MODELING AND DESIGN OF ECONOMICALLY BASED DOUBLE SAMPLING PLANS

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in partial fulfillment of the requirements
for the Degree of
DOCTOR OF PHILOSOPHY
July, 1981

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BASED DOUBLE SAMPLING PLANS

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PREFACE

The problem addressed in this dissertation is that of determining the optimum economically based double sampling plan. This topic is not covered in any textbook on statistical quality control. The purpose of this research is to provide the modeling and optimization technology as well as a new and well-developed tool in selecting the most cost effective double acceptance sampling plan.

The modified Guthrie-Johns model, including fixed costs, is developed. The methodology and an interactive computer program are developed to select the optimum double sample size pair and corresponding acceptance/rejection number vector which provide the minimum total expected cost. The model sensitivities are presented to determine relative economic advantages.

My graduate work was made possible by the financial support of the government of the Republic of China for three and a half years. I am grateful for the encouragement and support of the government.

I would like to express sincere appreciation to my major advisor, Professor Kenneth E. Case, whose timely advice and encouragement has always been welcome. His guidance during this research has been very helpful and is reflected in many places throughout this dissertation. Dr. Case has been a truly outstanding teacher and advisor. I would like to thank Professor Joe H. Mize for his inspiration and advice during my graduate study. It has been an invaluable experience to associate with such an outstanding individual as Dr. Mize. I would also like to express my gratitude to Dr. Lyle D. Broemeling, Dr. Carl B. Estes, and Dr. M. Palmer Terrell who have contributed advice, counsel, and encouragement throughout my graduate study. It has been a pleasure and privilege to have worked under such an outstanding doctoral committee.

Special credit is due Mrs. Margaret Estes for her excellent typing and advice in the preparation of this study. Mr. Eldon Hardy deserves recognition for his outstanding work in drawing the numerous figures and graphs herein.

I would like to thank my parents, Mr. and Mrs. C. C. Chen, for the inspiration and encouragement they have given.

I would also like to express appreciation to my wife, Karen, and son, Lee-Jen, for the many sacrifices they have made. I hope my effort in graduate school has been worthy of my family's effort.

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CHAPTER I

THE RESEARCH PROBLEM

Purpose

One of the most important aspects of quality control is acceptance sampling. Traditional acceptance sampling plans make accept/reject decisions based upon calculable statistical risks instead of cost considerations. Sometimes, the plans are economically good; however, at other times they are very costly. In order to obtain plans capable of low total expected costs, economically based double acceptance sampling by attributes plans are studied herein. The relevant economic model employs Bayesian decision theory.

The basic model to be used is that of Guthrie and Johns which includes the cost of sampling, lot acceptance, and lot rejection. Fixed cost factors are added in order to provide a more realistic model. Newly developed procedures for model optimization are required in order to select the appropriate sample sizes and acceptance and rejection numbers which provide the minimum total expected cost. An interactive computer program is developed, suitable for use by practitioners with a minimum of technical background. The double sampling model is then investigated in comparison to a comparable single sampling model in order to determine the relative economic advantage of double sampling.

Introduction

General

The selection of appropriate acceptance sampling plans is one of the most important jobs of the quality control engineer. Acceptance sampling is used to make accept/reject decisions on incoming parts, in-process items, and finished goods. Its purpose is to determine a course of action, not to estimate or control lot quality. It is specifically for the purpose of sentencing lots to either acceptance or rejection.

Inspection by attributes is inspection whereby either the unit of product is classified simply as defective or nondefective, or the number of defects in the unit of product is counted, with respect to a given requirement or set of requirements. There are several types of attributes sampling plans for lot-by-lot inspection. They include single-sampling, double-sampling, multiple sampling, and sequential sampling plans.

The most commonly used plans in industry are single-sampling and double-sampling. Double-sampling plans are known to have some advantages and some disadvantages with respect to single-sampling plans. One relatively unknown area of comparison is in regard to the degree of economic advantage achieved by double-sampling. A thorough Bayesian economic model for single-sampling now exists. The following double-sampling effort not only advances the leading edge of economically based sampling, but permits a valid assessment of the economic comparison between double- and single-sampling.

Single-Sampling

The single-sample fraction-defective sampling plan is very simple. It calls for a decision on the basis of evidence from one sample taken from a lot. It specifies the sample size (n) of items that should be taken randomly from the lot. If the number of defective items (x) in the sample is less than, or equal to the acceptance number (c), the lot is accepted; otherwise, it is rejected.

The single-sampling decision criterion is shown as follows:

Sample n: If $x \leq c$, accept

If $x > c$, reject

A flow chart of the procedures is presented in Figure I.1.

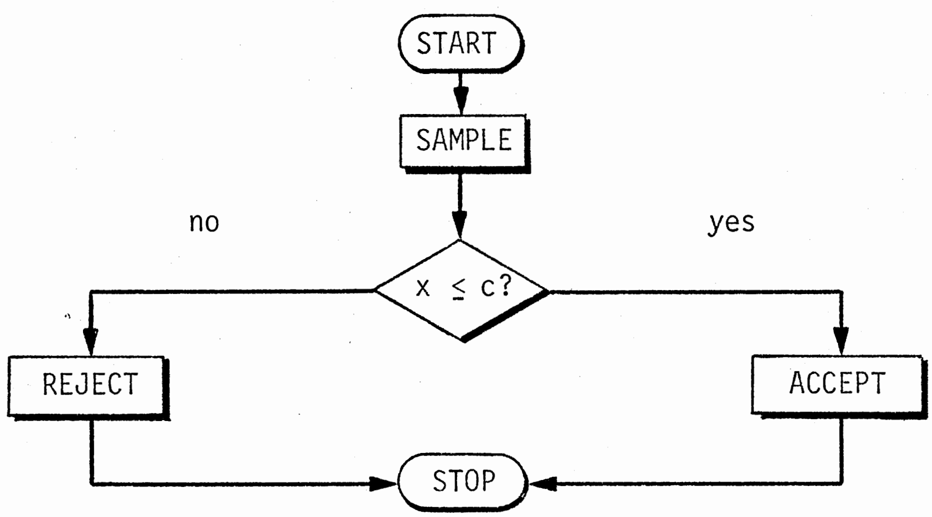


Figure I.1. Flow Chart of Single-Sampling

Double-Sampling

Double-sampling plans involve four possibilities. Acceptance or rejection may take place immediately following observation of the first sample. Alternatively, the decision may be deferred to where acceptance or rejection take place following the second sample.

The plan is designated by six numbers (n_1, n_2, c_1, r_1, c_2, and r_2), c_1 being less than r_1 and c_2+1 being equal to r_2. A sample of size n_1 items is taken from a given lot. If the number of defective items in the sample is less than or equal to the first acceptance number c_1, the lot is accepted. If the number of defective items in the sample equals or exceeds r_1, the lot is rejected. However, if the number of defective units is greater than c_1 but less than r_1, a second sample of size n_2 is taken from the remainder of the lot. If the number of defectives in the combined samples does not exceed the second acceptance number c_2, the lot is accepted. If there are more than c_2 defectives, the lot is rejected.

The double-sampling decision criteria are shown as follows:

Sample n_1: If $x_1 \leq c_1$, accept

If $x_1 \geq r_1$, reject

If $c_1 < x_1 < r_1$, take second sample.

Sample n_2: If $x_1 + x_2 \leq c_3$, accept

If $x_1 + x_2 > c_3$, reject

A flow chart of the procedures is presented in Figure I.2.

A double-sampling plan has some possible advantages over a single-sampling plan. First, it may reduce the total amount of inspection. Second, a double-sampling plan provides the psychological advantage of giving a lot a second chance. This advantage is, of course, purely psychological. It also provides a lower total expected cost of operation. The primary disadvantage of double-sampling is the difficulty with which it is administered in an actual inspection operation.

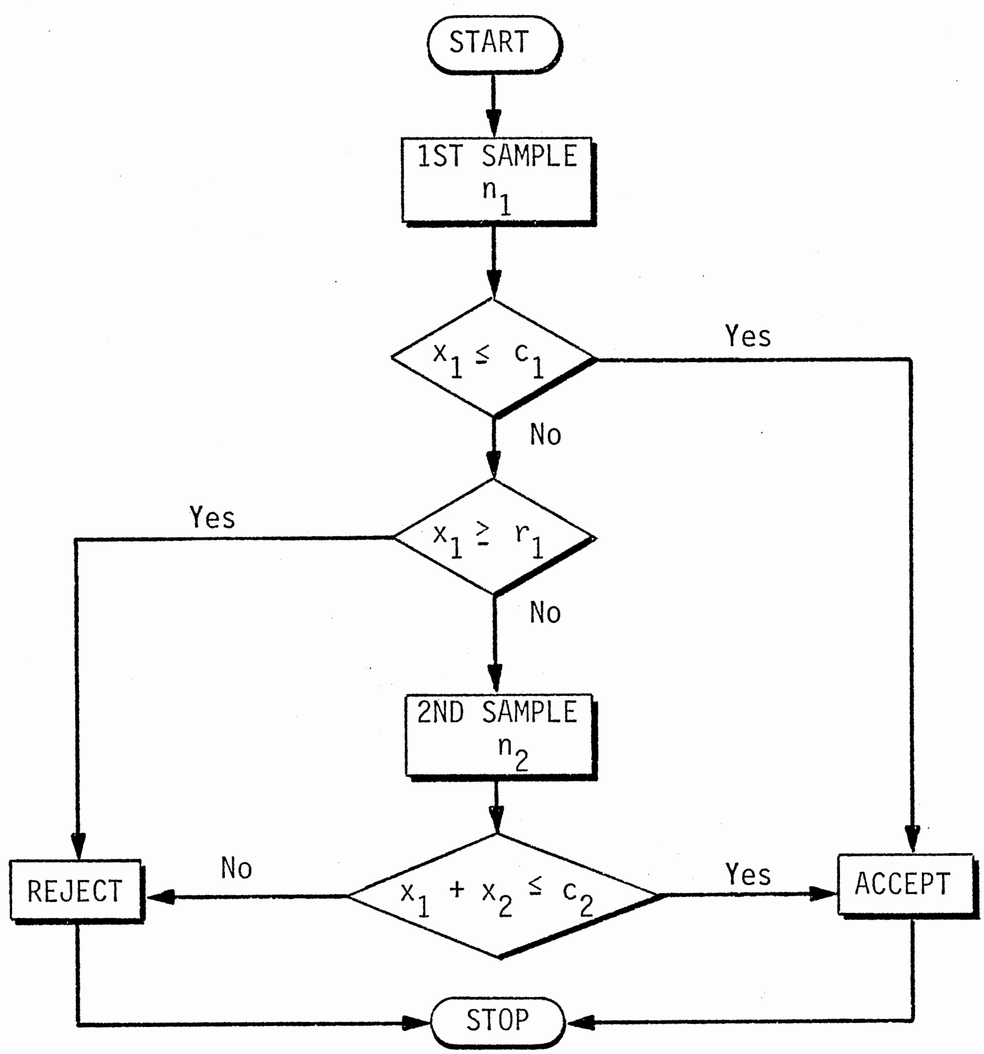


Figure I.2.. Flow Chart of Double-Sampling

Risks Associated With Acceptance Sampling

The classical risk-based sampling plan may be determined once certain criteria (e.g., AQL, LTPD, AOQ) have been satisfied. The most popular approach for designing a sampling plan is the "2-point design." That is, a producer's risk (α) is identified with a "good" fraction defective (p_1). A consumer's risk (β) is identified with a "poor" fraction defective (p_2). Those risks lead to a desired high probability of lot acceptance $1-\alpha$ when the lot has been formed from a process having a good fraction defective p_1. Also, the desired low probability of lot acceptance is β when the lot has been formed from a process having poor fraction defective p_2.

Put differently, a good sampling plan is one which provides a small producer's risk that lots of good quality will be rejected. Likewise, it provides a small consumer's risk that lots of poor quality will be accepted.

The classified methods are usually determined based upon a mental assessment of the risks inherent due to sampling. Unfortunately, it is very difficult to accurately mentally assess these risks and costs and arrive at a defensible set of criteria by which to determine an attributes acceptance sampling plan. Sometimes, the resulting risk-based plans are very costly due to either over- or under-inspection of lots.

Costs Associated With Acceptance Sampling

Generally speaking, the costs associated with acceptance sampling can be classified as (1) costs due to sampling and inspection, (2) costs due to rejecting good items, and (3) costs due to accepting bad items. These costs include variable and fixed cost components. The cost of sampling is dependent upon the number of items inspected, and the manpower required. The cost of rejecting good items consists of all monies lost as a result of the decision to reject a lot such as costs of sorting, repairing, and reinspecting. The cost of accepting and passing on bad items is the most important of all and includes costly handling, rework, repair, and paperwork processing. The details of cost items are discussed in the following section.

Economically Based Acceptance Sampling

Modeling

In an attempt to truly optimize the sampling effort and resultant risks for double-sampling plans, a stochastic mathematical economically-based acceptance sampling model is derived. The well known Guthrie-Johns model for single-sampling is redeveloped for double-sampling. It is also modified to contain nine cost elements rather than six, including three components each associated with the cost of sampling, lot acceptance, and lot rejection. The model is Bayesian in nature, requiring a "prior" distribution to express the user's pre-sampling beliefs about the quality of the lots, based either upon past data, personal feeling, or both.

The model describes the total expected cost per lot according to the decision criteria for double-sampling discussed in the previous section. In particular, it accounts for the cost of sampling, inspection, and rework of any defectives found therein. It also considers the downstream adverse effects of defective items which have either escaped in accepted lots or have been incorrectly classified as good. Finally, it allows for the cost of screening rejected lots and reworking any defective items found.

The model is a function of several variables. They include lot size (N), first and second sample sizes (n_1 and n_2), acceptance and rejection numbers for the first and second samples (c_1, r_1, c_2, r_2), number of defectives in the lot (X), and number of defectives in the first and second samples (x_1 and x_2). Often, the variable r_2 is omitted, because always in double sampling, $r_2 = c_2 + 1$. The variables n_1, n_2, c_1, r_1, c_2, and r_2 are decision variables under control of the user. The variables X, x_1, and x_2 are random variables over which the user has no control. The variable of lot size may or may not be under the user's control. It is assumed fixed in this research.

The model is reduced to the point that it is a function only of decision variables by taking the expectation over X, x_1, and x_2. Assuming that the lot size is fixed, total cost is a function of only the decision variables, $TC(n_1, n_2, c_1, r_1, c_2)$. Newly developed exact analytical and search procedures are developed for the model to be optimized by selecting the appropriate decision variables which provide a minimum total expected cost. The required developments are described subsequently.

Cost Elements

The Guthrie-Johns model, proposed in 1959, contains six cost elements, including two each concerned with the cost of sampling, lot acceptance, and lot rejection, has been referenced and used by numerous authors. It is a versatile Bayesian economically-based attributes acceptance sampling model for single-sampling. Unfortunately, it omits factors for fixed costs which do not vary in proportion to the quantity of sampled items or resultant defectives. For 20 years, no comments or modifications to include a complete set of fixed costs have been published in the open literature.

The Guthrie-Johns model, in addition to being modified for double-sampling, will contain not only the original six cost elements, but also three fixed factors for each cost. The cost elements are described as follows:

1. S_0 = fixed cost of sampling, inspection, and testing per lot. This includes lot handling, print review, inspection setup, incremental first item inspection, and any other cost per lot for sampling, inspection, and testing, regardless of the number of items to be considered.
2. S_1 = cost per item of sampling, inspecting, and testing. This includes manpower, overhead, inspection tool wear, materials used, and any other costs incurred during inspection and/or test.
3. S_2 = additional cost per defective item found during sampling, inspection, and testing. This includes rework/repair manpower, overhead, and materials. It also includes additional record keeping, reinspection, and related handling. Any extra expenditures per item due to the fact that the item was found defective during sampling are accounted for here.
4. A_0 = fixed cost of accepting a lot containing one or more defective items, when that lot is identified as defective downstream. This includes writing a reject tag, engineering fix, manufacturing corrective action writeup,

segregation, stores checking and reinspecting, etc. This is usually a substantial cost which should not be ignored.

5. A_1 = cost per item of the N-n items not inspected in an accepted lot. These items are considered the "norm." If good, they will go on to earn a profit for the company which is "expected." As such, this cost is usually taken as zero. If this portion of the lot requires "special handling," for example, A_1 may be greater than zero.

6. A_2 = additional cost per defective item later discovered in an accepted lot. This includes rework/repair manpower, overhead and materials. It includes damage, dismantling, lost goodwill, and work stoppage costs downstream. Also involved are reject tag processing costs, fix approval, reinspection and related handling. Any extra expenditures per item due to the fact that the item was found defective after having been accepted are accounted for here. This cost can be quite high.

7. R_0 = fixed cost of rejection per lot rejected on original inspection. This includes writing a rework, repair, or reject tag, handling of the rejected lot, and any other cost assessed per lot for a lot found defective and rejected in its own shop.

8. R_1 = cost per item of the N-n items in the rest of a rejected lot. This will normally be the cost per item of inspection and testing. This includes manpower, overhead,

inspection, tool wear, materials used, and any other costs incurred in treating a rejected lot. This cost is often less than or equal to S_1.

9. R_2 = additional cost per defective item found while inspecting and testing the rest of a rejected lot. This includes rework/repair manpower, overhead, and materials. It also includes additional record keeping, reinspection, and related handling. Any extra expenditures per item due to the fact that the item was found defective while inspecting and testing the rest of a rejected lot are accounted for here. This cost is often equal to S_2.

Bayesian Distributional Considerations

The methods utilized in this research are based upon Bayesian decision theory. Historical data and/or beliefs are used to predict the quality of a lot before it is observed. Then, the lot quality history is combined quantitatively with actual sample results to form an opinion about the lot after sampling. Based upon this latter opinion, the lot is either accepted or rejected.

The Bayesian approach to statistical inference is based upon a theorem first presented by Thomas Bayes (1702-1761). Bayes' basic theorem was later modified by Laplace, and this modified version is used today and is commonly referred to as Bayes' theorem.

In order to demonstrate the development of this theorem, the intersection probability of two events A and B is described as:

$$P(AB) = P(A)\ P(B|A) = P(B)\ P(A|B).$$

From this, conditional probability relations such as the following may be stated:

$$P(A|B) = P(A)\,\frac{P(B|A)}{P(B)}\ .$$

Here, P(A) is the prior probability of event A before the information about event B becomes available, and P(A|B) is the posterior probability of event A based upon the results of event B. This is similar to the version of Bayes' theorem used in this research.

The decision variables X, x_1, and x_2 represent the number of defectives in the lot and the number of defectives in the first and second samples, as discussed earlier. Considering only the first sample, the joint distribution of X and x_1 are the four probability distributions described previously may be expressed as follows:

$$J(X - x_1, x_1) = f_N(X)\ \ell_{n_1}(x_1|X) = g_{n_1}(x_1)\ h_{N-n_1}(X - x_1|x_1)$$

or

$$\begin{matrix}\text{Joint} \\ \text{Distribution}\end{matrix} = \begin{matrix}\text{Prior} \\ \text{Distribution}\end{matrix} \times \begin{matrix}\text{Sampling} \\ \text{Distribution}\end{matrix} = \begin{matrix}\text{Marginal} \\ \text{Distribution}\end{matrix} \times \begin{matrix}\text{Posterior} \\ \text{Distribution}\end{matrix}$$

The four non-joint distributions can be defined as follows:

Prior distribution $f_N(X)$ - This distribution represents the decision maker's beliefs, prior to sampling, concerning the probability of X defectives occurring in a lot of size N.

The prior distribution must be specified in advance to describe the user's beliefs prior to sampling about the quality of the lot. These beliefs may be based upon past data or "feel."

In the quality control job, when product items are grouped in batches of finite size prior to acceptance sampling, it is obvious that the lot fraction defective on each attribute must be discrete. As the lot size increases, the number of possible lot fractions defective increases. Often, a continuous density function is used to express the prior distribution. But a continuous distribution is only an approximation to the exact discrete distribution. The desire to use discrete prior distributions is more important with smaller lot sizes, due to the significantly poorer approximation capability of continuous distributions.

One of the most important discrete prior distributions is the mixed binomial mass function. It is a realistic and applicable prior distribution which represents the situation when many vendors supply incoming parts, with each vendor furnishing a proportion produced at each process fraction defective. Similarly, it may be used to describe product coming from different machine/material/operator sources when each is operating at a different process fraction defective.

Sampling distribution $\ell_{n_1}(x_1|X)$ - This distribution gives the probability of observing x_1 defectives in a random first sample of size n_1, given that there are X defectives in the lot. The appropriate distribution here is the hypergeometric.

Marginal distribution $g_{n_1}(x_1)$ - This distribution gives the unconditional probability of observing x_1 defectives in a random first sample of size n_1, taken from the lot.

Posterior distribution $h_{N-n_1}(X-x_1|x_1)$ - This distribution gives the probability of having X defectives in a lot of size N given that x_1 defectives were observed in a random first sample of size n_1 taken from the lot.

Design

The double-sampling plan and its total expected cost model had been discussed. It is a function of decision variables n_1, n_2, c_1, r_1, c_2, and r_2, as well as random variables X, x_1, x_2. It has been noted that four possible decision profiles can be formed with acceptance or rejection coming on either the first or second sample. These four possibilities are reflected in the mathematical model.

The cost function can be expressed in the following form:

$$TC\ (N, n_1, n_2, c_1, r_1, c_2, r_2, X, x_1, x_2) =$$

Case 1: Accept the lot after taking the first sample

$$= TCA_1\ (N, n_1, c_1, X, x_1) \qquad \text{if } x_1 \le c_1$$

Case 2: Reject the lot after taking the first sample

$$= TCR_1\ (N, n_1, r_1, X, x_1) \qquad \text{if } x_1 \ge r_1$$

Case 3: Accept the lot after taking the second sample

$$= TCA_2\ (N, n_1, n_2, c_1, r_1, r_2, X, x_1, x_2)$$

$$\text{if } c_1 < x_1 < r_1 \text{ and } x_1 + x_2 \le c_2$$

Case 4: Reject the lot after taking the second sample

$$= TCR_2\ (N, n_1, n_2, c_1, r_1, r_2, X, x_1, x_2)$$

$$\text{if } c_1 < x_1 < r_1 \text{ and}$$

$$x_1 + x_2 \ge r_2 = c_2 + 1$$

Since X, x_1, and x_2 are random variables, it is necessary to take the expectation over them in order to obtain a function of decision variables (n_1, n_2, c_1, r_1, c_2, r_2). These six decision variables are reduced to five by recognizing that $r_2 = c_2 + 1$. Selection of values for the remaining five unknowns to minimize total expected costs is required.

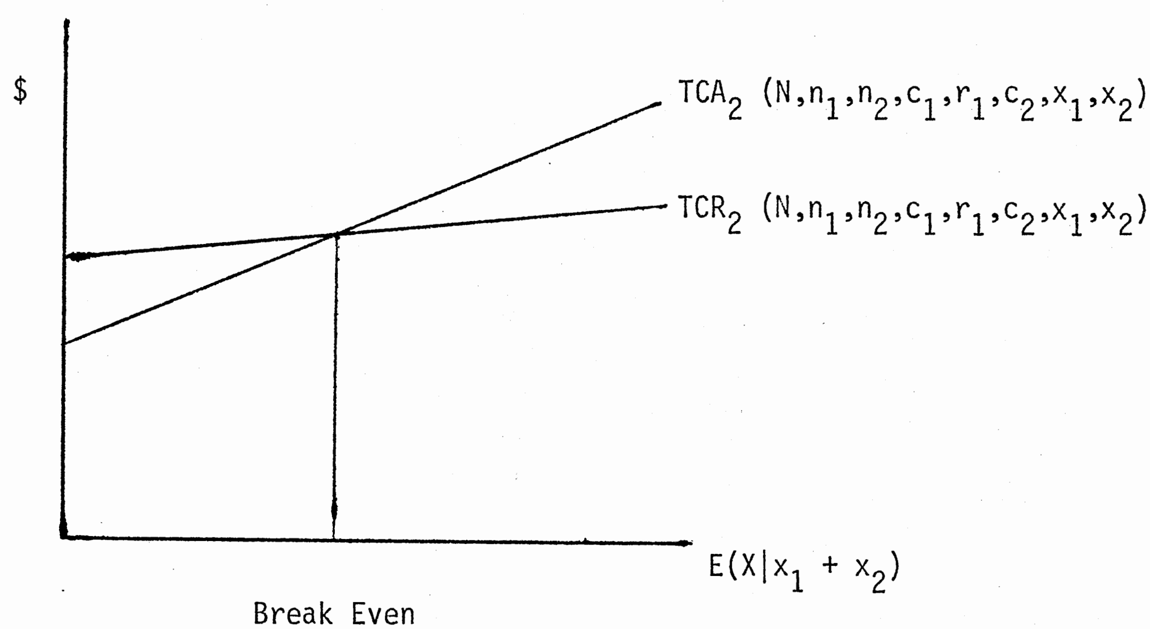
It is possible to equate TCA_2 (N, n_1, n_2, c_1, r_1, c_2, x_1, x_2) to TCR_2 (N, n_1, n_2, c_1, r_1, c_2, x_1, x_2) following the expectation of these cost formulas over X. This results in the ability to determine at what posterior value of X the two costs are equal, or at break even. From this, it is possible to determine the largest number of sampled defectives, $x_1 + x_2$, for which the least cost choice is to accept the lot. Such a value will be known as c_2. This approach is shown in Figure I.3.

The methodology outlined for determining c_2 is also applied to the selection of c_1 and r_1. This task is much more difficult; however, it utilizes the breakeven principle to select c_1 and r_1. Only n_1 and n_2 then remain to be determined. In practice, a fixed relationship often dictates $n_2 = n_1$ or $n_2 = 2n_1$, and thus a univariate search is used to select values which optimize total expected cost.

Research Objectives

Based upon the above discussions, the overall objective of this research can be stated.

OVERALL OBJECTIVE: To provide industry and government with a new and well-developed tool to assist in selecting the cost effective double acceptance sampling plan for a wide range of realistic situations.



Conditional Number of Defectives in Lot

Figure I.3. Acceptance and Rejection Costs as a Function of the Posterior Expectation of X

In order to accomplish this objective, several specific sub-objectives must be included as follows.

SUBOBJECTIVES:

(1) Development of the Guthrie-Johns model for use in double-sampling.

(2) Modification of the Guthrie-Johns model to include fixed cost components for sampling, rejection, and acceptance.

(3) Development of the theoretically exact analytical and search procedures for optimizing double-sampling plans using a discrete mathematical model with the fixed cost expansion.

(4) Development of an interactive computer program for double-sampling in a format suitable for use by industry and government.

(5) Comparison of optimum single and double-sampling plan total expected costs in order to determine the relative economic advantage of double-sampling.

Summary

The successful completion of this research provides benefits to both the theoretician and the practitioner in industry and government. Theoretically, the accomplishment of the objectives of this study fills several voids that now exist in the theory of economically based acceptance sampling for double-sampling plans. Many concepts involved are not presented in any textbooks on statistical quality control, but are of considerable and growing interest in the quality control area.

The practitioner will benefit from this research because it provides sound procedures for evaluating alternative sampling strategies.

Improved decision making capabilities will result from having the methodology to compare single-sampling vs. double-sampling, various first and second sample size relationships, and the sensitivity of total expected costs to economic components, distributional parameters, etc. The net result should be increased profitability through quality control.

CHAPTER II

LITERATURE REVIEW

Introduction

This chapter reviews developments in the literature relevant to the objectives of this research. General support for the research effort has been documented in Chapter I. This chapter elaborates on this support. In addition to economically based double-sampling work, other sources which communicate concepts relating to the objectives of this study will be presented.

This chapter is divided into three areas. These are:

(1) Attributes sampling plan design methodologies.

(2) Early origins of economically based acceptance sampling.

(3) Development of economically based acceptance sampling.

Attributes Sampling Plan Design Methodologies

Statistical quality control was introduced by Shewhart [65, 66, 67] in the 1920's and 1930's. These concepts and techniques have spread throughout the world, and Duncan [25] indicates that almost all industrialized nations use statistical quality control. Case [12] points out that quality control can be used by both large and small manufacturers. Perhaps the most widely used statistical quality control area is acceptance sampling. While traditional sampling plans have been based upon statistical risks, considerable effort and emphasis is being placed upon economically based sampling. Evidence of the widespread research of acceptance sampling schemes with emphasis on the economic aspect is given by a bibliography, contained in Wetherill and Chiu's [79] recent paper, which contains 246 references on this field. Their work indicates the most widely used acceptance sampling technique is attribute sampling.

Both single and double-sampling plans for statistically based acceptance sampling (SBAS) and economically based acceptance sampling (EBAS) have been discussed in Chapter I. According to Chen [17], due to (1) high precision technology, (2) multinational company organization and expenditures, (3) new management philosophies introduced, and (4) the energy crisis, all industries are facing an era of challenge with high competition. Sound ways to succeed against this challenge are to: (1) improve product quality, (2) reduce the cost of goods, and (3) get more efficient management. So, Case [13] predicts that during the 1980's, a fundamental change will be made by government and industry in the philosophy and design of attributes acceptance sampling. Statistically based sampling schemes, using techniques held sacred for 50 years, may be replaced and will surely be supplemented by economically based philosophies.

Case and Keats [15] indicate that attributes acceptance sampling plans may be categorized as in Figure II.1. This figure shows the four distinct breakdowns of sampling plan design methodology as published in the literature.

Category 1 describes the traditional approach to sampling plan design. It draws upon producer and consumer risks as depicted by the

	Risk-Based	Economically-Based
Non-Bayesian	1	2
Bayesian	3	4

Figure II.1. Classification of Attributes Sampling Plan Design Methodologies operating characteristic (OC) curve. MIL-STD-105D [51], ISO 2859 [48], the Dodge-Romig tables [24], much of Hald's work, and many other contributions belong to this area. The vast majority of practitioners today are applying Category 1 plans because they are widely available, widely accepted, and relatively easy to use.

Category 2 focuses upon the economic aspects of sampling and the literature through Breakwell [5], Brown et al. [6], Martin [54], Truscott [72], and van der Waerden [75]. This approach aims at minimizing costs or regrets without a prior knowledge of the process fraction defective. Usually, minimax principles are used here to choose the sampling plan. However, its acceptance in industry has been relatively limited. None of the sampling plans of Categories 1 or 2 require that the distribution of defectives from lot to lot be known.

During the past ten years there has been a dramatic increase in the number of papers using the Bayesian approach to sampling plan design. Plans using a Bayesian approach fall into Categories 3 and 4. Bayesian sampling plans require the user to explicitly specify the distribution of defectives from lot to lot. This distribution is known as the prior distribution. It expresses the user's pre-sampling beliefs about the quality of the lot, based either upon past data, personal feeling, or both. Decisions to accept or reject the lot are then based on a posterior distribution which combines the user's prior knowledge of lot to lot variation with the sampling inspection results.

In Category 3, the producer's and consumer's risks are associated with Bayes' theorem and are used for determining the sampling plan. The prior distributions are needed for decision-making, but the costs are not explicitly considered as are the statistical risks.

While there is limited work in Category 3 [Lauer [53], Moreno [57, 58, 59], Schafer [62], Hald [35, 36]], there are numerous published works in Category 4. These are discussed in detail in the following section. Also, recently, several large companies have begun implementing the economically based Bayesian plans of Category 4 on some product lines.

Early Origins of Economically-Based Acceptance Sampling

Among the early contributions relevant to Bayesian economically based acceptance sampling, one of the most important is by Mood [56], who states that "Sampling of lots drawn from a binomial population will provide no basis whatsoever for inferences concerning the remainder of the lot." The binomial population to which Mood refers is one in which the population fraction defective is constant. This implies that the number of defectives occurring from one lot to the next is independent and binomially distributed. It is most startling to discover that whenever this assumption is valid, then the sample obtained from any lot provides no information whatsoever about the quality of the unsampled portion of the lot. Barnard [1] states differently, if a process is in a perfect state of statistical control for fraction defective, it makes no sense to perform acceptance sampling on lots formed as a sequence of Bernoulli trials from the process.

Several studies of prior distributions applicable to economically based acceptance sampling were published in the early 1950's. The most well known studies are those of Sittig [68], Champernowne [16], Barnard [1], Horsell [47], Taylor [71], and Hamaker [43]. Sittig presents the power prior distribution

$$
\begin{aligned}
f(p) &= 1 - A, \quad p = 0 \\
&= A\,(s + 1)\,(1 - p)^s, \quad p > 0
\end{aligned}
$$

in his paper. Hamaker discusses various expressions and derives the optimum sample size using the minimax principle. These results may be successful in isolated cases, but do not lead to simple principles with a wide field of application as needed in industry.

In the later 1950's, Vagholkar and Wetherill present the applications of decision theory to sampling schemes in theses. Vagholkar [73] studies a two-ordinate process curve with a two component mixed binomial distribution for acceptance sampling problems. He also collaborates with Wetherill [74] on a binomial prior distribution in the Bayesian version of the sequential probability ratio test. Wetherill [76] investigates the mixed binomial prior distribution with more than two components (a_i, p_i), $(i = 1, 2, \ldots, k)$, $\sum_{i=1}^{k} a_i = 1$ which provides a method of obtaining a single-sampling scheme with minimum risk for the particular model. This gives a simple relationship between n and c. The optimum n is found by directly minimizing expected costs.

Development of Economically-Based Acceptance Sampling

Following the early origins of economically based acceptance sampling, more systematic treatments were forthcoming in this area of research. Guthrie and Johns [32] develop the theory for a versatile economic cost model for attributes sampling plan in their paper of 1959. Also, sampling tables which minimize the average costs for various prior distributions are derived by Hald [33] in his paper of 1960. Since then, the economic design of quality control models has been receiving much attention in the literature.

