CONTAMINANT SERVICE LIFE AND COST
OF HYDRAULIC SYSTEM

By

MASAO IWANAGA

Bachelor of Science in Engineering
Nihon University
Tokyo, Japan
1964

Certification
The National Defense Academy
Yokosuka, Japan
1970

Submitted to the Faculty of the Graduate College
of the Oklahoma State University
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for the Degree of
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Thesis Approved:

E C Fitch
Thesis Adviser

Bennett Basore

Kenneth E Case

W.O. Focks

Norman N Durham
Dean of the Graduate College

PREFACE

This thesis is concerned with the investigation of the contaminant service life and the cost of the hydraulic system consisting of a filter and a pump. Due to the complexity of the contamination problem, the accurate prediction of hydraulic components' lives is very difficult, the cost of the hydraulic system has hardly been analyzed.

Under such conditions, a design engineer must choose the hydraulic components with his experience and intuition in the design phase of a hydraulic system. However, deriving the system design from only the intuition of the design engineer is not only inadequate from the standpoint of system reliability, but also sometimes tremendously wasteful from the economic standpoint. This paper will present to the design engineer how to predict the contaminant service life of hydraulic components and how to design the most economical hydraulic system.

I would like to express my sincere appreciation to Dr. E. C. Fitch, who served as my research adviser. His advise, patience, counsel, encouragement and inspiration have been great contributions not only in completing this research work but also in advancing my knowledge as a professional researcher.

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CHAPTER I

INTRODUCTION

The hydraulic system is a most powerful but very compact device in a mechanical system. Due to this advantage, it has been used in various fields and for various purposes. At the same time, many technical innovations have been achieved.

Since Fitch [1] explored the importance of the contamination in a hydraulic system, contamination control is one of the most significant and interesting problems for the hydraulic engineer in the world today. Except for catastrophic failure due to the fatigue of a component, improper application etc., the service life of hydraulic components depends upon the contaminant concentration in the hydraulic system fluid and the contaminant wear tolerance of the elements exposed to the fluid. Furthermore, the effect of the contaminant on the life of hydraulic components is serious.

The life of a hydraulic component is defined as the operating time during which the degree of performance degradation is acceptable. The performance degradation of a component due to the presence of a specific contaminant distribution in the fluid is referred to as "contaminant sensitivity." The contaminant sensitivity reflects the inherent abrasive resistance properties of the component which can, therefore, be determined through laboratory testing.

In an actual hydraulic system, if the contaminant particle distribution in the fluid is known, the life of hydraulic components can be determined theoretically using their respective contaminant sensitivity values.

The contaminant particle distribution in the fluid depends upon the ingression rate of the contaminant and the actual filter medium used in the hydraulic system. The sources of ingressed contaminant are fabrication residue, external environment, active internal processes, and particle sloughing from filters (evasive particles). Ingressed contaminant from the external environment entering via rod seals and evasive particles generally dominate the others over a period of time. In order to decrease the contaminant ingression rate, an effective wiper seal is needed for the piston rod. The cost of the wiper seal ordinarily is inexpensive compared with other hydraulic components, therefore, from an economic or reliability standpoint, the higher the particle exclusion performance of the wiper seal used in the hydraulic system, the longer the component life, thus lower the total maintenance cost. Therefore, good wiper seals must be used to decrease the ingression rate of contaminant.

Many researchers not only at Fluid Power Research Center (FPRC), Oklahoma State University (OSU), but also in the world have worked to improve the contaminant characteristics of hydraulic components or hydraulic system. However, numerous aspects still remain to be investigated. The hydraulic components on which prominent research results have been revealed are only the pump, hydraulic motor, and filter. The research effort on the other components is just beginning.

The pump, however, is recognized as the most unreliable component and as the most expensive. While, the filter in a hydraulic system is known as the most influential component on the pump life. Therefore, the theoretical analysis of the contaminant service life of a pump and filter and their costs are very important from the reliability or the economic standpoint of a hydraulic system.

The purpose of this study is to analyze the contaminant service life of a pump and filter, and to establish a method which minimizes the maintenance cost of the hydraulic system consisting of a pump and filter. Although the theoretical analysis on the life and the cost of the hydraulic system is the primary purpose of this study, there are many secondary aspects which require investigation before the life and the cost can be analyzed.

The first thing to be investigated is "what kind of method is available to analyze the accurate contaminant wear coefficient of a pump?" The most worthy result on the pump life in the past is the theoretical study for the contaminant sensitivity derived by Fitch and Bensch [1]. This theory is the most fundamental to analyze the flow degradation or the life of pump due to contaminant. According to this theory, the flow degradation rate of a pump is proportional to the contaminant wear coefficient of the pump and the square of the contaminant concentration in the fluid upstream of the pump. The contaminant wear coefficient of a pump is analyzed from the contaminant sensitivity test data of the pump in a laboratory under the assumption that the leakage flow rate of the pump due to contaminant can be expressed as a function of the clearance space and the space cubed, which is called the cubic fit method. The cubic fit method is based upon the leakage flow theory of a small clearance. It seems to be a reasonable assumption. However, the contaminant sensitivity test data from many actual tests do not necessarily fit this theory, but also the contaminant wear coefficient of some pumps analyzed by using the cubic fit is negative. The contaminant wear coefficient should always be positive. Thus, the cubic fit method results in considerable error in the prediction of the hydraulic pump life.

In order to solve this problem, a considerable amount of pump data on the contaminant sensitivity test was plotted on various kinds of graph paper. It was found that the leakage flow ratio which is the ratio of the leakage flow rate to the rated flow of a pump can be expressed in a straight line on a Log-Log graph paper. The contaminant wear coefficient analyzed using this characteristic is always positive and reasonable. Therefore, the straight line approximation on a Log-Log graph paper of the leakage flow ratio was used to predict the pump life.

The second thing to be solved before the life and cost of the hydraulic system having a pump and filter concerns the evaluation of filter performance. The contaminant particles ingressed into a hydraulic system must be removed by the filter in the system. Filter performance is a major factor in the control of contamination level; the filtration or Beta ratio is used internationally to describe this performance. The Beta ratio, however, does not necessarily indicate the separation performance of the filter, because its value unfortunately varies with the contaminant particle distribution upstream of the filter. The contaminant particle distribution of a hydraulic system in the field is different for each system and may vary from time to time. Therefore, the Beta ratio itself is not suitable for use in estimating the life of hydraulic components.

As a substitute for the Beta ratio, a cumulative flow distribution of the filter (in this study, it is simply called flow distribution) has been developed. A filter medium consists of many pores having various diameters which allow the passage of hydraulic fluid. The flow distribution is defined as the ratio of the flow rate passing through pores smaller than a given diameter to the total flow rate through the filter. It is a true representation of the inherent separation properties of the filter and is not influenced by extraneous conditions. The flow distribution was investigated for many filters. It was found that the flow distribution has a Log-Normal distribution in the main flow rate region of the filter, and its standard deviation is a function of the mean. Therefore, if the mean of the flow distribution of a filter can be known, the performance of the filter can be completely analyzed.

Needless to say, the flow distribution varies with the time as the filter is exposed to particles while used in a hydraulic system. The variation rate is different with the form of the contaminant particle size distribution ingressing. If this variation versus time can be analyzed theoretically for every kind of contaminant particle size distribution ingressing, an exact contaminant particle size distribution in a hydraulic system can be predicted at any time while the filter is used and for any contaminant particle size distribution ingressing into the hydraulic system, so that the component life and cost for the system in the field can be theoretically estimated. For such a purpose, the theoretical equation was derived by which one can predict the variation of the flow distribution of a filter at any time and for any kind of contaminant particle size distribution ingressing into the system. Then, the multipass filter test was simulated with this theoretical equation. Unfortunately, however, the theoretical equation did not necessarily fit the experimental data. Therefore, the theoretical equation to predict the variation of the flow distribution was not used to calculate either pump life or filter life, but the experimental equation derived from the multipass filter test data using ACFTD was used. For such a reason, the life and cost analysis in this report is assumed that the contaminant particle size distribution ingressing is the same as that of ACFTD. If the distribution ingressing into a hydraulic system is different from the distribution of ACFTD, some errors will be included in the life and the cost of the filter and the pump using the theory advanced here.

The last problem to be solved is the particle size distribution of ACFTD. As mentioned previously, ACFTD is the test dust specified when the multipass filter test and the contaminant sensitivity test of a pump are conducted. If the nominal particle size distribution of ACFTD is incorrect, serious errors will be reflected on the life and cost prediction of a pump and filter.

So far, it has been commonly known that the particle size distribution of ACFTD is expressed with a straight line on a Log versus Log squared graphic paper. However, this currently used particle size distribution is considerably different, in the small size region, from that transformed from the weight distribution. In order to make the difference clear, the weight distribution and the particle size distribution of ACFTD were investigated experimentally. The weight distribution was transformed into the particle size distribution. The particle size distribution transformed was compared with that currently used. The particle size distribution currently used includes serious error in the small size region. Johnston [27] investigated the weight distribution of ACFTD and converted it into the particle size distribution. His result is very close to the author's analysis results in the small size region. However, it is incorrect in the large size region. Thus, the particle size distribution of ACFTD was revised into the correct one. The up-to-date particle size distribution was used to analyze the flow degradation characteristics of a pump and the filtration performance of a filter.

After the three above factors were analyzed, the lives of pumps were calculated and the life equation of a pump was derived. According to this analysis result, the pump life is influenced drastically by the performance of the filter. In other words, the improper selection of the filter results in a much different pump life from the expected one.

Even if the performance of a filter is influential to the pump life, the life analysis is not the sole purpose. In business, the main purpose is the pursuit of profit. The life extension or the reliability improvement of a device must be directed to making a profit except for a few special purposes such as a personal security. However, the majority of the engineers in industry tend to disregard the cost of system assessment and selection and evaluate only the performance. Such a viewpoint may be very wasteful in terms of cost. All of the decision-making should take into account the cost of the system over its full life. The improper selection of hydraulic components results in economic waste. The cost analysis of hydraulic systems is one of the most important jobs for a hydraulic engineer in the design phase. However, there are few reports concerned with the cost of a hydraulic system. The design engineer needs help in ways to minimize the cost of the hydraulic system.

Under such a background, the cost equation of the hydraulic system consisting of a filter and pump was derived. At the same time, the method of how to select the filter which minimizes the cost of the hydraulic system was developed for the case when the pump having a specified performance and the contaminant ingression rate were given. By using the cost equation and the method to determine the best filter, the cost of the hydraulic system can be minimized and the value determined.

Thus, to minimize the maintenance cost of the hydraulic system, it is necessary that the filter and pump be properly matched as to their contaminant characteristics. A matching methodology allows a design engineer to properly select the components which are necessary for optimal life cycle cost economics.

CHAPTER II

PARTICLE DISTRIBUTION IN AIR CLEANER FINE TEST DUST

ACFTD (Air Cleaner Fine Test Dust) has been widely used for many purposes in the world as well as at the FPRC, OSU. ACFTD is the test dust used most often at the FPRC. It has been used mainly for the multipass filter test to investigate the performance of a filter, or for the contaminant sensitivity test of a pump to investigate the flow degradation characteristics and for many other tests.

The particle distribution of ACFTD currently used was developed by Bench [4]. According to his work, the distribution can be expressed as a straight line on the Log-versus-Log squared scale paper. The number of particles greater than any specific diameter per 1.0 mg is represented by the following equation:

$$N = 10^{6.244 - 1.085 (\log D)^2} \qquad (1)$$

where: D = the nominal diameter of particle.

N = the number of particles greater than D per 1.0 mg.

However, the manufacture of ACFTD reports its weight distribution. When the number of particles corresponding to the diameter was transformed from the weight distribution and compared with the particle number, represented by Equation (1), serious differences were noted. That is, there is not much difference between the particle number expressed in Equation (1) and that transformed from the weight distribution in the large diameter region, but considerable difference is noted in the small diameter range. The contaminant particle distribution greatly affects the filtration performances of a filter and the flow degradation characteristics of a pump. Incorrect information regarding the particle distribution of the test dust can result in an erroneous assessment of the contaminant performance tests of hydraulic components. Johnston [27] measured the weight distribution of ACFTD and transformed it into a particle size distribution. According to his investigations, there were not any particles with diameters greater than 80 micrometers. This is obviously incorrect because ACFTD has particles with diameter greater than 200 micrometers.

Particles having large diameters are very influential on the performance degradation of various hydraulic components. A more accurate particle distribution of ACFTD was required to predict the precise lives of hydraulic components and their cost. For these reasons, the particle size distribution and the weight distribution of ACFTD were measured experimentally. The remainder of this chapter is devoted to analyzing the accuracy of the particle size distribution.

Weight Distribution and Particle Size Distribution Analysis of ACFTD With the Sedimentation Method

Several techniques have been developed for measuring the particle size distribution by sedimentation. However, the Andressen Pipette Method has reasonably good accuracy in small particle diameter range and high repeatability, and has been used successfully throughout the world for many years. This method is also inexpensive and extremely simple to operate.

The Andressen Pipette apparatus shown in Figure 1 consists of a settling cylinder and a capillary (pipette). The settling cylinder has an approximate inner diameter of 50 mm and a capacity of 500 mℓ when full. The pipette extends 20 cm below the top guage mark to about 4 cm from the bottom of the vessel. The pipette bulb has a capacity of 10 mℓ and has been provided with a three-way stopcock and spout for drainage. The fluid in the cylinder is sampled using the pipette during the experiment. The weight distribution analysis with this method is based principally on Stokes' law. The particles having larger diameters can fall faster than those having smaller diameter. If the shape of the particle is spherical, Stokes' law can be used to analyze the particle motion. However, the shape of ACFTD is not spherical but it is close to the ellipse of which the ratio of the longest diameter to the shortest diameter is about 1.5. Therefore, Stokes' law cannot be used directly in this case. A new theoretical equation in which Stokes' law is modified must be derived.

Derivation of the Theoretical Equation for an Elliptical Body Motion in a Fluid

In order to derive the theoretical equation describing the motion of ACFTD in a fluid, the following three assumptions were made:

1. Particles have elliptical bodies.
2. The particles fall according to Stokes' law.
3. The particles fall with such an attitude that their cross

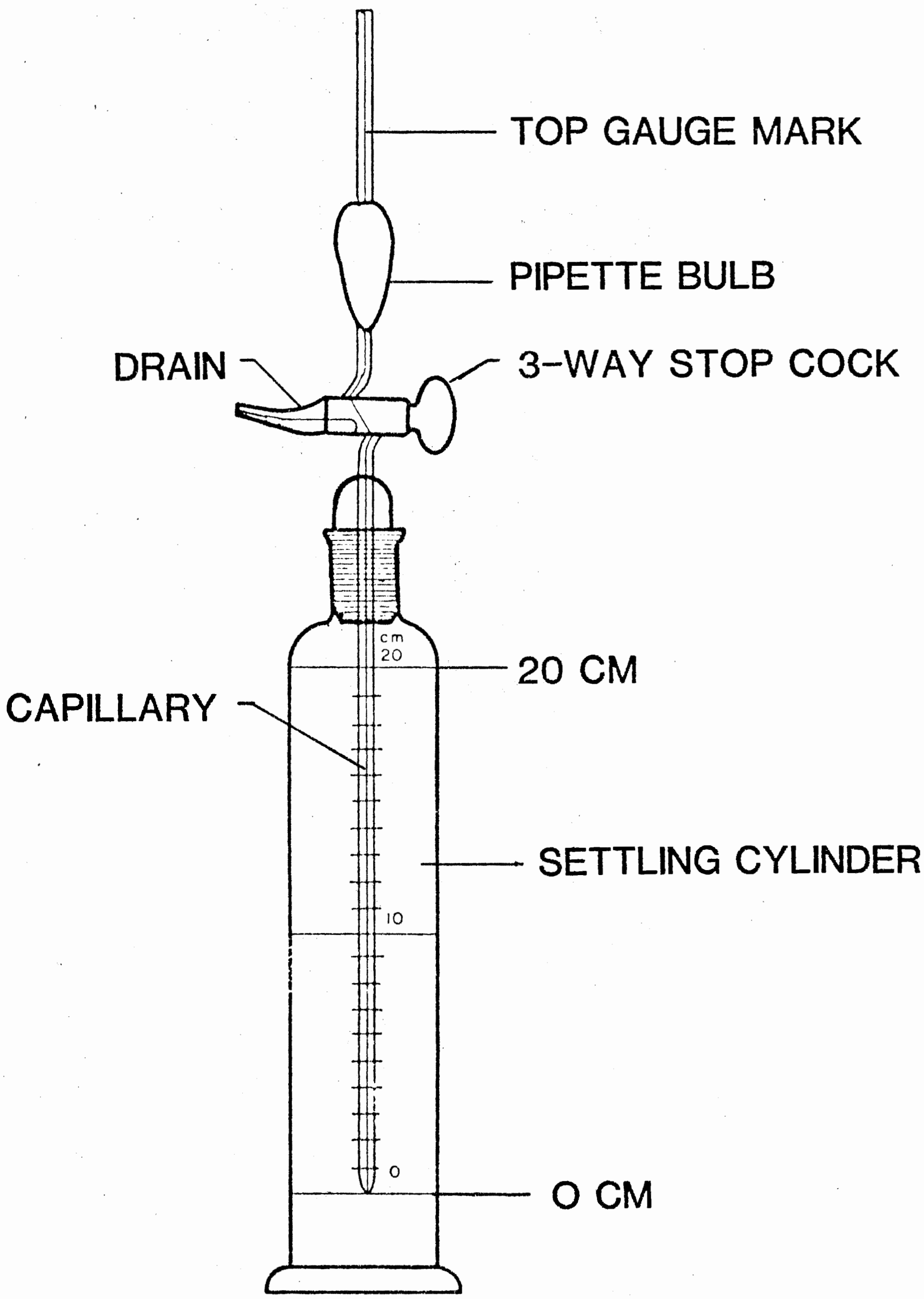

Figure 1. Andreasen Pipette Apparatus

sectional area, perpendicular to the direction of the motion, is minimum.

Under these assumptions, the theoretical equations are derived. The detail is explained in Appendix A. A particle suspended in a fluid falls through the fluid to the bottom of the cylinder. The velocity is given by the following equation:

$$U = \frac{ab}{18\mu_f} (\gamma_s - \gamma_f) \tag{2}$$

where: U = particle terminal velocity.

a = the longest diameter of the particle.

b = the shortest diameter of the particle.

μ_f = visosity of fluid.

γ_s = specific gravity of solid particle.

γ_f = specific gravity of fluid.

The above equation is valid only when the Reynolds' number, $R_e = bU\gamma_f/(\mu_f g)$, is less than 0.6, where g is the gravitational constant.

The time required for the particle to travel the distance of h is given by

$$t = \frac{h}{U} = \frac{18\mu_f h}{ab(\gamma_s - \gamma_f)} \tag{3}$$

where: h = the travel distance of a particle.

From microscopic investigation, the ratio of the longest diameter to the shortest diameter is given by

$$\frac{a}{b} = 1.5 \tag{4}$$

Substituting Equation (4) into Equation (3) yields

$$t = \frac{27\mu_f h}{a^2(\gamma_s - \gamma_f)} \tag{5}$$

If h is defined as the distance from the surface of the suspending fluid to any point in the fluid of interest, then Equation (5) physically means that at t seconds after the particles which were mixed homogeneously with the fluid in the cylinder are settled, any particles having the longest diameter greater than "a" will pass through the point where the distance from the fluid level is h. The fluid at this point contains the same quantity of particles smaller than and equal to the diameter of "a" as the fluid had initially. Therefore, if the fluid at this point is sampled, the weight of the contaminant having diameters smaller than "a" can be analyzed by measuring the weight of the contaminant in the fluid. In the experiment, distilled water containing two percent of sodium pyrophosphate ($Na_4P_2O_7 \cdot 10H_2O$) was used for the fluid. The amount of contaminant added was one percent of the fluid by weight. The bottle containing the suspending fluid and the contaminant was shaken for fifteen minutes so that they were mixed homogeneously. Then 10 mℓ of the fluid was sampled from the bottle in order to weigh the contaminant in the fluid.

Once the weight of the contaminant, W_s, in the fluid of 10 mℓ is measured, the weight percent, W_c, smaller than each particle size is calculated using the following formula:

$$W_c(\%) = \frac{V_t W_s}{10 M_t} \times 100 \tag{6}$$

where: W_c = weight percentage of the contaminant smaller than any specified diameter.

V_t = total volume of the fluid in the cylinder.

W_s = the contaminant weight in the sample fluid of 10 mℓ.

M_t = the total weight of the contaminant added initially.

The experimental results are plotted in Figure 2. Although this graph is presented on Weibull probability paper, the correlation between the weight of ACFTD smaller than a specified diameter and the particle diameter is expressed as a straight line in the graph. Therefore, the weight distribution of ACFTD is a Weibull distribution and its cumulative weight distribution and density function are given by

$$F_w(a) = 1 - \mathrm{Exp}\{ - (\frac{a}{19.5})^{0.84}\} \qquad (7)$$

$$f_w(a) = 0.0693\ a^{-0.16}\ \mathrm{Exp}\{ - (\frac{a}{19.5})^{0.84}\} \qquad (8)$$

where: $F_w(a)$ = cumulative weight distribution.

$f_w(a)$ = density function of weight distribution.

Transformation of Weight Distribution to Particles Size Distribution

Once the density function of a weight distribution is known, the particle size distribution and its density function can be derived. Suppose that the density function of a weight distribution was transformed into the density function of the particle size distribution, as shown in Figure 3 (A), (B).

