LIMIT AND DEAD CENTER POSITIONS OF

A GEARED FIVE-LINK MECHANISM

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PREFACE

In this thesis, the limit and dead center positions of a geared five-link mechanism are first derived analytically. A sample derivation for a gear train speed ratio of two is included to demonstrate the procedure. Secondly, a graphical solution is studied, and a simple graphical procedure is presented. Finally, the positioning of the instant centers of velocity of this mechanism are studied.

I would like to acknowledge the assistance and continuous encouragements received from my advisor, Dr. A. H. Soni. Thanks are also due to my friends: Louis Torfason, Dilip Kohli and Luis Loeff for their stimulative discussion on the subject.

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NOMENCLATURE

r_1	length of ground link MQ
r_2	length of input link MA
r_3	length of input link AB
r_4	length of coupler link BC
r_5	length of output link CQ
θ_1	displacement angle of ground link MQ
θ_2	displacement angle of input link MA
θ_3	displacement angle of input link AB
θ_4	displacement angle of coupler link BC
θ_5	displacement angle of output link CQ
G_1	gear fixed to the ground link MQ
G_2	gear fixed to the input link AB
R_1	radius of G_1
R_2	radius of G_2
ω_1	angular velocity of the fixed gear G_1
ω_2	angular velocity of input link MA
ω_3	angular velocity of input link AB
$\bar{V}_i$, $(i = 2,3)$	velocity vectors
GR	ratio of R_1/R_2
α	initial displacement angle of input link AB
$\tilde{r}_i$, $(i = 1,..,5)$	vectors
$\bar{u}_i$, $(i = 1,..,5)$	unit vectors

γ	any angle
ξ	small angle
IC	instantaneous center of velocity
N	gear train speed ratio

CHAPTER I

INTRODUCTION

There is considerable interest in the design and analysis of geared mechanisms [1-7]*. Freundenstein claims that the work on geared mechanisms dates back to the eighteenth century. An excellent contribution in the study of motion of geared five-link mechanisms is by Freundenstein and Primrose [2,3]. The dimensional synthesis of geared five-link mechanisms is considered by Sandor and his associates [4-6]. The study on the coupler cognate mechanism is conducted by Hartenberg [1] and by Soni and Pamidi [7].

An interesting topic and one that is of practical importance in design and analysis of geared five-link mechanisms is to develop the "Grashof Criteria" for these mechanisms. A systematic approach in the development of such criteria, however, requires one to develop analytical methods to determine the conditions for the existence of limit positions and dead center positions. Accordingly, the objective of this thesis is to develop mathematically the criteria for the existence of limit positions and dead center positions and show their relationships with the instantaneous centers of velocity of one of the inversions of a geared five-link mechanism.

Chapter II presents an analytical description of the geared

*Numbers in brackets designate references.

five-link mechanism under investigation.

In Chapter III, an analytical procedure is developed to obtain the existence conditions for the limit positions and dead center positions.

In Chapter IV, the geometric properties of the limit positions and dead center positions of the geared five-link mechanism are studied.

In Chapter V, the motion of the instantaneous centers of velocity is studied. It is observed that at the limit positions and dead center positions these instantaneous centers of velocity arrange themselves in predictable manners.

The significant contribution of this thesis is summarized below.

(1) Development of a generalized approach to determine the limit positions and dead center positions of any given mechanism.

(2) Development of the existence conditions for the limit positions and dead center positions of a geared five-link mechanism.

(3) Development of a graphical construction method to determine the limit positions and dead center positions of the geared five-link mechanism under investigation.

(4) Development of relationships between the instantaneous centers of velocity and the limit positions and dead center positions.

(5) Development of theorems on instantaneous centers of velocity and limit positions and dead center positions.

CHAPTER II

GENERAL DESCRIPTION OF THE GEARED FIVE-LINK MECHANISM IN STUDY

Figure 1 shows the geared five-link mechanism studied in this thesis.

In this figure, M, A, B, C, and Q are revolute pairs joining the links. r_i and θ_i, $i = 1,\ldots,5$, are the link lengths and displacement angles of the links. The links are:

$i = 1$, ground link MQ

$i = 2$, input link MA

$i = 3$, input link AB

$i = 4$, coupler link BC

$i = 5$, output link CQ

Also, R_1 is the radius of the gear G_1, which is fixed to the ground link MQ, and R_2 is the radius of the gear G_2 pivoted on input link MA at point A onto which is rigidly fixed input link AB.

The following definitions will prove to be helpful in the understanding of the contents of this thesis:

(a) Limit position: occurs when the output link reaches an extrema, and reverses its motion at this extrema.

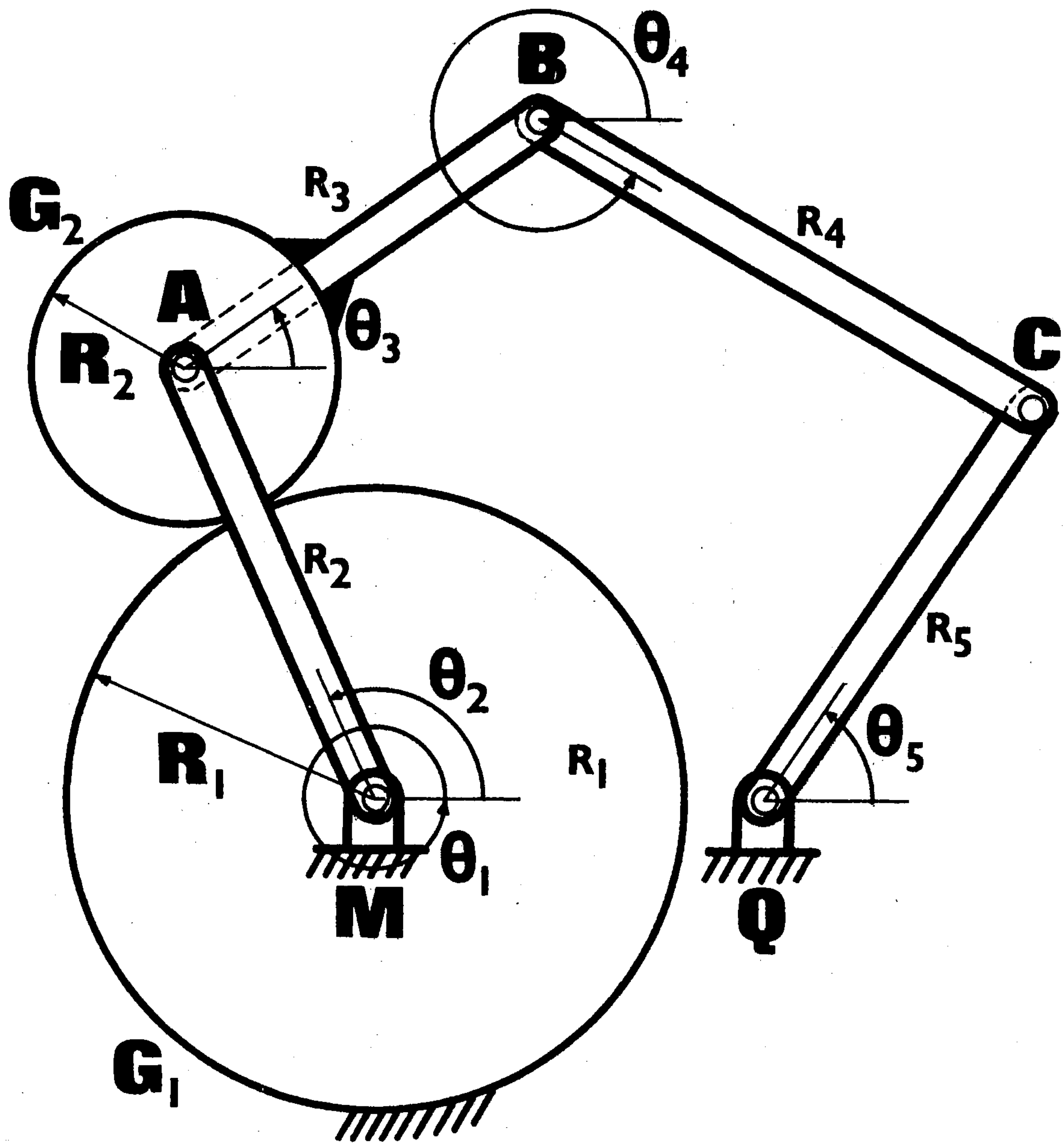


Figure 1. Illustration of Notation and Configuration of the Geared Five-Link Mechanism in Study

(b) Pseudo-limit position: occurs when the output link is at a dwell, and continues the motion in the same direction prior to the dwell.

(c) Dead center position: occurs when the input link reaches an extrema. The mechanism is permanently locked at this position. The mobility of the mechanism is restored by an external force. The dead center position of the output link is the limit position of the input link.

(d) Instantaneous centers of velocity (IC): are a pair of coincident points which have zero relative velocity.

CHAPTER III

ANALYTICAL DERIVATION OF THE LIMIT POSITIONS AND DEAD CENTER POSITIONS

The development of the mathematical conditions for the limit positions requires one to

(1) derive in a closed form the input-output displacement relationships;

(2) obtain in a closed form the relationship which describes $d\theta_5/d\theta_2 = 0$ where θ_2 and θ_5 are the input and output angular displacements;

(3) develop conditions for the limit positions by eliminating the unwanted output parameter θ_5.

The procedure discussed in this chapter can be used for any type of mechanism.

3.1 Displacement Analysis

The displacement analysis is accomplished by finding the vector loop-closure equation of the mechanism shown in Figure 2.

