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CHAPTER I

INTRODUCTION

The purpose of this dissertation is to determine a pricing policy which will optimize profits for an organization in which:

1. Several products are produced at the same plant that has a limit on its capacity.
2. The products are distributed through regional sales areas with different demand characteristics, some of which are other than purely competitive.
3. Cost and production functions are known.

The value of such a policy is shown in a statement the author has heard from several executives whose responsibilities include both the sales and manufacturing functions of a firm. The quote is:

> "A salesman gives away more over a cup of coffee than a manufacturing man can save in a year."

The basis for this statement is that the salesman, by establishing prices or negotiation, reduces the revenue much more than manufacturing costs can be reduced. The 2%-10-net 30 sales terms which are common business practice is an example. An equivalent reduction of 2% in total manufacturing costs of a product would be a Herculean task.

Pricing policy operates on the strategic factor when the goal is to improve profits. A typical firm, operating in monopolistic competition might have the following Income Statement:

Sales	$50,000,000
Cost of Sales	42,000,000
Gross Profit	$ 8,000,000
Other Expenses	6,000,000
Net Profit before Taxes	$ 2,000,000

A one percent increase in Sales to $50,500,000 would increase Net Profit before Taxes by $500,000 if other factors remain constant. If prices and a change in product mix were all that was required to obtain the revenue increase, it is likely that Cost of Goods Sold and Other Expenses would remain relatively constant. The result would be a 25% increase in profit as a result of a 1% increase in revenue. For these reasons, the author feels that the establishment of a pricing policy is indeed a most significant contribution to the state of the art of scientific management.

Determination of a policy differs from system theory. Systems theory is a study in which the researcher attempts to optimize a model which will be as generally applicable as possible. Policy determination is a study in which the researcher attempts to adapt the systems theory approach to a specific problem, the result of which will define operating policy for an organization.

A major difference between systems theory and policy determination is the requirement on input data. The data necessary to determine policy must be available to the user. Also, the cost of obtaining this data must not outweigh the savings that are realized through the application of the policy. An example of model optimization, which would not suffice as operating policy because input data is not available, is found in queueing theory. Here the cost of waiting is balanced against the cost of providing additional service. Unfortunately, the cost of waiting is not easily quantified. It is, therefore, necessary for an educated and experienced researcher to reduce the theory to operating policy.

Another major difference between systems theory and policy determination is selection of the criteria by which optimization will be judged. If, in fact, the model is to be used for policy determination this criterion must be in accord with the goals of the organization in which the policy is to be applied. An example of systems theory which may not be suitable for establishing policy is minimizing forecasting error by the least squares technique. If an organization's goal was to minimize the costs resulting from forecast error, least squares would only be applicable if the costs varied as the square of the forecasting error.

The criteria for the selection of independent variables through which optimization may be achieved is more constraining to the researcher who is determining policy. These variables must be easily maneuvered and understood by the organization which is to adopt the policy. An example of not selecting a suitable independent variable for optimization would be found in inventory theory. If the inventory model were optimized using the reorder point as an independent variable, the system could not be applied as an operating policy in an organization where the amount of goods in inventory could not be determined.

The actual construction of the model will also depend on the intention of the researcher. If one's aim is to develop system theory with no specific application intentions, the researcher would strive to achieve a high degree of generality. But, in order to make his model generally applicable, he must abstract from the particular problems of policy users. This approach is very natural and clearly desirable; however, the results of such an analysis is rarely applicable, but rather provides the theory by which operating policy can be developed.

The model constructed by the researcher intent on developing operating policy must be specifically tailored and adapted to the specific situation and individual problem which is to be investigated. These two requirements, generality and speciality, are diametrically opposed and hence affect model construction. An example of a generalized model which is not applicable to an operating policy would be a pricing policy which equates marginal revenue and marginal cost. A model of this type is studied in detail and determined not applicable as operating policy in the next chapter.

In the remainder of this dissertation the reader will be led through a brief review of classical economic theory which is relevant to the author's problem. Four specific problems will then be presented along with solutions which were found in the literature. The problems were selected because of their relation to the problem solved in this dissertation. The author will then show the shortcomings of the problems as either not applicable to the development of operating policy or not specific enough to simulate the real environment. A pricing policy will then be developed through the following steps:

1. Identification of the properties of the system.
2. Constructing a model.
3. Selecting independent variables which will define the policy.
4. Optimizing the model with respect to the independent variables.
5. Determining the applicability of the results.

In Chapters III and IV profit maximization models are developed and optimized. The models developed in Chapter III assumes that the quantity demanded at a particular price is known with certainty. Hence, these models are called Deterministic Models. The models developed in

Chapter IV considers the quantity demanded at a particular price to be a random variable. These models are entitled Probabilistic Models

Chapter V starts with a discussion of the relationships between model sophistication and the characteristics of organizations. These relationships dictate the level of model sophistication which would result in efficient policy. It is then determined that the models developed in Chapters III and IV have too high a degree of sophistication to be applicable in most organizations. The optimum price formulations developed in Chapter III are then explored to determine policys with lower degrees of sophistication.

Numerical examples are presented in Chapter VI to demonstrate the salient points previously developed. The dissertation is concluded with a discussion of conclusions and suggestions for continuing research.

CHAPTER II

CLASSICAL THEORY

The development of classical theory of the firm describes rational actions of an entrepreneur who:

1. Purchases inputs with which he produces outputs.
2. Determines how much is to be spent on inputs.
3. Allocates this total amount among various inputs.
4. Determines how much of each input will be allocated to each type of output.
5. Determines how much of each output the firm will produce.
6. Attempts to make these decisions so as to maximize his profits.

The study of how these decisions are made will require the development of three concepts - production function, cost function, and price function.

The concept of a production function can be developed by considering a firm in which n inputs are used, $(X_1, X_2, \ldots X_n)$, to produce m products, $(Z_1, Z_2, \ldots Z_m)$. Decisions on inputs and outputs can not be made independently, i.e., when j levels, j is less than m and n, of X_i and Z_i are selected, the choices of the levels of the remaining X's and Z's are thereby restricted. This interdependence of the input and output levels is summarized in the production function. If x_i and z_i represent the levels of X_i and Z_i respectively, the production function can be represented as

$$H(x_1, x_2, \ldots, x_n, z_1, z_2, \ldots, z_n) = 0$$

The cost function must express the cost of each specified level and mix of production. Micro-economic Theorists represent this cost as

$$C = u_1 x_1 + u_2 x_2 + \ldots + u_n x_n + C_F$$

where u_i = unit price of input x_i

and C_F = fixed cost of the firm.

The demand function is a representation of the relationship between selling price and demand quantity. This relationship is very much affected by the competitive structure of the market. Economists have classified industries or groups of firms into classifications depending on the nature of the competitive structure of the market. The three classifications of interest in this paper are defined below:

a) Pure Competition - a firm is operating under conditions of pure competition where

   1) a large number of firms make up the industry, none of which is large enough to affect the selling price by either entering or leaving the market.

   2) the products produced are identical.

3) entry or exit into the industry is not artifically burdened.

b) Pure Monopoly - a firm is operating under conditions of pure monopoly if it is the sole producer of some product which has no substitutes and the firm faces no imminent threat of competitors.

c) Monopolistic Competition - this is a market structure very similar to pure competition except the products are differentiated in the mind of the consumer.

The demand function reveals the relationship of demand for a certain product with price, all other micro and macro-economic factors are considered constant. If p_i represents the price of product i and q_i the quantity demanded, the demand function is:

$$q_i = f(p_i)$$

A useful characteristic of the demand function is the price elasticity of demand, ϵ_d.

$$\epsilon_d = - \frac{dq}{dp} \frac{p}{q}$$

ϵ_d is the negative ratio of the percentage change in quantity demanded to the percentage change in price.

Under conditions of pure competition, each firm is insignificant and hence can not affect the selling price. Such a demand curve is said to be perfectly elastic since ϵ_d is infinite. However, it is only under the restrictive definitions of pure competition that an entrepreneur can not affect his demand by price manipulation. Pure competition is virtually non-existent.

With the above excerpts from classical economic theory as a foundation, the author will present several problems and solutions by various authors in the field.

Baumol's Model

Baumol [3] discusses profit maximization for a firm with multiple inputs and outputs where limits are placed on the inputs. Baumol states:

> For a profit-maximizing decision which takes multiple commodities into account we have a marginal rule: Any limited input should be allocated between the two outputs in such a way that the marginal profit yield of each input in the production of each output is equal for all constraining input-output combinations.

Although not explicitly demonstrated, the reasoning behind this is straightforward. If the rule above is violated, the firm can increase its profits by diverting some portion of a scarce resource from an output where a lower return is realized to another output with a higher return.

Henderson and Quandt's Model

Henderson and Quandt [9] solve several maximization models, the most relevant of which will be discussed here. The model, constrained revenue maximization, has the following characteristics:

1. The firm manufactures two products, Q_1, and Q_2.
2. There is one limited input, X_1.
3. Prices are fixed, i.e., purely competitive market.

The firms revenue can be expressed as:

$$R = \begin{cases} p_1 q_1 + p_2 q_2 & h(q_1, q_2) \leq X^o \\ \text{Undetermined} & \text{Otherwise} \end{cases}$$

where:

R = revenue

p_i = price for output i $\quad i = 1,2$

q_i = demand for output i $\quad i = 1,2$

$h(q_1, q_2)$ = explicit solution of production function $H(q_1, q_2, x)$

X = limited resource

X^o = maximum level of limited resource

The solution of this model requires the use of the Lagrange multiplier.

$$W = p_1 q_1 + p_2 q_2 + \lambda\, [x^o - h(q_1, q_2)]$$

$$\frac{\partial W}{\partial q_1} = p_1 - \lambda h_1 = 0$$

$$\text{where } h_i = \frac{\partial h}{\partial q_i}$$

$$\frac{\partial W}{\partial q_2} = p_2 - \lambda h_2 = 0$$

$$\frac{\partial W}{\partial \lambda} = X^o - h(q_1, q_2) = 0$$

The necessary conditions for optimization are:

$$\frac{p_1}{h_1} = \frac{p_2}{h_2} \quad \text{and} \quad X^o = h(q_1, q_2)$$

Henderson and Quandt also show that the sufficient condition is that $h(q_1, q_2, x)$ be concave from below.

