AN APPLICATION OF COMPUTER-AIDED OPTIMIZATION IN THE DESIGN OF A RING-STIFFENED CYLINDRICAL SHELL

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PREFACE

This thesis is concerned with the development of a computer-aided optimization technique as a tool in the design of a ring-stiffened cylindrical shell under hydrostatic pressure based on least weight. Actually in stating the problem, it is desired to design a shell of least cost, but due to a past study it is felt that by using sound design criteria, the shell of least weight is also the shell of least cost.

The optimization phase consists of the development of different figures of merit and their evaluation. The actual optimization is done on the digital computer using a gradient search algorithm and FORTRAN IV programming. The figures of merit are evaluated to see how effective they are as mathematical models of the desired design conditions. Finally, they are compared and the best one is selected according to its effectiveness and ability to satisfy all the design conditions or any compromises that may develop.

I would like to take this opportunity to express my appreciation for the assistance given me by the following members of my committee: Dr. J. E. Bose, who headed my committee and introduced me to computer-aided design; Dr. H. R. Sebesta, for his interest and time in the development of this study; and Dr. J. R. Partin, for his suggestions and assistance in the preparation for this study.

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NOMENCLATURE

A_1 - Cross-sectional area of stiffener, in^2.

A_2 - Cross-sectional area of stiffener and shell of stiffner spacing width, in^2.

C_1 - Distance from neutral axis of stiffener and effective length of shell to outside surface of stiffener, in.

D_1 - Inside diameter of shell, in.

D_2 - Mean diameter of shell, in.

D_3 - Outside diameter of shell, in.

E - Young's modulus of elasticity: 30×10^6 lbs/in. for steel.

I_1 - Moment of inertia of effective length of shell and one stiffener, in^4.

I_2 - Moment of inertia of stiffener ring, in^4.

I_3 - Moment of inertia of unsupported span of shell, in^4.

L_1 - Stiffener spacing (center to center), in.

L_2 - Unsupported span of shell, in.

L_3 - Width of stiffener, in.

L_4 - Effective length of shell, in.

L_5 - Distance from the median surface of the shell to the neutral axis of stiffener cross section, in.

P - Design pressure, lbs/in^2.

P1 - Pressure that will cause shell failure due to yielding, psi.

P2 - Pressure that will cause shell failure due to elastic and plastic deformation, psi.

P3 - Pressure that will cause shell failure due to elementary two-lobed instability, psi.

P4 - Pressure that will cause shell failure due to general instability, psi.

R_1 - Inside radius of shell, in.

R_2 - Mean radius of shell, in.

R_3 - Shell radius + distance from inside of shell to the neutral axis of the stiffener and effective length of shell, in.

R_4 - Outside radius of shell, in.

S_b - Stress caused by bending moment due to imperfections in the shell, lbs/in^2.

S_c - Stress caused by compressive load due to external hydrostatic pressure, lbs/in^2.

S_y - Yield strength of material, lbs/in^2.

S1 - Stress in the shell at a location midway between stiffeners, psi.

S2 - Stress in the shell located at the stiffener, psi.

T_1 - Shell wall thickness, in.

T_2 - Thickness of stiffener, in.

v - Poisson's ratio: 0.3 for steel.

WO - Radial out-of-roundness, in.

CHAPTER I

INTRODUCTION

General Introduction

The need for a marine structure that is capable of withstanding great hydrostatic pressures is of ever-increasing importance. A new generation of offshore oil-drilling rigs includes submersible barges, semi-submersible platforms and drilling ships. The military submarine must operate at greater and greater depths because of the increased requirements of equipment and payload and because of the increased threat of enemy detection.

The cylindrical and spherical shells are popular geometric shapes used in subsurface structures. They are used as individual units or combined to form more elaborate structures. A unique use of the cylindrical shell under hydrostatic pressure is the vertical steel casing used in the Atomic Energy Commission's underground testing program in the state of Nevada. A hole 120" in diameter is drilled to depths greater than 5000 feet. A reinforced steel casing is used to line the hole.

In 1970, a casing design and cost study was made by the author using a design philosophy current at that time (1). The design called for the use of one grade of steel throughout the casing string (2). The grade was dictated by the strength requirements of the section of casing under the greatest pressure. Also, stiffener rings were used to reinforce the casing, and since a 2" x 6" rectangular ring was a popular size, the width and thickness of the rings were held constant during the initial part of the study.

The study showed that by varying five parameters, the wall thickness of the casing, the width and thickness of the rectangular stiffeners, the stiffener spacing and the grade of steel, the results were a significant savings in weight and cost. A 48" diameter casing for a 4560' hole was the subject of investigation. A weight savings of 4% (128,000 lbs.) and a cost savings of 6% ($37,000) was possible based on the assumptions made in the study.

Statement of Problem

The general problem considered may be stated as follows: "Given the overall length, inside diameter and material of construction, determine the least expensive cylindrical shell reinforced with rectangular ring stiffeners that can be used under hydrostatic pressure with a specified factor of safety and maximum depth." The problem of simply listing the necessary parameters may become overwhelming when considering such items as fabrication and shipping costs.

The purpose of this study was not to attempt to solve the general problem, but by using this problem as an example, to select a few design parameters and demonstrate a computer-aided optimization technique which could equally well be used in other fields of engineering. The "optimum" solution is a relative term, and criteria for selection of the optimum design must be stated for a given problem.

Literature Search

The previous work done in this field was divided into two parts. The first part was concerned with the development of the design equations for the reinforced cylindrical shell, and the second part dealt with optimization techniques.

It was found early in the research for this study that there are differing equations that have been developed for a reinforced cylindrical shell. Some of the equations are purely theoretical while some are combinations of theoretical and experimental work. Among the theoretical works reviewed is that of Timoshenko (3). This reference includes derivations for the failure of stiffened cylindrical shells under external pressure.

There is also literature on shell design based on experimental investigation of the theoretical equations. Among the early experimental work was that of Saunders and Windenburg, who recognized the limited scope of shell design equations used at that time (4). Windenburg, in 1937, published an article on the theoretical and empirical equations that were represented in the construction rules for unfired pressure vessels under external pressure (5).

A survey of the existing or commonly used shell design equations is presented in various references. Wenk outlines fundamental principles and mechanisms of submarine hull failure used during the ten years previous to 1961 (6). This reference also includes information on minimum weight design, factor of safety and structural toughness.

The most current and comprehensive survey of shell analysis and design equations was prepared under the direction of the Department of Naval Architecture and Marine Engineering at Massachusetts Institute of Technology (7). The material was printed in 1969 and covers various ocean engineering structures with about 25% of the printed matter devoted to shell analysis and design. Of particular interest for this study are topics on the design of stiffened and unstiffened cylindrical shells under hydrostatic pressure and different loading systems. The critical pressure and instability equations are given for the various modes of failure. The reference index includes more than 150 sources for information on pressure hull structures.

There is available some work done specifically on the design of ring-stiffened cylindrical shells used for casing deep holes. Russell and DeHart published an interim report to the United States Atomic Energy Commission in 1967 on the subject of deep hole casings (8). The reference includes a summary of the design method, loading conditions, modes of failure, factors of safety and materials. Computer programs used in the casing analysis are also listed.

In a cost evaluation study made in 1969 there were design equations for both plain and ring-stiffened casings (2). The equations were simplified and used by this author in an earlier study (1).

The second part of the literature search was concerned with optimization techniques. The word optimization as used in this study included the development of a valid figure of merit and its use with a suitable search algorithm to specify the values of selected variables such that the figure of merit is either maximized or minimized according to the algorithm used. The literature on figures of merit may be further broken down into natural and artifical figures of merit and will be discussed in a later chapter. Mischke has devoted a chapter to the discussion of natural figures of merit such as cost, reliability and time (9). The artifical figures of merit are given an interesting treatment by Henderson and White on the optimum design of spur gears (10).

There are many references dealing with the mathematical development of various searching algorithms. Of particular interest are the multidimensional searches. The gradient or steepest ascent method is used and discussed in Mischke (9) and Wilde (11). An application of optimization using the multidimensional search in the chemical engineering industry is given by Boas (12).

Scope of the Study

The scope of this study includes:

1. The preliminary research necessary to obtain the

governing equations for the design of a ring-stiffened cylindrical shell;

2. The basic theory necessary to develop figures of merit for specific design conditions; and,
3. The use of the figures of merit in a computer-aided technique to find the values of the variable parameters for the optimum design as specified by the design criteria.

CHAPTER II is devoted to shell theory and introduces the computer program used to calculate the critical pressures, stresses and weight for different designs. A list of observations was made based on how changes in the variable parameters affect dependent variables such as the shell weight. CHAPTER III deals with the optimization phase of the design and gives an introduction to the idea of figures of merit and their use. A mathematical derivation of the gradient search algorithm is given and some important terminology is discussed. The use of the FORTRAN IV computer subroutine for the gradient search algorithm using various figures of merit is demonstrated and the results are discussed. CHAPTER IV contains the summary, conclusions and recommendations for future studies.

It was found in this study that if the conditions imposed on a shell design are numerous or complicated that a compromise may result in the final design. The best figures of merit were based on the addition of individual factors, each of which described a design condition.

It was also found that for a shell design based on least weight, the collapse pressures should be kept as low as possible while the stress levels should be kept as high as possible.

CHAPTER II

SHELL THEORY

The design procedure is divided into three phases. The initial phase is the subject of this chapter and consists of:

1. developing the governing shell design equations;
2. calculating and tabulating the pressures, stresses and weight of different shell designs caused by varying selected parameters; and,
3. examining this tabulated data in order to list observations that reflect the behavior of the variables as the shell weight varies for the different designs.

The second and third phases deal with the optimization procedure and the specification of the final shell design and are covered in CHAPTER III.

The shell design equations used in this study are the result of the work of many contributors to the field of shell design. Some of the equations are purely theoretical while the rest are empirical relationships resulting from experimental testing. The pressures, stresses and weight of different shell designs are calculated using a digital computer. The computer application for this work is treated in a separate topic later in this chapter. The observations are important because they form the design criteria used in

CHAPTER III for the optimization phase. Since the factor of safety is an important consideration to the designer, a brief discussion defining the factor of safety used in this study is also included in this chapter.

Shell Design Equations

Critical Pressure Equations

Failure of a stiffened cylindrical shell by instability occurs in three regions according to the interaction between the shell and stiffener rings.

In Region 1, the ring stiffeners are assumed to be close enough that only the shell thickness-mean diameter ratio is important. Failure in Region 1 occurs as stresses approach the yield point of the material in the shell. The critical pressure in this region is given by the hoop stress formula as

$$P1 = 2\left(\frac{T_1}{D_3}\right)Sy. \tag{2-1}$$

Refer to Figure 1. This relationship is important when L_2/D_2 is approximately 0.2 or less (7).