The gear arrangement for the mechanism in Figure 2 is shown in Figure 3; from this figure we write

$$\bar{V}_2 = (\bar{k}\ \omega_2) \times (\bar{j}\ [R_1 + R_2]), \tag{3.1.1}$$

$$\bar{V}_3 = (\bar{k}\ \omega_3) \times (-\bar{j}\ R_2) \tag{3.1.2}$$

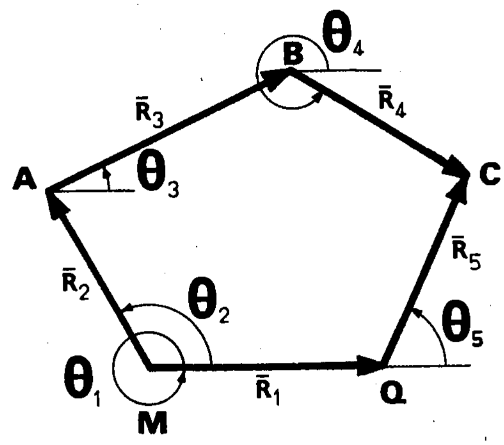


Figure 2. Vectorial Representation of the Geared Five-Link Mechanism in Study

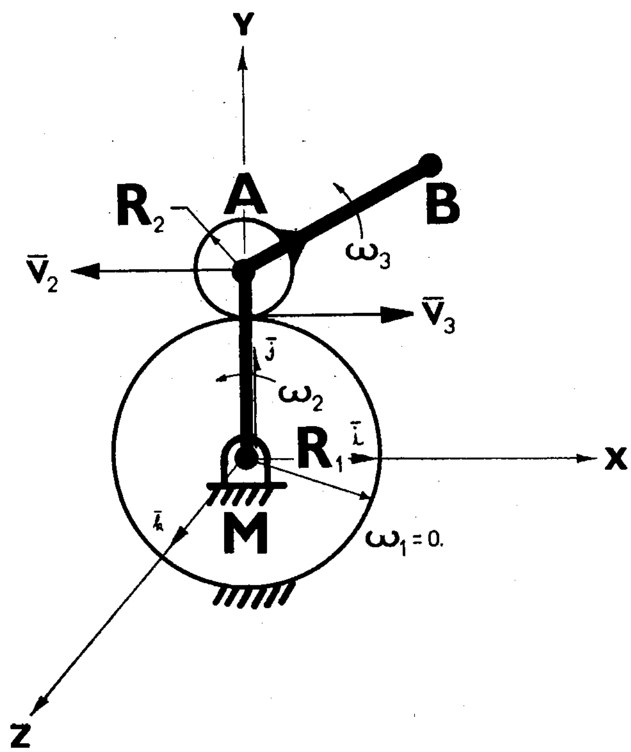


Figure 3. Input Velocity Vectors

where, $\bar{V}_2$ and $\bar{V}_3$ are velocity vectors,

ω_2 is the angular velocity of the input link MA,

ω_3 is the angular velocity of the input link AB,

$\bar{i}$, $\bar{j}$ and $\bar{k}$ are unit vectors on the x, y and z axis respectively.

Adding the velocity vectors, the following results:

$$\bar{V}_2 + \bar{V}_3 = 0;$$

that is,

$$-\bar{i}\,\omega_2\,(R_1 + R_2) + i\omega_3\,R_2 = 0.$$

Simplifying and rearranging the above equation, the following results:

$$\omega_3 = \left(\frac{R_1 + R_2}{R_2}\right)\omega_2,$$

or

$$\omega_3 = (GR + 1)\,\omega_2 \qquad (3.1.3)$$

Since the angular velocities are the first time derivatives of the displacements, then,

$$\frac{d\theta_3}{dt} = (GR + 1)\,\frac{d\theta_2}{dt}.$$

Integration of both sides gives the following:

$$\theta_3 = (GR + 1)\,\theta_2 + \alpha . \qquad (3.1.4)$$

Where, θ_2 and θ_3 are the displacements of input link MA and input link AB, GR is the ratio of R_1/R_2, and α is the initial displacement of the input link AB, or more simply,

$$\theta_3 = N\theta_2 + \alpha \, , \qquad (3.1.5)$$

where N is the gear train speed ratio (GR + 1).

Now the output displacement will be derived from Figure 2. The vectors are $\bar{r}_i = \bar{u}_i r_i$, $i = 1,\ldots,5$, where r_i are the link lengths and $\bar{u}_i$ are unit vectors corresponding to each r_i. They can be arranged in the following vector loop closure equation;

$$\bar{r}_2 + \bar{r}_3 + \bar{r}_4 - \bar{r}_1 - \bar{r}_5 = 0. \qquad (3.1.6)$$

The vectors can be represented as

$$\bar{r}_i = r_i e^{j\theta_i}; \; i = 1,\ldots,5.$$

From the theory of complex numbers,

$$r_i e^{j\theta_i} = r_i(\cos\theta_i + j\sin\theta_i), \; i = 1,\ldots,5. \qquad (3.1.7)$$

Equation (3.1.6), after expansion and separation of the real and complex parts, becomes,

$$r_2\cos\theta_2 + r_3\cos\theta_3 + r_4\cos\theta_4 - r_1\cos\theta_1 - r_5\cos\theta_5 = 0. \qquad (3.1.8)$$

$$r_2\sin\theta_2 + r_3\sin\theta_3 + r_4\sin\theta_4 - r_1\sin\theta_1 - r_5\sin\theta_5 = 0. \qquad (3.1.9)$$

To simplify the calculations θ_1 may be assumed equal to 360 degrees without any loss of generality. Since the output displacement θ_5 is desired, the coupler link displacement θ_4 must be eliminated from equations (3.1.8) and (3.1.9). Therefore, rearrangement of these equations gives the following:

$$r_2\cos\theta_2 + r_3\cos\theta_3 - r_1 - r_5\cos\theta_5 = -r_4\cos\theta_4 \qquad (3.1.10)$$

$$r_2 \sin\theta_2 + r_3 \sin\theta_3 - r_5 \sin\theta_5 = -r_4 \sin\theta_4 . \tag{3.1.11}$$

Squaring both sides of equations (3.1.10) and (3.1.11) and then adding them together gives the following:

$$\begin{aligned}
&r_1^2 + r_2^2 + r_3^2 - r_4^2 + r_5^2 + 2r_2 r_3 \cos(\theta_2 - \theta_3) - \\
&-2r_1 r_2 \cos\theta_2 - 2r_1 r_3 \cos\theta_3 - 2r_2 r_5 \cos\theta_2 \cos\theta_5 - \\
&-2r_3 r_5 \cos\theta_3 \cos\theta_5 + 2r_1 r_5 \cos\theta_5 - \\
&-2r_2 r_5 \sin\theta_2 \sin\theta_5 - 2r_3 r_5 \sin\theta_3 \sin\theta_5 = 0.
\end{aligned} \tag{3.1.12}$$

Rearrangement and collection of the terms results in:

$$A_1 \cos\theta_5 + A_2 \sin\theta_5 + A_3 = 0 \tag{3.1.13}$$

where,

$$\begin{aligned}
A_1 &= 2r_5 (r_1 - r_2 \cos\theta_2 - r_3 \cos\theta_3) \\
A_2 &= -2r_5 (r_2 \sin\theta_2 + r_3 \sin\theta_3) \\
A_3 &= r_1^2 + r_2^2 + r_3^2 - r_4^2 + r_5^2 + 2[r_2 (r_3 \cos(\theta_2 - \theta_3) - \\
&\quad -r_1 \cos\theta_2) - r_1 r_3 \cos\theta_3].
\end{aligned}$$

The following trigonometric relationships are used to find θ_5 explicitly:

$$\begin{aligned}
\sin\gamma &= \left(\frac{2 \tan(\gamma/2)}{1 + \tan^2(\gamma/2)}\right) \\
\cos\gamma &= \left(\frac{1 - \tan^2(\gamma/2)}{1 + \tan^2(\gamma/2)}\right)
\end{aligned} \tag{3.1.14}$$

for any γ. Substituting the relationships (3.1.14) in equation (3.1.13) gives

$$K_1 \tan^2\frac{\theta_5}{2} + K_2 \tan\frac{\theta_5}{2} + K_3 = 0 \tag{3.1.15}$$

where,

$$K_1 = A_2 - A_1'$$
$$K_2 = 2A_2'$$
$$K_3 = A_3 + A_1'$$

from where,

$$\theta_5 = 2 \text{ artan}\left(\frac{-K_2 \pm \sqrt{K_2^2 - 4K_1 K_3}}{2K_1}\right). \tag{3.1.16}$$

Two values of θ_5 are found to describe the normal and crossed configurations.

Now the displacement of the coupler link θ_4 will be found. To accomplish this, equation (3.1.11) is rearranged so that,

$$\theta_4 = \text{arsin}\left(\frac{1}{r_4}[r_5\sin\theta_5 - r_2\sin\theta_2 - r_3\sin\theta_3]\right) \tag{3.1.17}$$

There are two values of θ_4 corresponding to the two values of θ_5.

3.2 Limit Positions

At the limit positions of the output link, a reversal of motion can be observed. Hence, the velocity of the output link must be zero and the mathematical condition for the existence of the limit positions is obtained by setting $d\theta_5/d\theta_2 = 0$. That is,

$$\frac{d\theta_5}{d\theta_2} = \cos\theta_5[-r_5(r_2\sin\theta_2 + r_3\sin\theta_3)] + \sin\theta_5[r_5(r_2\cos\theta_2 + r_3\cos\theta_3)] + [r_2r_3\sin(\theta_2 - \theta_3) - r_1(r_2\sin\theta_2 + r_3\sin\theta_3)] = 0. \tag{3.2.1}$$

This can be written in a simpler form,

$$C_1\cos\theta_5 + C_2\sin\theta_5 + C_3 = 0 \tag{3.2.2}$$

where,

$$C_1 = -r_5(r_2\sin\theta_2 + r_3\sin\theta_3)$$

$$C_2 = r_5(r_2\cos\theta_2 + r_3\cos\theta_3)$$

$$C_3 = r_2r_3\sin(\theta_2 - \theta_3) - r_1(r_2\sin\theta_2 + r_3\sin\theta_3).$$

Using relationships (3.1.14), the above equation becomes,

$$K_4\tan^2\frac{\theta_5}{2} + K_5\tan\frac{\theta_5}{2} + K_6 = 0 \qquad (3.2.3)$$

where,

$$K_4 = C_3 - C_1$$

$$K_5 = 2C_2$$

$$K_6 = C_3 + C_1 .$$

Equation (3.2.3) forces the velocity of the output link to be zero. Equation (3.1.15) describes the loop closure condition which is also valid at the limit positions. Both of these relationships involve θ_2 and θ_5 and are not linearly related. To obtain the position of the input link MA corresponding to each of the limit positions of the output link CQ, θ_5 must be eliminated from these two independent equations. This elimination of θ_5 is accomplished using Sylvester's dyalitic eliminant technique. Application of this technique yields the determinant condition written below.

$$\begin{vmatrix} K_1 & K_2 & K_3 & 0 \\ 0 & K_1 & K_2 & K_3 \\ K_4 & K_5 & K_6 & 0 \\ 0 & K_4 & K_5 & K_6 \end{vmatrix} = 0. \qquad (3.2.4)$$

The above determinant when expanded and simplified yields a polynomial in $\tan \theta_2/2$.

The order of this polynomial will depend on the gear train speed ratio since K_i, $i = 1,\ldots,6$, are functions of θ_2 and θ_3, where $\theta_3 = N\theta_2 + \alpha$.

The above statement is valid for any integer or noninteger value of N. The noninteger value of N can be delt in the following manner:

$$\cos(\Delta_1 \theta_2) = \cos\left(\Delta_1 \Delta_2\left(\frac{\theta_2}{\Delta_2}\right)\right),$$
$$\cos(\theta_2 - \Delta_1 \theta_2) = \cos\left(\Delta_2 (1 - \Delta_1)\left(\frac{\theta_2}{\Delta_2}\right)\right), \quad \text{and}$$
$$\cos(\theta_2) = \cos\left(\Delta_2\left(\frac{\theta_2}{\Delta_2}\right)\right)$$

where,

Δ_1 = non integer number

Δ_2 = a number that makes $\Delta_1 \Delta_2$ and $\Delta_2(1 - \Delta_1)$ an integer.

