RIGID AND KINETO-ELASTO DYNAMIC STUDY OF MECHANISMS

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Dedicated to

My wife, Ligia R. de Hossne

and

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for their endurance and spiritual support

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NOMENCLATURE

a_n	linear acceleration
a, b, c, d	link lengths
A	cross-sectional area
(A)	vector of coefficients of the shape functions
[A]	translational gyroscopic matrix
$[A_D]$	total translational gyroscopic matrix
[B]	translational axial matrix
[C]	boundary condition matrix
[D]	damping matrix
E	modulus of elasticity
ET	sum of external moments with respect to center of mass of element
F_1, F_4	forces in the X direction
F_2, F_5	forces in the Y direction
F_3, F_6	moment or torque
g	acceleration of gravity
I	cross-sectional moment of inertia
I_n	rotational moment of inertia
[K]	translational stiffness matrix in local coordinates
$[K_D]$	total translational stiffness matrix in local coordinates
L	length of element
LF	Lagrangian function
m_n	mass of element

m	mass of element of link per unit length
[M]	translational mass matrix in local coordinates
$[M_D]$	total translational mass matrix in local coordinates
MG	parameter equal to the product of m x g
(Q)	right-hand vector of the equations of motion
(S)	deformation vector in local coordinates
(S), $(\dot{S})$, $(\ddot{S})$	deformation, velocity, and acceleration vectors of the equation of motion, respectively
(S_e)	total deformation vector in global coordinates of the mechanism in the joint deformation form
(S'_e)	Total deformation vector in global coordinates of the mechanism in the element deformation form
T	kinetic energy
u, u(x)	axial function
V	potential energy
w, w(x)	transverse function
XLD	length of the deformed link
$\dot{\theta}$	angular position of element
θ	angular velocity of element
$\ddot{\theta}$	angular acceleration of center of mass of element
∂	variation formulation

CHAPTER I

INTRODUCTION

To predict system performance to a higher degree of accuracy so that energy saving could be achieved on account of the required size of a mechanism means that an improved mathematical model of the system is needed. Nobody is interested in a mechanism that either vibrates or is too heavy in accordance to its need, reliability, failure, and performance. With a more precise mechanism the phenomenon could be better understood and quality predicted.

The demand for higher speeds in production and execution procedure in mechinery has created greater problems for designers. The elastic deformations of the machine components cause inaccuracies in position, fatigue, failure, and noise. Special effects are produced when a mechanism is driven at resonant speed of its spring and mass. The performance of mechanisms at high speeds cannot yet be determined accurately in the design stage because there is not sufficient knowledge yet about the elastic properties of the links and effect of backlash in the bearing (22).

A study of the dynamic behavior of a mechanism may begin logically with an investigation of its properties which relate deflections to inertial and applied forces. Force-deflection properties based on the static or vibrating equilibrium states are not necessarily applicable to the same mechanism in the dynamic state. In mechanisms which transmit relatively large forces or move at high speeds, there are often considerable differences between the theoretical and actual motions. For this reason, it would be desirable not only to take into consideration the elastic properties of individual links but also to attempt a kinematic synthesis including the effects of elastic deformations. The magnitude of the inertia forces may be several times as large as the static forces, and may, in addition, possess quite different directions. The term "inertia force" denotes a force which is equal in magnitude to the product of mass and acceleration of the body, and opposite in direction to the acceleration vector (1, 2, 22). Presently, experimental measurements of dynamic effects are particularly important because of the influences mentioned previously, which cannot yet be adequately taken into consideration at the design stage. Dynamic investigations of mechanisms, taking into account the effects of inertia forces, are very time-consuming since not only the deformations, displacements, velocities, and accelerations, but also the static forces in a mechanism must be taken into consideration. The total load on a machine element is due to the combined effects of static and dynamic forces.

A general approach for the dimensional synthesis with or without optimization of rigid mechanisms would be very advantageous to the designer. The existing methods, especially for planar or spherical mechanisms, are useful for the synthesis of particular cases. In linkage design, constraints are necessary to insure closure and to make sure that the mechanism will operate. For this reason, unconstrained minimization techniques are of no use to the designer. With the availability of optimization methods which could handle inequality, parameter or function constraints, such as the penalty function approach of Fiacco and McCormick

(56), Marquardt's Algorithm (53), Gauss-Newton method, and Kubicek's algorithm (58), mechanism optimization became feasible. A method is needed to optimize the dimensional synthesis of mechanisms. Such considerations require decisions to be made concerning the exact form of the input motion, the form of the output motion, and the criteria for judging what constitutes an optimum configuration. Hence, the variability of the number of parameters that take place is considerable according to every design requirement.

The study of the dynamics of a system of interconnected rigid bodies, that is, higher-order effect caused by the elasticity of individual members is neglected, has been carried out by different approaches such as the joint force method (48), vector calculus, d'Alembert's principle and the principle of virtual work (57), and Lagrange's equation. Since the advent of the finite element method, there have been great changes in providing versatile mathematical models in every area of science. The study of the dynamics of rigid bodies has not been approached yet by the finite element scheme. By this approach, it is believed that classical dynamics could well be revolutionized.

The increasing use of digital computers and its time-saving ability to apply numerical techniques encouraged the development and application of finite element techniques. They can be formulated for almost any type of engineering problem, and solutions by classical methods that were considered impossible in the past years can now be obtained. In general, the finite element method provides the most efficient procedure for expressing the displacements of arbitrary mechanism configurations by means of a discrete set of coordinates. Better accuracy can be achieved in a dynamic analysis for a given number of degrees of freedom by using the shape function method of idealization than by the lumped mass approach (3).

Objectives

The main objective of this study was to develop mathematical models for analysis of rigid link and elastic link mechanisms. There is a belief that the models developed represent the phenomena with versatility, applicability, and reality. The major objective of this study can be broken down into the following categories:

1. Development of a method for the synthesis of planar and spherical mechanisms with rigid links. The method is based upon solving a system of nonlinear equations which represents the configurations of the mechanisms. The system of equations could be either solved iteratively, or by using Marquardt's algorithm, or the Gauss-Newton method. Optimization is also obtained by using the former methods.

2. Development of a mathematical model approached by finite element analysis, to perform dynamic analysis of mechanisms with rigid links.

3. Development of a general formulation of kineto-elasto-dynamic analysis which is applicable to study planar and space mechanisms with elastic links. The development of the mathematical model is based on finite element formulation and Hamilton's principle.

4. Development of a new approach for performing stress analysis, length determination, and plotting of the deformed links of the mechanism. The approach is based upon the shape functions which represent the deformation behavior of the elements of the mechanism.

5. An experimental analysis was conducted to select representative information to establish comparisons with analytical results.

The problem analysis is demonstrated by analyzing a four-link-crank-rocker mechanism. The synthesis of rigid links might be extended up to spherical mechanisms.

CHAPTER II

LITERATURE REVIEW

The literature in kinematics and dynamics of mechanisms has grown so rapidly within the past decades that it is not possible to attempt to review all of them or even the major contributions in the field. In order to provide some focus, however, a few references in the areas related to this study are presented.

The four-bar linkage is chosen as the mechanism to validate the applicability of the mathematical models developed because it is a common element in many engineering mechanisms and studied extensively. Although its kinematics have been investigated widely, its dynamics--rigid or not--have been found less amenable to general treatment, possibly because of the length and complexity of the governing transcendental equations. However, now that the use of digital computers is routine, it is a simple matter to solve a variety of hitherto intractable dynamical problems by numerical means.

There are various principles and techniques available for formulating mathematical models for the analysis of rigid mechanisms. Paul (72) reviews various principles and techniques available for formulating the equations of motion. Among those techniques are the vector methods, joint force analysis, d'Alembert's Principle, Lagrange's equations, and Hamilton's equations. Smith's (66) method reduces the calculation of reaction forces for multi-degree of freedom, constrained, mechanical, dynamic systems based on Lagrange's equations with constraints. Yang (67) formulates a dynamic equation, where bearing reactions and inertia torques are obtained, based on dual vector and screw calculus. Andrews and Kesavan (68) describe a procedure for applying graph theory which is based on vector mechanics to the analysis of general, dynamic, lumped mechanical systems. Bagci (69) uses the joint force method for the analysis of planar mechanisms with Coulomb and viscous damping. Smith and Maunder (70) and Suh (48) also use the joint force method for the dynamic analysis of planar mechanisms. Woo and Freudenstein (71) use screw coordinates for the dynamic analysis of planar or space mechanisms.

A survey of literature which describes the kineto-elasto-dynamic behavior of mechanisms is discussed to present the state-of-the-art and establish comparisons. A supplementary bibliography that presents a survey of investigations in this area performed by Lowen and Jandrasits (94) and Erdman and Sandor (102) is recommended.

In their analysis or synthesis, most of the works reviewed do not consider axial deformation due to known problems of instability. Several researchers have introduced this intrinsic phenomenon. Eringen and Woinowsky (28, 29) consider the effect of axial forces on the vibration of elastic bars. It was observed that axial stress increases rapidly with the decrease of the ratio of a nonlinear period over a linear period or with the increase of frequency ratios. The vibration of an extensible bar, carrying no transverse load and having the ends fixed at the supports, causes axial tensile force with a period equal to the half-period of the vibration of the bar. Axial buckling is important and axial deformations should not be neglected in a stability analysis.

The study of partially elastic mechanisms from a linear point of view has been approached by several researchers. Broniareck and Sandor (100) studied the dynamics of a four-bar linkage with massless elastic coupler and rigid cranks, connecting two rotating masses mounted on elastic shafts by examining Mathieu's equation. Stability conditions of the system are determined with the use of the stability chart.

Sadler and Sandor (64) use harmonic analysis for the study of mechanisms whereby one link is regarded as elastic. The equation of motion is derived by using Newton's second law and Lagrange's equation. Finally, employing harmonic analysis, the steady-state solution of the differential equation is obtained, and the dynamic coupler point path is determined for various speeds and damping ratios.

Gandhi and Thompson (59) developed a finite element equation appropriate for the analysis of a flexible planar mechanical system using a mixed variational principle. The element is defined with four degrees of freedom to represent transverse deformation and rotation at the nodes. The inertial loading from the rigid body analysis is used to continuously update the right-hand side of the following equation:

$$[M](S) + [K](\ddot{S}) = (Q). \tag{2.1}$$

In their application they consider only one link to be flexible. The Newmark method was used to solve the system of linear differential equations.

Gayfer and Mills (95) made a theoretical study of the small amplitude vibrations of a four-bar linkage. The mechanism considered has two flexible links and a rigid coupler; all members are of uniform cross section. The natural frequencies are calculated by receptance methods and showed positive agreement with experimental results. The theoretical work is based on the assumption that only flexural vibrations are important. The theory deals with small amplitude, undamped flexural vibrations of the linkage by means of receptance methods and it is assumed that changes in potential energy due to gravity are negligible.

Badlani and Kleinhenz (90) conducted a study of the dynamic stability of a slider-crank mechanism with an undamped elastic connecting rod. The analysis is done by application of the Euler-Bernoulli and Timoshenko beam theories. It is concluded that regions of instability exist when rotary inertia and shear deformation effects are included in the analysis. Seveers and Yanz (103) also studied the dynamic stability of the slider-crank mechanism.

Jandrasits and Lowen (33) performed a theoretical analysis of elastic-dynamic behavior of a counter-weighted four-bar linkage rocker link which carries an overhanging mass. The equations of motion are obtained using Hamilton's integral and Kantorovich's method. Hill's equations were used to furnish the time portions of the solutions and the Floquet theory is adopted for stability considerations. In general, good qualitative and quantitative agreement between analytical and experimental results was found. The Runge-Kutta method was used to solve the system of differential equations.

The methods used for the quasi-static and vibrational analysis of structures have been extended to study mechanisms with elastic links by considering them as instantaneous structures. Ashok et al. (35) derive an iterative technique for analysis of elastic deformation of mechanisms which provides an approximate particular solution. The algorithm is time-step size independent. The total system mass and system stiffness matrices are constructed by utilizing the permutation vector method of structural analysis. The mechanism is regarded as an instantaneous structure at every position.

Smith and Maunder (79) determined the stability boundaries of the coupler of a four-bar linkage using a two-parameter perturbation method. Later, Smith (80) proposed a three-parameter perturbation method for the same problem. Starting from partial differential equations and assuming the shape function of the coupler to be a first mode sine function, they obtained a single undamped Hill's equation as the basis for the stability analysis.

Iman, Sandor, and Kramer (37) extended the permutation method of structural analysis to obtain linkage mass and stiffness matrices for the determination of deflection and rotation equations at the nodal points. They introduced the rate of change of the eigenvalues with respect to the motion of mechanisms. The rate of change of the eigensystems with respect to time has also been used for time efficiency (the modal analysis method was used to solve the system of differential equations) (61). The linkages are regarded as an instantaneous structure at every position. The equations of motion can be written as

$$[M](\ddot{S}) + [D](\dot{S}) + [K](S) = 0 \tag{2.2}$$

where

[M],[D],[K] = system mass, damping, and stiffness matrices;

(Q) = inertia forces due to the gross rigid body motion of the mechanism plus external forces; and

(S) = unknown generalized coordinates.

Bagci and Kalaycioglu developed a method for the elasto-dynamic analysis of planar mechanisms. The finite element method and lumped mass system are used to formulate the equations of motion of a mechanism:

$$(S) = [F_s][(Q) - [M](\ddot{S}) - [D](\dot{S})] \tag{2.3}$$

where

$(S),(\dot{S}),(\ddot{S})$ = generalized coordinate displacement vectors and its first and second time derivatives, respectively;

$[F_s]$ = generalized coordinate external flexibility matrix;

$[M]$ = diagonal mass matrix consisting of the masses and mass moments of inertia lumped to the generalized coordinates;

$[D]$ = damping matrix; and

(Q) = forcing vector which consists of any time dependent externally applied force components in the directions of the generalized coordinates.

The method also includes inertial forces and inertial torques due to kinematic gross motion of the mechanisms. Axial deformation of link members is neglected. The system of differential equations is solved by the matrix exponential method.

Sandor (63) develops a method whereby the kineto-elasto dynamic analysis of mechanisms is approached by the lumped parameter technique. The system of differential equations obtained is solved by the Runge-Kutta method.

Syed and Soni (19) conduct the static elastic analysis of a path-generating four-bar and its cognate mechanism. The flexibility method of structural analysis is employed to determine the elastic displacements of the coupler point. The deflections were observed to be nearly doubled when the speed was increased from 300 to 400 rpm.

$$(S) = [b]^T[F_s][b](Q)$$

where $[F_s]$ is the flexibility matrix. (Q) is the system force vector, and [b] is the force-transfer matrix.

Erdman (17) demonstrates how a flexibility matrix may be employed in setting up a system of coupled differential equations describing the vibrational behavior of a linkage. The link deformations are represented mathematically by an operator. Each element consists of six degrees of freedom. The differential equations of motion used are formed by introducing a mass matrix, based on the lumped method and a damping matrix:

$$[M](\ddot{S}) + [D](\dot{S}) + [K](S) = (Q) \tag{2.4}$$

The Runge-Kutta method was used to solve the system of differential equations. Erdman et al. (17, 32, 60, 90) develop a kineto-elasto dynamic equivalence approach using the flexibility matrix methods. Erdman, Sandor, and Oakberg (60) first applied the flexibility approach of structural analysis to a quasi-static deflection investigation of mechanisms. The method was extended to spatial mechanisms.

Ashok, Erdman, and Frohrib (39) develop a numerical closed-form algorithm applicable to the design of elastic-link mechanisms. During each time step, the system parameters (mass, damping,end stiffness) are assumed to remain constant in solving the equations of motion. The displacement finite element is used to develop the mass and stiffness matrices of the linkage. The Wilson method (18) is used to construct the damping matrix.

Alexander and Lawrence (30, 31) analyze two planar mechanisms with a mathematical model based upon the stiffness method of structural analysis, and obtain an analytical coupler and output link strain variation. The axial strains are determined to be less than five percent of the bending strains in a four-bar planar mechanism. The obtained results matched the experimental results. The Runge-Kutta method was used to solve the system of differential equations.

Midha et al. (23) demonstrate the effects of a multi-element idealization of the links of an elastic planar crank-rocker mechanism on modal frequencies. One-, two-, and four-element idealizations were used. Their method is based upon the stiffness method of matrix structural analysis, and determines the natural frequencies and normalized mode shapes of beam structures. A six-degree-of-freedom element was used. The undamped structural equations of motion are written as

$$(Q) = [K] - \omega^2 [M](S) \tag{2.5}$$

where

[K] = structural stiffness matrix;

[M] = structural mass matrix;

(S) = vector of unknown displacements;

ω = undamped natural frequency; and

(Q) = vector of applied forces on the structure.

Midha et al. conclude that a multi-element idealization of the linkage members was desirable.

Winfrey (25, 62) uses the stiffness approach of structural analysis to perform the analysis of elastic planar and spatial linkages. He also utilizes the reduction of coordinate techniques for determining a particular deflection in a mechanism (26). Elements with six degrees of freedom are considered. The equations of motion are

$$[M](\ddot{S}) + [K](S) = (Q) \tag{2.6}$$

where

$$[m]_{ij} = \int_0^{\ell} m(x)\phi_i(x)\phi_j(x)\,dx \tag{2.7}$$

$$[k]_{ij} = \int_0^{\ell} \ddot{\phi}_i \, EI_i(x) \, \ddot{\phi}_j(x) \, dx \tag{2.8}$$

where (Q) is the vector of inertial forces computed previously from the gross rigid motion. The modal analysis method was applied to solve the system of equations, and was extended to the analysis of the Bennett mechanism.

Midha et al. (18) presented a general approach for deriving the equations of motion of planar linkages. The finite element method by way of Lagrange's equations is employed to develop the mass and stiffness properties of an elastic linkage. Each element consists of six degrees of freedom. The equations are expressed as

$$[M](\ddot{S}) + [D](\dot{S}) + [K](S) = -[M](\ddot{S}r) \tag{2.9}$$

where $(\ddot{S}r)$ is the rigid body acceleration vector.

Nath and Ghosh (54) developed a method for the vibrational analysis of planar mechanisms. The equations of motion are developed by determining the element stiffness matrix, mass matrix, and load vector from the following equations (55):

$$[K] = \int_V b^T x b \, dv, \tag{2.10}$$

$$[M] = \int_V \rho a^T a \, dv \tag{2.11}$$

$$(\phi) = \int_V a^T q \, dv \tag{2.12}$$

where

b = matrix of strains due to unit displacements;

x = component of the matrix of stress;

ρ = density;

a = matrix function of the position coordinate (in general, one can expect only approximate expressions for a); and

Q = load vector.

Several examples were worked out but no comparisons were established.

Bahgat and Willmert (15) present the vibrational analysis of general planar mechanisms using a finite element approach. The analysis considers both axial and lateral vibrations using a high order hermite polynomial approximation. The result is a system of linear ordinary differential equations expressed as

$$[M](\ddot{S}) + [D](\dot{S}) + [K](S) = (Q) \tag{2.13}$$

where the mass, the stiffness and damping matrices, and the force function are all functions of the rigid body motion of the mechanism; therefore, the rigid body dynamics must be solved first. They consider eight degrees of freedom per element.

Boronkay and Mei (101) analyze a multiple flexible link mechanism using the finite element method in which the revolute joints are replaced by flexible joints.

Mahalingam (96) attempts to improve the results of Gayfer and Mills (97). He believes that the solution of the receptance equation for the entire range of configurations was tedious. He demonstrates that configurations in which natural frequencies are stationary are independent of the elastic characteristics of the input and output links, and depend only on the geometry of the system and mass distribution of the coupler.

Kohli et al. (93) consider the effects of elastic links, elastic supports, and elastic shafts of a slider-crank mechanism by elastodynamic analysis. The rigid displacements of the mechanism links due to deformations in the support are evaluated by the Taylor series approximation. The deformations of the links are approximated by using a finite number of terms in a Fourier series employing the Raleigh-Ritz method. The equations of motion of the slider-crank rocker are obtained by Lagrange equations. A set of linear ordinary differential equations is obtained. It is assumed that axial load does not vary over the length of the link:

$$[M_b](\ddot{S}) + [M_c](\dot{S}) + [D](S) + [M_k](S) = (Q) \qquad (2.14)$$

where matrices $[M_b]$, $[M_c]$, $[M_k]$, and vector (Q) are functions of the time-dependent rigid body motion and the inertia and stiffness properties of the mechanism links and shafts. The Runge-Kutta method was used to solve the system of equations.

The nonlinear analysis of elastic mechanisms has been approached by several researchers. Sadler and Sandor (38, 88, 89) develop an analytical lumped-parameter model for the nonlinear vibration analysis of mechanisms. Application of finite difference approximations to the Euler-Bernoulli beam theory leads to a system of coupled, damped, ordinary nonlinear differential equations. Axial deformations were ignored. The model is applicable for both periodic and nonperiodic mechanism motion and for both uniform and nonuniform cross-sectional links. The model is used to determine the bending deflections, dynamic stresses and strains of a four-bar-planar linkage.

Kohli and Sandor (92) extend the lumped parameter approach for kineto-elasto dynamic analysis of planar mechanisms to the analysis of elastic spatial mechanisms. The Runge-Kutta method is used to solve the system of equations.

Jasinski, Lee, and Sandor (82) derive the equations of motion of a slider-crank mechanism considering its connecting rod to be elastic. They neglect the nonlinear coupling term. The ratio of the length of the crank to the length of the connecting rod is considered to be less than one. Longitudinal and transverse vibrations are considered. Two simultaneous nonlinear periodically time-variant partial differential equations represent the model.

Chu and Pan (78) derive the equations of motion of a slider-crank mechanism with an elastic connecting rod. Assuming first mode sinusoidal shape functions, they use the method of Kantorovich and the method of weighted residuals. Stability criteria are presented based on the Floquet theory. The resulting equations are solved numerically by use of the piecewise polynomial method and the fourth-order Runge-Kutta method.

Viscomi and Ayre (83) study the nonlinear dynamic response of an elastic slider crank mechanism by assuming small displacements and neglecting axial deformation. The energy method is used to develop the equation of motion which is solved by using Hemming's modified predictor-corrector method.

Neubauer, Cohen, and Hall (81) examine the vibrations of the connecting rod of the slider-crank. Longitudinal deformations, Coriolis, and relative tangential and relative normal components of acceleration are neglected. The equations of motion are derived by making use of the D'Alembert principle and the Euler-Bernoulli equation. Leibnitz's rule is used for differentiation of the obtained integral. The fourth-order partial differential equations are solved by the finite-difference method. The equations are also linearized and solved using the Runge-Kutta method. The periodic coefficient was approximated by a cosine term.

Dressing (14) processes an algorithm to determine the periodic coefficients of the linear differential equations which describe the relations between dynamic forces and motions in planar mechanisms. The algorithm allows for calculations of vibrations in mechanisms with n degrees of freedom. The algorithm is represented by a system of nonlinear differential equations which are approached by means of a system of linear differential equations with periodic coefficients.

Sutherland (77), using a mode analysis approach, presents the derivation of equations of motion of a fully elastic four-bar linkage. The problem is approached by forming the Euler-Lagrange equations of motion for the system in a manner similar to that which would be used for a complex rigid body dynamic problem. A set of partial coupled ordinary differential equations is produced. Joint reaction forces are automatically eliminated because the Lagrange approach is used to determine the equations of motion. The effect of axial forces on the lateral deflections of each member is considered, but the actual axial equilibrium conditions for the members are not. The Runge-Kutta method is used to solve the system of equations. The validity of the mathematical model is then confirmed by the results of a physical experimentation.

Variational calculus has also been used by several researchers to approach the elastic study of mechanisms. Thompson et al. (59, 97) develop a variational approach whereby the physical system characteristics are embodied in a functional. The first variation of the functional yields the equations of motion of the complete mechanism.

Cleghorn, Fenton, and Tabarrok (13) obtained a procedure for determining the equations of motion of a mechanism with flexible links. The governing equations are derived using Hamilton's principle. The equations are discretized by the finite element method. The procedure considers a flexible-axially-rigid beam subjected to prescribed translations and rotations. Seven degrees of freedom are considered for the element. The equations obtained are:

$$[M](\ddot{S}) - 2\dot{\theta}[B](\dot{S}) + [[K] - \dot{\theta}^2[M] - \ddot{\theta}[B] + P_o[A^*] + P_1[B^*] + P_2[D]](S) = (Q) \quad (2.15)$$

where the mass, gyroscopic, and stiffness matrices are observed. The different values of P represent the coefficients of a second degree polynomial that represent the distribution of longitudinal load for rigid motion.

There is a wide interest in the synthesis of elastic mechanisms. Many approaches that present several models have been published. In general, the approaches discuss the difficulty of taking the inertia forces into consideration in the synthesis of a mechanism. The mechanism should first of all be designed by considering its members rigid in order to determine the distribution of mass among the various members. If unfavorable characteristics result, improvements can be made by changing the link dimensions (22).

Patwardhan and Soni (20) present a method for synthesizing a planar crank-rocker mechanism with elastic links. Synthesis equations are developed, and the equations of motion are determined by obtaining the mass matrix and stiffness matrix separately. The mass matrix is obtained by application of the stiffness method. Six degrees of freedom are considered for each element. The equations obtained are:

$$[M](\ddot{S}) + [D](\dot{S}) + [K](S) = (Q) \quad (2.16)$$

The synthesis equations are represented by three nonlinear algebraic equations in three unknown link proportions. The unknowns involve unknown deflections and their time rate of change. No calculated examples have been presented.

Khan and Willmert (21, 24, 27) introduce the concept of a constant length finite element technique for vibrational analysis of planar mechanisms. The technique can also be applied to links of a mechanism which have variable cross-sectional sizes along their lengths. They consider each element to be axially rigid. A six-degrees-of-freedom element is used with vertical deformation, slope, and curvature at each node. The resulting form of the differential equations describing the vibrational motion is

$$[M](\ddot{S}) + [K](S) = (Q) \qquad (2.17)$$

The solutions of these equations give the vibrational deformation, slope, and bending moment as functions of time at the ends of the finite element. The method determines the optimal cross-sectional size of the elements, and minimizes the total weight of the mechanism subjected to limitations on the stresses in the links using an optimality criterion technique. Based on the Kuhn-Tucker conditions for an optimal solution, a recursion relation is derived which is used to change the values of the design variables from one iteration to the next.

Erdman et al. (17, 32, 60, 91) present a synthesis method based on the Burmester theory, the complex number method, and the stretch rotation operator in an iterative synthesis-analysis-resynthesis algorithm.

Sandler and Sandor (38) introduce a scheme for minimizing the maximum stress level in an elastic member of a given length without increasing the total mass. The procedure is based on an iterative technique for finding the uniform strength shape where depth is the only variable.

Physical experimental studies have been performed by several researchers with the main objective of establishing comparisons with the analytical results. The following works have been reviewed and are presented according to the publishing date.

Gayfer and Mills (95) perform an experimental study of small amplitude vibrations of a four-bar linkage. The fundamental natural frequencies of the linkage are examined by free vibration tests, and fundamental resonant response curves are obtained under conditions of forcing. The input was a mechanical scotch yoke driven when links were flexible and an electromagnetic exciter operated when links were rigid. The two forms of instrumentation used were long exposure photography which give a blurred image of the amplitudes, and electrical capacitance gages whose signals were recorded through a multi-channel camera.

Alexander and Lawrence (30, 31) present experimental results of strain histories of coupler and rocker midpoints of a four-bar linkage containing three elastic links. Strain gages were mounted at three points on both the coupler and output links of the four-bar mechanism experimental model. At each point, the gages were mounted on both upper and lower surfaces of the beam so that bending and axial strains could be recorded separately. A photocell was employed to obtain cycle timing data. Strain data were recorded using an oscilloscope equipped with a type Q transducer and strain gage preamplifier plug-in unit and a camera.

Sutherland (77) investigates experimentally a constant speed elastic four-bar function generator. The prime mover was a master 1/3 hp ac induction motor with an integral 7.5-60:1 variable speed reduction unit. A five-step cone sheave V-belt drive (for course speed range changes) was used to drive a shaft which was connected by timing belts to the split crank. Each crank shaft had an 8 x 3/4 in. cast steel disk flywheel attached to reduce crank speed fluctuations. To obtain the angular position of the crank and follower shafts, 321 Servo Gamewell precision conductive plastic single turn continuous rotation potentiometers were used. The shaft potentiometers were charged by an SCR variable potential dc power supply, and output was observed on a Tektronix 502A oscilloscope. The recording was made on a Brush light-beam oscillograph. Good results were obtained by taking high speed 16mm motion pictures (1000 and 2000 frames per second).

Jandrasits and Lowen (33), in their experimental investigation, use a four-bar linkage consisting of rigid aluminum coupler and a thin brass rocker link with a counterweight m and end mass M. The crank is combined with the flywheel. The rocker link thickness and sizes of the end mass and counterweight represent the tradeoffs to obtain sufficient deflection in the experiment without exceeding the elastic limit. Strain gages (SR-4 350 ohm foil) are located at the top continuation of the rocker and middle of the rocker. The mechanism is directly driven by a variable speed 1/6 hp dc motor, and its speed is monitored by a tachometer generator with a 60 cycle/resolution ac output. The zero angle of the input is determined with the aid of a photoelectric transducer and a reflecting tape attached to the flywheel. The range of speed was between 110 and 200 rpm.

Bagci and Kelaycioglu (34) use a full scale model to perform their experiment. The response of the model is tested by measuring normal strain at certain joint locations on the output link with strain gages fixed at the center of the output link. Normal strains are recorded using a Tektronix cathode ray oscilloscope equipped with a type Q transducer, a strain gage amplifier plug-in unit, and a Polaroid camera. Different input crank speeds are used ranging from 85 rpm to 430 rpm.

CHAPTER III

SYNTHESIS OF PLANAR MECHANISMS WITH RIGID LINKS BY A GEOMETRIC ANALYTIC METHOD

The approach to be followed will be based upon the method developed in Reference (52) that was introduced for the design of spherical double rockers class I and class II linkages. The method consists in solving, either iteratively or by Marquardt and Gauss-Newton techniques, a set of trigonometric equations obtained from the triangular configurations that the mechanism forms at certain steps. The set of equations is also optimized by application of Marquardt's method and Gauss-Newton's technique (53). The triangular configurations for a four-bar planar mechanism are shown in Figure 1.

The existing methods consist in giving solutions to specific problems with specific conditions (50, 84, 86, 87). The problems they presented can be solved and, furthermore, optimized by the method presented here.

To develop the method, one has first of all to consider that there is a total of seventeen unknowns. Sixteen of the unknowns are obtained from the configurations of Figure 1 and one more that represents the oscillation angle of the output link c. It is only possible to obtain three independent trigonometric equations from each configuration, plus the equation that relates the two limit positions of the output link and its oscillation angle. Therefore, a total of thirteen independent

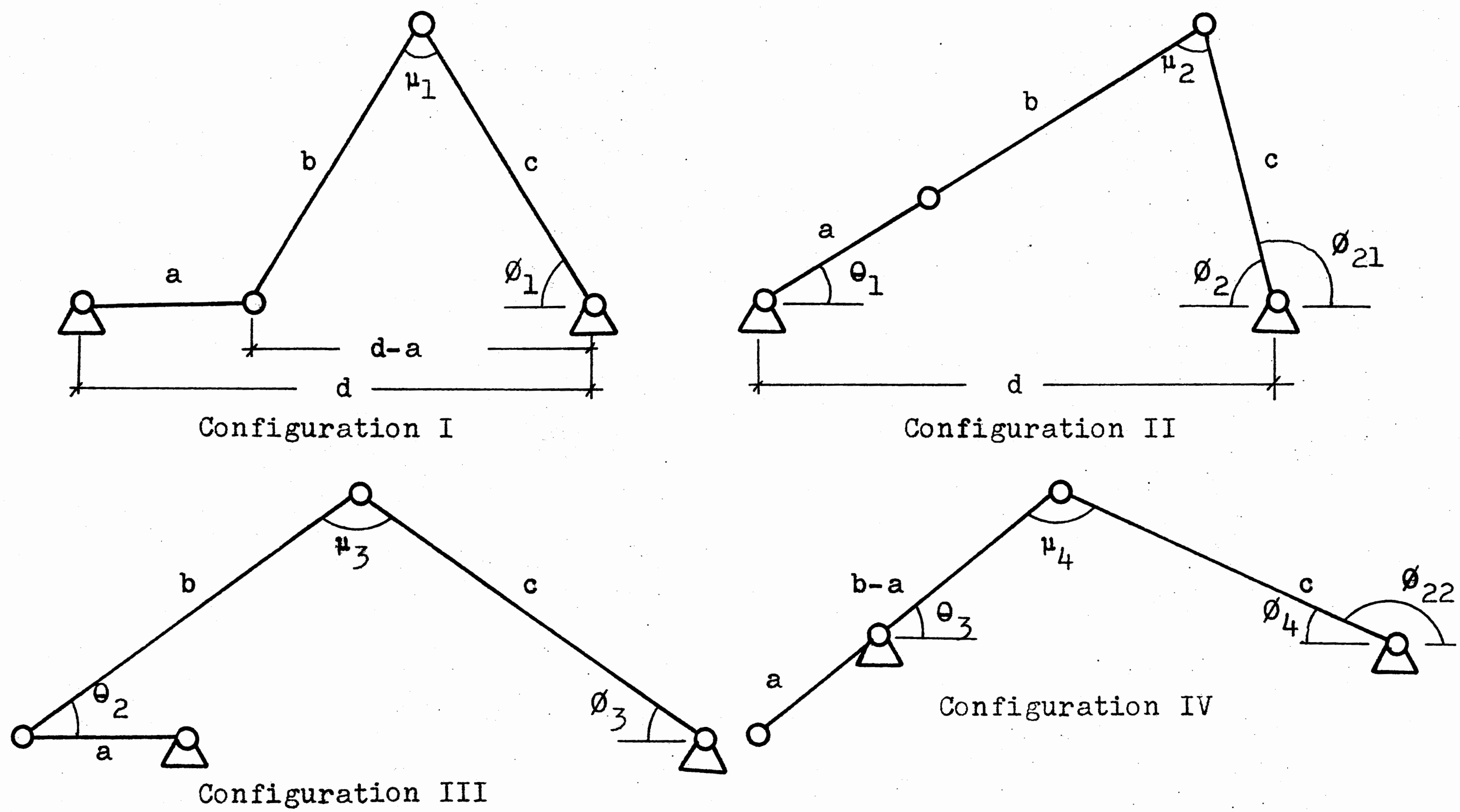


Figure 1. Triangular Configurations of a Four-Bar Mechanism

equations is obtained which means that four unknowns at least have to be prescribed so that the system of trigonometric equations might be solved.

It has been found that the number of trigonometric equations had to be equal to the number of unknowns so that a solution might be obtained. By Taylor series, every continuous and differentiable function may be expressed in a polynomial form; on the other hand, by Fourier series, any polynomial function may be expressed in terms of trigonometric functions.

Therefore, the set of equations to be used depends upon the prescribed conditions or assumptions for the problems to be either solved or optimized. The named thirteen equations could be used to define the complete synthesis of the four-bar planar mechanism, and the prescribed parameters might be chosen according to the designer's requirements. If optimization is required, then several different problems might be set up accordingly. Two cases are presented here in accordance with the needs of this study and to show the applicability of the method.

The optimization of the two cases presented is carried out either by assigning a range of variability to the maximum transmission angle in the Marquardt's algorithm or by introducing the following expression.

$$\mathrm{Min} \leq \mathrm{Tr} \leq \mathrm{Max} \tag{3.1}$$

$$\mathrm{Tr} = \frac{\mathrm{Max} - \mathrm{Min}}{2} \sin\theta + \mathrm{Mn} \tag{3.2}$$

where Tr represents the parameter upon which the optimization is based, Max and Min represent the limits of variation of Tr, Mn is normally equal to the positive difference between Max and Min, and θ is a new variable to be found.

Case Study I

The oscillation angle $\Delta\phi$ and one limit position of the output link, the minimum transmission angle μ_1, and the fixed link length are prescribed.

The system of equations to be used has to be obtained from Configurations I, II, and IV of Figure 1. Hence, a total of six independent equations is sufficient and necessary to design the mechanism. The vector of unknowns is represented by:

$$(\bar{X})^T = (a\ b\ c\ \phi_4\ \mu_2\ \theta_1) \tag{3.3}$$

and the system of equations is represented by:

$$F_1(\bar{X})^T = \phi_4 - \phi_2 + \Delta\phi \tag{3.4}$$

$$F_2(\bar{X})^T = \mu_2 + \theta_1 + \phi_2 + 180 \tag{3.5}$$

$$F_3(\bar{X})^T = c\ \sin\mu_2 - d\ \sin\theta_1 \tag{3.6}$$

$$F_4(\bar{X})^T = b - \frac{1}{2}\,((c^2 + d^2 - 2cd\ \cos\phi_2)^{1/2} + (c^2 + d^2 - 2cd\ \cos\phi_4)^{1/2}) \tag{3.7}$$

$$F_5(\bar{X})^T = a - d + (b^2 + c^2 - 2bc\ \cos\mu_1)^{1/2} \tag{3.8}$$

$$F_6(\bar{X})^T = \theta_1 - \text{arcos}\ ((d - c\ \cos\phi_2)/(b + a)). \tag{3.9}$$

Case Study II

The oscillation angle $\Delta\phi$ of the output link, the limit positions θ_1, and θ_3 of the input link and the fixed link are prescribed.

The system of equations to be used has to be obtained from Configurations II and IV of Figure 1. A total of seven independent equations is sufficient and necessary to design the mechanism. The vector of unknowns is represented by:

$$(\bar{X})^T = (a\ b\ c\ \phi_2\ \phi_4\ \mu_2\ \mu_4) \quad (3.10)$$

and the system of equations is represented by:

$$G_1(\bar{X})^T = \phi_4 - \phi_2 + \Delta\phi \quad (3.11)$$

$$G_2(\bar{X})^T = \mu_2 - 180 + \theta_1 + \phi_2 \quad (3.12)$$

$$G_3(\bar{X})^T = \mu_4 - \theta_1 + \theta_3 - \Delta\phi - \mu_2 \quad (3.13)$$

$$G_4(\bar{X})^T = c \sin\mu_4 - d \sin\theta_3 \quad (3.14)$$

$$G_5(\bar{X})^T = b - \frac{1}{2} c\ (\sin\phi_2/\sin\theta_1 + \sin\phi_4/\sin\theta_3) \quad (3.15)$$

$$G_6(\bar{X})^T = a - (d - c \cos\phi_2 - b \cos\theta_1)/\cos\theta_1 \quad (3.16)$$

$$G_7(\bar{X})^T = \sin\phi_4 - ((b - a) \sin\theta_3/c). \quad (3.17)$$

Results and Discussion

Two computing programs in FORTRAN were written to solve iteratively the system of equations for Case Study I and II, respectively. For the optimization procedure, Marquardt's method and Gauss-Newton's technique were used together so that a much more accurate solution could be obtained (see Appendix G). The checking out of the solutions was carried out by using Grashof's criteria (85).

Figures 2 and 3 show the design charts for the two cases presented. The solutions are optimized based on the transmission angle to range between 30 and 90 degrees. The two examples show the versatility and generality of the presented method, especially in the area of

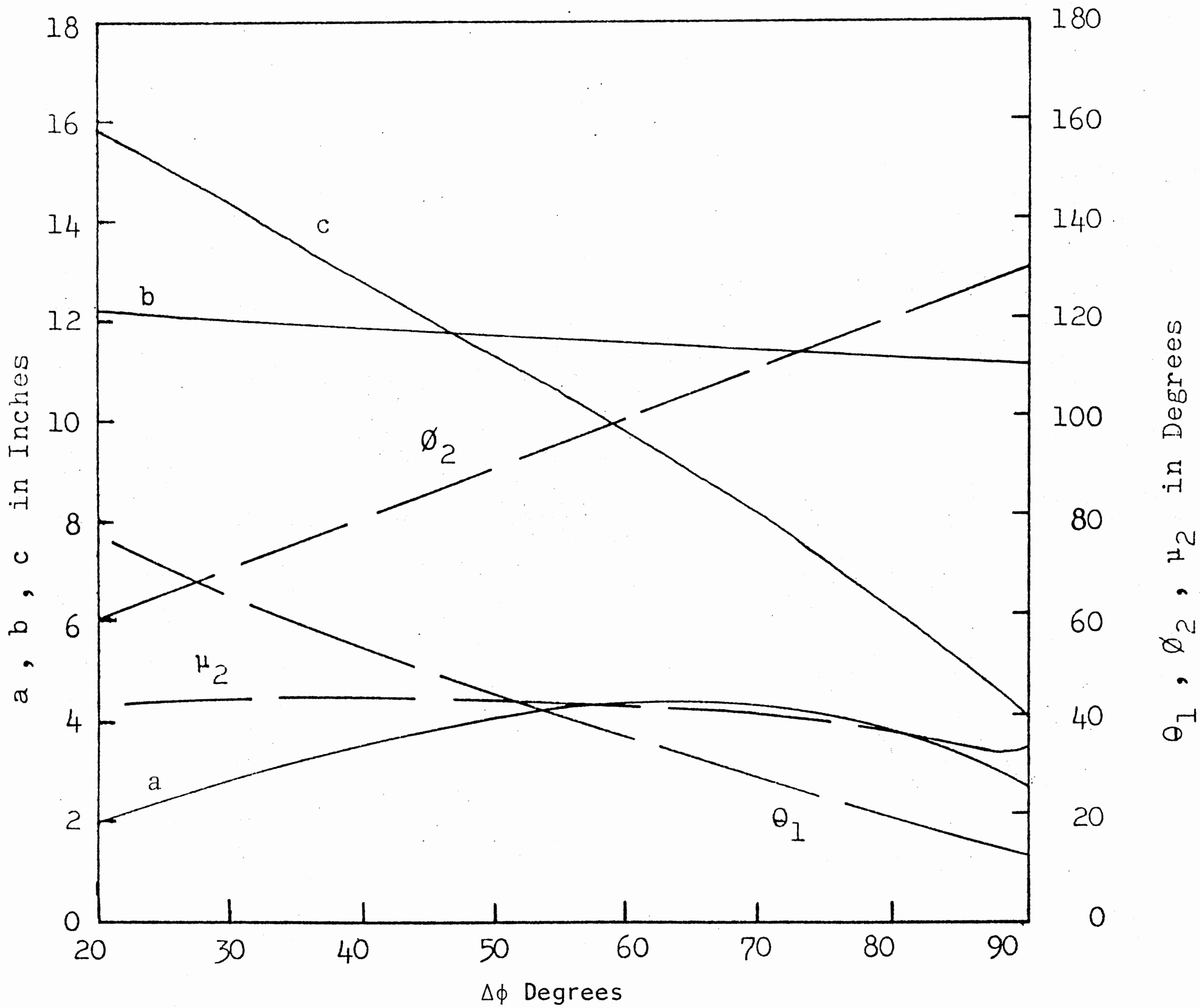


Figure 2. Design Chart for a Four-Bar Planar Mechanism, Given μ_1, $\Delta\phi$, ϕ_4, and d

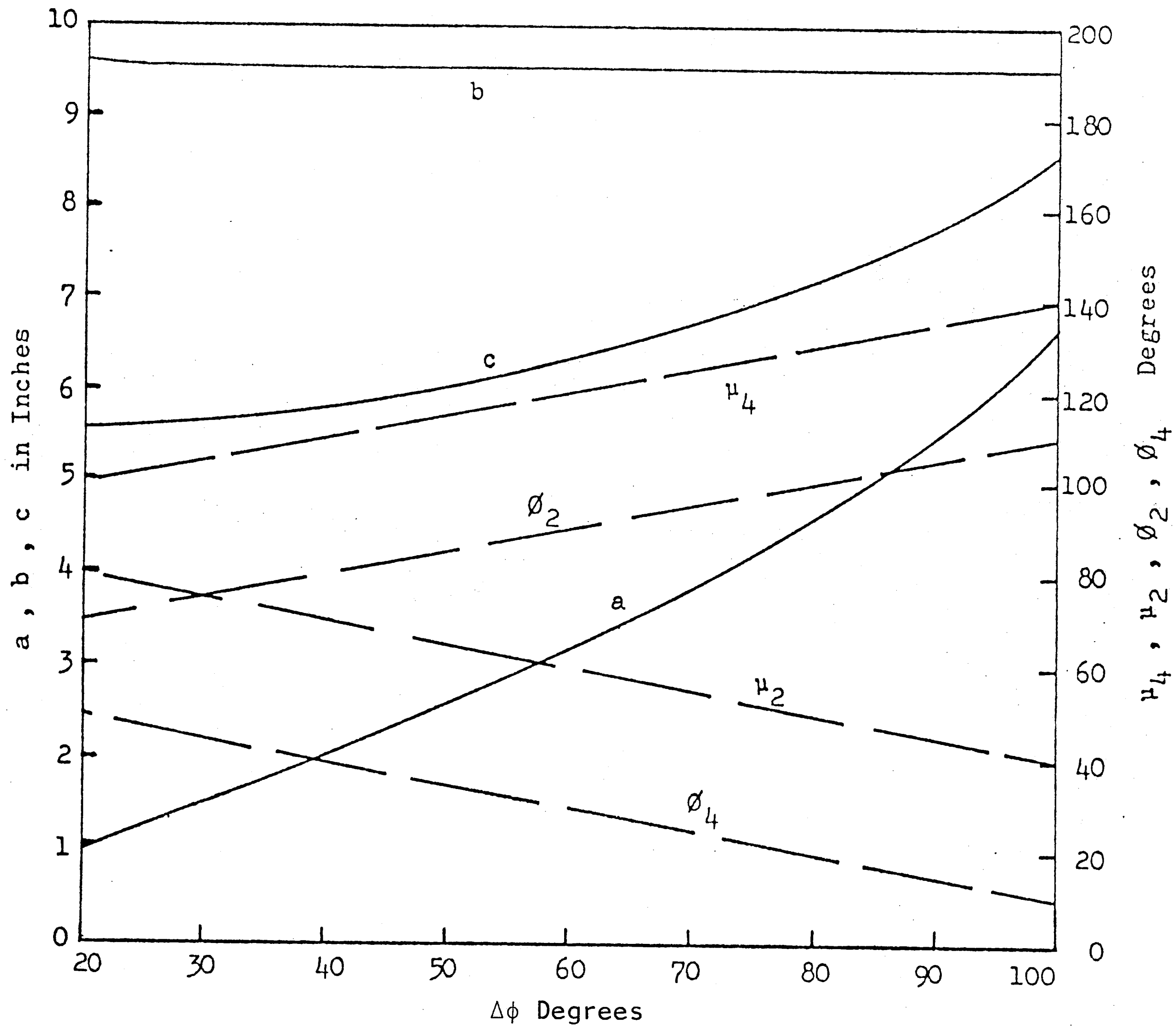


Figure 3. Design Chart of a Four-Bar Planar Mechanism, Given $\Delta\phi$, θ_1, θ_3, and d

optimization which could be done by using every type of constraints, whether it is either a parameter or a function constraint of any type.

CHAPTER IV

DYNAMIC MODEL FOR RIGID-LINK PLANAR MECHANISMS BY A FINITE ELEMENT APPROACH

To carry out the kineto-elasto-dynamic study of planar mechanisms it is necessary to have some general information concerning the rigid body motion. A finite element approach based on a variable cross-sectional element is introduced. This method is believed to be applied here for the first time in the area of rigid linkage design. The element has six degrees of freedom which represent forces and moments at the extreme nodes. The element is defined to have at its center of mass linear acceleration in the x and y directions, angular acceleration, external moment, and gravity force (see Figure 4). By analyzing such a figure, one can show the following relations to exist:

$$(F)^T = (F_1\ f_2\ F_3\ F_4\ F_5\ F_6) \tag{4.1}$$

$$(P)^T = (\ddot{G}_{jx}\ \ddot{G}_{jy}\ \ddot{\theta}_j\ ET_j\ MG_j) \tag{4.2}$$

$$(R)^T = ((X_k - X_i)/2\ (Y_k - Y_i)/2) \tag{4.3}$$

$$(H)^T = ((X_k = -X_i)/2\ (Y_k - Y_i)/2) \tag{4.4}$$

where for a constant cross-sectional element vectors $(R)^T$ and $(H)^T$ are equal.

A crank-rocker-four-bar planar mechanism has one degree of freedom for motion. Therefore, only one torque is necessary to put the mechanism into motion. Let us choose node M in Figure 5 to be the one where

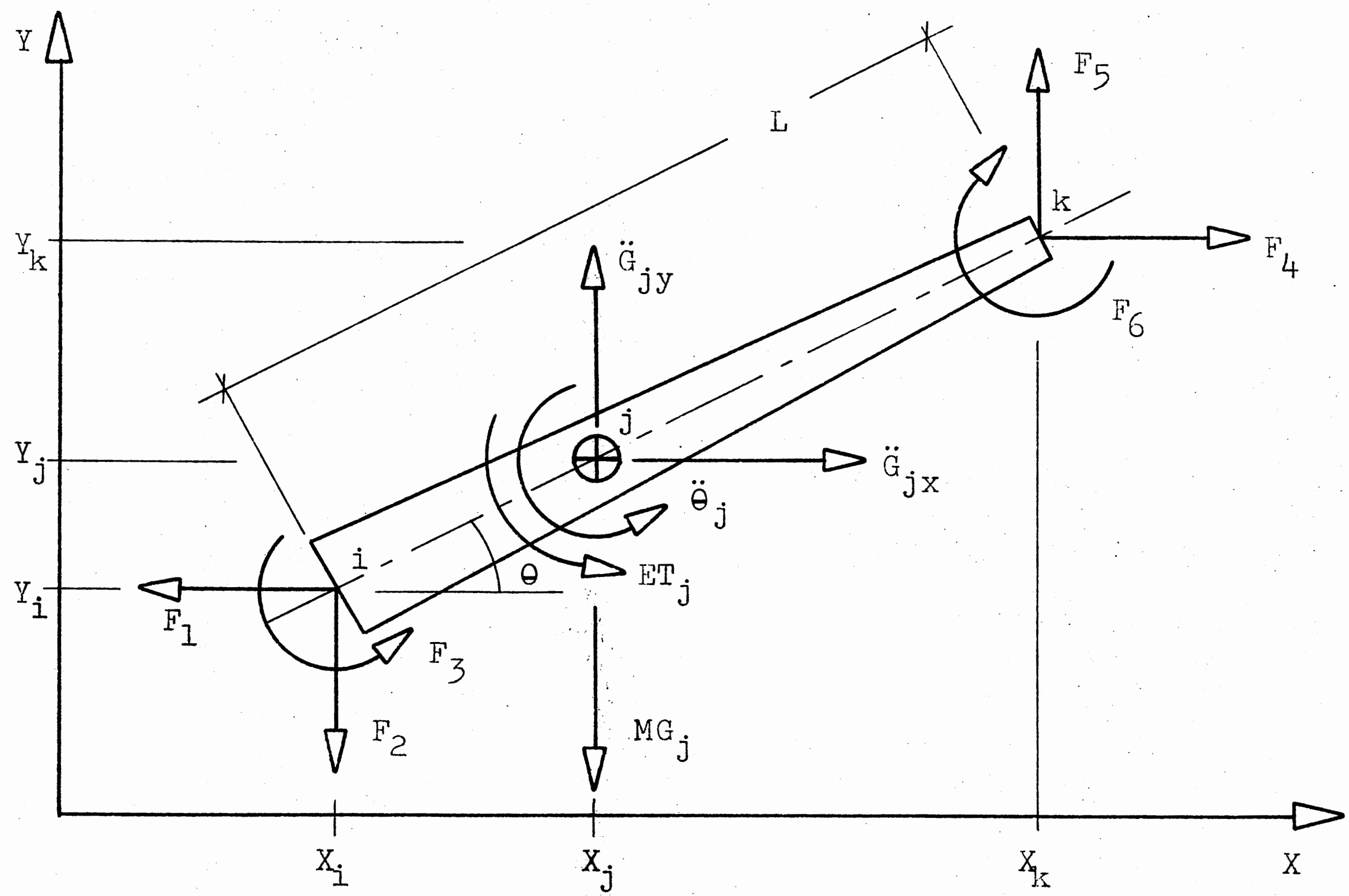


Figure 4. Rigid Element with Six Degrees of Freedom for the Dynamic Study of Rigid-Link Planar Mechanisms

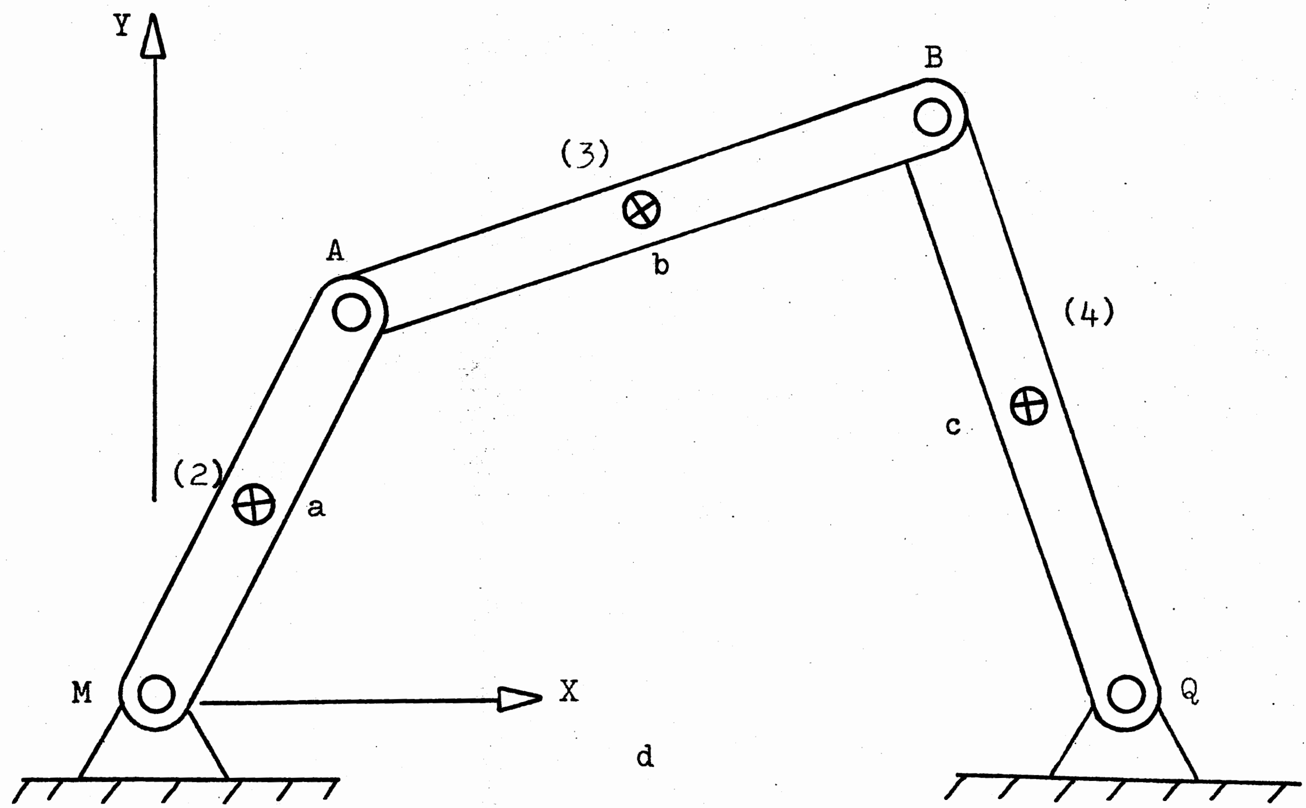


Figure 5. Ideal Four-Bar Planar Mechanism

torque is applied. Hence, the torques at A, B, and Q are assumed to be zero since the revolute pairs are assumed to be frictionless. According to the general equations of motion for a rigid body in planar motion, we have (6, 8, 48):

$$\Sigma F = m\ a \tag{4.5}$$

$$\Sigma M = I\ \ddot{\theta} \tag{4.6}$$

The general equations of motion for the element are given by

$$-F_1 + F_4 = m_n\ \ddot{G}_{nx} \tag{4.7}$$

$$-F_2 + F_5 = m_n\ \ddot{G}_{ny} + MG_n \tag{4.8}$$

$$-F_1 R_{ny} + F_2 R_{nx} + F_3 - F_4 S_{ny} + F_5 S_{nx} - F_6 = I_n \ddot{\theta}_n - ET_n \tag{4.9}$$

These equations are expressed in matrix form by Equation (4.10). There, n is the number of elements that form the mechanism; matrix [d] is the instantaneous geometric matrix; and vector (f) is the force vector and vector (e) is the inertial vector of the element.

$$\begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ -R_{ny} & R_{nx} & 1 & -S_{ny} & S_{nx} & -1 \end{bmatrix} \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{pmatrix} = \begin{pmatrix} m_n & \ddot{G}_{nx} & & \\ m_n & \ddot{G}_{ny} & + & MG_n \\ I_n & \ddot{\theta}_n & - & ET_n \end{pmatrix} \tag{4.10}$$

$$[d]\ (f) = (e) \tag{4.10a}$$

To apply Equation (4.10) to any planar mechanism, the assembling procedure will be shown through the dynamic study of a four-bar planar mechanism (see Figure 5).

One, two, and three elements per link are considered (see Figures 6, 7, and 8). The global instantaneous geometric matrix [D] for the mechanism is obtained by using the assembling Equation (4.11):

$$\underset{L\times K}{[D]} = \underset{L \times M}{\begin{bmatrix} d & & & & & \\ & d & & & & \\ & & d & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & d \end{bmatrix}} \underset{M \times K}{\begin{bmatrix} a & & & & & \\ & a & & & & \\ & & a & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & a \end{bmatrix}} \tag{4.11}$$

where

$$L = 3 \times n \tag{4.12}$$

$$M = 6 \times n \tag{4.13}$$

$$K = 3 \times n + 3 \tag{4.14}$$

and the assembling matrix [a] is given by

$$[a] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.15}$$

which has been obtained and generalized through the use of Figures 6, 7, and 8. When boundary conditions are applied, then matrix [D] is a square matrix. The global equations of motion for the mechanism are structured; therefore, the force vector can be found at any step of the mechanism position. The vector of boundary conditions always eliminates the moments at the pin joints, except at the input pin joint. Finally, Equation (4.16) is obtained.

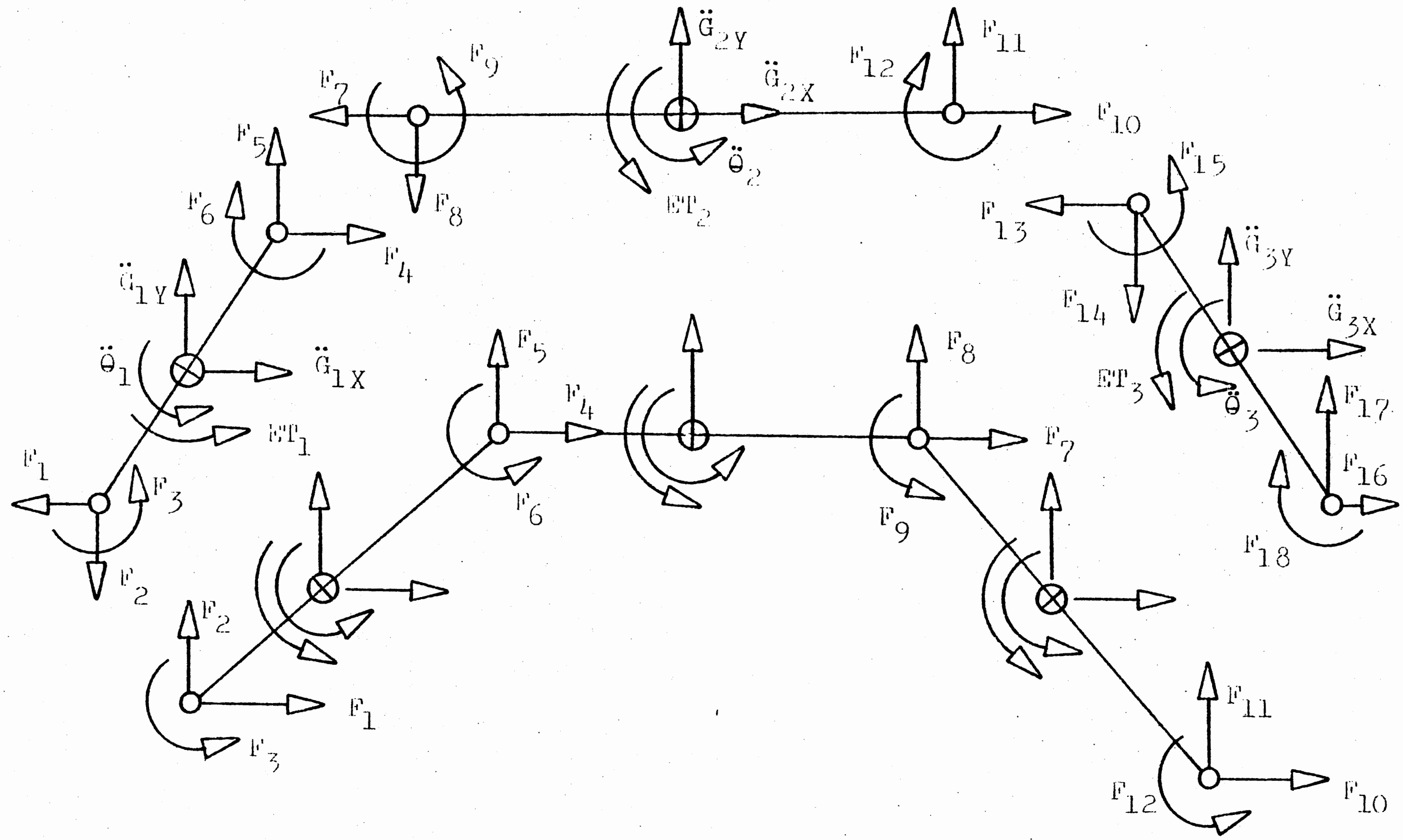


Figure 6. The Element Force System and the Joint Force System for a Four-Bar-Planar Mechanism. One Element Per Link

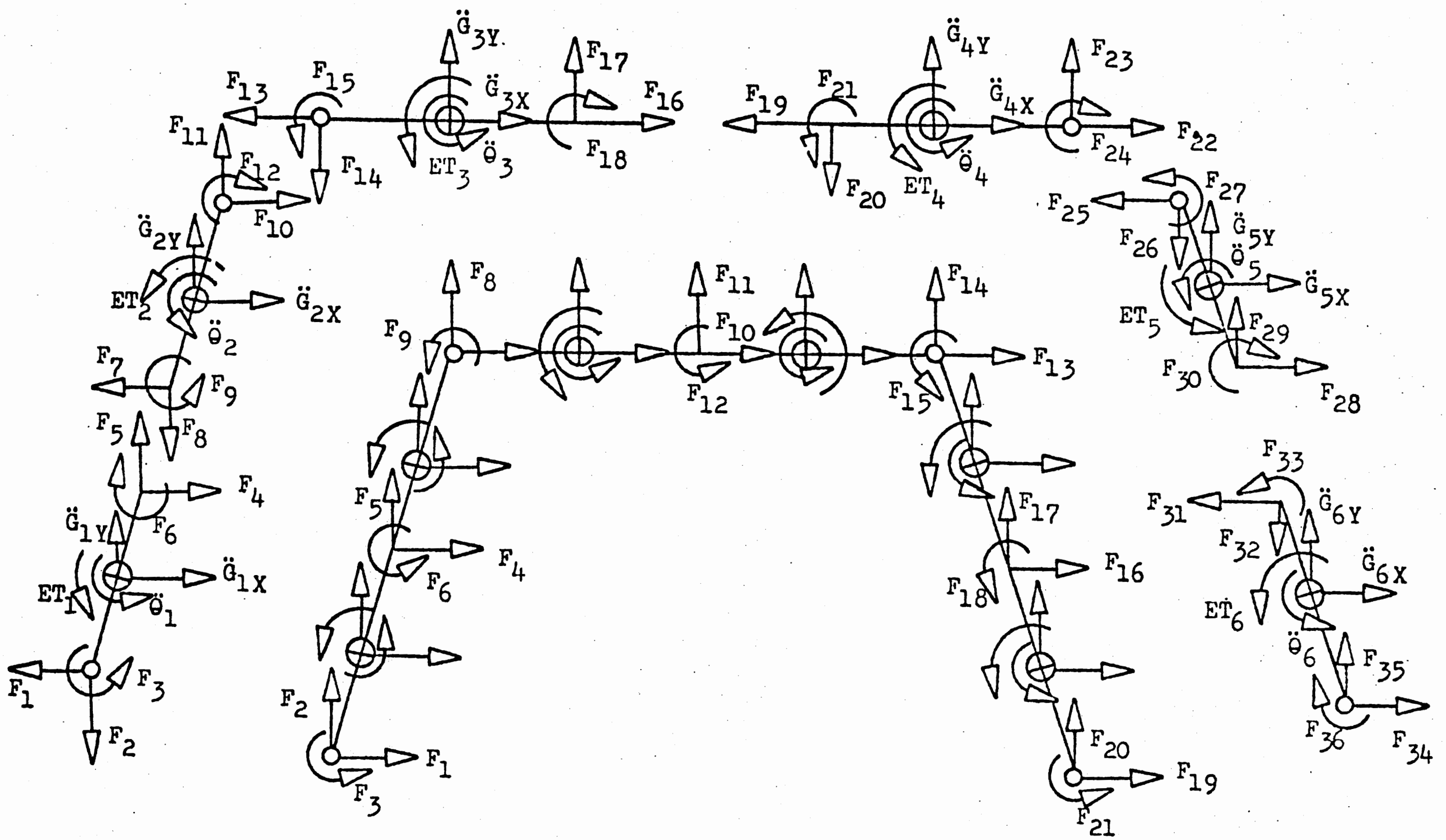


Figure 7. The Element Force System and the Joint Force System for a Four-Bar-Planar Mechanism. Two Elements Per Link

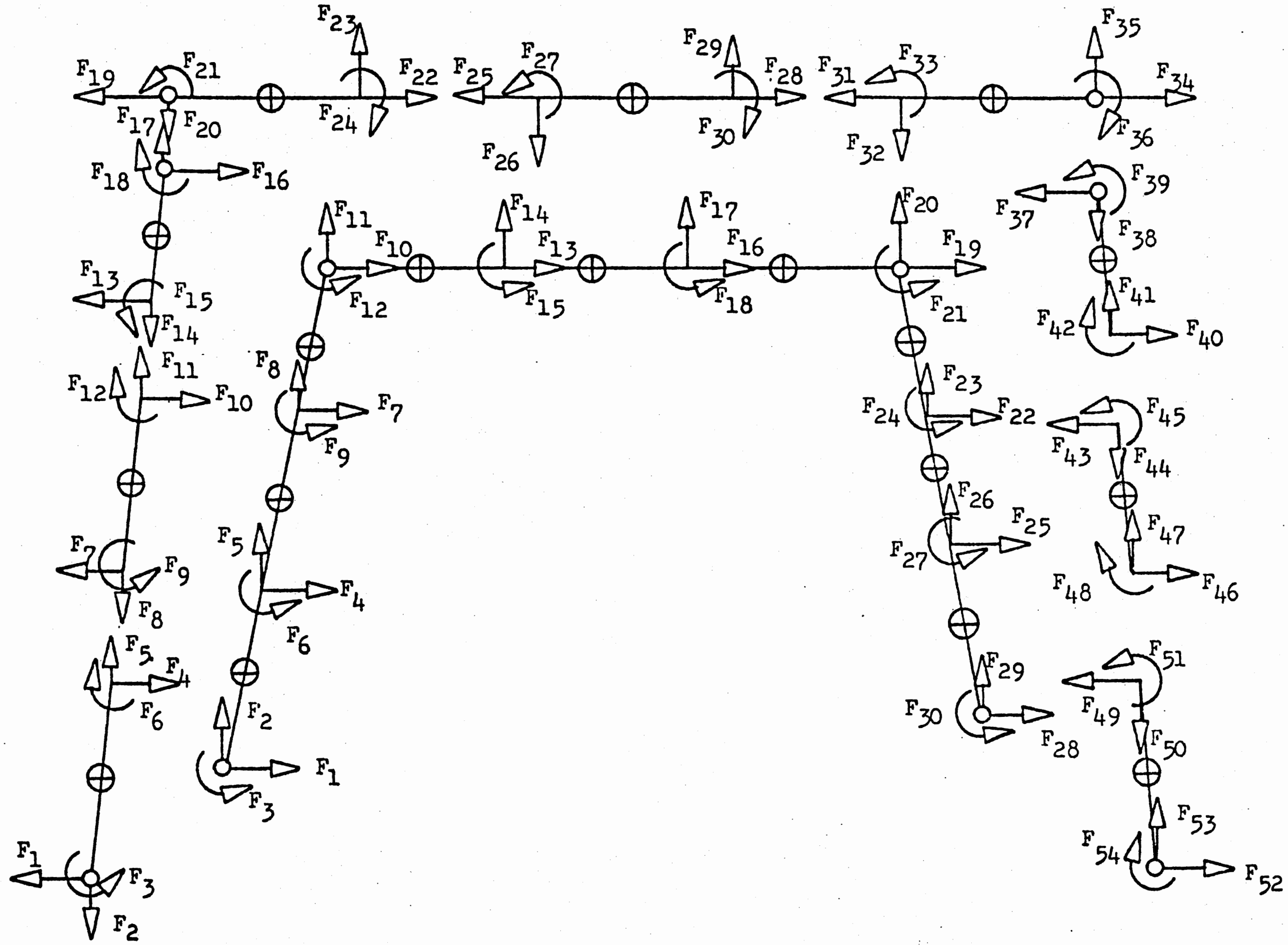


Figure 8. The Element Force System and the Joint Force System for a Four-Bar-Planar Mechanism. Three Elements Per Link

$$\underset{L\times L}{[D]}\ \underset{L}{(F)} = \underset{L}{(E)} \tag{4.16}$$

The information needed for the global inertial vector (E) of the mechanism is also needed in the kineto-elasto-dynamic study. To obtain this information, one starts first of all with Freudenstein's equation to determine the direction angle for each link (49, 50):

$$K_1 \cos\theta_3 - K_2 \cos\theta_1 + K_3 = \cos(\theta_1 - \theta_3) \tag{4.17}$$

where

$$K_1 = d/a \tag{4.18}$$

$$K_2 = d/c \tag{4.19}$$

$$K_3 = (a^2 - b^2 + c^2 + d^2)/2ac \tag{4.20}$$

By letting

$$K_4 = d/b \tag{4.21}$$

$$K_5 = (c^2 - d^2 - a^2 - b^2)/2ab \tag{4.22}$$

$$K_6 = \cos\theta_1 + K_3 - K_1 - K_2 \cos\theta_1 \tag{4.23}$$

$$K_7 = -2 \sin\theta_1 \tag{4.24}$$

$$K_8 = K_1 + K_3 - (1 + K_2)/\cos\theta_1 \tag{4.25}$$

$$K_9 = K_4 \cos\theta_1 + \cos\theta_1 + K_5 - K_1 \tag{4.26}$$

$$K_{10} = K_4 \cos\theta_1 - \cos\theta_1 + K_5 + K_1 \tag{4.27}$$

we have:

$$\theta_3 = 2 \arctan((-K_7 \pm (K_7^2 - 4K_6K_8)^{1/2})/2K_6) \tag{4.28}$$

$$\theta_2 = 2 \arctan((-K_7 \pm (K_7^2 - 4K_9K_{10})^{1/2})/2K_9) \tag{4.29}$$

The angular velocities and accelerations for the coupler and output links are given by (49):

$$\dot{\theta}_2 = (a\dot{\theta}_1 \sin(\theta_3 - \theta_1))/(b\sin(\theta_2 - \theta_3)) \tag{4.30}$$

$$\dot{\theta}_3 = (a\dot{\theta}_2 \sin(\theta_2 - \theta_3))/(c\sin(\theta_3 - \theta_2)) \tag{4.31}$$

$$\ddot{\theta}_2 = (H_1H_2 - H_3H_4)/(H_3H_5 - H_6H_2) \tag{4.32}$$

$$\ddot{\theta}_3 = (H_1H_5 - H_6H_4)/(H_3H_5 - H_6H_2) \tag{4.33}$$

where

$$H_1 = a\theta_1\sin\theta_1 + a\dot{\theta}_1^2\cos\theta_1 + b\dot{\theta}_2^2\cos\theta_2 - c\dot{\theta}_3^2\cos\theta_3 \tag{4.34}$$

$$H_2 = c\ \cos\theta_3 \tag{4.35}$$

$$H_3 = c\ \sin\theta_3 \tag{4.36}$$

$$H_4 = a\ddot{\theta}_1\cos\theta_1 - a\dot{\theta}_1^2\sin\theta_1 - b\dot{\theta}_2^2\sin\theta_2 + c\dot{\theta}_3^2\sin\theta_3 \tag{4.37}$$

$$H_5 = b\ \cos\theta_2 \tag{4.38}$$

$$H_6 = b\ \sin\theta_2 \tag{4.39}$$

The angular accelerations for nodes and center of mass are given by the following equations:

$$\begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} \begin{pmatrix} \dot{\theta}_1\dot{\theta}_2C_2 - \dot{\theta}_2^2C_1 - \dot{\theta}_3^2C_1 + \dot{\theta}_1\dot{\theta}_3C_3 \\ \dot{\theta}_1\dot{\theta}_3C_3 - \dot{\theta}_3^2C_2 - \dot{\theta}_1^2C_2 + \dot{\theta}_1\dot{\theta}_2C_1 \\ \dot{\theta}_1\dot{\theta}_3C_1 - \dot{\theta}_1^2C_3 - \dot{\theta}_2^2C_3 + \dot{\theta}_2\dot{\theta}_3C_3 \end{pmatrix} + \begin{pmatrix} \ddot{\theta}_2C_3 - \ddot{\theta}_3C_2 \\ \ddot{\theta}_3C_1 - \ddot{\theta}_1C_3 \\ \ddot{\theta}_1C_2 - \ddot{\theta}_2C_1 \end{pmatrix} \tag{4.40}$$

where S_i represents the points whose accelerations are to be found. T_i represents a point where the accelerations are known. The position vector $\vec{C}$ is given by:

$$\vec{C} = \vec{S} - \vec{T} \tag{4.41}$$

where $\vec{S}$ and $\vec{T}$ are the position vector of points S and T, respectively. The transformation equations to be used are as follows:

$$A_x = a \cos\theta_1 \tag{4.42}$$

$$A_y = a \sin\theta_1 \tag{4.43}$$

$$B_x = Q_x + c \cos\theta_3 \tag{4.44}$$

$$B_y = Q_y + c \sin\theta_3 \tag{4.45}$$

$$G_{1x} = (M_x + A_x)/2 \tag{4.46}$$

$$G_{1y} = (M_y + A_y)/2 \tag{4.47}$$

$$G_{2x} = (A_x + B_x)/2 \tag{4.48}$$

$$G_{2y} = (A_y + B_y)/2 \tag{4.49}$$

$$G_{3x} = (B_x + Q_x)/2 \tag{4.50}$$

$$G_{3y} = (B_y + Q_y)/2 \tag{4.51}$$

where symbols A, B, Q, and M represent the nodes of the ideal mechanism of Figure 5. The center of mass of each mechanism is represented by G. Subscripts x and y represent the coordinate direction.

Results and Discussion

The mechanism of Figure 5 was analyzed for one, two, and three elements per link. Figures 9, 10, and 11 show the results when the input angle is zero. The analysis was carried out through 360 degrees of rotation of the input link. The results obtained showed a pattern of behavior similar to the one shown in Figures 12, 13, and 14. Figure

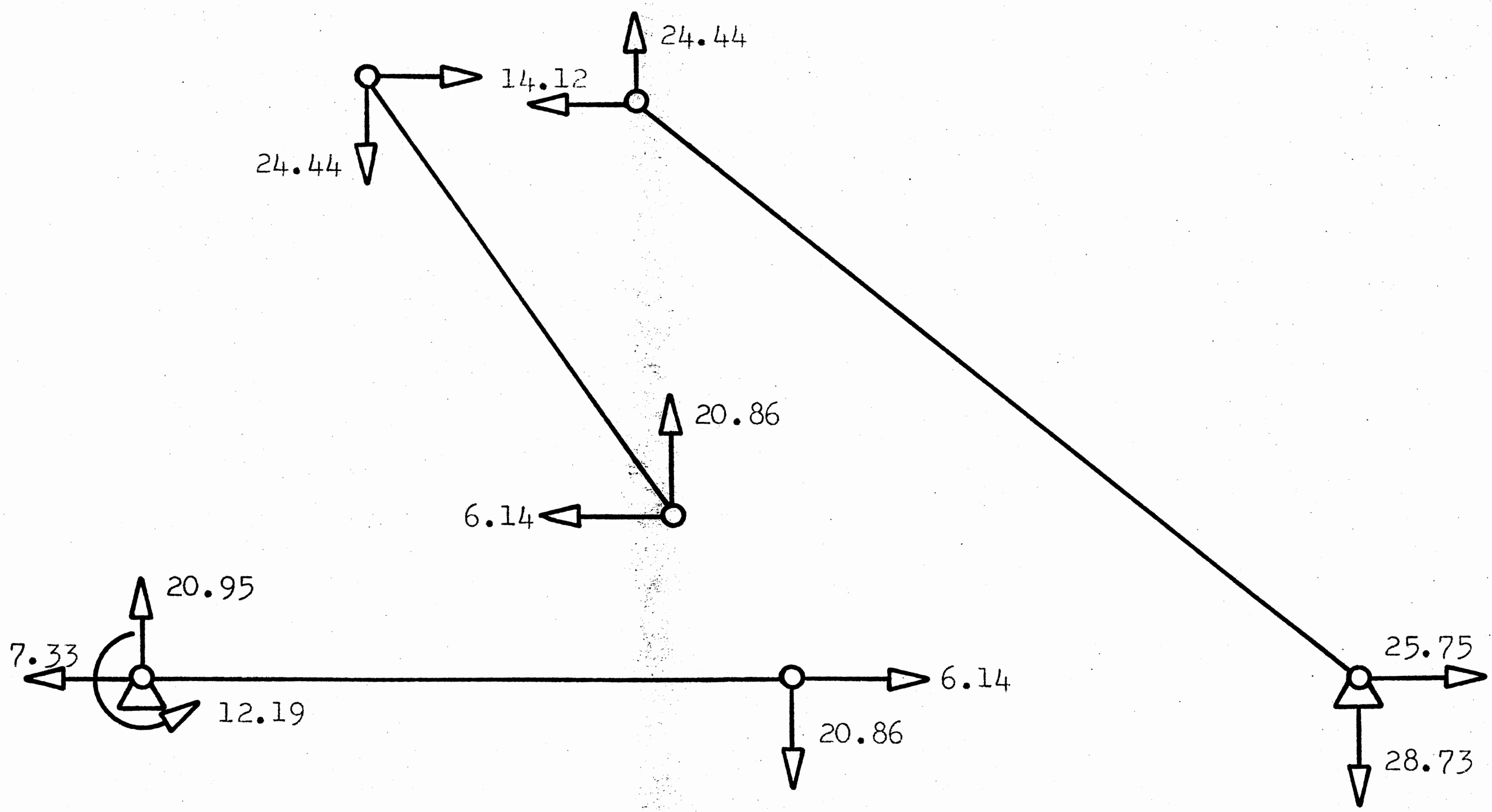


Figure 9. Solutions for a Four-Bar-Planar Mechanism when the Input Angle is Zero. One Element Per Link. Forces in lbf and Moments in lbf-ft. At 373 rpm of the Input Link.

