SEQUENTIAL PROCESS CONTROL WITH MEASUREMENT
OF PROCESS VARIABLES USING
ATTRIBUTE SAMPLING

By

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PREFACE

Sequential inspection techniques, though widely used for the control of quality, have had limited use for control of process parameters. This study is concerned with providing a methodology for the use of attribute inspection procedures for the detection and consequent estimation of changes in process parameters when such changes occur. Different models are suggested and a detailed study of one of the models is made. The theoretical foundation of the model and suggestion for practical use in a manufacturing environment are given.

The author wishes to express his appreciation to his major adviser, Dr. M. Palmer Terrell, for his guidance and assitance throughout the study. His assistance and encouragement have been invaluable both in the development of the model and in the preparation of the final manuscript.

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NOMENCLATURE

α	- Probability of rejection
ASN	- Average sample size
$AOQ_{g,h}$	- A specific average outgoing quality
β	- Probability of acceptance
$df_{g,h}$	- Fraction of items beyond specification limits with process at quality level g,h
g	- The number of standard deviations shift of the process average
h	- The number of standard deviations increase in process dispersion
K	- Intercept of rejection plane on the YZ plane
L	- Intercept of accept plane on the XY plane
m	- Specified dimension or value, desired value
MSN	- Maximum sample size
NT	- A total number of paths, NT = NE + NR
NE	- A number of paths that have not touched a barrier
NR	- A number of paths that have touched a barrier
O.C.	- Operating characteristic
p	- Probability of a step toward the accept barrier
PA	- Probability of acceptance
PR	- Probability of rejection
PRH	- Probability of rejection on the reject oversize barrier
PRV	- Probability of rejection on the reject undersize barrier

q - Probability of a step toward the reject oversize barrier

r - Probability of a step toward the reject undersize barrier

R - Range of values, difference between highest and lowest value of a sample

SPC - Sequential Process Control

SPCM-I - Sequential Process Control Model I

SPCM-II - Sequential Process Control Model II

SPCM-III - Sequential Process Control Model III

SPCM-IV - Sequential Process Control Model IV

σ_x - Process standard deviation

σ' - Standard deviation of a statistically controlled process

t - An allowable variation from m

t_g - A variation from m in the go-no go gauge

$\bar{X}'$ - Process average

$Y_{i,j}^{(n)}$ - Number of paths to point (n, i, j) that have not touched a barrier

CHAPTER I

SEQUENTIAL PROCESS CONTROL

A Historical Background

"Classical" sampling methods are based on the concept of a rigidly fixed sample size, and considerable effort is directed to making the best possible use of the prescribed amount of data. Gradually, there has developed a general awareness that in many situations it would be valuable to have a flexible method of deciding on the sample size -- a method that would allow the statistician to take into account information obtained in the course of the investigation itself.

More or less routine procedures for assessing the quality of manufactured items, collectively known as "acceptance sampling", were being used in the 1920's. It was recognized that if a batch was very good or very bad a relatively small sample size would suffice to show this. Larger samples need be used only in borderline cases. Dodge and Romig (1929) introduced a "double sampling" procedure in which a first sample of a fixed, pre-assigned size was taken from each batch as a matter of routine and the results of the observations analyzed. On specified conditions, when a decision on the quality of the batch is not reached, a second sample of a definite pre-assigned size is taken. This idea was developed by Bartley (1943) to include more than one optional sampling plan.

Another method of sampling, resulting in a variable sample size, drew considerable attention around the same time (1943). This method applies specifically to cases in which a "quantal" (0 or 1) variate is observed. In its simplest form, it calls for the total number of 1's observed rather than the total number of observations. Haldane (1945) advocated its use on the grounds that the unbiased estimation of the probability of occurrence'p' based on such samples have approximately constant coefficient of variation for all values of 'p'.

Starting with the work of A. Wald (1945), "Sequential Analysis" has developed into a well-established sector of statistical theory. Wald introduced the concept of sequentially sampling items, one at a time, from batches of mass-produced items. The only observation made is whether the item is "satisfactory" or "defective". A record of the observations is represented in a simple "inspection diagram". This contains two coordinate axes; one coordinate (usually measured in the X-axis) representing the number of items examined, the other representing the number of defective items observed. Any particular sequence of observations will then be represented by a path with vertical steps of unit height (when a defective item is observed) with the remainder of the path being horizontal. Sampling decisions are represented on the inspection diagram by three mutually exclusive regions: the acceptance region, the continuation region, and the rejection region. An acceptance boundary divides the acceptance and the continuation regions, and the rejection boundary divides the continuation and the rejection regions. When the result of an observation is plotted on the inspection diagram by a horizontal or a vertical line, the decision whether to continue sampling, accept, or reject the batch is determined by whether the line terminates in the continuation region, on the acceptance boundary, or on the rejection boundary.

After a short period of relative quiescence, "Sequential Analysis" began in the 1950's to attract a steadily increasing amount of interest. The first applications of sequential analysis were with reference to sampling inspection procedures but in spite of the large volume of work, there has been a distinct lack of work connected with its use in statistical control. Burr (1949) developed a truncated sequential inspection scheme; however, its later interest was focused on its truncation rather than its application to process control. More recently, Shamblin, Beightler, and Amstead (1964) developed the technique of Sequential Process Control for the control of the quality of a continuous manufacturing process. Sequential Process Control was based on sequential attribute inspection. The decision to accept or reject a sample was based on a certain attribute of the sample. However, the decision to change or continue with a process was based on a random walk in a closed region with acceptance and rejection barriers. Later Terrell and Beightler (1966) extended sequential process control techniques to include random walk in a closed region in three dimensions. This eliminated the disadvantage in the scheme proposed by Shamblin by providing control over the variance as well as the mean. However, this technique did not provide any measure of the deviations of the mean and the variance.

The Technique of Sequential Process Control

Though the concepts of Sequential Analysis were applied to attribute inspection and quality assurance, its application in the control of a process was first made by Burr (1949) who utilized the concepts in variables sampling. The inherent advantage of a small average sample size lent itself to the control of a process setting for a fluctuating process mean. Burr's scheme of inspection set confidence limits for variations in process average for specific sampling plans. Thus, for defined confidence limits, sequential sampling plans were provided having average sample numbers less than those for fixed sample size plans. The method showed the tendency of the process to produce 'oversize' or 'undersize' components. The disadvantages of the method are the need for a measurement each time and the necessity of maintaining a cumulative algebraic sum by the inspector. It also does not provide for truncation or a maximum sample size.

In 1964, a technique of Sequential Process Control was developed by Shamblin, Beightler, and Amstead. The technique was based on an item-by-item attribute sampling procedure to control the process average of a continuous manufacturing process. Basically, Sequential Process Control (SPC) is a two-dimensional random walk in a region containing the origin and closed by boundaries for acceptance and rejection. The reject oversize boundary is defined by the line $Y = K + X$ where K is a constant. The reject undersize boundary is defined by the lines $Y = X - K$. The accept boundaries are defined by lines $X = L, Y > K$, $Y = L, X > K$.

A modified form of go-no go gauge is used for inspection. The gauge is set at a fixed nominal dimension 'm'. A step in the X-direction is taken every time the gauge indicates a dimension greater than 'm'. A step in the Y-direction is taken every time the gauge indicates a dimension less than 'm'. The random walk terminates when it is absorbed in one of the barriers and a decision is made on the process as to whether it is oversize, undersize, or acceptable.

The SPC technique provides sensitive control over the deviation of the process average. It has the advantage of a small average sample size and a finite and small maximum sample size. However, the gauge as designed fails to control variations in the variance. Moreover, even in the control of a process average it does not give any estimate of the shift in the process average. It merely indicates whether the process average has increased or decreased.

In his dissertation (1964), Ferguson introduced the concept of three-dimensional random walk. His model consists of a three-dimensional grid and movements within the grid are in the positive direction determined by whether or not the sample is:

(1) below lower tolerance limit (movement in Z-axis),

(2) above upper tolerance limit (movement in Y-axis),

(3) within tolerance limits (movement in X-axis).

Inspection is performed until the random walk is terminated on either of three barriers:

(1) reject undersize defined by the plane $Z = C_1$,

(2) reject oversize defined by plane $Y = C_2$,

(3) accept defined by the plane $X = C_3$.

The model (Figure 1) is similar to curtailed inspection schemes with curtailment occurring on absorption in any one of the three barriers.

Terrell et al. extended the concept of Sequential Process Control to include besides the control of a process average, the control of dispersion. It employs the three-dimensional random walk on a three-dimensional grid (Figure 2). A go-no go gauge was designed which

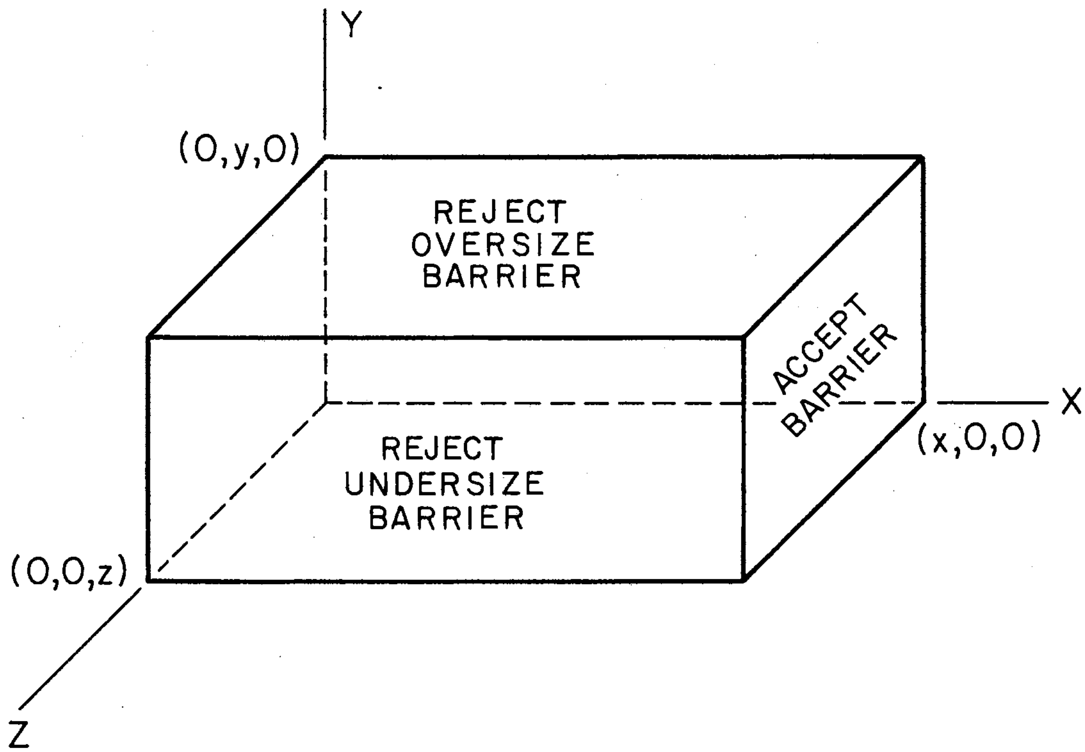

Figure 1. Reject Planes at 90°

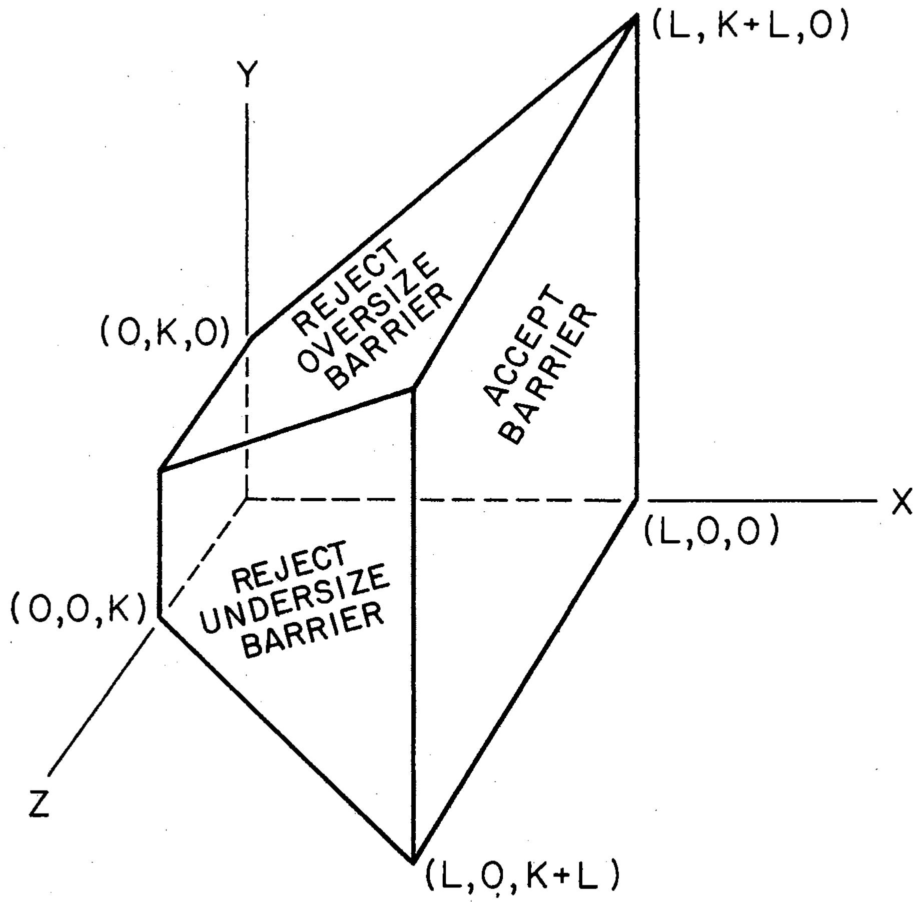

Figure 2. SPCM-I

classified the result of a test as one of three possible outcomes:

(1) reject oversize when the measurement is above the upper gauge limit,

(2) reject undersize when the measurement is below the lower gauge limit,

(3) accept when the measurement is within gauge limits.

