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A NUMERICAL APPROACH TO WET LINE CORRECTIONS IN

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Symbol	Nomenclature
F_D	Drag force, lb
ρ	Density of water
d	Diameter of wire, in.
v	Velocity of flow perpendicular to the wire, feet per second.
θ	Inclination of the wire to the vertical, degrees.
T	Tension in the wire, lb.
F'_D	Drag force on inclined wire, lb.
D	Depth of stream, ft.
S	Wet line length, ft.
r	Correlation coefficient.

A NUMERICAL APPROACH TO WET LINE CORRECTIONS IN STREAM FLOW MEASUREMENTS

CHAPTER I

INTRODUCTION

The water flowing in surface streams forms one of the most valuable natural assets of any country, and it is an important factor in the development of plans for irrigation, power, municipal water supply, and other projects involving the use of water from surface streams. It is, therefore, necessary to have data from which both the total flow of the stream and its distribution from day to day throughout the year can be obtained. The data necessary for obtaining such flows are the discharge measurements at various stages, from which a rating curve and a table can be prepared, giving the discharge for any stage. In general, measurements of discharge in open channels are done in two ways: (1) by means of weirs, and (2) velocity-area method. The weir, if its coefficient is accurately known, is probably the most accurate method of measuring discharge for small streams, whose channels do not have suitable sections for accurate current meter measurements. In practice, however, weirs rarely conform to the requirements imposed by the experimenter. If the crest of the weir is sharp, clean, and sufficiently high above the bottom of the channel, the end contractions are complete, and the velocity of approach is negligibly small, the empirical formulas used will give good results. On the other hand, if these essential conditions are not complied with, especially if the velocity approach is considerable, and contractions are imperfect, the weir method will not give accurate measurement of discharge. On rivers, however, the cost of a weir is so great as to be prohibitive.

The second method-the area-velocity method-is the one commonly used for determining the flow of streams, especially on large rivers. The discharge by means of the area-velocity method is obtained by determining the average velocity of the current and the area of cross section. Each of these factors must be measured at a sufficiently large number of points in the cross section to give the required degree of accuracy. The general formula for discharge per second is,

$$Q = A_1V_1 + A_2V_2 + A_3V_3 + \ldots\ldots\ldots\ldots + A_nV_n$$

where, A_1, A_2, ... A_n, are the component parts of the cross section, and V_1, V_2, ...V_n, are the mean velocities in each of these parts. If the width of these component parts is b, and the mean depth is d_1, d_2, d_3 . . . . , this can be written

$$Q = b\ (V_1d_1 + V_2d_2 + \ldots\ldots\ldots\ldots + V_nd_n).$$

The measurement of velocity is made at frequent intervals across the stream, and close enough to take account of any abrupt change in the velocity. For convenience, the velocities are usually observed in the same verticals at which soundings are made. Although the velocity of flow is best determined by current meter, on some streams fairly good measurement of velocities may be made by other methods, such as, floats in channels of uniform cross sectional area over a considerable distance.

Several methods are in use for obtaining the mean velocity in a vertical. The two principal methods in general use are: (1) the two-point method, and (2) six-tenths depth method. In the two-point method, observations of velocities are made in each vertical at 0.2 and 0.8 of the depth below the water surface, and the mean of velocities at these two points is taken as the average velocity for that vertical. This method is based on the theory that the vertical velocity curve is a part of a parabola with axis horizontal at the point of highest velocity. Actual observations under a wide range of conditions show that this method gives the mean velocity very closely for open conditions. Because of its greater reliability, it is generally preferred. Unusual or abnormal conditions in the movement of water have less serious effects on the accuracy of measurements if the observations are made by this method. It is very extensively used in the regular practice of the United States Geological Survey.

In very shallow depths, it is not always practicable to use current-meters at 0.2 and 0.8 depth positions because of proximity of these positions to the water surface or the stream bed. Under these circumstances, the 0.6 depth method may be preferable. This method consists of an observation of velocity in each of the vertical at 0.6 depth below the surface. It gives good results where there is a straight channel with little obstruction, and no sudden change in velocity.

Another method of obtaining the mean velocity in the vertical is the vertical velocity-curve method, in which a series of velocity determinations are made in each vertical at regular intervals. By plotting these velocities as abscissas and their depths as ordinates, and by drawing a smooth curve among the resulting points, the vertical velocity-curve is developed. The mean velocity is then obtained by dividing the area bounded by this velocity curve, and its axis by the depth. Although this method of obtaining the mean velocity is best known, its use is largely limited because of the length of time required to make a complete measurement.

When the mean velocities in the different verticals have been found, the average of two adjacent means is taken as the mean velocity of that individual section. The area of the section is computed by multiplying the width of the section by the mean depth. The discharge of each section is the product of the area multiplied by the mean velocity, and the total discharge of stream results from summing up the discharge of the individual sections. Considerable judgment is necessary in selecting points in the section where depth and velocity should be measured to secure a proper degree of accuracy.

A great many discharge measurements of natural and artificial channels have been made in various ways with various kinds of instruments. Very little appears to have been done in the way of determining the degree of accuracy of the measurements. The central problem in the discharge measurements is the determination of the depth (See Figure 1). The biggest errors usually result from the measurement of depth rather than of velocity in making discharge measurements with current meters. Unless the true depth is obtained using sonic depth finders, accurate determination of depth, and mean velocity in the vertical are of primary importance in obtaining accurate measurements of discharge. For this reason, the measuring of depth of stream, though it appears to be a quite simple operation, receives considerable attention. Since the product of depth, width and mean velocity is the discharge, an error in sounding of a station produces a corresponding error in the volume of discharge of that section. Moreover, it is in the more important gaging stations where, with great velocities and depths the discharge is largest, that this error is most likely to occur. At these stations accurate sounding is most imperative by reason of large magnitudes of flow. Any refinement in other factors that enter into problems of river gaging is annuled by less refinement in sounding.

When velocities are low, no serious difficulties are encountered, and the depths are easily measured. However, when the velocities are high, despite the use of a heavy weight, it is rarely possible to get true depths by sounding in swift water because the meter, and the weight are carried downstream by the current. Under these conditions the length of the line paid-out is greater than the true depth. The wire is curved and this curved line extends from the observer to the water surface, and forms a curve configuration below the water surface. Therefore, there are two sources of error: the dry line error and the wet line error. (See Figure 1). While the wet line correction methods in use at present, such as the one presented by F. C. Shenehon in the annual report, Chief of Engineers, U.S. Army 1900, are believed to be sufficiently accurate for purposes of ascertaining water resources, it is still important to know their probable accuracy. Increasing accuracy in this measurement is essential if we expect to maintain pace with improved instruments. The best solution would be to determine the equation of the suspending cable, and the position at any length of paid-out line could accurately be determined.

The purpose of this study was to investigate a mathematical relationship between the length of wet line paid-out, and the true depth used in arriving at the discharge at a section, while taking into account the forces acting on the wire. Numerical methods have paved the way to solve the resulting equation using the digital family of computers. The solution for the resulting equation is possible in terms of a hyperbolic cosine function. The equation thus derived may be used to solve any problem of this nature.

Literature Review

For many years, engineers have recognized the problem of obtaining the true depth from wet line length paid-out (See Figure 1), and have expended considerable time and thought to its solution. Most investigators directed their efforts towards an analytical approach to the problem. A brief evolution of the work that has been done in this field will be described.

There have been several discussions of the nature of the curve taken by the wire in the water, (See Figure 1) notably by De Marchi, Tonta, and by Gougenheim. The four curves which have been mentioned are the parabola, the catenary, a modified form of the catenary, and the circle. It is the opinion of most investigators that, for small angles and depths, the differences among these are not very important. For greater depths, the circle was found to give large corrections, and the corrections as found by the other three differ by about equal amounts from one curve to the other.

Captain L. Tanta (1) concludes that the form of equilibrium of the towed lead line is an arc of catenary, the symmetrical exis of which is horizontal, and the concavity of which is in the direction of movement. He compared the corrections for inclination obtained theoretically by Courtier, De Marchi and himself and showed that, in the three hypotheses considered, the correction for inclination, k, may be reduced to a single formula:

$$k = l\left[1 - \psi(\alpha)\right] + C_oB\ (\omega)\left[\psi(\omega) - \psi\ (\alpha)\right]$$

k = correction for inclination, in feet.

represents the length of the submerged line, in feet

α its angle from the vertical at the point at which it enters the water, in degrees.

ω its angle from the vertical at the point at which the lead is attached, in degrees.

and C_o, a constant of the wire system, approximately measuring the length of the arc of the curve of the sounding line produced from the lead to the apex of this curve.

M. A. Gougenheim (2) simplified the equation given by Tanta to determine inclination correction nomographically to:

$$k = \left[1 - \psi(\alpha)\right]\left[l + C_o\ (1 - A^2)\right]$$

where,

$$A = \frac{C_o}{C_o + l}$$

In this form, the product of a function of α by a function of l , the correction for inclination, may be obtained by means of transversal straight line nomogram. Further, he writes the above equation for k as,

$$k = \left[1 - \psi(\alpha)\right] \left(C_o + l \right) \left(1 - A^3 \right)$$

and finally, after neglecting the term A^3, with some reasoning he writes the above formula for inclination correction as,

$$k = \left[1 - \psi(\alpha)\right] \left(C_o + l \right)$$

and gives a nomographic solution for k. The diagrams constructed allow for inclination to be obtained within a much greater approximation than that due to uncertainty in α. In his conclusions he advises the reader, "as long as the determination of this angle is not improved, the transversal straight line nomogram be employed", on the sole condition that l is greater than C_o, if the nomographic method be used for obtaining the correction k.

H. F. Johnson (3) adopting a parobolic curve has given the following formula:

$$d = L\,C_\phi - h\,(\mathrm{Sec}\,\phi - 1)\,C_\phi - f\,(1 - C_\phi)$$

where,

d = true depth, in feet.

h = distance, spool to water surface, in feet.

L = length of wire out from the spool when the weight touches bottom, less h.

$g = f/C_\alpha$

g = length of wire which has the same horizontal resistance to the water as the weight, and f = its projection on the vertical.

C_{∞} = a Coeffieient

ϕ = the angle of wire with the vertical at the spool, in degrees.

α = the similar angle at the weight, in degrees.

C = ratio of its projection on the vertical to length of arc of curve.

$$C = \frac{1}{2}\left[\sec\phi + \cot\phi \log_e \tan\left(\frac{\phi}{2} + \frac{\pi}{4}\right)\right]$$

F. Embacher (4) assumes the shape of the wire under water as parabola and developed an equation for the true depth, h:

$$h = \frac{s - 0.4 \tan^2\phi\ m}{1 + 0.15\ a \tan^2\phi}$$ for $\tan\phi$ between 0 to 1. and for

$\tan\phi$ between 1 to 2

$$h = \frac{s - 0.4 \tan^{1.7}\phi\ m}{1 + 0.15\ a \tan^{1.7}\phi}$$

where:

s = wet line length, in feet.

m = vertical distance from the apex to water surface, in feet.

ϕ = the angle of wire with the vertical at the apex, in degrees.

a = a factor depending on the shape of the curve.

F.C. Shenehon (5) gives a table of corrections for wet line lengths evolved by the static method of sounding. The method depends on the elementary principles of mechanics: that, when a known horizontal force is applied to a weight suspended on a cord, the cord takes a position of rest at some angle with the vertical. In applying the above principle to conditions of measurement of depths of flowing water, it is assumed that with a properly designed sounding weight, the horizontal pressure on the weight in the comparatively still water near the bottom can be neglected. The distribution of total horizontal pressure along the sounding line is made in accordance with the variation of velocity from surface to bottom. The excess in length of the curved line over the vertical depth is the sum of the products of each tenth of depth and secants of the corresponding angles derived for each tenth of depth by means of tangent relation of the forces acting below any point. The table is general, not for any particular sounding weight, provided it is designed so as to present little current resistance. This is the table that is in great use by the United States Geological Survey.

L. Landweber and M. H. Protter (6) derived equations and curves by considering several cases of a towing cable.

CHAPTER II

THEORETICAL ANALYSIS OF THE PROBLEM

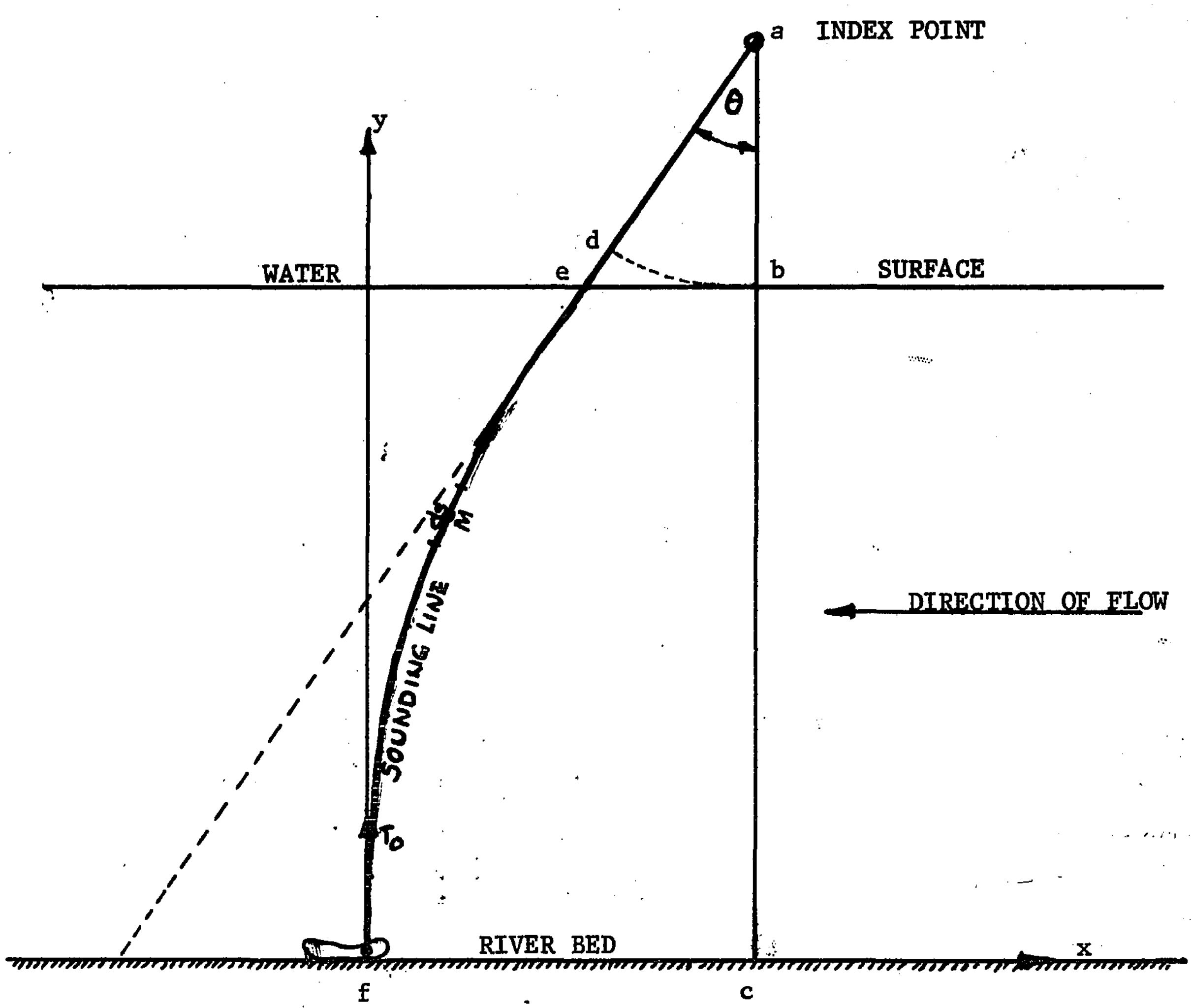

Fig. 1 Position of Sounding line in Swift water.

Figure 1, shows the position assumed by the sounding line and the weight, after it has been carried downstream, the distance of which is dependent on the weight, depth, and velocity of water. From this figure it is seen that the meter-line is straight from the Index Point to the water surface, and forms a curve underwater. The distance 'af' represents the amount of line let out by the reel during the process of lowering the weight to the channel bottom. The distance 'ae' is the product of the vertical depth 'ab' and the secant of the vertical angle θ . i.e. $ae = ab \sec\theta$. From the length of cable 'ae', the vertical distance 'ab' from the index point to the water surface must be deducted to obtain the so called air correction 'de'. The wet line length 'ef' is then the difference between the observed depth 'df', which is the amount of line let out by the reel during the process of lowering the weight from the surface to the river bottom, and the air correction 'de'.

In order to determine the depth 'bc', an attempt has been made, on the pages to follow, to establish a mathematical relationship between the wet line length 'ef' and vertical depth 'bc'.

Analysis

For purpose of analysis, a small differential element ds, of the cable at any point M, having coordinates (x, y) relative to the rectangular coordinate axes in the plane of the cable with the origin at the bottom of the cable, is shown in the Figure 2. The differential element ds, of the cable is in equilibrium under the action of a system of forces comprising:

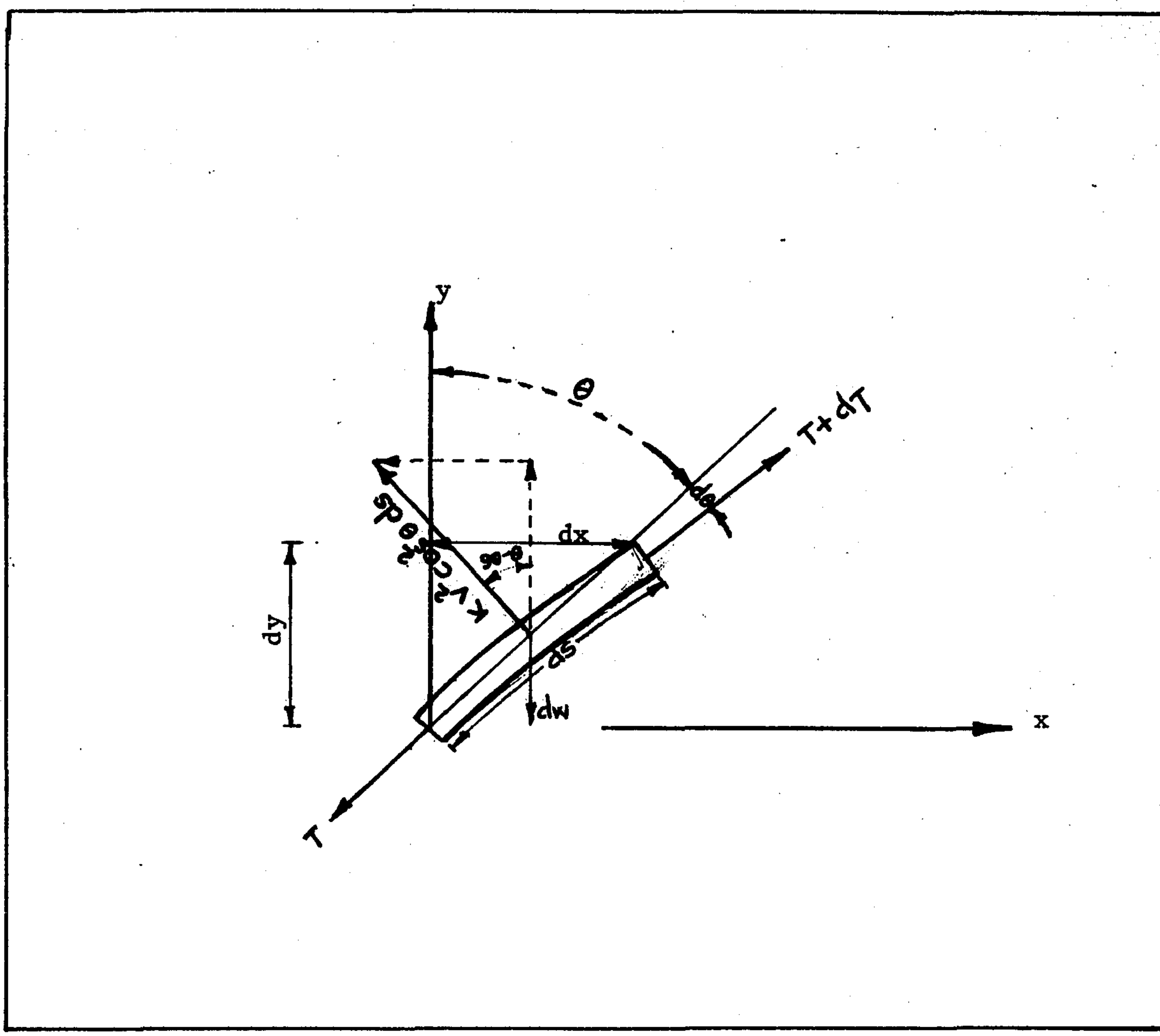

Fig. 2 Forces acting on an Element of a Cable.

(1) The tensions T, and T + dT at the two extremities of the part of the wire under consideration.

(2) The weight of the element, dw.

(3) The hydrodynamic force component perpendicular to the wire.