Failure in Region 2 is valid for an L_2/D_2 ratio approximately between 0.2 and 10.0. In this region not only is the L_2/D_2 ratio important, but the T_1/D_2 ratio is significant. Failure in this region is due to eleastic and plastic deformation, and the critical pressure is given by

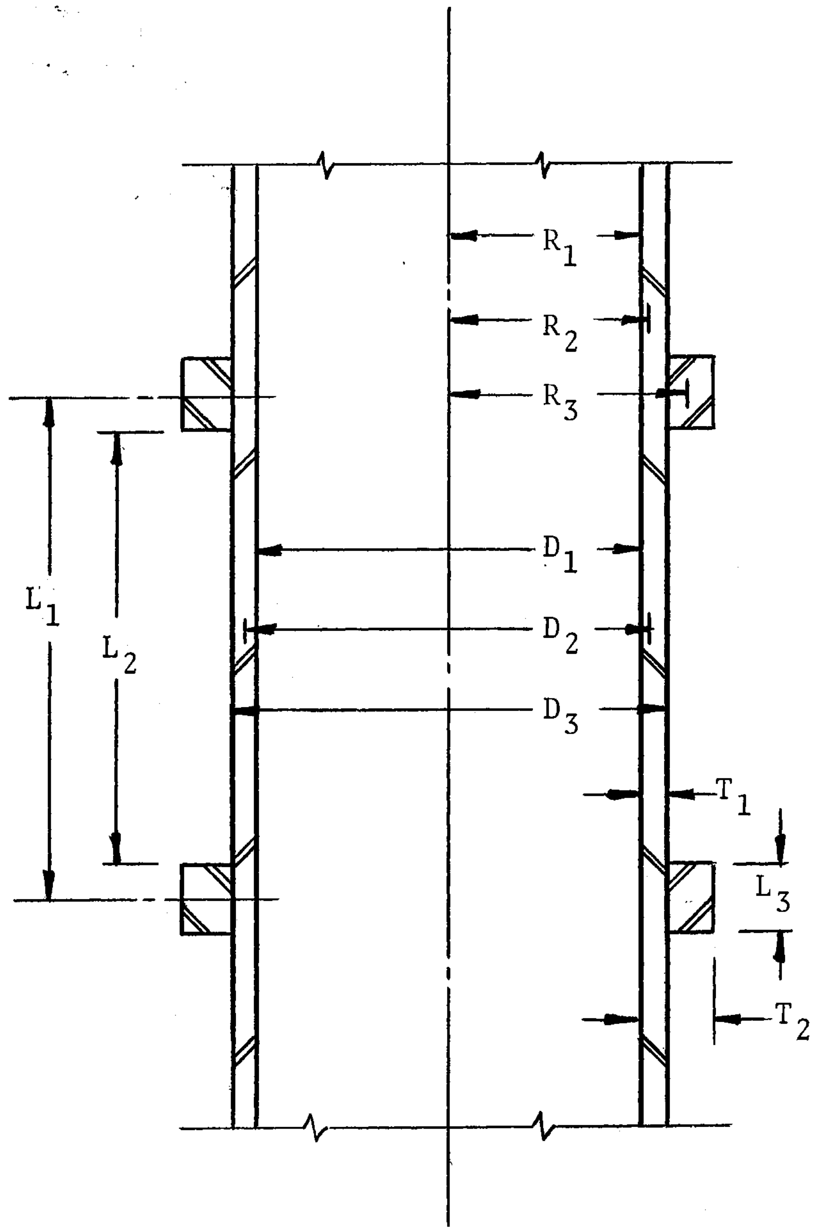

Figure 1. Shell Section

$$P2 = \frac{2.24\ E}{(1-v^2)^{3/4}} \frac{(T_1/D_2)^{5/2}}{L_2/D_2 - 0.45(T_1/D_2)^{1/2}} \quad (2\text{-}2)$$

or

$$= 2.60\ E \frac{(T_1/D_2)^{5/2}}{L_2/D_2 - 0.45(T_1/D_2)^{1/2}}$$

for steel. Equation (2-2) was developed by Windenburg and is in close agreement with previous equations for this relationship (7). P2 is taken to be the critical pressure required to collapse the shell at the mid-point between stiffeners.

Region 3 is characterized by the fact that the ring stiffeners are far enough apart that they have no influence in strengthening the shell. Failure in this region is elementary two-lobed instability collapse given as

$$P3 = \frac{2\ E}{(1-v^2)} (T_1/D_2)^3 \quad (2\text{-}3)$$

$$= 66 \times 10^6\ (T_1/D_2)^3$$

for steel (7). This expression is used when the L_2/D_2 ratio is greater than approximately 10.0.

The shell may also fail by general instability which is characterized by collapse of the shell and stiffener rings simultaneously. This overall elastic collapse pressure is given by

$$P4 = \frac{3\ E\ I_1}{(1-v^2)\ R_2\ R_3^2\ L_1} \quad (2\text{-}4)$$

where I_1 is the moment of inertia of an effective length of shell and one stiffener (8).

Wenk gives the expression for I_1 as

$$I_1 = \frac{A_1 (L_5 + T_1/2)^2}{1 + \frac{A_1}{L_1 \ T_1}} + I_2 + \frac{L_4 \ T_1^3}{12} \tag{2-5}$$

where I_2 is the moment of inertia of the stiffener rings (6).

For the rectangular stiffener ring the moment of inertia becomes

$$I_2 = \frac{L_3 \ T_2^3}{12} \tag{2-6}$$

Substituting this value of I_2 into Equation (2-5), the moment of inertia of the composite section becomes

$$I_1 = \frac{A_1 \ (L_5 + T_1/2)^2}{1 + \frac{A_1}{L_1 \ T_1}} + \frac{L_3 \ T_2^3 + L_4 \ T_1^3}{12} \tag{2-7}$$

The effective length of shell, L_4, is given by

$$L_4 = 1.56 \sqrt{R_2 \ T_1} \ X_3(B_2 L_1) \tag{2-8}$$

where

$$X_3(B_2 L_1) = \frac{\cosh(B_2 L_1) - \cos(B_2 L_1)}{\sinh(B_2 L_1) + \sin(B_2 L_1)}$$

and

$$B_2 = \frac{1.285}{\sqrt{R_2 \ T_1}}$$

and is taken from Pulos (13) and Russell and DeHart (8).

And if $L_1 >> 2\sqrt{R_2 T_1}$, then

$$L_4 = 1.57\sqrt{R_2 T_1} \tag{2-9}$$

as given by Wenk (6).

Stress Equations

The stress levels in the stiffened shell are caused from various types of loadings. The two considered here are external hydrostatic pressure and fabrication imperfections. The combined loading causes a maximum stress given by

$$S_{max} = S_c + S_b \tag{2-10}$$

where S_c is the stress caused by compression due to external hydrostatic pressure, and S_b is the stress caused by the bending moment due to initial out-of-roundness.

At the midpoint between stiffeners, the stress due to hydrostatic pressure is

$$S_c = \frac{P(R_2 + T_1/2)}{T_1} \tag{2-11}$$

and at the stiffener ring, the stress is

$$S_c + \frac{P(R_2 + T_1/2)L_1}{A_2}. \tag{2-12}$$

The value $(R_2 + T_1/2)$ is used because the pressure is applied on the outer surface. Equations (2-11) and (2-12) may be used assuming the shell to be perfectly circular. Any out-of-roundness would cause some bending moment circumferentially which would not exist otherwise.

The amplitude of the initial out-of-roundness may be taken as a fraction of the allowable difference between the major and minor axis such as

$$WO = 1/4\ (D_{max} - D_{min}) \tag{2-13}$$

or simply as a known value such as 1/8".

The term for the stress for radial out-of-roundness is given by Evans and Adamchak as

$$S_b = P(R_2 + T_1/2)L_2\ WO\ \frac{P}{P_{cr} - P}\ \frac{C}{I} \tag{2-14}$$

where C/I depends upon the section in question (7). A non-linear effect is introduced by the factor $P/(P_{cr} - P)$. It can be seen that as the hydrostatic design pressure, P, approaches the shell collapse pressure, P_{cr}, the term goes to infinity showing its significance.

Therefore, the maximum stress at the midpoint between stiffeners is

$$S1 = \frac{P\ R_4}{T_1} + P\ R_4\ L_2\ WO\ \frac{P}{P_{cr} - P}\ \frac{T_1}{2}\ \frac{12}{L_2\ T_1^3} \tag{2-15}$$

$$= \frac{P\ R_4}{T_1} + \frac{6P\ R_4}{T_1^2}\ WO\ \frac{P}{P_{cr} - P}$$

and the maximum stress at the composite shell and stiffener section is

$$S2 = \frac{P\ R_4\ L_1}{A_2} + P\ R_4\ L_2\ WO\ \frac{P}{P_{cr} - P}\ \frac{C_1}{I_1} \tag{2-16}$$

where $R_4 = R_2 + T_1/2$

and $C_1 = R_1 + T_1 + T_2 - R_3$

and I_1 is given in Equation (2-7).

Factor of Safety

There are several reasons why a factor of safety was considered in the design of a reinforced cylindrical shell. Wenk lists these as unknowns or limitations in 1) theory, 2) variables in materials, 3) imperfect workmanship or inspection, 4) degradation from corrosion, and 5) other unknowns in service loading systems (6).

The specific factor of safety used in submarine design is also used in this study. The former is defined as

$$FS = \frac{\text{collapse depth}}{\text{specified operating depth}}. \tag{2-17}$$

The collapse depth is predicted from theory and confirmed by model testing. Since the early 1960's, factors of safety of 1.5 to 2.0 have been used. If the shells were designed using the traditional pressure vessel factor of safety of 4.0, the corresponding weight would possibly be unacceptable. On the other hand, such low factors of safety demonstrate the tremendous responsibility in precision structural analysis and assembly. In additional support of low factors of safety, there are numerous design, fabrication and test operations. Final shell inspection including x-ray inspection of welds, sampling of materials used in construction and physical measurements for fabrication imperfections are costly but often used.

The factor of safety in this study is used to define a design pressure as follows:

$$P = FS \times MD \tag{2-18}$$

where P is the design pressure, and MD is the maximum depth of operation. According to the value of the L_2/D_2 ratio, the critical pressure that will collapse the shell is calculated using either Equation (2-1), (2-2), or (2-3). The pressure at which the shell will fail by general instability is calculated using Equation (2-4). If the design pressure was larger than the smallest pressure that would cause shell failure, the design was rejected. The design pressure is a parameter in calculating the stresses in the shell using Equations (2-15) and (2-16). If the stresses in the shell were larger than the yield strength of the material, the design was again rejected.

Computer Application in Calculating Various Shell Designs

The purpose of this phase is not to conduct an exhaustive search which would mean varying each parameter systematically through very small increments and tabulating the results. By using this approach, the optimum design would be found by examining the tabulated data, but this would also be the least economical scheme.