The upper gauge limit was set as $m = t_g$, and the lower gauge limit was set at $m - t_g$, m being the process average and t_g arbitrarily set at $1/2\,\sigma$, σ being the standard deviation of a controlled process.

Movement within the three-dimensional grid is always in the positive direction determined by the outcome of the gauge test. The random walk is terminated when absorption occurs on any one of three barriers:

(1) reject oversize barrier defined by the plane $Z = K + X$,

(2) reject undersize barrier defined by the plane $Y = K + X$,

(3) accept barrier defined by the plane $X = L$.

This technique known as SPC-II was a significant improvement over SPC. Besides control over the process mean, it achieved control over the dispersion and utilized a small average sample size. A large number of sampling plans with different values for K and L were developed for different values of α and β.

However, SPC-II has the disadvantage of not being capable of indicating any estimate of a magnitude shift in the process average or standard deviation. Though sensitive to the shift in the parameters the plans do not yield a quantifiable estimate of the shift in process parameters. Moreoever, when the inspection is terminated on any one of the absorption barriers it cannot be determined whether the process mean has shifted or if there is a change in the standard deviation.

Research Objectives

The purposes of this thesis are:

(1) to study the adequacy of the model proposed by Terrell for the measurement of process paremeters,

(2) to suggest and study other models which might be adequate for parameter measurement, and which might reduce average sample number for defined confidence limits,

(3) to provide an operational procedure for Sequential Process Control decisions,

(4) to develop an estimation procedure for measurement of process parameters,

(5) to provide an operational method for estimating the changes in mean and variance yielding an "out of control" condition.

CHAPTER II

GAUGE LIMITS -- SPC MODELS

Introduction to Models

In this thesis, four models have been investigated and compared. Two of these models have been studied in depth. The other two were examined, not in great detail, but sufficiently in depth to arrive at certain conclusions, given later. The accompanying figures shown in three-dimensional representation illustrate how the absorption barriers have been set. The first model considered is the same as the one developed by Terrell et al. in SPC-II, the only modification being to redefine the tolerance limits in the go-no go gauge used. Originally the gauge limits were arbitrarily set at $m \pm {}^{1}\!/2\,\sigma$. In the modified model, the gauge limits have been set at $m \pm 0.431\sigma$. A detailed description of the logic involved is given in Chapter III. The modified SPC-II model will be referred to as SPCM-I. (See Figure 2, Chapter I.)

The second model to be investigated referred to as SPCM-II is shown in Figure 3. The barriers are fixed by the planes:

$Z = K + X$ (reject undersize),

$Y = K + Z$ (reject oversize),

$X = Z - K$ (accept),

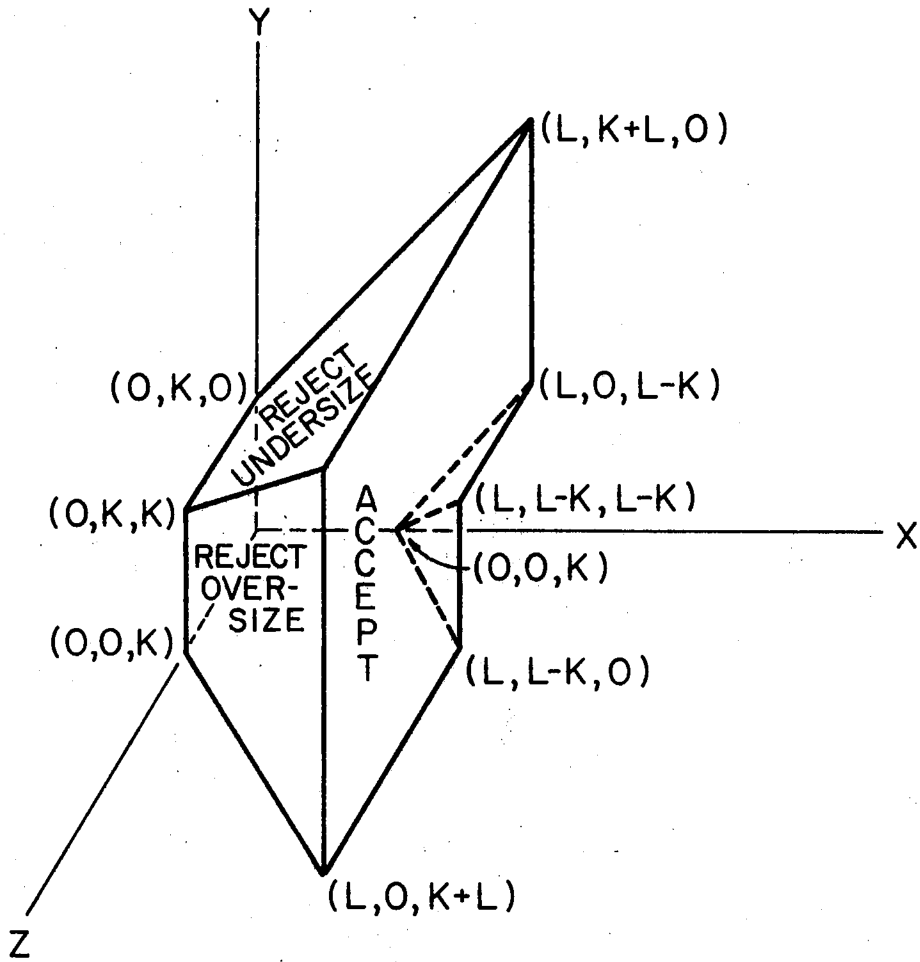

Figure 3. SPCM-II

$$Y = Z - K \quad \text{(accept)},$$

$$Z = L \quad \text{(accept)}.$$

For small values of the K/L ratio this model is a considerable improvement over SPCM-I in that the average sample size is considerably reduced with only a slight change in the operating characteristic curves. This occurs because acceptance is achieved faster for a process expected to be well within control.

The third model shown in Figure 4 is referred to as SPCM-III has boundaries defined as:

$$X + Y = K + Z \quad \text{(reject boundary)},$$

$$Z = L \quad \text{(accept boundary)}.$$

The gauge limits defined on this model are set at $m + 0.675\sigma$. Movement in the XYZ grid are made on the results of three possible outcomes:

$$M > m + 0.675\sigma \quad \text{(reject oversize)},$$

$$M < m + 0.675\sigma \quad \text{(reject undersize)},$$

$$m - 0.675\sigma < M < m + 0.675\sigma \quad \text{(accept)}.$$

Detailed comparisons of this model with the previous two are developed in considerable depth in later chapters. Here, it is sufficient to just describe the model.

The fourth model SPCM-IV shown in Figure 5 is similar to SPCM-III but with an added acceptance barrier. The boundaries in SPCM-IV are defined by the planes:

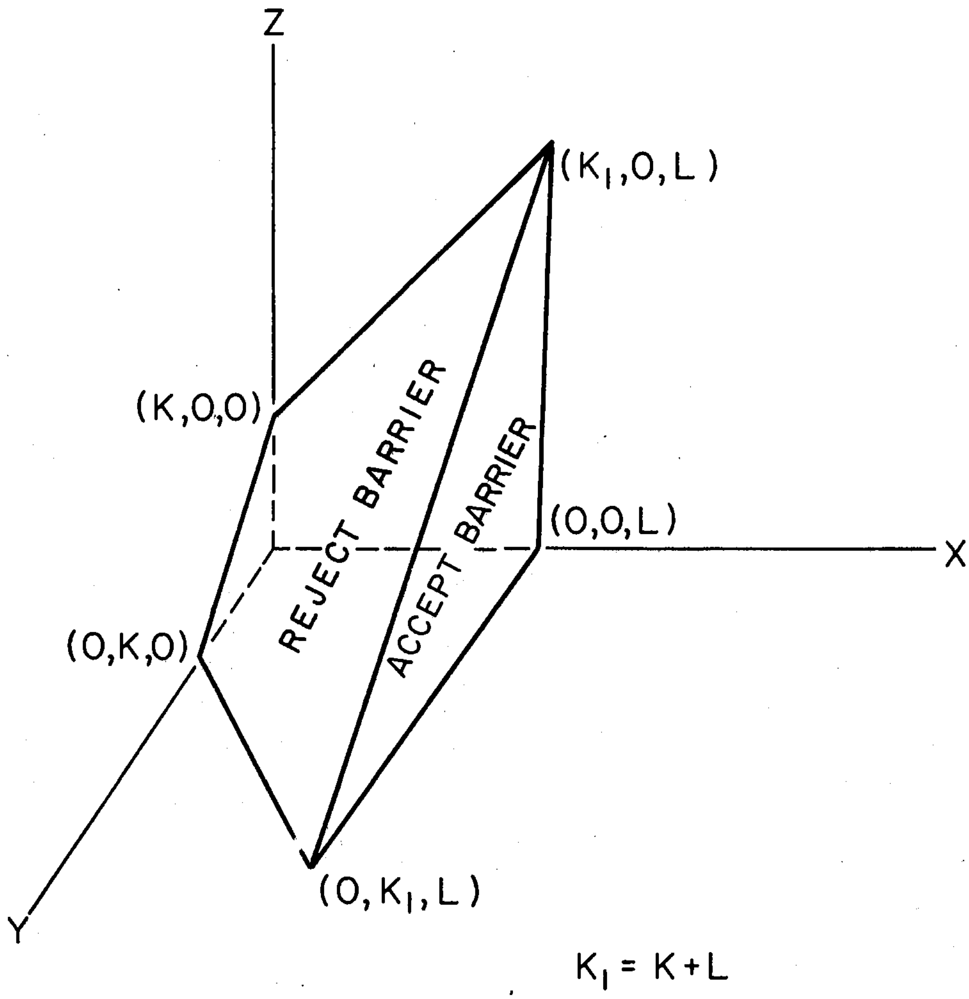

Figure 4. SPCM-III

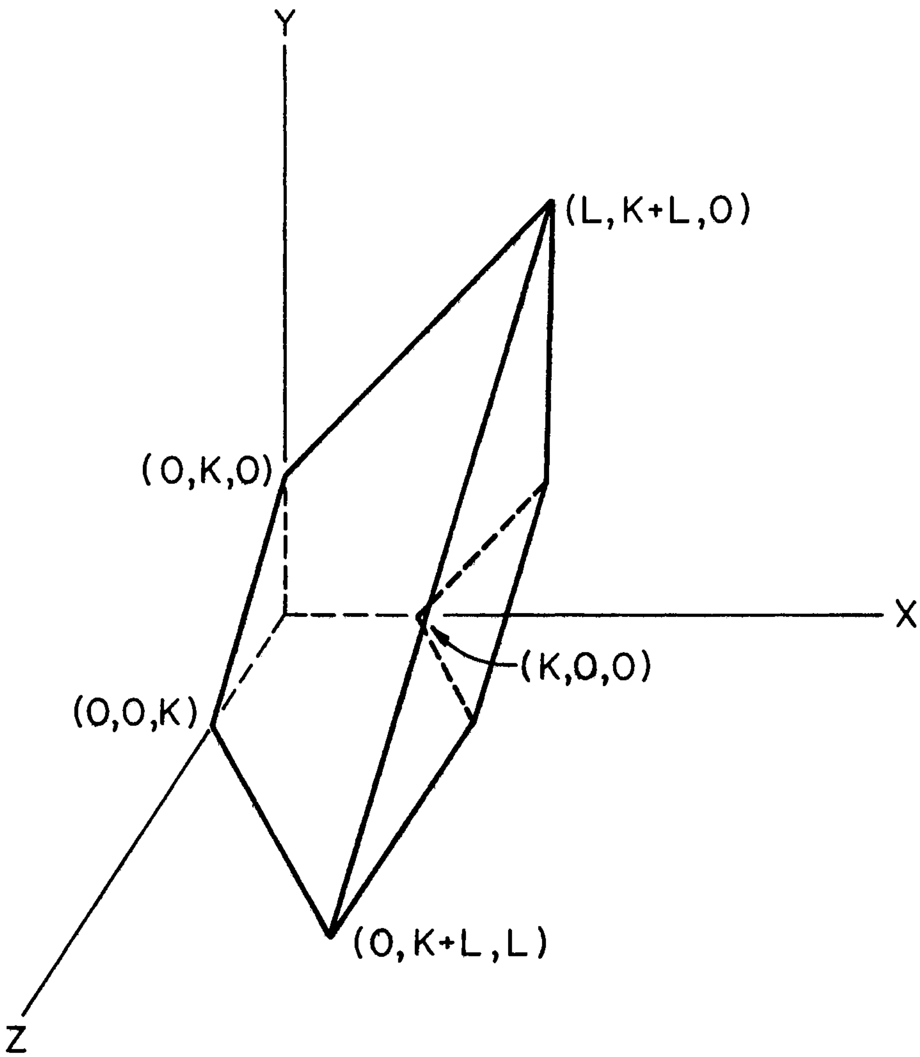

Figure 5. SPCM-IV

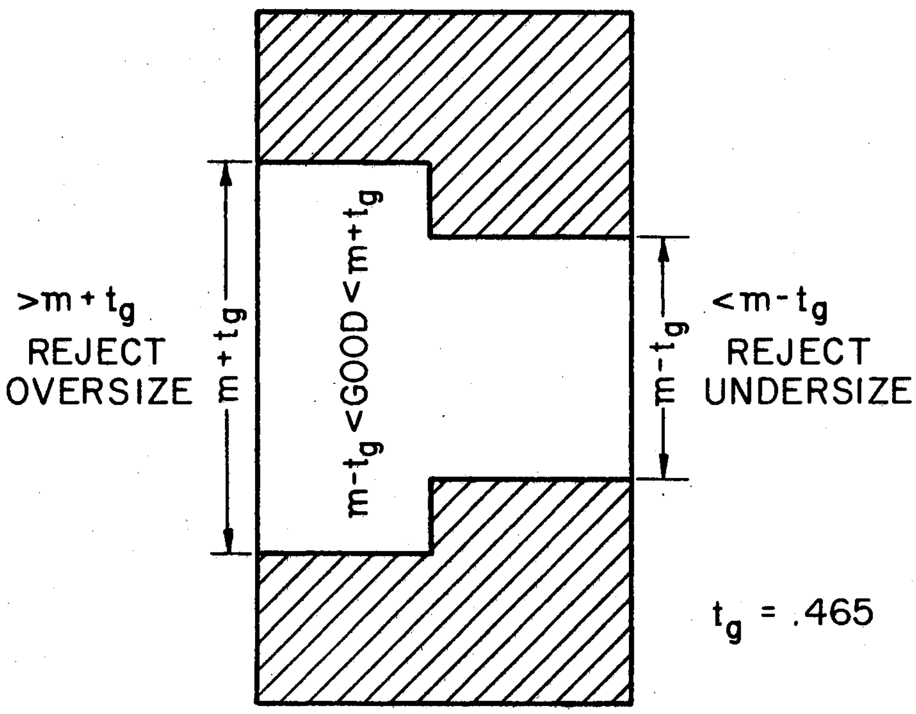

Figure 6. Go-No Go Gauge

$$X + Y = K + Z \text{ (reject)},$$

$$X + Y = Z - K \text{ (accept)},$$

$$Z = C \text{ (accept)}.$$

SPCM-IV is an improvement over SPCM-III in reducing average sample size only for K/L ratios smaller than 0.5.