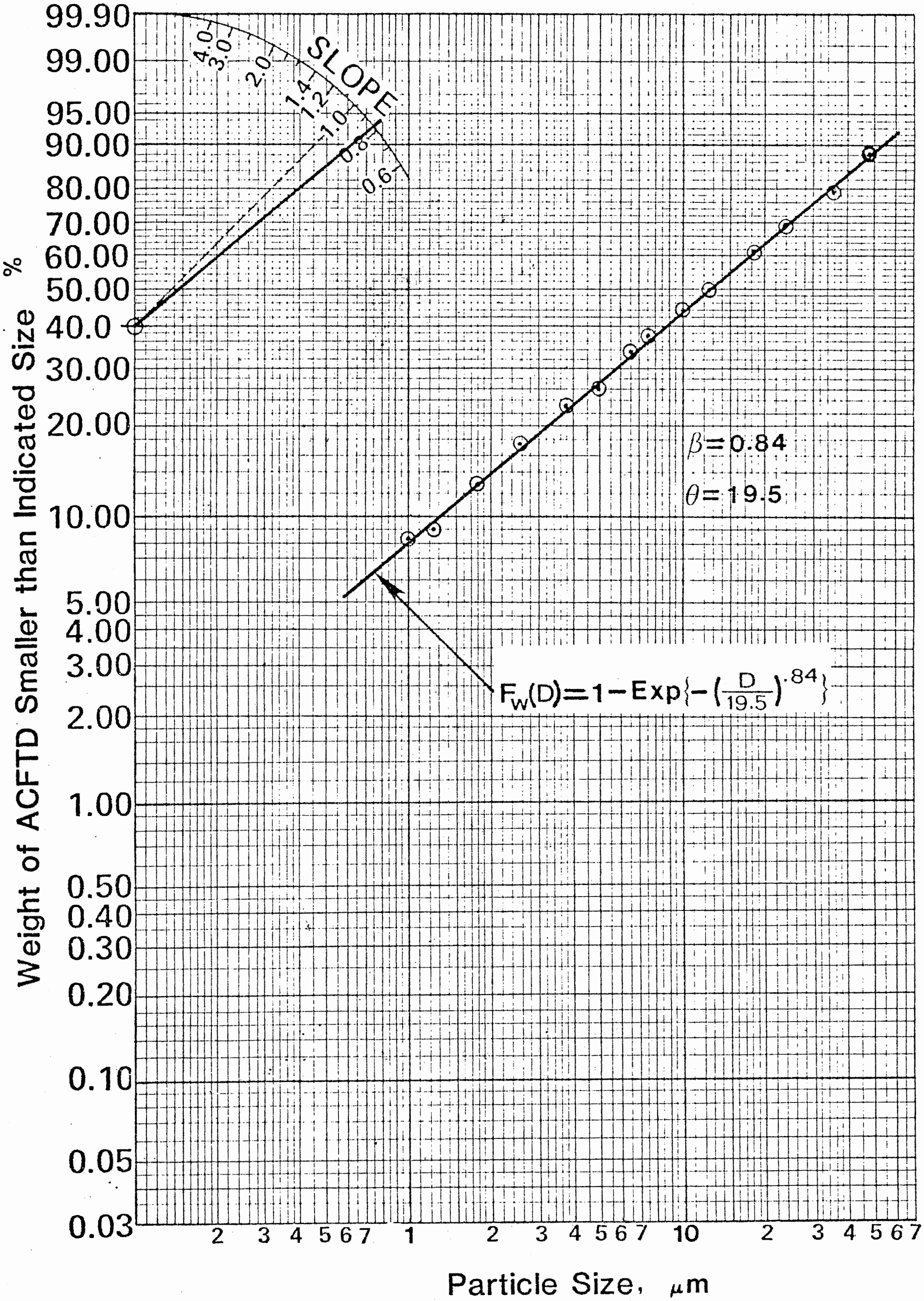

Figure 2. Weight Distribution of ACFTD

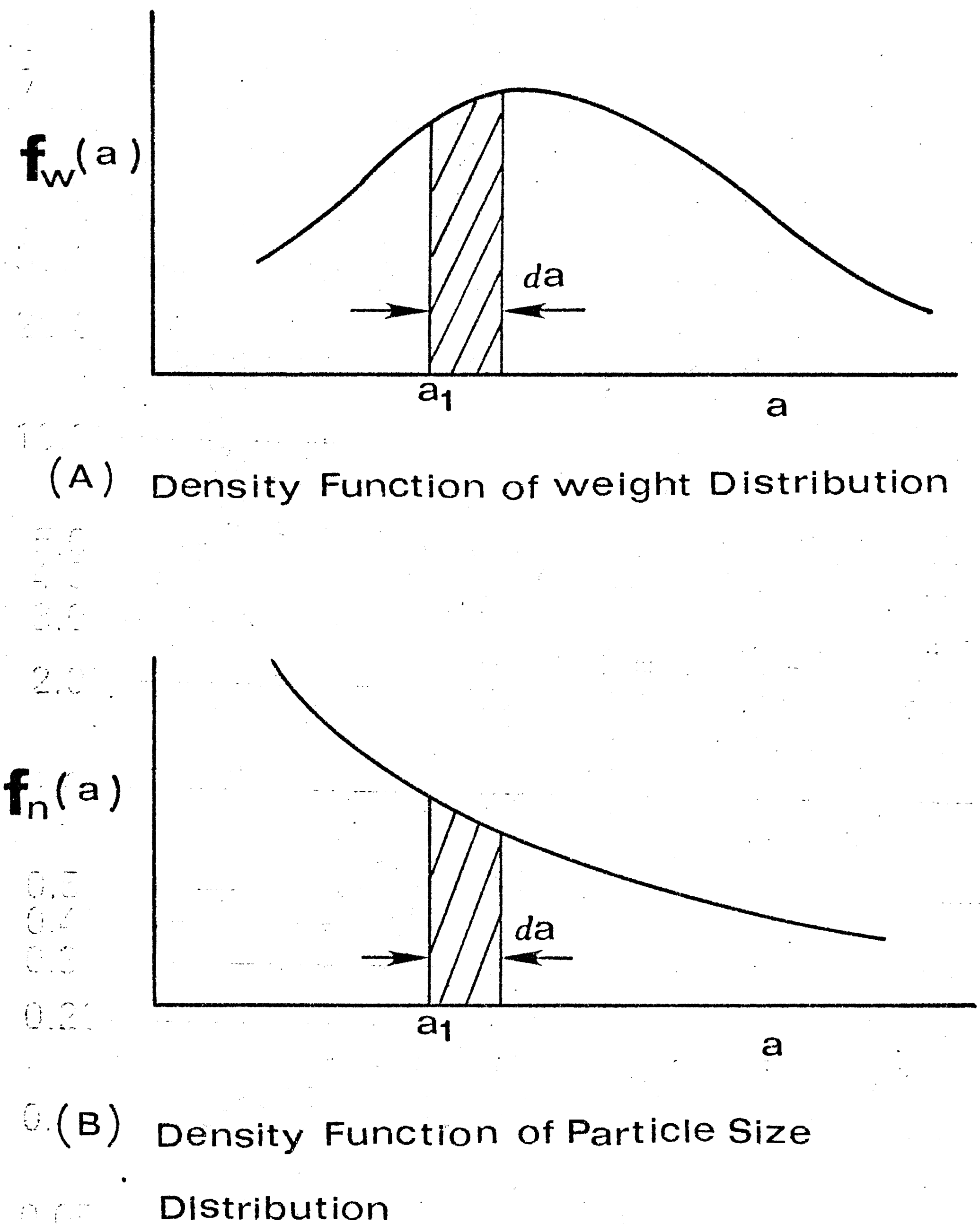

Figure 3. Correlation between the Density Function of Weight Distribution and that of Particle Size Distribution

The weight of particles which exists within a_1 and $a_1 + da$ must be equal for both density functions. Therefore, the following equation is described:

$$f_w(a)da = f_n(a)da\frac{\pi}{6}\, a^3\gamma_s\frac{1}{S_f}\ . \qquad (9a)$$

Therefore,

$$f_n(a) = \frac{6S_f}{\pi\gamma_s a^3}\, f_w(a) \qquad (9b)$$

where: $f_n(a)$ = density function of the particle size distribution.

S_f = shape factor of particle; the ratio of the volume of the sphere having the same diameter as the nominal diameter of a particle to the real volume of the particle; in this case, S_f = 2.25.

For ACFTD, γ_s = 2.80 g/cm^3 = 2.8 x 10^{-9} mg/cubic micrometer.

Therefore, Equation (9b) is modified as:

$$f_n(a) = 1.53 \times 10^9 a^{-3} f_w(a). \qquad (10)$$

Substituting Equation (8) into Equation (10) yields

$$f_n(a) = 1.06 \times 10^8 a^{-3.16}\mathrm{Exp}\{ - (\frac{a}{19.5})^{0.84}\}. \qquad (11)$$

Then,

$$F_n(a) = 1.06 \times 10^8 \int_{\infty}^{a} x^{-3.16}\mathrm{Exp}\{ - (\frac{x}{19.5})^{0.84}\}dx \qquad (12)$$

where: $F_n(a)$ = (inverse cumulative) particle size distribution.

x = dummy variable.

Equation (12) cannot be solved mathematically. The only way to find the solution of Equation (12) is to use a digital computer. By using a digital computer, the particle size distribution was solved numerically. The results are shown in Figure 3. At the same time, the data which was investigated by microscopic counting and the particle size distribution currently used are superimposed to make the comparison between them.

Particle Size Distribution with the Microscopic Method

If the particle number transformed from the weight distribution is reliable, the additional work required to analyze the particle size distribution is not needed. If the assumptions which were made when the theoretical equations for transformation were derived are correct, the particle number transformed from the weight distribution is also correct. But some assumption is not entirely correct. For example, although the first assumption was that particles have an elliptical shape, the particles of ACFTD are not perfectly elliptical. As a result, the actual downward velocity of the particles in the sedimentation test become different from the theoretically calculated value, and some error is noticed in the particle size distribution.

For these reasons, the particle numbers of ACFTD were counted microscopically. The experimental procedure and results for the microscopical counting will be shown in the following sections.

Experimental Procedure of Microscopical Analysis

In order to count the particle number of ACFTD, ACFTD of 10 mg was mixed with 400 cm^3 of clean oil (MIL-H-5606). After this original sample was shaken by a paint shaker for 15-20 minutes, a volume of 1-5 cm^3 (25-125 microgram of ACFTD), taken from the original sample was drafted into another bottle and was diluted with clean oil. The diluted sample was filtered with a Milipore Pad and a particle number on the pad was counted by using a microscope. Since it is a very time consuming job to measure particle diameters over the entire area of the pad, the particle number in a part of the area of the pad was counted per SAE/ARP 598.

Usually, the shape of ACFTD particles is not round. The longest particle dimension was measured to indicate the diameter of a particle. A particle number corresponding to the longest diameter was counted. For a couple of hundred particles, the shortest diameter, as well as the longest, was measured in order to get the ratio of the longest diameter to the shortest diameter.

As the particles having a large diameter are extremely few, an adequate confidence level is not achieved by the data in the large diameter region if the whole ACFTD distribution is used as the sample dust according to the above procedure. In order to achieve a high confidence level for the experimental data, the dust from which small particles were removed was used with the particle number in the large diameter region. That is, to count the particles between 2.5 micrometers and 15 micrometers, the whole ACFTD was used. When the particles between 15 micrometers and 80 micrometers were counted, an adjusted sample of ACFTD which is excluded of the particles smaller than 5 micrometers was used. Also, an adjusted sample of ACFTD which is excluded of the particles smaller than 50 micrometers was used to count the particles larger than 80 micrometers. These test dusts are listed in Table I. Thus, the particle numbers were counted using three different kinds of dust.

Experimental Result of Microscopic Counting

The particle number counted by the microscopic method is plotted in Figure 4. In the figure, the data transformed from the weight distribution and the particle size distribution currently used are superimposed.

Comparing the microscopically counted data and the data transformed from the weight, both data coincide well within the experimental error in the region of 1 micrometer up to 15 micrometers. However, in the range of the particle diameter larger than 15 micrometers, there are considerable differences. It is thought that this difference is mainly caused by the experimental error of the Andreasen pipette method. That is, some fluids are sampled from the settling cylinder. The contaminated fluid is sampled very soon after the dust and the fluid are mixed in order to analyze the particles in the large diameter region. But, occasionally, the fluid in the cylinder does not settle soon and some large particles move around in the cylinder, thus the error arises. However, for the small diameter particle, the settling time (the time between shaking and sampling) is very long. Usually, it is 35 minutes to 50 hours. Therefore, the error due to the initial turbulence of the

TABLE I

DIAMETER RANGE OF TEST DUST

Diameter Range Counted	Diameter Range of Test Dust
0 - 15 μm	$0 < D$
20 - 80 μm	$5 < D$
80 - 200 μm	$50 < D$

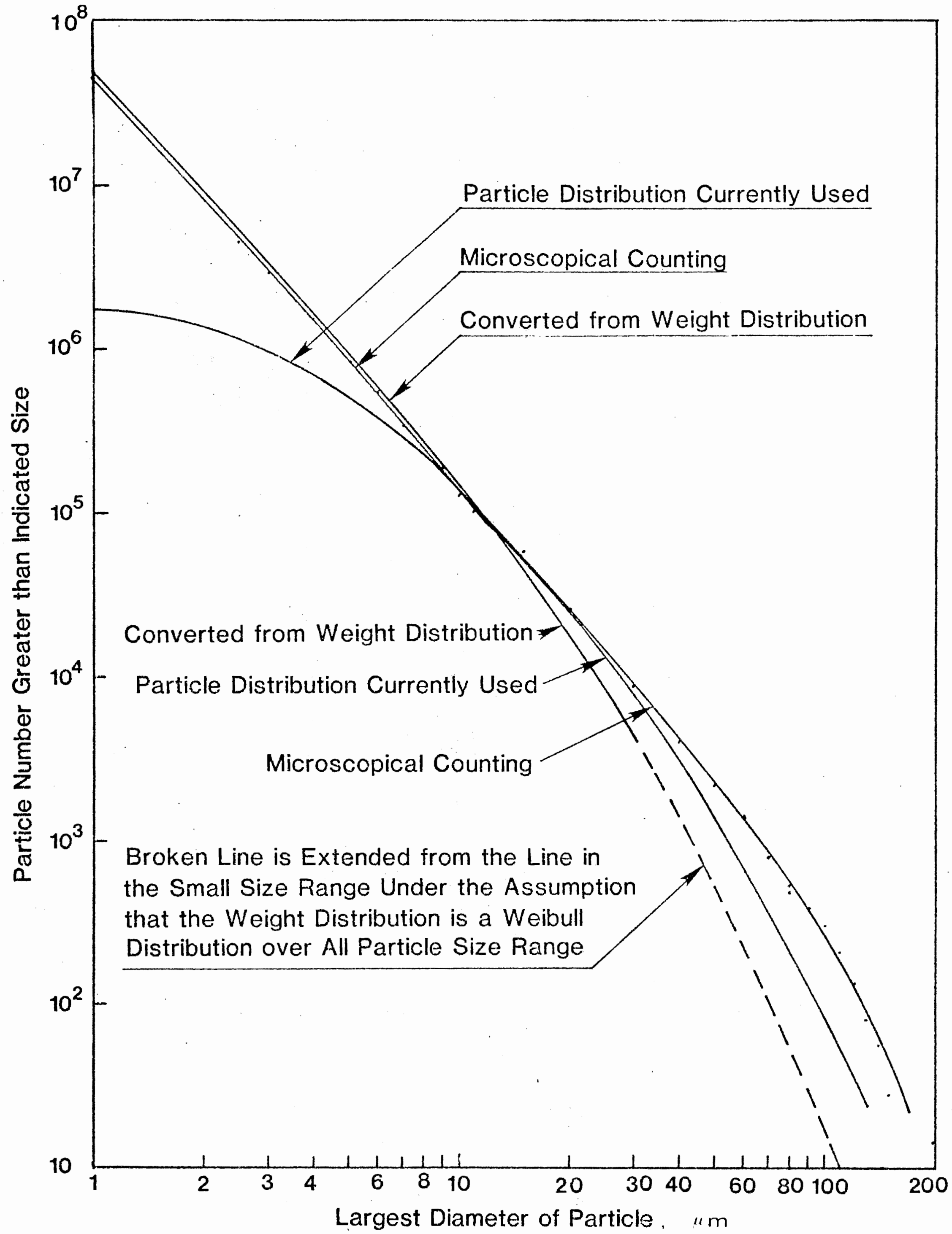

Figure 4. Comparison of Particle Size Distribution

fluid is eliminated. For this reason, the particle number transformed from the weight distribution is not accurate in the large diameter region.

If the particle size distribution currently used is compared with the others, the distribution currently used is considerably different in the small diameter region and in the very large diameter region. But there are minute differences between them in the middle diameter region of about 8 micrometers to 30 micrometers. The small particles are influential on the performance degradation of a high quality filter. Large particles severely effect the leakage characteristics of a pump when it is used with a filter having poor performance. For such reasons, the error of the particle number in the small diameter region results in an incorrect assessment of a high quality filter when it is tested with the multipass test procedure of the ISO standard. Also, the particle counting error in the large diameter region results in an incorrect prediction for pump life. Therefore, to use the correct particle distribution over the entire particle size range is very important for correctly assessing the performance of hydraulic components relative to the contaminant.

Which of the three lines represents the correct particle size distribution of ACFTD? In the small diameter region, the particle distribution counted microscopically coincides well with that transformed from the weight distribution. Therefore, it can be concluded that the distribution currently used is incorrect in this region. However, in the large diameter region, the three lines differ significantly from each other. As mentioned above, the weight distribution is not accurate in the large diameter region. Therefore, the distribution transformed from the weight distribution can be omitted from the consideration. Observing the remaining two lines, it can be said that the particle distribution counted microscopically presents the correct particle distribution of ACFTD for the following reasons:

1. The particle counting in the small diameter region was accurate enough. That in the large diameter region is easier and more reliable than in the small diameter region because the particle diameter is larger. Therefore, the particle counting in the large diameter region might be accurate enough.

2. The data for the particle size distribution counted microscopically was collected from three different kinds of dust (see Table I). But each data is plotted on the continuous line. Therefore, this data are reliable and the particle size distribution is correct.

Thus, it is decided that the particle size distribution counted microscopically represents the true particle distribution of ACFTD. In this paper, this up-to-date particle distribution is used to analyze the flow degradation characteristics of a pump in Chapter III and the filtration performance characteristics of a filter in Chapter IV.

The best fit curve for the up-to-date particle size distribution is drawn in Figure 4. The mathematical expression of the best fit curve is given by

$$N = 2.22 \times 10^5 (200 - D) D^{-2.464}, \quad 1 \leq D \leq 150 \text{ micrometers} \tag{13}$$

where: N = inverse cumulative particle size distribution; that is the particle number greater than indicated size.

D = the longest diameter of a particle.

This equation fits well the actual data in the range of 1 micrometer to 150 micrometers. The particle number greater than 150 micrometers must be referred to the graphical expression.

CHAPTER III

THEORETICAL ANALYSIS OF HYDRAULIC PUMP PERFORMANCE

Theoretical Derivation of Pump Life Equation

The performance of hydraulic components is degraded during normal operation by the contaminants entrained in the oil. The performance degradation depends upon the contaminant sensitivity of the component and the contaminant distribution in the oil. When the performance degradation exceeds some specified level, it is decreed that failure of the component has occurred. So far, little theoretical work has been accomplished concerning the effect of contaminants on hydraulic component life. A detailed analysis has been performed only on pumps and motors; analysis of other components is just beginning. The pump, however, is recognized as the most unreliable component and as the most expensive. Therefore, a theoretical analysis of the contaminant service life of the pump is very important from a reliability or economic standpoint.

When a pump is exposed to a full contaminant distribution, the flow degradation ratio is expressed by the following equation [1];

$$\frac{dQ}{Q} = - \left[\sum_{i=1}^{i_{max}} \alpha_i \, n_i^2 \, (t) \right] dt \qquad (14)$$

where: Q = flow rate of pump.

i = size of the particle; $i = 1 \ldots D = 0 \sim 5\mu m$,

$i = 2 \ldots D = 5 \sim 10 \mu m$, ..., and so on.

α_i = contaminant wear coefficient for size interval i.

n_i = particle concentration in the fluid in the size interval i.

If the above differential equation is solved, the time interval over which the flow rate decreases from its initial value to the failure level is given by:

$$T_p = \frac{- \ln(Q_T/Q_0)}{\sum_{i=i}^{imax} \alpha_i \, n_i^2 \, (t)} \tag{15}$$

where: T_p = life of pump.

Q_0 = initial flow rate.

Q_T = flow rate at $t = T$.

The above equation is called the service life equation of a pump. In this equation, if the ratio Q_T/Q_0 is constant, the life of the pump is a function of the contaminant wear coefficient, α_i, evaluated from laboratory tests and the particle concentration for the various size intervals, n_i. The particle concentration is influenced by the performance of the filter used in the hydraulic system and will be analyzed in the next chapter.

Flow Degradation Characteristics and Contaminant Wear Coefficient Analysis of Pump

As shown in Equation (15), the life of a pump is the function of the contaminant wear coefficient and the particle concentration in the oil. The contaminant wear coefficient is the inherent property of the pump and can be analyzed from the test data conducted by the "Contaminant Sensitivity Test for a Pump" specified in ISO standard. In this section, a new method to analyze the pump contaminant wear coefficient will be developed.

In order to analyze the flow degradation characteristics of the pump, several methods have been tried. A typical method is the cubic fit method [1]. This theory states that the flow degradation of the pump due to contaminant can be expressed as a function of the clearance space and the space cubed, as shown in the following equation:

$$Q_f = Q_r(1 - AD_c - BD_c^3) \tag{16}$$

where: Q_f = the flow rate after the pump is exposed to the classified test dust having the maximum diameter of D_c.

Q_r = the rated flow of pump.

A,B = constant coefficients.

D_c = maximum diameter of the classified test dust.

This approximation is based upon the leakage flow theory of a small clearance. Although it seems to be reasonable since it fits well with some pump data; it does not fit for all pumps. The constant coefficients A and B in Equation (16) and the contaminant wear coefficient α_i in Equation (15) must always be positive. But values of A and α_i are sometimes negative when they are analyzed by using Equation (16). Thus considerable error will arise in the prediction of the hydraulic pump life.

The other methods to analyze the flow degradation characteristics of a pump are straight line approximations on Normal or Log-Normal probability paper [37, 18]. The test data of a few pumps can be approximated by a straight line on this probability paper, but the majority of the data cannot. In addition, the cubic fit method is used to predict pump life. Principally, the cubic fit method and the straight line approximation on the above probability papers contradict each other. Therefore, the Normal or Log-Normal approximation includes many theoretical errors. Hence, in order to predict a pump life more accurately and easily, many approaches have been tried. As a result, the leakage flow ratio versus the maximum diameter of the classified test dust injected into the hydraulic system can be plotted as a straight line on a Log-Log graph paper up to at least 20 or 30 percent of the leakage flow ratio for most of the pumps tested according to ISO standards at the FPRC, OSU. Where the leakage flow ratio is defined as the ratio of the leakage flow to rated flow of the pump;

$$y = \frac{Q_r - Q_f}{Q_r} \tag{17}$$

where: y = the leakage flow ratio.
Examples of several kinds of the flow degradation characteristics are shown in Figure 5.

