PLANT ASSOCIATIONS AS SITE PREDICTORS IN THE PINE-HARDWOOD TENSION ZONES IN SOUTHEASTERN OKLAHOMA

By

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CHAPTER I

INTRODUCTION

It is a recognizable fact that certain species of woody plants occur together in the forest, just as it is recognizable that certain species do not occur together. Though many foresters and plant ecologists support this idea, much of their support is based on empirical data alone. It is relevant to substantiate this empirical knowledge with a quantifying analysis. Just how much confidence can be placed in these apparent plant associations or dissociations?

Site quality can be determined by a thorough investigation of the factors affecting it. Soil, moisture, temperature, slope, and aspect are the major factors affecting its quality. This rather involved procedure can be shortened, however, to a brief examination of the woody plants associated on the area. In short, certain species associations are in fact restricted to a particular site quality. With a knowledge of these associations, foresters may make accurate statements about site quality and thus relative value of land for commercial venture.

A ranking, as to the relative site requirements, of species common to the Gulf Coastal Plains has been proposed by Silker (1963). The ranking was based mostly upon observation. This ranking evaluates the sequence in which each species appears as the site quality increases from poor to excellent. In the pages that follow statistical support

is given to this ranking. It is believed that a very practical and reliable tool for the forester or plant ecologist results.

It is the intent of this paper to test given field data to determine the extent of the concept of site stratification or site determination based on the associations or dissociations of woody plant species, and to show the relationship of the species as site conditions change.

CHAPTER II

LITERATURE REVIEW

There has been considerable work done with plant associations since the concept was generated in Europe around the turn of the century. Much of the basic ground work was laid out in the early 1900's in Finland by Cajander (1926), who is generally credited with the development of the plant indicator system. He synthesized the concept of classifying the forest by type (associations), independent of any individual species. Cajander classified plant associations by those species which are abundant, those which are present but not frequent. and those which are never present. He suggested that site classification by plant indicators, in order to be practical, must be based on the climax species in areas where man has not interfered. Ilvessalo (1926), in support of Cajander's work, concluded from research in 1916, 1917, and 1918 that growth varied little within a site class, but varied greatly between different site classes.

Hodgkins (1960) used vegetative association as a measure of site potential for longleaf pine. He listed possible site indicator species and inventoried these species on all his test plots. A dominance factor was used to rank each species on each plot. This system was similar to one divised by Poulton (1959). In referring to plant communities Hodgkins stated that, "Communities can be grouped into societies and associations that in turn reflect site." He went on to

state that the challenge is to select the proper representative species to use as an evaluation of site quality. After developing his "plant indicator scale" he field tested it and found it acceptable for longleaf pine.

Spurr (1952), in an attempt to classify sites, used an "indicator plant spectrum" on northwestern spruce and fir. He rated species, or indicators, on each of four site classifications, as present, common, or abundant.

Silker (1963) reported on his study of plant communities in the Gulf Coastal Plain forests. He related plant associations to soil site characteristics, which in turn were related to silvicultural management tools. Silker (1963, 1965) suggested that the plant indicators, because of this relationship, could be used to determine the silvicultural tool best adapted for maintaining or gaining control of the site. He stressed the term "total site" in an attempt to correlate all of the relationships on a site. He developed a "wedge chart" to show the total site relationships and the silvicultural tool adaption for controlling certain associations of undesirable hardwoods. The chart also indicates the regeneration potential for southern pine (ie., shortleaf pine and loblolly pine). The associated species involved, and their competition with the southern pines, are also indicated. He used hardwoods as indicator species, a point of divergence with investigators up to 1960. Hardwoods are considered here as the climax vegetation and therefore less susceptible to temporary surface disturbance.

Daubenmire (1961) worked with vegetative indicators in ponderosa pine and used scattergrams for each "habitat type", depicting the

height growth obtainable on that site. The results obtained were subjected to regression analysis, and were classified into four categories based on the site potential for ponderosa pine. Each site was defined by the major climax woody plants occurring most frequently on that site. Daubenmire found that vegetative indicators have low value in eastern Washington and northern Idaho if site index curves are used in the application. If, however, the raw data is used for determination of site potential the correlations are stronger and the worth of plant indicators is much greater. Daubenmire also pointed out that plant indicators can prove to be a rapid and very economic means of mapping land classes and can also be helpful in predicting disease potential, (ie., Arceuthobium), in ponderosa pine.

In a discussion of the worth of plant indicators, Westveld (1954) made some summary recapitualtions of the concept. Plant communities are distinct entities which have developed according to definite biological laws. They are not distributed simply by chance. He further stated, "Since vegetation is the product of all the site factors operating on a site, it should provide a much sounder basis for site classification than would a single site factor or a group of site facors."

Quantification of research on plant associations has been for the most part left up to the plant ecologists, and much of their insight has come from quantifying animal populations. Gemborys and Hodgkins (1971) worked in western Alabama and approached a quantification analysis of vegetation in two different ways. They obtained good agreement between the two. One method they used and which they termed "dominant analysis" was based on earlier work done by Cottam (1949),

Curtis and McIntosh (1950, 1951) and Brown and Curtis (1952). They used "importance values" which were based on the sum of the relative density and relative dominance expressed as a percentage. These values were used to construct an ordination of the plots. Variation of vegetation and habitat factors were then studied in relation to this ordination. The second analysis, termed "correlation analysis", was used to develop a three-dimensional graph showing the relationships of species. Only presence and absence data were used in this analysis. A χ^2 test was used to test for association. For the construction of the three-dimensional "constellation" graph, the reciprocal of χ^2 was used. These graphs were used, then, to depict approximate inter-species relationships.

Hopkins (1957) also used presence and absence data in his work in an attempt to show species relationships. His work was done on the concept of "basic units", which he defined as a group of positively associated species within a plant community. These positively associated species were plotted in a method similar to one used by Gemborys and Hodgkins (1971) to show inter-species relationships. He concluded that his "basic unit" is of great ecological value for analyzing vegetation pattern and showing up ecological problems.

Goodall (1953), in his second of two publications dealing with the classification of vegetation, gave special attention to the value of indicator species. He used the ratio of the frequency of a particular species in a community to the frequency of that species in another community as an indicator of its worth as a site determinant species. He used a two-way table showing presence and absence of a species on two different sites. A χ^2 test and calculations of probability were

used to test the significance of the difference of the ratios of the frequencies on each of the sites. Goodall's methodology was based on the assumption that an index of fidelity which will reflect the continuous variation in the feature it measures, and which will be capable of reasonable exact treatment, should be based on only one type of measurement. Frequency was considered the most practical measurement.

There has been a great deal of work done by plant ecologists on the distributions of plant communities. Pielou (1959, 1960, 1961), Cole (1946), Romell (1930) and Greig-Smith (1964) are among those working with distributions. Most of these writers have concerned themselves with showing, by means of mathematical distributions, that plant communities do follow orderly and predictable patterns.

If any conclusions can be drawn from the work of these men and others in the field, the opinion that rarely are species distributed randomly is perhaps most important. Also of almost universal acceptance is the quadrat size is an important factor to consider in studying distributions. Pielou (1961) emphasized this point with the words, "It is meaningless to say that positive or negative association exists unless you state at what scale this association exists." This conclusion is in agreement with Greig-Smith (1964, chapter 4). Romell (1930) also believed that at some optimum sample size the organizms would appear to be randomly distributed.

Pielou (1961) tried to remedy the size of sample dilemma by the "concept of segregation". This concept is proposed to measure the association of two species in such a way that the result obtained is independent of quadrat size. The method used was based on a two-way table with a χ^2 test for association. Nearest-neighbor relationships

were used in this study and the results were good, but the work involved in establishing random points and then selecting the nearest neighbor was considered to be excessive. Pielou used an indicator of clumpiness in distribution which was determined by the equation:

$$S = 1 - \frac{\text{observed number of AB and BA nearest-neighbor relationships}}{\text{expected number of AB and Ba nearest-neighbor relationships}},$$

where A and B are two species. If the two species are segregated, the observed number of AB and BA nearest-neighbor relationships will be less than the number expected in an unsegregated population containing the same number of A's and B's; therefore when $S = 0$, an unsegregated population was indicated.

Pielou (1959), in another paper on pattern in plant populations used a point-to-plant distances method and sample density to test for randomness of distribution. The amount of work involved in the point-to-plant method was considerably less than that needed in the nearest-neighbor method. As an index on non-randomness Pielou used $\underline{a} = \pi D w$, where D is the density and w is the mean of the squares of the point-to-plant distances. Therefore $\underline{a}$ is equal to, less than, or greater than $(n - 1) / n$ depending on whether the population has a random, a regular, or an aggregated distribution. The significance of departure of $\underline{a}$ from this value is easily found since $2na$ is distributed like χ^2 with $2n$ degrees of freedom. Observed values of $\underline{a}$ from two non-random populations may be compared with a t-test.

CHAPTER III

PROCEDURE

Research plots in McIntire-Stennis project 1161 were established in southeastern part of Oklahoma. The plots were established over a period of four years from 1963 to 1966. Transects 50 feet in length were established. The area is a rolling plain of 400 to 600 feet elevation, cut into tilted, alternating Pennsylvanian-Mississippian sandstone and shale bedrock, with a few areas covered by a very thin mantle of Pliocene-Pleistocene alluvium. The areas receive 40 to 48 inches of precipitation annually and lie at the western edge of the pine-hardwood zone, just inland from the Coastal Plain. All woody plants two feet in height and taller were tallied on an area six feet either side of the transects. A total of 563 plots 12 X 50 feet were established, and each contained 600 square feet (.0137 acres). For the purposes of analysis, each plot was divided into two 6 X 50-foot plots. Each of the resulting plots had an area of 300 square feet (.0068 acres).

A standard form tally sheet was used for recording the information. Criteria of location, history, rainfall, species, diameter, and height were recorded. In addition, information of the location of each species in relationship to the stand canopy was observed. These data were then punched on IBM cards.

For purposes of this research the only information needed was

frequency of species on each plot. A summary punch was made so that each card represented one plot and contained only the frequency of each species. The summary card deck served to reduce the somewhat cumbersome size of the data deck to a readily useable size, and put it in a better form for computation.

The cards were sorted into three categories based on the empirical evaluation of site quality. The empirical stratifications were offered by Silker.[1] Stratification I is a post oak-blackjack oak site, and is represented by 118 plots at one location; stratification II is a post oak-blackjack oak-hickory site, and is represented by 626 plots at five locations; stratification III is a post oak-blackjack oak-hickory-red oak site, represented by 382 plots at three locations. The quality of the site steadily increases from stratification I to stratification III.

Univariate frequency tables were constructed for each stratification, using all the plots sampled. The frequency tables were constructed so as to list each species and the number of plots on which that species showed stem counts of 0, 1, 2, 3,..., times. The mean count was then determined as the $\Sigma fx / N$, and the standard deviation as $\sqrt{\Sigma(X - \bar{X})^2 / N - 1}$.

