LOW ORDER MULTI-PORT TIME DOMAIN MODELS

FOR FLUID POWER VALVES

By

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PREFACE

This report is part of a continuing effort to develop mathematical models of fluid power components, which are particularly suitable for system simulation. Component models as developed by classical techniques depend on design features to such an extent that it is difficult to establish a general method for their simplification. It was recognized that an entirely new approach to component modeling was necessary for developing component models. Valves were selected as the class of components for consideration as previous modeling work indicated that they contributed largely to the complexity of system equations. The multi-port concept, referred to in this study, is not entirely new; however, its use has been largely restricted to linear systems, where transmission matrices can be formulated. With the development of multi-input multi-output models for non-linear components, it is expected that the modeling of systems along similar lines will become practicable.

I am more than indebted to Dr. E. C. Fitch, Jr., for his guidance, encouragement, and support during my graduate program.

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CHAPTER I

INTRODUCTION

Advances in computational techniques during the past few years have given great impetus to the development and usage of mathematical models of dynamic systems. In their application to fluid power systems, however, emphasis has been more on the computational aspects and less on the non-mathematical areas of modeling. The conceptualizing of an ideal model from a study of the physical system, the process of deciding the degree of 'lumping' (combining elements) to be used and the evaluation of the model itself are examples of non-mathematical decision-making which cannot be ignored by the system designer.

Faced with the task of exercising judgment in these areas, the tendency is to develop as elaborate a model as possible with the hope firstly that it will be accurate and secondly that it will be amenable to simplification. The accuracy of complex models is difficult to ascertain if physical measurements pertain to only a few of the variables, and the process of simplification depends to a large extent on the model itself, especially for non-linear systems. An inspection of the modeling technique can, however, lead to the development of simple low order models more useful for a variety of purposes than more complex ones. Such low order models of components are particularly useful if they can be interfaced with other models for the synthesis of large systems.

Although there are a number of unifying concepts in systems analysis which form the basis for all dynamic modeling, there are a few problems which have stood in the way of the development of component models for fluid power systems. The first problem was that of 'lumping' since fluid power systems are inherently distributed-parameter in nature and the accuracy of the model dictates the degree of 'lumping' necessary. The second was that of isolating a component from the rest of the system. Because of these difficulties, models were more likely to be developed for systems rather than components.

The electro-hydraulic servo-mechanism was one of the first fluid power devices subjected to dynamic analysis (1) (2). Linear analysis was used as it offered the strong incentives of working in the frequency domain and using classical techniques for compensation, feedback, and stability analysis. The servo-valve, in particular was studied fairly exhaustively as it was the heart of the power-amplification stage (1) (2) (3). However, the analysis of such a component was carried out only after including a certain load and assuming certain upstream conditions. The results of analysis were not so much the response of the servo-valve as that of the system as a whole. While their usefulness cannot be questioned, to describe them as being characteristic of the servo-valve is somewhat misleading.

The concept of a system as being made up of components is useful in all areas of systems analysis. The modeling of components permits the synthesizing of system models without having to formulate a complex set of equations for the whole system. Component models, if they are general enough, can be interfaced with any other model and this gives the designer the advantage of checking alternative designs with minimum effort. As control elements, valves are used in almost every fluid power circuit and the usefulness of developing their models is obvious.

The application of the classical modeling technique for writing component models requires the following steps:

(1) Isolate the component from the rest of the system and decide which quantities are to be treated as inputs, outputs, and disturbances.

(2) Idealize and lump all elements involved and applying the laws of mechanics and other sciences where necessary, formulate mathematical relationships between variables and parameters.

(3) Impose the inputs to the model so that its behavior may be studied.

Since all valves perform the necessary control function by varying the area of one or more control orifices, the analysis from a classical standpoint involves the following tasks:

(1) Identifying flow paths in the valve and writing flow and continuity equations.

(2) Identifying forces on reaction elements and writing force balance equations.

(3) Identifying constraints on displacements and velocities imposed by geometrical features in the valve.

Performing these tasks leads, generally, to a set of coupled algebraic-cum-differential equations in which some state-variables are subject to constraints. Either analog or digital methods may be used for solving the equations. As will be shown in Chapter II, this approach, referred to as the "classical" method, has the following disadvantages:

(1) A large number of parameters are involved, and detailed working drawings of the component are necessary for proper evaluation. The effect of changes in parameters is not always obvious.

(2) Interfacing such a model with other component models is not straightforward. This is especially so, if the assumptions made in developing one model are incompatible with those for another.

(3) Identification of parameters can take excessive computer time if digital simulation is being used. The presence of non-linearities in the model often leads to problems in convergence to the correct values.

(4) Few of the state-variables in the model are amenable to measurement; each unknown state-variable contributes an unknown "parameter" in the form of an initial condition.

The purpose of this thesis is to present a new approach to the modeling of fluid power components. This approach, referred to in the paper as the "grey-box" approach, utilizes the multi-port concept of the development of general time domain models. Though the method of analysis is general, the development of models is illustrated for valves having a metering element under the action of a number of forces dependent on upstream and downstream pressures and flow. This includes valves which are self-regulatory in nature, as distinct from directional control or switching valves. Examples of self-regulatory valves are pressure relief valves, pressure reducing valves, and flow control valves. The models are developed using the multi-port concept by isolating the component and considering only the 'through' and 'across' variables that are amenable to measurement. The form of the models proposed is such that geometrical details of valves are unnecessary, though they could be used to improve the models. The parameters involved are few in number and can be identified from static and dynamic tests. To the extent that the models are not relatable to design features, fluid properties, and other parameters of a classical model, they are empirical. Furthermore, since they are presented as being suitable for an entire class of valves rather than a particular one, they are general models.

The next chapter presents a survey of the classical modeling technique and an example model development has been included in Appendix A. Chapter III presents an equivalent orifice representation for valves to permit the utilization of the multi-port concept. It also includes the development of the new modeling approach and illustrates its applicability by the formulation of relief valve models. The results of a comprehensive experimental effort are discussed in Chapter IV, while the general conclusions and recommendations are presented in Chapter V. Appendix B contains a discussion relative to the orifice-equation, and Appendix C describes the problems associated with flow measurements for dynamic tests.

CHAPTER II

A REVIEW OF THE CLASSICAL MODELING TECHNIQUE

Though the modeling of fluid power systems is not new it is only recently that the process has been systematized to a degree that an analyst can use a standard procedure and be reasonably sure that no important physical phenomena have been overlooked. One such procedure is given in the _Hydraulic Component Modeling Manual_ of the Basic Fluid Power Research Program (4). The modeling manual helps to describe the system in rigorous mathematical terminology. Once the model is described, the next step involves simplifying the mathematical expressions to a form suitable for machine computation or, if necessary, hand computation. The final step is to determine the behavior of the model when it is subjected to the inputs under consideration.

Some of the early work in the area of modeling of components was done prior to the general acceptance of time-domain analysis for the type of systems under consideration (1) (5) (6) (7). Emphasis was on linearization so that transfer functions could be written and frequency-domain analysis techniques used. One of the first studies was by Foster (5) who analyzed a two-stage relief valve. System equations were written in the usual manner but subsequently linearized about a steady-state operating condition. Analog simulation was accomplished using a step change in flow as the input. As many as 17 parameters were involved - most of them geometrical in nature. A reduction to nine parameters using linearization techniques was necessary before simulation could be performed. Upstream pressure was the only variable recorded and compared against the model, which in the simplified version consisted of two coupled fourth-order differential equations. The model was so inadequate that it could not predict the occurrence of sustained oscillations under certain operating conditions. It was conjectured that the oscillations were due to pump ripple which had been ignored in formulating the input. In the words of the authors, "The study presented can only be qualitative because of the large amount of guessing in the estimation of coefficients and in the amount of deduction used in analyzing the experimental results" (5, p. 215). A similar analysis is presented by Wolf (6) and Mrazek (8) gives a fifth-order model but no details regarding simulation or parameter identification.

Ma (7) presents an analysis for a single-stage pressure reducing valve. In this, apart from making all hydraulic resistances linear, the supply pressure is considered constant - an assumption not always justifiable. Transfer functions and block diagrams are presented to characterize the valve model. The incremental outflow and the spring preload were used as inputs and the controlled pressure was the output. Although the model is satisfactory for small perturbations, its validity for large inputs id doubtful. No less than 12 parameters were involved although the valve had only one moving element. The linearized model included six parameters which needed identification from experimental data.

It is, thus, seen that linear analysis while being a good beginning leaves much to be desired. Stability analysis using linearized equations in particular, is vulnerable to serious errors and the process of obtaining the static characteristics by setting all time derivatives to zero in such dynamic models may disguise the fact that the valve never reaches the steady state, due to the presence of non-linearities. Linear analysis also does not offer any special advantages for interfacing with other circuit components, except for single-input, single-output models.

The realization of the advantages of the classical time-domain analysis method and especially the adaptability of digital simulation to solving a set of coupled first-order differential equations laid the groundwork for non-linear analysis. The relative ease of setting up a dynamic model for systems and components is illustrated by Unruh (9). This formulation is one of the reasons for the widespread acceptance of system modeling (10) (11) (12) (13).

In the context of the present work, all attempts at component modeling (5) (8) (10) (7) have the following features:

(1) The component under study is not isolated from the rest of the system; i.e., the dynamic response portrayed is that of a valve in a particular system. Certain upstream and downstream characteristics (usually capacitances) are included as part of the model, sometimes explicitly, more often implicitly.

(2) The model is not the most general form in that certain inputs are ignored; e.g., downstream pressures are assumed constant for relief valves and upstream pressures are held constant for pressure reducing valves.

(3) A large number of parameters are involved (typically geometrical dimensions of moving parts, orifices, clearances,

and fluid properties) not all of which are amenable to direct measurement. The significance and relative importance of these parameters are not always obvious.

(4) A large number of coupled differential equations are needed to describe the system. Each moving element requires two first-order equations to characterize its dynamics, and continuity equations are generally of the first order when compressibility effects are considered. Long transmission lines may require the use of distributed parameter models (14).

Though digital simulation using the classical technique has gained wide acceptance, its drawbacks have not been overlooked. Computation time and effort have sometimes been large enough to induce the system analyst to look for a means of simplifying the model. Because of the presence of non-linearities it is not possible to segregate the system into fast-response and slow-response components. The presence of high frequency oscillations in certain state-variables has necessitated the selection of step sizes even as small as ten micro-seconds so that numerical integration routines would not go unstable. Simplification of the model to one of lower order or the formulation of approximate algebraic models could not proceed without an initial 'exact' analysis for comparison (11). Thus, for each design of a valve one had to develop the classical model before attempting to simplify it to a form where it could be conveniently interfaced with other circuit components.

Appendix A presents the digital simulation of a two-stage relief valve by the classical method. Both static and dynamic characteristics are included. The conclusions which can be drawn from this classical model are typical of any simulation using the same method, and can be summarized as follows:

(1) The dynamic model is a set of coupled differential-cum-algebraic equations and even if the only quantities of interest are the input and outputs, all the intermediate variables must be obtained.

(2) The significance of various physical parameters is not apparent. Repeated solutions would have to be performed changing one parameter at a time to evaluate their affect on performance.

(3) The static model is a set of non-linear equations, and neither pressure differential nor flow can be expressed explicitly in terms of the other. The solution of such equations requires iterative techniques.

(4) Computational time is prohibitively large, especially for dynamic simulation.

It must be emphasized, however, that classical models of varying degree of complexity can be developed by the exercise of suitable judgment on the part of the designer. In the context of this work, 'classical' refers to lumped-parameter models which would be obtained by following the analysis procedure detailed in reference (4).

CHAPTER III

DEVELOPMENT OF THE NEW MODELING APPROACH

General Considerations

The detracting features of the classical modeling technique summarized at the end of Chapter II and the absence of any established procedure for simplifying the models are enough incentive to review the philosophy underlying the procedure. It is noted that no attempt has been made to differentiate <u>component</u> modeling from <u>system</u> modeling. It can be argued that every component can be considered as a system in itself and there is no need to establish a distinction. Nevertheless there are some features, which when recognized early enough in the modeling process, yield valuable insight into the nature of problems confronting the development of component models. Some of these features are worthy of review.

In simulating a system, the inputs are usually obvious. For electro-hydraulic and hydraulic systems, electrical and mechanical inputs are the most common. These inputs are usually of a low power level and the output impedance on the actuator is small so that inputs of a special kind (e.g., step, ramp, or sinusoidal) can be readily imposed. Although these inputs are particularly convenient for linear analysis, their amenability to mathematical description makes them useful for more general analysis, using digital simulation. The choice of inputs, for components, is more complex. The process of isolating the components from the rest of the system leaves it with two or more energy ports, each of which introduces a through variable and an across variable for analysis. One of the variables has to be considered an input and for some ports the selection cannot be established on a cause-effect relationship. Thus, for valves with an internal feedback path either the pressure differential or flow may be treated as the input. The mathematical relations used for analysis make no distinction in the selection as long as a one-to-one correspondence exists between the variables.

The arbitrary selection of the input for an isolated model is also indicated by the fact that in experimental work the human or electrical input for dynamic testing is one imposed on the test <u>system</u> which, in turn, changes the through and across variables for the test <u>component</u>. The <u>system</u> characteristics (e.g., upstream capacitance, line resistance, etc.) influence the changes in flow and pressure so that it is difficult to impose arbitrary inputs on the component.

Another consequence of isolating the component is that models have to be formulated as for multi-input systems. Thus, analyses in which constant upstream or downstream pressures are assumed lack generality and are of limited use. Even though a particular system may warrant such assumptions a general analysis should not ignore any inputs.

Component models, if they are to be useful for system synthesis, must be such that the performance of the valve can be readily deduced from the model parameters. Comparison of alternative designs is easy if the same form of model is used for all designs. Using the classical technique for modeling gives a unique model for each design and comparison of designs can be made only after simulation under identical circumstances.

Any approach to the development of valve models can be considered promising if it includes the following considerations:

(1) Models should be general for the type of valves under consideration. Thus, all valves which a system designer would consider interchangeable should have the same form of model irrespective of individual differences in design.

(2) A minimum number of parameters must be involved. Hence, geometric parameters like spring rates, clearances, etc., which are design-dependent should be avoided.

(3) All valves of the same class must have the same inputs. This greatly facilitates interfacing with other system components.

(4) The models should permit refinement and the more accurate models should still use the parameters contained in initial forms of the model.

The idealistic black-box approach to modeling does have some of the above features. The primary objective of such an approach is to formulate relationships between the inputs and the outputs based entirely on experimental data. Since the component is treated as a black-box, details of internal construction do not influence the formulation of the model, nor is it possible to incorporate any variables, internal to the component, in the model. Since any number of dynamic relationships can be made to yield almost identical responses to a given input, the model needs to be validated by imposing different inputs and comparing predicted and actual responses. The variety of inputs that can, thus, be imposed is theoretically infinite, and as a consequence, judgment has to be exercised in selecting the appropriate inputs for verification.

Such black-box models, however, lack generality in that they have been developed with the objective of describing a specific valve. The grey-box approach proposed here, on the other hand, attempts to formulate general models for a given class of valves irrespective of design differences. Even though details of construction of the valves are not considered, the basic mechanicsm of action is used to develop the relationships between the inputs and the outputs. Thus, the incorporation of the fundamental mechanism insures the generality of the models. The parameters in the model have, in general, unknown functional relationships to the geometrical constants, fluid properties, and other quantities which would be present in a classical model. The grey-box approach does not attempt to establish these relationships. It, however, does make parameters in the models meaningful to the system designer.

Emphasis in this discussion has so far centered on dynamic performance. Nevertheless, static characteristics are also useful to the system designer. For self-regulatory valves these characteristics are flow-pressure relationships. It is shown in Appendix A that it is not possible to develop simple explicit expressions for either flow or pressure using the classical technique. However, the new approach leads to simple relationships which can be solved explicitly for any one of the variables. In fact it will be demonstrated that it is not necessary for the static model to be derived from the dynamic model. The basis for the grey-box approach is the relationship between the through and across variables of a multi-port element. In the case of a self-regulatory valve, this relationship can be described in terms of an equivalent orifice.

The Equivalent Orifice Representation

Since all fluid power valves perform the necessary control function by varying the resistance of a flow path, it is convenient to treat all such control elements with one inlet and one outlet as variable area orifices. Appendix B contains a discussion relative to the orifice equation and its use in the development of fluid power system models. For the discussion that follows, any established functional relationship for an orifice, which can be described by the following equation, is valid.

$$Q = f_1(P, A_x, C_d, \rho) \tag{3.1}$$

where

Q = Flow through the orifice

P = Pressure differential across the orifice

A_x = Cross-sectional area of orifice

ρ = Fluid density

C_d = Coefficient of discharge

and f_1 is the functional notation for the relation.

An equation such as Equation (3.1) can be represented in three-dimensional rectangular coordinate space with P, Q, and A_x along the three axes. It is convenient, at this stage, to consider C_d as a constant; in any case, it cannot be treated as an independent variable. Thus, even though it may not be possible to algebraically invert Equation (3.1) to the form

$$A_x = f_2(Q, P, C_d, \rho) \tag{3.2}$$

it is still possible to obtain the same result graphically.

Usually, there is only one moving element controlling the orifice area and considerations of the geometrical features of the valve will permit the writing of the equation

$$A_x = f_3(x) \tag{3.3}$$

where

x = displacement of the moving element

and f_3 is the functional notation describing the relation. It can be noted that as long as the orifice is effectively metering flow, the existence of the inverse relationship

$$x = f_3^{-1}(A_x)$$

is established. The geometrical relation described above may be written as

$$x = f_4(Q,\ P,\ C_d,\ \rho). \tag{3.4}$$

Consequently, the characteristic equation for the orifice can be represented uniquely in three-dimensional space with P, Q, and x as the axes (see Figure 1). The importance of this form of expression for the valve characteristics cannot be over-emphasized. It basically permits the description of the movement of the metering element in terms of the through and across variables for the valve. This, in turn, permits the development of the new approach to modeling, wherein attention is focused on the through and across variables rather than the dynamics of the internal parts of the valve.

