

STRUCTURAL SYNTHESIS AND ANALYSIS  
OF KINEMATIC CHAINS USING  
PATH MATRICES

By

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## PREFACE

This thesis presents a method of describing kinematic chains and mechanisms using matrices which represent paths or circuits in a chain. The path matrices developed permit the use of the digital computer in developing mechanisms from kinematic chains, synthesizing modified chains based on basic kinematic chains, and comparing kinematic chains. Computer programs using the path matrices were applied to the 230 ten-link chains and resulted in tremendous amounts of data which are summarized in the body of the thesis.

I would like to acknowledge my indebtedness to my adviser, Dr. A. H. Soni, whose inspiration and support were instrumental in the development of this thesis.

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## CHAPTER I

### INTRODUCTION

During the past several years, the problems of structural synthesis and analysis of kinematic chains have been approached in several ways. Davies and Crossley (1)\* utilized Franke's condensed notation, a method of representing kinematic chains as molecules, to obtain a census of seven, nine, ten, and eleven-bar kinematic chains. Hain (2,3,4,5) has done a great deal of work in this area of kinematics, working mainly with the six and eight-link chains. He has obtained censuses not only for the basic chains, but also for these chains with modifications such as the addition of cam pairs, prism pairs, and multiple joints. In addition, he has obtained the mechanisms derived from these chains and presented them schematically thus furnishing the designer with a large choice of solutions to a given problem. This work has been done mainly by visual inspection of the structural drawing of each chain.

The results obtained by the above individuals have relied heavily on the researcher's ability to visualize the various configurations a chain may assume and to recognize isomorphisms which may occur. As more complex chains are examined, this ability is heavily taxed and the tremendous number of chains and mechanisms makes a manual approach

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\*Numbers in parentheses refer to numbered references in the Bibliography.

impractical. Therefore, the need arises for an automated method of performing the structural analysis of kinematic chains.

Woo (6) has used graph theory in enumerating the plane kinematic chains with 13 revolute pairs and 10 links, thus verifying some of the results obtained by Davies and Crossley above. An incidence matrix was used to develop the chains and also served as the structural discriminant to detect isomorphisms. Similarly, Dobrjanskyj and Freudenstein (7) have developed an algorithm which applies graph theory to the comparison of kinematic chains and mechanisms. Buchsbaum (8) has further developed the use of graph theory for the structural classification and type synthesis of chains with multiple elements.

All of the preceding approaches which utilize graph theory have the advantage of being computerized and have contributed significantly to the analysis of kinematic chains.

The purpose of this thesis is to present an alternate approach to the problem of type synthesis and analysis of kinematic chains which does not rely on graph theory but utilizes a method of comparing paths or circuits in a chain. A Chain Path Matrix (CPM) is developed and from it, the Mechanism Path Matrix (MPM) is formed. The MPM serves as the structural discriminant in comparing mechanisms and chains. The CPM is useful in converting basic kinematic chains into chains with special elements such as pulleys and belts, cams, double joints, etc. The method is easily computerized and, consequently, fills the need for automated methods of processing chains with more than eight links. Furthermore, it has the advantage of requiring no mathematics in its derivation or application.

The path matrix approach has been applied to the analysis of the

230 ten-link chains with one degree of freedom (Appendix A). The amount of data generated by this analysis is enormous, and, consequently, has not been included in this thesis. The data has, however, been included in reference (9), a copy of which is available in the Oklahoma State University Library and summaries of this data are included at the end of each appropriate chapter herein.

In the chapters which follow, the development of the CPM and MPM will be described. The application of these matrices in synthesizing mechanisms, developing chains with special elements (cams, springs, etc.) and converting kinematic chains to gear trains will then be discussed.

## CHAPTER II

### DEFINITIONS OF NOTATION AND TERMS

In order to develop a basis for the following chapters, a brief discussion of notation and terms will be presented.

A kinematic link is a rigid, massless member and is classified according to the number of joints which it can accept. A binary link is a link which can accept two joints; a ternary link, three joints; and a k-nary link, k joints. The standard kinematic notation used to represent various links is illustrated in Figure 1.

A kinematic pair consists of two contacting elements and is used to connect two kinematic links. Three types of kinematic pairs will be considered here:

1. Revolute Pair - Permits only relative rotation between two connected links. Also called a turning pair or pin joint.
2. Prism Pair - Permits only relative translation between two connected links. Also called a sliding pair.
3. Cam Pair - Permits both rotary and translatory motion between two connected links.

In addition to the three pairs mentioned above, the double joint will also be used as a means of connecting kinematic links. The double joint is simply the merging of two revolute pairs to form a joint which permits rotational motion among three links.

The kinematic notation used to represent the various joints described above is illustrated in Figure 2.

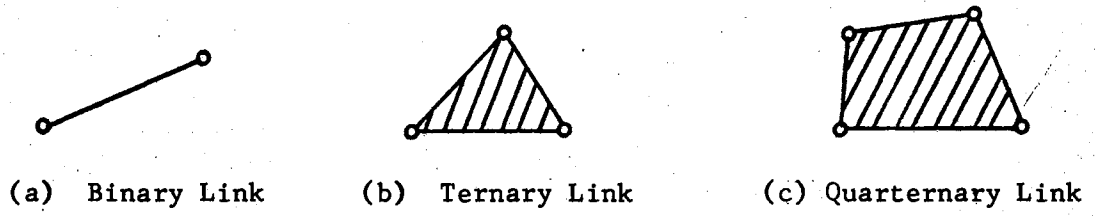


Figure 1. Representation of Kinematic Links

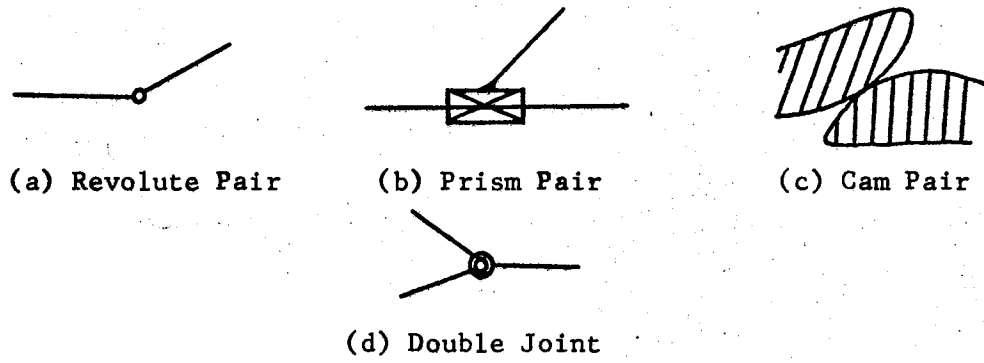


Figure 2. Representation of Kinematic Joints

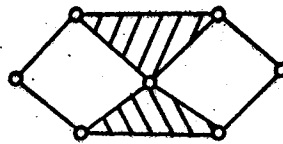


Figure 3. Six-Link Basic Kinematic Chain

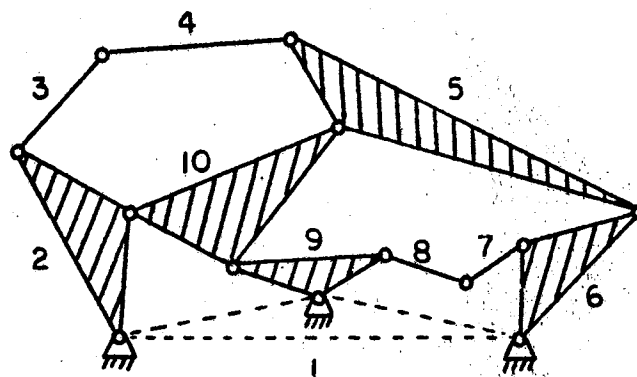


Figure 4. Ten-Link Mechanism



A kinematic chain is constructed using kinematic links and pairs to form one or more closed loops. Each link must be free to move relative to the links connected to it (e.g., a structure such as three binary links joined by three revolute pairs to form a triangle is not a kinematic chain). If a kinematic chain is constructed so that the motion of all links takes place in one plane or in parallel planes, the chain is called a planar kinematic chain. A basic kinematic chain is a planar kinematic chain with all links connected by revolute pairs (Figure 3).

A kinematic mechanism is defined as a kinematic chain which has one link fixed with respect to ground and which has a single degree of freedom. The degrees of freedom of a basic kinematic chain (and the mechanisms derived from it) can be determined from the formula  $F = 6(N-1) - 5P_1$  where  $F$  is the degrees of freedom,  $N$  is the number of links, and  $P_1$  is the number of revolute pairs. Basically, a mechanism (or chain) with one degree of freedom is one in which the motion of all links is constrained, that is, no link is free to move independently of the other links. Figure 4 illustrates a ten-link mechanism.

A path, as used here, will be defined as a sequence of connected elements encountered in traversing a chain (or mechanism) beginning at a given element and returning to that element without repeating any element. For example, Figure 5 illustrates a kinematic chain with four loops. The links have been numbered as a means of identification. One path beginning at link 1 could be described by the sequence of link numbers 1,2,10,9. This path represents the links encountered in traversing loop 1 in a clockwise direction. A second path can be represented by the sequence 1,2,10,5,6,7,8 and can be thought of as the path



generated by combining loops 1, 3, and 4. Several other paths may also be found which originate at link 1. Each one of these paths can be thought of as a combination of either loop 1 or loop 4 (loops which contain link 1) and one or more of the other loops of the chain. This point will be further illustrated in the next chapter.

The foregoing provides sufficient background for an understanding of the path comparison method. The following chapters will describe the development of the path matrices and their applications.

## CHAPTER III

### DEVELOPMENT OF THE CHAIN PATH MATRIX

In the previous chapter, a path was defined as the sequence of elements encountered in traversing a loop or combination of loops in a kinematic chain. In this chapter, the development of the Chain Path Matrix (CPM), which represents all possible paths in a chain, will be discussed and a computer program for generating the CPM will be described.

In order to derive any meaning from the numbers representing the links in a chain, it is desirable to be able to designate both the link type (binary, ternary, quaternary, etc.) and the link number. To do this, a three or four digit integer will be used in which the ones and tens places represent the link number and the hundreds and thousands places represent the link type. For example, link number 1 of chain 189 (Figure 6) is a ternary link and this fact will be reflected by placing a three in the hundreds place of its identifying number. Since it is also link number 1, the digits 01 will be placed in the tens and ones positions. The identifying number for link 1 will then be 301. Similarly, the identifying number for link 2 is 402 and for link 3 is 203. The thousands place of the identifying number will be used in later chapters to designate special elements inserted in the basic kinematic chain.

The development of the CPM is based on a matrix called the loop



matrix which represents the path around each loop of the chain. The path is developed in a clockwise direction as used here although the direction is immaterial as long as all paths are taken in the same direction. Using the identifying numbers described above, the loop matrix for chain 189 would appear as shown in Table I. The starting element for each row may be chosen at random since the beginning and end elements of each row are considered to be adjacent.

The GPM will be composed of all possible combinations of the basic loops described by the loop matrix. The GPM will then be formed as shown in Table II and will identify every path or circuit which can be formed in the kinematic chain. In order for two loops to combine, they must have at least two elements in common, otherwise, an element in the combined path would be repeated. Because of this restriction, some of the combinations shown in Table II cannot be realized. When this case occurs, the row of the GPM representing that combination will be filled with zeroes to indicate that the path represented by that row does not exist. Based on the preceding, the GPM for chain 189 would be as shown in Table III.

Technically, neither the loop matrix nor the GPM as shown are true matrices since they are not rectangular arrays. However, the arrays as used in the computer program are true matrices as shown below.

To develop the GPM by computer, the loop matrix is required as the initial input. In most cases, developing the loop matrix is a simple matter but, for a certain class of chains such as Group 64D as shown in Appendix A, problems arise. These chains have paths which cross one another and, consequently, basic loops are not available. The loop matrices actually used for this class of chains (Appendix B) were

TABLE I  
LOOP MATRIX FOR CHAIN 189

ROW 1	301	209	406	207	208
ROW 2	301	402	406	209	
ROW 3	402	210	305	406	
ROW 4	402	203	204	305	210

TABLE II  
COMBINATIONS OF LOOPS USED IN FORMING THE CPM

ROW #	Loop Combinations
1	Loop 1
2	Loop 2
3	Loop 3
4	Loop 4
5	Loops 1 and 2
6	Loops 1 and 3
7	Loops 1 and 4
8	Loops 2 and 3
9	Loops 2 and 4
10	Loops 3 and 4
11	Loops 1 and 2 and 3
12	Loops 1 and 2 and 4
13	Loops 1 and 3 and 4
14	Loops 2 and 3 and 4
15	Loops 1 and 2 and 3 and 4

TABLE III  
CPM FOR CHAIN 189

ROW #	Sequence
1	301 209 406 207 208
2	301 402 406 209
3	402 210 305 406
4	402 203 204 305 210
5	301 402 406 207 208
6	0 0 0 0 0
7	0 0 0 0 0
8	301 402 210 305 406 209
9	0 0 0 0 0
10	402 203 204 305 406
11	301 402 210 305 406 207 208
12	0 0 0 0 0
13	0 0 0 0 0
14	301 402 203 204 305 406 209
15	301 402 203 204 305 406 207 208

TABLE IV  
COMPUTER LOOP MATRIX FOR CHAIN 189

301 209 406 207 208	0	0	0	0	0	0	5
301 402 406 209 0	0	0	0	0	0	0	4
402 210 305 406 0	0	0	0	0	0	0	4
402 203 204 305 210	0	0	0	0	0	0	5



arrived at by trial and error and no rules governing their development have become apparent.

The loop matrix is read into the computer in the form of a matrix and forms the first four (in the case of the ten-link chains) rows of the CPM. Sufficient columns are available in the matrix to contain the longest row which may occur (10 elements in the case of the ten-link chains) plus two. The two extra columns are required for use in certain operations performed in comparing rows. Any elements of the matrix which are not defined are filled with zeroes. As the loop matrix is read in, the number of non-zero elements in each row is counted and this number is stored in the last column of that row. Thus, the loop matrix (and the first four rows of the CPM for chain 189) would appear as shown in Table IV.

To form the remaining rows of the CPM, combinations of the first rows must be taken. In combining rows, the following rules are observed:

1. If an element is common to both rows and the elements on both sides of it are also common to both rows, that element will not appear in the combined row. For example, if rows 1 and 2 of Table IV are being combined, link 209 will not appear in the combined row since it is flanked by links 301 and 406 which are also common to both rows. Note that in row 2, 301 is considered to be adjacent to 209.

2. If row A is to be combined with row B, the path sequence will begin with  $A_1$  (the first element in row A) and will continue to element  $A_m$ , the first element encountered which is common to both rows. If rule 1 does not apply and if  $A_{m+1}$  is the same as  $B_{n-1}$  (where  $B_n$  is the element in row B which is equivalent to  $A_m$ ), the sequence will transfer

to row B at element  $B_n$  and will continue in row B until similar conditions permit a transfer back to row A to complete the path.

3. If no transfer can be made between rows, the resulting combined row will be filled with zeroes.

To illustrate the above procedure, the combining of rows 2 and 3 of the loop matrix for chain 189 (Table IV) will be performed step by step.

1. The first element of row 2 (301) is not common to both rows and will thus become the first element of the combined row.

2. The second element of row 2 (402) is the same as the first element of row 3. Rule 2 is applied to determine if a transfer can be made. Since the third element in row 2 (406) is the same as the last element in row 3 (which is considered as preceding the first element in row 3) the transfer can be made. The second element in the combined row thus becomes 402.

3. The sequence continues with elements 2 (210) and 3 (305) of row 3 since they are not common to both rows. At this point, the combined row is:

301 402 210 305.

4. The next element in row 3 (406) is common to both rows. Rule 2 is applied and, since the element following 406 in row 3 (402) is the same as the element preceding 406 in row 2, the sequence transfers back to row 2 and 406 becomes the fifth element of the combined row.

5. The sequence then continues in row 2 until the starting element is encountered. (When the sequence reaches the last element of a row, it starts over at the first element. Therefore, the first element of the row will be encountered after the last element of the row.)

6. The remainder of the combined row is filled with zeroes and the number of non-zero elements stored in the last column. The resulting row is:

301 402 210 305 406 209 0 0 0 0 0 6.

This row agrees with row 8 of the CPM as shown in Table III.

Rows 5 through 10 of the CPM are similarly derived. Row 11 of the CPM represents the combination of rows 1, 2, and 3 of the loop matrix. This combination can most easily be obtained by combining row 1 of the CPM with row 8 of the CPM (the combination of rows 2 and 3) or by combining row 3 with row 5 (the combination of rows 1 and 2). Since one or more of the combined rows mentioned above may be zero, the computer tests the rows to be combined and uses the first pair which have non-zero entries in their first columns. The possible combinations to be tested are provided as input data for the program. Rows 11 through 15 of the CPM are generated in this manner. The row combinations used for the ten-link chains appear in Table V.

TABLE V  
ROW COMBINATIONS USED IN FORMING THE GPM FOR TEN-LINK CHAINS

ROW #	Combined Rows
5	1 and 2
6	1 and 3
7	1 and 4
8	2 and 3
9	2 and 4
10	3 and 4
11	1 and 8 or 5 and 3
12	4 and 5 or 7 and 2
13	10 and 1 or 3 and 7
14	8 and 4 or 2 and 10
15	5 and 10 or 6 and 9 or 7 and 8 or 12 and 3 or 13 and 2 or 14 and 1

## CHAPTER IV

### DEVELOPMENT AND USE OF THE MECHANISM PATH MATRIX

#### Theory

Once the GPM has been formed for a given chain, it represents the sequence of links encountered in traversing each possible path in the chain. The GPM could be used directly in comparing two chains but the procedure would involve checking each row in one matrix against each row in the other matrix and cycling one of the rows several times for each comparison. A method which will eliminate a great deal of the comparisons made in the above procedure is based on the premise that if a mechanism formed from one chain is identical to a mechanism formed from a second chain, the two chains are equivalent. Conversely, if a mechanism formed from one chain is not equivalent to any mechanism formed from the second chain, the chains are not equivalent. The problem thus becomes one of representing mechanisms and then, comparing two mechanisms.

As defined previously, a mechanism is created when one link of a kinematic chain is fixed with respect to ground. The configuration can be completely defined by defining all paths in the chain which include the fixed link. It will prove to be advantageous to use the fixed link as the starting element for each of the paths. A Mechanism Path Matrix (MPM) which describes the mechanism formed by fixing a specific link is developed by using only those rows in the GPM which contain the fixed

link and by cycling each row so that the fixed link is the first element in the row. Table VI shows the CPM formed for chain 189 and Table VII, the MPM representing the mechanism formed by fixing link 1 of chain 189.

When comparing two paths, the sequence of link types encountered in traversing the paths is the factor which determines whether or not the paths are equivalent. Consequently, once the MPM is formed, the link numbers may be dropped leaving only the matrix of link types. Table VIII shows the MPM of Table VII reduced to a matrix of link types. The term Mechanism Path Matrix (MPM) will mean the matrix of link types in all following discussion.

To proceed further, the definition of equivalent paths must be established. Two paths are equivalent if and only if the sequence of elements encountered in traversing the first path in a given direction is the same as the sequence of elements encountered in traversing the second path in the same or opposite direction. That is, when comparing rows of two MPM's, the path represented by the sequence 2 3 2 3 4 is equivalent to the path represented by the sequence 2 4 3 2 3.

The equivalency of two MPM's, and therefore two mechanisms, will be established if each row in one path matrix has an equivalent row in the other path matrix, each row being used only once. Obviously, then, for two MPM's to be equivalent they must have the same number of rows and equivalent rows must contain the same number of non-zero elements. Only after these two conditions are satisfied is it necessary to compare sequences within two rows.

To illustrate the comparison of two MPM's, mechanisms 1 and 5 of chain 189 will be compared. The MPM for mechanism 5 is shown in Table

TABLE VI  
COMPUTER CPM FOR CHAIN 189

---

301	209	406	207	208	0	0	0	0	0	0	5
301	402	406	209	0	0	0	0	0	0	0	4
402	210	305	406	0	0	0	0	0	0	0	4
402	203	204	305	210	0	0	0	0	0	0	5
301	402	406	207	208	0	0	0	0	0	0	5
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
301	402	210	305	406	209	0	0	0	0	0	6
0	0	0	0	0	0	0	0	0	0	0	0
402	203	204	305	406	0	0	0	0	0	0	5
301	402	210	305	406	207	208	0	0	0	0	7
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
301	402	203	204	305	406	209	0	0	0	0	7
301	402	203	204	305	406	207	208	0	0	0	8

---

TABLE VII  
MPM FOR MECHANISM 1 OF CHAIN 189

---

301	209	406	207	208	0	0	0	0	0	0	5
301	402	406	209	0	0	0	0	0	0	0	4
301	402	406	207	208	0	0	0	0	0	0	5
301	402	210	305	406	209	0	0	0	0	0	6
301	402	210	305	406	207	208	0	0	0	0	7
301	402	203	204	305	406	209	0	0	0	0	7
301	402	203	204	305	406	207	208	0	0	0	8

---

TABLE VIII  
MPM OF TABLE VII REDUCED TO MATRIX OF LINK TYPES

---

3	2	4	2	2	0	0	0	0	0	0	5
3	4	4	2	0	0	0	0	0	0	0	4
3	4	4	2	2	0	0	0	0	0	0	5
3	4	2	3	4	2	0	0	0	0	0	6
3	4	2	3	4	2	2	0	0	0	0	7
3	4	2	2	3	4	2	0	0	0	0	7
3	4	2	2	3	4	2	2	0	0	0	8

---

(1) Equal P  
(2)

IX and will be compared with the MPM in Table VIII.

First, the two matrices have the same number of rows thus justifying further comparisons. Row 1 of Table VIII contains 5 elements and, therefore, may be equivalent to either row 2 or row 4 of Table IX. It is found to be equivalent to row 2 of Table IX. Similarly, each row of Table VIII is found to be equivalent to a row in Table IX and vice versa. Therefore, the two mechanisms are equivalent.

As mentioned previously, two chains can be compared by comparing mechanisms formed from them. However, other initial comparisons will determine whether or not comparison of mechanisms is warranted. Obviously, the two chains must have the same number of links and the same number of each type of link. Also, the CPM's must have the same number of rows thus indicating that the two chains have the same number of paths. Once these conditions have been met, mechanisms from each chain should be chosen for comparison. The choice of mechanisms will often reduce the number of comparisons required.

(1) S  
L  
(2) S  
eal  
lin  
(3) C  
ne

For two mechanisms to be equivalent they must have the same type of fixed link. Thus, choosing the link type which appears least frequently in the chain will limit the number of mechanisms compared. For example, if two chains are to be compared and each chain has only one quaternary link, the only mechanisms which have to be compared are those formed from fixing the quaternary link.

#### Computer Development of the MPM

Subroutine MECH (Appendix D) uses the CPM to develop the MPM for each mechanism derived from a given chain. To do this, it takes each link (in numerical order) and forms the MPM from rows in the CPM which



TABLE IX  
MPM FOR MECHANISM 5 OF CHAIN 189

---

3	4	4	2	0	0	0	0	0	0	0	4
3	2	4	2	2	0	0	0	0	0	0	5
3	4	2	3	4	2	0	0	0	0	0	6
3	4	4	2	2	0	0	0	0	0	0	5
3	4	2	2	3	4	2	0	0	0	0	7
3	4	2	3	4	2	2	0	0	0	0	7
3	4	2	2	3	4	2	2	0	0	0	8

---

contain that link. As the MPM is formed, each row is cycled so that the fixed link appears as the first element. In addition, the number of rows in each MPM is determined and this number is stored in the (N-1)st column of the first row of the MPM. This permits a rapid determination of the number of rows when the MPM's are being compared.

#### Computer Comparison of MPM's

Subroutine MECCOM (Appendix E) compares MPM's using the comparison procedure described earlier in this chapter as may be seen from the flow chart in Appendix E. An array of MPM's such as that developed by Subroutine MECH serves as the input to MECCOM. Each MPM is compared with all other MPM's and the results stored in array IEQUIV.

Two features are incorporated in Subroutine MECCOM which are not used in comparing the MPM's as they have been described thus far.

1. Provisions are made for cycling one of two rows being compared. This feature is used in comparing modified MPM's described in following chapters.

2. Provisions are made for zeroing certain rows of array JSTORE and then compacting the non-zero rows. This feature is used in the development of the modified MPM's described in following chapters.

#### Computer Program for Determining the Ten-Link Mechanisms

The main program and subroutines (other than LOOP, MECH, and MECCOM) used in determining the ten link mechanisms are shown in Appendix F. One page of the output obtained for chains 1 through 25 is shown in Table X. In Table X, the columns represent link numbers one through ten and the rows represent the chain being described. An entry

TABLE X  
 SAMPLE OUTPUT FOR TEN-LINK MECHANISMS

CHAIN	LINK NUMBER										UNIQUE MECHANISMS	
	1	2	3	4	5	6	7	8	9	10		
1		1		3	1	1	3	3		9		3
2		1					3	4	6	5		5
3												10
4				3	2	1						7
5												10
6												10
7												10
8			2	1			6	5		9		5
9			2	1		5		5	5	7		4
10			2	1				5	6	7		5
11												10
12												10
13												10
14												10
15												10
16				3	2	1			7			6
17				2	1					9		7
18												10
19												10
20												10
21			2	1						9		7
22												10
23		1			4	3	3	4	4	3		3
24				1	2	3			8	7		5
25			1			5	4			8		6

in any column indicates that the link designated by that column is equivalent to the link number entered in the column. For example, in row 1 representing chain 1, links 2, 5, and 6 are equivalent to link 1, links 4, 7, and 8 are equivalent to link 3, and link 10 is equivalent to link 9. A blank entry indicates that the mechanism formed by fixing the link corresponding to the column number is a unique mechanism. For chain 1 there are three blank columns indicating that fixing links 1, 3, or 9 will give three unique mechanisms. Fixing any other link will result in a mechanism equivalent to one of the above three. The total number of unique mechanisms for each chain is reflected in the last column.

The entire program for developing the ten-link mechanisms took 6 minutes and 50 seconds to run on an IBM 360 Model 50 computer. The total number of unique mechanisms obtained was 1836.

## CHAPTER V

### CHAINS WITH SPRINGS

In Chapter I it was indicated that the CPM is useful in converting basic kinematic chains into chains with various elements such as springs and cams. In this chapter, the development of chains with springs will be covered.

#### Theory

Two binary links connected by a revolute pair may be replaced by a spring as shown in Figure 7. In the CPM, this configuration would appear as two adjacent binary links in one or more rows. If a spring is designated by a type number, for example 11, then the two numbers representing the binary links can be replaced by the single number representing the spring.

From Figure 6 it can be seen that links 3 and 4 and links 7 and 8 of chain 189 can be replaced by springs. This information is also apparent in Table VI which shows the CPM for chain 189. In the CPM, binary links 3 and 4 are seen to be adjacent in rows 4, 10, 14, and 15 and binary links 7 and 8 are seen to be adjacent in rows 1, 5, 11, and 15. Thus, from the CPM it would appear that chain 189 can yield two chains with one spring, one with links 3 and 4 replaced by a spring and one with links 7 and 8 replaced by a spring. It is evident from Figure 6 that these two chains are equivalent due to the symmetry of chain

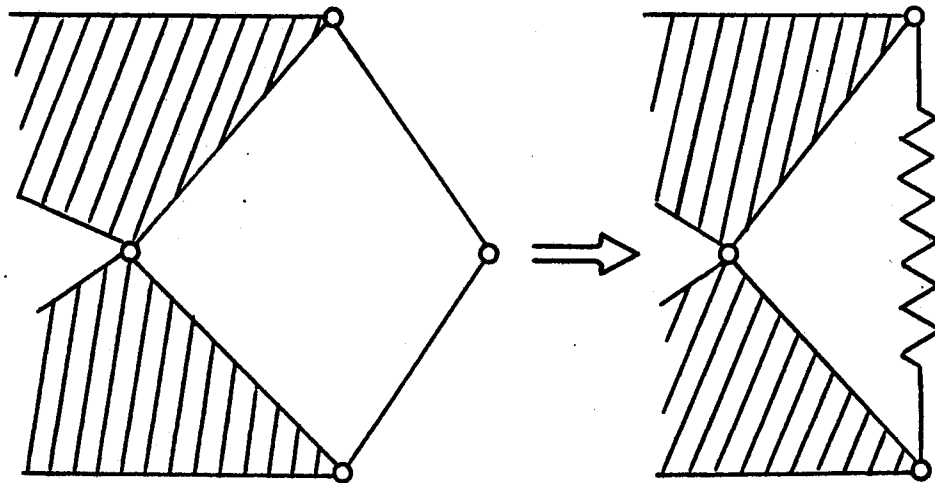


Figure 7. Two Binary Links Replaced by a Spring

189. However, a procedure must be devised to determine symmetry from the GPM.

If links 3 and 4 are replaced by a spring and an MPM formed as if the spring were a fixed link, the MPM would then represent all paths originating at the spring. Similarly, an MPM can be formed for the chain with links 7 and 8 replaced by a spring. The two MPM's would appear as shown in Tables XI and XII. The two MPM's are identical thus indicating that the two chains are equivalent.

To obtain a chain with two springs, both pairs of binary links are replaced with springs in the GPM. To compare the resulting chain with another chain containing two springs, the MPM's representing each spring could be compared. However, the same result may be obtained by using a modified MPM consisting of the combination of the individual MPM's for each spring. That is, the modified MPM will contain each row of the GPM which contains either spring or both springs. The modified MPM representing chain 189 is shown in Table XIII. Since the modified MPM does not represent the paths originating from a single element, the procedure for comparing two modified MPM's must be changed. Essentially, what is required is that, before deciding that two rows are not equivalent, a row containing two springs must be compared first with one spring as the first element and second, with the other spring as the first element. In Table XIII, only the last row contains two springs and the row would be the same regardless of which spring was used as the first element. However, for two rows, A and B, as shown in Table XIV, one of the rows must be cycled to obtain an equivalence. If row B is cycled as shown in the table, it becomes the same as row A read in reverse. Thus the two rows are equivalent.

TABLE XI

MPM FOR CHAIN 189 WITH LINKS 3 AND 4 REPLACED BY A SPRING

11	3	2	4	0	0	0	0	0	0	0	4
11	3	4	4	0	0	0	0	0	0	0	4
11	3	4	2	3	4	0	0	0	0	0	6
11	3	4	2	2	3	4	0	0	0	0	7

TABLE XII

MPM FOR CHAIN 189 WITH LINKS 7 AND 8 REPLACED BY A SPRING

11	3	2	4	0	0	0	0	0	0	0	4
11	3	4	4	0	0	0	0	0	0	0	4
11	3	4	2	3	4	0	0	0	0	0	6
11	3	4	2	2	3	4	0	0	0	0	7

TABLE XIII

MODIFIED MPM FOR CHAIN 189 WITH TWO SPRINGS

11	3	2	4	0	0	0	0	0	0	0	4
11	3	2	4	0	0	0	0	0	0	0	4
11	3	4	4	0	0	0	0	0	0	0	4
11	3	4	4	0	0	0	0	0	0	0	4
11	3	4	2	3	4	0	0	0	0	0	6
11	3	4	2	3	4	0	0	0	0	0	6
11	3	4	11	3	4	0	0	0	0	0	6

TABLE XIV

EQUIVALENCE OF ROWS WITH TWO SPRINGS

ROW A	11	3	4	11	4	3	3
ROW B	11	4	3	11	3	3	4
.....							
ROW B CYCLED	11	3	3	4	11	4	3



In the last chapter, it was mentioned that a certain portion of Subroutine MECCOM was used for cycling rows when required for the comparison of certain modified MPM's. That portion of MECCOM performs the cycling described above and permits comparison of modified MPM's representing any number of special elements. The number of times any row is cycled is limited to the number of identical special elements inserted in the chain. If a chain contained three springs, for example, a row would not be cycled more than three times.

#### Computer Program for Developing Chains With Springs

The computer program for developing chains with springs is shown in Appendix G along with a description of the variables used. The program performs the process of adding springs to a chain by proceeding as follows:

1. The loop matrix is read in and converted to the CPM. (Subroutine LOOP)
2. The CPM is examined and the pairs of links to be replaced by springs are assigned a number and stored in array LPAIR. (Subroutine SPRLOC)
3. The identification of the possible springs is printed out. (Subroutine ARRAY)
4. MPM's are developed for a single spring at each one of the locations determined in step 2 and are stored in array MX. (Subroutine SPRNG1)
5. The MPM's are compared and the equivalent chains identified. The results of the comparison are printed out and the unique chains, identified by the spring number assigned in step 2, are stored in array

JSTORE. (Subroutine MECCOM)

6. A second spring is added to the unique chains identified in step 5 and the modified MPM's formed. (Subroutine SPRING2)

7. The modified MPM's are compared and the unique chains identified as in step 5. (Subroutine MECCOM)

8. Additional springs are added and unique chains determined until the chain can accept no more springs. (Subroutines SPRNG3 and SPRNG4)

A sample print out for chain 189 with springs is shown in Table XV. A summary of results obtained from the ten-link chains is shown in Table XVI.