Mahoney's Model

Edward A. Mahoney presented an empirical solution to the profit maximization problem in "Marketing Strategy-A Case History," [14]. Major deviations from the above two models are:

1. Multiple region demand differences are recognized.

2. Price-quantity relationships are considered.

Mahoney estimates demand at various prices by measuring market penetration at various prices. Market penetration is the portion of the total demand held by the firm. An acceptable second order curve fit was found relating market penetration with price. When this technique was applied to thirty different sales regions it was found that the regions fell into two distinct groups -- sensitive and insensitive. Market penetration in price sensitive areas varied from .30 to .10 with a 10% price variation. Insensitive areas showed variation of only .12 to .10 with the same price change. This realization simplified calculations since only two regression curves were required and each region was classified as sensitive or insensitive.

Mr. Mahoney then used enumeration to evaluate the company's various pricing policies. Total company profits were established by:

1. Determining market penetration at given prices in sensitive and insensitive regions.

2. Multiplying this market penetration by estimated

industry sales in the different regions to obtain total sales.

3. Subtracting cost of sales to obtain profit.

The following table shows the results of the above calculations for various prices. The prices are ficticious but representative.

TABLE 1

RESULTS OF MAHONEY'S MODEL
"PROFIT IN DOLLARS"

		Prices in Insensitive Regions					
		$4.80	$4.90	$5.00	$5.10	$5.20	$5.30
Prices in Sensitive Regions	$4.80	122	125	127	130	126	124
	$4.90	123	127	130	133	128	126
	$5.00	121	123	126	128	124	123
	$5.10	119	120	122	123	121	120
	$5.20	118	119	121	121	120	119
	$5.30	117	118	119	120	119	118

The author feels that the above approach to price determination is an excellent example of problem solving in the face of insufficient information.

Mills' Model

Edwin S. Mills [17] introduces the concept of demand as a random variable. The model maximizes profit for an organization distributing one product in one region. The model is constructed as follows:

1. The level of demand, q, is represented as:

$$q = G(p) + u$$

   where $G(p)$ is the mean demand at price p and u is random variation with density function $f(u)$.

2. The production level for the product is z.

3. Revenue, R, can be represented as

$$R(z,p) = \begin{cases} pq & q \leq z \\ pz & q \geq z \end{cases}$$

$$E(R(z,p)) = \int_{-\infty}^{z-G(p)} pqf(u)\,du + \int_{z-G(p)} pz\,f(u)du$$

$$= p\int_{-\infty}^{z-G(p)} (G(p)+u)\,f(u)du + \int_{z-G(p)}^{\infty} pz\,f(u)du$$

$$= pG(p)\int_{-\infty}^{z-G(p)} f(u)du + p\int_{-\infty}^{z-G(p)} uf(u)du + pz\int_{z-G(p)} f(u)du$$

$$= pG(p)[F(z-G(p)] + pz[1-F(z-G(p))]+p\int_{-\infty}^{z-G(p)} uf(u)du$$

$$= pG(p) - p \int_{z-G(p)}^{\infty} (u - z + G(p))\, f(u)\, du$$

if $$D(z,p) = \int_{z-G(p)}^{\infty} (u - z + G(p))\, fu\, du$$

$$E(R) = pG(p) - pD(p,z)$$

profit, π, can then be expressed as

$$\pi = R(z,p) - C(z)$$

where $C(z)$ is the cost function, i.e., $C(z)$ shows the cost for any level of output.

$$E(\pi) = E(R(z,p)) - E(C(z))$$
$$= pG(p) - pD(z,p) - C(z)$$

Since price affects revenues only, maximization can be achieved by first maximizing revenue with respect to price for any level of output, then maximize profits with respect to z. When this procedure is accomplished, the necessary condition for maximization is

$$F(z - X(p)) = \frac{p - C'(z)}{p}$$

Expressed in words this means the firm's output policy should make the probability of a shortage equal to the ratio of marginal cost to price.

Model Evaluation

It is now necessary to compare each of the above models with the criteria established in Chapter I for operating policy. These criteria are:

1. Data Availability - The required input data must be available and capable of economical assembly.

2. Criterion Desirability - The criterion selected as the measure of optimality must be in accord with organization goals.

3. Variable Maneuverability - The independent variable which determines the necessary condition for optimization must be easily maneuverable and understood by the organization which is going to adopt the policy.

4. Model Practicality - The model must be constructed so as to reflect practical circumstances or is it too general for policy determination.

Marginal profit yield for each input, the necessary input data for Baumol's model, is not available from accounting records in most firms. Henderson and Quandt's model requires an explicit, continuous production function and the limiting level of input. These are either available or easily obtained. Mahoney's model requires a demand function for insensitive and sensitive sales regions, a method of classifying each sales region as either sensitive or insensitive, an estimate of total industry sales in each region and a production function for establishing cost of sales for different levels and mix of output. An illustration of this availability is General Electric

Company's application of Mr. Mahoney's model as policy. Mills' model requires the probability density function of shortages for different price-production level combinations and the production function. The probability density function, although obtainable from the same data as Mr. Mahoney's model, may not be economically feasible to obtain.

The only acceptable criteria for determining optimality in the opinion of the author is profit maximization. Any other criteria such as revenue maximization or cost minimization has the inherent danger of suboptimization. The models of Baumol, Mahoney and Mills are profit maximization models; Henderson's and Quandt's model is revenue optimization.

The independent variable used to determine necessary conditions for optimization for the models discussed in this chapter are:

1. Marginal profit yield for each input in Baumol's model.
2. Price in Henderson and Quandt's model.
3. Price in Mahoney's model.
4. Probability of shortage in Mill's model.

Price is the only variable which satisfies the author's criteria for operating policy; it is easily maneuverable and understood by the organization adopting the policy.

In the author's opinion, a model which does not recognize multiple sales regions, multiple products and demand-price relationships is too general for adoption as operating policy. The only model that recognizes all of these features is Mr. Mahoney's.

The above discussion of the applicability of each model as operating policy is summarized in the following table.

TABLE II

Applicability of Models as Operating Policy

Criteria for Operating Policy	Author of Model: Baumol	Henderson & Quandt	Mahoney	Mills
Data Availability	0	X	X	0
Criterion Desirability	X	0	X	X
Variable Maneuverability	0	X	X	0
Model Practicality	0	0	X	0

X - applicable

0 - not applicable

The results of this comparison are not surprising since only Mr. Mahoney designed his model for application. In the following chapters the author will apply the general theory of Baumol, Henderson, Quandt and Mills to developing operating policy. The first step in this development is to construct and optimize the Deterministic Models.

CHAPTER III

DEVELOPMENT OF DETERMINISTIC MODELS

In this chapter two models with a common characteristic, deterministic relationship between price and demand, will be constructed and optimized.

Model I

The properties of this model are:

1. A firm produces n products and has a limit of K hours of production capacity.
2. The products are distributed through m regional sales areas. The demand function, f_{ij}, for each product-sales region combination is known and deterministic. Where f_{ij} is a function of p_{ij} and q_{ij}, the price and quantity respectively of product j which will be distributed in region i where

   $i = 1,2,\ldots,m$ $\qquad$ $j = 1,2,\ldots,n$

3. The cost function for the plant is:

$$\text{Total Cost} = C_F + \sum_{j=1}^{n} (C_j \sum_{i=1}^{m} q_{ij})$$

where

C_F = fixed cost

C_j = variable cost per unit for product j

q_{ij} = quantity demanded of product j in region i

$i = 1,2,...,m \quad j = 1,2,...,n$

4. The production function for the plant is:

$$\text{Total Production Hours} = \sum_{j=1}^{m} (r_j \sum_{i=1}^{n} q_{ij})$$

r_j = production rate of product j in hours/unit

$i = 1,2,...,m \quad j = 1,2,...,n$

A model must now be constructed. The criterion which will define optimality shall be profit. The independent variable through which optimality will be achieved shall be the price.

The model can be stated as follows:

$$\text{Maximize} \left\{ [\sum_j \sum_i (p_{ij} - C_j)\, q_{ij}] - C_F \right\}$$

$$\text{Subject to:} \quad \sum_j \sum_i r_j q_{ij} \leq K$$

Given: $\quad q_{ij} = f(p_{ij}) \quad\quad i = 1,2,...,m$
$j = 1,2,...,n$

Solution:

$$F = [\sum_j \sum_i (p_{ij} - C_j) q_{ij}] - C_F + \lambda \left\{ [\sum_j \sum_i r_j q_{ij}] - K \right\}$$

$$\frac{\partial F}{\partial p_{ij}} = q_{ij} + \frac{d(f_{ij})}{dp_{ij}}(p_{ij} - C_j) + \lambda r_j \frac{d(f_{ij})}{dp_{ij}} \qquad \begin{array}{l} i=1,2,\ldots,m \\ j=1,2,\ldots,n \end{array} \quad \text{(I)}$$

$$\frac{\partial F}{\partial \lambda} = \sum_j \sum_i r_j q_{ij} - K \qquad \text{(II)}$$

For Linear Demand functions:

$$q_{ij} = \alpha_{ij} + \beta_{ij} p_{ij}$$

$$\frac{d(f_{ij})}{dp_{ij}} = \beta_{ij}$$

Substitute into equation set (I) and equate to zero:

$$\frac{\partial F}{\partial p_{ij}} = 0 = \alpha_{ij} + \beta_{ij} p_{ij} + \beta_{ij}(p_{ij} - C_j) + \lambda r_j \beta_{ij}$$

$$= p_{ij}(\beta_{ij} + \beta_{ij}) + \alpha_{ij} - \beta_{ij} C_j + \lambda r_j \beta_{ij}$$

$$p_{ij} = -\left(\frac{\alpha_{ij} - \beta_{ij} C_j + \lambda r_j \beta_{ij}}{2\beta_{ij}} \right)$$

$$p_{ij} = -\frac{1}{2}\left(\frac{\alpha_{ij}}{\beta_{ij}} - C_j + \lambda r_j \right)$$