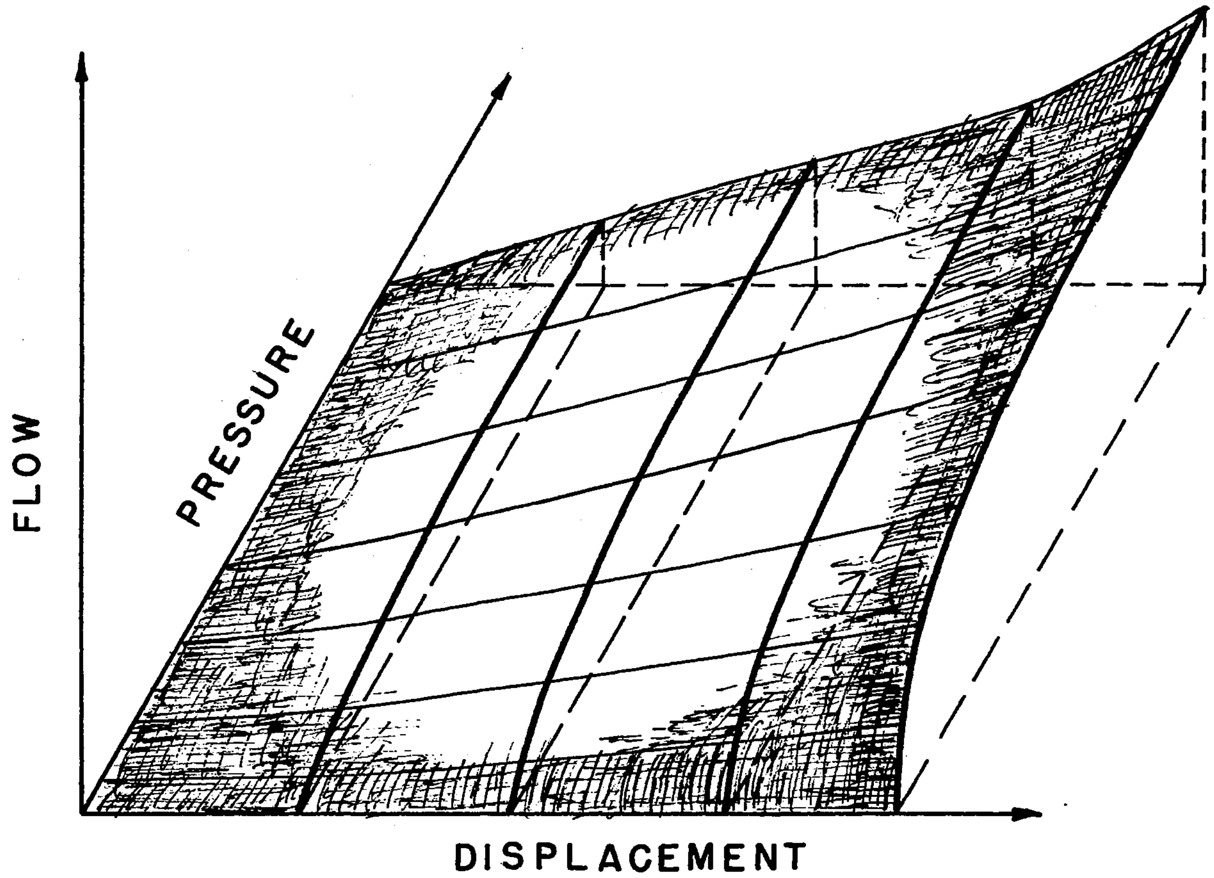


Figure 1. Characteristic Surface for a Variable-Area Orifice

The Grey-Box Modeling Approach

The objective in formulating the grey-box modeling approach was to obtain low-order relationships to describe the dynamics of system components. These relationships must reflect the inherent nonlinearities of fluid power components in order to be usable for the synthesis of realistic system models. In addition, the component models must possess forms which are particularly suited for interfacing with each other.

In order to apply the grey-box modeling approach to any system component, it is necessary to have sufficient expressions relating the measurable through and across variables, to mathematically eliminate all intermediate state-variables which cannot be measured.

In the case of a two-port self-regulating valve, the equivalent orifice representation must be one of the basic expressions in the model. The other expressions needed for the model may be derived from force balance considerations. For any moving element the forces can be categorized as follows:

(1) Pressure forces on the element - these are functions of upstream and downstream pressures.

(2) Spring forces - these are functions of displacement and are generally taken to be linear.

(3) Flow forces - these are dependent on pressure differentials and flow in addition to geometric features of the valve. The common assumption that steady-flow forces are equivalent to a linear spring, and unsteady forces to a linear damping effect are valid only for constant pressure differentials.

(4) Drag forces - these are generally considered to be velocity dependent.

Mathematical expressions for these operative forces ultimately need to be written in terms of measurable through and across variables. The elimination of the intermediate state-variables can be illustrated for the case of a viscous drag force in the following manner.

The viscous drag force on a moving element is classically represented by the expression:

$$f_{vd} = B\,\dot{x} \tag{3.5}$$

where

f_{vd} = drag force

B = drag coefficient

$\dot{x}$ = velocity of moving element.

The elimination of state-variable $\dot{x}$ can be accomplished by using the equivalent orifice expression (Equation 3.4). The differentiation of Equation 3.4 yields:

$$\dot{x} = \frac{\partial x}{\partial Q}\dot{Q} + \frac{\partial x}{\partial P}\dot{P}.$$

Now $\frac{\partial x}{\partial Q}$ and $\frac{\partial x}{\partial Q}$ are partial derivatives whose values can be obtained directly from the characteristic surface of the valve (Figure 1). Substituting this expression for $\dot{x}$ into Equation 3.5 yields:

$$f_{vd} = B\left(\frac{\partial x}{\partial Q}\dot{Q} + \frac{\partial x}{\partial P}\dot{P}\right). \tag{3.6}$$

From an idealistic standpoint, the values for the partial derivatives in Equation 3.6 should be obtained from the characteristic surface of the component as illustrated in Figure 1. However, for self-regulatory valves, the effect of feedback (interactions between the variables) is to cause the displacement to be dependent upon pressure and flow. Thus, this feedback precludes the development of the characteristic surface from measurement of through and across variables only. Consequently, it is necessary, for the case of self-regulatory valves, to utilize an auxiliary means of obtaining the values for the partial derivatives. This can be accomplished using the classical orifice equation in conjunction with the geometrical relationship Equation 3.3. In particular the orifice equation for flow can be written as

$$A_x = \frac{Q\sqrt{\rho/2}}{C_d\sqrt{P}} \tag{3.7}$$

and a geometrical relationship for Equation 3.3 of the form

$$x = KA_x . \tag{3.8}$$

$$\dot{X} = \frac{K\sqrt{\rho/2}}{C_d}\left[\frac{\dot{Q}}{\sqrt{P}}\,\theta - \frac{1}{2}\frac{Q\dot{P}}{P^{\frac{3}{2}}}\right] = \frac{K\sqrt{\rho/2}}{C_d}\left[\dot{Q}\sqrt{P} - \frac{1}{2}\frac{Q\dot{P}}{\sqrt{P}}\right]$$

where

K is a constant.

Equations (3.6), (3.7), and (3.8) can be combined to give

$$\dot{x} = \frac{k\sqrt{\rho/2}}{C_d}\left[\dot{Q}\sqrt{P} - \frac{Q\dot{P}}{2\sqrt{P}}\right]\frac{1}{P^2} . \tag{3.9}$$

The expression for the viscous drag force can now be written in terms of the through and across variables only, as

$$f_{vd} = \frac{BK\sqrt{\rho/2}}{C_d}\left[\dot{Q}\sqrt{P} - \frac{Q\dot{P}}{2\sqrt{P}}\right]\frac{1}{P^2} . \tag{3.10}$$

Similar expressions can be derived for any other force relationship. However, interest in component modeling, is not so much in evaluating velocities and drag forces, as in establishing relationships between the through and across variables, P and Q and their derivatives. The next two sections of this chapter illustrate the development of such grey-box expressions for a relief valve and the last, an example development of grey-box models for multi-port valves.

Static Grey-Box Models for Relief Valves

The static characteristics of relief valves are conveniently depicted by plots of pressure drop across the valve versus flow. For an ideal valve this would be a horizontal line at the set pressure (Figure 2) but actual valves depart from this ideal - especially so at very low and very high flow rates. At low flow rates, the valve begins to crack and the corresponding pressure may be much lower than the set pressure. At extremely high flow rates, the valve may function as an orifice of fixed area, although some valves show a tendency to go into sustained oscillations. Between these two extremes of flow conditions, most valves show a positive gradient of pressure versus flow.

In developing the static grey-box model for a relief valve the following assumptions will be made:

(1) Changes in fluid properties are negligible.

(2) All moving parts reach a state of equilibrium which is unique for the combination of flow rate and pressure differentials - this rules out limit cycling conditions.

(3) Flow occurs from the pressure port to the tank port only.

(4) Pressure forces on the moving element are directly proportional to the pressure differential across the valve.

(5) Flow forces are proportional to the flow rate and the square root of the pressure differential. This is, at

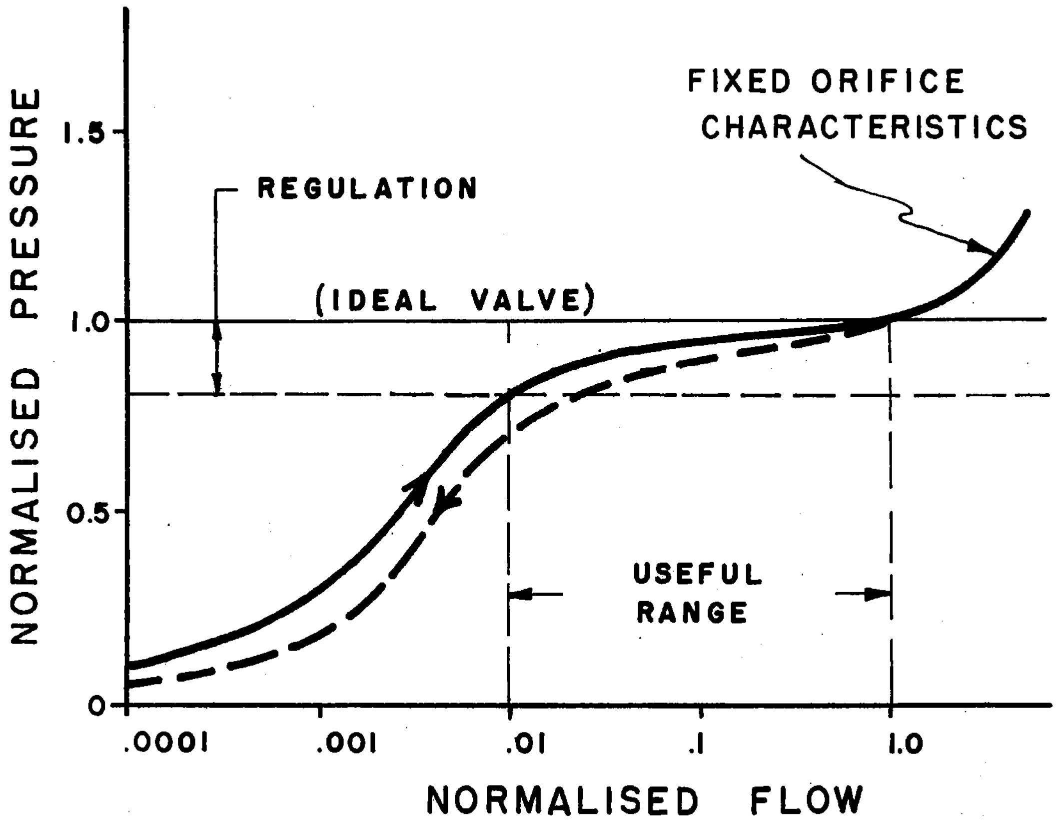


Figure 2. Typical Static Characteristic of a Relief Valve

best, an approximation for steady flow forces. A more rigorous expression can be written if details of construction of the valve are available.

(6) A linear spring furnishes the balancing force.

A force balance equation for the moving element can be written as:

$$\underset{\text{Pressure Force}}{\alpha_1 P} + \underset{\text{Flow Force}}{\alpha_2 Q\sqrt{P}} \pm \underset{\text{Spring Preload}}{\alpha_3} = \underset{\text{Spring Force}}{\alpha_4 x} \tag{3.11}$$

where α_1, α_2, α_3 and α_4 are constants of proportionality. Using Equations 3.7 and 3.8 the above equation can be re-written as

$$\alpha_1 P + \alpha_2 Q\sqrt{P} \pm \alpha_3 = \alpha_4 K\sqrt{\rho/2}\,\left(\frac{1}{C_d}\right)\frac{1}{\sqrt{P}}\,) \tag{3.12}$$

or

$$Q = \left(\frac{k_1 P + k_3}{\sqrt{\rho/2} - k_2 P}\right)\sqrt{P}$$

where k_1, k_2, and k_3 are empirical constants that can be identified from experimental data.

The results of a computer simulation of the static characteristics given by the algebraic model (Equation 3.12), using five sets of parameters, are presented in Figures 3 and 4. Table I summarizes the values of the parameters used. The quantity $(-k_3/k_1)$ is referred to as the cracking pressure of the valve and ρ/k_2 as the asymptotic pressure.

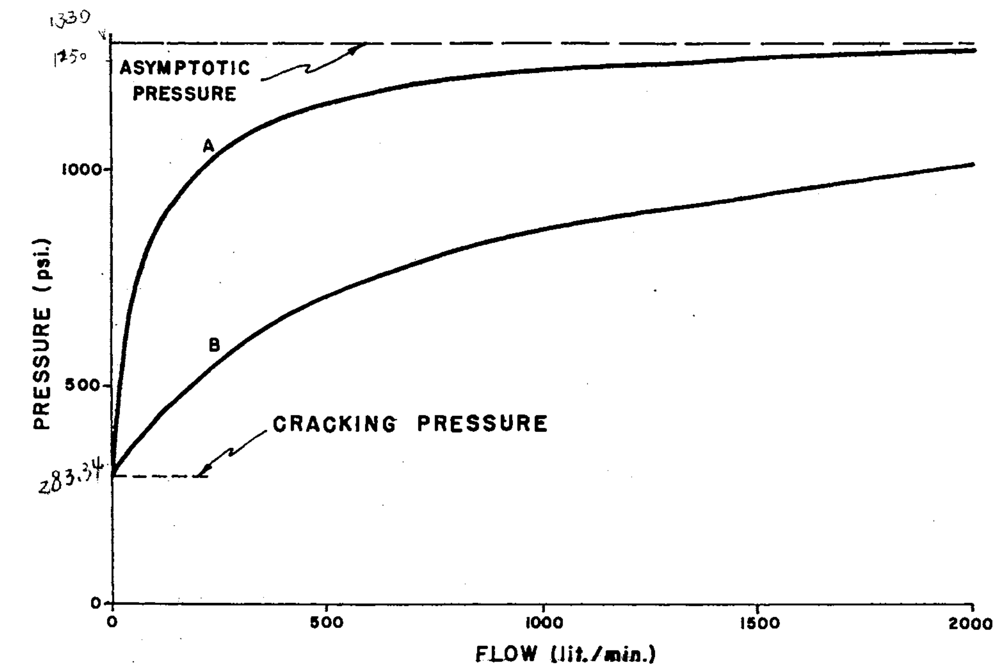


Figure 3. Simulation of Static Characteristics of a Relief Valve, With Constant Cracking Pressure

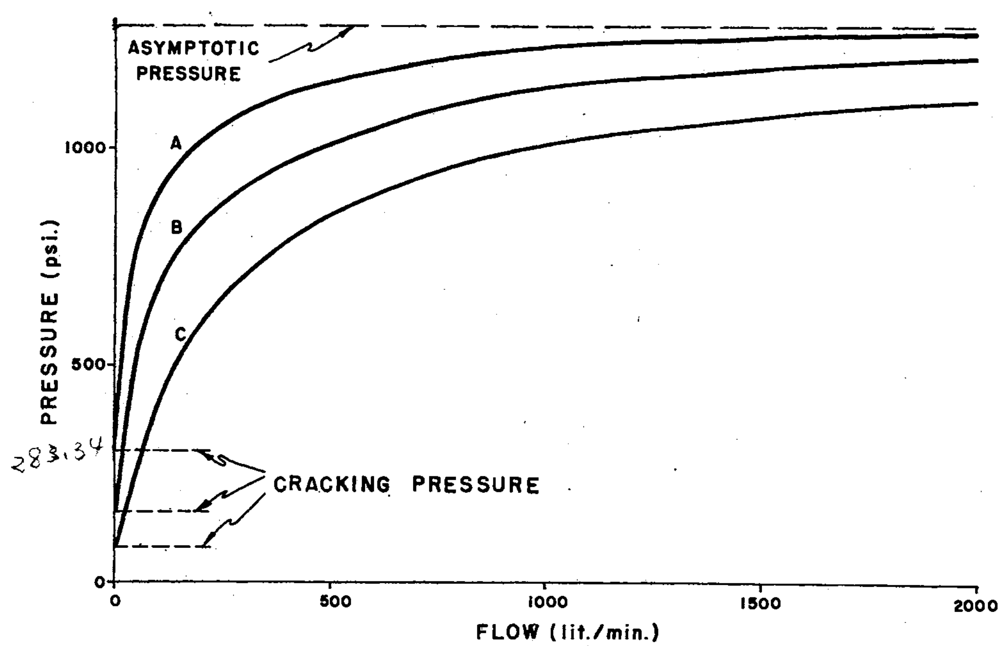


Figure 4. Simulation of Static Characteristics of a Relief Valve, With Various Values of k_1

TABLE I

PARAMETERS USED FOR SIMULATING STATIC CHARACTERISTICS FOR RELIEF VALVES

Figure	Curve	k_1	k_2	k_3
3	A	1.26×10^{-5}	4.85×10^{-6}	-3.57×10^{-3}
	B	2.52×10^{-5}	4.85×10^{-6}	-3.57×10^{-3}
	C	5.04×10^{-5}	4.85×10^{-6}	-3.57×10^{-3}
8	A	1.26×10^{-5}	4.85×10^{-6}	-3.57×10^{-3}
	B	1.26×10^{-4}	4.85×10^{-6}	-3.57×10^{-2}

It can be seen that smaller values of k_1 not only lead to higher cracking pressures but also to better regulation. For the same cracking pressure, numerically smaller values of k_1 and k_3 also lead to better regulation. It may be noted that k_2 was left unaltered so that the asymptotic pressure for high flows would be the same in all cases. It is interesting to note that by varying parameters k_1, k_2, and k_3 the model can be made to fit a wide variety of characteristics. The models given by Equations 3.12 compare favorably with actual characteristics for similar valves presented by Foster (5), Smolina (15), and Ford-OSU (11).

Dynamic Grey-Box Models for Relief Valves

Even though the usefulness of a static model is apparent, it can be asserted that a dynamic model gives the fullest possible description of the performance of a valve. Dynamic effects in self-regulatory valves arise due to two main effects: firstly the dynamics of the moving parts and secondly the capacitance and inertance of the fluid in the system. Though it is fairly easy to treat the former as lumped elements, the latter are more difficult to handle. Also, the interactions between the fluid and the moving elements can often be subtle. Thus, exceedingly small movements of an element can appreciably change a control volume and affect its capacitance seriously, and flow changes can drastically affect the balance of moving elements, even to the extent of driving them unstable.

