AN APPLICATION OF INTEGER LINEAR PROGRAMMING TO LIGHT OIL TERMINAL - TRUCK OPERATIONS

By

TUNCAY AKOGLU

Bachelor of Science

Middle East Technical University

Ankara, Turkey

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ose of Study: The purpose of this study is to determine the optim mix of company trucks and common carriers to be used for the operations of East Chicago, Indiana, light oil (gasoline, heating oil, etc.) terminal of Cities Service Oil Company (CITGO) in order to satisfy daily fluctuating demand requirements for the area servicec and to minimize the total cost of deliveries over a specific operation period. An Integer Linear Programming Model, with the objective of cost minimization which is subject to constraints related t demand, resource availability and company policy conditions, was developed and solved for this purpose.

lusions: The optimum mix of truck fleet in terms of daily strategies over a week operation period was determined for East Chicago terminal. The application of the model to any CITGO truck terminal was explained through developing a decision chart for management us The interpretation of the optimal solution and the ranging analyses which examine the influence of changes in cost, demand and resource availability - conditions were also presented to facilitate use of the model in decision making process.

SER'S APPROVAL Mitchell O. Locks

AN APPLICATION OF INTEGER LINEAR PROGRAMMING

TO LIGHT OIL TERMINAL - TRUCK OPERATIONS

Report Approved:

Report Adviser

Head, Department of Administrative Sciences

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CHAPTER I

INTRODUCTION

Description of a Light Oil Terminal

[T]he problem studied in this paper is concerned with the CITGO [...]ting Operation of a light oil terminal. A light oil terminal con- [...]s a large inventory of light oils (gasoline, heating oil, etc.) [...]h have been shipped to the terminal from the refinery by barge or [...]ine. From the light oil terminal the product goes to the retailer [...]timate user by truck. The user may pick up the product with his [...]trucks or common carrier trucks under his direction. The user may [...]ase the product F.O.B. destination where the terminal must provide [...]transportation to the user. The terminal may or may not use compan [...]s, depending on whether or not it is more economical to operate [...]ny trucks versus the common carrier rate at a given location. [U]sually the terminal will operate company trucks to deliver to its [s]ervice stations. The oil company must make a decision as to mix o [...]ny trucks and/or common carrier usage at a given terminal. There [...]eaks and valleys of demand for company directed trucking operation [...]usly the oil company cannot afford to have enough company trucks o at a given location to cover the maximum demand in a given period. [C]ompany may decide to use part company trucks and part common [...]er.

Approach Used in the Study

The problem of determining the optimum mix of company and common ier trucks in order to minimize cost of operations under demand, urce availability and company policy conditions is an integer line ramming problem. The optimum capacity in terms of the number of ks to be utilized should be determined as integers. The optimum tegy cannot be stated as 1.3 company trucks and 2.7 common carrier day. This study, therefore, uses mathematical optimization tech- e of integer programming.

The optimal solution and related optimum strategies are subject to ges in demand, cost, resource availability and company policy cond s. In order to check the sensitivity of the optimal solution ined, a ranging analysis is employed over the cost coefficients an tants of the model.

Summaries of the Chapters

A short background on linear and integer programming and survey of rature on the theory and applications of IP are summarized in ter II. Chapter III presents a description on the nature and use athematical Programming System Extended (MPSX) which is utilized i study. The control language program and data set for the problem rovided separately in the Appendix to the study.

Chapter IV discusses the definition of the problem studied in this r and presents the IP model formulation utilized to solve it.

Analysis of data on East Chicago terminal to calculate the coeffi- ts and the constants of the model is presented in Chapter V.

ıumerical model constructed in this way is solved by computer :zation and results are summarized in Chapter VI. Interpretation o ›ptimal solution in terms of daily optimum strategies is given in :er VII together with the sensitivity analysis. Chapter VII also ıdes an analytical chart to aid management use of the model. Limi- ›ns of the study and of the model developed are discussed as a last .on in Chapter VII.

CHAPTER II

SURVEY OF THE LITERATURE

,inear programming is a mathematical optimization technique designe .alyze the potentialities of alternate activities and to choose : that permit the best use of resources in the pursuit of a desir- objective. Linear programming models handle situations where an :tive should be minimized or maximized under linear constraints . are specified as the expressions of resource availabilities or of .tions to be satisfied in the realization of the stated objective. nteger programming is different than regular LP models only in of the requirement that variables in the problem and the optimal ion must be integer valued. Although the simplex method is the ing point to solve the integer programming problems, certain tech- s should be employed to reach the optimal integer solution after a nteger solution has been obtained.

inear programming and integer programming models, because of their sified capabilities, have many uses. They can be used to analyze al, raw materials, manpower, plant and storage facilities, and late their findings into minimum cost or maximum profits for users may be employed to allocate, assign, schedule, select or evaluate ver possibilities limited resources possess for different jobs. applications include to distribute, control, order, budget, bid, trim, price, purchase and plan in order to minimize costs or mize profits related to these operations subject to limited resourc or conditions to be satisfied. They can deduce the most profitable od of transporting foods from plant to warehouse to outlet.

Integer programming as a branch of LP has several versions. "Assig problem" deals with assigning men to machines, machines to jobs ar eneral with assignment and scheduling problems. "Transportation lem" is used to optimize transporting goods from factory to ware- e, from supply centers to demand centers, etc. "Zero-one program- " is a technique to handle integer programming models where the prc is to decide on an activity (or activities) to engage in or not.

Transportation problem was first interpreted by Hitchcock [38]. nans [42] studied and applied the same topic in 1947. "Hungarian od" to solve assignment problems was discussed by Flood [21]. nans and Beckmann's study [43] on the location of economic activiti Manne's study [51] on the job-shop scheduling problem are the clas- on this type of application. Baumol and Wolfe [8] utilized IP to house location problem. Machol [49] applied assignment problem oach in one of the classical studies, too. There are three classic ies of application of LP to oil industry. In 1951, Charles, Cooper Mellon made a study [13] to plan and program interdependent activi- in blending aviation gasolines in an integrated oil company. G. H nds, in 1953, applied LP to optimize refinery operations [58]. And 956, Alan S. Manne made a study [50] for scheduling petroleum refir operations through the utilization of linear programming.

Integer programming developed in theory and applications through th nsive contributions made in 1960's. Two main techniques were devel to handle integer programming problems. These are employed to rea optimal integer solution after a non-integer optimum solution has obtained. Gomory [34] developed the "cutting-plane" algorithm. ach-and-bound" method was originated by Land and Doig [44]. Comially used LP systems which can efficiently handle integer conints are usually designated as Mixed Integer Programming (MIP) ems. In these cases, a branch-and-bound solution with the integer ables is used to modify the optimum solution with non-integer ables.

Balas [2] with his "Implicit Enumeration-Algorithm" developed a od to handle special cases of IP called Zero-one programming. frion made contributions and improvements in this method [27, 28]. inkel and Nemhauser also developed [23] an enumerative algorithm fc partitioning problem in zero-one programming. A new approach to -one integer programming was also formulated by Cabot and Hurter . "Group Theoretic Algorithms" for integer programming problems been discussed by Shapiro [57].

Besides these above new approaches, Gomory's and Land and Doig's inal methods have been subject to many theoretical and practical ies. Glover, with his "primal integer programming algorithm" [31, extended the cutting plane method of Gomory. Land and Doig's h-and-bound technique was further studied by Lawler and Wood [45], iore and Nemhauser [11], Beale and Small [9].

A comprehensive survey of methods and uses of integer programming been discussed by Balinski [3, 4] and Dantzig [17]. Geoffrion and en presented a more recent review [29] in integer programming algo is. The present state and complete survey of the techniques of ger programming have been provided by Zionts [63].

'he above developments in theory and in computer utilization have ı the opportunity to utilize many diversified applications of inte-›rogramming in various studies. Woolsey [62] discusses four real : applications of IP: operator scheduling at a telephone company; ;n program of cutting stock for reinforcing bars; capital budgeting :esearch and development; and allocation of sales districts to ›men.

'he assignment problems and location studies is one of the areas ısive applications take place. Some of the examples may include: ; and Ray's study on capacitated facilities location problem [18] b :h-and-bound algorithm; Efroymson and Ray's study [20] on location .em dealing with assignment of facilities. Branch-and-bound algo-ı was also used by Gavett and Plyter [25] in their study on a loca-·assignment type problem.

nteger programming has been utilized in various scheduling problem r, Giglio and Glaser [61] made a study of preventive maintenance uling by IP. Pritsker, Watters and Wolfe [54] studied zero-one er programming of multiproject and job shop scheduling problems ding resource constraints such as due dates, job splitting, re-e substitutability and performance requirements. Ignall and ge [39] employed branch-and-bound technique to flow-shop schedul-roblems. A problem of minimum change-over scheduling of several cts on one machine was studied by Glassey [30]. Greenberg [37] ed branch-and-bound solution through mixed integer programming to , m machine scheduling problem.

elivery, transportation and truck assignment problems have been ed with the application of integer programming. Balinski and t [5] used IP for truck delivery problem. Determining a minimum kation fleet through an integer programming model has been studied Esope and Lefkowitz [19]. Rao and Zionts made a study [55] of ating transportation units to alternative trips at a minimum se under a set of trip commitments and availability of units. [64] utilized IP in a truck assignment problem similarly.

arious interesting subjects and real life problems were attacked b tilization of integer programming. Little [46] studied the syn- ization of traffic signals by mixed-integer linear programming. s, ReVelle and Lynn presented a model [48] for determining the t of wastewater treatment required to achieve at minimum cost for articular set of stream within a river basin. A study was made by es, DeVoe, Learner and Reinecke [14] to develop a model for media ing accounting for duplicating audiences over a variety of time ds. Senju and Toyoda [56] applied zero-one programming to choose jobbing firm the optimal package of orders from potential ones restrictions on available resources, working time of different ities, number of specialists and materials. For some diversified cations of integer programming, the following references and stud- an be pointed out. Kalvaitis and Posgay [40] employed mixed inte- rogramming to the direct mail industry. An approach utilized by [41] determined an efficient set of control methods for air tion abatement. Baugh, Ibaraki and Muroga [6] used Gomory's all er algorithm to design optimum logical networks for digital com- s. Integer programming was employed to study reliability optimiza- problems by Tillman and Liittschwager [59]. Project management, gh a method for simultaneous planning, scheduling and control of cts, was discussed by Crowston and Thompson [15] in their integer amming approach. A zero-one integer programming model was applied aring, Swart and Var [26] to determine the optimal investment y for tourism sector in Turkey.

he above references, nevertheless, are not a complete list of the cations of integer programming. The widespread use of computer ms and solution techniques gives the opportunity of handling fairl scale problems. This ability and their effect on reducing con-nce time for integer solutions (as discussed in [22, 63, 62]) will ve the expanding trend of integer programming applications on real problems.