Theoretical Foundation for Gauge Limits

General Considerations

One of the purposes of this dissertation is to establish more appropriate gauge specifications (Figure 6) so that there can be a more meaningful relationship between gauge specifications and rejection barriers. This goal must be achieved before one can continue by defining operating characteristic curves. The principle used in establishing the gauge specifications is this: that after any given number of experiments the distance to any of the rejection barriers is the same for a point of maximum likelihood when the process is in control. Consider a case when in the beginning of a sequence of experiments the path is set at the origin (0, 0, 0). If the path is to remain equidistant from any of the rejection barriers after three experiments, it should have reached (1, 1, 1), and after six experiments it should have reached the point (2, 2, 2). This is because the rejection barriers are inclined at a 45° angle to the axis. Now in designing the gauge specifications it should be such that the point of maximum likelihood after three experiments is (1, 1, 1) and after 3n experiments (n, n, n).

Specifications for SPCM-I and SPCM-II

For the specific cases of SPCM-I and SPCM-II this point of maximum likelihood has been derived.

The equations of the rejection barriers are given as:

$$Y = Z + K,$$

$$X = Z + K \ .$$

The point of maximum likelihood at the beginning of the sequence is the origin (0, 0, 0).

The smallest number of movements to reach both rejection barriers is K. Let X_i, Y_i, Z_i be the coordinates of a point after i experiments. Since it is symmetrically placed between the barriers after the sequence of experiments, the point of maximum likelihood would be such that

$$Z_i = Y_i \ .$$

$$\text{Now, } X_i + Y_i + Z_i = 3n \ .$$

The distance from the reject boundary is $(K + X_i) - Y_i$. This must be equal to K with

$$K + X_i - Y_i = K \ ,$$

$$X_i - Y_i = 0,$$

$$X_i = Y_i = Z_i \ .$$

By the maximum likelihood principle, the probability of movement in the X direction is given by the equation

$$p_i = \frac{Z_i}{X_i + Y_i + Z_i} = \frac{n}{3n} = 1/3 .$$

Similarly, the probability of movement in the Y direction is

$$q_i = \frac{Y_i}{X_i + Y_i + Z_i} = \frac{n}{3n} = 1/3$$

and the probability of movement in the Z direction is

$$r_i = \frac{X_i}{X_i + Y_i + Z_i} = \frac{n}{3n} = 1/3 .$$

Now in the design of the gauge the limits should be so set that the probability of reject oversize is equal to $1/3$, probability of reject undersize is $1/3$, and the probability of acceptance is $1/3$. For a normally distributed function, this would be at the 0.431σ points.

Thus the gauge specifications are specified as

$$m \pm 0.431\sigma$$

σ = standard deviation of the population.

Specifications for SPCM-III and SPCM-IV

The gauge specifications for models SPCM-III and SPCM-IV are as follows.

In this model, the points defining the reject barriers, starting from the origin, are given by

$$Z + Y = X + K.$$

Consider a sequence of 4n experiments starting from the origin such that a process in control satisfies

$$X_i + Y_i + Z_i = 4n \; .$$

The distance from the boundary is $(K + X_i) - (Z_i + Y_i) = K$. Solving for K yields

$$X_i = Z_i + Y_i \; .$$

Now $\quad Z_i = Y_i$ (being symmetrical),

$$X_i = 2n \; .$$

Then,

$$p_i = \frac{Z_i}{X_i + Y_i + Z_i} = 1/4; \; q_i = 1/4; \; r_i = 1/2 \; .$$

Thus,

the probability of a reject oversize = $1/4$,

the probability of a reject undersize = $1/4$,

the probability of acceptance = $1/2$.

The gauge limits for which these relationships hold are $m \pm 0.675$.

At this moment it would be in order to keep in mind certain important points:

The relationship of the gauge limits have been specified for a process with a normally distributed variate. The method can be used for other distributions as long as the gauge limits are so designed that the probability of a reject oversize = $1/3$ and the probability of reject undersize = $1/3$ for the models SPCM-I and SPCM-II; and corresponding probabilities of $1/4$ for the models SPCM-III and SPCM-IV.

Also, it should be emphasized that the σ used in the tolerance limits of the gauge refer to the population standard deviation for a process in statistical control. It does not refer to the specification tolerance limits to be satisfied by the process.

Having specified the tolerance limits on the gauge, an item is classified as reject oversize if its dimension is greater than $m + t_y \sigma$ and reject undersize if the dimension is less than $m - t_y \sigma$. This does not mean that the material should be rejected or that the process is out of statistical control. It just specifies that the item being representative of a subset of a total population has dimensions which are beyond the gauge tolerance limits. As a matter of fact, for a process under control, $2/3$ of the sample would be classified oversize or undersize by the gauge.

Relationship to Wald's Sequential Analysis

These same relationships can be derived by the principles of Wald's Sequential Analysis in the following manner.

Consider the two-dimensional plot of p and r for the models SPCM-I and SPCM-II

$$\bar{p} = \frac{p}{p + r} = \frac{1/3}{1/3 + 1/3} = 1/2 .$$

Using the equation established by Wald's Sequential Analysis, one obtains the expressions

$$S = \frac{\log \left(\frac{1 - p_1}{1 - p_2}\right)}{\log \left(\frac{p_2}{p_1}\right) \left(\frac{1 - p_1}{1 - p_2}\right)}$$

In the present case $p_1 = p$ and $p_2 = r$.

$$\text{Therefore, } p_1 = p_2 = 1/2 .$$

Substituting in the original expressions

$$S = \frac{\log\left(\frac{1 - 1/2}{1 - 1/2}\right)}{\log\left(\frac{1/2}{1/2}\right)\left(\frac{1 - 1/2}{1 - 1/2}\right)} = \frac{\log 1}{\log 1} = 1.$$

$$\text{Now } S = \frac{Y}{Z} = 1.$$

That is, when the probability of reject due to oversize is equal to the probability of acceptance, the slope of the reject barrier in the XZ axis is 45°.

The model thus in actuality is a truncated three-dimensional Wald's Sequential Analysis model.

A similar argument can be used to show the same results in the models SPCM-III and SPCM-IV, care being taken to observe that any point on the grid is always equidistant from the reject barrier both in the X and Y directions and there is only one rejection barrier. Thus, any rejection be it due to obersize or undersize is a step towards the rejection barriers in both the X and Y directions.

CHAPTER III

GRAPHICAL ANALYSIS OF SEQUENTIAL PROCESS CONTROL

Introduction

All the models consist of three-dimensional grids, with axes X, Y, and Z. A model has an origin point (0, 0, 0) and consists of only positive coordinates X, Y, and Z. Any movement in the three-dimensional grid results in the increment of one and only one of the coordinates. Thus, any movement from (1, 1, 1) would result in new coordinates (2, 1, 1), (1, 2, 1), or (1, 1, 2). The entire grid consists of three mutually exclusive sets of points, defined as follows:

(1) the continuous points consisting of points with at least one path from the origin and at least one path beyond,

(2) the boundary points consisting of points with at least one path from the origin and no path beyond,

(3) the inaccessible points, points which do not have a single feasible path from the origin.

Every sequence of experiments begins at the origin and takes a random walk within the grid until the path terminates on one of the boundary points, and a decision as regards the outcome of the sequence of experiments is determined. Between the origin and the boundary points all points on the path are continuous points.

Feasible Paths

The <u>feasible paths</u> to any point (continuous or boundary) are the number of paths from the origin to the point, having a sequence of points being continuous.

Assuming during experimentation that the process parameters are unchanged, the probability of moving in any one direction in the grid is constant.

Let

the probability of movement in the X direction = p

the probability of movement in the Y direction = q

and the probability of movement in the Z direction = r

where p + q + r = 1.

The probability of movement from the origin to a point i with coordinates X_i, Y_i, Z_i is given by the expression

$$P(i) = K(i)\ (p^{x_i})\ (q^{y_i})\ (r^{z_i}),$$

where K(i) is the number of feasible paths from the origin to i.

The Computer Technique to Calculate Probabilities

In the computation of the operating characteristic curve, a computer technique was used to make an actual count of all feasible paths to a boundary point and the expression

$$P(i) = K(i)\ (p^{x_i})\ (q^{y_i})\ (r^{z_i})$$

was used to determine the probability of reaching the point. Summing the probabilities for all the points in a boundary set, the probabilities of rejecting due to oversize, rejecting due to undersize, and the probabilities of accepting are determined. Development of the computer technique is as follows (See Appendix, Table II for computer program listing):

(1) Every movement in the grid results in the increment of one and only one of the coordinates. Let a movement occur from $i-1$ to i. If the coordinates of i are x, y, z then the coordinates of $i-1$ have to be either $x-1$, y, z or x, $y-1$, z or x, y, $z-1$. The point (x, y, z) can be reached from only one of these points. Thus, if $K(i)$ denotes the number of paths to a continuous point i, then one has the relationship

$$K(X, Y, Z) = K(X-1, Y, Z) + K(X, Y-1, Z) + K(X, Y, Z-1).$$

The basic counting procedure makes use of this recursive relationship. This, however, necessitates certain initializing statements.

(2) Once the boundary points are reached the path is terminated and no path leaves the boundary point. Hence, the number of paths to a boundary point b is defined as equal to zero and the actual number of paths to the boundary point is taken to be equal to the number of paths to the continuous point immediately ahead.

$$K(b) = 0$$

$$b \in B .$$

(3) As inaccessible points by definition have no paths from the origin, they are defined as zero.

(4) As redefinition of boundary points and inaccessible points can occur when the recursive relationship is used to calculate the number of paths to a point, the counting procedure within the three-dimensional grid is constrained to calculate for points i for all i in

the set of continuous points.

(5) In order that the recursive relationship begin, the origin point is set at (2, 2, 2) and K(0) is set equal to 1. All points on the planes $X + 1$, $Y = 1$, $Z = 1$ are set equal to 0, and

$K(i) = 0$ All values of Y and Z when $X = 1$

$K(i) = 0$ All values of X and Y when $Z = 1$

$K(i) = 0$ All values of X and Z when $Y = 1$.

(6) Next, the probabilities p, q, and r are defined, the probabilities for all points in the boundary region are computed, and the summations

$$\sum_{b_1} P(b_1), \quad \sum_{b_2} P(b_2), \quad \sum_{b_3} P(b_3)$$

$$b_1 \in B_1, \quad b_2 \in B_2, \quad b_3 \in B_3$$

are calculated, where

B_1 is the set of boundary points constituting reject due to oversize,

B_2 is the set of boundary points constituting reject due to undersize,

B_3 is the set of boundary points constituting acceptance.

These give the probabilities of reject oversize, reject undersize, and the probability of acceptance.

(7) The average sample number is given by the relation:

$$\text{ASN} = \frac{\sum_i \{(X_i + Y_i + Z_i)\ (P(i))\}}{\sum_i P(i)} \qquad i \in B_1 \cup B_2 \cup B_3.$$

As the number of continuous points within the grid is bounded, the grid is closed. The closure implies that the total probability of absorption on one of the boundary points is equal to unity, i.e.,

$$\Sigma P(b_1) + \Sigma P(b_2) + \Sigma P(b_3) = 1$$

$$b_1 \in B_1,\ b_2 \in B_2,\ b_3 \in B_3.$$

Thus, for any defined set of conditions for the boundary points such that the continuous points are closed one can compute the probabilities,

$$\Sigma P(b_1),\ \Sigma P(b_2),\ \text{and}\ \Sigma P(b_3)$$

by actually counting the number of feasible paths, given the probabilities p, q, and r as detailed earlier. As the continuous region is closed

$$\sum_i P(i) = 1$$

$$ASN = \sum_i \{(X_i + Y_i + Z_i)\ (P(i))\}$$

$$i \in B_1 \cup B_2 \cup B_3.$$

Steps 1, 4, 6, and 7 are common to all the SPCM models. In regard to steps 2, 3, and 5 each model is treated separately.

SPCM-I

In this model, the boundaries are the planes:

$$X = Z + Y$$

$$Y = Z + K$$

$$Z = L \ .$$

All points on the boundary are initialed as shown. Consider the point i:

For all values of Y

If $X = Z + K$ (boundary B_1)

For all values of X

If $Y = Z + K$ (boundary B_2)

For all values of X and Y

If $Z = L$ (boundary B_3.

K and L are varied and the probabilities of being absorbed on each of the boundaries $P(B_1)$, $P(B_2)$, and $P(B_3)$ along with the average sample number ASN is calculated.

Limits for the count are set by the relations:

$$Y_i < Z_i + K$$

$$X_i < Z_i + K$$

$$Z_i < L$$

all three relationships being satisfied.

