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THE DEVELOPMENT AND TESTING OF SUBJECT MATTER FOR A COURSE IN METHODS OF TEACHING SECONDARY SCHOOL MATHEMATICS

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TABLE OF CONTENTS

TABLE OF CONTENTS

LIST OF TABLES

LIST OF TABLES

THE DEVELOPMENT AND TESTING OF SUBJECT MATTER FOR A COURSE IN METHODS OF TEACHING SECONDARY SCHOOL MATHEMATICS

CHAPTER I

INTRODUCTION

Purpose of the Study

The twentieth century has witnessed the establishment of numerous commissions and committees whose purpose has been to study the optimum preparation which colleges and universities should provide future teachers of secondary school mathematics. The most recent and influential of these is the Committee on the Undergraduate Program in Mathematics whose recommendations were published by the Mathematical Association of America in 1961.

Generally these study groups recognized the existence of two basic problems. The first problem was the type of mathematics courses the future teacher should study while attending college and the second was how the individual can best be prepared to teach mathematics.

In response to the first problem, the groups, including the Committee on the Undergraduate Program in Mathematics, were quite diligent in outlining their recommendations concerning an appropriate sequence of mathematics courses and the content of those courses. In compliance with the Committee's recommendations, colleges and universities throughout the United States have modified their curriculum offerings in mathematics.

Many leaders in mathematics and mathematics education have expressed themselves regarding the problem of preparing the individual to teach mathematics. The comment by Donovan Johnson of the University of Minnesota is typical of their expression of opinion.

> All major groups establishing ideal programs of preparation for mathematics teaching have included student teaching and a methods course in their recommendations.[1]

A methods course, therefore, is considered to be an essential part of the preparation of future secondary school mathematics teachers. The critical issue here is that none of these groups has prepared a detailed syllabus for the methods course; therefore, its structure remains undefined in comparison with the content courses in mathematics.

Statement of the Problem

This study will deal with the selection and validation of subject matter to be included in the undergraduate methods course entitled "The Teaching of Secondary School Mathematics." In order to achieve this goal, the following must be accomplished.

1. The criteria for the selection of subject matter must be established.

2. The subject matter for the methods course must be selected.

3. The proposed methods course must be field tested.

4. The proposed methods course must be evaluated.

Significance of the Problem

In 1893 the first methods course in mathematics was taught at the

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[1]Donovan A. Johnson, "A Methods Course for Mathematics Teachers," The American Mathematical Monthly, LXXI (November, 1964), 1038.

University of Michigan.[1] Since then, numerous groups from the International Commission on the Teaching of Mathematics, established in 1908, to the Committee on the Undergraduate Program in Mathematics, which issued its report in 1961, have cited the need for a special methods course in mathematics. This need, as indicated in the literature and research in mathematics education during the past fifteen years, demonstrates that the mathematics community is aware of the importance of the methods course but is reluctant to propose the content and the teaching procedures for the course.

In order to illustrate this reluctance, the comments by groups of national prominence will be the first to be investigated. In discussing their proposed curriculum for secondary school mathematics teachers, the Committee on the Undergraduate Program in Mathematics suggests that everyone to whom we entrust the process of the education of our youth should have included a methods course as a part of his undergraduate preparation.[2]

The discussion of the mathematical and professional preparation of teachers by the Commission on Mathematics can be summarized as meaning that masterful scholarship in a body of relevant knowledge is an absolute for effective teaching, but it must be supplemented by a proficiency in the use of effective techniques of instruction.[3] In reference to the

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[1]International Commission on the Teaching of Mathematics, The American Report, Training of Teachers of Elementary and Secondary Mathematics (U. S. Bureau of Education, Bulletin No. 12, Washington: Government Printing Office, 1911), pp. 5-6.

[2]Committee on the Undergraduate Program in Mathematics, Recommendations for the Training of Teachers of Mathematics (Buffalo, New York: Mathematical Association of America, 1961), p. 14.

[3]Commission on Mathematics, Report of the Commission on Mathematics (New York: College Entrance Examination Board, 1959), pp. 50-58.

methods course, they state,

> . . . there is a need for a course which deals specifically with the teaching of mathematics in the secondary schools.[1]

In a study conducted by Eleanor Walters, a survey was made of mathematicians and mathematics educators to determine the needs which they felt were peculiar to teachers but which are not met by the usual content courses in college mathematics. A universal recommendation of the sample questioned by Walters was that a special methods course in mathematics should be included as a part of the pre-professional training of every future secondary school mathematics teacher.[2] Rosalind Chavier verified this conclusion in a questionnaire survey which she conducted in reference to the methods course offered by colleges and universities throughout the United States. One question she asked was,

> Do you feel that the methods course can best be classified as necessary, supplementary, or unnecessary in the preparation of future high school mathematics teachers?[3]

Fifty-eight institutions responded to this question and fifty-four said necessary, three said supplementary, and only one said unnecessary.[4]

The writings of William Gager of the University of Florida and Donovan Johnson of the University of Minnesota agree that the methods

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[1] Ibid., p. 57.

[2] Eleanor B. Walters, "Concept, Place, and Purpose of Professionalized Subject-Matter in the Education of Teachers of Secondary Mathematics" (unpublished Ed. D. doctoral project, Teachers College, Columbia University, 1955), p. 61.

[3] Rosalind Roper Chavier, "The Undergraduate Course in Methods of Teaching Secondary School Mathematics" (unpublished Ed. D. doctoral project, Teachers College, Columbia University, 1964), p. 269.

[4] Ibid.

course is essential for future teachers, but they are displeased with the way in which they know it is taught in many colleges and universities throughout the United States. When speaking about the education of teachers of secondary school mathematics, Gager says,

> Most of these teachers have not been sufficiently trained in . . . methods to teach the subject so that it means something to their pupils.[1]

Gager states his beliefs on mathematics teacher education like this:

> In addition to giving very special attention to those who are preparing to teach mathematics, each university mathematics department should realize that it also has an obligation to see that some of its members share in the improvement of the secondary mathematics materials and methods just as much as it has an obligation to see that some of its members engage in pure mathematics research and graduate teaching.[2]

Johnson echoes Gager's comments in reference to the role and function of the university mathematics department when he says,

> The better the teacher's background in mathematics, the greater his problem in selecting what he can hope to present to adolescents -- hence, the need for a methods course. . . . If as the Commission on Mathematics says, 'A poor curriculum well taught is better than a good curriculum badly taught,' then a methods course which improves teaching techniques is an essential course for every prospective mathematics teacher.[3]

The value of the methods course is also acutely realized by the classroom teacher. Thacker and Read polled high school mathematics teachers to ascertain which courses they felt were the most valuable for the future mathematics teacher. When all of the courses that a teacher

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[1]William A. Gager, "Is Your College Giving Proper Training for Teachers of Secondary School Mathematics?" The Mathematics Teacher, LV (October, 1962), 494.

[2]Ibid., p. 495.

[3]John A. Brown and John R. Mayor (eds.), "The Methods Course in Mathematics for Prospective Secondary School Teachers," The American Mathematical Monthly, LXVII (August-September, 1960), 689.

of mathematics takes as a part of his undergraduate preparation were ranked according to importance, the methods course was listed third. It was preceded only by College Algebra and Plane Trigonometry.[1]

In a similar study involving 951 in-service teachers, Small found that if teachers themselves were establishing a fifth year program in teacher education, they would include an advanced course in the methods and problems of teaching mathematics. The in-service teachers ranked this course first among those selected for the fifth year program.[2]

Based on comments and research by mathematicians, mathematics educators, classroom teachers, and committees and commissions of national prominence, the need for and the importance of the methods course in mathematics is clearly evident. The structure of the methods course has not, however, been made clear. Why has the structure for a methods course in mathematics been omitted by all the groups, commissions, and committees recommending it?

Because the significance of this course has been realized by the mathematics community since the turn of the present century, one might be led to conclude that many detailed syllabuses for such a course would have been proposed and that these in turn were taught, evaluated and revised many times in order to find a core of topics around which the methods course could be structured. To the present, however, there have been no concrete proposals put forth, tried out under experimental

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[1]G. R. Thacker and C. B. Read, "Courses Desirable for Training Teachers of High School Mathematics," School Science and Mathematics, XLIX (November, 1949), 615.

[2]Dwain Small, "The Fifth Year of Teacher Education for Teachers of Mathematics," The Mathematics Teacher, L (March, 1957), 199-203.

conditions, and evaluated for their effectiveness.

The absence of concrete proposals, as indicated above, is amplified by Henderson in his discussion on the research in reference to teaching secondary school mathematics. He says,

> One does not have to read much pedagogical theory before he realizes that such theory is essentially normative. Builders of pedagogical theory are interested primarily in influencing institutional practices regarding mathematics education or the classroom behavior of mathematics teachers. Hence the language of the theory (more accurately -- theories) is primarily advisory rather than descriptive.[1]

The Committee on the Undergraduate Program in Mathematics has exerted a profound and vital influence on the curriculum for the preparation of teachers of secondary school mathematics. Their recommendations have been published in two separate pamphlets. The first pamphlet, Recommendations for the Training of Teachers of Mathematics, prescribes the following preparation for prospective junior high school mathematics teachers; three courses in analysis, one course in abstract algebra, one course in probability and statistics, and one elective chosen from introduction to real variables, number theory, topology, history of mathematics, or numerical analysis.[2] The preparation program for senior high school school mathematics teachers would include the courses listed above for teachers in the junior high school, an additional elective, and one more course in each of the areas of probability and statistics, geometry, and abstract algebra.[3] Both groups would include as an integral

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[1]Kenneth B. Henderson, "Research on Teaching Secondary School Mathematics," in Handbook of Research on Teaching, edited by N. L. Gage (Chicago: Rand McNally & Company, 1963), pp. 1011-12.

[2]Committee on the Undergraduate Program in Mathematics, Recommendations . . . , pp. 11-12.

[3]Ibid., pp. 12-13.

part of their preparation a course in the methods of teaching secondary school mathematics.[1]

In order to further define their recommendations the Committee issued a second pamphlet, Course Guides for the Training of Teachers of Junior High and High School Mathematics. Here they say,

> The Panel on Teacher Training of the Committee on the Undergraduate Program in Mathematics has as one of its major concerns the development of college curricula needed as a preparation for future teachers of mathematics. We have given recommendations on the nature of the courses which we believe prospective teachers of mathematics, at each of several levels, should study. . . .
>
> Presented here are outlines of sample courses which might be used in connection with our Recommendations for the training of teachers of junior high and high school mathematics.
>
> Each outline includes a short description of the level and flavor of the suggested course, and there is a bibliography suitable for constructing such a course.[2]

Even though the methods course is prescribed by the Committee in their first pamphlet and described there as a course which "merits special attention,"[3] a suggested outline for its content is omitted from their second pamphlet.

Felder points out that while pre-service methods courses have been included in the curriculum for the education of teachers of secondary school mathematics for nearly seventy years, there exists no analysis per se of their place or purpose.[4]

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[1] Ibid., p. 14.

[2] Committee on the Undergraduate Program in Mathematics, Course Guides for the Training of Teachers of Junior High and Senior High School Mathematics (Buffalo, New York: Mathematical Association of America, 1961), p. 3.

[3] Committee on the Undergraduate Program in Mathematics, Recommendations . . . , pp. 11-12.

[4] Virginia Felder, "A Proposal for a Methods Course to be Used in the Education of Teachers of Secondary-School Mathematics" (unpublished Ed. D. doctoral project, Columbia University, 1959), p. 4.

Chavier's study demonstrates that the criticism the methods course has received has not been directed toward its inherent value but rather toward the determination of content for the course.[1]

In summarizing the research which he did in reference to the historical development of the methods course, Gordon Mock says,

> The point of this review is to indicate that the wisest choice of subject matter for a methods course is not determined by simple historical accretion or accident. There must be some basic philosophy concerning the content of the courses necessary for the future teacher, and this should determine the course syllabus. Not everyone will have the same beliefs, it hardly needs to be pointed out, but we need to generate and communicate some ideas in this area so that the methods course will be improved. . . . The difficulty lies in determining what topics to include since there are more than could possibly be adequately covered in a three semester hour course.[2]

In summary then, the mathematics community is aware of the need for and the importance of the methods course in the preparation of future secondary school mathematics teachers. There has been no formal attempt, however, to propose a syllabus for this course, to teach the proposed syllabus, and to evaluate its effectiveness. This fact may at least partially explain why all groups of individuals recommending it have not proposed a structure upon which it could be built. The present research will provide information which allows this highly significant but neglected problem in the education of future teachers of secondary school mathematics to be answered.

Terminology of the Study

College and university curricula designed to prepare teachers of

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[1]Chavier, p. 270.

[2]Gordon D. Mock, "The Methods Course," The Mathematics Teacher, LIV (January, 1961), 17-18.

secondary school mathematics have customarily included a course in which unique problems associated with the teaching of mathematics are discussed. When this course was first taught, the emphasis was placed upon "methods" of presenting the subject matter; thus, the title "methods course" became associated with it.[1]

With the passage of time, the scope of the methods course was broadened so that today it is thought of as the point in the teacher education curriculum where the subject matter to be taught converges with general educational theory and the professional aspects of teaching. As a result of this evolution, college mathematics departments gradually began to list this course in their catalogs as "The Teaching of Secondary School Mathematics."

In this study "the methods course in mathematics" and the course "the teaching of secondary school mathematics" shall be considered as being synonymous. Either will mean the course in the teacher education curriculum which deals exclusively with the techniques of, problems of, and responsibilities for teaching secondary school mathematics.

Scope of the Study

The first stage of this research dealt with the selection of the subject matter to be included in the experimental methods course. A survey was made of existing recommendations of individuals and study groups in addition to an analysis of the content of methods course textbooks and the structure of contemporary methods courses throughout the United States. These findings are described in Chapter II.

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[1]Gordon Duane Mock, "The Development of Methods Courses in the Teaching of Mathematics Since 1890" (unpublished Ph. D. dissertation, University of Wisconsin, 1959), p. 1.

An analysis of the findings in Chapter II led to the construction of a list of topics which was suitable for inclusion in a methods course for contemporary secondary school mathematics teachers. This list was then incorporated into a questionnaire (Appendix A) which was sent to specialists in mathematics education throughout the United States. A composite of the opinions of 200 of these specialists was used to design a one semester experimental methods course. The selection of the specialists, the results of the questionnaire survey, and the structure of the experimental methods course are discussed in Chapter III.

The second phase of this research dealt with the field testing of the experimental methods course. Three instructors taught the course during the first semester of the 1966-67 school year. On the basis of their experiences in teaching the various topics in the amount of time allotted to each of the topics, an evaluation of the experimental course's "teachability" was made. The nature and results of this evaluation are discussed in Chapter IV.

In order to compare the experimental course with current teaching practices in the methods course, a survey was made of thirty private colleges, state colleges and universities in Pennsylvania. A composite of their activities in the methods course was constructed and then compared with the experimental course. The composite of the thirty colleges and universities was used as a control group to which the topics and the time devoted to each of the topics in the experimental course was compared. These findings and comparisons are discussed in Chapter IV.

Since the methods course is designed as a pre-student teaching experience, the opinions of forty supervising teachers in the public

secondary schools of Pennsylvania were solicited to determine what topics they felt should be included in the methods course and the amount of class time which should be devoted to each of these topics. This information enabled a comparison to be made in Chapter IV between the opinions of the specialists in mathematics education and the supervising teachers in mathematics concerning the needs of prospective secondary school mathematics teachers.

During the sixth week of the second semester of the 1966-67 school year, the students who were enrolled in the experimental methods course and who were then student teaching were polled in reference to their opinions concerning the most appropriate structure for the methods course. This enabled a comparison to be made between the content of the experimental methods course and the composite opinion of the individuals for whom the course was designed. These results and comparisons are discussed in Chapter IV.

The final stage of this research involved the evaluation of the experimental methods course. Opinions and experiences of the instructors who taught the experimental course, of supervising teachers, and of student teachers were considered for this evaluation. On the basis of this evaluation, recommendations were made for the modification of the experimental course and an optimum methods course was proposed for contemporary secondary school mathematics teachers.

The students who participated in the experimental methods course can be described as follows:

1. They were enrolled in the course the semester prior to the one in which they did their student teaching.

2. Their background in mathematics was equivalent to that which is recommended by the Committee on the Undergraduate Program in Mathematics for future high school teachers.

3. Their background in professional education included the following courses or their equivalents: History and Philosophy of Education, General Psychology, Educational Psychology, Evaluative Methods, and Audio-Visual Education.