In the development of empirical models in which only through and across variables are to be present, the problem is to hypothesize a functional relationship

$$f(P, P^1, P^2 .. P^n, Q, Q^1, Q^2 .. Q^m) = 0$$

where

P = pressure differential

Q = flow

and superscripts 1 through n and m denote time derivatives. Setting these derivatives to zero would yield a static model.

Purely empirical black-box models can be derived by inspecting the observed characteristics and proposing models which may, for example, involve linear combinations of the measurable variables and their derivatives (11). Although they may give excellent correlation for the observed responses, they will carry more assurance of generality if considerations of the mechanism of the valve have been made in their selection. This is the rationale for the grey-box approach. Dynamic models, by their vary nature, have to include $\dot{P}$ and $\dot{Q}$. Even though it may be possible to develop models with higher derivatives, difficulties in measuring these quantities experimentally, precludes their use.

The approach taken in this investigation was to consider a single dynamic effect and translate it into a suitable relationship between measurable through and across variables. Since the effect was to be such that it related exclusively to the valve, considerations of up-stream and downstream capacitances and their changes were ruled out. The dynamics of the moving element were considered most suitable for use as the basis of the desired relationship.

Moving elements in a valve contribute both inertia and damping terms to a dynamic model. An expression for viscous drag was derived in Equation 3.6 requiring the partial derivatives $\frac{\partial x}{\partial Q}$ and $\frac{\partial x}{\partial P}$. It was also noted that for self-regulatory valves, with internal feedback, the displacement x is a function of P and Q, and the characteristic surface can be mapped only if there is provision to adjust the displacement of the moving element independent of P and Q. Since such adjustment is not permitted in grey-box identification, Equation 3.10 was used to give the drag force expression in terms of the measurable variables. It was also hypothesized that for slow changes in inputs the drag forces would be more dominant than inertia forces and the latter could therefore be omitted from consideration.

The above considerations led to the development of the following model:

$$\underbrace{\alpha_1 P}_{\text{Pressure Force}} - \underbrace{\alpha_2 \dot{x}}_{\text{Drag Force}} - \underbrace{\alpha_3 x}_{\text{Spring Force}} + \underbrace{\alpha_4}_{\text{Preload}} = 0 \qquad (3.13)$$

where α_1, α_2, α_3, and α_4 are constants of proportionality. Substituting for x and $\dot{x}$ in terms of β, Q, $\dot{P}$, and $\dot{Q}$ gives

$$\alpha_1 P - \alpha_2 K\sqrt{\rho}/2 \; \left(\dot{Q}\sqrt{P} - \frac{Q\dot{P}}{2\sqrt{P}}\right) \frac{1}{P} - \alpha_3 K\sqrt{\rho/2}\, \frac{Q}{\sqrt{P}} \mp \alpha_4 = 0. \qquad (3.14)$$

By setting the derivatives equal to zero, the following static model is obtained.

$$Q = \frac{1}{\alpha_3 K\sqrt{\rho/2}} \left(\alpha_1 P \mp \alpha_4\right) \sqrt{P} \qquad (3.15)$$

The form of the model given by Equation 3.15 differs from the static model derived earlier (Equation 3.12) in not having a term to account for flow forces. As a first approximation it can be considered that these flow forces are equivalent to a linear spring and experimental identification would include them in as 'Equivalent spring-rate' parameter.

Equation 3.14 can be solved for the pressure derivative by writing it as

$$\dot{P} = \frac{2\sqrt{P}}{Q} \left\{ \left[\alpha_3 \frac{Q}{\sqrt{P}} - \left(\frac{\alpha_1 P + \alpha_4}{K\sqrt{\rho/2}}\right)\right] \frac{P}{\alpha_2} + \dot{Q}\sqrt{P} \right\} \qquad (3.16)$$

This is a first order non-linear equation suitable for digital simulation. Since for a component model 'inputs' and 'outputs' are not clearly defined, the Equation 3.14 can be re-written as

$$\dot{Q} = \sqrt{P} \left\{ \left(\frac{\alpha_1 P + \alpha_4}{K\sqrt{\rho/2}} - \alpha_3 \frac{Q}{\sqrt{P}}\right) \frac{P}{\alpha_2} + \frac{Q\dot{P}}{2\sqrt{P}} \right\} \qquad (3.17)$$

It may be noted that Equations 3.16 and 3.17 involve only the measurable through and across variables and their first derivatives.

It may also be seen that if absolute pressures are to be calculated, the downstream pressure is an input to the model, in addition to Q.

Grey-Box Model for a Multi-Input Valve

To demonstrate that the grey-box approach is not limited in application to relief valves, or that force balance considerations necessarily have to be used for eliminating the intermediate state-variables, a system shown in Figure 5 will be analyzed. An upstream volume is included and flow occurs not only through the metering orifice but also past the moving element. It is convenient to treat the inflow and port pressure P_1 and P_2 as inputs to the system. The following notations will be used:

α_1, α_2, α_3, α_4, α_5	Empirical Constants.
β	Bulk Modulus of Fluid.
f_1, f_2	Functional Relationships.
P_1	Primary Port Pressure.
P_2	Secondary Port Pressure.
P_s	Upstream Pressure.
Q_{in}	Inflow.
Q_{pr}	Primary Port Flow.
Q_{sec}	Secondary Port Flow.
V_{up}	Upstream Volume.
x	Displacement of Moving Element.

A continuity equation for the upstream volume can be written as

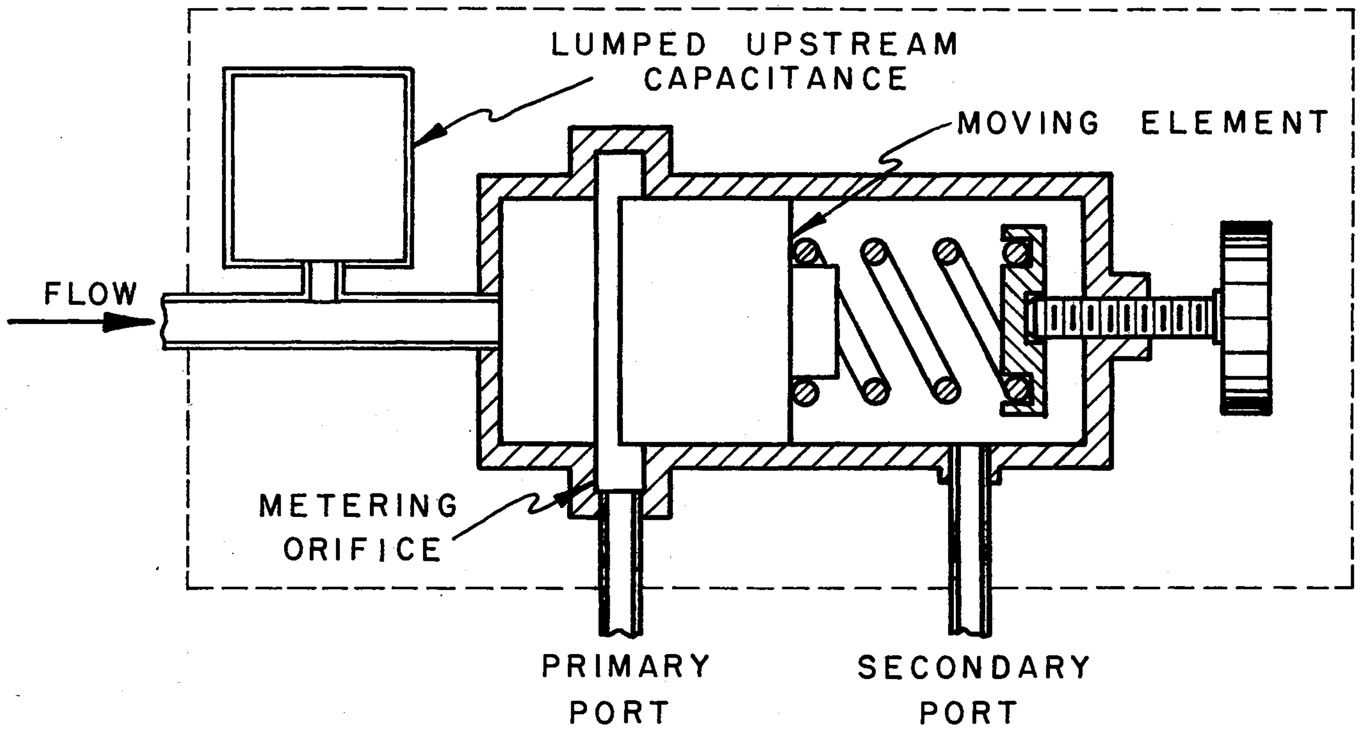


Figure 5. Schematic View of a Multi-Port Valve With One Moving Element

$$\dot{P}_s\left(\frac{V_{up}}{\beta}\right) = (Q_{in} - Q_{pr} - Q_{sec} - \alpha_1\dot{x}) \tag{3.18}$$

The last term on the right allows for the increase in the control volume due to the movement of the metering element. Flow equations can be written as

$$Q_{pr} = \alpha_2 x\sqrt{P_s - P_1} \tag{3.19}$$

$$Q_{sec} = \alpha_3\sqrt{P_x - P_2} \tag{3.20}$$

Both equations 3.19 and 3.20 are orifice flow equations wherein the coefficients of discharge and the fluid density are included in the empirical constants α_2 and α_3. If Q_{sec} is a flow through a very small clearance, the laminar flow equation could have been used in formulating Equation 3.20.

It may be noted that Q_{sec} is independent of movement of the metering element.

Equation 3.19 can be inverted subject to the assumption that the orifice effectively controls flow at all times. After inversion and differentiation it gives

$$\dot{x} = \frac{1}{2\alpha_2}\left[2\dot{Q}_{pr}(P_s-P_1) - Q_{pr}(\dot{P}_s-\dot{P}_1)\right]\frac{1}{(P_s-P_1)^{3/2}} \tag{3.21}$$

Substitution of expressions for Q_{sec} and $\dot{x}$ from Equations 3.20 and 3.21 into 3.18 gives

$$\dot{P}_s\left(\frac{V_{up}}{\beta}\right) = Q_{in} - Q_{pr} - \alpha_3\sqrt{P_s-P_2} - \frac{\alpha_1}{2\alpha_2(P_s-P_1)^{3/2}}\left[2\dot{Q}_{pr}(P_s-P_1)-Q_{pr}(\dot{P}_s-\dot{P}_1)\right] \tag{3.22}$$

Rearranging terms in Equation 3.22 gives

$$P_s\left(\alpha_4 - \frac{\alpha_1 Q_{pr}}{2\alpha_2(P_s-P_1)^{3/2}}\right) = Q_{in} - Q_{pr} - \alpha_3\sqrt{P_s-P_2} - \frac{\alpha_1(2\dot{Q}_{pr}(P_s-P_1) - Q_{pr}\dot{P}_1)}{2\alpha_2(P_s-P_1)^{3/2}} \qquad (3.23)$$

where

$$\alpha_4 = \frac{V_{up}}{\beta}\,.$$

Solving for rate of change of the supply pressure gives

$$\dot{P}_s = \frac{Q_{in} - Q_{pr} - \alpha_3\sqrt{P_s-P_2} - \frac{\alpha_5}{(P_s-P_1)^{3/2}}(2\dot{Q}_{pr}(P_s-P_1) + \dot{P}_1 Q_{pr})}{\alpha_4 - \alpha_5\frac{Q_{pr}}{(P_s-P_1)^{3/2}}} \qquad (3.24)$$

where

$$\alpha_5 = \frac{\alpha_1}{2\alpha_2}\,.$$

Equation 3.24 gives a relationship between the supply pressure rise rate $\dot{P}_s$ and the following quantities:

Q_{in} Inflow

P_1 Primary port pressure

P_2 Secondary port pressure.

Derivatives of the first two quantities are also required for purposes of simulation. It may be noted that α_5 is the upstream capacitance and not a part of the valve. Thus, the valve contributes only two parameters α_3 and α_4. It is thus demonstrated that the writing of a force balance equation is not necessary to establish the dynamic relationship between the through and across variables for the system.

The analysis also demonstrates that for any self-regulatory valve, it is possible to write dynamic equations relating the through and across variables for the energy ports. This grey-box approach can be extended to all valves having variable area orifices. It is not necessary that flow and pressure have to be the through and across variables, as has been the case for the valves in the development given above.

CHAPTER IV

EXPERIMENTAL VERIFICATION

The main purpose of the verification program was not so much to fit a model to a particular valve, as to confirm the validity of the form of the model. The black-box approach to modeling does not take into considerations any features of internal construction, with the result that there is no assurance that the empirical model developed for a valve will fit any other design. On the other hand, the grey-box models proposed here, are based on selected relationships used in classical modeling, and therefore, carry a higher assurance of generality.

Experimental Considerations

In order to put experimental verification in the proper perspective it is useful to consider some features influencing such work, not only for the models proposed but also for classical models.

Knowledge of Working Mechanism of the Valve

Classical models, being design-dependent, cannot be formulated without the help of working drawings indicating various geometrical parameters involved. A black-box approach, on the other hand, completely ignores such information. Grey-box models, as reported here, strike a compromise in that a qualitative description of the mode of operation is used as the basis for establishing the form of the model.

The grey-box model is capable of being refined by the inclusion of various design features, but is not dependent upon them.

Knowledge of the geometric relationship between the metering orifice area and the displacement of the moving element, is probably the most useful information in formulating the grey-box model. This is the basis for the development of the characteristic surface for a valve (Figure 1). Although it may be possible to establish such a relationship from geometrical considerations, it may be found useful to build a prototype valve for experimentally establishing the characteristic surface. It should be recognized that this does not mean that the prototype needs to behave like a regular design in the dynamic sense. In the absence of any information about the characteristic surface, a linear metering area-displacement relation is the most convenient to use. It is valid for poppet valves having small displacements, as well as for spool valves in the mid-range of their travel. This linear relationship has been used for the verification work.

Effect of Discontinuities in the Model

Classical models usually have constraints on state-variables in addition to discontinuities in algebraic relations. Thus, moving elements may have an idle travel before opening an orifice, springs may be in compression over only part of the travel, and most important of all, moving elements will have bounds for travel. Since all state variables are evaluated in the course of numerical integration, it is a comparatively easy matter to include constraints in the mathematical model for digital simulation. For grey-box models, the situation is more difficult in that no 'hard' constraints can be imposed on the variables (pressures and flow) present in the model except that neither can be negative. One way of ensuring that such negative quantities are not introduced in the simulation phase is to restrain all state-variables to a state-space region where a single model with no discontinuities exists. In effect this limits the magnitude of inputs which can be used for verification. The selection of inputs for this investigation was dictated by these considerations.

Tractability of Equations

The identification of parameters for classical models requires the solution of two types of equations: (1) complex algebraic equations in the case of static characteristics and (2) algebraic-cum-differential equations in the case of dynamic characteristics. Complex static models usually require iterative solution techniques or the use of algebraic minimization programs. Parameter identification from dynamic tests requires, in addition, the integration of all system differential equations. In contrast, static models developed by the grey-box approach lend themselves to conversion, to explicit relations for the parameters, in terms of the measured quantities. Also, the dynamic equations can be written as a set of algebraic equations with the parameters as unknown quantities. These can be solved to give a first approximation of the values of the parameters and, consequently, the time and effort used in dynamic simulation is reduced.

Instrumentation Limitations

Instrumentation for dynamic testing has received scant attention in the literature on valve modeling. Even where models of high order have been developed by the classical technique, verification has used only a few of the state-variables. The state-variables encountered in valve models are

(a) pressure

(b) flow

(c) displacement

(d) velocity.

Time derivatives of all four variables may be present in a dynamic model. It is, therefore, useful to consider the problems faced in measuring these quantities in fluid power systems.

The measurement of pressure, even under dynamic conditions, offers little difficulty. Transducers with diaphragm sensors are available with a frequency response that is flat to 10kHz and beyond, while piezo-electric transducers go far beyond this value. Dynamic testing often requires the measurement of fairly low pressure differentials (below 25 psi) and transducers which have been chosen to measure pressures of the order of 1000 psi are not too accurate for such low values. The measurement of pressures in small enclosed volumes such as those present in two-stage valves, requires the analysis of the change in the system parameters introduced by the instrumentation. None of the experimenters presenting simulation for two-stage valves considered the verification of state-variables other than outputs (5) (10)(11). The models proposed here avoid the issue by not considering such pressures or related flows, as they are internal to the valve, and do not appear in the relationships for the through and across variables, for the valve as a whole.

The occurrence of high frequency components superimposed on slower variations in the state variables is characteristic of most valves (5)

(11). The grey-box models developed here, aim to portray only the slower variations, and, thus, actual measurement can be restricted to frequencies below 1000 Hz. An important source of high frequency components is pump ripple. These fluctuations can be significant when pressure rates are being measured. Since the models are designed to ignore high frequency components, any rate measurements required for verification should remove these disturbances, otherwise they would contribute a large amount of background noise. Since pressure rate transducers normally do not have pre-filtering capacity, they are not useful for verification work.

Flow measurement is of crucial importance for the verification of the types of models under study, and the details of this aspect are contained in Appendix B. It will only be noted here that a target flowmeter was found most suitable for all dynamic tests. Numerical differentiation was used for obtaining the rate of change of flow.

The measurement of displacement and velocity is unnecessary for grey-box models of self-regulating valves, as they do not appear among the through or across variables at any port. Usually none of the active metering elements in such valves is accessible for instrumentation and the small size of the moving parts involved requires special care to ensure that the dynamics are not affected by the sensors. Such measurement, however, would be useful not only for refining grey-box models, but also for parameter identification in classical models.

All experimental activity for verification was directed towards the measurement of pressure and flow. Two main series of tests were conducted and the results are presented in the next two sections.