Substitute p_{ij} into equation II:

$$\sum_j r_j \sum_i (\alpha_{ij} + \beta_{ij} p_{ij}) = K$$

$$\sum_j r_j \sum_i \left\{ \alpha_{ij} + \beta_{ij} \left(- \frac{\alpha_{ij} - \beta_{ij} C_j + \lambda r_j \beta_{ij}}{2\beta_{ij}} \right) \right\} = K$$

$$\sum_{j} r_j \sum_{i} \left\{ \alpha_{ij} - \frac{\alpha_{ij} - \beta_{ij} c_j + \lambda r_j \beta_{ij}}{2} \right\} = K$$

$$\sum_{j} r_j \left\{ \frac{1}{2} \sum_{i} \alpha_{ij} + \frac{1}{2} \sum_{i} \beta_{ij} c_j - \frac{1}{2} \lambda r_j \sum_{i} \beta_{ij} \right\} = K$$

$$\frac{1}{2} \sum_{j} r_j \sum_{i} \alpha_{ij} + \frac{1}{2} \sum_{j} r_j c_j \sum_{i} \beta_{ij} - \frac{\lambda}{2} \sum_{j} r_j^{2} \sum \beta_{ij} = K$$

$$- \lambda \left(\frac{1}{2} \sum_{j} \sum_{i} r_j^{2} \beta_{ij} \right) = K - \frac{1}{2} \sum_{j} \sum_{i} r_j c_j \beta_{ij} - \frac{1}{2} \sum_{j} \sum_{i} r_j \alpha_{ij}$$

$$\lambda = \frac{-K + \frac{1}{2} \sum_{j} \sum_{i} r_j c_j \beta_{ij} + \frac{1}{2} \sum_{j} \sum_{i} r_j \alpha_{ij}}{\frac{1}{2} \sum_{j} \sum_{i} r_j^{2} \beta_{ij}}$$

Hence the optimum price for any product region combination can be calculated from:

$$p_{ij} = - \frac{\alpha_{ij} - \beta_{ij} c_j}{2 \beta_{ij}} - \frac{r_j}{2} \left\{ \frac{-K + \frac{1}{2} \sum_{j} \sum_{i} r_j c_j \beta_{ij} + \frac{1}{2} \sum_{j} \sum_{i} r_j \alpha_{ij}}{\frac{1}{2} \sum_{j} \sum_{i} r_j^{2} \beta_{ij}} \right\}$$

Note: β_{ij} will be assumed negative as the problem is formulated because almost all demand functions will have a negative slope.

Model II

The properties of this model are the same as Model I except an additional restriction is added. This restriction is that a price must be established for each product, i.e., price discrimination between regions is not allowed.

The model is now stated as:

$$\text{Maximize} \left\{ \left[\sum_j \sum_i (p_j - C_j) q_{ij}\right] - C_F \right\}$$

$$\text{Subject to:} \quad \sum_j \sum_i r_j \; q_{ij} \leq K$$

$$\text{Given:} \qquad q_{ij} = f_{ij}(p_j) \qquad \begin{array}{l} i=1,2,\ldots,m \\ j=1,2,\ldots,n \end{array}$$

Solution:

$$F = \left[\sum_j \sum_i (p_j - C_j)\; q_{ij}\right] - C_F + \lambda \left\{ \left[\sum_j \sum_i r_j q_{ij}\right] - K \right\}$$

$$\frac{\partial F}{\partial p_j} = \sum_i q_{ij} + (p_j - C_j)\,\frac{d(\sum_i f_{ij})}{dp_j} + \lambda r_j \,\frac{d(\sum_i f_{ij})}{dp_j} \qquad j=1,2,\ldots,n \text{(I)}$$

$$\frac{\partial F}{\partial \lambda} = \sum_j \sum_i r_j \; q_{ij} - K \qquad \text{(II)}$$

for linear demand functions:

$$f_{ij} = \alpha_{ij} + \beta_{ij} \, p_j$$

$$\sum_i f_{ij} = \sum_i \alpha_{ij} + \sum_i \beta_{ij} p_j$$

Define:

$$q_j \equiv \sum_i q_{ij} \quad . \; . \quad \alpha_j \equiv \sum_i \alpha_{ij} \qquad \beta_j \equiv \sum_i \beta_{ij}$$

Then:

$$\sum_i f_{ij} = \alpha_j + \beta_j p_j$$

$$\frac{d(\sum f_{ij})}{dp_j} = \beta_j$$

substitute into equation set I and equate to zero:

$$\frac{\partial F}{\partial p_j} = 0 = q_j + (p_j - C_j)\beta_j + \lambda r_j \beta_j \qquad j = 1, 2, \ldots, n$$

$$0 = q_j + \beta_j p_j - \beta_j C_j + \lambda r_j \beta_j$$

but $\quad q_j = \alpha_j + \beta_j \, p_j$

therefore,

$$0 = \alpha_j + \beta_j p_j + p_j \beta_j - \beta_j c_j + \lambda r_j \beta_j$$

$$p_j = -\left(\frac{\alpha_j - \beta_j c_j + \lambda r_j \beta_j}{2\beta_j}\right) \qquad j=1,2,\ldots,n$$

substitute p_j into equation II and equate to zero:

$$0 = \sum_j \sum_i r_j \; q_{ij} - K$$

$$= \sum_j r_j \; q_j - K$$

$$= \sum_j r_j(\alpha_j + \beta_j p_j) - K$$

$$= \sum_j r_j \left\{ \alpha_j - \beta_j \left[\frac{\alpha_j - \beta_j c_j + \lambda r_j \beta_j}{2\beta_j} \right] \right\} - K$$

$$0 = \sum_j r_j \alpha_j - \frac{1}{2} \sum_j r_j \alpha_j + \frac{1}{2} \sum_j \beta_j c_j r_j - \frac{\lambda}{2} \sum_j r_j^2 \beta_j - K$$

$$\frac{\lambda}{2} \sum_j r_j^2 \beta_j = -K + \frac{1}{2} \sum_j r_j \alpha_j + \frac{1}{2} \sum_j \beta_j c_j r_j$$

$$\lambda = \frac{-K + \frac{1}{2}\sum_j r_j \alpha_j + \frac{1}{2}\sum_j \beta_j c_j r_j}{\frac{1}{2}\sum_j r_j^{\,2}\beta_j}$$

Hence p_j for any product can be calculated from:

$$p_j = -\frac{\alpha_j - \beta_j c_j}{2\beta_j} - \frac{r_j}{2}\ \frac{-K + \frac{1}{2}\sum_j r_j \beta_j c_j + \frac{1}{2}\sum_j r_j \alpha_j}{\frac{1}{2}\sum_j r_j^{\,2}\beta_j}$$

In this chapter two models have been constructed and optimized. Both models have the characteristic that the quantity demanded is known with certainty for each price. The results of these models are extensively applied later in this dissertation. The reader should be aware of the assumptions inherent in the optimum price formulations of Models I and II. Of particular importance is the assumption of a linear demand. In the next chapter this assumption will be relaxed so that the quantity demanded becomes a random variable.

CHAPTER IV

DEVELOPMENT OF PROBABILISTIC MODELS

In this chapter two models will be constructed. These models have the common characteristic that the price-demand relationship is not known with certainty.

Model III

The properties of this model are:

1. A firm produces n product and has a limit of K hours of production capacity.

2. The products are distributed through m regional sales regions. The demand for each product-region combination is represented by:

$$q_{ij} = f_{ij}(p_{ij}) + \mu_{ij} \qquad \begin{array}{l} i = 1,2,\ldots,m \\ j = 1,2,\ldots,n \end{array}$$

q_{ij} = quantity of product j demanded in region i.

$f_{ij}(p_{ij})$ = mean quantity demanded of product j in region i.

μ_{ij} = random variation of demand of product j in region i.

3. The cost function for the firm is:

$$C = C_F + \sum_{j=1}^{n} z_j C_j$$

C = total cost

C_F = fixed cost

C_j = variable cost per unit for product j

z_j = scheduled production of product j

$$z_j = \sum_i z_{ij}$$

z_{ij} = scheduled production for product j in region i

4. The production function for the plant is

$$\text{Total Production Hours} = \sum_{j=1}^{n} r_j z_j$$

$$j = 1,2,\ldots,n$$

r_j = production rate of product j in hours per unit

The expected profit for the firm can be expressed as follows where profit and revenue are expressed as π and R respectively.

$$E(\pi) = E(R - C)$$
$$E(\pi) = E(R) - E(C)$$

The expected value of cost is

$$E(C) = E(C_F - \sum_{j=1}^{n} C_j z_j)$$

$$= C_F - \sum_{j=1}^{n} C_j z_j$$

The expected value of revenue is more difficult to construct. This will be accomplished by constructing the expected value of revenue for a particular product and summing over all products. With the properties as defined above for the probabilistic model, it is possible for demand,

$\sum_{i} q_{ij}$, to be greater or less than production, z_j, for any product. For the first case, $\sum_{i} q_{ij} < z_j$, the revenue will be $\sum_{i} p_{ij}q_{ij}$ since the total demand can be be satisfied. It is noteworthy that this does not require that the production intended for a particular region must be sold in that region. The only requirement is that the total demand for all regions is less than or equal to the production for that product.

An expression now must be obtained for revenue when $\sum_{i} q_{ij} > z_j$. This requires that the model explain how the scarce resource will be distributed throughout the i regions. Several methods of allocation could be suggested.

1. Allocate such that revenue will be maximized.
2. Allocate according to the forecast. Forecast is the planned production for any region, z_{ij}.

The firm will not, indeed can not, allocate the scarce product in any rational manner. The allocation will be dictated by the sequence in which the demands are received, as the firm will not know beforehand that its production of a particular product is insufficient to satisfy the demand. Since no rational manner of allocation can be determined, allocation according to forecast will be used since its representation is less complex. For this case the revenue will be $\sum_i p_{ij}z_{ij}$.