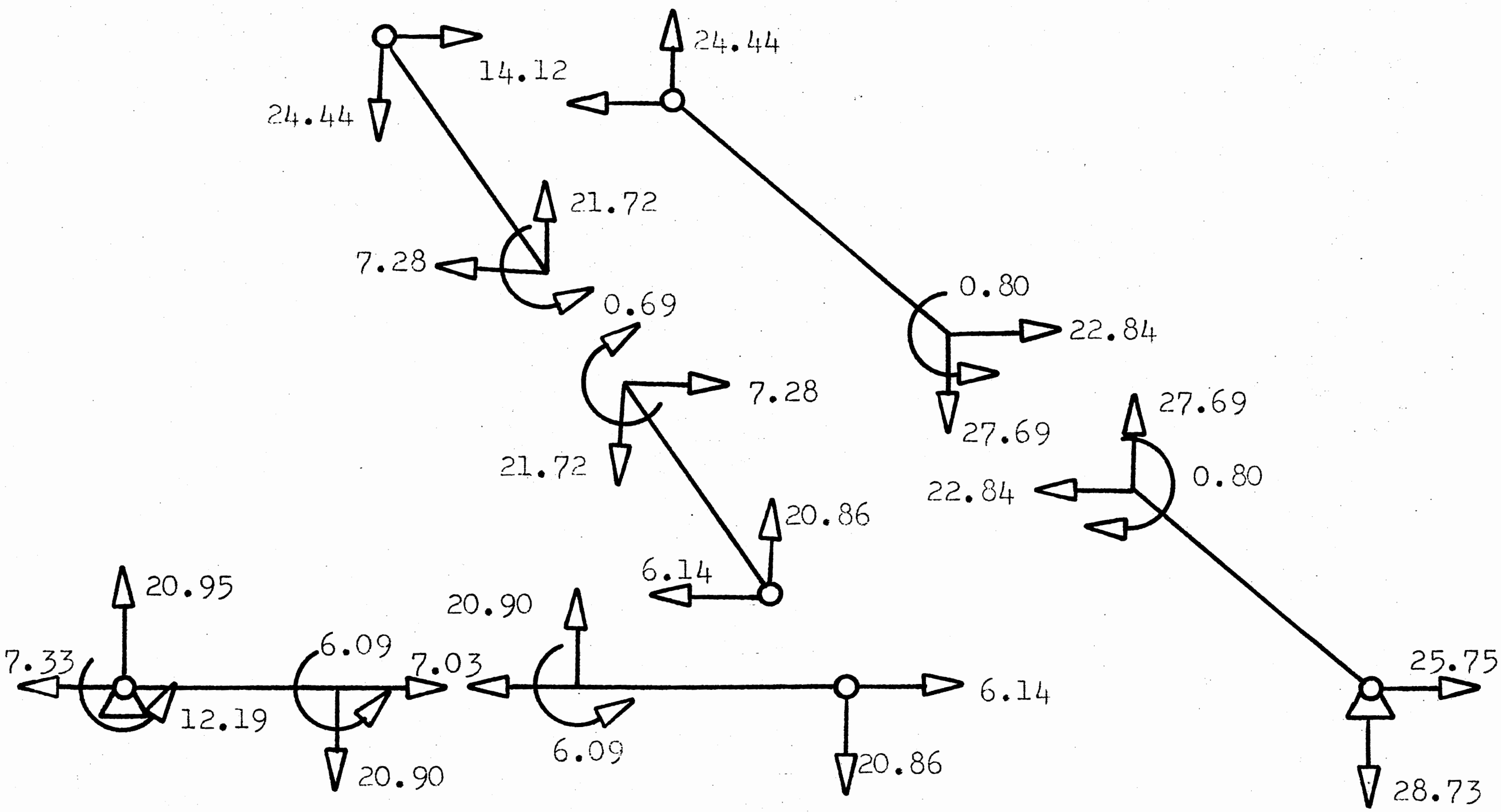


Figure 10. Solutions for a Four-Bar-Planar Mechanism when the Input Angle is Zero. Two Elements Per Link. Forces in lbf and Moments in lbf-ft. At 373 rpm of the Input Link.

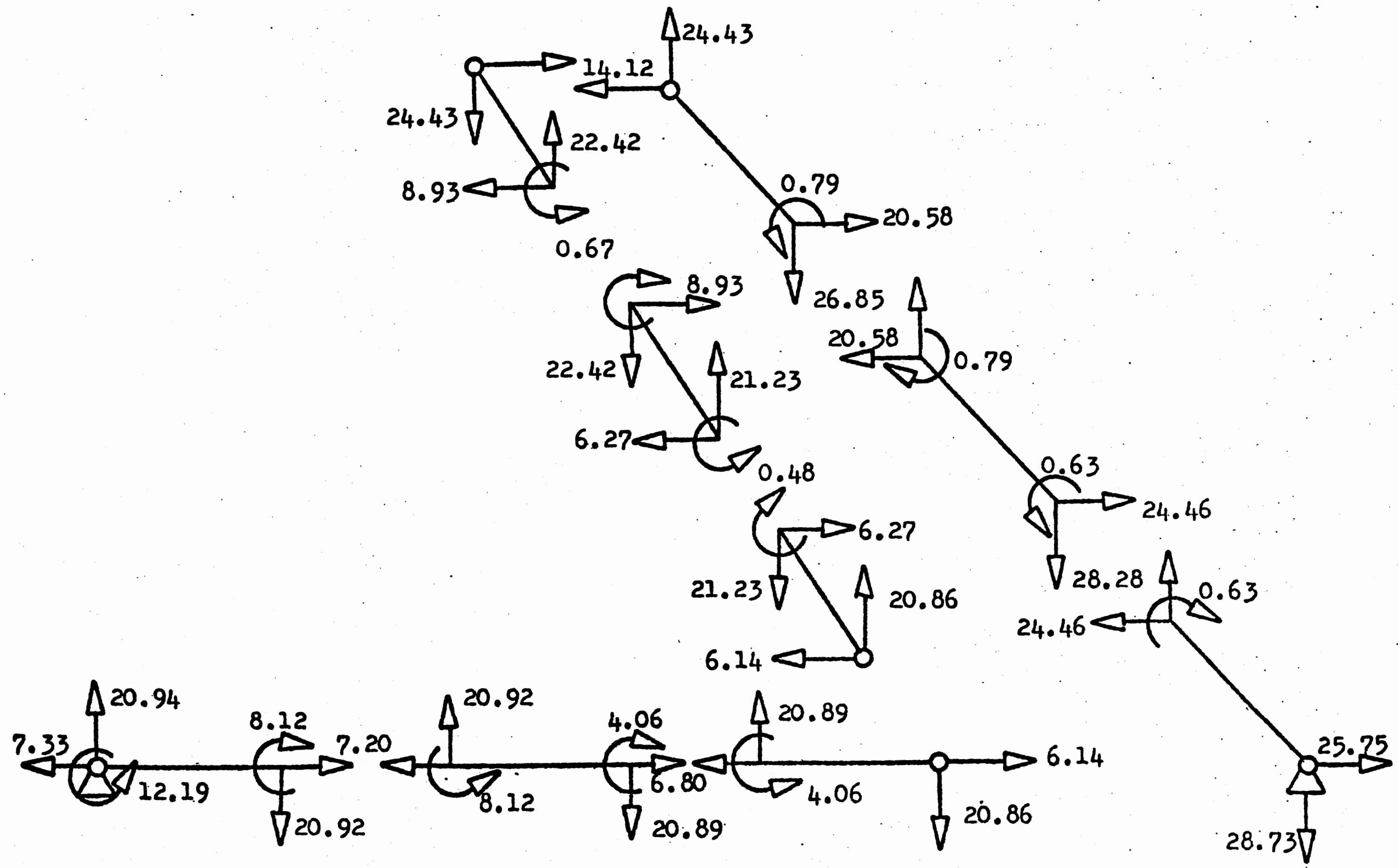


Figure 11. Solutions for a Four-Bar-Planar Mechanism When the Input Angle is Zero. Three Elements Per Link. Forces in lbf and Moments in lbf-ft. At 373 rpm of the Input Link

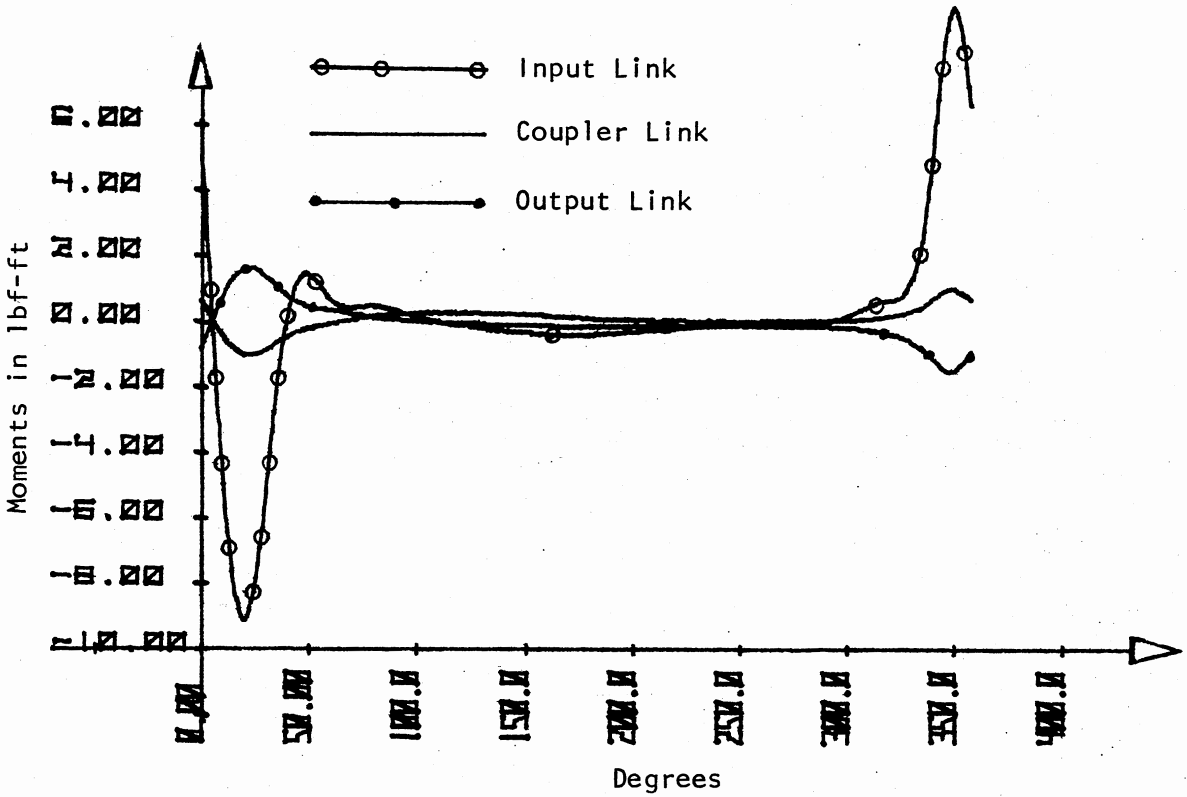


Figure 12. Moments at the Center of the Links of the Four-Bar Mechanism Rigidly Analyzed at 373 rpm of the Input Link

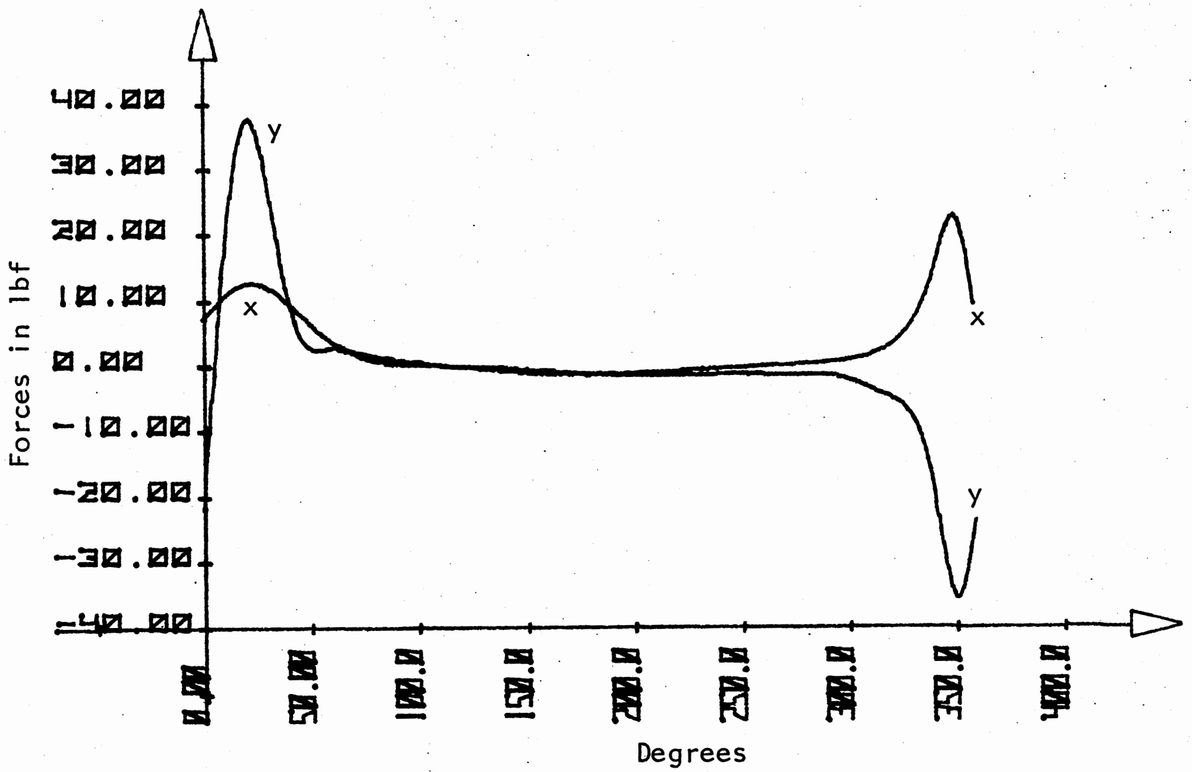


Figure 13. Forces in the x and y Directions at the Center of the Coupler Link of the Four-Bar Mechanism Rigidly Analyzed at 373 rpm of the Input Link

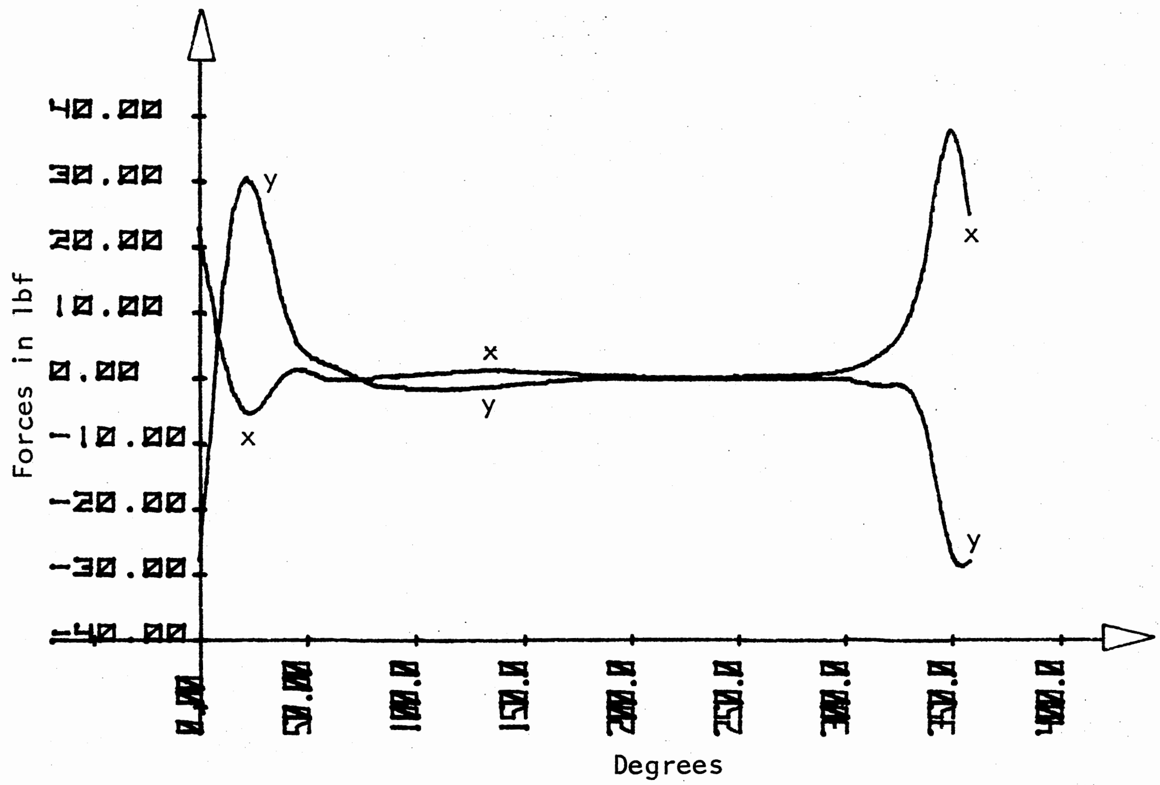


Figure 14. Forces in the x and y Directions at the Center of the Output Link of the Four-Bar Mechanism Rigidly Analyzed at 373 rpm of the Input Link

14 presents the results for the moments at the center of the links. The greatest peak values are observed at the center of the input link. Figures 13 and 14 show the results of the forces at the center of the coupler and output links. The greatest peak values are observed at the center of the output link. The results showed that the maximum inertial forces take place about -60 and 50 degrees of the input link motion. The biggest forces take place at -10 degrees of the input link.

CHAPTER V

SHAPE FUNCTIONS FOR THE ELEMENT

The line element has eight degrees of freedom, four per node (see Figure 15). Two shape functions are used with a total of eight unknown coefficients or a total of eight generalized coordinates. A fifth degree polynomial is used to represent the transverse deflection, rotation or slope and curvature. A first degree polynomial is used to represent the axial deflection. The strain energy due to transverse shear is negligible compared to the strain energy due to bending and axial deformations. Hence the effect of shear deformation is neglected. Rotary inertia and frictional forces at the joints are also neglected.

$$w(x) = A_0 + A_1 x + A_2 x^2 + A_3 x^3 + A_4 x^4 + A_5 x^5 \tag{5.1}$$

$$u(x) = B_0 + B_1 x \tag{5.2}$$

In matrix form:

$$w(x) = \begin{pmatrix} 0 & 1 & x & x^2 & 0 & x^3 & x^4 & x^5 \end{pmatrix} \begin{pmatrix} B_0 \\ A_0 \\ A_1 \\ A_2 \\ B_1 \\ A_3 \\ A_4 \\ A_5 \end{pmatrix} \tag{5.3}$$

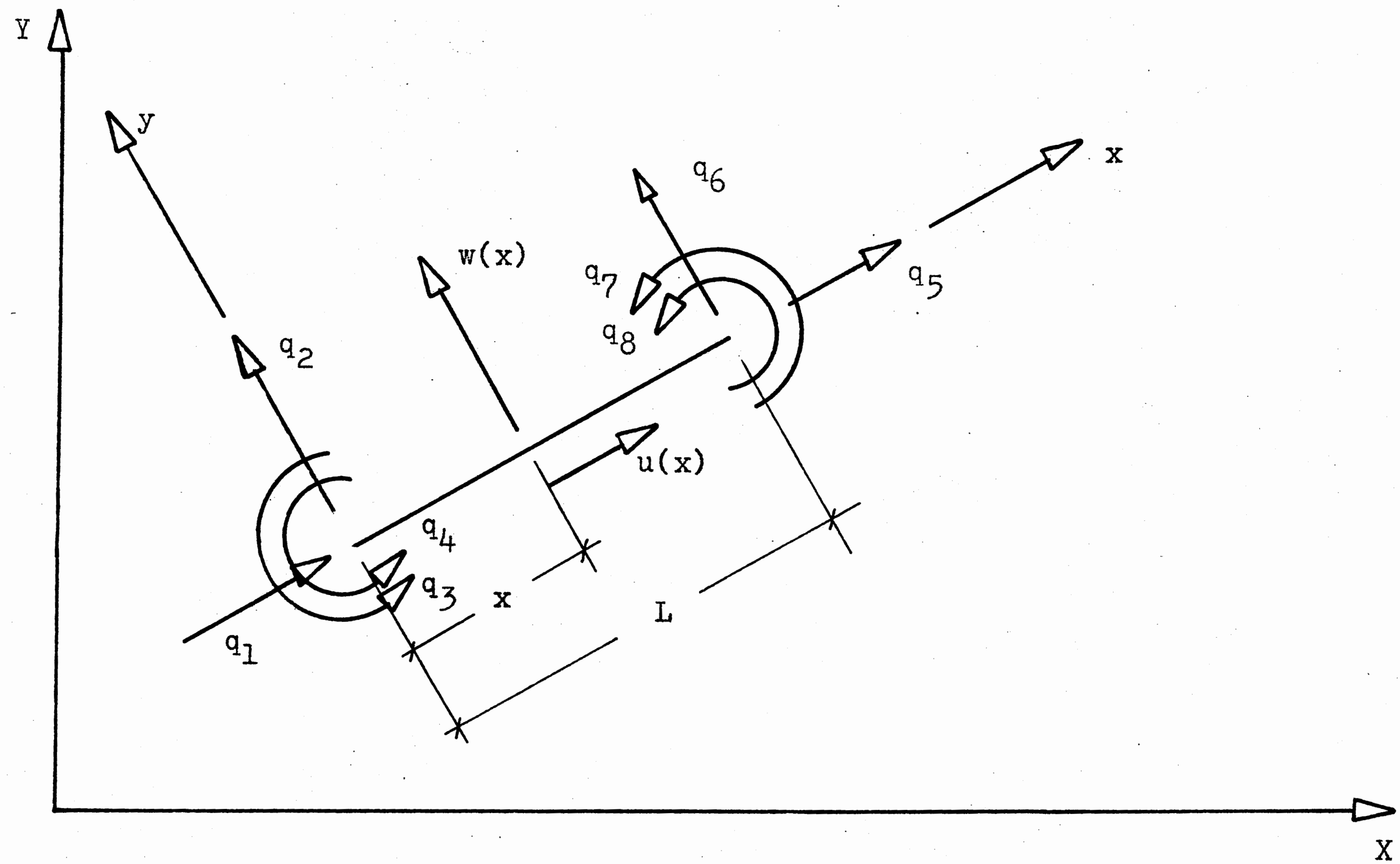


Figure 15. The Element and Its Eight Degrees of Freedom

or

$$w(x) = (X)^T (A) \tag{5.4}$$

and

$$u(x) = (1 \quad 0 \quad 0 \quad 0 \quad x \quad 0 \quad 0 \quad 0) \begin{pmatrix} B_0 \\ A_0 \\ A_1 \\ A_2 \\ B_1 \\ A_3 \\ A_4 \\ A_5 \end{pmatrix} \tag{5.5}$$

or

$$u(x) = (x)^T (A) \tag{5.6}$$

The deformation vector (S) is defined to be:

$$(S)^T = (q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5 \quad q_6 \quad q_7 \quad q_8) \tag{5.7}$$

whose dimensions are:

$$(S) = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{pmatrix} \begin{matrix} \text{Unit length} \\ \text{Unit length} \\ \text{Radians} \\ \text{Radians/Unit length} \\ \text{Unit length} \\ \text{Unit length} \\ \text{Radians} \\ \text{Radians/Unit length} \end{matrix}$$

The vectors (A) and (S) are related by the boundary conditions:

<u>At x = 0</u>:

$$q_1 = u(0) = B_0 \tag{5.8}$$

$$q_2 = w(0) = A_0 \tag{5.9}$$

$$q_3 = dw(0)/dx = A_1 \tag{5.10}$$

$$q_4 = d^2w(0)/dx^2 = 2A_2 \tag{5.11}$$

At x = L:

$$q_5 = u(L) = B_0 - B_1 L \tag{5.12}$$

$$q_6 = w(L) = A_0 + A_1 L + A_2 L^2 + A_3 L^3 + A_4 L^4 + A_5 L^5 \tag{5.13}$$

$$q_7 = dw(L)/dx = A_1 + 2A_2 L + 3A_3 L^2 + 4A_4 L^3 + 5A_5 L^4 \tag{5.14}$$

$$q_8 = d^2w(L)/dx^2 = 2A_2 + 6A_3 L + 12A_4 L^2 + 20A_5 L^3 \tag{5.15}$$

In matrix form we would have:

$$(S) = [C]\ (A) \tag{5.16}$$

$$(A) = [C]^{-1}\ (S) \tag{5.17}$$

where

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & L & 0 & 0 & 0 \\ 0 & 1 & L & L^2 & 0 & L^3 & L^4 & L^5 \\ 0 & 0 & 1 & 2L & 0 & 3L^2 & 4L^3 & 5L^4 \\ 0 & 0 & 0 & 2 & 0 & 6L & 12L^2 & 20L^3 \end{bmatrix} \begin{pmatrix} B_0 \\ A_0 \\ A_1 \\ A_2 \\ B_1 \\ A_3 \\ A_4 \\ A_5 \end{pmatrix} \tag{5.18}$$

The matrix $[C]^{-1}$ is obtained by partitioning and is shown by Matrix (6.23). From Equations (6.4), (6.6), and (6.17) we have:

$$w(x) = (X)^T [C]^{-1} (S) \tag{5.19}$$

$$u(x) = (X)^T [C]^{-1} (S) \tag{5.20}$$

$$\dot{w}(x) = (X)^T [C]^{-1} (\dot{S}) \tag{5.21}$$

$$\dot{u}(x) = (x)^T [C]^{-1} (\dot{S}) \tag{5.22}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 0 & 0 & 0 \\ -1/L & 0 & 0 & 0 & 1/L & 0 & 0 & 0 \\ 0 & -10/L^3 & -6/L^2 & -3/2L & 0 & 10/L^3 & -4/L^2 & 1/2L \\ 0 & 15/L^4 & 8/L^3 & 3/2L^2 & 0 & -15/L^4 & 7/L^3 & -1/L^2 \\ 0 & -6/L^5 & -3/L^4 & -1/2L^3 & 0 & 6/L^5 & -3/L^5 & 1/2L^3 \end{bmatrix} \tag{5.23}$$

CHAPTER VI

KINETIC AND POTENTIAL ENERGY OF AN ELASTIC LINK

It is to be understood that there are significant differences between a mechanism and a structure from an analysis standpoint. As far as the synthesis of elastic mechanisms is concerned, these differences are wider. The concept of instantaneous structure considers the mechanism as a structure with different mass, stiffness, and force matrices. The rigid body inertia forces due to gross motion of the mechanism which change in successive iterations due to elastic changes of the links, affect the deflection and stress characteristics of the mechanism. The rotation of the link about a principal axis produces an angular momentum which will keep the link rotating about that axis with constant angular velocity. If the torque on the link is not zero, the angular momentum experiences a change with respect to time which causes a procession of the instantaneous axis of rotation. This is known as the gyroscopic motion whose effect on the link's elastic behavior has been coined in this study as the gyroscopic effect. In the case of a structure, any increase in the areas of cross-section of members would generally reduce the deflections. In the case of a mechanism, however, increase of area increases the rigid body inertia forces and the gyroscopic effect, and therefore may increase the deflections (16).

Planar Displacement of an Elastic Link

Deformations such as transverse and axial deformations, rotation or slope and curvature characterize the deformation of a bar in planar motion. It is possible to observe in Figure 16 that the deformation of differential element Q represented by differential element $\bar{Q}$ in the deformed bar can be represented by vector $\vec{r}$ in local coordinates, that is:

$$\vec{r} = (\vec{i} \quad \vec{j}) \begin{pmatrix} x + u(x) \\ w(x) \end{pmatrix} \tag{6.1}$$

$$\vec{R} = \begin{pmatrix} \vec{X} \\ \vec{Y} \end{pmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{pmatrix} x + u(x) \\ w(x) \end{pmatrix} + \begin{pmatrix} X_1 \\ Y_1 \end{pmatrix} \tag{6.2}$$

and

$$\vec{R} = (\vec{i} \quad \vec{j}) \begin{pmatrix} x\cos\theta + u(x)\cos\theta - w(x)\sin\theta + X_1 \\ x\sin\theta + u(x)\sin\theta + w(x)\cos\theta + Y_1 \end{pmatrix} \tag{6.3}$$

Vector $\vec{R}$ is very important, since it will be later used to determine the limits of integration to find the length of the deformed element, and, furthermore, to determine the new position of the nodes in motion of the mechanism at any time (see Figure 17).

To obtain the velocity of any differential element of the deformed element, it is done by differentiation of the displacement vector $\vec{R}$ (see Equation (6.4) on page 59). By rotation of the velocity vector $\dot{\vec{R}}$, we have Equations (6.5) and (6.6) (see page 59). By simplification,

$$\dot{\vec{R}} = \begin{pmatrix} \dot{u}(x) - w(x)\dot{\theta} + \dot{X}_1\cos\theta + \dot{Y}_1\sin\theta \\ x\dot{\theta} + u(x)\dot{\theta} + \dot{w}(x) - \dot{X}_1\sin\theta + \dot{Y}_1\cos\theta \end{pmatrix} \tag{6.7}$$

and by letting

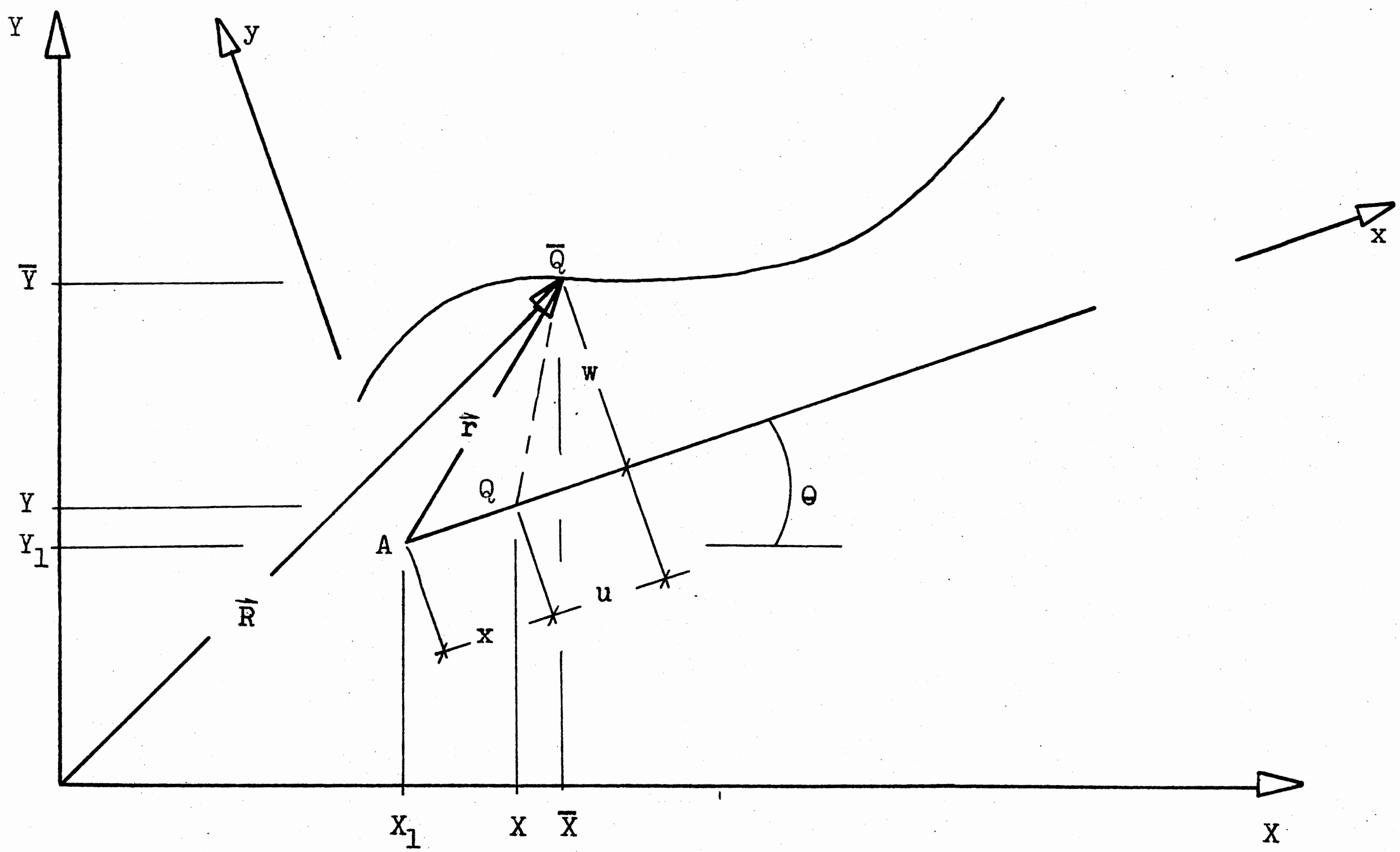


Figure 16. The Element in Deformed and Displaced Conditions

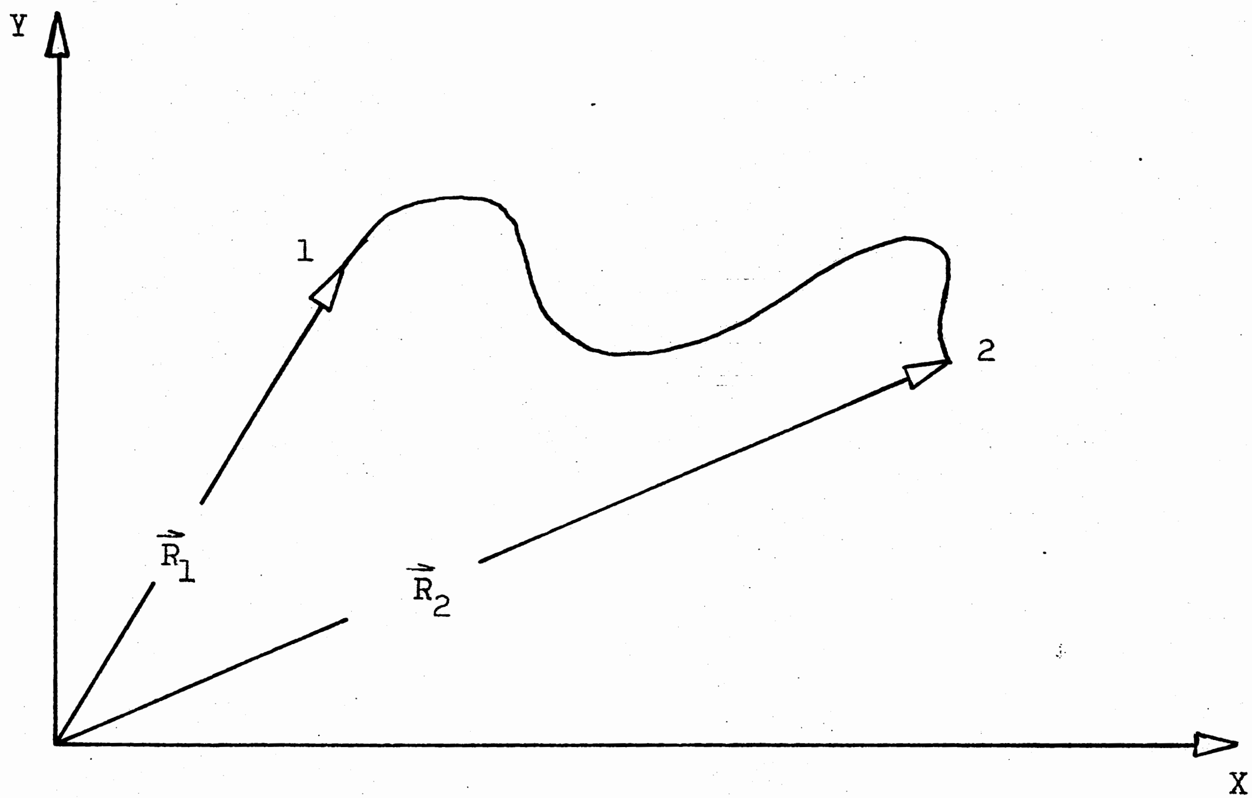


Figure 17. Two Consecutive Positions of a Node of the Deformed Link

$$\dot{\vec{R}} = (\vec{I} \quad \vec{J}) \begin{pmatrix} -x\dot{\theta}\sin\theta + \dot{u}(x)\cos\theta - u(x)\dot{\theta}\sin\theta - \dot{w}(x)\sin\theta - w(x)\dot{\theta}\cos\theta + \dot{X}_1 \\ x\dot{\theta}\cos\theta + \dot{u}(x)\sin\theta + u(x)\dot{\theta}\cos\theta + \dot{w}(x)\cos\theta - w(x)\dot{\theta}\sin\theta + \dot{Y}_1 \end{pmatrix} \tag{6.4}$$

$$\dot{\vec{R}}_1 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{pmatrix} -x\dot{\theta}\sin\theta + \dot{u}(x)\cos\theta - u(x)\dot{\theta}\sin\theta - \dot{w}(x)\sin\theta - w(x)\dot{\theta}\cos\theta + \dot{X}_1 \\ x\dot{\theta}\cos\theta + \dot{u}(x)\sin\theta + u(x)\dot{\theta}\cos\theta + \dot{w}(x)\cos\theta - w(x)\dot{\theta}\sin\theta + \dot{Y}_1 \end{pmatrix} \tag{6.5}$$

$$\dot{\vec{R}}_1 = \begin{pmatrix} -x\dot{\theta}\sin\theta\cos\theta + \dot{u}(x)\cos^2\theta - u(x)\dot{\theta}\sin\theta\cos\theta - \dot{w}(x)\sin\theta\cos\theta - w(x)\dot{\theta}\cos^2\theta + \dot{X}_1\cos\theta \\ + x\dot{\theta}\cos\theta\sin\theta + \dot{u}(x)\sin^2\theta + u(x)\dot{\theta}\cos\theta\sin\theta + \dot{w}(x)\cos\theta\sin\theta - w(x)\dot{\theta}\sin^2\theta + \dot{Y}_1\sin\theta \\ \\ x\dot{\theta}\sin^2\theta - \dot{u}(x)\cos\theta\sin\theta + u(x)\dot{\theta}\sin^2\theta + \dot{w}(x)\sin^2\theta + w(x)\dot{\theta}\cos\theta\sin\theta - \dot{X}_1\sin\theta \\ + \dot{x}\dot{\theta}\cos^2\theta + \dot{u}(x)\sin\theta\cos\theta + u(x)\dot{\theta}\cos^2\theta + \dot{w}(x)\cos^2\theta - w(x)\dot{\theta}\sin\theta\cos\theta + \dot{Y}_1\cos\theta \end{pmatrix} \tag{6.6}$$

$$\dot{x}_1 = \dot{X}_1\cos\theta + \dot{Y}_1\sin\theta \tag{6.8}$$

$$\dot{y}_1 = -\dot{X}_1\sin\theta + \dot{Y}_1\cos\theta \tag{6.9}$$

hence,

$$\dot{\vec{R}}_1 = (\vec{I} \quad \vec{J}) \begin{pmatrix} \dot{x}_1 + \dot{u}(x) - \dot{\theta}w(x) \\ \dot{y}_1 + x\dot{\theta} + \dot{\theta}u(x) + \dot{w}(x) \end{pmatrix} \tag{6.10}$$

Kinetic Energy of Elastic Links

The kinetic energy of a differential element is obtained from the following equation:

$$T = \frac{m}{2}\int_0^L |\dot{\vec{R}}_1|^2 \, dx \tag{6.11}$$

where rotational kinetic energy has been neglected, hence

$$T = \frac{m}{2}\int_0^L [(\dot{x}_1 + \dot{u}(x) - \dot{\theta}w(x))^2$$

$$+ (\dot{y}_1 + x\dot{\theta} + \dot{\theta}u(x) + \dot{w}(x))^2]dx \tag{6.12}$$

and by squaring, adding, arranging, and integrating, we have from Appendices A and B:

$$T = \frac{m}{2}[L\dot{x}_1^2 + L\dot{y}_1^2 + L^2\dot{y}_1\dot{\theta} + L^3\dot{\theta}^2/3 + 2\dot{x}_1(E)^T(\dot{S})$$

$$+ (\dot{S})^T[B](\dot{S}) - 2\dot{\theta}(\dot{S})^T[A](S) - 2\dot{x}_1\dot{\theta}(G)^T(S)$$

$$+ \dot{\theta}^2(S)^T[M](S) + 2\dot{y}_1\dot{\theta}(E)^T(S) + 2\dot{y}_1(G)^T(\dot{S})$$

$$+ 2\dot{\theta}^2(N)^T(S) + 2\dot{\theta}(L)^T(\dot{S}) + \dot{\theta}^2(S)^T[B](S)$$

$$+ 2\dot{\theta}(S)^T[A](\dot{S}) + (\dot{S})^T[M](\dot{S})] \tag{6.13}$$

Potential Energy of an Elastic Link

The potential energy of an elastic element is due to its spring behavior and gravitational position. The potential energy due to the elastic behavior of the element is given by:

$$V_1(t) = EA/2 \int_0^L (\delta u(x,t)/\delta x)^2 dx + EI/2 \int_0^L (\delta^2 w(x,t)/\delta x^2)^2 dx \tag{6.14}$$

The potential energy due to the element gravitational position is given by:

$$V_2(t) = mg/2\ (2Y_1 + L\sin\theta) \tag{6.15}$$

To determine $V_1(t)$ one needs to know the mode functions for transverse and axial deformation. We will start first with axial deformation. In Figure 18, we can observe the axial nodal deformations $q_1(t)$ and $q_5(t)$ subject to the joint forces $F_1(t)$ and $F_5(t)$. The axial displacement at any point x of the rod can be expressed as:

$$u(x,t) = \phi_1(x)\ q_1(t) + \phi_5(x)\ q_5(t) \tag{6.16}$$

where $\phi_1(x)$ and $\phi_5(x)$ are the mode functions that must satisfy given boundary conditions but are otherwise arbitrary. From Appendix C, we have:

$$\phi_1(x) = 1 - x/L \tag{6.17}$$

$$\phi_5(x) = x/L \tag{6.18}$$

hence:

$$u(x,t) = (1 - x/L)\ q_1(t) + xq_5(t)/L \tag{6.19}$$

and

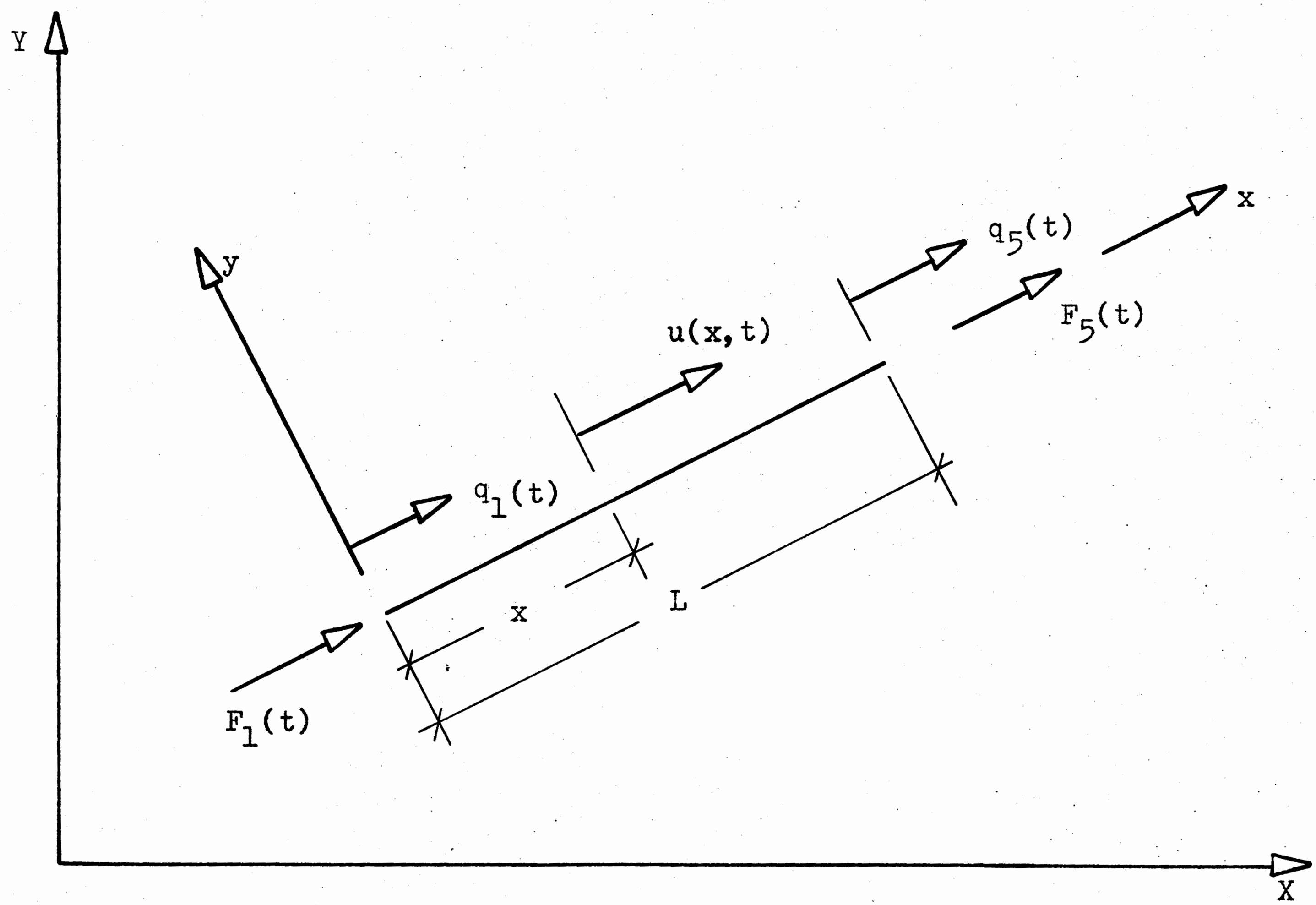


Figure 18. Graphical Representation of Mode Functions for Axial Deformation

$$EA/2 \int_0^L (\delta u(x,t)/\delta x)^2 dx = EA/2 \int_0^L (-q_1(t)/L + q_5(t)/L)^2 dx \tag{6.20}$$

integrating:

$$EA/2 \int_0^L (\delta u(x,t)/\delta x)^2 dx = EA/2L \; (q_1^2(t) - 2q_1(t)\; q_5(t) + q_5^2(t)) \tag{6.21}$$

For the case of transverse deformation, a given node can undergo translational and rotational deformation and curvature (see Figure 19). The displacement at any point in the element is given by:

$$\begin{aligned} w(x,t) &= \phi_2(x)\; q_2(t) + \phi_3(x)\; q_3(t) + \phi_4(x)\; q_4(t) \\ &\quad + \phi_6(x)\; q_6(t) + \phi_7(x)\; q_7(t) + \phi_8(x)\; q_8(t) \end{aligned} \tag{6.22}$$

where $\phi_i(x)$ ($i = 2, 3, 4, 6, 7, 8$) are the mode functions that must satisfy given boundary conditions (see Figure 19). From Appendix C we have:

$$\phi_2(x) = 1 - 10x^3/L^3 + 15x^4/L^4 - 6x^5/L^5 \tag{6.23}$$

$$\phi_3(x) = x - 6x^3/L^2 + 8x^4/L^3 - 3x^5/L^4 \tag{6.24}$$

$$\phi_4(x) = x^2/2 - 3x^2/2L + 3x^4/2L^2 - x^5/2L^3 \tag{6.25}$$

$$\phi_6(x) = 10x^3/L^3 - 15x^4/L^4 + 6x^5/L^5 \tag{6.26}$$

$$\phi_7(x) = -4x^3/L^2 + 7x^4/L^3 - 3x^5/L^4 \tag{6.27}$$

$$\phi_8(x) = x^3/2L - x^2/L^2 + x^5/2L^3 \tag{6.28}$$

Equations (5.23) through (5.28) are introduced into Equation (6.14). The integration is carried out to obtain:

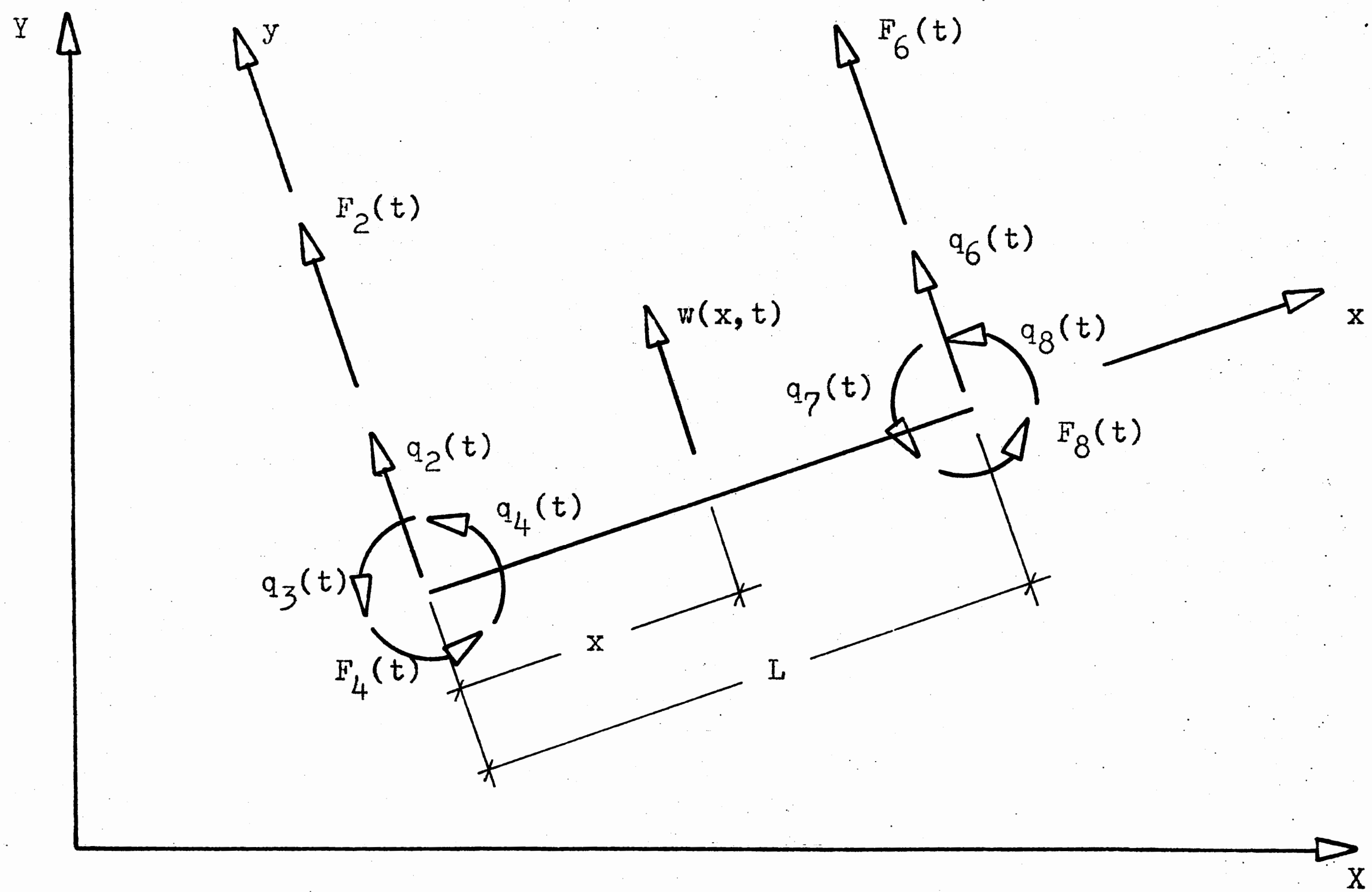


Figure 19. Graphical Representation of Mode Functions for Transverse Deformations

$$EI/2 \int_0^L (\delta^2 w(x,t)/\delta x^2)^2 dx = EI/2\,(D_0L + D_1L^2/2 + D_2L^3/3 + D_3L^4/4 + D_4L^5/5 + D_5L^6/6 + D_6L^7/7) \quad (6.29)$$

where the coefficients D_i $(i = 0, 1, 2, 3, 4, 5, 6)$ are shown in Appendix D. Hence:

$$\begin{aligned} V(t) = {} & EA/2L\,(q_1^2(t) - 2q_1(t)\,q_5(t) + q_5^2(t)) \\ & + EI/]\,(D_0L + D_1L^2/2 + D_2L^3/3 + D_3L^4/4 \\ & + D_4L^5/5 + D_5L^6/6 + D_6L^7/7) \\ & + mg/2\,(2Y_1 + L\sin\theta) \end{aligned} \quad (6.30)$$

Equation (6.30) has to be expressed as a function of the displacement vector (S). This might be done by application of the Taylor expansion, ignoring terms of order three and higher. Therefore, see Appendix D:

$$V(t) = (S)^T \frac{1}{2}[K_1](S) + (S)^T \frac{1}{2}[K_2](S) + mg/2\,(2Y_1 + L\sin\theta) \quad (6.31)$$

or

$$V(t) = (S)^T \frac{1}{2}[K](S) + mg/2\,(2Y_1 + L\sin\theta) \quad (6.32)$$

where

$$\frac{1}{2}[K] = \frac{1}{2}[K_1] + \frac{1}{2}[K_2] \quad (6.33)$$

CHAPTER VII

EQUATIONS OF MOTION OF AN ELASTIC ELEMENT

The calculus of variations (6, 9, 10, 11, 12, 65) is a vastly important area of classical mathematics with applications in science and engineering. The earliest work of what is currently named variational calculus was an attempt to extend the concepts of the calculus of Newton and Leibniz to the problem of finding the minimum of a functional. It has been observed that many of the laws which govern the phenomena of nature emanate from the principle of a path minimum time between two points. Problems of this class are of the following general form (6). Find y(x) such that:

$$g = \int_{x_1}^{x_2} \phi[x,\ y(x),\ y'(x)]\ dx \tag{7.1}$$

is a minimum. Hamilton's principle (1805-1865) is one of these principles. Hamilton's principle is a cornerstone to the variational approach to mechanics. It is a unifying influence on analytical thought and provides such a powerfully elegant tie between the mathematics of variational calculus and physics of natural systems (6).

The equations of motion are to be obtained by application of Hamilton's principle. In the discussion to follow, the variational problem will be supplied by what is known in classical mechanics as Hamilton's principle. Our system may be considered to be conservative; that is, each force that acts is derivable from a potential. Let T and V denote the total kinetic and potential energies, respectively. Introducing the Lagrangian function (6-12):

$$LF = T - V \tag{7.2}$$

Hamilton's principle states that the actual motion connecting two known states of the system, say at times t_1 and t_2, is the one that minimizes the integral (6, 7, 8):

$$\int_{t_1}^{t_2} (T - V)\, dt = \int_{t_1}^{t_2} LF\, dt \tag{7.3}$$

The general type of the Lagrangian has the form:

$$LF = LF\,(q_1, q_2, \ldots, q_n; \dot{q}_1, \dot{q}_2, \ldots, \dot{q}_n; t) \tag{7.4}$$

it follows that

$$\int_{t_1}^{t_2} LF\,(q_1, \ldots, q_n; \dot{q}_1, \ldots, \dot{q}_n; t)\, dt = \int_{t_1}^{t_2} (T - V)\, dt \tag{7.5}$$

has an extreme value, or:

$$\int_{t_1}^{t_2} \partial(LF(q_1, \ldots, q_n; \dot{q}_1, \ldots, \dot{q}_n; t))\, dt = 0 \tag{7.6}$$

where we denote by $\partial(LF)$ the variation in LF from its value in the actual motion at the instant t to its value at the same instant in the varied path (8, 10, 11, 12).

From previous chapters one knows T and V, and LF is obtained by subtraction:

$$LF = \frac{1}{2}\, m\, (L\dot{x}_1^2 + L\dot{y}_1^2 + L^2\dot{y}_1\dot{\theta} + L^3\dot{\theta}^2/3 + 2\dot{x}_1(E)^T(\dot{S})$$

$$+ (\dot{S})^T[B](\dot{S}) - 2\dot{\theta}(\dot{S})^T[A](S) - 2\dot{x}_1\dot{\theta}(G)^T(S)$$

$$+ \dot{\theta}^2(S)^T[M](S) + 2\dot{y}_1\dot{\theta}(E)^T(S) + 2\dot{y}_1(G)^T(\dot{S})$$

$$+ 2\dot{\theta}^2(N)^T(S) + 2\dot{\theta}(L)^T(\dot{S}) + \dot{\theta}^2(S)^T[B](S)$$

$$+ 2\dot{\theta}(S)^T[A](\dot{S}) + (\dot{S})^T[M](\dot{S})) - \frac{1}{2}(S)^T[K](S)$$

$$- \frac{1}{2}mg(2y_1 + L\sin\theta) \qquad (7.7)$$

Taking variation with respect to (S):

$$\partial(LF) = \frac{1}{2}m(2\dot{x}_1(E)^T\partial(\dot{S}) + (\dot{S})^T[B]\partial(\dot{S}) + \partial(\dot{S})^T[B](\dot{S})$$

$$- 2\dot{\theta}(\dot{S})^T[A]\partial(S) - 2\dot{\theta}\partial(\dot{S})^T[A](S) - 2\dot{x}_1\dot{\theta}(G)^T\partial(S)$$

$$+ \dot{\theta}^2(S)^T[M]\partial(S) + \dot{\theta}^2\partial(S)^T[M](S) + 2\dot{y}_1\dot{\theta}(E)^T\partial(S)$$

$$+ 2\dot{y}_1(G)^T\partial(\dot{S}) + 2\dot{\theta}^2(N)^T\partial(S) + 2\dot{\theta}(L)^T\partial(\dot{S})$$

$$+ \dot{\theta}^2(S)^T[B]\partial(S) + \dot{\theta}^2\partial(S)^T[B](S) + 2\dot{\theta}(S)^T[A]\partial(\dot{S})$$

$$+ 2\dot{\theta}\partial(S)^T[A](\dot{S}) + (\dot{S})^T[M]\partial(\dot{S}) + \partial(\dot{S})^T[M](\dot{S}))$$

$$- \frac{1}{2}(S)^T[K]\partial(S) - \frac{1}{2}\partial(S)^T[K](S) \qquad (7.8)$$

Adding equal terms and arranging, we have:

$$\partial(LF) = \frac{1}{2}m(2\dot{x}_1(E)^T\partial(\dot{S}) + 2(\dot{S})^T[B]\partial(\dot{S}) - 2\dot{\theta}(\dot{S})^T[A]\partial(S)$$

$$- 2\dot{\theta}(S)^T[A]^T\partial(\dot{S}) - 2\dot{x}_1\dot{\theta}(G)^T\partial(S) + 2\dot{\theta}^2(S)^T[M]\partial(S)$$

$$+ 2\dot{y}_1\dot{\theta}(E)^T\partial(S) + 2\dot{y}_1(G)^T\partial(\dot{S}) + 2\dot{\theta}^2(N)^T\partial(S)$$

$$+ 2\dot{\theta}(L)^T\partial(\dot{S}) + 2\dot{\theta}^2(S)^T[B]\partial(S) + 2\dot{\theta}(S)^T[A]\partial(\dot{S})$$

$$+ 2\dot{\theta}(\dot{S})^T[A]^T\partial(S) + 2(\dot{S})^T[M]\partial(\dot{S})) - (S)^T[K]\partial(S) \qquad (7.9)$$

The term $\partial(\dot{S})$ has to be eliminated so that Hamilton's integral might be applied. To do that, integration by part has to be carried out by using the following equation:

$$d[(u)^T(v)]/dt = d[(u)^T(v)]/dt + (u)^T[d(v)/dt]$$

or

$$d[(u)^T(v)]/dt - [d(u)^T/dt](v) = (u)^T[d(v)/dt]$$

By integration, we have:

$$\int((u)^T(d(u)/dt))dt = (u)^T(v) - \int(d((u)^T(v))/dt)dt \qquad (7.10)$$

Hence, by application of this equation to those terms with $\partial(\dot{S})$, we have:

1. $\int_{t_1}^{t_2} 2\dot{x}_1(E)^T\partial(\dot{S})dt$

$$(u)^T = 2\dot{x}_1(E)^T, \ (v) = \partial(S)$$

$$\int_{t_1}^{t_2} 2\dot{x}_1(E)^T\partial(\dot{S})dt = 2\dot{x}_1(E)^T\partial(S)\Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} 2\dot{x}_1(E)^T\partial(S)dt$$

$$\int_{t_1}^{t_2} 2\dot{x}_1(E)^T\partial(\dot{S})dt = -\int_{t_1}^{t_2} 2\ddot{x}_1(E)^T\partial(S)dt \qquad (7.11)$$

2. $\int_{t_1}^{t_2} 2(\dot{S})^T[B]\partial(\dot{S})dt$

$$(u)^T = 2(\dot{S})^T[B], \ (V) = \partial(S)$$

$$\int_{t_1}^{t_2} 2(\dot{S})^T[B]\partial(\dot{S})dt = 2(\dot{S})^T[B]\partial(S)\Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} 2(\ddot{S})^T[B]\partial(S)dt$$

$$\int_{t_1}^{t_2} 2(\dot{S})^T[B]\partial(\dot{S})dt = -\int_{t_1}^{t_2} 2(\ddot{S})^T[B]\partial(S)dt \qquad (7.12)$$

3. $\int_{t_1}^{t_2} 2\dot{\theta}(S)^T[A]^T\partial(\dot{S})dt$

$$(u)^T = 2\dot{\theta}(S)^T[A]^T, \quad (v) = \partial(S)$$

$$\int_{t_1}^{t_2} 2\dot{\theta}(S)^T[A]^T\partial(\dot{S})dt = 2\dot{\theta}(S)^T[A]^T\partial(S)\Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} (2\ddot{\theta}(S)^T[A]^T$$

$$+ 2\dot{\theta}(\dot{S})^T[A]^T)\partial(S)dt$$

$$\int_{t_1}^{t_2} 2\dot{\theta}(S)^T[A]^T\partial(\dot{S})dt = -\int_{t_1}^{t_2} (2\ddot{\theta}(S)^T[A]^T + 2\dot{\theta}(\dot{S})^T[A]^T)\partial(S)dt \tag{7.13}$$

4. $\int_{t_1}^{t_2} 2\dot{y}_1(G)^T\partial(\dot{S})dt$

$$(u)^T = 2\dot{y}_1(G)^T, \quad (v) = \partial(S)$$

$$\int_{t_1}^{t_2} 2\dot{y}_1(G)^T\partial(\dot{S})dt = 2\dot{y}_1(G)^T\partial(S)\Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} 2\ddot{y}_1(G)^T\partial(S)dt$$

$$\int_{t_1}^{t_2} 2\dot{y}_1(G)^T\partial(\dot{S})dt = -\int_{t_1}^{t_2} 2\ddot{y}_1(G)^T\partial(S)dt \tag{7.14}$$

5. $\int_{t_1}^{t_2} 2\dot{\theta}(L)^T\partial(\dot{S})dt$

$$(u)^T = 2\dot{\theta}(L)^T, \quad (v) = \partial(S)$$

$$\int_{t_1}^{t_2} 2\dot{\theta}(L)^T\partial(\dot{S})dt = 2\dot{\theta}(L)^T\partial(S)\Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} 2\ddot{\theta}(L)^T\partial(S)dt$$

$$\int_{t_1}^{t_2} 2\dot{\theta}(L)^T\partial(\dot{S})dt = -\int_{t_1}^{t_2} 2\ddot{\theta}(L)^T\partial(S)dt \tag{7.15}$$

6. $\int_{t_1}^{t_2} 2\dot{\theta}(S)^T[A]\partial(\dot{S})dt$

$$(u)^T = 2\dot{\theta}(S)^T[A], \quad (v) = \partial(S)$$

$$\int_{t_1}^{t_2} 2\dot{\theta}(S)^T[A]\partial(\dot{S})dt = 2\dot{\theta}(S)^T[a]\partial(S)\Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} (2\ddot{\theta}(S)^T[A]$$

$$+ 2\dot{\theta}(\dot{S})^T[A])\partial(S)dt$$

$$\int_{t_1}^{t_2} 2\dot{\theta}(S)^T[A]\partial(\dot{S})dt = -\int_{t_1}^{t_2} (2\ddot{\theta}(S)^T[A] + 2\dot{\theta}(\dot{S})^T[A])\partial(S)dt \quad (7.16)$$

7. $\int_{t_1}^{t_2} 2(\dot{S})^T[M]\partial(\dot{S})dt$

$(u)^T = 2(\dot{S})^T[M]$, $(v) = \partial(S)$

$$\int_{t_1}^{t_2} 2(\dot{S})^T[M]\partial(S)dt = 2(\dot{S})^T[M]\partial(S)\Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} 2(\ddot{S})^T[M]\partial(S)dt$$

$$\int_{t_1}^{t_2} 2(\dot{S})^T[M]\partial(\dot{S})dt = -\int_{t_1}^{t_2} 2(\ddot{S})^T[M]\partial(S)dt \quad (7.17)$$

Since all of the terms with $\partial(\dot{S})$ have been transformed to terms with $\partial(S)$, Hamilton's integral might be applied.

$$\int_{t_1}^{t_2} (\tfrac{1}{2} m(2\dot{x}_1(E)^T\partial(\dot{S}) + 2(\dot{S})^T[B]\partial(\dot{S}) - 2\dot{\theta}(\dot{S})^T[A]\partial(S)$$

$$- 2\dot{\theta}(S)^T[A]^T\partial(\dot{S}) - 2\dot{x}_1\dot{\theta}(G)^T\partial(S) + 2\dot{\theta}^2(S)^T[M]\partial(S)$$

$$+ 2\dot{y}_1\dot{\theta}(E)^T\partial(S) + 2\dot{y}_1(G)^T\partial(\dot{S}) + 2\dot{\theta}^2(N)^T\partial(S)$$

$$+ 2\dot{\theta}(L)^T\partial(\dot{S}) + 2\dot{\theta}^2(S)^T[B]\partial(S) + 2\dot{\theta}(S)^T[A]\partial(\dot{S})$$

$$+ 2\dot{\theta}(\dot{S})^T[A]^T\partial(S) + 2(\dot{S})^T[M]\partial(\dot{S}))$$

$$- (S)^T[K]\partial(S))dt = 0 \quad (7.18)$$

$$\int_{t_1}^{t_2} (\tfrac{1}{2} m(-2\ddot{x}_1(E)^T\partial(S) - 2(\ddot{S})^T[B]\partial(S) - 2\dot{\theta}(\dot{S})^T[A]\partial(S)$$

$$+ 2\ddot{\theta}(S)^T[A]^T\partial(S) + 2\dot{\theta}(\dot{S})^T[A]^T\partial(S) - 2\dot{x}_1\dot{\theta}(G)^T\partial(S)$$

$$+ 2\dot{\theta}^2(S)^T[M]\partial(S) + 2\dot{y}_1\dot{\theta}(E)^T\partial(S) - 2\ddot{y}_1(G)^T\partial(S)$$

$$+ 2\dot{\theta}^2(N)^T\partial(S) - 2\ddot{\theta}(L)^T\partial(S) + 2\dot{\theta}^2(S)^T[B]\partial(S)$$

$$- 2\ddot{\theta}(S)^T[A]\partial(S) - 2\dot{\theta}(\dot{S})^T[A]\partial(S) + 2\dot{\theta}(\dot{S})^T[A]^T\partial(S)$$

$$- 2(\ddot{S})^T[M]\partial(S)) - (S)^T[K]\partial(S))dt = 0 \qquad (7.19)$$

$$\int_{t_1}^{t_2} (\frac{1}{2} m(-2\ddot{x}_1(E)^T - 2(\ddot{S})^T[B] - 2\dot{\theta}(\dot{S})^T[A] + 2\ddot{\theta}(S)^T[A]^T$$

$$+ 2\dot{\theta}(\dot{S})^T[A]^T - 2\dot{x}_1\dot{\theta}(G)^T + 2\dot{\theta}^2(S)^T[M] + 2\dot{y}_1\dot{\theta}(E)^T$$

$$- 2\ddot{y}_1(G)^T + 2\dot{\theta}^2(N)^T - 2\ddot{\theta}(L)^T + 2\dot{\theta}^2(S)^T[B]$$

$$- 2\ddot{\theta}(S)^T[A] - 2\dot{\theta}(\dot{S})^T[A] + 2\dot{\theta}(\dot{S})^T[A]^T - 2(\ddot{S})^T[M])$$

$$- (S)^T[K])\partial(S)dt = 0 \qquad (7.20)$$

Applying Hamilton's integral, we have:

$$-m\ddot{x}_1(E)^T - m(\ddot{S})^T[B] - m\dot{\theta}(\dot{S})^T[A] + m\ddot{\theta}(S)^T[A]^T + m\dot{\theta}(\dot{S})^T[A]^T$$

$$- m\dot{x}_1\dot{\theta}(G)^T + m\dot{\theta}^2(S)^T[M] + m\dot{y}_1\dot{\theta}(E)^T - m\ddot{y}_1(G)^T + m\dot{\theta}^2(N)^T$$

$$- m\ddot{\theta}(L)^T + m\dot{\theta}^2(S)^T[B] - m\ddot{\theta}(S)^T[A] - m\dot{\theta}(\dot{S})^T[A]$$

$$+ m\dot{\theta}(\dot{S})^T[A]^T - m(\ddot{S})^T[M] - (S)^T[K] = 0 \qquad (7.21)$$

Ordering:

$$m(\ddot{S})^T[M] + m(\ddot{S})^T[B] + m\dot{\theta}(\dot{S})^T[A] - m\dot{\theta}(\dot{S})^T[A]^T + m\dot{\theta}(\dot{S})^T[A]$$

$$- m\dot{\theta}(\dot{S})^T[A]^T - m\ddot{\theta}(S)^T[A]^T - m\dot{\theta}^2(S)^T[M] - m\dot{\theta}^2(S)^T[B]$$

$$+ m\ddot{\theta}(S)^T[A] + (S)^T[K] = m\dot{y}_1\dot{\theta}(E)^T - m\ddot{x}_1(E)^T$$

$$- m\dot{x}_1\dot{\theta}(G)^T - m\ddot{y}_1(G)^T + m\dot{\theta}^2(N)^T - m\ddot{\theta}(L)^T \qquad (7.22)$$

$$(\ddot{S})^T(m[M] + m[B]) + (\dot{S})^T(m\dot{\theta}[A] - m\dot{\theta}[A]^T + m\dot{\theta}[A] - m\dot{\theta}[A]^T)$$
$$+ (S)^T([K] - m\ddot{\theta}[A]^T - m\dot{\theta}^2[M] - m\dot{\theta}^2[B] + m\ddot{\theta}[A])$$
$$= m\dot{y}_1\dot{\theta}(E)^T - m\ddot{x}_1(E)^T - m\dot{x}_1\dot{\theta}(G)^T - m\ddot{y}_1(G)^T + m\dot{\theta}^2(N)^T$$
$$- m\ddot{\theta}(L)^T \tag{7.23}$$

Let

$$[M_D] = m[M] + m[B] \tag{7.24}$$

$$[A_D] = m[A] - m[A]^T \tag{7.25}$$

$$[K_D] = [K] - \dot{\theta}^2[M_D] + \ddot{\theta}[A_D]$$

$$(Q)^T = m\dot{y}_1\dot{\theta}(E)^T - m\ddot{x}_1(E)^T - m\dot{x}_1\dot{\theta}(G)^T - m\ddot{y}_1(G)^T$$
$$+ m\dot{\theta}^2(N)^T - m\ddot{\theta}(L)^T \tag{7.26}$$

Hence

$$(\ddot{S})^T[M_D] + (\dot{S})^T2\dot{\theta}[A_D] + (S)^T[K_D] = (Q)^T \tag{7.27}$$

By taking the transpose, we have finally the equations of motion of an elastic element.

$$[M_D](\ddot{S}) - 2\dot{\theta}[A_D](\dot{S}) + [K_D](S) = (Q) \tag{7.28}$$

where

$$[K_D] = [K] - \dot{\theta}^2[M_D] - \ddot{\theta}[A_D] \tag{7.29}$$

Matrices $[M_D]$, $[A_D]$, and $[K_D]$ are shown in Appendix G. Matrix $[M_D]$ is called the total translational mass matrix, matrix $[K_D]$ is called the total translational stiffness matrix, and matrix $[A_D]$ is called the total translational gyroscopic matrix. The transformations to global coordinates are also carried out. See Appendices E and F. Those transformations are carried out based on the assumption that we are only dealing with small deformations of the element. It can be observed that it is not necessary to solve the rigid body dynamics to find the answers to Equation (7.28).

CHAPTER VIII

ASSEMBLING AND CONDENSING PROCEDURE LENGTH AND STRESS OF THE DEFORMED ELEMENT

Assembling

Since the equations of motion of an elastic element in planar motion have been defined, it is now necessary to present the procedure of how to assemble those elements so that a mechanism could be studied. The procedure to follow is developed based on the analysis of a four-bar planar mechanism by considering each link as one element. However, the procedure could easily be extended to as many elements per link as necessary so that accuracy could be achieved. Figure 20 shows a four-bar-planar mechanism defined by 20 degrees of freedom of deformation. These degrees of freedom have to be assembled from the element's degrees of freedom. The procedure to achieve the assembling is based on forming matrix [Z] which is obtained by using Figure 21. The Matrix [Z] is shown in Appendix E.

The assembling procedure is then carried out by using the following equation which is applied to the stiffness matrix expressed in global coordinates, but it is valid also for the other matrices forming the equations of motion:

$$\underset{20\times20}{[K]} = \underset{20\times24}{[Z]^T} \underset{24\times24}{\begin{bmatrix} [K_1] & 0 & 0 \\ 0 & [K_2] & 0 \\ 0 & 0 & [K_3] \end{bmatrix}} \underset{24\times20}{[Z]} \qquad (8.1)$$

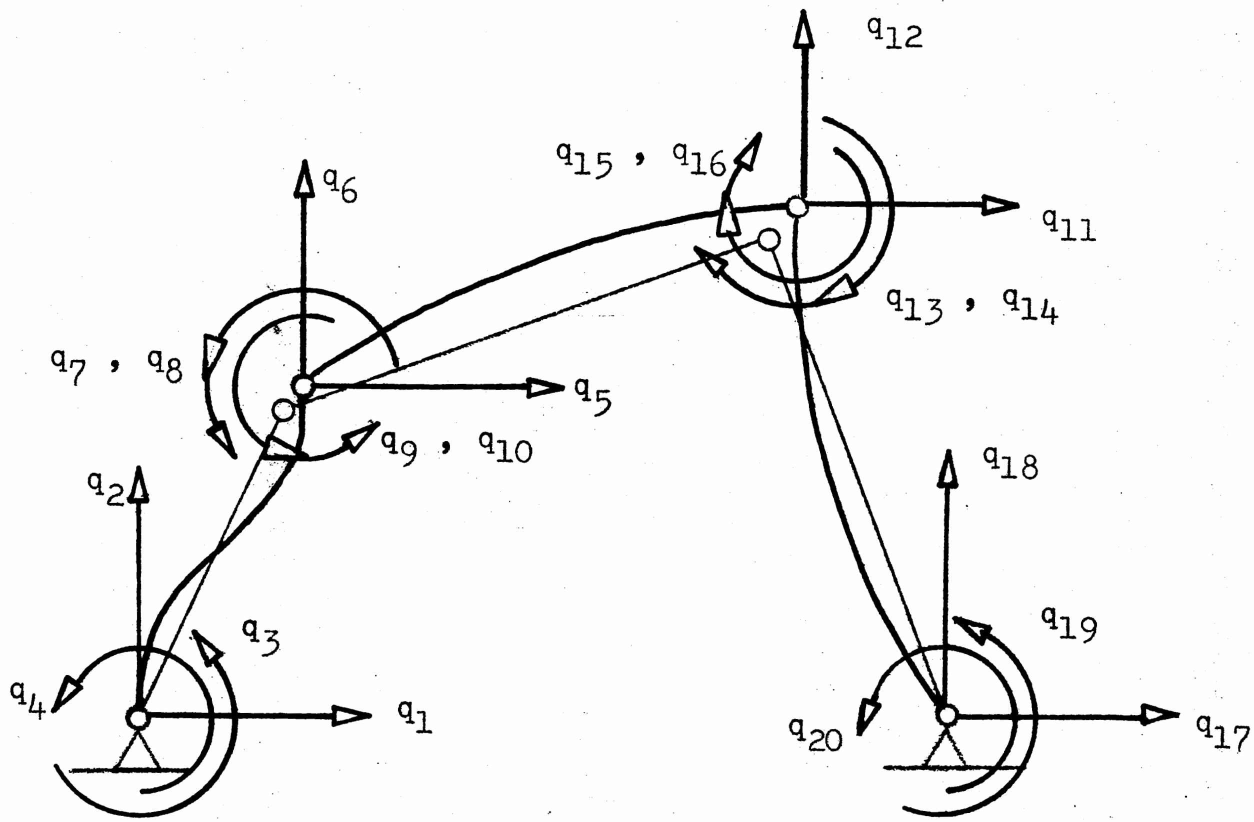


Figure 20. Elastic Degrees of Freedom in Global Coordinates of a Four-Link Planar Mechanism

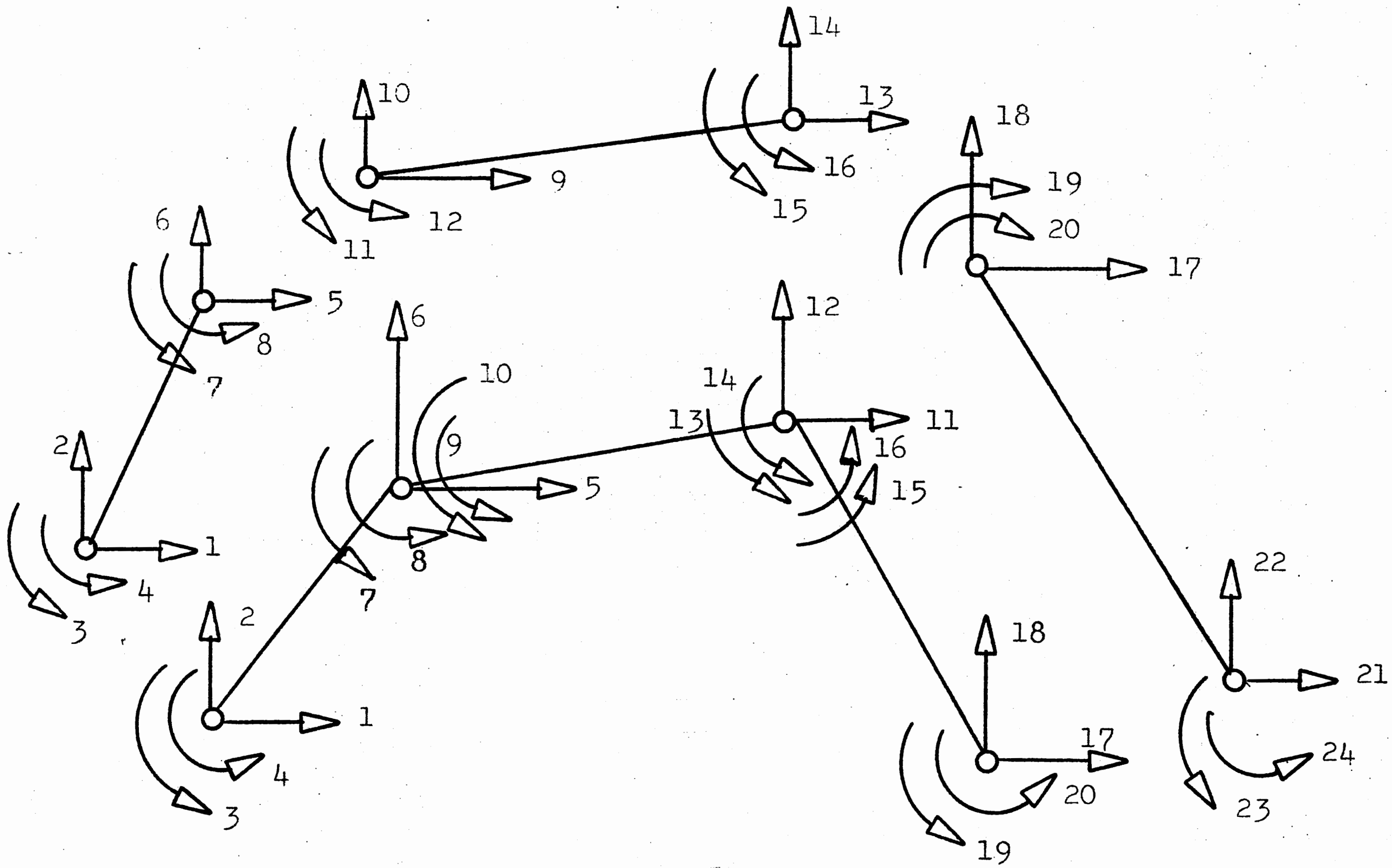


Figure 21. The Element Deformation System and the Joint Deformation System for a Four-Bar-Planar Mechanism. One Element Per Link.

where $[K_i]$ is the stiffness matrix for each element ($i = 1, n$), and n is the number of elements of the mechanism. The number of second order linear differential equations to describe the motion of the mechanism is obtained by using the following relation:

$$Ne = 4 \times Nd - 5 \tag{8.2}$$

where Ne is the number of second order linear differential equations, and Nd is the number of nodes of the mechanism after division into the number of elements required.

