MODIFICATION OF STIFFNESS ANALYSIS TO ACCOUNT FOR SHEAR DEFORMATIONS

By

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TABLE OF CONTENTS

DEFINITION OF VARIABLES

A	AREA
E	MODULUS OF ELASTICITY
G	MODULUS OF RIGIDITY
I	MOMENT OF INERTIA
IX	MOMENT OF INERTIA ABOUT THE X AXIS
IY	MOMENT OF INERTIA ABOUT THE Y AXIS
IZ	MOMENT OF INERTIA ABOUT THE Z AXIS
M_L	STRESS RESULTANT DUE TO REAL LOAD (FLEXURE)
M_U	STRESS RESULTANT DUE TO UNIT LOAD (FLEXURE)
Q	FIRST MOMENT OF AN AREA
V_L	STRESS RESULTANT DUE TO REAL LOAD (SHEAR)
V_U	STRESS RESULTANT DUE TO UNIT LOAD (SHEAR)
f_s	SHEAR FORM FACTOR

THE EFFECT OF SHEAR DEFORMATIONS IN STIFFNESS ANALYSIS

INTRODUCTION

The stiffness method of structural analysis assumes linear, elastic, prismatic, homogenous behavior. Within the frame work of these assumptions, the stiffness matrix normally used for analysis does not account for the effect of shear deformations. However within this frame work of linear, elastic, prismatic, homogenous behavior a stiffness matrix to account for the effect of shear deformations can be derived.

The stiffness analysis program, "Space Frame" written by Louis O. Bass and John O. Houston, was modified to account for the effect of shear deformations. The modification is now a users specified option in the modified version of "Space Frame".

SHEAR DEFORMATION

Shear deformations will be assumed not to be affected by member axial (Px) and torsional (Mx) forces, and to behave in a linearly elastic manner. These assumptions allow a simple modification to the basic stiffness matrix. Through the use of virtual work, the modifications are derived. By taking the internal work;

$$dW_{int} = (\sigma\, dy\ dz)(\epsilon\, dx) + (\tau\, dy\ dz)(\gamma\, dx)$$

substituting and integrating over the volume with the following terms.

$$\sigma = \frac{Mu\ Y}{I} \qquad \tau = \frac{Vu\ Q}{I\ b}$$

$$\epsilon = \frac{Ml\ Y}{E\ I} \qquad \gamma = \frac{Vl\ Q}{G\ I\ b}$$

Yielding

$$W_{int} = \int \frac{Mu\ Ml\ Y^2}{E\ I^2}\, dx\ dy\ dz + \int \frac{Vu\ Vl\ Q^2}{G\ I^2 b^2}\, dx\ dy\ dz$$

Noting that at a given cross section the quantites Mu, Ml, Vu, Vl, E, G, and I are constant. Simplifing

$$W_{int} = \int_L \frac{Mu\ Ml}{E\ I^2}\left[\int_A Y^2\, dy\ dz\right] dx + \int_L \frac{Vu\ Vl}{G\ I^2}\left[\int_A \frac{Q^2}{b^2}\, dy\ dz\right] dx$$

Letting L and A denote the intergration carried out over the length and the cross-sectional area respectively. The first term in the square brackets is the moment of inertia (I) and the second term is the shear form factor (f_s). The shear form factor is defined as follows;

$$f_s = \frac{A}{I^2} \int_A \frac{Q^2}{b^2} dA$$

Replacing the terms in the square brackets with I and $f_s\ I^2/A$ respectively and equating the external and internal work, deformations are given by;

$$\Delta = \int \frac{Mu\ Ml}{E\ I} dx + \int \frac{f_s\ Vu\ Vl}{G\ A} dx$$

By evulating the above expression the modified stiffness matrix accounting for shear deformations is found. For these terms see Appendix A.