4. They were not told or made aware that the course in which they were enrolled was experimental.

The proposed course is designed to be offered for three semester hours of credit or forty-five periods of classroom instruction. It is organized to meet the needs of secondary school mathematics teachers as these needs exist today. There is no attempt to suggest permanence in its structure since the constant metamorphosis which education undergoes will affect what is taught in this course in the future. The course will not guarantee good teaching, but it is designed to contribute to this goal.

CHAPTER II

REVIEW OF THE LITERATURE AND RELATED RESEARCH

Purpose of the Review

One of the necessary procedures in constructing and testing a mathematics methods course was the establishment of criteria for the selection of subject matter to be included. The first step in this process is to review the existing hypotheses which have been presented and build upon them. Since this study is concerned with proposing a methods course to serve the needs of contemporary high school mathematics teachers, only those recommendations dealing specifically with the content of the methods course and made since the beginning of the current revolution in mathematics (about 1955) will be considered here. This chapter, therefore, will review the recommendations made by committees, commissions, mathematicians, and mathematics educators relative to the methods course since 1955. The data gathered from this review were analyzed and that analysis permitted the initial selection of topics which were appropriate for inclusion in the methods course.

The Recommendations of Committees and Commissions

Highly significant is the fact that all of the committees and commissions established in the twentieth century to study the question of the collegiate education which mathematics teachers should receive

soundly endorsed the methods course as an integral part of the program.[1] It is somewhat disappointing, however, to note the extent and depth of their recommendations. The examples listed below will illustrate the varying degrees of definitiveness.

The current model for many college and university mathematics programs can be ascribed to the Committee on the Undergraduate Program in Mathematics. Their complete list of recommendations for the methods course includes the following items with which they feel the teacher must be familiar.

> 1. The objectives and content of the many proposals for change in our curriculum and texts.
> 2. The techniques, relative merits, and roles of such teaching procedures as the inductive and deductive approaches to new ideas.
> 3. The literature of mathematics and its teaching.
> 4. The underlying ideas of elementary mathematics and the manner in which they may provide a rational basis for teaching.
> 5. The chief applications which have given rise to various mathematical subjects. These applications will depend upon the level of mathematics to be taught, and are an essential part of the equipment of all mathematics teachers.[2]

The Committee further elaborated on these suggestions at their meeting in Seattle on April 6-7, 1961.

> One suggestion with regard to the mathematics methods course was that, if preceded by the suggested mathematics courses, it should not be necessary to dwell upon content but rather should cover principles of learning as they apply to the teaching of mathematics in high school, effective methods for testing, recent curricular developments, acquaintance with professional teaching organizations, selecting of texts, suggested reading texts for students, etc.[3]

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[1]Chavier, p. 269.

[2]Committee on the Undergraduate Program in Mathematics, Recommendations . . ., p. 14.

[3]Committee on the Undergraduate Program in Mathematics, Five Conferences on the Training of Mathematics Teachers, Report No. 1 (Buffalo, New York: Mathematical Association of America, 1961), no pp.

A surprising development at this meeting is the suggestion for the teaching of the methods course.

> . . . the high schools might be best equipped to give such courses inasmuch as they provide the natural laboratory situation for the training of teachers.[1]

Not many high schools would be willing to assume the financial and professional burden for presenting the methods course and not many are staffed with the type of individual whom the Committee suggests teach it.[2]

Eight college teachers participating in a National Science Foundation summer conference at Oklahoma State University attempted to clarify the five objectives for the methods course as stated by the Committee on the Undergraduate Program in Mathematics. Although half of the group had never taught the methods course before, their interpretation of the Committee's recommendations was comprehensive and explicit.[3]

In summarizing these recommendations, they suggested that the methods course should include a discussion of the historical development of the high school mathematics curriculum with emphasis on the current proposals of commissions and committees. The students would study these proposals and the current literature concerning these proposals and make a comparative analysis of them. The course should permit the student to function in a role similar to the high school teacher and to assume similar instructional responsibilities. This would mean that the student

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[1]Ibid.

[2]Committee on the Undergraduate Program in Mathematics, Recommendations . . ., p. 14.

[3]National Science Foundation Summer Conference for College Mathematics Teachers, "Methods for Teachers of Secondary School Mathematics," Report of Seminar Group I (Department of Mathematics, Oklahoma State University, 1961), pp. 1-24. (Mimeographed.)

should construct lesson plans, should teach from these plans, and should evaluate his teaching and student learning. An emphasis should be placed on independent reading and research, from which notes would be incorporated in a notebook including additional sections on his reading and analysis of contemporary literature in mathematics education, observations of high school mathematics classes, a review of current films and audio-visual aids, a survey of how mathematics has served the needs of man from the time of the Egyptian pyramid builders, and the mathematical content of contemporary junior and senior high school programs.[1]

Another set of recent suggestions which has influenced the collegiate training received by mathematics teachers is Guidelines for Preparation Programs of Teachers of Secondary School Science and Mathematics.[2] Guideline VII states "The Program should include preparation in the methods especially appropriate to the subject to be taught." The recommendations of the National Association of State Directors of Teacher Education and Certification and the American Association for the Advancement of Science for the content of the methods course are identical to the five listed earlier in this chapter and credited to the Committee on the Undergraduate Program in Mathematics.

The Commission on Mathematics soundly endorses a course which deals specifically with the teaching of mathematics in the secondary school.

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[1] Ibid.

[2] National Association of State Directors of Teacher Education and Certification and the American Association for the Advancement of Science, Guidelines for Preparation Programs of Teachers of Secondary School Science and Mathematics (Washington, D. C.: American Association for the Advancement of Science, 1961), p. 26.

> This course should trace enough of the history of the teaching of mathematics in the United States to provide background material for understanding present-day pressures for curriculum change. Following this, there should be a detailed discussion of some of the principal ideas of modern mathematics in light of secondary school teaching.[1]

The Cambridge Conference on School Mathematics was composed of twenty-nine outstanding mathematicians and natural scientists; this group hypothesized an optimum mathematics curriculum for grades K-12 in the year 1990. Their proposal, therefore, has some significant implications for teacher education, and they devote an entire section of their report to pedagogical principles and techniques. Although they do not describe a methods course per se, they do outline the activities of the teacher which have a direct bearing on the type of pedagogical preparation which future teachers would receive. A summary of their curriculum philosophy follows.

The mathematics program of the Cambridge group employs a spiral curriculum. Their goal is to foster independent and creative thinking which they feel is best accomplished through the use of the discovery technique in teaching. This approach to the learning process permits new skills to be learned by utilizing previously learned ones which are reinforced in the process of obtaining more interesting developments. The Cambridge group feels that the introduction of new topics can best be motivated by an historical approach, indicating an application of the concept or through the construction or utilization of appropriate models. The teacher must assist the students in the development of new ideas and construct examinations which foster the program's goal.[2]

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[1] Commission on Mathematics, p. 57.

[2] Cambridge Conference on School Mathematics, Goals for School Mathematics (Boston: Houghton Mifflin Company, 1963), pp. 15-22.

A methods course designed to equip students to teach the program advocated by the Cambridge group, therefore, would emphasize methods of lesson presentation (especially the discovery technique in reference to encouraging creative and independent thinking), the psychology of how students learn mathematics, motivational techniques, the historical development of mathematics, applications of mathematics in the physical and social sciences, audio-visual devices, test construction and evaluative techniques.

Individual Recommendations

As a result of reviewing the historical development of the methods course, Gordon Mock concludes,

> Two jobs, essentially are left for the methods course. The first, a maximum of one-third of the course, should deal with the history of the teaching of mathematics. . . .
> Then, the course should proceed to discuss the specific problems involved in teaching in the high school. Essentially, the content will be mathematical matter, that is, the mathematical material will form the structure upon which is built the background of high school material.[1]

Felder's suggestions for the course were based on the needs of the state of Mississippi as she interpreted them to be in 1959 and on the comments which she received in letters of opinion from seven mathematics educators. She feels that the course should be structured around the following topics: (1) the place of mathematics in the history of secondary education, (2) the processes involved in learning mathematics, (3) the structure of the secondary school mathematics curriculum, (4) textbook selection, (5) the mathematical content of high school courses, (6) visual aids, (7) evaluation, (8) mathematical recreations

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[1]Mock, p. 308.

and mathematics clubs, (9) professionalism, (10) individual differences, and (11) topics from modern mathematics.[1] She does not attempt to identify how much time should be devoted to each of these topics, but suggests that the time be adjusted to meet the needs of the class.[2]

Chavier's study involved a questionnaire survey of fifty-nine colleges to determine what was being taught in the methods course. Some of the items investigated were the textbooks used in the course, the construction of a syllabus for the course, attitudes of the instructor teaching the course, and the number of students enrolled in the course.[3] Chavier's analysis of the survey results led her to believe that the methods course should include the following topics: (1) the history of the teaching of mathematics, (2) the technique of teaching with meaning, (3) the literature on the teaching of mathematics, (4) topics from modern mathematics, (5) methods of teaching, (6) modes of presentation, and (7) evaluation.[4] She did not indicate what portion of the time allotted to the methods course should be devoted to each of the topics, nor is there evidence that an attempt was made to experimentally teach the course which she proposed.

Textbooks

A verifiable generalization in college teaching is that the syllabus for the course being taught parallels the content of the textbook being used in the course. A review of the composition of these texts,

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[1] Felder, pp. 175-89.

[2] Ibid., p. 175.

[3] Chavier, pp. 166-75.

[4] Ibid., p. 265.

therefore, is in order; the review will serve as an indicator of the content of the present methods courses being taught.

In discussing the eighteen methods texts that have been published since 1887, Mock summarized their content as including subsets of the following twelve categories:

> (1) values, (2) algebra, (3) geometry, (4) advanced high school mathematics, (5) general mathematics, (6) curriculum, (7) methods, (8) college mathematics, (9) evaluation, (10) psychological problems, (11) professionalism, and (12) visual aids.[1]

Since Mock's study, three new publications have appeared for use in the methods course. The first was a sixty-four page pamphlet by Aaron Hankin entitled Meaningful Mathematics Teaching.[2] The topics discussed in this pamphlet are organizing a typical mathematics lesson, using homework effectively in mathematics, planning and conducting the lesson, sample lesson plans in action, and rating pupils accurately.

A 1963 publication by Roy Dubisch entitled The Teaching of Mathematics[3] includes chapters on Teaching in General, The Aims of Teaching Mathematics, General Remarks on the Teaching of Mathematics, The Teaching of Algebra, The Teaching of Trigonometry and Logarithms, The Teaching of Analytic Geometry, The Teaching of Differential Calculus, and The Teaching of Integral Calculus.

The most recent publication for use in the methods course is Contemporary Teaching of Secondary School Mathematics by Stephen S.

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[1]Mock, The Mathematics Teacher, LIV, 18.

[2]Aaron Hankin, Meaningful Mathematics Teaching (Englewood Cliffs, New Jersey: Teachers Practical Press, 1961), pp. 3-4.

[3]Roy Dubisch, The Teaching of Mathematics (New York: John Wiley and Sons, Inc., 1963), pp. ix-xi.

Willoughby.[1] The chapters included in this text are Modern Mathematics; The History of Mathematics Education in the United States; Teaching and Learning; Hints for the Classroom; Arithmetic; Probability for Junior High School; Number Systems and Numeration Systems; Intuitive Geometry; Language, Logic, and Sets; Expanding the Number System; Algebra; Geometry; Circular Functions and Trigonometry; Probability and Statistical Inference; Calculus; and Teaching Secondary School Mathematics.

A review of the three previously mentioned texts is sufficient evidence that the statement by Mock still has validity today.

> One historical generalization that seems valid is that the number of topics discussed in the methods texts has increased. As new books appear, they generally retain all the old topics and add new ones.[2]

The Content of the Contemporary Methods Course

The information presented in this chapter has been concerned either directly or indirectly with proposals for the methods course from the viewpoint of committees, individuals, and textbook authors. A recent article in The American Mathematical Monthly[3] will serve as a basis for contrasting these proposals with what is actually being taught in the methods course in various institutions of higher learning throughout the United States.

Harold Trimble describes the content of the course at Iowa State Teachers College as follows.

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[1]Stephen S. Willoughby, Contemporary Teaching of Secondary School Mathematics (New York: John Wiley and Sons, Inc., 1967), p. vii.

[2]Mock, The Mathematics Teacher, LIV, 18.

[3]Brown and Mayor (eds.), The American Mathematical Monthly, LXVII, pp. 688-90.

> (a) review the objectives of instruction in mathematics; (b) look at typical organization of content, kindergarten through college; (c) question relative emphases on topics in light of changing emphases in mathematics and its applications; (d) identify contradictions, at least from the learner's point of view, in commonly taught language and concepts; (e) re-examine the roles of the teacher and the learner in light of new psychological insights into the learning process; (f) build hypothetical teaching sequences to illustrate and try out the principles evolved; (g) study critically the current available experimental or recommended materials of instruction.[1]

From the State University of Iowa, H. Vernon Price reports that the methods course consists of two basic parts. In the first, the secondary curriculum of the past, present, and future is discussed. The second part is devoted to assigned readings in mathematical content, special methods and techniques involved in teaching a variety of topics, study and evaluation of current literature, and a review of the proposals of experimental groups.[2]

Clifford Bell describes the course at the University of California in Los Angeles as "designed to give secondary teacher candidates the necessary maturity in algebra to make good high school teachers of algebra."[3] The course includes a development of the number system, a consideration of the field postulates, a discussion of proof, a review of the function concept, the theory of determinants and matrices, and topics from the theory of equations.

At the University of Minnesota, Donovan Johnson reports that the methods course includes the following topics:

> Motivating the learning of mathematics; guiding the student to discover new concepts; teaching the student how to study

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[1] Ibid., p. 688.

[2] Ibid.

[3] Ibid., pp. 688-89.

> mathematics; building skill in solving problems; providing a program of acceleration and enrichment for gifted students; planning an effective review, remedial and practice program; evaluating the learning of facts, skills, and problem solving; developing desirable attitudes and appreciations of mathematics; evaluating new curriculum recommendations; procuring and using new instructional materials; planning and demonstrating a variety of ways to present a mathematical topic; learning the goals of mathematics instruction; reading research in mathematics education.[1]

William C. Lowry reports the course at the University of Virginia is built around four units: "(1) Number and Computation, (2) Measurement, (3) Relationships and (4) Proof."[2] There is no textbook used in the course, but the students are required to do readings from various professional publications.

The variety of treatments which the methods course receives, as noted above, all occurred in the same year but at different institutions throughout the United States. Gordon Mock's comment, "The content of what has come to be called the methods course has not been uniform . . . ."[3] appears well founded here, if not a classic understatement!

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[1] Ibid., p. 689.

[2] Ibid., pp. 689-90.

[3] Mock, The Mathematics Teacher, LIV, 17.

CHAPTER III

STRUCTURING THE EXPERIMENTAL METHODS COURSE

The Selection of Appropriate Topics

In Chapter II of this study a review was made of individual and group recommendations for the methods course, of the content and teaching procedures of contemporary methods courses, and of the content of textbooks written for the methods course. This review permitted the construction of a list of topics that were appropriate for inclusion in a methods course for prospective high school mathematics teachers. An analysis of this list indicated that there is a nucleus of thirty-seven different topics studied or suggested for study in existing or hypothesized methods courses. Those topics are (1) the development of the mathematics curriculum to 1950, (2) curriculum experimentation in the 1950's and 60's, (3) curriculum implications for the future, (4) the literature of mathematics and its teaching, (5) the philosophy of mathematics, (6) methods of lesson presentation, (7) the unit plan, (8) the lesson plan, (9) the evolution of mathematical concepts in grades K-12, (10) teaching junior high school mathematics, (11) teaching algebra, (12) teaching geometry, (13) teaching advanced topics in high school mathematics, (14) individual differences, (15) enrichment materials, (16) the psychology of learning mathematics, (17) programmed instruction,

(18) homework, (19) test construction, (20) student evaluation, (21) the mathematics club, (22) presentation of lessons by students, (23) problem solving, (24) presentation of reports by students, (25) textbook evaluation, (26) techniques of observation, (27) audio-visual aids, (28) how to study mathematics, (29) motivation, (30) the bulletin board, (31) discipline, (32) the role of the mathematics supervisor, (33) the role and responsibilities of the student teacher, (34) applications of mathematics, (35) graduate work in mathematics and mathematics education, (36) research in mathematics education, and (37) professionalism and professional organizations.