The hydrodynamic force, arising from the pressure and frictional stresses developed by the flow of fluid around the wire, which is placed at right angles to the direction of flow, is proportional to the cross section of the wire, the density of water, and the square of the velocity. The hydrodynamic force acting on the inclined wire (7) can be expressed in terms of the force on a wire perpendicular to the stream, knowing the component of velocity of flow normal to the wire. If the inclination of the wire to the vertical is θ, the component of velocity normal to the inclined wire is, $v \cos\theta$, and its component parallel to the wire is $v \sin\theta$. The drag force on the wire per unit length in the direction of flow is given by

$$F_D = \frac{1}{2} C_d \rho d v^2$$

where:

C_d = drag coefficient.

ρ = density of water.

d = diameter of the wire.

v = velocity of flow perpendicular to the wire, in feet per second.

Or $F_D = k v^2$

where,

$$k = \frac{1}{2} C_d \rho d$$

The drag force per unit length on inclined wire is

$$F_D = \tfrac{1}{2}\, C_d\, \rho\, d \;(v \cos\theta)^2$$

$$= kv^2 \cos^2\theta$$

The component of velocity parallel to the wire, v sinθ, gives a frictional resistance and will be small, and it appears legitimate to consider as neglible.

It will be assumed that the weight of the cable is negligible, and with this assumption the forces which act on the element are reduced to hydraulic resistance, and to the difference of the tensions applied to the extremities, as shown in the Figure 2.

Resolving the forces in the X direction:

$$\Sigma F_x = 0$$

$$(T + dT) \sin(\theta + d\theta) - T \sin\theta - k\, v^2 \cos^3\theta\, ds = 0 \qquad \text{(Eqn. 2-1)}$$

$$T \sin(\theta + d\theta) + dT \sin(\theta + d\theta) - T \sin\theta - k\, v^2 \cos^3\theta\, ds = 0$$

$$T [\sin\theta \cos d\theta + \cos\theta \sin d\theta] + dT [\sin\theta \cos d\theta + \cos\theta \sin d\theta]$$

$$- T \sin\theta - k\, v^2 \cos^3\theta\, ds = 0$$

$$T \sin\theta \cos d\theta + T \cos\theta \sin d\theta + dT \sin\theta \cos d\theta +$$

$$dT \cos\theta \sin d\theta - T \sin\theta - k\, v^2 \cos^3\theta\, ds = 0$$

For small dθ, $\sin d\theta \approx d\theta$ and $\cos d\theta \approx 1$

Therefore,

$$T \sin\theta + T \cos\theta\, d\theta + dT \sin\theta + dT \cos\theta\, d\theta - T \sin\theta - k\, v^2 \cos^3\theta$$

$$ds = 0$$

$$dT (\sin\theta + \cos\theta\, d\theta) + T \cos\theta\, d\theta - k\, v^2 \cos^3\theta\, ds = 0$$

$$dT = \frac{k\, v^2 \cos^3\theta\, ds - T \cos\theta\, d\theta}{\sin\theta + \cos\theta\, d\theta} \qquad \text{(Eqn. 2-2)}$$

Resolving the forces in the Y direction

$\Sigma F_y = 0$

$$(T + dT)\cos(\theta + d\theta) + kv^2\cos^2\theta\, ds\cos(90 - \theta) - T\cos\theta = 0$$

(Eqn. 2-3)

$$T\cos(\theta + d\theta) + dT\cos(\theta + d\theta) + k\,v^2\cos^2\theta\sin\theta ds - T\cos\theta = 0$$

$$T\cos\theta\cos d\theta - T\sin\theta\sin d\theta + dT\cos\theta\cos d\theta - dT\sin\theta\sin d\theta$$
$$+ k\,v^2\cos^2\theta\sin\theta\, ds - T\cos\theta = 0$$

For small $d\theta$, $\sin d\theta \approx d\theta$ and $\cos d\theta \approx 1$

$$T\cos\theta - T\sin\theta\, d\theta + dT\cos\theta - dT\sin\theta\, d\theta + k\,v^2\cos^2\theta\sin\theta\, ds - T\cos\theta = 0$$

$$dT(\cos\theta - \sin\theta\ d\theta) = T\sin\theta\, d\theta - k\,v^2\cos^2\theta\sin\theta\, ds$$

$$dT = \frac{T\sin\theta\, d\theta - k\,v^2\cos^2\theta\sin\theta\, ds}{\cos\theta - \sin\theta\, d\theta} \qquad \text{(Eqn.2-4)}$$

From Equations (2-2) and (2-4) we have,

$$\frac{kv^2\cos^3\theta\, ds - T\cos\theta\, d\theta}{\sin\theta + \cos\theta d\theta} = \frac{T\sin\theta\, d\theta - k\,v^2\cos^2\theta\sin\theta ds}{\cos\theta - \sin\theta\, d\theta}$$

$$(k\,v^2\cos^3\theta\, ds - T\cos\theta\, d\theta)(\cos\theta - \sin\theta\, d\theta)$$
$$= (T\sin\theta\, d\theta - k\,v^2\cos^2\theta\sin\theta\, ds)(\sin\theta + \cos\theta\, d\theta)$$

$$k\,v^2\cos^4\theta\, ds - T\cos^2\theta\, d\theta - k\,v^2\cos^3\theta\, ds\sin\theta\, d\theta + T\cos\theta\, d\theta\sin\theta\, d\theta$$
$$= T\sin^2\theta\, d\theta - k\,v^2\cos^2\theta\sin^2\theta\, ds + T\sin\theta\, d\theta\cos\theta\, d\theta - kv^2\cos^3\theta\sin\theta\, ds\, d\theta$$

$$k\,v^2\cos^4\theta\, ds + k\,v^2\cos^2\theta\sin^2\theta\, ds - T\cos^2\theta\, d\theta - T\sin^2\theta\, d\theta = 0$$

$$k\,v^2\cos^2\theta\, ds(\cos^2\theta + \sin^2\theta) - T(\cos^2\theta + \sin^2\theta)\, d\theta = 0$$

$$k\,v^2\cos^2\theta\, ds - T\, d\theta = 0 \qquad \text{(Eqn. 2-5)}$$

It remains now to find a solution for Equation (2-5)

Rewriting Equation (2-5),

$$k\,v^2\cos^2\theta\, ds = T\, d\theta$$

Dividing both sides by $\cos\theta$, we have

$$k\,v^2\cos\theta\,ds = T\,\frac{d\theta}{\cos\theta}$$

Since,

$$\frac{dy}{ds} = \cos\theta$$

Therefore,

$$k\,v^2\,\frac{dy}{ds}\cancel{ds} = T\,\frac{d\theta}{\cos\theta}$$

$$k\,v^2\,dy = T\,\frac{d\theta}{\cos\theta}$$

$$k\int_0^y v^2dy = T\int_0^\theta \frac{d\theta}{\cos\theta}$$

$$k\int_0^y v^2dy = T\left[\ln\,(\sec\theta + \tan\theta)\right]$$

$$\frac{k}{T}\int_0^y v^2dy = \ln\,(\sec\theta + \tan\theta)$$

therefore,

$$\sec\theta + \tan\theta = e^{\frac{k}{T}\int_0^y v^2dy} \qquad \text{(Eqn. 2-6)}$$

now,

$$e^{-\frac{k}{T}\int_0^y v^2dy} = \frac{1}{e^{\frac{k}{T}\int_0^y v^2dy}}$$

$$= \frac{1}{\sec\theta + \tan\theta}$$

$$= \frac{(\sec\theta - \tan\theta)}{(\sec\theta - \tan\theta)}\;\frac{1}{(\sec\theta + \tan\theta)}$$

$$= \frac{\sec\theta - \tan\theta}{\sec^2\theta - \tan^2\theta}$$

$$= \sec\theta - \tan\theta$$

Therefore,

$$\sec\theta - \tan\theta = e^{-\frac{k}{T}\int_0^y v^2 dy} \qquad \text{(Eqn. 2-7)}$$

Adding Equations (2-6) and (2-7);

$$2\sec\theta = e^{\frac{k}{T}\int_0^y v^2 dy} + e^{-\frac{k}{T}\int_0^y v^2 dy}$$

Therefore,

$$\sec\theta = \frac{e^{\frac{k}{T}\int_0^y v^2 dy} + e^{-\frac{k}{T}\int_0^y v^2 dy}}{2}$$

$$\frac{ds}{dy} = \cosh\left(\frac{k}{T}\int_0^y v^2 dy\right)$$

$$\int_0^s ds = \int_0^y \left[\cosh\left(\frac{k}{T}\int_0^y v^2 dy\right)\right] dy$$

$$S = \int_0^y \left[\cosh\left(\frac{k}{T}\int_0^y v^2 dy\right)\right] dy \qquad \text{(Eqn. 2-8)}$$

Returning to Equations (2-1) and (2-3), for small $d\theta$ we may write

$$(T + dT)(\sin\theta + \cos\theta\, d\theta) - kv^2 \cos^3\theta\, ds - T\sin\theta = 0$$

$$(T + dT)(\cos\theta - \sin\theta\, d\theta) + kv^2 \cos^2\theta \sin\theta\, ds - T\cos\theta = 0$$

Simplifying, and neglecting the product of terms $d\theta\, ds$, we have,

$$T\cos\theta\, d\theta + dT\sin\theta - kv^2\cos^3\theta\, ds = 0$$

$$-T\sin\theta\, d\theta + dT\cos\theta + kv^2\cos^2\theta\sin\theta\, ds = 0$$

Multiplying the above equations by $\sin\theta$, and $\cos\theta$ respectively, and dividing by dy, we write,

$$T\cos\theta\sin\theta\frac{d\theta}{dy} + \sin^2\theta\frac{dT}{dy} - kv^2\cos^3\theta\sin\theta\frac{ds}{dy} = 0$$

$$- T \cos\theta \sin\theta \frac{d\theta}{dy} + \cos^2\theta \frac{dT}{dy} + kv^2 \cos^3\theta \sin\theta \frac{ds}{dy} = 0$$

solving for $\frac{dT}{dy}$, we have,

$$\frac{dT}{dy} (\cos^2\theta + \sin^2\theta) = 0$$

$$\frac{dT}{dy} = 0$$

from which,

$$T = \text{constant} = T_o$$

For all practical purposes, T_o may be taken to be equal to the magnitude of sounding weight minus buoyant force.

Vertical Velocity Distribution

Equation (2-8) demands a knowledge of the vertical distribution of velocity at a section before an attempt is made to integrate that equation. The relation between velocity and depth in a vertical section of a stream is, therefore, desirable. Also, from such a curve the ratio of the velocity at any depth to mean velocity can easily be found.

Much effort has been expended in investigating this relation. As might be expected, the results of the experiments in this direction do not agree. Each investigator adopts a different form of curve to fit his observations. H. J. Tracy and C. M. Lester found that each vertical-curve in the central region of flow is logarthmic from the surface to a point very close to floor of smooth rectangular channel. Henry found it to be an ellipse for the St. Clair river. For Mississippi River, Humphreys and Abbot found it to be a parabola, whose axis is three-tenths depth below the water surface. Racourt made observations on the Nava River, from which he concluded that this curve is an ellipse whose minor axis is a little below the surface.

The shape of theoretical velocity curve depends much on local conditions such as roughness of bed, slope, etc. The exact shape of the curve varies greatly, however, for different depths of streams and for different velocities. Moreover, the observations obtained with a single meter when plotted, will not fall on any curve, and it is possible to fit more or less a set of imperfect vertical velocity curves. Therefore, in open channel flow the velocity distribution problem cannot be placed on the precise basis which it enjoys in pipe flow. However, from long experience and thousands of measurements, as well as studies of vertical velocity curves made under widely different hydraulic conditions by the United States Geological Survey, it is shown, that their shapes usually resemble the part of a parabola, the axis of which is parallel to the surface of water, located at about twenty to twenty-five percent depth below the surface, coinciding, in general, with the filament of maximum velocity. The velocity decreases gradually from the axis to the surface, and downward to the bottom. The curvature of the vertical velocity curve decreases as the depth increases for the same mean velocity and different depths of water.

It appears, therefore, appropriate for our analysis, to assume that the vertical velocity distribution is parabolic having the form

$$v = A\,Y^2 + B\,Y + C$$

In order to solve for A, B, and C we make use of certain characteristics of velocity distribution established by the United States Geological Survey (8) which are amplified by the following statements:

(1) The location of maximum velocity is 0.25 D below the water surface.

(2) Average velocity is approximately eighty-five percent of the surface velocity.

(3) A reliable and more accurate means of obtaining the average velocity is by taking the numerical mean of the velocities at 0.2 D and 0.8 D below the water surface, where D is the depth of water below the water surface.

To locate the position of maximun velocity, differentiate v, with respect to Y, and equate the result to zero.

$$v = A\,Y^2 + B\,Y + C$$

$$\frac{\partial v}{\partial Y} = 2\,A\,Y + B = 0$$

$$2\,A\,Y = -\,B$$

therefore

$$-\frac{B}{2A} = Y$$

According to (1), of the characteristic velocity distribution, maximum velocity occurs at $Y = -\,0.25\text{ D}$

therefore,

$$-\frac{B}{2A} = -\,0.25\text{ D}$$

from which,

$$\frac{B}{A} = \frac{D}{2} = \frac{\alpha D}{\alpha 2}$$

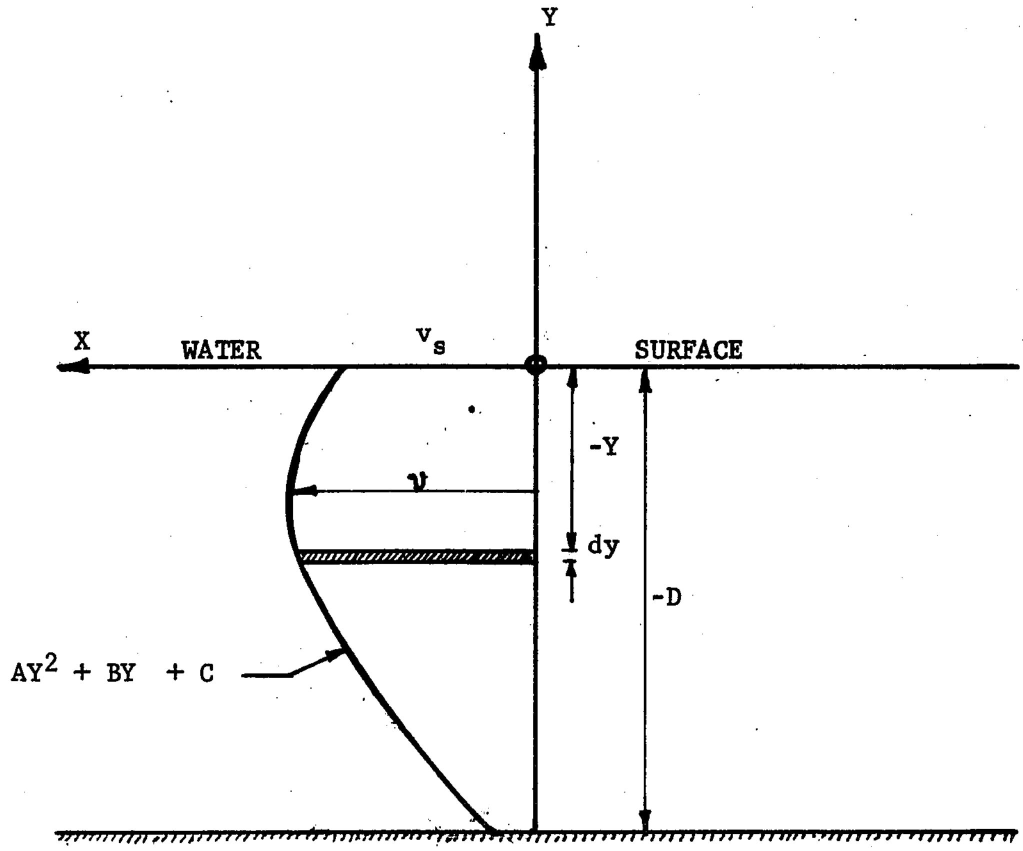

Fig. 3 Vertical Velocity profile

where,

$$\alpha = \text{the constant of proportionality}$$

therefore,

$$B = \alpha D$$

$$A = 2\alpha$$

With these values for A and B, the parabolic equation becomes

$$v = 2\alpha\, Y^2 + \alpha D\, Y + C$$

Referring to Figure 3,

$$v = v_s \quad \text{at } Y = 0$$

with these values for v and Y in the parabolic equation we have,

$$v_s = 0 + 0 + C$$

i.e., $v_s = C$

From (2), of the characteristic velocity distribution,

$$v_m = 0.85\ v_s = 0.85\ C \qquad \text{(Eqn. 2-9)}$$

From (3), of characteristics of velocity distribution

$$v_m = \frac{v\ (-0.2D) + v\ (-0.8D)}{2}$$

$$= \frac{2\alpha(-0.2D)^2 + \alpha D\ (-0.2D) + C + 2\alpha(-0.8D)^2 + \alpha D\ (-0.8D) + C}{2}$$

$$= \frac{0.36\alpha D^2 + 2\ C}{2}$$

$$= 0.18\alpha D^2 + C \qquad \text{(Eqn. 2-10)}$$

From Equations (2-9), and (2-10)

$$0.85\ C = 0.18 \alpha D^2 + C$$

therefore,

$$C = - \frac{0.18}{0.15}\ D^2 \alpha$$

$$= - 1.\ 2 \alpha D^2$$

from which,

$$v_s = - 1.2 \alpha D^2 = C \qquad \text{(Eqn. 2-11)}$$

and finally, the parabolic equation for v, can be written,

$$v = \alpha \left[2\ \ Y^2 + D\ Y - 1.2\ D^2 \right] \qquad \text{(Eqn. 2-12)}$$

From Equations (2-9 and (2-11),

$$v_m = 0.85\ (- 1.2 \alpha\ D^2)$$

$$= - 1.02 \alpha\ D^2 \qquad \text{(Eqn. 2-13)}$$

At this point a check for the mean velocity is in order. Referring to Figure 3, consider an elemental strip at a depth - Y, from the water surface.

Area of strip per food width, da = dY

Discharge per foot width of channel = dQ = v dY

therefore,

$$\int_0^Q dQ = \int_0^{-D} v\ dY$$

$$\int_{0}^{Q} dQ = \alpha \int_{0}^{-D} (2\,Y^2 + D\,Y - 1.2\,D^2)\;dY$$

$$Q = \alpha \left[\frac{2}{3}\,(-D)^3 + \frac{D}{2}\,(-D)^2 - 1.2\,D^2\,(-D) \right]$$

$$= 1.03\,\alpha\,D^3$$

But, discharge per foot width = $-D\,v_m$

therefore,

$$D\,v_m = -\,1.03\,\alpha\,D^3$$

$$v_m = -\,1.03\,D^2\alpha$$

This is in good agreement with the velocity v_m, of Equation (2-13).