SPCM-II

In this model, the boundaries are established by the relations,

$$Y_i = Z_i + K \quad \text{(boundary } B_1\text{)}$$

$$X_i = Z_i + K \quad \text{(boundary } B_2\text{)}$$

$$Y_i = Z_i - K \quad \text{(boundary } B_3\text{)}$$

$$X_i = Z_i - K \quad \text{(boundary } B_3\text{)}$$

$$Z_i = L \quad \text{(boundary } B_3\text{)}$$

and the count of the total number of feasible paths is constrained to include only continuous points derived by the relations,

$$X_i < Z_i + K$$

$$Y_i < Z_i + K$$

$$X_i > Z_i - K$$

$$Y_i > Z_i - K$$

$$Z_i < L \ .$$

SPCM-III

The boundaries in this model are given by the equalities:

$$X_i + Y_i = Z_i + K$$

$$Z_i = L$$

and the constraining inequalities for keeping the count of the total number of feasible paths to a boundary point are,

$$X_i + Y_i < Z_i + K,$$

$$Z_i < L.$$

SPCM-IV

The boundaries for this model are defined by the relations:

$$X_i + Y_i = Z_i + K$$

$$X_i + Y_i = Z_i - K$$

$$Z_i = L$$

and the constraining inequalities for the count are:

$$X_i + Y_i < Z_i + K ,$$

$$X_i + Y_i > Z_i - K ,$$

$$Z_i = L.$$

Results of Analysis

A preliminary investigation of the four models indicated that once the gauge limits are properly defined, any of the four models are adequate for the estimation of the parameters. For K/L ratios of 1 or less, there is no difference between SPCM-I and SPCM-II, and between SPCM-III and SPCM-IV. Only for K/L ratios of 3 or more does the difference become significant. For larger ratios, β the probability of accepting a process out of control increases. However, the average sample number decreases. This investigation considers low values of K/L and hence SPCM-II and SPCM-IV are not considered further. The analysis and the enumeration of probabilities of acceptance and rejection are quite similar and is easily extended to large K/L ratios and to the different geometry of SPCM-II and SPCM-IV. Comparing SPCM-I and SPCM-III, one observes that SPCM-I discriminates shifts in mean better, and SPCM-III detects shifts in variance better. This investigation continues as SPCM-I is studied in depth.

CHAPTER IV

OPERATING CHARACTERISTIC CURVES FOR SPCM-I

In sequential process control, as in all sampling plans, there exists the risk σ of rejecting a process producing good quality items, and the risk β of accepting a process producing poor quality items.

It is desirable that a sequential process control plan have a high probability of acceptance $(1-\alpha)$ for a manufacturing process that is in control, and a small probability of acceptance (β) for a manufacturing process not in control. For SPCM-I, this probability of acceptance, PA, is calculated as discussed in Chapter III.

A sequential process control plan is defined by two parameters, K and L. K is the number of steps from the axis-origin in either the Y or Z direction to the beginning of the reject planes, and L is the number of steps in the X direction to the accept barrier. The selection of these two parameters determines the relative frequency with which the plan will accept a process of various qualities.

For the purpose of process control using SPCM-I, the use of the word "quality" refers to the amount of variation of the process average $\overline{X}'$ from the desired value m, and the positive variation of the process standard deviation σ_x from the controlled value σ' as follows:

$$\overline{X}' = m \pm g\sigma'$$

$$\sigma_x = \sigma' + h\sigma'$$

where g and h are non-negative variations from the desired values. This concept allows SPCM-I to indicate shifts in the operating conditions of the process without the necessity of knowing the relationship between the "actual engineering specifications and tolerance" and the "natural statistical tolerances" of the manufacturing process. When necessary, any combination of shifts may be converted to per cent defective for any product specifications by reference to normal probability curves, or by "area under the normal curve" calculations.

An operating characteristic table (Tables III-VI, Appendix) for a given plan shows the relative frequencies of accepting, or rejecting, a process of any quality in the long run under the plan. If any quality is assumed for a shift in process average $\overline{X}'$ or for an increase in σ_x, then a unique operating characteristic curve is specified by a specific K and L.

Effect of Parameters on the Operating Characteristic Curves

K and L are the parameters controlling the shape of the operating characteristic curves. The effect of increasing K (for a fixed L) on the probability of acceptance for three manufacturing processes is shown in Figure 7. One process has its $\overline{X}'$ equal to the desired value m, and its standard deviation σ_x equal to the desired value σ'. For this process $g = 0$, $h = 0$. The second process has $\overline{X}'$ equal to the desired value m, but σ_x is increased to two times the desired value σ'. For this process, $g = 0$, $h = 1$. The third process is one in which $\overline{X}'$ has shifted from the desired value m by the amount $1\sigma'$ with σ_x remaining at the desired value σ'. For this process $g = 1$, $h = 0$.

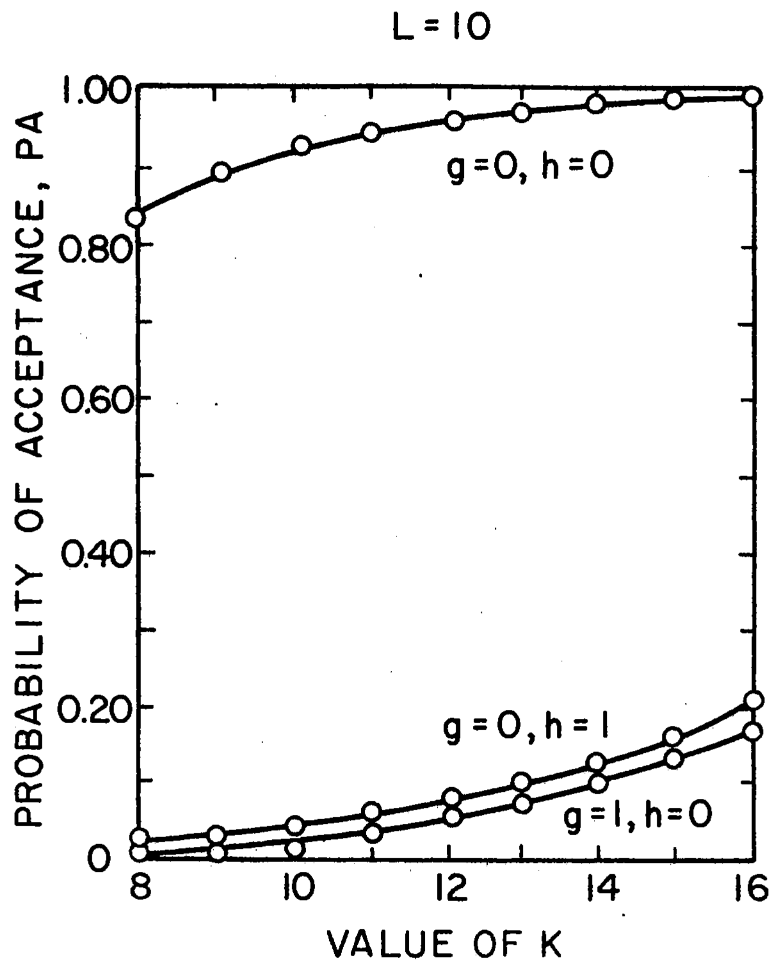

Figure 7. Effect of K on Probability of Acceptance

As K increases, the probability of acceptance increases for all processes, but at varying rates. In all processes, the PA approaches 1.0 so long as p is greater than zero.

SPCM-I can be considered as an absorbing Markov chain process by considering the coordinate points in the three-dimensional grid as states in the stochastic process. If one removes the reject barriers (equivalent to letting K approach infinity), and let all points on the accept barrier (the $[K + L - 2]^2$ points) be absorbing states (the only absorbing states), then so long as p is greater than zero (it must be possible to go to an absorbing state) the probability of absorption is equal to 1.0 (Parzen, 1962).

An increase in K shifts the entire operating characteristic curve in a direction of higher PA as can be noted in Figure 8 and 9. In Figure 8, it is shown that an increase in K causes a greater increase in PA for an acceptable quality level (g approaches 0) than for a poor quality level, $g > 0$. Figure 9 indicates, also, that an increase in K causes an increasingly greater decrease in PA as the process approaches a poor quality level, $h > 0$.

In Figure 7, it is seen that the curve for the process $g = 0$, $h = 1$ increases more rapidly than the curve for $g = 1$, $h = 0$. This difference in slope can also be observed by checking the differences in PA for $K = 6$ and $K = 15$ for Figure 8 and Figure 9 at $g = 1$, $h = 1$.

An increase in K causes a reduction of α error in both Figure 8 and Figure 9. The β error is, however, increased. This increase in β is not desirable, but it can be controlled by adjusting the parameter L.

The effect of L (for a fixed K) on the probability of acceptance for three manufacturing processes is shown in Figure 10.

