SYNTHESIS OF STEPHENSON TYPE II SIX-LINK FUNCTION GENERATOR FOR FINITELY AND INFINITESIMALLY SEPARATED POSITIONS

By

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PREFACE

The purpose of this study was the development of a method for synthesizing the Stephenson Type II six-link function generator for both finite and infinitesimally separated positions. That is, both the positions of the input and output link and the velocity, acceleration, jerk and kerk ratios of the input to the output link are possible design parameters. The procedure developed was incorporated into computer programs for ease of synthesis.

I wish to express my appreciation to my thesis adviser, Dr. A. H. Soni, for his encouragement, understanding and expertise in the field of mechanisms. Special thanks go to Dilip Kohli for his valuable advice. The committee members, Dr. M. Mamoun and Dr. Larry D. Zirkle, deserve recognition for giving both time and suggestions.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

One of the reasons for the existence of the field of mechanisms is the need for production of non-linear motion. Because the function generating mechanism can produce non-linear motion, it is a vital tool for industry. Therefore, procedures have been developed for synthesizing the planar six-link function generator.

McLarnan (1)* discussed the procedure for synthesizing six-link mechanisms with complex numbers and numerical techniques for from 6 to 11 finite points. Soni, Varma and Juneja (2) synthesized the six-link mechanism for three finitely separated positions, while Myklebust and Tesar (3) synthesized for five positions. Kaufman (4) synthesized for five positions, correlating coupler motion with input crank rotations. Kim, Hamid and Soni (5) synthesized the six-link mechanism for point path generation. Soni and others (6) made use of the matrix method of synthesis and synthesized for eleven precision positions by numerical methods.

The studies mentioned above indicate there is a need for a clear method for synthesizing the six-link function generator for all combinations of finitely and infinitesimally separated positions when one is

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*Numbers in parentheses refer to numbered references in the Bibliography.

obtaining a closed form solution. The matrix method set forth by Suh and Radcliffe (7) proves to be concise and easily adaptable in going from finitely separated position synthesis to infinitesimally separated position synthesis. It was chosen for this thesis work because the equations used for finitely separated position synthesis can be differentiated to obtain infinitesimally separated position synthesis of any degree with minimal change in synthesis procedure.

One may question why the six-link mechanism is practical, having two more linkages and three more revolute pairs than the four-link mechanism. After all, these add to the cost and maintenance involved for industrial practicality. But when one looks at the six-link mechanism's advantages of extra mobility, versatility and compactness over the four-link mechanism, it can be seen that the six-link mechanism is a necessary tool for industry.

All the six-link mechanisms with one degree of freedom come from two kinematic chains. These are shown in Figure 1. These two chains result in five six-link mechanisms, shown in Figure 2.

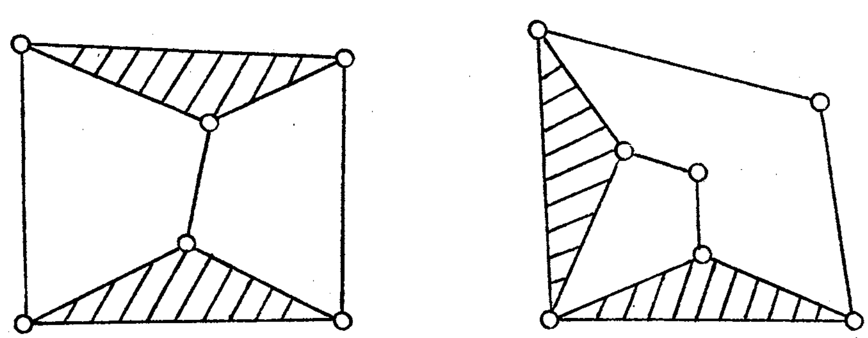

Figure 1. Six-Link Kinematic Chains

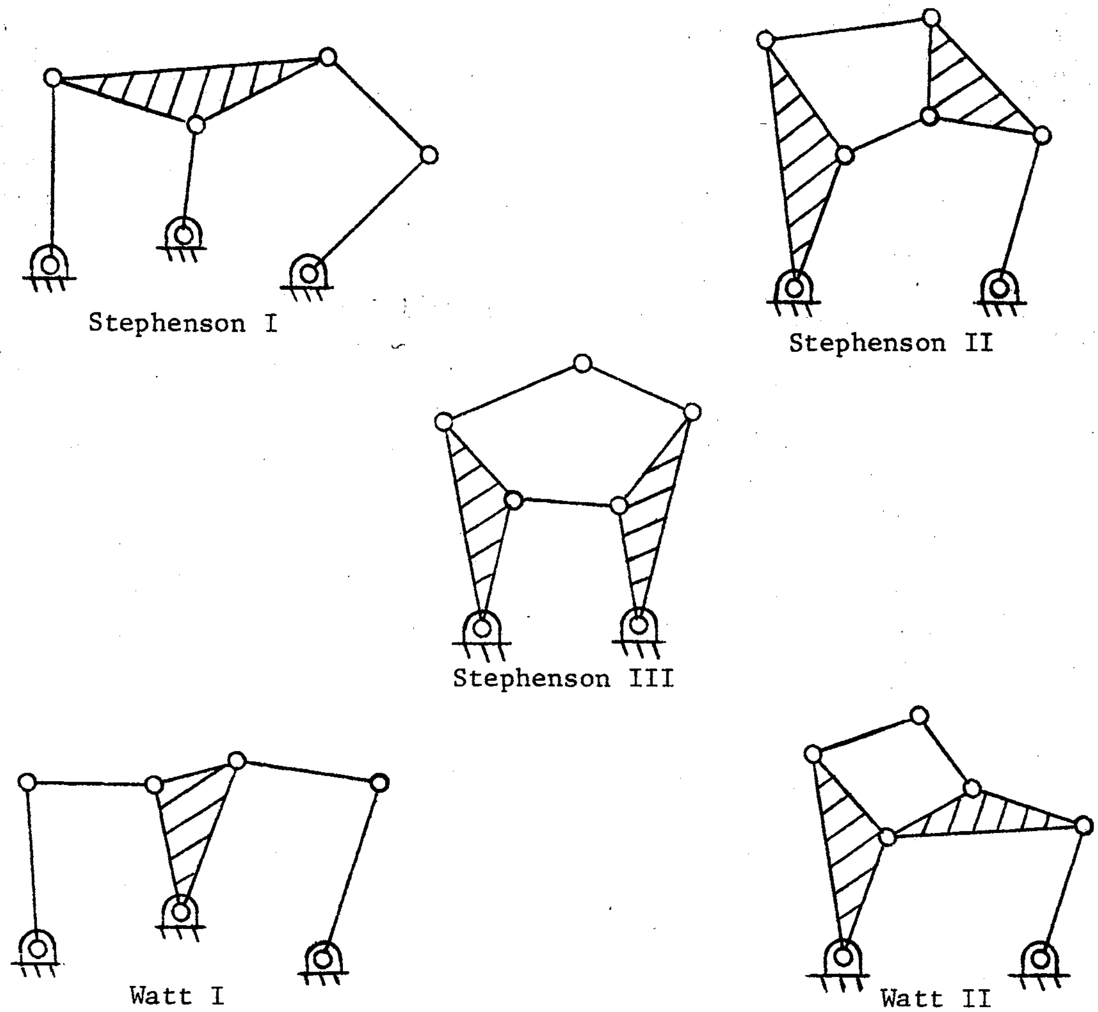

Figure 2. Five Six-Link Mechanisms

As can be seen from Figure 2, the Stephenson Type III mechanism and the Watt Type II mechanism have a basic four-link structure which limits their function generating capabilities to that of a four-link mechanism. The Watt Type I and Stephenson Type I mechanisms have been studied for all possible types of function generation problems. This is because they consist of the four-link mechanism with two binary links connected in series to them. The Stephenson Type I mechanism has the two binary links connected to the coupler point of the four-link mechanism, reducing the function generation problem to that of combining the coupler point motion of the four-link mechanism to the placement and length of the two binary links. For the case of the Watt Type I mechanism the desired function generation is obtained by connecting the output of the first four-link mechanism to the input of the second four-link mechanism. The Stephenson Type II mechanism has the same function generation flexibility as the Stephenson Type I and the Watt Type I mechanisms, but it has the advantage of having only two grounded revolute pairs, thus being applicable to a variety of situations in which three grounded revolute pairs would be impractical or unable to meet design criteria. However, a generalized method of synthesis for the Stephenson Type II mechanism has not been developed.

This thesis involves developing a closed form solution for the synthesis of the Stephenson Type II six-link mechanism function generator for five positions. The matrix approach to synthesis set forth by Suh and Radcliffe (7) and later developed by Kohli and Soni (8) will be used along with the principle of inversion. In order to specify the velocity, acceleration, jerk and kerk (time derivative of jerk) ratios of output link to input link, the principle of infinitesimally separated positions introduced by Mueller (9) and developed by Bottema (10) and Tesar (11, 12, 13) was used.

This thesis will set forth a method of synthesizing the Stephenson Type II six-link mechanism for five precision positions of function generation. Five precision positions were chosen because five is the maximum number of precision positions for which the synthesis equations can be made linear. From six to a maximum of eleven precision positions the synthesis equations are non-linear and have to be solved by numerical techniques. This method can have the problem of convergence, and the resulting mechanism is the closest solution--not necessarily the exact solution. For three precision positions the design equations are linear and can be solved directly, while for four and five positions the principle of linear superposition, discussed by Mohan Rao and Sandor (14) must be utilized in order to make the equations linear. Five precision positions obviously give the designer more flexibility and are necessary when considering the additional design specifications of velocity, acceleration, jerk and kerk ratios.

Tesar (13) uses a set of nomenclature for describing the possible combinations of finitely separated and infinitesimally separated displacements for five positions. All the possible combinations are:

P-P-P-P-P			
PP-P-P-P	P-PP-P-P	P-P-PP-P	P-P-P-PP
PP-PP-P	PP-P-PP	P-PP-PP	
PPP-P-P	P-PPP-P	P-P-PPP	
PPP-PP	PP-PPP		
PPPP-P	P-PPPP		
PPPPP			

Dashes indicate finitely separated points and no dash indicates infinitesimally separated points. This thesis presents an analytical method of synthesizing for all these motions. These combinations can be used to obtain a wide variety of function generation motions.

The synthesis procedure used in this thesis involves, first, using the principle of inversion to transform the synthesis problem into a rigid body guidance four-link mechanism synthesis problem. The matrix approach is used to design a closed form solution for the rigid body guidance problem. The result is the designed six-link mechanism. The same procedure is then used to synthesize the Stephenson Type II six-link mechanism for infinitesimally separated positions.

One motivation behind this thesis has been to give industry a simple method for using the six-link mechanism in specific design situations. To do this, computer programs were written to give solutions for any one of the above motion programs.

CHAPTER II

MATRIX METHOD OF SYNTHESIS FOR FINITELY SEPARATED POSITIONS

Mechanisms in one form or another have been used for many hundreds of years to obtain mechanical motion. As engineering and mathematics advanced, it became desirable to develop mathematical methods to synthesize mechanisms to perform desired motions. The computer opened up a whole new field of possible synthesis methods.

One of these methods was set forth by Suh (7), whose paper presents a method for using a generalized displacement matrix to describe rigid body motion. A rigid body is specified when a point on the rigid body is known in a specified coordinate system, and when the angle of rotation of the rigid body with respect to the point is known. As the rigid body executes planar motion these two values are specified as in Figure 3. The points in the rigid body are designated as C_1, C_2, and C_3, and the rotation of the rigid body between positions C_1 and C_2 is α_{12}, and between C_1 and C_3 is α_{13}. Point B_n is any point on the rigid body. The positions of B_2 and B_3 are described by multiplying the generalized displacement matrix times the first position matrix of B_1.

$$[b_n] = [D_{1n}]\ [b_1] \qquad (2\text{-}1)$$

The generalized displacement matrix is given by

$$[D_{1n}] = \begin{bmatrix} \cos\alpha_{1n} & -\sin\alpha_{1n} & X_{cn} - X_{c1}\cos\alpha_{1n} + Y_{c1}\sin\alpha_{1n} \\ \sin\alpha_{1n} & \cos\alpha_{1n} & Y_{cn} - X_{c1}\sin\alpha_{1n} - Y_{c1}\cos\alpha_{in} \\ 0 & 0 & 1 \end{bmatrix}. \quad (2\text{-}2)$$

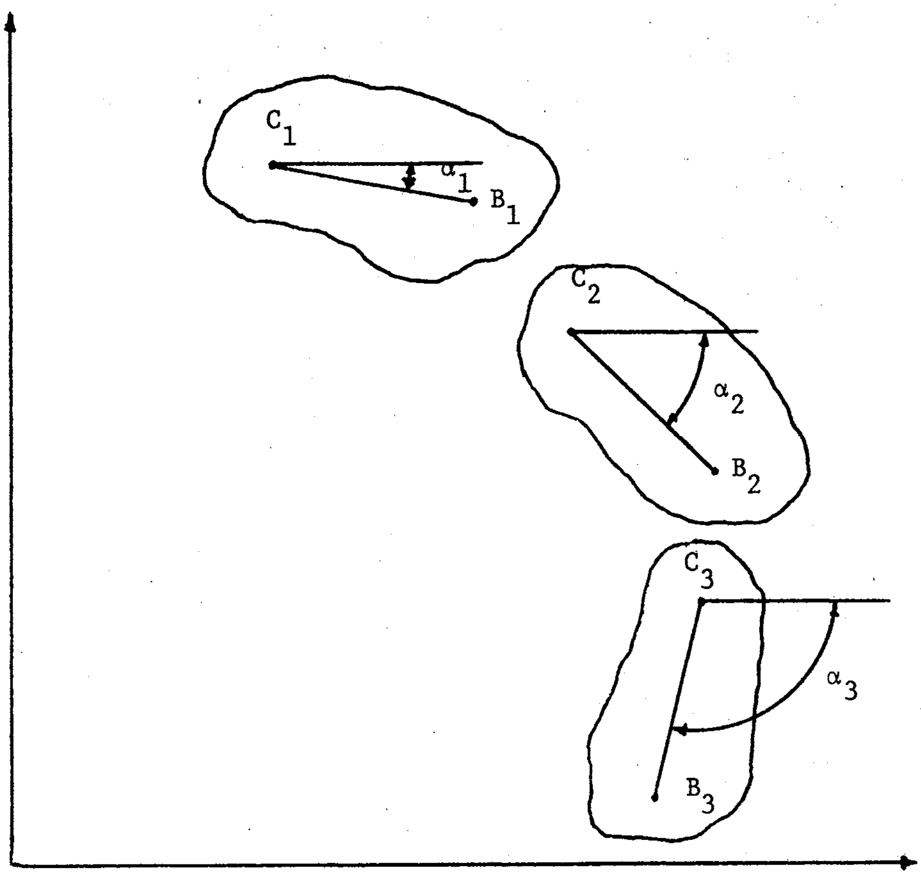

Figure 3. Description of Rigid Body Motion

According to the Burmester Theory for three and four finite positions of the rigid body there are an infinite number of circle points (designated as B_n) on the rigid body which follow a circular path. For five finite positions of the rigid body there are a maximum of four possible points on the rigid body that lie in a circle. A link with one pivot connected to the point B_n on the rigid body and another pivot connected to the fixed pivot A can be described by the equation of a circle, where (X_A, Y_A) are coordinates of fixed pivot A.

$$(X_{bn}-X_A)^2 + (Y_{bn}-Y_A)^2 = (X_{b1}-X_A)^2 + (Y_{b1}-Y_A)^2 \qquad (2\text{-}3)$$

The design equation is obtained by the substitution of Equation (2-1) into Equation (2-3).

For three finitely separated positions there are two design equations that are linear if any two of the four variables X_A, Y_A, X_{B1} and Y_{B1} are assumed. For four finitely separated positions there are three design equations; therefore, one of the four variables must be assumed. For five finitely separated positions there are four design equations and all four variables are to be determined. In the last two cases the equations are non-linear but can be made linear by applying the principle of linear superposition.

CHAPTER III

MATRIX METHOD OF SYNTHESIS FOR INFINITESIMALLY SEPARATED POSITIONS

In the previous chapter the matrix method of synthesizing a four-link mechanism for finitely separated positions of point C on the rigid body was discussed. The matrix method can also be used to synthesize the four-link mechanism for infinitesimally separated positions of point C on the rigid body. In order to utilize this method, a new matrix has been developed by Soni (15) which, when multiplied by a finite position of C, results in the infinitesimally separated position of C. To do this, point C on the rigid body and point B, a circle point, are described by vectors as in Figure 4.

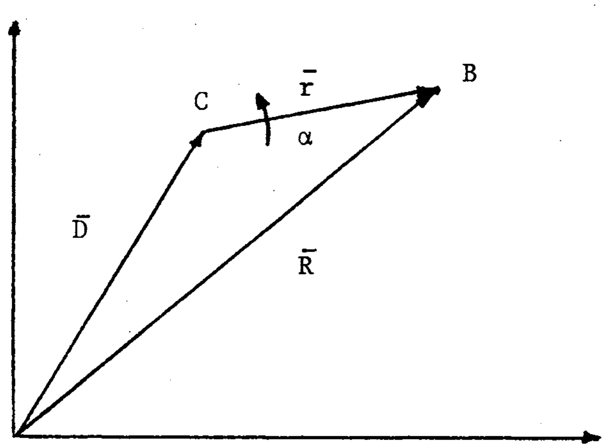

Figure 4. Vector Description of Rigid Body

Since $\bar{R} = \bar{D} + \bar{r}$, the velocity of point B is given by

$$\frac{d\bar{R}}{dt} = \frac{d\bar{D}}{dt} + \frac{d\alpha}{dt}\,\bar{K} \times \bar{r} \tag{3-1}$$

where

$$\begin{aligned} \bar{r} &= (X_B - X_C)\,\bar{i} + (Y_B - Y_C)\,\bar{j} \\ \bar{D} &= X_C\bar{i} + Y_C\bar{j} \\ \bar{R} &= X_B\bar{i} + Y_B\bar{j}\ . \end{aligned} \tag{3-2}$$

Substituting Equations (3-2) into Equation (3-1) results in

$$\frac{dX_B}{dt}\,\bar{i} + \frac{dY_B}{dt}\,\bar{j} = \frac{dX_C}{dt}\,\bar{i} + \frac{dY_C}{dt}\,\bar{j} + \bar{K}\frac{d\alpha}{dt} \times [(X_B - X_C)\bar{i} + (Y_B - Y_C)\bar{j}]. \tag{3-3}$$

By changing the independent parameter from t to α Equation (3-3) becomes

$$\frac{dX_B}{d\alpha}\,\bar{i} + \frac{dY_B}{d\alpha}\,\bar{j} = \frac{dX_C}{d\alpha}\,\bar{i} + \frac{dY_C}{d\alpha}\,\bar{j} + (X_B - X_C)\,\bar{j} - (Y_B - Y_C)\,\bar{i}. \tag{3-4}$$

By separating the i and j components Equation (3-4) becomes

$$\begin{aligned} \frac{dX_B}{d\alpha} &= \frac{dX_C}{d\alpha} - (Y_B - Y_C) \\ \frac{dY_B}{d\alpha} &= \frac{dY_C}{d\alpha} + (X_B - X_C). \end{aligned} \tag{3-5}$$

For the n^{th} position, in matrix form this becomes

$$\begin{bmatrix} \dot{X}_{Bn} \\ \dot{Y}_{Bn} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & \dfrac{dX_{Cn}}{d\alpha_{1n}} + Y_{Cn} \\ 1 & 0 & \dfrac{dY_{Cn}}{d\alpha_{1n}} - X_{Cn} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{Bn} \\ Y_{Bn} \\ 1 \end{bmatrix}. \tag{3-6}$$

In order to develop the first infinitesimally separated position synthesis equation, Equations (2-1) and (3-6) must be substituted into the derivative of Equation (2-3).

$$2\ (X_{Bn}-X_A)\ \dot{X}_{Bn} + 2\ (Y_{Bn}-Y_A)\ \dot{Y}_{Bn} = 0 \tag{3-7}$$

In order to find higher orders of infinitesimally separated positions, Equations (3-5) should be differentiated successively. The second order infinitesimally separated position is

$$\begin{aligned} \frac{d^2X_B}{d\alpha^2} &= \frac{d^2X_C}{d\alpha^2} + \frac{dY_C}{d\alpha} - \frac{dY_B}{d\alpha} \\ \frac{d^2Y_B}{d\alpha^2} &= \frac{d^2Y_C}{d\alpha^2} + \frac{dX_B}{d\alpha} - \frac{dX_C}{d\alpha}\ . \end{aligned} \tag{3-8}$$

Substituting Equations (3-5) into Equations (3-8) results in

$$\begin{aligned} \frac{d^2X_B}{d\alpha^2} &= \frac{d^2X_C}{d\alpha^2} + X_C - X_B \\ \frac{d^2Y_B}{d\alpha^2} &= \frac{d^2Y_C}{d\alpha^2} + Y_C - Y_B\ . \end{aligned} \tag{3-9}$$

The matrix equation is

$$\begin{bmatrix} \ddot{X}_{Bn} \\ \ddot{Y}_{Bn} \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & \dfrac{d^2X_{Cn}}{d\alpha_{1n}^2} + X_{Cn} \\ 0 & -1 & \dfrac{d^2Y_{Cn}}{d\alpha_{1n}^2} + Y_{Cn} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{Bn} \\ Y_{Bn} \\ 1 \end{bmatrix}. \tag{3-10}$$

Following the same procedure, the third order infinitesimally separated position matrix equation is

$$\begin{bmatrix} \dddot{X}_{Bn} \\ \dddot{Y}_{Bn} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & \dfrac{d^3X_{Cn}}{d\alpha_{1n}^3} - Y_{Cn} \\ -1 & 0 & \dfrac{d^3Y_{Cn}}{d\alpha_{1n}^3} + X_{Cn} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{Bn} \\ Y_{Bn} \\ 1 \end{bmatrix}. \tag{3-11}$$

and the fourth order infinitesimally separated position matrix is

$$\begin{bmatrix} \ddddot{X}_{Bn} \\ \ddddot{Y}_{Bn} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \dfrac{d^4X_{Cn}}{d\alpha_{1n}^4} - X_{Cn} \\ 0 & 1 & \dfrac{d^4Y_{Cn}}{d\alpha_{1n}^4} - Y_{Cn} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{Bn} \\ Y_{Bn} \\ 1 \end{bmatrix} . \qquad (3\text{-}12)$$

The third and fourth infinitesimally separated position design equations are obtained by substituting the appropriate matrix equation into the derivatives of Equation (3-7).

CHAPTER IV

SYNTHESIS PROCEDURE FOR FIVE FINITELY SEPARATED POSITIONS

For clarity's sake, a set of nomenclature will be established. The Stephenson Type II six-link mechanism is of the form shown in Figure 5, and will have revolute pairs at A, B, C, D, and E and grounded revolute pairs at M and Q. The input link will be the ternary link MAD while the output will be the link QC. Changes in rotation of link 1 from its first position will be designated as θ_{1n} while changes in rotation with respect to the first position of the output will be Φ_{1n}. Also, the rotation displacements of the ternary link about point C with respect to the first position will be α_{1n}.

To produce a specified function generation the synthesis procedure was changed to a rigid body guidance problem by means of inverting the mechanism. Inversion was taken about ternary link 1. Holding link 1 fixed, the once grounded link MQ was rotated a minus θ_{in} direction for each position of desired input rotation. At the same time, link QC was rotated at an angle $(\Phi_{1n}-\theta_{1n})$ for each position (Figure 6). The displacement equation which gives the positions of X_C, Y_C in terms of $-\theta_{1n}$ and $(\Phi_{1n}-\theta_{1n})$ is

$$X_{Cn} = (X_{C1}-X_{Q1}) \cos (\Phi_{1n}-\theta_{1n}) - (Y_{C1}-Y_{Q1}) \sin (\Phi_{1n}-\theta_{1n}) + X_{Q1} \cos (-\theta_{1n})-Y_{Q1}\sin (-\theta_{1n})$$

$$Y_{Cn} = (X_{C1}-X_{Q1}) \sin (\Phi_{1n}-\theta_{1n}) + (Y_{C1}-Y_{Q1}) \cos (\Phi_{1n}-\theta_{1n}) + X_{Q1}\sin (-\theta_{1n}) + Y_{Q1} \cos (-\theta_{1n}). \qquad (4\text{-}1)$$

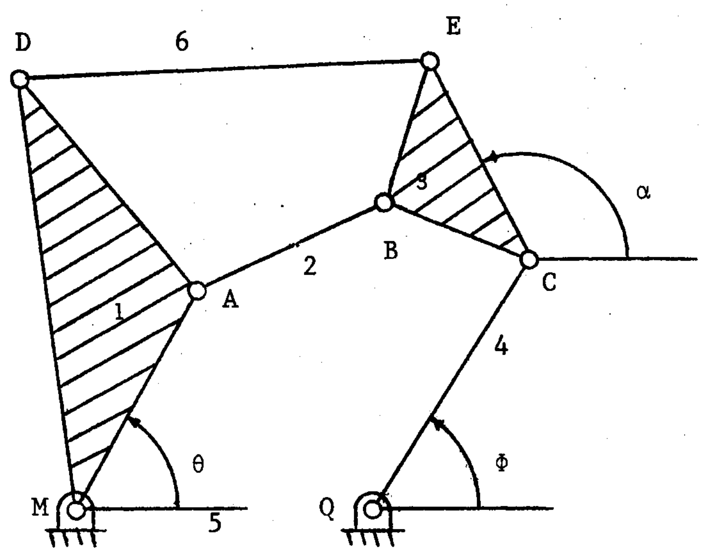

Figure 5. Stephenson Type II Six-Link Mechanism

In order to define the positions of X_C, Y_C in the inverted positions, the length of QC must be assumed in relation to the unit length of MQ, and the initial angle Φ_1 must be given. These two parameters are totally arbitrary and can be varied to obtain the optimum mechanism for a desired function, or can be specified to meet additional design criteria.

With the values of X_C, Y_C in the inverted positions known, the synthesis procedure now becomes a four-link rigid body guidance problem. By using the matrix approach described in Chapter II, a matrix was obtained which describes the positions of points B and E of the ternary link as the rigid body passes through the points X_C, Y_C and rotates through the inverted displacement angles of α_{1n}.