CHAPTER III

NATURE AND USE OF MPSX

he discussion in this chapter is based upon Chapter 5.S of [47]. PSX is the advanced version of MPS/360 (Mathematical Programming m/360) [65] which is used to obtain solutions for LP problems. is composed of a set of procedures and subroutines to solve intege: amming problems via its mixed-integer programming feature. The m can be used to process very large problems with hundreds of con- nts and variables. The problems can have minimization or maximiza· objectives with a mixture of constraints as $\leq$ inequalities, $\geq$ alities or equalities. Upper and/or lower bounds can be imposed any of the variables or row constraints.

he strategy for solving an integer programming problem is the ed execution of a series of the procedures and subroutines of the n. The user conveys the proposed strategy to MPSX via the MPSX ol language. The procedure, called statement of the control lan- , calls the LP procedures and transfers arguments to them. MPSX ol statements are preprocessed by the control program COMPILER. is the first step of an MPSX job. Both the syntax and use of the LER are fully described in the MPSX Control Language User's Manual 57].

ter processing by the compiler, the control language problem is ed out under the control of the EXECUTOR as a second job step.

ded in each job step are the data definition (DD) statements. Eacl atement describes a single device and specifies the type and other rties to be used by the LP procedures. The COMPILER is called by X.2.SYSIN DD* while the EXECUTOR performs the related steps by X.1.SYSIN DD* statement. The control language and data set used i roblem of this project is given in the Appendix to the study.

he LP procedures of MPSX use the bounded variable technique and ys the revised simplex method. Revised simplex method is based on act that the entire work matrix can be partitioned and expressed a ction of the basis matrix. Basis matrix can be defined through th fication of simplex method. If there are m constraints (rows) in onstraint matrix and these are linearly independent, then there is of m columns (variables or vectors) which are also linearly inde- nt. Hence, any right-hand side (constraint constant) can be ex- ed in terms of these m columns. This is called a basis. The sim- method uses these basic solutions, stepping from one to another (b nging one column in the basis with one column not in the basis on step or iteration), until a solution (called a basic feasible solu- is obtained that meets all of the criteria specified by con- nts, including the requirement that all column values be gative.

fter the non-integer optimal solution (satisfying above require- with a minimum or maximum value of objective function) is found, implex method steps along, examining one branch at a time in the ion tree in order to check for integer feasible solutions. To do MPSX calls the macro-system named OPTIMIX. Through the bounded ble technique, integer solutions in each possible branch are ned by this system to find one that satisfies the requirements and the value of the functional (or objective) row be a maximum or um under integer valued structural variables. This is called the al solution.

ot all LP problems have an optimal solution. If there is no solu- at all in nonnegative variables, or none that keeps the variables n their specified bounds, the LP problem is said to be "infeasible." feasible solution is found, but the constraint rows do not confine alue of the functional row to finite values, the LP problem is saic "unbounded."

fter the integer optimal solution is found, summary of the search ss and values of the variables in the optimal solution are printed y MPSX procedures. These outputs include the analyses of row and a vairables and value and effect of dual variables. RANGE proce- included in the control program performs sensitivity (ranging, ptimality) analysis on the values of the objective function coeffi- s and constraint constants. Detailed discussion of these outputs nalyses is given in Chapter VII through presenting the results of application to this study.

CHAPTER IV

METHODOLOGY

Definition of the Problem

he problem to be solved is determining the optimum mix of company trucks and common carriers over a specific operation period in to minimize total cost of truck operations (deliveries) at East ;o light oil terminal.

ince the number of company owned trucks to be used and common ers to be hired should be determined as integer and since the pro- is to minimize the total cost of truck operations under constraints ; from cost, demand and policy conditions, the model to obtain our ion utilizes Integer Linear Programming technique.

ır objective function in this problem is to minimize the total cost ;ht oil deliveries by company trucks and common carriers over a :ic operation period. The specification of the operation period tc ısidered for the problem will be discussed in the "formulation" s.

›nstraints setting boundaries to the above objective are:

. Daily demand requirements in the area serviced by East Chicago terminal.

. Limits specified on the number of deliveries, since a truck can handle a certain number of deliveries in a given work day.

- Number of company owned trucks (present truck fleet).
- Number of common carriers available to be hired in a given day.
- Capacity of a truck associated with the load that can be delivered per trip.
- Number of trips and trucks are integer variables.
- The capacity of the terminal from the supply side is not considered as a constraint in this problem.

Formulation of the Model

his section deals with the formulation of a general Integer Linear amming model to solve the problem defined earlier in terms of its tive function and constraints presenting boundaries to it.

irst consideration in the formulation process is deciding on the tion period over which the variables of the model, the objective of ninimization and the related constraints are to be built. After point has been decided on, variables, objective function, related raints and bounds to be established will be specified and then lated.

xamination of the historical data taken from the "Truck Production' Driver Report" files of Cities Service Oil Company in Tulsa, Okla- gives the following observation. The demand requirements faced by ruck operations at East Chicago terminal show a day-to-day, week- ek and month-to-month fluctuating pattern with considerable peaks rtain days of the week and in certain months of the year. A model lve the problem defined earlier should cover a reasonable operation l to reflect influences coming from the above fact.

ith this idea in mind, the model in this study was formulated to ize weekly cost of operations under daily fluctuating demand condi- for a six-work-day week. In this way, rather than having a one- ptimization model, weekly optimization will present a solution handles daily demand fluctuations and shows day-to-day changes in ptimum mix.

ne following short verbal discussion about the model is presented idea on the model formulation process and to familiarize the r with the concepts used in the model.

ariables used in the model represent the number of trips (deliver- by company trucks or by common carriers in a given day. The possi- y of overtime operations is also considered as a comparison to use mmon carriers. A constant which measures the cost of a delivery lation to distance, time and load per trip should appear in the tive function which is to be minimized. Since a truck can handle tain number of deliveries in a work day, optimum numbers of company s and common carriers in a given day can be determined from the al number of trips found in the solution.

nere are three types of structural variables in the model. These amely the number of trips by company owned trucks in straight-time, umber of trips by company trucks in overtime, and the number of by common carriers in a given day.

esides the costs associated with the above variables, there is er type of cost to be considered in the truck operations and to be led in the objective function of our model. This is the cost lated with the trips not made by company trucks in straight-time : in overtime. This cost which is called as "penalty cost" in our is composed of the fixed costs that have to be incurred regardless ether a delivery is made or not by a company truck. This discus- is specially important when the competitiveness of company trucks ommon carriers is compared in terms of their costs. A decision to ne less company truck or rather to make one less trip by a company in a given day has to have influence on the objective function. ncluding this discussion into the picture, the number of variables e formulation increases to five per day by adding a variable for not made by company trucks in straight-time and another variable rips not made by company trucks in overtime.

ne following expression is the portion of the objective function ne first day of the week.

$$C_1X_1 + C_2X_2 + C_3X_3 + C_4X_4 + C_5X_5 \qquad (4.1)$$

: Total number of deliveries (trips) to be made by all company trucks in a 20-hour work day (straight time).

: Total number of trips not made by company trucks in straight time.

: Total number of trips to be made by all common carriers in one day (20-hour work day).

: Total number of trips to be made by all company trucks in overtime.

: Total number of trips not made by company trucks in overtime.

.l the above variables refer to the first day.

: Unit cost per delivery by a company truck in straight time.

: Penalty cost per delivery not made by a company truck in straight time.

3: Unit cost per delivery made by a common carrier.

4: Unit cost per delivery made by a company truck in overtime.

5: Penalty cost per delivery not made by a company truck in overtime.

xpression (4.1) is the portion of the objective function for the day of the week and it is to be minimized. Since there are six lays in the operation period over which our model is based on, the number of variables in the objective function is 30 (five varia- per day x six days).

ierefore, the objective function of our model is:

$$\text{Minimize } Z = \sum_{i=1}^{30} C_i X_i \qquad (4.2)$$

ı expression (4.2), i between one and five denotes first day varia- 6-10 second day, 11-15 first day, and so on.

onstraints setting boundaries to the above objective function can ouped mainly in four sets of expressions.

irst, the total number of trips to be made by company trucks in ght time cannot exceed a maximum amount, since a company truck can a specified* number of trips which can be made by a company truck raight time work day. If maximum number of trips which can be made company truck in straight time work day is denoted by b and if the of company owned trucks available is denoted by k, then the total of deliveries that can be made by all company trucks in straight annot exceed b x k.

This and other arguments as to the specified limits referred to ghout this section will be quantified in the following chapter the analysis of data.

ıerefore, as an example for the first day of the week:

$$X_1 \leq b \times k \qquad (4.3)$$

ıen the number of trips not made by company trucks is also consid- expression (4.3) becomes

$$X_1 + X_2 = b \times k \qquad (4.4)$$

›tally, there are six constraints similar to expression (4.4) in ›mplete model accounting for each work day in the week.

ı interesting approach of our model could be seen when expressions and (4.4) are closely examined together. X_2 has the situation ıs known as slack variable in conventional linear programming. It ;lack variable because it is associated with the portion of the :ce which is not utilized, that is, the number of deliveries that be made by company trucks in straight time but not made. That is , converts expression (4.3) into an equality. At the same time we ›ur slack variable in the objective function with an associated :y cost (Expression (4.1)). This treatment of slack variable is :ent than examples and textbook treatment given for conventional : Programming models. The different approach utilized in our model to force the slack variable's influence on the objective function ·ms of its cost coefficient.

ıe second set of constraints is related to number of common carrier and availability of common carriers. The total number of trips made by all common carriers in a work day cannot exceed a maximum :, since a common carrier can handle a specified number of deliv- per day. If maximum number of deliveries that can be made by a ı carrier in a 20-hour work day is b and if the number of common ers available to hire in a given day is p, then the total number of eries that can be made by all common carriers in a work day cannot l b x p.

ierefore, as an example for the first day of the week:

$$X_3 \leq b \times p \qquad (4.5)$$

.y, there are six constraints similar to expression (4.5) in the ete model accounting for each work day in the week.

ie third set of constraints is related to overtime operations. The number of deliveries to be made by company trucks in overtime can- : more than a maximum amount, since a company truck can make only a :ied number of deliveries in overtime operation. If maximum number .iveries which can be made by a company truck in overtime is denote and if the number of company trucks available is denoted again by en the total number of deliveries that can be made by all company ; in overtime for a given day cannot exceed b' x k.

ierefore, as an example for the first day of the week:

$$X_4 \leq b' \times k \qquad (4.6)$$

$$X_4 + X_5 = b' \times k \qquad (4.7)$$

en from expression (4.7), when the number of deliveries not made by ey trucks in overtime is also considered, expression (4.6) becomes iality. This is the same discussion and treatment of slack vari- :oncept presented in detail earlier for expressions (4.3) and (4.4) .ation to constraints for straight time operations.

ie complete model includes six constraints similar to expression accounting for each work day in the week.