TABLE XV

## SAMPLE OUTPUT FOR CHAIN 189 WITH SPRINGS

CHAIN 189  
 \*\*\*\*\*

## IDENTIFICATION OF SPRINGS

SPRING#		1		2	
<hr/>					
LINK1, LINK2		3, 4		7, 8	

CHAINS WITH ONE SPRING (IDENTIFIED BY THE SPRING NUMBER ABOVE)  
 \*\*\*\*\*

## EQUIVALENT CHAINS

1 = 2

NUMBER OF UNIQUE CHAINS - 1

IDENTIFICATION OF CHAINS WITH TWO SPRINGS  
 \*\*\*\*\*

CHAIN#		1	
<hr/>			
SPRINGS		1,2	

## EQUIVALENT CHAINS

NCNE

NUMBER OF UNIQUE CHAINS - 1

NOTE: "LINK1, LINK-2" indicates the links replaced by the spring.

TABLE XVI  
TEN-LINK CHAINS WITH SPRINGS

---

Total number of 10-link chains with 1 spring	-	234
Total number of 10-link chains with 2 springs	-	83
Total number of 10-link chains with 3 springs	-	12
Total number of 10-link chains with 4 springs	-	1

---

## CHAPTER VI

### CHAINS WITH PULLEYS AND BELTS

#### Theory

A ternary link with two binary links connected to it can be converted to a pulley and belt as shown in Figure 8. In the CPM, such an arrangement of links would appear as a ternary link with a binary link on each side of it. If the ternary link is replaced by some type number designating a pulley, say 11 again, and the binary links are replaced by some type number representing a belt, say 12, the CPM will then represent the chain with one pulley. An MPM can be formed using the pulley as the starting element for each path and will serve as a means of comparing one chain with a pulley to a second chain with a pulley.

As an example of the above, consider link 1 of chain 189 (Figure 6). Since it is a ternary link connected to two binary links, it can be converted to a pulley and belt. This fact is reflected in row 1 of the CPM for chain 189 as shown in Table VI where 301 appears between 208 and 209. To develop the MPM with link 1 converted to a pulley, 301 must be converted to 11 everywhere it appears in the CPM. Similarly, links 208 and 209 must be converted to 12 wherever they appear. The MPM with the above conversions completed is shown in Table XVII.

Some kinematic chains may contain several pulleys just as some may contain several springs. However, some rules must be established to

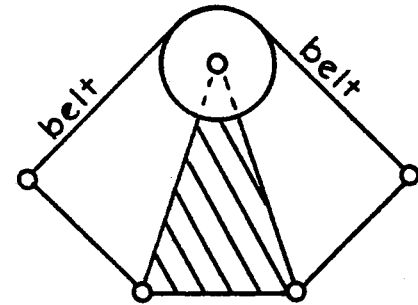
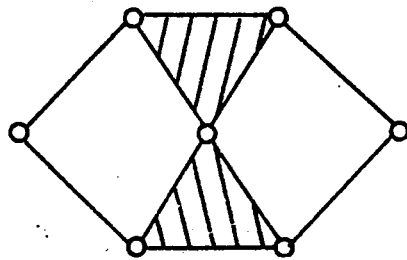


Figure 8. Ternary Link and Two Binary Links Converted to a Pulley and Belt

TABLE XVII

MPM FOR CHAIN 189 WITH LINK 1 CONVERTED TO A PULLEY  
AND LINKS 8 AND 9 CONVERTED TO A BELT

---

11	12	4	2	12	0	0	0	0	0	0	5
11	4	4	12	0	0	0	0	0	0	0	4
11	4	4	2	12	0	0	0	0	0	0	5
11	4	2	3	4	12	0	0	0	0	0	6
11	4	2	3	4	2	12	0	0	0	0	7
11	4	2	2	3	4	12	0	0	0	0	7
11	4	2	2	3	4	2	12	0	0	0	8

---

prevent the chain from becoming a structure. The rules established in converting the ten-link chains to chains with pulleys are given below and illustrated in the referenced figures.

1. A ternary link joined to three binary links may be converted to three different pulley/belt combinations. However, no two of these can exist simultaneously. (Figure 9)

2. Two pulleys may share a common belt, but two binary links connected by a revolute pair may not be converted to a single belt. (Figure 10)

#### Computer Program for Converting Basic Kinematic Chains to Chains with Pulleys and Belts

Essentially, the same procedure is followed in developing chains with pulleys as was followed in the preceding chapter where springs were used. The possible locations for pulleys are determined and then, the pulleys are added one at a time to establish the chains. Subroutine PULLOC, which determines the pulley/belt locations, and Subroutine PUL3, which develops the modified MPM for chains with three pulleys, are shown in Appendix H. Subroutines for adding more or less pulleys follow the same procedures as PUL3. The entire program for adding pulleys is the same as the program for adding springs (Appendix G) except that Subroutine PULLOC would replace Subroutine SPRLOC and Subroutines PUL1, PUL2, PUL3, and PUL4 would replace Subroutines SPRNG1, SPRNG2, SPRNG3, and SPRNG4. All other subroutines are unchanged except for format statements. The computer print out for chain 189 with pulleys is shown in Table XVIII. A summary of the results obtained for the ten-link chains with pulleys appears in Table XIX.



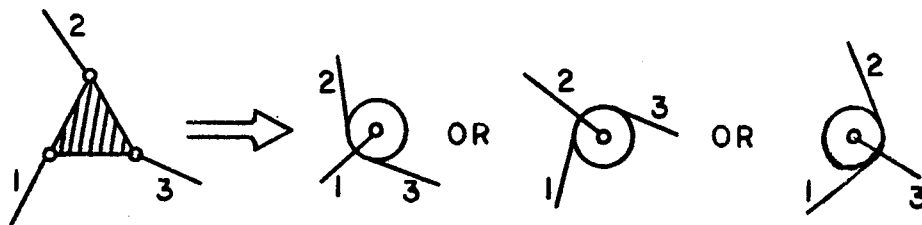


Figure 9. Ternary Link with 3 Binary Links  
Converted to Pulleys

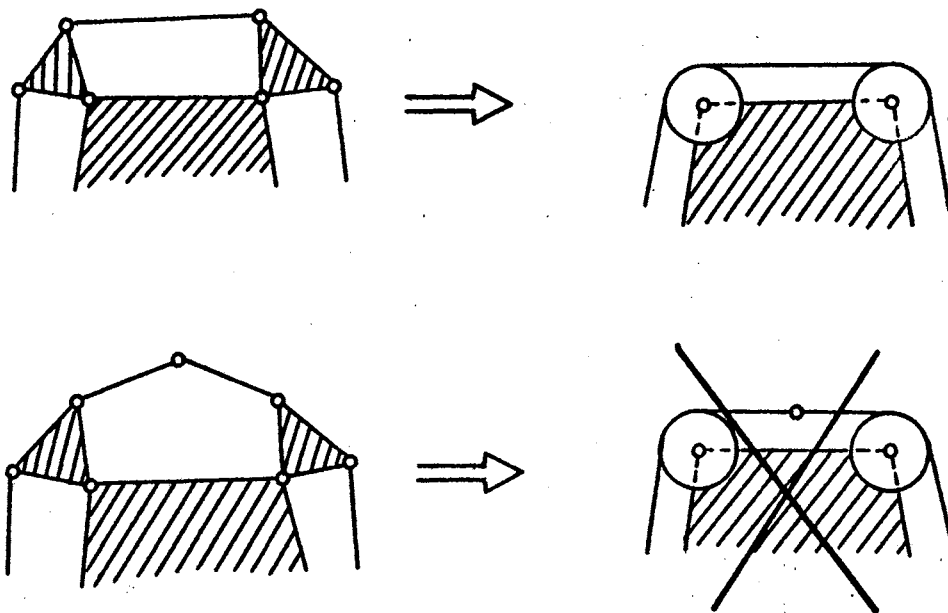


Figure 10. Ternary Links Sharing a Binary Link  
Converted to Pulleys and Belt

TABLE XVIII

## SAMPLE OUTPUT FOR CHAIN 189 WITH PULLEYS

CHAIN 189  
\*\*\*\*\*

## IDENTIFICATION OF PULLEYS

PULLEY#		1		2	
-----					
PULLEY(BELT,BELT)		1( 8, 9)		5( 4,10)	

CHAINS WITH ONE PULLEY (IDENTIFIED BY PULLEY NUMBER ABOVE)  
\*\*\*\*\*

## EQUIVALENT CHAINS

l= 2

NUMBER OF UNIQUE CHAINS - 1

IDENTIFICATION OF CHAINS WITH TWO PULLEYS  
\*\*\*\*\*

CHAIN#		1	
-----			
PULLEYS		1,2	

## EQUIVALENT CHAINS

NONE

NUMBER OF UNIQUE CHAINS - 1

NOTE: "PULLEY (BELT, BELT)" refers to the link replaced by a pulley and the two links replaced by a belt.

TABLE XIX  
TEN-LINK CHAINS WITH PULLEYS

---

Total number of 10-link chains with 1 pulley	-	358
Total number of 10-link chains with 2 pulleys	-	240
Total number of 10-link chains with 3 pulleys	-	58
Total number of 10-link chains with 4 pulleys	-	7

---

## CHAPTER VII

### CHAINS WITH CAM PAIRS

A binary link and two revolute pairs may be converted to a cam pair as shown in Figure 11. In a basic kinematic chain, all links are joined by revolute pairs, therefore, each binary link may be replaced by a cam pair. If a cam pair is represented by a type number, say 11, the CPM may be converted to an MPM representing a chain with one cam pair simply by replacing a binary link with the number 11. Additional cam pairs may be added and modified MPM's formed subject to the restriction that two adjacent binary links cannot be simultaneously converted to cam pairs. This means that, in Figure 11, links 2 and 3 cannot both be converted to cam pairs.

The computer program for developing chains with cam pairs follows the same pattern as those for springs and for pulleys and belts. Subroutine CAMLOC, which locates the cam positions, and Subroutine CAM3, which corresponds to SPRNG3 and PUL3 in previous chapters, are shown in Appendix I. The computer print out for chain 189 with cam pairs is shown in Table XX. A summary of the results obtained for the ten-link chains with cam pairs is shown in Table XXI.

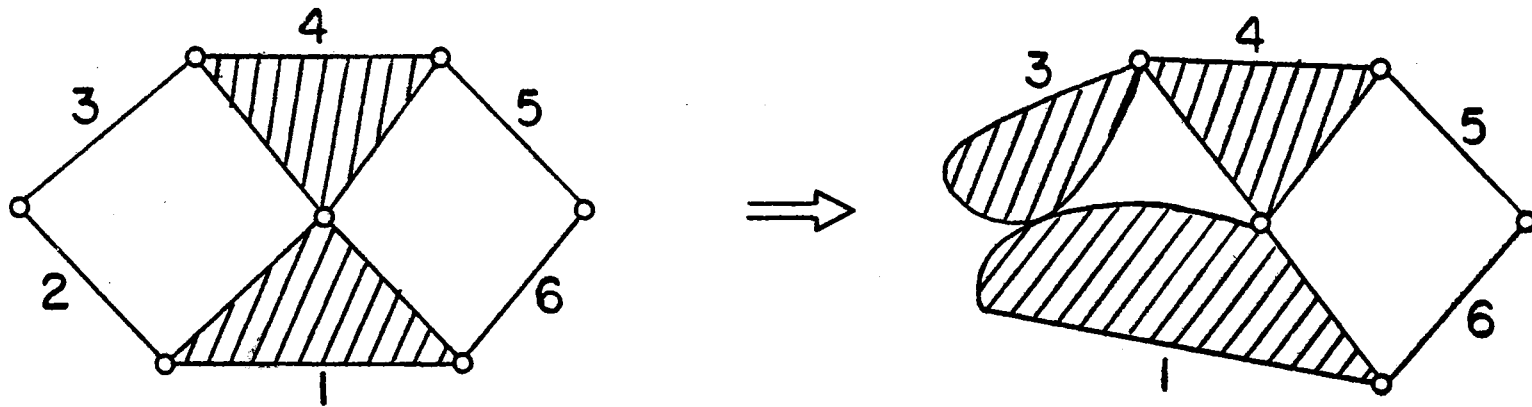


Figure 11. Binary Link Converted to a Cam Pair

TABLE XX

## SAMPLE OUTPUT FOR CHAIN 189 WITH CAM PAIRS

CHAIN 189  
\*\*\*\*\*

## IDENTIFICATION OF CAM PAIRS

CAM#		1		2		3		4		5		6	
-----													
LINK		3		4		7		8		9		10	

## CHAINS WITH ONE CAM PAIR (IDENTIFIED BY THE CAM NUMBER ABOVE)

\*\*\*\*\*

## EQUIVALENT CHAINS

1= 3  
2= 4  
5= 6

NUMBER OF UNIQUE CHAINS - 3

## IDENTIFICATION OF CHAINS WITH TWO CAM PAIRS

\*\*\*\*\*

CHAIN#		1		2		3		4		5	
-----											
CAMS		1,3		1,4		1,5		1,6		2,3	
CHAIN#		6		7		8		9			
-----											
CAMS		2,4		2,5		2,6		5,6			

## EQUIVALENT CHAINS

2= 5

NUMBER OF UNIQUE CHAINS - 8

## IDENTIFICATION OF CHAINS WITH THREE CAM PAIRS

\*\*\*\*\*

CHAIN#		1		2		3		4		5	
-----											
CAMS		1,3,5		1,3,6		1,4,5		1,4,6		1,5,6	
CHAIN#		6		7		8					
-----											
CAMS		2,4,5		2,4,6		2,5,6					

TABLE XX (Continued)

CHAIN 189 (CONTINUED)  
 \*\*\*\*\*

## EQUIVALENT CHAINS

1= 2  
 6= 7

NUMBER OF UNIQUE CHAINS - 6

IDENTIFICATION OF CHAINS WITH FOUR CAM PAIRS  
 \*\*\*\*\*

CHAIN#	1	2	3
CAMS	1,3,5,6	1,4,5,6	2,4,5,6

## EQUIVALENT CHAINS

NCNE

NUMBER OF UNIQUE CHAINS - 3

IDENTIFICATION OF CHAINS WITH FIVE CAM PAIRS  
 \*\*\*\*\*

NO POSSIBLE CHAINS

TABLE XXI  
TEN-LINK CHAINS WITH CAM PAIRS

---

Total number of 10-link chains with 1 cam pair	-	913
Total number of 10-link chains with 2 cam pairs	-	1661
Total number of 10-link chains with 3 cam pairs	-	1415
Total number of 10-link chains with 4 cam pairs	-	624
Total number of 10-link chains with 5 cam pairs	-	121
Total number of 10-link chains with 6 cam pairs	-	10

---



## CHAPTER VIII

### CHAINS WITH PRISM PAIRS

A prism pair may replace a revolute pair in a kinematic chain thus permitting linear motion rather than rotary motion. If a prism pair is represented by a specific type number, say 11 once more, converting the CPM to represent a chain with one prism pair becomes a simple matter of inserting the 11 between the two links to be joined by the prism pair. The MPM is then formed from each row of the CPM containing a prism pair. Chains with two prism pairs may also be formed in the same manner and a modified MPM used for comparison of those chains just as in previous chapters.

If more than two prism pairs are added to a chain, care must be taken to insure that the mobility of the chain is retained. If three prism pairs are inserted in a circuit or path with only four links, the mobility of the chain is no longer retained. Similarly, four prism pairs in a circuit consisting of five links will result in that circuit becoming underconstrained although constrained motion is achieved with three prism pairs (with non-parallel axes). Since every path represented in the CPM actually represents a complete circuit, the requirement for mobility reduces to the rule that no path may contain more than  $n-2$  prism pairs where  $n$  is the number of links in the path. Thus, to insure that mobility of a chain is maintained when adding more than two prism pairs, each row of the CPM must be checked to determine

if it will contain more than  $n-2$  prism pairs.

The computer program for developing chains with prism pairs is similar to those developed in previous chapters. Subroutine JOINT locates all joints in the chain and identifies them according to the links incident to that joint. For example, the joint connecting links 2 and 3 in a chain would be identified by the notation 2-3.

Subroutine PRISM2 demonstrates the logic used in developing chains with multiple prism pairs. As was the case with double joints, the number of chains with more than two prism pairs which may be developed from a single basic kinematic chain becomes very large and therefore, the program used for the ten-link chains was limited to two prism pairs. For this reason, the test for mobility was not included in the program. Subroutines JOINT and PRISM2 are shown in Appendix J. The computer print out for chain 189 with prism pairs appears in Table XXII. A summary of the results obtained for all ten-link chains with prism pairs is shown in Table XXIII.

TABLE XXII

## SAMPLE OUTPUT FOR CHAIN 189 WITH PRISM PAIRS

CHAIN 189  
\*\*\*\*\*

## IDENTIFICATION OF JOINTS

JOINT#	1	2	3	4	5	6	7
LINK1, LINK2	1, 2	1, 8	1, 9	2, 3	2, 6	2, 10	3, 4
JOINT#	8	9	10	11	12	13	
LINK1, LINK2	4, 5	5, 6	5, 10	6, 7	6, 9	7, 8	

CHAINS WITH ONE PRISM PAIR (IDENTIFIED BY THE JOINT NUMBER ABOVE)  
\*\*\*\*\*

## EQUIVALENT CHAINS

1= 9  
2= 8  
3=10  
4=11  
6=12  
7=13

NUMBER OF UNIQUE CHAINS - 7

IDENTIFICATION OF CHAINS WITH TWO PRISM PAIRS  
\*\*\*\*\*

CHAIN#	1	2	3	4	5
PULLEYS	1,2	1,3	1,4	1,5	1,6
CHAIN#	6	7	8	9	10
PULLEYS	1,7	1,8	1,9	1,10	1,11
CHAIN#	11	12	13	14	15
PULLEYS	1,12	1,13	2,3	2,4	2,5
CHAIN#	16	17	18	19	20
PULLEYS	2,6	2,7	2,8	2,9	2,10

NOTE: The word "PULLEYS" should read "JOINTS" in this table.

TABLE XXII (Continued)

## CHAIN 189 (CONTINUED)

\*\*\*\*\*

CHAIN#	21	22	23	24	25	
PULLEYS	2,11	2,12	2,13	3,4	3,5	
CHAIN#	26	27	28	29	30	
PULLEYS	3,6	3,7	3,8	3,9	3,10	
CHAIN#	31	32	33	34	35	
PULLEYS	3,11	3,12	3,13	4,5	4,6	
CHAIN#	36	37	38	39	40	
PULLEYS	4,7	4,8	4,9	4,10	4,11	
CHAIN#	41	42	43	44	45	
PULLEYS	4,12	4,13	5,6	5,7	5,8	
CHAIN#	46	47	48	49	50	
PULLEYS	5,9	5,10	5,11	5,12	5,13	
CHAIN#	51	52	53	54	55	
PULLEYS	6,7	6,8	6,9	6,10	6,11	
CHAIN#	56	57	58	59	60	
PULLEYS	6,12	6,13	7,8	7,9	7,10	
CHAIN#	61	62	63			
PULLEYS	7,11	7,12	7,13			

## EQUIVALENT CHAINS

4=46  
 7=19  
 9=29  
 10=38  
 11=53  
 12=59  
 15=45  
 20=28  
 21=37

## TABLE XXII (Continued)

## CHAIN 189 (CONTINUED)

\*\*\*\*\*

22=52

23=58

25=47

31=39

33=60

34=48

41=55

42=61

43=49

44=50

57=62

NUMBER OF UNIQUE CHAINS -43

TABLE XXIII  
TEN-LINK CHAINS WITH PRISM PAIRS

---

Total number of 10-link chains with 1 prism pair	- 2312
Total number of 10-link chains with 2 prism pairs	- 12969

---

## CHAPTER IX

### CHAINS WITH DOUBLE JOINTS

#### Theory

Ternary and higher links may be converted to links with double joints by reducing a side between two revolute pairs to zero. Some examples of this are shown in Figure 12. When a double joint is added, care must be taken to insure that the chain retains a single degree of freedom. Rules to prevent all or part of a chain from becoming a structure are as follows:

1. No loop may be reduced to only three links. (Figure 13)
2. No two paths, each containing only four links, may have three links in common. In Figure 14, this rule is demonstrated by loops 2 and 3. Initially, loop 2 contains 5 links, but, when a double joint is added such that links 7 and 5 are joined, loop 2 is reduced to four links. The portion of the chain composed of loops 2 and 3 becomes a structure which cannot move. (Although the figure shows an eight-link chain, this rule was derived from, and applied to, the ten-link chains).

In the discussion that follows, double joints will be identified by a set of numbers of the form  $l(2,3)$ . The first number represents the link which has one side reduced to zero and will be referred to as the base link. The two numbers in parentheses represent the two links joined to the base link at the double joint. For example, the double joint shown in Figure 12 (a) would be referred to as  $4(3,5)$ .

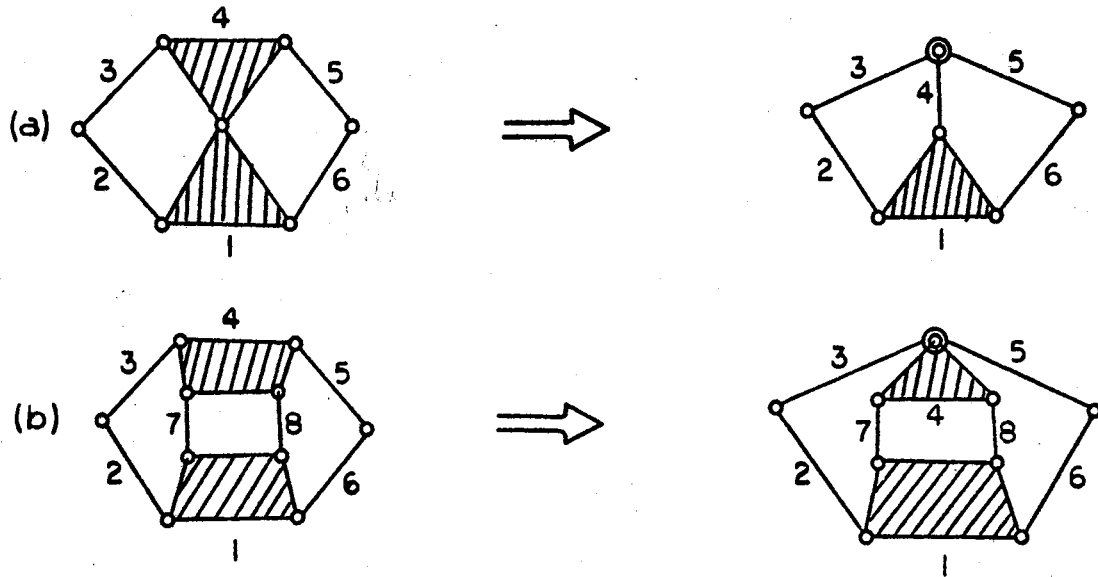


Figure 12. Addition of Double Joint to Basic Kinematic Chain

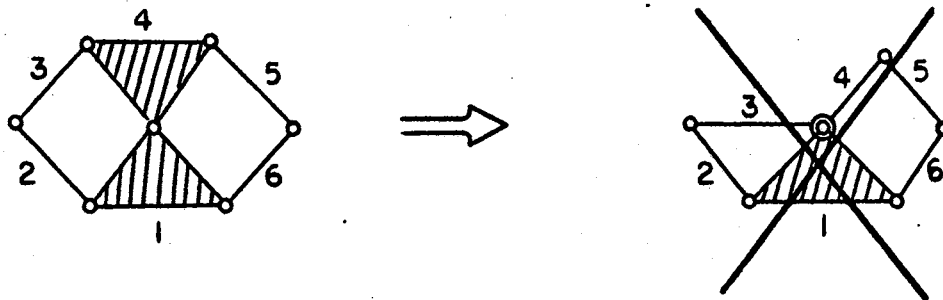


Figure 13. Demonstration of Rule 1 for Double Joints

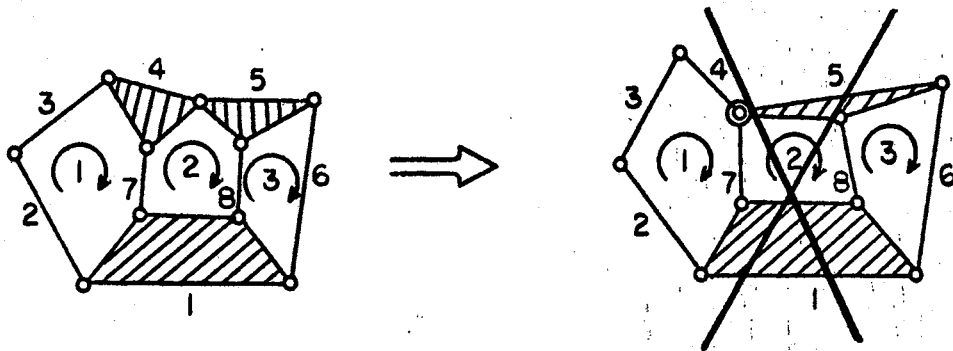


Figure 14. Demonstration of Rule 2 for Double Joints



Since double joints can be formed with any ternary or higher link, all possible double joints in a chain can be easily located by use of the CPM. In a row of the CPM, any link other than a binary link may be a base link, in which case, the two links adjacent to it become the other links incident at the double joint. For example, possible double joints found from the first two rows of the CPM for chain 189 (Table VI) are 1(8,9), 6(7,9), 1(2,9), 2(1,6), and 6(2,9). Some of these cannot actually be used due to rule 1 or 2 above. A double joint at position 1(2,9) would reduce row 2 of the CPM to a row with only three links (link 1 would effectively be removed from that path) thus violating rule 1. Positions 2(1,6) and 6(2,9) cannot be used for the same reason.

As in previous chapters, if a double joint is represented by a specific type number such as 11, an MPM may be developed which represents the chain with a double joint. In forming the MPM, the base link is replaced by the number 11 in those rows where it is flanked by the incident links. In other rows where the base link and only one of the incident links appear, the number 11 is inserted between the base link and the incident link. Table XXIV shows the MPM (with complete link identification numbers) for chain 189 with a double joint at position 1(8,9). The double joint is represented by 1100 to conform with the convention that the type number appears in the hundreds and thousands places.

Once the MPM has been established, chains with one double joint may be compared just as chains with other special elements. Additional double joints may also be added and modified MPM's formed. However, for each double joint added, the rules for mobility must be reapplied.

TABLE XXIV

MPM FOR CHAIN 189 WITH DOUBLE JOINT AT POSITION 1(8,9)

---

1100	209	406	207	208	0	0	0	0	0	0	5
1100	301	402	406	209	0	0	0	0	0	0	5
1100	301	402	406	207	208	0	0	0	0	0	6
1100	301	402	210	305	406	209	0	0	0	0	7
1100	301	402	210	305	406	207	208	0	0	0	8
1100	301	402	203	204	305	406	209	0	0	0	8
1100	301	402	203	204	305	406	207	208	0	0	9

---

In addition, a third restriction that arises when more than one double joint is used is that no two double joints may have two elements in common. That is, they may not share two links. For the chain shown in Figure 21 (b), joints 4(3,7) and 4(5,8) could exist simultaneously but 4(3,7) and 4(3,5) could not. The combination of position 4(3,7) and 4(3,8) would, in effect, create a triple joint which is not considered here.

#### Computer Program for Developing Chains With Double Joints

The basic program for developing chains with double joints follows the same pattern as previous programs. Subroutine DBLOC, which locates the positions of double joints, and subroutine DBL2, which constructs the modified MPM for chains with two double joints, appear in Appendix K. Subroutine DBL2 also tests for mobility by calling Subroutines LNKCNT and MOBCK. The computer print out for chain 189 with double joints appears in Table XXV.

The ten-link chains with more than two double joints were not developed due to the time required to perform the computations. In testing the program, it was found that a number of the ten-link chains would each produce in excess of 130 chains with three double joints. The computer time required to compare these chains runs into hours and was therefore not attempted. A summary of the results obtained for the ten-link chains with double joints appears in Table XXVI.

TABLE XXV

## SAMPLE OUTPUT FOR CHAIN 189 WITH DOUBLE JOINTS

CHAIN 189  
\*\*\*\*\*

## IDENTIFICATION OF DOUBLE JOINTS

JOINT#	1	2	3	4
BASE(LINK1,LINK2)	1( 8, 9)	1( 8, 2)	2( 1,10)	2( 1, 3)
JOINT#	5	6	7	8
BASE(LINK1,LINK2)	2( 6, 3)	2(10, 3)	5(10, 4)	5( 6, 4)
JOINT#	9	10	11	12
BASE(LINK1,LINK2)	6( 9, 7)	6( 9, 5)	6( 7, 2)	6( 7, 5)

CHAINS WITH ONE DOUBLE JOINT (IDENTIFIED BY THE JOINT NUMBER ABOVE)  
\*\*\*\*\*

## EQUIVALENT CHAINS

1= 7  
2= 8  
3=10  
4=12  
5=11  
6= 9

NUMBER OF UNIQUE CHAINS - 6

IDENTIFICATION OF CHAINS WITH TWO DOUBLE JOINTS  
\*\*\*\*\*

CHAIN#	1	2	3	4	5
JOINTS	1,3	1,4	1,5	1,6	1,7
CHAIN#	6	7	8	9	10
JOINTS	1,8	1,10	1,11	1,12	2,5
CHAIN#	11	12	13	14	15
JOINTS	2,6	2,7	2,8	2,9	2,10

TABLE XXV (Continued)

## CHAIN 189 (CONTINUED)

\*\*\*\*\*

CHAIN#	16	17	18	19	20
--------	----	----	----	----	----

JOINTS	2,12	3,5	3,7	3,8	3,9
--------	------	-----	-----	-----	-----

CHAIN#	21	22	23	24	25
--------	----	----	----	----	----

JOINTS	3,10	3,11	3,12	4,7	4,8
--------	------	------	------	-----	-----

CHAIN#	26	27	28	29	30
--------	----	----	----	----	----

JOINTS	4,9	4,10	4,11	4,12	5,7
--------	-----	------	------	------	-----

CHAIN#	31	32	33	34	35
--------	----	----	----	----	----

JOINTS	5,9	5,10	5,12	6,8	6,9
--------	-----	------	------	-----	-----

CHAIN#	36	37	38
--------	----	----	----

JOINTS	6,10	6,11	6,12
--------	------	------	------

## EQUIVALENT CHAINS

6=12  
 7=18  
 8=30  
 9=24  
 14=34  
 15=19  
 16=25  
 20=36  
 22=32  
 23=27  
 26=38  
 28=33  
 31=37

NUMBER OF UNIQUE CHAINS -25

TABLE XXVI

## TEN-LINK CHAINS WITH DOUBLE JOINTS

---

Total number of 10-link chains with 1 double joint	- 2039
Total number of 10-link chains with 2 double joints	- 7575

---

## CHAPTER X

### COMPARISON OF MECHANISMS DERIVED FROM CHAINS WITH SPECIAL ELEMENTS

Mechanisms derived from chains with special elements may be compared by use of the MPM. To demonstrate this and to further clarify the derivation of the CPM and MPM, three examples will be used. In these examples, the various special elements will be represented by type numbers as follows:

Pulley	- 12
Belt	- 13
Spring	- 14
Prism Pair	- 15
Double Joint	- 16
Cam Pair	- 17

#### Example 1

As the first example, chain number 4 from Appendix A will be used. Special elements will be added as follows:

1. Links 3 and 4 replaced by spring
2. Prism pair added at joint 8-9
3. Link 7 (and associated revolute pairs) replaced by a cam pair.

The resulting chain would appear as shown in Figure 15.