Observed joint frequency tables or bivariate frequency tables were constructed with the use of a computer program. These tables compared all possible two-way combinations of species and gave the number of plots which contained joint occurrences of each pair of species. An attempt was made at this point to determine whether or not individual

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[1] T. H. Silker, assistant professor of forestry, Oklahoma State University.

species appeared to be distributed according to the negative binomial frequency distribution. Expected distributions were calculated based on the negative binomial and χ^2 tests were made using the hypothesis that all species were distributed independently. An attempt was also made to predict the distribution with the Poisson expectation. As with the negative binomial, the expected distribution was generated and a χ^2 test was used to determine significance of fit.

In order to test for association between species, two-way contingency tables were built. These two-way tables showed the observed joint presence and absence values of all two-way combinations of species. Expected presence and absence values were obtained by probabilities and χ^2 tests were performed. Another attempt was made to show associations of species by a test developed by Walker (1970), called the "seven-region" analysis test. The significant results with the "seven-region" analysis were limited somewhat by the large number of zero count plots, and the small means in the univariate frequency tables.

It was first believed that the size of individual plants on a plot had a significant effect upon the positive or negative association of species on that plot. In an attempt to isolate the effect of the size of individuals on associations, separate χ^2 tests were made on a new set of two-way contingency tables. The two new sets of tables resulted from separating all the plots into: (1) those plots containing individuals > than 3.1 inches in diameter[2], and (2) those plots

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[2] Diameter at breast height for all trees 4.5 feet and taller. Diameter was measured 6 inches above the ground for trees less than 4.5 feet tall.

having no individuals as large as 3.1 inches in diameter.

A ratio of the mean number of trees per plot on the occupied to the mean number of trees for all plots was used to establish a quantitative measure of the validity of the "wedge chart" (Silker, 1963).

CHAPTER IV

RESULTS AND DISCUSSION

Selection of Indicator Species

Site evaluations may be made by observing the vegetative communities occupying a given site, and it is therefore possible to select the all-important species which distinguish one site from another. It is necessary to discover what species or groups of species occur or are absent on the given site. The presence or absence of certain species combinations may reveal the unique nature of a site.

Table I is a list of all of the species which were studied as possible indicator species. The species were tallied according to stipulations cited earlier in this paper. A total of 40 different species was recorded on the initial tallies of the plots. Of these 40 species only 32, chosen primarily on the basis of abundance or frequency, were selected for analysis. Species exhibiting low frequencies are difficult to analyze. Shortleaf pine was eliminated due to the fact that this species was the one for which the sites were being evaluated.

Subsequent to constructing the frequency tables it was found necessary to further reduce the species list to only 13 species. Only these 13 species were present with frequencies great enough to support analytical work. Not all of the 13 species were abundant or even present on all of the stratifications, but this fact in itself is an important piece of evidence on plant indicators.

TABLE I

SPECIES IDENTIFIED ON THE PLOTS

* post oak	Quercus stellata
* blackjack oak	Quercus marilandica
* hickory	Carya spp.
* tree huckleberry (tall)	Vaccineum arboreum
* flowering dogwood	Cornus florida
* black oak	Quercus velutina
* southern red oak	Quercus falcata
black haw	Viburnum refidulum
red gum	Liquidambar styraciflua
black gum	Nyssa sylvatica
white oak	Quercus alba
water oak	Quercus nigra
* sumac	Rhus glabra
* chittam wood	Bumelia lanuginosa
sassafras	Sassafras albidum
* wild plum	Prunus spp.
* winged elm	Ulmus alata
American holly	Ilex opaca
* red haw	Cretaegus spp.
* tree huckleberry (low)	Vaccineum spp.
fringe tree	Chionanthus virginicus
American elm	Ulmus americana
red maple	Acer rubrum
mulberry	Morus rubra
white ash	Fraxinus americana
red cedar	Juniperus virginiana
buckbrush	Symphoricarpos spp.
French mulberry	Morus spp.
redbud	Cercis canadensis
deciduous holly	Ilex spp.

*indicates species used in the analysis

Tests for Association With the Two-Way Contingency Tables

When an attempt is made to classify site potential using plant indicators, a decision must be made concerning which quantifying measure to use. Frequency appears to be the easiest and most adaptable to this research work. As mentioned by Goodall (1953), constancy and frequency are accurate variables for measuring the association among species.

It follows, then, that if the empirical stratifications set up in the initial research are accurate, the frequency of species in each stratification and the species themselves should vary. Stratification I, the "post oak-blackjack oak" site, is shown in table II. The univariate frequency table is constructed so that each entry represents the number of plots containing a particular count for that species, (ie., there are 31 plots, out of 118 total plots, containing no post oak). The corresponding means are shown at the bottom of each table. For example the average plot contained 2.093 post oaks. Stratifications II and III, "post oak-blackjack oak-hickory" and "post oak-blackjack oak-hickory-red oak" respectively, are shown in tables III and IV.

From the univariate frequency tables, two-way presence and absence tables were constructed. Walker (1970), Pielou (1961), Gemborys and Hodgkins (1971) and others have used two-way contingency tables in evaluating vegetative associations. Table V illustrates a two-way comparison of post oak and blackjack oak found in stratification I.

The total absence value for each species is readily obtained from the zero count position in the univariate frequency tables. Subtrac-

TABLE II

UNIVARIATE FREQUENCY TABLE FOR STRATIFICATION I

count (X)	pos. oak	blaj. oak	hic.	tr. huk.	flw. dog	bla. oak	so. red oak	bla. haw	red gum	bla. gum	wh. oak	wat. oak	sum.	chi. wd.	sas.	wil. pl.	win. elm	Am. elm	red haw	vac.	fri. tr.	Am. hol.	red map.	mul.	wh. ash	red ced.	bu. br.	Fre. mul.	red bud	dec. hol.
0	31	27	114	117	118	118	118	116	118	118	118	118	109	109	118	114	3	118	104	118	118	118	118	118	118	118	103	118	117	118
1	26	40	4	1				1					4	7		3	5		11								6		1	
2	24	21						0					3	1		0	8		2								1			
3	11	11						1					2	0		1	10		1								1			
4	10	8												0			13										2			
5	6	5												0			6										1			
6	3	2												1			4										1			
7	5	2															6										0			
8	1	1															8										0			
9	1																4										1			
10																	4										0			
11																	5										0			
12																	2										0			
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45																	1													
46																	0													
47																	0													
48																	1													
$\bar{X}$	2.09	1.82	0.03										0.14	0.13		0.05	12.50		0.15								0.87			

TABLE III

UNIVARIATE FREQUENCY TABLE FOR STRATIFICATION II

count (X)	pos. oak	blaj. oak	hic.	tr. huk.	flw. dog	bla. oak	so. red oak	bla. haw	red gum	bla. gum	wh. oak	wat. oak	sum.	chi. wd.	sas.	wil. pl.	win. elm	Am. elm	red haw	vac.	fri. tr.	Am. hol.	red map.	mul.	wh. ash	red ced.	bu. br.	Fre. mul.	red bud	dec. hol.
0	100	183	167	563	618	608	624	618	626	626	626	626	551	616	626	583	170	625	411	626	626	626	626	622	622	625	623	626	624	625
1	107	167	156	35	7	16	2	5					33	7		28	118	1	89					4	4	1	0		2	1
2	116	111	108	12	1	1		2					19	1		6	101		51								1			
3	97	72	70	7		1		1					13	1		1	62		38								2			
4	46	48	49	1									4	0		3	37		15											
5	45	19	34	2									2	0		1	38		6											
6	30	10	16	2									3	1		1	24		7											
7	19	8	13	3									1			1	10		3											
8	15	4	7	1												2	19		4											
9	16	0	2														12		1											
10	10	1	4														7		1											
11	8	3															5													
12	4																4													
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$\bar{X}$	3.32	1.73	2.00	0.22	0.01	0.08							0.26	0.03		0.14	2.97		0.81											

TABLE IV

UNIVARIATE FREQUENCY TABLE FOR STRATIFICATION III

count (X)	pos. oak	blaj. oak	hic.	tr. huk.	flw. dog	bla. oak	so. red oak	bla. haw	red gum	bla. gum	wh. oak	wat. oak	sum.	chi. wd.	sas.	wil. pi.	win. elm	Am. elm	red haw	vac.	fri. tr.	Am. hol.	red map.	mul.	wh. ash	red ced.	bu. br.	Pre. mul.	red bud	dec. hol.
0	77	319	14	339	262	221	282	361	382	378	382	382	227	375	366	342	228	381	298	381	379	382	382	373	377	381	382	376	382	382
1	86	43	51	20	77	82	70	16		2			48	7	10	29	101	1	55	1	2			8	5	1		6		
2	70	15	64	7	18	44	19	0		1			21		2	6	41		21		0			1						
3	55	3	76	8	10	17	9	0		0			15		3	4	7		4		1									
4	35	1	56	2	8	8	2	1		0			19		0	0	1		1											
5	19	0	32	1	2	7		2		0			9		1	0	2		1											
6	13	0	37	2	4	2		0		0			9			1	1		2											
7	11	1	29	2	1	1		1		1			6				0													
8	7		10	0				0					6				0													
9	2		8	0				0					4				1													
10	3		1	1				0					4																	
11	1		1					1					3																	
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31	0																													
32	0																													
33	1																													
$\bar{X}$	2.50	0.24	3.73	0.28	0.57	0.80	0.37						1.78			0.15	0.61		0.34											

tion then produces the total present for each species. Once the position of absent-absent for both species is set, the remainder of the table is produced by subtraction. The absent-absent value of seven was obtained from table VI, the bivariate frequency table for post oak and blackjack oak. The number of plots having no blackjack oak but having some post oak is 27 - 7 = 20; having no post oak but having blackjack oak is 31 - 7 = 24; having both post oak and blackjack oak is 91 - 24 = 67 or 87 - 20 = 67.

TABLE V

TWO-WAY CONTINGENCY TABLE COMPARING THE PRESENCE AND ABSENCE OF POST OAK AND BLACKJACK OAK IN STRATIFICATION I

		blackjack oak		
		Present	Absent	Total
post oak	Present	67 (a)	20 (b)	87
	Absent	24 (c)	7 (d)	31
	Total	91	27	118

From the observed values in the two-way contingency table the expectation probabilities can be calculated. A χ^2 test on the association between the two species may then be made. If the hypothesis is made that the two species are distributed independently, then the expected probabilities a, b, c, and d of their joint occurrence are the

product of their individual occurrence:

Pa = (P post oak present)(P blackjack oak present)

= (87 / 118)(91 / 118)

= .5686.

The expected number of plots is calculated as the product P(N) = .5686 (118) = 67.098.