The same is true for the sine terms; and the problem now will be to find the $\left(\frac{\theta_2}{\Delta_2}\right)$.

From relationships (3.1.14), it can be observed that the half tangent terms are squared for each corresponding cosine term. Since the solution of equation (3.2.4) results in a polynomial in terms of $\tan \frac{\theta_2}{2}$ only, the following theorem will be stated:

<u>Theorem #1</u>: The degree of the polynomial, solution of the Sylvester's dyalitic eliminant, is function of the gear train speed ratio only.

The real roots of the polynomial give the necessary conditions for a limit position to exist. These roots must be replaced in equation (3.1.16), which will give two values to θ_5, that must be checked with equation (3.2.1) to see which value of θ_5 make the equation equal to zero, thus obtaining the possible limit positions of the normal and crossed configurations.

To determine the existence of the limit positions, the second derivative of equation (3.1.12) must be obtained. That is:

$$\frac{d^2\theta_5}{d\theta_2{}^2} = \frac{\begin{array}{l}\cos\theta_5[-r_5(r_2\cos\theta_2 + r_3\cos\theta_3)] + \\ + \sin\theta_5[-r_5(r_2\sin\theta_2 + r_3\sin\theta_3)] - \\ - [r_2(-r_3\cos(\theta_2 - \theta_3) + r_1\cos\theta_2) + r_1 r_3\cos\theta_3]\end{array}}{\begin{array}{l}\cos\theta_5[-r_5(r_2\sin\theta_2 + r_3\sin\theta_3)] + \\ + \sin\theta_5[-r_5(r_1 - r_2\cos\theta_2 - r_3\cos\theta_3)]\end{array}} \quad (3.2.5)$$

If $d^2\theta_5/d\theta_2{}^2 \neq 0$, then θ_5 is at a limit position and this is a sufficient condition. If $d^2\theta_5/d\theta_2{}^2 = 0$, then further derivatives are to be taken to determine if θ_5 is a limit position or a pseudo-limit position.

Pseudo-limit position as previously defined is the condition in which the output during its motion has a dwell, and continues moving in the same direction as prior to the dwell [8]. The dwell can be instantaneous if $d^3\theta_5/d\theta_2{}^3 \neq 0$ or longer if $d^m\theta_5/d\theta^m{}_2 \neq 0$, m is odd.

In case that $d^m\theta_5/d\theta_2{}^m \neq 0$, m is even, then the output is at a limit position with a dwell occurring at that extrema.

If $d^2\theta_5/d\theta_2{}^2 = 0$ occurs, then a practical approach to know if the output is at a limit or pseudo-limit position is to check with $\theta_2 \pm \xi$, where ξ is a small angle, the displacement analysis, and observe if the output reverses its motion (limit position), or continues in the same direction (pseudo-limit position).

3.3 Dead Center Positions

In order to find the dead center positions, the first derivative of the input with respect to the output displacements, must be set equal to zero. This is accomplished by taking $d\theta_2/d\theta_5 = 0$ of equation (3.1.12) obtaining:

$$\frac{d\theta_2}{d\theta_5} = \cos\theta_5[-r_5(r_2\sin\theta_2 + r_3\sin\theta_3)] + \\ + \sin\theta_5[r_5(r_2\cos\theta_2 + r_3\cos\theta_3)] = 0 \qquad (3.3.1)$$

or,

$$C_4\cos\theta_5 + C_5\sin\theta_5 = 0 \qquad (3.3.2)$$

where,

$$C_4 = -r_5(r_2\sin\theta_2 + r_3\sin\theta_3)$$

$$C_5 = r_5(r_2\cos\theta_2 + r_3\cos\theta_3).$$

The above equation, with the use of relationships (3.1.14), becomes:

$$K_7\tan^2\frac{\theta_5}{2} + K_8\tan\frac{\theta_5}{2} + K_9 = 0 \qquad (3.3.3)$$

where,

$$K_7 = C_4$$

$$K_8 = -2C_5$$

$$K_9 = -C_4$$

Similarly as for the limit positions, a common root θ_5 must be found for equations (3.1.15) and (3.3.3). Here again, two equations with two unknowns are provided. In order to obtain an equation in terms of θ_2 only, Sylvester's dyalitic eliminant technique is used to obtain:

$$\begin{vmatrix} K_1 & K_2 & K_3 & 0 \\ 0 & K_1 & K_2 & K_3 \\ K_7 & K_8 & K_9 & 0 \\ 0 & K_7 & K_8 & K_9 \end{vmatrix} = 0. \qquad (3.3.4)$$

Solving, a polynomial in terms of $\tan \frac{\theta_2}{2}$ results. Theorem #1 applies here, too. The real roots of the polynomial provide the necessary and sufficient conditions for the existence of dead center positions, since the mechanism becomes a structure at these positions.

3.4 Sample Derivation of the Limit and Dead Center Positions of a Geared Five-Link Mechanism of Gear Train Speed Ratio = 2

The general approach presented in the previous section is reexamined to obtain numerical results for a geared five-link mechanism with a gear train speed ratio = 2. For this purpose, the displacement analysis will first be performed.

Since N = 2, equation (3.1.5) becomes

$$\theta_3 = 2\theta_2 + \alpha \qquad (3.4.1)$$

substituting θ_3 in equation (3.1.12), the following is obtained:

$$r_1^2 + r_2^2 + r_3^2 - r_4^2 + r_5^2 + 2r_2 r_3 \cos(\theta_2 + \alpha) - 2r_1 r_2 \cos\theta_2 -$$
$$-2r_1 r_3 \cos(2\theta_2 + \alpha) - 2r_2 r_5 \cos\theta_2 \cos\theta_5 -$$
$$-2r_3 r_5 \cos(2\theta_2 + \alpha)\cos\theta_5 + 2r_1 r_5 \cos\theta_5 -$$
$$-2r_2 r_5 \sin\theta_2 \sin\theta_5 - 2r_3 r_5 \sin(2\theta_2 + \alpha)\sin\theta_5 = 0; \qquad (3.4.2)$$

equation (3.1.13) becomes,

$$A_1 \cos\theta_5 + A_2 \sin\theta_5 + A_3 = 0 \qquad (3.4.3)$$

where,

$$A_1 = 2r_5[r_1 - r_2\cos\theta_2 - r_3(\cos\alpha(\cos^2\theta_2 - \sin^2\theta_2) -$$
$$-2\sin\theta_2\cos\theta_2\sin\alpha)]$$
$$A_2 = -2r_5[r_2\sin\theta_2 + r_3(2\sin\theta_2\cos\theta_2\cos\alpha +$$
$$+ \sin\alpha(\cos^2\theta_2 - \sin^2\theta_2))]$$
$$A_3 = r_1^2 + r_2^2 + r_3^2 - r_4^2 + r_5^2 + 2[r_2(r_3(\cos\theta_2\cos\alpha -$$
$$- \sin\theta_2\sin\alpha) - r_1\cos\theta_2) - r_1 r_3(\cos\alpha(\cos^2\theta_2 -$$
$$\sin^2\theta_2) - 2\sin\theta_2\cos\theta_2\sin\alpha)].$$

To obtain the conditions for the limit positions, the first derivative of the output with respect to the input displacement must be equated to zero. To accomplish this, take $d\theta_5/d\theta_2 = 0$ of equation (3.4.2), rearrange and collect terms. Thus,

$$\frac{d\theta_5}{d\theta_2} = C_1\cos\theta_5 + C_2\sin\theta_5 + C_3 = 0 \qquad (3.4.4)$$

where,

$$C_1 = r_5[r_2\sin\theta_2 + 2r_3(2\cos\theta_2\sin\theta_2\cos\alpha +$$
$$+ \sin\alpha(\cos^2\theta_2 - \sin^2\theta_2))]$$

$$C_2 = -r_5[r_2\cos\theta_2 + 2r_3(\cos\alpha(\cos^2\theta_2 - \sin^2\theta_2) -$$
$$- 2\sin\alpha\cos\theta_2\sin\theta_2)]$$
$$C_3 = -r_2r_3(\cos\alpha\sin\theta_2 + \sin\alpha\cos\theta_2) +$$
$$+ r_1[r_2\sin\theta_2 + 2r_3(2\cos\alpha\cos\theta_2\sin\theta_2 +$$
$$+ \sin\alpha(\cos^2\theta_2 - \sin^2\theta_2))].$$

Equations (3.1.15) and (3.2.3) are rewritten,

$$K_1\tan^2\frac{\theta_5}{2} + K_2\tan\frac{\theta_5}{2} + K_3 = 0 \qquad (3.4.5)$$

and

$$K_4\tan^2\frac{\theta_5}{2} + K_5\tan\frac{\theta_5}{2} + K_6 = 0 \qquad (3.4.6)$$

where,

$$K_1 = A_3 - A_1$$
$$K_2 = 2A_2$$
$$K_3 = A_3 + A_1$$
$$K_4 = C_3 - C_1$$
$$K_5 = 2C_2$$
$$K_6 = C_3 + C_1$$

with the A's and C's found in equations (3.4.3) and (3.4.4). In order to find a common root of θ_5 in the above equations, Sylvester's dyalitic eliminant technique is applied. This procedure yields a 16th degree polynomial;

$$V(17)t^{16} + V(16)t^{15} + V(15)t^{14} + V(14)t^{13} + V(13)t^{12} +$$
$$+ V(12)t^{11} + V(11)t^{10} + V(10)t^{9} + V(9)t^{8} + V(8)t^{7} + V(7)t^{6} +$$

$$+ V(6)t^5 + V(5)t^4 + V(4)t^3 + V(3)t^2 + V(2)t + V(1) = 0 \tag{3.4.7}$$

Where V(i), i = 1,...,17, are defined in Appendix A and $t^m = \tan^m \frac{\theta_2}{2}$, m = 1,...,16.

The real roots of the above polynomial must be introduced in equation (3.1.16) and then in (3.4.4) to find the possible limit positions of the normal and crossed configurations.

Introduce the values of the real roots in equation (3.2.5) and follow the procedure outlined in Section 3.2 to check the existance of a limit or a pseudo-limit position.