The global matrices of the equations of motion of the mechanism have to be reduced according to the boundary conditions. This is done in the way shown in Figure 22. If it is necessary to carry out a reduction of the number of degrees of freedom, the procedure is shown in Appendix E.

Length Strains and Stresses

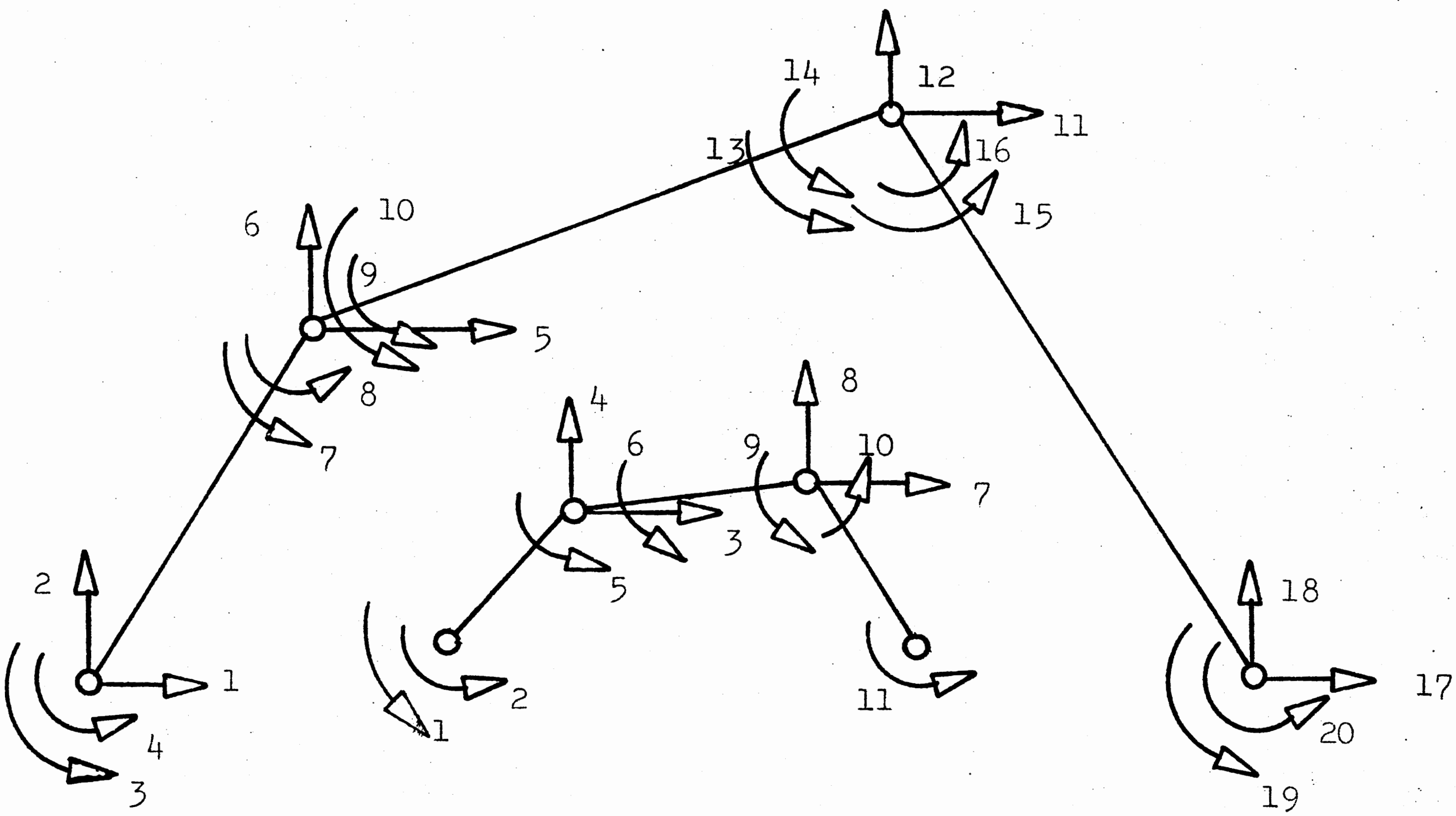
When the equations of motion are solved, the deformation vector (S) in generalized coordinates of the mechanism is obtained. To obtain the deformation vector (s_e) and the coefficient vector (A_e) of each element in local coordinates, the following procedure is used:

$$\underset{24\times1}{(S'_e)} = \underset{24\times20}{[Z]}\ \underset{20\times1}{(S_e)} \tag{8.3}$$

Then the deformation vector (S'_e) of the elements of the mechanism is partitioned to obtain the deformation vector $(S''_e)_i$ of each element in global coordinates:

$$(S''_e)_i = (S'_e) \qquad \text{by partition} \quad i = 1, n \tag{8.4}$$

Hence,



(B) = (1 2 8 10 14 16 17 18 20)

Figure 22. Vector of Boundary Conditions for the Four-Bar Mechanism. One Element Per Link.

$$(s_e)_i = [T_e]_i^T (S_e'')_i \tag{8.5}$$

where $(s_e)_i$ is the deformation vector for each element in local coordinates, and $[T_e]_i$ is the transformation matrix for each element shown in Appendix E. The coefficient vector $(A_e)_i$ of each element can now be obtained:

$$(A_e)_i = [C_e]_i^{-1} (s_e)_i \tag{8.6}$$

To obtain the strains and stresses of each element, it is known that

$$\sigma = Eh(d^2w(x)/dx^2) + Edu(x)/dx \tag{8.7}$$

where h is the cross-sectional depth, E is the modulus of elasticity, and w(x) and u(x) are shape functions. In matrix form:

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = E \begin{bmatrix} 0 & 0 & h & 2h & 0 & 6hx & 12hx^2 & 20hx^3 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} B_0 \\ A_0 \\ A_1 \\ A_2 \\ B_1 \\ A_3 \\ A_4 \\ A_5 \end{pmatrix} \tag{8.8}$$

$$\sigma = \sigma_1 + \sigma_2 \tag{8.9}$$

The positions of the joints of the mechanism that are in motion are obtained by using Equation (5.3) of Chapter V. Then the lengths of the deformed elements or links may be found. The lengths of the elements or links due to transverse or axial deformations are obtained by using the following equation:

$$XLD = \int_{R_1}^{R_2} \sqrt{1 + (d\phi(x)/dx)^2}\, dx \tag{8.10}$$

where $\phi(x)$ represents the shape functions.

CHAPTER IX

EXPERIMENTAL ANALYSIS

Since linearization and approximations are introduced in the equation derivations, it is desirable to have some means of comparisons. Due to the difficulty of obtaining results in the desired forms from other research experiences, a suitable physical model of a four-bar planar mechanism has been designed, constructed, and tested. The experimental work was carried out according to the available instrumentation.

It is of basic importance, if economy is to be obtained, to design mechanisms so that the full strength of the engineering materials is utilized within the margins of safety in order that waste resulting from inefficient design be reduced to a minimum. Many machines and mechanisms are constructed too heavy; in some there is a waste of considerable amounts of essential materials and space. An exact determination of the forces to which a mechanism is subjected, together with a rational analysis of the stresses and strains caused by these forces, will lead to a lighter design. This will not only save materials, but will reduce the cost of the product and may improve its performance. Such a rational consideration of actual service conditions will put strength where it is needed and will eliminate material where it does not serve a useful purpose.

The principle of the strain gage is based on the physical property, called strain sensitivity, of certain metallic alloys and carbon compounds to change their electrical resistance when subjected to strain. The ratio of unit resistance change to unit strain for the material is known as the strain-sensitivity factor.

Experimental Model

An in-line four-bar crank-rocker mechanism was designed and built. Its synthesis and dynamic analysis were carried out by the analytical methods introduced in this study. The mechanism was built with split links to maintain as much as possible an in-line motion. Figures 23 through 27 show the characteristics of the mechanism and its installation. The mechanism had the following physical characteristics:

Minimum transmission angle	17.76 degrees
Fixed link	11.00 in.
Input link	7.00 in.
Coupler link	10.00 in.
Output link	12.00 in.
Density	5.292 slugs/ft^3
Total width	1.00 in.
Thickness	1/8 in.
Modulus of elasticity	10.3 10^6 lb/in.^{2}

The mechanism was belt-driven by a variable speed 1/2 horsepower dc motor. Two belts were used for input motion placed on either side of the plane of the coupler link. This arrangement was used to ensure symmetry of input loading and thus maintain planar motion of the mechanism. Each crank shaft had 6 x 12/16 in. aluminum disk flywheel attached to reduce crank speed fluctuations. Aluminum shafts were used to reduce inertia. Fafnir PSD 1/2 power transmission units with eccentric collar number

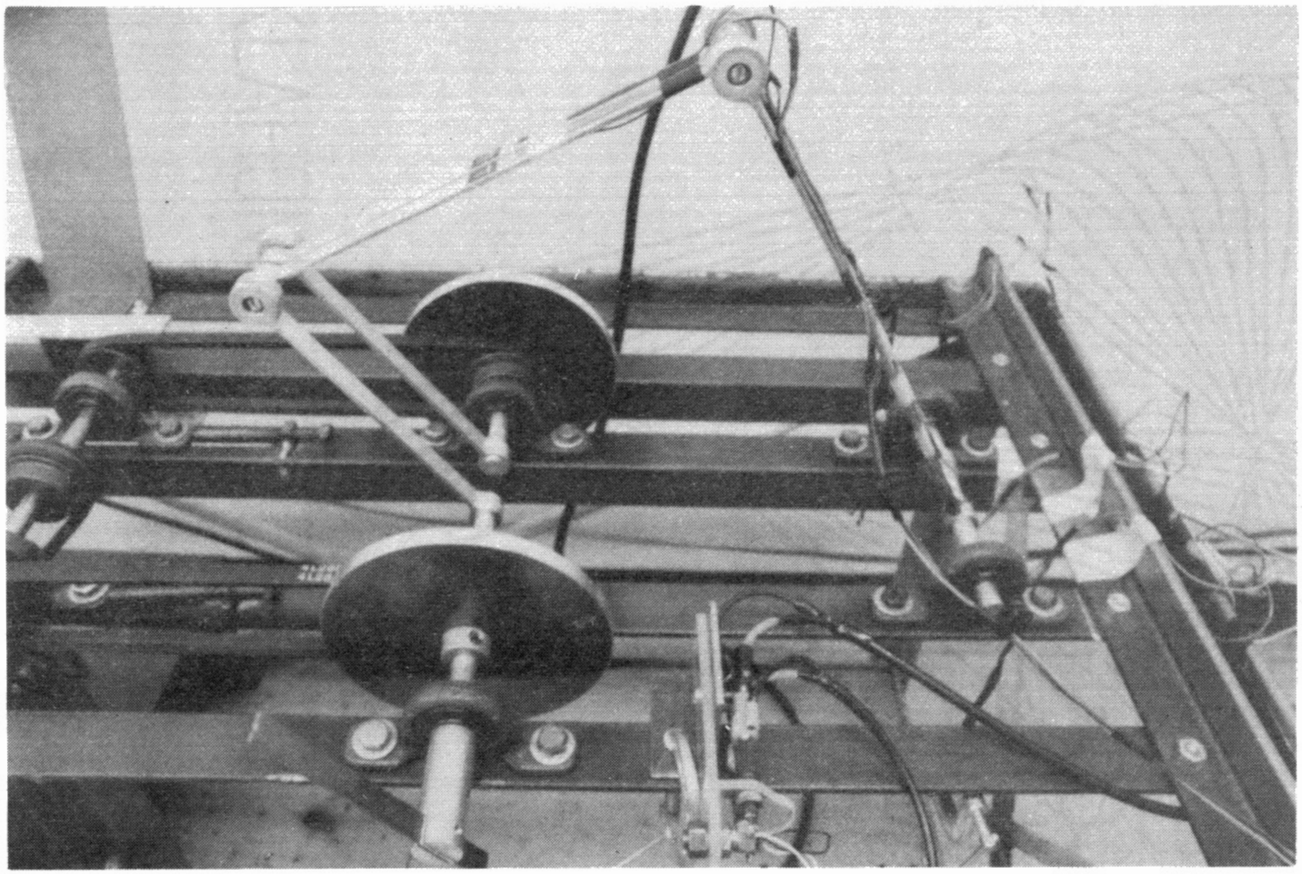


Figure 23. Four-Bar-Planar Mechanism Model

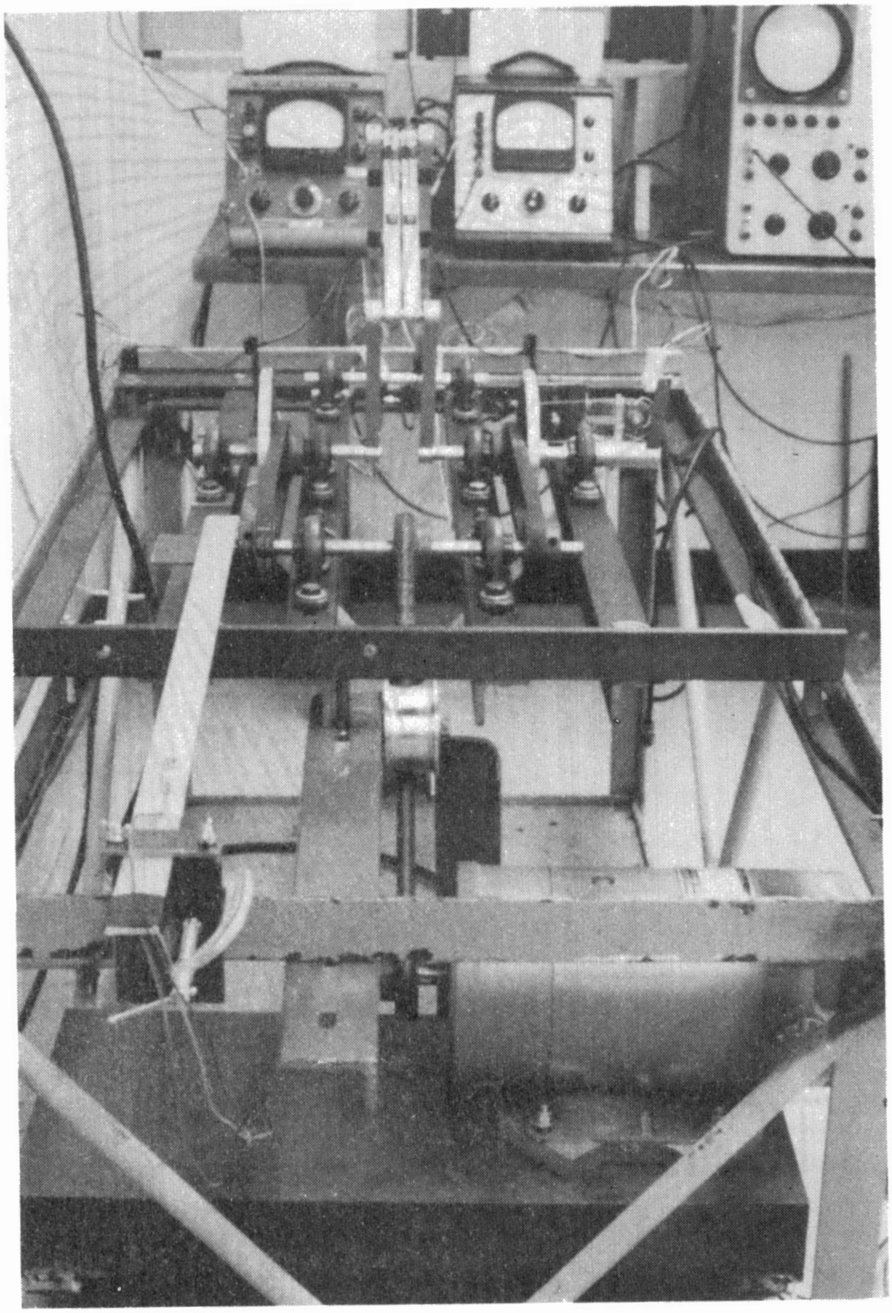


Figure 24. Side View of the Mechanism Setup

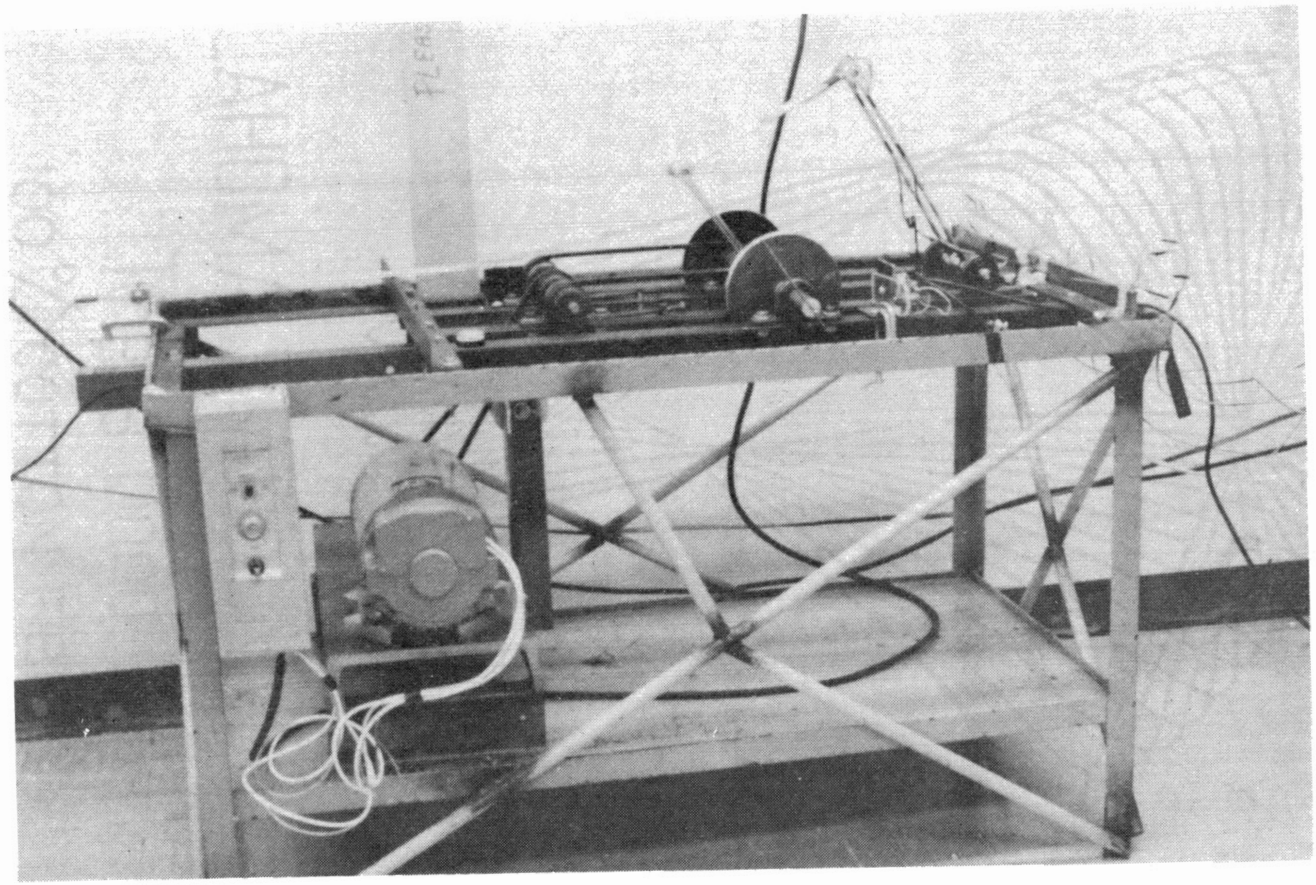


Figure 25. Front View of the Mechanism Setup

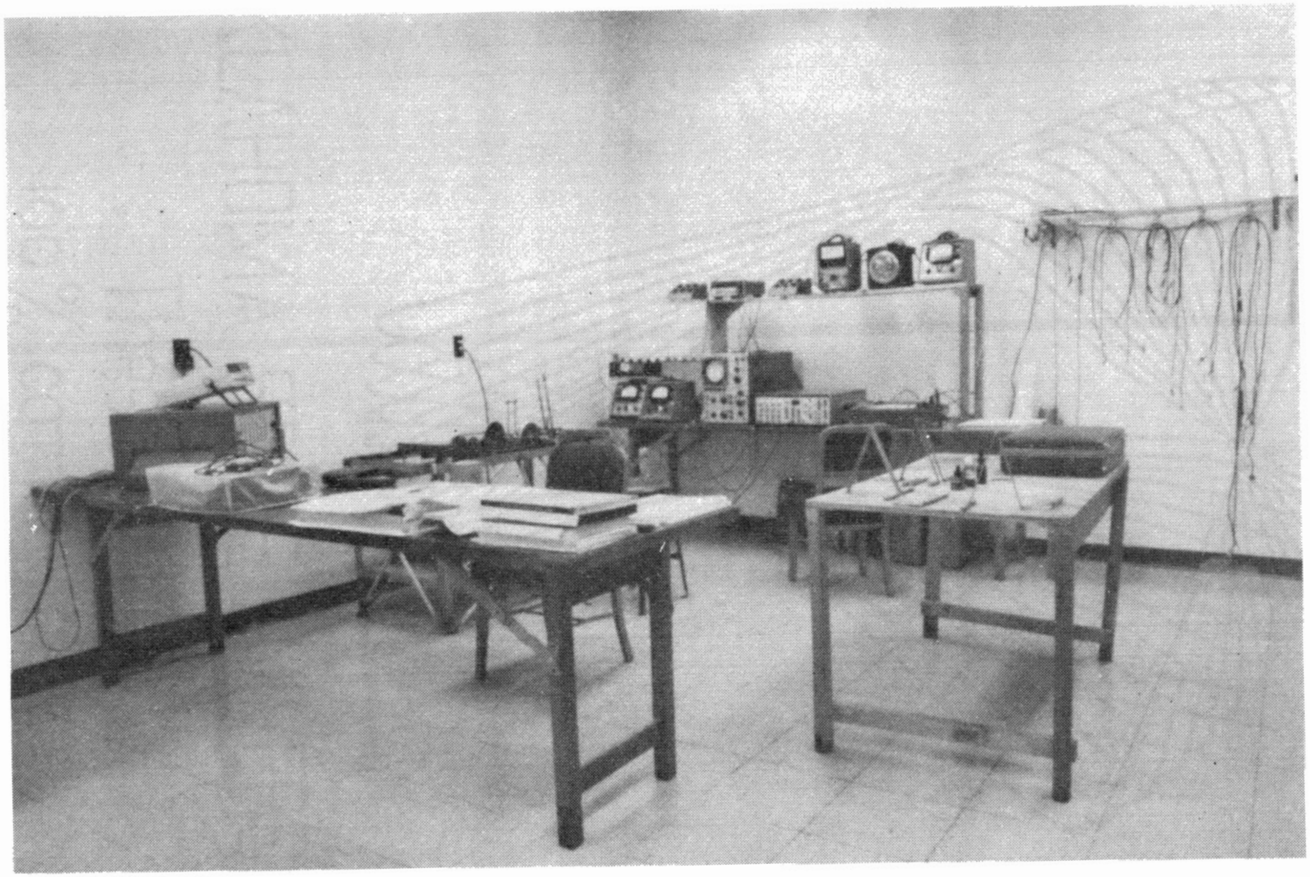


Figure 26. Instrumentation Setup

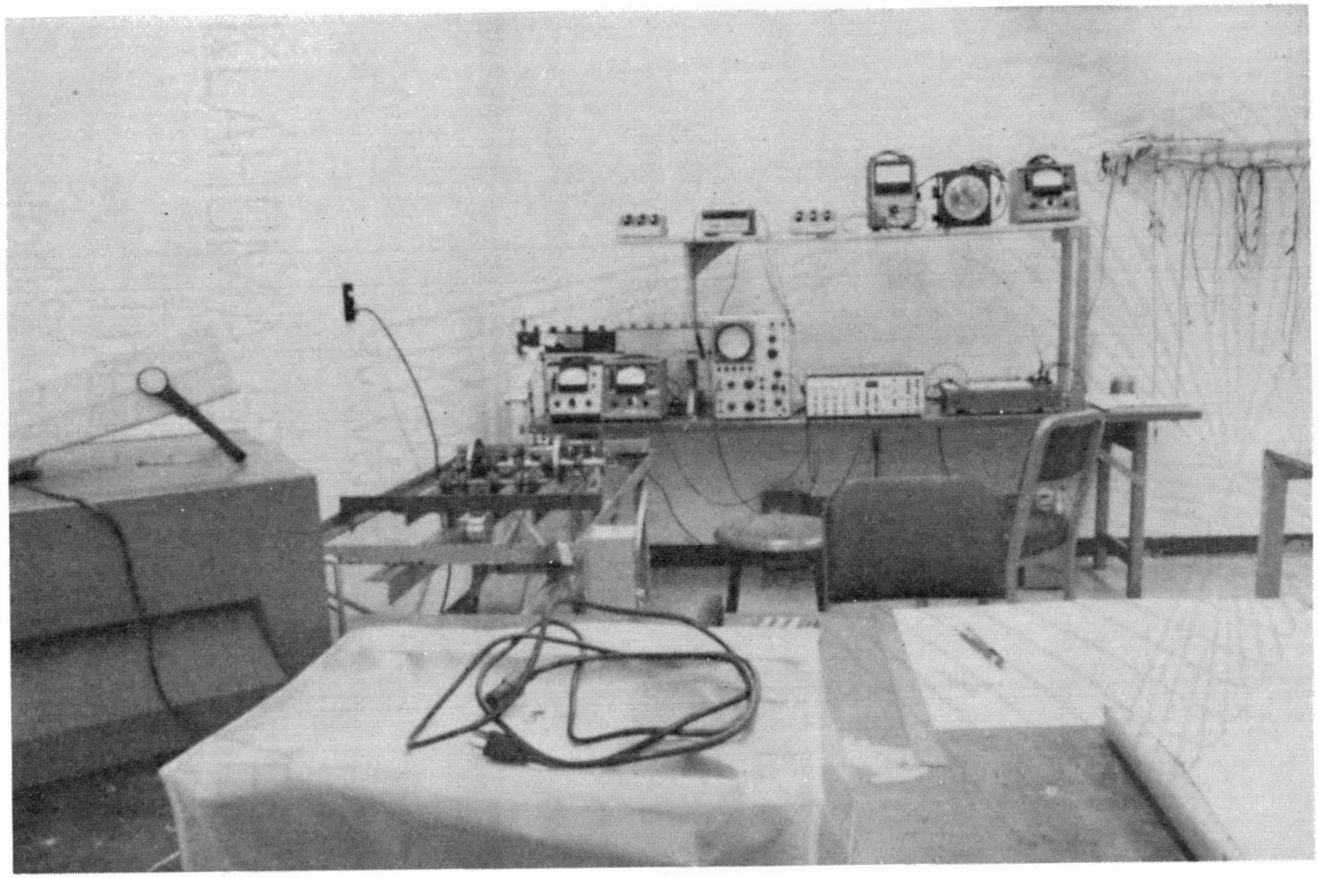


Figure 27. Instrumentation Setup

S1D08K were used for support of the shafts. Fafnir MS1K ball bearing units were used as the floating bearings. The total weight of the bearings with sleeves at each end of the coupler was 0.13128 lbf.

Instrumentation Setup

Figure 28 shows the instrumentation setup used in the experiment. Micromeasurements type ED-DY-375BG-350 strain gages were mounted at one point on both the coupler and output links, as shown in Figure 29. At each point the gages were mounted on both the upper and lower surfaces of the beam so that bending and axial strains could be recorded separately. The zero angle of the input link and the triggering system for the wave recorder were determined with the help of a magnetic sensor, which is a non-contact transducer which converts mechanical motion into electrical energy. The period of the mechanism was recorded by a universal counter. Strain data were registered and amplified by using bridge amplifier meter model BAM-1. The data were recorded in a wave recorder. The output was observed on a Tektronix 502A dual beam oscilloscope. Representative traces were then recorded on an X-Y recorder.

Results and Discussion

The first natural frequencies of the input, coupler, and output links, when considered as nonrotating uniform pinned end beams, were calculated to be 304.25 Hz of lateral vibration and 28697.58 Hz of longitudinal vibration for the input link, 124.73 Hz of lateral vibration and 5022.10 Hz of longitudinal vibration for the coupler, and 79.07 Hz of lateral vibration and 4185.08 Hz of longitudinal vibration for the output link.

The mechanism was operated at three different crank speeds controlled

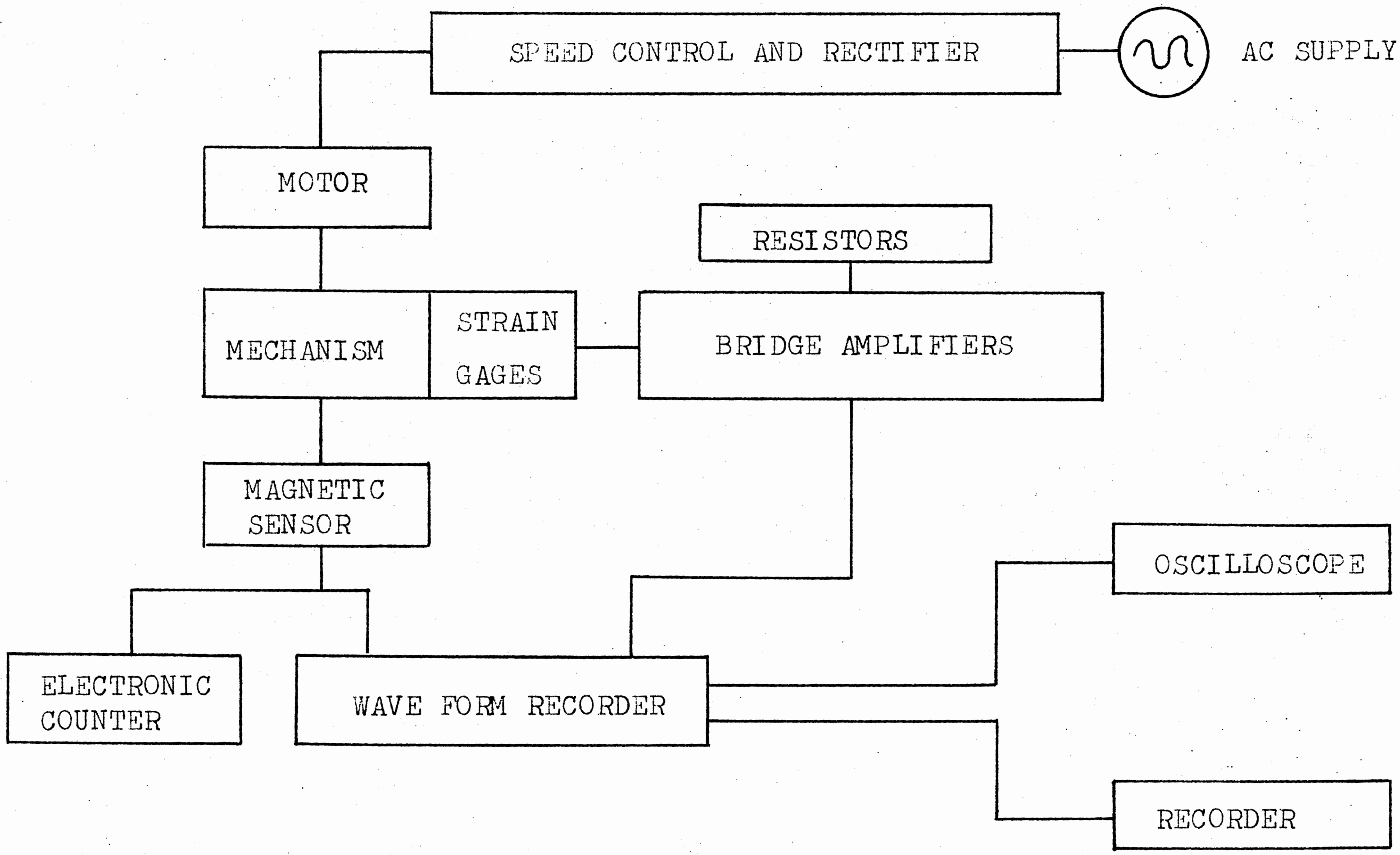


Figure 28. Instrumentation Setup

by a motor control, and experimental data were taken using the apparatus previously described. Experimentally, the natural frequencies of lateral vibration were found to be 125.0 Hz for the coupler and 78.13 Hz for the output link. Typical records are shown in Figures 30 through 41, where the bending and axial strains at gages A and B (middle position of the links; see Figure 29) are presented for input link speeds of 95, 241, and 373 rpm. The frequency component of the wave form shown for gages A and B extends from about -60° to 50°. Peak values are observed at about between 25° and 40°, and between -25° and -40° of the input link position angle. Figures 30, 31, 36, and 37 show the presence of noise in the output and coupler link records. This is mainly due to settings of the bridge amplifier meters and the wave recorder so that an enlarged figure could have been obtained. Figures 32 and 38 show also the presence of noise but at a lower rate than the previous observation.

A sweep of 0.2048 s/in. was used during the experimentation. The experimentally determined response frequencies shown in Figures 31, 33, and 35 resulted in about 78.125 Hz. Therefore, the first natural frequency of the output link was excited by the high acceleration of the link at the limit position in which the input link position angle is 44.66°. The beating effect is believed to be due to the small minimum transmission angle used. It is known that if the transmission angle becomes less than 40°, the linkage tends to bind because of friction in the joints; also, the coupler and output link tend to align and may lock. The experimentally determined response frequencies for the coupler resulted in the same effect as previously discussed.

Table I shows the average peak values of strains in μin./in. and percentage obtained for the output and coupler links of the four-bar planar

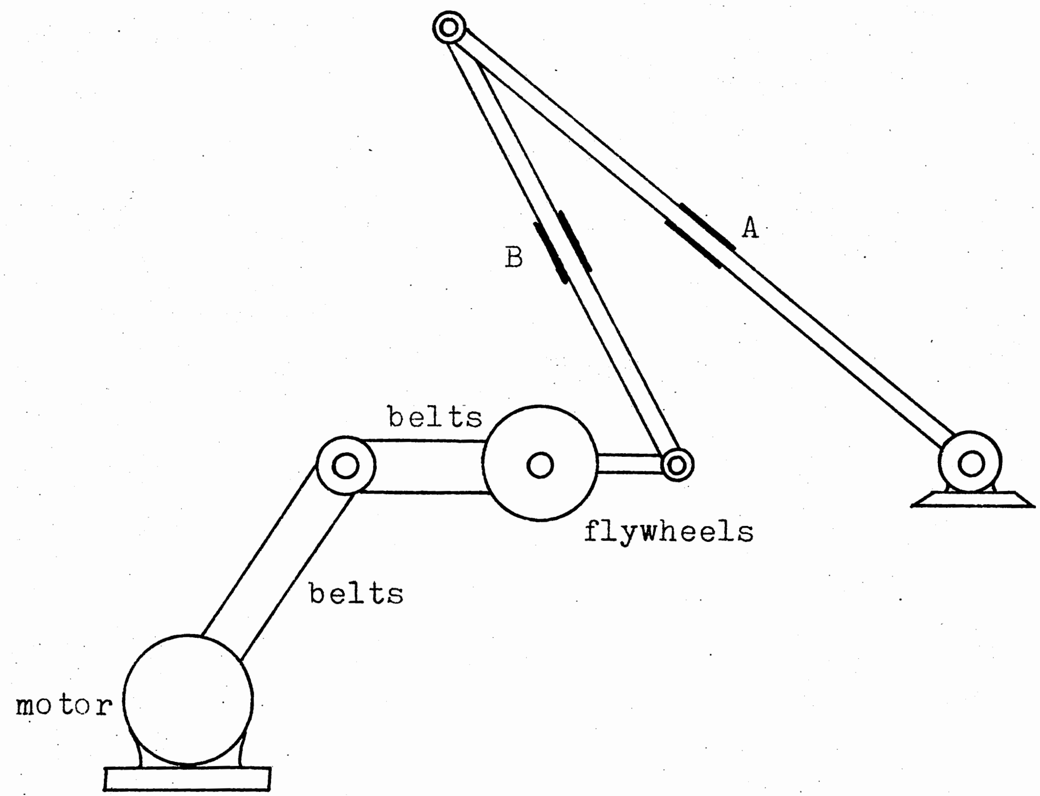


Figure 29. Schematic of Experimental Model

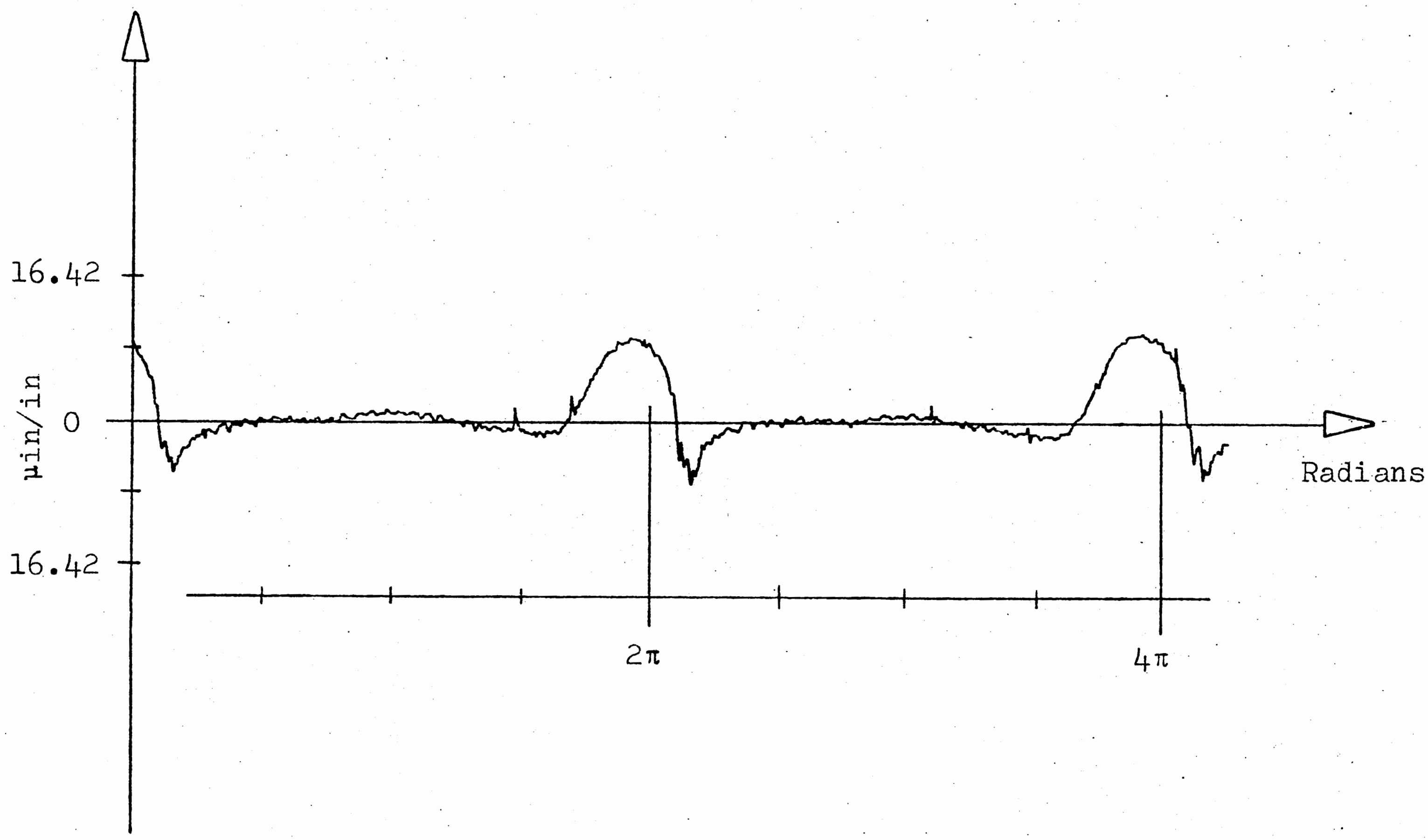


Figure 30. Axial Strains for the Output Link at 95 rpm of the Input Link. Strain Variation of 0.821 µin/in/mm

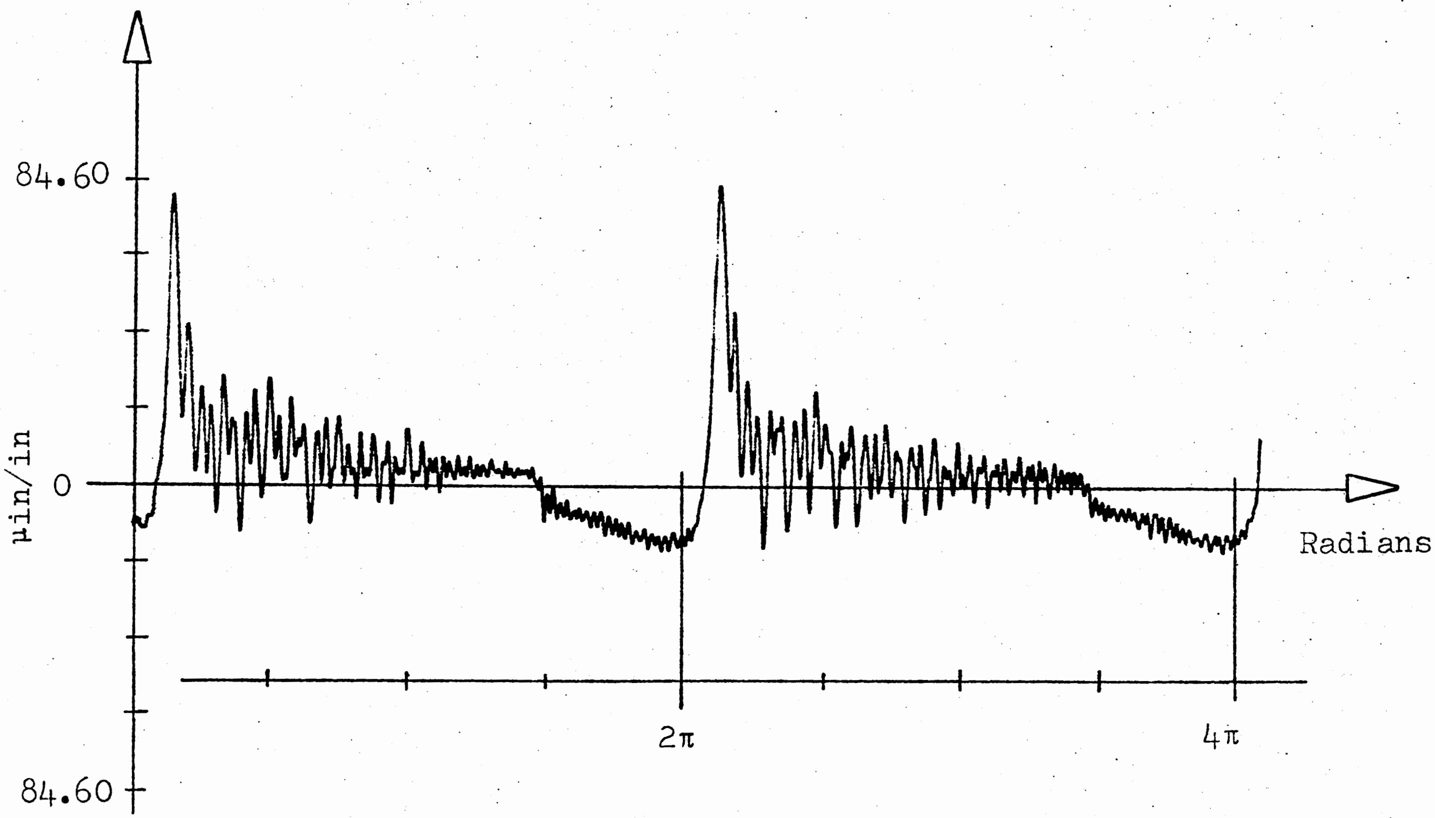


Figure 31. Bending Strains for the Output Link at 95 rpm of the Input Link. Strain Variation of 2.115 μin/in/mm

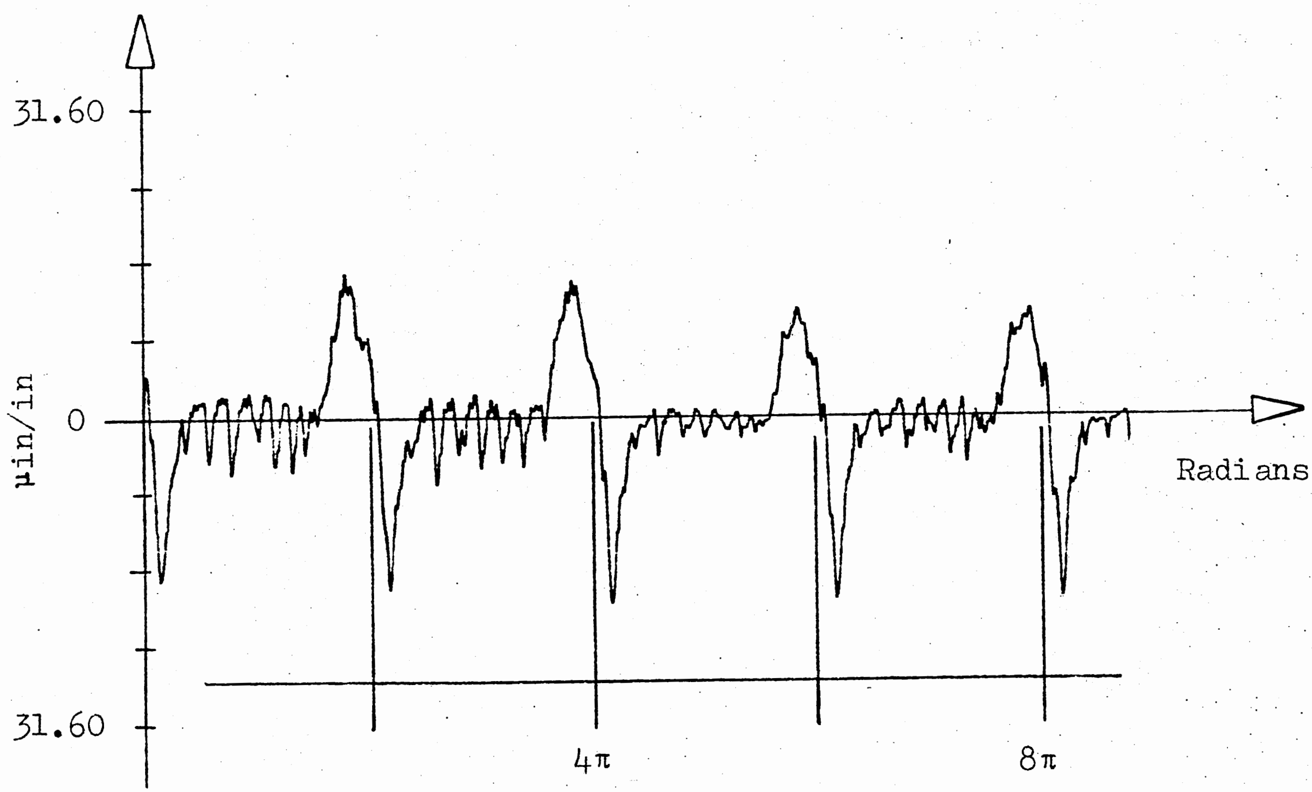


Figure 32. Axial Strains for the Output Link at 241 rpm of the Input Link. Strain Variation of 0.790 µin/in/mm

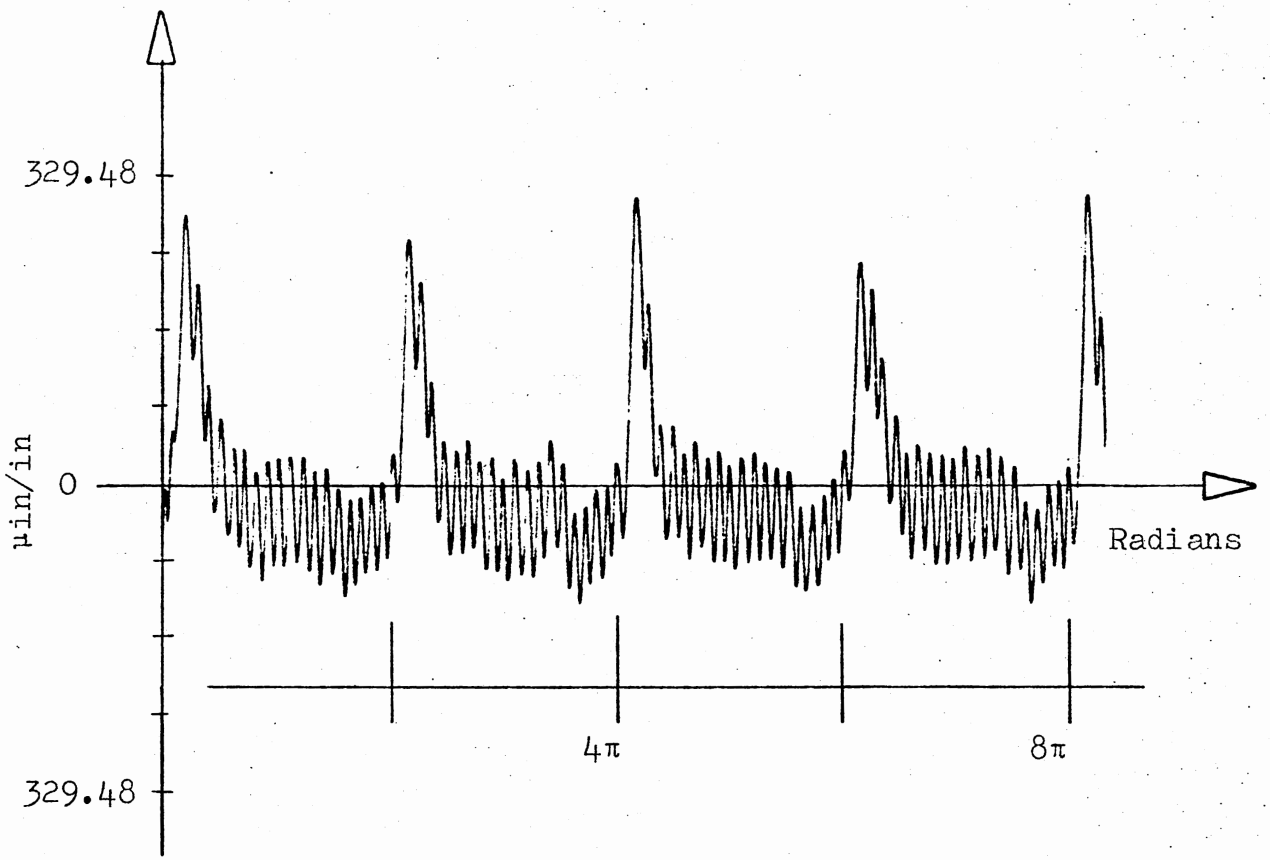


Figure 33. Bending Strains for the Output Link at 241 rpm of the Input Link. Strain Variation of 8.237 µin/in/mm

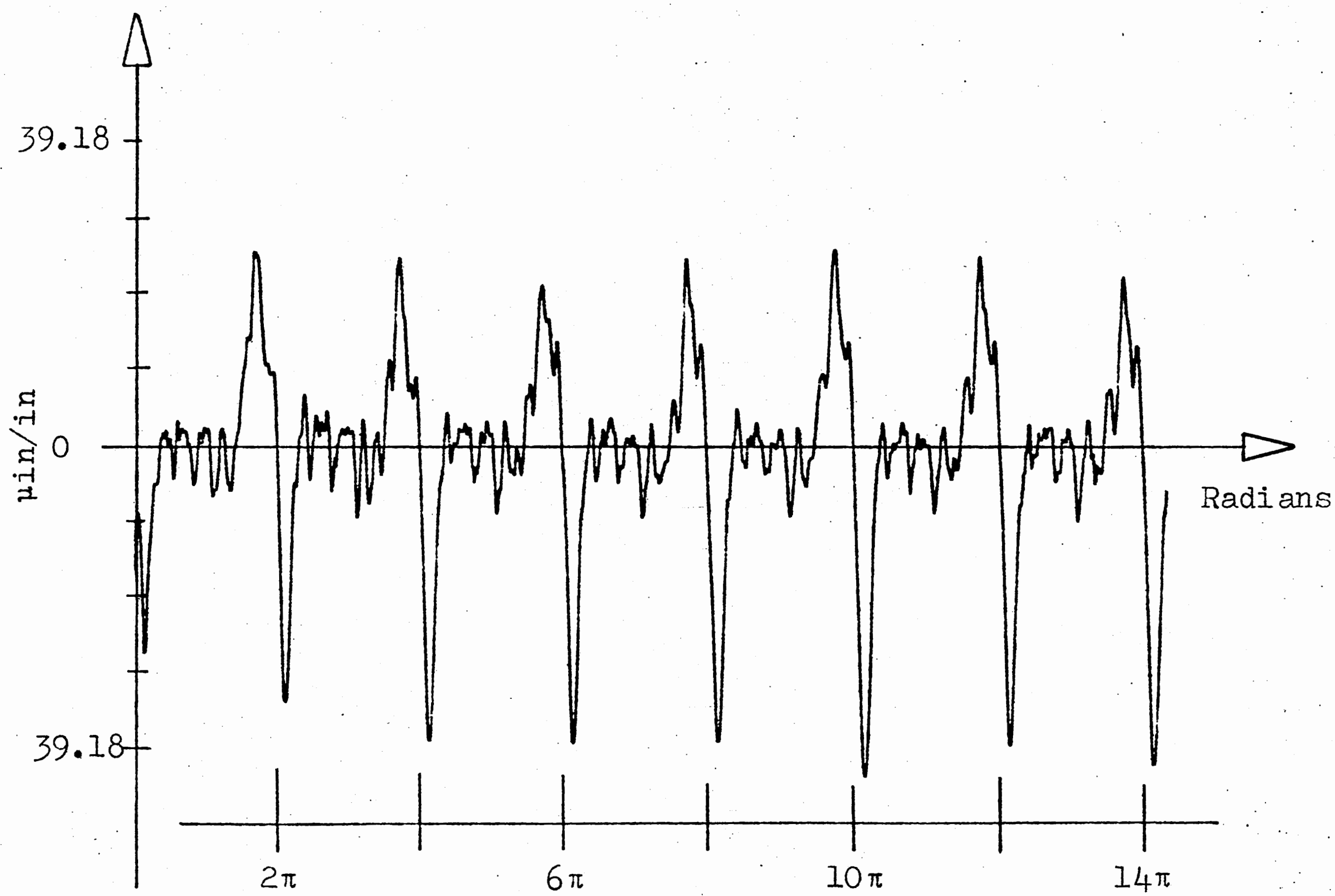


Figure 34. Axial Strains for the Output Link at 373 rpm of the Input Link.
Strain Variation of 0.980 µin/in/mm

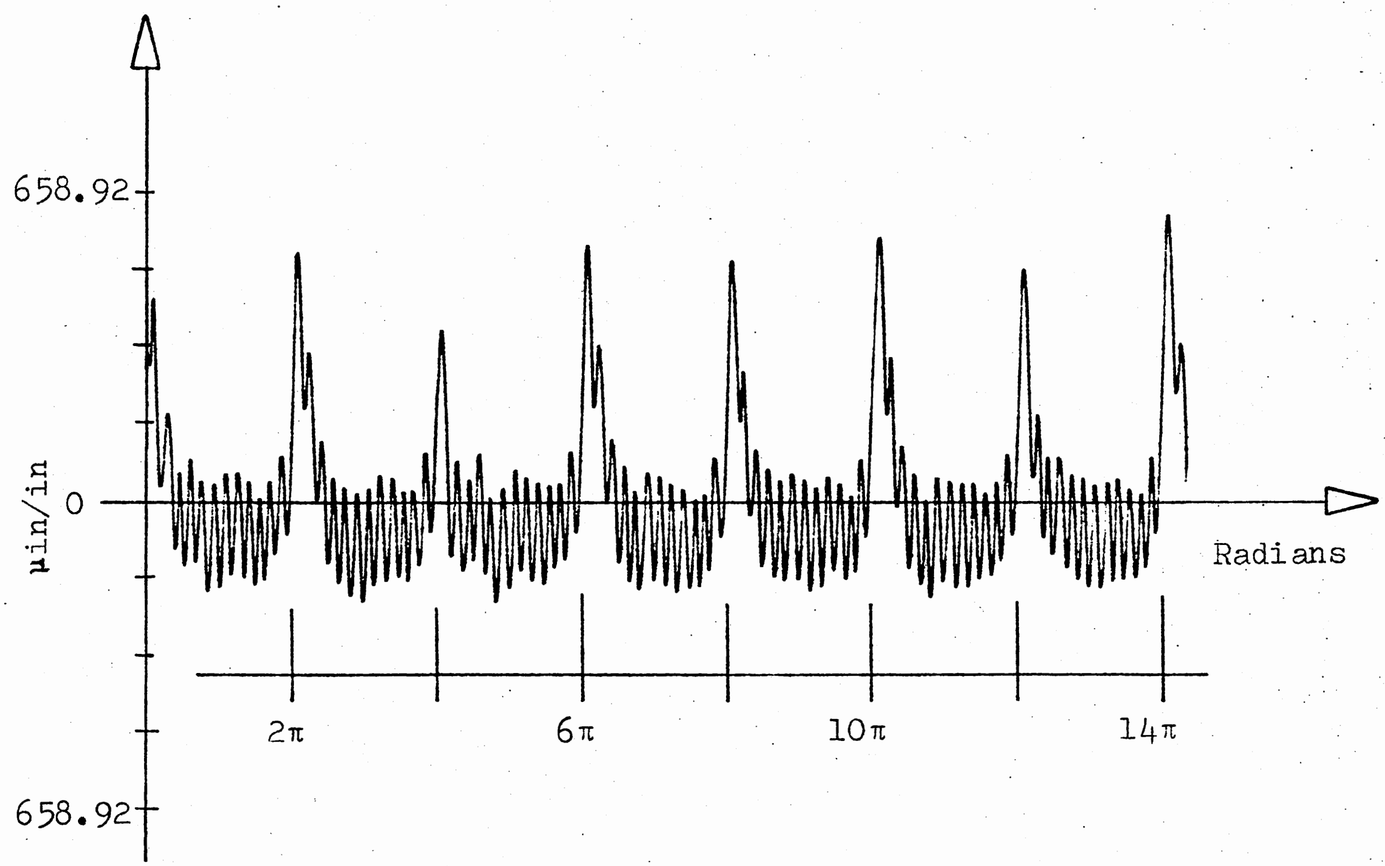


Figure 35. Bending Strains for the Output Link at 373 rpm of the Input Link. Strain Variation of 16.473 µin/in/mm

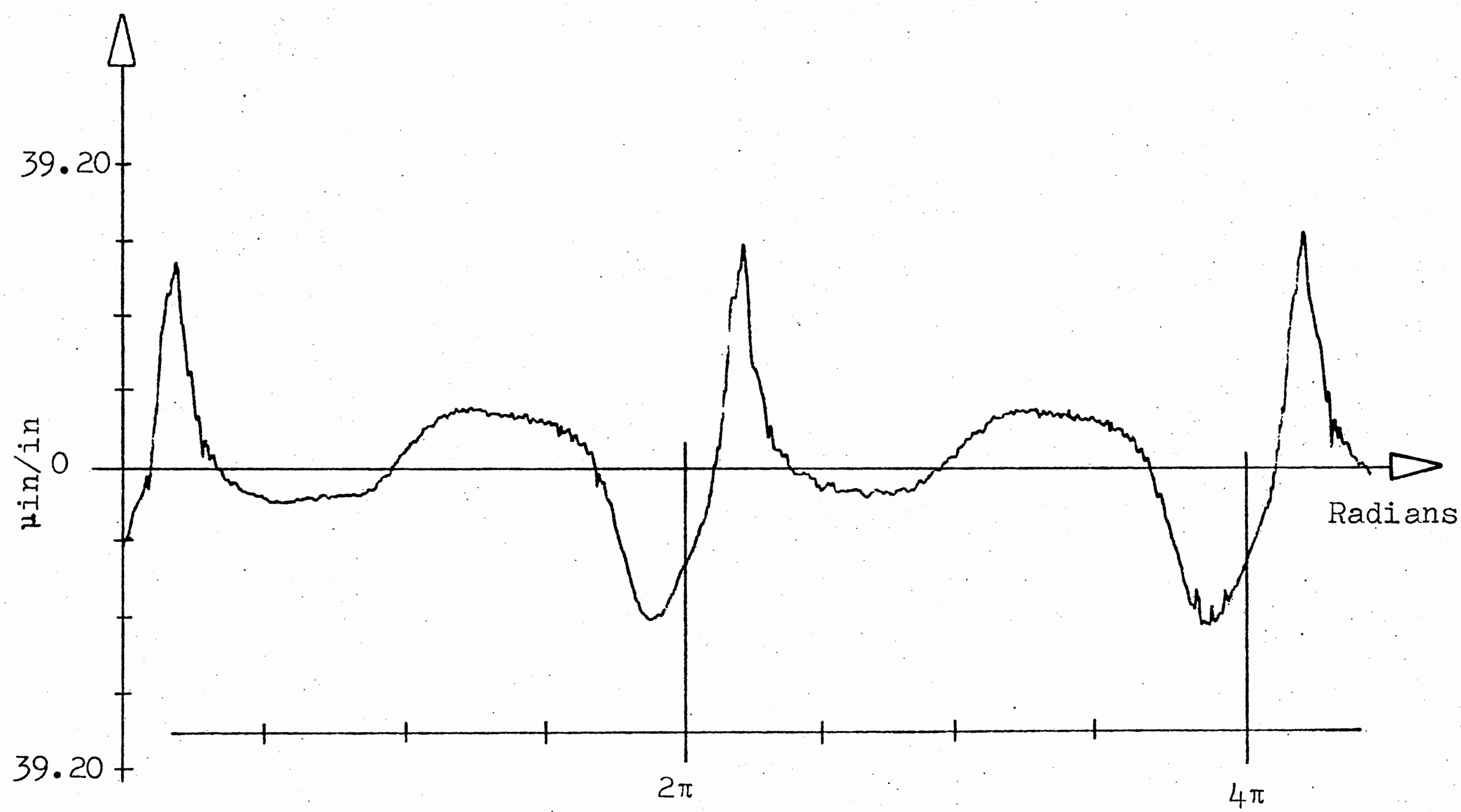


Figure 36. Axial Strains of the Coupler Link at 95 rpm of the Input Link. Strain Variation of 0.980 µin/in/mm

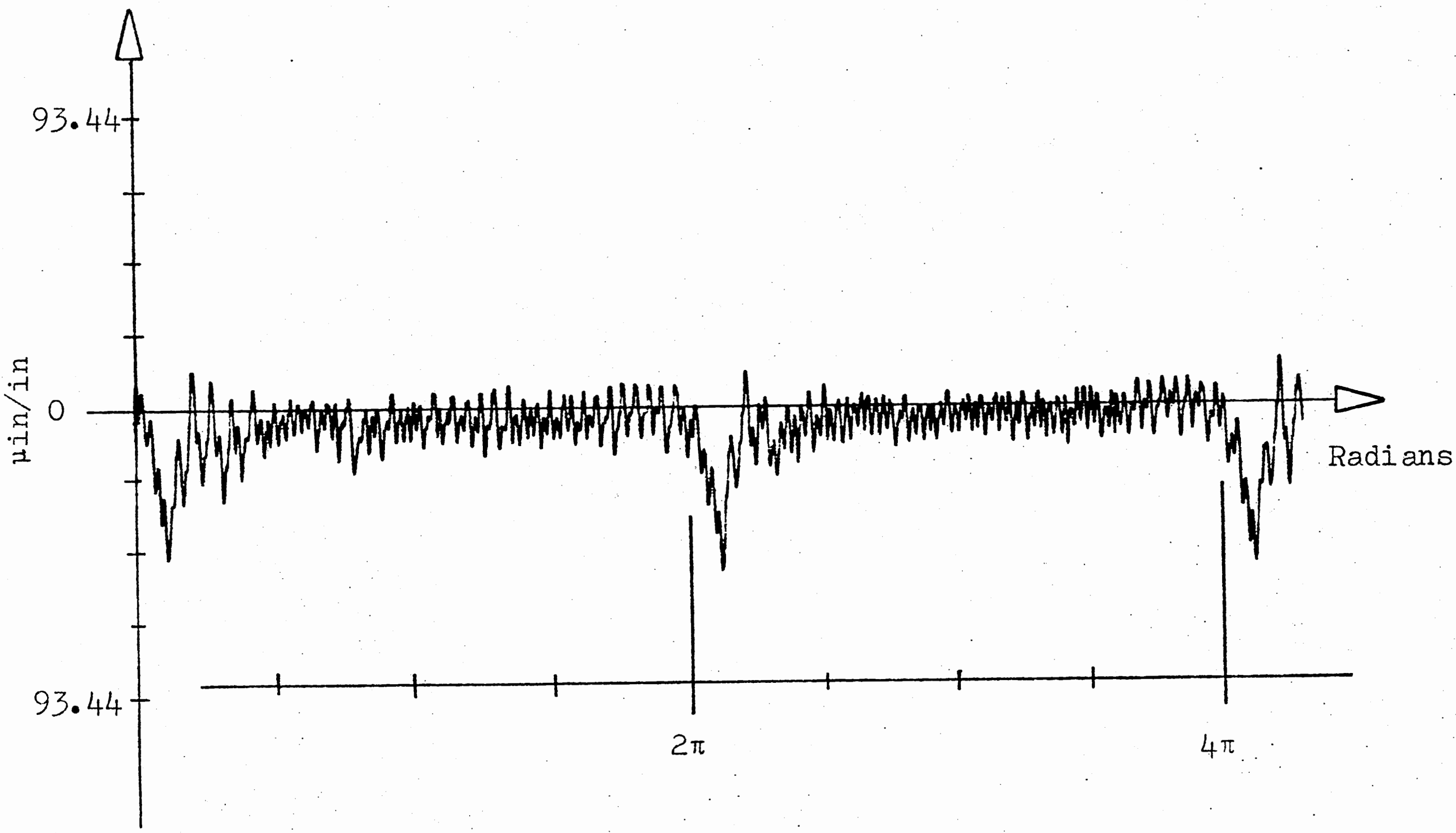


Figure 37. Bending Strains of the Coupler Link at 95 rpm of the Input Link. Strain Variation of 2.336 μin/in/mm

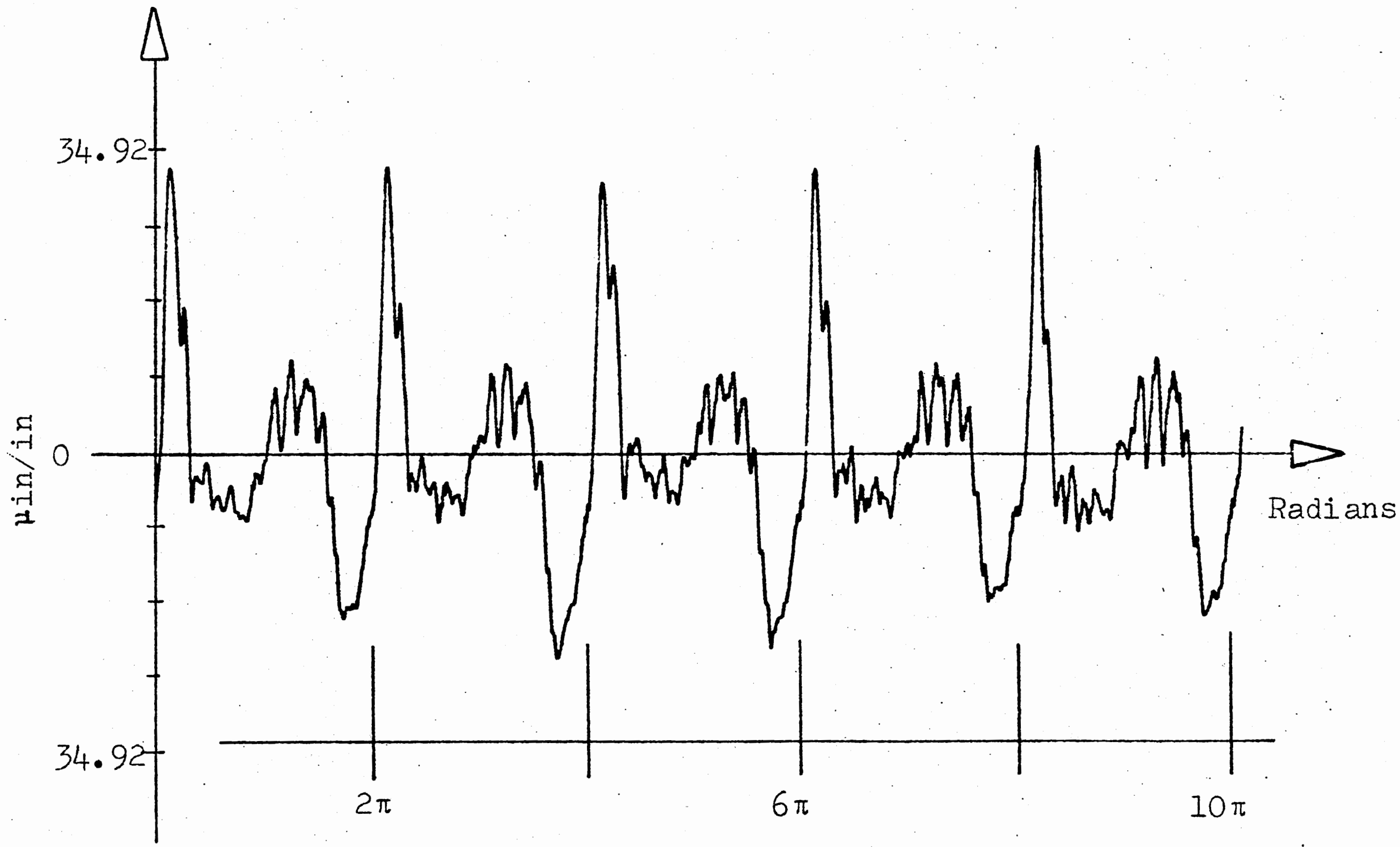


Figure 38. Axial Strains of the Coupler Link at 241 rpm of the Input Link.
Strain Variation of 0.873 µin/in/mm

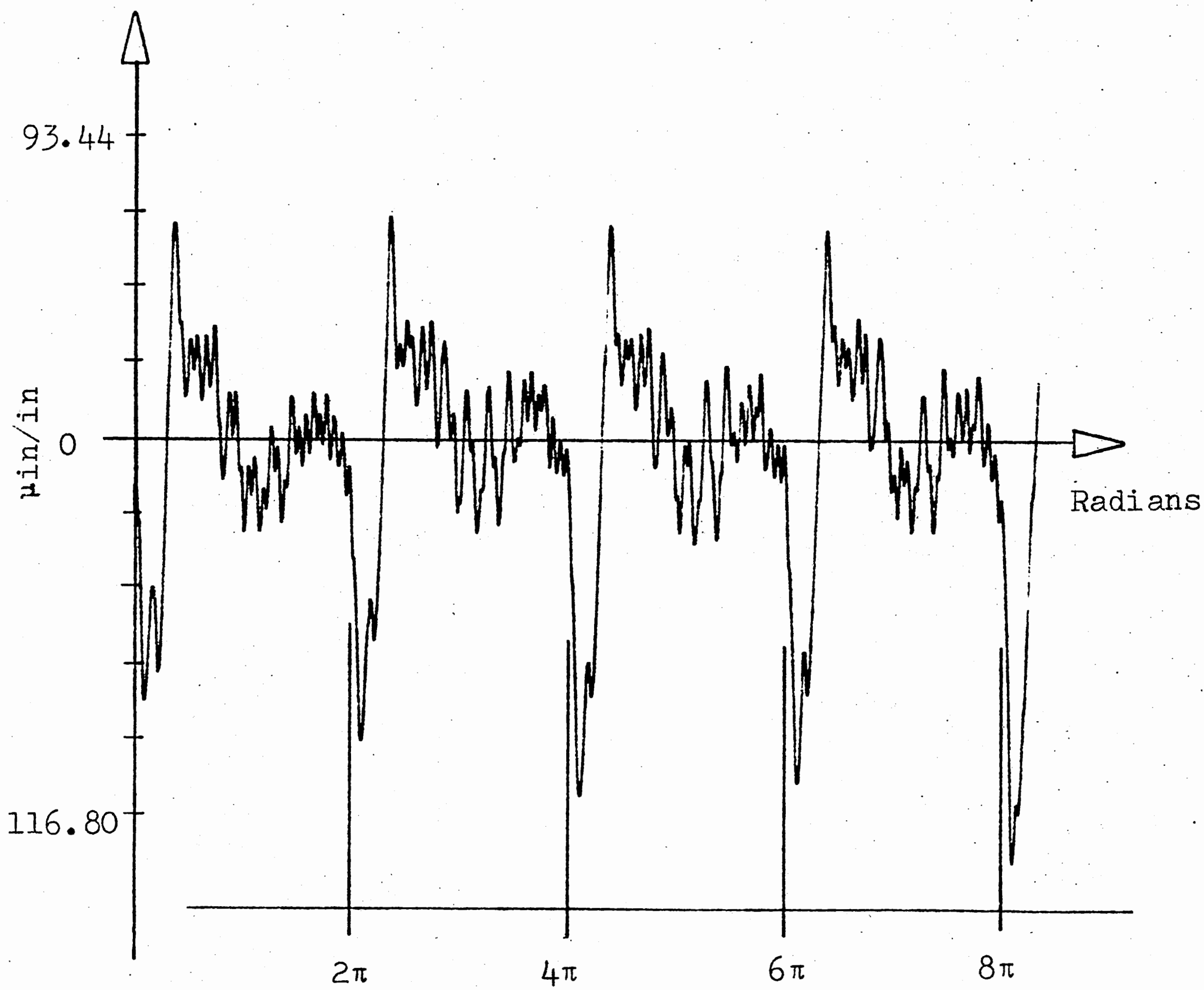


Figure 39. Bending Strains of the Coupler Link at 241 rpm of the Input Link. Strain Variation of 2.336 μin/in/mm

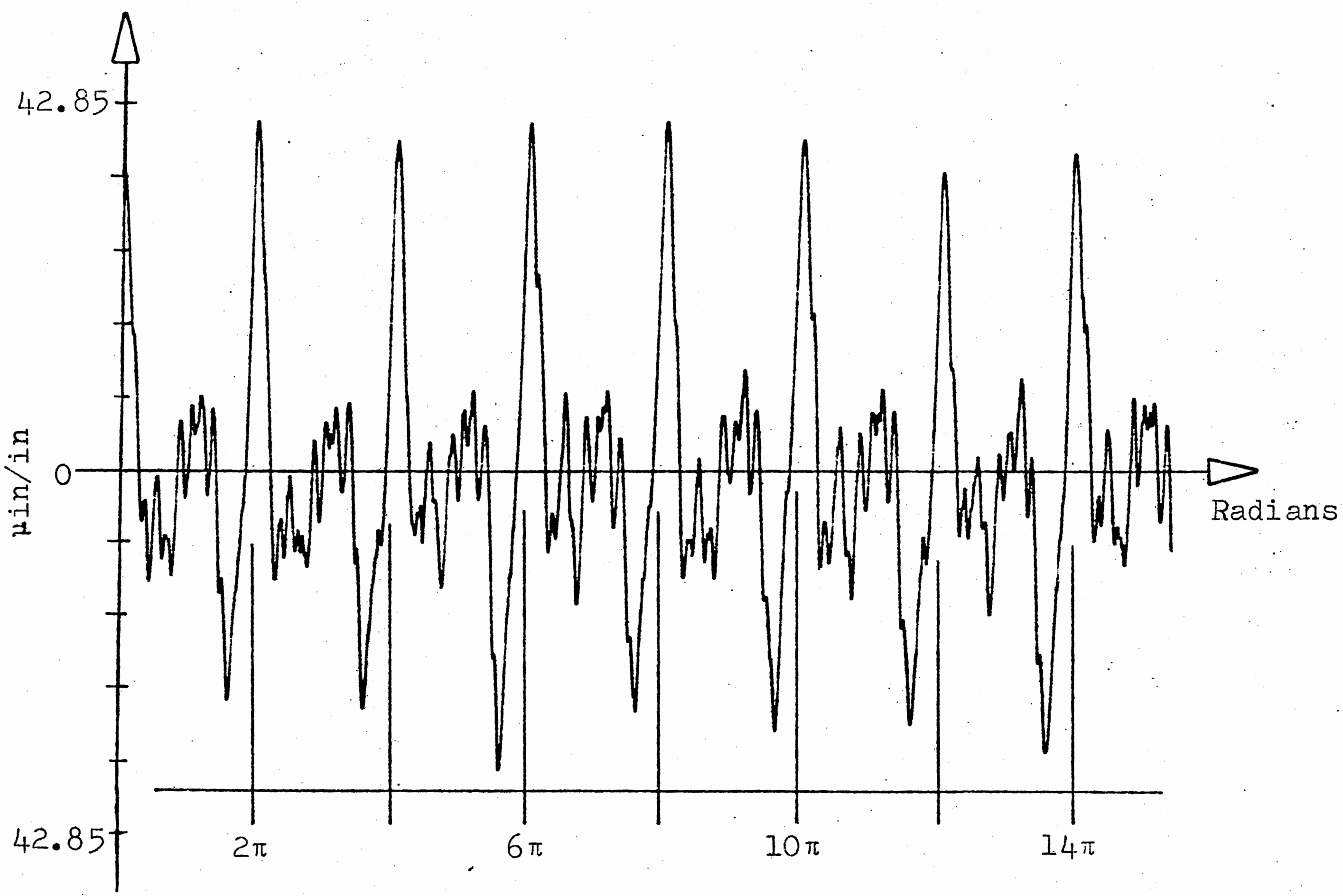


Figure 40. Axial Strains of the Coupler Link at 373 rpm of the Input Link. Strain Variation of 0.857 μin/in/mm

TABLE I

AVERAGE PEAK VALUES OF STRAINS IN μIN./IN. AND PERCENTAGE OBTAINED EXPERIMENTALLY FOR THE OUTPUT AND COUPLER LINKS OF A FOUR-BAR-PLANAR MECHANISM

Speed rpm	Strain	Output (+) μin./in.	Output Percent	Output (-) μin./in.	Output Percent	Coupler (+) μin./in.	Coupler Percent	Coupler (-) μin./in.	Coupler Percent
95	Bending	80.788	100.00	16.214	100.00	12.381	100.00	50.613	100.00
	Axial	9.359	11.59	6.568	40.51	28.583	230.86	16.660	32.92
241	Bending	281.541	100.00	118.407	100.00	66.284	100.00	104.186	100.00
	Axial	12.601	4.48	18.012	15.21	33.349	50.312	20.079	19.27
373	Bending	509.222	100.00	191.322	100.00	79.343	100.00	135.372	100.00
	Axial	23.030	4.52	37.056	19.368	38.458	48.47	30.301	22.38

mechanism model. It can be observed that a tendency of the bending strains is to increase very rapidly with respect to the axial strains as the speed of the mechanism increases.

The general effects of linkage geometry, which is intimately related to the minimum transmission angle and the operating speed, are clearly demonstrated. The wave forms reflect the fact that the natural frequency characteristics of the mechanisms change with crank position. The following observed characteristics are believed to directly or indirectly influence the results:

1. The arrangement of split links caused a counter-vibration phenomenon in the links.

2. The support system of the model.

3. The minimum transmission angle was too small (17.76°) and created vibrational reactions in the system.

4. The wire system of the strain gages--a slow rotation of the mechanism produced small but measurable bending strains due to flexure of the strain gage wires acting as a torsional spring between the coupler and output links.

