DEVELOPMENT OF AN ECONOMICALLY-BASED ASYMMETRIC CUMULATIVE SUM CHART WITH WEIBULL PROCESS FAILURE MECHANISM

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PREFACE

This research is concerned with the modeling and evaluation of the powerful process control scheme -- Cumulative Sum (Cusum) Chart. A special control chart methodology is introduced and incorporated into this model along with Weibull process failure mechanism.

The formulation of the model follows the same cost structure as in Duncan's economic $\overline{X}$ chart model. An optimization procedure is employed to economically design the decision variables of this asymmetric Cusum control chart. The results are then be compared and analyzed.

I wish to express sincere appreciation to my major adviser, Dr. Kenneth E. Case, for his constant encouragement, guidance, assistance throughout this research and during my doctoral program. Thanks also to my committee members, Dr. Carl B. Estes, Dr. Barry K. Moser, Dr. Allen C. Schuermann and Dr. M. Palmer Terrell, for their interest and assistance.

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CHAPTER I

THE RESEARCH PROBLEM

Purpose

Concepts of statistical quality control have been widely applied as tools for process control in various industrial sectors. Control charts, a powerful statistical process control (SPC) tool, are used for determining in-control/out of control status, troubleshooting processes, analyzing process capability, and maintaining statistical control. The most commonly used control chart is the Shewhart chart with 3-sigma control limits. It is designed to allow the inherent variability (or noise) of a process to roam randomly between control limits. It is assumed that an observed value that falls beyond control limits is an indication of the occurrence of an assignable cause in the process.

There are numerous modifications and extensions to Shewhart charts. One important development is the cumulative sum (Cusum) control chart, which is based upon sums of observations rather than upon individual observations. Some persons argue that the cumulative sum chart is more sensitive to process shifts than is the Shewhart chart. The use of any control chart is basically an economical problem.

The cost aspects of a process should be considered when any SPC procedure is utilized for process control.

The objective of this dissertation is to develop procedures for the design and optimization of a new and richer set of economically-based charts. This research deals with the design of Cusum control charts for the control of the mean of a process when the observations are independent. It extends process control charting by :

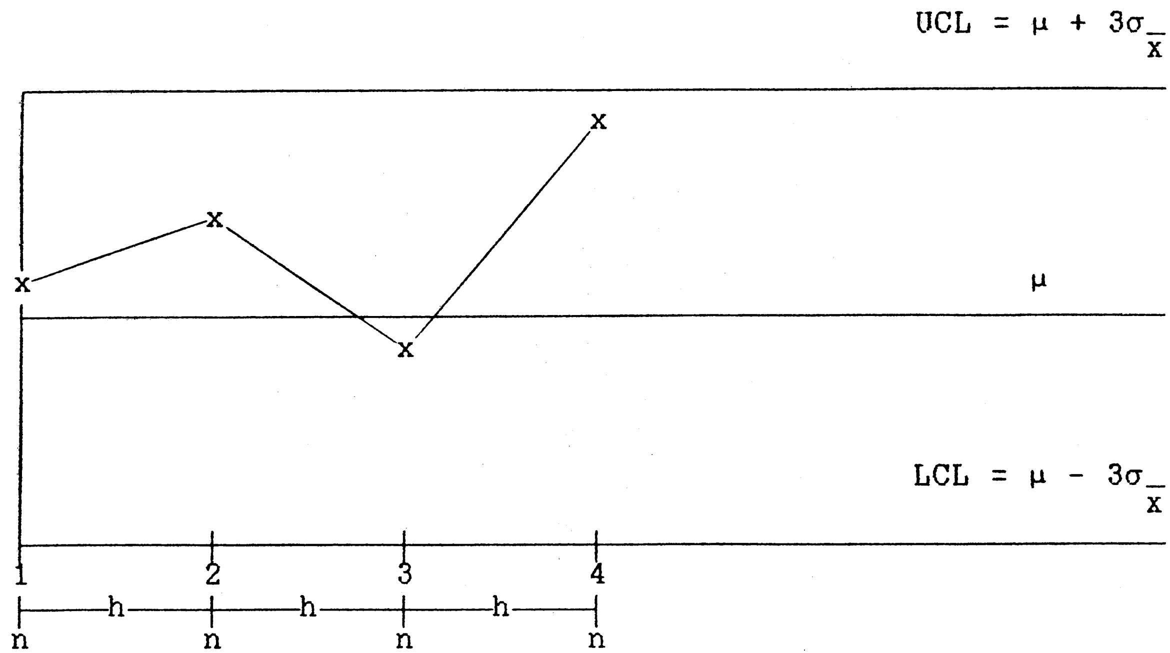
1. Defining and developing an economically-based Cusum control chart which explicitly recognizes asymmetric specification limits and asymmetric costs of being off-target.
2. Utilizing a process failure mechanism described by the Weibull distribution on the in-control time of the process (an exponential process failure mechanism is the most widely applied by researchers to date).
3. Developing an optimization procedure in which sample size n, sampling interval h, dead band values k_U and k_L, and decision intervals d_U and d_L are optimized.

The $\bar{X}$ Control Chart

A control chart is a statistical device principally used for the study and control of repetitive processes. At the basis of the theory of control charts is a differentiation of the causes of variation in quality (Duncan, 1974). One type of variability, produced by "chance causes", is inherent in a process and cannot be removed easily, if at all. In addition to this variability, there are sources of relatively large variation, called "assignable causes", which are attributed to variabilities in people, machines, materials, methods, and environments.

Shewhart suggests that samples of size n = 4 or 5 be taken from a process at regular intervals (every h hours) and the samples' averages ($\bar{X}$) be plotted on a chart. Being a sample result, $\bar{X}$ is subject to sampling fluctuations. The commonly used limits for an $\bar{X}$ control chart are located at the process mean plus or minus three standard deviations ($\pm 3\sigma_{\bar{x}}$) of the sample averages as depicted in Figure 1.1. If no assignable causes occur in the process, $\bar{X}$'s are approximately normally distributed. In other words, the inherent variability of a process or a statistic calculated from process data is expected to fluctuate within six standard deviations. Assuming the normal distribution applies, there is a very small, 0.00135, probability that a point will fall beyond the upper control limit; likewise, for the lower control limit. Therefore, if a point falls outside control limits, it should be inferred that one or more assignable causes exist in the process.

The introduction of the statistical design of the $\bar{X}$ chart provides a scientific approach for control of the process mean. However, the suggested values of sample size n = 4 or 5, and 3-sigma control limits might result in a control chart plan which is far from optimal in an



UCL = upper control limit

LCL = lower control limit

μ = process mean

$\sigma_{\bar{x}}$ = standard deviation of sample averages

h = time interval between subgroups

n = subgroup size

Figure 1.1 Design of An $\bar{X}$ Control Chart economical sense.

The Cumulative Sum Control Chart

The nature of Shewhart-type control charts, coupled with rules for reading them, is taking actions based on the last one or several plotted points. In order to increase the sensitivity of the control chart in detecting lack of control, Page (1954) proposes a procedure which adapts a rule for action based on sums of observations, rather than individual observations. This is done by the use of a cumulative sum (or Cusum) chart. The Cusum chart is a system of charting that is based upon all the data since the last process change. It is supposed to detect a sudden and persistent change in the process average more rapidly than a comparable Shewhart chart.

