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TABLE OF CONTENTS

Chapter Page

LIST OF TABLES

LIST OF ILLUSTRATIONS

AN ANALYSIS OF SOME OF THE MOST USEFUL QUANTITATIVE METHODS RELATING TO THE CONTROLLERSHIP FUNCTION

CHAPTER I

INTRODUCTION

Few people would challenge the conclusion that during the last twenty years there has been increased use of the quantitative approach to managerial decision making. This newer approach has had an impact on the training that a controller should receive in order to perform his duties effectively as an advisor to management in its planning and control functions.

Because of the greater use of quantitative analysis in decision making, many books and articles have been written on various quantitative methods. Although the different writings have been excellent for different groups of interested persons, they have not, in this writer's opinion, adequately served the needs of the controller in regard to some of the most useful quantitative methods.

A factor of prime importance to a writer is the background and needs of his reading audience. The reading audience which is composed of mathematicians, statisticians, industrial engineers, and mathematical economists will find that their training and requirements have been adequately considered in numerous readings on quantitative methods. Chief executives who may require only a slight knowledge of quantitative methods can find in popular business periodicals many articles written for their purpose.

However, it is this writer's opinion that the controller, and accountants in general, requires a facility with quantitative methods which lies somewhere between the slight knowledge needed by the chief executive and the depth or mastery of the techniques possessed by the mathematician. Mathematically oriented compositions on quantitative methods are often so complex that they are beyond the comprehension of the controller with limited mathematical training. On the other hand, brief and generalized statements prepared for chief executives are not quite detailed enough to meet the controller's requirements. Therefore, the purpose of this study is to analyze some of the most useful quantitative methods relating to the controllership function and to present them in a manner that will enable the controller to use them effectively in planning and control.

In planning and control, the controller will frequently use such statistical methods as capital budgeting techniques, break-even analysis, budgeting techniques, probability theory, compound interest and annuity formulas, linear programming, regression and correlation analysis, game theory, sampling techniques, ratio analysis, Program Evaluation and Review Techniques, variance analysis, quality control techniques, index numbers, as well as others. Three of these methods are described and illustrated in this study. They are linear programming, PERT, and game theory. The controller may use linear programming and game theory as planning tools, while PERT may be used for both planning and control purposes.

In this writer's opinion, the controller's training in quantitative methods is adequate under the following conditions. First, he should be able to recognize the situations in which a quantitative method may be correctly applied. Second, he should be able to use the method to solve the less complicated problems. Third, he should understand the logic of each step in the solution process. And fourth, he should be cognizant of the advantages and limitations of the method. Accordingly, the points just named have been emphasized in subsequent chapters.

The study begins by describing in Chapter II the growth and development of the controllership function. This historical account is centered around the influence that various organizations, such as the Financial Executives Institute and the Securities and Exchange Commission, have had upon the evolution of the controllership function.

The functions of the controller are examined in Chapter III. In this chapter it is observed that the quantitative methods of subsequent chapters are useful to the controller in performing his primary duties as an advisor to management. The specialized training, general education, and personal qualities of a successful controller are also presented in Chapter III.

Quantitative methods in general are discussed in Chapter IV. The steps that an analyst would take in solving a managerial problem are outlined and compared with those that would be taken by an executive who did not quantify his procedures. This chapter sets forth the advantages and limitations of quantitative analysis, and considerable attention is given to the problem of supplying accurate input data for quantitative models.

Chapter V is the first of three chapters that deal with specific quantitative methods. Linear programming is the first tool that is analyzed. The purpose of linear programming and the types of problems to which it may be applied are discussed. Two simplified planning problems are solved to illustrate the use of the simplex method. Major emphasis is placed on an explanation and interpretation of each step in the solution process. The advantages and limitations of linear programming are also reviewed.

A study of PERT is presented in Chapter VI. After an examination of the purpose and nature of PERT, a building project is utilized to illustrate the application of the PERT approach to managerial planning and control. The strong and the weak features of PERT are also discussed.

The theory of games is explored in Chapter VII. Following a statement of the purpose of game theory, a discussion is presented of the two-person, constant sum game with pure strategies and the two-person, constant sum game with mixed strategies. Other types of games and the problems associated with them are then considered. Finally, an evaluation of game theory is offered.

Chapter VIII, Summary and Conclusions, completes the study.

CHAPTER II

GROWTH AND DEVELOPMENT OF THE OFFICE OF THE CONTROLLER

Growth of the Controllership Function Prior to 1931

The growth and development of the controllership function represents one of the most important organizational developments in American business during the past three-quarters of a century.[1] The development, however, has not been one which has occurred in finite steps or specific divisions that can be easily defined and listed in chronological order. Instead, the growth has been more on the order of a confused evolution, particularly in the period preceding 1931.

The confusion or lack of direction which surrounded the early growth of the controllership function can be attributed to the fact that there was no organized and coordinated efforts on the part of controllers themselves to

[1] J. Hugh Jackson, "The Growth of the Controllership Function," Controllership in Modern Management, Thornton F. Bradshaw and Charles C. Hull, Editors (Chicago, Illinois: Richard D. Irwin, Inc., 1949), p. 11.

guide and direct their own development. Evidence of the lack of agreement on the controllership function can be seen by comparing the functions of the controllers in two of the earliest industries having controllers. These were railroads and department stores.

Railroad Industry

Beginning in 1880 numerous railroads added the office of controller to their organizational charts. The Atchison, Topeka and Santa Fe Railway System (1880), Southern Pacific Company (1885), New York, New Haven and Hartford Railroad Company (1887), the Illinois Central System (1890), and the Lehigh Valley Railroad Company (1890), were a few.[2] As an example of the duties of the early controllers in railroad companies, the duties of the controller of the Atchinson, Topeka and Santa Fe Railway System were quoted as follows:

> The duties of the Comptroller are largely financial and relate to the bonds, stocks, and securities owned by the company; the issuance, cancellation and re-issuance of stock certificates; the issuance of negotiable bonds of the company, and various other financial matters.[3]

Department Store Industry

While the duties of controllers in railroad companies were related primarily to the ownership and issuance

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[2] J. Hugh Jackson, The Controller: His Functions and Organization (Cambridge, Massachusetts: Harvard University Press, 1949), p. 7.

[3] *Ibid.*, p. 8.

of stocks and bonds, controllers in department stores were functioning as key men in the decision-making processes of their organizations. The controllership concept was adopted in department stores because of the thousands of items handled by the stores, the inventory and pricing problems which they presented, the turnover rates of items handled, and special budgeting problems.[4] Working with these problems gave controllers important positions in the management of department stores.

Lack of uniformity in the idea of controllership is shown not only by the variety of functions performed by controllers but is also shown by the variety of titles and positions held by controllers in the organizational structure of various firms. In some organizations the controller's position was combined with that of the treasurer; in others, he reported to the treasurer; while in still others, the controller was the senior officer and the treasurer reported to him. Occasionally, he was little more than a routine clerical worker.

Early Industrial Associations

The diversity of duties, titles, and opinions relating to the office of the controller may have actually

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[4]Arthur R. Tucker, "Organized Co-operation Among Controllers in the United States," <u>Controllership in Modern Management</u>, Thornton F. Bradshaw and Charles C. Hull, Editors (Chicago, Illinois: Richard D. Irwin, Inc., 1949), p. 32.

been fostered by the controllers themselves. Early in the history of the controllership function, controllers in various industries organized intra-industry associations of controllers. For example, controllers working for firms belonging to the National Retail Dry Goods Association formed the Controllers Congress. Controllers in banking institutions organized the National Conference of Bank Auditors and Controllers. Governmental Controllers formed the Municipal Finance Officers Association, and Controllers and accounting officers in railroad companies organized the Railway Accounting Officers Association.[5] It is not illogical for one to suspect that these intra-industry associations devoted their time and attention to the consideration of problems peculiar to their particular industry, while giving little or no concern to the coordinated effort needed for the overall development of the controllership function.

Although the intra-industry associations were not contributing to the organized growth of controllership, they were performing an important service which later led to organized growth. The associations, each in its own individual way, were planting firmly in the minds of their managements the need for controllership functions. Once the concept of controllership had been fully received, then the stage was

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[5]Arthur R. Tucker, "Organized Co-operation Among Controllers in the United States," Controllership in Modern Management, Thornton F. Bradshaw and Charles C. Hull, Editors (Chicago, Illinois: Richard D. Irwin, Inc., 1949), p. 31.

set for the application of organized and coordinated direction to the growth and development of the controllership function. This juncture had been reached by 1931.

Growth of the Controllership Function Following 1931

Influence of the Financial Executives Institute

The growth and development of the controllership function since 1931 has been closely interwoven with the history of the Controllers Institute of America, whose name was changed to the Financial Executives Institute in 1962. For some time prior to 1931, leading controllers in industry had recognized the need for an organization to give effective guidance to individual controllers who were concerned with the advancement of the controllership function.

Although the idea of an association such as the Financial Executives Institute may have been in the minds of various persons at various times, credit for the proposal to organize goes to Arthur Ray Tucker.[6] In the fall of 1931, Mr. Tucker took his idea of a controllers association to Charles Heiss, who was controller of the American Telephone and Telegraph Company. They decided to send letters to one hundred controllers, selected at random, to ask if they considered the organization to be appropriate. Responses to

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[6]George K. Dahl, "Arthur Ray Tucker: 1881-1959," The Controller, Volume XXVII (May, 1959), p. 212.

the letter were so enthusiastically in favor of the proposal that Mr. Tucker immediately made plans for an organizational meeting.[7]

At the request of Mr. Tucker, eight men gathered in New York City on December 29, 1931. During this meeting, they formally organized the Financial Executives Institute.[8] The first annual meeting was the following September with approximately one hundred and fifty controllers present.[9]

Objective of the Financial Executives Institute

From its inception, the objective of the Financial Executives Institute has been to educate businessmen, the public, and even controllers themselves in an understanding of the concept of controllership. The original founders of the Institute realized that for many people controllership was a completely unknown function and that to others it was a function whose full dimensions and worth had not been fully realized. These leaders also knew that the burden of promoting controllership in industry rested primarily with the individual members of the Institute. Through the activities

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[7]George K. Dahl, "Arthur Ray Tucker: 1881-1959," The Controller, Volume XXVII (May, 1959), p. 212.

[8]Arthur Ray Tucker, "Editorial Comment," The Controller, Volume XIV (December, 1946), p. 658.

[9]Arthur R. Tucker, "Organized Co-operation Among Controllers in the United States," Controllership in Modern Management, Thornton F. Bradshaw and Charles C. Hull, Editors (Chicago, Illinois: Richard D. Irwin, Inc., 1949), p. 34.

of the Institute, the members hoped to provide an atmosphere in which the controllership function could grow and thrive.

Program of Education

To provide this atmosphere, members of the Institute engaged upon a carefully prepared program of education. They sponsored local programs of lecture-discussions and encouraged members to speak on the services they had to offer management in the orderly conduct of business. Conferences provided places where controllers could share their ideas. Controllers were encouraged to write articles for publication, describing their duties, responsibilities, and value to management. To provide for the circulation of articles written by controllers, a monthly periodical, The Controller, was first published by the Institute in 1934. Arthur Ray Tucker, founder and first managing director of the Institute, was the first editor of The Controller.[10] The Controller was officially changed to the Financial Executive with the January, 1963 issue.

In addition to encouraging controllers in their individual efforts to enhance the concept of controllership, research projects requiring collective efforts were undertaken. For example, committees were appointed to conduct detailed studies of troublesome and controversial questions

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[10] George K. Dahl, "Arthur Ray Tucker: 1881-1959," The Controller, Volume XXVII (May, 1959), p. 212.

relating to the controllership function. Findings and recommendations of these committees were then published and circulated in the form of booklets, pamphlets, and reports.

As the number of research projects became larger and the value of research in general became more widely recognized, members of the Institute saw need for an auxiliary research organization. Therefore, in December, 1944, the formation of the Controllership Foundation, Inc.,--a research organization--was announced.[11] The purpose of the Foundation was to conduct research in the solution of far-reaching problems in business administration, economics, and controllership. Although some attention has been given to the economic, governmental, and technical aspects of problems of wide interest, primary attention has been given to those activities which had as their main objective the advancement of the profession of controllership.

At all times the objective of the Financial Executives Institute and the Controllership Foundation has been to communicate the controllership function to management, bankers, economists, educators, and the entire business community. It is impossible to measure the influence which these organizations have had on the development of the controllership concept.

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[11]Arthur Ray Tucker, "Editorial Comment," The Controller, Volume XII (December, 1944), p. 520.

Statements on Controllership

However, the greatest problem which has faced controllers has not been a lack of opportunities and ways of advocating controllership in business. Their most significant problem has been in trying to formulate a precise, specific, all-inclusive statement defining the controller. The lack of a definition of controllership and a statement clearly distinguishing controllership functions from purely accounting functions, left many people thinking that "controller" was merely a new title for the "chief accountant."

Throughout its history the Financial Executives Institute has had committees assigned to the task of writing a definition of controllership. A clear-cut definition was desired for two reasons. First, the controllers realized that it would be difficult to advance a concept without a statement of the concept. Secondly, a definition was needed in setting eligibility standards for membership in the Institute.

To date there is still no widely accepted, precise definition of controllership. Institute members' efforts to formulate a definition of controllership, however, have not been fruitless. Members of various committees of the Institute have drawn up and publicized at various times lists of duties which controllers should perform. These lists have been widely accepted by the business community. Therefore, controllers have had to be content with a statement which defines the controller as a man who performs the duties or most of the duties enumerated in the list of duties of the controller as prepared by the Financial Executives Institute.

Even a list of duties has been most difficult to enumerate. This difficulty has been primarily due to two reasons. First, the office of the controller is so relatively new that the duties of the office have not had time to become fixed as have the duties of other officers. Second, in many companies the duties assigned to the controller have depended largely upon the capabilities of the individual controller in the particular company. A comparison of the earliest list of duties of the controller with the latest list shows the change that has occurred in the concept of controllership.

The Institute's first statement of the responsibilities of the controller consisted of seventeen points as follows:

1. The installation and interpretation of all accounting records of the corporation.
2. The preparation and interpretation of the financial statements and reports of the corporation.
3. The continuous audit of all accounts and records of the corporation, wherever located.
4. The compilation of production costs.
5. The compilation of costs of distribution.
6. The taking and costing of all physical inventories.
7. The preparation and filing of tax returns and the supervision of all matters relating to taxes.
8. The preparation and interpretation of all statistical records and reports of the corporation.

9. The preparation, as budget director, in conjunction with other officers and department heads, of an annual budget covering all activities of the corporation, for submission to the board of directors prior to the beginning of the fiscal year. The authority of the Controller, with respect to the veto of commitments or expenditures not authorized by the budget, shall, from time to time, be fixed by the board of directors.

10. The ascertainment currently that the properties of the corporation are properly and adequately insured.

11. The initiation, preparation and issuance of standard practices relating to all accounting matters and procedures and the coordination of systems throughout the corporation, including clerical and office methods, records, reports, and procedures.

12. The maintenance of adequate records of authorized appropriations and the determination that all sums expended pursuant thereto are properly accounted for.

13. The ascertainment currently that financial transactions covered by minutes of the Board of Directors and/or the Executive Committee are properly executed and recorded.

14. The maintenance of adequate records of all contracts and leases.

15. The approval for payment (and/or countersigning) of all checks, promissory notes, and other negotiable instruments of the corporation which have been signed by the treasurer or such other officers as shall have been authorized by the by-laws of the corporation or from time to time designated by the board of directors.

16. The examination of all warrants for the withdrawal of securities from the vaults of the corporation and the determination that such withdrawals are made in conformity with the by-laws and/or regulations established from time to time by the board of directors.

17. The preparation or approval of the regulations or standard practices required to assure compliance with orders or regulations issued by duly constituted governmental agencies.[12]

Careful examination of the above listed responsibilities reveals that they deal almost exclusively with the firm's accounting records. As the concept of controllership developed and broadened to include advisory services to

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[12]John H. MacDonald, Controllership: Its Functions and Techniques (New York, New York: Controllers Institute of America, 1940), p. 7.

management, the need for a restatement of the controller's duties became evident. Although supervision of the accounting records is a part of controllership, it is by no means the only part and perhaps not even the most significant part.

In recognition of the newer and enlarged role of the controller in the management function, the Institute revised its list of the controller's responsibilities. The Institute's present concept of the function of controllership is as follows:

> 1. Planning for Control. To establish, coordinate, and administer, as an integral part of management, an adequate plan for the control of operations. Such a plan would provide, to the extent required in the business, profit planning, programs for capital investing and for financing, sales forecasts, expense budgets and cost standards, together with the necessary procedures to effectuate the plan.
> 2. Reporting and Interpreting. To compare performance with operating plans and standards, and to report and interpret the results of operations to all levels of management and to the owners of the business. This function includes the formulation of accounting policy, the coordination of systems and procedures, the preparation of operating data and of special reports as required.
> 3. Evaluating and Consulting. To consult with all segments of management responsible for policy or action concerning any phase of the operation of the business as it relates to the attainment of objectives and the effectiveness of policies, organization structure and procedures.
> 4. Tax Administration. To establish and administer tax policies and procedures.
> 5. Government Reporting. To supervise or coordinate the preparation of reports to government agencies.
> 6. Protection of assets. To assure protection for the assets of the business through internal control, internal auditing, and assuring proper insurance coverage.
> 7. Economic Appraisal. To continuously appraise

> economic and social forces and government influences, and to interpret their effect upon the business.[13]

A perusal of this most recent statement on the functions of controllership shows that the job is viewed as an important part of the management function. A comparison of early and late statements on the functions of controllership shows that controllership has expanded beyond the traditional duties of recording transactions, auditing accounts, authorizing payments, and issuing statements, and has become an important part of top management. Credit for this enlarged role must be attributed mainly to the influence of the Financial Executives Institute.

Influence of the Securities and Exchange Commission

Following the infamous McKesson and Robbins hearings in 1939, the attention of the accounting profession, government agencies, stock exchanges, and the investment community was focused sharply on the question of who was mainly responsible for the published financial reports of a business firm. After a lengthy hearing, involving hundreds of pages of testimony from numerous expert witnesses, various members of the Securities and Exchange Commission stated publicly that they looked primarily to corporate management for accuracy and completeness in the reports of listed companies,

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[13]Alexander L. Stott, "Controllership in Action," Financial Executive, Volume XXXII (September, 1964), p. 31.

and that they considered the certificate of the independent public accountant as a necessary but secondary check which does not in any way relieve management of its primary responsibility.[14]

Placing final and full responsibility for the correctness and completeness of accounting reports upon the shoulders of management meant, of course, placing it upon the shoulders of the controllers. The truth of this statement is evident in an address delivered by Commissioner Robert E. Healy of the Securities and Exchange Commission. Speaking before the Financial Executives Institute at Cleveland on May 15, 1939, Commissioner Healy said:

> But in a broader sense, it cannot be denied that the Controller is the man who holds the key to sound corporate accounting. It is his system upon which adequate corporate reporting ultimately rests. The auditor, of course, plays a significant role. But he is only the periodical check-up man.
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> . . . .
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> What we need, it seems to me, is a return to the recognition that the primary responsibility for proper accounting rests on the corporate management in the person of the Controller. Whether the books are audited or not, the stockholder has a right to look to the corporation's own accounting system for an adequate, intelligible, and honest reporting of its affairs.[15]

Such a statement from an authoritative source focused the attention of management and the public on the

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[14]David R. Anderson, "The Function of Industrial Controllership," The Accounting Review, Volume XIX (January, 1944), p. 59.

[15]Editorial, The Journal of Accountancy, Volume LXVIII (July, 1939), p. 2.

importance of the controllership function. These statements brought recognition, prestige, and above all, responsibility to the office of the controller. The influence of the Securities and Exchange Commission has been most significant in the growth and development of the office of the controller.

Influence of the New York Stock Exchange

At the same time that the Securities and Exchange Commission was conducting its study on the question of who was responsible for a firm's financial statements, stock exchanges were making similar studies.

The Committee on Stock List, New York Stock Exchange, appointed a Subcommittee on Independent Audits and Audit Procedure, and charged the members of the committee with the responsibility of reviewing developments and trends of thought in auditing matters with special reference to the independent audits required of listed companies by the Exchange. Although the Subcommittee's investigation and recommendations were primarily concerned with the independent audit, its report contained one recommendation of particular interest to controllers. Recognizing that independent audits cannot under complex business conditions give conclusive assurance against all possibilities of error and fraud, and that primary responsibility for the accuracy of accounting statements lies with corporate management, the report submitted by the Subcommittee recommended in part that:

> More emphasis should be placed on the responsibility of the Controller and the assurance to him of adequate authority and facilities. The scope of his responsibilities should be fixed by the board of directors, and he should report periodically to them, in addition to making his customary reports to the operating management.[16]

The Subcommittee's report was adopted on August 23, 1939, by the Board of Governors, New York Stock Exchange, and added one more significant step in the growth and development of the controllership function.