[he fourth and final set of constraints is related to daily demand irement. In order to satisfy the daily demand requirement, the sum eliveries made by company trucks (in straight time and in overtime) y common carriers should at least be equal to the daily demand.

[f the capacity of a truck per delivery is denoted by a, and if the id for a given day is denoted by d, then this above condition can b essed as (example of the first day of the week):

$$aX_1 + aX_3 + aX_4 \geq d_1 \qquad (4.8$$

Expression (4.8) formulates the demand constraint for the first day ie week. In the complete model, there are six constraints similar xpression (4.8) accounting for each of six working days.

'our sets of constraints for six days make up the total number of :raints in the model as 24. Therefore, the model formulated in thi ion tries to solve the problem defined with an objective function i iriables and subject to 24 constraints. In addition, all variables .d be determined as integers.

Jp to this point, variables and constants used in the formulation o iodel have been discussed and defined. Also, the constraints and .on of the objective function for the first day have been presented :ately in detail to enable the reader to follow the formulation pro before the complete model shown.

In order to cover weekly operation period, combining the expression :d earlier, the complete Integer LP model can be presented as follo

$$\text{Minimize } Z = \sum_{i=1}^{30} c_i X_i$$

or

$$\text{Minimize } Z = c_1X_1 + c_2X_2 + c_3X_3 + c_4X_4 + c_5X_5 + c_6X_6 + c_7X_7 + c_8X_8 + c_9X_9 + c_{10}X_{10} + c_{11}X_{11} + c_{12}X_{12} + c_{13}X_{13} + c_{14}X_{14} + c_{15}X_{15} + c_{16}X_{16} + c_{17}X_{17} + c_{18}X_{18} + c_{19}X_{19} + c_{20}X_{20} + c_{21}X_{21} + c_{22}X_{22} + c_{23}X_{23} + c_{24}X_{24} + c_{25}X_{25} + c_{26}X_{26} + c_{27}X_{27} + c_{28}X_{28} + c_{29}X_{29} + c_{30}X_{30}$$

ct to:

$X_1 + X_2 = b \times k$ | (M): $X_{16} + X_{17} = b \times k$

$X_3 \leq b \times p$ | (N): $X_{18} \leq b \times p$

$X_4 + X_5 = b' \times k$ | (O): $X_{19} + X_{20} = b' \times k$

$aX_1 + aX_3 + aX_4 \geq d_1$ | (P): $aX_{16} + aX_{18} + aX_{19} \geq d_4$

$X_6 + X_7 = b \times k$ | (Q): $X_{21} + X_{22} = b \times k$

$X_8 \leq b \times p$ | (R): $X_{23} \leq b \times p$

$X_9 + X_{10} = b' \times k$ | (S): $X_{24} + X_{25} = b' \times k$

$aX_6 + aX_8 + aX_9 \geq d_2$ | (T): $aX_{21} + aX_{23} + aX_{24} \geq d_5$

$X_{11} + X_{12} = b \times k$ | (U): $X_{26} + X_{27} = b \times k$

$X_{13} \leq b \times p$ | (V): $X_{28} \leq b \times p$

$X_{14} + X_{15} = b' \times k$ | (W): $X_{29} + X_{30} = b' \times k$

$aX_{11} + aX_{13} + aX_{14} \geq d_3$ | (Y): $aX_{26} + aX_{28} + aX_{30} \geq d_6$

and (Z): $X_i \geq 0$ and integer valued for $i = 1, \ldots, 30$.

X_i:

or $i = 1, 6, 11, 16, 21, 26$: Total number of straight time deliveries to be made by all company trucks in the first day when i = 1, second day when i = 6, etc.

or i = 2, 7, 12, 17, 22, 27: Total number of straight time deliveries not made by company trucks in the first day when i = 2, second day when i = 7, etc.

or i = 3, 8, 13, 18, 23, 28: Total number of deliveries to be made by all common carriers in the first day when i = 3, second day when i = 8, etc.

or i = 4, 9, 14, 19, 24, 29: Total number of overtime deliveries to be made by all company trucks in the first day when i = 4 second day when i = 9, etc.

or i = 5, 10, 15, 20, 25, 30: Total number of overtime deliveries not made by company trucks in the first day when i = 5, second day when i = 10, etc.

or i = 1, 6, 11, 16, 21, 26: Unit cost per straight time delivery made by a company truck in the first day when i = 1, second day when i = 6, etc.

or i = 2, 7, 12, 17, 22, 27: Penalty cost per straight time delivery not made by a company truck in first day when i = 2 second day when i = 7, etc.

or i = 3, 8, 13, 18, 23, 28: Unit cost per delivery made by a common carrier in the first day when i = 3, second day when i = 8, etc.

or i = 4, 9, 14, 19, 24, 29: Unit cost per overtime delivery to be made by a company truck in the first day when i = 4, second day when i = 9, etc.

ɔr i = 5, 10, 15, 20, 25, 30: Penalty cost per overtime delivery not made by a company truck in the first day when i = 5, second day when i = 10, etc.

: maximum number of deliveries that can be made by a truck (company owned or common carrier) in a 20-hour work day.

': maximum number of deliveries that can be made by a company truck in overtime.

: The number of company owned trucks available in the terminal.

: The number of common carriers available to be hired for the terminal.

: Capacity per truck per delivery (gallons).

: Daily demand requirement (gallons) in first day when i = 1, second day when i = 2, etc.

ɔme additional notes about the above complete model will make it : and better to picture. The first line in the objective function ;ents the first day operations, the second line represents the secıy operations, and so on. Similarly, the first group of constraint :ough D) in "subject to" section represent the first day conıts, the second group (E through H) represents the second day conıts, and so on.

ıe last necessary clarification about the model is on determining ımber of company trucks to be used in straight time and/or overtime ıe number of common carriers to be hired for a given day. Since .efined in the model represent the number of deliveries by truck .nd operation type, solution to the model will specify only the · of deliveries. In order to convert the optimum number of deli- : specified in the solution to the optimum number of trucks to be or hired, the following manipulations should be made:

:f,

X_i^* = optimum number of straight time deliveries by company trucks given in solution (i = 1, 6, 11, 16, 21, 26)

:hen

X_i^*/b (= next nearest integer) = optimum number of company trucks to be used in corresponding day (straight time).

:f,

X_i^* = optimum number of deliveries by common carriers given in the solution (i = 3, 8, 13, 18, 23, 28)

:hen

X_i^*/b (= next nearest integer) = optimum number of common carriers to be hired in corresponding day.

:f,

X_i^* = optimum number of overtime deliveries by company trucks given in the solution (i = 4, 9, 14, 19, 24, 29)

:hen

X_i^*/b' (= next nearest integer) = optimum number of company trucks to be used in corresponding day (overtime).

'he above manipulations are not more than a technicality of the to be applied to the optimum solution. That concludes our discus on the formulation of the model.

he following chapter will discuss how the coefficients, constants ounds on variables in this model are determined from the analysis oblem data on East Chicago terminal.

CHAPTER V

ANALYSIS OF PROBLEM DATA

his chapter deals with (1) calculation of the objective function coefficients, (2) specification of constants of the model (bounds uck availabilities and number of deliveries) and (3) estimation oi y and daily demand requirements. The general Integer Linear Pro- ing model presented in the previous chapter can be applied to any oil terminal truck operations of Cities Service by specifying the ant and coefficients for that particular terminal. In this sectic tempt to perform this process for East Chicago terminal.

Calculation of the Cost Coefficients

here are five types of costs in the objective function of the mode

. c_i (i = 1, 6, 11, 16, 21, 26) = Unit cost per straight time delivery made by a company truck.

ities Service Oil Company has a highly computerized processing of -date truck operations data. Originating at the driver-terminal and being evaluated at the decision-making levels of marketing- operations departments, historical data in terms of accounting an ting records are very accurate and complete.

:counting data about East Chicago terminal shows four tractors and :railers in the present fleet. One tractor unit and one trailer nakes up a truck on which our model is based. The data has every l on each tractor and trailer in terms of its costs and performanc acquisition and year to date. Performance for each unit is giver rms of gallons hauled, miles driven, hours worked, and deliveries Costs are given by the breakdown of maintenance (including fixed), operating (variable), depreciation and insurance expenses. Frc data, it is possible to calculate figures in terms of cost per mil ll as cost per gallon, cost per hour or per delivery for each unit urse, these figures are averages about what they measure.

rom the data, the hourly cost of operating each tractor and the y cost of operating each trailer were determined. Then the averag y cost of the tractors and the average hourly cost of the trailers combined together. This total figure shows the hourly cost of ting one truck for East Chicago terminal. And it is $7.05 per per hour.

his amount represents only one part of the total cost involved in a company truck for a straight time delivery. The second part from the driver costs.

he calculation of the annual cost per driver can be given as ws:

ght time payment[1] (10 hrs/day, 40 hrs/week, 52 eeks/year[2]) $13,624.C

e Benefits[3] 2,997.2

Present wage rate is $6.55 per hour. There are 2,080 hours in 52 weeks.

Which include four weeks of paid vacation, nine days of paid holi- paid birthday and five days paid sickness period.

22% of straight time payment.

itution for driver in vacation (40 hrs. x
weeks x $9.83/hr.[1]) $1,572.8

itution for sick driver (5 days x 10 hrs. x
9.83/hr.[1]) 491.5

itution for birthday (10 hrs. x $9.83/hr.[1]) 98.3

Total Annual Wage per Driver $18,783.8

ased on the above calculation, the average cost of a driver is
2 per hour.

xamination of the historical data proves that one delivery on the ge takes three hours[3] for East Chicago terminal operations.

ince a driver works 10 hours a day, he can at most handle three eries in his ten-hour operation period. Therefore,

$9.03/hr. x 10 hrs. = $90.30/day/driver

and

Driver Cost per Delivery = $30.10

ince a truck is in operation for 20 hours a day[4], one truck can e six deliveries in a 20-hour operation period. Therefore,

$7.05/hr. x 20 hrs. = $141.00/day/truck

and

Truck Cost per Delivery = $23.50

Time-and-a-half payment (overtime).