Static Tests

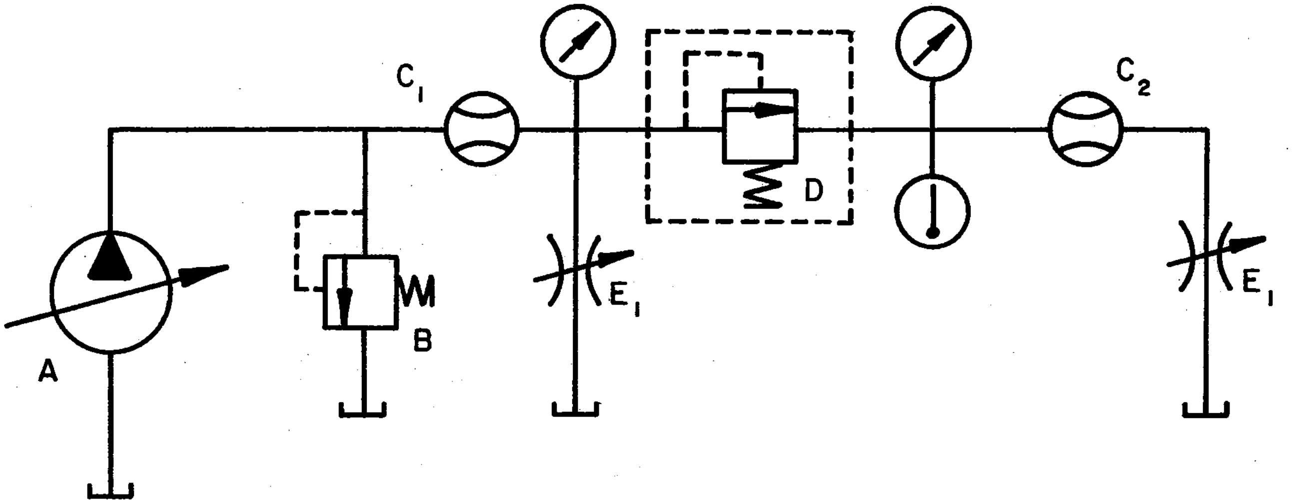
A schematic of the test set-up is shown in Figure 6 and a circuit diagram for the performance test-stand in Figure 7. Static tests were performed to obtain pressure differential-flow curves for a given pressure setting, as shown in Figure 2. Tests were conducted after the system achieved a steady-state operating temperature. Samples of fluid were frequently checked for cleanliness and a gravimetric level of 10 mg/liter and below was consistently maintained. The variable restriction $\mathbf{E}_1$ (Figure 6) and the pump displacement were adjusted to effect changes in flow rate through the test valve. Low flows (below 2 liters/minute) were measured by collecting fluid in a graduated cylinder over a known time interval. Care was taken to ensure that flow changes were made in one direction only; i.e., either increasing or decreasing, so that valve hysteresis would not invalidate the observations.

Since the proposed model, Equation (3-11) had the form

$$Pk_1 + Q\sqrt{P}\,k_2 + k_3 = \frac{Q}{\sqrt{P}} \tag{4.1}$$

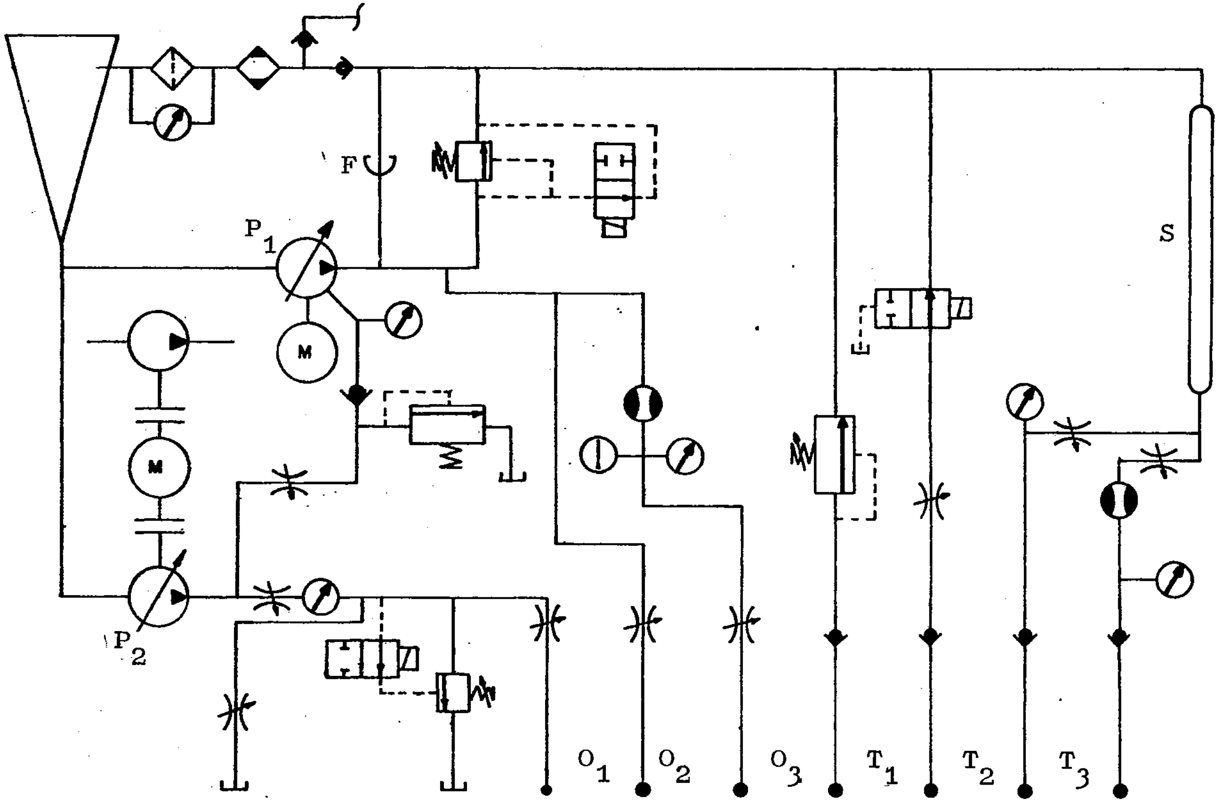
it was found that a least-squares fit of the experimental data gave unbiased values for the parameters k_1, k_2, k_3. With n observations of flow and the corresponding pressure, a matrix [A] and vectors $\underline{B}$ and $\underline{K}$ were set up as follows:

$$A = \begin{array}{ccc} P_1 & P_1\,Q & 1 \\ P_2 & P_2\,Q_2 & 1 \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ P_n & P_n\,Q_n & 1 \end{array}$$



A Variable Displacement Pump
B System Relief Valve
C_1, C_2 Flowmeters
D Test Valve
E_1, E_2 Needle Valves

Figure 6. Schematic for Static Tests



F	Hydraulic Fuse
P_1	Main Pump
P_2	Pilot Pump
S	Sight Glass
O_1, O_2, O_3	Outlets for Test Equipment
T_1, T_2, T_3	Return Connexions

Figure 7. Circuit Schematic for Valve Tests

$$\underline{B} = \begin{bmatrix} \frac{Q_1}{P_1} & \frac{Q_2}{P_2} & \cdots & \frac{Q_n}{P_n} \end{bmatrix}^T \quad \text{and} \quad \underline{K} = [k_1 \quad k_2 \quad k_3]^T$$

The IBM subroutine LLSQ (16) was used to identify the parameters.

For Valve #RV-22 a much better fit was obtained by solving three simultaneous equations using three sets of data from experimental results. Table II and Figures 8 to 10 present the results of static tests. The cracking pressure, P_c defined as the lowest pressure at which the valve allows flow, is given by

$$P_c = -(k_3/k_1)$$

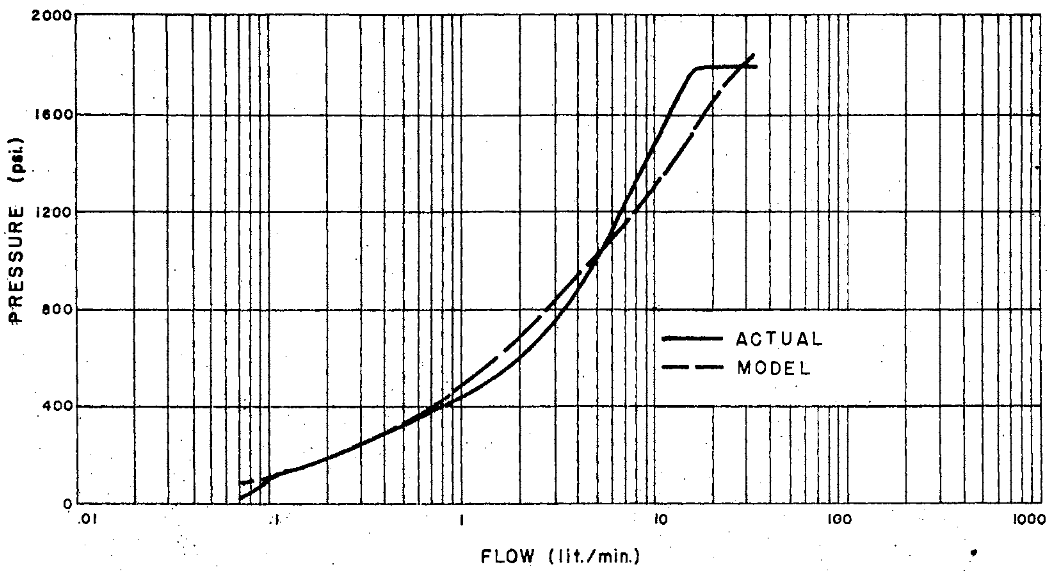
whereas the maximum pressure P_m is given by

$$P_m = 6.36 \times 10^{-6}/k_2.$$

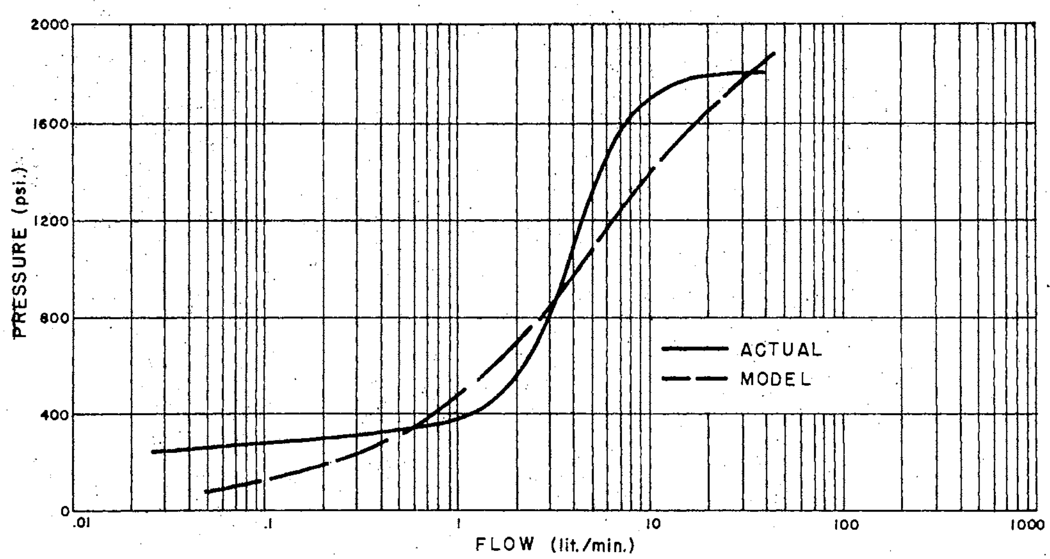
The model is valid only for pressures between these two values.

It is instructive to compare the test results for Valve #RV-21 with the simulation results presented in Table I and Figures 3 and 4. Values of parameters for curves labelled 'A' in the latter are the same as for Valve #RV-21.

Apart from exhibiting a very low cracking pressure, Valve #RV-9 also showed limit cycling at flow rates beyond 25 liters/minute. Valve #RV-22 indicated substantial hysteresis in that the cracking pressure was much less for decreasing flows than for increasing flows. Within the operating range of 10% to 100% of the rated flow, a maximum error of 10% (discrepancy in pressure, expressed as a fraction of actual pressure) is exhibited by the models. Such accuracy is within the limits of experimental error for measurements in the range of values handled. It should be noted that the static models do not consider any flow path

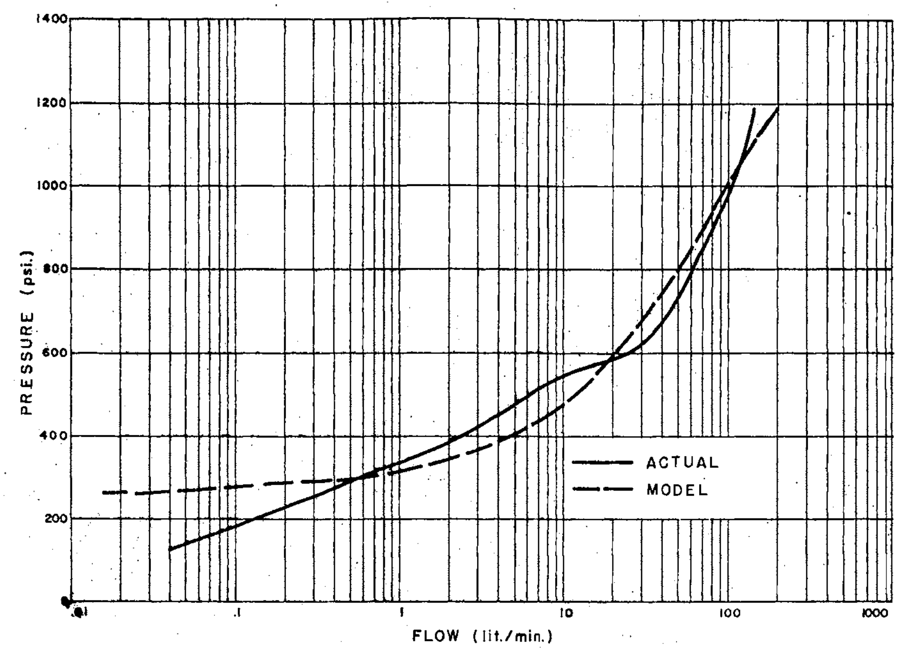


(a)

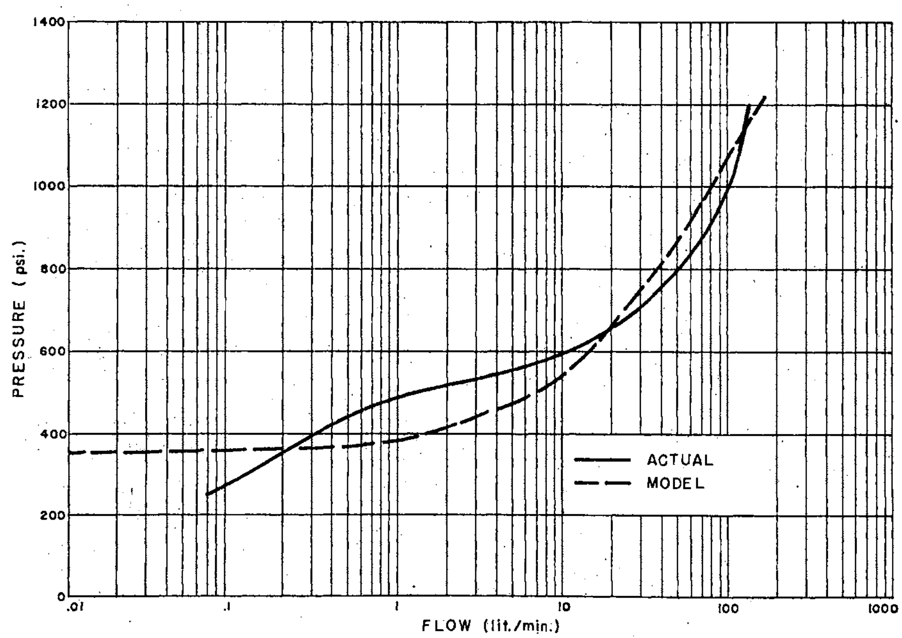


(b)

Figure 8. Static Characteristics for Valve #RV-9 for (a) Increasing Flow, and (b) Decreasing Flow

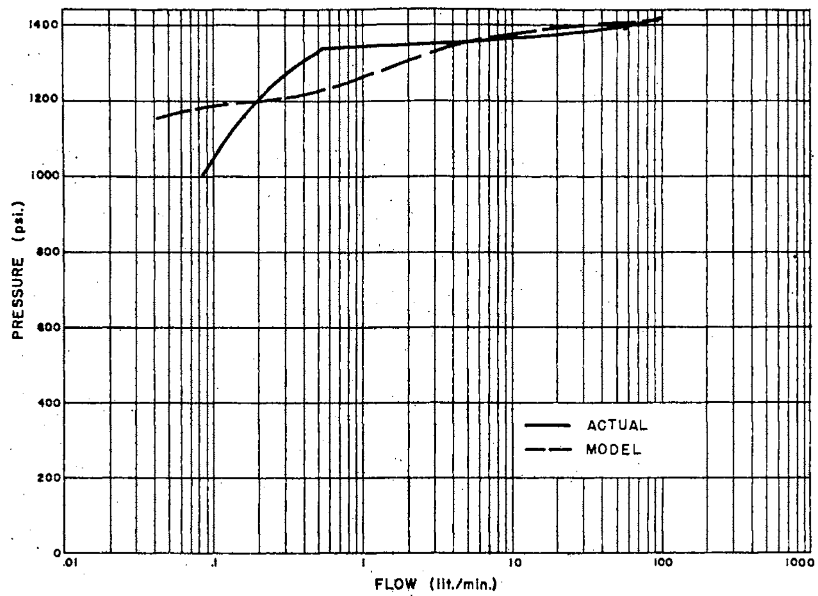


(a)

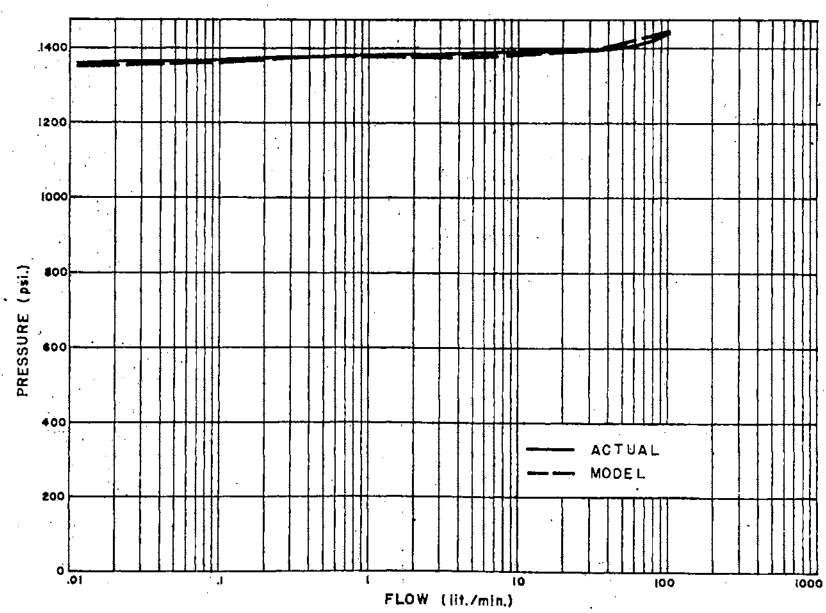


(b)

Figure 9. Static Characteristics for Valve #RV-21 for (a) Increasing Flow, and (b) Decreasing Flow



(a)



(b)

Figure 10. Static Characteristics for Valve #RV-22 for (a) Increasing Flow, and (b) Decreasing Flow

TABLE II

SUMMARY OF STATIC MODEL PARAMETERS

Valve	Flow	k_1	k_2	k_3	P_c*	P_m**
#RV-9	Increasing	4.24×10^{-7}	3.21×10^{-6}	0	0	2000
	Decreasing	4.82×10^{-7}	2.93×10^{-6}	0	0	1870
#RV-21	Increasing	1.26×10^{-5}	4.85×10^{-6}	-3.57×10^{-3}	283.3	1310
	Decreasing	1.26×10^{-5}	4.59×10^{-6}	-4.53×10^{-3}	360	1385
#RV-22	Increasing	9.09×10^{-6}	4.36×10^{-6}	-1.25×10^{-2}	1370	1460
	Decreasing	4.04×10^{-6}	3.43×10^{-6}	-5.05×10^{-3}	1250	1850

*P_c = Cracking pressure = $-k_3/k_1$

**P_m = Maximum pressure = $.00636/k_2$

other than the metering orifice. Also, a linear metering orifice-displacement relationship is assumed in the analysis. It is difficult, in the absence of detail drawings, to estimate the errors introduced by these assumptions.