Influence of Other Government Agencies

At various times governmental agencies have asked controllers to come to Washington to render expert testimony on matters of national interest. For example, the Director of Price Administration and members of the War Production Board called upon controllers for advice and consultation during World War II. The Secretary of the Treasury almost always asks controllers for their opinions on proposed tax laws. The Director of the Budget is a frequent solicitor of counsel from controllers. There may be a question as to whether government requests for services is more of a contribution to growth and development or a result of growth and development. However, it is certain that such requests gave prestige and confidence to controllers and assured them

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[16]Report of Subcommittee on Independent Audits and Audit Procedures, Committee on Stock List, New York Stock Exchange, The Journal of Accountancy, Volume LXVIII (October, 1939), p. 242.

that they were highly respected members of the business community.

Influence of Increasing Complexity of Business

One of the most important factors influencing the growth and development of controllership has been the increasing complexity of business. In early American history the business community was dominated by small business firms where one man was both owner and manager. This person either performed all functions of the business or personally supervised their performance. As business firms grew in size, there was a separation of ownership and management, with the latter becoming specialists in their functions. There was also a division and specialization of labor at all other levels of activity.

Although small business firms are still significant in terms of their total number, they account for only a small percentage of business transactions in terms of dollar volume. It has been estimated that seventy-five percent of the business receipts for goods and services go to corporations.[17] Many of these corporations have multiple plants, multiple lines, are diversified, and vertically integrated. These complex business organizations and operations mean

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[17]Raymond E. Glos and Harold A. Baker, Introduction to Business, third edition (Cincinnati, Ohio: Southwestern Publishing Company, 1955), p. 65.

that aid to management has become the most pressing need in the business community. Management can no longer plan on the basis of observation only nor control on the basis of personal supervision only.

Much of the aid required by the management of a complex business organization is in the form of financial data that has been properly analyzed, interpreted, and presented. Since the controller is the one who accumulates the financial data, it is logical for management to turn to him for analysis, interpretation, and recommendations. In those firms where the controller has been effective in analyzing and interpreting financial data and in making recommendations thereon, he has truly become an indispensable part of top management.

Position of the Controller in the Organizational Chart

According to the evidence available to this writer, there seems to be no exact location where controllers are placed in an organizational chart. This may not be so unfortunate as it first appears. It may be better to fit an organization to the abilities of the individuals concerned than to try to mold people to an organizational plan. In the opinion of this writer, duties, responsibilities, and authority should be delegated to a controller in accordance with the talents and capabilities of the particular man.

However, where the controller participates in the functions of top management and effectively performs all the duties which the Financial Executives Institute expects a controller to perform, he should be given a rank commensurate with his functions, responsibilities, and performance. In the opinion of this writer, such a rank should be at the policy making level, probably a vice-president. As the concept of controllership has grown and developed, there has been an increase in the number of controllers holding the rank of vice-president.

Summary

In this chapter, the growth and development of the controllership function was divided into two periods--the period preceding 1931 and the period following 1931.

Prior to 1931, there was very little organized efforts among controllers to guide their development. Controllers were found in the railroad industry and in the department store industry, although their functions in these industries were dissimilar. In the railroad companies, controllers were primarily concerned with the issuance and cancellation of stocks and bonds. In department stores, controllers participated in making decisions relating to inventories, pricing, budgeting, and other management problems.

Following 1931, growth and development of the controllership function increased rapidly. The increase was due primarily to the influence of the Financial Executives

Institute, formerly the Controllers Institute of America. From its inception in 1931, the objective of the Financial Executives Institute has been to educate businessmen, the public, and controllers in an understanding of the concept of controllership. This objective has been achieved through educational programs which included lectures, discussions, conferences, and publications. A most notable achievement of the Financial Executives Institute has been the formulation of a seven-point statement on the concept of controllership.

The Securities and Exchange Commission has also been influential in the growth and development of controllership. Following the McKesson and Robbins Case of 1939, various members of the Securities and Exchange Commission stated publicly that controllers were the key to sound corporate accounting.

At the same time that the Securities and Exchange Commissioners were emphasizing the role of the controller, stock exchange officials were making similar statements concerning controllership. In 1939, the Board of Governors of the New York Stock Exchange adopted a committee report which stated that controllers should have greater authority and facilities, and that more emphasis should be placed on their responsibilities.

Many government agencies have also had an influence on the growth and development of controllership. The

Director of the Budget, the Secretary of the Treasury, the Commissioner of Internal Revenue, the Director of Price Administration and other governmental officials have often asked controllers for advice on important matters.

One of the most important factors affecting the growth and development of controllership has been the increasing complexity of business. The modern day corporation is a complex business organization. To properly manage these complex business organizations, management must have financial data that has been correctly analyzed, interpreted, and presented. It has been logical for management to turn to controllership for assistance in problems of financial management.

The chapter was concluded with a discussion of the position of the controller in the organizational chart. In this writer's opinion, the controller should hold a rank at the policy making level.

CHAPTER III

FUNCTIONS OF AND REQUISITES FOR CONTROLLERSHIP

The Functions

Although there are general characteristics common to the position of controller in virtually all companies, the specific duties which he performs vary considerably from firm to firm, depending largely upon the abilities of the individual controller, as suggested in the previous chapter. However, his functions generally can be classified in three categories: general accounting, control, and planning. These aspects are so closely interrelated that it is somewhat difficult to discuss one without discussing the others. Therefore, the following discussion is not intended to suggest independence among the functions. The subdivisions are used to facilitate the orderly presentation of the subject matter.

General Accounting Function

Perhaps the oldest and best known duties of the controller have been those of chief accounting officer of a corporation. A reference to the Financial Executives Institute's first statement on the functions of controllership (page 15) shows that at that time (1939) the controller was concerned almost entirely with the accounting function. Even in the Institute's latest seven-point Concept of Controllership, published in 1959, (Chapter II, page 17) four of the seven points relate to general accounting.

The controller is required to assist management in reporting the results of operations to the owners of the business, in establishing and administering tax policies and procedures, in supervising preparation of reports to government, and in protecting the assets through adequate internal control, internal auditing, and insurance coverage. The performance of these traditional duties is basic and indispensable to the operation of a modern corporation, and it is not the purpose of this section to minimize their importance. Neither is it the purpose of this section to describe in detail the nature of the accounting function. The purpose of this section is to place the general accounting service in proper perspective. The need for a balanced perspective on general accounting is discussed in the following paragraph.

Although it is fundamental and indispensable to the operation of a business, the general accounting function is not the motivating force for controllership. Controllership's greatest opportunities for service are found in the area of assisting management in its planning and control functions. However, it appears that many controllers are neglecting planning and control. For example, some management consultants report that in their contacts with business firms they find that many firms have created executive positions which are responsible for planning and control. These executives often carry the title, "Manager of Corporate Planning and Control." It is with these executives, and not the controller, that management consultants often work.[1]

It is not illogical for one to suspect that in those cases where the Controller has been excluded from planning and control activities, his exclusion has been caused largely by his preoccupation with general accounting duties and his lack of time for adequately preparing himself for participation in planning and control. If this is indeed the case, it is sufficient reason for emphasizing the necessity of placing the general accounting function in its proper perspective.

In order that his role in planning and control will not be restricted by an over-emphasis on the general accounting function, it seems to the writer that the controller must take two steps in discharging his obligations as the firm's chief accounting officer. First, the controller must delegate to staff members such duties as recording transactions, authorizing payments, auditing accounts, issuing reports, and countless other record-keeping and related duties.

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[1]John T. Garrity, "Is the Role of the Controller Shrinking," The Controller, Volume XXIX (May, 1961), p. 220.

Sometimes, controllers who have spent years in auditing, taxes, and accounting are reluctant to delegate responsibility for those phases of accounting which they have long performed and which they may consider as the most important functions in the firm. The controller can also see the immediate results of his accounting activities, while the results of his time and energy devoted to planning and control are long-term results not immediately apparent. However, delegation of responsibility for maintaining routine, historical accounting records is an essential step for the controller who would fulfill his responsibilities in planning and control.

The second step necessary for the successful controller is one that may require him to look to public accounting firms and the academic ranks for answers to technical problems in the area of financial accounting and reporting. This may be true particularly for the small and medium-size firms. The controller is a businessman primarily interested in accounting as a managerial tool. He is not an accounting theorist.[2] The controller cannot, for example, afford to spend his time in studying, debating, and arguing the numerous questions surrounding the problem of profit determination. Public accounting firms, on the other

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[2]James L. Peirce, "Controllership and Accounting: A Contrast," <u>The Controller</u>, Volume XXI (September, 1953), p. 410.

hand, have, through years of experience and research, and with the aid of the academic profession, achieved considerable success in solving the problems of financial accounting. The opinions of members of public accounting firms are considered authoritative and objective by large segments of business and industry. Therefore, controllership in small and medium size firms should turn with confidence to public accounting firms for answers to questions relating to the reporting of periodic profits and financial position.

The versatile staffs of public accounting firms have also enabled them to assume leadership in the field of tax practice as well as in accounting theory. Although the controller is concerned with the tax implications of each managerial decision, he cannot afford to devote his time to routine tax reporting or to negotiations with tax authorities. Where the controller's staff is unable to handle tax problems, as may easily be the case in small and medium size firms, he should rely on the tax consulting service available through public accounting firms.[3]

To summarize, two steps are essential if the controller would not let an overload of general accounting activities endanger his planning and control functions. If the controller delegates routine accounting duties to

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[3]James L. Peirce, "Controllership and Accounting: A Contrast," The Controller, Volume XXI (September, 1953), p. 410.

subordinate staff members, and relies upon public accounting firms for advice in matters of financial and tax accounting, he will find himself free to participate fully in the most rewarding functions of controllership, namely, planning and control.

Control Function

In a field where principles have been developed to the extent achieved in business management, one would expect to find the terminology unambiguous. Yet, an examination of management literature reveals a lack of agreement among the writers as to the precise meaning of the term "control."

Definitions for the term "control" usually fall in one of two groups. Typical of one group is the definition given by Koontz and O'Donnell. They write:

> The Managerial function of control is the measurement and correction of the performance of subordinates in order to make sure that enterprise objectives and the plans devised to attain them are accomplished.[4]

According to this definition, control is concerned with both measuring and correcting. Koontz and O'Donnell are in agreement with Sherwin, who writes that control involves checking to see whether progress toward the objectives is being made, and acting, if necessary, to correct any deviations from the plan.[5]

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[4]Harold Koontz and Cyril O'Donnell, Principles of Management, third edition (New York: McGraw-Hill Book Company, 1964), p. 537.

[5]Douglas S. Sherwin, "The Meaning of Control," Dun's

Koontz and O'Donnell's measuring and correcting, and Sherwin's checking and acting, are characteristics often found in the definitions of control. A different meaning, however, is found in the term as used by Marshall E. Dimock. Dimock writes:

> Control is the analysis of present performance, in the light of fixed goals and standards, in order to determine the extent to which accomplishment measures up to executive orders and expectations. . . . The most concise description is to call it the checkup.[6]

Dimock uses the term to mean analyzing, measuring, checking. Nothing is said of correcting or acting.

Throughout this study the term "control" will be assumed to signify both measurement and correction. It is within this context that the role of the controller in the managerial function of control will be examined.

Control implies planning. Objectives and plans for achieving those objectives must be formulated before control can be exercised. After plans have been made, controlling those plans includes two elements. First, actual performance is measured against those plans and any differences between the two are reported and analyzed. Second, corrective action is taken, where necessary, to bring performance up to the plan or, in some cases, to alter the plan. In each of these--planning, measuring, and correcting--the controller

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Review and Modern Industry, Volume LXVII (January, 1956), p. 46.

[6]Marshall E. Dimock, The Executive in Action (New York: Harpers and Brothers Publishers, 1945), p. 217.

has an important function to perform, even though it is management who is finally responsible for the entire planning and control process.

The controller's role in planning, that is, formulating objectives and plans for accomplishing them, will be reserved for a subsequent section. It should be noted here, however, that the establishment of plans and the measurement of performance against those plans are closely related and intermingled. It is reasonable to assume that involvement by the controller in the making of plans sets the stage for more efficient performance by the controller in comparing actual results against those plans.

The controller's role in the first step in the managerial function of control, that of measuring performance, requires that he emphasize accuracy, correctness, and timeliness. The second point of the Financial Executives Institute's seven-point Concept of Controllership charges the controller with responsibility for comparing performance with operating plans and with reporting and interpreting the results of operations to all levels of management. The importance of accuracy and correctness in this measuring, reporting, and interpreting function is discussed below.

Although responsibility for taking corrective action lies with line management, the basis for their decisions and action is information. Although information of a non-quantitative nature is extremely important, a great deal of the data utilized by line management in making decisions is information quantified and interpreted by the controller. Since the information supplied by the controller becomes so significant in the control process, it is very important that he emphasize accuracy and correctness in the accumulation and reporting of information.

The office of the controller becomes a center for the accumulation of data. As a provider of information, the controller's work is composed of three phases. First, data must be gathered and analyzed. This results from comparing actual operations with planned operations. The comparisons may be made through detailed analysis. Consultations may be held with line officers for purposes of discussing and comparing actual and planned performance. A great part of the information, however, is the product of statistical analysis of operations.

For example, the controller can develop valuable information for control purposes by statistically analyzing the variance between planned employment of labor and actual employment of labor, or between planned consumption of materials and actual consumption of materials. He can aid management by analyzing relationships between categories of expenses and sales, or between profits and sales, or between profits and invested capital. The controller may wish to construct Index numbers, such as a Cost of Goods Sold Index, and observe their movements over time. Such statistical analyses may well be the controller's most vital contribution to managerial control.

Second, after gathering and analyzing data, the controller must report his findings to management. If the basis of control is information in the hands of management, then reporting is elevated to a level of considerable importance. Yet, the preparation of reports is a frequently neglected area of responsibility by controllers.

Many managers have a dislike for rows and columns of statistical data, and in those cases the Controller's statistical reports, although accurate, correct, and informative, may not receive proper attention. The fluent, articulate controller may achieve greater success in those cases through a verbal presentation of his analyses during staff meetings, conferences, and personal visits with the manager.

Whether his reports are presented in oral or written form, or both, the controller should observe all principles of good communications. His reports should not be so long as to remain unread nor so short as to leave gaps. He should exclude extraneous, irrelevant material, but include all known, essential facts. His written reports should be attractive, which means that charts and graphs may sometimes be preferred over statistical tables.

The controller who has long worked with external reports may fail to realize the distinct difference between preparing internal and external reports. In preparing his control reports the controller's only objective is to prepare reports which will be useful in taking corrective action. Since there is no danger of public liability in internal reporting, the restrictive influence of the criteria of conservatism, consistency, and objectivity give way to the criterion of usefulness.

In the reporting process, the controller is deeply concerned with the timeliness of his reports. Control, by its very nature, is an after-the-fact process. Performance is followed by measurement and analysis, which becomes the basis for corrective action. The controller's objective should be to minimize the time lag between performance and corrective action.

In fact, there may be occasions when a less accurate, early report would be preferred for control purposes over a later, more highly accurate report. The controller must be aware of those occasions, and when they arise, he must be willing to sacrifice detail for timeliness in reporting.

Third, in addition to measuring operations and preparing reports on his findings, the controller also has the responsibility of interpreting the results of operations to management. This function of controllership is written into the second point of the Financial Executives Institute's seven-point Concept of Controllership.

The authority to interpret always carries with it a heavy responsibility. The power to interpret is very close to the power to act. The controller who supports his interpretations of operations with reasons, well-founded, well-organized, and competently presented, may make the corrective action so obvious that it may appear that he, the controller, is taking the corrective action. Taking corrective action, though, is a function of line management. Therefore, in order not to be misunderstood, the controller must exercise care and caution in the presentation of his interpretation of operations.

The controller's role in the second step in the managerial function of control, taking corrective action, requires that he exercise his duties with extreme caution. In all his functions, particularly where action is being taken, the controller must remember that his position is a staff position and that his function is a service function. He cannot issue communications which give the appearance of demanding that specific corrective actions be taken. Taking corrective action is a line responsibility. Probably, the best way for the controller to lose the confidence and respect of the people he serves is for him to give the semblance of usurping the power and interfering with the performance of operating line executives.

The controller's proper role in this second phase of control, taking remedial action, is fully explained in the third point of the Financial Executives Institute's seven-point Concept of Controllership. A quick reference to that point will remind the controller that he is to consult (emphasis mine) with all levels of management responsible for policy or action relating to all phases of operations. When the controller goes beyond consulting, he goes beyond the proper boundaries of his office. When he goes beyond the boundaries of his office in matters of corrective action, he increases the probabilities that line officers may not accept as completely objective the controller's findings in the first step of control, that of measuring performance. In such cases, the controller will have created in the minds of operating executives a fear that the controller's findings in measuring performance will have been biased in order to support his desires in matters of corrective actions. Therefore, the guiding rule for the controller is to consult only, and to do that with care and caution.

If the controller is correct and timely in measuring and reporting the results of operations, if he is careful and cautious in recommending and consulting in questions of corrective action, the controller will have effectively participated in one of his most vital roles--that of control.

Planning Function

One of the major developments in the field of management during the past decade has been an increasing preoccupation with planning by business, government, schools, and other institutions. Newman writes that "planning is deciding in advance what is to be done. That is a plan is a projected course of action."[7] A similar definition is given by Anderson, who writes: "To plan is to propose a forward program for guiding the future affairs of an enterprise."[8]

The essence of planning, then, is the determination of what should be done, how it should be done, where action should be taken, who should be responsible for it, and why. Viewed in this light, planning becomes a challenging, intellectual process, replete with opportunities for creative thinking. Participation in this elevated, managerial planning function is the controller's newest and greatest opportunity to render a service to management.

Although informal, loose, haphazard planning is as old as man himself, the function as practiced today is one of the newest managerial activities. Planning is considered new only because it is conducted on a more formal, systematic, organized basis. This new approach employs new methods, techniques, and tools, many of which are statistical in nature. Because of the variety of new quantitative techniques and the newer integrated approach to planning, the planning function has become too large to be performed by one man alone. The chief executive who is responsible for planning

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[7]William H. Newman, Administrative Action: The Techniques of Organization and Management, second edition, (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1963), p. 15.

[8]Richard C. Anderson, Management Practices (New York: McGraw-Hill Book Company, Inc., 1960), p. 13.

must seek advice, counsel, and assistance from many persons in the organization. One of those persons is the controller.

The controller's role in the planning function is discussed in the following sentences. As stated in the section on the control function, planning and control are closely interrelated. In fact, planning is a prerequisite to control. Without plans there is nothing to control. In other words, plans prepare the way for control, and control often results in a change of plans. The two are inextricably woven. If the controller is to be a key figure in the control function, as suggested in the preceding section, then it is logical for one to conclude that the controller should also be a key man in the planning function.

In addition to facilitating control, planning offers other advantages. It unifies interdepartmental activities, substitutes deliberate decisions for snap judgments, and forces management to turn from immediate, pressing, daily problems to a consideration of the future. It concentrates management's attention on the company's objectives, and offsets change and uncertainty by anticipating them. Without planning, management decisions become random choices.

In dealing with the planning function most writers break down the process between three to eight steps. Any such outline of steps is arbitrary and probably oversimplified; however, perhaps it is necessary for presentation purposes. Therefore, the remaining part of this section will use an arbitrary, four-step breakdown, giving special attention to the controller's role in each step.

Establishment of Objectives

The first step in the planning function is the establishment of objectives for the entire enterprise. Without goals and objectives, the remaining steps in the planning process will be disorganized, uncoordinated, and sometimes even contradictory.

Traditionally, profit making has been declared the objective of a privately owned business firm in a free enterprise economy. In recent decades many, many volumes have been written on the profit motive, some defending it, others attacking it as against the public interest. Although the pursuit of profit is an essential, common element in all business firms, it does not seem correct to say that it is the only objective of business. There is no apparent reason why a firm cannot have multiple objectives, the same as an individual may have multiple objectives in planning his life.

The problem, then, is not to establish one and only one objective for the firm, but rather to maintain a proper balance among numerous goals. The contribution of the controller in this step of establishing objectives depends upon his ability to see them in their relationship to one another and to communicate his ideas on these relationships to other members of management. The following is an example of different objectives which might be uppermost in the minds of different officials.

In a business firm there may be officers whose primary concern is employee welfare. To them, all planning should be directed toward improving the employees' well-being. It is possible that such an objective could be so predominant as to be detrimental to the long-run financial health of the firm. In a discussion of employee welfare, profits must not be disregarded. Neither should employee welfare be subordinated in the search for profits.

Others in the same firm may be inordinately concerned with customer service or stockholder welfare. Some members of management may be chiefly concerned with their own job security. While the objective of certain officers might be to obtain a larger share of the market than their competitors, the aim of others may be to keep the business small, simple, and friendly, even if it means inefficiency and less profit.

In a group discussion of enterprise objectives various members of the management team may vigorously present their opinions on the proper objectives for the firm. In such cases someone in management must speak for balance among objectives. If the controller is one who insists on reasonable emphasis on many objectives and undue emphasis on none, then he will have made a worthy contribution in establishing unanimity and balance.