The data fit straight line well except for the small diameter region. The data dispersion in the small diameter region is the cause of experimental measuring error. The leakage flow ratio is very low in the small diameter region; thus, the percentage of error in this region is increased. From this figure it is assumed that the leakage flow ratio can be expressed as a straight line on Log-Log graph paper up to 30 percent of the leakage flow ratio. This characteristic is very

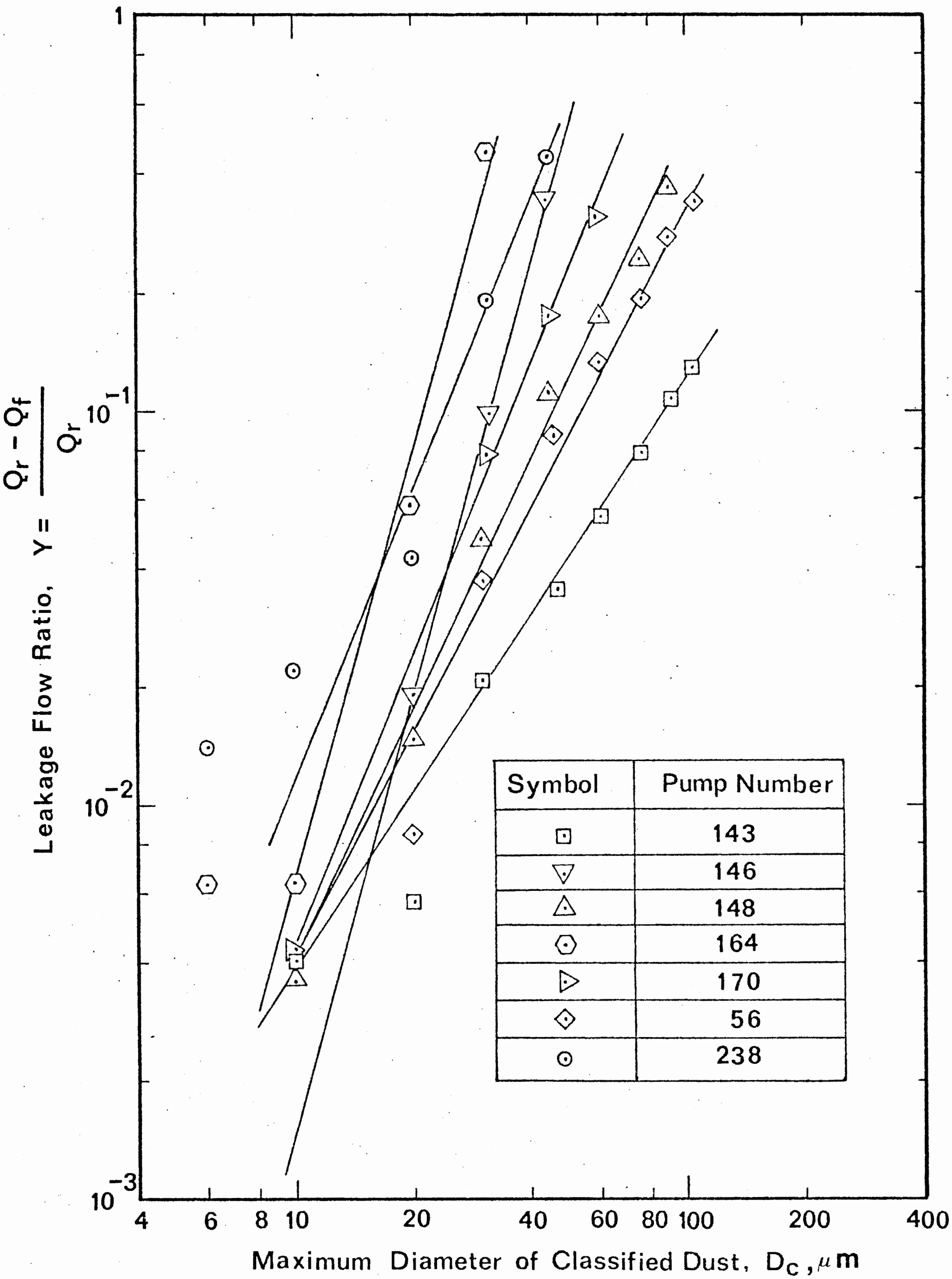

Figure 5. Leakage Flow Characteristics of Pumps

convenient to analyze the performance of a pump. The pump performance can be expressed exactly if two data points are given. From the point of view of accuracy and application to various kinds of pumps, the leakage flow ratios at 10 and 30 micrometers of the maximum diameter of the classified dust are used to evaluate the flow degradation characteristics of a pump. Then, the leakage flow ratio for any test dust having the maximum diameter of D_c is given by:

$$y = y_{10}\left(\frac{D_c}{10}\right)^{2.096 \log(y_{30}/y_{10})} \tag{18}$$

where: y_{10}, y_{30} = value of y at D_c = 10, 30 micrometers, respectively.
Combining Equation (17) and Equation (18), yields:

$$Q_f = Q_r\{1 - y_{10}\left(\frac{D_c}{10}\right)^{2.096 \log(y_{30}/y_{10})}\}. \tag{19}$$

The ACFTD used for the contaminant sensitivity test includes particles larger than 200 micrometers. If the leakage flow ratios in Figure 5 are extended upward, some of the leakage flow ratios may be greater than one in the large particle diameter region. In other words, the leakage flow rate (Q_r - Q_f) becomes greater than the rated flow Q_r. This is caused by the high gravimetric level of the system fluid (300 mg/ℓ). In the "Pump Contaminant Sensitivity Test" procedure in the ISO standard, a 300 mg/ℓ contamination level is specified to be used in order to determine the flow degradation in relatively short periods of time by testing at high stress level. If the contaminant concentration of the system fluid is small enough, the leakage flow rate would never be over the rated flow within 105 micrometers of the particle diameter. Thus with this concentration, the contaminant wear coefficient, α_i, can be analyzed over the contaminant size range injected into the test circuit of the pump. The leakage flow ratio, y, for 300 mg/ℓ contamination level must be converted into the new leakage flow ratio for lower contamination level. From Equation (15), a relationship between the particle concentration and flow degradation ratio, Q_f/Q_o. for a pump is given by:

$$\left(\frac{Q_f}{Q_o}\right)_u = \mathrm{Exp}\left\{\left(\frac{n_u}{n_v}\right)^2 \ell n\left(\frac{Q_f}{Q_o}\right)_v\right\} \qquad (20)$$

where: $\left(\frac{Q_f}{Q_o}\right)_u$, $\left(\frac{Q_f}{Q_o}\right)_v$ = final to initial flow ratio for particle concentration u, v mg/ℓ, respectively.

n_u, n_v = particle number in unit volume for particle concentration u, v mg/ℓ, respectively.

By using the above correlation, the normalized flow degradation ratio or the normalized leakage flow ratio is calculated. Then the normalized leakage flow rate, $(Q_r - Q_f)$, is always less than the rated flow.

When the leakage flow ratio is close to one, there is no evidence that the leakage flow ratio is still on the straight line extended from the lower leakage flow ratio. Perhaps, the leakage flow ratio close to one may not be on the extension of the straight line. Therefore, for the leakage flow ratio exceeding 20 percent, the gradient of the leakage flow characteristic line where it crosses the 20 percent level of the leakage flow ratio was extended after each leakage flow ratio had been normalized. For example, in Figure 6 the original flow ratio characteristic line derived by the "Pump Contaminant Sensitivity Test" of the ISO standard crosses the 20 percent level of the leakage flow

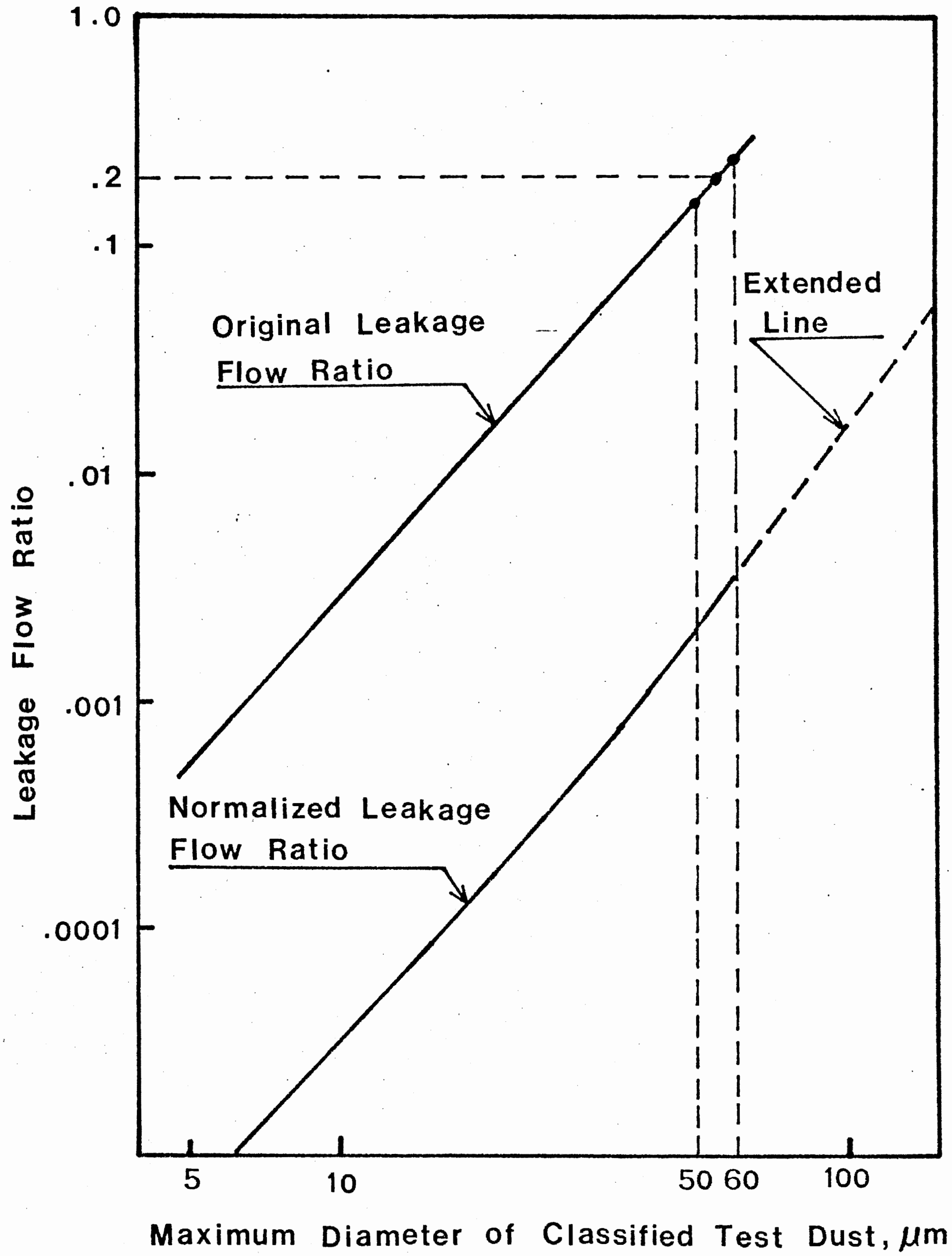

Figure 6. Schematic Explanation on the Normalization of Leakage Flow Ratio Close to One

ratio between 50 and 60 micrometers particle size. In order to obtain the normalized leakage flow ratio in the region of the particle diameter greater than 60 micrometers, the normalized leakage flow ratio characteristic lines between 50 and 60 micrometers were extended.

Once the normalized flow degradation ratio for the classified test dust is given, the contaminant wear coefficients for the particle size interval can be calculated by the following equations [1]:

$$\alpha_1 = \frac{-2}{\tau n_1^2} \ln\left(\frac{Q_f}{Q_o}\right)_1 , \quad \text{for dust of 0~5 micrometers} \tag{21}$$

$$\alpha_i = \frac{-2}{\tau n_{i/r}^2} \left\{ \ln\left[\left(\frac{Q_f}{Q_o}\right)_r - \sum_{j=1}^{i-1} \left(\mathrm{Exp}\left(\frac{\alpha_j \tau n_{j/r}^2}{2}\right) - 1 \right) \right] \right\}, \tag{22}$$

for dust above 5 micrometers

where: τ = the time constant of the particle destruction process in the test circuit, = 9 minutes.

r = contaminant size range; $r = 1$ ... D = 0~5, $r = 2$ ... D = 0~10, $r = 3$ ... D = 0~20, and so on.

$n_{i/r}$ = particle number in unit volume of a size interval "i" in the size range "r".

Thus, once the flow degradation characteristics of a pump are known by the test conducted according to the ISO test procedure, the contaminant wear coefficient can be analyzed by a computer manipulation. Then, if the particle distribution of the contaminant upstream of the pump is known, the life of the pump can be calculated by substituting these particle concentration values into Equation (15).

CHAPTER IV

DEVELOPMENT OF THE FILTER PERFORMANCE EQUATIONS AND THEIR CHARACTERISTICS

Development of the Flow and Pore Size Distribution

As shown in Equation (15), the life of a pump is a function of the square of the contaminant particle concentration in the hydraulic fluid. This concentration is influenced considerably by the performance of the filter used in the hydraulic system. The performance of the filter varies with time as it is used in the system. If the filter performance is inadequately evaluated, considerable error results when the life of the pump is estimated theoretically. For this reason, it is necessary to derive theoretical equations which can be used to accurately evaluate filter performance.

Currently, the Beta ratio is employed to rate and evaluate filter performance. However, this "figure-of merit" is influenced not only by the separation characteristics of the medium, but also by the particle distribution upstream of the filter. Thus, the Beta value cannot be used to precisely describe the performance of a filter. Therefore, a new parameter to evaluate filter performance is needed. This section will be devoted to deriving a new parameter that will precisely indicate filter performance.

Ingressed contamination is intended to be trapped by the hydraulic system filter media. The remaining contamination passes through the filter and circulates throughout the hydraulic system. If the performance of the filter is not good, the contaminant concentration in the hydraulic oil is increased and the hydraulic components are quickly worn out by the abrasive action of the contaminants. In order to derive the theoretical equations for the analysis of filter performance, the following assumptions are made:

1. The filter matrices or media are two-dimensional.
2. Particles having diameters equal to or greater than the diameter of a given pore are trapped and retained 100 percent of the time by the pore, and those smaller than the designated diameter of the pore escape 100 percent of the time.
3. Fluid flow through the filter pores is laminar.

The number of particles caught by the filter media can be expressed by:

$$\begin{aligned} Z_1 &= X_{11}\eta_{11} + X_{21}\eta_{21} + \cdots\cdots + X_{\lambda 1}\eta_{\lambda 1} \\ Z_2 &= X_{12}\eta_{12} + X_{22}\eta_{22} + \cdots\cdots + X_{\lambda 2}\eta_{\lambda 2} \\ &\text{- - - - - - - - - - - - - - - - - - -} \\ Z_{\phi} &= X_{1\phi}\eta_{1\phi} + X_{2\phi}\eta_{2\phi} + \cdots\cdots + X_{\lambda\phi}\eta_{\lambda\phi} \end{aligned} \tag{23}$$

where: Z_{ℓ} = the number of particles having a diameter of ℓ(μm) that are caught by the filter.

$X_{k\ell}$ = the number of particles having a diameter of ℓ come in contact with filter pores having a diameter of k.

$\eta_{k\ell}$ = the filtration efficiency which corresponds to k, ℓ.

λ, ϕ = the particle and the pore having the maximum diameter.

A general expression for the above equations is

$$Z_{\ell} = \sum_{k=1}^{\lambda} X_{k\ell}\eta_{k\ell} \tag{24}$$

The number of particles having a diameter of ℓ that attempt to pass through pores having a diameter of k, $X_{k,\ell}$ is expressed in Equation (25) as the product of the particle concentration in the oil, the flow rate, and the time interval.

$$X_{k\ell} = \Omega_{\ell} Q_k \Delta t \tag{25}$$

where: Ω_{ℓ} = the concentration of particles within the oil that have a diameter of ℓ.

Q_k = the partial flow rate; the flow rate that passes through pores of a given diameter. In reality, the pores within a filter may not be round. Q_k is the equivalent flow rate through round pores with a diameter of k.

Δt = a short time interval.

According to the second assumption, the filtration efficiency that corresponds to the particle diameter, ℓ, and pore diameter, k, can take a value of "1" or "0" as shown in Equation (26) and Figure 7.

$$\eta_{k\ell} = \begin{cases} 1 & \text{if} \quad \ell \geq k \\ 0 & \text{if} \quad \ell < k \end{cases} \tag{26}$$

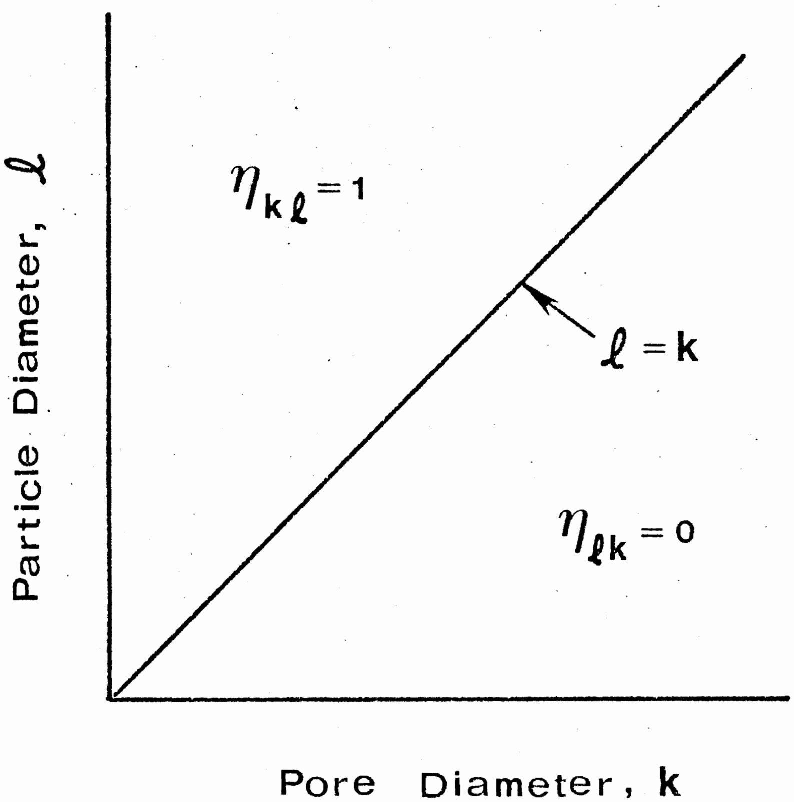

Figure 7. Illustration of Filtration Efficiency

By substituting Equation (25), (26), into Equation (24), the number of particles caught by the filter can be obtained by the following equation:

$$Z_{\ell} = \Omega_{\ell} \Delta t \sum_{k=1}^{\ell} Q_k. \tag{27}$$

An incremental filtration efficiency has also been used to express filter performance. The number of particles caught by a filter can be expressed by the following equation:

$$Z_{\ell} = \zeta_{\ell} \Omega_{\ell} Q_r \Delta t \tag{28}$$

where: ζ_{ℓ} = incremental filtration efficiency; the filtration efficiency for particles having a diameter of ℓ.

By equating Equation (27) and (28), the incremental filtration efficiency can be expressed by the following equation:

$$\zeta(\ell = D) = \frac{1}{Q_r} \sum_{k=1}^{\ell} Q_k. \tag{29}$$

In the continuous case, the partial flow rate Q_k is expressed as an integration of the flow density function over a specified interval such that

$$Q_k = Q_r \int_{k-\Delta k}^{k} q(x)dX \tag{30}$$

where: q = flow density function

X = dummy variable

Δk = small interval of pore size

when $\Delta k = 1$, Equation (29) is modified using Equation (30) as follows:

$$\zeta(D) = \frac{1}{Q_r}(Q_1 + Q_2 + \cdots\cdots + Q_D)$$

$$= \int_0^1 q(k)dk + \int_1^2 q(k)dk + \cdots\cdots + \int_{D-1}^D q(k)dk \qquad (31)$$

$$= \int_0^D q(k)dk.$$

Equation (31) shows that the incremental filtration efficiency, for contaminants having a diameter of D, $\zeta(D)$, is expressed as an integral of the flow density function, q(k), over a pore size range of zero through D. In other words, the incremental filtration efficiency for a particle diameter, D, equals the percentage of the total flow rate of oil that passes through pores having a diameter smaller than the particle diameter, D. If the flow density function is known, then the performance of a filter can be exactly specified.

The incremental filtration efficiency is the ratio of the number of particles caught by the filter over the number of particles upstream of the filter for a specified particle diameter. If test contaminants could be developed over a wide particle size range that have narrow increments of specific particle diameters, the incremental filtration efficiency could be very easily determined by experimentation. Unfortunately there are no such test contaminants available. It would be very convenient if the flow density function could be derived using multipass filter test data derived from tests conducted by ISO standards, since such test is very easy to conduct and produces stable data.

When the upstream and downstream particle distributions are known, the density functions of the particle concentrations are derived as follows:

$$Z_u(D) = -\frac{dN_u(D)}{dD}$$
$$Z_d(D) = -\frac{dN_d(D)}{dD} \qquad (32)$$

where: z = the probability density function of the particle concentration; the probability density of particles having a specified diameter per unit volume.

N(D) = the number of particles greater than a specified particle diameter; the inverse cumulative distribution of the contaminant concentration.

Subscripts u or d = upstream or downstream, respectively, of the filter media. By definition, the incremental filtration efficiency is expressed by the following equation:

$$\varsigma(D) = \lim_{\Delta D \to 0} \frac{Z_u(D)\Delta D - Z_d(D)\Delta D}{Z_u(D)\Delta D}$$
$$= 1 - \frac{Z_d(D)}{Z_u(D)} . \qquad (33)$$

The equating of Equation (31) and (33) results in the following:

$$q(k = D) = \frac{d\,\varsigma(D)}{dD}$$
$$= \frac{-d\{Z_u(D)/Z_d(D)\}}{dD} \qquad (34)$$

Equation (34) shows that the flow density function of a filter can be derived by using multipass filter test data.

Once the flow density function is derived, the cumulative flow distribution (it is simply called the flow distribution in this study) and the pore size distribution can be obtained. The flow distribution can be obtained by integrating the flow density function from a pore size of zero through any arbitrary pore size as shown in the following equation:

$$R(k) = \int_0^k q(x)dx \tag{35}$$

where: R = flow distribution.

By the second assumption, particles having diameters equal to or greater than the diameter of a given pore are trapped and retained by the pore 100 percent of the time while all other particles escape 100 percent of the time, the flow distribution expressed by Equation (35) is exactly equal to the incremental filtration efficiency expressed by Equation (31) when k = D.

$$R(k) = \zeta(D = k). \tag{36}$$

The flow through a very fine tube is restricted as reflected by the Hagen-Poiseulle law. This law is applicable for the oil flow through pores of a filter media. The number of pores having a specified diameter is expressed by the following equation:

$$M(k) = \frac{128\mu_f L\, Q(k)}{\pi k^4 P} \tag{37}$$

where: $M(k)$ = the number of pores of a filter media having a diameter of k.

L = the length of the pore.

P = the pressure drop across the filter media.

In order to facilitate the understanding of the theoretical analysis, the relationship of the above describing equations is shown in block diagram form in Figure 8. By using these theoretical equations, filter performance can be analyzed from multipass filter test data. The flow distribution and the pore size distribution of a filter derived by the above analysis techniques are inherent properties of a filter and are not influenced by any external conditions such as the particle size distribution of the contasminant.

On the other hand, the filtration ratio, β, which has been widely used in fluid power industries is affected by the particle size distribution of the contaminant used in the multipass filter test; the value of β is different when the contaminant having a different particle size distribution is used, even if the filter performance is the exactly same. A theoretical discussion follows that will attempt to explain the influence of the particle size distribution of the test contaminant on the filtration ratio.

The filtration ratio, β, is defined by the following equation:

$$\beta_\ell = \frac{N_u(\ell)}{N_d(\ell)} = \frac{\int_\ell^\infty Z_u(D)dD}{\int_\ell^\infty \{1 - \zeta(D)\}\, Z_u(D)dD} \qquad (38)$$

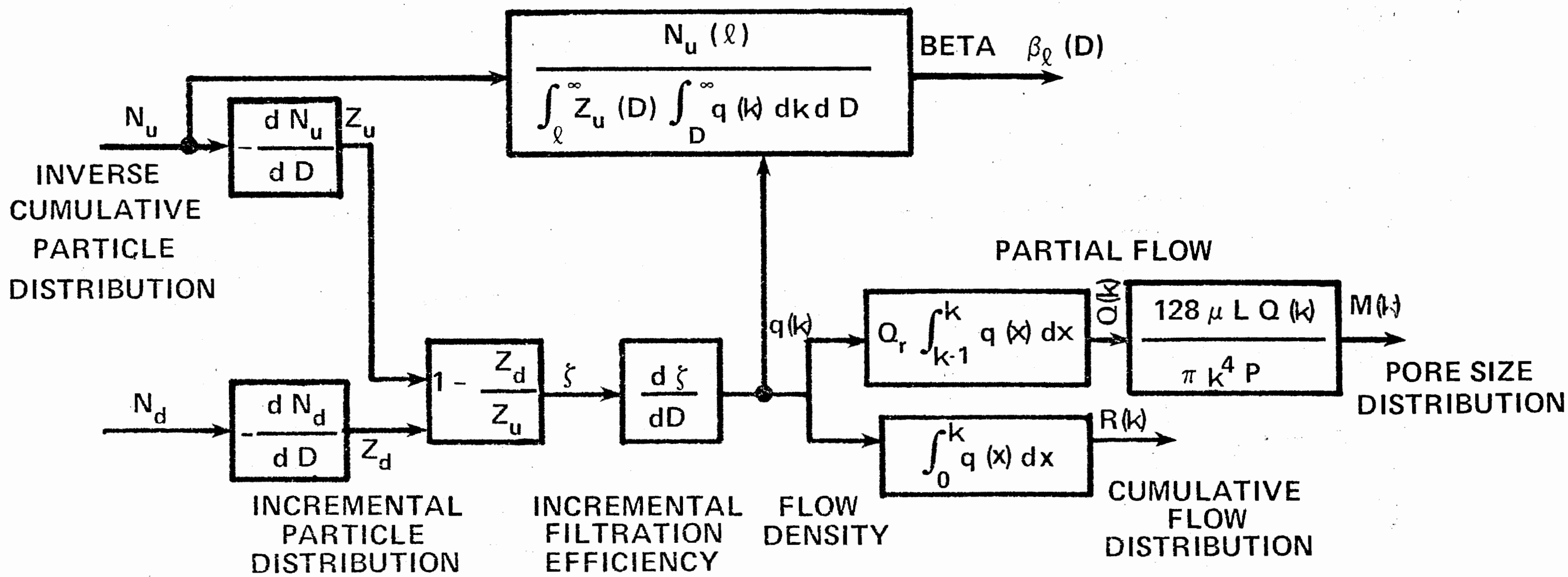

Figure 8. Block Diagram for Filter Performance Analysis

where: β_ℓ = the filtration ratio for particles greater than a diameter of ℓ.

