SIZE AND LOCATION OF CAPACITORS ON FEEDER CIRCUITS TO OPTIMIZE LOSS REDUCTION AND VOLTAGE IMPROVEMENT

By

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SIZE AND LOCATION OF CAPACITORS ON FEEDER

CIRCUITS TO OPTIMIZE LOSS REDUCTION

AND VOLTAGE IMPROVEMENT

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LIST OF SYMBOLS

P - Power loss in line section

$|I|$ - Magnitude of current

R - Total line feeder resistance in ohms

I_R - In-phase current

I_I - Quadrature current

L - Energy loss caused by quadrature current in feeder

D_n - Ratio between input quadrature current during time T_n and maximum input quadrature current

T_n - Time, in per unit of period of load cycle, during which the input quadrature current equals D_n.

$L.F.$ - Load factor, or average load divided by the peak load

$S.F.$ - Loss factor, or ratio of the average loss to the peak loss

a - Location of the first capacitor in per unit of total feeder length

I_c - Rated capacitor current in p.u. of maximum input quadrature current to feeder

ΔL - Change in energy loss in p.u. which is the difference before and after the Capacitor application divided by the loss before capacitor application

K - Total capacitive current divided by the quadrature current

V_S - Sending End Voltage

V_R - Receiving End Voltage

ℓ - Total length of the feeder

$\Delta V_{p.u.}$ - Voltage improvement in per unit on the line, which is the difference between the voltage drops before and after capacitor application, divided by the voltage drop before capacitor application

P.I. - Performance Index

α - Weighting constant

b - Location of the second capacitor in per unit or in feet

K_1 - The capacitive current of first capacitor divided by the quadrature current

K_2 - Capacitive current of the second capacitor divided by the quadrature current

h - In phase current divided by the quadrature current

p - Line resistance divided by the line inductance of the feeder

V - Total energy demand for the time period

u - Sum of the squared power increments times the time increment

w - Sum of the time increments and equals the total time period

K_T - Addition of ratios of capacitive to quadrature current of first and second capacitor

X - Inductance of the line

Z - Impedance of the line

$Z_{p.u.}$ - Magnitude of $|IZ|$ divided by the magnitude of $|V_R|$

CHAPTER I

INTRODUCTION

a. Description Of A Distrubution Primary Feeder Circuit

Primary feeder circuits, emanating from the distributive substations, carry the power throughout the congested areas. Power is ultimately delivered to the consumer through the use of distribution transformers which are located along the primary feeder circuits, and reduce the voltage from a primary feeder voltage level to the utilization level.

Primary feeders generally are radial circuits either single or three phase with loads distributed throughout their lengths. Common voltages at which primary feeders are operated range from 2400 to 20,000 volts line to ground, with a predominant value of 7200 volts.

The power factors of the loads distributed along a primary feeder range in value from 70% to 90% and nearly always are inductive. A typical overall power factor for the total load on a primary feeder is 80%.

b. Why Shunt Capacitors Are Used On Feeder Circuits

The flow of real power on a distribution system is accompanied by a flow of reactive power required for various types of loads, such as motors and flourescent lighting. As a result the power factor of distribution circuits may be relatively low, 70% to 80% or even lower.

Flow of reactive power on a distribution system loads up the cables, transformers and overhead wires, and causes the following disadvantages:

(1) Reactive power reduces the ability of the distribution system to carry larger amounts of active power. For a power factor of 70%, 42% of the useful capacity of the distribution systems handle the reactive current.

(2) For a given active power being supplied, the copper losses in the system is increased by the flow of reactive power, e.g. for a power factor of 70% the system copper losses is twice the loss for a power factor of 100%.

(3) Reactive power causes a larger voltage drop in the distribution system, especially on overhead lines. Therefore, in those cases where the capacity of the circuit is limited by voltage drop, the flow of reactive power reduces the amount of load which can be carried on the circuit.

Instead of supplying reactive power from the generators and transmitting it to the load through the distribution system, it is possible to obtain the necessary reactive power by using capacitors. They connect directly to the distribution circuits close to the load. Since capacitors operate at a leading power factor, neutralize the lagging reactive current required by the inductive loads on the system. Capacitors can be considered as generators of wattless current to supply the reactive power required by the system load. The use of capacitors on distribution systems in large numbers is being used for the last fifteen years. The cost of capacitors has been reduced materially in recent years, while the cost of other type of power system equipment has been increasing with the result that their use has been economical. Studies show it is economical to use capacitors in amounts of 20% to 30% of the total load on the distribution system.

Figure (1) shows a typical case of shunt capacitor in a distribution circuit. The currents in the system before and after installing the capacitors are shown. Addition of the capacitor greatly reduces the amount of current carried in the circuit and in this manner makes possible the supply of a larger kilowatts by the same circuit.

In addition to releiving the effective load burden from the equipment, the capacitors also reduce the energy loss in the power system caused by inductive load currents.

c. Shunt Capacitors For Voltage Control

Shunt capacitors are commonly applied on primary feeders for voltage control to provide a feeder voltage within prescribed maximum and minimum allowable values at light load and peak load conditions.

A primary feeder consisting of two shunt capacitors is shown in figure (2). Generally capacitors riase the voltage level of the feeder by an amount roughly equal to capacitor current multiplied by the line reactance from the regulated voltage source to the point of capacitor location.

Figures (3) and (4) illustrate that whether one large bank or two small banks (equal to one large bank) are used, the reduction in voltage spread remains the same. This is true for the condition where the capacitors are located for maximum loss reduction on the feeder with evenly distributed loads. Thus nothing is lost from the voltage standpoint by installing one big bank on the primary feeder rather than two small ones.

The most important voltage benefit of capacitors on a primary feeder

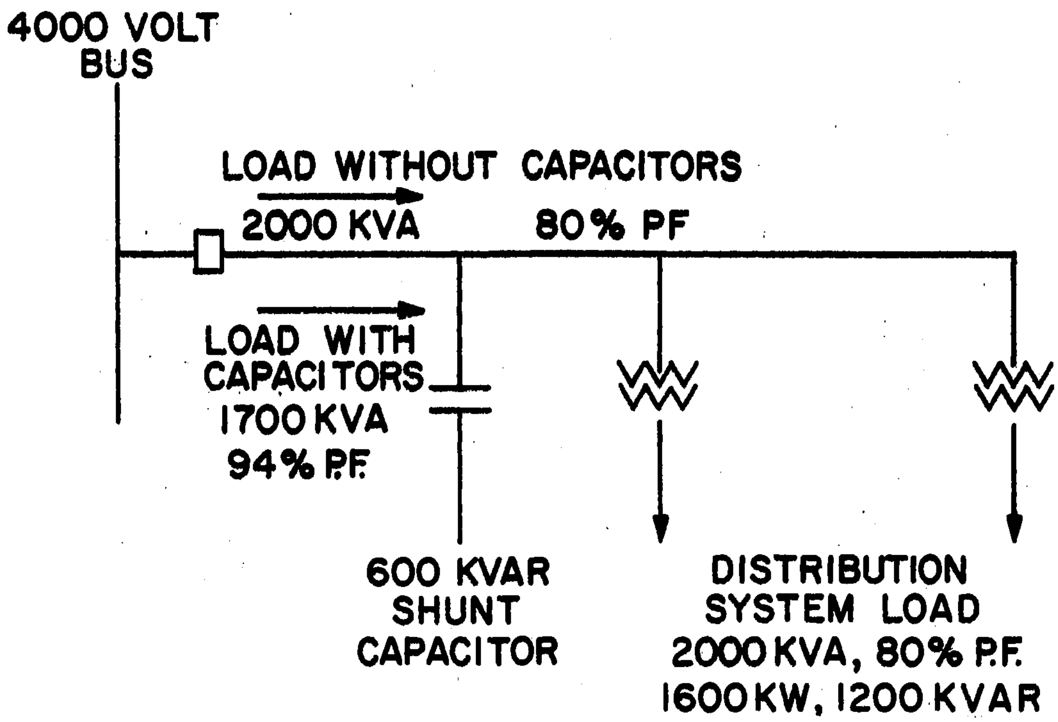


Figure 1. Distribution Feeder with Shunt Capacitors

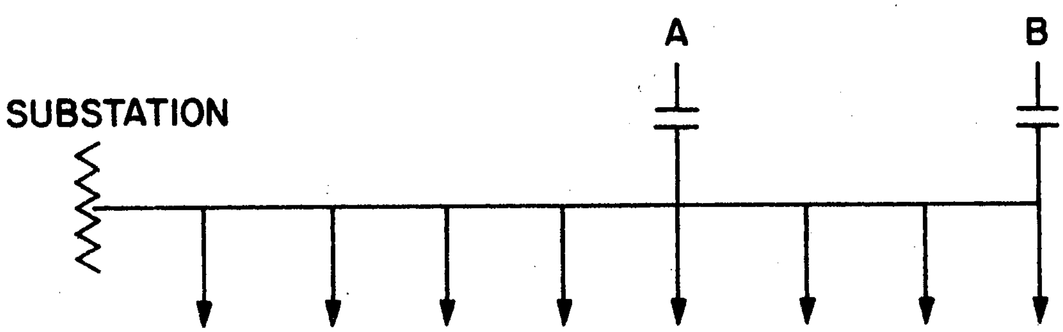


Figure 2. Uniformly distributed load with two capacitors

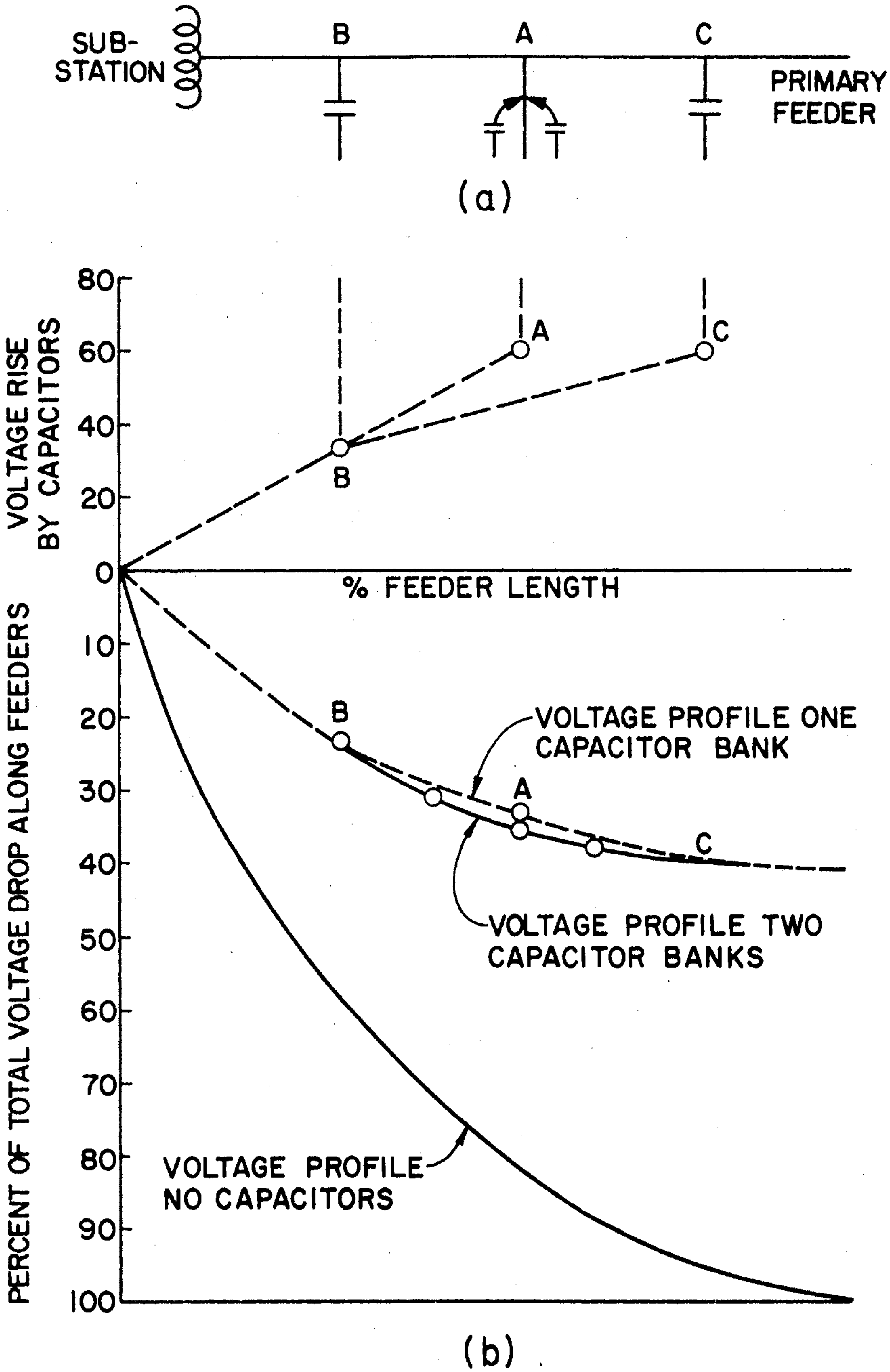


Figure 3. Voltage spread of two half size capacitor banks located at B and C or one capacitor at A on the feeder

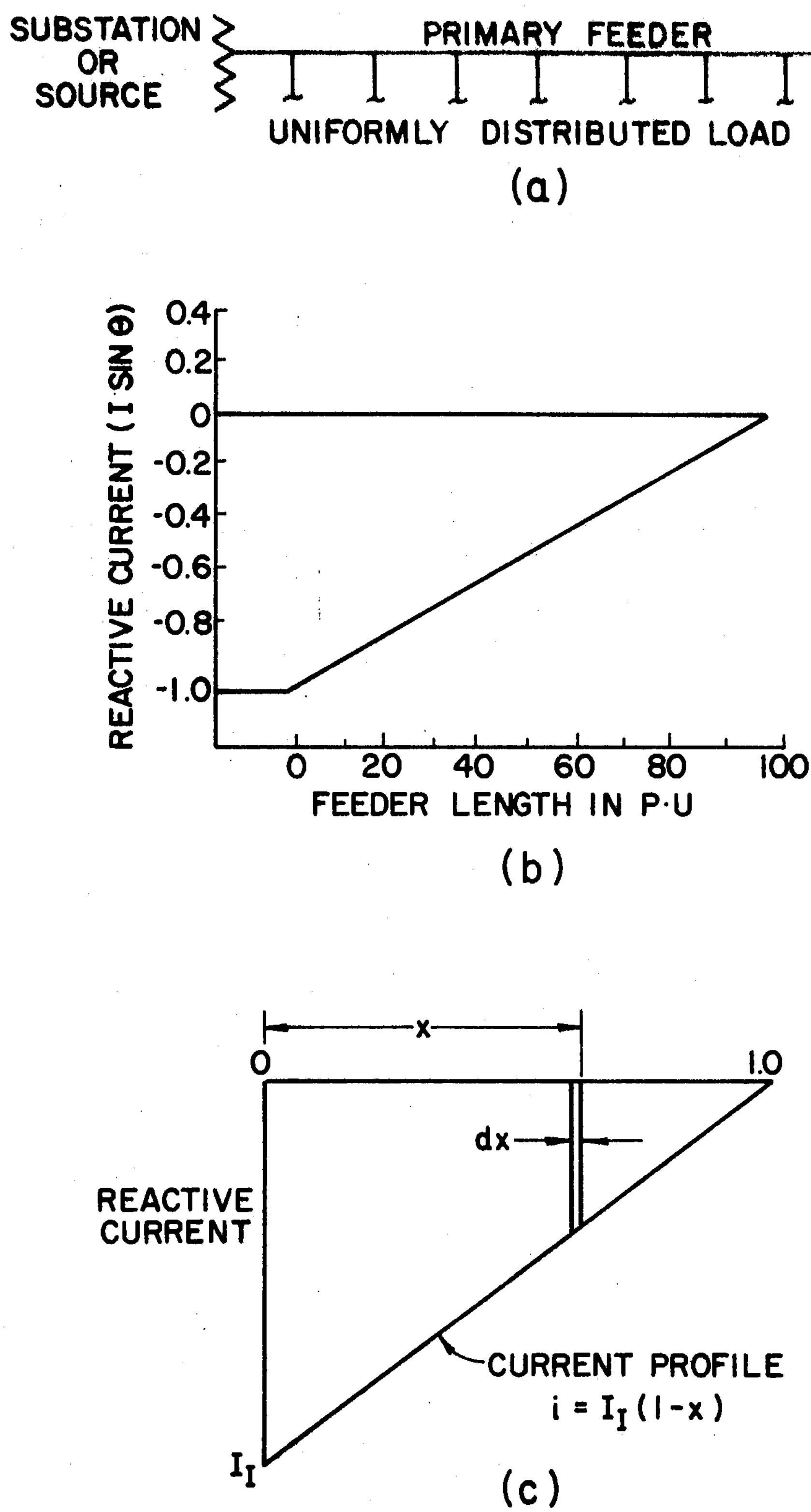


Figure 4. Primary feeder with uniformly distributed load and current profile

is the reduction of the voltage spread between first and last customer. The reduction of voltage spread allows more load on the voltage standpoint to be added to the feeder.

d. Review Of Present Method Of Applying Shunt Capacitors For Energy And Loss Reduction

The "TWO THIRDS" rule is one of the most widely used methods of applying Shunt Capacitors. This rule states that "at two-thirds of the way along the feeder locate an amount of CKVA (Capacitor Kva) equal to two-third of the Kvar (Reactive KVA) input to the feeder."

The use of this rule could result in a significant increase in the energy loss in a feeder even though the reduction of peak reactive load loss is optimum. Previous general capacitor applications were based on peak and energy feeder loss reductions, but no attempt was made to analytically include the voltage drop occurring on it.

e. Purpose Of The Thesis

The main problem involved is to develop the necessary analytical solution for the proper application of Shunt Capacitors to primary feeders for the purpose of peak power loss or energy loss reduction and voltage control. Voltage Control is considered for both the energy and peak loss reduction which is then optimized with the loss reduction. The proper application of shunt capacitors consists of specifying the number and ratings of capacitors and the location of each, to optimize a Performance Index, which combines the influence of minimizing the feeder power loss and maximizing the voltage improvement for voltage control.

CHAPTER II

GENERAL ANALYSIS OF PEAK LOSS REDUCTION WITH ONE FIXED CAPACITOR

a. Expansion Of Basic Peak Loss Equation

A feeder is considered to have a uniformly distributed load with I_I as the reactive component of the current flowing at its source. The total length of the feeder is 1.0p.u. A primary feeder for uniformly distributed load is as shown in figures (4a), and (4b).

The loss L due to reactive current may be calculated by integrating.

$$L = \int_0^1 i^2 R\,dx = \int_0^1 \left[I_I(1-x)\right]^2 R\,dx = \frac{I_I^2 R}{3} \qquad (2.1)$$

which is one-third the product of the square of the reactive current at substation and the total feeder resistance as shown by figure (4c). This derivation is important only in that the principle involved are used to illustrate how further work will be accomplished.

Figure (5) shows the location of capacitor bank along the feeder which causes break in the continuity of the reactive load profile.