Average Run Length, ARL

Page (1954) introduces the concept of the average run length for a Cusum chart. The value of the process mean and the Cusum chart decision variables determine the ARL. Suppose the cumulative sums are plotted for either the upward shift or the downward shift only. Then $ARLS_U$ represents the ARL of the process with an upward shift in the process mean; likewise, $ARLS_L$ represents the ARL of a downward shift.

Kemp (1961) presents a formula for computing the ARL of a two-sided Cusum chart. He considers a two-sided Cusum chart as a composition of two one-sided Cusum charts.

Letting $ARLS_1$ be the ARL of a two-sided Cusum chart with a shift in the process mean, it follows that $ARLS_1$ is given by the equation

$$\frac{1}{ARLS_1} = \frac{1}{ARLS_U} + \frac{1}{ARLS_L}.$$

Kemp declares that this relation is not strictly confined to symmetric Cusum charts. In this dissertation, an asymmetric model is developed. The ARL for a process with either an upward shift or a downward shift in the process mean will be developed in more detail in a later chapter.

Subgroup Size, n, and Sampling Interval, h

Two of the decision variables with which this research is concerned are the subgroup size n and sampling interval h. Since this study is conducted on an economical basis, the optimal subgroup size and the time interval between subgroups is sought. It is assumed that the subgroup size n and sampling interval h are constant throughout the operation of the Cusum chart.

Decision Interval, d

As noted earlier, chance variation is the random variation which is inherent in the process. Assignable variation is due to a real change in the process mean. The decision interval is used to help distinguish which is which. The rule for deciding when a real change has occurred is to compute the accumulated sum of deviations from some "dead band" value. If the accumulated sum exceeds d, it is concluded that the process mean has changed. The criterion for choosing d is a large ARL for the process operating at the acceptable quality level, μ_a, and a small ARL when the process is running at the rejectable quality level, μ_r. In this dissertation two values of d, d_U and d_L, will be required due to the asymmetry allowed by the model.

<u>Dead Band Value, k</u>

Ewan and Kemp (1960) report that the use of a "dead band" will provide advantages by not permitting the Cusum chart to react to small changes in the mean. The dead band value often used is $k \approx \frac{1}{2}(\mu_a + \mu_r)$. The value of k is obviously closely related to both μ_a and μ_r. The dead band value k requires that the sample statistic fall outside $\frac{1}{2}(\mu_a + \mu_r)$ before it adds to the cumulative sum; however, it can subtract from a positive cumulative sum even if it falls within the dead band.

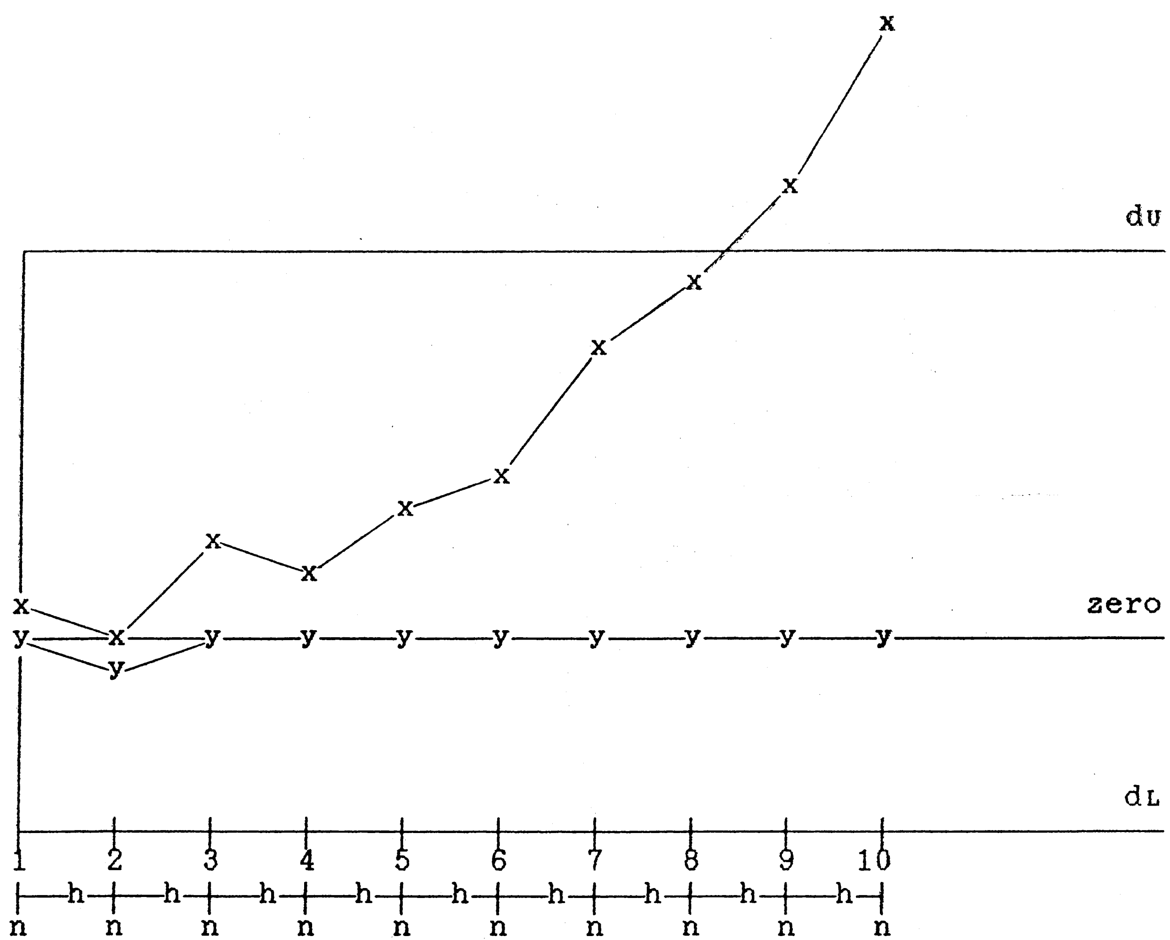
In this dissertation, $k = \frac{1}{2}(\mu_a + \mu_r)$ is used. Again, there must be two values of k, k_U and k_L, due to asymmetric conditions of the model.

Economically Based Cusum Charts

Traditionally, control charting is based on statistical criteria for process control. In recent years, attention has been focused on economical aspects of a Cusum chart, such as the cost of sampling, testing and maintaining process surveillance.

Taylor (1968) initiates economical design concepts into cumulative sum control charts. He develops a formula giving approximately the long-run average cost per unit of operating time as a function of the Cusum scheme's decision variables and design parameters. Goel and Wu (1973), who follow Duncan's approach for the economical design of $\overline{X}$ charts (1956), derive an economical model for Cusum charts. They employ the "pattern-search" method to determine the optimum values of the sample size, the sampling interval, the dead band value and the decision interval.

Only symmetric Cusum charts have been considered to date. An asymmetric Cusum scheme which better reflects reality is studied in this dissertation. In an asymmetric Cusum scheme the distance between the acceptable quality level and upper rejectable quality level is different from that of the acceptable quality level and lower rejectable quality level, as is the cost of reaching the upper or lower rejectable quality level. The concept of an asymmetric Cusum chart is illustrated in Figure 1.2. Based upon Duncan's concept, the best values of the decision variables subgroup size n, time interval between subgroups h, dead band values k_U and k_L, and decision intervals d_U and d_L will be determined using optimization techniques.



n = subgroup size

h = time interval between subgroups

x = cumulative sum in upper direction

y = cumulative sum in lower direction

d_U = upper decision interval

d_L = lower decision interval

Figure 1.2 Design of An Asymmetric Cusum Chart

Process Failure Mechanism

Assumptions about the behavior pattern of a process are required to formulate the economically-based design of Cusum charts. An important assumption is the nature of the occurrence of assignable causes which shift the process from an in-control state to an out of control state. Montgomery (1980) describes this characteristic as the "process failure mechanism".

