Paper presented at the meeting of the Southwestern Psychological Association, Dallas, May, 1973.

Joreskog, K. A generalized computer program for simultaneous factor analysis in several populations. Research Bulletin 70-62. Princeton, New Jersey: Educational Testing Service, 1970.

Kaiser, H. The varimax criterion for analytic rotation in factor analysis. Psychometrika, 1958, 23, 187-200.

Kaiser, H. Computer program for varimax rotation in factor analysis. Educational and Psychological Measurements, 1959, 19, 413-420.

Kelley, T. Essential traits of mental life. Harvard Studies in Education. Cambridge, Mass.: Harvard University Press, 1935.

McCloy, E., Metheny, E., & Knott, V. A comparison of the Thurstone method of multiple factors with the Hotelling method of principal components. Psychometrika, 1938, 3, 61-66.

McDougall, W. Psychology: The study of behavior. New York: Henry Holt and Co., 1912.

Mischel, W. Personality and assessment. New York: John Wiley and Sons, 1968.

Mosier, C. Influence of chance error on simple structure. Psychometrika, 1939, 4, 33-44.

Mulaik, S. The foundations of factor analysis. New York: McGraw-Hill, 1972.

Neuhaus, J., & Wrigley, C. The quartimax method: An analytic approach to orthogonal simple structure. British Journal of Statistical Psychology, 1954, 7, 81-91.

Pearson, K. On lines and planes of closest fit to systems of points in space. Philosophic Magazine, 1901, 6, 559-572.

Rao, C. R. Estimation and tests of significance in factor analysis. Psychometrika, 1955, 20, 93-111.

Reyburn, H. A., & Raath, M. J. Simple structure: A critical examination. British Journal of Psychology, Statistical Section, 1949, 2, 125-133.

Reyburn, H. A., & Taylor, J. G. On the interpretation of common factors: A criticism and a statement. Psychometrika, 1943, 8, 53-64.

Rummel, R. J. Applied factor analysis. Evanston: Northwestern University Press, 1970.

Schmid, J., & Leiman, J. M. The development of hierarchial factor solutions. Psychometrika, 1957, 22, 53-61.

Schonemann, P. H. A generalized solution of the orthogonal procrustes problem. Psychometrika, 1966, 31, 1-10.

Spearman, C. General intelligence, objectively determined and measured. American Journal of Psychology, 1904, 201-293.

Spearman, C. The abilities of man. New York: Macmillan Co., 1927.

Spearman, C. How G can disappear. Psychometrika, 1941, 6, 353-354.

Tenopyr, M. L., & Michael, W. B. A comparison of two computer based procedures of orthogonal analytic rotation with a graphical method when a general factor is present. Educational and Psychological Measurement, 1963, 23, 587-599.

Thomson, G. H. Hotelling's method modified to give Spearman's G. Journal of Educational Psychology, 1934, 25, 366-374.

Thomson, G. H. The factorial analysis of human ability. New York: Houghton Mifflin Co., 1951.

Thurstone, L. L. The vectors of the mind. Chicago: University of Chicago Press, 1935.

Thurstone, L. L. Primary mental abilities. Psychometrika Monographs, No. 1, Chicago: University of Chicago Press, 1938.

Thurstone, L. L. Multiple factor analysis. Chicago: University of Chicago Press, 1947.

Tryon, R. C. General dimensions of individual differences: Cluster vs. multiple factor analysis. Educational and Psychological Measurement, 1958, 18, 477-495.

Vernon, P. E. The structure of human abilities. New York: John Wiley and Sons, 1951.

Vernon, P. E. The measurement of abilities. London: University of London Press, 1956.

Wolfle, D. Factor analysis to 1940. Psychometric Monographs, No. 3, Chicago: University of Chicago Press, 1940.

Younger, K. The approximation of simple structure as a function of the distribution of the systematic factor variance. Unpublished manuscript, Oklahoma State University, 1973.