Total loss before capacitor application is:

$$L = \int_0^1 \left[I_I(1-x)\right]^2 R\,dx = \frac{I_I^2 R}{3} \qquad (2.2)$$

Total loss after capacitor application:

$$L = \int_0^a \left[I_I(1-x) - I_c\right]^2 R\,dx + \int_0^1 \left[I_I(1-x)\right]^2 R\,dx \qquad (2.3)$$

where "a" is the location of the first capacitor

Performing the operation on equation (3) above results in the following equation for peak loss:

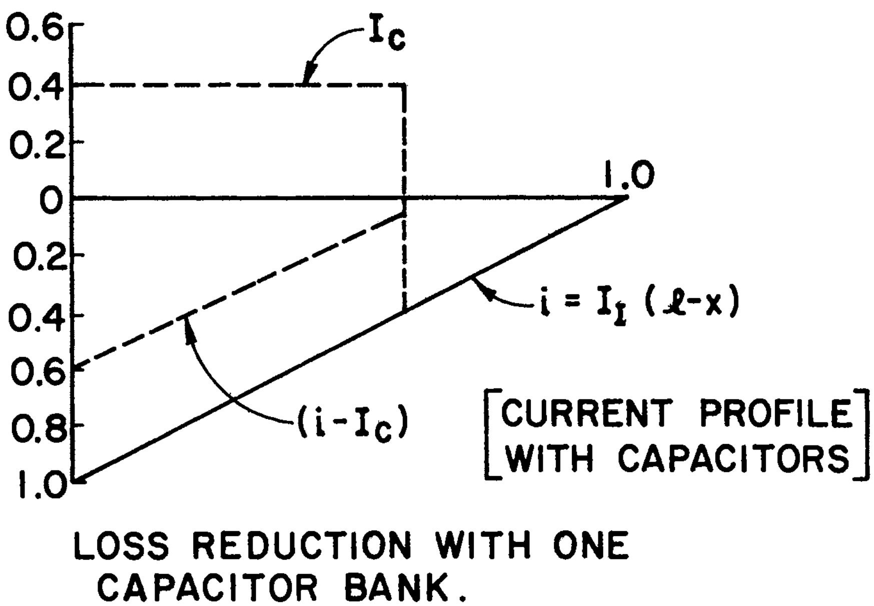


Figure 5. Loss reduction with one capacitor bank

$$L = \frac{I_I^2 R}{3} - 2 I_I I_c a R + I_c^2 a R + I_I I_c a^2 R \tag{2.4}$$

In order to make the resulting equation useful for any given primary circuit, a "PER UNIT" system is used such that all quantities are dimensionless. In order to normalize the peak loss in p.u. the losses before and after capacitor application are subtracted and divided by the loss before capacitor application i.e.

$$\Delta L_{p.u} = \frac{L_{before} - L_{after}}{L_{before}}$$

Therefore,

$$\Delta L_{p.u.} = \frac{3(2 I_I I_c a R - I_c^2 a R - I_I I_c a^2 R)}{I_I^2 R} \tag{2.5}$$

$$\Delta L_{p.u} = 3ak\left[2 - a - k\right] \tag{2.6}$$

The optimum location for any capacitor to result in minimum loss is found by setting the first partial derivatives of L with respect to "a" and "k" equal to zero.

$$\frac{\partial \Delta L}{\partial a} = 3k\left[2 - 2a - k\right] = 0 \tag{2.7}$$

$$\frac{\partial \Delta L}{\partial k} = 3a\left[2 - a - 2k\right] = 0 \tag{2.8}$$

Solving equations (7) and (8) simultaneously

$$a = \frac{2}{3}, \quad k = \frac{2}{3} \tag{2.9}$$

Equation (9) shows that the capacitor is situated at two-third of the distance from the beginning of the feeder.

Equations (7) and (8) give a solution for maximum loss reduction since the second derivatives of "a" and "k" are negative.

$$\frac{\partial \Delta L^2}{\partial a^2} = -6k \tag{2.10}$$

$$\frac{\partial \Delta L^2}{\partial k^2} = -6a \tag{2.11}$$

b. Voltage Drop Along Uniformly Loaded Feeder With Uniform Resistance

Voltage drop in a circuit depends upon the resistance, reactance, current flow and the time phase angle between current and voltage. The voltage drop is then defined as:

$$|V| = |V_S| - |V_R| \tag{2.12}$$

where $|V_S|$ = sending and voltage

$|V_R|$ = receiving and voltage

If a circuit contains only resistance and inductive reactance and serves an inductive load, the vector diagram of current and voltage will be similar to figure (6). Voltage at the source is OC, voltage at the load is OA, and voltage drop in the line is equal to the difference between OC and OA.

Figure (7) shows the voltage drop between the sending end and receiving end of the line. The voltage drop "V" of equation (12) is:

$$|V| = |IR| \cos\theta + |IX| \sin\theta + T \tag{2.13}$$

or

$$|V_S| = |V_R| + |IZ| \cos\beta + T \tag{2.14}$$

where 'T' is an approximation and can be neglected, which will be considered in the following:

$$|V_S|^2 = \left[|V_R| + |IZ| \cos\beta + T\right]^2 \tag{2.15}$$

The approximation 'T' is taken to be equal to:

$$T = e\,(|IZ| \cos\beta) \tag{2.16}$$

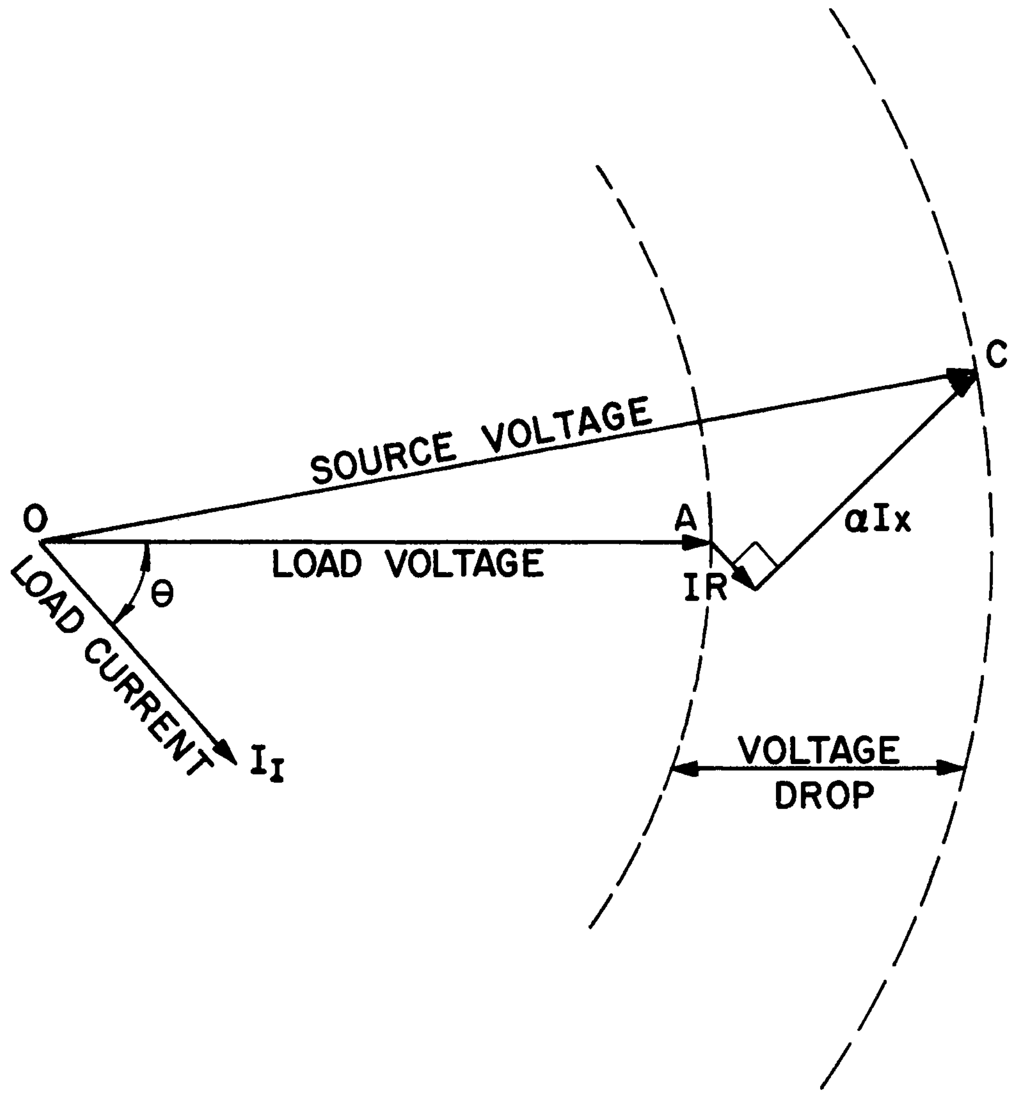
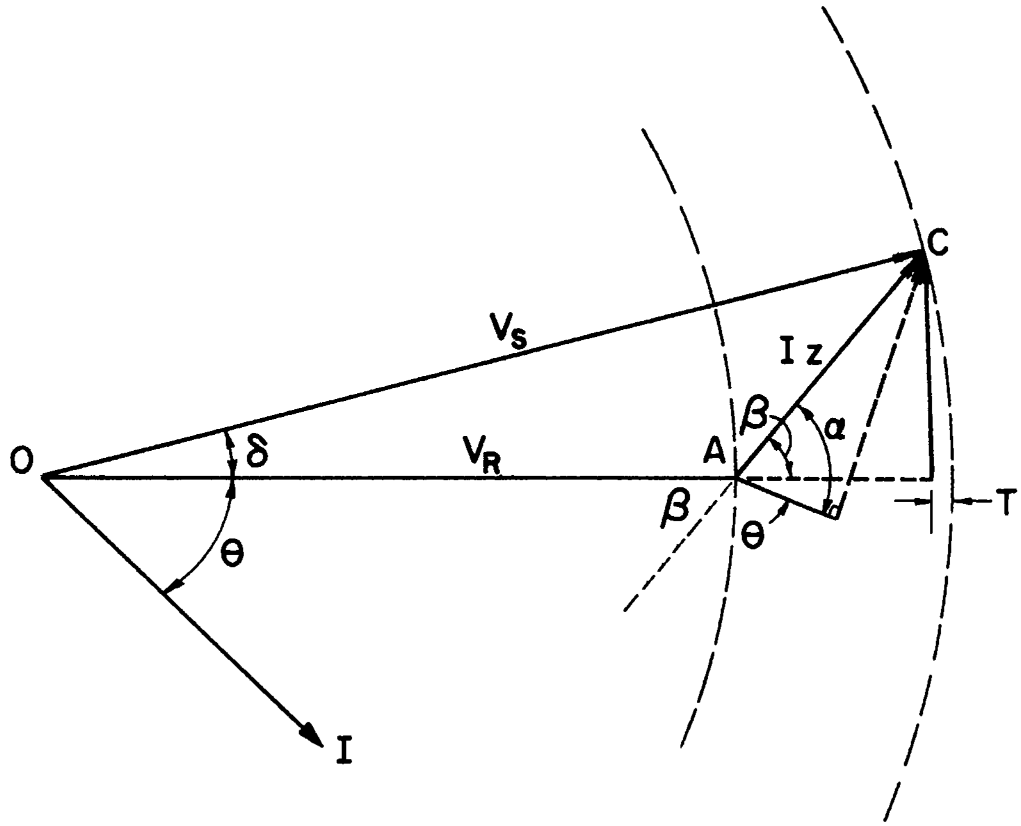


Figure 6. Phasor diagram showing total voltage drop occurring on the feeder



$$\tan \alpha = \frac{X}{R}$$

$$\beta = \alpha - \theta$$

Figure 7. Phasor diagram showing voltage drop and approximation T which is to be neglected

where 'e' is the percentage error. Equation (15) can also be written as:

$$|V_S|^2 = \left[(|V_R| + |IZ|\cos\beta)^2 + T^2 + 2T(|V_R| + |IZ|\cos\beta)\right] \quad (2.17)$$

Also from figure (7)

$$|V_S|^2 = \left[|V_R| + |IZ|\cos\beta\right]^2 + \left[|IZ|\sin\beta\right]^2 \quad (2.18)$$

Comparing equation (17) and (18) result in the following:

$$T^2 + 2T(|V_R| + |IZ|\cos\beta) = \left[|IZ|\sin\beta\right]^2 \quad (2.19)$$

which with equation (15) can also be written as:

$$e^2(|IZ|\cos\beta)^2 + 2e|IZ|\cos\beta\left[|V_R| + |IZ|\cos\beta\right] = \left[|IZ|\sin\beta\right]^2 \quad (2.20)$$

Equation (20) rearranged is:

$$(|IZ|\cos\beta)^2\left[e^2 + 2e\right] + 2e|V_R|\,|IZ|\cos\beta = (|IZ|\sin\beta)^2 \quad (2.21)$$

Simplifying equation (21) gives:

$$\frac{\cos\beta}{Z\ p.u.} = \frac{\sin^2\beta}{2e} - \frac{(e^2+2e)}{2e}\cos^2\beta \quad (2.22)$$

Let the error 'e' be 2% and the impedence of the line be 5%. Hence,

$$\frac{\cos\beta}{0.05} = \frac{\sin^2\beta}{0.04} - \frac{\cos^2\beta}{1} \quad (2.23)$$

or

$$0.8 = \tan\beta\sin\beta - 0.04\cos\beta \quad (2.24)$$

Solving for β it was found that $\beta = 48^\circ$, which is the maximum value of β,

Therefore,

$$\beta = \alpha - \theta \leqslant \beta max \qquad (2.25)$$

or,

$$\frac{\operatorname{Tan}\alpha - \operatorname{Tan}\theta}{1 + \operatorname{Tan}\alpha \operatorname{Tan}\theta} \leqslant \operatorname{Tan}\beta max. \qquad (2.26)$$

Since

$$p = \frac{R}{X} = 1/\tan\alpha$$

$$h = I_R/I_I = 1/\tan\theta$$

Hence equation (26) can be written as:

$$\frac{p-h}{1+hp} \leqslant \operatorname{Tan}\beta max \qquad (2.27)$$

or,

$$\frac{p-h}{1+hp} \leqslant 1.11 \qquad (2.28)$$

Solving equation (28) can result in values of 'h' and 'p' which are acceptable, which are shown in figure (8). Hence the approximation of 2% error is physically realiable and correct.

For higher power factor, resistance causes a greater proportionate part of the voltage drop than reactance. For low power factor reactance causes a greater proportionate part of the voltage drop than resistance. The power factor depends primarily on the relative proportion of resistance and reactance of the load. Figure (9a) shows the complete set up. Now for the case of uniformly distributed load the voltage drop per unit length is:

$$\frac{dv}{dx} = i(x)\frac{R}{l}\operatorname{Cos}\theta + i(x)\frac{X}{l}\operatorname{Sin}\theta \qquad (2.29)$$

The value of (R Cos θ + XSin θ) is called the effective impedence of the

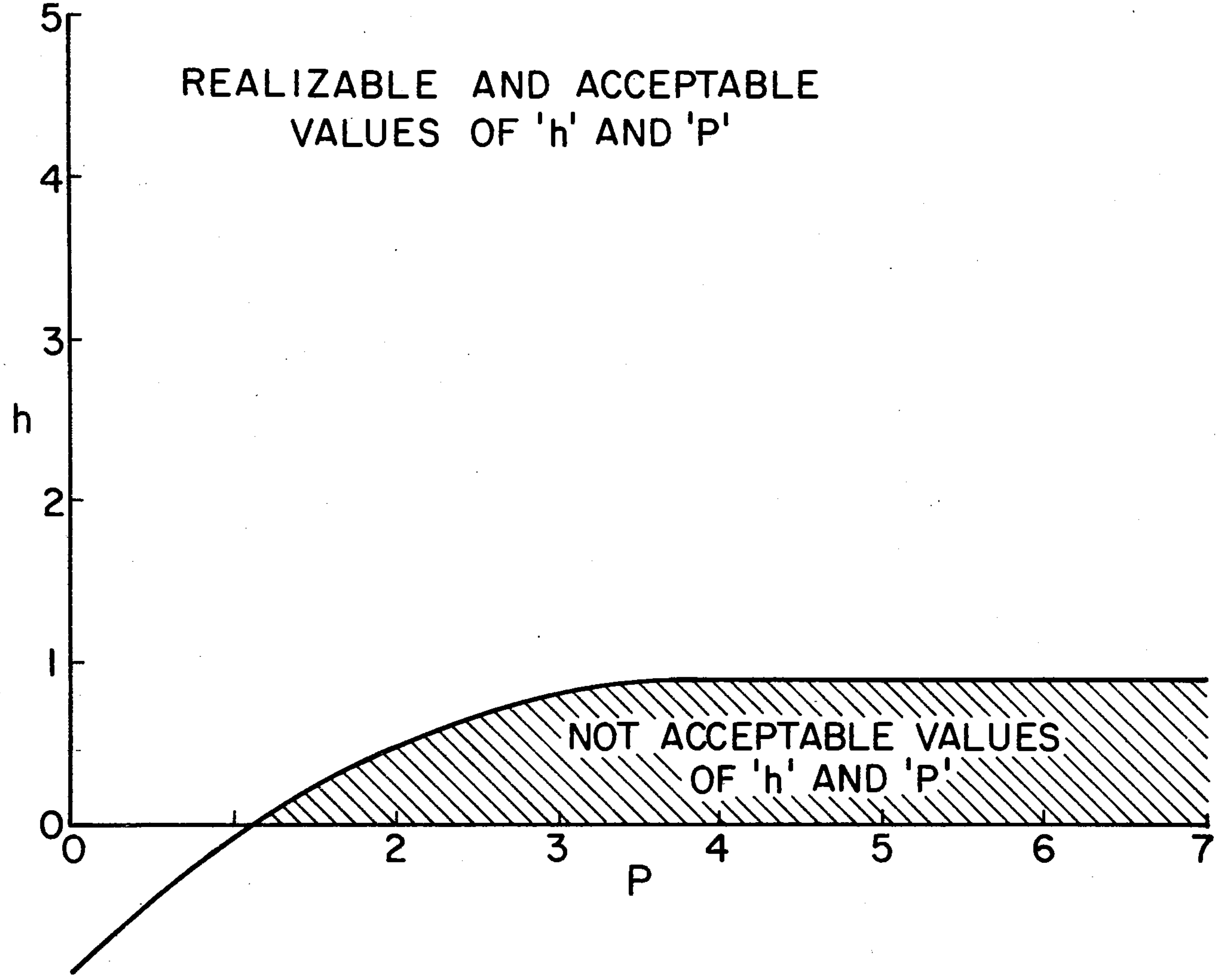


Figure 8. Graph showing values of 'h' and 'p' which are realizable and acceptable

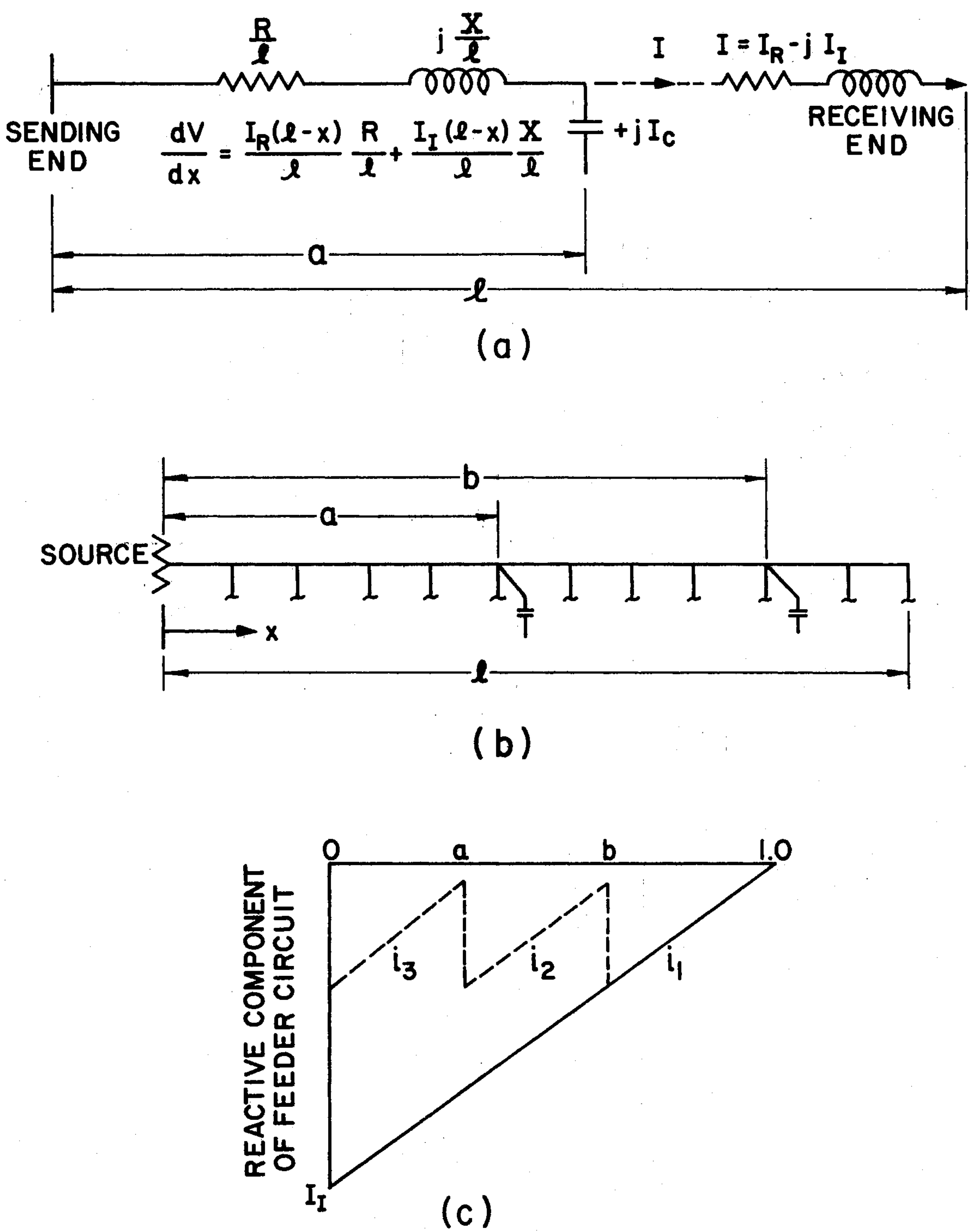


Figure 9. Primary feeder showing resistance and inductance with two capacitors

line, represents a constant for constant Θ. When multiplying by the current flowing in the line, it very closely approaches the voltage drop for a given power factor. The error is so small as to be negligible in distribution line calculations. Hence equation (29) transforms to:

$$\frac{dv}{dx} = i_R(x)\frac{R}{\ell} + i_I(x)\frac{X}{\ell} \tag{2.30}$$

Since the load is uniform along the length of the feeder, hence

$$i_R(x) = \frac{I_R(\ell - x)}{\ell}, \quad i_I(x) = \frac{I_I(\ell - x)}{\ell} \tag{2.31}$$