From Equation (2-13)

$$\alpha = -\,\frac{v_m}{1.02\,D^2}$$

therefore Equation (2-12) becomes,

$$= -\,\frac{v_m}{1.02\,D^2}\left[2\,Y^2 + D\,Y - 1.2\,D^2\right]$$

At this point, the transformation of coordinates is in order. The transformation equation from old coordinate system (Figure 2) to new coordinate system (Figure 3) is

$$y = Y + D$$

or

$$Y = y - D$$

and,

$$dy = dY$$

Therefore, v, (Eqn. 2-12) in old coordinate system is,

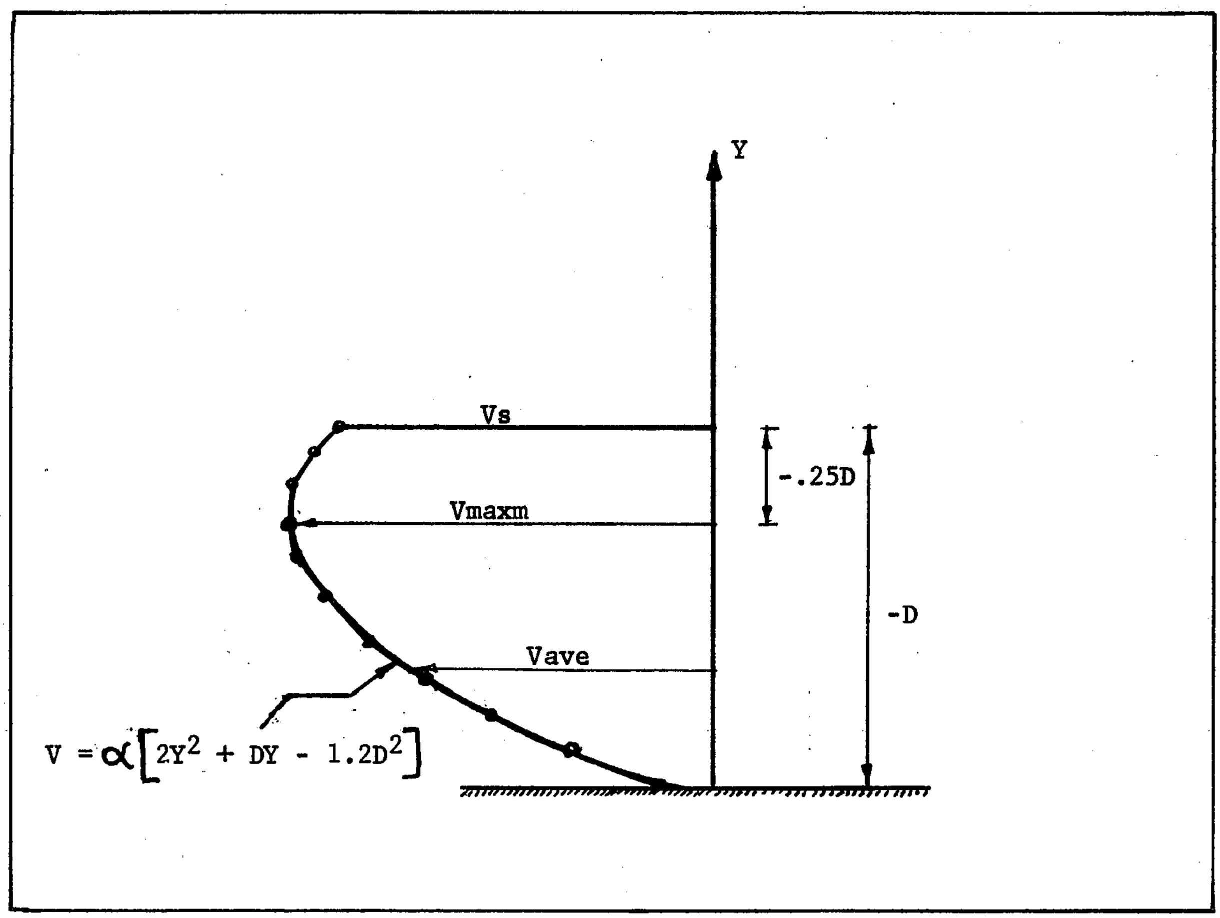

Fig. 4 Plot of Vertical Velocity distribution

$$v = -\frac{v_m}{1.02\ D^2}\left[2\ (y-D)^2 + D\ (y-D) - 1.2\ D^2\right]$$

and the term $\int_0^y v^2\ dy$, in Equation (2-8) can now be evaluated.

$$\int_0^y v^2\ dy = \frac{v^2{}_m}{(1.02D^2)^2}\int_0^y\left[2\ (y - D)^2 + D\ (y-D) - 1.2\ D^2\right]^2 dy$$

$$= \frac{v^2{}_m}{(1.02D^2)^2}\int_0^y\left[2\ (y^2 - 2yD + D^2) + Dy - D^2 - 1.2\ D^2\right]^2 dy$$

$$\int_0^y v^2\ dy = \frac{v_m^2}{(1.02\ D^2)^2}\int_0^y\left[2\ y^2 - 3\ y\ D - 0.2\ D^2\right]^2 dy$$

$$= \frac{v^2{}_m}{(1.02\ D^2)^2}\int_0^y\left[4\ y^4 - 12\ y^3\ D + 8.2\ y^2D^2 + 1.2\ y\ D^3 + \right.$$

$$\left. 0.04\ D^4\right] dy$$

$$= \frac{v^2{}_m}{(1.02\ D^2)^2}\left[\frac{4}{5}\ y^5 - \frac{12}{4}\ y^4\ D + \frac{8.2}{3}\ y^3 D^2 + \frac{1.2}{2}\ y^2 D^3 + \right.$$

$$\left. \frac{0.004}{1}\ y\ D^4\right]$$

$$= \frac{v^2{}_m}{(1.02\ D^2)^2}\left[0.8\ y^5 - 3\ y^4\ D + 2.7334\ y^3\ D^2 + 0.6\ y^2\ D^3 + \right.$$

$$\left. 0.04\ y\ D^4\right]$$

Equation (2-8) can now be written with reference to old coordinate system as,

$$S = \int_0^y \left[\text{Cosh} \left(\frac{k}{T} \frac{v^2_m}{1.0404\ D^4} \right. \right. \left(0.8\ y^5 - 3\ y^4\ D + 2.7334 y^3\ D^2 + 0.6 y^2 D^3 + 0.04\ y\ D^4\right)\Bigg)\Bigg]\ dy$$

In terms of new coordinate system, we have,

$$S = \int_0^y \left[\text{COSH} \left(\frac{k\ v^2_m}{1.0404\ D^4\ T} \right. \right. \left(0.8\ (Y + D)^5 - 3\ (Y + D)^4\ D + 2.7334\ (Y + D)^3\ D^2 + 0.6\ (Y + D)^2\ D^3 + 0.04\ (Y + D)\ D^4\right)\Bigg)\Bigg]\ dy$$

(Eqn. 2-14)

CHAPTER III

THE DRAG FORCE

Before an attempt is made to integrate Equation (2-14), a relation between the drag force on the suspension wire as a result of flow of water, and the velocity of water is essential to arrive at a value of k appearing in Equation (2-14).

The determination of the hydrodynamic force on a body in a flowing water is of primary importance in many engineering problems. In a few cases, this force can be evaluated by analytical methods. In most practical problems it is necessary to resort to a combination of analysis and experiment. A number of papers (9, 10, 11,) are available dealing with the drag forces. The results of these investigations have shown that the velocity gradient affects the drag coefficient. These studies have also indicated that velocity gradient effect is significant in a number of problems of engineering importance, and pose several questions requiring further investigation.

There is a considerable literature available concerning the drag of the circular cylinder as influenced by velocity gradient along its axis, but little has been contributed so far on the nature of drag on a wire. A variable velocity distribution and an unknown factor as a result of oscillation of wire are involved, and thus, makes it still extremely complex. The work in this direction by a few investigators illustrates the importance of velocity gradients on drag forces, but does not give information that can be used directly by designers. Much has yet to be done in this direction.

The latest literature available, in the opinion of author, on drag-velocity relationship with regard to a suspension wire is by Bruce B. Sharp (11) in his extensive investigation of displacement characteristics of thin wires suspended in an open channel.

Until further contributions to this important drag-velocity relationship are made we will borrow the drag-velocity relations extrapolated by Bruce B. Sharp to establish a value for k, on the basis that the drag force is proportional to the square of the perpendicular component of the speed V. i.e. $F = k'v^2$. From Figure 5, the average value for k' was found to be = 0.18.

Therefore,

$$F = 0.18\ v^2$$

$$\tfrac{1}{2}\ C_d \rho\ d\ y\ v^2 = 0.18\ v^2$$

$$\text{i.e.}\quad k = \frac{0.18}{y}$$

and

$$C_d = \frac{0.1855}{d\ y\ \rho}$$

Knowing the diameter of the cable, the magnitudes of k, and C_d may be computed for different depths.

Meter cables commonly used for suspension of current meters, and weights by the United States Geological Survey are:

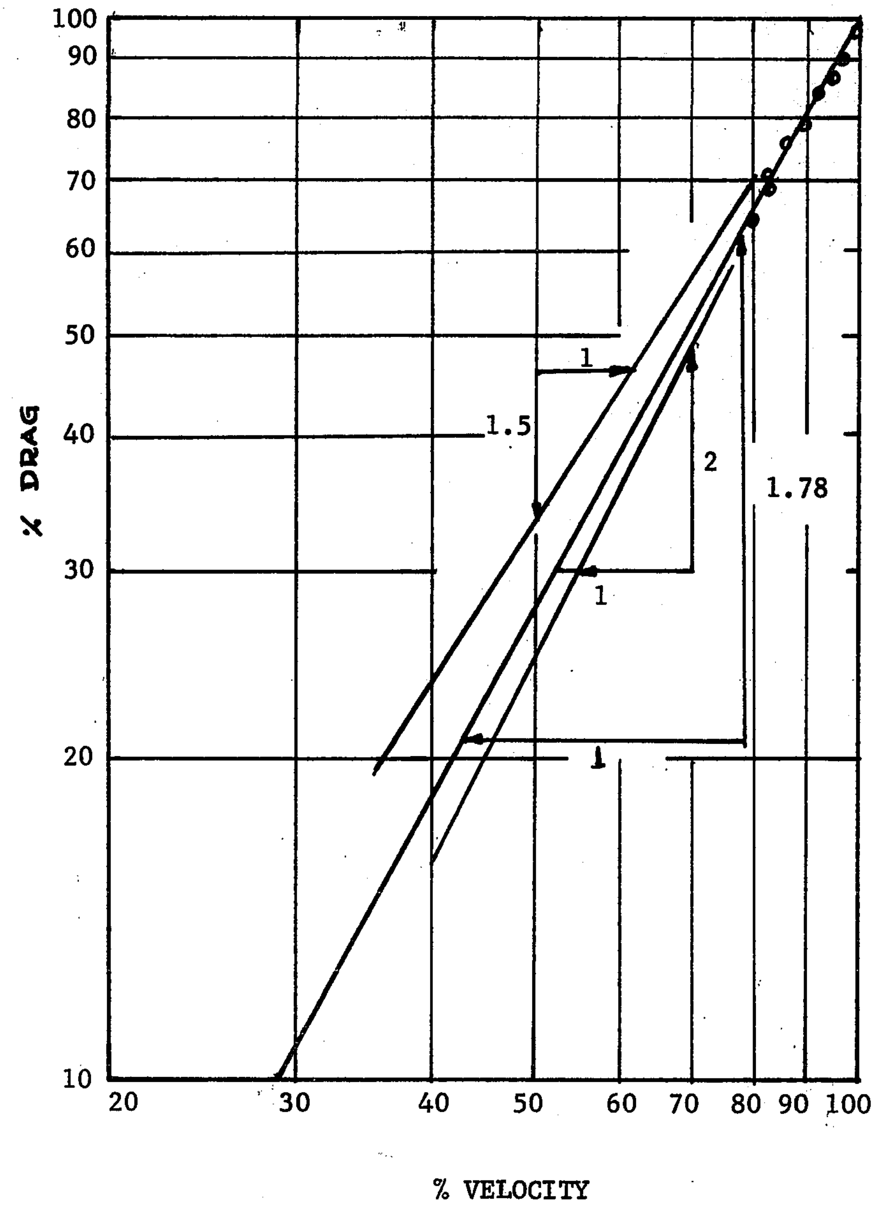

Fig. 5 Flume Test-Suspension Wire Probe

(As established by Bruce)

(1) Direct-lay cable, in two sizes having 0.09" and 0.11" diameters. (2) The riverse lay cable made of 0.10" and 0.125" diameters. Since 0.10" meter cable is the one mostly used in suspension of current meters in the United States in river gaging, we make use of this value for d whenever we encounter it in our work. The values for k and C_d with this value for d is shown in the Table 1.

The United States Geological Survey uses C type weights ranging in sizes of 15, 30, 50, 75, 100 and 150 lbs. For our analysis use is made of 50 lbs.

TABLE I

Values for k and C_d for different depths

D	k	C_d
12	0.01500	1.85
16	0.01126	1.39
20	0.009001	1.11
24	0.007501	0.93
28	0.006428	0.79
32	0.005628	0.69
36	0.005000	0.62
40	0.004500	0.56
44	0.004091	0.51
48	0.003751	0.46
52	0.003461	0.43
56	0.003213	0.40
60	0.003000	0.37
64	0.002813	0.35
68	0.002647	0.33
72	0.002500	0.31
76	0.002369	0.29
80	0.002250	0.28
84	0.002143	0.26
88	0.002046	0.25
92	0.001956	0.24
96	0.001875	0.23
100	0.001800	0.22

CHAPTER IV

NUMERICAL APPROACH

The definite integral, Equation (2-14), is difficult to evaluate by ordinary methods. Thus we turn to the method of numerical integration, the process of computing the value of a definite integral from a set of numerical values of the integrand. If it is applied to the integration of the function of one variable, the process is frequently called the mechanical quadrature. The problem of numerical integration is solved by replacing the integrand by an interpolation function, and then integrating this function between the desired limits. The repeated use of a simple numerical integration formula involving many steps is appropriate, especially if a computer is available for the routine calculations. The accuracy of the result depends upon the ability of interpolating function to represent the integrand over the interval of integration. It is frequently desirable to investigate this fact before embarking upon extensive integration.

Out of the quadrature formulas in ordinary use, Simpson's Rule gives a more accurate approximation to the integral of the function, and is well adapted for use, especially with electronic computers, Moreover, Simpson's Rule is exact for second degree polynomial or lower, and it turns out to be exact even for third degree or lower. The error of approximating the integral diminishes much more rapidly than the trapezoidal method.

The characteristic approach of Simpson's method is as follows: Suppose we wish to evaluate a definite integral

$$I = f\ (x)\ dx$$

where f(x) is a known function but whose integral is either not easily evaluated or cannot conveniently be expressed in a closed form, then we can evaluate I by dividing the interval a and b into an even number, of 2n intervals of legnth, $h = \frac{b-a}{2n}$ as shown in Figure 6, finding the approximate area of each of the strips, and summing their areas to arrive at the value of the definite integral I between the limits a and b.

The value of the definite integral is given by,

$$\int_a^b f(x)\ dx = \frac{h}{3}\Big[f(x_o) + 4\ f(x_1) + 2\ f(x_2) + 4\ f(x_3) + 2\ f(x_4) + \text{------}$$

$$\text{--------} + 2\ f(x_{2n-2}) + 4\ f(x_{2n-1}) + f(x_{2n})\Big]$$

$$= \frac{h}{3}\Big[y_o + 4\ y_1 + 2\ y_2 + 4\ y_3 + 2\ y_4 + \text{------------}$$

$$\text{-------} + 2\ y_{n-4} + 4\ y_{n-3} + 2\ y_{n-2} + 4\ y_{n-1} + y_n\Big]$$

We make use of Simpson's Rule to evaluate our definite integral S, Where f(x) is the right hand side of Equation (2-14), by writing a computer program that computes S at different depths. The program written for this problem is included in the Appendix A and the results are shown in Table 2 (a-k).

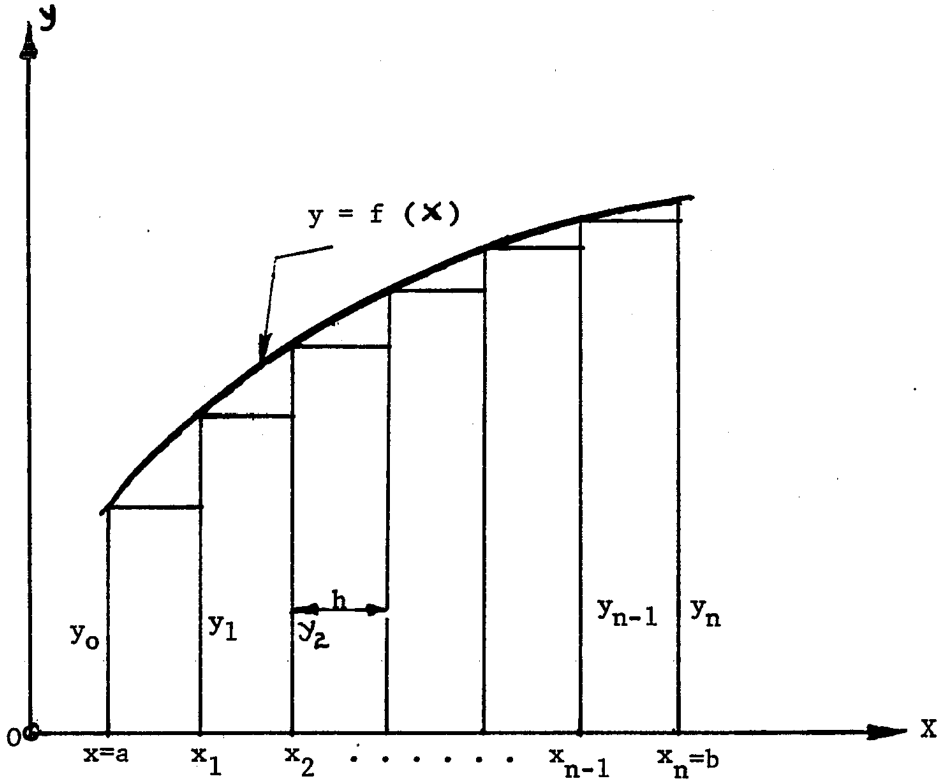

Fig. 6 Graphical Integration

TABLE 2 a

Mean Velocity: 2 ft./Second

Y (ft)	S (feet)																				
4	4.00	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000
8	8.00	8.001	8.001	8.001	8.001	8.001	8.001	8.001	8.001	8.001	8.001	8.001	8.001	8.001	8.001	8.001	8.001	8.001	8.001	8.001	8.001
12	12.002	12.002	12.002	12.002	12.002	12.002	12.002	12.002	12.002	12.002	12.002	12.002	12.002	12.002	12.002	12.002	12.002	12.002	12.002	12.001	12.001
16		16.003	16.003	16.003	16.003	16.003	16.003	16.003	16.003	16.003	16.003	16.003	16.002	16.002	16.002	16.002	16.002	16.002	16.002	16.002	16.002
20			20.004	20.004	20.004	20.004	20.004	20.004	20.004	20.004	20.004	20.003	20.003	20.003	20.003	20.003	20.003	20.003	20.003	20.003	16.003
24				24.005	24.005	24.005	24.005	24.005	24.005	24.004	24.004	24.004	24.004	24.004	24.004	24.004	24.004	24.004	24.004	24.004	24.004
28					28.006	28.006	28.005	28.005	28.005	28.005	28.005	28.005	28.005	28.005	28.005	28.005	28.005	28.005	28.005	28.005	28.005
32						32.007	32.006	32.006	32.006	32.006	32.006	32.006	32.006	32.006	32.006	32.006	32.006	32.006	32.006	32.005	32.005
36							36.008	36.007	36.007	36.007	36.007	36.007	36.007	36.007	36.007	36.007	36.007	36.007	36.006	36.006	36.006
40								40.008	40.008	40.008	40.008	40.008	40.008	40.008	40.008	40.007	40.007	40.007	40.007	40.007	40.007
44									44.01	44.01	44.01	44.01	44.01	44.01	44.01	44.01	44.008	44.008	44.007	44.007	44.008
48										48.01	48.01	48.01	48.01	48.01	48.01	48.01	48.009	48.009	48.009	48.01	48.01
52											52.01	52.01	52.01	52.01	52.01	52.01	52.01	52.01	52.01	52.01	52.01
56												56.01	56.01	56.01	56.01	56.01	56.01	56.01	56.01	56.01	56.01
60													60.01	60.01	60.01	60.01	60.01	60.01	60.01	60.01	60.01
64														64.01	64.01	64.01	64.01	64.01	64.01	64.01	64.01
68															68.01	68.01	68.01	68.01	68.01	68.01	68.01
72																72.01	72.01	72.01	72.01	72.01	72.01
76																	76.01	76.01	76.01	76.01	76.01
80																		80.02	80.02	80.01	80.01
84																			84.02	84.02	84.02
88																				88.02	88.02
92																				92.02	92.02
96																				96.02	96.02
100																					100.02

TABLE 2 b

Mean Velocity: 3 ft./Second

Y (ft)	S (feet)																			
4	4.003	4.003	4.003	4.003	4.003	4.003	4.003	4.003	4.003	4.003	4.003	4.003	4.003	4.003	4.003	4.003	4.002	4.002	4.002	4.002
8	8.008	8.007	8.007	8.007	8.007	8.007	8.007	8.006	8.006	8.006	8.006	8.006	8.006	8.006	8.006	8.006	8.006	8.006	8.005	8.005
12	12.01	12.01	12.01	12.01	12.01	12.01	12.01	12.01	12.01	12.01	12.01	12.01	12.01	12.01	12.01	12.01	12.01	12.01	12.01	12.01
16		16.02	16.02	16.02	16.02	16.01	16.01	16.01	16.01	16.01	16.01	16.01	16.01	16.01	16.01	16.01	16.01	16.01	16.01	16.01
20			20.02	20.02	20.02	20.02	20.02	20.02	20.02	20.02	20.02	20.02	20.02	20.02	20.02	20.02	20.01	20.01	20.01	20.01
24				24.03	24.02	24.02	24.02	24.02	24.02	24.02	24.02	24.02	24.02	24.02	24.02	24.02	24.02	24.02	24.02	24.02
28					28.03	28.03	28.03	28.03	28.03	28.03	28.03	28.03	28.02	28.02	28.02	28.02	28.02	28.02	28.02	28.02
32						32.03	32.03	32.03	32.03	32.03	32.03	32.03	32.03	32.03	32.03	32.03	32.03	32.03	32.03	32.03
36							36.04	36.04	36.04	36.04	36.04	36.04	36.03	36.03	36.03	36.03	36.03	36.03	36.03	36.03
40								40.04	40.04	40.04	40.04	40.04	40.04	40.04	40.03	40.03	40.04	40.04	40.04	40.03
44									44.05	44.05	44.04	44.04	44.04	44.04	44.04	44.04	44.04	44.04	44.04	44.04
48										48.05	48.05	48.05	48.05	48.05	48.05	48.05	48.05	48.05	48.04	48.04
52											52.06	52.05	52.05	52.05	52.05	52.05	52.05	52.05	52.05	52.05
56												56.06	56.06	56.06	56.06	56.06	56.05	56.05	56.05	56.05
60													60.06	60.06	60.06	60.06	60.06	60.06	60.05	60.06
64													64.07	64.07	64.07	64.06	64.07	64.06	64.06	64.06
68														68.07	68.07	68.07	68.07	68.07	68.06	68.06
72															72.08	72.08	72.07	72.07	72.07	72.07
76																76.08	76.08	76.08	76.08	76.07
80																	80.09	80.08	80.08	80.08
84																		84.09	84.09	84.08
88																			88.09	88.09
92																			92.10	92.10
96																			96.10	96.10
100																				100.11