$18,783.88/2,080 hrs. = $9.03/hr.

This is a very interesting observation, because monthly averages 73 data and weekly and daily averages of February, 1974 data all ate an average of three hours per trip with a very small insignifi variance.

A truck is driven by first driver in the first shift (10 hours) ar cond driver in the second shift (10 hours).

; a result, the unit cost of making a straight time delivery by a ıy truck is \$30.10 + \$23.50 = \$53.60 (or \$54/delivery). Based on :esult, c_i (i = 1, 6, 11, 16, 21, 26) = 54.

c_i (i = 2, 7, 12, 17, 22, 27) = Penalty cost per straight time delivery not made by a company truck.

ıis penalty costs consider only those costs that have to be incurr(:hough a delivery is not made by a company truck. In a convention: ıent, they more or less correspond to what is know as fixed costs. : analysis, they include part of the depreciation and maintenance ;es, insurance and driver costs. Research on the composition of operations expenses for East Chicago terminal provides enough evi· that an assumption of 40% of total truck costs as fixed cost is r realistic. Therefore,

Truck Cost per Delivery = \$23.50 x .40 = \$9.40

and

Driver cost per Delivery =		\$30.10
	Total	\$39.50

; a result, the penalty cost for not making a straight time deli- ›y a company truck is calculated as \$40. Based on this result, = 2, 7, 12, 17, 22, 27) = 40.

c_i (i = 3, 8, 13, 18, 23, 28) = Unit cost per delivery to be made by a common carrier.

ıe common carrier rates in the area serviced by East Chicago termi· :e charged according to the distance and load of the delivery in .on. Therefore, "zones" specified according to distance and ser- ;tation location determine the rates.

Examining the most recent data,[1] an average rate based on distance load has been determined. According to this analysis, the average on carrier rate per load of 1000 gallons is \$8.71. This figure sted for an average delivery of 7,800 gallons per trip gives us 94. Based on this analysis, the unit cost of making a delivery ugh a common carrier is assumed to be \$68. As a result, c_i (i = 3 3, 18, 23, 28) = 68.

4. c_i (i = 4, 9, 14, 19, 24, 29) = Unit cost per overtime delivery to be made by a company truck.

This cost is calculated on the basis of the figures presented ier for the cost type 1. For overtime operations truck cost is med to be 15% higher due to the expected increases in depreciation tenance and part of the operating expenses. Considering this fact time-and-a-half pay for drivers in overtime, the cost of an overti ation comes up to be \$18.10 per hour.

Overtime means four additional hours in a work day. As discussed specified earlier, a delivery takes three hours on the average. P ng enough time for shift changes and maintenance, a truck can hand one delivery during a four-hour overtime operation period.

Based on this analysis, the unit cost of making an overtime delive company truck is assumed to be \$18.10 x 4 = \$72.40 (or \$72). The , c_i (i = 4, 9, 14, 19, 24, 29) = 72.

5. c_i (i = 5, 10, 15, 20, 25) = Penalty cost per overtime deliver not made by a company truck.

Utilizing the same approach discussed for the cost type 2, only

---

[1] February 1974 data.

ime fixed costs and time-and-a-half payment for drivers are ded in this cost. Again based on the hourly costs for four hours ime operation, the associated penalty cost becomes \$52 per delivei ade.

herefore, the penalty cost for not making an overtime delivery by ny truck is \$52. As a result, c_i (i = 5, 10, 15, 20, 25, 30) = 52

Specification on the Constants of the Model

here are four different constants in the model.

. b = maximum number of deliveries that can be made by a truck (company owned or common carrier) in a 20-hour work day.

s stated earlier, the average time per delivery is assumed to be hours. Therefore, maximum number of deliveries[1] in a 20-hour day is six. Based on this result, b = 6.

. b' = maximum number of deliveries that can be made by a compan truck in overtime.

ince one delivery takes three hours, the number of deliveries that e handled by a company truck in a four-hour overtime operation is Therefore, b' = 1.

. k = the number of company owned trucks available in the terminal.

he present truck fleet for East Chicago terminal consists of four ors and four trailers available for operations. Definition of a is made as the combination of a tractor and a trailer. Therefore

---

Research, besides giving an average of three hours per delivery, an average of six deliveries per work day in the historical data ıber of company owned trucks available to be used in East Chicago .l is four. Based on this, k = 4.

p = number of common carriers available to be hired for the terminal.

order to form an upper bound in the number of common carriers le to be hired for the terminal, p is assumed originally to be . discussion about the manipulation of this bound will be provided n the next chapter under the heading of "Solutions." This dis- will analyze the sensitivity of solution (in terms of computer d cost) to various bounds tried on the number of common carriers le.

Bounds on the Company Trucks
As a Policy Condition

ginally, the model assumes no lower bounds on the number of com- ucks to be utilized. But again, similar to the above discussion or the bounds on common carriers, a series of utilization levels conditions) will be tried to test the sensitivity of the solu- ne and cost. In this attempt, 50% and then 75% utilization (two and three trucks respectively*) per day are set as policy ons. Results are summarized in the section dealing with ıs.

ıce the model uses the number of deliveries as a variable, a ɔund of two trucks (company) per day would mean at least 12 : time deliveries per day. These are actually lower bounds for responding X_i variables in the model.

Analysis of Demand Constraints

'here are two points to be considered in the demand constraints of ıodel. First, truck capacity per delivery and second, daily fluc-.ng demand requirement.

ınalysis of the historical data on the load hauled per delivery :ates an average of 7,800 gallons although the official truck capa is 8,000. This can be attributed to the fact that there are some 'eries which are made with less than full load due to demand condi ; in certain times or for certain service stations. Also, since : oil delivery includes premium and regular gasoline, sometimes : will be a difference between the types requested for the same 'ery to a service station. This will cause the total load to be than full per delivery in question.

:eeping these points in mind and to be on the safe side, truck cap per trip is assumed to be 7,800 gallons. Therefore, a = 7,800.

:n order to estimate the daily demand requirements, an analysis of ıistorical data in terms of fluctuating demand trend is necessary. ›rical data examined is about the supply side. But since the dail load can be assumed to be equal to the demand, fluctuations and ıations as to the amount delivered per day, per week establish bas ;pecifying d's in the model.

ılthough our application here assumes a set of daily demand requir ; estimated on an average weekly-daily basis, different d constant ›e applied to any terminal, any operation period under different ıd assumptions by user of the model. This is one of the attribute ır general model presented earlier.

For the East Chicago terminal, after examining the fluctuations, kly demand pattern distributed over six work days with a fluctuat ly demand has been determined. According to this, average daily and is assumed to be 200,000 gallons with the following fluctuati tern over a week: the first day of the week (Monday), 20% under rage; the second day, around average; the third day, 10% under av ; the fourth day, 10% above average; the fifth day, 5% above aver the sixth day, 15% above average. As seen from the assumption a fluctuations over a week, demand towards the end of the week is her (especially the last day) than at the beginning of the week. s can be considered as a pretty sharp assumption. But to repeat, el is flexible enough to be applied to any demand condition fores specified by the user. If the operation period in question over ch the model is to be applied exhibits a totally different patter ctuations and estimations, the related constants can be revised ily for Cities Service applications.

Based on our analysis, d constants for the model to be applied t t Chicago terminal are estimated (in gallons) as d_1 = 160,000, d_2 ,000, d_3 = 180,000, d_4 = 220,000, d_5 = 210,000 and d_6 = 230,000.

In the following chapter the complete numerical model for East cago terminal is presented.

To conclude our discussion on this analysis part, we would like t te the assumption which actually has been made throughout this tion.[1] This is the assumption that all company owned trucks are

[1]This is the reason we can work with averages to calculate our stants, especially our c_i coefficients, for the corresponding x_i iables.

valent in cost, volume hauled and time spent per delivery. Simila common carriers are equivalent in cost, volume hauled and time spe delivery.

CHAPTER VI

SOLUTIONS

Numerical Model

[T]he previous chapter specified the coefficients and the constants [m]odel. This section by substituting them into the theoretical mod[el] [pres]ents the numerical model which will be solved for the East Chicago [term]inal.

[T]he following system is the numerical model for a weekly operation [peri]od of the East Chicago terminal:[1]

[M]inimize $Z = 54X_1 + 40X_2 + 68X_3 + 72X_4 + 52X_5 +$

$$54X_6 + 40X_7 + 68X_8 + 72X_9 + 52X_{10} +$$
$$54X_{11} + 40X_{12} + 68X_{13} + 72X_{14} + 52X_{15} +$$
$$54X_{16} + 40X_{17} + 68X_{18} + 72X_{19} + 52X_{20} +$$
$$54X_{21} + 40X_{22} + 68X_{23} + 72X_{24} + 52X_{25} +$$
$$54X_{26} + 40X_{27} + 68X_{28} + 72X_{29} + 52X_{30}$$

[Subj]ect to:

(A): $X_1 + X_2 = 24$

(B): $X_3 \leq 12$

(C): $X_4 + X_5 = 4$

(D): $7800X_1 + 7800X_3 + 7800X_4 \geq 160000$

---

[1]Under the given assumptions fitting the theoretical model formu[late]d earlier.

(E): $X_6 + X_7 = 24$

(F): $X_8 \leq 12$

(G): $X_9 + X_{10} = 4$

(H): $7800X_6 + 7800X_8 + 7800X_9 \geq 200000$

(I): $X_{11} + X_{12} = 24$

(J): $X_{13} \leq 12$

(K): $X_{14} + X_{15} = 4$

(L): $7800X_{11} + 7800X_{13} + 7800X_{14} \geq 180000$

(M): $X_{16} + X_{17} = 24$

(N): $X_{18} \leq 12$

(O): $X_{19} + X_{20} = 4$

(P): $7800X_{16} + 7800X_{17} + 7800X_{18} \geq 220000$

(Q): $X_{21} + X_{22} = 24$

(R): $X_{23} \leq 12$

(S): $X_{24} + X_{25} = 4$

(T): $7800X_{21} + 7800X_{23} + 7800X_{24} \geq 210000$

(U): $X_{26} + X_{27} = 24$

(V): $X_{28} \leq 12$

(W): $X_{29} + X_{30} = 4$

(Y): $7800X_{26} + 7800X_{28} + 7800X_{30} \geq 230000$

(Z): $X_i \geq 0$ and integer valued for $i = 1, \ldots, 30$.

ll X_i's represent the variables defined earlier in the theoretical odel.

his system which is an Integer Linear Programming Problem in 30 bles and 24 constraints can be solved by Computer Utilization gh IBM's Extended Mathematical Programming System (MPSX). The wing section summarizes the results of this application.