THE SHEAR FORM FACTOR

The shear form factor (f_s) in the modification of the stiffness matrix is defined as;

$$f_s = \frac{A}{I^2} \int \frac{Q^2}{b^2} dA$$

A = area of the cross section

I = moment of inertia of the cross section

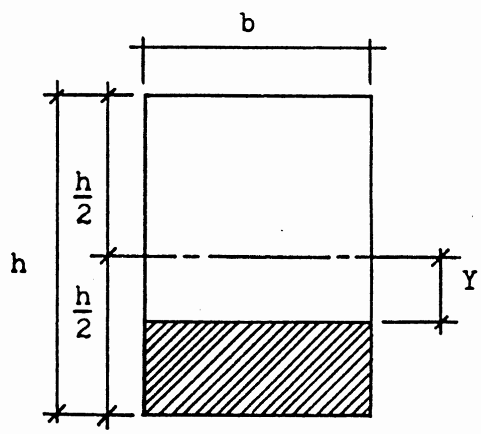
Q = the first moment of area for the cross section

b = the thickness of the cross section

For some standard shapes the shear form factor is given. These are defined as follows;

Rectangle	$f_s = 6/5 = 1.2$
Circle	$f_s = 10/9 = 1.11$
Thin tube	$f_s = 2.0$
Wide Flange or Box Section	f_s = Area Total/Area of the Web

or it may be calculated for the shape. To calculate the shear form factor Q and b need to be defined. For an example take a rectangle of width b and height h.



$$A = b\,h$$

$$I = \frac{b\,h^3}{12}$$

$$Q = \frac{b}{2}\left[\frac{h^2}{4} - Y^2\right]$$

$$b = b$$

Writing the equation for the shear form factor;

$$f_s = \frac{144}{b\,h^3} \int \frac{b^2}{4} \left[\frac{h^2}{4} - Y^2\right]^2 \frac{1}{b^2}$$

$$f_s = \frac{144}{b\,h^5} \int_{-h/2}^{h/2} \frac{1}{4} \left[\frac{h^2}{4} - Y^2\right]^2 b \; dy$$

Evaulating

$$f_s = \frac{6}{5} = 1.2$$

For other values of the first moment of area (Q) see Appendix B. If there is a question about which shear form factor to use it is desirable to calculate the shear form factor.

USER INSTRUCTIONS

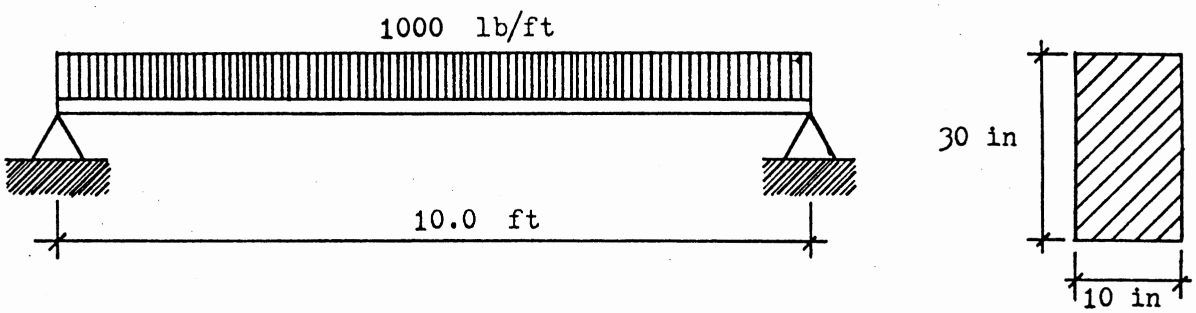
When executing "Space Frame" with shear deformations, it is necessary for the user to have a copy of the user's manual for "Space Frame" by L. O. Bass. First you will be asked several questions about basic program parameters, after answering these, the user will be asked if they want to consider shear deformations. By inputing a one (1), the question is answered with a yes, and by inputing a zero (0), the question is answered with a no. If answered no, the program continues with "Space Frame" and no other questions concerning shear deformations will be asked. When shear deformations are being considered (a yes answer), the user will be asked to input the shear form factor (SFF). The SFF is inputed with the member properties, the display will read A,IX,IY,IZ,SFF? these values are inputed with commas separting them, and the shear form factor is last. Once the member properties have been inputed, the program will not ask the user any more questions dealing with shear deformations. From here the program follows the "Space Frame" manual. For flow charts see Appendix C.