TABLE 2 c

Mean Velocity: 4 ft./Second

Y (ft)	S (feet)																				
4	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01
8	8.02	8.02	8.02	8.02	8.02	8.02	8.02	8.02	8.02	8.02	8.02	8.02	8.02	8.02	8.02	8.02	8.02	8.02	8.02	8.02	8.02
12	12.04	12.04	12.04	12.04	12.04	12.03	12.03	12.03	12.03	12.03	12.03	12.03	12.03	12.03	12.03	12.03	12.03	12.03	12.03	12.03	12.03
16		16.06	16.05	16.05	16.05	16.05	16.05	16.05	16.05	16.05	16.04	16.04	16.04	16.04	16.04	16.04	16.04	16.04	16.04	16.04	16.04
20			20.07	20.07	20.06	20.06	20.06	20.06	20.06	20.06	20.06	20.06	20.05	20.05	20.05	20.05	20.05	20.05	20.05	20.05	20.05
24				24.08	24.08	24.08	24.08	24.07	24.07	24.07	24.07	24.07	24.07	24.07	24.07	24.07	24.07	24.07	24.06	24.06	24.06
28					28.10	28.10	28.09	28.09	28.09	28.09	28.08	28.08	28.08	28.08	28.08	28.08	28.08	28.08	28.08	28.08	28.07
32						32.11	32.11	32.11	32.10	32.10	32.10	32.10	32.10	32.10	32.10	32.10	32.09	32.09	32.09	32.09	32.09
36							36.12	36.12	36.12	36.12	36.11	36.11	36.11	36.11	36.11	36.11	36.11	36.11	36.11	36.10	36.10
40								40.14	40.14	40.14	40.13	40.13	40.12	40.12	40.12	40.12	40.12	40.12	40.12	40.12	40.12
44									44.15	44.15	44.14	44.14	44.14	44.14	44.14	44.14	44.14	44.14	44.13	44.13	44.13
48										48.17	48.16	48.16	48.15	48.15	48.15	48.15	48.15	48.15	48.15	48.15	48.14
52											52.18	52.17	52.17	52.17	52.17	52.17	52.16	52.16	52.16	52.16	52.16
56												56.19	56.18	56.18	56.18	56.18	56.18	56.18	56.18	56.17	56.17
60													60.20	60.20	60.20	60.19	60.19	60.19	60.19	60.19	60.19
64														64.22	64.22	64.21	64.21	64.21	64.21	64.20	64.20
68															68.24	68.23	68.23	68.22	68.22	68.21	68.21
72																72.25	72.24	72.24	72.24	72.23	72.23
76																	76.27	76.26	76.25	76.25	76.24
80																		80.28	80.27	80.26	80.26
84																			84.29	84.27	84.27
88																				88.29	88.29
92																				92.31	92.31
96																				96.34	96.33
100																					100.35

TABLE 2 d

Mean Velocity: 5 ft./Second

Y (ft)	S (feet)																				
4	4.03	4.03	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02
8	8.06	8.06	8.06	8.05	8.05	8.05	8.05	8.05	8.05	8.05	8.05	8.05	8.05	8.05	8.05	8.04	8.04	8.04	8.04	8.04	8.04
12	12.10	12.10	12.10	12.09	12.09	12.08	12.08	12.08	12.08	12.08	12.08	12.08	12.07	12.07	12.07	12.07	12.07	12.07	12.07	12.07	12.07
16		16.14	16.13	16.12	16.12	16.12	16.11	16.11	16.11	16.11	16.11	16.11	16.10	16.10	16.10	16.10	16.10	16.10	16.10	16.10	16.10
20			20.17	20.16	20.16	20.15	20.15	20.15	20.15	20.15	20.14	20.14	20.14	20.14	20.13	20.13	20.13	20.13	20.13	20.12	20.12
24				24.20	24.19	24.19	24.19	24.18	24.18	24.18	24.18	24.18	24.17	24.17	24.17	24.17	24.16	24.16	24.16	24.16	24.16
28					28.24	28.23	28.22	28.22	28.22	28.22	28.21	28.21	28.21	28.20	28.20	28.20	28.20	28.20	28.19	28.19	28.19
32						32.28	32.26	32.25	32.25	32.25	32.25	32.24	32.24	32.24	32.24	32.23	32.23	32.23	32.22	32.22	32.22
36							36.31	36.29	36.29	36.29	36.28	36.28	36.27	36.27	36.27	36.27	36.27	36.26	36.26	36.25	36.25
40								40.34	40.33	40.33	40.32	40.32	40.31	40.31	40.30	40.30	40.30	40.30	40.30	40.29	40.28
44									44.38	44.38	44.36	44.35	44.35	44.34	44.34	44.34	44.34	44.33	44.33	44.32	44.32
48										48.41	48.41	48.40	48.38	48.38	48.37	48.37	48.37	48.37	48.36	48.36	48.36
52											52.45	52.45	52.42	52.41	52.41	52.41	52.40	52.40	52.40	52.39	52.39
56												56.48	56.46	56.45	56.45	56.44	56.44	56.44	56.43	56.43	56.43
60													60.52	60.50	60.49	60.48	60.48	60.47	60.47	60.46	60.46
64														64.55	64.53	64.52	64.52	64.51	64.51	64.50	64.50
68															68.58	68.57	68.56	68.55	68.54	68.53	68.53
72																72.62	72.60	72.59	72.58	72.57	72.56
76																	76.65	76.64	76.62	76.60	76.60
80																		80.70	80.67	80.64	80.64
84																			84.72	84.68	84.68
88																				88.73	88.72
92																				92.77	92.76
96																				96.83	96.81
100																					100.86

TABLE 2 e

Mean Velocity: 6 ft./Second

Y (ft)	S (feet)																				
4	4.06	4.05	4.05	4.05	4.05	4.05	4.05	4.05	4.05	4.05	4.05	4.05	4.05	4.04	4.04	4.04	4.04	4.04	4.04	4.04	4.04
8	8.13	8.12	8.12	8.11	8.11	8.11	8.11	8.10	8.10	8.10	8.10	8.10	8.10	8.10	8.10	8.10	8.10	8.09	8.09	8.09	8.09
12	12.21	12.20	12.20	12.18	12.18	12.18	12.17	12.17	12.16	12.16	12.16	12.16	12.16	12.15	12.15	12.15	12.15	12.15	12.15	12.14	12.14
16		16.29	16.26	16.26	16.25	16.25	16.24	16.24	16.23	16.23	16.23	16.22	16.22	16.21	16.21	16.21	16.21	16.21	16.21	16.20	16.20
20			20.36	20.33	20.32	20.32	20.31	20.31	20.30	20.30	20.30	20.29	20.29	20.28	20.28	20.28	20.27	20.27	20.27	20.26	20.26
24				24.43	24.40	24.39	24.39	24.38	24.38	24.37	24.37	20.36	24.36	24.35	24.35	24.34	24.34	24.34	24.33	24.32	24.32
28					28.50	28.47	28.46	28.45	28.45	28.44	28.44	28.43	28.43	28.42	28.42	28.41	28.41	28.41	28.40	28.39	28.39
32						32.57	32.54	32.53	32.52	32.52	32.51	32.51	32.50	32.50	32.50	32.49	32.48	32.48	32.47	42.46	32.45
36							36.64	36.61	36.60	36.59	36.58	36.58	36.57	36.57	36.56	36.56	36.55	36.55	36.54	36.53	36.53
40								40.71	40.68	40.67	40.66	40.65	40.65	40.64	40.64	40.63	40.63	40.62	40.62	40.60	40.60
44									44.79	44.75	44.74	44.73	44.72	44.71	44.71	44.70	44.70	44.69	44.69	44.67	44.67
48										48.86	48.83	48.81	48.79	48.79	48.78	48.77	48.77	48.76	48.76	48.74	48.74
52											52.93	52.90	52.87	52.86	52.85	52.85	52.84	52.84	52.83	52.82	52.81
57												57.00	56.97	56.95	56.93	56.92	56.92	56.91	56.90	56.89	56.88
60													61.07	61.04	61.02	61.00	60.99	60.98	60.97	60.96	60.96
64														65.15	65.11	65.08	65.07	65.06	65.05	65.03	65.03
68															69.22	69.18	69.16	69.14	69.13	69.10	69.10
72																73.29	73.25	73.23	73.21	73.18	73.17
76																	77.36	77.32	77.30	77.25	77.25
80																		81.43	81.40	81.33	81.32
84																			85.50	85.42	85.40
88																				89.51	89.50
92																				93.60	93.58
96																				97.72	97.68
100																					101.79

TABLE 2 f

Mean Velocity: 7 ft./Second

Y (ft)	S (feet)																				
4	4.11	4.10	4.10	4.10	4.09	4.09	4.09	4.09	4.09	4.08	4.08	4.08	4.08	4.08	4.08	4.08	4.08	4.08	4.08	4.08	4.08
8	8.24	8.23	8.22	8.21	8.21	8.20	8.20	8.19	8.19	8.19	8.18	8.18	8.18	8.18	8.18	8.18	8.18	8.18	8.17	8.17	8.17
12	12.39	12.36	12.35	12.34	12.33	12.33	12.32	12.31	12.31	12.30	12.30	12.29	12.29	12.29	12.28	12.28	12.28	12.28	12.28	12.27	12.27
16		16.53	16.49	16.48	16.47	16.46	16.45	16.44	16.43	16.43	16.42	16.41	16.41	16.40	16.40	16.40	16.39	16.39	16.39	16.38	16.38
20			20.66	20.62	20.60	20.59	20.58	20.57	20.56	20.56	20.55	20.54	20.53	20.53	20.52	20.52	20.51	20.51	20.50	20.49	20.48
24				24.80	24.75	24.73	24.72	24.71	24.70	24.69	24.68	24.67	24.66	24.65	24.65	24.64	24.64	24.63	24.62	24.61	24.60
28					28.93	28.88	28.86	28.84	28.84	28.83	28.81	28.80	28.80	28.79	28.78	28.77	28.76	28.76	28.75	28.73	28.72
32						33.06	33.01	32.98	32.97	32.96	32.95	32.94	32.93	32.92	32.91	32.90	32.90	32.89	32.88	32.86	32.85
36							37.19	37.14	37.11	37.10	37.08	37.07	37.06	37.05	37.05	37.04	37.03	37.02	37.01	36.99	36.98
40								41.33	41.27	41.24	41.22	41.21	41.20	41.19	41.18	41.17	41.16	41.15	41.15	41.11	41.11
44									45.46	45.40	45.37	45.35	45.33	45.33	45.32	45.31	45.30	45.29	45.28	45.25	45.24
48										49.60	49.53	49.50	49.48	49.46	49.45	49.44	49.43	49.42	49.41	49.38	49.37
52											53.73	53.66	53.63	53.61	53.59	53.58	53.57	53.56	53.55	53.52	53.51
56												57.86	57.79	57.76	57.73	57.71	57.70	57.69	57.68	57.65	57.64
60													61.99	61.93	61.89	61.86	61.84	61.83	61.82	61.79	61.78
64														66.13	66.06	66.02	65.99	65.97	65.96	65.92	65.91
68															70.26	70.19	70.15	70.12	70.10	70.05	70.05
72																74.39	74.33	74.28	74.25	74.20	74.18
76																	78.53	78.46	78.41	78.34	78.32
80																		82.66	82.59	82.48	82.46
84																			86.79	86.64	86.61
88																				90.80	90.76
92																				94.98	94.94
96																				99.19	99.12
100																					103.32

TABLE 2 h

Mean Velocity: 8 ft./Second

Y (ft)	S (feet)																				
4	4.18	4.17	4.17	4.16	4.16	4.16	4.15	4.15	4.15	4.15	4.15	4.15	4.14	4.14	4.14	4.14	4.14	4.14	4.14	4.14	4.14
8	8.41	8.39	8.38	8.37	8.36	8.35	8.34	8.33	8.33	8.32	8.32	8.32	8.31	8.31	8.31	8.31	8.31	8.30	8.30	8.29	8.29
12	12.68	12.63	12.60	12.59	12.57	12.56	12.55	12.54	12.53	12.52	12.51	12.51	12.50	12.50	12.50	12.48	12.48	12.48	12.47	12.46	12.46
16		16.91	16.84	16.82	16.81	16.79	16.77	16.76	16.75	16.73	16.72	16.71	16.70	16.69	16.69	16.68	16.67	16.67	16.66	16.65	16.64
20			21.14	21.06	21.04	21.02	21.00	20.99	20.97	20.96	20.94	20.93	20.92	20.91	20.89	20.88	20.88	20.87	20.86	20.84	20.83
24				25.36	25.28	25.25	25.23	25.22	25.20	25.19	25.17	25.15	25.14	25.13	25.11	25.10	25.10	25.08	25.07	25.09	25.03
28					29.59	29.51	29.47	29.49	29.43	29.42	29.40	29.38	29.34	29.34	29.34	29.32	29.31	29.30	29.28	29.25	29.24
32						33.82	33.73	33.69	33.66	33.69	33.63	33.61	33.60	33.58	33.56	33.55	33.54	33.52	33.51	33.47	33.46
36							38.05	37.95	37.91	37.88	37.86	37.84	37.83	37.81	37.80	37.78	37.76	37.75	37.73	37.69	37.68
40								42.28	42.18	42.13	42.10	42.07	42.06	42.04	42.02	42.01	41.99	41.98	41.96	41.92	41.90
48									46.51	46.40	46.35	46.31	46.29	46.27	46.25	46,24	46.22	46.21	46.19	46.14	46.13
52										50.74	50.63	50.57	50.55	50.51	50.49	50.47	50.45	50.44	50.42	50.37	50.36
56											54.96	54.85	54.79	54.75	54.72	54.70	54.68	54.67	54.65	54.60	54.59
60												59.19	59.08	59.01	58.97	58.94	58.92	58.90	58.88	58.83	58.82
64													63.42	63.30	63.23	63.19	63.16	63.13	63.11	63.06	63.05
68														67.75	67.53	67.45	67.41	67.38	67.35	67.29	67.28
72															71.87	71.75	71.68	71.63	71.59	71.52	71.51
76																76.10	75.98	75.91	75.81	75.76	75.74
80																	80.33	80.21	80.13	80.00	79.98
84																		84.56	84.44	84.25	84.22
88																			88.79	88.52	88.47
92																				92.80	92.74
96																				97.12	97.03
100																				101.47	101.34
																					105.70

TABLE 2 j

Mean Velocity: 9 ft./Second

Y (ft)	S (feet)																				
4	4.29	4.28	4.27	4.26	4.25	4.25	4.25	4.24	4.24	4.24	4.24	4.24	4.24	4.23	4.23	4.23	4.23	4.23	4.23	4.23	4.23
8	8.66	8.64	8.61	8.59	8.57	8.56	8.55	8.54	8.53	8.52	8.52	8.51	8.51	8.50	8.50	8.49	8.49	8.49	8.49	8.48	8.48
12	13.10	13.01	12.98	12.95	12.93	12.91	12.89	12.87	12.85	12.84	12.82	12.81	12.81	12.80	12.80	12.78	12.77	12.77	12.76	12.75	12.75
16		17.47	17.36	17.32	17.30	17.27	17.25	17.22	17.20	17.18	17.16	17.15	17.13	17.12	17.10	17.10	17.08	17.07	17.06	17.04	17.04
20			21.83	21.71	21.67	21.65	21.62	21.60	21.57	21.54	21.52	21.50	21.48	21.46	21.44	21.43	21.41	21.40	21.49	21.35	21.34
24				26.20	26.07	26.02	25.99	25.96	25.94	25.91	25.88	25.86	25.84	25.81	25.80	25.77	25.75	25.74	25.72	25.68	25.66
28					30.57	30.43	30.37	30.33	30.31	30.28	30.25	30.23	30.20	30.18	30.15	30.13	30.11	30.09	30.07	30.00	30.00
32						34.94	34.79	34.72	34.68	34.65	34.62	34.60	34.57	34.55	34.52	34.50	34.47	34.45	34.42	34.36	34.34
36							39.31	39.15	39.07	39.03	39.00	38.97	38.94	38.92	38.90	38.87	38.84	38.81	38.79	38.72	38.70
40								43.67	43.51	43.43	43.37	43.34	43.31	43.29	43.26	43.24	43.21	43.18	43.15	43.08	43.06
44									48.04	47.87	47.78	47.72	47.69	47.66	47.63	47.61	47.58	47.55	47.52	47.45	47.42
48										52.41	52.23	52.13	52.08	52.04	52.00	51.98	51.95	51.92	51.90	51.82	51.80
52											56.77	56.59	56.49	56.43	56.39	56.45	56.32	56.30	56.27	56.19	56.17
56												61.14	60.96	60.85	60.78	60.74	60.70	60.67	60.64	60.56	60.53
60													65.51	65.33	65.21	65.14	65.09	65.05	65.01	64.93	64.90
64														69.98	69.70	69.57	69.50	69.44	69.40	69.30	69.28
68															74.25	74.05	73.93	75.85	73.79	73.68	73.65
72																78.61	78.42	78.29	78.20	78.06	78.02
76																	82.98	82.78	82.65	82.45	82.40
80																		87.35	87.15	86.85	86.80
84																			91.72	91.28	91.20
88																				95.74	95.64
92																				100.24	100.01
96																				104.82	104.61
100																					109.18

TABLE 2 k

Mean Velocity: 10 ft./Second

Y (ft)	S (feet)																				
4	4.45	4.43	4.41	4.40	4.39	4.38	4.38	4.38	4.37	4.37	4.36	4.36	4.36	4.36	4.36	4.36	4.36	4.35	4.35	4.35	4.35
8	9.01	8.98	8.93	8.91	8.88	8.86	8.84	8.83	8.81	8.80	8.79	8.78	8.77	8.77	8.76	8.76	8.75	8.75	8.75	8.73	8.73
12	13.69	13.55	13.50	13.46	13.42	13.39	13.36	13.34	13.30	13.29	13.27	13.25	13.23	13.22	13.21	13.20	13.19	13.18	13.17	13.15	13.15
16		18.26	18.08	18.03	17.99	17.95	17.91	17.90	17.84	17.81	17.78	17.76	17.73	17.71	17.70	17.68	17.66	17.65	17.63	17.60	17.59
20			22.82	22.63	22.56	22.53	22.48	22.48	22.40	22.37	22.33	22.29	22.26	22.24	22.21	22.19	22.17	22.14	22.13	22.07	22.06
24				27.39	27.17	27.10	27.05	27.03	26.97	26.93	26.89	26.85	26.82	26.78	26.75	26.72	26.70	26.66	26.64	26.57	26.55
28					31.95	31.73	31.64	31.63	31.94	31.50	31.46	31.42	31.38	31.34	31.31	31.27	31.24	31.20	31.17	31.09	31.07
32						36.52	36.28	36.28	36.12	36.07	36.02	36.00	35.95	35.91	35.87	35.83	35.80	35.76	35.72	35.63	35.60
36							41.08	41.07	40.71	40.65	40.60	40.55	40.52	40.48	40.44	40.40	40.36	40.32	40.28	40.17	40.14
40								45.40	45.38	45.26	45.18	45.13	45.09	45.05	45.00	44.97	44.93	44.87	44.85	44.73	44.70
48									50.21	49.95	49.80	49.72	49.66	49.62	49.58	49.54	49.50	49.45	49.42	49.30	49.26
52										54.78	54.50	54.35	54.26	54.20	54.15	54.10	54.07	54.02	53.98	53.86	53.82
56											59.34	59.05	58.90	58.80	58.74	58.68	58.64	58.69	58.55	58.43	58.35
60												63.90	63.62	63.45	63.35	63.27	63.22	63.17	63.12	63.00	62.96
64													68.47	68.18	68.00	67.89	67.82	67.75	67.70	67.57	67.53
68														73.04	72.74	72.56	72.44	72.35	72.28	72.14	72.10
72															77.60	77.30	77.11	76.98	76.89	76.72	76.67
76																82.16	81.86	81.66	81.53	81.30	81.25
80																	86.73	86.42	86.22	85.90	85.83
84																		91.29	90.98	90.52	90.44
88																			95.86	95.18	95.06
92																				99.89	99.73
96																				104.67	104.44
100																				109.55	109.23
																					114.12

CHAPTER V

REGRESSION MODELS

From Table 2 (a-k) of the previous Chapter, it may be seen that at Y_i, $i = 1, \text{------}, k$, we have n_i observations s_{iv} , $v = 1, \text{---}, n_i$. It appears appropriate to fit a regression line of s, on Y. A plot of Y, and the mean of the corresponding distribution of s_{iv} , indicates that their relation is very nearly exact, and the data may be approximated well by a straight line passing through the origin. The plot also indicated that one regression line cannot fit all the data, and several regression Models are necessary to fit into the data. In this Chapter interest is centered on establishing regression Models, and to measure the degree to which the variables are associated in each Model by correlation analysis. Using these Models, the wet line corrections for different wet line lengths will be established.