At later points in this study the reader may want to know what objectives have been assumed for this study. Since profit is the essential, common element in all firms, and since the objectives of employee welfare, customer service, public relations, management security, and others, vary from management to management, the reader may assume, wherever an assumption is needed, that the chief interest of the firm is profit making.

Forecast of Environment

The second step in the planning function involves a forecast of the environment in which the firm will operate. It means that assumptions are made concerning the future. Koontz and O'Donnell refer to these predictions as "planning premises."[9]

Planning premises are made with reference to such topics as population growth, future price levels, political environment, and business cycles. Premises on changes in tax rates are made. Whether the nation will be in war or peace, whether or not there will be shortages of raw materials and supplies, are important issues to planners. Premises relating to the action of competitors must be made. Planning is affected by so many external and internal considerations, that it is impossible to make premises or

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[9]Harold Koontz and Cyril O'Donnell, Principles of Management, third edition (New York: McGraw-Hill Book Company, 1964), p. 82.

assumptions for all of them. Therefore, management will have done well if it does no more than make correct decisions on the most critical and strategic considerations.

The importance of accurate premises and assumptions can be easily seen. Plans which have been made on the premise that peace will prevail may have to be discarded, or at least altered, in case of war. Plans which provide for profits on the premise of steady, stable prices may result in losses during a period of changing price levels.

The importance of premises is so great that it is often advantageous to develop plans under alternative premises. For example, management may assume that its nearest competitor will follow one of two courses of action. Therefore, it will develop two different sets of plans for the two different premises, and let future events dictate their use.

The relevant question here, however, is whether or not the controller has a role in the formulation of premises. Reference to the last point in the Financial Executives Institute's seven-point Concept of Controllership shows that the controller must be able to appraise economic forces, social trends, and government influences, and to interpret their effects upon the business. This is a heavy responsibility, and perhaps the most difficult of the controllership assignments. It definitely charges the controller to study the economic, social, and political environment in which managerial plans are fulfilled.

In predicting the future environment the controller should be helped greatly by the use of statistical methods. For example, the controller may be able to correlate the sales of his firm's product with some known factors and then through the use of correlation analysis he can make valid premises on demand. This premise is fundamental to all other plans.

He may also employ methods of trend analysis to make premises on population growth and on general economic growth, In establishing premises relating to actions of his competitors he may utilize probability statistics and game theory. The use of quantitative analysis is discussed in Chapter IV.

However, the controller cannot rely entirely on quantitative analyses. Many of his premises will be based solely on qualitative value judgments. War or peace, legislative regulations affecting business, innovations that completely change the production process or the product market, are three examples of areas in which premises must be made and for which there are no helpful statistical methods. In such cases the accuracy of the controller's value judgments determines the worth of his contributions to the second step of the planning process.

Determination and Evaluation of Alternative Courses of Action

The third step in the planning function includes the determination and evaluation of alternative courses of action which, under the premises established in step two, will achieve the objectives formulated in step one. This is the core of planning. In this step the controller plays his most significant role.

It may be incorrect to assume that the role of the controller in establishing objectives and premises is any different from the role of any other member of the management team. In evaluating alternative courses of action, however, the controller undoubtedly occupies a unique position.

A considerable portion of the work in evaluating alternatives involves an analysis of financial data. Costs under various alternatives are analyzed and compared. Anticipated revenues of competing alternatives are studied. The effects of various alternatives on fixed capital and working capital needs are analyzed. Consideration is given to cash flows for each plan. Projected Balance Sheets and Operating Statements are prepared for each alternative. Such financial analyses are familiar to controllers. No other member of management has the experience with financial data that the controller has. Therefore, it is logical to conclude that the controller's position is unique in the step of evaluating alternative courses of action.

In this step of the planning function the controller will again make use of quantitative methods. For example, in planning for the optimum combination of limited resources to obtain a desired goal, the controller may employ the method of linear programming. If there are plans to reduce the time that customers wait in line for service, queuing theory may be useful. Program Evaluation and Review Techniques may be helpful in planning a project composed of distinct stages of work which can be started at various dates. In choosing among investment opportunities the controller may utilize capital budgeting techniques. Break-even analysis may help him to determine the effects on profits of a planned change in volume.

However, the controller cannot rely entirely on quantitative methods in evaluating alternatives. If all the variables in a plan could be quantified and precisely measured, then all that would be needed to evaluate that plan would be a statistician and a computer. But such is not the case. Plans are replete with uncertainties and intangible factors which make relatively simple problems difficult to evaluate. In such cases management must fall back on potentially fallible personal judgments. The controller whose judgments are sound and accurate, and who is proficient in methods of quantitative analysis, will prove himself indispensable in evaluating alternative courses of action.

Adoption of the Plan

The fourth and final step in the planning function is adoption of the plan. At this point the controller may suggest or advise that one plan be adopted over another. The decision, however, is the responsibility of the chief executive.

In closing this topic, a parallel may be drawn between the controller's participation in the control function and his participation in the planning function. In the control function the controller measures actual performance against planned performance and reports any deviations he finds. The decision to take corrective action, however, is the responsibility of the chief executive. In the planning function the controller evaluates alternative plans and reports his findings. The decision to adopt one plan over another, however, is the responsibility of the chief executive.

The Requisites

Throughout the history of controllership the functions of the office have varied widely from firm to firm, as suggested in the preceding chapter. This disparity may have been due in large measure to the differences in controllers--differences in their training, differences in their abilities, and differences in their personal characteristics. It is appropriate, therefore, to briefly consider at this point the training and personal qualities which characterize the successful controller.

Specialized Training

Seldom does one read an article criticizing controllers for a lack of specialized training. As a group, controllers are adequately prepared in technical subjects. Reference to the Financial Executives Institute's seven-point Concept of Controllership reminds one that several of the functions of the controller require that he have a considerable measure of specialized training. For example, the controller is responsible for the internal control system and the internal audit activities in the firm. He must establish and administer tax procedures. In addition, he supervises the preparation of special reports to government agencies. He is also responsible for the ledgers, journals, subsidiary records, and original documents which must be maintained in order to report periodic profits and financial condition to outsiders. In other words, the entire general accounting function, which was discussed earlier in this chapter, requires a high degree of proficiency in accounting principles and procedures.

In more recent years controllers have also seen a need for specialized training in statistical analysis. As a participant in managerial function of control, he utilizes such statistical methods as sampling techniques, quality control techniques, variance analysis, and ratio analysis. As a participant in the managerial function of planning, he employs such statistical methods as linear programming, game theory, probability theory, program evaluation and review techniques, break-even analysis, budgeting and capital budgeting techniques. The fact that extensive use of statistical analysis is relatively new is sufficient reason for suspecting that the controller's specialized training in statistics is less adequate than it is in accounting.

One other area in which the controller should find specialized knowledge helpful is in the area of computer applications. Computers are used in processing accounting data and in making the mathematical computations involved in statistical analyses. In fact, some statistical methods, although simple in concept, require such large volumes of arithmetical calculations that their use is impractical without the aid of a computer. A study of these three areas --accounting principles, statistical methods, and computer applications--should prepare the controller to perform the specialized aspects of his job effectively.

Technical competence, however, implies more than familiarity with complicated, complex techniques. It also implies judgment in applying those techniques to basic, practical problems. Sound judgments and an understanding of basic problems are not necessarily the product of a specialized education alone. They are more likely to develop from a broad, liberal education. Therefore, it is logical to conclude that the controller's specialized training should be built on a good foundation of general education.

General Education

A careful reading of the Financial Executives Institute's seven-point Concept of Controllership also reflects that the concept cannot be fully achieved by a controller with specialized training only. For example, point seven of that concept requires the controller to appraise the influence of economic, social, and government forces on business. This requires the controller to have a knowledge of economic theory, economic history, economic institutions, and economic systems. He should know something of sociology and psychology. A study of political history and political systems would be helpful. In other words, the controller needs to be a keen student of human nature. This kind of knowledge is gained from a study of the humanities and social sciences. Its purpose is to give the controller an understanding of the environment in which he and his firm operate.

Aside from point seven, there are other reasons for insisting that the controller's specialized training be supplemented by a broad, general education. As a member of the management team, the controller must see each part of the firm in relation to the whole, and then see each of the parts in relation to each other. That is, he must see the production department as a small part of the entire firm, the accounting department as a small part of the entire firm, and finally, see production and accounting as they relate to each other.

Such balanced views are not fostered by more and more technical training in either production or accounting. They are the outgrowth of a broad study of business in general. Therefore, the controller's training should include a study of marketing, transportation, industrial management, personnel management, business law, finance, and all other areas of business administration.

Perhaps the most important contribution of general education is in the area of communications. The ability to communicate is essential to the successful controller. The value of accounting data depends largely upon effective communications between accountants and non-accountants. The results of sophisticated, statistical analyses may be of no value unless properly communicated to non-statisticians. Therefore, it is not illogical to conclude that the controller's training in English composition and public speaking may be the most significant elements of his general education.

To summarize, general education enables the controller to view accounting as a small part of the total business firm. It enables him to see the business firm as a small part of the total world environment. It also enables him to communicate effectively with other men. General education provides a good educational foundation on which to build technical ability.

Personal Characteristics

Completion of the most highly specialized of all technical training programs, plus completion of the broadest of all general education programs, will not assure success for the controller. A well-educated, well-trained controller may fail in his functions because of a lack of necessary personal characteristics.

Since he works with people as well as figures, the controller should be an expert in the art of human relations. His temperament and emotional stability must withstand the wear and tear of give-and-take encounters with fellow workers. At times he must take the initiative in dealing with another member of the firm. For example, it may be necessary for him to discuss a variance analysis with a foreman. In such cases he must be diplomatic and tactful in his approach, fair in his assessment of the situation, and, above all, tolerant and understanding of the foreman's explanation for the variance.

At other times fellow workers will come to him with their problems. In such cases the controller must demonstrate interest, understanding, and willingness to help. He must be one on whom people can rely for assistance. His position demands that he be cooperative, friendly, courteous, practical, and enthusiastic. He must be confident in himself and instill confidence in others. This means that he completes his duties in as relaxed atmosphere as possible. The tense, impatient controller will not give the appearance of being in full command of his job. He will find his work easier if he also possesses a sense of humor. In short, he must enjoy people and have a genuine concern for their problems, their successes, and their reverses.

A controller with the specialized training, general education, and personal qualities described above would, of course, be an ideal controller. The ideal controller, like ideal workers of all kinds, is seldom, if ever, found. However, an occasional look at the blueprint for one should be helpful to everyone.

Summary

The functions of controllership were divided into three groups: general accounting, control, and planning. General accounting is the oldest of the controller's three functions. His general accounting duties includes the reporting of results of operations, establishing and administering tax policies and procedures, supervising preparation of reports to government, and protecting the assets through adequate internal control, internal auditing, and insurance coverage.

Although general accounting is necessary, it was observed that the controller should not let his traditional accounting duties prevent him from participating fully in planning and control. In order that the controller may have adequate time for planning and control, it was suggested that he should delegate most of his general accounting duties to his staff.

In the managerial function of control, actual performance is measured against planned performance and any needed corrective action is taken. The controller has a significant role in the control function since a large part of the data utilized in the control function are financial data. The controller accumulates the data, analyzes and interprets it, and presents it to management for corrective action.

The managerial function of planning implies that a projected course of action has been made. Participation in managerial planning is the controller's newest opportunity to render a service to management. Within the planning function, the controller should assist in establishing objectives, forecasting the environment, and determining and evaluating alternative courses of action. Adopting the plan is the responsibility of management.

The chapter was concluded with a discussion of the training and personal characteristics which a successful controller should possess. It was suggested that the controller should have specialized training in accounting principles, statistical methods, and computer applications. His specialized training should be built on a good foundation in general education. His general education should include courses in communications, psychology, economics, and general business.

In addition to his formal training, it was observed that the controller should be cooperative, friendly, courteous, practical, enthusiastic, diplomatic, and tactful. These fine personal characteristics should enable the controller to work effectively with all employees of the firm.

CHAPTER IV

THE USE OF QUANTITATIVE METHODS IN THE CONTROLLERSHIP FUNCTION

Throughout history man has wanted assistance in the task of making decisions. Among other things he has followed proverbs and rules of thumb. He has relied upon consultants and advisors to direct him into the right decisions. In other words, man has never completely trusted his own personal judgment in making decisions.

In recent years decision makers in business and industry have depended heavily upon quantitative methods to furnish them with the right answers in a problem situation. The evolution and purpose of this quantitative approach to decision-making, the contributions and limitations of the approach, and the use of accounting data in the approach, compose the subject matter of this chapter.

Evolution of Quantitative Methods

It has often been observed that military necessity accelerates the introduction of new concepts, new methods, new techniques. One example which may be given to support the validity of this observation was the introduction of quantitative methods to the military decision-making processes during World War II.[1]

At the beginning of World War II, British military commanders were faced with such problems as how to utilize a small number of fighter planes to defend against a much larger number of German bombers and fighter escorts. After radar reported the incoming bombers, there were the questions of where they would be after a given length of time, how long it would take British fighters to intercept them, and, above all, the probability of success in an air battle between a given German force and a given British force.

They also had the problem of finding the best location for a limited number of anti-aircraft weapons defending British cities. At sea there was the problem of searching for enemy submarines and firing depth charges in such a way as to insure the highest probability of destroying them.

Recognizing the complexity of these and other problems, the British high command employed teams of experts, often from the physical sciences, to study the problems in detail. The work of these groups has gone by various names, such as operations evaluations, operations analysis, and operations research.[2] Whatever the name, its primary

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[1] Annesta R. Gardner, "What Is Operations Research," Dun's Review and Modern Industry, Volume LXVI (December, 1955), p. 46.

[2] John F. Magee, "Operations Research and the Accountant," NACA Bulletin, Volume XXXVI (August, 1955), p. 1749.

characteristic was its quantitative nature.

The British teams of experts were highly successful in their efforts to assist British commanders in making military decisions. The achievements of the teams were made known to American military men. Consequently, when the United States entered the war in December, 1941, operations research teams were organized as part of the American military forces. American teams were also successful in their studies of military problems. One of their most notable accomplishments was the determination of a pattern for laying mines in Japanese waters. The pattern was designed for the purpose of achieving the maximum results from a limited number of mines.[3]

After the war, many of the scientists who had worked on military operations research teams returned to their peacetime activities with the conviction that the quantitative methods which they had employed to improve military operations could also be employed to improve business operations. The scientists were certain that since there had been a "best" location for a group of anti-aircraft weapons defending a given city, then there must also be a "best" location for a group of warehouses serving a given market area. Since there had been an "optimum" combination of pilots, planes, and bombs for producing the maximum

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[3]Ernest Dale, Management: Theory and Practice (New York: McGraw-Hill Book Company, 1965), p. 687.

destruction to the enemy, then there must also be an "optimum" combination of men, machines, and materials for producing the maximum economic goods and services for the consumer. The scientists were confident that the similarity between military and business problems was so great that both types of problems could be solved by the same methods.

Businessmen at first were very skeptical of the value of quantitative methods in evaluating business decisions. Even today, some executives still believe that a quantitative analysis is an intellectual process with little, if any, practical merit. Generally, however, management recognizes the contribution that a quantitative analysis can make to a complete, total, over-all analysis of a managerial problem. Therefore, executives have shown increasing interest in learning about the nature of quantitative methods, the advantages and limitations of the methods, and, above all, the types of practical problems to which quantitative methods can be most successfully applied.

As a member of the management group, controllers have also been concerned with the growth and development of quantitative methods of a managerial tool. Controllers in general have realized that if they are to contribute significantly to managerial planning and control decisions, they must acquire a knowledge of the nature and uses of quantitative methods.

Purpose and Nature of Quantitative Methods

One of the greatest misconceptions surrounding the use of quantitative methods relates to the purpose for using them. A quantitative analysis serves one main purpose. It provides information on which to base management decisions.[4] It does not furnish all the information management needs, but it provides an important part.

This main purpose, however, is often overlooked by many who have the misconception that quantitative methods supply the best solution to a business problem. This misconception is due in large measure to the fact that many books and articles on the subject of quantitative methods are written in such a way that misleading conclusions regarding the purpose of quantitative methods can be easily drawn.

Close observation will show that articles on quantitative methods are replete with such phrases as "optimum solution," "best answer," "right decision." It may be true that a quantitative analysis will produce an "optimum solution" only to a textbook problem. This may be true for three reasons.

First, the textbook problems make precise, specific assumptions which in real life would be subjective assumptions

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[4] Robert M. Trueblood, "Operations Research--A Challenge to Accounting," The Journal of Accountancy, Volume CIX (May, 1960), p. 48.

and, therefore, not so precise and specific. Second, the textbook problem may consider only the quantitative aspects. However, a quantitative analysis is only a partial analysis of the problem. Once qualitative factors are brought into consideration, the "optimum solution" as shown by the quantitative analysis may no longer be the truly "optimum solution." Third, the textbook problems are isolated problems. They are often solved alone and apart from all other problems of the firm. Yet, in real life there are no isolated problems. They are all interrelated, intermingled, and produce alternating impacts on each other. Failure to emphasize these three aspects of textbook problems has contributed to the erroneous conclusion by unwary readers that the purpose of quantitative methods is to produce an "optimum solution" in any problem situation.

The ability of quantitative methods to provide information and their inability to provide unfailingly optimal solutions can be seen through an examination of the characteristics of quantitative methods and their limitations. Quantitative methods are characterized by four essential features.

The first characteristic is a precise, specific statement in quantitative terms of the objective of the analysis. For example, the objective may be to find the operation that will maximize profits or minimize costs. The narrower the problem and the more specific the objective, the greater the benefits will be from a quantitative analysis.

For purposes of illustration, the following example will be carried through this description of the nature of quantitative analysis. Assume that a chief executive desires to increase the sales of his firm and that he requests the controller to make a quantitative analysis of the problem of how to achieve the increase. First, the objective must be stated in precise, quantitative terms. Assume that the objective is stated as: Increase sales by ten percent.

The second characteristic is the identification of critical, quantifiable variables in the solution. By discarding the variables of minor importance and focusing attention upon the most relevant factors, a tremendous quantity of work is eliminated and yet satisfactory results achieved. Returning to the illustration of increasing sales by ten percent, the controller now selects those quantitative variables which are significant to the level of sales. Assume that he determines that sales are affected by five important factors, namely, the amount of advertising expenditures, the number of retail outlets, the price of the product, the population of the market area, and the level of personal income in the market area.

The controller may arrange the critical variables in the form of an equation. His equation could be

$$S = f(A, O, Pr, Po, I)$$

where S = sales, f = function, A = advertising expenditures, O = retail outlets, Pr = price, Po = population of the area, and I = personal income of the area. The purpose of the equation or model is to facilitate the analysis of the problem.

The third characteristic is the quantitative description of the degree of relationship between each variable and the objective. Interrelationships among variables and joint effects of variables upon the objective are also determined and stated. Regression and correlation analysis is a most helpful tool in determining these relationships.

Returning to the illustration of increasing sales by ten percent, the controller will determine the functional relationship between advertising expenditures and sales, between the number of retail outlets and sales, between price and sales, between population and sales, and between the level of income and sales. He may also calculate joint effects, such as the effect of advertising and price upon sales.

The fourth characteristic of a quantitative analysis is the assignment of numerical values to each of the variables. By manipulating the values assigned to the variables the analysis may show several different ways of attaining the objective. Returning to the example of increasing sales by ten percent, the controller may now assign values to the variables under his control, namely, advertising expenditures, number of retail outlets, and price. The population of the area and the income of the area are variables beyond his control. The controller may decide to hold the price and number of retail outlets at their present level and alter advertising expenditures. Since he knows the functional relationship between all the variables and sales, he can determine the change in advertising expenditures necessary to increase sales by ten percent.

The controller may alternatively hold advertising expenditures and number of retail outlets constant and change prices. This step will give him the required change in prices that will effect a ten percent increase in sales. Then he may hold prices and advertising expenditures constant, and determine the change in the number of retail outlets necessary to increase sales ten percent. Finally, the controller may determine that sales may be increased ten percent by a simultaneous change in each of the three controllable variables.

After completing the analysis, the controller may submit a report to the chief executive precisely explaining that the sales may be increased ten percent in any of the following ways: first, increase advertising expenditures x percent; second, increase the number of retail outlets x percent; third, decrease prices x percent; or fourth, some combination, such as increase advertising x percent and decrease prices y percent.

With this information to supplement any other information available, the chief executive must make a decision. The chief executive may be primarily concerned with giving the consumer the lowest prices possible. He may be chiefly interested in providing employment in the community by opening as many retail stores as possible. He may be looking only for the least expensive way of increasing sales.

All these factors enter into the final solution of how to provide for the increase and support an earlier conclusion that the purpose of quantitative analysis is not to provide the "optimal solution" but to provide information on which to base decisions. That none of the solutions resulting from a quantitative analysis may be truly "optimum" is confirmed by an examination of the limitations of quantitative analysis.