The Questionnaire Survey

Since all thirty-seven of the topics listed above could not be satisfactorily treated in a three semester hour methods course, a decision had to be made about which ones to include. Inherent in arriving at this decision was a consideration of the following:

1. Which of the topics on the list are the most important to the inexperienced mathematics teacher?

2. Which of the topics selected for question 1 are most appropriately taught in the mathematics methods course?

3. What portion of the forty-five classroom periods in the methods course should be devoted to each of the topics selected in question 2?

4. Who is best qualified to make the selection of topics?

In order to have the best possible guidance about what topics should be included in an exemplary methods course, the jury technique was used. The jury was selected from the 1965 National Council of Teachers of Mathematics roster of Teacher Education Personnel. This list contains

the names of 754 specialists in mathematics education at colleges and universities throughout the United States. Since the number of specialists was known, the formula[1] developed by the Research Division of the National Education Association was employed to determine the size of the sample population. This formula indicated that a random sample of 200 of these specialists would predict within plus or minus five percentage points the population opinion at a confidence level of ninety percent. Each of the 754 specialists on the roster of Teacher Education Personnel was assigned a three digit numeral from 000-753 in order that a representative sample might be selected by consulting a table of random numbers. It was assumed that there would not be a 100 percent return of the questionnaires; therefore, 225 names were initially selected from the roster.

A questionnaire was constructed (Appendix A) on which appeared each of the thirty-seven topics indicated above. In the cover letter (Appendix B) which accompanied each questionnaire, each jury member was asked to select the topics which he felt should be included in the methods course by indicating opposite each of the topics the number of class periods, out of the total of forty-five available for the course, that he would devote to it. This procedure would permit an analysis to be made of the opinions of the jury members and would permit an experimental methods course to be constructed from their collective recommendations.

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[1]National Education Association Research Division, "Small Sample Techniques," National Education Association Research Bulletin, XXXVIII (December, 1960), 99.
Formula: $n = (X^2Np(1-p))/(d^2(N-1) + X^2p(1-p))$, where n is the sample size, X^2 is the table value for chi-square for one degree of freedom, N is the size of the population, p is the population proportion which it is desired to estimate, and d is the degree of accuracy expressed as a proportion.

In order to allow the specialists to include topics other than the thirty-seven mentioned on the questionnaire, a space was provided on the second page of the form for them to list any additional topics and to note the amount of the allotted class time that would be devoted to each of them.

The composite opinions of the jury members would determine the structuring of the experimental methods course in terms of the topics discussed and the number of class periods that would be devoted to each of these topics. Just as important as the classroom topics was a consideration of complementary outside class activities appropriate for the future teachers who were enrolled in the course. On the second page of the questionnaire, therefore, each jury member was invited to comment on "What learning experiences would you suggest for the students in the methods course other than the formal periods spent with you in the classroom?"

The questionnaire was mailed to each of the jury members during the third week of the second semester of the 1965-66 school year. Within three weeks, 159 of the questionnaires had been returned. During the sixth week of the same semester, a follow up letter (Appendix C) was mailed to the jury members who had not as yet responded. This procedure enabled the calculated number (200) of responses to be obtained.

Results of the Questionnaire Survey

A tabulation of the responses to the methods course survey questionnaire is indicated below in Table 1. The topics are entered in the table in accordance with the frequency (column 2) with which they were selected by the 200 jury members. Column 3 indicates the mean number of periods suggested by the jury for each topic; column 4, the

standard deviation, is an indicator of the variability existing in the jury's recommendations; and column 5 lists the maximum number of periods suggested by any one of these specialists for devotion to a particular topic.

TABLE 1

A SUMMARY OF THE 200 RESPONSES TO THE METHODS COURSE CONTENT SURVEY

Topic (1)	Number Selecting Topic (2)	Number of Periods		
		Mean (3)	S. D. (4)	Max. (5)
Teaching geometry	183	3.18	2.26	10
Teaching algebra	181	3.38	2.43	10
Curriculum experimentation in the 1950's and 60's	181	1.91	1.83	12
Teaching advanced topics in high school mathematics	172	2.55	2.28	17
Test construction	171	1.48	1.07	6
Methods of lesson presentation . .	169	1.96	1.84	13
Curriculum implications for the future	168	1.00	0.78	5
Teaching junior high school mathematics	165	2.58	2.17	10
Development of the mathematics curriculum to 1950	161	1.07	1.00	5
The evolution of mathematical concepts in grades K-12	160	2.34	3.31	30
Enrichment materials	160	1.14	0.99	5
Student evaluation	154	1.13	1.00	5
The lesson plan	154	0.90	0.79	5

TABLE 1 -- Continued

Topic (1)	Number Selecting Topic (2)	Number of Periods		
		Mean (3)	S. D. (4)	Max. (5)
Audio-visual aids	150	0.93	0.80	5
Professionalism and professional organizations	150	0.71	0.55	3
The literature of mathematics and its teaching	149	1.15	1.25	10
Textbook evaluation	144	0.88	0.49	5
Motivation	143	0.85	0.85	5
Problem solving	141	1.63	1.68	10
The psychology of learning mathematics	140	1.08	1.07	5
Individual differences	133	0.84	0.95	6
Presentation of lessons by students	128	3.12	3.96	17
Homework	125	0.58	0.58	3
Programmed instruction	124	0.60	0.61	3
How to study mathematics	121	0.77	0.83	4
The philosophy of mathematics . .	120	0.88	1.24	10
The unit plan	120	0.60	0.62	2
Applications of mathematics . . .	113	0.93	1.10	5
Research in mathematics education	111	0.69	0.93	5
The role and responsibilities of the student teacher	111	0.57	0.66	3
Discipline	93	0.44	0.59	3

TABLE 1 -- Continued

Topic (1)	Number Selecting Topic (2)	Number of Periods		
		Mean (3)	S. D. (4)	Max. (5)
Graduate work in mathematics and mathematics education . . .	84	0.38	0.52	2
The mathematics club	81	0.31	0.45	2
Presentation of reports by students	79	1.01	1.74	10
The role of the mathematics supervisor	77	0.31	0.44	2
Techniques of observation	76	0.42	0.70	4
The bulletin board	76	0.27	0.39	2

Table 2 presents a summary of the jury's responses to the question on page two of the form which asked them to list any additional topics not included among the thirty-seven on the first page of the questionnaire, but which they would include in an exemplary methods course. The twenty topics listed in this table resulted from recommendations of thirty-eight jury members. The most frequently mentioned topic, new subject matter, was indicated by twelve jury members or six percent of the sample population. The mean number of hours indicated opposite each topic is the amount of time which the jury members suggesting the topic would devote to it; i. e. the twelve jury members suggesting that new subject matter be taught in the methods course would devote an average of 9.8 of the forty-five class periods to this topic.

TABLE 2

ADDITIONAL TOPICS SUGGESTED FOR THE METHODS COURSE

Topic	Number Suggesting Topic	Mean Periods Suggested
New subject matter	12	9.8
The history of mathematics	7	2.6
Standardized tests in mathematics	4	1.5
Field and laboratory work in mathematics	3	2.0
Objectives of secondary education	3	2.0
Careers in mathematics	3	1.0
Mathematics contests	3	1.0
Teaching computer mathematics	2	1.5
Non-professional duties	2	1.0
Research in psychology and education	1	6.0
Encouraging creativity	1	5.0
Incidental learning	1	3.0
Using the chalkboard	1	2.0
Teaching general mathematics	1	2.0
Community relations	1	1.0
Research techniques in education	1	1.0
School politics	1	1.0
Self evaluation	1	1.0
Using the library	1	1.0
Teaching on television	1	1.0

One hundred fifty-seven jury members responded to the question, "What learning experiences would you suggest for the students in the methods course other than the formal periods spent with you in the classroom?" A category summary of their recommendations is given below in Table 3.

TABLE 3

A SUMMARY OF SUGGESTED EXTRA-CLASS LEARNING EXPERIENCES

Topic	Frequency
Classroom observations	86
Reading periodical mathematics literature	45
Reviewing published materials	32
Participation in the teaching process	28
A term project	21
Independent study of new subject matter	20
Familiarization with available visual-aids	13
Attendance at a professional meeting	8
Design and equip a mathematics laboratory	5
Study and observation of slow learners	4
Build a curriculum file	3
Write their philosophy of education	2
Informal visits with the methods course professor	2
Prepare a list of mathematical projects suitable for different grade and ability levels	1
Prepare a constitution for a high school mathematics club	1
Write a TV script on some topic in mathematics	1
Write a programmed unit	1

TABLE 3 -- Continued

Topic	Frequency
Prepare a list of paperbook books about mathematics	1
Attend a department meeting at a nearby high school	1
Visit an industry employing mathematicians	1
Investigate recent research in mathematics education	1
Participate in a mathematical debate	1

The following summary is a review of the five most frequently suggested extra-class learning experiences included in Table 3.

Classroom observations -- Classroom observations far outpolled any of the other suggested areas. The majority of the eighty-six jury members suggesting this learning experience qualified it by mentioning that the classroom of an outstanding high school mathematics teacher should be visited. Other suggestions in this area were that visitations should be made to classes where different techniques of lesson presentation were employed, that classes of different ability levels in the same grade should be visited, and finally that the methods course professor should teach a high school class and then discuss his presentation with the students in the methods course.

Reading periodical mathematics literature -- The periodicals which were most frequently suggested for student exploration were The Mathematics Teacher, The Arithmetic Teacher, School Science and Mathematics, and The American Mathematical Monthly. The students were to present evidence of their participation in the reading program by handing to the instructor each week note cards on which appeared a summary of the articles which they had read, by giving oral reports

to the class on assigned or supplementary reading, or by including sections in their notebooks summarizing and analyzing what they had read.

Reviewing published materials -- The published materials which were suggested for the student to review included standardized tests, elementary and secondary textbooks and workbooks, experimental programs, paperback mathematics books, monographs and pamphlets, and reference texts such as the yearbooks of the National Council of Teachers of Mathematics. The students could then present a written or oral report on how different authors treated the same unit of study; how various experimental groups agreed and disagreed in philosophy and content; the availability, validity, and reliability of standardized tests in mathematics; and the location and nature of supplementary material for various units of study at different grade levels.

Participation in the teaching process -- The theme that one learns best by doing was strongly echoed by all those jury members suggesting the students active participation in the classroom learning process. This experience could be incorporated into the student's program by having him intern at the local high school or campus laboratory school while he is enrolled in the methods course, by having him assist professors of freshmen mathematics courses with paper grading and tutoring activities, or by giving talks and demonstrating visual aids to high school mathematics clubs in the area of the college.

A term project -- A term project related to the teaching of mathematics was suggested by twenty-one jury members. Included among their suggestions for student projects were the construction of visual aids; research papers on the history of algebra, geometry, or trigonometry;

the location and classification of enrichment materials for junior and senior high school students; and the design and construction of a bulletin board to complement the teaching of a particular unit in mathematics.

Choosing the Topics to be Included in the Experimental Methods Course

When the tabulation of the survey data was completed, a decision had to be made about which topics should be included in the experimental methods course and how much class time should be devoted to each of the topics which was selected. The assumption was made that not all thirty-seven topics could be satisfactorily treated in a three semester hour course.

The first question then was how to rank the thirty-seven topics so that a subset of them might be selected for inclusion in the experimental methods course. This question was answered by ordering the topics on the basis of the number of times they were nominated for inclusion in the methods course by the jury members. Thus, the first topic selected was teaching geometry since 183 jury members nominated it for inclusion in the methods course; the second topic selected was teaching algebra, etc. following the same order in which the topics are entered in Table 1.

The second question was how much class time should be devoted to each of the topics which was selected for the experimental methods course. Using the mean number of periods (Table 1) would not suffice since this would imply teaching all thirty-seven of the topics and, therefore, would contradict the assumption with which this section began. The question was resolved by choosing the median number of periods suggested by the

jury for each of the topics which was included in the experimental methods course. The median rather than the mean was used to designate the number of classroom periods devoted to a particular topic because it is more practical and because it is more representative of the wishes of the majority of the jury since it is not as adversely affected by very large or very small values as is the mean.

By combining the procedures described in the last two paragraphs, the twenty-four topics listed below in Table 4 were selected for inclusion in the experimental methods course. Noted opposite each of these topics in the column headed "Periods" is the portion of the forty-five class periods that was devoted to each of the topics.

TABLE 4

THE EXPERIMENTAL METHODS COURSE

Topic	Periods
Teaching geometry	4
Teaching algebra	4
Curriculum experimentation in the 1950's and 60's	2
Teaching advanced topics in high school mathematics	3
Test construction	2
Methods of lesson presentation	2
Curriculum implications for the future	1
Teaching junior high school mathematics	3
Development of the mathematics curriculum to 1950	1
The evolution of mathematical concepts in grades K-12	3
Enrichment materials	1
Student evaluation	1

TABLE 4 -- Continued

Topic	Periods
The lesson plan	1
Audio-visual aids	1
Professionalism and professional organizations	1
The literature of mathematics and its teaching	2
Textbook evaluation	1
Motivation	1
Problem solving	2
The psychology of learning mathematics	2
Individual differences	1
Presentation of lessons by students	4
Homework	1
Programmed instruction	1

By using the criteria developed in this section for the inclusion of topics in the experimental methods course it will be noted that none of the additional topics listed in Table 2 was included in the methods course. This criteria demanded that a minimum of sixty-two percent of the jury nominate a topic before it would be included in the course. The most frequently mentioned "additional topic" was nominated by only six percent of the jury.

The Experimental Methods Course

The criteria of the previous section of this research determined the topics to be included and the time structure of the experimental methods course. This section deals with the instructor treatment of each topic and the student requirements for the course.

Since there was no textbook available that would satisfactorily treat all of the topics included in the experimental methods course, each student was required to keep a notebook which became his text for the course. Section I of this notebook contained a summary of each of the daily class activities and a summary of the readings which the student completed to complement the topics discussed in the course. The reading requirements for the experimental methods course are outlined in Appendix D. In addition every student was required to read and summarize one article in each of the current monthly issues of School Science and Mathematics, The Arithmetic Teacher, The Mathematics Teacher, and The American Mathematical Monthly. These summaries were included in Section II of his notebook. In Section III, the student outlined the construction of a visual aid and discussed its use as a teaching device. The students were each required to do a minimum of five classroom observations during the semester. These included one observation with his supervising teacher of the following semester, one observation at the high school from which he graduated, and one observation at the campus elementary school. Section IV consisted of a record of the classroom activities during these observations. In Section V each student was required to make a comparison of how a particular unit of study in mathematics was treated in (a) a commercial textbook published prior to 1955, (b) one of the experimental programs, and (c) a commercial textbook published since 1963.

All of the students joined the National Council of Teachers of Mathematics which provided an opportunity for them to continue their professional reading program at the conclusion of the methods course. They were also required to read The Process of Education by Jerome

Bruner, The Central Purpose of American Education by The Educational Policies Commission, and How to Solve It by G. Polya in order to provide a basis upon which critical discussions and comparisons could be made throughout the course.

During the semester, the students were asked to decide what in their opinion constitutes quality teaching in mathematics and present their thoughts in a formal paper which was handed in to the instructor during the last week of the course.

At the conclusion of the class discussions on test construction, each student was required to construct a final examination dealing with the subject matter of the methods course. He was asked to demonstrate his ability at writing quality test items of the true-false, completion, essay, multiple-choice, and problem types.

The activities described above required the students to participate in four out of the five most frequently suggested extra-class learning experiences included in Table 3. The opportunity for them to participate directly in the learning process was curtailed because Indiana University of Pennsylvania does not have an experimental laboratory school at the secondary level.

Indicated below are the topics which were included in the experimental methods course and the order in which they were presented. The sequence of lesson numbers corresponds to the number of periods devoted to each of the topics. The references indicated immediately following each topic are the ones used by the instructors as a basis for their class discussions and lectures. The reference material used by the students is included in Appendix D.

Lesson 1 Professionalism and professional organizations

Mathematics Teaching as a Career. A leaflet of The National Council of Teachers of Mathematics, Washington, D. C.

National Council of Teachers of Mathematics. A leaflet of The National Council of Teachers of Mathematics, Washington, D. C.

Central Association of Science and Mathematics Teachers. A leaflet of the Central Association of Science and Mathematics Teachers, Inc., Bloomington, Indiana.

The Mathematical Association of America. A leaflet of the Mathematical Association of America, Buffalo, New York.

Lesson 2 The development of the mathematics curriculum to 1950

Butler, Charles H., and Wren, F. Lynwood. Chapter I, "The Evolving Program of Secondary Mathematics." The Teaching of Secondary Mathematics. 4th ed. New York: McGraw-Hill Book Company, 1965.