The loop matrix for the basic chain appears in Table XXVII. To obtain the CPM for the modified chain, the CPM for the basic chain may be formed and the special elements added. An alternate approach would

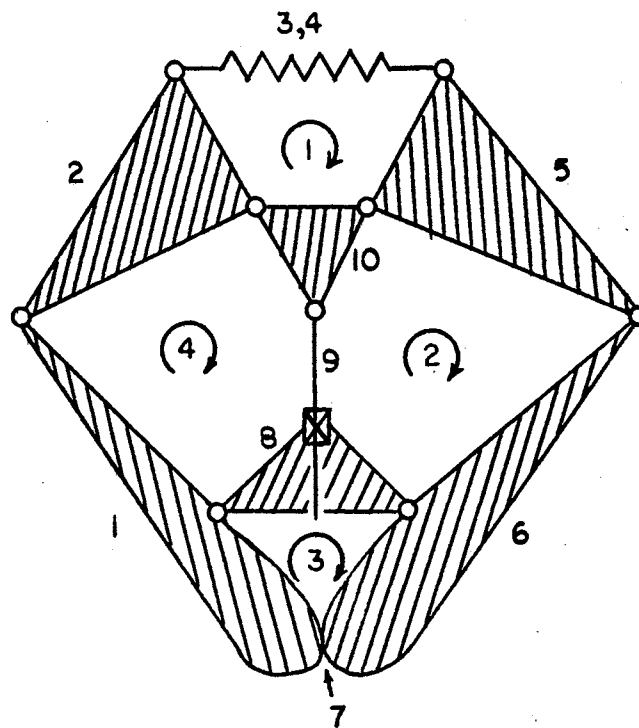


Figure 15. Chain 4 with Special Elements Added



TABLE XXVII  
LOOP MATRIX FOR CHAIN 4

---

302	203	204	305	310	0	0	0	0	0	0	5
305	306	308	209	310	0	0	0	0	0	0	5
306	207	301	308	0	0	0	0	0	0	0	4
301	302	310	209	308	0	0	0	0	0	0	5

---

TABLE XXVIII  
LOOP MATRIX FOR CHAIN 4 WITH SPECIAL ELEMENTS

---

302	1400	305	310	0	0	0	0	0	0	0	4
305	306	308	1500	209	310	0	0	0	0	0	6
306	1700	301	308	0	0	0	0	0	0	0	4
301	302	310	209	1500	308	0	0	0	0	0	6

---

TABLE XXIX  
CPM FOR CHAIN 4 WITH SPECIAL ELEMENTS

---

302	1400	305	310	0	0	0	0	0	0	0	4
305	306	308	1500	209	310	0	0	0	0	0	6
306	1700	301	308	0	0	0	0	0	0	0	4
301	302	310	209	1500	308	0	0	0	0	0	6
302	1400	305	306	308	1500	209	310	0	0	0	8
0	0	0	0	0	0	0	0	0	0	0	0
302	1400	305	310	209	1500	308	301	0	0	0	8
305	306	1700	301	308	1500	209	310	0	0	0	8
305	306	308	301	302	310	0	0	0	0	0	6
306	1700	301	302	310	209	1500	308	0	0	0	8
302	1400	305	306	1700	301	308	1500	209	310	0	10
302	1400	305	306	308	301	0	0	0	0	0	6
302	1400	305	310	209	1500	308	306	1700	301	0	10
305	306	1700	301	302	310	0	0	0	0	0	6
302	1400	305	306	1700	301	0	0	0	0	0	6

---

be to form the loop matrix for the modified chain and then use it to form the CPM. The latter approach will be used here.

The loop matrix for the modified chain appears in Table XXVIII. The special elements have been added according to the rules established in previous chapters. This loop matrix is then used to form the CPM (Table XXIX). The next step is to form the MPM's for all mechanisms which can be derived from the modified chain.

The basic chain could have been converted to 10 mechanisms (1 mechanism per link), however, in the modified chain, links 3, 4, and 7 have been replaced by elements which cannot be fixed. Consequently, mechanisms 3, 4, and 7 cannot exist. This fact is obvious from the CPM since link numbers 3, 4, and 7 do not appear. Forming the MPM's for the remaining links yields the results shown in Table XXX.

To compare the mechanisms, each MPM must be compared with all other MPM's. Starting with MPM's 1 and 2, it is seen that each has 11 rows and therefore, further comparison is justified. Comparing row 1 of MPM #1 with row 1 of MPM #2 (both have 4 elements) shows that the two mechanisms are not equivalent since these rows are not equivalent and no other rows with 4 elements exist in CPM #2. Checking all other matrices shows that only MPM #6 has a 4 element row that is equivalent to row 1 of MPM #1. Further comparisons show the following equivalent rows:

MPM # 1		MPM #6
2	=	1
3	=	3
4	=	6
5	=	5
6	=	4
7	=	9
8	=	8
9	=	7
10	=	10
11	=	11

TABLE XXX

MPM'S FOR CHAIN 4 WITH SPECIAL ELEMENTS

MPM # 1											MPM # 6										
3	3	3	17	0	0	0	0	0	0	4	3	3	15	2	3	3	0	0	0	0	6
3	3	3	2	15	3	0	0	0	0	6	3	17	3	3	0	0	0	0	0	0	4
3	3	14	3	3	2	15	3	0	0	8	3	3	15	2	3	3	14	3	0	0	8
3	3	15	2	3	3	3	17	0	0	8	3	17	3	3	15	2	3	3	0	0	8
3	3	3	3	3	3	0	0	0	0	6	3	3	3	3	3	3	0	0	0	0	6
3	3	3	2	15	3	3	17	0	0	8	3	17	3	3	3	2	15	3	0	0	8
3	3	15	2	3	3	14	3	3	17	10	3	17	3	3	15	2	3	3	14	3	10
3	3	14	3	3	3	0	0	0	0	6	3	3	3	3	14	3	0	0	0	0	6
3	3	14	3	3	2	15	3	3	17	10	3	17	3	3	14	3	3	2	15	3	10
3	3	3	3	3	17	0	0	0	0	6	3	17	3	3	3	3	0	0	0	0	6
3	3	14	3	3	17	0	0	0	0	6	3	17	3	3	14	3	0	0	0	0	6
.....											.....										
MPM # 2											MPM # 8										
3	14	3	3	0	0	0	0	0	0	4	3	15	2	3	3	3	0	0	0	0	6
3	3	2	15	3	3	0	0	0	0	6	3	3	17	3	0	0	0	0	0	0	4
3	14	3	3	3	15	2	3	0	0	8	3	3	3	3	2	15	0	0	0	0	6
3	14	3	3	3	15	3	3	0	0	8	3	15	2	3	3	14	3	3	0	0	8
3	3	3	3	3	3	0	0	0	0	6	3	3	3	14	3	3	2	15	0	0	8
3	3	2	15	3	3	17	3	0	0	8	3	15	2	3	3	3	17	3	0	0	8
3	14	3	3	17	3	3	15	2	3	10	3	3	3	3	3	3	0	0	0	0	6
3	14	3	3	3	3	0	0	0	0	6	3	3	17	3	3	3	2	15	0	0	8
3	14	3	3	2	15	3	3	17	3	10	3	15	2	3	3	14	3	3	17	3	10
3	3	3	3	17	3	0	0	0	0	6	3	3	3	14	3	3	0	0	0	0	6
3	14	3	3	17	3	0	0	0	0	6	3	3	17	3	3	14	3	3	2	15	10
.....											.....										
MPM # 5											MPM # 9										
3	3	3	14	0	0	0	0	0	0	4	2	3	3	3	3	15	0	0	0	0	6
3	3	3	15	2	3	0	0	0	0	6	2	15	3	3	3	3	0	0	0	0	6
3	3	3	15	2	3	3	14	0	0	8	2	3	3	14	3	3	15	0	0	0	8
3	3	2	15	3	3	3	14	0	0	8	2	15	3	3	3	14	3	3	0	0	8
3	3	17	3	3	15	2	3	0	0	8	2	3	3	3	17	3	3	15	0	0	8
3	3	3	3	3	3	0	0	0	0	6	2	15	3	3	17	3	3	3	0	0	8
3	3	17	3	3	15	2	3	3	14	10	2	3	3	14	3	3	17	3	3	15	10
3	3	3	3	3	14	0	0	0	0	6	2	15	3	3	17	3	3	14	3	3	10
3	3	2	15	3	3	17	3	3	14	10	.....										
3	3	17	3	3	3	0	0	0	0	6	MPM # 10										
3	3	17	3	3	14	0	0	0	0	6	3	3	14	3	0	0	0	0	0	0	4
.....											3	3	3	3	15	2	0	0	0	0	6
.....											3	2	15	3	3	3	0	0	0	0	6
.....											3	3	14	3	3	3	15	2	0	0	8
.....											3	2	15	3	3	3	14	3	0	0	8
.....											3	3	3	17	3	3	15	2	0	0	8
.....											3	3	3	3	3	3	0	0	0	0	6
.....											3	2	15	3	3	17	3	3	0	0	8
.....											3	3	14	3	3	17	3	3	15	2	10
.....											3	2	15	3	3	17	3	3	14	3	10
.....											3	3	3	17	3	3	0	0	0	0	6
.....											.....										

Since each row in MPM #1 is equivalent to a row in MPM #6, the two mechanisms, 1 and 6, are equivalent. Similarly, comparison of MPM's 2 and 5 shows that they are equivalent. No other equivalencies exist, hence, five unique mechanisms (1, 2, 8, 9, and 10) may be formed from chain 4 with the special elements as shown in Figure 15.

#### Example 2

The second example is based on chain number 48 from Appendix A. Special elements will be added as follows:

1. Link 4 replaced by a pulley
2. Links 3 and 8 replaced by a belt
3. Prism pairs added at joints 5-6 and 1-5

The resulting chain will appear as shown in Figure 16.

Since chain 48 is one of the chains with crossed links, the loops required to form the loop matrix are not easily identified. Therefore, the loop matrix for chain 48 as shown in Appendix B is used to form the loop matrix with special elements added (Table XXXI). Note that the special elements have been assigned "link numbers." This has been done in order to differentiate between elements of the same type so that incorrect row transfers will not take place when forming the CPM.

The CPM (Table XXXII) is formed by combining various rows of the loop matrix as explained in Chapter III. The process of combining rows is especially useful in this case since visual determination of the required paths is quite difficult.

The MPM's for all mechanisms which may be formed from chain 48 with the special elements added are shown in Table XXXIII. Note that MPM's 3, 4, and 8 do not appear since the corresponding links have been

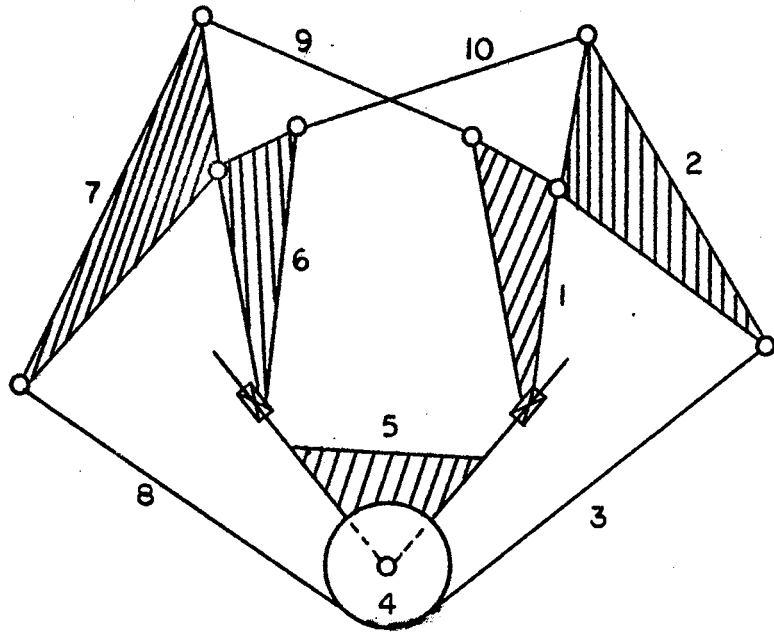


Figure 16. Chain 48 with Special Elements Added

TABLE XXXI

## LOOP MATRIX FOR CHAIN 48 WITH SPECIAL ELEMENTS

301	1514	305	1515	306	210	302	0	0	0	0	7
301	302	1312	1211	305	1514	0	0	0	0	0	6
1211	1313	307	306	1515	305	0	0	0	0	0	6
301	1514	305	1515	306	307	209	0	0	0	0	7

TABLE XXXII

## CPM FOR CHAIN 48 WITH SPECIAL ELEMENTS

301	1514	305	1515	306	210	302	0	0	0	0	0	0	7
301	302	1312	1211	305	1514	0	0	0	0	0	0	0	6
1211	1313	307	306	1515	305	0	0	0	0	0	0	0	6
301	1514	305	1515	306	307	209	0	0	0	0	0	0	7
305	1515	306	210	302	1312	1211	0	0	0	0	0	0	7
301	1514	305	1211	1313	307	306	210	302	0	0	0	0	9
0	0	0	0	0	0	0	0	0	0	0	0	0	0
301	302	1312	1211	1313	307	306	1515	305	1514	0	0	0	10
301	302	1312	1211	305	1515	306	307	209	0	0	0	0	9
1211	1313	307	209	301	1514	305	0	0	0	0	0	0	7
306	210	302	1312	1211	1313	307	0	0	0	0	0	0	7
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0
301	302	1312	1211	1313	307	209	0	0	0	0	0	0	7
305	1515	306	210	302	1312	1211	1313	307	209	301	1514	0	12

TABLE XXXIII

MPM'S FOR CHAIN 48 WITH SPECIAL ELEMENTS

MPM # 1												
3	15	3	15	3	2	3	0	0	0	0	0	7
3	3	13	12	3	15	0	0	0	0	0	0	6
3	15	3	15	3	3	2	0	0	0	0	0	7
3	15	3	12	13	3	3	2	3	0	0	0	9
3	3	13	12	13	3	3	15	3	15	0	0	10
3	3	13	12	3	15	3	3	2	0	0	0	9
3	15	3	12	13	3	2	0	0	0	0	0	7
3	3	13	12	13	3	2	0	0	0	0	0	7
3	15	3	15	3	2	3	13	12	13	3	2	12
.....												
MPM # 2												
3	3	15	3	15	3	2	0	0	0	0	0	7
3	13	12	3	15	3	0	0	0	0	0	0	6
3	13	12	3	15	3	2	0	0	0	0	0	7
3	3	15	3	12	13	3	3	2	0	0	0	9
3	13	12	13	3	3	15	3	15	3	0	0	10
3	13	12	3	15	3	3	2	3	0	0	0	9
3	13	12	13	3	3	2	0	0	0	0	0	7
3	13	12	13	3	2	3	0	0	0	0	0	7
3	13	12	13	3	2	3	15	3	15	3	2	12
.....												
MPM # 5												
3	15	3	2	3	3	15	0	0	0	0	0	7
3	15	3	3	13	12	0	0	0	0	0	0	6
3	12	13	3	3	15	0	0	0	0	0	0	6
3	15	3	3	2	3	15	0	0	0	0	0	7
3	15	3	2	3	13	12	0	0	0	0	0	7
3	12	13	3	3	2	3	3	15	0	0	0	9
3	15	3	3	13	12	13	3	3	15	0	0	10
3	15	3	3	2	3	3	13	12	0	0	0	9
3	12	13	3	2	3	15	0	0	0	0	0	7
3	15	3	2	3	13	12	13	3	2	3	15	12
.....												
MPM # 6												
3	2	3	3	15	3	15	0	0	0	0	0	7
3	15	3	12	13	3	0	0	0	0	0	0	6
3	3	2	3	15	3	15	0	0	0	0	0	7
3	2	3	13	12	3	15	0	0	0	0	0	7
3	2	3	2	15	3	12	13	3	0	0	0	9
3	15	3	15	3	3	13	12	13	3	0	0	10
3	3	2	3	3	13	12	3	15	0	0	0	9
3	2	3	13	12	13	3	0	0	0	0	0	7
3	2	3	13	12	13	3	2	3	15	3	15	12

MPM # 7												
3	3	15	3	12	13	0	0	0	0	0	0	6
2	2	3	15	3	15	3	0	0	0	0	0	7
3	3	2	3	3	15	3	12	13	0	0	0	9
3	3	15	3	15	3	3	13	12	13	0	0	10
3	2	3	3	13	12	3	15	3	0	0	0	9
3	2	3	15	3	12	13	0	0	0	0	0	7
3	3	2	3	13	12	13	0	0	0	0	0	7
3	2	3	3	13	12	13	0	0	0	0	0	7
3	2	3	15	3	15	3	2	3	13	12	13	12
.....												
MPM # 9												
2	3	15	3	15	3	3	0	0	0	0	0	7
2	3	3	13	12	3	15	3	3	0	0	0	9
2	3	15	3	12	13	3	0	0	0	0	0	7
2	3	3	13	12	13	3	0	0	0	0	0	7
2	3	15	3	15	3	2	3	13	12	13	3	12
.....												
MPM #10												
2	3	3	15	3	15	3	0	0	0	0	0	7
2	3	13	12	3	15	3	0	0	0	0	0	7
2	3	3	15	3	12	13	3	3	0	0	0	9
2	3	13	12	13	3	3	0	0	0	0	0	7
2	3	13	12	13	3	2	3	15	3	15	3	12

replaced by elements which may not be fixed.

Comparison of the MPM's will show that mechanisms 1 and 6 are equivalent, mechanisms 2 and 7 are equivalent, and mechanisms 9 and 10 are equivalent. Thus chain 48 with special elements as shown in Figure 16, may be converted into only four unique mechanisms, mechanisms 1, 2, 5, and 9.

### Example 3

The third example will be based on chain 206 from Appendix A. The following special elements will be added:

1. Link 5 replaced by a pulley
2. Links 4 and 6 replaced by a belt
3. Links 8 and 9 replaced by a spring
4. Prism pair placed at joint 3-10
5. Double joint placed at position 3(2,4)

The chain resulting from the above additions would appear as shown in Figure 17. Note that the pulley need not be circular.

The loop matrix for chain 206 without the special elements added is shown in Table XXXIV. As in the previous examples, the special elements are added to form the loop matrix shown in Table XXXV. This loop matrix is then used to generate the CPM in Table XXXVI. The MPM's appear in Table XXXVII and, after comparison, show that all five mechanisms are unique.

The previous examples demonstrate the capabilities of the path matrix approach in describing and comparing kinematic chains with all types of special elements. The following chapter will show how the CPM can be applied to the development of gear trains.



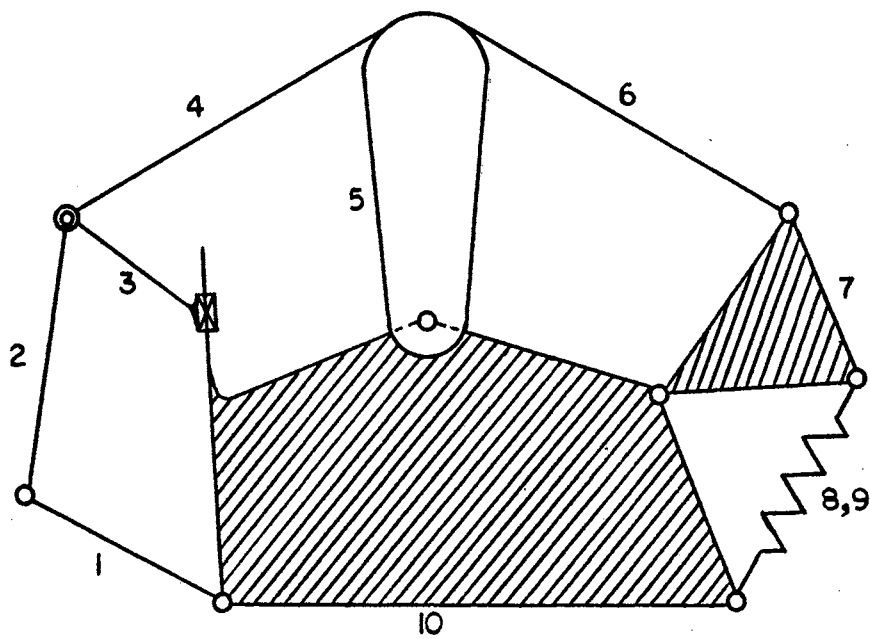


Figure 17. Chain 206 with Special Elements Added

TABLE XXXIV

## LOOP MATRIX FOR CHAIN 206

---

201	202	303	510	0	0	0	0	0	0	0	4
510	303	204	305	0	0	0	0	0	0	0	4
305	206	307	510	0	0	0	0	0	0	0	4
307	208	209	510	0	0	0	0	0	0	0	4

---

TABLE XXXV

## LOOP MATRIX FOR CHAIN 206 WITH SPECIAL ELEMENTS

---

201	202	1616	303	1515	510	0	0	0	0	0	6
510	1515	303	1616	1312	1211	0	0	0	0	0	6
1211	1313	307	510	0	0	0	0	0	0	0	4
307	1414	510	0	0	0	0	0	0	0	0	3

---

TABLE XXXVI

## CPM FOR CHAIN 206 WITH SPECIAL ELEMENTS

---

201	202	1616	303	1515	510	0	0	0	0	0	6
510	1515	303	1616	1312	1211	0	0	0	0	0	6
1211	1313	307	510	0	0	0	0	0	0	0	4
307	1414	510	0	0	0	0	0	0	0	0	3
201	202	1616	1312	1211	510	0	0	0	0	0	6
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
510	1515	303	1616	1312	1211	1313	307	0	0	0	8
0	0	0	0	0	0	0	0	0	0	0	0
1211	1313	307	1414	510	0	0	0	0	0	0	5
201	202	1616	1312	1211	1313	307	510	0	0	0	8
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
510	1515	303	1616	1312	1211	1313	307	1414	0	0	9
201	202	1616	1312	1211	1313	307	1414	510	0	0	9

---

TABLE XXXVII

MPM'S FOR CHAIN 206 WITH SPECIAL ELEMENTS

.....											
MPM # 1											
2	2	16	3	15	5	0	0	0	0	0	6
2	2	16	13	12	5	0	0	0	0	0	6
2	2	16	13	12	13	3	5	0	0	0	8
2	2	16	13	12	13	3	14	5	0	0	9
.....											
MPM # 2											
2	16	3	15	5	2	0	0	0	0	0	6
2	16	13	12	5	2	0	0	0	0	0	6
2	16	13	12	13	3	5	2	0	0	0	8
2	16	13	12	13	3	14	5	2	0	0	9
.....											
MPM # 3											
3	15	5	2	2	16	0	0	0	0	0	6
3	16	13	12	5	15	0	0	0	0	0	6
3	16	13	12	13	3	5	15	0	0	0	8
3	16	13	12	13	3	14	5	15	0	0	9
.....											
MPM # 7											
3	5	12	13	0	0	0	0	0	0	0	4
3	14	5	0	0	0	0	0	0	0	0	3
3	5	15	3	16	13	12	13	0	0	0	8
3	14	5	12	13	0	0	0	0	0	0	5
3	5	2	2	16	13	12	13	0	0	0	8
3	14	5	15	3	16	13	12	13	0	0	9
3	14	5	2	2	16	13	12	13	0	0	9
.....											
MPM #10											
5	2	2	16	3	15	0	0	0	0	0	6
5	15	3	16	13	12	0	0	0	0	0	6
5	12	13	3	0	0	0	0	0	0	0	4
5	3	14	0	0	0	0	0	0	0	0	3
5	2	2	16	13	12	0	0	0	0	0	6
5	15	3	16	13	12	13	3	0	0	0	8
5	12	13	3	14	0	0	0	0	0	0	5
5	2	2	16	13	12	13	3	0	0	0	8
5	15	3	16	13	12	13	3	14	0	0	9
5	2	2	16	13	12	13	3	14	0	0	9
.....											

## CHAPTER XI

### STRUCTURAL SYNTHESIS OF GEAR TRAINS

Johnson and Towfigh (10) have shown that certain kinematic chains may be converted to gear trains, both compound and epicyclic. In order to be convertible to a gear train, the chain must meet the following criteria:

1. Every link in the associated linkage must belong to at least one four-link path.
2. The chain must contain at least one multiple joint (double joint, triple joint, etc).
3. Any binary link incident to a multiple joint must also be connected to another binary link by a revolute pair.

Once the above criteria have been satisfied, the chain may be converted to an equivalent gear linkage. Gear trains may be formed from the gear linkage by fixing various elements with respect to ground, just as mechanisms are formed from kinematic chains by fixing various links.

To construct the equivalent gear linkage, each joint in the kinematic chain must be designated as either a rotation point (axis for a gear) or a base point (point on the edge of a gear). Certain joints are required to be either a base point or a rotation point according to the rules specified in Table XXXVIII. The remaining joints may be chosen as either a base point or a rotation point. However, as each one is chosen, the rules in Table XXXVIII must be applied to determine

TABLE XXXVIII  
MANDATORY BASE POINTS AND ROTATION POINTS

- 
- I. Mandatory Rotation Points
- a. Joints between higher links
  - b. Multiple joints
- II. Mandatory Base Points
- a. Joints between two binary links
  - b. Joint at other end of binary link with one rotation point
  - c. Both joints of a binary link connected to the link in b
  - d. Both joints on a binary link which connects two higher links
- 

TABLE XXXIX  
RULES APPLYING TO MANDATORY POINTS IN FIGURE 18

---

JOINT	TYPE POINT	RULE
1-6	R	Ia
3-6	R	Ia
3(4,9)	R	Ib
7-8	B	IIa
9-10	B	IIa
6-10	B	IIc
5-6	B	IIc
1-2	B	IIId
2-3	B	IIId

---

if other joints are affected by the choice. Where several choices exist, several gear linkages may be formed.

To illustrate this method, chain number 223 from Appendix A will be converted to a gear linkage, first, by observation of the chain itself, and second, by use of the CPM.

Figure 18 shows chain 223 with a double joint at position 3(4,9). The chain meets all criteria for conversion to a gear linkage. The mandatory rotation points and base points are shown in the figure. The rule pertaining to each mandatory point is shown in Table XXXIX. The only joints which remain free to choice are joints 6-7 and 1-8. If one of these is chosen to be a rotation point, the other automatically becomes a base point because of rule IIb. No symmetry exists for the chain as shown so that two unique gear linkages may be formed. The equivalent gear linkage formed by placing a rotation point at joint 6-7 is shown in Figure 19.

Gear trains may be formed by fixing any link containing one or more rotation points. Thus, gear trains could be formed by fixing any links of Figure 19 except links 2, 5, 8, and 10. Fixing links 4 or 9 would result in identical gear trains due to symmetry of the chain. Therefore, only five unique gear trains exist.

To apply the CPM to the development of gear trains, the following procedure would be followed:

1. Develop chains with one double joint.
2. Determine if each link appears in a path containing four links. This is done by simply taking the links in numerical order and determining if each one appears in at least one row of the CPM containing four links.

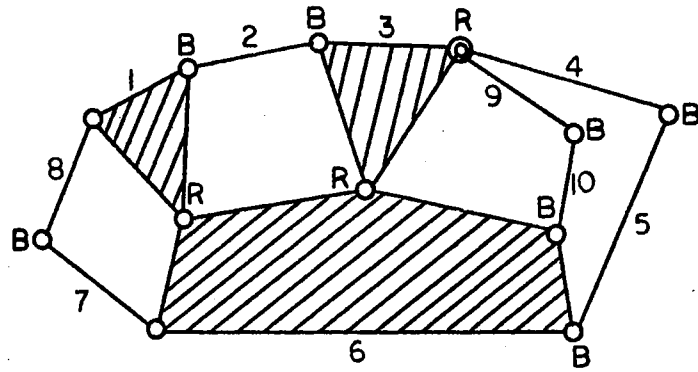


Figure 18. Chain 223 with Double Joint and Base and Rotation Points

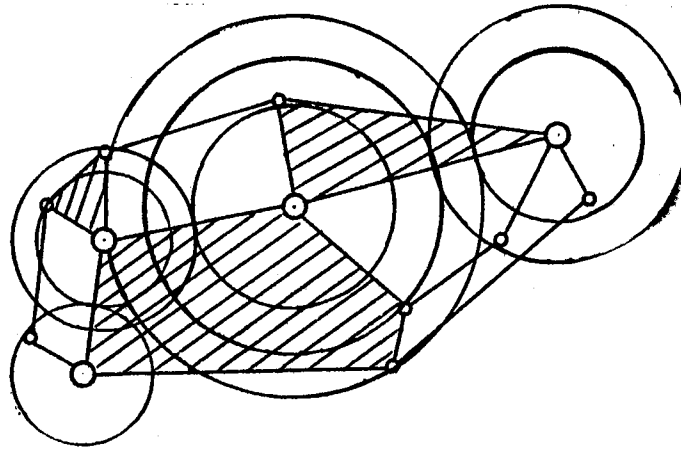


Figure 19. Equivalent Gear Linkage for Figure 18

3. Check each row in which a double joint appears to insure that if a binary link is next to the double joint, a second binary link is next to the first.

4. Identify the mandatory rotation points

a. Any joint where type numbers of the connected links are greater than 2.

b. The double joint

5. Identify the mandatory base points.

a. Any joint connecting binary links

b. Second joint on any binary link with a rotation point at the first joint.

c. Both joints of a binary link connected to the link in b.

d. Both joints of a binary link appearing between two higher links.

6. One at a time, make each remaining joint a rotation point and apply the rules for mandatory base points. The number of possible combinations resulting will yield the number of possible gear linkages. To determine whether or not two chains are equivalent, all that is required is to show that the optional joints converted to rotation points are equivalent. If, for example, joints A and B were found to be equivalent in the original chain (by methods used in Chapter VIII), then, placing a rotation point at joint A would result in the same gear linkage as that produced by placing a rotation point at joint B.

7. Gear trains may be formed by considering each link which contains a rotation point as being fixed. Equivalent trains may be found by determining which links are equivalent (by methods used in Chapter X).



Although a computer program has not yet been written to convert kinematic chains to gear trains, it appears that the CPM contains all the information necessary to perform the steps described above.

## CHAPTER XII

### SUMMARY AND CONCLUSIONS

The objective of this thesis has been to present the path matrix approach to the description and analysis of kinematic chains and mechanisms. It has been shown that a kinematic chain may be completely defined by the loop matrix which, in turn, is used to generate the Chain Path Matrix. Since the Chain Path Matrix numerically describes all paths or circuits in a kinematic chain, it serves as a powerful tool in evaluating, comparing, and modifying the chain with the aid of the high speed computer. The Mechanism Path Matrix, developed from the Chain Path Matrix, uniquely describes mechanisms derived from a kinematic chain and serves as a means of comparing chains and mechanisms, again using the computer.

The computer programs developed for use in analyzing the ten-link chains were initially tested on the six and eight-link chains. The results from these tests agreed with those obtained by Hain (2,3,4,5) through many months of work. These initial tests vividly illustrated the benefits to be gained by a computerized approach to the analysis of kinematic chains.

Once the programs had been proved on the six and eight-link chains, they were applied to the ten-link chains and catalogs of the following material were obtained (9):

1. Ten-link mechanisms

2. Ten-link chains with 1, 2, 3, and 4 springs
3. Ten-link chains with 1, 2, 3, and 4 pulleys
4. Ten-link chains with 1, 2, 3, 4, 5, and 6 cam pairs
5. Ten-link chains with 1 and 2 prism pairs
6. Ten-link chains with 1 and 2 double joints.

This material provides a great deal of data for further analysis by kinematicians. Moreover, it is felt that the possibilities of the path matrix approach have only been touched upon and that further development of the concept will prove to be of great value in the type synthesis and analysis of kinematic chains.

In addition to providing material for further research, the data obtained from the ten-link chains describes a tremendous number of kinematic chains, each of which can furnish a number of mechanisms. For the designer, this data provides many approaches to a given problem and may also serve to stimulate development along lines not previously considered.

To further develop the path matrix approach, it is recommended that research be conducted in the following areas:

1. Determination of the rules necessary for the development of loop matrices for chains with crossed links. This area will become of more importance in the analysis of chains with more than ten links.
2. Use of the path concept in the structural synthesis of plane kinematic chains. This area appears promising since higher order chains may be developed from chains of a given order by adding additional loops. If this can, in fact, be done, it should eliminate the problems encountered with the crossed linkages since the CPM could be developed directly without having to form the loop matrix.

3. Use of the path concept in the structural analysis and synthesis of spatial kinematic chains. If appropriate requirements for mobility are defined for paths containing various types of joints, it appears that the synthesis of multi-loop space mechanisms should be possible.

4. Development of a means of automatically sketching chains and mechanisms represented by a CPM. This would provide a much better reference for designers than the catalogs presently developed for the ten-link chains.

5. Research into the most advantageous means of cataloging the material obtained from analysis of higher order kinematic chains so as to be of greatest benefit to the designer, research kinematician, etc. As the data on the ten-link chains now stands, the chains may be classified according to the type of elements inserted in the basic chain. However, if chains with combinations of elements are developed, the classification of the chains will require some method which facilitates retrieval of the desired information.

6. Development of a computer program to synthesize gear trains based on kinematic chains. The procedure for this has been outlined in Chapter XI. The application of such a program to the ten-link chains would provide a valuable addition to the data already obtained.

#### A SELECTED BIBLIOGRAPHY

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**A P P E N D I C E S**

## APPENDIX A

### LINE DRAWINGS OF THE 230 TEN-LINK CHAINS

The line drawings shown in Figure 20 were taken from the Appendix of reference 6. The links of each chain have been numbered and minor corrections have been made in some of the drawings.