Limitations in the Use of Quantitative Methods

In their enthusiasm for methods of quantitative analysis, ardent supporters of the methods often overlook some of the serious limitations of the analysis. Although quantitative analysis plays a significant role in managerial decision-making, there are several reasons why the analysis may not produce results which point up the best decision.

The main limitation in the use of quantitative analysis is that it is limited to a study of tangible, measurable factors. Some of the most important factors affecting business decisions, however, are intangible or qualitative factors. Until these can be quantified, quantitative analysis will continue to have limited usefulness. Selection among alternative courses of action must continue to be ultimately based on managerial judgments and intuition.

In the example of increasing sales by ten percent, it may be that the critical factor in this example was the personality, appearance, and sales ability of the sales representatives. However, it could not be quantified and brought into the analysis. It is entirely possible that the most effective way to increase sales in this case would be to require all sales personnel to enroll in an adult education course in the art of salesmanship.

A second limitation relates to the selection of the critical variables in the solution. If too many variables are included, the model becomes unwieldy. If too few are included, the analysis provides little, if any, useful information. Therefore, the selection of the variables involves a compromise between completeness and simplicity. In the example of increasing sales by ten percent, the analyst selected five critical variables for study even though there may have been other variables. His decision to disregard the remaining variables was a subjective decision.

After the critical variables have been selected, a third limitation appears. It pertains to the determination of relationships among the variables and the objective. In the example of increasing sales by ten percent, the analyst may find it difficult, for example, to determine the relationship between advertising expenditures and sales. In fact, Robert Dorfman writes that "no one really knows what the relationship is" between advertising expenditures and sales.[5]

Obviously, a quantitative analysis is limited in its usefulness in all cases where the relationships between the critical variables and the objective are unknown. In such cases, the analyst may "proceed on the basis of a bold conjecture"[6] even though it is "an insubstantial foundation for an important decision."[7] A conjecture on the relationship between a variable and the objective is clearly a subjective judgment.

A fourth limiting factor in the use of quantitative methods is the lack of trained personnel to apply the methods to daily problems. No doubt there are many useful mathematical and statistical techniques ready for use. There is also no doubt that plenty of unsolved problems exist in the business world. However, between the abundance of tools on one hand, and the over-abundance of problems on the other hand,

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[5]Robert Dorfman, "Operations Research," The American Economic Review, Volume L (September, 1960), p. 614.

[6]Ibid.

[7]Ibid.

there appears to be a gap--a gap that can best be filled by personnel who are both theoretical and pragmatical.

In the past there has been a shortage of personnel who had both the ability to handle quantitative tools and an understanding of basic managerial problems. This made it necessary for analysts and managers to work together in teams on the same problems. This team effort pointed up a fifth limitation in the use of quantitative analysis, that is, a lack of effective communications between the analyst and executive.

The extent of this limitation depends upon the intelligence and breadth of knowledge of both the analyst and the executive. If the executive is willing to study and learn what the analyst needs in order to quantify a given problem, and if the analyst is willing to study and learn the practical facts surrounding a given problem, then the difficulty in communications can be diminished.

Finally, quantitative analyses carry implications of authority and finality. A refined, statistical analysis appears to be so valid and accurate that those with no training in quantitative methods are often led to believe that they provide the only and ultimate solution.

Contributions of Quantitative Methods

Notwithstanding the limitations described above, quantitative analysis contributes significantly to improve quality in managerial decisions. Four of these contributions should be emphasized.

First, quantitative analysis brings greater precision to the identification of significant variables and their relative importance. In a subjective study of the example of increasing sales by ten percent, the executive might make a generalized statement, such as, "advertising is quite important." The quantitative analyst, however, attempts to determine, as precisely as possible, the relationship between advertising and sales.

In a subjective evaluation of the influence of price upon sales the executive might make a generalized statement, such as, "People will buy a whole lot more if we cut the price some." The analyst, however, states as specifically as possible that "If prices are reduced <u>x</u> percent, sales will increase <u>y</u> percent." This substitution of precise, well-organized thinking for loose, generalized thoughts is one, if not the greatest, of the contributions of quantitative analysis.

Second, the use of quantitative methods requires that the problem be clearly defined and the objective precisely stated. If a quantitative analysis is considered only long enough to insure that the problem and objective are properly formulated, it will have made a major contribution to the solution of the problem.

Third, quantitative methods indicate clearly the kinds of data needed in analyzing a problem and provide an efficient framework within which data may be studied.

Finally, it has been suggested that the greatest contributions of quantitative analyses are by-products. Dorfman writes that businessmen have learned more about their businesses by answering an analyst's questions, supplying data for his analysis, and checking over the analysis with him than they have learned from his final reports.[8]

Careful reading of these contributions shows that no change has been effected by quantitative methods to the basic character of decision making. The contributions have simply been better organization and greater precision in the decision-making process.

Supplying Data for Quantitative Methods

Traditionally, accounting has been primarily concerned with providing objective, factual information relating to past operating activities and present financial status of the firm. It has been proper, therefore, for the profession to devote its attention largely to historical costs, accountability for assets, measurement of liabilities, and determination of net profit.

The typical accounting system has been designed to provide information required by such statutory and legal bodies as the Securities and Exchange Commission, the Internal Revenue Service, stock exchanges and various other

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[8] Robert Dorfman, "Operations Research," The American Economic Review, Volume L (September, 1960), p. 614.

commissions. The purpose of the conventional accounting system has been to ensure fulfillment of management's responsibility to creditors, stockholders, governmental regulatory and taxing agencies. These financial accounting systems have been and still are quite adequate for this purpose.

As a processor of financial information, the financial accounting system has been the main part of the firm's total information system. In fact, in many corporations the accounting system has constituted the entire formal information system. When corporations were small, static, and relatively simple, a formal information system larger than the financial accounting system was unnecessary. Interdepartmental and general information not conveyed by the formal accounting system could be conveyed through informal conversation among department heads.

In recent decades, however, corporations which were small, static, and relatively simple have become large, dynamic, and relatively complex. The increased complexity of business has necessitated a scientific approach to management and the use of quantitative methods in decision-making. The newer approach to management requires new types of information. The demand for information has become so great that it has been estimated that the cost of gathering, storing, manipulating, and organizing information in United

States industry is greater than the cost of direct factory labor.[9]

For planning purposes management found that it needed information about the future. It needed information expressed in non-financial terms, such as share of the market, productivity, quality levels, and adequacy of customer service. It needed information on external conditions, such as economic and political environment, and competitive activity. It also found that the traditional accounting system provided only a very small part of the information required by management.

Consequently, some corporations have created full-time management information departments, and charged them with the responsibility for identifying the information needs of all levels of management for both planning and control purposes, for developing the necessary system to fulfill these information needs, and for operating the data processing equipment necessary to generate the needed information.[10]

Daniel also wrote:

> To some extent these departments, reporting high in the corporate structure, have impinged on

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[9]Marshall K. Evans and Lou R. Hague, "Master Plan for Information System," Harvard Business Review, Volume XL (January-February, 1962), p. 92.

[10]D. Ronald Daniel, "Management Information Crisis," Harvard Business Review, Volume XXXIX (September-October, 1961), p. 120.

> responsibilities traditionally assigned to the accounting organization since they are concerned with financial as well as non-financial information.[11]

The development of management information departments, detached from the controller's office, and the changes occurring in the area of information technology are not minor changes and developments. They are major developments presenting a serious challenge to controllership, and in fact, to the entire accounting profession.

In a sense, accounting is an activity within the general activity of information processing. As such, accounting theory is a part of information theory.[12] From this view of accounting, the challenge which the growth of information technology presents to the accounting function can be easily stated. The challenge is: Will the expansion in information theory be accompanied by an expansion in the scope of accounting theory or will accounting theory retain its present scope with the result that the accounting system becomes a smaller and smaller part of the firm's information system?

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[11] D. Ronald Daniel, "Management Information Crisis," Harvard Business Review, Volume XXXIX (September-October, 1961), p. 120.

[12] C. West Churchman and Russell L. Ackoff, "Operational Accounting and Operations Research," The Journal of Accountancy, Volume IC (February, 1955), p. 33.

Summary

In recent years decision makers in business have relied heavily upon quantitative methods to provide them with information on which to make managerial decisions. The use of quantitative analysis in business decision making increased rapidly following the successful application of quantitative methods in military decision making during World War II.

In a quantitative analysis of a business problem, emphasis is placed upon the selection of the quantitative variables in the problem and establishing the relationship among these variables. The chief advantage stated for this process was that it brought better organization and greater precision to decision making.

Several limitations in the use of quantitative methods were discussed. The primary limitations was that the methods could not include intangible, immeasurable factors. Many of the limitations in the use of quantitative methods were not the fault of the methods directly. They were actually the fault of the users of the methods.

The final part of this chapter was a discussion of the data needed for quantitative methods. It was stated that controllers have traditionally provided data for purposes of external reporting. With increased use of quantitative methods by management, controllers have a new opportunity to provide quantitative data for these methods.

CHAPTER V

THE USE OF LINEAR PROGRAMMING IN PLANNING

Industrial managers have no problem in deciding which products to manufacture when plant capacity is so great that they can produce any products that can be sold. Traffic managers have no difficulty in selecting shipping routes when each customer can be supplied from the nearest plant. Processors of animal foods have no blending problems when all the ingredients are equal in content and price.

Management, however, is seldom confronted with situations so easily resolved. It is more likely that plant capacity will be limited, requiring a decision about which products to manufacture. Ingredients of processed foods are not equal in content and price, making it necessary to select ingredients which will most efficiently meet standards of quality and cost. If customer "A" is supplied from the warehouse nearest him, that warehouse may not be able to supply customer "B" who may also be closer to that warehouse than any other. Therefore, decisions must be made regarding the best shipping routes.

In these and similar cases, management requires a framework in which it can study the problem and find the best solution. Linear programming is such a method.

Origin of Linear Programming

According to the literature, the linear programming technique was first formulated and presented as a managerial tool by George B. Dantzig, Marshall Wood, and Associates, in 1947.[1] At that time Dantzig, Wood, and Associates, were studying the possibility of applying scientific and mathematical methods and techniques to Air Force planning problems. The question facing the Air Force was how to conduct procurement, recruitment, maintenance, training, and other activities so that Air Force goals would be achieved most efficiently.

Dantzig proposed that the interrelationship among Air Force activities be viewed as a linear-programming type of model. This proposal led to the formation of an Air Force research group called Project SCOOP (Scientific Computation of Optimum Programs) under the direction of Marshall K. Wood.[2] While a member of Project SCOOP, Dantzig prepared the initial mathematical statement of the linear programming

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[1]Giuseppe M. Ferrero di Roccaferrera, Operations Research Models for Business and Industry (Cincinnati: South-Western Publishing Company, 1964), p. 294.

[2]E. Leonard Arnoff and S. Sankar Sengupta, "Mathematical Programming," Progress in Operations Research, Russell L. Ackoff, editor, Two Volumes (New York: John Wiley & Sons, Inc., 1961), Volume I, p. 112.

problem. He also developed an outstanding technique--the simplex technique--for solving it. The basic paper in which Dantzig set forth the simplex method was circulated privately from 1947 to 1951 when it was finally published.[3]

Although optimization of linear functions subject to linear constraints had been studied previously, Dantzig's paper in 1947 initiated a period of intense interest in the study of linear programming. A conference on Linear Programming held at the University of Chicago in June, 1949, and a Symposium on Linear Programming held in Washington, D.C., in June, 1951, were the forerunners of many similar conferences. Research on the development and application of linear programming techniques were sponsored by the Operations Research Society of America, The Institute of Management Sciences, The Society for Industrial and Applied Mathematics, and many other professional organizations in applied mathematics and econometrics.

Of greater importance, from the businessman's point of view was the appearance of popularized articles on linear programming in such periodicals as Business Week, Fortune, and others. These non-technical articles, describing the merits of linear programming, attracted the attention of

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[3]The paper was published as G.D. Dantzig, "Maximization of a Linear Function of Variables Subject to Linear Inequalities," in T.C. Koopmans, editor, Activity Analysis of Production and Allocation (New York: John Wiley and Sons, Inc., 1951), p. 339.

business managers. Once convinced of its values, management has made widespread industrial applications of linear programming.

Applications of Linear Programming

The 1955 Conference on Business Statistics, sponsored by the American Statistical Association and the Wharton School of the University of Pennsylvania, included a two-day panel discussion on "The Current Uses of Linear Programming in American Business." The participants in the panel knew of approximately twenty examples of industrial applications of linear programming, and they estimated that only one-half of these had made important contributions to managerial decisions.[4]

Since 1955, however, the literature indicates that the number of applications of linear programming has grown rapidly. This has probably been due to increased publicity of the technique, additional refinements and extensions of the technique so that it may be applied to a greater variety of problems, and greater satisfaction on the part of businessmen with the results of linear programming.

The linear programming method may be successfully applied to component-mix problems, that is, determining the quantity of each component that should be used in blending

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[4]L. Wheaton Smith, Jr., "Current Status of the Industrial Use of Linear Programming," Management Science, Volume II (January, 1956), p. 156.

gasoline, paints, fertilizers, animal feeds, and similar products. It is very useful in solving problems of product-mix, that is, finding the portion of plant capacity that should be assigned to each product. Problems of allocating the outputs of a number of sources among a number of destinations are susceptible to solution by linear programming techniques.

Multiple period scheduling, assignment of skilled personnel, setting executive compensation, machine loading, selecting factory sites, and routing aircraft, are examples of linear programming type of problems.[5] In fact, linear programming can be used so frequently that it has probably become more closely associated with managerial planning than any other quantitative method.

Purpose of Linear Programming

Although its purpose has already been implied, it should be specifically stated that the objective of linear programming is "to allocate some kind of limited resource to competing demands in the most effective way."[6]

Business managers have always been confronted with

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[5]For an amusing article on the application of Linear Programming by a housewife, the reader is referred to Paul Gunther, "Use of Linear Programming in Capital Budgeting," Journal of the Operations Research Society of America, Volume III (May, 1955), p. 219.

[6]Elwood S. Buffa, Models for Production and Operations Management (New York: John Wiley and Sons, Inc., 1963), p. 291.

allocation decisions. Limited resources for the firm may include capital, personnel, raw materials, and equipment. The various products that constitute the output of the firm represent the competing demands to which the limited resources must be allocated. The most effective way may be one that maximizes profits, minimizes costs, or optimizes some other appropriate measure of desired performance.

In allocation problems involving only a few limited resources and competing demands, judgment and intuition may provide satisfactory decisions. However, where a large number of factors are included, linear programming techniques should be used. The following illustration demonstrates the application of one of the linear programming methods--the Simplex Method--to a simplified product-mix problem. The procedure would be the same for a case involving a larger number of variables.

Illustration of Maximization Problem

The Problem

Assume that a firm manufactures two products which are designated "X" and "Y". Assume that the manufacturing process involves three work centers--A, B, and C. A unit of product "X" requires two hours at work center "A", one hour at work center "B" and four hours at "C". A unit of product "Y" requires one hour at work center "A", three hours at "B", and two hours at "C".

During each planning period work center "A" has available 77 hours, work center "B" has available 85 hours and "C" has 90 hours. The marginal contribution (sales value minus variable cost) is $40 a unit for product "X" and $30 a unit for product "Y". The question facing the firm is how many units of each product should it produce in order to maximize marginal contribution. The assumption is made that the firm can sell all the units it manufactures.

The Objective

In order to employ the simplex method, algebraic statements representing the objective and constraints in the problem must be formulated. Since the marginal contribution is $40 for "X" and $30 for "Y", the objective function in this problem may be written as follows:

$$40X + 30Y = P \text{ (Maximize)}$$

where P = profits. Since the combination of "X" and "Y" which maximizes marginal contribution will also maximize profits, the objective function from this point forward will be stated as "maximize P".

The Constraints

Next, the constraints must be symbolically stated. The three constraints in this problem are the 77 hours of time at work center "A", the 85 hours at "B", and the 90 hours at "C". Since work center "A" must give two hours to each unit of product "X" and one hour to each unit of product "Y", the inequality constraint statement for work center "A" may be written as follows:

$$2X + Y \leq 77$$

Work center "B" contributes one hour to product "X" and three hours to product "Y". Therefore, the inequality constraint statement for that department is:

$$X + 3Y \leq 85$$

For work center "C" the inequality constraint is:

$$4X + 2Y \leq 90$$

The Conversion of Inequalities

The constraints above are necessarily written in inequality form inasmuch as it is possible to use less than the total time available in each work center. However, the mathematics of the simplex method requires the use of equations instead of inequations. Therefore, the inequalities must be converted to equalities by adding to the left side of the inequalities another variable which will represent the slack in each work center, that is, the difference between the time actually worked and the time available. With this change, the inequations above become the equations below.

$$2X + Y + S_a = 77$$

$$X + 3Y + S_b = 85$$

$$4X + 2Y + S_c = 90$$

where S_a = slack time or unused time in work center "A", S_b = slack time in "B", and S_c = slack in "C". Following conversion of the inequalities, the objective equation may be expanded as follows:

$$40X + 30Y + 0S_a + 0S_b + 0S_c = P \text{ (Maximize).}$$

Obviously, the coefficients for slack times should be zero, since slack times do not contribute to profit.

The problem as now formulated consists of four equations with five unknowns. For the reader's convenience these equations are summarized in one group below.

$$2X + Y + S_a = 77$$

$$X + 3Y + S_b = 85$$

$$4X + 2Y + S_c = 90$$

$$40X + 30Y + 0S_a + 0S_b + 0S_c = P \text{ (Maximize).}$$

The objective is to solve these four equations simultaneously, and find a unique value for X, Y, S_a, S_b, and S_c, which will satisfy all four equations.

Before proceeding with the solution, an additional point should be mentioned. It may be necessary to write other inequality constraints, such as $X \geqq 0$, $Y \geqq 0$, $S_a \geqq 0$, $S_b \geqq 0$, and $S_c \geqq 0$. At first, it may appear ridiculous to write a statement that negative units of "X" may not be produced. However, lengthy linear programming problems are usually programmed for computers. Unless they are "told" otherwise, computers do not hesitate to print out such answers as "produce 50 units of 'X' and -20 units of 'Y'." Therefore, all constraints, even those seemingly absurd, should be written into the program.

Returning to the problem presented above, a solution will be developed. The procedures for the simplex method have been reduced to a set of rigorous rules which can be applied rather mechanically. Meaningless mechanical procedures, though, do not create interest nor give understanding of the technique. Therefore, the following pages will place major emphasis upon a full explanation and interpretation of each step in the simplex method.

The Initial Matrix

To make the solution easier, the equations for the three work centers should be arranged in a matrix with the variables as column headings. This arrangement is shown in Table 1.

TABLE 1

INITIAL MATRIX (INCOMPLETE)

X	Y	S_a	S_b	S_c	Quantity
2	1	1	0	0	77
1	3	0	1	0	85
4	2	0	0	1	90

The first two columns in Table 1 form the body of the matrix. The third, fourth, and fifth columns, representing slack times, make an identity matrix. The coefficients in the first column indicate the hours required in each work center to produce one unit of product "X". Coefficients in the second column have similar meaning. These coefficients will be referred to as substitution rates and fully interpreted in a subsequent section.

Table 1 may now be expanded to include a product-mix column, a unit contribution column, and a check column. These are included in Table 2.

TABLE 2

INITIAL MATRIX (INCOMPLETE)

Unit Contribution	Product Mix	X	Y	S_a	S_b	S_c	Quantity	Check Column
0	S_a	2	1	1	0	0	77	81
0	S_b	1	3	0	1	0	85	90
0	S_c	4	2	0	0	1	90	97

The product-mix column indicates which products are being produced at all stages of the solution. At the beginning, the firm has nothing but slack time in all three work centers. The amount of slack time is shown in the quantity column. As the solution progresses, slack time will be used to produce products "X" and "Y". Therefore, "X" and "Y" will appear in the product-mix column and slack time will disappear.

The unit contribution column indicates the amount of profits contributed by each unit of each variable in the product-mix. In Table 2 above, the unit contribution of the slack variables which compose the product-mix are zero. As products "X" and "Y" enter the solution, their unit contributions will appear in the unit contribution column.

The check column is helpful but not necessary to the solution. It simply shows the sum of all the numbers on each row. For example, on row S_a, $2 + 1 + 1 + 0 + 0 + 77 = 81$. Any discrepancy between a total in the check column and the sum of the individual quantities on the corresponding row indicates that an arithmetical error has been made. Any algebraic operation performed on individual row items must also be performed on check column totals.[7]

It will be seen later that the arithmetic in the simplex method is very tedious. The quantities in the body and identity matrix usually include fractions. In those cases, a check column can be very useful in pointing up arithmetical errors.

Two more extensions will complete the initial matrix. First, the objective equation or function should be included in the matrix. For the reader's convenience, the objective equation is repeated here.

$$40X + 30Y + 0S_a + 0S_b + 0S_c = P \text{ (Maximize)}$$

It is advantageous to arrange the coefficients of the objective function at the top of the column headings to which they relate. This arrangement is shown in Table 3 and may be referred to as the objective row.