The method used was to hold certain parameters constant and vary the others only by representative amounts and tabulate this data. For example, if the shell thickness had a range of from 0.5 to 1.5 inches, then instead of varying the thickness in 0.001 inch increments as in an exhaustive search, the shell thickness was varied in 0.5 inch steps. The purpose then is to examine the tabulated data and to notice trends which would be helpful in developing figures of merit for the optimization phase.

The question of which parameters to hold constant and which to vary is left to the experience of the designer. TABLE I shows the parameters as either being constant or variable as used in this study.

TABLE I

LIST OF CONSTANT AND VARIABLE PARAMETERS

Constant Parameters	Variable Parameters
Length of shell	Shell thickness
Inside diameter of shell	Stiffener thickness
External pressure	Stiffener width
Factor of safety	Stiffener spacing
Type of material (Young's modulus, Poisson's ratio and density)	Strength of material
	Radial out-of-roundness

The parameters listed in TABLE I are independent variables. The dependent variables as discussed in this study are the pressures, stresses and weight.

All valid shell designs were tabulated under the following design assumptions:

1. no bending stresses;
2. no axial (end) loading; and,
3. the material in the stiffener ring is the same as that of the shell to which it is attached.

The FORTRAN IV SUBROUTINE SHELL1 was written to make the necessary calculations for the critical pressures, the maximum stresses and the total weight of a reinforced cylindrical shell under consideration. A main program is used to read the input data and to call SUBROUTINE SHELL1.

Input Data

Three data cards contained the required input information. FORTRAN IV computer language is used in this section to define the variables. The first data card contained the shell length, XL6, and the inside diameter, D1. Both were in inches and were read with FORMAT(2F10.0). The second data card contained the maximum depth at which the shell would be used, MD, and the design factor of safety, FS. The depth was in feet and the factor of safety was a real number such as 1.25. This data was read with FORMAT(I10,F10.0). The third data card contained Young's modulus of elasticity, ME, and Poisson's ratio, V, for the material of construction. Young's modulus was in pounds per square inch and Poisson's ratio was dimensionless. The data on this card was read with FORMAT(I10,F5.0).

Output Data

The input data was listed at the top of the output. The remaining output was in tabulalar form. Different designs were listed as the values of the variable independent parameters were varied over their specificed ranges. The tabulated data thus consisted of designs with the following information:

P4 - collapse pressure caused by general instability, psi.,
P5 - smaller of P4 and P6,
P6 - collapse pressure midway between stiffeners, psi.,
SY - strength of material, psi.,
S1 - stress midway between stiffeners, psi.,
S2 - stress at stiffener, psi.,
S3 - larger of S1 and S2,
T1 - shell thickness, in.,
T2 - stiffener thickness, in.,
WGT - weight of shell, lbs.,
WO - radial out-of-roundness, in.,
XL1 - stiffener spacing, in., and
XL3 - stiffener width, in.

Additional subroutines used with SUBROUTINE SHELL1 are listed below with their purposes briefly given.

SUBROUTINE WEIGHT - To calculate the weight in pounds of a cylindrical shell reinforced with rectangular ring stiffeners.

SUBROUTINE PRESS - To calculate the critical pressures that will cause the shell to fail by either yielding, buckling or general instability. Equations (2-1), (2-2), (2-3) and (2-4) were used.

SUBROUTINE STRESS - To calculate the maximum stresses in the shell. Equations (2-15) and (2-16) were used.

SUBROUTINE INERT1 - To calculate the moment of inertia of the effective length of shell and stiffener, taken as a composite section of one material. Equation (2-7) was used.

SUBROUTINE RNAXIS - To calculate the radius to the neutral axis of the effective length of shell and stiffener, taken as a composite section of one material.

SUBROUTINE EFFLEN - To calculate the effective length of shell. Equation (2-8) was used.

The complete program arrangement is listed in APPENDIX A.

Observations from Tabulated Data

The purpose of this topic is to review the different shell designs resulting from the use of SUBROUTINE SHELL1 and to record important trends in the parameters that might be used in developing figures of merit. Twelve sets of shell designs were computed using different values for the input data. In each of the twelve designs, four parameters were kept common and assigned the following values:

Factor of Safety	1.5
Length of Shell	960.0 in
Young's Modulus	30.0×10^6 lbs/in^2
Poisson's Ratio	0.3.

The input data that differed in the twelve designs is listed in TABLE II.

TABLE II

INPUT DATA FOR TWELVE SHELL DESIGN SETS

Shell Design Number	Maximum Depth (ft.)	Inside Diameter of Shell (in.)	Strength of Material (psi.)
1	500	48	30,000
2	500	48	50,000
3	500	48	70,000
4	1040	48	30,000
5	1040	48	50,000
6	1040	48	70,000
7	500	96	30,000
8	500	96	50,000
9	500	96	70,000
10	1040	96	30,000
11	1040	96	50,000
12	1040	96	70,000

The following observations were made only from the computed data but may also be valid for a wider range of parameter values:

1. Neither the collapse pressure of the shell caused by general instability or the collapse pressure at the mid-bay region due to buckling or yielding was completely dominant over the other throughout the design.
2. As a general trend, when the values of the collapse pressures individually approached the design pressure, the shell weight tended to decrease.
3. As a general trend, when the values of the collapse

pressures approached each other, the shell weight tended to decrease. Or in other words, as the difference in the values of the collapse pressures approached zero, the shell weight decreased.

4. Neither the shell stress at the midpoint between stiffeners or the stress at the stiffener section was completely dominant over the other throughout the design, although the higher stresses occurred more often between stiffeners.
5. As a general trend, when the values of the stresses individually approached the strength of the material, the shell weight tended to decrease.
6. As a general trend, when the values of the stresses approached each other, the shell weight tended to decrease. Or as the difference in the values of the stresses approached zero, the shell weight tended to decrease. For two similar designs, in which the difference in the stresses was approximately zero for both cases, the design in which the stress levels are nearer the strength of the material tends to yield a lighter shell assuming the collapse pressures of both shells are the same. The stress level in the shell for a given design increased as the out-of-roundness increased.
7. It would be more desirable to use a steel right up to its yield point before changing to another grade with a higher strength.

In CHAPTER III, the preceeding observations are used as design criteria to form the basis for developing different figures of merit in an attempt to find a suitable figure of merit for the design of a shell of least weight.

CHAPTER III

OPTIMIZATION PROCEDURE

The second and third phases of the design procedure are covered in this chapter. The second phase consisted of:

1. The development of various relationships, called figures of merit, which mathematically described the design criteria taken from the list of observations in CHAPTER II;
2. A mathematical introduction to the gradient search algorithm; and,
3. A computer application of using the algorithm with the various figures of merit to specify the values of the variable parameters.

The third phase deals with the evaluation of each figure of merit to determine how efficient the relationship was as a mathematical model.

The first topic covers the theory behind the development of figures of merit for the different design criteria. Examples are given to illustrate the mathematics involved. The concepts of natural and artifical figures of merit, weighting, scaling, and sensitivity are discussed. The computer program using the gradient search algorithm with the different figures of merit is introduced in the final topic, and the results of several investigations are tabulated. Each time a figure of merit was used, the computer program would calculate the values for the variable parameters corresponding to the optimum design. These values were then used as input values to calculate the pressures, stresses and weight of different shell designs for comparison purposes. The third design phase consisted of this evaluation and comparison of the various figures of merit.

General Figures of Merit

Mischke (9) states that a "figure of merit is simply a number whose magnitude is an index to the merit or desirability of a solution to a problem." The figure of merit, denoted Y, is a mathematical expression of n variables and is written as

$$Y = Y(X_1, X_2, \ldots, X_n). \qquad (3\text{-}1)$$

The n variables are not necessarily independent. There may exist functional relationships between the variables called functional constraints. Some or all of the variables in Equation (3-1) may be valid or defined only in certain regions. This type of constraint is called a regional or geometric constraint. The following example does not deal with shell design, but it illustrates the ideas of a figure of merit, functional and geometric constraints.

Example 1

It is required to fence a rectangular piece of land with a river acting as one side. It is desired to enclose the maximum area with 1000 feet of fencing. What are the dimensions to satisfy these requirements?

The first step is to list the necessary equations and constraints.

$$A = LW \qquad \text{(functional constraint)} \tag{3-2}$$

$$L + 2W = 1000 \qquad \text{(geometric constraint)} \tag{3-3}$$

Mathematically, to maximize the area taking the constraint into consideration, a figure of merit using a Lagrange multiplier is developed as

$$Y = LW + \lambda(L + 2W - 1000), \qquad \text{(figure of merit)} \tag{3-4}$$

where Y is the representation used to express a figure of merit.

The three unknowns in this equation are L, the length, W, the width, and λ, the Lagrange multiplier. To maximize the expression, the partial derivative of Y with respect to each unknown is set equal to zero giving three equations with three unknowns.

$$\frac{\partial Y}{\partial L} = W + \lambda = 0 \tag{3-5}$$

$$\frac{\partial Y}{\partial W} = L + 2\lambda = 0 \tag{3-6}$$

$$\frac{\partial Y}{\partial \lambda} = L + 2W - 1000 = 0 \tag{3-7}$$

Solving Equation (3-5) for λ and substituting this value into Equation (3-6) gives

$$L = 2W. \tag{3-8}$$

When Equation (3-8) is substituted into Equation (3-7), the length and width are found to be

$$L = 500 \text{ feet}$$
$$W = 250 \text{ feet.} \tag{3-9}$$

Natural or Direct Figures of Merit

The natural or direct figures of merit are simply ones in which the expressions or parameters used to build the figure of merit give a direct indication as to what condition is actually being maximized or minimized. The following three examples illustrate the idea of a natural or direct figure of merit.

Example 2

Suppose the objective of an optimization program is to find the dimensions of the rectangular stiffener which will maximize the collapse pressure of the shell due to general instability. The mathematical expression to maximize will be given by Equation (2-4) and written here as

$$Y = P4 \tag{3-10}$$

$$= \frac{3\ E\ I}{(1-v^2)\ R_2\ R_3^2\ L_1} ,$$

where

E - constant

v - constant

$I = I(T_1, T_2, L_1, L_3, R_2)$

$R_2 = R_2(T_1, R_1)$

$R_3 = R_3(T_1, T_2, L_1, R_1, R_2)$

L_1 - variable.

Since only the values of the stiffener width, L_3, and the thickness, T_2, are desired, constant values must be assigned to E, v, T_1, L_1 and R_1.

Geometric constraints are imposed on the stiffener dimensions, T_2 and L_3, due to plate thickness availability and machine fabrication limitations.