$N(\ell)$ = the number of particles with a diameter greater than ℓ.

In the numerator of the above equation, {1 - (D)} is modified by using Equation (31) as follows:

$$1 - \zeta(D) = 1 - \int_0^D q(k)dk$$
$$= \int_D^\infty q(k)dK. \tag{39}$$

Equation (39) is substituted into Equation (38) as follows:

$$\beta_\ell = \frac{\int_\ell^\infty Z_u(D)dD}{\int_\ell^\infty Z_u(D)\int_D^\infty q(K)dkdD} \tag{40}$$

As shown in Equation (40), the value of β is a function of the particle density function upstream, $z_u(D)$, and the flow density function of the filter media, q(k). Although the flow density function inherently expresses filter performance, the particle density function characterizes the contaminant. Therefore, the filtration ratio, β, is not only a function of the inherent transmission properties of the filter medium, but it is also influenced by the particle distribution of the contaminant. This equation is expressed in the block diagram of Figure 8.

Analysis of the Flow Distribution

By using the theoretical equations derived in the previous section and actual filter test data obtained by the FPRC at OSU using the ISO multipass filter test procedure, the flow distributions for several filters were analyzed. In order to explain and illustrate the analysis results, one filter identified as OSU No. 732 is used. The inverse cumulative upstream and downstream particle distributions of filter No. 732 are shown in Figure 9. In the graph, the data taken at two minutes after the beginning of the test and at 10, 40, and 80 percent of the maximum allowable differential pressures are plotted. The figure shows that the particle distributions both upstream and downstream of the filter obviously vary with time. This variation causes the performance of the filter to change.

From the particle distributions, the flow distributions for the filter were analyzed. The analysis results are plotted on Log-Normal probability paper in Figure 10. The flow distribution curves shown in Figure 10 reveal several important facts. First, the flow distribution varies with time. The gradient of the flow distribution is almost the same at any sampling point.

Second, the mean value of the flow distribution shifts with increases in pressure drop. Since the mean value is defined as the pore size having a flow distribution of 50%, an increase in the mean value indicates a decrease in filtration performance. Figure 10 shows that the mean value for the two-minute data point is 6.3 micrometers and the mean value for the 80 percent pressure drop is 10.5 micrometer. Therefore, one-half of the particles having a diameter of 6.3 micrometer are trapped by the filter at the two-minute data point, but only 8 percent of them are trapped at the 80 percent pressure drop point. Likewise, 90 percent of the particles having a 10.5 micrometer diameter are trapped at the two-minute data point, but only 50 percent

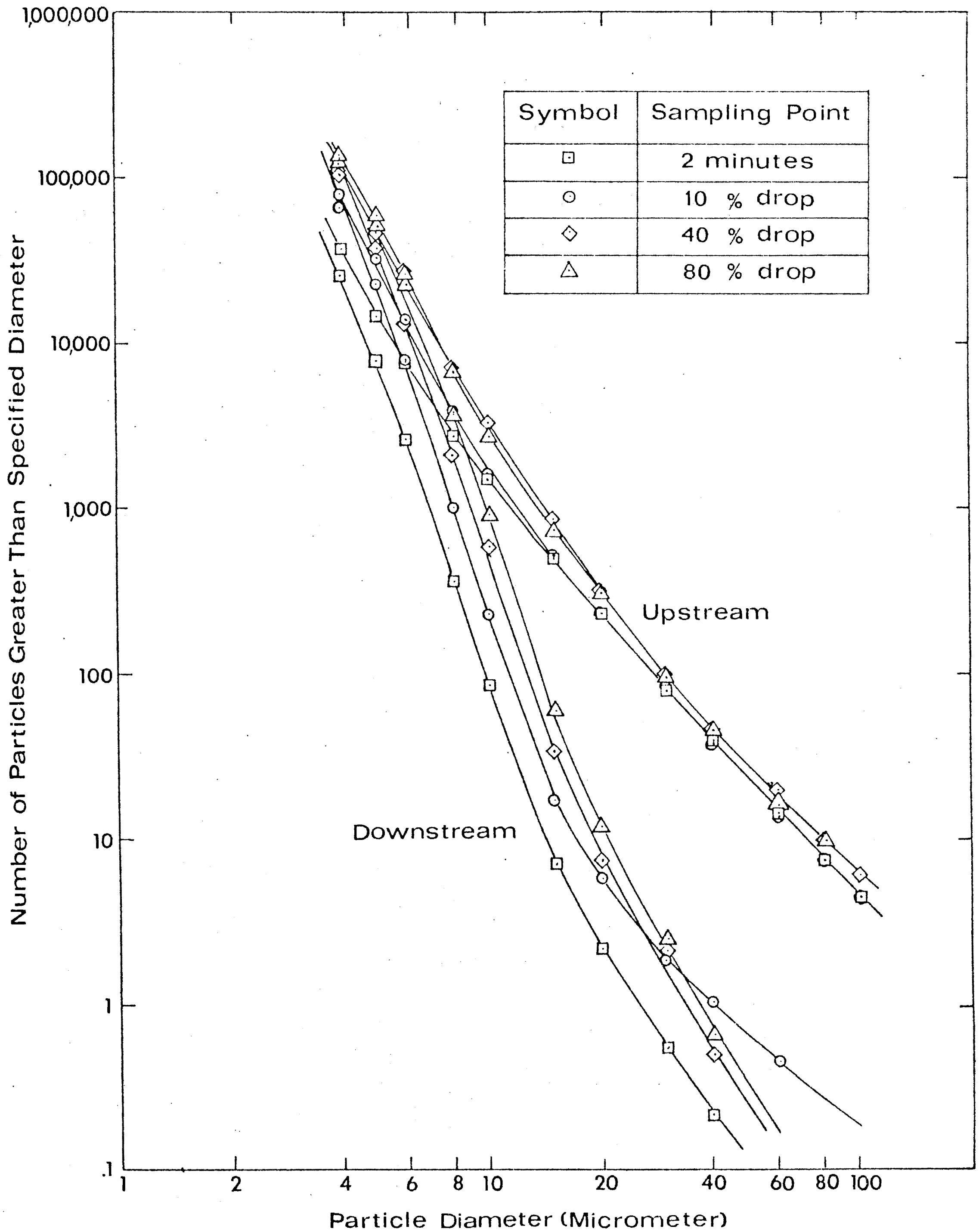

Figure 9. Particle Distribution of No. 732 Filter Test at Various Pressure Drops

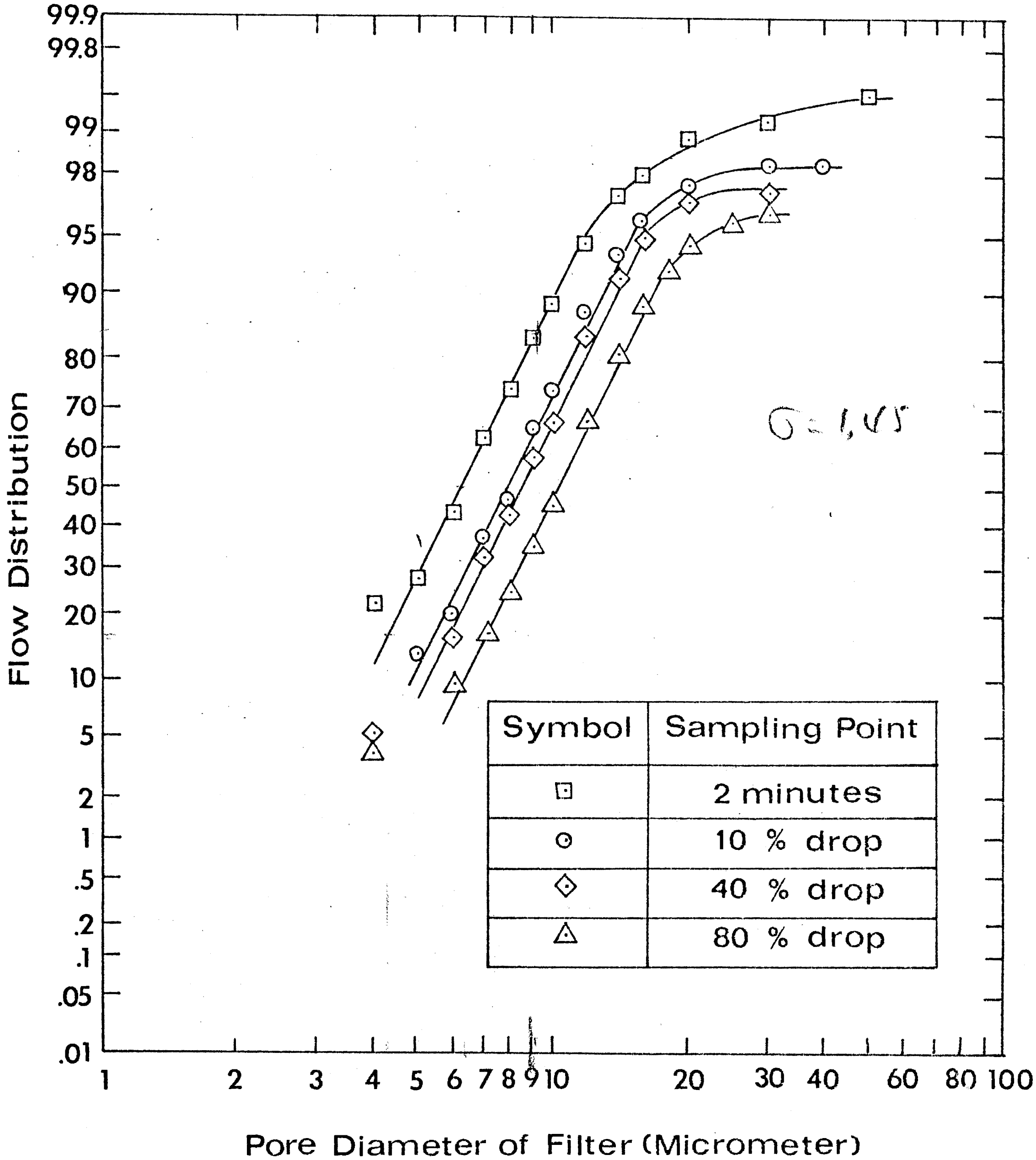

Figure 10. Flow Distribution of Filter 732

of the particles are captured at the 80 percent pressure drop data point. Therefore, the filtration efficiency of Filter 732 decreases with an increase in the pressure drop.

The last but most important observation gained by this study was the observation of the linearity of the flow distribution. As shown in Figure 10, the flow distributions can be expressed by straight lines for the main flow rate range. Although saturations appear in the large pore size range, the amount of the flow rate over the saturation point is not very much compared with the total flow rate. Such a saturation is not serious from the filtration performance standpoint. Therefore, the flow distribution for Filter 732 can be approximated as a Log-Normal distribution. Any such distribution can be described by only two parameters, standard deviation and mean value. If these parameters for a flow distribution are known, the performance of a filter can be described exactly and the number of contaminant particles downstream can be easily predicted by using these two parameters. The flow distribution and its density function are expressed in the following equations:

$$R(k) = \frac{1}{\sigma_n\sqrt{2\pi}} \int_0^k \frac{1}{x} \, \text{Exp}\left\{-\frac{1}{2}\left(\frac{\ln x - \mu_n}{\sigma_n}\right)^2\right\} dx \tag{41}$$

$$q(k) = \frac{1}{k\sigma_n\sqrt{2\pi}} \, \text{Exp}\left\{-\frac{1}{2}\left(\frac{\ln k - \mu_n}{\sigma_n}\right)^2\right\} \tag{42}$$

where: μ_n = the mean of the natural logarithm of the variable "x", $\mu_n = E[\ln k]$

σ_n = the standard deviation of the natural logarithm of the variable "x", $\sigma_n = \sqrt{V(\ln k)}$

In many engineering applications, common logarithms are used instead of natural logarithms and to reflect this, Equation (41) and Equation (42) can be transformed to Equation (43) and Equation (44). The details of the transformation are described in Appendix B.

$$R(k) = \frac{1}{2.303\sqrt{2\pi}\,\sigma_c} \int_0^k \frac{1}{x} \operatorname{Exp}\left\{-\frac{1}{2}\left(\frac{\log x - \mu_c}{\sigma_c}\right)^2\right\} dX \qquad (43)$$

$$q(k) = \frac{1}{2.303\sqrt{2\pi}\,\sigma_c k} \operatorname{Exp}\left\{-\frac{1}{2}\left(\frac{\log k - \mu_c}{\sigma_c}\right)^2\right\} \qquad (44)$$

where: $\mu_c = E\,[\log k]$ (45)

$\sigma_c = \sqrt{V(\log k)}$

For Filter 732, mean values of the flow distribution, μ_c are 6.3, 7.8, 8.5, and 10.5 micrometer at the two-minute, 10, 20, and 80 percent pressure drop data points, respectively. The standard deviation for each data point is almost the same value, about 0.174. In other words, the mean value of the flow distribution changes with time, but the standard deviations scarcely change with the time. These facts are applicable not only for Filter 732, but also for other filter. Variations of filter performances with time will be analyzed in detail in the next section.

The flow distribution of several filters were analyzed and the results at the two-minute data point are plotted in Figure 11. These results confirm that the flow distribution is a Log-Normal distribution.

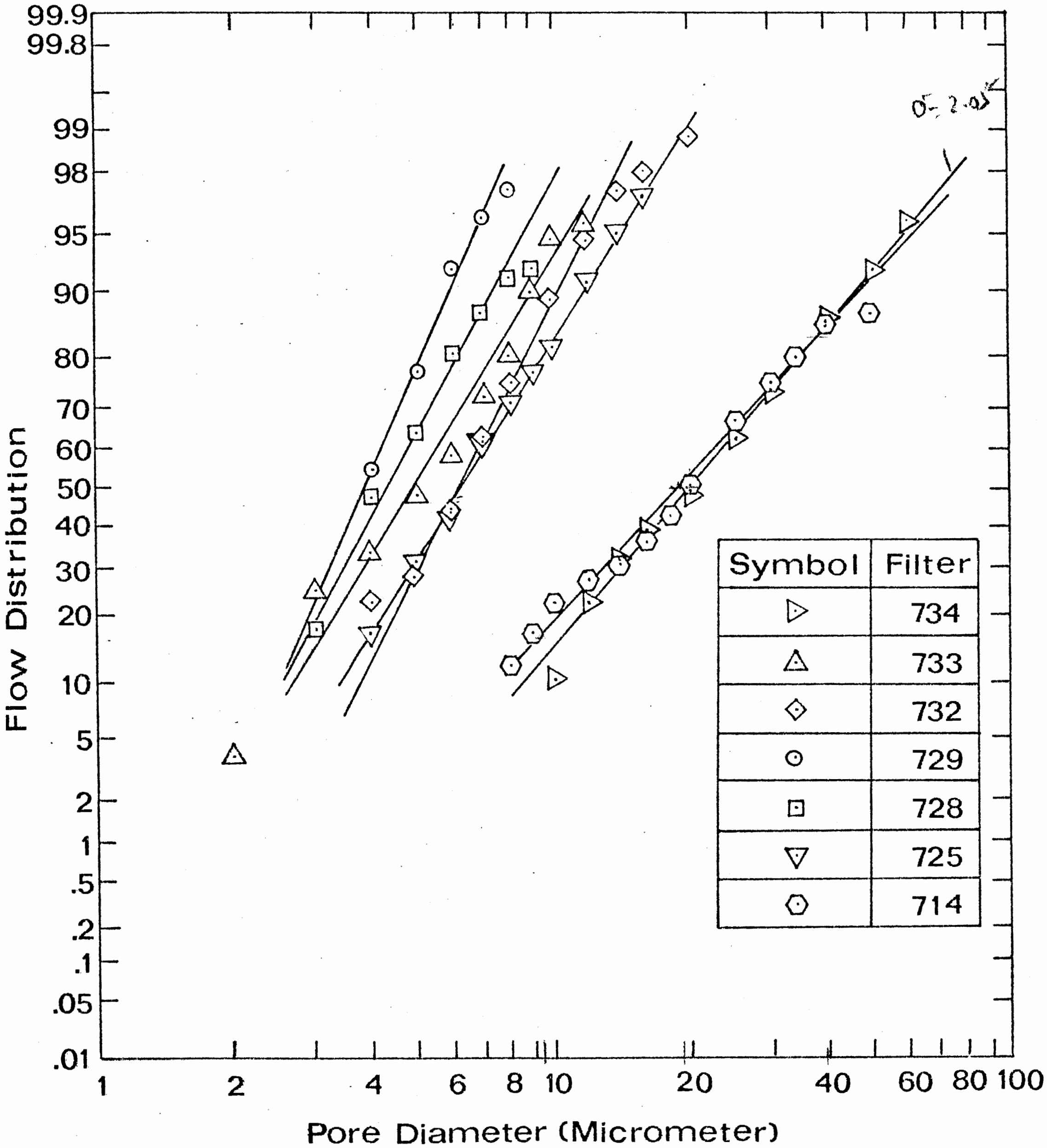

Figure 11. Flow Distribution at Two Minute Data Point for Filters

Furthermore, the slope of the flow distribution depends on the filter. The filter having a larger mean seems to have a larger standard deviation.

The correlations between the mean and the standard deviation at the 10 percent pressure drop data point for various kinds of filters are plotted in Figure 12. The correlation is expressed as a straight line; the standard deviation can be approximated in Equation (46) as a function of the mean.

$$\sigma_c = 0.111\ D_{m10}^{0.266} \tag{46}$$

where: D_{m10} = median pore size at 10 percent pressure drop,

$\mu_{c10} = \log D_{m10}$.

By using Equation (46), once the median value of the pore size is known, the flow distribution for a filter can be approximated and the particle distribution downstream of the filter can be predicted.

Performance Variation of a Filter

As shown in Figure 10, filter performance varies with time. As the pores of a filter media are plugged by trapped contaminant, the pore size distribution changes. The performance variation of a filter is mainly caused by the plugging of its pores.

The most remarkable indication of this change in performance with time of a filter is the change in pressure drop between upstream and downstream of the filter. In Figure 13, the correlation between pressure drop and time are plotted for several filters that were tested according to ISO standards at the FPRC, OSU. The ordinate of the graph

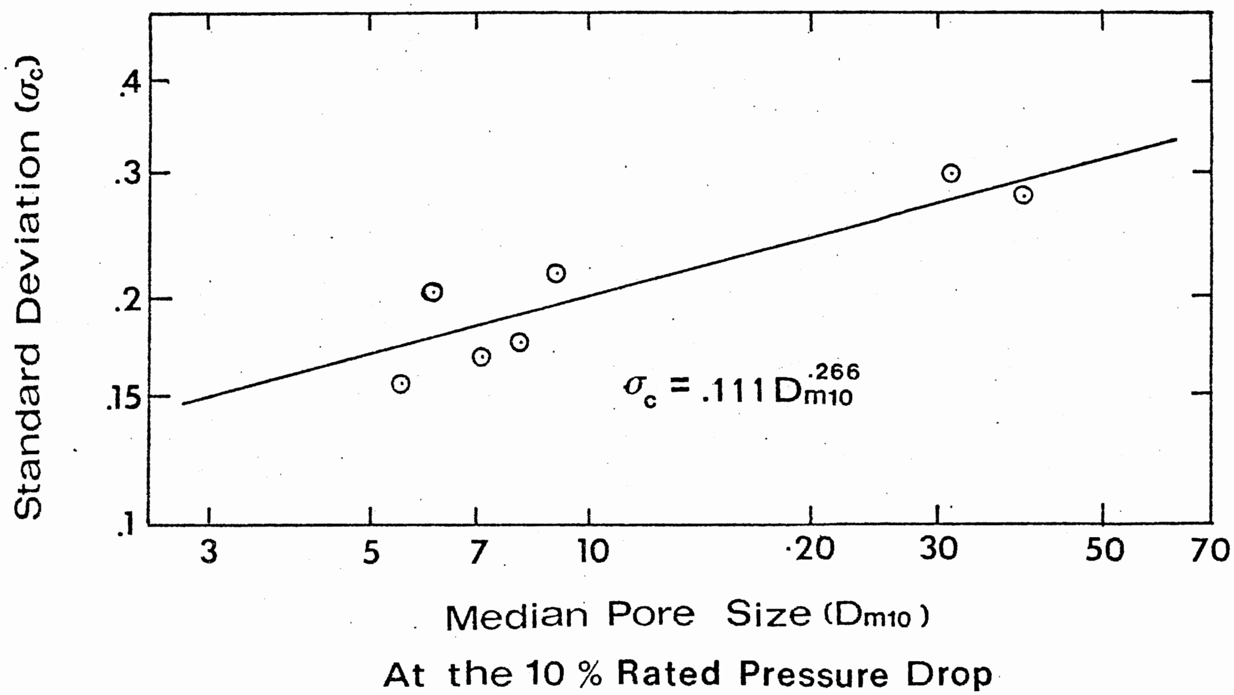

Figure 12. Correlation Between Median and Standard Deviation for Various Filters

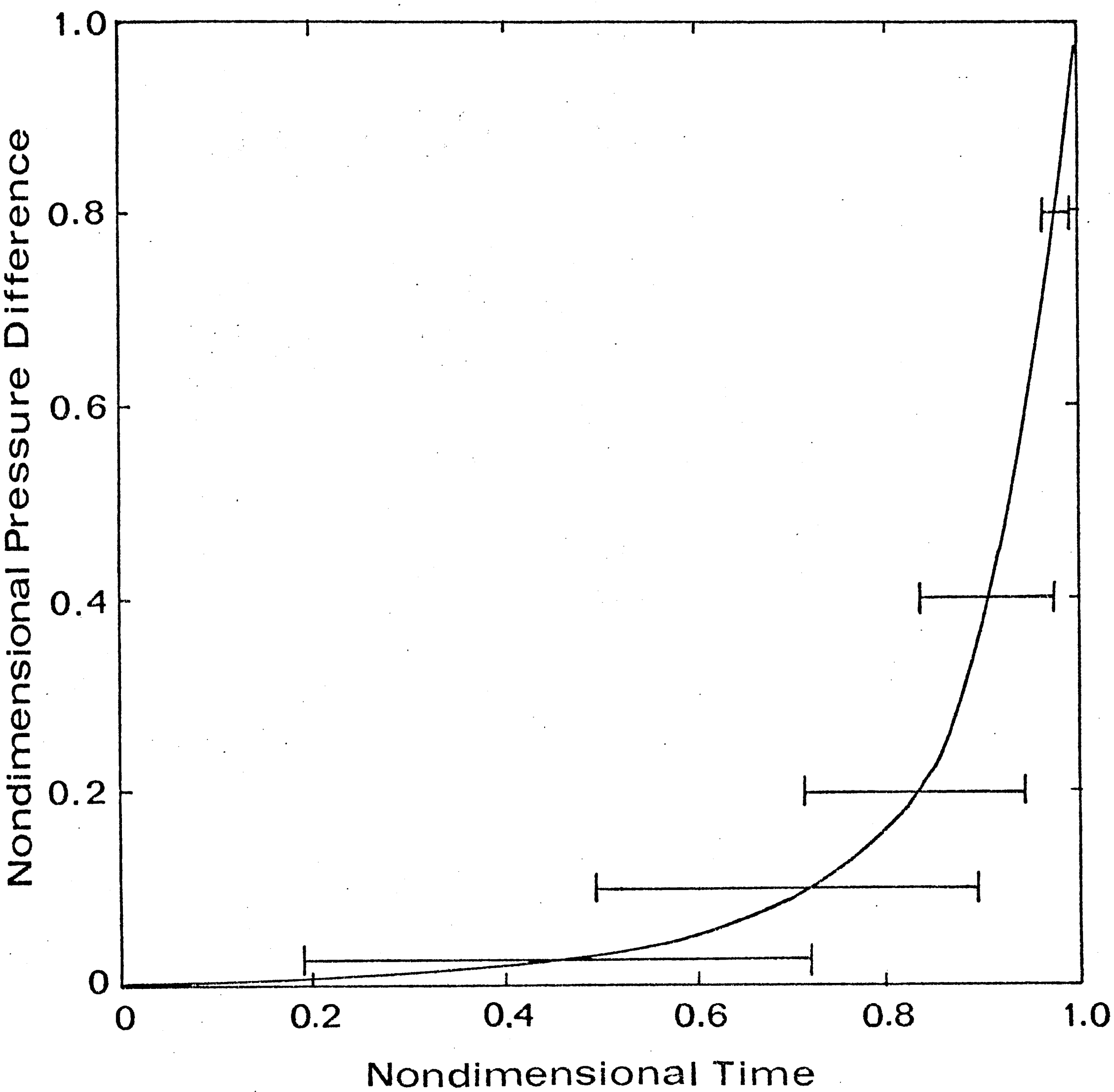

Figure 13. Correlation Between Pressure Variation and Time on Filter Test

represents a nondimensional pressure, the ratio of incremental pressure difference between upstream and downstream of a filter media to the maximum allowable pressure difference of the filter media, and the abscissa represents a nondimensional time, the ratio of the time difference between the start of the test and any arbitrary time during the test and the total time duration of the test (the filter life). The filter data plotted in Figure 13 were selected at random from the FPRC filter test data bank. Although this data is a little dispersed at the low pressure range, the variation is not significant. The curved solid line in the graph represents the mean value of the data. From the graph, it is obvious that the pressure drop increases suddenly in the region of the non-dimensional time greater than 0.8.