TABLE VI

JOINT FREQUENCY TABLE FOR POST OAK AND BLACKJACK OAK IN STRATIFICATION I

		Blackjack oak count										
		0	1	2	3	4	5	6	7	8	9	T
Post oak count	0	7	10	5	2	2	1	1	2	1		31
	1	3	11	7	1	1	2	1				26
	2	6	8	4	4	1	0	0	0	0	1	24
	3	1	4	1	2	3						11
	4	5	2	2	0	1						10
	5	3	1	1	0	0	1					6
	6	1	2									3
	7	1	2	1	1							5
	8	0	0	0	0	0	1					1
	9	0	0	0	1							1
	total	27	40	21	11	8	5	2	2	1	1	118

The basis for a χ^2 test are now present between the observed and expected in the following form:

post oak present, blackjack oak absent,

(87 / 118)(27 / 118) = .1687 (118) = 19.9066,

$\chi^2 = (20 - 19.9066)^2 / 19.9066 = .0004$;

blackjack oak but no post oak,

(91 / 118)(31 / 118) = .2026 (118) = 23.9068,

$\chi^2 = (24 - 23.9068)^2 / 23.9068 = .0004$;

no blackjack oak and no post oak,

(31 / 118)(27 / 118) = .0601 (118) = 7.0918,

$\chi^2 = (7 - 7.0918)^2 / 7.0918 = .0012$.

The total χ^2 value can be obtained by summing the χ^2 values of the four parts. The value, .0001 + .0004 + .0004 + .0012 = .0021, can be looked up in a table of the distribution of χ^2 with one degree of freedom, and the hypothesis of independent distribution can be either rejected or accepted. The tabulated value is 3.84 at the .05 percent level and the calculated χ^2 value is, for all practical purposes, zero. Therefore, the hypothesis would not be rejected, and it must be concluded that post oak and blackjack oak are distributed independently.

Positive or negative association can be predicted by looking at the present-present value of the observed and expected tables. If the present-present observed value is significantly greater than the present-present expected value, positive association is indicated. If this value is significantly less than expected, the association is negative. Non-significant positive or negative values form a range, within which there is little or no association.

A different measurement of positive and negative association is offered by Cole (1946). He uses the formula Na / (a+b)(a+c) as a value to indicate positive or negative association. If this value is greater than one, positive association is present. When this value is less than one, negative association is present. In the example between post oak and blackjack oak figured above, Cole's formula would yield:

$$118\ (67)\ /\ (67+27)(67+24) = .9242.$$

This value being less than one would indicate a negative association, but according to the previous test, the negative association is not statistically significant.

In table VII are listed all significant associations by stratifications. Signs indicate positive or negative association.

Interpretation of Two-Way Associations

The significant associations shown in table VII can give a great deal of insight into the site requirements of an individual species and into associations between species. Positive association tells that a species tends to occur with other members of its association rather than randomly throughout the plot. When negative association is indicated it suggests that the two species tend to occur in one-species groups rather than with associated species or randomly throughout the plot.

Association between post oak and blackjack oak is not significant in stratifications I and II but proves to be significant in stratification III. Based on empirical observations and upon the frequencies of both species, it is evident that the sites in stratification III are becoming unsuitable for their optimum occurrence. The frequencies have

peaked and are tapering off as competition from other species for the site increases. Comparison of the $\hat{k}$ values of the negative binomial distribution indicate that the species are becoming more contagiously distributed. (See table XXV for a comparison and explanation of the $\hat{k}$ values.) The fact that these two species are forced into clumps or groups gives supporting evidence to the significant association found.

The negative association found between post oak and flowering dogwood in stratification III indicates the tendency of post oak to clump together on the poorer end of the site and the tendency of flowering dogwood to clump on the better end of the site. The clumping of the two species on different microsites causes the observed negative frequencies to exceed the expected negative frequencies.

The negative association between blackjack oak and winged elm in stratification II is explained in the same manner. In stratification II blackjack oak exhibits only mildly contagious distribution, whereas winged elm is strongly contagious on this same stratification. The result is a negative association between the two species.

The $\hat{k}$ values of the negative binomial distribution suggest that hickory and tree huckleberry are negatively associated. However, a comparison of the frequencies of the two species in each of the stratifications suggests that both species are just beginning to establish themselves on stratification II. The fact that both species prefer stratification II supports the positive association found on that stratification.

Hickory and red haw are the only two species exhibiting positive association in one stratification and negative association in another. The positive association on stratification II is supported strongly by

the fact that both species tend to be contagiously distributed on this site. The reverse holds true in stratification III. Hickory tends to dominate this site with a very even distribution, while red haw is more clumped here than in stratification II. These relationships are borne out by the $\hat{k}$ values shown in table XXV.

TABLE VII

POSITIVE AND NEGATIVE ASSOCIATIONS BETWEEN SPECIES, BY STRATIFICATION (SIGNIFICANT χ^2 TESTS IN 2X2 CONTINGENCY TABLES)

Association	Stratification		
	I	II	III
post oak-blackjack oak			+
post oak-flowering dogwood			+
post oak-sumac	+		
blackjack oak-sumac		+	+
blackjack oak-wild plum		-	
blackjack oak-winged elm		-	
hickory-wild plum		+	
hickory-tree huckleberry		+	
hickory-red haw		+	-
tree huckleberry-flowering dogwood			-
tree huckleberry-southern red oak			+
tree huckleberry-sumac		+	
flowering dogwood-black oak			+
flowering dogwood-southern red oak			+
flowering dogwood-sumac			-
flowering dogwood-red haw			-
black oak-winged elm			-
southern red oak-sumac			-
sumac-wild plum		-	
sumac-winged elm		-	
sumac-red haw		-	+
chittam wood-symphoricarpos	+		
wild plum-winged elm		+	
wild plum-red haw		+	
winged elm-symphoricarpos	-		
winged elm-red haw		+	+

+ positive association, - negative association

The negative association shown by tree huckleberry and flowering dogwood is puzzling at first examination, but further study reveals a possible explanation. Both species exhibit a contagious distribution in stratification III, but these clumps could indicate two separate microclimates or niches. The flowering dogwood would therefore occupy the better of the two niches because it is at the threshold of its prefered site. On the other hand the tree huckleberry would occupy the poorer of the two niches. The reproductive characteristics of tree huckleberry could also help explain the very clumped distribution.

The positive association between tree huckleberry and southern red oak on stratification III follows the pattern already established. Both are highly contagious distributions with about the same site requirements. A comparison of the means of these two species on stratification III show that their frequencies are about equal.

The flowering dogwood and black oak association and the flowering dogwood and southern red oak association on stratification III also follow in the established pattern. The $\hat{k}$ values indicate highly contagious distributions. This particular site defines the threshold for all three species. The frequencies of the three in stratifications I and II are comparable, also indicating a close association.

Flowering dogwood and red haw exhibit a negative association in stratification III. The basis for this can be established by the decreasing frequency of red haw and the increasing frequency of flowering dogwood. Stratification III tends to favor the flowering dogwood, but not the red haw. They would each be contagiously distributed at opposite ends of the stratification, red haw occupying the poorer areas and flowering dogwood the better areas.

Symphoricarpos and chittam wood are strongly represented in stratification I, as evidenced by their high frequencies. Symphoricarpos, in particular, shows a significant mean in stratification I, but is completely absent in stratification III. The strong tendency for both species to occupy the same site gives support to the positive association found in stratification I.

Winged elm occurs most frequently in stratification I but is also present in stratification II. It is a minor component is stratification III. Red haw is prevalent in stratification II, and is a minor component is stratifications I and III. The lack of association in stratification I is supported by the low frequency of red haw on that site. Stratification II, however, is favorable to both species, and this is the probable reason for positive association between the two species on that site. Stratification III is not favorable to either species which would tend to force them to clump together on the poorer extreme of the site. The positive association between the two species in stratification III is evidence supporting the replacement of these species with others of higher site requirements. Both species exhibit relatively strong contagion wherever they occur.

Other associations are of lesser importance in terms of the objectives of this study. The associations and species singled out of the table are considered more reliable for site determination than are the other species. The chosen species largely are the major species of the "wedge chart" offered by Silker (1963, 1965).

Many of the species did not have the frequency of occurrence required to be of great value as an indicator species. Still other species are ignored for other reasons peculiar to them alone. One

species which is quite common but not used is sumac. Sumac is considered to be an unreliable species due to its characteristics of establishment. It tends to be contagiously distributed wherever it occurs. It also appears to be a temporary species which moves in rapidly after a disturbance, occupying the site in a very clumpy distribution, and then disappearing from the site.

The associations involving winged elm are also discredited heavily. Though winged elm shows a definite high frequency on stratification I, it can occur in sufficient numbers on all sites to generate positive associations with other species. Positive associations with winged elm, therefore, are of little significance in identifying site. Winged elm also demonstrates characteristics similar to sumac in that it invades a site, particularly a poorer one, with any disturbance.

Before attempting to draw any conclusions concerning the associations present or absent in any stratification it should be remembered that the forest is a continuum and distinct boundaries are ambiguous at best. By this, reference is made to the stratification boundaries. Stratifications are not separated by exact boundaries, but instead tend to overlap, making some associations difficult to delimit exactly. By the same reasoning the three stratifications cannot be defined by three separate points, but must be defined as ranges. Each range will vary from the poorer extreme to the better, and will overlap to some extent with the adjacent stratification.

Statistically, the significant associations in table VII indicate a particular site. Any of the three stratifications can, therefore, be predicted by one or more associations of species.

As a practical field tool, an experienced user can become fairly

adept at pinpointing a site based on plant indicators. It should be remembered that the total association should be examined and not only one or two species. One or two species can have a wide range of habitat which makes an accurate site appraisal difficult. If, however, all the associations are taken into account and the parameters of frequency and size are considered, an accurate site appraisal can be made. Frequency and size give an indication as to where in the range of a certain species, the site is located. That is to say, if a species normally occurs as large specimens distributed with high frequency, but is found in a specific condition only as small plants occurring infrequently, the probable cause is a site far from optimum for the species.

Joint Frequency and the Seven-Region Analysis

Joint frequency, or bivariate frequency, tables were constructed to get a better picture of the association between two species. These tables were constructed in the form indicated in table V. As stated by Walker (1970) in his study of diameter distribution, the two-way tables and their χ^2 test do not give an accurate picture of the joint frequency and they do not offer a coefficient of association. Walker's seven-region analysis test was developed in an effort to overcome these deficiencies.

The breakdown of the seven-region analysis test is as follows:

Region 1. Plots having less than the mean number of trees for both species.

Region 2. Plots having greater than the mean number of trees for

both species.

Region 3. Plots having greater than the mean number of trees for one species and less than the mean number of trees for the other species.

Region 4. Plots having less than the mean number of trees for one species and greater than a mean number for the other.

Region 5. Plots having the mean number of trees for both species.

Region 6. Plots having a mean number of trees for one species and a non-mean number of trees for the other.

Region 7. Same as region 6 but reversing the species.

An illustration of region breakdown is given in table VIII.

The construction of table IX, the seven-region analysis table, uses both the univariate frequency table and the bivariate frequency table. The values for each of the seven regions for the post oak and blackjack oak are tallied with relation to the respective means from the univariate frequency tables. For example, the means for post oak and blackjack oak in stratification I are approximately equal to two. (The means must be rounded to the nearest integer.) Tallies are then made for each category by combining the tallies in the indicated cells of the table.

The expected units for post oak and blackjack oak are determined by the product of their individual probabilities. The general form for the expected units is as follows: for category one,

($<\bar{X}$ post oak units / total post oak units)($<\bar{X}$ blackjack oak units / total blackjack oak units)(total number of plots, N) = expected number of units.

