

SYNTHESIS OF STEPHENSON TYPE II SIX-LINK  
FUNCTION GENERATOR FOR FINITELY AND  
INFINITESIMALLY SEPARATED  
POSITIONS

By

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## PREFACE

The purpose of this study was the development of a method for synthesizing the Stephenson Type II six-link function generator for both finite and infinitesimally separated positions. That is, both the positions of the input and output link and the velocity, acceleration, jerk and kerk ratios of the input to the output link are possible design parameters. The procedure developed was incorporated into computer programs for ease of synthesis.

I wish to express my appreciation to my thesis adviser, Dr. A. H. Soni, for his encouragement, understanding and expertise in the field of mechanisms. Special thanks go to Dilip Kohli for his valuable advice. The committee members, Dr. M. Mamoun and Dr. Larry D. Zirkle, deserve recognition for giving both time and suggestions.

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## CHAPTER I

### INTRODUCTION

One of the reasons for the existence of the field of mechanisms is the need for production of non-linear motion. Because the function generating mechanism can produce non-linear motion, it is a vital tool for industry. Therefore, procedures have been developed for synthesizing the planar six-link function generator.

McLarnan (1)\* discussed the procedure for synthesizing six-link mechanisms with complex numbers and numerical techniques for from 6 to 11 finite points. Soni, Varma and Juneja (2) synthesized the six-link mechanism for three finitely separated positions, while Myklebust and Tesar (3) synthesized for five positions. Kaufman (4) synthesized for five positions, correlating coupler motion with input crank rotations. Kim, Hamid and Soni (5) synthesized the six-link mechanism for point path generation. Soni and others (6) made use of the matrix method of synthesis and synthesized for eleven precision positions by numerical methods.

The studies mentioned above indicate there is a need for a clear method for synthesizing the six-link function generator for all combinations of finitely and infinitesimally separated positions when one is

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\*Numbers in parentheses refer to numbered references in the Bibliography.



obtaining a closed form solution. The matrix method set forth by Suh and Radcliffe (7) proves to be concise and easily adaptable in going from finitely separated position synthesis to infinitesimally separated position synthesis. It was chosen for this thesis work because the equations used for finitely separated position synthesis can be differentiated to obtain infinitesimally separated position synthesis of any degree with minimal change in synthesis procedure.

One may question why the six-link mechanism is practical, having two more linkages and three more revolute pairs than the four-link mechanism. After all, these add to the cost and maintenance involved for industrial practicality. But when one looks at the six-link mechanism's advantages of extra mobility, versatility and compactness over the four-link mechanism, it can be seen that the six-link mechanism is a necessary tool for industry.

All the six-link mechanisms with one degree of freedom come from two kinematic chains. These are shown in Figure 1. These two chains result in five six-link mechanisms, shown in Figure 2.

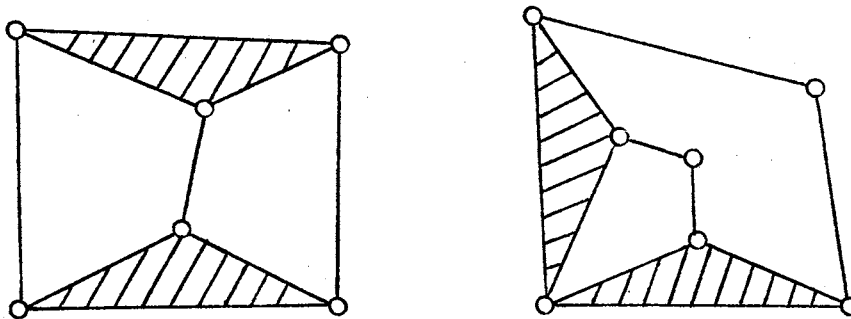


Figure 1. Six-Link Kinematic Chains

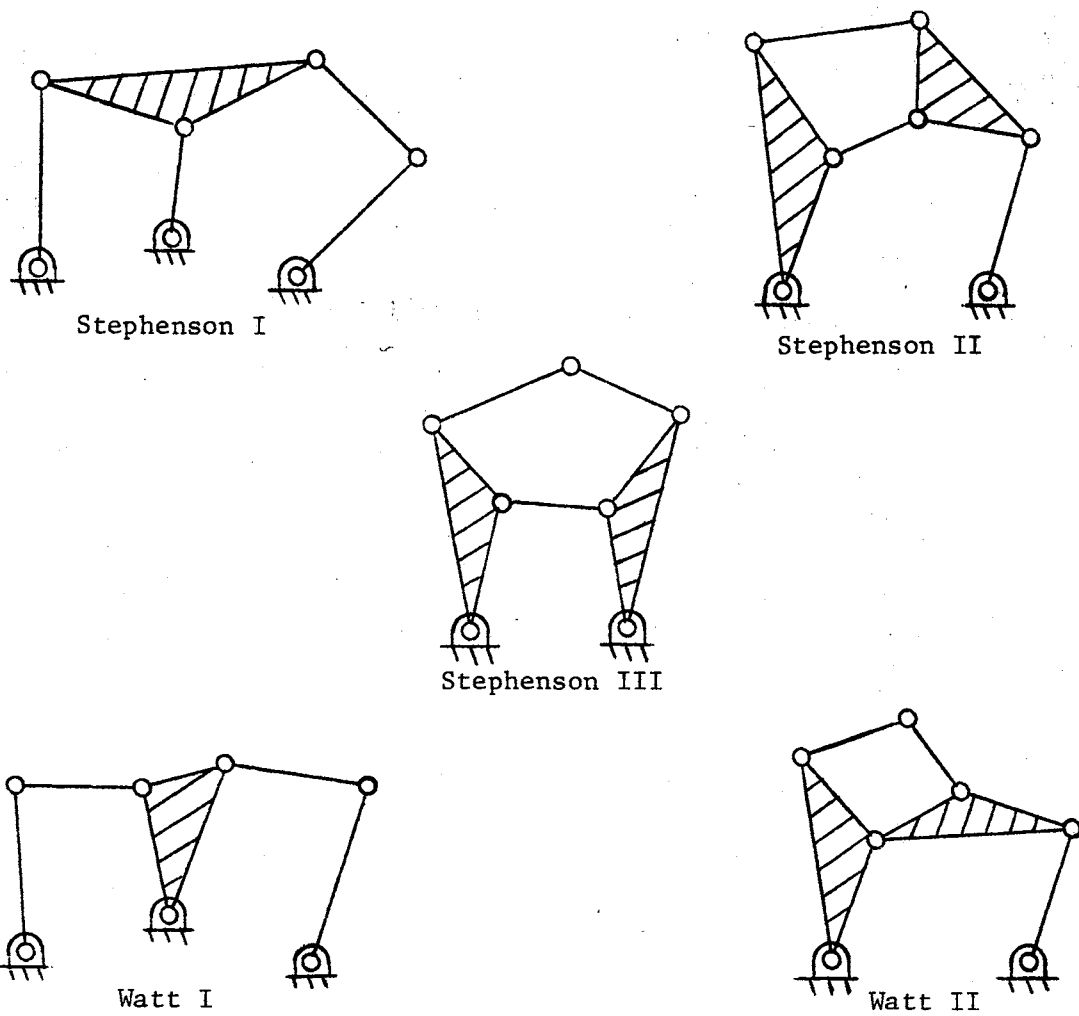


Figure 2. Five Six-Link Mechanisms

As can be seen from Figure 2, the Stephenson Type III mechanism and the Watt Type II mechanism have a basic four-link structure which limits their function generating capabilities to that of a four-link mechanism. The Watt Type I and Stephenson Type I mechanisms have been studied for all possible types of function generation problems. This is because they consist of the four-link mechanism with two binary

links connected in series to them. The Stephenson Type I mechanism has the two binary links connected to the coupler point of the four-link mechanism, reducing the function generation problem to that of combining the coupler point motion of the four-link mechanism to the placement and length of the two binary links. For the case of the Watt Type I mechanism the desired function generation is obtained by connecting the output of the first four-link mechanism to the input of the second four-link mechanism. The Stephenson Type II mechanism has the same function generation flexibility as the Stephenson Type I and the Watt Type I mechanisms, but it has the advantage of having only two grounded revolute pairs, thus being applicable to a variety of situations in which three grounded revolute pairs would be impractical or unable to meet design criteria. However, a generalized method of synthesis for the Stephenson Type II mechanism has not been developed.

This thesis involves developing a closed form solution for the synthesis of the Stephenson Type II six-link mechanism function generator for five positions. The matrix approach to synthesis set forth by Suh and Radcliffe (7) and later developed by Kohli and Soni (8) will be used along with the principle of inversion. In order to specify the velocity, acceleration, jerk and kerk (time derivative of jerk) ratios of output link to input link, the principle of infinitesimally separated positions introduced by Mueller (9) and developed by Bottema (10) and Tesar (11, 12, 13) was used.

This thesis will set forth a method of synthesizing the Stephenson Type II six-link mechanism for five precision positions of function generation. Five precision positions were chosen because five is the maximum number of precision positions for which the synthesis equations

can be made linear. From six to a maximum of eleven precision positions the synthesis equations are non-linear and have to be solved by numerical techniques. This method can have the problem of convergence, and the resulting mechanism is the closest solution--not necessarily the exact solution. For three precision positions the design equations are linear and can be solved directly, while for four and five positions the principle of linear superposition, discussed by Mohan Rao and Sandor (14) must be utilized in order to make the equations linear. Five precision positions obviously give the designer more flexibility and are necessary when considering the additional design specifications of velocity, acceleration, jerk and kerk ratios.

Tesar (13) uses a set of nomenclature for describing the possible combinations of finitely separated and infinitesimally separated displacements for five positions. All the possible combinations are:

✓ P-P-P-P-P			
✓ PP-P-P-P	✓ P-PP-P-P	P-P-PP-P	P-P-P-PP
✓ PP-PP-P	PP-P-PP	P-PP-PP	
✓ PPP-P-P	✓ P-PPP-P	P-P-PPP	
✓ PPP-PP	✓ PP-PPP		
✓ PPPP-P	P-PPPP		
✓ PPPPP			

Dashes indicate finitely separated points and no dash indicates infinitesimally separated points. This thesis presents an analytical method of synthesizing for all these motions. These combinations can be used to obtain a wide variety of function generation motions.

The synthesis procedure used in this thesis involves, first, using the principle of inversion to transform the synthesis problem into a rigid body guidance four-link mechanism synthesis problem. The matrix approach is used to design a closed form solution for the rigid body guidance problem. The result is the designed six-link mechanism. The

same procedure is then used to synthesize the Stephenson Type II six-link mechanism for infinitesimally separated positions.

One motivation behind this thesis has been to give industry a simple method for using the six-link mechanism in specific design situations. To do this, computer programs were written to give solutions for any one of the above motion programs.

## CHAPTER II

### MATRIX METHOD OF SYNTHESIS FOR FINITELY SEPARATED POSITIONS

Mechanisms in one form or another have been used for many hundreds of years to obtain mechanical motion. As engineering and mathematics advanced, it became desirable to develop mathematical methods to synthesize mechanisms to perform desired motions. The computer opened up a whole new field of possible synthesis methods.

One of these methods was set forth by Suh (7), whose paper presents a method for using a generalized displacement matrix to describe rigid body motion. A rigid body is specified when a point on the rigid body is known in a specified coordinate system, and when the angle of rotation of the rigid body with respect to the point is known. As the rigid body executes planar motion these two values are specified as in Figure 3. The points in the rigid body are designated as  $C_1$ ,  $C_2$ , and  $C_3$ , and the rotation of the rigid body between positions  $C_1$  and  $C_2$  is  $\alpha_{12}$ , and between  $C_1$  and  $C_3$  is  $\alpha_{13}$ . Point  $B_n$  is any point on the rigid body. The positions of  $B_2$  and  $B_3$  are described by multiplying the generalized displacement matrix times the first position matrix of  $B_1$ .

$$[b_n] = [D_{1n}] [b_1] \quad (2-1)$$

The generalized displacement matrix is given by

$$[D_{1n}] = \begin{bmatrix} \cos\alpha_{1n} & -\sin\alpha_{1n} & X_{cn} - X_{c1} \cos\alpha_{1n} + Y_{c1} \sin\alpha_{1n} \\ \sin\alpha_{1n} & \cos\alpha_{1n} & Y_{cn} - X_{c1} \sin\alpha_{1n} - Y_{c1} \cos\alpha_{1n} \\ 0 & 0 & 1 \end{bmatrix} \quad (2-2)$$

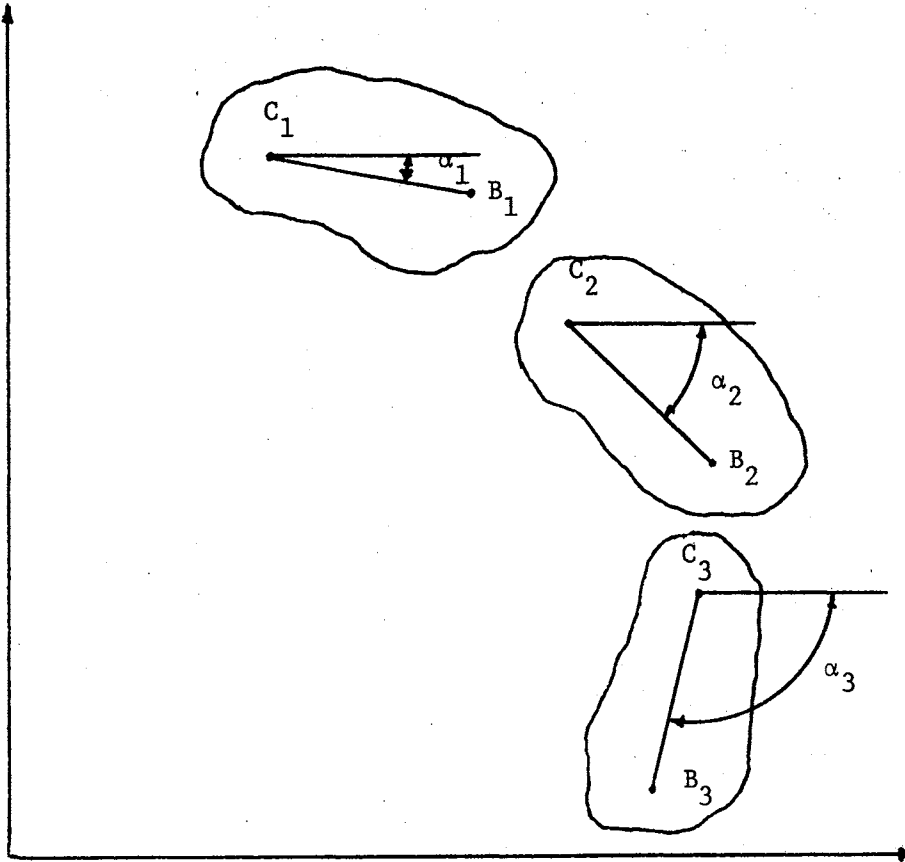


Figure 3. Description of Rigid Body Motion

According to the Burmester Theory for three and four finite positions of the rigid body there are an infinite number of circle points (designated as  $B_n$ ) on the rigid body which follow a circular path. For five finite positions of the rigid body there are a maximum of four

possible points on the rigid body that lie in a circle. A link with one pivot connected to the point  $B_n$  on the rigid body and another pivot connected to the fixed pivot A can be described by the equation of a circle, where  $(X_A, Y_A)$  are coordinates of fixed pivot A.

$$(X_{bn} - X_A)^2 + (Y_{bn} - Y_A)^2 = (X_{b1} - X_A)^2 + (Y_{b1} - Y_A)^2 \quad (2-3)$$

The design equation is obtained by the substitution of Equation (2-1) into Equation (2-3).

For three finitely separated positions there are two design equations that are linear if any two of the four variables  $X_A, Y_A, X_{B1}$  and  $Y_{B1}$  are assumed. For four finitely separated positions there are three design equations; therefore, one of the four variables must be assumed. For five finitely separated positions there are four design equations and all four variables are to be determined. In the last two cases the equations are non-linear but can be made linear by applying the principle of linear superposition.



### CHAPTER III

#### MATRIX METHOD OF SYNTHESIS FOR INFINITESIMALLY SEPARATED POSITIONS

In the previous chapter the matrix method of synthesizing a four-link mechanism for finitely separated positions of point C on the rigid body was discussed. The matrix method can also be used to synthesize the four-link mechanism for infinitesimally separated positions of point C on the rigid body. In order to utilize this method, a new matrix has been developed by Soni (15) which, when multiplied by a finite position of C, results in the infinitesimally separated position of C. To do this, point C on the rigid body and point B, a circle point, are described by vectors as in Figure 4.

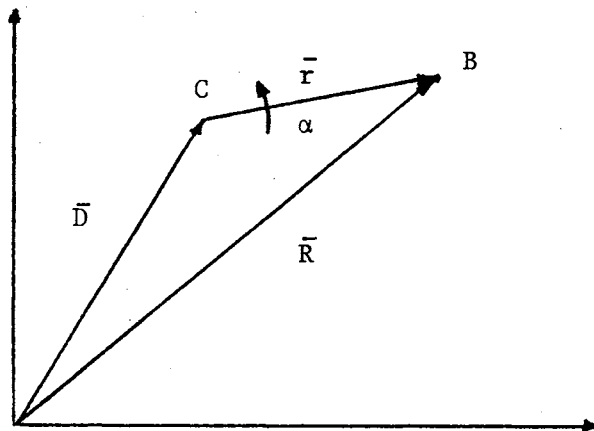


Figure 4. Vector Description of Rigid Body

Since  $\bar{R} = \bar{D} + \bar{r}$ , the velocity of point B is given by

$$\frac{d\bar{R}}{dt} = \frac{d\bar{D}}{dt} + \frac{d\alpha}{dt} \bar{K} \times \bar{r} \quad (3-1)$$

where

$$\begin{aligned} \bar{r} &= (X_B - X_C) \bar{i} + (Y_B - Y_C) \bar{j} \\ \bar{D} &= X_C \bar{i} + Y_C \bar{j} \\ \bar{R} &= X_B \bar{i} + Y_B \bar{j} . \end{aligned} \quad (3-2)$$

Substituting Equations (3-2) into Equation (3-1) results in

$$\frac{dX_B}{dt} \bar{i} + \frac{dY_B}{dt} \bar{j} = \frac{dX_C}{dt} \bar{i} + \frac{dY_C}{dt} \bar{j} + \bar{K} \frac{d\alpha}{dt} \times [(X_B - X_C) \bar{i} + (Y_B - Y_C) \bar{j}]. \quad (3-3)$$

By changing the independent parameter from  $t$  to  $\alpha$  Equation (3-3)

becomes

$$\frac{dX_B}{d\alpha} \bar{i} + \frac{dY_B}{d\alpha} \bar{j} = \frac{dX_C}{d\alpha} \bar{i} + \frac{dY_C}{d\alpha} \bar{j} + (X_B - X_C) \bar{j} - (Y_B - Y_C) \bar{i}. \quad (3-4)$$

By separating the  $i$  and  $j$  components Equation (3-4) becomes

$$\begin{aligned} \frac{dX_B}{d\alpha} &= \frac{dX_C}{d\alpha} - (Y_B - Y_C) \\ \frac{dY_B}{d\alpha} &= \frac{dY_C}{d\alpha} + (X_B - X_C). \end{aligned} \quad (3-5)$$

For the  $n^{\text{th}}$  position, in matrix form this becomes

$$\begin{bmatrix} \dot{X}_{Bn} \\ \dot{Y}_{Bn} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & \frac{dX_{Cn}}{d\alpha_{1n}} + Y_{Cn} \\ 1 & 0 & \frac{dY_{Cn}}{d\alpha_{1n}} - X_{Cn} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{Bn} \\ Y_{Bn} \\ 1 \end{bmatrix}. \quad (3-6)$$

In order to develop the first infinitesimally separated position synthesis equation, Equations (2-1) and (3-6) must be substituted into the derivative of Equation (2-3).

$$2 (X_{Bn} - X_A) \dot{X}_{Bn} + 2 (Y_{Bn} - Y_A) \dot{Y}_{Bn} = 0 \quad (3-7)$$

In order to find higher orders of infinitesimally separated positions, Equations (3-5) should be differentiated successively. The second order infinitesimally separated position is

$$\begin{aligned} \frac{d^2 X_B}{d\alpha^2} &= \frac{d^2 X_C}{d\alpha^2} + \frac{dY_C}{d\alpha} - \frac{dY_B}{d\alpha} \\ \frac{d^2 Y_B}{d\alpha^2} &= \frac{d^2 Y_C}{d\alpha^2} + \frac{dX_B}{d\alpha} - \frac{dX_C}{d\alpha} . \end{aligned} \quad (3-8)$$

Substituting Equations (3-5) into Equations (3-8) results in

$$\begin{aligned} \frac{d^2 X_B}{d\alpha^2} &= \frac{d^2 X_C}{d\alpha^2} + X_C - X_B \\ \frac{d^2 Y_B}{d\alpha^2} &= \frac{d^2 Y_C}{d\alpha^2} + Y_C - Y_B . \end{aligned} \quad (3-9)$$

The matrix equation is

$$\begin{bmatrix} \ddots \\ X_{Bn} \\ \ddots \\ Y_{Bn} \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & \frac{d^2 X_{Cn}}{d\alpha^2} + X_{Cn} \\ 0 & -1 & \frac{d^2 Y_{Cn}}{d\alpha^2} + Y_{Cn} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{Bn} \\ Y_{Bn} \\ 1 \end{bmatrix} . \quad (3-10)$$

Following the same procedure, the third order infinitesimally separated position matrix equation is

$$\begin{bmatrix} \dots \\ X_{Bn} \\ \dots \\ Y_{Bn} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & \frac{d^3 X_{Cn}}{d\alpha^3} - Y_{Cn} \\ -1 & 0 & \frac{d^3 Y_{Cn}}{d\alpha^3} + X_{Cn} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{Bn} \\ Y_{Bn} \\ 1 \end{bmatrix} . \quad (3-11)$$

and the fourth order infinitesimally separated position matrix is

$$\begin{bmatrix} \dots \\ X_{Bn} \\ \dots \\ Y_{Bn} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{d^4 X_{Cn}}{d\alpha_{1n}^4} - X_{Cn} \\ 0 & 1 & \frac{d^4 Y_{Cn}}{d\alpha_{1n}^4} - Y_{Cn} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{Bn} \\ Y_{Bn} \\ 1 \end{bmatrix} \quad (3-12)$$

The third and fourth infinitesimally separated position design equations are obtained by substituting the appropriate matrix equation into the derivatives of Equation (3-7).

## CHAPTER IV

### SYNTHESIS PROCEDURE FOR FIVE FINITELY SEPARATED POSITIONS

For clarity's sake, a set of nomenclature will be established. The Stephenson Type II six-link mechanism is of the form shown in Figure 5, and will have revolute pairs at A, B, C, D, and E and grounded revolute pairs at M and Q. The input link will be the ternary link MAD while the output will be the link QC. Changes in rotation of link 1 from its first position will be designated as  $\theta_{1n}$  while changes in rotation with respect to the first position of the output will be  $\phi_{1n}$ . Also, the rotation displacements of the ternary link about point C with respect to the first position will be  $\alpha_{1n}$ .

To produce a specified function generation the synthesis procedure was changed to a rigid body guidance problem by means of inverting the mechanism. Inversion was taken about ternary link 1. Holding link 1 fixed, the once grounded link MQ was rotated a minus  $\theta_{1n}$  direction for each position of desired input rotation. At the same time, link QC was rotated at an angle  $(\phi_{1n} - \theta_{1n})$  for each position (Figure 6). The displacement equation which gives the positions of  $X_C$ ,  $Y_C$  in terms of  $-\theta_{1n}$  and  $(\phi_{1n} - \theta_{1n})$  is

$$X_{Cn} = (X_{C1} - X_{Q1}) \cos (\phi_{1n} - \theta_{1n}) - (Y_{C1} - Y_{Q1}) \sin (\phi_{1n} - \theta_{1n}) + X_{Q1} \cos (-\theta_{1n}) - Y_{Q1} \sin (-\theta_{1n})$$

$$Y_{Cn} = (X_{C1} - X_{Q1}) \sin (\phi_{1n} - \theta_{1n}) + (Y_{C1} - Y_{Q1}) \cos (\phi_{1n} - \theta_{1n}) + X_{Q1} \sin (-\theta_{1n}) + Y_{Q1} \cos (-\theta_{1n}). \quad (4-1)$$

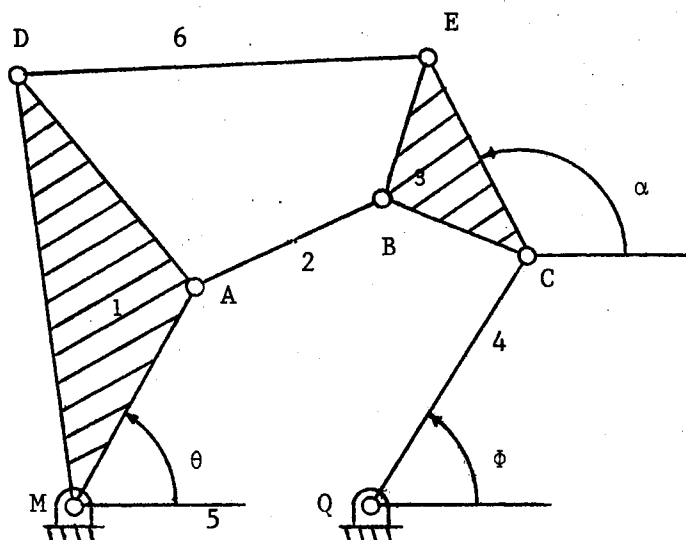


Figure 5. Stephenson Type II Six-Link Mechanism

In order to define the positions of  $X_C$ ,  $Y_C$  in the inverted positions, the length of  $QC$  must be assumed in relation to the unit length of  $MQ$ , and the initial angle  $\phi_1$  must be given. These two parameters are totally arbitrary and can be varied to obtain the optimum mechanism for a desired function, or can be specified to meet additional design criteria.

With the values of  $X_C$ ,  $Y_C$  in the inverted positions known, the synthesis procedure now becomes a four-link rigid body guidance problem. By using the matrix approach described in Chapter II, a matrix

was obtained which describes the positions of points B and E of the ternary link as the rigid body passes through the points  $X_C$ ,  $Y_C$  and rotates through the inverted displacement angles of  $\alpha_{1n}$ .