It is usually assumed that the process failure mechanism is an exponential random variable. This assumption considerably simplifies the algorithm for the development of economical models of Cusum schemes. Baker (1971) suggests that the choice of process failure mechanism has a somewhat significant impact on the optimally economical design of control charts. Gibra (1975) and Montgomery (1980) also suggest that it is necessary to investigate and recognize the physical failure pattern of the process so that the principle of economical design can be validly implemented. Saniga (1979) investigates the impacts of process failure mechanisms and the Markov property on the economical design of $\bar{X}$ and R charts. He infers that the misusage of the process failure mechanism will result in a substantial loss of cost. Qureishi (1964) points out that statisticians have questioned the validity of the assumption of the exponential distribution for the life times of the units put to a test. Several researchers point out the exponential approximation to life-data is only a fair approximation for practical purposes.

In this dissertation it is assumed that the nature of the occurrence of assignable causes is according to the Weibull distribution. The Weibull distribution is regarded as a better model for the process failure mechanism in the sense that it embraces a number of interesting situations. It can reduce to the exponential distribution or reduce to the Rayleigh distribution.

To avoid incorrect modeling, it is desirable to economically design a Cusum chart in which the process failure mechanism is administered by a more generalized distribution. Accordingly, the Weibull distribution is proposed rather than the exponential.

Summary of Research Objectives

Objective

The primary objective of this research is to : Provide an operational tool which will permit the cumulative sum chart to be used in an economically optimum manner as an alternative to Shewhart control charts for monitoring a process in a realistic environment.

Subobjectives

In order to accomplish this objective, several subobjectives have to be satisfied :

1. Develop an economically-based model for evaluating

Cusum process control plans.

2. Provide for asymmetric rejectable quality levels and resultant costs of asymmetric process shifts.
3. Incorporate a process failure mechanism which is Weibull distributed.
4. Develop a computer program which approximately optimizes, based upon economics, the subgroup size n, sampling interval h, dead band values k_U and k_L, and decision intervals d_U and d_L.

CHAPTER II

LITERATURE REVIEW

This chapter reviews developments in the literature pertaining to the objectives of this research. Substantiation for this particular research is elaborated upon. Furthermore, other sources which correspond with the general concepts relevant to this study are presented.

This chapter is divided into five parts :

1. Shewhart control charts and their enhancements and modifications.
2. Economical design of $\bar{X}$ control charts.
3. Cumulative sum control charts.
4. Economical design of cumulative sum control charts.
5. Process failure mechanisms.

Shewhart Control Charts and Their Enhancements and Modifications

Shewhart (1931) originated the control chart for determining the state of statistical control of a process. Statistical quality control chart techniques have been applied widely in various fields, such as manufactured products, delivery services, research works, and developmental environments. Duncan (1974) and Vance (1983) point out that

Shewhart control charts are fundamentally used for one of the following three purposes: (a) to determine the goal or standard for a process that management might strive to acquire, (b) to judge whether the goal has been achieved, and (c) to maintain current control of a process.

Shewhart Control Charts and Their Enhancements

Shewhart (1931) develops the use of 3-sigma control limits as action limits. Meanwhile, he suggests the use of sample sizes of 4 or 5 as being appropriate for $\bar{X}$ and R charts. The sampling interval is left to be determined by the quality control personnel or other concerned staff.

In the last four decades, many enhancements of Shewhart control charts have been suggested. For example, a run test on sample means has been widely used. Weiler (1953) suggests that to make use of consecutive runs for control charts for the process mean might significantly decrease inspection. Warning limits have also been proposed. Page (1962) adopts the concept of warning limits and demonstrates a scheme based on warning and action limits. In general, the scheme is superior to a scheme based on runs. The sensitivity of Shewhart control charts for detecting small shifts in the process mean from the specified or target value is investigated. Weindling et al. (1970) establishes a pair of warning limits, located inside the action limits, for detecting small shifts in process mean and indicating a possible out of control condition. Hillier (1969) develops a method for setting the control limits for $\bar{X}$ and R charts so that they can be reliably used regardless of how few subgroups have been inspected. Chung-How and Hillier (1970) provide guidance on what constants to use for mean and variance control chart limits if the power of the charts is of paramount importance, and computational considerations are secondary.

The background of computing limits on Shewhart control charts is built on a presumption of normality, justified by the Central Limit Theorem. Measurable quality characteristics often have non-normal distributions. The introduction of the assumption of non-normality is another enhancement to Shewhart control charts. Burr (1967) establishes tables which provide guidance on what constants to use for $\bar{X}$ and R charts if the parent population is markedly non-normally distributed. Schilling and Nelson (1976) facilitate a numerical method for determining the cumulative probabilities of the distribution of sample means which is non-normally distributed . Ferrell (1958) suggests that transformation is required when the underlying universe is badly skewed. Vasilopoulos and Stamboulis (1978) modify and extend the existing standard methodology by utilizing the time series analysis approach and by introducing dependence via a second order autoregressive process (AR(2) Model) when either independence and/or normality are not present.

Modifications of Shewhart Control Charts

The arithmetic mean and the subgroup range have been used to determine whether or not a state of statistical control exists for variables in Shewhart control charts. Moving Average, Moving Range, Median and Midrange, and the Geometric Moving Average (or Exponentially Weighted Moving average) charts represent general modifications of Shewhart control charts. The Cumulative Sum control charts are relevant to this classification, but they are presented in the next sections.

Moving Average and Moving Range control charts are used in situations where the time interval between subgroups is too great to collect sufficient samples as a rational subgroup. Or, they are used in continuous process manufacture (e.g., chemicals, refining, mining, etc.) where the smoothing effect of the moving average has an effect on the figures often similar to the effect on the product of the blending and mixing that happens in the remainder of the production process. The sensitivity of these control charts can be increased by allowing more successive points to be computed for the moving average. The more successive points averaged, the greater the smoothing effect and the more the curve emphasizes trends rather than point-to-point fluctuations.

Ferrell (1964) advocates the use of Median and Midrange charts using run-size subgroups for controlling certain processes. Nelson (1982) suggests the use of medians to reduce the burdensome calculation of a mean in Shewhart control charts. In his approach, the setting of control limits is based upon the average of the subgroup medians and the average of the ranges.

Roberts (1959, 1966) suggests a procedure for generating geometric moving averages. The author shows that tests based upon geometric moving averages are better than multiple run tests and moving average tests with regard to simplicity and statistical properties. Wortham et al. (1974) present an adaptive exponentially smoothed control system. The adaptive nature is achieved by varying the weighting factor according to the value of a tracking signal. The authors also illustrate an example of an adaptive control chart with associated sensitivity curves which present the probabilities of acceptance as a function of sampling periods after a change in a process occurs. Robinson and Ho (1978) present a numerical procedure for the tabulation of average run lengths (ARL's) of geometric moving average charts. Both one- and two-sided ARL's are given for various settings of the control limits, smoothing constant and shift in the nominal level of process mean. Hunter (1986) describes a procedure to establish the control limits for exponentially weighted moving average schemes. The author declares that the exponentially weighted moving average can be used as a dynamic process control tool to provide a forecast of where the process will be in the next instant of time.