In the example given above between post oak and blackjack oak the

results are:

(57 / 118)(67 / 118)(118) = 32 expected units in category one.

TABLE VIII

SOURCE OF VALUES FOR THE SEVEN REGIONS
(ASSUMING $\bar{X}$=2 AND $\bar{Y}$=3)

		species X						
		0	1	2	3	4	5	6
species Y	0	region 1		region 7	region 3			
	1							
	2							
	3	region 6		region 5	region 6			
	4	region 4		region 7	region 2			
	5							
	6							

The actual units for category one in the seven-region table are obtained from the bivariate table partitioned appropriately as indicated in table X. The partitioning is based on the general form given in table VIII with the appropriate means for post oak and blackjack oak drawn in. Summation of the partitioned cells yields the value indi-

cated in each category in table IX.

TABLE IX

SEVEN-REGION ANALYSIS AND χ^2 VALUE FOR THE ASSOCIATION BETWEEN POST OAK AND BLACKJACK OAK

region		post oak units		blackjack oak units	expected value units	observed value units	partial χ^2
1	$<\bar{X}$	57	$<\bar{X}$	67	32	31	.0312
2	$>\bar{X}$	37	$>\bar{X}$	30	9	10	.1111
3	$<\bar{X}$	57	$>\bar{X}$	30	14	14	0
4	$>\bar{X}$	37	$<\bar{X}$	67	21	22	.0476
5	$\bar{X}$	24	$\bar{X}$	21	4	4	0
6	$\bar{X}$	24	non $\bar{X}$	97	20	20	0
7	non $\bar{X}$	94	$\bar{X}$	21	17	17	0

total χ^2 = .1899

With the observed and expected values in category I a partial χ^2 value can be determined:

$$(31 - 32)^2 / 32 = .0321.$$

The partial values for each region are then summed to give a total χ^2 value which can be used to test for significance with four degrees of freedom. Again as in the two-way test the hypothesis is that the two species are distributed independently.

Positive or negative association can be indentified by comparing

the first four categories. Positive association is indicated when the actual units are larger than the expected units in categories one and two and less than the expected units in categories three and four. Negative association is indicated when the reverse of the above occurs.

TABLE X

BIVARIATE FREQUENCIES OF POST OAK AND BLACKJACK OAK PARTITIONED FOR SEVEN-REGION ANALYSIS

		blackjack oak (trees per plot)										
		0	1	2	3	4	5	6	7	8	9	10
post oak	0	7	10	5	2	2	1	1	2	1	0	0
	1	3	11	7	1	1	2	1	0	0	0	
	2	6	8	4	4	1	0	0	0	0	1	
	3	1	4	1	2	3						
	4	5	2	2	0	1						
	5	3	1	1	0	0	1					
	6	1	2	0								
	7	1	2	1	1							
	8	0	0	0	0	0	1					
	9	0	0	0	1							

Results of the seven-region analysis tests are given in table XI. The results are sketchy due to the limitation that the means of the

species involved must be rounded to the nearest integer value. Because of the high zero count and low frequency many associations could not be evaluated with this test. A comparison of the results obtained from the two-way tests with those obtained from the seven-region analysis tests is made in table XI. There is little difference between the values obtained in the two-way tests and those obtained in the seven-region analysis on the significant associations.

The species post oak and hickory show significant negative association in stratification III by the seven-region test, but no association by the two-way analysis. The negative association is borne out by the decreasing frequency of post oak on an unfavorable site and the increasing frequency of hickory on a site more favorable to its occurrence.

The blackjack oak and hickory association in stratification II also shows negative association in the seven-region analysis, but not in the two-way analysis. The fact that negative association exists also appears to be supported by the decreasing blackjack oak and increasing hickory frequencies.

Both of the above associations and the other associations which were tested with the seven-region analysis appear to follow the pattern previously established in the discussion of the two-way associations (pages 22 to 28).

Tests for Effect of Size of Plants on Limiting Association

It was believed that the size classes in a particular species and the frequency of occurrence of that species would have an effect on the

TABLE XI

COMPARISON BETWEEN TWO-WAY AND SEVEN-REGION χ^2 TESTS FOR SPECIES SHOWING SIGNIFICANT POSITIVE AND NEGATIVE ASSOCIATION

Association	Stratification		
	I	II	III
* post oak-blackjack oak			+
post oak-hickory			--
* post oak-flowering dogwood			-
* post oak-sumac	+		
post oak-winged elm		++	
* blackjack oak-sumac		+	+
blackjack oak-hickory		--	
* blackjack oak-wild plum		-	
blackjack oak-winged elm		- --	
* hickory-tree huckleberry		+	
* hickory-wild plum		+	
hickory-red haw		+ ++	-
* tree huckleberry-flowering dogwood			-
* tree huckleberry-southern red oak			+
* tree huckleberry-sumac		+	
* flowering dogwood-black oak			+
* flowering dogwood-southern red oak			+
* flowering dogwood-sumac			-
* flowering dogwood-red haw			-
* black oak-winged elm			-
* southern red oak-sumac			-
* sumac-wild plum		-	
* sumac-winged elm		-	
* sumac-red haw		-	+
* chittam wood-symphoricarpos	+		
* wild plum-winged elm		+	
* wild plum-red haw		+	
* winged elm-symphoricarpos	-		
winged elm-red haw		+ ++	+

\+ two-way positive association
\- two-way negative association
++ seven-region positive association
-- seven-region negative association
* associations which could not be tested with the seven-region analysis test

association of the species around it. There are numerous reasons for suspecting that a size factor might significantly affect association. Preemption of growing space, tolerance of shade, root area, trunk diameter, and competition for the nutrients and soil moisture are the primary factors which determine whether or not such modification of association will actually follow.

In an attempt to measure the effect the individual plant size had on a plot, it was necessary to subdivide the plots on the basis of a limiting size. The total plots in each stratification were sorted on the basis of plots containing individuals 3.1 inches in diameter and larger and those having no individuals as large as 3.1 inches in diameter. Diameter was measured at 4.5 feet above mean ground level for those individuals with sufficient height to qualify. For those plants between 2 feet and 4.5 feet in height the diameter was measured 6 inches above the ground. Diameter was recorded to the nearest tenth of an inch.

New univariate frequency tables were constructed, using the newly-sorted card decks. The 118 plots of stratification I were separated into 87 plots having individuals > 3.1 inches and 31 plots having no individuals as large as 3.1 inches in diameter. These frequencies are illustrated in tables XII and XIII respectively.

The 626 plots in stratification II were separated into 447 plots which had individuals > 3.1 inches in diameter and 177 plots which had no members this large. The two frequency tables are illustrated in tables XIV and XV respectively.

Finally, the 382 plots composing stratification III were subdivided into 296 plots containing individuals 3.1 inches in diameter and

larger and 86 plots having no such size classes. Tables XVI and XVII illustrate the two frequency tables.

A comparison of the two different size groups, > 3.1 diameter and < 3.1 diameter, was made and the associations were measured. The expectation was that the associations on the size-stratified groups would be different. In order to test for the difference in association, a series of two-way tables were constructed for each size group, just as was done for the whole sample. A χ^2 test was then used as before to determine the significance of associations. The comparisons are recorded in table XVIII.

Examination of the comparisons in association reveals that no effect on association is caused by the presence of the larger members. When the two size breakdowns are compared to the whole, a reference must be made to the number of plots used for calculating association.

The N values used in the < 3.1 inch diameter class are too small to yield an adequate sample. It should also be noted that due to the small N value involved in the whole of stratification I, any further breakdown of that stratification also yields an insufficient N count. With knowledge of these insufficient N values the differences in association can be explained.

The post oak-blackjack oak, post oak-flowering dogwood and tree huckleberry-southern red oak associations, when compared to the whole did not change in any of the stratifications.

There appeared to be a difference in association between the whole and the size-stratified groups among the associations of post oak-symphoricarpos and chittam wood-symphoricarpos. There also appeared to be a difference among the associations of post oak-chittam wood, post

TABLE XII

UNIVARIATE FREQUENCY TABLE FOR PLOTS HAVING SOME INDIVIDUALS > 3.1 INCHES IN DIAMETER (STRATIFICATION I)

count (X)	pos. oak	blaj. oak	hic.	tr. huk.	flw. dog	bla. oak	so. red oak	bla. haw	red gum	bla. gum	wh. oak	wat. oak	sum.	chi. wd.	sas.	wil. pl.	win. elm	Am. elm	red haw	vac.	fri. tr.	Am. hol.	red map.	mul.	wh. ash	red ced.	bu. br.	Fre. mul.	red bud	dec. hol.
0	13	17	83	86	87	87	87	86	87	87	87	87	80	79	87	84	3	87	78	87	87	87	87	87	87	87	74	87	86	87
1	19	27	4	1				1					4	6		2	4		8								6		1	
2	22	18											2	1		0	8		1								0			
3	10	10											1	0		1	13										1			
4	10	6												0			4										2			
5	6	4												0			2										1			
6	2	2												1			3										0			
7	3	1															5										0			
8	1	1															4										0			
9	1	1															3										1			
10																	4										0			
11																	2										0			
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42																	1													
43																	0													
44																	1													
45																	0													
46																	0													
47																	1													
48																	1													
$\bar{X}$	2.45	1.99											0.13	0.16			11.37		0.12								1.09			

TABLE XIII

UNIVARIATE FREQUENCY TABLE FOR PLOTS WITH NO MEMBERS AS LARGE AS 3.1 INCHES IN DIAMETER (STRATIFICATION I)

count (X)	pos. oak	blaj. oak	hic.	tr. huk.	flw. dog	bla. oak	so. red oak	bla. haw	red gum	bla. gum	wh. oak	wat. oak	sum.	chi. wd.	sas.	wil. pl.	win. elm	Am. elm	red haw	vac.	fri. tr.	Am. hol.	red map.	mul.	wh. ash	red ced.	bu. br.	Fre. mul.	red bud	dec. hol.
0	18	10	31	31	31	31	31	30	31	31	31	31	29	30	31	30	0	31	26	31	31	31	31	31	31	31	29	31	31	31
1	7	13						0					0	1		1	1		3								0			
2	2	3						0					1				0		1								1			
3	1	1						1					1				2										0			
4	0	2															0										0			
5	0	1															2										0			
6	1	0															2										1			
7	2	1															3													
8																	3													
9																	0													
10																	1													
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37																	0													
38																	0													
39																	1													
40																	1													
$\bar{X}$	1.10	1.36											0.16				15.65		0.26									0.26		