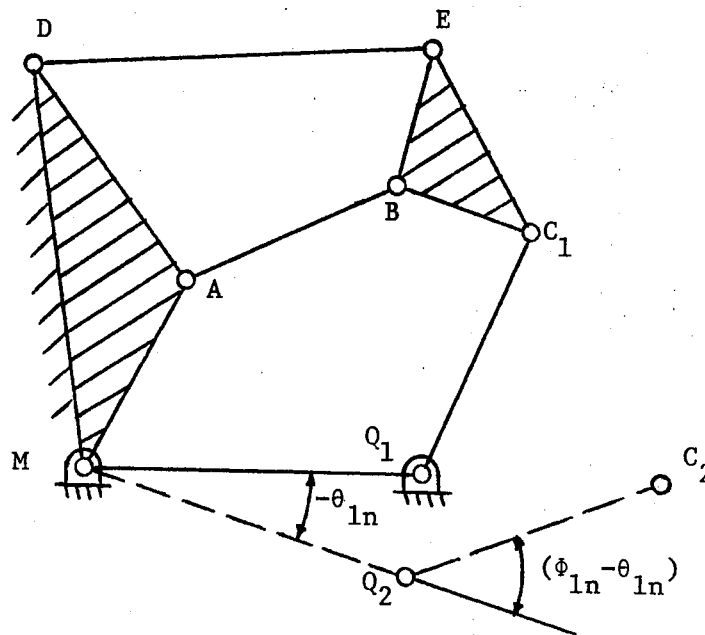


Figure 6. Inversion About Link 1

The values of  $\alpha_{1n}$  that the ternary link is to rotate are also assumed values and can be used to the designer's advantage when synthesizing the six-link function generator. That is, if the designer has a problem in which the ternary link rotations have to be specified, this can be done; or if it is totally arbitrary, its variations will produce different six-link mechanisms. As explained in Chapter I, the

four-link rigid body guidance synthesis equation (4-2) is non-linear when five positions are desired.

$$\begin{aligned} X_{A1} [C_{1n}] + Y_{A1} [C_{2n}] + X_{B1} [C_{3n}] + Y_{B1} [C_{4n}] &= [1 - \cos\alpha_{1n}] \\ [X_{A1} X_{B1} + Y_{A1} Y_{B1}] + \sin\alpha_{1n} [X_{A1} Y_{B1} - X_{B1} Y_{A1}] + \frac{1}{2}[C_{1n}^2 + C_{2n}^2] & \end{aligned} \quad (4-2)$$

where

$$\begin{aligned} C_{1n} &= X_{Cn} - X_{C1} \cos\alpha_{1n} + Y_{C1} \sin\alpha_{1n} \\ C_{2n} &= Y_{Cn} - Y_{C1} \cos\alpha_{1n} - X_{C1} \sin\alpha_{1n} \\ C_{3n} &= X_{C1} - X_{Cn} \cos\alpha_{1n} - Y_{Cn} \sin\alpha_{1n} \\ C_{4n} &= Y_{C1} - Y_{Cn} \cos\alpha_{1n} + X_{Cn} \sin\alpha_{1n}. \end{aligned}$$

To solve this equation in closed form for five positions, the principle of linear superposition must be used. Letting

$$\begin{aligned} \lambda_1 &= X_{A1} X_{B1} + Y_{A1} Y_{B1} \quad \text{and} \\ \lambda_2 &= X_{A1} Y_{B1} - X_{B1} Y_{A1} \end{aligned} \quad (4-3)$$

Equation (4-2) can be divided into three linear equations (4-4) which can be solved for five positions simultaneously.

$$\begin{aligned} r_1 [C_{1n}] + r_2 [C_{2n}] + r_3 [C_{3n}] + r_4 [C_{4n}] &= \frac{1}{2}[C_{1n}^2 + C_{2n}^2] \\ P_1 [C_{1n}] + P_2 [C_{2n}] + P_3 [C_{3n}] + P_4 [C_{4n}] &= 1 - \cos\alpha_{1n} \\ q_1 [C_{1n}] + q_2 [C_{2n}] + q_3 [C_{3n}] + q_4 [C_{4n}] &= \sin\alpha_{1n} \end{aligned} \quad (4-4)$$

$n = 2, 3, 4, 5$

where

$$\begin{aligned} X_{A1} &= r_1 + \lambda_1 P_1 + \lambda_2 q_1 \\ Y_{A1} &= r_2 + \lambda_1 P_2 + \lambda_2 q_2 \\ X_{B1} &= r_3 + \lambda_1 P_3 + \lambda_2 q_3 \\ Y_{B1} &= r_4 + \lambda_1 P_4 + \lambda_2 q_4. \end{aligned} \quad (4-5)$$



Substituting Equations (4-5) into the compatibility conditions given by Equations (4-3) will result in

$$\begin{aligned} F_1 \lambda_2^2 + (F_2 \lambda_1 + F_3) \lambda_2 + F_4 \lambda_1^2 + F_5 \lambda_1 + F_6 &= 0 \\ G_1 \lambda_2^2 + (G_2 \lambda_1 + G_3) \lambda_2 + G_4 \lambda_1^2 + G_5 \lambda_1 + G_6 &= 0 \end{aligned} \quad (4-6)$$

where

$$\begin{aligned} F_1 &= q_1 q_3 + q_2 q_4 \\ F_2 &= p_1 q_3 + q_1 p_3 + p_2 q_4 + q_2 p_4 \\ F_3 &= r_1 q_3 + q_1 r_3 + q_2 r_4 + q_4 r_2 \\ F_4 &= p_1 p_3 + p_2 p_4 \\ F_5 &= p_1 r_3 + p_3 r_1 + p_2 r_4 + p_4 r_2 - 1 \\ F_6 &= r_1 r_3 + r_2 r_4 \\ G_1 &= q_1 q_4 - q_2 q_3 \\ G_2 &= p_1 q_4 + q_1 p_4 - p_2 q_3 - q_2 p_3 \\ G_3 &= r_1 q_4 + q_1 r_4 - r_2 q_3 - q_2 r_3 - 1 \\ G_4 &= p_1 p_4 - p_2 p_3 \\ G_5 &= p_1 r_4 + r_1 p_4 - p_2 r_3 - p_3 r_2 \\ G_6 &= r_1 r_4 - r_2 r_3. \end{aligned} \quad (4-7)$$

Using the Sylvester technique results in a fourth order polynomial, Equation (4-8), the roots of which are the values of  $\lambda_1$ .

$$L_1 \lambda_1^4 + L_2 \lambda_1^3 + L_3 \lambda_1^2 + L_4 \lambda_1 + L_5 = 0 \quad (4-8)$$

where

$$L_1 = G_4^2 F_1^2 + G_2^2 F_1 F_4 + G_1 G_4 F_2^2 + G_1^2 F_4^2 - G_2 G_4 F_1 F_2 - \\ G_1 G_2 F_2 F_4 - 2 G_1 G_4 F_1 F_4$$

$$L_2 = 2 G_4 G_5 F_1^2 + G_2^2 F_1 F_5 + 2 G_2 G_3 F_1 F_4 - G_2 G_5 F_1 F_2 - \\ G_3 G_4 F_1 F_2 - G_2 G_4 F_1 F_4 + G_1 G_5 F_2^2 + 2 G_1 G_4 F_2 F_3 + 2 \\ G_1^2 F_4 F_5 - G_1 G_2 F_2 F_5 - G_1 G_3 F_2 F_4 - G_1 G_2 F_3 F_4 - 2 G_1 G_5 F_1 F_4 \\ - 2 G_1 G_4 F_1 F_5$$

$$L_3 = 2 G_4 G_6 F_1^2 + G_5^2 F_1^2 + G_2^2 F_1 F_6 + 2 G_2 G_3 F_1 F_5 + G_3^2 F_1 F_4 - \\ G_2 G_6 F_1 F_2 - G_3 G_5 F_1 F_2 - G_2 G_5 F_1 F_3 - G_3 G_4 F_1 F_3 + G_1 G_6 F_2^2 + \\ 2 G_1 G_5 F_2 F_3 + G_1 G_4 F_3^2 + 2 G_1^2 F_4 F_6 + G_1^2 F_5^2 - G_1 G_2 F_2 F_6 - \\ G_1 G_3 F_2 F_5 - G_1 G_2 F_3 F_5 - G_1 G_3 F_3 F_4 - 2 G_1 G_6 F_1 F_4 - 2 G_1 G_5 F_1 F_5 \\ - 2 G_1 G_4 F_1 F_6$$

(4-9)

$$L_4 = 2 G_5 G_6 F_1^2 + 2 G_2 G_3 F_1 F_6 + G_3^2 F_1 F_5 - G_3 G_6 F_1 F_2 - G_2 G_6 F_1 F_3 - \\ G_3 G_5 F_1 F_3 + 2 G_1 G_6 F_2 F_3 + G_1 G_5 F_3^2 + 2 G_1^2 F_5 F_6 - G_1 G_3 F_2 F_6 - \\ G_1 G_2 F_3 F_6 - G_1 G_3 F_3 F_5 - 2 G_1 G_5 F_1 F_6 - 2 G_1 G_6 F_1 F_5$$

$$L_5 = F_1^2 G_6^2 + G_3^2 F_1 F_6 - G_3 G_6 F_1 F_3 + G_1 G_6 F_3^2 + G_1^2 F_6^2 - \\ G_1 G_3 F_3 F_6 - 2 G_1 G_6 F_1 F_6$$

The  $\lambda_1$  roots are substituted into Equations (4-6), to yield roots for  $\lambda_2$ . With these two roots known, using Equations (4-5), the values of  $X_{A1}$ ,  $Y_{A1}$ ,  $X_{B1}$  and  $Y_{B1}$  can be found.

Since there are a maximum of four possible pairs of  $\lambda_1$  and  $\lambda_2$ , there are a maximum of four possible solutions for the center point and

$$L_1 = G_4^2 F_1^2 + G_2^2 F_1 F_4 + G_1 G_4 F_2^2 + G_1^2 F_4^2 - G_2 G_4 F_1 F_2 - \\ G_1 G_2 F_2 F_4 - 2 G_1 G_4 F_1 F_4$$

$$L_2 = 2 G_4 G_5 F_1^2 + G_2^2 F_1 F_5 + 2 G_2 G_3 F_1 F_4 - G_2 G_5 F_1 F_2 - \\ G_3 G_4 F_1 F_2 - G_2 G_4 F_1 F_4 + G_1 G_5 F_2^2 + 2 G_1 G_4 F_2 F_3 + 2 \\ G_1^2 F_4 F_5 - G_1 G_2 F_2 F_5 - G_1 G_3 F_2 F_4 - G_1 G_2 F_3 F_4 - 2 G_1 G_5 F_1 F_4 \\ - 2 G_1 G_4 F_1 F_5$$

$$L_3 = 2 G_4 G_6 F_1^2 + G_5^2 F_1^2 + G_2^2 F_1 F_6 + 2 G_2 G_3 F_1 F_5 + G_3^2 F_1 F_4 - \\ G_2 G_6 F_1 F_2 - G_3 G_5 F_1 F_2 - G_2 G_5 F_1 F_3 - G_3 G_4 F_1 F_3 + G_1 G_6 F_2^2 + \\ 2 G_1 G_5 F_2 F_3 + G_1 G_4 F_3^2 + 2 G_1^2 F_4 F_6 + G_1^2 F_5^2 - G_1 G_2 F_2 F_6 - \\ G_1 G_3 F_2 F_5 - G_1 G_2 F_3 F_5 - G_1 G_3 F_3 F_4 - 2 G_1 G_6 F_1 F_4 - 2 G_1 G_5 F_1 F_5 \\ - 2 G_1 G_4 F_1 F_6$$

(4-9)

$$L_4 = 2 G_5 G_6 F_1^2 + 2 G_2 G_3 F_1 F_6 + G_3^2 F_1 F_5 - G_3 G_6 F_1 F_2 - G_2 G_6 F_1 F_3 - \\ G_3 G_5 F_1 F_3 + 2 G_1 G_6 F_2 F_3 + G_1 G_5 F_3^2 + 2 G_1^2 F_5 F_6 - G_1 G_3 F_2 F_6 - \\ G_1 G_2 F_3 F_6 - G_1 G_3 F_3 F_5 - 2 G_1 G_5 F_1 F_6 - 2 G_1 G_6 F_1 F_5$$

$$L_5 = F_1^2 G_6^2 + G_3^2 F_1 F_6 - G_3 G_6 F_1 F_3 + G_1 G_6 F_3^2 + G_1^2 F_6^2 - \\ G_1 G_3 F_3 F_6 - 2 G_1 G_6 F_1 F_6$$

The  $\lambda_1$  roots are substituted into Equations (4-6), to yield roots for  $\lambda_2$ . With these two roots known, using Equations (4-5), the values of  $X_{A1}$ ,  $Y_{A1}$ ,  $X_{B1}$  and  $Y_{B1}$  can be found.

Since there are a maximum of four possible pairs of  $\lambda_1$  and  $\lambda_2$ , there are a maximum of four possible solutions for the center point and

the circle point. The number of solutions depends on whether  $\lambda_1$  and  $\lambda_2$  have imaginary roots. Only real roots for  $\lambda_1$  and  $\lambda_2$  result in possible solutions. A choice of any two of these center point and circle point combinations will result in a maximum of six solutions for a designed four-link mechanism.

TABLE I  
FIVE FINITE POSITIONS

---

INPUT ROTATION ANGLES ARE	-15.000	-25.000	-40.000	-60.000
OUTPUT ROTATION ANGLES ARE	-20.000	-30.000	-50.000	-75.000
TERNARY LINK ROTATIONS ARE	10.000	20.000	30.000	40.000

OUTPUT LINK IS 1.0 LONG AND AT AN ANGLE OF 100.0 DEGREES

	<u>SOLUTION</u>	
	X	Y
M	0.00000000	0.00000000
Q	1.00000000	0.00000000
C1	0.82635313	0.98480803

POSSIBLE SOLUTION FOR POINTS A1 AND B1 OR D1 AND E1

CENTER POINT		CIRCLE POINT	
X	Y	X	Y
0.03711897	0.09393525	0.21974057	0.34299600
-5.62751389	-8.39265823	-8.16039371	-5.00552368

ROOT IS IMAGINARY

ROOT IS IMAGINARY

---

Once this is done, the six-link mechanism is designed in its first position. The center points of the four-link mechanism are now points A and D of the input ternary link, and the corresponding circle points are the points B and E of the moving ternary link. Now by grounding link MQ the desired input-output relationship is obtained. A computer program that follows this procedure is listed in Appendix A, and an example problem is presented in Table I. The resulting six-link mechanism will go through the five desired input and output rotations, while the ternary link goes through a total rotation of  $(\alpha_{1n} + \theta_{1n})$ . The ternary link rotation is now relative to grounded link MQ.

## CHAPTER V

### SYNTHESIS PROCEDURE FOR FIRST INFINITESIMALLY SEPARATED POSITIONS

A continuation of the procedure set forth in Chapter IV is the development of a method that will specify the velocity ratio of the input to output at a certain finite position of the input and output links. In Chapter IV it was shown that the synthesis for five positions involved, first, inversion and then four-link rigid body guidance synthesis. The same procedure will be followed for synthesizing the Stephenson Type II six-link mechanism for velocity ratios. The displacement of the output link is a function of the input link, and both are a function of time; that is,  $\phi(t) = f(\theta(t))$ .

In order to obtain a specified velocity ratio, this relationship must be differentiated with respect to time, resulting in  $\frac{d\phi}{dt} = \frac{d\phi}{d\theta} \frac{d\theta}{dt}$ . It can be seen that the infinitesimal displacement  $\frac{d\phi}{d\theta}$ , can be specified by the ratio of  $\frac{d\phi}{dt}$  to  $\frac{d\theta}{dt}$ , both of which are design parameters. For the function  $\phi = f(\theta)$ , as shown in Figure 7, the infinitesimal displacement is the direction and magnitude of the slope at a point. In other words, the desired velocity ratios are now converted into an infinitesimal displacement problem.

For infinitesimal displacement for five positions, there are two basic combinations.

PP-P-P-P	PP-PP-P
P-PP-P-P	PP-P-PP
P-P-PP-P	P-PP-PP
P-P-P-PP	

That is, the two combinations are specifying four finite positions and an infinitesimal displacement at one of the four finite positions, or specifying three finite positions and an infinitesimal displacement at two of the finite positions.

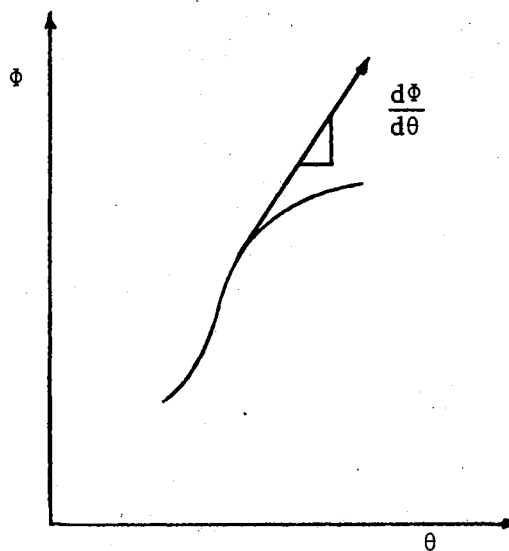


Figure 7. Infinitesimal Displacement

The principle of inversion is still valid for this case. The four finite positions can be defined by the procedure set forth in Chapter IV. To find the infinitesimal displacement of point C, the displacement Equation (4-1) will be differentiated with respect to  $\theta$ , resulting in

$$\begin{aligned}
\frac{dX_{Cn}}{d\theta_{1n}} &= (X_{C1} - X_{Q1}) \left[ \sin(\phi_{1n} - \theta_{1n}) - \sin(\phi_{1n} - \theta_{1n}) \frac{d\phi_{1n}}{d\theta_{1n}} \right] \\
&\quad + (Y_{C1} - Y_{Q1}) \left[ \cos(\phi_{1n} - \theta_{1n}) - \cos(\phi_{1n} - \theta_{1n}) \frac{d\phi_{1n}}{d\theta_{1n}} \right] \\
&\quad + X_{Q1} \sin(-\theta_{1n}) + Y_{Q1} \cos(-\theta_{1n}) \\
\frac{dY_{Cn}}{d\theta_{1n}} &= (X_{C1} - X_{Q1}) \left[ -\cos(\phi_{1n} - \theta_{1n}) + \cos(\phi_{1n} - \theta_{1n}) \frac{d\phi_{1n}}{d\theta_{1n}} \right] \\
&\quad + (Y_{C1} - Y_{Q1}) \left[ \sin(\phi_{1n} - \theta_{1n}) - \sin(\phi_{1n} - \theta_{1n}) \frac{d\phi_{1n}}{d\theta_{1n}} \right] \\
&\quad - X_{Q1} \cos(-\theta_{1n}) + Y_{Q1} \sin(-\theta_{1n}).
\end{aligned} \tag{5-1}$$

This results in a value for  $\frac{dX_{Cn}}{d\theta_{1n}}$ ,  $\frac{dY_{Cn}}{d\theta_{1n}}$  which is the infinitesimal displacement of point C with respect to  $\theta_{1n}$  at a certain point. As explained in Chapter III, infinitesimal displacements for rigid body guidance can be obtained by means of the matrix method. The infinitesimal displacement of point C in designing the four-link rigid body guidance mechanism is defined by rotations  $\alpha_{1n}$ . In other words, the infinitesimal displacement of point C given by  $\frac{dX_{Cn}}{d\alpha_{1n}}$ ,  $\frac{dY_{Cn}}{d\alpha_{1n}}$  is necessary for the four-link synthesis. Since

$$\frac{dX_{Cn}}{d\alpha_{1n}} = \frac{d\theta_{1n}}{d\alpha_{1n}} \frac{dX_{Cn}}{d\theta_{1n}} \quad \text{and} \quad \frac{dY_{Cn}}{d\alpha_{1n}} = \frac{d\theta_{1n}}{d\alpha_{1n}} \frac{dY_{Cn}}{d\theta_{1n}} \tag{5-2}$$

the infinitesimal movement of  $\frac{d\theta_{1n}}{d\alpha_{1n}}$  must be given. This is arbitrary and can be used to the designer's advantage.

In order to obtain the synthesis equations for synthesizing the four-link mechanism, the principle set forth in Chapter III may be utilized. The matrix Equations (2-1) and (3-6) are substituted into Equation (3-7) to obtain the design Equation (5-3).



$$\begin{aligned}
& X_{A1} \left( \frac{dX_{Cn}}{d\alpha_{1n}} + X_{C1} \sin\alpha_{1n} + Y_{C1} \cos\alpha_{1n} \right) + \\
& Y_{A1} \left( \frac{dY_{Cn}}{d\alpha_{1n}} + Y_{C1} \sin\alpha_{1n} - X_{C1} \cos\alpha_{1n} \right) + \\
& X_{B1} \left[ -\left( \frac{dX_{Cn}}{d\alpha_{1n}} + Y_{Cn} \right) \cos\alpha_{1n} - \left( \frac{dY_{Cn}}{d\alpha_{1n}} - X_{Cn} \right) \sin\alpha_{1n} \right] + \\
& Y_{B1} \left[ -\left( \frac{dY_{Cn}}{d\alpha_{1n}} - X_{Cn} \right) \cos\alpha_{1n} + \left( \frac{dX_{Cn}}{d\alpha_{1n}} + Y_{Cn} \right) \sin\alpha_{1n} \right] = \\
& \sin\alpha_{1n} (X_{A1} X_{B1} + Y_{A1} Y_{B1}) + \cos\alpha_{1n} (X_{A1} Y_{B1} - X_{B1} Y_{A1}) - \\
& \cos\alpha_{1n} \left( X_{C1} \frac{dX_{Cn}}{d\alpha_{1n}} + Y_{C1} \frac{dY_{Cn}}{d\alpha_{1n}} - Y_{C1} X_{Cn} + X_{C1} Y_{Cn} \right) + \\
& \sin\alpha_{1n} \left( Y_{C1} \frac{dX_{Cn}}{d\alpha_{1n}} - X_{C1} \frac{dY_{Cn}}{d\alpha_{1n}} + X_{C1} X_{Cn} + Y_{C1} Y_{Cn} \right) + \\
& X_{Cn} \frac{dX_{Cn}}{d\alpha_{1n}} + Y_{Cn} \frac{dY_{Cn}}{d\alpha_{1n}}
\end{aligned} \tag{5-3}$$

Please note that at this point a quicker and easier procedure for obtaining this equation may be followed. This procedure will be of particular use when developing higher order infinitesimal displacement synthesis equations. For infinitesimal movement, the changes in the positions of B and E are very small. For displacement analysis the equation of a circle is used in defining the movement of circle point B around center point A. Equation (2-1) is substituted into the equation of a circle, which results in the following equation.

$$f(X_{B1}, Y_{B1}, X_{A1}, Y_{A1}, \alpha_{1n}) = 0 \tag{5-4}$$

In order to obtain the infinitesimal movement, this equation must be differentiated. By means of the Taylor series, neglecting higher order

terms, the derivative of this function can be given by Equation (5-5).

$$f(X_{B1}, Y_{B1}, X_{A1}, Y_{A1}, \alpha_{1n}) + \frac{\partial f(X_{B1}, Y_{B1}, X_{A1}, Y_{A1}, \alpha_{1n})}{\partial \alpha_{1n}} \Delta \alpha_{1n} + \dots = 0 \quad (5-5)$$

The first part has been satisfied by the equation (5-4) and the second part has yet to be satisfied. Therefore, in order to obtain the infinitesimal displacement equation for synthesis, the finite position synthesis Equation (4-2) must be differentiated with respect to  $\alpha_{1n}$ .

Therefore, the first infinitesimal displacement synthesis equation may be obtained by differentiating the coefficients of the finite displacement synthesis Equation (4-2).

Since Equation (5-3) is of the same form as Equation (4-2), the linear superposition principle allows for Equation (4-2) to be substituted where desired. The four positions result in three displacement equations which become linear after applying the principle of linear superposition. One more equation is needed to solve for the four unknowns. The infinitesimal displacement equation is also linear and has the same unknowns, therefore it is the desired fourth equation. The method used in solving for the four unknowns will be identical to that used for solving the five finite displacement problem in Chapter IV.

A computer program has been written which results in the synthesized six-link Stephenson Type II mechanism for four finite positions and a specified velocity ratio at one of the finite positions. It is listed in Appendix B and an example is presented in Table II. A computer program has also been written for the case of three finite

positions and a velocity ratio specified at any two of the finite positions. It is listed in Appendix C, and an example is presented in Table III.

TABLE II

FOUR FINITE POSITIONS AND ONE VELOCITY RATIO  
(VELOCITY RATIO IS SPECIFIED AT POINT 4)

INPUT VELOCITY = 1.0	OUTPUT VELOCITY = 2.0		
INPUT ROTATION ANGLES ARE	-15.000	-30.000	-45.000
OUTPUT ROTATION ANGLES ARE	-20.000	-45.000	-75.000
TERNARY LINK ROTATIONS ARE	10.000	20.000	30.000

OUTPUT LINK IS 1.0 LONG AND AT AN ANGLE OF 100.0 DEGREES.

FIRST INFINITESIMAL DISPLACEMENTS

DPHDTH(J) = 2.0  
DTHDAL(J) = 1.0

SOLUTION

	X	Y
M	0.00000000	0.00000000
Q	1.00000000	0.00000000
C1	0.82635210	0.98480770

POSSIBLE SOLUTION FOR POINTS A1 AND B1 OR D1 AND E1

CENTER POINT		CIRCLE POINT	
X	Y	X	Y
-0.22540400	1.59108100	-0.25595210	1.53780500
-0.48618850	1.69824500	-0.40762280	1.74143700

ROOT IS IMAGINARY

ROOT IS IMAGINARY

TABLE III

THREE FINITE POSITIONS AND TWO VELOCITY RATIOS  
(VELOCITIES SPECIFIED AT POINTS 1 AND 3)

---

1	INPUT VELOCITY = 1.0	OUTPUT VELOCITY = 0.0
3	INPUT VELOCITY = 1.0	OUTPUT VELOCITY = 0.0
	INPUT ROTATION ANGLES ARE	-15.000      -30.000
	OUTPUT ROTATION ANGLES ARE	-20.000      -60.000
	TERNARY LINK ROTATIONS ARE	10.000      20.000

OUTPUT LINK IS 1.0 LONG AND AT AN ANGLE OF -30.0 DEGREES.

FIRST INFINITESIMAL DISPLACEMENTS      DPHDTH(J) = 0.0  
DTHDAL(J) = 1.0

FIRST INFINITESIMAL DISPLACEMENTS      DPHDTH(K) = 0.0  
DTHDAL(K) = 2.0

SOLUTION

	X	Y
M	0.00000000	0.00000000
Q	1.00000000	0.00000000
Cl	1.86602400	-0.49999999

POSSIBLE SOLUTION FOR POINTS A1 AND B1 OR D1 AND E1

CENTER POINT		CIRCLE POINT	
X	Y	X	Y
1.38239300	-0.72231820	1.58071100	-0.74633180
1.34985100	-1.81822200	1.25711400	-1.85000600
1.94092400	-2.69249400	1.79815800	-2.82720700
4.05168700	-1.16569700	4.92417600	-1.61335900

---

## CHAPTER VI

### SYNTHESIS PROCEDURE FOR SECOND INFINITESIMALLY SEPARATED POSITIONS

In addition to the velocity ratio at a finite point, another design parameter might be to specify an acceleration ratio for the input and output at the same finite point. The input acceleration is the second derivative of  $\theta$  with respect to time, while the output acceleration is the second derivative of  $\phi$  with respect to time. This ratio can be obtained by differentiating  $\phi(t) = f(\theta(t))$  twice. The result is

$$\frac{d^2\phi}{dt^2} = \frac{d^2\phi}{d\theta^2} \left(\frac{d\theta}{dt}\right)^2 + \frac{d\phi}{d\theta} \frac{d^2\theta}{dt^2}. \quad (6-1)$$

To find the second infinitesimal displacement  $\frac{d^2\phi}{d\theta^2}$ , all other terms must be specified. For a specific problem, this involves specifying

$\frac{d^2\phi}{dt^2}$  = the acceleration of the output link,

$\frac{d^2\theta}{dt^2}$  = the acceleration of the input link,

$\frac{d\theta}{dt}$  = the velocity of the input link, and

$\frac{d\phi}{d\theta}$  = the first infinitesimal displacement.

The term  $\frac{d\phi}{d\theta}$  has already been determined from specifying the velocity ratio at the same finite point.

For five positions the second infinitesimal displacement involves the following class of problems:

PPP-P-P	PPP-PP
P-PPP-P	PP-PPP
P-P-PPP	

That is, the two combinations are specifying three finite points and the first and second infinitesimal displacements at one of the finite points, or specifying two finite points and first and second infinitesimal displacements at one of the finite points and a first infinitesimal displacement at another finite point.

In order to synthesize the Stephenson Type II six-link mechanism for the above two cases, the same procedure employed in the previous chapters will be used. Inversion will be taken about the input link. The links MQ and QC are rotated according to the principle of inversion to find the finite points  $X_{Cn}$ ,  $Y_{Cn}$ . The first and the second derivatives of the displacement equations are obtained by differentiating Equations (4-1) with respect to  $\theta$  to find  $\frac{dX_{Cn}}{d\theta}$ ,  $\frac{dY_{Cn}}{d\theta}$ ,  $\frac{d^2X_{Cn}}{d\theta^2}$ , and  $\frac{d^2Y_{Cn}}{d\theta^2}$ . The first derivative Equations (5-1) are given in Chapter V and the second derivative equation is

$$\begin{aligned}
 \frac{d^2X_{Cn}}{d\theta_{1n}^2} = & (X_{C1} - X_{Q1}) (-\cos(\phi_{1n} - \theta_{1n}) + 2 \cos(\phi_{1n} - \theta_{1n}) \frac{d\phi_{1n}}{d\theta_{1n}} \\
 & - \cos(\phi_{1n} - \theta_{1n}) \left(\frac{d\phi_{1n}}{d\theta_{1n}}\right)^2 - \sin(\phi_{1n} - \theta_{1n}) \frac{d^2\phi_{1n}}{d\theta_{1n}^2}) \\
 & + (Y_{C1} - Y_{Q1}) (\sin(\phi_{1n} - \theta_{1n}) - 2 \sin(\phi_{1n} - \theta_{1n}) \frac{d\phi_{1n}}{d\theta_{1n}} \\
 & + \sin(\phi_{1n} - \theta_{1n}) \left(\frac{d\phi_{1n}}{d\theta_{1n}}\right)^2 - \cos(\phi_{1n} - \theta_{1n}) \frac{d^2\phi_{1n}}{d\theta_{1n}^2}) \\
 & - X_{Q1} \cos(-\theta_{1n}) + Y_{Q1} \sin(-\theta_{1n})
 \end{aligned} \tag{6-2}$$

$$\frac{d^2Y_{Cn}}{d\theta_{1n}^2} = (X_{C1} - X_{Q1}) (-\sin(\phi_{1n} - \theta_{1n}) + 2 \sin(\phi_{1n} - \theta_{1n}) \frac{d\phi_{1n}}{d\theta_{1n}} -$$

$$\begin{aligned}
& \sin (\phi_{1n} - \theta_{1n}) \left( \frac{d\phi_{1n}}{d\theta_{1n}} \right)^2 + \cos (\phi_{1n} - \theta_{1n}) \frac{d^2\phi_{1n}}{d\theta_{1n}^2} \\
& + (Y_{C1} - Y_{Q1}) (-\cos (\phi_{1n} - \theta_{1n})) + 2 \cos (\phi_{1n} - \theta_{1n}) \frac{d\phi_{1n}}{d\theta_{1n}} \\
& - \cos (\phi_{1n} - \theta_{1n}) \left( \frac{d\phi_{1n}}{d\theta_{1n}} \right)^2 - \sin (\phi_{1n} - \theta_{1n}) \frac{d^2\phi_{1n}}{d\theta_{1n}^2} \\
& - X_{Q1} \sin (-\theta_{1n}) - Y_{Q1} \cos (-\theta_{1n}).
\end{aligned}$$

With the first and second infinitesimal displacements of  $X_{Cn}$ ,  $Y_{Cn}$  known, the synthesis procedure now involves a four-link rigid body guidance synthesis problem with first and second infinitesimal displacements. Notice that the four-link synthesis procedure involves the first and second infinitesimal displacement of  $X_{Cn}$ ,  $Y_{Cn}$  with respect to the rotation of the ternary link. That is,  $\frac{dX_{Cn}}{d\alpha_{1n}}$ ,  $\frac{dY_{Cn}}{d\alpha_{1n}}$ ,  $\frac{d^2X_{Cn}}{d\alpha_{1n}^2}$  and  $\frac{d^2Y_{Cn}}{d\alpha_{1n}^2}$  are to be specified in synthesizing the four-link mechanism. Equations (5-2) must be differentiated with respect to  $\alpha_{1n}$ , resulting in

$$\begin{aligned}
\frac{d^2X_{Cn}}{d\alpha_{1n}^2} &= \frac{d^2\theta_{1n}}{d\alpha_{1n}^2} \frac{dX_{Cn}}{d\theta_{1n}} + \left( \frac{d\theta_{1n}}{d\alpha_{1n}} \right)^2 \frac{d^2X_{Cn}}{d\theta_{1n}^2} \\
\frac{d^2Y_{Cn}}{d\alpha_{1n}^2} &= \frac{d^2\theta_{1n}}{d\alpha_{1n}^2} \frac{dY_{Cn}}{d\theta_{1n}} + \left( \frac{d\theta_{1n}}{d\alpha_{1n}} \right)^2 \frac{d^2Y_{Cn}}{d\theta_{1n}^2}.
\end{aligned} \tag{6-3}$$

The ratio  $\frac{d^2\phi_{1n}}{d\theta_{1n}^2}$  has to be specified along with  $\frac{d\phi_{1n}}{d\theta_{1n}}$ , which was needed for the first infinitesimal displacement synthesis. Again, these are arbitrary values, or can be used to the designer's advantage.