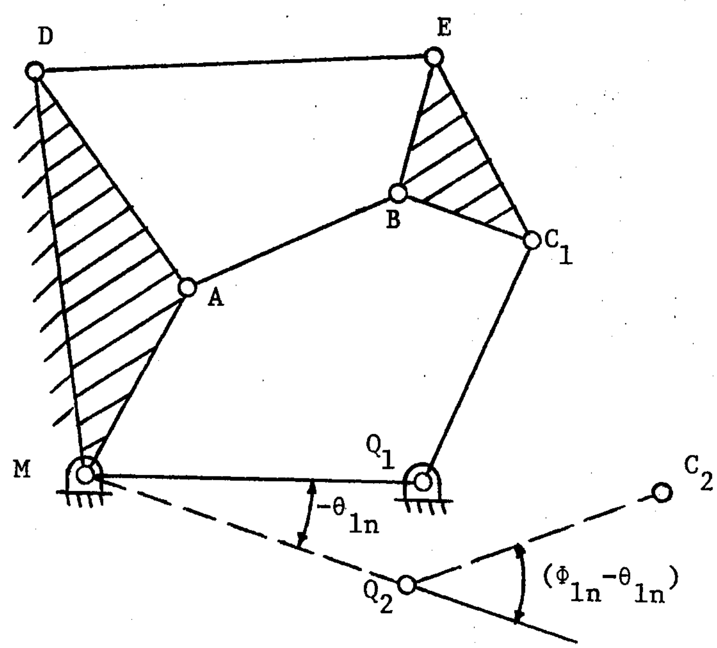

Figure 6. Inversion About Link 1

The values of α_{1n} that the ternary link is to rotate are also assumed values and can be used to the designer's advantage when synthesizing the six-link function generator. That is, if the designer has a problem in which the ternary link rotations have to be specified, this can be done; or if it is totally arbitrary, its variations will produce different six-link mechanisms. As explained in Chapter I, the four-link rigid body guidance synthesis equation (4-2) is non-linear when five positions are desired.

$$X_{A1}\,[C_{1n}] + Y_{A1}\,[C_{2n}] + X_{B1}\,[C_{3n}] + Y_{B1}\,[C_{4n}] = [1-\cos\alpha_{1n}]$$
$$[X_{A1}X_{B1} + Y_{A1}Y_{B1}] + \sin\alpha_{1n}\,[X_{A1}Y_{B1}-X_{B1}Y_{A1}] + \tfrac{1}{2}[C_{1n}^2 + C_{2n}^2] \qquad (4\text{-}2)$$

where

$$C_{1n} = X_{Cn} - X_{C1}\cos\alpha_{1n} + Y_{C1}\sin\alpha_{1n}$$
$$C_{2n} = Y_{Cn} - Y_{C1}\cos\alpha_{1n} - X_{C1}\sin\alpha_{1n}$$
$$C_{3n} = X_{C1} - X_{Cn}\cos\alpha_{1n} - Y_{Cn}\sin\alpha_{1n}$$
$$C_{4n} = Y_{C1} - Y_{Cn}\cos\alpha_{1n} + X_{Cn}\sin\alpha_{1n}.$$

To solve this equation in closed form for five positions, the principle of linear superposition must be used. Letting

$$\lambda_1 = X_{A1}\,X_{B1} + Y_{A1}\,Y_{B1} \quad \text{and}$$
$$\lambda_2 = X_{A1}\,Y_{B1} - X_{B1}\,Y_{A1} \qquad (4\text{-}3)$$

Equation (4-2) can be divided into three linear equations (4-4) which can be solved for five positions simultaneously.

$$r_1[C_{1n}] + r_2[C_{2n}] + r_3[C_{3n}] + r_4[C_{4n}] = \tfrac{1}{2}[C_{1n}^2 + C_{2n}^2]$$
$$P_1[C_{1n}] + P_2[C_{2n}] + P_3[C_{3n}] + P_4[C_{4n}] = 1 - \cos\alpha_{1n} \qquad (4\text{-}4)$$
$$q_1[C_{1n}] + q_2[C_{2n}] + q_3[C_{3n}] + q_4[C_{4n}] = \sin\alpha_{1n}$$
$$n = 2, 3, 4, 5$$

where

$$X_{A1} = r_1 + \lambda_1\,P_1 + \lambda_2\,q_1$$
$$Y_{A1} = r_2 + \lambda_1\,P_2 + \lambda_2\,q_2$$
$$X_{B1} = r_3 + \lambda_1\,P_3 + \lambda_2\,q_3 \qquad (4\text{-}5)$$
$$Y_{B1} = r_4 + \lambda_1\,P_4 + \lambda_2\,q_4.$$

Substituting Equations (4-5) into the compatibility conditions given by Equations (4-3) will result in

$$F_1 \lambda_2^2 + (F_2 \lambda_1 + F_3) \lambda_2 + F_4 \lambda_1^2 + F_5 \lambda_1 + F_6 = 0$$
$$G_1 \lambda_2^2 + (G_2 \lambda_1 + G_3) \lambda_2 + G_4 \lambda_1^2 + G_5 \lambda_1 + G_6 = 0 \qquad (4\text{-}6)$$

where

$$F_1 = q_1 q_3 + q_2 q_4$$
$$F_2 = p_1 q_3 + q_1 p_3 + p_2 q_4 + q_2 p_4$$
$$F_3 = r_1 q_3 + q_1 r_3 + q_2 r_4 + q_4 r_2$$
$$F_4 = p_1 p_3 + p_2 p_4$$
$$F_5 = p_1 r_3 + p_3 r_1 + p_2 r_4 + p_4 r_2 - 1$$
$$F_6 = r_1 r_3 + r_2 r_4$$
$$(4\text{-}7)$$
$$G_1 = q_1 q_4 - q_2 q_3$$
$$G_2 = p_1 q_4 + q_1 p_4 - p_2 q_3 - q_2 p_3$$
$$G_3 = r_1 q_4 + q_1 r_4 - r_2 q_3 - q_2 r_3 - 1$$
$$G_4 = p_1 p_4 - p_2 p_3$$
$$G_5 = p_1 r_4 + r_1 p_4 - p_2 r_3 - p_3 r_2$$
$$G_6 = r_1 r_4 - r_2 r_3.$$

Using the Sylvester technique results in a fourth order polynomial, Equation (4-8), the roots of which are the values of λ_1.

$$L_1 \lambda_1^4 + L_2 \lambda_1^3 + L_3 \lambda_1^2 + L_4 \lambda_1 + L_5 = 0 \qquad (4\text{-}8)$$

where

$$L_1 = G_4^2 F_1^2 + G_2^2 F_1 F_4 + G_1 G_4 F_2^2 + G_1^2 F_4^2 - G_2 G_4 F_1 F_2 -$$
$$G_1 G_2 F_2 F_4 - 2\, G_1 G_4 F_1 F_4$$

$$L_2 = 2\, G_4 G_5 F_1^2 + G_2^2 F_1 F_5 + 2\, G_2 G_3 F_1 F_4 - G_2 G_5 F_1 F_2 -$$
$$G_3 G_4 F_1 F_2 - G_2 G_4 F_1 F_4 + G_1 G_5 F_2^2 + 2\, G_1 G_4 F_2 F_3 + 2$$
$$G_1^2 F_4 F_5 - G_1 G_2 F_2 F_5 - G_1 G_3 F_2 F_4 - G_1 G_2 F_3 F_4 - 2\, G_1 G_5 F_1 F_4$$
$$- 2\, G_1 G_4 F_1 F_5$$

$$L_3 = 2\, G_4 G_6 F_1^2 + G_5^2 F_1^2 + G_2^2 F_1 F_6 + 2\, G_2 G_3 F_1 F_5 + G_3^2 F_1 F_4 -$$
$$G_2 G_6 F_1 F_2 - G_3 G_5 F_1 F_2 - G_2 G_5 F_1 F_3 - G_3 G_4 F_1 F_3 + G_1 G_6 F_2^2 +$$
$$2\, G_1 G_5 F_2 F_3 + G_1 G_4 F_3^2 + 2\, G_1^2 F_4 F_6 + G_1^2 F_5^2 - G_1 G_2 F_2 F_6 -$$
$$G_1 G_3 F_2 F_5 - G_1 G_2 F_3 F_5 - G_1 G_3 F_3 F_4 - 2\, G_1 G_6 F_1 F_4 - 2\, G_1 G_5 F_1 F_5$$
$$- 2\, G_1 G_4 F_1 F_6$$

(4-9)

$$L_4 = 2\, G_5 G_6 F_1^2 + 2\, G_2 G_3 F_1 F_6 + G_3^2 F_1 F_5 - G_3 G_6 F_1 F_2 - G_2 G_6 F_1 F_3 -$$
$$G_3 G_5 F_1 F_3 + 2\, G_1 G_6 F_2 F_3 + G_1 G_5 F_3^2 + 2\, G_1^2 F_5 F_6 - G_1 G_3 F_2 F_6 -$$
$$G_1 G_2 F_3 F_6 - G_1 G_3 F_3 F_5 - 2\, G_1 G_5 F_1 F_6 - 2\, G_1 G_6 F_1 F_5$$

$$L_5 = F_1^2 G_6^2 + G_3^2 F_1 F_6 - G_3 G_6 F_1 F_3 + G_1 G_6 F_3^2 + G_1^2 F_6^2 -$$
$$G_1 G_3 F_3 F_6 - 2\, G_1 G_6 F_1 F_6$$

The λ_1 roots are substituted into Equations (4-6), to yield roots for λ_2. With these two roots known, using Equations (4-5), the values of X_{A1}, Y_{A1}, X_{B1} and Y_{B1} can be found.

Since there are a maximum of four possible pairs of λ_1 and λ_2, there are a maximum of four possible solutions for the center point and

$$L_1 = G_4^2 F_1^2 + G_2^2 F_1 F_4 + G_1 G_4 F_2^2 + G_1^2 F_4^2 - G_2 G_4 F_1 F_2 - G_1 G_2 F_2 F_4 - 2 G_1 G_4 F_1 F_4$$

$$L_2 = 2 G_4 G_5 F_1^2 + G_2^2 F_1 F_5 + 2 G_2 G_3 F_1 F_4 - G_2 G_5 F_1 F_2 - G_3 G_4 F_1 F_2 - G_2 G_4 F_1 F_4 + G_1 G_5 F_2^2 + 2 G_1 G_4 F_2 F_3 + 2 G_1^2 F_4 F_5 - G_1 G_2 F_2 F_5 - G_1 G_3 F_2 F_4 - G_1 G_2 F_3 F_4 - 2 G_1 G_5 F_1 F_4 - 2 G_1 G_4 F_1 F_5$$

$$L_3 = 2 G_4 G_6 F_1^2 + G_5^2 F_1^2 + G_2^2 F_1 F_6 + 2 G_2 G_3 F_1 F_5 + G_3^2 F_1 F_4 - G_2 G_6 F_1 F_2 - G_3 G_5 F_1 F_2 - G_2 G_5 F_1 F_3 - G_3 G_4 F_1 F_3 + G_1 G_6 F_2^2 + 2 G_1 G_5 F_2 F_3 + G_1 G_4 F_3^2 + 2 G_1^2 F_4 F_6 + G_1^2 F_5^2 - G_1 G_2 F_2 F_6 - G_1 G_3 F_2 F_5 - G_1 G_2 F_3 F_5 - G_1 G_3 F_3 F_4 - 2 G_1 G_6 F_1 F_4 - 2 G_1 G_5 F_1 F_5 - 2 G_1 G_4 F_1 F_6 \qquad (4\text{-}9)$$

$$L_4 = 2 G_5 G_6 F_1^2 + 2 G_2 G_3 F_1 F_6 + G_3^2 F_1 F_5 - G_3 G_6 F_1 F_2 - G_2 G_6 F_1 F_3 - G_3 G_5 F_1 F_3 + 2 G_1 G_6 F_2 F_3 + G_1 G_5 F_3^2 + 2 G_1^2 F_5 F_6 - G_1 G_3 F_2 F_6 - G_1 G_2 F_3 F_6 - G_1 G_3 F_3 F_5 - 2 G_1 G_5 F_1 F_6 - 2 G_1 G_6 F_1 F_5$$

$$L_5 = F_1^2 G_6^2 + G_3^2 F_1 F_6 - G_3 G_6 F_1 F_3 + G_1 G_6 F_3^2 + G_1^2 F_6^2 - G_1 G_3 F_3 F_6 - 2 G_1 G_6 F_1 F_6$$

The λ_1 roots are substituted into Equations (4-6), to yield roots for λ_2. With these two roots known, using Equations (4-5), the values of X_{A1}, Y_{A1}, X_{B1} and Y_{B1} can be found.

Since there are a maximum of four possible pairs of λ_1 and λ_2, there are a maximum of four possible solutions for the center point and the circle point. The number of solutions depends on whether λ_1 and λ_2 have imaginary roots. Only real roots for λ_1 and λ_2 result in possible solutions. A choice of any two of these center point and circle point combinations will result in a maximum of six solutions for a designed four-link mechanism.

TABLE I

FIVE FINITE POSITIONS

INPUT ROTATION ANGLES ARE	-15.000	-25.000	-40.000	-60.000
OUTPUT ROTATION ANGLES ARE	-20.000	-30.000	-50.000	-75.000
TERNARY LINK ROTATIONS ARE	10.000	20.000	30.000	40.000

OUTPUT LINK IS 1.0 LONG AND AT AN ANGLE OF 100.0 DEGREES

SOLUTION

	X	Y
M	0.00000000	0.00000000
Q	1.00000000	0.00000000
C1	0.82635313	0.98480803

POSSIBLE SOLUTION FOR POINTS A1 AND B1 OR D1 AND E1

CENTER POINT		CIRCLE POINT	
X	Y	X	Y
0.03711897	0.09393525	0.21974057	0.34299600
-5.62751389	-8.39265823	-8.16039371	-5.00552368

ROOT IS IMAGINARY

ROOT IS IMAGINARY

Once this is done, the six-link mechanism is designed in its first position. The center points of the four-link mechanism are now points A and D of the input ternary link, and the corresponding circle points are the points B and E of the moving ternary link. Now by grounding link MQ the desired input-output relationship is obtained. A computer program that follows this procedure is listed in Appendix A, and an example problem is presented in Table I. The resulting six-link mechanism will go through the five desired input and output rotations, while the ternary link goes through a total rotation of $(\alpha_{1n} + \theta_{1n})$. The ternary link rotation is now relative to grounded link MQ.

CHAPTER V

SYNTHESIS PROCEDURE FOR FIRST INFINITESIMALLY SEPARATED POSITIONS

A continuation of the procedure set forth in Chapter IV is the development of a method that will specify the velocity ratio of the input to output at a certain finite position of the input and output links. In Chapter IV it was shown that the synthesis for five positions involved, first, inversion and then four-link rigid body guidance synthesis. The same procedure will be followed for synthesizing the Stephenson Type II six-link mechanism for velocity ratios. The displacement of the output link is a function of the input link, and both are a function of time; that is, $\Phi(t) = f\ (\theta(t))$.

In order to obtain a specified velocity ratio, this relationship must be differentiated with respect to time, resulting in $\frac{d\Phi}{dt} = \frac{d\Phi}{d\theta}\ \frac{d\theta}{dt}$. It can be seen that the infinitesimal displacement $\frac{d\Phi}{d\theta}$, can be specified by the ratio of $\frac{d\Phi}{dt}$ to $\frac{d\theta}{dt}$, both of which are design parameters. For the function $\Phi = f\ (\theta)$, as shown in Figure 7, the infinitesimal displacement is the direction and magnitude of the slope at a point. In other words, the desired velocity ratios are now converted into an infinitesimal displacement problem.

For infinitesimal displacement for five positions, there are two basic combinations.

PP-P-P-P	PP-PP-P
P-PP-P-P	PP-P-PP
P-P-PP-P	P-PP-PP
P-P-P-PP	

That is, the two combinations are specifying four finite positions and an infinitesimal displacement at one of the four finite positions, or specifying three finite positions and an infinitesimal displacement at two of the finite positions.

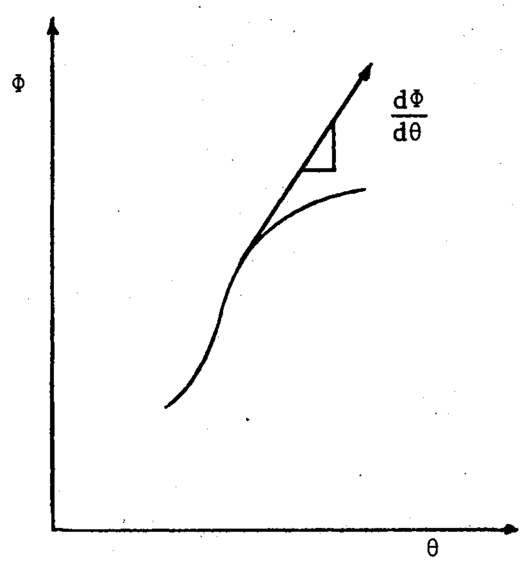

Figure 7. Infinitesimal Displacement

The principle of inversion is still valid for this case. The four finite positions can be defined by the procedure set forth in Chapter IV. To find the infinitesimal displacement of point C, the displacement Equation (4-1) will be differentiated with respect to θ, resulting in

$$\frac{dX_{Cn}}{d\theta_{1n}} = (X_{C1}-X_{Q1})\ [\sin\ (\Phi_{1n}-\theta_{1n}) - \sin\ (\Phi_{1n}-\theta_{1n})\ \frac{d\Phi_{1n}}{d\theta_{1n}}]$$

$$+ (Y_{C1}-Y_{Q1})\ [\cos\ (\Phi_{1n}-\theta_{1n}) - \cos\ (\Phi_{1n}-\theta_{1n})\ \frac{d\Phi_{1n}}{d\theta_{1n}}]$$

$$+ X_{Q1}\ \sin(-\theta_{1n}) + Y_{Q1}\ \cos\ (-\theta_{1n})$$

$$\frac{dY_{Cn}}{d\theta_{1n}} = (X_{C1}-X_{Q1})\ [-\cos\ (\Phi_{1n}-\theta_{1n}) + \cos\ (\Phi_{1n}-\theta_{1n})\ \frac{d\Phi_{1n}}{d\theta_{1n}}] \qquad (5\text{-}1)$$

$$+ (Y_{C1}-Y_{Q1})\ [\sin\ (\Phi_{1n}-\theta_{1n}) - \sin\ (\Phi_{1n}-\theta_{1n})\ \frac{d\Phi_{1n}}{d\theta_{1n}}]$$

$$-X_{Q1}\ \cos\ (-\theta_{1n}) + Y_{Q1}\ \sin(-\theta_{1n}).$$

This results in a value for $\frac{dX_{Cn}}{d\theta_{1n}}$, $\frac{dY_{Cn}}{d\theta_{1n}}$ which is the infinitesimal displacement of point C with respect to θ_{1n} at a certain point. As explained in Chapter III, infinitesimal displacements for rigid body guidance can be obtained by means of the matrix method. The infinitesimal displacement of point C in designing the four-link rigid body guidance mechanism is defined by rotations α_{1n}. In other words, the infinitesimal displacement of point C given by $\frac{dX_{Cn}}{d\alpha_{1n}}$, $\frac{dY_{Cn}}{d\alpha_{1n}}$ is necessary for the four-link synthesis. Since

$$\frac{dX_{Cn}}{d\alpha_{1n}} = \frac{d\theta_{1n}}{d\alpha_{1n}}\frac{dX_{Cn}}{d\theta_{1n}} \quad \text{and} \quad \frac{dY_{Cn}}{d\alpha_{1n}} = \frac{d\theta_{1n}}{d\alpha_{1n}}\frac{dY_{Cn}}{d\theta_{1n}} \qquad (5\text{-}2)$$

the infinitesimal movement of $\frac{d\theta_{1n}}{d\alpha_{1n}}$ must be given. This is arbitrary and can be used to the designer's advantage.

In order to obtain the synthesis equations for synthesizing the four-link mechanism, the principle set forth in Chapter III may be utilized. The matrix Equations (2-1) and (3-6) are substituted into Equation (3-7) to obtain the design Equation (5-3).

$$X_{A1}\left(\frac{dX_{Cn}}{d\alpha_{1n}} + X_{C1}\sin\alpha_{1n} + Y_{C1}\cos\alpha_{1n}\right) +$$

$$Y_{A1}\left(\frac{dY_{Cn}}{d\alpha_{1n}} + Y_{C1}\sin\alpha_{1n} - X_{C1}\cos\alpha_{1n}\right) +$$

$$X_{B1}\left[-\left(\frac{dX_{Cn}}{d\alpha_{1n}} + Y_{Cn}\right)\cos\alpha_{1n} - \left(\frac{dY_{Cn}}{d\alpha_{1n}} - X_{Cn}\right)\sin\alpha_{1n}\right] +$$

$$Y_{B1}\left[-\left(\frac{dY_{Cn}}{d\alpha_{1n}} - X_{Cn}\right)\cos\alpha_{1n} + \left(\frac{dX_{Cn}}{d\alpha_{1n}} + Y_{Cn}\right)\sin\alpha_{1n}\right] = \qquad (5-3)$$

$$\sin\alpha_{1n}\,(X_{A1}X_{B1} + Y_{A1}Y_{B1}) + \cos\alpha_{1n}\,(X_{A1}Y_{B1} - X_{B1}Y_{A1}) -$$

$$\cos\alpha_{1n}\left(X_{C1}\frac{dX_{Cn}}{d\alpha_{1n}} + Y_{C1}\frac{dY_{Cn}}{d\alpha_{1n}} - Y_{C1}X_{Cn} + X_{C1}Y_{Cn}\right) +$$

$$\sin\alpha_{1n}\left(Y_{C1}\frac{dX_{Cn}}{d\alpha_{1n}} - X_{C1}\frac{dY_{Cn}}{d\alpha_{1n}} + X_{C1}X_{Cn} + Y_{C1}Y_{Cn}\right) +$$

$$X_{Cn}\frac{dX_{Cn}}{d\alpha_{1n}} + Y_{Cn}\frac{dY_{Cn}}{d\alpha_{1n}}$$

Please note that at this point a quicker and easier procedure for obtaining this equation may be followed. This procedure will be of particular use when developing higher order infinitesimal displacement synthesis equations. For infinitesimal movement, the changes in the positions of B and E are very small. For displacement analysis the equation of a circle is used in defining the movement of circle point B around center point A. Equation (2-1) is substituted into the equation of a circle, which results in the following equation.

$$f\,(X_{B1}, Y_{B1}, X_{A1}, Y_{A1}, \alpha_{1n}) = 0 \qquad (5-4)$$

In order to obtain the infinitesimal movement, this equation must be differentiated. By means of the Taylor series, neglecting higher order terms, the derivative of this function can be given by Equation (5-5).

$$f\,(X_{B1}, Y_{B1}, X_{A1}, Y_{A1}, \alpha_{1n}) + \frac{\partial f\,(X_{B1}, Y_{B1}, X_{A1}, Y_{A1}, \alpha_{1n})}{\partial \alpha_{1n}} \Delta \alpha_{1n} + \ldots = 0 \qquad (5\text{-}5)$$

The first part has been satisfied by the equation (5-4) and the second part has yet to be satisfied. Therefore, in order to obtain the infinitesimal displacement equation for synthesis, the finite position synthesis Equation (4-2) must be differentiated with respect to α_{1n}. Therefore, the first infinitesimal displacement synthesis equation may be obtained by differentiating the coefficients of the finite displacement synthesis Equation (4-2).

Since Equation (5-3) is of the same form as Equation (4-2), the linear superposition principle allows for Equation (4-2) to be substituted where desired. The four positions result in three displacement equations which become linear after applying the principle of linear superposition. One more equation is needed to solve for the four unknowns. The infinitesimal displacement equation is also linear and has the same unknowns, therefore it is the desired fourth equation. The method used in solving for the four unknowns will be identical to that used for solving the five finite displacement problem in Chapter IV.

A computer program has been written which results in the synthesized six-link Stephenson Type II mechanism for four finite positions and a specified velocity ratio at one of the finite positions. It is listed in Appendix B and an example is presented in Table II. A computer program has also been written for the case of three finite positions and a velocity ratio specified at any two of the finite positions. It is listed in Appendix C, and an example is presented in Table III.

TABLE II

FOUR FINITE POSITIONS AND ONE VELOCITY RATIO
(VELOCITY RATIO IS SPECIFIED AT POINT 4)

INPUT VELOCITY = 1.0 OUTPUT VELOCITY = 2.0

INPUT ROTATION ANGLES ARE	-15.000	-30.000	-45.000
OUTPUT ROTATION ANGLES ARE	-20.000	-45.000	-75.000
TERNARY LINK ROTATIONS ARE	10.000	20.000	30.000

OUTPUT LINK IS 1.0 LONG AND AT AN ANGLE OF 100.0 DEGREES.

FIRST INFINITESIMAL DISPLACEMENTS DPHDTH(J) = 2.0
DTHDAL(J) = 1.0

SOLUTION

	X	Y
M	0.00000000	0.00000000
Q	1.00000000	0.00000000
C1	0.82635210	0.98480770

POSSIBLE SOLUTION FOR POINTS A1 AND B1 OR D1 AND E1

CENTER POINT		CIRCLE POINT	
X	Y	X	Y
-0.22540400	1.59108100	-0.25595210	1.53780500
-0.48618850	1.69824500	-0.40762280	1.74143700

ROOT IS IMAGINARY

ROOT IS IMAGINARY

TABLE III

THREE FINITE POSITIONS AND TWO VELOCITY RATIOS
(VELOCITIES SPECIFIED AT POINTS 1 AND 3)

1 INPUT VELOCITY = 1.0 OUTPUT VELOCITY = 0.0
3 INPUT VELOCITY = 1.0 OUTPUT VELOCITY = 0.0

INPUT ROTATION ANGLES ARE	-15.000	-30.000
OUTPUT ROTATION ANGLES ARE	-20.000	-60.000
TERNARY LINK ROTATIONS ARE	10.000	20.000

OUTPUT LINK IS 1.0 LONG AND AT AN ANGLE OF -30.0 DEGREES.

FIRST INFINITESIMAL DISPLACEMENTS DPHDTH(J) = 0.0
DTHDAL(J) = 1.0

FIRST INFINITESIMAL DISPLACEMENTS DPHDTH(K) = 0.0
DTHDAL(K) = 2.0

SOLUTION

	X	Y
M	0.00000000	0.00000000
Q	1.00000000	0.00000000
C1	1.86602400	-0.49999999

POSSIBLE SOLUTION FOR POINTS A1 AND B1 OR D1 AND E1

CENTER POINT		CIRCLE POINT	
X	Y	X	Y
1.38239300	-0.72231820	1.58071100	-0.74633180
1.34985100	-1.81822200	1.25711400	-1.85000600
1.94092400	-2.69249400	1.79815800	-2.82720700
4.05168700	-1.16569700	4.92417600	-1.61335900

CHAPTER VI

SYNTHESIS PROCEDURE FOR SECOND INFINITESIMALLY SEPARATED POSITIONS

In addition to the velocity ratio at a finite point, another design parameter might be to specify an acceleration ratio for the input and output at the same finite point. The input acceleration is the second derivative of θ with respect to time, while the output acceleration is the second derivative of Φ with respect to time. This ratio can be obtained by differentiating $\Phi(t) = f\ (\theta(t))$ twice. The result is

$$\frac{d^2\Phi}{dt^2} = \frac{d^2\Phi}{d\theta^2} \left(\frac{d\theta}{dt}\right)^2 + \frac{d\Phi}{d\theta}\frac{d^2\theta}{dt^2}. \tag{6-1}$$

To find the second infinitesimal displacement $\frac{d^2\Phi}{d\theta^2}$, all other terms must be specified. For a specific problem, this involves specifying

$\frac{d^2\Phi}{dt^2}$ = the acceleration of the output link,

$\frac{d^2\theta}{dt^2}$ = the acceleration of the input link,

$\frac{d\theta}{dt}$ = the velocity of the input link, and

$\frac{d\Phi}{d\theta}$ = the first infinitesimal displacement.

The term $\frac{d\Phi}{d\theta}$ has already been determined from specifying the velocity ratio at the same finite point.

For five positions the second infinitesimal displacement involves the following class of problems:

PPP-P-P　　PPP-PP
P-PPP-P　　PP-PPP
P-P-PPP

That is, the two combinations are specifying three finite points and the first and second infinitesimal displacements at one of the finite points, or specifying two finite points and first and second infinitesimal displacements at one of the finite points and a first infinitesimal displacement at another finite point.