Valve #RV-9 was a poppet valve, #RV-21 a spool valve, and #RV-22 a compound relief valve. The good correlation between the experimental results and the models, thus, confirms the validity of the form of the model for all commonly used designs of relief valves. It is possible to improve the models, by considering individual design features while developing the pertinent equations. It will be shown in the next section that it is possible and desirable to include parameters identified from static models in dynamic models.

Dynamic Tests

Apart from the instrumentation needed for the continuous recording of pressures and flows, the test-stand had to be arranged to permit the application of fast and slow inputs. Directional control valves (both solenoid and manually operated) were included in a bypass line as shown in Figure 11. An accumulator was introduced upstream in order to dampen pump ripple. Since only through and across variables for the test valve are present in the model it was not necessary to either measure or adjust upstream and downstream capacitances or resistances. This may be compared with the classical modeling technique in which upstream and downstream capacitances are usually part of the model and are estimated for simulation purposes.

As explained in Chapter III it is not possible to impose inputs of any arbitrary ~~desired~~ shape. The speed of operation of the solenoid

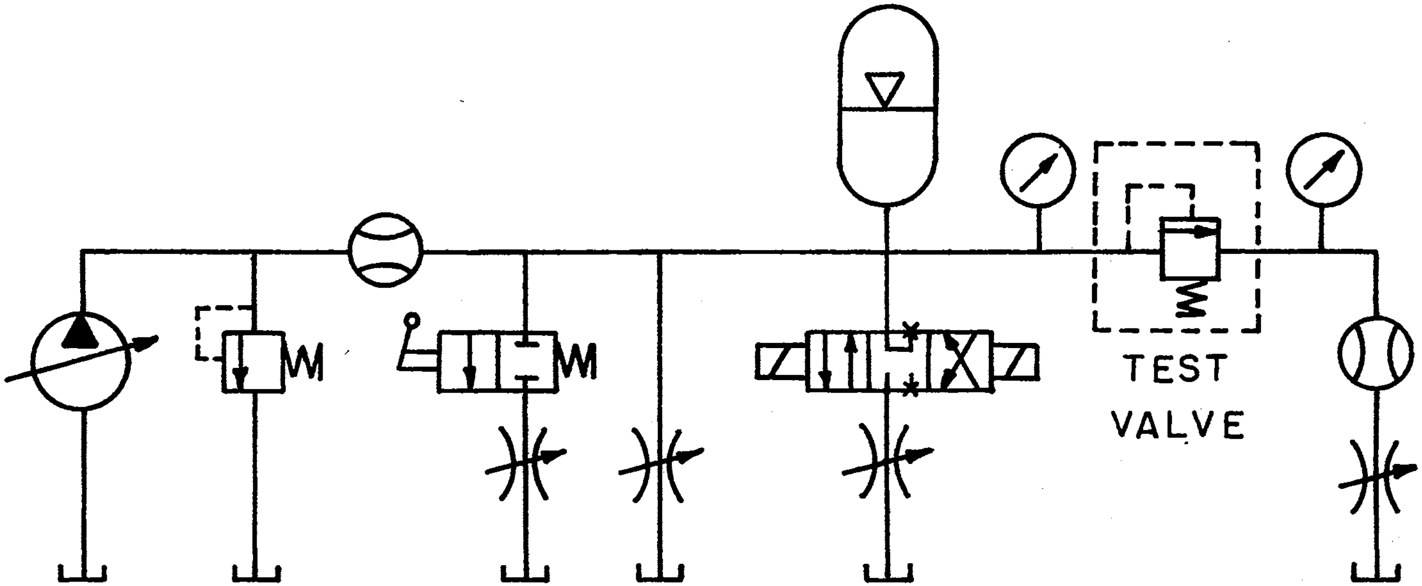


Figure 11. Circuit Schematic for Dynamic Tests

operated valve could not be adjusted and inputs were changed by changing the resistance of the parallel bleed-off. Apart from restrictions due to its dynamic response, the flow-meter could not be considered accurate for low flows (below 10 lit/min). Both flow and pressure were recorded continuously using a recording oscillograph. No attempt was made to process these signals to directly obtain their time derivatives.

Preliminary investigations showed that the magnitude of pressure changes during the transient phase was small as compared to flow changes, Thus, it was decided to treat flow and flow rate as the inputs to the dynamic model and pressure differential as the output. This reduced chances of numerical integration going unstable due to small errors in read-off from recordings. The target flow-meter behaves basically as an underdamped second-order system. The natural frequency of the one used was 200 Hz. Hence, a step size of 0.01 secs was considered to be the smallest which could be used for numerical differentiation. As the models to be verified were non-linear, there was no convenient yard-stick like a 'natural frequency' to select step sizes for integration. Inputs were chosen such that a steady state was reached in 0.20-0.30 secs. Rise time for 90% of steady-state value had to be kept above .04 secs as otherwise flow-meter readings were inaccurate. The results of the dynamic verification tests are presented for each valve below.

<u>Valve #RV-10</u>

The dynamic model given by Equation (3.14) can be re-written as

$$k_1 P - k_2 \sqrt{\rho/2}\,(\dot{Q}\sqrt{P} - Q\dot{P}/2\sqrt{P})/P + k_3 = Q/\sqrt{P} \qquad (4.3)$$

where

P = Pressure differential across valve

Q = Flow through valve

ρ = Fluid density

and k_1, k_2, and k_3 are empirical constants.

Assuming that downstream pressures are low and stationary, P can be considered as the upstream pressure. Using the recorded values of upstream pressures and flows, $\dot{P}$ and $\dot{Q}$ were obtained for the duration of the transient by applying a numerical differentiation program (DGT3) given in the SSP manual (16). The program DGT3 uses parabolic interpolation to evaluate derivatives. Subsequently k_1, k_2, and k_3 were identified using a least squares fit as was done for static characteristics. It was found that the selection of a number of points after the steady state had been reached tended to reduce the rank of the 'A'-matrix and the sub-routine would give erroneous results. Values of k_1, k_2, and k_3 thus identified were used as initial guesses for fitting to the dynamic model presented in Chapter III

$$\dot{P} = \frac{2\sqrt{P}}{Q}\left[\dot{Q}\sqrt{P} - \left(\frac{Q}{\sqrt{P}} - \frac{Pk_1 + k_3}{\sqrt{\rho/2}}\right)\frac{1}{k_2}\right] \tag{4.4}$$

Subroutine OPTIM of the digital simulation package DYSIMP (17) was used to identify the parameters to a higher degree of accuracy. An integral squared error performance index of the following form was used:

$$\text{P.I.} = \int_{t_o}^{t_f} (P_{meas} - P_{model})^2 dt \tag{4.5}$$

where

P.I. = Performance Index

P_{meas} = experimental value of upstream pressure

P_{model} = upstream pressure, as predicted by model

t_o = initial time

t_f = final time.

A typical result of the simulation process is summarized in Table III.

TABLE III

RESULTS OF PARAMETER IDENTIFICATION USING OPTIM

Parameter	Lower Limit for Search	Upper Limit for Search	Optimum Value
k_1	5.2×10^{-6}	5.3×10^{-6}	5.22×10^{-6}
k_2	2.13×10^{-5}	2.2×10^{-5}	2.18×10^{-5}
k_3	-4.1×10^{-3}	-3.9×10^{-3}	-4.095×10^{-3}
Fractional reduction in uncertainty			10%
Number of iterations			38
Computation time			24 secs
Real time for transient response			.21 secs

Figure 12 shows the actual and predicted responses for the 'best' set of parameters. A residual maximum error of 300 psi (20%) is noted. It was found more effective to use a simplified version of DYSIMP

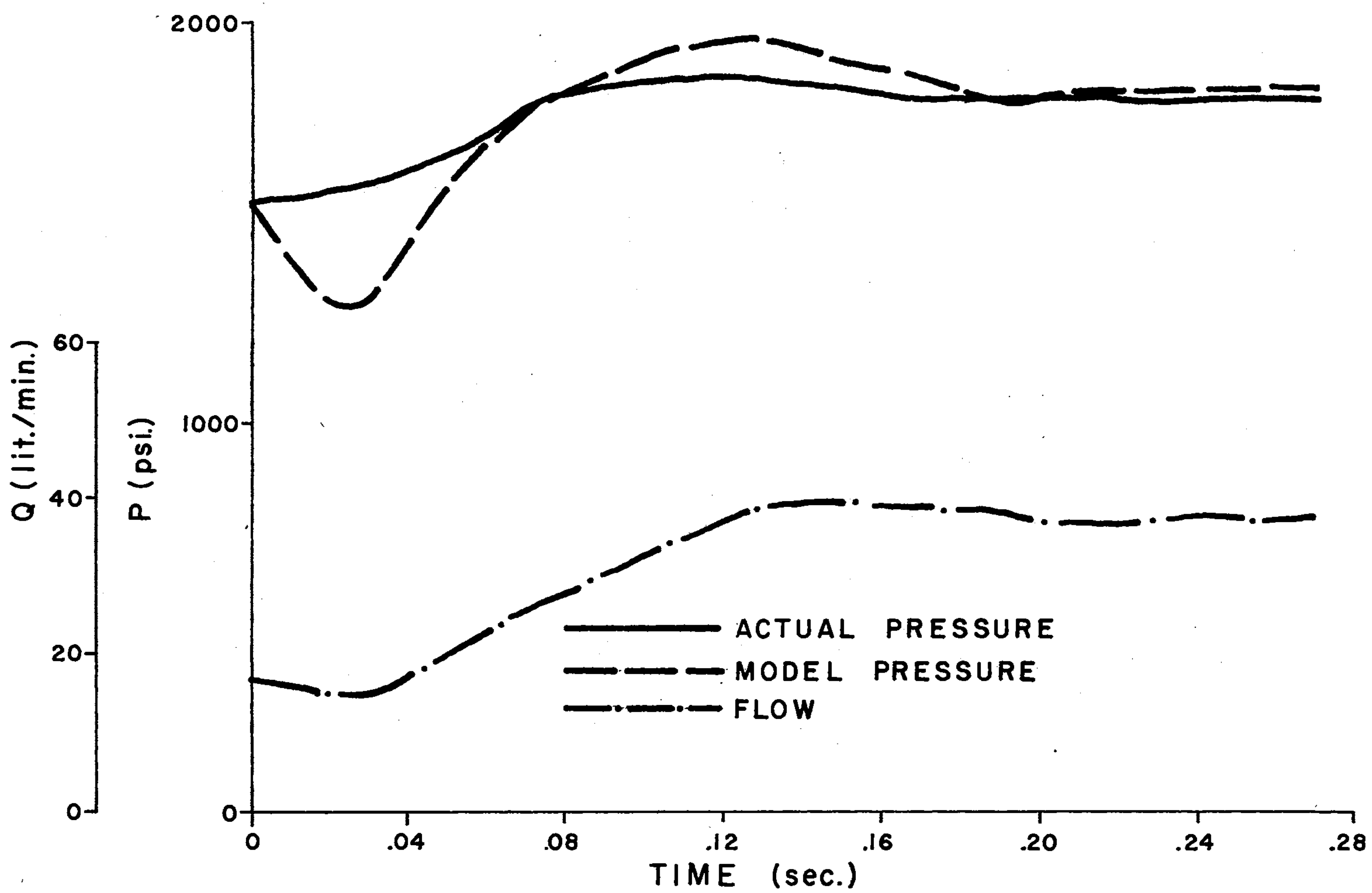


Figure 12. Results of Using OPTIM for Parameter Identification on Valve #RV-10

without the optimization routine which gave a maximum error of 11% for the largest input. It may be noted that Equation (4.4) is different from the dynamic model proposed earlier (Equation (3.15)) in that k_2 is divided by P. The difference is, however, not significant as can be seen from Table IV. Figures 13 to 15 give the nature of inputs and outputs for three of the test runs presented in Table IV.

TABLE IV

SIMULATION RESULTS FOR VALVE #RV-10

		Model I[1]		Model II[2]	
Run	Integration Time	Performance[3] Index	Maximum Error	Performance Index	Maximum Error
1[4]	.28	4.84×10^3	12.5%	1.95×10^3	11%
2	.40	7.95×10^3	14.2%	4.34×10^3	13.8%
3	.20	2.84×10^3	7.5%	1.04×10^3	7.5%
4	.20	3.3×10^3	6.5%	1.02×10^3	7.5%

[1]Model I has the form $\dot{P} = \frac{2\sqrt{P}}{Q}\left[\dot{Q}\sqrt{P} + \left(\frac{Q}{\sqrt{P}} - \frac{Pk_1+k_3}{\sqrt{\rho/2}}\right)\frac{1}{k_2}\right]$ with $k_1 = 5.32 \times 10^{-6}$, $k_2 = 1.57 \times 10^{-5}$, and $k_3 = -4.245 \times 10^{-3}$.

[2]Model II has the form $\dot{P} = \frac{2\sqrt{P}}{Q}\left[\dot{Q}\sqrt{P} + \left(\frac{Q}{\sqrt{P}} - \frac{Pk_1+k_2}{\sqrt{\rho/2}}\right)\frac{P}{k_2}\right]$ with $k_1 = 6.054 \times 10^{-6}$, $k_2 = 3.45 \times 10_{-2}$, and $k_3 = -5.71 \times 10^{-3}$.

[3]Performance Index is the integral squared error given by Equation (4.5).

[4]Run 1 was used to identify the parameters for both models.