The estimated Regression Equation is, $s = b\,Y$

The standard method of estimation of parameters in regression is the method of Least Squares; this is to use those values of b, which will minimize the sum of the squares of deviations, R, between the observed values s_{iv}, and the predictions of s_i in the estimated Equation. As far as the estimation of parameters is concerned, it does not require the assumption of normality. The method of Least Squares has the desirable properties that the estimates it gives are unbiased, and all unbiased linear estimates have the minimum variance.

The sum of squares of deviations between the observed values of s_{iv}, and the predicted values of s_i is,

$$R = \sum_{i}^{K} \sum_{v}^{n_i} (s_{iv} - s_i)^2$$

$$= \sum_{i}^{K} \sum_{v}^{n_i} (s_{iv} - b\,Y_i)^2$$

To find the value of b that minimizes R, we differentiate R with respect to b, and equate to zero.

$$\frac{\partial R}{\partial b} = 2 \sum_{i}^{K} \sum_{v}^{n_i} (s_{iv} - b\,Y_i)\,(-\,Y_i) = 0$$

$$\sum_{i}^{K} \sum_{v}^{n_i} b\,Y_i^2 - \sum_{i}^{K} \sum_{v}^{n_i} s_{iv}\,Y_i = 0$$

$$b \sum_{i}^{K} \sum_{v}^{n_i} Y_i^2 = \sum_{i}^{K} \sum_{v}^{n_i} s_{iv}\,Y_i$$

$$b = \frac{\sum_{i}^{K} \sum_{v}^{n_i} s_{iv}\,Y_i}{\sum_{i}^{K} \sum_{v}^{n_i} Y_i^2}$$

$$b = \frac{\sum_{i}^{K} \sum_{v}^{n_i} s_{iv}\,Y_i}{\sum_{i}^{K} n_i\,Y_i^2} \qquad \text{(Eqn 8)}$$

The values of b, for different model ranges, are reported in Appendix B. The Regression Models for these model ranges are shown in Table 3.

Using these Models, the wet line corrections are computed and included in Table 4.

Correlation Coefficient

Having established the Regression Models, it is necessary to analyse how clesely the variables s_i and Y_i are associated. The measure of the degree of association is called the correlation coefficient. For our work it is given by:

$$r = \frac{\sum_{i}^{k} \sum_{v}^{n_i} s_{iv} Y_i}{\sqrt{\left(\sum_{i}^{k} n_i Y_i^2\right)\left(\sum_{i}^{k}\sum_{v}^{n_i} s^2_{iv}\right)}}$$

The Correlation Coefficients for different Regression Models are shown in Table 5. From this Table it is readily seen that all the models have the values of r, which are close to one.

TABLE 3

REGRESSION MODELS

MODEL RANGE (feet)	MEAN VELOCITY IN FEET PER SECOND				
	1	2	3	4	5
1-20	S=Y	S=1.0001452 Y	S=1.0008188 Y	S=1.0030085 Y	S=1.00725 Y
20-40	S=Y	S=1.0001909 Y	S=1.0009312 Y	S=1.0031063 Y	S=1.00769 Y
40-60	S=Y	S=1.0001859 Y	S=1.0009780 Y	S=1.0032306 Y	S=1.00802 Y
60-80	S=Y	S=1.00016 Y	S=1.0010153 Y	S=1.0033297 Y	S=1.00819 Y
80-92	S=Y	S=1.0003101 Y	S=1.0016927 Y	S=1.003337 Y	S=1.00829 Y
92-100	S=Y	S=1.0003101 Y	S=1.0016927 Y	S=1.0034932 Y	S=1.00856 Y

TABLE 3 (Continued)

REGRESSION MODELS

MODEL RANGE (feet)	MEAN VELOCITY IN FEET PER SECOND				
	6	7	8	9	10
1-20	S=1.01486 Y	S=1.02743 Y	S=1.04656 Y	S=1.0748 Y	S=1.11334 Y
20-40	S=1.01587 Y	S=1.02932 Y	S=1.04879 Y	S=1.08096 Y	S=1.12322 Y
40-60	S=1.01655 Y	S=1.03022 Y	S=1.05241 Y	S=1.08438 Y	S=1.12979 Y
60-80	S=1.01701 Y	S=1.03155 Y	S=1.05397 Y	S=1.08697 Y	S=1.13342 Y
80-92	S=1.01703 Y	S=1.03155 Y	S=1.05469 Y	S=1.08842 Y	S=1.13535 Y
92-100	S=1.01777 Y	S=1.03298 Y	S=1.05654 Y	S=1.09113 Y	S=1.14000 Y

TABLE 4

WET LINE CORRECTIONS

WET LINE LENGTH (feet)	MEAN VELOCITY IN FEET PER SECOND									
	1	2	3	4	5	6	7	8	9	10
8	INSIGNIFICANT	0.0010	0.0065	0.024	0.060	0.117	0.213	0.356	0.556	0.814
9		0.0013	0.0073	0.027	0.065	0.131	0.240	0.400	0.626	0.916
10		0.0014	0.008	0.030	0.072	0.146	0.267	0.445	0.696	0.018
11		0.0016	0.009	0.033	0.079	0.161	0.293	0.489	0.765	0.120
12		0.0017	0.010	0.036	0.086	0.175	0.320	0.533	0.835	1.221
13		0.0018	0.010	0.039	0.093	0.190	0.347	0.578	0.904	1.323
14		0.0020	0.011	0.042	0.100	0.204	0.373	0.623	0.974	1.425
15		0.0021	0.012	0.045	0.108	0.219	0.400	0.667	1.044	1.527
16		0.0023	0.013	0.048	0.115	0.234	0.427	0.712	1.113	1.628
17		0.0025	0.014	0.051	0.122	0.249	0.454	0.756	1.183	1.730
18		0.0026	0.015	0.054	0.129	0.263	0.480	0.800	1.252	1.832
19		0.0027	0.0155	0.057	0.136	0.278	0.507	0.845	1.322	1.934
20		0.0029	0.016	0.060	0.144	0.293	0.534	0.889	1.391	2.036

TABLE 4 (Continued)

WET LINE CORRECTIONS

WET LINE LENGTH (feet)	MEAN VELOCITY IN FEET PER SECOND									
	1	2	3	4	5	6	7	8	9	10
21	INSIGNIFICANT	0.0030	0.017	0.063	0.151	0.310	0.561	0.934	1.461	2.137
22		0.0043	0.020	0.068	0.163	0.332	0.587	0.978	1.647	2.413
23		0.0046	0.021	0.071	0.175	0.359	0.634	1.069	1.722	2.523
24		0.0048	0.022	0.074	0.183	0.375	0.683	1.116	1.797	2.632
25		0.0049	0.023	0.077	0.191	0.390	0.712	1.163	1.872	2.742
26		0.0051	0.024	0.081	0.198	0.406	0.741	1.209	1.947	2.852
27		0.0053	0.025	0.084	0.206	0.421	0.769	1.256	2.022	2.962
28		0.0055	0.026	0.087	0.213	0.437	0.797	1.302	2.10	3.071
29		0.0057	0.027	0.090	0.221	0.453	0.826	1.349	2.171	3.181
30		0.0059	0.028	0.093	0.229	0.468	0.854	1.395	2.246	3.291
31		0.0061	0.029	0.096	0.236	0.484	0.883	1.442	2.322	3.400
32		0.0063	0.029	0.10	0.244	0.500	0.911	1.488	2.396	3.510
33		0.0065	0.030	0.102	0.252	0.515	0.940	1.535	2.471	3.620
34		0.0067	0.031	0.105	0.259	0.531	0.968	1.581	2.546	3.729

TABLE 4 (Continued)

WET LINE CORRECTIONS

WET LINE LENGTH (feet)	MEAN VELOCITY IN FEET PER SECOND									
	1	2	3	4	5	6	7	8	9	10
35		0.0067	0.032	0.108	0.267	0.546	0.997	1.628	2.621	3.839
36		0.0068	0.033	0.111	0.274	0.562	1.025	1.675	2.696	3.949
37		0.0071	0.034	0.114	0.282	0.578	1.054	1.721	2.771	4.058
38		0.0072	0.035	0.117	0.290	0.593	1.082	1.767	2.846	4.168
39		0.0074	0.036	0.120	0.297	0.609	1.111	1.814	2.921	4.278
40		0.0076	0.037	0.124	0.305	0.624	1.139	1.861	2.996	4.388
41		0.0078	0.038	0.127	0.313	0.647	1.167	1.910	3.070	4.497
42	INSIGNIFICANT	0.0081	0.041	0.135	0.327	0.670	1.20	2.091	3.268	4.824
43		0.0082	0.042	0.138	0.335	0.70	1.261	2.141	3.346	4.939
44		0.0084	0.043	0.141	0.350	0.716	1.290	2.191	3.424	5.054
45		0.0086	0.044	0.145	0.358	0.732	1.320	2.241	3.501	5.169
46		0.0088	0.0449	0.148	0.366	0.748	1.349	2.291	3.579	5.284
47		0.0089	0.046	0.151	0.374	0.765	1.378	2.340	3.657	5.399
48		0.0091	0.047	0.154	0.381	0.781	1.408	2.390	3.735	5.514

TABLE 4 (Continued)

WET LINE CORRECTIONS

WET LINE LENGTH (feet)	MEAN VELOCITY IN FEET PER SECOND									
	1	2	3	4	5	6	7	8	9	10
49	INSIGNIFICANT	0.0093	0.048	0.157	0.390	0.797	1.437	2.440	3.812	5.629
50		0.0095	0.049	0.160	0.398	0.814	1.466	2.490	3.890	5.743
51		0.0097	0.050	0.164	0.405	0.830	1.496	2.540	3.968	5.858
52		0.0099	0.051	0.167	0.414	0.846	1.525	2.589	4.046	5.973
53		0.0100	0.052	0.171	0.421	0.863	1.554	2.639	4.124	6.088
54		0.0103	0.053	0.174	0.429	0.879	1.584	2.689	4.201	6.203
55		0.0105	0.054	0.177	0.437	0.895	1.613	2.739	4.279	6.318
56		0.0107	0.0547	0.180	0.445	0.912	1.642	2.788	4.357	6.433
57		0.0109	0.055	0.183	0.453	0.928	1.672	2.838	4.435	6.548
58		0.0110	0.056	0.187	0.461	0.944	1.701	2.888	4.513	6.663
59		0.0112	0.058	0.190	0.469	0.960	1.730	2.938	4.591	6.777
60		0.0114	0.0586	0.193	0.477	0.976	1.760	2.988	4.668	6.892
61		0.0116	0.061	0.196	0.485	0.993	1.789	3.037	4.746	7.007

TABLE 4 (Continued)

WET LINE CORRECTIONS

WET LINE LENGTH (feet)	MEAN VELOCITY IN FEET PER SECOND									
	1	2	3	4	5	6	7	8	9	10
62	INSIGNIFICANT	0.0118	0.063	0.205	0.500	1.023	0.818	3.174	4.961	7.300
63		0.0120	0.064	0.210	0.510	1.054	1.926	3.226	5.041	7.410
64		0.0122	0.065	0.212	0.520	1.070	1.957	3.277	5.121	7.530
65		0.0124	0.066	0.216	0.528	1.087	1.990	3.328	5.20	7.651
66		0.0126	0.067	0.219	0.536	1.103	2.018	3.379	5.280	7.769
67		0.0128	0.068	0.222	0.544	1.120	2.049	3.431	5.361	7.886
68		0.0130	0.0689	0.225	0.552	1.137	2.079	3.482	5.440	8.004
69		0.0132	0.067	0.229	0.560	1.154	2.110	3.533	5.521	8.122
70		0.0134	0.071	0.232	0.568	1.170	2.141	3.584	5.601	8.240
71		0.0135	0.072	0.235	0.576	1.187	2.171	3.635	5.681	8.357
72		0.0137	0.073	0.239	0.584	1.204	2.202	3.686	5.761	8.475
73		0.0139	0.074	0.242	0.593	1.220	2.232	3.738	5.840	8.593
74		0.0141	0.075	0.245	0.601	1.237	2.263	3.789	5.921	8.710

TABLE 4 (Continued)

WET LINE CORRECTIONS

WET LINE LENGTH (feet)	MEAN VELOCITY IN FEET PER SECOND									
	1	2	3	4	5	6	7	8	9	10
75	INSIGNIFICANT	0.0143	0.076	0.249	0.609	1.254	2.294	3.840	6.000	8.829
76		0.0145	0.077	0.252	0.617	1.271	2.324	3.891	6.081	8.946
77		0.0147	0.078	0.255	0.625	1.287	2.355	3.942	6.161	9.064
78		0.0149	0.079	0.259	0.633	1.304	2.385	3.994	6.241	9.181
79		0.0151	0.080	0.262	0.641	1.321	2.416	4.045	6.321	9.299
80		0.0152	0.081	0.265	0.650	1.338	2.446	4.096	6.401	9.417
81		0.0251	0.136	0.269	0.661	1.355	2.477	4.147	6.481	9.534
82		0.0254	0.138	0.272	0.670	1.373	2.507	4.252	6.661	9.775
83		0.0257	0.140	0.276	0.678	1.390	2.538	4.303	6.742	9.894
84		0.026	0.142	0.279	0.690	1.406	2.569	4.355	6.824	10.014
85		0.0263	0.144	0.282	0.698	1.423	2.599	4.407	6.905	10.133
86		0.0266	0.145	0.285	0.707	1.440	2.630	4.459	6.986	10.252
87		0.0269	0.147	0.289	0.715	1.457	2.661	4.511	7.067	10.371

TABLE 4 (Continued)

WET LINE CORRECTIONS

WET LINE LENGTH (feet)	MEAN VELOCITY IN FEET PER SECOND									
	1	2	3	4	5	6	7	8	9	10
88	INSIGNIFICANT	0.0273	0.149	0.292	0.723	1.473	2.691	4.563	7.148	10.490
89		0.0275	0.150	0.296	0.732	1.490	2.722	4.615	7.230	10.610
90		0.0279	0.152	0.299	0.739	1.501	2.752	4.666	7.311	10.729
91		0.0282	0.154	0.302	0.748	1.523	2.783	4.718	7.392	10.848
92		0.0285	0.155	0.320	0.756	1.540	2.814	4.770	7.474	10.967
93		0.0288	0.157	0.324	0.764	1.567	2.844	4.822	7.555	11.086
94		0.0291	0.159	0.327	0.772	1.607	2.874	5.030	7.852	11.543
95		0.0294	0.160	0.331	0.781	1.650	2.905	5.083	7.934	11.666
96		0.0297	0.162	0.334	0.814	1.676	3.065	5.137	8.017	11.789
97		0.0300	0.164	0.337	0.823	1.693	3.097	5.191	8.101	11.912
98		0.0304	0.166	0.341	0.831	1.711	3.128	5.244	8.184	12.035
99		0.0306	0.167	0.344	0.840	1.730	3.160	5.298	8.268	12.157
100		0.0310	0.169	0.348	0.850	1.745	3.192	5.351	8.351	12.280

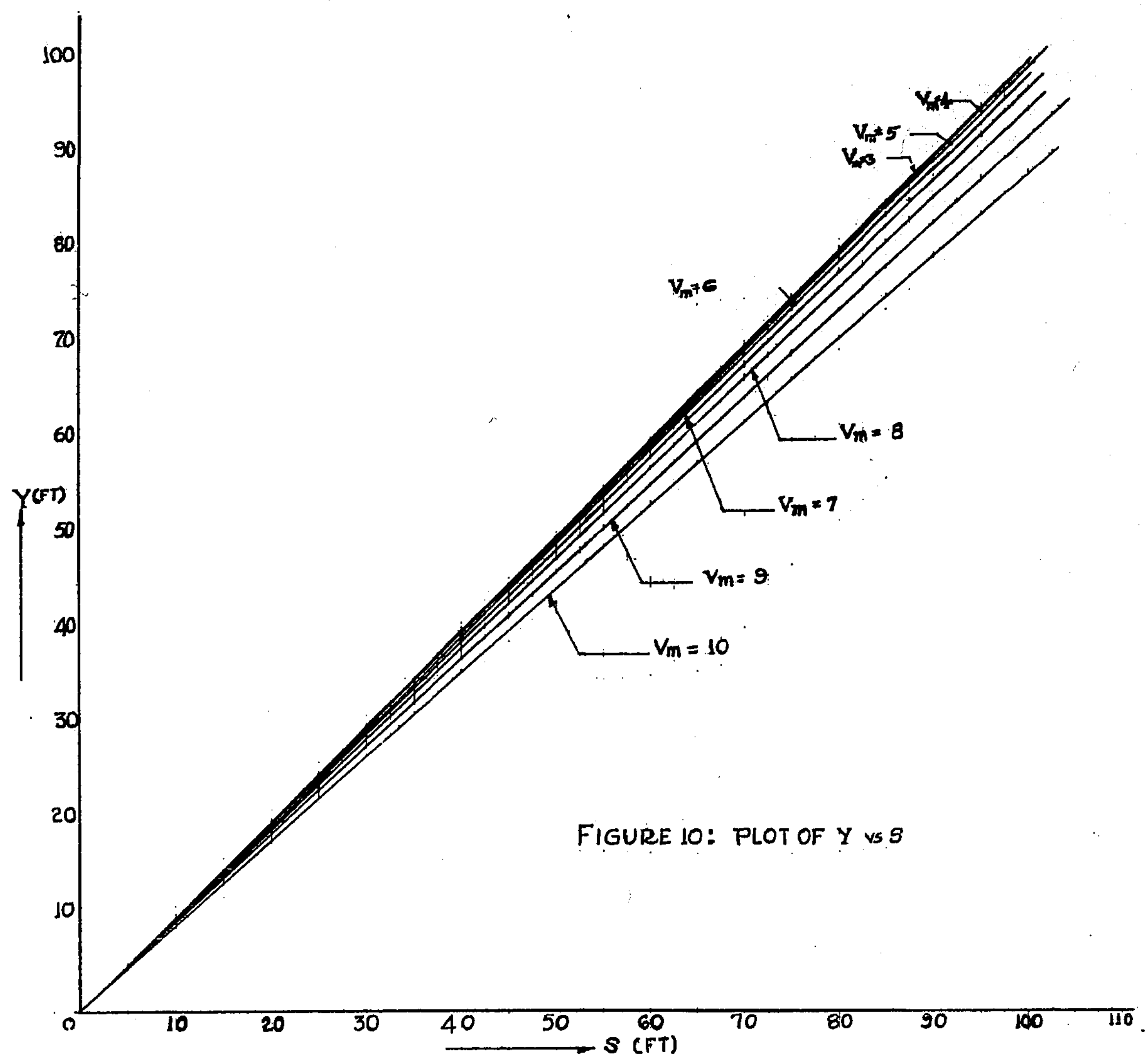

FIGURE 10: PLOT OF Y vs S

TABLE 5

Correlation Coefficients

Mean Vel. ft./Sec.	Model Range in feet 1-20	20-40	40-60	60-88	80-92	92-100
2	0.9997	0.9997	0.9990	1.000	0.9990	0.9990
3	0.9996	1.0000	0.9999	0.9997	0.9996	0.9996
4	0.9999	0.9991	0.9998	0.9998	0.9997	1.0000
5	0.9996	1.0000	0.9996	1.0000	0.9998	1.0000
6	1.0000	0.9995	0.9999	0.9817	1.0000	0.9998
7	1.0000	0.9998	0.9997	0.9997	0.9998	1.0000
8	0.9982	0.9991	1.0000	0.9883	0.9998	1.0000
9	1.0000	0.9998	0.9997	1.0000	1.0000	1.0000
10	0.9998	0.9997	1.0000	0.9997	1.0000	0.9997

CHAPTER VI

TEST RESULTS

Equipment

The tests were conducted to collect data, lending some support to the theoretical wet-line corrections.

The installation comprised a self-contained circulating water system. Water was supplied from the Laboratory sump by a 2 inch I. D. Steel pipe. The sump is 30 feet long, 15 feet wide, and 4.5 feet deep. A Worthington centrifugal pump driven by a variable speed General Electric motor was used.

The flume was of plexiglass, six feet long, one foot high and mounted inside a flume twelve feet long and 18" high to facilitate the out flow into a cylindrical receptical which returned the discharge to the sump. A schematic view of the flume is shown in Figure 7 and photographically as Figure 8.