In order to synthesize the Stephenson Type II six-link mechanism for the above two cases, the same procedure employed in the previous chapters will be used. Inversion will be taken about the input link. The links MQ and QC are rotated according to the principle of inversion to find the finite points X_{Cn}, Y_{Cn}. The first and the second derivatives of the displacement equations are obtained by differentiating Equations (4-1) with respect to θ to find $\frac{dX_{Cn}}{d\theta}$, $\frac{dY_{Cn}}{d\theta}$, $\frac{d^2X_{Cn}}{d\theta^2}$, and $\frac{d^2Y_{Cn}}{d\theta^2}$. The first derivative Equations (5-1) are given in Chapter V and the second derivative equation is

$$\begin{aligned}\frac{d^2X_{Cn}}{d\theta_{1n}^2} = {} & (X_{C1}-X_{Q1}) \Big(-\cos(\Phi_{1n}-\theta_{1n}) + 2\cos(\Phi_{1n}-\theta_{1n}) \frac{d\Phi_{1n}}{d\theta_{1n}} \\ & -\cos(\Phi_{1n}-\theta_{1n}) \left(\frac{d\Phi_{1n}}{d\theta_{1n}}\right)^2 - \sin(\Phi_{1n}-\theta_{1n}) \frac{d^2\Phi_{1n}}{d\theta_{1n}^2}\Big) \\ & + (Y_{C1}-Y_{Q1}) \Big(\sin(\Phi_{1n}-\theta_{1n}) - 2\sin(\Phi_{1n}-\theta_{1n}) \frac{d\Phi_{1n}}{d\theta_{1n}} \\ & + \sin(\Phi_{1n}-\theta_{1n}) \left(\frac{d\Phi_{1n}}{d\theta_{1n}}\right)^2 - \cos(\Phi_{1n}-\theta_{1n}) \frac{d^2\Phi_{1n}}{d\theta_{1n}^2}\Big) \\ & - X_{Q1}\cos(-\theta_{1n}) + Y_{Q1}\sin(-\theta_{1n})\end{aligned} \qquad (6-2)$$

$$\frac{d^2Y_{Cn}}{d\theta_{1n}^2} = (X_{C1}-X_{Q1}) \Big(-\sin(\Phi_{1n}-\theta_{1n}) + 2\sin(\Phi_{1n}-\theta_{1n}) \frac{d\Phi_{1n}}{d\theta_{1n}} -$$

$$\sin(\Phi_{1n}-\theta_{1n})\left(\frac{d\Phi_{1n}}{d\theta_{1n}}\right)^2 + \cos(\Phi_{1n}-\theta_{1n})\frac{d^2\Phi_{1n}}{d\theta_{1n}^2})$$

$$+ (Y_{C1}-Y_{Q1})(-\cos(\Phi_{1n}-\theta_{1n}) + 2\cos(\Phi_{1n}-\theta_{1n})\frac{d\Phi_{1n}}{d\theta_{1n}}$$

$$- \cos(\Phi_{1n}-\theta_{1n})\left(\frac{d\Phi_{1n}}{d\theta_{1n}}\right)^2 - \sin(\Phi_{1n}-\theta_{1n})\frac{d^2\Phi_{1n}}{d\theta_{1n}^2})$$

$$- X_{Q1}\sin(-\theta_{1n}) - Y_{Q1}\cos(-\theta_{1n}).$$

With the first and second infinitesimal displacements of X_{Cn}, Y_{Cn} known, the synthesis procedure now involves a four-link rigid body guidance synthesis problem with first and second infinitesimal displacements. Notice that the four-link synthesis procedure involves the first and second infinitesimal displacement of X_{Cn}, Y_{Cn} with respect to the rotation of the ternary link. That is, $\frac{dX_{Cn}}{d\alpha_{1n}}$, $\frac{dY_{Cn}}{d\alpha_{1n}}$, $\frac{d^2X_{Cn}}{d\alpha_{1n}^2}$ and $\frac{d^2Y_{Cn}}{d\alpha_{1n}^2}$ are to be specified in synthesizing the four-link mechanism. Equations (5-2) must be differentiated with respect to α_{1n}, resulting in

$$\frac{d^2X_{Cn}}{d\alpha_{1n}^2} = \frac{d^2\theta_{1n}}{d\alpha_{1n}^2}\frac{dX_{Cn}}{d\theta_{1n}} + \left(\frac{d\theta_{1n}}{d\alpha_{1n}}\right)^2\frac{d^2X_{Cn}}{d\theta_{1n}^2}$$

$$\frac{d^2Y_{Cn}}{d\alpha_{1n}^2} = \frac{d^2\theta_{1n}}{d\alpha_{1n}^2}\frac{dY_{Cn}}{d\theta_{1n}} + \left(\frac{d\theta_{1n}}{d\alpha_{1n}}\right)^2\frac{d^2Y_{Cn}}{d\theta_{1n}^2}. \tag{6-3}$$

The ratio $\frac{d^2\Phi_{1n}}{d\theta_{1n}^2}$ has to be specified along with $\frac{d\Phi_{1n}}{d\theta_{1n}}$, which was needed for the first infinitesimal displacement synthesis. Again, these are arbitrary values, or can be used to the designer's advantage.

As a result of the justification established in Chapter V, the coefficients of the design Equation (4-2) can be differentiated twice with respect to α_{1n} to obtain the second infinitesimal displacement four-link mechanism synthesis Equation (6-4).

$$X_{A1}\left(\frac{d^2X_{Cn}}{d\alpha_{1n}^2} + X_{C1}\cos\alpha_{1n} - Y_{C1}\sin\alpha_{1n}\right) +$$

$$Y_{A1}\left(\frac{d^2Y_{Cn}}{d\alpha_{1n}^2} + X_{C1}\sin\alpha_{1n} + Y_{C1}\cos\alpha_{1n}\right) +$$

$$X_{B1}\left[-\left(\frac{d^2X_{Cn}}{d\alpha_{1n}^2} + 2\frac{dY_{Cn}}{d\alpha_{1n}} - X_{Cn}\right)\cos\alpha_{1n} - \left(\frac{d^2Y_{Cn}}{d\alpha_{1n}^2} - 2\frac{dX_{Cn}}{d\alpha_{1n}} - Y_{Cn}\right)\sin\alpha_{1n}\right] +$$

$$Y_{B1}\left[-\left(\frac{d^2Y_{Cn}}{d\alpha_{1n}^2} - 2\frac{dX_{Cn}}{d\alpha_{1n}} - Y_{Cn}\right)\cos\alpha_{1n} + \left(\frac{d^2X_{Cn}}{d\alpha_{1n}^2} + 2\frac{dY_{Cn}}{d\alpha_{1n}} - X_{Cn}\right)\sin\alpha_{1n}\right] =$$

$$\cos\alpha_{1n}\,(X_{A1}X_{B1} + Y_{A1}Y_{B1}) - \sin\alpha_{1n}\,(X_{A1}Y_{B1} - X_{B1}Y_{A1}) +$$

$$\cos\alpha_{1n}\left(2Y_{C1}\frac{dX_{Cn}}{d\alpha_{1n}} - 2X_{C1}\frac{dY_{Cn}}{d\alpha_{1n}} - Y_{C1}\frac{d^2Y_{Cn}}{d\alpha_{1n}^2} - X_{C1}\frac{d^2X_{Cn}}{d\alpha_{1n}^2} + X_{C1}X_{Cn} + Y_{C1}Y_{Cn}\right) +$$

$$\sin\alpha_{1n}\left(2X_{C1}\frac{dX_{Cn}}{d\alpha_{1n}} + 2Y_{C1}\frac{dY_{Cn}}{d\alpha_{1n}} - X_{C1}\frac{d^2Y_{Cn}}{d\alpha_{1n}^2} + Y_{C1}\frac{d^2X_{Cn}}{d\alpha_{1n}^2} - Y_{C1}X_{Cn} + X_{C1}Y_{Cn}\right) + X_{Cn}\frac{d^2X_{Cn}}{d\alpha_{1n}^2} + Y_{Cn}\frac{d^2Y_{Cn}}{d\alpha_{1n}^2} +$$

$$\frac{dX_{Cn}}{d\alpha_{1n}}\frac{dX_{Cn}}{d\alpha_{1n}} + \frac{dY_{Cn}}{d\alpha_{1n}}\frac{dY_{Cn}}{d\alpha_{1n}} \qquad (6\text{-}4)$$

Since this equation is still of the same form as the finite displacement and first infinitesimal displacement equations, it can be used as a fourth equation that is needed due to the reduction of one finite position equation.

Two computer programs have been written which cover the two classes of problems discussed in this chapter. A computer program which synthesizes the Stephenson Type II mechanism for three finite positions and first and second infinitesimally separated positions of input to output specified at one of the finite positions is listed in Appendix D, and an example problem is presented in Table IV.

TABLE IV

THREE FINITE POSITIONS AND ONE ACCELERATION RATIO (ACCELERATION RATIO IS AT POINT 3)

```
INPUT VELOCITY = 1.0                  OUTPUT VELOCITY = 0.0
INPUT ACCELERATION = 2.0              OUTPUT ACCELERATION = 0.0

     INPUT ROTATION ANGLES ARE             -25.000    -50.000
     OUTPUT ROTATION ANGLES ARE            -10.000    -30.000
     TERNARY LINK ROTATIONS ARE              5.000     15.000

OUTPUT LINK IS 1.0 LONG AND AT AN ANGLE OF 100.0 DEGREES.

FIRST INFINITESIMAL DISPLACEMENTS          DPHDTH(J) = 0.0
                                           DTHDAL(J) = 1.0

SECOND INFINITESIMAL DISPLACEMENTS         DDPHTH(J) = 0.0
                                           DDTHAL(J) = 1.0

                          SOLUTION

                   X                        Y

     M      0.00000000               0.00000000
     Q      1.00000000               0.00000000
     C1     0.82635313               0.98480803

   POSSIBLE SOLUTION FOR POINTS A1 AND B1 OR D1 AND E1

          CENTER POINT                   CIRCLE POINT

        X             Y                X              Y

  -1.47581768   -0.56793118     0.37606907     1.30638123
   1.31134033    6.33950806   -10.17961121   -15.10237122

  ROOT IS IMAGINARY

  ROOT IS IMAGINARY
```

A computer program which synthesizes the Stephenson Type II mechanism for two finite positions and first and second infinitesimally separated positions of input to output at one of the finite points, as well as a first infinitesimally separated position specified at the other finite point is listed in Appendix E, and an example problem is presented in Table V.

TABLE V

TWO FINITE POSITIONS, ONE VELOCITY RATIO, AND ONE ACCELERATION RATIO (VELOCITY RATIO IS AT POINT 1 AND ACCELERATION RATIO IS AT POINT 2)

1	INPUT VELOCITY = 1.0	OUTPUT VELOCITY = 3.0
2	INPUT VELOCITY = 1.0	OUTPUT VELOCITY = 2.0
2	INPUT ACCELERATION = 0.0	OUTPUT ACCELERATION = 0.0

INPUT ROTATION ANGLE IS	30.000
OUTPUT ROTATION ANGLE IS	45.000
TERNARY LINK ROTATION IS	10.000

OUTPUT LINK IS 1.0 LONG AND AT AN ANGLE OF 60.0 DEGREES.

FIRST INFINITESIMAL DISPLACEMENTS	DPHDTH(K) = 3.0
	DTHDAL(K) = 1.0
SECOND INFINITESIMAL DISPLACEMENTS	DDPHTH(J) = 0.0
	DDTHAL(J) = 1.0
FIRST INFINITESIMAL DISPLACEMENTS	DPHDTH(J) = 2.0
	DTHDAL(J) = 1.0

SOLUTION

	X	Y
M	0.00000000	0.00000000
Q	1.00000000	0.00000000
C1	1.50000000	0.86602520

POSSIBLE SOLUTION FOR POINTS A1 AND B1 OR D1 AND E1

CENTER POINT		CIRCLE POINT	
X	Y	X	Y
1.74607800	-0.63277810	1.88640900	-0.50722500
2.98628100	-1.30977000	3.82497700	-1.56066700

ROOT IS IMAGINARY
ROOT IS IMAGINARY

CHAPTER VII

SYNTHESIS PROCEDURE FOR THIRD AND FOURTH INFINITESIMALLY SEPARATED POSITIONS

An additional design criterion might be the specification of the jerk and kerk ratios of input to output. For example, at a finite point it may be desirable not only to specify a zero acceleration ratio, but also to have a zero jerk and kerk ratio. The jerk and kerk ratios enable the designer to have greater control over his input-output linkage motion.

The same synthesis procedure set forth in the previous chapters will be used for these next two cases, which are

PPPP-P　　　　PPPPP
P-PPPP

Third Infinitesimally Separated Position

In order to solve the third infinitesimal displacement problem, one must realize that at a finite position, the first, second and third infinitesimal displacement parameters must be given. That is, $\frac{d\Phi}{d\theta}$ is first found from the specified velocity ratios and, in turn, is used in Equation (6-1) to find $\frac{d^2\Phi}{d\theta^2}$. Now the derivative of Equation (6-1) can be taken to solve for the third infinitesimally separated position $\frac{d^3\Phi}{d\theta^3}$.

$$\frac{d^3\Phi}{dt^3} = \frac{d^3\Phi}{d\theta^3}\left(\frac{d\theta}{dt}\right)^3 + 3\,\frac{d\theta}{dt}\cdot\frac{d^2\theta}{dt^2}\cdot\frac{d^2\Phi}{d\theta^2} + \frac{d^3\theta}{dt^3}\cdot\frac{d\Phi}{d\theta} \qquad (7\text{-}1)$$

These values are then used in Equations (5-1), (6-2) and (7-2) to find the values for $\frac{dX_{Cn}}{d\theta_{1n}}$, $\frac{dY_{Cn}}{d\theta_{1n}}$, $\frac{d^2X_{Cn}}{d\theta_{1n}^2}$, $\frac{d^2Y_{Cn}}{d\theta_{1n}^2}$, $\frac{d^3X_{Cn}}{d\theta_{1n}^3}$, and $\frac{d^3Y_{Cn}}{d\theta_{1n}^3}$.

$$\frac{d^3X_{Cn}}{d\theta_{1n}^3} = (X_{C1}-X_{Q1})\ [-\sin\ (\Phi_{1n}-\theta_{1n}) + 3\ \sin\ (\Phi_{1n}-\theta_{1n})\ \frac{d\Phi_{1n}}{d\theta_{1n}}$$

$$-\ 3\ \sin\ (\Phi_{1n}-\theta_{1n})\ (\frac{d\Phi_{1n}}{d\theta_{1n}})^2 + 3\ \cos\ (\Phi_{1n}-\theta_{1n})\ \frac{d^2\Phi_{1n}}{d\theta_{1n}^2}$$

$$-\ 3\ \cos\ (\Phi_{1n}-\theta_{1n})\ \frac{d\Phi_{1n}}{d\theta_{1n}} \cdot \frac{d^2\Phi_{1n}}{d\theta_{1n}^2} + \sin(\Phi_{1n}-\theta_{1n})(\frac{d\Phi_{1n}}{d\theta_{1n}})^3$$

$$-\ \sin\ (\Phi_{1n}-\theta_{1n})\ \frac{d^3\Phi_{1n}}{d\theta_{1n}^3}] +$$

$$(Y_{C1}-X_{Q1})\ [-\cos(\Phi_{1n}-\theta_{1n}) + 3\ \cos(\Phi_{1n}-\theta_{1n})\ \frac{d\Phi_{1n}}{d\theta_{1n}}$$

$$-\ 3\ \cos\ (\Phi_{1n}-\theta_{1n})(\frac{d\Phi_{1n}}{d\theta_{1n}})^2 - 3\ \sin(\Phi_{1n}-\theta_{1n})\ \frac{d^2\Phi_{1n}}{d\theta_{1n}^2}$$

$$+\ 3\ \sin\ (\Phi_{1n}-\theta_{1n})\ \frac{d\Phi_{1n}}{d\theta_{1n}} \cdot \frac{d^2\Phi_{1n}}{d\theta_{1n}^2} + \cos(\Phi_{1n}-\theta_{1n})(\frac{d\Phi_{1n}}{d\theta_{1n}})^3$$

$$-\ \cos(\Phi_{1n}-\theta_{1n})\frac{d^3\Phi_{1n}}{d\theta_{1n}^3}]\ -X_{Q1}\sin(-\Phi_{1n})-Y_{Q1}\cos(-\theta_{1n}) \qquad (7\text{-}2)$$

$$\frac{d^3Y_{Cn}}{d\theta_{1n}^3} = (X_{C1}-X_{Q1})[\cos(\Phi_{1n}-\theta_{1n}) - 3\ \cos(\Phi_{1n}-\theta_{1n})\ \frac{d\Phi_{1n}}{d\theta_{1n}}$$

$$+\ 3\ \cos(\Phi_{1n}-\theta_{1n})(\frac{d\Phi_{1n}}{d\theta_{1n}})^2 + 3\ \sin(\Phi_{1n}-\theta_{1n})\ \frac{d^2\Phi_{1n}}{d\theta_{1n}^2}$$

$$-\ 3\ \sin(\Phi_{1n}-\theta_{1n})\ \frac{d\Phi_{1n}}{d\theta_{1n}} \cdot \frac{d^2\Phi_{1n}}{d\theta_{1n}^2} - \cos(\Phi_{1n}-\theta_{1n})(\frac{d\Phi_{1n}}{d\theta_{1n}})^3$$

$$+\ \cos(\Phi_{1n}-\theta_{1n})\ \frac{d^3\Phi_{1n}}{d\theta_{1n}^3}] +$$

$$(Y_{C1}-Y_{Q1})[-\sin(\Phi_{1n}-\theta_{1n})+3\sin(\Phi_{1n}-\theta_{1n})\frac{d\Phi_{1n}}{d\theta_{1n}}$$

$$-3\sin(\Phi_{1n}-\theta_{1n})\left(\frac{d\Phi_{1n}}{d\theta_{1n}}\right)^2+3\cos(\Phi_{1n}-\theta_{1n})\frac{d^2\Phi_{1n}}{d\theta_{1n}^2}$$

$$-3\cos(\Phi_{1n}-\theta_{1n})\frac{d\Phi_{1n}}{d\theta_{1n}}\cdot\frac{d^2\Phi_{1n}}{d\theta_{1n}^2}+\sin(\Phi_{1n}-\theta_{1n})\left(\frac{d\Phi_{1n}}{d\theta_{1n}}\right)^3$$

$$-\sin(\Phi_{1n}-\theta_{1n})\frac{d^3\Phi_{1n}}{d\theta_{1n}^3}]+X_{Q1}\cos(-\theta_{1n})-Y_{Q1}\sin(-\theta_{1n})$$

These values are, in turn, changed to $\frac{dX_{Cn}}{d\alpha_{1n}}$, $\frac{dY_{Cn}}{d\alpha_{1n}}$, $\frac{d^2X_{Cn}}{d\alpha_{1n}^2}$, $\frac{d^2Y_{Cn}}{d\alpha_{1n}^2}$, $\frac{d^3X_{Cn}}{d\alpha_{1n}^3}$ and $\frac{d^3Y_{Cn}}{d\alpha_{1n}^3}$ by means of Equations (5-2), (6-3) and (7-3).

$$\frac{d^3X_{Cn}}{d\alpha_{1n}^3}=\frac{d^3\theta_{1n}}{d\alpha_{1n}^3}\cdot\frac{dX_{Cn}}{d\theta_{1n}}+3\frac{d\theta_{1n}}{d\alpha_{1n}}\cdot\frac{d^2\theta_{1n}}{d\alpha_{1n}^2}\cdot\frac{d^2X_{Cn}}{d\theta_{1n}^2}+\left(\frac{d\theta_{1n}}{d\alpha_{1n}}\right)^3\frac{d^3X_{Cn}}{d\theta_{1n}^3}$$

$$\frac{d^3Y_{Cn}}{d\alpha_{1n}^3}=\frac{d^3\theta_{1n}}{d\alpha_{1n}^3}\cdot\frac{dY_{Cn}}{d\theta_{1n}}+3\frac{d\theta_{1n}}{d\alpha_{1n}}\cdot\frac{d^2\theta_{1n}}{d\alpha_{1n}^2}\cdot\frac{d^2Y_{Cn}}{d\theta_{1n}^2}+\left(\frac{d\theta_{1n}}{d\alpha_{1n}}\right)^3\frac{d^3Y_{Cn}}{d\theta_{1n}^3} \tag{7-3}$$

At a finite point, the first, second and third infinitesimal displacement synthesis Equations (5-3), (6-4) and (7-4), respectively, can now be used to design the desired Stephenson Type II motion.

$$X_{A1}\left(\frac{d^3X_{Cn}}{d\alpha_{1n}^3}-X_{C1}\sin\alpha_{1n}-Y_{C1}\cos\alpha_{1n}\right)+$$

$$Y_{A1}\left(\frac{d^3Y_{n}}{d\alpha_{1n}^3}+X_{C1}\cos\alpha_{1n}-Y_{C1}\sin\alpha_{1n}\right)+$$

$$X_{B1}\left[-\left(\frac{d^3X_{Cn}}{d\alpha_{1n}^3}+3\frac{d^2Y_{Cn}}{d\alpha_{1n}^2}-3\frac{dX_{Cn}}{d\alpha_{1n}}-Y_{Cn}\right)\cos\alpha_{1n}\right.$$

$$\left.-\left(\frac{d^3Y_{Cn}}{d\alpha_{1n}^3}-3\frac{d2X_{Cn}}{d\alpha_{1n}^2}-3\frac{dY_{Cn}}{d\alpha_{1n}}+X_{Cn}\right)\sin\alpha_{1n}\right]+$$

$$Y_{B1}\Big[-\Big(\frac{d^3Y_{Cn}}{d\alpha_{1n}^3} - 3\,\frac{d^2X_{Cn}}{d^2\alpha_{1n}} - 3\,\frac{dY_{Cn}}{d\alpha_{1n}} + X_{Cn}\Big)\cos\alpha_{1n}$$

$$+\Big(\frac{d^3X_{Cn}}{d\alpha_{1n}^3} + 3\,\frac{d^2Y_{Cn}}{d\alpha_{1n}^2} - 3\,\frac{dX_{Cn}}{d\alpha_{1n}} - Y_{Cn}\Big)\sin\alpha_{1n}\Big] =$$

$$-\sin\alpha_{1n}(X_{A1}X_{B1} + Y_{A1}Y_{B1}) - \cos\alpha_{1n}(X_{A1}Y_{B1}-X_{B1}Y_{A1})$$

$$+\cos\alpha_{1n}\ \Big(-Y_{C1}\,\frac{d^3Y_{Cn}}{d\alpha_{1n}^3} - X_{C1}\,\frac{d^3X_{Cn}}{d\alpha_{1n}^3} + 3\,Y_{C1}\,\frac{d^2X_{Cn}}{d\alpha_{1n}^2} - 3\,X_{C1}\,\frac{d^2Y_{Cn}}{d\alpha_{1n}^2}$$

$$+ 3\,X_{C1}\,\frac{dX_{Cn}}{d\alpha_{1n}} + 3\,Y_{C1}\,\frac{dY_{Cn}}{d\alpha_{1n}} - Y_{C1}X_{Cn}+X_{C1}Y_{Cn}\Big) \qquad (7\text{-}4)$$

$$+\sin\alpha_{1n}\ \Big(-X_{C1}\,\frac{d^3Y_{Cn}}{d\alpha_{1n}^3} + Y_{C1}\,\frac{d^3X_{Cn}}{d\alpha_{1n}^3} + 3\,X_{C1}\,\frac{d^2X_{Cn}}{d\alpha_{1n}^2} + 3Y_{C1}\,\frac{d^2Y_{Cn}}{d\alpha_{1n}^2} - 3\,Y_{C1}\,\frac{dX_{Cn}}{d\alpha_{1n}} + 3X_{C1}\,\frac{dY_{Cn}}{d\alpha_{1n}}$$

$$- X_{C1}X_{Cn}-Y_{C1}Y_{Cn}\Big)$$

$$+ X_{Cn}\,\frac{d^3X_{Cn}}{d\alpha_{1n}^3} + Y_{Cn}\,\frac{d^3Y_{Cn}}{d\alpha_{1n}^3} + 3\,\frac{dX_{Cn}}{d\alpha_{1n}}\cdot\frac{d^2X_{Cn}}{d\alpha_{1n}^2} + 3\,\frac{dY_{Cn}}{d\alpha_{1n}}\cdot\frac{d^2Y_{Cn}}{d\alpha_{1n}^2}$$

Since the equations are all of the same form, the principle of linear superposition still applies.

A computer program which synthesizes the Stephenson Type II mechanism for two finite positions and first, second and third infinitesimally separated positions of input to output at one of the finite positions is listed in Appendix F, and an example problem is presented in Table VI.

TABLE VI

TWO FINITE POSITIONS AND ONE JERK RATIO
(JERK RATIO IS SPECIFIED AT POINT 2)

```
INPUT VELOCITY = 1.0                OUTPUT VELOCITY = -2.5
INPUT ACCELERATION = 0.0            OUTPUT ACCELERATION = -1.0
INPUT JERK = 0.0                    OUTPUT JERK = 0.0

           INPUT ROTATION ANGLE IS      30.000
           OUTPUT ROTATION ANGLE IS    -45.000
           TERNARY LINK ROTATION IS    -25.000

OUTPUT LINK IS 1.2 LONG AND AT AN ANGLE OF -30.0 DEGREES.

FIRST INFINITESIMAL DISPLACEMENTS          DPHDTH(J) = -2.5
                                           DTHDAL(J) =  1.0
SECOND INFINITESIMAL DISPLACEMENTS         DDPHTH(J) = -1.0
                                           DDTHAL(J) =  1.0
THIRD INFINITESIMAL DISPLACEMENTS          DDDPTH(J) =  0.0
                                           DDDTAL(J) =  1.0
```

SOLUTION

	X	Y
M	0.00000000	0.00000000
Q	1.00000000	0.00000000
C1	2.03922900	-0.59999970

POSSIBLE SOLUTION FOR POINTS A1 AND B1 OR D1 AND E1

CENTER POINT		CIRCLE POINT	
X	Y	X	Y
1.09700200	-0.47618860	2.43076600	-0.57500930
4.04580400	-4.92590300	5.21653500	-3.42272900

ROOT IS IMAGINARY

ROOT IS IMAGINARY

TABLE VII

ONE FINITE POSITION AND ONE KERK RATIO

INPUT VELOCITY	= 1.0	OUTPUT VELOCITY	= 2.0
INPUT ACCELERATION	= 0.0	OUTPUT ACCELERATION	= 3.0
INPUT JERK	= 0.0	OUTPUT JERK	= 0.0
INPUT KERK	= 0.0	OUTPUT KERK	= 0.0

OUTPUT LINK IS 1.5 LONG AND AT AN ANGLE OF 30.0 DEGREES.

FIRST INFINITESIMAL DISPLACEMENTS	DPHDTH(J) = 2.0
	DTHDAL(J) = 1.0
SECOND INFINITESIMAL DISPLACEMENTS	DDPHTH(J) = 3.0
	DDTHAL(J) = 1.0
THIRD INFINITESIMAL DISPLACEMENTS	DDDPTH(J) = 0.0
	DDDTAL(J) = 1.0
FOURTH INFINITESIMAL DISPLACEMENTS	DDDDPT(J) = 0.0
	DDDDTA(J) = 1.0

SOLUTION

	X	Y
M	0.00000000	0.00000000
Q	1.00000000	0.00000000
C1	2.29903793	0.74999970

POSSIBLE SOLUTIONS FOR POINTS A1 AND B1 OR D1 AND E1

CENTER POINT		CIRCLE POINT	
X	Y	X	Y
1.58438492	-0.57121086	2.80116749	1.10895443
4.47628021	0.35051918	7.80039597	0.81995583

ROOT IS IMAGINARY

ROOT IS IMAGINARY

Fourth Infinitesimally Separated Position

The procedure stated above can now be expanded to include the fourth infinitesimally separated position synthesis problem. The procedure will be the same, except that Equations (7-1), (7-2), (7-3) and (7-4) must be differentiated. Since there is an additional design equation for this case, the first, second, third and fourth infinitesimal displacements can be specified only at one finite point.

A computer program which synthesizes the Stephenson Type II mechanism for one finite point and first, second, third and fourth infinitesimally separated positions of input to output is listed in Appendix G, and an example problem is presented in Table VII.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

This thesis develops an approach for synthesizing the Stephenson Type II six-link function generator for five positions of input and output. These five positions consist of both finitely and infinitesimally separable positions. There are fourteen possible combinations of finite and infinitesimally separated positions for which function generation motion can be obtained.

The synthesis procedure consists of starting with desired finite displacements, velocity ratios, acceleration ratios, jerk ratios or kerk ratios, whichever are desired, and changing them to an n^{th} order infinitesimally separated position synthesis problem. The principle of inversion is used to transform the Stephenson Type II mechanism synthesis problem into a four-link rigid body guidance problem. The matrix method proved to be easily adaptable for synthesizing any one of the fourteen motion programs. In order to obtain the closed form solution for synthesis, the principle of linear superposition was utilized.

Once the synthesis procedure for the Stephenson Type II six-link function generator was developed, seven computer programs were written to perform any of the fourteen function generation motions. Feeding the desired design parameters into the programs will result in from 0 to 6 possible solutions for the six-link mechanism because from 0 to 4 center point and circle point combinations are possible for the rigid body motion.