Lessons 3-4 Curriculum experimentation in the 1950's and 60's

The National Council of Teachers of Mathematics. The Revolution in School Mathematics. Washington, D. C.: The National Council of Teachers of Mathematics, 1961.

The National Council of Teachers of Mathematics. An Analysis of New Mathematics Programs. Washington, D. C.: The National Council of Teachers of Mathematics, 1963.

Lesson 5 Curriculum implications for the future

The Report of the Cambridge Conference on School Mathematics. Goals for School Mathematics. Boston: Houghton Mifflin Co., 1963.

Lessons 6-8 The evolution of mathematical concepts in grades K-12

The National Council of Teachers of Mathematics. Chapter II, "The Concept of Number." Insights into Modern Mathematics. 23rd Yearbook. Washington, D. C.: The National Council of Teachers of Mathematics, 1957.

The National Council of Teachers of Mathematics. Chapter II, "Number and Operation." The Growth of Mathematical Ideas K-12. 24th Yearbook. Washington, D. C.: The National Council of Teachers of Mathematics, 1959.

Lessons 9-10 The literature of mathematics and its teaching

Schaaf, William L. The High School Mathematics Library. Washington, D. C.: The National Council of Teachers of Mathematics, 1960.

Hardgrove, Clarence Ethel. The Elementary and Junior High School Mathematics Library. Washington, D. C.: The National Council of Teachers of Mathematics, 1960.

Also included as a part of these lessons was a display of School Science and Mathematics, The Mathematics Teacher, The Arithmetic Teacher, The Mathematics Student Journal, Scientific American, Mathematics Magazine, Pi Mu Epsilon Journal, Science and Math Weekly, and the 21st through the 29th yearbooks of The National Council of Teachers of Mathematics. A discussion of the format of each of these publications complemented the display.

Lessons 11-12 Problem solving

Polya, G. How to Solve It. 2nd ed. Garden City, N. Y.: Doubleday and Co., Inc., 1957.

The National Council of Teachers of Mathematics. Chapter VIII, "Problem Solving in Mathematics." The Learning of Mathematics: Its Theory and Practice. 21st Yearbook. Washington, D. C.: The National Council of Teachers of Mathematics, 1953.

Lessons 13-14 The psychology of learning mathematics

The National Council of Teachers of Mathematics. Chapter X, "Implications of the Psychology of Learning for the Teaching of Mathematics." The Growth of Mathematical Ideas Grades K-12. 24th Yearbook. Washington, D. C.: The National Council of Teachers of Mathematics, 1959.

The National Council of Teachers of Mathematics. Chapter I, "Theories of Learning Related to the Field of Mathematics." The Learning of Mathematics: Its Theory and Practice. 21st Yearbook. Washington, D. C.: The National Council of Teachers of Mathematics, 1953.

Lesson 15 Individual differences

The National Council of Teachers of Mathematics. Chapter IX, "Provisions for Individual Differences." The Learning of Mathematics: Its Theory and Practice. 21st Yearbook. Washington, D. C.: The National Council of Teachers of Mathematics, 1953.

Lesson 16 Motivation

The National Council of Teachers of Mathematics. Chapter II, "Motivation for Education in Mathematics." The Learning of Mathematics: Its Theory and Practice. 21st Yearbook. Washington, D. C.: The National Council of Teachers of Mathematics, 1953.

Lesson 17 Audio-visual aids

Berger, Emil J., and Johnson, Donovan A. A Guide to the Use and Procurement of Teaching Aids for Mathematics. Washington, D. C.: The National Council of Teachers of Mathematics, 1959.

Krulik, Stephen, and Kaufman, Irwin. Multi-Sensory Techniques in Mathematics Teaching. Englewood Cliffs, N. J.: Teachers Practical Press, Inc., 1963.

Lessons 18-19 Methods of lesson presentation

Davis, David R. Chapter II, "Techniques of Teaching." The Teaching of Mathematics. Cambridge, Mass.: Addison-Wesley Press, Inc., 1951.

Lesson 20 The lesson plan

Reeve, William David. Chapter V, "How to Plan and Teach a Lesson in Mathematics." Mathematics for the Secondary School. New York: Holt, Rinehart and Winston, Inc., 1954.

The National Council of Teachers of Mathematics. Chapter X, "Planned Instruction." The Learning of Mathematics: Its Theory and Practice. 21st Yearbook. Washington, D. C.: The National Council of Teachers of Mathematics, 1953.

Lessons 21-23 Teaching junior high school mathematics

Butler, Charles H., and Wren, F. Lynwood. Chapter XI, "The Teaching of Arithmetic" and Chapter XII, "The Teaching of Further Topics in Arithmetic." The Teaching of Secondary Mathematics. 4th ed. New York: McGraw-Hill Book Company, 1965.

Lesson 24 First student lesson presentation

School Mathematics Study Group. Chapter II, Section 5, "Numerals in Base Seven." Mathematics for Junior High School. Vol. I, Part I. New Haven, Conn.: Yale University Press, 1961.

Lesson 25-28 Teaching algebra

College Entrance Examination Board. Section I, "Algebra." Appendices to the Report of the Commission on Mathematics. Princeton, N. J.: Educational Testing Service, 1959.

Lesson 29 Second student lesson presentation

School Mathematics Study Group. Chapter XVII, Section 1, "The Function Concept." First Course in Algebra. Text II. New Haven, Conn.: Yale University Press, 1961.

Lessons 30-33 Teaching geometry

College Entrance Examination Board. Section II, "Geometry." Appendices to the Report of the Commission on Mathematics. Princeton, N. J.: Educational Testing Service, 1959.

Lesson 34 Third student lesson presentation

School Mathematics Study Group. Chapter V, Section 4, "Writing Your Own Proofs." Geometry. Text I, Part I. New Haven, Conn.: Yale University Press, 1961.

Lesson 35 Homework

Hankin, Aaron. Chapter II, "Using Homework Effectively in Mathematics." Meaningful Mathematics Teaching. Englewood Cliffs, N. J.: Teachers Practical Press, Inc., 1961.

Lesson 36 Enrichment materials

The National Council of Teachers of Mathematics. Enrichment Mathematics for the Grades. 27th Yearbook. Washington, D. C.: The National Council of Teachers of Mathematics, 1963.

The National Council of Teachers of Mathematics. Enrichment Mathematics for High School. 28th Yearbook. Washington, D. C.: The National Council of Teachers of Mathematics, 1963.

Lessons 37-39 Teaching advanced topics in high school mathematics

College Entrance Examination Board. Section III, "Trigonometry." Appendices to the Report of the Commission on Mathematics. Princeton, N. J.: Educational Testing Service, 1959.

Lesson 40 Fourth student lesson presentation

School Mathematics Study Group. Chapter X, Section 5, "Definitions of the Trigonometric Functions." Intermediate Mathematics. Text II, Part II. New Haven, Conn.: Yale University Press, 1961.

Lesson 41 Textbook evaluation

Report of the Committee on Criteria for the Analysis of Instructional Materials. Aids for Evaluators of Mathematics Textbooks. Washington, D. C.: The National Council of Teachers of Mathematics, 1965.

Lessons 42-43 Test construction

The National Council of Teachers of Mathematics. Chapter IV, "Constructing Achievement Tests and Interpreting Scores," and Chapter V, "Analysis of Illustrative Test Items." Evaluation in Mathematics. 26th Yearbook. Washington, D. C.: The National Council of Teachers of Mathematics, 1961.

Lesson 44 Student evaluation

The National Council of Teachers of Mathematics. Chapter III, "Basic Principles of Evaluation." Evaluation in Mathematics.

26th Yearbook. Washington, D. C.: The National Council of Teachers of Mathematics, 1961.

Lesson 45 Programmed instruction

Phi Delta Kappan, XLIV (March, 1963). (A special issue on programmed instruction.)

May, Kenneth O. Programmed Learning and Mathematical Education. Buffalo, N. Y.: Mathematical Association of America, 1965.

In lessons 24, 29, 34, and 40, which dealt with the presentation of lessons by students, each student in the class was required to construct a lesson plan dealing with the particular topic under consideration. On the day of those class meetings one student was picked at random for the class presentation while all of the students handed in their lesson plans for evaluation purposes.

In the next chapter of this study the experimental methods course described above will be evaluated by the three instructors who taught the course, by the high school supervising teachers in mathematics, and by the student teachers who were enrolled in the course the semester prior to their student teaching assignment.

CHAPTER IV

THE EVALUATION OF THE EXPERIMENTAL METHODS COURSE

The Evaluation Procedure

A natural consequence of any experimental curriculum proposal is the program's evaluation. This chapter will deal with the evaluation of the experimental methods course described in Chapter III. The evaluation was carried out in four distinct phases.

The first phase consisted of a comparison between the experimental methods course and current teaching practices in the methods course at Pennsylvania colleges and universities. The information received from a survey of thirty private colleges, state colleges, and universities throughout Pennsylvania was utilized to construct a composite of their activities in the methods course. This procedure allowed a comparison to be made between what was theoretically the most desirable set of topics for the methods course, from the viewpoint of the jury, and the set of topics that are currently being taught in existing methods courses.

The second phase of the evaluation involved the surveying of the opinions of forty secondary school mathematics teachers who supervise future mathematics teachers during their semester of student teaching. The teachers surveyed were questioned about what they considered to be the most appropriate structure for the methods course. Since these teachers work regularly each semester with student teachers, they are

acutely aware of the needs of this group. The assumption was made that their opinions would reflect the immediate needs of the student teacher and serve as a basis for evaluating whether the experimental methods course was properly structured to meet these needs.

The third phase of the evaluation involved the surveying of the opinions of thirty senior students, who at the time of the survey were doing their student teaching in mathematics, in reference to the structuring of the methods course. All of these students, although not informed so, were enrolled in the experimental methods course the preceding semester. It was assumed that their opinions would be a reflection of their needs and would, therefore, serve as a basis for determining whether the structure of the experimental methods course was meeting these needs.

The final phase of the evaluation of the experimental methods course involved the opinions of the three instructors who taught the course during the first semester of the 1966-67 school year. An evaluation of the experimental course's "teachability" was formulated on the basis of their classroom experiences while teaching the proposed course.

Current Teaching Practices in the Methods Course

Since the experimental methods course was constructed from the composite opinions of 200 specialists in mathematics education at colleges and universities throughout the United States, it was deemed appropriate to ascertain how this hypothetical course compared with current teaching practices in the mathematics methods course. Here the jury's theoretical structure for an optimum methods course could be compared with the actual teaching practices of their peers.

To answer this question, a sample was selected from the institutions of higher learning throughout the Commonwealth of Pennsylvania. Included in this sample were those Pennsylvania schools listed in the Guidebook to Departments in the Mathematical Sciences in the United States and Canada[1] which had graduated ten or more mathematics majors in the 1965-66 school year and had offered a methods course in mathematics as ascertained by reviewing their respective college catalogs. There were thirty schools which satisfied these two criteria.

During the tenth week of the first semester of the 1966-67 school year, each mathematics department chairman at these thirty schools was mailed a questionnaire (Appendix A) and an accompanying cover letter (Appendix E) asking him to pass along the questionnaire to the instructor in his department who taught the methods course and for that individual to describe in reference to this questionnaire the current topics which he included in the methods course and the number of periods which he devoted to each of these topics.

A summary of the thirty returned questionnaires is included in Table 5. The topics are entered in the table in accordance with the frequency (column 2) with which they are included in the current methods courses at these schools. Column 3 indicates the mean number of periods that are devoted to each of these topics; column 4, the standard deviation is an indicator of the variability existing in the treatment of the various topics; and column 5 lists the maximum number of periods the topic is dealt with at any one of the thirty schools.

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[1] Raoul Hailpern (ed.), Guidebook to Departments in the Mathematical Sciences in the United States and Canada (Buffalo, New York: The Mathematical Association of America, 1966), pp. 50-52.

TABLE 5

A SUMMARY OF THE RESPONSES BY THIRTY METHODS COURSE INSTRUCTORS IN REFERENCE TO THE CONTENT OF THE METHODS COURSE AT THEIR SCHOOL

Topic (1)	Number Selecting Topic (2)	Number of Periods		
		Mean (3)	S. D. (4)	Max. (5)
Methods of lesson presentation . .	27	2.40	3.18	18
Curriculum experimentation in the 1950's and 60's	24	1.66	1.26	4
The lesson plan	24	1.27	1.30	6
Individual differences	24	1.18	1.32	6
Motivation	24	1.12	0.89	4
Teaching algebra	23	2.50	2.28	9
Audio-visual aids	23	1.16	0.96	3
Student evaluation	23	0.90	0.84	3
Enrichment materials	23	0.83	0.79	3
Teaching geometry	22	2.30	2.00	6
Test construction	22	1.10	1.10	4
Development of the mathematics curriculum to 1950	22	1.02	0.90	3
Homework	22	0.68	0.56	2
Presentation of lessons by students	21	7.12	7.84	27
Teaching junior high school mathematics	21	1.98	1.83	6
The literature of mathematics and its teaching	21	0.95	0.88	3
Discipline	20	0.78	0.74	2

TABLE 5 -- Continued

Topic (1)	Number Selecting Topic (2)	Number of Periods		
		Mean (3)	S. D. (4)	Max. (5)
Curriculum implications for the future	20	0.78	0.81	3
Teaching advanced topics in high school mathematics	19	1.62	1.77	6
The psychology of learning mathematics	19	0.98	0.94	3
Professionalism and professional organizations . .	19	0.52	0.55	2
Problem solving	18	1.38	1.77	7
The unit plan	18	1.03	1.96	10
Textbook evaluation	18	0.65	0.81	3
Programmed instruction	18	0.59	0.65	2
The philosophy of mathematics . .	17	0.92	1.67	9
The role and responsibilities of the student teacher	17	0.61	0.66	2
The evolution of mathematical concepts in grades K-12	15	1.59	4.19	23
Applications of mathematics . . .	14	1.21	3.93	22
Graduate work in mathematics and mathematics education . . .	14	0.45	0.76	3
Presentation of reports by students	13	1.25	2.10	9
How to study mathematics	13	0.51	0.75	3
Techniques of observation	13	0.48	1.00	2
The bulletin board	13	0.37	0.55	2

TABLE 5 -- Continued

Topic (1)	Number Selecting Topic (2)	Number of Periods		
		Mean (3)	S. D. (4)	Max. (5)
The role of the mathematics supervisor	11	0.43	0.73	3
Research in mathematics education	10	0.44	0.86	3
The mathematics club	9	0.20	0.40	2

In order to compare the jury's recommendations for the structure of the methods course with current teaching practices in the methods course at Pennsylvania colleges and universities, a composite of their activities was formulated into a course with forty-five class meetings using the same criteria that were applied to the jury's recommendations in Chapter III. This was accomplished by first ordering the topics in accordance with the reported number of times that each topic is included in the methods course at these schools, then associating with each of the ordered topics the median number of periods that the topic is dealt with in these existing methods courses, and, finally, proceeding from the most frequently selected topic, summing the median number of periods associated with each of the topics until the forty-five period course has been formulated.

By utilizing the above procedure, the twenty-eight topics listed below in Table 6 with the associated number of periods opposite each of the topics are assumed to be representative of the current mathematics methods course in Pennsylvania colleges and universities.

TABLE 6

TYPICAL CONTENT AND TIME STRUCTURE OF THE METHODS COURSE IN PENNSYLVANIA COLLEGES AND UNIVERSITIES

Topic	Periods
Methods of lesson presentation	2
Curriculum experimentation in the 1950's and 60's	2
The lesson plan	1
Individual differences	1
Motivation	1
Teaching algebra	3
Audio-visual aids	1
Student evaluation	1
Enrichment materials	1
Teaching geometry	3
Test construction	1
Development of the mathematics curriculum to 1950	1
Homework	1
Presentation of lessons by students	8
Teaching junior high school mathematics	2
The literature of mathematics and its teaching	1
Discipline	1
Curriculum implications for the future	1
Teaching advanced topics in high school mathematics	2
The psychology of learning mathematics	1
Professionalism and professional organizations	1
Problem solving	2
The unit plan	1

TABLE 6 -- Continued

Topic	Periods
Textbook evaluation	1
Programmed instruction	1
The philosophy of mathematics	1
The role and responsibilities of the student teacher	1
The evolution of mathematical concepts in grades K-12	2

When replying to the questionnaire, the methods course instructors were invited to indicate on the second page of the form any topics which were a part of the methods course which they taught, but which could not be included as a subset of the thirty-seven topics listed on page one of the questionnaire. Three instructors indicated that they included other topics in their methods courses. One taught logic for two hours, one taught statistics for two hours, and one taught topics from set theory for one hour.