It may be desirable to consider shear deformations on some members but not on others. By setting the shear form factor equal to zero (0). The modified stiffness matrix reverts back to the standard stiffness matrix, where SFF equal zero (0). Again it may be desirable to consider shear deformations on a frame, then run the same frame without shear deformations. To do this run the frame first with shear deformations; then rerun the frame without shear deformations, by setting all the SFF equal to zero(0) for all member types.

EXAMPLE PROBLEM

The PASSWORD for the modified version of "Space Frame" is [redacted]

For an example problem, take a ten foot (10 ft) concrete beam with simple supports and an uniformed load of 1000 pounds per foot.



$A = 30 \text{ in}^2$

$IX = 25{,}000 \text{ in}^4$

$IY = 2{,}500 \text{ in}^4$

$IZ = 22{,}500 \text{ in}^4$

$E = 3.64 \text{ E } 6 \text{ psi}$

$G = 1.46 \text{ E } 6 \text{ psi}$

$f_s = 1.2$ (SFF)

```
GET"FRAME"
RUN
DO NOT USE THIS SPACE FRAME  !!
PASSWORD?[redacted]
JOB NAME, DATE, ETC?TONY GOETZ SIMPLE SUPPORTES
COMMENTS-60 CHARACTERS MAX?30X10 CONCRETE BEAM (FC=4000 PSI)


  DATA ECHO                                                    PAGE #    1

  TONY GOETZ SIMPLE SUPPORTES
  30X10 CONCRETE BEAM (FC=4000 PSI)

  BASIC JOB PARAMETERS

#JOINTS?11
# MEMBERS?10
# MEMBER PROPERTY TYPES?1
# SUPPORTS?2
# LOAD CONDITIONS?1
# FINITE ELEMENTS?0
CONSIDER SHEAR DEFORMATIONS?1
  #JOINTS   #MEMBERS   #M.TYPES   #SUPPORTS   #LOAD COND'S   #ELEM'S
     11         10          1           2              1          0
  CONSIDERING EFFECTS OF SHEAR DEFORMATIONS
ANY SUPPORT RELEASES?1
ARE RELEASES IN GENERAL SYSTEM?1
  SUPPORT RELEASES OCCUR IN THE GENERAL SYSTEM

DATA OK?1

  JOINT CARTESIAN COORDINATES

COORDINATE LENGTH UNITS?12.0000FEET        FT
  UNITS =FEET
UNITS OK?1

  JOINT      -X-(FT)          -Y-(FT)          -Z-(FT)

X, Y, Z --- AT 1     ?0,0,0
    1        0.0000            0.0000           0.0000
X, Y, Z --- AT 2     ?1,0,0
    2        1.0000            0.0000           0.0000
X, Y, Z --- AT 3     ?2,0,0
    3        2.0000            0.0000           0.0000
X, Y, Z --- AT 4     ?3,0,0
    4        3.0000            0.0000           0.0000
X, Y, Z --- AT 5     ?4,0,0
    5        4.0000            0.0000           0.0000
X, Y, Z --- AT 6     ?5,0,0
    6        5.0000            0.0000           0.0000
X, Y, Z --- AT 7     ?6,0,0
    7        6.0000            0.0000           0.0000
X, Y, Z --- AT 8     ?7,0,0
    8        7.0000            0.0000           0.0000
X, Y, Z --- AT 9     ?8,0,0
    9        8.0000            0.0000           0.0000
X, Y, Z --- AT 10    ?9,0,0
   10        9.0000            0.0000           0.0000
X, Y, Z --- AT 11    ?10,0,0
   11       10.0000            0.0000           0.0000
ALL COORDINATES OK?1
```

```
JOINT SUPPORT DATA

NOTE:  PX,PY---MZ RELEASE CODES >>---> 0=FIXED, 1=RELEASED

    <-----------RELEASES--------------> <---------------LISTING--------------->
REF PPPMMM      ANGLES---UNITS=DEGREES                       OF
 #  XYZXYZ      ALFA       BETA       GAMA               JOINT SUPPORTS

SUPPORT LIST W/SAME CONSTANTS?1,11
PX,PY---MZ RELEASE (KEY)?PZ3
PX,PY---MZ RELEASE (KEY)?MX4
PX,PY---MZ RELEASE (KEY)?MY5
PX,PY---MZ RELEASE (KEY)?MZ6
PX,PY---MZ RELEASE (KEY)?0
  1  001111        0          0           0  1,11
SUPPORT LIST W/SAME CONSTANTS?0
ALL SUPPORT DATA OK?1
LENGTH UNITS, MEMBER PROPERTIES?1.00000INCHES        IN
FORCE UNITS MEMBER PROPERTIES?1000.00KIPS          K
ARE E & G CONSTANT?1
CONST'S E,G (K /IN↑2)?3.64E3,1.46E3

   MEMBER PROPERTY CATALOG
   CONSTANTS:   E = 3640      G = 1460
   UNITS INCHES       KIPS

   MEMB     AX         IX         IY         IZ       SFF     DESCRIPTION