Water was admitted to the flume through a vertical down pipe. Certain appurtenances were used to damp out any disturbances, such as turbulence and shock that developed. Slightly downstream, three groups of copper tubes, four inches long, ½ inch in diameter, one behind the other, were positioned to straighten the flow, and lessen or eliminate any surface irregularity that occured. In addition, to aid in the damping action, a wire mesh was placed in front of the copper tube system. This scheme proved to be quite effective. There was almost no surface waviness, and water filaments were virtually a straight line through the entire flume length.

The flow rate was metered by a 1.654 inch bore, flat orifice plate. The desired flow rate Q, was obtained from readings of water and mercury manometers that have been previously calibrated by standard weighing methods. The calibration curves for both mercury and water manometers are shown in the Appendix A.

A pitot tube was used for velocity measurements. It was incorporated in the flume at the end of working section to determine the velocity profile along a vertical section. The velocities are generally great enough to create an adequate velocity head in the manometer. Two different diameter probe openings were used. It was found that both these probe diameters (0.025 in. and 0.0625 in.) gave essentially the same readings. The average velocity was compared with that of the measured flow rate. A reasonable agreement (See Table 7, Appendix A) was obtained for the average velocity from pitot tube measurements that gave slightly higher results over that obtained from the measured flow. The velocity profiles for various flow rates are shown in Figure 9.

The wire of 0.01 inch diameter was used. It was suspended into the flow in the plane of the velocity measurement along with a cylindrical piece of lead, and subjected to the drag of water in the flume. This piece of lead was made to resemble approximately the shape of its prototype.

For each run, the wet line length was measured by lowering the cylindrical lead weight so that its bottom was at the water surface and noting the additional length of wire that had to be paid out to lower the lead weight to the bottom of the flume, and at intermediate points. The true vertical depth was measured by a scale fixed to the wall of the flume. After reading the manometers, the pitot-tube, the depth was increased by increasing the discharge at which time the entire cycle was repeated. The discharge was incrementally increased until maximum pump capacity was obtained.

The results of the suspension wire tests have been included in Appendix A and the Experimental wet line corrections are compared with that obtained by theoretical investigations in Table 6.

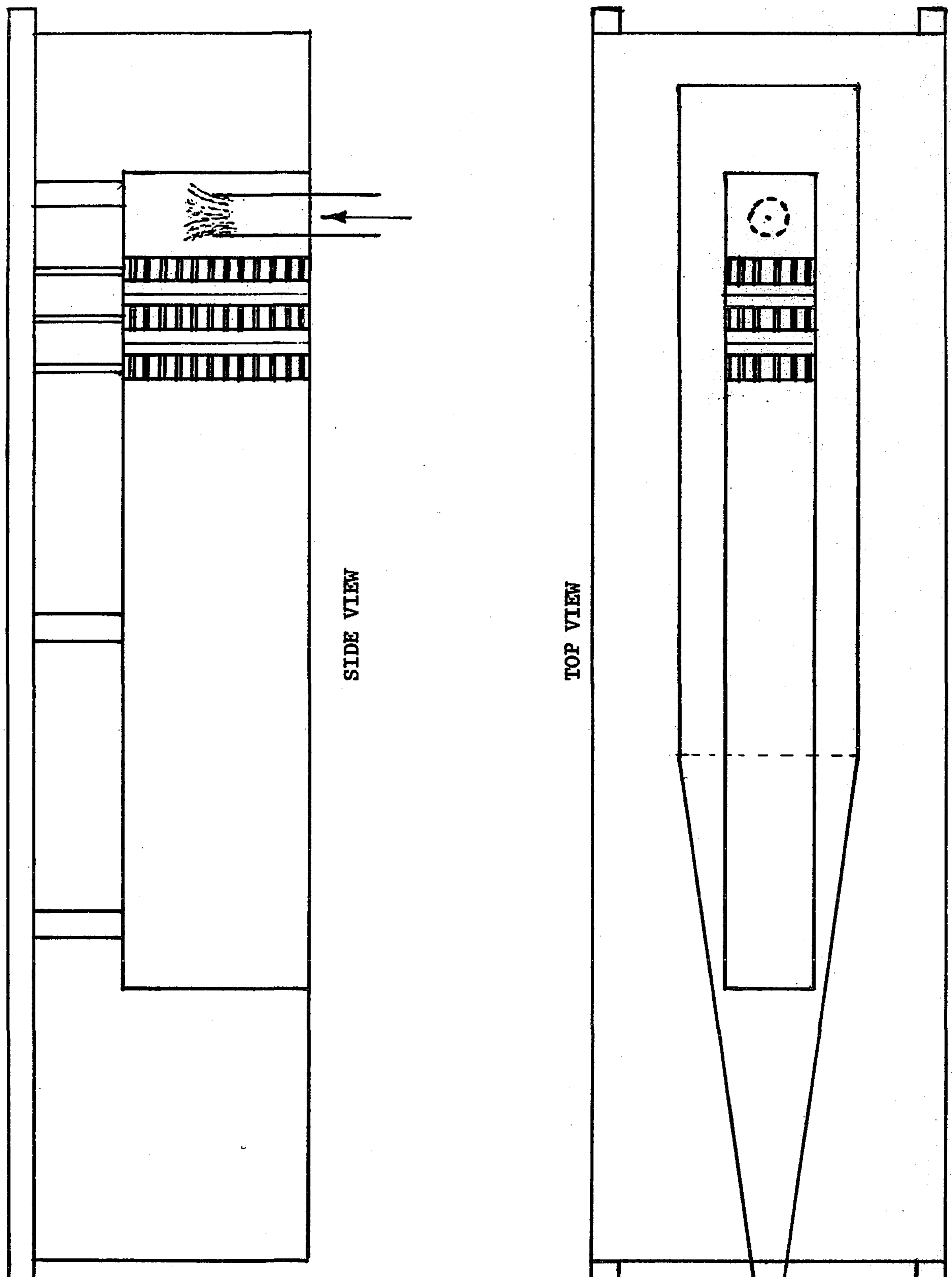

Fig. 8a Experimental Channel

Fig. 8 (b) Experimental Channel

VELOCITY PROFILES

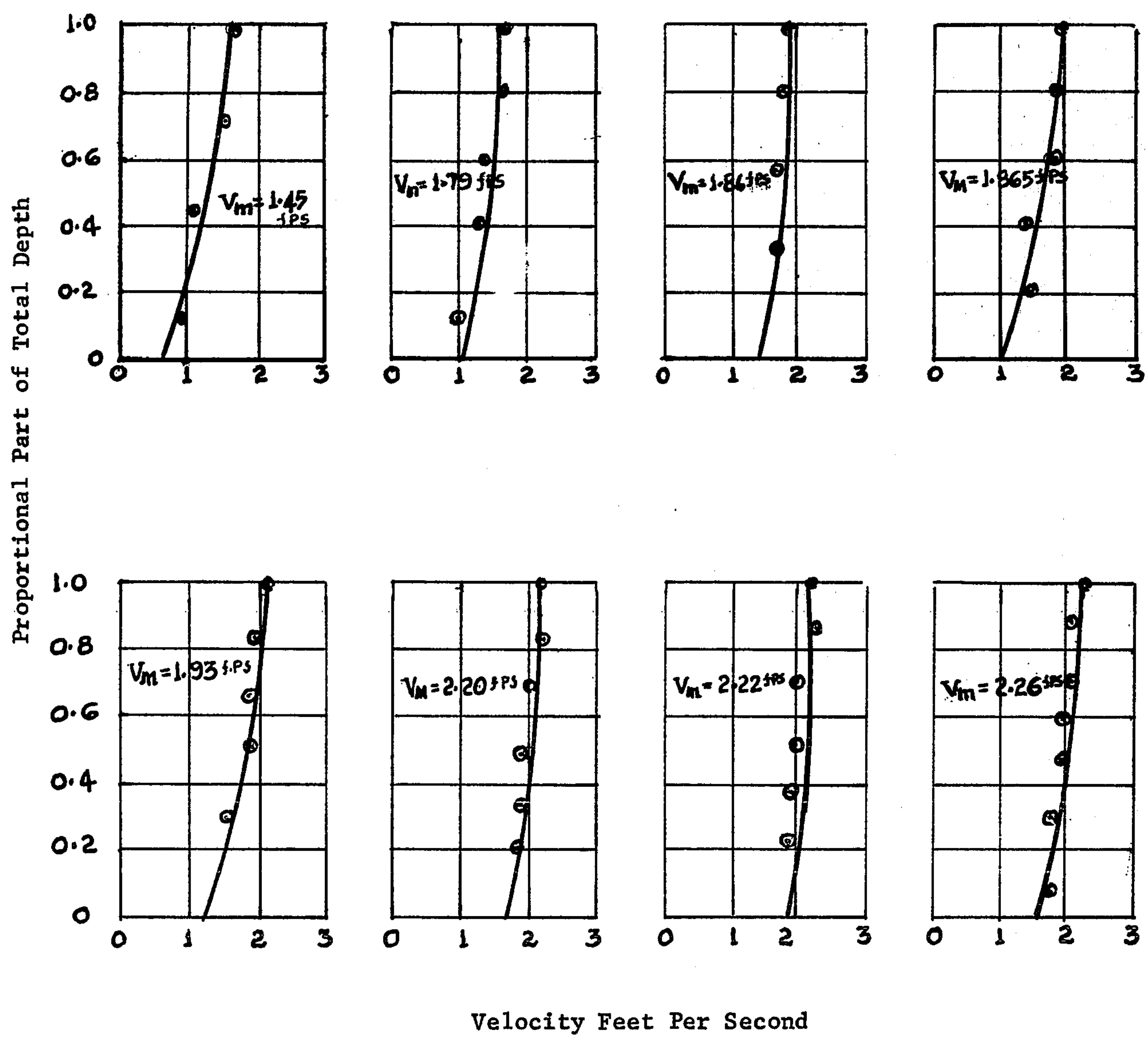

Fig. 9 Velocity Profiles

TABLE 6

THEORETICAL AND EXPERIMENTAL WET LINE

CORRECTIONS

D ft.	Y ft.	S ft.	Wet line corrections Theoretical	Experimental
2.917	2.917	2.942	0.020	0.026
	2.083	2.109	0.015	0.026
3.333	3.333	3.411	0.052	0.078
	2.500	2.552	0.040	0.052
	1.666	1.690	0.025	0.024
3.85	3.850	3.932	0.060	0.082
	3.020	3.073	0.045	0.053
	2.180	2.210	0.032	0.030
4.166	4.166	4.244	0.062	0.080
	3.333	3.385	0.049	0.059
	2.500	3.062	0.037	0.052
	1.666	1.692	0.025	0.026
4.583	4.583	4.661	0.078	0.078
	3.750	3.802	0.055	0.052
	2.916	2.942	0.043	0.036
	2.080	2.109	0.031	0.030
5.0	5.000	5.102	0.137	0.103
	4.160	4.245	0.110	0.085
	3.330	3.411	0.091	0.081
	2.500	2.550	0.068	0.050

TABLE 6 (Continued)

D ft.	Y ft.	S ft.	Wet line corrections Theoretical	Experimental
5.885	5.885	5.963	0.110	0.078
	5.052	5.130	0.138	0.078
	4.218	4.271	0.115	0.053
	3.385	3.437	0.093	0.052
	2.552	2.580	0.070	0.030

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

A theoretical formula has been developed which describes the relation between the length of wet line paid out, and the true depth. The method is particularly well suited for machine calculations, and reliable solutions can be obtained. General applicability seems justified in view of the correspondence between the observed and computed values of corrections for inclinations. However, the accuracy of theoretical wet line corrections has been purposely underestimated because of lack of direct correlation between the drag and velocity. Consequently, the wet line corrections in Table 4 are merely meant to be indicative of the procedure in applying the formula developed. Although some progress has been made over the years, more work needs to be done on this very important drag-velocity relationship which is significant in a number of problems of Engineering importance. Although the Test device provided a measure of wet line length, an error in the measurement of its magnitude was always involved because of the oscillations of the wire.

The inclination corrections are presented rather to show that the Equation thus derived provides a method by which the true depth may be computed knowing the wet line paid out. The real significance of the results obtained lies in the fact that a systematic evaluation of the formula can be undertaken with the aid of digital computers. The Equation thus derived may be applied to any problem of this nature.

Recommendations

As has been pointed out earlier, some minor aspects of the problem merit further research. Notably among these are:

(1) The weight of the cable and its effect on the mathematical model.
(2) The fabrication configurational twist and the stiffness in the wire.
(3) The force of bouyancy.
(4) The frictional resistance arising out of the component of velocity parallel to the wire.

If these were included, the resulting equation would indeed look formidable, and an analytical solution would be very difficult. Nevertheless, the matter of discipline and application may open the doors to solve such an equation by future investigators, utilizing the advantages furnished by computers.

A proper correlation between the drag and velocity has never been properly defined, although considerable effort has been expended by many investigators. Since the Drag force will always be involved in a problem of this nature, it is suggested that the Drag-velocity relation on suspension wires should be studied both analytically and experimentally. This work will have to be done before we can have a complete understanding of the problem.

In the Equation (2-14) the tension T is approximated to be equal to the magnitude of meter and the sounding weight. In the future investigation, a much better way of evaluating this T is needed in the problem.

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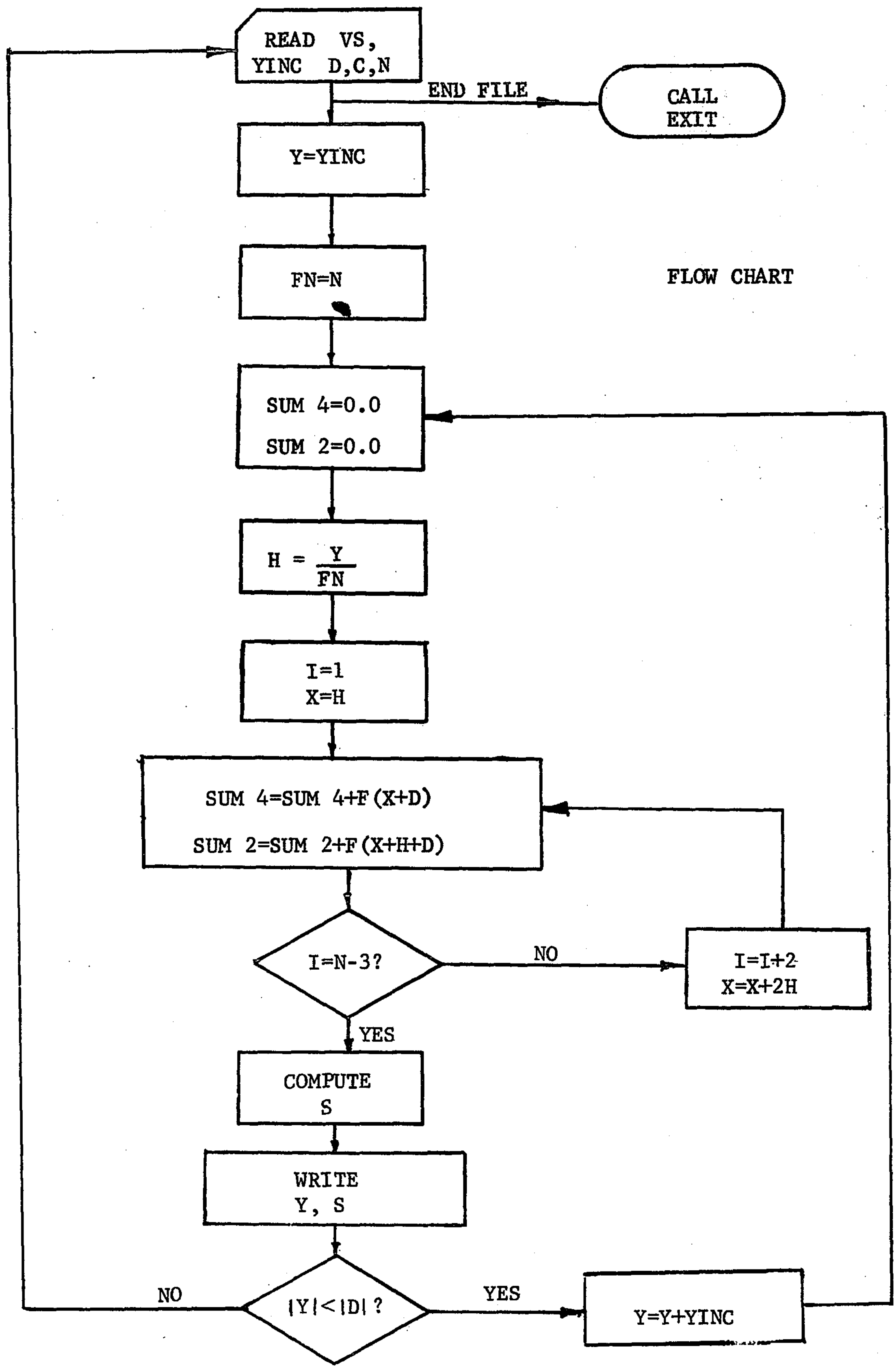
APPENDIX A

Computer Program, Experimental Data

and

Calibration Curves


READ VS,
YINC D,C,N
END FILE
CALL
EXIT
Y=YINC
FN=N
FLOW CHART
SUM 4=0.0
SUM 2=0.0
H = Y/FN
I=1
X=H
SUM 4=SUM 4+F(X+D)
SUM 2=SUM 2+F(X+H+D)
I=N-3?
NO
I=I+2
X=X+2H
YES
COMPUTE
S
WRITE
Y, S
NO
|Y|<|D|?
YES
Y=Y+YINC

```
C       WIRE SHAPE IN STREAM MEASUREMENTS      H. MALLAREDDY
        COSH(A)=(1.0-(TANH(A))**2)**(-0.5)
      9 READ(1.60)  VS,YINC,D,C,N
     60 FORMAT  (4F10.0, 14)
        WRITE(3.70)  VS,D,C
     70 FORMAT(//30X,2HV=, F3.0, 3X, 2HD=, F6.0, 3X, 2HK=, F8.4)
        WRITE(3,71)
     71 FORMAT (/32X,  1HY,   14X,  1HS)
        Y=YINC
        FN=N
    100 SUM4=0.0
        SUM2=0.0
        H=Y/FN
        I=I
        X=H
     12 SUM4=SUM4+COSH(C*VS*VS*(0.8*(X+D)**5-3.0*(X+D)**4*D
        1+2.7334*(X+D)**3*D*D+0.6*(X+D)**2*D**3+0.04*(X+D)*(D*D)**2)
        2/(52.02*(D*D)**2))
        SUM2=SUM2+COSH(C*VS*VS*(0.8*(X+H+D)**5-3.0*(X+H+D)**4*D
        1+2.7334*(X+H+D)**3*D*D+0.6*(X+H+D)**2*D**3+0.04*(X+H+D)*
        2(D*D)**2)/(52.02* (D+D)**2))
         IF(I-N+3) 21, 32, 32
        21 I=I+2
        X=X+2.*H
        GO TO 12
        32 S=H/3*(4.*SUM4+2.*SUM2+4.0*
        1              COSH(C*VS*VS*(0.8*(Y-H+D)**5-3.0*(Y-H+D)**4*D
```

```
     2+2.7334*(Y-H+D)**3*D*D+0.6*(Y-H+D)**2*D**3+0.04*(Y-H+D)*
     3(D*D)**2)/(52.02*(D*D)**2))
     4          +COSH(C*VS*VS*(0.8*(Y+D)**5-3.0*(Y+D)**4*D
     5+2.7334*(Y+D)**3*D*D+0.6*(Y+D)**2*D**3+0.04*(Y+D)*(D*D)**2)
     6/(52.02*(D*D)**2))
     7+COSH(1.1734*C*VS*VS*D/52.02))
      WRITE(3,61) Y,S
   61 FORMAT (F6.0,E20.8)
      IF (ABS(Y)-ABS(D)) 101,200,200
  101 Y=Y+YINC
      GO TO 100
      200 GO TO 9
      END
```

TABLE 7

EXPERIMENTAL DATA

Run No.	Discharge Q, cfs	Mean Velocity From Q	Mean Velocity Pitot Tube	Depth Y, in.	Wet Line S, in.
1	0.102	1.400	1.455	3.500	3.530
				2.500	2.530
2	0.139	1.680	1.790	4.000	4.094
				3.000	3.062
				2.000	2.031
3	0.169	1.760	1.860	4.625	4.718
				3.625	3.687
				2.625	2.655
4	0.187	1.869	1.8650	5.000	5.093
				4.000	4.060
				3.000	3.062
				2.000	2.047
5	0.224	1.960	1.930	5.500	5.594
				4.500	5.562
				3.500	3.531
				2.500	2.531
6	0.265	2.120	2.200	6.000	6.125
				5.000	5.094
				4.000	4.093
				3.000	3.062
7	0.293	2.170	2.200	6.500	6.656
				5.500	5.625
				4.500	4.594
				3.500	3.594
				2.500	2.562
8	0.316	2.350	2.260	7.062	7.156
				6.062	6.156
				5.062	5.125
				4.062	4.125
				3.062	3.094