With these seven basic programs, a wide variety of function generation problems can be solved. For example, the Stephenson Type II mechanism can be synthesized for five finite positions, where the output link goes through more than 180 degrees. If cutting action is desired, ternary link rotations can be specified along with the output link rotations, resulting in the desired relative motion between two connecting rigid bodies. Since it was necessary in the synthesis procedure for n^{th} order infinitesimally separated positions to specify $\frac{d\theta_{1n}}{d\alpha_{1n}}$, $\frac{d^2\theta_{1n}}{d\alpha_{1n}^2}$, etc., an additional parameter in which the ternary link infinitesimal rotations are related to the input infinitesimal rotations is given the designer. When an object is to be picked up at zero velocity and left off at zero velocity, this can be done for three finite positions by using the program in Appendix C. Or specific forces at a specified velocity and position can be obtained by achieving an appropriate acceleration of the output link. An example might be a stamping action. This can be accomplished, depending upon the other design parameters, by using programs in Appendices D, E, F or G. Another major area in which these programs could be used is finite dwell problems. Any of the programs in Appendices A, B, C, D, E or F can be used to obtain a finite dwell of the output while the input link rotates through finite positions. For five finite positions all five of the output rotations can be set equal to zero, while the input goes through the desired rotation angles. The program in Appendix C can be used to specify one finite position and a dwell at another finite position. The dwell would be developed by specifying two finite points and a zero velocity ratio at those two points. The same procedure can also be used for programs in Appendices B, D, E and F by setting the appropriate velocity, acceleration and jerk ratios equal to zero. These programs will, of course, result in an approximate finite dwell, but error can be minimized by determining which program is the best for the specific problem.

The synthesis procedure shown in this thesis can be used for synthesis of different mechanisms, not only for finite positions, but also for infinitesimally separated positions. It can be applied to both function generation synthesis problems and rigid body guidance synthesis problems. This versatile and concise method was ideal for synthesis of the Stephenson Type II six-link function generator, which resulted in a practical vehicle for solving design problems.

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APPENDIX A

SYNTHESIS PROGRAM FOR FIVE FINITE POSITIONS

```
0000000000111111111122222222223333333333444444444455555555556666666666777777777778
1234567890123456789012345678901234567890123456789012345678901234567890123456789 0
C ***********************************************************************
C *                                                                     *
C *                     SYNTHESIS OF SIX-LINK STEPHENSON 2              *
C *                              FUNCTION GENERATOR                     *
C *                                                                     *
C *                           FIVE FINITE POSITIONS                     *
C *                                                                     *
C *                                                                     *
C *                  INPUT FINITE ROTATIONS         TH1(I)              *
C *                                                                     *
C *                  OUTPUT FINITE ROTATIONS        PH1(I)              *
C *                                                                     *
C *                  ASSUMED ROTATICNS OF TERNARY   AL1(I)              *
C *                                                                     *
C *                  OUTPUT LINK LENGTH             QC                  *
C *                                                                     *
C *                  OUTPUT LINK INITIAL ANGLE      THE                 *
C *                                                                     *
C ***********************************************************************
C
      DIMENSICN XCOF(5),COF(5),ROOTR(4),ROOTI(4)
      DIMENSION XQ(5),YQ(5),XA(5),YA(5),XB(5),YB(5),XC(5),YC(5)
      DIMENSION TH1(5),PH1(5),AL1(5)
      DIMENSION H(16),H1(16),H2(16),R(4),P(4),Q(4)
      READ (5,10) TH1(2),TH1(3),TH1(4),TH1(5)
      READ (5,10) PH1(2),PH1(3),PH1(4),PH1(5)
      READ (5,10) AL1(2),AL1(3),AL1(4),AL1(5)
   10 FORMAT (4F10.0)
      REAC (5,20) QC,THE
   20 FORMAT (2F10.0)
      WRITE (6,6)
    6 FORMAT (//,25X,22H FIVE FINITE POSITIONS,//)
      WRITE (6,11) TH1(2),TH1(3),TH1(4),TH1(5)
   11 FORMAT (9X,26H INPUT ROTATION ANGLES ARE,2X,4F10.3,/)
      WRITE (6,12) PH1(2),PH1(3),PH1(4),PH1(5)
   12 FORMAT (9X,27H OLTPUT ROTATION ANGLES ARE,1X,4F10.3,/)
      WRITE (6,13) AL1(2),AL1(3),AL1(4),AL1(5)
   13 FORMAT (9X,23H TERNARY LINK ROTATIONS,5X,4F10.3,//)
      WRITE (6,14) QC,THE
   14 FORMAT (12X,15H OUTPUT LINK IS,F10.3,24H LONG AND AT AN ANGLE OF,F
     $10.3,//)
      XQ(1)=1.0
      YQ(1)=0.0
      XM=0.0
      YM=0.0
      THE=(THE*3.14159265)/180.0
      DO 21 I=2,5
      TH1(I)=(TH1(I)*3.14159265)/180.0
      PH1(I)=(PH1(I)*3.14159265)/180.0
   21 AL1(I)=(AL1(I)*3.14159265)/180.0
      XC(1)=1.0+QC*COS(THE)
      YC(1)=0.0+QC*SIN(THE)
      DO 30 I=2,5
      YQ(I)=1.0*SIN(-TH1(I))
```

```
000000000111111111122222222223333333333444444444455555555556666666666777777777778
123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890
      XQ(I)=1.0*COS(-TH1(I))
      XC(I)=XC(1)*COS(PH1(I)-TH1(I))-YC(1)*SIN(PH1(I)-TH1(I))+XQ(I)-XQ(1
     $)*COS(PH1(I)-TH1(I))+YQ(1)*SIN(PH1(I)-TH1(I))
   30 YC(I)=XC(1)*SIN(PH1(I)-TH1(I))+YC(1)*COS(PH1(I)-TH1(I))+YQ(I)-XQ(1
     $)*SIN(PH1(I)-TH1(I))-YQ(1)*COS(PH1(I)-TH1(I))
      WRITE (6,31)
   31 FORMAT (12X,3H XC,18X,3H YQ,18X,3H XC,18X,3H YC,/)
      DO 33 I=1,5
   33 WRITE (6,32) XQ(I),YQ(I),XC(I),YC(I)
   32 FORMAT (6X,F12.8,9X,F12.8,9X,F12.8,9X,F12.8,/)
      DO 40 I=1,4
   40 H(I)=XC(I+1)-XC(1)*COS(AL1(I+1))+YC(1)*SIN(AL1(I+1))
      DO 50 I=1,4
   50 H(I+4)=YC(I+1)-YC(1)*COS(AL1(I+1))-XC(1)*SIN(AL1(I+1))
      DO 60 I=1,4
   60 H(I+8)=-H(I)*COS(AL1(I+1))-H(I+4)*SIN(AL1(I+1))
      DO 70 I=1,4
   70 H(I+12)=-H(I+4)*COS(AL1(I+1))+H(I)*SIN(AL1(I+1))
      DO 72 I=1,16
   72 H1(I)=H(I)
      DO 74 I=1,16
   74 H2(I)=H(I)
      DO 80 I=1,4
   80 R(I)=0.5*(H(I)*H(I)+H(I+4)*H(I+4))
      DO 90 I=1,4
   90 P(I)=1.0-COS(AL1(I+1))
      DO 100 I=1,4
  100 Q(I)=SIN(AL1(I+1))
      CALL SIMQ(H,R,4,KS)
      CALL SIMQ(H1,P,4,KS1)
      CALL SIMQ(H2,Q,4,KS2)
      WRITE (6,99) KS
      WRITE (6,99) KS1
      WRITE (6,99) KS2
   99 FORMAT (1X,I1)
      F1=Q(1)*Q(3)+Q(2)*Q(4)
      F2=P(1)*Q(3)+Q(1)*P(3)+P(2)*Q(4)+Q(2)*P(4)
      F3=R(1)*Q(3)+Q(1)*R(3)+Q(2)*R(4)+Q(4)*R(2)
      F4=P(1)*P(3)+P(2)*P(4)
      F5=P(1)*R(3)+P(3)*R(1)+P(2)*R(4)+P(4)*R(2)-1.0
      F6=R(1)*R(3)+R(2)*R(4)
      G1=Q(1)*Q(4)-Q(2)*Q(3)
      G2=P(1)*Q(4)+Q(1)*P(4)-P(2)*Q(3)-Q(2)*P(3)
      G3=R(1)*Q(4)+Q(1)*R(4)-R(2)*Q(3)-Q(2)*R(3)-1.0
      G4=P(1)*P(4)-P(2)*P(3)
      G5=P(1)*R(4)+R(1)*P(4)-P(2)*R(3)-P(3)*R(2)
      G6=R(1)*R(4)-R(2)*R(3)
      XCOF(1)=F1*F1*G6*G6+G3*G3*F1*F6-G3*G6*F1*F3+G1*G6*F3*F3+G1*G1*F6*F
     $6-G1*G3*F3*F6-2.0*G1*G6*F1*F6
      XCOF(2)=2.0*G5*G6*F1*F1+2.0*G2*G3*F1*F6+G3*G3*F1*F5-G3*G6*F1*F2-G2
     $*G6*F1*F3-G3*G5*F1*F3+2.0*G1*G6*F2*F3+G1*G5*F3*F3+2.0*G1*G1*F5*F6-
     $G1*G3*F2*F6-G1*G2*F3*F6-G1*G3*F3*F5-2.0*G1*G5*F1*F6-2.0*G1*G6*F1*F
     $5
      XCOF(3)=2.0*G4*G6*F1*F1+G5*G5*F1*F1+G2*G2*F1*F6+2.0*G2*G3*F1*F5+G3
```

```
00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
      $*G3*F1*F4-G2*G6*F1*F2-G3*G5*F1*F2-G2*G5*F1*F3-G3*G4*F1*F3+G1*G6*F2
      $*F2+2.0*G1*G5*F2*F3+G1*G4*F3*F3+2.0*G1*G1*F4*F6+G1*G1*F5*F5-G1*G2*
      $F2*F6-G1*G3*F2*F5-G1*G2*F3*F5-G1*G3*F3*F4-2.0*G1*G6*F1*F4-2.0*G1*G
      $5*F1*F5-2.0*G1*G4*F1*F6
       XCOF(4)=2.0*G4*G5*F1*F1+G2*G2*F1*F5+2.0*G2*G3*F1*F4-G2*G5*F1*F2-G3
      $*G4*F1*F2-G2*G4*F1*F3+G1*G5*F2*F2+2.0*G1*G4*F2*F3+2.0*G1*G1*F4*F5-
      $G1*G2*F2*F5-G1*G3*F2*F4-G1*G2*F3*F4-2.0*G1*G5*F1*F4-2.0*G1*G4*F1*F
      $5
       XCOF(5)=G4*G4*F1*F1+G2*G2*F1*F4-G2*G4*F1*F2+G1*G4*F2*F2+G1*G1*F4*F
      $4-G1*G2*F2*F4-2.0*G1*G4*F1*F4
       CALL POLRT(XCOF,COF,4,ROOTR,ROOTI,IER)
       WRITE (6,109)
   109 FORMAT (/,27X,9H L1 ROOTS,/)
       DO 110 I=1,4
   110 WRITE (6,112) ROOTR(I),ROOTI(I)
   112 FORMAT (17X,E13.5,10X,E13.5,/)
       WRITE (6,114) IER
   114 FORMAT (1X,I1)
       WRITE (6,106)
   106 FORMAT (1H1,//)
   125 DO 215 I=1,4
       OA=F1
       OB=F2*ROOTR(I)+F3
       OC=F4*ROOTR(I)*ROOTR(I)+F5*ROOTR(I)+F6
       OD=G1
       OE=G2*ROOTR(I)+G3
       OF=G4*ROOTR(I)*ROOTR(I)+G5*ROOTR(I)+G6
       IF (ABS(ROOTI(I)).LT.0.000001) ROOTI(I)=0.0
       IF (ROOTI(I)) 200,130,200
   200 WRITE (6,201)
   201 FORMAT (5X,18H ROOT IS IMAGINARY,/)
       GO TO 215
   130 SA=OB*OB-4.0*OA*OC
       IF (SA) 200,140,140
   140 ROOTA1=(-OB+SQRT(SA))/(2.0*OA)
       ROOTA2=(-OB-SQRT(SA))/(2.0*OA)
       SB=OE*OE-4.0*OD*OF
       IF (SB) 200,150,150
   150 ROOTB1=(-OE+SQRT(SB))/(2.0*OD)
       ROOTB2=(-OE-SQRT(SB))/(2.0*OD)
       WRITE (6,155) ROOTA1,ROOTA2,ROOTB1,ROOTB2
   155 FORMAT (/,13X,8H ROOTA1=,F18.6,5X,8H ROOTA2=,F18.6,/,13X,8H ROOTB1
      $=,F18.6,5X,8H ROOTB2=,F18.6,/)
       IF (ABS(ROOTA1-ROOTB1).LT.0.05) GO TO 160
       IF (ABS(ROOTA1-ROOTB2).LT.0.05) GO TO 160
       IF (ABS(ROOTA2-ROOTB1).LT.0.05) GO TO 170
       IF (ABS(ROOTA2-ROOTB2).LT.0.05) GO TO 170
       GO TO 215
   160 WRITE (6,180) ROOTA1
   180 FORMAT (2X,16H SECOND ROOT IS ,F20.5,/)
       ROOT=ROOTA1
       GO TO 210
   170 WRITE (6,180) ROOTA2
       ROOT=ROOTA2
```

```
00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890

  210 WRITE (6,212)
  212 FORMAT (14X,13H CENTER POINT,31X,13H CIRCLE POINT,/,8X,3H XA,19X,3
     $H YA,19X,3H XB,19X,3H YB,/)
      XA(I)=R(1)+P(1)*ROOTR(I)+Q(1)*ROOT
      YA(I)=R(2)+P(2)*ROOTR(I)+Q(2)*ROOT
      XB(I)=R(3)+P(3)*ROOTR(I)+Q(3)*ROOT
      YB(I)=R(4)+P(4)*ROOTR(I)+Q(4)*ROOT
      WRITE (6,230) XA(I),YA(I),XB(I),YB(I)
  230 FORMAT (4F20.8,/,33X,20H XXXXXXXXXXXXXXXXXXX,//)
  215 CONTINUE
      STOP
      END
```

APPENDIX B

SYNTHESIS PROGRAM FOR FOUR FINITE POSITIONS AND ONE FIRST INFINITESIMAL DISPLACEMENT

```
0000000001111111111222222222233333333334444444444555555555566666666667777777778
1234567890123456789012345678901234567890123456789012345678901234567890123456789
C ***************************************************************************
C *                                                                         *
C *                     SYNTHESIS OF SIX-LINK STEPHENSON 2                  *
C *                            FUNCTION GENERATOR                           *
C *                                                                         *
C *               FOUR FINITE POSITIONS AND ONE VELOCITY RATIO              *
C *                                                                         *
C *                                                                         *
C *                 INPUT FINITE ROTATIONS             TH1(I)               *
C *                                                                         *
C *                 OUTPUT FINITE ROTATIONS            PH1(I)               *
C *                                                                         *
C *                 ASSUMED ROTATIONS OF TERNARY       AL1(I)               *
C *                                                                         *
C *                 FIRST INFINITESIMAL DISPLACEMENT   DPHDTH(J)            *
C *                                                                         *
C *                 OUTPUT LINK LENGTH                 QC                   *
C *                                                                         *
C *                 OUTPUT LINK INITIAL ANGLE          THE                  *
C *                                                                         *
C ***************************************************************************
C
      DIMENSION XCOF(5),COF(5),ROOTR(4),ROOTI(4)
      DIMENSION XQ(5),YQ(5),XA(5),YA(5),XB(5),YB(5),XC(5),YC(5)
      DIMENSION TH1(5),PH1(5),AL1(5)
      DIMENSION H(16),H1(16),H2(16),R(4),P(4),Q(4)
      DIMENSION DXC(5),DYC(5),PXC(5),PYC(5)
      DIMENSION DPHDTH(5),DTHDAL(5)
      READ (5,5) J
    5 FORMAT (I1)
      WRITE (6,6) J
    6 FORMAT (//,16X,38H VELOCITY RATIO IS SPECIFIED AT POINT ,I1,//)
      READ (5,10) TH1(2),TH1(3),TH1(4)
      READ (5,10) PH1(2),PH1(3),PH1(4)
      READ (5,10) AL1(2),AL1(3),AL1(4)
   10 FORMAT (3F10.0)
      READ (5,20) QC,THE
      READ (5,20) DPHDTH(J),DTHDAL(J)
   20 FORMAT (2F10.0)
      WRITE (6,11) TH1(2),TH1(3),TH1(4)
   11 FORMAT (9X,26H INPUT ROTATION ANGLES ARE,2X,3F10.3,/)
      WRITE (6,12) PH1(2),PH1(3),PH1(4)
   12 FORMAT (9X,27H OUTPUT ROTATION ANGLES ARE,1X,3F10.3,/)
      WRITE (6,13) AL1(2),AL1(3),AL1(4)
   13 FORMAT (9X,23H TERNARY LINK ROTATIONS,5X,3F10.3,//)
      WRITE (6,14) QC,THE
   14 FORMAT (9X,15H OUTPUT LINK IS,F10.3,24H LONG AND AT AN ANGLE OF,F1
     $0.3,/)
      WRITE (6,15) DPHDTH(J),DTHDAL(J)
   15 FORMAT (9X,11H DPHDTH(J)=,F10.3,10X,11H DTHDAL(J)=,F10.3,//)
      XQ(1)=1.0
      YQ(1)=0.0
      XM=0.0
      YM=0.0
```

```
000000000111111111122222222223333333333444444444455555555556666666666777777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
      TH1(1)=0.0
      PH1(1)=0.0
      AL1(1)=0.0
      THE=(THE*3.14159265)/180.0
      DO 21 I=2,4
      TH1(I)=(TH1(I)*3.14159265)/180.0
      PH1(I)=(PH1(I)*3.14159265)/180.0
   21 AL1(I)=(AL1(I)*3.14159265)/180.0
      XC(1)=1.0+QC*COS(THE)
      YC(1)=0.0+QC*SIN(THE)
      DO 30 I=2,4
      XQ(I)=1.0*COS(-TH1(I))
      YQ(I)=1.0*SIN(-TH1(I))
      XC(I)=XC(1)*COS(PH1(I)-TH1(I))-YC(1)*SIN(PH1(I)-TH1(I))+XQ(I)-XQ(1
     $)*COS(PH1(I)-TH1(I))+YQ(1)*SIN(PH1(I)-TH1(I))
   30 YC(I)=XC(1)*SIN(PH1(I)-TH1(I))+YC(1)*COS(PH1(I)-TH1(I))+YQ(I)-XQ(1
     $)*SIN(PH1(I)-TH1(I))-YQ(1)*COS(PH1(I)-TH1(I))
      DXC(J)=(XC(1)-XQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(J)-TH1(J))*DPHDTH
     $(J))+(YC(1)-YQ(1))*(COS(PH1(J)-TH1(J))-COS(PH1(J)-TH1(J))*DPHDTH(J
     $))+XQ(1)*SIN(-TH1(J))+YQ(1)*COS(-TH1(J))
      DYC(J)=(XC(1)-XQ(1))*(-COS(PH1(J)-TH1(J))+COS(PH1(J)-TH1(J))*DPHDT
     $H(J))+(YC(1)-YQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(J)-TH1(J))*DPHDTH(
     $J))-XQ(1)*COS(-TH1(J))+YQ(1)*SIN(-TH1(J))
      WRITE (6,17) DXC(J),DYC(J)
   17 FORMAT (9X,8H DXC(J)=,3X,F10.3,10X,8H DYC(J)=,3X,F10.3,///)
      PXC(J)=DTHCAL(J)*DXC(J)
      PYC(J)=DTHCAL(J)*DYC(J)
      WRITE (6,31)
   31 FORMAT (11X,3H XQ,18X,3H YQ,18X,3H XC,18X,3H YC,/)
      DO 33 I=1,4
   33 WRITE (6,32) XQ(I),YQ(I),XC(I),YC(I)
   32 FORMAT (6X,F12.8,9X,F12.8,9X,F12.8,9X,F12.8,/)
      DO 40 I=1,3
   40 H(I)=XC(I+1)-XC(1)*COS(AL1(I+1))+YC(1)*SIN(AL1(I+1))
      H(4)=PXC(J)+XC(1)*SIN(AL1(J))+YC(1)*COS(AL1(J))
      DO 50 I=1,3
   50 H(I+4)=YC(I+1)-YC(1)*COS(AL1(I+1))-XC(1)*SIN(AL1(I+1))
      H(8)=PYC(J)-XC(1)*COS(AL1(J))+YC(1)*SIN(AL1(J))
      DO 60 I=1,3
   60 H(I+8)=-H(I)*COS(AL1(I+1))-H(I+4)*SIN(AL1(I+1))
      H(12)=-PXC(J)*COS(AL1(J))-PYC(J)*SIN(AL1(J))+XC(J)*SIN(AL1(J))-YC(
     $J)*COS(AL1(J))
      DO 70 I=1,3
   70 H(I+12)=-H(I+4)*COS(AL1(I+1))+H(I)*SIN(AL1(I+1))
      H(16)=PXC(J)*SIN(AL1(J))-PYC(J)*COS(AL1(J))+XC(J)*COS(AL1(J))+YC(J
     $)*SIN(AL1(J))
      DO 72 I=1,16
   72 H1(I)=H(I)
      DO 74 I=1,16
   74 H2(I)=H(I)
      DO 80 I=1,3
   80 R(I)=0.5*(H(I)*H(I)+H(I+4)*H(I+4))
      R(4)=(YC(1)*XC(J)-XC(1)*YC(J)-YC(1)*PYC(J)-XC(1)*PXC(J))*COS(AL1(J
     $))+(XC(1)*XC(J)+YC(1)*YC(J)-XC(1)*PYC(J)+YC(1)*PXC(J))*SIN(AL1(J))
```

```
     000000000111111111122222222223333333333444444444455555555556666666666777777777778
     123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890
     $+XC(J)*PXC(J)+YC(J)*PYC(J)
      DO 90 I=1,3
   90 P(I)=1.0-COS(AL1(I+1))
      P(4)=SIN(AL1(J))
      DO 100 I=1,3
  100 Q(I)=SIN(AL1(I+1))
      Q(4)=COS(AL1(J))
      CALL SIMQ(H,R,4,KS)
      CALL SIMQ(H1,P,4,KS1)
      CALL SIMQ(H2,Q,4,KS2)
      WRITE (6,99) KS
      WRITE (6,99) KS1
      WRITE (6,99) KS2
   99 FORMAT (1X,I1)
      F1=Q(1)*Q(3)+Q(2)*Q(4)
      F2=P(1)*Q(3)+Q(1)*P(3)+P(2)*Q(4)+Q(2)*P(4)
      F3=R(1)*Q(3)+Q(1)*R(3)+Q(2)*R(4)+Q(4)*R(2)
      F4=P(1)*P(3)+P(2)*P(4)
      F5=P(1)*R(3)+P(3)*R(1)+P(2)*R(4)+P(4)*R(2)-1.0
      F6=R(1)*R(3)+R(2)*R(4)
      G1=Q(1)*Q(4)-Q(2)*Q(3)
      G2=P(1)*Q(4)+Q(1)*P(4)-P(2)*Q(3)-Q(2)*P(3)
      G3=R(1)*Q(4)+Q(1)*R(4)-R(2)*Q(3)-Q(2)*R(3)-1.0
      G4=P(1)*P(4)-P(2)*P(3)
      G5=P(1)*R(4)+R(1)*P(4)-P(2)*R(3)-P(3)*R(2)
      G6=R(1)*R(4)-R(2)*R(3)
      XCOF(1)=F1*F1*G6*G6+G3*G3*F1*F6-G3*G6*F1*F3+G1*G6*F3*F3+G1*G1*F6*F
     $6-G1*G3*F3*F6-2.0*G1*G6*F1*F6
      XCOF(2)=2.0*G5*G6*F1*F1+2.0*G2*G3*F1*F6+G3*G3*F1*F5-G3*G6*F1*F2-G2
     $*G6*F1*F3-G3*G5*F1*F3+2.0*G1*G6*F2*F3+G1*G5*F3*F3+2.0*G1*G1*F5*F6-
     $G1*G3*F2*F6-G1*G2*F3*F6-G1*G3*F3*F5-2.0*G1*G5*F1*F6-2.0*G1*G6*F1*F
     $5
      XCOF(3)=2.0*G4*G6*F1*F1+G5*G5*F1*F1+G2*G2*F1*F6+2.0*G2*G3*F1*F5+G3
     $*G3*F1*F4-G2*G6*F1*F2-G3*G5*F1*F2-G2*G5*F1*F3-G3*G4*F1*F3+G1*G6*F2
     $*F2+2.0*G1*G5*F2*F3+G1*G4*F3*F3+2.0*G1*G1*F4*F6+G1*G1*F5*F5-G1*G2*
     $F2*F6-G1*G3*F2*F5-G1*G2*F3*F5-G1*G3*F3*F4-2.0*G1*G6*F1*F4-2.0*G1*G
     $5*F1*F5-2.0*G1*G4*F1*F6
      XCOF(4)=2.0*G4*G5*F1*F1+G2*G2*F1*F5+2.0*G2*G3*F1*F4-G2*G5*F1*F2-G3
     $*G4*F1*F2-G2*G4*F1*F3+G1*G5*F2*F2+2.0*G1*G4*F2*F3+2.0*G1*G1*F4*F5-
     $G1*G2*F2*F5-G1*G3*F2*F4-G1*G2*F3*F4-2.0*G1*G5*F1*F4-2.0*G1*G4*F1*F
     $5
      XCOF(5)=G4*G4*F1*F1+G2*G2*F1*F4-G2*G4*F1*F2+G1*G4*F2*F2+G1*G1*F4*F
     $4-G1*G2*F2*F4-2.0*G1*G4*F1*F4
      CALL POLRT(XCOF,COF,4,ROOTR,ROOTI,IER)
      WRITE (6,109)
  109 FORMAT (/,27X,9H L1 ROOTS,/)
      DO 110 I=1,4
  110 WRITE (6,112) ROOTR(I),ROOTI(I)
  112 FORMAT (17X,E13.5,10X,E13.5,/)
      WRITE (6,114) IER
  114 FORMAT (1X,I1)
      WRITE (6,106)
  106 FORMAT (1H1,//)
  125 DO 215 I=1,4
```

```
0000000001111111111222222222233333333334444444444555555555566666666667777777778
1234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890
      OA=F1
      OB=F2*ROOTR(I)+F3
      OC=F4*ROOTR(I)*ROOTR(I)+F5*ROOTR(I)+F6
      OD=G1
      OE=G2*ROOTR(I)+G3
      OF=G4*ROOTR(I)*ROOTR(I)+G5*ROOTR(I)+G6
      IF (ABS(ROOTI(I)).LT.0.000001) ROOTI(I)=0.0
      IF (ROOTI(I)) 200,130,200
  200 WRITE (6,201)
  201 FORMAT (5X,18H ROOT IS IMAGINARY,/)
      GO TO 215
  130 SA=OB*OB-4.0*OA*OC
      IF (SA) 200,140,140
  140 ROOTA1=(-OB+SQRT(SA))/(2.0*OA)
      ROOTA2=(-OB-SQRT(SA))/(2.0*OA)
      SB=OE*OE-4.0*OD*OF
      IF (SB) 200,150,150
  150 ROOTB1=(-OE+SQRT(SB))/(2.0*OD)
      ROOTB2=(-OE-SQRT(SB))/(2.0*OD)
      WRITE (6,155) ROOTA1,ROOTA2,ROOTB1,ROOTB2
  155 FORMAT (/,13X,8H ROOTA1=,F18.6,5X,8H ROOTA2=,F18.6,/,13X,8H ROOTB1
     $=,F18.6,5X,8H ROOTB2=,F18.6,/)
      IF (ABS(ROOTA1-ROOTB1).LT.0.05) GO TO 160
      IF (ABS(ROOTA1-ROOTB2).LT.0.05) GO TO 160
      IF (ABS(ROOTA2-ROOTB1).LT.0.05) GO TO 170
      IF (ABS(ROOTA2-ROOTB2).LT.0.05) GO TO 170
      GO TO 215
  160 WRITE (6,180) ROOTA1
  180 FORMAT (2X,16H SECOND ROOT IS ,F20.5,/)
      ROOT=ROOTA1
      GO TO 210
  170 WRITE (6,180) ROOTA2
      ROOT=ROOTA2
  210 WRITE (6,212)
  212 FORMAT (14X,13H CENTER POINT,31X,13H CIRCLE POINT,/,8X,3H XA,19X,3
     $H YA,19X,3H XB,19X,3H YB,/)
      XA(I)=R(1)+P(1)*ROOTR(I)+Q(1)*ROOT
      YA(I)=R(2)+P(2)*ROOTR(I)+Q(2)*ROOT
      XB(I)=R(3)+P(3)*ROOTR(I)+Q(3)*ROOT
      YB(I)=R(4)+P(4)*ROOTR(I)+Q(4)*ROOT
      WRITE (6,230) XA(I),YA(I),XB(I),YB(I)
  230 FORMAT (4F20.8,/,33X,20H XXXXXXXXXXXXXXXXXXX,//)
  215 CONTINUE
      STOP
      END
```