Guthrie and Johns [32] propose a general linear cost model of the decision procedures and sample sizes which are optimal in the Bayes sense. They proceed to find explicit asymptotic characterizations for large batch sizes. Their model contains six cost elements, including two each associated with the cost of sampling, acceptance, and rejection. Similarly, Suzuki [70] considers and introduces Bayesian procedures into an inspection scheme with a beta prior distribution.

Hald [33] discusses single-sampling inspection plans in detail. His classic paper consists of two main parts. One studies the general theorem for the compound hypergeometric distribution and reproducibility. Properties of this distribution associated with rectangular, Polya, and mixed binomial prior distributions are investigated. The other part gives a general solution for determining the optimum sampling plan, i.e., his paper, presented in 1960, provides a theoretical and systematic foundation for research in this field.

A series of papers is published by Hald dating from 1960 to 1970 [34, 35, 36, 37, 38]. Two papers from 1967 on single-sampling plans based on the producer's and consumer's risk belong to Category 3 [35, 36]. In another 1967 paper, Hald proposes a twice differentiable prior distribution in an open interval about the break-even point, a general loss function, an operating characteristic written as an Edgeworth expansion, and sampling costs expressible as a polynomial in the sample size. This is a special case of asymptotic expressions for the Bayesian single-sampling plan [37].

In another two papers, Hald [39, 40] sets up a model based on a differentiable prior distribution, a linear loss function, an asymptotically normal sampling distribution and sampling costs proportional to the sample size. Asymptotic expressions are derived for sample sizes, acceptance and rejection criteria and minimum regret by minimizing the average regret for the sampling and decision procedure. The results for single, double, and multiple sampling plans are presented. He obtains a very interesting result in double sampling plans that the first sample should be proportional to $\ell n\ N$ and the second sample should be proportional to $\sqrt{N}$.

Pfanzagl and Shuler [61] make a model of acceptance inspection by which an objective comparison is made of sequential sampling plans in terms of costs. Pfanzagl [60], in another paper, suggests a double-sampling scheme where the second sample size (n_2) can depend on the outcome (x_1) of the first sample. The reason that one would prefer to choose the size of n_2 in advance is to ease administration of the sampling scheme.

Johansen [49] discusses asymptotic properties of the restricted Bayesian double-sampling plan. The lot size, cost function, and mixed binomial prior distribution are given. The optimal double-sampling plan is defined as the plan which minimizes the asymptotic expansion of the regret function between the five parameters: two sample sizes, two acceptance numbers, and one rejection number where the lot size approaches infinity. He indicates that the exact solution for double-sampling plan is very complicated. This is the reason why most authors study this problem by considering asymptotic behavior.

One of the most important mathematical acceptance sampling models is presented by Smith [69] in which he combines the basic concepts of the Guthrie-Johns and Hlad papers. He describes the total cost function by six elements--two each for inspection, acceptance, and rejection. He then takes the expectation over the number of defectives in both the lot and sample. Finally, the asymptotic formula is used to determine the approximate values for single-sampling plans. His work provides a long step toward rational economic decision making in sampling inspection. Similar modeling techniques are applied in another paper by Wortham and Wilson [82]. They apply a backward recursive technique (dynamic programming) for designing optimal sequential sampling plans. This method is based upon Bellman's principle of optimality and the Markovian property of sequential sampling plans.

Guenther [30] considers the degenerate, the beta, and the two-point distributions as prior distributions in the determination of single-sampling attribute plans based upon a linear cost model. He modifies Hald's work with these different prior distributions. Barnett [2] discusses the relationships of Bayesian decision theoretic methods applied to industrial problems in 1973. After that he proposes [3] a particular cost structure but no prior information. He uses the break-even quality for the loss function to choose the sample size and acceptance number which is economically most desirable for the batch. He also discusses the Bayesian solution when no process information is available. At the same time, Chiu [18] points out a new prior distribution other than the beta. Sampling tables are constructed using a model of a normally distributed quality characteristic, whose mean has a normal prior distribution. Asymptotic single attributes sampling plans using this new prior distribution are studied.

Schmidt and Bennett [63] and Case et al. [11] develop a mathematical model for economic multiattribute acceptance sampling. These papers develop and analyze models for which the cost components are influenced by a lot acceptance/rejection decision based upon the simultaneous assessment of several distinct and independent attributes. Each attribute is assumed to have its own sampling plan consisting of a sample size and an acceptance number (n_i, c_i, i = 1, 2, ..., m). Any item inspected on one attribute may be inspected on all other attributes, thus the total number of items sampled is max $\{n_1, n_2, \ldots, n_m\}$. The lot is accepted only if $x_1 \leq c_i$; i = 1, 2, ..., m. The first paper utilizes continuous density functions to approximate the number of defective items of each attribute in a lot. The second utilizes discrete prior mass functions to describe the system. Search techniques and sensitivity measures are investigated in those papers.

As the lot size becomes larger, the number of possible lot fractions defective on each attribute will be large, and it becomes reasonable to utilize continuous density functions to approximate the discrete system. But, when product items are grouped in batches of relatively small size prior to acceptance sampling, it is obvious that the lot fraction defective on each attribute must be discrete.

Case [11] concludes that, for large lot sizes, either continuous or discrete models may be used to determine the optimal sampling plan or as a predictor of total expected cost. Even for small lot sizes, the continuous approximate model is satisfactory to determine the optimal sampling plan. As a predictor of total cost, however, the deviation is quite sharp at low values of the lot size ($N \leq 20$).

Stewart et al. [64] presents an approximate model for the optimum economic design of double-sampling plans for attributes in 1979. Four decision variables (n_1, c_1, n_2, c_2) are used instead of five decision variables (n_1, c_1, r_1, n_2, c_2) in determining the minimum total cost. Total cost is assumed to consist of the cost of sampling, the cost of accepting defectives, and the cost of rejecting good items. Fixed costs of sampling inspection only are considered. The prior distribution of the process fraction defective used in this study is the beta distribution. Curtailment of the second sample, and model sensitivity are investigated.

More recent work is provided by Case [13]. An economically based single acceptance sampling plan is provided using the modified Guthrie-Johns model, including fixed cost elements. The Polya and mixed binomial distributions are available at the users option. Bayesian decision theory is applied to obtain the posterior expected value in order to find the minimum total expected cost. An exact model with a discrete prior distribution is presented.

Summary

This chapter presents a survey of the literature on the problems, contributions, and needs relative to the objectives of this research. This survey demonstrates the interest in the economic design of quality control models in the area of attributes acceptance sampling. Models using the discrete prior distribution for single and sequential sampling are well developed. But, all such models omit some fix cost factors. Also, there is no work toward developing double sampling plans utilizing a discrete distribution. A need has been cited for new methods of optimizing the total cost.

This survey indicates that in the case of economically based acceptance sampling for attributes, a need exists for the following:

(1) Inclusion of fixed cost factors in each of the types of costs for double-sampling plans.

(2) Newly developed optimization procedures for double-sampling plans, using exact discrete modeling.

(3) An interactive computer program suitable for use by practitioners with a minimum of technical background.

(4) Comparisons of model sensitivity to the cost coefficients and to potential misspecification of the parameters of the prior distribution, as well as the total cost of approximate and exact models.

The author believes that this research will complete an important gap that currently exists in the theory and application of economically-based acceptance sampling by attributes.

CHAPTER III

ECONOMICALLY BASED MODEL DEVELOPMENT

Introduction

The purposes of this chapter are to develop the Guthrie-Johns model for use in double-sampling and to modify the Guthrie-Johns (MGJ) model to include fixed cost components for sampling, rejection, and acceptance. The methods utilized in this chapter are based upon Bayesian decision theory. The prior, sampling, marginal, and posterior distributions dealing with double-sampling plans are used to derive the expected cost model. Nine situations using the MGJ model associated with four decisions for double-sampling plans are discussed.

The Polya and mixed binomial families are used as prior distributions in this study. These have been shown to describe well the actual lot quality in real situations. Reproducible properties of these priors permit the derivation of mathematical relationships for the cost modeling employed in this research. Methodology is developed to express a wide range of expected cost models for double-sampling plans. It is assumed that the reader has at least a basic understanding of acceptance sampling cost modeling.

Notations

This section defines the mathematical notations used in this research.

N = lot size.

n_1 = first sample size.

n_2 = second sample size.

X = number of defectives in the entire lot.

x_1 = number of defectives in the first sample.

x_2 = number of defectives in the second sample.

c_1 = acceptance number for first sample.

c_2 = acceptance number for second sample.

r_1 = rejection number for first sample.

r_2 = rejection number for second sample.

S_0 = fixed cost of sampling, inspection, and testing per lot.

S_1 = cost per item of sampling, inspecting, and testing.

S_2 = additional cost per defective item found during sampling, inspection, and testing.

A_0 = fixed cost of accepting a lot containing one or more defective items yet to be found downstream.

A_1 = cost per item of handling the items not inspected in an accepted lot.

A_2 = additional cost per defective item later discovered in an accepted lot.

R_0 = fixed cost of rejection per lot rejected on original inspection.

R_1 = cost per item of inspecting and testing the items in the rest of a rejected lot.

R_2 = additional cost per defective item found while inspecting and testing the rest of a rejected lot.

$f_N(X)$ = discrete "prior" distribution describing the probability of having X defectives in a lot of size N. $(X = 0, 1, 2, \ldots, N)$.

$\ell_{n_1}(x_1|X)$ = hypergeometric "sampling" distribution describing the probability of having x_1 defectives in a sample of size n_1 taken from a lot having X defectives $(x_1 = 0, 1, 2, \ldots, \min(n_1, X))$.

$\ell_{n_1+n_2}(x_1+x_2|X)$ = hypergeometric "sampling" distribution describing the probability of having x_1+x_2 defectives in a sample of size n_1+n_2 taken from a lot having X defectives $(x_1+x_2 = 0, 1, 2, \ldots, \min(n_1+n_2, X))$.

$g_{n_1}(x_1)$ = "marginal" (or unconditional) distribution describing the probability of having x_1 defectives in a sample of size n_1 taken from a lot. $(x_1 = 0, 1, 2, \ldots, n_1)$.

$g_{n_1+n_2}(x_1+x_2)$ = "marginal" (or unconditional) distribution describing the probability of having x_1+x_2 defectives in a combined sample of size n_1+n_2 taken from a lot $(x_1+x_2 = 0, 1, 2, \ldots, n_1+n_2)$.

$h_{N-n_1}(X-x_1|x_1)$ = "posterior" distribution describing the probability of having $X-x_1$ defectives in the rest of a lot of size $N-n_1$ given that x_1 defectives are observed in a sample n_1 taken from the lot. $(X-x_1 = 0, 1, 2, \ldots, N-n_1)$.

$h_{N-n_1-n_2}(X-x_1-x_2|x_1+x_2)$ = "posterior" distribution describing the probability of having $X-x_1-x_2$ defectives in the rest of a lot of size $N-n_1-n_2$ given that x_1+x_2 defectives are observed in a combined sample n_1+n_2 taken from the lot. $(X-x_1-x_2 = 0, 1, \ldots, N-n_1-n_2)$.

$h_{N-n_1}(X-x_1=0|x_1)$ = "posterior" distribution describing the probability of having no defectives in the rest of lot of size $N-n_1$ given that x_1 defectives were observed in a sample n_1 taken from the lot.

$h_{N-n_1-n_2}(X-x_1-x_2=0|x_1+x_2)$ = "posterior" distribution describing the probability of having no defectives in the rest of a lot of size $N-n_1-n_2$ given that x_1+x_2 defectives are observed in a combined sample n_1+n_2 taken from the lot.

$h_{n_2}(x_2|x_1)$ = "marginal" distribution describing the probability of having x_2 defectives in the second sampling with size n_2 given that x_1 defectives were observed in a sample n_1 had taken from the lot.

$E[X-x_1|x_1]$ = expected number of defectives in the rest of a lot, $X-x_1$, given the number of defectives x_1 in the first sample.

$E[X-x_1-x_2|x_1+x_2]$ = expected number of defectives in the rest of a lot, $X-x_1-x_2$, given the number of defectives x_1+x_2 in the combined first and second samples.

$TC_i(\cdot)$ = total expected cost on the i^{th} sample as a function of the variables in the argument.

$TCA_i(\cdot)$ = total expected cost of acceptance on the i^{th} sample as a function of the variables in the argument.

$TCR_i(\cdot)$ = total expected cost of rejection on the i^{th} sample as a function of the variables in the argument.

Basic Model

The nine situations of MGJ model for double sampling are described as follows:

1. Lot 100% inspected.
2. Lot accepted outright with no inspection; defectives found downstream.
3. Lot accepted outright with no inspection; no defectives found downstream.
4. First sample inspected; lot accepted; defectives found downstream.
5. First sample inspected; lot accepted; no defectives found downstream.
6. First sample inspected; lot rejected.
7. Second sample inspected; lot accepted; defectives found downstream.
8. Second sample inspected; lot accepted; no defectives found downstream.

9. Second sample inspected; lot rejected.

A flow chart of these nine situations is presented in Figure III.1.

The basic model is described mathematically as follows:

$TC(N, n_1, n_2, X, x_1, x_2, c_1, r_1, c_2, r_2)$:

1. $= S_0 + NS_1 + XS_2$ (3.1a)

(Lot 100% inspected)

2. $= A_0 + NA_1 + XA_2$ $\quad n_1 = 0$ (3.1b)

$X = 1, 2, \ldots, N$

(Lot accepted outright with no inspection; defectives found downstream)

3. $= NA_1$ $\quad n_1 = 0$ (3.1c)

$X = 0$

(Lot accepted outright with no inspection; no defectives found downstream)

4. $= S_0 + n_1S_1 + x_1S_2 + A_0 + (N - n_1)\ A_1 + (X - x_1)\ A_2$

$n_1 > 0$ (3.1d)

$x_1 \leq c_1$

$X - x_1 = 1, 2, \ldots, N-n_1$

(First sample inspected; lot accepted; defectives found downstream)

5. $= S_0 + n_1S_1 + x_1S_2 + (N - n_1)\ A_1$

$n_1 > 0$ (3.1e)

$x_1 \leq c_1$

$X - x_1 = 0$

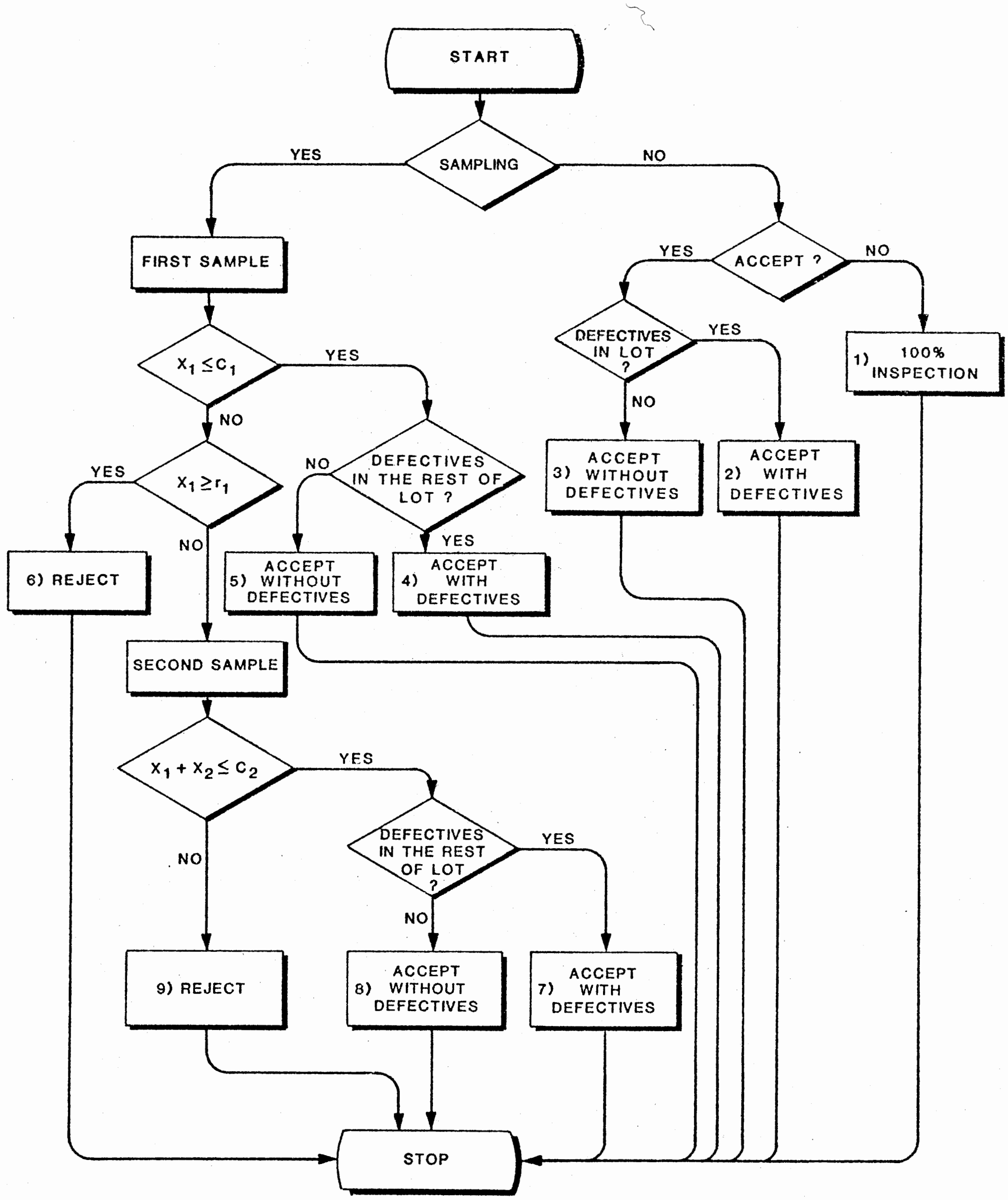


Figure III.1. Flow Chart of Nine Situations in the Basic MGJ Model for Double Sampling

(First sample inspected; lot accepted; no defectives found downstream)

6. $= S_0 + n_1S_1 + x_1S_2 + R_0 + (N - n_1)\ R_1 + (X - x_1)\ R_2$

$$n_1 > 0 \qquad (3.1f)$$
$$x_1 \geq r_1$$
$$X - x_1 = 0, 1, 2, \ldots, N-n_1$$

(First sample inspected; lot rejected)

7. $= S_0 + S_1\ (n_1 + n_2) + S_2\ (x_1 + x_2) + A_0 + (N - n_1 - n_2)\ A_1 + (X - x_1 - x_2)\ A_2$

$$n_1 > 0 \qquad (3.1g)$$
$$n_2 > 0$$
$$c_1 < x_1 < r_1$$
$$x_1 + x_2 \leq c_2$$
$$X - x_1 - x_2 = 1, 2, \ldots, N - n_1 - n_2$$

(Second sample inspected; lot accepted; defectives found downstream)

8. $= S_0 + S_1\ (n_1 + n_2) + S_2\ (x_1 + x_2) + (N - n_1 - n_2)\ A_1$

$$n_1 > 0 \qquad (3.1h)$$
$$n_2 > 0$$
$$c_1 < x_1 < r_1$$
$$x_1 + x_2 \leq c_2$$
$$X - x_1 - x_2 = 0$$

(Second sample inspected; lot accepted; no defectives found downstream)

9. $= S_0 + S_1\ (n_1 + n_2) + S_2\ (x_1 + x_2) + R_0 + (N - n_1 - n_2)\ R_1 + (X - x_1 - x_2)\ R_2$

$$n_1 > 0 \qquad (3.1i)$$

$$n_2 > 0$$

$$c_1 < x_1 < r_1$$

$$x_1 + x_2 \geq r_2$$

(Second sample inspected; lot rejected)

Those costs are seen to be a function of N, n_1, n_2, X, x_1, x_2, c_1, r_1, c_2, and r_2. Some of these variables (n_1, n_2, c_1, r_1, r_2) are "decision" variables under the control of the user; others are random variables (X, x_1, x_2) over which the user has no control. The variable N may or may not be under the user's control.

Distributional Properties

The relevant probability distributions for the first sample are shown in Chapter I as:

$$J(X - x_1, x_1) = f_N (X)\, \ell_{n_1} (x_1|X) = g_{n_1} (x_1)\, h_{N-n_1} (X - x_1|x_1) \qquad (3.2)$$

or

$$\begin{array}{c}\text{Joint}\\ \text{Distribution}\end{array} = \begin{array}{c}\text{Prior}\\ \text{Distribution}\end{array} \times \begin{array}{c}\text{Sampling}\\ \text{Distribution}\end{array} = \begin{array}{c}\text{Marginal}\\ \text{Distribution}\end{array} \times \begin{array}{c}\text{Posterior}\\ \text{Distribution}\end{array}$$

From this, the distributions for the second sample may be expressed as:

$$J(X - x_1 - x_2, x_1 + x_2) = h_{N-n_1} (X - x_1|x_1)\, \ell_{n_2} (x_2|X - x_1) \qquad (3.3)$$

$$= g_{n_2}(x_2) h_{N-n_1-n_2}(X - x_1 - x_2|x_1 + x_2)$$

The posterior distribution from the first sample becomes the prior distribution for the rest of the lot from which the second sample is taken.

Polya Distribution

The Polya prior distribution is described mathematically as:

$$f_N(X) = \binom{N}{X} \frac{\Gamma(s + X)}{\Gamma(s)} \frac{\Gamma(t + N - X)}{\Gamma(t)} \frac{\Gamma(s + t)}{\Gamma(s + t + N)}, \tag{3.4}$$

$$s, t > 0$$

$$X = 0, 1, \ldots, N$$

The mean of Polya distribution is:

$$E(X) = \frac{Ns}{s + t} \tag{3.5}$$

and its variance is:

$$\mathrm{Var}(X) = \frac{Nst}{(s + t)^2} \frac{(s + t + N)}{(s + t + 1)} \tag{3.6}$$

Proper selection of s and t will cause the Polya to become a discrete uniform, binomial, hypergeometric, or literally infinite other distributions. Since the Polya distribution is reproducible to hypergeometric sampling. This means that with a Polya prior and a hypergeometric sampling distribution, the marginal distribution is known to be a Polya distribution. The marginal distribution of the number of defectives observed in the first sample is:

$$g_{n_1}(x_1) = \binom{n_1}{x_1} \frac{\Gamma(s + x_1)}{\Gamma(s)} \frac{\Gamma(t + n_1 - x_1)}{\Gamma(t)} \frac{\Gamma(s + t)}{\Gamma(s + t + n_1)} \tag{3.7}$$

$$s, t > 0$$

$$x_1 = 0, 1, 2, \ldots, n_1$$

The posterior distribution considers both the prior parameters and the sample results to express the quality of the lot following sample inspection. The mathematical expression for the posterior distribution of defectives in the rest of the lot following the first sample $h_{N-n_1}(X-x_1|x_1)$, is found from Equation (3.2) as follows:

$$h_{N-n_1}(X-x_1|x_1) = \frac{f_N(X)\ \ell_{n_1}(x_1|X)}{g_{n_1}(x_1)}$$

$$= \frac{\binom{N}{X} \dfrac{\Gamma(s+X)\ \Gamma(t+N-X)\ \Gamma(s+t)}{\Gamma(s)\ \ \Gamma(t)\ \ \Gamma(s+t+N)}}{\binom{n_1}{x_1} \dfrac{\Gamma(s+x_1)\ \Gamma(t+n_1-x_1)\ \Gamma(s+t)}{\Gamma(s)\ \ \Gamma(t)\ \ \Gamma(s+t+n_1)}} \quad \frac{\binom{n_1}{x_1}\binom{N-n_1}{X-x_1}}{\binom{N}{X}}$$

$$= \binom{N-n_1}{X-x_1} \frac{\Gamma(s+X)\ \Gamma(t+N-X)\ \Gamma(s+t+n_1)}{\Gamma(s+x_1)\ \Gamma(t+n_1-x_1)\ \Gamma(s+t+N)}$$

$$X-x_1 = 0, 1, 2, \ldots, N-n_1 \tag{3.8}$$

The posterior distribution following the second sample is similar to the above expression:

$$h_{N-n_1-n_2}(X-x_1-x_2|x_1+x_2) = \binom{N-n_1-n_2}{X-x_1-x_2} \frac{\Gamma(s+X)\ \Gamma(t+N-X)\ \Gamma(s+t+n_1+n_2)}{\Gamma(s+x_1+x_2)\ \Gamma(t+n_1+n_2-x_1-x_2)\ \Gamma(s+t+N)}$$

$$X-x_1-x_2 = 0, 1, 2, \ldots, N-n_1-n_2 \tag{3.9}$$

Mixed Binomial Distribution

The mixed binomial prior distribution is very useful when it is likely that a lot s is formed from one of m process fractions defective, $p_1, p_2, \ldots, p_m$. The weights $w_1, w_2, \ldots, w_m$ correspond to an estimate of the fraction of product formed at the process fraction defective $p_1, p_2, \ldots, p_m$. The distribution is described mathematically as:

$$f_N(X) = \sum_{i=1}^{m} w_i \binom{N}{X} p_i^{\,X} (1-p_i)^{N-X}, \quad 0 < p_i < 1 \tag{3.10}$$

$$X = 0, 1, 2, \ldots, N$$

$$\sum_{i=1}^{m} w_i = 1$$

The mean of the mixed binomial distribution is:

$$E[X] = N\bar{p} = \sum_{i=1}^{m} w_i N p_i \tag{3.11}$$

and its variance is:

$$\mathrm{Var}[X] = \sum_{i=1}^{m} w_i N p_i (1-p_i) + \sum_{i=1}^{m} w_i N^2 (p_i - \bar{p})^2 \tag{3.12}$$

Hald has shown that the mixed binomial distribution is also "reproducible to hypergeometric sampling." Thus, the marginal distribution of the number of defectives in the first sample may be written directly as

$$g_{n_1}(x_1) = \sum_{i=1}^{m} w_i \binom{n_1}{x_1} p_i^{x_1} (1-p_i)^{n_1-x_1}, \quad x_1 = 0, 1, 2, \ldots, n_1 \tag{3.13}$$

The posterior distribution of the number of defectives in the rest of the lot given that x_1 defectives have been observed in the sample is:

$$h_{N-n_1}(X-x_1 | x_1) = \frac{\sum_{i=1}^{m} w_i \binom{N}{X} p_i^{X} (1-p_i)^{N-X}}{\sum_{i=1}^{m} w_i \binom{n_1}{x_1} p_i^{x_1} (1-p_i)^{n_1-x_1}} \quad \frac{\binom{n_1}{x_1}\binom{N-n_1}{X-x_1}}{\binom{N}{X}}$$

$$= \frac{f_N(X) \ell_{n_1}(x_1 | X)}{g_{n_1}(x_1)}$$

$$= \frac{\sum_{i=1}^{m} w_i \binom{N-n_1}{X-x_1} p_i^{X} (1-p_i)^{N-X}}{\sum_{i=1}^{m} w_i \; p_i^{x_1} (1-p_i)^{n_1-x_1}} \tag{3.14}$$

$$X-x_1 = 0, 1, 2, \ldots, N-n_1$$

The posterior distribution of the number of defectives in the rest of the lot given that x_1+x_2 defectives are observed in the combined sample is

$$h_{N-n_1-n_2}(X-x_1-x_2|x_1,x_2) = \frac{\sum_{i=1}^{m} w_i \binom{N-n_1-n_2}{X-x_1-x_2} p_i^{X} (1-p_i)^{N-X}}{\sum_{i=1}^{m} w_i \; p_i^{x_1+x_2} (1-p_i)^{n_1+n_2-x_1-x_2}} \tag{3.15}$$

$$X-x_1-x_2 = 0, 1, 2, \ldots, N-n_1-n_2$$

Expectation

In discussing the cost function, it is desired to express total cost as a function of only the decision variables, $TC(n_1, n_2, c_1, r_1, c_2, r_2)$. This requires taking expectations over x_1, x_2, X, and simply removing N from the argument as it is assumed fixed.

The nine situations of the MGJ model for double sampling can be classified into one of the following four decisions:

1. Lot 100% inspected.
2. Lot accepted outright.
3. Lot decision made following inspection of first sample.
   (a) Lot accepted
   (b) Lot rejected
4. Lot decision made following inspection of second sample.

(a) Lot accepted

(b) Lot rejected

Lot 100% Inspected

A valid action is to perform "100% inspection"; however, it is a special case which is treated separately. No decision variables and no random variables exist in the case of 100% inspection. Thus, the total cost remains at:

$$TC(N) = S_0 + NS_1 + XS_2 \tag{3.16}$$

The decision to inspect 100% will be attractive when either quality is usually very poor, or the cost consequences of passing on defectives is substantial.

Lot Accepted Outright With No Inspection

"No inspection" is another valid decision. It includes two possible outcomes: 1) defectives found downstream, and 2) no defectives found downstream.

Consider Equations (3.1b) and (3.1c) in which no inspection is performed and the lot is accepted. No decision variables exist in this case; therefore, it is only necessary to take the expectation with respect to X. The probabilities to be used in taking this expectation over X are described by the prior distribution, $f_N(X)$. The expected cost will be

$$TC(N) = NA_1 f_N(X=0) + \sum_{X=1}^{N} (A_0+NA_1+XA_2)\ f_N(X)$$

$$= \sum_{X=1}^{N} A_0\ f_N(X) + \sum_{X=0}^{N} NA_1\ f_N(X) + \sum_{X=1}^{N} XA_2\ f_N(X)$$

$$= A_0(1-f_N(X=0)) + NA_1 + A_2E\,[X]\ . \tag{3.17}$$

If a Polya prior distribution is used, it is known from Equation (3.4) that:

$$f_N(X=0) = \frac{\Gamma(t+N)}{\Gamma(t)}\ \frac{\Gamma(s+t)}{\Gamma(s+t+N)}\ . \tag{3.18}$$

Also, from Equation (3.5)

$$E[X] = \frac{Ns}{s+t} \tag{3.19}$$

Therefore, for a Polya prior distribution, the expected total cost of lot acceptance without inspection is:

$$TC(N) = A_0\left(1 - \frac{\Gamma(t+N)}{\Gamma(t)}\ \frac{\Gamma(s+t)}{\Gamma(s+t+N)}\right) + NA_1 + A_2\,\frac{Ns}{s+t}\ . \tag{3.20}$$

If a mixed binomial prior distribution is used, it is known from Equation (3.10) that:

$$f_N(X=0) = \sum_{i=1}^{m} w_i\ (1-p_i)^N\ . \tag{3.21}$$

Also, from Equation (3.11)

$$E(X) = \sum_{i=1}^{m} w_i\ Np_i\ . \tag{3.22}$$

Therefore, for a mixed binomial prior distribution, the expected total cost of lot acceptance without inspection is:

$$TC(N) = A_0\left(1 - \sum_{i=1}^{m} w_i\ (1-p_i)^N\right) + NA_1 + A_2\ \sum_{i=1}^{m} w_i\ Np_i\ . \tag{3.23}$$

The "no inspection" case may save considerable money when either quality is usually very good, or the cost consequence of passing on defectives is slight.

Lot Decision Made Following Inspection of First Sample

When a decision is made following inspection of the first sample, Equations (3.1d), (3.1e), and (3.1f) are appropriate. Expectation will take place over both random variables X and x_1, with X being first for computational reasons. When expecting over X, the weight to be used is the posterior probability of the number of defectives in the rest of the lot given the number of defectives in the first sample, $h_{N-n_1}(X-x_1|x_1)$. This decision includes three situations:

1. Lot accepted after first sample, defectives found downstream.
2. Lot accepted after first sample, no defectives found downstream.
3. Lot rejected after first sample.

An acceptance cost term and a rejection cost term are written separately, since the decision to accept depends on x_1 being less than or equal to c_1, while the decision to reject occurs if x_1 equals or exceeds r_1.

The acceptance cost term is

$$
\begin{aligned}
TCA_1(N,n_1,c_1,x_1) &= [S_0+n_1S_1+x_1S_2+(N-n_1)A_1]\ h_{N-n_1}(X-x_1=0|x_1) \\
&\quad + \sum_{X-x_1=1}^{N-n_1} [S_0+n_1S_1+x_1S_2+A_0+(N-n_1)A_1 \\
&\qquad + (X-x_1)A_2]\ h_{N-n_1}(X-x_1|x_1) \\
&= S_0+n_1S_1+x_1S_2+A_0[1-h_{N-n_1}(X-x_1=0|x_1)] \\
&\quad + (N-n_1)A_1 + A_2E[X-x_1|x_1] \qquad (3.24)
\end{aligned}
$$

where

$$n_1 > 0$$
$$x_1 \le c_1$$
$$X\text{-}x_1 = 0,\ 1,\ 2,\ .\ .\ .,\ N\text{-}n_1$$

The term $E[X-x_1|x_1]$ is the posterior expected value following the first sample. It stems from the expression

$$\sum_{X-x_1=0}^{N-n_1} (X-x_1)h_{N-n_1}(X-x_1|x_1)$$

which sums $X-x_1$ over its entire range, and uses as weights the posterior distribution.

The rejection cost term is

$$TCR_1(N,n_1,r_1,x_1) = \sum_{X-x_1=0}^{N-n_1} [S_0+n_1S_1+x_1S_2+R_0+(N-n_1)R_1 + R_2(X-x_1)]h_{N-n_1}(X-x_1|x_1)$$

$$= S_0+n_1S_1+x_1S_2+R_0+(N-n_1)R_1+R_2E[X-x_1|x_1] \qquad (3.25)$$

where

$$n_1 > 0$$
$$x_1 \ge r_1$$
$$X\text{-}x_1 = 0,\ 1,\ 2,\ .\ .\ .,\ N\text{-}n_1$$

Summarizing the total cost expression to this point,

$$TC_1(N,n_1,c_1,r_1,x_1) = TCA_1(N,n_1,c_1,r_1,x_1)$$

$$= S_0+n_1S_1+x_1S_2+A_0[1-h_{N-n_1}(X-x_1=0|x_1)] + (N-n_1)A_1+A_2E[X-x_1|x_1]\ , \qquad (3.26)$$

$$x_1 \le c_1$$

or

$$TCR_1(N,n_1,c_1,r_1,x_1) = S_0+n_1S_1+x_1S_2+R_0+(N-n_1)R_1+R_2E[X-x_1|x_1] \qquad (3.27)$$

$$\text{if } x_1 \geq r_1$$

The only random variable in these expressions is the number of defectives in the first sample. These relationships are later used in order to determine the optimum acceptance and rejection numbers for the first sample.