As a result of the justification established in Chapter V, the coefficients of the design Equation (4-2) can be differentiated twice with respect to  $\alpha_{1n}$  to obtain the second infinitesimal displacement four-link mechanism synthesis Equation (6-4).

$$\begin{aligned}
& X_{A1} \left( \frac{d^2 X_{Cn}}{d\alpha_{1n}^2} + X_{C1} \cos\alpha_{1n} - Y_{C1} \sin\alpha_{1n} \right) + \\
& Y_{A1} \left( \frac{d^2 Y_{Cn}}{d\alpha_{1n}^2} + X_{C1} \sin\alpha_{1n} + Y_{C1} \cos\alpha_{1n} \right) + \\
& X_{B1} \left[ - \left( \frac{d^2 X_{Cn}}{d\alpha_{1n}^2} + 2 \frac{dY_{Cn}}{d\alpha_{1n}} - X_{Cn} \right) \cos\alpha_{1n} - \left( \frac{d^2 Y_{Cn}}{d\alpha_{1n}^2} - 2 \frac{dX_{Cn}}{d\alpha_{1n}} \right. \right. \\
& \qquad \qquad \qquad \left. \left. - Y_{Cn} \right) \sin\alpha_{1n} \right] + \\
& Y_{B1} \left[ - \left( \frac{d^2 Y_{Cn}}{d\alpha_{1n}^2} - 2 \frac{dX_{Cn}}{d\alpha_{1n}} - Y_{Cn} \right) \cos\alpha_{1n} + \left( \frac{d^2 X_{Cn}}{d\alpha_{1n}^2} + 2 \frac{dY_{Cn}}{d\alpha_{1n}} - \right. \right. \\
& \qquad \qquad \qquad \left. \left. X_{Cn} \right) \sin\alpha_{1n} \right] = \\
& \qquad \qquad \qquad (6-4) \\
& \cos\alpha_{1n} (X_{A1} X_{B1} + Y_{A1} Y_{B1}) - \sin\alpha_{1n} (X_{A1} Y_{B1} - X_{B1} Y_{A1}) + \\
& \cos\alpha_{1n} \left( 2 Y_{C1} \frac{dX_{Cn}}{d\alpha_{1n}} - 2 X_{C1} \frac{dY_{Cn}}{d\alpha_{1n}} - Y_{C1} \frac{d^2 Y_{Cn}}{d\alpha_{1n}^2} - X_{C1} \frac{d^2 X_{Cn}}{d\alpha_{1n}^2} \right. \\
& \qquad \qquad \qquad \left. + X_{C1} X_{Cn} + Y_{C1} Y_{Cn} \right) + \\
& \sin\alpha_{1n} \left( 2 X_{C1} \frac{dX_{Cn}}{d\alpha_{1n}} + 2 Y_{C1} \frac{dY_{Cn}}{d\alpha_{1n}} - X_{C1} \frac{d^2 Y_{Cn}}{d\alpha_{1n}^2} + Y_{C1} \frac{d^2 X_{Cn}}{d\alpha_{1n}^2} \right. \\
& \qquad \qquad \qquad \left. - Y_{C1} X_{Cn} + X_{C1} Y_{Cn} \right) + X_{Cn} \frac{d^2 X_{Cn}}{d\alpha_{1n}^2} + Y_{Cn} \frac{d^2 Y_{Cn}}{d\alpha_{1n}^2} + \\
& \qquad \qquad \qquad \frac{dX_{Cn}}{d\alpha_{1n}} \frac{dX_{Cn}}{d\alpha_{1n}} + \frac{dY_{Cn}}{d\alpha_{1n}} \frac{dY_{Cn}}{d\alpha_{1n}}
\end{aligned}$$

Since this equation is still of the same form as the finite displacement and first infinitesimal displacement equations, it can be used as a fourth equation that is needed due to the reduction of one finite position equation.



Two computer programs have been written which cover the two classes of problems discussed in this chapter. A computer program which synthesizes the Stephenson Type II mechanism for three finite positions and first and second infinitesimally separated positions of input to output specified at one of the finite positions is listed in Appendix D, and an example problem is presented in Table IV.

TABLE IV

THREE FINITE POSITIONS AND ONE ACCELERATION  
RATIO (ACCELERATION RATIO IS AT POINT 3)

---

INPUT VELOCITY = 1.0	OUTPUT VELOCITY = 0.0
INPUT ACCELERATION = 2.0	OUTPUT ACCELERATION = 0.0
INPUT ROTATION ANGLES ARE	-25.000    -50.000
OUTPUT ROTATION ANGLES ARE	-10.000    -30.000
TERNARY LINK ROTATIONS ARE	5.000      15.000
OUTPUT LINK IS 1.0 LONG AND AT AN ANGLE OF 100.0 DEGREES.	
FIRST INFINITESIMAL DISPLACEMENTS	DPHDTH(J) = 0.0 DTHDAL(J) = 1.0
SECOND INFINITESIMAL DISPLACEMENTS	DDPHTH(J) = 0.0 DDTHAL(J) = 1.0

<u>SOLUTION</u>			
X		Y	
M	0.00000000	0.00000000	0.00000000
Q	1.00000000	0.00000000	0.00000000
C1	0.82635313	0.98480803	

POSSIBLE SOLUTION FOR POINTS A1 AND B1 OR D1 AND E1

CENTER POINT		CIRCLE POINT	
X	Y	X	Y
-1.47581768	-0.56793118	0.37606907	1.30638123
1.31134033	6.33950806	-10.17961121	-15.10237122

ROOT IS IMAGINARY
ROOT IS IMAGINARY

---

A computer program which synthesizes the Stephenson Type II mechanism for two finite positions and first and second infinitesimally separated positions of input to output at one of the finite points, as well as a first infinitesimally separated position specified at the other finite point is listed in Appendix E, and an example problem is presented in Table V.

TABLE V

TWO FINITE POSITIONS, ONE VELOCITY RATIO, AND ONE ACCELERATION RATIO (VELOCITY RATIO IS AT POINT 1 AND ACCELERATION RATIO IS AT POINT 2)

---

1	INPUT VELOCITY = 1.0	OUTPUT VELOCITY = 3.0
2	INPUT VELOCITY = 1.0	OUTPUT VELOCITY = 2.0
2	INPUT ACCELERATION = 0.0	OUTPUT ACCELERATION = 0.0

INPUT ROTATION ANGLE IS	30.000
OUTPUT ROTATION ANGLE IS	45.000
TERNARY LINK ROTATION IS	10.000

OUTPUT LINK IS 1.0 LONG AND AT AN ANGLE OF 60.0 DEGREES.

FIRST INFINITESIMAL DISPLACEMENTS	DPHDTH(K) = 3.0
	DTHDAL(K) = 1.0

SECOND INFINITESIMAL DISPLACEMENTS	DDPHTH(J) = 0.0
	DDTHAL(J) = 1.0

FIRST INFINITESIMAL DISPLACEMENTS	DPHDTH(J) = 2.0
	DTHDAL(J) = 1.0

SOLUTION

	X	Y
M	0.00000000	0.00000000
Q	1.00000000	0.00000000
C1	1.50000000	0.86602520

POSSIBLE SOLUTION FOR POINTS A1 AND B1 OR D1 AND E1

CENTER POINT		CIRCLE POINT	
X	Y	X	Y
1.74607800	-0.63277810	1.88640900	-0.50722500
2.98628100	-1.30977000	3.82497700	-1.56066700
ROOT IS IMAGINARY			
ROOT IS IMAGINARY			

---

## CHAPTER VII

### SYNTHESIS PROCEDURE FOR THIRD AND FOURTH INFINITESIMALLY SEPARATED POSITIONS

An additional design criterion might be the specification of the jerk and kerk ratios of input to output. For example, at a finite point it may be desirable not only to specify a zero acceleration ratio, but also to have a zero jerk and kerk ratio. The jerk and kerk ratios enable the designer to have greater control over his input-output linkage motion.

The same synthesis procedure set forth in the previous chapters will be used for these next two cases, which are

PPPP-P                      PPPPP  
P-PPPP

#### Third Infinitesimally Separated Position

In order to solve the third infinitesimal displacement problem, one must realize that at a finite position, the first, second and third infinitesimal displacement parameters must be given. That is,  $\frac{d\phi}{d\theta}$  is first found from the specified velocity ratios and, in turn, is used in Equation (6-1) to find  $\frac{d^2\phi}{d\theta^2}$ . Now the derivative of Equation (6-1) can be taken to solve for the third infinitesimally separated position  $\frac{d^3\phi}{d\theta^3}$ .

$$\frac{d^3\phi}{dt^3} = \frac{d^3\phi}{d\theta^3} \left(\frac{d\theta}{dt}\right)^3 + 3 \frac{d\theta}{dt} \cdot \frac{d^2\theta}{dt^2} \cdot \frac{d^2\phi}{d\theta^2} + \frac{d^3\theta}{dt^3} \cdot \frac{d\phi}{d\theta} \quad (7-1)$$

These values are then used in Equations (5-1), (6-2) and (7-2) to find the values for  $\frac{dX_{Cn}}{d\theta_{1n}}$ ,  $\frac{dY_{Cn}}{d\theta_{1n}}$ ,  $\frac{d^2X_{Cn}}{d\theta_{1n}^2}$ ,  $\frac{d^2Y_{Cn}}{d\theta_{1n}^2}$ ,  $\frac{d^3X_{Cn}}{d\theta_{1n}^3}$ , and  $\frac{d^3Y_{Cn}}{d\theta_{1n}^3}$ .

$$\begin{aligned}
\frac{d^3X_{Cn}}{d\theta_{1n}^3} &= (X_{C1} - X_{Q1}) \left[ -\sin(\phi_{1n} - \theta_{1n}) + 3 \sin(\phi_{1n} - \theta_{1n}) \frac{d\phi_{1n}}{d\theta_{1n}} \right. \\
&\quad - 3 \sin(\phi_{1n} - \theta_{1n}) \left( \frac{d\phi_{1n}}{d\theta_{1n}} \right)^2 + 3 \cos(\phi_{1n} - \theta_{1n}) \frac{d^2\phi_{1n}}{d\theta_{1n}^2} \\
&\quad - 3 \cos(\phi_{1n} - \theta_{1n}) \frac{d\phi_{1n}}{d\theta_{1n}} \cdot \frac{d^2\phi_{1n}}{d\theta_{1n}^2} + \sin(\phi_{1n} - \theta_{1n}) \left( \frac{d\phi_{1n}}{d\theta_{1n}} \right)^3 \\
&\quad \left. - \sin(\phi_{1n} - \theta_{1n}) \frac{d^3\phi_{1n}}{d\theta_{1n}^3} \right] + \\
(Y_{C1} - X_{Q1}) &\left[ -\cos(\phi_{1n} - \theta_{1n}) + 3 \cos(\phi_{1n} - \theta_{1n}) \frac{d\phi_{1n}}{d\theta_{1n}} \right. \\
&\quad - 3 \cos(\phi_{1n} - \theta_{1n}) \left( \frac{d\phi_{1n}}{d\theta_{1n}} \right)^2 - 3 \sin(\phi_{1n} - \theta_{1n}) \frac{d^2\phi_{1n}}{d\theta_{1n}^2} \\
&\quad + 3 \sin(\phi_{1n} - \theta_{1n}) \frac{d\phi_{1n}}{d\theta_{1n}} \cdot \frac{d^2\phi_{1n}}{d\theta_{1n}^2} + \cos(\phi_{1n} - \theta_{1n}) \left( \frac{d\phi_{1n}}{d\theta_{1n}} \right)^3 \\
&\quad \left. - \cos(\phi_{1n} - \theta_{1n}) \frac{d^3\phi_{1n}}{d\theta_{1n}^3} \right] - X_{Q1} \sin(-\phi_{1n}) - Y_{Q1} \cos(-\theta_{1n}) \quad (7-2)
\end{aligned}$$

$$\begin{aligned}
\frac{d^3Y_{Cn}}{d\theta_{1n}^3} &= (X_{C1} - X_{Q1}) \left[ \cos(\phi_{1n} - \theta_{1n}) - 3 \cos(\phi_{1n} - \theta_{1n}) \frac{d\phi_{1n}}{d\theta_{1n}} \right. \\
&\quad + 3 \cos(\phi_{1n} - \theta_{1n}) \left( \frac{d\phi_{1n}}{d\theta_{1n}} \right)^2 + 3 \sin(\phi_{1n} - \theta_{1n}) \frac{d^2\phi_{1n}}{d\theta_{1n}^2} \\
&\quad - 3 \sin(\phi_{1n} - \theta_{1n}) \frac{d\phi_{1n}}{d\theta_{1n}} \cdot \frac{d^2\phi_{1n}}{d\theta_{1n}^2} - \cos(\phi_{1n} - \theta_{1n}) \left( \frac{d\phi_{1n}}{d\theta_{1n}} \right)^3 \\
&\quad \left. + \cos(\phi_{1n} - \theta_{1n}) \frac{d^3\phi_{1n}}{d\theta_{1n}^3} \right] +
\end{aligned}$$

$$\begin{aligned}
& (Y_{C1} - Y_{Q1}) [-\sin(\phi_{1n} - \theta_{1n}) + 3 \sin(\phi_{1n} - \theta_{1n}) \frac{d\phi_{1n}}{d\theta_{1n}} \\
& - 3 \sin(\phi_{1n} - \theta_{1n}) \left(\frac{d\phi_{1n}}{d\theta_{1n}}\right)^2 + 3 \cos(\phi_{1n} - \theta_{1n}) \frac{d^2\phi_{1n}}{d\theta_{1n}^2} \\
& - 3 \cos(\phi_{1n} - \theta_{1n}) \frac{d\phi_{1n}}{d\theta_{1n}} \cdot \frac{d^2\phi_{1n}}{d\theta_{1n}^2} + \sin(\phi_{1n} - \theta_{1n}) \left(\frac{d\phi_{1n}}{d\theta_{1n}}\right)^3 \\
& - \sin(\phi_{1n} - \theta_{1n}) \frac{d^3\phi_{1n}}{d\theta_{1n}^3}] + X_{Q1} \cos(-\theta_{1n}) - Y_{Q1} \sin(-\theta_{1n})
\end{aligned}$$

These values are, in turn, changed to  $\frac{dX_{Cn}}{d\alpha_{1n}}$ ,  $\frac{dY_{Cn}}{d\alpha_{1n}}$ ,  $\frac{d^2X_{Cn}}{d\alpha_{1n}^2}$ ,  $\frac{d^2Y_{Cn}}{d\alpha_{1n}^2}$ ,  $\frac{d^3X_{Cn}}{d\alpha_{1n}^3}$

and  $\frac{d^3Y_{Cn}}{d\alpha_{1n}^3}$  by means of Equations (5-2), (6-3) and (7-3).

$$\begin{aligned}
\frac{d^3X_{Cn}}{d\alpha_{1n}^3} &= \frac{d^3\theta_{1n}}{d\alpha_{1n}^3} \cdot \frac{dX_{Cn}}{d\theta_{1n}} + 3 \frac{d\theta_{1n}}{d\alpha_{1n}} \cdot \frac{d^2\theta_{1n}}{d\alpha_{1n}^2} \cdot \frac{d^2X_{Cn}}{d\theta_{1n}^2} + \left(\frac{d\theta_{1n}}{d\alpha_{1n}}\right)^3 \frac{d^3X_{Cn}}{d\theta_{1n}^3} \\
\frac{d^3Y_{Cn}}{d\alpha_{1n}^3} &= \frac{d^3\theta_{1n}}{d\alpha_{1n}^3} \cdot \frac{dY_{Cn}}{d\theta_{1n}} + 3 \frac{d\theta_{1n}}{d\alpha_{1n}} \cdot \frac{d^2\theta_{1n}}{d\alpha_{1n}^2} \cdot \frac{d^2Y_{Cn}}{d\theta_{1n}^2} + \left(\frac{d\theta_{1n}}{d\alpha_{1n}}\right)^3 \frac{d^3Y_{Cn}}{d\theta_{1n}^3}
\end{aligned} \tag{7-3}$$

At a finite point, the first, second and third infinitesimal displacement synthesis Equations (5-3), (6-4) and (7-4), respectively, can now be used to design the desired Stephenson Type II motion.

$$\begin{aligned}
X_{A1} & \left( \frac{d^3X_{Cn}}{d\alpha_{1n}^3} - X_{C1} \sin\alpha_{1n} - Y_{C1} \cos\alpha_{1n} \right) + \\
Y_{A1} & \left( \frac{d^3Y_{Cn}}{d\alpha_{1n}^3} + X_{C1} \cos\alpha_{1n} - Y_{C1} \sin\alpha_{1n} \right) + \\
X_{B1} & \left[ -\left( \frac{d^3X_{Cn}}{d\alpha_{1n}^3} + 3 \frac{d^2Y_{Cn}}{d\alpha_{1n}^2} - 3 \frac{dX_{Cn}}{d\alpha_{1n}} Y_{Cn} \right) \cos\alpha_{1n} \right. \\
& \left. - \left( \frac{d^3Y_{Cn}}{d\alpha_{1n}^3} - 3 \frac{d^2X_{Cn}}{d\alpha_{1n}^2} - 3 \frac{dY_{Cn}}{d\alpha_{1n}} + X_{Cn} \right) \sin\alpha_{1n} \right] +
\end{aligned}$$

$$\begin{aligned}
& Y_{B1} \left[ - \left( \frac{d^3 Y_{Cn}}{d\alpha_{1n}^3} - 3 \frac{d^2 X_{Cn}}{d\alpha_{1n}^2} - 3 \frac{dY_{Cn}}{d\alpha_{1n}} + X_{Cn} \right) \cos\alpha_{1n} \right. \\
& \quad \left. + \left( \frac{d^3 X_{Cn}}{d\alpha_{1n}^3} + 3 \frac{d^2 Y_{Cn}}{d\alpha_{1n}^2} - 3 \frac{dX_{Cn}}{d\alpha_{1n}} - Y_{Cn} \right) \sin\alpha_{1n} \right] = \\
& - \sin\alpha_{1n} (X_{A1} X_{B1} + Y_{A1} Y_{B1}) - \cos\alpha_{1n} (X_{A1} Y_{B1} - X_{B1} Y_{A1}) \\
& + \cos\alpha_{1n} \left( -Y_{C1} \frac{d^3 Y_{Cn}}{d\alpha_{1n}^3} - X_{C1} \frac{d^3 X_{Cn}}{d\alpha_{1n}^3} + 3 Y_{C1} \frac{d^2 X_{Cn}}{d\alpha_{1n}^2} - \right. \\
& \qquad \qquad \qquad \left. 3 X_{C1} \frac{d^2 Y_{Cn}}{d\alpha_{1n}^2} \right) \\
& \qquad \qquad \qquad (7-4) \\
& \quad + 3 X_{C1} \frac{dX_{Cn}}{d\alpha_{1n}} + 3 Y_{C1} \frac{dY_{Cn}}{d\alpha_{1n}} - Y_{C1} X_{Cn} + X_{C1} Y_{Cn} \\
& + \sin\alpha_{1n} \left( -X_{C1} \frac{d^3 Y_{Cn}}{d\alpha_{1n}^3} + Y_{C1} \frac{d^3 X_{Cn}}{d\alpha_{1n}^3} + 3 X_{C1} \frac{d^2 X_{Cn}}{d\alpha_{1n}^2} + \right. \\
& \quad \left. + 3 Y_{C1} \frac{d^2 Y_{Cn}}{d\alpha_{1n}^2} - 3 Y_{C1} \frac{dX_{Cn}}{d\alpha_{1n}} + 3 X_{C1} \frac{dY_{Cn}}{d\alpha_{1n}} \right. \\
& \quad \left. - X_{C1} X_{Cn} - Y_{C1} Y_{Cn} \right) \\
& + X_{Cn} \frac{d^3 X_{Cn}}{d\alpha_{1n}^3} + Y_{Cn} \frac{d^3 Y_{Cn}}{d\alpha_{1n}^3} + 3 \frac{dX_{Cn}}{d\alpha_{1n}} \cdot \frac{d^2 X_{Cn}}{d\alpha_{1n}^2} + 3 \\
& \quad \frac{dY_{Cn}}{d\alpha_{1n}} \cdot \frac{d^2 Y_{Cn}}{d\alpha_{1n}^2}
\end{aligned}$$

Since the equations are all of the same form, the principle of linear superposition still applies.

A computer program which synthesizes the Stephenson Type II mechanism for two finite positions and first, second and third infinitesimally separated positions of input to output at one of the finite

positions is listed in Appendix F, and an example problem is presented in Table VI.

TABLE VI  
TWO FINITE POSITIONS AND ONE JERK RATIO  
(JERK RATIO IS SPECIFIED AT POINT 2)

---

INPUT VELOCITY = 1.0	OUTPUT VELOCITY = -2.5
INPUT ACCELERATION = 0.0	OUTPUT ACCELERATION = -1.0
INPUT JERK = 0.0	OUTPUT JERK = 0.0
INPUT ROTATION ANGLE IS	30.000
OUTPUT ROTATION ANGLE IS	-45.000
TERNARY LINK ROTATION IS	-25.000

OUTPUT LINK IS 1.2 LONG AND AT AN ANGLE OF -30.0 DEGREES.

FIRST INFINITESIMAL DISPLACEMENTS	DPHPTH(J) = -2.5
	DTHDAL(J) = 1.0
SECOND INFINITESIMAL DISPLACEMENTS	DDPHTH(J) = -1.0
	DDTHAL(J) = 1.0
THIRD INFINITESIMAL DISPLACEMENTS	DDDPH(J) = 0.0
	DDDTAL(J) = 1.0

SOLUTION

	X	Y		
M	0.00000000	0.00000000		
Q	1.00000000	0.00000000		
C1	2.03922900	-0.59999970		

POSSIBLE SOLUTION FOR POINTS A1 AND B1 OR D1 AND E1

CENTER POINT		CIRCLE POINT	
X	Y	X	Y
1.09700200	-0.47618860	2.43076600	-0.57500930
4.04580400	-4.92590300	5.21653500	-3.42272900

ROOT IS IMAGINARY

ROOT IS IMAGINARY

---

TABLE VII  
ONE FINITE POSITION AND ONE KERK RATIO

---

INPUT VELOCITY	= 1.0	OUTPUT VELOCITY	= 2.0
INPUT ACCELERATION	= 0.0	OUTPUT ACCELERATION	= 3.0
INPUT JERK	= 0.0	OUTPUT JERK	= 0.0
INPUT KERK	= 0.0	OUTPUT KERK	= 0.0

OUTPUT LINK IS 1.5 LONG AND AT AN ANGLE OF 30.0 DEGREES.

FIRST INFINITESIMAL DISPLACEMENTS	DPHPTH(J) = 2.0
	DTHDAL(J) = 1.0
SECOND INFINITESIMAL DISPLACEMENTS	DDPPTH(J) = 3.0
	DDTHAL(J) = 1.0
THIRD INFINITESIMAL DISPLACEMENTS	DDDPPTH(J) = 0.0
	DDDTAL(J) = 1.0
FOURTH INFINITESIMAL DISPLACEMENTS	DDDDPT(J) = 0.0
	DDDDTA(J) = 1.0

SOLUTION

	X	Y
M	0.00000000	0.00000000
Q	1.00000000	0.00000000
C1	2.29903793	0.74999970

POSSIBLE SOLUTIONS FOR POINTS A1 AND B1 OR D1 AND E1

CENTER POINT		CIRCLE POINT	
X	Y	X	Y
1.58438492	-0.57121086	2.80116749	1.10895443
4.47628021	0.35051918	7.80039597	0.81995583

ROOT IS IMAGINARY

ROOT IS IMAGINARY

---



#### Fourth Infinitesimally Separated Position

The procedure stated above can now be expanded to include the fourth infinitesimally separated position synthesis problem. The procedure will be the same, except that Equations (7-1), (7-2), (7-3) and (7-4) must be differentiated. Since there is an additional design equation for this case, the first, second, third and fourth infinitesimal displacements can be specified only at one finite point.

A computer program which synthesizes the Stephenson Type II mechanism for one finite point and first, second, third and fourth infinitesimally separated positions of input to output is listed in Appendix G, and an example problem is presented in Table VII.

## CHAPTER VIII

### SUMMARY AND CONCLUSIONS

This thesis develops an approach for synthesizing the Stephenson Type II six-link function generator for five positions of input and output. These five positions consist of both finitely and infinitesimally separable positions. There are fourteen possible combinations of finite and infinitesimally separated positions for which function generation motion can be obtained.

The synthesis procedure consists of starting with desired finite displacements, velocity ratios, acceleration ratios, jerk ratios or kerk ratios, whichever are desired, and changing them to an  $n^{\text{th}}$  order infinitesimally separated position synthesis problem. The principle of inversion is used to transform the Stephenson Type II mechanism synthesis problem into a four-link rigid body guidance problem. The matrix method proved to be easily adaptable for synthesizing any one of the fourteen motion programs. In order to obtain the closed form solution for synthesis, the principle of linear superposition was utilized.

Once the synthesis procedure for the Stephenson Type II six-link function generator was developed, seven computer programs were written to perform any of the fourteen function generation motions. Feeding the desired design parameters into the programs will result in from 0 to 6 possible solutions for the six-link mechanism because from 0 to 4

center point and circle point combinations are possible for the rigid body motion.

With these seven basic programs, a wide variety of function generation problems can be solved. For example, the Stephenson Type II mechanism can be synthesized for five finite positions, where the output link goes through more than 180 degrees. If cutting action is desired, ternary link rotations can be specified along with the output link rotations, resulting in the desired relative motion between two connecting rigid bodies. Since it was necessary in the synthesis procedure for  $n^{\text{th}}$  order infinitesimally separated positions to specify  $\frac{d\theta_{1n}}{d\alpha_{1n}}$ ,  $\frac{d^2\theta_{1n}}{d\alpha_{1n}^2}$ , etc., an additional parameter in which the ternary link infinitesimal rotations are related to the input infinitesimal rotations is given the designer. When an object is to be picked up at zero velocity and left off at zero velocity, this can be done for three finite positions by using the program in Appendix C. Or specific forces at a specified velocity and position can be obtained by achieving an appropriate acceleration of the output link. An example might be a stamping action. This can be accomplished, depending upon the other design parameters, by using programs in Appendices D, E, F or G. Another major area in which these programs could be used is finite dwell problems. Any of the programs in Appendices A, B, C, D, E or F can be used to obtain a finite dwell of the output while the input link rotates through finite positions. For five finite positions all five of the output rotations can be set equal to zero, while the input goes through the desired rotation angles. The program in Appendix C can be used to specify one finite position and a dwell at another finite position. The dwell would be developed by specifying two finite points

and a zero velocity ratio at those two points. The same procedure can also be used for programs in Appendices B, D, E and F by setting the appropriate velocity, acceleration and jerk ratios equal to zero. These programs will, of course, result in an approximate finite dwell, but error can be minimized by determining which program is the best for the specific problem.