The values of T_2 and L_3 are allowed to vary as shown.

$$1'' \leq T_2 \leq 6'' \qquad \text{(geometric constraint)} \qquad (3\text{-}11)$$

$$2'' \leq L_3 \leq 6'' \qquad \text{(geometric constraint)} \qquad (3\text{-}12)$$

Optimization is carried out by searching the range of values of the two variables in a fashion determined by the gradient search scheme, and as should be expected, the dimensions of the stiffener are 6" x 6". It should be noted that cost or weight are not involved in this example, and as far as shell cost is concerned, the above figure of merit will probably produce a shell that is too heavy and expensive.

Example 3

As another example, suppose that it is the objective of the optimization program to maximize the collapse pressures of the shell at the midpoint between stiffeners and at the stiffener. It is also desired that the collapse pressure at the stiffener be 10% higher than that in the midbay region. The design equations are listed as functions of their parameters.

Collapse pressure in midbay region - dependent upon L_2/D_2 ratio.

$$P1 = P1(T_1, D_3, Sy) \tag{3-13}$$

$$P2 = P2(E, v, T_1, D_2, L_2) \tag{3-14}$$

$$P3 = P3(E, v, T_1, D_2) \tag{3-15}$$

Collapse by general instability is given by

$$P4 = P4(E, v, I_1, R_2, R_3, L_1). \tag{3-16}$$

Not all the parameters are independent in each expression. For example, D_3 in Equation (3-13) is defined as

$$D_3 = D_1 + 2T_1 \tag{3-17}$$

where

$$D_1 = 2R_1.$$

By examination of each parameter in Equations (3-13) through (3-16), the parameters E, v, Sy, and R_1 could be held constant allowing T_1, T_2, L_1 and L_3 to vary.

A figure of merit that will satisfy the requirements is given as

$$Y = 1.10\ P4 + P6 \tag{3-18}$$

where P6 is the suitable expression for midbay failure according to the L_2/D_2 ratio used. Geometric constraints similar to those in Equations (3-11) and (3-12) are also required.

The figure of merit of Equation (3-18) is more complicated than that of Equation (3-11) in two respects. First of all, more calculations are required because there are more equations with more variables. Secondly, weighting factors have been introduced. The factor of 1.10 preceding the expression P4 is a mathematical approach to weigh P4 by 10% over the weight given the expression P5.

Example 4

In addition to the requirements of Example 3, it is desired that the weight of the shell be minimized. Therefore, an additional expression for W, the shell weight, would be developed, and the figure of merit will become

$$Y = \frac{1.10\ P4 + P6}{W}. \tag{3-19}$$

Since Y is to be maximized, W is placed in the denominator because a decrease in W would increase the value of Y. Therefore, the expressions or parameters to be maximized are placed in the numerator while the expressions or parameters to be minimized are placed in the denominator.

Artificial or Indirect Figures of Merit

In contrast to the natural or direct figures of merit are the artificial or indirect figures of merit. Examination of the organization of such a figure of merit gives at best only superficial insight into what is actually being maximized or minimized. The following example will illustrate this idea.

Example 5

From the observations made in CHAPTER II, it is found that as the difference in the values of the two collapse pressures approach zero, the shell weight tends to decrease. Therefore, a basis is formed for an artificial figure of merit.

A figure of merit based on this observation will include the factor P4 - P6 or P6 - P4. If the first factor is used with P4 equal to 100 and P6 equal to 50, then the value of the factor will be +50. If instead, the values of P4 and P6 are 50 and 100 respectively, then the value of the factor will be -50. Although the signs of the factor are different for the two cases, the magnitudes are the same. To solve this dilemma, the absolute value of the difference in the values of the two collapse pressures will show that both cases have the same merit. A possible figure of merit could be written as

$$Y = \frac{1}{|P4 - P5| + 1} , \quad (3\text{-}20)$$

and the search routine will return the values of the variable parameters for the case when the absolute value of the difference in collapse pressures is at a minimum. By adding 1 in the denominator, division by zero is prevented.

Equation (3-20) is an example of an artificial or indirect figure of merit because a minimum weight condition was indirectly specified by the use of pressure terms.

Several terms used in the study of figures of merit are weighting, scaling and sensitivity. Each will be given a brief discussion.

Weighting

The weighting of parameters in a figure of merit may simply be thought of as assigning a degree of priority or importance to each parameter. Included in a paper by White and Henderson are several examples of weighting (10). The following example concerning the design of spur gears was taken from this paper.

Example 6

Suppose the design criteria suggests that the bending stress of the gear be favored by 5% over the bending stress of the pinion and that the bending lives be as high as possible. The figure of merit used could be

$$Y = \frac{A + B}{(Sp - 1.05\ Sg) + 1} \tag{3-21}$$

where

A, B - bending lives of the pinion and gear, respectively,

Sp, Sg - bending stress, pinion and gear, respectively.

Another example of weighting was shown in Equation (3-18).

Scaling

Wilde states that it is desirable "to select scales of measurement in which a unit change in one factor <u>at the optimum</u> gives the same change in the dependent variable as a unit change in any other factor" (11). The efficiency of the search is directly related to the choice of scales.

<u>Example 7</u>

In the observations made in CHAPTER II, it was found that as the difference in the two stresses approached zero, the shell weight tended to decrease. A figure of merit that could be used is

$$Y = \frac{1}{|S1 - S2| + 1}. \tag{3-22}$$

In addition, suppose that it was desired to reduce the radial out-of-roundness, WO, Equation (3-22) could be changed to give

$$Y = \frac{1}{|S1 - S2| + 1} + \frac{1}{WO}. \qquad (3\text{-}23)$$

At the optimum design, the first factor would be very near unity while the value of the second factor would be near 16 if WO was allowed to vary between 1/16" and 1/4".

Therefore, a more suitable figure of merit to satisfy the requirements would be scaled to give

$$Y = \frac{1}{|S1 - S2| + 1} + \frac{1}{16\ WO}. \qquad (3\text{-}24)$$

Sensitivity

Sensitivities answer the question of how much of a change in one parameter is equal in value to a change in another parameter (9).

Example 8

Suppose that the area of a rectangular piece of land is 1000 square feet. The length and width are given as 100 feet and 10 feet respectively. How sensitive is the area to a change in width? This question is answered by taking the partial derivative of the area with respect to the width,

$$\frac{\partial A}{\partial W} = \frac{\Delta A}{\Delta W} = L. \qquad (3\text{-}25)$$

or

$$\Delta A = 100\ \Delta W. \qquad (3\text{-}26)$$

The value of the partial derivative in Equation (3-25) is known as the sensitivity and shows that a unit change in the width has a corresponding change in area of 100 square feet.

The question of how dependent is a particular figure of merit on a certain parameter is found in a similar fashion and is illustrated in the following example.

Example 9

Suppose that the figure of merit of Equation (3-22) is to be used with the additional desire that the sensitivity of the figure of merit to out-of-roundness be reduced. S1 and S2 are both dependent upon WO as shown in Equations (2-15) and (2-16). The sensitivity of Equation (3-22) to radial out-of-roundness is defined by partial derivatives as

$$\frac{\partial Y}{\partial WO} = S. \tag{3-31}$$

To take the partial derivative, it was necessary to note that

$$Y = Y(S_1, S_2) \tag{3-32}$$

where

$$S_1 = S_1(WO) \tag{3-33}$$

and

$$S_2 = S_2(WO) \tag{3-34}$$

Therefore,

$$\frac{\partial Y}{\partial WO} = \left(\frac{\partial Y}{\partial S_1}\right) \left(\frac{\partial S_1}{\partial WO}\right) + \left(\frac{\partial Y}{\partial S_2}\right) \left(\frac{\partial S_2}{\partial WO}\right) \qquad (3\text{-}35)$$

by the use of the chain rule. The expression for the sensitivity is dependent upon the magnitudes of S_1 and S_2 and using Equation (3-31) is given as

$$S = - \frac{1}{(|S_1 - S_2| + 1)^2} \frac{6 \, P \, R_4}{T_1^2} \frac{P}{P_{cr} - P}$$

$$+ \frac{1}{(|S_1 - S_2| + 1)^2} P \, R_4 \, L_2 \frac{P}{P_{cr} - P} \frac{C_1}{I_1}$$

or

$$S = K \left[\frac{L_2 \, C_1}{I_1} - \frac{6}{T_1^2} \right] \quad , \quad S_1 > S_2 \qquad (3\text{-}36)$$

and

$$S = - K \left[\frac{L_2 \, C_1}{I_1} - \frac{6}{T_1^2} \right] \quad , \quad S_1 < S_2 \qquad (3\text{-}37)$$

and

$$S = \text{undefined}, \quad S_1 = S_2 \qquad (3\text{-}38)$$

where

$$K = \frac{1}{(|S_1 - S_2| + 1)^2} \frac{P^2 \, R_4}{P_{cr} - P} \qquad (3\text{-}39)$$

Therefore, the figure of merit of Equation (3-22) could be modified in at least three logical ways as follows:

$$Y = \frac{1}{|S_1 - S_2| + 1} + \frac{1}{|S| + 1}, \tag{3-40}$$

or

$$Y = \frac{1}{|S| + |S_1 - S_2| + 1}, \tag{3-41}$$

or

$$Y = \frac{1}{|S||S_1 - S_2| + 1}. \tag{3-42}$$

These three figures of merit would have to be evaluated to find which one actually was more efficient in reducing the sensitivity of the figure of merit to out-of-roundness.

Remarks on Figures of Merit

The task of developing a figure of merit for a particular problem should be taken in logical steps as follows:

1. Become familiar with the problem by understanding the governing equations.
2. List the criteria expected in the final design.
3. List the regional constraints (limits) for the parameters and the functional relationships that may exist between parameters.
4. Begin to build a figure of merit by developing individual terms or factors for each condition listed

in the design criteria.

5. Proceed by building several figures of merit as a result of multiplying or adding different terms together.
6. Test each figure of merit by using it in an optimization program. During the testing phase, the computer output should include values for each of the conditions listed in the design criteria to be used in comparison with other figures of merit.
7. Select the figure of merit that will best satisfy the requirements of the problem.

Later in this chapter, these steps are followed while a figure of merit using the observations of CHAPTER II is developed.

Gradient Search Algorithm

The gradient searching method, often called the method of steepest ascent, is used in the optimization phase of this study. Mischke gives a complete mathematical derivation of the gradient algorithm, but only a brief development is given here (9).

Mathematical Development

A three dimensional space will be used for the development, but is easily expanded to n space. Suppose that a merit surface, defined by the function $f(x_1,x_2)$, lies above the x_1x_2 plane and that an arbitrary ordinate to the merit surface is evaluated at the point (a, b) as shown on Figure 2. This initial functional evaluation is called m_o. Two additional functional evaluations are made with the following definitions:

m_o = elevation at point p_o, $p_o(a, b)$.

m_1 = elevation at a point a small distance (Δx_1) from point p_o in the x_1-direction, $p_1(a + \Delta x_1, b)$.

m_2 = elevation at a point a small distance (Δx_2) from point p_o in the x_2-direction, $p_2(a, b + \Delta x_2)$.