   TYPE  (IN↑2)    (IN↑4)     (IN↑4)     (IN↑4)
MBR 1 DESCRIPTION?30X10 BEAM
MBR 1  A,IX,IY,IZ,SF?30,25000,2500,22500,1.2
   1     30.00 25000.00   2500.00  22500.00  1.20  30X10 BEAM
ALL TYPES OK?1
MBR ROTATION UNITS (DEG OR RAD)?.01745329252DEGREES
MBR LENGTH UNITS?12.0000FEET        FT
ALL MBR PROPERTIES IDENTICAL?1
ANY MEMBER END RELEASES?0
ANY GAMA ROTATIONS?0
```

```
TONY GOETZ SIMPLE SUPPORTES
30X10 CONCRETE BEAM (FC=4000 PSI)


MEMBER DATA, UNITS = FEET           DEGREES
NO MBR END RELEASES    ALL MBR GAMA'S=0    ALL MBR PROPERTIES=TYPE #1

 MBR  LE#  RE# TYPE       ALFA        BETA        GAMA       LENGTH    LE CODE   RE CODE
DATA OK?1
-END#, +END#; MBR 1     ?1,2
   1    1    2    1     0.000       0.000       0.000       1.000     000000    000000
-END#, +END#; MBR 2     ?2,3
   2    2    3    1     0.000       0.000       0.000       1.000     000000    000000
-END#, +END#; MBR 3     ?3,4
   3    3    4    1     0.000       0.000       0.000       1.000     000000    000000
-END#, +END#; MBR 4     ?4,5
   4    4    5    1     0.000       0.000       0.000       1.000     000000    000000
-END#, +END#; MBR 5     ?5,6
   5    5    6    1     0.000       0.000       0.000       1.000     000000    000000
-END#, +END#; MBR 6     ?6,7
   6    6    7    1     0.000       0.000       0.000       1.000     000000    000000
-END#, +END#; MBR 7     ?7,8
   7    7    8    1     0.000       0.000       0.000       1.000     000000    000000
-END#, +END#; MBR 8     ?8,9
   8    8    9    1     0.000       0.000       0.000       1.000     000000    000000
-END#, +END#; MBR 9     ?9,10
   9    9   10    1     0.000       0.000       0.000       1.000     000000    000000
-END#, +END#; MBR 10    ?10,11
  10   10   11    1     0.000       0.000       0.000       1.000     000000    000000
CHANGE MEMBER DATA?0
LENGTH UNITS FOR MOMENT LOADS?12.0000FEET          FT
FORCE UNITS FOR LOADS (KEY)?1000.00KIPS           K
   LOAD UNITS FEET          KIPS
IS TEMP CHANGE CENTIGRADE?0
   TEMPERATURE UNITS = DEGREES FAHRENHEIT
ANY FOUNDATION SETTLEMENTS?0
UNITS OK?1
ANY SELF WEIGHT LOADINGS?0
IS LOAD TEMPLATE IN PLACE?1

   LOAD CONDITION # 1

   JT&MBR LOAD    DIM        PX          PY          PZ          MX          MY          MZ
   SETTLEMENTS               DX          DY          DZ          0X          0Y          0Z

MEMBER OR JOINT LOADS?MBR8-1
MBR LIST?1-10
   MBR LIST 1-10
LIST OK?1
CONC,UNIF,LIN OR TEMP?UNI20
JNT,MBR,DIS,FRA,PX,PY--MZ ?MBR8-1
JNT,MBR,DIS,FRA,PX,PY--MZ ?PY2
LOAD PY . K /FT?1.0
JNT,MBR,DIS,FRA,PX,PY--MZ ?0
   UNI M                0.000       1.000       0.000      -0.000       0.000       0.00
LOADS OK?1
CONC,UNIF,LIN OR TEMP?0
MEMBER OR JOINT LOADS?0
THRU WITH LOAD COND 1      ?1
IS UNITS TEMPLATE IN PLACE?1
```

```
DATA INPUT ECHO                                                PAGE #   2

TONY GOETZ SIMPLE SUPPORTES
30X10 CONCRETE BEAM (FC=4000 PSI)


OUTPUT UNITS SPECIFICATIONS

UTPUT LENGTH UNITS FOR MOMENT?1.00000INCHES      IN
UTPUT UNITS FOR FORCE?1000.00KIPS          K
UTPUT LENGTH UNITS,REACTIONS?1.00000INCHES      IN
UTPUT FORCE UNITS,REACTIONS?1000.00KIPS       K
ENGTH UNITS FOR DEFORMATIONS?1.00000INCHES      IN
/O ROTATION UNITS,DEFORMATIONS?.01745329252DEGREES
  MEMBER FORCES & MOMENTS = INCHES      KIPS
  REACTIONS               = INCHES      KIPS
  DEFORMATIONS            = INCHES      DEGREES

NITS OK?1
ONSIDER REBAND OR REPACK?0

  END OF DATA INPUT

  PROGRAMS AVAILABLE FOR CONTINUED PROCESSING

    ECHO------REPRINT DATA
    PLOT------PLOT PERSPECTIVE OF THE FRAME
    ISOMET----PLOT ISOMETRIC OF THE FRAME
    FRAME-----TO MAKE DATA CHANGES
    BANDER----SOLUTION INITIALIZATION--REQ'D BEFORE SOLVE OR REBAND
    REBAND----RENUMBER JOINTS FOR NEW SOLUTION SEQUENCE


ET"BANDER"
```