In Table 7 a comparison is made between the structure of the experimental methods course and the typical structure of the methods course at Pennsylvania colleges and universities. In Section I of this table are indicated those topics which were given identical time consideration in the experimental methods course and the typical methods course at Pennsylvania colleges and universities; Section II lists those topics which were allotted a greater number of periods in the experimental methods course; and Section III indicates those topics which were allotted a greater number of periods in the typical methods course now being taught at colleges and universities throughout the Commonwealth of Pennsylvania.

TABLE 7

A COMPARISON OF THE EXPERIMENTAL METHODS COURSE AND THE TYPICAL CONTENT OF THE METHODS COURSE AT PENNSYLVANIA COLLEGES AND UNIVERSITIES

Topic	Periods Exp.	Periods Penna.
Section I		
Curriculum experimentation in the 1950's and 60's	2	2
Methods of lesson presentation	2	2
Problem solving	2	2
Curriculum implications for the future	1	1
The development of the mathematics curriculum to 1950	1	1
Enrichment materials	1	1
Student evaluation	1	1
The lesson plan	1	1
Audio-visual aids	1	1
Professionalism and professional organizations	1	1
Textbook evaluation	1	1
Motivation	1	1
Individual differences	1	1
Homework	1	1
Programmed instruction	1	1
Section II		
Teaching geometry	4	3
Teaching algebra	4	3
Teaching advanced topics in high school mathematics	3	2
Teaching junior high school mathematics	3	2
The evolution of mathematical concepts in grades K-12	3	2

TABLE 7 -- Continued

Topic	Periods	
	Exp.	Penna.
Test construction	2	1
The literature of mathematics and its teaching	2	1
The psychology of learning mathematics	2	1
Section III		
The presentation of lessons by students	4	8
The philosophy of mathematics	0	1
The unit plan	0	1
The role and responsibilities of the student teacher .	0	1
Discipline	0	1

The thirty methods course instructors were also asked to indicate on page two of the questionnaire any learning experiences which they had structured for their students in addition to the formal classroom meetings. Twenty-one instructors responded to this question with seventeen of them indicating that they required their students to observe a master high school mathematics teacher; five instructors required their students to participate in the teaching process at the campus laboratory school, at the local high school, or in freshmen mathematics classes; three instructors had their students participate in a tutoring program in the local high school or with college freshmen; two instructors required their students to attend professional meetings which were held in the college area; and one instructor who was also responsible for visiting student teachers had each student in his methods course make one trip with him to a local high school to observe a student teacher. All of

the above topics except the last one were also suggested by the 200 member jury.

Appropriate Structure for the Methods Course from the Viewpoint of High School Supervising Teachers

This phase of the evaluation of the experimental methods course involved a comparison between the structure of the experimental methods course and the appropriate structure for such a course from the viewpoint of secondary school mathematics teachers who also function as supervisors of student teachers. This group of high school mathematics teachers have distinguished themselves as master teachers in their subject area by being chosen by their administrators and the university mathematics faculty to serve as both model and critic for the student teacher. It was assumed that this group knew the needs of the beginning teacher; therefore, a hypothesis which they would formulate in reference to the structure of the methods course could be used as a basis for evaluating whether the experimental methods course was appropriately designed to meet these needs.

The forty supervising teachers surveyed in reference to their opinion of the most desirable content for the methods course all serve regularly as supervising teachers for the mathematics student teachers of Indiana University of Pennsylvania. Each member of this group of teachers has had a minimum of five years of teaching experience in the public schools, all of them have undergraduate degrees with a major in mathematics, and thirty-three of them either have a master's degree or are actively engaged in the process of completing the requirements for a master's degree.

During the tenth week of the first semester of the 1966-67 school year, each of these forty teachers was mailed a questionnaire (Appendix A) and an accompanying cover letter (Appendix F) asking him to indicate on the questionnaire the topics and the number of class periods that should be devoted to each of these topics in an exemplary methods course. A summary of the forty responses to the questionnaire is included below in Table 8. The topics are entered in the table in accordance with the frequency (column 2) with which they were nominated for inclusion in the methods course by these supervising teachers. Column 3 indicates the mean number of periods suggested by this group for devotion to a particular topic; column 4, the standard deviation, is an indicator of the variability that existed within the individual recommendations; and column 5 lists the maximum number of periods any one of the forty teachers suggested be devoted to a particular topic.

TABLE 8

A SUMMARY OF THE RESPONSES BY FORTY SUPERVISING TEACHERS IN REFERENCE TO THE CONTENT OF THE METHODS COURSE

Topic (1)	Number Selecting Topic (2)	Number of Periods		
		Mean (3)	S. D. (4)	Max. (5)
The lesson plan	40	2.60	1.20	6
The unit plan	39	2.00	1.12	5
The role and responsibilities of the student teacher	37	1.55	0.95	4
Methods of lesson presentation . .	36	3.83	3.06	14
Test construction	36	1.78	1.19	5

TABLE 8 -- Continued

Topic (1)	Number Selecting Topic (2)	Number of Periods		
		Mean (3)	S. D. (4)	Max. (5)
Motivation	34	1.88	1.59	6
Homework	33	0.90	0.59	2
Teaching geometry	32	1.98	1.71	6
Teaching algebra	32	1.93	1.77	7
Discipline	32	1.75	1.71	9
Student evaluation	32	1.46	1.33	6
Enrichment materials	32	1.11	0.79	3
Individual differences	30	1.28	1.28	6
Techniques of observation	30	1.06	1.04	6
Teaching junior high school mathematics	29	1.68	1.44	5
Curriculum implications for the future	28	0.90	0.87	3
Professionalism and professional organizations	28	0.76	0.60	2
The evolution of mathematical concepts in grades K-12	27	1.35	1.76	10
Audio-visual aids	27	0.88	0.80	3
Teaching advanced topics in high school mathematics	26	1.36	1.45	5
Textbook evaluation	26	0.81	0.74	3
Applications of mathematics	25	1.08	1.47	7
Curriculum experimentation in the 1950's and 60's	25	1.04	1.37	5
How to study mathematics	25	0.86	0.94	4

TABLE 8 -- Continued

Topic (1)	Number Selecting Topic (2)	Number of Periods		
		Mean (3)	S. D. (4)	Max. (5)
Problem solving	24	1.04	1.23	6
The bulletin board	24	0.90	1.07	5
The philosophy of mathematics . .	24	0.64	0.67	3
The psychology of learning mathematics	22	0.78	0.82	3
Presentation of lessons by students	21	2.49	3.85	15
The literature of mathematics and its teaching	21	0.51	0.76	2
Development of the mathematics curriculum to 1950	20	0.57	0.76	3
The role of the mathematics supervisor	20	0.56	0.65	2
The mathematics club	15	0.43	0.87	5
Presentation of reports by students	14	0.55	0.91	3
Programmed instruction	13	0.26	0.40	1
Research in mathematics education	12	0.31	0.54	2
Graduate work in mathematics and mathematics education . . .	9	0.19	0.37	1

In order to compare the structure of the experimental methods course with the recommendations made by the supervising teachers for its content, a composite of the supervising teachers' recommendations was formulated into a forty-five class period course. The criteria

used to structure this theoretical course were the same that were used to structure the experimental methods course.

By utilizing this procedure, the thirty topics listed below in Table 9 with the associated number of classroom periods opposite each of the topics are assumed to be representative of an exemplary methods course as designed by master teachers to meet the needs of the beginning teacher.

TABLE 9

THE CONTENT AND TIME STRUCTURE OF AN EXEMPLARY METHODS COURSE FROM THE VIEWPOINT OF FORTY SUPERVISING TEACHERS

Topic	Periods
The lesson plan	3
The unit plan	2
The role and responsibilities of the student teacher	2
Methods of lesson presentation	4
Test construction	2
Motivation	2
Homework	1
Teaching geometry	2
Teaching algebra	2
Discipline	2
Student evaluation	1
Enrichment materials	1
Individual differences	1
Techniques of observation	1
Teaching junior high school mathematics	2
Curriculum implications for the future	1

TABLE 9 -- Continued

Topic	Periods
Professionalism and professional organizations	1
The evolution of mathematical concepts in grades K-12	1
Audio-visual aids	1
Teaching advanced topics in high school mathematics	1
Textbook evaluation	1
Applications of mathematics	1
Curriculum experimentation in the 1950's and 60's	1
How to study mathematics	1
Problem solving	1
The bulletin board	1
The philosophy of mathematics	1
The psychology of learning mathematics	1
Presentation of lessons by students	3
The literature of mathematics and its teaching	1

In Table 10 a comparison is made between the structure of the experimental methods course and the structure for the methods course as suggested by the supervising teachers in mathematics. In Section I of this table are indicated those topics which were given identical time consideration in the experimental methods course and the methods course suggested by the supervising teachers; Section II lists those topics which were allotted a greater number of periods in the experimental methods course; and Section III indicates those topic areas which were allotted a greater number of periods in the methods course hypothesized by the supervising teachers.

TABLE 10

A COMPARISON OF THE EXPERIMENTAL METHODS COURSE WITH THE CONTENT SUGGESTED FOR THE METHODS COURSE BY SUPERVISING TEACHERS

Topic	Periods	
	Exp.	Super.
Section I		
Test construction	2	2
Curriculum implications for the future	1	1
Enrichment materials	1	1
Student evaluation	1	1
Audio-visual aids	1	1
Professionalism and professional organizations	1	1
Textbook evaluation	1	1
Individual differences	1	1
Homework	1	1
Section II		
Teaching geometry	4	2
Teaching algebra	4	2
Teaching advanced topics in high school mathematics	3	1
The evolution of mathematical concepts in grades K-12	3	1
Presentation of lessons by students	4	3
Teaching junior high school mathematics	3	2
Curriculum experimentation in the 1950's and 60's	2	1
The literature of mathematics and its teaching	2	1
Problem solving	2	1
The psychology of learning mathematics	2	1
Programmed instruction	1	0

TABLE 10 -- Continued

Topic	Periods	
	Exp.	Super.
The development of the mathematics curriculum to 1950 .	1	0
Section III		
Methods of lesson presentation	2	4
The lesson plan	1	3
Motivation	1	2
The role and responsibilities of the student teacher . .	0	2
The unit plan	0	2
Discipline	0	2
How to study mathematics	0	1
The philosophy of mathematics	0	1
Applications of mathematics	0	1
Techniques of observation	0	1
The bulletin board	0	1

When replying to the questionnaire, the supervising teachers were invited to indicate on the second page of the form any topics which they felt should be included in the methods course that were not listed among the thirty-seven topics on the first page of the questionnaire. Five teachers indicated five different topics for inclusion in the methods course. They were the history of mathematics, two hours; topics from modern mathematics, two hours; the teaching of business mathematics, one hour; solid geometry, one hour; and recreational mathematics, one hour.

The forty supervising teachers were also asked to indicate on page two of the questionnaire any outside class learning experiences

which they felt would complement the formal periods which the methods course student spends in the college classroom. Seventeen teachers responded to this question with twelve of them indicating that it would be desirable for the students to observe various ability level high school mathematics classes prior to doing their student teaching; two teachers suggested that the methods course students tutor low ability high school students; two teachers suggested that the students who have completed their student teaching requirement visit the methods class and discuss with the class members the various aspects of student teaching; and one teacher suggested that the methods class should take field trips to various industries where mathematicians are employed. All of the above topics except the one, where students who have completed their student teaching requirement would visit the methods class, were suggested by the 200 member jury.

Appropriate Structure for the Methods Course from the Viewpoint of Student Teachers

The third phase of the evaluation of the experimental methods course involved a comparison between the structure of the experimental methods course and the appropriate structure for such a course based upon a survey conducted among thirty college seniors. The thirty seniors who participated in the survey during the second semester of the 1966-67 school year were all regularly enrolled students at Indiana University of Pennsylvania and at the time of the survey were culminating their college careers with a semester of student teaching in mathematics. In addition, all thirty of these students were enrolled, although not informed of this fact, in the experimental methods course the previous semester.

During the sixth week of the second semester of the 1966-67 school year, each of these thirty student teachers was mailed a questionnaire (Appendix A) and an accompanying cover letter (Appendix G) asking him to indicate in the column beside each topic the amount of available classroom time that he felt should be devoted to the particular topic in an exemplary methods course. A summary of the thirty responses to the questionnaire are included in Table 11. The topics are entered in the table in accordance with the frequency (column 2) with which they were nominated for inclusion in the methods course by the student teachers. Column 3 indicates the mean number of periods suggested by this group for devotion to a particular topic; column 4, the standard deviation is an indicator of the variability that existed within the individual recommendations; and column 5 lists the maximum number of periods that any one of the student teachers suggested be devoted to a particular topic.

TABLE 11

A SUMMARY OF THE RESPONSES BY THIRTY STUDENT TEACHERS IN REFERENCE TO THE CONTENT OF THE METHODS COURSE

Topic (1)	Number Selecting Topic (2)	Number of Periods		
		Mean (3)	S. D. (4)	Max. (5)
Methods of lesson presentation . .	30	2.90	1.60	8
The lesson plan	30	2.40	1.40	7
The unit plan	30	1.45	0.69	3
The role and responsibilities of the student teacher	29	1.85	0.98	5
Motivation	28	1.57	0.73	3

TABLE 11 -- Continued

Topic (1)	Number Selecting Topic (2)	Number of Periods		
		Mean (3)	S. D. (4)	Max. (5)
Textbook evaluation	28	1.23	0.62	3
Discipline	27	1.93	1.42	6
Curriculum experimentation in the 1950's and 60's	26	1.44	0.87	3
Teaching advanced topics in high school mathematics	26	1.25	0.78	3
Homework	26	1.00	0.66	3
Student evaluation	25	1.40	0.92	4
Individual differences	25	1.38	0.83	3
Teaching algebra	25	1.20	0.82	3
Teaching geometry	25	1.17	0.81	3
Curriculum implications for the future	25	0.99	0.68	3
Teaching junior high school mathematics	24	1.35	0.96	3
The philosophy of mathematics . .	24	1.07	0.92	2
Techniques of observation	24	1.00	0.89	5
Professionalism and professional organizations . .	23	0.92	0.66	3
Development of the mathematics curriculum to 1950	22	1.03	0.94	4
Test construction	21	1.23	1.02	3
The literature of mathematics and its teaching	21	1.07	1.02	4
Enrichment materials	21	1.00	0.87	3

TABLE 11 -- Continued

Topic (1)	Number Selecting Topic (2)	Number of Periods		
		Mean (3)	S. D. (4)	Max. (5)
Problem solving	20	1.30	1.10	3
Applications of mathematics . . .	19	1.07	1.00	3
The evolution of mathematical concepts in grades K-12	19	0.80	0.76	3
The bulletin board	19	0.66	0.63	2
Presentation of lessons by students	18	3.57	2.98	15
The psychology of learning mathematics	18	0.93	0.94	3
Graduate work in mathematics and mathematics education . . .	18	0.68	0.72	3
How to study mathematics	16	0.77	0.80	2
Research in mathematics education	16	0.70	0.81	3
Audio-visual aids	16	0.64	0.78	3
The role of the mathematics supervisor	14	0.57	0.82	3
Programmed instruction	13	0.35	0.43	1
Presentation of reports by students	12	0.93	1.52	5
The mathematics club	9	0.23	0.38	1

It was assumed that a poll conducted among the student teachers would serve two purposes. First, it would be an indicator of the needs of this group in their initial teaching experience, and secondly it would

provide a basis whereby the structure of the experimental methods course could be evaluated to determine whether it was properly designed to meet these needs.

In order to compare the structure of the experimental methods course with the recommendations made by the student teachers for its content, a composite of the student teachers' recommendations was formulated into a forty-five class period course. The criteria used to structure this theoretical course were the same that were used to structure the experimental methods course. By utilizing this procedure, the thirty topics listed below in Table 12 with the associated number of classroom periods opposite each of the topics are assumed to be representative of an exemplary methods course designed by student teachers to meet their needs.