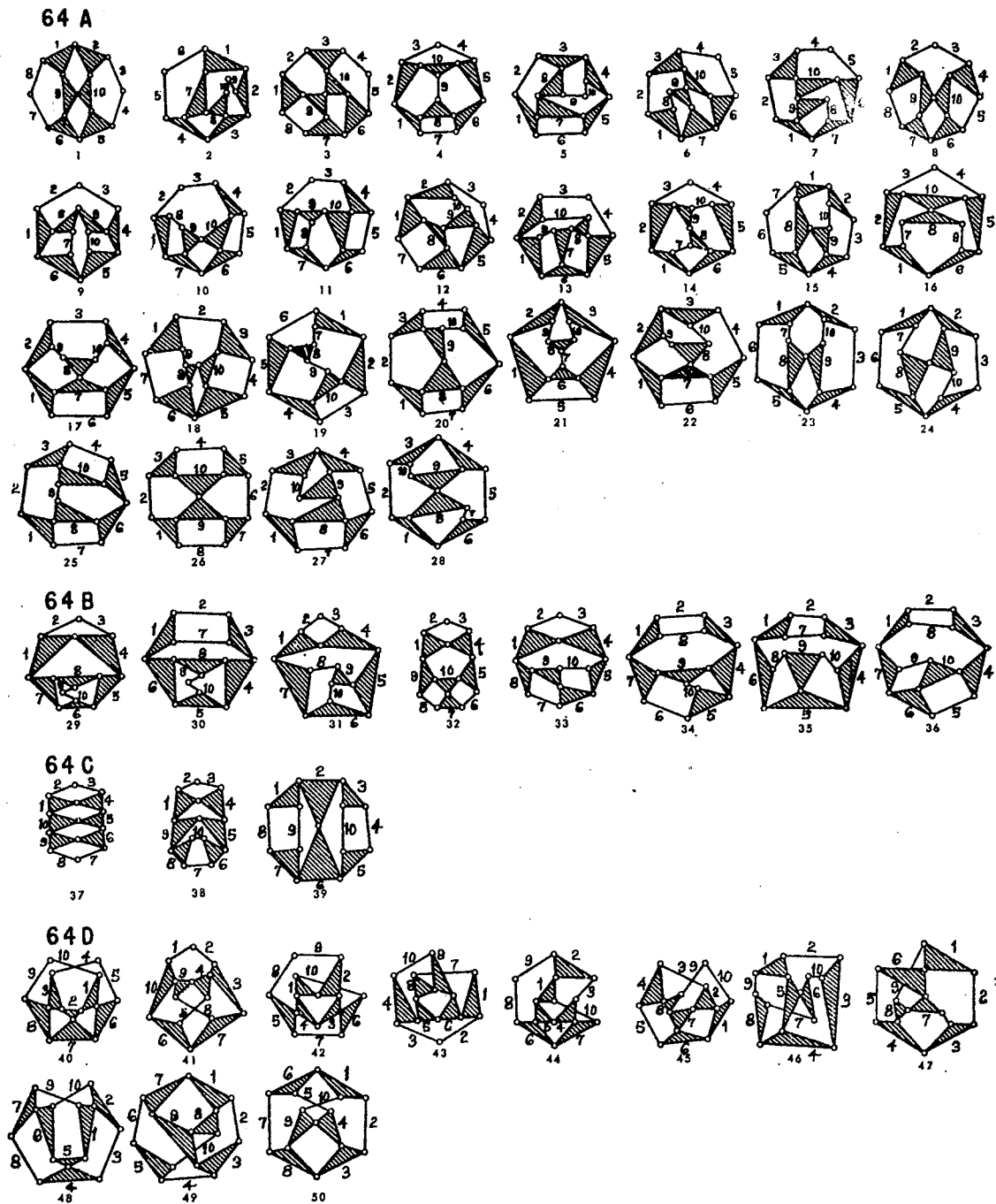
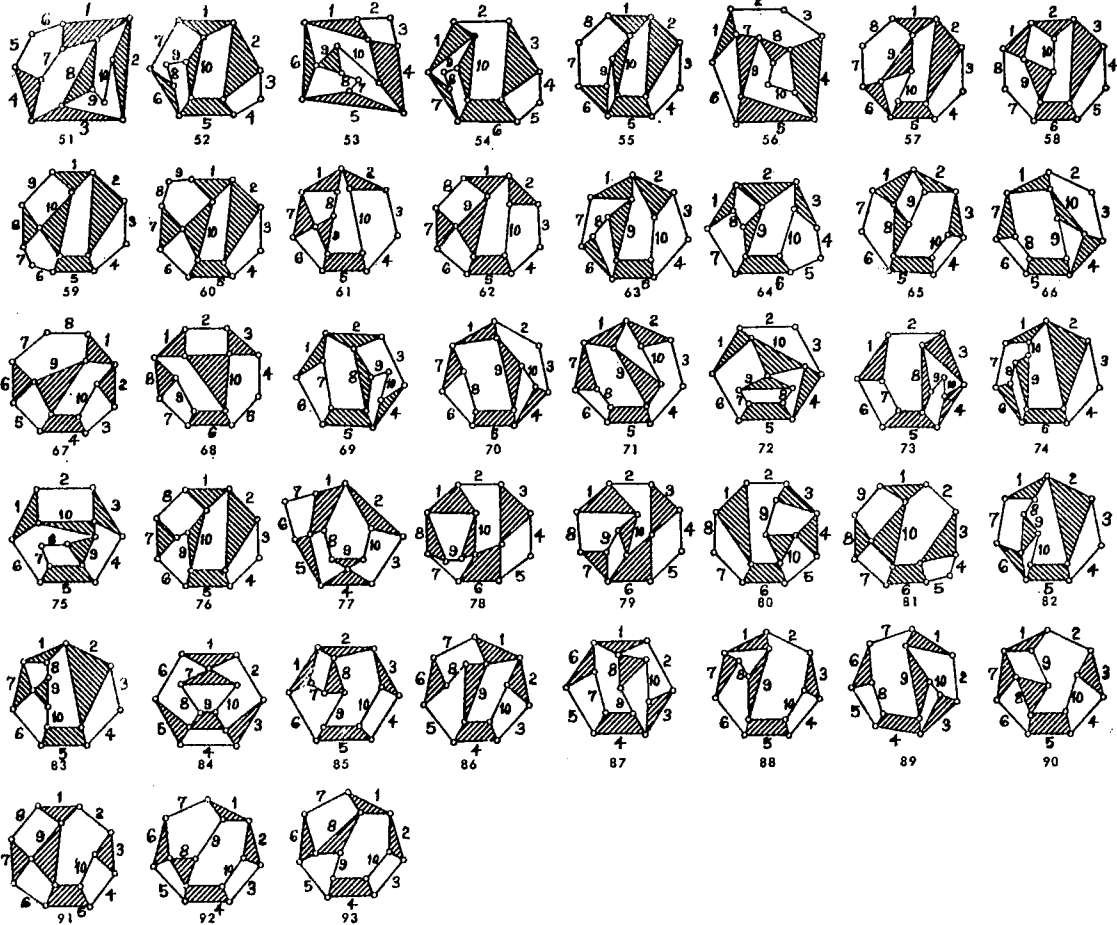


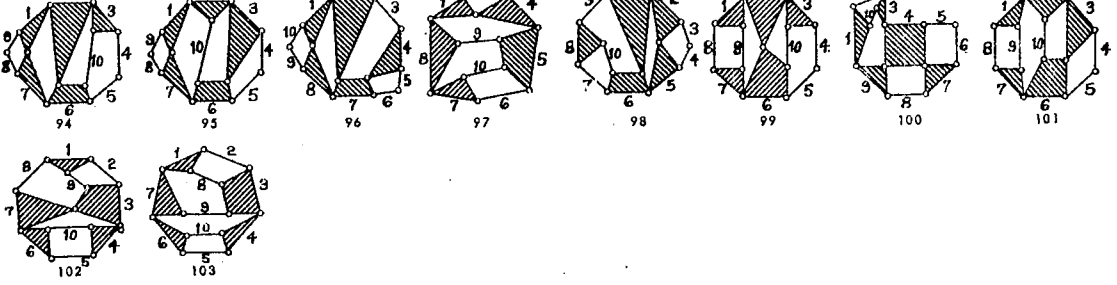
Figure 20. Line Drawings of Ten-Link Chains



145 A



145 B



145 C

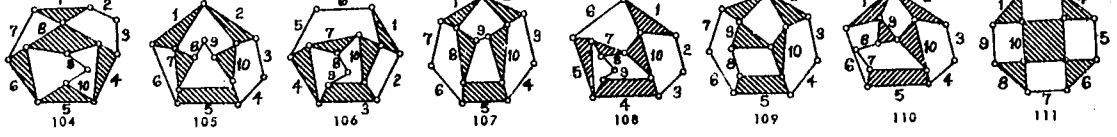


Figure 20 (Continued)

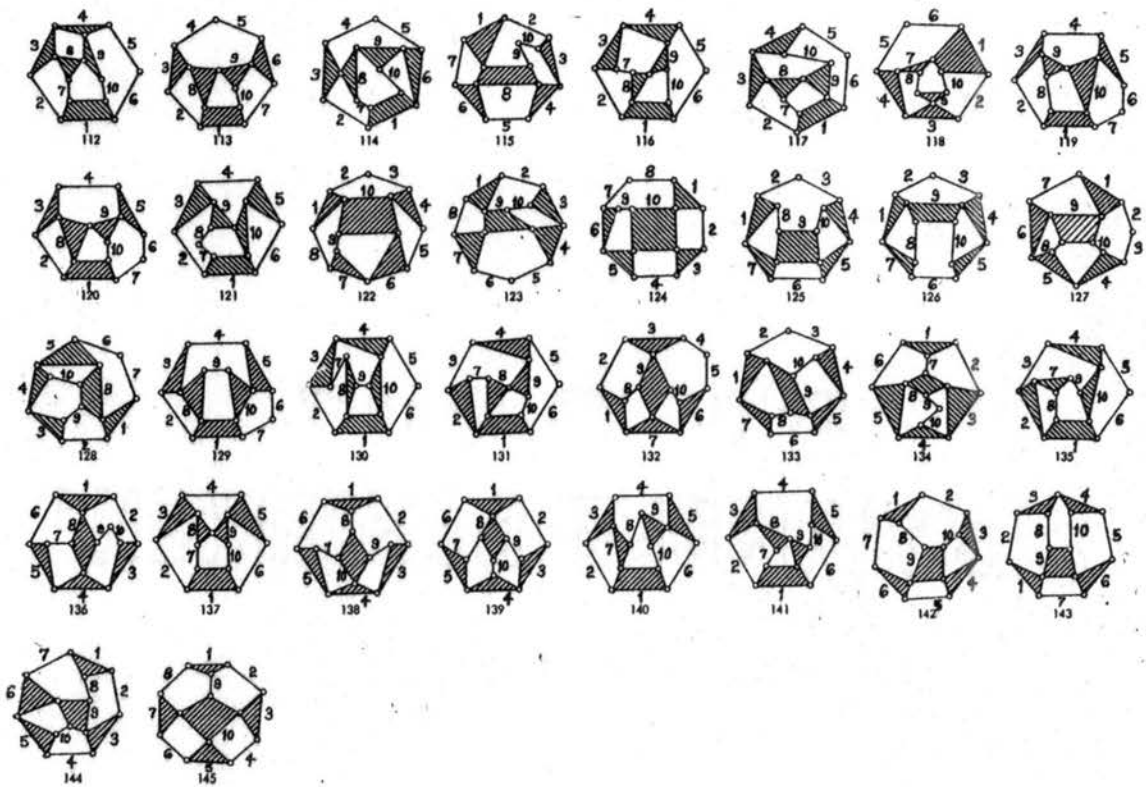
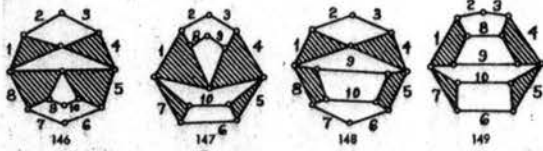
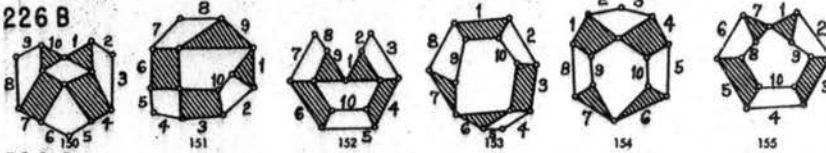


Figure 20 (Continued)

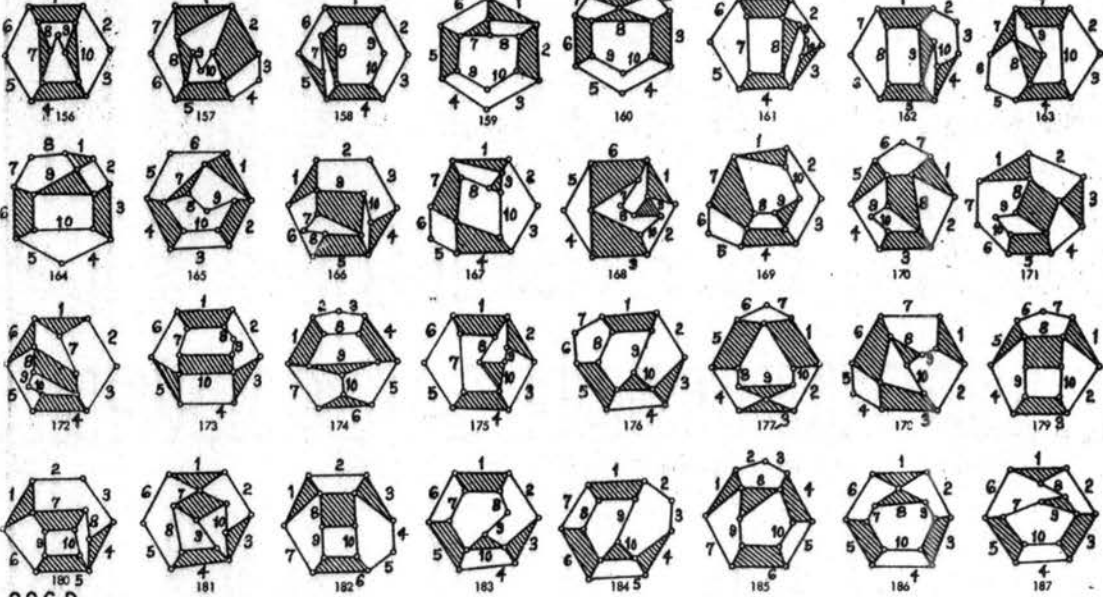
226 A



226 B



226 C



226 D

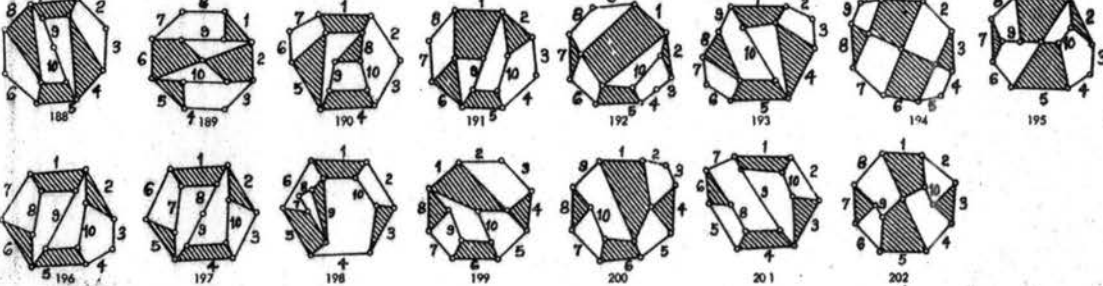


Figure 20 (Continued)

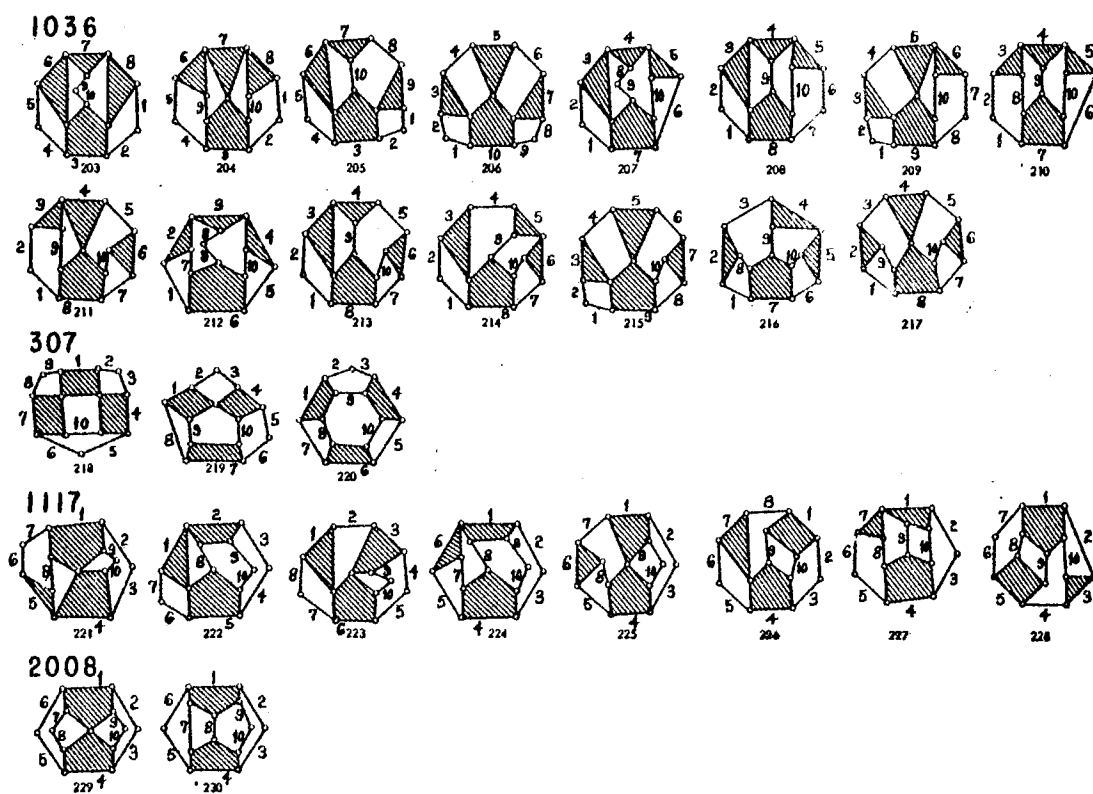


Figure 20 (Continued)

## APPENDIX B

### LOOP MATRICES FOR TEN-LINK CHAINS WITH CROSSED LINKS

Table XL shows the loop matrices used for the ten-link chains in Group 64D of Appendix A. As mentioned in Chapter III, these loop matrices were determined by trial and error.

TABLE XL  
LOOP MATRICES FOR GROUP 64D OF APPENDIX A

---

CHAIN #40							
	109	110	301	306	307	308	
	301	302	307	306			
	302	303	308	307			
	303	104	105	306	307	308	
CHAIN #41							
	310	306	105	304	303	102	101
	310	101	102	303	307	306	
	310	309	108	307	303	102	101
	310	101	102	303	304	309	
CHAIN #42							
	301	302	303	306	110		
	301	110	306	107	305	108	109 302
	301	304	305	107	306	110	
	301	110	306	303	304		
CHAIN #43							
	301	306	305	304	103	102	
	301	102	103	304	110	309	306
	310	107	308	309	110	304	103 102
	301	102	103	304	305	308	107
CHAIN #44							
	301	302	103	304	305		
	301	305	306	108	109	302	
	301	110	307	306	305		
	301	305	304	307	110		
CHAIN #45							
	301	306	307	308	109	110	
	301	110	109	308	304	105	306
	301	302	103	304	308	109	110
	301	110	109	308	307	302	
CHAIN #46							
	301	102	303	304	305		
	304	303	110	306	305		
	304	305	306	107	308		
	301	305	304	308	109		

---

Note: A type number of 1 represents a binary link.

TABLE XL (Continued)

---

**CHAIN #47**

301	306	307	110	303	102
301	102	303	304	105	306
304	308	307	306	105	
301	306	307	308	109	

**CHAIN #48**

301	305	306	110	302	
301	302	103	304	305	
304	108	307	306	305	
301	305	306	307	109	

**CHAIN #49**

301	308	110	305	104	303	102
301	102	303	104	305	106	307
307	309	308	301			
301	308	309	303	102		

**CHAIN #50**

301	110	309	304	303	102
301	102	303	304	105	306
306	107	308	309	110	301
301	110	309	308	303	102

---

## APPENDIX C

### SUBROUTINE LOOP

Subroutine LOOP is used to develop the GPM from the loop matrix as described in Chapter III. The variables used in subroutine LOOP are shown in Table XLI. A flow chart for the subroutine is shown in Figure 21 and the subroutine itself appears in Table XLII.



TABLE XLI  
IDENTIFICATION OF VARIABLES USED IN SUBROUTINE LOOP

---

Array M	-	Chain Path Matrix (CPM)
NB	-	Number of Bars (Links) in Chain
NL	-	Number of Loops in Chain
NR	-	Number of Rows in CPM
NCOL	-	Number of Columns in CPM
NK	-	NR-NL (Number of rows formed by combining rows of Loop Matrix)
KM & LM	-	Primary row combinations used in forming rows NL+1 through NR-1 of the CPM
KS & LS	-	Secondary row combinations used in forming rows 11 through 14 (for 10-link chains) of the CPM
KT & LT	-	Row combinations required for row 15 (for 10-link chains) of the CPM

---

TABLE XLII  
SUBROUTINE LOOP

```

FORTRAN IV G LEVEL 1, MOD 4          LOOP          DATE = 70191          20/49/09          PAGE 0001

0001          SUBROUTINE LOOP
0002          IMPLICIT INTEGER*2(I-N)
0003          COMMON/ALL/M(15,15),NB,NL,NR,NCOL,NK,NCH,NELEM,NSTORE
0004          COMMON/LOC/KM(11),LM(11),KS(11),LS(11),KF(5),LF(5)
C
C READ IN LOOPS
C
0005          DO 10 I=1,NL
0006             READ(5,1)(M(I,J),J=1,NCOL)
0007             1 FORMAT(20I4)
0008             10 CONTINUE
0009             DO 12 IA=1,NL
0010                ISUM=0
0011                DO 11 IB=1,NCOL
0012                   IF(M(IA,IB).GT.9000) M(IA,IB)=M(IA,IB)-9000
0013                   IF(M(IA,IB).NE.0) ISUM=ISUM+1
0014                11 CONTINUE
0015                12 M(IA,NCOL)=ISUM
0016                DO 200 I=1,NK
0017                   IS=1
0018                   K=KM(I)
0019                   L=LM(I)
0020                   JL=1
0021                   13 J=1
0022                      IF(I+NL.EQ.NR.AND.(M(K,1).EQ.0.OR.M(L,1).EQ.0)) GOTO 400
0023                      IF(I+NL.GE.NK.AND.(M(K,1).EQ.0.OR.M(L,1).EQ.0)) GOTO 300
C
C START WITH LOOP K
C
C CHECK TO SEE IF ELEMENT J EXISTS IN BOTH LOOPS
C
0024          20 KN=M(K,NCOL)
0025             LN=M(L,NCOL)
0026             IF(LN.EQ.0) GOTO 100
0027             22 DO 21 IL=1,LN
0028                IF(M(K,J).EQ.M(L,IL)) GOTO 30
0029             21 CONTINUE
0030             M(I+NL,JL)=M(K,J)
0031             J=J+1
0032             JL=JL+1
0033             IF(M(K,J).EQ.0) GOTO 100
0034             GOTO 22
C
C CHECK ELEMENTS ON BOTH SIDES OF POSSIBLE TRANSFER ELEMENT
C
0035          30 IF(IL.EQ.1) GOTO 21
0036             IF(J.EQ.KN) GOTO 32
0037             IF(M(K,J+1).EQ.M(L,IL-1)) GOTO 40
0038             M(I+NL,JL)=M(K,J)
0039             J=J+1
0040             IF(M(K,J).EQ.0) GOTO 100
0041             JL=JL+1
0042             GOTO 22
0043          31 IF(J.EQ.KN) GOTO 33
0044             IF(M(K,J+1).EQ.M(L,LN)) GOTO 40
0045             M(I+NL,JL)=M(K,J)
0046             J=J+1
0047             IF(M(K,J).EQ.0) GOTO 100

```

## TABLE XLII (Continued)

```

FORTRAN IV G LEVEL 1, MOD 4      LOOP      DATE = 70191      20/49/09      PAGE 0002

0048      JL=JL+1
0049      GOTO 22
0050      32 IF(M(K,1).EQ.M(L,IL-1)) GOTO 40
0051      GOTO 100
0052      33 IF(M(K,1).EQ.M(L,LN)) GOTO 40
0053      GOTO 100
C
C NEXT ELEMENT IN FIRST LOOP IS COMMON TO BOTH LOOPS. CHECK TO SEE IF LAST
C ELEMENT WAS COMMON. IF IT WAS, PROCEED TO NEXT ELEMENT IN FIRST LOOP. IF NOT
C TRANSFER TO SECOND LOOP.
C
0054      40 IF(J.EQ.1) GOTO 41
0055      IF(IL.EQ.LN) GOTO 42
0056      IF(M(K,J-1).NE.M(L,IL+1)) GOTO 50
0057      J=J+1
0058      IF(M(K,J).EQ.0) GOTO 100
0059      GOTO 22
0060      41 IF(IL.EQ.LN) GOTO 43
0061      IF(M(K,KN).NE.M(L,IL+1)) GOTO 50
0062      J=J+1
0063      IF(M(K,J).EQ.0) GOTO 100
0064      GOTO 22
0065      42 IF(M(K,J-1).NE.M(L,1)) GOTO 50
0066      J=J+1
0067      IF(M(K,J).EQ.0) GOTO 100
0068      GOTO 22
0069      43 IF(M(K,KN).NE.M(L,1)) GOTO 50
0070      J=J+1
0071      IF(M(K,J).EQ.C) GOTO 100
0072      GOTO 22
C
C TRANSFER TO SECOND LOOP
C
0073      50 M(I+NL,JL)=M(L,IL)
0074      51 JL=JL+1
0075      IL=IL+1
0076      IF(M(L,IL).EQ.0) IL=1
0077      DO 53 J=1,KN
0078      IF(M(L,IL).EQ.M(K,J)) GOTO 60
0079      53 CONTINUE
0080      M(I+NL,JL)=M(L,IL)
0081      GOTO 51
C
C CHECK ELEMENTS ON BOTH SIDES OF POSSIBLE TRANSFER ELEMENT
C
0082      60 IF(J.EQ.1) GOTO 61
0083      IF(IL.EQ.LN) GOTO 62
0084      IF(M(L,IL+1).EQ.M(K,J-1)) GOTO 70
0085      GOTO 100
0086      61 IF(IL.EQ.LN) GOTO 63
0087      IF(M(L,IL+1).EQ.M(K,KN)) GOTO 70
0088      GOTO 100
0089      62 IF(M(L,1).EQ.M(K,J-1)) GOTO 70
0090      GOTO 100
0091      63 IF(M(L,1).EQ.M(K,KN)) GOTO 70
0092      GOTO 100
C
C NEXT ELEMENT IN SECOND LOOP IS COMMON TO BOTH LOOPS. TRANSFER BACK TO FIRST

```

```

FORTRAN IV G LEVEL 1, MOD 4      LOOP      DATE = 70191      20/49/09      PAGE 0003

C LOOP .
C
0093      70 IF(M(K,J).EQ.M(I+NL,1)) GOTO 110
0094      M(I+NL,JL)=M(K,J)
0095      J=J+1
0096      IF(M(K,J).EQ.0) J=1
0097      JL=JL+1
0098      GOTO 70
0099      300 K=KS(1)
0100      L=LS(1)
0101      GOTO 20
0102      400 K=KI(1S)
0103      L=LI(1S)
0104      IS=IS+1
0105      IF(IS.GT.5) GOTO 100
0106      GOTO 13
0107      100 JL=1
0108      110 DO 111 J=JL,NCOL
0109      111 M(I+NL,J)=0
0110      M(I+NL,NCOL)=JL-1
0111      200 CONTINUE
0112      RETURN
0113      END

```



## APPENDIX D

### SUBROUTINE MECH

Subroutine MECH develops the MPM's for mechanisms formed from a kinematic chain as described in Chapter IV. Most of the variables used in MECH are the same as those used in Subroutine LOOP. The only variables used in LOOP are MY, which holds the CPM reduced to an array of link numbers, and MX, which holds the MPM's that are generated. The flow chart for Subroutine MECH appears in Figure 22 and the subroutine itself in Table XLIII.

TABLE XLIII  
SUBROUTINE MECH

```

FORTRAN IV G LEVEL 1, MOD 4          MECH          DATE = 70192          09/43/16          PAGE 0001

0001          SUBROUTINE MECH
0002          IMPLICIT INTEGER*2(I-N)
0003          COMMON/ALL/M(15,20),NB,NL,NR,NCOL,NK,NCH,NELEM
0004          COMMON/MCH/MX(10,15,15)
0005          DIMENSION MY(15,15)

C
C STORE M IN MY AFTER REDUCING TO LINK NUMBERS
C
0006          DO 11 I=1,NR
0007             NE=M(I,NCOL)
0008             IF(NE.EQ.0) GOTO 13
0009             DO 12 J=NE,NCOL
0010                12 MY(I,J)=M(I,J)
0011            DO 10 J=1,NE
0012                10 MY(I,J)=M(I,J)-(M(I,J)/100)*100
0013            GOTO 11
0014            13 MY(I,1)=0
0015            MY(I,2)=0
0016            11 CONTINUE

C
C DEVELOP MECHANISM MATRICES
C
0017          DO 100 I=1,NB
C
C SEARCH FOR LINK I IN EACH ROW OF MY
C
0018             L=1
0019             J=1
0020             21 K=1
0021             KI=1
0022             22 IF(MY(J,K).EQ.1) GOTO 30
0023             K=K+1
0024             IF(MY(J,K).NE.0) GOTO 22
0025             J=J+1
0026             IF(J.GT.NR) GOTO 100
0027             GOTO 21

C
C STORE ROW IN MX STARTING WITH LINK I
C
0028             30 MX(I,L,KI)=M(I,K)
0029             K=K+1
0030             KI=KI+1
0031             IF(M(I,K).EQ.0) K=1
0032             IF(M(I,K).EQ.MX(I,L,1)) GOTO 40
0033             GOTO 30

C
C ALL LINKS HAVE BEEN STORED IN MX. COMPLETE ROW WITH CORRESPONDING PORTION OF
C M.
C
0034             40 DO 41 J=KI,NCOL
0035                41 MX(I,L,J)=M(I,J)
0036             L=L+1
0037             J=J+1
0038             IF(J.GT.NR) GOTO 100
0039             GOTO 21

C
C ALL ROWS CONTAINING LINK I HAVE BEEN PROPERLY STORED IN MX
C

FORTRAN IV G LEVEL 1, MOD 4          MECH          DATE = 70192          09/43/16          PAGE 0002

0040          100 MX(I,1,NCOL-1)=L
0041          NCH=NB
0042          RETURN
0043          END

```

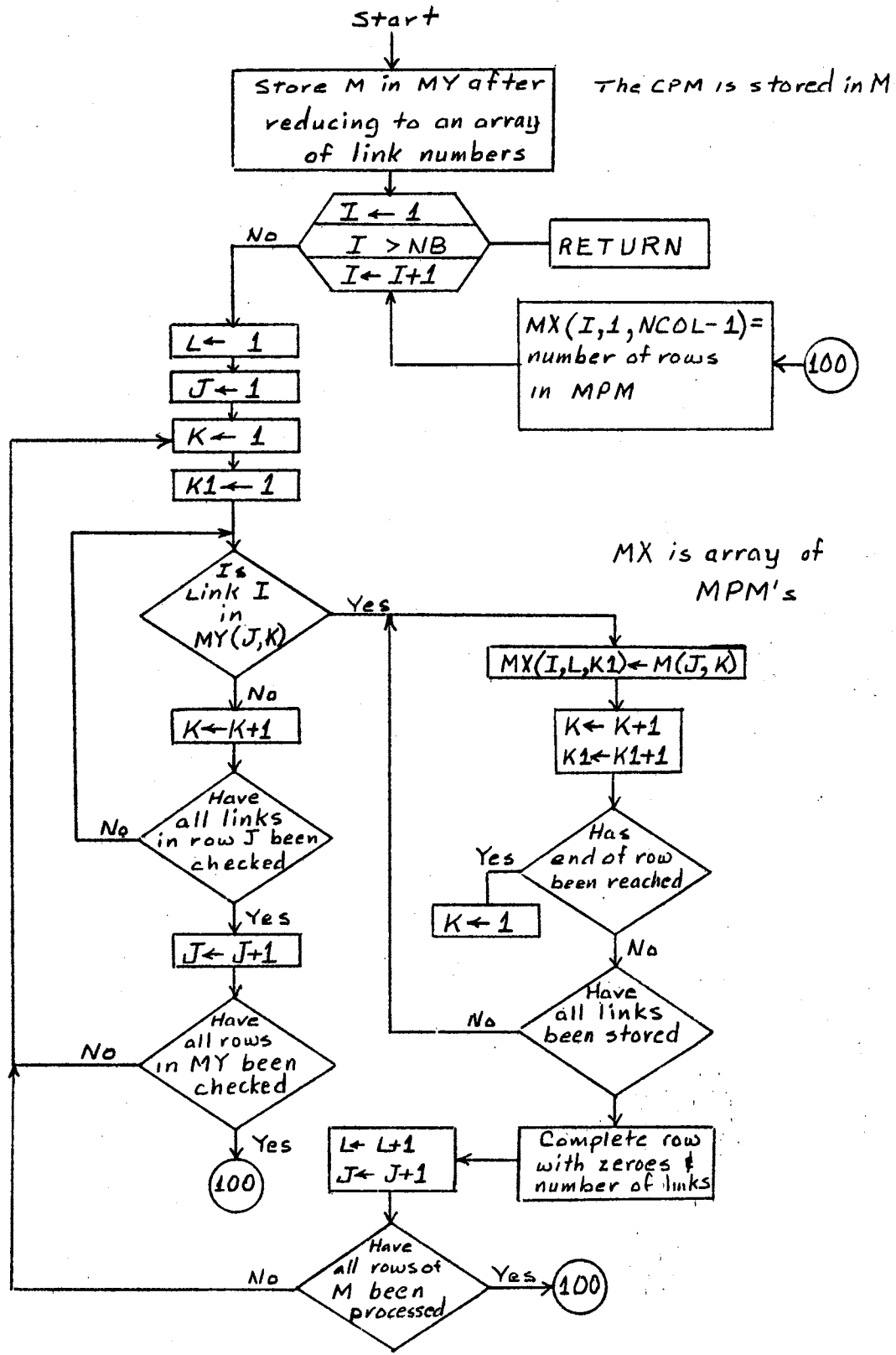


Figure 22. Flow Chart for Subroutine MECH

## APPENDIX E

### SUBROUTINE MECCOM

Subroutine MECCOM compares MPM's as described in Chapter IV. A flow chart for this subroutine is shown in Figure 23. A list of the variables used in MECCOM appears in Table XLIV and the subroutine itself is in Table XLV.