As mentioned in the previous section, the mean value of the flow distribution varies with time as the filter is being used; whereas, the standard deviation hardly changes with time. Although the standard deviation of a few filters slightly decrease with an increase in the pressure, the standard deviation of most filters is constant regardless of the mean value. The standard deviation of the flow distribution at any pressure drop data point can be assumed to be the same as the standard deviation at the two-minute data point.

The variations of the median value of the pore size with time are shown in Figure 14. This figure shows that the median of the pore size increases almost linearly with increasing pressure drop when depicted on Log-Log graph paper. The slopes of these lines are almost identical. Therefore, the correlation between the mean value and the pressure drop can be expressed by the following equations:

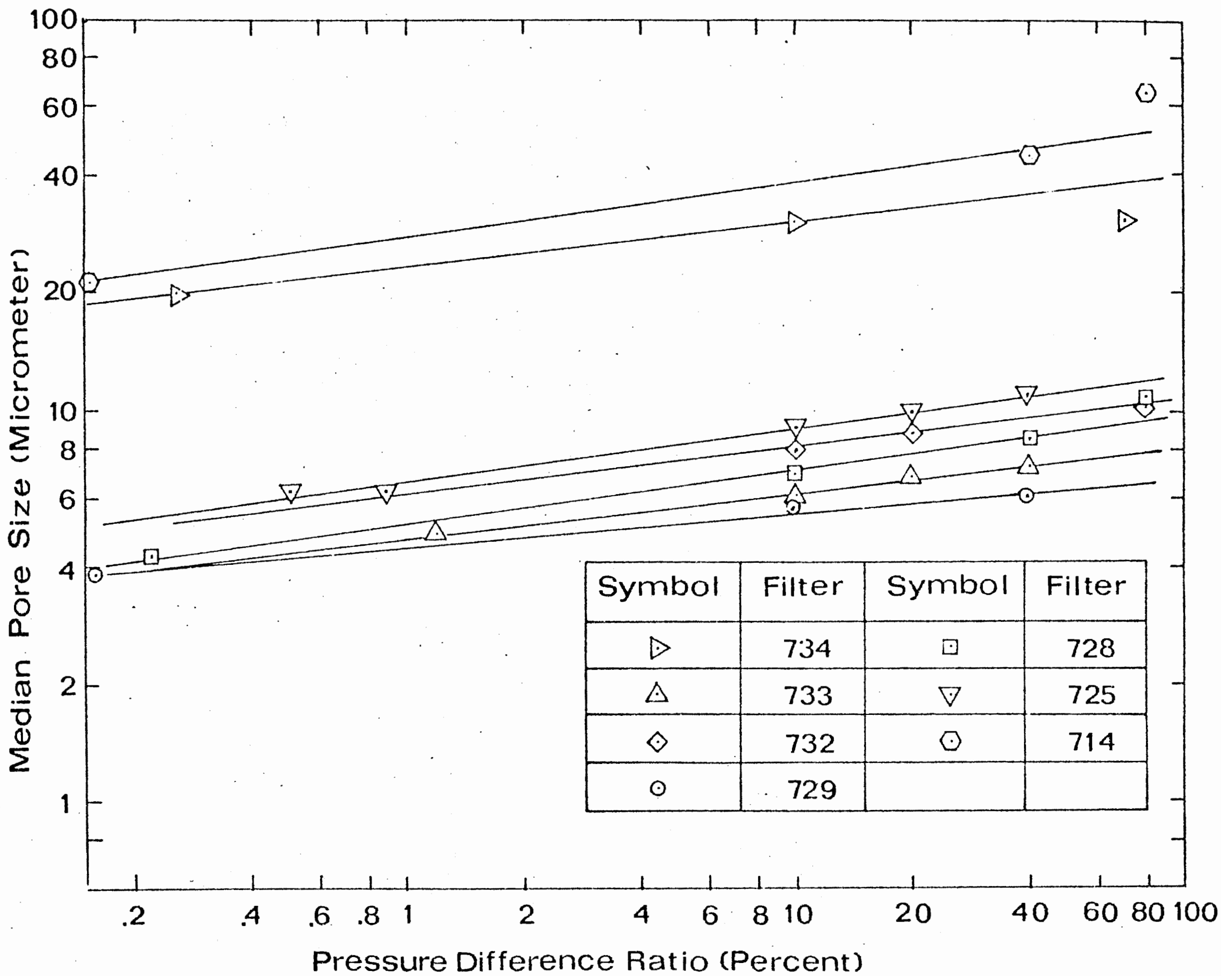

Figure 14. Variation of Median Value versus Pressure Drop

$$\mu_c = \log\left\{ D_{m10}\left(\frac{P}{10}\right)^{0.130} \right\} \tag{47}$$

$$D_m = D_{m10}\left(\frac{P}{10}\right)^{0.130} \tag{48}$$

where: D_{m10} = the median pore size at 10 percent of the rated pressure drop.

P = the pressure drop across the filter expressed as a percentage of the rated pressure drop.

Thus, the standard deviation can be expressed in terms of the median pore size, and the value of median at any time is the function of the pressure drop between upstream and downstream of the filter and of the median value at 10 percent pressure drop. Also the pressure drop is a unique function of the nondimensional time. Therefore, if the median pore size of the flow distribution at the 10 percent pressure drop can be known, the filter performance and the particle size distribution downstream can be predicted at any time during the life of the filter.

Particle Concentration in Fluid

A schematic of a typical hydraulic system is shown in Figure 15. There are many places where contaminant is either generated or ingressed. But the majority of this contaminant enters through the cylinder rod wiper seal. If the ingression rate of the contaminant and the filter performance are given, the contaminant concentration in the hydraulic system can be determined. The density function of the particle concentration downstream is expressed by Equation (49) a combination of Equation (33) and Equation (36).

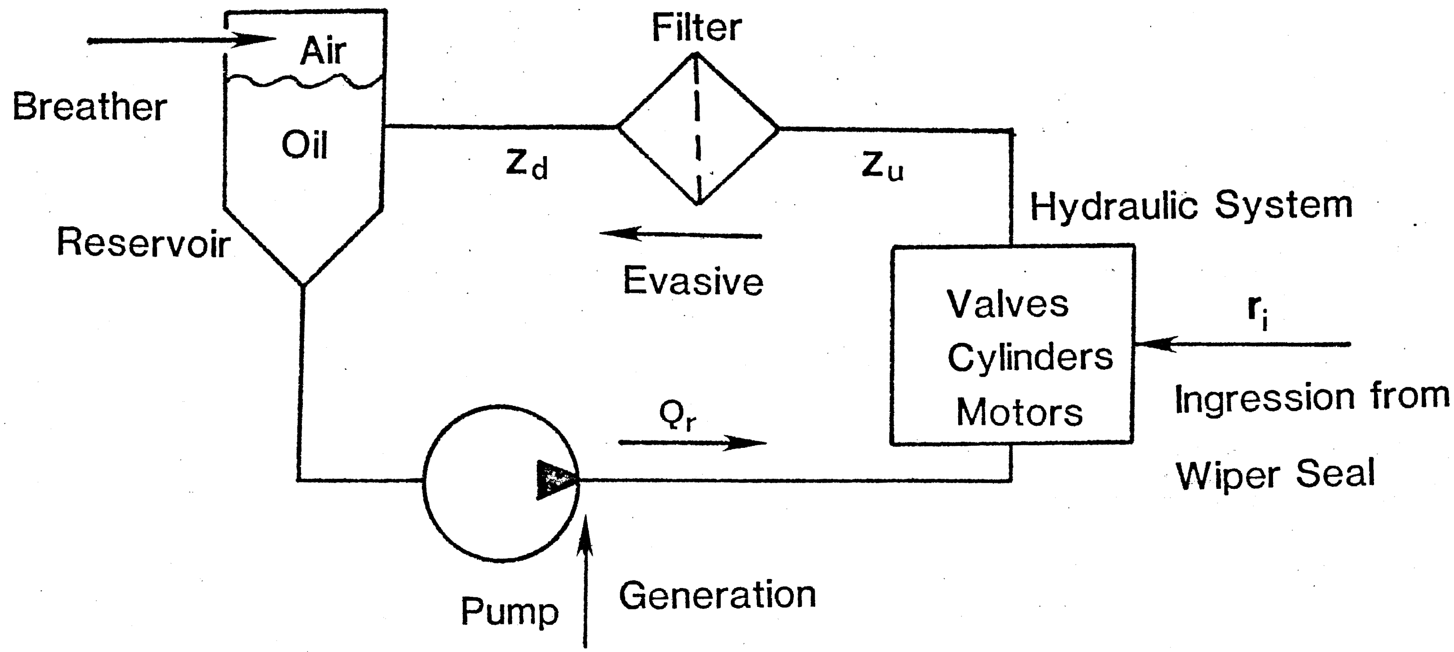

Figure 15. Schematic of Hydraulic System

$$Z_d(D) = Z_u(D)\{1 - R(k = D)\} \tag{49}$$

Z_u in Equation (49) is given in terms of ingression rate of particle and Z_d;

$$Z_u(D) = \frac{r_i}{Q_r} + Z_d(D) \tag{50}$$

where: r_i = density function of contaminant ingression rate.

The number of particles per milliliter in the size interval i, n_i, is found by integrating the density function of the particle concentration downstream expressed in Equation (49) over any specified size range, D_1 and D_2;

$$n_i = \int_{D_2}^{D_1} Z_d(D)dD \ . \tag{51}$$

Equation (51) is the ultimate goal of this chapter. All equations in this chapter were derived and modified in order to obtain the number of particles in a given size interval, n_i. By substituting Equation (51) and Equation (21) or Equation (22) into Equation (15), the life of a pump can now be analyzed. The numerical calculation will be shown in Chapter V.

CHAPTER V

LIFE AND COST OF PUMP - FILTER SYSTEM

Pump life can be expressed as a function of the contaminant wear coefficient of the pump and the contaminant concentration upstream of the pump. A new method to calculate the contaminant wear coefficient was developed in Chapter III. The new method showed that the contaminant wear coefficient of a pump can be analyzed if only two data points from the contaminant sensitivity test were given.

On the other hand, the contaminant concentration depends upon the filter performance and the contaminant ingression rate. In Chapter IV, the concept of the flow distribution was introduced and it was shown that the mean value of the flow distribution could be used for rating the absolute filter performance.

The analysis results from Chapters III and IV are briefly shown in a diagramatic form in Figure 16. In this chapter, the life of a pump is analyzed by using the equations derived previously. The life equation of a pump will be expressed in terms of the performance of a pump, the performance of a filter and a contaminant ingression rate. Then, the service cost of a hydraulic system consisting of a filter and pump is analyzed.

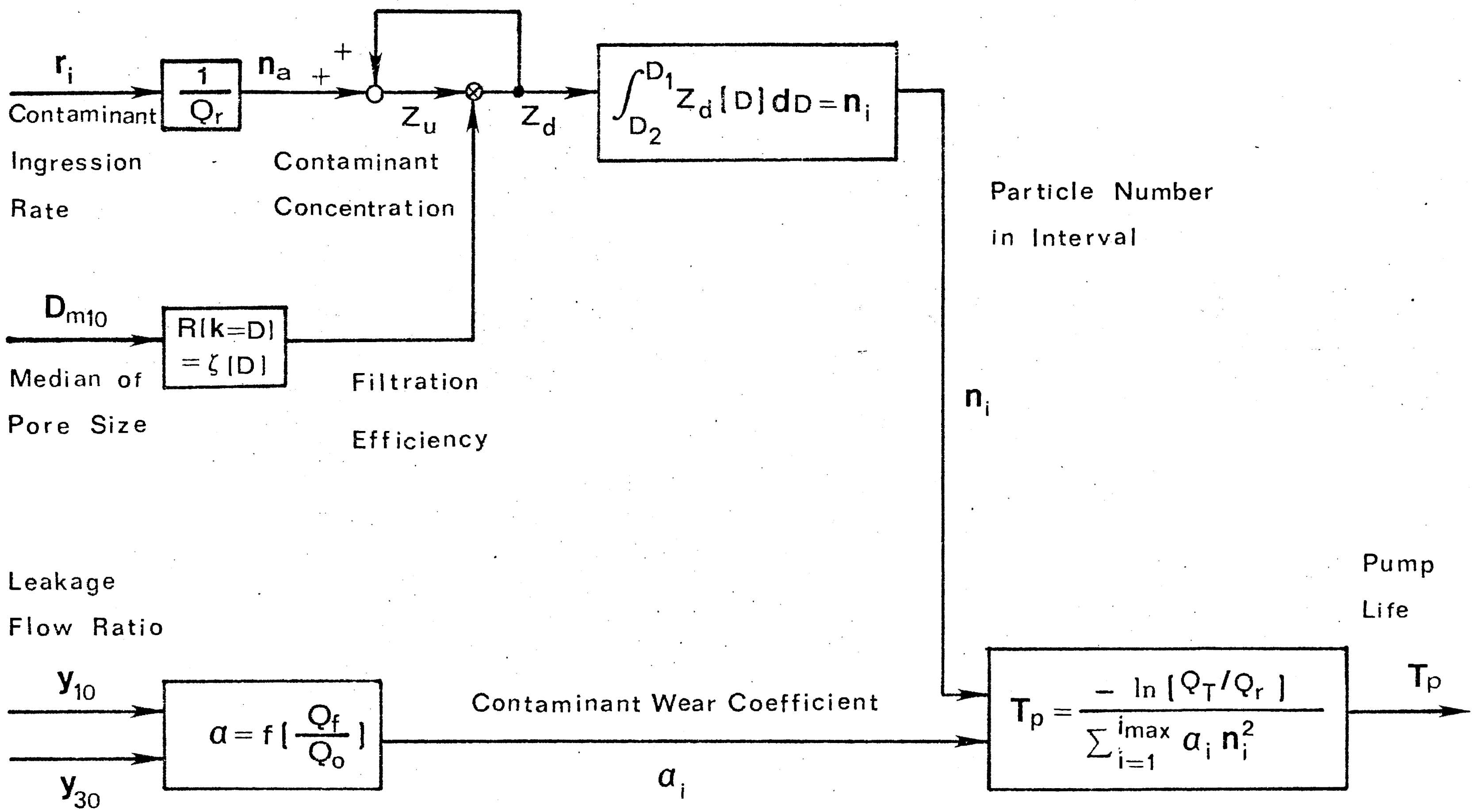

Figure 16. Correlation Between Pump Life and the Influential Parameters

Derivation of Pump Life Equation

The blockdiagram shown in Figure 16 shows concisely the correlation between the life of a pump and the factors influencing the life, as well as the relationship between the contaminant concentration ingressed into the hydraulic system, the flow distribution of the filter and the contaminant wear coefficient of the pump. The arrow on the right side of the diagram represents the life of the pump. Four arrows on the left side of the block diagram are the factors which influence directly the pump life. That is, the life of a pump is influenced by four factors: the contaminant concentration ingressing into the hydraulic system, r_i/Q_r; the median pore size of the filter at 10 percent pressure drop, D_{m10}; and the leakage flow ratios for the classified test dust 0∿10, 0∿30 micrometers, y_{10}; y_{30}, respectively.

Based on these four factors, the effect of the contaminant concentration on pump life has been analyzed [1]. The results show that pump life is inversely proportional to the square of the contaminant concentration. Therefore, if the pump life for a specific contaminant concentration is known, then for any other contaminant concentration, pump life is predicted with a very simple calculation. By using this characteristic, the contaminant concentration can be fixed on the life calculation of a pump and taken off from the influential factors to the pump life in the first analysis. In this study, 1000 particles per milliliter greater than a 10-micrometer diameter of the contaminant concentration ingressing is used to calculate the pump life. Thus, the influential factors are reduced to three. This reduction makes the life analysis of a pump enormously simple. In order to calculate the pump life by using the theoretical equations derived previously, further considerations are needed. As shown in Chapter IV, the flow distribution representing the performance of a filter can be approximated by Log-Normal distribution - the parameters of which are a function of the pressure differences ratio of the filter media. The pressure differences ratio of the filter in the laboratory test varies with the non-dimensional time of the test as expressed in Figure 13. This characteristic is expected to be the same in the field as in the laboratory test, if the ingressing contaminant distribution is the same as ACFTD. Therefore, the filter performance is a function of the non-dimensional time of the filter. The life of a filter in the field is expressed by the following equation:

$$T_f = T_{f\ell}\frac{G_{i\ell}}{G_i} = \frac{W_f}{G_i} \tag{52}$$

where: T_f = the filter life in the field.

$T_{f\ell}$ = the filter life in the laboratory test.

G_i = the contaminant ingression rate in the field.

$G_{i\ell}$ = the contaminant ingression rate in the laboratory test.

W_f = the contaminant capacity of the filter, = $T_{f\ell}G_{i\ell}$.

The flow degradation concerning the filter life is illustrated in Figure 17. A filter in a hydraulic system is replaced by a new one when the pressure drop across the filter media reaches a given pressure. Suppose that the flow rate of a pump is degraded, due to contaminant, from a flow delivery of Q_0 to Q_1 while a filter is provided in the hydraulic system. As soon as the filter life is terminated the old filter is replaced with a new one of the same type as the old one.

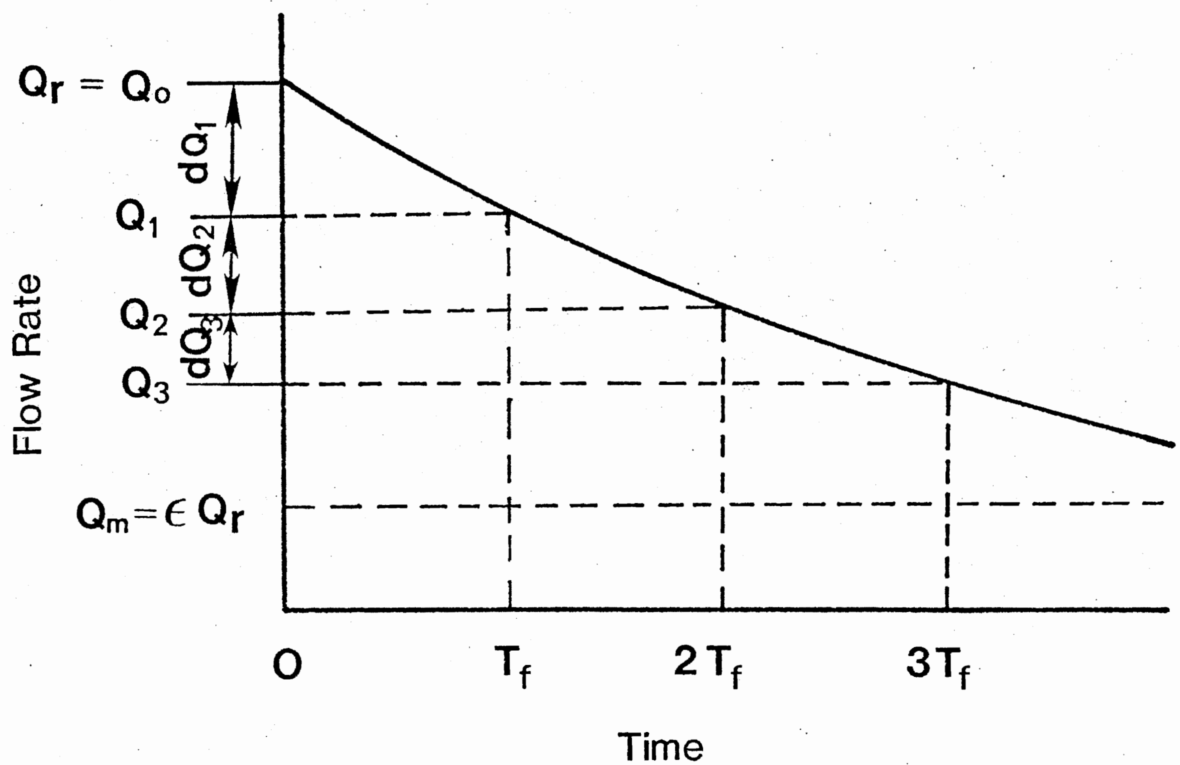

Figure 17. Schematic of Flow Degradation

During the life of the new filter, the pump is degraded from Q_1 to Q_2, and so on. If m filters are replaced before the pump is degraded into $Q_m = \varepsilon Q_0$, the following equations are derived from Equation (15);

$$
\begin{aligned}
Q_1 &= Q_0 \mathrm{Exp}(KT_f) \\
Q_2 &= Q_1 \mathrm{Exp}(KT_f) \\
&- - - - - - - - - \\
&- - - - - - - - - \\
Q_m &= Q_{m-1} \mathrm{Exp}(KT_f)
\end{aligned}
\tag{53}
$$

where: K = the average value of $\{ - \sum_{i=1}^{imax} \alpha_i n_i^2 \}$.

m = the number of the filter replaced before the pump is degraded into εQ_0.

As steady state, the value of "K" is assumed constant for each iteration. Therefore,

$$\frac{Q_i}{Q_{i-1}} = \text{Constant}, \; i = 1, 2, \ldots, m \; . \tag{54}$$

Combining Equation (53) and Equation (54) yields

$$\frac{Q_m}{Q_0} = \left(\frac{Q_1}{Q_0}\right)^m . \tag{55}$$

If the life of a pump is defined as the time when the flow rate of the pump decreases from the rated flow Q_r to εQ_r (in this case, $Q_0 = Q_r$, $Q_m = \varepsilon Q_r$), then,

$$\varepsilon = \frac{Q_m}{Q_0} = \left(\frac{Q_1}{Q_r}\right)^m \tag{56}$$

where: ε = the flow rate ratio of the rated flow of a pump to the flow rate which the pump is judged as a failure.