The synthesis procedure shown in this thesis can be used for synthesis of different mechanisms, not only for finite positions, but also for infinitesimally separated positions. It can be applied to both function generation synthesis problems and rigid body guidance synthesis problems. This versatile and concise method was ideal for synthesis of the Stephenson Type II six-link function generator, which resulted in a practical vehicle for solving design problems.

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APPENDIX A  
SYNTHESIS PROGRAM FOR FIVE  
FINITE POSITIONS

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CARD
1 C *****
2 C *
3 C *           SYNTHESIS OF SIX-LINK STEPHENSON 2
4 C *           FUNCTION GENERATOR
5 C *
6 C *           FIVE FINITE POSITIONS
7 C *
8 C *
9 C *           INPUT FINITE ROTATIONS           TH1(I)
10 C *
11 C *          OUTPUT FINITE ROTATIONS           PHI(I)
12 C *
13 C *          ASSUMED ROTATIONS OF TERNARY     AL1(I)
14 C *
15 C *          OUTPUT LINK LENGTH                QC
16 C *
17 C *          OUTPUT LINK INITIAL ANGLE        THE
18 C *
19 C *****
20 C
21   DIMENSION XCOF(5),COF(5),ROOTR(4),ROOTI(4)
22   DIMENSION XQ(5),YQ(5),XA(5),YA(5),XB(5),YB(5),XC(5),YC(5)
23   DIMENSION TH1(5),PHI(5),AL1(5)
24   DIMENSION H(16),H1(16),H2(16),R(4),P(4),Q(4)
25   READ (5,10) TH1(2),TH1(3),TH1(4),TH1(5)
26   READ (5,10) PHI(2),PHI(3),PHI(4),PHI(5)
27   READ (5,10) AL1(2),AL1(3),AL1(4),AL1(5)
28   10 FORMAT (4F10.0)
29   READ (5,20) QC,THE
30   20 FORMAT (2F10.0)
31   WRITE (6,6)
32   6 FORMAT (//,25X,22H FIVE FINITE POSITIONS,/)
33   WRITE (6,11) TH1(2),TH1(3),TH1(4),TH1(5)
34   11 FORMAT (9X,26H INPUT ROTATION ANGLES ARE,2X,4F10.3,/)
35   WRITE (6,12) PHI(2),PHI(3),PHI(4),PHI(5)
36   12 FORMAT (9X,27H OUTPUT ROTATION ANGLES ARE,1X,4F10.3,/)
37   WRITE (6,13) AL1(2),AL1(3),AL1(4),AL1(5)
38   13 FORMAT (9X,23H TERNARY LINK ROTATIONS,5X,4F10.3,/)
39   WRITE (6,14) QC,THE
40   14 FORMAT (12X,15H OUTPUT LINK IS, F10.3, 24H LONG AND AT AN ANGLE OF, F
41   $10.3,/)
42   XQ(1)=1.0
43   YQ(1)=0.0
44   XM=0.0
45   YM=0.0
46   THE=(THE*3.14159265)/180.0
47   DO 21 I=2,5
48   TH1(I)=(TH1(I)*3.14159265)/180.0
49   PHI(I)=(PHI(I)*3.14159265)/180.0
50   21 AL1(I)=(AL1(I)*3.14159265)/180.0
51   XC(1)=1.0+QC*COS(THE)
52   YC(1)=0.0+QC*SIN(THE)
53   DO 30 I=2,5
54   YQ(I)=1.0*SIN(-TH1(I))

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55      XQ(I)=1.0*COS(-T+1(I))
56      XC(I)=XC(1)*COS(PH1(I)-TH1(I))-YC(1)*SIN(PH1(I)-TH1(I))+XQ(I)-XQ(1
57      $)*COS(PH1(I)-TH1(I))+YQ(1)*SIN(PH1(I)-TH1(I))
58      3C YC(I)=XC(1)*SIN(PH1(I)-TH1(I))+YC(1)*COS(PH1(I)-TH1(I))+YQ(I)-XQ(1
59      $)*SIN(PH1(I)-TH1(I))-YQ(1)*COS(PH1(I)-TH1(I))
60      WRITE (6,31)
61      31 FORMAT (12X,3H XC,18X,3H YQ,18X,3H XC,18X,3H YC,/)
62      DO 33 I=1,5
63      33 WRITE (6,32) XQ(I),YQ(I),XC(I),YC(I)
64      32 FORMAT (6X,F12.8,9X,F12.8,9X,F12.8,9X,F12.8,/)
65      DO 40 I=1,4
66      40 H(I)=XC(I+1)-XC(1)*COS(AL1(I+1))+YC(1)*SIN(AL1(I+1))
67      DO 50 I=1,4
68      50 H(I+4)=YC(I+1)-YC(1)*COS(AL1(I+1))-XC(1)*SIN(AL1(I+1))
69      DO 60 I=1,4
70      60 H(I+8)=-H(I)*COS(AL1(I+1))-H(I+4)*SIN(AL1(I+1))
71      DO 70 I=1,4
72      70 H(I+12)=-H(I+4)*COS(AL1(I+1))+H(I)*SIN(AL1(I+1))
73      DO 72 I=1,16
74      H1(I)=H(I)
75      DO 74 I=1,16
76      74 H2(I)=H(I)
77      DO 80 I=1,4
78      80 R(I)=0.5*(H(I)*H(I)+H(I+4)*H(I+4))
79      DO 90 I=1,4
80      90 P(I)=1.0-COS(AL1(I+1))
81      DO 100 I=1,4
82      100 Q(I)=SIN(AL1(I+1))
83      CALL SIMQ(H,R,4,KS)
84      CALL SIMQ(H1,P,4,KS1)
85      CALL SIMQ(H2,Q,4,KS2)
86      WRITE (6,99) KS
87      WRITE (6,99) KS1
88      WRITE (6,99) KS2
89      99 FORMAT (1X,I1)
90      F1=Q(1)*Q(3)+Q(2)*Q(4)
91      F2=P(1)*Q(3)+Q(1)*P(3)+P(2)*Q(4)+Q(2)*P(4)
92      F3=R(1)*Q(3)+Q(1)*R(3)+Q(2)*R(4)+Q(4)*R(2)
93      F4=P(1)*P(3)+P(2)*P(4)
94      F5=P(1)*R(3)+P(3)*R(1)+P(2)*R(4)+P(4)*R(2)-1.0
95      F6=R(1)*R(3)+R(2)*R(4)
96      G1=Q(1)*Q(4)-Q(2)*Q(3)
97      G2=P(1)*Q(4)+Q(1)*P(4)-P(2)*Q(3)-Q(2)*P(3)
98      G3=R(1)*Q(4)+Q(1)*R(4)-R(2)*Q(3)-Q(2)*R(3)-1.0
99      G4=P(1)*P(4)-P(2)*P(3)
100     G5=P(1)*R(4)+R(1)*P(4)-P(2)*R(3)-P(3)*R(2)
101     G6=R(1)*R(4)-R(2)*R(3)
102     XCDF(1)=F1*F1*G6*G6+G3*G3*F1*F6-G3*G6*F1*F3+G1*G6*F3*F3+G1*G1*F6*F6
103     $6-G1*G3*F3*F6-2.0*G1*G6*F1*F6
104     XCDF(2)=2.0*G5*G6*F1*F1+2.0*G2*G3*F1*F6+G3*G3*F1*F5-G3*G6*F1*F2-G2
105     $*G6*F1*F3-G3*G5*F1*F3+2.0*G1*G6*F2*F3+G1*G5*F3*F3+2.0*G1*G1*F5*F6-
106     $G1*G3*F2*F6-G1*G2*F3*F6-G1*G3*F3*F5-2.0*G1*G5*F1*F6-2.0*G1*G6*F1*F
107     $5
108     XCDF(3)=2.0*G4*G6*F1*F1+G5*G5*F1*F1+G2*G2*F1*F6+2.0*G2*G3*F1*F5+G3
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109      $*G3*F1*F4-G2*G6*F1*F2-G3*G5*F1*F2-G2*G5*F1*F3-G3*G4*F1*F3+G1*G6*F2
110      $*F2+2.0*G1*G5*F2*F3+G1*G4*F3*F3+2.0*G1*G1*F4*F6+G1*G1*F5*F5-G1*G2*
111      $F2*F6-G1*G3*F2*F5-G1*G2*F3*F5-G1*G3*F3*F4-2.0*G1*G6*F1*F4-2.0*G1*G
112      $5*F1*F5-2.0*G1*G4*F1*F6
113      XCOF(4)=2.0*G4*G5*F1*F1+G2*G2*F1*F5+2.0*G2*G3*F1*F4-G2*G5*F1*F2-G3
114      $*G4*F1*F2-G2*G4*F1*F3+G1*G5*F2*F2+2.0*G1*G4*F2*F3+2.0*G1*G1*F4*F5-
115      $G1*G2*F2*F5-G1*G3*F2*F4-G1*G2*F3*F4-2.0*G1*G5*F1*F4-2.0*G1*G4*F1*F
116      $5
117      XCOF(5)=G4*G4*F1*F1+G2*G2*F1*F4-G2*G4*F1*F2+G1*G4*F2*F2+G1*G1*F4*F
118      $4-G1*G2*F2*F4-2.0*G1*G4*F1*F4
119      CALL POLRT(XCOF,COF,4,ROOTR,ROOTI,IER)
120      WRITE (6,109)
121      109 FORMAT (/ ,27X,9H L1 RGOTS,/)
122      DO 110 I=1,4
123      110 WRITE (6,112) RCCTR(I),ROOTI(I)
124      112 FORMAT (17X,E13.5,10X,E13.5,/)
125      WRITE (6,114) IER
126      114 FORMAT (1X,I1)
127      WRITE (6,106)
128      106 FORMAT (1H1,/)
129      125 DO 215 I=1,4
130      OA=F1
131      OB=F2*ROOTR(I)+F3
132      OC=F4*ROOTR(I)*ROOTR(I)+F5*ROOTR(I)+F6
133      OD=G1
134      OE=G2*ROOTR(I)+G3
135      OF=G4*ROOTR(I)*ROOTR(I)+G5*ROOTR(I)+G6
136      IF (ABS(ROOTI(I)).LT.0.000001) ROOTI(I)=0.0
137      IF (ROOTI(I)) 2CC,13C,200
138      200 WRITE (6,201)
139      201 FORMAT (5X,18H RGOT IS IMAGINARY,/)
140      GO TO 215
141      130 SA=OB*OB-4.0*OA*OC
142      IF (SA) 200,140,140
143      140 ROOTA1=(-OB+SQRT(SA))/(2.0*OA)
144      ROOTA2=(-OB-SQRT(SA))/(2.0*OA)
145      SB=OE*OE-4.0*OD*OF
146      IF (SB) 200,150,150
147      150 ROOTB1=(-OE+SQRT(SB))/(2.0*OD)
148      ROOTB2=(-OE-SQRT(SB))/(2.0*OD)
149      WRITE (6,155) RCCTA1,ROOTA2,ROOTB1,ROOTB2
150      155 FORMAT (/ ,13X,8H ROOTA1=,F18.6,5X,8H ROCTA2=,F18.6,/,13X,8H ROOTB1
151      $=,F18.6,5X,8H ROOTB2=,F18.6,/)
152      IF (ABS(ROOTA1-RCCTB1).LT.0.05) GO TO 160
153      IF (ABS(ROOTA1-ROOTB2).LT.0.05) GO TO 160
154      IF (ABS(ROOTA2-ROOTB1).LT.0.05) GO TO 170
155      IF (ABS(ROOTA2-RCOTB2).LT.0.05) GO TO 170
156      GO TO 215
157      160 WRITE (6,180) ROOTA1
158      180 FORMAT (2X,16H SECCND ROOT IS ,F20.5,/)
159      ROOT=ROOTA1
160      GO TO 210
161      170 WRITE (6,180) RCCTA2
162      ROOT=ROOTA2
```

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CARD
163 210 WRITE (6,212)
164 212 FORMAT (14X,13H CENTER POINT,31X,13H CIRCLE POINT,/,8X,3H XA,19X,3
165 $H YA,19X,3H XB,15X,3H YB,/)
166 XA(I)=R(1)+P(1)*RODTR(I)+Q(1)*ROOT
167 YA(I)=R(2)+P(2)*RODTR(I)+Q(2)*ROOT
168 XB(I)=R(3)+P(3)*RODTR(I)+Q(3)*ROOT
169 YB(I)=R(4)+P(4)*RODTR(I)+Q(4)*ROOT
170 WRITE (6,230) XA(I),YA(I),XB(I),YB(I)
171 230 FORMAT (4F20.8,/,33X,20H XXXXXXXXXXXXXXXXXXXX,/)
172 215 CONTINUE
173 STOP
174 END
```

APPENDIX B

SYNTHESIS PROGRAM FOR FOUR FINITE POSITIONS  
AND ONE FIRST INFINITESIMAL  
DISPLACEMENT

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CARD
1 C *****
2 C *
3 C *           SYNTHESIS OF SIX-LINK STEPHENSON 2           *
4 C *           FUNCTION GENERATOR                           *
5 C *
6 C *           FOUR FINITE POSITIONS AND ONE VELOCITY RATIO *
7 C *
8 C *
9 C *           INPUT FINITE ROTATIONS           TH1(I)      *
10 C *
11 C *          OUTPLT FINITE ROTATICNS          PH1(I)      *
12 C *
13 C *          ASSUMED ROTATIONS OF TERNARY     AL1(I)      *
14 C *
15 C *          FIRST INFINITESIMAL DISPLACEMENT DPHDTH(J) *
16 C *
17 C *          OUTPLT LINK LENGTH              QC           *
18 C *
19 C *          OUTPUT LINK INITIAL ANGLE        THE         *
20 C *
21 C *****
22 C
23     DIMENSION XCOF(5),COF(5),ROQTR(4),ROOTI(4)
24     DIMENSION XQ(5),YQ(5),XA(5),YA(5),XB(5),YB(5),XC(5),YC(5)
25     DIMENSION TH1(5),PH1(5),AL1(5)
26     DIMENSION H(16),H1(16),H2(16),R(4),P(4),Q(4)
27     DIMENSION DXC(5),DYC(5),PXC(5),PYC(5)
28     DIMENSION DPHDTH(5),DTHDAL(5)
29     READ (5,5) J
30     5 FORMAT (I1)
31     WRITE (6,6) J
32     6 FORMAT (//,16X,28H VELOCITY RATIO IS SPECIFIED AT POINT ,I1,/)
33     READ (5,10) TH1(2),TH1(3),TH1(4)
34     READ (5,10) PH1(2),PH1(3),PH1(4)
35     READ (5,10) AL1(2),AL1(3),AL1(4)
36     10 FORMAT (3F10.0)
37     READ (5,20) QC,THE
38     READ (5,20) DPHDTH(J),DTHDAL(J)
39     20 FORMAT (2F10.0)
40     WRITE (6,11) TH1(2),TH1(3),TH1(4)
41     11 FORMAT (9X,26H INPUT ROTATION ANGLES ARE,2X,3F10.3,/)
42     WRITE (6,12) PH1(2),PH1(3),PH1(4)
43     12 FORMAT (9X,27H OUTPUT ROTATION ANGLES ARE,1X,3F10.3,/)
44     WRITE (6,13) AL1(2),AL1(3),AL1(4)
45     13 FORMAT (9X,23H TERNARY LINK ROTATIONS,5X,3F10.3,/)
46     WRITE (6,14) QC,THE
47     14 FORMAT (9X,15H CLTPUT LINK IS,F10.3,24H LONG AND AT AN ANGLE OF,F1
48     $0.3,/)
49     WRITE (6,15) DPHDTH(J),DTHDAL(J)
50     15 FORMAT (9X,11H DPHDTH(J)=,F10.3,10X,11H DTHDAL(J)=,F10.3,/)
51     XQ(1)=1.0
52     YQ(1)=0.0
53     XM=0.0
54     YM=0.0