The cost term $TC_1(N,n_1,c_1,r_1,x_1)$ may be reduced to $TC_1(N,n_1,c_1,r_1)$. This is performed by taking the expectation over x_1 in Equations (3.26) and (3.27) using the marginal probability function $g_{n_1}(x_1)$ for the weighting probabilities. That is,

$$TC_1(N,n_1,c_1,r_1) = \sum_{x_1=0}^{c_1} TCA_1(N,n_1,c_1,r_1,x_1)g_{n_1}(x_1)$$

$$+ \sum_{x_1=r_1}^{n_1} TCR_1(N,n_1,c_1,r_1,x_1)g_{n_1}(x_1)$$

$$= \sum_{x_1=0}^{c_1} \{S_0+n_1S_1+x_1S_2+A_0[1-h_{N-n_1}(X-x_1=0|x_1)]+(N-n_1)A_1$$

$$+ A_2E[X-x_1|x_1]\}g_{n_1}(x_1)$$

$$+ \sum_{x_1=r_1}^{n_1} \{S_0+n_1S_1+x_1S_2+R_0+(N-n_1)R_1+R_2E[X-x_1|x_1]\}g_{n_1}(x_1)$$

$$(3.28)$$

Lot Decision Made Following Inspection of Second Sample

If the number of defectives in the first sample x_1 is greater than c_1 but less than r_1, a second sample is taken. If in the combined samples there are c_2 or fewer defective units, the lot is accepted. If there are more than c_2 defective units, the lot is rejected. This decision also includes three situations:

1. Lot accepted after second sample; with defectives found downstream.
2. Lot accepted after second sample; no defectives found downstream.
3. Lot rejected after second sample.

When a second sample is inspected, Equations (3.1g), (3.1h), and (3.1i) are appropriate. An acceptance cost term and a rejection cost term are written separately.

The acceptance cost term is

$$
\begin{aligned}
TCA_2(N,n_1,n_2,c_1,r_1,c_2,r_2,x_1,x_2) &= [S_0+(n_1+n_2)S_1+(x_1+x_2)S_2 \\
&\qquad + (N-n_1-n_2)A_1]h_{N-n_1-n_2}(X-x_1-x_2=0|x_1,x_2) \\
&\quad + \sum_{X-x_1-x_2=0}^{N-n_1-n_2} [S_0+(n_1+n_2)S_1+(x_1+x_2)S_2 \\
&\qquad + A_0+(N-n_2-n_2)A_1+(X-x_1-x_2)A_2] \\
&\quad \cdot h_{N-n_1-n_2}(X-x_1-x_2|x_1,x_2) \\
&= S_0+(n_1+n_2)S_1+(x_1+x_2)S_2 \\
&\qquad +A_0[1-h_{N-n_1-n_2}(X-x_1-x_2=0|x_1,x_2)] \\
&\quad + (N-n_1-n_2)A_1+E[X-x_1-x_2|x_1,x_2]A_2
\end{aligned}
\tag{3.29}
$$

$$n_1 > 0$$
$$n_2 > 0$$
$$c_1 < x_1 < r_1$$
$$x_1 + x_2 \leq c_2$$
$$X - x_1 - x_2 = 0, 1, 2, \ldots, N - n_1 - n_2$$

The term $E[X-x_1-x_2|x_1,x_2]$ is the posterior expected value following the second sample. It stems from the expression

$$\sum_{X-x_1-x_2=0}^{N-n_1-n_2} (X-x_1-x_2)h_{N-n_1-n_2}[X-x_1-x_2|x_1,x_2]$$

which sums $X-x_1-x_2$ over its entire range from 0 to $N-n_1-n_2$, and uses as weights the posterior distribution.

The rejection cost term is

$$\begin{aligned} TCR_2(N,n_1,n_2,c_1,r_1,c_2,r_2,x_1,x_2) &= \sum_{X-x_1-x_2=0}^{N-n_1-n_2} [S_0+(n_1+n_2)S_1+(x_1+x_2)S_2+R_0 \\ &\quad + R_1(N-n_1-n_2) \\ &\quad + R_2(X-x_1-x_2)]h_{N-n_1-n_2}(X-x_1-x_2|x_1,x_2) \\ &= S_0+(n_1+n_2)S_1+(x_1+x_2)S_2+R_0+R_1(N-n_1-n_2) \\ &\quad + R_2E[X-x_1-x_2|x_1,x_2] \qquad (3.30) \end{aligned}$$

$$n_1 > 0$$
$$n_2 > 0$$
$$c_1 < x_1 < r_1$$
$$x_1 + x_2 \geq r_2$$
$$X - x_1 - x_2 = 0, 1, 2, \ldots, N - n_1 - n_2$$

Summarizing the total cost expression to this point,

$$TC_2(N,n_1,n_2,c_1,r_1,c_2,r_2,x_1,x_2) = TCA_2(N,n_1,n_2,c_1,r_1,c_2,r_2,x_1,x_2)$$

$$= S_0+(n_1+n_2)S_1+(x_1+x_2)S_2$$

$$+ A_0[1-h_{N-n_1-n_2}(X-x_1-x_2=0|x_1+x_2)]$$

$$+ (N-n_1-n_2)A_1+E[X-x_1-x_2|x_1+x_2]\ A_2 \tag{3.31}$$

$$\text{if} \quad x_1+x_2 \le c_2$$

or

$$= TCR_2(N,n_1,n_2,c_1,r_1,c_2,r_2,x_1,x_2)$$

$$= S_0+(n_1+n_2)S_1+(x_1+x_2)S_2+R_0+R_1(N-n_1-n_2)$$

$$+R_2E[X-x_1-x_2|x_1+x_2] \tag{3.32}$$

$$\text{if} \quad x_1+x_2 \ge r_2$$

and for all above

$$n_1 > 0$$

$$n_2 > 0$$

$$c_1 < x_1 < r_1$$

$$X - x_1 - x_2 = 0,\ 1,\ 2,\ .\ .\ .,\ N - n_1 - n_2$$

The cost term $TC_2(N,n_1,n_2,c_1,r_1,c_2,r_2,x_1,x_2)$ may be reduced to $TC_2(N,n_1,n_2,c_1,r_1,c_2,r_2)$. This is performed by taking the expectation with respect to x_1 and x_2 in Equations (3.31) and (3.32) using the marginal probability function $g_{n_1}(x_1)$ and conditional distribution $h_{n_2}(x_2|x_1)$:

$$TC_2(N,n_1,n_2,c_1,r_1,c_2,r_2) = \sum_{x_1=c_1+1}^{r_1-1} \left\{ \sum_{x_2=0}^{c_2-x_1} TCA_2(N,n_1,n_2,c_1,r_1,c_2,r_2,x_1,x_2) \cdot h_{n_2}(x_2|x_1) + \sum_{x_2=r_2-x_1}^{n_2} TCR_2(N,n_1,n_2,c_1,r_1,c_2,r_2,x_1,x_2) \cdot h_{n_2}(x_2|x_1) \right\} g_{n_1}(x_1)$$

$$= \sum_{x_1=c_1+1}^{r_1-1} \left\{ \sum_{x_2=0}^{c_2-x_1} [S_0+(n_1+n_2)S_1+(x_1+x_2)S_2 + A_0(1-h_{N-n_1-n_2}(X-x_1-x_2=0|x_1+x_2) + (N-n_1-n_2)A_1+A_2E[X-x_1-x_2|x_1+x_2]] \cdot h_{n_2}(x_2|x_1) + \sum_{x_2=r_2-x_1}^{n_2} [S_0+(n_1+n_2)S_1+(x_1+x_2)S_2+R_0 + R_1(N-n_1-n_2)+R_2E[X-x_1-x_2|x_1+x_2]] \cdot h_{n_2}(x_2|x_1) \right\} g_{n_1}(x_1) \qquad (3.33)$$

Equation (3.33) completes development of the objective cost function for the second sample. The only decision variables remaining include the first sample size (n_1), second sample size (n_2), and the acceptance and rejection numbers for first and second sampling (c_1, r_1, c_2, r_2). The total cost depends upon the values selected for those decision variables.

Summary

The cost model developed in this chapter utilizes the basic Guthrie-Johns model for economically based sampling. The GJ model has been modified for use in double sampling and includes fixed cost components for sampling, rejection, and acceptance. The Modified Guthrie-Johns model for double sampling includes nine situations described within four decisions: Lot 100% inspected; lot accepted outright; lot decision made following inspection of first sample; and lot decision made following inspection of second sample. These decisions and their mathematical cost functions are summarized in Table III.1. Two general families of prior distributions, the Polya and mixed binomial families, have been used to describe actual lot quality. The model developed in this research entertains the selection of all possible decision variables (n_1, n_2, c_1, r_1, c_2, r_2); the total expected cost is a function of these. Optimization of these decision variables is discussed in the next chapter.

TABLE III.1

SUMMARIZED MGJ MODEL DECISIONS AND THEIR MATHEMATICAL COST FUNCTIONS

Decision	Decision and Random Variables	Limitation	Cost Function
Lot 100% Inspected	N, X		$S_0 + NS_1 + XS_2$
Lot Accepted Outright With No Inspection	N		$A_0[1-f_N(X=0)] + A_1 N + A_2\ E[X]$
Lot Decision Made Following Inspection of First Sample	N, n_1, c_1, r_1, x_1	$n_1>0$ $x_1 \le c_1$ $X-x_1=0,1,\ldots,N-n_1$	$S_0 + S_1 n_1 + S_2 x_1 + A_0[1-h_{N-n_1}(X-x_1=0 \mid x_1)]$ $+ A_1(N-n_1) + A_2\ E[X-x_1 \mid x_1]$
		$n_1>0$ $x_1 \ge r_1$ $X-x_1=0,1,\ldots,N-n_1$	$S_0 + S_1 n_1 + S_2 x_1 + R_0 + R_1(N-n_1) + R_2\ E[X-x_1 \mid x_1]$
Lot Decision Made Following Inspection of Second Sample	$N, n_1, n_2, c_1,$ $r_1, c_2, r_2, x_1,$ x_2	$n_1>0$ $n_2>0$ $c_1<x_1<r_1$ $x_1+x_2 \le c_2$ $X-x_1-x_2=0,1,\ldots,$ $N-n_1-n_2$	$S_0 + S_1(n_1+n_2) + S_2(x_1+x_2) + A_0[1-h_{N-n_1-n_2}(X-x_1$ $-x_2=0 \mid x_1,x_2)]$ $+ A_1(N-n_1-n_2) + A_2\ E[X-x_1-x_2 \mid x_1,x_2]$
		$n_1>0$ $n_2>0$ $c_1<x_1<r_1$ $x_1+x_2 \ge r_2$ $X-x_1-x_2=0,1,\ldots,$ $N-n_1-n_2$	$S_0 + S_1(n_1+n_2) + S_2(x_1+x_2) + R_0 + R_1(N-n_1-n_2)$ $+ R_2\ E[X-x_1-x_2 \mid x_1,x_2]$

CHAPTER IV

COST MODEL OPTIMIZATION

Introduction

The purpose of this chapter is to develop the methodology for determining the optimum values of the decision variables, including sample sizes and acceptance and rejection numbers (n_1*, n_2*, c_1*, r_1*, c_2*, r_2*). Theoretically, it is possible to evaluate all combinations of (n_1, n_2, c_1, r_1, c_2, r_2) in the total cost model. Practically, however, it is time consuming even for small lot size and infeasible for large lot sizes.

Due to the large number of decision variables, it is desirable to determine the optimum acceptance and rejection numbers (c_1, r_1, c_2, r_2), given any sample size pair (n_1, n_2). Those optimum acceptance and rejection numbers can be determined by considering and comparing the posterior expected costs of (1) accepting after the first sample, (2) taking a second sample and making an accept/reject decision on the lot, and (3) rejecting after the first sample. Once the acceptance and rejection numbers are determined, the total cost of the double sampling plan may be determined. Then, other sample size pairs and their corresponding optimum acceptance and rejection numbers may be evaluated. An appropriate heuristic search procedure over only (n_1, n_2) may then be used to determine the economically optimum double sampling plan.

Optimum Acceptance and Rejection Number for

Second Sample

The cost equations (3.31) (3.32) are utilized to decide upon the optimum acceptance and rejection numbers for a complete double sample. It is reasonable to assume that if the total number of defectives observed in the combined samples $(x_1 + x_2)$ causes the expected acceptance cost term for the combined sample to be less than or equal to the expected rejection cost term for the combined sample $(TCA_2 \le TCR_2)$, the logical decision is to accept the lot. Conversely, the lot should be rejected if $TCA_2 > TCR_2$. For any given sample size pair, it is possible to determine the highest value of $x_1 + x_2$ such that $TCA_2 \le TCR_2$. This value of $x_1 + x_2$ will be designated the acceptance number for the combined number of defectives following the second sample (c_2). The corresponding rejection number (r_2) is $c_2 + 1$.

Based upon the above logic, it is desired to find the largest value of $x_1 + x_2$ such that:

$$TCA_2(N,n_1,n_2,c_1,r_1,c_2,r_2,x_1,x_2) \le TCR_2(N,n_1,n_2,c_1,r_1,c_2,r_2,x_1,x_2) \tag{4.1}$$

or

$$S_0+(n_1+n_2)S_1+(x_1+x_2)S_2+A_0[1-h_{N-n_1-n_2}(X-x_1-x_2=0|x_1,x_2)]+A_1(N-n_1-n_2)$$
$$+A_2E[X-x_1-x_2|x_1,x_2] \quad \le$$

$$S_0+(n_1+n_2)S_1+(x_1+x_2)S_2+R_0+R_1(N-n_1-n_2)+R_2E[X-x_1-x_2|x_1,x_2] \tag{4.2}$$

This results in

$$(A_2-R_2)E[X-x_1-x_2|x_1,x_2] \le R_0-A_0[1-h_{N-n_1-n_2}(X-x_1-x_2=0|x_1,x_2)]+(R_1-A_1)(N-n_1-n_2) \tag{4.3}$$

or

$$E[X-x_1-x_2|x_1,x_2] \leq \frac{R_0-A_1[1-h_{N-n_1-n_2}(X-x_1-x_2=0|x_1,x_2)]+(R_1-A_1)(N-n_1-n_2)}{A_2-R_2} \tag{4.4}$$

It is easy to find the largest value of x_1+x_2 satisfying inequality (4.4) if only the expressions for $h_{N-n_1-n_2}(X-x_1-x_2=0|x_1,x_2)$ and $E[X-x_1-x_2|x_1,x_2]$ are known. These expressions depend upon whether the Polya or mixed binomial prior distribution is being used.

Finding $h_{N-n_1-n_2}(X-x_1-x_2=0|x_1,x_2)$

The term $h_{N-n_1-n_2}(X-x_1-x_2=0|x_1,x_2)$ is the probability that the number of defectives in the entire lot is the same as the number actually found in the combined samples. That is, it is the probability that all of the lot defectives are found in the first and second samples. This probability will usually be quite small, except in the case where quality is extremely good and there are no defectives found in the sample because there are none in the lot.

For the Polya prior distribution, this probability may be found utilizing Equation (3.9) for $h_{N-n_1-n_2}(X-x_1-x_2|x_1,x_2)$ and letting $X=x_1+x_2$:

$$h_{N-n_1-n_2}(X-x_1-x_2=0|x_1,x_2) = \frac{\Gamma(t+N-x_1-x_2)}{\Gamma(t+n_1+n_2-x_1-x_2)} \frac{\Gamma(s+t+n_1+n_2)}{\Gamma(s+t+N)} \tag{4.5}$$

For a mixed binomial prior distribution, this probability may be found utilizing Equation (3.15):

$$h_{N-n_1-n_2}(X-x_1-x_2=0|x_1,x_2) = \frac{\sum_{i=1}^{m} w_i\, p_i^{x_1+x_2}\, (1-p_i)^{N-x_1-x_2}}{\sum_{i=1}^{m} w_i\, p_i^{x_1+x_1}\, (1-p_i)^{n_1+n_2-x_1-x_2}} \tag{4.6}$$

Those equations in this form correspond with the computer program written to perform these and other calculations.

Finding $E[X-x_1-x_2|x_1,x_2]$

This term is the expected value of the number of defectives remaining in the lot $(X-x_1-x_2)$ given that x_1+x_2 defectives have actually been observed in the combined sample. Hald [33] has shown that, for both the Polya and mixed binomial distributions, the posterior expectation for single sampling is:

$$E[X-x|x] = \frac{(N-n)(x+1)}{(n+1)} \frac{g_{n+1}(x+1)}{g_n(x)} \tag{4.7}$$

It follows that:

$$E[X-x_1-x_2|x_1,x_2] = \frac{[N-n_1-n_2](x_1+x_2+1)}{(n_1+n_2+1)} \frac{g_{n_1+n_2+1}(x_1+x_2+1)}{g_{n_1+n_2}(x_1+x_2)} \tag{4.8}$$

For the Polya prior distribution, the posterior expectation is:

$$E[X-x_1-x_2|x_1,x_2]$$

$$= \frac{(N-n_1-n_2)(x_1+x_2+1)}{(n_1+n_2+1)} \frac{\binom{n_1+n_2+1}{x_1+x_1+1} \frac{\Gamma(s+x_1+x_2+1)}{\Gamma(s)} \frac{\Gamma(t+n_1+n_2-x_1-x_1)}{\Gamma(t)} \frac{\Gamma(s+t)}{\Gamma(s+t+n_1+n_2+1)}}{\binom{n_1+n_2}{x_1+x_2} \frac{\Gamma(s+x_1+x_2)}{\Gamma(s)} \frac{\Gamma(t+n_1+n_2-x_1-x_2)}{\Gamma(t)} \frac{\Gamma(s+t)}{\Gamma(s+t+n_1+n_2)}}$$

$$= \frac{(N-n_1-n_2)(s+x_1+x_2)}{(s+t+n_1+n_2)} \tag{4.9}$$

For the mixed binomial prior distribution, the posterior expectation is:

$$E[X-x_1-x_2\,|x_1,x_2] = \frac{(N-n_1-n_2)(x_1+x_2+1)}{(n_1+n_2+1)} \; \frac{\sum_{i=1}^{m} w_i \binom{n_1+n_2+1}{x_1+x_2+1} p_i^{\,x_1+x_2+1} (1-p_i)^{n_1-n_2-x_1-x_2}}{\sum_{i=1}^{m} w_i \binom{n_1+n_2}{x_1+x_2} p_i^{\,x_1+x_2} (1-p_i)^{n_1-n_2-x_1-x_2}} \tag{4.10}$$

where $\sum_{i=1}^{m} w_i = 1$, $0 \le p_i \le 1$

These equations are presented in this form to correspond with the computer program written to perform these and other calculations.

Optimum Acceptance and Rejection Number for the First Sample

The same methodology outlined in the above section for determining c_2 and r_2 is also applied to the selection of the optimum acceptance and the rejection numbers for the first sample (c_1 and r_1). There exists a logical relationship between the total expected cost of acceptance following the first sample (TCA_1), the total expected cost of acceptance or rejection following the second sample (TCC_2, $TCC_2 = TCA_2 + TCR_2$), and the total expected cost of rejection following the first sample (TCR_1). If the number of defectives in the first sample is 0, this is often an indication that the lot may be good and acceptance should take place immediately. In this case, TCA_1 will be less than or equal to TCC_2 or TCR_1. This reason will hold for any value of x_1 from 0 through some value, later to be designated c_1. As the number of defectives in the first sample (x_1) increases, there is uncertainty about the desirability of the lot and a decision is made to consider a second sample. In this case, TCC_2 will be the smallest among the three expected costs. When x_1 reaches a sufficiently large value, later to be designated r_1, the expected cost TCR_1 becomes smallest, indicating the desirability of rejecting on the first sample.

Using the above reasoning, it is desired to accept the lot following the first sample as long as x_1 results in $TCA_1 \le TCC_2 \le TCR_1$. If x_1 is such that $TCA_1 > TCC_2$, and $TCC_2 \le TCR_1$, then a second sample is observed. Finally, if $TCA_1 \ge TCC_2 > TCR_1$, the decision is made to reject the lot following the first sample.

Since the optimum acceptance and rejection numbers for the combined first and second samples (c_2 and r_2) have already been decided, it is possible to calculate TCC_2 by considering all possible values which x_2 may assume, splitting the calculation into two parts (TCA_2 and TCR_2). Then, by comparing TCA_1 against TCC_2, for any given first sample size (n_1), it is possible to determine the highest value of x_1 such that $TCA_1 \le TCC_2$. This value of x_1 is the optimum first sample acceptance number, c_1.

Using the same logic, comparing the TCC_2 against TCR_1, the smallest value of x_1 may be found such that $TCC_2 > TCR_1$. This number of defectives in the first sample (x_1) is designated the rejection number for the first sample (r_1). The cost function (3.24), (3.31), and (3.32) are reconsidered, on the basis of above logic, to determine the largest value of x_1 such that:

$$TCA_1(N,n_1,c_1,x_1) \le \sum_{x_2=0}^{c_2-x_1} TCA_2(N,n_1,n_2,c_1,r_1,c_2,x_1,x_2)h_{n_2}(x_2|x_1)$$

$$+ \sum_{x_2=c_2-x_1+1}^{n_2} TCR_2(N,n_1,n_2,c_1,r_1,r_2,x_1,x_2)h_{n_2}(x_2|x_1) \qquad (4.11)$$

That is,

$$S_0+n_1S_1+x_1S_2+A_0[1-h_{N-n_1}(X-x_1=0|x_1)]+(N-n_1)A_1+E[X-x_1|x_1]A_2$$

$$\le \sum_{x_2=0}^{c_2-x_1} \{S_0+(n_1+n_2)S_1+(x_1+x_2)S_2+A_0[1-h_{N-n_1-n_2}(X-x_1-x_2=0|x_1,x_2)]$$

$$+ (N-n_1-n_2)A_1+E[X-x_1-x_2|x_1,x_2]A_2\} \; h_{n_2}(x_2|x_1)$$

$$+ \sum_{x_2=c_2-x_1+1}^{n_2} \{S_0+(n_1+n_2)S_1+(x_1+x_2)S_2+R_0+(N-n_1-n_2)R_1+E[X-x_1-x_2|x_1,x_2]R_2\}$$

$$\cdot \; h_{n_2}(x_2|x_1) \qquad (4.12)$$

The above inequality may be determined once the values of $h_{N-n_1}(X-x_1=0|x_1)$, $E[X-x_1|x_1]$, $h_{N-n_1-n_2}(X-x_1-x_2=0|x_1,x_2)$, $E[X-x_1-x_2|x_1,x_2]$, and $h_{n_2}(x_2|x_1)$ have been decided. Then, the largest value of x_1 satisfying inequality (4.12) may be found; its value is designated c_1.

The cost functions (3.25), (3.31), and (3.32) are reconsidered to form another inequality used to determine the smallest value of x_1 such that:

$$\sum_{x_2=0}^{c_2-x_1} TCA_2(N,n_1,n_2,c_1,r_1,c_2,x_1,x_2)h_{n_2}(x_2|x_1)$$

$$+ \sum_{x_2=c_2-x_1+1}^{n_2} TCR_2(N,n_1,n_2,c_1,r_1,r_2,x_1,x_2)h_{n_2}(x_2|x_1)$$

$$> TCR_1(N,n_1,c_1,x_1) \qquad (4.13)$$

That is

$$\sum_{x_2=0}^{c_2-x_1} \{S_0+(n_1+n_2)S_1+(x_1+x_2)S_2+[1-h_{N-n_1-n_2}(X-x_1-x_2=0|x_1,x_2)]A_0$$

$$+ (N-n_1-n_2)A_1+E[X-x_1-x_2|x_1,x_2]A_2\}\ h_{n_2}(x_2|x_1)$$

$$+ \sum_{x_2=c_2-x_1+1}^{n_2} \{S_0+(n_1+n_2)S_1+(x_1+x_2)S_2+R_0+(N-n_1-n_2)R_1+E[X-x_1-x_2|x_1,x_2]R_2\}$$

$$\cdot\ h_{n_2}(x_2|x_1)$$

$$> \quad S_0+n_1S_1+x_1S_2+R_0+(N-n_1)R_1+E[X-x_1|x_1]R_2 \qquad (4.14)$$

The smallest value of x_1 satisfying inequality (4.14) is the optimum rejection number designated r_1.

The values of $h_{N-n_1-n_2}(X-x_1-x_2=0|x_1,x_2)$ and $E[X-x_1-x_2|x_1,x_2]$ have been established in the previous section. The values remaining undecided are $h_{N-n_1}(X-x_1=0|x_1)$, $E[X-x_1|x_1]$, and $h_{n_2}(x_2|x_1)$.

Find $h_{N-n_1}(X-x_1=0|x_1)$

The term $h_{N-n_1}(X-x_1=0|x_1)$ is the probability that the number of defectives in the entire lot is the same as the actual number found in the first sample. That is, it is the posterior probability of having no defectives in the rest of a lot of size $N-n_1$ given that x_1 defectives are observed in the first sample. This probability is usually quite small, except in the case where quality is extremely good and there are no defectives found in the sample because there are none in the lot.

For a Polya prior distribution, this probability may be found using Equation (3.8) for $h_{N-n_1}(X-x_1|x_1)$ and letting $X=x_1$:

$$h_{N-n_1}(X-x_1=0|x_1) = \frac{\Gamma(t+N-x_1)}{\Gamma(t+n_1-x_1)} \; \frac{\Gamma(s+t+n_1)}{\Gamma(s+t+N)} \; . \tag{4.15}$$

For a mixed binomial prior distribution, this probability may be found using Equation (3.14):

$$h_{N-n_1}(X-x_1=0|x_1) = \frac{\sum_{i=1}^{m} w_i \, p_i^{x_1} \, (1-p_i)^{N-x_1}}{\sum_{i=1}^{m} w_i \, p_i^{x_1} \, (1-p_i)^{n_1-x_1}} \tag{4.16}$$

<u>Finding $E[X-x_1|x_1]$</u>

This term is the expected value of the number of defectives remaining in the lot $(X-x_1)$ given that x_1 defectives have actually been observed in the first sample. Adapting Equation (4.7),

$$E[X-x_1|x_1] = \frac{(N-n_1)(x_1+1)}{(n_1+1)} \; \frac{g_{n_1+1}(x_1+1)}{g_{n_1}(x_1)} \tag{4.17}$$

For the Polya prior distribution, the posterior expectation is:

$$E[X-x_1|x_1] = \frac{(N-n_1)(x_1+1)}{(n_1+1)} \; \frac{\binom{n_1+1}{x_1+1} \frac{\Gamma(s+x_1+1)}{\Gamma(s)} \; \frac{\Gamma(t+n_1-x_1)}{\Gamma(t)} \; \frac{\Gamma(s+t)}{\Gamma(s+t+n_1+1)}}{\binom{n_1}{x_1} \frac{\Gamma(s+x_1)}{\Gamma(s)} \; \frac{\Gamma(t+n_1-x_1)}{\Gamma(t)} \; \frac{\Gamma(s+t)}{\Gamma(s+t+n_1)}}$$

$$= \frac{(N-n_1)(s+x_1)}{(s+t+n_1)} \tag{4.18}$$

For the mixed binomial prior distribution, the posterior expectation is:

$$E[X-x_1|x_1) = \frac{(N-n_1)(x_1+1) \; \sum_{i=1}^{m} w_i \binom{n_1+1}{x_1+1} p_i^{x_1+1} \; (1-p_i)^{n_1-x_1}}{(n_1+1) \; \sum_{i=1}^{m} w_i \binom{n_1}{x_1} \; p_i^{x_1} \; (1-p_i)^{n_1-x_1}} \tag{4.19}$$

Finding $h_{n_2}(x_2|x_1)$

This probability is the conditional distribution of the number of defectives found in a second sample x_2, given x_1 defectives are found in the first sample. In order to solve for $h_{n_2}(x_2|x_1)$, it is necessary to realize that the posterior distribution of first sample will be the prior distribution of the second sample.

For the Polya distribution, Equation (3.8) is the posterior following the first sample and the prior preceding the second sample. That is:

$$h_{N-n_1}(X-x_1|x_1) = \binom{N-n_1}{X-x_1} \frac{\Gamma(s+X)\Gamma(t+N-X)\Gamma(s+t+n_1)}{\Gamma(x+x_1)\Gamma(t+n_1-x_1)\Gamma(s+t+N)}$$

Appendix B shows that the conditional distribution for the second sample is:

$$h_{n_2}(x_2|x_1) = \binom{n_2}{x_2} \frac{\Gamma(s+x_1+x_2)\Gamma(t+n_1+n_2-x_1-x_2)\Gamma(s+t+n_1)}{\Gamma(s+x_1) \quad \Gamma(t+n_1-x_1) \qquad \Gamma(s+t+n_1+n_2)} \tag{4.20}$$

For the mixed binomial distribution, Equation (3.14) is the posterior following the first sample and the prior preceding the second sample. That is,

$$h_{N-n_1}(X-x_1|x_1) = \frac{\sum_{i=1}^{m} w_i \binom{N-n_1}{X-x_1} p_i^{X} (1-p_i)^{N-X}}{\sum_{i=1}^{m} w_i \; p_i^{x_1}(1-p_i)^{n_1-x_1}}$$

Appendix B shows that the conditional distribution for the second sample is:

$$h_{n_2}(x_2|x_1) = \sum_{i=1}^{m} \frac{w_i}{\sum_{i=1}^{m} w_i} \binom{n_2}{x_2} p_i^{\,x_2}(1-p_i)^{n_2-x_2} \tag{4.21}$$

Now that all terms in inequalities (4.12) and (4.14) are explained, the optimum first sample acceptance and rejection numbers (c_1,r_1) for any sample size pair (n_1,n_2) may be found. The value c_1 is the largest value of x_1 for which inequality (4.12) is satisfied; r_1 is the smallest value of x_1 for which inequality (4.14) is satisfied.

Using the above inequalities and a simple search procedure, the optimum acceptance and rejection numbers for the first and second samples can be found explicitly for any sample size pair (n_1,n_2) of interest. There is no need to include decision variables (c_1,r_1,c_2,r_2) in an extensive and time consuming search.

Optimum Sample Size Pair

Optimizing the sample size pair involves finding the values of n_1 and n_2, with their corresponding vector (c_1,r_1,c_2,r_2) that minimize the total expected cost function (3.33). This might be done by trying every possible sample size pair, determining the optimum c_1,r_1,c_2,r_2 for each as outlined previously, and evaluating each set of decision variables in Equation (3.33). This, however, is time consuming and likely infeasible. Normally, double sampling plans have a consistent relationship between n_1 and n_2 such that n_2 = Constant x n_1. If this condition is accepted, the only decision variable remaining to be solved is the first sample size n_1. A search procedure follows for selecting the optimal sampling plan.

Cost Surface

It is unknown whether the total cost surface as a function of n_1 is truly convex. Yet, it is reasonably well behaved as shown in Figure IV.1. The value of $TC(n_1,n_2,c_1,r_1,c_2,r_2)$ makes successive dips, each dip being associated with a particular acceptance/rejection number vector.

The minimum point of each dip becomes lower and lower, up to a point (the global optimum) at which time it begins to increase. It has been observed in this and previous research that the locus of TC values associated with an acceptance/rejection number vector is nearly convex, occasionally having only a small ripple containing, say, two local minima. It is suspected that these minor ripples are due to computer roundoff mechanisms. Since these are always so close in total cost, nothing practical is lost by treating each dip as strictly convex with but one local optimum. Also, the locus of the local minima have but one global optimum over all possible sample sizes. Finally, another observed property is that the sample size n_1 at the global minimum total cost occurs approximately midway between the sample sizes at which the next lower and higher acceptance/rejection number vectors become optimum. Utilizing these properties, a search procedure follows for finding the optimum double sampling plan $(n_1,n_2,c_1,r_1,c_2,r_2)$.