5. The effect of gravity.

6. The frictional moments in the journal bearings.

7. The enlargements at the bearing locations, which influence both the frequency and mass of the links.

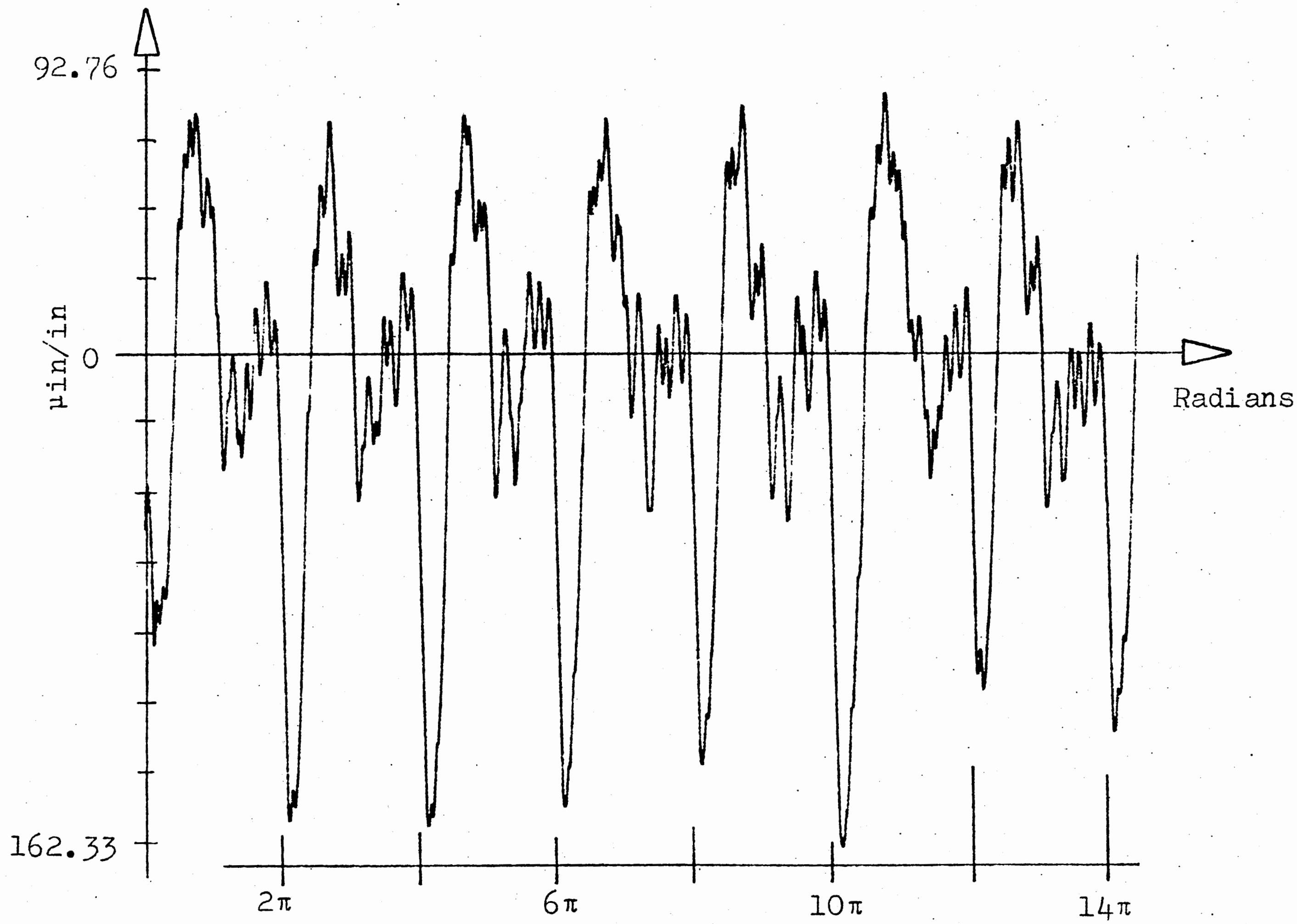


Figure 41. Bending Strains of the Coupler Link at 373 rpm of the Input Link.
Strain Variation of 2.319 μin/in/mm

CHAPTER X

RESULTS AND DISCUSSION FOR THE KINETO-ELASTO-DYNAMIC STUDY OF A FOUR-BAR-PLANAR MECHANISM

The finite element model developed in this study for the analysis of elastic mechanisms has been applied in the elastic analysis of the four-bar-planar mechanism that has been rigidly and experimentally analyzed.

The solution of Equation (7.28) might consist of: (1) solution of the eigenvalue problem, (2) transformation into a set of uncoupled equations, and (3) determination of the transient response of the system by a direct integration method. The third approach has been carried out in this study by utilizing numerical procedures. The use of numerical methods for solving differential equations generally yields solutions which differ from the true solutions. The difference between the numerical solution and the true solution, at any given step, is known as the total error at that step.

The total error at any step results from: (1) a roundoff error which is due to a limited number of significant digits, (2) a truncation error due to the use of approximate formulas, and (3) an error is present at a given step because of errors introduced in preceding steps. The roundoff error introduced at each step is generally very small for most methods. However, if an unstable method is used and if the integration involves a large number of steps, the cumulative effect can lead to serious total error. The use of higher precision is an effective means of controlling total roundoff error. The truncation error at each step is minimum in methods which employ formulas having truncation errors of higher order. This can be reduced, in any method, by reducing the step size. However, in reducing the per-step truncation error by decreasing the size, a limit is reached at which further reduction in step size increases the total number of steps to a point where roundoff error becomes dominant, and the total error will increase with further reduction in step size. The cumulative effect of small per-step truncation errors and their magnification in calculating subsequent steps can lead to serious total errors.

The resulting coupled second order ordinary differential equations of this study are very unstable. Unconditionally stable integration schemes like the Wilson θ method and the Newmark method were used. Conditionally stable integration schemes like the Gears method and the Runge-Kutta-Verner fifth and sixth order method were also employed. The system of equations with its eleven degrees of freedom could not be solved by the named methods due to roundoff error and an instability problem. A reduction of coordinates was carried out to eliminate axial deformation that represented the high frequency parameters. The resulting coupled linear second order ordinary differential equations with eight degrees of freedom were finally solved numerically by use of the Runge-Kutta-Verner method.

First, the problem was solved without taking into account the end masses of joints in motion due to the bearing system. The results showed good solutions as far as the coupler link was concerned. However, the solutions for the output were too small. The problem was then solved by considering the masses of the joints as lumped masses and added to the system. The step size used was 0.16 degrees or $7.14924\ 10^{-5}$ seconds of the input link. Two computing programs in FORTRAN were written to determine the deformations, strains, stresses, lengths of links, and deformed positions of the nonrigid joints of the mechanism. The results obtained are shown in Tables I through VI and Figures 42 through 44.

Table II presents the bending strains at the center of the links of the mechanism at 373 rpm of the input link. Figures 42, 43, and 44 show the graphs of the bending strains of the mechanism determined at the center of the links. The pattern followed by the curves is similar to those obtained experimentally. The response curves obtained analytically and experimentally are out of phase. This disagreement is not too detrimental from a design standpoint since the amplitudes and their positions correspond fairly well. The experimental and analytical response curve for the coupler link shows a fair similarity in peak values. However, a wide difference in the positive peak value between the experimental and analytical response curves is observed. The difference can be explained through the conclusions of the experimental analysis (Chapter IX). The analytical results show peak values around 350 degrees of the crank.

Tables III and IV show how the joints that connect the input link with the coupler and the coupler with the output link move with respect to the rigid position due to the degrees of freedom of deformation of the mechanism. Axial rigidity of the input link can be observed at 180 and 360 degrees of the position of the crank.

Table V shows the eight degrees of freedom of deformations in global coordinates of the mechanism at 373 rpm of the input link. The deformations are presented with respect to the position angle of the crank link. It could be observed that the largest values of most degrees of freedom

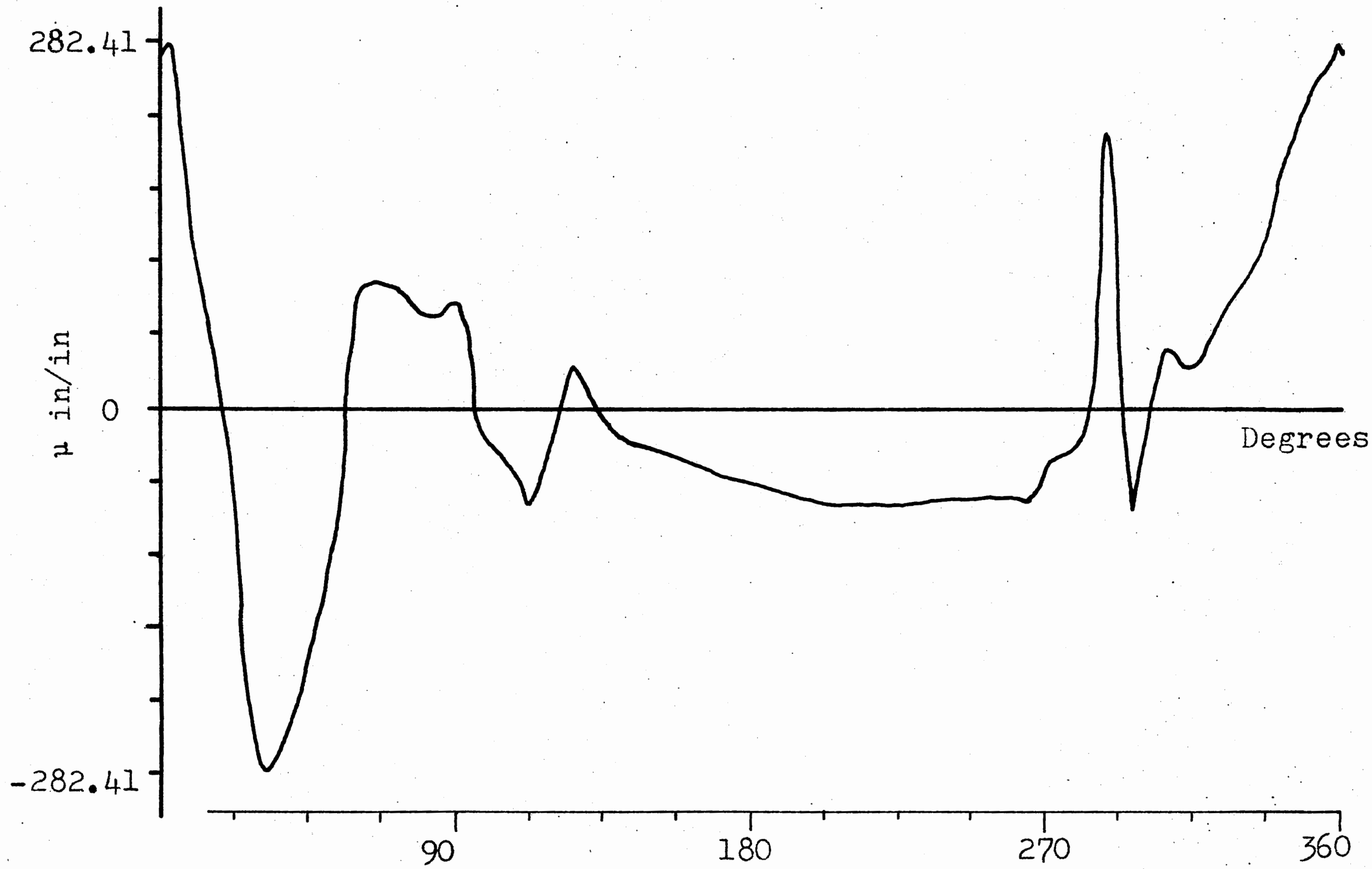


Figure 42. Bending Strains at the Center of the Input Link of the Four-Bar-Planar Mechanism at 373 rpm of the Crank

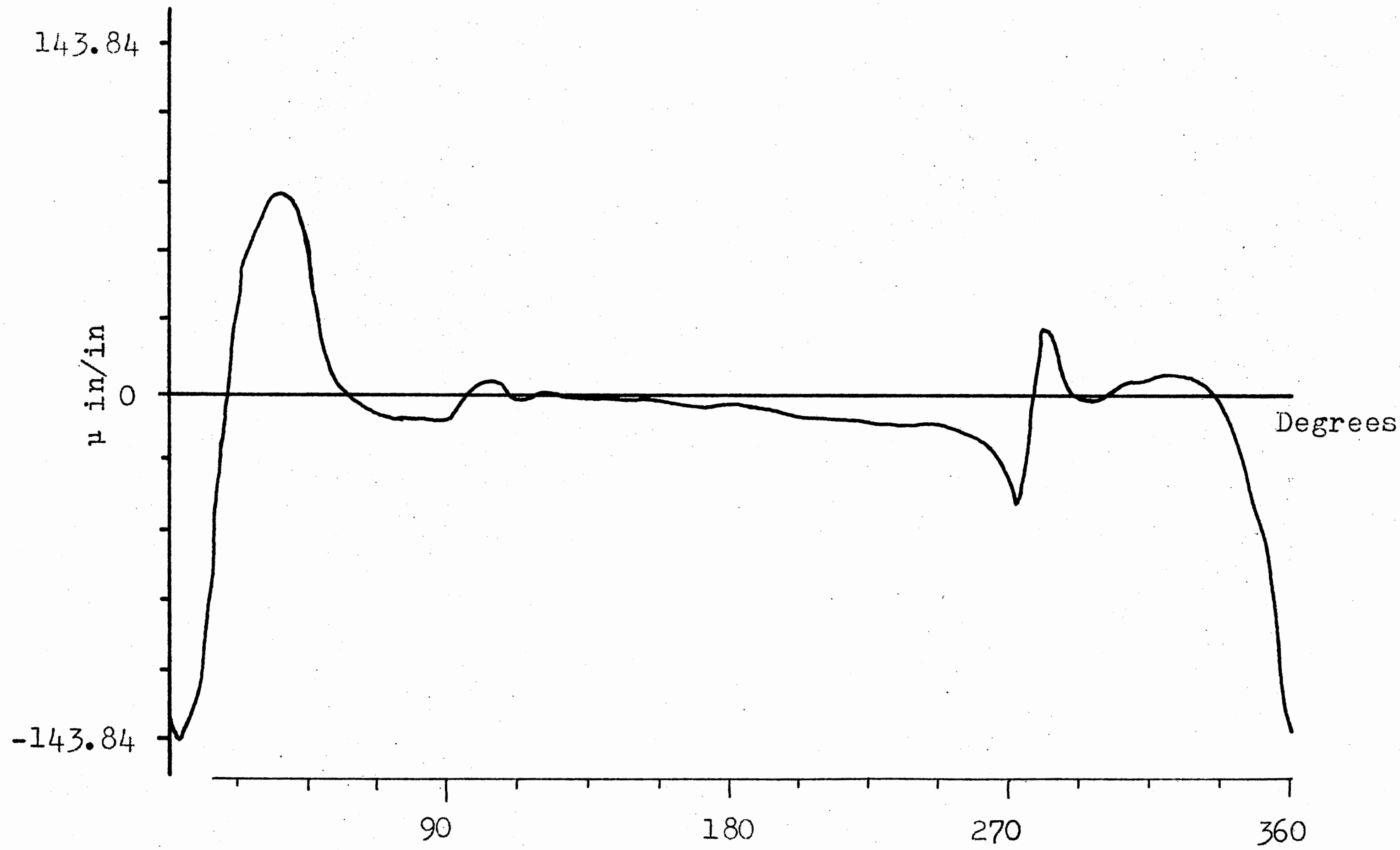


Figure 43. Bending Strains at the Center of the Coupler Link of the Four-Bar-Planar Mechanism at 373 rpm of the Crank.

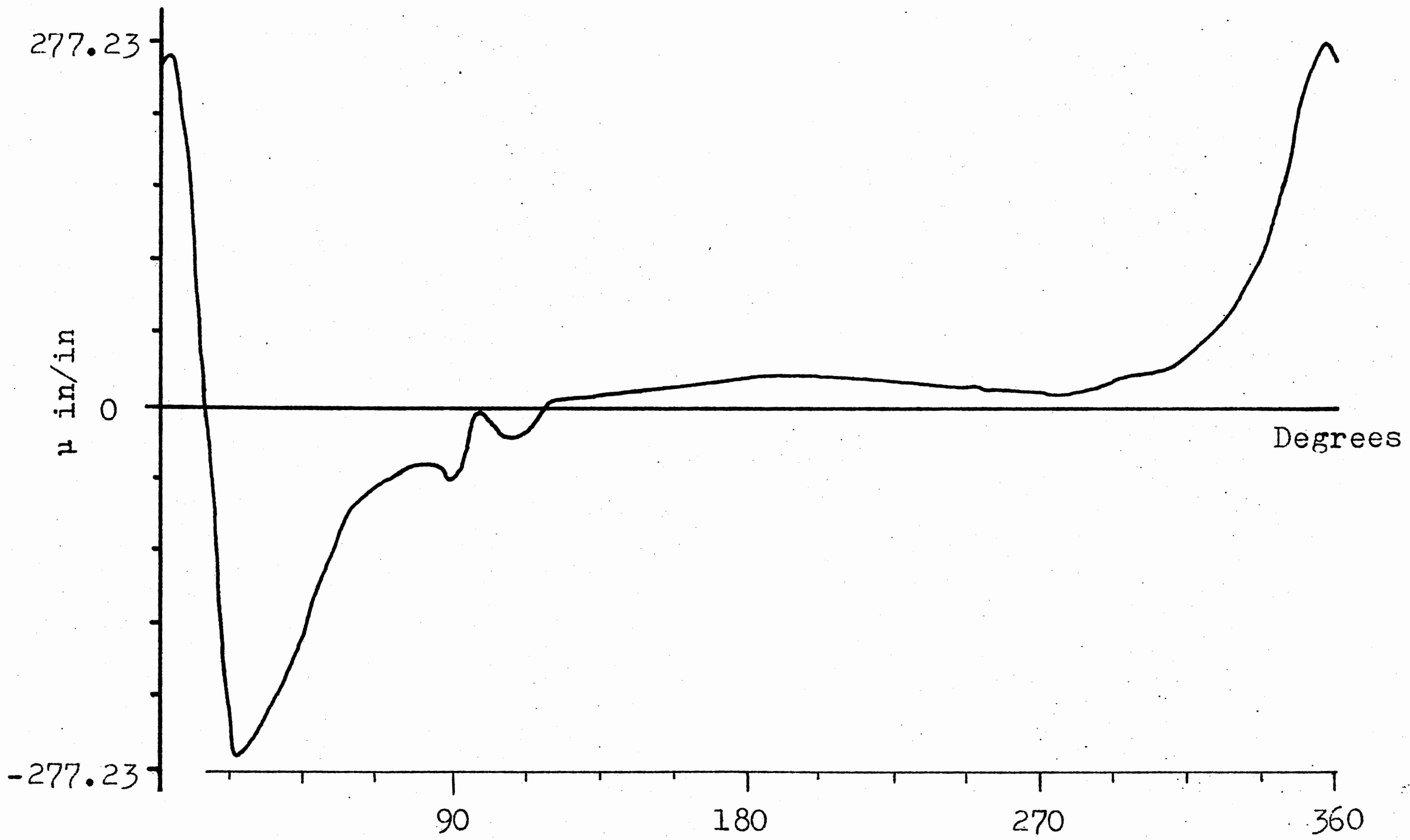


Figure 44. Bending Strains at the Center of the Output Link of the Four-Bar-Planar Mechanism at 373 rpm of the Crank

TABLE II

BENDING STRAINS AT THE CENTER OF THE LINKS
OF THE FOUR-BAR-PLANAR MECHANISM
AT 373 RPM OF THE CRANK

Crank (Degrees)	Input Link (μ in./in.)	Coupler Link (μ in./in.)	Output Link (μ in./in.)
0	277.072	-132.519	261.481
8	197.584	-113.038	75.360
16	29.568	-21.284	-174.031
24	-155.690	56.253	-265.965
32	-279.064	81.629	-238.412
40	-269.612	63.081	-188.416
48	-140.062	27.265	-136.177
56	3.211	2.175	-91.227
64	96.397	-7.803	-64.847
72	94.334	-9.898	-50.927
80	75.256	-10.189	-43.504
88	83.260	-23.851	-58.333
96	-2.198	0.530	-9.668
104	-38.356	4.435	-23.905
112	-70.001	-1.929	-18.604
120	-32.130	0.031	1.253
128	32.281	-0.185	5.329
136	-15.443	-0.465	6.812
144	-17.933	-0.773	10.206
152	-24.223	-1.374	13.709
160	-34.199	-2.217	16.888
168	-43.226	-3.341	19.617
176	-51.507	-4.661	21.878
184	-58.500	-6.100	23.302
192	-64.867	-7.621	23.752
200	-69.273	-9.090	23.432
208	-72.072	-10.304	22.486
216	-73.473	-11.292	21.032
224	-73.629	-12.039	19.379

TABLE II (Continued)

Crank (Degrees)	Input Link (μ in./in.)	Coupler Link (μ in./in.)	Output Link (in./in.)
232	-72.367	-12.430	17.799
240	-70.424	-12.585	16.365
248	-69.271	-12.775	15.218
256	-72.273	-13.676	14.481
264	-75.113	-19.464	14.337
272	-38.151	-44.667	10.805
280	-32.492	26.802	14.186
288	218.182	1.690	16.712
296	-68.759	-1.188	20.287
304	42.275	2.537	25.842
312	28.986	3.451	34.695
320	53.470	4.340	49.181
328	88.416	3.607	73.216
336	137.909	-3.551	113.341
344	203.977	-27.739	177.209
352	265.292	-79.748	255.296
360	277.072	-132.519	261.481

TABLE III

COORDINATES OF JOINT "A" OF THE FOUR-BAR-PLANAR MECHANISM IN RIGID AND DEFORMED STATES AT 373 RPM OF THE CRANK

Crank Position (Degrees)	Rigid Abscissa (10^{-4} ft)	Deformed Abscissa (10^{-4} ft)	Rigid Ordinate (10^{-4} ft)	Deformed Ordinate (10^{-4} ft)
0	5833.330	5833.330	0.000	-52.965
8	5776.560	5782.107	811.843	772.375
16	5607.360	5609.803	1607.880	1599.359
24	5329.020	5318.071	2372.630	2397.222
32	4946.950	4919.242	3091.200	3135.543
40	4468.590	4434.386	3749.590	3790.353
48	3903.260	2881.542	4335.010	4354.565
56	3261.960	3261.249	4836.050	4836.529
64	2557.170	2573.301	5242.970	5235.102
72	1802.600	1859.294	5547.830	5539.157
80	1012.950	1049.732	5744.710	5738.224
88	203.580	215.535	5829.780	5825.870
96	-609.749	-619.291	5801.380	5800.377
104	-1411.210	-1407.071	5660.060	5661.092
112	-2185.210	-2185.572	5408.570	5408.424
120	-2916.670	-2916.844	5051.810	5051.710
128	-3591.360	-3591.671	4796.730	4596.487
136	-4196.150	-4197.191	4052.170	4051.092
144	-4719.270	-4720.991	3428.750	3426.381
152	-5150.530	-5152.658	2738.580	2734.579
160	-5481.540	-5483.664	1995.120	1989.284
168	-5737.500	-5738.998	1053.030	1044.866
176	-5819.120	-5819.785	406.913	397.401
184	-5819.120	-5818.360	-406.913	-417.780
192	-5705.860	-5703.345	-1212.820	-1224.652
200	-5481.540	-5477.077	-1995.120	-2007.381
208	-5150.530	-5144.144	-2738.580	-2750.591
216	-4719.270	-4711.099	-3428.750	-3439.996
224	-4196.150	-4186.415	-4052.170	-4062.251

TABLE III (Continued)

Crank Position (Degrees)	Rigid Abscissa (10^{-4} ft)	Deformed Abscissa (10^{-4} ft)	Rigid Ordinate (10^{-4} ft)	Deformed Ordinate (10^{-4} ft)
232	-3591.360	-3580.457	-4596.730	-4605.249
240	-2916.670	-2904.952	-5051.810	-5058.575
248	-2185.210	-2172.804	-5408.570	-5413.582
256	-1411.210	-1397.567	-5660.060	-6553.462
264	-609.749	-590.161	-5801.380	-5803.439
272	203.580	198.033	-5829.780	-5830.672
280	1012.950	1006.357	-5744.710	-5745.873
288	1802.600	1801.475	-5547.830	-5548.195
296	2557.170	2555.909	-5242.970	-5243.585
304	3261.960	3259.320	-4836.050	-4837.831
312	3903.260	3898.983	-4335.010	-4338.861
320	4468.590	4462.410	-3749.590	-3756.955
328	4946.950	4938.683	-3091.200	-3104.429
336	5329.020	5318.969	-2372.630	-2395.205
344	5607.360	5597.164	-1607.880	-1643.437
352	5776.560	5769.729	-811.843	-860.446
360	5833.330	5833.330	0.000	-52.965

TABLE IV

COORDINATES OF JOINT "B" OF THE FOUR-BAR-PLANAR MECHANISM IN RIGID AND DEFORMED STATES AT 373 RPM OF THE CRANK

Crank Position (Degrees)	Rigid Abscissa (10^{-4} ft)	Deformed Abscissa (10^{-4} ft)	Rigid Ordinate (10^{-4} ft)	Deformed Ordinate (10^{-4} ft)
0	2916.670	2771.600	7806.250	7699.081
8	5050.570	4942.647	9113.470	9064.097
16	7156.710	7148.941	9795.920	9789.329
24	8709.420	8756.732	9989.540	9988.277
32	9621.990	9670.671	9989.630	9982.211
40	10021.300	10046.222	9963.410	9951.360
48	10051.000	10057.863	9960.820	9949.141
56	9819.230	9827.774	9978.690	9967.360
64	9401.230	9417.663	9997.250	9988.958
72	8850.100	8872.364	9994.990	9993.331
80	8205.240	8234.492	9953.680	9960.058
88	7497.680	7596.196	9859.740	9880.150
96	6753.080	6724.236	9704.360	9739.761
104	5993.400	5992.028	9483.160	9494.878
112	5237.790	5224.530	9195.860	9220.986
120	4502.880	4490.156	8845.850	8870.290
128	3802.910	3789.578	8439.800	8464.613
136	3149.610	3134.839	7987.180	8011.726
144	2552.010	2536.170	7499.750	7522.584
152	2016.260	1999.576	6990.830	7011.354
160	1545.550	1528.436	6424.450	6492.135
168	1066.650	1049.576	5864.270	5878.128
176	797.375	780.792	5473.110	5484.394
184	512.701	497.053	5010.880	5019.189
192	279.796	265.356	4585.140	4590.813
200	91.611	78.701	4200.400	4203.763
208	-58.979	-70.206	3858.430	3860.013
216	-178.619	-188.146	3558.880	3559.131
224	-273.142	-280.834	3300.000	3299.221

TABLE IV (Continued)

Crank Position (Degrees)	Rigid Abscissa (10^{-4} ft)	Deformed Abscissa (10^{-4} ft)	Rigid Ordinate (10^{-4} ft)	Deformed Ordinate (10^{-4} ft)
232	-347.425	-353.356	3079.290	3077.888
240	-405.380	-409.472	2894.120	2892.355
248	-450.029	-452.289	2742.110	2740.220
256	-483.608	-483.558	2621.490	2619.606
264	-507.648	-502.797	2531.330	2529.449
272	-523.025	-536.118	2471.820	2460.635
280	-529.941	-499.000	2444.550	2450.456
288	-527.799	-532.000	2453.030	2451.763
296	-514.902	-519.572	2503.440	2501.469
304	-487.810	-489.889	2605.970	2604.472
312	-440.019	-444.624	2776.980	2772.926
320	-359.140	-367.687	3042.860	3033.819
328	-220.687	-236.419	3446.390	3427.251
336	25.990	-3.603	4055.620	4016.917
344	487.585	430.698	4967.250	4895.337
352	1368.650	1264.247	6260.270	6150.851
360	2916.670	2771.600	7806.250	7699.081

TABLE V

DEFORMATIONS IN GLOBAL COORDINATES OF THE FOUR-BAR-PLANAR MECHANISM
AT 373 RPM OF THE INPUT LINK

Crank (Degrees)	q_1 (Rad)	q_2 (Rad/ft)	q_3 (Ft)	q_4 (Rad)	q_5 (Rad)	q_6 (Rad)	q_7 (Rad)	q_8 (Rad)
0	-0.8444E-2	-0.9481E-5	-0.5297E-2	-0.9801E-2	-0.3366E-1	-0.4485E-2	-0.2727E-2	0.3256E-1
8	-0.6968E-2	0.3731E-4	-0.3947E-2	-0.6665E-2	0.2660E-1	0.4144E-3	0.1254E-1	0.1382E-1
16	-0.2376E-2	0.2470E-4	-0.8521E-3	-0.5345E-3	0.3654E-2	0.2584E-2	0.2294E-1	-0.1463E-1
24	0.3446E-2	0.2140E-4	0.2459E-2	0.5957E-2	-0.1378E-1	0.1535E-2	0.2251E-1	-0.2654E-1
32	0.8024E-2	-0.2045E-4	0.4434E-2	0.1004E-1	-0.2058E-1	0.1988E-2	0.1951E-1	-0.2401E-1
40	0.9001E-2	-0.3248E-4	0.4076E-2	0.9264E-2	-0.1705E-1	0.3291E-2	0.1845E-1	-0.1797E-1
48	0.5428E-2	-0.1012E-4	0.1955E-2	0.4546E-2	-0.8335E-2	0.3266E-2	0.1534E-1	-0.1232E-1
56	0.5919E-3	-0.2195E-5	0.4793E-4	-0.3559E-3	-0.2148E-2	0.2883E-2	0.9930E-2	-0.8367E-2
64	-0.2722E-2	0.6170E-5	-0.7868E-3	-0.3485E-2	0.9069E-3	0.1959E-2	0.5376E-2	-0.6568E-2
72	-0.4560E-2	0.1219E-4	-0.8673E-3	-0.5115E-2	0.2580E-2	0.5473E-3	0.2343E-2	-0.5738E-2
80	-0.6223E-2	0.4008E-4	-0.6486E-3	-0.6640E-2	0.3781E-2	-0.7800E-3	-0.1399E-5	-0.5569E-2
88	0.3638E-2	-0.1010E+0	0.1527E-4	0.1016E-2	0.2982E-2	-0.5056E-2	0.4767E-2	0.2005E-2
96	0.3198E-2	-0.3320E-1	-0.1463E-4	0.3756E-3	0.4772E-2	-0.5675E-2	0.8334E-3	-0.2104E-2
104	0.9211E-3	-0.3366E-2	-0.1316E04	0.1101E02	0.5553E-2	-0.5867E02	-0.7922E-5	-0.1017E-2
112	0.2392E02	-0.1049E01	-0.1004E-4	0.1861E02	0.5907E02	-0.5511E-2	0.2472E-3	0.2428E-3
120	0.4569E-3	0.5246E-2	-0.9202E-5	0.6073E-3	0.5683E-2	-0.5708E-2	0.1806E-3	0.5065E-3
128	-0.7248E-3	-0.1759E-2	-0.2431E-4	-0.6290E-3	0.5929E-2	-0.5641E-2	-0.3006E-3	0.5819E-3
136	0.2393E-3	0.1160E-2	-0.1078E-3	0.4794E-3	0.6054E-2	-0.5767E-2	-0.5000E-3	0.7053E-3
144	0.4096E-3	0.3391E-3	-0.2369E-3	0.6452E-3	0.6087E-2	-0.5646E-2	-0.7199E-3	0.1067E-2
152	0.6303E-3	-0.2529E-4	-0.4001E-3	0.9072E-3	0.6058E-2	-0.5441E-2	-0.9036E-3	0.1454E-2
160	0.9108E-3	-0.1967E-4	-0.5836E-3	0.1253E-2	0.5990E-2	-0.5125E-2	-0.1068E-2	0.1858E-2
168	0.1177E-2	-0.5071E-5	-0.7780E-3	0.1578E-2	0.5910E-2	-0.4727E-2	-0.1214E-2	0.2106E-2
176	0.1435E-2	-0.4355E-5	-0.9512E-3	0.1868E-2	0.5790E-2	-0.4251E-2	-0.1368E-2	0.2345E-2
184	0.1660E-2	0.2686E-4	-0.1087E-2	0.2109E-2	0.5663E-2	-0.3754E-2	-0.1993E-2	0.2485E-2
192	0.1852E-2	0.2165E-4	-0.1183E-2	0.2334E-2	0.5548E-2	-0.3257E-2	-0.1552E-2	0.2523E-2
200	0.2016E-2	-0.1657E-4	-0.1226E-2	0.2486E-2	0.5392E-2	-0.2748E-2	-0.1571E-2	0.2476E-2
208	0.2124E-2	0.6818E-5	-0.1201E-2	0.2573E-2	0.5195E-2	-0.2270E-2	-0.1562E-2	0.2358E-2

TABLE V (Continued)

Crank (Degrees)	q_1 (Rad)	q_2 (Rad/ft)	q_3 (Ft)	q_4 (Rad)	q_5 (Rad)	q_6 (Rad)	q_7 (Rad)	q_8 (Rad)
216	0.2178E-2	0.1737E-4	-0.1125E-2	0.2618E-2	0.4971E-2	-0.1839E-2	-0.1510E-2	0.2189E-2
224	0.2224E-2	0.7921E-6	-0.1008E-2	0.2610E-2	0.4701E-2	-0.1423E-2	-0.1445E-2	0.1999E-2
232	0.2217E-2	-0.1950E-5	-0.8519E-3	0.2554E-2	0.4366E-2	-0.1038E-2	-0.1390E-2	0.1815E-2
240	0.2190E-2	-0.3428E-5	-0.6766E-3	0.2474E-2	0.4003E-2	-0.6862E-3	-0.1343E-2	0.1647E-2
248	0.2190E-2	-0.5024E-5	-0.5013E-3	0.2420E-2	0.3640E-2	-0.3316E-3	-0.1313E-2	0.1510E-2
256	0.2337E-2	-0.1063E-4	-0.3402E-3	0.2507E-2	0.3361E-2	0.1045E-3	-0.1306E-2	0.1418E-2
264	0.3323E-2	-0.5523E-4	-0.2059E-3	0.3454E-2	0.3653E-2	0.1059E-2	-0.1305E-2	0.1400E-2
272	0.3880E-2	-0.7580E-4	-0.6059E-3	0.8065E-2	0.7565E-2	0.1056E-2	-0.2025E-2	0.1405E-2
280	-0.7601E-2	0.7851E-1	-0.1163E-3	-0.6278E-5	0.5090E-2	0.6297E-2	-0.1924E-2	0.1175E-2
288	-0.3502E-2	-0.1822E-1	-0.3654E-4	-0.4937E-2	0.1034E-2	-0.8792E-3	-0.1983E-2	0.1478E-2
296	0.2171E-2	-0.5751E-2	-0.6148E-4	0.1521E-2	-0.8817E-3	-0.9997E-3	-0.2263E-2	0.1842E-2
304	-0.8658E-3	-0.1142E-2	-0.1781E-3	-0.1781E-3	-0.3202E-3	-0.4355E-3	-0.2818E-2	0.2368E-2
312	-0.7456E-3	0.7596E-3	-0.3851E-3	-0.1161E-2	-0.6450E-3	-0.9691E-3	-0.3678E-2	0.3215E-2
320	-0.1297E-2	0.1260E-3	-0.7365E-3	-0.2024E-2	-0.5597E-3	-0.1795E-2	-0.4952E-2	0.4644E-2
328	-0.2152E-2	-0.7790E-4	-0.1323E-2	-0.3302E-2	— 0.3297E-3	-0.3124E-2	-0.6766E-2	0.7117E-2
336	-0.3444E-2	-0.2863E-4	-0.2258E-2	-0.5141E-2	0.3012E-2	-0.5083E-2	-0.9065E-2	0.1149E-1
344	-0.5310E-2	-0.2685E-4	-0.3556E-2	-0.7527E-2	0.9751E-2	-0.7188E-2	-0.1115E-1	0.1897E-1
352	-0.7354E-2	-0.8469E-5	-0.4860E-2	-0.9635E-2	0.2239E-1	-0.7809E-2	-0.1062E-1	0.2914E-1
360	-0.8444E-2	-0.9481E-5	-0.5297E-2	-0.9801E-2	0.3366E-1	-0.4485E-2	-0.2727E-2	0.3256E-1

TABLE VI

VARIATION OF THE POSITION ANGLE OF THE INPUT LINK OF THE FOUR-BAR-PLANAR MECHANISM AT 373 RPM OF THE CRANK

Rigid Degrees	Actual Degrees	Rigid Degrees	Actual Degrees
0.00	-0.48	8.00	7.60
16.00	15.86	24.00	24.20
32.00	32.46	40.00	40.52
48.00	48.31	56.00	56.03
64.00	63.84	72.00	71.74
80.00	79.64	88.00	88.21
96.00	96.18	104.00	104.05
112.00	112.14	120.00	120.03
128.00	127.96	136.00	136.01
144.00	144.02	152.00	152.04
160.00	160.05	176.00	176.08
184.00	184.10	192.00	192.11
200.00	200.12	208.00	208.12
216.00	216.12	224.00	224.13
232.00	232.13	240.00	240.13
248.00	248.13	256.00	256.13
264.00	264.19	272.00	272.22
280.00	279.56	288.00	287.80
296.00	296.12	304.00	303.95
312.00	311.96	320.00	319.93
328.00	327.88	336.00	335.80
344.00	343.70	352.00	351.58
360.00	359.52		

occur between about -30 and 30 degrees of the input link. This pattern of behavior is confirmed by the rigid and experimental analysis.

Table VI shows the elastic variation of the position angle of the input link. The variation of the position angle of the crank, as every other degree of freedom, is measured from the rigid position of the links and joints. The input link moves from its rigid position up to almost one degree from its rigid position.

CHAPTER XI

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary and Conclusions

An investigation was undertaken to develop simplified, advanced, and general procedures for the synthesis and optimization of a rigid link mechanism and the application of the finite element method to the analysis of either rigid or elastic link mechanisms. The accomplishments of this investigation may be summarized as follows:

1. An analytic model consisting of a system of equations that represents the geometric configurations a mechanism forms during its motion is introduced for the synthesis of rigid-link-planar linkages. The method could be applied with or without optimization. The optimization constraints could be either functions, parameters, or inequalities. The geometric-analytic method that was introduced was successful by means of the calculated examples. Its applicability could be so extended to all planar and spherical mechanisms that general procedures might be established for their synthesis.

2. The dynamic analysis of rigid link mechanisms has been, for the first time, approached by the finite element method. A dynamic-rigid-planar element with nonuniform cross-sectional area was defined and proved to be suitable for the analysis of planar mechanisms. The investigation of a four-bar-planar mechanism was analyzed by considering its links to be formed by one, two, and three elements, respectively. An assembling

procedure for the elements of the mechanism is introduced and generalized. The method proved to be successful and it is believed that by defining other planar elements, the study of planar rigid link mechanisms could be dynamically studied by a general finite element approach. The method could well be extended to the analysis of space mechanisms.

3. The finite element method and Hamilton's principle were employed to derive a mathematical model of a dynamic planar element in motion, for the study of planar mechanisms. The model proved to be an effective and evolved tool in determining the deformations and elastic characteristics of mechanisms. The defined dynamic vibrating element, according to the literature reviewed, is shown to be the most advanced mathematical algorithm in comparison to the existing ones. It was also evidenced that solving the system of differential equations that results from the finite element method is a separate problem that deserves more investigation. The elastic responses of the mechanism with its eleven degrees of freedom element may be determined by direct numerical integration by using either higher computational precision like quadruple precision or by decoupling the system of linear second order ordinary differential equations or by employing a piecewise polynomial approach. The four-bar-planar mechanism was analyzed by conducting a reduction of coordinates to eliminate the high frequency motions associated with axial vibration of the links. The analytical results compared to the experimental results showed good compatibility which asseverates the validity of the analytical model. However, further refinements in the experimental model and in solving the system of differential equations are necessary to broaden and extend these conclusions.

4. A new procedure for determining strains, stress, and length of

the deformed element is introduced. This is achieved through the use of Equations (5.3), (6.17), and (8.8). Their applicability and versatility were demonstrated by means of the calculated example.

5. An experiment was conducted to obtain axial and bending strains at the middle of the coupler and output links of a four-bar-planar mechanism model. The general effects of linkage geometry, which is intimately related to the minimum transmission angle, the operating speeds, and the bearing system of the joints in motion, are clearly demonstrated. The experimental results show a poor correlation with those presented by the proposed theory. This poor correlation is attributed to the physical parameters that govern the behavior system, to a possible oversimplification in the mathematical model, and computational difficulties in obtaining a reasonably acceptable solution of the governing equations.

Recommendations

Based on the observations made during the course of this study, the following recommendations are made:

1. The procedure developed in the present study for the synthesis of rigid mechanisms should be extended so that a general method, with optimization, can be produced for the synthesis of planar and spherical mechanisms. The input method of constraints should provide facilities for those constraints to be parameters, functions, or inequalities.

2. The results of the previous recommendation will be useful in developing design charts for various situations which will eventually replace the existing ones that are based upon direct synthesis without optimization.

3. The application of the finite element method which has been introduced into the analysis of rigid mechanisms should be extended to the analysis of all planar and space mechanisms. This could be achieved by defining more appropriate elements and the generalization of assembling procedures. Methods like the Lagrange's equations, Hamilton's principle, dual numbers, and screw calculus may be employed for defining advanced elements.

4. The finite element technique described in this study for the analysis of elastic-element mechanisms should be extended to the analysis of space mechanisms. Elements with nonuniform cross-sectional area and more degrees of freedom should be defined. Among those degrees of freedom there should be included rotary inertia, shear deformation, and bearing friction. Gravity, external forces, and moments should be physical characteristics of the element.

5. It has been illustrated by the literature review and observations made in this study that solving the system of differential equations resulting from the finite element technique is a distinct topic. Therefore, it is highly recommended that investigations should be conducted to determine better methods or to improve and adjust existing ones. It should be taken into account that instability due to truncation and roundoff errors is the main problem, and it increases with the increase of degrees of freedom. Higher order computational precision is advised. The step size along the X or Y axis might have to be replaced by a step size along the curve.

6. The results of the previous recommendation will be useful in extending the finite element technique to the variable length element so that accuracy can be achieved efficiently. It will also facilitate the

extension of the method to the analysis of space mechanisms and the synthesis of elastic link mechanisms.

7. The synthesis of elastic link mechanisms by an iterative procedure might be established without much handicap via the use of the equations obtained in this study. However, this is a very costly and time consuming procedure. Instead, it is possible to use Equations (6.20) and (7.28) for optimization procedures.

8. More experimental work should be conducted. The conclusions generated in this study should be considered. Experimental equations could be developed that would serve for design purposes and for establishing comparisons with analytical models. The investigation should be extended to space mechanisms.

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APPENDIX A

KINETIC ENERGY FORMULA

We have that:

$$[(\dot{x}_1 + \dot{u}(x)) - w(x)\dot{\theta}]^2 = (\dot{x}_1 + \dot{u}(x))^2 - 2w(x)\ \dot{\theta}(\dot{x}_1 + \dot{u}(x))$$

$$+ (w(x))^2\ \dot{\theta}^2$$

$$= \dot{x}_1^2 + 2\dot{x}_1\dot{u}(x) + (\dot{u}(x))^2$$

$$- 2w(x)\ \dot{\theta}\dot{x}_1 - 2\dot{\theta}\dot{u}(x)\ w(x)$$

$$+ (w(x))^2\ \dot{\theta}^2 \tag{A.1}$$

$$[(\dot{y}_1 + x\dot{\theta}) + (u(x)\dot{\theta} + \dot{w}(x))]^2 = (\dot{y}_1 + x\dot{\theta})^2 + 2(\dot{y}_1 + x\dot{\theta})(u(x)\dot{\theta}$$

$$+ w(x)) + (u(x)\dot{\theta} + \dot{w}(x))^2$$

$$= \dot{y}_1^2 + 2\dot{y}_1 x\dot{\theta} + x^2\dot{\theta}^2$$

$$+ w\dot{y}_1 u(x) + 2\dot{y}_1\dot{w}(x)$$

$$+ 2xu(x)\dot{\theta}^2 + 2x\dot{\theta}\dot{w}(x)$$

$$+ (u(x))^2\dot{\theta}^2 + 2u(x)\dot{\theta}\ \dot{w}(x)$$

$$+ (\dot{w}(x))^2 \tag{A.2}$$

Hence:

$$T = \frac{1}{2}m\int_0^L [\dot{x}_1^2 + \dot{y}_1^2 + 2\dot{y}_1 x\dot{\theta} + x^2\dot{\theta}^2 + 2\dot{x}_1\dot{u}(x) + (\dot{u}(x))^2$$

$$- 2\dot{\theta}\dot{u}(x)\ w(x) - 2w(x)\dot{\theta}\ \dot{x}_1 + (w(x))^2\dot{\theta}^2 + 2\dot{y}_1 u(x)\dot{\theta}$$

$$+ 2\dot{y}_1\dot{w}(x) + 2xu(x)\dot{\theta}^2 + 2x\dot{\theta}w(x) + (u(x))^2\dot{\theta}^2$$

$$+ 2u(x)\dot{\theta}\dot{w}(x) + (\dot{w}(x))^2]dx \tag{A.3}$$

Carrying out the integration of each term, we have

$$\int_0^L \dot{x}_1^2 dx = L\dot{x}_1^2 \tag{A.4}$$

$$\int_0^L \dot{y}_1^2 dx = L\dot{y}_1^2 \tag{A.5}$$

$$\int_0^L 2\dot{y}_1\theta x dx = L^2\dot{y}_1\theta \tag{A.6}$$

$$\int_0^L \dot{\theta}^2 x^2 dx = L^3\dot{\theta}^2/3 \tag{A.7}$$

$$\int_0^L 2\dot{x}_1 u(x) dx = \int \ 2\dot{x}_1 (x)^T \ [D] \ (\dot{S}) dx \tag{A.8}$$

$$\int_0^L (\dot{u}(x))^2 dx = \int_0^L ((x)^T \ [D] \ (\dot{S}))^2 dx$$

$$\int_0^L (((x)^T \ [D] \ (\dot{S}))^T \ ((x)^T \ [D] \ (\dot{S}))) dx$$

$$\int_0^L ((\dot{S})^T \ [D] \ (x)(x)^T \ [D] \ (\dot{S})) dx \tag{A.9}$$

$$\int_0^L 2\dot{\theta}\dot{u}(x) \ w(x) dx = \int_0^L (2\dot{\theta}(x)^T \ [D] \ (\dot{S})(X)^T \ [D] \ (S)) dx$$

$$\int_0^L 2\dot{\theta}[(x)^T \ [D] \ (\dot{S})]^T \ (x)^T \ [D] \ (S) dx$$

$$\int_0^L 2\dot{\theta}(\dot{S})^T \ [D]^T \ (x)(X)^T \ [D] \ (S) dx \tag{A.10}$$

$$\int_0^L 2x_1\dot{\theta}w(x) dx = \int_0^L 2\dot{x}_1\dot{\theta}(X)^T \ [D] \ (S) dx \tag{A.11}$$

$$\int_0^L \dot{\theta}^2 (w(x))^2 dx = \int_0^L \dot{\theta}^2 [(X)^T \ [D] \ (S)]^2 dx$$

$$\int_0^L \dot{\theta}^2 [(X)^T \ [D] \ (S)]^T \ (X)^T \ [D] \ (S) dx$$

$$\int_0^L \dot{\theta}^2 (S)^T \ [D]^T \ (X)(X)^T \ [D] \ (S) dx \tag{A.12}$$

$$\int_0^L 2\dot{y}_1\dot{\theta}u(x) dx = \int_0^L 2\dot{y}_1\dot{\theta}(x)^T \ [D] \ (S) dx \tag{A.13}$$

$$\int_0^L 2\dot{y}_1\dot{w}(x) dx = \int_0^L 2\dot{y}_1 (X)^T \ [D] \ (\dot{S}) dx \tag{A.14}$$

$$\int_0^L 2\dot{\theta}^2 x u(x) dx = \int_0^L 2\dot{\theta}^2 x(x)^T \ [D] \ (S) dx \tag{A.15}$$

$$\int_0^L 2\dot{\theta} x\dot{w}(x) dx = \int_0^L 2\dot{\theta} x(X)^T \ [D] \ (\dot{S}) dx \tag{A.16}$$

$$\int_0^L \dot{\theta}^2 (u(x))^2 dx = \int_0^L \dot{\theta}^2 (S)^T \, [D]^T \, (x)(x)^T \, [D] \, (S) dx \qquad (A.17)$$

$$\int_0^L 2\dot{\theta} u(x) \, \dot{w}(x) dx = \int_0^L 2\dot{\theta} (x)^T \, [D] \, (S)(X)^T \, [D] \, (\dot{S}) dx$$

$$\int_0^L 2\dot{\theta} (S)^T \, [D]^T \, (x)(X)^T \, [D] \, (\dot{S}) dx \qquad (A.18)$$

$$\int_0^L (\dot{w}(x))^2 dx = \int_0^L (\dot{S})^T \, [D]^T \, (X)(X)^T \, [D] \, (\dot{S}) dx \qquad (A.19)$$

APPENDIX B

DEVELOPMENT AND INTEGRATION OF MATRICES

(E), [B], [A], (G), [M], (N), (L)

From Appendix A we have the following integrals:

$$(E)^T = \int_0^L (x)^T [D]dx \tag{B.1}$$

$$[B] = \int_0^L [D]^T (x)(x)^T [D]dx \tag{B.2}$$

$$[A] = \int_0^L [D]^T (x)(X)^T [D]dx \tag{B.3}$$

$$(G)^T = \int_0^L (X)^T [D]dx \tag{B.4}$$

$$[M] = \int_0^L [D]^T (X)(X)^T [D]dx \tag{B.5}$$

$$(N)^T = \int_0^L x(x)^T [D]dx \tag{B.6}$$

$$(L)^T = \int_0^L x(X)^T [D]dx \tag{B.7}$$

$$[D] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & F & 0 & 0 & 0 & 0 \\ -G & 0 & 0 & 0 & G & 0 & 0 & 0 \\ 0 & -H & -J & -K & 0 & H & -Z & B \\ 0 & C & D & U & 0 & -C & I & -P \\ 0 & -M & -Q & -N & 0 & M & -Q & N \end{bmatrix} \tag{B.8}$$

$F = \frac{1}{2}$ $\quad G = 1/L$ $\quad H = 10/L^3$ $\quad J = 6/L^2$ $\quad K = 3/2L$

$Z = 4/L^2$ $\quad B = 1/2L$ $\quad C = 15/L^4$ $\quad D = 8/L^3$ $\quad U = 3/2L^2$

$I = 7/L^3$ $\quad P = 1/L^2$ $\quad Q = 3/L^4$ $\quad N = 1/2L^3$ $\quad M = 6/L^5$

Matrix (E)

$$(\bar{E})^T = (1 \quad 0 \quad 0 \quad 0 \quad x \quad 0 \quad 0 \quad 0) \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & F & 0 & 0 & 0 & 0 \\ -G & 0 & 0 & 0 & G & 0 & 0 & 0 \\ 0 & -H & -J & -K & 0 & H & -Z & B \\ 0 & C & D & U & 0 & -C & I & -P \\ 0 & -M & -Q & -N & 0 & M & -Q & N \end{bmatrix} \tag{B.9}$$

$$(\bar{E})^T = ((1 - x/L) \quad 0 \quad 0 \quad 0 \quad x/L \quad 0 \quad 0 \quad 0) \tag{B.10}$$

$$(E)^T = \int_0^L (\bar{E})^T dx \tag{B.11}$$

$$(E)^T = (L/2 \quad 0 \quad 0 \quad 0 \quad L/2 \quad 0 \quad 0 \quad 0) \tag{B.12}$$

Matrix [B]

$$[\bar{B}] = \begin{bmatrix} 1 & 0 & 0 & 0 & -G & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -H & C & -M \\ 0 & 0 & 1 & 0 & 0 & -J & D & -Q \\ 0 & 0 & 0 & F & 0 & -K & U & -N \\ 0 & 0 & 0 & 0 & G & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & H & -C & M \\ 0 & 0 & 0 & 0 & 0 & -Z & I & -Q \\ 0 & 0 & 0 & 0 & 0 & B & -P & N \end{bmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ x \\ 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1-x/L \\ 0 \\ 0 \\ 0 \\ x/L \\ 0 \\ 0 \\ 0 \end{pmatrix} (1 \quad 0 \quad 0 \quad 0 \quad x \quad 0 \quad 0 \quad 0)$$

$$\rightarrow \begin{bmatrix} 1-x/L & 0 & 0 & 0 & x-x^2/L & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ x/L & 0 & 0 & 0 & x^2/L & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & F & 0 & 0 & 0 & 0 \\ -G & 0 & 0 & 0 & G & 0 & 0 & 0 \\ 0 & -H & -J & -K & 0 & H & -Z & B \\ 0 & C & D & U & 0 & -C & I & -P \\ 0 & -M & -Q & -N & 0 & M & -Q & N \end{bmatrix} =$$

$$
\begin{bmatrix}
(1 - 2x/L + x^2/L^2) & 0 & 0 & 0 & (x/L - x^2/L^2) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(x/L - x^2/L^2) & 0 & 0 & 0 & x^2/L^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
[B] = \int_0^L [\bar{B}]dx = m \begin{bmatrix}
L/3 & 0 & 0 & 0 & L/6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
L/6 & 0 & 0 & 0 & L/3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \quad (B.13)
$$

Matrix [A]

$$[\bar{A}] = \begin{bmatrix} 1 & 0 & 0 & 0 & -G & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -H & C & -M \\ 0 & 0 & 1 & 0 & 0 & -J & D & -Q \\ 0 & 0 & 0 & F & 0 & -K & U & -N \\ 0 & 0 & 0 & 0 & G & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & H & -C & M \\ 0 & 0 & 0 & 0 & 0 & -Z & I & -Q \\ 0 & 0 & 0 & 0 & 0 & B & -P & N \end{bmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ x \\ 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1-x/L \\ 0 \\ 0 \\ 0 \\ x/L \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & x & x^2 & 0 & x^3 & x^4 & x^5 \end{pmatrix} \rightarrow$$

$$\begin{bmatrix} 0 & (1-x/L) & (x-x^2/L) & (x^2-x^3/L) & 0 & (x^3-x^4/L) & (x^4-x^5/L) & (x^5-x^6/L) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & x/L & x^2/L & x^3/L & 0 & x^4/L & x^5/L & x^6/L \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & F & 0 & 0 & 0 & 0 \\ -G & 0 & 0 & 0 & G & 0 & 0 & 0 \\ 0 & -H & -J & -K & 0 & H & -Z & B \\ 0 & C & D & U & 0 & -C & I & -P \\ 0 & -M & -Q & -N & 0 & M & -Q & N \end{bmatrix}$$

$$
\begin{bmatrix}
0 & Y1 & Y2 & Y3 & 0 & Y4 & Y5 & Y6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & Z1 & Z2 & Z3 & 0 & Z4 & Z5 & Z6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$Y1 = 1 - x/L - 10\ x^3/L^3 + 25\ x^4/L^4 - 21\ x^5/L5 + 6\ x^6/L6$$

$$Y2 = x - x^2/L - 6\ x^3/L^2 + 14\ x^4/L^3 - 11\ x^5/L^4 + 3\ x^6/L^5$$

$$Y3 = x^2/2 - 2\ x^3/L + 3\ x^4/L^2 - 2\ x^5/L^3 + x^6/2L^4$$

$$Y4 = 10\ x^3/L^3 - 25\ x^4/L^4 + 21\ x^5/L^5 - 6\ x^6/L^6$$

$$Y5 = -4\ x^3/L^2 + 11\ x^4/L^3 - 10\ x^5/L^4 + 3\ x^5/L^5$$

$$Y6 = x^3/2L - 3x^4/2L^2 + 3\ x^5/2L^3 - x^6/2L^4$$

$$Z1 = x/L - 10\ x^4/L^4 + 15\ x^5/L5 - 6\ x^6/L^6 \qquad Z2 = x^2/L - 6\ x^4/L^3 + 8\ x^5/L^4 - 3\ x^6/L^5$$

$$Z3 = x^3/2L - 3\ x^4/2L^2 + 3\ x^5/2L^3 - x^6/2L^4 \qquad Z4 = 10\ x^4/L^4 - 15\ x^5/L^5 + 6\ x^6/L^6$$

$$Z5 = -4\ x^4/L^3 + 7\ x^5/L^4 - 3\ x^6/L^5 \qquad Z6 = x^4/2L^2 - x^5/L^3 + x^6/2L^4$$

$$
[A] = \int_0^L [\bar{A}]dx =
\begin{bmatrix}
0 & 5\ L/14 & 13\ L^2/210 & L^3/210 & 0 & L/7 & -4\ L^2/105 & L^3/280 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & L/7 & 4\ L^2/280 & L^3/280 & 0 & 5\ L/14 & -13\ L^2/210 & L^3/210 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\qquad (B.14)
$$

$$[A]^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5\,L/14 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 13\,L^2/210 & 0 & 0 & 0 & 4\,L^2/105 & 0 & 0 & 0 \\ L^3/210 & 0 & 0 & 0 & L^3/280 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ L/7 & 0 & 0 & 0 & 5\,L/14 & 0 & 0 & 0 \\ -4\,L^2/105 & 0 & 0 & 0 & -13\,L^2/210 & 0 & 0 & 0 \\ L^3/280 & 0 & 0 & 0 & L^3/210 & 0 & 0 & 0 \end{bmatrix} \tag{B.15}$$

Matrix (G)

$$(\bar{G})^T = (0 \quad 1 \quad x \quad x^2 \quad 0 \quad x^3 \quad x^4 \quad x^5) \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & F & 0 & 0 & 0 & 0 \\ -G & 0 & 0 & 0 & G & 0 & 0 & 0 \\ 0 & -H & -J & -K & 0 & H & -Z & B \\ 0 & C & D & U & 0 & -C & I & -P \\ 0 & -M & -Q & -N & 0 & M & -Q & N \end{bmatrix}$$

$$= (0 \quad X1 \quad X2 \quad X3 \quad 0 \quad X4 \quad X5 \quad X6)$$

$$X1 = 1 - 10\ x^3/L^3 + 15\ x^4/L^4 - 6\ x^5/L^5 \qquad X2 = x - 6\ x^3/L^2 + 8\ x^4/L^3 - 3\ x^5/L^4$$

$$X3 = x^2/2 - 3\ x^3/2L + 3\ x^4/2L^2 - x^5/2L^3 \qquad X4 = 10\ x^3/L^3 - 15\ x^4/L^4 + 6\ x^5/L^5$$

$$X5 = -4\ x^3/L^2 + 7\ x^4/L^3 - 3\ x^5/L^4 \qquad X6 = x^3/2L - x^4/L^2 + x^5/2L^3$$

$$(G)^T = \int_0^L (\bar{G})^T dx = (0 \quad L/2 \quad L^2/10 \quad L^3/120 \quad 0 \quad L/2 \quad -L^2/10 \quad L^3/120) \ . \qquad (B.16)$$

Matrix [M]

$$[\bar{M}] = \begin{bmatrix} 1 & 0 & 0 & 0 & -G & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -H & C & -M \\ 0 & 0 & 1 & 0 & 0 & -J & D & -Q \\ 0 & 0 & 0 & F & 0 & -K & U & -N \\ 0 & 0 & 0 & 0 & G & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & H & -C & M \\ 0 & 0 & 0 & 0 & 0 & -Z & I & -Q \\ 0 & 0 & 0 & 0 & 0 & B & -P & N \end{bmatrix} \begin{pmatrix} 0 \\ 1 \\ x \\ x^2 \\ 0 \\ x^3 \\ x^4 \\ x^5 \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} 0 \\ 1 - 10\, x^3/L^3 + 15\, x^4/L^4 - 6\, x^5/L^5 \\ x - 6\, x^3/L^2 + 8\, x^4/L^3 - 3\, x^5/L^4 \\ x^2/2 - 3\, x^3/2L + 3\, x^4/2L^2 - x^5/2L^3 \\ 0 \\ 10\, x^3/L^3 - 15\, x^4/L^4 + 6\, x^5/L^5 \\ -4\, x^3/L^2 + 7\, x^4/L^3 - 3\, x^5/L^4 \\ x^3/2L - x^4/L^2 + x^5/2L^3 \end{pmatrix} (0 \quad 1 \quad x \quad x^2 \quad 0 \quad x^3 \quad x^4 \quad x^5) \rightarrow$$

$$
\rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & T1 & T2 & T3 & 0 & T4 & T5 & T6 \\ 0 & S1 & S2 & S3 & 0 & S4 & S5 & S6 \\ 0 & R1 & R2 & R3 & 0 & R4 & R5 & R6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I1 & I2 & I3 & 0 & I4 & I5 & I6 \\ 0 & J1 & J2 & J3 & 0 & J4 & J5 & J6 \\ 0 & K1 & K2 & K3 & 0 & K4 & K5 & K6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & F & 0 & 0 & 0 & 0 \\ -G & 0 & 0 & 0 & G & 0 & 0 & 0 \\ 0 & -H & -J & -K & 0 & H & -Z & B \\ 0 & C & D & U & 0 & -C & I & -P \\ 0 & -M & -Q & -N & 0 & M & -Q & N \end{bmatrix}
$$

$$
= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Y1 & Y2 & Y3 & 0 & Y4 & Y5 & Y6 \\ 0 & X1 & X2 & X3 & 0 & X4 & X5 & X6 \\ 0 & Z1 & Z2 & Z3 & 0 & Z4 & Z5 & Z6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A1 & A2 & A3 & 0 & A4 & A5 & A6 \\ 0 & B1 & B2 & B3 & 0 & B4 & B5 & B6 \\ 0 & C1 & C2 & C3 & 0 & C4 & C5 & C6 \end{bmatrix} = [\bar{M}]
$$

$$T1 = 1 - 10\,x^3/L^3 + 15\,x^4/L^4 - 6\,x^5/L^5$$
$$T2 = x - 10\,x^4/L^3 + 15\,x^5/L^4 - 6\,x^6/L^5$$
$$T3 = x^2 - 10\,x^5/L^3 + 15\,x^6/L^4 - 6\,x^7/L^5$$
$$T4 = x^3 - 10\,x^6/L^3 + 15\,x^7/L^4 - 6\,x^8/L^5$$
$$T5 = x^4 - 10\,x^7/L^3 + 15\,x^8/L^4 - 6\,x^9/L^5$$
$$T6 = x^5 - 10\,x^8/L^3 + 15\,x^9/L^4 - 6\,x^{10}/L^5$$

$$S1 = x - 6\,x^3/L^2 + 8\,x^4/L^3 - 3\,x^5/L^4$$
$$S2 = x^2 - 6\,x^4/L^2 + 8\,x^5/L^3 - 3\,x^6/L^4$$
$$S3 = x^3 - 6\,x^5/L^2 + 8\,x^6/L^3 - 3\,x^7/L^4$$
$$S4 = x^4 - 6\,x^6/L^2 + 8\,x^7/L^3 - 3\,x^8/L^4$$
$$S5 = x^5 - 6\,x^7/L^2 + 8\,x^8/L^3 - 3\,x^9/L^4$$
$$S6 = x^6 - 6\,x^8/L^2 + 8\,x^9/L^3 - 3\,x^{10}/L^4$$

$$R1 = x^2/2 - 3\,x^3/2L + 3\,x^4/2L^2 - x^5/2L^3$$
$$R2 = x^3/2 - 3\,x^4/2L + 3\,x^5/2L^2 - x^6/2L^3$$
$$R3 = x^4/2 - 3\,x^5/2L + 3\,x^6/2L^2 - x^7/2L^3$$
$$R4 = x^5/2 - 3\,x^6/2L + 3\,x^7/2L^2 - x^8/2L^3$$
$$R5 = x^6/2 - 3\,x^7/2L + 3\,x^8/2L^2 - x^9/2L^3$$
$$R6 = x^7/2 - 3\,x^8/2L + 3\,x^9/2L^2 - x^{10}/2L^3$$

$$I1 = 10\,x^3/L^3 - 15\,x^4/L^4 + 6\,x^5/L^5$$
$$I2 = 10\,x^4/L^3 - 15\,x^5/L^4 + 6\,x^6/L^5$$
$$I3 = 10\,x^5/L^3 - 15\,x^6/L^4 + 6\,x^7/L^5$$
$$I4 = 10\,x^6/L^3 - 15\,x^7/L^4 + 6\,x^8/L^5$$
$$I5 = 10\,x^7/L^3 - 15\,x^8/L^4 + 6\,x^9/L^5$$
$$I6 = 10\,x^8/L^3 - 15\,x^9/L^4 + 6\,x^{10}/L^5$$

$$J1 = -4\,x^3/L^2 + 7\,x^4/L^3 - 3\,x^5/L^4$$
$$J2 = -4\,x^4/L^2 + 7\,x^5/L^3 - 3\,x^6/L^4$$
$$J3 = -4\,x^5/L^2 + 7\,x^6/L^3 - 3\,x^7/L^4$$
$$J4 = -4\,x^6/L^2 + 7\,x^7/L^3 - 3\,x^8/L^4$$
$$J5 = -4\,x^7/L^2 + 7\,x^8/L^3 - 3\,x^9/L^4$$
$$J6 = -4\,x^8/L^2 + 7\,x^9/L^3 - 3\,x^{10}/L^4$$

$$K1 = x^3/2L - x^4/L^2 + x^5/2L^3$$
$$K2 = x^4/2L - x^5/L^2 + x^6/2L^3$$
$$K3 = x^5/2L - x^6/L^2 + x^7/2L^3$$
$$K4 = x^6/2L - x^7/L^2 + x^8/2L^3$$
$$K5 = x^7/2L - x^8/L^2 + x^9/2L^3$$
$$K6 = x^8/2L - x^9/L^2 + x^{10}/2L^3$$

$$Y1 = 1 - 20x^3/L^3 + 30x^4/L^4 - 12x^5/L^5 + 100x^6/L^6 - 300x^7/L^7 + 345x^8/L^8 - 180x^9/L^9 + 36x^{10}/L^{10}$$

$$Y2 = x - 6x^3/L^2 - 2x^4/L^3 + 12x^5/L^4 + 54x^6/L^5 - 170x^7/L^6 + 186x^8/L^7 - 93x^9/L^8 + 18x^{10}/L^9$$

$$Y3 = x^2/2 - 3x^3/2L + 3x^4/2L^2 - 11x^5/2L^3 + 45x^6/2L^4 - 81x^7/2L^5 + 73x^8/2L^6 - 33x^9/2L^7 + 3x^{10}/L^8$$

$$Y4 = 10x^3/L^3 - 15x^4/L^4 + 6x^5/L^5 - 100x^6/L^6 + 300x^7/L^7 - 345x^8/L^8 + 180x^9/L^9 - 36x^{10}/L^{10}$$

$$Y5 = -4x^3/L^2 + 7x^4/L^3 - 3x^5/L^4 + 40x^6/L^5 - 130x^7/L^6 + 159x^8/L^7 - 87x^9/L^8 + 18x^{10}/L^9$$

$$Y6 = x^3/2L - x^4/L^2 + x^5/2L^3 - 5x^6/L^4 + 35x^7/2L^5 - 23x^8/L^6 + 27x^9/2L^7 - 3x^{10}/L^8$$

$$X1 = x - 6x^3/L^2 - 2x^4/L^3 + 12x^5/L^4 + 54x^6/L^5 - 170x^7/L^6 + 186x^8/L^7 - 93x^9/L^8 + 18x^{10}/L^9$$

$$X2 = x^2 - 12x^4/L^2 + 16x^5/L^3 + 30x^6/L^4 - 96x^7/L^5 + 100x^8/L^6 - 48x^9/L^7 + 9x^{10}/L^8$$

$$X3 = x^3/2 - 3x^4/2L - 3x^5/2L^2 + 25x^6/2L^3 - 45x^7/2L^4 + 39x^8/2L^5 - 17x^9/2L^6 + 3x^{10}/2L^7$$

$$X4 = 10x^4/L^3 - 15x^5/L^4 - 54x^6/L^5 + 170x^7/L^6 - 186x^8/L^7 + 93x^9/L^8 - 18x^{10}/L^9$$

$$X5 = -4x^4/L^2 + 7x^5/L^3 + 21x^6/L^4 - 74x^7/L^5 + 86x^8/L^6 - 45x^9/L^7 + 9x^{10}/L^8$$

$$X6 = x^4/2L - x^5/L^2 - 5x^6/2L^3 + 20x^7/2L^4 - 25x^8/2L^5 + 14x^9/2L^6 - 3x^{10}/2L^7$$

$$Z1 = x^2/2 - 3x^3/2L + 3x^4/2L^2 - 11x^5/2L^3 + 45x^6/2L^4 - 81x^7/2L^5 + 73x^8/2L^6 - 33x^9/2L^7 + 6x^{10}/2L^8$$

$$Z2 = x^3/2 - 3x^4/2L - 3x^5/2L^2 + 25x^6/2L^3 - 45x^7/2L^4 + 39x^8/2L^5 - 17x^9/2L^6 + 3x^{10}/2L^7$$

$$Z3 = x^4/4 - 6x^5/4L + 15x^6/4L^2 - 20x^7/4L^3 + 15x^8/4L^4 - 6x^9/4L^5 + x^{10}/4L^6$$

$$Z4 = 10x^5/2L^3 - 45x^6/2L^4 + 81x^7/2L^5 - 73x^8/2L^6 + 33x^9/2L^7 - 6x^{10}/2L^8$$

$$Z5 = -4x^5/2L^2 + 19x^6/2L^3 - 36x^7/2L^4 + 34x^8/2L^5 - 16x^9/2L^6 + 3x^{10}/2L^7$$

$$Z6 = x^5/4L - 5x^6/4L^2 + 10x^7/4L^3 - 10x^8/4L^4 + 5x^9/4L^5 - x^{10}/4L^6$$

$$A1 = 10x^3/L^3 - 15x^4/L^4 + 6x^5/L^5 - 100x^6/L^6 + 300x^7/L^7 - 345x^8/L^8 + 180x^9/L^9 - 36x^{10}/L^{10}$$

$$A2 = 10x^4/L^3 - 15x^5/L^4 - 54x^6/L^5 + 170x^7/L^6 - 186x^8/L^7 + 93x^9/L^8 - 18x^{10}/L^9$$

$$A3 = 10x^5/2L^3 - 45x^6/2L^4 + 81x^7/2L^5 - 73x^8/2L^6 + 33x^9/2L^7 - 6x^{10}/2L^8$$

$$A4 = 100x^6/L^6 - 300x^7/L^7 + 345x^8/L^8 - 180x^9/L^9 + 36x^{10}/L^{10}$$

$$A5 = -40x^6/L^5 + 130x^7/L^6 - 159x^8/L^7 + 87x^9/L^8 - 18x^{10}/L^9$$

$$A6 = 10x^6/2L^4 - 35x^7/2L^5 + 46x^8/2L^6 - 27x^9/2L^7 + 6x^{10}/2L^8$$

$$B1 = -4x^3/L^2 + 7x^4/L^3 - 3x^5/L^4 + 40x^6/L^5 - 130x^7/L^6 + 159x^8/L^7 - 87x^9/L^8 + 18x^{10}/L^9$$

$$B2 = -4x^4/L^2 + 7x^5/L^3 + 21x^6/L^4 - 74x^7/L^5 + 86x^8/L^6 - 45x^9/L^7 + 9x^{10}/L^8$$

$$B3 = -4x^5/2L^2 + 19x^6/2L^3 - 36x^7/2L^4 + 34x^8/2L^5 - 16x^9/2L^6 + 3x^{10}/2L^7$$

$$B4 = -40x^6/L^5 + 130x^7/L^6 - 159x^8/L^7 + 87x^9/L^8 - 18x^{10}/L^9$$

$$B5 = 16x^6/L^4 - 56x^7/L^5 + 73x^8/L^6 - 42x^9/L^7 + 9x^{10}/L^8$$

$$B6 = -4x^6/2L^3 + 15x^7/2L^4 - 21x^8/2L^5 + 13x^9/2L^6 - 3x^{10}/2L^7$$

$$C1 = x^3/2L - x^4/L^2 + x^5/2L^3 - 10x^6/2L^4 + 35x^7/2L^5 - 46x^8/2L^6 + 27x^9/2L^7 - 6x^{10}/2L^8$$

$$C2 = x^4/2L - x^5/L^2 - 5x^6/2L^3 + 20x^7/2L^4 - 25x^8/2L^5 + 14x^9/2L^6 - 3x^{10}/2L^7$$

$$C3 = x^5/4L - 5x^6/4L^2 + 10x^7/4L^3 - 10x^8/4L^4 + 5x^9/4L^5 - x^{10}/4L^6$$

$$C4 = 10x^6/2L^4 - 35x^7/2L^5 + 46x^8/2L^6 - 27x^9/2L^7 + 6x^{10}/2L^8$$

$$C5 = -4x^6/2L^3 + 15x^7/2L^4 - 21x^8/2L^5 + 13x^9/2L^6 - 3x^{10}/2L^7$$

$$C6 = x^6/4L^2 - 2x^7/2L^3 + 6x^8/4L^4 - 2x^9/2L^5 + x^{10}/4L^6$$

$$[M] = \int_0^L [\bar{M}]dx$$

$$[M] = m \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A & B & C & 0 & D & -E & F \\ 0 & B & G & I & 0 & E & -K & L \\ 0 & C & I & H & 0 & F & -L & J \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & D & E & F & 0 & A & -B & C \\ 0 & -E & -K & -L & 0 & -B & G & -I \\ 0 & F & L & J & 0 & C & -I & H \end{bmatrix} \qquad (B.17)$$

$A = 181\ L/462$ $\quad B = 311\ L^2/4620$ $\quad C = 281\ L^3/55440$ $\quad D = 25\ L/231$ $\quad E = 151\ L^2/4620$

$F = 181\ L^3/55440$ $\quad G = 52\ L^3/3465$ $\quad H = L^5/9240$ $\quad I = 23\ L^4/18480$ $\quad K = 19\ L^3/1980$

$L = 13\ L^4/13860$ $\quad J = L^5/11088$

Matrix (N)

$$(N)^T = \int_0^L x(\bar{E})dx = \int_0^L ((x - x^2/L) \quad 0 \quad 0 \quad 0 \quad x^2/L \quad 0 \quad 0 \quad 0)\ dx \tag{B.18a}$$

$$(N)^T = (L^2/6 \quad 0 \quad 0 \quad 0 \quad L^2/3 \quad 0 \quad 0 \quad 0) \tag{B.18b}$$

Matrix (L)

$$(L) = \int_0^L x(G)^T dx = \int_0^L \begin{pmatrix} 0 \\ x - 10x^4/L^3 + 15x^5/L^4 - 6x^6/L5 \\ x^2 - 6x^4/L^2 + 8x^5/L^3 - 3x^6/L^4 \\ x^3/2 - 3x^4/2L + 3x^5/2L^2 - x^6/2L^3 \\ 0 \\ 10x^4/L^3 - 15x^5/L^4 + 6x^6/L^5 \\ -4x^4/L^2 + 7x^5/L^3 - 3x^6/L^4 \\ x^4/2L - x^5/L^2 + x^6/2L^3 \end{pmatrix} dx$$

$$(L)^T = (0 \quad L^2/7 \quad 4L^3/105 \quad L^4/280 \quad 0 \quad 5L^2/14 \quad -13L^3/210 \quad L^4/210) \tag{B.19}$$

APPENDIX C

MODE FUNCTIONS FOR AXIAL AND TRANSVERSE DEFORMATIONS

Axial Deformation

$$u(x,t) = \phi_1(x)\ q_1(t) + \phi_5(x)\ q_5(t) \tag{C.1}$$

Since:

$$u(0,t) = q_1(t) \quad \text{at } x = 0 \tag{C.2}$$

$$u(L,t) = q_5(t) \quad \text{at } x = L \tag{C.3}$$

hence:

$$\phi_1(0) = 1 \qquad \phi_5(0) = 0 \quad \text{at } x = 0 \tag{C.4}$$

$$\phi_1(L) = 0 \qquad \phi_5(L) = 1 \quad \text{at } x = L \tag{C.5}$$

From Equation (6.2), we have

$$u(x) = B_0 + B_1 x \tag{C.6}$$

where, by inserting the boundary conditions, it gives:

$$\phi_1(0) = B_0 = 1 \tag{C.7}$$

$$\phi_1(L) = B_0 = B_1 L = 0 \tag{C.8}$$

hence:

$$B_0 = 1, \quad B_1 = -1/L \tag{C.9}$$

hence:

$$\phi_1(x) = 1 - x/L \tag{C.10}$$

For mode function $\phi_5(x)$, we have:

$$\phi_5(0) = B_0 = 0 \tag{C.11}$$

$$\phi_5(L) = B_0 + B_1 L = 1 \tag{C.12}$$

$$\phi_5(x) = x/L \tag{C.13}$$

Transverse Deformations

$$w(x,t) = \phi_2(x)\ q_2(t) + \phi_3(x)\ q_3(t) + \phi_4(x)\ q_4(t) + \phi_6(x)\ q_6(t)$$
$$+ \phi_7(x)\ q_7(t) + \phi_8(x)\ q_8(t) \tag{C.14}$$

From Figure 9 (page 43), we have:

$w(0,t) = q_2(t)$ $\quad$ $\delta w(0,t)/\delta x = q_3(t)$

$w(L,t) = q_6(t)$ $\quad$ $\delta w^2(0,t)/\delta x^2 = q_4(t)$

$\delta w(L,t)/\delta x = q_7(t)$ $\quad$ $\delta w^2(L,t)/\delta x^2 = q_8(t)$

Hence, the mode function $\phi_2(x)$ must satisfy the boundary conditions:

$\phi_2(0) = 1$ $\quad$ $d\phi_2(0)/dx = 0$ $\quad$ $d^2\phi_2(0)/dx^2 = 0$

$\phi_2(L) = 0$ $\quad$ $d\phi_2(L)/dx = 0$ $\quad$ $d^2\phi_2(L)/dx^2 = 0$

Introducing this information into the following equation:

$$w(x) = A_0 + A_1x + A_2x^2 + A_3x^3 + A_4x^4 + A_5x^5 \tag{C.15}$$

and by solving the system of six equations, we have:

$$\phi_2(x) = 1 - 10\ x^3/L^3 + 15\ x^4/L^4 - 6\ x^5/L^5 \tag{C.16}$$

The mode function $\phi_3(x)$ must also satisfy the boundary conditions:

$\phi_3(0) = 0$ $\quad$ $d\phi_3(0)/dx = 1$ $\quad$ $d^2\phi_3(0)/dx^2 = 0$

$\phi_3(L) = 0$ $\quad$ $d\phi_3(L)/dx = 0$ $\quad$ $d^2\phi_3(L)/dx^2 = 0$

hence, from the system of six equations, we have:

$$\phi_3(x) = x - 6\, x^3/L^2 + 8\, x^4/L^3 - 3\, x^5/L^4 \tag{C.17}$$

The mode function $\phi_4(x)$ must also satisfy the boundary conditions:

$$\phi_4(0) = 0 \qquad d\phi_4(0)/dx = 0 \qquad d^2\phi_4(0)/dx^2 = 1$$

$$\phi_4(L) = 0 \qquad d\phi_4(L)/dx = 0 \qquad d^2\phi_4(L)/dx^2 = 0$$

and

$$\phi_4(x) = x^2/2 - 3\, x^3/2L + 3\, x^4/2L^2 - x^5/2L^3 \tag{C.18}$$

The mode function $\phi_6(x)$ must also satisfy the boundary conditions:

$$\phi_6(0) = 0 \qquad d\phi_6(0)/dx = 0 \qquad d^2\phi_6(0)/dx^2 = 0$$

$$\phi_6(L) = 1 \qquad d\phi_6(L)/dx = 0 \qquad d^2\phi_6(L)/dx^2 = 0$$

and

$$\phi_6(x) = 10\, x^3/L^3 - 15\, x^4/L^4 + 6\, x^5/L^5 \tag{C.19}$$

The mode function $\phi_7(x)$ must also satisfy the boundary conditions:

$$\phi_7(0) = 0 \qquad d\phi_7(0)/dx = 0 \qquad d^2\phi_7(0)/dx^2 = 0$$

$$\phi_7(L) = 0 \qquad d\phi_7(L)/dx = 1 \qquad d^2\phi_7(L)/dx^2 = 0$$

and

$$\phi_7(x) = -4\, x^3/L^2 + 7\, x^4/L^3 - 3\, x^5/L^4 \tag{C.20}$$

The mode function $\phi_8(x)$ must also satisfy the boundary conditions:

$$\phi_8(0) = 0 \qquad d\phi_8(0)/dx = 0 \qquad d^2\phi_8(0)/dx^2 = 0$$

$$\phi_8(L) = 0 \qquad d\phi_8(L)/dx = 0 \qquad d^2\phi_8(L)/dx^2 = 1$$

and

$$\phi_8(x) = x^3/2L - x^4/L^2 + x^5/2L^3 \ . \tag{C.21}$$

APPENDIX D

POTENTIAL ENERGY FOR TRANSVERSE DEFORMATION

We have that:

$$w(x,t) = (1 - 10x^3/L^3 + 15x^4/L^4 - 6x^5/L^5)\ q_2(t) + (x - 6x^3/L^2 + 8x^4/L^3 - 3x^5/L^4)\ q_3(t)$$
$$+ (x^2/2 - 3x^3/2L + 3x^4/2L^2 - x^5/2L^3)\ q_4(t) + (10x^3/L^3 - 15x^4/L^4 - 6x^5/L^5)\ q_6(t)$$
$$+ (-4x^3/L^2 + 7x^4/L^3 - 3x^5/L^4)\ q_7(t) + (x^3/2L - x^4/L^2 + x^5/2L^3)\ q_8(t) \qquad \text{(D.1)}$$

Differentiating with respect to x, we have:

$$\delta w(x,t)/\delta x = (-30x^2/L^3 + 60x^3/L^4 - 30x^4/L^5)\ q_2(t) + (1 - 18x^2/L^2 + 32x^3/L^3 - 15x^4/L^4)\ q_3(t)$$
$$+ (x - 9x^2/2L + 12x^3/2L^2 - 5x^4/2L^3)\ q_4(t) + (30x^2/L^3 - 60x^3/L^4 + 30x^4/L^5)\ q_6(t)$$
$$+ (-12x^2/L^2 + 28x^3/L^3 - 15x^4/L^4)\ q_7(t) + (3x^2/2L - 4x^3/L^2 + 5x^4/2L^3)\ q_8(t) \qquad \text{(D.2)}$$

$$\delta w^2(x,t)/\delta x^2 = (-60x/L^3 + 180x^2/L^4 - 120x^3/L^5)\ q_2(t) + (-36x/L^2 + 96x^2/L^3 - 60x^3/L^4)\ q_3(t)$$
$$+ (1 - 9x/L + 18x^2/L^2 - 10x^3/L^3)\ q_2(t) + (60x/L^3 - 180x^2/L^4 + 120x^3/L^5)\ q_6(t)$$
$$+ (-24x/L^2 + 84x^2/L^3 - 60x^3/L^4)\ q_7(t) + (3x/L - 12x^2/L^2 + 10x^3/L^3)\ q_8(t) \qquad \text{(D.3)}$$

Ordering it gives:

$$\delta w^2(x,t)/\delta x^2 = q_4(t) + (-60\ q_2(t)/L^3 - 36\ q_3(t)/L^2 - 9\ q_4(t)/L + 60\ q_6(t)/L^3 - 24\ q_7(t)/L^2$$
$$+ 3\ q_8(t)/L)x + (180\ q_2(t)/L^4 + 96\ q_3(t)/L^3 + 18\ q_4(t)/L^2 - 180\ q_6(t)/L^4$$
$$+ 84\ q_7(t)/L^3 - 12\ q_8(t)/L^2)x^2 + (-120\ q_2(t)/L^5 - 60\ q_3(t)/L^4 - 10\ q_4(t)/L^3$$
$$+ 120\ q_6(t)/L^5 - 60\ q_7(t)/L^4 + 10\ q_8(t)/L^3)x^3 \qquad \text{(D.4)}$$

Let

$$a_0 = q_4 \quad \text{(D.5)}$$

$$a_1 = -60\, q_2/L^3 - 36\, q_3/L^2 - 9\, q_4/L + 60\, q_6/L^3 - 24\, q_7/L^2 + 3\, q_8/L \quad \text{(D.6)}$$

$$a_2 = 180\, q_2/L^4 + 96\, q_3/L^3 + 18\, q_4/L^2 - 180\, q_6/L^4 + 84\, q_7/L^3 - 12\, q_8/L^2 \quad \text{(D.7)}$$

$$a_3 = -120\, q_2/L^5 - 60\, q_3/L^4 - 10\, q_4/L^3 + 120\, q_6/L^5 - 60\, q_7/L^4 + 10\, q_8/L^3 \quad \text{(D.8)}$$

The coefficients of the product of two polynomials are:

$$D_0 = a_0^2 \quad \text{(D.9)}$$

$$D_1 = 2a_0a_1 \quad \text{(D.10)}$$

$$D_2 = 2a_0a_2 + a_1^2 \quad \text{(D.11)}$$

$$D_3 = 2a_0a_3 + 2a_1a_2 \quad \text{(D.12)}$$

$$D_4 = 2a_1a_3 + a_2^2 \quad \text{(D.13)}$$

$$D_5 = 2a_2a_3 \quad \text{(D.14)}$$

$$D_6 = a_3^2 \quad \text{(D.15)}$$

Hence:

$$D_0 = q_4^2(t) \quad \text{(D.16)}$$

$$D_1 = -120\ q_2q_4/L^3 - 72\ q_3q_4/L^2 - 18\ q_4^2/L + 120\ q_4q_6/L^3 - 48\ q_4q_7/L^2 + 6\ q_4q_8/L \qquad \text{(D.17)}$$

$$D_2 = 3600\ q_2^2/L^6 + 1296\ q_3^2/L^4 + 117\ q_4^2/L^2 + 3600\ q_6^2/L^6 + 576\ q_7^2/L^4 + 9\ q_8^2/L^2 + 4320\ q_2q_3/L^5$$

$$+ 1440\ q_2q_4/L^4 - 7200\ q_2q_6/L^6 + 2880\ q_2q_7/L^5 - 360\ q_2q_8/L^4 + 840\ q_3q_4/L^3 - 4320\ q_3q_6/L^5$$

$$+ 1728\ q_3q_7/L^4 - 216\ q_3q_8/L^3 - 1440\ q_4q_6/L^4 + 600\ q_4q_7/L^3 - 78\ q_4q_8/L^2 - 2880\ q_6q_7/L^5$$

$$+ 360\ q_6q_8/L^4 - 144\ q_7q_8/L^3 \qquad \text{(D.18)}$$

$$D_3 = -21600\ q_2^2/L^7 - 6912\ q_3^2/L^5 - 344\ q_4^2/L^3 - 21600\ q_6^2/L^7 - 4032\ q_7^2/L^5 - 72\ q_8^2/L^3$$

$$- 24480\ q_2q_3/L^6 - 5640\ q_2q_4/L^5 + 43200\ q_2q_6/L^7 - 18720\ q_2q_7/L^6 + 2520\ q_2q_8/L^5$$

$$- 3144\ q_3q_4/L^4 + 24480\ q_3q_6/L^6 - 10565\ q_3q_7/L^5 + 1440\ q_3q_8/L^4 + 5640\ q_4q_6/L^5$$

$$- 2496\ q_4q_7/L^4 + 344\ q_4q_8/L^3 + 18720\ q_6q_7/L^6 - 2520\ q_6q_8/L^5 + 1080\ q_7q_8/L^4 \qquad \text{(D.19)}$$

$$D_4 = 46800\ q_2^2/L^8 + 13536\ q_3^2/L^6 + 504\ q_4^2/L^4 + 46800\ q_6^2/L^8 + 9936\ q_7^2/L^6 + 204\ q_8^2/L^4$$

$$+ 50400\ q_2q_3/L^7 + 9840\ q_2q_4/L^6 - 93600\ q_2q_6/L^8 + 43200\ q_2q_7/L^7 - 6240\ q_2q_8/L^6$$

$$+ 5256\ q_3q_4/L^5 - 50400\ q_3q_6/L^7 + 23328\ q_3q_7/L^6 - 3384\ q_3q_8/L^5 - 9840\ q_4q_6/L^6$$

$$+ 4584\ q_4q_7/L^5 - 672\ q_4q_8/L^4 - 43200\ q_6q_7/L^7 - 6240\ q_6q_8/L^6 - 2856\ q_7q_8/L^5 \qquad \text{(D.20)}$$

$$D_5 = -43200\ q_2^2/L^9 - 11520\ q_3^2/L^7 - 360\ q_4^2/L^5 - 43200\ q_6^2/L^9 - 10080\ q_7^2/L^7 - 240\ q_8^2/L^5$$

$$- 44640\ q_2q_3/L^8 - 7920\ q_2q_4/L^7 + 86400\ q_2q_6/L^9 - 41760\ q_2q_7/L^8 + 6480\ q_2q_8/L^7$$

$$- 4080\ q_3q_4/L^6 + 44640\ q_3q_6/L^8 - 21600\ q_3q_7/L^7 + 3360\ q_3q_8/L^6 + 7920\ q_4q_6/L^7$$

$$- 3840\ q_7q_7/L^6 + 600\ q_4q_8/L^5 + 41760\ q_8q_7/L^8 - 6480\ q_6q_8/L^7 + 3120\ q_7q_8/L^6 \qquad (D.21)$$

$$D_6 = 14400\ q_2^2/L^{10} + 3600\ a_3^2/L^8 + 100\ q_4^2/L^6 + 14400\ q_6^2/L^{10} + 3600\ q_7^2/L^8 + 100\ q_8^2/L^6$$

$$+ 14400\ q_2q_3/L^9 + 2400\ q_2q_4/L^8 - 28800\ q_q{}^2{}_6/L^{10} + 14400\ q_2q_7/L^9 - 2400\ q_2q_8/L^8$$

$$+ 1200\ q_3q_4/L^7 - 14400\ q_3q_6/L^9 + 7200\ q_3q_7/L^8 - 1200\ q_3q_8/L^7 - 2400\ q_4q_6/L^8$$

$$+ 1200\ q_4q_7/L^7 - 200\ q_4q_8/L^6 - 14400\ q_6q_7/L^9 + 2400\ q_6q_8/L^8 - 1200\ q_7q_8/L^7 \qquad (D.22)$$

Hence, the resulting polynomial is:

$$D(x) = D_0 + D_1x + D_2x^2 + D_3x^3 + d_4x^4 + D_5x^5 + D_6x^6 \qquad (D.23)$$

and

$$(\delta^2 w(x,t)/\delta x^2)^2 = D(x) \qquad (D.24)$$

Introducing D(x) into the equation of the potential energy, we have:

$$V_2(t) = \frac{1}{2}\int_0^L EI\ (\delta^2 w(x,t)/\delta x^2)^2\ dx \qquad (D.25)$$

$$V_2(t) = \frac{1}{2} EI \int_0^L (D_0 + D_1x + D_2x^2 + D_3x^3 + D_4x^4 + D_5x^5 + D_6x^6)\,dx \qquad (D.26)$$

Integrating:

$$V_2(t) = \frac{1}{2} EI \,(D_0L + D_1L^2/2 + D_2L^3/3 + D_3L^4/4 + D_4L^5/5 + D_5L^6/6 + D_6L^7/7) \qquad (D.27)$$

$$(S)^T \frac{1}{2} \begin{bmatrix} X & 0 & 0 & 0 & -X & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -X & 0 & 0 & 0 & X & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} (S) \;, \quad \text{or:} \quad (S)^T \frac{1}{2} [K_1](S)$$

$$(S)^T \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & B & C & F & 0 & -B & C & -F \\ 0 & C & G & J & 0 & -C & N & -M \\ 0 & F & J & H & 0 & -F & M & P \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -B & -C & -F & 0 & B & -C & F \\ 0 & C & N & M & 0 & -C & G & -J \\ 0 & -F & -M & P & 0 & F & -J & H \end{bmatrix} (S) \;, \quad \text{or:} \quad (S)^T \frac{1}{2} [K_2](S)$$

Where:

$X = EA/L$ $B = 120\ EI/7L^3$ $C = 60\ EI/7L^2$ $F = 3\ EI/7L$

$G = 192\ EI/35L$ $H = 3\ EI\ L/35$ $J = 11\ EI/35$ $N = 108\ EI/35L$

$M = 4\ EI/35$ $P = EI\ L/70$

From matrix $[K_1]$ and $[K_2]$, one obtains matrix [K] which is shown by Equation (D.28):

$$[K] = \begin{bmatrix} X & 0 & 0 & -F & -X & 0 & 0 & 0 \\ 0 & B & C & F & 0 & -B & C & -F \\ 0 & C & G & J & 0 & -C & N & -M \\ 0 & F & J & H & 0 & -F & M & P \\ -X & 0 & 0 & 0 & X & 0 & 0 & 0 \\ 0 & -B & -C & -F & 0 & B & -C & F \\ 0 & C & N & M & 0 & -C & G & -J \\ 0 & -F & -M & P & 0 & F & -J & H \end{bmatrix} \tag{D.28}$$

APPENDIX E

TRANSFORMATIONS

Transformation Matrix

Let [t] denote the transformation matrix to be used in this study. It should be understood that there are different transformation matrices [t] for different elements. If the elements have some of their generalized coordinates in the same orientation in space, then the local coordinates corresponding to these orientations are parallel. Hence, in this case we have that (43, 44):

$$[t] = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{E.1}$$

$$\begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix} = [t] \begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ w_1 \end{pmatrix} \tag{E.2}$$

$$\begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ w_1 \end{pmatrix} \tag{E.3}$$

Figure 45 shows the graphical representation used in this study. The above equations might be combined so as to apply it to the entire element by writing simply:

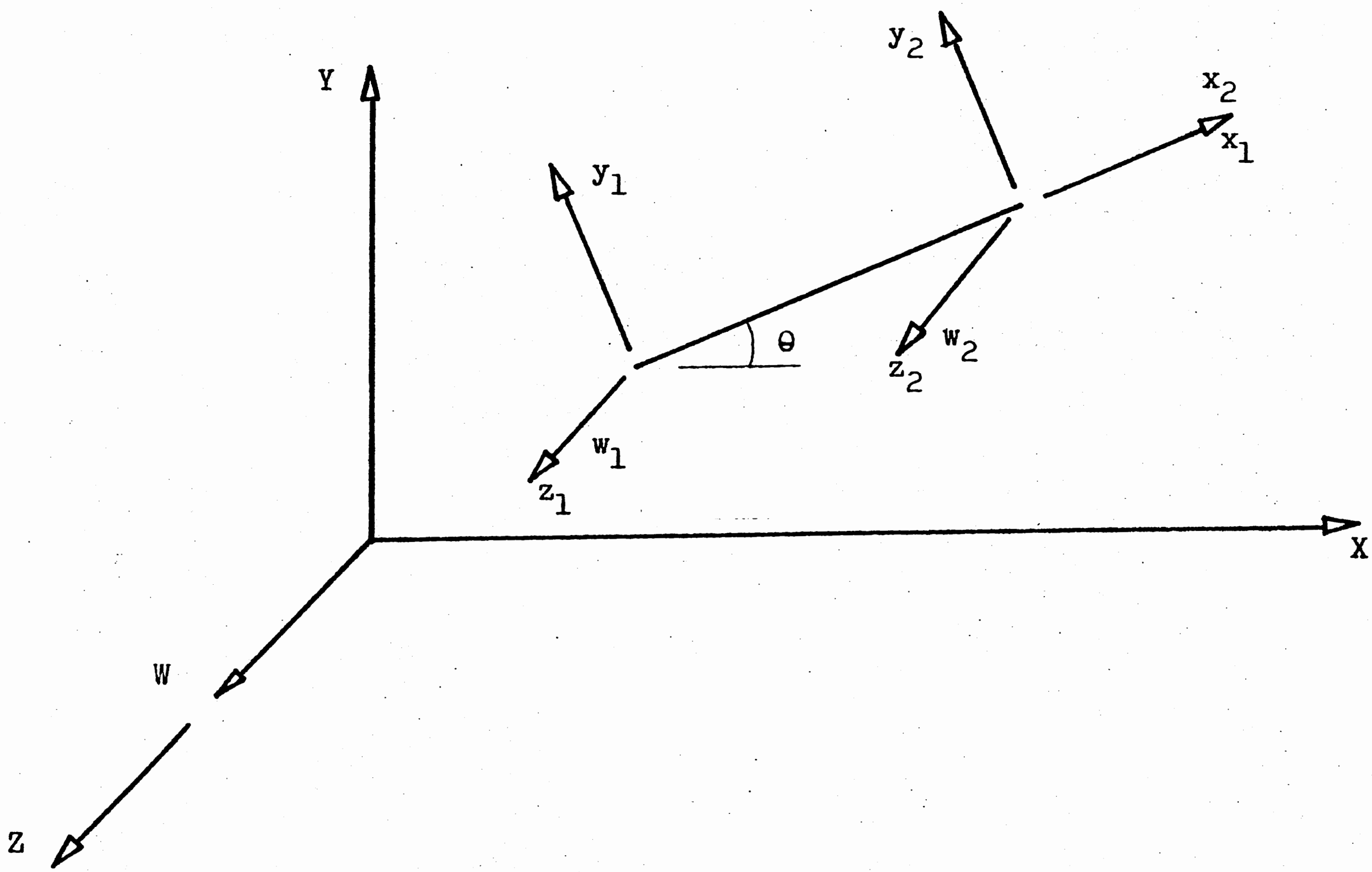

Figure 45. Graphical Representation of the Transformation Used in This Study

$$\begin{pmatrix} X \\ Y \\ Z \\ W \\ X \\ Y \\ Z \\ W \end{pmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 & 0 & 0 & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\theta & -\sin\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ w_1 \\ x_2 \\ y_2 \\ z_2 \\ w_2 \end{pmatrix} \tag{E.4}$$

or

$$(U) = [T](u) \tag{E.5}$$

where

$$[T] = \begin{bmatrix} t & 0 \\ 0 & t \end{bmatrix} . \tag{E.6}$$

Transformation of the Kinetic and Potential Energy

$$(S) = [T](s) \tag{E.7}$$

where (S) and (s) represent the global and local deformation vectors, respectively (43, 44). The kinetic energy T(t) might be written in the form of:

$$T(t) = \frac{1}{2} (s)^T [m](s) \tag{E.8}$$

where

$$(s) = [T]^T (S) \tag{E.9}$$

hence

$$T(t) = \frac{1}{2} (S)^T [T][m][T]^T (S) \tag{E.10}$$

$$T(t) = \frac{1}{2} (S)^T [M](S) \tag{E.11}$$

where

$$[M] = [T][m][T]^T \tag{E.12}$$

The potential energy can be written as:

$$V(t) = \frac{1}{2} (s)^T [k](s) \tag{E.13}$$

where

$$(s) = [T]^T (s) \tag{E.14}$$

hence

$$V(t) = \frac{1}{2} (S)^T [T][k][T]^T (S) \tag{E.15}$$

$$V(t) = \frac{1}{2} (S)^T [K](S) \tag{E.16}$$

where

$$[K] = [T][k][T]^T \tag{E.17}$$

For the right-hand side vector or force vector, we know that the virtual work has the expression:

$$\delta W = (s)^T (q) \tag{E.18}$$

hence

$$\delta W = (S)^T [T](q) \tag{E.19}$$

where

$$(Q) = [T](q) \tag{E.20}$$

is recognized as the vector of the joint forces in terms of the global components.

Reduction of Degrees of Freedom

The applications for analyses of computerized finite-element methods have evolved to a high level of sophistication and accuracy and are now accepted as necessary tools in the analysis of extremely complex structures and mechanisms. The current state-of-the-art is such that system idealizations with many degrees of freedom may be systematically assembled by programming techniques based on linear matrix analysis (75, 76). At the same time, the extension of these analyses to solve mechanism dynamical problems for natural vibration modes, or transient responses by modal techniques, has been hampered by difficulties in accurately and efficiently handling eigenvalue problems or degrees of freedom of comparable large size.

A typical structure or mechanism may have too many degrees of freedom for economical treatment. Bagci and Kalaycioglu (34), working on kineto-elastodynamic analysis of mechanisms experienced that for four cycles at 125 rpm, using a step size h = 0.00001s and ten terms in the matrix exponential series, the Xerox Sigma 6 digital computer required 18.11 hours CPU time. Accordingly, we call on the eigenvalue economizer (73, 74, 75, 76) to eliminate many or most degrees of freedom from the problem. Let:

$$\underset{n\times 1}{(D)} = \underset{n\times m}{[T]}\,\underset{m\times 1}{(D_r)} \qquad \text{(E.21)}$$

where

n = total degrees of freedom;

m = reduced degrees of freedom;

(D) = complete set of n degrees of freedom;

(D_r) = condensed set of m degrees of freedom; and

[T] = condensing matrix.

we have (1):

$$([K] - \omega^2[M])(D) = 0 \qquad \text{(E.22)}$$

From Equations (E.1) and (E.2)

$$([K] - \omega^2[M])[T](D_r) = 0 \qquad \text{(E.23)}$$

multiplying by $[T]^T$

$$[T]^T([K] - \omega^2[M])[T](D_r) = 0$$

$$([T]^T[K][T] - \omega^2[T]^T[M][T])(D_r) = 0 \qquad \text{(E.24)}$$

hence

$$\underset{m\times m}{[K_r]} = \underset{m\times n}{[T]^T}\,\underset{n\times n}{[K]}\,\underset{n\times m}{[T]} \qquad \text{(E.25)}$$

$$\underset{m\times m}{[M_r]} = [T]^T\underset{n\times n}{[M]}[T] \qquad \text{(E.26)}$$

In the same way, it could be shown that:

$$\underset{m\times m}{[A_r]} = [T]^T\underset{n\times n}{[A]}[T] \qquad \text{(E.27)}$$

$$\underset{m\times 1}{(Q_r)} = [T]^T\underset{n\times 1}{(Q)} \qquad \text{(E.28)}$$

It has been shown (26, 73, 74, 98) that matrix [T] has the form

$$[T] = \begin{bmatrix} & I & \\ -C_D^{-1} & & C_1 \end{bmatrix} \qquad \text{(E.29)}$$

where

[I] = identity matrix;

$[C_I]$ = matrix of independent coordinates; and

$[C_D]$ = matrix of dependent coordinates.

For the four-bar mechanism of Figure 46, we have:

$\bar{U}_2 = 0$

$\bar{U}_3 = 0$

$\bar{U}_4 = 0$

where

$$\begin{pmatrix} U_2 \\ V_2 \end{pmatrix} = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{pmatrix} q_3 \\ q_4 \end{pmatrix} \tag{E.30}$$

$$\begin{pmatrix} U'_3 \\ V'_3 \end{pmatrix} = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{pmatrix} q_3 \\ q_4 \end{pmatrix} \tag{E.31}$$

$$\begin{pmatrix} U''_3 \\ V''_3 \end{pmatrix} = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{pmatrix} q_7 \\ q_8 \end{pmatrix} \tag{E.32}$$

$$\begin{pmatrix} U_4 \\ V_4 \end{pmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{pmatrix} q_7 \\ q_8 \end{pmatrix} \tag{E.33}$$

hence

$$U_2 = q_3 \cos\theta_1 + q_4 \sin\theta_1 \tag{E.34}$$

$$V_2 = -q_3 \sin\theta_1 + q_4 \cos\theta_1 \tag{E.35}$$

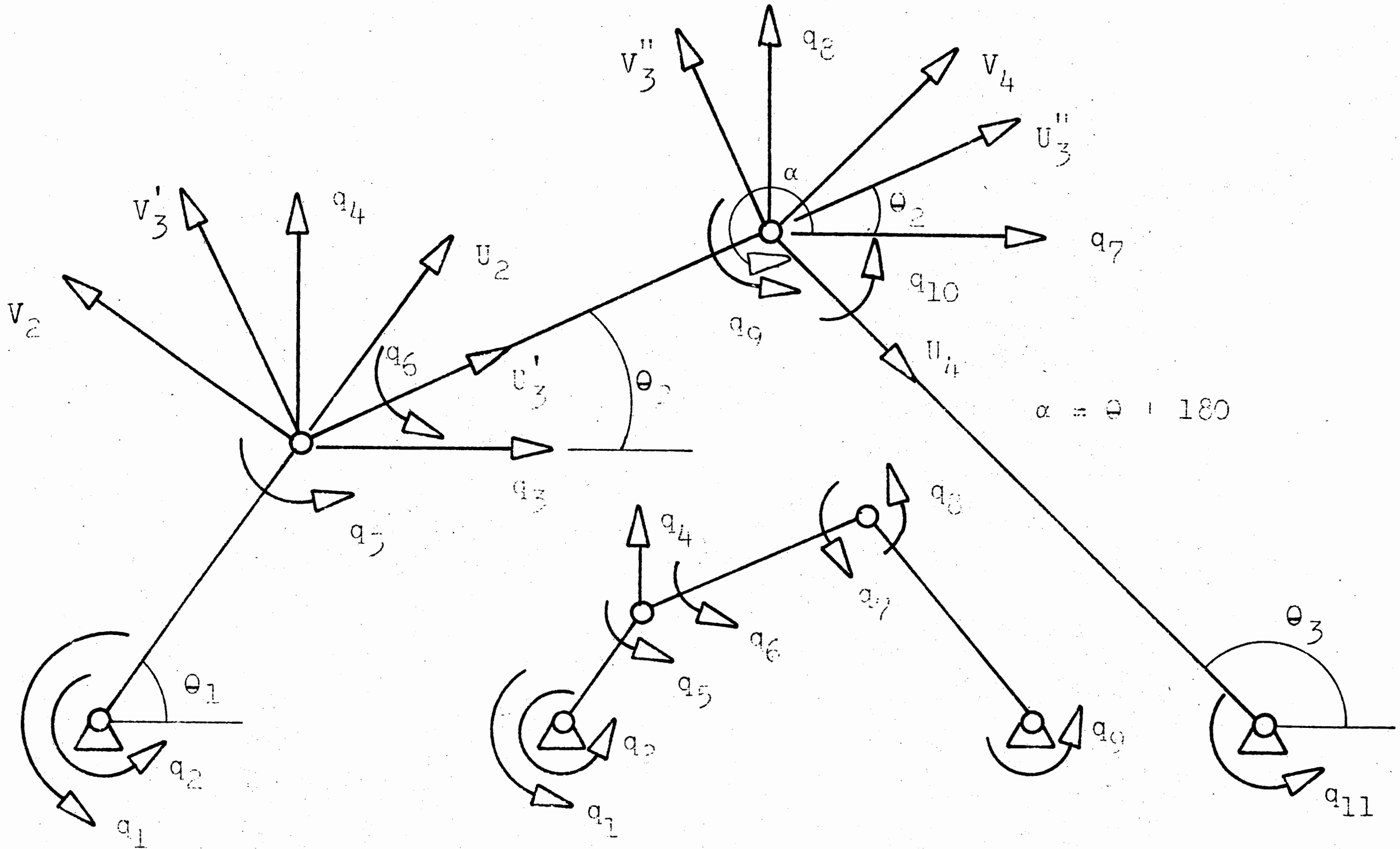

Figure 46. Graphical Representation of the Independent and Dependent Generalized Coordinates in the Condensation Procedure

$$U_3' = q_3 \cos \theta_2 + q_4 \sin \theta_2 \tag{E.36}$$

$$V_3' = -q_3 \sin \theta_2 + q_4 \cos \theta_2 \tag{E.37}$$

$$U_4 = q_7 \cos \alpha + q_8 \sin \alpha \tag{E.38}$$

$$V_4 = -q_7 \sin \alpha + q_8 \cos \alpha \tag{E.39}$$

and

$$q_3 \cos \theta_1 + q_4 \sin \theta_1 = 0 \tag{E.40}$$

$$q_3 \cos \theta_2 + q_4 \sin \theta_2 = q_7 \cos \theta_2 + q_8 \sin \theta_2 \tag{E.41}$$

$$q_7 \cos \theta_3 + q_8 \sin \theta_3 = 0 \tag{E.42}$$

If q_4 is chosen as the independent generalized coordinate, we obtain the constraint relation:

From Equation (E.40)

$$q_3 = -q_4 \tan \theta_1 \tag{E.43}$$

From Equations (E.40), (E.41), and (E.42)

$$-q_4 \tan \theta_1 \cos \theta_2 + q_4 \sin \theta_2 = q_7 \cos \theta_2 + q_8 \sin \theta_2$$

$$-q_4 \tan \theta_1 \cos \theta_2 + q_4 \sin \theta_2 = q_7 \cos \theta_2 - \frac{\cos \theta_3}{\sin \theta_3} \sin \theta_2$$

or

$$q_7 = (q_4 \sin \theta_2 - q_4 \tan \theta_1 \cos \theta_2)/(\cos \theta_2 - \frac{\cos \theta_3}{\sin \theta_3} \sin \theta_2)$$

$$q_7 = (q_4 \sin \theta_2 \sin \theta_3 - q_4 \tan \theta_1 \cos \theta_2 \sin \theta_3)/\sin(\theta_3 - \theta_2) \tag{E.44}$$

and

$$q_8 = -(q_4 \sin\theta_2 \cos\theta_3 - q_4 \tan\theta_1 \cos\theta_2 \cos\theta_3)/\sin(\theta_3 - \theta_2)$$

or

$$q_8 = (q_4 \tan\theta_1 \cos\theta_2 \cos\theta_3 - q_4 \sin\theta_2 \cos\theta_3)/\sin(\theta_3 - \theta_2) \tag{E.45}$$

in matrix form:

$$\begin{pmatrix} q_3 \\ q_4 \\ q_7 \\ q_8 \end{pmatrix} = \begin{pmatrix} -\tan\theta_1 \\ 1 \\ X \\ Y \end{pmatrix} q_4 \tag{E.46}$$

where

$$X = (\sin\theta_2 \sin\theta_3 - \tan\theta_1 \cos\theta_2 \sin\theta_3)/\sin(\theta_3 - \theta_2)$$

$$Y = (\tan\theta_1 \cos\theta_2 \cos\theta_3 - \sin\theta_2 \cos\theta_3)/\sin(\theta_3 - \theta_2)$$

and matrix [T] could be constructed (see Equation (E.47) below).

$$
\begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \\ q_9 \\ q_{10} \\ q_{11} \end{Bmatrix} = \underset{[T]}{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\tan\theta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & X & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}} \begin{Bmatrix} q_1 \\ q_2 \\ q_4 \\ q_5 \\ q_6 \\ q_9 \\ q_{10} \\ q_{11} \end{Bmatrix} \qquad \text{(E.47)}
$$

$$
[Z]_{24\times20} = \begin{bmatrix}
1 \\
&1 \\
&&1 \\
&&&1 \\
&&&&1 \\
&&&&&1 \\
&&&&&&1 \\
&&&&&&&1 \\
&&&&1 \\
&&&&&1 \\
&&&&&&&&1 \\
&&&&&&&&&1 \\
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&&&&&&&&&&&&&&&1 \\
&&&&&&&&&&&&&&&&1 \\
&&&&&&&&&&&&&&&&&1 \\
&&&&&&&&&&&&&&&&&&1 \\
&&&&&&&&&&&&&&&&&&&1
\end{bmatrix}
\qquad
[Z]^T_{20\times24} = \begin{bmatrix}
1 \\
&1 \\
&&1 \\
&&&1 \\
&&&&1&&&&1 \\
&&&&&1&&&&1 \\
&&&&&&1 \\
&&&&&&&1 \\
&&&&&&&&&&1 \\
&&&&&&&&&&&1 \\
&&&&&&&&&&&&1&&&&1 \\
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&&&&&&&&&&&&&&&1 \\
&&&&&&&&&&&&&&&&&&1 \\
&&&&&&&&&&&&&&&&&&&1 \\
&&&&&&&&&&&&&&&&&&&&1 \\
&&&&&&&&&&&&&&&&&&&&&1 \\
&&&&&&&&&&&&&&&&&&&&&&1 \\
&&&&&&&&&&&&&&&&&&&&&&&1
\end{bmatrix}
\qquad (E.48)
$$

APPENDIX F

MATRICES $[M_D]$, $[M_T]$, $[A_D]$, $[A_T]$, $[K_D]$, $[K_T]$, (Q), AND (Q_T)

$$[M_D] = ([M] + [B])m$$

$$[M_D] = \begin{bmatrix} A & 0 & 0 & 0 & B & 0 & 0 & 0 \\ 0 & D & E & F & 0 & G & -H & J \\ 0 & E & L & M & 0 & H & -N & P \\ 0 & F & M & Q & 0 & J & -P & W \\ B & 0 & 0 & 0 & A & 0 & 0 & 0 \\ 0 & G & H & J & 0 & D & -E & F \\ 0 & -H & -N & -P & 0 & -E & L & -M \\ 0 & J & P & W & 0 & F & -M & Q \end{bmatrix} \qquad \text{(F.1)}$$

$A = (L/3)\ m$ $\qquad$ $B = (L/6)\ m$

$D = (181\ L/462)\ m$ $\qquad$ $E = (311\ L^2/4620)\ m$

$F = (281\ L^3/55440)\ m$ $\qquad$ $G = (25\ L/231)\ m$

$H = (151\ L^2/4620)\ m$ $\qquad$ $J = (181\ L^3/55440)\ m$

$L = (52\ L^3/3465)\ m$ $\qquad$ $M = (23\ L^4/18480)\ m$

$N = (19\ L^3/1980)\ m$ $\qquad$ $P = (13\ L^4/13860)\ m$

$Q = (L^5/9240)\ m$ $\qquad$ $W = (L^5/11088)\ m$

$$
\begin{bmatrix}
C & -S & 0 & 0 & 0 & 0 & 0 & 0 \\
S & C & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & C & -S & 0 & 0 \\
0 & 0 & 0 & 0 & S & C & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
A & 0 & 0 & 0 & B & 0 & 0 & 0 \\
0 & D & E & F & 0 & G & -H & J \\
0 & E & L & M & 0 & H & -N & P \\
0 & F & M & Q & 0 & J & -P & W \\
B & 0 & 0 & 0 & A & 0 & 0 & 0 \\
0 & G & H & J & 0 & D & -E & F \\
0 & -H & -N & -P & 0 & -E & L & -M \\
0 & J & P & W & 0 & F & -M & Q
\end{bmatrix}
\longrightarrow
$$

$$
\begin{bmatrix}
AC & -DS & -ES & -FS & BC & -GS & HS & -JS \\
AS & DC & EC & FC & BS & GC & -HC & JC \\
0 & E & L & M & 0 & H & -N & P \\
0 & F & M & Q & 0 & J & -P & W \\
BC & -GS & -HS & -JS & AC & -DS & ES & -FS \\
BS & GC & HC & JC & AS & DC & -EC & FC \\
0 & -H & -N & -P & 0 & -E & L & -M \\
0 & J & P & W & 0 & F & -M & Q
\end{bmatrix}
\begin{bmatrix}
C & S & 0 & 0 & 0 & 0 & 0 & 0 \\
-S & C & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & C & S & 0 & 0 \\
0 & 0 & 0 & 0 & -S & C & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
=
$$

$$[M_T] = \begin{bmatrix} AC^2 + DS^2 & ACS - DCS & -ES & -FS & BC^2 + GS^2 & BCS - GCS & HS & -JS \\ ACS - DCS & AS^2 + DC^2 & EC & FC & BCS - GCS & BS^2 + GC^2 & -HC & JC \\ -ES & EC & L & M & -HS & HC & -N & P \\ -FS & FC & M & Q & -JS & JC & -P & W \\ BC^2 + GS^2 & BCS - GCS & -HS & -JS & AC^2 + DS^2 & ACS - DCS & ES & -FS \\ BCS - GCS & BS^2 + GC^2 & HC & JC & ACS - DCS & AS^2 + DC^2 & -EC & FC \\ HS & -HC & -N & -P & ES & -EC & L & -M \\ -JS & JC & P & W & -FS & FC & -M & Q \end{bmatrix} \qquad (F.2)$$

where

$C = \cos\theta$

$S = \sin\theta$

$$[A_D] = [[A] - [A]^T]\ m$$

$$[A_D] = \begin{bmatrix} 0 & A & B & H & 0 & D & -E & F \\ -A & 0 & 0 & 0 & -D & 0 & 0 & 0 \\ -B & 0 & 0 & 0 & -E & 0 & 0 & 0 \\ -H & 0 & 0 & 0 & -F & 0 & 0 & 0 \\ 0 & D & E & F & 0 & A & -B & H \\ -D & 0 & 0 & 0 & -A & 0 & 0 & 0 \\ E & 0 & 0 & 0 & B & 0 & 0 & 0 \\ -F & 0 & 0 & 0 & -H & 0 & 0 & 0 \end{bmatrix} \qquad (F.3)$$

$A = (5\ L/14)\ m$

$B = (13\ L^2/210)\ m$

$H = (L^3/210)\ m$

$D = (L/7)\ m$

$E = (4\ L^2/105)\ m$

$F = (L^3/280)\ m$

$$\begin{bmatrix} C & -S & 0 & 0 & 0 & 0 & 0 & 0 \\ S & C & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C & -S & 0 & 0 \\ 0 & 0 & 0 & 0 & S & C & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & A & B & H & 0 & D & -E & F \\ -A & 0 & 0 & 0 & -D & 0 & 0 & 0 \\ -B & 0 & 0 & 0 & -E & 0 & 0 & 0 \\ -H & 0 & 0 & 0 & -F & 0 & 0 & 0 \\ 0 & D & E & F & 0 & A & -B & H \\ -D & 0 & 0 & 0 & -A & 0 & 0 & 0 \\ E & 0 & 0 & 0 & B & 0 & 0 & 0 \\ -F & 0 & 0 & 0 & -H & 0 & 0 & 0 \end{bmatrix} \longrightarrow$$

$$\begin{bmatrix} AS & AC & BC & HC & DS & DC & -EC & FC \\ -AC & AS & BS & HS & -DC & DS & -ES & FS \\ -B & 0 & 0 & 0 & -E & 0 & 0 & 0 \\ -H & 0 & 0 & 0 & -F & 0 & 0 & 0 \\ DS & DC & EC & FC & AS & AC & -BC & HC \\ -DC & DS & ES & FS & -AC & AS & -BS & HS \\ E & 0 & 0 & 0 & B & 0 & 0 & 0 \\ -F & 0 & 0 & 0 & -H & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} C & S & 0 & 0 & 0 & 0 & 0 & 0 \\ -S & C & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C & S & 0 & 0 \\ 0 & 0 & 0 & 0 & -S & C & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$[A_T] = \begin{bmatrix} 0 & AS^2 + AC^2 & BC & HC & 0 & DS^2 + DC^2 & -EC & FC \\ -AC^2 - AS^2 & 0 & BS & HS & -DC^2 - DS^2 & 0 & -ES & FS \\ -BC & -BS & 0 & 0 & -EC & -ES & 0 & 0 \\ -HC & -HS & 0 & 0 & -FC & -FS & 0 & 0 \\ 0 & DS^2 + DC^2 & EC & FC & 0 & AS^2 + AC^2 & -BC & HC \\ -DS^2 - DC^2 & 0 & ES & FC & -AC^2 - AS^2 & 0 & -BS & HS \\ EC & ES & 0 & 0 & BC & BS & 0 & 0 \\ -FC & -FS & 0 & 0 & -HC & -HS & 0 & 0 \end{bmatrix} \qquad (F.4)$$

where

$C = \cos\theta$

$S = \sin\theta$

$$[K_D] = [K] - \theta^2 [M_D] - \theta [A_D]$$

$$[K_D] = \begin{bmatrix} A & -B & -D & -E & -F & -H & I & -J \\ B & K & L & M & H & -N & P & -Q \\ D & L & R & U & I & -P & V & -Z \\ E & M & U & X & J & -Q & Z & Y \\ -F & -H & -I & -J & A & -B & D & -E \\ H & -N & -P & -Q & B & K & -L & M \\ -I & P & V & Z & -D & -L & R & -U \\ J & -Q & -Z & Y & E & M & -U & X \end{bmatrix} \qquad (F.5)$$

$A = EA/L - (\dot{\theta}^2 L/3)\ m$

$B = (5\ \ddot{\theta} L/14)\ m$

$D = (13\ \ddot{\theta} L^2/210)\ m$

$E = (\ddot{\theta} L^3/210)\ m$

$F = EA/L + (\dot{\theta}^2 L/6)\ m$

$H = (\ddot{\theta} L/7)\ m$

$I = (4\ \ddot{\theta} L^2/105)\ m$

$J = (\ddot{\theta} L^3/280)\ m$

$K = 120\ EI/7L^3 - (181\ \dot{\theta}^2 L/462)\ m$

$L = 60\ EI/7L^2 - (311\ \dot{\theta}^2 L^2/4620)\ m$

$M = 3\ EI/7L - (281\ \dot{\theta}^2 L^3/55440)\ m$

$N = 120\ EI/7L^3 + (25\ \dot{\theta}^2 L/231)\ m$

$P = 60\ EI/7L^2 + (151\ \dot{\theta}^2 L^2/4620)\ m$

$Q = 3\ EI/7L + (181\ \dot{\theta}^2 L^3/55440)\ m$

$R = 192\ EI/35L - (52\ \dot{\theta}^2 L^3/3465)\ m$

$U = 11\ EI/35 - (23\ \dot{\theta}^2 L^4/18480)\ m$

$V = 108\ EI/35L + (19\ \dot{\theta}^2 L^3/1980)\ m$

$Z = 4\ EI/35 + (13\ \dot{\theta}^2 L^4/13860)\ m$

$X = 3\ LEI/35 + (\dot{\theta}^2 L^5/9240)\ m$

$Y = LEI/70 - (\dot{\theta}^2 L^5/11088)\ m$

$$
\begin{bmatrix}
C & -S & 0 & 0 & 0 & 0 & 0 & 0 \\
S & C & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & C & -S & 0 & 0 \\
0 & 0 & 0 & 0 & S & C & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
A & -B & -D & -E & -F & -H & I & -J \\
B & K & L & M & H & -N & P & -Q \\
D & L & R & U & I & -P & V & -Z \\
E & M & U & X & J & -Q & Z & Y \\
-F & -H & -I & -J & A & -B & D & -E \\
H & -N & -P & -Q & B & K & -L & M \\
-I & P & V & Z & -D & -L & R & -U \\
J & -Q & -Z & Y & E & M & -U & X
\end{bmatrix}
\longrightarrow
$$

$$
\begin{bmatrix}
AC-BS & -BC-KS & -DC-LS & -EC-MS & -FC-HS & -HC+NS & IC-PS & -JC+QS \\
AS+BC & -BS+KC & -DS+LC & -ES+MC & -FS+HC & -HS+NC & IS+PC & -JS+QC \\
D & L & R & U & I & -P & V & -Z \\
E & M & U & X & J & -Q & Z & Y \\
-FC-HS & -HC+NS & -IC+PS & -JC+QS & AC-BS & -BC-KS & DC+LS & -EC-MS \\
-FS+HC & -HS-NC & -IS-PC & -JS-QC & AS+BC & -BS+KC & DS-LC & -ES+MC \\
-I & P & V & Z & -D & -L & R & -U \\
J & -Q & -Z & Y & E & M & -U & X
\end{bmatrix}
\begin{bmatrix}
C & S & 0 & 0 & 0 & 0 & 0 & 0 \\
-S & C & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & C & S & 0 & 0 \\
0 & 0 & 0 & 0 & -S & C & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
=
$$

Matrix $[K_T]$

$$
\begin{bmatrix}
AC^2 + KS^2 & ACS - B - KCS & -DC - LS & -EC - MS & -FC^2 - NS^2 & -FCS - + - NCS & IC - PS & -JC + QS \\
ACS + B - KCS & AS^2 + KC^2 & -DS + LC & -ES + MC & -FCS + H + NCS & -FS^2 - NC^2 & IS + PC & -JS - QC \\
DC - LS & DS + LC & R & U & IC + PS & IS - PC & V & -Z \\
EC - MS & ES + MC & U & X & JC + QS & JS - QC & Z & Y \\
-FC^2 - NS^2 & -FCS - H + NCS & -IC + PS & -JC + QS & AC^2 + KS^2 & ACS - B - KCS & DC + LS & -EC - MS \\
-FCS + H + NCS & -FS^2 - NC^2 & -IS - PC & -JS - QC & ACS + B - KCS & AS^2 + KC^2 & DS - LC & -ES + MC \\
-IC - PS & -IS + PC & V & Z & -DC + LS & -DS - LC & R & -U \\
JC + QS & JS - QC & -Z & Y & EC - MS & ES + MC & -U & X
\end{bmatrix}
\qquad (F.6)
$$

where

$C = \cos\theta$

$S = \sin\theta$

$$\begin{pmatrix} \frac{\dot{Y}_1 \dot{\theta} L}{2} \\ -\frac{\dot{X}_1 \dot{\theta} L}{2} \\ -\frac{\dot{X}_1 \dot{\theta} L^2}{10} \\ -\frac{\dot{X}_1 \dot{\theta} L^3}{120} \\ \frac{\dot{Y}_1 \dot{\theta} L}{2} \\ -\frac{\dot{X}_1 \dot{\theta} L}{2} \\ \frac{\dot{X}_1 \dot{\theta} L^2}{10} \\ -\frac{\dot{X}_1 \dot{\theta} L^3}{120} \end{pmatrix} \begin{matrix} - \\ - \\ - \\ - \\ - \\ - \\ + \\ - \end{matrix} \begin{pmatrix} \frac{\ddot{X}_1 L}{2} \\ \frac{\ddot{Y}_1 L}{2} \\ \frac{\ddot{Y}_1 L^2}{10} \\ \frac{\ddot{Y}_1 L^3}{120} \\ \frac{\ddot{X}_1 L}{2} \\ \frac{\ddot{Y}_1 L}{2} \\ \frac{\ddot{Y}_1 L^2}{10} \\ \frac{\ddot{Y}_1 L^3}{120} \end{pmatrix} \begin{matrix} + \\ - \\ - \\ - \\ + \\ - \\ + \\ - \end{matrix} \begin{pmatrix} \frac{\dot{\theta}^2 L^2}{6} \\ \frac{\ddot{\theta} L^2}{7} \\ \frac{4 \ddot{\theta} L^3}{105} \\ \frac{\ddot{\theta} L^4}{280} \\ \frac{\dot{\theta}^2 L^2}{3} \\ \frac{5 \ddot{\theta} L^2}{14} \\ \frac{13 \ddot{\theta} L^3}{210} \\ \frac{\ddot{\theta} L^4}{210} \end{pmatrix} \qquad (F.7)$$

$$
(Q_T) = \begin{bmatrix} C & -S & 0 & 0 & 0 & 0 & 0 & 0 \\ S & C & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C & -S & 0 & 0 \\ 0 & 0 & 0 & 0 & S & C & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} A \\ B \\ D \\ E \\ F \\ G \\ H \\ I \end{pmatrix} \begin{pmatrix} AC - BS \\ AS + BC \\ D \\ E \\ FC - GS \\ FS + GC \\ H \\ I \end{pmatrix} \tag{F.8}
$$

$A = (\dot{Y}_1\dot{\theta}L/2 - \ddot{X}_1L/2 + \dot{\theta}^2L^2/6)\ m$

$B = (-\dot{X}_1\dot{\theta}L/2 - \ddot{Y}_1L/2 - \ddot{\theta}L^2/7)\ m$

$C = \cos\theta$

$D = (-\dot{X}_1\dot{\theta}L^2/10 - \ddot{Y}_1L^2/10 - 4\ddot{\theta}L^3/105)\ m$

$E = (-\dot{X}_1\dot{\theta}L^3/120 - \ddot{Y}_1L^3/120 - \ddot{\theta}L^4/280)\ m$

$F = (\dot{Y}_1\dot{\theta}L/2 - \ddot{X}_1L/2 + \dot{\theta}^2L^2/3)\ m$

$G = (-\dot{X}_1\dot{\theta}L/2 - \ddot{Y}_1L/2 - 5\ddot{\theta}L^2/14)\ m$

$H = (\dot{X}_1\dot{\theta}L^2/10 + \ddot{Y}_1L^2/10 + 13\ddot{\theta}L^3/210)\ m$

$I = (-\dot{X}_1\dot{\theta}L^3/120 - \ddot{Y}_1L^3/120 - \ddot{\theta}L^4/210)\ m$