TABLE XLIV  
VARIABLES USED IN SUBROUTINE MECCOM

---

MX	-	Array of MPM's
NCH	-	Number of MPM's to be compared
IEQUIV	-	Array indicating equivalent MPM's
ISAMEM	-	Array which indicates whether an MPM is already equivalent to another MPM
ISAMEY	-	Array which indicates whether a row is already equivalent to another row
IHOLD	-	Array used in cycling rows
ITAG	-	Counter used to determine how many times a row has been cycled
JSTORE	-	Array in which descriptions of unique chains are stored
NSTORE	-	Number of unique chains stored in JSTORE

All variables in COMMON/ALL/ are described in Appendix C in conjunction with Subroutine LOOP.

---

## TABLE XLV

## SUBROUTINE MECCOM

```

FORTRAN IV G LEVEL 1, MCD 4          MECCOM          DATE = 70191          20/49/09          PAGE 0001

0001          SUBROUTINE MECCOM
0002          IMPLICIT INTEGER*2(I-N)
0003          COMMON/ALL/MI(15,15),NB,NL,NR,NCOL,NK,NCH,NELEM,NSTORE
0004          COMMON/MEC/IEQUIV(50,50)
0005          COMMON/DBLMX/MZ(15,15),MZ1(15,15),MZ2(15,15),MZ3(15,15),MZ4(15,15)
0006          COMMON/DBALL/JSTORE(100,6),JTEP(100,6)
0007          DIMENSION ISAMEM(50),ISAMEY(50),IHOLD(20)

C CONVERT TO ARRAYS OF LINK TYPES
0008          ITAG=0
0009          DO 10 I=1,NCH
0010          DO 5 J=1,NCH
0011          5 IEQUIV(I,J)=0
0012          ISAMEM(I)=0
0013          NROW=MX(I,1,NCOL-1)-1
0014          MX(I,1,NCOL-1)=NROW
0015          DO 10 J=1,NROW
0016          NCLM=MX(I,J,NCOL)
0017          DO 10 K=1,NCLM
0018          10 MX(I,J,K)=MX(I,J,K)/100
0019          DO 11 I=1,NR
0020          11 ISAMEY(I)=0

C COMPARE EACH MECHANISM WITH ALL OTHER MECHANISMS
0021          IX=1
0022          19 IY=IX+1
C CHECK TO SEE IF MX(IY) IS ALREADY EQUIVALENT
0023          20 IF(ISAMEM(IY).EQ.1) GOTO 70
C CHECK TO SEE IF MX(IY) HAS SAME NUMBER OF ROWS AS MX(IX)
0024          IF(MX(IX,1,NCOL-1).NE.MX(IY,1,NCOL-1)) GOTO 70
C COMPARE EACH ROW OF MX(IX) WITH ROWS OF MX(IY)
0025          IXROW=1
0026          IYROW=1
0027          30 IF(MX(IX,IXROW,NCOL).NE.MX(IY,IYROW,NCOL)) GOTO 90
C CHECK TO SEE IF MX(IY, IYROW) IS ALREADY EQUIVALENT TO A ROW IN MX(IX)
0028          IF(ISAMEY(IYROW).EQ.1) GOTO 90
C COMPARE ELEMENTS OF MX(IX, IXROW) WITH MX(IY, IYROW) IN FORWARD DIRECTION
0029          IXCOL=1
0030          40 IF(MX(IX,IXROW,IXCOL).NE.MX(IY,IYROW,IXCOL)) GOTO 110
0031          IXCOL=IXCOL+1
0032          IF(IXCOL.LE.MX(IX,IXROW,NCOL)) GOTO 40
0033          ISAMEY(IYROW)=1
0034          GOTO 100
C INCREMENT IY
0035          70 IY=IY+1
0036          IF(IY.GT.NCH) GOTO 80
0037          GOTO 20
C INCREMENT IX
0038          80 IX=IX+1
0039          IF(IX.GT.NCH-1) GOTO 210
0040          GOTO 19
C INCREMENT IYROW
0041          90 IYROW=IYROW+1
0042          ITAG=0
0043          IF(IYROW.GT.MX(IX,1,NCOL-1)) GOTO 100
0044          GOTO 30
C INCREMENT IXROW
0045          100 IXROW=IXROW+1
0046          IF(IXROW.GT.MX(IX,1,NCOL-1)) GOTO 200

```

## TABLE XLV (Continued)

```

FORTRAN IV G LEVEL 1, MOD 4      MECCOM      DATE = 70191      20/49/09      PAGE 0002

0047      IYROW=J
0048      GOTO 30
      C COMPARE MX(IY,IYROW) IN REVERSE
0049      110 IXCOL=1
0050      IYCOL=MX(IY,IYROW,NCOL)+1
0051      MX(IY,IYROW,IYCOL)=MX(IY,IYROW,1)
0052      111 IF(MX(IX,IXROW,IXCOL).NE.MX(IY,IYROW,IYCOL)) GOTO 120
0053      IXCOL=IXCOL+1
0054      IYCOL=IYCOL-1
0055      IF(IXCOL.LE.MX(IX,IXROW,NCCL)) GOTO 111
0056      ISAMEY(IYROW)=1
0057      GOTO 100
0058      120 NE=MX(IY,IYROW,NCOL)
0059      IF(ITAG.EQ.NELEM-1) GOTO 90
0060      DO 130 IA=2,NE
0061      IF(MX(IY,IYROW,IA).EQ.MX(IY,IYROW,1)) GOTO 131
0062      130 CONTINUE
0063      GOTO 90
      C CYCLE ROW TO NEXT ELEMENT
0064      131 ISTOP=IA
0065      JH=1
0066      132 IHOLD(JH)=MX(IY,IYROW,IA)
0067      IA=IA+1
0068      IF(IA.EQ.ISTOP) GOTO 133
0069      IF(IA.GT.NE) IA=1
0070      JH=JH+1
0071      GOTO 132
0072      133 DO 134 IH=1,JH
0073      134 MX(IY,IYROW,IH)=IHOLD(IH)
0074      ITAG=ITAG+1
0075      IXCOL=1
0076      GOTO 40
      C ALL ROWS HAVE BEEN COMPARED. CHECK TO SEE IF MATRICES ARE EQUIVALENT.
0077      200 NROW=MX(IX,1,NCOL-1)
0078      ISUM=0
0079      DO 201 I=1,NROW
0080      IF(ISAMEY(I).EQ.1) ISUM=ISUM+1
0081      201 ISAMEY(I)=0
0082      IF(ISUM.EQ.NROW) GOTO 220
0083      GOTO 70
0084      220 IEQUIV(IX,IY)=IY
0085      ISAMEY(IY)=1
0086      DO 1000 IW=1,NELEM
0087      1000 JSTORE(IY,IW)=0
0088      GOTO 70
      C WRITE EQUIVALENCE
0089      210 CALL PRINT
0090      DO 1001 IW=2,NCH
0091      IF(JSTORE(IW,1).EQ.0) GOTO 1002
0092      1001 CONTINUE
0093      NSTORE=NCH
0094      RETURN
0095      1002 IZERO=IW
0096      IWPI=IW+1
0097      DO 1006 IW=IWPI,NCH
0098      IF(JSTORE(IW,1).NE.0) GOTO 1004
0099      GOTO 1006
0100      1004 DO 1005 IX=1,NELEM

```

```

FORTRAN IV G LEVEL 1, MOD 4      MECCOM      DATE = 70191      20/49/09      PAGE 0003

0101      1005 JSTORE(IZERO,IX)=JSTORE(IW,IX)
0102      IZERC=IZERO+1
0103      1006 CONTINUE
0104      NSTORE=IZERO-1
0105      RETURN
0106      END

```

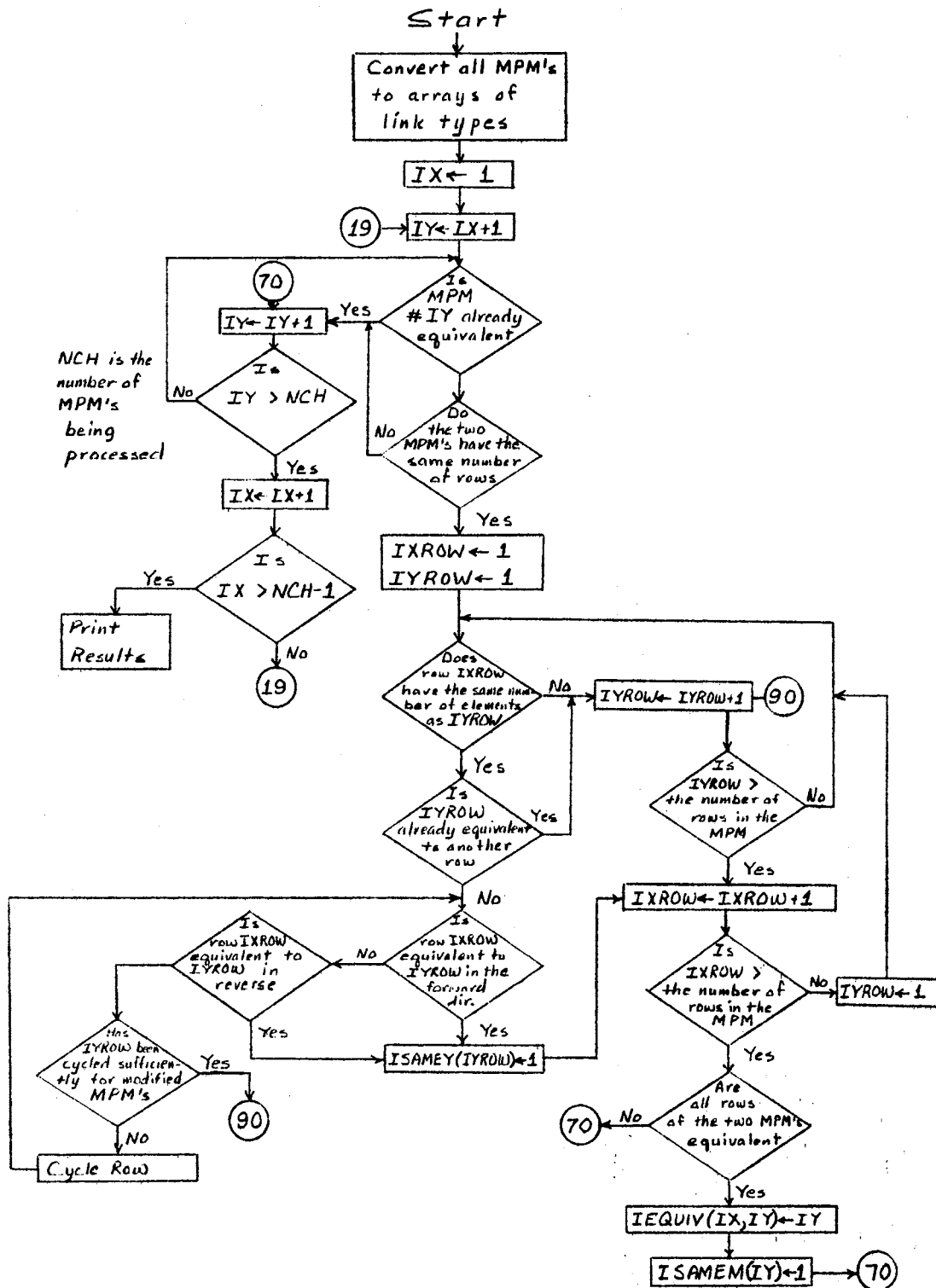


Figure 23. Flow Chart for Subroutine MECCOM

## APPENDIX F

### PROGRAM FOR DETERMINING THE TEN-LINK MECHANISMS

The program shown in Table XLVI was used to generate the ten-link mechanisms. The input data required for the program (and all other programs) is described in Table XLVII.

TABLE XLVI

## PROGRAM FOR DETERMINING TEN-LINK MECHANISMS

```

FORTRAN IV G LEVEL 1, MOD 4          MAIN          DATE = 70191          20/59/33          PAGE 0001

C
C TEN-LINK CHAINS
C
C DEFINITION OF INPUT DATA
C
C NL - NUMBER OF LOOPS
C NB - NUMBER OF BARS
C NR - NUMBER OF ROWS IN LOOP MATRIX
C NCOL - NUMBER OF COLUMNS IN LOOP MATRIX
C NK - NUMBER OF LOOPS IN ADDITION TO BASIC LOOPS
C NC - NUMBER OF CHAINS
C KN&LN- SEQUENCES FOR DERIVING LOOPS
C KS&LS- SECONDARY SEQUENCES FOR DERIVING LOOPS
C KT&LT- TERTIARY SEQUENCES FOR DERIVING LOOPS
C
C *****
0001      [IMPLICIT INTEGER*2(I-N)
0002      COMMON/ALL/M(15,20),NB,NL,NR,NCOL,NK,NCH,NELEM
0003      COMMON/LOO/KM(11),LM(11),KS(11),LS(11),KT(5),LT(5)
0004      COMMON/MCH/MX(10,15,15)
0005      COMMON/PRI/ITOTAL,I,IROW
0006      COMMON/MEC/IEQUIV(10,10)
0007      READ(5,1) NL,NB,NR,NCOL,NC
0008      NK=NR-NL
0009      1 FORMAT(15I5)
0010      READ(5,1)(KM(I),I=1,NK)
0011      READ(5,1)(LM(I),I=1,NK)
0012      NELEM=1
0013      NS=NK-NL
0014      READ(5,1)(KS(I),I=NS,NK)
0015      READ(5,1)(LS(I),I=NS,NK)
0016      READ(5,1)(KT(I),I=1,5)
0017      READ(5,1)(LT(I),I=1,5)
0018      ITOTAL=0
0019      CALL PRINT1
0020      DO 2 I=1,NC
0021      CALL LOOP
0022      CALL MECH
0023      CALL MECCOM
0024      IF(IROW.GT.25) CALL PRINT1
0025      2 CONTINUE
0026      WRITE(6,4) ITOTAL
0027      4 FORMAT('TOTAL NUMBER OF 10-LINK MECHANISMS - ',
0028      1 15)
0029      WRITE(6,1000)
0030      1000 FORMAT('1')
0031      STOP
      END

```

## TABLE XLVI (Continued)

```

FORTRAN IV G LEVEL 1, MOD 4          PRINT          DATE = 70191          20/59/33          PAGE 0001

0001      SUBROUTINE PRINT
0002      IMPLICIT INTEGER*2(I-N)
0003      COMMON/ALL/M(15,20),NB,NL,NR,NCOL,NK,NCH,NELEM
0004      COMMON/MEC/IEQUIV(10,10)
0005      COMMON/PRI/ITOTAL,1,IROW
0006      DIMENSION ANUM(10),EQUIV(10)
0007      DATA BLANK/' '/
0008      DATA EX/' X'/
0009      DATA ANUM/' 1',' 2',' 3',' 4',' 5',' 6',' 7',' 8',' 9','10'/
0010      1 FORMAT('+',T11,10(A2,2X))
0011      2 FORMAT('T10,11('' '))
0012      3 FORMAT('IX,80('---'))
0013      6 FORMAT(' ',13,' |')
0014      7 FORMAT('+',T9,' |',10(' | '),T50,' | |')
0015      8 FORMAT('+',T51,' |',T59,12)
0016      ISUM=10
0017      WRITE(6,3)
0018      WRITE(6,7)
0019      WRITE(6,2)
0020      WRITE(6,6) 1
0021      DO 20 IX=1,10
0022      DO 10 IY=1,10
0023      EQUIV(IY)=BLANK
0024      IF(IEQUIV(IX,IY).EQ.99) EQUIV(IX)=EX
0025      IF(IEQUIV(IX,IY).EQ.1Y)EQUIV(IY)=ANUM(IX)
0026      IF(EQUIV(IY).NE.BLANK) ISUM=ISUM-1
0027      10 CONTINUE
0028      WRITE(6,1) (EQUIV(J),J=1,10)
0029      20 CONTINUE
0030      WRITE(6,8) ISUM
0031      ITOTAL=ITOTAL+ISUM
0032      IROW=IROW+1
0033      RETURN
0034      END

```

```

FORTRAN IV G LEVEL 1, MOD 4          PRINT1         DATE = 70191          20/59/33          PAGE 0001

0001      SUBROUTINE PRINT1
0002      IMPLICIT INTEGER*2(I-N)
0003      COMMON/PRI/ITOTAL,1,IROW
0004      DIMENSION ANUM(10)
0005      DATA ANUM/' 1',' 2',' 3',' 4',' 5',' 6',' 7',' 8',' 9','10'/
0006      1 FORMAT('+',T11,10(A2,2X))
0007      2 FORMAT('T10,11('' '))
0008      4 FORMAT('1',////,T25,'LINK NUMBER')
0009      5 FORMAT('+'CHAIN ['T51,'] UNIQUE MECHANISMS')
0010      WRITE(6,4)
0011      WRITE(6,2)
0012      WRITE(6,1)(ANUM(IA),IA=1,10)
0013      WRITE(6,5)
0014      IROW=1
0015      RETURN
0016      END

```

## TABLE XLVII

## INPUT DATA FOR COMPUTER PROGRAMS

---

**Card 1**

NL - Number of loops in chains  
NB - Number of bars (links) in chains  
NR - Number of rows in CPM  
NCOL - Number of columns in CPM  
NC - Number of chains to be processed

Data is read in with FORMAT (15I5).

**Card 2 KM(I)****Card 3 LM(I)**

KM and LM are the rows to be combined in forming other rows of the CPM. For the ten-link chains, to form row 5, rows 1 and 2 of the CPM are combined. Therefore, KM(1) would be 1 and LM(1) would be 2. Table V shows all combinations used for the ten-link chains. The data on cards 2 and 3 is read in with FORMAT (15I5).

**Card 4 KS(I)****Card 5 LS(I)**

KS and LS are the second row combinations to be used if the primary combinations fail. For the ten-link chains, the secondary combinations are required only for rows 9 through 15 of the CPM, therefore, only 7 entries are required on each card. The data on cards 4 and 5 is read in with FORMAT (15I5).

**Card 6 KT(I)****Card 7 LT(I)**

KT and LT are additional row combinations used only for row 15 of the CPM for ten-link chains. They are read in with FORMAT (15I5).

**Cards 8 -**

These cards are read in by Subroutine LOOP and furnish the loop matrices. For the ten-link chains, 4 cards are required for each chain, one card per loop. Each card provides one row of the loop matrix. (Do not insert number showing the count of the elements in the row). These cards are read in with FORMAT (20I4).

---



TABLE XLVII (Continued)

---

The input cards for the 10-link chains appeared as follows:

Card 1)	4	10	15	15	230						
Card 2)	1	1	4	2	2	3	1	4	10	8	5
Card 3)	2	3	1	3	4	4	8	5	1	4	10
Card 4)	5	7	3	2	6						
Card 5)	3	2	7	10	9						
Card 6)	8	4	3	2	1						
Card 7)	7	11	12	13	14						
Card 8)	301	312	310	309							
Card 9)	302	103	104	305	310						
Card 10)	305	306	309	310							
Card 11)	306	107	108	301	309						

NOTE: The programs were set up for binary links being represented by a type number of 1 (as in Franke's notation) instead of 2. Therefore, the numbers 103 and 104 on card 9 would represent binary links 3 and 4.

---

## APPENDIX G

### PROGRAM FOR DETERMINING THE TEN-LINK CHAINS WITH SPRINGS

The complete program for determining the ten-link chains with springs is shown in Table XLVIII. Flow charts for Subroutines SPRLOC and SPRNG3 appear in Figures 24 and 25 respectively. This program serves as the model for all programs which follow.

## TABLE XLVIII

## PROGRAM FOR DETERMINING TEN-LINK CHAINS WITH SPRINGS

FORTRAN IV G LEVEL 1, MOD 4                      MAIN                      DATE = 70191                      20/49/09                      PAGE 0001

```

C
C TEN-LINK CHAINS
C
C DEFINITION OF INPUT DATA
C
C NL - NUMBER OF LOOPS
C NB - NUMBER OF BARS
C NR - NUMBER OF ROWS IN LCCP MATRIX
C NCCL - NUMBER OF COLUMNS IN LOOP MATRIX
C NK - NUMBER OF LCOPS IN ADDITION TO BASIC LOOPS
C NC - NUMBER OF CHAINS
C KM&LM- SEQUENCES FOR DERIVING LOOPS
C KS&LS- SECONDARY SEQUENCES FOR DERIVING LOOPS
C KT&LT- TERTIARY SEQUENCES FOR DERIVING LOOPS
C
C*****
0001 IMPLICIT INTEGER*(1-N)
0002 COMMON/ALL/M(15,15),NB,NL,NR,NCCL,NK,NCH,NELEM,NSTORE
0003 COMMON/LOD/KM(11),LM(11),KS(11),LS(11),KT(5),LT(5)
0004 COMMON/DBL/JLOC(20,3),JORDW(20,15),JROW(20,15)
0005 COMMON/DBL/MX/PZ(15,15),MZ1(15,15),MZ2(15,15),MZ3(15,15),MZ4(15,15)
      *PZ5(15,15),MX(20,15,15)
0006 COMMON/PRI/ITCTAL(6)
0007 READ(5,1) NL,NB,NR,NCCL,NC
0008 AK=NR-NL
0009 1 FORMAT(15I5)
0010 READ(5,1)(KM(I),I=1,NK)
0011 READ(5,1)(LM(I),I=1,NK)
0012 NS=NK-NL
0013 READ(5,1)(KS(I),I=NS,AK)
0014 READ(5,1)(LS(I),I=NS,AK)
0015 READ(5,1)(KT(I),I=1,5)
0016 READ(5,1)(LT(I),I=1,5)
0017 NCHS=1
0018 DO 2 I=1,NC
0019 NELEM=1
0020 CALL LUP?
0021 CALL SPALOC
0022 IF(NCHS.EQ.0.AND.NCH.EQ.0) GOTO 40
0023 WRITE(6,3) I
0024 3 FORMAT('1',//,' CHAIN ',I3,/,I3,11('**'))
0025 GOTO 41
0026 40 WRITE(6,5) I
0027 5 FORMAT(/,/, ' CHAIN ',I3,/,I3,11('**'))
0028 GOTO 31
0029 41 NCHS=NCH
0030 IF(NCH.EQ.0) GOTO 31
0031 CALL ARRAY
0032 GOTO 32
0033 31 WRITE(6,8)
0034 8 FORMAT(15,'NO POSSIBLE CHAINS')
0035 GOTO 2
0036 32 IF(NCHS.LT.NELEM) GOTO 2
0037 NCH=NCHS
0038 GOTO (100,200,300,400),NELEM
0039 100 CALL SPRNG1
0040 GOTO 33
0041 200 CALL SPRNG2

```

FORTRAN IV G LEVEL 1, MOD 4                      MAIN                      DATE = 70191                      20/49/09                      PAGE 0002

```

0042 GOTO 33
0043 300 CALL SPRNG3
0044 GOTO 33
0045 400 CALL SPRNG4
0046 33 IF(NCH-1)31,35,34
0047 34 CALL MECCCM
0048 GOTO 36
0049 35 CALL PRINT
0050 36 NELEM=NELEM+1
0051 IF(NELEM-4) 32,32,2
0052 2 CONTINUE
0053 NELEM=NELEM-1
0054 WRITE(6,1000)
0055 DO 6 I=1,4
0056 6 WRITE(6,4) I,ITCTAL(I)
0057 4 FORMAT('TOTAL NUMBER OF 10-LINK CHAINS WITH',I3,' SPRING(S) - ',
      * 15)
0058 WRITE(6,1000)
0059 1000 FORMAT('1')
0060 STOP
0061 END

```

## TABLE XLVIII (Continued)

```

FORTRAN IV G LEVEL 1, MOD 4          SPRLOC          DATE = 70191          20/49/09          PAGE 0001

0001          SUBROUTINE SPRLOC
0002          IMPLICIT INTEGER*(I-A)
0003          COMMON/ALL/M(15,15),NB,NL,NR,NCOL,NK,NCH,NELEP,NSTORE
0004          COMMON/SPL/LPAIR(20,2),LROW(20,15),LINK2(10)
0005          ISPRNG=1
0006          DO 1 I=1,NB
0007          1 LINK2(I)=0
0008          DO 2 J=1,4
0009          LPAIR(J,1)=0
0010          DO 2 I=1,NR
0011          2 LROW(J,1)=0

C
C SEARCH FOR ADJACENT BINARY LINKS
C
0012          DO 10 I=1,NB
0013          IF(LINK2(I).EQ.1) GOTO 10
0014          DO 20 J=1,NL
0015          NE=P(J,ACCL)
0016          DO 20 K=1,NE
0017          IF(M(J,K)-(M(J,K)/100)*100.EQ.1) GOTO 40
0018          20 CONTINUE
0019          GOTO 10
0020          40 IF(M(J,K)/100.EQ.1) GOTO 50
0021          IF((M(J,K)-9000)/100.EQ.1) GOTO 60
0022          GOTO 10

C
C LINK 1 IS A BINARY LINK. CHECK ADJACENT LINKS.
C
0023          50 IF(M(J,K+1)/100.EQ.1) GOTO 70
0024          IF(K.EQ.1) GOTO 51
0025          IF(M(J,K-1)/100.EQ.1) GOTO 71
0026          GOTO 10
0027          51 IF(M(J,NE)/100.EQ.1) GOTO 72
0028          GOTO 10
0029          60 IF((M(J,K+1)-9000)/100.EQ.1) GOTO 70
0030          IF(K.EQ.1) GOTO 61
0031          IF((M(J,K-1)-9000)/100.EQ.1) GOTO 71
0032          GOTO 10
0033          61 IF((M(J,NE)-9000)/100.EQ.1) GOTO 72
0034          GOTO 10

C
C LINK 1 IS ADJACENT TO ANOTHER BINARY LINK.
C
0035          70 LPAIR(ISPRNG,1)=M(J,K)
0036          LPAIR(ISPRNG,2)=M(J,K+1)
0037          L=(M(J,K+1)-(M(J,K+1)/100)*100
0038          LINK2(L)=1
0039          GOTO 80
0040          71 LPAIR(ISPRNG,1)=M(J,K)
0041          LPAIR(ISPRNG,2)=M(J,K-1)
0042          L=(M(J,K-1)-(M(J,K-1)/100)*100
0043          LINK2(L)=1
0044          GOTO 80
0045          72 LPAIR(ISPRNG,1)=M(J,K)
0046          LPAIR(ISPRNG,2)=M(J,NE)
0047          L=(M(J,NE)-(M(J,NE)/100)*100
0048          LINK2(L)=1

C

FORTRAN IV G LEVEL 1, MOD 4          SPRLOC          DATE = 70191          20/49/09          PAGE 0002

C LOCATE ROWS CONTAINING ADJACENT BINARY LINKS.
C
0049          80 DO 81 IA=1,NR
0050          NE=M(IA,NCOL)
0051          IF(NE.EQ.0) GOTO 81
0052          DO 82 IB=1,NE
0053          IF(M(IA,IB).EQ.LPAIR(ISPRNG,1)) LROW(ISPRNG,IA)=IA
0054          82 CONTINUE
0055          81 CONTINUE
0056          ISPRNG=ISPRNG+1
0057          10 CONTINUE
0058          NCH=ISPRNG-1
0059          RETURN
0060          END

```

## TABLE XLVIII (Continued)

```

FORTRAN IV G LEVEL 1, MOD 4          CHAIN          DATE = 70191          20/49/09          PAGE 0001

0001          SUBROUTINE CHAIN
0002          IMPLICIT INTEGER*2(I-N)
0003          COMMON/ALL/M(15,15),NB,NL,NR,ACOL,NK,NCH,NELEM,NSTORE
0004          COMMON/DBALL/JSTORE(100,6),JTEMP(100,6)
0005          COMMON/CHN/IDBL,KSTCRE,KGUNT
0006          DIMENSION ANUM(10),ARRAY(8,12)
0007          DATA C/'---',D1/'-',C/'',D/'',/
0008          DATA ANUM/'0','1','2','3','4','5','6','7','8','9',BLANK/' '/
0009          WRITE(6,1)(B,I,I=KSTORE,IDBL),B
0010          1 FORMAT(5,'CHAIN ',B(A1,4X,12,6X),A1)
0011          WRITE(6,2)(D,D,D,D,D,D,D,D,D,D,D,D,I=KSTORE,IDBL)
0012          2 FORMAT(5,'-----',B(6A2,A1))
0013          I=0
0014          DO 10 IA=KSTORE,IDBL
0015          ISUM=0
0016          I=I+1
0017          DO 11 J=1,NELEM
0018          IF(JSTORE(IA,J).GE.10) GOTO 12
0019          ISUM=ISUM+2
0020          GOTO 11
0021          12 ISUM=ISUM+3
0022          11 CONTINUE
0023          IT=(12-ISUM)/2
0024          DO 13 J=1,IT
0025          13 ARRAY(I,J)=BLANK
0026          IT=IT+1
0027          DO 14 J=1,NELEM
0028          NUM=JSTORE(IA,J)
0029          NDIG=2
0030          IF(NUM.LT.10) NDIG=1
0031          IF(NDIG.EQ.2) GOTO 15
0032          ARRAY(I,IT)=ANUM(NUM+1)
0033          IT=IT+1
0034          IF(J.EQ.NELEM) GOTO 14
0035          GOTO 16
0036          15 NUM1=NUM/10
0037          ARRAY(I,IT)=ANUM(NUM1+1)
0038          NUM2=NUM-NUM1*10
0039          IT=IT+1
0040          ARRAY(I,IT)=ANUM(NUM2+1)
0041          IT=IT+1
0042          IF(J.EQ.NELEM) GOTO 14
0043          16 ARRAY(I,IT)=C
0044          IT=IT+1
0045          14 CONTINUE
0046          DO 17 J=IT,12
0047          17 ARRAY(I,J)=BLANK
0048          10 CONTINUE
0049          KGUNT=KGUNT-1
0050          WRITE(6,3)(B,(ARRAY(I,J),J=1,12),I=1,KGUNT),B
0051          3 FORMAT(5,'SPRINGS',B(13A1),A1,/)
0052          KCUAT=1
0053          KSTORE=IDBL+1
0054          RETURN
0055          END

FORTRAN IV G LEVEL 1, MOD 4          SPRADD          DATE = 70191          20/49/09          PAGE 0001

0001          SUBROUTINE SPRADD(NR,ACOL,MY,MZ,ISPRNG)
0002          IMPLICIT INTEGER*2(I-N)
0003          COMMON/SPL/LPAIR(20,2),LROW(20,15),LINK2(10)
0004          DIMENSION MY(15,20),MZ(15,20)
0005          DO 1 I=1,NR
0006          NE=MY(I,ACOL)
0007          IF(NE.EQ.0) NE=1
0008          DO 1 J=NE,NCOL
0009          1 MZ(I,J)=0
0010          L1=LPAIR(ISPRNG,1)
0011          L2=LPAIR(ISPRNG,2)
0012          C LOCATE LINK L1
0013          C
0014          DO 10 I=1,NR
0015          NE=MY(I,NCOL)
0016          IF(NE.EQ.0) GOTO 10
0017          IF(LROW(ISPRNG,I).NE.T) GOTO 9
0018          DO 20 J=1,NE
0019          IF(MY(I,J).EQ.L1.OR.MY(I,J).EQ.L2) GOTO 30
0020          20 CONTINUE
0021          30 IF(MY(I,J+1).EQ.L2.OR.MY(I,J+1).EQ.L1) GOTO 40
0022          GOTO 50
0023          40 ICOL=1
0024          JS=J
0025          MZ(I,ICOL)=1111
0026          J=J+2
0027          IF(J.GT.NE) J=J-NE
0028          41 ICOL=ICOL+1
0029          MZ(I,ICOL)=MY(I,J)
0030          J=J+1
0031          IF(J.GT.NE) J=1
0032          IF(J.EQ.JS) GOTO 70
0033          GOTO 41
0034          50 J=NE
0035          GCTG 40
0036          70 MZ(I,NCOL)=NE-1
0037          GOTO 10
0038          9 DO 8 J=1,NE
0039          MZ(I,J)=MY(I,J)
0040          8 MZ(I,NCOL)=NE
0041          10 CONTINUE
0042          RETURN
0043          END