Computer Utilization and Solutions

his section summarizes the process experienced in the computer zation. It includes a discussion on the sensitivity of solution and computer cost to the changes manipulated on the bounds over umber of company trucks to be utilized and common carriers to be in a given day (policy conditions). Finally, as computer output, ptimal solution is given to the problem defined and formulated er for the East Chicago terminal. Interpretation of the optimal ion and ranging analysis in which ranges of values and conditions iscussed to determine the sensitivity of this optimal solution wil esented in the following chapter.

he system stated in the previous section was put into the standard for MPSX utilization.* Table I summarizes the process through the computer determined the optimal solution under each policy tion. It also shows the sensitivity of computer time and cost in ion to each bounding decision. The optimal solution to all of the ems is the same because the bounding decisions on common carriers

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Refer to Appendix to the study--use of MPSX control program and ration of input data for the problem.

TABLE I

COMPUTER UTILIZATION

em r*	Functional Value (Objective Function)	Number of Iterations Until Optimality	Branches Abandoned While Computing	Processor Time (in hours)	Tota Compu Cos
	$9452.00	4637	505	.11987	$81.(
	9452.00	2441	773	.06295	42.7
	9452.00	1247	387	.03328	22.(
	9452.00	1247	387	.03214	21.(

Problem No. 1--An upper bound of six on common carriers available red per day (consequently, an upper bound of 36 on X_i for i = 3, 8 8, 23, 28.

--No lower limit on the number of company trucks to b zed (no policy condition on company truck utilization level.)

Problem No. 2--An upper bound of five on common carriers.

--A lower limit of two on company trucks to be utili ay (50% utilization level as a policy condition). Consequently, a bound of 12 on X_i, i = 1, 6, 11, 16, 21, 26.

Problem No. 3--An upper bound of four on common carriers.

--A lower limit of three on company trucks (75% zation level).

Problem No. 4--An upper bound of two on common carriers.

--A lower limit of three on company trucks.

:he company truck utilization have no effect on the solution for tl Chicago terminal problem due to the cost structure.

:n the search process summarized above, the computer found eight .ble integer solutions to the problem. Every time the computer ; a feasible integer solution in the branch it is searching, it ; the corresponding node for that iteration and goes on to another :h to look for a better solution if it exists. The eight feasible ;er solutions to the problem are given in Table II.

'he optimal solution given in Table III is the solution to Problem :ble I. Since all the optimal solutions obtained Problems 1 throu; .er different bounds are the same, only this one is stated here. ppendix to the study and the discussion in the following chapter refer to Problem 4 and this optimal solution.

'here are two points about Tables II and III that should be clari- here. First, MPSX, in its internal system, numbers the original ıbles starting with m + 1 up to m + n where m is the number of row ;traints) and n is the number of basic variables in the problem. is why 30 variables of our problem are numbered from 26 to 55. ıdly, "node" with its specified number is the name given to a solu point in a certain branch in the search process. Since some of t :hes and nodes are abandoned in this process, iteration number is :he same as a corresponding node number.

'he following chapter will discuss the optimal solution given abov discussion will include also interpretation of the solution and .ng analyses on the values over which the problem itself and the :ion are specified.

FEASIBLE SOLUTIONS

NODE	66	289	314	480	747	923	1097	1133
FUNCTIONAL	10118.0000	10100.0000	9776.0000	9728.0000	9680.0000	9632.0000	9584.0000	9452.0000
ESTIMATION	INTEGER	INTEGER	INTEGER	INTEGER	INTEGER	INTEGER	INTEGER	INTEGER
26= X1	18.0000	21.0000	21.0000	21.0000	21.0000	21.0000	21.0000	21.0000
27= X2	6.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
28= X3	.	.	.	.	.	.	.	.
29= X4	3.0000	.	.	.	.	.	.	.
30= X5	1.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
31= X6	24.0000	24.0000	24.0000	24.0000	24.0000	24.0000	24.0000	24.0000
32= X7	.	.	.	.	.	.	.	.
33= X8	.	.	.	.	.	.	.	.
34= X9	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
35= X10	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
36= X11	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	24.0000
37= X12	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	.
38= X13	6.0000	6.0000	6.0000	5.0000	4.0000	3.0000	2.0000	.
39= X14	.	.	.	1.0000	2.0000	3.0000	4.0000	.
40= X15	4.0000	4.0000	4.0000	3.0000	2.0000	1.0000	.	4.0000
41= X16	18.0000	18.0000	24.0000	24.0000	24.0000	24.0000	24.0000	24.0000
42= X17	6.0000	6.0000	.	.	.	.	.	.
43= X18	7.0000	7.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
44= X19	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
45= X20	.	.	.	.	.	.	.	.
46= X21	24.0000	24.0000	24.0000	24.0000	24.0000	24.0000	24.0000	24.0000
47= X22	.	.	.	.	.	.	.	.
48= X23	.	.	.	.	.	.	.	.
49= X24	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
50= X25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
51= X26	24.0000	24.0000	24.0000	24.0000	24.0000	24.0000	24.0000	24.0000
52= X27	.	.	.	.	.	.	.	.
53= X28	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
54= X29	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
55= X30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE III

OPTIMAL SOLUTION

NODE	1133
FUNCTIONAL	9452.0000
ESTIMATION	INTEGER
26= X1	21.0000
27= X2	3.0000
28= X3	.
29= X4	.
30= X5	4.0000
31= X6	24.0000
32= X7	.
33= X8	.
34= X9	2.0000
35= X10	2.0000
36= X11	24.0000
37= X12	.
38= X13	.
39= X14	.
40= X15	4.0000
41= X16	24.0000
42= X17	.
43= X18	1.0000
44= X19	4.0000
45= X20	.
46= X21	24.0000
47= X22	.
48= X23	.
49= X24	3.0000
50= X25	1.0000
51= X26	24.0000
52= X27	.
53= X28	2.0000
54= X29	4.0000
55= X30	.

CHAPTER VII

ANALYSIS

Interpretation of the Solutions

This sections analyzes the optimal solution found to the problem in previous chapter. It interprets the X_i values to determine the ategy for each day in terms of the number of company trucks to be l in straight time and overtime and the number of common carriers to nired. The optimal solution gives the number of deliveries as the imum values for the corresponding X_i variables. Therefore, the consion process stated in the model formulation section should be perned to determine the number of trucks to be used for each correspond- X_i.[1]

First Day:

$X_1^* = 21$; The number of company trucks to be used in the first day to make 21 straight time deliveries is four.

$X_2^* = 3$; Fourth company truck will make only 6 - 3 = 3 straight time deliveries.

$X_3^* = 0$; No common carriers.

$X_4^* = 0$, $X_5^* = 4$; No overtime use of trucks.

---

[1]Throughout this chapter, variables will be stated only by their scripts in order to avoid the repetition of the same expressions and initions for various X_i's interpreted. Each X_i represents and denote variable defined in the model formulation section.

Second Day:

$X_6^* = 24$; Four company trucks will be used in straight time.

$X_7^* = 0$; Four company trucks will be used in straight time.

$X_8^* = 0$; No common carriers.

$X_9^* = 2$; Two company trucks will be used in overtime.

$X_{10}^* = 2$; Two company trucks will be used in overtime.

Third Day:

$X_{11}^* = 24$; Four company trucks/straight time.

$X_{12}^* = 0$; Four company trucks/straight time.

$X_{13}^* = 0$; No common carriers.

$X_{14}^* = 0$; No overtime use of trucks.

$X_{15}^* = 4$; No overtime use of trucks.

Fourth Day:

$X_{16}^* = 24$; Four company trucks/straight time.

$X_{17}^* = 0$; Four company trucks/straight time.

$X_{18}^* = 1$; One common carrier to make one delivery.

$X_{19}^* = 4$; Four company trucks/overtime.

$X_{20}^* = 0$; Four company trucks/overtime.

Fifth Day:

$X_{21}^* = 24$; Four company trucks/straight time.

$X_{22}^* = 0$; Four company trucks/straight time.

$X_{23}^* = 0$; No common carrier.

$X_{24}^* = 3$; Three company trucks/overtime.

$X_{25}^* = 1$; Three company trucks/overtime.

Sixth Day:

$X_{26}^* = 24$; Four company trucks/straight time.

$X_{27}^* = 0$; Four company trucks/straight time.

X_{28}^{*} = 2; One common carrier to make two deliveries.

X_{29}^{*} = 4; Four company trucks/overtime.

X_{30}^{*} = 0; Four company trucks/overtime.

According to these above optimum strategies, the value of the func- ıal (value of the objective function = minimum weekly cost of truck rations) is $9,452. Of course, this is true under the assumptions conditions as to demand and cost figures stated earlier in the el.

Further analyses of the optimal solution can be made by the examina- ı of computer outputs on Rows and Columns sections provided in Table ıd Table II.

Rows Table

Table IV gives for each row (constraint) the optimum activity level; ck activity (unused portion of the resource associated with that row) er limit and upper limit specified for that row; and dual activity, ential cost increase in relation to per unit increase in the asso- ted resource availability. For example, let us take row (constraint) Since the activity level is set at UL, corresponding slack activity 0. If the RHS constant (resource availability) which is the number straight time deliveries to be made by company trucks in the first is increased by one, the cost will decrease by $40. (Note that this the cost associated with the slack activity - X_2.)

For row B, however, the situation is different. Activity level is C slack activity is 12. Increasing the resource availability (increas number of common carriers available to hire) will not influence the

ROWS - OUTPUT

- ROWS

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	VALUE	BS	9452.00000	9452.00000-	NONE	NONE	1.00000
2	A	EQ	24.00000	.	24.00000	24.00000	40.00000-
3	B	BS	.	12.00000	NONE	12.00000	.
4	C	EQ	4.00000	.	4.00000	4.00000	52.00000-
5	D	BS	163800.00000	3800.00000-	160000.00000	NONE	.
6	E	EQ	24.00000	.	24.00000	24.00000	40.00000-
7	F	BS	.	12.00000	NONE	12.00000	.
8	G	EQ	4.00000	.	4.00000	4.00000	52.00000-
9	H	BS	202799.99999	2800.00000-	200000.00000	NONE	.
10	I	EQ	24.00000	.	24.00000	24.00000	40.00000-
11	J	BS	.	12.00000	NONE	12.00000	.
12	K	EQ	4.00000	.	4.00000	4.00000	72.00000-
13	L	BS	187200.00000	7200.00000-	180000.00000	NONE	.
14	M	EQ	24.00000	.	24.00000	24.00000	40.00000-
15	N	BS	1.00000	11.00000	NONE	12.00000	.
16	O	EQ	4.00000	.	4.00000	4.00000	52.00000-
17	P	BS	226199.99999	6200.00000-	220000.00000	NONE	.
18	Q	EQ	24.00000	.	24.00000	24.00000	40.00000-
19	R	BS	.	12.00000	NONE	12.00000	.
20	S	EQ	4.00000	.	4.00000	4.00000	52.00000-
21	T	BS	210599.99999	600.00000-	210000.00000	NONE	.
22	U	EQ	24.00000	.	24.00000	24.00000	40.00000-
23	V	BS	2.00000	10.00000	NONE	12.00000	.
24	W	EQ	4.00000	.	4.00000	4.00000	52.00000-
25	Y	BS	233999.99999	4000.00000-	230000.00000	NONE	.