APPENDIX C

SYNTHESIS PROGRAM FOR THREE FINITE POSITIONS AND TWO FIRST INFINITESIMAL DISPLACEMENTS

```
0000000001111111111222222222233333333334444444444555555555566666666667777777777 8
1234567890123456789012345678901234567890123456789012345678901234567890123456789 0
C ***************************************************************************
C *                                                                         *
C *                     SYNTHESIS OF SIX-LINK STEPHENSON 2                  *
C *                            FUNCTION GENERATOR                           *
C *                                                                         *
C *                THREE FINITE POSITIONS AND TWO VELOCITY RATIOS           *
C *                                                                         *
C *                                                                         *
C *                  INPUT FINITE ROTATIONS              TH1(I)             *
C *                                                                         *
C *                  OUTPUT FINITE ROTATIONS             PH1(I)             *
C *                                                                         *
C *                  ASSUMED ROTATIONS OF TERNARY        AL1(I)             *
C *                                                                         *
C *                  FIRST INFINITESIMAL DISPLACEMENT    DPHDTH(J)          *
C *                                                                         *
C *                  OUTPUT LINK LENGTH                  QC                 *
C *                                                                         *
C *                  OUTPUT LINK INITIAL ANGLE           THE                *
C *                                                                         *
C ***************************************************************************
C
      DIMENSION XCOF(5),COF(5),ROOTR(4),ROOTI(4)
      DIMENSION XQ(5),YQ(5),XA(5),YA(5),XB(5),YB(5),XC(5),YC(5)
      DIMENSION TH1(5),PH1(5),AL1(5)
      DIMENSION H(16),H1(16),H2(16),R(4),P(4),Q(4)
      DIMENSION DXC(5),DYC(5),PXC(5),PYC(5)
      DIMENSION DPHDTH(5),DTHDAL(5)
      READ (5,5) J,K
    5 FORMAT (I1,I1)
      WRITE (6,6) J,K
    6 FORMAT (//,16X,34H VELOCITY RATIOS ARE AT POSITIONS ,I1,5H AND ,I1
     $,//)
      READ (5,10) TH1(2),TH1(3)
      READ (5,10) PH1(2),PH1(3)
      READ (5,10) AL1(2),AL1(3)
   10 FORMAT (2F10.0)
      READ (5,20) QC,THE
      READ (5,20) DPHDTH(J),DTHDAL(J)
      READ (5,20) DPHDTH(K),DTHDAL(K)
   20 FORMAT (2F10.0)
      WRITE (6,11) TH1(2),TH1(3)
   11 FORMAT (9X,26H INPUT ROTATION ANGLES ARE,2X,2F10.3,/)
      WRITE (6,12) PH1(2),PH1(3)
   12 FORMAT (9X,27H OUTPUT ROTATION ANGLES ARE,1X,2F10.3,/)
      WRITE (6,13) AL1(2),AL1(3)
   13 FORMAT (9X,23H TERNARY LINK ROTATIONS,5X,2F10.3,//)
      WRITE (6,14) QC,THE
   14 FORMAT (9X,15H OUTPUT LINK IS,F10.3,24H LONG AND AT AN ANGLE OF,F1
     $0.3,/)
      WRITE (6,15) DPHDTH(J),DTHDAL(J)
   15 FORMAT (9X,11H DPHDTH(J)=,F10.3,10X,11H DTHDAL(J)=,F10.3,/)
      WRITE (6,16) DPHDTH(K),DTHDAL(K)
   16 FORMAT (9X,11H DPHDTH(K)=,F10.3,10X,11H DTHDAL(K)=,F10.3,//)
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     000000000111111111122222222223333333333444444444455555555556666666666777777777778
     123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
          XQ(1)=1.0
          YQ(1)=0.0
          XM=0.0
          YM=0.0
          TH1(1)=0.0
          PH1(1)=0.0
          AL1(1)=0.0
          THE=(THE*3.14159265)/180.0
          DO 21 I=2,3
          TH1(I)=(TH1(I)*3.14159265)/180.0
          PH1(I)=(PH1(I)*3.14159265)/180.0
       21 AL1(I)=(AL1(I)*3.14159265)/180.0
          XC(1)=1.0+QC*COS(THE)
          YC(1)=0.0+QC*SIN(THE)
          DO 30 I=2,3
          XQ(I)=1.0*COS(-TH1(I))
          YQ(I)=1.0*SIN(-TH1(I))
          XC(I)=XC(1)*COS(PH1(I)-TH1(I))-YC(1)*SIN(PH1(I)-TH1(I))+XQ(I)-XQ(1
         $)*COS(PH1(I)-TH1(I))+YQ(1)*SIN(PH1(I)-TH1(I))
       30 YC(I)=XC(1)*SIN(PH1(I)-TH1(I))+YC(1)*COS(PH1(I)-TH1(I))+YQ(I)-XQ(1
         $)*SIN(PH1(I)-TH1(I))-YQ(1)*COS(PH1(I)-TH1(I))
          DXC(J)=(XC(1)-XQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(J)-TH1(J))*DPHDTH
         $(J))+(YC(1)-YQ(1))*(COS(PH1(J)-TH1(J))-COS(PH1(J)-TH1(J))*DPHDTH(J
         $))+XQ(1)*SIN(-TH1(J))+YQ(1)*COS(-TH1(J))
          DYC(J)=(XC(1)-XQ(1))*(-COS(PH1(J)-TH1(J))+COS(PH1(J)-TH1(J))*DPHDT
         $H(J))+(YC(1)-YQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(J)-TH1(J))*DPHDTH(
         $J))-XQ(1)*COS(-TH1(J))+YQ(1)*SIN(-TH1(J))
          DXC(K)=(XC(1)-XQ(1))*(SIN(PH1(K)-TH1(K))-SIN(PH1(K)-TH1(K))*DPHDTH
         $(K))+(YC(1)-YQ(1))*(COS(PH1(K)-TH1(K))-COS(PH1(K)-TH1(K))*DPHDTH(K
         $))+XQ(1)*SIN(-TH1(K))+YQ(1)*COS(-TH1(K))
          DYC(K)=(XC(1)-XQ(1))*(-COS(PH1(K)-TH1(K))+COS(PH1(K)-TH1(K))*DPHDT
         $H(K))+(YC(1)-YQ(1))*(SIN(PH1(K)-TH1(K))-SIN(PH1(K)-TH1(K))*DPHDTH(
         $K))-XQ(1)*COS(-TH1(K))+YQ(1)*SIN(-TH1(K))
          WRITE (6,17) DXC(J),DYC(J)
       17 FORMAT (9X,8H DXC(J)=,3X,F10.3,10X,8H DYC(J)=,3X,F10.3,/)
          WRITE (6,18) DXC(K),DYC(K)
       18 FORMAT (9X,8H DXC(K)=,3X,F10.3,10X,8H DYC(K)=,3X,F10.3//)
          PXC(J)=DTHDAL(J)*DXC(J)
          PYC(J)=DTHDAL(J)*DYC(J)
          PXC(K)=DTHDAL(K)*DXC(K)
          PYC(K)=DTHDAL(K)*DYC(K)
          WRITE (6,31)
       31 FORMAT (11X,3H XQ,18X,3H YQ,18X,3H XC,18X,3H YC,/)
          DO 33 I=1,3
       33 WRITE (6,32) XQ(I),YQ(I),XC(I),YC(I)
       32 FORMAT (6X,F12.8,9X,F12.8,9X,F12.8,9X,F12.8,/)
          DO 40 I=1,2
       40 H(I)=XC(I+1)-XC(1)*COS(AL1(I+1))+YC(1)*SIN(AL1(I+1))
          H(3)=PXC(J)+XC(1)*SIN(AL1(J))+YC(1)*COS(AL1(J))
          H(4)=PXC(K)+XC(1)*SIN(AL1(K))+YC(1)*COS(AL1(K))
          DO 50 I=1,2
       50 H(I+4)=YC(I+1)-YC(1)*COS(AL1(I+1))-XC(1)*SIN(AL1(I+1))
          H(7)=PYC(J)-XC(1)*COS(AL1(J))+YC(1)*SIN(AL1(J))
          H(8)=PYC(K)-XC(1)*COS(AL1(K))+YC(1)*SIN(AL1(K))
```

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     000000000111111111122222222223333333333444444444455555555556666666666777777777778
     123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890
      DO 60 I=1,2
   60 H(I+8)=-H(I)*COS(AL1(I+1))-H(I+4)*SIN(AL1(I+1))
      H(11)=-PXC(J)*COS(AL1(J))-PYC(J)*SIN(AL1(J))+XC(J)*SIN(AL1(J))-YC(
     $J)*COS(AL1(J))
      H(12)=-PXC(K)*COS(AL1(K))-PYC(K)*SIN(AL1(K))+XC(K)*SIN(AL1(K))-YC(
     $K)*COS(AL1(K))
      DO 70 I=1,2
   70 H(I+12)=-H(I+4)*COS(AL1(I+1))+H(I)*SIN(AL1(I+1))
      H(15)=PXC(J)*SIN(AL1(J))-PYC(J)*COS(AL1(J))+XC(J)*COS(AL1(J))+YC(J
     $)*SIN(AL1(J))
      H(16)=PXC(K)*SIN(AL1(K))-PYC(K)*COS(AL1(K))+XC(K)*COS(AL1(K))+YC(K
     $)*SIN(AL1(K))
      DO 72 I=1,16
   72 H1(I)=H(I)
      DO 74 I=1,16
   74 H2(I)=H(I)
      DO 80 I=1,2
   80 R(I)=0.5*(H(I)*H(I)+H(I+4)*H(I+4))
      R(3)=(YC(1)*XC(J)-XC(1)*YC(J)-YC(1)*PYC(J)-XC(1)*PXC(J))*COS(AL1(J
     $))+(XC(1)*XC(J)+YC(1)*YC(J)-XC(1)*PYC(J)+YC(1)*PXC(J))*SIN(AL1(J))
     $+XC(J)*PXC(J)+YC(J)*PYC(J)
      R(4)=(YC(1)*XC(K)-XC(1)*YC(K)-YC(1)*PYC(K)-XC(1)*PXC(K))*COS(AL1(K
     $))+(XC(1)*XC(K)+YC(1)*YC(K)-XC(1)*PYC(K)+YC(1)*PXC(K))*SIN(AL1(K))
     $+XC(K)*PXC(K)+YC(K)*PYC(K)
      DO 90 I=1,2
   90 P(I)=1.0-COS(AL1(I+1))
      P(3)=SIN(AL1(J))
      P(4)=SIN(AL1(K))
      DO 100 I=1,2
  100 Q(I)=SIN(AL1(I+1))
      Q(3)=COS(AL1(J))
      Q(4)=COS(AL1(K))
      CALL SIMQ(H,R,4,KS)
      CALL SIMQ(H1,P,4,KS1)
      CALL SIMQ(H2,Q,4,KS2)
      WRITE (6,99) KS
      WRITE (6,99) KS1
      WRITE (6,99) KS2
   99 FORMAT (1X,I1)
      F1=Q(1)*Q(3)+Q(2)*Q(4)
      F2=P(1)*Q(3)+Q(1)*P(3)+P(2)*Q(4)+Q(2)*P(4)
      F3=R(1)*Q(3)+Q(1)*R(3)+Q(2)*R(4)+Q(4)*R(2)
      F4=P(1)*P(3)+P(2)*P(4)
      F5=P(1)*R(3)+P(3)*R(1)+P(2)*R(4)+P(4)*R(2)-1.0
      F6=R(1)*R(3)+R(2)*R(4)
      G1=Q(1)*Q(4)-Q(2)*Q(3)
      G2=P(1)*Q(4)+Q(1)*P(4)-P(2)*Q(3)-Q(2)*P(3)
      G3=R(1)*Q(4)+Q(1)*R(4)-R(2)*Q(3)-Q(2)*R(3)-1.0
      G4=P(1)*P(4)-P(2)*P(3)
      G5=P(1)*R(4)+R(1)*P(4)-P(2)*R(3)-P(3)*R(2)
      G6=R(1)*R(4)-R(2)*R(3)
      XCOF(1)=F1*F1*G6*G6+G3*G3*F1*F6-G3*G6*F1*F3+G1*G6*F3*F3+G1*G1*F6*F
     $6-G1*G3*F3*F6-2.0*G1*G6*F1*F6
      XCOF(2)=2.0*G5*G6*F1*F1+2.0*G2*G3*F1*F6+G3*G3*F1*F5-G3*G6*F1*F2-G2
```

```
00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
     $*G6*F1*F3-G3*G5*F1*F3+2.0*G1*G6*F2*F3+G1*G5*F3*F3+2.0*G1*G1*F5*F6-
     $G1*G3*F2*F6-G1*G2*F3*F6-G1*G3*F3*F5-2.0*G1*G5*F1*F6-2.0*G1*G6*F1*F
     $5
      XCOF(3)=2.0*G4*G6*F1*F1+G5*G5*F1*F1+G2*G2*F1*F6+2.0*G2*G3*F1*F5+G3
     $*G3*F1*F4-G2*G6*F1*F2-G3*G5*F1*F2-G2*G5*F1*F3-G3*G4*F1*F3+G1*G6*F2
     $*F2+2.0*G1*G5*F2*F3+G1*G4*F3*F3+2.0*G1*G1*F4*F6+G1*G1*F5*F5-G1*G2*
     $F2*F6-G1*G3*F2*F5-G1*G2*F3*F5-G1*G3*F3*F4-2.0*G1*G6*F1*F4-2.0*G1*G
     $5*F1*F5-2.0*G1*G4*F1*F6
      XCOF(4)=2.0*G4*G5*F1*F1+G2*G2*F1*F5+2.0*G2*G3*F1*F4-G2*G5*F1*F2-G3
     $*G4*F1*F2-G2*G4*F1*F3+G1*G5*F2*F2+2.0*G1*G4*F2*F3+2.0*G1*G1*F4*F5-
     $G1*G2*F2*F5-G1*G3*F2*F4-G1*G2*F3*F4-2.0*G1*G5*F1*F4-2.0*G1*G4*F1*F
     $5
      XCOF(5)=G4*G4*F1*F1+G2*G2*F1*F4-G2*G4*F1*F2+G1*G4*F2*F2+G1*G1*F4*F
     $4-G1*G2*F2*F4-2.0*G1*G4*F1*F4
      CALL POLRT(XCOF,COF,4,ROOTR,ROOTI,IER)
      WRITE (6,109)
  109 FORMAT (/,27X,9H L1 RCOTS,/)
      DO 110 I=1,4
  110 WRITE (6,112) RCCTR(I),ROOTI(I)
  112 FORMAT (17X,E13.5,10X,E13.5,/)
      WRITE (6,114) IER
  114 FORMAT (1X,I1)
      WRITE (6,106)
  106 FORMAT (1H1,//)
  125 DO 215 I=1,4
      OA=F1
      OB=F2*ROOTR(I)+F3
      OC=F4*ROOTR(I)*RCCTR(I)+F5*ROOTR(I)+F6
      OD=G1
      OE=G2*ROOTR(I)+G3
      OF=G4*ROOTR(I)*ROOTR(I)+G5*ROOTR(I)+G6
      IF (ABS(ROOTI(I)).LT.0.000001) ROOTI(I)=0.0
      IF (ROOTI(I)) 200,130,200
  200 WRITE (6,201)
  201 FORMAT (5X,18H ROOT IS IMAGINARY,/)
      GO TO 215
  130 SA=OB*OB-4.0*OA*OC
      IF (SA) 200,140,140
  140 ROOTA1=(-OB+SQRT(SA))/(2.0*OA)
      ROOTA2=(-OB-SQRT(SA))/(2.0*OA)
      SB=OE*OE-4.0*OD*CF
      IF (SB) 200,150,150
  150 ROOTB1=(-OE+SQRT(SB))/(2.0*OD)
      ROOTB2=(-OE-SQRT(SB))/(2.0*OD)
      WRITE (6,155) ROOTA1,ROOTA2,ROOTB1,ROOTB2
  155 FORMAT (/,13X,8H ROOTA1=,F18.6,5X,8H ROOTA2=,F18.6,/,13X,8H ROOTB1
     $=,F18.6,5X,8H RCCTB2=,F18.6,/)
      IF (ABS(ROOTA1-ROOTB1).LT.0.05) GO TO 160
      IF (ABS(ROOTA1-ROOTB2).LT.0.05) GO TO 160
      IF (ABS(ROOTA2-ROOTB1).LT.0.05) GO TO 170
      IF (ABS(ROOTA2-ROOTB2).LT.0.05) GO TO 170
      GO TO 215
  160 WRITE (6,180) ROOTA1
  180 FORMAT (2X,16H SECOND ROOT IS ,F20.5,/)
```

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         00000000011111111112222222222333333333344444444445555555555666666666677777777778
         12345678901234567890123456789012345678901234567890123456789012345678901234567890
               ROOT=ROOTA1
               GO TO 210
           170 WRITE (6,180) ROOTA2
               ROOT=ROOTA2
           210 WRITE (6,212)
           212 FORMAT (14X,13H CENTER POINT,31X,13H CIRCLE POINT,/,8X,3H XA,19X,3
              $H YA,19X,3H XB,19X,3H YB,/)
               XA(I)=R(1)+P(1)*ROOTR(I)+Q(1)*ROOT
               YA(I)=R(2)+P(2)*ROOTR(I)+Q(2)*ROOT
               XB(I)=R(3)+P(3)*ROOTR(I)+Q(3)*ROOT
               YB(I)=R(4)+P(4)*ROOTR(I)+Q(4)*ROOT
               WRITE (6,230) XA(I),YA(I),XB(I),YB(I)
           230 FORMAT (4F20.8,/,33X,20H XXXXXXXXXXXXXXXXXXX,//)
           215 CONTINUE
               STOP
               END
```

APPENDIX D

SYNTHESIS PROGRAM FOR THREE FINITE POSITIONS AND ONE SECOND INFINITESIMAL DISPLACEMENT

```
00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
C **********************************************************************
C *                                                                    *
C *                  SYNTHESIS OF SIX-LINK STEPHENSON 2                *
C *                         FUNCTION GENERATOR                         *
C *                                                                    *
C *              THREE FINITE POSITIONS AND ONE ACCELERATION RATIO     *
C *                                                                    *
C *                INPUT FINITE ROTATIONS            TH1(I)            *
C *                                                                    *
C *                OUTPUT FINITE ROTATIONS           PH1(I)            *
C *                                                                    *
C *                ASSUMED ROTATIONS OF TERNARY      AL1(I)            *
C *                                                                    *
C *                SECOND INFINITESIMAL DISPLACEMENT DDPHTH(J)         *
C *                                                                    *
C *                OUTPUT LINK LENGTH                QC                *
C *                                                                    *
C *                OUTPUT LINK INITIAL ANGLE         THE               *
C *                                                                    *
C **********************************************************************
C
      DIMENSION XCOF(5),COF(5),ROOTR(4),ROOTI(4)
      DIMENSION XQ(5),YQ(5),XA(5),YA(5),XB(5),YB(5),XC(5),YC(5)
      DIMENSION TH1(5),PH1(5),AL1(5)
      DIMENSION H(16),H1(16),H2(16),R(4),P(4),Q(4)
      DIMENSION DPHDTH(5),DDPHTH(5),DTHDAL(5),DDTHAL(5)
      DIMENSION DXC(5),DYC(5),DDXC(5),DDYC(5)
      DIMENSION PXC(5),PYC(5),PPXC(5),PPYC(5)
      READ (5,5) J
    5 FORMAT (I1)
      WRITE (6,6) J
    6 FORMAT (//,10X,32H ACCELERATION RATIO IS AT POINT ,I1,//)
      READ (5,10) TH1(2),TH1(3)
      READ (5,10) PH1(2),PH1(3)
      READ (5,10) AL1(2),AL1(3)
   10 FORMAT (2F10.0)
      READ (5,20) QC,THE
      READ (5,20) DPHDTH(J),DTHDAL(J)
      READ (5,20) DDPHTH(J),DDTHAL(J)
   20 FORMAT (2F10.0)
      WRITE (6,11) TH1(2),TH1(3)
   11 FORMAT (9X,26H INPUT ROTATION ANGLES ARE,2X,2F10.3,/)
      WRITE (6,12) PH1(2),PH1(3)
   12 FORMAT (9X,27H OUTPUT ROTATION ANGLES ARE,1X,2F10.3,/)
      WRITE (6,13) AL1(2),AL1(3)
   13 FORMAT (9X,23H TERNARY LINK ROTATIONS,5X,2F10.3,//)
      WRITE (6,14) QC,THE
   14 FORMAT (6X,15H OUTPUT LINK IS,F10.3,24H LONG AND AT AN ANGLE OF,F1
     $0.3,/)
      WRITE (6,15) DPHDTH(J),DTHDAL(J)
   15 FORMAT (9X,11H DPHDTH(J)=,F10.3,10X,11H DTHDAL(J)=,F10.3,/)
      WRITE (6,16) DDPHTH(J),DDTHAL(J)
   16 FORMAT (9X,11H DDPHTH(J)=,F10.3,10X,11H DDTHAL(J)=,F10.3,//)
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000C0CC001111111111222222222233333333334444444444555555555566666666667777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
      XQ(1)=1.0
      YQ(1)=0.0
      XM=0.0
      YM=0.0
      TH1(1)=0.0
      PH1(1)=0.0
      AL1(1)=0.0
      THE=(THE*3.14159265)/180.0
      DO 21 I=2,3
      TH1(I)=(TH1(I)*3.14159265)/180.0
      PH1(I)=(PH1(I)*3.14159265)/180.0
   21 AL1(I)=(AL1(I)*3.14159265)/180.0
      XC(1)=1.0+QC*COS(THE)
      YC(1)=0.0+QC*SIN(THE)
      DO 30 I=2,3
      XQ(I)=1.0*COS(-TH1(I))
      YQ(I)=1.0*SIN(-TH1(I))
      XC(I)=XC(1)*COS(PH1(I)-TH1(I))-YC(1)*SIN(PH1(I)-TH1(I))+XQ(I)-XQ(1
     $)*COS(PH1(I)-TH1(I))+YQ(1)*SIN(PH1(I)-TH1(I))
   30 YC(I)=XC(1)*SIN(PH1(I)-TH1(I))+YC(1)*COS(PH1(I)-TH1(I))+YQ(I)-XQ(1
     $)*SIN(PH1(I)-TH1(I))-YQ(1)*COS(PH1(I)-TH1(I))
      DXC(J)=(XC(1)-XQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(J)-TH1(J))*DPHDTH
     $(J))+(YC(1)-YQ(1))*(COS(PH1(J)-TH1(J))-COS(PH1(J)-TH1(J))*DPHDTH(J
     $))+XQ(1)*SIN(-TH1(J))+YQ(1)*COS(-TH1(J))
      DYC(J)=(XC(1)-XQ(1))*(-COS(PH1(J)-TH1(J))+COS(PH1(J)-TH1(J))*DPHDT
     $H(J))+(YC(1)-YQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(J)-TH1(J))*DPHDTH(
     $J))-XQ(1)*COS(-TH1(J))+YQ(1)*SIN(-TH1(J))
      DDXC(J)=(XC(1)-XQ(1))*(-COS(PH1(J)-TH1(J))+2.0*COS(PH1(J)-TH1(J))*
     $DPHDTH(J)-COS(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-SIN(PH1(J)-TH1(J)
     $)*DDPHTH(J))+(YC(1)-YQ(1))*(SIN(PH1(J)-TH1(J))-2.0*SIN(PH1(J)-TH1(
     $J))*DPHDTH(J)+SIN(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-COS(PH1(J)-TH
     $1(J))*DDPHTH(J))-XQ(1)*COS(-TH1(J))+YQ(1)*SIN(-TH1(J))
      DDYC(J)=(XC(1)-XQ(1))*(-SIN(PH1(J)-TH1(J))+2.0*SIN(PH1(J)-TH1(J))*
     $DPHDTH(J)-SIN(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)+COS(PH1(J)-TH1(J)
     $)*DDPHTH(J))+(YC(1)-YQ(1))*(-COS(PH1(J)-TH1(J))+2.0*COS(PH1(J)-TH1
     $(J))*DPHDTH(J)-COS(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-SIN(PH1(J)-T
     $H1(J))*DDPHTH(J))-XQ(1)*SIN(-TH1(J))-YQ(1)*COS(-TH1(J))
      PXC(J)=DTHDAL(J)*DXC(J)
      PYC(J)=DTHDAL(J)*DYC(J)
      PPXC(J)=DTHDAL(J)*DTHDAL(J)*DDXC(J)+DDTHAL(J)*DXC(J)
      PPYC(J)=DTHDAL(J)*DTHDAL(J)*DDYC(J)+DDTHAL(J)*DYC(J)
      WRITE (6,17) DXC(J),DYC(J)
   17 FORMAT (9X,8H DXC(J)=,3X,F10.3,10X,8H DYC(J)=,3X,F10.3,/)
      WRITE (6,18) DDXC(J),DDYC(J)
   18 FORMAT (9X,9H DDXC(J)=,2X,F10.3,10X,9H DDYC(J)=,2X,F10.3,//)
      WRITE (6,31)
   31 FORMAT (11X,3H XQ,18X,3H YQ,18X,3H XC,18X,3H YC,/)
      DO 33 I=1,3
   33 WRITE (6,32) XQ(I),YQ(I),XC(I),YC(I)
   32 FORMAT (6X,F12.8,9X,F12.8,9X,F12.8,9X,F12.8,/)
      DO 40 I=1,2
   40 H(I)=XC(I+1)-XC(1)*COS(AL1(I+1))+YC(1)*SIN(AL1(I+1))
      H(3)=PXC(J)+XC(1)*SIN(AL1(J))+YC(1)*COS(AL1(J))
      H(4)=PPXC(J)+XC(1)*COS(AL1(J))-YC(1)*SIN(AL1(J))
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      DO 50 I=1,2
   50 H(I+4)=YC(I+1)-YC(1)*COS(AL1(I+1))-XC(1)*SIN(AL1(I+1))
      H(7)=PYC(J)-XC(1)*COS(AL1(J))+YC(1)*SIN(AL1(J))
      H(8)=PPYC(J)+XC(1)*SIN(AL1(J))+YC(1)*COS(AL1(J))
      DO 60 I=1,2
   60 H(I+8)=-H(I)*COS(AL1(I+1))-H(I+4)*SIN(AL1(I+1))
      H(11)=-PXC(J)*COS(AL1(J))-PYC(J)*SIN(AL1(J))+XC(J)*SIN(AL1(J))-YC(
     $J)*COS(AL1(J))
      H(12)=-PPXC(J)*COS(AL1(J))-PPYC(J)*SIN(AL1(J))+2.0*PXC(J)*SIN(AL1(
     $J))-2.0*PYC(J)*COS(AL1(J))+XC(J)*COS(AL1(J))+YC(J)*SIN(AL1(J))
      DO 70 I=1,2
   70 H(I+12)=-H(I+4)*COS(AL1(I+1))+H(I)*SIN(AL1(I+1))
      H(15)=PXC(J)*SIN(AL1(J))-PYC(J)*COS(AL1(J))+XC(J)*COS(AL1(J))+YC(J
     $)*SIN(AL1(J))
      H(16)=PPXC(J)*SIN(AL1(J))-PPYC(J)*COS(AL1(J))+2.0*PXC(J)*COS(AL1(J
     $))+2.0*PYC(J)*SIN(AL1(J))-XC(J)*SIN(AL1(J))+YC(J)*COS(AL1(J))
      DO 72 I=1,16
   72 H1(I)=H(I)
      DO 74 I=1,16
   74 H2(I)=H(I)
      DO 80 I=1,2
   80 R(I)=0.5*(H(I)*H(I)+H(I+4)*H(I+4))
      R(3)=(YC(1)*XC(J)-XC(1)*YC(J)-YC(1)*PYC(J)-XC(1)*PXC(J))*COS(AL1(J
     $))+(XC(1)*XC(J)+YC(1)*YC(J)-XC(1)*PYC(J)+YC(1)*PXC(J))*SIN(AL1(J))
     $+XC(J)*PXC(J)+YC(J)*PYC(J)
      DD1=2.0*YC(1)*PXC(J)-2.0*XC(1)*PYC(J)-YC(1)*PPYC(J)-XC(1)*PPXC(J)+
     $XC(1)*XC(J)+YC(1)*YC(J)
      DD2=2.0*XC(1)*PXC(J)+2.0*YC(1)*PYC(J)-XC(1)*PPYC(J)+YC(1)*PPXC(J)-
     $YC(1)*XC(J)+XC(1)*YC(J)
      R(4)=COS(AL1(J))*DD1+SIN(AL1(J))*DD2+XC(J)*PPXC(J)+YC(J)*PPYC(J)+P
     $XC(J)*PXC(J)+PYC(J)*PYC(J)
      DO 90 I=1,2
   90 P(I)=1.0-COS(AL1(I+1))
      P(3)=SIN(AL1(J))
      P(4)=COS(AL1(J))
      DO 100 I=1,2
  100 Q(I)=SIN(AL1(I+1))
      Q(3)=COS(AL1(J))
      Q(4)=-SIN(AL1(J))
      CALL SIMQ(H,R,4,KS)
      CALL SIMQ(H1,P,4,KS1)
      CALL SIMQ(H2,Q,4,KS2)
      WRITE (6,99) KS
      WRITE (6,99) KS1
      WRITE (6,99) KS2
   99 FORMAT (1X,I1)
      F1=Q(1)*Q(3)+Q(2)*Q(4)
      F2=P(1)*Q(3)+Q(1)*P(3)+P(2)*Q(4)+Q(2)*P(4)
      F3=R(1)*Q(3)+Q(1)*R(3)+Q(2)*R(4)+Q(4)*R(2)
      F4=P(1)*P(3)+P(2)*P(4)
      F5=P(1)*R(3)+P(3)*R(1)+P(2)*R(4)+P(4)*R(2)-1.0
      F6=R(1)*R(3)+R(2)*R(4)
      G1=Q(1)*Q(4)-Q(2)*Q(3)
      G2=P(1)*Q(4)+Q(1)*P(4)-P(2)*Q(3)-Q(2)*P(3)
```