The revenue for a particular product, j, is then:

$$R_j = \begin{cases} \sum_i p_{ij}q_{ij} & \sum_i q_{ij} < z_j \\ \sum_i p_{ij}z_{ij} & \sum_i q_{ij} > z_j \end{cases}$$

$$R = \sum_j R_j$$

$$E(R) = E(\sum_j R_j)$$

$$= \int_{-\infty}^{z_j} (\sum_i p_{ij}q_{ij})\, g(\sum_i q_{ij})\, d(\sum_i q_{ij})$$

$$+ \int_{z_j}^{\infty} (\sum_i p_{ij} z_{ij}) g(\sum_i q_{ij}) d(\sum_i q_{ij})$$

since $$q_{ij} = f_{ij}(p_{ij}) + \mu_{ij}$$

this can be restated as

$$E(R) = \sum_j \left\{ \int_{-\infty}^{z_j - \sum_i f_{ij}(p_{ij})} [\sum_i p_{ij}(f_{ij}(p_{ij}) + \mu_{ij})] g(S) dS + \int_{z_j - \sum_i f_{ij}(p_{ij})}^{\infty} [\sum_i p_{ij} z_{ij}] \; g(S) dS \right\}$$

where $$S = \sum_i \mu_{ij}$$

expanding and simplifying the first term

$$E(R) = \sum_j \left\{ \int_{-\infty}^{z_j - \sum_i f_{ij}(p_{ij})} [\sum_i p_{ij} f_{ij}(p_{ij})] \; g(S) dS \right.$$

$$+ \int_{-\infty}^{z_j - \sum_i f_{ij}(p_{ij})} [\sum_i p_{ij}\mu_{ij}]\ g(S)dS$$

$$\left. + \int_{z_j - \sum_i f_{ij}(p_{ij})}^{\infty} [\sum_i p_{ij}z_{ij}]\ g(S)dS \right\}$$

rearranging the terms and expressing $\int_{-\infty}^{a} g(S)d(S)=F(a)$

$$E(R) = \sum_j \left\{ [\sum_i p_{ij}f_{ij}(p_{ij})]\ [F(z_j - f_{ij}(p_{ij}))] \right.$$

$$+ [\sum_i p_{ij}z_{ij}]\ [1-F(z_j - f_{ij}(p_{ij}))]$$

$$\left. + \int_{-\infty}^{z_j - f_{ij}(p_{ij})} [\sum_i p_{ij}\mu_{ij}]\ g(S)dS \right\}$$

The last term in this expression is not integrable in closed form since to do so requires expressing $\sum_i p_{ij}\mu_{ij}$ as a function of S.

Note that if price did not vary with the sales region the last term may be expressed as:

$$p_j \int_{-\infty}^{z_j - \sum_i f_{ij}(p_j)} S\ g(S)dS$$

since $$\sum_i p_j\mu_{ij} = p_j \sum_i \mu_{ij} = p_j S$$

This leads directly to Model IV.

<u>Model IV</u>

Model IV has the same properties as Model III except price discrimination among regions is not allowed. The expected value of profit for the firm is:

$$\begin{aligned} E(\pi) &= E(R - C) \\ &= E(R) - E(C) \end{aligned}$$

and $$\begin{aligned} E(C) &= E(C_F - \sum_j c_j z_j) \\ &= C_F - \sum_j c_j z_j \end{aligned}$$

After the argument put forth for Model III the revenue for the jth product may be expressed as:

$$R_j = \begin{cases} p_j \sum_i q_{ij} & \sum_i q_{ij} < z_j \\ p_j z_j & \sum_i q_{ij} > z_j \end{cases}$$

$$R = \sum_j R_j$$

$$E(R) = \sum_j E(R_j)$$

$$E(R) = \sum_j \left\{ \int_{-\infty}^{z_j} [\sum_i p_j q_{ij}] \; g(\sum_i q_{ij}) \; d(\sum_i q_{ij}) + \int_{z_j}^{\infty} [\sum_i p_j z_{ij}] \; g(\sum_i q_{ij}) \; d(\sum_i q_{ij}) \right\}$$

This can be restated as

$$E(R) = \sum_j \left\{ p_j \int_{-\infty}^{z_j - \sum_i f_{ij}(p_j)} [\sum_i q_{ij}] \, g(\sum_i \mu_{ij}) \, d(\sum_i \mu_{ij}) + \int_{z_j - \sum_i f_{ij}(p_{ij})}^{\infty} [\sum_i z_{ij}] \, g(\sum_i \mu_{ij}) \, d(\sum_i \mu_{ij}) \right\}$$

or by expanding the first term and substitution $S = \sum_i \mu_{ij}$

$$E(R) = \sum_j p_j \left\{ \int_{-\infty}^{z_j - \sum_i f_{ij}(p_j)} [\sum_i (f_{ij}(p_j) + \mu_{ij})] \, g(S)dS + \int_{z_j - \sum_i f_{ij}(p_j)}^{\infty} [\sum_i z_{ij}] \, g(S)dS \right\}$$

$$E(R) = \sum_j p_j \left\{ \left[\sum_i f_{ij}(p_j) \right] \left[\int_{-\infty}^{z_j - \sum_i f_{ij}(p_j)} g(S)dS \right] \right.$$

$$+ \int_{-\infty}^{z_j - \sum_i f_{ij}(p_j)} S \; g(S)dS$$

$$\left. + \left[\sum_i z_{ij} \right] \left[\int_{z_j - \sum_i f_{ij}(p_j)}^{\infty} g(S)dS \right] \right\}$$

if $$I(p_j) = \int_{-\infty}^{z_j - \sum_i f_{ij}(p_j)} \left[z_j - \sum_i f_{ij}(p_j) - S \right] g(S)d(S)$$

$$E(R) = \sum_j p_j \left\{ z_j - I(p_j) \right\}$$

Note that $I(p_j)$ has a physical meaning. It is the expected amount of inventory remaining after the demand has been satisfied.

Therefore, the expected profits for the firm may be expressed as:

$$E(\pi) = \sum_j p_j \left\{ z_j - I(p_j) \right\} - C_F - \sum_j C_j z_j$$

The problem can then be stated as:

$$\text{Maximize} \left\{ \sum_j p_j \left[z_j - I(p_j) \right] - C_F - \sum_j C_j z_j \right\}$$

$$\text{Subject to:} \qquad \sum_j z_j r_j \leq K$$

Note that neither the cost function nor the capacity constraint contain any p_j terms. The maximization can, therefore, be accomplished by maximizing the expected value of revenue with respect to the p's for any combination of z's and then maximizing the profit subject to the capacity constraint with respect to the z's.

$$E(R) = \sum_j p_j \left[z_j - I(p_j) \right]$$

$$= \sum_j p_j z_j - \sum_j p_j I(p_j)$$

$$\frac{\partial (E(R))}{\partial p_j} = z_j - \sum_j p_j I'(p_j) - \sum_j I(p_j) \quad j=1,2,\ldots,n$$

where $$I'(p_j) - \frac{\partial (I(p_j))}{\partial p_j}$$

The solution of the following n implicit equations will determine the optimum price for each product, p^*_j, as a function of the z_j's.

$$0 = \sum_j p^*_j \; I'(p^*_j) - \sum_j I(p^*_j) \quad j=1,2,\ldots,n$$

The problem can now be represented as

$$\text{Maximize} \quad \sum_j p^*_j \; [z_j - I(p^*_j)] - C_F - \sum_j C_j z_j$$

$$\text{Subject to:} \quad \sum_j z_j r_j \leq K$$

By forming the following Lagrange function optimization can be achieved:

$$L(\vec{z}, \lambda) = \sum_j p^*_j \, [z_j - I(p^*_j)] - C_F - \sum_j C_j z_j + \lambda \, [\sum_j r_j z_j - K]$$

$$\frac{\partial L}{\partial z_j} = - I'(p^*_j) \, p^*_j + [z_j - I(p^*_j)] - C_j z_j + \lambda \, r_j z_j$$

$$j = 1, 2, \ldots, n$$

$$\frac{\partial L}{\partial \lambda} = \sum_j r_j z_j - K$$

By equating these expressions to zero and solving, the optimum z's can be obtained. These z^*'s can be used in the p^* functions to obtain optimum prices for each product. Although this model can theoretically be solved, the actual solution is very difficult. The author attempted to solve a two product-two region example, similar to the example shown in Chapter VI. Even for this simple case the solution of the z_j's would have required solving two simultaneous fifth order equations.

Probabilistic and deterministic models have now been constructed and optimized. In Chapter V these models will be examined to determine if they are applicable as a basis for operating policy.

CHAPTER V

DETERMINATION OF POLICY

Within the preceding two chapters of this dissertation the author has progressed through the first four steps of policy determination for three different models. The steps are:

1. Identification of the properties of the system.
2. Selecting independent variables which will define the policy.
3. Constructing a model.
4. Optimizing the model with respect to the independent variables.

The last step in determination of policy remains - determine the applicability of the policy. The applicability of the policy will depend upon the individual characteristics of the using organization.

The discussion of the interface between a particular organization and operating policy designed for that organization can best be presented with reference to Figure .1. The Figure represents the costs incurred by an organization which performs a necessary service for the firm but can not by its nature contribute to profits. Hence, the

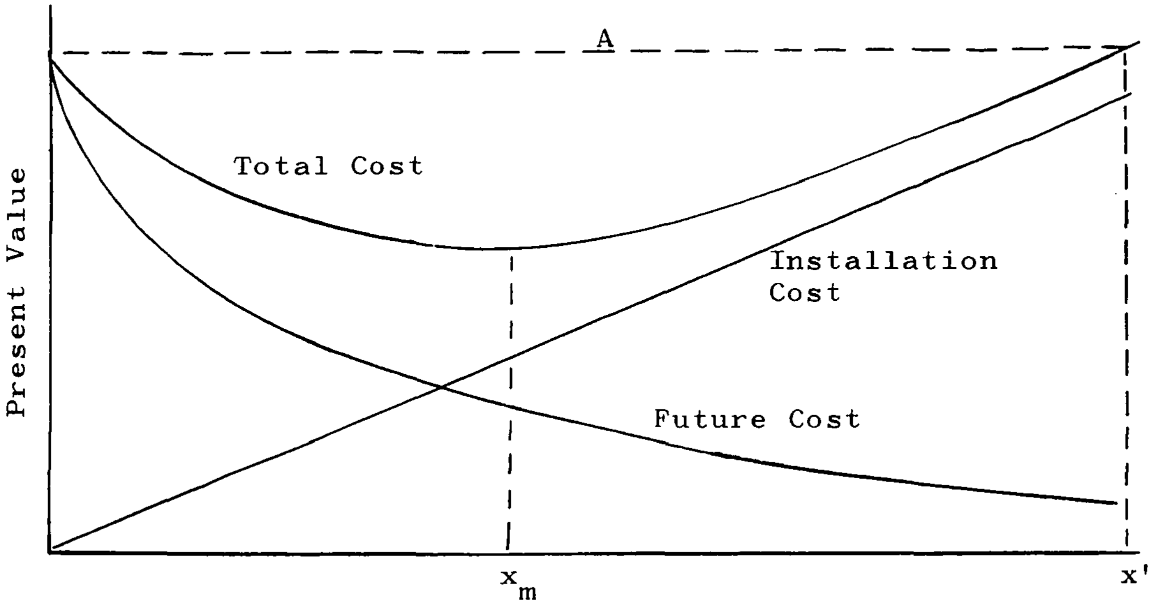

Figure 1.--Policy Effectiveness

goal of the organization would be to perform the service while incurring a minimum cost. An example of such an organization would be the management of inventories. The horizontal line, A, represents the cost of providing the service with the current policy or lack of policy. The curve entitled future cost represents the present value of the cost of providing the service for different levels of model sophistication. This future cost will vary inversely with the degree of sophistication. The curve entitled installation cost represents the present value of the cost of developing, installing and maintaining the model which will determine policy. The curve entitled total cost is the sum of the other curves. The degrees of sophistication shown, x_m and x', are important parameters in developing operating policy. x' is the degree of sophistication which will result in no reduction in the total cost to the firm. Effective policy will be defined as any policy resulting from a model with a degree of sophistication below x'. x_m is the degree of sophistication which will result in the minimum total cost for the firm. Efficient policy will be defined as policy resulting from a model with x_m level of sophistication.