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[7]The relationship between the check totals and the corresponding individual amounts is analogous to the relationship between a control account balance and the corresponding individual accounts.

TABLE 3

INITIAL MATRIX

Unit Contribution	Product Mix	40 X	30 Y	0 S_a	0 S_b	0 S_c	Quantity	Check Column
0	S_a	2	1	1	0	0	77	81
0	S_b	1	3	0	1	0	85	90
0	S_c	4	2	0	0	1	90	97
		-40	-30	0	0	0	0	-70

Second, an index row is included as the last row of Table 3. The purpose of this row is explained as follows. As stated earlier, the objective of later algebraic operations will be to replace variables in the product-mix with other variables which will contribute more to profits. In order that the best substitutions may be made, the analyst needs an indicator which will point out those variables that may be profitably substituted for those already in the product-mix. The index row is that indicator.

Essentially, the index row consists of numbers which reveal the difference between the contribution rates of the variables in the solution with each variable outside. The index is calculated by weighting the unit contributions in the initial matrix by the coefficients in the column for which the index is being calculated. After the unit contributions are weighted, the products are summed. From that sum, the contribution rate of the variable under consideration is subtracted. The difference is the index number for that particular variable.

As an example, the index number for product "X" in Table 3 is calculated as follows. The unit contribution of each variable in the product-mix is zero. The coefficients in column "X" are 2, 1, and 4. Using these coefficients as weights, a measure of the contribution of the initial solution is computed as follows: (2 x 0) + (1 x 0) + (4 x 0) = 0. The contribution of product "X" ($40) is now subtracted. 0 - 40 = -40. Minus forty is the index number for column "X."

Index numbers are similarly calculated for the body, identity, and quantity columns. The only difference is in the use of different coefficients.

It will be noticed that the index row is the same as the objective row, except for signs. This will be the case <u>only</u> when every variable in the product-mix has a unit contribution of zero. Therefore, one must be cautious of such shortcut rules as "change the signs of the objective row and enter in the index row."

Upon the completion of the index row, the analyst can quickly see ways of increasing profits. Any negative number in the index row means that one or more of the variables in the solution are contributing less to profits than would be contributed by the variable at the heading of that column. In other words, negative amounts represent opportunity losses. Removal of opportunity losses would obviously increase profits. The objective of later algebraic operations will be to eliminate all negative figures from the index row. When that has been achieved, the zero now at the intersection of the index row and quantity column will have been converted to a figure of maximum profits.

The Second Matrix

The completed initial matrix is ready for conversion to a second matrix. The second matrix will show a different product-mix, less opportunity loss, and more profit. An important characteristic of the simplex method is that each succeeding matrix--except in the case of degeneracy discussed later--offers an improvement over the previous matrix.

To prepare the second matrix, it is necessary to identify the key column, key row, and key number of the initial matrix. Table 4 below reproduces the initial matrix with the key column, key row, and key number identified with a "K."

TABLE 4

INITIAL MATRIX WITH KEY COLUMN, KEY ROW, AND KEY NUMBER IDENTIFIED

Unit Contribution	Product Mix	40 X	30 Y	0 S_a	0 S_b	0 S_c	Quantity	Check Column
0	S_a	1	1	1	0	0	77	81
0	S_b	1	3	0	1	0	85	90
0	S_c	K 4	2	0	0	1	90	97 K
		-40	-30	0	0	0	0	-70
		K						

The key column may be any one of the columns with negative numbers. However, the column with the greatest negative number is usually selected. The reason for this follows:

It will be recalled that the negative amounts in the index row represent opportunity losses. The firm may eliminate $40 of opportunity loss by producing one unit of "X". It may remove $30 of loss by producing one unit of "Y". All other things being equal, the firm would rather begin by manufacturing "X". Therefore, "X" is brought into production and column "X" becomes the key column.

The key row is found by dividing each positive, non-zero amount in the key column into the corresponding amount in the quantity column. The row with the smallest positive quotient becomes the key row. In this problem, S_c is the key row, since 90/4 produces a smaller quotient than 85/1 or 77/2. The logic in this step is explained below.

As stated earlier, the coefficients in column "X" represent the hours needed in each work center to produce one unit of product "X". The amounts in the quantity column indicate the hours available in each work center. The division operation performed above reveals the maximum number of units of "X" that each work center can handle. Naturally, the work center that can handle the smallest number of units becomes the limiting factor and the key row. The key number is found at the intersection of the key row and the key column.

Substitutions are now made into the second matrix. Product "X" (key column) is substituted for S_c (key row) in the product-mix column. This substitution necessitates new coefficients for the new main row of the second matrix. The coefficients are calculated by dividing all quantities in the old S_c row by the key number 4. The new main row (M) is shown in Table 5 below.

TABLE 5

SECOND MATRIX (INCOMPLETE)

Unit Contri-bution	Prod-uct Mix	30 Y	0 S_a	0 S_b	0 S_c	Quantity	Check Column
0	S_a						
0	S_b						
40	X 1	1/2	0	0	1/4	22 1/2	24 1/4 (M)

At this point it is apparent that the firm's product-mix is composed of an unknown quantity of slack time in work centers "A" and "B", and 22 1/2 units of product "X". The check figure proves that the arithmetic is correct.

Column "X" of the initial matrix does not exist in the second matrix. It has become a row item. Therefore, the coefficients in column "X" of the initial matrix are not needed in the second matrix. They must be reduced to zero.

Looking to the S_b row of the initial matrix, a 1 is observed in the "X" column. It can be reduced to zero by subtracting the corresponding 1 in the new main row of the second matrix. This may be performed, providing the entire new main row is subtracted from the entire old S_b row. The operation is effected and the new S_b row is shown in Table 6.

TABLE 6

SECOND MATRIX (INCOMPLETE)

Unit Contribution	Product Mix		30 Y	0 S_a	0 S_b	0 S_c	Quantity	Check Column
0	S_a							
0	S_b	0	2 1/2	0	1	-1/4	62 1/2	65 3/4
40	X	1	1/2	0	0	1/4	22 1/2	24 1/4 (M)

Substitutions must be made into the S_a row of the second matrix. Again, the objective is to eliminate the digit 2 shown in the old S_a row and "X" column of the initial matrix. This may be accomplished by multiplying each element in the new main row by 2 and subtracting from the corresponding element in old S_a row of the initial matrix. The new S_a row is shown in Table 7 below:

TABLE 7

SECOND MATRIX (INCOMPLETE)

Unit Contribution	Product Mix		30 Y	0 S_a	0 S_b	0 S_c	Quantity	Check Column
0	S_a	0	0	1	0	-1/2	32	32 1/2
0	S_b	0	2 1/2	0	1	-1/4	62 1/2	65 3/4
40	X	1	1/2	0	0	1/4	22 1/2	24 1/4 (M)

One more step will complete the second matrix. The -40 in column "X" of the initial matrix must be reduced to zero. This is achieved by multiplying each quantity in the new main row of the second matrix by 40 and adding to each corresponding quantity in the old index row of the initial matrix. This new index row and the completed second matrix is shown in Table 8 below.

TABLE 8

SECOND MATRIX

Unit Contribution	Product Mix		30 Y	0 S_a	0 S_b	0 S_c	Quantity	Check Column
0	S_a	0	0	1	0	-1/2	32	32 1/2
0	S_b	0	2 1/2	0	1	-1/4	62 1/2	65 3/4
40	X	1	1/2	0	0	1/4	22 1/2	24 1/4 (M)
		0	-10	0	0	10	900	900

The second matrix is interpreted as follows. The firm may produce 22 1/2 units of product "X" and earn $900 profits. At this level of production the firm will have 32 hours of slack time in work center "A," 62 1/2 hours of slack time in center "B," and zero slack time in "C." It also discloses that this level of production is not optimal. The negative figure in the index row indicates that opportunity losses are still present. The firm may wish to develop a third matrix which will eliminate the opportunity losses and increase profits.

The Third Matrix

The procedure for converting the second matrix into the third is identical with that for building the second, and will not be repeated here. The completed third matrix is presented in Table 9.

TABLE 9

THIRD MATRIX

Unit Contribution	Product Mix			0 S_a	0 S_b	0 S_c	Quantity	Check Column
0	S_a	0	0	1	0	-1/2	32	32 1/2
30	Y	0	1	0	2/5	-1/10	25	26 3/10
40	X	1	0	0	-1/5	3/10	10	11 1/10
		0	0	0	4	9	1150	1163

The significant facts of the third matrix are as follows. The firm's product-mix is composed of products "X" and "Y" and 32 hours of slack time in work center "A." There is no slack time in work centers "B" and "C." Any variable which does not appear in the product-mix column is zero.

The firm may produce 25 units of "Y" and 10 units of "X," and earn $1150 profit. It will be noticed that the second matrix showed that 24 units of "X" could be produced. The reduction in the production of "X" was necessary in order to produce some units of "Y." The net effect was an increase in profits by $250. Since no negative indicators exist, the optimum solution has been found.

Degeneracy

In concluding this problem perhaps degeneracy should be briefly discussed. Degeneracy may be recognized when two or more rows tie for the key row. If the wrong row is selected, the problem may form a cycle or loop. If cycling develops, the problem never terminates.

In order to avoid cycling, a tie for the key row should be broken in the following manner. New quotients should be found by using as divisors the figures on the tied rows and in the first slack column--S_a in this problem. If the quotients are still equal, the figures on the tied rows and second slack column should be used as divisors, and so on. As soon as the tied rows show unequal quotients, the row with the smaller quotient is the key row.

According to Arnoff and Sengupta, the question of cycling is not serious outside the classroom. They have written that:

> In practice, the problem of degeneracy is not an important one. . . . While degeneracy is an important theoretical consideration . . . it can virtually be completely ignored in all practical problems since . . . out of practical [e.g., industrial] problems, not one of them has been known to have recycled. . . . While examples have been given to demonstrate cycling and, hence, the failure of the simplex technique, the only available examples were artifically constructed for that purpose. . . .[8]

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[8]E. Leonard Arnoff and S. Sankar Segupta, "Mathematical Programming," Progress in Operations Research, Russell L. Ackoff, editor, Two Volumes (New York: John Wiley and Sons, Inc., 1961), Volume I, p. 123.

Illustration of Minimization Problem

Except for a few important differences, the procedures just described apply equally to problems of both maximization and minimization. Therefore, the following minimization problem will not be explained in detail. Instead, attention will be focused on the difference between the two procedures and on a few additional points which apply to both types of problems but which were not included in the maximization solution.

Before presenting an illustration, perhaps it would be well to outline the one great contrast between the maximization and minimization procedures. It will be recalled that the starting point in the previous maximization process was at zero production. The solution contained no products, no profits--only slack time. Profitwise, it was an unreasonable solution. Yet, it served as a starting position from which the simplex method in a finite number of steps found the point of maximum profit.

Likewise, the following cost minimization solution must begin at some unreasonable starting position. In contrast, though, the unreasonable place will be some extremely high point from which the simplex method in a finite number of steps will find the point of minimum cost.

The Problem

The following exercise will illustrate the solution of a cost minimization problem. Assume that a customer orders 1000 pounds of a chemical mixture which contains two ingredients, "X" and "Y." Assume that the customer specifies that the 1000 pounds contain not more than 400 pounds of ingredient "X" and at least 200 pounds of "Y." Assume that the firm blending the mixture can purchase ingredient "X" for $2 per pound and ingredient "Y" for $3 per pound. The firm wants to determine the least cost combination of ingredients which will also meet the customer's requirements.

The Objective

The objective function can be stated symbolically as follows:

$$2X + 3Y = \text{Cost (Minimize)}$$

The Constraints

The inequality constraints are the requirements specified by the customer. They are written as follows:

$$X \leq 400$$

$$Y \geq 200$$

$$X + Y = 1000$$

The Conversion of Inequalities

As stated earlier, the simplex method requires that all inequality constraints be converted to equations. Therefore, the inequality constraints are rewritten.

$$X + S_1 = 400$$

$$Y - S_2 = 200$$

$$X + Y = 1000$$

where S_1 and S_2 are slack variables that must be added or subtracted in order to change the inequalities to equations.

The simplex method requires that each equation have one and only one entry in the identity and that this entry must have a coefficient of +1. An inspection of the three equations above reveals that only one of the three has a slack variable with a coefficient of +1. This means the other two must be expanded to include an artificial slack variable which will appear in the identity. All three equations, two of them expanded, are shown below.

$$X + S_1 = 400$$

$$Y - S_2 + U_1 = 200$$

$$X + Y + U_2 = 1000$$

where U_1 and U_2 are artificial variables. S_2 will now appear in the body of the matrix.

The interpretation of U_1 and U_2 is that they are ingredients which may be used to produce the mixture. By assigning them a high cost, such as \$100 per pound, it is assured that they will not be present in the optimum solution.[9] By including the high priced U_1 and U_2 in the initial solution, the initial matrix will display the unreasonably high starting position mentioned earlier.

After including slack variables and artificial variables, the objective equation may be expanded as follows:

$$2X + 3Y + 0S_1 - 0S_2 + 100U_1 + 100U_2 = \text{Cost (Minimize)}$$

where 100 is the arbitrarily assigned coefficient of U_1 and U_2.

The Initial Matrix

Rearranging the equations so that the identity is on the right, the initial matrix is shown in Table 10 below.

TABLE 10

INITIAL MATRIX

Unit Cost	Component Mix	2 X	3 Y	0 S_2	0 S_1	100 U_1	100 U_2	Quantity	Check Column
0	S_1	1	0	0	1	0	0	400	402
100	U_1	0	1	-1	0	1	0	200	201
100	U_2	1	1	0	0	0	1	1000	1003
		98	197	-100	0	0	0	120000	120195

The initial matrix discloses that the present cost of ingredients for the 1000 pounds of mixture is $120,000.

[9]If an artificial product, U_1, should be included in an equation in a profit maximization problem, the U_1 product should be assigned an extremely low unit contribution or maybe even a unit loss. This would insure that U_1 would fall out during the solution process and the firm would not produce it.

Worse than that, the mixture is a 'make believe' mixture composed of slack and artificial ingredients. During the solution process, the high cost artificial components, U_1 and U_2, will be replaced by lower cost variables "X" and "Y," and a minimum cost will be determined.

There is one difference between solving this initial matrix and the initial matrix of the preceding maximization problem. In the maximization solution the objective was to eliminate all negative amounts from the index row, since the negative figures indicated that the variables in the solution were contributing less to profits than the variables outside. In the minimization solution the objective is to eliminate all positive figures from the index row, since the positive numbers indicate that the variables in the solution are contributing more to cost than the variables outside.

The Solution

Procedures for finding the key row, the key number, and making substitutions into succeeding matrices are identical with those for maximization problems. Therefore, they are not repeated here. The solution, however, is presented in Table 11.

TABLE 11

SECOND MATRIX

Unit Cost	Component Mix	2 X		0 S_2	0 S_1	100 U_1	100 U_2	Quantity	Check Column
0	S_1	1	0	0	1	0	0	400	402
3	Y	0	1	-1	0	1	0	200	201
100	U_2	1	0	1	0	-1	1	800	802
		98	0	97	0	-197	0	80600	80598

THIRD MATRIX

Unit Cost	Component Mix			0 S_2	0 S_1	100 U_1	100 U_2	Quantity	Check Column
2	X	1	0	0	1	0	0	400	402
3	Y	0	1	-1	0	1	0	200	201
100	U_2	0	0	1	-1	-1	1	400	400
		0	0	97	-98	-197	0	41400	41202

FOURTH MATRIX

Unit Cost	Component Mix				0 S_1	100 U_1	100 U_2	Quantity	Check Column
2	X	1	0	0	1	0	0	400	402
3	Y	0	1	0	-1	0	1	600	601
0	S_2	0	0	1	-1	-1	1	400	400
		0	0	0	-1	-100	-97	2600	2402

According to the fourth matrix the least-cost combination of ingredients is $2600. This consists of 400 pounds of "X" at $2 per pound, and 600 pounds of "Y" at $3 per pound. Four hundred pounds of "X" meets the customer's specification that the mixture contain 400 pounds or less of "X". Six hundred pounds of "Y" satisfies his requirement that 200 or more pounds of "Y" must be used.

The answers to the original questions in the two illustrations above have been found. But that is not the only significant information provided by linear programming. Other important data which it furnishes are discussed in the contributions of linear programming.

Contributions of Linear Programming

The primary advantage of linear programming is its ability to maximize or minimize a given function subject to given constraints. This advantage is already apparent at this point. Therefore, attention is directed to a secondary advantage which is very significant and yet is often overlooked in the literature.

In the maximization problem discussed previously, the linear programming technique was primarily concerned with the two outputs--products "X" and "Y." In finding the optimum combination of the outputs, the method also generated some enlightening information relating to the three inputs--labor in work center "A," labor in work center "B," and labor in work center "C." In order to fully understand the information relating to inputs, an interpretation of the coefficients in a matrix must be given.

The coefficients in each column of each matrix represents the changes that must be made in the row variables in order to acquire from internal sources one additional unit of the input variable at the column heading. A positive coefficient means the row item must be given up; a negative coefficient means that the row variable must be increased. An example follows, using the S_c column of Table 9, page 96.

The S_c column in Table 9 has coefficients of -1/2, -1/10, 3/10, and 9. The interpretation is: If the firm is willing to give up 3/10 of a unit of "X," increase production of "Y" by 1/10 of a unit, and add 1/2 hour of slack time in work center "A," then the firm will have generated from internal sources one additional hour of slack time in work center "C." In doing so, the firm will lose $9. Proof of this interpretation is given below.

	Changes in available times in each work center. (hours)		
Changes in row variables:	A	B	C
Giving up 3/10 unit of "X" increases slack time by	6/10	3/10	1 2/10
Increasing "Y" by 1/10 unit uses slack time equal to	1/10	3/10	2/10
Difference	5/10	0	1
Eliminate 1/2 hour increase in work center "A's" slack time	5/10	0	0
Net difference	0	0	1

Proof that the firm will lose $9 in making the changes is shown as follows: Giving up 3/10 unit of "X" loses $12. (3/10 x 40). Producing 1/10 unit of "Y" gains

\$3. (1/10 x 30). \$12 - \$3 = \$9. Hence, the firm's loss of nine dollars.

The calculations above indicate that the coefficients are substitution rates, that is, they reveal the substitutions that must be made in row items (outputs) in order to release one unit of a column item (input). However, management is seldom concerned with the question of how to create additional inputs from internal sources. It is more likely to be concerned with the question of whether to obtain an additional unit of input from external sources.

If one additional unit of input S_c (1 hour of labor) is acquired from outside the firm, it will have an effect on the row variables that is just opposite to the effect that was produced when the additional unit of S_c was generated from internal sources. In other words, the coefficients change signs.

If one additional hour of labor is employed in work center "C," the effects can be explained by changing the coefficients in column S_c to 1/2, 1/10, -3/10, and -9. This is proved as follows:

Changes in row variables:	Changes in available times in each work center. (hours) A	B	C
Employ one additional unit of S_c which will increase times by	0	0	1
Giving up 1/10 unit of "Y" increases time by	1/10	3/10	2/10
Giving up 1/2 hour slack time in work center "A" increases work time by	5/10	0	0
Totals	6/10	3/10	1 2/10
Producing 3/10 unit of "X" uses slack time equal to	6/10	3/10	1 2/10
Net difference	0	0	0

As a result of these changes, the firm will gain \$9. The proof follows. Increasing the production of "X" by 3/10 unit will mean \$12 gain. (3/10 x 40). Reducing the production of "Y" by 1/10 unit will mean \$3 loss. (1/10 x 30). \$12 - \$3 = \$9. This proves the \$9 gain.

One other method may be used to prove that the firm would gain \$9 by utilizing one more hour of labor in work center "C." One slight change could be made in the initial matrix in Table 3, page 89. Quantity of labor available in row S_c could be increased from 90 to 91 hours. With that change only, the final matrix would show profits of \$1159 instead of \$1150 as now shown in Table 9, page 96.

The discussion above leads to the conclusion that management should hire more labor for work center "C."

There would, of course, be a saturation point for input S_c.

The problem of determining the optimum level of labor in work center "C" is an old problem. Marginal analysis has always considered this question and has correctly taught that labor in "C" should be employed up to the point where its marginal cost was equal to its marginal revenue. Finding that point, though, has been difficult for businessmen. The method of linear programming--subject to the limitations discussed later--seems to be a very practical tool for locating the point beyond which the employment of labor in center "C" would be unprofitable.

The discussion above relating to work center "C" is equally applicable to centers "A" and "B." The S_a column, Table 9, page 96, shows that an additional hour of labor in "A" would earn the firm zero profits. This is easy to understand in view of the fact that 32 hours of unused time already exists in center "A." According to Table 9, the firm could profitably transfer resources from "A" to "C." This assumes that the workers in center "A" are able to perform the duties required of workers in "C."