These three points define a plane. Actually, this plane is an approximation to the plane tangent to the merit surface, and as $\Delta x_1 \to 0$ and $\Delta x_2 \to 0$, this plane becomes coincident with the tangent plane to the merit surface at point $p_o(a, b)$. The tangent plane at $p_o(a, b)$ is defined by the equation

$$p(x_1, x_2) = p_o + p_1 x_1 + p_2 x_2. \tag{3-43}$$

The constants p_o, p_1 and p_2 are calculated from the simultaneous solution of the above equation at the three known ordinates m_o, m_1 and m_2.

$$m_o = p_o + p_1 a + p_2 b \tag{3-44}$$

$$m_1 = p_o + p_1 (a + \Delta x_1) + p_2 b$$

$$m_2 = p_o + p_1 a + p_2 (b + \Delta x_2) \tag{3-46}$$

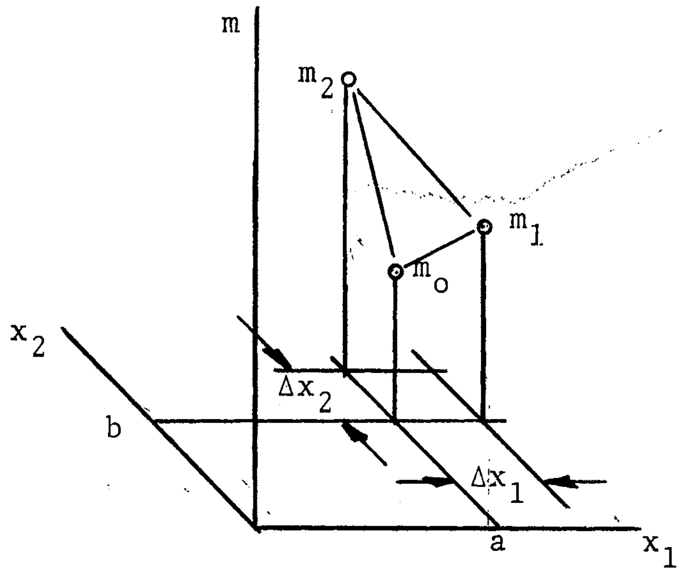

Figure 2. Three Function Evaluations in a Three-Dimensional Space

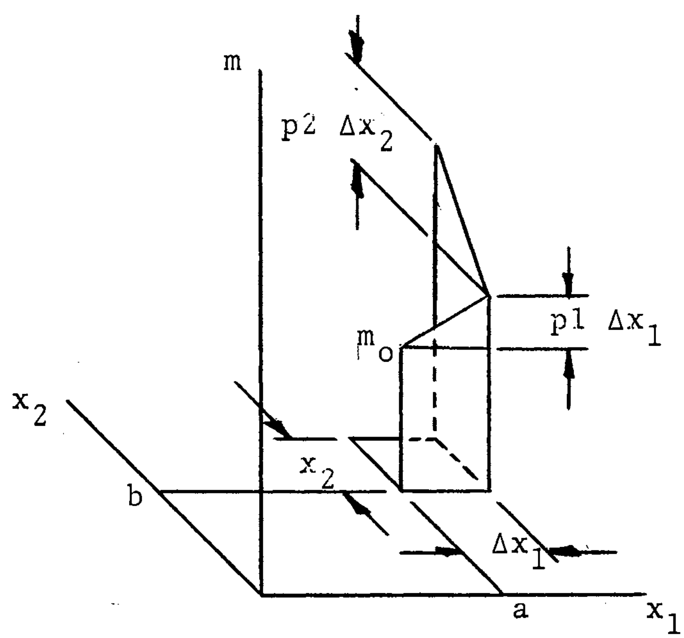

Figure 3. The Resulting Increase in Elevation Due to Positive Movements in the x_1 and x_2 Directions

Subtracting Equation (3-45) from Equation (3-44) yields $m_o - m_1 = -p_1 \Delta x_1$, or

$$p_1 = \frac{m_1 - m_o}{\Delta x_1} \ . \tag{3-47}$$

Subtracting Equation (3-46) from Equation (3-44) gives $m_o - m_2 = p_2 \Delta x_2$, or

$$p_2 = \frac{m_2 - m_o}{\Delta x_2} \ . \tag{3-48}$$

The slopes of the plane in the x_1- and x_2-directions are given by p_1 and p_2.

$$\frac{\partial p}{\partial x_1} = \text{slope in } x_1\text{-direction} = p_1 = \frac{m_1 - m_o}{\Delta x_1} \tag{3-49}$$

$$\frac{\partial p}{\partial x_2} = \text{slope in } x_2\text{-direction} = p_2 = \frac{m_2 - m_o}{\Delta x_2} \tag{3-50}$$

If p_1 and p_2 are positive, then the plane is uphill both in the x_1- and x_2-directions. The change Δp, due to a movement Δx_1 and Δx_2, is given by

$$\Delta p = \frac{\partial p}{\partial x_1} \Delta x_1 + \frac{\partial p}{\partial x_2} \Delta x_2 = p_1 \Delta x_1 + p_2 \Delta x_2 \ . \tag{3-51}$$

Refer to Figure 3. The slope of the diagonal line from point (a, b) to point $(a + \Delta x_1, b + \Delta x_2)$ is

$$S = \frac{\Delta p}{\sqrt{\Delta x_1^2 + \Delta x_2^2}} = \frac{p_1\ \Delta x_1 + p_2\ \Delta x_2}{\sqrt{\Delta x_1^2 + \Delta x_2^2}} \ . \tag{3-52}$$

The slope is greatest when

$$\frac{\partial S}{\partial x_1} = 0 \tag{3-53}$$

and

$$\frac{\partial S}{\partial x_2} = 0 \tag{3-54}$$

which leads to

$$\Delta x_2 = \frac{p_2\ \Delta x_1}{p_1} \ . \tag{3-55}$$

And when the correct choice for Δx_1 and Δx_2 is made, a new point is selected on the path of steepest ascent.

The algorithm is easily expanded to n space called a hyperspace. The plane tangent to the hypersurface is called a hyperplane. The mathematics is covered in the literature and will not be presented in this study.

Unimodal and Multimodal Functions

Mischke states that "a unimodal function has a single peak in a given interval, and each successive ordinate is progressively larger than the last until the peak is reached; then each successive ordinate is progressively less than the last" (9). A unimodal function can be thought of as having a single peak, whereas a multimodal function has more than one peak. The gradient search may converge to either peak depending upon where the search is started. A suspicion of multimodality should be checked by starting the search at several different points to see if it converges to the same value each time. Wilde suggests that in a multimodal situation, the peaks should be isolated and explored individually (11).

Computer Application of the Gradient Search Algorithm Using Different Figures of Merit

The purpose of this section is to examine different figures of merit using the gradient search and digital computer. The technique will be demonstrated using some of the simple figures of merit developed in this chapter. The ultimate purpose, however, is to develop the best figure of merit using the observations of CHAPTER II as design criteria for a shell of least weight.

The FORTRAN IV SUBROUTINE GRAD4 determines the extreme ordinate of a unimodal hypersurface of up to eight independent variables. The subroutine will terminate search after the number of evaluations of the figure of merit is equal to 100 times the number of independent variables. The subroutine calls a user-supplied SUBROUTINE MERIT4 from which an ordinate Y is returned when the column vector of abscissa X is tendered. Mischke documents and lists these subroutines for benefit of the user (9).

The arguments used in SUBROUTINE GRAD4 are defined by the use of the following calling statement:

CALL GRAD4(I1, I2, I3, A4, A5, A6, A7, A8, A9, B1, B2, J3, J4, B5, B6),

where I1 - number of independent variables in search (8 or less).
- I2 - 0, convergence monitor will not print.
- I2 - 1, convergence monitor print every 1st survey step.
- I2 - 2, convergence monitor print every 2nd survey step.

.
.
.

- I3 - 1, commence search centrally in domain of uncertainty.
- I3 - 2, commence search in "lower corner" of domain of uncertainty.
- I3 - 3, commence search in "upper corner" of domain of uncertainty.
- I3 - 4, commence search at location specified in column vector B2.
- A4 - initial exploration step size.
- A5 - step size growth multiplier (a little more than unity).
- A6 - fractional reduction in domain of uncertainty desired.
- A7 - survey pattern increment.
- A8 - lower bound of search domain, column vector.
- A9 - upper bound of search domain, column vector.
- B1 - extreme ordinate found during search.
- B2 - column vector of abscissas corresponding to extreme ordinate.
- J3 - 1, largest ordinate is an extreme.
- J3 - 2, largest ordinate is a maximum.
- J3 - 3, largest ordinate is in a plateau.
- J3 - 4, error returned, search truncated.
- J4 - number of function evaluations expended during search.
- B5 - column vector of forward slopes of hyperspace at extreme.
- B6 - column vector of backward slopes of hyperspace at extreme.

SUBROUTINE MERIT4 contains the figure of merit and necessary calculations for the pressures, stresses and weight. APPENDIX B gives the program arrangement using GRAD4 and APPENDIX C lists a sample SUBROUTINE MERIT4.

Different figures of merit will be evaluated and compared in the next topic of this chapter. The range for each variable parameter is given below.

Shell wall thickness, in.	$0.5 \leq T1 \leq 2.0$
Stiffener thickness, in.	$1.0 \leq T2 \leq 6.0$
Stiffener width, in.	$2.0 \leq L3 \leq 12.0$
Stiffener spacing, in.	$24.0 \leq L1 \leq 216.0$
Out-of-roundness, in.	$0.0625 \leq WO \leq 0.250$
Strength of material,psi.	$36,000 \leq Sy \leq 80,000$

The following parameters were held constant throughout the optimization phase:

Maximum operating depth	1040 ft.
Factor of safety	1.50
Length of shell	960 in.
Inside diameter of shell	96 in.
Young's modulus of elasticity	30.0×10^6 psi.
Poisson's ratio	0.3

As a means of evaluation, SUBROUTINE SHELL3 was written. Input to the subroutine includes all values of the constant parameters and the discrete values of the variable parameters obtained from SUBROUTINE GRAD4. The output of this subroutine is a complete listing of the input data plus the values for the collapse pressures, stresses and the shell weight. This information was used to check the effectiveness of the figure of merit to see if it had satisfied the desired requirements. APPENDIX D contains the program arrangement using SUBROUTINE SHELL3.

Development of Figure of Merit for Least Weight Shell

The list of observations in CHAPTER II was used as a set of design criteria. The shell design was sought that would best meet the design criteria and give least weight.

Equation (3-20) was used as the first figure of merit to start the optimization procedure and was rewritten here.

$$Y = \frac{1}{|P4 - P6| + 1} \tag{3-56}$$

The final results are listed in TABLE III.