For the case of one shunt capacitor connected to the feeder the differential voltage equation in that section of the feeder containing the capacitor current becomes:

$$\frac{dv}{dx} = i_R\frac{R}{\ell} + i_I\frac{X}{\ell} - I_c\frac{X}{\ell} \tag{2.32}$$

$$V_{after} = \int_0^a \left[\frac{I_I(\ell - x)}{\ell} - I_c\right]\frac{X}{\ell}dx + \int_a^\ell \left[\frac{I_I(\ell - x)}{\ell}\right]\frac{X}{\ell}dx + \int_0^\ell \left[\frac{I_R(\ell - x)}{\ell}\right]\frac{R}{\ell}dx \tag{2.33}$$

Substituting $\ell = 1$ and calculating equation (33) leads to the expression:

$$V_{after} = \frac{I_I X}{2} - I_c a X + \frac{I_R R}{2} \tag{2.34}$$

In order to normalize the voltage drop in per unit the voltage drop before and after the capacitor application are subtracted and divided by the voltage drop without the capacitor application i.e.

$$\Delta V_{p.u.} = \frac{V_{before} - V_{after}}{V_{before}} \tag{2.35}$$

$$V_{before} = \int_0^\ell \left[\frac{I_I(\ell - x)}{\ell}\right]\frac{X}{\ell}dx + \int_0^\ell \left[\frac{I_R(\ell - x)}{\ell}\right]\frac{R}{\ell}dx = \frac{I_I X}{2} + \frac{I_R R}{2} \tag{2.36}$$

Voltage before can also be obtained by using equation (33) and setting $I_c = 0$. Now combining equation (34) and (37) will yield the normalized per unit voltage improvement across the feeder due to a capacitor located at a point 'a' from the sending end:

$$\Delta V_{p.u.} = \frac{2 I_C a X}{I_I X + I_R R} \tag{2.37}$$

c. <u>Selection Of Performance Criterion Considering Peak Loss And Voltage Drop In Per Unit</u>

The Performance Index Criterion combines the peak loss and voltage as shown in the following equations:

$$\Delta P.I._{p.u.} = \Delta L_{p.u} + \alpha (\Delta V)_{p.u.} \tag{2.38}$$

where α is some weighting constant.

$$\Delta P.I_{p.u.} = \frac{6 I_I I_C a R - 3 I_C^2 a R - 3 I_I I_C a^2 R}{I_I^2 R} + \alpha \left\{ \frac{2 I_C a X}{I_I X + I_R R} \right\} \tag{2.39}$$

In order to optimize the Performance Index and to obtain the optimum location of the capacitor on the feeder to minimize losses, and maximize the voltage rise the first partial derivatives of "a" and "k" are taken

$$\frac{\partial \Delta P.I.}{\partial a} = 3 \frac{I_C}{I_I} \left\{ \frac{2 - I_C - 2a}{I_I} \right\} + \alpha \left\{ \frac{2 I_C X}{I_I X + I_R R} \right\} \tag{2.40}$$

or,

$$\frac{\partial \Delta P.I.}{\partial a} = \left(6k - 3k^2 - 6ak\right) + \left(\frac{2k\alpha}{1 + hp}\right) \tag{2.41}$$

where $h = \frac{I_R}{I_I}$ and $p = \frac{R}{X}$

Similarly,

$$\frac{\partial \Delta P.I.}{\partial K} = \left(6aK - 3aK^2 - 3a^2K\right) + \alpha\left(\frac{2aK}{1+hp}\right) \qquad (2.42)$$

Setting equation (40) and (42) to zero and solving simultaneously give the results that

$$K = a, \quad a = \frac{2}{3} + \frac{2\alpha}{9(1+hp)} \qquad (2.43)$$

The results in equation (43) are only true in the case when $0 < a \leq 1$ If the voltage correction is considered to be negligible then $\alpha = 0$ and the capacitor location "a" comes out to be 2/3, which checks out with the previous result when only the peak loss was considered.

CHAPTER III

GENERAL ANALYSIS OF PEAK LOSS REDUCTION WITH TWO FIXED CAPACITORS

a. Expansion Of Basic Loss Equation

Figure (9b, c) depicts the peak loss and location of two banks on a feeder. It will be shown that the economical location for each capacitor bank, starting from the end of the feeder, is still where the capacitor Kva is twice the size the Kvar at the point of capacitor installation.

Hence, peak loss after application of capacitors "a" and "b" is given by the following development.

$$L = \int_0^a \left[I_I(1-x) - (I_{ca}+I_{cb}) \right]^2 R\,dx + \int_a^b \left[I_I(1-x) - I_{cb} \right]^2 R\,dx + \int_b^1 \left[I_I(1-x) \right]^2 R\,dx \qquad (3.1)$$

Evaluating the integrals will result in the following:

$$L = -2 I_I I_{ca} aR + I_{ca}^2 aR + 2 I_{ca} I_{cb} aR + I_I I_{ca} a^2 R - 2 I_I I_{cb} bR + I_I I_{cb} b^2 R + I_{cb}^2 bR + \frac{I_I^2 R}{3} \qquad (3.2)$$

As in the previous one capacitor case the peak loss in per unit in this case is obtained in equations (3) and (4).

$$\Delta L\ p.u. = \frac{L_{before} - L_{after}}{L_{before}} \qquad (3.3)$$

The loss is p.u. before the capacitor application is the same as in the one capacitor case i.e.

$$I_I^2 R/3$$

Hence, the loss in the two capacitor case is:

$$\Delta L_{p.u.} = 3\left[2\frac{I_{ca}}{I_I}a - \left(\frac{I_{ca}}{I_I}\right)^2 a - \frac{2 I_{ca} I_{cb} a}{I_I^2} - \left(\frac{I_{ca}}{I_I}\right)a^2 + 2\frac{I_{cb} b}{I_I} - \frac{I_{cb}}{I_I}b^2 - \left(\frac{I_{cb}}{I_I}\right)^2 b\right] \quad (3.4)$$

Substituting $K_1 = \frac{I_{ca}}{I_I}$ and $K_2 = \frac{I_{cb}}{I_I}$ yields

$$\Delta L_{p.u.} = 3\left[2ak_1 - ak_1^2 - 2ak_1k_2 - a^2k_1 + 2bk_2 - b^2k_2 - bk_2^2\right] \quad (3.5)$$

Taking the first partial derivative of equation (5) seperately with respect to "a", "b", "k_1", and "k_2", and setting the result to zero will yield the following equations:

$$\frac{\partial \Delta L}{\partial a} = 0 = \left[2k_1 - k_1^2 - 2k_1k_2 - 2ak_1\right] \quad (3.6)$$

$$\frac{\partial \Delta L}{\partial b} = 0 = \left[2k_2 - k_2^2 - 2k_2 b\right] \quad (3.7)$$

$$\frac{\partial \Delta L}{\partial k_1} = 0 = \left[2a - 2ak_1 - 2ak_2 - a^2\right] \quad (3.8)$$

$$\frac{\partial \Delta L}{\partial k_2} = 0 = \left[2b - 2ak_1 - 2bk_2 - b^2\right] \quad (3.9)$$

Simplifying and solving equations (6), (7), (8), and (9) simultaneously give the results that:

$$k_1 = a = \frac{b}{2} \quad (3.10)$$

Now, substituting equation (10) into equation (7), results in a quadratic equation in k_1, with roots $k_1 = b$ and $k_2 = b/2$. The first result $k_1 = b$ is physically not realizable as on solving it "a" comes out to be greater than one, which is of course not possible since the total length of the feeder is one p.u. Also k_2 comes out to be a negative number which is not possible. Hence, we can neglect this value of k_1 and calculate only with $k_1 = b/2$. The results obtained from this substitution are neither negative nor greater than one:

$$k_1 = k_2 = \frac{2}{5} \quad (3.11)$$

$$a = k_1 = \frac{b}{2} \quad (3.12)$$

This is reasonable enough as "b" is less than one p.u. and is situated at twice that of 'a'. The loss reduction can observed to be maximum if the second partial derivatives of equations (6), (7), (8), and (9) are taken which yield negative results.

b. Voltage Drop Along Feeder With Uniform Distributed Load

It is assumed that the feeder is of the same wire size throughout its length. Even if a smaller sized wire is used near the end of the feeder, the effect is not too pronounced on the line losses. The voltage drop after the application of both the capacitors is as under:

$$V = \int_0^a \left[\frac{I_I(l-x)}{l} - (I_{ca} + I_{cb})\right]\frac{X}{l}dx + \int_a^b \left[\frac{I_I(l-x)}{l} - I_{cb}\right]\frac{X}{l}dx + \int_b^l \left[\frac{I_I(l-x)}{l}\right]\frac{X}{l}dx + \int_0^l \left[\frac{I_R(l-x)}{l}\right]\frac{R}{l}dx \qquad (3 \cdot 13)$$

Simplifying by taking $l = 1$ p.u. and performing the necessary operations above give the following equation:

$$V = \frac{I_I X}{2} - I_{ca} a X - I_{cb} b X + \frac{I_R R}{2} \qquad (3 \cdot 14)$$

Voltage drop before the application of capacitors "a" and "b" can be expressed as:

$$V_{before} = \int_0^a \left[I_I - I_I x\right] X dx + \int_a^b \left[I_I - I_I X\right] X dx + \int_b^1 \left[I_I - I_I x\right] X dx + \int_0^1 \left[I_R(1-x)\right] R dx \qquad (3 \cdot 15)$$

$$V_{before} = \frac{I_I X}{2} + \frac{I_R R}{2} \qquad (3 \cdot 16)$$

The improvement in voltage drop across the feeder by the application of capacitors at "a" and "b" is

$$\Delta V_{p.u.} = \frac{V_{before} - V_{after}}{V_{before}} \qquad (3 \cdot 17)$$

which upon substitution and simplification gives equation (18).

$$\Delta V\ p.u = \frac{I_{ca}aX + I_{cb}bX}{\frac{I_I X}{2} + \frac{I_R R}{2}} \qquad (3.18)$$

c. Performance Criterion Considering Peak Loss And Voltage Drop In Per Unit

The performance index is expressed as the addition of the loss equation and the product of the voltage drop and weighing factor.

Mathematically this may be expressed as:

$$\Delta P.I.\ p.u. = \Delta L\ p.u. + \alpha(\Delta V)\ p.u \qquad (3.19)$$

$$\Delta P.I.\ p.u. = 3\left[2ak_1 - ak_1^2 - 2ak_1k_2 - a^2k_1 + 2bk_2 - b^2k_2 - bk_2^2\right] + \alpha\left[\frac{I_{ca}aX + I_{cb}bX}{\frac{I_I X}{2} + \frac{I_R R}{2}}\right] \qquad (3.20)$$

Taking the first partial derivatives seperately with respect to "a", "b", "k_1", and "k_2" yields

$$\frac{\partial \Delta P.I.}{\partial a} = 3\left[2k_1 - k_1^2 - 2k_1k_2 - 2ak_1\right] + \alpha\left[\frac{2I_{ca}X}{I_I X + I_R R}\right] \qquad (3.21)$$

$$\frac{\partial \Delta P.I.}{\partial k_1} = 3\left[2a - 2ak_1 - 2ak_2 - a^2\right] + \alpha\left[\frac{2a}{1+h\rho}\right] \qquad (3.22)$$

where $h = \frac{I_R}{I_I}$ and $\rho = \frac{R}{X}$

$$\frac{\partial \Delta P.I.}{\partial b} = 3\left[2k_2 - 2bk_2 - k_2^2\right] + \alpha\left[\frac{2k_2}{1+h\rho}\right] \qquad (3.23)$$

$$\frac{\partial \Delta P.I.}{\partial k_2} = 3\left[2b - b^2 - 2bk_2 - 2ak_1\right] + \alpha\left[\frac{2b}{1+h\rho}\right] \qquad (3.24)$$

Setting equations (21) and (22) to zero and solving gives the result

$$a = k_1 \qquad (3.25)$$

Also solving equations (23) and (22) set equal to zero gives a quadratic equation in "k_1" with roots $k_1 = b/2$ or $k_1 = b$. As before the only realistic solution is $k_1 = b/2$. Substituting it into the above equation and simplifying give the following results:

$$K_1 = \frac{2}{5} + \frac{2\alpha}{15(1+\hbar p)} \qquad (3.26)$$

$$K_2 = \frac{2}{5} + \frac{2\alpha}{15(1+\hbar p)} \qquad (3.27)$$

Hence $a = k_1 = b/2$. (3.28)

Equation (28) gives the same result as was obtained in equation (12).

CHAPTER IV

BASIC ENERGY LOSS EQUATION FOR ONE FIXED CAPACITOR

a. Energy Loss Equation With Time Intervals, Load And Loss Factors

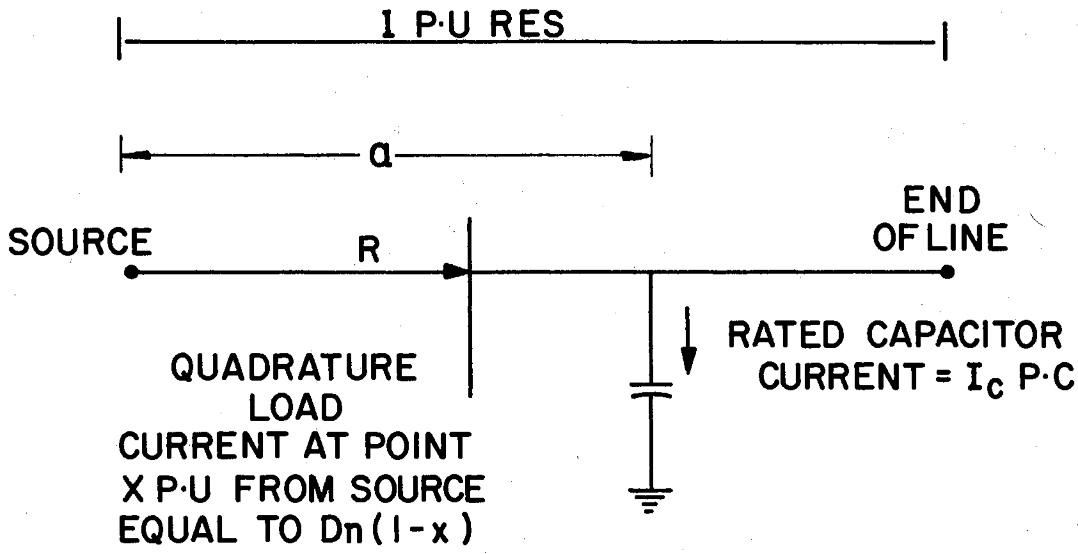
Consider a radial circuit that has reactive load uniformly distributed along the line. A plot of per unit quadrature current versus per unit line resistance is shown in figures (10) and (11).

In order to take into account a general load cycle, the p.u. peak quadrature line current at any point in the line is given by (1 - x). During time T_1 the p.u. quadrature line current is D_1 (1 - x) and during time T_n, the p.u. quadrature line current is D_n (1 - x), shown in figure (11). By considering a large number of time T_1 through T_n, where each time is measured in p.u. of the period of the load cycle, a completely general load cycle of any shape and duration is taken into account.

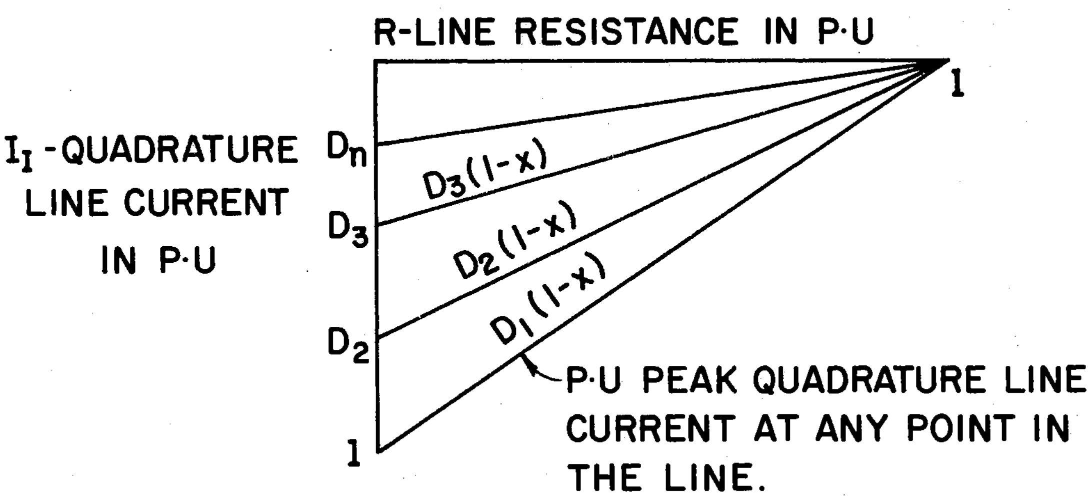
The energy loss in the feeder is given by equation (1). Included in the loss equation is the effect of a capacitor with rated current I_c. The capacitor location on the line is designated by "a". The effect of the capacitor current is to subtract from the quadrature line current up to the capacitor location "a".

$$L = \int_0^a \Big[\{D_1 I_I(1-x) - I_c\}^2 RT_1 + \{D_2 I_2 (1-x) - I_c\}^2 RT_2 + \text{-----} \\ \text{---} + \{D_n I_I (1-x) - I_c\}^2 RT_n \Big] dx + \int_a^1 \Big[\{D_1 I_I (1-x)\}^2 RT_1 + \\ + \{D_2 I_I (1-x)\} RT_2 + \text{---} + \{D_n I_I (1-x)\}^2 RT_n \, dx \qquad (4\cdot 1)$$

where L = energy loss caused by quadrature current in feeder.