TABLE 12

THE CONTENT AND TIME STRUCTURE OF AN EXEMPLARY METHODS COURSE FROM THE VIEWPOINT OF THIRTY STUDENT TEACHERS

Topic	Periods
Methods of lesson presentation	3
The lesson plan	2
The unit plan	2
The role and responsibilities of the student teacher	2
Motivation	2
Textbook evaluation	1
Discipline	2
Curriculum experimentation in the 1950's and 60's	2
Teaching advanced topics in high school mathematics	1
Homework	1

TABLE 12 -- Continued

Topic	Periods
Student evaluation	2
Individual differences	1
Teaching algebra	1
Teaching geometry	1
Curriculum implications for the future	1
Teaching junior high school mathematics	2
The philosophy of mathematics	1
Techniques of observation	1
Professionalism and professional organizations	1
Development of the mathematics curriculum to 1950	1
Test construction	2
The literature of mathematics and its teaching	1
Enrichment materials	1
Problem solving	1
Applications of mathematics	1
The evolution of mathematical concepts in grades K-12	1
The bulletin board	1
Presentation of lessons by students	5
The psychology of learning mathematics	1
Graduate work in mathematics and mathematics education	1

In Table 13 a comparison is made between the structure of the experimental methods course and the structure recommended for the methods course by the student teachers in mathematics. In Section I of this table are indicated those topics which were given identical time

consideration in the experimental methods course and in the methods course designed by the student teachers; Section II lists those topics which were allotted a greater number of periods in the experimental methods course; and Section III indicates those topics which were allotted a greater number of periods in the methods course designed by the student teachers.

TABLE 13

A COMPARISON OF THE EXPERIMENTAL METHODS COURSE AND THE METHODS COURSE DESIGNED BY STUDENT TEACHERS

Topic	Periods	
	Exp.	Student
Section I		
Curriculum experimentation in the 1950's and 60's	2	2
Test construction	2	2
Curriculum implications for the future	1	1
The development of the mathematics curriculum to 1950	1	1
Enrichment materials	1	1
Professionalism and professional organizations	1	1
Textbook evaluation	1	1
Individual differences	1	1
Homework	1	1
Section II		
Teaching geometry	4	1
Teaching algebra	4	1
Teaching advanced topics in high school mathematics	3	1
The evolution of mathematical concepts in grades K-12	3	1
Teaching junior high school mathematics	3	2

TABLE 13 -- Continued

Topic	Periods	
	Exp.	Student
The literature of mathematics and its teaching	2	1
Problem solving	2	1
The psychology of learning mathematics	2	1
Audio-visual aids	1	0
Programmed instruction	1	0
Section III		
Presentation of lessons by students	4	5
Methods of lesson presentation	2	3
Student evaluation	1	2
The lesson plan	1	2
Motivation	1	2
The unit plan	0	2
The role and responsibilities of the student teacher	0	2
Discipline	0	2
The philosophy of mathematics	0	1
Applications of mathematics	0	1
Graduate work in mathematics and mathematics education	0	1
Techniques of observation	0	1
The bulletin board	0	1

On the second page of the questionnaire to which the student teachers responded, they were asked to comment on three items. The first was: Are there any additional topics which you feel should be included in the methods course that were not among the thirty-seven listed on the

first page of the questionnaire? All thirty of the student teachers agreed that there was a sufficient number of topics included on the first page of the questionnaire around which to structure the methods course and no additional topics were nominated.

The second question to which the student teachers were asked to respond querried them in regards to the learning experiences which should be structured for the students in the methods course in addition to the formal class meetings. Twenty-six of the seniors responded to this question with twenty-one of them indicating that the observation of high school mathematics classes prior to the student teaching experience would be a very desirable learning experience. One student said,

> The greatest value to a future teacher is the knowledge of what to expect in a classroom situation. The only way to acquire this information is by actual classroom observations.

Another student commented,

> I believe a prospective teacher should observe as many teachers as he can before and during his student teaching assignment. Arrangements should be made with schools in the area to permit the student to see first hand the 'difficulties' and the methods used by experienced teachers to handle these difficulties.

Ten of the student teachers indicated that some provision should be made whereby the students in the methods course could participate in an actual teaching situation. Two of the suggestions offered for the implementation of this idea were,

> Arrangements should be made with the local high schools for the students to teach several lessons while they are enrolled in the methods course. This would be a 'real' learning experience.

and,

> I feel that one learning experience that would be particularly helpful to future teachers would be to have them participate in the tutoring service at the college. In my own

> case this offered valuable experience by teaching me the correct way to explain mathematical concepts. This experience has been a great help to me as a student teacher.

Seven seniors suggested that the methods course students write a research paper dealing with a problem in the teaching of mathematics about which they are personally concerned. Five of the seniors suggested that an intensive reading program should be correlated with the activities of the methods course. Here one student commented,

> I think that the best experience for me was going to the library and doing the readings for the methods course. In this way I became more closely associated with the ideas and thoughts of other people. These readings were presented in such a way that they were correlated with the topics which we were discussing in class and I feel that these readings helped me immensely in my student teaching.

It is significant to note that all of the learning experiences which were suggested by the student teachers were a part of or paralleled the learning experiences associated with the experimental methods course in which they had been enrolled.

The final question which the student teachers were asked to respond to was: "We would appreciate learning of any thoughts which you might have concerning the methods course." Here the response was directed toward the methods course which they had just completed and was overwhelmingly complementary. One student said,

> When I finished the methods course, I not only felt capable of teaching a high school mathematics class, but I felt for the first time an honest desire to teach.

The experimental methods course differed markedly from what is generally considered typical for the course in the amount of time devoted to the presentation of lessons by students (four hours as opposed to twenty-seven in some of the methods courses investigated) and the required

reading program. In reference to the presentation of lessons by students, one student commented,

> I am glad that in the methods course we did not spend more time on student lessons. When a student presents a lesson, it amounts to about the same thing as giving a speech in English class. There are no problems -- everyone understands the material and you always get a correct response to your questions. You are not teaching -- you are merely presenting material. Any resemblance between this activity and actual teaching is purely coincidental.

As noted above, several students commented on the importance of an individual reading program. One student said,

> As much as I hated going to the library, I see now that I am teaching that it was all worthwhile. If I had it to do again, I certainly would begin reading about the teaching of mathematics long before my last semester on the campus.

This section will be concluded with the following student comment.

> After filling out this questionnaire, I have come to the conclusion that forty-five periods isn't enough time to do all the things that should be done in the methods course.

The Teachability of the Experimental Methods Course

The final phase of the evaluation of the experimental methods course involved the concept of its teachability from the viewpoint of the three instructors[1] who taught the experimental methods course during the first semester of the 1966-67 school year. In order to evaluate the course's teachability, the instructors employed the following guidelines:

1. Are the topics appropriate for inclusion in the mathematics methods course?

2. Are too many or too few topics attempted?

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[1]Dr. Howard L. Prouse, Professor, Department of Mathematics and Astronomy, Mankato State College, Mankato, Minnesota; and Mr. Wallace F. Morrell, Associate Professor, Department of Mathematics, Indiana University of Pennsylvania, Indiana, Pennsylvania.

3. What topics should be eliminated and/or added?

4. How would the time devoted to each of the topics be adjusted in reference to the changes which you suggested in guideline 3?

5. What is the preferred ordering of the topics to be presented in the methods course?

6. Does the structure of the course permit sufficient student activity and participation?

7. Is the structure of the course such that it encourages student self motivation?

Each of these guidelines will be considered individually and the instructor's as well as the investigator's reaction to them noted.[1]

In reference to guideline 1, "Are the topics appropriate for inclusion in the mathematics methods course?" all three professors agreed that they were. Here Prouse commented,

> I feel that the topics selected by the jury for inclusion in the course are appropriate.

Morrell said,

> . . . the course is extremely well planned. I learned a lot personally -- many vague ideas of philosophy were crystalized for me. The next time that I teach the methods course I intend to follow a similar program.

The researcher concurs with the jury on their identification of a nucleus of topics around which the methods course can be structured. Although several topic areas such as audio-visual aids, individual differences, and test construction are dealt with in other courses in the undergraduate curriculum, their inclusion in the methods course permits the student to to strengthen his skills in these areas and offers him the opportunity

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[1] Permission to quote Dr. Prouse and Mr. Morrell was obtained.

to see their application in his chosen professional speciality.

Guideline 2, "Are too many or too few topics attempted?" and guideline 3, "What topics should be eliminated and/or added?" will be treated simultaneously. Dr. Prouse and the investigator would add some topic areas to those included in the experimental methods course while Mr. Morrell would add some topics and delete one topic because of time limitations. Prouse said,

> Specific topics which I believe should be included are (a) the bulletin board, (b) applications of mathematics, (c) research in mathematics education, and (d) observation techniques.
>
> I am not attempting to be dogmatic. However, I do believe that these topics should be treated in a secondary methods course. One's personal concepts of mathematics education and teacher education will certainly determine where emphasis will be placed.

Morrell commented,

> Although I feel that all the topics which we included in the experimental methods course can be justifiably taught in the methods course, my past experience with student teachers forces me to eliminate some, add others, and alter the time devoted to the remaining ones. I feel that the role and responsibilities of the student teacher, techniques of observation, discipline, the unit plan, and current research in mathematics education are topics which should be dealt with in the methods course. In light of introducing these new topics and increasing the time devoted to others I would significantly reduce the time devoted to the areas of teaching junior high school mathematics, teaching geometry, and teaching algebra. I would discuss these topics as follow ups to lessons presented by the students in these areas. In addition, the time allotment for the course would force me to eliminate a discussion of programmed instruction.

In the opinion of the researcher four additional topic areas should be added to the twenty-four which were included in the experimental methods course. The first of these is the unit plan. Hopefully, a discussion of this topic would impress the future teacher with the importance of long range planning and serve to help eliminate that mathematics teacher who

is one lesson ahead of the students. Secondly, it is very important to discuss techniques of observation. The student is required to make several classroom observations during the methods course and ordinarily spends the first two weeks of his student teaching semester observing other teachers. If this experience is to have meaning for the student, he must be made aware of the techniques and procedures which he should be observing. Third on the list is the topic of discipline. One would be overly optimistic to conclude that this area would not concern the new teacher or is a totally nonexistent problem in the typical public school. The time spent discussing the techniques of discipline could pay large dividends for the new teacher in helping him to make the proper start in his teaching career. The fourth area would concern the role and responsibilities of the student teacher. One of the principal purposes of the methods course is to prepare the student to enter successfully into his student teaching experience. Certainly we should orient him to what is expected of him as a student teacher.

The instructors' responses to guideline 4, "How would the time devoted to each of the topics be adjusted in reference to the changes which you suggested in guideline 3?" are summarized below in Table 14. Column 1 indicates the time devoted to each of the topics in the experimental methods course while column 2, 3, and 4 represent the opinions of the three instructors who taught the experimental methods course in reference to guideline 4. Column 5 represents the consensus opinion of the three instructors. It was constructed by including the topics which all three instructors recommended for inclusion in the methods course and then assigning to each of these topics the median number of suggested teaching periods.

TABLE 14

THE RECOMMENDATIONS OF THE INSTRUCTORS IN REFERENCE TO THE TOPIC CONTENT AND TIME STRUCTURE OF THE METHODS COURSE

Topic	Exp. (1)	Prouse (2)	Morrell (3)	Shafer (4)	Cons. (5)
Professionalism and professional organizations	1	0.5	1	1	1
Development of the mathematics curriculum to 1950	1	1	1	1	1
Curriculum experimentation in the 1950's and 60's	2	1	2	2	2
Curriculum implications for the future	1	1	1	1	1
The evolution of mathematical concepts in grades K-12 . . .	3	1.5	2	2	2
The literature of mathematics and its teaching	2	3	3	2	3
Problem solving	2	2.5	3	3	3
The psychology of learning mathematics	2	2	2	2	2
Individual differences	1	2	1	2	2
Motivation	1	1	1	1	1
Audio-visual aids	1	1	1	1	1
Methods of lesson presentation .	2	3	1	3	3
The lesson plan	1	1	1	2	1
Teaching junior high school mathematics	3	2	1	2	2
Teaching algebra	4	2	1	2	2
Teaching geometry	4	2	1	2	2

TABLE 14 -- Continued

Topic	Exp. (1)	Prouse (2)	Morrell (3)	Shafer (4)	Cons. (5)
Presentation of lessons by students	4	7	9	4	7
Homework	1	0.5	1	1	1
Enrichment materials	1	2	1	1	1
Teaching advanced topics in high school mathematics . . .	3	2	2	1	2
Textbook evaluation	1	1	1	1	1
Test construction	2	1	2	2	2
Student evaluation	1	1	1	1	1
Programmed instruction	1	1	0	1	0
Techniques of observation	0	0.5	1	1	1
Research in mathematics education	0	1	1	0	0
The unit plan	0	0	1	1	0
Discipline	0	0	1	1	0
The role and responsibilities of the student teacher	0	0	1	1	0
Applications of mathematics . . .	0	1	0	0	0
The bulletin board	0	0.5	0	0	0

The topics are entered in Table 14 in the order in which they were presented in the experimental methods course with the exception of presentation of lessons by students. The four class periods which were devoted to this topic were included in the course sequence in the following

pattern. Lesson one was presented after the topic teaching junior high school mathematics, lesson two was presented after teaching algebra, lesson three was presented after teaching geometry, and the final lesson was presented after teaching advanced topics in high school mathematics. The preferred ordering of the topics in the methods course is the next guideline to receive consideration.

In reference to guideline 5, "What is the preferred ordering of the topics to be presented in the methods course?" the three instructors concurred that the order in which the topics were presented was generally satisfactory. Concerning this guideline, Prouse felt that it would be better to conclude the methods course with the lesson presentations by the students. The four additional topic areas which he suggested for inclusion in the methods course would be incorporated into the program in the following manner: the bulletin board would follow lesson five, curriculum implications for the future; research in mathematics education would follow lesson ten, the literature of mathematics and its teaching; techniques of observation would follow lesson fourteen, the psychology of learning mathematics; and applications of mathematics would follow lesson sixteen, motivation.

Concerning guideline 5, Morrell suggested that the same sequence of topics be followed with the exception that beginning with the sixth Friday (assuming Monday, Wednesday, Friday class meetings in a fifteen week long semester) and continuing each Friday thereafter the period be devoted to the presentation of lessons by students. The five additional topics which he suggested for inclusion in the methods course would be inserted into the course sequence in the following manner: research in mathematics education would follow lesson five, curriculum implications

for the future; techniques of observation would follow lesson ten, the literature of mathematics and its teaching; discipline would follow lesson fifteen, individual differences; the unit plan would follow lesson twenty, the lesson plan; and the role and responsibilities of the student teacher would be discussed during the last class meeting.

The investigator would follow the sequential presentation of topics that was followed in the experimental methods course with one exception. The literature of mathematics and its teaching would be presented after lesson one, professionalism and professional organizations. A large portion of the literature which is important to the beginning teacher is published by the various professional organizations which were discussed during the first class meeting; therefore, consideration of the topic at that time would serve to complement the first class meeting. Secondly, since no one methods text is used in the course and many library sources are consulted, it would set the stage for the individual reading program which is essential for the student's professional growth in this course. The additional four topics suggested by the researcher for inclusion in the methods course would be inserted into the course sequence as follows: techniques of observation would follow lesson eight, the evolution of mathematical concepts in grades K-12. It is included this early in the course since the students are required and encouraged to make many observations throughout the semester and they should have some guidelines to follow in making these observations. Discipline would be discussed after lesson fifteen, individual differences; the unit plan would follow lesson twenty, the lesson plan; and a discussion of the role and responsibilities of the student teacher would be a most appropriate topic with which to conclude the methods course.

In considering guideline 6, "Does the structure of the course permit sufficient student activity and participation?" Prouse reported,

> I feel that it does. Working with the literature of mathematics education forces the student to read and digest many important ideas. Hopefully, he reflects upon these. The teaching opportunities allow for student activity. Problem solving and concept formation can be treated by involving the students in situations in which they are forced to consider carefully what their instructor has said and what the literature has said in order to satisfactorily resolve the situations. The discovery lesson can be illustrated by actually having the class members work toward uncovering some previously unknown idea from an area of subject matter which they have already studied.

In reference to this guideline, Morrell commented,

> In my opinion the students always felt that they were a part of what was happening in class. However, we need more student involvement in the making of lesson plans and, particularly, the teaching of these lessons.

The investigator feels that the structure of the methods course encouraged student activity and participation. The purpose of the course was to prepare the student to enter successfully and intelligently into his student teaching experience. The classroom consideration and discussion of the various topic areas concerned with the teaching of mathematics supplemented by a comprehensive reading program, field investigations, and personal projects all served to help make the transition from student to teacher a natural process. The students were always busy, but they were busy in the pursuit of professional growth.

Concerning guideline 7, "Is the structure of the course such that it encourages student self motivation?" Prouse said,

> As the course is set up, I feel the answer must be yes. Probably one is too much of an idealist and not enough realist many times but I must subscribe to the idealists' position. Enough broad areas have been sampled to give the student some insight into the complex communication system which constitutes teaching.