For the dead center positions, equation (3.3.2) becomes,

$$\frac{d\theta_2}{d\theta_5} = C_4 \cos\theta_5 + C_5 \sin\theta_5 = 0 \tag{3.4-8}$$

where,

$$C_4 = r_5[r_2 \sin\theta_2 + r_3(2\cos\theta_2 \sin\theta_2 \cos\alpha + \\ + \sin\alpha(\cos^2\theta_2 - \sin^2\theta_2))]$$

$$C_5 = r_5[-r_2\cos\theta_2 + r_3(\cos\alpha(\sin^2\theta_2 - \cos^2\theta_2) + \\ + 2\cos\theta_2 \sin\theta_2 \sin\alpha) + r_1].$$

Equation (3.3.3) is rewritten,

$$K_7 \tan^2 \frac{\theta_5}{2} + K_8 \tan\frac{\theta_5}{2} + K_9 = 0 \tag{3.4.9}$$

where,

$$K_7 = C_4$$

$$K_8 = -2C_5$$

$$K_9 = -C_4$$

with the C's found in equation (3.4.8). In order to find a common root of θ_5 in equations (3.4.5) and (3.4.9), Sylvester's dyalitic eliminant technique is applied. This procedure yields a 16th degree polynomial;

$$W(17)t^{16} + W(16)t^{15} + W(15)t^{14} + W(14)t^{13} + W(13)t^{12} + \\ + W(12)t^{11} + W(11)t^{10} + W(10)t^{9} + W(9)t^{8} + W(8)t^{7} + \\ + W(7)t^{6} + W(6)t^{5} + W(5)t^{4} + W(4)t^{3} + W(3)^{2} + W(2)t + \\ + W(1) = 0 \qquad (3.4.10)$$

where, W(i), i = 1,...,17, are defined in Appendix A.

The real roots of the above equation give the necessary and sufficient conditions for the existance of the dead center positions.

Examples of limit, pseudo-limit and dead center positions are presented in Table I; in this table,

r_i, i = 1,...,5 = link lengths

Alpha = initial input link AB displacement

Theta 2 = input link MA displacement

Theta 3 = input link AB displacement

Theta 4N = coupler link BC displacement - mechanism in normal configuration

Theta 5N = output link CQ displacement - mechanism in normal configuration

Theta 4C = coupler link BC displacement - mechanism in crossed configuration

Theta 5C = output link CQ displacement - mechanism in crossed configuration

Position;

1111 = limit position

2222 = pseudo-limit position

3333 = dead center position

TABLE I

NUMERICAL EXAMPLES

R1	R2	R3	R4	R5	ALPHA	THETA 2	THETA 3	THETA 4N	THETA 5N	THETA 4C	THETA 5C	POSITION
5.00	1.00	1.25	3.25	2.75	75.00	348.7	52.4			35.5	102.8	1111
						51.1	177.3	334.4	191.9			1111
						312.1	339.2	331.6	263.7			1111
						69.2	213.5			14.4	157.5	1111
						119.9	314.8	324.4	224.1			1111
						145.7	6.4			26.9	127.8	1111
						265.1	245.2	20.7	200.9	20.9	200.7	3333
						185.0	85.0	348.1	169.8	349.7	167.9	3333
1.50	2.00	1.75	3.00	2.25	75.00	82.9	240.7			36.2	81.9	1111
						110.2	295.3	302.1	274.0			1111
						133.2	341.4			10.0	39.2	1111
						185.5	86.1	297.9	209.2			1111
1.00	2.50	1.25	3.00	2.25	90.00	87.3	264.7			356.0	27.7	2222
						90.0	270.0	278.7	229.7			1111
						90.0	270.0			356.1	27.7	2222
						90.0	270.0	278.7	229.7			1111
						90.0	270.0			356.1	27.7	2222
						153.9	37.8	275.8	209.8			1111
1.00	2.50	0.95	3.00	2.25	90.00	97.2	284.3	283.9	216.9			1111
						141.0	11.9	272.4	213.0			1111
1.00	2.50	1.50	3.00	2.25	90.00	89.9	269.8	270.6	242.7			1111
						71.3	232.6			0.8	32.9	1111
						106.9	303.8			354.0	21.7	1111
						158.7	47.4	275.4	205.7			1111

TABLE I (CONTINUED)

R1	R2	R3	R4	R5	ALPHA	THETA 2	THETA 3	THETA 4N	THETA 5N	THETA 4C	THETA 5C	POSITION
1.75	1.00	2.00	3.25	2.75	270.00	272.0	94.0	274.7	234.7			1111
						192.9	295.8			78.7	155.0	1111
						238.8	27.6	349.8	349.4	354.0	354.4	3333
						232.8	15.6	29.1	28.8	33.1	33.5	3333
4.00	1.00	0.00	2.00	3.25	0.0	143.6	287.2	323.5	190.5			1111
						216.4	72.8			36.4	169.5	1111
						307.0	254.0	307.0	227.5			1111
						53.0	106.0			53.0	132.5	1111
1.00	4.00	0.00	3.25	2.00	0.0	NO EXTREMAS OR DWELLS EXIST FOR THIS CASE						
2.00	3.25	0.00	1.00	4.00	0.0	290.9	221.8	290.9	263.0			1111
						69.1	138.2			69.1	97.0	1111
						140.4	280.9	320.4	159.0			1111
						219.6	79.1			39.6	201.0	1111
						64.7	129.3	71.6	103.5	84.7	100.2	3333
						295.3	230.7	275.3	259.8	288.4	256.5	3333
						143.4	286.8	333.9	157.9	340.3	156.3	3333
						216.6	73.2	19.7	203.7	26.1	202.1	3333
2.00	3.25	0.00	4.00	1.00	0.0	64.7	129.3	280.2	275.3	283.5	288.4	3333
						295.3	230.7	76.5	71.6	79.8	84.7	3333
						143.4	286.8	337.2	157.3	337.2	157.1	3333
						216.6	73.2	22.8	202.9	22.8	202.7	3333
1.00	0.00	3.25	4.00	2.00	0.0	NO EXTREMAS OR DWELLS EXIST FOR THIS CASE						

CHAPTER IV

GRAPHICAL STUDY OF THE LIMIT AND DEAD CENTER POSITIONS

In actual design work, in many instances a designer is interested in a quick and simple technique to check the motion characteristics of the output of a mechanism.

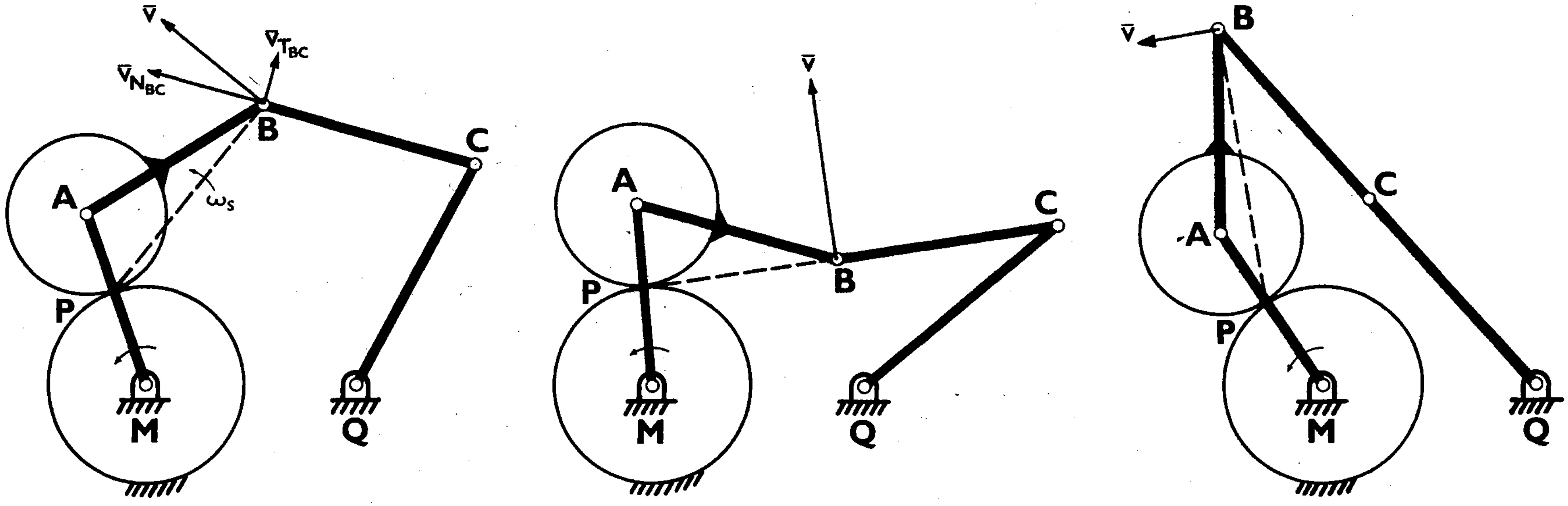
In this chapter a graphical procedure is explained that will enable the designer to find all the limit and dead center positions of a geared five-link mechanism for both normal and crossed configurations.

4.1 General Motions of Trochoids

A trochoid is a curve traced by a point on a circle when the circle rolls on another fixed circle. The number of convolutions and geometry of this curve depends on the ratio of the radii of both circles, and the distance of the point generating the trochoid from the center of the rolling circle. In figure 3, point B generates a trochoid.

4.2 Limit Positions

In order to understand the occurrence of the limit positions of a geared five-link mechanism, consider a vector $\bar{V}$ (see Figure 4a) which is the velocity vector corresponding to the angular velocity (ω_s) at which point B moves about the rolling contact point P of the two gears.



(a) (b) (c)

Figure 4. Velocity at Revolute Pair B when ω_2 is the Input Angular Velocity; (a) General Configuration; (b) Limit Position; (c) Dead Center Position

That is $\bar{V} = \omega_s \times \bar{PB}$, where $\bar{PB}$ is the distance between point P and B, or simply $\bar{V}$ is perpendicular to line PB at point B. In the same figure, vectors $\bar{V}n_{BC}$ and $\bar{V}t_{BC}$ are the normal and tangential components of $\bar{V}$ with respect to the coupler link BC.

At a limit position, $\bar{V}n_{BC}$ must be zero, and after this condition occurs, $\bar{V}n_{BC}$ must have opposite direction as prior to the occurence of this condition, otherwise it is at a dwell (pseudo-limit position). For $\bar{V}n_{BC}$ to become zero. line $\bar{PB}$ must be collinear to coupler link BC see Figure 4b.

Any gear train with several gears has a speed ratio equivalent to a gear train with only two gears; these two gears will be named. equivalent gears.

The following theorem can be stated:

<u>Theorem #2</u>: A limit or pseudo-limit position of a geared five-link mechanism with any gear train speed ratio exists only when the coupler link or its prolongation passes through the point of contact of the two equivalent gears.