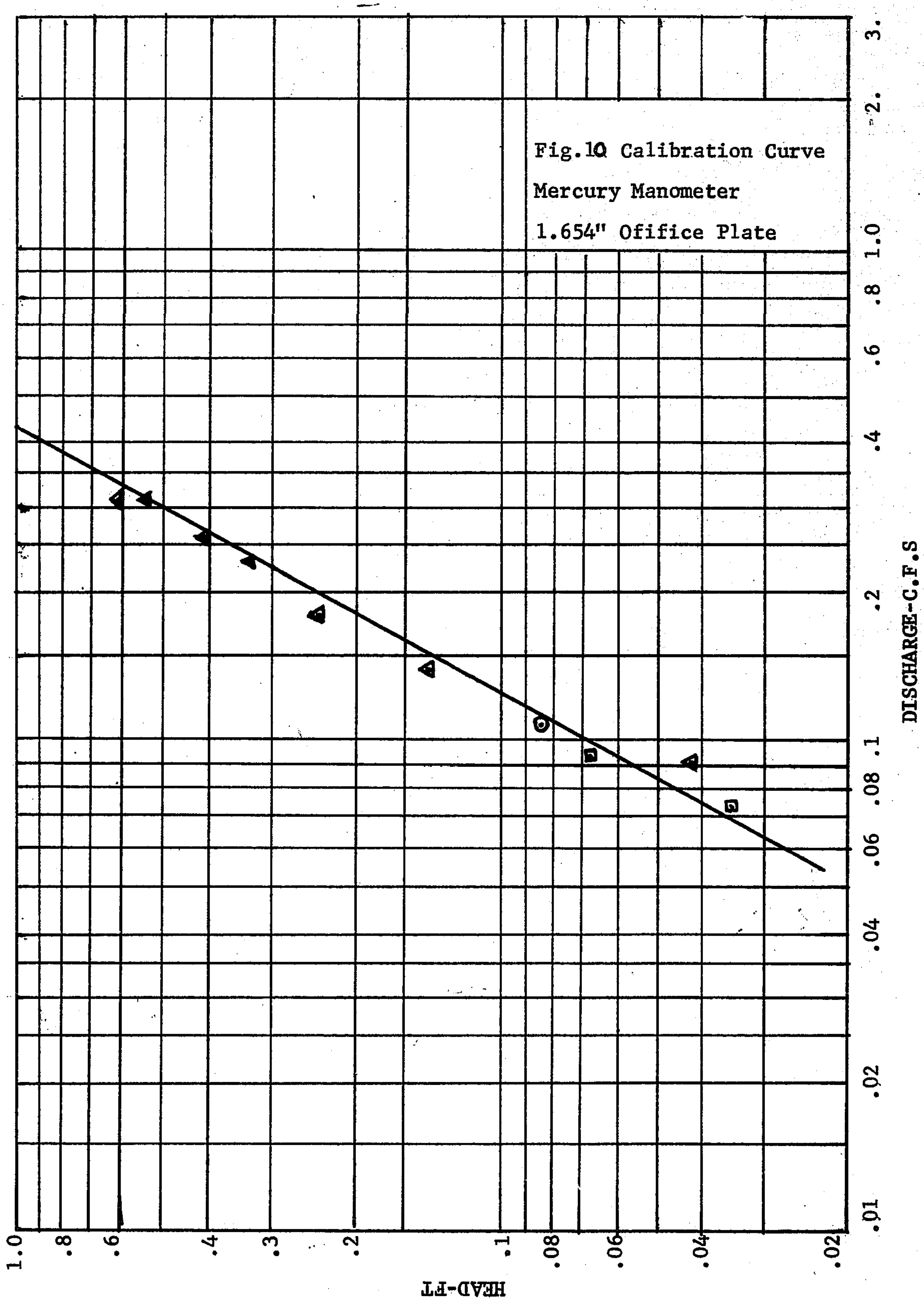


Fig.10 Calibration Curve
Mercury Manometer
1.654" Ofifice Plate

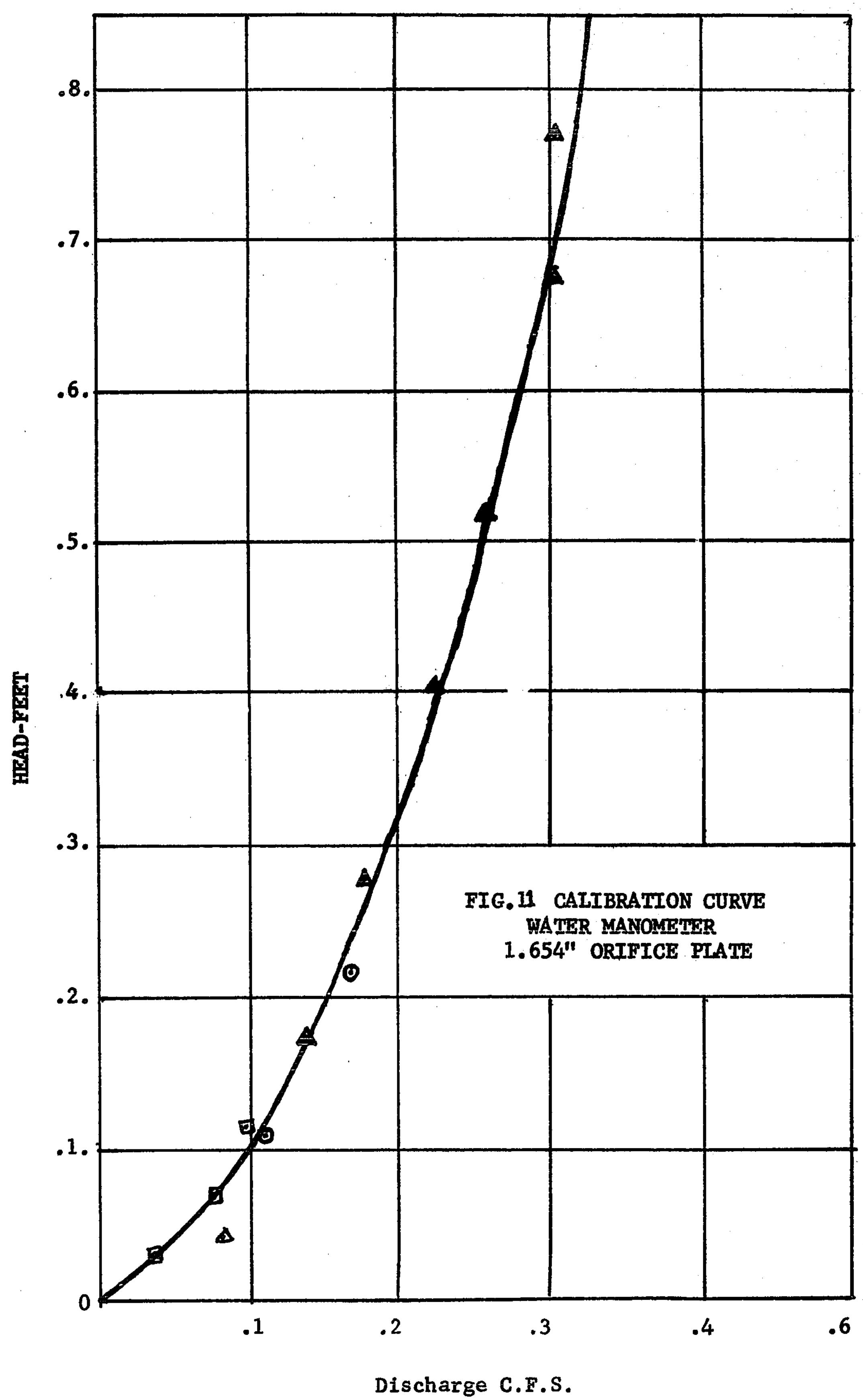


FIG. 11 CALIBRATION CURVE
WATER MANOMETER
1.654" ORIFICE PLATE

APPENDIX B

Regression parameters b, and Correlation coefficient

for different Model ranges.

TABLE 8-2

Mean Velocity: 2 ft./Second

Y_i	4	8	12	16	20
S_{iv}	4.00	8.001	12.002	16.003	20.004
			12.001	16.002	20.003
n_i	1	1	2	2	2

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 1680.244$$

$$\sum_{i}^{k} n_i Y_i^2 = 1680.00$$

$$b = 1.0001452$$

$$r = \frac{1680.244}{\sqrt{1680 \times 1680.53}} = 0.9997$$

TABLE 8-2 (Continued)

Mean Velocity: 2 ft./Second

Y_i	24	28	32	36	40
	24.005	28.006	32.007	36.008	40.008
S_{iv}	24.004	28.005	32.005	36.007	40.007
				36.006	
n_i	2	2	2	3	2

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 11858.264$$

$$\sum_{i}^{k} n_i Y_i^2 = 11856.00$$

$$b = 1.0001908$$

$$r = \frac{11858.264}{\sqrt{11856 \times 11860.52}} = 0.9997$$

TABLE 8-2 (Continued)

Mean Velocity: 2 ft./Second

Y_i	44	48	52	56	60
	44.01	48.01	52.01	56.01	60.01
S_{iv}	44.008	48.009			
	44.007				
n_i	3	2	1	1	1

$$\sum_i^k \sum_v^{n_i} S_{iv} Y_i = 19859.692$$

$$\sum_i^k n_i Y_i^2 = 19856.00$$

$$b = 1.0001859$$

$$r = \frac{19859.692}{\sqrt{19856 \times 19863.385}} = 0.999$$

TABLE 8-2 (Continued)

Mean Velocity: 2 ft./Second

Y_i	64	68	72	76	80
S_{iv}	64.01	68.01	72.01	76.01	80.02
					80.01
n_i	1	1	1	1	2

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv}\ Y_i = 32485.2$$

$$\sum_{i}^{k} n_i\ Y_i^2 = 32480.00$$

$$b = 1.00016$$

$$r = \frac{32485.2}{\sqrt{32480 \times 32490.4}} = 1.000$$

TABLE 8-2 (Continued)

Mean Velocity: 2 ft./Second

Y_i	84	88	92	96	100
S_{iv}	84.02	88.05	92.05	96.02	100.02
		88.02	92.02		
n_i	1	2	2	1	1

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 58706.2$$

$$\sum_{i}^{k} n_i Y_i^2 = 58688.00$$

$$b = 1.0003101$$

$$r = \frac{58706.2}{\sqrt{58688 \times 58724.41}} = 0.999$$

TABLE 8-3

Mean Velocity: 3 ft./Second

Y_i	4	8	12	16	20
S_{iv}	4.003	8.008	12.01	16.02	20.02
	4.002	8.007		16.01	20.01
		8.006			
		8.005			
n_i	2	4	1	2	2

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 1745.428$$

$$\sum_{i}^{k} n_i Y_i^2 = 1744.00$$

$$b = 1.0008188$$

$$r = \frac{1745.428}{\sqrt{1744 \times 1746.857}} = 0.9996$$

TABLE 8-3 (Continued)

Mean Velocity: 3 ft./Second

Y_i	24	28	32	36	40
S_{iv}	24.03	28.03	32.03	36.04	40.04
	24.02	28.02		36.03	40.03
n_i	2	2	1	2	2

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 9544.88$$

$$\sum_{i}^{k} n_i Y_i^2 = 9536.00$$

$$b = 1.0009312$$

$$r = \frac{9544.88}{\sqrt{9536 \times 9553.768}} = 1.0000$$

TABLE 8-3 (Continued)

Mean Velocity; 3 ft./Second

Y_i	44	48	52	56	60
S_{iv}	44.05	48.05	52.06	56.06	60.06
	44.04	48.04	52.05	56.05	60.05
n_i	2	2	2	2	2

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 27386.76$$

$$\sum_{i}^{k} n_i Y_i^2 = 27360.00$$

$$b = 1.0009780$$

$$r = \frac{27386.76}{\sqrt{27360 \times 27413.65}} = 0.9999$$

TABLE 8-3 (Continued)

Mean Velocity: 3 ft./Second

Y_i	64	68	72	76	80
S_{iv}	64.07	68.07	72.08	76.08	80.09
	64.06	68.06	72.07	76.07	80.08
n_i	2	2	2	2	2

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 52212.96$$

$$\sum_{i}^{k} n_i Y_i^2 = 52160.00$$

$$b = 1.0010153$$

$$r = \frac{52212.96}{\sqrt{52160.00 \times 52265.97}} = 0.9997$$

TABLE 8-3 (Continued)

Mean Velocity: 3 ft./Second

Y_i	84	88	92	96	100
	84.09	88.27	92.28	96.10	100.11
S_{iv}	84.08	88.26	92.10		
		88.09			
n_i	2	3	2	1	1

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 73612.40$$

$$\sum_{i}^{k} n_i Y_i^2 = 73488.00$$

$$b = 1.0016927$$

$$r = \frac{73612.40}{\sqrt{73488 \times 73737.07}} = 0.9996$$

TABLE 8-4

Mean Velocity: 4 ft./Second

Y_i	4	8	12	16	20
	4.01	8.02	12.04	16.06	20.07
S_{iv}	4.009		12.03	16.05	20.06
				16.04	20.05
n_i	2	1	2	3	3

$$\sum_{i}^{k} \sum_{v}^{n_i} S \quad Y = 2359.076$$

$$\sum_{i}^{k} n_i Y_i^2 = 2352.00$$

$$b = 1.0030085$$

$$r = \frac{2359.076}{\sqrt{2352 \times 2366.17}} = 0.9993$$

TABLE 8-4 (Continued)

Mean Velocity: 4 ft./Second

Y_i	24	28	32	36	40
	24.08	28.10	32.11	36.12	40.14
	24.07	28.09	32.10	36.11	40.13
S_{iv}	24.06	28.08	32.09	36.10	40.12
		28.07			
n_i	3	4	3	3	3

$$\sum_{i}^{k}\sum_{v}^{n_i} S_{iv}Y_i = 16675.64$$

$$\sum_{i}^{k} n_i\, Y_i^2 = 16624.00$$

$$b = 1.0031063$$

$$r = \frac{16675.64}{\sqrt{16624 \times 16727.44}} = 0.9991$$

TABLE 8-4 (Continued)

Mean Velocity: 4 ft./Second

Y_i	44	48	52	56	60
	44.15	48.17	52.18	56.19	60.20
	44.14	48.16	52.17	56.18	60.19
S_{iv}	44.13	48.15	52.16	56.17	
		48.14			
n_i	3	4	3	3	2

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 39872.40$$

$$\sum_{i}^{k} n_i Y_i^2 = 39744.00$$

$$b = 1.0032306$$

$$r = \frac{39872}{\sqrt{39744 \times 40001.22}} = 0.9998$$

TABLE 8-4 (Continued)

Mean Velocity: 4 ft./Second

Y_i	64	68	72	76	80
S_{iv}	64.22	68.24	72.25	76.27	80.28
	64.21	68.23	72.24	76.25	80.27
	64.20	68.22	72.23	76.24	80.26
		68.21			
n_i	3	4	3	3	3

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 83139.920$$

$$\sum_{i}^{k} n_i Y_i^2 = 82864.00$$

$$b = 1.0033297$$

$$r = \frac{83139.92}{\sqrt{82864 \times 83416.76}} = 0.9998$$

TABLE 8-4 (Continued)

Mean Velocity: 4 ft./Second

Y_i	84	88	92
S_{iv}	84.29 84.27	88.29	92.31
n_i	2	1	1

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 30421.08$$

$$\sum_{i}^{k} n_i Y_i^2 = 30320.00$$

$$b = 1.0033337$$

$$r = \frac{30421.08}{\sqrt{30320 \times 30522.49}} = 0.9997$$

TABLE 8-4 (Continued)

Mean Velocity: 4 ft./Second

Y_i	96	100
S_{iv}	96.34 96.33	100.35
n_i	2	1

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 28531.32$$

$$\sum_{i}^{k} n_i Y_i^2 = 28432.00$$

$$b = 1.0034932$$

$$r = \frac{28531.32}{\sqrt{28432 \times 28630.98}} = 1.000$$

TABLE 8-5

Mean Velocity: 5 ft./Second

Y_i	4	8	12	16	20
	4.03	8.06	12.10	16.14	20.17
	4.02	8.05	12.09	16.13	20.16
S_{iv}		8.04	12.08	16.12	20.15
			12.07	16.11	20.14
				16.10	20.13
					20.12
n_i	2	3	4	5	6

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 4512.48 \qquad \sum_{i}^{k} n_i Y_i^2 = 4480.00$$

$$b = 1.00725$$

$$r = \frac{4512.48}{\sqrt{4480.0 \times 4545.20}} = 0.9996$$

TABLE 8-5 (Continued)

Mean Velocity: 5 ft./Second

Y_i	24	28	32	36	40
S_{iv}	24.20 24.19 24.18 24.17 24.16	28.24 28.23 28.22 28.21 28.20 28.19	32.28 32.26 32.25 32.24 32.23 32.22	36.31 36.29 36.28 36.27 36.26 36.25	40.34 40.33 40.32 40.31 40.30 40.29 40.28
n_i	5	6	6	6	7

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 32955.64 \qquad \sum_{i}^{k} n_i Y_i^2 = 32704.00$$

$$b = 1.00769$$

$$r = \frac{32955.64}{\sqrt{32704.00 \times 33209.23}} = 1.000$$

TABLE 8-5 (Continued)

Mean Velocity: 5 ft./Second

Y_i	44	48	52	56	60
	44.38	48.41	52.45	56.48	60.52
	44.36	48.40	52.42	56.46	60.50
	44.35	48.38	52.41	56.45	60.49
S_{iv}	44.34	48.37	52.40	56.44	60.48
	44.33	48.36	52.39	56.43	60.47
	44.32				60.46
n_i	6	5	5	5	6

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 74529.08 \qquad \sum_{i}^{k} n_i Y_i^2 = 73936.00$$

$$b = 1.00802$$

$$r = \frac{74529.08}{\sqrt{73936 \times 75126.93}} = 0.9996$$

TABLE 8-5 (Continued)

Mean Velocity: 5 ft./Second

Y_i	64	68	72	76	80
	64.55	68.58	72.62	76.64	80.70
	64.53	68.57	72.60	76.62	80.67
	64.52	68.56	72.59	76.60	80.64
S_{iv}	64.51	68.55	72.58		
	64.50	68.54	72.57		
		68.53	72.56		
n_i	5	6	6	3	3

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 116805.08 \qquad \sum_{i}^{k} n_i Y_i^2 = 115896.00$$

$$b = 1.00819$$

$$r = \frac{116805.08}{\sqrt{115856 \times 117761.94}} = 1.000$$

TABLE 8-5 (Continued)

Mean Velocity: 5 ft./Second

Y_i	84	88	92
S_{iv}	84.72	88.73	92.77
	84.68	88.72	92.76
n_i	2	2	2

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 46913.95 \qquad \sum_{i}^{k} n_i Y_i^2 = 46528{\cdot}00$$

$$b = 1.00829$$

$$r = \frac{46913.5}{\sqrt{46528 \times 47303.12}} = 0.9998$$

TABLE 8-5 (Continued)

Mean Velocity: 5 ft./Second

Y_i	96	100
S_{iv}	96.83 96.81	100.86
n_i	2	1

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 28675.44 \qquad \sum_{i}^{k} n_i Y_i^2 = 28432.00$$

$$b = 1.00856$$

$$r = \frac{28675.44}{\sqrt{28432 \times 28920.96}} = 1.000$$

TABLE 8-6

Mean Velocity: 6 ft./Second

Y_i	4	8	12	16	20
	4.06	8.13	12.21	16.29	20.36
	4.05	8.12	12.20	16.26	20.33
		8.11	12.18	16.25	20.32
		8.10	12.17	16.24	20.31
S_{iv}		8.09	12.16	16.23	20.30
			12.15	16.22	20.29
			12.14	16.21	20.28
				16.20	20.27
					20.26
n_i	2	5	7	8	9

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 7124.16 \qquad \sum_{i}^{k} n_i Y_i^2 = 7008.00$$

$$b = 1.01486$$

$$r = \frac{7124.16}{\sqrt{7008 \times 7217.88}} = 1.0$$

TABLE 8-6 (Continued)

Mean Velocity: 6 ft./ Second

Y_i	24	28	32	36	40
S_{iv}	24.43	28.50	32.57	36.64	40.71
	24.40	28.47	32.54	36.61	40.68
	24.39	28.46	32.53	36.60	40.67
	24.38	28.45	32.52	36.59	40.66
	24.37	28.44	32.51	36.58	40.65
	24.36	28.43	32.50	36.57	40.64
	24.35	28.42	32.49	36.56	40.63
	24.34	28.41	32.48	36.55	40.62
	24.33	28.40	32.47	36.54	40.60
	24.32	28.39	32.46	36.53	
			32.45		
n_i	10	10	11	10	9

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 53053.20 \qquad \sum_{i}^{k} n_i Y_i^2 = 52224.00$$

$$b = 1.01587$$

$$r = \frac{53053.20}{\sqrt{52224 \times 53997.06}} = 0.9995$$

TABLE 8-6 (Continued)