(a)



(b)

Figure 10. Per Unit feeder with one fixed capacitor line resistance versus line current

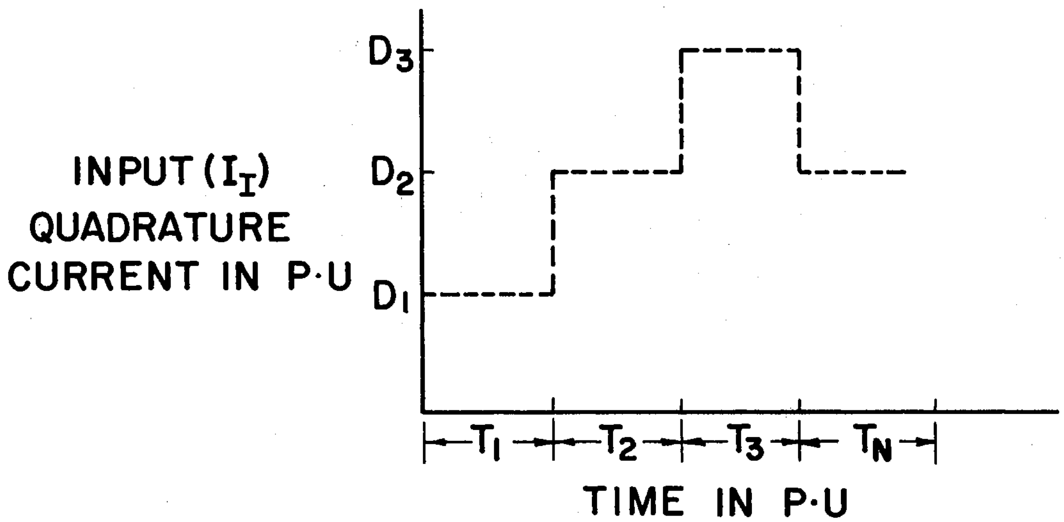


Figure 11. Portion of the reactive load cycle

Performing the indicated operations on equation (1) results in the following equation for energy loss:

$$L = a^2 I_I I_c R (D_1T_1 + D_2T_2 + \text{--------} + D_nT_n) - $$
$$-2 I_I I_c a R (D_1T_1 + D_2T_2 + \text{-------} + D_nT_n) +$$
$$+ a I_c^2 R (T_1 + T_2 + \text{--------} + T_n) +$$
$$+ \frac{1}{3}\left(D_1^2 I_I^2 RT_1 + D_2^2 I_I^2 RT_2 + \text{-----} + D_n^2 I_I^2 RT_n\right) \quad (4.2)$$

The p.u. change in quadrature current energy loss caused by the effect of adding a capacitor with a current I_c at a location "a" is found by subtracting the term not dependent on I_c (which is the energy loss with no capacitor on the line). Hence:

$$\Delta L\,p.u. = \frac{L_{before} - L_{after}}{L_{before}} \qquad (4.3)$$

The energy loss before the capacitor application is found by taking $I_c = 0$.

$$L_{before} = \int_0^a \Big[\{D_1 I_I (1-x)\}^2 RT_1 + \{D_2 I_I (1-x)\}^2 RT_2 + \text{-----}$$
$$\text{--} + \{D_n I_I (1-x)\}^2 R T_n\Big] dx + \int_a^1 \Big[\{D_1 I_I (1-x)\}^2 RT_1 +$$
$$+ \{D_2 I_I (1-x)\}^2 RT_2 + \text{---} + \{D_n I_I (1-x)\}^2 RT_n \, dx\Big] \quad (4.4)$$

Solving equation (4) leads to results as shown in equation (5).

$$L_{before} = \left[\frac{D_1^2 I_I^2 RT_1}{3} + \frac{D_2^2 I_I^2 RT_2}{3} + \text{-----} + \frac{D_n^2 I_I^2 RT_n}{3}\right] \quad (4.5)$$

Substituting equations (2) and (5) into (3) gives equation (6).

$$\Delta L\,p.u. = \frac{3\left[2a I_I I_c R (D_1T_1 + D_2T_2 + \text{---} + D_nT_n) - a^2 I_I I_c R (D_1T_1 + D_2T_2 + \text{--} + D_nT_n) - a I_c^2 R (T_1 + T_2 + \text{-} T_n)\right]}{I_I^2 R (D_1^2T_1 + D_2^2T_2 + \text{-----} + D_n^2 T_n)} \quad (4.6)$$

or

$$\Delta L_{p.u.} = 3\left[2ak\frac{V}{U} - a^2 k\frac{V}{U} - ak^2\frac{W}{U}\right] \qquad (4.7)$$

where:

$$V = (D_1T_1 + D_2T_2 + \text{------} + D_nT_n)$$

$$U = (D_1^2T_1 + D_2^2T_2 + \text{-----} + D_n^2T_n)$$

$$W = (T_1 + T_2 + \text{--------} + T_n)$$

The optimum location for any capacitor to result in minimum loss is found by setting the first partial derivatives with respect to "a" and "k" equal to zero,

$$\frac{\partial \Delta L}{\partial a} = 0 = 3\left[2k\frac{V}{U} - 2ak\frac{V}{U} - k^2\frac{W}{U}\right] = \left[2V - 2aV - kW\right] \qquad (4.8)$$

$$\frac{\partial \Delta L}{\partial k} = 0 = 3\left[2a\frac{V}{U} - a^2\frac{V}{U} - 2ak\frac{W}{U}\right] = \left[2V - aV - 2kW\right] \qquad (4.9)$$

Solving equations (8) and (9) simultaneously result in equations (10) and (11)

$$a = \frac{2}{3} \qquad (4.10)$$

$$K = \frac{2}{3}\,(L.F) \qquad (4.11)$$

where L.F is the Load Factor and is equal to v/w, and is defined as "the ratio of the average power to the maximum demand".

If the load factor is equal to one then the results of "a" and "k" are exactly identical to those obtained based on peak loss only.

b. Voltage Drop Along Feeder In Per Unit

The voltage drop caused by energy loss in feeder can be expressed as under:

$$V = \int_0^a \{D_1 I_I(1-x) - I_c\} X T_1 + \{D_2 I_I(1-x) - I_c\} X T_2 + --- ---- + \{D_n I_I(1-x) - I_c\} X T_n \, dx + \int_a^1 \{D_1 I_I(1-x)\} X T_1 + \{D_2 I_I(1-x)\} X T_2 + --- + \{D_n I_I(1-x)\} X T_n + + \int_0^1 \{D_1 I_R(1-x)\} R T_1 + \{D_2 I_R(1-x)\} R T_2 + -- + \{D_n I_R(1-x)\} R T_n \tag{4.12}$$

Performing the operations indicated in equation (12) will result in the following:

$$V = \frac{I_I X}{2}\left[D_1 T_1 + D_2 T_2 + -------- + D_n T_n\right] - I_c a X\left[T_1 + + T_2 + ------ + T_n\right] + \frac{I_R R}{2}\left[D_1 T_1 + D_2 T_2 + ---- + D_n T_n\right] \tag{4.13}$$

As before, the per-unit voltage drop across the feeder is found by subtracting the voltage drop after capacitor application from the voltage drop before application of the capacitor, divided by voltage before.

$$\Delta V \text{ p.u.} = \frac{V_{before} - V_{after}}{V_{before}} \tag{4.14}$$

The voltage drop before the application of the capacitor is found by taking $I_c = 0$ and is expressed as:

$$V_{before} = \int_0^a \Big[\{D_1 I_I(1-x)\} X T_1 + \{D_2 I_I(1-x)\} X T_2 + ---- --- + \{D_n I_I(1-x)\} X T_n\Big] dx + \int_a^1 \Big[\{D_1 I_I(1-x)\} X T_1 + + \{D_2 I_I(1-x)\} X T_2 + ------ + \{D_n I_I(1-x)\} X T_n\Big] dx + \int_0^1 \Big[\{D_1 I_R(1-x)\} R T_1 + \{D_2 I_R(1-x)\} R T_2 + ---- + \{D_n I_R(1-x)\} R T_n\Big] dx. \tag{4.15}$$

Equation (15) results in the following:

$$V_{before} = \frac{I_I X}{2}\left\{D_1T_1 + D_2T_2 + \text{-----} + D_nT_n\right\} + \\ + \frac{I_R R}{2}\left\{D_1T_1 + D_2T_2 + \text{-----} + D_nT_n\right\} \quad (4.16)$$

Substituting equations (16) and (13) in equation (14) results in the normalized voltage drop in per-unit, which is shown as under in equation (17).

$$\Delta V_{p.u.} = \frac{2 I_C a X\left[T_1 + T_2 + \text{------} + T_n\right]}{I_I X\left[D_1T_1 + D_2T_2 + \text{---} + D_nT_n\right] + I_R R\left[D_1T_1 + D_2T_2 + \text{--} D_nT_n\right]} \quad (4.17)$$

or

$$\Delta V_{p.u.} = \frac{2 I_C a X}{L.F\,(I_I X + I_R R)} \quad (4.18)$$

If the load factor is taken to be equal to one in equation (18) this results in the identical equation to the peak loss voltage drop equation.

c. Performance Criterion Considering Energy Loss And Voltage Drop In P.U.

The performance index criterion combines the energy loss and the voltage drop which is as shown below:

$$\Delta P.I._{p.u} = \Delta L_{p.u} + \alpha(\Delta V)_{p.u} \quad (4.19)$$

where α is some weighting constant

$$\Delta P.I_{p.u} = 6ak\frac{v}{u} - 3a^2k\frac{v}{u} - 3ak^2\frac{w}{u} + 2\alpha\left\{\frac{akw}{v(1+hp)}\right\} \quad (4.20)$$

where $LF = \frac{v}{w}$, $SF = \frac{u}{w}$, $\frac{v}{u} = \frac{LF}{SF}$, $k = \frac{I_C}{I_I}$, $h = \frac{I_R}{I_I}$, $p = \frac{R}{X}$

In order to optimize the performance index and to obtain the optimum location of the capacitor on the feeder to give the minimum losses, the first partial derivatives of "a" and "k" are then:

$$\frac{\partial \Delta P.I}{\partial a} = 6K\frac{V}{u} - 6aK\frac{V}{u} - 3K^2\frac{w}{u} + 2\alpha\left\{\frac{Kw}{V(1+hp)}\right\} \tag{4.21}$$

Similarly

$$\frac{\partial \Delta P.I}{\partial K} = 6a\frac{V}{u} - 3a^2\frac{V}{u} - 6aK\frac{w}{u} + 2\alpha\left\{\frac{aw}{V(1+hp)}\right\} \tag{4.22}$$

Solving equations (21) and (22) simultaneously the values of 'a' and 'k' are obtained which are:

$$a = \frac{2}{3} + \frac{2\alpha}{9(1+hp)}\frac{(SF)}{(LF)^2} \tag{4.23}$$

$$K = \frac{2}{3}(LF) + \frac{2\alpha}{9(1+hp)}\frac{(SF)}{(LF)} \tag{4.24}$$

where SF = u/w and is defined as the ratio of average loss to peak loss. It can be observed from equations (23) and (24) that these are identical to equation (2:43) in the peak loss case when the load factor and the loss factor are to be taken equal to one.

CHAPTER V

ENERY LOSS EQUATION FOR TWO FIXED CAPACITORS

a. <u>Energy Loss Equation With Time Intervals And Loss And Load Factors</u>

The energy loss in the two capacitor case is shown in equation (1) below:

$$
\begin{aligned}
L = \int_0^a & \left[D_1 T_1 (1-x) - (I_{ca}+I_{cb})\right]^2 RT_1 + \left[D_2 I_I (1-x) - (I_{ca}+I_{cb})\right]^2 RT_2 \\
& + --- + \left[D_n I_I (1-x) - (I_{ca}+I_{cb})\right]^2 RT_n \, dx + \int_a^b \left[D_1 I_I (1-x) - I_{cb}\right]^2 RT_1 \\
& + \left[D_2 I_I (1-x) - I_{cb}\right]^2 RT_2 \, dx + -- \left[D_n I_I (1-x) - I_{cb}\right]^2 RT_n \, dx \\
& + \int_b^1 \left[D_1 I_I (1-x)\right]^2 RT_1 + \left[D_2 I_I (1-x)\right]^2 RT_2 + --- + \left[D_n I_I (1-x)\right]^2 RT_n \, dx
\end{aligned}
\qquad (5.1)
$$

where the location of the first capacitor from the source is "a" and "b" is the location of the second capacitor from the source. I_{ca} and I_{cb} are the rated capacitor currents in capacitors "a" and "b" respectively.

Equation (2) is the result of equation (1) after the operations have been performed in it.

$$
\begin{aligned}
L = & -2 I_{ca} I_I a R \left[D_1T_1 + D_2T_2 + --- + D_nT_n\right] + I_I I_{ca} a^2 R \left[D_1T_1 + \right. \\
& \left. + D_2T_2 + ---- + D_nT_n\right] + I_{ca}^2 a R \left[T_1 + T_2 + --- + T_n\right] + \\
& + 2 I_{ca} I_{cb} a R \left[T_1 + T_2 + --- + T_n\right] - 2 I_{cb} I_I b R \left[D_1T_1 + \right. \\
& \left. + D_2T_2 + --- + D_nT_n\right] + I_I I_{cb} b^2 R \left[D_1T_1 + D_2T_2 + - \right. \\
& \left. ---- + D_nT_n\right] + I_{cb}^2 b R \left[T_1 + T_2 + --- + T_n\right] \\
& + \frac{1}{3} I_I^2 R \left[D_1^2 T_1 + D_2^2 T_2 + ------ + D_n^2 T_n\right]
\end{aligned}
\qquad (5.2)
$$

The per- unit change in quadrature current energy loss caused by the affect of adding capacitors "a" and "b" with ratings I_{ca} and I_{cb} at locations "a" and "b" is found by subtracting the term not dependent on I_{ca} and I_{cb} (which gives the energy loss with no capacitor on the line) and is shown below.

$$\Delta L_{p.u.} = \frac{L_{before} - L_{after}}{L_{before}} \tag{5.3}$$

The energy loss before the application of capacitors at locations "a" and "b" is:

$$L_{before} = \int_0^a \left[D_1 I_I (1-x)\right]^2 RT_1 + \left[D_2 I_I (1-x)\right]^2 RT_2 + \cdots \left[D_n I_I (1-x)\right]^2 RT_n\,dx$$
$$+ \int_a^b \left[D_1 I_I (1-x)\right]^2 RT_1 + \left[D_2 I_I (1-x)\right]^2 RT_2 + \cdots + \left[D_n I_I (1-x)\right]^2 RT_n\,dx$$
$$+ \int_b^1 \left[D_1 I_I (1-x)\right]^2 RT_1 + \left[D_2 I_I (1-x)\right]^2 RT_2 + \cdots + \left[D_n I_I (1-x)\right]^2 RT_n\,dx \tag{5.4}$$

Equation (4) when solved leads to equation (5).

$$L_{before} = \frac{1}{3}\left[D_1^2 I_I^2 RT_1 + D_2^2 I_I^2 RT_2 + \cdots + D_n^2 I_I^2 RT_n\right]$$
$$= \frac{I_I^2 R}{3}\left[D_1^2 T_1 + D_2^2 T_2 + \cdots + D_n^2 T_n\right]$$
$$= \frac{I_I^2 RU}{3} \tag{5.5}$$

Substituting equations (5) and (2) in equation (3) results in the following equation:

$$\Delta L_{p.u} = 3\left[2ak_1\frac{V}{U} - a^2 k_1 \frac{V}{U} - a k_1^2 \frac{W}{U} - 2a k_1 k_2 \frac{W}{U} + 2k_2 b\frac{V}{U} - b^2 K_2 \frac{V}{U} - K_2^2 b\frac{W}{U}\right] \tag{5.6}$$

The optimum location for any capacitor to result in minimum loss is found by setting the first partials of "a", "k_1", "k_2" and "b" equal to zero, as was shown in the peak loss case for two capacitors.

$$\frac{\partial \Delta L}{\partial a} = 3\left[2k_1\frac{V}{u} - 2ak_1\frac{V}{u} - k_1^2\frac{w}{u} - 2k_1k_2\frac{w}{u}\right] = 0 \quad (5.7)$$

$$\frac{\partial \Delta L}{\partial k_1} = 3\left[2a\frac{V}{u} - a^2\frac{V}{u} - 2k_1a\frac{w}{u} - 2ak_2\frac{w}{u}\right] = 0 \quad (5.8)$$

$$\frac{\partial \Delta L}{\partial b} = 3\left[2k_2\frac{V}{u} - 2bk_2\frac{V}{u} - k_2^2\frac{w}{u}\right] = 0 \quad (5.9)$$

$$\frac{\partial \Delta L}{\partial k_2} = 3\left[2b\frac{V}{u} - 2ak_1\frac{w}{u} - b^2\frac{V}{u} - 2k_2b\frac{w}{u}\right] = 0 \quad (5.10)$$

Simplifying and solving equations (7) and (8) the values of "a" and "k" are found to be

$$a = k_1\frac{w}{V}\ ; \qquad k_1 = a\frac{V}{w} \qquad (5.11)$$

when substituted back into equation (8) and subtracted from (10) two values of "b" are obtained i.e.

$$b = 2k_1\frac{w}{V}\ ; \qquad b = k_1\frac{w}{V} \qquad (5.12)$$

These values are substituted in equation (9) and for "b" = $2k_1w/v$ the location of the first capacitor comes out to be greater than one and also k_2 comes out to be negative, hence this value of "b" is physically not realiable and is discarded for capacitor operations. The other values of "b" give good results which are:

$$k_1 = k_2 = \frac{2V}{5w} = \frac{2}{5}(LF) \qquad (5.13)$$

and $a = k_1 = b/2$. (5.14)

When the load factor is taken to be equal to one, then the results are identical to the results obtained in the peak loss of two capacitors.

b. Voltage Drop Along Feeder In Per Unit

The voltage drop in the two capacitor case involves the combination of capacitor currents at 'a' and 'b' and then subtracted from the reactive load currents to give the equation shown below:

$$V=\int_0^a \left[D_1 I_I(1-x)-(I_{ca}+I_{cb})\right] X T_1+\left[D_2 I_I(1-x)-(I_{ca}+I_{cb})\right] X T_2+ ---- +\left[D_n I_I(1-x)-(I_{ca}+I_{cb})\right] X T_n\,dx+\int_a^b \left[D_1 I_I(1-x)-I_{cb}\right] X T_1$$
$$+\left[D_2 I_I(1-x)-I_{cb}\right] X T_2+ ---- \left[D_n I_I(1-x)-I_{cb}\right] X T_n dx$$
$$+\int_b^1 \left[D_1 I_I(1-x)\right] X T_1+\left[D_2 I_I(1-x)\right] X T_2+ -- \left[D_n I_I(1-x)\right] X T_n dx$$
$$+\int_0^1 \left[D_1 I_R(1-x)\right] R T_1+\left[D_2 I_R(1-x)\right] R T_2+ -- \left[D_n I_R(1-x)\right] R T_n dx \tag{5.15}$$

After the indicated operations have been performed in equation (15) the result is shown in equation (16).

$$V=-I_{ca}\,a X\left[T_1+T_2+ ---- +T_n\right]-I_{cb}\,b X\left[T_1+T_2+ -- +T_n\right]$$
$$+\frac{I_I X}{2}\left[D_1 T_1+D_2 T_2+ -- +D_n T_n\right]+\frac{I_R R}{2}\left[D_1 T_1+D_2 T_2+ -- +D_n T_n\right] \tag{5.16}$$

The voltage drop before application of the capacitors is found by taking $I_{ca} = 0$ and $I_{cb} = 0$ which is then as shown:

$$V_{before}=\int_0^a \left[D_1 I_I(1-x)\right] X T_1+\left[D_2 I_I(1-x)\right] X T_2+ -- \left[D_n I_I(1-x)\right] X T_n dx$$
$$+\int_a^b \left[D_1 I_I(1-x)\right] X T_1+\left[D_2 I_I(1-x)\right] X T_2+ -- \left[D_n I_I(1-x)\right] X T_n dx$$
$$+\int_b^1 \left[D_1 I_I(1-x)\right] X T_1+\left[D_2 I_I(1-x)\right] X T_2+ -- \left[D_n I_I(1-x)\right] X T_n dx$$

$$+\int_0^1 \left[D_1 I_R(1-x)\right] RT_1 + \left[D_2 I_R(1-x)\right] + \cdots + \left[D_n I_R(1-x)\right] RT_n dx \qquad (5.17)$$

Integrating and evaluating the limits results in equation (18).

$$V_{before} = \frac{I_I X}{2}\left[D_1T_1 + D_2T_2 + \cdots + D_nT_n\right] + \frac{I_R R}{2}\left[D_1T_1 + D_2T_2 + \cdots + D_nT_n\right] \qquad (5.18)$$

The per unit voltage drop is then:

$$\Delta V_{p.u.} = \frac{V_{before} - V_{after}}{V_{before}} \qquad (5.19)$$

Substituting equation (16 and 18) in (19) will result in per unit values of the voltage drop

$$\Delta V_{p.u} = \frac{2 I_{ca} a X}{LF(I_I X + I_R R)} + \frac{2 I_{cb} b X}{LF(I_I X + I_R R)} \qquad (5.20)$$

If the load factor is taken to be equal to one then the voltage drop equation is similar to the peak loss equation in the two capacitor case.

c. <u>Performance Criterion Considering Energy Loss And Voltage Drop In Per Unit</u>

The performance criterion, as seen before combines the energy loss and the voltage drop which is as below:

$$\Delta P.I._{p.u} = \Delta L_{p.u} + \alpha (\Delta V)_{p.u} \qquad (5.21)$$

When the energy loss equation in per unit and voltage drop equation which is also in per unit are combines, the P.I. equation is:

$$\Delta P.I\ p.u = 3\Big[2ak_1\frac{v}{u} - a^2k_1\frac{v}{u} - ak_1^2\frac{w}{u} - 2ak_1k_2\frac{w}{u} + 2k_2 b\frac{v}{u} - b^2k_2\frac{v}{u} - k_2^2 b\frac{w}{u}\Big] + \alpha\Big[\frac{2I_{ca}aXw}{V(I_IX+I_RR)} + \frac{2I_{cb}bXw}{V(I_IX+I_RR)}\Big] \quad (5.22)$$

where: $v/u = L.F/S.F$, $K_1 = I_{ca_1}/I_I$, $K_2 = I_{ca_2}/I_I$.