TABLE III

RESULTS OF USING EQUATION (3-56) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
1.3704	3.4463	120.0000	6.9889	0.2500	58,000

Dependent Variables				
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)
1536	1531	46,392	30,179	129,464

From the data in TABLE III, it is shown that P4 was very close to P6 although the values of the stresses were not very close to each other at that point. It is also shown that neither of the values of the collapse pressures was near the design pressure.

The next step was to develop and test two additional figures of merit. The first was an expression to force P4 to approach the design pressure and to minimize this difference and was written as

$$Y = \frac{1}{P4 - P + 1} . \tag{3-57}$$

If the design pressure ever became larger than the collapse pressure of the shell, a negative value of the figure of merit would be returned. TABLE IV lists the results of using Equation (3-57).

TABLE IV

RESULTS OF USING EQUATION (3-57) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
0.6880	3.0427	120.0000	6.9137	0.2500	58,000
Dependent Variables					
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)	
676	272	-126,772	7276	70,066	

P4 was very close to the design pressure as desired. The negative value of the stress, S1, was due to the design pressure being larger than the smallest collapse pressure of the shell, P6, in this case. Failure would occur by buckling or yielding.

While the expression appeared to be useless, it was saved for later use in the final figure of merit.

The second figure of merit developed was an expression that forced P6 to approach the design pressure and also required the difference to be minimized. It was given as

$$Y = \frac{1}{P6 - P + 1} . \tag{3-58}$$

The results of using this figure of merit are shown as TABLE V.

TABLE V

RESULTS OF USING EQUATION (3-58) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
0.9942	3.5000	120.0000	6.9988	0.2500	58,000

Dependent Variables				
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)
1214	685	3,524,245	912,437	97,973

Reviewing this data, it is seen that while the desired collapse pressure was very close to the design pressure, P4 was too large. More important was the fact that the stress levels were much too high and would cause the shell to fail by yielding.

While still dealing with pressures only, the next step was to combine the Equations (3-56), (3-57), and (3-58), either by addition or multiplication or a combination of both.

The first trial consisted of simply adding two terms with a figure of merit given as

$$Y = \frac{1}{P4 - P + 1} + \frac{1}{P6 - P + 1} . \qquad (3\text{-}59)$$

The results are shown in TABLE VI.

TABLE VI

RESULTS OF USING EQUATION (3-59) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
0.9951	3.4818	120.0000	6.9952	0.1250	58,000

Dependent Variables				
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)
1200	687	1,536,261	412,293	97,958

By reviewing TABLE VI, it is shown that the second term had a dominant effect. The stresses were also much too high.

The next step was to multiply the two terms of Equation (3-59) instead of adding. The figure of merit for this requirement was

$$Y = \frac{1}{(P4 - P + 1)\ (P6 - P + 1)}. \tag{3-60}$$

The results are listed in TABLE VII.

TABLE VII

RESULTS OF USING EQUATION (3-60) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
0.9896	3.4649	120.000	6.9919	0.1563	58,000
Dependent Variables					
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)	
1182	677	very large	3,163,603	97,410	

This figure of merit appeared to be no better than the figure of merit of Equation (3-59).

Two other figures of merit evaluated were the result of adding Equation (3-57) and Equation (3-58) to Equation (3-56) individually and were given as

$$Y = \frac{1}{|P4 - P6| + 1} + \frac{1}{P4 - P + 1} \qquad (3\text{-}61)$$

and

$$Y = \frac{1}{|P4 - P6| + 1} + \frac{1}{P6 - P + 1} \; . \qquad (3\text{-}62)$$

The results are given in TABLES VIII and IX.

TABLE VIII

RESULTS OF USING EQUATION (3-61) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
1.3688	3.4432	120.0000	6.9883	0.2500	58,000

Dependent Variables				
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)
1532	1532	45,579	30,267	129,313

Both figures of merit appeared to have the same characteristics with the first term of each being dominant.

The possibilities were nearly exhausted except for adding Equations (3-56), (3-57) and (3-58). This figure of merit was written as

$$Y = \frac{1}{|P4 - P6| + 1} + \frac{1}{P4 - P + 1} + \frac{1}{P6 - P + 1} \qquad (3\text{-}63)$$

with the results being listed in TABLE X.

TABLE IX

RESULTS OF USING EQUATION (3-62) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
1.3673	3.4408	120.0000	6.9878	0.2500	58.000

Dependent Variables				
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)
1528	1522	45,756	30,348	129,173

TABLE X

RESULTS OF USING EQUATION (3-63) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
1.3632	3.4344	120.0000	6.9862	0.2500	58,000

Dependent Variables				
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)
1518	1511	46,249	30,571	128,793

Thus far in the evaluation program, the only valid figures of merit were those described by Equations (3-56), (3-61), (3-62) and (3-63). Throughout the program, Equation (3-56) appeared to be the most dominant factor because it equalized the two collapse pressures and kept the stresses within acceptable levels. The shell weights for all four figures of merit were nearly the same.

Perhaps at this point, by weighting the factors and trying different combinations, a more desirable figure of merit could have been obtained. This was not tried because when factors satisfying the observations on stress levels were combined with the pressure terms, there may be no need for weighting.

Figures of merit dealing with the observations on stresses were evaluated. The first one forced both stresses to an equal value and minimized this value. It was written as

$$Y = \frac{1}{|S1 - S2| + 1} \tag{3-64}$$

and the results are given in TABLE XI.

This figure of merit appeared to do a very good job of equalizing the stresses although the weight was higher than when using other figures of merit.

To force the stresses in the shell to approach the strength of the material Equations (3-65) and (3-66) were written and evaluated.

$$Y = \frac{1}{SY - S1 + 1} \qquad (3\text{-}65)$$

The results are listed in TABLE XII.

TABLE XI

RESULTS OF USING EQUATION (3-64) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
1.9944	2.5848	120.0000	6.5060	0.1920	58,000

Dependent Variables				
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)
1284	3896	27,809	27,806	177,899

TABLE XII

RESULTS OF USING EQUATION (3-65) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
1.2384	3.5000	120.0000	7.0000	0.1798	58,000

Dependent Variables				
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)
1451	1188	57,729	34,059	118,564

This figure of merit was effective in forcing S1 to the value of the material strength.

$$Y = \frac{1}{SY - S2 + 1} \, . \tag{3-66}$$

The results are listed in TABLE XIII.

TABLE XIII

RESULTS OF USING EQUATION (3-66) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
1.1247	3.4906	120.0000	6.9956	0.2500	58,000
Dependent Variables					
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)	
1330	933	132,686	56,837	108,912	

Equation (3-66), used as a figure of merit, forced S2 to approach the material strength but S1 was much too high. Therefore, a combination of Equations (3-65) and (3-66) was defined by the following figure of merit:

$$Y = \frac{1}{SY - S1 + 1} + \frac{1}{SY - S2 + 1} \, . \tag{3-67}$$

The results are listed in TABLE XIV.

TABLE XIV

RESULTS OF USING EQUATION (3-67) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
1.2383	3.5000	120.0000	6.9999	0.1798	58,000

Dependent Variables				
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)
1451	1188	57,745	34,065	118,555

Comparing the results of TABLE XIV with those of TABLE XII, it was shown that the first term of Equation (3-67) was dominant.

Another figure of merit was developed by multiplying the terms of Equations (3-65) and (3-66) and was written as

$$Y = \frac{1}{SY - S1 + 1} \; \frac{1}{SY - S2 + 1}. \tag{3-68}$$

The results of this figure of merit are listed in TABLE XV.

This figure of merit was unacceptable as it was written because the stresses were much too high.

Finally, a figure of merit, adding the terms in Equations (3-64), (3-65) and (3-66), was developed and given as

$$Y = \frac{1}{|S1 - S2| + 1} + \frac{1}{SY - S1 + 1} + \frac{1}{SY - S2 + 1}. \tag{3-69}$$

The results of this figure of merit are listed in TABLE XVI.

TABLE XV

RESULTS OF USING EQUATION (3-68) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
1.2384	3.4999	120.0000	6.9999	1.7987	58,000

Dependent Variables				
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)
1451	1188	335,572	133,066	118,563

TABLE XVI

RESULTS OF USING EQUATION (3-69) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
1.2386	3.5000	120.0000	6.9999	0.1800	58,000

Dependent Variables				
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)
1451	1188	57,707	34,053	118,580

It appeared that with some weighting and scaling that this figure of merit could be usable. The work needed would include raising the value of S2 and lowering the values of P4 and P6.

Up to this point there was no combination of pressure and stress terms to satisfy the design criteria. Therefore, a figure of merit composed of adding Equations (3-64) and (3-56) was written as

$$Y = \frac{1}{|P4 - P6| + 1} + \frac{1}{|S1 - S2| + 1} \qquad (3\text{-}70)$$

with the results listed in TABLE XVII.

TABLE XVII

RESULTS OF USING EQUATION (3-70) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
1.3705	3.4465	120.0000	6.9889	0.1557	58,000

Dependent Variables				
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)
1537	1531	37,446	26,801	129,473

This figure of merit keeps the pressures and stresses within tolerance, but the weight was some higher than in previous trials.

To show the reason why the weight of the shell was not used as a natural figure of merit, the following scaled figure of merit was defined and evaluated:

$$Y = \frac{100,000}{WGT} . \tag{3-71}$$

The results are listed in TABLE XVIII.

TABLE XVIII

RESULTS OF USING EQUATION (3-71) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
0.5000	1.000	120.8400	2.0000	0.1759	58,000

Dependent Variables				
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)
16	116	-76,096	-255,591	42,312

Equation (3-71) was found to be undesirable as a figure of merit without some check on the pressures and stresses.

A composite figure of merit was then developed using the desirable traits of Equations (3-56) through (3-71) and was written initially as

$$Y = \frac{1}{|P4 - P6| + 1} + \frac{1}{P4 - P + 1} + \frac{1}{P6 - P + 1}$$

$$+ \frac{1}{|S1 - S2| + 1} + \frac{1}{SY - S1 + 1}$$

$$+ \frac{1}{SY - S2 + 1} + \frac{100,000}{WGT} \tag{3-72}$$

with the results being listed in TABLE XIX.

TABLE XIX

RESULTS OF USING EQUATION (3-72) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
0.5000	3.3112	120.0000	6.9403	0.1610	58,000

Dependent Variables				
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)
675	122	-89,107	33,097	53,586

It is seen that the figure of merit of Equation (3-72) would not only cause shell failure due to general instability, but it would cause shell yielding in the midbay region between stiffeners. The term involving weight was the probable cause of the low pressures and negative stresses.

The figure of merit that formed the basis for the rest of the evaluation program was written as

$$Y = \frac{1}{|P4 - P6| + 1} + \frac{1}{P4 - P - 1} + \frac{1}{P6 - P + 1} + \frac{1}{|S1 - S2| + 1} + \frac{1}{SY - S1 + 1} + \frac{1}{SY - S2 + 1}. \qquad (3\text{-}73)$$

The results of using this figure of merit, listed in TABLE XX, show that it had good qualities in equalizing the collapse pressure and a fair ability in equalizing the stresses. Also, the stresses and pressures were kept within the acceptable range.