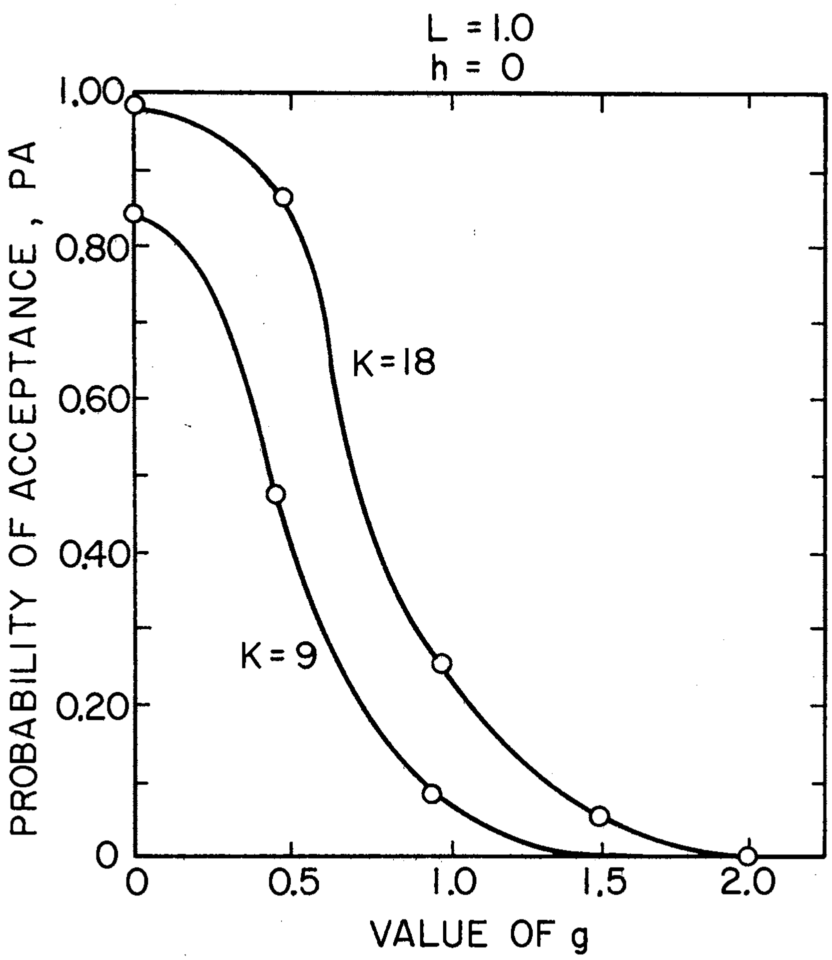

Figure 8. Effect of K on O.C. Curve for g

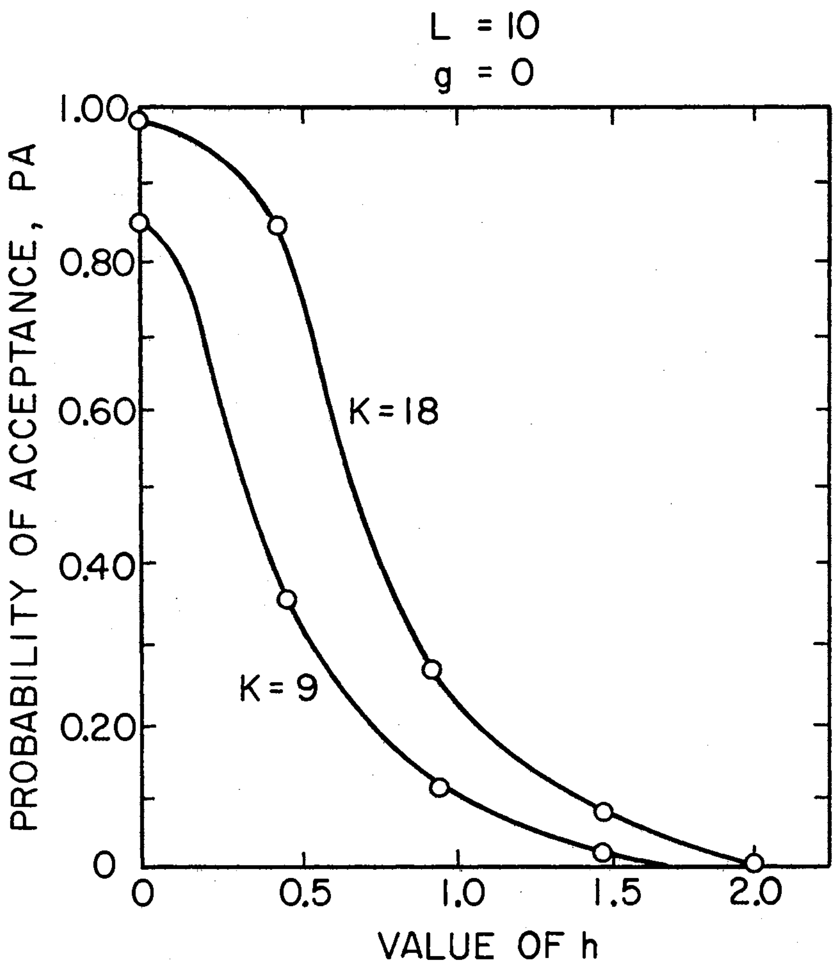

Figure 9. Effect of K on O.C. Curve
for h

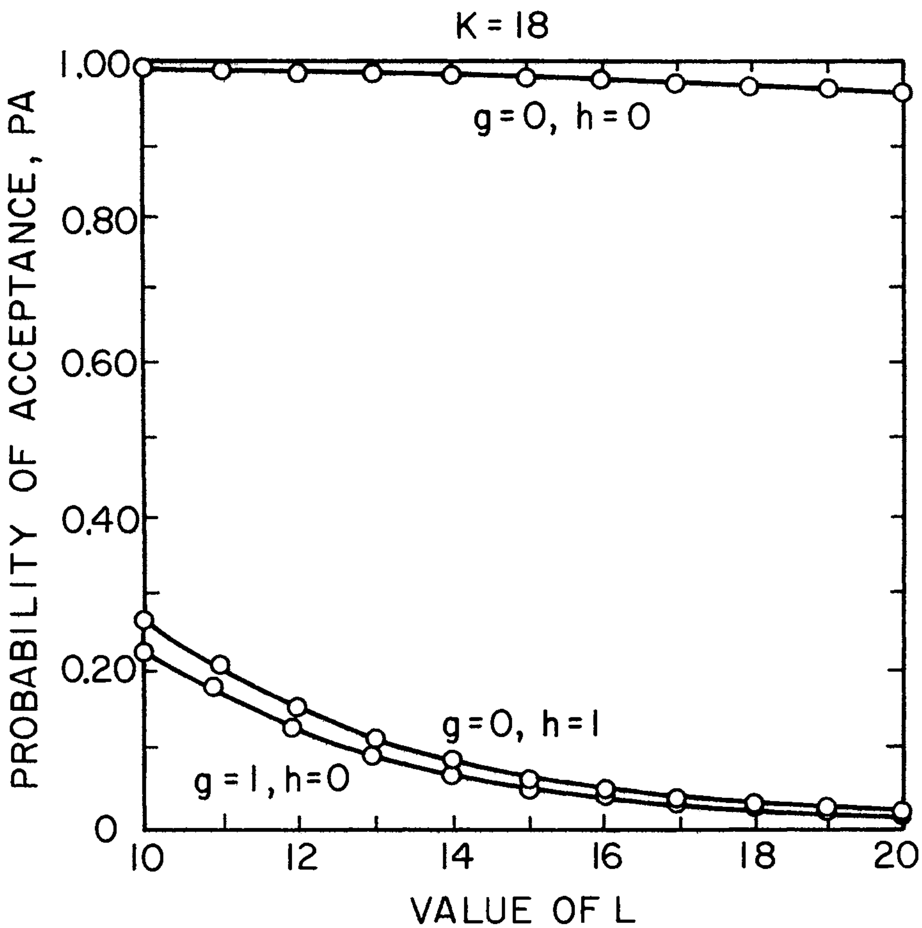

Figure 10. Effect of L on Probability of Acceptance

The probability of acceptance decreases as L increases. This is true for all three processes: (1) $g = 0$, $h = 0$; (2) $g = 0$, $h = 1$; and (3) $g = 1$, $h = 0$. The decrease in PA is more rapid for the process associated with a poor quality level, $g \neq 0$ and/or $h \neq 0$.

An increase in L shifts the entire operating characteristic curve in a direction of lower PA as can be noted in Figure 11 and Figure 12. In Figure 11, it may be seen that an increase in L causes a greater decrease in PA as the process approaches a poor quality level, $g > 0$. Figure 12 demonstrates that an increase in L causes an even greater decrease in the PA for a poor quality level, $h > 0$. This is desirable since it corrects for the previous increase in PA caused by an increase in K as was illustrated in Figure 9.

In both Figure 11 and 12, an increase in L is shown to increase the α error and decrease the β error. The decrease in the β error is greater than the increase in the α error.

By proper selection of the value of K and L, the quality control engineer may place the reject and accept barriers in positions that control the shape of the operating characteristic curve.

Increases in K reduce the risk α of rejecting a manufacturing process at an acceptable quality level, while increases in L reduce the risk β of accepting a process that is operating at a poor quality level.

Increasing both K and L steepens the operating characteristic curve, thereby making the inspection plan more sensitive for distinguishing between an acceptable and rejectionable process.

In Figure 13 for $h = 0$, and Figure 14 for $g = 0$, the operating characteristic curves for several plans illustrate the conclusions discussed above.

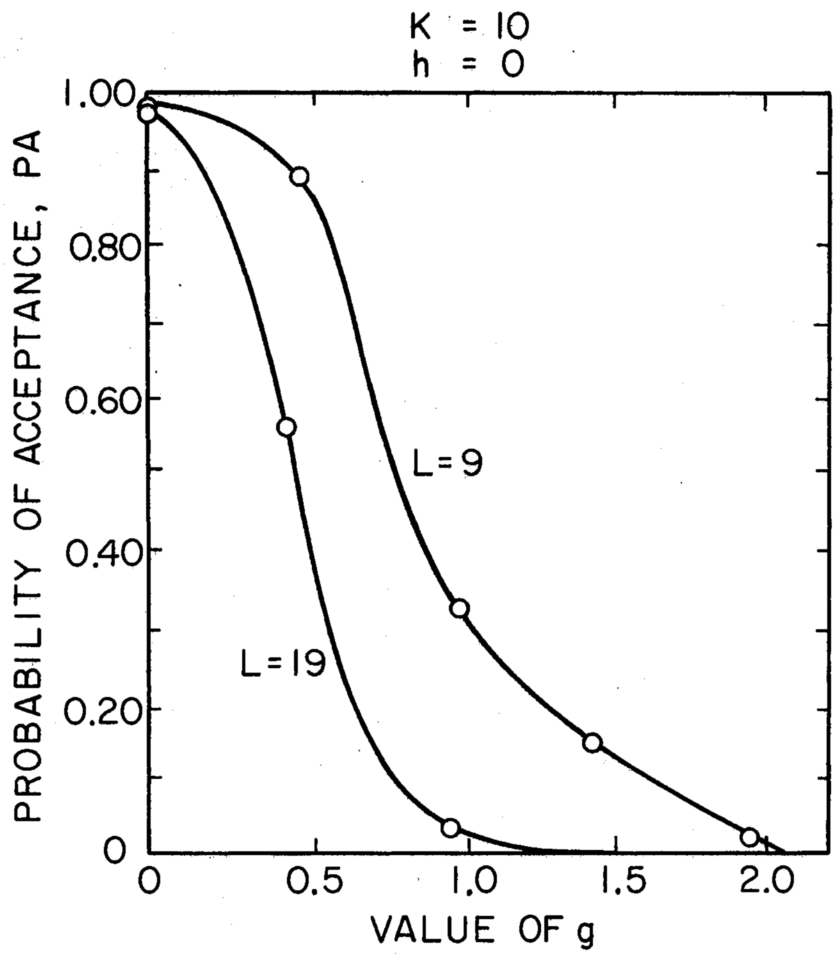

Figure 11. Effect of L on O.C. Curve for g

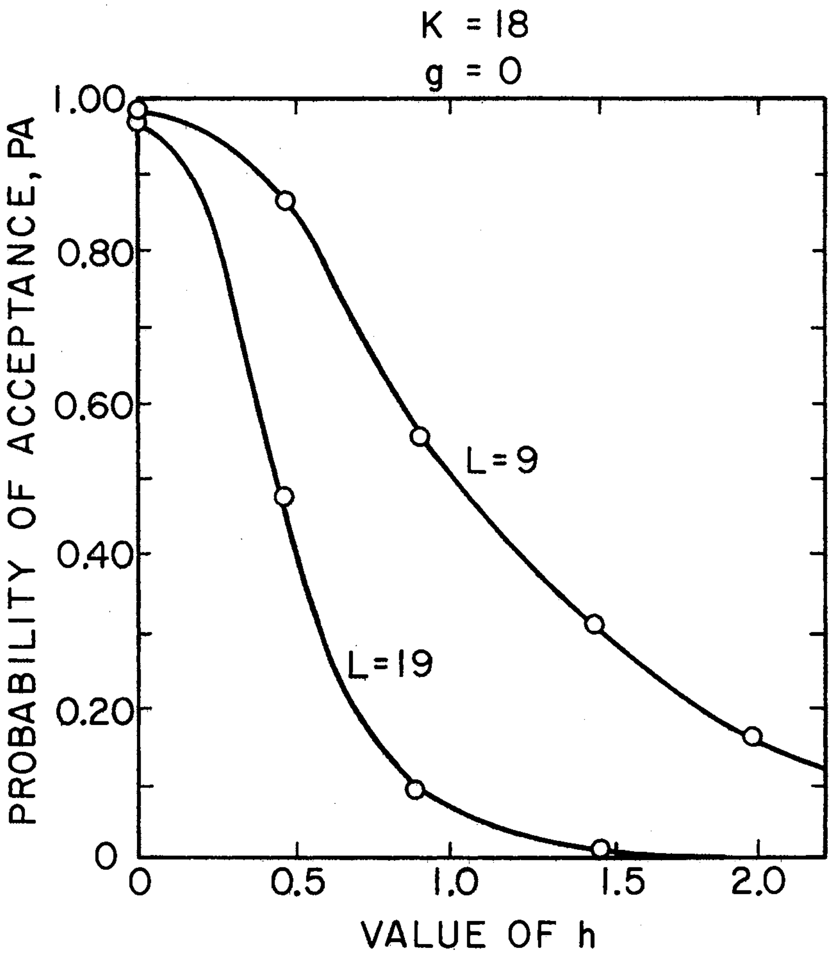

Figure 12. Effect of L on O.C. Curve for h

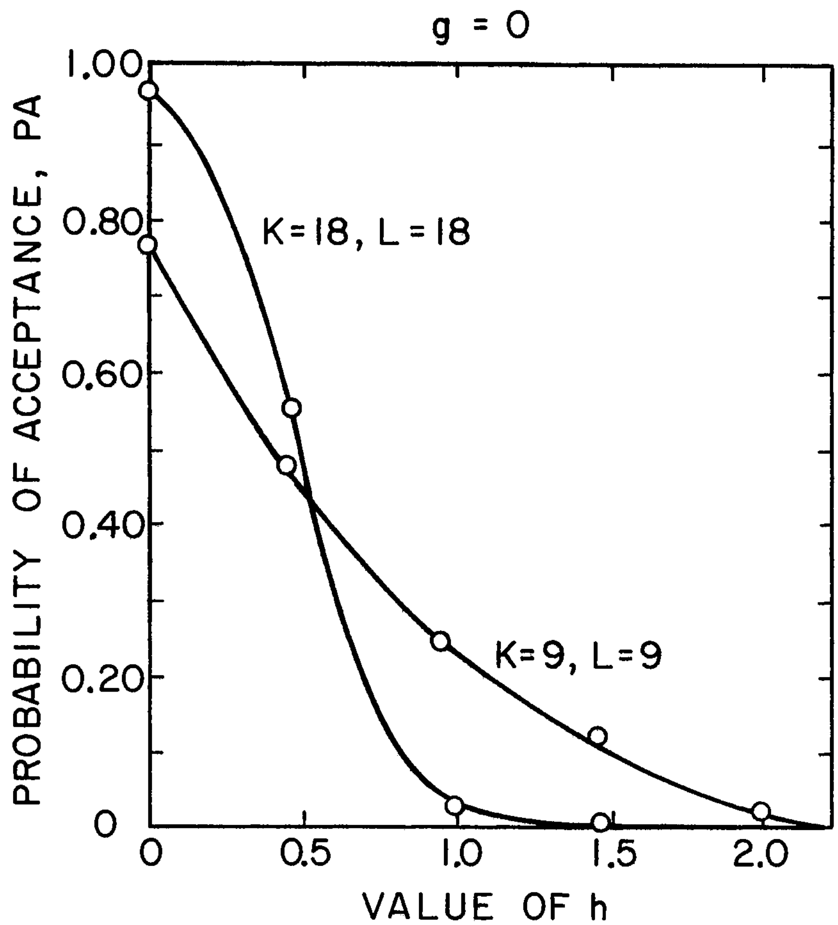

Figure 13. Effect of K and L on O.C. Curve for h

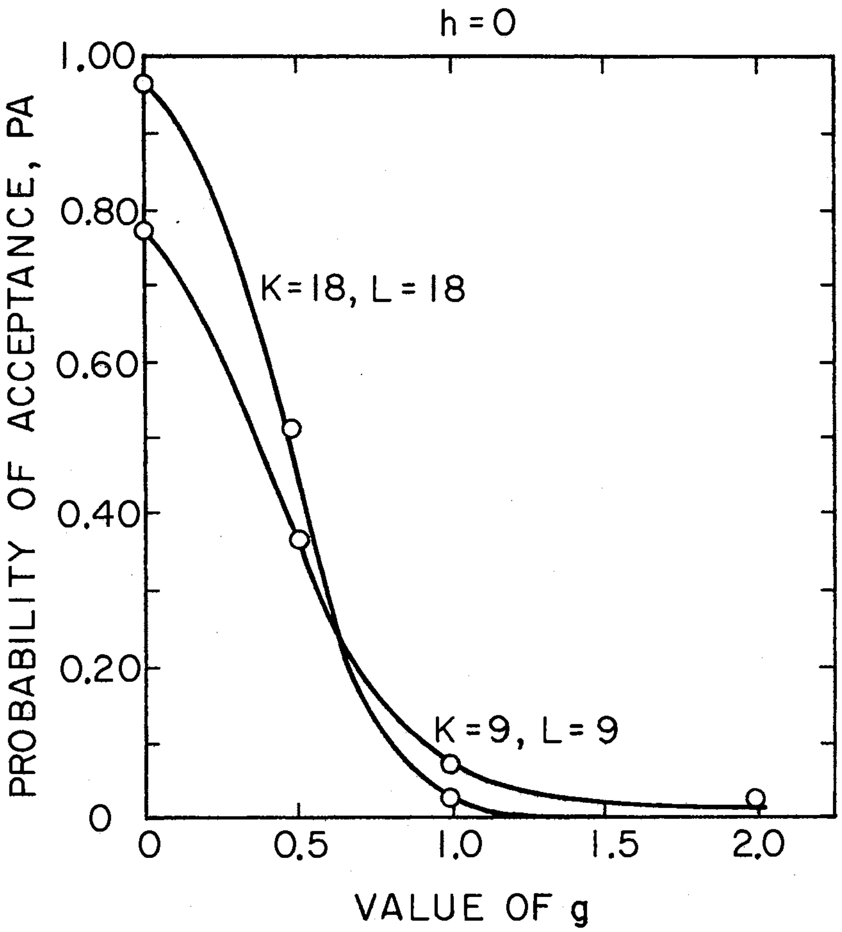

Figure 14. Effect of K and L on O.C. Curve for g

Effect of Parameters on Average Sample Size

The number of items inspected in the go-no go gauge increases as K and L are increased. The average sample size, ASN, for various quality levels for a fixed K and L is computed as shown in Chapter III.