TABLE XIV

UNIVARIATE FREQUENCY TABLE FOR PLOTS HAVING SOME INDIVIDUALS > 3.1 INCHES IN DIAMETER (STRATIFICATION II)

count(X)	pos. oak	blaj. oak	hic.	tr. huk.	flw. dog	bla. oak	so. red oak	ble. haw	red gum	bla.gum	wh. oak	wat. oak	sum.	chi. wd.	sas.	wil. pl.	win. elm	Am. elm	red haw	vac.	fri. tr.	Am. hol.	red map.	mul.	wh. ash	red cod.	bu. br.	Pre. mul.	red bud	dec. hol.
0	61	122	56	395	439	435	445	440	447	447	447	447	393	441	447	414	115	446	284	447	447	447	447	444	443	446	446	447	446	446
1	73	125	109	30	7	10	2	5					23	5		22	84	1	63					3	4	1	0		1	1
2	88	82	80	12	1	1		2					17	0		6	70		40								0			
3	71	53	57	4		1							10	0		1	47		31								1			
4	36	30	39	0									1	0		1	30		12											
5	35	14	30	2									1	0		1	33		5											
6	19	8	15	1									1	1		0	15		5											
7	14	7	11	2									1			1	8		2											
8	14	3	6	1												1	10		3											
9	13	0	1														11		1											
10	10	0	3														4		1											
11	5	3															4													
12	2																2													
13	1																2													
14	0																0													
15	0																3													
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32																	0													
33																	0													
34																	0													
35																	0													
36																	0													
37																	0													
38																	1													
$\bar{X}$	3.37	1.72	2.24	0.23									0.24			0.14	3.02		0.86											

TABLE XV

UNIVARIATE FREQUENCY TABLE FOR PLOTS WITH NO MEMBERS AS LARGE AS 3.1 INCHES IN DIAMETER (STRATIFICATION II)

count (X)	pos. oak	blaj. oak	hic.	tr. huk.	flw. dog	bla. oak	so. red oak	bla. haw	red gum	bla. gum	wh. oak	wat. oak	sum.	chi. wd.	sas.	wil. pl.	win. elm	Am. elm	red haw	vac.	fri. tr.	Am. hol.	red map.	mul.	wh. ash	red ced.	bu. br.	Fre. mul.	red bud	dec. hol.
0	39	61	71	166	177	171	177	176	177	177	177	177	157	173	177	168	55	177	125	177	177	177	177	176	177	177	175	177	176	177
1	34	42	47	5		6		0					10	2		6	34		26					1			0		1	
2	28	29	28	0				0					2	1		0	31		11								1			
3	26	18	13	3				1					2	1		0	15		7								1			
4	9	17	9	1									3			2	7		3											
5	10	5	4	0									1			0	5		1											
6	11	2	1	1									2			2	9		2											
7	5	1	1	1													2		1											
8	1	1	1														8		1											
9	3	0	1														1													
10	0	1	1														3													
11	2																1													
12	2																2													
13	0																1													
14	1																1													
15	1																0													
16	2																0													
17	1																0													
18	0																0													
19	1																0													
20	0																0													
21	0																0													
22	1																0													
23																	0													
24																	0													
25																	0													
26																	2													
$\bar{X}$	3.14	1.61	1.35	0.18									0.28			0.14	2.71		0.64											

TABLE XVI

UNIVARIATE FREQUENCY TABLE FOR PLOTS HAVING SOME INDIVIDUALS > 3.1 INCHES IN DIAMETER (STRATIFICATION III)

count (X)	pos. oak	blej. oak	hic.	tr. huk.	flw. dog	bla. oak	so. red oak	bla. haw	red gum	bla. gum	wh. oak	wat. oak	sum.	chi. wd.	sas.	wil. pl.	win. elm	Am. elm	red haw	vac.	fri. tr.	Am. hol.	red map.	mul.	wh. ash	red ced.	bu. br.	Fre. mul.	red bud	dec. hol.
0	56	251	6	264	197	167	211	281	296	292	296	296	181	290	281	267	175	295	236	295	293	296	296	287	292	295	296	294	296	296
1	68	30	40	14	63	62	60	11		2			38	6	10	20	77	1	38	1	2			8	4	1		2		
2	56	12	47	5	15	36	16	0		1			14		1	5	37		18		0			1						
3	46	1	63	5	9	13	8	0		0			8		3	4	3		1		1									
4	26	1	41	2	6	8	1	1		0			18		0		1		1											
5	17	0	26	1	2	7		2		0			7		1		1		1											
6	8	0	28	2	3	2		0		0			6				1		1											
7	9	1	23	2	1	1		0		1			4				0													
8	4		9	0				0					5				0													
9	0		8	0				0					2				1													
10	3		1	1				0					4																	
11	1		1					1					3																	
12	1		3										1																	
13	0												2																	
14	0												0																	
15	1												1																	
16													1																	
17													0																	
18													0																	
19													0																	
20													1																	
$\bar{X}$	2.43	0.23	3.88	0.30	0.61	0.88	0.41	0.12					1.63		0.09	0.14	0.62		0.31											

TABLE XVII

UNIVARIATE FREQUENCY TABLE FOR PLOTS WITH NO MEMBERS AS LARGE AS 3.1 INCHES IN DIAMETER (STRATIFICATION III)

count (X)	pos. oak	blaj. oak	hic.	tr. huk.	flw. dog	bla. oak	so. red oak	bla. haw	red gum	bla. gum	wh. oak	wat. oak	sum.	chi. wd.	sas.	wil. pl.	win. elm	Am. elm	red haw	vac.	fri. tr.	Am. hol.	red map.	mul.	wh. ash	red ced.	bu. br.	Fre. mul.	red bud	dec. hol.
0	21	68	8	75	65	54	71	80	86	86	86	86	46	85	85	75	53	86	62	86	86	86	86	86	85	86	86	82	86	86
1	18	13	11	6	14	20	10	5					10	1	0	9	24		17						1			4		
2	14	3	17	2	3	8	3	0					7		1	1	4		3											
3	9	2	13	3	1	4	1	0					7			0	4		3											
4	9		15		2		1	0					1			0	0		0											
5	2		6		2			0					2			0	1		0											
6	5		9		0			0					3			1			1											
7	2		6		1			1					2																	
8	3		1										1																	
9	2												2																	
10	0												0																	
11	0												0																	
12	0												0																	
13	0												0																	
14	0												1																	
15	0												1																	
16	0												0																	
17	0												2																	
18	0												0																	
19	0												0																	
20	0												1																	
21	0																													
22	0																													
23	0																													
24	0																													
25	0																													
26	0																													
27	0																													
28	0																													
29	0																													
30	0																													
31	0																													
32	0																													
33	1																													
$\bar{X}$	2.77	0.29	3.23	0.22	0.43	0.56	0.27	0.14					2.33			0.20	0.57		0.44											

TABLE XVIII

COMPARISON OF ASSOCIATIONS AMONG ALL PLOTS, THOSE WITH SOME MEMBERS > 3.1 INCHES DIA., AND THOSE WITH ALL MEMBERS < 3.1 INCHES DIA.

Association	Stratification								
	I			II			III		
post oak-blackjack oak							+	++	+++
post oak-hickory						+++			
post oak-flowering dogwood							-	--	---
post oak-sumac	+	--							
post oak-chittam wood		--							
post oak-winged elm		++							
post oak-red haw		--							
post oak-symphoricarpos		--							
blackjack oak-sumac				+	++		+	++	
blackjack oak-wild plum				-	--				
blackjack oak-winged elm				-	--				
blackjack oak-symphoricarpos		++	+++						
hickory-tree huckleberry				+	++				
hickory-sumac						+++			
hickory-wild plum				+	++				
hickory-winged elm									---
hickory-red haw				+	++		-		
tree huckleberry-flowering dogwood							-	--	
tree huckleberry-southern red oak							+	++	+++
tree huckleberry-sumac				+	++				
tree huckleberry-winged elm									---
flowering dogwood-black oak							+		
flowering dogwood-southern red oak							+	++	
flowering dogwood-sumac							-	--	---
flowering dogwood-red haw							-	--	
black oak-sumac									---
black oak-winged elm							-	--	
southern red oak-sumac							-	--	
sumac-wild plum				-	--				
sumac-winged elm				-	--				
sumac red haw				-	--		+	++	
chittam wood-symphoricarpos	+	++							
wild plum-winged elm				+		+++			
wild plum-red haw				+	++				
winged elm-red haw				+	++	+++	+	++	
winged elm-symphoricarpos	-	--							

+, - positive or negative association on all plots

++, -- positive or negative association on plots with 3.1 inch dia. plants present

+++, ---positive or negative association on plots with no plants as large as 3.1 inches in dia.

oak-red haw and blackjack oak-symphoricarpos. Little significance is attributed to this difference due to the insufficient N count in the size-stratified groups.

Too few plots are available for testing association in the other important associations, such as hickory-tree huckleberry, hickory-red haw, tree huckleberry-flowering dogwood, flowering dogwood-southern red oak, flowering dogwood-red haw and chittam wood-symphoricarpos.

An exception to the above-mentioned lack of sufficient sample units occurs in stratification III in the associations between hickory-red haw and flowering dogwood-black oak. The N count for the > 3.1 inch diameter class should have been sufficient to get an adequate association evaluation.

It is believed that, while the 3.1 inch diameter limitation yielded no significant size factor relationships, an alternative larger size-class limit would produce some significant relationships, provided that sufficient sample units were available to support statistically sound measurements. More research is needed to determine if size is a limiting factor in the association of species.

A Quantified Ranking of Species With Respect to Site Quality

Figure 1, called a "wedge chart", represents a "total site classification" offered by Silker (1963). The chart consists of three parts: (A) Recommendations are made for the best adaptable silviculture tool for gaining control and maintaining control of the undesirable hardwoods; and surface soil depths and the plant associations to which these tools are appropriately assigned for control work. (B) The