If the problem had been expanded to include equations for different types of inputs, for example various kinds of raw materials, the same interpretations would be made to determine which inputs to employ more extensively and which one should be used in less quantities. If management uses linear programming for finding the optimum combination of outputs, and fails to analyze the information it provides in relation to the optimum level of inputs, it will be overlooking one of the important contributions of linear programming to managerial planning.

Limitations of Linear Programming

Rigorous methods of analysis appear to be so highly accurate that overreliance upon the results is a subtle danger. For this reason, one must always be aware of the limitations of the method in question. In regard to linear programming, the serious limitations are two. They are the assumption of linearity and the assumption of certainty.

The Assumption of Linearity

A linear relationship is assumed between the cost of inputs and the quantity of inputs used, between the quantities of inputs and outputs, and also between the quantity of outputs and the price of outputs. With reference to the maximization problem above, this means that if the first hour of labor in work center "A" cost $5, then the 1,000,000th hour employed cost $5. It means that if two hours of labor in center "A" is required to produce the first unit of "X," it will also take two hours in "A" to produce the 1,000,000th unit of "X." If the first unit of "X" contributes $40 to profits, the 1,000,000th unit put on the market will contribute $40, too.

These straight line relationships are not easy to accept, especially for the controller who has studied U-shaped cost curves, product transformation curves, indifference curves, curving isoquants, and other curves that permeate economic theory. For two very practical reasons, though, the linear relationships are assumed.

First, it makes the problem manageable. Second, and more important, sometimes functions which are nonlinear over their entire range may be reasonably approximated by a straight line in that portion of the curve which is of practical significance. In those cases, linear programming can furnish useful information to management despite the error in assuming linearity.

The Assumption of Certainty

The assumption of certainty relates to the coefficients of substitution. They are assumed to be correct. In blending problems where the coefficients represent chemical measurements, they are highly accurate. When the coefficients are operation times and the operating methods are reasonably standardized and stable, the input data are generally acceptable.

However, reliable information may not always be obtainable. Records of the firm may not provide input data for quantitative models. Even where they do, new methods, new equipment, new operations may cause historical data to be inapplicable to the future. In those cases, approximations may still give results that are useful in managerial planning.

Other Methods of Programming

Since the introduction of linear programming and the simplex method, many other programming models have been developed. Some have been designed for specific types of problems, and others have grown out of the need to eliminate the limitations of linear programming.

Curvilinear programming, dynamic programming, integer programming, parametric and stochastic programming have some special characteristic not found in the simplex method of linear programming. The simplex method itself has the following variations: standard simplex technique, generalized simplex method, composite simplex algorithm, primal-duel algorithm, multiplex method, and possibly others. Vogel's approximation method and the modified distribution method are variations of the transportion model used to solve distribution type problems.

Extensive knowledge of each method just named would surely be required of a programming specialist. However, it is maintained here that the simplex method as discussed in this study adequately serves the needs of controllers who should be interested in linear programming but have no need to become specialists with the tool.

Summary

As an advisor to management, the controller who has a knowledge of linear programming should be more competent in counseling management on such problems as selecting the best shipping routes from warehouses to customers, choosing the best combination of ingredients to blend into a product, deciding which products to produce, loading machines, scheduling production, and many other planning problems.

The linear programming technique was first formulated and presented as a managerial tool by George B. Dantzig, Marshall Wood, and Associates, in 1947. Since that time linear programming has become so widely used that it is probably more closely associated with managerial planning than any other quantitative method.

Two illustrations were used in this chapter to show the way a controller might apply linear programming to managerial planning problems. One illustration was concerned with finding the best combination of outputs in order to produce the largest profit. The second illustration was concerned with finding the best combination of inputs in order to incur the least costs. Both problems, of course, had specific constraints within which the controller had to work.

In addition to maximizing or minimizing a given function, it was shown that linear programming could provide useful secondary information. The limitations of linear programming were also discussed. The two primary limitations were the assumption of certainty and the assumption of linearity.

Finally, it was observed that many programming models have been developed. It was maintained, however, that the simplex method as discussed in this chapter adequately served the needs of controllers who were interested in linear programming as a planning device but had no need to become specialists with the method.

CHAPTER VI

THE USE OF PERT IN PLANNING

When a board chairman calls a meeting, he first prepares an agenda. When a college student writes a term report, he first makes an outline. Similarly, when an industrial manager undertakes a new project, he should first assemble and organize all the information pertinent to the project. These initial acts of the board chairman, the college student, and the industrial manager can be described in one word: planning.

Management is always seeking techniques that will make planning more effective, particularly where a complex set of activities and relationships is involved. A major step in the direction of more effective planning resulted from the innovation of PERT network analysis. The succeeding discussion centers around the origin of PERT, the purpose and nature of PERT, and the contributions and limitations of the PERT approach to managerial planning and control.

Origin of PERT

PERT is the military's acronym for Program Evaluation and Review Technique. PERT was originally designed as a tool to assist the Navy in planning and controlling the more than 70,000 parts of the Fleet Ballistic Missile Program, better known as the Polaris-Submarine project.[1]

At the beginning of the Polaris project Vice Admiral W.F. Raborn, then Director of the Navy's Special Projects Office, recognized that the managerial problems associated with planning and controlling the Polaris program were equal to or greater than the project's scientific and technological problems. Therefore, in 1956 Admiral Raborn established a Program Evaluation Division and charged it with the responsibility for continuously appraising the over-all performance and progress of the Polaris program.

By late 1957 personnel in the Program Evaluation Division were fully convinced that none of the then current managerial tools for planning and control were adequate for effective program evaluation and decision making for the Polaris project. After a series of high level discussions, an agreement was made to employ outside organizations to assist the Special Projects office in designing a system for program evaluation that would be adequate for such an extraordinarily complex program as the Polaris project.

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[1]"Shortcut for Project Planning," Business Week (July 7, 1962), p. 104.

In December, 1957, Booz, Allen, and Hamilton, management consultants, and The Lockheed Missile and Space Division were chosen to participate in an operations research assignment with the objective of developing a method of continuous program evaluation.[2] Many individuals served as members of the operations research team and each contributed to the development of PERT. Major credit for the PERT system, however, frequently goes to Admiral Raborn. Without his support, backing, and consent, the PERT concept could never have been tested and proven on the Polaris project.

In recognizing the work of Admiral Raborn and the entire operations research team, special recognition should also go to one individual member of the team, Dr. C.E. Clark, a mathematician.[3] Early in the team's study of the problem, Dr. Clark conceived the idea of a network to represent the thousands of parts of the Polaris project. In true classroom style he made a blackboard presentation of his idea to the PERT team. He drew circles to represent the events that must be accomplished in the Polaris program. He connected the circles with lines representing the sequence and interdependency of events. He explained that the time required to go from one event to another was variable, and that the interconnection and criss-crossing of lines depicted

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[2] Willard Frazer, "The Origin of PERT," The Controller, Volume XXX (December, 1962), p. 598.

[3] Ibid.

the complexity of the Polaris program.

The PERT team considered Dr. Clark's network concept to be worthy of extensive study. As study progressed, the PERT approach was presented to scientists, technical directors, and other high level executives in the Special Projects Office. Finally, in April, 1958, the team made a formal presentation of the PERT concept to Admiral Raborn, who immediately approved a pilot test of the technique. The test was so successful that full-scale application of PERT to the Polaris program was ordered by Admiral Raborn in October, 1958.[4]

The success of the Polaris program is well known. This success, no doubt, was due in part to effective managerial planning and control of the program. The effective managerial planning and control, no doubt, was due in part to the origination of PERT.

Purpose and Nature of PERT

PERT may be described as a managerial planning and control tool which graphically displays the essential relationships among various tasks comprising a complex program. Its purpose is to assist management in organizing the multiplicity of jobs involved in projects where the number, interrelatedness, and complexity of jobs are so great that organization by an unaided human mind is impossible. By

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[4]Willard Frazer, "The Origin of PERT," The Controller, Volume XXX (December, 1962), p. 598.

using the PERT approach, management is able to integrate numerous tasks into a single, over-all, coordinated plan.

The fundamental characteristic of PERT is a network representing the tasks to be performed. Therefore, the heart of good PERT management is good networking capability. The construction of a good PERT network and the collection and processing of data relevant to the network are characterized by the five following steps.

The Objective

First, the objective of the project must be stated and every individual event and activity that must occur in the given project must be identified and defined. An event represents an accomplishment at a specific point in time, while an activity represents the time and resources required to move from one event to another. This step makes it necessary for the individual preparing the PERT network to be extremely familiar with the scope of the project.

Sequence and Interdependency Among Events

Second, the sequence and interdependency among events are determined. This is then presented graphically by means of a flow chart or network. Frequently the PERT analyst may first prepare a skeleton network which depicts only the main parts of the project. The purpose of the skeleton network is to establish the logical procedure for completing the program. The skeleton network can then be expanded into a complete, detailed network of the total project.

Time-Estimates

Third, time-estimates must be assigned to each activity in the network. Although some methods of time network analysis employ only one time-estimate, the PERT method employs three time-estimates for each activity. Originally, the three time-estimates were considered necessary for two reasons. First, three time-estimates made it possible to evaluate the uncertainty associated with research and development projects which were the principal projects to which PERT was applied. Second, they provided means of determining statistically the probability of meeting project scheduling dates.

In recent years, however, PERT has been used extensively in types of work where there is little uncertainty in the time required to complete an activity. For example, many jobs performed by bricklayers, painters, electricians, and other construction workers have productivity levels specifically established by labor contracts. Therefore, there is now some doubt concerning the need for three time-estimates.[5]

The three time-estimates are: first, optimistic time, which is the time required to perform an activity where good

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[5]Harry F. Evarts, Introduction to PERT (Boston: Allyn & Bacon, 1964), p. 24.

luck is experienced. It is the time that is likely to occur one out of a hundred times. Second, most likely time, which is the time required where normal working conditions prevail. Third, pessimistic time, which is the time required where bad luck is experienced. It is also likely to occur one out of a hundred times. Generally, time-estimates are solicited by oral rather than written communications, and by skipping around the network rather than following a specific path. They are made by engineers, equipment manufacturers, architects, contractors, and line managers.

Expected Time

Fourth, an expected time is calculated for each activity. The expected time is calculated by the following formula:

$$t = (a + 4m + b)/6$$

where t = expected time, a = optimistic time, m = most likely time, and b = pessimistic time.

Some discussion has surrounded the use of the above formula. Early users of PERT plotted the three time-estimates as a probability distribution which would occur if the activity were to recur a large number of times. They located the optimistic and pessimistic times, each of which has a probability of occurring one out of a hundred times, at the low ends of a probability distribution and located the most likely time at the peak of the distribution. These early users noted some similarity between their distribution of time-estimates and another distribution--the Beta distribution. Although there were differences between the two distributions, it was decided that the similarity was great enough to permit the use of the Beta distribution as a model for the three time-estimates of PERT.

However, the calculation of expected time through an analysis of the Beta distribution was tedious and required mathematical sophistication. After empirical investigation, the early users of PERT found that the simple formula above gives results approximating the results obtained through rigorous analysis of the Beta distribution. It was decided that the use of this formula would introduce no significant error in the calculation of expected times. Therefore, the above equation has become the standard equation for calculating expected time of an activity.[6]

Critical Path

Fifth, the critical path through the network is determined. Expected times along every possible path in the network are summed and the longest distance in time between the beginning and end of the project is identified as the critical path. Any variation in the time required for an activity on the critical path will have an effect on the total time required to complete the project.

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[6]Harry F. Evarts, Introduction to PERT (Boston: Allyn & Bacon, 1964), p. 49.

All paths other than the critical path are known as slack paths. On each slack path there is slack time or spare time. Calculation of slack time helps management to know the amount of time by which activities on slack paths may be delayed without affecting the total time required to complete the project.

Upon completion of the five steps above management will have an integrated plan showing the sequence in which activities must be completed, the expected time required for each activity, and the total time required for the entire project. An application of these five steps is made in the following illustration.

Illustration of PERT

Assume that management decides to construct a building and to use the PERT approach in planning the building project. Management's first step is to define the objective and identify all events that must be accomplished in order to attain the objective. In practice, the objective in this case would include detailed, written specifications of the building. In this illustration, however, the objective is simply to "complete the building."

The events leading to that objective are identified in Table 12 below. In identifying the events to be accomplished, the PERT planner may make the list of events as detailed or condensed as he desires. For example, event number eight, plumbing completed, could be subdivided into plumbing materials purchased, plumbers employed, lower plumbing completed, and upper plumbing completed. For purposes of this illustration, the events in Table 12 are considered to be sufficiently detailed.

TABLE 12

EVENTS THAT MUST BE ACCOMPLISHED TO COMPLETE A BUILDING

Event Number	Event
0	Decision to build
1	Architect selected
2	Contractor selected
3	Site selected
4	Architect's plans accepted
5	Abstract completed and purchase closed
6	Building loan secured
7	Foundation completed
8	Plumbing completed
9	Frame completed
10	Siding completed
11	Roofing completed
12	Wiring completed
13	Walls completed
14	Painting completed
15	Building completed

Management's second step is to prepare a flow diagram or network showing the sequence in which events must be accomplished. The network for this illustration is given in Figure 1 below.

The third step is solicitation of time estimates. In this illustration the time-estimates for accomplishing each event are shown in Figure 1 on the line or lines preceding the events, and also in Columns 3, 4, and 5 of Table 13 below. The first time-estimate (a) is the optimistic time, the second (m) is the most likely time, and

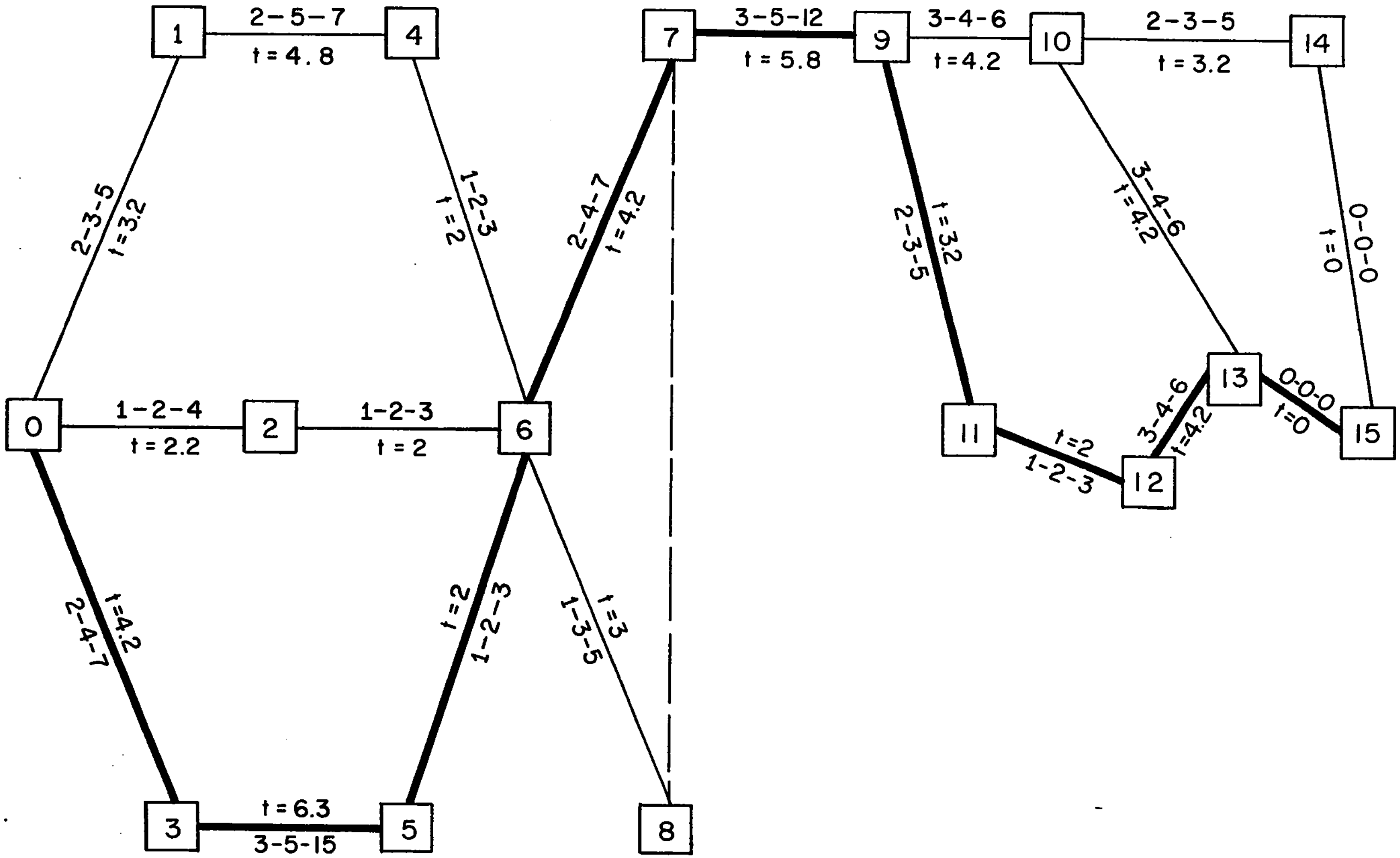

Figure 1
PERT NETWORK
EVENTS THAT MUST BE ACCOMPLISHED TO COMPLETE A BUILDING

the third (b) is the pessimistic time. All time-estimates are in terms of weeks.

The fourth step is calculation of expected time (t) to accomplish each event. Expected times are shown in Column 6 of Table 13 below, and also in Figure 1. Also shown in Table 13 are earliest starting time (ES), latest starting time (LS), earliest finishing time (EF), and latest finishing time (LF) for activities leading to each event. Total slack time (TS) and free slack time (FS) are also shown, and are defined below.

The final step is identification of the critical path, which is the longest path through the network. The critical path is identified in Figure 1 by a heavier line and is 31.9 weeks in length.

To aid the reader in his study of Table 13 the following explanations are given. The first column, Event Number, is a listing of events that must be accomplished to complete the building. The listing is taken directly from Table 12.

Column number 2, Preceding Event, shows the events that must be accomplished before any event in question may be accomplished. For example, activities leading to event number 6 may not be started until events number 2, 4, and 5 have been accomplished.

Columns number 3, 4, and 5 show optimistic time, most likely time, and pessimistic time, respectively, for accomplishing each event. For example, event number 1 may be accomplished in 2 weeks at the best, 5 weeks at the worst, but most likely in 3 weeks.

Column number 6 gives the expected time for accomplishing each event. It is calculated from the data in columns number 3, 4, and 5, according to the formula:

$$t = (a + 4m + b)/6.$$

Substituting time estimates for event number 1, the expected time for accomplishing that event is 3.2 weeks.

Columns number 7 and 8 show the earliest starting time and earliest finishing time for accomplishing each event. These times are found by starting at the beginning of a network and working forward. The earliest starting time for an activity is the summation of the expected times for the longest sequence of preceding events. For example, the earliest starting time for accomplishing event number 1 is zero time since no time consuming activities precede event number 1. The earliest starting time for accomplishing event number 4 is the time required to accomplish number 1, that is, 3.2 weeks, since event number 1 is the only event that must precede number 4.

Greater care, however, must be exercised in determining the earliest starting time for accomplishing event number 6 or any other event into which more than one path leads. Figure 1 shows 3 paths leading to event number 6. In one path, events 1 and 4, requiring a total of 8 weeks,

TABLE 13

PERT WORKSHEET

Event No. (1)	Preceding Event (2)	a (3)	m (4)	b (5)	t (6)	ES (7)	EF (8)	LS (9)	LF (10)	TS (11)	FS (12)
0	0	0	0	0	0	0	0	0	0	0	0
1	0	2	3	5	3.2	0	3.2	2.5	5.7	2.5	0
2	0	1	2	4	2.2	0	2.2	8.3	10.5	8.3	8.3
3	0	2	4	7	4.2	0	4.2	0	4.2	0	0
4	1	2	5	7	4.8	3.2	8.0	5.7	10.5	2.5	2.5
5	3	3	5	15	6.3	4.2	10.5	4.2	10.5	0	0
6	2,4,5	1	2	3	2.0	10.5	12.5	10.5	12.5	0	0
7	6	2	4	7	4.2	12.5	16.7	12.5	16.7	0	0
8	6	1	3	5	3.0	12.5	15.5	13.7	16.7	1.2	1.2
9	7,8	3	5	12	5.8	16.7	22.5	16.7	22.5	0	0
10	9	3	4	6	4.2	22.5	26.7	23.5	27.7	1.0	1.0
11	9	2	3	5	3.2	22.5	25.7	22.5	25.7	0	0
12	11	1	2	3	2.0	25.7	27.7	25.7	27.7	0	0
13	10,12	3	4	6	4.2	27.7	31.9	27.7	31.9	0	0
14	10	2	3	5	3.2	26.7	29.9	28.7	31.9	2.0	2.0
15	13,14	0	0	0	0	31.9	31.9	31.9	31.9	0	0

KEY: a = Optimistic time
m = Most likely time
b = Pessimistic time
t = Mean time
ES = Earliest starting time
EF = Earliest finishing time
LS = Latest starting time
LF = Latest finishing time
TS = Total slack time
FS = Free slack time

must be completed. In another path, only event number 2, requiring 2.2 weeks, must be completed. A third path shows that events 3 and 5, requiring 10.5 weeks, must be completed. The largest of these total times, which is 10.5 weeks, becomes the earliest starting time for activity number 6.