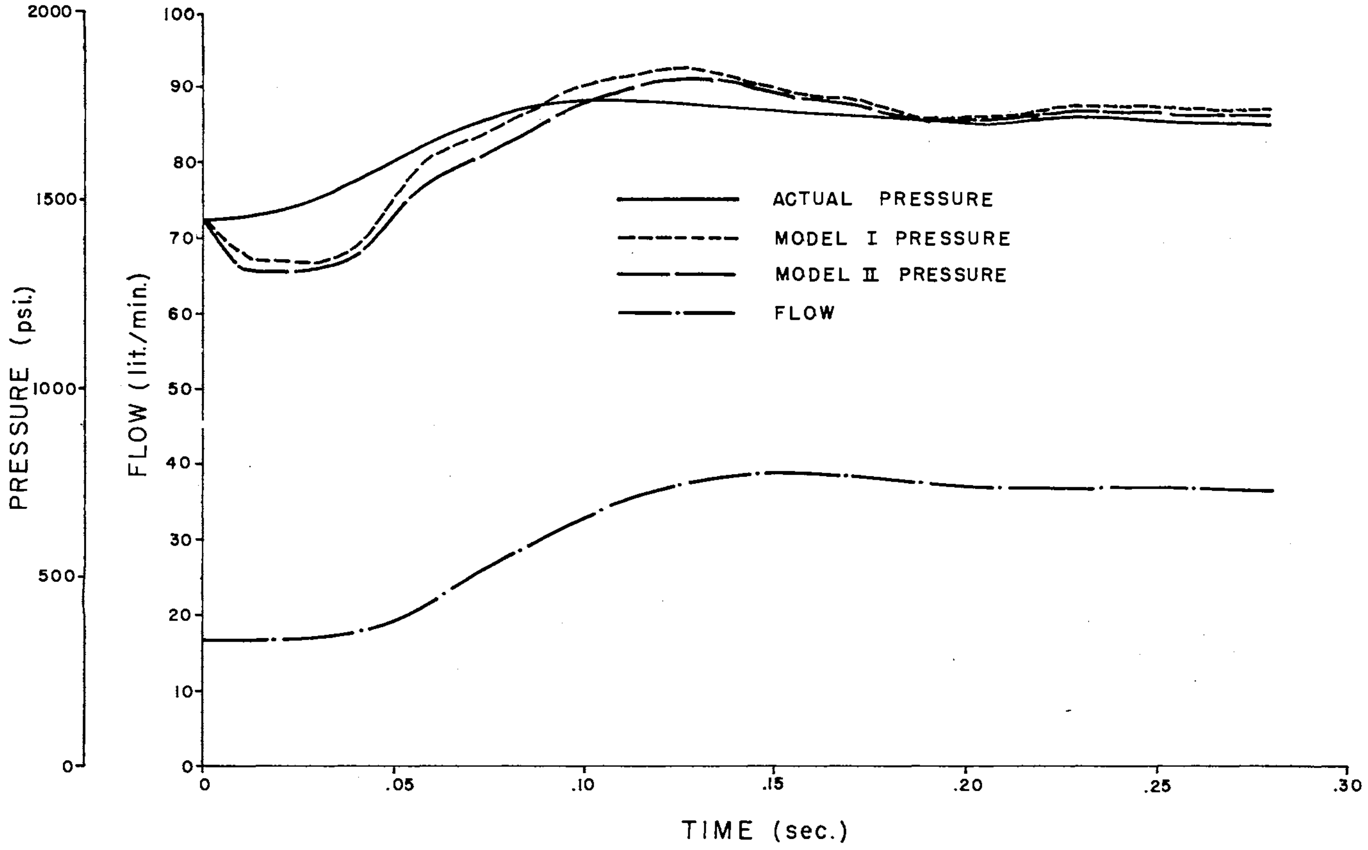


Figure 13. Dynamic Simulation for #RV-10; Run #1

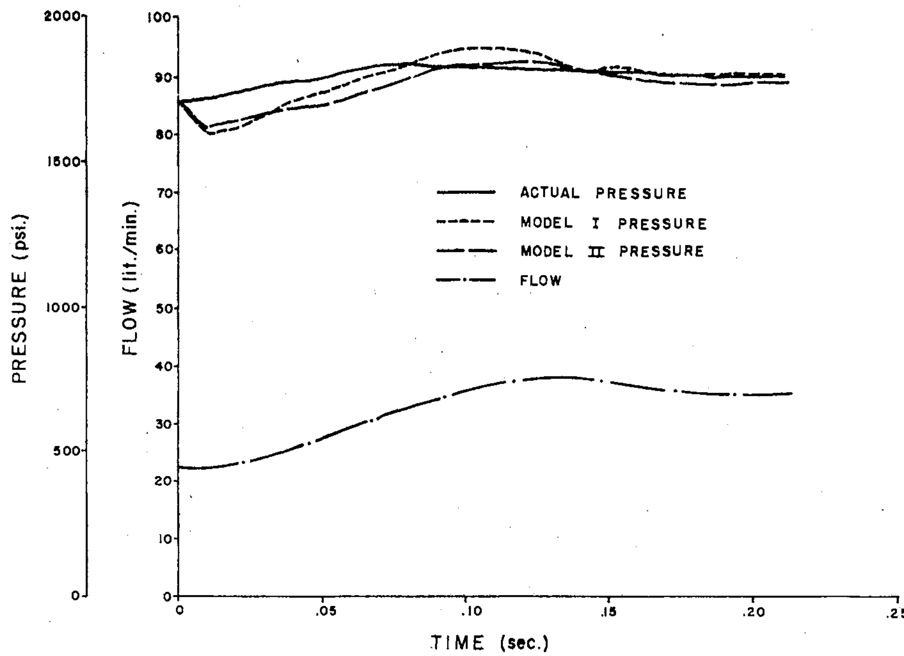


Figure 14. Dynamic Simulation for #RV-10; Run #2

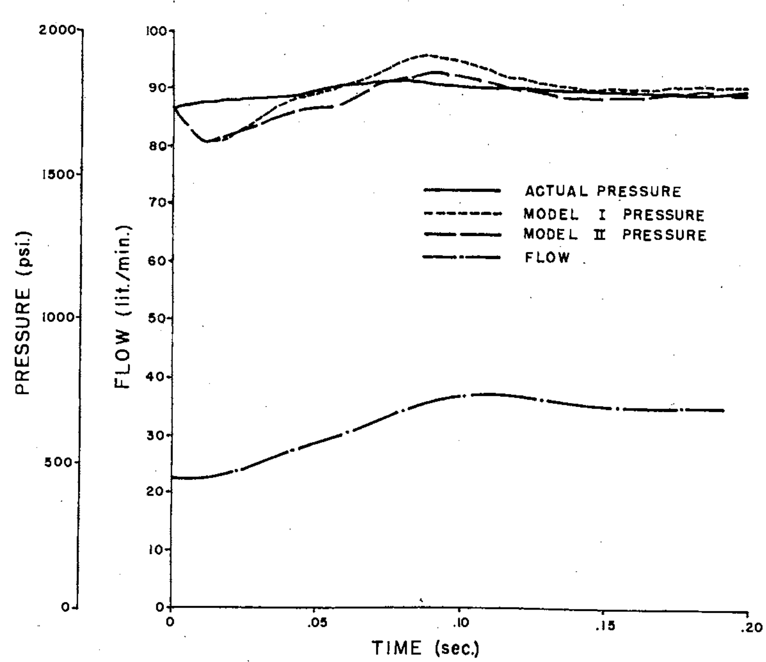


Figure 15. Dynamic Simulation for #RV-10; Run #3

It may be noted that the major discrepancy occurs at the beginning of the transient phase. Two reasons are advanced for this deviation. Firstly, the static model is itself subject to some error and the effect of imposing initial conditions different from that given by the model is to cause the dynamic model to reach a pseudo-steady-state at the predicted value before following the input. Secondly, the downstream pressure changes appreciably during this phase even though it may drop to a low value during steady-state operation. This condition has been confirmed by tests on Valve #RV-22.

Valve #RV-22

In view of the results obtained with #RV-10, both upstream and downstream pressures were recorded continuously for tests on this valve. Even though this was a compound relief valve, it was decided to use the dynamic equation, derived in Chapter III, for valves with one moving element Equation (3.15). It was also decided to use the results of a static test to identify parameters k_1 and k_3 in the equation

$$k_1 P - k_2\sqrt{\rho/2}\ (\dot{Q}\sqrt{P} - Q\ \dot{P}/2\sqrt{P})1/P^2 + k_3 = Q/\sqrt{P} \qquad (4.6)$$

It is seen from Figure 16 that when the static characteristic given by the model is made to agree with experimental results at zero flow and rated flow (100 lit/min) a maximum error of only 3% remains. Solving for k_1 and k_3 is particularly easy in this case as

$$k_1(1020) + k_3 = 106 \times (8.1 \times 10^{-5}/2)/1020 \qquad (4.7)$$

and

$$k_1(910) + k_3 = 0 \qquad (4.8)$$

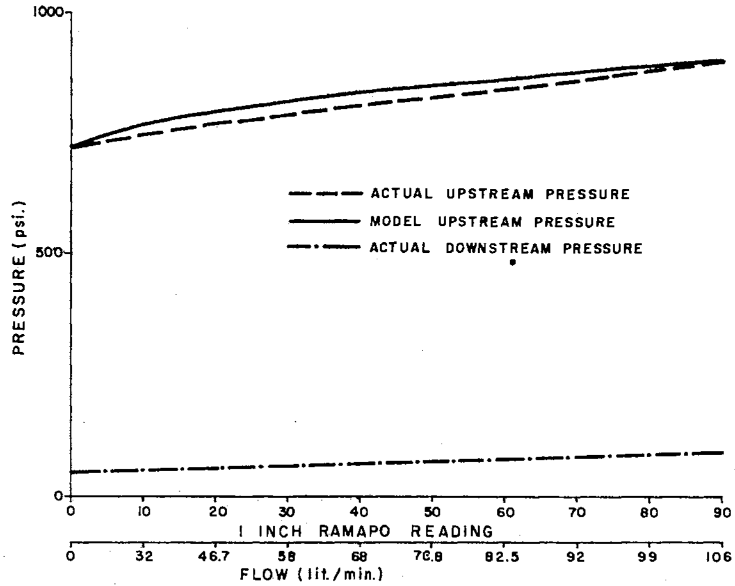


Figure 16. Static Characteristics of Valve #RV-22

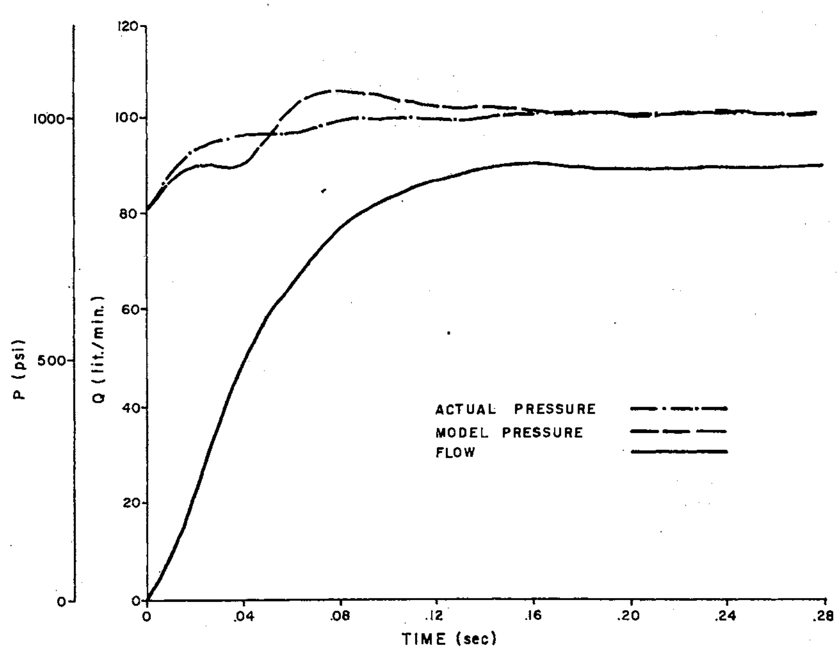


Figure 17. Dynamic Simulation for #RV-22; Run #1

giving

$$k_1 = 1.2 \times 10^{-4}$$

and

$$k_3 = -1.02 \times 10^{-1}.$$

Thus, the only parameter left to be identified from dynamic tests is k_2. The effort involved in this identification is much less than for the first valve. DYSIMP was again used and a few iterations were enough to establish a value for k_2 which gave an error of only 10% for the largest input (Figures 17 to 20). However, the error was larger for faster changes in input as shown in Figure 21.

The presence of valve hysteresis usually necessitates the development and use of two models - one for increasing flows and one for decreasing flows. With valve #RV-22, the hysteresis was found to be negligible and the model developed for increasing flow was found to be extremely good for decreasing flows (Figures 22 and 23).

In reviewing the experimental work for the dynamic tests, it must be kept in mind that the objective was not to find the best empirical models for the valves tested. The goal was rather, to develop forms of models which would:

(1) Be general enough to fit most general purpose valves of the class tested - in this case relief valves.

(2) Yield parameters which could guide a system designer in selecting valves to interface with other circuit components.

(3) Permit refinements if more accurate models for specific valves were required.

The experimental work illustrates the difficulties in imposing inputs of any desired shape. Since the models are nonlinear, the

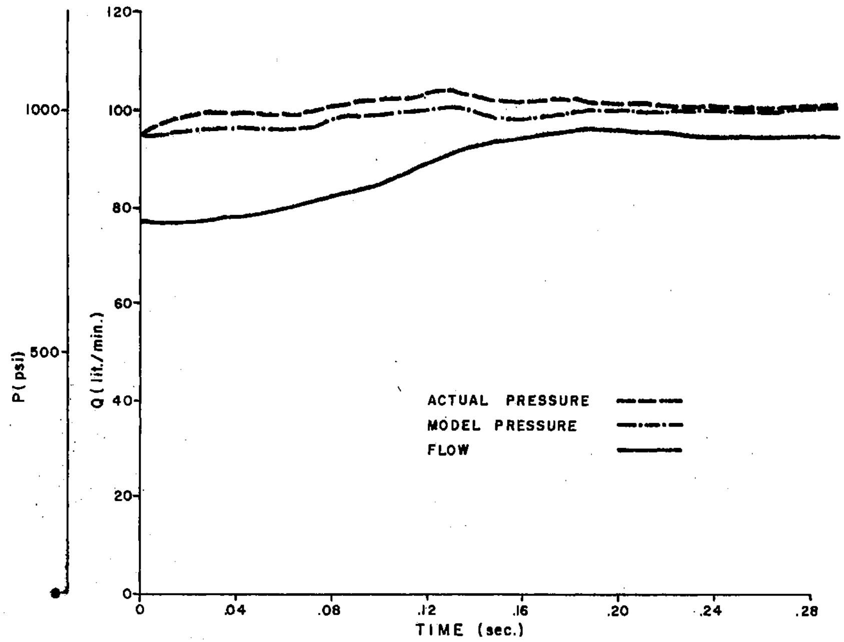


Figure 18. Dynamic Simulation for #RV-22; Run #2

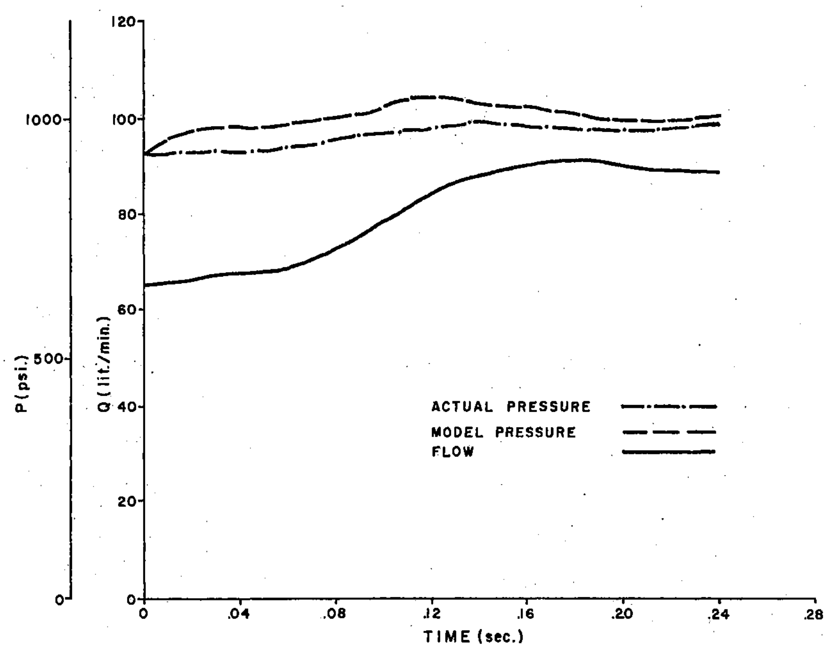


Figure 19. Dynamic Simulation for #RV-22; Run #3

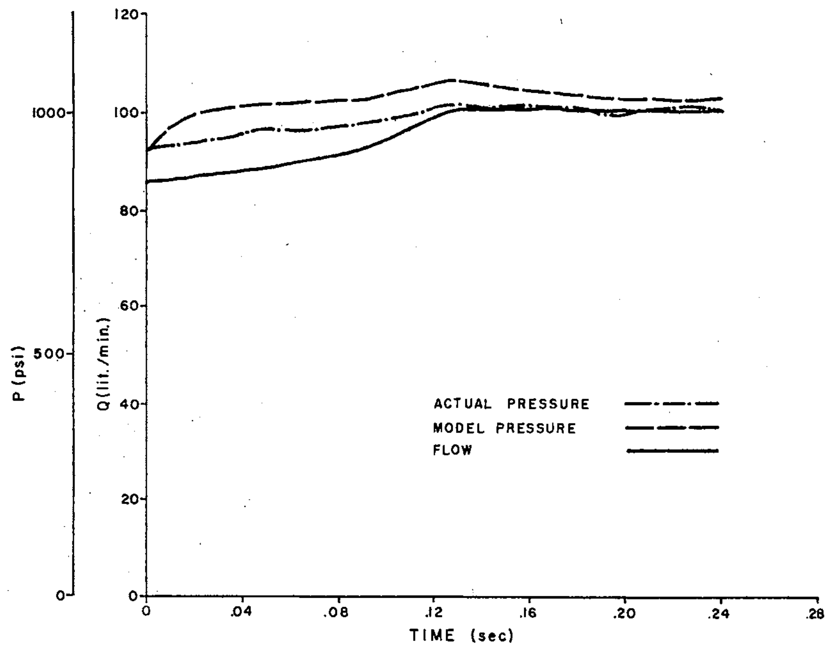


Figure 20. Dynamic Simulation for #RV-22; Run #4

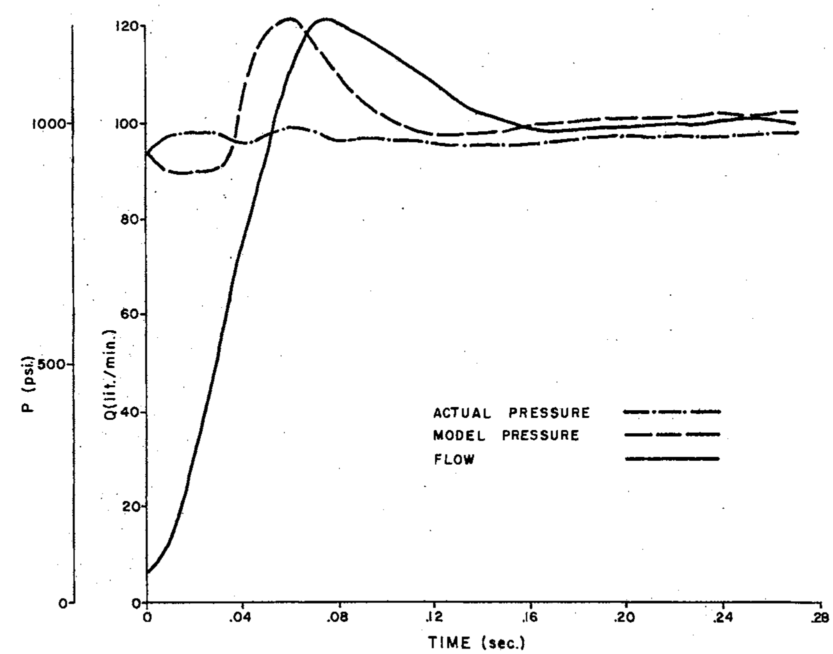


Figure 21. Dynamic Simulation for #RV-22; Run #5

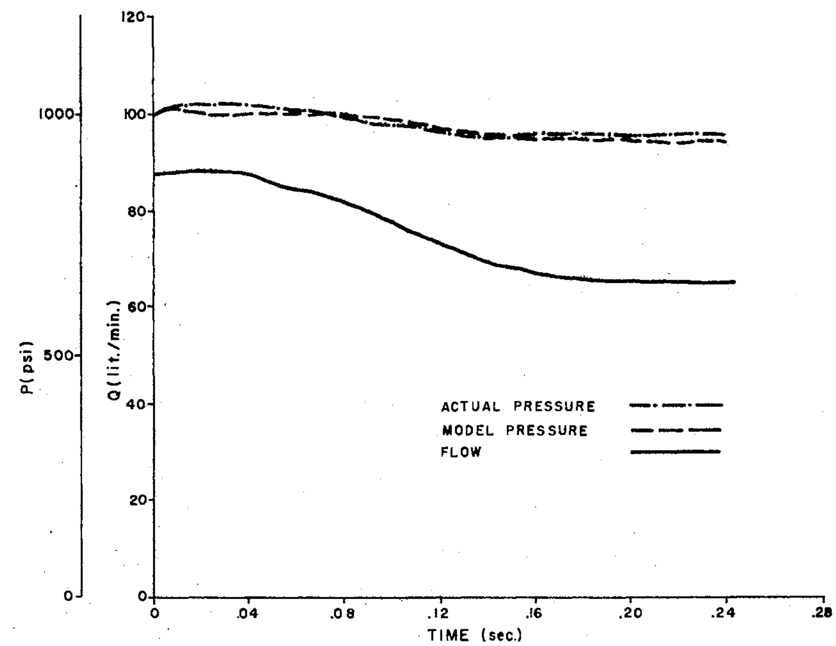


Figure 22. Dynamic Simulation for #RV-22; Run #6

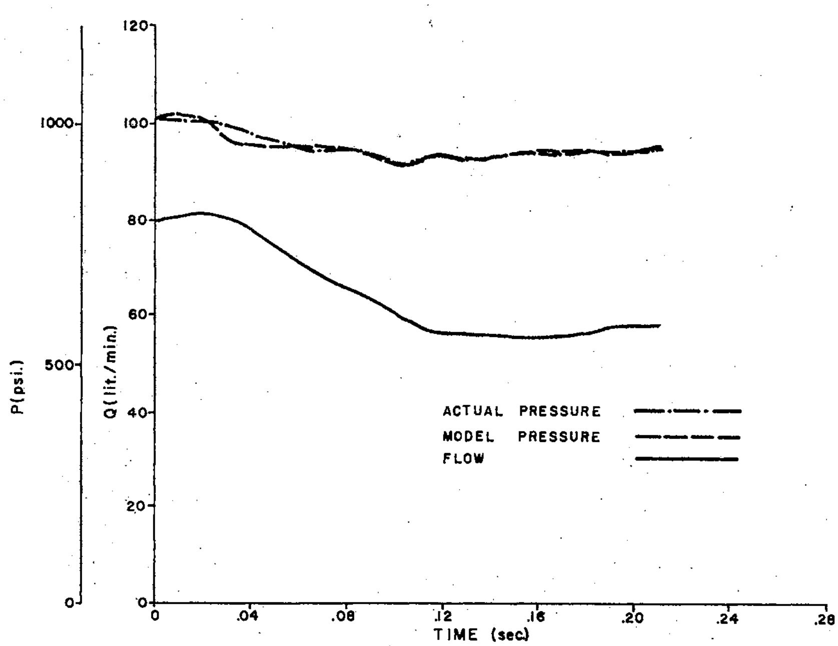


Figure 23. Dynamic Simulation for #RV-22, Run #7

TABLE V

SUMMARY OF RESULTS OF DYNAMIC SIMULATION ON VALVE #RV-22

Test	Input (Flow in Lit/Min) Initial Value	Final Value	Diff.	Average Rate of Change LIT/MIN/SEC	Integration Time (secs)	P.I.*	Max Error	Final Value Error
1**	9.3	89	79.7	568	.28	1000	9.3%	0%
5	7.0	100.3	92.7	1900	.26	1021	22%	-4.9%
3	77	94	16.9	106	.28	19	4%	-0.7%
4	65.8	88.6	22.8	163	.23	100	6%	-2.1%
2	85.6	107.8	22.2	158.5	.23	500	6%	-2.9%
6	99.7	78	23.7	297	.20	12.8	1.95%	.7%
7	107.8	84.4	23.4	167	.23	45	2.52%	2.1%

*Performance Index as given by Equation (4.5).