Search Procedure

The procedure developed and programmed is to find the midpoint of the range of sample sizes for which the first acceptance/rejection number vector (c_1,r_1,c_2,r_2) is optimum. Then, the sample size n_1 is increased (as is n_2) until the midpoint of the range of sample sizes for the next acceptance/rejection number vector is determined. At each

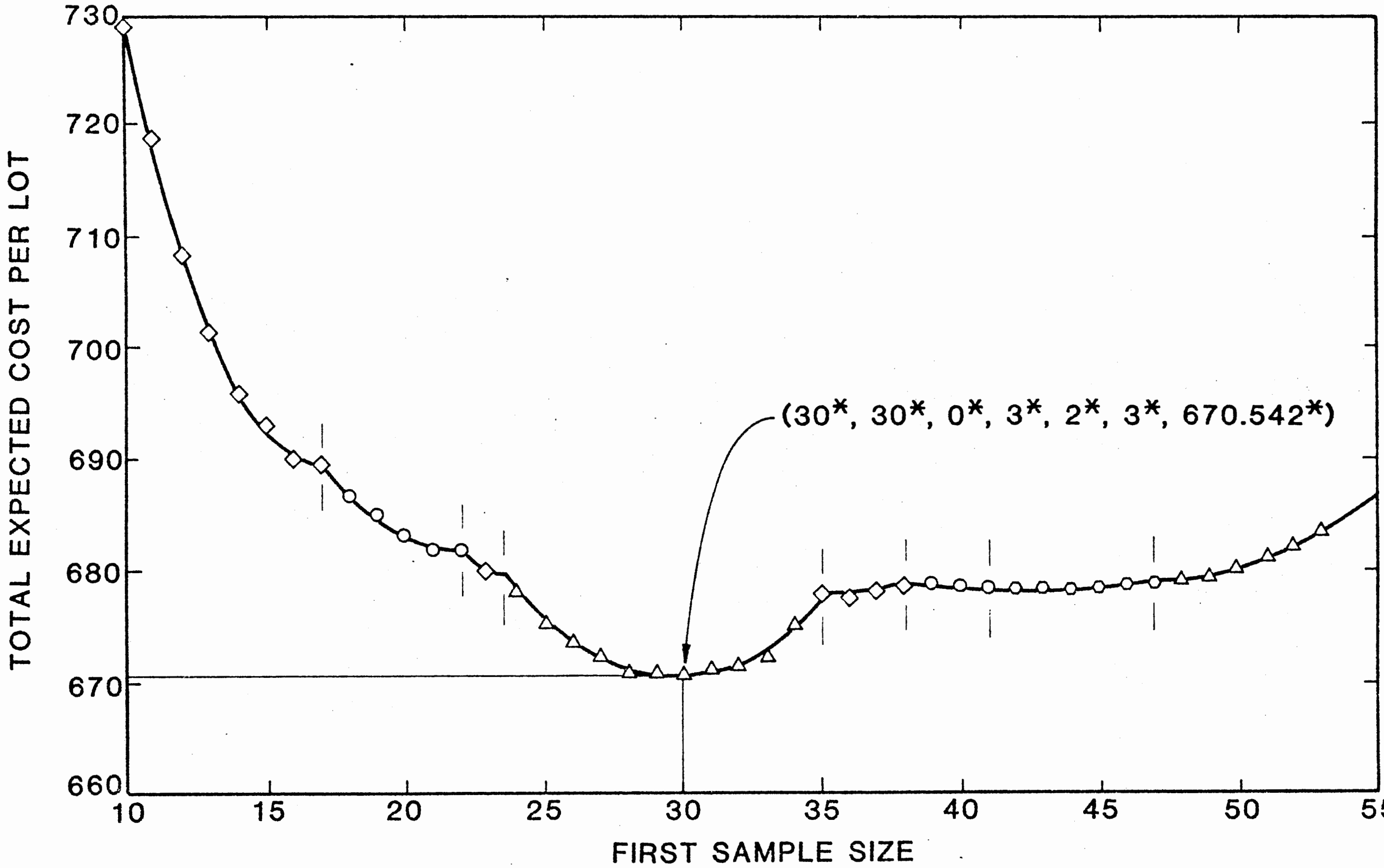


Figure IV.1. Total Expected Cost Per Lot Response Surface as a Function of First Sample Size, Using Optimum Acceptance/Rejection Number Vector for Each Sample Size Pair (The parameters $(w_1,w_2,w_3,f_1,f_2,f_3)=(.6,.3,.1,.01,.1,.3)$ $(S_0,S_1,S_2,A_0,A_1,A_2,R_0,R_1,R_2)=(3.0,2.5,1.9,10.0,6.0,40.0,5.0,2.0,1.9)$.)

midpoint, the total cost is evaluated. This procedure continues until the total cost at a midpoint just begins to increase over that at the previous midpoint.

The search procedure then returns to the range of the last three acceptance/rejection number vectors. One by one, the sample sizes around each midpoint are evaluated and compared until the lowest cost is found. This minimum cost is then taken as the global optimum. An interactive computer program performs these calculations, within a format suitable for use by industry and government.

Summary

This chapter develops the theoretically exact analytical and search procedures for economically optimizing a double-sampling plan using a discrete mathematical model with the fixed cost expansion. Based upon the analysis and design in this chapter, the following results may be determined:

(1) Optimum acceptance and rejection numbers for the combined first and second samples (c_2* and r_2*).

(2) Optimum acceptance and rejection numbers for the first sample (c_1* and r_1*).

(3) Optimum double-sampling size pair and corresponding acceptance/rejection number vector (n_1*, n_2*, c_1*, r_1*, c_2*, r_2*).

The above original methodology is developed using a break-even approach and an appropriate search procedure. An interactive computer program is established for use by government and industry; its operation is covered in the next chapter.

CHAPTER V

USING THE INTERACTIVE COMPUTER PROGRAM

Introduction

This chapter details the use of an interactive computer program which permits easy utilization of the design, and evaluation methodology presented in Chapters III and IV. The actual FORTRAN program is documented and appears in Appendix A. It has been implemented on an IBM 370/168 using various time share terminals.

The entire program is interactive, and the user is prompted for all necessary inputs by the computer. Many typical or often-used values of inputs are pre-programmed. These are presented to the user for either verification or change. If the user changes any values, they are again presented for verification or change. Only when a set of inputs has been verified does the program continue.

When several values are to be input, they need only be separated by a comma or a space. With the prompting and verification feature, the input mechanism is virtually self-explanatory, as long as the user understands the terms being input and their mathematically feasible ranges. All relevant mathematical and computer terms and notation are explained in Chapters III and IV.

Overview

The modified Guthrie-Johns computer program provides the capability for three major activities:

(1) Design an economically based sampling plan.

(2) Design the optimum acceptance/rejection number vector, given the sample size pair.

(3) Evaluate the expected cost of a sampling plan.

The flowchart of all major activities is presented in Figure V.1.

Designing an economically based sampling plan refers to the selection of the sample sizes (n_1, n_2), acceptance numbers (c_1, c_2), and rejection numbers (r_1, r_2) needed to minimize the expected total cost per lot. Designing the optimum acceptance/rejection number vector refers to minimizing the expected total cost per lot given a prespecification of the sample size pair (n_1, n_2). Evaluating the expected total cost of a sampling plan refers to calculating the expected total cost per lot for any desired double sampling plan.

The program begins by stating the three tasks for which it may be used:

```
THIS PROGRAM PERMITS YOU TO DO THE FOLLOWING THINGS:
(1) DESIGN AN ECONOMICALLY BASED SAMPLING PLAN
(2) DESIGN THE OPT ACC/REJ VECTOR,GIVEN SAMP SIZE PAIR
(3) EVALUATE THE EXPECTED COST OF A SAMPLING PLAN
WHICH DO YOU WANT TO DO ? ENTER 1 , 2 ,OR 3
?
1
```

The user has entered a "1," indicating a desire to design an economically based sampling plan.

Designing An Economically Based Sampling Plan

Before proceeding with sampling plan design, the program verifies the user's selection:

```
YOU WANT TO DESIGN AN ECONOMICALLY BASED SAMPLING PLAN  !
CORRECT ? NO (0) OR YES (1)
?
1
```

The user responds by confirming the desire to design an economically

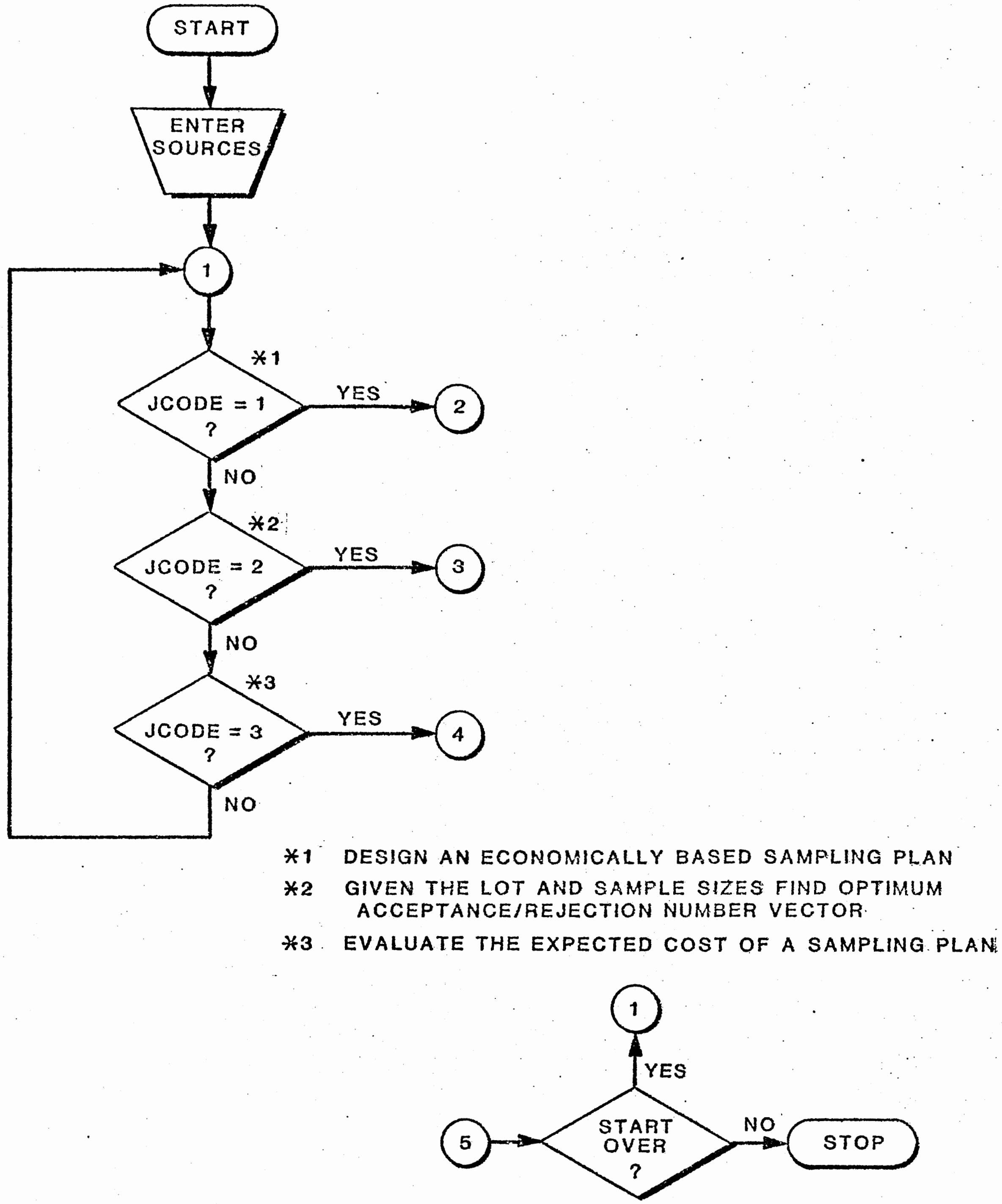
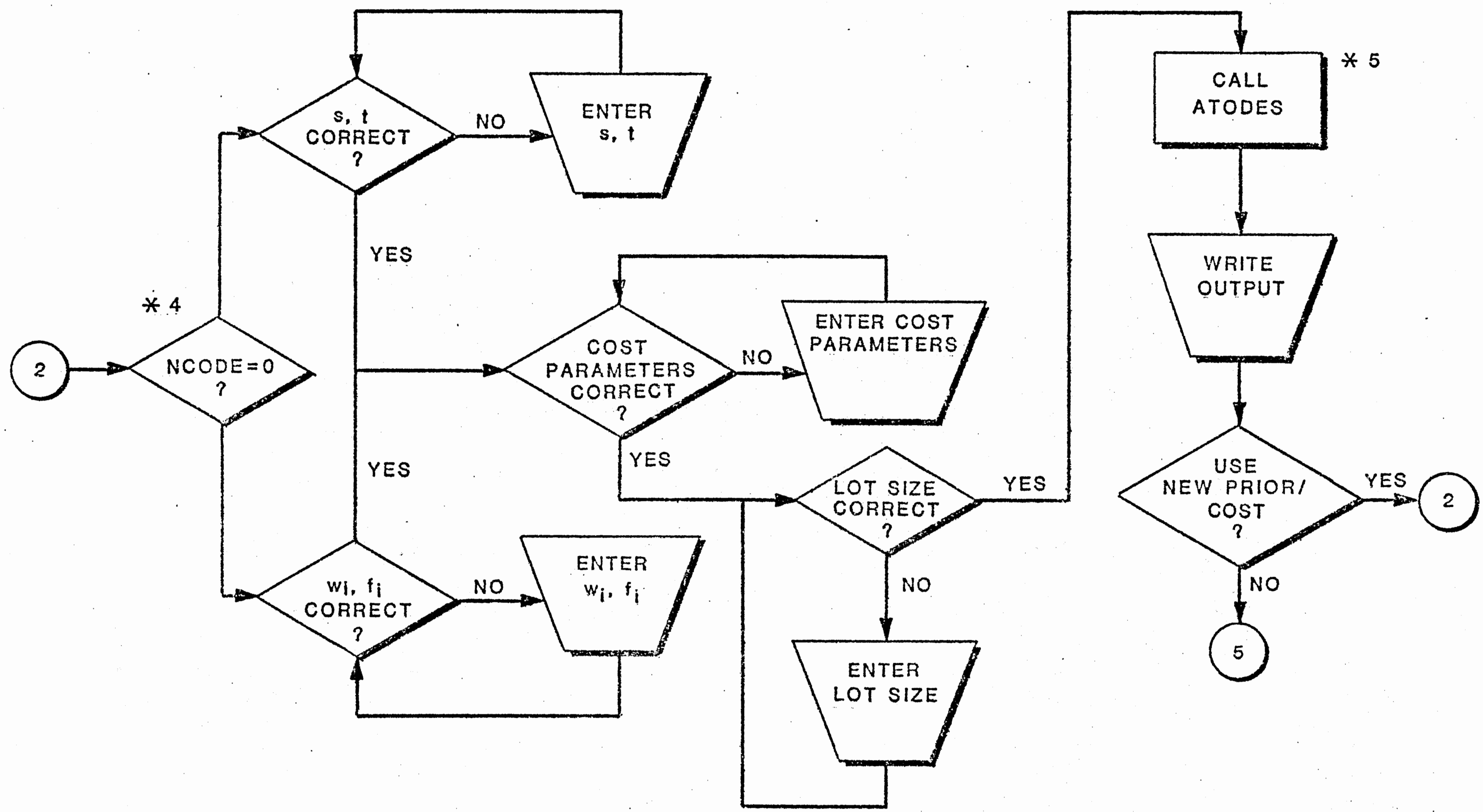


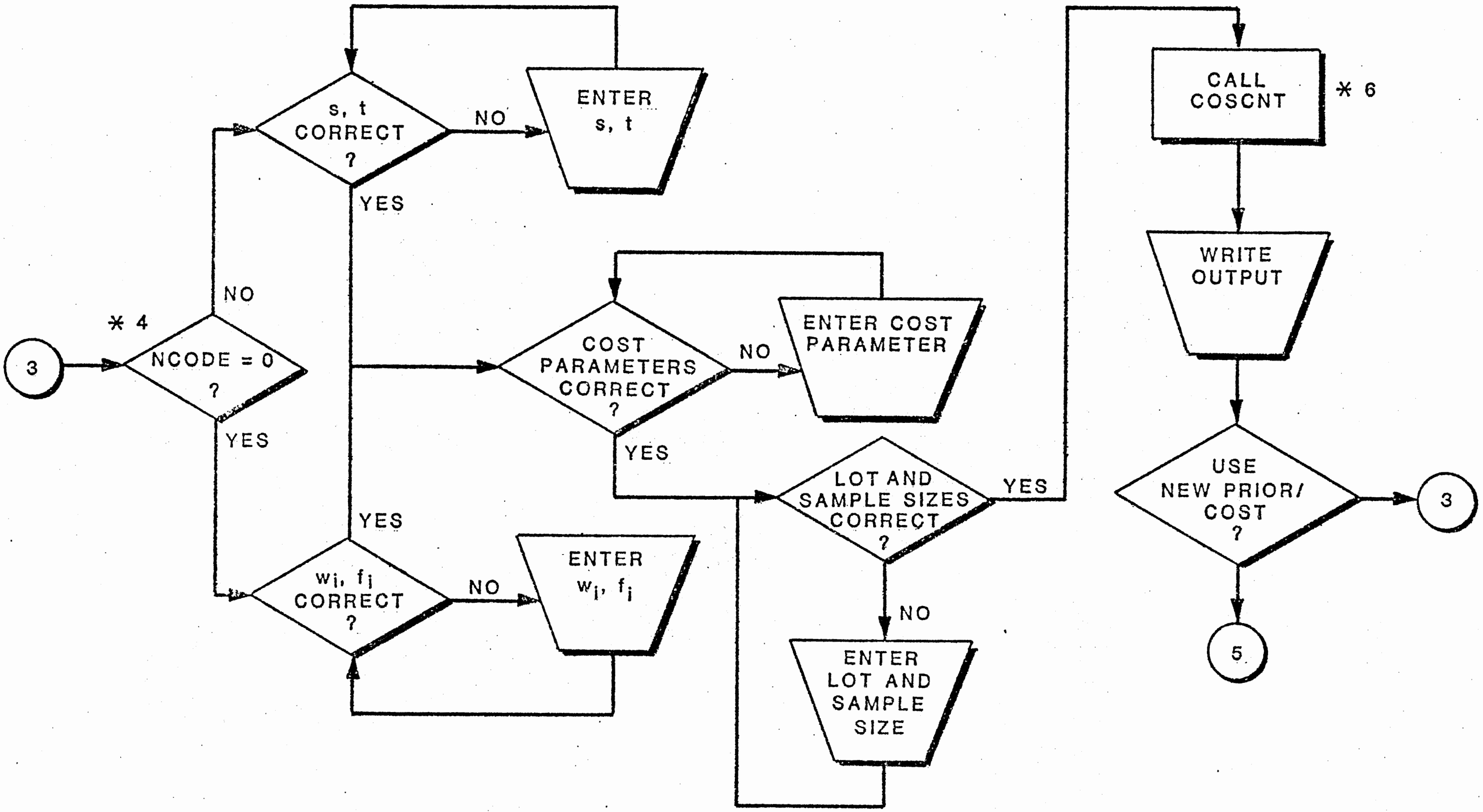
Figure V.1. Flow Chart of All Major Activities in Modified Guthrie-Johns Computer Program



*4 IF NCODE=0, PRIOR DISTRIBUTION IS MIXED BINOMIAL; OTHERWISE, IT IS POLYA

*5 ATODES AUTOMATICALLY DESIGN AN ECONOMICALLY BASED SAMPLING PLAN

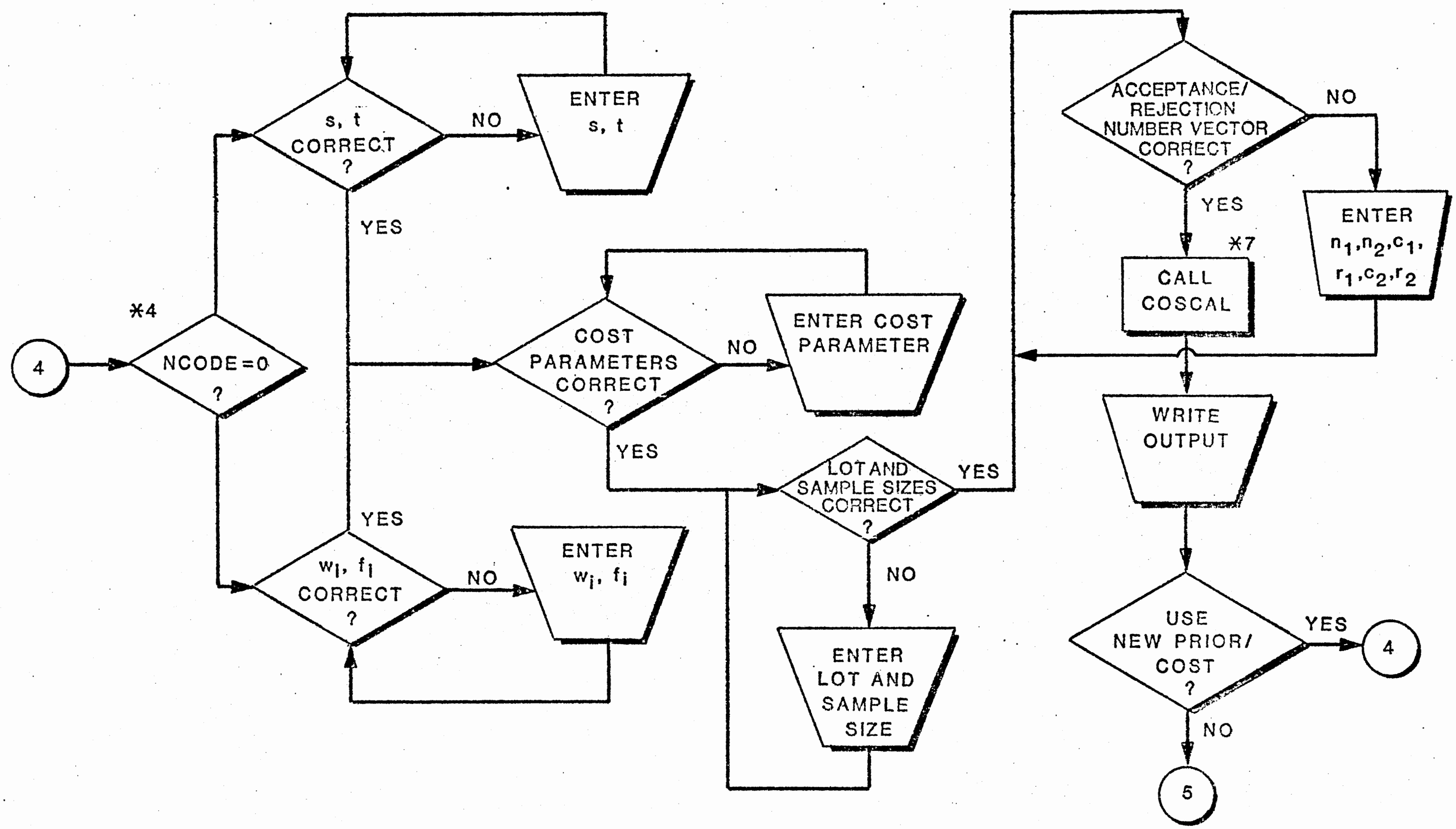
Figure V.1. (Continued)



* 4 IF NCODE = 0, PRIOR DISTRIBUTION IS MIXED BINOMIAL; OTHERWISE, IT IS POLYA

* 6 COSCNT AUTOMATICALLY DESIGNS AN ECONOMICALLY BASED DOUBLE SAMPLING PLAN GIVEN THE SAMPLE SIZE PAIR

Figure V.1. (Continued)



*4 IF NCODE = 0, PRIOR DISTRIBUTION IS MIXED BINOMIAL; OTHERWISE, IT IS POLYA
*7 COSCAL CALCULATES THE EXPECTED TOTAL COST OF A DOUBLE SAMPLING PLAN

Figure V.1. (Continued)

based sampling plan. Had an error been made, the user would input a "0" and the program would start over automatically.

The user is next asked whether the Bayesian prior is a mixed binomial or Polya. In the following illustration, a Polya distribution is selected:

```
WHICH IS THE PRIOR DISTRIBUTION???
     MIXED BINOMIAL(0) OR POLYA(1)
?
1
```

The current parameters of the Polya distribution are then displayed for verification. In the following illustration, the Polya parameters are correct:

```
POLYA PARAMETERS ARE S=  0.462103   T=  6.539455
CORRECT??? NO(0) OR YES(1)
?
1
```

During subsequent runs of the program, the Polya parameters will remain fixed at these values unless changed. The nine cost values are next displayed for verification. In the following illustration, the cost factors are correct:

```
COST VALUES ARE SO=  3.00   S1=  2.50   S2=  1.90   AO= 10.00
A1=  0.0    A2= 40.00   RO=  5.00   R1=  2.00   R2=  1.90
CORRECT??? NO(0) OR YES(1)
?
1
```

The constant factor is 1:

```
CONSTANT FACTOR =  1.00
CORRECT??? NO(0) OR YES(1)
?
1
```

The lot size is next displayed for verification. In the following illustration, the lot size is correct:

```
LOT SIZE =    500.00
CORRECT??? NO(0) OR YES(1)
?
1
```

At this point, all necessary data have been entered in order to design the economically optimum double sampling plan. Output of the results begins with a statement that this is an economically based sampling plan design. The lot size, optimal sample sizes (n_1*, n_2*), distribution parameters, and cost values are then listed to provide the user with a permanent record of all relevant input. Next, the optimum acceptance and rejection numbers (c_1*, r_1*, c_2*, r_2*) are listed. The last item output is the minimum expected total cost per lot.

```
**********************************************************************
**********************************************************************
**********************************************************************

ECONOMICALLY BASED DOUBLE SAMPLING PLAN DESIGN
LOT SIZE =    500.0 1ST SAMP SIZE =    26.0 2ND SAMP SIZE =    26.0
POLYA PARAMETERS ARE S=  0.462103    T=  6.539455
COST VALUES ARE SO=  3.00    S1=  2.50    S2=  1.90    AO= 10.00
A1=  0.0     A2= 40.00    RO=  5.00    R1=  2.00    R2=  1.90
ACC NO 1 =    0.0 RJ NO 1 =    3.0 ACC NO 2 =    2.0 RJ NO 2 =    3.0
TOTAL COST =          712.344

**********************************************************************
**********************************************************************
**********************************************************************
```

The opportunity to design another sampling plan, changing the prior distribution, parameters, and/or costs, is then offered. In the following illustration, the user does exercise this option:

```
 WANT TO DESIGN PLAN USING NEW PRIOR/COST
    PARAMETERS??? NO(0) OR YES(1)
?
1
```

At this time, the program again requests the user to input the prior distribution. In the following illustration, a Polya distribution is requested again.

```
 WHICH IS THE PRIOR DISTRIBUTION???
      MIXED BINOMIAL(0) OR POLYA(1)
?
1
```

The current parameters of the Polya prior distribution are then displayed for verification. In the following illustration, the Polya parameters are not correct:

```
 POLYA PARAMETERS ARE S=  0.462103    T=  6.539455
 CORRECT??? NO(0) OR YES(1)
?
0
```

The user is then told to enter the two Polya parameters. These values must be non-negative; however, they need not be integers. The entries must be separated by a comma or a space:

```
ENTER S,T
?
.679445,7.89941
```

The Polya parameters are again displayed for verification and found to be correct. During subsequent runs of the program, the Polya parameters will remain fixed at these values unless changed:

```
POLYA PARAMETERS ARE S=  0.679445    T=  7.899410
CORRECT??? NO(0) OR YES(1)
?
1
```

From this point, the program operates exactly as described previously, providing the opportunity to modify cost values, constant factor, and the lot size. Then, the results are presented:

```
COST VALUES ARE SO=  3.00    S1=  2.50    S2=  1.90    AO= 10.00
A1=  0.0     A2= 40.00    RO=  5.00    R1=  2.00    R2=  1.90
CORRECT??? NO(0) OR YES(1)
?
1
CONSTANT FACTOR =  1.00
CORRECT??? NO(0) OR YES(1)
?
1
LOT SIZE =     500.00
CORRECT??? NO(0) OR YES(1)
?
1

**********************************************************************
**********************************************************************
**********************************************************************

ECONOMICALLY BASED DOUBLE SAMPLING PLAN DESIGN
LOT SIZE =    500.0 1ST SAMP SIZE =    27.0 2ND SAMP SIZE =    27.0
POLYA PARAMETERS ARE S=  0.679445    T=  7.899410
COST VALUES ARE SO=  3.00    S1=  2.50    S2=  1.90    AO= 10.00
A1=  0.0     A2= 40.00    RO=  5.00    R1=  2.00    R2=  1.90
ACC NO 1 =    0.0 RJ NO 1 =    3.0 ACC NO 2 =    2.0 RJ NO 2 =    3.0
TOTAL COST =           838.851

**********************************************************************
**********************************************************************
**********************************************************************
```

Again, the opportunity to design a sampling plan using a new prior distribution and/or cost parameters is offered. In the following illustration, a Polya distribution and different cost parameters are presented:

```
 WANT TO DESIGN PLAN USING NEW PRIOR/COST
    PARAMETERS??? NO(0) OR YES(1)
?
1
 WHICH IS THE PRIOR DISTRIBUTION???
      MIXED BINOMIAL(0) OR POLYA(1)
?
1
 POLYA PARAMETERS ARE S=  0.679445   T=  7.899410
 CORRECT??? NO(0) OR YES(1)
?
1
 COST VALUES ARE SO=  3.00    S1=  2.50    S2=  1.90   AO= 10.00
 A1=  0.0     A2= 40.00    RO=  5.00    R1=  2.00    R2=  1.90
 CORRECT??? NO(0) OR YES(1)
?
0
 ENTER SO,S1,S2,AO,A1,A2,RO,R1, AND R2
?
3,2.5,1.56,10,.,40,5,2,1.56
 COST VALUES ARE SO=  3.00    S1=  2.50    S2=  1.56    AO= 10.00
 A1=  0.0     A2= 40.00    RO=  5.00    R1=  2.00    R2=  1.56
 CORRECT??? NO(0) OR YES(1)
?
1
 CONSTANT FACTOR =  1.00
 CORRECT??? NO(0) OR YES(1)
?
1
 LOT SIZE =     500.00
 CORRECT??? NO(0) OR YES(1)
?
1
 *********************************************************************
 *********************************************************************
 *********************************************************************

 ECONOMICALLY BASED DOUBLE SAMPLING PLAN DESIGN
 LOT SIZE =    500.0 1ST SAMP SIZE =    29.0 2ND SAMP SIZE =    29.0
 POLYA PARAMETERS ARE S=  0.679445   T=  7.899410
 COST VALUES ARE SO=  3.00    S1=  2.50    S2=  1.56    AO= 10.00
 A1=  0.0     A2= 40.00    RO=  5.00    R1=  2.00    R2=  1.56
 ACC NO 1 =    0.0 RJ NO 1 =    3.0 ACC NO 2 =    2.0 RJ NO 2 =    3.0
 TOTAL COST =          827.382

 *********************************************************************
 *********************************************************************
 *********************************************************************
```

Again, the opportunity to design a sampling plan using new prior/cost parameters is offered. In the following illustration, a mixed binomial distribution is selected:

```
 WANT TO DESIGN PLAN USING NEW PRIOR/COST
    PARAMETERS??? NO(0) OR YES(1)
?
1
 WHICH IS THE PRIOR DISTRIBUTION???
      MIXED BINOMIAL(0) OR POLYA(1)
?
0
```

The current parameters of the mixed binomial distribution are then displayed for verification. In the following illustration, the mixed binomial parameters are not correct:

```
MIXED BINOMIAL PARAMETERS ARE W1=0.6000   W2=0.3000
W3=0.1000   F1= 0.0100000   F2= 0.1000000   F3= 0.3000000
CORRECT??? NO(0) OR YES(1)
?
0
```

The user is then told to enter the six mixed binomial parameters. First, however, the user is reminded that the three weights (w_1, w_2, w_3) must sum to 1 and all must be positive. Also, the three process fractions defective (f_1, f_2, f_3) must be between 0 and 1, but not 0 or 1. A value of 0.0 would indicate a perfectly operating process, and would normally be a legitimate entry; however, certain mathematical operations disallow the use of a 0, and a .0000001 is recommended instead. Similarly, a .9999999 is recommended in place of a 1. Even if one or two of the weights are 0, the corresponding process fraction defective must be entered:

```
REMEMBER, W1+W2+W3=1.0 AND ALL MUST BE POSITIVE
ALSO, F1, F2, AND F3 MUST BE BETWEEN 0 AND 1, BUT NOT 0 OR 1
ENTER W1,W2,W3,F1,F2,F3
?
.58,.3,.12,.01,.1,.3
```

The mixed binomial parameters are again displayed for verification and found to be correct. During subsequent runs of the program the mixed binomial parameters will remain fixed at these values unless changed:

```
MIXED BINOMIAL PARAMETERS ARE W1=0.5800   W2=0.3000
W3=0.1200   F1= 0.0100000   F2= 0.1000000   F3= 0.3000000
CORRECT??? NO(0) OR YES(1)
?
1
```

The nine cost values and lot size are next displayed for verification. In the following illustration, both of them are correct:

```
COST VALUES ARE SO=  3.00   S1=  2.50   S2=  1.56   AO= 10.00
A1=  0.0    A2= 40.00   RO=  5.00   R1=  2.00   R2=  1.56
CORRECT??? NO(O) OR YES(1)
?
1
CONSTANT FACTOR =  1.00
CORRECT??? NO(O) OR YES(1)
?
1
LOT SIZE =    500.00
CORRECT??? NO(O) OR YES(1)
?
1
*********************************************************************
*********************************************************************
*********************************************************************

ECONOMICALLY BASED DOUBLE SAMPLING PLAN DESIGN
LOT SIZE =   500.0 1ST SAMP SIZE =   30.0 2ND SAMP SIZE =   30.0
MIXED BINOMIAL PARAMETERS ARE W1=0.5800   W2=0.3000
W3=0.1200   P1= 0.0100000   P2= 0.1000000   P3= 0.3000000
COST VALUES ARE SO=  3.00   S1=  2.50   S2=  1.56   AO= 10.00
A1=  0.0    A2= 40.00   RO=  5.00   R1=  2.00   R2=  1.56
ACC NO 1 =   0.0 RJ NO 1 =   3.0 ACC NO 2 =   2.0 RJ NO 2 =   3.0
TOTAL COST =        679.475

*********************************************************************
*********************************************************************
*********************************************************************
```

Again, the opportunity to design a sampling plan using new prior/ cost parameters is offered. In the following illustration, this opportunity is declined:

```
WANT TO DESIGN PLAN USING NEW PRIOR/COST
   PARAMETERS??? NO(O) OR YES(1)
?
0
```

The user is then given the opportunity to begin the program over; this option is accepted:

```
WANT TO START OVER ??? NO(O) OR YES(1)
?
1
```