In reference to guideline 7, Morrell commented,

> I felt that the program was quite satisfactory on this point. The reading program which the students pursued in relation to the course helped them to formulate a healthy philosophy towards mathematics teaching.

In the opinion of the researcher, the answer to the question posed by guideline 7 deserves a resounding yes. The structure of the experimental methods course permitted the student to participate both in class and outside of class in direct proportion to his desire and enthusiasm to prepare for the teaching profession. The student was made aware in the beginning of the structure of the methods course and the interrelationship of the topics which were to be presented in the course. He realized what his needs were and that this course would serve to fulfill these needs and help him to make a smooth transition from his role as a student to his role as a teacher.

In this chapter a comparison was made between the structure of the experimental methods course and current teaching practices in the methods course at thirty institutions of higher learning throughout the Commonwealth of Pennsylvania. Presentation was made of a poll conducted among forty supervising teachers and thirty student teachers in reference to their suggestions for the topic content and time structure for the methods course and these recommendations were then compared with the jury's proposal for the methods course. Finally, the evaluation of the experimental course's teachability by the three instructors who taught the course was presented. These comparisons, recommendations, and evaluations will serve as a basis for designing the exemplary methods course which is presented in the final chapter of this study.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

The purpose of this study was the selection and validation of subject matter to be included in the undergraduate methods course entitled "The Teaching of Secondary School Mathematics." This objective was achieved in this research by employing the following methods.

A survey was made of the recommendations of individuals and study groups in addition to an analysis of the content of textbooks written for the methods course and the structure of contemporary methods courses at colleges and universities throughout the United States. As a result of this survey, thirty-seven different topic areas were identified as being included or suggested for inclusion in existing or hypothesized methods courses. These thirty-seven topics were incorporated into a questionnaire which was mailed to specialists in mathematics education throughout the United States. On the basis of the responses made by 200 of these specialists, a three semester hour experimental methods course which included twenty-four of the original thirty-seven topics was designed.

During the first semester of the 1966-67 school year, the experimental methods course was field tested with two groups of students at Indiana University of Pennsylvania and one group of students at Mankato

State College, Mankato, Minnesota. The three instructors who were involved in teaching the experimental methods course evaluated it in terms of its "teachability" and made recommendations for its modification in terms of the topics which were covered and the amount of class time that was devoted to each of these topics.

The second phase of the validation of the subject matter included in the experimental methods course involved a poll conducted among thirty colleges and universities in the Commonwealth of Pennsylvania. This poll permitted the determination of the structure of existing mathematics methods courses. This procedure enabled a comparison to be made between the design of the experimental methods course and the current teaching practices in Pennsylvania used in the methods course. Similarities and differences in the treatment of the various topics were noted also.

The third phase of the validation process consisted of a survey conducted among forty supervisors of student teachers in mathematics to determine their recommendations for the topic content and time structure of the methods course. Since this group is keenly aware of the needs of the beginning teacher, the survey enabled a comparison to be made between their recommendations and the structure of the experimental methods course as it helped to fulfill these needs.

The final phase of the validation process involved polling thirty student teachers for an expression of their opinion as to what the topic content and time structure of the methods course should be. Since all thirty of these student teachers had been enrolled in the experimental methods course the semester prior to the one in which the survey was made, it was possible to evaluate and compare their recommendations for

the methods course with the structure of the experimental methods course and to ascertain whether the experimental methods course was appropriately designed to meet the needs of the beginning teacher.

Conclusions

When the data were tabulated from the survey conducted among the specialists in mathematics education, the methods course instructors at Pennsylvania colleges and universities, the supervisors of student teachers in mathematics, the mathematics student teachers, and the instructors who had taught the experimental methods course, a composite of each of these group's recommendations for the methods course was formulated into a three semester hour course. A summary of these recommendations is included in Table 15. Column 1 contains the thirty-seven topics that were included on the questionnaire; column 2 indicates the topic content and time structure of the experimental methods course; column 3 represents the typical topic content and time structure of existing methods courses at Pennsylvania colleges and universities; column 4 lists the recommendations of the supervising teachers in mathematics in reference to the structuring of the methods course; column 5 lists the recommendations of the student teachers in mathematics in reference to the structuring of the methods course; and column 6 is a composite of the reactions of the three instructors who taught the experimental methods course in reference to the appropriate structure for the methods course. The numerical entries in Table 15 opposite each of the thirty-seven topics indicates the number of class periods that each of these groups suggested be devoted to that topic in the methods course.

TABLE 15

A SUMMARY OF THE RECOMMENDATIONS FOR THE METHODS COURSE

Topic (1)	Exp (2)	Penna (3)	Sup-T (4)	Stu-T (5)	Prof (6)
Teaching geometry	4	3	2	1	2
Teaching algebra	4	3	2	1	2
Curriculum experimentation in the 1950's and 60's	2	2	1	2	2
Teaching advanced topics in high school mathematics	3	2	1	1	2
Test construction	2	1	2	2	2
Methods of lesson presentation . .	2	2	4	3	3
Curriculum implications for the future	1	1	1	1	1
Teaching junior high school mathematics	3	2	2	2	2
Development of the mathematics curriculum to 1950	1	1	0	1	1
The evolution of mathematical concepts in grades K-12	3	2	1	1	2
Enrichment materials	1	1	1	1	1
Student evaluation	1	1	1	2	1
The lesson plan	1	1	3	2	1
Audio-visual aids	1	1	1	0	1
Professionalism and professional organizations . .	1	1	1	1	1
The literature of mathematics and its teaching	2	1	1	1	3
Textbook evaluation	1	1	1	1	1

TABLE 15 -- Continued

Topic (1)	Exp (2)	Penna (3)	Sup-T (4)	Stu-T (5)	Prof (6)
Motivation	1	1	2	2	1
Problem solving	2	2	1	1	3
The psychology of learning mathematics	2	1	1	1	2
Individual differences	1	1	1	1	2
Presentation of lessons by students	4	8	3	5	7
Homework	1	1	1	1	1
Programmed instruction	1	1	0	0	0
How to study mathematics	0	0	1	0	0
The philosophy of mathematics	0	1	1	1	0
The unit plan	0	1	2	2	0
Applications of mathematics	0	0	1	1	0
Research in mathematics education	0	0	0	0	0
The role and responsibilities of the student teacher	0	1	2	2	0
Discipline	0	1	2	2	0
Graduate work in mathematics and mathematics education	0	0	0	1	0
The mathematics club	0	0	0	0	0
Presentation of student reports	0	0	0	0	0
Role of the mathematics supervisor	0	0	0	0	0
Techniques of observation	0	0	1	1	1
The bulletin board	0	0	1	1	0

An analysis of the data included in Table 15 indicates that a core of topics around which to structure the methods course has been identified. The jury recommended that the methods course be structured around twenty-four topic areas. When the recommendations of the other four groups are compared with the recommendations of the jury, one notes that these five groups unanimously agree that twenty-one of the twenty-four topics which were included in the experimental methods course should be included in the mathematics methods course.

Investigation of the jury nominated topics which did not win unanimous approval for inclusion in the methods course indicates that the supervising teachers excluded the development of the mathematics curriculum to 1950 from their list of recommendations, and the student teachers excluded a consideration of audio-visual aids. The exclusion of audio-visual aids by the student teachers can be attributed to the fact that each one of them had completed a semester long course in this area. Programmed instruction was excluded from the recommendations made by the supervising teachers, the student teachers, and the instructors of the experimental methods course. The consensus of opinion was that programmed instruction can best be handled through reading assignments and individual work with programmed materials.

It is significant to note the similarities that existed between the number of periods which these five groups would devote to the twenty-one topics unanimously nominated for inclusion in the methods course. Five topic areas were given identical time consideration by all of the groups. In seven topic areas the difference between the maximum and minimum time recommendation was only one period, and in six topic areas the difference between the maximum and minimum time recommendation was

two periods. The difference between the maximum and minimum time recommendations for teaching algebra and teaching geometry was three periods. The greatest difference of opinion occurred concerning the amount of time that should be devoted to the presentation of lessons by students. Current teaching practices in the methods course indicate that an average of eight periods is spent on this topic, whereas the combined recommendations of the supervising teachers indicate that it would be sufficient to devote a total of three periods to the presentation of lessons by students. Two comments received by the researcher in conjunction with the questionnaire survey are noted below to indicate this difference of opinion. A methods course professor at a Pennsylvania college commented,

> Even though it is very time consuming, each student is required to give a demonstration lesson before the class. I have often thought of discontinuing the demonstration lessons, but reports from our student teachers indicate they feel they gain much from doing them. Indeed, some report it is the most valuable part of the course.

On the other hand a supervising teacher commented,

> You will note that I indicated zero hours to be devoted to the presentation of lessons by students. There are so many other topics on this questionnaire that you could spend time on in the methods course that would be of more benefit to these students in preparing them for their teaching assignment. After all, the students are going to devote a whole semester to this topic under the direction of their critic teacher and under actual classroom conditions!

The investigator feels that if one were attempting to affect a compromise between these five groups in reference to the amount of time to be devoted to the topics unanimously nominated for inclusion in the methods course, the question of how much time to devote to the presentation of lessons by students would be a difficult problem to resolve. Clearly, this is an area which merits some additional research.

Although the prime consideration of this research was with the identification of a core of topics around which to structure the methods course and then to determine the amount of available class time that should be devoted to each of the topics in this core, appropriate activities for the students in addition to the formal class meetings were also considered. The extra-class learning experience that was nominated by the greatest number of teachers in each of the groups polled was classroom observations. A summarization of the opinions of the individuals nominating this experience would be that the most desirable model for the beginning teacher to emulate is an outstanding high school mathematics teacher. Therefore, the opportunity for the student in the methods course to observe the learning process at any level should be an integral part of every methods course.

There was no single commercial textbook used in the experimental methods course. To compensate for this the students pursued a reading program of current literature (Appendix D) coordinated with the topics discussed in the methods course. The reaction of the students to this procedure was quite favorable. On the thirty returned questionnaires not one student teacher suggested the termination of this procedure; in fact, several called for an expansion of the reading program. The methods course instructor, therefore, should consider the advantages of substituting an enlightning reading program of current literature in mathematics education for the traditional textbook. The instructor who follows this procedure must keep his reading assignments current and must attempt to expose his students to both sides of the question in areas such as curriculum experimentation in the 1950's and 60's and curriculum implications for the future. As a part of the reading program, the student can

incorporate other suggested learning experiences like reviewing published materials, familiarization with visual aid materials, and locating enrichment materials into his background. The student should also be given the opportunity to explore in depth some topic of significant concern to himself and to present the results of his research in a formal paper.

Another extra-class learning experience which was soundly endorsed by the groups participating in this survey was the participation of the methods course student in the learning process. The attitude of the individuals who suggested this experience was that the students were learning about teaching mathematics to high school students, but they were totally divorced from the realities of the experience in the methods course classroom. The suggestions which the survey participants offered for incorporating this activity into the students' learning experiences included having the students tutor high school students and college freshmen, having the students intern at the local high school, having the students teach several lessons to college freshmen, and having the students give talks to high school mathematics classes.

The investigator concurs that participation in the learning process would be a valuable and meaningful experience for the students enrolled in the methods course and that the activities described above would serve to accomplish this goal. The implementation of many of these suggestions, however, would cause some administrative problems at the high school and at the college. As an alternative, the researcher proposes the following program for the involvement of the methods course student in the learning process and for making the methods course a more meaningful experience in the future teacher's undergraduate program. This program involves offering the methods course concurrently with the student teaching assignment.

The first two weeks that the student teacher spends with his supervising teacher is largely consumed by classroom observations. The suggested program does not deviate from the wisdom of this procedure but rather serves to complement it. During the first two weeks of the semester, the student would devote three hours each morning to attending methods course classes. In the afternoon during these two weeks the student would observe high school mathematics classes, assist with routine classroom duties, and have conferences with the supervising teacher. This would allow the student to gradually break into the routine of student teaching and provide him with a frame of reference for the topics which are discussed during these two weeks in the methods course.

Beginning with the third week and continuing for the remainder of the semester, the student would spend every day at the high school. Each Saturday the student would return to the campus where the topics included in the methods course would continue to be discussed along with the teaching problems encountered by the students during the week. The instructor in this program would have the responsibility of visiting the classroom of each student teacher and holding individual conferences with him concerning his classroom techniques and professional progress.

This program would permit the methods course instructor to have sixty-nine contact periods with the students in the methods course. The additional twenty-four periods would be used to share ideas and to discuss and propose solutions to teaching problems that the students had encountered during the week. This program would certainly involve the student in the learning process in conjunction with his enrollment in the methods course. The researcher feels that by combining the methods course and the student teaching assignment in this fashion, they would complement

each other. The advantages of this procedure as opposed to the traditional organization of offering the methods course the semester prior to the student teaching assignment would be a suitable question for further research.

Recommendations

This research was not undertaken with the intention of finding a formula into which one could fit the methods course. There is no subject in the undergraduate curriculum which should be taught as if its content were fixed. Although complete rigidity of course content is one extreme, the other is that it would be impossible to structure course content.

The results of this study have indicated that there is a core of topics around which to structure the methods course. Based upon the recommendations of 200 specialists in mathematics education at colleges and universities throughout the United States, thirty methods course instructors at Pennsylvania colleges and universities, forty high school supervising teachers, thirty student teachers, and two colleagues who were involved in teaching the experimental methods course, the researcher proposes the following topic content and time structure for the methods course in mathematics. This proposal is based upon the combined recommendations of the five groups as indicated in Table 15. Of the twenty-eight topics which are suggested for inclusion in the methods course, twenty-one received the unanimous endorsement of all five groups, two were suggested by four of the five groups, and five were suggested by three of the groups. The time that is suggested for devotion to each of these twenty-eight topics is the median number of periods recommended by the groups which nominated the particular topic. This procedure has the

affect of eliminating any extreme time recommendations and more accurately representing the combined expression of the 303 individuals involved in this study. The topics are entered in Table 16 in the order in which the researcher suggests that they be presented in the methods course.

TABLE 16

THE METHODS COURSE

Topic	Periods
Professionalism and professional organizations	1
The literature of mathematics and its teaching	1
The development of the mathematics curriculum to 1950	1
Curriculum experimentation in the 1950's and 60's	2
Curriculum implications for the future	1
The evolution of mathematical concepts in grades K-12	2
Techniques of observation	1
The philosophy of mathematics	1
Problem solving	2
The psychology of learning mathematics	1
Individual differences	1
Discipline	2
Motivation	1
Audio-visual aids	1
Methods of lesson presentation	3
The lesson plan	1
First student lesson presentation	1
The unit plan	2
Teaching junior high school mathematics	2

TABLE 16 -- Continued

Topic	Periods
Second student lesson presentation	1
Teaching algebra	2
Third student lesson presentation	1
Teaching geometry	2
Fourth student lesson presentation	1
Homework	1
Enrichment materials	1
Teaching advanced topics in high school mathematics	2
Fifth student lesson presentation	1
Textbook evaluation	1
Test construction	2
Student evaluation	1
The role and responsibilities of the student teacher	2

The investigator hopes that this study will be of value to instructors who are concerned with the structuring of the methods course in terms of its content and the optimum amount of time that is to be allocated to each of the topics in a three semester hour course. It is also hoped that the suggestions and recommendations which were proposed will be evaluated experimentally and that the results of these experiments will be reported to the mathematics community in future research studies.

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Small, Dwain. "The Fifth Year of Teacher Education for Teachers of Mathematics," The Mathematics Teacher, L (March, 1957), 199-203.

Thacker, G. R., and C. B. Read. "Courses Desirable for Training Teachers of High School Mathematics," School Science and Mathematics, XLIX (November, 1949), 611-19.

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Cambridge Conference on School Mathematics. Goals for School Mathematics. Boston: Houghton Mifflin Company, 1963.

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Committee on the Undergraduate Program in Mathematics. Course Guides for the Training of Teachers of Junior High and High School Mathematics. Buffalo, New York: The Mathematical Association of America, 1961.

Committee on the Undergraduate Program in Mathematics. Five Conferences on the Training of Mathematics Teachers. Report No. 1. Buffalo, New York: The Mathematical Association of America, 1961.

Committee on the Undergraduate Program in Mathematics. Recommendations for the Training of Teachers of Mathematics. Buffalo, New York: The Mathematical Association of America, 1961.

International Commission on the Teaching of Mathematics, The American Report. Training of Teachers of Elementary and Secondary Mathematics (U. S. Bureau of Education, Bulletin No. 12). Washington: Government Printing Office, 1911.