**This run was used to identify parameter k_2 of the dynamic model.

advantages of special inputs, e.g., step or ramp, would have been marginal. However, the inputs used are representative of those observed in actual physical systems.

The importance of considering static and dynamic characteristics together is illustrated by the analysis of Valve #RV-22. Considerable effort was saved by using the results of static tests to identify parameters in the dynamic model.

Since #RV-22 was a compound relief valve and, thus, had more than one metering element, the accuracy of fit obtained for dynamic responses shows that the model, developed for a valve with one metering element, is still reasonably accurate. A classical model, as given in Appendix A, would have been of a much higher order.

It is, thus, demonstrated that grey-box models give accurate results for a wide variety of inputs, at the same time reducing the order of the model to the bare minimum. Such models are, therefore, eminently suitable for synthesizing system models without introducing either a large number of parameters for identification, or raising the order of the dynamic model to an excessive degree.

It must be emphasized, however, that grey-box models need to be used with caution if the system conditions under which the valve is used differ widely from that on the test stand used to identify the parameters. Identification of parameters under actual working conditions would yield the best model.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

The synthesis of system models using component models as building-blocks, should be considered a breakthrough in the field of fluid power. This requires the development of self-contained component models. Classical modeling techniques, however, have failed to develop such models, as they have not differentiated between system models and component models. Consequently, the same methods of analysis have been used for both types of models. Thus, classical modeling techniques have resulted in the development of component models which: (1) are not isolated from the rest of the system; (2) cannot be easily interfaced with other component models; and (3) often lack generality due to the assumptions made in their derivation.

The development of models for components is more complex than for systems because components are basically multi-port elements. Hence, only multi-input and multi-output models of components are general enough to be useful for pursuing the module concept for system synthesis. Classical modeling techniques applied in the development of single-input models (as are normally required for system models) requires the inclusion of certain upstream and downstream characteristics. Consequently, these techniques defeated the very purpose of developing component models. Classical models, in the case of valves, are also of high order and involve a large number of parameters - many of which need experimental identification.

This investigation was prompted by the necessity of developing component models which would be particularly suitable for interfacing. It addressed itself to the formulation of an approach to developing simple dynamic models for a class of fluid power valves. This class, referred to as self-regulatory valves, is characterized by the presence of two energy ports and a single metering element controlling the resistance of the flow path. By suitable feedback, the dynamics of this moving member are made dependent on upstream and downstream pressures and flow. Using the force balance on the moving member as the foundation, it has been shown that it is possible to derive dynamic equations relating the through and across variables for any self-regulatory valve. The relationships can be made to give static characteristics by setting the time derivatives to zero. The approach is semi-empirical in that the parameters involved have to be experimentally identified. However, since no geometric parameters are involved, the models are not design-dependent. Also, the number of parameters in the models are significantly lower than in classical models. Such models are, therefore, especially advantageous to the system designer in assessing the performance of alternate designs as part of a system and the influence of parameters on final system performance.

The experimental verification had as its objective the validation of the grey-box approach rather than the fitting and refinement of individual models. Static tests were conducted on relief valves of different types and the models exhibited excellent correlation. The verification of dynamic models posed serious problems especially in the areas of flow measurement and choice of inputs. However, the results of the simulation showed extraordinarily good agreement with experimental data. Specifically, a maximum transient error of 15% and a maximum steady-state error of 5% were exhibited for all valves in the study. Much more significant was the reduction in simulation time by a factor of 500 due to the use of low-order models. The simulation of large complex systems is impracticable unless such low-order models are available for all components used.

The contributions to the fluid power engineering field resulting from this study are as follows:

(1) The need for differentiating component models from system models has been established.

(2) The recognition of component models as being basically multi-input and multi-output has been highlighted. System synthesis from component models is extremely difficult unless this concept is utilized in developing general models.

(3) A new approach to the modeling of self-regulatory valves has been developed. This semi-empirical method, referred to as the grey-box approach, permits the analysis of an entire class of valves without necessitating the detailed study of each individual design. Models thus developed, nevertheless, have provision for refinement as more design information becomes available.

Although this study limited itself to the development of low order, multi-port time-domain models for valves, the grey-box approach is considered general enough to be applied to almost all dynamic components in fluid power systems.

Recommendations for Further Study

There are three main areas of investigation which were brought to light during this investigation.

(1) All modeling of fluid power valves uses the turbulent orifice-flow equation. Sufficient investigation has not been conducted to warrant its uninhibited use under all conditions of flow. Investigations need to be carried out to confirm its validity for unsteady flow or to formulate more accurate flow-pressure relationships.

(2) A major assumption made in the development of the grey-box models was the linearity of the metering area - displacement relationship. Such an assumption can be avoided if the characteristic surface of the valve can be developed, as indicated in Chapter III. This would necessitate the building of a prototype valve in which the displacement of the moving member could be adjusted and measured. Since the characteristic surface can be developed by static tests, no elaborate instrumentation is required. The results would definitely lead to improved models.

(3) The actual synthesis of a system using the building-block approach needs to be undertaken to verify the extent of simplification offered by grey-box modeling. As a prerequisite, it is necessary to develop suitable multi-input, multi-output models for all circuit components.

An area of study which, though not central to the theme of this thesis, nevertheless is extremely important to its development, is that of measurement of fluid flow. The development of suitable instrumentation for measuring unsteady flows and rate-of-change of flow could make the difference between success and failure of the grey-box approach.

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APPENDIX A

ANALYSIS OF A COMPOUND RELIEF VALVE BY THE CLASSICAL METHOD

The objective in presenting this analysis is to illustrate the salient features of the classical analysis technique, as applied to component modeling. The two-stage relief valve is particularly suited for this purpose as it is typical of fast-response components having small moving parts. Such components are characterized by static models given by implicit functions, and dynamic models having a number of coupled differential-cum-algebraic equations. For the purposes of classical analysis, it is also common to include certain upstream and downstream characteristics in the "system" analyzed. With this arrangement it is possible to have just one input for the valve.

Figure 24 shows the schematic arrangement of the valve analyzed. Although individual designs differ in details of manufacture, the complexity of the system equations is representative for all relief valves using two moving elements. The following assumptions, which are conventionally made for developing classical models, were made for this analysis (8) (10) (11):

(1) Downstream capacitance is infinite, so that downstream pressure is constant.

(2) Fluid capacitance and inertance are treated as lumped elements.

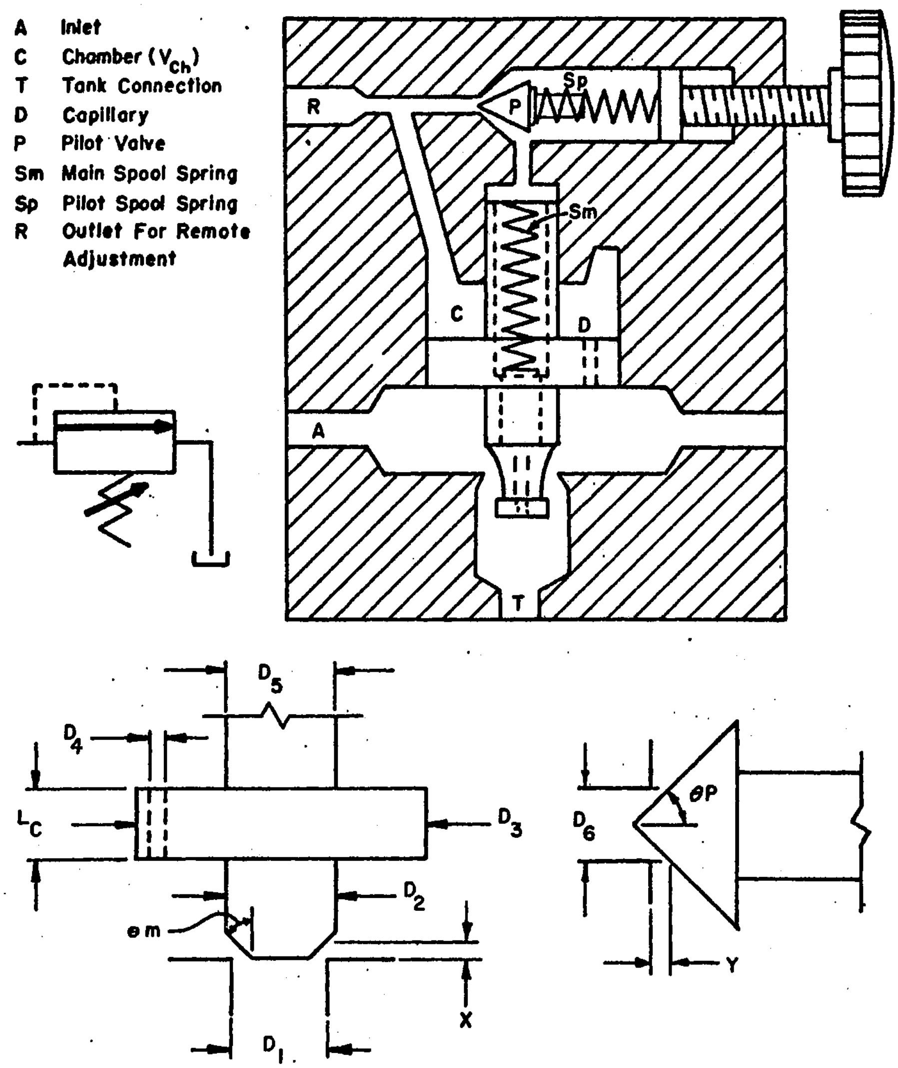

Figure 24. Schematic View of a Two-Stage Relief Valve

(3) Linear damping provides the drag force on both moving elements.

(4) The turbulent flow orifice equation is valid for all types of flow. Coefficients of discharge are constant.

(5) Fluid properties, such as viscosity and bulk modulus are constant.

An additional assumption made in this analysis is that the areas of flow of the metering orifices are proportional to the displacement of the relevant metering element.

The <u>Hydraulic Component Modeling Manual</u> (4) was used to develop the system equations and the terminology used therein is retained.

Notation		Numerical Value Used
A_p	= Area of Pilot Valve on Which Differential Pressure Acts.	
A_{s_1}	= Area of Main Spool on Which Pressure P_1 Acts.	1.436 in^2
A_{s_2}	= Area of Main Spool on Which Pressure P_2 Acts.	1.436 in^2
A_{sp}	= Peripheral Area of Main Spool.	
A_x	= Area of Flow Through Main Valve.	
A_y	= Area of Flow Through Pilot Valve Opening.	
β	= Effective Bulk Modulus of Upstream Volume.	20×10^{-5} lbs/in^2
B	= Damping Factor for Main Spool	
B_p	= Damping on Pilot Valve.	
c	= Peripheral Spool Clearance.	.004 ins
C_{dm}	= Coefficient of Discharge for Main Valve.	.8
C_{dp}	= Coefficient of Discharge of Pilot Valve Opening.	.75

Notation (Continued)

Symbol		Description	Numerical Value Used
$D_1 \ldots D_6$	=	Diameters - See Figure 24.	D_1=.5, D_2=.65, D_3=1.5 D_4=.01, D_5=.65, D_6=.2
K_m	=	Stiffness of Main Spool Spring.	100 lbs/in
K_p	=	Stiffness of Pilot Valve Spring	100 lbs/in
l_c	=	Length of Capillary in Main Spool.	.45 ins
l_1, l_2	=	Damping Lengths for Main Valve.	1,0.15 ins.
l_3, l_4	=	Damping Lengths for Pilot Valve.	0.1,0.1 ins.
m	=	Mass of Main Spool.	.5 lbs
m_p	=	Pilot Valve Mass.	.1 lbs
θ_m	=	Semi-apical Angle of Main Valve.	67^o
θ_p	=	Semi-apical Angle of Pilot Valve.	17^o
P_1	=	Upstream Pressure.	
P_2	=	Chamber Pressure (see Figure 24).	
P_o	=	Outlet/Tank Pressure (Assumed to be 0 psig).	
Q_{cap}	=	Flow Through Capillary in Main Spool.	
Q_{in}	=	Flow Through Relief Valve.	
Q_{main}	=	Flow Through Main Valve (of Relief Valve).	
Q_{pilot}	=	Flow Through Pilot Valve.	
Q_{supply}	=	Flow Put Out by Upstream Components (Input to Valve)	250 cu.ins/sec
μ	=	Viscosity of Fluid.	10 cSt.
V	=	Upstream Volume.	250 cu.ins
V_{ch}	=	Chamber Volume (see Figure 24).	1 cu. in
w	=	Specific Weight of Fluid.	60 lbs/cu.ft.
x_o	=	Initial Compression of Main Spool Spring.	1.0 ins
y_o	=	Initial Compression of Pilot Valve Spring.	0.2 ins

Notation (Continued) | Numerical Value Used

x	= Displacement of Main Valve.	
y	= Displacement of Pilot Valve.	
x_{max}	= Maximum Displacement of Main Valve.	.25 ins
y_{max}	= Maximum Displacement of Pilot Valve.	.12 ins

Steady-State Analysis

All equations have been categorized as below for convenience.

I. Continuity Equations.

A. $Q_{supply} = Q_{in}$

B. $Q_{in} = Q_{main} + Q_{pilot}$

C. $Q_{pilot} = Q_{cap}$

II. Area Equations.

A. $A_x = \pi D_1 x \text{ Sin } \theta_m$

B. $A_y = \pi D_6 y \text{ Sin } \theta_p$

III. Flow Equations.

A. $Q_{main} = C_{dm} A_x \sqrt{\frac{2(P_1-P_o)}{w}}$

B. $Q_{cap} = \frac{\pi}{128\mu} \left(\frac{D_4^4}{l_c}\right) (P_1-P_2)$

C. $Q_{pilot} = C_{dp} A_y \sqrt{\frac{2(P_2-P_o)}{w}}$

IV. Force Balance.

A. Main Valve.

$$(P_1 A_{s_1} - P_2 A_{s_2}) = K_m(x+x_o) + (P_1-P_o)\, C_{dm} \pi D_1 x \left(\frac{\text{Sin } 2\theta_m}{2}\right. -$$

$$-\frac{4\ C_{dm}\ \mathrm{Sin}^2\ \theta_m}{D_1}\Big)\ \frac{(P_1-P_o)}{|P_1-P_o|}$$

The left hand side gives the hydrostatic force; the first term on the right is the spring force, and the second is the steady flow force.

B. Pilot Valve.

$$P_2A_p = K_p(y+y_o) + (P_2-P_o)\ C_{dp}\pi D_6 y\left(\frac{\mathrm{Sin}\ (2\theta_p)}{2} - \right.$$

$$\left.\frac{4\ C_{dp}y\ \mathrm{Sin}^2\ \theta_p}{D_6}\right)\ \frac{(P_2-P_o)}{|P_2-P_o|}$$

Although it is possible to combine some of the equations written above, there is little to be gained by this as an explicit relation between the flow Q_{supply} and the pressure P_1 cannot be developed. For purposes of simulation, a trial value of y was used to initiate a process of iteration, to calculate the flow corresponding to a given value of upstream pressure. Results of the simulation are shown in Figure 25.

Dynamic Analysis

All equations have been categorized as below for convenience.