:, since the associated dual cost is 0.* In explaining Table IV, ʳ the above two examples are given. The other rows can be inter- :ed depending on their situation in the same fashion as either the ;t or second example.

Columns Table

Table V gives for each structural variable the optimum activity :l; the input cost; lower limit and upper limit (LL is zero unless :rwise specified, because of nonnegativity constraints on the struc- :l variables); and reduced cost, the optimum value of the correspon- ; dual slack variable (cost associated with the unused portion of a ›urce). A nonbasic structural variable (variable which is not in the imal solution = activity level is 0 for that variable) is at either upper limit (UL) or the lower limit (LL) and has a nonzero dual ıe (example X_3 in the table). X_3 with a \$68 per delivery input cost, ıt LL, zero, with a dual value of \$68; that is, a delivery by a commo :ier in the first day would cause the cost to increase \$68.

On the other hand, a basic structural variable (a variable with a ltive activity level = variable in the optimal solution) with a re- :d cost figure can be interpreted like this: As an example let us : X_6. Since it is at UL, 24, with a reduced cost of \$14; within a ;itivity range, each additional straight time delivery by a company

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*To emphasize this very interesting result we would like to point ou t this was exactly what happened when we tried to impose various ıds on X_i, i = 3, 8, 13, 18, 23, 28, in the computer utilization pro- ; discussed earlier. Since the minimum cost and optimal solution did change according to resource variations on the associated constraint common carrier availability, the optimal solution values remained same in all of the cases.

COLUMNS - OUTPUT

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
26	X1	IV	21.00000	54.00000	18.00000	24.00000	14.00000
27	X2	IV	3.00000	40.00000	.	6.00000	.
28	X3	IV	.	68.00000	.	12.00000	68.00000
29	X4	IV	.	72.00000	.	4.00000	20.00000
30	X5	IV	4.00000	52.00000	.	4.00000	.
31	X6	IV	24.00000	54.00000	18.00000	24.00000	14.00000
32	X7	IV	.	40.00000	.	6.00000	.
33	X8	IV	.	68.00000	.	12.00000	68.00000
34	X9	IV	2.00000	72.00000	.	4.00000	20.00000
35	X10	IV	2.00000	52.00000	.	4.00000	.
36	X11	IV	24.00000	54.00000	18.00000	24.00000	14.00000
37	X12	IV	.	40.00000	.	6.00000	.
38	X13	IV	.	68.00000	.	12.00000	68.00000
39	X14	IV	.	72.00000	.	4.00000	.
40	X15	IV	4.00000	52.00000	.	4.00000	20.00000-
41	X16	IV	24.00000	54.00000	18.00000	24.00000	14.00000
42	X17	IV	.	40.00000	.	6.00000	.
43	X18	IV	1.00000	68.00000	.	12.00000	68.00000
44	X19	IV	4.00000	72.00000	.	4.00000	20.00000
45	X20	IV	.	52.00000	.	4.00000	.
46	X21	IV	24.00000	54.00000	18.00000	24.00000	14.00000
47	X22	IV	.	40.00000	.	6.00000	.
48	X23	IV	.	68.00000	.	12.00000	68.00000
49	X24	IV	3.00000	74.00000	.	4.00000	22.00000
50	X25	IV	1.00000	52.00000	.	4.00000	.
51	X26	IV	24.00000	54.00000	18.00000	24.00000	14.00000
52	X27	IV	.	40.00000	.	6.00000	.
53	X28	IV	2.00000	68.00000	.	12.00000	68.00000
54	X29	IV	4.00000	72.00000	.	4.00000	20.00000

:k in the second day could be made with a net saving of \$14 (of :se if the associated resource condition permitted). Corresponding :k variable for X_6 is X_7. As seen from the table, X_7 is not in the :mal solution since X_6 is at UL and has a dual value of 0.

In explaining Table V only the above examples are given. The ɜr columns and corresponding variables can be interpreted depending :heir situation either as the first or second example.

The changes discussed as the examples to the interpretation of the ımal solution in Tables IV and V are true within a sensitivity range. following section, by examining the computer outputs of RANGE proce- ɜ, tries to present the sensitivity of the cost coefficients and ›urce specifications of the model.

Ranging Analysis

Output of RANGE procedure used in the program gives four different Les: Row variables at limit level (rows which are at either UL or LL :he optimal solution); Column variables at limit level (variables :h are at either UL or LL in the optimal solution); Row variables at ɜrmediate level (rows that are in the optimal solution with a value veen their UL and LL); and Column variables at intermediate level :iables which are in the optimal solution with a value between their ınd LL).

Variables at Limit Levels (Table VI)

Row A, which is subject to an equality constraint, is 24 (activity), ı a shadow price (the change in the objective function per unit of rease or increase in row activity) of \$40 per delivery. If the

ROWS AT LIMIT LEVEL

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT. ..UPPER LIMIT.	LOWER ACTIVITY UPPER ACTIVITY	...UNIT COST.. ...UNIT COST..	..UPPER COST.. ..LOWER COST..	LIMITING PROCESS.	A A
2	A	EQ	23.99998	.	23.99998	23.99998	40.00003-		X2	L
					23.99998	23.99998	40.00003		X2	L
4	C	EQ	4.00000	.	4.00000	4.00000	52.00001-		X5	L
					4.00000	4.00000	52.00001		X5	L
6	E	EQ	23.99998	.	23.99998	23.99998	40.00003-		X7	L
					23.99998	23.99998	40.00003		X7	L
8	G	EQ	4.00000	.	4.00000	4.00000	52.00001-		X10	L
					4.00000	4.00000	52.00001		X10	L
10	I	EQ	23.99998	.	23.99998	23.99998	40.00003-		X12	L
					23.99998	23.99998	40.00003		X12	L
12	K	EQ	4.00000	.	4.00000	4.00000	72.00001-		X14	L
					4.00000	4.00000	72.00001		X14	L
14	M	EQ	23.99998	.	23.99998	23.99998	40.00003-		X17	L
					23.99998	23.99998	40.00003		X17	L
16	O	EQ	4.00000	.	4.00000	4.00000	52.00001-		X20	L
					4.00000	4.00000	52.00001		X20	L
18	Q	EQ	23.99998	.	23.99998	23.99998	40.00003-		X22	L
					23.99998	23.99998	40.00003		X22	L
20	S	EQ	4.00000	.	4.00000	4.00000	52.00001-		X25	L
					4.00000	4.00000	52.00001		X25	L
22	U	EQ	23.99998	.	23.99998	23.99998	40.00003-		X27	L
					23.99998	23.99998	40.00003		X27	L
24	W	EQ	4.00000	.	4.00000	4.00000	52.00001-		X30	L
					4.00000	4.00000	52.00001		X30	L

irement associated with row A were less than 24 (lower activity), limiting process in row 1) would leave the basis (optimal solution) L (0). If it were more than 24 (upper activity), again X_2 (limiting ess in row 2) would leave the basis at LL (0). In a simple expres- : if the number of deliveries that can be made by company trucks in ight time is reduced/increased by one, the cost will increase/de- se by \$40. But X_2 is the limiting process in this situation. There- , any change in the constant associated with constraint A will cause optimal solution to change.

Row C can be interpreted in a similar way. If the requirement (re- ce availability = the number of overtime deliveries that can be made ompany trucks) is less than 4 (or more than 4), the cost will in- se by \$52 per delivery (or decrease by \$52 per delivery). But X_5 is limiting process in this situation. If the row activity is changed e more than 4 or less than 4, X_5 will leave the basis changing the mal solution.

The other rows in Table VI can be explained similarly for each vity and daily strategy variable.

mns at Limit Level (Table VII)

X_1 is in the optimal solution with an activity level of 21. The low price associated with X_1 is \$14. The entry "INFINITY" for upper implies that any increase in X_1's input cost (\$54) would not change basis. The lower cost of \$40 implies that if the input cost were iced from \$52 to \$40, X_2 would leave the basis at LL (0). This shows the input cost of X_1 can be lowered up to \$40. Any unit cost under

					..UPPER LIMIT.	UPPER ACTIVITY	...UNIT COST...	..LOWER COST..	.PROCESS.	
26	X1	EQ	20.99999	54.00000	20.99999	20.99999	14.00000-	INFINITY	X2	LL
					20.99999	20.99999	14.00000	40.00000	X2	LL
28	X3	EQ	.	68.00000	.	.48718-	68.00000-	INFINITY	D	LL
					.	12.00000	68.00000	.	B	UL
29	X4	EQ	.	72.00000	.	.	20.00000-	INFINITY	X5	LL
					.	.	20.00000	52.00000	X5	LL
31	X6	EQ	24.00000	54.00000	24.00000	24.00000	14.00000-	INFINITY	X7	LL
					24.00000	24.00000	14.00000	40.00000	X7	LL
33	X8	EQ	.	68.00000	.	.35897-	68.00000-	INFINITY	H	LL
					.	12.00000	68.00000	.	F	UL
34	X9	EQ	2.00000	72.00000	2.00000	2.00000	20.00000-	INFINITY	X10	LL
					2.00000	2.00000	20.00000	52.00000	X10	LL
36	X11	EQ	24.00000	54.00000	24.00000	24.00000	14.00000-	INFINITY	X12	LL
					24.00000	24.00000	14.00000	40.00000	X12	LL
38	X13	EQ	.	68.00000	.	.92308-	68.00000-	INFINITY	L	LL
					.	12.00000	68.00000	.	J	UL
40	X15	EQ	4.00000	52.00001	4.00000	4.00000	20.00000	72.00002	X14	LL
					4.00000	4.00000	20.00000-	INFINITY-	X14	LL
41	X16	EQ	24.00000	54.00000	24.00000	24.00000	14.00000-	INFINITY	X17	LL
					24.00000	24.00000	14.00000	40.00000	X17	LL
43	X18	EQ	1.00000	68.00000	1.00000	.20513	68.00000-	INFINITY	P	LL
					1.00000	12.00000	68.00000	.	N	UL
44	X19	EQ	4.00000	72.00000	4.00000	4.00000	20.00000-	INFINITY	X20	LL
					4.00000	4.00000	20.00000	52.00000	X20	LL
46	X21	EQ	24.00000	54.00000	24.00000	24.00000	14.00000-	INFINITY	X22	LL
					24.00000	24.00000	14.00000	40.00000	X22	LL
48	X23	EQ	.	68.00000	.	.07692-	68.00000-	INFINITY	T	LL
					.	12.00000	68.00000	.	R	UL
49	X24	EQ	3.00000	74.00000	3.00000	3.00000	22.00000-	INFINITY	X25	LL
					3.00000	3.00000	22.00000	52.00001	X25	LL
51	X26	EQ	24.00000	54.00000	24.00000	24.00000	14.00000-	INFINITY	X27	LL
					24.00000	24.00000	14.00000	40.00000	X27	LL
53	X28	EQ	2.00000	68.00000	2.00000	1.48718	68.00000-	INFINITY	Y	LL
					2.00000	12.00000	68.00000	.	V	UL

would cause the optimal solution to change.[1]