Economical Design of $\bar{X}$ Control Charts

Duncan (1956) has established a model for the optimum economical design of the $\bar{X}$ control chart. His paper was the first to deal with a fully economical model of a Shewhart-type control chart. Duncan's paper leads the way to study in this area. In this model, the following assumptions are made about the process :

1. The process begins in a state of statistical control.
2. The process standard deviation (σ) remains the same in spite of the shifting mean of the process.
3. Due to an assignable cause the process mean may randomly shift to $\mu_0 \pm \delta\sigma$ and stay there until corrected.
4. The process is not shut down while searching for the assignable cause.
5. Neither the cost of adjustment or repair, nor the cost of bringing the process back into a state of statistical control after the assignable cause is discovered, is introduced into the economical model.
6. The specification limits are assumed to be symmetrically spaced about the desired process mean.
7. The loss-cost of a shift from μ_0 to either $\mu_0 + \delta\sigma$ or $\mu_0 - \delta\sigma$ is assumed to be the same.

The process is monitored by an $\bar{X}$ chart with central line at μ_0 and upper and lower control limits at $\mu_0 \pm k\sigma/\sqrt{n}$, respectively. Samples are taken at intervals of h hours. The assignable cause is assumed to occur according to a Poisson process with an expectation of λ occurrences per hour. The parameters μ_0, δ, and σ are assumed known, while sample size n, the control limit spread k, and the sampling interval h are decision variables. The expected time the process will be out of control is the sum of three components :

1. The average number of sampling intervals necessary for detecting the shift times the length of each interval, minus the average time of occurrence of the assignable cause within an interval between samples.
2. The delay in plotting a point, which is assumed to be a linear function of the sample size.
3. The average time taken to find the assignable cause.

A production cycle time is defined as the interval of time from the start of production in a state of statistical control to the detection and elimination of the assignable cause. The cycle, therefore, consists of the expected time the process will be in control and the expected time the process will be out of control.

Duncan presents a design criterion to minimize the loss-cost per unit of time. Cost incurred in the process contains four elements :

1. The loss of defective products being produced.
2. The average cost of a false alarm.

3. The average cost of a real alarm.
4. The average cost for sampling and maintaining control charts.

Several numerical approximations are used in the optimization of this model which essentially represent a sensitivity analysis for anticipated changes in the parameters of the model.

Goel et al. (1968) develop an iterative procedure to produce the exact optimal solution to Duncan's model (1956) by computer. Comparison is made between Duncan's approximate method and the developed procedure. The procedure is superior to Duncan's approximate optimization technique in some situations. However, in many cases the difference is insignificant.

Knappenberger and Grandage (1969) develop a method for choosing the decision variables n, h, and k in order to minimize the expected cost per unit produced. They assume that the time the process remains in control is an exponential random variable. In addition, it is assumed that the process mean is a continuous random variable which can be satisfactorily approximated by a discrete random variable. One value of the discrete random variable is associated with the in-control value of the process mean and the remaining values are associated with out of control values of the process mean. The expected total cost, per unit of product, associated with a quality control test procedure is similar to Duncan's model.

Optimization of the cost function is not developed analytically. Rather a two-stage numerical method is developed for determining the optimal decision variables, n, h, and k, of the $\bar{X}$ chart. In the first stage, the expected cost is computed for a wide variety of decision variables, cost coefficients, and for the desired values of a priori distribution parameters. In the second stage, the preliminary estimates obtained from the first stage are used as the starting point for a search method designed to locate the optimal values of the decision variables within any desired accuracy.

Gibra (1971) develops a model for determining the optimal $\bar{X}$ chart parameters for maintaining economical control of a process under practical conditions. These parameters are again n, h, and k. A cost function is formulated based upon Duncan's model. However, there is a difference. The sum of times required to take and inspect a sample, compute and plot a sample average, and to discover and eliminate the assignable cause has an Erlangian distribution. Gibra gives several examples to show how the formulated model can be applied and how the relevant cost function is minimized.

Chiu and Wetherill (1974) propose a simple, approximate procedure for optimizing Duncan's model. The principle for the choice of parameters is to minimize the average loss-cost, subject to a constraint on the OC-curve. One is free to choose a consumer's risk point on the OC-curve to acquire a desired protection against inferior quality. One may then determine the values of the sample size and the control limit coefficients from a table, by a rule of thumb. The value of a sampling interval is calculated by an algebraic formula. Chiu and Wetherill declare that this method permits a rapid determination of the control parameters which generally yield an average cost close to the exact minimum. Furthermore, they show that in most cases, despite its simplification of the problem, the developed method gives better solutions than Duncan's more involved procedure (1956) with the added advantage that the OC-curve can be partly controlled by the user.

Baker (1971) develops two discrete-time models in which a sample of size n is taken at the end of each period and the computed statistic plotted on a control chart with k-sigma limits. In the first process model the geometric distribution is applied to model the number of periods the process remains in the in-control state. In the second model any discrete probability function can be used to model the characteristic of the time to failure of a process. The author studies a Poisson time to failure and compares it to the usual geometric process model. It is shown that the former process results in smaller sample sizes and narrower control limits than will be economically optimal in the latter case.

Jones and Case (1981) develop an economical model to design a joint $\overline{X}$ and R control charts with a minimum cost.

Duncan's model (1956) is used as a basis for subsequent economical model development. The decision variables are sample size, width of $\bar{X}$ control chart limits, width of R control chart limits, and sampling interval in hours. Jones and Case emphasize the estimation of the expected time the process will be operating in an out of control condition. They assume that when a process is out of control, the resultant effect is an increase in the number of defective items produced which will cause additional economical losses. These losses are assumed to be dependent upon the types of out of control conditions and the length of time the process remains in each. Four control conditions are discussed in the model. That the mean and variance of the process are in control is defined as the in-control condition. The out of control conditions occur when either the process mean, the process variance, or both, are out of control. The four conditions form seven types of out-of-control states.

Lorenzen and Vance (1986) present a general process for determining n, h, and k for the designs of the economical models of $\bar{X}$, p, and u charts. A general process model is considered, and the hourly cost function is derived. Numerical techniques to minimize this cost function are discussed, and sensitivity analyses are performed. They also illustrate an example to reveal the potential savings of this technique of designing control charts.

Duncan (1971) has generalized his assignable cause model to the situation when there are several assignable causes. Each assignable cause produces a shift of known magnitude in the process mean. The occurrence times of the assignable causes are assumed to be independent exponential random variables. Duncan uses the direct search method to locate the local minimum of the cost function. The solutions to several example problems and a sensitivity analysis of the model are presented.

Cumulative Sum Control Charts

The control chart techniques mentioned in the previous sections are based on the rule, proposed by Shewhart, of taking action when a point falls outside of the "control limits," usually 3-sigma limits. It is a natural step to adopt a rule for action that is based upon sums of observations rather than the last few samples. This is done by the use of a cumulative sum control chart or "Cusum chart" as it has come to be called. The Cusum chart makes use of the historical data and provides an approach which is able to detect shifts in the process mean, especially if the shift is not large. It may also indicate the time of shifting more clearly by reviewing the trend of the cumulative sum.