By taking the logarithm of Equation (56)

$$m = \frac{\log \varepsilon}{\log \left(\frac{Q_1}{Q_r}\right)} . \tag{57}$$

Thus, the life of a pump is expressed by the following equation:

$$\begin{aligned} T_p &= mT_f \\ &= \frac{T_f \log \varepsilon}{\log \left(\frac{Q_1}{Q_r}\right)} . \end{aligned} \tag{58}$$

As shown in Equation (58), the life of a pump can be calculated if the filter life and the flow degradation of the pump during which the filter is used are given. It is not necessary to calculate the flow degradation of the pump from Q_r to Q_m. Only the calculation from Q_r to Q_1 is needed.

In Equation (58), the life of a pump, T_p, seems to be the function of the filter life, T_f. Actually, T_p is independent from T_f. Because when using a filter having a longer life is used, the flow degradation corresponding to the filter life is increased. The above statement can be easily proven. From Equation (53),

$$\frac{Q_m}{Q_0} = \{Exp(KT_f)\}^m. \tag{59}$$

Combining Equation (59) and Equation (56) yields

$$\varepsilon = \{Exp(KT_f)\}^m . \tag{60}$$

Therefore,

$$m = \frac{\ln \varepsilon}{KT_f} . \tag{61}$$

Thus the life of a pump is given by the following equation:

$$\begin{aligned} T_p &= mT_f \\ &= \frac{\ln \varepsilon}{K} . \end{aligned} \tag{62}$$

The above equation is another expression of the pump life. However, calculating K in Equation (62) is more complicated than calculating the flow degradation in Equation (58). For this reason, Equation (58) was used to calculate the pump life.

In the numerical calculation, the value of ε in Equation (58) can be selected arbitrarily. In this paper, $\varepsilon = 0.8$ is selected because it is commonly used by hydraulic engineers.

By using these theoretical equations, pump life was analyzed for various combinations of three parameters, the median value of pore size, D_{m10}, the leakage flow ratios of the pump for 10 and 30 micrometers of the test dust, y_{10} and y_{30}. The pump life was calculated under the assumption that the filter was used up to its terminal life. The analysis results are shown in Figure 18. In this figure, the pump life, T_p, versus the median value of pore size, D_{m10}, are plotted for various leakage flow ratios of pump, y_{10}, y_{30}.

The results of the analysis reflected in the figure show several important and interesting facts. The first and the most important fact is that the filter performance influences tremendously the pump life regardless of the quality of the pump. For example, comparing the pump lives corresponding to D_{m10} = 5 and D_{m10} = 40, although the intensity of D_{m10} depends upon the quality of the pump, there are differences of 300 to more than 1,000 times on the lives of the pumps.

The second interesting fact is that the larger the leakage flow ratio of pump, y_{10}, y_{30}, the shorter the pump life. Although there are complex relationships between the pump life and the parameters, the pump life becomes shorter when the leakage flow ratios of pump are larger.

The last interesting fact in this figure concerns the effectiveness of the leakage flow ratio. The leakage flow ratio, y_{30}, is more influential on the pump life in the large D_{m10} region than in the small D_{m10} region, while y_{10} is less influential than y_{30} except for very small D_{m10} region. Therefore, a combination of the pump having a high leakage flow ratio at a 30 micrometers diameter, y_{30}, and the filter having a large median size makes the pump life drastically short. Such combinations should not be used in an actual hydraulic system from not only the economical standpoint but also the reliability standpoint. A good filter must be used in order to protect a bad performance pump. The above facts will be illustrated numerically later.

By a best fit analysis, the pump life in Figure 18 can be approximated in terms of y_{10}, y_{30}, and D_{m10}:

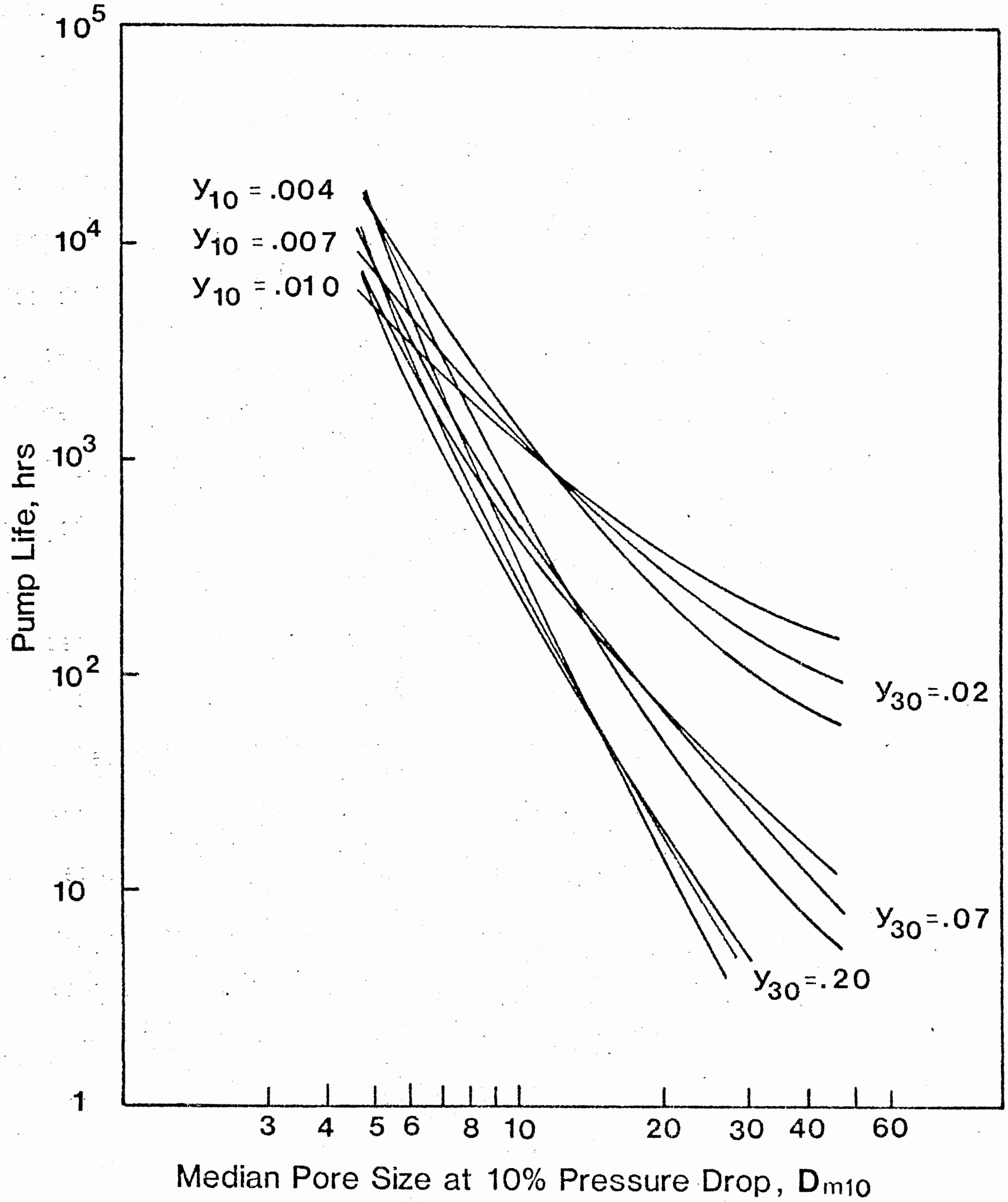

Figure 18. Analysis Results on Pump Life

$$T_{p1} = T_{p0}\left(\frac{y_{30}}{y_0}\right)^{-E} \qquad (63)$$

where: T_{p1} = pump life for 1000 particles/cm^3 greater than 10 micrometers contaminant ingression.

$T_{p0} = 14{,}000 \times 10^{\{-74.5(y_{10} - 0.004)\}}$.

$y_0 = \frac{1}{3} y_{10} - \frac{7}{12} \times 10^{-3}$, $y_{10} \geq 2.0 \times 10^{-3}$,

$E = 97.2(y_{10} + 0.0423)$ $(\log \log D_{m10} + 0.156)$, $D_{m10} > 1$.

Equation (63) is valid when $y_{10} \geq 0.002$ and $D_{m10} > 1$. Very few pumps exhibit zero flow degradation or extremely small flow degradation on the contaminant sensitivity test. If a pump has no flow degradation, there is no trouble in the calculation of the contaminant service life because it has an infinite contaminant service life. If a pump has an extremely small flow degradation, the leakage flow ratio, y_{10}, of the pump becomes less than 0.002. In this case, Equation (63) cannot be applied to predict the contaminant service life; hence, the life must be calculated by extrapolation using Figure 18. The pump life calculated with Equation (63) is very close to the value in Figure 18 within the restriction of y_{10}. Only a few percent of error are included in the region where the pump life is shorter than 10 hours. Usually, the pump having the contaminant service life shorter than 10 hours is not realistic. Therefore, Equation (63) represents accurately the contaminant service life of a real pump in the field.

The pump life represented by Equation (63) is for 1,000 particles per milliliter greater than a 10-micrometer diameter of contaminant ingression rate. If the hydraulic system has a different contaminant ingression rate than the above, Equation (63) must be modified. The correlation between pump life and contaminant concentration was explained previously, and it was expressed by Equation (15) or Equation (20). By using these correlations and Equation (63), a general expression of the pump life is given in the following equation:

$$\begin{aligned} T_p &= T_{p1}\left(\frac{1000}{n_a}\right)^2 \\ &= T_{p0}\left(\frac{y_{30}}{y_0}\right)^{-E}\left(\frac{1000}{n_a}\right)^2 \end{aligned} \tag{64}$$

where: T_p = pump life for contaminant concentration, n_a.

n_a = particles number per milliliter greater than 10 micrometer ingressing into the hydraulic system.

Sometimes the nomographic expression is very convenient for the practicing engineer or the design engineer because the pump life can be calculated with a pencil and ruler. Calculators are not required to predict the pump life if the nomograph is used. For such purposes, nomographs were developed and are presented in Appendix C [21]. Although the nomographic prediction of pump life includes a little more error than Equation (63), its simplicity is very useful.

In order to illustrate Equation (64), the expected life of pumps having OSU numbers 143, 164, 170 were calculated when the filtration performance was given by OSU number 714, 725, 729 and contaminant ingression rate is 1000 particles per milliliter greater than 10 micrometer. The performance parameters of the pumps and the filters, and the expected pump lives are shown in Table II, Table III, and Table IV, respectively. The combination of Filter No. 729 and pump No.

TABLE II

LEAKAGE FLOW RATIO OF THE PUMPS USED FOR ILLUSTRATION

Pump I. D. Number	y_{10}	y_{30}
143	0.004	0.02
170	0.0045	0.065
164	0.0065	0.33

TABLE III

MEDIAN PORE SIZE OF THE FILTER USED FOR ILLUSTRATION

Filter I. D. Number	D_{m10}
729	5.5
725	9.0
734	30.0

TABLE IV

EXPECTED PUMP LIFE FOR VARIOUS FILTERS FOR THE CONTAMINANT INGRESSION RATE = 1,000 PARTICLES/CM^3 GREATER THAN 10 μm

		Pump I.D. Number		
		143	170	164
Filter	729	9,600	7,800	4,800
I.D.	725	1,900	930	290
Number	734	110	23	2.4

143 indicates good performance and life, while the combination of the Filter No. 734 and the Pump No. 164 shows very poor performance.

Although Table IV illustrates numerically Figure 18, several important facts which were not explained previously are included in this table. The first important fact is the difference between the longest life and the shortest one. The pump life for the best combination of filter and pump is 9600 hours, while the life for the worst combination is only 2.4 hours. From this fact, it is obvious that the selection of pump and filter is very important. The second important fact concerns the effectiveness of filter and pump performance on pump life. As shown in Table IV, as long as a good performance filter is used, there are not large differences of pump life between the pump performing well and pump performing poorly. However, if a poor performance filter is allowed in the hydraulic system, a big difference results in the lives between a good pump and a poor one. This is true since a pump having poor performance is affected much more by filter performance than a pump having good performance.

A pump can fail due to causes other than contaminant, such as fatigue of the component. There is not any published data on pump life using clean oil. However, by rule of thumb, pump life with clean oil is supposed to be about 4000-7000 hours. If it is true, the filter having the OSU ID number of 729 is almost a complete filter at the ingression rate of contaminant used in this analysis. This filter can protect pumps for such long hours even if the performance of the pump is poor.

Cost Equation of Pump - Filter Hydraulic System

Once the lives of a filter and pump are known, their costs are given by the following equation:

$$C_f = \frac{V_f}{T_f} \tag{65}$$

$$C_p = \frac{V_p}{T_p} \tag{66}$$

and

$$C_t = C_f + C_p \tag{67}$$

where: C = cost per unit time.

V = price of filter or pump.

Subscript: t = total

f = filter

p = pump.

The life of a filter and the filter price on a unit weight of contaminant removal basis are given by

$$T_f = \frac{W_f}{G_i} \tag{68}$$

$$V_f = v_f W_f \tag{69}$$

where: W_f = the filter capacity for contaminant, g.

G_i = gravimetric level of the contaminant ingressing, g/time.

v_f = the filter price to remove a unit weight of a contaminant, \$/g.

Substituting Equation (68) and Equation (69) into Equation (65) yields

$$C_f = v_f G_i \tag{70}$$

That is, the filter cost is proportional to the ingression rate of the contaminant.

Finally, by substituting Equation (66) and Equation (70) into Equation (67), the total cost is given by

$$C_t = v_f G_i + \frac{V_p}{T_p} \tag{71}$$

Minimum Cost Analysis

When There is a Limited Selection of Possible Pump/Filter Combination

In the design phase of a hydraulic system, one of the most important jobs for a design engineer is to minimize the total cost of the system as expressed by Equation (71). If the number of the pumps and the filters which are usable is minimal, then the total cost can be calculated for each combination of pump and filter. Then, a pump and filter set which can make the total cost minimum can be selected.

For example, suppose that there are three pumps and three filters. Their performances and prices are listed in Table V and Table VI. For each combination of pump and filter, the life of the pump was calculated with Equation (64) and the total cost was calculated using Equation (71). In this analysis, the contaminant concentration ingressing into the hydraulic system, n_a, was assumed to be 500 particles greater than 10 μm per cubic centimeter, and the rated flow of the pump, Q_r, was assumed to be 37.35 liter per minute (10 GPM). Therefore, the con-

TABLE V

THE PRICES AND THE MEDIAN PORE SIZES
OF THE FILTER USED AS EXAMPLE

Filter I.D. \ Price and Median	D_{m10} µm	V_f \$/Hrs
1	7	2
2	10	1
3	20	0.4

n_a = 500 part/cm^3, Q_r = 10 GPM

TABLE VI

THE PRICES AND THE LEAKAGE FLOW RATIOS
OF THE PUMPS USED AS EXAMPLE

Filter I.D. \ Price and Performance	y_{10}	y_{30}	y_0
1	0.004	0.02	1,200
2	0.0041	0.042	1,000
3	0.0065	0.34	800

n_a = 500 part/cm^3, Q_r = 10 GPM

taminant ingression rate, G_i, is given by

$$G_i = \frac{n_a Q_r \times 10^3 \text{ Particles/min.}}{158 \times 10^6 \text{ Particles/gram}} \tag{72}$$

$$= 0.120 \text{ g/min.}$$

The lives of the pumps and the costs for each combination are calculated. The results are tabulated in Table VII and are shown schematically in Figure 19. From the table and the figure, it is obvious that the pump costs for No. 1 filter (which provides good performance but is high priced) is very cheap, although the filter cost is higher than the others. While the pump costs for No. 3 filter (which provides poor performance, but is low priced) is higher than for the No. 1 filter. As a result, a combination of No. 1 pump and No. 3 filter has the lowest total cost in this example. Thus, if the number of the pumps and the filters which are usable in the hydraulic system being designed is small, the design engineer can choose the pump and the filter so as to minimize the total cost by using Equation (71).

In the Case of Many Alternatives of Pump and Filter

When the number of pumps and filters which is usable for the hydraulic system being designed is small, the best combination of them to minimize the total cost could be determined by calculating the cost on each combination using Equation (71). However, when there are many usable pumps and filters for the hydraulic system, a large number of the combinations arise. Thus, a tedious calculation is required of the

TABLE VII

PUMP LIVES AND COSTS FOR VARIOUS COMBINATIONS OF PUMP AND FILTER

		Pump I. D.		
		1 (V_p = $1,200)	2 (V_p = $1,000)	3 (V_p = $800)
Filter	1 ($V_f = 2$) ($/g)	T_p = 16,000 C_p = 0.08 C_f = 14.4 C_t = 14.5	T_p = 12,000 C_p = 0.08 C_f = 14.4 C_t = 14.5	T_p = 4,400 C_p = 0.18 C_f = 14.4 C_t = 14.6
	2 ($V_f = 1$)	T_p = 5,600 C_p = 0.21 C_f = 7.2 C_t = 7.4	T_p = 3,300 C_p = 0.30 C_f = 7.2 C_t = 7.5	T_p = 690 C_p = 1.16 C_f = 7.2 C_t = 8.4
	3 ($V_f = 0.4$)	T_p = 1,000 C_p = 1.2 C_f = 2.9 C_t = 4.1	T_p = 430 C_p = 2.33 C_f = 2.9 C_t = 5.2	T_p = 37 C_p = 21.6 C_f = 2.9 C_t = 24.5

n_a = 500 particle/cm^3

(Pump Life : Hours, Cost : $/Hour)

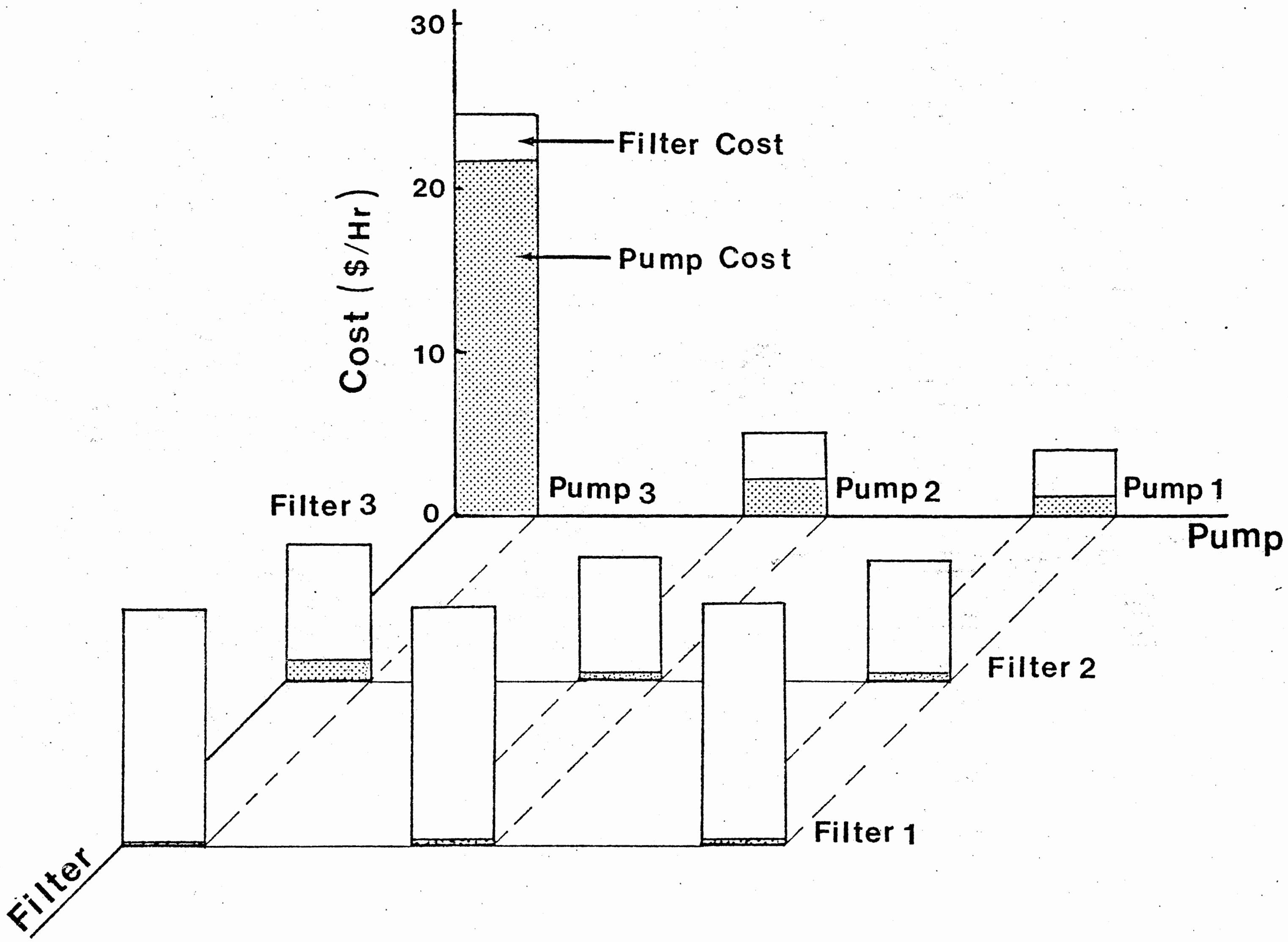

Figure 19. Schematic of Costs

design engineer. To facilitate this process, a new method to determine the economical combination was developed. The remainder of this section is devoted to deriving a new technique capable of minimizing the total cost.

Generally, the price of the filter which performs well is much more expensive than the filter which does not perform well. Therefore, the filter price to remove a unit weight of contaminant, v_f, in Equation (71), is a function of the performance. Suppose that the filter prices, v_f, can be plotted in a straight line on a Log-Log scale paper for their performances as shown in Figure 20 (correlations which cannot be approximated by a straight line will be discussed later). Then v_f can be expressed by the following equation:

$$v_f = v_{f10} \left(\frac{D_{m10}}{10} \right)^{-S} \tag{73}$$

where: v_{f10} = value of v_f at D_{m10} = 10 micrometers.

S = slope of the line on Log-Log scale paper.

Also, the gravimetric level of the contaminant concentration ingressing can be expressed in terms of the contaminant concentration ingressing and the rated flow of the pump.

$$G_i = a_1 n_a Q_r \tag{74}$$

where: a_1 = the conversion factor of the contaminant from the number of the particle into the weight, = $1/(1.58 \times 10^8)$ g/particle $\geq$ 10 micrometers.