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      G3=R(1)*Q(4)+Q(1)*R(4)-R(2)*Q(3)-Q(2)*R(3)-1.0
      G4=P(1)*P(4)-P(2)*P(3)
      G5=P(1)*R(4)+R(1)*P(4)-P(2)*R(3)-P(3)*R(2)
      G6=R(1)*R(4)-R(2)*R(3)
      XCOF(1)=F1*F1*G6*G6+G3*G3*F1*F6-G3*G6*F1*F3+G1*G6*F3*F3+G1*G1*F6*F
     $6-G1*G3*F3*F6-2.0*G1*G6*F1*F6
      XCOF(2)=2.0*G5*G6*F1*F1+2.0*G2*G3*F1*F6+G3*G3*F1*F5-G3*G6*F1*F2-G2
     $*G6*F1*F3-G3*G5*F1*F3+2.0*G1*G6*F2*F3+G1*G5*F3*F3+2.0*G1*G1*F5*F6-
     $G1*G3*F2*F6-G1*G2*F3*F6-G1*G3*F3*F5-2.0*G1*G5*F1*F6-2.0*G1*G6*F1*F
     $5
      XCOF(3)=2.0*G4*G6*F1*F1+G5*G5*F1*F1+G2*G2*F1*F6+2.0*G2*G3*F1*F5+G3
     $*G3*F1*F4-G2*G6*F1*F2-G3*G5*F1*F2-G2*G5*F1*F3-G3*G4*F1*F3+G1*G6*F2
     $*F2+2.0*G1*G5*F2*F3+G1*G4*F3*F3+2.0*G1*G1*F4*F6+G1*G1*F5*F5-G1*G2*
     $F2*F6-G1*G3*F2*F5-G1*G2*F3*F5-G1*G3*F3*F4-2.0*G1*G6*F1*F4-2.0*G1*G
     $5*F1*F5-2.0*G1*G4*F1*F6
      XCOF(4)=2.0*G4*G5*F1*F1+G2*G2*F1*F5+2.0*G2*G3*F1*F4-G2*G5*F1*F2-G3
     $*G4*F1*F2-G2*G4*F1*F3+G1*G5*F2*F2+2.0*G1*G4*F2*F3+2.0*G1*G1*F4*F5-
     $G1*G2*F2*F5-G1*G3*F2*F4-G1*G2*F3*F4-2.0*G1*G5*F1*F4-2.0*G1*G4*F1*F
     $5
      XCOF(5)=G4*G4*F1*F1+G2*G2*F1*F4-G2*G4*F1*F2+G1*G4*F2*F2+G1*G1*F4*F
     $4-G1*G2*F2*F4-2.0*G1*G4*F1*F4
      CALL POLRT(XCOF,COF,4,ROOTR,ROOTI,IER)
      WRITE (6,109)
  109 FORMAT (/,27X,9H L1 RCOTS,/)
      DO 110 I=1,4
  110 WRITE (6,112) RCCTR(I),ROOTI(I)
  112 FORMAT (17X,E13.5,10X,E13.5,/)
      WRITE (6,114) IER
  114 FORMAT (1X,I1)
      WRITE (6,106)
  106 FORMAT (1H1,//)
  125 DO 215 I=1,4
      OA=F1
      OB=F2*ROOTR(I)+F3
      OC=F4*ROOTR(I)*ROOTR(I)+F5*ROOTR(I)+F6
      OD=G1
      OE=G2*ROOTR(I)+G3
      OF=G4*ROOTR(I)*ROOTR(I)+G5*ROOTR(I)+G6
      IF (ABS(ROOTI(I)).LT.0.000001) ROOTI(I)=0.0
      IF (ROOTI(I)) 200,130,200
  200 WRITE (6,201)
  201 FORMAT (5X,18H ROOT IS IMAGINARY,/)
      GO TO 215
  130 SA=OB*OB-4.0*OA*OC
      IF (SA) 200,140,140
  140 ROOTA1=(-OB+SQRT(SA))/(2.0*OA)
      ROOTA2=(-OB-SQRT(SA))/(2.0*OA)
      SB=OE*OE-4.0*OD*OF
      IF (SB) 200,150,150
  150 ROOTB1=(-OE+SQRT(SB))/(2.0*OD)
      ROOTB2=(-OE-SQRT(SB))/(2.0*OD)
      WRITE (6,155) ROOTA1,ROOTA2,ROOTB1,ROOTB2
  155 FORMAT (/,13X,8H ROOTA1=,F18.6,5X,8H ROOTA2=,F18.6,/,13X,8H ROOTB1
     $=,F18.6,5X,8H ROOTB2=,F18.6,/)
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1234567890123456789012345678901234567890123456789012345678901234567890123456789 0
      IF (ABS(ROOTA1-ROOTB1).LT.0.05) GO TO 160
      IF (ABS(ROOTA1-ROOTB2).LT.0.05) GO TO 160
      IF (ABS(ROOTA2-ROOTB1).LT.0.05) GO TO 170
      IF (ABS(ROOTA2-ROOTB2).LT.0.05) GO TO 170
      GO TO 215
  16C WRITE (6,180) RCCTA1
  180 FORMAT (2X,16H SECOND ROOT IS ,F20.5,/)
      ROOT=ROOTA1
      GO TO 210
  170 WRITE (6,180) ROOTA2
      RCOT=ROOTA2
  210 WRITE (6,212)
  212 FORMAT (14X,13H CENTER POINT,31X,13H CIRCLE POINT,/,8X,3H XA,19X,3
     $H YA,19X,3H XB,19X,3H YB,/)
      XA(I)=R(1)+P(1)*ROOTR(I)+Q(1)*ROOT
      YA(I)=R(2)+P(2)*ROOTR(I)+Q(2)*ROOT
      XB(I)=R(3)+P(3)*ROOTR(I)+Q(3)*ROOT
      YB(I)=R(4)+P(4)*ROOTR(I)+Q(4)*ROOT
      WRITE (6,230) XA(I),YA(I),XB(I),YB(I)
  230 FORMAT (4F20.8,/,33X,20H XXXXXXXXXXXXXXXXXXX,//)
  215 CONTINUE
      STOP
      END
```

APPENDIX E

SYNTHESIS PROGRAM FOR TWO FINITE POSITIONS, ONE FIRST INFINITESIMAL DISPLACEMENT AND ONE SECOND INFINITESIMAL DISPLACEMENT

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123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890
C ***********************************************************************
C *                                                                     *
C *                  SYNTHESIS OF SIX-LINK STEPHENSON 2                 *
C *                          FUNCTION GENERATOR                         *
C *                                                                     *
C *                        TWO FINITE POSITIONS,                        *
C *            ONE VELOCITY RATIO,AND ONE ACCELERATION RATIO            *
C *                                                                     *
C *                                                                     *
C *             INPUT FINITE ROTATION                TH1(I)             *
C *                                                                     *
C *             OUTPUT FINITE ROTATION               PH1(I)             *
C *                                                                     *
C *             ASSUMED ROTATION OF TERNARY          AL1(I)             *
C *                                                                     *
C *             FIRST INFINITESIMAL DISPLACEMENT     DPHDTH(K)          *
C *                                                                     *
C *             SECOND INFINITESIMAL DISPLACEMENT    DDPHTH(J)          *
C *                                                                     *
C *             OUTPUT LINK LENGTH                   QC                 *
C *                                                                     *
C *             OUTPUT LINK INITIAL ANGLE            THE                *
C *                                                                     *
C *                                                                     *
C ***********************************************************************
C
      DIMENSION XCOF(5),COF(5),ROOTR(4),ROOTI(4)
      DIMENSION XQ(5),YQ(5),XA(5),YA(5),XB(5),YB(5),XC(5),YC(5)
      DIMENSION TH1(5),PH1(5),AL1(5)
      DIMENSION H(16),H1(16),H2(16),R(4),P(4),Q(4)
      DIMENSION DPHDTH(5),DDPHTH(5),DTHDAL(5),DDTHAL(5)
      DIMENSION DXC(5),DYC(5),DDXC(5),DDYC(5)
      DIMENSION PXC(5),PYC(5),PPXC(5),PPYC(5)
      READ (5,5) J,K
    5 FORMAT (I1,I1)
      WRITE (6,6) K,J
    6 FORMAT (//,5X,32H  VELOCITY RATIO IS AT POSITION ,I1,39H AND ACCEL
     $ERATION RATIO IS AT POSITION ,I1,//)
      READ (5,10) TH1(2)
      READ (5,10) PH1(2)
      READ (5,10) AL1(2)
   10 FORMAT (F10.0)
      READ (5,20) QC,THE
      READ (5,20) DPHDTH(J),DTHDAL(J)
      READ (5,20) DDPHTH(J),DDTHAL(J)
      READ (5,20) DPHDTH(K),DTHDAL(K)
   20 FORMAT (2F10.0)
      WRITE (6,11) TH1(2)
   11 FORMAT (9X,24H INPUT ROTATION ANGLE IS,2X,F10.3,/)
      WRITE (6,12) PH1(2)
   12 FORMAT (9X,25H OUTPUT ROTATION ANGLE IS,1X,F10.3,/)
      WRITE (6,13) AL1(2)
   13 FORMAT (9X,22H TERNARY LINK ROTATION,4X,F10.3,/)
      WRITE (6,14) QC,THE
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123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890
   14 FORMAT (6X,15H OLTPUT LINK IS,F10.3,24H LONG AND AT AN ANGLE OF,F1
     $0.3,/)
      WRITE (6,15) DPHDTH(J),DTHDAL(J)
   15 FORMAT (9X,11H DPHDTH(J)=,F10.3,10X,11H DTHDAL(J)=,F10.3,/)
      WRITE (6,16) DDPHTH(J),DDTHAL(J)
   16 FORMAT (9X,11H DDPHTH(J)=,F10.3,10X,11H DDTHAL(J)=,F10.3,/)
      WRITE (6,19) DPHDTH(K),DTHDAL(K)
   19 FORMAT (9X,11H DPHDTH(K)=,F10.3,10X,11H DTHDAL(K)=,F10.3,//)
      XQ(1)=1.0
      YQ(1)=0.0
      XM=0.0
      YM=0.0
      TH1(1)=0.0
      PH1(1)=0.0
      AL1(1)=0.0
      THE=(THE*3.14159265)/180.0
      I=2
      TH1(I)=(TH1(I)*3.14159265)/180.0
      PH1(I)=(PH1(I)*3.14159265)/180.0
      AL1(I)=(AL1(I)*3.14159265)/180.0
      XC(1)=1.0+QC*COS(THE)
      YC(1)=0.0+QC*SIN(THE)
      XQ(I)=1.0*COS(-TH1(I))
      YQ(I)=1.0*SIN(-TH1(I))
      XC(I)=XC(1)*COS(PH1(I)-TH1(I))-YC(1)*SIN(PH1(I)-TH1(I))+XQ(I)-XQ(1
     $)*COS(PH1(I)-TH1(I))+YQ(1)*SIN(PH1(I)-TH1(I))
      YC(I)=XC(1)*SIN(PH1(I)-TH1(I))+YC(1)*COS(PH1(I)-TH1(I))+YQ(I)-XQ(1
     $)*SIN(PH1(I)-TH1(I))-YQ(1)*COS(PH1(I)-TH1(I))
      DXC(J)=(XC(1)-XQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(J)-TH1(J))*DPHDTH
     $(J))+(YC(1)-YQ(1))*(COS(PH1(J)-TH1(J))-COS(PH1(J)-TH1(J))*DPHDTH(J
     $))+XQ(1)*SIN(-TH1(J))+YQ(1)*COS(-TH1(J))
      DYC(J)=(XC(1)-XQ(1))*(-COS(PH1(J)-TH1(J))+COS(PH1(J)-TH1(J))*DPHDT
     $H(J))+(YC(1)-YQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(J)-TH1(J))*DPHDTH(
     $J))-XQ(1)*COS(-TH1(J))+YQ(1)*SIN(-TH1(J))
      DDXC(J)=(XC(1)-XQ(1))*(-COS(PH1(J)-TH1(J))+2.0*COS(PH1(J)-TH1(J))*
     $DPHDTH(J)-COS(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-SIN(PH1(J)-TH1(J)
     $)*DDPHTH(J))+(YC(1)-YQ(1))*(SIN(PH1(J)-TH1(J))-2.0*SIN(PH1(J)-TH1(
     $J))*DPHDTH(J)+SIN(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-COS(PH1(J)-TH
     $1(J))*DDPHTH(J))-XQ(1)*COS(-TH1(J))+YQ(1)*SIN(-TH1(J))
      DDYC(J)=(XC(1)-XQ(1))*(-SIN(PH1(J)-TH1(J))+2.0*SIN(PH1(J)-TH1(J))*
     $DPHDTH(J)-SIN(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)+COS(PH1(J)-TH1(J)
     $)*DDPHTH(J))+(YC(1)-YQ(1))*(-COS(PH1(J)-TH1(J))+2.0*COS(PH1(J)-TH1
     $(J))*DPHDTH(J)-COS(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-SIN(PH1(J)-T
     $H1(J))*DDPHTH(J))-XQ(1)*SIN(-TH1(J))-YQ(1)*COS(-TH1(J))
      DXC(K)=(XC(1)-XQ(1))*(SIN(PH1(K)-TH1(K))-SIN(PH1(K)-TH1(K))*DPHDTH
     $(K))+(YC(1)-YQ(1))*(COS(PH1(K)-TH1(K))-COS(PH1(K)-TH1(K))*DPHDTH(K
     $))+XQ(1)*SIN(-TH1(K))+YQ(1)*COS(-TH1(K))
      DYC(K)=(XC(1)-XQ(1))*(-COS(PH1(K)-TH1(K))+COS(PH1(K)-TH1(K))*DPHDT
     $H(K))+(YC(1)-YQ(1))*(SIN(PH1(K)-TH1(K))-SIN(PH1(K)-TH1(K))*DPHDTH(
     $K))-XQ(1)*COS(-TH1(K))+YQ(1)*SIN(-TH1(K))
      WRITE (6,17) DXC(J),DYC(J)
   17 FORMAT (9X,8H DXC(J)=,3X,F10.3,10X,8H DYC(J)=,3X,F10.3,/)
      WRITE (6,21) DDXC(J),DDYC(J)
   21 FORMAT (9X,9H DDXC(J)=,2X,F10.3,10X,9H DDYC(J)=,2X,F10.3,/)
```

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00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
      WRITE (6,18) DXC(K),DYC(K)
   18 FORMAT (9X,8H DXC(K)=,3X,F10.3,10X,8H DYC(K)=,3X,F10.3,///)
      PXC(J)=DTHDAL(J)*DXC(J)
      PYC(J)=DTHDAL(J)*DYC(J)
      PPXC(J)=DDTHAL(J)*DXC(J)+DTHDAL(J)*DTHDAL(J)*DDXC(J)
      PPYC(J)=DDTHAL(J)*DYC(J)+DTHDAL(J)*DTHDAL(J)*DDYC(J)
      PXC(K)=DTHDAL(K)*DXC(K)
      PYC(K)=DTHDAL(K)*DYC(K)
      WRITE (6,31)
   31 FORMAT (11X,3H XQ,18X,3H YQ,18X,3H XC,18X,3H YC,/)
      DO 33 I=1,2
   33 WRITE (6,32) XQ(I),YQ(I),XC(I),YC(I)
   32 FORMAT (6X,F12.8,9X,F12.8,9X,F12.8,9X,F12.8,/)
      I=1
      H(I)=XC(I+1)-XC(1)*COS(AL1(I+1))+YC(1)*SIN(AL1(I+1))
      H(2)=PXC(J)+XC(1)*SIN(AL1(J))+YC(1)*COS(AL1(J))
      H(3)=PPXC(J)+XC(1)*COS(AL1(J))-YC(1)*SIN(AL1(J))
      H(4)=PXC(K)+XC(1)*SIN(AL1(K))+YC(1)*COS(AL1(K))
      H(I+4)=YC(I+1)-YC(1)*COS(AL1(I+1))-XC(1)*SIN(AL1(I+1))
      H(6)=PYC(J)-XC(1)*COS(AL1(J))+YC(1)*SIN(AL1(J))
      H(7)=PPYC(J)+XC(1)*SIN(AL1(J))+YC(1)*COS(AL1(J))
      H(8)=PYC(K)-XC(1)*COS(AL1(K))+YC(1)*SIN(AL1(K))
      H(I+8)=-H(I)*COS(AL1(I+1))-H(I+4)*SIN(AL1(I+1))
      H(10)=-PXC(J)*COS(AL1(J))-PYC(J)*SIN(AL1(J))+XC(J)*SIN(AL1(J))-YC(
     $J)*COS(AL1(J))
      H(11)=-PPXC(J)*COS(AL1(J))-PPYC(J)*SIN(AL1(J))+2.0*PXC(J)*SIN(AL1(
     $J))-2.0*PYC(J)*COS(AL1(J))+XC(J)*COS(AL1(J))+YC(J)*SIN(AL1(J))
      H(12)=-PXC(K)*COS(AL1(K))-PYC(K)*SIN(AL1(K))+XC(K)*SIN(AL1(K))-YC(
     $K)*COS(AL1(K))
      H(I+12)=-H(I+4)*COS(AL1(I+1))+H(I)*SIN(AL1(I+1))
      H(14)=PXC(J)*SIN(AL1(J))-PYC(J)*COS(AL1(J))+XC(J)*COS(AL1(J))+YC(J
     $)*SIN(AL1(J))
      H(15)=PPXC(J)*SIN(AL1(J))-PPYC(J)*COS(AL1(J))+2.0*PXC(J)*COS(AL1(J
     $))+2.0*PYC(J)*SIN(AL1(J))-XC(J)*SIN(AL1(J))+YC(J)*COS(AL1(J))
      H(16)=PXC(K)*SIN(AL1(K))-PYC(K)*COS(AL1(K))+XC(K)*COS(AL1(K))+YC(K
     $)*SIN(AL1(K))
      DO 72 I=1,16
   72 H1(I)=H(I)
      DO 74 I=1,16
   74 H2(I)=H(I)
      I=1
      R(I)=0.5*(H(I)*H(I)+H(I+4)*H(I+4))
      R(2)=(YC(1)*XC(J)-XC(1)*YC(J)-YC(1)*PYC(J)-XC(1)*PXC(J))*COS(AL1(J
     $))+(XC(1)*XC(J)+YC(1)*YC(J)-XC(1)*PYC(J)+YC(1)*PXC(J))*SIN(AL1(J))
     $+XC(J)*PXC(J)+YC(J)*PYC(J)
      ZS1=2.0*YC(1)*PXC(J)-2.0*XC(1)*PYC(J)-YC(1)*PPYC(J)-XC(1)*PPXC(J)+
     $XC(1)*XC(J)+YC(1)*YC(J)
      ZS2=2.0*XC(1)*PXC(J)+2.0*YC(1)*PYC(J)-XC(1)*PPYC(J)+YC(1)*PPXC(J)-
     $YC(1)*XC(J)+XC(1)*YC(J)
      R(3)=COS(AL1(J))*ZS1+SIN(AL1(J))*ZS2+XC(J)*PPXC(J)+YC(J)*PPYC(J)+P
     $XC(J)*PXC(J)+PYC(J)*PYC(J)
      R(4)=(YC(1)*XC(K)-XC(1)*YC(K)-YC(1)*PYC(K)-XC(1)*PXC(K))*COS(AL1(K
     $))+(XC(1)*XC(K)+YC(1)*YC(K)-XC(1)*PYC(K)+YC(1)*PXC(K))*SIN(AL1(K))
     $+XC(K)*PXC(K)+YC(K)*PYC(K)
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00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
      P(I)=1.0-COS(AL1(I+1))
      P(2)=SIN(AL1(J))
      P(3)=COS(AL1(J))
      P(4)=SIN(AL1(K))
      Q(I)=SIN(AL1(I+1))
      Q(2)=COS(AL1(J))
      Q(3)=-SIN(AL1(J))
      Q(4)=COS(AL1(K))
      CALL SIMQ(H,R,4,KS)
      CALL SIMQ(H1,P,4,KS1)
      CALL SIMQ(H2,Q,4,KS2)
      WRITE (6,99) KS
      WRITE (6,99) KS1
      WRITE (6,99) KS2
   99 FORMAT (1X,I1)
      F1=Q(1)*Q(3)+Q(2)*Q(4)
      F2=P(1)*Q(3)+Q(1)*P(3)+P(2)*Q(4)+Q(2)*P(4)
      F3=R(1)*Q(3)+Q(1)*R(3)+Q(2)*R(4)+Q(4)*R(2)
      F4=P(1)*P(3)+P(2)*P(4)
      F5=P(1)*R(3)+P(3)*R(1)+P(2)*R(4)+P(4)*R(2)-1.0
      F6=R(1)*R(3)+R(2)*R(4)
      G1=Q(1)*Q(4)-Q(2)*Q(3)
      G2=P(1)*Q(4)+Q(1)*P(4)-P(2)*Q(3)-Q(2)*P(3)
      G3=R(1)*Q(4)+Q(1)*R(4)-R(2)*Q(3)-Q(2)*R(3)-1.0
      G4=P(1)*P(4)-P(2)*P(3)
      G5=P(1)*R(4)+R(1)*P(4)-P(2)*R(3)-P(3)*R(2)
      G6=R(1)*R(4)-R(2)*R(3)
      XCOF(1)=F1*F1*G6*G6+G3*G3*F1*F6-G3*G6*F1*F3+G1*G6*F3*F3+G1*G1*F6*F
     $6-G1*G3*F3*F6-2.0*G1*G6*F1*F6
      XCOF(2)=2.0*G5*G6*F1*F1+2.0*G2*G3*F1*F6+G3*G3*F1*F5-G3*G6*F1*F2-G2
     $*G6*F1*F3-G3*G5*F1*F3+2.0*G1*G6*F2*F3+G1*G5*F3*F3+2.0*G1*G1*F5*F6-
     $G1*G3*F2*F6-G1*G2*F3*F6-G1*G3*F3*F5-2.0*G1*G5*F1*F6-2.0*G1*G6*F1*F
     $5
      XCOF(3)=2.0*G4*G6*F1*F1+G5*G5*F1*F1+G2*G2*F1*F6+2.0*G2*G3*F1*F5+G3
     $*G3*F1*F4-G2*G6*F1*F2-G3*G5*F1*F2-G2*G5*F1*F3-G3*G4*F1*F3+G1*G6*F2
     $*F2+2.0*G1*G5*F2*F3+G1*G4*F3*F3+2.0*G1*G1*F4*F6+G1*G1*F5*F5-G1*G2*
     $F2*F6-G1*G3*F2*F5-G1*G2*F3*F5-G1*G3*F3*F4-2.0*G1*G6*F1*F4-2.0*G1*G
     $5*F1*F5-2.0*G1*G4*F1*F6
      XCOF(4)=2.0*G4*G5*F1*F1+G2*G2*F1*F5+2.0*G2*G3*F1*F4-G2*G5*F1*F2-G3
     $*G4*F1*F2-G2*G4*F1*F3+G1*G5*F2*F2+2.0*G1*G4*F2*F3+2.0*G1*G1*F4*F5-
     $G1*G2*F2*F5-G1*G3*F2*F4-G1*G2*F3*F4-2.0*G1*G5*F1*F4-2.0*G1*G4*F1*F
     $5
      XCOF(5)=G4*G4*F1*F1+G2*G2*F1*F4-G2*G4*F1*F2+G1*G4*F2*F2+G1*G1*F4*F
     $4-G1*G2*F2*F4-2.0*G1*G4*F1*F4
      CALL POLRT(XCOF,COF,4,ROOTR,ROOTI,IER)
      WRITE (6,109)
  109 FORMAT (/,27X,9H L1 ROOTS,/)
      DO 110 I=1,4
  110 WRITE (6,112) RCCTR(I),ROOTI(I)
  112 FORMAT (17X,E13.5,10X,E13.5,/)
      WRITE (6,114) IER
  114 FORMAT (1X,I1)
      WRITE (6,106)
  106 FORMAT (1H1,//)
```

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000000000111111111122222222223333333333444444444455555555556666666666777777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
  125 DO 215 I=1,4
      OA=F1
      OB=F2*ROOTR(I)+F3
      OC=F4*ROOTR(I)*ROOTR(I)+F5*ROOTR(I)+F6
      OD=G1
      OE=G2*ROOTR(I)+G3
      OF=G4*ROOTR(I)*ROOTR(I)+G5*ROOTR(I)+G6
      IF (ABS(ROOTI(I)).LT.0.000001) ROOTI(I)=0.0
      IF (ROOTI(I)) 200,130,200
  200 WRITE (6,201)
  201 FORMAT (5X,18H ROOT IS IMAGINARY,/)
      GO TO 215
  130 SA=OB*OB-4.0*OA*OC
      IF (SA) 200,140,140
  140 ROOTA1=(-OB+SQRT(SA))/(2.0*OA)
      ROOTA2=(-OB-SQRT(SA))/(2.0*OA)
      SB=OE*OE-4.0*OD*OF
      IF (SB) 200,150,150
  150 ROOTB1=(-OE+SQRT(SB))/(2.0*OD)
      ROOTB2=(-OE-SQRT(SB))/(2.0*OD)
      WRITE (6,155) ROOTA1,ROOTA2,ROOTB1,ROOTB2
  155 FORMAT (/,13X,8H ROOTA1=,F18.6,5X,8H ROOTA2=,F18.6,/,13X,8H ROOTB1
     $=,F18.6,5X,8H ROOTB2=,F18.6,/)
      IF (ABS(ROOTA1-ROOTB1).LT.0.05) GO TO 160
      IF (ABS(ROOTA1-ROOTB2).LT.0.05) GO TO 160
      IF (ABS(ROOTA2-ROOTB1).LT.0.05) GO TO 170
      IF (ABS(ROOTA2-ROOTB2).LT.0.05) GO TO 170
      GO TO 215
  160 WRITE (6,180) ROOTA1
  180 FORMAT (2X,16H SECOND ROOT IS ,F20.5,/)
      ROOT=ROOTA1
      GO TO 210
  170 WRITE (6,180) ROOTA2
      ROOT=ROOTA2
  210 WRITE (6,212)
  212 FORMAT (14X,13H CENTER POINT,31X,13H CIRCLE POINT,/,8X,3H XA,19X,3
     $H YA,19X,3H XB,19X,3H YB,/)
      XA(I)=R(1)+P(1)*ROOTR(I)+Q(1)*ROOT
      YA(I)=R(2)+P(2)*ROOTR(I)+Q(2)*ROOT
      XB(I)=R(3)+P(3)*ROOTR(I)+Q(3)*ROOT
      YB(I)=R(4)+P(4)*ROOTR(I)+Q(4)*ROOT
      WRITE (6,230) XA(I),YA(I),XB(I),YB(I)
  230 FORMAT (4F20.8,/,33X,20H XXXXXXXXXXXXXXXXXX,//)
  215 CONTINUE
      STOP
      END
```