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55 TH1(1)=0.0
56 PH1(1)=0.0
57 AL1(1)=0.0
58 THE=(THE*3.14159265)/180.0
59 DO 21 I=2,4
60 TH1(I)=(TH1(I)*3.14159265)/180.0
61 PH1(I)=(PH1(I)*3.14159265)/180.0
62 21 AL1(I)=(AL1(I)*3.14159265)/180.0
63 XC(1)=1.0+QC*COS( THE )
64 YC(1)=0.0+QC*SIN( THE )
65 DO 30 I=2,4
66 XQ(I)=1.0*COS(-TH1(I))
67 YQ(I)=1.0*SIN(-TH1(I))
68 XC(I)=XC(1)*COS(PH1(I)-TH1(I))-YC(1)*SIN(PH1(I)-TH1(I))+XQ(I)-XQ(1)
69 $)*COS(PH1(I)-TH1(I))+YQ(1)*SIN(PH1(I)-TH1(I))
70 30 YC(I)=XC(1)*SIN(PH1(I)-TH1(I))+YC(1)*COS(PH1(I)-TH1(I))+YQ(I)-XQ(1)
71 $)*SIN(PH1(I)-TH1(I))-YQ(1)*COS(PH1(I)-TH1(I))
72 DXC(J)=(XC(1)-XQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(I)-TH1(I)))*DPHDT
73 $ (J))+YC(1)-YQ(1))*(COS(PH1(J)-TH1(J))-COS(PH1(I)-TH1(I)))*DPHDT
74 $ (J))+XQ(1)*SIN(-TH1(J))+YQ(1)*COS(-TH1(J))
75 DYC(J)=(XC(1)-XQ(1))*(-COS(PH1(J)-TH1(J))+COS(PH1(I)-TH1(I)))*DPHDT
76 $ (J))+YC(1)-YQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(I)-TH1(I)))*DPHDT
77 $ (J))-XQ(1)*COS(-TH1(J))+YQ(1)*SIN(-TH1(J))
78 WRITE (6,17) DXC(J),DYC(J)
79 17 FORMAT (9X,8H DXC(J)=,3X,F10.3,10X,8H DYC(J)=,3X,F10.3,///)
80 PXC(J)=DTHCAL(J)*DXC(J)
81 PYC(J)=DTHCAL(J)*DYC(J)
82 WRITE (6,31)
83 31 FORMAT (11X,3H XQ,18X,3H YQ,18X,3H XC,18X,3H YC,/)
84 DO 33 I=1,4
85 33 WRITE (6,32) XQ(I),YQ(I),XC(I),YC(I)
86 32 FORMAT (6X,F12.8,9X,F12.8,9X,F12.8,9X,F12.8,/)
87 DO 40 I=1,3
88 40 H(I)=XC(I+1)-XC(1)*COS(AL1(I+1))+YC(1)*SIN(AL1(I+1))
89 H(4)=PXC(J)+XC(1)*SIN(AL1(J))+YC(1)*COS(AL1(J))
90 DO 50 I=1,3
91 50 H(I+4)=YC(I+1)-YC(1)*COS(AL1(I+1))-XC(1)*SIN(AL1(I+1))
92 H(8)=PYC(J)-XC(1)*COS(AL1(J))+YC(1)*SIN(AL1(J))
93 DO 60 I=1,3
94 60 H(I+8)=-H(I)*COS(AL1(I+1))-H(I+4)*SIN(AL1(I+1))
95 H(12)=-PXC(J)*COS(AL1(J))-PYC(J)*SIN(AL1(J))+XC(J)*SIN(AL1(J))-YC(
96 $)*COS(AL1(J))
97 DO 70 I=1,3
98 70 H(I+12)=-H(I+4)*COS(AL1(I+1))+H(I)*SIN(AL1(I+1))
99 H(16)=PXC(J)*SIN(AL1(J))-PYC(J)*COS(AL1(J))+XC(J)*COS(AL1(J))+YC(J)
100 $)*SIN(AL1(J))
101 DO 72 I=1,16
102 72 H1(I)=H(I)
103 DO 74 I=1,16
104 74 H2(I)=H(I)
105 DO 80 I=1,3
106 80 R(I)=0.5*(H(I)+H(I+4))+H(I+4)*H(I+4))
107 R(4)=(YC(1)*XC(J)-XC(1)*YC(J)-YC(1)*PYC(J)-XC(1)*PXC(J))*COS(AL1(J)
108 $))+XC(1)*XC(J)+YC(1)*YC(J)-XC(1)*PYC(J)+YC(1)*PXC(J))*SIN(AL1(J))
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CARD
109      $+XC(J)*PXC(J)+YC(J)*PYC(J)
110      DO 90 I=1,3
111      90 P(I)=1.0-COS(AL1(I+1))
112      P(4)=SIN(AL1(J))
113      DO 100 I=1,3
114      100 Q(I)=SIN(AL1(I+1))
115      Q(4)=COS(AL1(J))
116      CALL SIMQ(H,R,4,KS)
117      CALL SIMQ(H1,P,4,KS1)
118      CALL SIMQ(H2,Q,4,KS2)
119      WRITE (6,99) KS
120      WRITE (6,99) KS1
121      WRITE (6,99) KS2
122      99 FORMAT (1X,I1)
123      F1=Q(1)*Q(3)+Q(2)*Q(4)
124      F2=P(1)*Q(3)+Q(1)*P(3)+P(2)*Q(4)+Q(2)*P(4)
125      F3=R(1)*Q(3)+Q(1)*R(3)+Q(2)*R(4)+Q(4)*R(2)
126      F4=P(1)*P(3)+P(2)*P(4)
127      F5=P(1)*R(3)+P(3)*R(1)+P(2)*R(4)+P(4)*R(2)-1.0
128      F6=R(1)*R(3)+R(2)*R(4)
129      G1=Q(1)*Q(4)-Q(2)*Q(3)
130      G2=P(1)*Q(4)+Q(1)*P(4)-P(2)*Q(3)-Q(2)*P(3)
131      G3=R(1)*Q(4)+Q(1)*R(4)-R(2)*Q(3)-Q(2)*R(3)-1.0
132      G4=P(1)*P(4)-P(2)*P(3)
133      G5=P(1)*R(4)+R(1)*P(4)-P(2)*R(3)-P(3)*R(2)
134      G6=R(1)*R(4)-R(2)*R(3)
135      XCOF(1)=F1*F1*G6*G6+G3*G3*F1*F6-G3*G6*F1*F3+G1*G6*F3*F3+G1*G1*F6*F6
136      $6-G1*G3*F3*F6-2.0*G1*G6*F1*F6
137      XCOF(2)=2.0*G5*G6*F1*F1+2.0*G2*G3*F1*F6+G3*G3*F1*F5-G3*G6*F1*F2-G2
138      $*G6*F1*F3-G3*G5*F1*F3+2.0*G1*G6*F2*F3+G1*G5*F3*F3+2.0*G1*G1*F5*F6-
139      $G1*G3*F2*F6-G1*G2*F3*F6-G1*G3*F3*F5-2.0*G1*G5*F1*F6-2.0*G1*G6*F1*F
140      $5
141      XCOF(3)=2.0*G4*G6*F1*F1+G5*G5*F1*F1+G2*G2*F1*F6+2.0*G2*G3*F1*F5+G3
142      $*G3*F1*F4-G2*G6*F1*F2-G3*G5*F1*F2-G2*G5*F1*F3-G3*G4*F1*F3+G1*G6*F2
143      $*F2+2.0*G1*G5*F2*F3+G1*G4*F3*F3+2.0*G1*G1*F4*F6+G1*G1*F5*F5-G1*G2*
144      $F2*F6-G1*G3*F2*F5-G1*G2*F3*F5-G1*G3*F3*F4-2.0*G1*G6*F1*F4-2.0*G1*G
145      $5*F1*F5-2.0*G1*G4*F1*F6
146      XCOF(4)=2.0*G4*G5*F1*F1+G2*G2*F1*F5+2.0*G2*G3*F1*F4-G2*G5*F1*F2-G3
147      $*G4*F1*F2-G2*G4*F1*F3+G1*G5*F2*F2+2.0*G1*G4*F2*F3+2.0*G1*G1*F4*F5-
148      $G1*G2*F2*F5-G1*G3*F2*F4-G1*G2*F3*F4-2.0*G1*G5*F1*F4-2.0*G1*G4*F1*F
149      $5
150      XCOF(5)=G4*G4*F1*F1+G2*G2*F1*F4-G2*G4*F1*F2+G1*G4*F2*F2+G1*G1*F4*F
151      $4-G1*G2*F2*F4-2.0*G1*G4*F1*F4
152      CALL POLRT(XCOF,COF,4,ROOTR,ROOTI,IER)
153      WRITE (6,109)
154      109 FORMAT (/ ,27X,9H L1 ROOTS,/)
155      DO 110 I=1,4
156      110 WRITE (6,112) RCCTR(I),ROOTI(I)
157      112 FORMAT (17X,E13.5,10X,E13.5,/)
158      WRITE (6,114) IER
159      114 FORMAT (1X,I1)
160      WRITE (6,106)
161      106 FORMAT (1H1,/)
162      125 DO 215 I=1,4
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CARD
163      OA=F1
164      OB=F2*ROOTR(I)+F3
165      OC=F4*RCOTR(I)*RCOTR(I)+F5*ROOTR(I)+F6
166      OD=G1
167      OE=G2*ROOTR(I)+G3
168      OF=G4*ROOTR(I)*RCOTR(I)+G5*ROOTR(I)+G6
169      IF (ABS(ROOTI(I)).LT.0.000001) ROOTI(I)=0.0
170      IF (ROOTI(I)) 200,130,200
171      200 WRITE (6,201)
172      201 FORMAT (5X,18H ROOT IS IMAGINARY,/)
173      GO TO 215
174      130 SA=OB*OB-4.0*OA*OC
175      IF (SA) 200,140,140
176      140 ROOTA1=(-OB+SQR(T(SA)))/(2.0*OA)
177      ROOTA2=(-OB-SQR(T(SA)))/(2.0*OA)
178      SB=OE*OE-4.0*OD*OF
179      IF (SB) 200,150,150
180      150 ROOTB1=(-OE+SQR(T(SB)))/(2.0*OD)
181      ROOTB2=(-OE-SQR(T(SB)))/(2.0*OD)
182      WRITE (6,155) RCCTA1,ROOTA2,ROOTB1,ROOTB2
183      155 FORMAT (/ ,13X,8H ROOTA1=,F18.6,5X,8H ROOTA2=,F18.6,/,13X,8H ROOTB1
184      $=,F18.6,5X,8H ROOTB2=,F18.6,/)
185      IF (ABS(ROOTA1-RCCTB1).LT.0.05) GO TO 160
186      IF (ABS(ROOTA1-ROOTB2).LT.0.05) GO TO 160
187      IF (ABS(ROOTA2-RCCTB1).LT.0.05) GO TO 170
188      IF (ABS(ROOTA2-ROOTB2).LT.0.05) GO TO 170
189      GO TO 215
190      160 WRITE (6,180) RCCTA1
191      180 FORMAT (2X,16H SECOND ROOT IS ,F20.5,/)
192      ROOT=ROOTA1
193      GO TO 210
194      170 WRITE (6,180) ROOTA2
195      ROOT=ROOTA2
196      210 WRITE (6,212)
197      212 FORMAT (14X,13H CENTER POINT,31X,13H CIRCLE POINT,/,8X,3H XA,19X,3
198      $H YA,19X,3H XB,19X,3H YB,/)
199      XA(I)=R(1)+P(1)*ROOTR(I)+Q(1)*ROOT
200      YA(I)=R(2)+P(2)*ROOTR(I)+Q(2)*ROOT
201      XB(I)=R(3)+P(3)*ROOTR(I)+Q(3)*ROOT
202      YB(I)=R(4)+P(4)*ROOTR(I)+Q(4)*ROOT
203      WRITE (6,230) XA(I),YA(I),XB(I),YB(I)
204      230 FORMAT (4F20.8,/,33X,20H XXXXXXXXXXXXXXXXXXXX,/)
205      215 CONTINUE
206      STOP
207      END
```



APPENDIX C

SYNTHESIS PROGRAM FOR THREE FINITE POSITIONS  
AND TWO FIRST INFINITESIMAL  
DISPLACEMENTS

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CARD
1 C *****
2 C *
3 C *           SYNTHESIS OF SIX-LINK STEPHENSON 2           *
4 C *           FUNCTION GENERATOR                           *
5 C *
6 C *           THREE FINITE POSITIONS AND TWO VELOCITY RATIOS *
7 C *
8 C *
9 C *           INPUT FINITE ROTATIONS           TH1(I)       *
10 C *
11 C *          OUTPUT FINITE ROTATIONS          PH1(I)       *
12 C *
13 C *          ASSUMED ROTATIONS OF TERNARY     AL1(I)       *
14 C *
15 C *          FIRST INFINITESIMAL DISPLACEMENT DPHDTH(J)  *
16 C *
17 C *          OUTPUT LINK LENGTH                QC            *
18 C *
19 C *          OUTPUT LINK INITIAL ANGLE         THE           *
20 C *
21 C *****
22 C
23   DIMENSION XCOF(5),COF(5),ROOTR(4),ROOTI(4)
24   DIMENSION XQ(5),YQ(5),XA(5),YA(5),XB(5),YB(5),XC(5),YC(5)
25   DIMENSION TH1(5),PH1(5),AL1(5)
26   DIMENSION H(16),H1(16),H2(16),R(4),P(4),Q(4)
27   DIMENSION DXC(5),DYC(5),PXC(5),PYC(5)
28   DIMENSION DPHDTH(5),DTHDAL(5)
29   READ (5,5) J,K
30   5 FORMAT (I1,I1)
31   WRITE (6,6) J,K
32   6 FORMAT (//,16X,34H VELOCITY RATIOS ARE AT POSITIONS ,I1,5H AND ,I1
33   $,/)
34   READ (5,10) TH1(2),TH1(3)
35   READ (5,10) PH1(2),PH1(3)
36   READ (5,10) AL1(2),AL1(3)
37   10 FORMAT (2F10.0)
38   READ (5,20) QC,THE
39   READ (5,20) DPHDTH(J),DTHDAL(J)
40   READ (5,20) DPHDTH(K),DTHDAL(K)
41   20 FORMAT (2F10.0)
42   WRITE (6,11) TH1(2),TH1(3)
43   11 FORMAT (9X,26H INPUT ROTATION ANGLES ARE,2X,2F10.3,/)
44   WRITE (6,12) PH1(2),PH1(3)
45   12 FORMAT (9X,27H OUTPUT ROTATION ANGLES ARE,1X,2F10.3,/)
46   WRITE (6,13) AL1(2),AL1(3)
47   13 FORMAT (9X,23H TERNARY LINK ROTATIONS,5X,2F10.3,/)
48   WRITE (6,14) QC,THE
49   14 FORMAT (9X,15H OUTPUT LINK IS,F10.3,24H LONG AND AT AN ANGLE OF,F1
50   $0.3,/)
51   WRITE (6,15) DPHDTH(J),DTHDAL(J)
52   15 FORMAT (9X,11H DPHDTH(J)=,F10.3,10X,11H DTHDAL(J)=,F10.3,/)
53   WRITE (6,16) DPHDTH(K),DTHDAL(K)
54   16 FORMAT (9X,11H DPHDTH(K)=,F10.3,10X,11H DTHDAL(K)=,F10.3,/)

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CARD
55      XQ(1)=1.0
56      YQ(1)=0.0
57      XM=0.0
58      YM=0.0
59      TH1(1)=0.0
60      PH1(1)=0.0
61      AL1(1)=0.0
62      THE=(THE*3.14159265)/180.0
63      DO 21 I=2,3
64      TH1(I)=(TH1(I)*3.14159265)/180.0
65      PH1(I)=(PH1(I)*3.14159265)/180.0
66      21 AL1(I)=(AL1(I)*3.14159265)/180.0
67      XC(1)=1.0+QC*COS(THE)
68      YC(1)=0.0+QC*SIN(THE)
69      DO 30 I=2,3
70      XQ(I)=1.0*CCS(-TH1(I))
71      YQ(I)=1.0*SIN(-TH1(I))
72      XC(I)=XC(1)*COS(PH1(I)-TH1(I))-YC(1)*SIN(PH1(I)-TH1(I))+XQ(I)-XQ(I
73      $)*COS(PH1(I)-TH1(I))+YQ(1)*SIN(PH1(I)-TH1(I))
74      30 YC(I)=XC(1)*SIN(PH1(I)-TH1(I))+YC(1)*COS(PH1(I)-TH1(I))+YQ(I)-XQ(I
75      $)*SIN(PH1(I)-TH1(I))-YQ(1)*COS(PH1(I)-TH1(I))
76      DXC(J)=(XC(1)-XQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(J)-TH1(J))*DPHDT
77      $(J))+YC(1)-YQ(1))*(COS(PH1(J)-TH1(J))-COS(PH1(J)-TH1(J))*DPHDT
78      $(J))+XQ(1)*SIN(-TH1(J))+YQ(1)*COS(-TH1(J))
79      DYC(J)=(XC(1)-XQ(1))*(-COS(PH1(J)-TH1(J))+COS(PH1(J)-TH1(J))*DPHDT
80      $(J))+YC(1)-YQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(J)-TH1(J))*DPHDT
81      $(J))-XQ(1)*COS(-TH1(J))+YQ(1)*SIN(-TH1(J))
82      DXC(K)=(XC(1)-XQ(1))*(SIN(PH1(K)-TH1(K))-SIN(PH1(K)-TH1(K))*DPHDT
83      $(K))+YC(1)-YQ(1))*(COS(PH1(K)-TH1(K))-COS(PH1(K)-TH1(K))*DPHDT
84      $(K))+XQ(1)*SIN(-TH1(K))+YQ(1)*COS(-TH1(K))
85      DYC(K)=(XC(1)-XQ(1))*(-COS(PH1(K)-TH1(K))+COS(PH1(K)-TH1(K))*DPHDT
86      $(K))+YC(1)-YQ(1))*(SIN(PH1(K)-TH1(K))-SIN(PH1(K)-TH1(K))*DPHDT
87      $(K))-XQ(1)*COS(-TH1(K))+YQ(1)*SIN(-TH1(K))
88      WRITE (6,17) DXC(J),DYC(J)
89      17 FORMAT (9X,8H DXC(J)=,3X,F10.3,10X,8H DYC(J)=,3X,F10.3,/)
90      WRITE (6,18) DXC(K),DYC(K)
91      18 FORMAT (9X,8H DXC(K)=,3X,F10.3,10X,8H DYC(K)=,3X,F10.3,/)
92      PXC(J)=DTHDAL(J)*DXC(J)
93      PYC(J)=DTHCAL(J)*DYC(J)
94      PXC(K)=DTHCAL(K)*DXC(K)
95      PYC(K)=DTHDAL(K)*DYC(K)
96      WRITE (6,31)
97      31 FORMAT (11X,3H XQ,18X,3H YQ,18X,3H XC,18X,3H YC,/)
98      DO 33 I=1,3
99      33 WRITE (6,32) XQ(I),YQ(I),XC(I),YC(I)
100     32 FORMAT (6X,F12.8,9X,F12.8,9X,F12.8,9X,F12.8,/)
101     DO 40 I=1,2
102     40 H(I)=XC(I+1)-XC(1)*COS(AL1(I+1))+YC(1)*SIN(AL1(I+1))
103     H(3)=PXC(J)+XC(1)*SIN(AL1(J))+YC(1)*COS(AL1(J))
104     H(4)=PXC(K)+XC(1)*SIN(AL1(K))+YC(1)*COS(AL1(K))
105     DO 50 I=1,2
106     50 H(I+4)=YC(I+1)-YC(1)*COS(AL1(I+1))-XC(1)*SIN(AL1(I+1))
107     H(7)=PYC(J)-XC(1)*COS(AL1(J))+YC(1)*SIN(AL1(J))
108     H(8)=PYC(K)-XC(1)*COS(AL1(K))+YC(1)*SIN(AL1(K))

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CARD
109      DO 60 I=1,2
110      60 H(I+8)=-H(I)*COS(AL1(I+1))-H(I+4)*SIN(AL1(I+1))
111      H(11)=-PXC(J)*CCS(AL1(J))-PYC(J)*SIN(AL1(J))+XC(J)*SIN(AL1(J))-YC(
112      $J)*COS(AL1(J))
113      H(12)=-PXC(K)*COS(AL1(K))-PYC(K)*SIN(AL1(K))+XC(K)*SIN(AL1(K))-YC(
114      $K)*COS(AL1(K))
115      DO 70 I=1,2
116      70 H(I+12)=-H(I+4)*COS(AL1(I+1))+H(I)*SIN(AL1(I+1))
117      H(15)=PXC(J)*SIN(AL1(J))-PYC(J)*COS(AL1(J))+XC(J)*COS(AL1(J))+YC(J
118      $)*SIN(AL1(J))
119      H(16)=PXC(K)*SIN(AL1(K))-PYC(K)*COS(AL1(K))+XC(K)*COS(AL1(K))+YC(K
120      $)*SIN(AL1(K))
121      DO 72 I=1,16
122      72 H1(I)=H(I)
123      DO 74 I=1,16
124      74 H2(I)=H(I)
125      DO 80 I=1,2
126      80 R(1)=0.5*(H(I)*H(I)+H(I+4)*H(I+4))
127      R(3)=(YC(1)*XC(J)-XC(1)*YC(J)-YC(1)*PYC(J)-XC(1)*PXC(J))*COS(AL1(J
128      $))+ (XC(1)*XC(J)+YC(1)*YC(J)-XC(1)*PYC(J)+YC(1)*PXC(J))*SIN(AL1(J))
129      $+XC(J)*PXC(J)+YC(J)*PYC(J)
130      R(4)=(YC(1)*XC(K)-XC(1)*YC(K)-YC(1)*PYC(K)-XC(1)*PXC(K))*COS(AL1(K
131      $))+ (XC(1)*XC(K)+YC(1)*YC(K)-XC(1)*PYC(K)+YC(1)*PXC(K))*SIN(AL1(K))
132      $+XC(K)*PXC(K)+YC(K)*PYC(K)
133      DO 90 I=1,2
134      90 P(I)=1.0-COS(AL1(I+1))
135      P(3)=SIN(AL1(J))
136      P(4)=SIN(AL1(K))
137      DO 100 I=1,2
138      100 Q(I)=SIN(AL1(I+1))
139      Q(3)=COS(AL1(J))
140      Q(4)=COS(AL1(K))
141      CALL SIMQ(H,R,4,KS)
142      CALL SIMQ(H1,P,4,KS1)
143      CALL SIMQ(H2,Q,4,KS2)
144      WRITE (6,99) KS
145      WRITE (6,99) KS1
146      WRITE (6,99) KS2
147      99 FORMAT (1X,11)
148      F1=Q(1)*Q(3)+Q(2)*Q(4)
149      F2=P(1)*Q(3)+Q(1)*P(3)+P(2)*Q(4)+Q(2)*P(4)
150      F3=R(1)*Q(3)+C(1)*R(3)+Q(2)*R(4)+Q(4)*R(2)
151      F4=P(1)*P(3)+P(2)*P(4)
152      F5=P(1)*R(3)+P(3)*R(1)+P(2)*R(4)+P(4)*R(2)-1.0
153      F6=R(1)*R(3)+R(2)*R(4)
154      G1=Q(1)*Q(4)-Q(2)*Q(3)
155      G2=P(1)*Q(4)+Q(1)*P(4)-P(2)*Q(3)-Q(2)*P(3)
156      G3=R(1)*Q(4)+Q(1)*R(4)-R(2)*Q(3)-Q(2)*R(3)-1.0
157      G4=P(1)*P(4)-P(2)*P(3)
158      G5=P(1)*R(4)+R(1)*P(4)-P(2)*R(3)-P(3)*R(2)
159      G6=R(1)*R(4)-R(2)*R(3)
160      XCDF(1)=F1*F1*G6*G6+G3*G3*F1*F6-G3*G6*F1*F3+G1*G6*F3*F3+G1*G1*F6*F6
161      $6-G1*G3*F3*F6-2.0*G1*G6*F1*F6
162      XCDF(2)=2.0*G5*G6*F1*F1+2.0*G2*G3*F1*F6+G3*G3*F1*F5-G3*G6*F1*F2-G2

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CARD
163      $*G6*F1*F3-G3*G5*F1*F3+2.0*G1*G6*F2*F3+G1*G5*F3*F3+2.0*G1*G1*F5*F6-
164      $G1*G3*F2*F6-G1*G2*F3*F6-G1*G3*F3*F5-2.0*G1*G5*F1*F6-2.0*G1*G6*F1*F
165      $5
166      XCOF(3)=2.0*G4*G6*F1*F1+G5*G5*F1*F1+G2*G2*F1*F6+2.0*G2*G3*F1*F5+G3
167      $*G3*F1*F4-G2*G6*F1*F2-G3*G5*F1*F2-G2*G5*F1*F3-G3*G4*F1*F3+G1*G6*F2
168      $*F2+2.0*G1*G5*F2*F3+G1*G4*F3*F3+2.0*G1*G1*F4*F6+G1*G1*F5*F5-G1*G2*
169      $*F2*F6-G1*G3*F2*F5-G1*G2*F3*F5-G1*G3*F3*F4-2.0*G1*G6*F1*F4-2.0*G1*G
170      $*F1*F5-2.0*G1*G4*F1*F6
171      XCOF(4)=2.0*G4*G5*F1*F1+G2*G2*F1*F5+2.0*G2*G3*F1*F4-G2*G5*F1*F2-G3
172      $*G4*F1*F2-G2*G4*F1*F3+G1*G5*F2*F2+2.0*G1*G4*F2*F3+2.0*G1*G1*F4*F5-
173      $G1*G2*F2*F5-G1*G3*F2*F4-G1*G2*F3*F4-2.0*G1*G5*F1*F4-2.0*G1*G4*F1*F
174      $5
175      XCOF(5)=G4*G4*F1*F1+G2*G2*F1*F4-G2*G4*F1*F2+G1*G4*F2*F2+G1*G1*F4*F
176      $4-G1*G2*F2*F4-2.0*G1*G4*F1*F4
177      CALL POLRT(XCOF,COF,4,ROOTR,ROOTI,IER)
178      WRITE (6,109)
179      109 FORMAT (/ ,27X,9H LI RCOTS,/)
180      DO 110 I=1,4
181      110 WRITE (6,112) RCCTR(I),ROOTI(I)
182      112 FORMAT (17X,E13.5,10X,E13.5,/)
183      WRITE (6,114) IER
184      114 FORMAT (1X,I1)
185      WRITE (6,106)
186      106 FORMAT (1H1,/)
187      125 DO 215 I=1,4
188      DA=F1
189      DB=F2*ROOTR(I)+F3
190      DC=F4*ROOTR(I)*RCCTR(I)+F5*ROOTR(I)+F6
191      DD=G1
192      DE=G2*ROOTR(I)+G3
193      OF=G4*ROOTR(I)*ROOTR(I)+G5*ROOTR(I)+G6
194      IF (ABS(ROOTI(I)).LT.0.000001) ROOTI(I)=0.0
195      IF (ROOTI(I)) 200,130,200
196      200 WRITE (6,201)
197      201 FORMAT (5X,18H ROOT IS IMAGINARY,/)
198      GO TO 215
199      130 SA=DB*DB-4.0*DA*DC
200      IF (SA) 200,140,140
201      140 ROOTA1=(-DB+SQRT(SA))/(2.0*DA)
202      ROOTA2=(-DB-SQRT(SA))/(2.0*DA)
203      SB=DE*DE-4.0*DD*CF
204      IF (SB) 200,150,150
205      150 ROOTB1=(-DE+SQRT(SB))/(2.0*DD)
206      ROOTB2=(-DE-SQRT(SB))/(2.0*DD)
207      WRITE (6,155) ROOTA1,ROOTA2,ROOTB1,ROOTB2
208      155 FORMAT (/ ,13X,8H ROOTA1=,F18.6,5X,8H ROOTA2=,F18.6,/,13X,8H ROOTB1
209      $=,F18.6,5X,8H ROOTB2=,F18.6,/)
210      IF (ABS(ROOTA1-ROOTB1).LT.0.05) GO TO 160
211      IF (ABS(ROOTA1-ROOTB2).LT.0.05) GO TO 160
212      IF (ABS(ROOTA2-ROOTB1).LT.0.05) GO TO 170
213      IF (ABS(ROOTA2-ROOTB2).LT.0.05) GO TO 170
214      GO TO 215
215      160 WRITE (6,180) ROOTA1
216      180 FORMAT (2X,16H SECOND ROOT IS ,F20.5,/)

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CARD

```
217      ROOT=ROOTA1
218      GO TO 210
219      170 WRITE (6,180) ROOTA2
220      ROOT=ROOTA2
221      210 WRITE (6,212)
222      212 FORMAT (14X,13H CENTER POINT,31X,13H CIRCLE PCINT,/,8X,3H XA,19X,3
223      $H YA,19X,3H XB,19X,3H YB,/)
224      XA(I)=R(1)+P(1)*RCOTR(I)+Q(1)*ROOT
225      YA(I)=R(2)+P(2)*ROOTR(I)+Q(2)*ROOT
226      XB(I)=R(3)+P(3)*ROOTR(I)+Q(3)*ROOT
227      YB(I)=R(4)+P(4)*ROOTR(I)+Q(4)*ROOT
228      WRITE (6,230) XA(I),YA(I),XB(I),YB(I)
229      230 FORMAT (4F20.8,/,33X,20H XXXXXXXXXXXXXXXXXXXX,/)
230      215 CONTINUE
231      STOP
232      END
```

APPENDIX D

SYNTHESIS PROGRAM FOR THREE FINITE POSITIONS  
AND ONE SECOND INFINITESIMAL  
DISPLACEMENT

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CARD

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1 C *****
2 C *
3 C *           SYNTHESIS OF SIX-LINK STEPHENSON 2           *
4 C *           FUNCTION GENERATOR                           *
5 C *
6 C *           THREE FINITE POSITIONS AND ONE ACCELERATION RATIO *
7 C *
8 C *
9 C *           INPUT FINITE ROTATIONS           TH1(I)      *
10 C *
11 C *          OUTPUT FINITE ROTATIONS          PH1(I)      *
12 C *
13 C *          ASSUMED ROTATIONS OF TERNARY      AL1(I)      *
14 C *
15 C *          SECOND INFINITESIMAL DISPLACEMENT DDPH(J)   *
16 C *
17 C *          OUTPUT LINK LENGTH                QC           *
18 C *
19 C *          OUTPUT LINK INITIAL ANGLE        THE          *
20 C *
21 C *****
22 C
23     DIMENSION XCOF(5),COF(5),ROOTR(4),ROOTI(4)
24     DIMENSION XQ(5),YQ(5),XA(5),YA(5),XB(5),YB(5),XC(5),YC(5)
25     DIMENSION TH1(5),PH1(5),AL1(5)
26     DIMENSION H(16),F1(16),H2(16),R(4),P(4),Q(4)
27     DIMENSION DPHDT(15),DDPTH(5),DTHDAL(5),DDTHAL(5)
28     DIMENSION DXC(5),DYC(5),DDXC(5),DDYC(5)
29     DIMENSION PXC(5),PYC(5),PPXC(5),PPYC(5)
30     READ (5,5) J
31     5 FORMAT (I1)
32     WRITE (6,6) J
33     6 FORMAT (//,10X,22H ACCELERATION RATIO IS AT POINT ,I1,/)
34     READ (5,10) TH1(2),TH1(3)
35     READ (5,10) PH1(2),PH1(3)
36     READ (5,10) AL1(2),AL1(3)
37     10 FORMAT (2F10.0)
38     READ (5,20) QC,THE
39     READ (5,20) DPHDT(J),DTHDAL(J)
40     READ (5,20) DDPH(J),DDTHAL(J)
41     20 FORMAT (2F10.0)
42     WRITE (6,11) TH1(2),TH1(3)
43     11 FORMAT (9X,26H INPUT ROTATION ANGLES ARE,2X,2F10.3,/)
44     WRITE (6,12) PH1(2),PH1(3)
45     12 FORMAT (9X,27H OUTPUT ROTATION ANGLES ARE,1X,2F10.3,/)
46     WRITE (6,13) AL1(2),AL1(3)
47     13 FORMAT (9X,23H TERNARY LINK ROTATIONS,5X,2F10.3,/)
48     WRITE (6,14) QC,THE
49     14 FORMAT (6X,15H CLTPUT LINK IS,F10.3,24H LONG AND AT AN ANGLE OF,F1
50     $0.3,/)
51     WRITE (6,15) DDPH(J),DTHDAL(J)
52     15 FORMAT (9X,11H DPHDT(J)=,F10.3,10X,11H DTHDAL(J)=,F10.3,/)
53     WRITE (6,16) DDPH(J),DDTHAL(J)
54     16 FORMAT (9X,11H DDPH(J)=,F10.3,10X,11H DDT HAL(J)=,F10.3,/)

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CARD

```
55 XQ(1)=1.0
56 YQ(1)=0.0
57 XM=0.0
58 YM=0.0
59 TH1(1)=0.0
60 PH1(1)=0.0
61 AL1(1)=0.0
62 THE=(THE*3.14159265)/180.0
63 DO 21 I=2,3
64 TH1(I)=(TH1(I)*3.14159265)/180.0
65 PH1(I)=(PH1(I)*3.14159265)/180.0
66 21 AL1(I)=(AL1(I)*3.14159265)/180.0
67 XC(1)=1.0+QC*COS(THE)
68 YC(1)=0.0+QC*SIN(THE)
69 DO 30 I=2,3
70 XQ(I)=1.0*COS(-TH1(I))
71 YQ(I)=1.0*SIN(-TH1(I))
72 XC(I)=XC(1)*COS(PH1(I)-TH1(I))-YC(1)*SIN(PH1(I)-TH1(I))+XQ(I)-XQ(1)
73 $)*COS(PH1(I)-TH1(I))+YQ(1)*SIN(PH1(I)-TH1(I))
74 30 YC(I)=XC(1)*SIN(PH1(I)-TH1(I))+YC(1)*COS(PH1(I)-TH1(I))+YQ(I)-XQ(1)
75 $)*SIN(PH1(I)-TH1(I))-YQ(1)*COS(PH1(I)-TH1(I))
76 DXC(J)=(XC(1)-XQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(J)-TH1(J))*DPHDT
77 $(J))+YC(1)-YQ(1))*(COS(PH1(J)-TH1(J))-COS(PH1(J)-TH1(J))*DPHDT
78 $(J))+XQ(1)*SIN(-TH1(J))+YQ(1)*COS(-TH1(J))
79 DYC(J)=(XC(1)-XQ(1))*(-COS(PH1(J)-TH1(J))+COS(PH1(J)-TH1(J))*DPHDT
80 $(J))+YC(1)-YQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(J)-TH1(J))*DPHDT
81 $(J))-XQ(1)*COS(-TH1(J))+YQ(1)*SIN(-TH1(J))
82 DDXC(J)=(XC(1)-XQ(1))*(-COS(PH1(J)-TH1(J))+2.0*COS(PH1(J)-TH1(J))*
83 $DPHDT$(J)-COS(PH1(J)-TH1(J))*DPHDT$(J)*DPHDT$(J)-SIN(PH1(J)-TH1(J)
84 $)*DDPTH$(J))+YC(1)-YQ(1))*(SIN(PH1(J)-TH1(J))-2.0*SIN(PH1(J)-TH1(
85 $J))*DPHDT$(J)+SIN(PH1(J)-TH1(J))*DPHDT$(J)*DPHDT$(J)-COS(PH1(J)-TH
86 $1(J))*DDPTH$(J))-XQ(1)*COS(-TH1(J))+YQ(1)*SIN(-TH1(J))
87 DDYC(J)=(XC(1)-XQ(1))*(-SIN(PH1(J)-TH1(J))+2.0*SIN(PH1(J)-TH1(J))*
88 $DPHDT$(J)-SIN(PH1(J)-TH1(J))*DPHDT$(J)*DPHDT$(J)+COS(PH1(J)-TH1(J)
89 $)*DDPTH$(J))+YC(1)-YQ(1))*(-COS(PH1(J)-TH1(J))+2.0*COS(PH1(J)-TH1
90 $(J))*DPHDT$(J)-COS(PH1(J)-TH1(J))*DPHDT$(J)*DPHDT$(J)-SIN(PH1(J)-T
91 $H1(J))*DDPTH$(J))-XQ(1)*SIN(-TH1(J))-YQ(1)*COS(-TH1(J))
92 PXC(J)=DTHDAL(J)*DXC(J)
93 PYC(J)=DTHDAL(J)*DYC(J)
94 PPXC(J)=DTHDAL(J)*DTHDAL(J)*DDXC(J)+DTHAL(J)*DXC(J)
95 PPYC(J)=DTHDAL(J)*DTHDAL(J)*DDYC(J)+DTHAL(J)*DYC(J)
96 WRITE(6,17) DXC(J),DYC(J)
97 17 FORMAT(9X,8H DXC(J)=,3X,F10.3,10X,8H DYC(J)=,3X,F10.3,/)
98 WRITE(6,18) DDXC(J),DDYC(J)
99 18 FORMAT(9X,9H DCXC(J)=,2X,F10.3,10X,9H DDYC(J)=,2X,F10.3,/)
100 WRITE(6,31)
101 31 FORMAT(11X,3H XC,18X,3H YQ,18X,3H XC,18X,3H YC,/)
102 DO 33 I=1,3
103 33 WRITE(6,32) XQ(I),YQ(I),XC(I),YC(I)
104 32 FORMAT(6X,F12.8,9X,F12.8,9X,F12.8,9X,F12.8,/)
105 DO 40 I=1,2
106 40 H(I)=XC(I+1)-XC(1)*COS(AL1(I+1))+YC(1)*SIN(AL1(I+1))
107 H(3)=PXC(J)+XC(1)*SIN(AL1(J))+YC(1)*COS(AL1(J))
108 H(4)=PPXC(J)+XC(1)*COS(AL1(J))-YC(1)*SIN(AL1(J))
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109 DO 50 I=1,2
110 50 H(I+4)=YC(I+1)-YC(I)*COS(AL1(I+1))-XC(I)*SIN(AL1(I+1))
111 H(7)=PYC(J)-XC(I)*COS(AL1(J))+YC(I)*SIN(AL1(J))
112 H(8)=PPYC(J)+XC(I)*SIN(AL1(J))+YC(I)*COS(AL1(J))
113 DO 60 I=1,2
114 60 H(I+8)=-H(I)*COS(AL1(I+1))-H(I+4)*SIN(AL1(I+1))
115 H(11)=-PXC(J)*COS(AL1(J))-PYC(J)*SIN(AL1(J))+XC(J)*SIN(AL1(J))-YC
116 $J)*COS(AL1(J))
117 H(12)=-PPXC(J)*COS(AL1(J))-PPYC(J)*SIN(AL1(J))+2.0*PXC(J)*SIN(AL1
118 $J)-2.0*PYC(J)*COS(AL1(J))+XC(J)*COS(AL1(J))+YC(J)*SIN(AL1(J))
119 DO 70 I=1,2
120 70 H(I+12)=-H(I+4)*COS(AL1(I+1))+H(I)*SIN(AL1(I+1))
121 H(15)=PXC(J)*SIN(AL1(J))-PYC(J)*COS(AL1(J))+XC(J)*COS(AL1(J))+YC(J
122 $)*SIN(AL1(J))
123 H(16)=PPXC(J)*SIN(AL1(J))-PPYC(J)*COS(AL1(J))+2.0*PXC(J)*COS(AL1(J)
124 $)+2.0*PYC(J)*SIN(AL1(J))-XC(J)*SIN(AL1(J))+YC(J)*COS(AL1(J))
125 DO 72 I=1,16
126 72 H1(I)=H(I)
127 DO 74 I=1,16
128 74 H2(I)=H1(I)
129 DO 80 I=1,2
130 80 R(I)=0.5*(H(I)*H(I)+H(I+4)*H(I+4))
131 R(3)=(YC(I)*XC(J)-XC(I)*YC(J)-YC(I)*PYC(J)-XC(I)*PXC(J))*COS(AL1(J
132 $))+XC(I)*XC(J)+YC(I)*YC(J)-XC(I)*PYC(J)+YC(I)*PXC(J))*SIN(AL1(J))
133 $+XC(J)*PXC(J)+YC(J)*PYC(J)
134 DD1=2.0*YC(I)*PXC(J)-2.0*XC(I)*PYC(J)-YC(I)*PPYC(J)-XC(I)*PPXC(J)+
135 $XC(I)*XC(J)+YC(I)*YC(J)
136 DD2=2.0*XC(I)*PXC(J)+2.0*YC(I)*PYC(J)-XC(I)*PPYC(J)+YC(I)*PPXC(J)-
137 $YC(I)*XC(J)+XC(I)*YC(J)
138 R(4)=COS(AL1(J))*DD1+SIN(AL1(J))*DD2+XC(J)*PPXC(J)+YC(J)*PPYC(J)+P
139 $XC(J)*RXC(J)+PYC(J)*PYC(J)
140 DO 90 I=1,2
141 90 P(I)=1.0-COS(AL1(I+1))
142 P(3)=SIN(AL1(J))
143 P(4)=COS(AL1(J))
144 DO 100 I=1,2
145 100 Q(I)=SIN(AL1(I+1))
146 Q(3)=COS(AL1(J))
147 Q(4)=-SIN(AL1(J))
148 CALL SIMQ(H,R,4,KS)
149 CALL SIMQ(H1,P,4,KS1)
150 CALL SIMQ(H2,Q,4,KS2)
151 WRITE (6,99) KS
152 WRITE (6,99) KS1
153 WRITE (6,99) KS2
154 99 FORMAT (1X,I1)
155 F1=Q(1)*Q(3)+Q(2)*Q(4)
156 F2=P(1)*Q(3)+Q(1)*P(3)+P(2)*Q(4)+Q(2)*P(4)
157 F3=R(1)*Q(3)+Q(1)*R(3)+Q(2)*R(4)+Q(4)*R(2)
158 F4=P(1)*P(3)+P(2)*P(4)
159 F5=P(1)*R(3)+P(3)*R(1)+P(2)*R(4)+P(4)*R(2)-1.0
160 F6=R(1)*R(3)+R(2)*R(4)
161 G1=Q(1)*Q(4)-Q(2)*Q(3)
162 G2=P(1)*Q(4)+Q(1)*P(4)-P(2)*Q(3)-Q(2)*P(3)
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163      G3=R(1)*Q(4)+Q(1)*R(4)-R(2)*Q(3)-Q(2)*R(3)-1.0
164      G4=P(1)*P(4)-P(2)*P(3)
165      G5=P(1)*R(4)+R(1)*P(4)-P(2)*R(3)-P(3)*R(2)
166      G6=R(1)*R(4)-R(2)*R(3)
167      XCOF(1)=F1*F1*G6*G6+G3*G3*F1*F6-G3*G6*F1*F3+G1*G6*F3*F3+G1*G1*F6*F
168      $6-G1*G3*F3*F6-2.0*G1*G6*F1*F6
169      XCOF(2)=2.0*G5*G6*F1*F1+2.0*G2*G3*F1*F6+G3*G3*F1*F5-G3*G6*F1*F2-G2
170      $*G6*F1*F3-G3*G5*F1*F3+2.0*G1*G6*F2*F3+G1*G5*F3*F3+2.0*G1*G1*F5*F6-
171      $G1*G3*F2*F6-G1*G2*F3*F6-G1*G3*F3*F5-2.0*G1*G5*F1*F6-2.0*G1*G6*F1*F
172      $5
173      XCOF(3)=2.0*G4*G6*F1*F1+G5*G5*F1*F1+G2*G2*F1*F6+2.0*G2*G3*F1*F5+G3
174      $*G3*F1*F4-G2*G6*F1*F2-G3*G5*F1*F2-G2*G5*F1*F3-G3*G4*F1*F3+G1*G6*F2
175      $*F2+2.0*G1*G5*F2*F3+G1*G4*F3*F3+2.0*G1*G1*F4*F6+G1*G1*F5*F5-G1*G2*
176      $F2*F6-G1*G3*F2*F5-G1*G2*F3*F5-G1*G3*F3*F4-2.0*G1*G6*F1*F4-2.0*G1*G
177      $5*F1*F5-2.0*G1*G4*F1*F6
178      XCOF(4)=2.0*G4*G5*F1*F1+G2*G2*F1*F5+2.0*G2*G3*F1*F4-G2*G5*F1*F2-G3
179      $*G4*F1*F2-G2*G4*F1*F3+G1*G5*F2*F2+2.0*G1*G4*F2*F3+2.0*G1*G1*F4*F5-
180      $G1*G2*F2*F5-G1*G3*F2*F4-G1*G2*F3*F4-2.0*G1*G5*F1*F4-2.0*G1*G4*F1*F
181      $5
182      XCOF(5)=G4*G4*F1*F1+G2*G2*F1*F4-G2*G4*F1*F2+G1*G4*F2*F2+G1*G1*F4*F
183      $4-G1*G2*F2*F4-2.0*G1*G4*F1*F4
184      CALL POLRT(XCOF,COF,4,ROOTR,ROOTI,IER)
185      WRITE (6,109)
186      109 FORMAT (/ ,27X,9H L1 RCOTS,/)
187      DO 110 I=1,4
188      110 WRITE (6,112) RCCTR(I),ROOTI(I)
189      112 FORMAT (17X,E13.5,10X,E13.5,/)
190      WRITE (6,114) IER
191      114 FORMAT (1X,I1)
192      WRITE (6,106)
193      106 FORMAT (1H1,/)
194      DO 215 I=1,4
195      OA=F1
196      OB=F2*ROOTR(I)+F3
197      OC=F4*ROOTR(I)*ROOTR(I)+F5*ROOTR(I)+F6
198      OD=G1
199      OE=G2*ROOTR(I)+G3
200      OF=G4*ROOTR(I)*ROOTR(I)+G5*ROOTR(I)+G6
201      IF (ABS(ROOTI(I)).LT.0.000001) ROOTI(I)=0.0
202      IF (ROOTI(I)) 20C,130,200
203      200 WRITE (6,201)
204      201 FORMAT (5X,18H ROOT IS IMAGINARY,/)
205      GO TO 215
206      130 SA=OB*OB-4.0*OA*OC
207      IF (SA) 200,140,140
208      140 ROOTA1=(-OB+SQRT(SA))/(2.0*OA)
209      ROOTA2=(-OB-SQRT(SA))/(2.0*OA)
210      SB=OE*OE-4.0*OD*OF
211      IF (SB) 200,150,150
212      150 ROOTB1=(-OE+SQRT(SB))/(2.0*OD)
213      ROOTB2=(-OE-SQRT(SB))/(2.0*OD)
214      WRITE (6,155) ROOTA1,ROOTA2,ROOTB1,ROOTB2
215      155 FORMAT (/ ,13X,8H ROOTA1=,F18.6,5X,8H ROOTA2=,F18.6,/,13X,8H ROOTB1
216      $=,F18.6,5X,8H ROOTB2=,F18.6,/)