The values of K, L, and the quality level influence the average sample size.

The effect of K (for a fixed L) on ASN is illustrated in Figure 15. For an acceptable process, such as $g = 0$, $h = 0$, the ASN approaches a constant value. For the other two processes representing shifts toward an unacceptable range, ASN decreases. In the process, $g = 0$, $h = 1$, $q = r$, and $p = 1 - q - r$. An increase in σ_x causes an equal increase in q and r and a corresponding decrease in p. The random walk for this case is more of a three-dimensional walk than for the process $g = 1$, $h = 0$. In this case, $r = 1 - p - q$, is very small and the random walk tends to approximate closely a two-dimensional walk since the small probability r reduces the number of steps taken in the third direction. In each case, as the quality level gets worse, ASN decreases.

For $g = 0$, $h = 0$, p gets smaller and ASN lies between K and $2K - 1$. For $g > 0$, $h = 0$, r approaches 0, and ASN approaches K.

The effect of L (for a fixed K) on the average sample size and on the maximum sample size is shown in Figure 16. For the processes $g = 0$, $h = 1$, and $g = 1$, $h = 0$, the ASN tends to approach a finite value. The concept of increasing L (toward infinity) will, in the limit, produce an absorbing Markov process with the absorbing states as the coordinate points on the two reject barriers. The probability of rejection of the process will be 1.0 as long as q and/or r are greater than zero, and p approaches zero. Therefore, the ASN will approach a finite value in both processes: $g = 0$, $h = 1$ and $g = 1$, $h = 0$, as is illustrated in Figure 16.

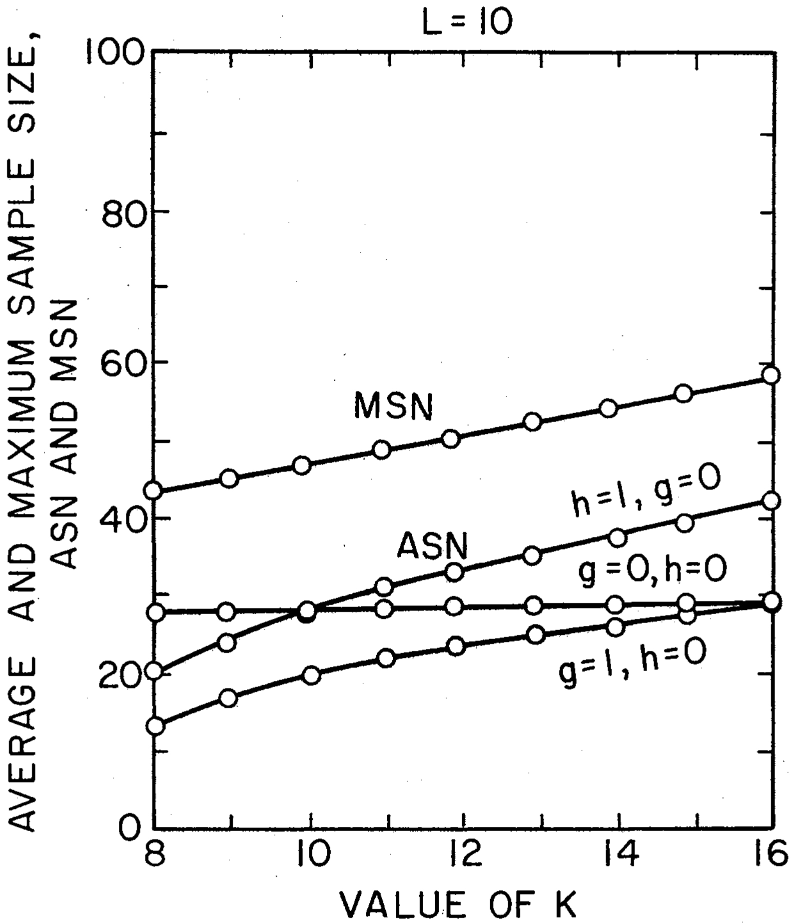

Figure 15. Effect of K on Sample Size

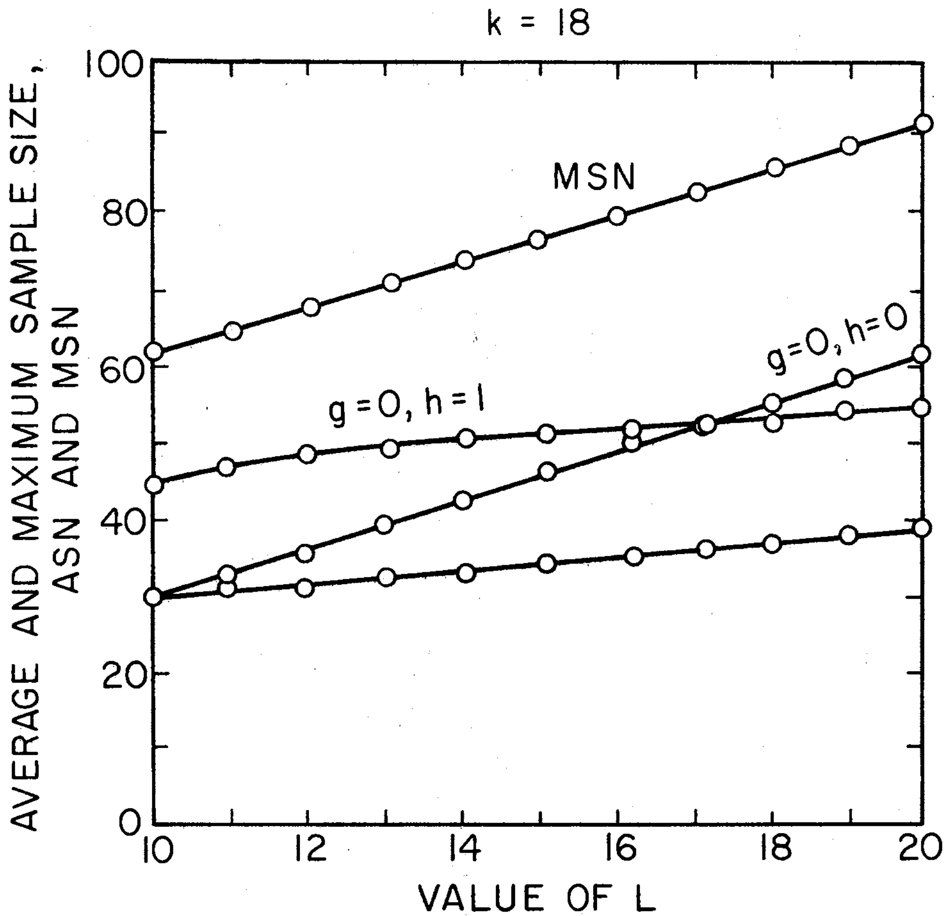

Figure 16. Effect of L on Sample Size

CHAPTER V

SELECTION OF SAMPLING PLAN AND OPERATING PROCEDURES

SPCM has been developed to determine statistically the acceptance or rejection of a manufacturing process whose parameters $\overline{X}'$ and σ' are susceptible to assignable causes of variation, one at a time or simultaneously. Tables III-VI, Appendix are arranged according to the sampling plan parameters K and L to show PA and ASN for various quality levels.

Selecting the SPCM Plan

In order to apply SPCM, it is necessary to define several terms.

$\alpha_{0,0}$ = the probability of rejecting a perfect process: $g = 0$, $h = 0$. (Also equal to $1 - PA_{0,0}$.)

$\beta_{g,0}$ = the probability of accepting a process when the process average shifts to an unacceptable level by the amount $\pm\, g\sigma'$.

$\beta_{0,h}$ = the probability of accepting a process when the process process standard deviation increases to an unacceptable level by the amount $+\, h\sigma'$.

Before selecting a sampling plan from the table, $\alpha_{0,0}$, $\beta_{g,0}$', and $\beta_{0,h}$ (the α and β risks inherent in any sampling plan) must be determined.

For example, assume that the following requirements have been formulated:

$$\alpha_{0,0} = 0.10$$

$$\beta_{1,0} = 0.10$$

$$\beta_{0,1} = 0.10 \; .$$

The first plan meeting these requirements is $K = 10$, $L = 9$. Normally, the first plan meeting the α and β requirements would be selected since the plan will provide lower ASNs than later plans in the table.

For this plan:

$$\alpha_{0,0} = 0.0899 \qquad\qquad \text{ASN} = 25.94$$

$$\beta_{1,0} = 0.0929$$

$$\beta_{0,1} = 0.0961$$

If management follows the practice of inspecting all items of a rejected process 100 per cent, and removes all items not meeting specification, the average outgoing quality level can be calculated for the manufacturing process.

The average outgoing quality is the expected fraction defective that will continue through the production process under the control of a particular SPCM plan as the process operates at a particular quality level.

The average outgoing quality is

$$\text{AOQ}_{g,h} = (\text{PA}_{g,h})\;(\text{df}_{g,h})$$

where:

$PA_{g,h}$ = the probability of acceptance of a process with quality level g,h.

$df_{g,h}$ = fraction of items beyond engineering specification limits when the process is at the quality level g,h.

Use on the Manufacturing Floor

SPCM-I may be used on the manufacturing floor in one of three forms.

(1) One form consists of a table having accept numbers and reject numbers which depend on K, L, and the cumulative inspection results. To use this table, it would be necessary to accumulate the sum of acceptable items, reject oversize items, and reject undersize items. Though an acceptable procedure, this method is susceptible to arithmetic errors, and requires the preparation of a different table for each sampling plan.

(2) A graphical representation eliminates the need for such additional tables and the accompanying possibility of arithmetic errors. It is a procedure easily learned and understood by operating personnel. The graphical control chart for SPCM-I plan $K = 10$, $L = 9$ is illustrated in Figure 17.

To use the graphical procedure, individual items are selected from the manufacturing process and checked in the go-no go gauge. If the gauge indicates reject oversize, one step is plotted in the oversize direction on the upper one-half of the chart, and the chart circle is either filled in or crossed out. If a gauge inspection indicates reject undersize, one step is plotted in the undersize direction on the

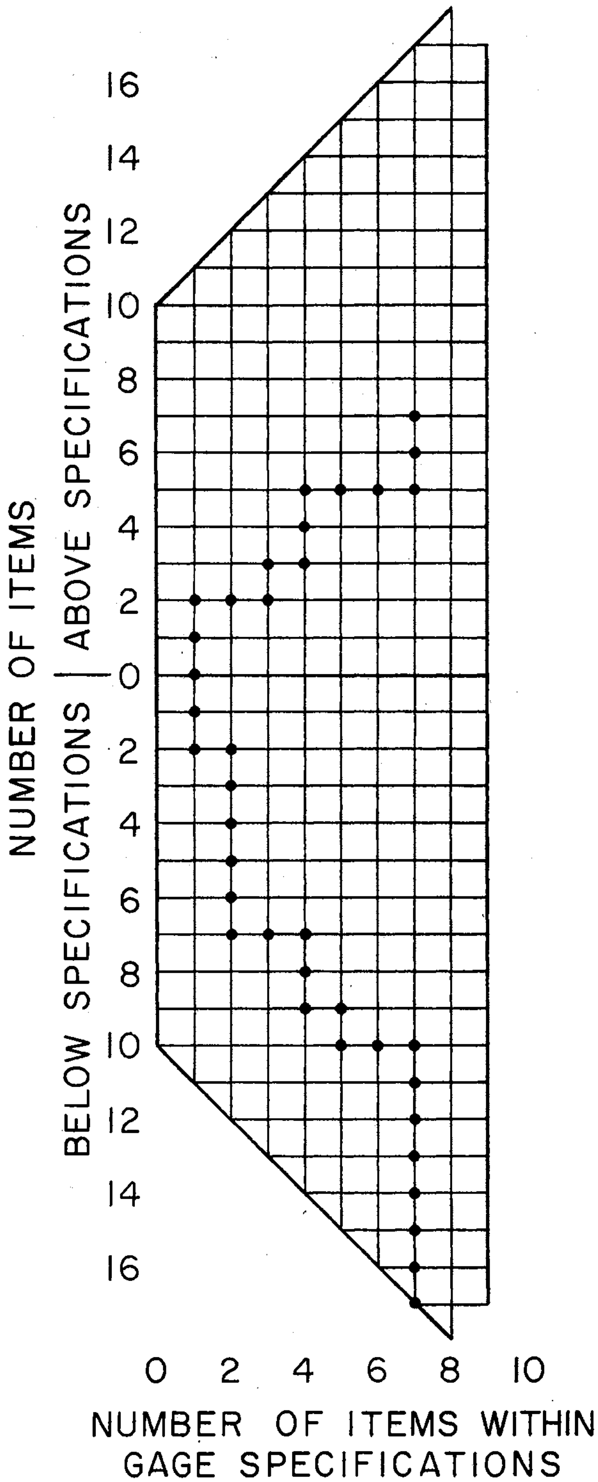

Figure 17. SPCM-I Control Chart Example

lower one-half of the chart, independent of the upper one-half of the chart. If an item checks acceptable, each point (in the two halves of the chart) is advanced one step in the accept direction. Thus, if a gauge reject occurs, the appropriate point moves one step toward its reject barrier. If a gauge accept occurs, both points move jointly in the horizontal direction toward the accept barrier. A decision is reached when one of the points reaches a reject barrier, or when both points reach the accept barrier.