To optimize the performance index the first partial derivative of 'a', 'k_1', 'k_2' and 'b' are found, and the optimum location of the capacitors on the feeder can be found by setting the following equations to zero.

$$\frac{\partial\Delta P.I}{\partial a} = 6k_1\frac{v}{u} - 6ak_1\frac{v}{u} - 3k_1^2\frac{w}{u} - 6k_1k_2\frac{w}{u} + \Big[\frac{2\alpha k_1 w}{V(1+hp)}\Big] \quad (5.23)$$

$$\frac{\partial\Delta P.I}{\partial k_1} = 6a\frac{v}{u} - 3a^2\frac{v}{u} - 6ak_1\frac{w}{u} - 6ak_2\frac{w}{u} + \Big[\frac{2\alpha a w}{V(1+hp)}\Big] \quad (5.24)$$

$$\frac{\partial\Delta P.I.}{\partial b} = 6k_2\frac{v}{u} - 6bk_2\frac{v}{u} - 3k_2^2\frac{w}{u} + \Big[\frac{2\alpha k_2 w}{V(1+hp)}\Big] \quad (5.25)$$

$$\frac{\partial\Delta P.I}{\partial k_2} = -6ak_1\frac{w}{u} + 6b\frac{v}{u} - 3b^2\frac{v}{u} - 6k_2\frac{w}{u} + \Big[\frac{2\alpha b w}{V(1+hp)}\Big] \quad (5.26)$$

Solving equation (23) and (24) simultaneously, then

$$a = \frac{k_1 w}{V} \quad (5.27)$$

Solving equations (27) in (24) and subtracting from (26), then two roots are obtained:

$$b = 2k_1\frac{w}{V} \ ; \ b = \frac{k_1 w}{V} \quad (5.28)$$

Also substituting values of 'b' in equation (25) yields two results. The result obtained when 'b' was taken to be equal to k_1w/v found to be greater than one. With $b = 2k_1w/v$, the following results are obtained:

$$K_1 = \frac{2}{5}(LF) + \frac{2\alpha}{15(1+hp)}\left(\frac{SF}{LF}\right) \qquad (5.29)$$

where L.F = v/w, S.F = u/w

$$a = \frac{2}{5} + \frac{2\alpha}{15(1+hp)} \frac{(SF)}{(LF)^2} \qquad (5.30)$$

and

$$b = \frac{4}{5} + \frac{4\alpha}{15(1+hp)} \frac{(SF)}{(LF)^2} \qquad (5.31)$$

Hence, if S.F and L.F are equal to one, then $k_1 = a = b/2$, which agrees with previous results in the peak loss case.

From equation (25) k_2 can be solved as shown below:

$$2\frac{v}{u} - 2b\frac{v}{u} - k_2\frac{w}{u} + \frac{2}{3}\alpha\left(\frac{w}{v(1+hp)}\right) = 0 \qquad (5.32)$$

$$K_2 = 2\frac{v}{w} - 2b\frac{v}{w} + \frac{2}{3}\alpha\left(\frac{u}{v(1+hp)}\right) = 0 \qquad (5.33)$$

Substituting $b = 2k_1 w/v$ will give the result:

$$K_2 = 2\frac{v}{w} - 4K_1 + \frac{2}{3}\alpha\left(\frac{u}{v(1+hp)}\right) \qquad (5.34)$$

Now k_2 can be substituted from equation (29) resulting in equation (34)

$$K_2 = 2\frac{v}{w} - 4\left\{\frac{2}{5}\frac{v}{w} + \frac{2\alpha u}{15v(1+hp)}\right\} + \frac{2\alpha u}{3v(1+hp)} \qquad (5.35)$$

$$K_2 = 2(L.F.) - \frac{8}{5}(L.F) - \frac{8\alpha}{15(1+hp)}\left(\frac{SF}{LF}\right) + \frac{2}{3(1+hp)}\left(\frac{SF}{LF}\right) \qquad (5.36)$$

where u/v = SF/LF

$$K_2 = \frac{2}{5}(L.F.) + \frac{2\alpha}{15(1+hp)}\left(\frac{SF}{LF}\right) \qquad (5.37)$$

The results of equation (29) and (37) are the same and hence it can be concluded that: $K_1 = K_2$

d. Size Of Capacitors For Maximum Loss Reduction

If a possible relationship is to be considered between the ratings of the fixed capacitors, where multiple capacitors are used on a primary feeder, the result in equation (38) clearly indicates that both capacitors must be of the same rating.

Let k_T = the total fixed capacitor rating,

then $k_1 = k_T/2$ (5.39)

and $k_2 = k_T/2$ (5.40)

hence $k_1 + k_2 = k_T$ (5.41)

CHAPTER VI

ENERGY LOSS EQUATION FOR "N" FIXED CAPACITORS

a. Energy Loss Equation For "N" Capacitors With Time Variations of Load

The general energy loss equation for fixed capacitors can be found be rewriting equation (5.1) for the two capacitor case for "N" capacitors, taking into account that for maximum loss reduction all capacitors should have the same rating I_c.

$$L = \int_0^{a_1} \left[\sum_{i=1}^{i=n} (D_i I_I - D_i I_I x - N I_c)^2 R T_i \right] dx +$$
$$\int_{a_1}^{a_2} \left[\sum_{i=1}^{i=n} (D_i I_I - D_i I_I x - (N-1) I_c)^2 R T_i \right] dx + ---$$
$$--+ \int_{a_N}^{1} \left[\sum_{i=1}^{i=n} (D_i I_I - D_i I_I x)^2 R T_i \right] dx \qquad (6.1)$$

Performing the required operations on equation (1) yield equation (2) which is of course the result after the capacitors have been applied:

$$L = \sum_{i=1}^{i=n} R T_i \Big\{ \Big[-2 D_i I_I I_c \left(a_1 - \frac{a_1^2}{2}\right) + I_c^2 a_1 \big(1 + 2(N-1)\big) -$$
$$- 2 D_i I_I I_c \left(a_2 - \frac{a_2^2}{2}\right) + I_c^2 a_2 \big(1 + 2(N-2)\big) + --$$
$$--- - 2 D_i I_I I_c \left(a_K - \frac{a_K^2}{2}\right) + I_c^2 a_K \big(1 + 2(N-K)\big)$$
$$--- - 2 D_i I_I I_c \left(a_N - \frac{a_N^2}{2}\right) + I_c^2 a_N (1) + \frac{D_i^2 I_I^2}{3} \Big\} \qquad (6.2)$$

The per unit change in quadrature current energy loss by the effect of adding 'N' capacitors with the same ratings is found as under:

$$\Delta L_{p.u} = \frac{L_{before} - L_{after}}{L_{before}} \qquad (6.3)$$

The energy loss before the application of 'N' capacitors is found to be:

$$L_{before} = \sum_{i=1}^{i=n} \frac{D_i^2 I_I^2 R T_i}{3} \qquad (6.4)$$

Substituting equations (4) and (2) in (3) will result in equation (5).

$$\begin{aligned}\Delta L_{p.u.} = 3\Big[&2K_c(SF)\left(a_1 - \frac{a_1^2}{2}\right) - K_c^2 a_1\left(\frac{SF}{LF}\right)\left(1+2(N-1)\right)\\ &+2K_c(SF)\left(a_2 - \frac{a_2^2}{2}\right) - K_c^2 a_2\left(\frac{SF}{LF}\right)\left(1+2(N-2)\right)\\ &- - - - - - - - - - - - - - -\\ &+2K_c(SF)\left(a_g - \frac{a_g^2}{2}\right) - K_c^2 a_g\left(\frac{SF}{LF}\right)\left(1+2(N-g)\right)\\ &- - - - - - - - - - - - - - -\\ &+2K_c(SF)\left(a_N - \frac{a_N^2}{2}\right) - K_c^2 a_N\left(\frac{SF}{LF}\right)(1)\Big]\end{aligned} \qquad (6.5)$$

The optimum locations for these capacitors to result in minimum loss is

found by setting the first partials of a_1, a_z, a_k, a_n, etc: equal to zero:

$$\frac{\partial \Delta L}{\partial a_1} = 6K_c(SF)(1-a_1) - 3K_c^2\left(\frac{SF}{LF}\right)\left\{1+2(N-1)\right\} = 0 \qquad (6.6)$$

$$a_1 = 1 - \frac{K_c}{2(LF)}(2N-1) \qquad (6.7)$$

$$\frac{\partial \Delta L}{\partial a_2} = 3\left[2K_c(SF)(1-a_2) - K_c^2\left(\frac{SF}{LF}\right)(2N-3)\right] = 0 \qquad (6.8)$$

$$a_2 = 1 - \frac{K_c}{2(LF)}(2N-3) \qquad (6.9)$$

Similarly, $$a_g = 1 - \frac{K_c}{2(LF)}\left\{1+2(N-g)\right\} \qquad (6.10)$$

and $$a_N = 1 - \frac{K_c}{2(LF)}(1) \qquad (6.11)$$

where k is actually k_c or the capacitive current per capacitor i.e.

$$K/N = K_c$$

where N = total number of capacitors.

b. Voltage Drop Along Feeder In Per Unit

The voltage drop in the 'N' capacitor case is the combination of capacitor currents at various locations of the capacitors, and is shown below in the equation:

$$V = \int_0^{a_1} \sum_{i=1}^{i=n} \left(D_i I_I - D_i I_I x - N I_c\right) X T_i \, dx + \int_{a_1}^{a_2} \sum_{i=1}^{i=n} \left(D_i I_I - D_i I_I x - (N-1) I_c\right) X T_i \, dx + \cdots + \int_{a_N}^{1} \sum_{i=1}^{i=n} \left(D_i I_I - D_i I_I x\right) X T_i \, dx + \int_0^1 \sum_{i=1}^{i=n} \left(D_i I_R - D_i I_R x\right) R T_i \, dx \qquad (6.12)$$

Integrating and evaluating with the proper limits, the result in equation (13) is obtained

$$V = -\omega I_c a_1 X - \omega X I_c a_2 - \cdots - \omega I_c a_K X - \cdots - \omega I_c a_N X + \frac{R I_R V}{2} + \frac{X I_I V}{2} \qquad (6.13)$$

The per unit voltage is as under:

$$\Delta V p.u = \frac{V_{before} - V_{after}}{V_{before}} \qquad (6.14)$$

The voltage drop before the capacitor application is found by the capacitor current = 0.

$$V_{before} = \frac{R I_R V}{2} + \frac{X I_I V}{2} \qquad (6.15)$$

Hence the p.u. voltage drop results in the following:

$$\Delta V\,\text{p.u} = \frac{2K_c}{LF(1+hp)}\left[a_1 + a_2 + \text{-----} a_K + \text{--} a_N\right] \tag{6.16}$$

c. <u>Performance Criterion Considering Energy Loss And Voltage Drop In Per Unit</u>

The performance index criterion combines the energy loss and the voltage drop p.u. which is as shown below:

$$\Delta P.I\,\text{p.u.} = \Delta L\,\text{p.u.} + \alpha(\Delta V)\,\text{p.u} \tag{6.17}$$

$$\begin{aligned}\Delta P.I\,\text{p.u} = {} & 6K_c(SF)\left(a_1 - \frac{a_1^2}{2}\right) - 3K_c^2 a_1\left(\frac{SF}{LF}\right)\left[1+2(N-1)\right] + \\ & + 6K_c(SF)\left(a_2 - \frac{a_2^2}{2}\right) - 3K_c^2 a_2\left(\frac{SF}{LF}\right)\left[1+2(N-2)\right] + \\ & + \text{-------------------------} \\ & + 6K_c(SF)\left(a_g - \frac{a_g^2}{2}\right) - 3K_c^2 a_g\left(\frac{LF}{SF}\right)\left[1+2(N-g)\right] + \\ & + 6K_c(SF)\left(a_N - \frac{a_N^2}{2}\right) - 3K_c^2 a_N\left(\frac{LF}{SF}\right)(1) + \\ & + \alpha\left[\frac{2K_c}{LF(1+hp)}\left(a_1 + a_2 + \text{--}\, a_g + \text{--}\, a_N\right)\right]\end{aligned} \tag{6.18}$$

To optimize the performance index the first partial derivatives of a_1, a_2, a_3, a_4, ----------a_k, ----------a_n, are taken and the optimum location of the capacitors on the feeder to give minimum losses is analyzed.

$$\frac{\partial\,\Delta P.I.}{\partial a_1} = 6K_c(SF)(1-a_1) - 3K_c^2\left(\frac{SF}{LF}\right)\left[1+2(N-1)\right] + \alpha\left[\frac{2K_c}{LF(1+hp)}(1)\right] = 0 \tag{6.19}$$

$$a_1 = 1 - \frac{K_c}{2(LF)}(2N-1) + \frac{\alpha}{3(LF)(SF)(1+hp)} \tag{6.20}$$

$$\frac{\partial \Delta P.I.}{\partial a_2} = 6k_c(SF)(1-a_2) - 3k_c^2\left(\frac{SF}{LF}\right)\left[1+2(N-2)\right] \qquad (6.21)$$

$$a_2 = 1 - \frac{k_c}{2(LF)}(2N-3) + \frac{\alpha}{3(LF)(SF)(1+hp)} \qquad (6.22)$$

Similarly for a_n, the result is:

$$a_N = 1 - \frac{k_c}{2(LF)} + \frac{\alpha}{3(LF)(SF)(1+hp)} \qquad (6.23)$$

where $k_c = k/n$.

CHAPTER VII

RESULTS

The loss reduction obtained by the installation of capacitors on the feeder circuit are influenced in some way or the other by certain factors. By increasing or decreasing the L.F., h, p, and k will result in the increase or decrease of the performance of the system.

Hence the following results obtained show the parameters and their influence on the system.

a. Energy Loss When Loss Factor And Load Factor Are Constant And For Various Values Of 'K'

1. One Capacitor Case

The energy loss equation as developed in chapter (4) equation (7) is:

$$\Delta L\, p.u. = 3\left[2aK\frac{v}{u} - a^2K\frac{v}{u} - aK^2\frac{w}{u}\right] \qquad (7.1)$$

Equation (1) can also be written as:

$$\Delta L\, p.u. = 3\left[2ak\left(\frac{LF}{SF}\right) - a^2k\left(\frac{LF}{SF}\right) - \frac{ak^2}{(SF)}\right] \qquad (7.2)$$

Load Factor (L.F.) and (S.F.) were taken to be maximum or equal to unity. Therefore, the loss equation becomes:

$$\Delta L\, p.u. = 3\left[2ak - a^2k - ak^2\right] \qquad (7.3)$$

By taking different values of 'k' ranging from zero to one, various equations can be formed which are given below:

$$K = 0.4 \qquad \Delta L\,p.u = 1.92a - 1.2a^2$$

$$k = 0.67 \qquad \Delta L\,p.u = 2.66a - 2.0a^2 \qquad (7.4)$$

$$k = 0.80 \qquad \Delta L\,p.u. = 2.88a - 2.4a^2$$

$$k = 1.0 \qquad \Delta L\,p.u. = 3.00a - 3.0a^2$$

Using values of 'a' ranging from zero to one the curve shown in figure (12) is obtained which shows the energy loss in the one capacitor case.

The curve for the optimum value of 'k' which is 0.67 is seen to be the maximum, which agrees the result that when one capacitor is installed it should be located at two-thirds of the distance from the beginning of the feeder. When the value of 'K' is below the optimum, then it can be seen that the maximum loss reduction occurs when the capacitor is located closer to the end of the line and is naturally smaller than the optimum. If the value of 'k' is taken to be greater than the optimum 'k' value, then it can be seen that the maximum loss reduction curve rises and now occurs when the capacitor is closer to the beginning of the line. By decreasing the loss and load factors the curves tend to flatten out.

2. Two Capacitor Case.

The energy loss equation for the two capacitor case as developed in chapter (5) equation (6) is:

$$\Delta L\,p.u. = 3\left[2k_1 a \frac{V}{u} - a^2 k_1 \frac{V}{u} - a k_1^2 \frac{w}{u} - 2a k_1 k_2 \frac{w}{u} + 2k_2 b \frac{V}{u} - b^2 k_2 \frac{V}{u} - k_2^2 b \frac{w}{u}\right] \qquad (7.5)$$

Equation (5) can be written in the following way be taking the loss and the load factors equal to unity.

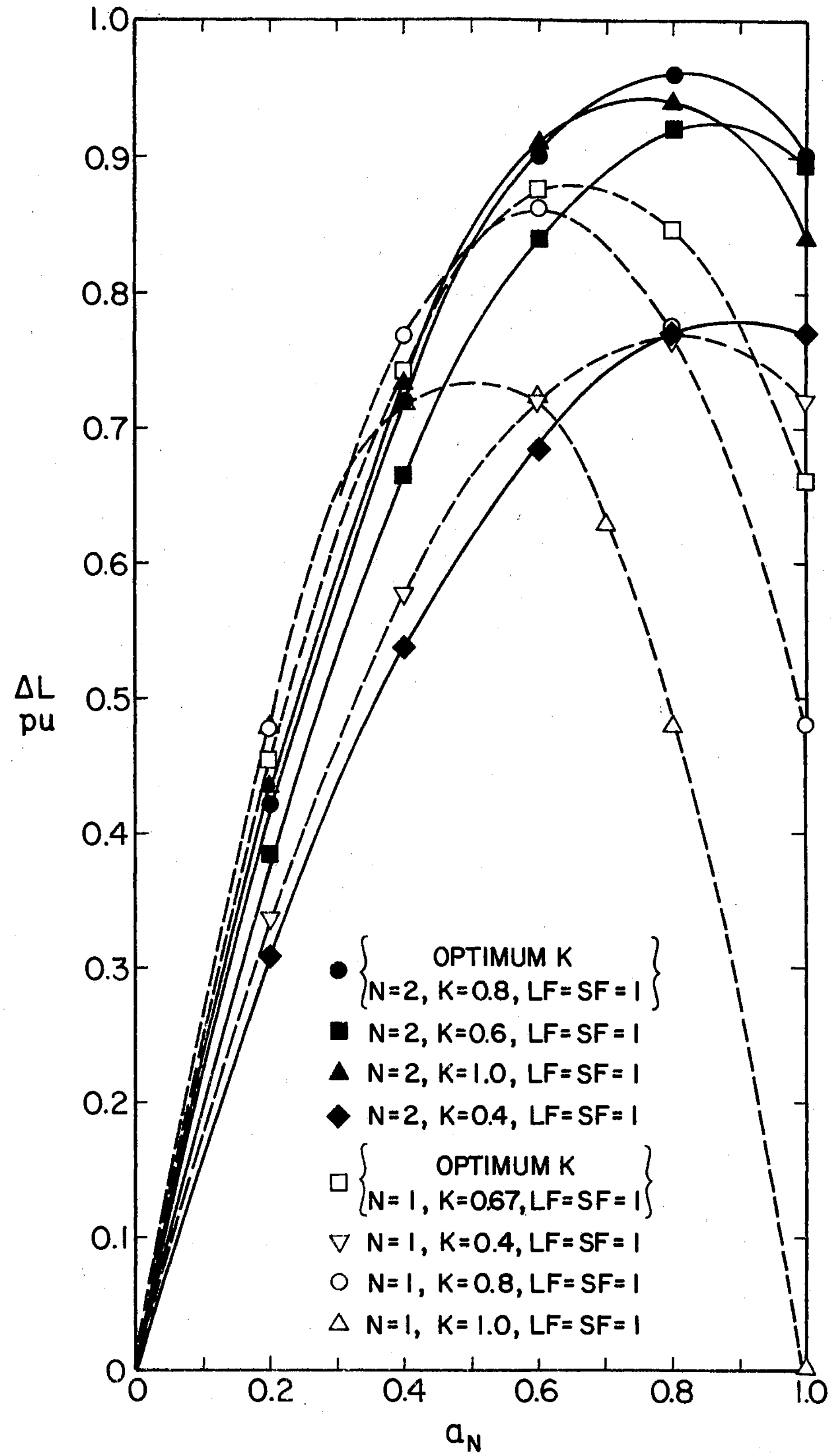


Figure 12. Peak loss and location of capacitors for various values of 'k'

$$\Delta L\,p.u. = 3\left[2ak_1 - a^2k_1 - ak_1^2 - 2ak_1k_2 + 2k_2b - b^2k_2 - k_2^2\right] \qquad (7.6)$$

In this case 'k' which is the ratio of the total capacitive current to the total reactive current is taken to be constant i.e.

$$k_1 = k_2 = k/2 \qquad (7.7)$$

To obtain the distance between the location of the capacitors a general equation was derived before, which is:

$$a_g = 1 - \frac{k_c}{2(LF)}\left[1 + 2(N-g)\right] \qquad (7.8)$$

where a_g is the location of a representative capacitor a_1, a_2, a_3, ---- a_n. From equation (8), a_1 and a_2 can be found which are:

$$a = a_1 = 1 - \frac{3k}{4} \qquad (7.9)$$

$$b = a_2 = 1 - \frac{k}{4} \qquad (7.10)$$

Hence,

$$\frac{a}{b} = \frac{4-3k}{4-k} \qquad (7.11)$$

or,

$$a = \left(\frac{4-3k}{4-k}\right)b \qquad (7.12)$$

Substituting these values in equation (6) the energy loss in obtained as:

$$\Delta L\,p.u. = 3\left[k\left(\frac{4-3k}{4-k}\right)b - \left(\frac{4-3k}{4-k}\right)^2 b^2\frac{k}{2} - \left(\frac{4-3k}{4-k}\right)\frac{bk^2}{4} - \left(\frac{4-3k}{4-k}\right)\frac{k^2b}{2} + kb - \frac{kb^2}{2} - \frac{k^2b}{4}\right] \qquad (7.13)$$

By giving different values to 'K' various values of ΔL can be obtained which are shown below:

$$\begin{aligned} K=0.4 \quad & \Delta L_{p.u.} = 1.731b - 0.954\,b^2 \\ K=0.6 \quad & \Delta L_{p.u.} = 2.172b - 1.278\,b^2 \\ K=0.8 \quad & \Delta L_{p.u.} = 2.40b - 1.50\,b^2 \\ K=1.0 \quad & \Delta L_{p.u.} = 2.50b - 1.66\,b^2 \end{aligned} \tag{7.14}$$

With the value of ΔL obtained the curve shown in figure (12) for N=2 is drawn by giving different values of 'b' ranging from b = 0 to b = 1.