National Association of State Directors of Teacher Education Certification and the American Association for the Advancement of Science. Guidelines for Preparation Programs of Teachers of Secondary School Science and Mathematics. Washington, D. C.: American Association for the Advancement of Science, 1961.

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Felder, Virginia Isabelle. "A Proposal for a Methods Course to be Used in the Education of Teachers of Secondary-School Mathematics." Unpublished Ed. D. doctoral project, Teachers College, Columbia University, 1959.

Mock, Gordon Duane. "The Development of Methods Courses in the Teaching of Mathematics Since 1890." Unpublished Ph. D. dissertation, University of Wisconsin, 1959.

National Science Foundation Summer Conference for College Mathematics Teachers. "Methods for Teachers of Secondary School Mathematics." Report of Seminar Group I. Stillwater, Oklahoma: Oklahoma State University, 1961. (Mimeographed.)

Walters, Eleanor B. "Concept, Place, and Purpose of Professionalized Subject-Matter in the Education of Teachers of Secondary Mathematics." Unpublished Ed. D. doctoral project, Teachers College, Columbia University, 1955.

APPENDICES

APPENDIX A -- MATHEMATICS METHODS COURSE SURVEY

Name _____

Address _____

Directions: Select the topics that you include in the mathematics methods course by indicating opposite each topic the total number of class periods that you devote to it. Space is provided on the second page of this form for you to list additional topics. There are 45 classroom periods available for this course.

TOPIC	PERIODS	TOPIC	PERIODS
The lesson plan	___	Development of the mathematics curriculum to 1950	___
The unit plan	___	Curriculum experimentation in the 1950's and 60's	___
Teaching algebra	___	Curriculum implications for the future	___
Motivation	___	The literature of mathematics and its teaching	___
Homework	___	The evolution of mathematical concepts in grades K-12	___
Discipline	___	Teaching advanced topics in high school mathematics	___
Problem solving	___	The psychology of learning mathematics	___
Audio-visual aids	___	The role of the mathematics supervisor	___
Test construction	___	Graduate work in mathematics and mathematics education	___
Student evaluation	___	Professionalism and professional organizations	___
Textbook evaluation	___	The role and responsibilities of the student teacher	___
The bulletin board	___	The philosophy of mathematics	___
Enrichment materials	___	Methods of lesson presentation	___
The mathematics club	___	Teaching Jr-Hi mathematics	___
Programmed instruction	___	Techniques of observation	___
Individual differences	___	Applications of mathematics	___
Teaching geometry	___	Presentation of reports by students	___
How to study mathematics	___	Presentation of lessons by students	___
Research in mathematics education	___		

APPENDIX A -- Continued

MATHEMATICS METHODS COURSE SURVEY -- Page 2

ADDITIONAL TOPICS	PERIODS
_____	_____
_____	_____
_____	_____
_____	_____

_____ Place a check mark here if you desire to have a summary of this survey upon its completion.

WE WOULD APPRECIATE YOUR REACTION TO THE FOLLOWING OR LEARNING OF ANY THOUGHTS WHICH YOU MIGHT HAVE CONCERNING THE METHODS COURSE

What learning experiences would you suggest for the students in the methods course other than the formal periods spent with you in the classroom?

APPENDIX B

JURY COVER LETTER

Dear Prof. _____ :

We are sincerely interested in your opinion as to what the subject matter content of the methods course for future secondary school mathematics teachers should be. Your inclusion on the N. C. T. M. Roster of Teacher Education Personnel indicates your unique qualifications to assist us with this problem.

We are in the process of experimentally structuring a one semester methods course that will best prepare students to enter successfully into their student teaching experience the following semester. There are 45 periods of classroom time available for this course.

On the enclosed questionnaire you will find a list of possible topics to be included in the methods course. If you feel the topic should be included, indicate so by stating opposite it the number of course periods that you would devote to it. Otherwise, leave the column headed periods opposite the topic blank. Please indicate on the second page of the form any additional topics which you would include in the methods course and the amount of time which you would devote to each of them.

The second page of this questionnaire is especially designed to tap your creative resources. Will you please take a few minutes to fill out this form and return it in the enclosed stamped, self-addressed envelope. If you would like to receive a copy of the results of this national survey, please indicate so by checking the appropriate space on the second page of this form.

Thank you for helping us to push forward the frontier in mathematics education.

Sincerely yours,

Dale M. Shafer, Director
Methods Course Survey

APPENDIX C

JURY FOLLOW-UP LETTER

Dear Prof. _____ :

Recently we asked for your assistance in helping us to determine what the content of the methods course should be for future secondary school mathematics teachers. To this date we have not received your reply.

It is only through the assistance of the leaders in the field of mathematics education that we can succeed in this endeavor. Would you please take a few minutes to fill out the enclosed form and return it to us in the stamped self-addressed envelope.

We are looking forward to hearing and learning from you.

Sincerely,

Dale M. Shafer, Director
Methods Course Survey

APPENDIX D

READING ASSIGNMENTS

Listed below are the professional reading assignments which you will explore in the methods course. A summary of each of these reading assignments will be included in Section I of your notebook to complement our class discussion of each topic.

In addition to the professional readings listed below you will also read during the first week of the course The Central Purpose of American Education by The Educational Policies Commission. During the second week of the course you will read The Process of Education by Jerome Bruner. These books will provide a basis for our discussions and comparisons throughout the remainder of the course.

Lesson 1 Professionalism and professional organizations

Mathematics Teaching as a Career. A leaflet of The National Council of Teachers of Mathematics, Washington, D. C.

Lesson 2 The development of the mathematics curriculum to 1950

Butler, Charles H., and Wren, F. Lynwood. Chapter I, "The Evolving Program of Secondary Mathematics." The Teaching of Secondary Mathematics. 4th ed. New York: McGraw-Hill Book Company, 1965.

Lessons 3-4 Curriculum experimentation in the 1950's and 60's

The National Council of Teachers of Mathematics. The Revolution in School Mathematics. Washington, D. C.: The National Council of Teachers of Mathematics, 1961.

Kline, Morris. "The Ancients Versus the Moderns, a New Battle of the Books," The Mathematics Teacher, LI (October, 1958), 418-27.

Meder, Albert E., Jr. "The Ancients Versus the Moderns -- A Reply," The Mathematics Teacher, LI (October, 1958), 428-33.

Lesson 5 Curriculum implications for the future

Adler, Irving. "The Cambridge Report: Blueprint or Fantasy?" The Mathematics Teacher, LIX (March, 1966), 210-17.

Stone, Marshall. "Review of Goals for School Mathematics: The Report of the Cambridge Conference on School Mathematics," The Mathematics Teacher, LIX (April, 1965), 353-60.

Lessons 6-8 The evolution of mathematical concepts in grades K-12

Freitag, Herta T., and Freitag, Arthur H. The Number Story. Washington, D. C.: The National Council of Teachers of Mathematics, 1960.

Lessons 9-10 The literature of mathematics and its teaching

Archer, Allene. How to Use Your Library in Mathematics. Washington, D. C.: The National Council of Teachers of Mathematics, 1958.

Schmidt, Roland L. "Using the Library in Junior High School Mathematics Classes," The Mathematics Teacher, LVI (January, 1963), 40-42.

Lessons 11-12 Problem solving

Polya, G. How to Solve It. 2nd ed. Garden City, New York: Doubleday and Company, Inc., 1957.

Lessons 13-14 The psychology of learning mathematics

Bruner, Jerome S. "On Learning Mathematics," The Mathematics Teacher, LIII (December, 1960), 610-19.

Gagne, Robert M. "Learning and Proficiency in Mathematics," The Mathematics Teacher, LVI (December, 1963), 620-26.

Lesson 15 Individual differences

Fehr, Howard F. "General Ways to Identify Students with Scientific and Mathematical Potential," The Mathematics Teacher, XLI (April, 1953), 230-34.

Jarvis, Oscar T. "An Analysis of Individual Differences in Arithmetic," The Arithmetic Teacher, XI (November, 1964), 471-74.

Lesson 16 Motivation

Bernstein, Allen L. "Motivations in Mathematics," School Science and Mathematics, LXIV (December, 1964), 749-52.

Holtan, Boyd. "Motivation and General Mathematics Students," The Mathematics Teacher, LVII (January, 1964), 20-25.

Lesson 17 Audio-visual aids

Bernstein, Allen L. "Use of Manipulative Devices in Teaching Mathematics," The Arithmetic Teacher, X (May, 1963), 380-83.

Spross, Patricia. "Considerations in the Selection of Learning Aids," The Arithmetic Teacher, XI (May, 1964), 350-53.

Lessons 18-19 Methods of lesson presentation

Hendrix, Gertrude. "Learning by Discovery," The Mathematics Teacher, LIV (May, 1961), 290-99.

Rupkey, Brother John Bosco, F. S. C. "Inductive Teaching Versus Deductive Teaching," The Arithmetic Teacher, XII (March, 1966), 218-20.

Lesson 20 The lesson plan

Reeve, William David. Chapter V, "How to Plan and Teach a Lesson in Mathematics." Mathematics for the Secondary School. New York: Holt, Rinehart and Winston, Inc., 1954.

Lessons 21-23 Teaching junior high school mathematics

Lowry, William C. "Helping Pupils Develop Their Ideas," The Mathematics Teacher, LII (October, 1959), 488-90.

Swineford, Edwin J. "Ninety Suggestions on the Teaching of Mathematics in the Junior High School," The Mathematics Teacher, LIV (March, 1961), 145-48.

Willerding, Margaret F. "Stimulating Interest in Junior High Mathematics," The Mathematics Teacher, LII (March, 1959), 197-201.

Lesson 24 First student lesson presentation

School Mathematics Study Group. Chapter II, Section 5, "Numerals in Base Seven." Mathematics for Junior High School. Vol. I, Part I. New Haven, Connecticut: Yale University Press, 1961.

Lessons 25-28 Teaching algebra

Snyder, Henry D. "An Impromptu Discovery Lesson in Algebra," The Mathematics Teacher, LVII (October, 1964), 415-16.

Swain, Robert L. "The Equation," The Mathematics Teacher, LV (April, 1962), 226-36.

Williams, Kenneth C. "The Three Faces of (-)," The Mathematics Teacher, LV (December, 1962), 668-69.

Lesson 29 Second student lesson presentation

School Mathematics Study Group. Chapter XVII, Section 1, "The Function Concept." First Course in Algebra. Text II. New Haven, Connecticut: Yale University Press, 1961.

Lessons 30-33 Teaching geometry

Daus, Paul H. "Why and How We Should Correct the Mistakes of Euclid," The Mathematics Teacher, LII (November, 1960), 576-81.

Fawcett, H. P. "Quod Erat Demonstrandum," The Mathematics Teacher, XLIX (January, 1956), 2-6.

Heinke, Clarence H. "Variation: A Process of Discovery in Geometry," The Mathematics Teacher, L (February, 1957), 146-54.

Lesson 34 Third student lesson presentation

School Mathematics Study Group. Chapter V, Section 4, "Writing Your Own Proofs." Geometry. Text I, Part I. New Haven, Connecticut: Yale University Press, 1961.

Lesson 35 Homework

Koch, Elmer A. "Homework in Arithmetic," The Arithmetic Teacher, XII (January, 1965), 12-13.

Lesson 36 Enrichment materials

Harwood, E. Hallie. "Enrichment for All," School Science and Mathematics, LXIII (May, 1963), 415-21.

Johnson, Donovan A. "Enriching Mathematics Instruction with Creative Activities," The Mathematics Teacher, LV (April, 1962), 238-42.

Lessons 37-39 Teaching advanced topics in high school mathematics

Pieters, Richard S., and Vance, E. P. "The Advanced Placement Program in Mathematics," The Mathematics Teacher, LIV (April, 1961), 201-08.

Rosenberg, Herman. "Modern Applications of Trigonometric Identities," Updating Mathematics. Book III. Croft Educational Services, 1964, 173-76.

Willerding, Margaret F. "Infinity and Its Presentation at the High School Level," School Science and Mathematics, LXIII (June, 1963), 463-74.

Lesson 40 Fourth student lesson presentation

School Mathematics Study Group. Chapter X, Section 5, "Definitions of the Trigonometric Functions." Intermediate Mathematics. Text II, Part II. New Haven, Connecticut: Yale University Press, 1961.

Lesson 41 Textbook evaluation

Peak, Philip. "Aids for Evaluators of Mathematics Textbooks," The Mathematics Teacher, LVIII (May, 1965), 467-78.

Lessons 42-43 Test construction

Buelow, Elsa H. "Tips on Testing," School Science and Mathematics, LXIV (November, 1964), 707-14.

Kline, William E. "Making a Good Unit Test in Algebra," Updating Mathematics. Book III. Croft Educational Services, 1964, 69-72.

Lesson 44 Student evaluation

Brydegaard, Marguerite. "How Do You as a Classroom Teacher Evaluate New Learnings?" The Arithmetic Teacher, XII (April, 1965), 251-52.

Kalin, Robert. "Some Comments on Grading Procedures," The Mathematics Teacher, LII (April, 1959), 303-06.

Lesson 45 Programmed instruction

Forbes, Jack E. "Programmed Instructional Materials -- Past, Present, and Future," The Mathematics Teacher, LVI (April, 1963), 224-27.

APPENDIX E

COVER LETTER TO DEPARTMENT CHAIRMEN

Dear Prof. _____ :

We are experimenting here at Indiana with the structure of the methods course in mathematics which our students enroll in the semester prior to their student teaching experience.

You can help us in this endeavor by letting us know what is being taught in this course at your school. We hope to achieve as one of our results a composite of what is taught in the methods course throughout the state.

Would you please pass the enclosed questionnaire on to the instructor who is currently teaching the methods course. On this form he will find a list of the topics which are generally included in a mathematics methods course. If he includes the topic in the methods course at your school, we ask him to indicate so by noting opposite it the number of class periods which he devotes to it. On the second page of this form there is a space to list the topics which are included in your methods course but which are not listed on page one. The note on the questionnaire concerning the 45 class periods which are available for this course is an approximation based on a three semester hour course.

We would appreciate it if your methods course instructor would fill out this form and return it in the enclosed stamped, self-addressed envelope. We thank both of you for helping us to push forward the frontier in mathematics education.

Sincerely yours,

Dale M. Shafer, Director
Methods Course Survey

APPENDIX F

COVER LETTER TO HIGH SCHOOL SUPERVISING TEACHERS

Dear Mr. _____ :

We are in the process of experimentally structuring a one semester methods course that will best prepare students to enter successfully into their student teaching experience the following semester. As one of our supervising teachers you are uniquely qualified to help us in this endeavor.

On the enclosed questionnaire you will find a list of possible topics to be included in the methods course. We ask that you consider these topics in terms of what you know to be the immediate needs of student teachers and first year mathematics teachers. If you feel the topic should be included, indicate so by stating opposite it the number of course periods that you would devote to it. Otherwise, leave the column headed periods opposite the topic blank. On the second page of the form we ask that you indicate any additional topics which you would include in this course and the number of periods which you would devote to each of them. There are 45 periods of classroom time available for this course.

The second page of this questionnaire is especially designed to tap your creative resources. Will you please take a few minutes to fill out this form and return it in the enclosed stamped, self-addressed envelope. If you would like to receive a copy of the results of this survey, please indicate so by checking the appropriate space on the second page of this form.

Thank you for helping us to push forward the frontier in mathematics education.

Sincerely yours,

Dale M. Shafer, Director
Methods Course Survey

APPENDIX G

COVER LETTER TO STUDENT TEACHERS

Dear Mr. _____ :

We are sincerely interested in your opinion as to what the subject matter content of the methods course for future secondary school mathematics teachers should be. Since you have recently completed such a course and are now in the midst of your student teaching experience, you are uniquely qualified to assist us with this problem.

We are asking you to place yourself in the role of the methods course instructor and decide what is the most appropriate topic content for this course so that it will (1) help the student to successfully enter into his student teaching experience and (2) equip him to assume his professional role and responsibilities as a full time teacher.

On the enclosed questionnaire you will find a list of possible topics to be included in the methods course. If you feel that the topic should be included, indicate so by noting opposite it the number of classroom periods that you would devote to it. There are 45 periods of classroom time available for this course. On the second page of the form, space is provided for you to indicate additional topics which you would include in the methods course which are not listed on page one.

Will you please take a few minutes to fill out this questionnaire and return it in the enclosed stamped, self-addressed envelope. If you would like to receive a copy of the results of this survey, please indicate so by checking the appropriate space on the second page of the form.

Thank you for helping us to push forward the frontier in mathematics education.

Sincerely yours,

Dale M. Shafer, Director
Methods Course Survey