Designing the Optimum Acceptance/Rejection Number Vector

The program again lists the three activities permitted and request the user's choice. In the following illustration, design of the optimum acceptance/rejection number vector given the sample size pair is selected and verified:

```
 THIS PROGRAM PERMITS YOU TO DO THE FOLLOWING THINGS:
 (1) DESIGN AN ECONOMICALLY BASED SAMPLING PLAN
 (2) DESIGN THE OPT ACC/REJ VECTOR,GIVEN SAMP SIZE PAIR
 (3) EVALUATE THE EXPECTED COST OF A SAMPLING PLAN
 WHICH DO YOU WANT TO DO ? ENTER 1 , 2 ,OR 3
?
2
 YOU WANT TO DESIGN OPT ACC/REJ VECTOR GIVEN SAMPLE SIZE PAIR !
 CORRECT ? NO (0) OR YES (1)
?
1
```

The prior distribution and costs are again presented for verification and possible modification:

```
 WHICH IS THE PRIOR DISTRIBUTION???
      MIXED BINOMIAL(0) OR POLYA(1)
?
0
 MIXED BINOMIAL PARAMETERS ARE W1=0.5800   W2=0.3000
 W3=0.1200   F1= 0.0100000   F2= 0.1000000   F3= 0.3000000
 CORRECT??? NO(0) OR YES(1)
?
1
 COST VALUES ARE SO=  3.00   S1=  2.50   S2=  1.56   AO= 10.00
 A1=  0.0    A2= 40.00   RO=  5.00   R1=  2.00   R2=  1.56
 CORRECT??? NO(0) OR YES(1)
?
1
```

Next, the lot size, and sample sizes are presented for verification. In the following illustration, the sample sizes are to be changed:

```
 LOT SIZE =   500.0 1ST SAMP SIZE =   60.0 2ND SAMP SIZE =   60.0
 CORRECT??? NO(0) OR YES(1)
?
0
```

The user is then told to enter the new lot size and sample sizes:

```
 ENTER LOT SIZE,1ST SAMP SIZE,AND 2ND SAMP SIZE
?
500,40,40
```

These values are again presented for verification:

```
 LOT SIZE =   500.0 1ST SAMP SIZE =   40.0 2ND SAMP SIZE =   40.0
 CORRECT??? NO(0) OR YES(1)
?
1
```

At this point, all necessary data have been entered in order to find an optimum acceptance/rejection number vector. Output of the results begins with a statement that this optimum acceptance/rejection number vector design. The lot size, sample sizes, prior distribution, cost parameters, and optimum acceptance and rejection numbers are presented. Four additional cost terms are primarily useful for someone attempting to compare numerical results with the model formulation presented in Chapters III and IV. The general user will be interested only in the expected total cost:

```
*******************************************************************
*******************************************************************
*******************************************************************

OPTIMUM ACC/REJ NUMBER VECTOR DESIGN
LOT SIZE =   500.0 1ST SAMP SIZE =   40.0 2ND SAMP SIZE =   40.0
MIXED BINOMIAL PARAMETERS ARE W1=0.5800   W2=0.3000
W3=0.1200   F1= 0.0100000   F2= 0.1000000   F3= 0.3000000
COST VALUES ARE S0=  3.00   S1=  2.50   S2 =  1.56   A0 = 10.00
A1 =  0.0    A2 = 40.00   R0 =  5.00   R1 =  2.00   R2 =  1.56
ACC NO 1 =   1.0 REJ NO 1 =   3.0 ACC NO 2 =   3.0 REJ NO 2 =   4.0
ACC 1ST SAMP COST =    209.18   REJ 1ST SAMP COST =    413.12
ACC 2ND SAMP COST =     17.67   REJ 2ND SAMP COST =     46.03
TOTAL COST =          685.990

*******************************************************************
*******************************************************************
*******************************************************************
```

The user is given the opportunity to evaluate another lot size or/and sample sizes, but maintaining the same prior distribution, parameters, and costs. In the following illustration, the user does exercise this option:

```
WANT TO EVAL ANOTHER SAMP PLAN USING LOT, SAMP SIZES    ???
NO(0) OR YES(1)
?
1
```

The new lot size and sample sizes are requested:

```
ENTER LOT SIZE,1ST SAMP SIZE,AND 2ND SAMP SIZE
?
400,25,25
```

and verified:

```
LOT SIZE =   400.0 1ST SAMP SIZE =   25.0 2ND SAMP SIZE =   25.0
CORRECT??? NO(0) OR YES(1)
?
1
```

The new results are again printed:

```
**********************************************************************
**********************************************************************
**********************************************************************

OPTIMUM ACC/REJ NUMBER VECTOR DESIGN
LOT SIZE =    400.0 1ST SAMP SIZE =    25.0 2ND SAMP SIZE =    25.0
MIXED BINOMIAL PARAMETERS ARE W1=0.5800   W2=0.3000
W3=0.1200    F1= 0.0100000    F2= 0.1000000    F3= 0.3000000
COST VALUES ARE SO=  3.00    S1=  2.50    S2 =  1.56    AO = 10.00
A1 =  0.0     A2 = 40.00    RO =  5.00    R1 =  2.00    R2 =  1.56
ACC NO 1 =    0.0 REJ NO 1 =    3.0 ACC NO 2 =    2.0 REJ NO 2 =    3.0
ACC 1ST SAMP COST =    135.63  REJ 1ST SAMP COST =    242.95
ACC 2ND SAMP COST =     67.87  REJ 2ND SAMP COST =    111.20
TOTAL COST =          557.661

**********************************************************************
**********************************************************************
**********************************************************************
```

Again, the opportunity to evaluate a new lot or sample sizes is offered and declined:

```
WANT TO EVAL ANOTHER SAMP PLAN USING LOT, SAMP SIZES    ???
NO(0) OR YES(1)
?
0
```

The opportunity to evaluate using new prior/cost parameters is offered. In the following illustration the user does exercise this option:

```
WANT TO DO ECON EVAL USING NEW PRIOR/COST
     PARAMETERS ??? NO(0) OR YES(1)
?
1
```

At this time, the program requests the user to input the prior distribution. In the following illustration, a Polya distribution is requested:

```
WHICH IS THE PRIOR DISTRIBUTION???
     MIXED BINOMIAL(0) OR POLYA(1)
?
1
```

Note that the most recent parameters of Polya distribution are again displayed for verification and possible change, as are the costs, lot size, and sample sizes. Any of these may be changed if desired. The results are then printed:

```
POLYA PARAMETERS ARE S=  0.679445    T=  7.899410
CORRECT??? NO(0) OR YES(1)
?
0
```

```
ENTER S,T
?
.462103,6.539455
 POLYA PARAMETERS ARE S=  0.462103   T=  6.539455
 CORRECT??? NO(0) OR YES(1)
?
1
 COST VALUES ARE SO=  3.00   S1=  2.50   S2=  1.56   AO= 10.00
 A1=  0.0    A2= 40.00   RO=  5.00   R1=  2.00   R2=  1.56
 CORRECT??? NO(0) OR YES(1)
?
1
 LOT SIZE =   400.0 1ST SAMP SIZE =   25.0 2ND SAMP SIZE =   25.0
 CORRECT??? NO(0) OR YES(1)
?
0
 ENTER LOT SIZE,1ST SAMP SIZE,AND 2ND SAMP SIZE
?
500,30,30
 LOT SIZE =   500.0 1ST SAMP SIZE =   30.0 2ND SAMP SIZE =   30.0
 CORRECT??? NO(0) OR YES(1)
?
1
 *******************************************************************
 *******************************************************************
 *******************************************************************

 OPTIMUM ACC/REJ NUMBER VECTOR DESIGN
 LOT SIZE =   500.0 1ST SAMP SIZE =   30.0 2ND SAMP SIZE =   30.0
 POLYA PARAMETERS ARE S=  0.462103   T=  6.539455
 COST VALUES ARE SO=  3.00   S1=  2.50   S2 =  1.56   AO = 10.00
 A1 =  0.0    A2 = 40.00   RO =  5.00   R1 =  2.00   R2 =  1.56
 ACC NO 1 =   0.0 REJ NO 1 =   4.0 ACC NO 2 =   3.0 REJ NO 2 =   4.0
 ACC 1ST SAMP COST =    142.18  REJ 1ST SAMP COST =    237.58
 ACC 2ND SAMP COST =   164.99   REJ 2ND SAMP COST =    162.81
 TOTAL COST =          707.561

 *******************************************************************
 *******************************************************************
 *******************************************************************
```

Again, the opportunity to find a new plan, or even use new prior/ cost parameters is offered. In the following illustration, these opportunities are declined:

```
 WANT TO EVAL ANOTHER SAMP PLAN USING LOT, SAMP SIZES    ???
 NO(0) OR YES(1)
?
0
 WANT TO DO ECON EVAL USING NEW PRIOR/COST
      PARAMETERS ??? NO(0) OR YES(1)
?
0
```

The user is then given the opportunity to start the program over again. In the following illustration, this option is accepted:

```
 WANT TO START OVER ??? NO(0) OR YES(1)
?
1
```

Evaluating the Expected Cost of a Sampling Plan

Once again, the program will list three activities permitted and request the user's choice. In the following illustration, evaluating the expected cost of a sampling plan is selected and verified.

```
 THIS PROGRAM PERMITS YOU TO DO THE FOLLOWING THINGS:
 (1) DESIGN AN ECONOMICALLY BASED SAMPLING PLAN
 (2) DESIGN THE OPT ACC/REJ VECTOR,GIVEN SAMP SIZE PAIR
 (3) EVALUATE THE EXPECTED COST OF A SAMPLING PLAN
 WHICH DO YOU WANT TO DO ? ENTER 1 , 2 ,OR 3
?
3
 YOU WANT TO EVALUATE THE EXPECTED COST OF A SAMPLING PLAN !
 CORRECT ? NO (0) OR YES (1)
?
1
```

The prior distribution and costs are again presented for verification and possible modification. The cost parameters are not correct and all to be changed or verified:

```
 WHICH IS THE PRIOR DISTRIBUTION???
      MIXED BINOMIAL(0) OR POLYA(1)
?
0
 MIXED BINOMIAL PARAMETERS ARE W1=0.5800   W2=0.3000
 W3=0.1200   F1= 0.0100000   F2= 0.1000000   F3= 0.3000000
 CORRECT??? NO(0) OR YES(1)
?
1
 COST VALUES ARE SO=  3.00   S1=  2.50   S2=  1.56   AO= 10.00
 A1=  0.0    A2= 40.00   RO=  5.00   R1=  2.00   R2=  1.56
 CORRECT??? NO(0) OR YES(1)
?
0
 ENTER SO,S1,S2,AO,A1,A2,RO,R1, AND R2
?
3,2.5,1.9,10,0,40,5,2,1.9
 COST VALUES ARE SO=  3.00   S1=  2.50   S2=  1.90   AO= 10.00
 A1=  0.0    A2= 40.00   RO=  5.00   R1=  2.00   R2=  1.90
 CORRECT??? NO(0) OR YES(1)
?
1
```

Next, the lot size, sample sizes, acceptance, and rejection numbers are presented for verification:

```
 LOT SIZE =   500.0 1ST SAMP SIZE =   30.0 2ND SAMP SIZE =   30.0
 CORRECT??? NO(0) OR YES(1)
?
1
 ACC/REJ NUMBERS ARE C1=   0.0 R1=   4.0 C2=   3.0 R2=   4.0
 CORRECT??? NO(0) OR YES(1)
?
1
```

At this point, all necessary data have been entered in order to evaluate the expected cost of a sampling plan. Output is listed as follows:

```
*********************************************************************
*********************************************************************
*********************************************************************

EXPECTED COST EVALUATION
LOT SIZE =   500.0 1ST SAMP SIZE =   30.0 2ND SAMP SIZE =   30.0
MIXED BINOMIAL PARAMETERS ARE W1=0.5800   W2=0.3000
W3=0.1200   F1= 0.0100000   F2= 0.1000000   F3= 0.3000000
COST VALUES ARE SO=  3.00   S1=  2.50   S2 =  1.90   AO = 10.00
A1 =  0.0    A2 = 40.00   RO =  5.00   R1 =  2.00   R2 =  1.90
ACC NO 1 =   0.0 REJ NO 1 =   4.0 ACC NO 2 =   3.0 REJ NO 2 =   4.0
ACC 1ST SAMP COST =    143.42  REJ 1ST SAMP COST =    273.25
ACC 2ND SAMP COST =   114.58   REJ 2ND SAMP COST =    171.03
TOTAL COST =          702.278

*********************************************************************
*********************************************************************
*********************************************************************
```

The user is given the opportunity to evaluate expected cost using a new lot size and/or sampling plan, but maintaining the same prior distribution, parameters, and costs. In the following illustration, the user does exercise this option:

```
 WANT TO EVAL ANOTHER SAMP PLAN USING LOT,SAMP SIZES     ???
 NO(0) OR YES(1)
?
1
```

The new lot size and sample sizes are requested:

```
 ENTER LOT SIZE, 1ST SAMP SIZE,AND 2ND SAMP SIZE
?
500,27,27
```

and verified:

```
 LOT SIZE =   500.0 1ST SAMP SIZE =   27.0 2ND SAMP SIZE =   27.0
 CORRECT??? NO(0) OR YES(1)
?
1
```

The acceptance and rejection numbers are verified:

```
 ACC/REJ NUMBERS ARE C1=   0.0 R1=   4.0 C2=   3.0 R2=   4.0
 CORRECT??? NO(0) OR YES(1)
?
1
```

The new results are again printed:

```
*********************************************************************
*********************************************************************
*********************************************************************

EXPECTED COST EVALUATION
LOT SIZE =    500.0 1ST SAMP SIZE =    27.0 2ND SAMP SIZE =    27.0
MIXED BINOMIAL PARAMETERS ARE W1=0.5800    W2=0.3000
W3=0.1200    F1= 0.0100000    F2= 0.1000000    F3= 0.3000000
COST VALUES ARE SO=  3.00    S1=  2.50    S2 =  1.90    AO = 10.00
A1 =  0.0    A2 = 40.00    RO =  5.00    R1 =  2.00    R2 =  1.90
ACC NO 1 =    0.0 REJ NO 1 =    4.0 ACC NO 2 =    3.0 REJ NO 2 =    4.0
ACC 1ST SAMP COST =    153.67  REJ 1ST SAMP COST =    247.75
ACC 2ND SAMP COST =    136.02  REJ 2ND SAMP COST =    174.56
TOTAL COST =          711.994

*********************************************************************
*********************************************************************
*********************************************************************
```

Again the opportunity to evaluate expected cost using a new lot size and/or sample sizes is offered and declined:

```
 WANT TO EVAL ANOTHER SAMP PLAN USING LOT,SAMP SIZES     ???
 NO(0) OR YES(1)
?
0
```

The opportunity to evaluate expected cost, changing the prior distribution, parameters, and/or costs, is then offered. In the following illustration the user does exercise this option:

```
 WANT TO CALCULATE COST USING NEW PRIOR/COST
      PARMETERS ??? NO(0) OR YES(1)
?
1
```

At this time, the program again requests the user to input the prior distribution. In the following illustration, a Polya distribution is requested:

```
 WHICH IS THE PRIOR DISTRIBUTION???
      MIXED BINOMIAL(0) OR POLYA(1)
?
1
```

Note that the most recent parameters of the Polya distribution are again displayed for verification and possible change, as are the costs, lot size, sampling sizes, and acceptance and rejection numbers. Any of these may be changed if desired. In the following illustration, the sampling plan decision variables are not correct and must be changed and verified. The results are then printed:

```
 POLYA PARAMETERS ARE S=  0.462103    T=  6.539455
 CORRECT??? NO(0) OR YES(1)
?
1
 COST VALUES ARE SO=  3.00    S1=  2.50    S2=  1.90    AO= 10.00
 A1=  0.0     A2= 40.00    RO=  5.00    R1=  2.00    R2=  1.90
 CORRECT??? NO(0) OR YES(1)
?
1
 LOT SIZE =    500.0 1ST SAMP SIZE =    27.0 2ND SAMP SIZE =    27.0
 CORRECT??? NO(0) OR YES(1)
?
0
 ENTER LOT SIZE, 1ST SAMP SIZE,AND 2ND SAMP SIZE
?
500,30,30
 LOT SIZE =    500.0 1ST SAMP SIZE =    30.0 2ND SAMP SIZE =    30.0
 CORRECT??? NO(0) OR YES(1)
?
1
 ACC/REJ NUMBERS ARE C1=   0.0 R1=   4.0 C2=   3.0 R2=   4.0
 CORRECT??? NO(0) OR YES(1)
?
0
 ENTER C1,R1,C2,AND R2
?
0,2,2,3
 ACC/REJ NUMBERS ARE C1=   0.0 R1=   2.0 C2=   2.0 R2=   3.0
 CORRECT??? NO(0) OR YES(1)
?
1
 *********************************************************************
 *********************************************************************
 *********************************************************************

 EXPECTED COST EVALUATION
 LOT SIZE =    500.0 1ST SAMP SIZE =    30.0 2ND SAMP SIZE =    30.0
 POLYA PARAMETERS ARE S=  0.462103    T=  6.539455
 COST VALUES ARE SO=  3.00    S1=  2.50    S2 =  1.90    AO = 10.00
 A1 =  0.0     A2 = 40.00    RO =  5.00    R1 =  2.00    R2 =  1.90
 ACC NO 1 =    0.0 REJ NO 1 =    2.0 ACC NO 2 =    2.0 REJ NO 2 =    3.0
 ACC 1ST SAMP COST =      142.18  REJ 1ST SAMP COST =     441.05
 ACC 2ND SAMP COST =       78.39  REJ 2ND SAMP COST =      59.17
 TOTAL COST =            720.793

 *********************************************************************
 *********************************************************************
 *********************************************************************
```

The user is again given the opportunity to evaluate the expected cost of using a new lot size, sampling plan, prior distribution, parameters, and/or costs, or to start the program from the beginning. In the following illustration, all options are declined:

```
 WANT TO EVAL ANOTHER SAMP PLAN USING LOT,SAMP SIZES     ???
 NO(0) OR YES(1)
?
0
```

```
 WANT TO CALCULATE COST USING NEW PRIOR/COST
      PARMETERS ??? NO(0) OR YES(1)
?
0
 WANT TO START OVER ??? NO(0) OR YES(1)
?
0
```

At this time the user is finished with the program and may log off.

Summary

Nearly every feature of the program has been illustrated in this chapter. It is a powerful tool for designing an economically based sampling plan finding an optimum acceptance/rejection number vector for a given sample size pair, and evaluating the total cost of a sampling plan. It is directly usable in industrial and governmental situations as well as in teaching, where the underlying model and assumptions are applicable.

CHAPTER VI

SENSITIVITY ANALYSIS

Introduction

The purpose of this chapter is to present a wide array of sensitivity analyses relevant to this research. Among the different situations discussed are the following:

(1) Sensitivity to sample size and different constant factors.

(2) Sensitivity to the cost coefficients.

(3) Sensitivity to the prior distribution.

(4) Comparison with optimum single sampling plan.

Tables are displayed to show the sensitivity properties in each case.

Sensitivity to Sample Size and Different Constant Factors

In actual industrial and government application, the second sample size n_2 is nearly always set equal to some constant multiple of the sample size n_1. In order to study the sensitivity of the cost model to sample size variations and different constant factors, suppose that the lot size is N=500, the cost components of the original model are $(S_0, S_1, S_2, A_0, A_1, A_2, R_0, R_1, R_2)$ = (3.0, 2.5, 1.9, 10.0, 0.0, 40.0, 5.0, 2.0, 1.9), with mean and variance for the prior distribution (μ, σ^2) = (32.999971, 1952.98924). For the mixed binomial distribution, the distribution, the parameters are $(w_1, w_2, w_3, f_1, f_2, f_3) = (0.6, 0.3, 0.1, 0.01, 0.1, 0)$; for the Polya distribution, the parameters are $(s, t) = (0.4621, 6.5394)$. The interactive computer program presented in Appendix A results in data summarized in Table VI.1.

In Table VI.1, within each constant factor and prior distribution, the optimal sampling plan $(n_1^*, n_2^*, c_1^*, r_1^*, c_2^*, r_2^*)$ and expected total cost TC* are presented. Given the optimal sample sizes, and maintaining the same constant factor of 2, other sample sizes are chosen which vary both ±10% and ±20% from the optimum. Using these new sample sizes, the optimal acceptance/rejection number vector and the resulting expected total cost are calculated. Table VI.1 illustrates the fact that values in the neighborhood of optimum are very close in total cost. Although the sample sizes are as much as ±20% off of optimum, the expected total cost difference never exceeds 2% for the examples considered.

Another important fact is that the optimal expected total cost occurs when the constant factor is 2. That is, of those constant factors considered, a second sample size twice that of the first sample size ($n_2 = 2 \times n_1$) is the best choice. It should be remembered that only gross (typical) constant factors (CF) are used, including values of CF = .75, 1, 1.5, 2, 2.5, and 3. Sensitivity measures over the different constant factors considered, but within a prior distribution indicate that the expected total cost varies by only 1% from optimum.

Sensitivity to the Cost Coefficients

A number of additional problems are solved using various values of the cost coefficients assuming a lot size of N = 500. The mixed binomial and Polya prior distributions, using the cost coefficients of

TABLE VI.1

SENSITIVITY OF THE EXPECTED TOTAL COST TO DIFFERENT SAMPLE SIZES AND DIFFERENT CONSTANT FACTORS

N = 500 $S_0 = 3.0$ $A_0 = 10.0$ $R_0 = 5.0$

$S_1 = 2.5$ $A_1 = 0.0$ $R_1 = 2.0$

$S_2 = 1.9$ $A_2 = 40.0$ $R_2 = 1.9$

$(\mu, \sigma^2) = (32.999971, 1952.96924)$

For the Mixed Binomial Distribution, the Parameters Are

$w_1 = .6$ $w_2 = .3$ $w_3 = .1$

$f_1 = .01$ $f_2 = .1$ $f_3 = .3$

For the Polya Distribution, the Parameters Are

s = 0.4621026 t = 6.53945446

Constant Factor CF	Prior Distribution	Sampling Plan n_1	n_2	c_1	r_1	c_2	r_2	Expected Total Cost TC	% Cost Increase Over Optimum Within CF and Prior Dist.	% Cost Increase Over Optimum Within Prior Dist.
0.5	Mixed Binomial	28	14	0	2	1	2	581.880	0.85%	
		32	16	0	3	2	3	679.666	0.52%	
		36	18	0	3	2	3	676.130		1.05%
		40	20	0	3	2	3	678.175	0.30%	
		44	22	1	3	2	3	679.740	0.34%	
	Polya	25	12	0	2	1	2	719.035	0.19%	
		28	14	0	3	2	3	719.470	0.25%	
		31	15	0	3	2	3	717.669		0.88%
		34	17	0	3	2	3	718.321	0.09%	
		37	18	0	3	2	3	721.121	0.48%	
0.75	Mixed Binomial	27	20	0	3	2	3	678.406	0.85%	
		30	22	0	3	2	3	673.933	0.18%	
		33	24	0	3	2	3	672.699		0.54%
		36	27	0	3	2	3	674.100	0.21%	
		39	29	0	3	2	3	678.634	0.88%	
	Polya	23	17	0	2	1	2	718.555	0.56%	
		26	19	0	3	2	3	714.969	0.06%	
		29	21	0	3	2	3	714.554		0.44%
		32	24	0	3	2	3	716.709	0.30%	
		35	26	0	3	2	3	719.713	0.72%	

TABLE VI.1 (Continued)

Constant Factor CF	Prior Distribution	Sampling Plan n_1	n_2	c_1	r_1	c_2	r_2	Expected Total Cost TC	% Cost Increase Over Optimum Within CF and Prior Dist.	% Cost Increase Over Optimum Within Prior Dist.
1.00	Mixed Binomial	24	24	0	3	2	3	677.223	1.00%	
		27	27	0	3	2	3	671.721	0.17%	
		30	30	0	3	2	3	670.542		0.22%
		33	33	0	3	2	3	672.535	0.30%	
		36	36	0	3	2	3	675.031	0.67%	
	Polya	20	20	0	2	1	2	718.960	0.93%	
		23	23	0	3	2	3	714.076	0.24%	
		26	26	0	3	2	3	712.344		0.13%
		29	29	0	3	2	3	714.625	0.32%	
		32	32	0	3	3	4	716.185	0.54%	
1.50	Mixed Binomial	20	30	0	3	2	3	678.209	1.28%	
		23	34	0	3	2	3	671.181	0.23%	
		26	39	0	3	2	3	669.655		0.09%
		29	43	0	3	2	3	671.074	0.21%	
		32	48	0	3	3	4	671.617	0.29%	
	Polya	20	30	0	3	2	3	713.660	0.28%	
		23	34	0	3	2	3	712.898	0.17%	
		26	39	0	3	3	4	711.665		0.04%
		29	43	0	3	3	4	713.495	0.26%	
		32	48	0	3	3	4	717.383	0.80%	
2.00	Mixed Binomial	20	40	0	3	2	3	674.444	0.80%	
		23	46	0	3	2	3	672.094	0.45%	
		26	52	0	3	3	4	669.086		
		29	58	0	3	3	4	671.327	0.33%	
		32	64	0	3	4	5	675.294	0.93%	
	Polya	18	38	0	3	2	3	716.334	0.69%	
		21	42	0	3	3	4	712.811	0.20%	
		24	48	0	3	3	4	711.409		
		27	54	0	3	4	5	713.647	0.31%	
		30	60	0	3	4	5	716.572	0.73%	
2.5	Mixed Binomial	17	42	0	3	2	3	681.570	1.63%	
		20	50	0	3	3	4	675.761	0.77%	
		23	57	0	3	3	4	670.633		0.23%
		26	65	0	3	3	4	671.250	0.09%	
		29	72	0	3	4	5	674.433	0.57%	

TABLE VI.1 (Continued)

Constant Factor CF	Prior Distribution	n_1	n_2	c_1	r_1	c_2	r_2	Expected Total Cost TC	% Cost Increase Over Optimum Within CF and Prior Dist.	% Cost Increase Over Optimum Within Prior Dist.
		18	45	0	3	3	4	716.206	0.51%	
		21	52	0	3	3	4	712.836	0.03%	
2.5	Polya	24	60	0	3	4	5	712.609		0.17%
		27	67	0	3	4	5	715.173	0.36%	
		30	75	0	3	5	6	719.283	0.94%	
		17	54	0	3	3	4	682.562	1.44%	
		19	57	0	3	3	4	676.710	0.57%	
	Mixed Binomial	21	63	0	3	3	4	672.891		0.57%
		23	69	0	3	3	4	672.893	0.00%	
		25	75	0	3	4	5	673.501	0.09%	
3.0										
		18	54	0	3	3	4	716.027	0.38%	
		20	60	0	3	4	5	714.379	0.15%	
	Polya	22	66	0	3	4	5	713.297		0.27%
		25	75	0	3	5	6	715.611	0.32%	
		27	81	0	3	5	6	717.488	0.59%	

the "original" model $(S_0, S_1, S_2, A_0, A_2, R_0, R_1, R_2) =$ (3.0, 2.5, 1.9, 10.0, 0.0, 40.0, 5.0, 2.0, 1.9) with mean and variance (μ, σ^2) = (32.999971, 1952.96924), are reconsidered. The cost values of S_0, S_1, A_0, A_1, and R_0 are held fixed, while values of R_1, R_2, S_2, and A_2 are varied and presented. The constant factor between n_1 and n_2 is held and assumed to be 1. Two sensitivity measures, Δ_1 and Δ_2, have been developed to help to show the sensitivity properties of the expected total cost expression to different cost coefficients.

The first measure, Δ_1, is expressed as

$$\Delta_1 = \frac{TC_{CC}(\underline{P}_0, \underline{C}_C, \underline{P}_C^*) - TC_{00}(\underline{P}_0, \underline{C}_0, \underline{P}_0^*)}{TC_{00}(\underline{P}_0, \underline{C}_0, \underline{P}_0^*)} \times 100\%$$

where

$\underline{C}_0(\underline{C}_C)$ = the original (changed) cost parameter vector

$\underline{P}_0^*(\underline{P}_C^*)$ = the optimum original (changed) decision variable vector including sample sizes and acceptance and rejection numbers as optimized in the original (changed) cost environment

$\underline{P}_0(\underline{P}_C)$ = the original (changed) prior distribution parameter vector

$TC_{CC}(\underline{P}_0, \underline{C}_C, \underline{P}_C^*)$ = the total expected cost predicted by the original prior distribution, under a changed cost vector, using the sampling plan determined to be optimal under the changed cost vector.

$TC_{00}(\underline{P}_0, \underline{C}_0, \underline{P}_0^*)$ = the total expected cost predicted by the original prior distribution, under the original cost vector, using the sampling plan determined to be optimal under the original cost vector

Thus, Δ_1 represents a measure of inaccuracy of the changed cost model when used to determine what is believed to be an optimal sampling plan which is then evaluated to predict total expected cost.

The second measure, Δ_2, is expressed as

$$\Delta_2 = \frac{TC_{OC}(\underline{P}_0, \underline{C}_0, \underline{P}_C^*) - TC_{OO}(\underline{P}_0, \underline{C}_0, \underline{P}_0^*)}{TC_{OO}(\underline{P}_0, \underline{C}_0, \underline{P}_0^*)} \times 100\%$$

where

$TC_{OC}(\underline{P}_0,\underline{C}_0,\underline{P}_C^*)$ = the total expected cost predicted by the original prior distribution, under the original cost vector, using the sampling plan determined to be optimal under the changed cost vector.

Thus, Δ_2 expressed a measure of how costly it will be to use the changed cost model's optimum plan in the original cost model environment. That is, Δ_2 is a measure which compares the two models as selectors of the optimal sampling plan.

The value of each changed cost parameter in the set (A_2, S_2, R_2) is varied up and down ±20% from the "original" values. Cost parameter R_1 differs ±10% and ±20% from the "original" model (i.e., 1.6, 1.8, 2.0, 2.2, 2.4 over five cases). The optimal sampling plans, total expected costs, and sensitivity measures are as shown in Table VI.2.