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30X10 CONCRETE BEAM (FC=4000 PSI)

DEFORMATIONS UNITS: INCHES DEGREES

JOINT	LC	DX	DY	DZ	θX	θY	θZ
1	1	0.0000	0.0000	0.0000	0.00000	0.00000	0.0042
2	1	0.0000	0.0023	0.0000	0.00000	0.00000	0.0039
3	1	0.0000	0.0043	0.0000	0.00000	0.00000	0.0033
4	1	0.0000	0.0057	0.0000	0.00000	0.00000	0.0023
5	1	0.0000	0.0066	0.0000	0.00000	0.00000	0.0012
6	1	0.0000	0.0069	0.0000	0.00000	0.00000	0.0000
7	1	0.0000	0.0066	0.0000	0.00000	0.00000	-0.0012
8	1	0.0000	0.0057	0.0000	0.00000	0.00000	-0.0023
9	1	0.0000	0.0043	0.0000	0.00000	0.00000	-0.0033
10	1	0.0000	0.0023	0.0000	0.00000	0.00000	-0.0039
11	1	0.0000	0.0000	0.0000	0.00000	0.00000	-0.0042

MEMBER FORCES AND MOMENTS UNITS: INCHES KIPS

MEMBER	LC	END	PX	PY	PZ	MX	MY	MZ
1	1	1	0.000	-5.000	0.000	0.000	0.000	-0.000
		2	0.000	4.000	0.000	0.000	0.000	-54.000
2	1	2	0.000	-4.000	0.000	0.000	0.000	54.000
		3	0.000	3.000	0.000	0.000	0.000	-96.000
3	1	3	0.000	-3.000	0.000	0.000	0.000	96.000
		4	0.000	2.000	0.000	0.000	0.000	-126.000
4	1	4	0.000	-2.000	0.000	0.000	0.000	126.00
		5	0.000	1.000	0.000	0.000	0.000	-144.00
5	1	5	0.000	-1.000	0.000	0.000	0.000	144.00
		6	0.000	0.000	0.000	0.000	0.000	-150.00
6	1	6	0.000	-0.000	0.000	0.000	0.000	150.00
		7	0.000	-1.000	0.000	0.000	0.000	-144.00
7	1	7	0.000	1.000	0.000	0.000	0.000	144.00
		8	0.000	-2.000	0.000	0.000	0.000	-126.00
8	1	8	0.000	2.000	0.000	0.000	0.000	126.00
		9	0.000	-3.000	0.000	0.000	0.000	-96.00