APPENDIX F

SYNTHESIS PROGRAM FOR TWO FINITE POSITIONS AND ONE THIRD INFINITESIMAL DISPLACEMENT

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     000000000111111111122222222223333333333444444444455555555556666666666777777777778
     12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
     C ***********************************************************************
     C *                                                                     *
     C *                 SYNTHESIS OF SIX-LINK STEPHENSON 2                  *
     C *                        FUNCTION GENERATOR                           *
     C *                                                                     *
     C *               TWO FINITE POSITIONS AND ONE JERK RATIO               *
     C *                                                                     *
     C *                                                                     *
     C *               INPUT FINITE ROTATION               TH1(I)            *
     C *                                                                     *
     C *               OUTPUT FINITE ROTATION              PH1(I)            *
     C *                                                                     *
     C *               ASSUMED ROTATION OF TERNARY         AL1(I)            *
     C *                                                                     *
     C *               THIRD INFINITESIMAL DISPLACEMENT    DDDPTH(J)         *
     C *                                                                     *
     C *               OUTPUT LINK LENGTH                  QC                *
     C *                                                                     *
     C *               OUTPUT LINK INITIAL ANGLE           THE               *
     C *                                                                     *
     C ***********************************************************************
     C
           DIMENSION XCOF(5),COF(5),ROOTR(4),ROOTI(4)
           DIMENSION XQ(5),YQ(5),XA(5),YA(5),XB(5),YB(5),XC(5),YC(5)
           DIMENSION TH1(5),PH1(5),AL1(5)
           DIMENSION H(16),H1(16),H2(16),R(4),P(4),Q(4)
           DIMENSION DPHDTH(5),DDPHTH(5),DDDPTH(5)
           DIMENSION DTHDAL(5),DDTHAL(5),DDDTAL(5)
           DIMENSION DXC(5),DYC(5),DDXC(5),DDYC(5),DDDXC(5),DDDYC(5)
           DIMENSION PXC(5),PYC(5),PPXC(5),PPYC(5),PPPXC(5),PPPYC(5)
           READ (5,5) J
         5 FORMAT (I1)
           WRITE (6,6) J
         6 FORMAT (//,15X,31H JERK RATIO SPECIFIED AT POINT ,I1,//)
           READ (5,10) TH1(2)
           READ (5,10) PH1(2)
           READ (5,10) AL1(2)
        10 FORMAT (F10.0)
           READ (5,20) QC,THE
           READ (5,20) DPHDTH(J),DTHDAL(J)
           READ (5,20) DDPHTH(J),DDTHAL(J)
           READ (5,20) DDDPTH(J),DDDTAL(J)
        20 FORMAT (2F10.0)
           WRITE (6,11) TH1(2)
        11 FORMAT (13X,24H INPUT ROTATION ANGLE IS,2X,F10.3,/)
           WRITE (6,12) PH1(2)
        12 FORMAT (13X,25H OUTPUT ROTATION ANGLE IS,1X,F10.3,/)
           WRITE (6,13) AL1(2)
        13 FORMAT (13X,22H TERNARY LINK ROTATION,4X,F10.3,//)
           WRITE (6,14) QC,THE
        14 FORMAT (4X,15H OUTPUT LINK IS,F10.3,24H LONG AND AT AN ANGLE OF,F1
          $0.3,//)
           WRITE (6,15) DPHDTH(J),DTHDAL(J)
        15 FORMAT (7X,11H DPHDTH(J)=,F10.3,10X,11H DTHDAL(J)=,F10.3,/)
```

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000000000111111111122222222223333333333444444444455555555556666666666777777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
      WRITE (6,16) DDPHTH(J),DDTHAL(J)
   16 FORMAT (7X,11H DDPHTH(J)=,F10.3,10X,11H DDTHAL(J)=,F10.3,/)
      WRITE (6,17) DDDPTH(J),DDDTAL(J)
   17 FORMAT (7X,11H DDDPTH(J)=,F10.3,10X,11H DDDTAL(J)=,F10.3,//)
      XQ(1)=1.0
      YQ(1)=0.0
      XM=0.0
      YM=0.0
      TH1(1)=0.0
      PH1(1)=0.0
      AL1(1)=0.0
      THE=(THE*3.14159265)/180.0
      I=2
      TH1(I)=(TH1(I)*3.14159265)/180.0
      PH1(I)=(PH1(I)*3.14159265)/180.0
      AL1(I)=(AL1(I)*3.14159265)/180.0
      XC(1)=1.0+QC*COS(THE)
      YC(1)=0.0+QC*SIN(THE)
      XQ(I)=1.0*COS(-TH1(I))
      YQ(I)=1.0*SIN(-TH1(I))
      XC(I)=XC(1)*COS(PH1(I)-TH1(I))-YC(1)*SIN(PH1(I)-TH1(I))+XQ(I)-XQ(1
     $)*COS(PH1(I)-TH1(I))+YQ(1)*SIN(PH1(I)-TH1(I))
      YC(I)=XC(1)*SIN(PH1(I)-TH1(I))+YC(1)*COS(PH1(I)-TH1(I))+YQ(I)-XQ(1
     $)*SIN(PH1(I)-TH1(I))-YQ(1)*COS(PH1(I)-TH1(I))
      DXC(J)=(XC(1)-XQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(J)-TH1(J))*DPHDTH
     $(J))+(YC(1)-YQ(1))*(COS(PH1(J)-TH1(J))-COS(PH1(J)-TH1(J))*DPHDTH(J
     $))+XQ(1)*SIN(-TH1(J))+YQ(1)*COS(-TH1(J))
      DYC(J)=(XC(1)-XQ(1))*(-COS(PH1(J)-TH1(J))+COS(PH1(J)-TH1(J))*DPHDT
     $H(J))+(YC(1)-YQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(J)-TH1(J))*DPHDTH(
     $J))-XQ(1)*COS(-TH1(J))+YQ(1)*SIN(-TH1(J))
      DDXC(J)=(XC(1)-XQ(1))*(-COS(PH1(J)-TH1(J))+2.0*COS(PH1(J)-TH1(J))*
     $DPHDTH(J)-COS(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-SIN(PH1(J)-TH1(J)
     $)*DDPHTH(J))+(YC(1)-YQ(1))*(SIN(PH1(J)-TH1(J))-2.0*SIN(PH1(J)-TH1(
     $J))*DPHDTH(J)+SIN(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-COS(PH1(J)-TH
     $1(J))*DDPHTH(J))-XQ(1)*COS(-TH1(J))+YQ(1)*SIN(-TH1(J))
      DDYC(J)=(XC(1)-XQ(1))*(-SIN(PH1(J)-TH1(J))+2.0*SIN(PH1(J)-TH1(J))*
     $DPHDTH(J)-SIN(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)+COS(PH1(J)-TH1(J)
     $)*DDPHTH(J))+(YC(1)-YQ(1))*(-COS(PH1(J)-TH1(J))+2.0*COS(PH1(J)-TH1
     $(J))*DPHDTH(J)-COS(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-SIN(PH1(J)-T
     $H1(J))*DDPHTH(J))-XQ(1)*SIN(-TH1(J))-YQ(1)*COS(-TH1(J))
      DDD1=-SIN(PH1(J)-TH1(J))+3.0*SIN(PH1(J)-TH1(J))*DPHDTH(J)-3.0*SIN(
     $PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)+3.0*COS(PH1(J)-TH1(J))*DDPHTH(J
     $)-3.0*COS(PH1(J)-TH1(J))*DPHDTH(J)*DDPHTH(J)+SIN(PH1(J)-TH1(J))*DP
     $HDTH(J)*DPHDTH(J)*DPHDTH(J)-SIN(PH1(J)-TH1(J))*DDDPTH(J)
      DDD2=-COS(PH1(J)-TH1(J))+3.0*COS(PH1(J)-TH1(J))*DPHDTH(J)-3.0*COS(
     $PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-3.0*SIN(PH1(J)-TH1(J))*DDPHTH(J
     $)+3.0*SIN(PH1(J)-TH1(J))*DPHDTH(J)*DDPHTH(J)+COS(PH1(J)-TH1(J))*DP
     $HDTH(J)*DPHDTH(J)*DPHDTH(J)-COS(PH1(J)-TH1(J))*DDDPTH(J)
      DDD3=COS(PH1(J)-TH1(J))-3.0*COS(PH1(J)-TH1(J))*DPHDTH(J)+3.0*COS(P
     $H1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)+3.0*SIN(PH1(J)-TH1(J))*DDPHTH(J)
     $-3.0*SIN(PH1(J)-TH1(J))*DPHDTH(J)*DDPHTH(J)-COS(PH1(J)-TH1(J))*DPH
     $DTH(J)*DPHDTH(J)*DPHDTH(J)+COS(PH1(J)-TH1(J))*DDDPTH(J)
      DDD4=-SIN(PH1(J)-TH1(J))+3.0*SIN(PH1(J)-TH1(J))*DPHDTH(J)-3.0*SIN(
     $PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)+3.0*COS(PH1(J)-TH1(J))*DDPHTH(J
```

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00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
     $)-3.0*COS(PH1(J)-TH1(J))*DPHDTH(J)*DDPHTH(J)+SIN(PH1(J)-TH1(J))*DP
     $HDTH(J)*DPHDTH(J)*DPHDTH(J)-SIN(PH1(J)-TH1(J))*DDDPTH(J)
      DDDXC(J)=(XC(1)-XQ(1))*DDD1+(YC(1)-YQ(1))*DDD2-XQ(1)*SIN(-TH1(J))-
     $YQ(1)*COS(-TH1(J))
      DDDYC(J)=(XC(1)-XQ(1))*DDD3+(YC(1)-YQ(1))*DDD4+XQ(1)*COS(-TH1(J))-
     $YQ(1)*SIN(-TH1(J))
      WRITE (6,21) DXC(J),DYC(J)
   21 FORMAT (7X,8H DXC(J)=,3X,F10.3,10X,8H DYC(J)=,3X,F10.3,/)
      WRITE (6,22) DDXC(J),DDYC(J)
   22 FORMAT (7X,9H DDXC(J)=,2X,F10.3,10X,9H DDYC(J)=,2X,F10.3,/)
      WRITE (6,23) DDDXC(J),DDDYC(J)
   23 FORMAT (7X,10H DDDXC(J)=,1X,F10.3,10X,10H DDDYC(J)=,1X,F10.3,//)
      PXC(J)=DTHDAL(J)*DXC(J)
      PYC(J)=DTHDAL(J)*DYC(J)
      PPXC(J)=DDTHAL(J)*DXC(J)+DTHDAL(J)*DTHDAL(J)*DDXC(J)
      PPYC(J)=DDTHAL(J)*DYC(J)+DDTHAL(J)*DDTHAL(J)*DDYC(J)
      PPPXC(J)=DDDTAL(J)*DXC(J)+3.0*DTHDAL(J)*DDTHAL(J)*DDXC(J)+DTHDAL(J
     $)*DTHDAL(J)*DTHDAL(J)*DDDXC(J)
      PPPYC(J)=DDDTAL(J)*DYC(J)+3.0*DTHDAL(J)*DDTHAL(J)*DDYC(J)+DTHDAL(J
     $)*DTHDAL(J)*DTHDAL(J)*DDDYC(J)
      WRITE (6,31)
   31 FORMAT (11X,3H XQ,18X,3H YQ,18X,3H XC,18X,3H YC,/)
      DO 33 I=1,2
   33 WRITE (6,32) XQ(I),YQ(I),XC(I),YC(I)
   32 FORMAT (6X,F12.8,9X,F12.8,9X,F12.8,9X,F12.8,/)
      I=1
      H(I)=XC(I+1)-XC(1)*COS(AL1(I+1))+YC(1)*SIN(AL1(I+1))
      H(2)=PXC(J)+XC(1)*SIN(AL1(J))+YC(1)*COS(AL1(J))
      H(3)=PPXC(J)+XC(1)*COS(AL1(J))-YC(1)*SIN(AL1(J))
      H(4)=PPPXC(J)-XC(1)*SIN(AL1(J))-YC(1)*COS(AL1(J))
      H(I+4)=YC(I+1)-YC(1)*COS(AL1(I+1))-XC(1)*SIN(AL1(I+1))
      H(6)=PYC(J)-XC(1)*COS(AL1(J))+YC(1)*SIN(AL1(J))
      H(7)=PPYC(J)+XC(1)*SIN(AL1(J))+YC(1)*COS(AL1(J))
      H(8)=PPPYC(J)+XC(1)*COS(AL1(J))-YC(1)*SIN(AL1(J))
      H(I+8)=-H(I)*COS(AL1(I+1))-H(I+4)*SIN(AL1(I+1))
      H(10)=-PXC(J)*COS(AL1(J))-PYC(J)*SIN(AL1(J))+XC(J)*SIN(AL1(J))-YC(
     $J)*COS(AL1(J))
      H(11)=-PPXC(J)*COS(AL1(J))-PPYC(J)*SIN(AL1(J))+2.0*PXC(J)*SIN(AL1(
     $J))-2.0*PYC(J)*COS(AL1(J))+XC(J)*COS(AL1(J))+YC(J)*SIN(AL1(J))
      H(12)=-PPPXC(J)*COS(AL1(J))-PPPYC(J)*SIN(AL1(J))+3.0*PPXC(J)*SIN(A
     $L1(J))-3.0*PPYC(J)*COS(AL1(J))+3.0*PXC(J)*COS(AL1(J))+3.0*PYC(J)*S
     $IN(AL1(J))-XC(J)*SIN(AL1(J))+YC(J)*COS(AL1(J))
      H(I+12)=-H(I+4)*COS(AL1(I+1))+H(I)*SIN(AL1(I+1))
      H(14)=PXC(J)*SIN(AL1(J))-PYC(J)*COS(AL1(J))+XC(J)*COS(AL1(J))+YC(J
     $)*SIN(AL1(J))
      H(15)=PPXC(J)*SIN(AL1(J))-PPYC(J)*COS(AL1(J))+2.0*PXC(J)*COS(AL1(J
     $))+2.0*PYC(J)*SIN(AL1(J))-XC(J)*SIN(AL1(J))+YC(J)*COS(AL1(J))
      H(16)=PPPXC(J)*SIN(AL1(J))-PPPYC(J)*COS(AL1(J))+3.0*PPXC(J)*COS(AL
     $1(J))+3.0*PPYC(J)*SIN(AL1(J))-3.0*PXC(J)*SIN(AL1(J))+3.0*PYC(J)*CO
     $S(AL1(J))-XC(J)*COS(AL1(J))-YC(J)*SIN(AL1(J))
      DO 72 I=1,16
   72 H1(I)=H(I)
      DO 74 I=1,16
   74 H2(I)=H(I)
```

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00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
      I=1
      R(I)=0.5*(H(I)*H(I)+H(I+4)*H(I+4))
      R(2)=(YC(1)*XC(J)-XC(1)*YC(J)-YC(1)*PYC(J)-XC(1)*PXC(J))*COS(AL1(J
     $))+(XC(1)*XC(J)+YC(1)*YC(J)-XC(1)*PYC(J)+YC(1)*PXC(J))*SIN(AL1(J))
     $+XC(J)*PXC(J)+YC(J)*PYC(J)
      DD1=2.0*YC(1)*PXC(J)-2.0*XC(1)*PYC(J)-YC(1)*PPYC(J)-XC(1)*PPXC(J)+
     $XC(1)*XC(J)+YC(1)*YC(J)
      DD2=2.0*XC(1)*PXC(J)+2.0*YC(1)*PYC(J)-XC(1)*PPYC(J)+YC(1)*PPXC(J)-
     $YC(1)*XC(J)+XC(1)*YC(J)
      R(3)=COS(AL1(J))*DD1+SIN(AL1(J))*DD2+XC(J)*PPXC(J)+YC(J)*PPYC(J)+P
     $XC(J)*PXC(J)+PYC(J)*PYC(J)
      DDD5=-YC(1)*PPPYC(J)-XC(1)*PPPXC(J)+3.0*YC(1)*PPXC(J)-3.0*XC(1)*PP
     $YC(J)+3.0*XC(1)*PXC(J)+3.0*YC(1)*PYC(J)-YC(1)*XC(J)+XC(1)*YC(J)
      DDD6=-XC(1)*PPPYC(J)+YC(1)*PPPXC(J)+3.0*XC(1)*PPXC(J)+3.0*YC(1)*PP
     $YC(J)-3.0*YC(1)*PXC(J)+3.0*XC(1)*PYC(J)-XC(1)*XC(J)-YC(1)*YC(J)
      R(4)=COS(AL1(J))*DDD5+SIN(AL1(J))*DDD6+XC(J)*PPPXC(J)+YC(J)*PPPYC(
     $J)+3.0*PXC(J)*PPXC(J)+3.0*PYC(J)*PPYC(J)
      P(I)=1.0-COS(AL1(I+1))
      P(2)=SIN(AL1(J))
      P(3)=COS(AL1(J))
      P(4)=-SIN(AL1(J))
      Q(I)=SIN(AL1(I+1))
      Q(2)=COS(AL1(J))
      Q(3)=-SIN(AL1(J))
      Q(4)=-COS(AL1(J))
      CALL SIMQ(H,R,4,KS)
      CALL SIMQ(H1,P,4,KS1)
      CALL SIMQ(H2,Q,4,KS2)
      WRITE (6,99) KS
      WRITE (6,99) KS1
      WRITE (6,99) KS2
   99 FORMAT (1X,I1)
      F1=Q(1)*Q(3)+Q(2)*Q(4)
      F2=P(1)*Q(3)+Q(1)*P(3)+P(2)*Q(4)+Q(2)*P(4)
      F3=R(1)*Q(3)+Q(1)*R(3)+Q(2)*R(4)+Q(4)*R(2)
      F4=P(1)*P(3)+P(2)*P(4)
      F5=P(1)*R(3)+P(3)*R(1)+P(2)*R(4)+P(4)*R(2)-1.0
      F6=R(1)*R(3)+R(2)*R(4)
      G1=Q(1)*Q(4)-Q(2)*Q(3)
      G2=P(1)*Q(4)+Q(1)*P(4)-P(2)*Q(3)-Q(2)*P(3)
      G3=R(1)*Q(4)+Q(1)*R(4)-R(2)*Q(3)-Q(2)*R(3)-1.0
      G4=P(1)*P(4)-P(2)*P(3)
      G5=P(1)*R(4)+R(1)*P(4)-P(2)*R(3)-P(3)*R(2)
      G6=R(1)*R(4)-R(2)*R(3)
      XCOF(1)=F1*F1*G6*G6+G3*G3*F1*F6-G3*G6*F1*F3+G1*G6*F3*F3+G1*G1*F6*F
     $6-G1*G3*F3*F6-2.0*G1*G6*F1*F6
      XCOF(2)=2.0*G5*G6*F1*F1+2.0*G2*G3*F1*F6+G3*G3*F1*F5-G3*G6*F1*F2-G2
     $*G6*F1*F3-G3*G5*F1*F3+2.0*G1*G6*F2*F3+G1*G5*F3*F3+2.0*G1*G1*F5*F6-
     $G1*G3*F2*F6-G1*G2*F3*F6-G1*G3*F3*F5-2.0*G1*G5*F1*F6-2.0*G1*G6*F1*F
     $5
      XCOF(3)=2.0*G4*G6*F1*F1+G5*G5*F1*F1+G2*G2*F1*F6+2.0*G2*G3*F1*F5+G3
     $*G3*F1*F4-G2*G6*F1*F2-G3*G5*F1*F2-G2*G5*F1*F3-G3*G4*F1*F3+G1*G6*F2
     $*F2+2.0*G1*G5*F2*F3+G1*G4*F3*F3+2.0*G1*G1*F4*F6+G1*G1*F5*F5-G1*G2*
     $F2*F6-G1*G3*F2*F5-G1*G2*F3*F5-G1*G3*F3*F4-2.0*G1*G6*F1*F4-2.0*G1*G
```

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000000000111111111122222222223333333333444444444455555555556666666666777777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
     $5*F1*F5-2.0*G1*G4*F1*F6
      XCOF(4)=2.0*G4*G5*F1*F1+G2*G2*F1*F5+2.0*G2*G3*F1*F4-G2*G5*F1*F2-G3
     $*G4*F1*F2-G2*G4*F1*F3+G1*G5*F2*F2+2.0*G1*G4*F2*F3+2.0*G1*G1*F4*F5-
     $G1*G2*F2*F5-G1*G3*F2*F4-G1*G2*F3*F4-2.0*G1*G5*F1*F4-2.0*G1*G4*F1*F
     $5
      XCOF(5)=G4*G4*F1*F1+G2*G2*F1*F4-G2*G4*F1*F2+G1*G4*F2*F2+G1*G1*F4*F
     $4-G1*G2*F2*F4-2.0*G1*G4*F1*F4
      CALL POLRT(XCCF,COF,4,ROOTR,ROOTI,IER)
      WRITE (6,109)
  109 FORMAT (/,27X,9H L1 ROOTS,/)
      DO 110 I=1,4
  110 WRITE (6,112) ROOTR(I),ROOTI(I)
  112 FORMAT (17X,E13.5,10X,E13.5,/)
      WRITE (6,114) IER
  114 FORMAT (1X,I1)
      WRITE (6,106)
  1C6 FORMAT (1H1,//)
  125 DO 215 I=1,4
      OA=F1
      OB=F2*ROOTR(I)+F3
      OC=F4*ROOTR(I)*ROOTR(I)+F5*ROOTR(I)+F6
      OD=G1
      OE=G2*ROOTR(I)+G3
      OF=G4*ROOTR(I)*ROOTR(I)+G5*ROOTR(I)+G6
      IF (ABS(ROOTI(I)).LT.0.000001) ROOTI(I)=0.0
      IF (ROOTI(I)) 2CC,130,200
  200 WRITE (6,201)
  201 FORMAT (5X,18H ROOT IS IMAGINARY,/)
      GO TO 215
  130 SA=OB*OB-4.0*OA*CC
      IF (SA) 200,140,140
  140 ROOTA1=(-OB+SQRT(SA))/(2.0*OA)
      ROOTA2=(-OB-SQRT(SA))/(2.0*OA)
      SB=OE*OE-4.0*OD*OF
      IF (SB) 200,150,150
  150 ROOTB1=(-OE+SQRT(SB))/(2.0*OD)
      ROOTB2=(-OE-SQRT(SB))/(2.0*OD)
      WRITE (6,155) RCCTA1,ROOTA2,ROOTB1,ROOTB2
  155 FORMAT (/,13X,8H ROOTA1=,F18.6,5X,8H ROOTA2=,F18.6,/,13X,8H ROOTB1
     $=,F18.6,5X,8H ROOTB2=,F18.6,/)
      IF (ABS(ROOTA1-ROCTB1).LT.0.05) GO TO 160
      IF (ABS(ROOTA1-ROOTB2).LT.0.05) GO TO 160
      IF (ABS(ROOTA2-ROOTB1).LT.0.05) GO TO 170
      IF (ABS(ROOTA2-ROOTB2).LT.0.05) GO TO 170
      GO TO 215
  160 WRITE (6,180) ROOTA1
  180 FORMAT (2X,16H SECOND ROOT IS ,F20.5,/)
      ROOT=ROOTA1
      GO TO 210
  170 WRITE (6,180) RCCTA2
      ROOT=ROOTA2
  210 WRITE (6,212)
  212 FORMAT (14X,13H CENTER POINT,31X,13H CIRCLE POINT,/,8X,3H XA,19X,3
     $H YA,19X,3H XB,19X,3H YB,/)
```

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000000000111111111122222222223333333333444444444455555555556666666666777777777778
123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890
        XA(I)=R(1)+P(1)*ROOTR(I)+Q(1)*ROOT
        YA(I)=R(2)+P(2)*ROOTR(I)+Q(2)*ROOT
        XB(I)=R(3)+P(3)*ROOTR(I)+Q(3)*ROOT
        YB(I)=R(4)+P(4)*ROOTR(I)+Q(4)*ROOT
        WRITE (6,230) XA(I),YA(I),XB(I),YB(I)
    230 FORMAT (4F20.8,/,33X,20H XXXXXXXXXXXXXXXXXXX,//)
    215 CONTINUE
        STOP
        END
```

APPENDIX G

SYNTHESIS PROGRAM FOR ONE FINITE POSITION AND ONE FOURTH INFINITESIMAL DISPLACEMENT

```
000000000111111111122222222223333333333444444444455555555556666666666777777777778
123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
C ***************************************************************************
C *                                                                         *
C *                 SYNTHESIS OF SIX-LINK STEPHENSON 2                      *
C *                        FUNCTION GENERATOR                               *
C *                                                                         *
C *                ONE FINITE POSITION AND ONE KERK RATIO                   *
C *                                                                         *
C *                                                                         *
C *             FOURTH INFINITESIMAL DISPLACEMENT DDDDPT(J)                 *
C *                                                                         *
C *             OUTPUT LINK LENGTH                  QC                      *
C *                                                                         *
C *             OUTPUT LINK INITIAL ANGLE           THE                     *
C *                                                                         *
C ***************************************************************************
C
      DIMENSION XCOF(5),COF(5),ROOTR(4),ROOTI(4)
      DIMENSION XQ(5),YQ(5),XA(5),YA(5),XB(5),YB(5),XC(5),YC(5)
      DIMENSION TH1(5),PH1(5),AL1(5)
      DIMENSION H(16),H1(16),H2(16),R(4),P(4),Q(4)
      DIMENSICN DPHDTH(5),DDPHTH(5),DDDPTH(5),DDDDPT(5)
      DIMENSION DTHDAL(5),DDTHAL(5),DDDTAL(5),DDDDTA(5)
      DIMENSION DXC(5),DYC(5),DDXC(5),DDYC(5)
      DIMENSION DDDXC(5),DDDYC(5),DDDDXC(5),DDDDYC(5)
      DIMENSION PXC(5),PYC(5),PPXC(5),PPYC(5)
      DIMENSION PPPXC(5),PPPYC(5),PPPPXC(5),PPPPYC(5)
      J=1
      WRITE (6,6)
    6 FORMAT (//,12X,32H KERK RATIO SPECIFIED AT POINT 1,//)
      READ (5,20) QC,THE
      READ (5,20) DPHDTH(J),DTHDAL(J)
      READ (5,20) DDPHTH(J),DDTHAL(J)
      READ (5,20) DDDPTH(J),DDDTAL(J)
      READ (5,20) DDDDPT(J),DDDDTA(J)
   20 FORMAT (2F10.0)
      WRITE (6,14) QC,THE
   14 FORMAT (4X,15H OUTPUT LINK IS,F10.3,24H LONG AND AT AN ANGLE OF,F1
     $0.3,//)
   15 FORMAT (7X,11H DPHDTH(J)=,F10.3,10X,11H DTHDAL(J)=,F10.3,/)
      WRITE (6,15) DPHDTH(J),DTHDAL(J)
      WRITE (6,16) DDPHTH(J),DDTHAL(J)
   16 FORMAT (7X,11H DDPHTH(J)=,F10.3,10X,11H DDTHAL(J)=,F10.3,/)
      WRITE (6,17) DDDPTH(J),DDDTAL(J)
   17 FORMAT (7X,11H DDDPTH(J)=,F10.3,10X,11H DDDTAL(J)=,F10.3,/)
      WRITE (6,18) DDDDPT(J),DDDDTA(J)
   18 FORMAT (7X,11H DDDDPT(J)=,F10.3,10X,11H DDDDTA(J)=,F10.3,//)
      XQ(1)=1.0
      YQ(1)=0.0
      XM=0.0
      YM=0.0
      TH1(1)=0.0
      PH1(1)=0.0
      AL1(1)=0.0
      THE=(THE*3.14159265)/180.0
```