The earliest finishing time for any activity is the earliest starting time plus the expected time for accomplishing the event. For example, the earliest finishing time for event 6 is 12.5 weeks--earliest starting time of 10.5 weeks plus expected time of 2 weeks.

Columns number 9 and 10 show the latest starting time and latest finishing time for accomplishing each event and still complete the project within the time required by the critical path. These times are determined by starting with the final event and working back through the network. For example, event number 15 cannot be completed earlier than 31.9 weeks according to the earliest finishing time shown in column 8. This earliest finishing time is also assumed to be the latest finishing time. Since no time is required for accomplishing event 15, the latest finishing time for event 15 is also its latest starting time.

If 31.9 weeks is the latest starting time for accomplishing event 15, then 31.9 weeks must also be the latest finishing time for all events immediately preceding event 15. If 31.9 weeks is the latest finishing time for activity 14, and if 3.2 weeks are required to accomplish event 14, then 28.7 weeks must be the latest starting time for activity 14. Similarly, if 31.9 weeks is the latest finishing time for activity 13, and if 4.2 weeks are required to perform the activity, then 27.7 weeks must be the latest starting time for activity 13.

Greater care must be exercised, however, in determining the latest starting and finishing times for activity 10 or any other event from which more than one path emerges. To find the latest finishing time for activity 10, one must observe the latest starting time for activities 13 and 14. The earlier of these latest starting times becomes the latest finishing time for activity 10. It has already been determined that 28.7 weeks is the latest starting time for activity 14, while 27.7 weeks is the latest starting time for activity 13. Therefore, 27.7 is the latest finishing time for activity 10. If 27.7 weeks is the latest finishing time for activity 10, and if 4.2 weeks is the expected time for activity 10, then 23.5 weeks is the latest starting time for activity 10.

Columns number 11 and 12 are concerned with total slack and free slack. Total slack is related to the slack available on a path, although it is usually assigned to each event on the path. Total slack on any given path is the difference between the summation of expected times on the given path and the critical path. For example, path 0-1-4-6 requires an expected time of 10 weeks. Path 0-3-5-6 requires 12.5 weeks. Therefore, total slack time on path 0-1-4-6 is 2.5 weeks. In preparing tables such as Table 13, the 2.5 weeks of total slack is assigned to each event on the path. That is, event 1 has 2.5 weeks total slack, and event 4 has 2.5 weeks of total slack. However, should event 1 be accomplished in a time 2.5 weeks longer than the expected time, then no slack time would be available for event 4.

It is possible that the time-estimates for activities 1 and 4 were underestimated. If the time required for accomplishing events 1 and 4 should become greater than the time required for events 3 and 5, then path 0-1-4-6 would become the critical path and some level of total slack time would be available on path 0-3-5-6. Arithmetically, total slack time is calculated by either subtracting earliest starting time from latest starting time or subtracting earliest finishing time from latest finishing time for each event.

Free slack is the difference between the earliest starting time of a successor activity and the earliest finishing time of the activity in question. For example, the earliest starting time for activity 4 is 3.2 weeks, while the earliest finishing time for activity 1 is 3.2 weeks. Therefore, free slack for event 1 is zero.

The distinction between total slack and free slack can be clearly stated. Total slack time refers to the time by which an activity can be delayed without affecting events on the critical path. Free slack time refers to the time by which an activity can be delayed without affecting any other activity--on or off the critical path.

Thus, activity 1 in Table 13 has 2.5 weeks of total slack time but no free slack. This means that activity 1 can be delayed 2.5 weeks without affecting the critical path, but it cannot be delayed any length of time without affecting the start of activity 4 by an equal length of time. It is clear, therefore, that free slack occurs only in activities immediately preceding merge points.

Leaving Table 13 and returning to Figure 1, the reader's attention is directed to the dashed line between events 8 and 7. This is often called a dummy or constraint. It does not require any time or effort and is something on the order of a "go-ahead" signal. Its logic can be seen by the following reasoning.

Assume that the dummy between events 8 and 7 is eliminated and a line is drawn directly between events 8 and 15. Since activity 8 requires only three weeks of time, this would mean that the latest starting time for accomplishing event 8, plumbing completed, would be three weeks before event 15, building completed. By then, the frame (9), roofing (11), siding (10), wiring (12), and portions of the walls would be completed. Obviously, plumbing activities should not wait until then to be started. Therefore, the constraint is drawn between events 8 and 7. In effect, it means that no work should proceed beyond event 7 without a "go-ahead" signal from event 8.

One other important use of PERT data should be considered. It pertains to the use of PERT data for determining the probability of completing the project within a given length of time. Baker and Eris have written:

> The management use of probability information in PERT has not been explored by most users of the technique; however, probability figures can provide useful information for evaluating schedules.[7]

Despite the controversy surrounding the use of measures of probability for events that occur only once, an attempt will be made at this point to show that management can use probability theory to great advantage in evaluating schedule completion dates. Assume that the management in our illustration desires to have the building completed in thirty days. The probability of completing the building in thirty days is determined in the following paragraphs.

Completion of the building project is composed of fifteen separate and distinct activities, each of which has an optimistic time, a most likely time, and a pessimistic time for completion. If each activity were repeated numerous times, it is assumed that the distribution of times for

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[7]Bruce N. Baker and Rene L. Eris, An Introduction to PERT-CPM (Homewood, Illinois: Richard D. Irwin, 1964), p. 12.

each activity would approach the normal distribution. From that distribution a mean expected time could be computed for each activity. These mean activity times have already been calculated and the reader is referred to them in column 6, Table 13.

Also with each distribution of activity times, there is a variance and standard deviation. The problem, though, is to find the mean and variance for the entire project, not each activity. On the basis of the Central Limit Theorem, however, this problem is not difficult.

In the context of PERT, the Central Limit Theorem may be stated in the following way. The mean time for the project is equal to the sum of the mean times of the activities. The variance of project time is equal to the sum of the variances for activity times.

Mean activity times are shown in column 6, Table 13. If the mean activity times along the critical path are summed, the results will be the mean project time. That result is 31.9 weeks as determined earlier.

The variance of project time must now be determined. It will equal the sum of the variances of the activity times along the critical path. The variance for each activity may be found by the formula:

$$S^2 = (b - a)^2/(6)^2$$

where $\underline{S}^2$ = variance, $\underline{b}$ = pessimistic time, and $\underline{a}$ = optimistic time.

Computation of project variance may be facilitated by construction of a table such as Table 14 below. By substitution, the formula shows $S^2 = 8.36$.
$S^2 = (b - a/6)^2 = 301/36 = 8.36$. If the project variance is 8.36, then the standard deviation is 2.89. (Square root of 8.36)

TABLE 14

VARIANCE COMPUTATIONS FOR CRITICAL PATH ACTIVITIES

Activity	a	b	(b-a)	$(b-a)^2$
0-3	2	7	5	25
3-5	3	15	12	144
5-6	1	3	2	4
6-7	2	7	5	25
7-9	3	12	9	81
9-11	2	5	3	9
11-12	1	3	2	4
12-13	3	6	3	9
13-15	0	0	0	0
				301

With mean time and the standard deviation of time determined, the probability of completing the project in any given time may now be calculated. To do this, the given time must be measured in units of the standard deviations. This is achieved by using the formula:

$$(T_s - T_e)/S$$

where T_s = given time for completing the project, T_e = mean time for completing the project, and S = standard deviation.

Returning to the problem of finding the probability of completing the project in thirty weeks, substitutions can be made. T_s is the 30 weeks. T_e is the 31.9 weeks, S is 2.89. Thus (30 - 31.9)/2.89 = -1.9/2.89 = -.66. This tells management that the scheduled time of thirty days is .66 standard deviations to the left of the mean time. Referring to a table of normal distributions, the -.66 standard deviations will show that the probability of completing the building project in thirty weeks or less is approximately 25%.

Similarly, probabilities can be determined for any number of weeks by substituting the desired number for T_s. T_e and S will remain the same in all calculations.

Calculations of probabilities provide useful information. If the probability of completing the project on a given date is low, then no agreement or contract should be made to deliver at the given date. If probabilities are high, then consideration could be given to transferring resources to some other project.

Contributions of PERT

Greater Knowledge of the Project

An appraisal of PERT indicates that significant advantages accrue from its use. The most important advantage is that a greater knowledge of the project results from a PERT approach to planning. The process of building the network, which requires that all events and activities be identified, sequenced and related, gives the PERT planner a greater detailed knowledge of the project than is given by any other planning approach. There is no place in PERT for planners who have only partial knowledge of the programs for which they construct PERT networks. If PERT is employed only long enough to insure a thorough understanding of the program, it will have served a most useful purpose.

Designates Responsibilities

A second advantage of PERT is that it precisely designates responsibilities. The manager of each activity clearly understands the interrelationship of his activity and all others. He knows the schedules and estimates of his work. He knows where he has coordinating responsibilities with other managers. He knows the final results expected of him and the effect which his failures will have on other managers. Thus, PERT removes vagueness in designating responsibilities.

Identifies Critical Path

The third advantage in PERT is that it determines for management a path of critical activities. Without a knowledge of critical and non-critical activities, management may needlessly emphasize non-critical activities while overlooking activities that affect completion of the program on time. No other managerial tool defines such a critical path. By knowing which activities are critical, management can better distribute normal working hours among the jobs and can use overtime to better advantage.

Basis for Control

The fourth advantage of PERT is that it is an excellent basis for control of the project. Often, corrective action is taken only after a control report has provided management with historical information showing that actual performance has not measured up to planned performance. The PERT network, however, lays the basis for anticipating trouble spots long before historical reports point them out. Therefore, the contribution of PERT to managerial control must be ranked along with its contributions to managerial planning.

Limitations of PERT

Although the PERT concept is relatively simple and highly useful, its application to real-life situations involves difficulties and limitations. These may be presented in two parts.

First, fallible human judgment is at the source of all that PERT can do. For example, if optimism or pessimism permeates the time-estimates prepared by individuals, then the entire time schedule will be skewed. PERT is simply a device for accumulating and integrating human judgments, and as long as those judgments are generally optimistic, pessimistic, or incompetent, no amount of quantification can remove the human error.

It seems reasonable, therefore, that primary concern should be for the accuracy of judgments which serve as source inputs for PERT networks. There is evidence, however, that primary concern has been for the development of more sophisticated methods of integrating the judgments. In less than five years after the origin of PERT, a leading periodical reported more than 30 versions of PERT.[8] Some of the versions had expanded beyond time data analysis and included other data, such as cost data.

It is not illogical to maintain that great value may be derived from simplicity. Management, therefore, should retain the simple version of PERT and direct its attention to the minimization of PERT's greatest limitation, namely, erroneous input data.

A second factor limiting the use of PERT is the cost of implementation. It has been reported that the initial cost of PERT planning is about twice the cost of conventional planning.[9] The increased cost is due to the fact that PERT calls for a higher degree of planning skill and a greater amount of detail than conventional planning methods. Also, if the project includes a large number of activities and

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[8] "Shortcut for Project Planning," <u>Business Week</u> (July 7, 1962), p. 106.

[9] Robert W. Miller, "How to Plan and Control with PERT," <u>Harvard Business Review</u>, Volume XXXX (March-April, 1962), p. 98.

events, computer time is required to up-date the network as work progresses.

Larger firms, which already own computers and have personnel adequately trained in PERT programming, may find that the benefits of PERT outweigh the costs. Many small companies, however, find that the cost of renting computers and employing specialists prohibit them from utilizing the PERT approach to planning.

Applications of PERT

Although PERT had its origin in military projects, and subsequently required of all prime defense contractors, the concept has been increasingly adopted in non-defense projects. In fact, one study disclosed that 62% of the users of PERT used it on industrial projects exclusively.[10]

Although PERT is used most frequently in construction type projects, a wide variation in the projects to which PERT has been applied can be seen from the following examples of the use of PERT. PERT has been used to keep track of the more than 200 steps involved in the production of a broadway play.[11] It has been used for sequencing the activities

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[10]Peter P. Schoderbek, "A Study of the Applications of PERT," Academy of Management Journal, Volume VIII (September, 1965), p. 201.

[11]George A.W. Boehm, "Helping the Executive to Make Up His Mind," Fortune, Volume LXV (April, 1962), p. 222.

and events which result in a budget or profit plan.[12] PERT may be used to program the activities and events in an audit engagement.[13] A book publisher used a PERT network to explain to the author, the editor, and the printer how minor delays could upset publishing schedules.[14]

PERT has been used for shutting down and restarting chemical plants, blast furnaces, and oil refineries where periodic maintenance shutdowns are required; preparing bids; installing and programming computers; conducting advertising programs; and introducing new products.[15] One of the most interesting applications of PERT is in precrisis planning. This simply means the preparation of a program that a utility would follow when a hurricane strikes; or a program of action for a company in case of a labor strike; or a sequence of activities which an automobile manufacturer would take if its transmission plant should burn.[16] These PERT programs

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[12] Gordon B. Davis, "Network Techniques and Accounting--With an Illustration," NAA Bulletin, Volume XXXXIV (May, 1963), p. 16.

[13] Gordon B. Davis, "The Application of Network Techniques (PERT/CPM) to the Planning and Control of an Audit," Journal of Accounting Research, Volume I (Spring, 1963), p. 96.

[14] Peter H. Burgher, "PERT and the Auditor," The Accounting Review, Volume XXXIX (January, 1964), p. 106.

[15] Harry F. Evarts, Introduction to PERT (Boston: Allyn & Bacon, 1964), p. 4.

[16] J.W. Pocock, "PERT as an Analytical Aid for Program Planning--Its Payoff and Problems," Operations Research, Volume X (November-December, 1962), p. 896.

are held on a stand-by basis, ready to be used should the crisis occur.

Although numerous examples can be given of PERT applications, there is also evidence indicating that on the whole PERT is not used extensively by American business. A survey of 200 of the largest industrial corporations in the United States, as listed in the Fortune Directory, disclosed that less than 45% of them utilized PERT as a managerial planning and control device.[17] If less than one-half of the largest corporations in America use PERT, it is not illogical to suspect that on the whole American business makes very little formal use of the PERT approach to planning and control.

Summary

As an advisor in managerial planning, the controller may be asked to assist in planning the introduction of a new product or in planning an advertising program. His counsel is necessary in identifying the sequence of steps to be followed in the preparation of a budget or internal audit. In planning for the steps involved in these managerial undertakings, PERT is a most useful tool. Therefore, PERT is quite relevant to the controllership function.

PERT was developed in the late 1950's as a managerial device to assist the Navy in planning and controlling

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[17]Peter P. Schoderbek, "Overcoming Resistance to PERT," Business Topics, Volume XIV (Spring, 1966), p. 51.

the thousands of activities included in the Polaris Submarine project. PERT network analysis was described in this chapter as a very effective tool which a controller could use in planning and control, particularly where a complex set of activities and relationships were involved.

A building project was used to show how a controller could use PERT for purposes of planning and control. The objective of the project was presented. Then all activities included in the project were listed and their sequence of completion was depicted in a network. The expected time for completion of each activity was calculated, and a critical path through the network was determined.

It was observed that PERT contributed to better management because it identified the critical path, gave the planner greater knowledge of the project, designated responsibilities, and formed a basis for control. The cost of implementation and having human judgments as inputs were given as the two greatest limitations of PERT.

Although considerable use has been made of PERT in large corporations, it was maintained that American business in general has made very little formal use of PERT.

CHAPTER VII

THE USE OF GAME THEORY IN PLANNING

One of the most perplexing problems of the businessman is that of predicting the actions of his competitors. If management could accurately forecast the activities of its rivals, a large part of its planning would become easier and more effective. Not all planning, of course, requires that competitors' actions be considered. For example, the PERT planner in the previous chapter did not face an anti-PERT planner whose every move was designed to counter some act of the first PERT planner.

In those situations, however, where consideration must be given to competitors' actions, management demands some type of framework in which to study its rivals' possible moves and the effects of those moves upon its own decisions. This section offers a discussion of one such framework, namely, the theory of games.

Purpose of Game Theory

The purpose of game theory is to facilitate the determination of rational decisions in situations of conflict. It deals with problems where the individual decision-maker does not have control of all factors affecting the outcome. Essentially, it is a theory of strategy.

A player of parlor games, such as chess, bridge, and poker, a military commander whose forces face an enemy, a seller of products who must compete with other sellers of the same products, management-labor negotiators, are all involved in struggles which may be classified as game situations. The outcome in these conflicts depends in large measure upon the selection of proper strategical moves.

Although some work was done in game theory in the 1920's, the subject received its greatest recognition in 1944 with the publication of Theory of Games and Economic Behavior by John von Neumann and Oskar Morgenstern. Von Neumann and Morgenstern developed the theory specifically for application to economic problems. Although parlor games and military battles may offer a better context in which to develop the logic of games, a simplified managerial planning problem will be used at this point to demonstrate the use of game theory in planning.

Two-Person, Constant-Sum Game with Pure Strategies

The game most easily resolved is the two-person, constant-sum game. Games with only two participants include military battles, games such as chess, and some economic situations where only two firms are involved. The game is a constant-sum or zero-sum game when the winnings of one player is equal to the losses of the other. The players follow pure strategies when some single strategy is the most rational decision among all their alternatives.

Assume that firm "A" and firm "B" are the only two firms in an industry and that each is concerned with the problem of increasing its share of the market. Since the size of the market remains constant at 100%, and since only two firms are included, management may analyze the problem within the framework of a two-person, constant-sum game in the following manner.

In order that the best decision may be more easily determined, a payoff matrix showing the results of all combinations of all strategies should be constructed. Assume that firm "A" and firm "B" have the same three strategies, and that strategy 1 is TV advertising, strategy 2 is radio advertising, and strategy 3 is no advertising.

Table 15 below is the payoff matrix showing the share of market going to firm "A" under each of the nine combinations of the three strategies of firm "A" and firm "B." Given Table 15, and assuming that firm "B" has the same information available and will use it to its own advantage, firm "A" would make its decision according to the following line of reasoning.

If firm "A" selects strategy 1, TV advertising, it may obtain 60%, 61%, or 62% of the market, depending on firm B's action. Firm "A" assumes that firm "B" will do all it can to hold firm "A's" share at a minimum. Therefore, firm "A" assumes that if it selects strategy 1, TV advertising, firm "B" will select strategy 1 and "A's" share of the market will be 60%. The 60% is enclosed in parentheses.

TABLE 15

PAYOFF MATRIX

		Firm B's Strategies		
		1	2	3
	1	(60%)*	61%*	62%
Firm A's Strategies	2	(58%)	60%	63%*
	3	57%	(56%)	60%

Examining its strategy 2, firm "A" finds that its share of the market may be 58%, 60%, or 63%, depending on firm "B's" response. Assuming that firm "B" will attempt to keep firm "A's" share at the lowest level, it will adopt strategy 1. Therefore, 58% is bracketed.

Similar reasoning reveals that 56% is the minimum share that firm "A" can expect if it should select strategy 3, no advertising. Firm "A" has now determined that it can expect a minimum of 60%, 58%, or 56% of the market depending on its own choice of alternatives. Obviously, firm "A" prefers the maximum of the minimum values. Therefore, management makes the decision always to advertise via TV--pure strategy 1.

Turning to firm "B," the same type of reasoning is followed by its management. If firm "B" chooses its strategy 1, TV advertising, then firm "A" may acquire 60%, 58%, or 57% of the market, depending on firm "A's" reaction to firm "B's" choice of strategy 1. Assuming that the worst will happen, an asterisk is placed by the 60% figure in column 1. This means that if firm "B" advertises over TV, it can hold firm "A" to a maximum of 60% of the market.

Upon an inspection of strategy 2, firm "B" management observes that firm "A" can obtain no more than a maximum of 61% of the market if firm "B" should select its strategy 2. This is indicated with an asterisk by 61% in column 2.

Likewise, 63% is the maximum share of the market which firm "A" may gain if firm "B" should select strategy 3, no advertising. An asterisk is placed by 63% in column 3.

Firm "B" management has now established that it can restrain firm "A" from receiving more than a maximum of 60%, 61%, or 63% of the market, depending upon firm "B's" own decision. Naturally, firm "B" prefers the minimum of these maximum values. Therefore, the decision reached by firm "B" management is to always advertise over TV--pure strategy 1.

Each firm has made a decision after taking into consideration the possible moves of its competitor. Careful study of Table 15 shows that neither firm can afford to change its decision. As long as firm "A" follows its strategy 1, firm "B" cannot leave its strategy 1 without giving up 1% or 2% of the market. As long as firm "B" follows its strategy 1, firm "A" cannot move from its strategy 1 without losing 2% or 3% of the market.