4.3 Graphical Procedure to Obtain the Limit Positions

A simple graphical procedure to obtain the limit positions is as follows:

1) Plot circle C_1 of length r_2 (input link MA) about P_1, that is,

$$X_2 = r_2 \cos\theta_2 \qquad (4.3.1)$$

$$Y_2 = r_2 \sin\theta_2 \qquad (4.3.2)$$

2) On the same graph plot the trochoid T_1 corresponding to the gear ratio and link lengths selected as inputs (r_2 and r_3) that is,

$$X_3 = X_2 + r_3 \cos\theta_3 \tag{4.3.3}$$

$$Y_3 = Y_2 + r_3 \sin\theta_3 \tag{4.3.4}$$

3) On the same graph plot two new trochoids T_2 and T_3, with the same data as step (2); except that for T_2, r_3 becomes $r_3 + r_4$ and for T_3, r_3 becomes $r_3 - r_4$; that is,

$$X_{31} = X_2 + (r_3 + r_4)\cos\theta_3 \tag{4.3.5}$$

$$Y_{31} = Y_2 + (r_3 + r_4)\sin\theta_3 \tag{4.3.6}$$

$$X_{32} = X_2 + (r_3 - r_4)\cos\theta_3 \tag{4.3.7}$$

$$Y_{32} = Y_2 + (r_3 - r_4)\sin\theta_3 \tag{4.3.8}$$

4) On the same graph, select a point at a distance r_1 (gound link MQ) from the center of rotation P_1 of circle C_1. Then with a length r_5 (output link CQ) as radius, plot circle C_2 about P_2 as fixed center.

5) All the points P_{CL} where C_2 cuts trochoids T_2 and T_3 are one of the ends of the coupler link BC. In order to find the other end, draw a line L_1 perpendicular to the trochoid T_2 or T_3 where the intersection with C_2 occurred until L_1 cuts trochoid T_1 perpendicularly at point P_{BL}; this will be the other end of the coupler link BC.

6) Now to find the position of the input link MA, draw a circle C_3 with radius corresponding to the input link AB length (r_3) with center at P_{BL}; it will cut circle C_1 at two points (or at one point if tangent to C_1). Draw another circle C_4 with radius corresponding to the equivalent fixed gear (R_1) about

point P_1. Draw line L_2 collinear with L_1 until it cuts circle C_4 (it may cut C_4 at two points). The point P_3 of the intersection of C_3 and C_1, which is collinear with the point P_4 of the intersection of L_2 with C_4, and point P_1 will be the point P_3 determining the location of input link MA.

7) Repeat steps (5) and (6) until all input positions are found. All these input positions correspond to limit or pseudo-limit position of the output link.

Figure 5a shows one of the limit positions which was obtained using the procedure just outlined.

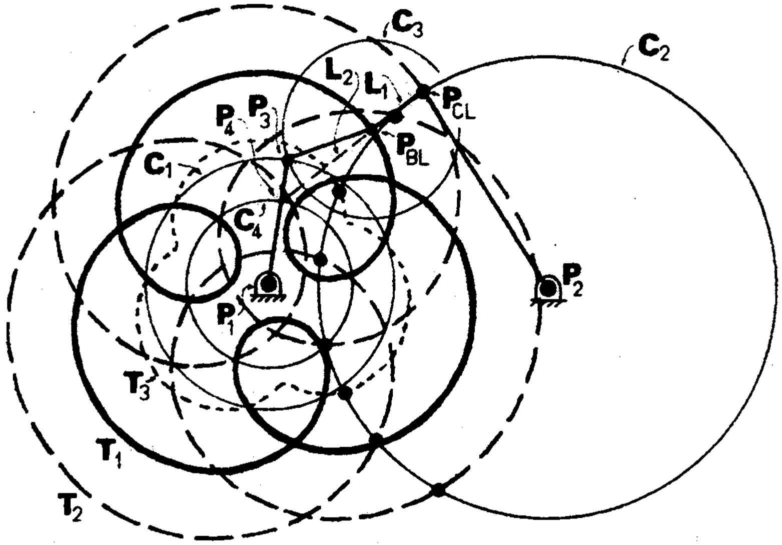
4.4 Dead Center Positions

In order to understand the occurrence of the dead center positions of a geared five-link mechanism see Figure 4a where $\bar{V}$, $\bar{V}n_{BC}$ and $\bar{V}t_{BC}$ are as previously defined in Section 4.2. For a dead center position to occur, the velocity vector $\bar{V}$ must become zero; this will only occur if the geometric configuration of the links force the input links to become stationary. The geometric configuration (see Figure 4c) that will hinder the input link MA from rotating occurs when links BC and CQ (coupler and output links) are collinear.

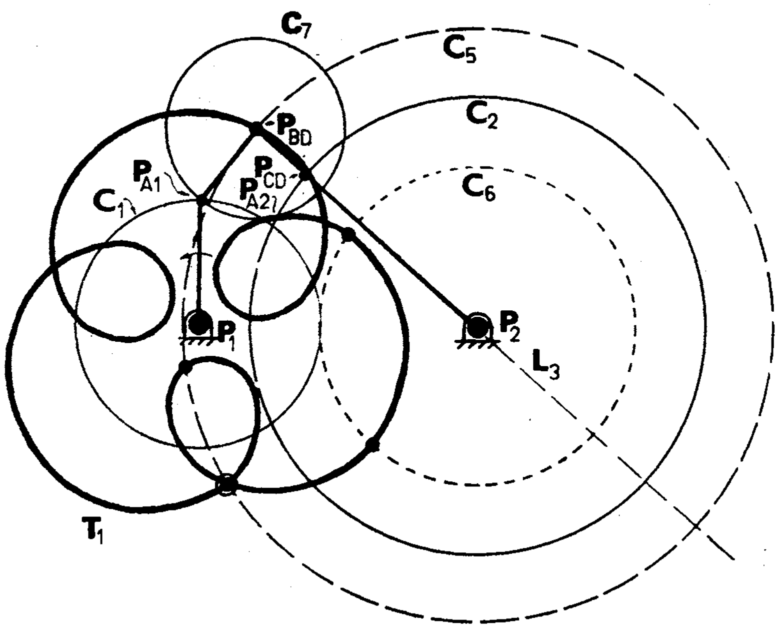
As explained in Section 4.2, any gear train with several gears has a speed ratio equivalent to a gear train with only two gears; and these two gears are named equivalent gears.

The following theorem can be stated:

<u>Theorem #3</u>: A dead center position of a geared five-link mechanism with any gear train speed ratio exists



(a)



(b)

Figure 5. Graphical Procedure; (a) Sample Limit Position; (b) Sample Dead Center Position

only when the coupler and the output links are collinear.

4.5 Graphical Procedure to Obtain the Dead Center Positions

A simple graphical procedure to obtain the dead center positions is as follows:

1) Carry out steps (1), (2), and (4) of Section 4.3 (graphical procedure to obtain the limit positions). Then circle C_1, trochoid T_1, circle C_2 and fixed points P_1 and P_2 are given.
2) On the same graph, plot two new circles C_5 and C_6 with radius $r_5 + r_4$ and $r_5 - r_4$ respectively with center at point P_2; that is,

$$X_{51} = r_1 + (r_5 + r_4)\cos\theta_5 \tag{4.5.1}$$

$$Y_{51} = (r_5 + r_4)\sin\theta_5 \tag{4.5.2}$$

$$X_{52} = r_1 + (r_5 - r_4)\cos\theta_5 \tag{4.5.3}$$

$$Y_{52} = (r_5 - r_4)\sin\theta_5 \tag{4.5.4}$$

3) All the points P_{BD} where circles C_5 and C_6 cut trochoid T_1 are one of the ends of the coupler link BC. In order to find the other end draw a line L_3 passing through P_{BD} and P_2; this line L_3 will cut circle C_5 in two places. The point P_{CD} on line L_3 which is at a length r_4 from point P_{BD} is the other end of the coupler link BC.
4) To obtain the position of the two input links MA and AB, draw circle C_7 with radius r_3 of the input link AB about P_{BD}; circle C_7 will cut circle C_1 at two points (or at one point if

tangent to C_1). One of these two points P_{A1} or P_{A2} will give the position of the input link MA. To determine the appropriate position, first assume P_{A1} is the solution, and try to rotate input link MA in the direction of ω_2 (clockwise or counterclockwise, depending on which one was selected for the mechanism);

a. If the coupler link BC could move, then the other point P_{A2} is the position of the input link.

b. If the coupler link could not move, that is, the mechanism remains in locked position then point P_{A1} is the initial position of the input link.

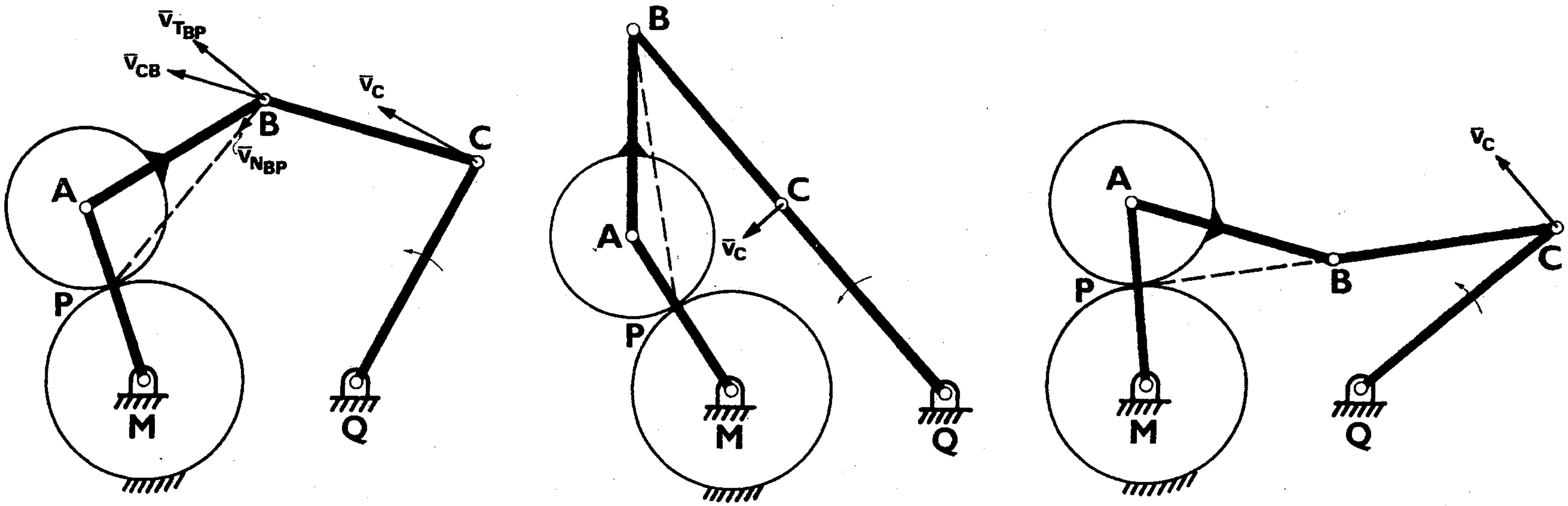
5) Repeat steps (3) and (4) until all input positions are found. This input position corresponds to dead center positions of the output link.

Figure 5b shows in the same example as Section 4.3 one of the dead center positions using the procedure just outlined.

4.6 Study of the Inverse Mechanism

In this section the motion of the geared five-link mechanism is studied for the case when θ_5 is the input displacement, θ_2 is the output displacement and ω_5 is the input angular velocity.