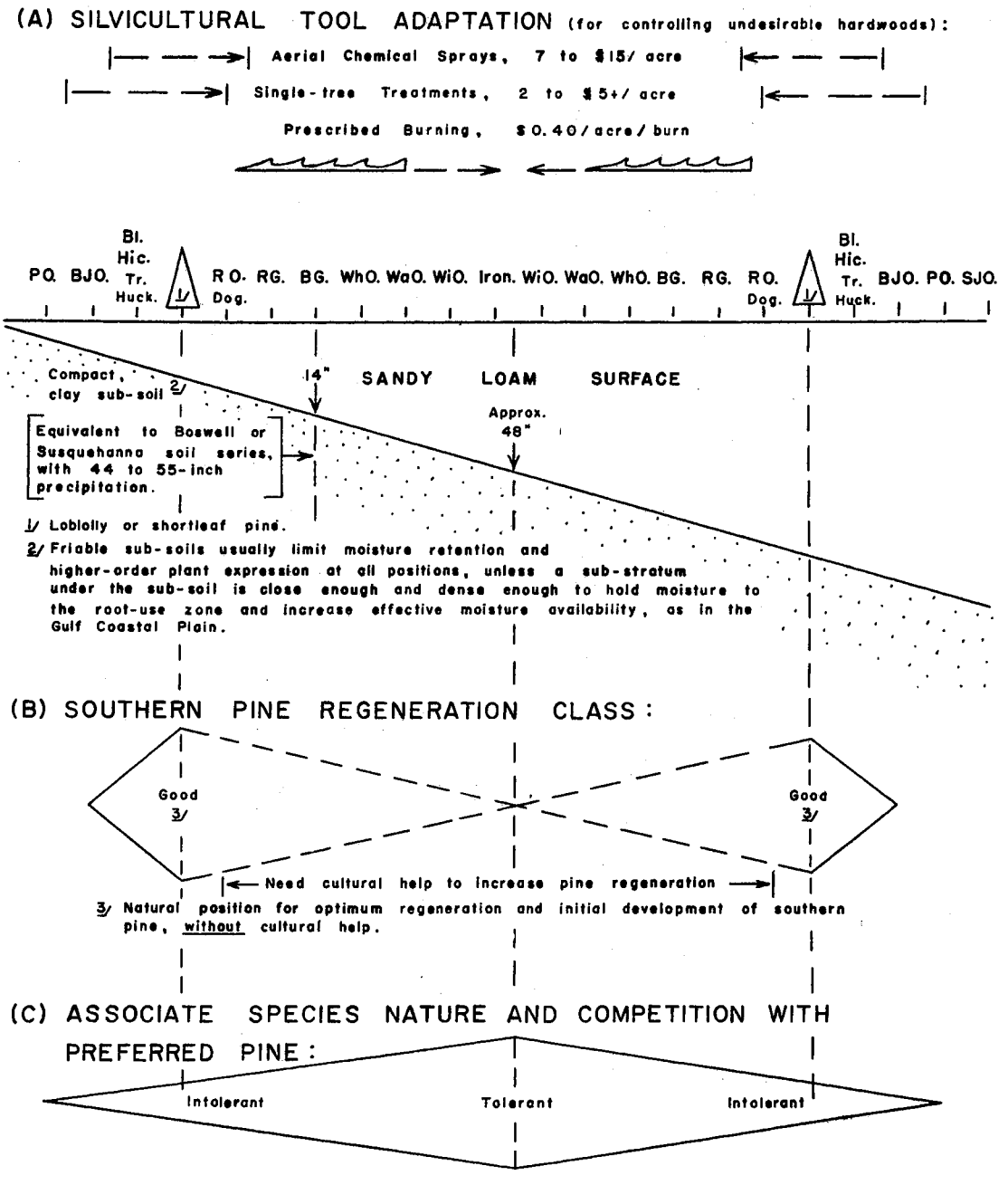

Figure 1. "Total Site Classification" by the Use of Plant Indicator Sequence. Tentative Rating and Relative Position of Predominant and Common Hardwoods in Reflecting Soil Moisture Availability

southern pine regeneration class as indicated by soil depth and related plant association. (C) The degree of competition encountered with the associated hardwoods is indentified. The ranking of species above the increasing soil depth indicates when a species is able to come in and establish itself naturally, with no cultural aid. This ranking is based on soil depth and moisture retention of the soil.

In reading the chart it may be observed that there are two relative positions where shortleaf and loblolly pine regeneration is optimum, without cultural help. The pine will thrive on the red oak and better sites, but will not occur frequently and naturally unless there is site disturbance. These pine must have silvicultural help to regenerate on the site.

The importance of the chart to this research rests primarily in the ranking of the species. Again, the positions indicated on the chart reflect not a single point, but rather a range, within which the species establishes itself. The site quality is indicated by the total plant association on the site. That is to say, site quality is determined by examining each species in relation to the other species. Frequency and the highest "demand species" on the site supply the information for determining site quality.

In an attempt to quantify the wedge chart, two parameters were examined for their possible usefulness. The first of these is the mean frequency of each species on each of the three stratifications. Frequency indicates the adaptability of that species for a particular stratification. Secondly, an "evenness of distribution ratio" may express the clumpiness or contagion of a species on a stratification, or it may indicate the uniformity with which the species is distributed.

Mean frequencies are given in table XIX for ease in comparison with the "evenness of distribution ratios", which are shown in table XX.

The "eveness of distribution ratios" are expressed by the ratio of the mean frequency of the occupied plots to the mean frequency of all the plots. Thus, for post oak in stratification I the following ratio is obtained:

$$\text{evenness of distribution ratio} = \bar{X} \text{ occupied plots} / \bar{X} \text{ all plots}$$
$$= 2.839 / 2.093$$
$$= 1.356.$$

A value approximating 1.0 indicates that the species is evenly distributed on the site. Values greater than 1.0 indicate clumpy distributions, the ratio increasing with increasing clumpiness or contagion. The evenness of distribution ratio should be distinguished from the moment estimate of k of the negative binomial distribution series, though the intent of both is to some degree intended to show contagion. However, the agreement between the $\hat{k}$ values for estimating contagion, table XXV, and the evenness of distribution ratio for estimating contagion should be noted. Table XX is a comparison of the evenness of distribution ratios by stratification. Where meaningful means were not determinable but where slight traces of the species occurred, the word trace was inserted. If a species was completely absent, it was so indicated.

Using mean frequency and the evenness of distribution ratios it is possible to identify the "areas" which are more suitable to a given species. A high frequency and a low evenness of distribution ratio reveals where that species is more favorably located. (The evenness of

distribution ratio indicates where a species is more evenly distributed, hence no micro-niche clumping, indicating an adverse site condition.) The term "area", rather than "stratification" is used in the above discussion. A species may tend to favor the overlapping zone rather than centering within a stratification.

TABLE XIX

MEAN FREQUENCIES BY STRATIFICATION

Species	Stratification		
	I	II	III
	the number of trees per average plot		
post oak	2.093	3.316	2.503
blackjack oak	1.822	1.730	.243
hickory	.033	1.995	3.730
tree huckleberry	.008	.216	.280
flowering dogwood		.014	.565
black oak		.080	.804
southern red oak		.003	.374
black haw	.034	.019	.126
black gum			.029
sumac	.136	.257	1.783
chittam wood	.127	.029	.018
sassafras			.073
wild plum	.051	.142	.154
winged elm	12.500	2.966	.610
red haw	.153	.813	.340
mulberry		.006	.026
white ash		.006	.013
symphoricarpos	.873		

Most species can be placed in their respective stratifications using the above methods. Exceptions arise when the values are not

sufficiently different to ascertain the site preference of one species compared to another. Such is the case with post oak and blackjack oak. It is not discernible which stratification is prefered by either species. The jump in the evenness of distribution ratio and the drop in mean value of blackjack oak in stratification III is the only suggestion that blackjack oak does not prefer stratification III. It is evident that both species prefer stratifications I and II, but as to which site is prefered by which species no definite conclusions can be made.

Stratifications II and III definitely tend to favor hickory, with stratification III being much more nearly an optimum site.

Tree huckleberry parallels the hickory occurrence but does not occur as frequently as hickory on stratification III.

Stratification III very strongly favors flowering dogwood, black oak, southern red oak and black haw.

Stratification I is prefered by symphoricarpos and chittam wood, but chittam wood in particular is found also on the other stratifications.

Stratification II appears to be only slightly better than stratification III for the occurrence of red haw.

These species and other lesser species are given in table XXI, which locates each species in the stratification it apparently prefers.

Quantitative rankings of the species may be developed, based on the assumption that the lower the evenness of distribution ratio, the better suited the site is for that species. The stratification ranking in table XXI represents the initial position of each species. It is assumed then, that those species which appear to favor stratifica-

tion I will establish themselves on poorer sites than the species which appear to favor stratification II and III. The problem now lies with positioning each of the species within the stratifications. This problem may be solved by comparing the magnitude of change from one site to the next with respect to frequency and evenness of distribution ratio. Therefore, a species which obviously prefers one site over another, as revealed by a low evenness of distribution ratio on the preferred site, would be ranked in a position ahead of a species which exhibits no strong preference. A ranking of the species based on these relationships is presented in table XXII.

It should be noted that the difference in the values between post oak and blackjack oak are so small that their relative position is not readily discernible. Actual observations from the field suggest that post oak be placed just ahead of blackjack oak. Quantitative analysis, however, suggests that blackjack oak, however slight the difference, should be placed ahead of post oak. This position is supported by Gemborys and Hodgkins (1971). They found that post oak sites possessed a slightly better site index for longleaf pine. Additional research is needed to establish the exact relationship of these two species.

Redbud, deciduous holly, American elm, vaccinium and red cedar were omitted from the evaluations because only traces were recorded and no direct comparisons could be made.

The table suggests that the sites become progressively better in advancing from the winged elm position to the fringe tree position. The agreement of this quantitative chart with Silker's wedge chart, figure 1, may be readily observed. The ranking in the wedge chart is from left to right.

TABLE XX

EVENNESS OF DISTRIBUTION RATIOS BY STRATIFICATION
($\bar{X}$ OCCUPIED PLOTS / $\bar{X}$ ALL PLOTS)

Species	Stratification		
	I	II	III
post oak	1.356	1.190	1.252
blackjack oak	1.297	1.413	6.074
hickory	30.303	1.364	1.038
tree huckleberry	125.000	9.291	8.866
flowering dogwood	absent	80.357	3.186
black oak	absent	14.587	2.372
southern red oak	absent	333.333	3.824
black haw	58.824	78.947	18.143
black gum	absent	absent	94.824
sumac	13.074	8.354	2.464
chittam wood	13.126	62.069	55.556
sassafras	absent	absent	23.973
wild plum	39.216	14.577	9.578
winged elm	1.026	1.373	2.480
American elm	absent	trace	trace
red haw	8.405	2.924	4.553
vaccinium	absent	absent	trace
fringe tree	absent	absent	128.230
mulberry	absent	166.670	42.731
white ash	absent	166.670	76.923
red cedar	absent	trace	trace
symphoricarpos	7.866	158.850	absent
French mulberry	absent	absent	62.500
deciduous holly	absent	trace	absent

The quantitative ranking includes some species which do not appear on the wedge chart just as the wedge chart includes a few species which did not appear with sufficient frequency for an analysis in this study. The principle species indicated on both are, however, in excellent agreement. According to Silker, lesser species which have been included in the quantitative chart, while not present in the wedge chart,

would be in agreement had they been shown.

TABLE XXI

STRATIFICATION PREFERENCE OF SPECIES BASED ON THEIR FREQUENCY AND EVENNESS OF DISTRIBUTION RATIOS

Species	Stratification		
	I	II	III
post oak	x	x	
blackjack oak	x	x	
hickory		x	x
tree huckleberry		x	x
black oak			x
southern red oak			x
black haw			x
black gum			x
chittam wood	x		
sassafras			x
wild plum		x	x
winged elm	x		
red haw		x	
mulberry			x
white ash			x
fringe tree			x
French mulberry			x

The relative positions of post oak and blackjack oak alone are at variance in the two charts. It is suggested that further study be employed to establish the disputed positions of the two species. Since both species are positioned on the poorest sites it is difficult to place either one before the other. It is possible that had sites poorer than stratification I been evaluated, one species or the other

would show a tendency to establish itself ahead of the other.