TABLE XX

RESULTS OF USING EQUATION (3-73) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
1.3730	3.4468	120.0000	6.9883	0.1630	58,000
Dependent Variables					
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)	
1539	1538	37,853	26,975	129,685	

A total of forty figures of merit were defined and evaluated. Those using Equation (3-73) as a basis differed mainly by the use of weighting factors in each of the terms. Equation (3-73) was scaled as it was defined without the use of scaling factors because near the optimum design each term was expected to be very close to unity.

In the evaluation of the figures of merit there was a design trade-off formed. Stated briefly, there was no figure of merit found that would meet all the requirements of the design criteria. The pressures could be equalized, but when there was an attempt to equalize the stresses, the difference in the values of the two pressures would grow. Also, when the stresses were forced to approach the strength of the material, the difference in the pressure values would grow. So the trade-off consisted basically of how much the designer was willing to compromise certain points in the design. The following two figures of merit illustrated the situation:

$$Y = \frac{1}{|P4 - P6| + 1} + \frac{5}{P4 - P + 1} + \frac{5}{P6 - P + 1}$$

$$+ \frac{7}{|S1 - S2| + 1} + \frac{1}{SY - S1 + 1} + \frac{SY}{SY - S2 + 1}. \qquad (3\text{-}74)$$

The results are listed in TABLE XXI.

The second figure of merit was written to show the effect of forcing S1 to the value of the strength of the material and was given as

$$Y = \frac{1}{|P4 - P6| + 1} + \frac{5}{P4 - P + 1} + \frac{5}{P6 - P + 1}$$

$$+ \frac{7}{|S1 - S2| + 1} + \frac{10}{SY - S1 + 1} + \frac{5}{SY - S2 + 1} \quad . \qquad (3\text{-}75)$$

The results of using this figure of merit are listed in TABLE XXII.

TABLE XXI

RESULTS OF USING EQUATION (3-74) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
1.3640	3.4282	120.0000	6.9842	0.1646	58,000

Dependent Variables				
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)
1513	1513	38,735	27,409	128,828

It is seen that higher stress levels were a definite factor in decreasing the weight of the shell. There was not found a figure of merit that would equalize the stresses near the material strength and at the same time, give collapse pressures that were acceptable.

Using a figure of merit defined by Equation (3-74) would yield a design that had some good structual toughness qualities. It would collapse in both modes at the same time which is a desired criteria in submarine hull design. It also had fairly equal resistance to yielding or buckling, but it was the heavier design. Using Equation (3-75) as a figure of merit yielded a shell design with collapse pressures being almost equal, but with the stress capacity much higher at the midbay between the stiffener rings than at the rings themselves. But this design was nearly 12,000 pounds lighter. Thus, the optimum design would be based upon a re-evaluation of the design criteria with the assignment of priorities.

TABLE XXII

RESULTS OF USING EQUATION (3-75) AS A FIGURE OF MERIT

Optimized Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
1.2212	3.4926	120.0000	6.9977	0.1604	58,000

Dependent Variables				
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)
1427	1147	57,994	34,105	117,071

The optimized parameters of TABLES XXI and XXII were changed slightly to be more representative of the real world. These results are listed in TABLES XXIII and XXIV.

TABLE XXIII

RESULTS OF USING EQUATION (3-74) AS A FIGURE OF MERIT WITH MODIFIED DATA

Modified Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
1.375	3.500	120.0000	7.000	0.1875	60,000

Dependent Variables				
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)
1592	1544	39,710	27,572	130,127

TABLE XXIV

RESULTS OF USING EQUATION (3-75) AS A FIGURE OF MERIT WITH MODIFIED DATA

Modified Parameters					
T_1(in.)	T_2(in.)	L_1(in.)	L_3(in.)	WO(in.)	Sy(psi.)
1.250	3.500	120.0000	7.000	0.1875	60,000

Dependent Variables				
P4(psi.)	P6(psi.)	S1(psi.)	S2(psi.)	WGT(lbs.)
1463	1216	56,573	33,708	119,545

CHAPTER IV

SUMMARY AND CONCLUSIONS

Summary

The purpose of this study was to select a set of cylindrical shell design equations and to develop a computer-aided optimization procedure in which the value of the variable parameters could be specified relative to a set of design conditions as set forth by the designer at the outset of the study. The design conditions consisted of a figure of merit with necessary geometric and functional constraints placed on the variables.

In this study, it was shown how to develop different figures of merit for various design specifications. Numerous examples were given to illustrate the mathematical treatment. Many figures of merit were developed and evaluated for the design of a ring-stiffened cylindrical shell of least weight. The results of several designs using different figures of merit were tabulated to show the variation in parameters.

Using a set of specified design criteria, it was found that an engineering trade-off existed because each condition in the design criteria could not be met without unfavorably affecting the others. Therefore, it was found that if an optimum design was possible, there would have to be a re-evaluation of the design criteria with the assignment of priorities to specific conditions.

Conclusions

There were several important conclusions made as a result of this study. Basically, it was found that the use of the computer-aided optimization technique was a valuable tool in the field of shell design. The use of figures of merit to mathematically describe the design criteria may be effective in satisfying the requirements of the design, but if the design conditions are numerous or complicated, the use of a figure of merit may lead to a trade-off. From the examination of the figures of merit used in this study, it was concluded that a better figure of merit would result if it was built on a summation of individual terms, each of which described a specific design condition.

From observations made in this study, the following conditions should be used as general guidelines in future shell design based on least weight:

1. The collapse pressures of the shell must always be higher than the design pressure. The weight tends to decrease as the values of the collapse pressures become equal and approach the design pressure.
2. The shell stresses must always be less than the strength of the material used for construction. The weight tends to decrease as the values of the

stresses become equal and approach the value for the material strength.

3. It was more desirable to use a material right up to its yield point before changing to another grade with a higher strength.

Recommendations for Future Studies

Further research on this subject should include a more comprehensive study of figures of merit. Such a study should give special attention to the fact that different shell sizes and materials could be used, and the best figure of merit would not be sensitive to such changes.

In this study, a stiffener ring was assumed to be of the same material to which it was attached. An investigation into the use of a different material for the rings may result in a further reduction in shell weight.

Finally, a stiffener with a geometry different than the rectangular cross-section used in this study should be investigated.

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APPENDIX A

PROGRAM ARRANGEMENT FOR CALCULATING THE CRITICAL PRESSURES, STRESSES AND WEIGHTS FOR VARIOUS SHELL DESIGNS

For this phase of the study, the parameters held constant and those that were allowed to vary are listed in TABLE I. Listed below are the variable parameters, the ranges in which they were allowed to vary and the increments by which they were varied.

Variable Parameter	Range	Increment
Shell thickness, in.	0.5 - 1.5	0.5
Stiffener thickness, in.	1.0 - 4.0	1.0
Stiffener spacing, in.	24.0 - 216.0	48.0
Stiffener width, in.	3.0 - 6.0	1.0
Out-of-roundness, in.	0.000 - 0.250	0.125
Strength of material, psi.	30,000 - 70,000	20,000

The output included the values of the critical pressures, maximum stresses and weight for various shell designs. Refer to Figure 4.

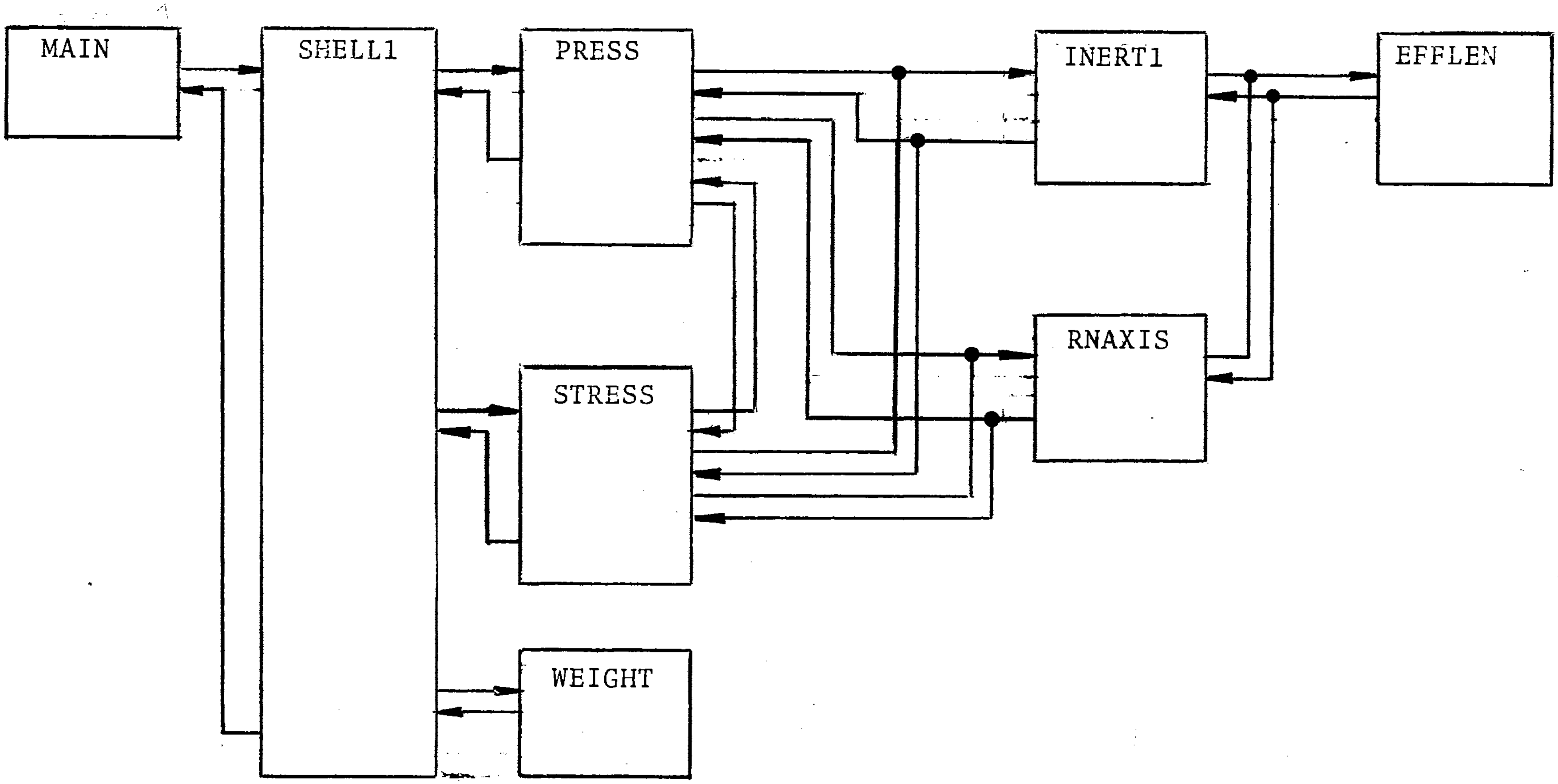

Figure 4. Program Arrangement for Calculating the Critical Pressures, Stresses and Weights for Various Shell Designs

APPENDIX B

PROGRAM ARRANGEMENT FOR DETERMINING THE OPTIMUM VALUES OF THE VARIABLE PARAMETERS

In the optimization phase, the variable parameters were allowed to vary in increments of approximately 0.01. Listed below are the variable parameters and the ranges in which they were allowed to vary.