Consider the vectors shown in Figure 6a where $\bar{V}_C$ is the vector corresponding to ω_5 (input angular velocity) crossed with link length r_5 (input link CQ), $\bar{V}_{CB}$ is the projection of $\bar{V}_C$ on coupler link BC, $\bar{V}n_{BP}$ and $\bar{V}t_{BP}$ are the normal and tangential components of velocity vector $\bar{V}_{CB}$ on line $\overline{BP}$ at point B.



(a) (b) (c)

Figure 6. Velocity Vectors at Revolute Pair C when ω_5 is the Input Angular Velocity; (a) General Configuration; (b) Limit Position; (c) Dead Center Position

A limit or pseudo-limit position occurs when $\bar{V}_{CB}$ is zero, that is, $\bar{V}_C$ is perpendicular to link BC; this occurs when the input link CQ is collinear with coupler link BC see Figure 6b. If the direction of $\bar{V}_{CB}$ after a small increment of ω_5 is opposite to prior to $\bar{V}_{CB} = 0$, then the mechanism at $\bar{V}_{CB} = 0$ is at a limit position; otherwise, it is at a pseudo-limit position.

A dead center position may only occur if the vector $\bar{V}_C$ becomes zero. This happens when the geometric configuration of the links force the input link CQ to become stationary; that is, line PB must be collinear with coupler link BC see Figure 6c. If, for an increment of ω_5 the mechanism remains locked, then it is at a dead center position. Let

MECH 1, when θ_2 = input, and θ_5 = output, and

MECH 2, when θ_5 = input, and θ_2 = output,

then, it can be concluded that,

1) When MECH 1 is at a limit position, MECH 2 is at a dead center position.
2) When MECH 1 is at a dead center position, MECH 2 is at a limit position.
3) When MECH 1 is at a pseudo-limit position, MECH 2 is at a dead center position.
4) When Mech 2 is at a pseudo-limit position Mech 1 is at a dead center position.

In other words the analysis of the limit, pseudo-limit and dead center positions is independent of which link MA or CQ is considered as input.

CHAPTER V

INSTANTANEOUS CENTERS OF VELOCITY IN THE ANALYSIS OF THE LIMIT AND DEAD CENTER POSITIONS

Once some specific geometric configurations of the limit and dead center positions are available, the next logical step is to analyze the instantaneous centers of velocity (represented as IC all throughout this chapter); with the objectives to understand other additional geometrical properties of limit and dead center positions.

5.1 General Notions of Instantaneous Centers of Velocity

Instantaneous centers of velocity as defined previously are a pair of coincident points having zero relative velocity [1].

In the study of the IC's in this chapter the concepts of Kennedy's theorem will be used. The theorem reads: "The instantaneous centers of velocity of any three rigid bodies having planar motion lie on the same straight line."

5.2 Instantaneous Centers of Velocity of a Geared Five-Link Mechanism

The number of IC of a mechanism can be determined by,

$$N = \frac{M(M-1)}{2} \tag{5.2.1}$$

where, N = number of IC, and M = number of links of the mechanism. The use of equation (5.2.1), with M = 5 (five-link), gives ten IC. These are:

$$
\begin{array}{cccc}
IC_{12} & IC_{13} & IC_{14} & IC_{15} \\
 & IC_{23} & IC_{24} & IC_{25} \\
 & & IC_{34} & IC_{35} \\
 & & & IC_{45}
\end{array}
$$

Figure 7 presents all the IC, and for ease of understanding the IC of the frame of the mechanism, and the IC corresponding to the point of contact of the gears will be named fixed IC. The other ones will be named floating IC. Thus, IC_{12}, IC_{23}, IC_{34}, IC_{45}, IC_{15}, and IC_{13} are fixed IC, and IC_{14}, IC_{24}, IC_{25} and IC_{35} are floating IC.

The fixed IC are defined geometrically as follows, (see Figure 7):

$$IC_{23}: \quad X_{23} = r_2 \cos\theta_2 \qquad (5.2.2)$$

$$Y_{23} = r_2 \sin\theta_2 \qquad (5.2.3)$$

$$IC_{45}: \quad X_{45} = r_1 + r_5 \cos\theta_5 \qquad (5.2.4)$$

$$Y_{45} = r_5 \sin\theta_5 \qquad (5.2.5)$$

$$IC_{12}: \quad X_{12} = 0. \qquad (5.2.6)$$

$$Y_{12} = 0. \qquad (5.2.7)$$

$$IC_{15}: \quad X_{15} = r_1 \qquad (5.2.8)$$

$$Y_{15} = 0. \qquad (5.2.9)$$

$$IC_{34}: \quad X_{34} = X_{23} + r_3 \cos\theta_3 \qquad (5.2.10)$$

$$Y_{34} = Y_{23} + r_3 \sin\theta_3 \qquad (5.2.11)$$

$$IC_{13}: \quad X_{13} = R_1 \cos\theta_2 \qquad (5.2.12)$$

$$Y_{13} = R_1 \sin\theta_2 \qquad (5.2.13)$$

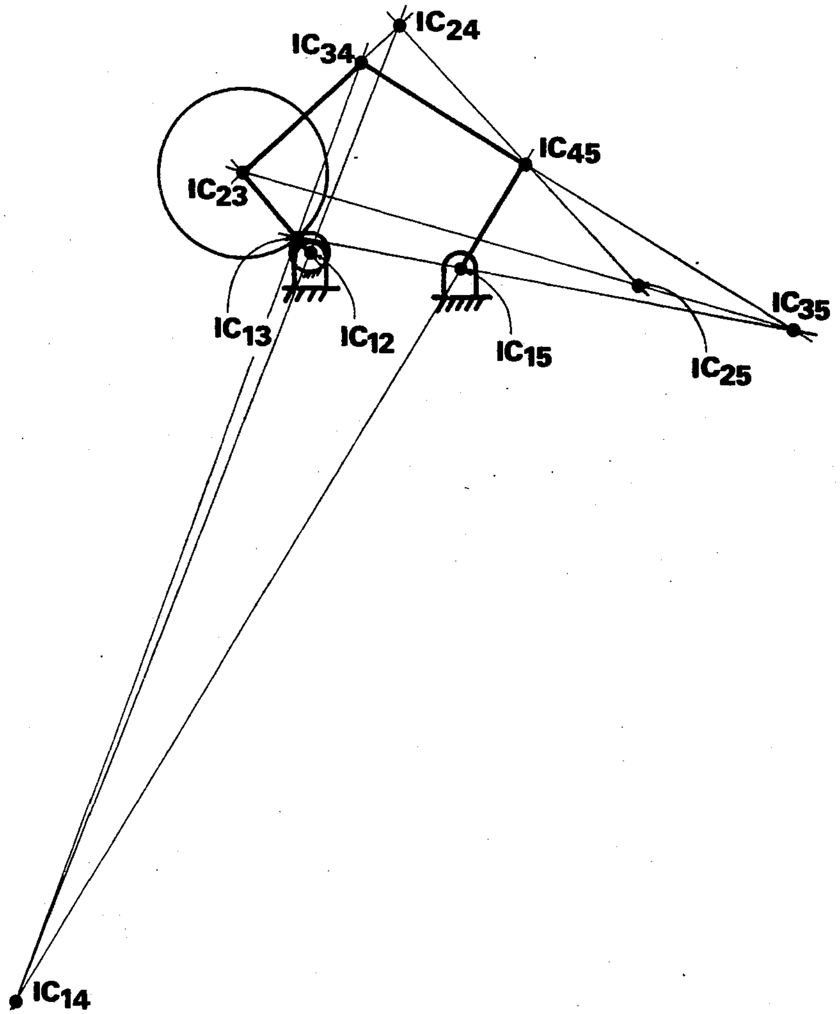


Figure 7. Instantaneous Centers of Velocity of a Geared Five-Link Mechanism

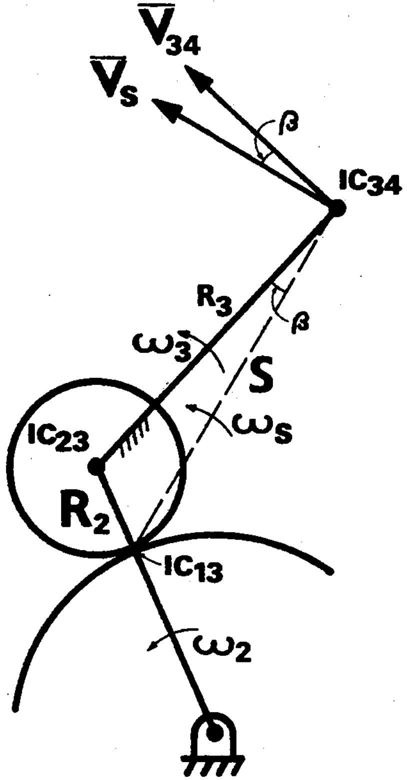


Figure 8. Velocity Vectors at IC_{34}

and the floating IC are defined in Appendix B.

Now that all the IC have been defined, the next step is to find the angular velocity of each link.

Using Kennedy's theorem and simple geometry , the relative velocity vector of any two links can be found [1].

The angular velocities are:

1) Ground link MQ: $\omega_1 = 0$ (5.2.14)

2) Input link MA: ω_2 = input velocity (5.2.15)

3) Input link AB: $\omega_3 = (1 + GR)\omega_2$ (5.2.16)

(for a derivation see Section 3.1)

4) Coupler link BC:

$$\omega_4 = \omega_s \frac{(IC_{13} -- IC_{34})}{(IC_{14} -- IC_{34})} \tag{5.2.17}$$

Where ω_s is found from the following derivation in Figure 8, r_3 and R_2 are known, and S is the distance from IC_{13} to IC_{34}. Therefore,

$$\cos\beta = \frac{r_3^2 + S^2 - R_2^2}{2r_3 S} , \tag{5.2.18}$$

$$\bar{V}_{34} = \omega_3 r_3 \tag{5.2.19}$$

and,

$$\bar{V}s = \frac{\bar{V}_{34}}{\cos\beta} \tag{5.2.20}$$

so;

$$\omega s = \frac{\bar{V}s}{S} \tag{5.2.21}$$

5) Output link CQ:

$$\omega_5 = \omega_4 \frac{(IC_{14} -- IC_{45})}{(IC_{15} -- IC_{45})} \quad . \tag{5.2.22}$$

ω_4 and ω_5 may also be obtained by taking other IC in consideration, but the procedure above is the simplest.

5.3 Limit Positions

Since the limit and the pseudo-limit positions occur when the output link has zero velocity, and since ω_2, ω_3 and ω_4 are in motion while the limit or pseudo-limit position occurs. The only way ω_5 in equation (5.2.22) can become zero is if the distance IC_{14} to IC_{45} becomes zero. See Figure 9a where the limit position presented in Figure 4b is redrawn. It is interesting to observe that as a result of Kennedy's theorem, the IC_{13} coincide with IC_{35}, and IC_{12} coincide with IC_{25}.