Mean Velocity: 6 ft./Second

Y_i	44	48	52	56	60
	44.79	48.86	52.93	57.00	61.07
	44.75	48.83	52.90	56.97	61.04
	44.74	48.81	52.87	56.95	61.02
	44.73	48.79	52.86	56.93	61.00
S_{iv}	44.72	48.78	52.85	56.92	60.99
	44.71	48.77	52.84	56.91	60.98
	44.70	48.76	52.83	56.90	60.97
	44.69	48.74	52.82	56.89	60.96
	44.67		52.81	56.88	
n_i	9	8	9	9	8

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 119156.64 \qquad \sum_{i}^{k} n_i Y_i^2 = 117216.00$$

$$b = 1.01655$$

$$r = \frac{119156.64}{\sqrt{117216 \times 121129.46}} = 0.9999$$

TABLE 8-6 (Continued)

Mean Velocity: 6 ft./Second

Y_i	64	68	72	76	80
	65.15	69.22	73.29	77.36	81.43
	65.11	69.18	73.25	77.32	81.40
	65.08	69.16	73.23	77.30	81.33
S_{iv}	65.07	69.14	73.23	77.25	81.32
	65.06	69.13	73.21		
	65.05	69.10	73.18		
	65.03		73.17		
n	7	6	7	4	4

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 138542.08 \qquad \sum_{i}^{k} n_i Y_i^2 = 136224.00$$

$$b = 1.01701$$

$$r = \frac{138542.08}{\sqrt{136224 \times 146262.28}} = 0.9817$$

TABLE 8-6 (Continued)

Mean Velocity: 6 ft./Second

Y_i	84	88	92
	85.50	89.51	93.60
S_{iv}	85.42	89.50	93.50
	85.40		
n_i	3	2	2

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 54496.96 \qquad \sum_{i}^{k} n_i Y_i^2 = 53584.00$$

$$b = 1.01703$$

$$r = \frac{54496.96}{\sqrt{53584 \times 55425.42}} = 1.00$$

TABLE 8-6 (Continued)

Mean Velocity: 6 ft./Second

Y_i	96	100
S_{iv}	97.72 97.68	101.79
n_i	2	1

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 28937.40 \qquad \sum_{i}^{k} n_i Y_i^2 = 28432.00$$

$$b = 1.01777$$

$$r = \frac{28937.40}{\sqrt{28432 \times 29451.78}} = 0.9998$$

TABLE 8-7

Mean Velocity: 7 ft./Second

Y_i	4	8	12	16	20
	4.11	8.24	12.39	16.53	20.66
	4.10	8.23	12.36	16.49	20.62
	4.09	8.22	12.35	16.48	20.60
	4.08	8.21	12.34	16.47	20.59
		8.20	12.33	16.46	20.58
		8.19	12.32	16.45	20.57
		8.18	12.31	16.44	20.56
S_{iv}		8.17	12.30	16.43	20.55
			12.29	16.42	20.54
			12.28	16.41	20.53
			12.27	16.40	20.52
				16.39	20.51
				16.38	20.50
					20.49
					20.48
n_i	4	8	11	13	15

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 11803.12 \qquad \sum_{i}^{k} n_i Y_i^2 = 11488.00$$

$$b = 1.02743$$

$$r = \frac{11803.12}{\sqrt{11488 \times 12126.96}} = 1.000$$

TABLE 8-7 (Continued)

Mean Velocity: 7 ft./Second

Y_i	24	28	32	36	40
	24.80	28.93	33.06	37.19	41.33
	24.75	28.88	33.01	37.14	41.27
	24.73	28.86	32.98	37.11	41.24
	24.72	28.84	32.97	37.10	41.22
	24.71	28.83	32.96	37.08	41.21
	24.70	28.81	32.95	37.07	41.20
	24.69	28.80	32.94	37.06	41.19
S_{iv}	24.68	28.79	32.93	37.05	41.18
	24.67	28.78	32.92	37.04	41.17
	24.66	28.77	32.91	37.03	41.16
	24.65	28.76	32.90	37.02	41.15
	24.64	28.75	32.89	37.01	41.14
	24.63	28.73	32.88	36.99	41.11
	24.62	28.72	32.86	36.98	
	24.60		32.85		
n_i	15	14	15	14	13

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 76087.44 \qquad \sum_{i}^{k} n_i Y_i^2 = 73920.00$$

$$b = 1.02932$$

$$r = \frac{76087.44}{\sqrt{73920 \times 78318.67}} = 0.9998$$

TABLE 8-7 (Continued)

Mean Velocity: 7 ft./Second

Y_i	44	48	52	56	60
S_{iv}	45.46	49.60	53.73	57.86	61.99
	45.40	49.53	53.66	57.79	61.93
	45.37	49.50	53.63	57.76	61.89
	45.35	49.48	53.61	57.73	61.86
	45.33	49.46	53.59	57.71	61.84
	45.32	49.45	53.58	57.70	61.83
	45.31	49.44	53.57	57.69	61.82
	45.30	49.43	53.56	57.68	61.79
	45.29	49.42	53.55	57.65	61.78
	45.28	49.41	53.52	57.64	
	45.25	49.38	52.51		
	45.24	49.37			
n_i	12	12	11	10	9

$$\sum_{i}^{k}\sum_{v}^{n_i} S_{iv}\,Y_i = 148748.24 \qquad \sum_{i}^{k} n_i\,Y_i^2 = 144384.00$$

$$b = 1.03022$$

$$r = \frac{148748.24}{\sqrt{144384.0 \times 153245.76}} = 0.9997$$

TABLE 8-7 (Continued)

Mean Velocity: 7 ft./Second

Y_i	64	68	72	76	80
S_{iv}	66.13	70.26	74.39	78.53	82.66
	66.04	70.19	74.33	78.46	82.59
	66.02	70.15	74.28	78.41	82.48
	65.99	70.12	74.25	78.34	82.46
	65.97	70.10	74.20	78.32	
	65.96	70.05	74.18		
	65.92				
	65.91				
n_i	8	6	6	5	4

$$\sum_i^k \sum_v^{n_i} S_{iv} Y_i = 150705.72 \qquad \sum_i^k n_i Y_i 2 = 146096.00$$

$$b = 1.03155$$

$$r = \frac{150705.72}{\sqrt{146096 \times 155458.41}} = 0.9997$$

TABLE 8-7 (Continued)

Mean Velocity: 7 ft./Second

Y_i	84	88	92
S_{iv}	86.79	90.81	94.78
	86.64	90.76	94.74
	86.61		
n_i	3	2	2

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 55274.88 \qquad \sum_{i}^{k} n_i Y_i^2 = 53584.00$$

$$b = 1.03155$$

$$r = \frac{55274.88}{\sqrt{53584 \times 56983.03}} = 0.9998$$

TABLE 8-7 (Continued)

Mean Velocity: 7 ft./Second

Y_i	96	100
S_{iv}	99.19 99.12	103.32
n_i	2	1

$$\sum_{i}^{k}\sum_{v}^{n_i} S_{iv}\,Y_i = 29369.76 \qquad \sum_{i}^{k} n_i\,Y_i^2 = 28432.0$$

$$b = 1.03298$$

$$r = \frac{29369.76}{\sqrt{28432 \times 30338.45}} = 1.00$$

TABLE 8-8

Mean Velocity: 8 ft./Second

Y_i	4	8	12	16	20
	4.18	8.41	12.68	16.91	21.14
	4.17	8.39	12.63	16.84	21.06
	4.16	8.38	12.60	16.82	21.04
	4.15	8.37	12.59	16.81	21.02
	4.14	8.36	12.57	16.79	21.00
		8.35	12.56	16.77	20.99
		8.34	12.55	16.76	20.97
		8.33	12.54	16.73	20.96
S_{iv}		8.32	12.53	16.72	20.94
		8.31	12.52	16.71	20.93
		8.30	12.51	16.70	20.92
		8.29	12.50	16.69	20.91
			12.48	16.68	20.89
			12.47	16.67	20.88
			12.46	16.66	20.87
				16.64	20.86
					20.84
					20.83
n_i	5	12	15	16	18

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 14970.08 \qquad \sum_{i}^{k} n_i Y_i^2 = 14304.00$$

$$b = 1.04656$$

$$r = \frac{14970.08}{\sqrt{14304.00 \times 15738.74}} = 0.9982$$

TABLE 8-8 (Continued)

Mean Velocity: 8 ft./Second

Y_i	24	28	32	36	40
	25.36	29.59	33.82	38.05	42.28
	25.28	29.51	33.73	37.95	42.18
	25.25	29.47	33.69	37.91	42.13
	25.23	29.45	33.66	37.88	42.10
	25.22	29.43	33.65	37.86	42.07
	25.20	29.42	33.63	37.84	42.06
	25.19	29.40	33.61	37.83	42.04
	25.17	29.38	33.60	37.81	42.02
	25.15	29.34	33.58	37.80	42.01
S_{iv}	25.14	29.32	33.56	37.78	41.99
	25.13	29.31	33.55	37.76	41.98
	25.11	29.30	33.54	37.75	41.96
	25.10	29.28	33.52	37.73	41.92
	25.08	29.24	33.51	37.69	41.90
	25.07		33.47	37.68	
	25.05		33.46		
	25.03				
n_i	17	14	16	15	14

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 82846.24 \qquad \sum_{i}^{k} n_i Y_i^2 = 78992.00$$

$$b = 1.04879$$

$$r = \frac{82846.24}{\sqrt{78992.00 \times 87057.51}} = 1.04879$$

TABLE 8-8 (Continued)

Mean Velocity: 8 ft./Second

Y_i	44	48	52	56	60
	46.51	50.74	54.96	59.19	63.42
	46.40	50.63	54.85	59.08	63.30
	46.35	50.57	54.79	59.01	63.23
	46.31	50.53	54.75	58.97	63.19
	46.29	50.51	54.72	58.94	63.16
	46.27	50.49	54.70	58.92	63.13
S_{iv}	46.25	50.47	54.68	58.90	63.11
	46.24	50.45	54.67	58.88	63.06
	46.22	50.44	54.65	58.83	63.05
	46.21	50.42	54.60	58.82	
	46.19	50.37	54.59		
	46.14	50.36			
	46.13				
n_i	13	12	11	10	9

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 153988.64 \qquad \sum_{i}^{k} n_i Y_i^2 = 146320.0$$

$$b = 1.05241$$

$$r = \frac{153988.64}{\sqrt{146320 \times 162059.84}} = 1.000$$

TABLE 8-8 (Continued)

Mean Velocity: 8 ft./Second

Y_i	64	68	72	76	80
S_{iv}	67.65	71.87	76.10	80.33	84.56
	67.53	71.75	75.98	80.21	84.44
	67.45	71.68	75.91	80.13	84.25
	67.41	71.63	75.85	80.00	84.22
	67.38	71.59	75.76	79.98	
	67.35	71.52	75.74		
	67.29	71.51			
	67.28				
n_i	8	7	6	5	4

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 158854.64 \qquad \sum_{i}^{k} n_i Y_i^2 = 154342.64$$

$$b = 1.05397$$

$$r = \frac{158854.64}{\sqrt{154342.64 \times 167428.81}} = 0.9883$$

TABLE 8-8 (Continued)

Mean Velocity: 8 ft./Second

Y_i	84	88	92
	88.79	92.80	97.12
S_{iv}	88.52	92.74	97.03
	88.47		
n_i	3	2	2

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 56514.84 \qquad \sum_{i}^{k} n_i Y_i^2 = 53584.0$$

$$b = 1.05469$$

$$r = \frac{56514.84}{\sqrt{53584 \times 59606.06}} = 0.9998$$

TABLE 8-8 (Continued)

Mean Velocity: 8 ft./Second

Y_i	96	100
S_{iv}	101.47 101.34	105.7
n_i	2	1

$$\sum_{i}^{k}\sum_{v}^{n_i} S_{iv} Y_i = 30039.76 \qquad \sum_{i}^{k} n_i Y_i^2 = 28432.00$$

$$b = 1.05654$$

$$r = \frac{30039.76}{\sqrt{28432 \times 31738.45}} = 1.00$$

TABLE 8-9

Mean Velocity: 9 ft./Second

Y_i	4	8	12	16	20
	4.29	8.66	13.10	17.47	21.83
	4.28	8.64	13.01	17.36	21.71
	4.27	8.61	12.98	17.32	21.67
	4.26	8.59	12.95	17.30	21.65
	4.25	8.57	12.93	17.27	21.62
	4.24	8.56	12.91	17.25	21.60
	4.23	8.55	12.89	17.22	21.57
		8.54	12.87	17.20	21.54
S_{iv}		8.53	12.85	17.18	21.52
		8.52	12.84	17.16	21.50
		8.51	12.82	17.15	21.48
		8.50	12.81	17.14	21.46
		8.49	12.80	17.13	21.44
		8.48	12.78	17.12	21.43
			12.77	17.10	21.41
			12.76	17.08	21.40
			12.75	17.07	21.39
				17.06	21.35
				17.04	21.34
n_i	7	14	17	19	19

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 17110.92 \qquad \sum_{i}^{k} n_i Y_i^2 = 15920.00$$

$$b = 1.0748$$

$$r = \frac{17110.92}{\sqrt{15920.0 \times 18385.92}} = 1.000$$

TABLE 8-9 (Continued)

Mean Velocity: 9 ft./Second

Y_i	24	28	32	36	40
	26.20	30.57	34.94	39.31	43.67
	26.07	30.43	34.79	39.15	43.51
	26.02	30.37	34.72	39.07	43.43
	25.99	30.33	34.68	39.03	43.37
	25.96	30.31	34.65	39.00	43.34
	25.94	30.28	34.62	38.97	43.31
	25.91	30.25	34.60	38.94	43.29
	25.88	30.23	34.57	38.92	43.26
S_{iv}	25.86	30.20	34.55	38.90	43.24
	25.84	30.18	34.52	38.87	43.21
	25.81	30.15	34.50	38.84	43.18
	25.80	30.13	34.47	38.81	43.15
	25.77	30.11	34.45	38.79	43.08
	25.75	30.09	34.42	38.72	43.06
	25.74	30.07	34.36	38.70	
	25.72	30.00	34.34		
	25.68				
	25.66				
n_i	18	16	16	15	14

$$\sum_{i}^{k} \sum_{v}^{n_i} S \quad Y = 87705.28 \qquad \sum_{i}^{k} n_i Y_i^2 = 81136.00$$

$$b = 1.08096$$

$$r = \frac{87705.28}{\sqrt{81136 \times 94808.64}} = 0.9998$$

TABLE 8-9 (Continued

Mean Velocity: 9 ft./Second

Y_i	44	48	52	56	60
	48.04	52.41	56.77	61.14	65.51
	47.87	52.23	56.59	60.96	65.33
	47.78	52.13	56.49	60.85	65.21
	47.72	52.08	56.43	60.78	65.14
	47.69	52.04	56.39	60.74	65.09
	47.66	52.00	56.35	60.70	65.05
S_{iv}	47.63	51.98	56.32	60.67	65.01
	47.61	51.95	56.30	60.64	64.93
	47.58	51.92	56.27	60.56	64.90
	47.55	51.90	56.19	60.53	
	47.52	51.82	56.17		
	47.45	51.80			
	47.42				
n_i	13	12	11	10	9

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 158667.12 \qquad \sum_{i}^{k} n_i Y_i^2 = 146320.00$$

$$b = 1.08438$$

$$r = \frac{158667.12}{\sqrt{146320 \times 172067.40}} = 0.9997$$

TABLE 8-9 (Continued)

Mean Velocity: 9 ft./Second

Y_i	64	68	72	76	80
S_{iv}	69.88 69.70 69.57 69.50 69.44 69.40 69.30 69.28	74.25 74.04 73.93 73.85 73.79 73.68 73.65	78.61 78.42 78.29 78.20 78.06 78.02	82.98 82.78 82.65 82.45 82.40	87.35 87.15 86.85 86.80
n_i	8	7	6	5	4

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 163829.04 \qquad \sum_{i}^{k} n_i Y_i^2 = 150720.00$$

$$b = 1.08697$$

$$r = \frac{163829.04}{\sqrt{150720 \times 178078.08}} = 1.00$$

TABLE 8-9 (Continued)

Mean Velocity: 9 ft./Second

Y_i	84	88	92
S_{iv}	91.72	95.74	100.24
	91.28	95.64	100.21
n_i	2	2	2

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 58322.36 \qquad \sum_{i}^{k} n_i Y_i^2 = 53584.00$$

$$b = 1.08842$$

$$r = \frac{58322.36}{\sqrt{53584 \times 55147.85}} = 1.00$$

TABLE 8-9 (Continued)

Mean Velocity: 9 ft./Second

Y_i	96	100
S_{iv}	104.82 104.61	109.18
n_i	2	1

$$\sum_{i}^{k}\sum_{v}^{n_i} S_{iv}\ Y_i = 28432.0 \qquad \sum_{i}^{k} n_i\ Y_i^2 = 20105.00$$

$$b = 1.09113$$

$$r = \frac{28432.00}{\sqrt{20105 \times 33850.75}} = 1.000$$

TABLE 8-10

Mean Velocity: 10 ft./Second

Y_i	4	8	12	16	20
	4.45	9.01	13.69	18.28	22.82
	4.43	8.98	13.55	18.08	22.63
	4.41	8.93	13.50	18.03	22.56
	4.40	8.91	13.46	17.99	22.53
	4.39	8.88	13.42	17.95	22.48
	4.38	8.86	13.39	17.91	22.40
	4.37	8.84	13.36	17.84	22.37
	4.36	8.81	13.30	17.81	22.33
	4.35	8.80	13.29	17.78	22.29
		8.79	13.27	17.76	22.26
		8.78	13.25	17.73	22.24
S_{iv}		8.77	13.23	17.71	22.21
		8.76	13.22	17.70	22.19
		8.75	13.21	17.68	22.17
		8.73	13.20	17.66	22.14
			13.19	17.65	22.13
			13.18	17.63	22.07
			13.17	17.60	22.06
			13.15	17.59	[illegible]
n_i	9	15	19	19	18

$$\sum_{i}^{k}\sum_{v}^{n_i} S_{iv}\,Y_i = 17706.68 \qquad \sum_{i}^{k} n_i\,Y_i^2 = 15904.0$$

$$b = 1.113344$$

$$r = \frac{17706.68}{\sqrt{19715.75 \times 15904}} = 0.9998$$

TABLE 8-10 (Continued)

Mean Velocity: 10 ft./Second

Y_i	24	28	32	36	40
S_{iv}	27.39	31.95	36.52	41.08	45.38
	27.17	31.73	36.28	40.71	45.36
	27.10	31.64	36.12	40.65	45.18
	27.05	31.54	36.07	40.60	45.13
	26.97	31.50	36.02	40.55	45.09
	26.93	31.46	36.00	40.52	45.05
	26.89	31.42	35.95	40.48	45.00
	26.85	31.38	35.91	40.44	44.97
	26.82	31.34	35.87	40.40	44.93
	26.78	31.31	35.83	40.36	44.89
	26.75	31.27	35.80	40.32	44.85
	26.72	31.24	35.76	40.28	44.73
	26.70	31.20	35.72	40.17	44.70
	26.66	31.17	35.63	40.14	
	26.64	31.09	35.60		
	26.57	31.07			
	26.55				
n_i	17	16	15	14	13

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 86083.80 \qquad \sum_{i}^{k} n_i Y_i^2 = 76640.00$$

$$b = 1.12322$$

$$r = \frac{86083.80}{\sqrt{76640 \times 96695.46}} = .9997$$

TABLE 8-10 (Continued)

Mean Velocity: 10 ft./Second

Y_i	44	48	52	56	60
	50.21	54.78	59.34	63.90	68.47
	49.95	54.50	59.05	63.62	68.18
	49.80	54.35	58.90	63.45	68.00
	49.72	54.26	58.80	63.35	67.89
	49.66	54.20	58.74	63.27	67.82
	49.62	54.15	58.68	63.22	67.75
	49.58	54.10	58.64	63.17	67.70
S_{iv}	49.54	54.07	58.59	63.12	67.57
	49.50	54.02	58.55	63.00	67.53
	49.45	53.98	58.43	62.96	
	49.42	53.86	58.39		
	49.30	53.82			
	49.26				
n_i	13	12	11	10	9

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 165311.84 \qquad \sum_{i}^{k} n_i Y_i^2 = 146320.00$$

$$b = 1.12979$$

$$r = \frac{165311.84}{\sqrt{146320 \times 186108.303}} = 1.000$$

TABLE 8-10 (Continued)

Mean Velocity: 10 ft./Second

Y_i	64	68	72	76	80
S_{iv}	73.04	77.60	82.16	86.73	91.29
	72.74	77.30	81.86	86.12	90.98
	72.56	77.11	81.66	86.22	90.52
	72.44	76.98	81.53	85.90	90.44
	72.35	76.89	81.30	85.83	
	72.28	76.72	81.25		
	72.14	76.67			
	72.10				
n_i	8	7	6	5	4

$$\sum_{i}^{k} \sum_{v}^{n_i} S_{iv} Y_i = 170829.88 \qquad \sum_{i}^{k} n_i Y_i^2 = 150720.00$$

$$b = 1.13342$$

$$r = \frac{170829.88}{\sqrt{150720 \times 193626}} = 0.9997$$