Page (1954) initiates the Cusum chart scheme. Starting from a process revision and restart, all subsequent plots contain information from the whole set of observations up to, and including, the plotted point. That is, the ordinate of the i^{th} point in a Cusum chart equals that of the $(i-1)^{th}$

point plus the statistic value computed from the i^{th} sample. Page introduces the average run length (ARL) to develop rules that use all the observations and that are suitable for detecting any magnitude of shift in the mean parameter. The inspection process developed permits detection of parameter variation in one and two directions. The value of the process mean determines the ARL of a Cusum scheme. Generally, the two specified mean levels are the acceptable quality level μ_a and the rejectable quality level μ_r, and the ARL at these quality levels are denoted by ARL_0 and ARL_1, respectively.

Page (1961) examines the practice of Cusum charts. He declares that the cumulative sum schemes are much more sensitive than the ordinary Shewhart control chart. Johnson and Leone (1962) give a complete description of Cusum charts with some basic deviations. Ewan (1963) outlines the variety of continuous graphical control schemes and the types of processes for which Cusum charts are most appropriate. He compares Cusum charts with Shewhart and weighted mean charts. Ewan concludes that Cusum charts are more effective than Shewhart control charts in detecting sustained changes in the process mean in the region 0.5-sigma to 2.0-sigma. Ewan also discusses the practical scale problems, the use of exact decision procedures, sample size, sampling interval and detection of trends.

Bakir and Reynolds (1979) develop a nonparametric procedure based on Wilcoxon signed-rank statistics where ranking is within groups. The procedure combines a Cusum chart with Wilcoxon statistics for quickly detecting any shift in the mean of a sequence of observations from a specified control value.

Johnson and Bagshaw (1974) study the effects of serial correlation on the performance of one-sided Cusum charts. Later, they (1975) develop another approximation to the cumulative sum charts which allows one to study the run length distribution after a change in level has occurred. They emphasize the effects on the run length distribution caused by the presence of serial correlation. Lucas and Crosier (1982) evaluate a standard Cusum control scheme and four modified Cusum control schemes for robustness. The average run length for each scheme is evaluated using a contaminated normal distribution, a distribution that has longer tails than the normal. They conclude that a Cusum control scheme that ignores the first suspected outlier, but gives an out of control signal for two successive outliers is found to perform well. Bissel (1984a) makes a comparison of the run length properties for Cusum schemes, Shewhart charts, and control charts with warning limits when there is a linear trend in the underlying mean.

Lucas (1985) and Vardeman and Ray (1985) describe design and implementation procedures for utilizing Cusum charts for attributes where the observations are Poisson or exponential random variables.

Woodall (1985, 1986) develops a method for designing quality control charts on the basis of their statistical performance over specified in-control and out of control regions of control limit spreads. He divides and defines the control limit spread of a two-sided Cusum chart as in-control, indifference, and out of control states.

Although a change in trend on a cumulative sum chart will indicate that a change has occurred in the process, it is desirable to have a visual record of data in both directions, upward and downward, for indicating where the change occurs and when it needs an action. The use of a V-shaped mask is implemented for this purpose. The vertex of the mask is placed a distance, called the lead distance, ahead of the last plotted point. The process is considered to be in a state of statistical control as long as all previously plotted points fall within the arms of the mask. Johnson and Leone (1962) show how to determine the dimensions and the significant characteristics of the V-mask. Lucas (1976) discusses practical aspects of the design and the use of V-mask control schemes. He recommends for plotting purposes a scale of one sample unit on the abscissa equaling two standard deviations of the process (2σ) on the ordinate. Lucas also presents a computational form of the V-mask. He declares that this form is especially helpful when the data arrive rapidly or when many parameters are being controlled simultaneously.

Ewan (1963) first proposes the use of two or more V-masks simultaneously to improve the sensitivity of the Cusum schemes to large shifts in the process mean. Later, Lucas (1973), Bissell (1979), and Rowlands et al. (1982) also advocate changes in the shape of the V-mask near its vertex, introducing a parabolic section. Lucas (1982) proposes a combined Cusum-Shewhart quality control scheme which will be classified as a modified V-mask.

Economical Design of Cumulative Sum Control Charts

Taylor (1968) first introduces economical design concepts into cumulative sum control charts. He studies the economical design of Cusum charts for controlling the process mean having normally distributed quality characteristics with known variance. The costs of repairing the process, of operating out of control, and of maintaining the control chart are assumed known. The process is shut down while searching for the assignable cause. If the assignable cause is not a false alarm, then adjustment time and cost are added to the process. In his research, Taylor finds no statistical significance and no practical difference in the run lengths as the number of samples taken when the process leaves control varies between 0 and 50. Thus, he assumes that the average time between the first sample after the occurrence of the assignable cause and the last sample prior to its detection equals the product of the sampling interval times the value ($ARL_1-\frac{1}{2}$). He develops a formula giving approximately the long-run average cost per unit of operating time as a function of the sample size n, sampling interval h, and the Cusum scheme's design parameters.

Taylor utilizes the expressions, derived by Goldsmith and Whitfield (1961), for ARL for in-control and out-of-control states to find by trial and error the values of the Cusum scheme's design parameters.

Goel and Wu (1973) follow Duncan's approach for the economical design of $\bar{X}$ charts (1956) to derive their economical model of Cusum charts which gives the long-run average cost as a function of decision variables, n, h, k, and decision interval d. The value k is defined as half of the sum of the desired and the shifted process means. In addition, the expected elapsed time between the first sample after the occurrence of the assignable cause and the last sample prior to its detection is determined using the results derived by Taylor (1968). Goel and Wu assign an integer value to n and then employ the "pattern-search" technique to determine the optimum values of the sampling interval h and the decision interval d. They also investigate numerically the cost surfaces, the effects of shifts in parameters, cost factors and the expected time for an assignable cause to occur on the loss-cost surfaces and the optimum designs, which provide information about the neighborhood of the optimum.

Chiu (1974) uses the decision interval criterion to develop the economical model of a Cusum chart for quality surveillance. He follows the general modeling strategy of

Duncan's $\bar{X}$ chart model but shuts down the process and makes a search for the assignable cause when the decision interval is exceeded. Chiu employs the Fibonacci search technique in two-dimensional space to find the optimum value of decision interval h, given sample size n. He also derives a simplified version of the algorithm which gives control plans close to optimum. A brief sensitivity analysis and a discussion of an extension of the model to a multiple cause system are given.

Goel (1968) makes a comparison for the economically optimal $\bar{X}$ and Cusum charts. He shows that the Cusum chart is very efficient in detecting a lack of control where the shift in the process level is close to the value for which it is designed. If the actual shift is much smaller or much larger, an $\bar{X}$ chart seems to be better. In general, more sampling will be required when using an $\bar{X}$ chart while keeping both ARL_0 and ARL_1 equal for the two charts. Furthermore, the optimum loss-cost for the Cusum chart is slightly less than that of the $\bar{X}$ chart. When a smaller than optimum sample size is used, the loss-cost difference makes the Cusum chart become more favorable. The variation in loss-cost for shifts smaller or larger than the designed value also shows that the Cusum chart is more economical than the $\bar{X}$ chart.

Woodall (1986) studies the methods of designing Cusum quality control charts. He shows that the statistical performance of control procedures obtained using economical models is often unsatisfactory. A numerical example is given to indicate that the more traditional Cusum procedure produces few false alarms, yet provides much more rapid detection of small shifts in the mean than the economically designed Cusum charts. Woodall declares that a major weakness of the economical models is that the shift that is assumed to occur when the process goes out of control usually corresponds to a substantial loss of quality and profit.