TONY GOETZ SIMPLE SUPPORTES
30X10 CONCRETE BEAM (FC=4000 PSI)

MEMBER FORCES AND MOMENTS UNITS: INCHES KIPS

MEMBER	LC	END	PX	PY	PZ	MX	MY	MZ
9	1	9	0.000	3.000	0.000	0.000	0.000	96.00
		10	0.000	-4.000	0.000	0.000	0.000	-54.00
10	1	10	0.000	4.000	0.000	0.000	0.000	54.00
		11	0.000	-5.000	0.000	0.000	0.000	0.00

REACTIONS AT SUPPORTS UNITS: INCHES KIPS

JOINT	LC	PX	PY	PZ	MX	MY	MZ
1	1	0.000	-5.000	0.000	0.000	0.000	-0.00
11	1	0.000	-5.000	0.000	0.000	0.000	0.00

SUM OF ALL REACTIONS

	LC	PX	PY	PZ	MX	MY	MZ
	1	0.000	-10.000	0.000	0.000	0.000	0.00

APPENDIX A

MODIFICATIONS TO THE STIFFNESS MATRIX

MODIFICATIONS TO THE STIFFNESS MATRIX

K_{ij} denotes the Stiffness Matrix and the position of the term.

$K_{2,2}$ and $K_{3,3}$ $$\frac{12\ EIGA}{GAL^3 + 12\ f_s EIL}$$

$K_{2,6}$ and $K_{6,2}$ $$\frac{6\ EIGA}{GAL^2 + 12\ f_s EI}$$

$K_{3,5}$ and $K_{5,3}$ $$-\frac{6\ EIGA}{GAL^2 + 12\ f_s EI}$$

$K_{5,5}$ and $K_{6,6}$ $$\frac{4(GAL^2 + 3\ f_s EI)(EI)^2}{GAEIL^3 + 12\ f_s L(EI)^2}$$