(3) The third method of using SPCM-I in a manufacturing environment is by use of a mechanical device illustrated in Figure 18. The device consists of three parallel scales A, B, and C. The middle scale B slides between the two outer ones, A and C. All the scales are linear and graduated with constant intervals. There are three pointers AP, BP, and CP which move on scales A, B, and C, respectively. Another pointer T, the termination pointer, moves on scale B. When a suitable plan has been selected (in this example with $K = 10$, $L = 9$), the point of origin in scale B is set ten graduations (corresponding to $K = 10$) to the right of the origin point in scale A and C by sliding the middle scale B. The termination pointer is set nine graduations (corresponding to $L = 9$) to the right of the origin in scale B on scale B.

At the beginning of a sequence of experiments, the pointers AP, BP, and CP are set at the origin. The result of an experiment on a sample will be one of the following three cases:

Case 1 - sample is oversize

Case 2 - sample is within tolerance limits

Case 3 - sample is undersize

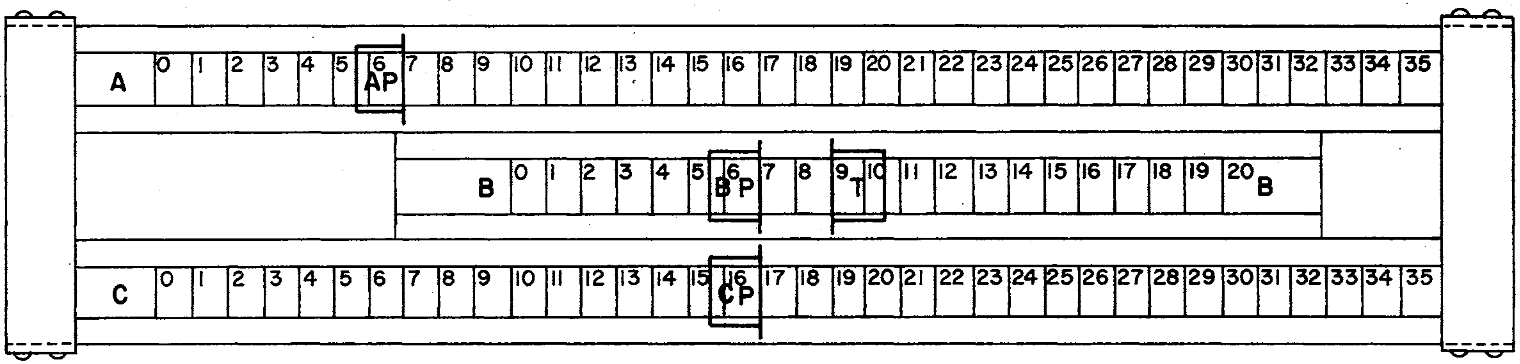

Figure 18. Mechanical Device Illustration

For Case 1 the pointer AP is moved one graduation to the right on scale A

For Case 2 the pointer BP is moved one graduation to the right on scale B

For Case 3 the pointer CP is moved one graduation to the right on scale C.

After this, a decision is made whether to continue sampling or to terminate the sequence of experiments. The sequence of experiments is terminated when one of the three results happens first:

(1) Pointer BP reaches T

(2) Pointer AP reaches BP

(3) Pointer CP reaches BP.

If it is the first result, then the process is under control and the pointers are moved back to the origin. For the second and third results, the process is out of control and the shifts in process parameters are estimated. The estimation procedure is given in Chapter VI.

An example of the use of the graphical technique and the mechanical device is given for $K = 10$, $L = 9$. Table I gives the results of sampling using a random number generator. The results of experiments on Table I are shown by Figures 17 and 18 to illustrate both the graphical technique and the technique using a mechanical device.

TABLE I

DATA FOR SPCM-I SAMPLING EXAMPLE IN CHAPTER VI

Item Number	Random Number	Decision
1	6539	A
2	8763	RO
3	0863	RU
4	0852	RU
5	8985	RO
6	4229	A
7	1640	RU
8	2558	RU
9	1453	RU
10	1447	RU
11	3225	RU
12	6109	A
13	8552	RO
14	1671	RU
15	4638	A
16	8869	RO
17	0813	RU
18	8263	RO
19	5665	A
20	2287	RU
21	4918	A
22	4404	A
23	3329	RU
24	7721	RO
25	8709	RO
26	2315	RU
27	1792	RU
28	0653	RU
29	1933	RU
30	2719	RU
31	1313	RU
	Reject at (7, 7, 11)	

RO - plot in reject oversize direction.

A - plot in accept direction.

RU - plot in reject undersize direction.

CHAPTER VI

ESTIMATION OF PROCESS PARAMETERS

Maximum Likelihood Principle

In all of the attribute sequential inspection schemes devised for process control, there is no systematic procedure to estimate the process parameters. In this thesis, a procedure has been developed to estimate the mean and the variance of a normally distributed variate using the maximum likelihood principles.

After a sequence of i experiments, let the process be absorbed in one of the reject boundaries. Let the coordinates of the point be X_i, Y_i, Z_i. Then, by the maximum likelihood principle, the probability of movement in the X direction is given by the expression:

$$P_x = \frac{X_i}{X_i + Y_i + Z_i}$$

Similarly,

$$P_y = \frac{Y_i}{X_i + Y_i + Z_i}$$

and

$$P_z = \frac{Z_i}{X_i + Y_i + Z_i} .$$

Now P_x is the probability that a randomly drawn sample from the new normally distributed variate population is above the upper gauge limit,

G_{upper}. Also P_y is the probability that a randomly drawn sample from the new population is below the lower gauge limit, G_{lower}.

A normal distribution is completely determined by the two parameters, the mean and the variance. Hence, knowing P_x and P_y one has two equations with two unknowns. Solving for them, one would get the new values of the mean and variance. This is accomplished very simply using normal probability paper and a graphical technique.

Use of Graphical Technique

The graphical computation of the shift in parameters is done using a nomograph illustrated in Figure 19. Probability graph paper is used with probability values graduated normally in the Y-axis and X-axis. The origin in the X-axis is shifted to near the middle of the paper and positive and negative values of X ranging from $+2.0\sigma$ to -2.0σ are shown. Parallel lines UU and OO run parallel to the Y-axis and intersect the X-axis at $+.431\sigma$ and $-.431\sigma$, respectively. The line MM runs parallel to the X-axis intersecting the Y-axis at 50%. Dotted lines PP and QQ are drawn parallel to the X-axis and intersect the Y-axis at 15% and 85%, respectively.

Taking the example shown earlier, one has after 31 experiments:

the probability of undersize $= \frac{7}{31}(100) = 22.8\%$

the probability of oversize $= \frac{17}{31}(100) = 54.8\%$.

The probability of oversize is then plotted on the line OO at O′ and the complement of the probability of undersize is plotted on UU at U′. A straight line through U′ and O′ is drawn cutting the line MM at M′ and the lines QQ and PP at Q′ and P′, respectively. The position of M′

corresponding to the scale on the X-axis gives the shift in the mean. In this case, it is $+.65\sigma$. The distance on the Y-axis corresponding to $M'Q'$ or $M'P'$ gives the new σ' which in this case is 1.505σ.

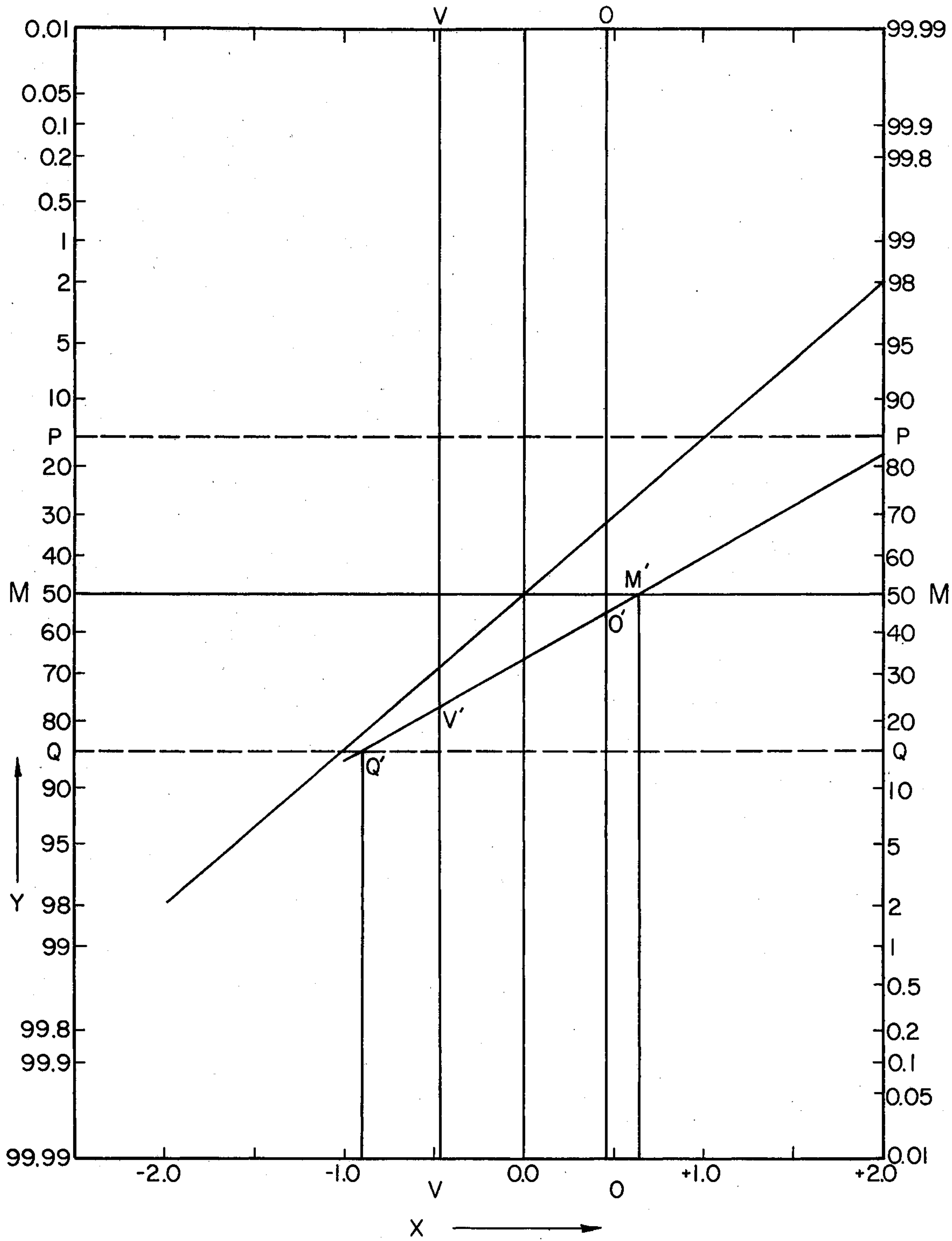

Figure 19. Probability Nomograph

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

Statistical and engineering literature has emphasized the application of sequential sampling plans in the field of quality assurance. The use of such plans in process control has been quite limited, and the available models can only detect changes in process parameters. The primary purpose of this research has been to develop an estimation procedure for measuring the changes in process parameters using attribute sampling.

Summary

Preliminary investigation of four models encompassing a three-dimensional grid has been accomplished, followed by an additional detailed study of the SPCM-I model. A go-no-go gauge has been designed in order to effectively classify the sampled data of a physical system into three attributes. The inspection results are portrayed as a random walk in the three-dimensional grid. Sequential inspection continued until a random walk terminated in one of the absorption barriers defined by the model. The relationship between gauge limits and the absorption barriers for the various models are studied, and gauge limits for each of the models are defined.

Operating characteristic curves are developed for various model parameters by completely enumerating the probabilities of acceptance and rejection for each point in the absorption barriers. This establishes the relationship between statistical errors α and β, and the different sampling plans defined by model parameters.

Using the relationship obtained for gauge limits and the principle of the maximum likelihood estimate, the shift in the process parameters is mathematically defined.

Conclusions

Analysis of the results of this research shows that a specific relation exists between the slope of the planes defining the boundaries of the Sequential Process Control models and the gauge limits which define a variable as being in one of three attribute conditions. The relationship between the absorption barriers and the gauge limits is mathematically established using the principles of Wald's Sequential Analysis and extending it to three dimensions. As the gauge transforms a measurable variable into an attribute, definition of the gauge limits establishes the correspondence between the variable and the attribute. The gauge limits for SPCM-I and SPCM-II are shown to be $m \pm .4316\sigma$, and the gauge limits for SPCM-III and SPCM-IV are $m \pm .6756\sigma$. A reduction in average sample number shown by Terrell, in three-dimensional sequential inspection, is achieved by SPCM-I. Further, by correcting the gauge limits to $m \pm .4316\sigma$, "β" errors are reduced for the same average sample number. The concept of the maximum likelihood estimate has been used to estimate the probabilities of reject oversize, reject undersize, and acceptance using these probabilities and the correspondence between the variable and the gauge limits the new parameters are established. Using the probability distribution paper, a graphical procedure is developed where by plotting two points and drawing a straight line through them, changes in the process parameters can be determined from the graph plot. For use in the operating environment a mechanical device similar to the slide rule has been developed.

Recommendations

SPCM-I has been thoroughly studied. For further research, the study of SPCM-II, III, and IV is recommended, particularly SPCM-III. It is conjectured that SPCM-III would reduce "β" errors for shifts in variance, compared with SPCM-I. Tables for sampling plans similar to the tables given in this study can be constructed using very similar computational algorithms. SPCM-II and SPCM-IV would increase "β" errors a little; however, it will reduce the average sample number. This relationship might also be studied. Another area of study might be to study the error associated with using the maximum likelihood estimate in computing the probabilities of rejection and acceptance. Girshick, Mosteller and Savage (1946) have shown that the unbiased estimate in assigning probabilities for such a random walk is given by the ratio:

$$P = \frac{\text{Number of feasible paths from point } (1, 0, 0)}{\text{Number of feasible paths from point } (0, 0, 0)} .$$

These estimates could then be compared with the estimates obtained by the maximum likelihood estimate. It is likely that the relationship between the error and the values of the estimate could be monotomic. An area of further research is to consider the study of other distributions. It is unlikely that considerable revisions in the algorithm would be required.