X_3 has a 0 activity level (X_3 = 0 in the optimal solution, at LL) has a shadow price of $68, in the range -.48718 (clearly this is ›thetical because LL = 0) to 12.0. As long as the specification on number of deliveries by common carriers is between 0 and 12, the .mal solution will remain optimal. If the number of deliveries that be made by common carriers is more than 12 (or 0), row D (or B) .d leave the basis and X_3 would enter. This is again the reason why various bounds on common carrier availability <u>for this problem</u> do influence the optimal <u>values</u> as discussed in the earlier sections. The interpretations of the other entires in this table and in Tables : and IX are very similar; to avoid unnecessary repetitions they are :ted from the discussion.

Only one fact about the demand constraints should be emphasized here. :e all the optimal values of the variables in the model are dependent :he demand specifications, the sensitivity of the optimal values are ' high to changes in demand conditions. Because of this fact, as the : section discusses, the main attention point on the specification of model constants is from the demand side.

Development of a Decision Table To Aid Management Use

The following analytical chart is presented to aid the decision .ng process through the application of the model formulated in this

---

[1]This is actually a logical result, because a unit cost for X_1 which .ess than the penalty cost of X_2 will cause X_2 to leave the basis.

ROWS AT INTERMEDIATE LEVEL

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT. ..UPPER LIMIT.	LOWER ACTIVITY UPPER ACTIVITY	...UNIT COST.. ...UNIT COST..	..UPPER COST.. ..LOWER COST..	LIMITING PROCESS.	A A
3	B	BS	.	12.00000	NONE 12.00000	. .	INFINITY INFINITY		NONE NONE	
5	C	BS	163799.84372	3799.99773-	159999.84599 NONE	163799.84372 163799.84372	INFINITY INFINITY		NONE NONE	
7	F	BS	.	12.00000	NONE 12.00000	. .	INFINITY INFINITY		NONE NONE	
9	H	BS	202799.89463	2799.99823-	199999.89640 NONE	202799.89463 202799.89463	INFINITY INFINITY		NONE NONE	
11	J	BS	.	12.00000	NONE 12.00000	. .	INFINITY INFINITY		NONE NONE	
13	L	BS	187199.86750	7199.99631-	179999.87119 NONE	187199.86750 187199.86750	INFINITY INFINITY		NONE NONE	
15	N	BS	1.00000	11.00000	NONE 12.00000	1.00000 1.00000	INFINITY INFINITY		NONE NONE	
17	P	BS	226199.79987	6199.99681-	219999.80305 NONE	226199.79987 226199.79987	INFINITY INFINITY		NONE NONE	
19	R	BS	.	12.00000	NONE 12.00000	. .	INFINITY INFINITY		NONE NONE	
21	T	BS	210599.78988	599.99942-	209999.79045 NONE	210599.78988 210599.78988	INFINITY INFINITY		NONE NONE	
23	V	BS	2.00000	10.00000	NONE 12.00000	2.00000 2.00000	INFINITY INFINITY		NONE NONE	
25	Y	BS	233999.81366	3999.99800-	229999.81566 NONE	233999.81366 233999.81366	INFINITY INFINITY		NONE NONE	

- COLUMNS AT INTERMEDIATE LEVEL

COLUMNS AT INTERMEDIATE LEVEL

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT. ..UPPER LIMIT.	LOWER ACTIVITY UPPER ACTIVITY	...UNIT COST.. ...UNIT COST..	..UPPER COST.. ..LOWER COST..	LIMITING PROCESS.	A' A'
27	X2	BS	3.00000	40.00003	3.00000	3.00000	INFINITY	INFINITY	NONE	
					3.00000	3.00000	INFINITY	INFINITY-	NONE	
30	X5	BS	4.00000	52.00001	4.00000	4.00000	INFINITY	INFINITY	NONE	
					4.00000	4.00000	INFINITY	INFINITY-	NONE	
32	X7	BS	.	40.00003	.	.	INFINITY	INFINITY	NONE	
					.	.	INFINITY	INFINITY-	NONE	
35	X10	BS	2.00000	52.00001	2.00000	2.00000	INFINITY	INFINITY	NONE	
					2.00000	2.00000	INFINITY	INFINITY-	NONE	
37	X12	BS	.	40.00003	.	.	INFINITY	INFINITY	NONE	
					.	.	INFINITY	INFINITY-	NONE	
39	X14	BS	.	72.00000	.	.	INFINITY	INFINITY	NONE	
					.	.	INFINITY	INFINITY-	NONE	
42	X17	BS	.	40.00003	.	.	INFINITY	INFINITY	NONE	
					.	.	INFINITY	INFINITY-	NONE	
45	X20	BS	.	52.00001	.	.	INFINITY	INFINITY	NONE	
					.	.	INFINITY	INFINITY-	NONE	
47	X22	BS	.	40.00003	.	.	INFINITY	INFINITY	NONE	
					.	.	INFINITY	INFINITY-	NONE	
50	X25	BS	1.00000	52.00001	1.00000	1.00000	INFINITY	INFINITY	NONE	
					1.00000	1.00000	INFINITY	INFINITY-	NONE	
52	X27	BS	.	40.00003	.	.	INFINITY	INFINITY	NONE	
					.	.	INFINITY	INFINITY-	NONE	
55	X30	BS	.	52.00001	.	.	INFINITY	INFINITY	NONE	
					.	.	INFINITY	INFINITY-	NONE	

. The chart (Table X) is based on the concepts and attributes of odel and on the dimensions and results obtained in the ranging sis.[1] It tries to present an analytical step-by-step approach to tilization of the model and to facilitate its application and inter- tion on other terminals or under different cost, availability, y and demand conditions.

Limitations

imitations of this study can be summarized around the following s:

.. The most important aspect in the CITGO's truck operations is the daily, weekly and monthly fluctuating trend of demand. Although this study tries to optimize the mix of trucks subject to specified daily demand requirements, the nature of fluctuating demand could only be reflected in this daily demand requirement by observation of trends in historical data and by making assumptions in terms of peak times and averages. The sensitivity of the optimal solution and daily strategies are directly related to fluctuations in daily demand requirements. Some sophisticated techniques on demand projections can be utilized to improve the accuracy of demand constants to be used in the model. This will help to produce results which are less sensitive to fluctuations in daily demand requirements.

The ranges and their influences on the variables cannot be the same lifferent cost and demand conditions. Because of this fact, each nal and each different condition will be subject to different sen- ity levels.

TABLE X

AN ANALYTICAL CHART FOR DECISION MAKING PROCESS AND APPLICATION OF THE MODEL

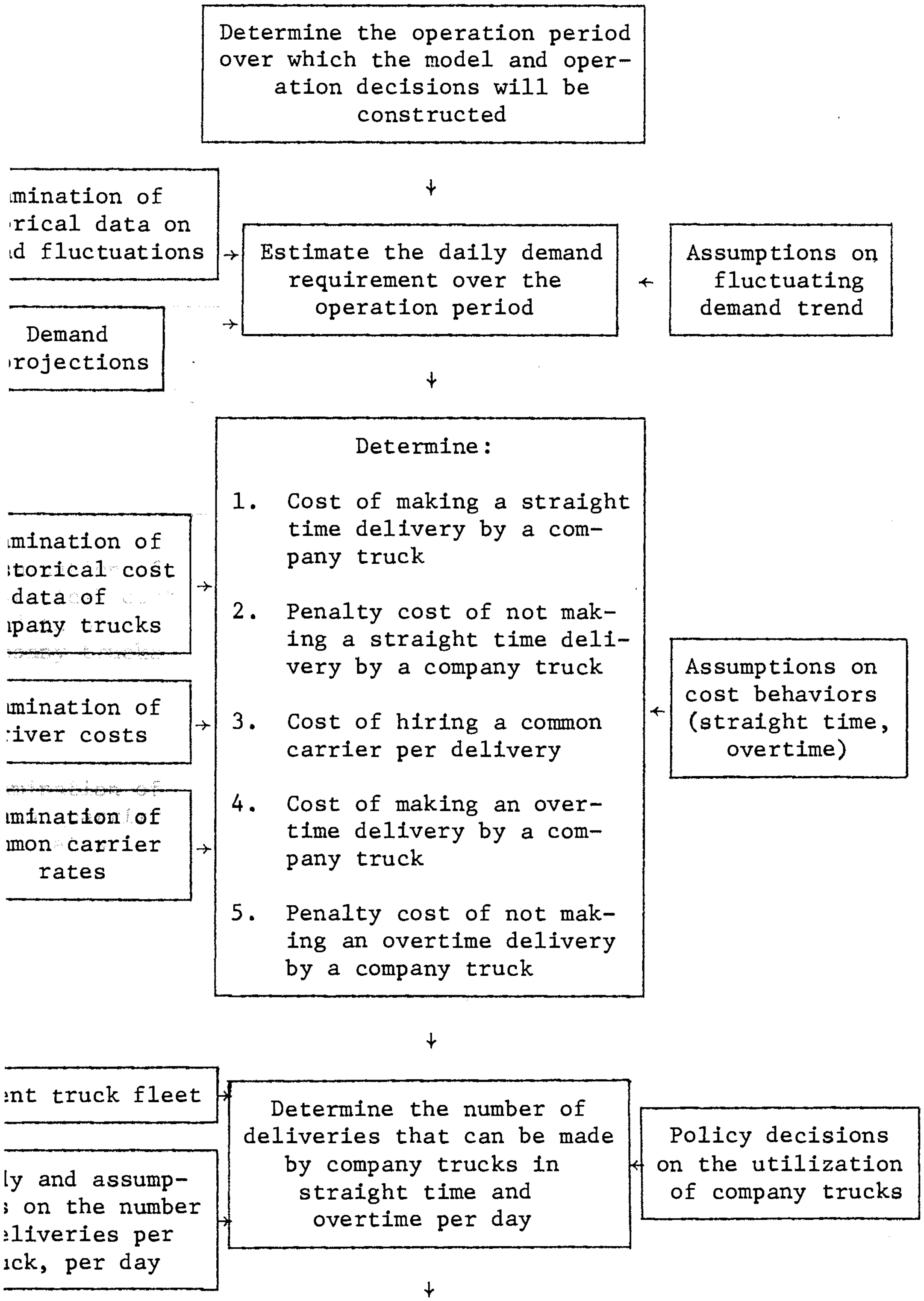

TABLE X (continued)