```

## TABLE XLVIII (Continued)

```

FORTRAN IV G LEVEL 1, MCO 4          SPRNG2          DATE = 70191          20/49/09          PAGE 0001

0001      SUBROUTINE SPRNG2
0002      IMPLICIT INTEGER*2(I-N)
0003      COMMON/ALL/M(15,15),NB,AL,AR,NCOL,NK,NCH,NELEM,NSTORE
0004      COMMON/OBALL/JSTORE(100,6),JTEMP(100,6)
0005      COMMON/OBLMX/MZ(15,15),MZ1(15,15),MZ2(15,15),MZ3(15,15),MZ4(15,15)
          *,MZ5(15,15),MX(20,15,15)
0006      COMMON/SPL/LPAIR(20,2),LROW(20,15),LINK2(10)
0007      COMMON/CHN/ISPRNG,KSTORE,KCUNT
0008      ISPRNG=1
0009      KCUNT=1
0010      KSTORE=1
0011      WRITE(6,1)
0012      1 FORMAT(/,'IDENTIFICATION OF CHAINS WITH TWO SPRINGS',/,IX,
          *41('*'),/)
0013      DO 100 I=1,NSTORE
0014      100 JTEMP(I,1)=JSTORE(I,1)
0015      DO 10 IZ=1,NSTORE
0016      9 I=JTEMP(IZ,1)
0017      CALL SPRADD(AR,NCOL,P,MZ1,I)
0018      IPI=I+1
0019      IF(IPI.GT.NCH) GOTC 10
0020      DO 30 J=IPI,NCH
0021      JSTORE(ISPRNG,1)=I
0022      JSTORE(ISPRNG,2)=J
0023      KOUNT=KOUNT+1
0024      IF(KOUNT.GT.8) CALL CHAIN
0025      CALL SPRADD(AR,NCOL,MZ1,MZ,J)
          C FILL MX
0026      IROW=1
0027      DO 20 IA=1,NR
0028      IF(LROW(1,IA).EQ.0.AND.LROW(J,IA).EQ.0) GOTO 20
0029      DO 40 IB=1,NCOL
0030      40 MX(ISPRNG,IROW,IB)=MZ(IA,IB)
0031      IROW=IROW+1
0032      20 CONTINUE
0033      MX(ISPRNG,1,NCOL-1)=IROW
0034      ISPRNG=ISPRNG+1
0035      30 CONTINUE
0036      10 CONTINUE
0037      ISPRNG=ISPRNG-1
0038      IF(ISPRNG.GE.KSTORE) CALL CHAIN
0039      NCH=ISPRNG
0040      RETURN
0041      END

```

```

FORTRAN IV G LEVEL 1, MCO 4          SPRNG1          DATE = 70191          20/49/09          PAGE 0001

0001      SUBROUTINE SPRNG1
0002      IMPLICIT INTEGER*2(I-N)
0003      COMMON/ALL/M(15,15),NB,NL,NK,NCOL,NK,NCH,NELEM,NSTORE
0004      COMMON/SPL/LPAIR(20,2),LROW(20,15),LINK2(10)
0005      COMMON/OBALL/JSTORE(100,6),JTEMP(100,6)
0006      COMMON/OBLMX/MZ(15,15),MZ1(15,15),MZ2(15,15),MZ3(15,15),MZ4(15,15)
          *,MZ5(15,15),MX(20,15,15)
0007      WRITE(6,1)
0008      1 FORMAT(/,'CHAINS WITH ONE SPRING (IDENTIFIED BY THE SPRING NUMBER
          * ABOVE)',/,IX,62('*'),/)
0009      DO 10 I=1,NCH
0010      ISPRNG=I
0011      JSTORE(I,1)=I
0012      CALL SPRADD(AR,NCOL,P,MZ,1)
          C FILL MX
0013      IROW=1
0014      DO 20 IA=1,NR
0015      IF(LROW(1,IA).EQ.0) GOTO 20
0016      DO 40 IB=1,NCOL
0017      40 MX(ISPRNG,IROW,IB)=MZ(IA,IB)
0018      IROW=IROW+1
0019      20 CONTINUE
0020      MX(ISPRNG,1,NCOL-1)=IROW
0021      10 CONTINUE
0022      NCH=ISPRNG
0023      RETURN
0024      END

```

## TABLE XLVIII (Continued)

```

FORTRAN IV G LEVEL 1, MOD 4          SPRNG3          DATE = 70191          20/49/09          PAGE 0001

0001          SUBROUTINE SPRNG3
0002          IMPLICIT INTEGER*2(I-N)
0003          COMMON/ALL/M(15,15),NB,NL,NR,NCOL,NK,NCH,NELEM,NSTORE
0004          COMMON/DBALL/JSTORE(100,6),JTEMP(100,6)
0005          COMMON/DBLMX/MZ(15,15),MZ1(15,15),MZ2(15,15),MZ3(15,15),MZ4(15,15)
          *M5(15,15),MX(20,15,15)
0006          COMMON/SPL/LPAIR(20,2),LRC(120,15),LINK2(10)
0007          COMMON/CHN/ISPRNG,KSTORE,KOUNT
0008          ISPRNG=1
0009          KOUNT=1
0010          KSTORE=1
0011          WRITE(6,1)
0012          1 FORMAT(/,'IDENTIFICATION OF CHAINS WITH THREE SPRINGS',/,IX,
          *43('*'),/)
0013          DO 100 I=1,NSTORE
0014          JTEMP(1,2)=JSTORE(1,2)
0015          100 JTEMP(1,1)=JSTORE(1,1)
0016          DO 10 IZ=1,NSTORE
0017          IF(IZ.EQ.1) GOTO 9
0018          IF(JTEMP(IZ,1).NE.JTEMP(IZ-1,1)) GOTO 9
0019          GOTO 8
0020          9 I=JTEMP(IZ,1)
0021          CALL SPRADD(INR,NCOL,M,MZ1,I)
0022          8 J=JTEMP(IZ,2)
0023          CALL SPRADD(INR,NCOL,MZ1,MZ2,J)
0024          JPI=J+1
0025          IF(JPI.GT.NCH) GOTO 10
0026          DO 30 K=JPI,NCH
0027          JSTORE(ISPRNG,1)=I
0028          JSTORE(ISPRNG,2)=J
0029          JSTORE(ISPRNG,3)=K
0030          KOUNT=KOUNT+1
0031          IF(KOUNT.GT.8) CALL CHAIN
0032          CALL SPRADD(INR,NCOL,MZ2,MZ,K)
          *C FILL MX
0033          IROW=1
0034          DO 20 IA=1,NR
0035          IF(LROW(1,IA).EQ.0.AND.LROW(J,IA).EQ.0.AND.LROW(K,IA).EQ.0) GOTO
          * 20
0036          DO 40 IB=1,NCOL
0037          40 MX(ISPRNG,IROW,IB)=MZ(IA,IB)
0038          IROW=IROW+1
0039          20 CONTINUE
0040          MX(ISPRNG,1,NCOL-1)=IROW
0041          ISPRNG=ISPRNG+1
0042          30 CONTINUE
0043          10 CONTINUE
0044          ISPRNG=ISPRNG-1
0045          IF(ISPRNG.GE.KSTORE) CALL CHAIN
0046          NCH=ISPRNG
0047          RETURN
0048          CC4E          END

```

## TABLE XLVIII (Continued)

```

FORTRAN IV G LEVEL 1, MOD 4          SPRNG4          DATE = 70191          20/49/09          PAGE 0001

0001      SUBROUTINE SPRNG4
0002      IMPLICIT INTEGER*2(I-N)
0003      COMMON/ALL/M(15,15),NB,NL,NR,NCOL,NK,NCH,NELEM,NSTORE
0004      COMMON/DBALL/JSTORE(100,6),JTEMP(100,6)
0005      COMMON/DBLMX/MZ(15,15),MZ1(15,15),MZ2(15,15),MZ3(15,15),MZ4(15,15)
          * ,MZ5(15,15),MX(20,15,15)
0006      COMMON/SPL/LPAIR(20,2),LRCH(20,15),LINK2(10)
0007      COMMON/CHW/ISPRNG,KSTORE,KCUNT
0008      ISPRNG=1
0009      KCUNT=1
0010      KSTORE=1
0011      WRITE(6,1)
0012      1 FORMAT(/,'IDENTIFICATION OF CHAINS WITH FOUR SPRINGS',/,IX,
          *42('+',/))
0013      DO 100 I=1,NSTORE
0014      JTEMP(1,3)=JSTORE(I,3)
0015      JTEMP(1,2)=JSTORE(I,2)
0016      100 JTEMP(1,1)=JSTORE(I,1)
0017      DO 10 IZ=1,NSTORE
0018      IF(IZ.EQ.1) GOTO 9
0019      IF(IJTEMP(IZ,1).NE.JTEMP(IZ-1,1)) GOTO 9
0020      IF(IJTEMP(IZ,2).NE.JTEMP(IZ-1,2)) GOTO 8
0021      GOTO 7
0022      9 I=JTEMP(IZ,1)
0023      CALL SPRAODINR,NCOL,M,MZ1,I)
0024      8 J=JTEMP(IZ,2)
0025      CALL SPRAODINR,NCOL,MZ1,MZ2,J)
0026      7 K=JTEMP(IZ,3)
0027      CALL SPRAODINR,NCOL,MZ2,MZ3,K)
0028      KCUNT=KCUNT+1
0029      IF(KCUNT.GT.NCH) GOTO 10
0030      DO 30 L=KPI,NCH
0031      JSTORE(ISPRNG,1)=I
0032      JSTORE(ISPRNG,2)=J
0033      JSTORE(ISPRNG,3)=K
0034      JSTORE(ISPRNG,4)=L
0035      KCUNT=KCUNT+1
0036      IF(KCUNT.GT.8) CALL CHAIN
0037      CALL SPRAODINR,NCOL,MZ3,MZ4,L)
          C FILL MX
0038      IROW=1
0039      DO 20 IA=1,NR
          IF(LROW(1,IA).EQ.0.AND.LROW(J,IA).EQ.0.AND.LROW(K,IA).EQ.0.AND.
          * LROW(L,IA).EQ.0) GOTO 20
0041      DO 40 IB=1,NCOL
0042      40 MX(ISPRNG,IROW,IB)=MZ(IA,IB)
0043      IROW=IROW+1
0044      20 CONTINUE
0045      MX(ISPRNG,1,NCOL-1)=IROW
0046      ISPRNG=ISPRNG+1
0047      30 CONTINUE
0048      10 CONTINUE
          ISPRNG=ISPRNG-1
0049      IF(ISPRNG.GE.KSTORE) CALL CHAIN
0050      NCH=ISPRNG
0051      RETURN
0052      END
0053

```



## TABLE XLVIII (Continued)

```

FORTRAN IV G LEVEL 1, MOD 4          PRINT          DATE = 70191          20/49/09          PAGE 0001
0001          SUBROUTINE PRINT
0002          IMPLICIT INTEGER*2(I-N)
0003          INTEGER IEQ
0004          COMMON/ALL/M(15,15),NB,NL,NR,NCOL,NK,NCH,NELEM,NSTORE
0005          COMMON/PRI/ITCTAL(6)
0006          COMMON/MEC/IEQLV(50,50)
0007          DIMENSION IPRINT(20)
0008          DATA IEQ/'-'/
0009          ISUBT=NCH
0010          WRITE(6,3)
0011          3 FORMAT('0',T5,'EQUIVALENT CHAINS',/)
0012          IF(INC-EQ.1) GOTO 40
0013          DO 30 IX=1,NCH
0014             ISUM=0
0015             DO 20 IY=1,NCH
0016                IF(IEQUIV(IX,IY).EQ.0) GOTO 20
0017                ISUM=ISUM+1
0018                IPRINT(ISUM)=IEQUIV(IX,IY)
0019          20 CONTINUE
0020          IF(ISUM.EQ.0) GOTO 30
0021          WRITE(6,1) IX,(IEQ,IPRINT(I),I=1,ISUM)
0022          1 FORMAT(T10,I2,10(A1,I2))
0023          ISUBT=ISUBT-ISUM
0024          30 CONTINUE
0025          IF(ISUBT.EQ.NCH) GOTO 40
0026          31 WRITE(6,2) ISUBT
0027          2 FORMAT('0',T5,'NUMBER OF UNIQUE CHAINS -',I2)
0028          ITCTAL(NELEM)=ITOTAL(NELEM)+ISUBT
0029          RETURN
0030          40 WRITE(6,4)
0031          4 FORMAT(T10,'NONE')
0032          GOTO 31
0033          END

FORTRAN IV G LEVEL 1, MOD 4          ARRAY          DATE = 70191          20/49/09          PAGE 0001
0001          SUBROUTINE ARRAY
0002          IMPLICIT INTEGER*2(I-A)
0003          COMMON/ALL/M(15,15),NB,NL,NR,NCOL,NK,NCH,NELEM,NSTORE
0004          COMMON/SPL/LPAIR(20,2),LRCW(20,15),LINK2(10)
0005          DIMENSION L(100,3)
0006          DATA C/'-'/,PL/'( ',PR/' )',C/' ',/
0007          DO 10 I=1,NCH
0008             L(I,1)=LPAIR(I,1)-(LPAIR(I,1)/100)*100
0009             10 L(I,2)=LPAIR(I,2)-(LPAIR(I,2)/100)*100
0010             WRITE(6,1)
0011             1 FORMAT('0IDENTIFICATION OF SPRINGS')
0012             WRITE(6,2) I,I=1,NCH)
0013             2 FORMAT('0',T5,'SPRING#',T18,11('| ',I2,3X))
0014             WRITE(6,3) (D,C,D,D,I=1,NCH)
0015             3 FORMAT(T5,13(' - '),10(4A2))
0016             WRITE(6,4) (L(I,1),C,L(I,2),I=1,NCH)
0017             4 FORMAT(T5,'LINK1, LINK2 ',11('| ',I2,A1,I2,1X),//)
0018             RETURN
0019             END

FORTRAN IV G LEVEL 1, MOD 4          BLK DATA          DATE = 70191          20/49/09          PAGE 0001
0001          BLECK DATA
0002          IMPLICIT INTEGER*2(I-A)
0003          COMMON/DGALL/JSTORE(100,6),JTEMP(100,6)
0004          COMMON/PRI/ITOTAL(6)
0005          DATA ITOTAL/6*C/
0006          DATA JSTORE/600*0/,JTEMP/600*0/
0007          END

```

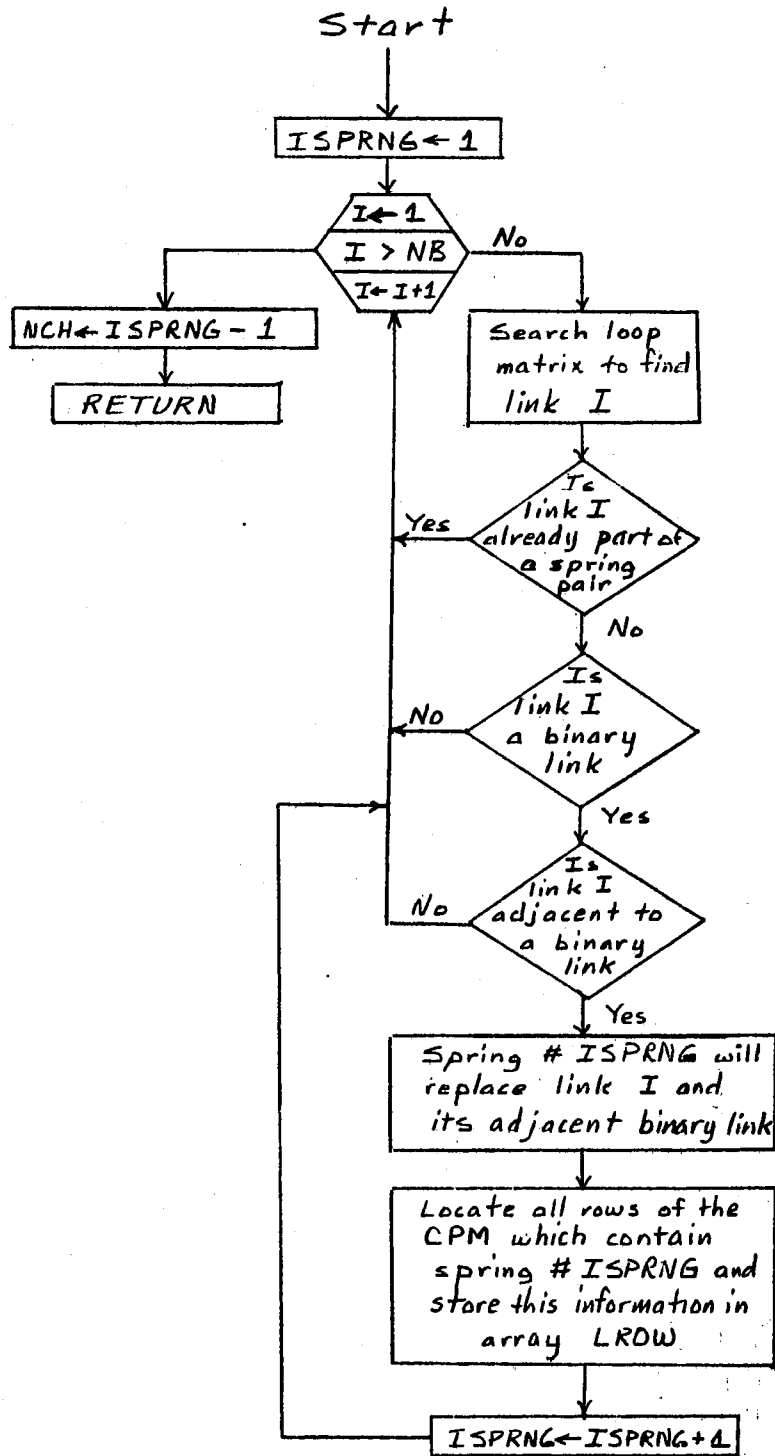


Figure 24. Flow Chart for Subroutine SPRLOC

*NSTORE* is the number of unique chains with two springs. The spring combinations for these chains are stored in array *JSTORE*

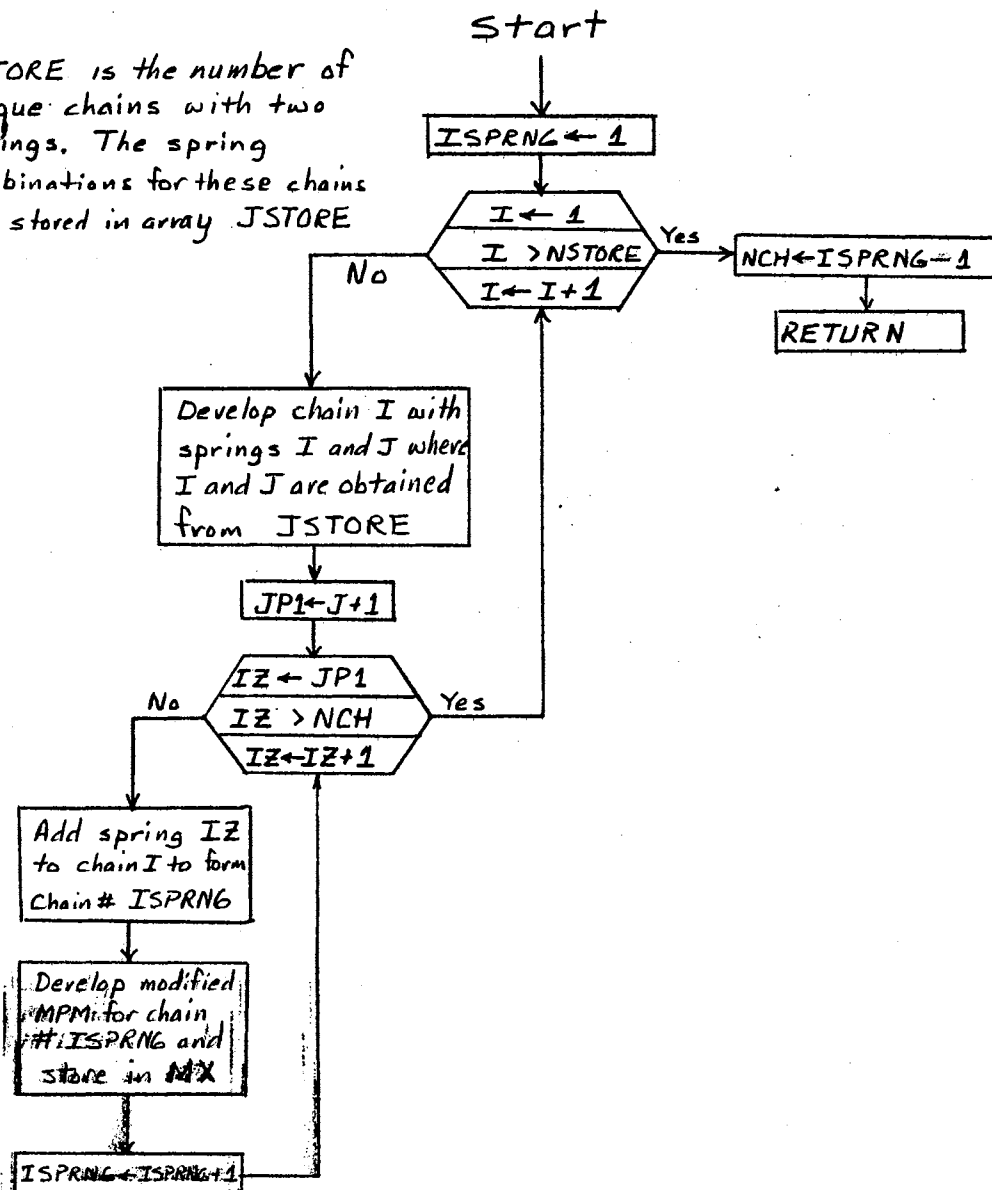


Figure 25. Flow Chart for Subroutine SPRNG3

## APPENDIX H

### SUBROUTINES USED IN DETERMINING THE TEN-LINK CHAINS WITH PULLEYS AND BELTS

The program for determining the chains with pulleys and belts is the same as that shown in Appendix G with Subroutine SPRLOC being replaced by Subroutine PULLOC and Subroutine SPRNG1 through SPRNG4 being replaced by PUL1 through PUL4. Also, some format statements in the output subroutines require changes.

Subroutine PULLOC is shown in Table XLIX and its flow chart in Figure 26. Subroutine PUL3, which demonstrates the logic for all PUL subroutines, appears in Table L and its flow chart is found in Figure 27.

TABLE XLIX  
SUBROUTINE PULLOC

```

FORTRAN IV G LEVEL 1, MOD 4      PULLOC      DATE = 70192      09/56/51      PAGE 0001

0001      SUBROUTINE PULLOC
0002      IMPLICIT INTEGER*2(I-N)
0003      COMMON/ALL/M(15,15),NB,NL,NR,NCOL,NK,NCM,NELEN,NSTORE
0004      COMMON/PUL/JPUL(10,6),JPROW(10,15)
0005      IPUL=1

C
C SEARCH FOR TERNARY LINKS
C
0006      DO 100 K=1,NB
0007      DO 10 I=1,NL
0008      NE=M(I,NCOL)
0009      IF(NE.EQ.0) GOTO 10
0010      DO 11 J=1,NE
0011      IF(M(I,J)-(M(I,J)/100)*100.EQ.K) GOTO 20
0012      11 CONTINUE
0013      10 CONTINUE
0014      20 IF(M(I,J)/100.NE.3) GOTO 100

C
C DETERMINE IF TERNARY LINK IS JOINED TO TWO BINARY LINKS
C
0015      DO 30 IA=1,NR
0016      NE=M(IA,NCOL)
0017      IF(NE.EQ.0) GOTO 30
0018      DO 40 IB=1,NE
0019      IF(M(IA,IB).EQ.M(I,J)) GOTO 41
0020      40 CONTINUE
0021      GOTO 30
0022      41 IY=IB-1
0023      IZ=IB+1
0024      IF(IB.EQ.1) IY=NE
0025      IV=IY-1
0026      IF(IV.EQ.1) IV=NE
0027      IF(IB.EQ.NE) IZ=1
0028      IZ=IZ+1
0029      IF(IZ.EQ.NE) IZ=1
0030      IF(M(IA,IV)/100.EQ.1.AND.M(IA,IZ)/100.EQ.1) GOTO 42
0031      GOTO 30
0032      42 JPUL(IPUL,1)=M(IA,IB)
0033      JPUL(IPUL,2)=M(IA,IV)
0034      JPUL(IPUL,3)=M(IA,IV)
0035      JPUL(IPUL,4)=M(IA,IZ)
0036      JPUL(IPUL,5)=M(IA,IZ)
0037      IPUL=IPUL+1
0038      GOTO 43
0039      30 CONTINUE
0040      GOTO 100

C
C DETERMINE IF A THIRD BINARY LINK IS CONNECTED TO THE TERNARY LINK
C
0041      43 DO 60 IC=1,NR
0042      NE=M(IC,NCOL)
0043      IF(NE.EQ.0) GOTO 60
0044      DO 50 ID=1,NE
0045      IF(M(IC,ID).EQ.M(I,J)) GOTO 61
0046      50 CONTINUE
0047      GOTO 60
0048      61 I1=ID-1
0049      I2=ID+1

```

## TABLE XLIX (Continued)

```

FORTRAN IV G LEVEL 1, MOD 4          PULLOC          OATE = 70192          09/56/51          PAGE 0002

0050          IF(I0.EQ.1) I1=NE
0051          I11=I1-1
0052          IF(I1.EQ.1) I11=NE
0053          IF(I0.EQ.NE) I2=1
0054          I21=I2+1
0055          IF(I2.EQ.NE) I21=1
0056          IF(M(IC,I1).NE.JPUL(IPUL-1,2).AND.M(IC,I1).NE.JPUL(IPUL-1,4))
0057          1 GOTO 70
          IF(M(IC,I2).NE.JPUL(IPUL-1,2).AND.M(IC,I2).NE.JPUL(IPUL-1,4))
0058          2 GOTO 71
0059          60 CONTINUE
0060          70 JPUL(IPUL-1,6)=M(IC,I1)
0061          IF(M(IC,I1)/100.NE.1) GOTO 100
0062          JPUL(IPUL,2)=M(IC,I1)
0063          JPUL(IPUL,3)=M(IC,I11)
0064          JPUL(IPUL+1,2)=M(IC,I1)
0065          JPUL(IPUL+1,3)=M(IC,I11)
0066          GOTO 72
0067          71 JPUL(IPUL-1,6)=M(IC,I2)
0068          IF(M(IC,I2)/100.NE.1) GOTO 100
0069          JPUL(IPUL,2)=M(IC,I2)
0070          JPUL(IPUL,3)=M(IC,I21)
0071          JPUL(IPUL+1,2)=M(IC,I2)
0072          JPUL(IPUL+1,3)=M(IC,I21)
0073          72 JPUL(IPUL,1)=M(I,J)
0074          JPUL(IPUL,4)=JPUL(IPUL-1,2)
0075          JPUL(IPUL,5)=JPUL(IPUL-1,3)
0076          JPUL(IPUL,6)=JPUL(IPUL-1,4)
0077          IPUL=IPUL+1
0078          JPUL(IPUL,1)=M(I,J)
0079          JPUL(IPUL,4)=JPUL(IPUL-2,4)
0080          JPUL(IPUL,5)=JPUL(IPUL-1,5)
0081          JPUL(IPUL,6)=JPUL(IPUL-2,2)
0082          IPUL=IPUL+1
0083          100 CONTINUE
          NCH=IPUL-1

C
C DETERMINE ROWS CONTAINING PULLEY
C
0084          IF(NCH.EQ.0) RETURN
0085          DO 110 I=1,NCH
0086          DO 110 J=1,NR
0087          JPROW(I,J)=0
0088          NE=M(I,NCOL)
0089          IF(NE.EQ.0) GOTO 110
0090          DO 120 K=1,NE
0091          IF(M(I,K).EQ.JPUL(I,1)) GOTO 121
0092          120 CONTINUE
0093          GOTO 110
0094          121 JPROW(I,J)=J
0095          110 CONTINUE
0096          RETURN
0097          END