From Table VI.2, it is seen that as $S_2 = R_2$ increases while other coefficients remain fixed, the optimal expected total costs increase while the sample sizes either decrease or remain the same. Thus, increasing S_2 and R_2 causes the plans to be less discriminating. As cost A_2 increases for fixed other coefficients, the optimal expected total cost increases while the sample sizes increase and the acceptance/ rejection number vector either decreases or remains the same. Thus,

TABLE VI.2

SENSITIVITY TO THE COST COEFFICIENTS

Optimal Double Sampling Plans for N = 500, Selected Cost Coefficients, and Mixed Binomial and Polya Priors With Mean = 32.999971 and Variance = 1952.96924

Prior Distribution	$S_0 = 3.0$, $S_1 = 2.5$		$A_0 = 10.0$, $A_1 = 0.0$	$R_0 = 5.0$, CF = 1
			$R_1 = 1.6$	
	A_2 \ $S_2=R_2$	1.52	1.90	2.28
Mixed Binomial	32	28,28,0,3,2,3 TC_{CC}=556.428 Δ_1=-17.02% TC_{OC}=670.604 Δ_2=0.009%	28,28,0,3,2,3 TC_{CC}=567.342 Δ_1=-15.39% TC_{OC}=670.604 Δ_2=0.009%	26,26,0,3,2,3 TC_{CC}=578.112 Δ_1=-13.78% TC_{OC}=672.360 Δ_2=0.27%
	40	31,31,0,3,2,3 TC_{CC}=587.373 Δ_1=-12.40% TC_{OC}=670.547 Δ_2=0.0007%	31,31,0,3,2,3 TC_{CC}=598.465 Δ_1=-10.75% TC_{OC}=670.547 Δ_2=0.0007%	31,31,0,3,2,3 TC_{CC}=609.557 Δ_1=-9.10% TC_{OC}=670.547 Δ_2=0.0007%
	48	33,33,0,3,2,3 TC_{CC}=615.509 Δ_1=-8.21% TC_{OC}=672.535 Δ_2=0.30%	33,33,0,3,2,3 TC_{CC}=626.703 Δ_1=-6.54% TC_{OC}=672.535 Δ_2=0.30%	33,33,0,3,2,3 TC_{CC}=637.896 Δ_1=-4.87% TC_{OC}=672.535 Δ_2=0.30%
Polya	32	22,22,0,2,2,3 TC_{CC}=587.604 Δ_1=-17.51% TC_{OC}=715.214 Δ_2=0.4%	22,22,0,2,2,3 TC_{CC}=597.787 Δ_1=-16.08% TC_{OC}=715.214 Δ_2=0.4%	22,22,0,2,2,3 TC_{CC}=607.970 Δ_1=-14.65% TC_{OC}=715.214 Δ_2=0.4%

TABLE VI.2 (Continued)

	R_1 = 1.6			
	A_2 \ S_2=R_2	1.52	1.90	2.28
Polya	40	24,24,0,2,1,2 TC_{CC}=627.849 Δ_1=-11.86% TC_{OC}=712.809 Δ_2-0.07%	24,24,0,2,1,2 TC_{CC}=638.639 Δ_1=-10.35% TC_{OC}=712.809 Δ_2=0.07%	24,24,0,2,1,2 TC_{CC}=649.428 Δ_1=-8.83% TC_{OC}=712.809 Δ_2=0.07%
	48	29,29,0,2,1,2 TC_{CC}=658.389 Δ_1=-7.57% TC_{OC}=714.626 Δ_2=0.32%	29,29,0,2,1,2 TC_{CC}=669.526 Δ_1=-6.01% TC_{OC}=714.626 Δ_2=0.32%	29,29,0,2,1,2 TC_{CC}=680.662 Δ_1=-4.45% TC_{OC}=614.626 Δ_2=0.32%
	R_1 = 1.8			
	A_2 \ S_2=R_2	1.52	1.90	2.28
Mixed Binomial	32	26,26,0,3,2,3 TC_{CC}=590.924 Δ_1=-11,87% TC_{OC}=672.360 Δ_2=0.27%	26,26,0,3,2,3 TC_{CC}=601.678 Δ_1=-10.27% TC_{OC}=672.360 Δ_2=0.27%	26,26,0,3,2,3 TC_{CC}=612.423 Δ_1=-8.67% TC_{OC}=672.360 Δ_2=0.27%
	40	31,31,0,3,2,3 TC_{CC}=623.414 Δ_1=-7.03% TC_{OC}=670.547 Δ_2=0.0007%	31,31,0,3,2,3 TC_{CC}=634.506 Δ_1=-5.37% TC_{OC}=670.547 Δ_2=0.0007%	31,31,0,3,2,3 TC_{CC}=645.598 Δ_1=3.72% TC_{OC}=670.547 Δ_2=0.0007%

TABLE VI.2 (Continued)

		R_1 = 1.8		
	A_2 \ S_2=R_2	1.52	1.90	2.28
Mixed Binomial	48	33,33,0,3,2,3 TC_{CC}=652.051 Δ_1=-2.76% TC_{OC}=672.535 Δ_2=0.30%	33,33,0,3,2,3 TC_{CC}=663.245 Δ_1=-1.09% TC_{OC}=672.535 Δ_2=0.30%	33,33,0,3,2,3 TC_{CC}=674.438 Δ_1=0.58% TC_{OC}=672.535 Δ_2=0.30%
Polya	32	22,22,0,3,2,3 TC_{CC}=620.729 Δ_1=-12.86% TC_{OC}=715.214 Δ_2=0.4%	22,22,0,3,2,3 TC_{CC}=630.702 Δ_1=-11.46% TC_{OC}=715.214 Δ_2=0.4%	22,22,0,3,2,3 TC_{CC}=640.676 Δ_1=-10.06% TC_{OC}=615.214 Δ_2=0.4%
	40	27,27,0,3,2,3 TC_{CC}=666.453 Δ_1=-6.44% TC_{OC}=713.038 Δ_2=0.10%	26,26,0,3,2,3 TC_{CC}=676.952 Δ_1=-4.97% TC_{OC}=712.344 Δ_2=0.0%	26,26,0,3,2,3 TC_{CC}=687.356 Δ_1=-3.51% TC_{OC}=712.344 Δ_2=0.0%
	48	31,31,0,2,2,3 TC_{CC}=701.382 Δ_1=-1.54% TC_{OC}=717.311 Δ_2=0.70%	31,31,0,2,2,3 TC_{CC}=712.336 Δ_1=-0.001% TC_{OC}=717.311 Δ_2=0.70%	31,31,0,2,2,3 TC_{CC}=723.291 Δ_1=1.54% TC_{OC}=717.311 Δ_2=0.70%

TABLE VI.2 (Continued)

	A_2 \ $S_2=R_2$	$R_1 = 2.0$: 1.52	1.90	2.28
Mixed Binomial	32	26,26,0,3,2,3 TC_{CC}=625.235 Δ_1=-6.76% TC_{OC}=672.360 Δ_2=0.27%	26,26,0,3,2,3 TC_{CC}=635.984 Δ_1=-5.15% TC_{OC}=672.360 Δ_2=0.27%	26,26,0,3,2,3 TC_{CC}=646.733 Δ_1=-3.55% TC_{OC}=672.360 Δ_2=0.27%
	40	31,31,0,3,2,3 TC_{CC}=659.455 Δ_1=-1.65% TC_{OC}=670.547 Δ_2=0.0007%	30,30,0,3,2,3 TC_{CC}=670.542	28,28,0,3,2,3 TC_{CC}=681.524 Δ_1=1.64% TC_{OC}=670.604 Δ_2=0.009%
	48	33,33,0,3,2,3 TC_{CC}=688.593 Δ_1=2.69% TC_{OC}=672.535 Δ_2=0.30%	31,31,0,3,2,3 TC_{CC}=699.720 Δ_1=4.35% TC_{OC}=670.547 Δ_2=0.0007%	31,31,0,3,2,3 TC_{CC}=710.812 Δ_1=6.01% TC_{OC}=670.547 Δ_2=0.0007%
Polya	32	20,20,0,3,2,3 TC_{CC}=652.264 Δ_1=-8.43% TC_{OC}=720.148 Δ_2=1.09%	19,19,0,3,2,3 TC_{CC}=661.819 Δ_1=-7.09% TC_{OC}=723.475 Δ_2=1.56%	19,19,0,3,2,3 TC_{CC}=671.327 Δ_1=-5.76% TC_{OC}=723.475 Δ_2=1.56%
	40	26,26,0,3,2,3 TC_{CC}=701.940 Δ_1=-1.46% TC_{OC}=712.344 Δ_2=0.0%	26,26,0,3,2,3 TC_{CC}=712.344	26,26,0,3,2,3 TC_{CC}=722.748 Δ_1=1.46% TC_{OC}=712.344 Δ_2=0.0%

TABLE VI.2 (Continued)

		$R_1 = 2.0$		
	A_2 \ $S_2=R_2$	1.52	1.90	2.28
Polya	48	31,31,0,3,2,3 TC_{CC}=740.650 Δ_1=3.97% TC_{OC}=717.311 Δ_2=0.70%	31,31,0,3,2,3 TC_{CC}=751.463 Δ_1=5.49% TC_{OC}=717.311 Δ_2=0.70%	31,31,0,3,2,3 TC_{CC}=762.276 Δ_1=7.01% TC_{OC}=717.311 Δ_2=0.70%
		$R_1 = 2.2$		
	A_2 \ $S_2=R_2$	1.52	1.90	2.28
Mixed Binomial	32	23,23,0,3,2,3 TC_{CC}=658.965 Δ_1=-1.73% TC_{OC}=679.729 Δ_2=1.37%	23,23,0,3,2,3 TC_{CC}=669.408 Δ_1=-0.17% TC_{OC}=679.729 Δ_2=1.37%	23,23,0,3,2,3 TC_{CC}=679.880 Δ_1=1.39% TC_{OC}=679.729 Δ_2=1.37%
	40	28,28,0,3,2,3 TC_{CC}=694.811 Δ_1=3.62% TC_{OC}=670.604 Δ_2=0.009%	28,28,0,3,2,3 TC_{CC}=705.731 Δ_1=5.25% TC_{OC}=670.604 Δ_2=0.009%	28,28,0,3,2,3 TC_{CC}=716.652 Δ_1=6.88% TC_{CC}=670.604 Δ_2=0.009%
	48	31,31,0,3,2,3 TC_{CC}=724.669 Δ_1=8.07% TC_{OC}=670.547 Δ_2=0.0007%	31,31,0,3,2,3 TC_{CC}=745.404 Δ_1=11.16% TC_{OC}=670.547 Δ_2=0.0007%	31,31,0,3,2,3 TC_{CC}=746.853 Δ_1=11.38% TC_{OC}=670.547 Δ_2=0.0007%

TABLE VI.2 (Continued)

		$R_1 = 2.2$		
	A_2 \ $S_2=R_2$	1.52	1.90	2.28
Polya	32	19,19,0,3,2,3 TC_{CC}=681.935 Δ_1=-4.27% TC_{OC}=723.475 Δ_2=1.56%	19,19,0,3,2,3 TC_{CC}=691.442 Δ_1=-2.93% TC_{OC}=723.475 Δ_2=1.56%	19,19,0,3,2,3 TC_{CC}=700.949 Δ_1=-1.60% TC_{OC}=723.475 Δ_2=1.56%
	40	24,24,0,3,2,3 TC_{CC}=736.573 Δ_1=3.40% TC_{OC}=712.809 Δ_2=0.07%	24,24,0,3,2,3 TC_{CC}=746.779 Δ_1=4.83% TC_{OC}=712.809 Δ_2=0.07%	24,24,0,3,2,3 TC_{CC}=756.986 Δ_1=6.27% TC_{OC}=712.809 Δ_2=0.07%
	48	29,29,0,3,2,3 TC_{CC}=778.437 Δ_1=9.28% TC_{OC}=714.626 Δ_2=0.32%	29,29,0,3,2,3 TC_{CC}=798.387 Δ_1=12.08% TC_{OC}=714.626 Δ_2=0.32%	29,29,0,3,2,3 TC_{CC}=799.782 Δ_1=12.27% TC_{OC}=714.626 Δ_2=0.32%
		$R_1 = 2.4$		
	A_2 \ $S_2=R_2$	1.52	1.90	2.28
Mixed Binomial	32	21,21,0,3,2,3 TC_{CC}=691.357 Δ_1=3.10% TC_{OC}=687.653 Δ_2=2.51%	21,21,0,3,2,3 TC_{CC}=701.513 Δ_1=4.62% TC_{OC}=687.653 Δ_2=2.51%	21,21,0,3,2,3 TC_{CC}=711.670 Δ_1=6.13% TC_{OC}=687.653 Δ_2=2.51%

TABLE VI.2 (Continued)

		$R_1 = 2.4$		
	A_2 \ $S_2=R_2$	1.52	1.90	2.28
Mixed Binomial	40	28,28,0,3,2,3 TC_{CC}=729.939 Δ_1=8.86% TC_{OC}=670.604 Δ_2=0.009%	28,28,0,3,2,3 TC_{CC}=740.859 Δ_1=10.49% TC_{OC}=670.604 Δ_2=0.009%	26,26,0,3,2,3 TC_{CC}=751.730 Δ_1=12.11% TC_{OC}=672.360 Δ_2=0.27%
	48	31,31,0,3,2,3 TC_{CC}=760.710 Δ_1=13.45% TC_{OC}=670.547 Δ_2=0.007%	31,31,0,3,2,3 TC_{CC}=771.802 Δ_1=15.10% TC_{OC}=670.547 Δ_2=0.0007%	31,31,0,3,2,3 TC_{CC}=782.894 Δ_1=16.77% TC_{OC}=670.547 Δ_2=0.0007%
Polya	32	22,22,0,4,3,4 TC_{CC}=708.491 Δ_1=-0.54% TC_{OC}=715.214 Δ_2=0.4%	22,22,0,4,3,4 TC_{CC}=717.697 Δ_1=0.75% TC_{OC}=715.214 Δ_2=0.4%	22,22,0,4,3,4 TC_{CC}=726.902 Δ_1=2.04% TC_{OC}=715.214 Δ_2=0.4%
	40	27,27,0,4,3,4 TC_{CC}=768.985 Δ_1=7.95% TC_{OC}=713.038 Δ_2=0.10%	26,26,0,4,3,4 TC_{CC}=778.814 Δ_1=9.33% TC_{OC}=712.344 Δ_2=0.0%	26,26,0,4,3,4 TC_{CC}=788.574 Δ_1=10.70% TC_{OC}=712.344 Δ_2=0.0%
	48	33,33,0,4,3,4 TC_{CC}=815.265 Δ_1=14.45%	26,26,0,4,3,4 TC_{CC}=825.767 Δ_1=15.92% TC_{OC}=712.344 Δ_2=0.0%	26,26,0,4,3,4 TC_{CC}=836.170 Δ_1=17.38% TC_{OC}=712.344% Δ_2=0.0%

Thus, increasing A_2 often makes the plans more discriminating, causing fewer lots to be accepted. Finally, if cost R_1 is increased while other coefficients remain fixed, the optimal expected total cost increases while the sample sizes decrease and the decision variables either increase or remain the same. This causes plans to again be less discriminating.

The Δ_1 measurement, and hence the model as a predictor of total expected cost, is most sensitive to changes in the cost coefficient R_1. That is, the inaccuracy of the changed cost model is relatively high when first used to determine what is believed to be an optimal sampling plan, and when then used to evaluate what is believed to be optimal total expected cost. A 10% change in the value of R_1 causes about a 5.3% change in the value of Δ_1 for the mixed binomial prior, and a 4.7% change for the Polya prior case. A 20% change in the value of A_2 causes a change of about 4-5% in Δ_1 for the mixed binomial prior case, and a 5-7% change for the Polya prior case. The performance measure Δ_1 is least sensitive to changes in the cost coefficients S_2 and R_2. A 20% change in the values of either S_2 or R_2 causes about 1.3-1.8% change in performance measure Δ_1 for both the mixed binomial and Polya prior distribution cases.

Changes in cost coefficients do not have a significant effect in selection of a sampling plan which is then evaluated in the correct cost environment. That is, to use the changed cost model's optimum plan in the original cost model environment is not terribly costly under reasonable circumstances. From Table VI.2, the Δ_2 values are usually below 1%; sometimes Δ_2 goes to 2%. In any case, there is no big difference in using the changed cost model's optimum plan in the original cost model, so long as incorrect estimates of the cost coefficients R_1,

R_2, S_2, and A_2 are within 20% of the correct value.

Sensitivity to the Prior Distribution

Additional problems are solved to investigate the sensitivity of the total expected cost to the parameters of the prior distribution. Only the Polya prior distribution with lot size N = 500 and three different sets of cost parameters are considered. The values of the mean and standard deviation $(\mu, \sigma) = (3.3, 44)$. The constant factor relating n_1 and n_2 is assumed to be 1. Two other sensitivity measures, Δ_3 and Δ_4, have been developed for studying the sensitivity to the prior distribution.

The first measure, Δ_3, may be expressed as

$$\Delta_3 = \frac{TC_{CC}(\underline{P}_C, \underline{C}_0, \underline{P}_C^*) - TC_{00}(\underline{P}_0, \underline{C}_0, \underline{P}_0^*)}{TC_{00}(\underline{P}_0, \underline{C}_0, \underline{P}_0^*)} \times 100\%$$

where

$TC_{CC}(\underline{P}_C, \underline{C}_0, \underline{P}_C^*)$ = the total expected cost calculated, using the changed prior distribution and the sampling plan determined to be optimal using the changed prior distribution while holding the cost vector at the original values.

$TC_{00}(\underline{P}_0, \underline{C}_0, \underline{P}_0^*)$ = the total expected cost calculated using the original prior distribution and the sampling plan determined to be optimal using the original prior distribution while holding the cost vector at the original values.

Thus, Δ_3 expresses a measure of inaccuracy of the model when using the changed prior distribution to determine what is believed to be an optimal sampling plan which is then evaluated to predict total expected cost.

The other measure, Δ_4, may be expressed as

$$\Delta_4 = \frac{TC_C\ (\underline{P}_0,\ \underline{C}_0,\ \underline{P}_C^*) - TC_{00}(\underline{P}_0,\ \underline{C}_0,\ \underline{P}_0^*)}{TC_{00}(\underline{P}_0,\ \underline{C}_0,\ \underline{P}_0^*)} \times 100\%$$

where

$TC_{0C}(\underline{P}_0, \underline{C}_0, \underline{P}_C^*)$ = the total expected cost predicted using the original prior distribution, but also using the sampling plan determined to be optimal under the changed prior distribution. The cost vector is held at its original values.

Thus, Δ_4 represents a measure of how costly it will be to use the changed prior distribution's optimum plan in the original prior distribution environment. That is, Δ_4 is a measure which compares the two models as selectors of the optimal sampling plan.

The optimal sampling plans, total expected costs, and sensitivity measures, Δ_3 and Δ_4, are as shown in Table VI.3. Calculations are made under three different sets of cost parameters.

It is found that as the prior mean increases, the sample sizes increase and the acceptance/rejection number vector either increases or remains the same. For increases in the prior standard deviation, the sample sizes decrease and the acceptance/rejection number vector either decreases or remains the same.

The total expected cost is more sensitive to a changed prior mean than it is to a changed prior standard deviation, as evidenced by the Δ_3 measurement. For example, a ±20% change in the prior mean, while holding constant the value of the standard deviation, produces approximately a ±17-20% change in total expected cost. A ±20% change in the prior standard deviation, while holding the value of the mean constant, causes about a 12-14% change in the total expected cost. That is, the changed

TABLE VI.3

SENSITIVITY TO THE PRIOR DISTRIBUTION

Optimum Double Sampling Plans for Different Polya Priors, N = 500

	S_0 = 3.0 S_1 = 2.5 S_2 = 1.9	A_0 = 10.0 A_1 = 0.0 A_2 = 40.0	R_0 = 5.0 R_1 = 2.0 R_2 = 1.9
Mean / Standard Deviation	26.4	33.0	39.6
35	s=0.496236265 t=8.90217590 25,25,0,3,2,3 TC_{CC}=670.379 Δ_3=-6.17% TC_{OC}=714.849 Δ_4=0.05%	s=0.784033477 t=11.0952587 28,28,0,3,2,3 TC_{CC}=818.061 Δ_3=14.50% TC_{OC}=715.453 Δ_4=0.14%	s=1.13327789 t=13.1757803 36,36,0,3,3,4 TC_{CC}=934.881 Δ_3=30.85% TC_{OC}=721.822 Δ_4=1.03%
44	s = 0.29196322 t=5.23764229 23,23,0,3,2,3 TC_{CC}=589.304 Δ_3=-20.32% TC_{OC}=716.235 Δ_4=0.25%	s=0.466807723 t=6.60603619 26,26,0,3,2,3 TC_{CC}=714.475	s=0.679445982 t=7.89940834 27,27,0,3,2,3 TC_{CC}=838.852 Δ_3=17.41% TC_{OC}=838.852 Δ_4=0.09%
53	s=0.183852911 t=3.29820919 22,22,0,3,2,3 TC_{CC}=483.527 Δ_3=-32.32% TC_{OC}=717.399 Δ_4=0.41%	s=0.299380422 t=4.23668575 23,23,0,3,2,3 TC_{CC}=622.638 Δ_3=-12.85% TC_{OC}=716.235 Δ_4=0.27%	s=0.440567017 t=5.12214565 27,27,0,3,2,3 TC_{CC}=747.632 Δ_3=4.64% TC_{OC}=715.495 Δ_4=0.09%

TABLE VI.3 (Continued)

	$S_0 = 3.0$ $S_1 = 2.5$ $S_2 = 1.52$	$A_0 = 10.0$ $A_1 = 0.0$ $S_2 = 32.0$	$R_0 = 5.0$ $R_1 = 1.6$ $R_2 = 1.52$
Standard Deviation \ Mean	26.4	33.0	39.6
35	s=0.496236265 t=8.90217590 23,23,0,3,2,3 TC_{CC}=553.538 Δ_3=-5.79% TC_{OC}=529.566 Δ_4=0.33%	s=0.784033477 t=11.0952587 24,24,0,2,2,3 TC_{CC}=673.778 Δ_3=14.67% TC_{OC}=589.621 Δ_4=0.34%	s=1.13327789 t=13.1757803 26,26,0,2,2,3 TC_{CC}=769.309 Δ_3=30.92% TC_{OC}=591.226 Δ_4=0.62%
44	s=0.29196322 t=5.23764229 18,18,0,2,1,2 TC_{CC}=469.853 Δ_3=-20.04% TC_{OC}=590.913 Δ_4=0.56%	s=0.466807723 t=6.60603619 22,22,0,2,2,3 TC_{CC}=587.604	s=0.679445982 t=7.89940834 25,25,0,2,2,3 TC_{CC}=690.325 Δ_3=17.48% TC_{OC}=590.555 Δ_4=0.50%
53	s=0.183852911 t=3.29820919 16,16,0,2,1,2 TC_{CC}=398.069 Δ_3=-32.25% TC_{OC}=593.757 Δ_4=1.05%	s=0.299380422 t=4.23668575 19,19,0,2,1,2 TC_{CC}=513.258 Δ_3=-12.65% TC_{OC}=590.282 Δ_4=0.46%	s=0.440567017 t=5.12214565 24,24,0,2,2,3 TC_{CC}=615.586 Δ_3=4.76% TC_{OC}=589.621 Δ_4=0.34%

TABLE VI.3 (Continued)

	S_0 = 3.0 S_1 = 2.5 S_2 = 2.28	A_0 = 10.0 A_1 = 0.0 A_2 = 48.0	R_0 = 5.0 R_1 = 2.4 R_2 = 2.28
Standard Deviation \ Mean	26.4	33.0	39.6
35	s=0.496236265 t=8.90217590 25,25,0,3,2,3 TC_{CC}=786.217 Δ_3=-6.25% TC_{OC}=839.768 Δ_4=0.13%	s=0.784033477 t=11.0952587 35,35,0,4,3,4 TC_{CC}=957.512 Δ_3=14.17% TC_{OC}=839.202 Δ_4=0.06%	s=1.13327789 t=13.1757803 38,38,0,4,3,4 TC_{CC}=1094.418 Δ_3=30.17% TC_{OC}=842.152 Δ_4=0.42%
44	s=0.29196322 t=5.23764229 24,24,0,3,2,3 TC_{CC}=567.176 Δ_3=-20.45% TC_{OC}=840.526 Δ_4=0.22%	s=0.466807723 t=6.60603619 28,28,0,3,2,3 TC_{CC}=838.653	s=0.679445982 t=7.89940834 36,36,0,4,3,4 TC_{CC}=983.239 Δ_3=17.24% TC_{OC}=840.189 Δ_4=0.18%
53	s=0.183852911 t=3.29820919 22,22,0,3,2,3 TC_{CC}=565.767 Δ_3=-32.54% TC_{OC}=844.691 Δ_4=0.72%	s=0.299380422 t=4.23668575 26,26,0,3,2,3 TC_{CC}=730.048 Δ_3=-12.95% TC_{OC}=838.706 Δ_4=0.006%	s=0.440567017 t=5.12214565 33,33,0,4,3,4 TC_{CC}=877.979 Δ_3=4.69% TC_{OC}=838.628 Δ_4=-0.003%

prior distribution's optimum plan, evaluated in the original prior distribution environment, is quite good so long as the mean and standard deviations are estimated within ±20% of their correct values.

Comparison With Optimum Single Sampling and Tabulated Sampling Plans

It is instructive to compare economically optimum double sampling plans to economically optimum single sampling plans, as well as both single and double sampling plans obtained from Military Standard 105D. Table VI.4 lists several economically optimal single sampling plans, their expected costs, the corresponding cost of the optimal double sampling plans, and the percent savings attained by using double sampling. The optimal double sampling plans are determined by using the interactive program described in Chapter V and listed in Appendix A. The optimal single sampling plans are derived using the program developed by Case [13].

From Table VI.4, the savings under double sampling using the mixed binomial prior distribution range from 2.67% to 3.41%. Savings using the Polya prior distribution range from 2.07% to 3.17% for various different values of cost and prior parameters evaluated. For increases in the value of the prior mean and decreases in the prior standard deviation, the economic advantage of double sampling relative to single sampling becomes more significant. In contrast, for decreases in the value of the prior mean and increases in the prior standard deviation, the advantage of double sampling over single sampling becomes less significant.

For comparison purposes, sampling plans from Military Standard 105D are presented using lot size N = 500, the original prior and cost terms

TABLE VI.4

COMPARISON OF OPTIMAL DOUBLE SAMPLING PLANS AND OPTIMAL SINGLE SAMPLING PLANS

Prior Distribution	Prior Parameters		Costs									Optimal Single Sampling			Optimal Double Sampling							Percent Differences
	μ	σ^2	S_0	S_1	S_2	A_0	A_1	A_2	R_0	R_1	R_2	n_1	c_1	Expected Cost	n_1	n_2	c_1	r_1	c_2	r_2	Expected Cost	
Mixed Binomial	32.9999	44.1924	3.0	2.5	1.9	10.0	0.0	40.0	5.0	1.6	1.9	38	1	617.85	31	31	0	3	2	3	598.465	3.24
Polya	32.9999	44.1924	3.0	2.5	1.9	10.0	0.0	40.0	5.0	1.6	1.9	36	1	652.39	24	24	0	2	1	2	638.639	2.15
Mixed Binomial	32.9999	44.1924	3.0	2.5	1.9	10.0	0.0	40.0	5.0	1.8	1.9	37	1	655.14	31	31	0	3	2	3	634.506	3.25
Polya	32.9999	44.1924	3.0	2.5	1.9	10.0	0.0	40.0	5.0	1.8	1.9	34	1	690.95	26	26	0	3	2	3	676.952	2.07
Mixed Binomial	32.9999	44.1924	3.0	2.5	1.9	10.0	0.0	40.0	5.0	2.0	1.9	36	1	692.03	30	30	0	3	2	3	670.542	3.20
Polya	32.9999	44.1924	3.0	2.5	1.9	10.0	0.0	40.0	5.0	2.0	1.9	32	1	728.37	26	26	0	3	2	3	712.344	2.25
Mixed Binomial	32.9999	44.1924	3.0	2.5	1.9	10.0	0.0	40.0	5.0	2.2	1.9	34	1	728.36	28	28	0	3	2	3	705.731	3.21
Polya	32.9999	44.1924	3.0	2.5	1.9	10.0	0.0	40.0	5.0	2.2	1.9	29	1	764.56	24	24	0	3	2	3	746.779	2.38
Mixed Binomial	32.9999	44.1924	3.0	2.5	1.9	10.0	0.0	40.0	5.0	2.4	1.9	49	2	762.39	28	28	0	3	2	3	740.859	2.91
Polya	32.9999	44.1924	3.0	2.5	1.9	10.0	0.0	40.0	5.0	2.4	1.9	39	2	797.17	26	26	0	4	3	4	778.814	2.36
Mixed Binomial	32.9999	44.1924	3.0	2.5	1.9	10.0	0.0	32.0	5.0	2.0	1.9	28	1	653.00	26	26	0	3	2	3	635.984	2.67
Polya	32.9999	44.1924	3.0	2.5	1.9	10.0	0.0	32.0	5.0	2.0	1.9	24	1	677.67	19	19	0	3	2	3	661.819	2.34
Mixed Binomial	32.9999	44.1924	3.0	2.5	1.9	10.0	0.0	48.0	5.0	2.0	1.9	40	1	723.61	31	31	0	3	2	3	699.720	3.41
Polya	32.9999	44.1924	3.0	2.5	1.9	10.0	0.0	48.0	5.0	2.0	1.9	38	1	767.29	31	31	0	3	2	3	751.563	2.11
Mixed Binomial	32.9999	44.1924	3.0	2.5	1.52	10.0	0.0	40.0	5.0	2.0	1.52	36	1	681.09	31	31	0	3	2	3	659.455	3.28
Polya	32.9999	44.1924	3.0	2.5	1.52	10.0	0.0	40.0	5.0	2.0	1.52	32	1	717.89	26	26	0	3	2	3	701.940	2.27
Mixed Binomial	32.9999	44.1924	3.0	2.5	2.28	10.0	010	40.0	5.0	2.0	2.28	35	1	702.91	28	28	0	3	2	3	681.524	3.13
Polya	32.9999	44.1924	3.0	2.5	2.28	10.0	0.0	40.0	5.0	2.0	2.28	31	1	738.81	26	26	0	3	2	3	722.748	2.22
Polya	26.4	35.0	3.0	2.5	1.9	10.0	0.0	40.0	5.0	2.0	1.9	30	1	686.55	25	25	0	3	2	3	670.379	2.41
Polya	39.6	35.0	3.0	2.5	1.9	10.0	0.0	40.0	5.0	2.0	1.9	53	2	948.15	36	36	0	3	3	4	934.881	1.42
Polya	26.4	53.0	3.0	2.5	1.9	10.0	0.0	40.0	5.0	2.0	1.9	26	1	498.84	22	22	0	3	2	3	483.527	3.17
Polya	39.6	53.0	3.0	2.5	1.9	10.0	0.0	40.0	5.0	2.0	1.9	32	1	763.00	27	27	0	3	2	3	747.632	2.06

TABLE VI.5

COMPARISON OF OPTIMAL DOUBLE SAMPLING PLANS AND SAMPLING PLANS FROM MILITARY STANDARD 105D

Mixed Binomial Prior

Classification			n_1	n_2	c_1	r_1	c_2	r_2	Total Expected Cost	Percent Difference
Economically Based		Double	30	30	0	3	2	3	670.542	
		Single	36		1				692.030	3.21%
AQL	1.0	Double	32	32	0	2	1	2	699.564	4.33%
		Single	50		1				708.640	5.68%
	1.5	Double	32	32	0	3	3	4	676.853	0.94%
		Single	50		2				694.970	3.64%

Polya Prior

Classification			n_1	n_2	c_1	r_1	c_2	r_2	Total Expected Cost	Percent Difference
Economically Based		Double	26	26	0	3	2	3	712.344	
		Single	32		1				7281360	2.25%
AQL	1.0	Double	32	32	0	2	1	2	738.144	3.62%
		Single	50		1				751.240	5.46%
	1.5	Double	32	32	0	3	3	4	716.185	0.54%
		Single	50		2				735.920	3.31%

(μ, σ, S_0, S_1, S_2, A_0, A_1, A_2, R_0, R_1, R_2) = (32.9999, 44.1924, 3.0, 2.5, 1.9, 10.0, 0.0, 40.0, 5.0, 2.0, 1.9), inspection level II, normal inspection, and AQL values of 1.0% and 1.5%. The results, in comparison with mixed binomial and Polya prior distributions are displayed in Table VI.5. For those examples, the savings range from about 0.5% to 6%. It is general to find that the economically based double sampling plans are considerably more cost-effective than those obtained from Military Standard 105D.

Summary

The purpose of this chapter is to present a wide array of sensitivity analyses for this research. This study considers variations in the optimum sample sizes as well as the constant factor relationship between first and second sample. This study also demonstrates the effects of incorrectly estimating the costs, and the prior distribution parameters. It also compares optimum single sampling as well as plans from Military Standard 105D. Both the mixed binomial and Polya prior distribution are considered in these analyses.

The following conclusions are drawn from this study:

1. As sample sizes vary up to ±20% from optimal sampling plan, the changes in cost compared to the optimal cost are never over 2%.
2. The best constant factor between n_1 and n_2 is 2 (i.e., $n_2 = 2 \times n_1$).
3. The sample sizes are most sensitive in cost parameters A_2 and R_1. If A_2 increases, then the sample sizes will increase. Contrarily, an increase in R_1 will decrease the sample size.

4. It is no great disadvantage in using the changed cost model's optimum plan then used in the changed environment to predict the total expected cost, sensitivity is most sensitive in R_1, A_2 next, S_2 and R_2 are the least ones.

5. Increases in the prior mean will increase the optimal sample sizes. Increases in the prior standard deviation will decrease the optimal sample size.

6. It is no large disadvantage in using either the changed cost vector's or prior distribution's optimum plan in the original cost or prior, respectively, model environment, provided changes in cost terms R_1, R_2, S_2, and A_2, and prior mean and standard deviation are within ±20%.

7. Economically based double sampling is more cost effective than either economically based single sampling or those plans obtained from Military Standard 105D. In this study, the savings range from 2% to 4%, and 0.5% to 6%, respectively.

It should be noted that these conclusions are based only upon the various parameter values selected for study herein.

CHAPTER VII

SUMMARY AND CONCLUSION

The overall objective of this research has been to provide industry and government with a new and well-developed tool to assist in selecting the most effective double acceptance sampling plan for a wide range of realistic situations.

Several specific subobjectives have been to:

1. Develop the Guthrie-Johns model for use in double sampling, including nine situations which depend on four decisions: lot 100% inspected; lot accepted outright without inspection; lot decision made following inspection of first sample; lot decision made following inspection of second sample.
2. Modify the Guthrie-Johns model to include fixed cost components for sampling, rejection, and acceptance. The cost terms developed are used to model and evaluate the cost of different decision variables and sampling outcomes.
3. Develop the theoretically exact analytical and search procedures for economically optimizing a double-sampling plan using a discrete mathematical model with the fixed cost expansion. The methodology is developed using an original break-even approach and an appropriate search procedure to determine the optimum double sample size pair and corresponding acceptance/rejection number vector. Two general

families of prior distributions, the Polya and mixed binomial, have been used to describe the actual lot quality.

4. Develop an interactive computer program for double sampling in a format suitable for use in industrial and governmental situations as well as in teaching. The program developed permits easy utilization of the design and evaluation methodologies for economically based double sampling.
5. Compare the optimum single and double-sampling plan total expected cost in order to determine the relative economic advantage of double-sampling. Sensitivity analyses were performed to determine the effects of changes in sample sizes, constant factors, cost coefficients, and prior distribution parameters on the total expected cost per lot. Also, economically based single sampling plans as well as tabulated double-sampling plans were evaluated for comparison purposes.

Based on the results obtained in this research, the following statements can be made:

a. The locus of total expected cost associated with a given acceptance/rejection vector is nearly, if not exactly, convex with a rather flat total cost surface as a function of sample size in the neighborhood of the optimum. Also, the locus of the local minima have but one global optimum over all possible sample sizes. The values of the total expected cost near the optimal sampling plan, even with different acceptance/rejection number vectors, are sufficiently close as to form a very flat shape so that there is little difference between the optimal total expected cost and the total expected cost for it's neighbor sample size pairs.

b. The economically based double-sampling plan has cost advantages over economically based single sampling. But, the savings is not significant. In this research, the savings range from 2.0% to 4.0% for different combinations of cost and prior distribution parameters.

c. The economically based double-sampling plan is more cost-effective than plans obtained from Military Standard 105D. The savings range from 0.5% to 6.0%.

d. In this research, it was determined that the best choice the second sample size n_2 is twice that of the first sample size n_1. Also, however, there is little difference between the constant factors 1, 1.5, 2, 2.5, and 3.

e. The optimal sample size pair and the total expected cost are very sensitive to cost coefficients A_2 and R_1, compared with the other cost parameters.

f. An increase in the prior mean will increase the optimal sample size pair. An increase in the prior variance will decrease the sample size pair.

Future research should consider the following:

1. A logical extension of this research is to apply techniques developed herein to economically based multiple-sampling. The success with economically based double-sampling plans may be extended to multiple-sampling. In fact, double-sampling plan is one special case of multiple-sampling when the number of stages equals two. All concepts of the cost model formulation and optimization from this research are applicable to this extension.

2. Economically based sequential sampling with fixed cost considerations should be evaluated. Economically based sequential sampling using a Bayesian prior distribution has already been developed; however, it omits the fixed cost factors.

3. Type 1 and type 2 inspection errors may be considered in an extension to this work. For this research, perfect inspection is assumed. However, inspection is well known to be imperfect. Thus, their effects should be considered.

4. Other prior distribution families should be studied. This author has been successful in using the Polya and mixed binomial families as prior distributions. Other priors may better describe actual lot quality in some situations.