Process Failure Mechanisms

Duncan (1956) assumes that the occurrence of assignable causes during an interval between samples is according to a Poisson process. In other words, the time to failure is an exponential random variable. This assumption simplifies considerably the development of the economical model. Montgomery (1980) calls the characteristic of the occurrence of assignable causes the "process failure mechanism". Baker (1971) proposes a model that allows the probability function of the time to failure of a process to be any discrete probability function. He reports that a non-Markovian process with a Poisson failure mechanism results in smaller sample sizes and narrower control limits than will be economically optimal in the geometric case. Baker concludes that the choice of process failure mechanism has a somewhat significant impact on the optimal economical design of control charts.

Gibra (1975) and Montgomery (1980) suggest that it is essential to examine and understand the physical behavior of the deterioration process so that the principle of economical design can be validly implemented. Saniga (1979) investigates the effects of process failure mechanisms and the Markov property (the memoryless property) on the economical design of $\bar{X}$ and R charts. He applies the long-run average time cost function developed by Baker (1971) to geometric, Poisson, and logarithmic series models. Numerical results are presented. These results indicate that both the Markov assumption and the process failure mechanisms are important determinants to the economically-based designs of $\bar{X}$ and R control charts. Saniga infers that the use of an incorrectly specified process failure mechanism will result in a substantial loss of cost.

Johnson (1966) describes a method for construction of cumulative sum control charts for controlling the mean of sequences of independent variables each having the same Weibull distribution. He points out that a Weibull distribution often gives a markedly more accurate representation than the exponential. Johnson presents several results to show the use of such charts when a non-exponential Weibull distribution would be more appropriate.

Summary

A literature survey of the problems, contributions, and needs related to the objectives of this research is presented. In the previous economically-based models of two-sided Cusum charts discussed above, all the researchers assume symmetric control limit spreads, symmetric decision intervals, and equal costs for either upward or downward shifts in the process mean. There has been no work done for seeking an optimum condition to the economically-based two-sided Cusum chart scheme which is associated with asymmetric control limit spreads, asymmetric decision intervals, and unequal costs for a shift in either the upward or downward direction. Further, this survey substantiates that most of the currently available economical models assume that the occurrence of the assignable cause is according to a Poisson process. The task of formulating an economical model of the cumulative sum control chart with a Weibull distributed process failure mechanism is yet to be accomplished.

This survey indicates that a need exists to:

1. Provide an economically-based cumulative sum control chart model in which the process failure mechanism is Weibull distributed.
2. Introduce asymmetric rejectable quality levels, asymmetric process shifts, and unequal costs into this economically-based cumulative sum control chart.
3. Develop appropriate procedures for the optimal design of the proposed model.
4. Adopt decision variables, sample size n, sampling interval h, dead band values k_U and k_L, decision

intervals d_U and d_L for modeling and optimization purposes.

CHAPTER III

MODEL DEVELOPMENT OF AN ASYMMETRICAL ECONOMICALLY-BASED CUSUM CHART

Introduction

This chapter analyzes the asymmetric cumulative sum chart and develops an economically-based model that is used to optimize the design of cumulative sum charts when associated with the Weibull process failure mechanism. The general economically-based modeling concepts developed by Duncan (1956) are applied in this research. However, they are applied to a Cusum chart, with an improvement on the assumption of the process failure mechanism to have a Weibull distribution of time to failure. This provides a more realistic model of the process environment. Concise assumptions and notation are presented to facilitate model development.

Assumptions

In order to develop the asymmetric cumulative sum chart, the following assumptions are made :

1. The asymmetric cumulative sum chart is applied to monitor and help maintain the statistical control of a process.

2. The process begins in a state of statistical control at a mean level μ_0.

3. The process standard deviation σ remains the same in spite of mean shifts in the process.

4. The process mean may randomly shift, due to an assignable cause, to $\mu_0 + \delta_U\sigma$ or $\mu_0 - \delta_L\sigma$ and stay there until corrected.

5. The occurrence of the process mean shift is instantaneous; the process will not drift from the in-control state, such as is the case with tool wear.

6. The process is not shut down while searching for the assignable cause.

7. As soon as the assignable cause is found, it is fixed instantly.

8. Neither the cost of adjustment or repair, nor the cost of bringing the process back into a state of statistical control after the assignable cause is discovered, is introduced into the economic model.

9. The hourly cost of sampling, measuring, computing and plotting the control chart has a linear relationship with subgroup size.

10. The occurrence times for the assignable causes are independent and follow a Weibull distribution.

The assumption of an exponential failure mechanism is a special case of assumption number 10. The other assumptions are similar to those used in Duncan's model (Duncan, 1956).

Notation

The following notation is introduced and will be employed throughout the entire dissertation.

- n : The number of individual measurements or samples that comprise a subgroup.
- h : The time interval between subgroups; subgroups of size n are taken from the process every h hours.
- d_U : The decision interval in the upward direction; cumulative sums beyond this value indicate a process mean shift.
- d_L : The decision interval in the downward direction; cumulative sums beyond this value indicate a process mean shift.
- k_U : The "dead band" value for detecting upward shifts; subgroup averages must be beyond the "dead band" to begin adding to the cumulative sum.
- k_L : The "dead band" value for detecting downward shifts; subgroup averages must be beyond the "dead band" to begin adding to the cumulative sum.
- θ , S : The parameters related to the time of occurrence of the assignable cause. The distribution of the process in control is Weibull distributed with a mean time $\theta\Gamma(1+1/S)$, where $\theta > 0$ is the scale parameter and $S > 0$ is the shape

parameter. The density function of the Weibull distribution is

$$f(t) = (S/\theta)(t/\theta)^{S-1} \exp(-(t/\theta)^{S}); \ t \geq 0. \qquad (3.1)$$

$E(f(t))$: The expected value of a function of variable t.

μ, μ_0 : The process mean μ has the standard or desired value μ_0 before any shifting occurs.

σ : The standard or desired process standard deviation which remains the same in spite of the occurrence of any shift.

δ_U : The magnitude of an upward shift in the process mean, expressed in multiples of σ ($\delta_U\sigma$); an upward shift will occur from μ_0 to $\mu_0 + \delta_U\sigma$.

δ_L : The magnitude of a downward shift in the process mean, expressed in multiples of σ ($\delta_L\sigma$); a downward shift will occur from μ_0 to $\mu_0 - \delta_L\sigma$.

V_0 : The hourly income which accrues from operation of the process in-control at mean level μ_0.

V_U : The hourly income which accrues from operation of the process out of control at mean level $\mu_0 + \delta_U\sigma$.

V_L : The hourly income which accrues from operation of the process out of control at mean level $\mu_0 - \delta_L\sigma$.

M_U : The diminution of hourly income attributed to the occurrence of an upward mean shift from μ_0 to $\mu_0 + \delta_U\sigma$; $M_U = V_0 - V_U$.

M_L : The diminution of hourly income attributed to the occurrence of a downward mean shift from μ_0 to $\mu_0 - \delta_L\sigma$; $M_L = V_0 - V_L$.

b : The cost per subgroup of sampling, measuring, computing, plotting, and making the acceptance/ rejection decision that is independent of the subgroup size.

c : The cost per unit of sampling, measuring, computing and plotting that is related to the subgroup size; the relationship is assumed to be linear.