```

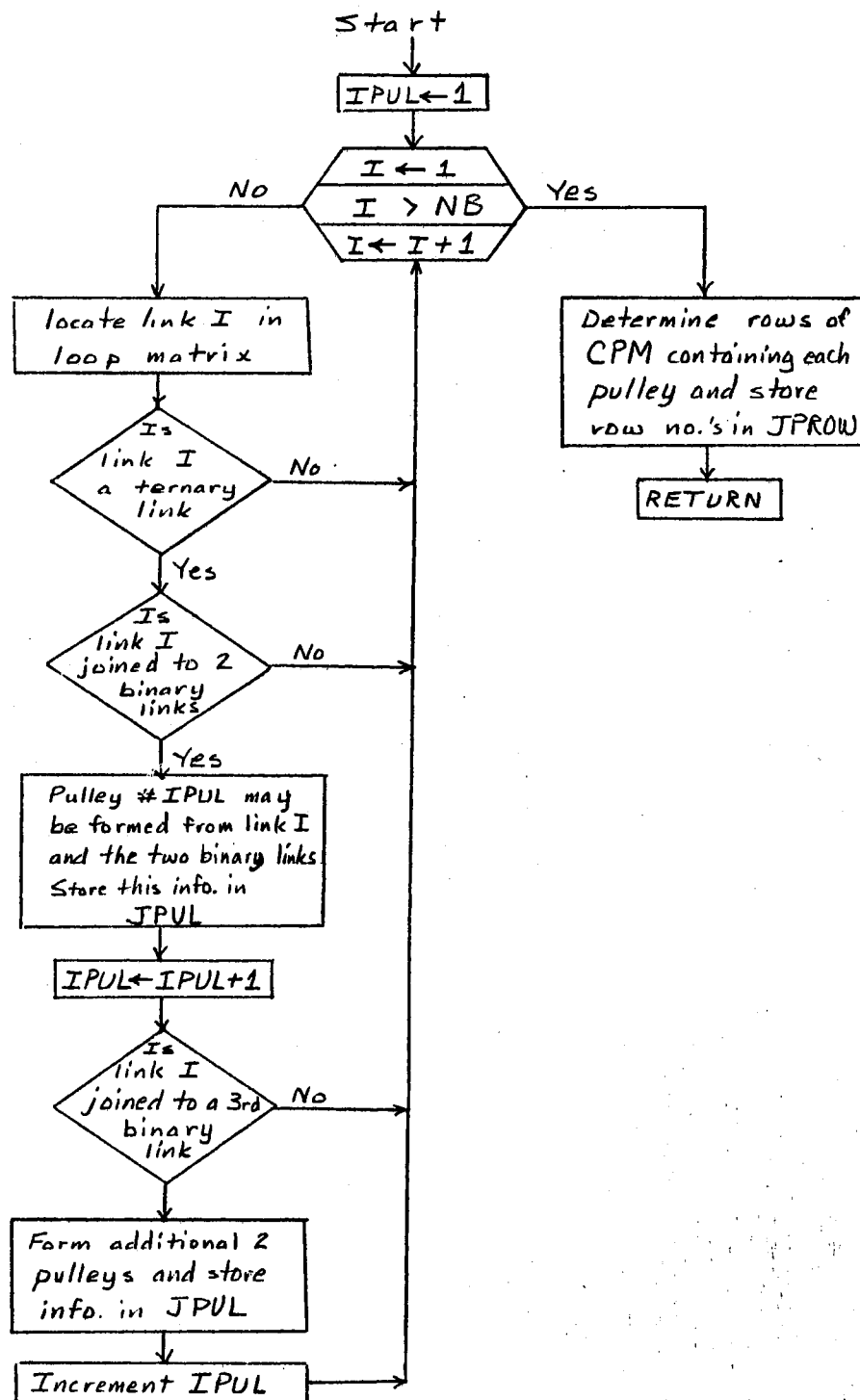


Figure 26. Flow Chart for Subroutine PULLOC

TABLE L  
SUBROUTINE PUL3

```

FORTRAN IV G LEVEL 1, MOD 4          PUL3          DATE = 70192          09/56/51          PAGE 0001

0001      SUBROUTINE PUL3
0002      IMPLICIT INTEGER*2(I-N)
0003      COMMON/ALL/M(15,15),NB,NL,NR,NCOL,NK,NCH,NELEM,NSTORE
0004      COMMON/DBALL/JSTORE(100,6),JTEMP(100,6)
0005      COMMON/DBLMX/MZ(15,15),MZ1(15,15),MZ2(15,15),MZ3(15,15),MZ4(15,15)
          * ,MZ5(15,15),MX(15,15,15)
0006      COMMON/PUL/JPUL(10,6),JPROW(10,15)
0007      COMMON/CHN/IPUL,KSTORE,KOUNT
0008      COMMON/LIN/LINE,IPAGE
0009      DIMENSION L1(5),L2(5),L3(5),L4(5)
0010      IPUL=1
0011      KOUNT=1
0012      KSTORE=1
0013      IF(LINE.GT.47) CALL PAGE
0014      WRITE(6,1)
0015      1 FORMAT(/,'IDENTIFICATION OF CHAINS WITH THREE PULLEYS',/,1X,
          * 43('+',/))
          LINE=LINE+5
0016      DO 100 I=1,NSTORE
0017      JTEMP(I,2)=JSTORE(I,2)
0018      JTEMP(I,1)=JSTORE(I,1)
0019      100 DO 10 IZ=1,NSTORE
0020      IF(IZ.EQ.1) GOTO 9
0021      IF(JTEMP(IZ,1).NE.JTEMP(IZ-1,1)) GOTO 9
0022      GOTO 8
0023      9 I=JTEMP(IZ,1)
0024      DO 91 IA=1,5
0025      L1(IA)=JPUL(I,IA)
0026      CALL PULADD(I,NR,NCOL,M,MZ1)
0027      8 J=JTEMP(IZ,2)
0028      DO 81 IA=1,5
0029      L2(IA)=JPUL(J,IA)
0030      CALL PULADD(J,NR,NCOL,MZ1,MZ2)
0031      JPI=J+1
0032      IF(JPI.GT.NCH) GOTO 10
0033      DO 30 K=JPI,NCH
0034      DO 71 IA=1,5
0035      L3(IA)=JPUL(K,IA)
0036      IF(L1(1).EQ.L3(1)) GOTO 30
0037      IF(L1(2).EQ.L3(3).OR.L1(2).EQ.L3(5)) GOTO 30
0038      IF(L1(4).EQ.L3(3).OR.L1(4).EQ.L3(5)) GOTO 30
0039      IF(L2(1).EQ.L3(1)) GOTO 30
0040      IF(L2(2).EQ.L3(3).OR.L2(2).EQ.L3(5)) GOTO 30
0041      IF(L2(4).EQ.L3(3).OR.L2(4).EQ.L3(5)) GOTO 30
0042      JSTORE(IPUL,1)=I
0043      JSTORE(IPUL,2)=J
0044      JSTORE(IPUL,3)=K
0045      KOUNT=KOUNT+1
0046      IF(KOUNT.GT.5) CALL CHAIN1
0047      CALL PULADD(K,NR,NCOL,MZ2,MZ)
0048      C FILL MX
0049      IROW=1
0050      DO 20 IA=1,NR
0051      IF(JPROW(IA,IA).NE.IA.AND.JPROW(J,IA).NE.IA.AND.JPROW(K,IA).NE.IA)
          * GOTO 20
0052      DO 40 IB=1,NCOL
0053      40 MX(IPUL,IROW,IB)=MZ(IA,IB)
0054      IROW=IROW+1

```

```

FORTRAN IV G LEVEL 1, MOD 4          PUL3          DATE = 70192          09/56/51          PAGE 0002

0055      20 CONTINUE
0056      MX(IPUL,1,NCOL-1)=IROW
0057      IPUL=IPUL+1
0058      30 CONTINUE
0059      10 CONTINUE
0060      IPUL=IPUL-1
0061      IF(IPUL.GE.KSTORE) CALL CHAIN1
0062      NCH=IPUL
0063      NSTORE=NCH
0064      RETURN
0065      END

```



## TABLE L (Continued)

```

FORTRAN IV G LEVEL 1, MOD 4          PULADD          DATE = 70192          14/06/25          PAGE 0001
0001          SUBROUTINE PULADD(I,NR,NCOL,MY,MZ)
0002          IMPLICIT INTEGER*2(I-N)
0003          COMMON/PUL/JPUL(10,6),JPROW(10,15)
0004          DIMENSION MY(15,20),MZ(15,20)
0005          L1=JPUL(I,1)
0006          L2=JPUL(I,2)
0007          L3=JPUL(I,4)
0008          DO 10 J=1,NR
0009          IF(JPROW(I,J).NE.J) GOTO 2
0010          NE=MY(J,NCOL)
0011          DO 20 K=1,NE
0012          IF(MY(J,K).EQ.L1) GOTO 30
0013          20 CONTINUE
0014          30 ISTOP=K
0015          ICOL=1
0016          MZ(J,I)=1111
0017          31 ICOL=ICOL+1
0018          K=K+1
0019          IF(K.GT.NE)K=1
0020          IF(K.EQ.ISTOP) GOTO 3
0021          MZ(J,ICOL)=MY(J,K)
0022          IF(MY(J,K).EQ.L2.OR.MY(J,K).EQ.L3) MZ(J,ICOL)=1211
0023          GOTO 31
0024          3 K=NE+1
0025          DO 4 KK=K,NCOL
0026          4 MZ(J,KK)=MY(J,KK)
0027          GOTO 10
0028          2 DO 5 K=1,NCOL
0029          5 MZ(J,K)=MY(J,K)
0030          10 CONTINUE
0031          RETURN
0032          END

```

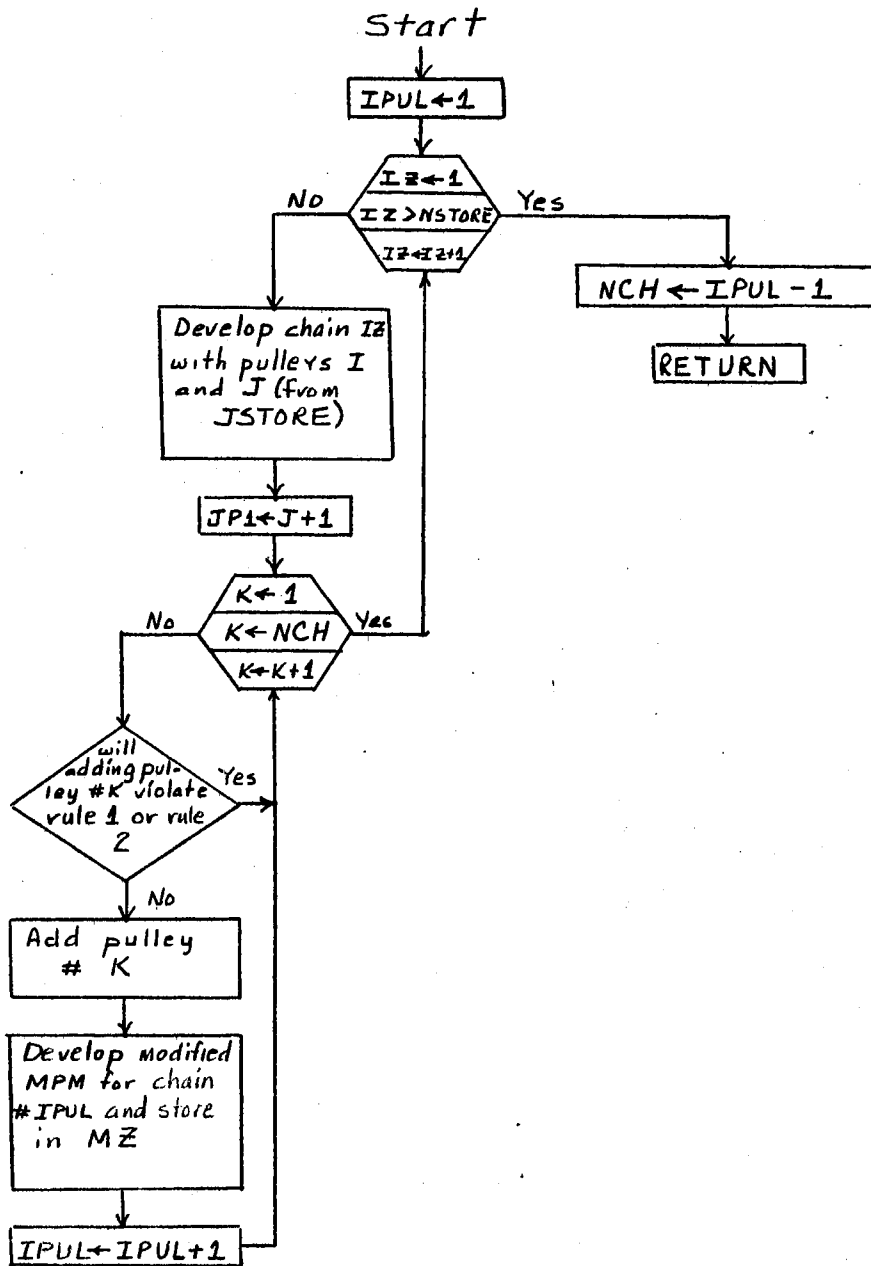


Figure 27. Flow Chart for Subroutine PUL3

## APPENDIX I

### SUBROUTINES USED IN DETERMINING THE TEN-LINK CHAINS WITH CAM PAIRS

As with the program for determining chains with pulleys (Appendix H), this program follows the basic form shown in Appendix G. Subroutine CAMLOC, which locates possible cam pairs, appears in Table LI and its flow chart in Figure 28. Subroutine CAM3, which develops the chains with 3 cam pairs, is shown in Table LII and its flow chart in Figure 29.

TABLE LI  
SUBROUTINE CAMLOC

```

FORTRAN IV G LEVEL 1, MOD 4          CAMADD          DATE = 70193          14/26/56          PAGE 0001

0001      SUBROUTINE CAMADD(K,NR,NCOL,MY,MZ)
0002      IMPLICIT INTEGER*2(I-N)
0003      COMMON/CAL/JCAM(8,3),JCROW(8,15)
0004      DIMENSION MY(15,15),MZ(15,15)
0005      LI=JCAM(K,1)
0006      DO 20 I=1,NR
0007      IF(JCROW(K,I).NE.1) GOTO 25
0008      NE=MY(I,NCOL)
0009      DO 30 J=1,NE
0010      IF(MY(I,J).EQ.LI) GOTO 40
0011      30 CONTINUE
0012      40 ISTOP=J
0013      ICOL=1
0014      MZ(I,ICOL)=1111
0015      41 ICOL=ICOL+1
0016      J=J+1
0017      IF(J.GT.NE)J=1
0018      IF(J.EQ.ISTOP) GOTO 42
0019      MZ(I,ICOL)=MY(I,J)
0020      GOTO 41
0021      42 J=NE+1
0022      43 DO 50 L=J,NCOL
0023      50 MZ(I,L)=MY(I,L)
0024      GOTO 20
0025      25 J=1
0026      GOTO 43
0027      20 CONTINUE
0028      RETURN
0029      END

```

```

FORTRAN IV G LEVEL 1, MOD 4          CAMLOC          DATE = 70193          14/26/56          PAGE 0001

0001      SUBROUTINE CAMLOC
0002      IMPLICIT INTEGER*2(I-N)
0003      COMMON/ALL/M(15,15),NB,NL,NR,NCOL,NK,NCH,NELEM,NSTORE
0004      COMMON/CAL/JCAM(8,3),JCROW(8,15)
0005      ICAM=1
0006      C
0007      C SEARCH FOR BINARY LINKS
0008      C
0009      DO 100 K=1,NB
0010      DO 10 I=1,NL
0011      JCROW(ICAM,I)=0
0012      NE=M(I,NCOL)
0013      DO 10 J=1,NE
0014      IF(M(I,J)-(M(I,J)/100)*100.EQ.K) GOTO 20
0015      10 CCNTINUE
0016      20 IF(M(I,J)/100.NE.1) GOTO 100
0017      JCAM(ICAM,1)=M(I,J)
0018      IX=J+1
0019      IY=J-1
0020      IF(J.EQ.NE)IX=1
0021      IF(J.EQ.1)IY=NE
0022      JCAM(ICAM,2)=M(I,IY)
0023      JCAM(ICAM,3)=M(I,IX)
0024      C
0025      C DETERMINE ROWS CONTAINING BINARY LINK
0026      C
0027      JCROW(ICAM,I)=1
0028      IPI=I+1
0029      DO 30 IA=IPI,NR
0030      JCROW(ICAM,IA)=0
0031      NE=M(IA,NCOL)
0032      IF(NE.EQ.0) GOTO 30
0033      DO 31 IB=1,NE
0034      IF(M(IA,IB).EQ.JCAM(ICAM,1)) GOTO 32
0035      31 CCNTINUE
0036      GOTO 30
0037      32 JCROW(ICAM,IA)=IA
0038      30 CCNTINUE
0039      ICAM=ICAM+1
0040      100 CONTINUE
0041      NCH=ICAM-1
0042      RETURN
0043      END

```

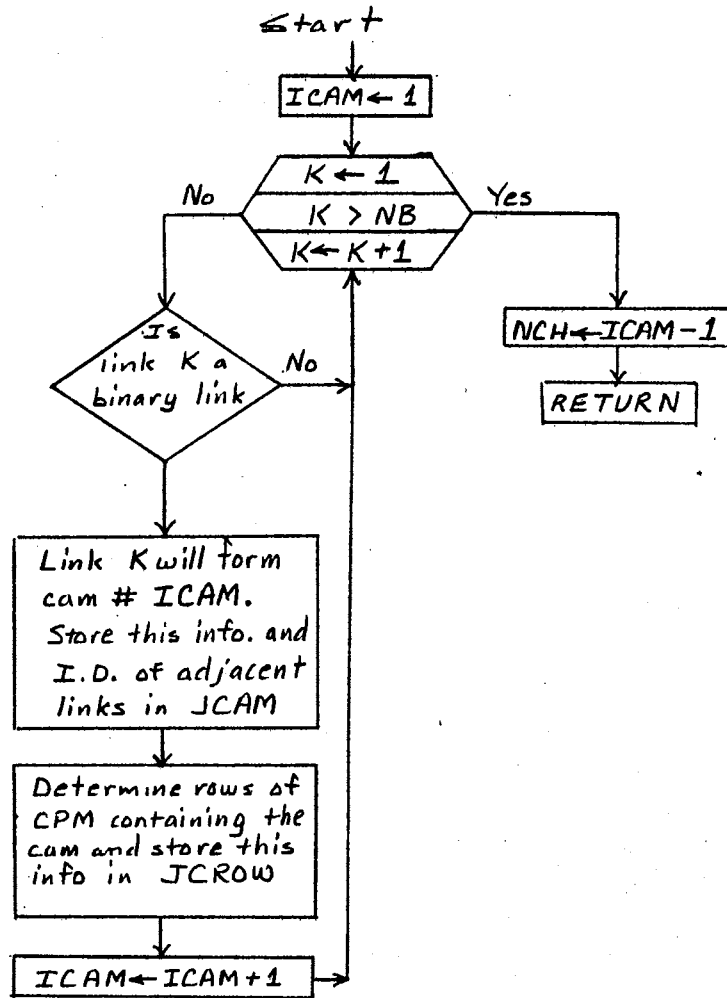


Figure 28. Flow Chart for Subroutine CAMLOC

TABLE LII  
SUBROUTINE CAM3

```

FORTRAN IV G LEVEL 1, MOD 4          CAM3          DATE = 70193          14/26/56          PAGE 0001

0001      SUBROUTINE CAM3
0002      IMPLICIT INTEGER*2(I-N)
0003      COMMON/ALL/M(15,15),NB,NL,NR,NCOL,NK,NCH,NELEM,NSTORE
0004      COMMON/DBALL/JSTORE(100,6),JTEMP(100,6)
0005      COMMON/DBLMX/MZ(15,15),MZ1(15,15),MZ2(15,15),MZ3(15,15),MZ4(15,15)
          *MZ5(15,15),MX(50,15,15)
0006      COMMON/CAL/JCAM(8,3),JCROW(8,15)
0007      COMMON/CHN/ICAM,KSTORE,KOUNT
0008      COMMON/LIN/LINE,IPAGE
0009      ICAM=1
0010      KOUNT=1
0011      KSTORE=1
0012      IF(LINE.GT.47) CALL PAGE
0013      WRITE(6,1)
0014      1 FORMAT(/,'IDENTIFICATION OF CHAINS WITH THREE CAM PAIRS',/,1X,
          *45('*'),/)
          LINE=LINE+5
0015      DO 100 I=1,NSTORE
0016      JTEMP(I,2)=JSTORE(I,2)
0017      100 JTEMP(I,1)=JSTORE(I,1)
0018      DO 10 IZ=1,NSTORE
0019      IF(IZ.EQ.1) GOTO 9
0020      IF(JTEMP(IZ,1).NE.JTEMP(IZ-1,1)) GOTO 9
0021      GOTO 8
0022      9 I=JTEMP(IZ,1)
0023      J1=JCAM(I,1)
0024      CALL CAMADD(I,NR,NCOL,M,MZ1)
0025      8 J=JTEMP(IZ,2)
0026      J1=JCAM(J,1)
0027      CALL CAMADD(J,NR,NCOL,MZ1,MZ2)
0028      JP1=J+1
0029      IF(JP1.GT.NCH) GOTO 10
0030      DO 30 K=JP1,NCH
0031      K2=JCAM(K,2)
0032      K3=JCAM(K,3)
0033      IF(I1.EQ.K2.OR.I1.EQ.K3) GOTO 30
0034      IF(J1.EQ.K2.OR.J1.EQ.K3) GOTO 30
0035      JSTORE(ICAM,1)=I
0036      JSTORE(ICAM,2)=J
0037      JSTORE(ICAM,3)=K
0038      KOUNT=KOUNT+1
0039      IF(KOUNT.GT.5) CALL CHAIN1
0040      CALL CAMADD(K,NR,NCOL,MZ2,MZ1)
0041      C FILL MX
          IROW=1
0042      DO 20 IA=1,NR
0043      IF(JCROW(IA,IA).NE.IA.AND.JCROW(J,IA).NE.IA.AND.JCROW(K,IA).NE.IA)
0044      * GOTO 20
          DO 40 IB=1,NCOL
0045      MX(ICAM,IROW,IB)=MZ(IA,IB)
0046      IROW=IROW+1
0047      20 CONTINUE
          MX(ICAM,1,NCOL-1)=IROW
0048      ICAM=ICAM+1
0049      30 CONTINUE
          10 CONTINUE
          ICAM=ICAM-1
0050      IF(ICAM.GE.KSTORE) CALL CHAIN1
          0051
          0052
          0053
          0054
FORTRAN IV G LEVEL 1, MOD 4          CAM3          DATE = 70193          14/26/56          PAGE 0002

0055      NCH=ICAM
0056      NSTORE=NCH
0057      RETURN
0058      END

```

NOTE: Subroutine CAMADD appears in Table LVI.

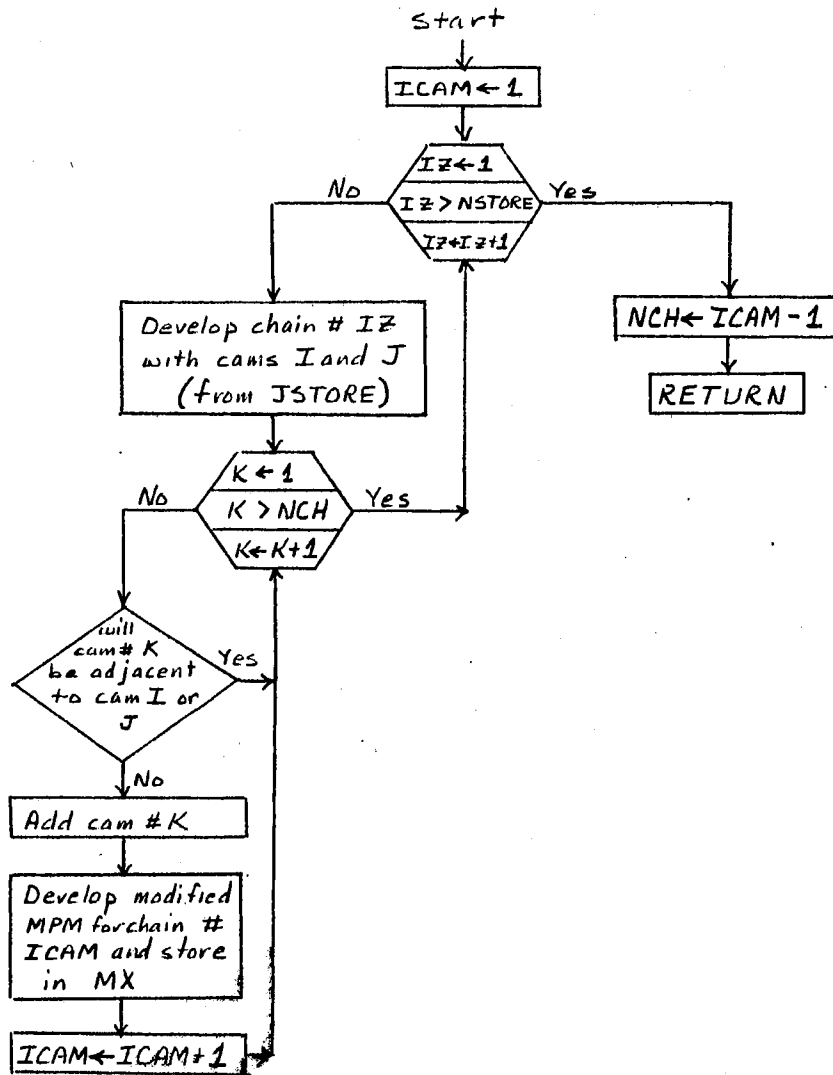


Figure 29. Flow Chart for Subroutine CAM3

## APPENDIX J

### SUBROUTINES USED IN DETERMINING THE TEN-LINK CHAINS WITH PRISM PAIRS

The program for prism pairs follows the same form as all previous programs. Subroutines JOINT (which locates all joints) and PRISM2 (which adds 2 prism pairs) are shown in Tables LIII and LIV respectively. Their flow charts appear in Figures 30 and 31.



TABLE LIII  
SUBROUTINE JOINT

```

FORTRAN IV G LEVEL 1, MOD 4          JOINT          DATE = 70195          02/53/33          PAGE 0001

0001          SUBROUTINE JOINT
0002          IMPLICIT INTEGER*2(I-N)
0003          COMMON/ALL/M(15,15),NB,NL,NR,NCOL,NK,NCH,NELEM,NSTORE
0004          COMMON/JO1/JLOC(13,2),JROW(13,15)
0005          NCH=13
0006          L1=1
0007          L2=2
0008          JNUM=1
0009          DO 1 I=1,13
0010             DO 1 J=1,15
0011                1 JROW(I,J)=0
C
C SEARCH FOR EITHER LINK L1 OR LINK L2.
C
0012          2 ISUM=0
0013             DO 10 I=1,NR
0014                NE=M(I,NCOL)
0015                IF(NE.EQ.0) GOTO 10
0016                DO 20 J=1,NE
0017                   IF(M(I,J)-(M(I,J)/100)*100.EQ.L1) GOTO 30
0018                   IF(M(I,J)-(M(I,J)/100)*100.EQ.L2) GOTO 40
0019                20 CONTINUE
0020                GOTO 10
0021                30 IF(M(I,J+1)-(M(I,J+1)/100)*100.EQ.L2) GOTO 50
0022                   IF(J.NE.1) GOTO 10
0023                   IF(M(I,NE)-(M(I,NE)/100)*100.EQ.L2) GOTO 60
0024                   GOTO 10
0025                40 IF(M(I,J+1)-(M(I,J+1)/100)*100.EQ.L1) GOTO 70
0026                   IF(J.NE.1) GOTO 10
0027                   IF(M(I,NE)-(M(I,NE)/100)*100.EQ.L1) GOTO 80
0028                   GOTO 10
C
C LINKS L1 AND L2 ARE ADJACENT IN ROW I.
C
0029          50 JLGC(JNUM,1)=M(I,J)
0030             JLOC(JNUM,2)=M(I,J+1)
0031             GOTO 90
0032          60 JLGC(JNUM,1)=M(I,J)
0033             JLOC(JNUM,2)=M(I,NE)
0034             GOTO 90
0035          70 JLOC(JNUM,1)=M(I,J+1)
0036             JLOC(JNUM,2)=M(I,J)
0037             GOTO 90
0038          80 JLOC(JNUM,1)=M(I,NE)
0039             JLOC(JNUM,2)=M(I,J)
0040          90 JROW(JNUM,1)=I
0041             ISUM=1
0042          10 CONTINUE
0043             IF(ISUM.EQ.0) GOTO 100
0044             JNUM=JNUM+1
0045             IF(JNUM.GT.13) RETURN
0046          100 L2=L2+1
0047             IF(L2.GT.NB) GOTO 110
0048             GOTO 2
0049          110 L1=L1+1
0050             L2=L1+1
0051             IF(L2.GT.NB) RETURN
0052             GOTO 2

```

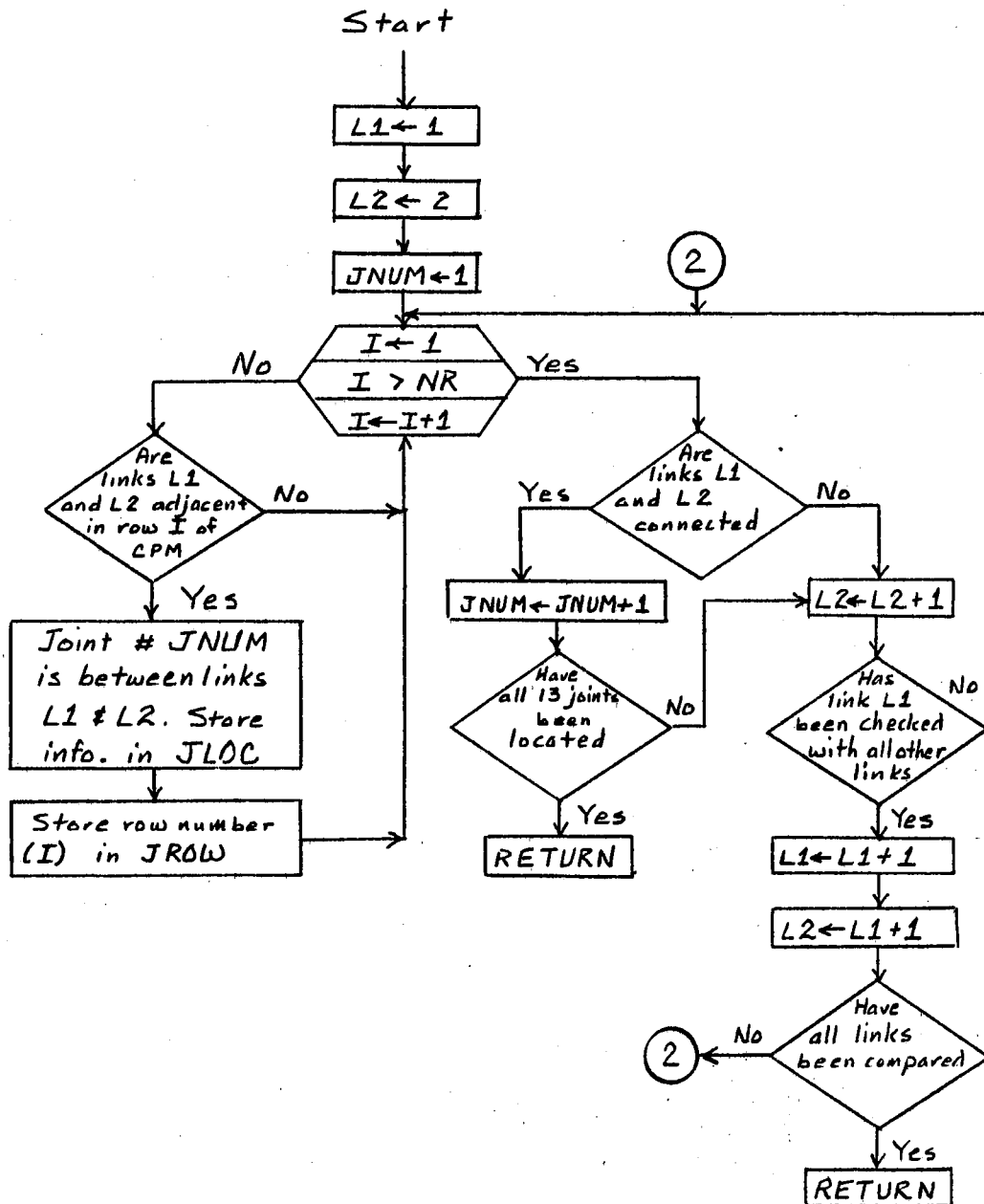


Figure 30. Flow Chart for Subroutine JOINT

TABLE LIV  
SUBROUTINE PRISM2

```

FORTRAN IV G LEVEL 1, MOD 4          PRISM2          DATE = 70195          02/53/33          PAGE 0001

0001      SUBROUTINE PRISM2
0002      IMPLICIT INTEGER*2(I-N)
0003      COMMON/ALL/M(15,15),NB,NL,NR,NCOL,NK,NCH,NELEM,NSTORE
0004      COMMON/OBALL/JSTORE(100,6),JTEMP(100,6)
0005      COMMON/PRISM/MZ(15,15),MZ1(15,15),MX(100,15,15)
0006      COMMON/JOI/LPAIR(13,2),LRCW(13,15)
0007      COMMON/CHN/IPRISM,KSTORE,KOUNT
0008      COMMON/LIN/LINE,IPAGE
0009      IPRISM=1
0010      KOUNT=1
0011      KSTORE=1
0012      IF(LINE.GT.47) CALL PAGE
0013      WRITE(6,1)
0014      1 FORMAT(/,'IDENTIFICATION OF CHAINS WITH TWO PRISM PAIRS',/,IX,
0015      *45('+',/),/),
0016      LINE=LINE+5
0017      DO 100 I=1,NSTORE
0018      JTEMP(I,1)=JSTORE(I,1)
0019      DU 10 IZ=1,NSTORE
0020      9 I=JTEMP(IZ,1)
0021      CALL LOCATE(I,NR,NCOL,M,MZ1)
0022      TP1=1+1
0023      IF(TP1.GT.NCH) GOTO 10
0024      DO 30 J=IP1,NCH
0025      JSTORE(IPRISM,1)=I
0026      JSTORE(IPRISM,2)=J
0027      KOUNT=KOUNT+1
0028      IF(KOUNT.GT.5) CALL CHAIN1
0029      CALL LOCATE(J,NR,NCOL,MZ1,MZ)
0030      C FILL MX
0031      IROW=1
0032      DO 20 IA=1,NR
0033      IF(LROW(IA,IA).EQ.0.AND.LRCW(J,IA).EQ.0) GOTO 20
0034      DO 40 IB=1,NCOL
0035      40 MX(IPRISM,IROW,IB)=MZ(IA,IB)
0036      IROW=IROW+1
0037      20 CONTINUE
0038      MX(IPRISM,1,NCOL-1)=IROW
0039      IPRISM=IPRISM+1
0040      30 CONTINUE
0041      10 CONTINUE
0042      IPRISM=IPRISM-1
0043      IF(IPRISM.GE.KSTORE) CALL CHAIN1
0044      NCH=IPRISM
0045      RETURN
0046      END