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CARD
217     IF (ABS(ROOTA1-ROOTB1).LT.0.05) GO TO 160
218     IF (ABS(ROOTA1-ROOTB2).LT.0.05) GO TO 160
219     IF (ABS(ROOTA2-ROOTB1).LT.0.05) GO TO 170
220     IF (ABS(ROOTA2-ROOTB2).LT.0.05) GO TO 170
221     GO TO 215
222     160 WRITE (6,180) RCCTA1
223     180 FORMAT (2X,16H SECOND ROOT IS ,F20.5,/)
224     ROOT=ROOTA1
225     GO TO 210
226     170 WRITE (6,180) RCOTA2
227     ROOT=ROOTA2
228     210 WRITE (6,212)
229     212 FORMAT (14X,13H CENTER POINT,31X,13H CIRCLE POINT,/,8X,3H XA,19X,3
230     $H YA,19X,3H XB,19X,3H YB,/)
231     XA(I)=R(1)+P(1)*ROOTR(I)+Q(1)*ROOT
232     YA(I)=R(2)+P(2)*ROOTR(I)+Q(2)*ROOT
233     XB(I)=R(3)+P(3)*ROOTR(I)+Q(3)*ROOT
234     YB(I)=R(4)+P(4)*ROOTR(I)+Q(4)*ROOT
235     WRITE (6,230) XA(I),YA(I),XB(I),YB(I)
236     230 FORMAT (4F20.8,/,33X,20H XXXXXXXXXXXXXXXXXXXX,/)
237     215 CONTINUE
238     STOP
239     END
```

APPENDIX E

SYNTHESIS PROGRAM FOR TWO FINITE POSITIONS,  
ONE FIRST INFINITESIMAL DISPLACEMENT  
AND ONE SECOND INFINITESIMAL  
DISPLACEMENT

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```

CARD
1 C *****
2 C *
3 C *           SYNTHESIS OF SIX-LINK STEPHENSON 2           *
4 C *           FUNCTION GENERATOR                           *
5 C *
6 C *           TWO FINITE POSITIONS,                         *
7 C *           ONE VELOCITY RATIO, AND ONE ACCELERATION RATIO *
8 C *
9 C *
10 C *          INPUT FINITE ROTATION             TH1(I)      *
11 C *
12 C *          OUTPUT FINITE ROTATION            PH1(I)      *
13 C *
14 C *          ASSUMED ROTATION OF TERNARY       AL1(I)      *
15 C *
16 C *          FIRST INFINITESIMAL DISPLACEMENT DPHDTH(K)   *
17 C *
18 C *          SECOND INFINITESIMAL DISPLACEMENT DDPHTH(J)  *
19 C *
20 C *          OUTPLT LINK LENGTH                 QC          *
21 C *
22 C *          OUTPUT LINK INITIAL ANGLE         THE          *
23 C *
24 C *
25 C *****
26 C
27   DIMENSION XCOF(5),COF(5),ROOTR(4),ROOTI(4)
28   DIMENSION XQ(5),YQ(5),XA(5),YA(5),XB(5),YB(5),XC(5),YC(5)
29   DIMENSION TH1(5),PH1(5),AL1(5)
30   DIMENSION H(16),H1(16),H2(16),R(4),P(4),Q(4)
31   DIMENSION DPHDTH(5),DDPHTH(5),DTHDAL(5),DDTHAL(5)
32   DIMENSION DXC(5),DYC(5),DDXC(5),DDYC(5)
33   DIMENSION PXC(5),PYC(5),PPXC(5),PPYC(5)
34   READ (5,5) J,K
35   5 FORMAT (I1,I1)
36   WRITE (6,6) K,J
37   6 FORMAT (//,5X,32H VELOCITY RATIO IS AT POSITION ,I1,39H AND ACCEL
38   $ERATION RATIO IS AT POSITION ,I1,/)
39   READ (5,10) TH1(2)
40   READ (5,10) PH1(2)
41   READ (5,10) AL1(2)
42   10 FORMAT (F10.0)
43   READ (5,20) QC,THE
44   READ (5,20) DPHDTH(J),DTHDAL(J)
45   READ (5,20) DDPHTH(J),DDTHAL(J)
46   READ (5,20) DPHDTH(K),DTHDAL(K)
47   20 FORMAT (2F10.0)
48   WRITE (6,11) TH1(2)
49   11 FORMAT (9X,24H INPUT ROTATION ANGLE IS,2X,F10.3,/)
50   WRITE (6,12) PH1(2)
51   12 FORMAT (9X,25H OUTPUT ROTATION ANGLE IS,1X,F10.3,/)
52   WRITE (6,13) AL1(2)
53   13 FORMAT (9X,22H TERNARY LINK ROTATION,4X,F10.3,/)
54   WRITE (6,14) QC,THE
  
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CARD
55 14 FORMAT (6X,15H OUTPUT LINK IS,F10.3,24H LONG AND AT AN ANGLE OF,F1
56 $0.3,/)
57 WRITE (6,15) DP+DTH(J),DTHDAL(J)
58 15 FORMAT (9X,11H DPHDTH(J)=,F10.3,10X,11H DTHDAL(J)=,F10.3,/)
59 WRITE (6,16) DDP+TH(J),DDTHAL(J)
60 16 FORMAT (9X,11H DDPHTH(J)=,F10.3,10X,11H DDTHAL(J)=,F10.3,/)
61 WRITE (6,19) DPHDTH(K),DTHDAL(K)
62 19 FORMAT (9X,11H CP+OTH(K)=,F10.3,10X,11H DTHDAL(K)=,F10.3,/)
63 XQ(1)=1.0
64 YQ(1)=0.0
65 XM=0.0
66 YM=0.0
67 TH1(1)=0.0
68 PH1(1)=0.0
69 AL1(1)=0.0
70 THE=(THE*3.14159265)/180.0
71 I=2
72 TH1(I)=(TH1(I)*3.14159265)/180.0
73 PH1(I)=(PH1(I)*3.14159265)/180.0
74 AL1(I)=(AL1(I)*3.14159265)/180.0
75 XC(1)=1.0+QC*COS(THE)
76 YC(1)=0.0+QC*SIN(THE)
77 XQ(I)=1.0*COS(-TH1(I))
78 YQ(I)=1.0*SIN(-TH1(I))
79 XC(I)=XC(1)*COS(PH1(I)-TH1(I))-YC(1)*SIN(PH1(I)-TH1(I))+XQ(I)-XQ(I
80 $)*COS(PH1(I)-TH1(I))+YQ(1)*SIN(PH1(I)-TH1(I))
81 YC(I)=XC(1)*SIN(PH1(I)-TH1(I))+YC(1)*COS(PH1(I)-TH1(I))+YQ(I)-XQ(I
82 $)*SIN(PH1(I)-TH1(I))-YQ(1)*COS(PH1(I)-TH1(I))
83 DXC(J)=(XC(1)-XQ(1))*SIN(PH1(J)-TH1(J))-SIN(PH1(J)-TH1(J))*DPHDTH
84 $(J))+YC(1)-YQ(1))*(COS(PH1(J)-TH1(J))-COS(PH1(J)-TH1(J))*DPHDTH(J
85 $))+XQ(1)*SIN(-TH1(J))+YQ(1)*COS(-TH1(J))
86 DYC(J)=(XC(1)-XQ(1))*(-COS(PH1(J)-TH1(J))+COS(PH1(J)-TH1(J))*DPHD
87 $H(J))+YC(1)-YQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(J)-TH1(J))*DPHDTH(
88 $J))-XQ(1)*COS(-TH1(J))+YQ(1)*SIN(-TH1(J))
89 DDXC(J)=(XC(1)-XQ(1))*(-COS(PH1(J)-TH1(J))+2.0*COS(PH1(J)-TH1(J))*
90 $DPHDTH(J)-COS(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-SIN(PH1(J)-TH1(J)
91 $)*DDPHTH(J))+YC(1)-YQ(1))*(SIN(PH1(J)-TH1(J))-2.0*SIN(PH1(J)-TH1(
92 $J))*DPHDTH(J)+SIN(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-COS(PH1(J)-TH
93 $I(J))*DDPHTH(J))-XQ(1)*COS(-TH1(J))+YQ(1)*SIN(-TH1(J))
94 DDYC(J)=(XC(1)-XQ(1))*(-SIN(PH1(J)-TH1(J))+2.0*SIN(PH1(J)-TH1(J))*
95 $DPHDTH(J)-SIN(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)+COS(PH1(J)-TH1(J)
96 $)*DDPHTH(J))+YC(1)-YQ(1))*(-COS(PH1(J)-TH1(J))+2.0*COS(PH1(J)-TH1
97 $(J))*DPHDTH(J)-COS(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-SIN(PH1(J)-T
98 $H1(J))*DDPHTH(J))-XQ(1)*SIN(-TH1(J))-YQ(1)*COS(-TH1(J))
99 DXC(K)=(XC(1)-XQ(1))*SIN(PH1(K)-TH1(K))-SIN(PH1(K)-TH1(K))*DPHDTH
100 $(K))+YC(1)-YQ(1))*(COS(PH1(K)-TH1(K))-COS(PH1(K)-TH1(K))*DPHDTH(K
101 $))+XQ(1)*SIN(-TH1(K))+YQ(1)*COS(-TH1(K))
102 DYC(K)=(XC(1)-XQ(1))*(-COS(PH1(K)-TH1(K))+COS(PH1(K)-TH1(K))*DPHD
103 $H(K))+YC(1)-YQ(1))*(SIN(PH1(K)-TH1(K))-SIN(PH1(K)-TH1(K))*DPHDTH(
104 $K))-XQ(1)*COS(-TH1(K))+YQ(1)*SIN(-TH1(K))
105 WRITE (6,17) DXC(J),DYC(J)
106 17 FORMAT (9X,8H DXC(J)=,3X,F10.3,10X,8H DYC(J)=,3X,F10.3,/)
107 WRITE (6,21) DDXC(J),DDYC(J)
108 21 FORMAT (9X,9H DCXC(J)=,2X,F10.3,10X,9H DDYC(J)=,2X,F10.3,/)
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CARD
109      WRITE (6,18) CXC(K),DYC(K)
110  18 FORMAT (9X,8H DXC(K)=,3X,F10.3,10X,8H DYC(K)=,3X,F10.3,///)
111      PXC(J)=DTHDAL(J)*DXC(J)
112      PYC(J)=DTHCAL(J)*DYC(J)
113      PPXC(J)=DDTHAL(J)*DXC(J)+DTHDAL(J)*DTHDAL(J)*DDXC(J)
114      PPYC(J)=DDTHAL(J)*DYC(J)+DTHDAL(J)*DTHDAL(J)*DDYC(J)
115      PXC(K)=DTHCAL(K)*DXC(K)
116      PYC(K)=DTHDAL(K)*DYC(K)
117      WRITE (6,31)
118  31 FORMAT (11X,3H XQ,18X,3H YQ,18X,3H XC,18X,3H YC,/)
119      DO 33 I=1,2
120  33 WRITE (6,32) XQ(I),YQ(I),XC(I),YC(I)
121  32 FORMAT (6X,F12.8,9X,F12.8,9X,F12.8,9X,F12.8,/)
122      I=1
123      H(I)=XC(I+1)-XC(I)*COS(AL1(I+1))+YC(I)*SIN(AL1(I+1))
124      H(2)=PXC(J)+XC(I)*SIN(AL1(J))+YC(I)*COS(AL1(J))
125      H(3)=PPXC(J)+XC(I)*COS(AL1(J))-YC(I)*SIN(AL1(J))
126      H(4)=PXC(K)+XC(I)*SIN(AL1(K))+YC(I)*COS(AL1(K))
127      H(I+4)=YC(I+1)-YC(I)*COS(AL1(I+1))-XC(I)*SIN(AL1(I+1))
128      H(6)=PYC(J)-XC(I)*COS(AL1(J))+YC(I)*SIN(AL1(J))
129      H(7)=PPYC(J)+XC(I)*SIN(AL1(J))+YC(I)*COS(AL1(J))
130      H(8)=PYC(K)-XC(I)*COS(AL1(K))+YC(I)*SIN(AL1(K))
131      H(I+8)=-H(I)*COS(AL1(I+1))-H(I+4)*SIN(AL1(I+1))
132      H(10)=-PXC(J)*COS(AL1(J))-PYC(J)*SIN(AL1(J))+XC(J)*SIN(AL1(J))-YC(
133      $J)*COS(AL1(J))
134      H(11)=-PPXC(J)*COS(AL1(J))-PPYC(J)*SIN(AL1(J))+2.0*PXC(J)*SIN(AL1(
135      $J))-2.0*PYC(J)*COS(AL1(J))+XC(J)*COS(AL1(J))+YC(J)*SIN(AL1(J))
136      H(12)=-PXC(K)*COS(AL1(K))-PYC(K)*SIN(AL1(K))+XC(K)*SIN(AL1(K))-YC(
137      $K)*COS(AL1(K))
138      H(I+12)=-H(I+4)*COS(AL1(I+1))+H(I)*SIN(AL1(I+1))
139      H(14)=PXC(J)*SIN(AL1(J))-PYC(J)*COS(AL1(J))+XC(J)*COS(AL1(J))+YC(J
140      $)*SIN(AL1(J))
141      H(15)=PPXC(J)*SIN(AL1(J))-PPYC(J)*COS(AL1(J))+2.0*PXC(J)*COS(AL1(J
142      $))+2.0*PYC(J)*SIN(AL1(J))-XC(J)*SIN(AL1(J))+YC(J)*COS(AL1(J))
143      H(16)=PXC(K)*SIN(AL1(K))-PYC(K)*COS(AL1(K))+XC(K)*COS(AL1(K))+YC(K
144      $)*SIN(AL1(K))
145      DO 72 I=1,16
146  72 H1(I)=H(I)
147      DO 74 I=1,16
148  74 H2(I)=H(I)
149      I=1
150      R(I)=0.5*(H(I)*H(I)+H(I+4)*H(I+4))
151      R(2)=(YC(I)*XC(J)-XC(I)*YC(J)-YC(I)*PYC(J)-XC(I)*PXC(J))*COS(AL1(J
152      $))+(XC(I)*XC(J)+YC(I)*YC(J)-XC(I)*PYC(J)+YC(I)*PXC(J))*SIN(AL1(J))
153      $+XC(J)*PXC(J)+YC(J)*PYC(J)
154      ZS1=2.0*YC(I)*PXC(J)-2.0*XC(I)*PYC(J)-YC(I)*PPYC(J)-XC(I)*PPXC(J)+
155      $XC(I)*XC(J)+YC(I)*YC(J)
156      ZS2=2.0*XC(I)*PXC(J)+2.0*YC(I)*PYC(J)-XC(I)*PPYC(J)+YC(I)*PPXC(J)-
157      $YC(I)*XC(J)+XC(I)*YC(J)
158      R(3)=COS(AL1(J))*ZS1+SIN(AL1(J))*ZS2+XC(J)*PPXC(J)+YC(J)*PPYC(J)+P
159      $XC(J)*PXC(J)+PYC(J)*PYC(J)
160      R(4)=(YC(I)*XC(K)-XC(I)*YC(K)-YC(I)*PYC(K)-XC(I)*PXC(K))*COS(AL1(K
161      $))+(XC(I)*XC(K)+YC(I)*YC(K)-XC(I)*PYC(K)+YC(I)*PXC(K))*SIN(AL1(K))
162      $+XC(K)*PXC(K)+YC(K)*PYC(K)
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CARD
163      P(1)=1.0-COS(AL1(I+1))
164      P(2)=SIN(AL1(J))
165      P(3)=COS(AL1(J))
166      P(4)=SIN(AL1(K))
167      Q(1)=SIN(AL1(I+1))
168      Q(2)=COS(AL1(J))
169      Q(3)=-SIN(AL1(J))
170      Q(4)=COS(AL1(K))
171      CALL SIMQ(H,R,4,KS)
172      CALL SIMQ(H1,P,4,KS1)
173      CALL SIMQ(H2,Q,4,KS2)
174      WRITE (6,99) KS
175      WRITE (6,99) KS1
176      WRITE (6,99) KS2
177      99 FORMAT (1X,I1)
178      F1=Q(1)*Q(3)+Q(2)*Q(4)
179      F2=P(1)*Q(3)+Q(1)*P(3)+P(2)*Q(4)+Q(2)*P(4)
180      F3=R(1)*Q(3)+Q(1)*R(3)+Q(2)*R(4)+Q(4)*R(2)
181      F4=P(1)*P(3)+P(2)*P(4)
182      F5=P(1)*R(3)+P(3)*R(1)+P(2)*R(4)+P(4)*R(2)-1.0
183      F6=R(1)*R(3)+R(2)*R(4)
184      G1=Q(1)*Q(4)-Q(2)*Q(3)
185      G2=P(1)*Q(4)+Q(1)*P(4)-P(2)*Q(3)-Q(2)*P(3)
186      G3=R(1)*Q(4)+Q(1)*R(4)-R(2)*Q(3)-Q(2)*R(3)-1.0
187      G4=P(1)*P(4)-P(2)*P(3)
188      G5=P(1)*R(4)+R(1)*P(4)-P(2)*R(3)-P(3)*R(2)
189      G6=R(1)*R(4)-R(2)*R(3)
190      XCOF(1)=F1*F1*G6*G6+G3*G3*F1*F6-G3*G6*F1*F3+G1*G6*F3*F3+G1*G1*F6*F6
191      $6-G1*G3*F3*F6-2.0*G1*G6*F1*F6
192      XCOF(2)=2.0*G5*G6*F1*F1+2.0*G2*G3*F1*F6+G3*G3*F1*F5-G3*G6*F1*F2-G2
193      $*G6*F1*F3-G3*G5*F1*F3+2.0*G1*G6*F2*F3+G1*G5*F3*F3+2.0*G1*G1*F5*F6-
194      $G1*G3*F2*F6-G1*G2*F3*F6-G1*G3*F3*F5-2.0*G1*G5*F1*F6-2.0*G1*G6*F1*F
195      $5
196      XCOF(3)=2.0*G4*G6*F1*F1+G5*G5*F1*F1+G2*G2*F1*F6+2.0*G2*G3*F1*F5+G3
197      $*G3*F1*F4-G2*G6*F1*F2-G3*G5*F1*F2-G2*G5*F1*F3-G3*G4*F1*F3+G1*G6*F2
198      $*F2+2.0*G1*G5*F2*F3+G1*G4*F3*F3+2.0*G1*G1*F4*F6+G1*G1*F5*F5-G1*G2*
199      $F2*F6-G1*G3*F2*F5-G1*G2*F3*F5-G1*G3*F3*F4-2.0*G1*G6*F1*F4-2.0*G1*G
200      $5*F1*F5-2.0*G1*G4*F1*F6
201      XCOF(4)=2.0*G4*G5*F1*F1+G2*G2*F1*F5+2.0*G2*G3*F1*F4-G2*G5*F1*F2-G3
202      $*G4*F1*F2-G2*G4*F1*F3+G1*G5*F2*F2+2.0*G1*G4*F2*F3+2.0*G1*G1*F4*F5-
203      $G1*G2*F2*F5-G1*G3*F2*F4-G1*G2*F3*F4-2.0*G1*G5*F1*F4-2.0*G1*G4*F1*F
204      $5
205      XCOF(5)=G4*G4*F1*F1+G2*G2*F1*F4-G2*G4*F1*F2+G1*G4*F2*F2+G1*G1*F4*F4
206      $4-G1*G2*F2*F4-2.0*G1*G4*F1*F4
207      CALL POLRT(XCOF,COF,4,ROOTR,ROOTI,IER)
208      WRITE (6,109)
209      109 FORMAT (/ ,27X,9H L1 ROOTS,/)
210      DO 110 I=1,4
211      110 WRITE (6,112) RCCTR(I),ROOTI(I)
212      112 FORMAT (17X,E13.5,10X,E13.5,/)
213      WRITE (6,114) IER
214      114 FORMAT (1X,I1)
215      WRITE (6,106)
216      106 FORMAT (1H1,//)
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CARD
217 125 DO 215 I=1,4
218     OA=F1
219     OB=F2*ROOTR(I)+F3
220     OC=F4*RCOTR(I)*RCOTR(I)+F5*ROOTR(I)+F6
221     OD=G1
222     OE=G2*ROOTR(I)+G3
223     OF=G4*ROOTR(I)*RCOTR(I)+G5*ROOTR(I)+G6
224     IF (ABS(ROOTR(I)).LT.0.000001) ROOTI(I)=0.0
225     IF (ROOTI(I)) 200,130,200
226 200 WRITE (6,201)
227 201 FORMAT (5X,18H ROOT IS IMAGINARY,/)
228     GO TO 215
229 130 SA=OB*OB-4.0*OA*OC
230     IF (SA) 200,140,140
231 140 ROOTA1=(-OB+SQRT(SA))/(2.0*OA)
232     ROOTA2=(-OB-SQRT(SA))/(2.0*OA)
233     SB=OE*OE-4.0*OD*OF
234     IF (SB) 200,150,150
235 150 ROOTB1=(-OE+SQRT(SB))/(2.0*OD)
236     ROOTB2=(-OE-SQRT(SB))/(2.0*OD)
237     WRITE (6,155) RCCTA1,ROOTA2,ROOTB1,ROOTB2
238 155 FORMAT (/ ,13X,8H ROOTA1=,F18.6,5X,8H ROOTA2=,F18.6,/,13X,8H ROOTB1
239     $=,F18.6,5X,8H ROOTB2=,F18.6,/)
240     IF (ABS(ROOTA1-ROOTB1).LT.0.05) GO TO 160
241     IF (ABS(ROOTA1-ROOTB2).LT.0.05) GO TO 160
242     IF (ABS(ROOTA2-ROOTB1).LT.0.05) GO TO 170
243     IF (ABS(ROOTA2-ROOTB2).LT.0.05) GO TO 170
244     GO TO 215
245 160 WRITE (6,180) RCCTA1
246 180 FORMAT (2X,16H SECOND ROOT IS ,F20.5,/)
247     ROOT=ROOTA1
248     GO TO 210
249 170 WRITE (6,180) ROOTA2
250     ROOT=ROOTA2
251 210 WRITE (6,212)
252 212 FORMAT (14X,13H CENTER POINT,31X,13H CIRCLE POINT,/,8X,3H XA,19X,3
253     $H YA,19X,3H XB,19X,3H YB,/)
254     XA(I)=R(1)+P(1)*ROOTR(I)+Q(1)*ROOT
255     YA(I)=R(2)+P(2)*ROOTR(I)+Q(2)*ROOT
256     XB(I)=R(3)+P(3)*ROOTR(I)+Q(3)*ROOT
257     YB(I)=R(4)+P(4)*ROOTR(I)+Q(4)*ROOT
258     WRITE (6,230) XA(I),YA(I),XB(I),YB(I)
259 230 FORMAT (4F20.8,/,33X,20H XXXXXXXXXXXXXXXXXXXX,/)
260 215 CONTINUE
261 STOP
262 END
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APPENDIX F

SYNTHESIS PROGRAM FOR TWO FINITE POSITIONS  
AND ONE THIRD INFINITESIMAL  
DISPLACEMENT

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CARD
1 C *****
2 C *
3 C *           SYNTHESIS OF SIX-LINK STEPHENSON 2
4 C *           FUNCTION GENERATOR
5 C *
6 C *           TWO FINITE POSITIONS AND ONE JERK RATIO
7 C *
8 C *
9 C *           INPUT FINITE ROTATION           TH1(I)
10 C *
11 C *          OUTPUT FINITE ROTATION          PHI(I)
12 C *
13 C *          ASSUMED ROTATION OF TERNARY     AL1(I)
14 C *
15 C *          THIRTEEN INFINITESIMAL DISPLACEMENT DDDPTH(J)
16 C *
17 C *          OUTPUT LINK LENGTH              QC
18 C *
19 C *          OUTPUT LINK INITIAL ANGLE       THE
20 C *
21 C *****
22 C
23   DIMENSION XCOF(5),COF(5),ROOTR(4),ROOTI(4)
24   DIMENSION XQ(5),YQ(5),XA(5),YA(5),XB(5),YB(5),XC(5),YC(5)
25   DIMENSION TH1(5),PHI(5),AL1(5)
26   DIMENSION H(16),H1(16),H2(16),R(4),P(4),Q(4)
27   DIMENSION DPHDTH(5),DDPHTH(5),DDDPHTH(5)
28   DIMENSION DTHDAL(5),DDTHAL(5),DDDTAL(5)
29   DIMENSION DXC(5),DYC(5),DDXC(5),DDYC(5),DDDXC(5),DDDYC(5)
30   DIMENSION PXC(5),PYC(5),PPXC(5),PPYC(5),PPPPXC(5),PPPPYC(5)
31   READ (5,5) J
32   5 FORMAT (I1)
33   WRITE (6,6) J
34   6 FORMAT (//,15X,31H JERK RATIO SPECIFIED AT POINT ,11,/)
35   READ (5,10) TH1(2)
36   READ (5,10) PHI(2)
37   READ (5,10) AL1(2)
38   10 FORMAT (F10.0)
39   READ (5,20) QC,THE
40   READ (5,20) DPHDTH(J),DTHDAL(J)
41   READ (5,20) DDPHTH(J),DDTHAL(J)
42   READ (5,20) DDDPTH(J),DDDTAL(J)
43   20 FORMAT (2F10.0)
44   WRITE (6,11) TH1(2)
45   11 FORMAT (13X,24H INPUT ROTATION ANGLE IS,2X,F10.3,/)
46   WRITE (6,12) PHI(2)
47   12 FORMAT (13X,25H OUTPUT ROTATION ANGLE IS,1X,F10.3,/)
48   WRITE (6,13) AL1(2)
49   13 FORMAT (13X,22H TERNARY LINK ROTATION,4X,F10.3,/)
50   WRITE (6,14) QC,THE
51   14 FORMAT (4X,15H OUTPUT LINK IS,F10.3,24H LONG AND AT AN ANGLE OF,F1
52   $0.3,/)
53   WRITE (6,15) DPHDTH(J),DTHDAL(J)
54   15 FORMAT (7X,11H DPHDTH(J)=,F10.3,10X,11H DTHDAL(J)=,F10.3,/)

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CARD
55      WRITE (6,16) DDPH(J),DDTHAL(J)
56      16 FORMAT (7X,11H DDPH(J)=,F10.3,10X,11H DDTAL(J)=,F10.3,/)
57      WRITE (6,17) DDCPTH(J),DDDTAL(J)
58      17 FORMAT (7X,11H DDCPTH(J)=,F10.3,10X,11H DDDTAL(J)=,F10.3,/)
59      XQ(1)=1.0
60      YQ(1)=0.0
61      XM=0.0
62      YM=0.0
63      TH1(1)=0.0
64      PH1(1)=0.0
65      AL1(1)=0.0
66      THE=(THE*3.14159265)/180.0
67      I=2
68      TH1(I)=(TH1(I)*3.14159265)/180.0
69      PH1(I)=(PH1(I)*3.14159265)/180.0
70      AL1(I)=(AL1(I)*3.14159265)/180.0
71      XC(1)=1.0+QC*COS(THE)
72      YC(1)=0.0+CC*SIN(THE)
73      XQ(I)=1.0*COS(-TH1(I))
74      YQ(I)=1.0*SIN(-TH1(I))
75      XC(I)=XC(1)*COS(PH1(I)-TH1(I))-YC(1)*SIN(PH1(I)-TH1(I))+XQ(I)-XQ(1)
76      YC(I)=YC(1)*COS(PH1(I)-TH1(I))+XQ(I)*SIN(PH1(I)-TH1(I))
77      YC(I)=XC(1)*SIN(PH1(I)-TH1(I))+YC(1)*COS(PH1(I)-TH1(I))+YQ(I)-XQ(1)
78      XQ(I)=XQ(1)*SIN(PH1(I)-TH1(I))-YC(1)*COS(PH1(I)-TH1(I))
79      DXC(J)=(XC(1)-XQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(I)-TH1(I)))*DPHOTH
80      $ (J)+(YC(1)-YQ(1))*(COS(PH1(J)-TH1(J))-COS(PH1(I)-TH1(I)))*DPHOTH(J)
81      $)+XQ(1)*SIN(-TH1(J))+YQ(1)*COS(-TH1(J))
82      DYC(J)=(XC(1)-XQ(1))*(-COS(PH1(J)-TH1(J))+COS(PH1(I)-TH1(I)))*DPHOTH
83      $ (J)+(YC(1)-YQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(I)-TH1(I)))*DPHOTH(J)
84      $)-XQ(1)*COS(-TH1(J))+YQ(1)*SIN(-TH1(J))
85      DDXC(J)=(XC(1)-XQ(1))*(-COS(PH1(J)-TH1(J))+2.0*COS(PH1(I)-TH1(I))*
86      $DPHOTH(J)-COS(PH1(J)-TH1(J))*DPHOTH(J)*DPHOTH(J)-SIN(PH1(J)-TH1(J)
87      $)*DDPHTH(J)+(YC(1)-YQ(1))*(SIN(PH1(J)-TH1(J))-2.0*SIN(PH1(I)-TH1(I)
88      $J))*DPHOTH(J)+SIN(PH1(J)-TH1(J))*DPHOTH(J)*DPHOTH(J)-COS(PH1(J)-TH1
89      $I(J))*DDPHTH(J)-XQ(1)*COS(-TH1(J))+YQ(1)*SIN(-TH1(J))
90      DDYC(J)=(XC(1)-XQ(1))*(-SIN(PH1(J)-TH1(J))+2.0*SIN(PH1(I)-TH1(I))*
91      $DPHOTH(J)-SIN(PH1(J)-TH1(J))*DPHOTH(J)*DPHOTH(J)+COS(PH1(J)-TH1(J)
92      $)*DDPHTH(J)+(YC(1)-YQ(1))*(-COS(PH1(J)-TH1(J))+2.0*COS(PH1(I)-TH1(I)
93      $J))*DPHOTH(J)-COS(PH1(J)-TH1(J))*DPHOTH(J)*DPHOTH(J)-SIN(PH1(J)-TH1
94      $I(J))*DDPHTH(J)-XQ(1)*SIN(-TH1(J))+YQ(1)*COS(-TH1(J))
95      DDD1=-SIN(PH1(J)-TH1(J))+3.0*SIN(PH1(I)-TH1(I))*DPHOTH(J)-3.0*SIN(
96      $PH1(J)-TH1(J))*DPHOTH(J)*DPHOTH(J)+3.0*COS(PH1(J)-TH1(J))*DDPHTH(J)
97      $)-3.0*COS(PH1(J)-TH1(J))*DPHOTH(J)*DDPHTH(J)+SIN(PH1(J)-TH1(J))*DP
98      $HOTH(J)*DPHOTH(J)*DPHOTH(J)-SIN(PH1(J)-TH1(J))*DDDPH(J)
99      DDD2=-COS(PH1(J)-TH1(J))+3.0*COS(PH1(I)-TH1(I))*DPHOTH(J)-3.0*COS(
100     $PH1(J)-TH1(J))*DPHOTH(J)*DPHOTH(J)-3.0*SIN(PH1(J)-TH1(J))*DDPHTH(J)
101     $)+3.0*SIN(PH1(J)-TH1(J))*DPHOTH(J)*DDPHTH(J)+COS(PH1(J)-TH1(J))*DP
102     $HOTH(J)*DPHOTH(J)*DPHOTH(J)-COS(PH1(J)-TH1(J))*DDDPH(J)
103     DDD3=COS(PH1(J)-TH1(J))-3.0*COS(PH1(I)-TH1(I))*DPHOTH(J)+3.0*COS(P
104     $H1(J)-TH1(J))*DPHOTH(J)*DPHOTH(J)+3.0*SIN(PH1(J)-TH1(J))*DDPHTH(J)
105     $)-3.0*SIN(PH1(J)-TH1(J))*DPHOTH(J)*DDPHTH(J)-COS(PH1(J)-TH1(J))*DPH
106     $OTH(J)*DPHOTH(J)*DPHOTH(J)+COS(PH1(J)-TH1(J))*DDDPH(J)
107     DDD4=-SIN(PH1(J)-TH1(J))+3.0*SIN(PH1(I)-TH1(I))*DPHOTH(J)-3.0*SIN(
108     $PH1(J)-TH1(J))*DPHOTH(J)*DPHOTH(J)+3.0*COS(PH1(J)-TH1(J))*DDPHTH(J)
```