D : The average time taken to find the assignable cause, once an out of control condition is detected.

e : The per unit average time for sampling, measuring, computing and plotting; this time is assumed proportional to the subgroup size n.

T : The average cost per event of searching for an assignable cause when none exists.

W : The average cost per event of searching for an assignable cause when one does exist.

α : The conditional probability that if there is a shift in the mean, the shift will be in the upward direction.

$1-\alpha$: The conditional probability that if there is a shift in the mean, the shift will be in the downward direction.

Γ_0 : The proportion of time the process is in a state of statistical control ($\mu = \mu_0$).

Γ_U : The proportion of time the process is out of control in the upward diretion ($\mu = \mu_0 + \delta_U\sigma$).

Γ_L : The proportion of time the process is out of control in the downward direction ($\mu = \mu_0 - \delta_L\sigma$).

T_{in} : The expected length of time a process is in-control at the acceptable quality level.

ARL_0 : The expected number of subgroups taken until a false alarm is indicated when a process is in-control at the acceptable quality level.

ARL_1 : The average number of subgroups taken before a shift in the process mean from μ_0 to either $\mu_0 + \delta_U\sigma$ or $\mu_0 - \delta_L\sigma$ is detected by virtue of exceeding either the upper decision interval or the lower decision interval.

$ARLA_U(\delta_U)$: The average number of subgroups taken following an upward shift from μ_0 to $\mu_0 + \delta_U\sigma$ before detection by virtue of the cumulative sum exceeding decision interval d_U.

$ARLA_U(\delta_L)$: The average number of subgroups taken following a downward shift from μ_0 to $\mu_0 - \delta_L\sigma$ before detection by virtue of the cumulative sum exceeding decision interval d_U.

$ARLA_L(\delta_U)$: The average number of subgroups taken following an upward shift from μ_0 to $\mu_0 + \delta_U\sigma$ before detection by virtue of the cumulative sum exceeding decision interval d_L.

$ARLA_L(\delta_L)$: The average number of subgroups taken following a downward shift from μ_0 to $\mu_0 - \delta_L\sigma$ before detection by virtue of the cumulative sum exceeding decision interval d_L.

$ARLA_1(\delta_U)$: The average number of subgroups taken before an upward shift from μ_0 to $\mu_0 + \delta_U\sigma$ will be detected by virtue of exceeding either the upper decision interval or lower decision interval.

$ARLA_1(\delta_L)$: The average number of subgroups taken before an upward shift from μ_0 to $\mu_0 - \delta_L\sigma$ will be detected by virtue of exceeding either the upper decision interval or lower decision interval.

T_1 : The average time elapsed from the time the process mean shifts from μ_0 to either $\mu_0 + \delta_U\sigma$ or $\mu_0 - \delta_L\sigma$ until the detecting subgroup is taken.

T_2 : The average time elapsed for sampling, measuring, computing and plotting a sample statistic and finding an assignable cause.

ATOWI : The expected time of occurrence of a process shift within a particular interval between subgroups.

ETOPS : The expected time of occurrence of a process shift within the interval between subgroups, over all intervals between subgroups.

ENSIN : The expected number of subgroups taken during the period of the process in control.

T_{out} : The expected length of time a process is out of control at a rejectable quality level.

T_{cycle} : The average time for one in-control, out of control cycle.

Model Formulation

General Structure

The operation of a two-sided Cusum control scheme for surveilling the process mean comprises three basic procedures: (1) sampling and measuring subgroups of size n at regularly spaced intervals of h hours, (2) computing and plotting the cumulative sums

$$S_{jU} = \text{Max } (0, \bar{X}_j - k_U + S_{(j-1)U})$$

and

$$S_{jL} = \text{Max } (0, k_L - \bar{X}_j + S_{(j-1)L}).$$

for subgroup j ($S_{0U} = S_{0L} = 0$), and (3) comparing the cumulative sums S_{jU} and S_{jL} to the decision intervals d_U and d_L, respectively. Whenever the computed value S_{jU} of a plotted point is greater than or equal to the upper decision interval, d_U, it indicates the likely occurrence of an upward shift in the process mean. Similarly, if the computed value S_{jL} of a plotted point is greater than or equal to the lower decision interval, d_L, it indicates a likely downward shift in the process mean. In other words, a decision that the process mean has shifted from the desired value is reached when either the upper or the lower decision interval is exceeded. Therefore, the subgroup size n, time interval between subgroups h, dead band values k_U and k_L, and decision intervals d_U and d_L are the decision variables required for implementing a two-sided Cusum control chart.

Average Run Length (ARL)

The run length of a control scheme is the number of subgroups necessitated before there is an out of control signal. An out of control signal indicates that an assignable cause has probably occurred in the process and that action should be taken to search for and remove the assignable cause. The ARL is used as a performance measure to evaluate the Cusum control chart. The decision variables n, h, k_U, k_L, d_U, and d_L of the Cusum chart determine values of ARL_0 and ARL_1 at acceptable and rejectable quality levels, respectively. In general, a good control chart scheme has a very large value of ARL_0, when the process is in-control, and a very small ARL_1, when the process mean has shifted.

The desired values of ARL_0 and ARL_1 at the acceptable and rejectable quality levels, respectively, are generally specified, in order to determine the decision variables of a Cusum control scheme. The decision variables are then formed by using nomograms of Ewan and Kemp (1960), Goel (1968) or Geol and Wu (1973) to satisfy, approximately, the specified ARL_0 and ARL_1. This approach of designing the Cusum control scheme does not, however, take into consideration the cost aspects of the process and the time interval between subgroups, h, which has to be determined by some rule of thumb. In general, nomograms are inconvenient and not precise.

Economically designed Cusum control schemes require repeated ARL computations to minimize an expected cost function. Vance (1986) presents a computer program for evaluating the ARL. This program is used to avoid the drawbacks of nomograms. However, Vance's ARL program produces an ARL value for one-sided Cusum control schemes. Fortunately, one may consider a two-sided Cusum control scheme as a synthesis of two one-sided Cusum control schemes. An asymmetric two-sided Cusum control scheme will have to deal with the magnitudes of an upward shift $\delta_U\sigma$ and a downward shift $\delta_L\sigma$ in the process mean, upper and lower dead band values, k_U and k_L, and upper and lower decision intervals, d_U and d_L. Recall that $ARLA_1(\delta_U)$ is the average number of subgroups taken before a magnitude of upward shift δ_U will be detected by virtue of exceeding either the upper decision interval or lower decision interval. Kemp (1961) shows that

$$\frac{1}{ARLA_1(\delta_U)} = \frac{1}{ARLA_U(\delta_U)} + \frac{1}{ARLA_L(\delta_U)}.$$

Likewise, recall that $ARLA_1(\delta_L)$ is the average number of subgroups taken before a magnitude of downward shift δ_L will be detected by virtue of exceeding either the upper decision interval or lower decision interval. Then,

$$\frac{1}{ARLA_1(\delta_L)} = \frac{1}{ARLA_U(\delta_L)} + \frac{1}{ARLA_L(\delta_L)}.$$

It is assumed that there is a possibility α that the process mean will shift upwardly. Then the ARL, ARL_1, of the two-sided Cusum control scheme is given by

$$ARL_1 = \alpha * ARLA_1(\delta_U) + (1-\alpha) * ARLA_1(\delta_L).$$

Nature of the Process and Cycle Time

The process starts at time t = 0 in a state of statistical control with a mean value μ_0 and a known standard deviation σ which remains constant. An assignable cause occurs randomly and causes a shift in the process mean of a known magnitude, either $\delta_U\sigma$ or $\delta_L\sigma$. Therefore, the shifted process mean is either $\mu = \mu_0 + \delta_U\sigma$ or $\mu = \mu_0 - \delta_L\sigma$, depending on the direction of shift. The process stays at this level until the shift is detected and adjustments are made to bring the process back to the desired mean value, μ_0. Then it stays in an in-control condition until the next assignable cause occurs.