```

```

FORTRAN IV G LEVEL 1, MOD 4          LOCATE          DATE = 70195          02/53/33          PAGE 0001

0001      SUBROUTINE LOCATE(JN,NR,NCOL,MY,MZ)
0002      IMPLICIT INTEGER*2(I-N)
0003      COMMON/JOI/JLOC(13,2),JROW(13,15)
0004      DIMENSION MY(15,20),MZ(15,20)
0005      DO 10 I=1,NR
0006      IF(JROW(JN,I).EQ.0) GOTO 18
0007      C SEARCH FOR LINKS INCIDENT TO JOINT JN
0008      C
0009      NE=MY(I,NCOL)
0010      DO 20 J=1,NE
0011      IF(MY(I,J).EQ.JLOC(JN,1)) GOTO 40
0012      IF(MY(I,J).EQ.JLOC(JN,2)) GOTO 50
0013      20 CONTINUE
0014      40 IF(MY(I,J+1).EQ.JLOC(JN,2)) GOTO 60
0015      GOTO 70
0016      50 IF(MY(I,J+1).EQ.JLOC(JN,1)) GOTO 60
0017      GOTO 70
0018      C INSERT PRISM PAIR IN PROPER LOCATION
0019      C
0020      70 KS=NE
0021      J=0
0022      GOTO 61
0023      60 KS=J
0024      61 MZ(I,1)=1011
0025      NE=NE+1
0026      J=J+1
0027      DO 62 K=2,NE
0028      MZ(I,K)=MY(I,J)
0029      J=J+1
0030      IF(J.EQ.KS+1) GOTO 63
0031      IF(J.EQ.NE) J=1
0032      62 CONTINUE
0033      63 K=K+1
0034      DO 64 IK=K,NCOL
0035      64 MZ(I,IK)=0
0036      MZ(I,NCOL)=NE
0037      GOTO 10
0038      18 DO 19 J=1,NCOL
0039      19 MZ(I,J)=MY(I,J)
0040      10 CONTINUE
0041      RETURN
0042      END

```

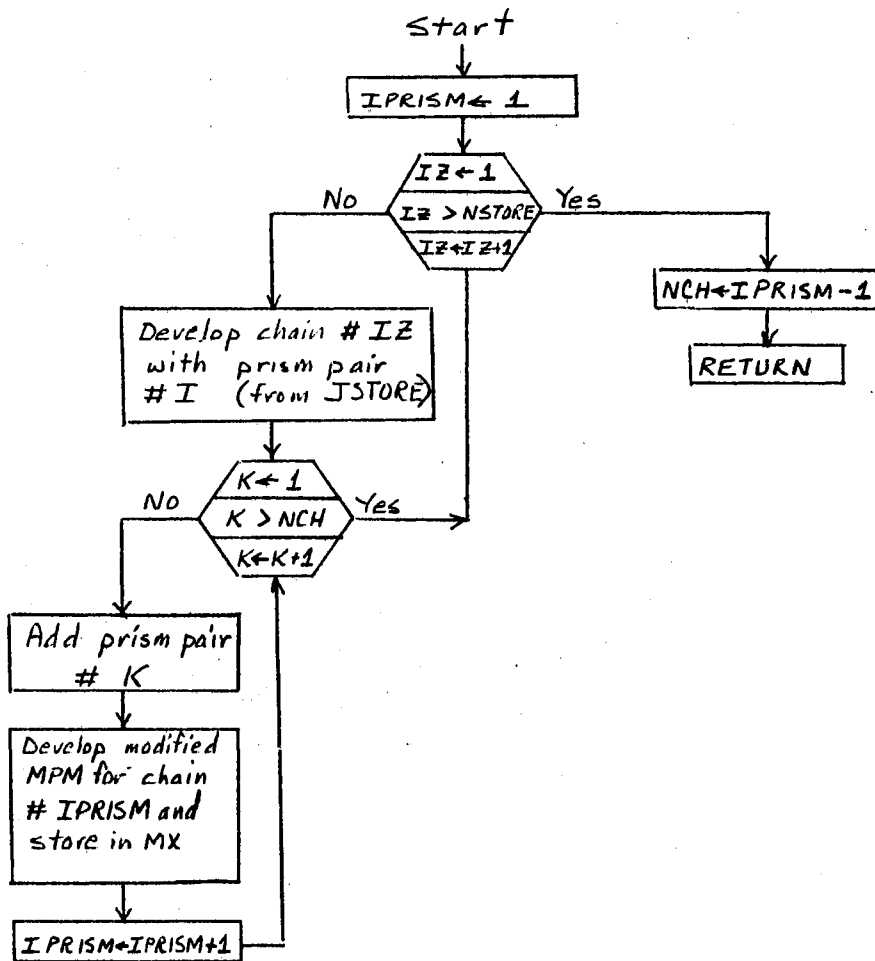


Figure 31. Flow Chart for Subroutine PRISM2

## APPENDIX K

### SUBROUTINES USED IN DETERMINING THE TEN-LINK CHAINS WITH DOUBLE JOINTS

This program follows the same form as all previous programs. Subroutine DBLOC (Table LV) locates all possible double joints. The flow chart for Subroutine DBLOC appears in Figure 32. Subroutine DBL2, which forms modified MPM's for chains with two double joints, is shown in Table LVI and its flow chart in Figure 33. In addition, Subroutines LNKCNT and MOBCK, which check for mobility of the chain by applying rules 1 and 2 of Chapter X, are shown in Table LVII. Their flow charts appear in Figure 34.

TABLE LV  
SUBROUTINE DBLOC

```

FORTRAN IV G LEVEL 1, MCD 4          DBLOC          DATE = 70199          08/35/08          PAGE 0001

0001      SUBROUTINE DBLOC
0002      IMPLICIT INTEGER*(I-A)
0003      COMMON/ALL/M(15,15),NB,NL,NR,NCOL,NK,NCH,NELEP,NSTORE
0004      COMMON/DBL/JLOC(20,3),JROW(20,15),JROW(20,15)
0005      DIMENSION LSTCRE(15)
0006      C SEARCH FOR BASE LINKS
0007      NCH=1
0008      DO 10 LA=1,NB
0009      DO 11 I=1,NL
0010      NE=M(I,NCOL)
0011      DO 11 J=1,NE
0012      IF(M(I,J)-M(I,J)/100)*100.NE.LN) GOTO 11
0013      IF(M(I,J)/100.LI.3) GOTO 10
0014      GOTO 12
0015      11 CONTINUE
0016      C LINK LN IS A BASE LINK. FIND ALL LINKS JOINED TO BASE LINK
0017      12 IBASE=M(I,J)
0018      DO 13 IS=1,5
0019      LSTORE(IS)=0
0020      IS=1
0021      20 IX=J-1
0022      IY=J+1
0023      IF(J.EQ.1) IX=NE
0024      IF(J.EQ.NE) IY=1
0025      DO 21 IA=1,IS
0026      IF(M(I,IX).EQ.LSTCRE(IA)) GOTO 22
0027      21 CONTINUE
0028      LSTCRE(IS)=M(I,IX)
0029      IS=IS+1
0030      IF(IS.GT.IBASE/100) GOTO 100
0031      22 DO 23 IB=1,IS
0032      IF(M(I,IY).EQ.LSTORE(IB)) GOTO 30
0033      23 CONTINUE
0034      LSTORE(IS)=M(I,IY)
0035      IS=IS+1
0036      IF(IS.GT.IBASE/100) GOTO 100
0037      30 II=I+1
0038      DO 36 I=II,NR
0039      NE=M(I,NCOL)
0040      IF(NE.EQ.0) GOTO 36
0041      DO 35 J=1,NE
0042      IF(M(I,J).EQ.IBASE) GOTO 20
0043      35 CONTINUE
0044      36 CONTINUE
0045      C ALL LINKS JOINED TO BASE LINK HAVE BEEN FOUND. DETERMINE ALL COMBINATIONS
0046      C OF THESE LINKS TAKEN TWO AT A TIME.
0047      100 IS=IS-1
0048      ISM1=IS-1
0049      DO 110 IA=1,ISM1
0050      IAP1=IA+1
0051      DO 110 IB=IAP1,IS
0052      C INSURE THAT AT LEAST FOUR LINKS WILL REMAIN IN EACH LOOP CONTAINING THE
0053      C DOUBLE JOINT
0054      CC 101 JA=1,NR
0055      IF(M(JA,NCOL).GT.4) GOTO 101
0056      ISUM=0
0057      DO 102 JB=1,4
0058      IZ=M(JA,JB)

```

```

FORTRAN IV G LEVEL 1, MCD 4          DBLOC          DATE = 70199          08/35/08          PAGE 0002

0053      102 IF(IZ.EQ.IBASE.OR.IZ.EQ.LSTCRE(IA).OR.IZ.EQ.LSTORE(IB)) ISUM=ISUM+1
0054      IF(ISUM.EQ.3) GOTO 110
0055      101 CONTINUE
0056      JLOC(NCH,1)=IBASE
0057      JLOC(NCH,2)=LSTORE(IA)
0058      JLOC(NCH,3)=LSTORE(IB)
0059      C LOCATE ALL ROWS INVOLVING JOINT NCH.
0060      JN=NCH
0061      DO 132 I=1,NR
0062      JROW(JN,I)=0
0063      JRCW(JN,I)=0
0064      NE=M(I,NCOL)
0065      IF(NE.EQ.0) GOTO 132
0066      DO 130 J=1,NE
0067      IF(M(I,J).NE.JLOC(JN,1)) GOTO 130
0068      IX=J-1
0069      IY=J+1
0070      IF(J.EQ.1) IX=NE
0071      IF(J.EQ.NE) IY=1
0072      IF(M(I,IX).EQ.JLOC(JN,2).AND.M(I,IY).EQ.JLOC(JN,3)) GOTO 133
0073      IF(M(I,IX).EQ.JLOC(JN,3).AND.M(I,IY).EQ.JLOC(JN,2)) GOTO 133
0074      IF(M(I,IX).EQ.JLOC(JN,2).OR.M(I,IX).EQ.JLOC(JN,3)) GOTO 131
0075      IF(M(I,IY).EQ.JLOC(JN,2).OR.M(I,IY).EQ.JLOC(JN,3)) GOTO 131
0076      GOTO 130
0077      133 JROW(JN,I)=I
0078      131 JROW(JN,I)=I
0079      130 CONTINUE
0080      132 CONTINUE
0081      CALL M0BCK(NCH,M,NR,NCOL,M0B)
0082      IF(M0B.EQ.0) GOTO 110
0083      NCH=NCH+1
0084      110 CONTINUE
0085      10 CONTINUE
0086      NCH=NCH-1
0087      RETURN
0088      END

```

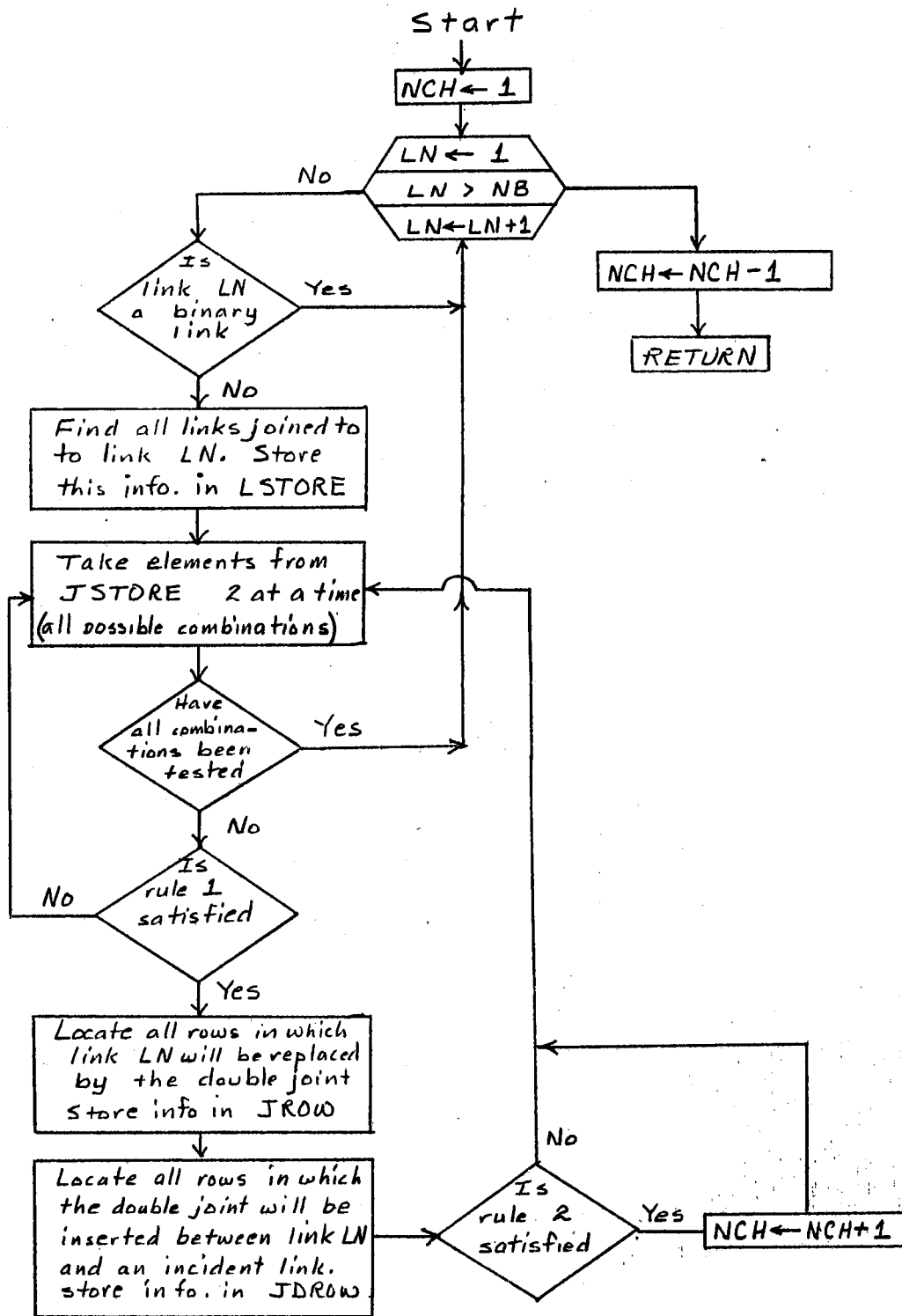


Figure 32. Flow Chart for Subroutine DBLOC

TABLE LVI  
SUBROUTINE DBL2

```

FORTRAN IV G LEVEL 1, MCD 4          CBL2          DATE = 70199          08/35/08          PAGE 0001

0001      SUBROUTINE DBL2
0002      IMPLICIT INTEGER*2(1-A)
0003      COMMON/ALL/M(15,15),NB,NL,NR,NCOL,NK,NCH,NELEK,NSTORE
0004      COMMON/DBALL/JSTORE(100,6),JTEMP(100,6)
0005      COMMON/DBLPX/MZ(15,15),MZ1(15,15),MZ2(15,15),MZ3(15,15),MZ4(15,15)
0006      COMMON/DBL/JLCC(20,3),JRCW(20,15),JROW(20,15)
0007      COMMON/CHN/IDBL,KSTORE,KOUNT
0008      COMMON/DBL6A/I(3),J(3),K(3),L(3),M(3),N(3)
0009      COMMON/LIN/LINE,IPAGE
0010      IOBL=1
0011      KCLAT=1
0012      KSTORE=1
0013      IF(LINE.GT.47) CALL PAGE
0014      WRITE(6,1)
0015      1 FORMAT(/,'IDENTIFICATION OF CHAINS WITH TWO DOUBLE JOINTS',/,1X,
0016      * 47('*,',/))
0017      LINE=LINE+5
0018      DO 100 I=1,NSTORE
0019      100 JTEMP(I,1)=JSTORE(I,1)
0020      DO 10 IZ=1,NSTORE
0021      9 J=JTEMP(IZ,1)
0022      CALL CBLADD(I,NA,NCOL,N,MZ1)
0023      IPI=I+1
0024      IF(IPI.GT.NCH) GOTO 10
0025      DO 90 IA=1,3
0026      90 I(IA)=JLOC(I,IA)
0027      DO 30 J=IPI,NCH
0028      ISUM=C
0029      DO 91 IA=1,3
0030      91 J(IA)=JLCC(J,IA)
0031      DO 91 IB=1,3
0032      91 IF(J(I(IA),EQ.I(II(1B))) ISUM=ISUM+1
0033      IF(ISUM.GT.1) GOTO 30
0034      CALL LNKCNT(MZ1,J,IPASS)
0035      IF(IPASS.EQ.1) GOTO 30
0036      CALL MOBCK(J,MZ1,NR,NCOL,MOB)
0037      IF(MOB.EQ.0) GOTO 30
0038      JSTORE(IDBL,1)=I
0039      JSTORE(IDBL,2)=J
0040      KOUNT=KOUNT+1
0041      IF(KOUNT.GT.5) CALL CHAINI
0042      CALL CBLADD(J,NA,NCOL,MZ1,MZ)
0043      C FILL MX
0044      IROW=1
0045      DO 20 IA=1,NR
0046      20 IF(JOROW(I,IA).NE.IA.AND.JOROW(J,IA).NE.IA) GOTO 20
0047      CALL MXFILL(IDBL,IA,IROW)
0048      IROW=IROW+1
0049      10 CONTINUE
0050      IOBL=IOBL+1
0051      IF(IOBL.GE.KSTORE) CALL CHAINI
0052      NCH=NCH+1
0053      NSTORE=NSTORE+1
0054      RETURN
0055

```



## TABLE LVI (Continued)

```

FCRTRAN IV G LEVEL 1, MOD 4          OBLAOD          DATE = 70199          08/35/08          PAGE 0001
0001      SUBROUTINE OBLAOD(I, NR, NCCL, MY, MZ)
0002      IMPLICIT INTEGER*2(I-N)
0003      COMMON/OBL/JLOC(20,3), JROW(20,15), JROW(20,15)
0004      DIMENSION MY(15,20), MZ(15,20)
0005      IBASE=JLOC(I,1)
0006      DO 10 IA=1, NR
0007      IF(JROW(I, IA).NE. IA) GOTC 11
0008      NE=MY(IA, NCOL)
0009      DD 20 IB=1, NE
0010      IF(MY(IA, IB).EQ. IBASE) GOTC 21
0011      20 CONTINUE
0012      21 IX=IB-1
0013      IY=IB+1
0014      IF(IB.EQ.1) IX=NE
0015      IF(IB.EQ.NE) IY=1
0016      IF(MY(IA, IX).EQ. JLOC(I,2).AND. MY(IA, IY).EQ. JLOC(I,3)) GOTC 30
0017      IF(MY(IA, IX).EQ. JLOC(I,3).AND. MY(IA, IY).EQ. JLOC(I,2)) GOTC 30
0018      IF(MY(IA, IX).EQ. JLOC(I,2).OR. MY(IA, IX).EQ. JLOC(I,3)) GOTC 40
0019      IF(MY(IA, IY).EQ. JLOC(I,2).OR. MY(IA, IY).EQ. JLOC(I,3)) GOTC 50
0020      30 DO 31 IC=1, NE
0021      MZ(IA, IC)=MY(IA, IC)
0022      31 IF(MZ(IA, IC).EQ. JLOC(I,1)) MZ(IA, IC)=1211
0023      K=NE+1
0024      DO 32 IC=K, NCOL
0025      32 MZ(IA, IC)=MY(IA, IC)
0026      GOTO 10
0027      40 ICCL=1
0028      DO 41 IC=1, NE
0029      MZ(IA, ICCL)=MY(IA, IC)
0030      ICCL=ICOL+1
0031      IF(ICCL.EQ. IX+1) GOTC 42
0032      GOTO 41
0033      42 MZ(IA, ICCL)=1211
0034      ICCL=ICOL+1
0035      41 CONTINUE
0036      DO 43 IC=ICOL, NCOL
0037      43 MZ(IA, IC)=0
0038      MZ(IA, NCOL)=NE+1
0039      GOTO 10
0040      50 IX=IB
0041      GOTO 40
0042      11 DO 12 IB=1, NCOL
0043      12 MZ(IA, IB)=MY(IA, IB)
0044      10 CONTINUE
0045      RETURN
0046      END

```

```

FORTRAN IV G LEVEL 1, MOD 4          MXFILL          DATE = 70199          08/35/08          PAGE 0001
0001      SUBROUTINE MXFILL(I, IA, IROW)
0002      IMPLICIT INTEGER*2(I-N)
0003      COMMON/ALL/M(15,15), NB, NL, NR, NCOL, NK, NCH, NELEM, NSTORE
0004      COMMON/DBL/MX/MZ(15,15), MZ1(15,15), MZ2(15,15), MZ3(15,15), MZ4(15,15)
0005      * , MZ5(15,15), MX(100,15,15)
0006      NE=MZ(IA, NCOL)
0007      DO 30 IB=1, NE
0008      IF(MZ(IA, IB).EQ. 1211) GOTC 40
0009      30 CONTINUE
0010      40 ICCL=1
0011      ISTOP=IB
0012      41 MX(I, IROW, ICCL)=MZ(IA, IB)
0013      ICCL=ICOL+1
0014      IB=IB+1
0015      IF(IB.GT.NE) IB=1
0016      IF(IB.EQ. ISTOP) GOTC 50
0017      GOTO 41
0018      50 DO 51 IB=ICOL, NCOL
0019      51 MX(I, IROW, IB)=MZ(IA, IB)
0020      IROW=IROW+1
0021      RETURN
0022      END

```

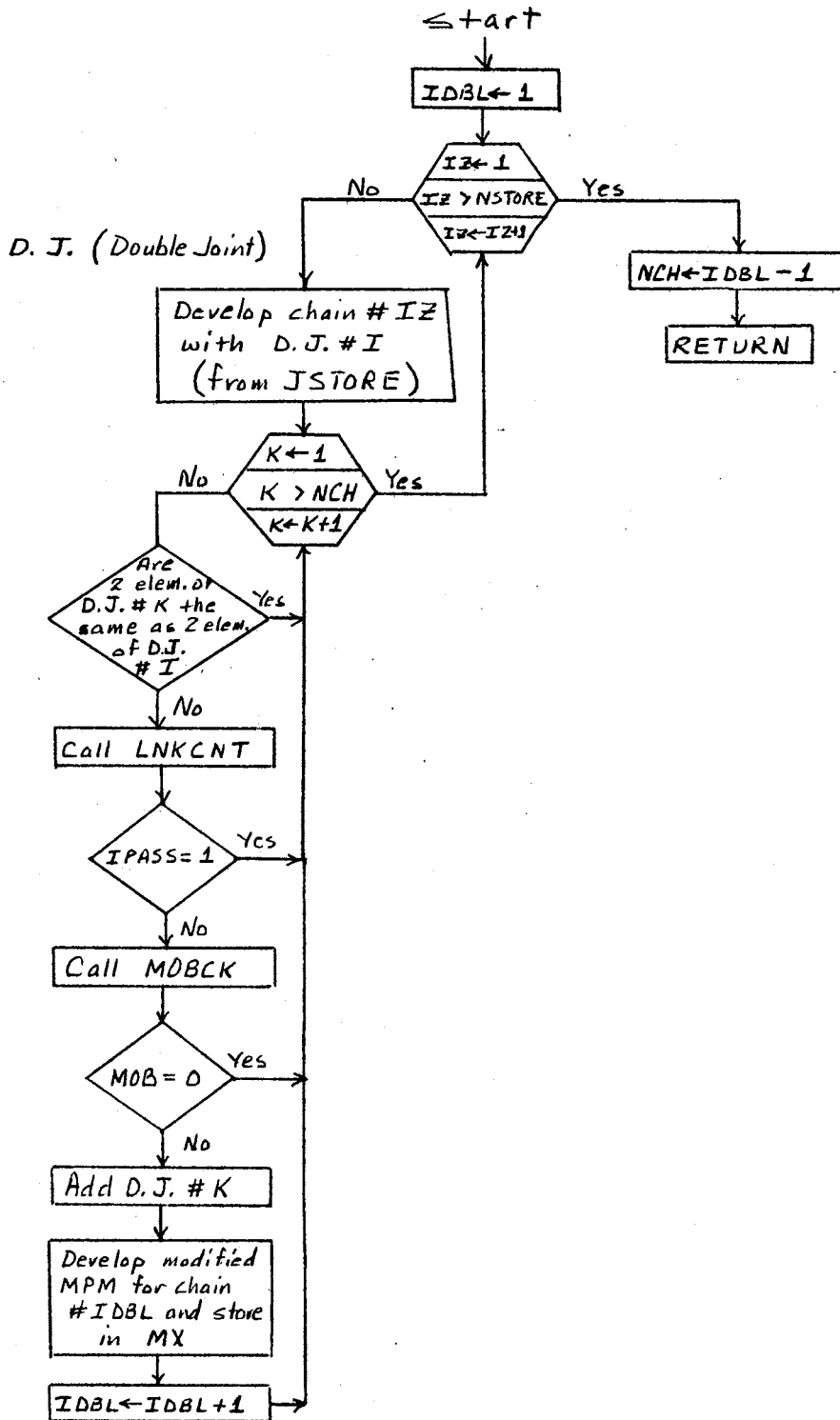


Figure 33. Flow Chart for Subroutine DBL2

## TABLE LVII

## SUBROUTINES LNKCNT AND MOBCK

```

FORTRAN IV G LEVEL 1, MOD 4          MOBCK          DATE = 70199          08/35/08          PAGE 0001

0001      SUBROUTINE MOBCK(N,M,AR,NCOL,MOB)
0002      IMPLICIT INTEGER*2(I-N)
0003      COMMON/DBL/JLCC(20,3),JCRCH(20,15),JROW(20,15)
0004      DIMENSION M(15,15)
          C CHECK ALL ROWS CONTAINING DOUBLE JOINT TO INSURE THAT NO TWO ROWS WHICH WILL
          C CONTAIN ONLY FOUR LINKS, HAVE THREE LINKS IN COMMON.
0005      L1=JLCC(I,1)
0006      L2=JLCC(I,2)
0007      L3=JLCC(I,3)
0008      NRPI=NR-1
0009      DO 10 I=1,NRPI
0010      IF(JROW(I),NE.I) GOTO 10
          C COUNT NUMBER OF LINKS EXCLUSIVE OF L1
0011      NE1=M(I,NCOL)
0012      ISUM=0
0013      DO 20 J=1,NE1
0014      20 IF(M(I,J).LT.1000.AND.M(I,J).NE.L1) ISUM=ISUM+1
0015      IF(ISUM.GT.4) GOTO 10
0016      IP1=I+1
0017      DO 30 J=IP1,NR
0018      IF(JROW(N,J).NE.J) GOTO 30
          C COUNT NUMBER OF LINKS EXCLUSIVE OF L1
0019      NE2=M(J,NCOL)
0020      ISUM=0
0021      DO 40 K=1,NE2
0022      40 IF(M(J,K).LT.1000.AND.M(J,K).NE.L1) ISUM=ISUM+1
0023      IF(ISUM.GT.4) GOTO 30
          C DETERMINE IF THE TWO LOOPS HAVE MORE THAN TWO LINKS IN COMMON
0024      ISUM=0
0025      DO 50 K=1,NE1
0026      IF(M(I,K).GT.1000.OR.M(I,K).EQ.L1) GOTO 50
0027      DO 60 L=1,NE2
0028      60 IF(M(I,K).EQ.M(J,L)) ISUM=ISUM+1
0029      50 CONTINUE
0030      MOB=C
0031      IF(ISUM.GT.2) RETURN
0032      30 CONTINUE
0033      10 CONTINUE
0034      MOB=1
0035      RETURN
0036      END

```

```

FORTRAN IV G LEVEL 1, MOD 4          LNKCNT          DATE = 70199          08/35/08          PAGE 0001

0001      SUBROUTINE LNKCNT(MZ,L,IPASS)
0002      IMPLICIT INTEGER*2(I-N)
0003      COMMON/ALL/M(15,15),NB,NL,AR,NCOL,NK,NCH,NELEN,NSTORE
0004      COMMON/DBL/JLCC(20,3),JCRCH(20,15),JROW(20,15)
0005      DIMENSION MZ(15,15)
0006      L1=JLCC(L,1)
0007      L2=JLCC(L,2)
0008      L3=JLCC(L,3)
0009      IPASS=0
0010      DO 10 I=1,NR
0011      IF(MZ(I,NCOL).GT.NELEN+3) GOTO 10
0012      NE=MZ(I,NCOL)
0013      IF(NE.EQ.0) GOTO 10
0014      ISUM=0
0015      JSUM=0
0016      DO 5 J=1,NE
0017      K=MZ(I,J)
0018      IF(K.GT.1000) JSUM=JSUM+1
0019      5 IF(K.EQ.L1.OR.K.EQ.L2.OR.K.EQ.L3) ISUM=ISUM+1
0020      IF(ISUM.EQ.3) JSUM=JSUM+1
0021      IF(NE-JSUM.LE.3) IPASS=1
0022      IF(IPASS.EQ.1) RETURN
0023      10 CONTINUE
0024      RETURN
0025      END

```

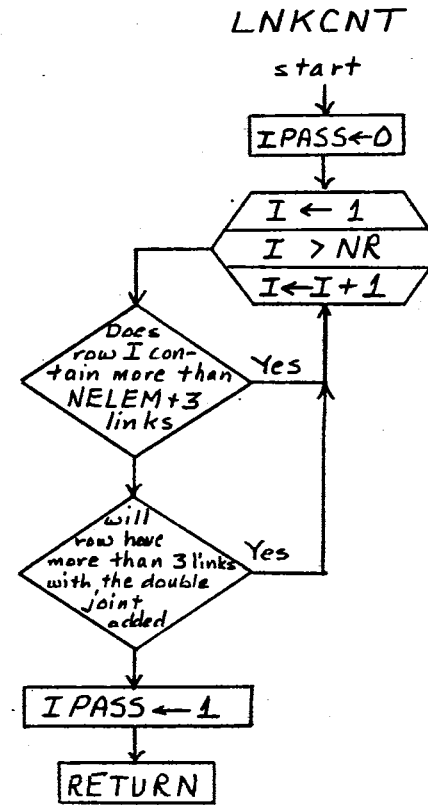
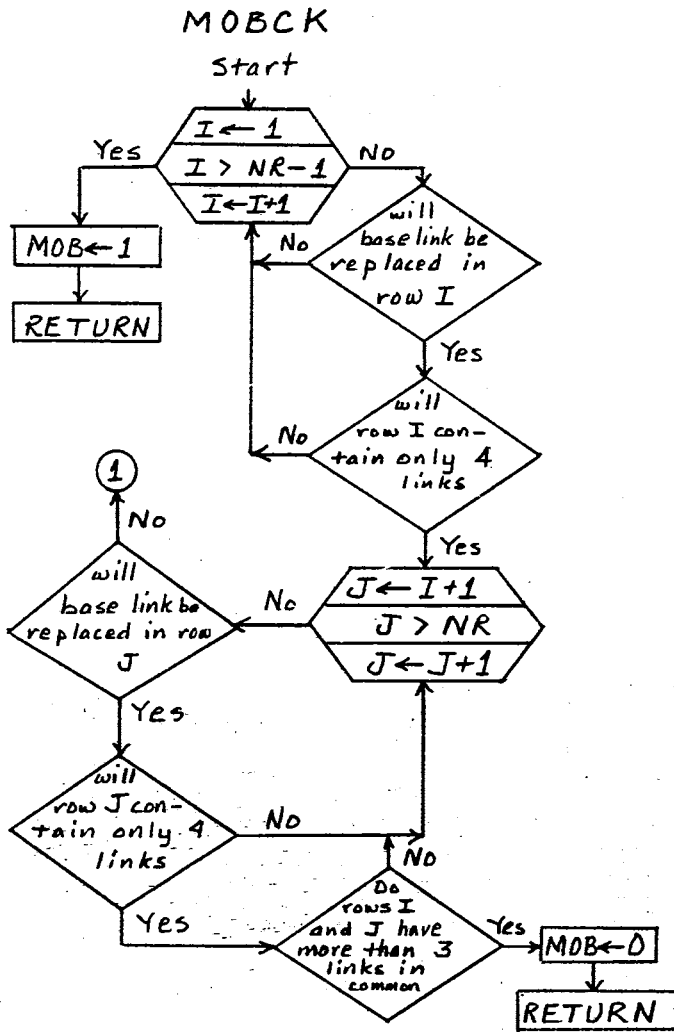


Figure 34. Flow Charts for Subroutines LNKCNT and MOBCK

VITA<sub>2</sub>

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