TABLE XXII

A QUANTITATIVE RANKING OF SPECIES BASED ON THEIR FREQUENCY AND EVENNESS OF DISTRIBUTION RATIOS

winged elm
blackjack oak
symphoricarpos
chittam wood
post oak
red haw
hickory
tree huckleberry
black oak
flowering dogwood
southern red oak
wild plum
black haw
sassafras
mulberry
French mulberry
white ash
black gum
fringe tree

In order to evaluate the quantitative chart in terms of site index, an appropriate site index value for each stratification must be determined. The mean site index values for shortleaf pine based on an age of fifty years is obtainable from a similar study done by Endicott (1971). He found that the mean site indexes for shortleaf pine associated with certain hardwood groups were as follows: (1) hickory association equals 50.7 feet, (2) hickory-tree huckleberry equals 54.2

feet, and (3) southern red oak equals 60.7 feet. Thus, applying these relationships to the current study the indicated site index can be predicted for each of the three stratifications. Stratification I would produce a site index of less than 50.7 feet. No exact site index can be given because rarely does pine occur naturally on this stratification. Stratification II would range from a site index of 50.7 feet to a site index of 54.2 feet. Stratification II begins approximately where hickory comes in but it terminates before the site is of sufficient quality to support red oak. Stratification III, which begins with the establishment of red oak, consists of site indexes 60.7 feet and greater.

It becomes clear that while exact site index values cannot be assigned for each species or each stratification, a range of site indexes can be determined. With a knowledge of the approximate boundaries and the appropriate range of site indexes for each stratification, the forester can, after a familiarization of the frequencies encountered for each species, indentify the approximate location within the stratification, and predict a reasonably accurate site index for that site.

Comparison of the Observed Distribution to the Poisson Expectation

An attempt was made to determine if the distributions of species encountered in this work could be described by Poisson expectation. The Poisson distribution assumes that any species can occur on a given sample and that any sample provides an equal chance for that species to occur. In general, the Poisson appears to fit more closely to the

observed distribution when the mean and the variance of the distribution are approximately equal.

The empty plot frequency in the Poisson distribution is obtained with:

$$f_0 = N\ e^{-\mu}$$

where,

f_0 = the expected frequency of the zero count;

N = the number of sample units;

e = base of the natural logarithms;

μ = the mean of the population.

Having obtained a value for f_0, succeeding frequencies are determined by a successive multiplication of the preceeding frequency times the $\overline{X}$ / X, or: f_0 ($\overline{X}$ / 1), f_1 ($\overline{X}$ / 2), f_2 ($\overline{X}$ / 3), etc. (X in this case is the count, 0, 1, 2, 3, ..., etc.)

In the example of post oak in stratification I, the expected frequencies are as follow:

$$f_0 = N\ e^{-\mu}$$

$$\log f_0 = \log N - \mu \log e$$

$$\log f_0 = \log 118 - 2.093 \log 2.71828$$

$$\log f_0 = 2.07188 - 2.093\ (.4342944)$$

$$\log f_0 = 1.1629$$

$$f_0 = 14.5514.$$

Succeeding frequencies are calculated as:

$$f_1 = f_0\ (\overline{X} / X)$$

$$f_1 = 14.5514\ (2.093 / 1)$$

$$f_1 = 30.4560,$$

and

$$f_2 = 30.4560 \ (2.093 \ / \ 2) = 31.8723.$$

Table XXIII gives the expected distribution and actual distribution.

TABLE XXIII

POISSON AND ACTUAL DISTRIBUTIONS OF POST OAK IN STRATIFICATION I

species count, X	$\bar{X}$ / X	Expected frequency	Actual frequency
0		14.5514	31
1	2.093/1	30.4560	26
2	2.093/2	31.8723	24
3	2.093/3	22.2362	11
4	2.093/4	11.6351	10
5	2.093/5	4.8705	6
6	2.093/6	1.6990	3
7	2.093/7	.5080	5
8	2.093/8	.1329	1
9	2.093/9	.0309	1

Under the assumption of Poisson distribution a χ^2 test can be made. The χ^2 value for this particular species becomes, then, 33.43. With the tabulated value under four degrees of freedom the hypothesis must be rejected.

None of the species tested were found to exhibit the Poisson distribution. Some species may have indicated a Poisson distribution had all been tested, but due to the large zero count and low frequency of many of the species, they could not be tested. The use of the χ^2 test requires that at least three entries, represented by three count

levels, be made. A minimum of three entries is needed to give a degree of freedom for the χ^2 test. The results of the Poisson distribution tests are given in table XXIV.

TABLE XXIV

COMPARISON OF THE CALCULATED χ^2 WITH THE χ^2 NECESSARY TO INDICATE A POISSON DISTRIBUTION AT THE 95% CONFIDENCE LEVEL

Species	Stratification I		II		III	
	cal. χ^2	table χ^2	cal. χ^2	table χ^2	cal. χ^2	table χ^2
post oak	33.43	9.49	63.61	15.51	158.03	12.59
blackjack oak	23.59	9.49	13.49	11.07	21.63	3.84
hickory			20.44	11.07	45.28	15.21
tree huckleberry			59.91	3.84	56.67	3.84
flowering dogwood					52.79	5.99
black oak					81.41	7.81
southern red oak					13.23	3.84
sumac			97.29	3.84	666.91	11.07
wild plum			34.92	3.84	18.51	3.84
winged elm	56.02	21.03	894.95	14.07	7.46	5.99
red haw			252.37	7.81	20.92	3.84
symphoricarpos	11.16	5.99				

The lack of agreement with the Poisson distribution is not surprising. In all the species distributions studied, the variances proved to be larger than the means, indicating that the negative binomial expectation might fit better the observed data. Furthermore, the $\hat{k}$ values of the negative binomial series, when computed from the

observed data, indicated a considerable degree of contagion or clumpiness in the distributions.

Fitting the Negative Binomial Distribution to the Observed Distribution

None of the species studied in this project were fitted satisfactorily with Poisson expectation, therefore none show random dispersion. After discovering that the distributions involved were not distributed Poisson, the negative binomial distribution was examined for fit. As mentioned earlier, few species have been found to be distributed randomly. Most tend to show patterns of "overdispersion" and contagious or clumpy distributions. The negative binomial series was developed specifically for use with such frequency distributions; therefore, trials with this mathematical expectation are made for the species involved here.

Calculating the expected frequencies of the negative binomial is laborious, but the work is handled readily with the aid of a computer. The series is based upon the expansion of the term $(P - Q)^{-n}$, where $P - Q = 1$. The equation for determining the frequency of the zero count appears as:

$$f_0 = N / q^{k}$$

where,

f_0 = the frequency of zero count plots;

N = the number of sample units;

$\hat{k}$ = a constant (moment estimate, or $\overline{X}^2 / s^2 - \overline{X}$);

$q = (1 + \mu / \hat{k})$.

The log form of the equation is prefered for ease in calculation:

$$\log f_0 = \log N - \hat{k} \log q.$$

Once the frequency of the zero count is determined, the remaining frequencies are successive arithmetic operations using f_{x-1}:

$$f_1 = \hat{k} \cdot R \cdot f_0, \text{ and the frequency of succeeding counts,}$$

$$f_i = \frac{(\hat{k} + i - 1)}{i} R (f_{i-1}), \; i = 2, 3, 4, \ldots, \text{etc.}$$

where

$$R = \frac{\mu}{\hat{k} + \mu}.$$

Substituting the values for post oak in stratification I into the equations the following expected frequencies are obtained:

$$\log f_0 = \log N - \hat{k} \log q$$

$$= 2.07188 - 1.915 \; (.32076)$$

$$= 1.45763$$

$$f_0 = 28.6832,$$

and

$$f_1 = (1.915)(.52221)(28.6832)$$

$$= 28.6839,$$

and

$$f_2 = \frac{(1.915 + 2 - 1)}{2} (.52221)(28.6839)$$

$$= 21.8317.$$

In the above illustration and in all the calculations concerning the negative binomial the $\bar{X}$ of the sample is used as an estimate of the μ for the population. The moment estimate, $\hat{k} = \frac{\bar{X}^2}{s^2 - \bar{X}}$, is used in this report. It is recognized, as pointed out by Walker (1970), that the maximum likelihood method for calculating $\hat{k}$ is more accurate, but in this research the $\hat{k}$ values are large enough that the moment estimate

will suffice. The $\hat{k}$ value is critical where $\hat{k}$ is less than 1.0. The $\hat{k}$ value is of lesser importance when $\hat{k}$ is between 1.0 and 2.0, and of little importance when $\hat{k}$ is in excess of 2.0. These relationships arise from the fact that the larger the variance the smaller the $\hat{k}$ value. Therefore, if a large variance (small $\hat{k}$ value) is present, care must be taken that $\hat{k}$ closely approximates k. Comparisons of the $\hat{k}$ values for species in the three stratifications are made in table XXV.

TABLE XXV

COMPARISON, BY STRATIFICATION, OF THE MOMENT ESTIMATE OF k

Species	Stratification		
	I	II	III
post oak	1.915	1.324	1.167
blackjack oak	2.807	1.873	.282
hickory		1.862	8.230
tree huckleberry		.088	.096
flowering dogwood			.446
black oak			.874
southern red oak			.846
sumac	.136	.140	.305
wild plum		.051	
winged elm	1.205	.647	1.039
red haw	.373	.440	.386
symphoricarpos	.044		

The $\hat{k}$ values reflect the degree of contagion present in a distribution. In general, the smaller the $\hat{k}$ value the more contagious the distribution. This fact is noted by Walker (1970), and is based on the

amount of variance in a population. A population which possesses a high variance suggests that, rather than being distributed evenly, the population tends to clump in irregular groups. The $\hat{k}$ values less than 1.0 indicate a high degree of contagion and values between 1.0 and 2.0 indicate a moderate amount of contagion.

After all of the expected frequencies were calculated, χ^2 tests were made, using the hypothesis that distributions were negative binomial. Expected frequencies in the tails of the distributions were combined so as to produce a minimum value five (Bliss, 1953). N - 3 degrees of freedom were used to establish the tabulated χ^2 value at the 95% level. In table XXVI is listed the species which exhibit negative binomial distributions.

TABLE XXVI

PROBABILITY VALUES OF SPECIES WHICH SHOW SIGNIFICANT NEGATIVE BINOMIAL DISTRIBUTIONS

Species	Stratification		
	I	II	III
post oak	.70		
blackjack oak	.30	.80	.25
hickory		.80	.15
tree huckleberry		.90	.20
flowering dogwood			.10
black oak			.50
southern red oak			.40
sumac			.30
wild plum		.05	
red haw		.05	.25

The species shown in table XXVI are those with sufficient number of entries to yield at least one degree of freedom for the χ^2 test. A blank in the table, therefore, represents either a species with a low frequency in a stratification or a species with a non-significant χ^2 value. It should be noted that some of the χ^2 values were of borderline significance. A more accurate estimate of k, such as would have been provided by the maximum liklihood method, might have elevated some of these χ^2 tests into the significant range. Wild plum in stratification II, red haw in stratification II, and post oak in stratification III are examples. Many of the other χ^2 tests, although not significant, still were close enough to indicate a satisfactory fitting with the negative binomial.

Fitting the Bivariate Negative Binomial

Since the frequency distributions of many of the species were found to display negative binomial characteristics, it was decided to see if there could be an association between the species, predictable by the negative binomial. With this in mind the bivariate negative binomial was studied.