$S = \sin\theta$

APPENDIX G

PROGRAMS

```
C
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     X                                                                  X
C     X PROGRAM FOR THE SYNTHESIS OF A FOUR-BAR-PLANE MACHANISM BY AN    X
C     X                    ITERATIVE METHOD                              X
C     X                      CRANK-ROCKER                                X
C     X              GIVEN U1 , DELAL , PH4D AND DD                      X
C     X                                                                  X
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
C
C       NOTATIONS :
C
C       AA      = INPUT LINK
C       BB      = COUPLER LINK
C       CC      = OUTPUT LINK
C       DD      = FIXED LINK
C       DELAL   = OSCILLATION ANGLE OF OUTPUT LINK FOR CRANK-ROCKER
C                 MECHANISM , RADIANS
C       DEL     = DELAL , INDEGREES
C       PH4     = MINIMUP OUTPUT ANGLE FOR CRANK-ROCKER MECHANISM , RADIANS
C       PH2     = MAXIMUP OUTPUT ANGLE FOR CRANK-ROCKER MECHANISM , RADIANS
C       DTR     = PARAMETER FOR DEGREE TO RADIANS CONVERTER
C       RTD     = PARAMETER FOR RADIAN TO DEGREE CONVERTER
C       U1      = MINIMUP TRANSMISSION ANGLE
C       U1D     = U1 IN DEGREES
C       PH4D    = PH4 IN DEGRESS
C       PH2D    = PH2 IN DEGREES
C       L       = NUMBER OF OSCILLATION ANGLES PROCESSED
C       FUNCT   = VALUES OF THE FUNCTIONS , IT SHOULD BE EQUAL TO OR
C                 NEAR TO ZERO FOR ACCURACY , FUNCT(5)
C       NSTEP1  = FLAG FOR PRINTING OUTPUT OF ITERATIONS , FOR NSTEP1
C                 .GT. 0 THERE WILL BE NO PRINTING OUT
C
C
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION AA(200) , BB(200) , CC(200) , U2(200) , U2D(200) ,
       1          T1(200) , T1D(200) , U4(200)
        COMMON AA,BB,CC,DD,PH4,PH2,PH4D,PH2D,DELAL,U1,RTD,PI,T1,U2,U2D,U4,
       1U1D,T1D,FUNCT(5),NSTEP1
        DATA NUM/46/
C
C
C       INITIALIZE BY INCORPORATING INITIAL INFORMATION AND PRINTING
C       THEM OUT
C
        PI = 4.0D0 * DATAN(1.0D0)
        DTR = PI/180.0
        RTD = 180.0/PI
        NSTEP1 = 0
        DD = 11.0D0
        DEL = 8.0D0
        U1D = 35.0D0
        U1 = U1D * DTR
        PH4D = 40.0D0
        PH4 = PH4D * DTR
C       INTRODUCE INITIAL VALUES OF UNKNOWNS
        T1D(1) = 89.707380
        T1(1) = T1D(1) * DTR
C
        PRINT 99
   99   FORMAT (1H1)
        PRINT 100
  100   FORMAT (///)
        PRINT 101
  101   FORMAT (47X,'INPUT INFORMATION AND INITIAL CONDITION',/)
        PRINT 102
  102   FORMAT (35X,'DD',14X,'DEL',12X,'U1D',12X,'PH4D',11X,'T1D',/)
        PRINT 103, DD , DEL , U1D , PH4D  , T1D(1)
  103   FORMAT (25X,5F15.4)
C
C       CALL SUBROUTINE SUCIB TO CARRY OUT THE SYNTHESIS OF A CRANK-
C       ROCKER-PLANE MECHANISM GIVEN DELAL , UR AND CC
C
        PRINT 99
        PRINT 100
        DO 5 L=1,NUM
        DEL = DEL + 2.0D0
        DELAL = DEL * DTR
        CALL SUCIB(J)
        PRINT 100
        PRINT 110
  110   FORMAT (22X,'L', 9X,'PH2D',11X,'U2D',12X,'AA',13X,'BB',13X,'CC',16
       1X,'DEL',/)
        PRINT 111, L , PH2D , U2D(J) , AA(J) , BB(J) , CC(J) , DEL
  111   FORMAT (18X,I5, 5X,5D15.6,F10.2)
        PRINT 100
        PRINT 120
  120   FORMAT (59X,'FUNCTION VALUES',/)
        PRINT 121, (FUNCT(N),N=1,5)
  121   FORMAT (20X,5D15.6)
        IF (NSTEP1 .GT. 0) GO TO 5
        PRINT 99
        PRINT 100
    5   CONTINUE
C
        PRINT 99
        STOP
        END
CEJECT
C
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
        SUBROUTINE SUCIB(I)
C
C       THIS SUBROUTINE SOLVE IN AN ITERATIVE WAY A SYSTEM OF FIVE
```

```
C     TRIGONOMETRIC EQUATIONS , THIS SYSTEM REPRESENT THE SYNTHESIS
C     EQUATIONS FOR A CRANK-ROCKER MECHANISM PRESCRIBING THE OSCILLATION
C     ANGLE OF OUTPUT LINK , LENGTH OF OUTPUT LINK AND MINIMUM
C     TRANSMISSION ANGLE
C
C     NOTATIONS :
C
C     A          = INPUT LINK LENGTH
C     B          = COUPLED LINK LENGTH
C     C          = OUTPUT LINK LENGTH
C     D          = FIXED LINK LENGTH
C     P4         = MINIMUM OUTPUT ANGLE , RADIANS
C     P2         = MAXIMUM OUTPUT ANGLE , RADIANS
C     DPH        = OSCILLATION ANGLE OF OUTPUT LINK , RADIANS
C     U1         = MINIMUM TRANSMISSION ANGLE
C     P4D        = P4 IN DEGREES
C     P2D        = P2 IN DEGREES
C     C1         = RADIANS TO DEGREES CONVERTER PARAMETER
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(200) , B(200) , C(200) , U2(200) , U2D(200) ,
     1          T1(200) , T1D(200) , U4(200)
      COMMON A,B,C,D,P4,P2,P4D,P2D,DPH,U1,C1,PI,T1,U2,U2D,U4,U1D,T1D,
     1FUNCT(5),NSTEP1
C
      R = 1C.0D-07
      P2 = P4 + DPH
      IF (NSTEP1 .GT. 6) GO TC 136
      PRINT 135
  135 FORMAT (27X,'I', 9X,'U2D',12X,'T1D',12X,'A',14X,'B',14X,'C',/)
  136 CONTINUE
C
C     INITI ALIZE DO LCOP TO SOLVE SYSTEM OF EQUATIONS
C
      DO 5 I=1,195
      U2(I) = PI - T1(I) - P2
      C(I) = B*DSIN(T1(I))/DSIN(U2(I))
      B(I) = 0.5*(DSQRT(C(I)**2+D**2-2.0*C(I)*D*DCOS(P2))+DSQRT(C(I)**2+
     1D**2-2.0*C(I)*D*DCOS(P4)))
      A(I) = B - DSQRT(B(I)**2+C(I)**2-2.0*B(I)*C(I)*DCOS(U1))
      T1(I+1) = DARCOS((D-C(I)*DCOS(P2))/(B(I)+A(I)))
C
      P2D = P2 * C1
      P4D = P4 * C1
      U2D(I) = U2(I) * C1
      T1D(I) = T1(I) * C1
      IF (NSTEP1 .GT. 6) GO TO 150
      PRINT 140, I , U2D(I) , T1D(I) , A(I) , B(I) , C(I)
  140 FORMAT (23X,I5, 5X,5D15.6)
  150 CONTINUE
C
      IF (I .EQ. 1) GO TO 5
      Y1 = DABS(A(I)-A(I-1))
      Y2 = DABS(B(I)-B(I-1))
      Y3 = DABS(C(I)-C(I-1))
      Y4 = DABS(T1(I)-T1(I-1))
      Y5 = DABS(U2(I)-U2(I-1))
      IF (Y1 .LE. R .AND. Y2 .LE. R .AND. Y3 .LE. R .AND. Y4 .LE. R .AND
     1. Y5 .LE. R) GO TO 7
C
    5 CONTINUE
C
    7 CONTINUE
      I = I - 1
      K = I
      FUNCT(1) = U2(K) - PI + T1(K) + P2
      FUNCT(2) = C(K) - D*DSIN(T1(K))/DSIN(U2(K))
      FUNCT(3) = B(K) - 0.5D0*(DSQRT(C(K)**2+D**2-2.0D0*C(K)*D*DCOS(P2))
     1           + DSQRT(C(K)**2+D**2-2.0D0*C(K)*D*DCOS(P4)))
      FUNCT(4) = A(K) - B + DSQRT(B(K)**2+C(K)**2-2.0D0*B(K)*C(K)*
     1           DCOS(U1))
      FUNCT(5) = T1(K) - DARCOS((D-C(K)*DCOS(P2))/(B(K)+A(K)))
      T1(1) = T1(I-1)
      RETURN
      END
C
$ENDLIST
```

```
C
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     X                                                                     X
C     X PROGRAM FOR THE SYNTHESIS OF A FOUR-BAR-PLANE MACHANISM BY AN      X
C     X                    ITERATIVE METHOD                                 X
C     X              GIVEN DELAL , TH1D , TH3D AND D                        X
C     X                      CRANK-ROCKER                                   X
C     X                                                                     X
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
C
C       NOTATIONS :
C
C       A        = INPUT LINK
C       B        = COUPLER LINK
C       C        = OUTPUT LINK
C       D        = FIXED LINK
C       DELAL    = OSCILLATION ANGLE OF OUTPUT LINK
C       PI       = SYMBOL PI
C       DTR      = PARAMETER FOR CONVERTION OF DEGREE TO RADIANS
C       RTD      = PARAMETER FOR CONVERTION OF RADIAN TO DEGREES
C       PH2      = MAXIMUM OUTPUT ANGLE , IN RADIANS
C       PH4      = MINIMUM OUTPUT ANGLE , IN RADIANS
C       PH2D     = PH2 IN DEGREES
C       PH4D     = PH4 IN DEGREES
C       FUNCT    = VALUES OF THE FUNCTIONS , IT SHOULD BE EQUAL TO OR
C                  NEAR TO ZERO FOR ACCURACY , FUNCT(7)
C       NSTEP1   = FLAG FOR PRINTING OUTPUT OF ITERATIONS ; FOR NSTEP1
C                  .GT. 0 THERE WILL BE NO PRINTING OUT
C
C
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION A(200) , B(200) , C(200) , PH2(200) , PH2D(200) ,
       1          PH4(200) , PH4D(200) , U1(200) , U1D(200) , U2(200) ,
       2          U2D(200) , U4(200) , U4D(200)
        COMMON A,B,C,D,TH1,TH3,PH2,PH2D,PH4,FH4D,U1,U1D,U2,U2D,U4,U4D,DTR,
       1         RTD,DELAL,PI,FUNCT(7),NSTEP1
C
C       INITIALIZE BY INCORPORATING INITIAL INFORMATION
C
        PI = 4.0D0 * DATAN(1.0D0)
        DTR = PI/180.0D+00
        RTD = 180.0D+00/PI
        NUM = 46
        NSTEP1 = 0
        TH1 = 30.0D0 * DTR
        TH3 = 30.0D0 * DTR
        DEL = 8.0D0
        D = 11.0D0
C       INTRODUCE INITIAL VALUES OF UNKNOWNS
        PH4D( 1) = 57.0D0
        PH4(1) = PH4D( 1) * DTR
C
C       CALL SUBROUTINE NEGRO TO CARRY OUT THE SYNTHESIS OF
C       CRANK-ROCKER-PLANAR MECHANISM GIVEN DELAL , TH1D AND TH3D
C
        PRINT 99
   99   FORMAT (1H1)
        PRINT 100
  100   FORMAT (///)
        DO 5 L=1,NUM
        DEL = DEL + 2.0
        DELAL = DEL * DTR
        CALL NEGRO(I)
        PRINT 100
        PRINT 105
  105   FORMAT ( 5X,'L', 7X,'PH2D',11X,'PH4D',11X,'L1D',12X,'A',14X,'B',14
       1X,'C',14X,'DEL',/)
        PRINT 107, L , PH2D(I) , PH4D(I) , U1D(I) , A(I) , B(I) , C(I) ,
       1 DEL
  107   FORMAT (3X,I3, 3X,7D15.6)
        PRINT 100
        PRINT 110
  110   FORMAT (59X,'FUNCTION VALUES',/)
        PRINT 111, (FUNCT(K),K=1,7)
  111   FORMAT (15X,7D15.6)
        IF (NSTEP1 .GT. 0) GO TO 5
        PRINT 99
        PRINT 100
    5   CONTINUE
C
C
        PRINT 99
        STOP
        END
CSEJECT
C
C       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.
C
        SUBROUTINE NEGRO(M)
C
C       THIS SUBROUTINE SOLVE IN AN ITERATIVE WAY A SYSTEM OF EIGHT
C       TRIGONOMETRIC EQUATIONS , THIS SYSTEM REPRESENT THE SYNTHESIS
C       EQUATIONS FOR A CRANK-ROCKER MECHANISM PRESCRIBING THE OSCILLATION
C       ANGLE OF OUTPUT LINK , AND INPUT ANGLES AT LIMIT POSITIONS OF THE
C       INPUT LINK
C
C       NOTATIONS :
C
C       AA       = INPUT LINK
C       BB       = COUPLER LINK
C       CC       = OUTPUT LINK
C       DD       = FIXED LINK
C       DELAL    = OSCILLATION ANGLE OF OUTPUT LINK
C       PI       = SYMBOL PI
C       DTR      = PARAMETER FOR CONVERTION OF DEGREE TO RADIANS
```

```
C     RTD      = PARAMETER FOR CONVERSION OF RADIAN TO DEGREES
C     R8       = ACCURACY FOR THE SOLUTIONS
C     P4       = MINIMUM OUTPUT ANGLE
C     P2       = MAXIMUM OUTPUT ANGLE
C     P4D      = P4 IN DEGREES
C     P2D      = P2 IN DEGREES
C     FUNCT    = VALUES OF THE FUNCTIONS ; IT SHOULD BE EQUAL TO OR
C                NEAR TO ZERO FOR ACCURACY . FUNCT(7)
C     NSTEP1   = FLAG FOR PRINTING OUTPUT OF ITERATIONS ; FOR NSTEP1
C                .GT. 0 THERE WILL BE PRINTING OUT
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION AA(200) , BB(200) , CC(200) , P2(200) , P2D(200) ,
     1          P4(200) , P4D(200) , U1(200) , U1D(200) , U2(200) ,
     2          U2D(200) , U4(200) , U4D(200)
      COMMON AA,BB,CC,DD,T1,T3,P2,P2D,P4,P4D,U1,U1D,U2,U2D,U4,U4D,DTR,
     1       RTD,DELAL,PI,FUNCT(7),NSTEP1
C
C
      R8 = 10.0D-08
      IF (NSTEP1 .GT. 0) GO TO 100
      PRINT 199
  199 FORMAT ( 5X,'I', 7X,'P2D',12X,'P4D',12X,'U2D',12X,'U4D',14X,'A',14
     1X,'B',14X,'C',/)
  100 CONTINUE
C
C     INITIALIZE DO LOOP TO SOLVE SYSTEM OF EQUATIONS
C
      DO 5 I=1,195
      P2(I) = P4(I) + DELAL
      U2(I) = PI - T1 - P2(I)
      U4(I) = T1 - T3 + DELAL + U2(I)
      CC(I) = DD*DSIN(T3)/DSIN(U4(I))
      BB(I) = 0.5*CC(I)*(DSIN(P2(I))/DSIN(T1)+DSIN(P4(I))/DSIN(T3))
      AA(I) = (DD-CC(I)*DCOS(P2(I))-BB(I)*DCOS(T1))/DCOS(T1)
      P4(I+1) = DARSIN((BB(I)-AA(I))*DSIN(T3)/CC(I))
C
      P4D(I) = P4(I) * RTD
      P2D(I) = P2(I) * RTD
      U2D(I) = U2(I) * RTD
      U4D(I) = U4(I) * RTD
      IF (NSTEP1 .GT. 0) GO TO 201
      PRINT 200, I , P2D(I) , P4D(I) , U2D(I) , U4D(I) , AA(I) , BB(I) ,
     1CC(I)
  200 FORMAT ( 3X,I3, 3X,7D15.6)
  201 CONTINUE
C
      IF (I .EQ. 1) GO TO 6
      Y1 = DABS(AA(I)-AA(I-1))
      Y2 = DABS(BB(I)-BB(I-1))
      Y3 = DABS(CC(I)-CC(I-1))
      Y4 = DABS(P4(I)-P4(I-1))
      Y5 = DABS(P2(I)-P2(I-1))
      Y6 = DABS(U2(I)-U2(I-1))
      Y7 = DABS(U4(I)-U4(I-1))
C
      IF (Y1 .LE. R8 .AND. Y2 .LE. R8 .AND. Y3 .LE. R8 .AND. Y4 .LE. R8
     1   .AND. Y5 .LE. R8 .AND. Y6 .LE. R8 .AND. Y7 .LE. R8 ) GO TO 7
    6 CONTINUE
    5 CONTINUE
    7 CONTINUE
C
      N = I - 1
      I = I - 1
      U1(I) = DARCOS((BB(I)**2+CC(I)**2-(DD-AA(I))**2)/(2.0*BB(I)*CC(I))
     1)
      U1D(I) = U1(I) * RTD
C     INITIALIZE DETERMINING THE VALUES OF THE FUNCTIONS
      FUNCT(1) = P2(I) - P4(I) - DELAL
      FUNCT(2) = U2(I) - PI + T1 + P2(I)
      FUNCT(3) = U4(I) - T1 + T3 - DELAL - U2(I)
      FUNCT(4) = CC(I) - DD*DSIN(T3)/DSIN(U4(I))
      FUNCT(5) = BB(I) - 0.5D0*CC(I)*(DSIN(P2(I))/DSIN(T1)+DSIN(P4(I))/
     1           DSIN(T3))
      FUNCT(6) = AA(I) - (DD - CC(I)*DCOS(P2(I)) - BB(I)*DCOS(T1))/
     1           DCOS(T1)
      FUNCT(7) = P4(I) - DARSIN((BB(I) - AA(I))*DSIN(T3)/CC(I))
C
      P4(1) = P4(N)
C
      RETURN
      END
C
```

```
C
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     X                                                                  X
C     X      PROGRAM FOR THE SYNTHESIS OF A FOUR-BAR-PLANE MECHANISM     X
C     X                  BY OPTIMIZATION TECHNIQUES                      X
C     X                      MARQUARDT' S METHOD                         X
C     X                      GAUSS-NEWTON METHOD                         X
C     X                         CRANK-ROCKER                             X
C     X                GIVEN DPHD , TH1D , TH3D AND D                    X
C     X                                                                  X
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
C
C     NOTATIONS :
C
C     A          = LENGTH OF INPUT LINK , UNKNOWN
C     B          = LENGTH OF COUPLER LINK , UNKNOWN
C     C          = LENGTH OF OUTPUT LINK , UNKNOWN
C     D          = LENGTH OF FIXED LINK , KNOWN
C     DPH,DPHD   = OSCILLATION ANGLE OF OUTPUT LINK
C                  KNOWN , RADIANS AND DEGREES
C     PH4,PH4D   = INTERNAL ANGULAR POSITION AT CONFIGURATION
C                  IV OF OUTPUT LINK , UNKNOWN , RADIANS AND
C                  DEGREES
C     PH2,PH2D   = INTERNAL ANGULAR POSITION AT CONFIGURATION
C                  II OF OUTPUT LINK , UNKNOWN , RADIANS AND
C                  DEGREES
C     F1TM       = FUNCTION VALUES , F1TM(I)
C     TH1,TH1D   = INTERNAL ANGLE OF INPUT LINK AT CONFIGURATION
C                  II , KNOWN , RADIANS AND DEGREES
C     U2,U2D     = MAXIMUM TRANSMISSION ANGLE , UNKNOWN ,
C                  RADIANS AND DEGREES
C     U4,U4D     = TRANSMISSION ANGLE AT CONFIGURATION IV ,
C                  UNKNOWN , RADIANS AND DEGREES
C     PI         = SYMBOL FOR PI
C     LP         = OUTPUT
C     IPRINT     = PRINTITNG FLAG , FOR IPRINT .GT. 0 THERE IS
C                  PRINTING
C     DTR        = PARAMETER TO CONVERT DEGREES TO RADIANS
C                  EQUAL TO PI/180
C     RTD        = PARAMETER TO CONVERT RADIANS TO DEGREES
C                  EQUAL TO 180/PI
C     NUMUN      = NUMBER OF UNKNOWNS
C     F          = INITIAL SOLUTIONS FOR UNKNOWNS , F(NUMUN)
C     MATRX      = PRINTITNG OUTPUT FOR UNKNOWNS
C     PHI        = SUM OF SQUARES , THE SMALLER PHI THE MORE
C                  ACCURATE THE SOLUTIONS
C     NTRAC      = PRINTING OUTPUT FROM STEPT , NTRAC = 0 , NO
C                  PRINTING , NTRAC .GT. 0 , PRINTING ALL OF THE
C                  ITERATIONS
C     RELOF      = ACCURACY FOR CONVERGENCE
C     NNN        = NUMBER OF ITERATIONS CARRIED OUT BY CHANGING DPHD
```

```
C     MASK       = THE UNKNOWNS CAN BE FIXED BY ASSIGNING MASK A VALUE
C                  OF 1 , SAY MASK(4) = 1 , THIS MEANS THAT VARIABLE
C                  4 HAS BEEN FIXED AND THE VALUE BE GIVEN
C
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/CSTEP/F(20),XMAX(20),XMIN(20),DELTX(20),DELMN(20),
     1             ERR(20,21),FOBJ,NV,NTRAC,MATRX,MASK(20),
     2             NFMAX,NFLAL,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW
      COMMON/NLLS3/FLAPB,FNU,RELOF,METHD,KALCP,KORDF,MAXIT,LEQU,
     1             MXSUB,MXUPD
      COMMON/CDAT/FIT(300),W(300),YSIG(300),NPTS
      COMMON A,B,C,D,TH1,PH4,PH2,DPH,U2,TH3,U4,PI,IPRINT,LP
      EXTERNAL FUNK
      DATA IN,LP/5,6/
C
C
C     INTRODUCE CONSTANT INFORMATION AND VALUES OF THE
C     KNOWN PARAMETERS
C
      PI = 4.0D0 * DATAN(1.0D0)
      DTR = PI/180.0D0
      RTD = 180.0D0/PI
      NPR = 0
      NUMUN = 7
      DPHD = 8.0D0
      TH1D = 30.0D0
      TH1 = TH1D * DTR
      TH3D = 30.0D0
      TH3 = TH3D * DTR
      D = 11.0D0
      IPRINT = 0
C
C     INITIALIZE DO LOOP TO FIND SOLUTIONS FOR THE DIFFERENT
C     VALUES OF THE OSCILLATION ANGLE DPHD
C
      NNN = 40
      DO 1000 KK=1,NNN
C
C     CALL SUBROUTINE STSET FOR ALLOCATION MEMORY OF ARRAY
C     INTRODUCE THE VARIATION OF DPHD
C     INTRODUCE INITIAL VALUES OF UNKNOWNS
C
      CALL STSET
      DPHD = DPHD + 2.0D0
      DPH = DPHD * DTR
      IF ( KK .GT. 1)  GO TO 199
      F(1) = 6.481188D0
      F(2) = 9.52628D0
      F(3) = 5.52101D0
      F(4) = 85.0D0 * DTR
      F(5) = 95.0D0 * DTR
      F(6) = 65.0D0 * DTR
      F(7) = 55.0D0 * DTR
```

```
  199 CONTINUE
C
C     PRINT OUT INPUT INFORMATION
C
      WRITE (LP,200)
  200 FORMAT (1H1)
      WRITE (LP,201)
  201 FORMAT (////)
      WRITE (LP,202)
  202 FORMAT (59X,'INPUT INFORMATION')
      WRITE (LP,203)
  203 FORMAT (59X,'XXXXXXXXXXXXXXXXX')
      WRITE (LP,201)
      WRITE (LP,204)
  204 FORMAT (65X,'NUMUN',/)
      WRITE (LP,205) NUMUN
  205 FORMAT (63X,I5)
      WRITE (LP,201)
      WRITE (LP,208)
  208 FORMAT (60X,'KNOWN PARAMETERS')
      WRITE (LP,209)
  209 FORMAT (45X,'D',14X,'TH1D',11X,'TH3D',11X,'DPHD',/)
      WRITE (LP,210) D , TH1D , TH3D , DPHD
  210 FORMAT (35X,4F15.5)
      WRITE (LP,201)
      WRITE (LP,213)
  213 FORMAT (59X,'UNKNOWN PARAMETERS')
      WRITE (LP,214)
  214 FORMAT (37X,'A', 9X,'B', 9X,'C', 9X,'U2D', 7X,'U4D', 7X,'PH2D', 6X
     1,'PH4D',/)
      Y1 = F(4) * RTD
      Y2 = F(5) * RTD
      Y3 = F(6) * RTD
      Y4 = F(7) * RTD
      WRITE (LP,215) (F(I),I=1,3) , Y1 , Y2 , Y3 , Y4
  215 FORMAT (31X,7F10.4)
C
C     START CALLING SUBROUTINES TO SOLVE OR OPTIMIZE SET OF EQUATIONS
C
      KORDF = 2
      RELDF = 1.0D-08
      MATRX = 0
      NTRAC = 0
      NPTS = NUMUN
      NV = NUMUN
      DO 50 J=1,NPTS
      Y(J) = 0.0D0
      YSIG(J) = 1.0D0
   50 CONTINUE
C
C     INITIALIZE MARQUARDT'S METHOD
C
      XMIN(4) = 30.0 * DTR
      XMAX(4) = 90.0 * DTR
      CALL STEPT (FUNK)
C
C     INITIALIZE GAUSS-NEWTON METHOD
C
      METHD = 0
      CALL STEPT (FUNK)
 1000 CONTINUE
C
      WRITE (LP,200)
      STOP
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE FUNK (F1TM)
C
C     THIS SUBROUTINE HAS THE SET OF EQUATIONS TO
C     BE OPTIMIZED
C
C     NOTATIONS :
C
C     A           = LENGTH OF INPUT LINK , UNKNOWN
C     B           = LENGTH OF COUPLER LINK , UNKNOWN
C     C           = LENGTH OF OUTPUT LINK , UNKNOWN
C     D           = LENGTH OF FIXED LINK , KNOWN
C     DPH,DPHD    = OSCILLATION ANGLE OF OUTPUT LINK
C                   KNOWN , RADIANS AND DEGREES
C     PH4,PH4D    = INTERNAL ANGULAR POSITION AT CONFIGURATION
C                   IV OF OUTPUT LINK , UNKNOWN , RADIANS AND
C                   DEGREES
C     PH2,PH2D    = INTERNAL ANGULAR POSITION AT CONFIGURATION
C                   II OF OUTPUT LINK , UNKNOWN , RADIANS AND
C                   DEGREES
C     F1TM        = FUNCTION VALUES , F1TM(1)
C     TH1,TH1D    = INTERNAL ANGLE OF INPUT LINK AT CONFIGURATION
C                   II , KNOWN , RADIANS AND DEGREES
C     U2,U2D      = MAXIMUM TRANSMISSION ANGLE , UNKNOWN ,
C                   RADIANS AND DEGREES
C     U4,U4D      = TRANSMISSION ANGLE AT CONFIGURATION IV ,
C                   UNKNOWN , RADIANS AND DEGREES
C     PI          = SYMBOL FOR PI
C     LP          = OUTPUT
C     IPRINT      = PRINITING FLAG , FOR IPRINT .GT. 0 THERE IS
C                   NO PRINTING
C
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION F1TM(1)
      COMMON/CSTEP/F(20),XMAX(20),XMIN(20),DELTX(20),DELMN(20),
     1             ERR(20,21),FOBJ,NV,NTRAC,MATRX,MASK(20),
     2             NFMAX,NFLAL,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW
      COMMON A,B,C,D,TH1,PH4,PH2,DPH,U2,TH3,U4,PI,IPRINT,LP
C
      A = F(1)
```

```
      B = F(2)
      C = F(3)
      U2 = F(4)
      U4 = F(5)
      PH2 = F(6)
      PH4 = F(7)
C
C     INTRODUCE EQUATIONS FOR OPTIMIZATIONS
C
      X1 = DSIN(TH3)
      X2 = DSIN(TH1)
      X3 = BCOS(TH1)
C
      FITN(1) = PH4 - PH2 + DPH
      FITN(2) = U2 - PI + TH1 + PH2
      FITN(3) = U4 - TH1 + TH3 - DPH - U2
      FITN(4) = C*DSIN(U4) - B*X1
      FITN(5) = B - 0.5D0*C*(DSIN(PH2)/X2 + DSIN(PH4)/X1)
      FITN(6) = A - (B - C*DCOS(PH2) - B*X3)/X3
      FITN(7) = DSIN(PH4) - ((B-A)*X1/C)
C
      IF (IPRINT .GT. 0) WRITE(LP,100) (FITN(K),K=1,7)
  100 FORMAT (7D18.6)
C
      RETURN
      END
$ENDLIST
```

```
C
C
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     X                                                                      X
C     X     PROGRAM FOR THE SYNTHESIS OF A FOUR-BAR-PLANE MECHANISM           X
C     X                  BY OPTIMIZATION TECHNIQUES                          X
C     X                     MARQUARDT ' S METHOD                             X
C     X                     GAUSS-NEWTON METHOD                              X
C     X                        CRANK-ROCKER                                  X
C     X                  GIVEN U1 , DPH , PH4 AND D                          X
C     X                                                                      X
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
C
C     NOTATIONS :
C
C     A          = LENGHT OF INPUT LINK , UNKNOWN
C     B          = LENGTH OF COUPLER LINK , UNKNOWN
C     C          = LENGTH OF OUTPUT LINK , UNKNOWN
C     D          = LENGTH OF FIXED LINK , KNOWN
C     PH2,PH2D   = INTERNAL ANGULAR POSITION AT CONFIGURATION I OF
C                  OUTPUT LINK , UNKNOWN , RADIANS AND DEGREES
C     PH4,PH4D   = INTERNAL ANGULAR POSITION AT CONFIGURATION IV OF
C                  OUTPUT LINK , KNOWN , RADIANS AND DEGREES
C     TH1,TH1D   = INTERNAL ANGULAR POSITION AT CONFIGURATION II OF
C                  INPUT LINK , UNKNOWN , RADIANS AND DEGREES
C     DPH,DPHD   = DIFFERENCE BETWEEN PH4 AND PH2 , KNOWN
C     U2,U2D     = MAXIMUM TRASMISION ANGLE , UNKNOWN
C     PI         = PARAMETER PI EQUAL TO 4*DATAN(1.0)
C     U1,U1D     = MINIMUM TRASMISSION ANGLE , KNOWN
C     DTR        = PARAMETER TO CONVERT DEGREES TO RADIANS , IT IS
C                  EQUAL TO PI/180
C     RTD        = PARAMETER RO CONVERT RADIANS TO DEGREES , IT IS
C                  EQUAL TO 180/PI
C     NUMUN      = NUMBER OF UNKNOWNS
C     F          = INITIAL SOLUTIONS FOR UNKNOWNS , F(NUMUN)
C     MATRX      = PRINTING OUTPUTS FROM STEPT , MATRX = 0 , NO PRINTING
C     PHI        = SUM OF SQUARES , THE SMALLER PHI THE MORE ACCURATE
C                  THE SOLUTIONS
C     NTRAC      = PRINTING OUTPUTS FROM STEPT , NTRAC = 0 , NO PRINTING
C                  NTRAC .GT. 0 , PRINTING ALL OF THE ITERATIONS
C     NPR        = FLAG FOR PRINTING VALUES OF F1TM , NPR .GT. 0 THERE
C                  ARE PRINTING
C     F1TM       = FUNCTION VALUES , F1TM(1)
C     RELDF      = ACCURACY FOR CONVERGENCE
C     NNN        = NUMBER OF ITERATIONS CARRIED OUT BY CHANGING DPHD
C     MASK       = THE VALUE OF THE UNKNOWN CAN BE FIXED BY ASSIGNING
C                  MASK A VALUE OF 1 , SAY MASK(5)=1 , THIS MEANS
C                  THAT VARIABLE 5 HAS BEEN FIXED AND THE VALUE
C                  BE GIVEN
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/CSTEP/F(20),XMAX(20),XMIN(20),DELTX(20),DELMN(20),
     1             ERR(20,21),FOBJ,NV,NTRAC,MATRX,MASK(20),
     2             NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW
      COMMON/NLLS3/FLAPB,FNU,RELDF,METHD,KALCP,KORDF,MAXIT,LEQU,
     1             MXSUB,MXUPD
      COMMON/CDAT/FIT(300),W(300),YSIG(300),NPTS
      COMMON A,B,C,D,TH1,U2,PH4,DPH,PH2,U1,PI,NPR,LP
      EXTERNAL FUNK
      DATA IN,LP/5,6/
C
C
C     INTRODUCE CONSTANT INFORMATION AND  VALUES OF THE
C     KNOWN PARAMETERS
C
      PI = 4.0D0 * DATAN(1.0D0)
      DTR = PI/180.0D0
      RTD = 180.0D0/PI
      NPR = 0
      NUMUN = 6
      DPHD = 0.0D0
      D = 11.0D0
      PH4D = 40.0D0
      PH4 = PH4D * DTR
      U1D = 35.0D0
      U1 = U1D * DTR
C
C     INITIALIZE DO LOOP TO FIND SOLUTIONS FOR THE DIFFERENT VALUES
C     OF THE OSCILLATION ANGLE DPHD
C
      NNN = 40
      DO 1000 KK=1,NNN
C
C     CALL SUBROUTINE STSET FOR ALLOCATION MEMORY OF ARRAY
C     INTRODUCE THE VARIATION OF DPHD
C     INTRODUCE INITIAL VALUES OF UNKNOWNS
C
      CALL STSET
      DPHD = DPHD + 2.0D0
      DPH = DPHD * DTR
      IF (KK .GT. 1) GO TO 195
      F(1) = 0.1009123D+01
      F(2) = 0.1215485D+02
      F(3) = 0.1711296D+02
      F(4) = 20.0D0
      F(4) = F(4) * DTF
      F(5) = 40.0D0
      F(5) = F(5) * DTR
      F(6) = 50.0D0
      F(6) = F(6) * DTR
  199 CONTINUE
C
C     PRINT OUT INPUT INFORMATION
C
      WRITE (LP,200)
```

```
  200 FORMAT (1H1)
      WRITE (LP,201)
  201 FORMAT (////)
      WRITE (LP,202)
  202 FORMAT (59X,'INPUT INFORMATION')
      WRITE (LP,203)
  203 FORMAT (59X,'XXXXXXXXXXXXXXXXX')
      WRITE (LP,201)
      WRITE (LP,204)
  204 FORMAT (65X,'NUMUN',/)
      WRITE (LP,205) NUMUN
  205 FORMAT (63X,I5)
      WRITE (LP,201)
      WRITE (LP,208)
  208 FORMAT (60X,'KNOWN PARAMETERS')
      WRITE (LP,209)
  209 FORMAT (45X,'D',14X,'PH4D',11X,'U1D',12X,'DPHD',/)
      WRITE (LP,210) D , PH4D , U1D , DPHD
  210 FORMAT (35X,4F15.5)
      WRITE (LP,201)
      WRITE (LP,213)
  213 FORMAT (59X,'UNKNOWN PARAMETERS')
      WRITE (LP,214)
  214 FORMAT (42X,'A', 9X,'B', 9X,'C', 9X,'TH1', 7X,'U2', 8X,'PH2',/)
      AD = F(1)
      BD = F(2)
      CD = F(3)
      TH1D = F(4) * RTD
      U2D = F(5) * RTD
      PH2D = F(6) * RTD
      WRITE (LP,215) AD , BD , CD , TH1D , U2D , PH2D
  215 FORMAT (36X,6F10.4)
C
C     START CALLING SUBROUTINE TO SOLVE AND MINIMIZE SET OF EQUATIONS
C
      KORDF = 2
      RELDF = 1.0D-08
      MATRX = 0
      NTRAC = 0
      NPTS = NUMUN
      NV = NUMUN
      DO 50 J=1,NPTS
      Y(J) = 0.0D0
      YSIG(J) = 1.0D0
   50 CONTINUE
C
C     INITIALIZE MARQUARDT'S METHOD
C
      XMIN(5) = 30.0D0 * DTR
      XMAX(5) = 90.0D0 * DTR
      CALL STEPT (FUNK)
C
C     INITIALIZE GAUSS - NEWTON METHOD
C
      METHD = 0

      CALL STEPT (FUNK)
 1000 CONTINUE
C
      WRITE (LP,200)
      STOP
      END
```

```
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE FUNK (F1TM)
C
C     THIS SUBROUTINE HAS THE SET OF EQUATIONS
C
C     NOTATIONS :
C
C     A          = LENGHT OF INPUT LINK , UNKNOWN
C     B          = LENGTH OF COUPLER LINK , UNKNOWN
C     C          = LENGTH OF OUTPUT LINK , UNKNOWN
C     D          = LENGTH OF FIXED LINK , KNOWN
C     PH2,PH2D   = INTERNAL ANGULAR POSITION AT CONFIGURATION II OF
C                  OUTPUT LINK , UNKNOWN , RADIANS AND DEGREES
C     PH4,PH4D   = INTERNAL ANGULAR POSITION AT CONFIGURATION IV OF
C                  OUTPUT LINK , KNOWN , RADIANS AND DEGREES
C     TH1,TH1D   = INTERNAL ANGULAR POSITION AT CONFIGURATION II OF
C                  INPUT LINK , UNKNOWN , RADIANS AND DEGREES
C     DPH,DPHD   = DIFFERENCE BETWEEN PH4 AND PH2 , KNOWN
C     U2,U2D     = MAXIMUM TRANSMISION ANGLE , UNKNOWN
C     PI         = PARAMETER PI EQUAL TO 4*DATAN(1.0)
C     U1,U1D     = MINIMUM TRANSMISSION ANGLE , KNOWN
C     F1TM       = FUNCTION VALUES , F1TM(1)
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION F1TM(1)
      COMMON/CSTEP/F(20),XMAX(20),XMIN(20),DELTX(20),DELMN(20),
     1             ERR(20,21),FOBJ,NV,NTRAC,MATRX,MASK(20),
     2             NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW
      COMMON A,B,C,D,TH1,U2,PH4,DPH,PH2,U1,PI,NPR,LP
C
      A = F(1)
      B = F(2)
      C = F(3)
      TH1 = F(4)
      U2 = F(5)
      PH2 = F(6)
C
C     INTRODUCE SET OF EQUATIONS FOR OPTIMIZATION
C
      X1 = PH2 - DPH
      X2 = PH2 - PI
      X3 = DCOS(PH2)
      X4 = DCOS(U1)
C
      F1TM(1) = X1 - PH4
      F1TM(2) = U2 + TH1 + X2
      F1TM(3) = C * DSIN(U2) - D * DSIN(TH1)
      F1TM(4) = B - 0.5D0*DSQRT(C**2+D**2-2.0D0*C*D*X3)
     1          - 0.5D0*DSQRT(C**2+D**2-2.0D0*C*D*DCOS(PH4))
      F1TM(5) = (A-B)**2 - D**2 - C**2 + 2.0D0*B*C*X4
      F1TM(6) = (B+A)*DCOS(TH1) - D + C*X3

C
      IF (NPR .GT. 0) WRITE(LP,1001)(F1TM(K),K=1,6)
 1001 FORMAT (6D20.6)
C
      RETURN
      END
```

```
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE STEPT (FUNK)
C
C  INTERFACE TO MAKE MARQ LOOK LIKE STEPT.
C
C  TO USE THIS ROUTINE, SET THE VALUE OF LPCOL AND THE DIMENSIONS OF
C  THE ARRAYS P, FITSV, FIT, Y, AND YSIG.  THE DIMENSIONS ARE....
C       P(LPCOL,NVMAX), FITSV(LPCOL), FIT(LPCOL), Y(LPCOL), YSIG(LPCOL)
C  WHERE LPCOL IS THE MAXIMUM VALUE OF NPTS AND NVMAX IS THE MAXIMUM
C  VALUE OF NV.  (IF LEQU.EQ.1, YSIG MAY BE DIMENSIONED YSIG(1).)
C  COMMON/CDAT/ DOES NOT APPEAR IN ANY ROUTINE OF THE MARQ PACKAGE
C  OTHER THAN THIS ONE, SO THAT THIS IS THE ONLY ONE WHICH MUST BE
C  RECOMPILED WHEN LPCOL AND THE DIMENSIONS OF THE ARRAYS ARE CHANGED.
C
C  THE FOLLOWING EXTERNAL STATEMENT IS REQUIRED BY SOME COMPILERS
C  (WATFIV, FOR EXAMPLE) AND FORBIDDEN BY OTHERS (MODCOMP II).
      EXTERNAL FUNK
C
      DOUBLE PRECISION P
      DOUBLE PRECISION Y,YSIG
      DOUBLE PRECISION FIT,FITSV
C
      DIMENSION P(100,10),FITSV(100)
      COMMON /CDAT/ FIT(300),Y(300),YSIG(300),NPTS
C
      LPCOL=1
      LPCOL=100
C
      CALL MARQ (FUNK,Y,YSIG,NPTS,FIT,FITSV,P,LPCOL)
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE STSET
C
C  STSET SETS SOME INPUT QUANTITIES TO DEFAULT VALUES, FOR MARQ.
C
C  NOTE,.........
C  THIS VERSION OF STSET MAY ALSO BE USED WITH STEPIT, SIMPLEX,
C  STP, KAUPE, AND MINF.
C
C  USAGE,....
C  CALL STSET,  THEN SET SOME INPUT QUANTITIES (NV AND NPTS, AT LEAST)
C  AND RESET ANY OF THOSE SET IN STSET (BETTER VALUES OF X, ETC.)
C  BEFORE CALLING MARQ OR THE STEPT-MARQ INTERFACE ROUTINE.
C
      DOUBLE PRECISION XMAX,XMIN,DELTX,DELMN,ERR,FOBJ,FLAMB,FNU,
     X     RELDF,RZERO,HUGE
      DOUBLE PRECISION X
C
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),DELMN(20),
     *     ERR(20,21),FOBJ,NV,NTRAC,MATRX,MASK(20),
     *     NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW
      COMMON /NLLS3/ FLAMB,FNU,RELDF,METHD,KALCP,KORDF,MAXIT,LEQU,
     X     MXSUB,MXUPD
C
C                              KW ...  LOGICAL UNIT NUMBER OF THE PRINTER
      KW=6
      NVMAX=20
C                              THE USER MUST SET NV AFTER CALLING STSET.
      NV=-1
      HUGE=1.E30
      RZERO=0.
      NTRAC=0
      NFMAX=32767
      MAXIT=20
      MXSUB=25
      METHD=1
      KALCP=0
      KORDF=1
      LEQU=0
      NFLAT=1
      MATRX=106
      NXTRA=0
      FLAMB=1.
      FNU=10.
      RELDF=1.E-8
C
      DO 10 JX=1,NVMAX
C       X(JX)=RZERO
        XMAX(JX)=HUGE
        XMIN(JX)=-HUGE
        DELTX(JX)=RZERO
        DELMN(JX)=RZERO
C
        MASK(JX)=0
   10 CONTINUE
C
      RETURN
      END
```

```
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE FOFX (JPT,NV,X,F)
C
C  THIS IS A DUMMY VERSION OF SUBROUTINE FOFX.
C  A NON-DUMMY VERSION OF FOFX MAY BE USED (OPTIONALLY) TO SUPPLY
C  TO MARQ VALUES OF THE FUNCTION BEING FITTED, INSTEAD OF USING A
C  -FUNK- SUBROUTINE TO DO THIS.  THE USE OF FOFX REQUIRES
C  SUBSTANTIALLY MORE OVERHEAD TIME DURING EXECUTION, BUT SAVES
C  CONSIDERABLE STORAGE BY NOT REQUIRING THAT THE JACOBIAN MATRIX, P,
C  BE STORED.  THE MINI-MARQ PACKAGE REQUIRES THE USE OF FOFX.
C  TO SEE HOW TO USE A NON-DUMMY FOFX, SEE THE LISTING OF MARQ.
C
      DOUBLE PRECISION X,F
      DIMENSION X(20)
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE CALCD (JPT,P,LPCOL)
C
C  THIS IS A DUMMY VERSION OF SUBROUTINE CALCD.
C  A NON-DUMMY VERSION OF CALCD MAY BE USED (OPTIONALLY) TO SUPPLY
C  TO MARQ OR MINI-MARQ ANALYTIC VALUES OF THE ELEMENTS OF THE
C  JACOBIAN MATRIX, INSTEAD OF CALCULATING THEM USING FINITE
C  DIFFERENCES.  MOST USERS USE FINITE DIFFERENCES MOST OF THE TIME.
C  TO SEE HOW TO USE A NON-DUMMY CALCD, SEE THE LISTING OF MARQ.
C
      DIMENSION P(LPCOL,1)
      RETURN
      END
```

```
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE MARQ (FUNK,Y,YSIG,NPTS,FIT,FITSV,P,LPCOL)
C
C  MARQ 2.7         A.N.S.I. STANDARD FORTRAN          JULY 1979
C  COPYRIGHT (C) 1979 J. P. CHANDLER
C
C  J. P. CHANDLER AND LEON W. JACKSON,
C                DEPARTMENT OF COMPUTING AND INFORMATION SCIENCES
C                OKLAHOMA STATE UNIVERSITY, STILLWATER, OKLAHOMA 74074
C
C  MARQ PERFORMS A NONLINEAR LEAST SQUARES FIT OF A USER-SUPPLIED
C  FUNCTION TO A GIVEN SET OF DATA, USING MARQUARDT-S METHOD, OR THE
C  GAUSS-NEWTON METHOD, OR A MODIFIED GAUSS-NEWTON METHOD.
C  D. W. MARQUARDT, J.SOC.IND.APPL.MATH. 11 (1963) 431-441
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  INPUT QUANTITIES.....  FUNK,X(*),XMAX(*),XMIN(*),DELMN(*),NF,NTRAC,
C                         MASK(*),MATRX,NFMAX,NFLAT,KW,
C                         Y(*),YSIG(*),NPTS,LPCOL,
C                         FLAMB,FNU,RELDF,METHD,KALCP,KORDF,
C                         MAXIT,LEQU,MXSUB
C  OUTPUT QUANTITIES....  X(*),FOBJ,ERR(*,*),KFLAG,FIT(*)
C  UNUSED QUANTITIES (INCLUDED FOR COMPATIBILITY WITH STEPIT COMMON)....
C                         DELTX(*),JVARY,NXTRA,NOREP,KERFL
C
C     FUNK        --  THE NAME OF THE FUNCTION CALLED TO OBTAIN
C                       THE FITTED VALUES IF KALCP=0
C     X(JX)       --  THE JX-TH PARAMETER
C     XMAX(JX)    --  THE UPPER LIMIT FOR X(JX)
C     XMIN(JX)    --  THE LOWER LIMIT FOR X(JX)
C     DELMN(JX)   --  THE CONVERGENCE TOLERANCE FOR X(JX)
C     FOBJ        --  RETURNS THE FINAL VALUE OF PHI, THE WEIGHTED
C                       SUM OF SQUARES (PHI IS BEING MINIMIZED AS A
C                       FUNCTION OF THE X(JX))
C     NV          --  THE NUMBER OF PARAMETERS
C     NTRAC       --  USER PRINT CONTROL
C                       =-3, NO OUTPUT EXCEPT FATAL ERROR MESSAGES
C                       =-2, NO OUTPUT EXCEPT DIAGNOSTIC MESSAGES
C                       =-1, STANDARD OUTPUT EXCEPT FINAL FIT VALUES
C                       = 0, STANDARD OUTPUT
C                       = 1, ALSO PRINTS RESULTS OF EACH ITERATION
C                       = 2, ALSO PRINTS THE COEFFICIENT MATRIX, QSAV
C                       = 3, ALSO PRINTS THE JACOBIAN MATRIX, P
C     MATRX       --  NONZERO IF STATISTICAL ERRORS ARE TO BE COMPUTED
C     MASK(JX)    --  NONZERO IF X(JX) IS TO BE HELD FIXED
C     NFMAX       --  THE MAXIMUM NUMBER OF FUNCTION COMPUTATIONS
C     NFLAT       --  NONZERO IF THE SEARCH IS TO TERMINATE WHEN TWO
C                       ITERATIONS GIVE IDENTICAL VALUES OF PHI
C     KW          --  THE LOGICAL UNIT NUMBER OF THE PRINTER
C     ERR(JX,KX)  --  RETURNS THE ERROR MATRIX
C     KFLAG       --  RETURNED .GT. ZERO FOR A NORMAL EXIT,
```

```
C                     RETURNED .LT. ZERO FOR AN ABNORMAL EXIT
C
C     Y(JPT)      --  THE JPT-TH DATA ORDINATE
C     YSIG(JPT)   --  THE EXPECTED ERROR IN Y(JPT)
C     NPTS        --  THE NUMBER OF DATA OBSERVATIONS
C     FIT(JPT)    --  THE JPT-TH FITTED VALUE
C     FITSV       --  A SCRATCH VECTOR OF NPTS VALUES
C     P(JPT,JX)   --  THE FIRST PARTIAL DERIVATIVE
C                         OF FIT(JPT) WITH RESPECT TO X(JX)
C     LPCOL       --  THE FIRST DIMENSION OF THE ARRAY CONTAINING P
C                       (LPCOL MUST BE .GE. NPTS IF KALCP IS .GE. ZERO)
C     FLAMB       --  MARQUARDT-S LAMBDA, THE RELATIVE AMOUNT BY WHICH
C                         THE DIAGONAL COEFFICIENTS OF THE NORMAL
C                         EQUATIONS ARE AUGMENTED
C     FNU         --  MARQUARDT-S NU, THE FACTOR BY WHICH FLAMB IS
C                         CHANGED
C     RELDF       --  DETERMINES THE MAGNITUDE OF THE DIFFERENCING STEP
C     METHD       --  DETERMINES THE METHOD USED
C                         =-1, MODIFIED GAUSS-NEWTON METHOD
C                         = 0, GAUSS-NEWTON METHOD
C                         = 1, MODIFIED FORM OF MARQUARDT-S METHOD
C                         = 2, MARQUARDT-S METHOD
C     KALCP       --  DETERMINES WHICH ROUTINE IS CALLED TO COMPUTE THE
C                         JACOBIAN MATRIX OF FIRST PARTIAL DERIVATIVES
C                         =-1, ONE ROW OF P RETURNED BY DERIV OR CALCD,
C                                DERIV CALLS FOFX
C                         = 0, ALL OF P RETURNED BY DERIV OR CALCD,
C                                DERIV CALLS FUNK
C                         = 1, ALL OF P RETURNED BY DERIV OR CALCD,
C                                DERIV CALLS FOFX
C     KORDF       --  DETERMINES HOW P IS TO BE CALCULATED
C                         = 1, FIRST ORDER APPROXIMATION USED BY DERIV
C                         = 2, SECOND ORDER APPROXIMATION USED BY DERIV
C                         = 3, ANALYTICAL DERIVATIVE SUPPLIED BY CALCD
C     MAXIT       --  THE MAXIMUM NUMBER OF ITERATIONS TO BE PERFORMED
C     LEQU        --  SAVES STORAGE IF ALL YSIG(JPT) ARE THE SAME
C                         =1 IF ALL YSIG(JPT) ARE EQUAL (IN THIS CASE
C                              ONLY YSIG(1) IS REFERENCED)
C                         =0 OTHERWISE
C     MXSUB       --  THE MAXIMUM NUMBER OF SUBITERATIONS PERMITTED
C
C  MODIFICATIONS TO MARQUARDT-S METHOD...
C     1.  THE QUANTITY (1+LAMBDA) IS NOT ALLOWED TO INCREASE BY MORE
C         THAN A FACTOR OF FNULM, IN ANY SINGLE CHANGE.
C     2.  WHEN CUTSTEPS ARE USED IN A LINE SEARCH (WHEN THE ANGLE
C         GAMMA .LT. GAMMA SUB ZERO), THE VALUE OF LAMBDA IS INCREASED.
C  IN BOTH THE MODIFIED MARQUARDT-S METHOD AND THE MODIFIED
C  GAUSS-NEWTON METHOD, A HALF STEP IS ATTEMPTED FOLLOWING EACH
C  SUCCESSFUL STEP.  THIS AVOIDS ONE FORM OF VERY SLOW CONVERGENCE.
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  THE FOLLOWING EXTERNAL STATEMENT IS REQUIRED BY SOME COMPILERS
C  (WATFIV, FOR EXAMPLE) AND FORBIDDEN BY OTHERS (MODCOMP II).
      EXTERNAL FUNK
C
C  ON A MACHINE HAVING LESS THAN ABOUT TEN SIGNIFICANT DIGITS IN
C  SINGLE PRECISION (FOR EXAMPLE THE IBM 360 OR 370), IF P IS
C  COMPUTED USING FINITE DIFFERENCES (KORDF.LT.3), THIS PART OF THE
C  COMPUTATION SHOULD BE DONE IN DOUBLE PRECISION.  TO ACCOMPLISH
C  THIS, ACTIVATE THE DOUBLE PRECISION STATEMENT IN EACH SUBROUTINE.
C  ORDINARILY THE OTHER COMPUTATIONS MAY BE DONE IN SINGLE PRECISION.
C
      DOUBLE PRECISION P
      DOUBLE PRECISION Y,YSIG,H,SCALE,XMAX,XMIN,DELTX,DELMN,
     X       ERR,FOBJ,FLAMB,FNU,RELDF,QSQRT,ARG,CRIT,FLDEF,RELMN,
     X       RLTOL,FACCL,HUGE,RZERO,RUNIT,RTWO,DELN,FNGN,FCUT,
     X       STFAC,PTERM,SCALJ,SA,PIVCT,EM,SLM,COSIN,SB,SC,MM,
     X       FRMIN,XMX,XMN,DENOM,FRAC,UPFAC,RLFAC,RSFAC,DIF,RMSDV,
     X       SDVMX
      DOUBLE PRECISION X,XSAVE,XTEMP,GRAD,FIT,FITSV,
     *                 XSAV,SIG,RTERM,YY,PHI,PHNEW,PHALF,XLIM
C
C  THE DIMENSIONS OF THE VECTORS AND MATRICES (AS OPPOSED TO ARRAYS)
C  ARE...
C        P(NPTS,NACTV) (OR P(1,NACTV) IF KALCP.EQ.-1),
C        FITSV(NPTS) (OF FITSV(1) IF KALCP.EQ.-1),
C        X(NV),XMAX(NV),XMIN(NV),DELMN(NV),ERR(NV,NV+1),
C        XSAVE(NV),H(NV),MASKT(NV),
C        GRAD(NACTV),SCALE(NACTV),
C        Y(NPTS),FIT(NPTS),YSIG(NPTS) (OR YSIG(1) IF LEQU.NE.0),
C  WHERE NACTV IS THE NUMBER OF ACTIVE (UNMASKED) X(J).
C
      DIMENSION P(LPCOL,1)
      DIMENSION Y(1),YSIG(1),FIT(1),FITSV(1)
      DIMENSION XSAVE(20),H(20),GRAD(20),MASKT(20),XTEMP(20)
C
C  USER COMMON.....
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),DELMN(20),
     *       ERR(20,21),FOBJ,NV,NTRAC,MATRX,MASK(20),
     *       NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW
C
C  MARQ COMMON.....
      COMMON /NLLS3/ FLAMB,FNU,RELDF,METHD,KALCP,KORDF,MAXIT,LEQU,
     *       MXSUB,MXUPD
C
C  SET THE LIBRARY FUNCTION FOR SINGLE PRECISION (SQRT) OR FOR
C  DOUBLE PRECISION (DSQRT).  NO OTHER FUNCTIONS ARE USED, EITHER
C  EXTERNAL OR INTRINSIC.
C  THE ONLY SUBROUTINES CALLED ARE FUNC, DERIV, CALCD, AND MOERR.
C
C     QSQRT(ARG)= SQRT(ARG)
      QSQRT(ARG)=DSQRT(ARG)
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  SET FIXED QUANTITIES ....
C
C  NVMAX IS THE MAXIMUM PERMISSIBLE VALUE OF NV.  IT IS ALSO THE
C  DIMENSION OF THE ARRAYS X, XMAX, XMIN, MASK, DELMN.
```

```
C  XSAVE, H, MASKT, GRAD, AND SCALE, AND THE FIRST DIMENSION OF ERR,
C  THE SECOND DIMENSION OF ERR IS NVMAX+1.
C
      NVMAX=20
C                              CRIT ...  COSINE OF MARQUARDT-S CRITICAL
C                                                ANGLE, GAMMA SUB ZERO
      CRIT=.70711
C                              FLDEF ...  DEFAULT VALUE FOR FLAMB
      FLDEF=1.
C                              RELMN ...  USED TO SET DEFAULT VALUE OF
C                                              DELMN
      RELMN=1.E-5
C                              RLTOL ...  TOLERANCE FOR A WARNING MESSAGE
      RLTOL=1.E-4
C                              FACCL ...  ACCELERATION FACTOR FOR FCUT
      FACCL=2.
C                              FNULM ...  LIMIT ON THE FACTOR BY WHICH
C                                   (1+FLAMB) MAY CHANGE
      FNULM=2.
C                              HUGE ...  A VERY LARGE REAL NUMBER
C                                   (DEFAULT VALUE FOR XMAX AND -XMIN)
      HUGE=1.E30
C
      RZERO=0.
      RUNIT=1.
      RTWO=2.
C
C  NO REAL CONSTANTS ARE USED BEYOND THIS POINT.
C
      NTRPT=NTRAC-(-2)
      NTRPU=NTRAC-(-1)
      NTRMU=NTRAC-1
      KFLAG=0
      NOREP=0
      ITER=0
      PHI=HUGE
C
      IF(NTRPU)30,10,10
   10 WRITE(KW,20)
   20 FORMAT(49H1MARQ....  BEGIN NONLINEAR LEAST SQUARES SOLUTION)
   30 NACTV=0
      IF(NV)80,80,40
   40 IF(NV-NVMAX)50,50,80
   50 IF(NPTS)80,80,60
   60 IF(KALCP)90,70,70
   70 IF(NPTS-LPCOL)90,90,80
   80 KFLAG=-1
      GO TO 220
C
C  CHECK SOME INPUT QUANTITIES, AND SET THEM TO DEFAULT VALUES IF
C  DESIRED.
C                              NACTV ...  NUMBER OF ACTIVE X(J)
   90 DO 200 JX=1,NV
        DELMN=DELMN(JX)
        IF(MASK(JX))200,100,200
  100 NACTV=NACTV+1
      IF(DELMN)130,110,140
  110 DELMN=RELMN*X(JX)
      IF(DELMN)130,120,140
  120 DELMN=RELMN
      GO TO 140
  130 DELMN=-DELMN
  140 IF(XMAX(JX)-XMIN(JX))150,150,160
  150 XMAX(JX)=HUGE
      XMIN(JX)=-HUGE
C
C  SET X(JX) TO AMAX1(AMIN1(JX),AMIN1(XMAX(JX),X(JX))).
C
  160 IF(X(JX)-XMAX(JX))180,180,170
  170 X(JX)=XMAX(JX)
  180 IF(X(JX)-XMIN(JX))190,200,200
  190 X(JX)=XMIN(JX)
  200 DELMN(JX)=DELMN
C
      IF(NACTV)210,210,240
  210 KFLAG=-2
  220 CONTINUE
      WRITE(KW,230)NV,NVMAX,NACTV,NPTS,LPCOL,KALCP
  230 FORMAT(//29H ILLEGAL INPUT VALUE IN MARQ..,4X,5H NV =,I3,4X,
     *      8H NVMAX =,I5,4X,8H NACTV =,I3/ 10X,7H NPTS =,I5,4X,
     *      8H LPCOL =,I5,4X,8H KALCP =,I3)
      GO TO 2000
  240 CONTINUE
      IF(NTRPU)320,250,250
  250 WRITE(KW,260)(MASK(J),J=1,NV)
  260 FORMAT(/10H MASK     = ,I7,7I13/(4X,8I13))
      WRITE(KW,270)(X(J),J=1,NV)
  270 FORMAT(/10H X        = ,8E13.5/(10X,8E13.5))
      WRITE(KW,280)(XMAX(J),J=1,NV)
  280 FORMAT(/10H XMAX     = ,8E13.5/(10X,8E13.5))
      WRITE(KW,290)(XMIN(J),J=1,NV)
  290 FORMAT(/10H XMIN     = ,8E13.5/(10X,8E13.5))
      WRITE(KW,300)(DELMN(J),J=1,NV)
  300 FORMAT(/10H DELMN    = ,8E13.5/(10X,8E13.5))
      WRITE(KW,310)NV,NPTS,LPCOL,NTRAC,METHD,KALCP,KORDF,NFLAT,
     *      NFMAX,MAXIT,MXSUB,CRIT,RELDF
  310 FORMAT(//6H NV = ,I4,5X,8H NPTS = ,I6,5X,9H LPCOL = ,I6,
     *      5X,9H NTRAC = ,I2,5X,9H METHD = ,I2,5X,9H KALCP = ,I2,
     *      5X,9H KORDF = ,I2//9H NFLAT = ,I2,5X,9H NFMAX = ,I7,
     *      5X,9H MAXIT = ,I5,5X,9H MXSUB = ,I5,5X,
     *      28H COSINE OF CRITICAL ANGLE = ,E12.5//9H RELDF = ,E12.5)
  320 JVARY=0
C
C  SET FMGN, IF NECESSARY, RESET FLAMB AND/OR FACCL.
C
      FMGN=RUNIT
      UPNU=FNU
      DOWNU=FNU
      IF(METHD)330,330,340
  330 FLAMB=RZERO
```

```
      GO TO 380
  340 IF(FLAMB)350,350,380
  350 FLAMB=FLDEF
  360 IF(METHD-1)370,380,370
  370 FACCL=RUNIT
C
C  COMPUTE THE INITIAL GOODNESS OF FIT OF THE MODEL TO THE DATA.
C  CALL FUNC TO CALCULATE THE VECTOR OF FITTED VALUES.
C
  380 CALL FUNC (FUNK,Y,YSIG,NPTS,FIT,PHI)
C
C                              NF ...  EQUIVALENT NUMBER OF CALLS TO FUNC
      NF=1
      IF(NTRPU)410,390,390
  390 WRITE(KW,400)PHI,FLAMB
  400 FORMAT(//20H PHI (THE SUM OF SQUARES) = ,E15.8,56X,9H LAMBDA =,
     *      E12.5//1H )
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  BEGIN THE NEXT ITERATION.
C  THIS IS THE ENTRY POINT AFTER A SUCCESSFUL STEP IF THE CONVERGENCE
C  CRITERION IS NOT MET.
C
  410 JSUB=0
      FCUT=RTWO
      ITER=ITER+1
      IF(NTRMU)440,420,420
  420 WRITE(KW,430)ITER,FMGN,FLAMB
  430 FORMAT(//17H BEGIN ITERATION ,I4,44X,7H FMGN =,E12.5,15X,
     *      9H LAMBDA =,E12.5)
  440 IF(NTRAC-3)470,450,450
  450 WRITE(KW,460)
  460 FORMAT(/28H P (THE JACOBIAN MATRIX).../1H )
C
C                              INITIALIZE FOR THIS ITERATION.
  470 STFAC=RUNIT
      DO 480 JX=1,NACTV
        GRAD(JX)=RZERO
        DO 480 KX=1,NACTV
  480     ERR(JX,KX)=RZERO
C
C  CALL DERIV (OR CALCD) TO COMPUTE THE JACOBIAN MATRIX, P.
C  DERIV IS CALLED NPTS TIMES IF KALCP=-1.
C
      SIG=YSIG(1)
      DO 620 JPT=1,NPTS
        KPT=JPT
        IF(KALCP)500,450,490
  490   IF(JPT-1)540,510,540
  500   KPT=1
  510   IF(KORDF-2)530,530,520
  520   CALL CALCD (JPT,F,LPCOL)
        GO TO 540
  530   CALL DERIV (JPT,FUNK,NPTS,FIT,FITSV,P,LPCOL)
  540   CONTINUE
        IF(NTRAC-3)570,550,550
  550   WRITE(KW,560)JPT,(P(KPT,JX),JX=1,NACTV)
  560   FORMAT(1X,I3,2X,6E15.7/(6X,6E15.7))
C
C  COMPUTE QSAV AND GRAD.
C  QSAV, WHICH IS STORED IN ONE HALF OF THE ARRAY ERR(*,*), IS PT*P.
C  GRAD IS EQUAL TO HALF THE NEGATIVE OF THE GRADIENT VECTOR.
C
  570   IF(LEQU)590,580,590
  580   SIG=YSIG(JPT)
  590   YY=Y(JPT)
        RTERM=(FIT(JPT)-YY)/SIG**2
        DO 610 JX=1,NACTV
          GRAD(JX)=GRAD(JX)-P(KPT,JX)*RTERM
          PTERM=P(KPT,JX)/SIG**2
          DO 600 KX=1,JX
  600       ERR(JX,KX)=ERR(JX,KX)+P(KPT,KX)*PTERM
  610     CONTINUE
  620   CONTINUE
C                              RESTORE FIT IF IT WAS DESTROYED IN DERIV.
      IF(KORDF-2)660,630,660
  630 IF(KALCP)650,640,650
  640 CALL FUNC (FUNK,Y,YSIG,NPTS,FIT,PHI)
      NF=NF+1
  650 NF=NF+NACTV
  660 NF=NF+NACTV
C
C  COMPUTE THE SCALE FACTORS AND STORE THEM IN DELTX(*).  SCALE GRAD.
C
      DO 690 JX=1,NACTV
        SCALJ=QSQRT(ERR(JX,JX))
        IF(SCALJ)680,670,680
  670   SCALJ=RUNIT
  680   DELTX(JX)=SCALJ
  690   GRAD(JX)=GRAD(JX)/SCALJ
      IF(NTRMU)720,700,700
  700 WRITE(KW,710)(GRAD(JX),JX=1,NACTV)
  710 FORMAT(/2X,19H SCALED GRADIENT = ,6E15.7/(21X,6E15.7))
C
C  SCALE QSAV.  THE DIAGONAL ELEMENTS OF QSAV ARE SCALED TO UNITY.
C
  720 DO 800 JX=1,NACTV
        DO 800 KX=1,JX
          SA=ERR(JX,KX)/(DELTX(JX)*DELTX(KX))
          IF(KX-JX)750,730,750
  730     IF(SA)740,770,740
  740     SA=RUNIT
          GO TO 800
  750     IF(SA-(RUNIT-RLTOL))760,770,770
  760     IF(SA+(RUNIT-RLTOL))770,770,800
  770     CONTINUE
          IF(NTRPT)800,780,780
  780     WRITE(KW,790)JX,KX,SA
  790     FORMAT(38H ********  POSSIBLY DANGEROUS VALUE OF ,
```

```
     *          16H COEFFICIENT....,5X,6H QSAV(,I3,1H,,I3,4H) = ,E14.7)
 800       ERR(JX,KX)=SA
      IF(NTRAC-2)840,810,810
 810  WRITE(KW,820)
 820  FORMAT(/49H QSAV (PT*P, SCALED, WHERE P IS THE JACOBIAN)..../1H )
      DO 830 JX=1,NACTV
 830    WRITE(KW,560)JX,(ERR(JX,KX),KX=1,JX)
 840  DO 850 JX=1,NV
 850    XSAVE(JX)=X(JX)
C                               INITIALIZE MASKT AND NACT.
 860  NACT=NACTV
      DO 870 JX=1,NV
 870    MASKT(JX)=MASK(JX)
C
C  COPY QSAV INTO Q AND GRAD INTO H, AND SET THE DIAGONAL ELEMENTS OF Q.
C  THIS IS THE ENTRY POINT FOR SUBITERATIONS IN WHICH FLAMB IS
C  INCREASED OR CONSTRAINTS ARE IMPOSED.
C
 880  KRANK=0
      JQ=0
      JT=0
      DO 970 JX=1,NV
        IF(MASK(JX))970,890,970
 890    JQ=JQ+1
        IF(MASKT(JX))970,900,970
 900    JT=JT+1
        H(JT)=GRAD(JQ)
        KQ=0
        KT=0
        DO 960 KX=1,JX
          IF(MASK(KX))960,910,960
 910      KQ=KQ+1
          IF(MASKT(KX))960,920,960
 920      KT=KT+1
          SA=ERR(JQ,KQ)
          IF(KX-JX)950,930,950
 930      IF(SA)940,950,940
 940      SA=RUNIT+FLAMB
          KRANK=KRANK+1
 950      ERR(KT,JT+1)=SA
 960      CONTINUE
 970    CONTINUE
C
C  SOLVE THE NORMAL EQUATIONS FOR H, THE CORRECTION VECTOR.
C  THE METHOD USED IS GAUSSIAN ELIMINATION WITHOUT PIVOTING.
C  (PIVOTING IS NOT NECESSARY FOR A POSITIVE DEFINITE MATRIX.)
C  ONLY ABOUT N**3/6 MULTIPLICATIONS ARE DONE.
C  THE CHOICE OF GAUSSIAN ELIMINATION RATHER THAN CHOLESKY
C  DECOMPOSITION IS INTENTIONAL.
C
      NSMAL=0
      NMU=NACT-1
      IF(NMU)1130,1040,980
C                          REDUCE THE SYSTEM TO TRIANGULAR FORM,
C                               UTILIZING THE SYMMETRY OF THE MATRIX.
 980  DO 1030 J=1,NMU
        PIVOT=ERR(J,J+1)
        IF(PIVOT)990,1030,990
 990    JPU=J+1
        DO 1020 K=JPU,NACT
          EM=ERR(J,K+1)/PIVOT
          IF(EM)1000,1020,1000
1000      DO 1010 L=K,NACT
1010        ERR(K,L+1)=ERR(K,L+1)-ERR(J,L+1)*EM
1020      H(K)=H(K)-H(J)*EM
1030    CONTINUE
C                                     DO THE BACK SOLUTION.
1040  DO 1120 JINV=1,NACT
        J=(NACT+1)-JINV
        PIVOT=ERR(J,J+1)
        IF(PIVOT)1050,1050,1060
1050    NSMAL=NSMAL+1
1060    IF(PIVOT)1080,1070,1080
1070    H(J)=RZERO
        GO TO 1120
1080    SUM=RZERO
        IF(J-NACT)1090,1110,1110
1090    JPU=J+1
        DO 1100 K=JPU,NACT
1100      SUM=SUM+ERR(J,K+1)*H(K)
1110    H(J)=(H(J)-SUM)/PIVOT
1120    CONTINUE
C
1130  MRANK=NACT-NSMAL
C
C  IF THE COEFFICIENT MATRIX HAS RANK DEFICIENT, PRINT A MESSAGE.
C
      IF(MRANK-NACT)1140,1210,1140
1140  COSIN=HUGE
      IF(NTRPT)1170,1150,1150
1150  WRITE(KW,1160)MRANK,NACT
1160  FORMAT(/41H RANK-DEFICIENT NORMAL EQUATIONS IN MARQ.,9X,
     *       7H RANK =,I3,7X,18H ORDER OF MATRIX =,I3)
1170  IF(MRANK)1180,1180,1190
1180  KFLAG=-4
      GO TO 2500
1190  IF(METHD)1210,1210,1200
1200  IF(MRANK-KRANK)1800,1210,1210
C
C  UNPACK AND DE-SCALE THE CORRECTION VECTOR H.
C
C  COMPUTE THE INNER PRODUCTS SA, SB, AND SC.
1210  SA=RZERO
      SB=RZERO
      SC=RZERO
      KX=NV
      KQ=NACTV
      KT=NACT
      DO 1260 JX=1,NV
        HH=RZERO
```

```
      IF(MASKT(KX))1220,1230,1220
 1220 IF(MASK(KX))1250,1240,1250
 1230 HH=H(KT)
      SA=SA+HH*GRAD(K0)
      SB=SB+HH*HH
      SC=SC+GRAD(K0)**2
      HH=HH*FNGN/DELTX(K0)
      KT=KT-1
 1240 K0=K0-1
 1250 H(KX)=HH
 1260 KX=KX-1
C
C  ADD THE CORRECTION VECTOR TO THE PARAMETER VECTOR AND
C  INSURE THAT NO CONSTRAINTS ARE VIOLATED.
C  THIS IS THE ENTRY POINT FOLLOWING A CUTSTEP.
C
 1270 CONTINUE
      IF(NTRMU)1300,1280,1280
 1280 WRITE(KW,1290)(H(JX),JX=1,NV)
 1290 FORMAT(/7X,14H CORRECTION = ,6E15.7/(21X,6E15.7))
 1300 NACSV=NACT
      FRMIN=RUNIT
      NLOOP=0
      JXLIM=0
 1310 DO 1530 JX=1,NV
      IF(MASKT(JX))1530,1320,1530
 1320 XSAV=XSAVE(JX)
      XMX=XMAX(JX)
      XMN=XMIN(JX)
      HH=H(JX)
      IF(XSAV-XMX)1340,1330,1330
 1330 IF(HH)1340,1340,1360
 1340 IF(XSAV-XMN)1350,1350,1390
 1350 IF(HH)1360,1390,1390
 1360 MASKT(JX)=1
      NACT=NACT-1
      X(JX)=XSAV
      IF(NTRPU)1530,1370,1370
 1370 WRITE(KW,1380)JX,XSAV
 1380 FORMAT(/8H FIX  X(,I3,4H) = ,E12.5,
     *        46H TEMPORARILY, TO AVOID VIOLATING A CONSTRAINT. )
      GO TO 1530
 1390 XLIM=XSAV+HH*FRMIN
      IF(JX-JXLIM)1410,1400,1410
 1400 IF(KBND)1440,1420,1420
 1410 X(JX)=XLIM
      IF(XLIM-XMX)1430,1430,1420
 1420 X(JX)=XMA
      JBND=1
      GO TO 1450
 1430 IF(XLIM-XMN)1440,1530,1530
 1440 X(JX)=XMN
      JBND=-1
 1450 IF(JX-JXLIM)1450,1460,1490
 1460 CONTINUE
      IF(NTRPU)1530,1470,1470
 1470 WRITE(KW,1480)JX,X(JX),FRMIN
 1480 FORMAT(/26H CONSTRAINT VIOLATED BY X(,I3,
     *        20H),    VALUE RESET TO ,E15.8,24H USING CUTSTEP FACTOR = ,
     *        E12.5 )
      GO TO 1530
 1490 IF(NLOOP)1530,1500,1530
 1500 DENOM=XLIM-XSA)
      IF(DENOM)1510,1530,1510
 1510 FRAC=(X(JX)-XSAV)/DENOM
      IF(FRAC-FRMIN)1520,1530,1530
 1520 FRMIN=FRAC
      JXLIM=JX
      KBND=JBND
 1530 CONTINUE
C
C  IF THE PROPOSED STEP WOULD VIOLATE ANY ALREADY ACTIVE CONSTRAINTS,
C  FIX THOSE COMPONENTS OF H EQUAL TO ZERO AND RECOMPUTE THE
C  OTHER COMPONENTS.
C
      IF(NACT)1540,1540,1570
 1540 KFLAG=3
      IF(NTRPT)2580,1550,1550
 1550 WRITE(KW,1560)
 1560 FORMAT(////47H APPARENT CONSTRAINED OPTIMUM LIES IN A CORNER. )
      GO TO 2580
 1570 IF(NACT-NACSV)880,1580,1580
 1580 IF(NLOOP)1600,1590,1600
 1590 NLOOP=1
      IF(JXLIM)1310,1600,1310
 1600 CONTINUE
      IF(NTRMU)1630,1610,1610
 1610 WRITE(KW,1620)(X(JX),JX=1,NV)
 1620 FORMAT(/16X,5H X = ,6E15.7/(21X,6E15.7))
C
C  CALCULATE THE NEW FITTED VALUES.
C
 1630 CALL FUNC (FUNK,Y,YSIG,NPTS,FIT,PHNEW)
      NF=NF+1
      IF(PHNEW-PHI)1980,1640,1680
C
C  THE NEW VALUE OF PHI IS EXACTLY EQUAL TO THE OLD VALUE.
C  CHECK FOR CONVERGENCE UNDER THE NFLAT OPTION.
C
 1640 IF(NFLAT)1650,1980,1650
 1650 KFLAG=2
      IF(NTRPU)1760,1660,1660
 1660 WRITE(KW,1670)
 1670 FORMAT(//45H CONVERGENCE ACHIEVED UNDER THE NFLAT OPTION.)
      GO TO 1760
C
C  THE NEW VALUE OF PHI IS GREATER THAN THE OLD VALUE.
C
 1680 CONTINUE
      IF(NTRMU)1710,1650,1690
```

```
 1690 WRITE(KW,1700)PHI,PHNEW
 1700 FORMAT(/33X,11H OLD PHI = ,E15.8,4X,11H NEW PHI = ,E15.8)
C
C  INSURE THAT JSUB HAS NOT EXCEEDED MXSUB.
C
 1710 JSUB=JSUB+1
      IF(JSUB-MXSUB)1720,1720,1730
 1720 IF(METHD)1890,2310,1780
 1730 KFLAG=1
      IF(NTRPU)1760,1740,1740
 1740 WRITE(KW,1750)MXSUB
 1750 FORMAT(//44H EXCEEDED MAXIMUM NUMBER OF SUBITERATIONS = ,I5,
     *        9H IN MARC.)
C                                   RESTORE X TO THE BASE POINT.
 1760 DO 1770 JX=1,NV
 1770   X(JX)=XSAVE(JX)
      CALL FUNC (FUNK,Y,YSIG,NPTS,FIT,PHI)
      GO TO 2500
C
C  THE NEW FIT IS WORSE THAN THE OLD FIT.  COMPUTE COSIN, THE COSINE
C  OF THE ANGLE BETWEEN THE SCALED GRADIENT AND THE SCALED CORRECTION
C  VECTOR.
C
 1780 DENOM=SO*SC
      IF(DENOM)1800,1800,1790
 1790 COSIN=SA/QSQRT(DENOM)
      IF(COSIN-CRIT)1800,1800,1890
C
C  COSIN IS NOT GREATER THAN CRIT.  INCREASE THE VALUE OF LAMBDA.
C
 1800 UPFAC=UPNU
      UPNU=UPNU*RTWO
      IF(UPNU-FNU)1820,1820,1810
 1810 UPNU=FNU
 1820 IF(METHD-1)1850,1830,1850
 1830 FLIM=FNULM+RUNIT/FLAMB
      IF(UPFAC-FLIM)1850,1850,1840
 1840 UPFAC=FLIM
 1850 FLAMB=FLAMB*UPFAC
      IF(NTRMU)1880,1860,1860
 1860 WRITE(KW,1870)JSUB,COSIN,FLAMB
 1870 FORMAT(/18H **** SUBITERATION,I3,4X,17H INCREASE LAMBDA.,
     *     4X,14H COS(GAMMA) = ,E12.5,27X,5H LAMBDA =,E12.5)
 1880 CONTINUE
C                                   GO BACK AND FORM THE NORMAL EQUATIONS
C                                         USING A LARGER VALUE OF LAMBDA.
      GO TO 860
C
C  COSIN IS GREATER THAN CRIT.  CUT THE MAGNITUDE OF THE STEP, H.
C
 1890 STFAC=STFAC/FCUT
      IF(METHD)1900,1910,1910
 1900 FNGN=FNGN/FCUT
      GO TO 1930
 1910 IF(METHD-1)1930,1920,1930
 1920 FLAMB=FLAMB*RTWO
 1930 DO 1940 JX=1,NV
 1940   H(JX)=(X(JX)-XSAVE(JX))/FCUT
      FCUT=FCUT*FACCL
      IF(NTRMU)1970,1950,1950
 1950 WRITE(KW,1960)JSUB,COSIN,STFAC
 1960 FORMAT(/18H **** SUBITERATION,I3,4X,16H TAKE CUT STEPS.,4X,
     *        14H COS(GAMMA) = ,E12.5,4X,17H CUTSTEP FACTOR =,E12.5 )
 1970 CONTINUE
C                                   GO BACK AND TRY A SMALLER CUTSTEP.
      GO TO 1270
C
C  THE VALUE OF PHI HAS DECREASED.  TRY A HALF STEP.
C
 1980 IF(METHD)2000,2310,1990
 1990 IF(METHD-2)2000,2310,2000
 2000 DO 2050 JX=1,NV
        XTEMP(JX)=X(JX)
        IF(MASK(JX))2050,2010,2050
 2010   X(JX)=XSAVE(JX)+(X(JX)-XSAVE(JX))/RTWO
        IF(X(JX)-XMAX(JX))2030,2030,2020
 2020   X(JX)=XMAX(JX)
 2030   IF(X(JX)-XMIN(JX))2040,2050,2050
 2040   X(JX)=XMIN(JX)
 2050   CONTINUE
      DO 2060 JPT=1,NPTS
 2060   FITSV(JPT)=FIT(JPT)
      CALL FUNC (FUNK,Y,YSIG,NPTS,FIT,PHALF)
      NF=NF+1
C
C  USE QUADRATIC INTERPOLATION, IN ORDER TO TRY TO REFINE THE
C  POSITION OF THE MINIMUM OF PHI.
C
      RLFAC=RUNIT
      DENOM=RTWO*((PHNEW-PHALF)-(PHALF-PHI))
      IF(DENOM)2080,2080,2070
 2070 STFAC=(PHI-PHNEW)/DENOM
      RSFAC=(RUNIT+STFAC)/RTWO
C
C                                   DO NOT EXTRAPOLATE.
      IF(STFAC-RUNIT)2090,2080,2080
 2080 STFAC=RZERO
 2090 DO 2100 JX=1,NV
        XSAVE(JX)=X(JX)
 2100   X(JX)=X(JX)+(XTEMP(JX)-X(JX))*STFAC
      IF(PHALF-PHNEW)2110,2170,2170
 2110 RLFAC=RUNIT/RTWO
      JSUB=JSUB+1
      DO 2120 JX=1,NV
 2120   XTEMP(JX)=XSAVE(JX)
      DO 2130 JPT=1,NPTS
 2130   FITSV(JPT)=FIT(JPT)
      IF(NTRMU)2160,2140,2140
 2140 WRITE(KW,2150)PHNEW,PHALF
 2150 FORMAT(/21H HALF STEP SUCCEEDED.,15X,8H PHNEW =,E15.8,18X,
```

```
     *       8H PHALF =,E15.8)
 2160 PHNEW=PHALF
 2170 IF(STFAC)2180,2150,2180
 2180 CALL FUNC (FUNK,Y,YSIG,NPTS,FIT,PHI)
      NF=NF+1
      IF(PHI-PHNEW)2250,2190,2190
 2190 DO 2200 JX=1,NV
 2200   X(JX)=XTEMP(JX)
      DO 2210 JPT=1,NPTS
 2210   FIT(JPT)=FITSV(JPT)
      IF(STFAC)2220,2280,2220
 2220 CONTINUE
      IF(NTRMU)2280,2230,2230
 2230 WRITE(KW,2240)RSFAC,PHI
 2240 FORMAT(/25H QUADRATIC INTERPOLATION.,22X,8H RSFAC =,E12.5,
     *       12X,6H PHI =,E15.8)
      GO TO 2280
 2250 RLFAC=RSFAC
      PHNEW=PHI
      IF(NTRMU)2280,2260,2260
 2260 WRITE(KW,2270)RLFAC,PHI
 2270 FORMAT(/35H QUADRATIC INTERPOLATION SUCCEEDED.,12X,8H RLFAC =,
     *       E12.5,12X,6H PHI =,E15.8)
 2280 IF(RLFAC)2310,2310,2290
 2290 FLAMB=FLAMB/RLFAC
      IF(METHD)2300,2310,2310
 2300 FMGN=FMGN*RLFAC
C
C  THE STEP IS ACCEPTED.  TEST FOR CONVERGENCE IF NO CONSTRAINT
C  BECAME ACTIVE DURING THIS ITERATION.
C
 2310 CONTINUE
      IF(NTRMU)2340,2320,2320
 2320 WRITE(KW,2330)ITER,PHNEW
 2330 FORMAT(/17H END    ITERATION ,I4,58X,6H PHI =,E15.8)
 2340 PHI=PHNEW
      IF(JXLIM)2350,2350,2420
 2350 DO 2390 JX=1,NV
        IF(MASK(JX))2390,2360,2390
 2360   DIF=X(JX)-XSAVE(JX)
        IF(DIF)2370,2380,2380
 2370   DIF=-DIF
 2380   IF(DIF-DELMN(JX))2390,2390,2420
 2390   CONTINUE
      KFLAG=1
      IF(NTRPU)2580,2400,2400
 2400 WRITE(KW,2410)
 2410 FORMAT(//30H CONVERGED WHEN THE STEP BECAME SMALL.)
      GO TO 2580
C
C  THE ITERATION HAS NOT YET CONVERGED.
C
 2420 IF(ITER-MAXIT)2450,2430,2430
 2430 KFLAG=-6
      WRITE(KW,2440)MAXIT
 2440 FORMAT(//46H MAXIMUM NUMBER OF ITERATIONS REACHED IN MARQ.,5X,
     *       9H MAXIT = ,I4 )
      GO TO 2580
C
C  IF SUBITERATIONS WERE NOT PERFORMED THIS ITERATION, DECREASE LAMBDA.
C
 2450 IF(NF-NFMAX)2460,2560,2560
 2460 IF(JSUB)2470,2470,2530
 2470 FMGN=FMGN*RTWO
      IF(FMGN-RUNIT)2480,2490,2480
 2480 FMGN=RUNIT
 2490 SCALJ=RUNIT+FLAMB
      IF(SCALJ-RUNIT)2510,2510,2500
 2500 FLAMB=FLAMB/DOWNL
 2510 UPNU=DOWNU
      DOWNU=DOWNU*RTWO
      IF(DOWNU-FNU)2550,2550,2520
 2520 DOWNU=FNU
      GO TO 2550
 2530 IF(METHD-1)2550,2540,2550
 2540 UPNU=FNULM
      DOWNU=FNULM
 2550 CONTINUE
C                                  GO BACK AND DO ANOTHER ITERATION.
      GO TO 410
 2560 KFLAG=-7
      WRITE(KW,2570)NFMAX
 2570 FORMAT(//24H NF HAS REACHED NFMAX = ,I7,9H IN MARQ. )
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  THE ITERATION HAS TERMINATED.
C  PRINT OUT THE DATA, FITTED VALUES, AND RESIDUALS.
C  COMPUTE AND PRINT THE STANDARD DEVIATION OF THE DATA FROM THE FIT.
C
 2580 CONTINUE
      IF(NTRPU)2780,2590,2590
 2590 WRITE(KW,2600)ITER,NF,PHI,FMGN,FLAMB
 2600 FORMAT(/1X,I4,11H ITERATIONS,7X,5H NF =,I5,9X,6H PHI =,E15.8,
     *       10X,7H FMGN =,E12.5,7X,9H LAMBDA =,E12.5)
      WRITE(KW,1620)(X(JX),JX=1,NV)
      IF(NTRAC)2630,2610,2610
 2610 WRITE(KW,2620)
 2620 FORMAT(///14X,1HJ,9X,5H Y(J),14X,7H FIT(J),10X,
     *       12H Y(J)-FIT(J),7X,8H YSIG(J),11X,13H (Y-FIT)/YSIG/1H )
 2630 SIG=YSIG(1)
      RMSDV=RZERO
      SDVMX=RZERO
      DO 2720 JPT=1,NPTS
        IF(LEQU)2650,2640,2650
 2640   SIG=YSIG(JPT)
 2650   YY=Y(JPT)
        PTERM=YY-FIT(JPT)
        RTERM=PTERM/SIG
        IF(NTRAC)2680,2660,2660
```

```
 2660    WRITE(KW,2670)JPT,YY,FIT(JPT),PTERM,SIG,RTERM
 2670    FORMAT(10X,I5,5X,E15.8,5X,E15.8,5X,E12.5,5X,E12.5,10X,E12.5)
 2680    RMSDV=RMSDV+RTERM**2
         IF(RTERM)2690,2700,2700
 2690    RTERM=-RTERM
 2700    IF(RTERM-SDVMX)2720,2720,2710
 2710    SDVMX=RTERM
 2720    CONTINUE
      DENOM=NPTS-NACTV
      WRITE(KW,2730)DENOM
 2730 FORMAT(//32H NUMBER OF DEGREES OF FREEDOM = ,E12.5 )
      IF(DENOM)2760,2760,2740
 2740 RMSDV=QSQRT(RMSDV/DENOM)
      WRITE(KW,2750)RMSDV
 2750 FORMAT(/43H R.M.S. SCALED DEVIATION OF DATA FROM FIT =,E12.5)
 2760 CONTINUE
      WRITE(KW,2770)SDVMX
 2770 FORMAT(/27H MAXIMUM SCALED DEVIATION =,E12.5)
C
C  CALL FUNC TO SET THE FINAL VALUES.
C
 2780 CALL FUNC (FUNK,Y,YSIG,NPTS,FIT,PHI)
      FOBJ=PHI
C
C  CALL MQERR TO PRINT THE PARAMETER ERRORS AND CORRELATIONS.
C  A DUMMY ROUTINE MAY BE SUBSTITUTED FOR MQERR IF THESE ARE NOT NEEDED.
C
      IF(MATRX)2800,2800,2790
 2790 CALL MQERR (NACTV,NPTS)
C
 2800 RETURN
C  END MARQ.
      END
```

```
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE MQERR (NACTV,NPTS)
C
C  MQERR 1.0          A.N.S.I. STANDARD FORTRAN          JULY 1979
C
C  MQERR IS CALLED BY MARQ TO COMPUTE AND PRINT APPROXIMATE VALUES OF
C  THE PARAMETER ERRORS AND CORRELATIONS.
C
C  FOR THE MEANING OF THE -MAXIMUM VARIANCE INFLATION FACTOR- BELOW,
C  SEE...      D. W. MARQUARDT AND R. D. SNEE,
C              RIDGE REGRESSION IN PRACTICE,
C              THE AMERICAN STATISTICIAN 29 (1975) 3-20
C
C  INPUT QUANTITIES.....  KW,ERR(*,*),NACTV,DELTX(*),NPTS,NV,NTRAC,
C                         MASK(*),FOBJ
C  OUTPUT QUANTITIES....  ERR(*,*)
C
      DOUBLE PRECISION H,SCALE,XMAX,XMIN,DELTX,DELMN,ERR,FOBJ,
     X     QSQRT,ARG,RZERO,RUNIT,HUGE,PIVOT,G,VIFMX,ER,TEMP,
     X     DENOM,SCFAC,RESCL
      DOUBLE PRECISION X
C
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),DELMN(20),
     *     ERR(20,21),FOBJ,NV,NTRAC,MATRX,MASK(20),
     *     NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW
C
C     QSQRT(ARG)= SQRT(ARG)
      QSQRT(ARG)=DSQRT(ARG)
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
      RZERO=0.
      RUNIT=1.
      HUGE=1.E30
C                              PRINT QSAV.
      IF(NTRAC-(-1))50,10,10
   10 WRITE(KW,20)
   20 FORMAT(///18H SUBROUTINE MQERR.//26H QSAV (PT*P, SCALED, WHERE,
     *     23H P IS THE JACOBIAN)....)
      DO 40 JX=1,NACTV
        WRITE(KW,30)JX,(ERR(JX,KX),KX=1,JX)
   30   FORMAT(/1X,I3,2X,6E15.7/(6X,6E15.7))
   40   CONTINUE
C
C  COMPUTE THE SCALED ERROR MATRIX, WHICH IS THE INVERSE OF QSAV.
C  INVERT QSAV USING THE GAUSS-JORDAN METHOD WITHOUT PIVOTING.
C  (PIVOTING IS NOT NECESSARY FOR A POSITIVE DEFINITE MATRIX.)
C  ONLY ABOUT N**3/2 MULTIPLICATIONS ARE DONE.
C  F. L. BAUER AND C. REINSCH, P. 45 IN -LINEAR ALGEBRA-
C  BY J. H. WILKINSON AND C. REINSCH (SPRINGER-VERLAG, 1971)
C  THE INVERSE IS NOT GUARANTEED TO BE POSITIVE DEFINITE, DUE TO
C  ROUNDOFF ERROR.  NEVERTHELESS, THE CHOICE OF THE GAUSS-JORDAN METHOD
```

```
C  RATHER THAN CHOLESKY DECOMPOSITION IS INTENTIONAL.
C  ERR(*,NV+1) IS USED AS A SCRATCH VECTOR.
C
   50 NVPLU=NV+1
      NSMAL=0
      DO 210 LINV=1,NACTV
        L=(NACTV+1)-LINV
        PIVOT=ERR(1,1)
        IF(PIVOT)60,60,70
   60   NSMAL=NSMAL+1
   70   IF(NACTV-2)150,80,80
   80   DO 140 K=2,NACTV
          Q=ERR(K,1)
          IF(PIVOT)100,90,100
   90     ERR(K,NVPLU)=RZERO
          GO TO 130
  100     IF(K-L)110,110,120
  110     ERR(K,NVPLU)=-Q/PIVOT
          GO TO 130
  120     ERR(K,NVPLU)=Q/PIVOT
  130     DO 140 M=2,K
  140       ERR(K-1,M-1)=ERR(K,M)+Q*ERR(M,NVPLU)
  150   IF(PIVOT)170,160,170
  160   ERR(NACTV,NACTV)=RZERO
        GO TO 180
  170   ERR(NACTV,NACTV)=RUNIT/PIVOT
  180   IF(NACTV-2)210,190,190
  190   DO 200 K=2,NACTV
  200     ERR(NACTV,K-1)=ERR(K,NVPLU)
  210   CONTINUE
C
      NRANK=NACTV-NSMAL
      IF(NRANK-NACTV)220,240,240
  220 CONTINUE
      WRITE(KW,230)NRANK,NACTV
  230 FORMAT(//44H THE SECOND DERIVATIVE MATRIX IS SINGULAR IN,
     *       7H MQERR.,9X,7H RANK =,I3,7X,8H ORDER =,I3//
     *       45H THEREFORE ALL PARAMETER ERRORS ARE INFINITE. )
C
C                              UNPACK THE ERROR MATRIX INTO THE UPPER
C                                  TRIANGLE OF ERR(*,*), DE-SCALING IT.
  240 JV=0
      VIFMX=RZERO
      DO 390 JX=1,NV
        IF(MASK(JX))260,250,260
  250   JV=JV+1
  260   KV=0
        DO 380 KX=1,JX
          ER=RZERO
          IF(MASK(JX))380,270,380
  270     IF(MASK(KX))380,280,380
  280     KV=KV+1
          TEMP=RZERO
          DENOM=DELTX(JV)*DELTX(KV)
          IF(DENOM)290,300,290
  290     TEMP=ERR(JV,KV)
          ER=TEMP/DENOM
  300     IF(JX-KX)380,310,380
  310     IF(TEMP)320,320,360
  320     CONTINUE
          IF(NTRAC-(-2))350,330,330
  330     WRITE(KW,340)JX,KX,TEMP
  340     FORMAT(/6H THE (,I3,1H,,I3,23H) ELEMENT OF QSAV**-1 =,E12.5,
     *      5X,45H THEREFORE ALL PARAMETER ERRORS ARE INFINITE. )
  350     TEMP=-TEMP
  360     IF(TEMP-VIFMX)380,380,370
  370     VIFMX=TEMP
  380     ERR(KX,JX+1)=ER
  390   CONTINUE
C
C                              COMPUTE AND PRINT THE STANDARD ERRORS.
      NDF=NPTS-NACTV
      SCFAC=HUGE
      IF(NDF)410,410,400
  400 SCFAC=NDF
      SCFAC=QSQRT(FOBJ/SCFAC)
  410 RESCL=NDF+NDF
      IF(RESCL)430,430,420
  420 RESCL=QSQRT(RESCL)
  430 CONTINUE
      IF(NTRAC-(-1))620,440,440
  440 WRITE(KW,450)NDF,NDF,RESCL,FOBJ,SCFAC
  450 FORMAT(///41H NUMBER OF DEGREES OF FREEDOM (N.D.F.) = ,
     *       15H(NPTS-NACTV) = ,I5//24H EXPECTED VALUE OF PHI = ,
     *       39H N.D.F. PLUS OR MINUS SQRT(2*N.D.F.) = ,I5,
     *       16H  PLUS OR MINUS ,E12.5//23H ACTUAL VALUE OF PHI = ,E12.5
     *       //39H RESCALING FACTOR = SQRT(PHI/N.D.F.) = ,E12.5 )
      WRITE(KW,460)VIFMX
  460 FORMAT(///36H MAXIMUM VARIANCE INFLATION FACTOR =,E12.5///
     *        32H APPROXIMATE STANDARD ERRORS.../68X,9H RESCALED/
     *       12X,1HJ,6X,7HMASK(J),9X,4HX(J),14X,5HERROR,12X,5HERROR )
      RESCL=HUGE
      DO 540 JX=1,NV
        SCALJ=RUNIT
        ER=ERR(JX,JX+1)
        IF(ER)470,490,480
  470   ER=-QSQRT(-ER)
        SCALJ=-ER
        GO TO 490
  480   ER=QSQRT(ER)
        SCALJ=ER
  490   IF(NDF)520,520,500
  500   IF(NRANK-NACTV)520,510,510
  510   RESCL=SCFAC*ER
  520   DELTX(JX)=SCALJ
        WRITE(KW,530)JX,MASK(JX),X(JX),ER,RESCL
  530   FORMAT(/10X,I3,I10,6X,E16.8,4X,E13.5,4X,E13.5 )
  540   CONTINUE
C                              COMPUTE AND PRINT THE CORRELATIONS.
      IF(NV-2)620,550,550
```

```
  550 CONTINUE
      WRITE(KW,560)(K,K=1,NV)
  560 FORMAT(///45H LOWER TRIANGLE OF THE CORRELATION MATRIX....//
     *    12X,7H K ....,I5,7I13/(11X,8I13))
      WRITE(KW,570)(MASK(K),K=1,NV)
  570 FORMAT(/7X,12H MASK(K)....,I5,7I13/(11X,8I13))
      WRITE(KW,580)
  580 FORMAT(/3X,1HJ,4X,7HMASK(J) )
      DO 610 JX=1,NV
        DO 590 KX=1,JX
  590     ERR(KX,1)=ERR(KX,JX+1)/(DELTX(JX)*DELTX(KX))
        WRITE(KW,600)J),MASK(JX),(ERR(KX,1),KX=1,JX)
  600   FORMAT(/1X,I3,18,5X,8E13.5/(17X,8E13.5))
  610   CONTINUE
C                               RESCALE ERR AND SYMMETRIZE IT.
  620 SCFAC=SCFAC**2
      IF(SCFAC-RUNIT)630,640,640
  630 SCFAC=RUNIT
  640 DO 650 JX=1,NV
        DO 650 KX=1,JX
          ERR(JX,KX)=ERR(KX,JX+1)*SCFAC
  650     ERR(KX,JX)=ERR(JX,KX)
C
      RETURN
C  END MOERR.
      END
```

```
C
C      XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C      XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE FUNC (FLMR,Y,YSIG,NPTS,FIT,PHI)
C
C  FUNC CALLS FUNK OR FOFX TO COMPUTE THE ARRAY OF FITTED VALUES FITM.
C
      DOUBLE PRECISION Y,YSIG,XMAX,XMIN,DELTX,DELPN,ERR,FOBJ,
     X      FLAMB,FNU,RELDF
      DOUBLE PRECISION X,FIT,F,PHI,SIG,YY
C
      DIMENSION Y(1),YSIG(1),FIT(1)
C
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),DELMN(20),
     *      ERR(20,21),FOBJ,NV,NTRAC,MATRX,MASK(20),
     *      NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NOREP,KERFL,KW
      COMMON /NLLS3/ FLAPB,FNL,RELDF,METHD,KALCP,KORDF,MAXIT,LEQU,
     X      MXSUB,MXUPD
C
      RZERO=0.
      IF(KALCP)5050,5040,5050
C
 5040 CALL FUNK (FIT)
      GO TO 5070
C
 5050 DO 5060 JPT=1,NPTS
        CALL FOFX(JPT,NV,X,F)
 5060   FIT(JPT)=F
C
 5070 PHI=RZERO
      SIG=YSIG(1)
      DO 5130 JPT=1,NPTS
        IF(LEQU)5090,5080,5090
 5080   SIG=YSIG(JPT)
C                               CHECK FOR AN ILLEGAL VALUE OF SIG.
 5090   IF(SIG)5100,5100,5120
 5100   WRITE(KW,5110)LEQU,JPT,SIG
 5110   FORMAT(/28H ERROR IN PARQ....    LEQU = ,I1,5X,6HJPT = ,I5,5X,
     *      7HYSIG = ,E12.5,14H IS .LE. ZERO.)
        STOP
C
 5120   YY=Y(JPT)
 5130   PHI=PHI+((FIT(JPT)-YY)/SIG)**2
C
      RETURN
C  END FUNC.
      END
```

```
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE DERIV (JPT,FUNK,NPTS,FIT,FITSV,P,LPCOL)
C
C  DERIV 4.0          A.N.S.I. STANDARD FORTRAN         JUNE 1975
C
C  DERIV COMPUTES THE JACOBIAN MATRIX P USING FINITE DIFFERENCES.
C
C  P(J,K) IS THE PARTIAL DERIVATIVE OF FIT(J) WITH RESPECT TO X(K).
C  IF KORDF.EQ.1, DERIV USES A NONCENTRAL DIFFERENCE FORMULA.
C  IF KORDF.EQ.2, DERIV USES A CENTRAL DIFFERENCE FORMULA.
C  KORDF.EQ.1 IS ABOUT TWICE AS FAST AS KORDF.EQ.2, BUT LESS ACCURATE.
C
C  ON A MACHINE HAVING LESS THAN ABOUT TEN SIGNIFICANT DIGITS IN
C  SINGLE PRECISION (FOR EXAMPLE THE IBM 360 OR 370), THE
C  DIFFERENCING SHOULD BE DONE IN DOUBLE PRECISION.  TO ACCOMPLISH
C  THIS, ACTIVATE THE DOUBLE PRECISION STATEMENT BELOW.
C
      DOUBLE PRECISION P
      DOUBLE PRECISION Y,YSIG,XMAX,XMIN,DELTX,DELMN,ERR,FOBJ,
     X      FLAMB,FNU,RELDF,RZERO
      DOUBLE PRECISION X,FIT,FITSV,DEL,TWODL,XSAVE,FX0,FX1
C
      DIMENSION FIT(1),FITSV(1),P(LPCOL,1)
C
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),DELMN(20),
     *      ERR(20,21),FOBJ,NV,NTRAC,MATRX,MASK(20),
     *      NFMAX,NFLAT,JVARY,NXTRA,KFLAG,NCREP,KERFL,KW
      COMMON /NLLS3/ FLAMB,FNU,RELDF,METHD,KALCP,KORDF,MAXIT,LEQU,
     X      MXSUB,MXUPD
C
      JVARY=0
C
C  SAVE FIT IF KALCP.GE.0 .
      IF(KALCP)6020,6000,6000
 6000 DO 6010 J=1,NPTS
 6010   FITSV(J)=FIT(J)
C
C  LOOP OVER THE ACTIVE PARAMETERS X(JX).
C
 6020 KK=0
      DO 6210 JX=1,NV
        IF(MASK(JX))6210,6030,6210
 6030   KK=KK+1
        DEL=RELDF*X(JX)
        IF(DEL)6050,6040,6050
 6040   DEL=RELDF
 6050   XSAVE=X(JX)
        X(JX)=XSAVE+DEL
        TWODL=DEL+DEL
        IF(KALCP)6160,6060,6120
C
C  KALCP.EQ.0 .  COMPUTE P, ONE COLUMN AT A TIME.
C
 6060   CALL FUNK (FIT)
        IF(KORDF-2)6070,6090,6240
 6070   DO 6080 J=1,NPTS
 6080     P(J,KK)=(FIT(J)-FITSV(J))/DEL
        GO TO 6200
C
C  KALCP.EQ.0 AND KORDF.EQ.2 .  IN THIS CASE, THE INPUT VALUES OF
C  FIT(J) WILL BE DESTROYED.
C
 6090   X(JX)=XSAVE-DEL
        DO 6100 J=1,NPTS
 6100     FITSV(J)=FIT(J)
        JVARY=JX
        CALL FUNK (FIT)
        JVARY=0
        DO 6110 J=1,NPTS
 6110     P(J,KK)=(FITSV(J)-FIT(J))/TWODL
        GO TO 6200
C
C  KALCP.GT.0 .  COMPUTE P, ONE ELEMENT AT A TIME.
C
 6120   DO 6150 J=1,NPTS
          CALL FOFX (J,NV,X,FX1)
          IF(KORDF-2)6130,6140,6240
 6130     P(J,KK)=(FX1-FITSV(J))/DEL
          GO TO 6150
 6140     X(JX)=XSAVE-DEL
          CALL FOFX (J,NV,X,FX0)
          P(J,KK)=(FX1-FX0)/TWODL
          X(JX)=XSAVE+DEL
 6150     CONTINUE
        GO TO 6200
C
C  KALCP.LT.0 .  COMPUTE ONE ROW OF P, ONE ELEMENT AT A TIME.
C
 6160   FITSV(1)=FIT(JPT)
        CALL FOFX (JPT,NV,X,FX1)
        IF(KORDF-2)6170,6180,6170
 6170   P(1,KK)=(FX1-FITSV(1))/DEL
        GO TO 6190
 6180   X(JX)=XSAVE-DEL
        CALL FOFX (JPT,NV,X,FX0)
        P(1,KK)=(FX1-FX0)/TWODL
 6190   FIT(JPT)=FITSV(1)
 6200   X(JX)=XSAVE
 6210   CONTINUE
C
      IF(KALCP)6240,6220,6220
 6220 DO 6230 J=1,NPTS
 6230   FIT(J)=FITSV(J)
C
 6240 RETURN
C  END DERIV.
      END
```

```
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     X                                                                  X
C     X   THIS PROGRAM PERMITS THE DYNAMIC ANALYSIS OF A FOUR-BAR-PLANE  X
C     X            MECHANISM BY A FINITE ELEMENT APPROACH                X
C     X                                                                  X
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
C
C       NOTATIONS :
C
C       AA        = LENGTH OF INPUT LINK
C       BB        = LENGTH OF COUPLER LINK
C       CC        = LENGTH OF OUTPUT LINK
C       DD        = LENGTH OF FIXED LINK
C       TH1,TH1D = INITIAL POSITION FOR INPUT LINK , RADIANS AND DEGREES
C       TH2,TH2D = ANGULAR POSITION FOR COUPLER LINK , RADIANS AND DEGREES
C       TH3,TH3D = ANGULAR POSITION FOR OUTPUT LINK , RADIANS AND DEGREES
C       OM1       = ANGULAR VELOCITY FOR INPUT LINK , RAD/S
C       AL1       = ANGULAR ACCELERATION FOR INPUT LINK , RAD/S**2
C       AL2       = ANGULAR ACCELERATION FOR COUPLER LINK , RAD/S**2
C       AL3       = ANGULAR ACCELERATION FOR OUTPUT LIN2 , RAD/S**2
C       R1X,R1Y = POSITION VECTOR OF REVOLUTE M
C       R2X,R2Y = POSITION VECTOR OF REVOLUTE A
C       R3X,R3Y = POSITION VECTOR OF REVOLUTE B
C       R4X,R4Y = POSITION VECTOR OF REVOLUTE Q
C       RTD       = PARAMETER FOR RADIANS TO DEGREES CONVERTER
C       DTR       = PARAMETER FOR DEGREES TO RADIANS CONVERTER
C       NUMEL     = NUMBER OF ELEMENT PER LINK
C       NUMSEL    = NUMBER OF SUBELEMENTS FORMED BY ALL THE POINTS OF EACH
C                   LINK
C       NUP       = NUMBER OF POINTS OF MECHANISM AFTER THE DIVISION INTO
C                   ELEMENTS
C       NN        = SIZE OF DOLOOP FOR MOTION OF MECHANISM ,
C                   NN = (360.0 DEG)/(STEP SIZE IN DEG)
C       R1        = DISTANCE FROM REVOLUTE "I" TO POINT OF STUDY OF LINK
C       R2        = DISTANCE FROM POINT OF STUDY TO REVOLUTE "J" OF LINK
C       XMA       = MAXIMUM RATIO DISTANCE FROM POINT TWO OF BAR OR LINK
C       XMI       = MAXIMUM RATIO DISTANCE FROM POINT ONE OF BAR OR LINK
C       PPX       = POSITION VECTOR IN THE X DIRECTION OF DIVIDING POINTS
C       PPY       = POSITION VECTOR IN THE Y DIRECTION OF DIVIDING POINTS
C       ACX       = LINEAR ACCELERATION IN THE X DIRECTION OF DIVIDING
C                   POINTS
C       ACY       = LINEAR ACCELERATION IN THE Y DIRECTION OF DIVIDING
C                   POINTS
C       STEPDE    = STEPS IN DEGREES WHERE THE MECHANISM IS ANALYSED
C       NUMELM    = NUMBER OF ELEMENTS OF MECHANISM
C       M21       = NUMBER OF ROWS OF V22 , M21 = NUMELM + 5
C       M22       = PARAMETER EQUAL TO 3 * NUMELM
C       M22       + PARAMETER EQUAL TO M22 + 2
C       V22       = TRANSFORMATION MATRIX TO OBTAIN THE DEFINITE GLOBAL
C                   POSITION MATRIX OF MECHANISM , V22(M21,M22)
C       DT        = DEFINITE GLOBAL POSITION MATRIX OF MECHANISM ,
C                   DT(M22,M22)
C       DT1       = REDUCED MATRIX AFTER APPLYING BOUNDARY CONDITIONS TO
C                   MATRIX DT , DT1(M22,MASIR)
C       MASIR     = SIZE OF MATRIX DTR , MASIR = M22 - NUMBOU
C       NBOUNR    = BOUNDARY CONDITIONS APPLIED TO THE ROWS OF DT ,
C                   NBOUNR(M22)
C       NBOUNC    = BOUNDARY CONDITIONS APPLIED TO THE COLUMNS OF DT ,
C                   NBOUNC(M22)
C       NUMBOU    = NUMBER OF BOUNDARY
C       LC        = SPECIFIED BOUNDARY CONDITIONS FOR COLUMNS OF DT ,
C                   LC(NUMBOU)
C       LR        = SPECIFIED BOUNDARY CONDITIONS FOR ROWS OF DT ,
C                   LR(NUMBOU)
C       TAS       = INETIAL VECTOR , TAS(M22)
C       TASR      = INERTIAL VECTOR REDUCED DUE TO BOUNDARY CONDITIONS ,
C                   TASR(MASIR)
C       XMM       = MASS OF EACH ELEMENT , XMM(NUMELM)
C       XMII      = MOMENT OF INERTIA OF EACH ELEMENT , XMI(NUMELM)
C       GM        = APPLIED TORQUE , IF ANY , AT NODE J OF EACH ELEMENT ,
C                   IT COULD BE APPLIED AT THE REVOLUTE J OF LINK ,
C                   GM(NUMELM)
C       AIA       = VECTOR FORMED BY THE ANGULAR ACCELERATIONS OF THE LINKS.
C                   IN THIS WAY THE ACCELERATION FOR EACH ELEMENT IS
C                   OBTAINED , ELEMENTS OF THE SAME LINK HAVE THE SAME
C                   ANGULAR ACCELERATION , AIA(NUMELM)
C       K         = COUNTER FOR ACX , ACY , PPX , PPY
C       FFF       = FORVEC , FFF(NN,MASIR)
C       FORVEC    = SOLUTION VECTOR OR FORCE VECTOR FOR THE MECHANISM ,
C                   FORVEC(MASIR)
C       VV        = INSTANTANEOUS GEOMETRIC MATRIX FOR EVERY ELEMENT FOR
C                   EVERY STEPDE , VV(NUMELM,4,6)
C       X1L       = LENGTH OF ELEMENTS , X1L(NUMELM)
C       RO        = SPECIFIC VOLUME OF EACH ELEMENT , RO(NUMELM)
C       WIDE      = WIDTH OF EACH ELEMENT , WIDE(NUMELM)
C       THICK     = THICKNESS OF EACH ELEMENT , THICK(NUMELM)
C       IT        = PARAMETER USED FOR FORMING MATRIX V22 , IT IS
C                   EQUAL TO "1" FOR A SYMETRIC UNIT MATRIX , IT IS
C                   EQUAL TO "2" FOR A NON-SYMETRIX UNIT MATRIX , IT(NUMELM)
C       T         = GLOBAL POSITION MATRIX OF THE ASSEMBLED ELEMENTS OF
C                   MECHANISM , T(M22,M21)
C
C
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION FFF(37,27) , SS(19,37) , ACX(37,19) , ACY(37,19) ,
     1          PPX(37,19) , PPY(37,19)
      DIMENSION RX(19) , RY(19) , SX(19) , SY(19)
      DIMENSION LR(9) , LC(9) , XMM(9) , XMII(9) , GM(9) , AIA(9)
      DIMENSION FORVEC(27) , DT1(27,27) , TASR(27)
      DIMENSION NBOUNR(27) , NBOUNC(30) , DT(27,30) , TAS(27)
      DIMENSION V22(54,30) , T(27,54) , VV(9,3,6)
      DIMENSION X1L(9) , RO(9) , WIDE(9) , THICK(9) , IT(9)
```

```
      COMMON/SUB1/DTR,RTD,TH1D,R1X,R1Y,R2X,R2Y,R3X,R3Y,R4X,R4Y,TH1,TH2,
     1              TH3,TH2D,TH3D
      COMMON/SUB2/AA,BE,CC,DD
      COMMON/SUB3/R1,R2
C
C
C
C     READ IN AND PRINT OUT THE GIVEN INFORMATION TO CARRY OUT ALL OF
C     THE REQUIRED INFORMATION
C
      READ 100, NUMEL , NUP , NN , NUMELM , NUMBOU , STEPDE , XMA , XMI
  100 FORMAT (5I2,3F5.0)
      READ 105, AA , BB , CC , DD , TH1D , OM1 , AL1
  105 FORMAT (4D20.0,/,3D15.0)
      DO 1 I=1,NUMBOU
      READ 107, LR(I) , LC(I)
  107 FORMAT (2I3)
    1 CONTINUE
      DO 2 I=1,NUMELM
      READ 109, RO(I) , WIDE(I) , THICK(I) , GM(I)
  109 FORMAT (4E20.0)
    2 CONTINUE
      READ 110, (IT(I),I=1,NUMELM)
  110 FORMAT (9I3)
      DO 3 I=1,NUMELM
      READ 113, X1L(I) , XMM(I) , XMII(I)
  113 FORMAT (3D20.0)
    3 CONTINUE
C
      PRINT 130
  130 FORMAT (1H1)
      PRINT 131
  131 FORMAT (////)
      PRINT 132
  132 FORMAT (61X,'INPUT DATA')
      PRINT 133
  133 FORMAT (61X,'XXXXXXXXXX')
      PRINT 131
      PRINT 135
  135 FORMAT (11X,'NUMEL',11X,'NUP',12X,'NN',11X,'NUMELM', 9X,'NUMBOU',
     110X,'STEPDE',11X,'XMA',12X,'XMI',/)
      PRINT 136, NUMEL , NUP , NN , NUMELM , NUMBOU , STEPDE , XMA , XMI
  136 FORMAT ( 9X,I5,10X,I5,10X,I5,10X,I5,10X,I5,3F15.2)
      PRINT 131
      PRINT 140
  140 FORMAT (20X,'AA',10X,'BB',10X,'CC',10X,'DD',10X,'TH1D', 8X,'OM1',
     19X,'AL1',/)
      PRINT 141, AA , BB , CC , DD , TH1D , OM1 , AL1
  141 FORMAT (21X,7F12.4)
      PRINT 131
      PRINT 142
  142 FORMAT (61X,'LR', 6X,'LC',/)
      DO 10 I=1,NUMBOU
      PRINT 143, LR(I) , LC(I)
  143 FORMAT (53X,2I10)
   10 CONTINUE
      PRINT 131
      PRINT 145
  145 FORMAT (26X,'ELEMENT', 8X,'RO',13X,'WIDE',11X,'THICK',10X,'GM',16X
     1,'IT',/)
      DO 11 I=1,NUMELM
      PRINT 146, I , RO(I) , WIDE(I) , THICK(I) , GM(I) , IT(I)
  146 FORMAT (25X,I5,5X,4E15.4,5X,I6)
   11 CONTINUE
      PRINT 131
      PRINT 200
  200 FORMAT (38X,'ELEMENT', 8X,'X1L',12X,'XMM',12X,'XMII',/)
      DO 12 I=1,NUMELM
      PRINT 201, I , X1L(I) , XMM(I) , XMII(I)
  201 FORMAT (37X,I5,5X,3E15.4)
   12 CONTINUE
C
C
C
C
C     INTRODUCE CONSTANT INFORMATION
C
      XMI  =  1.0D0/6.0D0
      XMA = 5.0D0 * XMI
      PI = 4.0 * DATAN(1.0D 0)
      DTR = PI/180.0D+00
      RTD = 180.0D+00/PI
      NUMSEL = NUMEL * 2
      M22 = 3 * NUMELM
      M21 = 6 * NUMELM
      M22 = M22 + 3
      OM1 = OM1 * 2.0D0 * PI / 60.0D0
      MASIR = M22 - NUMBOU
      CALL UNIT (NUMELP,M21,M22,IT,V22)
      DO 14 LM=1,M22
      DO 14 LN=1,M21
      T(LM,LN) = 0.0
   14 CONTINUE
      DO 16 K=1,M22
      NBOUNR(K) = 0
   16 CONTINUE
      DO 17 K=1,M22
      NBOUNC(K) = 0
   17 CONTINUE
      DO 18 K=1,NUMBOU
      IF (LR(K) .NE. 0) NBOUNR(LR(K))=LR(K)
      NBOUNC(LC(K)) = LC(K)
   18 CONTINUE
C
C
C     INITIALIZE DO LOOP THROUGH 360 DEGREES OF INPUT LINK MOTION
C
C
      DO 30 I=1,NN
C
```

```
      CALL LARGB(AA,BB,CC,DD)
      PPX(I,1) = R1X
      PPY(I,1) = R1Y
      ACX(I,1) = 0.0
      ACY(I,1) = 0.0
      CALL PIEDRA(TH1,TH2,TH3,OM1,AL1,OM2,AL2,OM3,AL3)
      SS(1,I) = TH1D
      SS(2,I) = TH2D
      SS(3,I) = TH3D
      SS(4,I) = AL2
      SS(5,I) = AL3
      SS(6,I) = OM2
      SS(7,I) = OM3
      R1 = 0.0
      K = 0
      DO 31 J=1,NUMSEL
      K = K + 1
      R2 = XMA - R1
      R1 = XMI + R1
      CALL ACCP(ACX(I,K),ACY(I,K),OM1,AL1,R1X,R1Y,R2X,R2Y,PPX(I,K),PPY(I
     1,K),PPX(I,K+1),PPY(I,K+1),ACX(I,K+1),ACY(I,K+1))
   31 CONTINUE
C
      R1 = 0.0
      DO 32 J=1,NUMSEL
      K = K + 1
      R2 = XMA - R1
      R1 = XMI + R1
      CALL ACCP(ACX(I,K),ACY(I,K),OM2,AL2,R2X,R2Y,R3X,R3Y,PPX(I,K),PPY(I
     1,K),PPX(I,K+1),PPY(I,K+1),ACX(I,K+1),ACY(I,K+1))
   32 CONTINUE
C
      R1 = 0.0
      DO 33 J=1,NUMSEL
      K = K + 1
      R2 = XMA - R1
      R1 = XMI + R1
      CALL ACCP(ACX(I,K),ACY(I,K),OM3,AL3,R3X,R3Y,R4X,R4Y,PPX(I,K),PPY(I
     1,K),PPX(I,K+1),PPY(I,K+1),ACX(I,K+1),ACY(I,K+1))
   33 CONTINUE
C
      CALL POSMAT(PPX,PPY,I,K,NUMELM,NUP,V22,SX,SY,RX,RY,VV,M21,M22,
     1               M22,T,DT,NN)
      CALL PARED (M22,P22,MASIR,NBOUNR,NBOLNC,DT,DT1)
      CALL MSSM(NUMEL,NUMELM,M22,MASIR,K,XMM,XMII,ACX,ACY,AL1,AL2,AL3,GM
     1             ,TAS,TASR,NBOUNR,A1A,I,NN)
      CALL SMEQ1 (DT1,TASR,FORVEC,MASIR,MASIR)
      DO 34 K1K=1,MASI6
      FFF(I,K1K) = FORVEC(K1K)
   34 CONTINUE
C
      TH1D = TH1D + STEPDE
   30 CONTINUE
C
C     INITIALIZE THE PRINTING OUT OF SOME OF THE CALCULATIONS CARRIED
C     OUT
C
      PRINT 130
      PRINT 131
      PRINT 600
  600 FORMAT (45X,'SOME RESULTS OBTAINED IN THE CALCULATIONS')
      PRINT 601
  601 FORMAT (45X,'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX')
      PRINT 131
      PRINT 604
  604 FORMAT (30X,'POSITION ANGLES , ANGULAR ACCELERATIONS AND ANGULAR V
     1ELOCITIES OF LINKS',//)
      PRINT 606
  606 FORMAT (27X,'TH1', 9X,'TH2', 9X,'TH3', 9X,'AL2', 9X,'AL3', 9X,'OM2
     1', 9X,'OM3',/)
      DO 40 L=1,NN
   40 PRINT 610, (SS(K,L),K=1,7)
  610 FORMAT (20X,7F12.2)
      PRINT 130
      PRINT 131
      PRINT 613
  613 FORMAT (43X,'ACCELERATIONS OF SOME POINTS IN THE X DIRECTION',//)
      PRINT 617, ((ACX(M,L),L=1,13),M=1,NN)
  617 FORMAT (13F10.2)
      PRINT 130
      PRINT 131
      PRINT 618, ((ACX(M,L),L=14,NUP),M=1,NN)
  618 FORMAT (6F10.2)
      PRINT 130
      PRINT 131
      PRINT 630
  630 FORMAT (43X,'ACCELERATIONS OF SOME POINTS IN THE Y DIRECTION',//)
      PRINT 617, ((ACY(M,L),L=1,13),M=1,NN)
      PRINT 130
      PRINT 131
      PRINT 618, ((ACY(M,L),L=14,NUP),M=1,NN)
      PRINT 130
      PRINT 131
      PRINT 633
  633 FORMAT (43X,'POSITION VECTOR OF SOME POINTS IN THE X DIRECTION',/)
      PRINT 617, ((PPX(M,L),L=1,13),M=1,NN)
      PRINT 130
      PRINT 131
      PRINT 618, ((PPX(M,L),L=14,NUP),M=1,NN)
      PRINT 130
      PRINT 131
      PRINT 635
  635 FORMAT (43X,'POSITION VECTOR OF SOME POINTS IN THE Y DIRECTION',/)
      PRINT 617, ((PPY(M,L),L=1,13),M=1,NN)
      PRINT 130
      PRINT 131
      PRINT 618, ((PPY(M,L),L=14,NUP),M=1,NN)
      PRINT 130
      PRINT 131
      PRINT 638
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 638 FORMAT (42X,'RESULTANT FORCES AND TORQUE AT JOINTS OF ELEMENTS',//
    1)
      PRINT 645, ((FFF(L,K),K=1,16),L=1,NN)
 645 FORMAT (16F8.2)
      PRINT 130
      PRINT 131
      PRINT 646, ((FFF(L,K),K=17,MASIR),L=1,NN)
 646 FORMAT (11F8.2)
C
      PRINT 130
      STOP
      END
C$EJECT
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE UNIT( NUMELE, M1,M2, IT,F)
C
C     THIS SUBROUTINE FORMS THE TRANSFORMATION MATRIX TO OBTAIN
C     THE DEFINITE GLOBAL POSITION MATRIX OF MECHANISM
C
C     NOTATIONS :
C
C     M1       = NUMBER OF ROWS OF F , M1 = 5 * NUMELE
C     M2       = NUMBER OF COLUMNS OF F , M2 = 4 * NUMELE - 1
C     NUMELE   = NUMBER OF ELEMENT OF ENCHANISM
C     F        = TRANSFORMATION MATRIX
C
C
      IMPLICIT REAL*8( A-H,O-Z )
      DIMENSION F(M1,M2) , IT(NUMELE)
C
C
C     INITIALIZE SPACE FOR MATRIX F
C
      DO 1 I=1,M1
      DO 2 J=1,M2
      F(I,J) = 0.0
    2 CONTINUE
    1 CONTINUE
C
C     INITIALIZE DETERMINATION OF UNIT COEFFICIENTS FOR MATRIX F
C
      L = 1
      J = 0
      M = 6
      K = 9
      DO 3 I=1,M1
      J = J + 1
      IF (IT(L) .EQ. 2 .AND. I .EQ. K) J = J + 1
      F(I,J) = 1.0
      IF (I .EQ. M) J = J - 3
      IF (I .EQ. M) L = L + 1
      IF (I .EQ. M) M = M + 6
      IF (I .EQ. K) K = K + 6
    3 CONTINUE
C
      RETURN
      END
C$EJECT
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE LARGO(A,B,C,D)
C
C     THIS SUBROUTINE DETERMINES THE DIRECTION ANGLES FOR
C     COUPLER LINK AND OUTPUT LINK, CARRIES OUT TRANSFORMATION
C     OF REVOLUTE POSITIONS , DETERMINES POSITION VECTORS OF POINTS
C
C     NOTATIONS :
C
C     A        = INPUT LINK LENGTH
C     B        = COUPLER LINK LENGTH
C     C        = OUTPUT LINK LENGTH
C     D        = FIXED LINK LENGTH
C     C1       = DEGREES TO RADIANS CONVERTER
C     C2       = RADIANS TO DEGREES CONVERTER
C     A1,A1D   = POSITION ANGLES OF INPUT LINK , RADIANS AND DEGREES
C     A2,A2D   = POSITION ANGLES OF OUTPUT LINK , RADIANS AND DEGREES
C     T1:T10   = COEFFICIENTS
C     E1X,E1Y  = POSITION VECTOR OF REVOLUTE P
C     E2X,E2Y  = POSITION VECTOR OF REVOLUTE A
C     E3X,E3Y  = POSITION VECTOR OF REVOLUTE B
C     E4X,E4Y  = POSITION VECTOR OF REVOLUTE Q
C
C
C
      IMPLICIT REAL*8( A-H,O-Z )
      COMMON/SUB1/C1,C2,A1D,E1X,E1Y,E2X,E2Y,E3X,E3Y,E4X,E4Y,A1,A2,A3,A2D
     1            ,A3D
C
      A1 = A1D * C1
C
C     INITIALIZE THE CALCULATIONS FOR DIRECTION ANGLES FOR COUPLER
C     LINK AND OUTPUT LINK
C
      T1 = D/A
      T2 = D/C
      T3 = (A**2 - B**2 + C**2 + D**2)/(2.0D+00*A*C)
      T4 = D/B
      T5 = (C**2 - D**2 - A**2 - B**2)/(2.0D+00*A*B)
      T6 = DCOS(A1) + T3 - T1 - T2 * DCOS(A1)
      T7 = - 2.0D+00 * DSIN(A1)
      T8 = T1 + T3 - (1.0D+00 + T2) * DCOS(A1)
      T9 = T4 * DCOS(A1) + DCOS(A1) + T5 - T1
      T10 = T4 * DCOS(A1) - DCOS(A1) + T5 + T1
C
      A2 = 2.0D+00 * DATAN((-T7 - DSQRT(T7**2 - 4.0D+00*T9*T10))/
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```
     1      (2.0D+00*T9))
      A3 = 2.0D+00 * DATAN((-T7 - DSURT(T7**2 - 4.0D+00*T6*T8))/
     1      (2.0D+00*T6))
      A2D = A2 * C2
      A3D = A3 * C2
C
C     INITIALIZE TRANSFORMATION TO FIND REVOLUTE POSITION VECTORS
C
      E1X = 0.0D+00
      E1Y = 0.0D+00
      E2X = A * DCOS(A1)
      E2Y = A * DSIN(A1)
      E4X = B
      E4Y = 0.0D+00
      E3X = E4X + C*DCCS(A3)
      E3Y = E4Y + C*DSIN(A3)
C
      RETURN
      END
CSEJECT
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE PIEDRA(O2,O3,O4,R2,AA2,R3,AA3,R4,AA4)
C
C     THIS SUBROUTINE DETERMINES THE ANGULAR VELOCITIES AND ANGULAR
C     ACCELERATIONS OF COUPLER AND OUTPUT LINKS RESPECTIVELY
C
C     NOTATIONS :
C
C     O2       = POSITION ANGLE OF INPUT LINK , RADIANS
C     O3       = POSITION ANGLE OF COUPLER LINK , RADIANS
C     O4       = POSITION ANGLE OF OUTPUT LINK , RADIANS
C     R2       = ANGULAR VELOCITY OF INPUT LINK , RAD/S
C     R3       = ANGULAR VELOCITY OF COUPLER LINK , RAD/S
C     R4       = ANGULAR VELOCITY OF OUTPUT LINK , RAD/S
C     AA2      = ANGULAR ACCELERATION OF INPUT LINK , RAD/S**2
C     AA3      = ANGULAR ACCELERATION OF COUPLER LINK , RAD/S**2
C     AA4      = ANGULAR ACCELERATION OF OUTPUT LINK , RAD/S**2
C     X6       = LENGTH OF INPUT LINK
C     X7       = LENGTH OF COUPLER LINK
C     X8       = LENGTH OF OUTPUT LINK
C     X9       = LENGTH OF FIXED LINK
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/SUB2/X6,X7,X8,X9
C
C     FIND THE VALUES FOR R3 AND R4
C
      R3 = (X6*R2*DSIN(O4-O2)) / (X7*DSIN(C3-O4))
      R4 = (X6*R2*DSIN(O2-O3)) / (X8*DSIN(O4-O3))
C
C     FIND THE VALUES OF AA3 AND AA4
C
      T4 = X6*AA2*DSIN(O2) + X6*R2**2*DCOS(O2) + X7*R3**2*DCOS(O3) -
     1     X8*R4**2*DCOS(O4)
      T5 = X8*DCOS(O4)
      T6 = X8*DSIN(O4)
      T7 = X6*AA2*DCOS(O2) - X6*R2**2*DSIN(O2) - X7*R3**2*DSIN(O3) +
     1     X8*R4**2*DSIN(O4)
      T8 = X7*DCOS(O3)
      T9 = X7*DSIN(O3)
C
      AA3 = (T4*T5 - T6*T7) / (T6*T8 - T9*T5)
      AA4 = (T4*T8 - T9*T7) / (T6*T8 - T9*T5)
C
      RETURN
      END
CSEJECT
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE ACCP(BX,BY,WZ,AAZ,X1,Y1,X2,Y2,B1,B2,A1,A2,AX,AY)
C
C     THIS SUBROUTINE DETERMINES THE ACCELERATION AT A
C     POINT "A" IF THE ACCELERATION OF POINT "B" IS KNOWN
C     OF A RIGID BODY IN THE PLANE
C
C     NOTATIONS :
C
C     AX       = ACCELERATION OF THE REQUIRED POINT "A" IN
C                THE X DIRECTION
C     AY       = ACCELERATION OF THE REQUIRED POINT "A" IN
C                THE Y DIRECTION
C     BX,BY    = GIVEN ACCELERATION OF POINT "B"
C     WZ       = GIVEN ANGULAR VELOCITY OF BAR "AB"
C     AAZ      = GIVEN ANGULAR ACCELERATION OF BAR "AB"
C     A1,A2    = POSITION VECTOR OF POINT "A"
C     B1,B2    = POSITION VECTOR OF POINT "B"
C     CX,CY    = VECTOR "BA"
C     R1       = DISTANCE FROM REVOLUTE "I" TO POINT OF STUDY OF LINK
C     R2       = DISTANCE FROM POINT OF STUDY TO REVOLUTE "J" OF LINK
C
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/SUB3/R1,R2
C
C
C     INITIALIZE THE DIVISION OR PARTINTION INTO ELEMENTS OF THE LINK
C     AND FIND VECTOR C
C
      A1 = (X1*R2 + X2*R1)/(R1 + R2)
      A2 = (Y1*R2 + Y2*R1)/(R1 + R2)
      CX = A1 - B1
      CY = A2 - B2
C
```

```
C     INITIALIZE DETERMINATION OF ACCELERATIONS
C
      AX = BX - WZ**2*CX - AA2*CY
      AY = BY - WZ**2*CY + AA2*CX
C
      RETURN
      END
C$EJECT
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE POSMAT (PX,PY,J,K,N1,M1,U,SX,SY,RX,RY,V,I2,J2,K2,T,DT,
     1                   NN)
C
C     THIS SUBROUTINE DETERMINES THE POSITION MATRIX OF THE MECHANISM
C
C     NOTATIONS :
C
C     N1        = NUMBER OF ELEMENTS IN THE MECHANISM
C     M1        = NUMBER OF POINTS OF MECHANISM AFTER THE  DIVISION INTO
C                 ELEMENTS
C     NN        = SIZE OF DOLOOP FOR MOTION OF MECHANISM
C     V         = POSITION MATRIX OF EACH ELEMENT , V(N1,4,6)
C     PX        = POSITION VECTOR IN THE X DIRECTION OF DIVIDING POINTS
C     PY        = POSITION VECTOR IN THE Y DIRECTION OF DIVIDING POINTS
C     RX,RY     = POSITION VECTOR OF CENTER OF MASS OF ELEMENT TO NODE I
C     SX,SY     = POSITION VECTOR OF CENTER OF MASS OF ELEMENT TO NODE J
C     T         = GLOBAL POSITION MATRIX OF THE ASSEMBLED ELEMENTS OF
C                 MECHANISM , T(K2,I2)
C     DT        = DEFINITE GLOBAL POSITION MATRIX OF MECHANISM , DT=T*U
C     U         = TRANSFORMATION MATRIX TO OBTAIN MATRIX DT
C     I2        = NUMBER OF ROWS OF MATRIX U
C     J2        = NUMBER OF COLUMNS OF MATRIX U
C     K2        = PARAMETER EQUAL TO M22 IN THE PROGRAM , EQUAL TO
C                 4 * (NUMSEL+NUMEL) + 2
C
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION PX(NN,P1) , PY(NN,M1) , RX(N1) , RY(N1) , SX(N1) ,
     1          SY(N1) , V(N1,3,6) , U(I2,J2) , T(K2,I2) , DT(K2,J2)
C
C
C     FIND VECTORS RX , RY , SX AND SY
C
      M2 = M1 - 2
      I = 0
      DO 1 K1=1,M2,2
      I = I + 1
      RX(I) = (PX(J,K1+2)-PX(J,K1))/2.0
      RY(I) = (PY(J,K1+2)-PY(J,K1))/2.0
      SX(I) = (PX(J,K1+2)-PM(J,K1))/2.0
      SY(I) = (PY(J,K1+2)-PY(J,K1))/2.0
    1 CONTINUE
C
C     FIND THE POSITION MATRIX OF EACH ELEMENT
C
      DO 2 I=1,N1
      V(I,1,1) = - 1.0
      V(I,1,2) =   0.0
      V(I,1,3) =   0.0
      V(I,1,4) =   1.0
      V(I,1,5) =   0.0
      V(I,1,6) = 0.0
      V(I,2,1) =   0.0
      V(I,2,2) = - 1.0
      V(I,2,3) =   0.0
      V(I,2,4) =   0.0
      V(I,2,5) =   1.0
      V(I,2,6) = 0.0
      V(I,3,1) = - RY(I)
      V(I,3,2) =   RX(I)
      V(I,3,3) =   1.0
      V(I,3,4) = - SY(I)
      V(I,3,5) =   SX(I)
      V(I,3,6) = - 1.0
    2 CONTINUE
C
C     INITIALIZE ARRANGEPENT TO OBTAIN MATRIX T
C
      LL = 1
      MM = 3
      JJ = 1
      II = 6
C
      DO 3 I=1,N1
      N = 0
      DO 4 M=LL,MM
      N = N + 1
      N2 = 0
      DO 5 L=JJ,II
      N2 = N2 + 1
      T(M,L) = V(I,N,N2)
    5 CONTINUE
    4 CONTINUE
      JJ = JJ + 6
      II = II + 6
      MM =  MM + 3
      LL = LL + 3
    3 CONTINUE
C
C     INITIALIZE CALCULATION TO OBTAIN MATRIX DT
C
      DO 6 L=1,K2
      DO 6 M=1,J2
      DT(L,M) = 0.0
      DO 6 KJ=1,I2
      DT(L,M) = DT(L,M) + T(L,KJ)*U(KJ,M)
    6 CONTINUE
```

```
C
      RETURN
      END
C$EJECT
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE MARED (MASIZ,MASIZ1,MASIR,NBOUNR,NBOUNC,A,B)
C