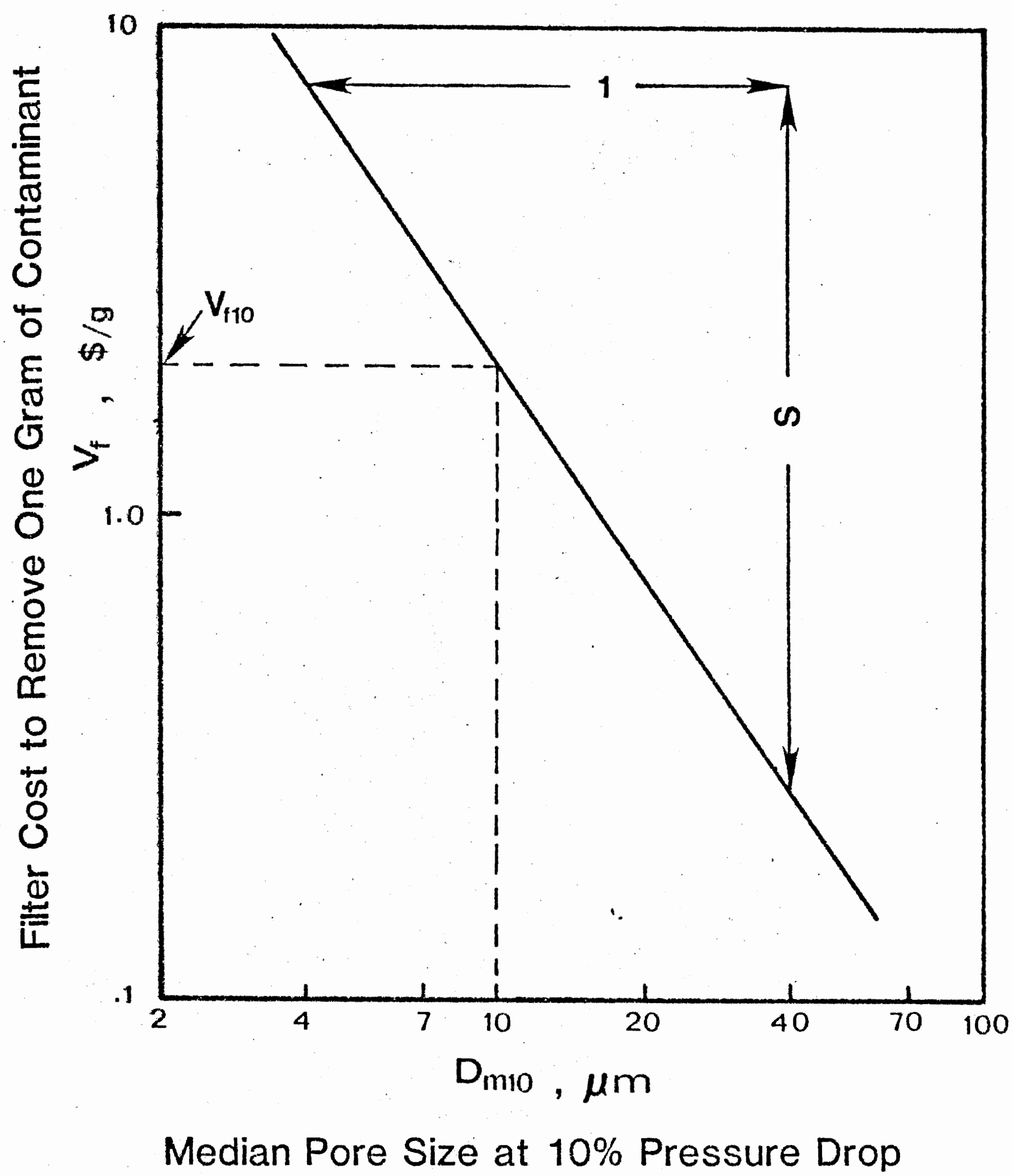

Figure 20. Straight Line Approximation of Filter Price versus Filter Performance

Substituting Equation (73) and Equation (74) into (70) yields

$$C_f = a_1 n_a Q_r v_{f10} \left(\frac{D_{m10}}{10}\right)^{-S}. \tag{75}$$

Thus, the filter cost could be expressed in terms of the filter performance parameter, D_{m10}.

On the other hand, the pump cost can be expressed in terms of D_{m10} by substituting Equation (64) into Equation (66)

$$C_p = \frac{V_p}{T_p} = \frac{V_p}{T_{po}\left(\frac{1000}{n_a}\right)^2} \left(\frac{y_{30}}{y_0}\right)^E$$

$$= A_1 A_2^E \tag{76}$$

where: $$A_1 = \frac{V_p}{T_{po}\left(\frac{1000}{n_a}\right)^2} \tag{76a}$$

$$A_2 = \frac{y_{30}}{y_0}\ . \tag{76b}$$

Remember that the E in Equation (76) is the function of D_{m10} which is given by

$$E = A_3(\log \log D_{m10} + A_4) \tag{77}$$

where: $$A_3 = 97.2(y_{10} + 0.0423) \tag{77a}$$

$$A_4 = 0.156. \tag{77b}$$

Substituting Equation (75) and Equation (76) into Equation (67) yields

$$C_t = a_1 n_a Q_r v_{f10}\left(\frac{D_{m10}}{10}\right)^{-S} + A_1 A_2^{A_3(\log\log D_{m10} + A_4)} . \qquad (78)$$

Thus, the total cost is expressed in terms of D_{m10}.

The purpose of this section is to find the filter which will minimize the total cost. Such a filter can be found by differentiating Equation (78) or Equation (67) in terms of D_{m10} and by equating it to zero. That is

$$\frac{dC_t}{dD_{m10}} = \frac{dC_f}{dD_{m10}} + \frac{dC_p}{dD_{m10}} = 0. \qquad (79)$$

From Equation (75)

$$\frac{dC_f}{dD_{m10}} = -10^S\ a_1 n_a Q_r v_{f10} S D_{m10}^{-(s+1)}$$

$$= -A_5 D_{m10}^{-(s+1)} \qquad (80)$$

where: $A_5 = 10^S\ a_1 n_a Q_r v_{f10} S$. (80a)

Also, from Equation (76) and Equation (77),

$$\frac{dC_p}{dD_{m10}} = \frac{dC_p}{dE}\ \frac{dE}{dD_{m10}}$$

$$= A_1 \ln A_2 A_2^E A_3 (\log e)^2\ \frac{1}{D_{m10}\log D_{m10}}$$

$$= A_1 A_3 \log A_2 \log e\ \frac{A_2^{A_3(\log\log D_{m10} + A_4)}}{D_{m10}\log D_{m10}} . \qquad (81)$$

Substituting Equation (80) and Equation (81) into Equation (79), and rearranging yields

$$D_{m10} = B_2(\log D_{m10})^{-B_1} \tag{82}$$

where: $$B_1 = \frac{A_3(\log A_2 - 1)}{S} \tag{82a}$$

$$B_2 = \{\frac{A_5}{A_1 A_3 A_2^{A_3 A_4} \log A_2 \log e}\}. \tag{82b}$$

If Equation (82) is solved mathematically in term of D_{m10}, it is a simple matter to obtain the filter which will make the total cost a minimum. However, it usually cannot be solved in a mathematical way. Equation (82) must be solved numerically using a digital computer.

The right hand side of Equation (82) monotonously increases with increasing D_{m10}, while the left hand side monotonously decreases with the increasing of D_{m10}. Therefore, Equation (82) has a unique solution as shown in Figure 21. And this solution minimizes the total cost, since the second derivative of the total cost in terms of D_{m10} is always positive for real hydraulic systems. The second derivative of the cost equation is given by

$$\frac{d^2C_p}{d^2D_{m10}} = A_5(S+1)D_{m10}^{-(s+2)} + \frac{A_1 A_3 \log A_2 \log e A_2^{A_3(\log\log D_{m10} + A_4)} \{A_3 \log A_2 \log(eD_{m10})\}}{(D_{m10}\log D_{m10})^2} \tag{84}$$

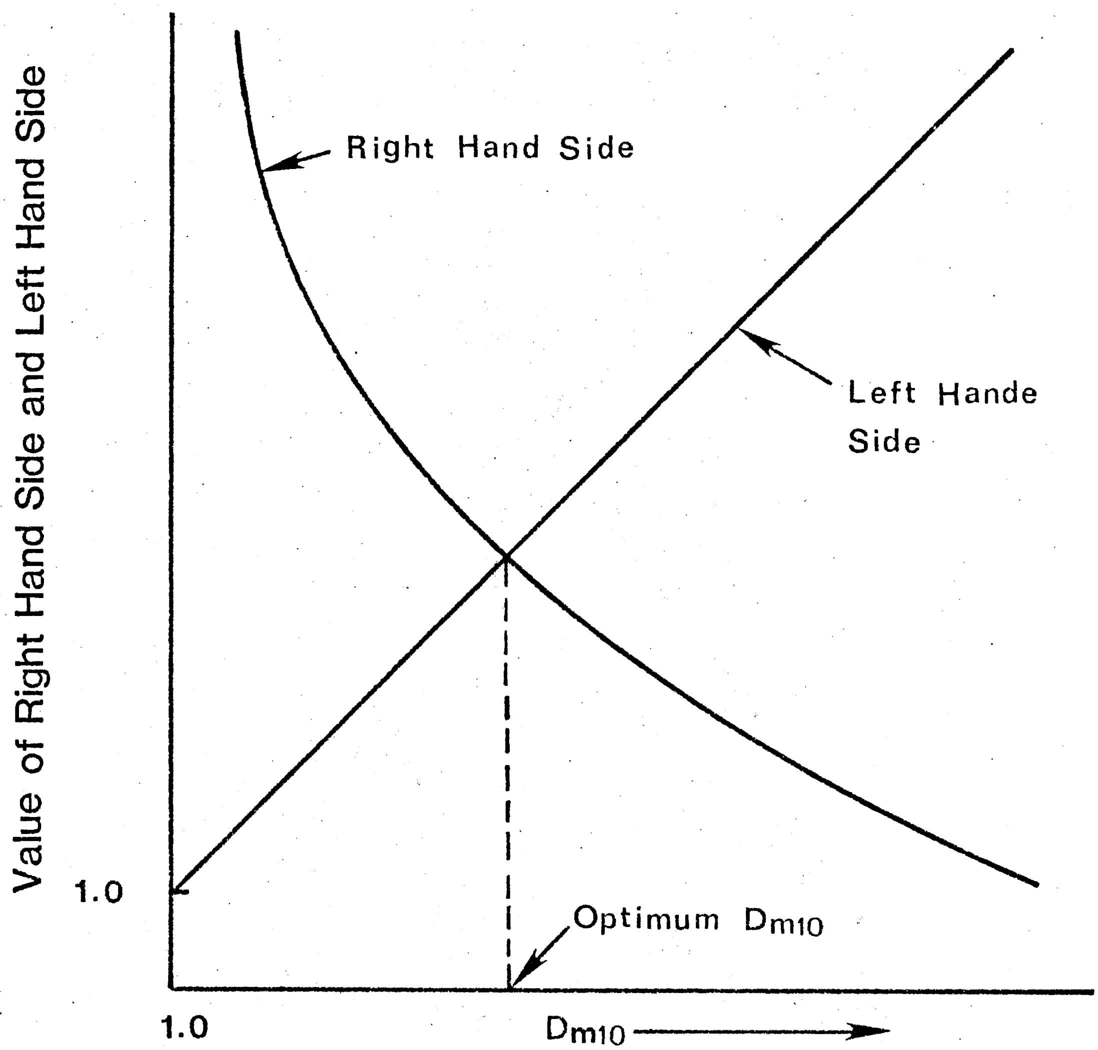

Figure 21. Schematic Expression of Equation (82)

In this equation, all constant coefficient, $A_1 \sim A_5$, and the slope, S, are positive. Therefore, the first terms of the right-hand side is positive. If the term in the bracket, { }, in the numerator of the second term is positive, then the second term becomes positive. For actual hydraulic systems, the term in the bracket is always positive. Thus, the second derivative of the cost equation in terms of D_{m10} is always positive, the filter having a D_{m10} which satisfies Equation (82) can minimize the total cost of the hydraulic system consisting of a pump and a filter.

Equation (82) was solved numerically using a digital computer. The analysis results are shown in Figure 22. By using this graph, a design engineer can obtain the most economical filter for a given hydraulic system. That is, if the leakage flow ratios of a pump, y_{10} and y_{30}, the rated flow of the pump, Q_r, the ingression rate of the contaminant, G_i, the price of the pump, V_p, and the straight-line approximation of the price versus the performance of filters, S and v_{f10}, are known, the constant parameters B_1 and B_2 can be calculated. Then, the D_{m10} corresponding to these B_1 and B_2 can be derived from Figure 22. Thus, D_{m10} makes the total cost of the hydraulic system minimum for a given pump. If the value of D_{m10} found in the graph is outside of the range of D_{m10} which is being investigated, the filter having the D_{m10} which is closest to the calculated D_{m10} must be selected. For example, suppose that the values of D_{m10} of the filter being investigated is 7, 10, 15, 20, and the calculated D_{m10} is 5. In such a case, the filter having D_{m10} = 7 must be selected. Since, as explained previously, the total cost expressed in Equation (78) has a unique minimum point. In other words, Equation (78) makes a convex

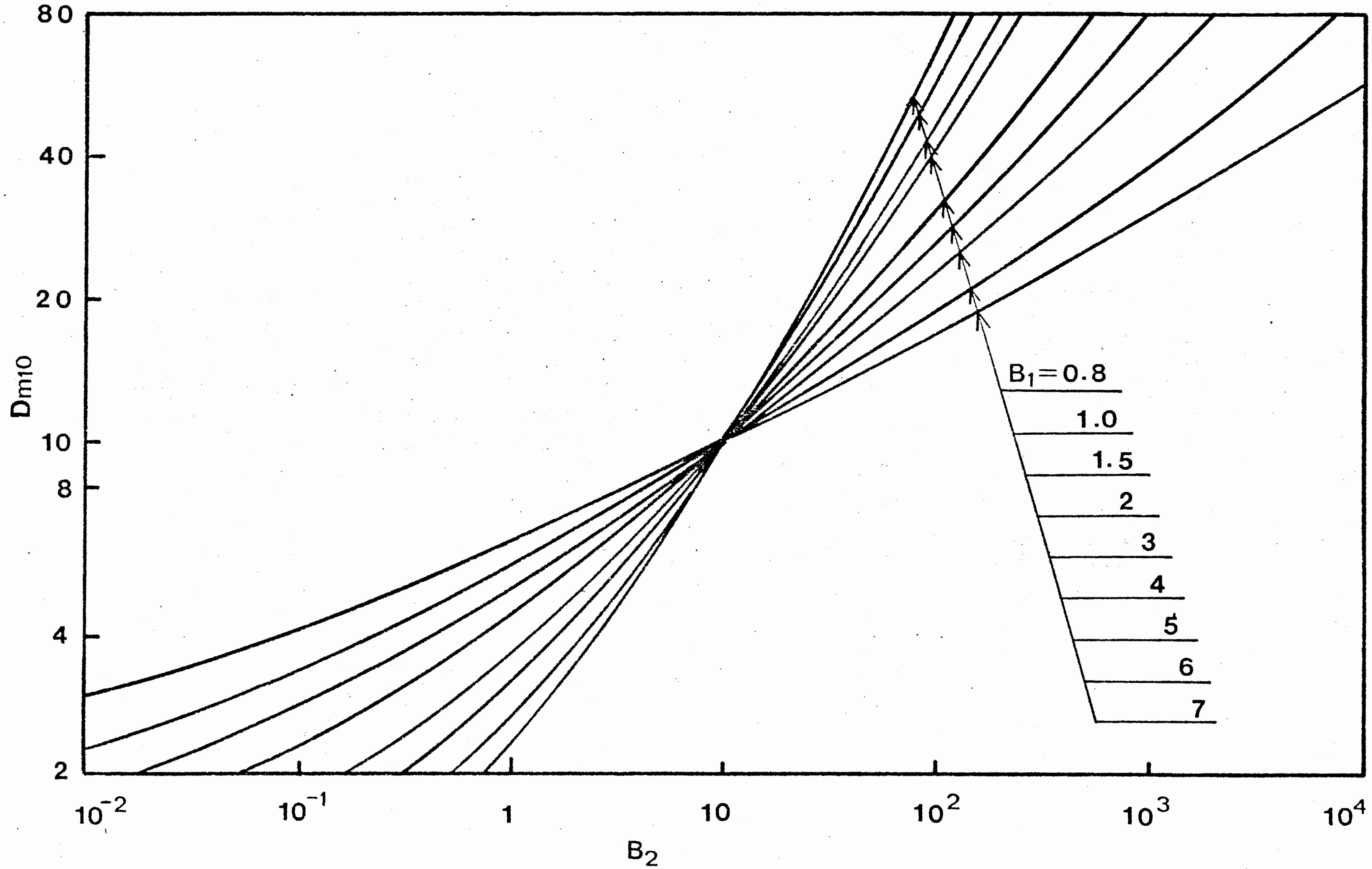

Figure 22. Value of D_{m10} which Minimize the Total Cost

curve for the actual hydraulic components. Therefore, the closer to the optimum value of D_{m10}, the lower the total cost.

Also, if the optimum D_{m10} analyzed by the above procedure is located between the D_{m10}'s of the filters which are being investigated, the total costs for these two filters must be calculated. Then, the filter corresponding to the lower total cost can be selected as the best. Thus, the best filter with a usable pump for the hydraulic system is chosen. In the same way, the best filters for other pump combinations can be selected. When the total cost of each set of filter and pump are calculated using Equation (71), and they are compared to each other, the set having the lowest cost is the most economical combination of pump and filter.

The above cost minimization method is effective only when the correlation between the filter price per one gram of the contaminant removed and the value of D_{m10} is approximated by a straight line on a Log-Log graph. If the correlation is not approximated in a straight line, then it must be linearized with two or more lines. Then, the above method can be used for each line, the most economical sets of pump and filter for each line are selected. After that, the set having the lowest cost out of the sets selected for each line will be the best set to minimize the total cost of all the pumps and filters which are usable for the hydraulic system.

Thus, if the prices and the performances of filters and pumps, and the contaminant ingression rate for the hydraulic system are given, a design engineer can select the most economical combination of pump and filter.

When the Equations (76a), (76b), (77a), (77b), and (80a) are substituted into Equations (82a) and (82b), parameters B_1, B_2 are expressed in terms of the leakage flow ratios of a pump and the other factors

$$B_1 = \frac{1}{S} f_1(y_{10}, y_{30}) \tag{84}$$

$$B_2 = \{10^S \; Sa_1 \frac{Q_r v_{f10}}{V_p n_a} f_2(y_{10}, y_{30})\}^{1/s} \tag{85}$$

where: f_1, f_2 = the function of the leakage flow ratios of a pump, y_{10}, y_{30}.

As shown in the above equations, the contaminant ingression rate, the rated flow of pump, and the prices of pump and filter affect only the parameter B_2. Also, it is obvious from Equations (84), (85), and Figure 22 that the smaller the Q_r (the rated flow) and v_{f10} (the filter price to remove a unit weight of a contaminant at D_{m10} = 10 micrometer) or the larger the V_p (price of pump) and n_a (contaminant ingression rate), the smaller the optimum D_{m10}.

CHAPTER VI

SUMMARY AND CONCLUSION

Summary

ADFTD, which is used to assess the performance of hydraulic components in the laboratory, was evaluated using the sedimentation method and the microscopic method. The experimental result of the sedimentation method showed that the weight distribution of ACFTD is a Weibull distribution in the small particle size region.

The particle size distribution of ACFTD currently used was compared with the one calculated from the weight distribution and the one counted microscopically. The particle size distribution currently used exhibits a considerable error in the small particle diameter region. The microscopically counted distribution obtained in this study was adopted as the up-to-date particle size distribution. Using this up-to-date particle size distribution, the performance of both pumps and filters was analyzed.

How a pump's performance changes when pumping contaminated fluid is assessed by monitoring the flow degradation of the life. In order to predict the life of pumps used in the field from the laboratory data, a new method (i.e., the leakage flow ratio) was developed. This is the ratio of the rated flow to the leakage flow rate due to an injection of classified test dust versus the maximum diameter of the classified test dust. It can be expressed as a straight line on

Log-Log graph paper. Therefore, the performance of a pump can be expressed by using two points on the leakage flow ratio curve. By using these two ratios, the contaminant wear coefficients of the pumps were analyzed. Thus, the pump life can be expressed in terms of the contaminant wear coefficient and the contaminant concentration of the system fluid.

The contaminant concentration of the system fluid is a function of the performance of the filter used in the system and the rate of contaminant ingressing rate into the system. The Beta ratio which is used currently to describe filter performance is influenced by the particle distribution in the fluid upstream of the filter. As a substitute for the Beta ratio, the flow distribution of the filter was introduced to evaluate the performance of the filter. The flow distribution of the filter has a Log-Normal distribution, its standard deviation is a function of the median pore size of the filter. The median pore size of a filter can be expressed in terms of the non-dimensionalized pressure drop across the filter media; its standard deviation hardly changing while the filter is in use. Therefore, the performance of a filter can be evaluated by the median pore size at any pressure drop. In this paper, the median pore size at 10 percent of the rated pressure drop was selected as the performance indicator of the filter. If the median pore size at this 10 percent pressure drop and the contaminant ingression rate are given, then the particle concentration of the hydraulic system can be predicted as well as the life of the filter.

Thus, the life of a pump depends upon only four parameters: two pump leakage flow ratios, the filter median pore size at 10 percent of the rated pressure drop, and the contaminant ingression rate. The pump life was calculated for various combinations of these parameters using a digital computer. The life equation for a pump was derived from the calculated results. Also, the life equation of a filter in the field was presented in terms of the contaminant ingression rate and the laboratory test data of the filter.

Finally, the cost equation for the hydraulic system consisting of a pump and filter was derived. If the lives and the prices of the pump and the filter are given, the total maintenance cost of the hydraulic system can be calculated using the cost equation. Furthermore, a new approach to minimize the total cost was developed. By using this approach, the filter which realizes the lowest total cost for a given pump can be selected.

Conclusion

From the research investigation described in the preceding chapters, several noteworthy contributions can be listed as follows:

1. Weight distribution of ACFTD is a Weibull distribution with a shape parameter of 0.84 and a scale parameter of 19.5 micrometers.
2. Particle size distribution of ACFTD currently used deviates considerably from the correct value; therefore, it must be revised.
3. The leakage flow ratio representing the flow degradation characteristics of pumps due to contaminant is expressed as a straight line on Log-Log paper. Therefore, the value of the leakage flow ratio at any two data points can be a pump performance indicator.

4. The Beta rating which is currently used to evaluate filter performance is affected by the particle size distribution upstream of the filter. Therefore, the filtration ratio, β, does not represent the basic transmission property of the filter medium.
5. The flow distribution developed in this study can represent the inherent transmission property. It is not influenced by external conditions.
6. The standard deviation of the flow distribution is a function of the mean. Also, its value changes very little while the filter is used. Therefore, the mean of the flow distribution or the median of the original variable can be used as a performance indicator of the filter media.
7. As a result, the particle concentration downstream of the filter can be expressed exactly in terms of the median at 10 percent of the rated pressure drop and the particle concentration upstream.
8. The pump life equation was developed. This equation is a function of only four parameters, the contaminant concentration in the hydraulic fluid, the median pore size of the flow distribution of filter, two leakage flow ratios obtained from the 10 micrometers and 30 micrometers classified dust tests. Therefore, if these four parameters are given, the pump life can be predicted exactly.
9. A filter having a small median value can protect the system regardless of its performance. However, a filter having a large median value (poor quality filter) will reduce the pump

life drastically. The combination of a poor quality filter and a poor quality pump should not be used from the standpoint not only of the life of the pump, but also of the economy of the hydraulic system.

10. Filter life in the field can be predicted by laboratory test data of the filter and the ingression rate of contaminant in the field.
11. The cost equation for a hydraulic system consisting of a pump and filter was developed. The minimum maintenance cost for this hydraulic system can be found by using the cost equation.
12. Since there are many different kinds of filters having various performance and price, a method to select the filter which makes the total maintenance cost a minimum was established. By using this method, a design engineer can design the most economical hydraulic system.

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APPENDIX A

MOTION OF A PARTICLE HAVING ELLIPTICAL BODY IN FLUID

A particle moving in a fluid receives, in a stationary condition, three kinds of forces--gravity, buoyancy, and friction. These three forces must always balance. The schematic of the forces is shown in Figure 23. The balance between the forces gives the following equation;

$$W_p = R_f + B_p \tag{A-1}$$

where; W_p = weight of particle.

R_f = resistant force due to friction between particle and fluid.

B_p = buoyancy of particle.

The weight of particle is expressed as the product of the volume of the particle and the specific gravity of the particle;

$$W_p = \frac{\pi}{6} ab^2 \gamma_s \tag{A-2}$$

where; a = the longest diameter of particle.

b = the shortest diameter of particle.

The resistant force due to friction between particle and fluid is given by;

$$R_f = \frac{1}{2} C_D \frac{\gamma_f}{g} b^2 U^2 S_c$$

$$= \frac{1}{2g} C_D \gamma_f b^2 U^2 \tag{A-3}$$

where; S_c = cross-section area of particle perpendicular to moving direction.