The basic mathematical formula used to generate the expected bivariate probability can be written in logrithmic form as:

$$\log P_{x,y}(m,n) = \log \frac{\Gamma(\alpha + m + n)}{m!\, n!\, \Gamma\, \alpha} + \alpha \log \beta + m \log A - (\alpha + m + n) \log (\beta + A + 1).$$

This equation was rewritten by Walker (1970) from earlier work involving a gama function equation designed by Bates and Neyman (1952). In this equation x and y represent the variables of the joint distribution and where:

m,n = the rows and columns of the desired joint count;

$\hat{\alpha}$ = the moment estimate of k of the combined distribution;

$\hat{\beta} = \hat{\alpha} / \overline{X}$;

$A = \overline{Y} / \overline{X}$.

The combined estimates of the mean and combined estimates of the variance must be made in order to calculate $\hat{k}$.

Table VI, the bivariate frequency of post oak and blackjack oak in stratification I, is used to obtain the combined distribution of the two species. The combined zero count becomes m,n (0,0) = 7; the combined one count becomes m,n (0,1; 1,0) = 13; the combined two count becomes m,n (0,2; 1,1; 2,0) = 22; etc.

From the combined distribution a new variance can be calculated:

$$s^2 \text{ combined} = \frac{f(x)^2 - \frac{(\Sigma fx)^2}{N}}{N - 1}$$

$$= \frac{2640 - \frac{(462)^2}{118}}{117}$$

$$= 7.10387.$$

The combined mean is calculated by summing the two individual means:

$$\overline{X} \text{ combined} = \overline{X} \text{ post oak} + \overline{X} \text{ blackjack oak}$$

$$= 2.093 + 1.872$$

$$= 3.965.$$

With a new mean and a new variance the combined moment estimate of k can be calculated:

$$\hat{k} \text{ combined} = \frac{(\overline{X} \text{ combined})^2}{s^2 \text{ combined} - \overline{X} \text{ combined}}$$

$$= \frac{(3.965)^2}{7.10387 - 3.965}$$

$$= 5.00856.$$

The values for $\hat{\beta}$ and A can be calculated:

$$\hat{\beta} = \hat{\alpha} \; / \; \overline{X} = 5.00856 \; / \; 1.872$$

$$= 2.67551$$

and

$$A = \overline{Y} \; / \; \overline{X} = 2.093 \; / \; 1.872$$

$$= 1.11805.$$

Substituting the values into the logrithmic equation the expected bivariate probabilities can be derived:

$$\log P_{1,1}(m,n) = \log \frac{\Gamma (5.00856 + 1 + 1)}{1 \; ! \; 1! \; \Gamma (5.00856)} + 5.00856 \log$$

$$2.67551 + 1 \log 1.11805 - (5.00856 + 1 + 1)$$

$$\log (2.67551 + 1.11805 + 1).$$

$$\Gamma (5.00856 + 1 + 1) = \Gamma (7.00856)$$

$$= (6.00856)(5.00856)(4.00856)(3.00856)$$

$$(2.00856)(1.00856) \; \Gamma (5.00856),$$

and $1 \; ! \; 1 \; ! \; \Gamma (5.00856) = \Gamma (5.00856).$

Therefore,

$$\frac{\Gamma (5.00856 + 1 + 1)}{1 \; ! \; 1 \; ! \; \Gamma (5.00856)} = \frac{735.21929 \; \Gamma (5.00856)}{\Gamma (5.00856)} = 735.21929.$$

$$\text{Log } P_{1,1}(m,n) = 735.21929 + 5.00856 \log 2.67551 + \log$$

$$1.11805 - (5.00856 + 1 + 1) \log (2.67551 +$$

$$1.11805 + 1),$$

$$= - 1.10279; \; P_{1,1} = .07983; \text{ and}$$

$$f_{1,1} = .07983 \; (118) = 9.42.$$

With the aid of a computer program the bivariate negative binomial distribution is produced. Table XXVII represents the final form for the bivariate table.

The upper figures in each block represent the observed joint

frequency and the lower figures represent the negative binomial expectation. It is obvious from the close relationship of the two sets of figures that the bivariate negative binomial produces a good expectation of the true joint frequencies for this case.

To provide statistical proof of the fit, two methods may be used. The bivariate table "seven-region" analysis can be made. A statistical measure of the goodness of fit may then be obtained with a χ^2 test. For the preceeding example, with four degrees of freedom, the calculated χ^2 is 7.25, which is smaller than the tabulated value of 9.49.

Another χ^2 test can be made using the individual cells of the table. In this case the expected frequencies must sum to three in the distribution tails. In the example above the calculated χ^2 was determined to be 11.5595 with 11 degrees of freedom. This also is much smaller than the tabulated χ^2 of 19.68.

Based on the assumption that either method would produce an adequate test, the latter was used because it is readily incorporated into the computer program.

The results of predicting bivariate negative binomial frequencies to fit the observed distributions were shown to be of little significant value. In stratification I many of the associations do appear to fit the bivariate negative binomial but the same did not hold true for stratifications II and III. One possible reason is that the species are less contagiously distributed on stratifications II and III. Stratification I is an adverse site for most of the species, causing them to clump with each other on the better micro-sites available. This extreme contagion appears to be responsible for the significant associations found on this stratification.

TABLE XXVII

THE OBSERVED AND EXPECTED FREQUENCIES OF POST OAK AND BLACKJACK OAK IN STRATIFICATION I BASED ON THE BIVARIATE NEGATIVE BINOMIAL

		Blackjack oak							
		0	1	2	3	4	5	6	7
Post oak	0	7.00	10.00	5.00	2.00	2.00	1.00	1.00	2.00 *
		6.73	6.76	4.10	1.94	.79	.29	.10	0.00
	1	3.00	11.00	7.00	1.00	1.00	2.00	1.00	0.00
		7.76	9.42	6.70	3.64	1.67	.69	.26	.09
	2	6.00	8.00	4.00	4.00	1.00	0.00	0.00	0.00
		5.41	7.69	6.27	3.85	1.97	.89	.37	.14
	3	1.00	4.00	1.00	2.00	3.00	0.00	0.00	0.00
		2.95	4.80	4.20	3.03	1.70	.84	.37	.15
	4	5.00	2.00	2.00	0.00	1.00	0.00	0.00	0.00
		1.38	2.54	2.60	1.96	1.21	.65	.31	.14
	5	3.00	1.00	1.00	0.00	0.00	1.00	0.00	0.00
		.58	1.19	1.35	1.11	.74	.43	.22	.10
	6	1.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00
		.23	.52	.64	.57	.41	.25	.14	0.00
	7	1.00	2.00	1.00	1.00	0.00	0.00	0.00	0.00
		.08	.21	.28	.27	.21	.14	0.00	0.00

* The upper figure in each cell represents the observed joint occurrence.

CHAPTER V

SUMMARY

This research paper has two purposes: first, to determine if there is an association between species which can reflect site potential for southern pine; second, to determine if there is statistical support for the observed ranking of species offered by Silker (1963) in his wedge chart.

To accomplish the first objective, univariate and bivariate frequency tables were constructed. From these tables two-way presence and absence tables were built. A χ^2 test was made on the observed and expected values in the two-way tables in order to determine if association was present and whether or not such association was positive or negative. It was found that certain associations are unique to a given site. It was determined that a significant positive or negative association of two species can be used to identify the site stratification on which the species occur. The stratifications involved were: I, the post oak-blackjack oak sites; II, the post oak-blackjack oak-hickory sites; III, the post oak-blackjack oak-hickory-red oak sites. A seven-region analysis was also performed where possible, in order to lend support to the significant findings of the two-way analysis. The two approaches were correlated and the findings were considered to be in agreement. Problems of species distribution within micro-niches and an overlapping of stratifications were encountered. It is suggest-

ed that additional research be invested to clarify these problems.

An attempt was made to discover whether or not the size of the individuals on a plot affected the association encountered. Plots having trees > 3.1 inches in diameter were separated from those containing no individuals as large as 3.1 inches diameter. Separate series of two-way tables were then obtained. The significant finding was that size did not appear to alter the association. The suggestion was made that perhaps a different size factor would alter the results. Also the separation of plots in stratification I reduced the sample size to a point that accurate analysis became impossible.

It was decided to examine the distribution of each species to see if the observed distribution could be predicted by a mathematical formulation. The Poisson series was examined and it was found that none of the species exhibited the Poisson characteristics. This lack of agreement was expected due to the high degree of contagion indicated by variances exceeding the means. This factor led to the examination of the negative binomial series. Satisfactory fit was obtained for many of the species with the negative binomial expectation.

The second objective of the research was to determine if statistical analysis would support the ranking of species offered in Silker's wedge chart (1963). The statistical support was accomplished by the relationship of two parameters. The first parameter was simply the observed frequency of each species as the sites became progressively better. The second, and probably the more important, parameter studied was termed the evenness of distribution ratio. This value was expressed as the ratio of the $\overline{X}$ of the occupied plots to the $\overline{X}$ of all the plots. A ratio value of 1.0 indicates an even distribution. Increas-

ing values above 1.0 indicate increasingly contagious, or clumpy, distributions. A species which is evenly distributed is one which is well adapted to the site. A quantitative ranking was proposed which had excellent agreement with the observed ranking offered by Silker (1963). The only obvious disagreement between the two proposed rankings was in the relation of post oak and blackjack oak. The question is, does post oak or blackjack oak predominate on poorer sites? More research in this area is recommended. Studies of plant frequency on poorer sites than those studied would be profitable.

It was concluded that a valuable field tool for the forester and plant ecologist was obtained with this study.

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APPENDIX

Abbreviations of Species

post oak	Quercus stellata	pos. oak
blackjack oak	Quercus marilandica	blaj. oak
hickory	Carya spp.	hic.
tree huckleberry	Vaccineum arboreum	tr. huk.
flowering dogwood	Cornus florida	flw. dog.
black oak	Quercus velutina	bla. oak
southern red oak	Quercus falcata	so. red oak
black haw	Viburnum refidulum	bla. haw
red gum	Liquidambar styraciflua	red gum
black gum	Nyssa sylvatica	bla. gum
white oak	Quercus alba	wh. oak
water oak	Quercus nigra	wat. oak
sumac	Rhus glabra	sum.
chittam wood	Bumelia lanuginosa	chi. wd.
sassafras	Sassafras albidum	sas.
wild plum	Prunus spp.	wil. pl.
winged elm	Ulmus alata	win. elm
American elm	Ulmus americana	Am. elm
red haw	Cretagus spp.	red haw
tree huckleberry (low)	Vaccineum spp.	vac.
fringe tree	Chionanthus virginicus	fri. tr.
American holly	Ilex opaca	Am. hol.
red maple	Acer rubrum	red map.
mulberry	Morus rubra	mul.
white ash	Fraxinus americana	wh. ash
red cedar	Juniperus virginiana	red ced.
buckbrush	Symphoricarpos spp.	bu. br.
French mulberry	Morus spp.	Fre. mul.
redbud	Cercis canadensis	redbud
deciduous holly	Ilex spp.	dec. hol.

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