The curve obtained when k = 0.4 shows that the furthest out capacitator is near the end of the line to maximize the loss reduction.

For two capacitors the optimum loss reduction occurs at a = 0.8 for k = 0.8. It can be observed that it is advisable to use one capacitor when the ratio of the capacitive current to the total reactive is less than 0.4, and if it is greater than this value, then the maximum loss reduction will be achieved by the installation of two capacitors on the feeder circuit.

b. <u>Voltage Drop In One Capacitor Case With "K" Fixed For Various Values Of "K".</u>

1. One Capacitor Case

The equation developed for one capacitor case in chapter (4) equation (18) is as follows:

$$\Delta V_{p.u.} = \frac{2 I_c a X}{LF(I_I X + I_R R)} \tag{7.15}$$

which may also be written as:

$$\Delta V_{p.u.} = \frac{2ak}{LF(1+hp)} \qquad (7.16)$$

Here 'h' and 'p' which are the ratios of I_R/I_I and R/X respectively are taken to be equal to one, and 'k' which is fixed is taken to be 2/3. Substituting these values in equation (16) yields the following:

$$\Delta V_{p.u.} = 0.66a \qquad (7.17)$$

Giving different values to 'a' which is the location of the capacitor if only one capacitor was to be located on the feeder circuit. Plotting the capacitor location versus the voltage drop on the feeder gives a straight line. The optimum amount of voltage improvement that can be obtained is when the capacitor is located at the furthest end of the feeder circuit. If the assumed values of 'h' and 'p' are taken to be less than one, than a better voltage improvement of the feeder circuit could be obtained.

Hence, as can be seen from the comparative study of figure (13), that if the ratio of the capacitive current to the inductive current was 0.67, it would be best to install one capacitor on the line. If the value of 'k' was decreased then the voltage improvement will not be as much as in the existing case plotted in figure (13). Conversely, if the value of 'K' is increased then the voltage improvement also rises.

2. Two Capacitor Case

The general voltage drop equation obtained is given below:

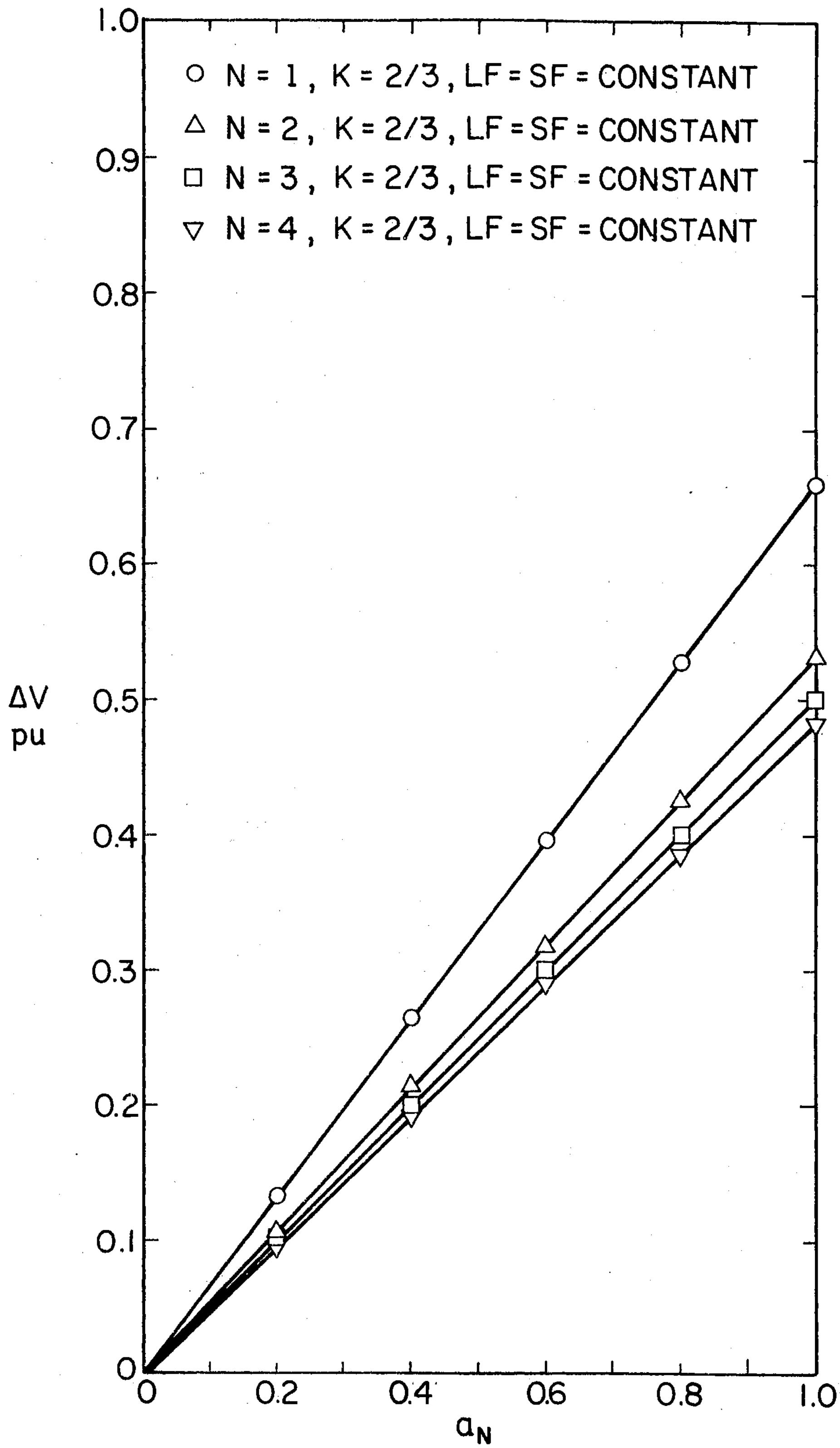


Figure 13. Voltage improvement and location of capacitors for fixed 'k'

$$\Delta V_{p.u.} = \frac{2K_c}{LF(1+hp)}\left[a_1 + a_2 + \text{---} + a_K + \text{--} a_N\right] \quad (7.18)$$

and the general equation for finding the location of the capacitors on the feeder circuit was found to be:

$$a_g = 1 - \frac{K_c}{2(LF)}\left[1 + 2(N-g)\right] \quad (7.19)$$

Therefore, calculating for both capacitors,

$$a_1 = 1 - \frac{3K}{4} \quad (7.20)$$

$$a_2 = 1 - \frac{K}{4} \quad (7.21)$$

Hence,

$$a_1 = \left(\frac{4-3K}{4-K}\right) a_2 \quad (7.22)$$

Substituting this equation in equation (18) gives the following:

$$\Delta V_{p.u} = \frac{K}{2}\left[\left(\frac{4-3K}{4-K}\right) a_2 + a_2\right] \quad (7.23)$$

As 'K' was fixed to be two-third yields equation (23) as:

$$\Delta V_{p.u.} = 0.53 a_2 \quad (7.24)$$

Giving different values to 'a_2' gives the straight line shown in figure (13). Here also the values of 'h' and 'p' were taken to be greater than one, then the voltage improvement would be less.

Figure (14) shows the case when 'k' is increased to 0.8, which clearly indicates that if 'k' is increased then the voltage improvement

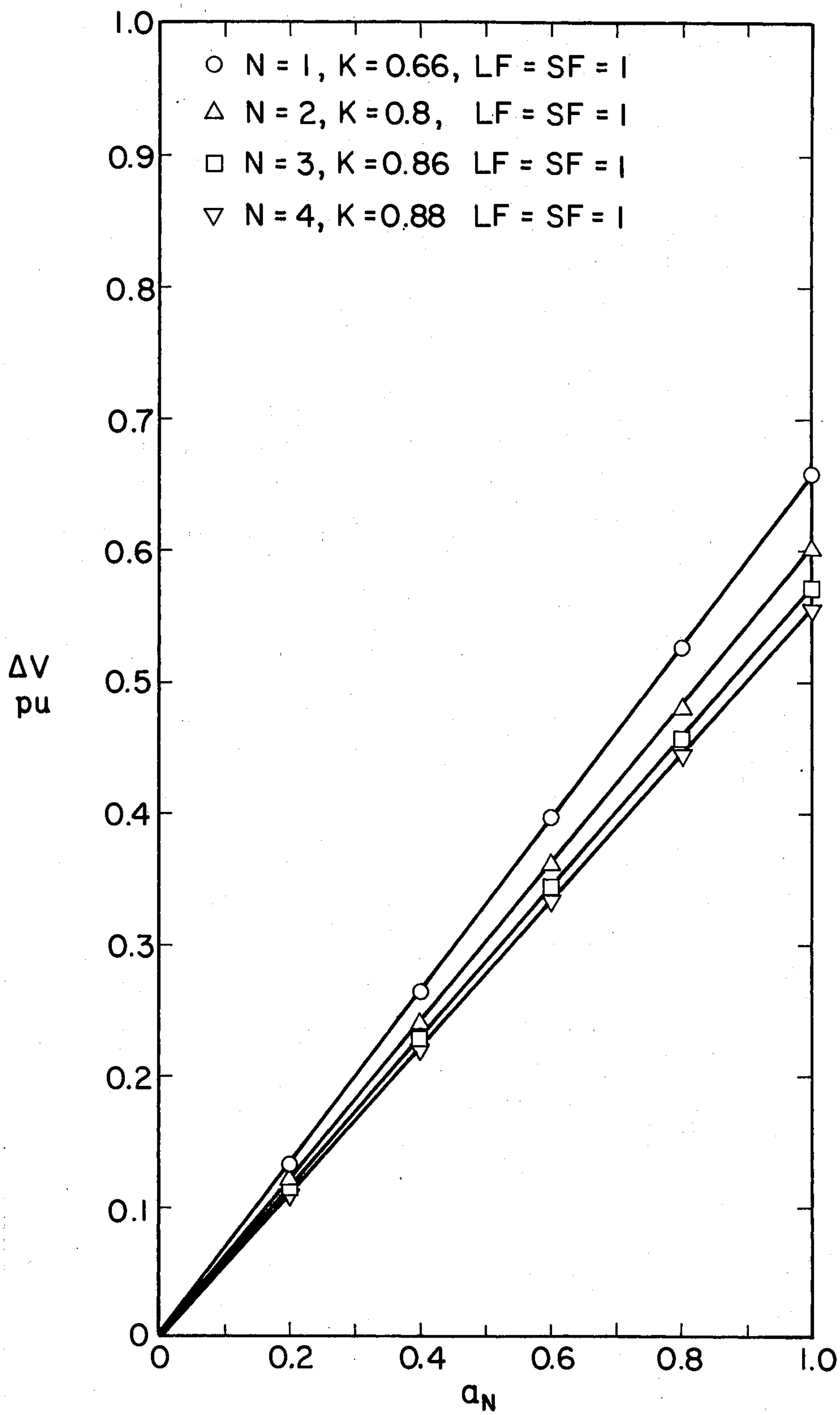


Figure 14. Voltage improvement and location of capacitors for various values of 'k'

also increases and holds true for Kmax $\leqslant$ 1. It is therefore advisable to use one capacitor which gives the best voltage improvement. If the load factor is decreased the voltage improvement would decrease. In fact the straight line curves of figures (2) and (3) depend on any one of these factors namely k, p, h, and the load factor.

3. Three Capacitor Case

The voltage drop equation for the nth case is as follows:

$$\Delta V_{p.u.} = \frac{2K_c}{LF(1+hp)}\left[a_1 + a_2 + \cdots a_K + \cdots a_N\right] \quad (7.25)$$

and the general equation for locating the capacitors is as under:

$$a_g = 1 - \frac{K_c}{2(LF)}\left[1 + 2(N-g)\right] \quad (7.26)$$

Hence,

$$a_1 = 1 - \frac{5K}{6} \quad (7.27)$$

$$a_2 = 1 - \frac{3K}{6} \quad (7.28)$$

$$a_3 = 1 - K/6 \quad (7.29)$$

Therefore,

$$a_1 = \left(\frac{6-5K}{6-K}\right)a_3 \quad (7.30)$$

$$a_2 = \left(\frac{6-3K}{6-K}\right)a_3 \quad (7.31)$$

Substituting these equations in equation (25) yields the following ΔV equation:

$$\Delta V_{p.u.} = \frac{K}{3}\left[\left(\frac{6-5K}{6-K}\right)a_3 + \left(\frac{6-3K}{6-K}\right)a_3 + a_3\right] \quad (7.32)$$

Since 'K' was fixed to be two-thirds, which on substituting in equation (12) gives $\Delta V = 0{\cdot}53$. (7.33)

Giving different values to 'a_3' gives the straight line graph in figures (13) and (14) for n = 3. Also h, p, and the load factor were taken to be equal to one. If h, p, or the load factor were to be less than one, then the voltage improvement would be less.

Comparison of figures (13) and (14) proves that when the value of 'k' is increased to 0.86 then the voltage improvement also increases.

4. Four Capacitor Case

The voltage drop equation for the nth case is:

$$\Delta V\, p.u. = \frac{2Kc}{LF(1+hp)}\left[a_1 + a_2 + a_3 + \cdots a_k + \cdots a_N\right] \quad (7.34)$$

and the general formula for obtaining the location of the capacitors can be found as:

$$a_g = 1 + \frac{Kc}{2(LF)}\left[1 + 2(N-g)\right] \quad (7.35)$$

Therefore,

$$a_1 = 1 - \frac{7k}{8} \quad (7.36)$$

$$a_2 = 1 - \frac{5k}{8} \quad (7.37)$$

$$a_3 = 1 - \frac{3k}{8} \quad (7.38)$$

$$a_4 = 1 - \frac{k}{8} \quad (7.39)$$

Hence,

$$a_1 = \left(\frac{8-7k}{8-k}\right) a_4 \quad (7.40)$$

$$a_2 = \left(\frac{8-5k}{8-k}\right) a_4 \quad (7.41)$$

$$a_3 = \left(\frac{8-3k}{8-k}\right)a_4 \qquad (7.42)$$

Substituting these equations in equation (34) yields the voltage drop equation in terms of 'a_4' and 'k'. Since 'k' is taken to be fixed at two-thirds, on substitution yields equation (44).

$$\Delta V_{p.u.} = \frac{k}{4}\left[\left(\frac{8-7k}{8-k}\right)a_4 + \left(\frac{8-5k}{8-k}\right)a_4 + \left(\frac{8-3k}{8-k}\right)a_4 + a_4\right] \qquad (7.43)$$

$$\Delta V_{p.u.} = 0.484\, a_4 \qquad (7.44)$$

Substituting different values to 'a_1' from 0 to 1 yields the straight line graph of figure (13) for n = 4. The load factor, 'h' and 'p' were assumed to be equal to one, but if they were taken to be less than one then the voltage improvement will increase, and if 'h' and 'p' were taken to be greater than one then the voltage improvement will be less:

Figure (14) shows that when 'k' is increased 0.88 then the voltage improvement also increases.

c. Relation Of Optimum 'K' Versus The Energy Loss

Figure (15) shows the energy loss in per unit (ΔLpu) versus 'k' which is the ratio of the capacitive current to the reactive current on the feeder. The values of ΔL are the optimum obtained from figure (12) at different values of 'k' and are plotted in figure (15). For the one capacitor case the curve is rather symmetrical and the optimum occurs at 0.66. The curve obtained for n = 2 is rather smooth and decreases very slowly which indicates that the energy loss is less as compared to the one capacitor case. When n = 3 and n = 4 the curves are relatively nearer as compared to one and two capacitor case. Hence if there were infinite number of capacitors to be used on the line then the curve would

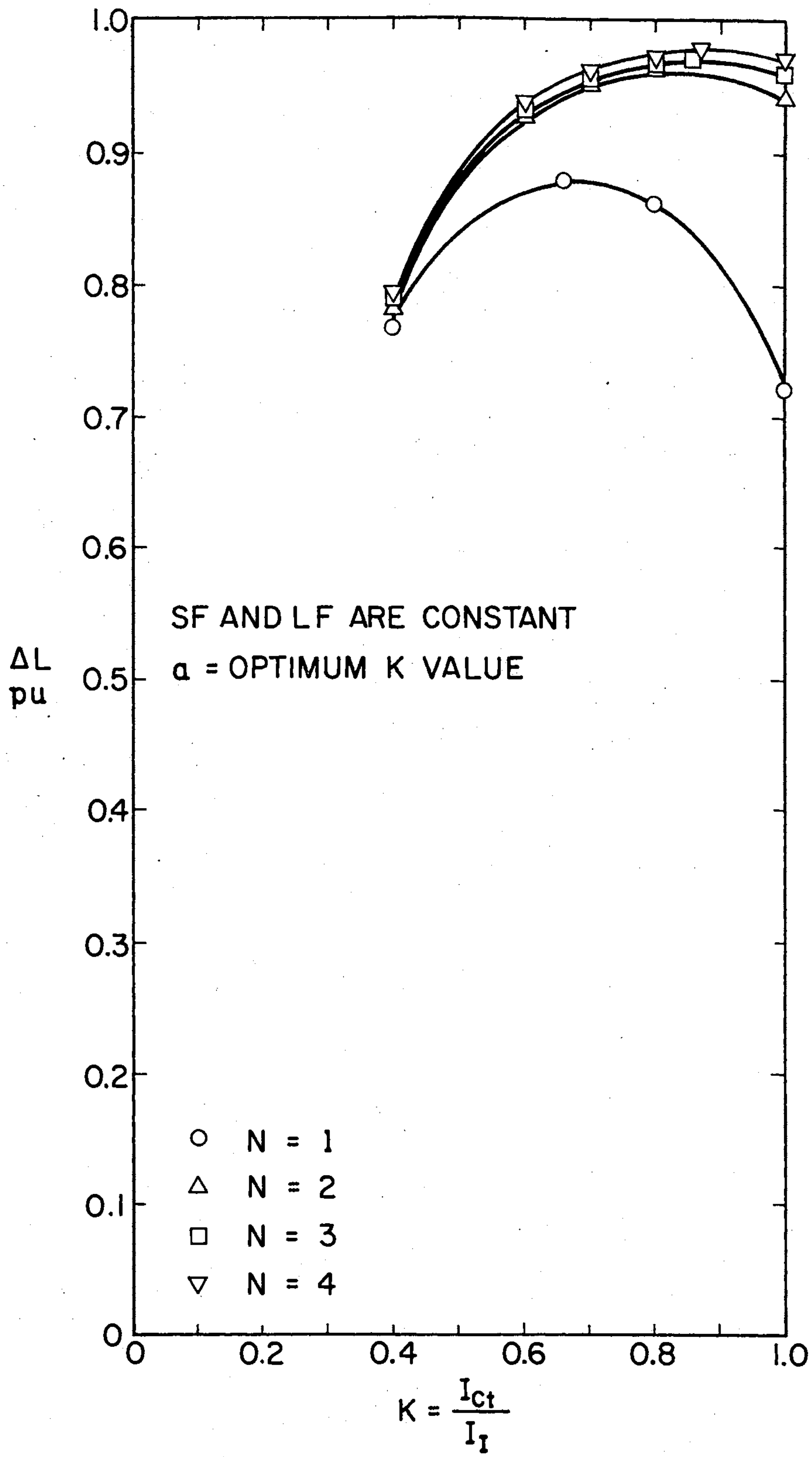


Figure 15. Peak loss versus 'k' with loss and load factors constant

touch the maximum loss improvement and tend to go towards one.