The shape of the curves in Figure 1 are peculiar to the personality of the firm and the organization. Although not quantifiable, the concept of effective and efficient policy must be apparent to the researcher who is intent on establishing operating policy.

A very significant influence upon the shape of the curves in Figure 1 is what the author calls the level of competence of the firm. This is in general the degree to which the firm is capable of applying scientific management techniques. The level of competence of the firm depends on such things as:

1. The interest of top management in scientific principles.
2. The ability of the firm and using organization to adapt to change.
3. The capability of the organization to adopt and apply new policy.

It is of significant importance that while the level of competence of a firm is a characteristic of the firm, it depends on the individuals within the firm. Since these individuals have the ability to learn and adapt to a changing environment, the shapes of the curves in Figure 1 will change with time as scientific management principles are applied. These changes will affect both the future cost curve and the installation cost curve. The future cost curve will shift downward as the users of the new policy learn new methods of reducing costs. The installation cost curve will also shift downward as the user's resistance to change decreases. The result of these shifts will move x_m to the right, representing higher levels of model sophistication. This phenomenon is similar to the well-known theory of manufacturing progress function.

The above concepts are the basis for a cardinal rule of the researcher who attempts to establish policy - start simple. In the author's opinion, the model used for the establishment of operating policy should never reach an x_m degree of sophistication. But rather, the model should be improved as the firm's level of competence increases, in a manner such that x_m is approached but never achieved. This approach to policy determination is suggested since the researcher should be at least as greatly concerned with the cultivation of the firm's level of competence as with the sophistication of the model.

If the input information is available from the firm's historical records and the firm's level of competence is compatible, any of the three models which were optimized in Chapters III and IV of this dissertation could be adopted as operating policy. Unfortunately, very few if any firms have the above characteristics. In other words, the degree of sophistication of all of the models is too high. The researcher must then adapt the concepts of these models to the degree of competence of the firm with the hopes of increasing the sophistication of the model as the firm adapts to scientific management.

In order to establish pricing policy with a lower level of sophistication, the optimum price formulations for Model I and Model II will be explored. The optimum price for Model I is:

$$p_{ij} = -\frac{\alpha_{ij}}{2\beta_{ij}} + \frac{c_j}{2} - \frac{r_j \lambda}{2}$$

The optimum price for Model II is:

$$p_j = -\frac{\alpha_j}{2\beta_j} + \frac{c_j}{2} - \frac{r_j \lambda}{2}$$

It is interesting to note that these optimum price formulations have three distinct components:

1. $-\frac{\alpha}{2\beta}$ - This component depends on the components of the demand function and will be referred to as the demand component of price.
2. C/2 - This component depends on the variable manufacturing costs of the products and will be referred to as the cost component of price.

3. $r\lambda/2$ - This component depends on the characteristics of the manufacturing process and will be referred to as the manufacturing component of price.

Demand Component of Price

The demand component of price is the only component which is influenced by the demand characteristics of the different sales regions. Any advantage a firm may enjoy by practicing price discrimination must be the result of the demand characteristic of price. It would be interesting to estimate the increase in profits one might expect if price discrimination is allowed. The procedure for obtaining this estimate will be to subtract the maximum profit which would result from the prices determined in Model II from the maximum profit obtainable if Model I were applied. From the equation for optimum price in Model I

$$p_{ij} = -\frac{\alpha_{ij}}{2\beta_{ij}} + \frac{c_j}{2} - \frac{r_j\lambda}{2}$$

The quantity demanded in region i is

$$q_{ij} = \alpha_{ij} + \beta_{ij}\,p_{ij}$$

or on substitution

$$q_{ij} = \alpha_{ij} - \frac{\alpha_{ij}}{2} + \frac{c_j\beta_{ij}}{2} - \frac{r_j\beta_{ij}\lambda}{2}$$

$$= \frac{1}{2}(\alpha_{ij} + c_j\beta_j - r_j\beta_{ij}\lambda)$$

The variable cost for producing q_{ij} units is:

$$C = C_j \, (q_{ij})$$

$$= \frac{C_j}{2} \, (\alpha_{ij} + C_j \beta_{ij} + r_j \beta_{ij} \lambda)$$

Total profit resulting from the application of Model I is:

$$\pi_I = \sum_j \sum_i p_{ij} q_{ij} - \sum_j \sum_i C_j q_{ij}$$

$$= \sum_j \sum_i \left[-\frac{\alpha_{ij}}{2\beta_{ij}} + \frac{C_j}{2} + \frac{r_j \lambda}{2} \right] \left[\frac{\alpha_{ij}}{2} + \frac{r_j \beta_{ij}}{2} - \frac{r_j \beta_{ij} \lambda}{2} \right]$$

$$- \sum_j \sum_i \frac{C_j}{2} [\alpha_{ij} + C_j \beta_{ij} + \lambda \beta_{ij}]$$

$$= \frac{1}{4} \left\{ \sum_j \sum_i - \frac{\alpha_{ij}^2}{\beta_{ij}} + \sum_j \sum_i \alpha_{ij} C_j + \sum_j \sum_i \alpha_{ij} \lambda r_j \right.$$

$$+ \sum_j \sum_i C_j \alpha_{ij} + \sum_j \sum_i C_j^2 \beta_{ij}$$

$$- \sum_j \sum_i C_j r_j \lambda \beta_{ij} - \sum_j \sum_i r_j \lambda \alpha_{ij}$$

$$-\sum_j\sum_i r_j \lambda c_j \beta_{ij} + \sum_j\sum_i r_j^2 \lambda^2 \beta_{ij} \Bigg\}$$

$$- \frac{1}{2}\left\{\sum_j\sum_i c_j \alpha_{ij} + \sum_j\sum_i c_j^2 \beta_{ij} + \sum_j\sum_i c_j r_j \lambda \beta_{ij} \right\}$$

By the same procedure the maximum profit obtainable through the application of Model II is:

$$\pi_{II} = \frac{1}{4}\Bigg\{\sum_j - \frac{\alpha_j^2}{\beta_j} + \sum_j c_j \alpha_j + \sum_j \alpha_j r_j \lambda + \sum_j c_j \alpha_j$$

$$+ \sum_j c_j^2 \beta_j - \sum_j c_j r_j \lambda \beta_j - \sum_j r_j \lambda \alpha_{ij} - \sum_j r_j \lambda c_j \beta_{ij}$$

$$+ \sum_j r_j^2 \lambda^2 \beta_{ij}\Bigg\} - \frac{1}{2}\left\{\sum_j c_j \alpha_j + \sum_j c_j^2 \beta_j + \sum_j c_j r_j \lambda \beta_{ij}\right\}$$

If the equation for π_{II} is subtracted from the equation for π_I term for term only the first term in each expression will remain.

$$\pi_I - \pi_{II} = \sum_j\sum_i - \frac{\alpha_{ij}^2}{4\beta_{ij}} - \sum_j - \frac{\alpha_j^2}{4\beta_j}$$

If the parameters, α and β, of the demand functions can be estimated, the increase in maximum profits can be determined. However, even if an exact determination of α and β is not feasible information can be drawn from the above expression. The physical meaning of α is the demand for the product if no price is charged -- How many can be given away? The parameter β is reflective of the amount of competition in the regions. If the market is highly competitive, β is small and vice versa. The difference in maximum profits increases greatly if α / β changes among regions. An example of a product which would have this characteristic would be air conditioners which are distributed throughout the country. Ideal conditions for price discrimination are that the regions with the greatest demands be highly competitive. Since this is generally true, competition is greater in regions with large demands, the salient market characteristic which makes price discrimination attractive is differences in total demand among regions.

Although the entrepreneur might have a feeling for the relative size of α for different regions, estimating α would be a difficult task. Consider again the equation derived above:

$$\pi_I - \pi_{II} = \sum_j \sum_i - \frac{\alpha_{ij}^2}{4\beta_{ij}} - \sum_j - \frac{\alpha_j^2}{4\beta_j}$$

$$= -\frac{1}{4}\sum_j \left[\sum_i \frac{\alpha_{ij}^2}{\beta_{ij}} - \frac{\alpha_j}{\beta_j}\right].$$

If $\frac{\alpha_{ij}}{\beta_{ij}}$ were constant for all regions say $\alpha_{ij}/\beta_{ij}=e$, then

$$\pi_I - \pi_{II} = -\frac{1}{4}\sum_j \left(\sum_i e\,\alpha_{ij} - e\,\alpha_j\right)$$

$$= -\frac{e}{4}\sum_j (\alpha_j - \alpha_j)$$

$$= 0.$$

If α/β is constant for all regions, no increase in profits can result from price discrimination. Since demands are considered a linear function of price in both Model I and Model II, a simple test can be derived to determine if price discrimination can increase the maximum profits possible to a firm.

$$q_{ij} = \alpha_{ij} + \beta_{ij}\, p_{ij}$$

$$\frac{q_{ij}}{\beta_{ij}} = \frac{\alpha_{ij}}{\beta_{ij}} + p_{ij}$$

$$\left(\frac{q_{ij}}{\beta_{ij}} - p_{ij}\right) = \frac{\alpha_{ij}}{\beta_{ij}}$$

Therefore, if $[(q_{ij}/\beta_{ij})-p_{ij}]$ is constant for all regions, price discrimination will not increase maximum profits.

The above test is quite powerful since it does not require a determination of the demand function. The only requirement is that the demand function can be represented by some arbitrary linear function with a known slope. However, the mere fact that profit potential can be increased is of little importance to the entrepreneur unless he also has some policy by which he can realize an increase in profits.