Determine the number of deliveries that can be made by common carriers per day ← Upper bounds on the availability of common carriers

↓

Substitute the constants and coefficients of the model specified under the above assumptions and estimations

↓

Solve the problem to obtain the optimum number of deliveries by truck and operation time ← Utilization of computer

↓

Determine the optimum number of company trucks to be used and common carriers to be hired for each day in the operation period

↓

›wledge on ꞇ programming sensitivity ꞁng) analysis → Determine the ranges to which the solution obtained to the model is sensitive in terms of cost, availability, company policy and demand conditions ← Computer Utilization

↓

ꞇy to predict ɜrform possi- ɿanges in op- mix for day- ɜy decisions ↔ Estimate and predict the possible changes in the optimum mix due to cost, availability and demand conditions ↔ Ability to predict what changes in these conditions can be undertaken

↓

TABLE X (continued)

When some observed changes occur in cost, demand and availability data, revise the constants and coefficients of the model

↓

Model is applicable to other terminals under different conditions and assumptions, by performing the above steps

The common carrier rate used to determine the hiring cost/ delivery of a common carrier is estimated on an average basis in this study. In practice, the common carrier rates are based on distance of deliveries to be made. Rate differentiation according to distance can be employed to create more realistic comparisons of company trucks with common carriers.

Costing of company trucks and determination of the cost per delivery are based on an analysis of hourly averages from the historical data. A more precise costing of company trucks according to their performance and conditions (age, depreciation, maintenance, etc.) may improve the accuracy of the model.

The model presented in this study does not take into account the possibility (or practice) of sharing company trucks in neighboring terminals. This may be considered to utilize idle capacity for a terminal in certain times against hiring common carriers for another terminal's excess demand.

Truck capacity per delivery is based on the official truck size and on an average delivery load. And it is assumed to be equal for each truck and delivery. Multiple sizes of trucks can be included as a possible option of the model.

The model presented in this study considers only the present truck fleet and its availability in terms of resource conditions. Options in terms of acquisitions and trade-ins can be included in the model by expanding it to determination of the optimal long run mix. Operation period to be considered in the model should be extended to at least 12 months in this case. Lead time for ordering and associated costs with these alternative actions should be examined and included in the model, too.

The expansion of the model to include even some of the above points :ds much more research effort and time. At the same time, increasing : size of the model means adding a considerable amount of variables . constraints to the model. In Integer Linear Programming problems, : size of the model, number of variables and constraints have a dras- . effect on computer time and costs. This is equally important in : discussion of limitations, too.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

An integer linear programming model was formulated and solved to letermine the optimum mix of company trucks and common carriers to be ised for the operations of East Chicago terminal of CITGO. The model vas built with the objective of weekly cost minimization and under th constraints related to demand, resource availability and policy conditions.

The optimal solution determined in terms of daily strategies was checked against the changes in the cost and demand conditions of the truck operations. Ranging analysis performed for this purpose showed that even a 5% change in daily demand would have an effect on the opt num mix of company and common carriers predetermined. For example, a 5% increase (from the average) in demand for a given day would necess tate the utilization of all company trucks in straight time and three company trucks in overtime with no common carrier usage. But a 10% increase would necessitate the utilization of all company trucks in straight time and overtime and one common carrier for that given day.

Ranging analysis on the cost coefficients shows that if the avail ability of company trucks increases further, cost reductions are possi in the operations, as long as the cost of making a delivery by a comp truck is less than the combined figure of "penalty cost" associated w not making that trip and common carrier cost per delivery.

These results are interpreted from the solution for East Chicago ιinal operations. Different cost, demand and availability condi- ıs for the application of integer programming model developed in · study may result in different optimum strategies and ranges for :r CITGO terminals.

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APPENDIX

USE OF MPSX CONTROL PROGRAM AND

INPUT DATA FOR THE PROBLEM

The following list of the procedures and statements used on the rol program and input data set of the problem is provided here to future users of the model and similar applications. This pre-ation gives only the list of the cards used in the program to e the particular problem of this study. As the basic source on procedures and guidance on the use of MPSX, refer to IMB lcations given in [66] and [67].

THE CONTROL PROGRAM AND INPUT

DATA FOR THE PROBLEM

```
// EXEC MPSX360,REGION.MPSX2=300K
//MPSX1.SYSIN DD *
PROGRAM
INITIALZ
MOVE(XDATA,'TRUCK')
MOVE(XPBNAME,'PBFILE')
CONVERT('SUMMARY')
BCDOUT
SETUP('MIN','BOUND','ST01','NODES',2000)
MOVE(XOBJ,'VALUE')
MOVE(XRHS,'GO')
CRASH
PRIMAL
SOLUTION
OPTIMIX('COST',0.,0,0,1)
RANGE
PUNCH
EXIT
PEND
/*
//MPSX2.SYSIN DD *
NAME           TRUCK
ROWS
 N  VALUE
 E  A
 L  B
 E  C
 G  D
 E  E
 L  F
 E  G
 G  H
 E  I
 L  J
 E  K
 G  L
 E  M
 L  N
 E  O
 G  P
 E  Q
 L  R
 E  S
 G  T
```

```
 U
 V
 W
 Y
.UMNS
 TF1       'MARKER'                'INTCRG'
 X1        VALUE        54.00000   A                1.000
 X1        D          7800.00000
 X2        VALUE        40.00000   A                1.000
 X3        VALUE        68.00000   B                1.000
 X3        D          7800.00000
 X4        VALUE        72.00000   C                1.000
 X4        D          7800.00000
 X5        VALUE        52.00000   C                1.000
 X6        VALUE        54.00000   E                1.000
 X6        H          7800.00000
 X7        VALUE        40.00000   E                1.000
 X8        VALUE        68.00000   F                1.000
 X8        H          7800.00000
 X9        VALUE        72.00000   G                1.000
 X9        H          7800.00000
 X10       VALUE        52.00000   G                1.000
 X11       VALUE        54.00000   I                1.000
 X11       L          7800.00000
 X12       VALUE        40.00000   I                1.000
 X13       VALUE        68.00000   J                1.000
 X13       L          7800.00000
 X14       VALUE        72.00000   K                1.000
 X14       L          7800.00000
 X15       VALUE        52.00000   K                1.000
 X16       VALUE        54.00000   M                1.000
 X16       P          7800.00000
 X17       VALUE        40.00000   M                1.000
 X18       VALUE        68.00000   N                1.000
 X18       P          7800.00000
 X19       VALUE        72.00000   O                1.000
 X19       P          7800.00000
 X20       VALUE        52.00000   O                1.000
 X21       VALUE        54.00000   Q                1.000
 X21       T          7800.00000
 X22       VALUE        40.00000   Q                1.000
 X23       VALUE        68.00000   R                1.000
 X23       T          7800.00000
 X24       VALUE        74.00000   S                1.000
 X24       T          7800.00000
```

25	VALUE	52.00000	S	1.00000
26	VALUE	54.00000	U	1.00000
26	Y	7800.00000		
27	VALUE	40.00000	U	1.00000
28	VALUE	68.00000	V	1.00000
28	Y	7800.00000		
29	VALUE	72.00000	W	1.00000
29	Y	7800.00000		
30	VALUE	52.00000	W	1.00000
=2	'MARKER'		'INTEND'	
]	A	24.00000	B	12.00000
]	C	4.00000	D	160000.0000
]	E	24.00000	F	12.00000
[	G	4.00000	H	200000.0000
]	I	24.00000	J	12.00000
]	K	4.00000	L	180000.0000
:	M	24.00000	N	12.00000
]	O	4.00000	P	220000.0000
]	Q	24.00000	R	12.00000
:	S	4.00000	T	210000.0000
]	U	24.00000	V	12.00000
]	W	4.00000	Y	230000.0000
;				
[01	X1	18.00000		
'01	X1	24.00000		
[C1	X2	6.00000		
'01	X3	12.00000		
'C1	X4	4.00000		
'C1	X5	4.00000		
'01	X6	18.00000		
01	X6	24.00000		
'C1	X7	6.00000		
01	X8	12.00000		
C1	X9	4.00000		
C1	X10	4.00000		
01	X11	18.00000		
01	X11	24.00000		
C1	X12	6.00000		
01	X13	12.00000		
01	X14	4.00000		
C1	X15	4.00000		
C1	X16	18.00000		
01	X16	24.00000		
C1	X17	6.00000		
C1	X18	12.00000		
01	X19	4.00000		
C1	X20	4.00000		

```
 LO  STO1          X21              18.00000
 UP  STO1          X21              24.00000
 UP  STO1          X22               6.00000
 UP  STO1          X23              12.00000
 UP  STO1          X24               4.00000
 UP  STO1          X25               4.00000
 LO  STO1          X26              18.00000
 UP  STO1          X26              24.00000
 UP  STO1          X27               6.00000
 UP  STO1          X28              12.00000
 UP  STO1          X29               4.00000
 UP  STO1          X30               4.00000
ENDATA
/*
//
```

VITA

Tuncay Akoglu

Candidate for the Degree of

Master of Business Administration

ort: AN APPLICATION OF LINEAR INTEGER PROGRAMMING TO LIGHT OIL TERMINAL - TRUCK OPERATIONS

or Field: Business Administration

graphical:

Personal Data: Born in Edirne, Turkey, April 9, 1947, the son of Mr. and Mrs. Celal Akoglu.

Education: Graduated from Konya High School, Konya, Turkey, in June, 1965; attended Middle East Technical University in 1965-1971; received the Bachelor of Science in Administrative Sciences in November, 1971; completed requirements for the Master of Business Administration degree at Oklahoma State University in July, 1974.

Professional Experience: Assistant Expert in the Economic Analysis Department of the State Investment Bank of Turkey.