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CARD
109      $)-3.0*CGS(PH1(J)-TH1(J))*DPHDT H(J)*DDPHT H(J)+SIN(PH1(J)-TH1(J))*DP
110      $HDT H(J)*DPHDT H(J)*DPHDT H(J)-SIN(PH1(J)-TH1(J))*DDDPHT H(J)
111      DDDXC(J)=(XC(1)-XQ(1))*DDD1+(YC(1)-YQ(1))*DDD2-XQ(1)*SIN(-TH1(J))-
112      $YQ(1)*COS(-TH1(J))
113      DCCYC(J)=(XC(1)-XQ(1))*DDD3+(YC(1)-YQ(1))*DDD4+XQ(1)*COS(-TH1(J))-
114      $YQ(1)*SIN(-TH1(J))
115      WRITE (6,21) DXC(J),DYC(J)
116      21 FORMAT (7X,8H DXC(J)=,3X,F10.3,10X,8H DYC(J)=,3X,F10.3,/)
117      WRITE (6,22) DDXC(J),DDYC(J)
118      22 FORMAT (7X,9H DCXC(J)=,2X,F10.3,10X,9H DDYC(J)=,2X,F10.3,/)
119      WRITE (6,23) DDCXC(J),DDDYC(J)
120      23 FORMAT (7X,10H DDDXC(J)=,1X,F10.3,10X,10H DDDYC(J)=,1X,F10.3,/)
121      PXC(J)=DTHDAL(J)*DXC(J)
122      PYC(J)=DTHDAL(J)*DYC(J)
123      PPXC(J)=DDTHAL(J)*DXC(J)+DTHDAL(J)*DTHDAL(J)*DDXC(J)
124      PPYC(J)=DDTHAL(J)*DYC(J)+DTHDAL(J)*DTHDAL(J)*DDYC(J)
125      PPPXC(J)=DDDTAL(J)*DXC(J)+3.0*DTHDAL(J)*DDTHAL(J)*DDXC(J)+DTHDAL(J)
126      $)*DTHDAL(J)*DTHDAL(J)*DDDXC(J)
127      PPPYC(J)=DDDTAL(J)*DYC(J)+3.0*DTHDAL(J)*DDTHAL(J)*DDYC(J)+DTHDAL(J)
128      $)*DTHDAL(J)*DTHDAL(J)*DDDYC(J)
129      WRITE (6,31)
130      31 FORMAT (11X,3H XQ,18X,3H YQ,18X,3H XC,18X,3H YC,/)
131      DO 33 I=1,2
132      33 WRITE (6,32) XQ(I),YQ(I),XC(I),YC(I)
133      32 FORMAT (6X,F12.8,9X,F12.8,9X,F12.8,9X,F12.8,/)
134      I=1
135      H(1)=XC(I+1)-XC(1)*COS(AL1(I+1))+YC(1)*SIN(AL1(I+1))
136      H(2)=PXC(J)+XC(1)*SIN(AL1(J))+YC(1)*COS(AL1(J))
137      H(3)=PPXC(J)+XC(1)*COS(AL1(J))-YC(1)*SIN(AL1(J))
138      H(4)=PPPXC(J)-XC(1)*SIN(AL1(J))-YC(1)*COS(AL1(J))
139      H(I+4)=YC(I+1)-YC(1)*COS(AL1(I+1))-XC(1)*SIN(AL1(I+1))
140      H(6)=PYC(J)-XC(1)*COS(AL1(J))+YC(1)*SIN(AL1(J))
141      H(7)=PPYC(J)+XC(1)*SIN(AL1(J))+YC(1)*COS(AL1(J))
142      H(8)=PPPYC(J)+XC(1)*COS(AL1(J))-YC(1)*SIN(AL1(J))
143      H(I+8)=-H(I)*COS(AL1(I+1))-H(I+4)*SIN(AL1(I+1))
144      H(10)=-PXC(J)*COS(AL1(J))-PYC(J)*SIN(AL1(J))+XC(J)*SIN(AL1(J))-YC(
145      $J)*COS(AL1(J))
146      H(11)=-PPXC(J)*COS(AL1(J))-PPYC(J)*SIN(AL1(J))+2.0*PXC(J)*SIN(AL1(
147      $J))-2.0*PYC(J)*COS(AL1(J))+XC(J)*COS(AL1(J))+YC(J)*SIN(AL1(J))
148      H(12)=-PPPXC(J)*COS(AL1(J))-PPPYC(J)*SIN(AL1(J))+3.0*PPXC(J)*SIN(A
149      $L1(J))-3.0*PPYC(J)*COS(AL1(J))+3.0*PXC(J)*COS(AL1(J))+3.0*PYC(J)*S
150      $IN(AL1(J))-XC(J)*SIN(AL1(J))+YC(J)*COS(AL1(J))
151      H(I+12)=-H(I+4)*COS(AL1(I+1))+H(I)*SIN(AL1(I+1))
152      H(14)=PXC(J)*SIN(AL1(J))-PYC(J)*COS(AL1(J))+XC(J)*COS(AL1(J))+YC(J)
153      $)*SIN(AL1(J))
154      H(15)=PPXC(J)*SIN(AL1(J))-PPYC(J)*COS(AL1(J))+2.0*PXC(J)*COS(AL1(J)
155      $)+2.0*PYC(J)*SIN(AL1(J))-XC(J)*SIN(AL1(J))+YC(J)*COS(AL1(J))
156      H(16)=PPPXC(J)*SIN(AL1(J))-PPPYC(J)*COS(AL1(J))+3.0*PPXC(J)*COS(AL
157      $1(J))+3.0*PPYC(J)*SIN(AL1(J))-3.0*PXC(J)*SIN(AL1(J))+3.0*PYC(J)*CO
158      $S(AL1(J))-XC(J)*COS(AL1(J))-YC(J)*SIN(AL1(J))
159      DO 72 I=1,16
160      72 H1(I)=H(I)
161      DO 74 I=1,16
162      74 H2(I)=H(I)
```

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CARD

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I=1
163 R(1)=0.5*(H(I)*H(I)+H(I+4)*H(I+4))
164 R(2)=(Y(1)*XC(J)-XC(1)*Y(J)-Y(1)*PYC(J)-XC(1)*PXC(J))*COS(AL1(J)
165 $)+(XC(1)*XC(J)+Y(1)*Y(J)-XC(1)*PYC(J)+Y(1)*PXC(J))*SIN(AL1(J))
166 $+XC(J)*PXC(J)+Y(J)*PYC(J)
167 DD1=2.0*Y(1)*PXC(J)-2.0*XC(1)*PYC(J)-Y(1)*PPYC(J)-XC(1)*PPXC(J)+
168 $XC(1)*XC(J)+Y(1)*Y(J)
169 DD2=2.0*XC(1)*PXC(J)+2.0*Y(1)*PYC(J)-XC(1)*PPYC(J)+Y(1)*PPXC(J)-
170 $Y(1)*XC(J)+XC(1)*Y(J)
171 R(3)=COS(AL1(J))*DD1+SIN(AL1(J))*DD2+XC(J)*PPXC(J)+Y(J)*PPYC(J)+P
172 $XC(J)*PXC(J)+PYC(J)*PYC(J)
173 DDD5=-Y(1)*PPPYC(J)-XC(1)*PPXC(J)+3.0*Y(1)*PPXC(J)-3.0*XC(1)*PP
174 $YC(J)+3.0*XC(1)*PXC(J)+3.0*Y(1)*PYC(J)-Y(1)*XC(J)+XC(1)*Y(J)
175 DDD6=-XC(1)*PPPYC(J)+Y(1)*PPXC(J)+3.0*XC(1)*PPXC(J)+3.0*Y(1)*PP
176 $YC(J)-3.0*Y(1)*PXC(J)+3.0*XC(1)*PYC(J)-XC(1)*XC(J)-Y(1)*Y(J)
177 R(4)=CJS(AL1(J))*DDD5+SIN(AL1(J))*DDD6+XC(J)*PPXC(J)+Y(J)*PPYC(J)
178 $J)+3.0*PXC(J)*PPXC(J)+3.0*PYC(J)*PPYC(J)
179 P(1)=1.0-COS(AL1(I+1))
180 P(2)=SIN(AL1(J))
181 P(3)=COS(AL1(J))
182 P(4)=-SIN(AL1(J))
183 Q(1)=SIN(AL1(I+1))
184 Q(2)=COS(AL1(J))
185 Q(3)=-SIN(AL1(J))
186 Q(4)=-COS(AL1(J))
187 CALL SIMQ(H,R,4,KS)
188 CALL SIMQ(H1,P,4,KS1)
189 CALL SIMQ(H2,Q,4,KS2)
190 WRITE(6,99) KS
191 WRITE(6,99) KS1
192 WRITE(6,99) KS2
193 99 FDRMAT(1X,11)
194 F1=Q(1)*Q(3)+Q(2)*Q(4)
195 F2=P(1)*Q(3)+Q(1)*P(3)+P(2)*Q(4)+Q(2)*P(4)
196 F3=R(1)*Q(3)+Q(1)*R(3)+Q(2)*R(4)+Q(4)*R(2)
197 F4=P(1)*P(3)+P(2)*P(4)
198 F5=P(1)*R(3)+P(3)*R(1)+P(2)*R(4)+P(4)*R(2)-1.0
199 F6=R(1)*R(3)+R(2)*R(4)
200 G1=Q(1)*Q(4)-Q(2)*Q(3)
201 G2=P(1)*Q(4)+Q(1)*P(4)-P(2)*Q(3)-Q(2)*P(3)
202 G3=R(1)*Q(4)+Q(1)*R(4)-R(2)*Q(3)-Q(2)*R(3)-1.0
203 G4=P(1)*P(4)-P(2)*P(3)
204 G5=P(1)*R(4)+R(1)*P(4)-P(2)*R(3)-P(3)*R(2)
205 G6=R(1)*R(4)-R(2)*R(3)
206 XCOF(1)=F1*F1*G6*G6+G3*G3*F1*F6-G3*G6*F1*F3+G1*G6*F3*F3+G1*G1*F6*F
207 $6-G1*G3*F3*F6-2.0*G1*G6*F1*F6
208 XCOF(2)=2.0*G5*G6*F1*F1+2.0*G2*G3*F1*F6+G3*G3*F1*F5-G3*G6*F1*F2-G2
209 $*G6*F1*F3-G3*G5*F1*F3+2.0*G1*G6*F2*F3+G1*G5*F3*F3+2.0*G1*G1*F5*F6-
210 $G1*G3*F2*F6-G1*G2*F3*F6-G1*G3*F3*F5-2.0*G1*G5*F1*F6-2.0*G1*G6*F1*F
211 $5
212 XCOF(3)=2.0*G4*G6*F1*F1+G5*G5*F1*F1+G2*G2*F1*F6+2.0*G2*G3*F1*F5+G3
213 $*G3*F1*F4-G2*G6*F1*F2-G3*G5*F1*F2-G2*G5*F1*F3-G3*G4*F1*F3+G1*G6*F2
214 $*F2+2.0*G1*G5*F2*F3+G1*G4*F3*F3+2.0*G1*G1*F4*F6+G1*G1*F5*F5-G1*G2*
215 $F2*F6-G1*G3*F2*F5-G1*G2*F3*F5-G1*G3*F3*F4-2.0*G1*G6*F1*F4-2.0*G1*G
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CARD
217      $5*F1*F5-2.0*G1*G4*F1*F6
218      XCOF(4)=2.0*G4*G5*F1*F1+G2*G2*F1*F5+2.0*G2*G3*F1*F4-G2*G5*F1*F2-G3
219      $*G4*F1*F2-G2*G4*F1*F3+G1*G5*F2*F2+2.0*G1*G4*F2*F3+2.0*G1*G1*F4*F5-
220      $G1*G2*F2*F5-G1*G3*F2*F4-G1*G2*F3*F4-2.0*G1*G5*F1*F4-2.0*G1*G4*F1*F
221      $5
222      XCOF(5)=G4*G4*F1*F1+G2*G2*F1*F4-G2*G4*F1*F2+G1*G4*F2*F2+G1*G1*F4*F
223      $4-G1*G2*F2*F4-2.0*G1*G4*F1*F4
224      CALL POLRT(XCCF,COF,4,ROOTR,ROOTI,IER)
225      WRITE (6,109)
226      109 FORMAT (/ ,27X,9H L1 ROOTS,/)
227      DO 110 I=1,4
228      110 WRITE (6,112) ROOTR(I),ROOTI(I)
229      112 FORMAT (17X,E13.5,10X,E13.5,/)
230      WRITE (6,114) IER
231      114 FORMAT (1X,I1)
232      WRITE (6,106)
233      106 FORMAT (1H1,/)
234      125 DO 215 I=1,4
235          OA=F1
236          OB=F2*ROOTR(I)+F3
237          OC=F4*ROOTR(I)*ROOTR(I)+F5*ROOTR(I)+F6
238          OD=G1
239          OE=G2*ROOTR(I)+G3
240          OF=G4*ROOTR(I)*ROOTR(I)+G5*ROOTR(I)+G6
241          IF (ABS(ROOTI(I)).LT.0.000001) ROOTI(I)=0.0
242          IF (ROOTI(I)) 2CC,130,200
243      200 WRITE (6,201)
244      201 FORMAT (5X,18H ROOT IS IMAGINARY,/)
245      GO TO 215
246      130 SA=OB*OB-4.0*OA*CC
247          IF (SA) 200,140,140
248      140 ROOTA1=(-OB+SQRT(SA))/(2.0*OA)
249          ROOTA2=(-OB-SQRT(SA))/(2.0*OA)
250          SB=OE*OE-4.0*OD*OF
251          IF (SB) 200,150,150
252      150 ROOTB1=(-OE+SQRT(SB))/(2.0*OD)
253          ROOTB2=(-OE-SQRT(SB))/(2.0*OD)
254          WRITE (6,155) RCCTA1,ROOTA2,ROOTB1,ROOTB2
255      155 FORMAT (/ ,13X,8H ROOTA1=,F18.6,5X,8H ROOTA2=,F18.6,/,13X,8H ROOTB1
256          $=,F18.6,5X,8H ROOTB2=,F18.6,/)
257          IF (ABS(ROOTA1-ROOTB1).LT.0.05) GO TO 160
258          IF (ABS(ROOTA1-ROOTB2).LT.0.05) GO TO 160
259          IF (ABS(ROOTA2-ROOTB1).LT.0.05) GO TO 170
260          IF (ABS(ROOTA2-ROOTB2).LT.0.05) GO TO 170
261      GO TO 215
262      160 WRITE (6,180) ROOTA1
263      180 FORMAT (2X,16H SECCND ROOT IS ,F20.5,/)
264          ROOT=ROOTA1
265      GO TO 210
266      170 WRITE (6,180) RCCTA2
267          ROOT=ROOTA2
268      210 WRITE (6,212)
269      212 FORMAT (14X,13H CENTER POINT,31X,13H CIRCLE POINT,/,8X,3H XA,19X,3
270          $H YA,19X,3H XB,19X,3H YB,/)

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CARD
271      XA(I)=R(1)+P(1)*ROOTR(I)+Q(1)*ROOT
272      YA(I)=R(2)+P(2)*ROOTR(I)+Q(2)*ROOT
273      XB(I)=R(3)+P(3)*ROOTR(I)+Q(3)*ROOT
274      YB(I)=R(4)+P(4)*ROOTR(I)+Q(4)*ROOT
275      WRITE (6,230) XA(I),YA(I),XB(I),YB(I)
276      230  FORMAT (4F20.8,/,33X,20H 'XXXXXXXXXXXXXXXXXXXX',//)
277      215  CONTINUE
278      STOP
279      END
```