APPENDIXES

APPENDIX A

SCALE ABBREVIATIONS FOR PERSONALITY VARIABLES

ABBREVIATION		SCALE
MMPI	Sc	Schizophrenia
	Hs	Hypochondriasis
	D	Depression
	Pt	Psychasthenia
	Pa	Paranoia
	Hy	Hysteria
	Pd	Psychopathic deviate
	F	Validity
	L	Lie
	K	K
	Ma	Hypomania
Rotter	IE	Internal-External
MAACL	A	Anxiety
	D	Depression
	H	Hostility
FIRO-B	Ie	Expressed Inclusion
	Aw	Wanted Affection
	Iw	Wanted Inclusion
	Ae	Expressed Affection
	Ce	Expressed Control
	Cw	Wanted Control
GPA		Grade Point Average

APPENDIX B

CORRELATION AND RESIDUAL MATRIX FOR

PERSONALITY VARIABLES

Variable	1	2	3	4	5	6	7	8	9	10	11
1. MMPI Sc		71	63	73	66	37	38	78	02	56	52
2. MMPI Hs	-02		61	63	54	56	19	56	-11	44	38
3. MMPI D	-04	03		72	52	55	45	48	20	30	24
4. MMPI Pt	07	01	03		68	45	51	35	30	09	03
5. MMPI Pa	01	-04	-10	04		49	42	39	21	26	28
6. MMPI Hy	-10	13	00	-12	-07		58	18	23	09	17
7. MMPI Pd	05	-11	-05	-02	-08	-04		09	55	-17	-13
8. MMPI F	03	-05	-02	-03	-06	-05	05		-28	79	76
9. MMPI Ma	07	-10	-01	02	-01	-16	02	05		-47	-44
10. MMPI L	-04	-03	04	-08	-02	04	-01	02	-01		86
11. MMPI K	-05	09	01	-09	01	10	-01	01	-01	08	
12. Rotter IE	08	-04	-08	02	02	-05	04	-01	04	-08	-04
13. MAACL A	-02	-02	-02	-07	02	03	00	00	-01	03	07
14. MAACL D	-05	00	-01	-04	00	07	00	01	-06	03	-09
15. MAACL H	01	02	01	01	02	00	07	05	03	01	-01
16. FIRO-B Ie	02	07	-06	05	03	-02	-02	-01	07	-02	-02
17. FIRO-B Aw	04	00	-02	-04	-02	06	-05	00	-09	-02	01
18. FIRO-B Iw	01	06	05	03	00	03	-07	00	-05	-02	-03
19. FIRO-B Ae	-01	-02	-09	-03	-06	01	08	-02	-02	02	00
20. FIRO-B Ce	06	03	06	04	03	-05	00	00	04	01	-03
21. FIRO-B Cw	02	-06	00	00	-02	00	-03	00	08	00	00
22. GPA	03	-08	-07	-09	-01	00	-03	04	00	10	05

The decimal points have been omitted. Correlations are above the diagonal, and residuals are below.

	Variable	12	13	14	15	16	17	18	19	20	21	21
1.	MMPI Sc	73	20	27	06	-23	-24	-10	09	-14	27	-10
2.	MMPI Hs	52	22	30	09	-19	-17	-03	06	-27	19	-09
3.	MMPI D	30	36	38	17	-39	-15	-03	-05	-14	18	-06
4.	MMPI Pt	41	37	39	21	-34	-16	-10	-01	-26	24	-04
5.	MMPI Pa	35	24	25	01	-20	-07	-03	06	-12	10	-01
6.	MMPI Hy	02	28	30	-01	-29	02	-05	04	-14	-07	-06
7.	MMPI Pd	-11	39	35	12	-25	-01	-03	09	08	-03	-13
8.	MMPI F	72	-01	15	-01	-08	-22	-03	11	-09	22	-09
9.	MMPI Ma	-35	22	06	06	-03	12	01	-01	06	-02	10
10.	MMPI L	62	-14	01	-14	-02	-25	-06	14	-07	16	-02
11.	MMPI K	59	-19	-05	-26	08	-14	01	18	-01	09	-09
12.	Rotter IE		03	17	04	-14	-26	-12	06	-24	34	-03
13.	MAACL A	-02		79	69	-47	-20	-16	-26	-12	10	-11
14.	MAACL D	00	01		70	-47	-24	-16	-27	-12	09	-11
15.	MAACL H	-01	-04	-04		-29	-32	-21	-28	-08	10	-19
16.	FIRO-B Ie	-01	-04	-05	07		41	58	42	30	13	09
17.	FIRO-B Aw	00	04	-03	-02	-08		63	50	14	09	32
18.	FIRO-B Iw	-01	-02	-02	-06	-02	02		33	34	22	17
19.	FIRO-B Ae	-01	03	03	08	-03	00	-18		11	23	28
20.	FIRO-B Ce	-05	-07	-10	-07	-10	-13	-12	-04		-16	02
21.	FIRO-B Cw	-02	-06	-07	-07	02	-12	-04	-05	-01		19
22.	GPA	-02	-06	07	02	-10	-05	-05	-03	-16	-15	

The decimal points have been omitted. Correlations are above the diagonal, and residuals are below.

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