Of course, there are many other related areas in which work remains to be initiated or extended. While this dissertation is certainly only a small study with respect to the entire area, it is hoped that it represents a significant contribution to quality control.

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APPENDIXES

APPENDIX A

MODIFIED GUTHRIE-JOHNS COMPUTER PROGRAM

FOR DOUBLE SAMPLING PLAN (FORTRAN

Computer Porgram Listing

Including Documentation)

```
C
C
C
C *****************************************************************
C
C      THIS PROGRAM ESTABLISHES THE INTENT OF THE USER AND DIRECTS
C      THE PROGRAM TO THE APPROPRIATE CONTROL SUBROUTINE. IT ALSO
C      INITIALIZES ALL INPUT DATA.
C
C *****************************************************************
C
C
C
       IMPLICIT REAL*8(A-H,O-X)
       DIMENSION DIDPT1(200),TCC(200),MIDPT1(200),STPT1(200),NPT1(200)
       DIMENSION SAC1(200),SRJ1(200),SAC2(200),SRJ2(200)
C**** INITIALIZE INPUT DATA
       DNLS=500.
       DNS1=60.
       DNS2=60.
       S=.462103
       T=6.539455
       W1=.6
       W2=.3
       W3=.1
       F1=.01
       F2=.1
       F3=.3
       S0=3
       S1=2.5
       S2=1.9
       A0=10
       A1=0
       A2=40
       R0=5
       R1=2
       R2=1.9
       CF=1
C**** WRITE CAPABILITIES OF PROGRAM
1      WRITE(6,101)
101    FORMAT(2X,'THIS PROGRAM PERMITS YOU TO DO THE FOLLOWING THINGS.',
      1/,2X,'(1) DESIGN AN ECONOMICALLY BASED SAMPLING PLAN',
      2/,2X,'(2) DESIGN THE OPT ACC/REJ VECTOR GIVEN SAMP SIZE PAIR',
      3/,2X,'(3) EVALUATE THE EXPECTED COST OF A SAMPLING PLAN')
C**** DETERMINE INTENT OF USER
       WRITE(6,2)
2      FORMAT(2X,'WHICH DO YOU WANT TO DO ? ENTER 1 , 2 ,OR 3')
       READ(5,*) JCODE
C**** TRANSFER TO APPROPRIATE PORTION OF PROGRAM
       IF(JCODE.LE.1) GO TO 4
```

```
      IF(JCODE.LE.2) GO TO 6
      IF(JCODE.LE.3) GO TO 70
C**** START OVER IF OTHER THAN 1,2,OR 3  ENTERED
      WRITE (6,3)
3     FORMAT(2X,'YOU DID NOT ENTER A 1,2, OR 3 ')
      GO TO 1
C**** STATE INTENT OF USER
4     WRITE(6,5)
5     FORMAT(2X,'YOU WANT TO DESIGN AN ECONOMICALLY BASED SAMPLING PLAN
     1 ! ')
      GO TO 8
6     WRITE(6,7)
7     FORMAT(2X,'YOU WANT TO DESIGN OPT ACC/REJ VECTOR GIVEN SAMPLE SIZE
     1 PAIR !  ')
      GO TO 8
70     WRITE(6,71)
71     FORMAT(2X,'YOU WANT TO EVALUATE THE EXPECTED COST OF A SAMPLING P
     1LAN ! ')
      GO TO 8
C**** CHECK INTENT OF USER
8     WRITE(6,9)
9     FORMAT(2X,'CORRECT ? NO (0) OR YES (1) ')
      READ(5,*) ICODE
      IF(ICODE.EQ.0) GO TO 1
C**** OBTAIN PRIOR DISTRIBUTION
47    WRITE(6,25)
25    FORMAT (2X, 'WHICH IS THE PRIOR DISTRIBUTION???  ',/,
     1 '         MIXED BINOMIAL(0) OR POLYA(1) ')
      READ(5,*) NCODE
      IF (NCODE.LE.0) GO TO 26
C**** WRITE POLYA PARAMETERS
32    WRITE(6,27) S,T
27    FORMAT(2X,'POLYA PARAMETERS ARE S=',F10.6,3X,'T=',F10.6)
C**** CHECK TO SEE IF PARAMETERS ARE CORRECT
      WRITE (6,28)
28    FORMAT (2X,'CORRECT??? NO(0) OR YES(1)')
      READ(5,*) ICODE
      IF (ICODE.EQ.1) GO TO 29
C**** INPUT NEW VALUES
44    WRITE(6,30)
30    FORMAT(2X,'ENTER S,T')
      READ(5,*) S,T
C**** CHECK TO ASSURE THAT NEW ENTRIES ARE RIGHT
      GO TO 32
C**** WRITE MIXED BINOMIAL PARAMETERS
26    WRITE(6,33) W1,W2,W3,F1,F2,F3
33    FORMAT (2X,'MIXED BINOMIAL PARAMETERS ARE W1=',F6.4,3X,'W2=',
     1 F6.4,3X,/,'  W3=',F6.4,3X,'F1=',F10.7,3X,'F2=',F10.7,3X,'F3=',
     2 F10.7)
C**** CHECK TO SEE IF NEW VALUES ARE DESIRED
```

```
      WRITE (6,34)
34    FORMAT (2X,'CORRECT??? NO(0) OR YES(1)')
      READ(5,*) ICODE
      IF (ICODE.EQ.1) GO TO 29
C**** WRITE WARNING
59    WRITE (6,60)
60    FORMAT (2X,'REMEMBER, W1+W2+W3=1.0 AND ALL MUST BE POSITIVE')
      WRITE (6,61)
61    FORMAT (2X,'ALSO, F1, F2, AND F3 MUST BE BETWEEN 0 AND 1, BUT ',
     1 'NOT 0 OR 1')
52    WRITE(6,35)
C**** INPUT NEW VALUES
35    FORMAT (2X,'ENTER W1,W2,W3,F1,F2,F3')
      READ(5,*) W1,W2,W3,F1,F2,F3
C**** CHECK THAT NEW ENTRIES ARE RIGHT
      GO TO 26
C**** WRITE COSTS
29    CONTINUE
      WRITE(6,37) S0,S1,S2,A0
37    FORMAT (2X,'COST VALUES ARE S0=',F6.2,3X,'S1=',F6.2,3X,'S2=',
     1 F6.2,3X,'A0=',F6.2)
53    WRITE(6,38) A1,A2,R0,R1,R2
38    FORMAT (2X, 'A1=',F6.2,3X,'A2=',F6.2,3X,'R0=',F6.2,
     1 3X, 'R1=',F6.2,3X,'R2=', F6.2)
C**** CHECK TO SEE IF COSTS ARE CORRECT
      WRITE(6,39)
39    FORMAT (2X,'CORRECT??? NO(0) OR YES(1)')
      READ(5,*) ICODE
      IF (ICODE.EQ.1) GO TO 40
C**** INPUT NEW VALUES
54    WRITE(6,41)
41    FORMAT (2X,'ENTER S0,S1,S2,A0,A1,A2,R0,R1, AND R2')
      READ(5,*) S0,S1,S2,A0,A1,A2,R0,R1,R2
C**** CHECK TO ASSURE THAT NEW ENTRIES ARE RIGHT
      GO TO 29
40    IF (JCODE.EQ.1) GO TO 43
      IF (JCODE.EQ.2) GO TO 75
C**** CALL COST CALCULATION SUBROUTINE
      CALL COSCAL(DNLS,DNS1,DNS2,NCODE,W1,W2,W3,F1,F2,F3,S,T,
     1S0,S1,S2,R0,R1,R2,A0,A1,A2,AC1,RJ1,AC2,RJ2,TACP1,TACP2,
     2TRJP2,TRJP1,TCC)
76    WRITE(6,77)
C**** GIVE OPPORTUNITY TO RUN NEW PRIOR/COST PARAMETERS
77    FORMAT(2X,'WANT TO CALCULATE COST USING NEW PRIOR/COST'
     1,/,'          PARMETERS ??? NO(0) OR YES(1)')
      READ(5,*) ICODE
      IF(ICODE.EQ.0) GO TO 46
      GO TO 47
C**** CALL AUTOMATIC PARTIAL DESIGN SUBROUTINE
75    CALL COSCNT(DNLS,DNS1,DNS2,NCODE,W1,W2,W3,F1,F2,F3,S,T,
```

```
         1S0,S1,S2,R0,R1,R2,A0,A1,A2,AC1,RJ1,AC2,RJ2,TACP1,TACP2,TRJP2,
         2TRJP1,TCC)
 55       WRITE (6,45)
C**** GIVE OPPORTUNITY TO RUN NEW PRIOR/COST PARAMETERS
 45       FORMAT (2X,'WANT TO DO ECON EVAL USING NEW PRIOR/COST',/,
         1  '          PARAMETERS ??? NO(0) OR YES(1)')
          READ(5,*) ICODE
          IF (ICODE.EQ.0) GO TO 46
          GO TO 47
C**** CALL AUTOMATIC DESIGN SUBROUTINE
 43       CALL ATODES(DNLS,NCODE,W1,W2,W3,F1,F2,F3,S,T,S0,S1,S2,
         1A0,A1,A2,R0,R1,R2,DOPTS1,DOPTS2,AC1,RJ1,AC2,RJ2,TCMIN,CF)
C**** GIVE OPPORTUNITY TO RUN NEW PRIOR/COST PARAMETERS
 56       WRITE(6,48)
 48       FORMAT (2X,'WANT TO DESIGN PLAN USING NEW PRIOR/COST ',/,
         1 '      PARAMETERS??? NO(0) OR YES(1)')
          READ(5,*) ICODE
          IF (ICODE.EQ.0) GO TO 46
          GO TO 47
C**** GIVE OPPORTUNITY TO EVALUATE NEW SAMPLING PLAN
 57       WRITE(6,49)
 49       FORMAT(2X,'WANT TO EVAL STAT PERF MEASURES OF ANOTHER PLAN ???',
         1 ' NO(0) OR YES(1)')
          READ(5,*) ICODE
          IF (ICODE.EQ.0) GO TO 46
C**** GIVE OPPORTUNITY TO START OVER
 46       WRITE(6,50)
 50       FORMAT (2X,'WANT TO START OVER ??? NO(0) OR YES(1)')
          READ(5,*) ICODE
          IF (ICODE.EQ.0) GO TO 51
          GO TO 1
 51       STOP
          END
```

```
C
C
C
C
C*****************************************************************
C
C       SUBROUTINE ATODES IS CALLED TO AUTOMATICALLY DESIGN AN
C       ECONOMICALLY BASED DOUBLE SAMPLING PLAN . IT BEGINS WITH
C       A FIRST SAMPLE SIZE (DNS1) AND A SECOND SIZE (DNS2, DNS2=
C       CONST*DNS1) OF ZERO . THE COST LOOP FOR FIXED DECISION
C       VARIABLES( AC1,RJ1.AC2,RJ2 )IS IDENTIFIED BY INCREASED THE
C       SAMPLE SIZES UNTIL ANY ONE OF THE DECISION VARIABLES CHANGES.
C       THE TOTAL COST ASSOCIATED WITH THE SAMPLE SIZES (RMID1 AND
C       RMID2 ) IN THE MIDDLE OF SEQUENTIAL COST LOOPS ARE CALCULATED,
C       STORED ,AND COMPARED . THIS PROCEDURE CONTINUES UNTIL THE
C       TOTAL COST BEGINS TO RISE . IT THEN SEARCHES FOR THE OPTIMUM
C       SAMPLE SIZES (DOPTS1 AND DOPTS2) WITHIN THE COST LOOP
C       ASSOCIATED WITH THE MINIMUM TOTAL COST . THE OPTIMUM
C       TOTAL COST IS (TCMIN) .
C
C*****************************************************************
C
C
C
        SUBROUTINE ATODES(DNLS,NCODE,W1,W2,W3,F1,F2,F3,S,T,SO,S1,
       1S2,AO,A1,A2,RO,R1,R2,DOPTS1,DOPTS2,AC1,RJ1,AC2,RJ2,TCMIN,CF)
        IMPLICIT REAL*8(A-H,O-X)
        DIMENSION DIDPT1(200),TCC(200),MIDPT1(200),STPT1(200),NPT1(200)
        DIMENSION SAC1(200),SRJ1(200),SAC2(200),SRJ2(200)
C****   WRITE CONSTANT FACTOR
120     WRITE(6,110) CF
110     FORMAT(2X,'CONSTANT FACTOR = ',F5.2)
C****   CHECK TO SEE IF CORRECT
        WRITE(6,111)
111     FORMAT(2X,'CORRECT??? NO(0) OR YES(1)')
        READ(5,*) ICODE
        IF(ICODE.EQ.1) GO TO 11
C****   INPUT NEW VALUES
        WRITE(6,113)
113     FORMAT(2X,'ENTER CONSTANT FACTOR')
        READ(5,*) CF
C****   CHECK TO ASSURE THAT NEW ENTRY IS RIGHT
        GO TO 120
C****   WRITE LOT SIZE
11      WRITE(6,10) DNLS
10      FORMAT(2X,'LOT SIZE = ',F9.2)
C****   CHECK TO SEE IF CORRECT
        WRITE(6,12)
12      FORMAT(2X,'CORRECT??? NO(0) OR YES(1)')
        READ(5,*) ICODE
        IF(ICODE.EQ.1) GO TO 18
```

```
C**** INPUT NEW VALUES
      WRITE(6,13)
13    FORMAT(2X,'ENTER LOT SIZE')
      READ(5,*) DNLS
C**** CHECK TO ASSURE THAT NEW ENTRY IS RIGHT
      GO TO 11
C**** INITIALIZE VARIABLES
18    DO 50 I=1,200
      NPT1(I)=5
C**** SEARCH FOR FEASIBLE POINT
1     NPT1(I)=NPT1(I)+1
      NPT2=CF*NPT1(I)
      DNPT1=NPT1(I)
      DNPT2=NPT2
      CALL S2S(DNLS,DNPT1,DNPT2,NCODE,W1,W2,W3,F1,F2,F3,S,T,SO,
     1S1,S2,AO,A1,A2,RO,R1,R2,AC2,RJ2)
C**** CHECK THE SECOND ACCEPTANCE NUMBER TO DETERMINE WEATHER OR
C**** NOT IT IS LESS THAN ZERO
      IF(AC2.LT.O.)GO TO 1
      CALL S1S(DNLS,DNPT1,DNPT2,NCODE,W1,W2,W3,F1,F2,F3,S,T,SO,S1,S2.
     1AO,A1,A2,RO,R1,R2,AC1,RJ1,AC2,RJ2)
C**** CHECK THE FIRST ACCEPTANCE NUMBER TO DETERMINE WEATHER OR
C**** NOT IT IS LESS THAN ZERO
      IF(AC1.LT.O.) GO TO 1
C**** GET THE INITIAL POINT FOR FIRST COST LOOP
      LOOP=1
      STPT1(1)=DNPT1
      SAC1(1)=AC1
      SRJ1(1)=RJ1
      SAC2(1)=AC2
      SRJ2(1)=AC2+1
C**** SEARCH FOR THE NEXT LOOP
2      NPT1(I)=NPT1(I)+1
      NPT2=CF*NPT1(I)
      DNPT1=NPT1(I)
      DNPT2=NPT2
      CALL S2S(DNLS,DNPT1,DNPT2,NCODE,W1,W2,W3,F1,F2,F3,S,T,SO,
     1S1,S2,AO,A1,A2,RO,R1,R2,AC2,RJ2)
      CALL S1S(DNLS,DNPT1,DNPT2,NCODE,W1,W2,W3,F1,F2,F3,S,T,SO,S1,S2,
     1AO,A1,A2,RO,R1,R2,AC1,RJ1,AC2,RJ2)
      IF(AC1.EQ.SAC1(LOOP).AND.RJ1.EQ.SRJ1(LOOP).AND.AC2.EQ.
     1SAC2(LOOP)) GO TO 2
C**** NEXT LOOP HAS BEEN FOUND,CHECK MIDDLE POINT
      MIDPT1(LOOP)=(DNPT1-1+STPT1(LOOP))/2
      DIDPT1(LOOP)=MIDPT1(LOOP)
      CALL COSEVL(DNLS,DIDPT1(LOOP),CF*DIDPT1(LOOP),NCODE,W1,W2,
     1W3,F1,F2,F3,S,T,SO,S1,S2,RO,R1,R2,AO,A1,A2,SAC1(LOOP),SRJ1(LOOP)
     2,SAC2(LOOP),SRJ2(LOOP),TACP1,TACP2,TRJP2,TRJP1,TCC(LOOP))
C**** DETERMINE WHETHER OR NOT TO STOP SEARCH IN THE NEXT LOOP
      IF(LOOP.EQ.1) GO TO 3
```

```
      READ(5,*) ICODE
      IF (ICODE.EQ.1) GO TO 5
C**** INPUT NEW VALUES
22    WRITE(6,6)
6     FORMAT (2X,'ENTER LOT SIZE,1ST SAMP SIZE,AND 2ND SAMP SIZE')
      READ(5,*) DNLS,DNS1,DNS2
C**** CHECK TO ASSURE THAT NEW ENTRIES ARE RIGHT
      GO TO 1
C**** BEGIN OUTPUT
5     WRITE (6,4)
      WRITE (6,4)
      WRITE(6,4)
4     FORMAT(2X,68('*'))
C**** WRITE TITLE
      WRITE (6,23)
23    FORMAT (/,2X,'OPTIMUM ACC/REJ NUMBER VECTOR DESIGN')
      WRITE(6,2) DNLS,DNS1,DNS2
      IF (NCODE.EQ.0) GO TO 10
C**** WRITE POLYA PARAMETERS
      WRITE(6,11) S,T
11    FORMAT (2X,'POLYA PARAMETERS ARE S=',F10.6,3X,'T=',F10.6)
      GO TO 12
C**** WRITE MIXED BINOMIAL PARAMETERS
10    WRITE (6,13) W1,W2,W3,F1,F2,F3
13    FORMAT (2X,'MIXED BINOMIAL PARAMETERS ARE W1=',F6.4,3X,'W2=',
     1 F6.4,3X,/,'  W3=',F6.4,3X,'F1=',F10.7,3X,'F2=',F10.7,3X,'F3=',
     2 F10.7)
C**** WRITE COSTS
12    WRITE(6,14) SO,S1,S2,AO
14    FORMAT (2X,'COST VALUES ARE SO=',F6.2,3X,'S1=',F6.2,3X,
     C 'S2 =',F6.2,3X,'AO =',F6.2)
      WRITE(6,15) A1,A2,RO,R1,R2
15    FORMAT (2X,'A1 =',F6.2,3X,'A2 =',F6.2,3X,'RO =',F6.2,3X,
     C 'R1 =',F6.2,3X,'R2 =',F6.2)
C**** CALL DECISION VARIABLES SUBROUTINES S2S,S1S
      CALL S2S(DNLS,DNS1,DNS2,NCODE,W1,W2,W3,F1,F2,F3,S,T,
     1SO,S1,S2,AO,A1,A2,RO,R1,R2,AC2,RJ2)
      CALL S1S(DNLS,DNS1,DNS2,NCODE,W1,W2,W3,F1,F2,F3,S,T,
     1SO,S1,S2,AO,A1,A2,RO,R1,R2,AC1,RJ1,AC2,RJ2)
      CALL COSEVL(DNLS,DNS1,DNS2,NCODE,W1,W2,W3,F1,F2,F3,S,T,
     1SO,S1,S2,RO,R1,R2,AO,A1,A2,AC1,RJ1,AC2,RJ2,TACP1,TACP2,
     2TRJP2,TRJP1,TCC)
      WRITE(6,55)AC1,RJ1,AC2,RJ2
55    FORMAT(2X,'ACC NO 1 = ',F5.1,' REJ NO 1 = ',F5.1,
     1' ACC NO 2 = ',F5.1,' REJ NO 2 = ',F5.1)
      WRITE(6,19) TACP1,TRJP1,TACP2,TRJP2,TCC
19    FORMAT(2X,'ACC 1ST SAMP COST = ',F9.2,2X,
     1'REJ 1ST SAMP COST = ',F9.2,/,2X,'ACC 2ND SAMP COST ='
     2,F9.2,2X,' REJ 2ND SAMP COST = ',F9.2,/,2X,
     3'TOTAL COST = ',F15.3,/)
```

```
      WRITE (6,4)
      WRITE (6,4)
      WRITE(6,4)
C**** GIVE OPPORTUNITY TO RUN NEW SAMPLING PLAN
24    WRITE(6,20)
20    FORMAT(2X,'WANT TO EVAL ANOTHER SAMP PLAN USING LOT, SAMP SIZES
     1 ???',/,'  NO(0) OR YES(1)')
      READ(5,*) ICODE
      IF (ICODE.EQ.0) GO TO 21
      GO TO 22
21    RETURN
      END
C
C
C
C*******************************************************************
C
C     SUBROUTINE COSCAL CALCULATES THE TOTAL COST OF DOUBLE
C     SAMPLING PLAN . THE INPUTS ARE LOT,1ST,AND 2ND SAMPLE
C     SIZE, ALL DECISION VARIABLES . THE OUTPUT IS TOTAL COST.
C
C*******************************************************************
C
C
C
      SUBROUTINE COSCAL(DNLS,DNS1,DNS2,NCODE,W1,W2,W3,F1,F2,F3,S,T,
     1S0,S1,S2,R0,R1,R2,A0,A1,A2,AC1,RJ1,AC2,RJ2,TACP1,
     2TACP2,TRJP2,TRJP1,TCC)
      IMPLICIT REAL*8(A-H,O-X)
C**** WRITE LOT SIZE, 1ST SAMPLE SIZE, AND 2ND SAMPLE SIZE
1     WRITE(6,2) DNLS,DNS1,DNS2
2     FORMAT(2X,'LOT SIZE = ',F7.1,' 1ST SAMP SIZE = ',F6.1,
     1' 2ND SAMP SIZE = ',F6.1)
C**** CHECK TO SEE IF CORRECT
      WRITE(6,3)
3     FORMAT (2X,'CORRECT??? NO(0) OR YES(1)')
      READ(5,*) ICODE
      IF (ICODE.EQ.1) GO TO 32
C**** INPUT NEW VALUES
22    WRITE(6,6)
6     FORMAT (2X,'ENTER LOT SIZE, 1ST SAMP SIZE,AND 2ND SAMP SIZE')
      READ(5,*) DNLS,DNS1,DNS2
C**** CHECK TO ASSURE THAT NEW ENTRIES ARE RIGHT
      GO TO 1
32    WRITE(6,30) AC1,RJ1,AC2,RJ2
30    FORMAT(2X,'ACC/REJ NUMBERS ARE C1= ',F5.1,' R1= ',
     1F5.1,' C2= ',F5.1,' R2= ',F5.1)
C**** CHECK TO SEE IF CORRECT
      WRITE(6,53)
53     FORMAT (2X,'CORRECT??? NO(0) OR YES(1)')
```

```
      READ(5,*) ICODE
      IF (ICODE.EQ.1) GO TO 5
C**** INPUT NEW VALUES
      WRITE(6,31)
31    FORMAT(2X,'ENTER C1,R1,C2,AND R2')
      READ(5,*) AC1,RJ1,AC2,RJ2
      GO TO 32
C**** BEGIN OUTPUT
5     WRITE (6,4)
      WRITE (6,4)
      WRITE(6,4)
4     FORMAT(2X,68('*'))
C**** WRITE TITLE
      WRITE (6,23)
23    FORMAT (/,2X,'EXPECTED COST EVALUATION')
      WRITE(6,2) DNLS,DNS1,DNS2
      IF (NCODE.EQ.0) GO TO 10
C**** WRITE POLYA PARAMETERS
      WRITE(6,11) S,T
11    FORMAT (2X,'POLYA PARAMETERS ARE S=',F10.6,3X,'T=',F10.6)
      GO TO 12
C**** WRITE MIXED BINOMIAL PARAMETERS
10    WRITE (6,13) W1,W2,W3,F1,F2,F3
13    FORMAT (2X,'MIXED BINOMIAL PARAMETERS ARE W1=',F6.4,3X,'W2=',
     1 F6.4,3X,/,'  W3=',F6.4,3X,'F1=',F10.7,3X,'F2=',F10.7,3X,'F3=',
     2 F10.7)
C**** WRITE COSTS
12    WRITE(6,14) SO,S1,S2,AO
14    FORMAT (2X,'COST VALUES ARE SO=',F6.2,3X,'S1=',F6.2,3X,
     C 'S2 =',F6.2,3X,'AO =',F6.2)
      WRITE(6,15) A1,A2,RO,R1,R2
15    FORMAT (2X,'A1 =',F6.2,3X,'A2 =',F6.2,3X,'RO =',F6.2,3X,
     C 'R1 =',F6.2,3X,'R2 =',F6.2)
C**** CAL COST EVALUATION SUBROUTINE COSEVL
      CALL COSEVL(DNLS,DNS1,DNS2,NCODE,W1,W2,W3,F1,F2,F3,S,T,
     1SO,S1,S2,RO,R1,R2,AO,A1,A2,AC1,RJ1,AC2,RJ2,TACP1,TACP2,
     2TRJP2,TRJP1,TCC)
      WRITE(6,55)AC1,RJ1,AC2,RJ2
55    FORMAT(2X,'ACC NO 1 = ',F5.1,' REJ NO 1 = ',F5.1,
     1' ACC NO 2 = ',F5.1,' REJ NO 2 = ',F5.1)
      WRITE(6,19) TACP1,TRJP1,TACP2,TRJP2,TCC
19    FORMAT(2X,'ACC 1ST SAMP COST = ',F9.2,2X,
     1'REJ 1ST SAMP COST = ',F9.2,/,2X,'ACC 2ND SAMP COST ='
     2,F9.2,2X,' REJ 2ND SAMP COST = ',F9.2,/,2X,
     3'TOTAL COST = ',F15.3,/)
      WRITE (6,4)
      WRITE (6,4)
      WRITE(6,4)
C**** GIVE OPPORTUNITY TO RUN NEW SAMPLING PLAN
24    WRITE(6,20)
```

```
 20     FORMAT(2X,'WANT TO EVAL ANOTHER SAMP PLAN USING LOT,SAMP SIZES
       1 ???',/,'  NO(0) OR YES(1)')
        READ(5,*) ICODE
        IF (ICODE.EQ.0) GO TO 21
        GO TO 22
 21     RETURN
        END
C
C
C
C*****************************************************************
C
C       SUBROUTINE COSEVL CALCULATES THE TOTAL COST PER LOT OF
C       THE SAMPLING PLAN (TCC) . IT INCLUDES THE COST OF ACCEPTANCE
C       ON THE FIRST SAMPLE (TACP1) . THE COST OF ACCEPTANCE ON
C       THE SECOND SAMPLE (TACP2) , THE COST OF REJECTON ON THE
C       SECOND SAMPLE (TRJP2) , AND THE COST OF REJECTION ON THE
C       FIRST SAMPLE (TRJP1).  TCC = TACP1+TACP2+TRJP2+TACP1.
C
C*****************************************************************
C
C
C
        SUBROUTINE COSEVL(DNLS,DNS1,DNS2,NCODE,W1,W2,W3,F1,F2,F3,S,T,
       1S0,S1,S2,R0,R1,R2,A0,A1,A2,AC1,RJ1,AC2,RJ2,TACP1,TACP2,
       2TRJP2,TRJP1,TCC)
        IMPLICIT REAL*8(A-H,O-X)
        CUMGNX=0.
C****   THIS PART CALCULATES THE COST FOR ACCEPTING FIRST SAMPLE
C****   (TACP1)
        TACP1=0.
        K=AC1+1.001
        IF(K.LE.0) GO TO 70
C****   CONSIDER ACCEPTANCE RANGE
 1      DO 3 J=1,K
        DND1=J-1
        IF(NCODE.EQ.1) GO TO 2
        CALL MB1S(DNLS,DNS1,DND1,W1,W2,W3,F1,F2,F3,
       1 EXX1GX,HNXX1X)
        CALL MB1SM(DNS1,DND1,W1,W2,W3,F1,F2,F3,GN1X1)
        GO TO 1550
 2      CALL PL1S(DNLS,DNS1,DND1,S,T,EXX1GX,HNXX1X)
        CALL PL1SM(DNS1,DND1,S,T,GN1X1)
C****   CALCULATE COST
 1550   TACP1=TACP1+(S0+S1*DNS1+DND1*S2+A0*(1-HNXX1X)
       1+A1*(DNLS-DNS1)+A2*EXX1GX)*GN1X1
C****   ACCUMULATE THE MARGINAL TERM
        CUMGNX=CUMGNX+GN1X1
 3      CONTINUE
C****   THIS PART CALCULATES THE TOTAL COST FOR SECOND SAMPLE
```

```
C**** . IT INCLUDES THE COST FOR ACCEPTING SECOND SAMPLING
C**** (TACP2) AND THE COST FOR REJECTING SECOND SAMPLING
C**** (TRJP2) .
C**** CALCULATE THE ACCEPTING SECOND SAMPLE PART(TACP2)
70     TACP2=0.
       TRJP2=0.
       K1=AC1+2
       L1=RJ1
C**** CONSIDER FIRST SAMPLE RESULTS WHICH WILL REQUIRE  SECOND
C**** SAMPLE
       DO 22 J1=K1,L1
       DND1=J1-1
       IF(NCODE.EQ.1) GO TO 5
       CALL MB1SM(DNS1,DND1,W1,W2,W3,F1,F2,F3,GN1X1)
C**** CALCULATE THE MARGINAL TERM
       CUMGNX=CUMGNX+GN1X1
       GO TO 6
5      CALL PL1SM(DNS1,DND1,S,T,GN1X1)
C**** CALCULATE THE MARGINAL TERM
       CUMGNX=CUMGNX+GN1X1
6      K2=DNS2+1.001
       CUMH21=0.
C**** CONSIDER EACH POSSIBLE DEFECTIVE IN SECOND SAMPLE
       DO 13 J2=1,K2
       DND2=J2-1
C**** CALCULATE COMBINED SAMPLE DEFECTIVES
       TX=DND1+DND2
       IF(NCODE.EQ.1) GO TO 7
       CALL MB2S(DNLS,DNS1,DNS2,TX,W1,W2,W3,F1,F2,F3,EXX2GX,HNXX2X)
       CALL MB12S(DNLS,DNS1,DNS2,DND1,DND2,W1,W2,W3,F1,F2,F3,HX2X1)
       GO TO 8
7      CALL PL2S(DNLS,DNS1,DNS2,TX,S,T,EXX2GX,HNXX2X)
       CALL PL12S(DNLS,DNS1,DNS2,DND1,DND2,S,T,HX2X1)
C**** IF NUMBER OF DEFECTIVES IN SECOND SAMPLE EXCEED ALLOWABLE
C**** NUMBER
8      IF(DND2.GT.(AC2-DND1)) GO TO 20
C**** CALCULATE THE COST OF REJTION ON THE SECOND SAMPLE
       TACP2=TACP2+((SC+S1*(DNS1+DNS2)+S2*(DND1+DND2)+A0*(1-HNXX2X)
      1+A1*(DNLS-DNS1-DNS2)+A2*EXX2GX)*HX2X1)*GN1X1
C**** ACCUMULATE POSTERIOR OF DND2 GIVEN DND1
       CUMH21=CUMH21+HX2X1
       GO TO 13
C**** CALCULATE THE REJECTING SECOND SAMPLE PART(TRJP2)
20     TRJP2=TRJP2+((SO+S1*(DNS1+DNS2)+S2*(DND1+DND2)
      1+RO+R1*(DNLS-DNS1-DNS2)+R2*EXX2GX)*HX2X1)*GN1X1
C**** ACCUMULATE POSTERIOR OF DND2 GIVEN DND1
       CUMH21=CUMH21+HX2X1
       IF(CUMH21.GT..999) GO TO 22
13     CONTINUE
22     CONTINUE
```

```
C**** CALCULATES THE COST FOR REJECTING FIRST SAMPLE
21     TRJP1=0.
      K=RJ1+1
      L=DNS1+1
C*** CONSIDER REJECTION RANGE
14    DO 24 J=K,L
      DND1=J-1
      IF(NCODE.EQ.1) GO TO 15
      CALL MB1S(DNLS,DNS1,DND1,W1,W2,W3,F1,F2,F3,EXX1GX,HNXX1X)
      CALL MB1SM(DNS1,DND1,W1,W2,W3,F1,F2,F3,GN1X1)
      GO TO 16
15    CALL PL1S(DNLS,DNS1,DND1,S,T,EXX1GX,HNXX1X)
      CALL PL1SM(DNS1,DND1,S,T,GN1X1)
C**** CALCULATE THE COST OF REJECTION ON FIRST SAMPLE
16    TRJP1=TRJP1+(SO+S1*DNS1+DND1*S2+RO+R1*(DNLS-DNS1)
     1+R2*EXX1GX)*GN1X1
C**** ACCUMULATE THE MARGINAL TERM
      CUMGNX=CUMGNX+GN1X1
      IF(CUMGNX.GT..999) GO TO 25
24    CONTINUE
C**** CALCULATE TOTAL COST PER LOT
25    TCC=TACP1+TACP2+TRJP2+TRJP1
      RETURN
      END
C
C
C
C**************************************************************
C
C     SUBROUTINE S2S CALCULATES THE ACCEPTANCE NUMBER (AC2)
C     AND THE REJECTION NUMBER (RJ2) IN THE SECOND SAMPLE
C     BY COMPAREING THE BREAK EVEN POINT ( BE ) AND THE EXPECTED
C     NUMBER OF DEFECTIVES IN THE REST OF LOT GIVEN THE NUMBER OF
C     DEFECTIVES IN THE FIRST AND SECOND SAMPLES ( EXX2GX )
C
C**************************************************************
C
C
C
      SUBROUTINE S2S(DNLS,DNS1,DNS2,NCODE,W1,W2,W3,F1,F2,F3,
     1S,T,SO,S1,S2,AO,A1,A2,RO,R1,R2,AC2,RJ2)
      IMPLICIT REAL*8(A-H,O-X)
      IF (NCODE.EQ.1) GO TO 11
      TX=-1.
 7    TX=TX+1.0
      CALL MB2S(DNLS,DNS1,DNS2,TX,W1,W2,W3,F1,F2,F3,EXX2GX,HNXX2X)
C
C**** CALCULATE BREAK EVEN
      BE=(RO-AO*(1.-HNXX2X)+(R1-A1)*(DNLS-DNS1-DNS2))/(A2-R2)
      IF (EXX2GX.LE.BE) GO TO 7
```

```
      IF (EXX2GX.GT.BE) GO TO 9
11    TX=-1.
C**** INCREMENT COMBINED SAMPLE DEFECTIVES
 8    TX=TX+1.0
      CALL PL2S(DNLS,DNS1,DNS2,TX,S,T,EXX2GX,HNXX2X)
C
C**** CALCULATE BREAK EVEN
      BE=(RO-AO*(1.-HNXX2X)+(R1-A1)*(DNLS-DNS1-DNS2))/(A2-R2)
      IF (EXX2GX.LE.BE) GO TO 8
C**** DECREMENT COMBINED SAMPLE ACCEPTANCE NUMBER
 9    AC2=TX-1.
C**** SET COMBINED SAMPLE REJECTION NUMBER
      RJ2=AC2+1.
      RETURN
      END
C
C
C
C*****************************************************************
C
C     SUBROUTINE S1S CALCULATES THE ACCEPTANCE (AC1) AND
C     REJECTION (RJ1) NUMBERS FOR THE FIRST SAMPLE . ACCEPTANCE
C     NUMBER AC1 IS DECIDED USING A BREAK EVEN ANALYSIS BETWEEN
C     THE COST PER LOT OF ACCEPTANCE ON THE FIRST SAMPLE (TAC1)
C     AND THE COST PER LOT OF MAKING A DECISION BASED UPON A
C     COMBINED SAMPLE (TC2) . REJECTION NUMBER RJ1 IS DECIDED
C     USING A BREAK EVEN ANALYSIS BETWEEN THE COST PER LOT OF
C     REJECTION ON THE FIRST SAMPLE (TRJ1) AND THE COST PER LOT
C     OF MAKING A DECISION BASED UPON A COMBINED SAMPLE (TC2).
C     THE TOTAL COST OF THE SECOND SAMPLE (TC2) IS THE SUM OF
C     THE TOTAL COST OF ACCEPTANCE (TAC2) AND REJECTION (TRJ2)
C     FOLLOWING THE SECOND SAMPLE.
C
C*****************************************************************
C
C
C
      SUBROUTINE S1S(DNLS,DNS1,DNS2,NCODE,W1,W2,W3,F1,F2,F3,S,T,S0,S1,S2
     1,A0,A1,A2,R0,R1,R2,AC1,RJ1,AC2,RJ2)
      IMPLICIT REAL*8(A-H,O-X)
C**** DETERMINE THE VALUE OF AC1
C**** CALCULATE TAC1
      TX=0.
      DND1=-1.
29    DND1=DND1+1.
      TAC1=0.
      TAC2=0.
      TRJ2=0.
      TC2=0.
      IF(NCODE.EQ.1) GO TO 30
```

```
      CALL MB1S(DNLS,DNS1,DND1,W1,W2,W3,F1,F2,F3,
     1 EXX1GX,HNXX1X)
      GO TO 31
30    CALL PL1S(DNLS,DNS1,DND1,S,T,EXX1GX,HNXX1X)
31    TAC1=S0+S1*DNS1+DND1*S2+A0*(1-HNXX1X)
     1+A1*(DNLS-DNS1)+A2*EXX1GX
C**** CALCULATE TAC2
      K=AC2-DND1+1.001
C**** CONSIDER ACCEPTANCE RANGE
      DO 32 J=1,K
      DND2=J-1
      TX=DND1+DND2
      IF(NCODE.EQ.1) GO TO 33
      CALL MB2S(DNLS,DNS1,DNS2,TX,W1,W2,W3,F1,F2,F3,EXX2GX,
     1 HNXX2X)
      CALL MB1S(DNLS,DNS1,DND1,W1,W2,W3,F1,F2,F3,
     1 EXX1GX,HNXX1X)
      CALL MB12S(DNLS,DNS1,DNS2,DND1,DND2,W1,W2,W3,
     1 F1,F2,F3,HX2X1)
      GO TO 32
33    CALL PL2S(DNLS,DNS1,DNS2,TX,S,T,EXX2GX,HNXX2X)
      CALL PL1S(DNLS,DNS1,DND1,S,T,EXX1GX,HNXX1X)
      CALL PL12S(DNLS,DNS1,DNS2,DND1,DND2,S,T,HX2X1)
C**** CALCULATE COST
32    TAC2=TAC2+(S0+S1*(DNS1+DNS2)+S2*(DND1+DND2)+A0*(1-HNXX2X)
     1+A1*(DNLS-DNS1-DNS2)+A2*EXX2GX)*HX2X1
C**** CALCULATE TRJ2
      K=AC2-DND1+2.001
      L=DNS2+1.001
C**** CONSIDER REJECTION RANGE
      DO 35 J=K,L
      DND2=J-1
      TX=DND1+DND2
      IF(NCODE.EQ.1) GO TO 34
      CALL MB2S(DNLS,DNS1,DNS2,TX,W1,W2,W3,F1,F2,F3,EXX2GX,HNXX2X)
      CALL MB1S(DNLS,DNS1,DND1,W1,W2,W3,F1,F2,F3,
     1 EXX1GX,HNXX1X)
      CALL MB12S(DNLS,DNS1,DNS2,DND1,DND2,W1,W2,W3,
     1 F1,F2,F3,HX2X1)
      GO TO 35
34    CALL PL2S(DNLS,DNS1,DNS2,TX,S,T,EXX2GX,HNXX2X)
      CALL PL1S(DNLS,DNS1,DND1,S,T,EXX1GX,HNXX1X)
      CALL PL12S(DNLS,DNS1,DNS2,DND1,DND2,S,T,HX2X1)
C**** CALCULATE COST
35    TRJ2=TRJ2+(S0+S1*(DNS1+DNS2)+(DND1+DND2)*S2+R0+
     1R1*(DNLS-DNS1-DNS2)+R2*EXX2GX)*HX2X1
C**** CALCULATE TOTAL COST
      TC2=TAC2+TRJ2
C
      IF (TAC1.LE.TC2) GO TO 29
```

```
C**** SET ACCEPTANCE NUMBER AC1
      AC1=DND1-1.
C**** DETERMINE THE VALUE OF RJ1
C**** CALCULATE TRJ1
      TX=0.
      DND1=AC1
39    DND1=DND1+1.
      TRJ1=0.
      TAC2=0.
      TRJ2=0.
      TC2=0.
      IF (NCODE.EQ.1) GO TO 40
      CALL MB1S(DNLS,DNS1,DND1,W1,W2,W3,F1,F2,F3,
     1 EXX1GX,HNXX1X)
      GO TO 41
40    CALL PL1S(DNLS,DNS1,DND1,S,T,EXX1GX,HNXX1X)
41    TRJ1=SO+DNS1*S1+DND1*S2+RO+(DNLS-DNS1)*R1+R2*EXX1GX
C**** CALCULATE TAC2
      K=AC2-DND1+1.001
      IF(K.LT.1) GO TO 60
C**** CONSIDER ACCEPTANCE RANGE
      DO 42 J=1,K
      DND2=J-1
      TX=DND1+DND2
      IF(NCODE.EQ.1) GO TO 43
      CALL MB2S(DNLS,DNS1,DNS2,TX,W1,W2,W3,F1,F2,F3,EXX2GX,HNXX2X)
      CALL MB1S(DNLS,DNS1,DND1,W1,W2,W3,F1,F2,F3,
     1 EXX1GX,HNXX1X)
      CALL MB12S(DNLS,DNS1,DNS2,DND1,DND2,W1,W2,W3,
     1 F1,F2,F3,HX2X1)
      GO TO 42
43    CALL PL2S(DNLS,DNS1,DNS2,TX,S,T,EXX2GX,HNXX2X)
      CALL PL1S(DNLS,DNS1,DND1,S,T,EXX1GX,HNXX1X)
C
42    TAC2=TAC2+(SO+S1*(DNS1+DNS2)+S2*(DND1+DND2)+AO*(1-HNXX2X)
     1+A1*(DNLS-DNS1-DNS2)+A2*EXX2GX)*HX2X1
C**** CALCULATE TRJ2
60    K=AC2-DND1+2.001
      L=DNS2+1.001
      DO 45 J=K,L
      DND2=J-1
      TX=DND1+DND2
      IF (NCODE.EQ.1) GO TO 44
      CALL MB2S(DNLS,DNS1,DNS2,TX,W1,W2,W3,F1,F2,F3,EXX2GX,HNXX2X)
      CALL MB1S(DNLS,DNS1,DND1,W1,W2,W3,F1,F2,F3,
     1 EXX1GX,HNXX1X)
      CALL MB12S(DNLS,DNS1,DNS2,DND1,DND2,W1,W2,W3,F1,F2,F3
     1,HX2X1)
      GO TO 45
44    CALL PL2S(DNLS,DNS1,DNS2,TX,S,T,EXX2GX,HNXX2X)
```

```
      CALL PL1S(DNLS,DNS1,DND1,S,T,EXX1GX,HNXX1X)
      CALL PL12S(DNLS,DNS1,DNS2,DND1,DND2,S,T,HX2X1)
45    TRJ2=TRJ2+(S0+S1*(DNS1+DNS2)+(DND1+DND2)*S2+RO
     1+R1*(DNLS-DNS1-DNS2)+R2*EXX2GX)*HX2X1
      TC2=TAC2+TRJ2
      ROTC=(TRJ1-TC2)/TRJ1
      IF(ROTC.GT..001) GO TO 39
C**** SET REJECTION NUMBER RJ1
      RJ1=DND1
      RETURN
      END
C
C
C
C*******************************************************************
C
C     SUBROUTINE MB2S CALCULATES TERMS FOR MIXED BINOMIAL
C     PRIOR DISTRIBUTION . THE TERMS ARE RELEVANT FOLLOWING
C     THE SECOND SAMPLE . THE TERMS INCLUDE THE EXPECTED
C     NUMBER OF DEFECTIVES IN THE REST OF THE LOT GIVEN THE
C     NUMBER OF DEFECTIVES IN THE FIRST AND SECOND SAMPLES
C     ( EXX2GX ) , AND THE POSTERIOR PROBABILITY THAT THERE
C     ARE NO ADDITIONAL DEFECTIVES IN THE LOT GIVEN THE NUMBER
C     OF DEFECTIVES IN THE FIRST AND SECOND SAMPLES ( HNXX2X ).
C
C*******************************************************************
C
C
C
      SUBROUTINE MB2S(DNLS,DNS1,DNS2,TX,W1,W2,W3,F1,F2,F3,
     1 EXX2GX,HNXX2X)
      IMPLICIT REAL*8(A-H,O-X)
C**** CALCULATE THE DENOMINATOR TERMS FOR EXX2GX AND
C**** HNXX2X .
      G21L=DLOG(W1)+(TX)*DLOG(F1)+(DNS1+DNS2-TX)*DLOG(1.-F1)
      IF (G21L.LE.-174.) G21L=-174.
      G21=DEXP(G21L)
      G22L=DLOG(W2)+(TX)*DLOG(F2)+(DNS1+DNS2-TX)*DLOG(1.-F2)
      IF (G22L.LE.-174.) G22L=-174.
      G22=DEXP(G22L)
      G23L=DLOG(W3)+(TX)*DLOG(F3)+(DNS1+DNS2-TX)*DLOG(1.-F3)
      IF (G23L.LE.-174.) G23L=-174.
      G23=DEXP(G23L)
C**** CALCULATE THE TERMS FOR EXX2GX
      W21E=DLOG(F1)+G21L
      IF(W21E.LE.-174.) W21E=-174.
      W21=DEXP(W21E)
      W22E=DLOG(F2)+G22L
      IF(W22E.LE.-174.) W22E=-174.
      W22=DEXP(W22E)
```

```
      W23E=DLOG(F3)+G23L
      IF(W23E.LE.-174.) W23E=-174.
      W23=DEXP(W23E)
C**** CALCULATE  EXX2GX
      W2T=(W21+W22+W23)/(G21+G22+G23)
      EXX2GX=(DNLS-DNS1-DNS2)*W2T
C****CALCULATE THE TERMS FOR HNXX2X
      H21L=DLOG(W1)+TX*DLOG(F1)+(DNLS-TX)*DLOG(1.-F1)
      IF(H21L.LE.-174.) H21L=-174.
      H21=DEXP(H21L)
      H22L=DLOG(W2)+TX*DLOG(F2)+(DNLS-TX)*DLOG(1.-F2)
      IF(H22L.LE.-174.) H22L=-174.
      H22=DEXP(H22L)
      H23L=DLOG(W3)+TX*DLOG(F3)+(DNLS-TX)*DLOG(1.-F3)
      IF(H23L.LE.-174.) H23L=-174.
      H23=DEXP(H23L)
C**** CALCULATE  HNXX2X
      HNXX2X=(H21+H22+H23)/(G21+G22+G23)
      RETURN
      END
C
C
C
C*********************************************************************
C
C      SUBROUTINE MB1S CALCULATES TERMS FOR THE MIXED BINOMIAL
C      PRIOR DISTRIBUTION . THE TERMS ARE RELEVANT FOLLOWING
C      THE FIRST SAMPLE . THE TERMS INCLUDE THE EXPECTED NUMBER
C      OF DEFECTIVES IN THE REST OF THE LOT GIVEN THE NUMBER OF
C      DEFECTIVES IN THE FIRST SAMPLE ( EXX1GX ) , THE POSTERIOR
C      PROBABILITY THAT THERE ARE NO ADDITIONAL DEFECTIVES IN
C      THE LOT GIVEN THE NUMBER OF DEFECTIVES IN THE FIRST SAMPLE
C      ( HNXX1X ) .
C
C*********************************************************************
C
C
C
      SUBROUTINE MB1S(DNLS,DNS1,DND1,W1,W2,W3,
     1 F1,F2,F3,EXX1GX.HNXX1X)
      IMPLICIT REAL*8(A-H,O-X)
C**** CALCULATE THE DENOMINATOR TERMS FOR EXX1GX
      G11L=DLOG(W1)+DND1*DLOG(F1)+(DNS1-DND1)*DLOG(1.-F1)
      IF(G11L.LE.-174.) G11L=-174.
      G11=DEXP(G11L)
      G12L=DLOG(W2)+DND1*DLOG(F2)+(DNS1-DND1)*DLOG(1.-F2)
      IF(G12L.LE.-174.) G12L=-174.
      G12=DEXP(G12L)
      G13L=DLOG(W3)+DND1*DLOG(F3)+(DNS1-DND1)*DLOG(1.-F3)
      IF(G13L.LE.-174.) G13L=-174.
```

```
      G13=DEXP(G13L)
C**** CALCULATE THE TERMS FOR EXX1GX
      W11E=DLOG(F1)+G11L
      IF(W11E.LE.-174.) W11E=-174.
      W11=DEXP(W11E)
      W12E=DLOG(F2)+G12L
      IF(W12E.LE.-174.) W12E=-174.
      W12=DEXP(W12E)
      W13E=DLOG(F3)+G13L
      IF(W13E.LE.-174.) W13E=-174.
      W13=DEXP(W13E)
C**** CALCULATE  EXX1GX
      W1T=(W11+W12+W13)/(G11+G12+G13)
      EXX1GX=(DNLS-DNS1)*W1T
C**** CALCULATE THE  TERMS FOR HNXX1X
      H11L=DLOG(W1)+DND1*DLOG(F1)+(DNLS-DND1)*DLOG(1.-F1)
      IF(H11L.LE.-174.) H11L=-174.
      H11=DEXP(H11L)
      H12L=DLOG(W2)+DND1*DLOG(F2)+(DNLS-DND1)*DLOG(1.-F2)
      IF(H12L.LE.-174.) H12L=-174.
      H12=DEXP(H12L)
      H13L=DLOG(W3)+DND1*DLOG(F3)+(DNLS-DND1)*DLOG(1.-F3)
      IF(H13L.LE.-174.)H13L=-174.
      H13=DEXP(H13L)
C**** CALCULATE  HNXX1X
      HNXX1X=(H11+H12+H13)/(G11+G12+G13)
      RETURN
      END
C
C
C
C*******************************************************************
C
C     SUBROUTINE MB12S CALCULATES FOR THE MIXED BINOMIAL
C     PRIOR DISTRIBUTION . THE TERM IS RELEVANT FOLLOWING
C     FIRST AND SECOND SAMPLE . IT IS THE CONDITIONAL
C     PROBABILITY OF NUMBER OF DEFECTIVES IN THE SECOND
C     SAMPLE GIVEN THE FIRST SAMPLE ( HX2X1 )
C
C*******************************************************************
C
C
C
      SUBROUTINE MB12S(DNLS,DNS1,DNS2,DND1,DND2,W1,W2,W3,
     1 F1,F2,F3,HX2X1)
      IMPLICIT REAL*8(A-H,O-X)
      G11L=DLOG(W1)+DND1*DLOG(F1)+(DNS1-DND1)*DLOG(1.-F1)
      IF(G11L.LE.-174.) G11L=-174.
      G11=DEXP(G11L)
      G12L=DLOG(W2)+DND1*DLOG(F2)+(DNS1-DND1)*DLOG(1.-F2)
```

```
      IF(G12L.LE.-174.) G12L=-174.
      G12=DEXP(G12L)
      G13L=DLOG(W3)+DND1*DLOG(F3)+(DNS1-DND1)*DLOG(1.-F3)
      IF(G13L.LE.-174.) G13L=-174.
      G13=DEXP(G13L)
C**** CALCULATE THE TERMS FOR HX2X1
      COMBL2=DLGAMA(DNS2+1.)-DLGAMA(DND2+1.)-DLGAMA(DNS2-DND2+1.)
      A21L=DND2*DLOG(F1)+(DNS2-DND2)*DLOG(1.-F1)+COMBL2+
     1 DLOG(G11/(G11+G12+G13))
      IF(A21L.LE.-174.) A21L=-174.
      A21=DEXP(A21L)
      A22L=DND2*DLOG(F2)+(DNS2-DND2)*DLOG(1.-F2)+COMBL2+
     1 DLOG(G12/(G11+G12+G13))
      IF(A22L.LE.-174.) A22L=-174.
      A22=DEXP(A22L)
      A23L=DND2*DLOG(F3)+(DNS2-DND2)*DLOG(1.-F3)+COMBL2
     1 +DLOG(G13/(G11+G12+G13))
      IF(A23L.LE.-174.) A23L=-174.
      A23=DEXP(A23L)
C**** CALCULATE  HX2X1
      HX2X1=A21+A22+A23
      RETURN
      END
C
C
C
C*******************************************************************
C
C     SUBROUTINE MB1SM CALCULATES TERM FOR THE MIXED BINOMIAL
C     PRIOR DISTRIBUTION . THE TERM IS THE MARGINAL DISTRIBUTION
C     OF NUMBER OF DEFECTIVES IN THE FIRST SAMPLE ( GN1X1) .
C
C*******************************************************************
C
C
C
      SUBROUTINE MB1SM(DNS1,DND1,W1,W2,W3,F1,F2,F3,
     1 GN1X1)
      IMPLICIT REAL*8(A-H,O-X)
C**** CALCULATE THE TERMS OF GN1X1
      G11L=DLOG(W1)+DND1*DLOG(F1)+(DNS1-DND1)*DLOG(1.-F1)
      G12L=DLOG(W2)+DND1*DLOG(F2)+(DNS1-DND1)*DLOG(1.-F2)
      G13L=DLOG(W3)+DND1*DLOG(F3)+(DNS1-DND1)*DLOG(1.-F3)
      COMBL1=DLGAMA(DNS1+1.)-DLGAMA(DND1+1.)-DLGAMA(DNS1-DND1+1.)
      E11L=G11L+COMBL1
      IF(E11L.LE.-174.) E11L=-174.
      E11=DEXP(E11L)
      E12L=G12L+COMBL1
      IF(E12L.LE.-174.) E12L=-174.
      E12=DEXP(E12L)
      E13L=G13L+COMBL1
```

```
      IF(E13L.LE.-174.) E13L=-174.
      E13=DEXP(E13L)
C**** CALCULATE  GN1X1
      GN1X1=E11+E12+E13
      RETURN
      END
C
C
C
C******************************************************************
C
C       SUBROUTINE PL2S CALCULATES TERMS FOR THE POLYA PRIOR
C       DISTRIBUTION . THE TERMS ARE RELEVANT FOLLOWING THE
C       SECOND SAMPLE . THE TERMS INCLUDE THE EXPECTED NUMBER
C       OF DEFECTIVES IN THE REST OF THE LOT GIVEN THE NUMBER
C       OF DEFECTIVES IN THE FIRST AND SECOND SAMPLES (EXX2GX )
C       AND THE POSTERIOR PROBABILITY THAT THERE ARE NO ADDITIONAL
C       DEFECTIVES IN THE LOT GIVEN THE NUMBER OF DEFECTIVES IN
C       THE FIRST AND SECOND SAMPLES ( HNXX2X ) .
C
C******************************************************************
C
C
C
      SUBROUTINE PL2S(DNLS,DNS1,DNS2,TX,S,T,EXX2GX,HNXX2X)
      IMPLICIT REAL*8(A-H,O-X)
C**** CALCULATE THE EXX2GX
      EXX2GX=(DNLS-DNS1-DNS2)*(S+TX)/(S+T+DNS1+DNS2)
C**** CALCULATE THE HNXX2X
      H21P=DLGAMA(T+DNLS-TX)-DLGAMA(T+DNS1+DNS2-TX)
     1+DLGAMA(S+T+DNS1+DNS2)-DLGAMA(S+T+DNLS)
      IF(H21P.LE.-174.) H21P=-174.
      HNXX2X=DEXP(H21P)
      RETURN
      END
C
C
C
C******************************************************************
C
C     SUBROUTINE PL1S CALCULATES TERMS FOR THE POLYA PRIOR
C     DISTRIBUTION . THE TERMS ARE RELEVANT FOLLOWING THE
C     FIRST SAMPLE. THE TERMS INCLUDE THE EXPECTED NUMBER
C     OF DEFECTIVES IN THE REST OF THE LOT GIVEN THE NUMBER
C     OF DEFECTIVES IN THE FIRST SAMPLE ( EXX1GX ). THE
C     POSTERIOR PROBABILITY THAT THERE ARE NO ADDITIONAL
C     EDFECTIVES IN THE LOT GIVEN THE NUMBER OF DEFECTIVES
C     IN THE FIRST SAMPLE ( HNXX1X ).
C
C******************************************************************
```

```
C
C
C
      SUBROUTINE PL1S(DNLS,DNS1,DND1,S,T,EXX1GX,HNXX1X)
      IMPLICIT REAL*8(A-H,O-X)
C**** CALCULATE EXX1GX
      EXX1GX=(DNLS-DNS1)*(S+DND1)/(S+T+DNS1)
C**** CALCULATE HNXX1X
      H11P=DLGAMA(T+DNLS-DND1)-DLGAMA(T+DNS1-DND1)
     1     +DLGAMA(S+T+DNS1)-DLGAMA(S+T+DNLS)
      IF (H11P.LE.-174.) H11P=-174.
      HNXX1X=DEXP(H11P)
      RETURN
      END
C
C
C
C******************************************************************
C
C     SUBROUTINE PL12S CALCULATES TERM FOR THE POLYA PRIOR
C     DISTRIBUTION . THE TERM IS RELEVENT FOLLOWING THE FIRST
C     AND SECOND SAMPLE . IT IS THE CONDITIONAL PROBABILITY
C     OF NUMBER OFDEFECTIVES IN THE SECOND SAMPLE GIVEN THE
C     FIRST SAMPLE ( HX2X1 ) .
C
C******************************************************************
C
C
C
      SUBROUTINE PL12S(DNLS,DNS1,DNS2,DND1,DND2,S,T,HX2X1)
      IMPLICIT REAL*8(A-H,O-X)
C**** CALCULATE HX2X1
      COMBL2=DLGAMA(DNS2+1)-DLGAMA(DND2+1)-DLGAMA(DNS2-DND2+1)
      A21L=DLGAMA(S+DND1+DND2)-DLGAMA(S+DND1)
      A22L=DLGAMA(T+DNS1+DNS2-DND1-DND2)-DLGAMA(T+DNS1-DND1)
      A23L=DLGAMA(S+T+DNS1)-DLGAMA(S+T+DNS1+DNS2)
      A2T=A21L+A22L+A23L+COMBL2
      IF(A2T.LE.-174.) A2T=-174.
      HX2X1=DEXP(A2T)
      RETURN
      END
C
C.
C
C******************************************************************
C
C     SUBROUTINE PL1SM CALCULATES TERM FOR POLYA PRIOR
C     DISTRIBUTION . THE TERM IS THE MARGINAL DISTRIBUTION
C     OF NUMBER OF DEFECTIVES IN THE FIRST SAMPLE (GN1X1)
C
C******************************************************************
```

```
C
C
C
      SUBROUTINE PL1SM(DNS1,DND1,S,T,GN1X1)
      IMPLICIT REAL*8(A-H,O-X)
      COMBL1=DLGAMA(DNS1+1)-DLGAMA(DND1+1)-DLGAMA(DNS1-DND1+1)
      B11L=DLGAMA(S+DND1)-DLGAMA(S)
      B12L=DLGAMA(T+DNS1-DND1)-DLGAMA(T)
      B13L=DLGAMA(S+T)-DLGAMA(S+T+DNS1)
      B1T=B11L+B12L+B13L+COMBL1
      IF (B1T.LE.-174.) B1T=-174.
      GN1X1=DEXP(B1T)
      RETURN
      END
```