I. Continuity Equations.

A. $Q_{supply} = Q_{in} + \dot{P}_q \dfrac{V}{\beta}$

B. $Q_{in} = Q_{main} + Q_{pilot}$

C. $Q_{pilot} = Q_{cap} + A_s\dot{x} - \dfrac{\dot{P}_2 V}{\beta}$

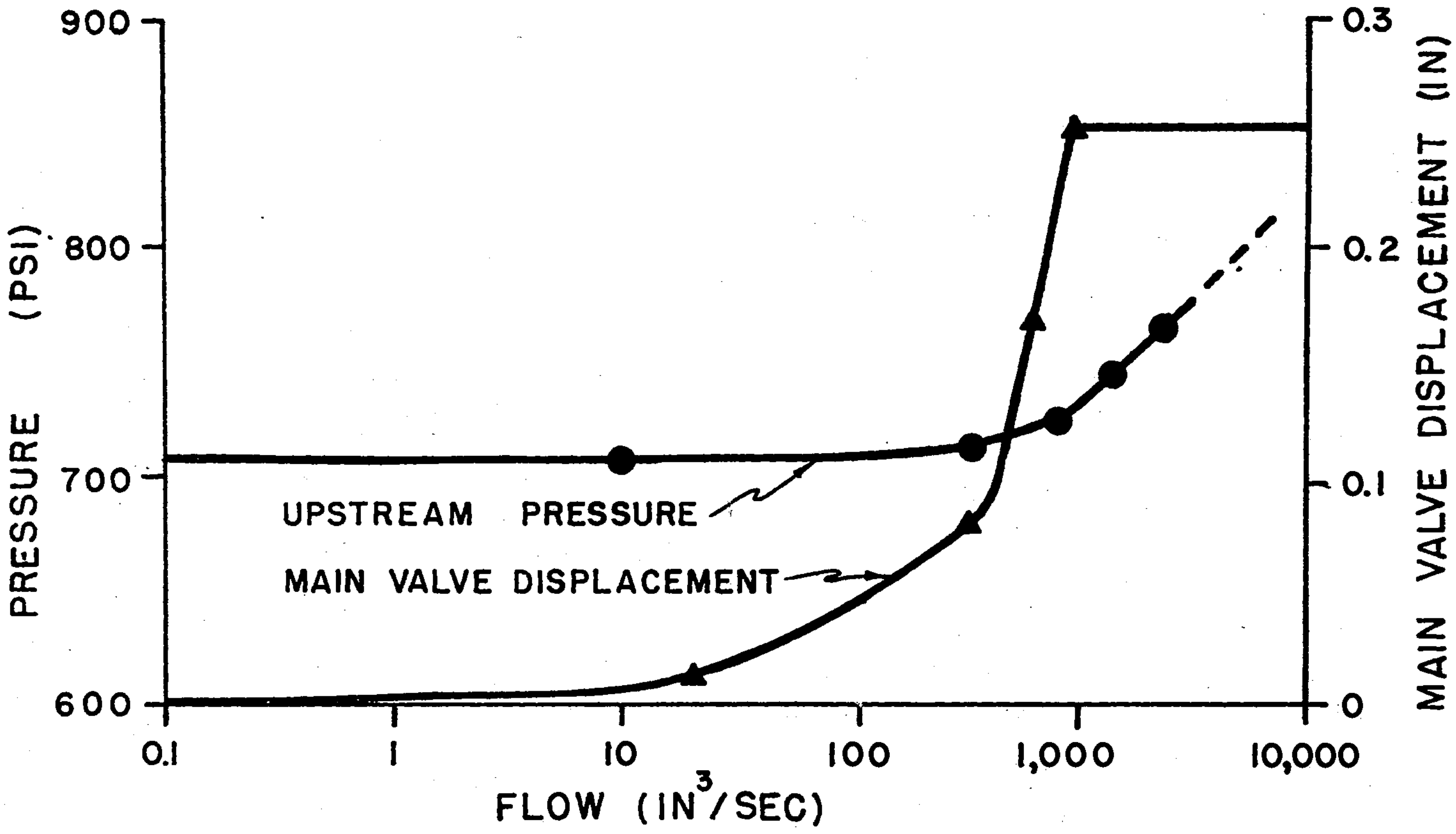

Figure 25. Static Characteristics of a Two-Stage Relief Valve, as Given by the Classical Model

II. Area Equations.

A. $A_x = \pi D_1 x \sin \theta_m$

B. $A_y = \pi D_6 y \sin \theta_p$

III. Flow Equations.

A. $Q_{main} = C_{dm} A_x \sqrt{\frac{2(P_1 - P_o)}{w}}$

B. $Q_{pilot} = C_{dp} A_y \sqrt{\frac{2(P_2 - P_o)}{w}}$

C. $Q_{cap} = \frac{\pi}{128\mu} \left(\frac{D_4^4}{l_c}\right) (P_1 - P_2)$

IV. Force Balance on Main Valve. The following forces are considered.

A. Hydrostatic Force.

$$f_p = P_1 A_{s_1} - P_2 A_{s_2}$$

B. Steady Flow Force.

$$f_{fs} = (P_1 - P_o)\, C_{dm} \pi D_1 x \left(\frac{\sin 2\theta_m}{2} - \frac{4\, C_{dm} x \sin^2 \theta_m}{D_1}\right) \frac{(P_1 - P_o)}{|P_1 - P_o|}$$

C. Unsteady Flow Force.

$$f_{fu} = x\left(\pi w l_1 D_1 \sin \theta_m \dot{P}_1 / 2\sqrt{\frac{2(P_1 - P_o)}{w}}\right) + w \cos \theta_m \left(\frac{\pi}{4} D_1^2 + \pi D_1 l_2 \sin \theta_m\right) \left\{ \frac{2(P_1 - P_o)}{w} \dot{x} + x\dot{P}_1 / 2\sqrt{\frac{2}{w}(P_1 - P_o)} \right\}$$

D. Spring Force.

$$f_{sl} = K_m (x + x_o)$$

E. Viscous Drag Force.

$$f_{vd} = \frac{\mu A_{sp}}{c} \dot{x}$$

F. Inertia Force.

$$f_a = m \ddot{x}$$

Force balance equation is

$$f_p = f_{fs} + f_{fu} + f_{sl} + f_{vd} + f_a$$

V. Force Balance on Pilot Valve. The following forces are considered.

A. Hydrostatic Force.

$$f_p = P_2 A_p$$

B. Steady Flow Force.

$$f_{fs} = (P_2 - P_o) C_{dp} \pi D_6 y \left\{ \frac{\sin 2\theta_p}{2} - \frac{4 C_{dp}}{D_6} \cdot y \sin^2 \theta_p \right\} \frac{(P_2 - P_o)}{|P_2 - P_o|}$$

C. Unsteady Flow Force.

$$f_{fu} = y\left(\frac{w l_3 D_6 \sin \theta_p \dot{P}_2}{2\sqrt{\frac{2P_2}{w}}} \right) + w \cos \theta_p \left(\frac{\pi}{4} D_6^2 + \pi D_6 l_4 \sin \theta_p \left(\sqrt{\frac{2P_2}{w}}\, \dot{y} + y \frac{\dot{P}_2}{2\sqrt{\frac{2P_2}{w}}} \right) \right.$$

D. Spring Force.

$$f_{sl} = K_p (y + y_o)$$

E. Viscous Drag.

$$f_{vd} + B_p \dot{y}$$

F. Inertia Force.

$$f_a = m_p \ddot{y}$$

Once again the force balance equation is

$$f_p = f_{fs} + f_{fu} + f_{sl} + f_{vd} + f_a$$

VI. Constraints. The simulation program included the following constraints in the set of first-order equations.

A. P_1 and P_2 cannot go below zero.

B. Main Valve Displacement cannot go below zero.

C. Pilot Valve Displacement cannot go below zero.

D. Main Valve Displacement cannot exceed x_{max}.

E. Pilot Valve Displacement cannot exceed y_{max}.

F. When the pilot valve is closed, rate of rise of pressure in the chamber is same as in the main line and upstream volume.

Computer simulation runs were performed on the dynamic model using DYSIMP (17) by imposing a step change in flow as input. It is known that the selection of a proper step size for numerical integration is no easy task. A rule of thumb is to use a step size between 1/20th and 1/100th the time period of the highest 'natural frequency' of the system. The introduction of small enclosed fluid volumes and the consideration of their compressibility raises this frequency to a value much higher than that of any mechanical springs in the system. Consideration of the enclosed volume 'V_{ch}' as a liquid spring gave a step size 3×10^{-6} secs. During simulation runs, however, it was found that a step size of 5×10^{-6} gave correct results for the first 5×10^{-3} secs, but beyond this, the pilot valve motion was not accurately portrayed. A value of 10^{-6} secs was finally selected.

The results obtained are fairly typical for the type of component analyzed. Thus, the OSU-Ford Report (11) mentions that a step size of

10^{-5} secs had to be used while Ebbesen (18) gives a range of 10^{-6} - 10^{-7} secs for a dual stage relief valve. The pressure oscillation with a frequency of 130 Hz observed in Figure 26 is comparable with the 180 Hz reported in the OSU-Ford Report.

It is instructive to note that the time required for simulating .04 secs of real time on the computer was 3 minutes and 32 secs., although the system was only of the sixth order. Any parameter identification requiring repeated solutions of the trajectory, for a real time of 0.1 to 0.5 secs would thus be prohibitively expensive. Not only is this kind of simulation lengthy and laborious, but it also does not give insight into the physical phenomena predicted - it is not possible after a cursory survey to guess which parameters may be the most effective in improving the performance in a given direction. It should be particularly noted that the manner in which upstream and downstream characteristics (in this case, capacitances) are introduced in the model make it difficult to assess their effect on the dynamic response.

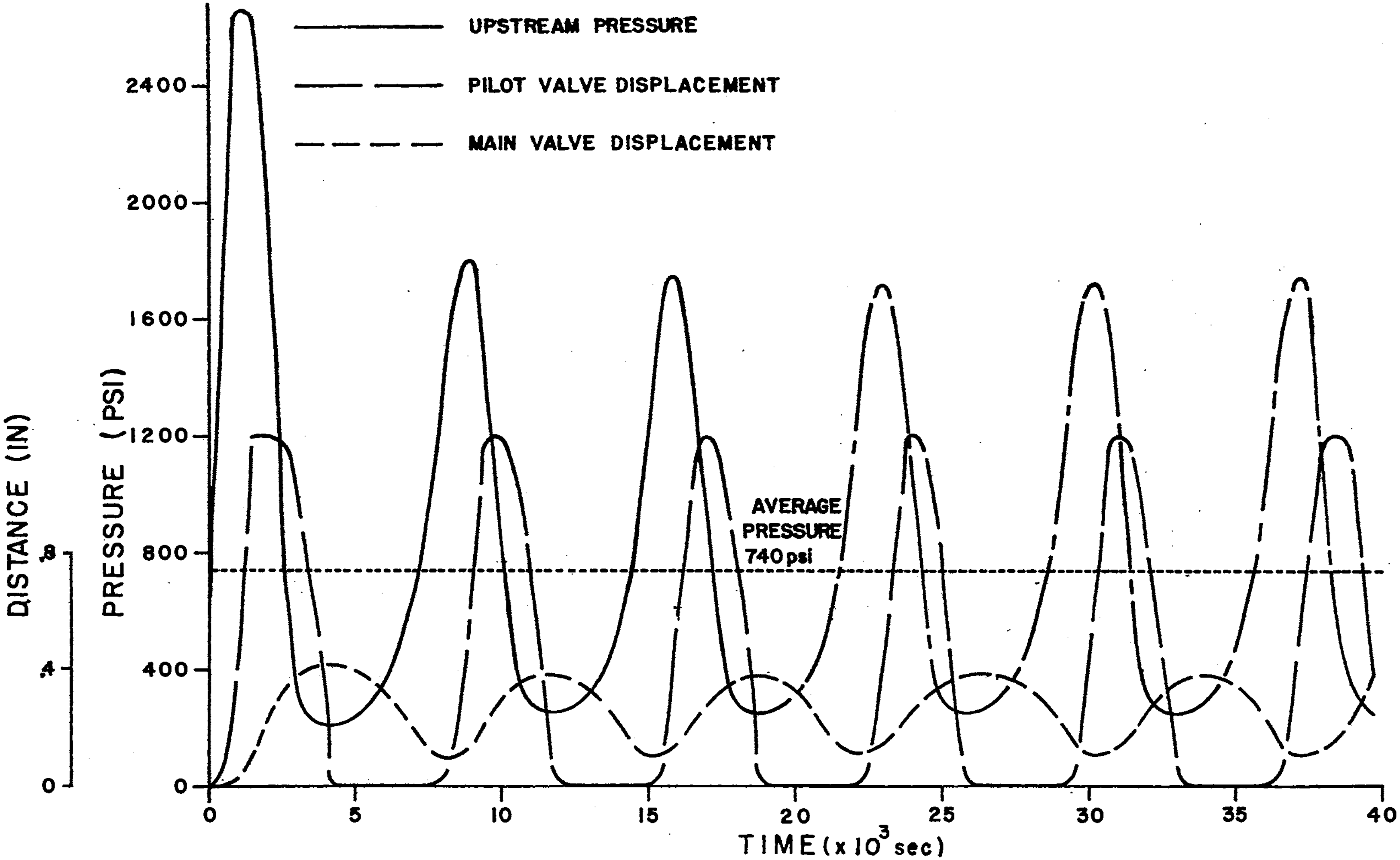

Figure 26. Simulated Response of a Two-Stage Relief Valve, as Given by the Classical Model

APPENDIX B

THE ORIFICE FLOW EQUATION

The investigation of flow through passages is the realm of fluid mechanics and the fluid power system designer is interested only to the extent the results can be applied for analyzing system or component models. A number of fluid power systems use a constant displacement pump and the only way of modulating power is by relieving excess flow through a relief valve or the tank port of an open-center valve. The variable area orifice is thus an indispensable part of most systems. A system designer should, therefore, be thoroughly familiar with the implications of using any relationship for orifices.

The turbulent flow orifice is usually described by the equation:

$$Q = C_d A \sqrt{\frac{2P}{\left[1 - \left(\frac{d_2}{d_1}\right)^2\right]} \frac{1}{\rho}}$$

where

Q = flow through orifice

P = pressure differential across orifice

C_d = coefficient of discharge

d_1 = pipe diameter

d_2 = orifice diameter

A = area of orifice

ρ = fluid density.

This equation is derived by applying Bernoulli's equation to a sudden contraction in a pipe with adequate straight lengths upstream and downstream.

The fluid power analyst usually simplifies this equation to

$$Q = K \sqrt{P}$$

where K is an empirical constant comparable to the admittance in an electrical network.

Fairly extensive investigations have been conducted to ascertain C_d for various orifices and flow regimes (19). A change in C_d from 0.1 in the laminar flow region to 0.6 for turbulent flow is not unusual for both sharp-edged and finite length orifices. It is reported that for a sharp-edged orifice this coefficient also varies with upstream configurations, from 0.6 to 1 (2). Thus, even for sharp-edged orifice of fixed area, there can be appreciable changes in C_d, and any analysis that ignores this can lead to erroneous results.

If the establishing of a pressure-flow relationship for a fixed restriction is considered complicated, that for a control orifice in a fluid power valve is even more so. Firstly, the geometry is rarely as simple as that used for experimental set-ups reported by various investigators. Secondly, self-regulatory valves are characterized by the presence of a control orifice which varies in shape and size from instant to instant, and thirdly, flow through such control valves is rarely steady. It is difficult to establish bounds on the error in formulating system equations, introduced by these factors.

The investigation of unsteady flow in closed conduits has been focused in two directions: firstly, the effect of accelerated and decelerated flow on line and orifice resistence has been studied, and secondly, the effects of pulsating flow on orifice characteristics has been investigated (20) (21) (22). In both cases it has been found difficult to isolate the orifice proper from upstream and downstream characteristics. It is reported that there is a time-lag between the imposition of a pressure differential and the establishment of the corresponding flow, but no convenient dynamic models have been presented, to the best of the authors knowledge (2). Steady fluid flow through an orifice is characterized by convective acceleration of the fluid to maintain continuity, but unsteady flow requires the addition of local acceleration. If the flow changes slowly, the local accelerations can be neglected, and the steady-flow equations used, but otherwise they can be expected to change the orifice-flow characteristics appreciably.

An additional complication which can also affect orifice performance is cavitation. This is ignored in most analysis, although its effect on orifice coefficients could be significant (23). Zielke (14) reports the development of an algorithm for modeling cavitation in return lines of airplane hydraulic systems.

Differentiating the turbulent flow orifice equation yields

$$\dot{Q} = K\frac{\dot{P}}{\sqrt{P}}$$

which implies that for low pressures $\dot{Q}$ can be extremely large. Since at low pressure differentials, the flow is no longer turbulent, the use of this equation (as is done in the calculation of unsteady flow forces) could lead to serious errors. Any modifications in the flow equation to account for changes in the flow regimes (from laminar to transistion to turbulent) will introduce additional parameters in the flow equation which will, consequently, cause it to lose simplicity.

Thus, it is seen that it is only for lack of a better model that the steady-flow orifice equation is used for modeling valves. It is but proper that the system designer be familiar with the divergence between the conditions under which the equation is derived and those under which it is used in dynamic analysis.

APPENDIX C

MEASUREMENT OF FLOW FOR DYNAMIC TESTS

Flow measurement is both an art and a science, and great ingenuity has been exercised in developing measuring instruments to meet the wide variety of conditions in industry. Fluid power systems usually handle flow rates varying from 1 milliliter per minute or less, in the case of leakage through seals, to as high as 600 liters per minute for large systems. Unlike pressure, however, flow is not usually monitored continuously on commercial systems.

Experimental work in fluid power systems, commonly utilizes the variable-area constant pressure-drop meter (rotameter) and the turbine flow-meter for flow measurements. The former are not only contaminant insensitive, but can also be made direct reading and insensitive to viscosity changes (24). The turbine type is approximately linear if flow is turbulent, but it is sensitive to viscosity changes (24) (25). The usual way of sensing the speed of the turbine rotor is with a magnetic pick-up and is eminently suitable for digital read-out.

Dynamic tests require the continuous monitoring of flow, which may vary from zero to the output of the pump in the system. Transient flows may reach even higher values. Most commercially available flow-meters, of any type whatsoever, have a range of ten to one, thus imposing a limitation on the type of flow inputs or outputs for dynamic tests. The presence of moving parts of appreciable size makes the dynamic response of the rotameter slow, while the digital nature of the output from the turbine flow-meter effectively leads to the same result. Hence, neither could be used for dynamic tests.

A target flow-meter was selected for such tests as it had a range most suitable for the valves tested, it gave an analog output, and its natural frequency was higher than that of all other alternatives. The target flow-meter measures the drag force on a body suspended in the flowing stream and an approximate expression for the output (which is usually a voltage) is

$$v = kQ^2$$

where

v = voltage output

Q = flow rate

k = a constant.

The dynamic response of the flow-meter depends on the stiffness of the mechanical member supporting the sensor and is basically second-order and under-damped in nature (24). The natural frequency of the meter used was 200 Hz, and this imposed a ceiling on the dynamic inputs during testing.

The measurement of rate-of-change of flow has received little or no attention in the literature. Yet, in the modeling of fluid power systems, this quantity is as important as the rate-of-change of pressure. In the absence of suitable measuring instruments, the only alternative, for this investigation was to differentiate the flow. Since the target flow-meter was nonlinear and the signal contained high frequency noise analog differentiation was considered infeasible. Consequently, numerical differentiation was used to obtain the rate-of-change of flow.

VITA

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