The following theorem can be stated:

<u>Theorem #4</u>: A limit or pseudo-limit position of a geared five-link mechanism exists if the instantaneous center of of velocity corresponding to the input and output links coincides with the instantaneous center of velocity coincident with the output moving pivot.

5.4 Dead Center Positions

A dead center position will occur when input link MA reaches an extrema (ω_2 becomes zero). That is, the mechanism becomes locked. This can be explained as follows; a dead center position occurs only

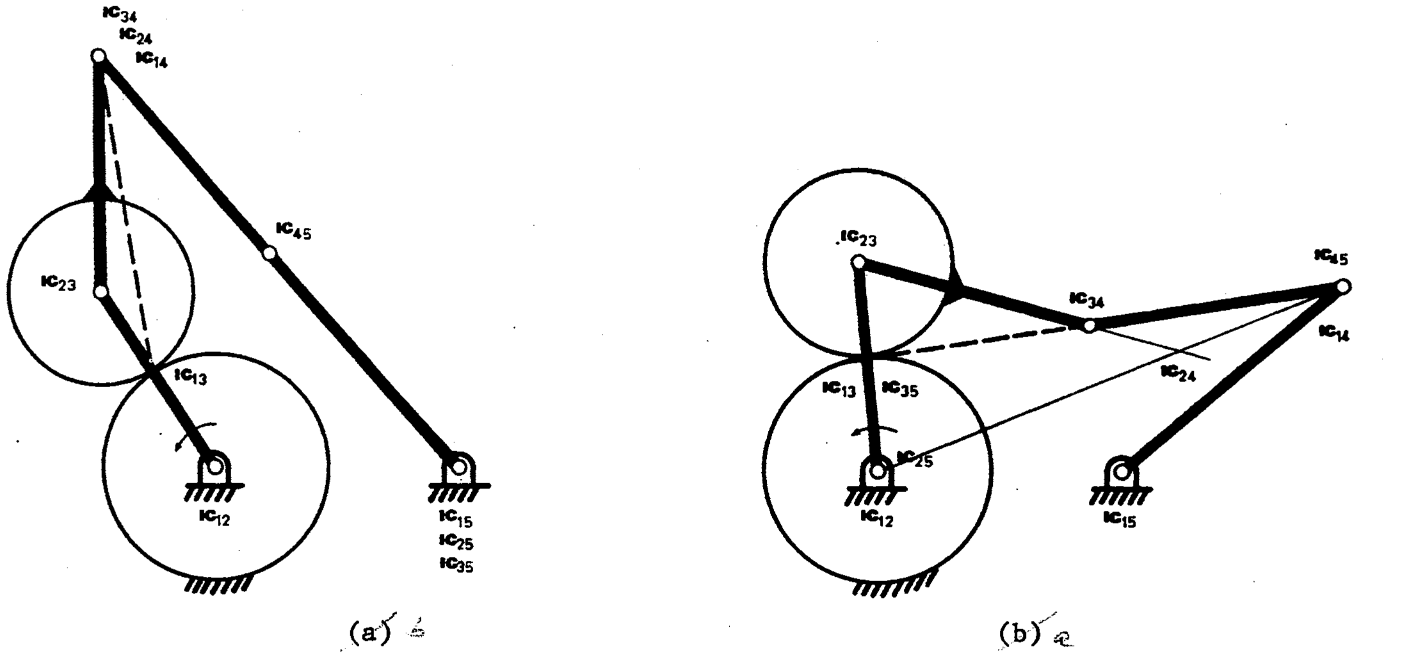


Figure 9. Position of the Instantaneous Centers of Velocity; (a) Limit Position; (b) Dead Center Position

when ω_4 in equation (5.2.22) is zero, therefore, ω_5 is zero. For ω_4 to become zero in equation (5.2.17) the IC_{14} must coincide with the IC_{34} at a dead center position. Thus, ω_4 would be ∞, but this condition can only occur when ω_3 is zero (that is, $\omega_S = 0$), therefore ω_4 is zero. Since ω_3 is zero, this implies that ω_2 is zero from equation (5.2.16).

It is interesting to observe that as a result of Kennedy's theorem, IC_{24} is also coincident with IC_{14} and IC_{34}; this in turn, will make the IC_{25} and IC_{35} coincide with IC_{15}. See Figure 9b where the dead center position presented in Figure 4c is redrawn.

The following theorem can now be stated:

Theorem #5. A dead center position of a geared five-link mechanism exists if the instantaneous centers of velocity corresponding to the input and output links are coincident with the instantaneous center of velocity of the input's moving pivot.

In other words when IC_{14} coincides with IC_{34}, a dead center position has been achieved (IC_{14} can only be coincident with IC_{24} at IC_{34}, because of the geometry involved in the construction of the IC).

CHAPTER VI

SUMMARY AND CONCLUSIONS

In this thesis a generalized approach is developed to find the limit and dead center positions of a geared five-link mechanism. This approach is applied to study the existence of limit and dead center positions of a geared five-link mechanism with a gear train speed ratio of two.

The analytical study show that the first and second derivative relationships give the necessary and sufficient conditions for the existance of the limit and dead center positions. It was found that the degree of the polynomials resulting from the expansion of Sylvester's dyalitic eliminant technique to determine the extremas is function of the gear train speed ratio of the mechanism.

The graphical approach is proposed to study to specific geometric configurations at the limit and dead center positions.

The motion of the instantaneous centers of velocity of geared five-link mechanisms show that they (IC) position themselves in a specific predictable manner at the extermas.

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APPENDIX A

DERIVATION OF THE COEFFICIENTS OF THE POLYNOMIALS

In the following sections the coefficients of the polynomials that provide the necessary conditions for the limit and dead center positions of a geared five-link mechanism with gear train speed ratio = 2 are derived.

A.1 Expansion of the Sylvester's Dyalitic Eliminants

The expansion of the Sylvester's dyalitic eliminants (3.2.4) and (3.3.4) give respectively for limit positions;

$$K_1^2 K_6^2 + K_1 K_3 K_5^2 - 2K_1 K_3 K_4 K_6 - K_1 K_2 K_5 K_6 + K_2^2 K_4 K_6 + K_3^2 K_4^2 - K_2 K_3 K_4 K_5 = 0 \tag{A.1.1}$$

for dead center positions,

$$K_1^2 K_9^2 + K_1 K_3 K_8^2 - 2K_1 K_3 K_7 K_9 - K_1 K_2 K_8 K_9 + K_2^2 K_7 K_9 + K_3^2 K_7^2 - K_2 K_3 K_7 K_8 = 0 \tag{A.1.2}$$

where the Kj, j = 1,...,9, are defined as follows:

A.2 Definition of the K_j

In order to simplify the data supplied for the computer K(j) = K_j, j = 1,..,9. Permute the following equation,

$$K(J) = S(I)t^4 + S(I + 1)t^3 + S(I + 2)t^2 +$$

$$S(I + 3)t + S(I + 4) \qquad (A.2.1)$$

with,

J = 1,2,3,4,5,6,7,8,9

I = 1,6,11,16,21,26,31,36,41

where the S(I), I = 1,..,45 as defined as follows:

A.3 Definition of the S(I)

The S(I)'s are:

S(1) = P(1) - P(2) + P(6)

S(2) = 2[P(4) - P(5)]

S(3) = 2[2P(3) - P(1) + P(6)]

S(4) = 2[P(4) + P(5)]

S(5) = P(1) + P(2) + P(6)

S(6) = P(7)

S(7) = 2[P(9) - P(10)]

S(8) = 2[2P(8) - P(7)]

S(9) = 2[P(9) + (10)]

S(10) = P(7)

S(11) = P(11) - P(12) + P(16)

S(12) = 2[P(14) - P(15)]

$$S(13) = 2[2P(13) - P(11) + P(16)]$$

$$S(14) = 2[P(14) + P(15)]$$

$$S(15) = P(11) + P(12) + P(16)$$

$$S(16) = P(17) - P(18)$$

$$S(17) = 2[P(20) - P(21)]$$

$$S(18) = 2[2P(19) - P(17)]$$

$$S(19) = 2[P(20) + P(21)]$$

$$S(20) = P(17) + P(18)$$

$$S(21) = P(22) - P(23)$$

$$S(22) = -2P(25)$$

$$S(23) = 2[2P(24) - P(22)]$$

$$S(24) = 2P(25)$$

$$S(25) = P(22) + P(23)$$

$$S(26) = P(26) - P(27)$$

$$S(27) = 2[P(29) - P(30)]$$

$$S(28) = 2[2P(28) - P(26)]$$

$$S(29) = 2[P(29) + P(30)]$$

$$S(30) = P(26) + P(27)$$

$$S(31) = P(31)$$

$$S(32) = 2[P(33) - P(34)]$$

$$S(33) = 2[2P(32) - P(31)]$$

$$S(34) = 2[P(33) + P(34)]$$

$$S(35) = P(31)$$

$$S(36) = P(35) - P(36) + P(39)$$

$$S(37) = -2P(38)$$

$$S(38) = 2[2P(37) - P(35) + P(39)]$$

$$S(39) = 2P(38)$$

S(40) = P(35) + P(36) + P(39)

S(41) = -S(31)

S(42) = -S(32)

S(43) = -S(33)

S(44) = -S(34)

S(45) = -S(35)

Where the P(K), K = 1,...,39 are defined as follows:

A.4 Definition of the P(K)

The P(K)'s are:

$P(1) = 2r_3(r_5 - r_1)\cos\alpha$

$P(2) = 2r_2(r_3 \cos \alpha - r_1 + r_5)$

$P(3) = 2r_3(r_1 - r_5)\cos\alpha$

$P(4) = -2r_2 r_3 \sin\alpha$

$P(5) = 4r_3(r_1 - r_5)\sin\alpha$

$P(6) = r_1^2 + r_2^2 + r_3^2 - r_4^2 + r_5^2 - 2r_1 r_5$

$P(7) = -4r_3 r_5 \sin\alpha$

$P(8) = -P(7)$

$P(9) = -4r_2 r_5$

$P(10) = -8r_3 r_5 \cos\alpha$

$P(11) = -2r_3(r_1 + r_5)\cos\alpha$

$P(12) = 2r_2(r_3 \cos \alpha - r_1 - r_5)$

$P(13) = 2r_3(r_1 + r_5)\cos\alpha$

$P(14) = -2r_2 r_3 \sin\alpha$

$P(15) = 4r_3(r_1 + r_5)\sin\alpha$

$P(16) = r_1^2 + r_2^2 + r_3^2 - r_4^2 + r_5^2 + 2r_1 r_5$

$P(17) = 4r_3(r_1 - r_5)\sin\alpha$

$P(18) = -2r_2r_3\sin\alpha$

$P(19) = 4r_3(r_5 - r_1)\sin\alpha$

$P(20) = 2r_2(r_1 - r_3\cos\alpha - r_5)$

$P(21) = 8r_3(r_1 - r_5)\cos\alpha$

$P(22) = -8r_3r_5\cos\alpha$

$P(23) = -4r_2r_5$

$P(24) = 8r_3r_5\cos\alpha$

$P(25) = 16r_3r_5\sin\alpha$

$P(26) = 4r_3(r_1 + r_5)\sin\alpha$

$P(27) = -2r_2r_3\sin\alpha$

$P(28) = -4r_3(r_1 + r_5)\sin\alpha$

$P(29) = 2r_2(r_1 - r_3\cos\alpha + r_5)$

$P(30) = 8r_3(r_1 + r_5)\cos\alpha$

$P(31) = 2r_3r_5\sin\alpha$

$P(32) = -P(31)$

$P(33) = 2r_2r_5$

$P(34) = 4r_3r_5\cos\alpha$

$P(35) = P(34)$

$P(36) = 2P(33)$

$P(37) = -P(34)$

$P(38) = -4P(31)$

$P(39) = -4r_1r_5$

Where r_1, r_2, r_3, r_4, r_5, and α are the initial conditions (link lengths and initial position of input link AB).