C     THIS SUBROUTINE REDUCES A MATRIX "A" TO A MATRIX "B" OF LOWER
C     ORDER BY ELIMINATING ROWS AND COLUMNS ACCORDING TO NBOUNR AND
C     NBOUNC
C
C     NOTATIONS :
C
C     MASIZ   = ROWS OF MATRIX "A"
C     MASIR   = SIZE OF MATRIX "B"
C     MASIZ1  = SIZE OF VECTOR "NBOUNC" AND THE COLUMNS OF "A"
C     NBOUNR  = BOUNDARY FOR ROWS OF "A"
C     NBOUNC  = BOUNDARY FOR COLUMNS OF "A"
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(MASIZ,MASIZ1) , B(MASIR,MASIR) , NBOUNC(MASIZ1) ,
     1          NBOUNR(MASIZ)
C
C
C
      N = 0
      DO 1 I=1,MASIZ
      IF (I .EQ. NBOUNR(I)) GO TO 30
      N = N + 1
      M = 0
      DO 2 J=1,MASIZ1
      IF (J .EQ. NBOUNC(J)) GO TO 31
      M = M + 1
      B(N,M) = A(I,J)
   31 CONTINUE
    2 CONTINUE
   30 CONTINUE
    1 CONTINUE
C
      RETURN
      END
C$EJECT
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE MSSX(NU,N,M,L,K,XM,XMI,AX,AY,A1,A2,A3,GM,X,XR,NBOUNR,
     1          AA,M23,NN)
C
C     THIS SUBROUTINE FORMS THE INERTIAL VECTOR GIVEN THE MASS ,
C     MOMENT OF INERTIA , LINEAR ACCELERATIONS , ANGULAR ACCELERATIONS
C     AND APPLIED TORQUE OF EACH ELEMENT
C
C     NOTATIONS :
C
C     NU       = NUMBER OF ELEMENTS PER LINK
C     M        = SIZE OF VECTOR X , M = 4 * N
C     N        = NUMBER OF ELEMENTS OF MECHANISM
C     NN       = SIZE OF DOLOOP FOR MOTION OF MECHANISM
C     XM       = MASS OF EACH ELEMENT , XM(N)
C     AX,AY    = ACCELERATIONS IN THE X AND Y DIRECTIONS OF CENTER OF
C                MASS OF EACH ELEMENT , AX(1,K) , AY(1,K)
C     A1,A2,A3 = ANGULAR ACCELERATION OF EACH LINK ; WHICH IS THE SAME
C                FOR EVERY ELEMENT IN THE LINK
C     GM         APPLIED TORQUE  , IF ANY , AT NODE J OF EACH ELEMENT ,
C                IT COULD BE APPLIED AT THE REVOLUTE , GM(N)
C     M23      = VALUE OF DO-LOOP AT WHERE THE MECHANISM IS ANALYSED ,
C                M23 = 1,NN
C     XMI      = MOMENT OF INERTIA OF THE ELEMENT , XMI(N)
C     X        = INERTIAL VECTOR , X(M)
C     XR       = REDUCED VECTOR X DUE TO BOUNDARY CONDITIONS ; XR(L)
C     L        = SIZE OF VECTOR XR
C     K        = NUMBER OF POINTS OF MECHANISM AFTER THE DIVISION
C                INTO ELEMENTS
C     NBOUNR   = BOUNDARY CONDITIONS APPLIED TO THE ROWS OF X ,
C                NBOUNR(M)
C     AA       = VECTOR FORMED BY THE ANGULAR ACCELERATIONS OF THE
C                LINKS , IN THIS WAY THE ACCELERATION FOR EACH ELEMENT
C                IS OBTAINED , ELEMENTS OF THE SAME LINK HAVE THE
C                SAME ANGULAR ACCELERATION ; AA(N)
C
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(M) , XM(N) , AX(NN,K) , AY(NN,K) , XMI(N) , AA(N) ,
     1          GM(N) , XR(L) , NBOUNR(M)
C
C
C     INITIALIZE FORMING VECTOR AA
C
      L1 = 0
      M1 = 0
      DO 1 I1=1,NU
      M1 = M1 + 1
      L1 = L1 + 1
    1 AA(I1) = A1
      DO 2 I1=1,NU
      L1 = L1 + 1
    2 AA(M1+I1) = A2
      DO 3 I1=1,NU
    3 AA(L1+I1) = A3
C
C     INITIALIZE FORMING VECTOR X
C
      I = 0
      DO 4 J=1,N
```

```
      I = I + 1
      X(I) = XM(J) + AX(M23,2*(J+1)-2)
      I = I + 1
      X(I) = XM(J) + AY(M23,2*(J+1)-2) + XM(J)*32.2
      I = I + 1
      X(I) = XMI(J) * AA(J) - GM(J)
    4 CONTINUE
C
C     INITIALIZE APPLYING BOUNDARY CONDITIONS
C
      K12 = 0
      DO 5 J=1,M
      IF (J .EQ. NBOUND(J)) GO TO 5
      K12 = K12 + 1
      XR(K12) = X(J)
    5 CONTINUE
C
      RETURN
      END
C$EJECT
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE SMEQ1  (A,B,Z,N,KR)
C
C     THIS SUBROUTINE PERMITS TO SOLVE A SYSTEM OF NXN LINEAR EQUATIONS BY THE
C     GAUSS ELIMINATION METHOD.
C
C
C     SOLUTION OF LINEAR SIMULTANEOUS ALGEBRAIC EQUATIONS. A*Z = B
C     GAUSS ELIMINATION METHOD
C     LARGEST PIVOTAL DIVISOR IS USED TO AVOID DIVISION BY SMALL NUMBERS
C
C     BORROWED FROM THE FORMA LIBRARY OF MARTIN MARIETTA
C
C     SUBROUTINE ARGUMENTS :
C
C     A    = INPUT  SQUARE MATRIX OF COEFFICIENTS. SIZE(N,N). *DESTROYED*
C     B    = INPUT  RIGHT HAND SIDE VECTOR. SIZE(N). *DESTROYED*
C     Z    = OUTPUT RESULT VECTOR. SIZE(N).
C     N    = INPUT  NUMBER OF EQUATIONS.
C     KR   = INPUT  ROW DIMENSION OF A IN CALLING PROGRAM.
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(KR,1),B(1),Z(1)
      DATA TOL /1.0D-15/
C
      IF (N .GT. 1) GO TO 5
                                                       NERROR = 1
      IF (DABS(A(1,1)) .LE. TOL) GO TO 999
      Z(1) = B(1)/A(1,1)
      RETURN
C
C     FORWARD SOLUTION
```

```
    5 DO 25 L=1,N
      AMAX = TOL
      DO 10 I=L,N
      IF (DABS(A(I,L)) .LT. DABS(AMAX)) GO TO 10
      AMAX = A(I,L)
      IMAX = I
   10 CONTINUE
                                                       NERROR = 2
      IF (DABS(AMAX) .LE. TOL) GO TO 999
      DO 15 J=L,N
      SAVE = A(IMAX,J)
      A(IMAX,J) = A(L,J)
   15 A(L,J) = SAVE/AMAX
      SAVE = B(IMAX)
      B(IMAX) = B(L)
      B(L) = SAVE/AMAX
      IF (L .EQ. N)  GO TO 40
      LP1 = L + 1
      DO 25 I=LP1,N
      DO 20 J=LP1,N
   20 A(I,J) = A(I,J) - A(I,L)*A(L,J)
   25 B(I) = B(I) - A(I,L)*B(L)
C
C     BACK SOLUTION
   40 Z(N) = B(N)
      NM1 = N - 1
      DO 45  L=1,NM1
      I = N - L
      Z(I) = B(I)
      IP1 = I + 1
      DO 45  J=IP1,N
   45 Z(I) = Z(I) - A(I,J)*Z(J)
      RETURN
C
  999 WRITE (6,201)  NERROR
  201 FORMAT (1H1//10X,6HSMEQ1 ,10X,8HNERROR =,I2)
      RETURN
      END
```

```
C
C
C
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     X                                                                 X
C     X    THIS PROGRAM PERMITS THE KINETC-ELASTCDYNAMIC ANALYSIS       X
C     X               CF A FOUR-BAR-PLANE MECHANISM BY                  X
C     X                     FINITE ELEMENT ANALYSIS                     X
C     X                                                                 X
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
C
C       NOTATICNS :
C
C       AA          = LENGTH OF  INPUT LINK
C       BB          = LENGTH OF CCUPLER LINK
C       CC          = LENGTH OF OUTPUT LINK
C       DD          = LENGTH OF FIXED LINK
C       TH1,TH1D    = INITIAL PCSITICN FOR INPUT LINK . RADIANS AND DEGREES
C       OM1         = ANGULAR VELCCITY FOR INPUT LINK . RAD/S
C       AL1         = ANGULAR ACCELERATION FOR INPUT LINK . RAD/S**2
C       AL2         = ANGULAR ACCELERATICN FCR CCUPLER LINK . RAD/S**2
C       AL3         = ANGULAR ACCELERATICN FCR OUTPUT LINK . RAD/S**2
C       R1X,R1Y     = PCSITICN VECTCR CF REVCLUTE M
C       R2X,R2Y     = POSITION VECTOR OF REVCLUTE A
C       R3X,R3Y     = PCSITICN VECTCR CF REVCLUTE B
C       R4X,R4Y     = PCSITION VECTOR OF REVCLUTE C
C       NUMEL       = NUMBER OF ELEMENTS PER LINK
C       IN          = INPUT READER
C       LP          = LINE PRINTER
C       NUMS        = SIZE CF DC-LOOP FOR MCTION OF MECHANISM
C       NUP         = NUMBER OF PCINTS OF MECHANISM AFTER THE
C                     DIVISION INTC ELEMENTS
C       NUMELM      = NUMBER OF ELEMENTS OF MECHANISM
C       STEPDE      = STEPS IN DEGREES WHERE THE MECHANISM IS ANALYSED
C       RTD         = RADIANS TC DEGREES CONVERTER
C       DTR         = DEGREES TC RADIANS CONVERTER
C       TH2,TH2D    = ANGULAR PCSITICN OF CCUPLER LINK . RADIANS AND DEGREES
C       TH3,TH3D    = ANGULAR PCSITICN OF OUTPUT LINK . RADIANS AND DEGREES
C       R1          = DISTANCE FRCM REVOLUTE "I" TC PCINT CF STUDY CF LINK
C       R2          = DISTANCE FRCM POINT OF STUDY TC REVOLUTE "J" CF LINK
C       XMA         = MAXIMUM RATIC DISTANCE FRCM PCINT TWC OF BAR CR LINK
C       XMI         = MAXIMUM RATIC DISTANCE FRCM PCINT CNE OF BAR OR LINK
C       PPX,PPY     = POSITICN VECTCR IN THE X AND Y DIRECTION OF DIVIDING
C                     PCINTS . PPX(NUP) . PPY(NUP)
C       ACX,ACY     = LINEAR ACCELERATIONS IN THE X AND Y DIRECTION OF
C                     DIVIDING POINTS . ACX(NUP) . ACY(NUP)
C       NUMSEL      = NUMBER OF SUBELEMENTS FORMED BY ALL CF THE POINTS
C                     CF EACH LINK
C       XLT         = LENGTH OF EACH ELEMENT . XLT(NUMEL)
C       XMS         = MASS OF EACH ELEMENT . XMS(NUMEL)
C       THS         = PCSITION ANGLE OF EACH ELEMENT IN RADIANS . THS(NUMEL)
C
C       OMS         = ANGULAR VELOCITY OF EACH ELEMENT . RAD/S . OMS(NUMEL)
C       ALS         = ANGULAR ACCELERATION CF EACH ELEMENT . RAD/S**2
C                     ALS(NUMEL)
C       CSA         = CROSS - SECTICNAL AREA OF EACH ELEMENT . CSA(NUMEL)
C       EM          = MODULUS OF ELASTICITY CF EACH ELEMENT . EM(NUMEL)
C       XMIS        = MOMENT OF INERTIA OF THE CROSS - SECTIONAL AREA OF
C                     EACH ELEMENT . XMIS(NUMEL)
C       XMASL       = TRANSLATICNAL MASS MATRIX CF EACH ELEMENT IN GLOBAL
C                     COORDINATES . XMASL(8,8)
C       GYRO        = TRANSLATICNAL GYROSCOPIC MATRIX CF EACH ELEMENT IN
C                     GLOBAL CCCRDINATES . GYRO(8,8)
C       STIFL       = TRANSLATICNAL STIFNESS MATRIX CF EACH ELEMENT IN
C                     GLOBAL CCCRDINATES . STIFL(8,8)
C       E1          = XMASL . E1(NUMEL,8,8)
C       E2          = GYROG . E2(NUMEL,8,8)
C       E3          = STIFL . E3(NUMEL,8,8)
C       OM2         = ANGULAR VELCCITY FCR CCUPLER LINK . RAD/S
C       OM3         = ANGULAR VELCCITY FOR CUTPUT LINK . RAD/S
C       M1M         = SIZE OF MATRIX AME . M1M = 8 * NUMEL
C       M2M         = SIZE OF MATRICES IT AND IB . M2M = M1M - 4
C       XMASG       = GLOBAL TRANSLATIONAL MASS MATRIX OF MECHANISM .
C                     XMASG(M2M,M2M)
C       GYROG       = GLOBAL TRANSLATICNAL GYRCSCCPIC MATRIX OF MECHANISM .
C                     GYRCG(M2M,M2M)
C       STIFG       = GLOBAL TRANSLATICNAL STIFFNESS MATRIX OF MECHANISM .
C                     STIFG(M2M,M2M)
C       SMA         = STEP MATRIX TC CBTAIN MATRICES XMASG . GYROG AND
C                     STIFG . SMA(M1M,M2M)
C       AME         = ARRANGEMENT CF MATRICES E1 . E2 OR E3 IN DIAGCNAL
C                     FCRM . AME(M1M,M1M)
C       IT,IB       = VECTCRS FCRMED FROM THE ASSEMBLAGE PATTERN OF
C                     ELEMENTS . IT(M2M) . IB(M2M)
C       NUMBOU      = NUMBER OF BOUNDARY CCCRDINATES
C       MASIR       = SIZE OF THE REDUCED MATRIX ACCCRDING TO THE NUMBER
C                     CF BOUNDARY CCNDITIONS . MASIR = M2M - NUMBOU
C       VX,VY       = LINEAR VELOCITIES OF NODE I IN THE X AND Y DIRECTICN
C                     VX(NUMEL) . VY(NUMEL)
C       CAX,CAY     = LINEAR ACCELERATICNS CF NODE I IN THE X AND Y
C                     DIRECTION . ACX(NUMEL) . ACY(NUMEL)
C       FORVE       = INERTIAL FORCE VECTOR FOR EACH ELEMENT IN GLOBAL
C                     CCCRDINATES . FORVE(8)
C       FORVE1      = TRANSFER PARAMETER EQUAL TC NUMEL*FORVE . FORVE1(M1M)
C       FORVEG      = GLOBAL INERTIAL FORCE VECTCR . FORVEG(M2M)
C       FORVER      = REDUCED VECTCR FORVEG DUE TC BOUNDARY CONDITICNS .
C                     FORVEG(MASIR)
C       XMASGR      = REDUCED MATRIX XMASG DUE TC BOUNDARY CONDITIONS
C                     XMASGR(MASIR)
C       GYROGR      = REDUCED MATRIX GYROG DUE TC BOUNDARY CONDITIONS
C                     GYRCGR(MASIR)
C       STIFGR      = REDUCED MATRIX STIFG DUE TC BOUNDARY CONDITIONS
C                     STIFGR(MASIR)
C       NBOUNR      = VECTCR WITH BOUNDARY CCNDITICNS FOR RCWS OF MATRICES .
C                     NBOUNR(MASIR)
C       NBOUNC      = VECTCR WITH BOUNDARY CCNDITICNS FOR COLUMNS OF
C                     MATRICES . NBOUNC(MASIR) . NBOUNR = NBOUNC
C
```

```
C       LR          = BOUNDARY CONDITIONS FOR ROWS . LR(NUMBOU)
C       LC          = BOUNDARY CONDITIONS FOR COLUMNS . LC(NUMBOU)
C       FORG2       = VECTOR EQUAL TO FORVEF . FORG2(NUMS,MASIR)
C       UL          =
C       X           = INDEPENDENT VARIABLE IN SUBROUTINE DVERK . ON INPUT
C                     X SUPPLIES THE INITIAL VALUE . ON OUTPUT X IS REPLACED
C                     WITH XEND
C       Y           = DEPENDENT VARIABLE IN SUBROUTINE DVERK . ON INPUT
C                     Y SUPPLIES INITIAL VALUES . ON OUTPUT Y SUPPLIES AN
C                     APPROXIMATE SOLUTION AT XEND . Y(M2M)
C       XEND        = VALUE OF X AT WHICH SOLUTION IS DESIRED
C       TOL         = TOLERANCE FOR ERROR CONTROL IN DVERK
C       FCN1        = SUBROUTINE FOR EVALUATING FUNCTIONS IN DVERK
C       IND         = INDICATOR FOR SUBROUTINE DVERK
C       W           = WORKSPACE MATRIX FOR DVERK . W(M2M,9)
C       IER         = ERROR PARAMETER FOR DVERK
C       J1J         = PRINTING PARAMETER . STEPS CHOSEN FOR PRINTING OUTPUT
C       NUMS1       = NUMBER OF STEPS IN SOLVING SYSTEM OF EQUATIONS
C                     WHEN USING THE SUBROUTINE TO DO SO
C       N2N         = PRINTING PARAMETER . STEPS CHOSEN FOR PRINTING OUTPUT
C       L1L         = NUMBER OF FIRST ORDER LINEAR DIFFERENTIAL EQUATIONS
C       M1M         = NUMBER OF D.O.F. IN LOCAL COORDINATES
C       C           = ERROR FLAG FOR SUBROUTINE DVERK
C       K7          = COUNTER FOR THE USE OF SUBROUTINE FCN1
C
C
C
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION IWK(20) , WK(620)
        DIMENSION XMASL(8,8) , GYRC(8,8) , STIFL(8,8) , FORVE(8)
        DIMENSION PPX( 7) , PPY( 7) , ACX( 7) , ACY( 7)
        DIMENSION XL1(3) , XM5(3) , TH5(3) , OM5(3) , AL5(3) , CSA(3) ,
     1            EM(3) , VX(3) , VY(3) , CAX(3) , CAY(3) , XMI5(3)
        DIMENSION E1(3,8,8) , E2(3,8,8) , E3(3,8,8)
        DIMENSION IT(20) , IB(20) , AME(24,24) , SMA(24,20) ,
     1            XMASG(20,20) , GYROG(20,20) , STIFG(20,20) ,
     2            FORVEG(20) , NBOUNR(20) , NBOUNC(20) , Y(22)
        DIMENSION FORVE1(24) , C(24) , W(16,9)
        DIMENSION FORVER(11) , XMASGR(11,11) , GYROGR(11,11) ,
     1            STIFGR(11,11) , LR(11) , LC(11) , T22(8)
        DIMENSION AXYA(2) , EXYB(2)
        DIMENSION XMASGC(8,8) , GYROGC(8,8) , STIFGC(8,8) , FORVEC(8) ,
     1            WT(8,11)
        COMMON/SUB1/DTR,RTD,TH1D,F1X,R1Y,F2X,R2Y,R3X,R3Y,R4X,F4Y,TH2D,TH3D
        COMMON/SUB2/OM1,AL1,OM2,AL2,OM3,AL3
        COMMON/SUB12/AA,BB,CC,DD,TH1,TH2,TH3
        COMMON/SUB3/R1,R2
        COMMON/B12/NEQUNF,M2M
        COMMON/B14/XMASGC,STIFGC,GYROGC,FORVEC,T22,MASIC,K7
        COMMON/CSPEC/UL(8,8),XX(8),LMETH
        COMMON/RED/FZZ,FXX,FYY
        EXTERNAL FCN1
        EXTERNAL FCNJ
        DATA IN,LP/5,6/
C
C
C
C       READ IN AND WRITE OUT THE GIVEN INFORMATION TO CARRY OUT ALL OF
C       THE REQUIRED CALCULATIONS
C
C
        READ (IN,150) NUMEL , NUMS , NUP , NUMELM , STEPDE , XMA , XMI
    150 FORMAT (4I6,2D10.0)
        READ (IN,153) M1M , M2M , NUMBOU , IND , L1L , N2N , J1J ,
     1                NUMS1 , X , TOL
    153 FORMAT (8I4,2D10.0)
        READ (IN,155) AA , BB , CC , DD , TH1D , OM1 , AL1
    155 FORMAT (4D20.0,/,3D15.0)
        DO 1 L=1,NUMELM
        READ (IN,158) XM5(L) , CSA(L) , EM(L) , XMI5(L)
    158 FORMAT (4D15.0)
      1 CONTINUE
        READ (IN,160) (IT(L),L=1,M2M)
        READ (IN,160) (IB(L),L=1,M2M)
    160 FORMAT (20I2)
        DO 3 L=1,NUMBOU
        READ (IN,163) LR(L) , LC(L)
    163 FORMAT (2I4)
      3 CONTINUE
        READ (IN,165) (Y(L),L=1,L1L)
    165 FORMAT (4D20.0,/,4D20.0,/,4D20.0,/,4D20.0)
C
        WRITE (LP,250)
    250 FORMAT (1H1)
        WRITE (LP,251)
    251 FORMAT (///)
        WRITE (LP,252)
    252 FORMAT (60X,'INPUT DATA')
        WRITE (LP,253)
    253 FORMAT (60X,'XXXXXXXXXX')
        WRITE (LP,251)
        WRITE (LP,256)
    256 FORMAT (19X,'NUMEL',11X,'NUMS',11X,'NUP',10X,'NUMELM', 8X,'STEPDE'
       1,11X,'XMA',12X,'XMI',/)
        WRITE (LP,257) NUMEL , NUMS , NUP , NUMELM , STEPDE , XMA , XMI
    257 FORMAT (17X,I5,10X,I5,10X,I5,10X,I5,3F15.4)
        WRITE (LP,251)
        WRITE (LP,258)
    258 FORMAT (18X,'M1M', 7X,'M2M', 5X,'NUMBOU', 6X,'IND', 7X,'L1L', 7X,
       1'N2N', 7X,'J1J', 6X,'NUMS1', 6X,'X', 6X,'TOL',/)
        WRITE (LP,259) M1M , M2M , NUMBOU , IND , L1L , N2N , J1J ,
     1                NUMS1 , X , TOL
    259 FORMAT (10X,8I10,F10.1,D12.3)
        WRITE (LP,251)
        WRITE (LP,260)
    260 FORMAT (28X,'AA',10X,'BB',10X,'CC',10X,'DD',10X,'TH1D', 8X,'OM1',
       19X,'AL1',/)
        WRITE (LP,261) AA , BB , CC , DD , TH1D , OM1 , AL1
    261 FORMAT (21X,7F12.4)
        WRITE (LP,251)
        WRITE (LP,262)
```

```
262 FORMAT (39X,'XMS',12X,'CSA',12X,'EM',13X,'XMIS',/)
    DO 30 L=1,NUMELM
    WRITE (LP,263) XMS(L) , CSA(L) , EM(L) , XMIS(L)
263 FORMAT (33X,4D15.5)
 30 CONTINUE
    WRITE (LP,251)
    WRITE (LP,265)
265 FORMAT (58X,'THE "IT" VECTOR',/)
    WRITE (LP,266) (IT(L),L=1,M2M)
    WRITE (LP,251)
    WRITE (LP,267)
267 FORMAT (58X,'THE "IB" VECTOR',/)
    WRITE (LP,266) (IB(L),L=1,M2M)
266 FORMAT ( 3X,20I6)
    WRITE (LP,250)
    WRITE (LP,251)
    WRITE (LP,268)
268 FORMAT (55X,'BOUNDARY CONDITIONS',/,59X,'LR', EX,'LC',/)
    DO 33 L=1,NUMBCU
    WRITE (LP,269) LR(L) , LC(L)
269 FORMAT (50X,2I10)
 33 CONTINUE
    WRITE (LP,251)
    WRITE (LP,272)
272 FORMAT (48X,'VECTOR Y(L) OF INITIAL CONDITIONS',/)
    DO 36 L=1,LIL
    WRITE (LP,273) Y(L)
273 FORMAT (55X,D15.4)
 36 CONTINUE
C
C
C     INTRODUCE CONSTANT INFORMATION .
C
C
    PI = 4.0D0 * DATAN(1.0D0)
    DTR = PI/180.0D0
    RTD = 180.0D0/PI
    NUMSEL = NUMEL * 2
    MASIF = M2M - NUMBCU
    MASIC = MASIF - 3
    XLT(1) = AA
    XLT(2) = BB
    XLT(3) = CC
    DO 50 L=1,M2M
    NBCUNR(L) = 0
    NBCUNC(L) = 0
 50 CONTINUE
    DO 55 L=1,NUMBOL
    NBOUNR(LR(L)) = LR(L)
    NBCUNC(LC(L)) = LC(L)
 55 CONTINUE
    DO 60 L=1,M2M
    DO 60 M=1,M2M
    XMASG(L,M) = 0.0D0
    GYROG(L,M) = 0.0D0
    STIFG(L,M) = 0.0D0
 60 CONTINUE
    OM1 = CM1 * 2.0D0*PI/60.0D0
    XEND2 = (STEPDE * DTR)/OM1
    DO 70 L=1,M1M
    DO 70 M=1,M1M
    AME(L,M) = 0.0D0
 70 CONTINUE
    DO 71 L=1,9
    C(L) = 0.0D0
 71 CONTINUE
C
C     INFORMATION FOR DGEAR
C
    HG = 1.0D-09
    TOLG = 1.0D-05
    METH = 2
    MITER = 1
    INDEX = 1
C
C
C
C     INITIALIZE DO-LOOP THROUGH 'STEPDE' OF INPUT LINK MOTION
C
C
    DO 100 I=1,NUMS
C
    XEND1 = XEND2 * DFLOAT(I)
    XEND = XEND1
    CALL LARGC
    PPX(1) = R1X
    PPY(1) = R1Y
    ACX(1) = 0.0D0
    ACY(1) = 0.0D0
    CALL PIEDRA
    THS(1) = TH1
    THS(2) = TH2
    THS(3) = TH3 + FI
    OMS(1) = OM1
    OMS(2) = OM2
    OMS(3) = OM3
    ALS(1) = AL1
    ALS(2) = AL2
    ALS(3) = AL3
    R1 = 0.0D0
    K = 0
    DO 115 J=1,NUMSEL
    K = K + 1
    R2 = XMA - R1
    R1 = XM1 + R1
    CALL ACCR(ACX(K),ACY(K),OM1,AL1,R1X,R1Y,R2X,R2Y,PPX(K),PPY(K),
   1PPX(K+1),PPY(K+1),ACX(K+1),ACY(K+1))
115 CONTINUE
    AXYA(1) = PPX(K+1)
    AXYA(2) = PPY(K+1)
```

```
C
      R1 = 0.0D0
      DO 116 J=1,NUMSEL
      K = K + 1
      R2 = XMA - R1
      R1 = XMI + R1
      CALL ACCP(ACX(K),ACY(K),CM2,AL2,R2X,R2Y,R3X,R3Y,PPX(K),PPY(K),
     1PPX(K+1),PPY(K+1),ACX(K+1),ACY(K+1))
  116 CONTINUE
      BXYB(1) = PPX(K+1)
      BXYB(2) = PPY(K+1)
C
      R1 = 0.0D0
      DO 117 J=1,NUMSEL
      K = K + 1
      R2 = XMA - R1
      R1 = XMI + R1
      CALL ACCP(ACX(K),ACY(K),CM3,AL3,R3X,R3Y,R4X,R4Y,PPX(K),PPY(K),
     1PPX(K+1),PPY(K+1),ACX(K+1),ACY(K+1))
  117 CONTINUE
C
      DO 120 J=1,NUMELM
      CALL MASMAT (XLT(J),THS(J),XMS(J),XMASL)
      CALL SKEW (XLT(J),THS(J),CMS(J),XMS(J),GYRO)
      CALL STIFF (XLT(J),THS(J),XMS(J),CSA(J),EM(J),XMIS(J),OMS(J),
     1             ALS(J),STIFL)
      DO 121 LL=1,8
      DO 121 MM=1,8
      E1(J,LL,MM) = XMASL(LL,MM)
      E2(J,LL,MM) = GYRO(LL,MM)
      E3(J,LL,MM) = STIFL(LL,MM)
  121 CONTINUE
  120 CONTINUE
      CALL ASSEMB (NUMELM,M1M,M2M,E1,IT,IB,AME,SMA,XMASG)
      CALL ASSEMB (NUMELM,M1M,M2M,E2,IT,IB,AME,SMA,GYROG)
      CALL ASSEMB (NUMELM,M1M,M2M,E3,IT,IB,AME,SMA,STIFG)
C
C     INITIALIZE THE FORMATION OF THE FORCE VECTOR FOR EACH
C     ELEMENT AND FOR THE MECHANISM
C
      VX(1) = 0.0D0
      VX(2) = AA*CM1*DSIN(TH2-TH1)
      VX(3) = 0.0D0
      VY(1) = 0.0D0
      VY(2) = AA*CM1*DCOS(TH2-TH1)
      VY(3) = - CC*CM3
      CAX(1) = 0.0D0
      CAX(2) = ACX(3)*DCOS(TH2) + ACY(3)*DSIN(TH2)
      CAX(3) = - ACX(5)*DCOS(TH3) - ACY(5)*DSIN(TH3)
      CAY(1) = 0.0D0
      CAY(2) = - ACX(3)*DSIN(TH2) + ACY(3)*DCOS(TH2)
      CAY(3) = ACX(5)*DSIN(TH3) - ACY(5)*DCOS(TH3)
C
      K = 0
      DO 123 J=1,NUMELM
      CALL FORCE (XLT(J),THS(J),CMS(J),ALS(J),VX(J),VY(J),CAX(J),
     1             CAY(J),XMS(J),FORVE)
      DO 124 L=1,8
      K = K + 1
      FORVE1(K) = FORVE(L)
  124 CONTINUE
  123 CONTINUE
      CALL FORCE1 ( FORVE1,FORVEG,FORVER)
C
      CALL MARED (M2M,MASIR,NBOUNR,NBOUNC,XMASG,XMASGR)
      CALL MARED (M2M,MASIR,NBOUNR,NBOUNC,GYROG,GYROGR)
      CALL MARED (M2M,MASIR,NBOUNR,NBOUNC,STIFG,STIFGR)
C
C
C     INITIALIZE CONDENSATION
C     FIND CONDENSING PARAMETERS FZZ , FXX AND FYY .
C     CONDENSE VECTOR FORCE FORVER TO FORVEC
C
      FZZ = - DTAN(TH1)
      FXX = (DSIN(TH2)*DSIN(TH3)-DTAN(TH1)*DCOS(TH2)*DSIN(TH3))/
     1        DSIN(TH3-TH2)
      FYY = (DTAN(TH1)*DCOS(TH2)*DCOS(TH3)-DSIN(TH2)*DCOS(TH3))/
     1        DSIN(TH3-TH2)
      FORVEC(1) = FORVER(1)
      FORVEC(2) = FORVER(2)
      FORVEC(3) = FORVER(3)*FZZ+FORVER(4)+FORVER(7)*FXX+FORVER(8)*FYY
      FORVEC(4) = FORVER(5)
      FORVEC(5) = FORVER(6)
      FORVEC(6) = FORVER(9)
      FORVEC(7) = FORVER(10)
      FORVEC(8) = FORVER(11)
C
      CALL REDUCE (MASIR,MASIC,XMASGR,XMASGC,WI)
      CALL REDUCE (MASIR,MASIC,GYROGR,GYROGC,WI)
      CALL REDUCE (MASIR,MASIC,STIFGR,STIFGC,WI)
C
      K7 = 0
      CALL DECOMP (MASIC,XMASGC,MASIC,UL,MASIC)
      CALL DVERK (L1L,FCN1,X,Y,XEND,TOL,IND,C,L1L,W,IER)
      IF (IER .GT. 128) GO TO 888
      IF (IND .LT. 0 .OR. IER .GT. 0) GO TO 888
C
C
C     INITIALIZE THE PRINTING OUT OF THE OUTPUT
C
      IF (I .NE. J1J) GO TO 310
      J1J = J1J + N2N
      WRITE(LP,251)
      WRITE(LP,599)
  599 FORMAT (56X,'X',17X,'TH1D')
      WRITE (LP,600) XEND1 , TH1D
  600 FORMAT (53X,D15.8,F10.3,/)
      WRITE (LP,601)
  601 FORMAT (27X,'TH1D', 8X,'TH2D', 8X,'TH3D', 8X,'AL2', 9X,'AL3', 9X,'
     1OM2', 9X,'OM3')
```

```
      WRITE (LP,602) THID , TH2C , TH3D , AL2 , AL3 , CM2 , OM3
  602 FORMAT (20X,7F12.2,/)
      WRITE (LP,610)
  610 FORMAT (40X,'ACCELERATICN CF SCME FCINTS IN THE X ANC Y DIRECTICN'
     1)
      WRITE (LP,612) (ACX(KKK),KKK=1,NUF)
      WRITE (LP,612) (ACY(KKK),KKK=1,NUP)
  612 FORMAT (28X,7F10.2)
      WRITE (LP,616)
  616 FORMAT (38X,'PCSITION VECTCR OF SCME PCINTS IN THE X AND Y DIRECTI
     1ONS')
      WRITE (LP,617) (FPX(KKK),KKK=1,NUF)
      WRITE (LP,617) (PPY(KKK),KKK=1,NUP)
  617 FORMAT (28X,7F10.2)
      WRITE (LP,625)
  629 FORMAT (//,41X,'INERTIAL',12X,'DEFCRMATICN', 9X,'VELOCITY')
      DO 300 JK=1,MASIC
      WRITE (LP,630) FORVEC(JK) , Y(JK) , Y(JK+MASIC)
  630 FCRMAT (33X,3D20.8)
  300 CONTINLE
C     STCRE SOME CATA CN DISC TC BE
C     USED BY ANCTHER PRCGRAM
      WRITE (10,2000) (Y(I8I),I8I=1,8)
 2000 FORMAT (8D15.6)
      WRITE(11,2001) (TH5(I8I),I8I=1,3)
 2001 FORMAT (3D15.6)
      WRITE(12,2002) AXYA(1) , AXYA(2)
      WRITE(13,2002) BXYB(1) , BXYB(2)
 2002 FORMAT (2D15.6)
  310 CONTINLE
C
C
      THID = STEPDE * CFLOAT(I)
  100 CONTINLE
C
C
      WRITE (LP,250)
      STCP
  888 CONTINLE
C
C     HANDLE IND .LT. 0 .OR. IER .GT. 0
C     ITEMS THAT MAY HELP DIAGNCSE THE PRCBLEM SHOULD BE
C     OUTPUT HERE
C     IND,TCL,M2M,M,Y(I),XNEC,C(IE
      WRITE (LP,889) IND , IER
  889 FORMAT (10X,2I10)
C
      WRITE (LP,250)
      STOP
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE LARGE
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```
C
C     THIS SUBROUTINE DETERMINES THE DIRECTICN ANGLES FOR
C     COUPLER LINK AND OUTPUT LINK. CARRIES CUT TRANSFCRMATION
C     OF REVCLUTE FCSITICNS . DETERMINES PCSITICN VECTCRS OF POINTS
C
C     NOTATICNS :
C
C     A        = INPUT LINK LENGTH
C     B        = COUPLER LINK LENGTH
C     C        = OUTPUT LINK LENGTH
C     D        = FIXED LINK LENGTH
C     C1       = DEGREES TO RADIANS CONVERTER
C     C2       = RADIANS TO DEGREES CONVERTER
C     A1,A1D   = PCSITICN ANGLES OF INPUT LINK . RADIANS AND DEGREES
C     A2,A2D   = POSITICN ANGLES OF OUTPUT LINK . RADIANS AND DEGREES
C     T1:T10   = CCEFFICIENTS
C     E1X,E1Y  = FCSITICN VECTCR CF REVCLUTE M
C     E2X,E2Y  = POSITICN VECTOR OF REVOLUTE A
C     E3X,E3Y  = FCSITICN VECTOR OF REVOLUTE B
C     E4X,E4Y  = PCSITICN VECTCR CF REVOLUTE C
C
C
C
      IMPLICIT REAL*8(A-H,C-Z)
      CCMMCN/SUB1/C1,C2,A1D,E1X,E1Y,E2X,E2Y,E3X,E3Y,E4X,E4Y,A2D,A3D
      COMMCN/SUB12/A,B,C,D,A1,A2,A3
C
      A1 = A1D * C1
C
C
C     INITIALIZE THE CALCULATICNS FOR DIRECTICN ANGLES FOR COUPLER
C     LINK AND OUTPUT LINK
      T1 = D/A
      T2 = D/C
      T3 = (A**2 - B**2 + C**2 + D**2)/(2.0D+00*A*C)
      T4 = D/B
      T5 = ((**2 - C**2 - A**2 - B**2)/(2.CD+0C*A*E)
      T6 = DCOS(A1) + T3 - T1 - T2 * DCCS(A1)
      T7 = - 2.0D+CC * DSIN(A1)
      T8 = T1 + T3 - (1.0D+00 + T2) * DCCS(A1)
      T9 = T4 * DCCS(A1) + DCCS(A1) + T5 - T1
      T10 = T4 * DCCS(A1) - DCCS(A1) + T5 + T1
C
      A2 = 2.0D+00 * DATAN((-T7 - DSQRT(T7**2 - 4.CD+0C*T9*T10))/
     1        (2.0D+00*T9))
      A3 = 2.0D+00 * DATAN((-T7 - DSQRT(T7**2 - 4.CD+00*T6*T8))/
     1        (2.0D+00*T6))
      A2D = A2 * C2
      A3D = A3 * C2
C
C     INITIALIZE TRANSFORMATICN TC FIND REVCLUTE PCSITICN VECTORS
C
      E1X = C.0D+0C
      E1Y = C.0D+00
      E2X = A * CCCS(A1)
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```
      E2Y = A * DSIN(A1)
      E4X = D
      E4Y = C.C0+CC
      E3X = E4X + C*DCOS(A2)
      E3Y = E4Y + C*DSIN(A3)
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE PIEDRA
C
C     THIS SUBROUTINE DETERMINES THE ANGULAR VELOCITIES AND ANGULAR
C     ACCELERATIONS OF COUPLER AND OUTPUT LINKS RESPECTIVELY
C
C     NOTATIONS :
C
C     O2        = POSITION ANGLE OF INPUT LINK . RADIANS
C     O3        = POSITION ANGLE OF COUPLER LINK . RADIANS
C     O4        = POSITION ANGLE OF OUTPUT LINK . RADIANS
C     R2        = ANGULAR VELOCITY OF INPUT LINK . RAD/S
C     R3        = ANGULAR VELOCITY OF COUPLER LINK . RAD/S
C     R4        = ANGULAR VELOCITY OF OUTPUT LINK . RAD/S
C     AA2       = ANGULAR ACCELERATION OF INPUT LINK . RAD/S**2
C     AA3       = ANGULAR ACCELERATION OF COUPLER LINK . RAD/S**2
C     AA4       = ANGULAR ACCELERATION OF OUTPUT LINK . RAD/S**2
C     X6        = LENGTH OF INPUT LINK
C     X7        = LENGTH OF COUPLER LINK
C     X8        = LENGTH OF OUTPUT LINK
C     X9        = LENGTH OF FIXED LINK
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/SUB2/R2,AA2,R3,AA3,R4,AA4
      COMMON/SUB12/X6,X7,X8,X9,O2,O3,O4
C
C     FIND THE VALUES FOR R3 AND R4
C
      R3 = (X6*R2*DSIN(O4-O2)) / (X7*DSIN(O3-O4))
      R4 = (X6*R2*DSIN(O2-O3)) / (X8*DSIN(O4-O3))
C
C     FIND THE VALUES OF AA3 AND AA4
C
      T4 = X6*AA2*DSIN(O2) + X6*R2**2*DCOS(O2) + X7*R3**2*DCOS(O3) -
     1     X8*R4**2*DCOS(O4)
      T5 = X8*DCOS(O4)
      T6 = X8*DSIN(O4)
      T7 = X6*AA2*DCOS(O2) - X6*R2**2*DSIN(O2) - X7*R3**2*DSIN(O3) +
     1     X8*R4**2*DSIN(O4)
      T8 = X7*DCOS(O3)
      T9 = X7*DSIN(O3)
C
      AA3 = (T4*T5 - T6*T7) / (T6*T8 - T9*T5)
      AA4 = (T4*T8 - T9*T7) / (T6*T8 - T9*T5)
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE ACCP(BX,BY,WZ,AAZ,X1,Y1,X2,Y2,B1,B2,A1,A2,AX,AY)
C
C     THIS SUBROUTINE DETERMINES THE ACCELERATION AT A
C     POINT "A" IF THE ACCELERATION OF POINT "B" IS KNOWN
C     OF A RIGID BODY IN THE PLANE
C
C     NOTATIONS :
C
C     AX        = ACCELERATION OF THE REQUIRED POINT "A" IN
C                 THE X DIRECTION
C     AY        = ACCELERATION OF THE REQUIRED POINT "A" IN
C                 THE Y DIRECTION
C     BX,BY     = GIVEN ACCELERATION OF POINT "B"
C     WZ        = GIVEN ANGULAR VELOCITY OF BAR "AB"
C     AAZ       = GIVEN ANGULAR ACCELERATION OF BAR "AB"
C     A1,A2     = POSITION VECTOR OF POINT "A"
C     B1,B2     = POSITION VECTOR OF POINT "B"
C     CX,CY     = VECTOR "BA"
C     R1        = DISTANCE FROM REVOLUTE "I" TO POINT OF STUDY OF LINK
C     R2        = DISTANCE FROM POINT OF STUDY TO REVOLUTE "J" OF LINK
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/SUB3/R1,R2
C
C
C     INITIALIZE THE DIVISION OF PARTINTION INTO ELEMENTS OF THE LINK
C     AND FIND VECTOR C
C
      A1 = (X1*R2 + X2*R1)/(R1 + R2)
      A2 = (Y1*R2 + Y2*R1)/(R1 + R2)
      CX = A1 - B1
      CY = A2 - B2
C
C     INITIALIZE DETERMINATION OF ACCELERATIONS
C
      AX = BX - WZ**2*CX - AAZ*CY
      AY = BY - WZ**2*CY + AAZ*CX
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE MASMAT(Z,Y,W,X)
```

```
C
C     THIS SUBROUTINE DETERMINES THE MASS MATRIX OF EACH ELEMENT IN
C     GLOBAL COORDINATES
C
C     NOTATIONS :
C
C     X        = MASS MATRIX FOR EACH ELEMENT . X(8,8)
C     Y        = DIRECTION ANGLE FOR EACH ELEMENT
C     Z        = LENGTH OF EACH ELEMENT
C     W,P        = MASS OF EACH ELEMENT
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(8,8)
C
C     INITIALIZE THE FORMATION OF THE MASS MATRIX OF EACH ELEMENT
C
C
      C = DCOS(Y)
      S = DSIN(Y)
      P = W
C
      X(1,1) = (Z*P*C**2)/3.0D0 + (181.0D0*Z*P*S**2)/462.0D0
      X(1,2) = (Z*P*C*S)/3.0D0 - (181.0D0*Z*P*C*S)/462.0D0
      X(1,3) = - (311.0D0*Z**2*P*S)/4620.0D0
      X(1,4) = - (281.0D0*Z**3*P*S)/55440.0D0
      X(1,5) = (Z*P*C**2)/6.0D0 + (25.0D0*Z*P*S**2)/231.0D0
      X(1,6) = (Z*P*C*S)/6.0D0 - (25.0D0*Z*P*C*S)/231.0D0
      X(1,7) = (151.0D0*Z**2*P*S)/4620.0D0
      X(1,8) = - (181.0D0*Z**3*P*S)/55440.0D0
      X(2,1) = X(1,2)
      X(2,2) = (Z*P*S**2)/3.0D0 + (181.0D0*Z*P*C**2)/462.0D0
      X(2,3) = (311.0D0*Z**2*P*C)/4620.0D0
      X(2,4) = (281.0D0*Z**3*P*C)/55440.0D0
      X(2,5) = X(1,6)
      X(2,6) = (Z*P*S**2)/6.0D0 + (25.0D0*Z*P*C**2)/231.0D0
      X(2,7) = - (151.0D0*Z**2*P*C)/4620.0D0
      X(2,8) = (181.0D0*Z**3*P*C)/55440.0D0
      X(3,1) = X(1,3)
      X(3,2) = X(2,3)
      X(3,3) = (52.0D0*Z**3*P)/3465.0D0
      X(3,4) = (23.0D0*Z**4*P)/18480.0D0
      X(3,5) = - X(1,7)
      X(3,6) = - X(2,7)
      X(3,7) = - (19.0D0*Z**3*P)/1980.0D0
      X(3,8) = (13.0D0*Z**4*P)/13860.0D0
      X(4,1) = X(1,4)
      X(4,2) = X(2,4)
      X(4,3) = X(3,4)
      X(4,4) = (Z**5*P)/9240.0D0
      X(4,5) = X(1,8)
      X(4,6) = X(2,8)
      X(4,7) = - X(3,8)
      X(4,8) = (Z**5*P)/11089.0D0
      X(5,1) = X(1,5)
      X(5,2) = X(2,5)
      X(5,3) = X(3,5)
      X(5,4) = X(4,5)
      X(5,5) = X(1,1)
      X(5,6) = X(1,2)
      X(5,7) = - X(1,3)
      X(5,8) = X(1,4)
      X(6,1) = X(1,6)
      X(6,2) = X(2,6)
      X(6,3) = X(3,6)
      X(6,4) = X(4,6)
      X(6,5) = X(5,6)
      X(6,6) = X(2,2)
      X(6,7) = - X(2,3)
      X(6,8) = X(2,4)
      X(7,1) = X(1,7)
      X(7,2) = X(2,7)
      X(7,3) = X(3,7)
      X(7,4) = X(4,7)
      X(7,5) = X(5,7)
      X(7,6) = X(6,7)
      X(7,7) = X(3,3)
      X(7,8) = - X(3,4)
      X(8,1) = X(1,8)
      X(8,2) = X(2,8)
      X(8,3) = X(3,8)
      X(8,4) = X(4,8)
      X(8,5) = X(5,8)
      X(8,6) = X(6,8)
      X(8,7) = X(7,8)
      X(8,8) = X(4,4)
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE SKEW(Z,Y,A,W,X)
C
C     THIS SUBROUTINE DETERMINES THE GYROSCOPIC MATRIX OF EACH
C     ELEMENT IN GLOBAL COORDINATES
C
C     NOTATIONS :
C
C     X        = GYROSCOPIC MATRIX FOR EACH ELEMENT . X(8,8)
C     Y        = DIRECTION ANGLE FOR EACH ELEMENT
C     Z        = LENGTH OF EACH ELEMENT
C     A        = ANGULAR VELOCITY OF EACH ELEMENT
C     W          = MASS OF EACH ELEMENT
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(8,8)
C
```

```
C       INITIALIZE THE FORMATION OF THE GYROSCOPIC MATRIX
C
        C = DCOS(Y)
        S= DSIN(Y)
        B = 2.0D0 * A  * W
C
        X(1,1) = 0.0D0
        X(1,2) = (5.0D0*Z*B*S**2)/14.0D0 + (5.0D0*Z*B*C**2)/14.0D0
        X(1,3) = (13.0D0*Z**2*B*C)/210.0D0
        X(1,4) = (Z**3*B*C)/210.0D0
        X(1,5) = 0.0D0
        X(1,6) = (Z*B*S**2)/7.0D0 + (Z*B*C**2)/7.0D0
        X(1,7) = - (4.0D0*Z**2*B*C)/105.0D0
        X(1,8) = (Z**3*B*C)/280.0D0
        X(2,1) = - X(1,2)
        X(2,2) = 0.0D0
        X(2,3) = (13.0D0*Z**2*B*S)/210.0D0
        X(2,4) = (Z**3*B*S)/210.0D0
        X(2,5) = - X(1,6)
        X(2,6) = 0.0D0
        X(2,7) = - (4.0D0*Z**2*B*S)/105.0D0
        X(2,8) = (Z**3*B*S)/280.0D0
        X(3,1) = - X(1,3)
        X(3,2) = - X(2,3)
        X(3,3) = 0.0D0
        X(3,4) = 0.0D0
        X(3,5) = X(1,7)
        X(3,6) = X(2,7)
        X(3,7) = 0.0D0
        X(3,8) = 0.0D0
        X(4,1) = - X(1,4)
        X(4,2) = - X(2,4)
        X(4,3) = 0.0D0
        X(4,4) = 0.0D0
        X(4,5) = - X(1,8)
        X(4,6) = - X(2,8)
        X(4,7) = 0.0D0
        X(4,8) = 0.0D0
        X(5,1) = 0.0D0
        X(5,2) = X(1,6)
        X(5,3) = - X(1,7)
        X(5,4) = X(1,8)
        X(5,5) = 0.0D0
        X(5,6) = X(1,2)
        X(5,7) = - X(1,3)
        X(5,8) = X(1,4)
        X(6,1) = - X(1,6)
        X(6,2) = 0.0D0
        X(6,3) = - X(2,7)
        X(6,4) = X(2,8)
        X(6,5) = - X(1,2)
        X(6,6) = 0.0D0
        X(6,7) = - X(2,3)
        X(6,8) = X(2,4)
        X(7,1) = - X(1,7)
        X(7,2) = - X(2,7)
        X(7,3) = 0.0D0
        X(7,4) = 0.0D0
        X(7,5) = - X(5,7)
        X(7,6) = X(2,3)
        X(7,7) = 0.0D0
        X(7,8) = 0.0D0
        X(8,1) = - X(1,8)
        X(8,2) = - X(2,8)
        X(8,3) = 0.0D0
        X(8,4) = 0.0D0
        X(8,5) = - X(1,4)
        X(8,6) = - X(2,4)
        X(8,7) = 0.0D0
        X(8,8) = 0.0D0
C
        RETURN
        END
C
C       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
        SUBROUTINE STIFF(Z,Y,W,A,E,D,V,U,X)
C
C       THIS SUBROUTINE DETERMINES THE STIFFNESS MATRIX OF EACH
C       ELEMENT IN GLOBAL COORDINATES
C
C       NOTATIONS :
C
C       X        = STIFFNESS MATRIX FOR EACH ELEMENT , X(8,8)
C       Y        = DIRECTION ANGLE FOR EACH ELEMENT
C       Z        = LENGTH OF EACH ELEMENT
C       W,P,R    = MASS OF EACH ELEMENT
C       A        = CROSS SECTIONAL AREA OF EACH ELEMENT
C       E        = MODULUS OF ELASTICITY OF EACH ELEMENT
C       V        = ANGULAR VELOCITY OF CENTER OF MASS OF EACH ELEMENT
C       U        = ANGULAR ACCELERATION OF CENTER OF MASS OF EACH ELEMENT
C       D        = MOMENT OF INERTIA OF EACH ELEMENT
C
C
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION X(8,8)
C
C       INITIALIZE THE FORMATION OF THE STIFFNESS MATRIX OF EACH ELEMENT
C
C
        C = DCOS(Y)
        S = DSIN(Y)
        P = 1.0D0
        R = W
C
        X(1,1) = (E*A/(P*Z)-V**2*Z*R/3.0D0)*C**2 + (12.0D0*E*D/(7.0D0*P*
     1          Z**3)-181.0D0*V**2*Z*R/462.0D0)*S**2
        X(1,2) = (E*A/(P*Z)-V**2*Z*R/3.0D0-120.0D0*E*D/(7.0D0*P*Z**3)+
     1          181.0D0*V**2*Z*R/462.0D0)*C*S - 5.0D0*U*Z*R/14.0D0
```

```
      X(1,3) = - 13.0D0*U*Z**2*C*R/210.0D0 - (60.0D0*E*D/(7.0D0*P*Z**2)
     1         -311.0D0*V**2*Z**2*R/4620.0D0)*S
      X(1,4) = - U*Z**3*C*R/210.0D0 - (3.0D0*E*D/(7.0D0*P*Z)
     1         -281.0D0*V**2*Z**3*R/55440.0D0)*S
      X(1,5) = - (E*A/(P*Z)+V**2*Z*R/6.0D0)*C**2 - (120.0D0*E*D/
     1         (7.0D0*P*Z**3)+25.0D0*V**2*Z*R/231.0D0)*S**2
      X(1,6) = - (E*A/(P*Z)+V**2*Z*R/6.0D0-120.0D0*E*D/(7.0D0*P*Z**3)
     1         -25.0D0*V**2*Z*R/231.0D0)*C*S - U*Z*R/7.0D0
      X(1,7) = 4.0D0*U*Z**2*C*R/105.0D0 - (60.0D0*E*D/(7.0D0*P*Z**2)
     1         +151.0D0*V**2*Z**2*R/4620.0D0)*S
      X(1,8) = - U*Z**3*C*R/280.0D0 + (3.0D0*E*D/(7.0D0*P*Z)
     1         +181.0D0*V**2*Z**3*R/55440.0D0)*S
      X(2,1) = (E*A/(P*Z)-V**2*Z*R/3.0D0-120.0D0*E*D/(7.0D0*P*Z**3)
     1         +181.0D0*V**2*Z*R/462.0D0)*C*S + 5.0D0*U*Z*R/14.0D0
      X(2,2) = (E*A/(P*Z)-V**2*Z*R/3.0D0)*S**2 + (120.0D0*E*D/
     1         (7.0D0*P*Z**3)-181.0D0*V**2*Z*R/462.0D0)*C**2
      X(2,3) = - 13.0D0*U*Z**2*S*R/210.0D0 + (60.0D0*E*D/(7.0D0*P*Z**2)
     1         -311.0D0*V**2*Z**2*R/4620.0D0)*C
      X(2,4) = - U*Z**3*S*R/210.0D0 + (3.0D0*E*D/(7.0D0*P*Z)
     1         -281.0D0*V**2*Z**3*R/55440.0D0)*C
      X(2,5) = - (E*A/(P*Z)+V**2*Z*R/6.0D0-120.0D0*E*D/(7.0D0*P*Z**3)
     1         -25.0D0*V**2*Z*R/231.0D0)*C*S + U*Z*R/7.0D0
      X(2,6) = - (E*A/(P*Z)+V**2*Z*R/6.0D0)*S**2 - (120.0D0*E*D/
     1         (7.0D0*P*Z**3)+25.0D0*V**2*Z*R/231.0D0)*C**2
      X(2,7) = 4.0D0*U*Z**2*S*R/105.0D0 + (60.0D0*E*D/(7.0D0*P*Z**2)
     1         +151.0D0*V**2*Z**2*R/4620.0D0)*C
      X(2,8) = - U*Z**3*S*R/280.0D0 - (3.0D0*E*D/(7.0D0*P*Z)
     1         +181.0D0*V**2*Z**3*R/55440.0D0)*C
      X(3,1) = 13.0D0*U*Z**2*C*R/210.0D0 - (60.0D0*E*D/(7.0D0*P*Z**2)
     1         -311.0D0*V**2*Z**2*R/4620.0D0)*S
      X(3,2) = 13.0D0*U*Z**2*S*R/210.0D0 + (60.0D0*E*D/(7.0D0*P*Z**2)
     1         -311.0D0*V**2*Z**2*R/4620.0D0)*C
      X(3,3) = 192.0D0*E*D/(35.0D0*P*Z) - 52.0D0*V**2*Z**3*R/3465.0D0
      X(3,4) = 11.0D0*E*D/(35.0D0*P) - 23.0D0*V**2*Z**4*R/18480.0D0
      X(3,5) = 4.0D0*U*Z**2*C*R/105.0D0 + (60.0D0*E*D/(7.0D0*P*Z**2)
     1         +151.0D0*V**2*Z**2*R/4620.0D0)*S
      X(3,6) = 4.0D0*U*Z**2*S*R/105.0D0 - (60.0D0*E*D/(7.0D0*P*Z**2)
     1         +151.0D0*V**2*Z**2*R/4620.0D0)*C
      X(3,7) = 108.0D0*E*D/(35.0D0*P*Z) + 19.0D0*V**2*Z**3*R/1980.0D0
      X(3,8) = - 4.0D0*E*D/(35.0D0*P) - 13.0D0*V**2*Z**4*R/13860.0D0
      X(4,1) = U*Z**3*C*R/210.0D0 - (3.0D0*E*D/(7.0D0*P*Z)
     1         -281.0D0*V**2*Z**3*R/55440.0D0)*S
      X(4,2) = U*Z**3*S*R/210.0D0 + (3.0D0*E*D/(7.0D0*P*Z)
     1         -281.0D0*V**2*Z**3*R/55440.0D0)*C
      X(4,3) = X(3,4)
      X(4,4) = 3.0D0*Z*E*D/(35.0D0*P) - V**2*Z**5*R/9240.0D0
      X(4,5) = U*Z**3*C*R/280.0D0 + (3.0D0*E*D/(7.0D0*P*Z)
     1         +181.0D0*V**2*Z**3*R/55440.0D0)*S
      X(4,6) = U*Z**3*S*R/280.0D0 - (3.0D0*E*D/(7.0D0*P*Z)
     1         +181.0D0*V**2*Z**3*R/55440.0D0)*C
      X(4,7) = - X(3,8)
      X(4,8) = Z*E*D/(7.0D0*P) - V**2*Z**5*R/11088.0D0
      X(5,1) = X(1,5)
      X(5,2) = X(1,6)
      X(5,3) = - X(1,7)
      X(5,4) = X(1,8)
      X(5,5) = X(1,1)
      X(5,6) = X(1,2)
      X(5,7) = - X(1,3)
      X(5,8) = X(1,4)
      X(6,1) = X(2,5)
      X(6,2) = X(2,6)
      X(6,3) = - X(2,7)
      X(6,4) = X(2,8)
      X(6,5) = X(2,1)
      X(6,6) = X(2,2)
      X(6,7) = - X(2,3)
      X(6,8) = X(2,4)
      X(7,1) = - X(3,5)
      X(7,2) = - X(3,6)
      X(7,3) = X(3,7)
      X(7,4) = - X(3,8)
      X(7,5) = - X(3,1)
      X(7,6) = - X(3,2)
      X(7,7) = X(3,3)
      X(7,8) = - X(3,4)
      X(8,1) = X(4,5)
      X(8,2) = X(4,6)
      X(8,3) = X(3,8)
      X(8,4) = X(4,8)
      X(8,5) = X(4,1)
      X(8,6) = X(4,2)
      X(8,7) = X(7,8)
      X(8,8) = X(4,4)
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE ASSEMB(N1,J2,J1,E,IT,IB,T,TA,ATTA)
C
C     THIS SUBROUTINE ASSEMBLES ALL OF THE MATRICES OF THE GRID
C
C     NOTATIONS :
C
C     N1         = NUMBER OF ELEMENTS IN THE MECHANISM
C     J2         = SIZE OF MATRIX T . J2 = N1*8 . NUMBER OF COLUMNS OF
C                  MATRIX T AND A
C     J1         = NUMBER OF ROWS OF MATRIX TA AND C1 . J1 = N1*8 - 4
C     E          = ELEMENT MATRIX IN GLOBAL COORDINATES . E(N1,8,8)
C     T          = ARRANGEMENT OF MATRICES E IN DIAGONAL FORM . T(J2,J2)
C
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION E(N1,8,8) , T(J2,J2) , TA(24,20)  , ATTA(20,20)
      INTEGER IT(20) , IB(20)
C
```

```
      LL = 1
      MM = 8
      JJ = 1
C
      DO 1 I=1,N1
      N = 0
      DO 2 K=LL,MM
      N = N + 1
      N2 = 0
      DO 3 L=JJ,MM
      N2 = N2 + 1
      T(K,L) = E(I,N,N2)
    3 CONTINUE
    2 CONTINUE
      JJ = JJ + 8
      MM = MM + 8
      LL = LL + 8
    1 CONTINUE
C
C
      DO 25 I=1,24
      DO 25 J=1,20
      SUM = 0.0D0
      IF (IB(J) .NE. 0) SUM = T(I,IB(J))
   25 TA(I,J) = SUM + T(I,IT(J))
C
      DO 26 I=1,20
      DO 26 J=1,20
      SUM = 0.0D0
      IF (IB(I) .NE. 0) SUM = TA(IB(I),J)
   26 ATTA(I,J) = SUM + TA(IT(I),J)
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE FORCE(Z,Y,V,U,P,R,S,T,W,X)
C
C     THIS SUBROUTINE DETERMINES THE FORCE VECTOR OF EACH
C     ELEMENT IN GLOBAL COORDINATES
C
C     NOTATIONS :
C
C     X        = FORCE VECTOR FOR EACH ELEMENT . X(8)
C     Z        = LENGTH OF EACH ELEMENT
C     Y        = DIRECTION ANGLE FOR EACH ELEMENT
C     V        = ANGULAR VELOCITY OF CENTER OF MASS OF EACH ELEMENT
C     U        = ANGULAR ACCELERATION OF CENTER OF MASS OF EACH ELEMENT
C     P        = LINEAR VELOCITY OF NODE I IN X DIRECTION
C     R        = LINEAR VELOCITY OF NODE I IN Y DIRECTION
C     S        = LINEAR ACCELERATION OF NODE I IN X DIRECTION
C     T        = LINEAR ACCELERATION OF NODE I IN Y DIRECTION
C     W,A        = MASS OF EACH ELEMENT
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(8)
C
C     INITIALIZE THE FORMATION OF THE FORCE VECTOR FOR EACH
C     ELEMENT IN GLOBAL COORDINATES
C
      CC = DCOS(Y)
      SS = DSIN(Y)
      A = W
C
      X(1) = (R*V*Z*A/2.0D0-S*Z*A/2.0D0+V**2*Z**2*A/6.0D0)*CC -
     1        (-P*V*Z*A/2.0D0-T*Z*A/2.0D0-U*Z**2*A/7.0D0)*SS
      X(2) = (R*V*Z*A/2.0D0-S*Z*A/2.0D0+V**2*Z**2*A/6.0D0)*SS +
     1        (-P*V*Z*A/2.0D0-T*Z*A/2.0D0-U*Z**2*A/7.0D0)*CC
      X(3) = - P*V*Z**2*A/10.0D0-T*Z**2*A/10.0D0-4.0D0*U*Z**3*A/105.0D0
      X(4) = - P*V*Z**3*A/120.0D0-T*Z**3*A/120.0D0-U*Z**4*A/280.0D0
      X(5) = (R*V*Z*A/2.0D0-S*Z*A/2.0D0+V**2*Z**2*A/3.0D0)*CC -
     1        (-P*V*Z*A/2.0D0-T*Z*A/2.0D0-5.0D0*U*Z**2*A/14.0D0)*SS
      X(6) = (R*V*Z*A/2.0D0-S*Z*A/2.0D0+V**2*Z**2*A/3.0D0)*SS +
     1        (-P*V*Z*A/2.0D0-T*Z*A/2.0D0-5.0D0*U*Z**2*A/14.0D0)*CC
      X(7) = P*V*Z**2*A/10.0D0+T*Z**2*A/10.0D0+13.0D0*U*Z**3*A/210.0D0
      X(8) = - P*V*Z**3*A/120.0D0-T*Z**3*A/120.0D0-U*Z**4*A/210.0D0
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE FORCE1(Y,X,XR)
C
C     THIS SUBROUTINE FORMS THE GLOBAL INERTIAL FORCE VECTOR X,
C     THEN ZEROES THOSE ROWS OF X ACCORDING TO BOUNDARY CONDITIONS
C     TO OBTAIN VECTOR XR
C
C     NOTATIONS :
C
C     X        = GLOBAL INERTIAL FORCE VECTOR . X(MASIZ)
C     XR       = REDUCED GLOBAL INERTIAL FORCE VECTOR DUE TO BOUNDARY
C                CONDITIONS . XR(N)
C     Y        = TRANSFER PARAMETER . REPRESENTS ALL OF THE FORCES
C                ORDERED OF THE MECHANISM ACCORDING TO THE
C                ORDER OF THE ELEMENTS . 8 PER ELEMENT . Y(24)
C     LECO     = BOUNDARY CONDITIONS APPLIED TO THE ROWS OF X . THIS
C                PERMITS THE REDUCTION OF VECTOR X(M) TO XR(N)
C     M        = NUMBER OF ROWS OF X
C     N        = NUMBER OF ROWS OF XR
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(20) , Y(24) , LECO(20) , XR(11)
      COMMON/B12/LECO,MASIZ
C
C     INITIALIZE THE FORMATION OF VECTOR X
```

```
C
      X(1) = Y(1)
      X(2) = Y(2)
      X(3) = Y(3)
      X(4) = Y(4)
      X(5) = Y(5) + Y(9)
      X(6) = Y(6) + Y(10)
      X(7) = Y(7)
      X(8) = Y(8)
      X(9) = Y(11)
      X(10) = Y(12)
      X(11) = Y(13) + Y(17)
      X(12) = Y(14) + Y(18)
      X(13) = Y(15)
      X(14) = Y(16)
      X(15) = Y(19)
      X(16) = Y(20)
      X(17) = Y(21)
      X(18) = Y(22)
      X(19) = Y(23)
      X(20) = Y(24)
C
      K12 = C
      DO 1 I=1,MASIZ
      IF (I .EQ. LECO(I)) GC TC 2
      K12 = K12 + 1
      XR(K12) = X(I)
    2 CCNTINLE
    1 CONTINLE
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUPFOLTINE MAFED(MASIZ,MASIF,NEOUNF,NBCUNC,A,B)
C
C     THIS SLBFOUTINE REDUCES A MATRIX "A" TC A MATRIX "B" OF LOWER
C     ORDER BY ELIMINATING FCWS AND CCLUMNS ACCORDING TC NBOUNR AND
C     NBCUNC
C
C     NOTATICNS :
C
C     MASIZ   = SIZE CF MATFIX "A"
C     MASIR   = SIZE CF MATFIX "B"
C     NBCUNF  = ECUNDARY FOF RCWS CF "A"
C     NBCUNR  = ECUNDARY FCF CCLUMNS OF "A"
C
C
      IMPLICIT REAL*8(A-H,C-Z)
      DIMENSION A(MASIZ,MASIZ) , B(MASIF,MASIR) , NBCUNF(MASIZ) ,
     1            NBCUNC(MASIZ)
C
C
      N = 0
C
      DO 1 I=1,MASIZ
      IF (I .EQ. NBCUNF(I)) GO TC 30
      N = N + 1
      M = 0
      DO 2 J=1,MASIZ
      IF (J .EQ. NBCUNC(J)) GC TC 31
      M = M + 1
      B(N,M) = A(I,J)
   31 CONTINUE
   32 CCNTINLE
   30 CONTINLE
    1 CONTINLE
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUFROLTINE REDUCE (N,M,S,WF,WI)
C
C     THIS SLBROLTINE CONDENSES DEGREES CF FREEDCM OF A SYSTEM
C     OF GENEFARALIZED COORDINATES
C
C     NOTATICNS :
C
C     T1          = ANGLE CF INPUT LINK , FADIANS
C     T2          = ANGLE OF COUPLEF LINK , FADIANS
C     T3          = ANGLE CF CUTPUT LINK , RADIANS
C     WI          = MATRIX ECUAL TC T(T)*S , WI(M,N)
C     S           = MATRIX TC BE CCNDENSEC , S(N,N)
C     T,T(T)      = CCNDENSING MATRIX , T(N,M) , T(T)(M,N)
C     WF          = CCNDENSED MATRIX ECUAL TC WI*T , WF(M,N)
C     N           = SIZE OF MATRIX S
C     M           = SIZE CF MTRIX WF
C
C
C
      IMPLICIT REAL*8(A-H,C-Z)
      DIMENSICN S(N,N) , WF(M,M) , WI(M,N)
      CCMMCN/RED/Z,X,Y
C
C
C
      DO 25 I=1,N
      WI(1,I) = S(1,I)
      WI(2,I) = S(2,I)
      WI(3,I) = S(3,I)*Z + S(4,I) + S(7,I)*X + S(8,I)*Y
      WI(4,I) = S(5,I)
      WI(5,I) = S(6,I)
      WI(6,I) = S(9,I)
      WI(7,I) = S(10,I)
      WI(8,I) = S(11,I)
```

```
   25 CONTINUE
C
      DO 30 I=1,M
      WF(I,1) = WI(I,1)
      WF(I,2) = WI(I,2)
      WF(I,3) = WI(I,3)*Z + WI(I,4) + WI(I,7)*X + WI(I,8)*Y
      WF(I,4) = WI(I,5)
      WF(I,5) = WI(I,6)
      WF(I,6) = WI(I,9)
      WF(I,7) = WI(I,10)
      WF(I,8) = WI(I,11)
   30 CONTINUE
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE FCN1(N,X,Y,YPRIME)
C
C     THIS SUBROUTINE EVALUATES FUNCTIONS . FCN1 SHOULD EVALUATE
C     YPRIME(1),.....YPRIME(N) GIVEN N, X AND Y(1).....Y(N) .
C
C     NOTATIONS :
C
C     M        = SIZE OF MATRICES
C     N        = NUMBER OF EQUATIONS . N = 2*M
C     X        = INDEPENDENT VARIABLE . INPUT AND OUTPUT
C                ON INPUT , X SUPPLIES THE INITIAL VALUE
C                ON OUTPUT , X IS REPLACED WITH XEND UNLESS
C                ERROR CONDITIONS ARISE . SEE THE DESCRIPTION OF
C                PARAMETER IND
C     Y        = DEPENDENT VARIABLES . VECTOR OF LENGTH N
C                INPUT AND OUTPUT
C                ON INPUT , Y(1)....Y(N) SUPPLY INITIAL VALUES
C                ON OUTPUT , Y(1)....Y(N) ARE REPLACED WITH AN
C                APPROXIMATE SOLUTION AT XEND UNLESS ERROR CONDITIONS
C                ARISE
C     YPRIME   = YPRIME(I) ARE THE FIRST DERIVATIVES OF Y(I) WITH
C                RESPECT TO X
C     GX       = REDUCED MASS MATRIX IN GLOBAL COORDINATES . GX(M,M)
C     GW       = REDUCED GYROSCOPIC MATRIX IN GLOBAL COORDINATES .
C                GW(M,M)
C     GY       = REDUCED STIFFNESS MATRIX IN GLOBAL COORDINATES . GY(M,M)
C     GZ       = REDUCED RIGHT HAND VECTOR OF INERTIAL FORCES IN GLOBAL
C                COORDINATES . GZ(M)
C     A        = GX
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION Y(N) , YPRIME(N) , GX(8,8) , GY(8,8) , GW(8,8) , GZ(8) ,
     1          SUM(8)
      COMMON/B14/GX,GY,GW,GZ,SUM,M,K7
      COMMON/CSPEC/UL(8,8),XX(8),LMETH
C
C     INITIALIZE SYSTEM OF DIFFERENTIAL EQUATIONS
C
      K7 = K7 + 1
      DO 3 K1=1,M
      YPRIME(K1) = Y(M+K1)
      SUM(K1) = GZ(K1)
    3 CONTINUE
C
      DO 6 I=1,M
      S = 0.0D0
      DO 4 J=1,M
      S = S - GY(I,J)*Y(J) + GW(I,J)*Y(J+M)
    4 CONTINUE
      SUM(I) = SUM(I) + S
    6 CONTINUE
C
      CALL SOLVE (M,UL,M,SUM,XX)
C
      DO 5 L=1,M
      YPRIME(L+M) = XX(L)
    5 CONTINUE
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE DECOMP(NN,A,LA,UL,LUL)
C
C  DECOMPOSES A INTO THE PRODUCT A=L*U, WHERE L IS A MONIC LOWER
C  TRIANGULAR MATRIX AND U IS UPPER TRIANGULAR.  STORES L-I AND U IN
C  THE ARRAY UL.
C  DECOMP PERFORMS ABOUT N**3/3 MULTIPLICATIONS.
C  NN MUST NOT EXCEED THE DIMENSION OF SCALES OR IPS.
C
C  G. E. FORSYTHE AND C. B. MOLER, -COMPUTER SOLUTION OF LINEAR
C      ALGEBRAIC SYSTEMS- (PRENTICE-HALL, 1967)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(LA,NN),UL(LUL,NN)
      DIMENSION SCALES(50)
      COMMON /COMIF/ IPS(50)
      DABS(ARG)=DABS(ARG)
C                                 NMAX IS THE DIMENSION OF SCALES AND IPS.
      NMAX=50
      ZERO = 0.0D0
      UNITY = 1.0D0
      N=NN
      IF(N.GT.NMAX) STOP
C
C  INITIALIZE IPS, UL, AND SCALES.
C
      DO 20 I=1,N
```

```
      IPS(I)=I
      ROWNRM=ZERO
      DO 10 J=1,N
        UL(I,J)=A(I,J)
C                              ROWNRM=AMAX1(ROWNRM,ABS(UL(I,J)))
        ABSUL=DABS(UL(I,J))
        IF(ABSUL.GT.ROWNRM) ROWNRM=ABSUL
   10   CONTINUE
      SCALES(I)=ZERO
      IF(ROWNRM.NE.ZERO) SCALES(I)=UNITY/ROWNRM
   20 CONTINUE
C
C  GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING...
C
     IF(N.EQ.1) RETURN
     NM1=N-1
     DO 50 K=1,NM1
       BIG=ZERO
       DO 30 I=K,N
         IP=IPS(I)
         SIZE=DABS(UL(IP,K)*SCALES(IP))
         IF(SIZE.LE.BIG) GO TO 30
         BIG=SIZE
         IDXPIV=I
   30    CONTINUE
       J=IPS(K)
       IPS(K)=IPS(IDXPIV)
       IPS(IDXPIV)=J
       KP=IPS(K)
       PIVOT=UL(KP,K)
       KP1=K+1
       DO 40 I=KP1,N
         IP=IPS(I)
         EM=ZERO
         IF(PIVOT.NE.ZERO) EM=-UL(IP,K)/PIVOT
         UL(IP,K)=-EM
         IF(EM.EQ.ZERO) GO TO 50
         DO 40 J=KP1,N
           UL(IP,J)=UL(IP,J)+EM*UL(KP,J)
   40      CONTINUE
   50  CONTINUE
C
     RETURN
     END
C
C    XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C    XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
     SUBROUTINE SOLVE(NN,UL,LUL,B,X)
C
C  SOLVES A*X=B USING UL FROM DECOMP.
C  SOLVE PERFORMS ONLY ABOUT N**2 MULTIPLICATIONS.
C  NN MUST NOT EXCEED THE DIMENSION OF IPS.
C
C  G. E. FORSYTHE AND C. B. MOLER, -COMPUTER SOLUTION OF LINEAR
C        ALGEBRAIC SYSTEMS- (PRENTICE-HALL, 1967)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION UL(LUL,NN),B(NN),X(NN)
      COMMON /COMIP/ IPS(50)
C
      RZERO = 0.0D0
      N=NN
      NP1=N+1
C
      IP=IPS(1)
      X(1)=B(IP)
      IF(N.EQ.1) GO TO 25
      DO 20 I=2,N
        IP=IPS(I)
        IM1=I-1
        SUM=B(IP)
        DO 10 J=1,IM1
   10     SUM=SUM-UL(IP,J)*X(J)
   20   X(I)=SUM
C
   25 IP=IPS(N)
      IF(UL(IP,N).NE.RZERO) GO TO 30
      X(N)=RZERO
      GO TO 40
   30 X(N)=X(N)/UL(IP,N)
   40 IF(N.EQ.1) RETURN
      DO 60 IBACK=2,N
        I=NP1-IBACK
        IP=IPS(I)
        IP1=I+1
        SUM=X(I)
        DO 50 J=IP1,N
   50     SUM=SUM-UL(IP,J)*X(J)
        X(I)=RZERO
        IF(UL(IP,I).NE.RZERO) X(I)=SUM/UL(IP,I)
   60   CONTINUE
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE FCNJ (N,X,Y,PD)
C
C     DUMMY SUBROUTINE
C
      IMPLICIT REAL*8(A-H,O-Z)
      RETURN
      END
$ENDLIST
```

```
C
C
C
C
C       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C       X                                                                   X
C       X               THIS PROGRAM PERMITS TO DETERMINE                   X
C       X                             STRAIN                                X
C       X                            STRESSES                               X
C       X                          DEFORMATIONS                             X
C       X                              FROM                                 X
C       X                   THE KINETO-ELASTODYNAMICS                       X
C       X                                                                   X
C       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
C
C
C       NOTATIONS :
C
C       DELG       = DEFORMATION VECTOR OF MECHANISM IN GLOBAL
C                    COORDINATES . DELG(NDM)
C       NDM        = NUMBER OF DEGREES OF FREEDOMS OF MECHANISM
C       DELEG      = DEFORMATION VECTOR OF ELEMENTS IN GLOBAL
C                    COORDINATES . DELEG(NDTE)
C       NDTE       = TOTAL NUMBER OF DEGREES OF FREEDOM OF ELEMENTS
C       DELEL      = DEFORMATION VECTOR OF ELEMENTS IN GLOBAL COORDINATES
C                    DELEL = DELEG . DELEL(NE,NDE)
C                    LOCAL COORDINATES . DELEL(NE,NDE)
C       NDE        = NUMBER OF DEGREES OF FREEDOMS OF EACH ELEMENT
C       NE         = NUMBER OF ELEMENT OF MECHANISM
C       CV         = COEFFICIENT VECTOR . CV(NE,NDE)
C       VV         = TRANSFERING PARAMETER . VV(NDE)
C                    EEE(8)
C       BB         = DEFORMATION VECTOR OF ELEMENTS IN LOCAL COORDINATES.
C       CC         = TRANSFERING PARAMETER . CC(8)
C       C          = INVERSE OF THE GEOMETRIC MATRIX . C(NDE,NDE)
C       EXI1       = STRAIN DUE TO TRANSVERSE FORCES AND MOMENTS .
C                    EXI1(NE)
C       EXI2       = STRAIN DUE TO AXIAL FORCES . EXI2(NE)
C       SIGM1      = STRESS DUE TO EXI1 . SIGM1(NE)
C       SIGM2      = STRESS DUE TO EXI2 . SIGM2(NE)
C       SIGM3      = EQUAL TO SIGM1 + SIGM2 . SIGM3(NE)
C       DEPTH      = CROSS-SECTIONAL DEPTH . DEPTH(NE)
C       POSI       = POSITION OF STRAIN AND STRESS IN THE ELEMENT .
C                    POSI(NE)
C       F          = ASSEMBLING MATRIX . F(NDTE,NDM)
C       IT         = FLAG USED TO FORM MATRIX F . IT(NE)
C       XIX,YIY    = X AND Y COORDINATES IN GLOBAL COORDINATES OF JOINT I
C                    OF ELEMENT
C       THT        = POSITION ANGLE OF COUPLER LINK . IN RADIANS
C       R1P,R2P    = POSITION VECTOR OF DEFORMED MOVING JOINTS OF
C                    MECHANISM . R1P(2) . R2P(2)
C       OXC,OYC    = POSITION VECTOR OF JOINT O OF MECHANISM
C       XLNEW      = DISTANCE BETWEEN JOINTS OF DEFORMED MECHANISM .
C                    XLNEW(NE)
C       AXIAL      = AXIAL DEFORMATION OF EACH ELEMENT . AXIAL(NE)
C       TR         = TRANSPOSE OF THE TRANSFORMATION MATRIX . TR(8,8)
C       AXIAL1     = NEW LENGTH OF EACH ELEMENT DUE TO AXIAL DEFORMATION
C                    AXIAL1(NEMEL)
C       AXIAL2     = AXIAL DEFORMATION OF EACH ELEMENT . AXIAL(NUMEL)
C       EM         = MODULUS OF ELASTICITY OF EACH ELEMENT . EM(NE)
C       NSTEP1     = STEPS IN THE ANALYSIS OF THE MECHANISM
C       NUMPOS     = SIZE OF ANALYSIS FOR THE MECHANISM . EQUAL TO THE
C                    NUMBER OF DEGREES THAT THE MECHANISM WAS RUN
C       NPRINT     = FLAG FOR PRINTING OUT DATA ON DISC IN SUBROUTINE DATA
C                    FOR NPRINT .GT. 0 THERE WILL BE PRINTING
C       RESAL      = TOTAL LENGTH OF LINKS DUE TO TRANSVERSE AND AXIAL
C                    FORCES . RESAL(NE)
C       AINTE1     = LENGTH OF LINKS DUE TO TRANSVERSE FORCES . AINTE1(NE)
C       ND1        = NUMBER OF GENERALIZED COORDINATES IN STUDY
C       NUM1       = SIZE OF VECTOR Y2
C
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION F(24,20) , DELG(20) , DELEG(24) , DELEL(3,8) , IT(3)
      DIMENSION CV(3,8) , TR(8,8) , VV(8) , C(8,8) , XL(3)
      DIMENSION EXI1(3) , EXI2(3) , SIGM1(3) , SIGM2(3) , SIGM3(3) ,
     1          DEPTH(3) , POSI(3) , EM(3) , BB(8) , CC(8)
      DIMENSION XXL(2) , R1P(2) , R2P(2)
      DIMENSION XLNEW(3) , AXIAL1(3) , AXIAL2(3) , AINTE1(3) , RESAL(3)
      DIMENSION ZZ(90,3,6) , UU(90,8) , DECR(90)
      DIMENSION Y1(8) , Y2(11) , THE(3) , AXYA(2) , EXYE(2)
      COMMON D,E,G,C1,C2
      COMMON/CON/FXX,FYY,FZZ
      EXTERNAL F1
      DATA IN,LP/5,6/
C
C
C     ******************************************************************
C     INITIALIZE READING AND PRINTING OUT GIVEN INFORMATION FROM THIS
C     PROGRAM
C     ******************************************************************
C
C
      READ (IN,1000) NDM , NDTE , NE , NDE , NUM1 , ND1 , NUMS , NUMS1 ,
     1NPRINT , N2N , STEPDE
 1000 FORMAT (10I6,D10.0)
      DO 1 K=1,NE
      READ (IN,1010) XL(K) , DEPTH(K) , EM(K) , POSI(K)
 1010 FORMAT (4D20.0)
    1 CONTINUE
      READ (IN,1020) OXC , OYC
 1020 FORMAT (2D20.0)
      READ (IN,1025) (IT(I),I=1,NE)
 1025 FORMAT (3I3)
C
      WRITE (LP,2000)
```

```
 2000 FORMAT (1H1)
      WRITE (LP,2001)
 2001 FORMAT (//)
      WRITE (LP,2002)
 2002 FORMAT (60X,'INPUT DATA')
      WRITE (LP,2003)
 2003 FORMAT (60X,'XXXXXXXXXX')
      WRITE (LP,2001)
      WRITE (LP,2004)
 2004 FORMAT (10X,'NDM', 8X,'NDTE', 8X,'NE', 9X,'NDE', 8X,'NUM1', 7X,'ND
     11', 6X,'NUMS', 8X,'NUMS1', 5X,'NPRINT', 6X,'N2N', 6X,'STEPDE',/)
      WRITE (LP,2005) NDM , NDTE , NE , NDE , NUM1 , ND1 , NUMS , NUMS1
     1, NPRINT , N2N , STEPDE
 2005 FORMAT ( 2X,10I11, 3X,D10.3)
      WRITE (LP,2001)
      WRITE (LP,2008)
 2008 FORMAT (32X,'XL',18X,'DEPTH',15X,'EM',18X,'PCSI',/)
      DO 2 K=1,NE
      WRITE (LP,2009) XL(K) , DEPTH(K) , EM(K) , PCSI(K)
 2009 FORMAT (24X,4D20.8)
    2 CONTINUE
      WRITE (LP,2001)
      WRITE (LP,2012)
 2012 FORMAT (52X,'QXO',17X,'QYO',/)
      WRITE (LP,2013) QXO , QYO
 2013 FORMAT (44X,2D20.8)
C
C
C
C     INITIALIZE CONSTANT INFORMATION
C
      PI = 4.0D0 * DATAN(1.0D0)
      NUM2 = 11
      CALL UNIT1 (NE,NDTE,NDM,IT,F)
C
C
C
C     ****************************************************************
C     INITIALIZE DO LOOP
C     ****************************************************************
C
C
      DO 9999 M=1,NUMS
C
C
C     ****************************************************************
C     READ OUT INFORMATION FROM DISC
C     ****************************************************************
      READ (10,2040) (Y1(MIM),MIM=1,ND1)
 2040 FORMAT (8D15.6)
      READ (11,2041) (THE(MIM),MIM=1,3)
 2041 FORMAT (3D15.6)
      READ (12,2042) AXYA(1) , AXYA(2)
      READ (13,2042) BXYB(1) , BXYB(2)
 2042 FORMAT (2D15.6)
C     ****************************************************************
C     PRINT OUT INFORMATION FROM DISC IF NECESSARY
C     ****************************************************************
      IF (NPRINT .EQ. 0) GO TO 2043
      WRITE (LP,2044) (Y1(MIM),MIM=1,ND1)
 2044 FORMAT (11D11.3)
      WRITE (LP,2045) (THE(MIM),MIM=1,3)
 2045 FORMAT (30X,3D15.6)
      WRITE (LP,2046) AXYA(1) , AXYA(2)
      WRITE (LP,2046) BXYB(1) , BXYB(2)
 2046 FORMAT (40X,2D15.6)
 2043 CONTINUE
C     ****************************************************************
C     FIND TRANSFORMATION PARAMETERS
C     ****************************************************************
      FXX = (DSIN(THE(2))*DSIN(THE(3)-PI)-DTAN(THE(1))*DCOS(THE(2))*
     1      DSIN(THE(3)-PI))/DSIN(THE(3)-PI-THE(2))
      FYY = (DTAN(THE(1))*DCOS(THE(2))*DCOS(THE(3)-PI)-DSIN(THE(2))*
     1      DCOS(THE(3)-PI))/DSIN(THE(3)-PI-THE(2))
      FZZ = - DTAN(THE(1))
C     ****************************************************************
C     CARRY OUT TRANSFORMATION OF INDEPENDENT TO DEPENDENT
C     GENERALIZED COORDINATES
C     ****************************************************************
      CALL CONDE (Y1,ND1,NUM1,Y2)
C     ****************************************************************
C     FORM VECTOR DELG OF DEFORMATIONS OF MECHANISM
C     IN GLOBAL COORDINATES
C     ****************************************************************
      CALL BUILD (Y2,THE,AXYA,DELG,X1X,Y1Y,THT)
C     ****************************************************************
C     FORM VECTOR DELEG OF DEFORMATIONS OF ELEMENTS
C     IN GLOBAL COORDINATES
C     ****************************************************************
      CALL MLLMTV (F,DELG,DELEG,NDTE,NDM)
C     ****************************************************************
C     FORM VECTOR DELEL OF DEFORMATIONS OF EACH
C     ELEMENT SEPARATELY IN GLOBAL COORDINATES
C     ****************************************************************
      CALL REDUCE (NE,NDTE,DELEG,DELEL)
      WRITE (LP,2000)
      WRITE (LP,2001)
      WRITE (LP,2048)
 2048 FORMAT (41X,'DEFORMATION VECTOR OF ELEMENTS IN GLOBAL COORDINATES'
     1)
      WRITE (LP,2050) ((DELEL(J,I),I=1,NDE),J=1,NE)
 2050 FORMAT ( 5X,8D15.6)
C     ****************************************************************
C     FORM VECTOR BB OF DEFORMATIONS OF EACH
C     ELEMENT IN LOCAL COORDINATES
C     ****************************************************************
      WRITE (LP,2001)
      WRITE (LP,2052)
 2052 FORMAT (42X,'DEFORMATION VECTOR OF ELEMENTS IN LOCAL COORDINATES')
      DO 9 I=1,NE
```

```
      DO 10 J=1,NDE
      VV(J) = DELFL(I,J)
   10 CONTINUE
      CALL TRAN (THE(I),TR)
      CALL MULMTV (TR,VV,BB,NDE,NDE)
      WRITE (LP,2050) (BB(K),K=1,NDE)
C     ******************************************************************
C     FORM VECTOR CC OF COEFFICIENTS IN LOCAL COORDINATES
C     ******************************************************************
      CALL UCC (C,XL(I),NDE)
      CALL MULMTV (C,BB,CC,NDE,NDE)
      DO 15 K=1,NDE
      CV(I,K) = CC(K)
   15 CONTINUE
    9 CONTINUE
      WRITE (LP,2001)
      WRITE (LP,2080)
 2080 FORMAT (42X,'COEFFICIENT VECTOR OF ELEMENTS IN LOCAL COORDINATES')
      WRITE (LP,2050) ((CV(J,I),I=1,NDE),J=1,NE)
C     ******************************************************************
C     FIND STRAIN AND STRESSES
C     ******************************************************************
      WRITE (LP,2001)
      WRITE (LP,2200)
 2200 FORMAT (22X,'EXI1',16X,'EXI2',15X,'SIGM1',15X,'SIGM2',15X,'SIGM3')
      DO 20 I=1,NE
      DO 21 J=1,NDE
      VV(J) = CV(I,J)
   21 CONTINUE
      CALL STRESS (VV,DEPTH(I),POSI(I),EM(I),EXI1(I),EXI2(I),SIGM1(I),
     1             SIGM2(I),SIGM3(I))
      WRITE (LP,2280) EXI1(I) , EXI2(I) , SIGM1(I) , SIGM2(I) , SIGM3(I)
 2280 FORMAT (12X,5D20.6)
      ZZ(M,I,1) = EXI1(I) * 1.0D+06
      ZZ(M,I,2) = EXI2(I) * 1.0D+06
      ZZ(M,I,3) = SIGM1(I)
      ZZ(M,I,4) = SIGM2(I)
      ZZ(M,I,5) = SIGM3(I)
   20 CONTINUE
C
C     ******************************************************************
C     INITIALIZE THE FINDING OF POSITION VECTORS OF MOVING JOINTS OF
C     DEFORMED MECHANISM , NEW DISTANCES BETWEEN JOINTS AND AXIAL
C     DEFORMATION OF ELEMENTS
C     ******************************************************************
C
      XXL(1) = 0.0D0
C
      XXL(2) = XL(2)
      WRITE (LP,2001)
      WRITE (LP,2289)
 2289 FORMAT ( 8X,'AXYA(1)', 9X,'R1R(1)', 9X,'AXYA(2)', 8X,'R2R(1)', 9
     1X,'BXYB(1)', 8X,'R1R(2)', 9X,'BXYB(2)', 8X,'R2R(2)')
      DO 25 I=1,8
      VV(I) = CV(2,I)
   25 CONTINUE
      CALL PERRO (XXL,X1X,Y1Y,THT,VV,R1R,R2R,CXC,CYC,XLNEW)
      WRITE (LP,2300) AXYA(1) , R1R(1) , AXYA(2) , R2R(1) , BXYB(1) ,
     1R1R(2) , BXYB(2) , R2R(2)
 2300 FORMAT ( 4X,8D15.7)
      UU(M,1) = AXYA(1) * 1.0D+04
      UU(M,2) = R1R(1) * 1.0D+04
      UU(M,3) = AXYA(2) * 1.0D+04
      UU(M,4) = R2R(1) * 1.0D+04
      UU(M,5) = BXYB(1) * 1.0D+04
      UU(M,6) = R1R(2) * 1.0D+04
      UU(M,7) = BXYB(2) * 1.0D+04
      UU(M,8) = R2R(2) * 1.0D+04
      WRITE (LP,2001)
      WRITE (LP,2349)
 2349 FORMAT (64X,'XLNEW')
      WRITE (LP,2350) (XLNEW(J),J=1,NE)
 2350 FORMAT (34X,3D20.8)
C
      WRITE (LP,2001)
      WRITE (LP,2379)
 2379 FORMAT (29X,'AXIAL1',59X,'AXIAL2')
      CALL PERRA (XL,CV,XLNEW,NE,AXIAL1,AXIAL2)
      WRITE (LP,2380) (AXIAL1(J),J=1,NE) , (AXIAL2(K),K=1,NE)
 2380 FORMAT (     3D20.8, 5X,3D20.8)
C
C     ******************************************************************
C     INITIALIZE CALCULATIONS TO FIND THE LENGTH OF THE DEFORMED
C     ELEMENTS ACCORDING TO THE SHAPE FUNCTIONS
C     ******************************************************************
C
      WRITE (LP,2001)
      WRITE (LP,2399)
 2399 FORMAT (44X,'ELEMENT', 5X,'AINTE1',14X,'RESAL')
      H = 0.0D0
      RERR = 0.0D0
      AERR = 1.0D-05
      DO 40 I=1,NE
      B = XLNEW(I)
      D = CV(I,3)
      P = CV(I,4)
      G = CV(I,6)
      C1 = CV(I,7)
      C2 = CV(I,8)
      CCI = DCADRE (FI,H,B,AERR,RERR,ERROR,IER,D,P,G,C1,C2)
      AINTE1(I) = CCI
      RESAL(I) = AINTE1(I) + AXIAL2(I)
      WRITE (LP,2400) I , AINTE1(I) , RESAL(I)
 2400 FORMAT (33X,I15,2D20.8)
      ZZ(M,I,6) = RESAL(I) * 1.0D+02
   40 CONTINUE
 9999 CONTINUE
C
C     ******************************************************************
C     INITIALIZE THE PRINTING OUT OF TABLES
```

```
C      ************************************************************
C
       WRITE (LP,2000)
       WRITE (LP,2001)
       WRITE (LP,2498)
 2498 FORMAT (17X,'DEGR',17X,'EXI1',11X,'EXI2',10X,'SIGM1',10X,'SIGM2',1
     10X,'SIGM3',10X,'RESAL')
       WRITE (LP,2499)
 2499 FORMAT (38X,'10-6',11X,'10-6',11X,'10+0',11X,'10+0',11X,'10+0',11X
     1,'10-2')
       DEGR(1) = STEPDE * DFLOAT(N2N)
       DO 60 K=1,NUMS
       WRITE (LP,2500) DEGR(K) , (ZZ(K,1,L),L=1,6)
 2500 FORMAT (17X,F7.3,5X,5F15.3,F15.4)
       DEGR(K+1) = DEGR(1) + STEPDE*DFLOAT(K)*DFLOAT(N2N)
   60 CONTINUE
       WRITE (LP,2000)
       WRITE (LP,2001)
       WRITE (LP,2498)
       WRITE (LP,2499)
       DO 61 K=1,NUMS
       WRITE (LP,2500) DEGR(K) , (ZZ(K,2,L),L=1,6)
   61 CONTINUE
       WRITE (LP,2000)
       WRITE (LP,2001)
       WRITE (LP,2498)
       WRITE (LP,2499)
       DO 62 K=1,NUMS
       WRITE (LP,2500) DEGR(K) , (ZZ(K,3,L),L=1,6)
   62 CONTINUE
       WRITE (LP,2000)
       WRITE (LP,2001)
       WRITE (LP,2548)
 2548 FORMAT ( 6X,'DEGR',10X,'AXYA(1)', 8X,'R1R(1)', 9X,'AXYA(2)', 8X,'R
     12R(1)', 9X,'EXYB(1)', 9X,'R1R(2)', 9X,'EXYB(2)', 8X,'R2R(2)')
       WRITE (LP,2549)
 2549 FORMAT (20X,'10-4',11X,'10-4',11X,'10-4',11X,'10-4',11X,'10-4',11X
     1,'10-4',11X,'10-4',11X,'10-4')
       DO 63 K=1,NUMS
       WRITE (LP,2550) DEGR(K) , (UU(K,L),L=1,8)
 2550 FORMAT ( 4X,F7.3,2X,8F15.3)
   63 CONTINUE
C
       WRITE (LP,2000)
       STOP
       END
C
C      XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C      XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
       SUBROUTINE UNIT1 (NUMELE,M1,M2,IT,F)
C
C      THIS SUBROUTINE FORMS THE ASSEMBLING MATRIX
C      TO OBTAIN GLOBAL MATRICES
C
C      NOTATIONS :
C
C      M1          = ROWS OF MATRIX "F" . M1 = 8 * NUMELE
C      NUMELE      = NUMBER OF ELEMENTS OF MECHANISM
C      M2          = COLUMNS OF MATRIX "F" . M2 = 5 * NODE
C      NODE        = NODES OF MECHANISM
C      F           = ASSEMBLING MATRIX . F(M1,M2)
C      IT          = FLAG USED TO FORM MATRIX F . IT(NUMELE)
C                    IT IS EQUAL TO "1" FOR SYMETRIC UNIT MATRIX
C                    IT IS EQUAL "2" FOR AN UNSYMETRIC ONE
C
C
       IMPLICIT REAL*8(A-H,O-Z)
       DIMENSION F(M1,M2) , IT(NUMELE)
C
C
C      INITIALIZE SPACE FOR MATRIX "F"
C
       DO 1 I=1,M1
       DO 2 J=1,M2
       F(I,J) = 0.0D0
     2 CONTINUE
     1 CONTINUE
C
C      INITIALZIE DETERMINATION OF UNIT COEFFICIENTS
C
       J = 0
       M = 8
       L = 1
       K = 11
       DO 3 I=1,M1
       J = J + 1
       IF (IT(L) .EQ. 2 .AND. I .EQ. K) J=J+2
       F(I,J) = 1.0D0
       IF (I .EQ. K) J=J-4
       IF (I .EQ. M) L=L+1
       IF (I .EQ. M) M=M+8
       IF (I .EQ. K) K=K+8
     3 CONTINUE
C
       RETURN
       END
C
C      XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C      XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
       SUBROUTINE CONDE (Y,M,N,YY)
C
C      THIS SUBROUTINE FINDS THE DEPENDENT VECTOR OF DEFORMATIONS
C      FROM THE INDEPENDENT VECTOR OF REDUCED GENERALIZED COORDINATES
C
C      NOTATIONS :
C
C      M           = SIZE OF VECTOR Y
C      N           = SIZE OF VECTOR YY
```

```
C       Y           = VECTOR OF REDUCED COORDINATES , Y(M)
C       YY          = VECTOR OF DEPENDENT COORDINATES , YY(N)
C       X1,X2,X3    = PARAMETERS FOR TRANSFORMATION
C
C
C
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION Y(M) , YY(N)
        COMMON/CON/X1,X2,X3
C
C       INITIALIZE FORMATION OF VECTOR YY
C
        YY(1) = Y(1)
        YY(2) = Y(2)
        YY(3) = X3 * Y(3)
        YY(4) = Y(3)
        YY(5) = Y(4)
        YY(6) = Y(5)
        YY(7) = X1 * Y(3)
        YY(8) = X2 * Y(3)
        YY(9) = Y(6)
        YY(10) = Y(7)
        YY(11) = Y(8)
C
        RETURN
        END
C
C       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
        SUBROUTINE BUILD (Y10,Y11,Y12,A,CX,CY,B1)
C
C       THIS SUBROUTINE BUILDS THE DEFORMATION VECTOR OF THE
C       MECHANISM , POSITION-ANGLE-VECTOR OF MECHANISM , COORDINATES OF
C       JOINT I OF COUPLER AND POSITION ANGLE OF COUPLER
C
C       NOTATIONS :
C
C       Y10         = DEFORMATION VECTOR OF MECHANISM IN GLOBAL COORDINATES
C                     Y10(11)
C       Y11         = POSITION ANGLES OF MECHANISM , Y11(3)
C       Y12         = POSITION VECTOR IN GLOBAL COORDINATES OF JOINT I
C                     OF COUPLER , Y12(2)
C       A           = DEFORMATION VECTOR OF MECHANISM IN GLOBAL
C                     COORDINATES , A(20)
C       CX,CY       = X AND Y COORDINATES IN GLOBAL COORDINATES OF
C                     JOINT I OF COUPLER
C       B1          = POSITION ANGLE OF COUPLER LINK
C       K           = SIZE OF DO LOOP
C
C
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION Y10(11) , Y11(3) , Y12(2) , A(20)
C
C       INITIALIZE FORMING VECTOR A
C
        A(1) = 0.0D0
        A(2) = 0.0D0
        A(3) = Y10(1)
        A(4) = Y10(2)
        A(5) = Y10(3)
        A(6) = Y10(4)
        A(7) = Y10(5)
        A(8) = 0.0D0
        A(9) = Y10(6)
        A(10) = 0.0D0
        A(11) = Y10(7)
        A(12) = Y10(8)
        A(13) = Y10(9)
        A(14) = 0.0D0
        A(15) = Y10(10)
        A(16) = 0.0D0
        A(17) = 0.0D0
        A(18) = 0.0D0
        A(19) = Y10(11)
        A(20) = 0.0D0
C
C
C       INITIALIZE FORMING CX , CY AND B1
C
        CX = Y12(1)
        CY = Y12(2)
        B1 = Y11(2)
C
        RETURN
        END
C
C       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
        SUBROUTINE MULMTV (A,V,R,M1,M2)
C
C       THIS SUBROUTINE PREMULTIPLY A VECTOR "V" BY A
C       MATRIX "A" TO OBTAIN VECTOR "R"
C
C       NOTATIONS :
C
C       A           = MATRIX , A(M1,M2)
C       M1          = ROWS OF A
C       M2          = COLUMNS OF A
C       V           = VECTOR , V(M2)
C       R           = RESULTANT VECTOR , R(M1)
C
C
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION A(M1,M2) , V(M2) , R(M1)
C
C       INITIALIZE MULTIPLICATION
C
        DO 1 I=1,M1
```

```
      R(I) = 0.0D0
      DO 1 J=1,M2
      R(I) = R(I) + A(I,J) * V(J)
    1 CONTINUE
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE REDUCE (N,M,D,C)
C
C     THIS SUBROUTINE CARRIES OUT PARTITIONING OF VECTOR D INTO
C     THREE VECTORS C
C
C     NOTATIONS :
C
C     N          = NUMBER OF ELEMENTS OF MECHANISM
C     M          = SIZE OF VECTOR D
C     D          = DEFORMATION VECTOR OF MECHANISM IN GLOBAL
C                  COORDINATES
C     C          = DEFORMATION VECTOR OF EACH ELEMENT IN GLOBAL
C                  COORDINATES
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION D(M) , C(N,8)
C
C     INITIALIZE PARTITIONING
C
      L = 0
      DO 1 I=1,N
      DO 2 J=1,8
      L = L + 1
      C(I,J) = D(L)
    2 CONTINUE
    1 CONTINUE
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CC     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE TRAN (TH,T)
C
C     THIS SUBROUTINE FORMS THE TRANSPOSE OF THE TRANSFORMATION
C     MATRIX "T" TO BE USED IN THE EQUATION (DEL) = (T) * (DEG)
C
C     NOTATIONS :
C
C     TH         = POSITION ANGLE OF ELEMENT , RADIANS
C     T          = TRANSPOSE OF THE TRANSFORMATION MATRIX , T(8,8)
C     DEL        = DEFORMATION VECTOR FOR EACH ELEMENT IN LOCAL
C                  COORDINATES
C     DEG        = DEFORMATION VECTOR OF ELEMENT IN GLOBAL
C                  COORDINATES
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION T(8,8)
C
C     INITIALIZE SPACE FOR T
C
      DO 1 I=1,8
      DO 1 J=1,8
      T(I,J) = 0.0D0
    1 CONTINUE
C
      X = DCOS(TH)
      Y = DSIN(TH)
C
      T(1,1) =   X
      T(1,2) =   Y
      T(2,1) = - Y
      T(2,2) =   X
      T(3,3) = 1.0D0
      T(4,4) = 1.0D0
      T(5,5) =   X
      T(5,6) =   Y
      T(6,5) = - Y
      T(6,6) =   X
      T(7,7) = 1.0D0
      T(8,8) = 1.0D0
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE UCC (C,XL,N)
C
C     THIS SUBROUTINE FORMS THE INVERSE GEOMETRIC MATRIX OF
C     THE ELEMENTS
C
C     NOTATIONS :
C
C     C          = INVERSE OF THE GEOMETRIC MATRIX , C(N,N)
C     N          = NUMBER OF DEGREES OF FREEDOM OF ELEMENT
C     XL         = LENGTH OF ELEMENT
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION C(N,N)
C
C     INITIALIZE FORMATION OF MATRIX "C"
C
      DO 1 I=1,N
```

```
      DO 1 J=1,N
      C(I,J) = 0.0D0
    1 CONTINUE
      X = XL**2
      Y = XL**3
      Z = XL**4
      W = XL**5
      C(1,1) = 1.0D0
      C(2,2) = 1.0D0
      C(3,3) = 1.0D0
      C(4,4) = 0.5D0
      C(5,1) = - 1.0D0/XL
      C(5,5) = - C(5,1)
      C(6,2) = - 10.0D0/Y
      C(6,3) = - 6.0D0/X
      C(6,4) = - 3.0D0/(2.0D0*XL)
      C(6,6) = - C(6,2)
      C(6,7) = - 4.0D0/X
      C(6,8) = 1.0D0/(2.0D0*XL)
      C(7,2) = 15.0D0/Z
      C(7,3) = 8.0D0/Y
      C(7,4) = 3.0D0/(2.0D0*X)
      C(7,6) = - C(7,2)
      C(7,7) = 7.0D0/Y
      C(7,8) = - 1.0D0/X
      C(8,2) = - 6.0D0/W
      C(8,3) = - 3.0D0/Z
      C(8,4) = - 1.0D0/(2.0D0*Y)
      C(8,6) = - C(8,2)
      C(8,7) = - 3.0D0 /W
      C(8,8) = - C(8,4)
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE STRESS (C,H,X,E,EX1,EX2,SIG1,SIG2,SIG3)
C
C     THIS SUBROUTINE DETERMINES STRAINS AND STRESSES
C
C     NOTATIONS :
C
C     C          = VECTOR WITH THE COEFFICIENTS OF THE SHAPE
C                  FUNCTIONS , C(8)
C     H          = CROSS-SECTIONAL DEPTH
C     X          = POSITION OF STRAINS AND STRESSES
C     E          = MODULUS OF ELASTICITY
C     EX1        = STRAIN DUE TO TRANSVERSE FORCES
C     EX2        = STRAIN DUE TO AXIAL FORCES
C     SIG1       = STRESS DUE TO EX1
C     SIG2       = STRESS DUE TO EX2
C     SIG3       = STRESS EQUAL TO SIG1 + SIG2
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION C(8)
C
C     INITIALIZE CALCULATIONS
C
      EX1 = C(3)*H + C(4)*2.0D0*H + C(6)*6.0D0*H*X + C(7)*12.0D0*H*X**2
     1      + C(8)*20.0D0*H*X**3
C
      EX2 = C(5)
C
      SIG1 = EX1 * E
      SIG2 = EX2 * E
      SIC3 = SIG1 + SIG2
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE REFRC (XL,X1,Y1,TH,C,R1,R2,QX,QY,XLNEW)
C
C     THIS SUBROUTINE FINDS THE POSITIONS OF THE MOVING JOINTS
C     OF THE DEFORMED MECHANISM AND THE DISTANCES BETWEEN JOINTS
C     OF THE DEFORMED MECHANISM
C
C     NOTATIONS :
C
C     XL         = COORDINATES IN THE X DIRECTION IN LOCAL COORDINATES
C                  OF ELEMENT . XL = 0 AND XL = LENGTH OF ELEMENT .
C                  XL(2)
C     X1         = X COORDINATE IN GLOBAL COORDINATE OF JOINT I OF
C                  ELEMENT
C     Y1         = Y COORDINATE IN GLOBAL COORDINATE OF JOINT I OF
C                  ELEMENT
C     TH         = POSITION ANGLE OF ELEMENT
C     C          = COEFFICIENT VECTOR . C(8)
C     R1         = POSITION VECTOR OF JOINT I OF DEFORMED ELEMENT . R1(2)
C     R2         = POSITION VECTOR OF JOINT J OF DEFORMED ELEMENT . R2(2)
C     QX,QY      = POSITION VECTOR OF JOINT Q OF MECHANISM
C     X          = XL
C     XLNEW      = DISTANCES BETWEEN JOINTS OF THE DEFORMED
C                  MECHANISM . XLNEW(NUMELM)
C     NUMELM     = NUMBER OF ELEMENTS
C
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION C(8) , R1(2) , R2(2) , XL(2) , XLNEW(3)
C
C
C     INITIALIZE CALCULATION TO FIND VECTORS R1 AND R2
C
      COSTH = DCOS(TH)
```

```
      SINTH = DSIN(TH)
C
      DO 1 I=1,2
      X = XL(I)
      WX = C(2) + C(3)*X + C(4)*X**2 + C(6)*X**3 + C(7)*X**4 + C(8)*X**5
      UX = C(1) + C(5)*X
      R1(I) = X*COSTH + UX*COSTH - WX*SINTH + X1
      R2(I) = X*SINTH + UX*SINTH + WX*COSTH + Y1
    1 CONTINUE
C
C     INITIALIZE CALCULATIONS TO FIND NEW LENGTH OF ELEMENTS
C
      XLNEW(1) = DSQRT(R1(1)**2 + R2(1)**2)
      XLNEW(2) = DSQRT((R1(2)-R1(1))**2 + (R2(2)-R2(1))**2)
      XLNEW(3) = DSQRT((R1(2)-CX)**2 + (R2(2)-CY)**2)
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      SUBROUTINE REFRA (DL,C,R,NUMEL,AXIAL1,AXIAL2)
C
C     THIS SUBROUTINE DETERMINES THE AXIAL DEFORMATION
C     OF EACH ELEMENT
C
C     NOTATIONS :
C
C     C          = COEFFITIEN VECTOR . C(NUMEL,8)
C     NUMEL      = NUMBER OF ELEMENT OF MECHANISM
C     R          = UPPER INTEGRAL LIMIT . THE LOWER LIMIT IS ZERO
C     AXIAL1     = NEW LENGTH OF EACH ELEMENT DUE TO AXIAL DEFORMATION
C                  AXIAL1(NEMEL)
C     AXIAL2     = AXIAL DEFORMATION OF EACH ELEMENT . AXIAL(NUMEL)
C     DL         = RIGID LENGTH OF EACH ELEMENT . DL(NEMEL)
C
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION C(NUMEL,8) , R(NUMEL) , AXIAL1(NUMEL) , AXIAL2(NUMEL) ,
     1          DL(NUMEL)
C
C
C     INITIALIZE THE FINDING OF AXIAL1 AND AXIAL2
C
      DO 1 I=1,NUMEL
      AXIAL1(I) = DSQRT(1.0+C(I,5)**2) * R(I)
      AXIAL2(I) = AXIAL1(I) - DL(I)
    1 CONTINUE
C
      RETURN
      END
C
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
      DOUBLE PRECISION FUNCTION F1(X)
      COMMON D,P,G,D1,C2
      DOUBLE PRECISION X,D,P,G,C1,C2
      F1 = DSQRT(1.0+(D+2.0*P*X+3.0*G*X**2+4.0*C1*X**3+5.0*D2*X**4)**2)
      RETURN
      END
C   IMSL ROUTINE NAME   - DCADRE
C
C-----------------------------------------------------------------------
C   COMPUTER            - IBM/DOUBLE
C
C   LATEST REVISION     - JANUARY 1, 1978
C
C                             CAUTIOUS ADAPTIVE ROMBERG EXTRAPOLATION
C
C   USAGE               - FUNCTION DCADRE (F,A,B,AERR,RERR,ERROR,IER)
C
C   ARGUMENTS    DCADRE - ESTIMATE OF THE INTEGRAL OF F(X) FROM A TO B.
C                           (OUTPUT).
C                F      - A SINGLE-ARGUMENT REAL FUNCTION SUBPROGRAM
C                           SUPPLIED BY THE USER. (INPUT)
C                          F MUST BE DECLARED EXTERNAL IN THE
C   PURPOSE             - NUMERICAL INTEGRATION OF A FUNCTION USING
C                          CALLING PROGRAM.
C                A,B    - THE TWO ENDPOINTS OF THE INTERVAL OF
C                           INTEGRATION. (INPUT)
C                AERR   - DESIRED ABSOLUTE ERROR IN THE ANSWER. (INPUT)
C                RERR   - DESIRED RELATIVE ERROR IN THE ANSWER. (INPUT)
C                ERROR  - ESTIMATED BOUND ON THE ABSOLUTE ERROR OF
C                           THE OUTPUT NUMBER, DCADRE. (OUTPUT)
C                IER    - ERROR PARAMETER. (OUTPUT)
C                         WARNING ERROR(WITH FIX)
C                           IER = 65 IMPLIES THAT ONE OR MORE
C                             SINGULARITIES WERE SUCCESSFULLY HANDLED.
C                           IER = 66 IMPLIES THAT, IN SOME
C                             SUBINTERVAL(S), THE ESTIMATE OF THE
C                             INTEGRAL WAS ACCEPTED MERELY BECAUSE THE
C                             ESTIMATED ERROR WAS SMALL, EVEN THOUGH NO
C                             REGULAR BEHAVIOR WAS RECOGNIZED.
C                         TERMINAL ERROR
C                           IER = 131 INDICATES FAILURE DUE TO
C                             INSUFFICIENT INTERNAL WORKING STORAGE.
C                           IER = 132 INDICATES FAILURE BE DUE TO
C                             TOO MUCH NOISE IN THE FUNCTION (RELATIVE
C                             TO THE GIVEN ERROR REQUIREMENTS) OR
C                             DUE TO AN ILL-BEHAVED INTEGRAND.
C                           IER = 133 INDICATES THAT RERR IS GREATER
C                             THAN 0.1, OR RERR IS LESS THAN 0.0, OR
C                             RERR IS TOO SMALL FOR THE PRECISION OF
C                             THE MACHINE.
C
C   PRECISION/HARDWARE  - SINGLE AND DOUBLE/H32
C                       - SINGLE/H36,H48,H60
```

```
C
C   REQD. IMSL ROUTINES - UERTST,UGETIO
C
C   NOTATION             - INFORMATION ON SPECIAL NOTATION AND
C                            CONVENTIONS IS AVAILABLE IN THE MANUAL
C                            INTRODUCTION OR THROUGH IMSL ROUTINE UHELP
C
C   REMARKS  1.  DCADRE CAN, IN MANY CASES, HANDLE JUMP
C                DISCONTINUITIES. SEE DOCUMENT REFERENCE FOR FULL
C                DETAILS.
C            2.  THE RELATIVE ERROR PARAMETER RERR MUST BE IN THE
C                INTERVAL (0.0,0.1) INCLUSIVELY. FOR EXAMPLE,
C                RERR = 0.1 INDICATES THAT THE ESTIMATE OF THE
C                INTEGRAL IS TO BE CORRECT TO ONE DIGIT, WHEREAS
C                RERR = .0001 CALLS FOR FOUR DIGITS OF ACCURACY.
C                IF DCADRE DETERMINES THAT THE RELATIVE ACCURACY
C                REQUIREMENTS CANNOT BE SATISFIED, IER IS SET TO
C                133 (RERR SHOULD BE LARGE ENOUGH THAT, WHEN ADDED
C                TO 100.0, THE RESULT IS A NUMBER GREATER THAN
C                100.0).
C            3.  THE ABSOLUTE ERROR PARAMETER, AERR, SHOULD BE NON-
C                NEGATIVE. IN ORDER TO GIVE A REASONABLE VALUE FOR
C                AERR, THE USER MUST KNOW THE APPROXIMATE MAGNITUDE
C                OF THE INTEGRAL BEING COMPUTED. IN MANY CASES IT IS
C                SATISFACTORY TO USE AERR = 0.0. IN THIS CASE, ONLY
C                THE RELATIVE ERROR REQUIREMENT IS SATISFIED IN THE
C                COMPUTATION.
C            4.  EVEN WHEN IER IS NOT EQUAL TO 0, DCADRE RETURNS THE
C                BEST ESTIMATE THAT HAS BEEN COMPUTED.
C                QUOTING FROM THE DOCUMENT REFERENCE; A VERY CAUTIOUS
C                MAN WOULD ACCEPT DCADRE ONLY IF IER IS 0 OR 65. THE
C                MERELY REASONABLE MAN WOULD KEEP THE FAITH EVEN IF
C                IER IS 66. THE ADVENTUROUS MAN IS QUITE OFTEN RIGHT
C                IN ACCEPTING DCADRE EVEN IF IER IS 131 OR 132.
C
C   COPYRIGHT            - 1978 BY IMSL, INC. ALL RIGHTS RESERVED.
C
C   WARRANTY             - IMSL WARRANTS ONLY THAT IMSL TESTING HAS BEEN
C                            APPLIED TO THIS CODE. NO OTHER WARRANTY,
C                            EXPRESSED OR IMPLIED, IS APPLICABLE.
C
C-----------------------------------------------------------------------
C
      DOUBLE PRECISION FUNCTION DCADRE (F,A,B,AERR,RERR,ERROR,IER,A3,A4,
     1A5,A6,A7)
C                                  SPECIFICATIONS FOR ARGUMENTS
      INTEGER            IER
      DOUBLE PRECISION   F,A,B,AERR,RERR,ERROR
      DOUBLE PRECISION A3,A4,A5,A6,A7
C                                  SPECIFICATIONS FOR LOCAL VARIABLES
      INTEGER            IBEGS(30),MAXTS,MAXTBL,MXSTGE,IBEG,II,NNLEFT
      INTEGER            I,N2,III,ISTEP2,IEND,ISTEP,L,LM1,IT,ISTAGE,N
      DOUBLE PRECISION   T(10,10),R(10),AIT(10),DIF(10),RN(4),TS(2049)
      DOUBLE PRECISION   BEGIN(30),FINIS(30),EST(30)
      DOUBLE PRECISION   H2TOL,AITTOL,LENGTH,JUMPTL,ZERO,P1,HALF,ONE
      DOUBLE PRECISION   TWO,FOUR,FOURP5,TEN,HUN,CADRE,AITLOW
      DOUBLE PRECISION   STEPMN,STEPNM,STAGE,CUREST,FNSIZE,HRERR
      DOUBLE PRECISION   PREVER,BEG,FBEG,EDN,FEND,STEP,ASTEP,TABS,HOVN
      DOUBLE PRECISION   FN,SUM,SUMABS,VINT,TABTLM,ERGL,ERGOAL
      DOUBLE PRECISION   ERRA,ERRR,FEXTRP,ERRER,DIFF,SING,FEXTM1
      DOUBLE PRECISION   H2NEXT,SINGNX,SLOPE,FBEG2,ERRET,H2TFEX,FI
      LOGICAL            H2CONV,AITKEN,RIGHT,REGLAR,REGLSV(30)
      DATA               AITLOW,H2TOL,AITTOL,JUMPTL,MAXTS,MAXTBL,MXSTGE
     1                   /1.1D0,.15D0,.1D0,.01D0,2049,10,30/
      DATA               RN(1),RN(2),RN(3),RN(4)/
     1                   .7142005D0,.3466282D0,.843751D0,.1263305D0/
      DATA               ZERO,P1,HALF,ONE,TWO,FOUR,FOURP5,TEN,HUN
     1                   /0.0D0,0.1D0,0.5D0,1.0D0,2.0D0,4.0D0,
     2                   4.5D0,10.0D0,100.0D0/
C                                  FIRST EXECUTABLE STATEMENT
      IER = 0
      CADRE = ZERO
      ERROR = ZERO
      CUREST = ZERO
      CUREST = ZERO
      VINT = ZERO
      LENGTH = DABS(B-A)
      IF (LENGTH .EQ. ZERO) GO TO 215
      IF (RERR .GT. P1 .OR. RERR .LT. ZERO) GO TO 210
      HRERR = RERR+HUN
      IF (AERR .EQ. ZERO .AND. HRERR .LE. HUN) GO TO 210
      ERRR = RERR
      ERRA = DABS(AERR)
      STEPMN = LENGTH/(TWO**MXSTGE)
      STEPNM = DMAX1(LENGTH,DABS(A),DABS(B))*TEN
      STAGE = HALF
      ISTAGE = 1
      FNSIZE = ZERO
      PREVER = ZERO
      REGLAR = .FALSE.
C                                  THE GIVEN INTERVAL OF INTEGRATION
C                                    IS THE FIRST INTERVAL CONSIDERED.
      BEG = A
      FBEG = F(BEG)*HALF
      TS(1) = FBEG
      IBEG = 1
      EDN = B
      FEND = F(EDN)*HALF
      TS(2) = FEND
      IEND = 2
    5 RIGHT = .FALSE.
C                                  INVESTIGATION OF A PARTICULAR
C                                    SUBINTERVAL BEGINS AT THIS POINT.
   10 STEP = EDN - BEG
      ASTEP = DABS(STEP)
      IF (ASTEP .LT. STEPMN) GO TO 205
      HRERR = STEPNM+ASTEP
      IF (HRERR .EQ. STEPNM) GO TO 205
      T(1,1) = FBEG + FEND
      TABS = DABS(FBEG) + DABS(FEND)
```

```
      L = 1
      N = 1
      H2CONV = .FALSE.
      AITKEN = .FALSE.
   15 LM1 = L
      L = L + 1
C                                       CALCULATE THE NEXT TRAPEZOID SUM,
C                                         T(L,1), WHICH IS BASED ON *N2* + 1
C                                         EQUISPACED POINTS. HERE,
C                                         N2 = N*2 = 2**(L-1).
      N2 = N+N
      FN = N2
      ISTEP = (IEND - IBEG)/N
      IF (ISTEP .GT. 1) GO TO 25
      II = IEND
      IEND = IEND + N
      IF (IEND .GT. MAXTS) GO TO 200
      HOVN = STEP/FN
      III = IEND
      FI = ONE
      DO 20 I=1,N2,2
         TS(III) = TS(II)
         TS(III-1) = F(BEGIN - FI * HOVN)
         FI = FI+TWO
         III = III-2
         II = II-1
   20 CONTINUE
      ISTEP = 2
   25 ISTEP2 = IBEG + ISTEP/2
      SUM = ZERO
      SUMABS = ZERO
      DO 30 I=ISTEP2,IEND,ISTEP
         SUM = SUM + TS(I)
         SUMABS = SUMABS + DABS(TS(I))
   30 CONTINUE
      T(L,1) = T(L-1,1)*HALF+SUM/FN
      TABS = TABS*HALF+SUMABS/FN
      N = N2
C                                       GET PRELIMINARY VALUE FOR *VINT*
C                                         FROM LAST TRAPEZOID SUM AND UPDATE
C                                         THE ERROR REQUIREMENT *ERGOAL*
C                                         FOR THIS SUBINTERVAL.
      IT = 1
      VINT = STEP*T(L,1)
      TABTLM = TABS*TEN
      FNSIZE = DMAX1(FNSIZE,DABS(T(L,1)))
      ERGL = ASTEP*FNSIZE*TEN
      ERGOAL = STAGE*DMAX1(ERRA,ERRR*DABS(CUREST+VINT))
C                                       COMPLETE ROW L AND COLUMN L OF *T*
C                                         ARRAY.
      FEXTRP = ONE
      DO 35 I=1,LM1
         FEXTRP = FEXTRP*FOUR
         T(I,L) = T(L,I) - T(L-1,I)
         T(L,I+1) = T(L,I) + T(I,L)/(FEXTRP-ONE)
   35 CONTINUE
      ERRER = ASTEP*DABS(T(1,L))
C                                       PRELIMINARY DECISION PROCEDURE
C                                         IF L = 2 AND T(2,1) = T(1,1),
C                                         GO TO 135 TO FOLLOW UP THE
C                                         IMPRESSION THAT INTEGRAND IS
C                                         STRAIGHT LINE.
      IF (L .GT. 2) GO TO 40
      HRERR = TABS+P1*DABS(T(1,2))
      IF (HRERR .EQ. TABS) GO TO 135
      GO TO 15
C                                       CACULATE NEXT RATIOS FOR
C                                         COLUMNS 1,...,L-2 OF T-TABLE
C                                         RATIO IS SET TO ZERO IF DIFFERENCE
C                                         IN LAST TWO ENTRIES OF COLUMN IS
C                                         ABOUT ZERO
   40 DO 45 I=2,LM1
         DIFF = ZERO
         HRERR = TABTLM+DABS(T(I-1,L))
         IF (HRERR .NE. TABTLM) DIFF = T(I-1,LM1)/T(I-1,L)
         T(I-1,LM1) = DIFF
   45 CONTINUE
      IF (DABS(FOUR-T(1,LM1)) .LE. H2TOL) GO TO 60
      IF (T(1,LM1) .EQ. ZERO) GO TO 55
      IF (DABS(TWO-DABS(T(1,LM1))) .LT. JUMPTL) GO TO 130
      IF (L .EQ. 3) GO TO 15
      H2CONV = .FALSE.
      IF (DABS((T(1,LM1)-T(1,L-2))/T(1,LM1)) .LE. AITTOL) GO TO 75
   50 IF (REGLAR) GO TO 55
      IF (L .EQ. 4) GO TO 15
      HRERR = ERGL+ERRER
   55 IF (ERRER .GT. ERGOAL .AND. HRERR .NE. ERGL) GO TO 175
      GO TO 145
C                                       CAUTIOUS ROMBERG EXTRAPOLATION
   60 IF (H2CONV) GO TO 65
      AITKEN = .FALSE.
      H2CONV = .TRUE.
   65 FEXTRP = FOUR
   70 IT = IT + 1
      VINT = STEP*T(L,IT)
      ERRER = DABS(STEP/(FEXTRP-ONE)*T(IT-1,L))
      IF (ERRER .LE. ERGOAL) GO TO 160
      HRERR = ERGL+ERRER
      IF (HRERR .EQ. ERGL) GO TO 160
      IF (IT .EQ. LM1) GO TO 125
      IF (T(IT,LM1) .EQ. ZERO) GO TO 70
      IF (T(IT,LM1) .LE. FEXTRP) GO TO 125
      IF (DABS(T(IT,LM1)/FOUR-FEXTRP)/FEXTRP .LT. AITTOL)
     1       FEXTRP = FEXTRP*FOUR
      GO TO 70
C                                       INTEGRAND MAY HAVE X**ALPHA TYPE
C                                         SINGULARITY.
C                                         RESULTING IN A RATIO OF *SING*  =
C                                         2**(ALPHA + 1)
   75 IF (T(1,LM1) .LT. AITLOW) GO TO 175
```

```
      IF (AITKEN) GO TO 80
      H2CONV = .FALSE.
      AITKEN = .TRUE.
   80 FEXTRP = T(L-2,LM1)
      IF (FEXTRP .GT. FOUR65) GO TO 65
      IF (FEXTRP .LT. AITLOW) GO TO 175
      IF (DABS(FEXTRP-T(L-3,LM1))/T(1,LM1) .GT. H2TOL) GO TO 175
      SING = FEXTRP
      FEXTM1 = ONE/(FEXTRP - ONE)
      AIT(1) = ZERO
      DO 85 I=2,L
         AIT(I) = T(I,1) + (T(I,1)-T(I-1,1))*FEXTM1
         R(I) = T(1,I-1)
         DIF(I) = AIT(I) - AIT(I-1)
   85 CONTINUE
      IT = 2
   90 VINT = STEP*AIT(L)
      ERRER = ERRER*FEXTM1
      HRERR = ERGL+ERRER
      IF (ERRER .GT. ERGOAL .AND. HRERR .NE. ERGL) GO TO 95
      IER = MAX0(IER,65)
      GO TO 160
   95 IT = IT + 1
      IF (IT .EQ. LM1) GO TO 125
      IF (IT .GT. 3) GO TO 100
      H2NEXT = FOUR
      SINGNX = SING+SING
  100 IF (H2NEXT .LT. SINGNX) GO TO 105
      FEXTRP = SINGNX
      SINGNX = SINGNX+SINGNX
      GO TO 110
  105 FEXTRP = H2NEXT
      H2NEXT = FOUR*H2NEXT
  110 DO 115 I=IT,LM1
         R(I+1) = ZERO
         HRERR = TABTLM+DABS(DIF(I+1))
         IF (HRERR .NE. TABTLM) R(I+1) = DIF(I)/DIF(I+1)
  115 CONTINUE
      H2TFEX = -H2TOL*FEXTRP
      IF (R(L) - FEXTRP .LT. H2TFEX) GO TO 125
      IF (R(L-1)-FEXTRP .LT. H2TFEX) GO TO 125
      ERRER = ASTEP*DABS(DIF(L))
      FEXTM1 = ONE/(FEXTRP - ONE)
      DO 120 I=IT,L
         AIT(I) = AIT(I) + DIF(I)*FEXTM1
         DIF(I) = AIT(I) - AIT(I-1)
  120 CONTINUE
      GO TO 90
C                                 CURRENT TRAPEZOID SUM AND RESULTING
C                                   EXTRAPOLATED VALUES DID NOT GIVE
C                                   A SMALL ENOUGH *ERRER*.
C                                   NOTE -- HAVING PREVER .LT. ERRER
C                                   IS AN ALMOST CERTAIN SIGN OF
C                                   BEGINNING TROUBLE WITH IN THE FUNC-
C                                   TION VALUES. HENCE, A WATCH FOR,
C                                   AND CONTROL OF, NOISE SHOULD
C                                   BEGIN HERE.
  125 FEXTRP = DMAX1(PREVER/ERRER,AITLOW)
      PREVER = ERRER
      IF (L .LT. 5) GO TO 15
      IF (L-IT .GT. 2 .AND. ISTAGE .LT. MXSTGE) GO TO 170
      ERRET = ERRER/(FEXTRP**(MAXTBL-L))
      HRERR = ERGL+ERRET
      IF (ERRET .GT. ERGOAL .AND. HRERR .NE. ERGL) GO TO 170
      GO TO 15
C                                 INTEGRAND HAS JUMP (SEE NOTES)
  130 HRERR = ERGL+ERRER
      IF (ERRER .GT. ERGOAL .AND. HRERR .NE. ERGL) GO TO 170
C                                 NOTE THAT  2*FN = 2**L
      DIFF = DABS(T(1,L))*(FN+FN)
      GO TO 160
C                                 INTEGRAND IS STRAIGHT LINE
C                                   TEST THIS ASSUMPTION BY COMPARING
C                                   THE VALUE OF THE INTEGRAND AT
C                                   FOUR *RANDOMLY CHOSEN* POINTS WITH
C                                   THE VALUE OF THE STRAIGHT LINE
C                                   INTERPOLATING THE INTEGRAND AT THE
C                                   TWO END POINTS OF THE SUB-INTERVAL.
C                                   IF TEST IS PASSED, ACCEPT *VINT*
  135 SLOPE = (FEND-FBEG)*TWO
      FBEG2 = FBEG+FBEG
      DO 140 I=1,4
         DIFF = DABS(F(BEG+RN(I)*STEP) - FBEG2-RN(I)*SLOPE)
         HRERR = TABTLM+DIFF
         IF(HRERR .NE. TABTLM) GO TO 155
  140 CONTINUE
      GO TO 160
C                                 NOISE MAY BE DOMINANT FEATURE
C                                   ESTIMATE NOISE LEVEL BY COMPARING
C                                   THE VALUE OF THE INTEGRAND AT
C                                   FOUR *RANDOMLY CHOSEN* POINTS WITH
C                                   THE VALUE OF THE STRAIGHT LINE
C                                   INTERPOLATING THE INTEGRAND AT THE
C                                   TWO ENDPOINTS. IF SMALL ENOUGH,
C                                   ACCEPT *VINT*
  145 SLOPE = (FEND-FBEG)*TWO
      FBEG2 = FBEG+FBEG
      I = 1
  150 DIFF = DABS(F(BEG+RN(I)*STEP) - FBEG2-RN(I)*SLOPE)
  155 ERRER = DMAX1(ERRER,ASTEP*DIFF)
      HRERR = ERGL+ERRER
      IF (ERRER .GT. ERGOAL .AND. HRERR .NE. ERGL) GO TO 175
      I = I+1
      IF (I .LE. 4) GO TO 150
      IER = 66
C                                 INTEGRATION OVER CURRENT SUB-
C                                   INTERVAL SUCCESSFUL
C                                   ADD *VINT* TO *DCADRE* AND *ERRER*
C                                   TO *ERROR*, THEN SET UP NEXT SUB-
C                                   INTERVAL, IF ANY.
```