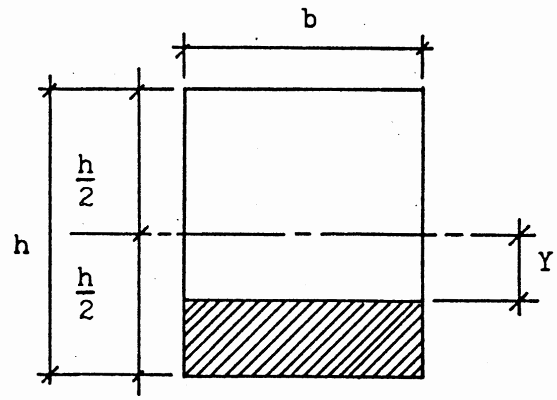
APPENDIX B

THE FIRST MOMENT OF AREA (Q) FOR SELECTED SHAPES

THE FIRST MOMENT OF AREA FOR SELECTED SHAPES

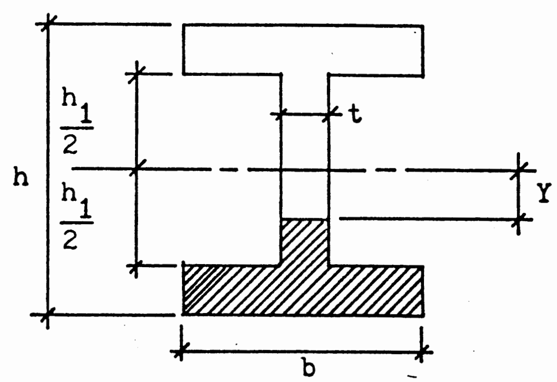
Rectangle

$$Q = \frac{b}{2}\left[\frac{h^2}{4} - y^2\right]$$



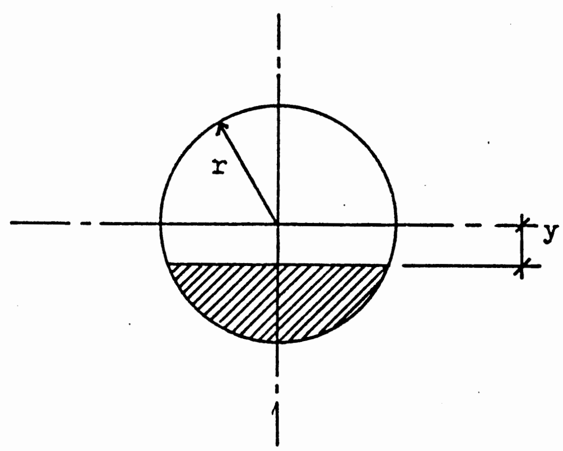
Wide Flange

$$Q = \frac{b}{2}\left[\frac{h^2}{4} - \frac{h_1^2}{4}\right] + \frac{t}{2}\left[\frac{h_1^2}{4} - y^2\right]$$



Circular (Solid)