APPENDIX B

DERIVATION OF THE CONDITIONAL PROBABILITY DISTRIBUTION OF THE NUMBER OF DEFECTIVES FOUND IN A SECOND SAMPLE, GIVEN THE NUMBER OF DEFECTIVES FOUND IN THE FIRST SAMPLE

This appendix refers to the derivation of the conditional probability distribution of the number of defectives found in a second sample x_2, given the number of defectives found in the first sample x_1. Both the mixed binomial and Polya cases are derived.

Mixed Binomial Distribution Case

Consider a mixed binomial distribution with the following prior probability function:

$$f_N(X) = \sum_{i=1}^{m} W_i \binom{N}{X} p_i^{X} (1-p_i)^{N-X}$$

where

$$0 < p < 1$$

$$\sum_{i=1}^{m} W_i = 1$$

$$X = 0, 1, 2, \ldots, N$$

The conditional probability distribution of the number of defectives found in a second sample, given the number of defectives in the first sample, is:

$$h_{n_2}(x_2 | x_1) = \sum_{i=1}^{m} \frac{W_i}{\sum_{i=1}^{m} W_i} \binom{n_2}{x_2} p_i^{x_2} (1-p_i)^{n_2-x_2}$$

where

$$0 < p_i < 1$$

$$\sum_{i=1}^{m} W_i = 1$$

$$x_2 = 0, 1, 2, \ldots, n_2$$

Logic:

Consider the prior:

$$f_N(X) = \sum_{i=1}^{m} W_i \binom{N}{X} p_i^{X} (1-p_i)^{N-X}$$

From Equation (3.14),

$$h_{N-n_1}(X-x_1|x_1) = \frac{\sum_{i=1}^{m} W_i \binom{N-n_1}{X-x_1} p_i^{X} (1-p_i)^{N-X}}{\sum_{i=1}^{m} W_i \quad p_i^{x_1}(1-p_i)^{n_1-x_1}}$$

Then,

$$h_{n_2}(x_2|x_1) = \sum_{X-x_1} \ell_{n_2}(x_2\ x_1, X-x_1)\ h_{N-n_1}(X-x_1|x_1)$$

$$= \sum_{X-x_1} \frac{\binom{n_2}{x_2}\binom{N-n_1-n_2}{X-x_1-x_2} \sum_{i=1}^{m} \binom{N-n_1}{X-x_1} W_i p_i^{X} (1-p_i)^{N-X}}{\binom{N-n_1}{X-x_1} \sum_{i=1}^{m} W_i p_i^{x_1}(1-p_i)^{n_1-x_1}}$$

$$= \sum_{i=1}^{m} \frac{W_i}{\sum_{i=1}^{m} W_i} p_i^{x_2} (1-p_i)^{n_2-x_2} .$$

$$\sum_{X-x_1} \binom{N-n_1-n_2}{X-x_1-x_2} p_i^{X-x_1-x_2} (1-p_i)^{N-n_1-n_2+x_1+x_2}$$

Since

$$\sum_{X-x_1} \binom{N-n_1-n_2}{X-x_1-x_2} p_i^{X-x_1-x_2} (1-p_i)^{N-n_1-n_2+x_1+x_2} = 1,$$

therefore

$$h_{n_2}(x_2|x_1) = \sum_{i=1}^{m} \frac{W_i}{\sum_{i=1}^{m} W_i} \binom{n_2}{x_2} p_i^{x_2} (1-p_i)^{n_2-x_2}$$

Polya Distribution Case

Consider a Polya distribution with the following prior probability function:

$$f_N(X) = \binom{N}{X} \frac{\Gamma(s+X)\Gamma(t+N-X)\Gamma(s+t)}{\Gamma(s)\ \ \Gamma(t)\quad \Gamma(s+t+N)}$$

where $s, t < 0$

$X = 0, 1, 2, \ldots, N$

The conditional probability distribution of the number of defectives found in a second sample, given the number of defectives in the first sample, is:

$$h_{n_2}(x_2|x_1) = \binom{n_2}{x_2}\frac{\Gamma(s+x_1+x_2)\Gamma(t+n_1+n_2-x_1-x_2)\Gamma(s+t+n_1)}{\Gamma(s+x_1)\quad\Gamma(t+n_1-x_1)\qquad\Gamma(s+t+n_1+n_2)}$$

where $s, t < 0$

$x_1 = 0, 1, 2, \ldots, n_1$

$x_2 = 0, 1, 2, \ldots, n_2$

Logic:

Consider the prior:

$$f_N(X) = \binom{N}{X}\frac{\Gamma(s+X)\Gamma(t+N-X)\Gamma(s+t)}{\Gamma(s)\quad\Gamma(t)\qquad\Gamma(s+t+N)}$$

From Equation (3.8),

$$h_{N-n_1}(X-x_1|x_1) = \binom{N-n_1}{X-x_1}\frac{\Gamma(s+X)\Gamma(t+N-X)\Gamma(s+t+n_1)}{\Gamma(s+x_1)\Gamma(t+n_1-x_1)\Gamma(s+t+N)}$$

Let

$M = N - n_1$

$Y = X - x_1$

$s' = s + x_1$

$t' = t + n_1 - x_1$

Then,

$$h_{N-n_1}(X-x_1|x_1) = \binom{M}{Y}\frac{\Gamma(s'+Y)\Gamma(t'+M-Y)\Gamma(s'+t')}{\Gamma(s')\quad\Gamma(t')\qquad\Gamma(s'+t'+M)}$$

since

$$\sum_{X-x_1} \ell_{n_2}(x_2|x_1,X-x_1)\ h_{N-n_1}(X-x_1|x_1) = h_{n_2}(x_2|x_1)$$

Therefore

$$\sum_{Y} \ell_{n_2}(x_2|x_1,Y)\ h_M(Y\ x_1) = h_{n_2}(x_2|x_1)$$

Thus,

$$h_{n_2}(x_2|x_1) = \sum_{Y-x_2=0}^{M-n_2} \frac{\binom{n_2}{x_2}\binom{M-n_2}{Y-x_2}}{\binom{M}{Y}} \binom{M}{Y} \frac{(s'+Y)}{(s')} \frac{(t'+M-Y)}{(t')} \frac{(s'+t')}{(s'+t'+M)}$$

$$= \binom{n_2}{x_2} \frac{\Gamma(s'+t')\Gamma(s'+x_2)\Gamma(t'+n_2-x_2)}{\Gamma(s')\quad \Gamma(t')\quad \Gamma(s'+t'+n_2)} \,.$$

$$\sum_{Y-x_2} \binom{M-n_2}{Y-x_2} \frac{\Gamma(s'+Y)\Gamma(t'+M-Y)\Gamma(s'+t'+n_2)}{\Gamma(s'+x_2)\Gamma(t'+n_2-x_2)\Gamma(s'+t'+M)}$$

Again, let

$$L = M - n_2$$

$$Z = Y - x_2$$

$$s'' = s' + x_2$$

$$t'' = t' + n_2 - x_2$$

Then,

$$\sum_{Y-x_2} \binom{M-n_2}{Y-x_2} \frac{\Gamma(s'+Y)\Gamma(t'+M-Y)\Gamma(s'+t'+n_2)}{\Gamma(s'+x_2)\Gamma(t'+n_2-x_2)\Gamma(s'+t'+M)}$$

$$= \sum_{Z} \binom{L}{Z} \frac{\Gamma(s''+Z)\Gamma(t''+L-Z)\Gamma(s''+t'')}{\Gamma(s'')\quad \Gamma(t'')\quad \Gamma(s''+t''+L)} = 1$$

Therefore,

$$h_{n_2}(x_2|x_1) = \binom{n_2}{x_2} \frac{\Gamma(s'+t')\Gamma(s'+x_2)\Gamma(t'+n_2-x_2)}{\Gamma(s') \quad \Gamma(t') \quad \Gamma(s'+t'+n_2)}$$

$$= \binom{n_2}{x_2} \frac{\Gamma(s+t+n_1)\Gamma(s+x_1+x_2)\Gamma(t+n_1+n_2-x_1-x_2)}{\Gamma(s+x_1) \quad \Gamma(t+n_1-x_1)\Gamma(s+t+n_1+n_2)}$$

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