Consider an ordered array of the optimum price formulas under Model I for one product and k regions. Since only one product is considered the j subscripts are ignored. C represents the cost component and manufacturing component of price which are constant for any one product.

$$p_1 = -\frac{\alpha_1}{2\beta_1} + C$$

$$p_2 = -\frac{\alpha_2}{2\beta_2} + C$$

$$\vdots$$

$$p_m = -\frac{\alpha_m}{2\beta_m} + C$$

$$\vdots$$

$$p_k = -\frac{\alpha_k}{2\beta_k} + C$$

The subscripts 1,m and k denote the maximum, median and minimum optimum price respectively.

If a median price, $\overline{P}$, around which the regional prices will be established and a price range, Δ, the maximum price discrimination among regions can be determined, a method of determining regional prices can be established such that if $\overline{P}=p_m$ and $\Delta=p_1-p_k$, the prices will be optimum.

$$p'_i = \overline{P} + \left[\frac{\left(\frac{\alpha_m}{\beta_m} - \frac{\alpha_i}{\beta_i}\right)}{\left(\frac{\alpha_k}{\beta_k} - \frac{\alpha_1}{\beta_1}\right)}\right] \Delta$$

where p'_i = new price

It can be demonstrated that p'_i is optimum if $\overline{P}=p_m$ and $\Delta = p_i-p_k$

$$p_i = \overline{P} + \Delta \frac{\left(\frac{\alpha_m}{\beta_m} - \frac{\alpha_i}{\beta_i}\right)}{\left(\frac{\alpha_k}{\beta_k} - \frac{\alpha_i}{\beta_i}\right)} \qquad i=1,2,\ldots,m,\ldots,k$$

If $\Delta = p_1 - p_k$

$$= -\frac{\alpha_1}{2\beta_1} + C - \left(\frac{-\alpha_k}{\beta_k}\right) - C$$

$$= \frac{1}{2}\left(\frac{\alpha_k}{\beta_k} - \frac{\alpha_1}{\beta_1}\right)$$

and $\overline{P} = p_m$

$$= -\frac{\alpha_m}{2\beta_m} + C$$

Then

$$p'_i = -\frac{\alpha_m}{2\beta_m} + C + \frac{1}{2}\left(\frac{\alpha_k}{\beta_k} - \frac{\alpha_1}{\beta_1}\right)\left(\frac{\frac{\alpha_m}{\beta_m} - \frac{\alpha_i}{\beta_i}}{\frac{\alpha_k}{\beta_k} - \frac{\alpha_1}{\beta_1}}\right)$$

$$= -\frac{\alpha_m}{2\beta_m} + C + \frac{1}{2}\left(\frac{\alpha_m}{2\beta_m} - \frac{\alpha_i}{\beta_i}\right)$$

$$= -\frac{\alpha_i}{2\beta_i} + C$$

$$= p_i \text{ (optimum price)}$$

This procedure allows a firm to enter into price discrimination without estimating the demand function since the slope of the demand function can be estimated any time the price is changed and $\alpha_i/\beta_i = \frac{q_i}{\beta_i} - p_i$. The median price, $\overline{P}$, can be estimated as the industry wide average price. Δ must be established by management, however, political considerations might well dictate the range of price discrimination even if enough information were available to determine optimum prices using Model I. The appeal of this method of establishing prices lies in the fact that the input parameters, $\overline{P}$ and Δ, are easily understood by the entrepreneur, while α and β are not understood unless one has a mathematical background.

Cost Component of Price

The cost component of price, $C_j/2$, is affected only by the variable cost of manufacturing the product. Consider again the optimum price formulation for Model II:

$$p_j = -\frac{\alpha_j}{2\beta_j} + \frac{C_j}{2} - \frac{r_j\lambda}{2}$$

If manufacturing capacity is not a constraint, this can be reduced to:

$$p_j = -\frac{\alpha_j}{2\beta_j} + \frac{C_j}{2}$$

It appears that a demand function could be formulated such that optimum profits would require selling a product at less than its variable manufacturing cost. For this to be the case

$$c_j > p_j$$

$$> -\frac{\alpha_j}{2\beta_j} + c_j/2$$

$$\frac{c_j}{2} > -\frac{\alpha_j}{2\beta_j}$$

$$c_j > -\alpha_j/\beta_j$$

$$\beta_j c_j < -\alpha_j$$

since β is assumed negative

$$|\beta_j c_j| > \alpha_j$$

$|\beta_j C_j|$ is the reduction in demand resulting from an increase of C_j in the price. The inequality shows that in order to sell at less than the variable manufacturing cost, the demand function must be such that the reduction in demand resulting from an increase in price equivalent to the variable manufacturing cost is greater than the demand if no price is charged. In other words the product is such that if a price equivalent to C_j was charged, none could be sold. This is clearly an unrealistic case, therefore, the optimum price formulation will never result in a price less than the variable manufacturing cost.

The fact that optimum price is a function of variable manufacturing costs is interesting since the "cost plus" pricing policy is prevalent in business. The "cost plus" policy can be described as:

$$p_j = C_j\ (1 + d)$$

where d is a constant.

Under what market conditions would "cost plus" pricing result in optimum pricing? This will be determined by equating to two pricing formulations and solving for d^*, the optimum mark up. The manufacturing component of price will not be included, which is equivalent to not having a capacity constraint.

$$C(1+d^*) = -\frac{\alpha}{2\beta} + \frac{C}{2}$$

$$C+Cd^* = -\frac{\alpha}{2\beta} + \frac{C}{2}$$

$$Cd^* = -\frac{\alpha}{2\beta} - \frac{C}{2}$$

$$d^* = \frac{1}{C}\left(-\frac{\alpha}{2\beta} - \frac{C}{2}\right)$$

$$d^* = -\frac{1}{2} - \frac{\alpha}{2\beta C}$$

Therefore, "cost plus" pricing, if the mark-up is constant for all products, can never result in optimum prices except when $(\frac{\alpha}{\beta C})$ is constant for all products. Note also that an optimum price mark-up, d^*, varies inversely with the variable manufacturing cost. This would mean that a Cadillac should have a lower mark-up than a Chevrolet if the demand functions were identical.

If the variable manufacturing cost, C, is increased by ΔC, how much should the optimum mark-up, Δd^*, be increased?

$$\Delta d = d^*_1 - d^*_2$$

where d^*_1 is the optimum price mark-up before the cost increase and d^*_2 is the optimum mark-up after the cost increase.

$$\Delta d = \left[-\frac{\alpha}{2\beta C} - \frac{1}{2}\right] - \left[-\frac{\alpha}{2\beta(C+\Delta C)} - \frac{1}{2}\right]$$

$$\Delta d = -\frac{\alpha}{2\beta C} + \frac{\alpha}{2\beta(C+\Delta C)}$$

$$4\beta^2 C\Delta d(C+\Delta C) = -2\beta\alpha(C+\Delta C) + 2\beta\alpha C$$

$$4\beta^2\Delta d(C^2+C\Delta C) = -2\beta\alpha\Delta C$$

$$2\beta\Delta d(C^2+(\Delta C) = -\alpha\Delta C$$

$$\Delta d = -\frac{\alpha}{2\beta}\left(\frac{\Delta c}{c^2 + c\Delta c}\right)$$

$$= -\frac{\alpha}{2\beta}\left(\frac{\Delta c/c^2}{(1+\frac{\Delta c}{c})}\right)$$

$$= -\frac{\alpha}{2\beta c}\left(\frac{\frac{\Delta c}{c}}{1+\frac{\Delta c}{c}}\right)$$

However, if the mark-up before the price increase were optimum

$$d_1{}^* = -\frac{\alpha}{2\beta c} - \frac{1}{2}$$

$$-\frac{\alpha}{2\beta c} = d_1{}^* + \frac{1}{2}$$

Therefore

$$\Delta d = (d_1{}^* + \tfrac{1}{2})\left(\frac{\frac{\Delta c}{c}}{1+\frac{\Delta c}{c}}\right)$$

This is a quite powerful tool since it allows the entrepreneur to adjust his price with cost changes toward an optimum price without knowing the demand function.

If an increase in variable costs is incurred, under what conditions should the percent increase in mark-up be greater than the percent increase in price? This question will be answered as follows:

$$\frac{\Delta d}{d^*} > \frac{\Delta c}{c}$$

$$\Delta d > \frac{\Delta c}{c} d^*$$

$$(d^*) + \frac{1}{2}\left(\frac{\frac{\Delta c}{c}}{1 + \frac{\Delta c}{c}}\right) > \frac{\Delta c}{c} d^*$$

$$(d^* + \frac{1}{2}\frac{\Delta c}{c}) > \frac{\Delta c}{c} d^* (1 + \frac{\Delta c}{c})$$

$$(d^* + \frac{1}{2}) > d^* (1 + \frac{\Delta c}{c})$$

$$\frac{1}{2d^*} > \frac{\Delta c}{c}$$

Hence, the percent increase in mark-up should be greater than the percent increase in costs when $1/(2d^*)$ is greater than the percent increase in price. This is quite interesting since if d^* were approximately 0.2, any cost increase less than 225% would result in a higher percent increase in mark-up than the percent increase in cost.