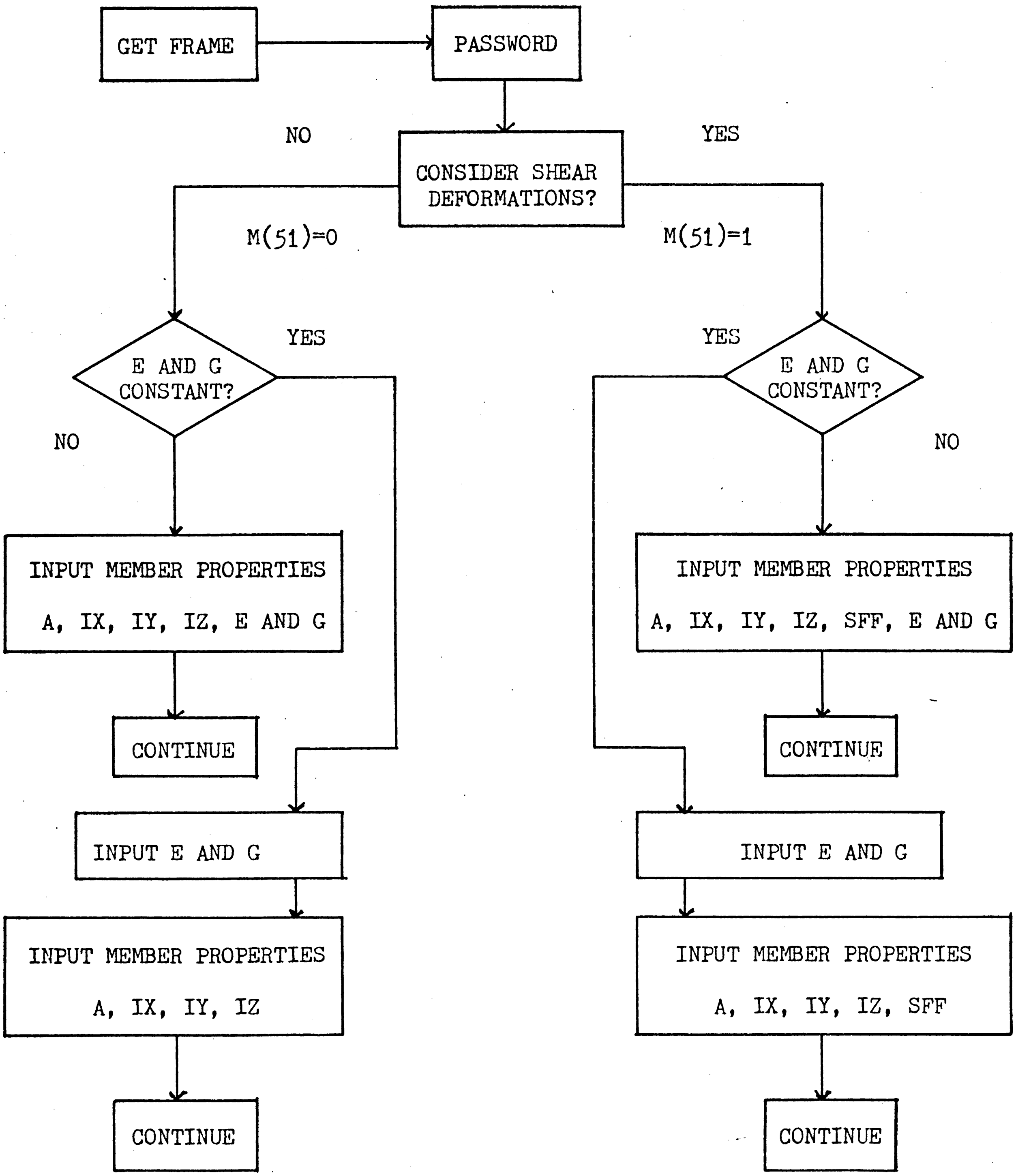
$$Q = \frac{2}{3}(r^2 - y^2)^{3/2}$$



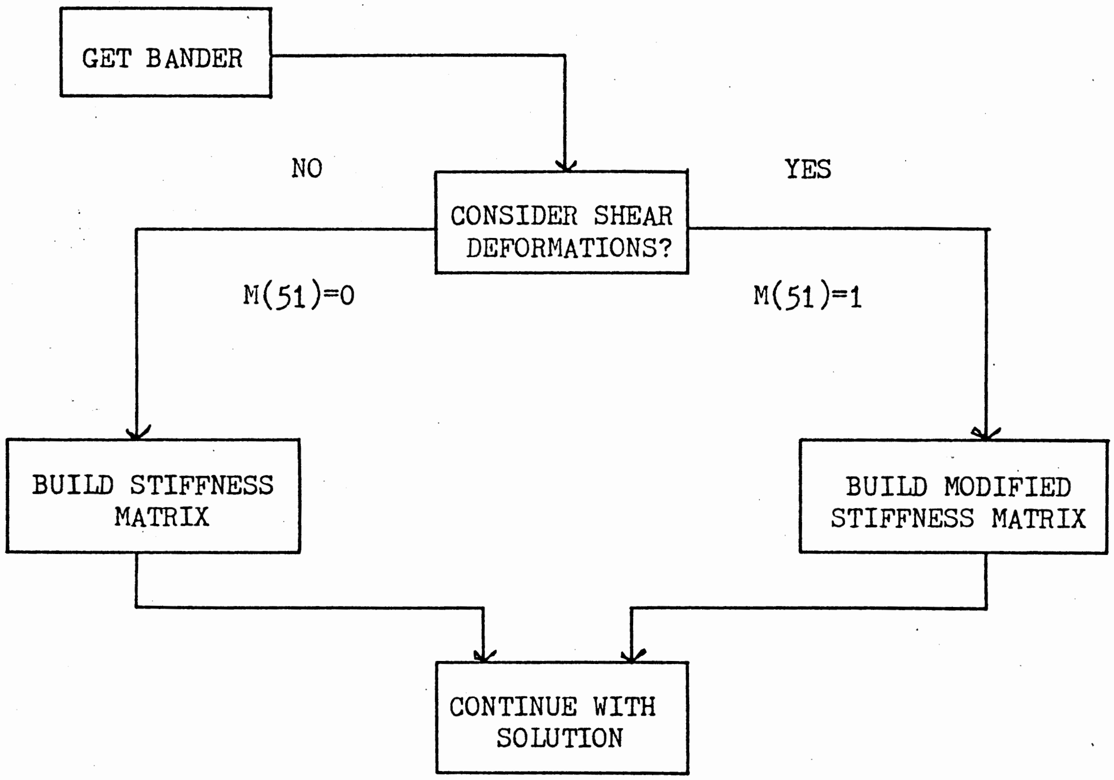
APPENDIX C

FLOW CHARTS

FLOW CHART FOR INPUT SEQUENCE



FLOW CHART FOR SOLUTION



BIBLIOGRAPHY

Ketter, Robert L., George C. Lee, and Sherwood P. Prawel, Jr. Structural Analysis and Design. New York: McGraw-Hill book Company, 1979.

Przemieniecki, J. S. Theory of Matrix Structural Analysis. New York: McGraw-Hill Book Company, 1968.

Timoshenko, S. P. Strength of Materials Part 1. Huntington, New York: Robert E. Krieger Publishing Company, 1955.

Timoshenko, S. P., and James M. Gere. Mechanics of Materials. New York: D. Van Nostrand Company, 1972.