APPENDIX G

SYNTHESIS PROGRAM FOR ONE FINITE POSITION  
AND ONE FOURTH INFINITESIMAL  
DISPLACEMENT

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CARD
1 C *****
2 C *
3 C *           SYNTHESIS OF SIX-LINK STEPHENSON 2           *
4 C *           FUNCTION GENERATOR                           *
5 C *
6 C *           ONE FINITE POSITION AND ONE KERK RATIO         *
7 C *
8 C *
9 C *           FOURTH INFINITESIMAL DISPLACEMENT DDDOPT(J) *
10 C *
11 C *           OUTPUT LINK LENGTH                QC          *
12 C *
13 C *           OUTPUT LINK INITIAL ANGLE        THE          *
14 C *
15 C *****
16 C
17 DIMENSION XCOF(5),COF(5),ROOTR(4),ROOTI(4)
18 DIMENSION XQ(5),YQ(5),XA(5),YA(5),XB(5),YB(5),XC(5),YC(5)
19 DIMENSION TH1(5),PH1(5),AL1(5)
20 DIMENSION H(16),H1(16),H2(16),R(4),P(4),Q(4)
21 DIMENSION DPHDT(5),CCPTH(5),DDOPT(5),DDDDPT(5)
22 DIMENSION DTHDAL(5),DDTHAL(5),DDDTAL(5),DDDDTA(5)
23 DIMENSION DXC(5),DYC(5),DDXC(5),DDYC(5)
24 DIMENSION DDDXC(5),DDDYC(5),DDDDXC(5),DDDDYC(5)
25 DIMENSION PXC(5),PYC(5),PPXC(5),PPYC(5)
26 DIMENSION PPPXC(5),PPPYC(5),PPPPXC(5),PPPPYC(5)
27 J=1
28 WRITE (6,6)
29 6 FORMAT (//,12X,32H KERK RATIO SPECIFIED AT POINT 1,/)
30 READ (5,20) QC,THE
31 READ (5,20) DPHDT(J),DTHDAL(J)
32 READ (5,20) DDPHTH(J),DDTHAL(J)
33 READ (5,20) DDDPTH(J),DDDTAL(J)
34 READ (5,20) DDDOPT(J),DDDDTA(J)
35 20 FORMAT (2F10.0)
36 WRITE (6,14) QC,THE
37 14 FORMAT (4X,15H OUTPUT LINK IS,F10.3,24H LONG AND AT AN ANGLE OF,F1
38 $0.3,/)
39 15 FORMAT (7X,11H DPHDT(J)=,F10.3,10X,11H DTHDAL(J)=,F10.3,/)
40 WRITE (6,15) DPHDT(J),DTHDAL(J)
41 WRITE (6,16) DDPHTH(J),DDTHAL(J)
42 16 FORMAT (7X,11H CCPTH(J)=,F10.3,10X,11H DDTHAL(J)=,F10.3,/)
43 WRITE (6,17) DDDPTH(J),DDDTAL(J)
44 17 FORMAT (7X,11H DDDPTH(J)=,F10.3,10X,11H DDDTA(J)=,F10.3,/)
45 WRITE (6,18) DDDOPT(J),DDDDTA(J)
46 18 FORMAT (7X,11H DDDOPT(J)=,F10.3,10X,11H DDDDTA(J)=,F10.3,/)
47 XQ(1)=1.0
48 YQ(1)=0.0
49 XM=0.0
50 YM=0.0
51 TH1(1)=0.0
52 PH1(1)=0.0
53 AL1(1)=0.0
54 THE=(THE*3.14159265)/180.0
```

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CARD  
55 XC(1)=1.0+QC\*COS(THE)  
56 YC(1)=0.0+QC\*SIN(THE)  
57 DXC(J)=(XC(1)-XQ(1))\*(SIN(PHI(J)-TH1(J))-SIN(PHI(J)-TH1(J))\*DPHDT  
58 \$(J))+YC(1)-YQ(1))\*(COS(PHI(J)-TH1(J))-COS(PHI(J)-TH1(J))\*DPHDT  
59 \$(J))+XQ(1)\*SIN(-TH1(J))+YQ(1)\*COS(-TH1(J))  
60 DYC(J)=(XC(1)-XQ(1))\*(-COS(PHI(J)-TH1(J))+COS(PHI(J)-TH1(J))\*DPHDT  
61 \$(J))+YC(1)-YQ(1))\*(SIN(PHI(J)-TH1(J))-SIN(PHI(J)-TH1(J))\*DPHDT  
62 \$(J))-XQ(1)\*COS(-TH1(J))+YQ(1)\*SIN(-TH1(J))  
63 DDXC(J)=(XC(1)-XQ(1))\*(-COS(PHI(J)-TH1(J))+2.0\*COS(PHI(J)-TH1(J))\*  
64 \$DPHDT\$(J)-COS(PHI(J)-TH1(J))\*DPHDT\$(J)\*DPHDT\$(J)-SIN(PHI(J)-TH1(J))  
65 \$)\*DDPHT\$(J))+YC(1)-YQ(1))\*(SIN(PHI(J)-TH1(J))-2.0\*SIN(PHI(J)-TH1(  
66 \$J))\*DPHDT\$(J)+SIN(PHI(J)-TH1(J))\*DPHDT\$(J)\*DPHDT\$(J)-COS(PHI(J)-TH  
67 \$1(J))\*DDPHT\$(J))-XQ(1)\*COS(-TH1(J))+YQ(1)\*SIN(-TH1(J))  
68 DDYC(J)=(XC(1)-XQ(1))\*(-SIN(PHI(J)-TH1(J))+2.0\*SIN(PHI(J)-TH1(J))\*  
69 \$DPHDT\$(J)-SIN(PHI(J)-TH1(J))\*DPHDT\$(J)\*DPHDT\$(J)+COS(PHI(J)-TH1(J))  
70 \$)\*DDPHT\$(J))+YC(1)-YQ(1))\*(-COS(PHI(J)-TH1(J))+2.0\*COS(PHI(J)-TH1  
71 \$(J))\*DPHDT\$(J)-COS(PHI(J)-TH1(J))\*DPHDT\$(J)\*DPHDT\$(J)-SIN(PHI(J)-T  
72 \$H1(J))\*DDPHT\$(J))-XQ(1)\*SIN(-TH1(J))-YQ(1)\*COS(-TH1(J))  
73 DDD1=-SIN(PHI(J)-TH1(J))+3.0\*SIN(PHI(J)-TH1(J))\*DPHDT\$(J)-3.0\*SIN(  
74 \$PH1(J)-TH1(J))\*DPHDT\$(J)\*DPHDT\$(J)+3.0\*COS(PHI(J)-TH1(J))\*DDPHT\$(J  
75 \$)-3.0\*COS(PHI(J)-TH1(J))\*DPHDT\$(J)\*DDPHT\$(J)+SIN(PHI(J)-TH1(J))\*DP  
76 \$HDT\$(J)\*DPHDT\$(J)\*DPHDT\$(J)-SIN(PHI(J)-TH1(J))\*DDDPHT\$(J)  
77 DDD2=-COS(PHI(J)-TH1(J))+3.0\*COS(PHI(J)-TH1(J))\*DPHDT\$(J)-3.0\*COS(  
78 \$PH1(J)-TH1(J))\*DPHDT\$(J)\*DPHDT\$(J)-3.0\*SIN(PHI(J)-TH1(J))\*DDPHT\$(J  
79 \$)+3.0\*SIN(PHI(J)-TH1(J))\*DPHDT\$(J)\*DDPHT\$(J)+COS(PHI(J)-TH1(J))\*DP  
80 \$HDT\$(J)\*DPHDT\$(J)\*DPHDT\$(J)-COS(PHI(J)-TH1(J))\*DDDPHT\$(J)  
81 DDD3=COS(PHI(J)-TH1(J))-3.0\*COS(PHI(J)-TH1(J))\*DPHDT\$(J)+3.0\*COS(P  
82 \$H1(J)-TH1(J))\*DPHDT\$(J)\*DPHDT\$(J)+3.0\*SIN(PHI(J)-TH1(J))\*DDPHT\$(J  
83 \$)-3.0\*SIN(PHI(J)-TH1(J))\*DPHDT\$(J)\*DDPHT\$(J)-COS(PHI(J)-TH1(J))\*DPH  
84 \$DTH(J)\*DPHDT\$(J)\*DPHDT\$(J)+COS(PHI(J)-TH1(J))\*DDDPHT\$(J)  
85 DDD4=-SIN(PHI(J)-TH1(J))+3.0\*SIN(PHI(J)-TH1(J))\*DPHDT\$(J)-3.0\*SIN(  
86 \$PH1(J)-TH1(J))\*DPHDT\$(J)\*DPHDT\$(J)+3.0\*COS(PHI(J)-TH1(J))\*DDPHT\$(J  
87 \$)-3.0\*COS(PHI(J)-TH1(J))\*DPHDT\$(J)\*DDPHT\$(J)+SIN(PHI(J)-TH1(J))\*DP  
88 \$HDT\$(J)\*DPHDT\$(J)\*DPHDT\$(J)-SIN(PHI(J)-TH1(J))\*DDDPHT\$(J)  
89 DDCXC(J)=(XC(1)-XQ(1))\*DDD1+(YC(1)-YQ(1))\*DDD2-XQ(1)\*SIN(-TH1(J))-  
90 \$YQ(1)\*COS(-TH1(J))  
91 DDDYC(J)=(XC(1)-XQ(1))\*DDD3+(YC(1)-YQ(1))\*DDD4+XQ(1)\*COS(-TH1(J))-  
92 \$YQ(1)\*SIN(-TH1(J))  
93 DDDD1=COS(PHI(J)-TH1(J))-4.0\*COS(PHI(J)-TH1(J))\*DPHDT\$(J)+6.0\*COS(  
94 \$PH1(J)-TH1(J))\*DPHDT\$(J)\*DPHDT\$(J)+6.0\*SIN(PHI(J)-TH1(J))\*DDPHT\$(J  
95 \$)-12.0\*SIN(PHI(J)-TH1(J))\*DPHDT\$(J)\*DDPHT\$(J)+6.0\*SIN(PHI(J)-TH1(J)  
96 \$))\*DPHDT\$(J)\*DPHDT\$(J)\*DDPHT\$(J)-4.0\*COS(PHI(J)-TH1(J))\*DPHDT\$(J)\*  
97 \$DPHDT\$(J)\*DPHDT\$(J)+4.0\*COS(PHI(J)-TH1(J))\*DDDPHT\$(J)-4.0\*COS(PHI(J  
98 \$)-TH1(J))\*DPHDT\$(J)\*DDDPHT\$(J)-3.0\*COS(PHI(J)-TH1(J))\*DDPHT\$(J)\*DDP  
99 \$HTH(J)+COS(PHI(J)-TH1(J))\*(DPHDT\$(J)\*\*4.0)-SIN(PHI(J)-TH1(J))\*DDDD  
100 \$PT(J)  
101 DDDD2=-SIN(PHI(J)-TH1(J))+4.0\*SIN(PHI(J)-TH1(J))\*DPHDT\$(J)-6.0\*SIN  
102 \$(PH1(J)-TH1(J))\*DPHDT\$(J)\*DPHDT\$(J)+6.0\*COS(PHI(J)-TH1(J))\*DDPHT\$(J  
103 \$J)-12.0\*COS(PHI(J)-TH1(J))\*DPHDT\$(J)\*DDPHT\$(J)+6.0\*COS(PHI(J)-TH1(  
104 \$J))\*DPHDT\$(J)\*DPHDT\$(J)\*DDPHT\$(J)+4.0\*SIN(PHI(J)-TH1(J))\*DPHDT\$(J  
105 \$)\*DPHDT\$(J)\*DPHDT\$(J)-4.0\*SIN(PHI(J)-TH1(J))\*DDDPHT\$(J)+4.0\*SIN(PHI(  
106 \$J)-TH1(J))\*DPHDT\$(J)\*DDDPHT\$(J)+3.0\*SIN(PHI(J)-TH1(J))\*DDPHT\$(J)\*DD  
107 \$PHT\$(J)-SIN(PHI(J)-TH1(J))\*(DPHDT\$(J)\*\*4.0)-COS(PHI(J)-TH1(J))\*DDDD  
108 \$DPT(J)

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CARD
109      DDDD3=S IN(PH1(J)-TH1(J))-4.0*SIN(PH1(J)-TH1(J))*DPHDTH(J)+6.0*SIN(
110      $PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-6.0*COS(PH1(J)-TH1(J))*DDPHTH(J
111      $)+12.0*COS(PH1(J)-TH1(J))*DPHDTH(J)*CDPHTH(J)-6.0*COS(PH1(J)-TH1(J)
112      $))*DPHDTH(J)*DPHCTH(J)*DDPHTH(J)-4.0*SIN(PH1(J)-TH1(J))*DPHDTH(J)*
113      $DPHDTH(J)*DPHDTH(J)+4.0*SIN(PH1(J)-TH1(J))*DDDPHTH(J)-4.0*SIN(PH1(J)
114      $)-TH1(J))*DPHDTH(J)*DDDPHTH(J)-3.0*SIN(PH1(J)-TH1(J))*DDPHTH(J)*DDP
115      $HTH(J)+SIN(PH1(J)-TH1(J))*(DPHDTH(J)**4.0)+COS(PH1(J)-TH1(J))*DDDD
116      $PT(J)
117      DDDD4=COS(PH1(J)-TH1(J))-4.0*COS(PH1(J)-TH1(J))*DPHDTH(J)+6.0*COS(
118      $PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)+6.0*SIN(PH1(J)-TH1(J))*DDPHTH(J
119      $)-12.0*SIN(PH1(J)-TH1(J))*DPHDTH(J)*DDPHTH(J)+6.0*SIN(PH1(J)-TH1(J)
120      $))*DPHDTH(J)*DPHCTH(J)*DDPHTH(J)-4.0*COS(PH1(J)-TH1(J))*DPHDTH(J)*
121      $DPHDTH(J)*DPHDTH(J)+4.0*COS(PH1(J)-TH1(J))*DDDPHTH(J)-4.0*COS(PH1(J)
122      $)-TH1(J))*DPHDTH(J)*DDDPHTH(J)-3.0*COS(PH1(J)-TH1(J))*DDPHTH(J)*DDP
123      $HTH(J)+COS(PH1(J)-TH1(J))*(DPHDTH(J)**4.0)-SIN(PH1(J)-TH1(J))*DDDD
124      $PT(J)
125      DDDDXC(J)=(XC(1)-XQ(1))*DDDD1+(YC(1)-YQ(1))*DDDD2+XQ(1)*COS(-TH1(J)
126      $)-YQ(1)*SIN(-TH1(J))
127      DDDDYC(J)=(XC(1)-XQ(1))*DDDD3+(YC(1)-YQ(1))*DDDD4+XQ(1)*SIN(-TH1(J)
128      $)+YQ(1)*COS(-TH1(J))
129      PXC(J)=DTHCAL(J)*DXC(J)
130      PYC(J)=DTHDAL(J)*DYC(J)
131      PPXC(J)=DDTHAL(J)*DXC(J)+DTHDAL(J)*DTHDAL(J)*DDXC(J)
132      PPYC(J)=DDTHAL(J)*DYC(J)+DDTHAL(J)*DDTHAL(J)*DDYC(J)
133      PPPXC(J)=DDDTAL(J)*DXC(J)+3.0*DTHDAL(J)*DCTHAL(J)*DXXC(J)+DTHDAL(J)
134      $)*DTHDAL(J)*DTHDAL(J)*DDDXC(J)
135      PPPYC(J)=DDDTAL(J)*DYC(J)+3.0*DTHDAL(J)*DCTHAL(J)*DDYC(J)+DTHDAL(J)
136      $)*DTHDAL(J)*DTHDAL(J)*DDDYC(J)
137      PPPXC(J)=DDDDTA(J)*DXC(J)+3.0*DDTHAL(J)*DDTHAL(J)*DDXC(J)+4.0*DTH
138      $DAL(J)*DDDTAL(J)*DDXC(J)+6.0*DTHDAL(J)*DTHDAL(J)*DDTHAL(J)*DDDXC(J)
139      $)+DTHDAL(J)*DTHCAL(J)*DTHDAL(J)*DTHDAL(J)*DDDXC(J)
140      PPPYC(J)=DDDDTA(J)*DYC(J)+3.0*DDTHAL(J)*DDTHAL(J)*DDYC(J)+4.0*DTH
141      $DAL(J)*DDDTAL(J)*DDYC(J)+6.0*DTHDAL(J)*DTHDAL(J)*DDTHAL(J)*DDDYC(J)
142      $)+DTHDAL(J)*DTHCAL(J)*DTHDAL(J)*DTHDAL(J)*DDDDYC(J)
143      WRITE(6,21) DXC(J),DYC(J)
144      21 FORMAT(7X,8H DXC(J)=,3X,F10.3,10X,8H DYC(J)=,3X,F10.3,/)
145      WRITE(6,22) CDXC(J),DDYC(J)
146      22 FORMAT(7X,9H DDXC(J)=,2X,F10.3,10X,9H DDYC(J)=,2X,F10.3,/)
147      WRITE(6,23) DDDXC(J),DDDYC(J)
148      23 FORMAT(7X,10H DDDXC(J)=,1X,F10.3,10X,10H DDDYC(J)=,1X,F10.3,/)
149      WRITE(6,24) DDDDXC(J),DDDDYC(J)
150      24 FORMAT(7X,11H DDDDXC(J)=,F10.3,10X,11H DDDDYC(J)=,F10.3,/)
151      WRITE(6,31)
152      31 FORMAT(11X,3H XQ,18X,3H YQ,18X,3H XC,18X,3H YC,/)
153      I=1
154      WRITE(6,32) XQ(I),YQ(I),XC(I),YC(I)
155      32 FORMAT(6X,F12.8,9X,F12.8,9X,F12.8,9X,F12.8,/)
156      H(1)=PXC(J)+XC(1)*SIN(AL1(J))+YC(1)*COS(AL1(J))
157      H(2)=PPXC(J)+XC(1)*COS(AL1(J))-YC(1)*SIN(AL1(J))
158      H(3)=PPXC(J)-XC(1)*SIN(AL1(J))-YC(1)*COS(AL1(J))
159      H(4)=PPXC(J)-XC(1)*COS(AL1(J))+YC(1)*SIN(AL1(J))
160      H(5)=PYC(J)-XC(1)*COS(AL1(J))+YC(1)*SIN(AL1(J))
161      H(6)=PPYC(J)+XC(1)*SIN(AL1(J))+YC(1)*COS(AL1(J))
162      H(7)=PPYC(J)+XC(1)*COS(AL1(J))-YC(1)*SIN(AL1(J))
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CARD  
163 H(8)=PPPPYC(J)-XC(1)\*SIN(AL1(J))-YC(1)\*CCS(AL1(J))  
164 H(9)=-PXC(J)\*COS(AL1(J))-PYC(J)\*SIN(AL1(J))+XC(J)\*SIN(AL1(J))-YC(J)  
165 \$)\*COS(AL1(J))  
166 H(10)=-PPXC(J)\*COS(AL1(J))-PPYC(J)\*SIN(AL1(J))+2.0\*PXC(J)\*SIN(AL1(J))  
167 \$J)-2.0\*PYC(J)\*COS(AL1(J))+XC(J)\*COS(AL1(J))+YC(J)\*SIN(AL1(J))  
168 H(11)=-PPXC(J)\*COS(AL1(J))-PPYC(J)\*SIN(AL1(J))+3.0\*PPXC(J)\*SIN(AL1(J))  
169 \$L1(J))-3.0\*PPYC(J)\*COS(AL1(J))+3.0\*PXC(J)\*COS(AL1(J))+3.0\*PYC(J)\*SIN(AL1(J))  
170 \$IN(AL1(J))-XC(J)\*SIN(AL1(J))+YC(J)\*COS(AL1(J))  
171 H(12)=-PPPPXC(J)\*COS(AL1(J))-PPPPYC(J)\*SIN(AL1(J))+4.0\*PPPPXC(J)\*SIN(AL1(J))  
172 \$N(AL1(J))-4.0\*PPPPYC(J)\*COS(AL1(J))+6.0\*PPXC(J)\*CCS(AL1(J))+6.0\*PPYC(J)\*SIN(AL1(J))  
173 \$C(J)\*SIN(AL1(J))-4.0\*PXC(J)\*SIN(AL1(J))+4.0\*PYC(J)\*COS(AL1(J))-XC(J)\*COS(AL1(J))-YC(J)\*SIN(AL1(J))  
174 H(13)=PXC(J)\*SIN(AL1(J))-PYC(J)\*COS(AL1(J))+XC(J)\*COS(AL1(J))+YC(J)\*SIN(AL1(J))  
175 \$)\*SIN(AL1(J))  
176 H(14)=PPXC(J)\*SIN(AL1(J))-PPYC(J)\*COS(AL1(J))+2.0\*PXC(J)\*COS(AL1(J))  
177 \$J)+2.0\*PYC(J)\*SIN(AL1(J))-XC(J)\*SIN(AL1(J))+YC(J)\*COS(AL1(J))  
178 H(15)=PPXC(J)\*SIN(AL1(J))-PPYC(J)\*CCS(AL1(J))+3.0\*PPXC(J)\*COS(AL1(J))  
179 \$I1(J))+3.0\*PPYC(J)\*SIN(AL1(J))-3.0\*PXC(J)\*SIN(AL1(J))+3.0\*PYC(J)\*COS(AL1(J))  
180 \$S(AL1(J))-XC(J)\*CCS(AL1(J))-YC(J)\*SIN(AL1(J))  
181 \$S(AL1(J))-XC(J)\*CCS(AL1(J))-YC(J)\*SIN(AL1(J))  
182 H(16)=PPPPXC(J)\*SIN(AL1(J))-PPPPYC(J)\*CCS(AL1(J))+4.0\*PPPPXC(J)\*COS(AL1(J))  
183 \$+4.0\*PPPPYC(J)\*SIN(AL1(J))-6.0\*PPXC(J)\*SIN(AL1(J))+6.0\*PPYC(J)\*COS(AL1(J))  
184 \$-4.0\*PXC(J)\*CCS(AL1(J))-4.0\*PYC(J)\*SIN(AL1(J))+XC(J)\*SIN(AL1(J))-YC(J)\*COS(AL1(J))  
185 \$)\*SIN(AL1(J))-YC(J)\*COS(AL1(J))  
186 DO 72 I=1,16  
187 72 H1(I)=H(I)  
188 DO 74 I=1,16  
189 74 H2(I)=H(I)  
190 R(1)=(YC(1)\*XC(J)-XC(1)\*YC(J)-YC(1)\*PYC(J)-XC(1)\*PXC(J))\*COS(AL1(J))  
191 \$)+(XC(1)\*XC(J)+YC(1)\*YC(J)-XC(1)\*PYC(J)+YC(1)\*PXC(J))\*SIN(AL1(J))  
192 \$+XC(J)\*PXC(J)+YC(J)\*PYC(J)  
193 DD1=2.0\*YC(1)\*PXC(J)-2.0\*XC(1)\*PYC(J)-YC(1)\*PPYC(J)-XC(1)\*PPXC(J)+  
194 \$XC(1)\*XC(J)+YC(1)\*YC(J)  
195 DD2=2.0\*XC(1)\*PXC(J)+2.0\*YC(1)\*PYC(J)-XC(1)\*PPYC(J)+YC(1)\*PPXC(J)-  
196 \$YC(1)\*XC(J)+XC(1)\*YC(J)  
197 R(2)=COS(AL1(J))\*DD1+SIN(AL1(J))\*DD2+XC(J)\*PPXC(J)+YC(J)\*PPYC(J)+P  
198 \$XC(J)\*PXC(J)+PYC(J)\*PYC(J)  
199 DDD5=-YC(1)\*PPPPYC(J)-XC(1)\*PPPPXC(J)+3.0\*YC(1)\*PPXC(J)-3.0\*XC(1)\*PP  
200 \$YC(J)+3.0\*XC(1)\*PXC(J)+3.0\*YC(1)\*PYC(J)-YC(1)\*XC(J)+XC(1)\*YC(J)  
201 DDD6=-XC(1)\*PPPPYC(J)+YC(1)\*PPPPXC(J)+3.0\*XC(1)\*PPXC(J)+3.0\*YC(1)\*PP  
202 \$YC(J)-3.0\*YC(1)\*PXC(J)+3.0\*XC(1)\*PYC(J)-XC(1)\*XC(J)-YC(1)\*YC(J)  
203 R(3)=COS(AL1(J))\*DDD5+SIN(AL1(J))\*DDD6+XC(J)\*PPPPXC(J)+YC(J)\*PPPPYC  
204 \$J)+3.0\*PXC(J)\*PPXC(J)+3.0\*PYC(J)\*PPYC(J)  
205 DDDD5=-XC(1)\*PPPPXC(J)-YC(1)\*PPPPYC(J)+4.0\*YC(1)\*PPXC(J)-4.0\*XC(1)  
206 \$)\*PPPPYC(J)+6.0\*XC(1)\*PPXC(J)+6.0\*YC(1)\*PPYC(J)-4.0\*YC(1)\*PXC(J)+4.  
207 \$0\*XC(1)\*PYC(J)-XC(1)\*XC(J)-YC(1)\*YC(J)  
208 DDDD6=-XC(1)\*PPPPYC(J)+YC(1)\*PPPPXC(J)+4.0\*XC(1)\*PPXC(J)+4.0\*YC(1)  
209 \$)\*PPPPYC(J)-6.0\*YC(1)\*PPXC(J)+6.0\*XC(1)\*PPYC(J)-4.0\*XC(1)\*PXC(J)-4.  
210 \$0\*YC(1)\*PYC(J)+YC(1)\*XC(J)-XC(1)\*YC(J)  
211 R(4)=COS(AL1(J))\*EDDD5+SIN(AL1(J))\*DDDD6+XC(J)\*PPPPXC(J)+YC(J)\*PP  
212 \$PYC(J)+4.0\*PXC(J)\*PPPPXC(J)+4.0\*PYC(J)\*PPPPYC(J)+3.0\*PPXC(J)\*PPXC(J)  
213 \$+3.0\*PPYC(J)\*PPYC(J)  
214 P(1)=SIN(AL1(J))  
215 P(2)=COS(AL1(J))  
216 P(3)=-SIN(AL1(J))

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CARD
217      P(4)=-COS(AL1(J))
218      Q(1)=COS(AL1(J))
219      Q(2)=-SIN(AL1(J))
220      Q(3)=-COS(AL1(J))
221      Q(4)=SIN(AL1(J))
222      CALL SIMQ(H,R,4,KS)
223      CALL SIMQ(H1,P,4,KS1)
224      CALL SIMQ(H2,Q,4,KS2)
225      WRITE (6,99) KS
226      WRITE (6,99) KS1
227      WRITE (6,99) KS2
228      99 FORMAT (1X,I1)
229      F1=Q(1)*Q(3)+Q(2)*Q(4)
230      F2=P(1)*Q(3)+Q(1)*P(3)+P(2)*Q(4)+Q(2)*P(4)
231      F3=R(1)*Q(3)+Q(1)*R(3)+Q(2)*R(4)+Q(4)*R(2)
232      F4=P(1)*P(3)+P(2)*P(4)
233      F5=P(1)*R(3)+P(3)*R(1)+P(2)*R(4)+P(4)*R(2)-1.0
234      F6=R(1)*R(3)+R(2)*R(4)
235      G1=Q(1)*Q(4)-Q(2)*Q(3)
236      G2=P(1)*Q(4)+Q(1)*P(4)-P(2)*Q(3)-Q(2)*P(3)
237      G3=R(1)*Q(4)+Q(1)*R(4)-R(2)*Q(3)-Q(2)*R(3)-1.0
238      G4=P(1)*P(4)-P(2)*P(3)
239      G5=P(1)*R(4)+R(1)*P(4)-P(2)*R(3)-P(3)*R(2)
240      G6=R(1)*R(4)-R(2)*R(3)
241      XCOF(1)=F1*F1*G6*G6+G3*G3*F1*F6-G3*G6*F1*F3+G1*G6*F3*F3+G1*G1*F6*F
242      $6-G1*G3*F3*F6-2.0*G1*G6*F1*F6
243      XCOF(2)=2.0*G5*G6*F1*F1+2.0*G2*G3*F1*F6+G3*G3*F1*F5-G3*G6*F1*F2-G2
244      $*G6*F1*F3-G3*G5*F1*F3+2.0*G1*G6*F2*F3+G1*G5*F3*F3*2.0*G1*G1*F5*F6-
245      $G1*G3*F2*F6-G1*G2*F3*F6-G1*G3*F3*F5-2.0*G1*G5*F1*F6-2.0*G1*G6*F1*F
246      $5
247      XCOF(3)=2.0*G4*G6*F1*F1+G5*G5*F1*F1+G2*G2*F1*F6+2.0*G2*G3*F1*F5+G3
248      $*G3*F1*F4-G2*G6*F1*F2-G3*G5*F1*F2-G2*G5*F1*F3-G3*G4*F1*F3+G1*G6*F2
249      $*F2+2.0*G1*G5*F2*F3+G1*G4*F3*F3+2.0*G1*G1*F4*F6+G1*G1*F5*F5-G1*G2*
250      $F2*F6-G1*G3*F2*F5-G1*G2*F3*F5-G1*G3*F3*F4-2.0*G1*G6*F1*F4-2.0*G1*G
251      $5*F1*F5-2.0*G1*G4*F1*F6
252      XCOF(4)=2.0*G4*G5*F1*F1+G2*G2*F1*F5+2.0*G2*G3*F1*F4-G2*G5*F1*F2-G3
253      $*G4*F1*F2-G2*G4*F1*F3+G1*G5*F2*F2+2.0*G1*G4*F2*F3+2.0*G1*G1*F4*F5-
254      $G1*G2*F2*F5-G1*G3*F2*F4-G1*G2*F3*F4-2.0*G1*G5*F1*F4-2.0*G1*G4*F1*F
255      $5
256      XCOF(5)=G4*G4*F1*F1+G2*G2*F1*F4-G2*G4*F1*F2+G1*G4*F2*F2+G1*G1*F4*F4
257      $4-G1*G2*F2*F4-2.0*G1*G4*F1*F4
258      CALL POLRT(XCOF,COF,4,ROOTR,ROOTI,IER)
259      WRITE (6,109)
260      109 FORMAT (/ ,27X,9F L1 ROOTS,/)
261      DO 110 I=1,4
262      110 WRITE (6,112) ROOTR(I),ROOTI(I)
263      112 FORMAT (17X,E13.5,10X,E13.5,/)
264      WRITE (6,114) IER
265      114 FORMAT (1X,I1)
266      WRITE (6,106)
267      106 FORMAT (1H1,/)
268      DO 125 I=1,4
269      125 DO 215 I=1,4
270      215 DO 269 I=1,4
271      269      OA=F1
272      270      OB=F2*ROOTR(I)+F3
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0000000011111111122222222233333333334444444445555555556666666667777777778  
 12345678901234567890123456789012345678901234567890123456789012345678901234567890

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CARD
271      QC=F4*RCOTR(I)*ROCTR(I)+F5*ROOTR(I)+F6
272      OD=G1
273      OE=G2*RODTR(I)+G3
274      OF=G4*ROCTR(I)*ROOTR(I)+G5*ROOTR(I)+G6
275      IF (ABS(ROOTI(I)).LT.0.000001) ROOTI(I)=0.0
276      IF (ROOTI(I)) 200,130,200
277      200 WRITE (6,201)
278      201 FORMAT (5X,18H RCOT IS IMAGINARY,/)
279      GO TO 215
280      130 SA=OB*OB-4.0*OA*OC
281      IF (SA) 200,140,140
282      140 ROOTA1=(-OB+SQRT(SA))/(2.0*OA)
283      ROOTA2=(-OB-SQRT(SA))/(2.0*OA)
284      SB=OE*OE-4.0*CD*CF
285      IF (SB) 200,150,150
286      150 ROOTB1=(-OE+SQRT(SB))/(2.0*OD)
287      ROOTB2=(-OE-SQRT(SB))/(2.0*OD)
288      WRITE (6,155) ROOTA1,ROOTA2,ROOTB1,ROOTB2
289      155 FORMAT (/ ,13X,8H ROOTA1=,F18.6,5X,8H ROOTA2=,F18.6,/,13X,8H ROOTB1
290      $=,F18.6,5X,8H RCCTB2=,F18.6,/)
291      IF (ABS(ROOTA1-ROOTB1).LT.0.05) GO TO 160
292      IF (ABS(ROOTA1-ROOTB2).LT.0.05) GO TO 160
293      IF (ABS(ROOTA2-ROOTB1).LT.0.05) GO TO 170
294      IF (ABS(ROOTA2-ROOTB2).LT.0.05) GO TO 170
295      GO TO 215
296      160 WRITE (6,180) ROOTA1
297      180 FORMAT (2X,16H SECCND ROOT IS ,F20.5,/)
298      ROOT=ROOTA1
299      GO TO 210
300      170 WRITE (6,180) RCCTA2
301      ROOT=ROOTA2
302      210 WRITE (6,212)
303      212 FORMAT (14X,13H CENTER POINT,31X,13H CIRCLE POINT,/,8X,3H XA,19X,3
304      $H YA,19X,3H XB,19X,3H YB,/)
305      XA(I)=R(1)+P(1)*ROCTR(I)+Q(1)*ROOT
306      YA(I)=R(2)+P(2)*ROCTR(I)+Q(2)*ROOT
307      XB(I)=R(3)+P(3)*ROCTR(I)+Q(3)*ROOT
308      YB(I)=R(4)+P(4)*ROCTR(I)+Q(4)*ROOT
309      WRITE (6,230) XA(I),YA(I),XB(I),YB(I)
310      230 FORMAT (4F20.8,/,33X,20H XXXXXXXXXXXXXXXXXXXX,/)
311      215 CONTINUE
312      STOP
313      END

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VITA 3

William Richard Coutant, Jr.

Candidate for the Degree of

Master of Science

Thesis: SYNTHESIS OF STEPHENSON TYPE II SIX-LINK FUNCTION GENERATOR  
FOR FINITELY AND INFINITESIMALLY SEPARATED POSITIONS

Major Field: Mechanical Engineering

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