The Manufacturing Component of Price

The manufacturing component of price, $(r_j/2)\lambda$, is completely determined by the properties of the manufacturing process. The first step in studying the manufacturing component of price will be to investigate the construction of λ.

$$\lambda = \frac{-K + \frac{1}{2}\sum_j \sum_i r_j c_j \beta_{ij} + \frac{1}{2}\sum_j \sum_i r_j \alpha_{ij}}{\frac{1}{2}\sum_j \sum_i r_j^2 \beta_{ij}}$$

The numerator of λ can be rearranged into the following form:

$$- K + \frac{1}{2}\sum_{j} r_j \sum_{i} (c_j \beta_{ij} + \alpha_{ij})$$

Note that the last sum:

$$\sum_{i} (c_j \beta_{ij} + \alpha_{ij})$$

is the quantity demanded at a price equal to the variable manufacturing cost. Hence:

$$\frac{1}{2}\sum_{j} r_j \sum_{i} (c_j \beta_{ij} - \alpha_{ij})$$

is one-half of the production hours required to satisfy the demand that would result if all products were priced at the variable manufacturing cost. Recalling that K is the available hours of production capacity, the numerator has dimension, hours, and is equal to the difference between available hours and one-half of the hours required to satisfy demand at price C_j. If H is defined as the production capacity required to produce the demand that would result if the price were C_j, λ may be expressed as:

$$\lambda = \frac{\frac{1}{2} H - K}{\frac{1}{2}\sum_{j}\sum_{i} r_j^2 \beta_{ij}}$$

To determine the effect λ has on the optimum price of a product, the formula for optimum price under Model II will be examined:

$$p_j = -\frac{\alpha_j}{2\beta_j} + \frac{c_j}{2} - \frac{r_j\lambda}{2}$$

If the demand component of price and the cost component of price are considered fixed and their sum represented by C, then:

$$p_j = C - \frac{r_j}{2} \frac{\frac{1}{2}H-K}{\frac{1}{2}\sum_j\sum_i r_j^2 \beta_{ij}}$$

Note that the sign of the manufacturing component of price is determined by $(\frac{1}{2}H-K)$ and that since β_{ij} is always negative, the manufacturing component of price will:

1. increase p_j if $\frac{1}{2}H$ is greater than K
2. decrease p_j if $\frac{1}{2}H$ is less than K

Before an interpretation can be made of the above results, the effect of the manufacturing component of price must be considered. The term involving the Lagrange multiplier is the foundation of the sufficient condition for optimality which will adjust the independent variable so that the constraint will be exactly satisfied. If, in fact, the constraint is not tight, the term involving the Lagrange multiplier will adjust the value of the independent variable away from optimality until the constraint is exactly satisfied. The manufacturing component of price is such a term. The sign of the manufacturing component of price will reveal whether or not the manufacturing constraint is tight. If the price is decreased by the manufacturing component of price, the expression is requiring that all available manufacturing capacity be utilized, resulting in a lower total profit. The numerical example in Chapter VI illustrates this effect. The following conclusions may be drawn:

If the manufacturing component of price reduces p_j then $\frac{1}{2}H$ is less than K, hence, optimum profit will be reached without utilizing all available capacity.

If the manufacturing component of price increases p_j then $\frac{1}{2}H$ is greater than K, hence, manufacturing capacity is a constraint and must be fully used to realize optimum profits.

The above indicates that prices should not be reduced in order to utilize excess manufacturing capacity unless the available capacity is more than twice as great as that which would be required to satisfy demand if $p_j = C_j$.

It is also quite interesting that the above conclusions indicate the optimum size of a plant is such that $\frac{1}{2}H = K$. In other words, if an entrepreneur is considering building a new plant or increasing the capacity of an existing plant, the new plant should have enough capacity to produce one-half of the demand which would result if $p_j = C_j$. This result is at first surprising since the fixed cost of the plant has no effect on the optimum size, however, this is reasonable since the criteria for optimization is profits, not rate of return on investment.

The value of λ has a physical meaning. The dimensional analysis which follows demonstrates this meaning:

The denominator of λ is:

$$\frac{1}{2} \sum_j \sum_i r_j^{\,2} \beta_{ij}$$

Since r_j has dimensions of [hours] per [quantity] and β_{ij} has dimensions of [quantity] per [dollars], the dimension of the denominator of λ is:

$$\frac{[\text{Hours}]^2 \quad [\text{Quantity}]}{[\text{Quantity}]^2 \; [\text{Dollars}]}$$

or

$$\frac{[\text{Hours}]^2}{[\text{Quantity}] \quad [\text{Dollars}]}$$

Since the dimension of the numerator has been shown to be [hours], the dimension of λ is

$$\frac{[\text{Hours}] \; [\text{Quantity}] \; [\text{Dollars}]}{[\text{Hours}]^2}$$

or

$$\frac{[\text{Quantity}] \quad [\text{Dollars}]}{[\text{Hours}]}$$

or

$$\frac{[\text{Revenue}]}{[\text{Hours}]}$$

In other words, λ is an indication of the change in revenue per hour of production capacity.

In the next chapter salient points which have been previously covered will be demonstrated by a numerical example.

CHAPTER VI

NUMERICAL EXAMPLE

A numerical example will be used to demonstrate the use of the results of the models which have been optimized. There are two products and two sales regions with the following properties:

1. Variable cost for product A=$ 5.00/1000 units
2. Variable cost for product B=$15.00/1000 units
3. Production rate of product A = 700 units/hour
4. Production rate of product B = 200 units/hour
5. There are 200 hours of manufacturing capacity available.
6. The demand schedules for the two products and two regions are as follows: p is expressed in dollars per 1000 units and q is expressed in thousands of units.

	Regions 1	Regions 2
Products A	$q_{A1} = 150-15\ p_{A1}$	$q_{A2} = 50-5\ p_{A2}$
Products B	$q_{B1} = 50-2\ p_{B1}$	$q_{B2} = 40-2\ p_{B2}$

Figure 2.--Demand Functions

The parameters used in the models have the following values:

$\alpha_{A1}=150$	$\beta_{A1}=-15$	$r_A=1/.7=1.429$ hrs/1000 units
$\alpha_{A2}=50$	$\beta_{A2}=-5$	$r_B=1/.2=5$ hrs/1000 units
$\alpha_A=200$	$\beta_A=-20$	$C_A=\$\ 5.00$
$\alpha_{B1}=50$	$\beta_{B1}=-2$	$C_B=\$15.00$
$\alpha_{B2}=40$	$\beta_{B2}=-2$	$K=200$
$\alpha_B=90$	$\beta_B=-4$	

The optimum prices if manufacturing capacity were not a constraint would be

$$p_{ij} = -\frac{\alpha_{ij}}{2\beta_{ij}} + \frac{C_j}{2}$$

$$p_{A1} = -\frac{150}{2(-15)} + \frac{5}{2}$$

$$= 5 + 2.50 = 7.50$$

$$p_{A2} = \frac{-50}{2(-5)} + \frac{5}{2}$$

$$= 5 + 2.50 = 7.50$$

$$p_{B1} = \frac{-50}{2(-2)} + \frac{15}{2}$$

$$= 12.50 + 7.50 = 20.00$$

$$p_{B2} = \frac{-40}{2(-2)} + \frac{15}{2}$$

$$= 10 + 7.50 = 17.50$$

The quantities demanded at these prices would be

$$q_{A1} = 150 + (-15)\ 7.50$$
$$= 150 - 112.50 = 37.50$$
$$q_{A2} = 50 + (-5)\ 7.50$$
$$= 50 - 37.50 = 12.50$$
$$q_{B1} = 50 + (-2)\ 20$$
$$= 50 - 40 = 10$$
$$q_{B2} = 40 + (-2)\ 17.50$$
$$= 40 - 35 = 5$$

The production capacity required to satisfy this demand would be

$$\sum_j r_j \left(\sum_i q_{ij}\right)$$

$$1.429(37.50 + 12.50) + 5(10+5)$$
$$1.429\ (50) + 5(15)$$
$$71.45 + 75$$
$$146.45$$

The profit which would result from marketing at these prices would be

$$\sum_j \sum_i (p_{ij} - c_j) q_{ij}$$

$$(7.50-5)\ 37.50+(7.50-5)\ 12.50+(20-15)\ 10+(17.50-15)5$$
$$2.50(50) + 5(10) + 2.50(5)$$
$$125 + 50 + 12.50$$
$$187.50$$

The operation of the firm at this policy would result in a profit of $187.50 and 53.55 hours of unused manufacturing capacity.

It is interesting to see what would happen if the firm applied the results of Model I without first checking the manufacturing capacity constraint

$$\lambda = \frac{-K + \frac{1}{2}\sum_j\sum_i r_j c_j \beta_{ij} + \frac{1}{2}\sum_j\sum_i r_j \alpha_{ij}}{\frac{1}{2}\sum_j\sum_i \beta_{ij} r_j^{\,2}}$$

Hence

$$\frac{1}{2}\sum_j\sum_i r_j c_j \beta_{ij} = \frac{1}{2}\left\{1.429(5)(-15-5)+5(15)(-2-2)\right\}$$
$$= -\frac{1}{2}\left\{142.9 + 300\right\}$$
$$= -221.45$$

$$\frac{1}{2}\sum_j\sum_i r_j \alpha_{ij} = \frac{1}{2}\left\{1.429(150 + 50) + 5(50 + 40)\right\}$$

$$= \frac{1}{2}\left\{285.50 + 450\right\}$$

$$= \quad 367.90$$

$$\frac{1}{2}\sum_j\sum_i r^2\,\beta_{ij} = \frac{1}{2}\left\{(1.429)^2(-15-5)+(5)^2(-2-2)\right\}$$

$$= \frac{1}{2}\left\{(2.042)(-20) + 25(-4)\right\}$$

$$= \frac{1}{2}\left\{-40.82 - 100\right\}$$

$$= \; - 70.41$$

Therefore

$$\lambda = \frac{-200 - 221.45 + 367.90}{-70.41} = \frac{53.55}{70.41} = .7605$$

For Model I using the following formula for optimum price:

$$p_{ij} = -\frac{\alpha_{ij}}{2\beta_{ij}} + \frac{c_j}{2} - \frac{r_j\lambda}{2}$$

$$p_{A1} = -\frac{150}{2(-15)} + \frac{5}{2} - \frac{1.429(.7605)}{2}$$

$$= 5 + 2.5 - .54$$

$$= \quad \$6.96$$

$$p_{B1} = \frac{-50}{2(-2)} + \frac{5}{2} - \frac{5(.7605)}{2}$$

$$= 12.50 + 7.50 - 1.90$$

$$= 20 - 1.90 = \$18.10$$

$$p_{B2} = \frac{-40}{2(-2)} + \frac{15}{2} - \frac{5(.7605)}{2}$$

$$= 10 + 7.50 - 1.90$$

$$= \$15.60$$

In order to determine the quantity sold of each product in each region the following is applied:

$$q_{ij} = \alpha_{ij} + \beta_{ij}p_{ij}$$

$$q_{A1} = 150 + (-15)(6.96)$$

$$= 150 - 104.40$$

$$= 45.60$$

$$q_{A2} = 50 + (-5)(6.96)$$

$$= 50 - 34.80$$

$$= 15.20$$

$$q_{B1} = 50 + (-2)(18.10)$$

$$= 50 - 36.20$$

$$= 13.80$$

$$q_{B2} = 40 + (-2)(15.60)$$

$$= 40 - 31.20$$

$$= 8.80$$

Hence profit is

$$\pi_1 = (6.96-5.00)45.60+(6.96-5.00)15.20+(18.10-15.00)13.80 +(15.60-15.00)8.80$$

$$= 1.96(60.80) + 3.10(13.80) + .60(8.80)$$

$$= 119.17 + 42.78 + 5.28$$

$$= 167.23$$

Therefore, by imposing the artificial restriction that all the available manufacturing capacity must be used, the firm reduced its profits by $20.27, over 10%!