Variable Parameter	Range
Shell thickness, in.	0.5 - 2.0
Stiffener thickness, in.	1.0 - 6.0
Stiffener spacing, in.	24 - 216
Stiffener width, in.	2.0 - 12.0
Out-of-roundness, in.	0.0625 - 0.2500
Strength of material, in.	36,000 - 80,000

The output included the values of the parameters (optimized parameters) that would maximize the figure of merit being used. Refer to Figure 5.

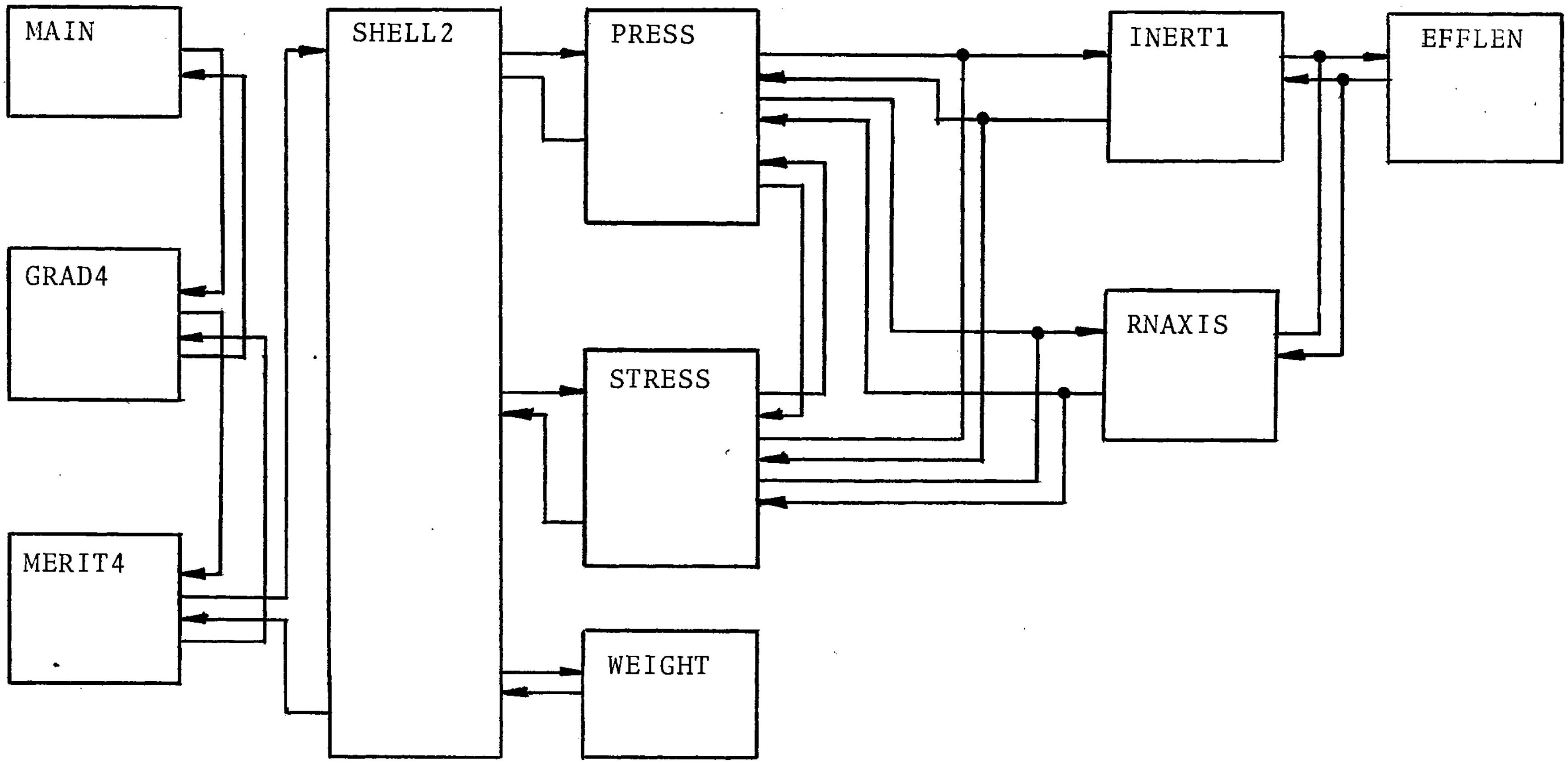

Figure 5. Program Arrangement for Determining the Optimum Values of the Variable Parameters

APPENDIX C

TABLE XXV

COMPUTER LISTING OF A SAMPLE SUBROUTINE MERIT4*

```
      SUBROUTINE MERIT4(X,Y)
      DIMENSION X(9)
C
      XL3 = X(1)
       T2 = X(2)
       WO = X(3)
       T1 = X(4)
      XL1 = X(5)
       SY = X(6)
C
      CALL SHELL2(T1,T2,XL1,XL3,WO,SY,B**,P4,P5,P6,S1,S2,S3,WGT)
C
C     USER-SUPPLIED FIGURE OF MERIT
C
      AA=1.0/(ABS(P4-P6)+1.0/(P4-B**+1.0)+1.0/(P6-B**+1.0)
      BB=1.0/(ABS(S1-S2)+1.0/(SY-S1+1.0)+1.0/(SY-S2+1.0)
      Y=AA+BB
C
      RETURN
      END
```

*The computer listing given in this Appendix is for use with Equation (3-73) as a figure of merit.

**Where B is defined as the design pressure in the optimization phase.

APPENDIX D

PROGRAM ARRANGEMENT FOR CALCULATING THE CRITICAL PRESSURES, STRESSES AND WEIGHT OF A SHELL DESIGN FOR EVALUATING SPECIFIC FIGURES OF MERIT

SUBROUTINE SHELL3 was written for the evaluation and comparison of different figures of merit. The calling program reads the values of the constant parameters and the values of the optimized parameters. The output includes the values of the critical pressures, maximum stresses and weight of a shell using a specific figure of merit. The figure of merit may be evaluated by examining the output and comparing it with the desired conditions as specified in the figure of merit. Refer to Figure 6.

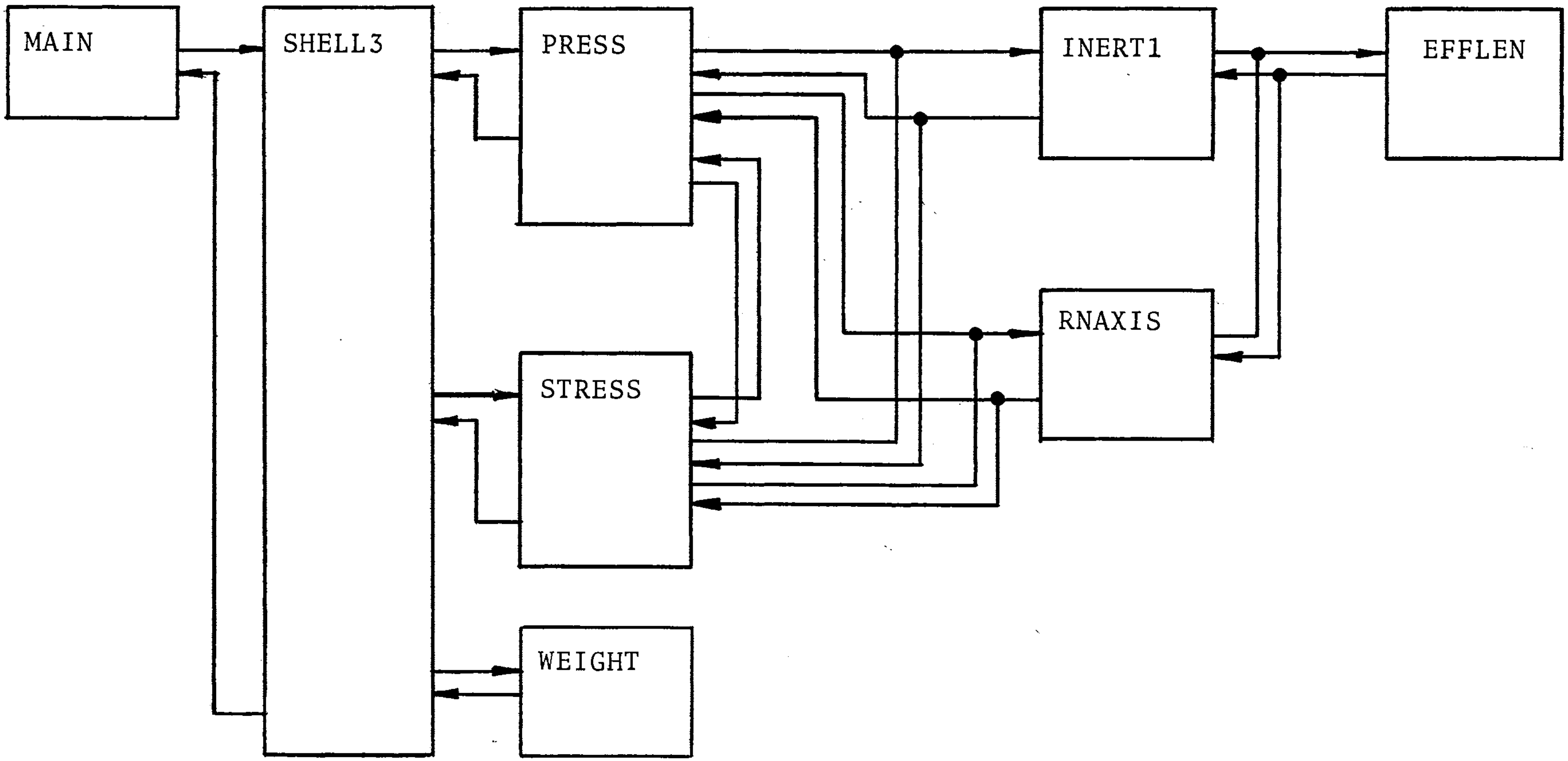

Figure 6. Program Arrangement for Calculating the Critical Pressures, Stresses and Weight of a Shell Design for Evaluating Specific Figures of Merit

VITA

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