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00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
      XC(1)=1.0+QC*COS(THE)
      YC(1)=0.0+QC*SIN(THE)
      DXC(J)=(XC(1)-XQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(J)-TH1(J))*DPHDTH
     $(J))+(YC(1)-YQ(1))*(COS(PH1(J)-TH1(J))-COS(PH1(J)-TH1(J))*DPHDTH(J
     $))+XQ(1)*SIN(-TH1(J))+YQ(1)*COS(-TH1(J))
      DYC(J)=(XC(1)-XC(1))*(-COS(PH1(J)-TH1(J))+COS(PH1(J)-TH1(J))*DPHDT
     $H(J))+(YC(1)-YQ(1))*(SIN(PH1(J)-TH1(J))-SIN(PH1(J)-TH1(J))*DPHDTH(
     $J))-XQ(1)*COS(-TH1(J))+YQ(1)*SIN(-TH1(J))
      DDXC(J)=(XC(1)-XC(1))*(-COS(PH1(J)-TH1(J))+2.0*COS(PH1(J)-TH1(J))*
     $DPHDTH(J)-COS(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-SIN(PH1(J)-TH1(J)
     $)*DDPHTH(J))+(YC(1)-YQ(1))*(SIN(PH1(J)-TH1(J))-2.0*SIN(PH1(J)-TH1(
     $J))*DPHDTH(J)+SIN(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-COS(PH1(J)-TH
     $1(J))*DDPHTH(J))-XQ(1)*COS(-TH1(J))+YQ(1)*SIN(-TH1(J))
      DDYC(J)=(XC(1)-XC(1))*(-SIN(PH1(J)-TH1(J))+2.0*SIN(PH1(J)-TH1(J))*
     $DPHDTH(J)-SIN(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)+COS(PH1(J)-TH1(J)
     $)*DDPHTH(J))+(YC(1)-YQ(1))*(-COS(PH1(J)-TH1(J))+2.0*COS(PH1(J)-TH1
     $(J))*DPHDTH(J)-COS(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-SIN(PH1(J)-T
     $H1(J))*DDPHTH(J))-XQ(1)*SIN(-TH1(J))-YQ(1)*COS(-TH1(J))
      DDD1=-SIN(PH1(J)-TH1(J))+3.0*SIN(PH1(J)-TH1(J))*DPHDTH(J)-3.0*SIN(
     $PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)+3.0*COS(PH1(J)-TH1(J))*DDPHTH(J
     $)-3.0*COS(PH1(J)-TH1(J))*DPHDTH(J)*DDPHTH(J)+SIN(PH1(J)-TH1(J))*DP
     $HDTH(J)*DPHDTH(J)*DPHDTH(J)-SIN(PH1(J)-TH1(J))*DDDPTH(J)
      DDD2=-COS(PH1(J)-TH1(J))+3.0*COS(PH1(J)-TH1(J))*DPHDTH(J)-3.0*COS(
     $PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-3.0*SIN(PH1(J)-TH1(J))*DDPHTH(J
     $)+3.0*SIN(PH1(J)-TH1(J))*DPHDTH(J)*DDPHTH(J)+COS(PH1(J)-TH1(J))*DP
     $HDTH(J)*DPHDTH(J)*DPHDTH(J)-COS(PH1(J)-TH1(J))*DDDPTH(J)
      DDD3=COS(PH1(J)-TH1(J))-3.0*COS(PH1(J)-TH1(J))*DPHDTH(J)+3.0*COS(P
     $H1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)+3.0*SIN(PH1(J)-TH1(J))*DDPHTH(J)
     $-3,0*SIN(PH1(J)-TH1(J))*DPHDTH(J)*DDPHTH(J)-COS(PH1(J)-TH1(J))*DPH
     $DTH(J)*DPHDTH(J)*DPHDTH(J)+COS(PH1(J)-TH1(J))*DDDPTH(J)
      DDD4=-SIN(PH1(J)-TH1(J))+3.0*SIN(PH1(J)-TH1(J))*DPHDTH(J)-3.0*SIN(
     $PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)+3.0*COS(PH1(J)-TH1(J))*DDPHTH(J
     $)-3.0*COS(PH1(J)-TH1(J))*DPHDTH(J)*DDPHTH(J)+SIN(PH1(J)-TH1(J))*DP
     $HDTH(J)*DPHDTH(J)*DPHDTH(J)-SIN(PH1(J)-TH1(J))*DDDPTH(J)
      DDDXC(J)=(XC(1)-XQ(1))*DDD1+(YC(1)-YQ(1))*DDD2-XQ(1)*SIN(-TH1(J))-
     $YQ(1)*COS(-TH1(J))
      DDDYC(J)=(XC(1)-XQ(1))*DDD3+(YC(1)-YQ(1))*DDD4+XQ(1)*COS(-TH1(J))-
     $YQ(1)*SIN(-TH1(J))
      DDDD1=COS(PH1(J)-TH1(J))-4.0*COS(PH1(J)-TH1(J))*DPHDTH(J)+6.0*COS(
     $PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)+6.0*SIN(PH1(J)-TH1(J))*DDPHTH(J
     $)-12.0*SIN(PH1(J)-TH1(J))*DPHDTH(J)*DDPHTH(J)+6.0*SIN(PH1(J)-TH1(J
     $))*DPHDTH(J)*DPHDTH(J)*DDPHTH(J)-4.0*COS(PH1(J)-TH1(J))*DPHDTH(J)*
     $DPHDTH(J)*DPHDTH(J)+4.0*COS(PH1(J)-TH1(J))*DDDPTH(J)-4.0*COS(PH1(J
     $)-TH1(J))*DPHDTH(J)*DDDPTH(J)-3.0*COS(PH1(J)-TH1(J))*DDPHTH(J)*DDP
     $HTH(J)+COS(PH1(J)-TH1(J))*(DPHDTH(J)**4.0)-SIN(PH1(J)-TH1(J))*DDDD
     $PT(J)
      DDDD2=-SIN(PH1(J)-TH1(J))+4.0*SIN(PH1(J)-TH1(J))*DPHDTH(J)-6.0*SIN
     $(PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)+6.0*COS(PH1(J)-TH1(J))*DDPHTH(
     $J)-12.0*COS(PH1(J)-TH1(J))*DPHDTH(J)*DDPHTH(J)+6.0*COS(PH1(J)-TH1(
     $J))*DPHDTH(J)*DPHDTH(J)*DDPHTH(J)+4.0*SIN(PH1(J)-TH1(J))*DPHDTH(J)
     $*DPHDTH(J)*DPHDTH(J)-4.0*SIN(PH1(J)-TH1(J))*DDDPTH(J)+4.0*SIN(PH1(
     $J)-TH1(J))*DPHDTH(J)*DDDPTH(J)+3.0*SIN(PH1(J)-TH1(J))*DDPHTH(J)*DD
     $PHTH(J)-SIN(PH1(J)-TH1(J))*(DPHDTH(J)**4.0)-COS(PH1(J)-TH1(J))*DDD
     $DPT(J)
```

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000000000111111111122222222223333333333444444444455555555556666666666777777777778
123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890
      DDDD3=SIN(PH1(J)-TH1(J))-4.0*SIN(PH1(J)-TH1(J))*DPHDTH(J)+6.0*SIN(
     $PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)-6.0*COS(PH1(J)-TH1(J))*DDPHTH(J
     $)+12.0*COS(PH1(J)-TH1(J))*DPHDTH(J)*DDPHTH(J)-6.0*COS(PH1(J)-TH1(J
     $))*DPHDTH(J)*DPHDTH(J)*DDPHTH(J)-4.0*SIN(PH1(J)-TH1(J))*DPHDTH(J)*
     $DPHDTH(J)*DPHDTH(J)+4.0*SIN(PH1(J)-TH1(J))*DDDPTH(J)-4.0*SIN(PH1(J
     $)-TH1(J))*DPHDTH(J)*DDDPTH(J)-3.0*SIN(PH1(J)-TH1(J))*DDPHTH(J)*DDP
     $HTH(J)+SIN(PH1(J)-TH1(J))*(DPHDTH(J)**4.0)+COS(PH1(J)-TH1(J))*DDDD
     $PT(J)
      DDDD4=COS(PH1(J)-TH1(J))-4.0*COS(PH1(J)-TH1(J))*DPHDTH(J)+6.0*COS(
     $PH1(J)-TH1(J))*DPHDTH(J)*DPHDTH(J)+6.0*SIN(PH1(J)-TH1(J))*DDPHTH(J
     $)-12.0*SIN(PH1(J)-TH1(J))*DPHDTH(J)*DDPHTH(J)+6.0*SIN(PH1(J)-TH1(J
     $))*DPHDTH(J)*DPHDTH(J)*DDPHTH(J)-4.0*COS(PH1(J)-TH1(J))*DPHDTH(J)*
     $DPHDTH(J)*DPHDTH(J)+4.0*COS(PH1(J)-TH1(J))*DDDPTH(J)-4.0*COS(PH1(J
     $)-TH1(J))*DPHDTH(J)*DDDPTH(J)-3.0*COS(PH1(J)-TH1(J))*DDPHTH(J)*DDP
     $HTH(J)+COS(PH1(J)-TH1(J))*(DPHDTH(J)**4.0)-SIN(PH1(J)-TH1(J))*DDDD
     $PT(J)
      DDDDXC(J)=(XC(1)-XQ(1))*DDDD1+(YC(1)-YQ(1))*DDDD2+XQ(1)*COS(-TH1(J
     $))-YQ(1)*SIN(-TH1(J))
      DDDDYC(J)=(XC(1)-XQ(1))*DDDD3+(YC(1)-YQ(1))*DDDD4+XQ(1)*SIN(-TH1(J
     $))+YQ(1)*COS(-TH1(J))
      PXC(J)=DTHDAL(J)*DXC(J)
      PYC(J)=DTHDAL(J)*DYC(J)
      PPXC(J)=DDTHAL(J)*DXC(J)+DTHDAL(J)*DTHDAL(J)*DDXC(J)
      PPYC(J)=DDTHAL(J)*DYC(J)+DDTHAL(J)*DDTHAL(J)*DDYC(J)
      PPPXC(J)=DDDTAL(J)*DXC(J)+3.0*DTHDAL(J)*DDTHAL(J)*DDXC(J)+DTHDAL(J
     $)*DTHDAL(J)*DTHDAL(J)*DDDXC(J)
      PPPYC(J)=DDDTAL(J)*DYC(J)+3.0*DTHDAL(J)*DDTHAL(J)*DDYC(J)+DTHDAL(J
     $)*DTHDAL(J)*DTHDAL(J)*DDDYC(J)
      PPPPXC(J)=DDDDTA(J)*DXC(J)+3.0*DDTHAL(J)*DDTHAL(J)*DDXC(J)+4.0*DTH
     $DAL(J)*DDDTAL(J)*DDXC(J)+6.0*DTHDAL(J)*DTHDAL(J)*DDTHAL(J)*DDDXC(J
     $)+DTHDAL(J)*DTHDAL(J)*DTHDAL(J)*DTHDAL(J)*DDDDXC(J)
      PPPPYC(J)=DDDDTA(J)*DYC(J)+3.0*DDTHAL(J)*DDTHAL(J)*DDYC(J)+4.0*DTH
     $DAL(J)*DDDTAL(J)*DDYC(J)+6.0*DTHDAL(J)*DTHDAL(J)*DDTHAL(J)*DDDYC(J
     $)+DTHDAL(J)*DTHDAL(J)*DTHDAL(J)*DTHDAL(J)*DDDDYC(J)
      WRITE (6,21) DXC(J),DYC(J)
   21 FORMAT (7X,8H DXC(J)=,3X,F10.3,10X,8H DYC(J)=,3X,F10.3,/)
      WRITE (6,22) DDXC(J),DDYC(J)
   22 FORMAT (7X,9H DDXC(J)=,2X,F10.3,10X,9H DDYC(J)=,2X,F10.3,/)
      WRITE (6,23) DDDXC(J),DDDYC(J)
   23 FORMAT (7X,10H DDDXC(J)=,1X,F10.3,10X,10H DDDYC(J)=,1X,F10.3,/)
      WRITE (6,24) DDDDXC(J),DDDDYC(J)
   24 FORMAT (7X,11H DDDDXC(J)=,F10.3,10X,11H DDDDYC(J)=,F10.3,//)
      WRITE (6,31)
   31 FORMAT (11X,3H XQ,18X,3H YQ,18X,3H XC,18X,3H YC,/)
      I=1
      WRITE (6,32) XQ(I),YQ(I),XC(I),YC(I)
   32 FORMAT (6X,F12.8,9X,F12.8,9X,F12.8,9X,F12.8,/)
      H(1)=PXC(J)+XC(1)*SIN(AL1(J))+YC(1)*COS(AL1(J))
      H(2)=PPXC(J)+XC(1)*COS(AL1(J))-YC(1)*SIN(AL1(J))
      H(3)=PPPXC(J)-XC(1)*SIN(AL1(J))-YC(1)*COS(AL1(J))
      H(4)=PPPPXC(J)-XC(1)*COS(AL1(J))+YC(1)*SIN(AL1(J))
      H(5)=PYC(J)-XC(1)*COS(AL1(J))+YC(1)*SIN(AL1(J))
      H(6)=PPYC(J)+XC(1)*SIN(AL1(J))+YC(1)*COS(AL1(J))
      H(7)=PPPYC(J)+XC(1)*COS(AL1(J))-YC(1)*SIN(AL1(J))
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      H(8)=PPPPYC(J)-XC(1)*SIN(AL1(J))-YC(1)*CCS(AL1(J))
      H(9)=-PXC(J)*COS(AL1(J))-PYC(J)*SIN(AL1(J))+XC(J)*SIN(AL1(J))-YC(J
     $)*COS(AL1(J))
      H(10)=-PPXC(J)*COS(AL1(J))-PPYC(J)*SIN(AL1(J))+2.0*PXC(J)*SIN(AL1(
     $J))-2.0*PYC(J)*COS(AL1(J))+XC(J)*COS(AL1(J))+YC(J)*SIN(AL1(J))
      H(11)=-PPPXC(J)*COS(AL1(J))-PPPYC(J)*SIN(AL1(J))+3.0*PPXC(J)*SIN(A
     $L1(J))-3.0*PPYC(J)*COS(AL1(J))+3.0*PXC(J)*COS(AL1(J))+3.0*PYC(J)*S
     $IN(AL1(J))-XC(J)*SIN(AL1(J))+YC(J)*COS(AL1(J))
      H(12)=-PPPPXC(J)*COS(AL1(J))-PPPPYC(J)*SIN(AL1(J))+4.0*PPPXC(J)*SI
     $N(AL1(J))-4.0*PPPYC(J)*COS(AL1(J))+6.0*PPXC(J)*CCS(AL1(J))+6.0*PPY
     $C(J)*SIN(AL1(J))-4.0*PXC(J)*SIN(AL1(J))+4.0*PYC(J)*COS(AL1(J))-XC(
     $J)*COS(AL1(J))-YC(J)*SIN(AL1(J))
      H(13)=PXC(J)*SIN(AL1(J))-PYC(J)*COS(AL1(J))+XC(J)*COS(AL1(J))+YC(J
     $)*SIN(AL1(J))
      H(14)=PPXC(J)*SIN(AL1(J))-PPYC(J)*COS(AL1(J))+2.0*PXC(J)*COS(AL1(J
     $))+2.0*PYC(J)*SIN(AL1(J))-XC(J)*SIN(AL1(J))+YC(J)*COS(AL1(J))
      H(15)=PPPXC(J)*SIN(AL1(J))-PPPYC(J)*CCS(AL1(J))+3.0*PPXC(J)*COS(AL
     $1(J))+3.0*PPYC(J)*SIN(AL1(J))-3.0*PXC(J)*SIN(AL1(J))+3.0*PYC(J)*CO
     $S(AL1(J))-XC(J)*CCS(AL1(J))-YC(J)*SIN(AL1(J))
      H(16)=PPPPXC(J)*SIN(AL1(J))-PPPPYC(J)*CCS(AL1(J))+4.0*PPPXC(J)*COS
     $(AL1(J))+4.0*PPPYC(J)*SIN(AL1(J))-6.0*PPXC(J)*SIN(AL1(J))+6.0*PPYC
     $(J)*COS(AL1(J))-4.0*PXC(J)*CCS(AL1(J))-4.0*PYC(J)*SIN(AL1(J))+XC(J
     $)*SIN(AL1(J))-YC(J)*COS(AL1(J))
      DO 72 I=1,16
   72 H1(I)=H(I)
      DO 74 I=1,16
   74 H2(I)=H(I)
      R(1)=(YC(1)*XC(J)-XC(1)*YC(J)-YC(1)*PYC(J)-XC(1)*PXC(J))*COS(AL1(J
     $))+(XC(1)*XC(J)+YC(1)*YC(J)-XC(1)*PYC(J)+YC(1)*PXC(J))*SIN(AL1(J))
     $+XC(J)*PXC(J)+YC(J)*PYC(J)
      DD1=2.0*YC(1)*PXC(J)-2.0*XC(1)*PYC(J)-YC(1)*PPYC(J)-XC(1)*PPXC(J)+
     $XC(1)*XC(J)+YC(1)*YC(J)
      DD2=2.0*XC(1)*PXC(J)+2.0*YC(1)*PYC(J)-XC(1)*PPYC(J)+YC(1)*PPXC(J)-
     $YC(1)*XC(J)+XC(1)*YC(J)
      R(2)=COS(AL1(J))*DD1+SIN(AL1(J))*DD2+XC(J)*PPXC(J)+YC(J)*PPYC(J)+P
     $XC(J)*PXC(J)+PYC(J)*PYC(J)
      DDD5=-YC(1)*PPPYC(J)-XC(1)*PPPXC(J)+3.0*YC(1)*PPXC(J)-3.0*XC(1)*PP
     $YC(J)+3.0*XC(1)*PXC(J)+3.0*YC(1)*PYC(J)-YC(1)*XC(J)+XC(1)*YC(J)
      DDD6=-XC(1)*PPPYC(J)+YC(1)*PPPXC(J)+3.0*XC(1)*PPXC(J)+3.0*YC(1)*PP
     $YC(J)-3.0*YC(1)*PXC(J)+3.0*XC(1)*PYC(J)-XC(1)*XC(J)-YC(1)*YC(J)
      R(3)=COS(AL1(J))*DDD5+SIN(AL1(J))*DDD6+XC(J)*PPPXC(J)+YC(J)*PPPYC(
     $J)+3.0*PXC(J)*PPXC(J)+3.0*PYC(J)*PPYC(J)
      DDDD5=-XC(1)*PPPPXC(J)-YC(1)*PPPPYC(J)+4.0*YC(1)*PPPXC(J)-4.0*XC(1
     $)*PPPYC(J)+6.0*XC(1)*PPXC(J)+6.0*YC(1)*PPYC(J)-4.0*YC(1)*PXC(J)+4.
     $0*XC(1)*PYC(J)-XC(1)*XC(J)-YC(1)*YC(J)
      DDDD6=-XC(1)*PPPPYC(J)+YC(1)*PPPPXC(J)+4.0*XC(1)*PPPXC(J)+4.0*YC(1
     $)*PPPYC(J)-6.0*YC(1)*PPXC(J)+6.0*XC(1)*PPYC(J)-4.0*XC(1)*PXC(J)-4.
     $0*YC(1)*PYC(J)+YC(1)*XC(J)-XC(1)*YC(J)
      R(4)=COS(AL1(J))*DDDD5+SIN(AL1(J))*DDDD6+XC(J)*PPPPXC(J)+YC(J)*PPP
     $PYC(J)+4.0*PXC(J)*PPPXC(J)+4.0*PYC(J)*PPPYC(J)+3.0*PPXC(J)*PPXC(J)
     $+3.0*PPYC(J)*PPYC(J)
      P(1)=SIN(AL1(J))
      P(2)=COS(AL1(J))
      P(3)=-SIN(AL1(J))
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      P(4)=-COS(AL1(J))
      Q(1)=COS(AL1(J))
      Q(2)=-SIN(AL1(J))
      Q(3)=-COS(AL1(J))
      Q(4)=SIN(AL1(J))
      CALL SIMQ(H,R,4,KS)
      CALL SIMQ(H1,P,4,KS1)
      CALL SIMQ(H2,Q,4,KS2)
      WRITE (6,99) KS
      WRITE (6,99) KS1
      WRITE (6,99) KS2
   99 FORMAT (1X,I1)
      F1=Q(1)*Q(3)+Q(2)*Q(4)
      F2=P(1)*Q(3)+Q(1)*P(3)+P(2)*Q(4)+Q(2)*P(4)
      F3=R(1)*Q(3)+Q(1)*R(3)+Q(2)*R(4)+Q(4)*R(2)
      F4=P(1)*P(3)+P(2)*P(4)
      F5=P(1)*R(3)+P(3)*R(1)+P(2)*R(4)+P(4)*R(2)-1.0
      F6=R(1)*R(3)+R(2)*R(4)
      G1=Q(1)*Q(4)-Q(2)*Q(3)
      G2=P(1)*Q(4)+Q(1)*P(4)-P(2)*Q(3)-Q(2)*P(3)
      G3=R(1)*Q(4)+Q(1)*R(4)-R(2)*Q(3)-Q(2)*R(3)-1.0
      G4=P(1)*P(4)-P(2)*P(3)
      G5=P(1)*R(4)+R(1)*P(4)-P(2)*R(3)-P(3)*R(2)
      G6=R(1)*R(4)-R(2)*R(3)
      XCOF(1)=F1*F1*G6*G6+G3*G3*F1*F6-G3*G6*F1*F3+G1*G6*F3*F3+G1*G1*F6*F
     $6-G1*G3*F3*F6-2.0*G1*G6*F1*F6
      XCOF(2)=2.0*G5*G6*F1*F1+2.0*G2*G3*F1*F6+G3*G3*F1*F5-G3*G6*F1*F2-G2
     $*G6*F1*F3-G3*G5*F1*F3+2.0*G1*G6*F2*F3+G1*G5*F3*F3+2.0*G1*G1*F5*F6-
     $G1*G3*F2*F6-G1*G2*F3*F6-G1*G3*F3*F5-2.0*G1*G5*F1*F6-2.0*G1*G6*F1*F
     $5
      XCOF(3)=2.0*G4*G6*F1*F1+G5*G5*F1*F1+G2*G2*F1*F6+2.0*G2*G3*F1*F5+G3
     $*G3*F1*F4-G2*G6*F1*F2-G3*G5*F1*F2-G2*G5*F1*F3-G3*G4*F1*F3+G1*G6*F2
     $*F2+2.0*G1*G5*F2*F3+G1*G4*F3*F3+2.0*G1*G1*F4*F6+G1*G1*F5*F5-G1*G2*
     $F2*F6-G1*G3*F2*F5-G1*G2*F3*F5-G1*G3*F3*F4-2.0*G1*G6*F1*F4-2.0*G1*G
     $5*F1*F5-2.0*G1*G4*F1*F6
      XCOF(4)=2.0*G4*G5*F1*F1+G2*G2*F1*F5+2.0*G2*G3*F1*F4-G2*G5*F1*F2-G3
     $*G4*F1*F2-G2*G4*F1*F3+G1*G5*F2*F2+2.0*G1*G4*F2*F3+2.0*G1*G1*F4*F5-
     $G1*G2*F2*F5-G1*G3*F2*F4-G1*G2*F3*F4-2.0*G1*G5*F1*F4-2.0*G1*G4*F1*F
     $5
      XCOF(5)=G4*G4*F1*F1+G2*G2*F1*F4-G2*G4*F1*F2+G1*G4*F2*F2+G1*G1*F4*F
     $4-G1*G2*F2*F4-2.0*G1*G4*F1*F4
      CALL POLRT(XCOF,COF,4,ROOTR,ROOTI,IER)
      WRITE (6,109)
  109 FORMAT (/,27X,9H L1 ROOTS,/)
      DO 110 I=1,4
  110 WRITE (6,112) ROOTR(I),ROOTI(I)
  112 FORMAT (17X,E13.5,10X,E13.5,/)
      WRITE (6,114) IER
  114 FORMAT (1X,I1)
      WRITE (6,106)
  106 FORMAT (1H1,//)
  125 DO 215 I=1,4
      OA=F1
      OB=F2*ROOTR(I)+F3
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      OC=F4*ROOTR(I)*ROOTR(I)+F5*ROOTR(I)+F6
      OD=G1
      OE=G2*ROOTR(I)+G3
      OF=G4*ROOTR(I)*ROOTR(I)+G5*ROOTR(I)+G6
      IF (ABS(ROOTI(I)).LT.0.000001) ROOTI(I)=0.0
      IF (ROOTI(I)) 200,130,200
  200 WRITE (6,201)
  201 FORMAT (5X,18H ROOT IS IMAGINARY,/)
      GO TO 215
  130 SA=OB*OB-4.0*OA*OC
      IF (SA) 200,140,140
  140 ROOTA1=(-OB+SQRT(SA))/(2.0*OA)
      ROOTA2=(-OB-SQRT(SA))/(2.0*OA)
      SB=OE*OE-4.0*OD*OF
      IF (SB) 200,150,150
  150 ROOTB1=(-OE+SQRT(SB))/(2.0*OD)
      ROOTB2=(-OE-SQRT(SB))/(2.0*OD)
      WRITE (6,155) ROOTA1,ROOTA2,ROOTB1,ROOTB2
  155 FORMAT (/,13X,8H ROOTA1=,F18.6,5X,8H ROOTA2=,F18.6,/,13X,8H ROOTB1
     $=,F18.6,5X,8H ROOTB2=,F18.6,/)
      IF (ABS(ROOTA1-ROOTB1).LT.0.05) GO TO 160
      IF (ABS(ROOTA1-ROOTB2).LT.0.05) GO TO 160
      IF (ABS(ROOTA2-ROOTB1).LT.0.05) GO TO 170
      IF (ABS(ROOTA2-ROOTB2).LT.0.05) GO TO 170
      GO TO 215
  160 WRITE (6,180) ROOTA1
  180 FORMAT (2X,16H SECOND ROOT IS ,F20.5,/)
      ROOT=ROOTA1
      GO TO 210
  170 WRITE (6,180) ROOTA2
      ROOT=ROOTA2
  210 WRITE (6,212)
  212 FORMAT (14X,13H CENTER POINT,31X,13H CIRCLE POINT,/,8X,3H XA,19X,3
     $H YA,19X,3H XB,19X,3H YB,/)
      XA(I)=R(1)+P(1)*ROOTR(I)+Q(1)*ROOT
      YA(I)=R(2)+P(2)*ROOTR(I)+Q(2)*ROOT
      XB(I)=R(3)+P(3)*ROOTR(I)+Q(3)*ROOT
      YB(I)=R(4)+P(4)*ROOTR(I)+Q(4)*ROOT
      WRITE (6,230) XA(I),YA(I),XB(I),YB(I)
  230 FORMAT (4F20.8,/,33X,20H XXXXXXXXXXXXXXXXXX,//)
  215 CONTINUE
      STOP
      END
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VITA

William Richard Coutant, Jr.

Candidate for the Degree of

Master of Science

Thesis: SYNTHESIS OF STEPHENSON TYPE II SIX-LINK FUNCTION GENERATOR FOR FINITELY AND INFINITESIMALLY SEPARATED POSITIONS

Major Field: Mechanical Engineering

Biographical:

Personal Data: Born in Manhattan, Kansas, July 31, 1950, the son of Dr. and Mrs. W. R. Coutant.

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