If only 125 hours of manufacturing capacity were available, the constraint would clearly be active. λ could be recalculated as:

$$\lambda = \frac{-125 - 221.45 + 367.50}{-70.41}$$

$$= + \frac{21.15}{70.41}$$

$$= .300$$

$$p_{A1} = 7.50 + \frac{1.429(.300)}{2}$$

$$= 7.50 + .21 = 7.71$$

$$p_{A2} = 7.50 + \frac{1.429(.3)}{2}$$

$$= 7.50 + .21 = 7.71$$

$$p_{B1} = 20 + \frac{5\ (.300)}{2}$$

$$= 20 + .75 = 20.75$$

$$p_{B2} = 17.50 + \frac{5\ (.300)}{2}$$

$$= 17.50 + .75 = 18.25$$

Quantities which would be demanded at those prices would be:

$$q_{A1} = 150 + (-15)(7.71)$$

$$= 150 - 115.65$$

$$= 34.35$$

$$q_{A2} = 50 - 38.55$$

$$= 11.45$$

$$q_{B1} = 50 + (-2)(20.75)$$

$$= 50 - 41.50$$

$$= 8.50$$

$$q_{B2} = 40 + (-2)(18.25)$$

$$= 40 - 36.50$$

$$= 3.50$$

Hence profit is

$$\pi_1 = (7.71-5.00)34.35+7.71-5.00)11.45+(20.75-15.00)8.50$$

$$+(18.25-15.00)3.50$$

$$= 2.71(45.80) + 5.75(8.50) + 3.25(3.50)$$

$$= 124.12 + 48.88 + 11.37$$

$$= 184.37$$

For Model II:

$$p_j = -\frac{\alpha_j}{2\beta_j} + \frac{c_j}{2} - \frac{r_j\lambda}{2}$$

$$p_A = -\left(\frac{200}{2(-20)}\right) + \frac{5}{2} + \frac{1.429(.300)}{2}$$

$$= 5 + 2.50 + 21$$

$$= \$7.71$$

$$p_B = -\left(\frac{90}{2(-4)}\right) + \frac{15}{2} + \frac{5(.300)}{2}$$

$$= 11.25 + 7.50 + .75$$

$$= \$19.50$$

q_{A1} and q_{A2} are the same as Model I since the prices are the same

$$q_{B1} = 50 + (-2)\ (19.50)$$

$$= 50 - 39$$

$$= 11$$

$$q_{B2} = 40 + (-2)\ (19.50)$$

$$= 40 - 39$$

$$= 1$$

Therefore profit for Model II is

$$\pi_2 = 124.12 + (19.50 - 15.00)\ 12$$

$$= 124.12 + (4.50)\ 12$$

$$= 124.12 + 54$$

$$= 178.12$$

The profit given up as a result of not allowing price discrimination is \$184.37 - \$178.12 = \$6.25. In Chapter V it was stated that the maximum profit given up as a result of not applying price discrimination was:

$$\pi_I - \pi_{II} = \sum_j \sum_i - \frac{\alpha_{ij}^2}{4\beta_{ij}} - \sum_j - \frac{\alpha_j^2}{4\beta_j}$$

or

$$= \frac{(150)^2}{60} + \frac{50^2}{20} + \frac{50^2}{8} + \frac{40^2}{8} - \frac{200^2}{80} - \frac{90^2}{16}$$

$$= \frac{22500}{60} + \frac{2500}{20} + \frac{2500}{8} + \frac{1600}{8} - \frac{40000}{80} - \frac{8100}{16}$$

$$= 375 + 125 + 312.50 + 200 - 500 - 506.25$$

$$= 512.50 - 506.25$$

$$= 6.25$$

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS FOR CONTINUING RESEARCH

Initial Policy

As stated earlier, the level of competence of most firms is not great enough to allow the initial application of any of the models derived in Chapters III and IV of this dissertation. Also, the data necessary to estimate the demand functions is not available in most firms. The researcher must initially apply policy which is more easily understood by the using organization and which does not require as much input data.

The purpose of studying the components of the optimum price was to make generally applicable policy statements which would improve profits while cultivating the firm's level of competence and which would not require complete definition of demand schedules as input data. While these policy statements will not obtain optimum profits, they will improve profits while mechanisms are established for collecting the necessary data to apply a more sophisticated model in the future. The statements of fact provide the foundation for pricing policies with a low level of sophistication.

1. Optimum prices are constructed from three distinct components which are the demand component, cost component and manufacturing component of price.

2. The demand component of price is the only component of an optimum price which adjusts prices among regional sales area to increase profits by price discrimination.

   a) The size of the increase in profits as a result of price discrimination is primarily dependent upon the differences in total demand among sales regions. Total demand is used here to mean the demand which would result if no price was charged for the goods.

   b) If the slope of the demand function for the different regions is known the following test can be applied to determine if price discrimination would result in higher profits:

   If the quantity $[(q/\beta)-p]$ is constant for all regions, profits would not be improved through price discrimination. p, q and β are respectively the price, demand and slope of the demand curve for a region.

   c) A portion of the potential profit increases as a result of price discrimination can be enjoyed without complete knowledge of the demand functions. If a median price, $\overline{P}$, and the maximum difference in prices among regions, Δ , can be established, the following iterative process can be used to adjust prices toward optimum:

$$p'_i = \overline{P} + \Delta \left(\frac{\frac{q_m}{\beta_m} - \frac{q_i}{\beta_i} + p_m - p_i}{\frac{q_k}{\beta_k} - \frac{q_l}{\beta_l} + p_k - p_l} \right)$$

p'_i is the adjusted price and p_i is the existing price in region i. The k, l, and m subscripts refer to the regions which have the minimum, maximum and median existing prices respectively. The q, p, and β are the existing demand, price, and slope of the demand function respectively. Δ should be established very small and gradually increased with time.

3. The cost component of price is equal to one-half of the variable manufacturing cost.

   a) A cost plus fixed mark-up percentage pricing policy can never result in optimum prices unless the demand functions are proportional for all products.

   b) Mark-up percentages should be adjusted when variable costs change. In particular, if an optimum mark-up percentage is in effect and the variable costs of the product change, the new mark-up percentage which will result in optimum profits can be calculated as follows:

$$d^*_2 = d^*_1 + \Delta d$$

   and

$$\Delta d = (d^*_1 + \tfrac{1}{2}) \left(\frac{\frac{\Delta c}{c}}{1 + \frac{\Delta c}{c}} \right)$$

   d^*_2 is the new optimum mark-up percentage and

   d^*_1 is the old optimum mark-up percentage.

   Δd and Δc are the changes in mark-up percentage and variable manufacturing cost respectively. C is the old variable manufacturing cost. This would be an excellent method of establishing $\overline{P}$, the median price used in the iterative process for adjusting prices to allow price discrimination.

4. The manufacturing component of price adjusts the demand and cost components according to the characteristics of the firms manufacturing process to reach an optimum price for that particular firm.

a) If the available manufacturing capacity is more than would be required to satisfy half of the demand that would result if only variable costs were the established prices, the utilization of full capacity will result in a reduction of profits. This is true because prices must be reduced below optimum to move the additional product. If a firm finds itself in the above position, an addition to the product line should be considered.

b) If a firm is considering a plant expansion or the construction of a new plant, the resulting capacity should be such that:

$$\frac{1}{2}H=K$$

H is the capacity required to produce the demand which would result if the price charged were the variable manufacturing cost. K is the new plant capacity.

The above facts coupled with the models developed in Chapters III and IV should provide a sufficient range of model sophistication to enable the researcher to fit a pricing policy to a particular firm. The most salient consideration in fitting the pricing policy to the firm is the cultivation of the firm's level of competence. The initial level of sophistication of the model from which pricing policy is determined must be effective and should not be high enough to be efficient.

Recommendations for Continuing Research

The greatest need in the area of pricing policy determination is the measurement of the demand function. It is clear that all models developed in this dissertation could be applied if the parameters are known. The difficulty which arises when determination of demand functions is attempted is that macro and micro-economic market conditions are changing as the data is being collected. It is not known if these changes in market conditions significantly affects the demand. Hence, when a new demand is measured after a price shift, the portion of the new demand which is a result of the changing market conditions can not be differentiated from the change in demand as a result of the price shift.

The method used by Edward Mahoney, [14], for estimating the demand functions tends to eliminate the effects of the dynamic market. Mr. Mahoney estimates total industry demand and the firm's market penetration independently. The firm's demand is then estimated as the product of the two estimates. It is Mr. Mahoney's contention that the primary effect of price changes is measured by market penetration. If this contention could be verified, the models derived herein could be applied with the slight change that the demand function would be a quadratic rather than a linear equation. The author would welcome an opportunity to apply the results of this dissertation under these conditions.

The probabilistic models, which were considered in Chapter IV, introduce an interesting field of study which is untouched in the author's opinion. Because optimum expected profit is a function of the z's, the planned production mix, and the p's, the prices, the models can be used to investigate the interfaces of the heretofore mutually exclusive areas of Marketing and Production. The inclusion of a shortage cost function for each regional area would allow the researcher to study the effects of production mix, inventories and sales policies on expected profits.

Investigation of the probabilistic models would also reveal the effects of errors in demand estimates on profits. This relationship is very important indeed since the sensitivity of profits to forecast errors will determine the effort which should be spent on improving demand forecasts.

A shortcoming of the models developed in this dissertation is the criteria through which optimization is achieved. Profits are rates of growth and hence must be measured over some period of time. The period of time over which profits are maximized in this dissertation is the period of time between price revisions. This could result in unstable profits, i.e., 10% in the first period, -4% in the second period, 30% in the third period, etc. Such a result may be undesirable to the Directors of the firm. This problem could be alleviated by the addition of an additional constraint that the change in profits should not exceed some predetermined constant. This could be expressed mathmatically as follows:

$$\sum_j \sum_i (p_{ij} - C_j)\, q_{ij} \leq L - C_F$$

where L is the predetermined limit on profit changes.

Another fruitful area for research would be the development of pricing policies with low level of sophistication and which do not require estimation of the demand functions as input data. This was done for the deterministic models in Chapter V of this dissertation. In the author's opinion, the information obtained thereby is the most valuable contribution in this dissertation. A similar study of the probabilistic models or other models which might be constructed may result in useful extensions to the art of price determination.

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