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APPENDIX

OPERATING CHARACTERISTIC DATA

TABLE II

COMPUTER LISTING

```
00101         DIMENSION SUMOV(10),SUMOK(10),SUMUN(10)
00103         DIMENSION ASNUN(10);ASNOK(10),ASNOV(10)
00104         DIMENSION TY(50,50,1),Y(50,50),PTY(50,50,10)
00105         DIMENSION PROBX(10),PROBY(10),PROBZ(10)
00106         DO 21 I=1,10
00111         READ(5,102)PROBX(I),PROBY(I),PROBZ(I)
00116         WRITE(6,103)I,PROBX(I),PROBY(I),PROBZ(I)
00124     102 FORMAT(3(F6.4,1X))
00125     103 FORMAT(1X,10X,I2,5X,3(F6.4,2X))
00126      21 CONTINUE
00130         DO 16 K=5,22
00133         KL=K+2
00134
00134
00134
00134
00134         DO 10 NX=1,50
00137         DO 10 NY=1,50

00142         Y(NX,NY)=0.0
00143         TY(NX,NY,1)=0.0
00144
00144      10 CONTINUE
00147         Y(2,2)=1.0
00150         TY(2,2,1)=1.0
00151         DO 25 J2=1,10
00154         SUMOV(J2)=0.0
00155         SUMUN(J2)=0.0
00156         ASNOV(J2)=0.0
00157         ASNUN(J2)=0.0
00160      25 CONTINUE
00162         DO 12 NZ=1,KL
00165         WRITE(6,105)K,NZ
00171     105 FORMAT(1X,//10X,5H  K= ,2X,I2,5X,5H  L= ,2X,I4,////)
00172
00172         NK=NZ+K
00173         NG=NK
00174         DO 11 NX=2,NK
00177         DO 11 NY=2,NK
00202         Y(NX,NY)=Y(NX-1,NY)+Y(NX,NY-1)+TY(NX,NY,1)
00203         TY(NX,NY,1)=Y(NX,NY)
00204         DO 26 J=1,10
00207         PX=PROBX(J)
00210         PY=PROBY(J)
00211         PZ=PROBZ(J)
00212         PTY(NX,NY,J)=(PX**(NX-2))*(PY**(NY-2))*(PZ**(NZ-1))
00212        1*TY(NX,NY,1)
00213      26 CONTINUE
00215      11 CONTINUE
00220         DO 28 J=1,10
00223         DO 13 NX=2,NG
00226         ASNOV(J)=ASNOV(J)+(NX+NK+NZ-4)*PTY(NX,NK,J)*PROBY(J)
00227      13 CONTINUE
00231         DO 14 NY=2,NG
00234         ASNUN(J)=ASNUN(J)+(NY+NK+NZ-4)*PTY(NK,NY,J)*PROBX(J)
00235      14 CONTINUE
00237         DO 27 J2=1,10
00242         SUMOK(J2)=0.0
00243         ASNOK(J2)=0.0
00244      27 CONTINUE
00246         DO 15 NX=2,NK
00251         DO 15 NY=2,NK
00254         SUMOK(J)=SUMOK(J)+PROBZ(J)*PTY(NX,NY,J)
00255         ASNOK(J)=ASNOK(J)+(NX+NY+NZ-4)*PTY(NX,NY,J)*PROBZ(J)
00256      15 CONTINUE
00261         ASN=ASNOV(J)+ASNUN(J)+ASNOK(J)
00262         SUMA=SUMOK(J)
00263         SUMR=1.0-SUMA
00264         WRITE(6,101)J,SUMR,SUMA,ASN
00272     101 FORMAT(1X,I5,3E15.6)
00273      28 CONTINUE
00275      12 CONTINUE
00277
00277      16 CONTINUE
00301         STOP
00302         END

   END OF UNIVAC 1108 FORTRAN V COMPILATION.      0 *DIAGNOSTIC* MESSAGE(S)
```

TABLE III

PROBABILITY OF ACCEPTANCE FOR g = O, h = O

L	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
K																								
5	9465	8831	8225	7678	7193	6766	6388																	
6	9725	9330	8902	8479	8078	7706	7363	7048																
7	9860	9622	9334	9026	8715	8411	8120	7843	8013															
8	9928	9790	9604	9389	9158	8922	8687	8455	8230	8013														
9	9963	9884	9767	9622	9458	9282	9099	8914	8729	8545	8365													
10	9981	9936	9865	9770	9657	9530	9393	9249	9101	8952	8802	8653												
11	9990	9965	9922	9862	9785	9696	9596	9488	9374	9256	9135	9012	8888											
12	9995	9981	9956	9918	9867	9806	9735	9656	9570	9479	9384	9285	9184	9082										
13	9997	9990	9975	9951	9919	9878	9828	9772	9708	9640	9566	9489	9409	9326	9241									
14	9998	9994	9986	9971	9951	9924	9890	9850	9805	9754	9698	9639	9576	9510	9442	9372								
15	9999	9997	9992	9983	9970	9953	9930	9903	9870	9833	9792	9748	9699	9648	9594	9538	9479							
16	9999	9998	9995	9990	9982	9971	9956	9937	9915	9888	9859	9825	9789	9750	9708	9663	9617	9568						
17	9999	9999	9997	9994	9989	9982	9972	9960	9944	9926	9905	9880	9853	9824	9792	9757	9721	9681	9624					
18	9999	9999	9998	9996	9994	9989	9983	9974	9964	9951	9936	9919	9899	9877	9853	9826	9798	9761	9702	9595				
19	9999	9999	9999	9998	9996	9993	9989	9984	9977	9968	9958	9945	9931	9915	9897	9877	9853	9817	9753	9638	9443			
20	9999	9999	9999	9999	9997	9996	9993	9990	9985	9979	9972	9963	9953	9941	9928	9913	9892	9855	9786	9664	9461	9153		
21	9999	9999	9999	9999	9998	9997	9996	9993	9990	9986	9981	9975	9968	9960	9950	9938	9917	9879	9807	9679	9471	9159	8721	
22	9999	9999	9999	9999	9999	9998	9997	9996	9994	9991	9988	9984	9979	9973	9965	9954	9934	9894	9818	9687	9476	9161	8723	8150

TABLE IV

PROBABILITY OF ACCEPTANCE FOR g = 1, h = 0

L / K	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
5	7223	4795	4857	1979	1264	8084	5184																	
6	7851	5596	3820	2554	1690	1112	0731	0480																
7	8337	6301	4519	3147	3154	1459	0982	0659	0441															
8	8713	6912	5178	3745	2647	1843	1271	0870	0593	0402														
9	9004	7436	5789	4333	3157	2257	1592	1113	0772	0533	0366													
10	9229	7881	6347	4902	3674	2694	1944	1386	9791	0686	0479	0332												
11	9403	8255	6849	5444	4189	3146	2320	1686	1212	0864	0611	0430	0301											
12	9538	8569	7297	5952	4694	3607	2715	2012	1471	1065	0765	0545	0386	0273										
13	9642	8830	7693	6424	5183	4069	3125	2358	1754	1289	0939	0678	0487	0347	0247									
14	9723	9046	8039	6857	5651	4527	3543	2721	2058	1536	1134	0830	0603	0435	0312	0223								
15	9786	9224	8340	7251	6093	4975	3965	3097	2380	1803	1350	1001	0736	0537	0389	0281	0202							
16	9834	9371	8600	7607	6507	5408	4385	3482	2717	2089	1585	1190	0886	0654	0479	0349	0253	0182						
17	9871	9490	8823	7925	6892	5824	4300	3872	3066	2391	1839	1398	1052	0785	0581	0428	0313	0228	0165					
18	9900	9588	9014	8203	7246	6218	5205	4262	3424	2706	2109	1623	1236	0932	0698	0518	0382	0281	0205	0149				
19	9923	9668	9177	8458	7570	6590	5597	4649	3787	3033	2394	1865	1436	1095	0828	0621	0462	0342	0252	0185	0135			
20	9940	9732	9314	8678	7864	6938	5973	5029	4152	3368	2691	2121	1652	1273	0972	0736	0553	0413	0307	0226	0166	0122		
21	9954	9785	9430	8870	8129	7261	6332	5400	4515	3709	2999	2391	1883	1466	1131	0864	0656	0494	0370	0275	0203	0150	0110	
22	9964	9827	9528	9037	8367	7559	6671	5760	4875	4052	3315	2673	2127	1674	1304	1006	0770	0585	0441	0331	0247	0183	0135	0099

TABLE V

PROBABILITY OF ACCEPTANCE FOR $g = 0, h = 1$

L / K	1	2	3	4	5	6	7	8	9	10	11	12
5	2.839	5.488	7.955	10.25	12.41	14.44	16.36					
6	2.917	5.715	8.335	10.92	13.35	15.66	17.87	19.98				
7	2.957	5.843	8.643	11.35	13.96	16.48	18.92	21.27	23.55			
8	2.978	5.914	8.795	11.61	14.35	17.03	19.64	22.17	24.64	27.04		
9	2.988	5.953	8.833	11.76	14.60	17.39	20.12	22.79	25.41	27.97	30.48	
10	2.993	5.974	8.933	11.86	14.76	17.61	20.43	23.21	25.94	28.62	31.26	33.86
11	2.996	5.985	8.962	11.92	14.85	17.73	20.64	23.48	26.30	29.07	31.81	34.51
12	2.998	5.991	8.978	11.95	14.91	17.85	20.77	23.67	26.54	29.38	32.19	34.98
13	2.998	5.995	8.987	11.97	14.94	17.91	20.85	23.78	26.70	29.59	32.46	35.30
14	2.999	5.996	8.992	11.98	14.96	17.94	20.91	23.86	26.80	29.73	32.64	35.53
15	2.999	5.997	8.994	11.98	14.98	17.96	20.94	23.91	26.87	29.82	32.76	35.68
16	2.999	5.998	8.996	11.99	14.98	17.97	20.96	23.94	26.91	29.88	32.84	35.78
17	2.999	5.998	8.997	11.99	14.99	17.98	20.97	23.96	26.94	29.92	32.89	35.85
18	2.999	5.998	8.997	11.99	14.99	17.99	20.98	23.97	26.96	29.94	32.93	35.90
19	2.999	5.998	8.997	11.99	14.99	17.99	20.98	23.98	26.97	29.96	32.95	35.93
20	2.999	5.998	8.998	11.99	14.99	17.99	20.99	23.98	26.98	29.97	32.96	35.95
21	2.999	5.998	8.998	11.99	14.99	17.99	20.99	23.99	26.98	29.98	32.97	35.96
22	2.999	5.998	8.998	11.99	14.99	17.99	20.99	23.99	26.99	29.98	32.98	35.97

TABLE V (Continued)

L	13	14	15	16	17	18	19	20	21	22	23	24
K												
5												
6												
7												
8												
9												
10												
11	37.18											
12	37.73	40.46										
13	38.13	40.92	43.69									
14	38.40	41.25	44.08	46.90								
15	38.59	41.48	44.36	47.22	50.07							
16	38.72	41.65	44.56	47.46	50.34	53.21						
17	38.81	41.76	44.69	47.62	50.54	53.41	56.09					
18	38.87	41.83	44.79	47.74	50.67	53.51	56.11	58.25				
19	38.91	41.88	44.85	47.82	50.74	53.54	56.05	58.08	59.36			
20	38.94	41.92	44.90	47.86	50.76	53.52	55.96	57.91	59.11	59.34		
21	38.95	41.94	44.92	47.88	50.76	53.47	55.87	57.75	58.90	59.09	58.12	
22	38.97	41.96	44.94	47.89	50.75	53.42	55.77	57.63	58.74	58.91	57.93	55.67

TABLE VI

AVERAGE SAMPLE NUMBER FOR $g = 0$, $h = 0$

L	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
K																								
5	7453	5170	3517	2385	1623	1110	0764																	
6	8142	6104	4418	3149	2231	1579	1118	0793																
7	8649	6894	5262	3922	2885	2107	1533	1114	0894															
8	9020	7549	6029	4676	3561	2680	2002	1488	1103	0816														
9	9291	8082	6707	5390	4237	3280	2512	1910	1444	1088	0817													
10	9488	8510	7296	6050	4896	3891	3052	2370	1827	1401	1069	0814												
11	9630	8849	7799	6648	5523	4498	3608	2860	2247	1752	1359	1049	0807											
12	9733	9116	8222	7181	6109	5089	4169	3370	2696	2138	1683	1318	1028	0798										
13	9808	9324	8574	7647	6646	5652	4723	3890	3166	2551	2040	1620	1279	1005	0787									
14	9862	9485	8863	8051	7132	6182	5261	4410	3649	2987	2423	1951	1560	1241	0983	0775								
15	9901	9610	9098	8396	7565	6672	5776	4922	4137	3438	2829	2308	1869	1505	1205	0960	0762							
16	9928	9705	9289	8689	7947	7120	6262	5419	4623	3897	3251	2687	2203	1794	1453	1170	0938	0749						
17	9949	9778	9442	8934	8280	7524	6713	5894	5101	4359	3684	3083	2559	2108	1725	1403	1136	0915	0708					
18	9963	9833	9564	9138	8567	7884	7129	6344	5563	4816	4122	3492	2932	2442	2020	1660	1357	1099	0809	0510				
19	9973	9875	9660	9306	8814	8203	7508	6764	6006	5264	4560	3908	3318	2794	2335	1938	1599	1272	0890	0538	0284			
20	9981	9906	9736	9444	9023	8483	7849	7152	6426	5698	4992	4327	3714	3160	2668	2236	1856	1421	0951	0557	0289	0134		
21	9986	9930	9796	9557	9119	8725	8154	7509	6819	6113	5415	4744	4115	3537	3015	2552	2092	1538	0995	0569	0291	0135	0056	
22	9990	9948	9843	9648	9346	8935	8423	7832	7184	6507	5823	5154	4516	3920	3374	2874	2289	1626	1024	0576	0293	0135	0057	0021

VITA

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