A.5 Definition of the K(L)K(M)

Permute the following equation:

$$K(L)K(M) = Q(K)t^8 + Q(K+1)t^7 + Q(K+2)t^6 + Q(K+3)t^5 + Q(K+4)t^4 + Q(K+5)t^3 + Q(K+6)t^2 + Q(K+7)t + Q(K+8) \tag{A.5.1}$$

with,

$L = 1,2,3,4,5,6,7,8,9,1,1,2,4,4,5,7,7,8$

$M = 1,2,3,4,5,6,7,8,9,2,3,3,5,6,6,8,9,9$

$I = 1,6,11,16,21,26,31,36,41,1,1,6,16,16,21,31,31,36$

$J = 1,6,11,16,21,26,31,36,41,6,11,11,21,26,26,36,41,41$

$K = 1,10,19,28,37,46,55,64,73,82,91,100,109,118,127,136,145,154$

where Q(K)'s are:

$$Q(K) = S(I)S(J)$$

$$Q(K+1) = S(I)S(J+1) + S(I+1)S(J)$$

$$Q(K+2) = S(I)S(J+2) + S(I+1)S(J+1) + S(I+2)S(J)$$

$$Q(K+3) = S(I)S(J+3) + S(I+1)S(J+2) + S(I+2)S(J+1) + S(I+3)S(J)$$

$$Q(K+4) = S(I)S(J+4) + S(I+1)S(J+3) + S(I+2)S(J+2) + S(I+3)S(J+1) + S(I+4)S(J)$$

$$Q(K+5) = S(I+1)S(J+4) + S(I+2)S(J+3) + S(I+3)S(J+2) + S(I+4)S(J+1)$$

$$Q(K+6) = S(I+2)S(J+4) + S(I+3)S(J+3) + S(I+4)S(J+2)$$

$$Q(K + 7) = S(I + 3)S(J + 4) + S(I + 4)S(J + 3)$$

$$Q(K + 8) = S(I + 4)S(J + 4)$$

where the S(I), I = 1,..,45, and the S(J), J = 1,..,45, are defined in Section A.3.

A.6 Definition of the K(L)K(M)K(N)K(Y)

Permute the following equation:

$$\begin{aligned} K(L)K(M)K(N)K(Y) = {} & T(K)t^{16} + T(K + 1)t^{15} + T(K + 2)t^{14} + \\ & T(K + 3)t^{13} + T(K + 4)t^{12} + T(K + 5)t^{11} + \\ & T(K + 6)t^{10} + T(K + 7)t^{9} + T(K + 8)t^{8} + \\ & T(K + 9)t^{7} + T(K + 10)t^{6} + T(K + 11)t^{5} + \\ & T(K + 12)t^{4} + T(K + 13)t^{3} + T(K + 14)t^{2} + \\ & T(K + 15)t + T(K + 16) \end{aligned} \tag{A.6.1}$$

with,

L = 1,1,1,1,2,3,2,1,1,1,1,2,3,2

M = 1,3,3,2,2,3,3,1,3,3,2,2,3,3

N = 6,5,4,5,4,4,4,9,8,7,8,7,7,7

Y = 6,5,6,6,6,4,5,9,8,9,9,9,7,8

I = 1,91,91,82,10,19,100,1,91,91,82,10,19,100

J = 46,37,118,127,118,28,109,73,64,145,154,145,55,136

K = 1,18,35,52,69,86,103,120,137,154,171,188,205,222

where the T(K)'s are:

$$T(K) = Q(I)Q(J)$$

$$T(K + 1) = Q(I)Q(J + 1) + Q(I + 1)Q(J)$$

$$T(K + 2) = Q(I)Q(J + 2) + Q(I + 1)Q(J + 1) + Q(I + 2)Q(J)$$

$$T(K + 3) = Q(I)Q(J + 3) + Q(I + 1)Q(J + 2) + Q(I + 2)Q(J + 1) + Q(I + 3)Q(J)$$

$$T(K + 4) = Q(I)Q(J + 4) + Q(I + 1)Q(J + 3) + Q(I + 2)Q(J + 2) + Q(I + 3)Q(J + 1) + Q(I + 4)Q(J)$$

$$T(K + 5) = Q(I)Q(J + 5) + Q(I + 1)Q(J + 4) + Q(I + 2)Q(J + 3) + Q(I + 3)Q(J + 2) + Q(I + 4)Q(J + 1) + Q(I + 5)Q(J)$$

$$T(K + 6) = Q(I)Q(J + 6) + Q(I + 1)Q(J + 5) + Q(I + 2)Q(J + 4) + Q(I + 3)Q(J + 3) + Q(I + 4)Q(J + 2) + Q(I + 5)Q(J + 1) + Q(I + 6)Q(J)$$

$$T(K + 7) = Q(I)Q(J + 7) + Q(I + 1)Q(J + 6) + Q(I + 2)Q(J + 5) + Q(I + 3)Q(J + 4) + Q(I + 4)Q(J + 3) + Q(I + 5)Q(J + 2) + Q(I + 6)Q(J + 1) + Q(I + 7)Q(J)$$

$$T(K + 8) = Q(I)Q(J + 8) + Q(I + 1)Q(J + 7) + Q(I + 2)Q(J + 6) + Q(I + 3)Q(J + 5) + Q(I + 4)Q(J + 4) + Q(I + 5)Q(J + 3) + Q(I + 6)Q(J + 2) + Q(I + 7)Q(J + 1) + Q(I + 8)Q(J)$$

$$T(K + 9) = Q(I + 1)Q(J + 8) + Q(I + 2)Q(J + 7) + Q(I + 3)Q(J + 6) + Q(I + 4)Q(J + 5) + Q(I + 5)Q(J + 4) + Q(I + 6)Q(J + 3) + Q(I + 7)Q(J + 2) + Q(I + 8)Q(J + 1)$$

$$T(K + 10) = Q(I + 2)Q(J + 8) + Q(I + 3)Q(J + 7) + Q(I + 4)Q(J + 6) + Q(I + 5)Q(J + 5) + Q(I + 6)Q(J + 4) + Q(I + 7)Q(J + 3) + Q(I + 8)Q(J + 2)$$

$$T(K + 11) = Q(I + 3)Q(J + 8) + Q(I + 4)Q(J + 7) + Q(I + 5)$$
$$Q(J + 6) + Q(I + 6)Q(J + 5) + Q(I + 7)Q(J + 4) +$$
$$Q(I + 8)Q(J + 3)$$

$$T(K + 12) = Q(I + 4)Q(J + 8) + Q(I + 5)Q(J + 7) + Q(I + 6)$$
$$Q(J + 6) + Q(I + 7)Q(J + 5) + Q(I + 8)Q(J + 4)$$

$$T(K + 13) = Q(I + 5)Q(J + 8) + Q(I + 6)Q(J + 7) + Q(I + 7)$$
$$Q(J + 6) + Q(I + 8)Q(J + 5)$$

$$T(K + 14) = Q(I + 6)Q(J + 8) + Q(I + 7)Q(J + 7) + Q(I + 8)$$
$$Q(J + 6)$$

$$T(K + 15) = Q(I + 7)Q(J + 8) + Q(I + 8)Q(J + 7)$$

$$T(K + 16) = Q(I + 8)Q(J + 8)$$

The Q(I)'s and Q(J)'s are defined in Section A.5.

A.7 Definition of the coefficients V(j) and W(j)

These are the coefficients of the polynomials (3.4.5) and (3.4.7). Permute the following equation:

$$V(18 - I) = T(I) + T(I + 17) - 2T(I + 34) - T(I + 51) + T(I + 68) + T(I + 85) - T(I + 102) \quad (A.7.1)$$

$$W(18 - I) = T(I + 119) + T(I + 136) - 2T(I + 153) - T(I + 170) + T(I + 187) + T(I + 204) - T(I + 221) \quad (A.7.2)$$

with I = 1,2,3,....17, where the T(I)'s are defined in Section A.6.

APPENDIX B

GEOMETRICAL DEFINITION OF THE FLOATING INSTANTANEOUS CENTERS OF VELOCITY OF A GEARED FIVE-LINK MECHANISM

A floating IC is the intersection point of two lines with equation of this form:

$$Y = mX + b \tag{B.1.1}$$

where, m is the slope of the line and b is the x-intercept.

The point of intersection of two lines is at,

$$X = \frac{b_2 - b_1}{m_1 - m_2} \tag{B.1.2}$$

$$Y = m_1 X + b_1 \tag{B.1.3}$$

where,

$$m_1 = \frac{A_1y - A_2y}{A_1x - A_2x}$$

$$m_2 = \frac{B_1y - B_2y}{B_1x - B_2x}$$

$$b_1 = A_1y - m_1A_1x$$

$$b_2 = B_1y - m_2B_1x$$

and, $(A_1x,\ A_1y)$, $(A_2x,\ A_2y)$, $(B_1x,\ B_1y)$, and $(B_2x,\ B_2y)$ are the XY coordinates of points A_1, A_2, B_1, and B_2 respectively. Therefore, to find the floating IC, permute with equations (B.1.2) and (B.1.3), the following:

Floating IC	A_1	A_2	B_1	B_2
IC_{14}	IC_{34}	IC_{13}	IC_{45}	IC_{15}
IC_{24}	IC_{12}	IC_{14}	IC_{34}	IC_{23}
IC_{35}	IC_{34}	IC_{45}	IC_{13}	IC_{15}
IC_{25}	IC_{24}	IC_{45}	IC_{23}	IC_{35}

where, IC_{12}, IC_{13}, IC_{23}, IC_{34}, IC_{45} and IC_{15} are fixed IC, and were defined in Section 5.2.

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