When two firms operate at a point from which neither can be induced to move, they are operating at a "saddle point." A "saddle point" occurs whenever the maximum of firm "A's" row minima coincides with the minimum of firm "B's" column maxima. A game with a "saddle point" is often called a strictly determined game. The quantity found at the "saddle point" is referred to as the value of the game. When a firm follows the same strategy at all times, it has chosen a pure strategy.

The game presented in Table 15 sheds some light on the manner in which firms move towards the equilibrium or saddle point. Assume there is no advertising by either firm. Thus, firm "A" has 60% of the market and firm "B" has 40%. Management in firm "A" sees an opportunity to capture 63% of the market by switching from strategy 3, no advertising, to strategy 2, radio advertising. Firm "A" makes the move.

Recognizing that firm "A" has increased its market share, the management in firm "B" is forced to make a counter move. Firm "B" observes that if firm "A" remains in its strategy 2, then firm "B" can reduce firm "A's" share from

63% to 58% by moving from strategy 3, no advertising, to strategy 1, TV advertising. Firm "B" makes the move.

Now, it is firm "A's" turn again. In studying firm "B's" latest strategy, firm "A" learns that if firm "B" stays in strategy 1, firm "A" can enlarge its share of the market from 58% to 60% by using strategy 1, TV advertising. Firm "A" makes the move.

After moves and countermoves, each firm is exercising strategy 1, TV advertising. Firm "A" has 60% of the market and firm "B" has 40%. This is exactly the same share each firm had when there was no advertising. Yet, neither firm dares not advertise.

Two-Person, Constant-Sum Game with Mixed Strategies

Not all games have saddle points or equilibrium values which can be attained by playing any single strategy. This can be illustrated by slightly changing the previous example.

Assume that firm "A" and firm "B" each desire to advertise and that the only advertising media are TV and radio. Assume that strategy 1 is TV advertising and strategy 2 is radio advertising. Their payoff matrix in terms of market shares is shown in Table 16 below.

Firm "A" may select strategy 2, radio advertising, hoping that firm "B" will select strategy 2 and thereby give firm "A" 60% of the market. Firm "B," however, reacts by selecting strategy 1, TV advertising, with the result that firm "A" has only 45% of the market. To counter firm "B's" selection of strategy 1, firm "A" moves to its strategy 1 and increases its share of the market from 45% to 50%.

TABLE 16

PAYOFF MATRIX

		Firm B's Strategies 1	Firm B's Strategies 2	
Firm A's Strategies	1	50%	40%	$p = 3/5$
	2	45%	60%	$1-p = 2/5$
		q $4/5$	$1-q$ $1/5$	

Knowing that firm "A" is using strategy 1, firm "B" switches to strategy 2, and firm "A's" share diminishes to 40%. Obviously, firm "A" will choose strategy 2 now, since that will give it 60% of the market. But it will also give firm "B" sufficient reason for switching strategies again--and so the game continues indefinitely.

It is evident that there is no pure strategy which produces a saddle point in this game. The game is like many economic situations--unstable. In an unstable state the best strategy is one that keeps the competition guessing. Therefore, each firm will find it advantageous to devise a mixed strategy, that is, a strategy that follows strategy 1 some of the time and strategy 2 the rest of the time. If a mixed strategy is properly proportioned and randomly played, the firm may expect a greater share of the market than it can expect from playing its best pure strategy.

To calculate the proper mixed strategy for firm "A," the following computations should be made. Let "p" represent the portion of time that firm "A" used strategy 1, TV advertising. Then "1-p" will represent the time it follows strategy 2, radio advertising. When firm "B" is using strategy 1, the value of the game, that is, share of the market, for firm "A" will be:

$$V = 50(p) + 45(1-p)$$

where V = value of the game. When "B" follows strategy 2, the value to firm "A" will be:

$$V = 40(p) + 60(1-p).$$

Firm "A," of course, has no way of knowing in advance when firm "B" will exercise strategy 1 or 2. Therefore, firm "A" attempts to devise a mixed strategy which will give it the same expected value regardless of which strategy firm "B" selects. Firm "A" can accomplish this by setting the value of the game when firm "B" is using strategy 1 equal to the value of the game when firm "B" is following strategy 2. In other words, let

$$50(p) + 45(1-p) = 40(p) + 60(1-p).$$

Solution of the equation shows that $p = 3/5$ and $1-p = 2/5$.

These computations show management in firm "A" that it should use strategy 1, TV advertising, 3/5 of the time, and strategy 2, radio advertising, 2/5 of the time. By playing this mixed strategy, firm "A" can expect to receive 48% of the market. This is shown in the following calculations.

Assume that firm "B" plays pure strategy 1, then the value of the game to firm "A" would be:

$$50(3/5) + 45(2/5) = 48.$$

Assume that firm "B" plays pure strategy 2, then the value to firm "A" would be:

$$40(3/5) + 60(2/5) = 48.$$

Assume that firm "B" plays a mixed strategy, for example, strategy 1, TV advertising, 1/2 of the time and strategy 2, radio advertising, 1/2 of the time. In that case the value of the game to firm "A" would be:

$$3/5[50(1/2) + 40(1/2)] + 2/5[45(1/2) + 60(1/2)] = 48.$$

Hence, regardless of firm "B's" decisions, firm "A" can expect 48% of the market by playing a mixed strategy of 3/5 and 2/5.

Comparison of the mixed strategy with pure strategies shows the advantage of playing the mixed strategy. If firm "A" used pure strategy 1 consistently, it could expect firm "B" to react in such a way that firm "A" would take only 40% of the market. If firm "A" followed pure strategy 2 consistently, firm "B's" response would hold firm "A" to 45% of the market. With the mixed strategy, however, firm "A" can keep the competition guessing about its strategy, and will play a game with an expected value of 48%.

Now that the mixture has been properly proportioned, firm "A" must select the strategies at random. If a pattern is established in the use of strategies 1 and 2, and if the competition detects that pattern, then firm "A's" mixed strategy will be no better than a pure strategy played consistently.

As shown, there is no need to be concerned with firm "B's" choices of strategy as long as firm "A" plays a correctly weighted, mixed strategy selected by chance. However, firm "B" has no assurance that the management in firm "A" has calculated such an optimal mixed strategy. Therefore, firm "B" attempts to play its strategy in such a way as to minimize firm "A's" share of the market.

This can best be accomplished by computing a mixed strategy for firm "B." Letting "q" equal the portion of time that firm "B" follows strategy 1, and "1-q" equal the portion of time it uses strategy 2, firm "B's" mixed strategy becomes 4/5 and 1/5. The procedure for calculating these strategies is identical with that used above for firm "A." By using the mixed strategy 4/5 and 1/5, firm "B" can expect to prevent firm "A" from obtaining more than 48% of the market.

The necessity for randomly making the choices in a mixed strategy for firm "A" is equally applicable to firm "B." When both firms follow their best mixed strategies, the value of the game to firm "A" is 48%.

$$3/5[50(4/5) + 40(1/5)] + 2/5[45(4/5) + 60(1/5)] = 48.$$

Other Types of Games

The theory of games is in a far less satisfactory state outside the area of two-person, constant-sum games.[1] This is unfortunate since the great majority of competitive situations are too complex to fit neatly into the two-person, constant-sum framework.

Two variations of the two-person, constant-sum game are immediately apparent. First, the sum of the game may be non-constant, and, second, it may have more than two players. Returning to the advertising problem discussed above in the two-person, constant-sum game with pure strategies, the reader will recall that the problem was analyzed in terms of market shares. Obviously, the size of the market must remain at 100% and whatever one firm gains, the other must necessarily lose.

However, it was not essential that the conflict be studied in terms of market shares. Management could have desired to analyze the level of profits where different strategies were followed. With reference to profits, though, one firm's gain is not necessarily the other's loss, and vice versa. Cut-throat competition may result in everybody

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[1]William J. Baumol, Economic Theory and Operations Research, Second edition (Englewood Cliffs, New Jersey: Prentice-Hall, 1965), p. 542.

losing, while collusion may be followed by everyone winning. In those cases, constant-sum theory no longer applies.

Another type of game, which is observed as frequently as the non-constant variety, is the many-person game--a game which, so far, has "proved rather intractable to analysis."[2] A study of many-person games is essentially a study of coalitions.[3] The game is reduced to two-persons--each person being a coalition. The theory then attempts to answer such questions as which coalitions will form and how the extra winnings will be divided.

The number of coalitions become extremely large, even when there are only a few persons. For example, only ten firms may form 511 different coalitions. Even though there is realism in the many-person game, the number of computations quickly become unweildy.

Evaluation of Game Theory

Although the game approach to competitive situations has some advantages, use of the theory in management decision making is still quite restricted.[4] Limited use is

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[2]William J. Baumol, Economic Theory and Operations Research, Second edition (Englewood Cliffs, New Jersey: Prentice-Hall, 1965), p. 544.

[3]Robert Dorfman, Paul A. Samuelson, and Robert M. Solow, Linear Programming and Economic Analysis (New York: McGraw Hill Book Company, Inc., 1958), p. 444.

[4]Richard I. Levin and C.A. Kirkpatrick, Quantitative Approaches to Management (New York: McGraw Hill Book Company, 1965), p. 304.

probably due in large measure to the inability of businessmen to construct a payoff matrix with accurate values.

Returning to the advertising example in Table 15, the payoff shows that if firm "A" follows strategy 1 and firm "B" follows strategy 2, firm "A" will take 61% of the market. The accuracy of that percentage, as well as the others, is highly questionable. With erroneous input data to work on, the method cannot be expected to yield optimal strategies.

Limited use of the theory is also due to the fact that the only part which produces convincing solutions, that is, two-person, constant-sum solutions, is not applicable to many business conflicts. Economic problems and the interaction of businessmen with different economic goals are far too complex to be analyzed as easily as a checker game.

In those cases where the game is presented more realistically, that is, with many persons and non-constant sum, the solution is so questionable that its use is of limited value. Therefore, the merits of game theory must be found in something other than the quantitative answers which it provides.

Ironically, the advantages of this quantitative method are qualitative in nature. The concept of strategies, the importance of mixing strategies and thereby keeping the competition guessing about the next move, are forcefully demonstrated in the game approach to problem solving.

Representing the payoffs or outcomes in a matrix helps management to see more easily the effects that his competitors' actions have upon his decisions and his payoff. The theory of games, therefore, provides an excellent, conceptual framework for a qualitative discussion of the problem, the strategies, and the outcomes in a competitive situation.

Summary

In planning situations where management must take into consideration the action of its competitors, the theory of games is a useful planning tool. For example, in a pricing decision, in deciding upon the size of the advertising budget, in choosing whether to be active or passive in labor negotiations, game theory is a useful tool.

As an advisor to management in its planning function, the controller's contributions to planning may be much greater if he has considerable knowledge of the theory of games. The position has been taken, therefore, that the theory of games is highly relevant to the controllership function.

Game theory was described as a planning tool which a controller may use in planning a course of action in situations of conflict. Two examples were used to illustrate the two-person, constant-sum game with pure strategies, and the two-person, constant-sum game with mixed strategies.

The theory of games is less satisfactorily developed outside the two-person, constant-sum game. This is very unfortunate since most economic situations do not fit into the two-person, constant-sum framework. Although the use of game theory has been quite restricted, it has the advantage of providing management with a framework in which to discuss problems, strategies, and outcomes in competitive situations.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

Summary

The purpose of this study was to analyze some of the most useful quantitative methods relating to the controllership function and to present them in a manner that would enable the controller to use them effectively in planning and control. Linear programming, PERT, and game theory were the quantitative methods chosen for study.

Linear programming was described as a planning tool which the controller would find useful in advising management in problems relating to allocation of resources. The controller should find game theory to be helpful in assisting management to find solutions to problems where competitor's actions must be considered such as in pricing decisions. PERT was presented as a statistical tool for both planning and control. For example, the controller could use PERT as a tool to facilitate the planning and controlling of internal audit activities.

In Chapter II the growth and development of the controllership function was reviewed. This was done by dividing the evolution into two time periods--the period preceding 1931 and the period following 1931. Prior to 1931, there was little organized effort among controllers to guide their own developments. Consequently, there was no uniformity in the duties performed by controllers. This was shown by comparing the duties of controllers in various industries prior to 1931.

The first two industries to employ controllers were railroads and department stores. The duties of controllers in railroads related to the issuance and cancellation of stocks and bonds, and other financial matters. In department stores, controllers worked with problems of inventory, pricing, and budgeting, and participated, to some extent, in the decision making process.

There was a noticeable lack of uniformity in the positions held by the controller in the organizational charts of the firms. In some firms the controller reported to the treasurer. In others, the treasurer reported to the controller. In still others, the offices of treasurer and controller were combined.

Prior to 1931, professional associations of controllers were organized along industry lines. Controllers in banking institutions, controllers in railroad companies, governmental controllers, and controllers in retail dry goods companies formed industry-wide associations. However, these intra-industry organizations did not provide the over-all coordinated direction that was needed to advance the concept of controllership. Organized guidance was not achieved until the formation of the Financial Executives Institute in 1931.

Following 1931, growth and development of the controllership function has been closely interwoven with the Financial Executives Institute. In 1931, Arthur Ray Tucker and a small group of controllers formed the Controllers Institute of America, whose name was later changed (1962) to the Financial Executives Institute. The objective of the Financial Executives Institute has been to educate controllers and businessmen in the important contribution that controllers can make to prudent business management.

One of the most notable works of the Financial Executives Institute has been the formulation of statements describing the controllership function. In 1939, a statement listing seventeen duties of the controller was issued by the Institute. The duties related almost entirely to the firm's accounting records. As the concept of controllership expanded to include advisory services to management, revisions were made in the list of duties of the controller.

The latest statement on the concept of controllership was adopted by the Institute in 1959. It consisted of seven points which emphasized the controller's role as an advisor to management in its planning and control functions.

The Securities and Exchange Commission and the New

York Stock Exchange also exerted an influence on the growth and development of controllership. Following an investigation of the McKesson and Robbins case, various members of the SEC stated publicly that the primary responsibility for accuracy and completeness in accounting reports rested upon management in the person of the controller.

At the same time, the New York Stock Exchange conducted a study on the question of who was responsible for accuracy in accounting statements. A report adopted in 1939 by the Board of Governors, New York Stock Exchange, reflected that more emphasis should be placed on the responsibility of controllers for accuracy and completeness in financial records.

Controllers have often been asked to testify before government agencies on matters of national interest. The Office of Price Administration, the War Production Board, the Treasury Department, and the Director of the Budget have been solicitors of counsel from controllers. This has also influenced the growth and development of controllership.

The increasing complexity of business has also affected the growth of controllership. As business firms have increased in size and complexity, management has sought more information and advice. As controllers have provided sound advice, they have grown in importance on the management team. Chapter II was concluded with a discussion of the position of the controller in the organizational chart of the firm. This writer was unable to find in the literature any statement which set forth the proper location of the controller in the organizational chart. However, it was often recommended that he should hold the rank of vice-president. In this writer's opinion, the controller should rank at the policy making level and preferably hold the title of Vice-President. The title of Vice-President would be appropriate since he is an advisor to the president, and vice presidents within the firm. The title of Vice-President would also give more prestige to his office and his counsel would receive more serious attention.

The functions of controllership and the requisites necessary for the controller were considered in Chapter III. The functions were classified in three groups--the general accounting function, the control function, and the planning function.

The general accounting function included the preparation of financial accounting reports, administration of tax policies, preparation of government reports, and protection of assets. Although these duties are indispensable to the firm, it was suggested in this chapter that the controller should delegate these responsibilities in order that he could devote his time to the more important functions of controllership, namely, planning and control.

The managerial function of control consists of measuring performance and taking corrective action. In measuring performance, the controller must analyze variances between actual performance and planned performance and report his findings to management. It was observed in this section that the controller could profitably use such quantitative methods as variance analysis and correlation analysis.

The managerial function of planning consists of deciding in advance what is to be done. Participation in the planning function is the controller's newest opportunity to render a service to management. It was emphasized in this section that the controller could also use many quantitative methods in performing his planning duties.

The personal qualities of the controller were discussed under three headings. First, it was suggested that the controller should have specialized training in accounting principles, statistical analysis, and computer applications.

Second, it was recommended that the controller should have a broad, liberal education. His general education should include courses in economics, sociology, psychology, history, and government. It should include studies in all areas of business administration such as marketing, business law, finance, and management. Great emphasis was also placed on the ability to communicate.

Third, the personal characteristics of the controller were discussed. Since the controller works with people throughout the firm, it was observed that the controller should be an expert in the art of human relations. He must be diplomatic and tactful in his relations with other people and must possess all the personal qualities which earn the respect of others.

The nature of quantitative analysis available for use by the controller was discussed in Chapter IV. The chapter began by examining the reasons for the evolution of quantitative methods. It was explained that quantitative methods were used successfully by British and American commanders in analyzing military problems in World War II. Since the war, businessmen have applied the same methods to economic problems.

Next, characteristics of quantitative analysis were discussed. A practical problem was used to illustrate the way in which the characteristics are applied. It was also emphasized that quantitative analysis may not find the optimal solution to a problem, but it does provide information on which management can base decisions. Following the review of the characteristics of quantitative analysis, the limitations and contributions of quantitative methods were presented.

Finally, considerable thought was given to the question of supplying data for quantitative models. Some firms have organized management information departments which have been given responsibility for identifying the information needs of management and for supplying those needs. These information departments often encroach upon the duties traditionally assigned to the controller's office.

The creation of information departments, detached from the office of the controller, raises a pertinent question for controllers and accountants in general. The question is whether the accounting function should collect all quantitative data needed in the firm or only the required financial data. In the opinion of this writer, the office of the controller should collect all quantitative data needed in the firm.

In Chapter V the writer illustrated linear programming as a quantitative tool available to the controller. Used originally as a planning tool by the United States Air Force, linear programming has become so widely used in business that it is probably more closely identified with managerial planning than any other quantitative method.

The purpose of linear programming techniques is to maximize or minimize a given function subject to given constraints. The procedure for maximizing a given function was illustrated with a simplified problem in which management wanted to know the optimum combination of products that would yield maximum profits where certain constraints were present.

Also, minimization procedures were illustrated by a problem in which management desired to find the least-cost combination of ingredients which would produce a mixture that would satisfy requirements specified by a customer. The simplex method was used in both illustrations.

Emphasis was also placed upon the significance of information provided by linear programming in relation to the employment of resources. Proper analysis and interpretation of the data relating to inputs aids management in planning the optimum utilization of resources.

Two limitations of linear programming were discussed. They were the assumption of linearity, which relates to the relationships among the variables, and the assumption of certainty which pertains to the substitution rates.

Program Evaluation and Review Technique, commonly called PERT, which is another quantitative tool available to the controller, was investigated in Chapter VI. The chapter began with a historical sketch of the development of PERT as a managerial planning and control tool in connection with the Polaris submarine project.

The purpose of PERT is to organize the multiplicity of jobs involved in complex projects. Through use of a PERT network, management graphically displays the essential relationships among the various tasks. The characteristics of PERT were discussed and an illustration was presented showing the manner in which PERT should be applied.

Contributions and limitations of PERT were also presented. The most important advantage of PERT is that greater knowledge of the project results from a PERT approach to planning. The greatest limitation is erroneous input data due to optimism or pessimism on the part of individuals who prepare the time estimates used in PERT.

The chapter was concluded with a discussion of the extent to which American business has employed the PERT method of planning and control. Although PERT has been used in many cases, there is reason to believe that on the whole American businessmen have made little formal use of PERT.

The theory of games, another quantitative tool the controller may find useful, was studied in Chapter VII. Game theory was defined as a framework in which rational decisions in situations of conflict were determined. The theory is useful to military commanders, players of chess and poker, management-labor negotiators, and businessmen in competitive situations.

The games which can be most readily solved are the two-person, constant-sum game with pure strategies, and the two-person, constant-sum game with mixed strategies. Illustrations of these two types of games were presented, along with detailed explanations of the solutions.

The most realistic economic games, however, include more than two persons and are not constant-sum games. The theory for many persons, non-constant-sum games has not been developed to the point where it can contribute significantly to the solution of practical problems.

An evaluation of game theory concluded the chapter.

The greatest limitation of game theory is that its application is not practical in many cases. Even where it is practical, inability to construct an accurate payoff matrix limits its usefulness. The advantage in using game theory is that it offers a framework in which to conduct a discussion of the problem, the strategies, and outcomes in situations of conflict.

Conclusions

It is this writer's opinion that considerable training in quantitative methods is a necessary part of education for controllership. It is also the writer's opinion that this training will become even more necessary in the future.

This writer also believes that the controller must give serious attention to integrating new types of information with the present accounting information. Much of the data required by quantitative analyses are not provided by the present accounting systems. If controllership meets the challenge of supplying the data needed for quantitative analysis, controllers in the future will preside over a much broader information system, an integrated system encompassing all quantitative data in the firm.

No doubt the future will see increased application of quantitative analysis in managerial decisions and an increased need for newer types of information. It may well be, however, that the controller's contribution to managerial planning and control will depend more upon his ability to provide required information than upon his competence with quantitative models.

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