C_D = resistance coefficient of particle.

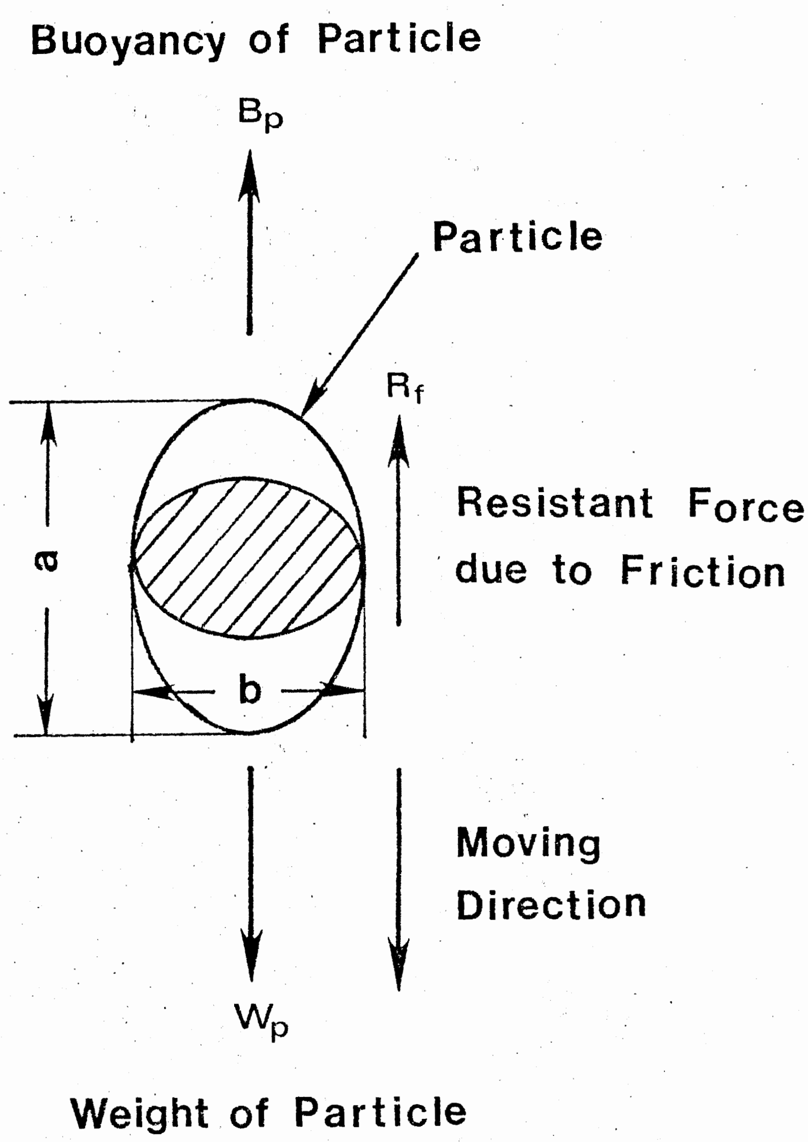


Figure 23. Schematic of Force Balance on Particle Motion

C_D is a function of Reynolds number, and is given by;

$$C_D = \frac{24}{R_e}$$

$$= \frac{24\mu g}{bU\gamma_f} \quad . \qquad \text{(A-4)}$$

Combining of Equation (A-4) and Equation (A-3) yields;

$$R_f = 3\pi \ \mu_f \ bU \quad . \qquad \text{(A-5)}$$

The buoyancy of a particle is given by;

$$B_p = \frac{\pi}{6} a \ b^2 \ \gamma_f \qquad \text{(A-6)}$$

Substituting Equation (A-2), Equation (A-5), and Equation (A-6) into Equation (A-1) yields;

$$U = \frac{ab}{18\mu_f} (\gamma_s - \gamma_f) \quad . \qquad \text{(A-7)}$$

APPENDIX B

TRANSFDRMATION OF NATURAL LOGARITHM TO

COMMON LOGARITHM

The flow distribution and flow density functions are described by the following equation;

$$R(k) = \frac{1}{\sigma_n \sqrt{2\pi}} \int_0^k \frac{1}{x} \text{Exp}\left\{ -\frac{1}{2}\left(\frac{\ln x - \mu_n}{\sigma_n}\right)^2 \right\} dX \qquad \text{(B-1)}$$

$$q(k) = \frac{1}{k\sigma_n \sqrt{2\pi}} \text{Exp}\left\{ -\frac{1}{2}\left(\frac{\ln k - \mu_n}{\sigma_n}\right)^2 \right\} \qquad \text{(B-2)}$$

where;

$$\mu_n = E[\ell n x]. \qquad \text{(B-3)}$$

$$\sigma_n = \sqrt{V[\ell n x]}. \qquad \text{(B-4)}$$

For any variable, the natural logarithm and the common logarithm have the following correlation;

$$\ell n x = 2.303 \log \qquad \text{(B-5)}$$

Substituting Equation (B-5) into Equation (B-3) and Equation (B-4) yields the correlation between these logarithms for the mean and standard deviation;

$$\mu_n = E[2.303 \log x] = 2.303\, E[\log x]$$
$$= 2.303\mu_c \qquad \text{(B-6)}$$
$$\sigma_n = \sqrt{V(2.303 \log x)} = \sqrt{(2.303)^2\, V[\log x]}$$
$$= 2.303\sigma_c \qquad \text{(B-7)}$$

where;

$$\mu_c = E[\log x] \qquad \text{(B-8)}$$

$$\sigma_c = \sqrt{V(\log x)} \qquad \text{(B-9)}$$

Substituting Equation (B-5) through Equation (B-7) into Equation (B-1) and Equation (B-2) yields the flow distribution and its density function for the common logarithm;

$$R(k) = \frac{1}{2.303\sqrt{2\pi}\,\sigma_c} \int_0^k \frac{1}{x}\, \mathrm{Exp}\left\{-\frac{1}{2}\left(\frac{\log x - \mu_c}{\sigma_c}\right)^2\right\} dX \qquad \text{(B-10)}$$

$$q(k) = \frac{1}{2.303\sqrt{2\pi}\,\sigma_c\, k}\; \mathrm{Exp}\left\{-\frac{1}{2}\left(\frac{\log k - \mu_c}{\sigma_c}\right)^2\right\}. \qquad \text{(B-11)}$$

APPENDIX C

NOMOGRAPHIC PREDICTION OF PUMP LIFE

Equation (63) is expressed in nomographic form in Figure 24a, 24b, 24c and 24d. As the equations modeled by the nomograph are very complicated, the nomograph is divided into four sub-parts in order to decrease the operating errors. A step-by-step procedure for the use of these nomographs is as follows:

1. On Figure 24a, mark the median pore size D_{m10} and y_{10} values line that intersects the E scale.
2. On Figure 24b, mark the y_{30} and y_{10} values on the appropriate scales. Connect these two points with a line that intersects the X scale.
3. On Figure 24c, mark the absolute value of E from Figure 24a and the value of X from Figure 24b on the appropriate scales. Connect these two points with a line that intersects the Y scale.
4. On Figure 24d, mark the Y scale with the appropriate value from Figure 24c. Choose the appropriate y_{10} scale according to the sign of E and mark the y_{10} value. Connect these two points with a line that intersects the T_p scale. Read the appropriate gradations of T_p according to the sign of E. T_p represents the predicted pump life in hours.

The pump life obtained by the above procedures is for an ingression rate of one thousand particles greater than a 10 micrometers in diameter of the contaminant per milliliter. If the ingression rate of the hydraulic system in the field is different from this value, the above pump life must be modified using Equation (64).

The nomographic pump life is quite accurate except for the region where the pump life is shorter than twenty hours. However, the pump life shorter than twenty hours is not realistic. Therefore, the nomographic pump life is useful when a hydraulics engineer designs an actual hydraulic system.

This nomographic procedure may, at first, seem to be cumbersome. The corresponding equations, however, are very complex. With some practice, the nomographs provide a relatively quick method of predicting the life expectancy of a pump.

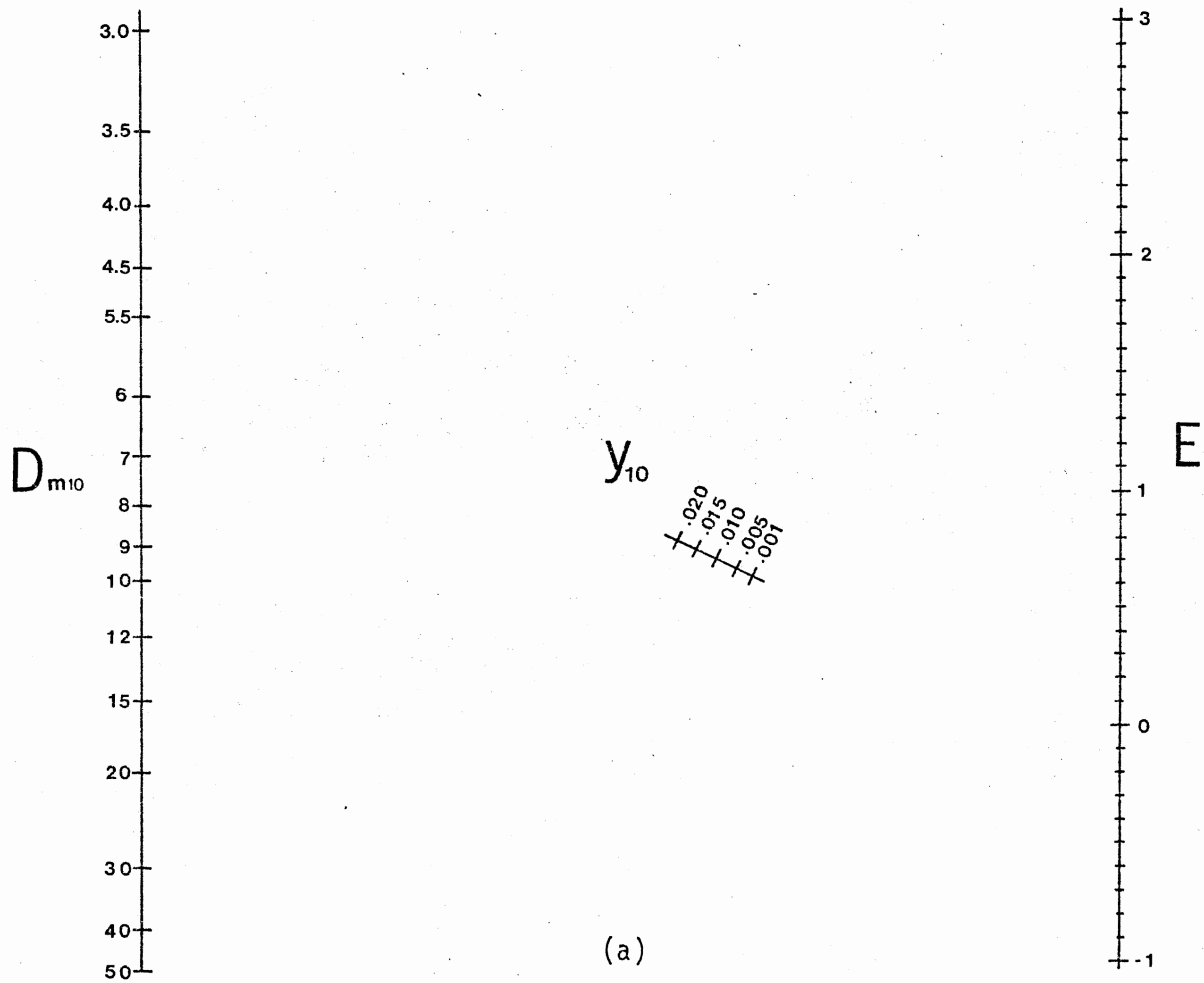
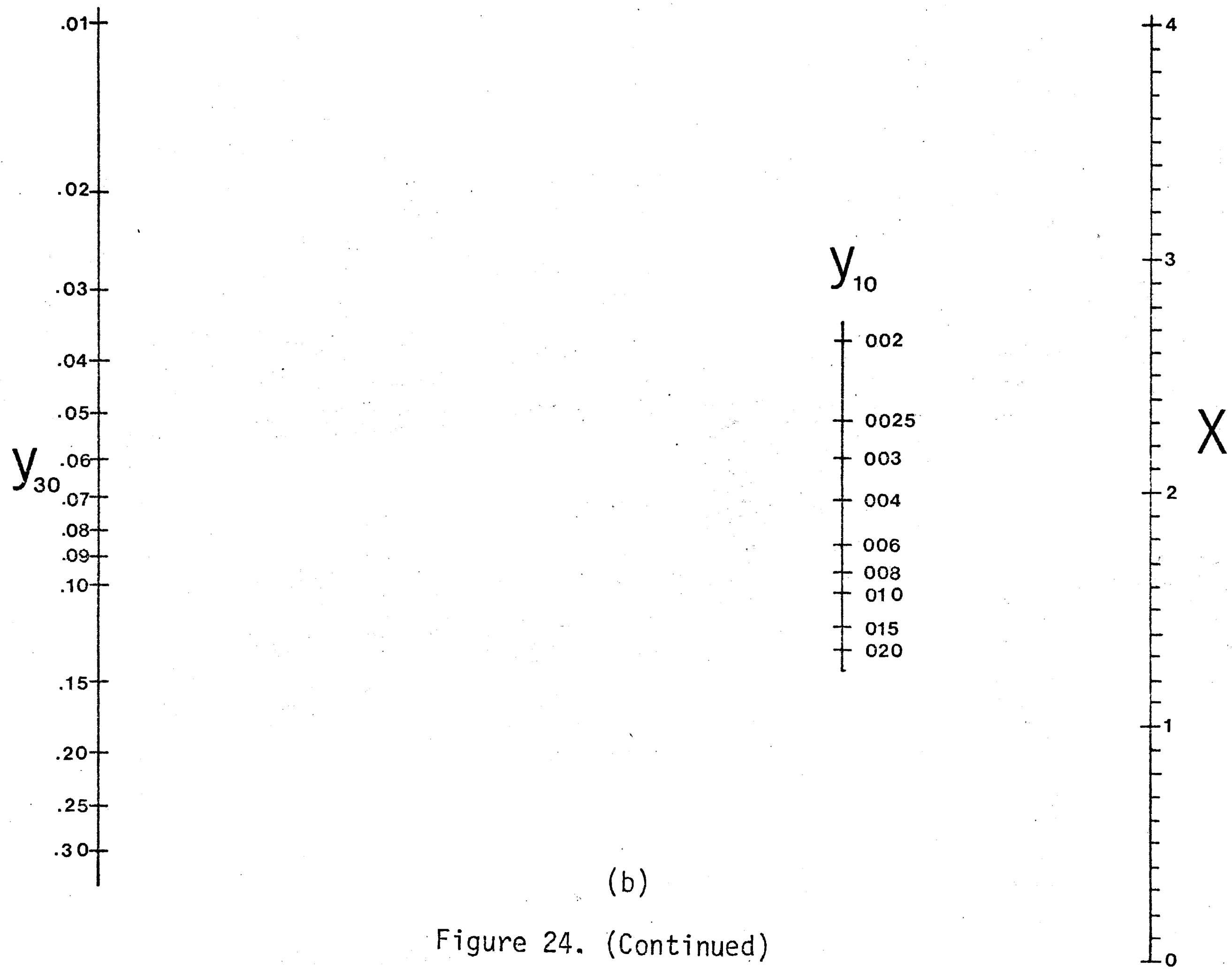
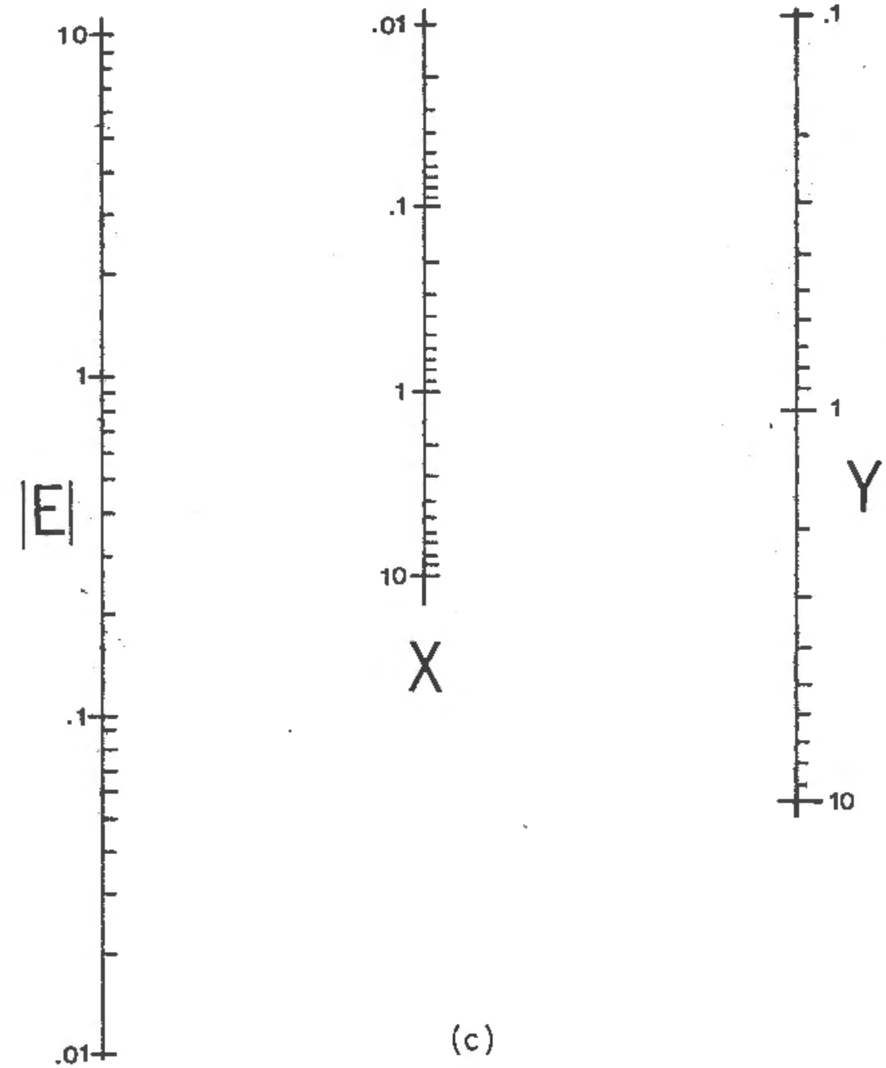


Figure 24. Nomograph Pump Life



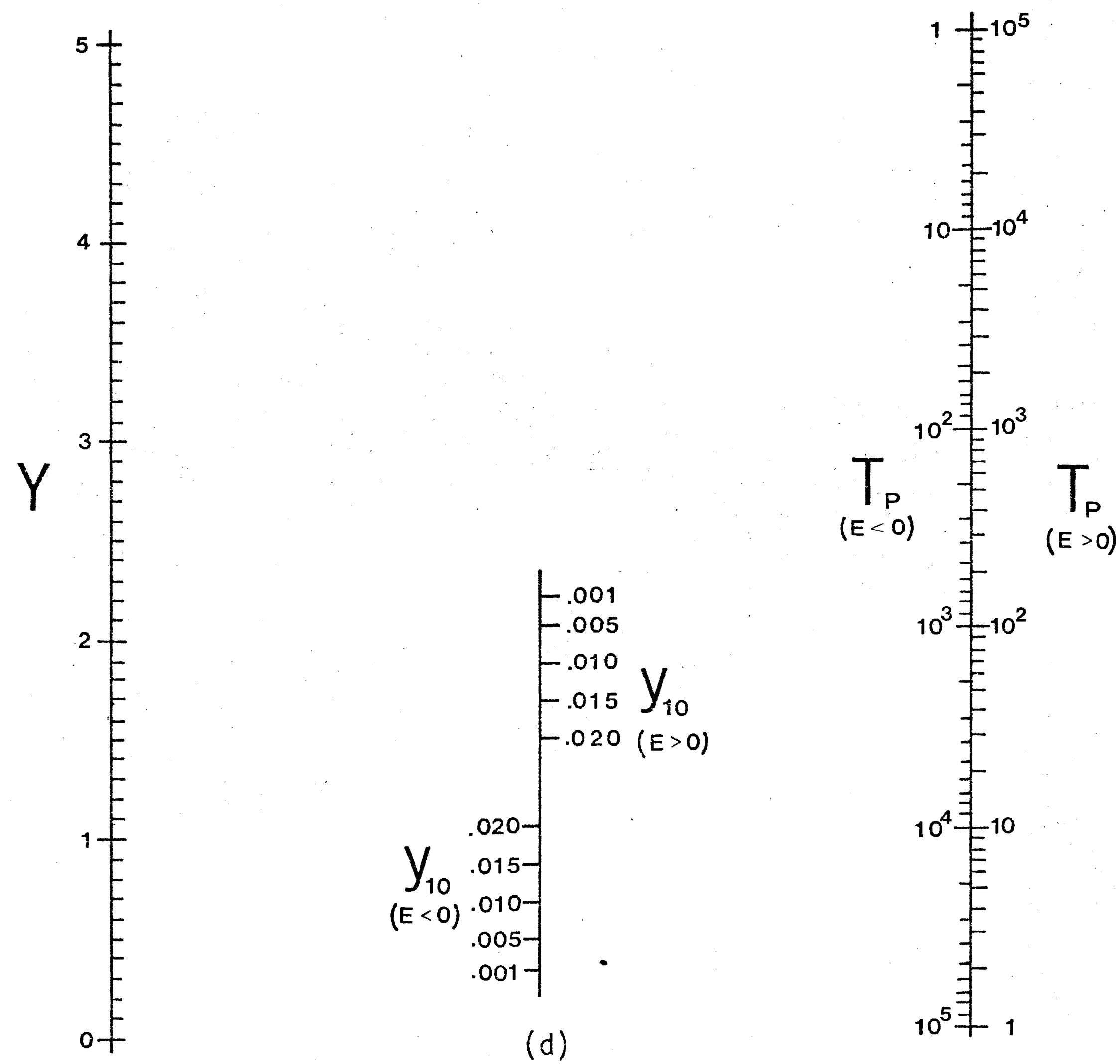
(b)

Figure 24. (Continued)



(c)

Figure 24. (Continued)



(d)

Figure 24. (Continued)

VITA

Masao Iwanaga

Candidate for the Degree of

Doctor of Philosophy

Thesis: CONTAMINANT SERVICE LIFE AND COST OF HYDRAULIC SYSTEM

Major Field: General Engineering

Biographical:

Personal Data: Born in Nagasaki, Japan, January 3, 1941, the son of Mr. and Mrs. Hajime Iwanaga. Married in Tokyo, Japan, May 1, 1967, to Michiko Nakamura. Beget Hiromi Iwanaga, July 2, 1968, and Seiya Iwanaga, October 18, 1970.

Education: Graduated from Nagasaki City High School, Nagasaki, Japan, in March, 1960; received the Bachelor of Science in Engineering degree from Nihon University, Tokyo, Japan, in 1964, with a major in Mechanical Engineering; received certification (Master of Science degree equivalent) from The National Defense Academy, Yokosuka, Japan, in 1970, with a major in Aerospace Engineering; completed requirements for the Doctor of Philosophy degree at Oklahoma State University in May, 1980.

Professional Experience: Research Engineer, Third Research Center, Technical Research and Development Institute, Japan Defense Agency, Tokyo, from April, 1964, to August, 1976; Project Associate, at the Fluid Power Research Center, Oklahoma State University, September, 1976, to July, 1979; Project Engineer, at Fluid Power Research Center, Oklahoma State University, July, 1979, to present.

Professional Affiliations: Japan Society of Mechanical Engineering, Japan Hydraulic and Pneumatic Association.

Publications: More than thirty papers and published research reports in the area of Hydraulic Control, Cavitation, Filtration Performance, and Hydraulic Reservoir.