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  160 CADRE = CADRE + VINT
      ERROR = ERROR + ERRER
      IF (RIGHT) GO TO 165
      ISTAGE = ISTAGE - 1
      IF (ISTAGE .EQ. 0) GO TO 220
      REGLAR = REGLSV(ISTAGE)
      BEG = BEGIN(ISTAGE)
      EDN = FINIS(ISTAGE)
      CUREST = CUREST - EST(ISTAGE+1) + VINT
      IEND = IBEG - 1
      FEND = TS(IEND)
      IBEG = IBEGS(ISTAGE)
      GO TO 180
  165 CUREST = CUREST + VINT
      STAGE = STAGE+STAGE
      IEND = IBEG
      IBEG = IBEGS(ISTAGE)
      EDN = BEG
      BEG = BEGIN(ISTAGE)
      FEND = FBEG
      FBEG = TS(IBEG)
      GO TO 5
C                                    INTEGRATION OVER CURRENT SUBINTERVAL
C                                      IS UNSUCCESSFUL. MARK SUBINTERVAL
C                                      FOR FURTHER SUBDIVISION. SET UP
C                                      NEXT SUBINTERVAL.
  170 REGLAR = .TRUE.
  175 IF (ISTAGE .EQ. MXSTGE) GO TO 205
      IF (RIGHT) GO TO 185
      REGLSV(ISTAGE+1) = REGLAR
      BEGIN(ISTAGE) = BEG
      IBEGS(ISTAGE) = IBEG
      STAGE = STAGE*HALF
  180 RIGHT = .TRUE.
      BEG = (BEG+EDN)*HALF
      IBEG = (IBEG+IEND)/2
      TS(IBEG) = TS(IBEG)*HALF
      FBEG = TS(IBEG)
      GO TO 10
  185 NNLEFT = IBEG - IBEGS(ISTAGE)
      IF (IEND+NNLEFT .GE. MAXTS) GO TO 200
      III = IBEGS(ISTAGE)
      II = IEND
      DO 190 I=III,IBEG
         II = II + 1
         TS(II) = TS(I)
  190 CONTINUE
      DO 195 I=IBEG,II
         TS(III) = TS(I)
         III = III + 1
  195 CONTINUE
      IEND = IEND + 1
      IBEG = IEND - NNLEFT
      FEND = FBEG
      FBEG = TS(IBEG)
      FINIS(ISTAGE) = EDN
      EDN = BEG
      BEG = BEGIN(ISTAGE)
      BEGIN(ISTAGE) = EDN
      REGLSV(ISTAGE) = REGLAR
      ISTAGE = ISTAGE + 1
      REGLAR = REGLSV(ISTAGE)
      EST(ISTAGE) = VINT
      CUREST = CUREST + EST(ISTAGE)
      GO TO 5
C                                    FAILURE TO HANDLE GIVEN INTEGRA-
C                                      TION PROBLEM
  200 IER = 131
      GO TO 215
  205 IER = 132
      GO TO 215
  210 IER = 133
  215 CADRE = CUREST + VINT
  220 DCADRE = CADRE
 9000 CONTINUE
      IF (IER .NE. 0) CALL UERTST (IER,6HDCADRE)
 9005 RETURN
      END
$ENDLIST
```

VITA [2]

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