The loss and load factors are taken to be equal to one and 'a' values are taken whenever the curve is optimum. Figure (16) shows the influence of dividing the capacitors into different locations. The number of capacitors are plotted against the energy loss which is somewhat stable in the three or four capacitor case. These curves were drawn in the case of optimum 'K', and 'a' values.

d. Performance Index In Per Unit

1. One Capacitor Case

The performance index equation is given as:

$$\Delta P.I.\ p.u. = \Delta L\ p.u. + \alpha\,(\Delta V)\ p.u. \qquad (7.45)$$

Hence for one capacitor the values of ΔPI p.u. = ΔL p.u. + ΔV p.u. are known for various values of 'a'. Let α be taken as equal to one

a	ΔL p.u.	ΔV p.u.	Σ = ΔPI p.u.	$\Sigma/2$
0•2	0•452	0•133	0•585	0•296
0•4	0•744	0•266	1•010	0•505
0•6	0•876	0•400	1•276	0•638
0•67	0•888	0•440	1•320	0•660
0•80	0•848	0•533	1•381	0•690
1•0	0•660	0•660	1•320	0•660

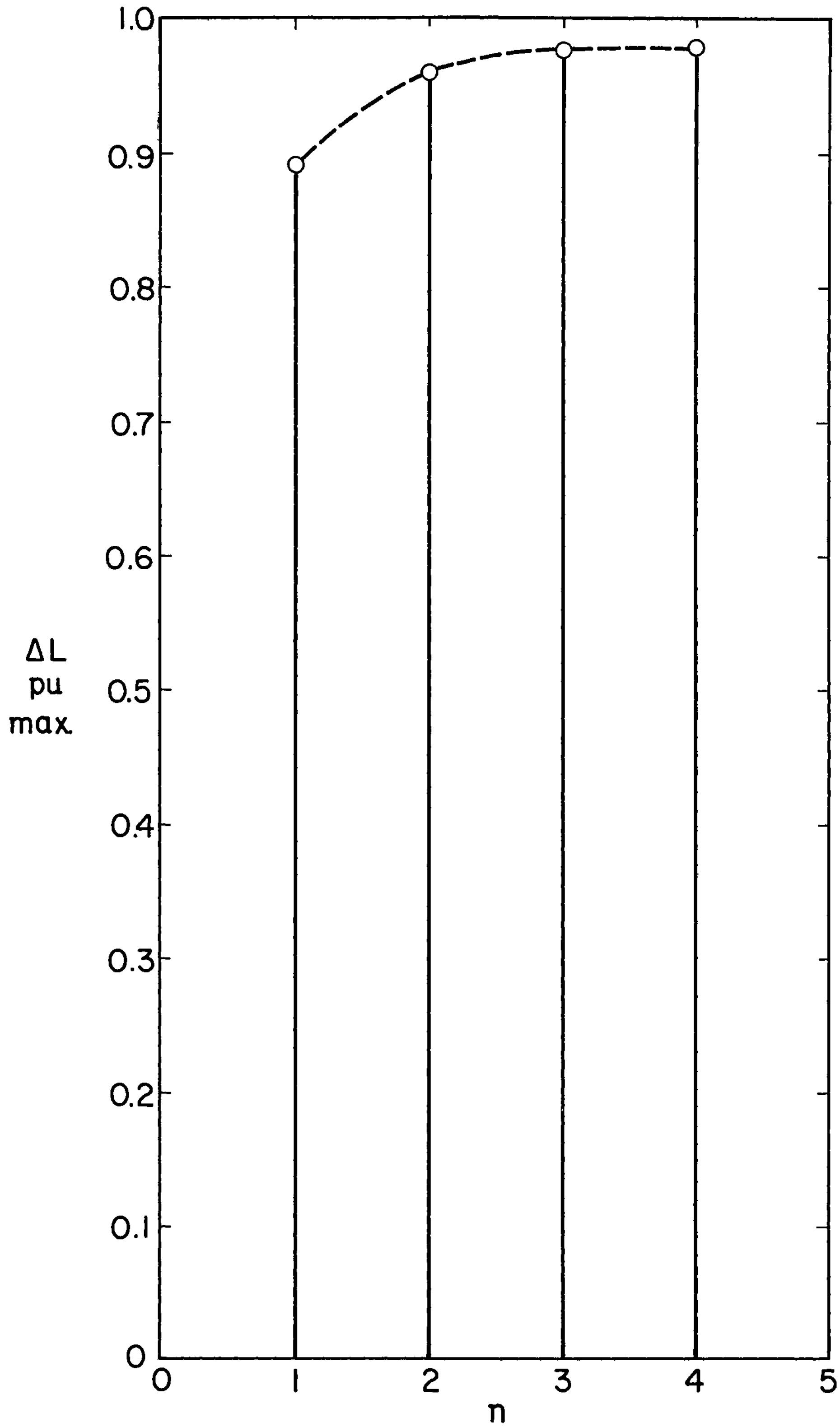


Figure 16. Influence of dividing capacitors into different locations versus peak loss

In this case the maximum value of ΔL and ΔV can be equal to one. Hence the equation becomes = $1 + \alpha$. Therefore, the performance index ΔPI p.u. maximum < 2. Verification of ΔPI mathematically can be achieved by the equation derived in chapter (6) equation (20).

$$a_1 = 1 - \frac{K_c}{2(LF)}(2N-1) + \frac{\alpha}{3(LF)(SF)(1+hp)} \qquad (7.46)$$

$$a_1 = 1 - \frac{K_c}{2(LF)}(2N-1) + \frac{\alpha}{3(1+hp)} \qquad (7.47)$$

$$a_1 = 0.833 \qquad (7.48)$$

Hence the location of the maximum performance index is found to be o.8333 and $\frac{\Delta PI \text{ maximum}}{2}$ is at 0.69. The performance index curve is shown in figure (17) along with the energy loss and voltage drop curves. In the P.I. curve α is taken to be equal to one and if it is taken to be less than one the P.I. curve tends to be lower than the present curve, and if α is taken to be more than one then the P.I. curve will be much more higher and the performance index would occur at a larger value of 'a'. On the other hand if h, p, L., F., and S., F., were taken to be less than one then it will be less.

Hence the location of $\Delta PI_{\text{maximum}}$ is satisfied mathematically and graphically.

2. Two Capacitor Case

The P.I. equation is given as under:

$$\Delta P.I \text{ p.u.} = \Delta L \text{ p.u.} + \alpha(\Delta V) \text{ p.u.} \qquad (7.49)$$

Let the value of α be equal to one as before and find the value of 'K' is at optimum value of 0.8 (L.F.)

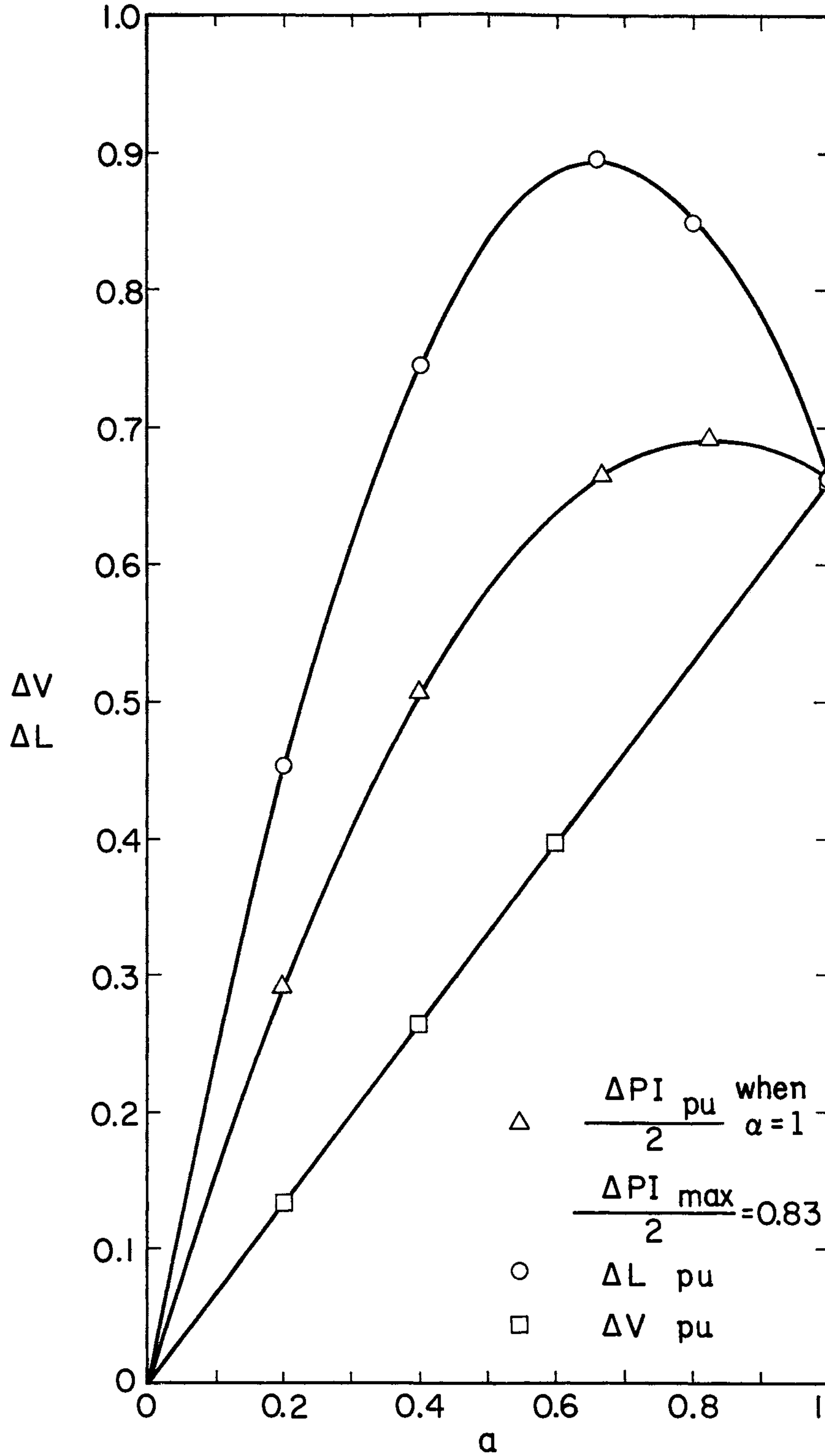


Figure 17. Peak loss, voltage improvement and the Performance Index curves plotted against the location of one capacitor on the feeder

a	ΔL p.u.	ΔV p.u.	Σ = ΔPI p.u.	Σ/2
0·2	0·42	0·12	0·54	0·27
0·4	0·72	0·24	0·96	0·48
0·6	0·90	0·36	1·26	0·63
0·8	0·96	0·48	1·44	0·72
1·0	0·90	0·60	1·50	0·75

Verification of ΔPI mathematically can be achieved by the equation derived in chapter (6) equation (21).

$$a_2 = 1 - \frac{K_c}{2(LF)}(2N-3) + \frac{\alpha}{3(LF)(SF)(1+R_p)} \qquad (7.50)$$

$$a_2 = 0{\cdot}966 \qquad (7.51)$$

Hence $PI_{maximum}$ occurs at $a_2 = {\cdot}966$ and has a vlue of 0.75.

Figure (18) shows the curves for the energy loss, the voltage drop and the P.I. curve for the case when 'K' is taken to be at its optimum value of 0.8. The loss and the load factors are taken to be equal to one and the curves would vary if they were different than this value. For the P.I. curve the value of α is taken to be equal to one and if it was taken to be below this value then the curve would have been lower than that shown, and if α is more than one, then the curve would have been higher.

The optimum value of , at which optimum P.I. occured was calculated both mathematically and graphically and came out to be the same.

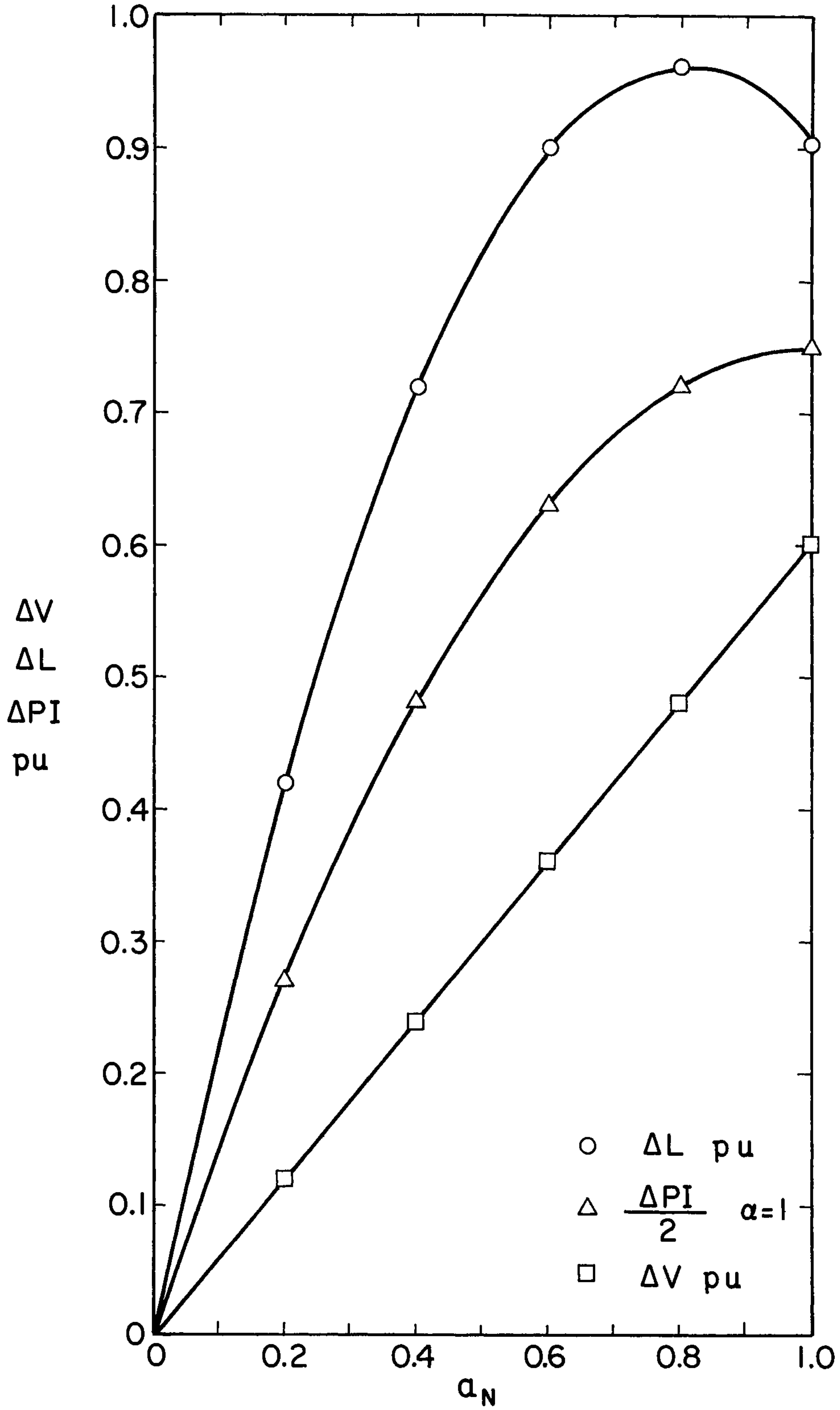


Figure 18. Peak loss, voltage improvement and the Performance Index curves plotted against the location of two capacitors on the feeder

CHAPTER VIII

EXAMPLE

The following example was furnished by Oklahoma Gas and Electric Company, and will be used to compare the procedure developed this thesis for loss reduction. Example: Determine the size and location of a three phase capacitor bank to improve the power factor from 79% to 99% for a certain feeder circuit 4.16 K.V. It will be assumed in the solution of this problem that the load is uniformly distributed throughout the length of the feeder. The size of the wire is also the same and no. 4AS8 was chosen which has a resistance of 0.483 Ω/1000 feet and inductive reactance of .1605 Ω/1000 feet.

The total load of 29 KVA is assumed which when multiplied with the assumed power factor gives 232 KW load. The desired power factor of 0.99 yields a correction factor of 0.634 (obtained from O.G.E. catalog) and hence gives the total Kvar.

$$\text{Total Kvar} = 0.634 \times 232 = 150 \text{ Kvar}$$

In this thesis, for multiple capacitors, all the capacitors were found to be of the same rating, hence for a three capacitor case each capacitor has a rating of 50 Kvar.

The general formula for the location of the three capacitors on a line 7000 feet long is:

$$a_g = 1 - \frac{K_c}{2(LF)}\left[1 + 2(N-g)\right] \qquad (8.1)$$

Hence with L.F. = 1

$$a_1 = 1 - \frac{5k}{6} \qquad (8.2)$$

$$a_2 = 1 - \frac{3k}{6} \qquad (8.3)$$

$$a_3 = 1 - \frac{k}{6} \qquad (8.4)$$

Substituting $k = I_c/I_I$ gives the following results:

$$a_1 = 2086 \text{ ft}$$

$$a_2 = 4046 \text{ ft} \qquad (8.5)$$

$$a_3 = 6020 \text{ ft}$$

The voltage improvement as developed in equation (6.16) was found to be:

$$\Delta V \text{ p.u.} = \frac{2K_c}{LF(1+hp)} \left[a_1 + a_2 + a_3 + \cdots a_k + \cdots a_N \right] \qquad (8.6)$$

Taking L.F. = 1 and $h = I_R/ I_I = 32.8/24.7$, and $p = R/X = .438/.16$, gives equation (6), which is the voltage improvement to be equal to 0.405.

a. Voltage Drop Without Capacitors

In order to plot the voltage profile with and without the capacitors, the voltage drop needs to be calculated. Figure (19) shows the secondary line voltage with and without the capacitors.

First the voltage drop without the capacitors placed on the line is calculated. The basic voltage drop equation developed in chapter (1) is as follows:

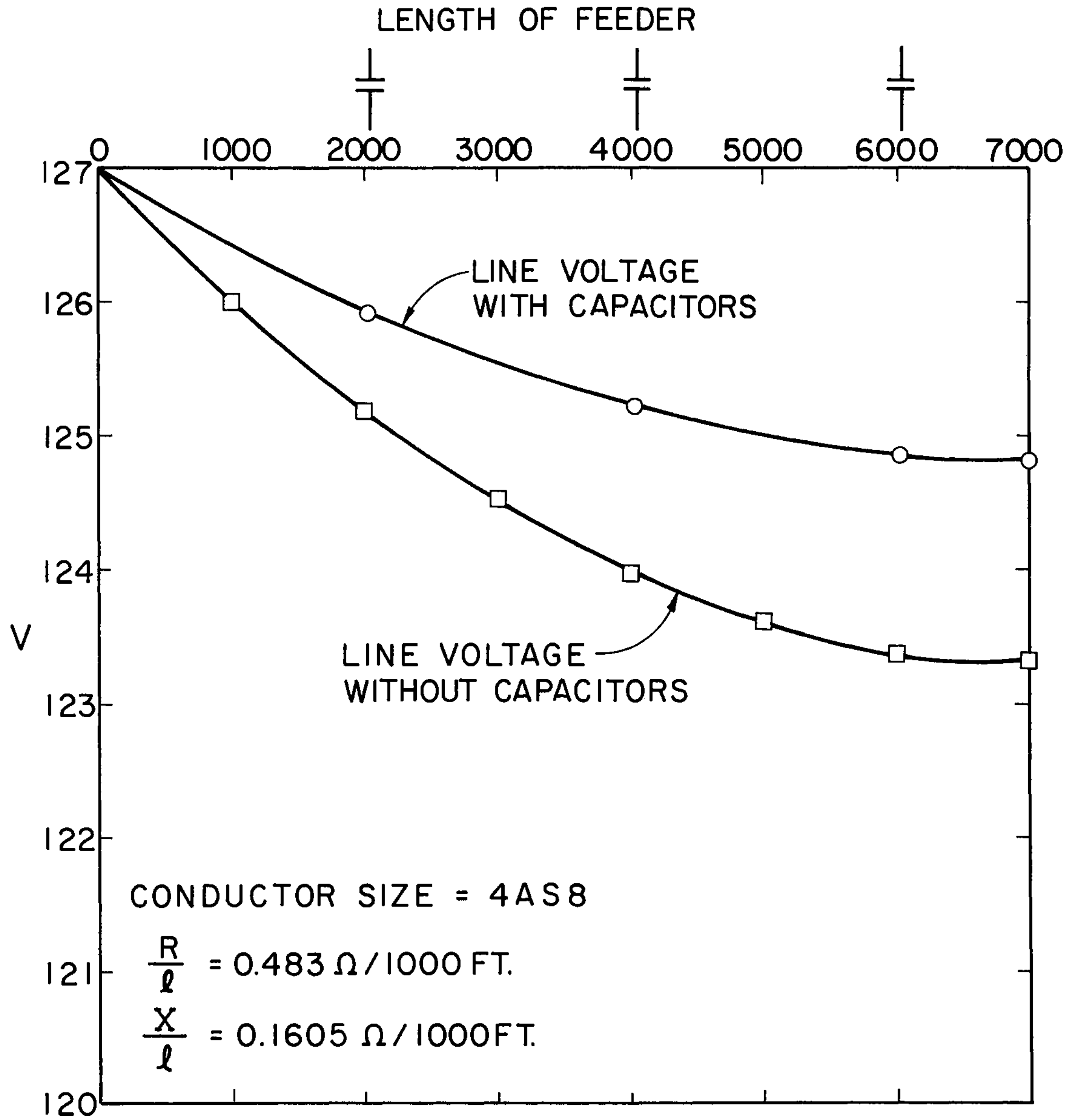


Figure 19. Line voltage with and without capacitors and the voltage improvement

$$V_{WITHOUT} = \int_0^S \left[\frac{I_I(l-x)}{l} \right] \frac{X}{l} dx + \int_0^S \left[\frac{I_R(l-x)}{l} \right] \frac{R}{l} dx \quad (8.7)$$

where $0 \leqslant S \leqslant l$, which when integrated and solved gives:

Voltage drop without capacitors:

$$V_{WITHOUT} = \frac{X}{l}$$

where 's' is any point along the line.

The capacitive current can be determined as:

$$I_I = \frac{Kvar}{\sqrt{3}(V_L)} = \frac{178\ Kvar}{\sqrt{3} \times 4.16 kV} = 24.7A \quad (8.9)$$

Taking the distances to be 1000 feet at each point and calculating equating (8) gives the voltage profile without capacitor application as shown in figure (19).

b. Voltage Drop With Capacitors

The voltage drop equations for each of the three capacitors are as follows:

$$V_{1WITH} = \int_0^S \left[\frac{I_I(l-x)}{l} - \left\{ I_{ca} + I_{cb} + I_{cc} \right\} \frac{X}{l} \right] dx + \int_0^S \left[\frac{I_I(l-x)}{l} \right] \frac{R}{l} dx \quad (8.10)$$

where $0 \leqslant S \leqslant S_1$

$$V_{2WITH} = \Delta V_{1WITH} + \int_{S_1}^S \left[\frac{I_I(l-x)}{l} - \left\{ I_{ca} + I_{cc} \right\} \frac{X}{l} \right] dx + \int_{S_1}^S \left[\frac{I_I(l-x)}{l} \right] \frac{R}{l} dx \quad (8.11)$$

where $S_1 \leq S \leq S_2$.

$$\Delta V_{3W} = \Delta V_{2W} + \int_{S_2}^{S}\left[\frac{I_I(l-x)}{l} - I_{cc}\right]\frac{X}{l}\,dx + \int_{S_2}^{S}\left[\frac{I_I(l-x)}{l}\right]\frac{R}{l}\,dx \qquad (8.12)$$

where $S_2 \leq S \leq S_3$

and also the voltage drop from the third capacitor to the end of the line is:

$$V_4 = \Delta V_{3W} + \int_{S_3}^{S}\left[\frac{I_I(l-x)}{l}\right]\frac{R}{l}\,dx \qquad (8.13)$$

where $S_3 \leq S \leq l$

Solving equation (10), (11), (12), and (13) give the voltage drops of the first, second, and third capacitors respectively which are:

$$\begin{aligned} V_{1W} &= 1.08\,V \\ V_{2W} &= 1.795\,V \\ V_{3W} &= 2.150\,V \\ V_{4W} &= 2.190\,V \end{aligned} \qquad (8.14)$$

Plotting these values on the graph as shown in figure (19) gives the voltage improvement which is also defined by the equation:

$$\Delta V = \frac{V_{before} - V_{after}}{V_{before}} = \frac{3.68 - 2.19}{3.68} = .405 \quad (8.15)$$

Hence the voltage improvement obtained graphically is found to be 0.405, while mathematically it was found to be 0.405, which proves that the equations developed were correct.

The method thus evolved in this thesis is comparable to the result obtained by O.G.E. and simpler use. It can be applied to non uniformally distributed loads as well as to any feeder with any wire size.

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APPENDIX A

APPENDIX A

ENERGY LOSS FOR 'N' CAPACITORS

$$L = \int_{0}^{a_1} \Big\{ \sum_{i=1}^{i=n} \left(D_i I_I - D_i I_I x - N I_c \right)^2 R T_i \, dx \; +$$

$$+ \int_{a_1}^{a_2} \sum_{i=1}^{i=n} \left(D_i I_I - D_i I_I x - (N-1) I_c \right)^2 R T_i \, dx + ----$$

$$+ \int_{a_n}^{1} \sum_{i=1}^{i=n} \left(D_i I_I - D_i I_I x \right)^2 R T_i \, dx$$

$$L = \sum_{i=1}^{i=n} \Big[D_i^2 I_I^2 a_1 - D_i^2 I_I^2 a_1^2 - 2 D_i I_I I_c N a_1 + \frac{D_i^2 I_I^2 a_1^3}{3}$$

$$+ D_i I_I I_c N a_1^2 + N^2 I_c^2 a_1 + D_i^2 I_I^2 a_2 - D_i^2 I_I^2 a_1$$

$$- D_i^2 I_I^2 a_2^2 + D_i^2 I_I^2 a_1^2 - 2 D_i I_I I_c N a_2 \; +$$

$$+ 2 D_i I_I I_c a N + 2 D_i I_I I_c a_2 - 2 D_i I_I I_c a_1 \; +$$

$$+ \frac{D_i^2 I_I^2 a_2^3}{3} - \frac{D_i^2 I_I^2 a_1^3}{3} + D_i I_I I_c N a_2^2 \; -$$

$$- D_i I_I I_c a_1^2 + N^2 I_c^2 a_2 - N^2 I_c^2 a_1 - 2 N I_c^2 a_2$$

$$+ 2 N I_c^2 a_1 + I_c^2 a_2 - I_c^2 a_1 + ------$$

$$+ D_i^2 I_I^2 - D_i^2 I_I^2 a_N - D_i^2 I_I^2 + D_i^2 I_I^2 a_N^2 \; +$$

$$+ \frac{D_i^2 I_I^2}{3} - \frac{D_i^2 I_I^2 a_n^3}{3} \Big]$$

$$L = \sum_{i=1}^{i=n} \Big[D_i^2 I_I^2 a_2 - D_i^2 I_I^2 a_2^2 - 2 D_i I_I I_c N a_2 +$$
$$+ 2 D_i I_I I_c a_2 - 2 D_i I_I I_c a_1 + \frac{D_i^2 I_I^2 a_2^3}{3} + D_i I_I I_c N a_2^2$$
$$- D_i I_I I_c a_2^2 + D_i I_I I_c a_1^2 + N^2 I_c^2 a_2^2 - 2 N I_c^2 a_2$$
$$+ 2 N I_c^2 a_1 + I_c^2 a_2 - I_c^2 a_1 + --- + D_i^2 I_I^2 a_N +$$
$$+ D_i^2 I_I^2 a_N^2 + \frac{D_i^2 I_I^2}{3} - \frac{D_i^2 \cdot I_I^2 a_N^3}{3} \Big]$$

$$G.I = \sum_{J=0}^{n} \int_{a_J}^{a_{J+1}} \Big[D_i^2 I_I^2 (1-x)^2 - 2 D_i I_I I_c (1-x)(N-J)$$
$$+ I_c^2 (N-J)^2 \Big] dx$$

$$= \sum_{J=0}^{N} \Big[D_i^2 I_I^2 \left(x - x^2 + \frac{x^3}{3}\right) - 2 D_i I_I I_c \left(x - \frac{x^2}{2}\right)(N-J)$$
$$+ I_c^2 (N-J)^2 x \Big] \Big|_{a_J}^{a_{J+1}}$$

where $J = 0, 1, 2, --- N$ and $a_0 = 0,\ a_{N+1} = 1$

solving

$J = 0, \ell = 0$

where $\ell =$ lower limit and $u =$ upper limit

$J=0,u \quad - 2 D_i I_I I_c \left(a_1 - \frac{a_1^2}{2}\right) N + I_c^2 a_1 (N)^2$

$J=1,\ell \quad + 2 D_i I_I I_c \left(a_1 - \frac{a_1^2}{2}\right)(N-1) - I_c^2 a_1 (N-1)^2$

$J=1,u \quad - 2 D_i I_I I_c \left(a_2 - \frac{a_2^2}{2}\right)(N-1) + I_c^2 a_2 (N-1)^2$

--- ---------------------------

$J=N-1,\ell \quad + 2 D_i I_I I_c \left(a_{N-1} - \frac{a_{N-1}^2}{2}\right)(1) - I_c^2 a_{N-1} (1)$

$J=N-1,u \quad - 2 D_i I_I I_c \left(a_N - \frac{a_N^2}{2}\right)(1) + I_c^2 a_N (1).$

$J = N, l \qquad 0$

$J = N, u \qquad \dfrac{D_i^2 I_I^2}{3}$

$$L = \sum_{i=1}^{n} RT_i \sum_{J=1}^{n} -2D_i I_I I_c \left(a_J - \frac{a_J^2}{2}\right) + I_c^2 a_J (1+2(N-J)) + \sum_{i=1}^{n} \frac{D_i^2 I_I^2 RT_i}{3}$$

$$L_{after} = \sum_{i=1}^{n} RT_i \Big[-2D_i I_I I_c \left(a_1 - \frac{a_1^2}{2}\right) + I_c^2 a_1 \{1+2(N-1)\}$$
$$-2D_i I_I I_c \left(a_2 - \frac{a_2^2}{2}\right) + I_c^2 a_2 [1+2(N-2)]$$
$$+ --- 2D_i I_I I_c \left(a_K - \frac{a_K^2}{2}\right) + I_c^2 a_K [1+2(N-K)]$$
$$+ --- 2D_i I_I I_c \left(a_N - \frac{a_N^2}{2}\right) + I_c^2 a_N [1] + \frac{D_i^2 I_I^2}{3} \Big]$$

$$L_{before} = \sum_{i=1}^{n} \frac{D_i^2 I_I^2 R T_i}{3}$$

$$\Delta L_{p.u.} = \frac{L_{before} - L_{after}}{L_{before}}$$

Since, $\displaystyle\sum_{i=1}^{n} D_i^2 T_i = [D_1^2 T_1 + D_2^2 T_2 + --- + D_n^2 T_n] = u$

$$\sum_{i=1}^{n} T_i D_i = [D_1 T_1 + D_2 T_2 + -- + D_n T_n] = V$$

$$\Delta L_{p.u} = 3 \Big[2K(SF)\left(a_1 - \frac{a_1^2}{2}\right) - K^2 a_1 \left(\frac{SF}{LF}\right)[1+2(N-1)]$$
$$+ 2K(SF)\left(a_2 - \frac{a_2^2}{2}\right) - K^2 a_2 \left(\frac{SF}{LF}\right)[1+2(N-2)]$$
$$+ -----------------$$
$$+ 2K(SF)\left(a_K - \frac{a_K^2}{2}\right) - K^2 a_K \left(\frac{SF}{LF}\right)[1+2(N-K)]$$
$$+ 2K(SF)\left(a_N - \frac{a_N^2}{2}\right) - K^2 a_N \left(\frac{SF}{LF}\right)(1)$$

taking partial derivative with respect to a_1, a_2, a_k, an etc.

$$a_1 = 1 - \frac{K}{2(LF)}(2N-1)$$

$$a_2 = 1 - \frac{K}{2(LF)}(2N-3)$$

$$a_K = 1 - \frac{K}{2(LF)}\left[1+2(N-K)\right]$$

$$a_N = 1 - \frac{K}{2(LF)}(1)$$

VOLTAGE IMPROVEMENT

$$V_{WITH} = \int_0^{a} \sum_{i=1}^{i=n} \left(D_i I_I + D_i I_I x - N I_c\right) X T_i \, dx +$$

$$+ \int_{a_1}^{a_2} \sum_{i=1}^{i=N} \left(D_i I_I - D_i I_I x - (N-1) I_c\right) X T_i \, dx$$

$$+ - - - - - - - - - - - - - -$$

$$+ - - \int_{a_N}^{1} \sum_{i=1}^{i=n} \left(D_i I_I - D_i I_I x\right) X T_i \, dx +$$

$$+ \sum_{i=1}^{i=n} \int_0^1 \left[D_i I_R - D_i I_R x\right] R T_i \, dx$$

$$G.E = \sum_{J=0}^{i=n} X T_i \left\{ D_i I_I \left(x - \frac{x^2}{2}\right) - I_c x (N-J) \right\} \Bigg|_{a_J}^{a_{J+1}}$$

$$+ \sum_{i=1}^{i=n} R T_i \left\{ D_i I_R \left(x - \frac{x^2}{2}\right) \right\} \Bigg|_0^1$$

where $J = 0, 1, 2, - - - N$ and $a_0 = 0, a_{N+1} = 1$.

$$J = 0, 4 \quad \sum_{i=1}^{i=n} X T_i \left[D_i I_I \left(a_1 - \frac{a_1^2}{2}\right) - I_c a_1 (N) \right] +$$

$$+ \sum_{i=1}^{i=n} \frac{R T_i D_i I_R}{2}$$

$$J=0,\ell \quad \sum_{i=1}^{i=n} -XT_i\left[D_i I_I\left(a_0-\frac{a_0^2}{2}\right)+I_c a_0(N)\right]+\sum_{i=1}^{i=n}\frac{RT_i D_i I_R}{2}$$

$$J=N,\ell \quad \sum_{i=1}^{i=n} -XT_i\left[D_i I_I\left(a_N-\frac{a_N^2}{2}\right)+I_c a_N(0)\right]+\sum_{i=0}^{i=N}\frac{RT_i D_i I_R}{2}$$

$$J=N,u \quad \sum_{i=1}^{i=n} +XT_i\left[D_i I_I\left(a_{N+1}-\frac{a_{N+1}^2}{2}\right)-I_c a_{N+1}(0)+\right.$$

$$+\sum_{i=1}^{i=n}\frac{RT_i D_i I_R}{2}$$

$$V_{after}=\sum_{J=1}^{N} -\omega I_c X a_J+\frac{RI_R V}{2}+\frac{XI_I V}{2}$$

$$V_{after}=-\omega I_c X a_1-\omega X I_c a_2 - - - -\omega X I_c a_K$$

$$- - - - -\omega X I_c a_N+\frac{RI_R V}{2}+\frac{XI_I V}{2}$$

$$V_{before}=\frac{RI_R V}{2}+\frac{XI_R V}{2}$$

Hence,

$$\Delta V\ p.u.=\frac{2K}{LF(1+h_p)}\left[a_1+a_2+ - - - a_K+ - - +a_N\right]$$

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$$\Delta P.I\ p.u.=\Delta L\ p.u+\alpha(\Delta V)p.u.$$

$$\Delta P.I\,p.u.=6K(SF)\left(a_1-\frac{a_1^2}{2}\right)-3K^2a_1\left(\frac{SF}{LF}\right)\left[1+2(N-1)\right]$$

$$+6K(SF)\left(a_2-\frac{a_2^2}{2}\right)-3K^2a_2\left(\frac{SF}{LF}\right)\left[1+2(N-2)\right]$$

$$+ - - - - - - - - - -$$

$$+6K(SF)\left(a_K-\frac{a_K^2}{2}\right)-3K^2a_K\left(\frac{SF}{LF}\right)\left[1+2(N-K)\right]$$

$$+ 6K(SF)\left(a_N - \frac{a_N^2}{2}\right) - 3K^2 a_N \left(\frac{SF}{LF}\right)\left[1 + 2(N-K)\right]$$
$$+ \alpha \left[\frac{2K}{LF(1+hp)} \left\{a_1 + a_2 + \text{--} + a_K + \text{--}\ a_N\right\}\right]$$

Taking partials with respect to a_1, a_2, - - - -

$$a_1 = 1 - \frac{K}{2(LF)}(2N-1) + \frac{\alpha}{3(LF)(SF)(1+hp)}$$

$$a_2 = 1 - \frac{K}{2(LF)}(2N-3) + \frac{\alpha}{3(LF)(SF)(1+hp)}$$

$$a_N = 1 - \frac{K}{2(LF)} + \frac{\alpha}{3(LF)(SF)(1+hp)}$$

APPENDIX B

APPENDIX B

VOLTAGE DROP WITHOUT CAPACITORS

$$V_{WITHOUT} = \int_0^s \left[\frac{I_I(l-x)}{l}\right]\frac{X}{l}dx + \int_0^s \left[\frac{I_R(l-x)}{l}\right]\frac{R}{l}dx$$

where $0 \leq s \leq l$.

$$V_{WITHOUT} = \frac{I_I}{l}\left(lx - \frac{x^2}{2}\right)\frac{X}{l}\Big|_0^s + \frac{I_R}{l}\left(lx - \frac{x^2}{2}\right)\frac{R}{l}\Big|_0^s$$

$$= \frac{X}{l}\left[\frac{I_I}{l}\left(ls - \frac{s^2}{2}\right)\right] + \frac{R}{l}\left[\frac{I_R}{l}\left(ls - \frac{s^2}{2}\right)\right]$$

$$\frac{R}{l} =$$

$$\frac{X}{l} =$$

Ratio of primary to secondary $= \frac{4.16}{\sqrt{3} \times 127} \times 10^3 = 18.9 \text{ V}.$

VOLTAGE DROP WITH CAPACITORS.

$$V_{1\,WITH} = \int_0^s \left[\frac{I_I(l-x)}{l} - \left\{I_{ca} + I_{cb} + I_{cc}\right\}\right]\frac{X}{l}dx + \int_0^s \left\{\frac{I_I(l-x)}{l}\right\}\frac{R}{l}dx.$$

where $0 \leq s \leq s_1$

$$V_{1\,WITH} = \frac{X}{\ell}\left[\frac{I_I}{\ell}\left(\ell s_1 - \frac{s_1^2}{2}\right) - \left\{I_{ca} s_1 + I_{cb} s_1 + I_{cc} s_1\right\} + \frac{I_I R}{\ell}\left(\ell s_1 - \frac{s_1^2}{2}\right)\right]$$

where $0 \leq s \leq s_1$

$$V_{1\,WITH} = \frac{X}{\ell}\left[\frac{I_I}{\ell}\left(\ell s_1 - \frac{s_1^2}{2}\right) - \left\{I_{ca} s_1 + I_{cb} s_1 + I_{cc} s_1\right\} + \frac{I_I R}{\ell}\left(\frac{\ell s_1 - \frac{s_1^2}{2}}{\ell}\right)\right]$$

Similarly

$$V_{2\,WITH} = V_{1W} + \int_{s_1}^{s}\left[\frac{I_I(\ell-x)}{\ell} - \left\{I_{cb} + I_{cc}\right\}\right]\frac{X}{\ell}\,dx + \int_{s_1}^{s}\left[\frac{I_I(\ell-x)}{\ell}\right]\frac{R}{\ell}\,dx$$

where $s_1 \leq s \leq s_2$

$$V_{2W} = V_{1W} + \frac{X}{\ell}\left[I_I\left(\ell s_2 - \ell s_1 - \frac{s_2^2}{2} + \frac{s_1^2}{2}\right) - \right.$$
$$\left. - \left[\frac{I_{cb} s_2 - I_{cb} s_1 + I_{cc} s_2 - I_{cc} s_1}{\ell}\right] + \right.$$
$$\left. + I_I\left[\left(\frac{\ell s_2 - \ell s_1 + \frac{s_2^2}{2} + \frac{s_1^2}{2}}{\ell}\right)\right]\frac{R}{\ell}\right.$$

Also,

$$V_{3W} = V_{2W} + \int_{s_2}^{s}\left[\frac{I_I(\ell-x)}{\ell} - I_{cc}\right]\frac{X}{\ell}\,dx + \int_{s_2}^{s}\left[\frac{I_I(\ell-x)}{\ell}\right]\frac{R}{\ell}\,dx$$

where $s_2 \leq s \leq s_3$

$$V_{3W} = V_{2W} + \frac{X}{\ell}\left[\frac{I_I}{\ell}\left(\ell s_3 - \ell s_2 - \frac{s_3^2}{2} + \frac{s_2^2}{2}\right) - \left(I_{cc}s_2 - I_{cc}s_3\right)\right.$$
$$\left. + \frac{I_I}{\ell}\left[\left(\ell s_3 - \ell s_2 - \frac{s_3^2}{2} + \frac{s_2^2}{2}\right)\right]\frac{R}{\ell}\right.$$

Hence,

$$V_{4W} = V_{3W} + \int_{s_3}^{s}\left[\frac{I_I\,(\ell - x)}{\ell}\right]\frac{R}{\ell}\,dx$$

where $s_3 \leq s \leq \ell$.

VITA

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