The cycle time of the process is defined as the total time of the process, starting from an in-control state, shifting to an out of control condition, detecting the lack of control and finding the assignable cause. In other words, cycle time is composed of durations in-control, out of control before detection of the assignable cause, and while searching for the assignable cause. An illustration of cycle time is given in Figure 3.1.

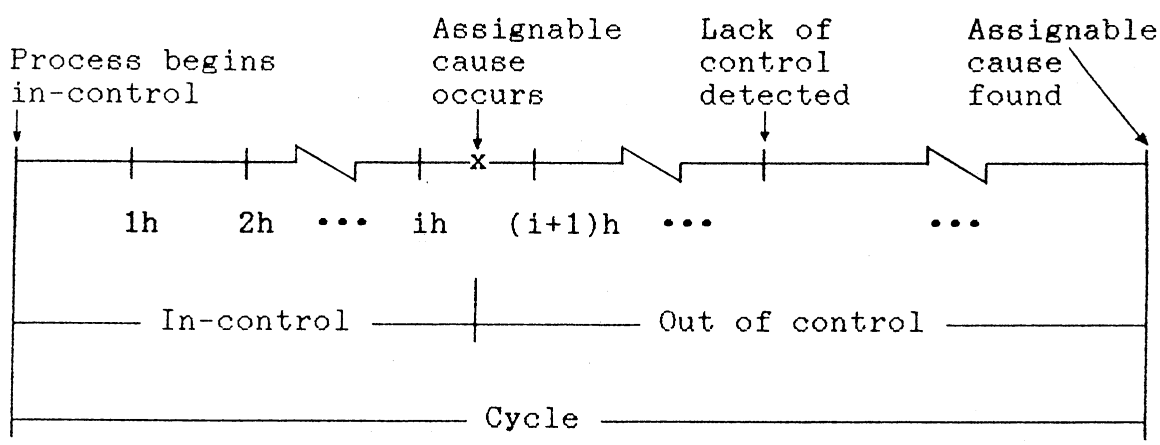


Figure 3.1. Cycle Time

Derivation of the Economic Model

Average cycle time plays an important role in determining the cost components of the model. When the average cycle time is determined, then the cost components can be converted to an hourly cost basis. A diagrammatic explanation of the procedures involved in the derivation is given in Figure 3.2.

Average In-control, Out of control and Cycle Time

As illustrated in Figure 3.2, the average cycle time is developed as follows:

		(1)		(2)
Average cycle time	=	Average in-control time	+	Average time the process is out of control before a detecting subgroup is taken

	(3)		(4)
+	Average time for sampling, measuring, computing and plotting a subgroup	+	Average time seeking for the assignable cause

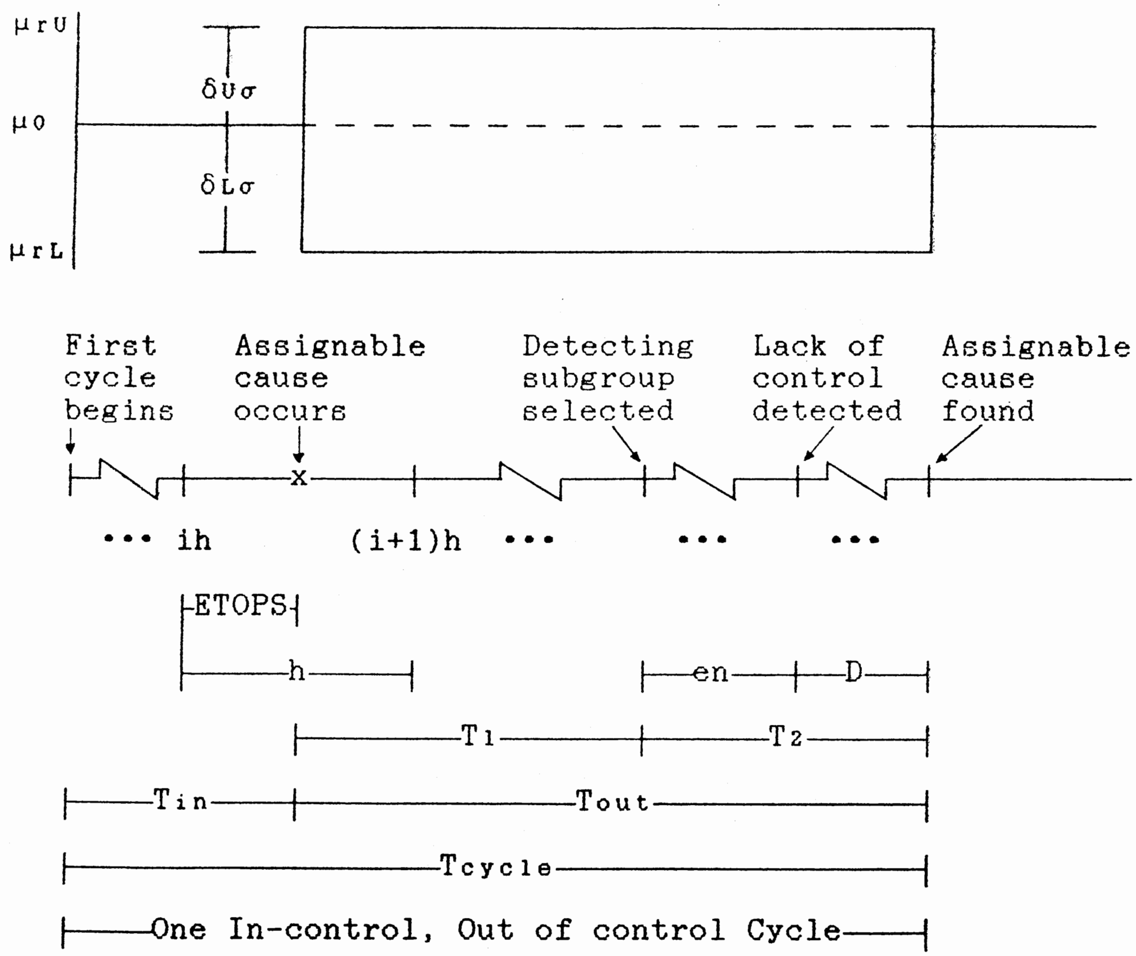
(1) From Eq. (3.1), the probability that an assignable cause occurs in the interval t to t+Δt is approximately

$$f(t)\Delta t = (S/\theta)(t/\theta)^{S-1} \exp(-(t/\theta)^{S}) t.$$

The average time required for the assignable cause to occur is

$$E(f(t)) = \int_0^\infty tf(t)dt = \theta\Gamma(1+1/S).$$

The time period the process remains in the in-control state, given that it begins in-control, is equal to the



$T_{in} = \theta\Gamma(1+1/S)$ $\quad T_1 = ARL_1 * h - ETOPS$

$T_{out} = T_1 + T_2$ $\quad T_2 = en + D$

$T_{cycle} = T_{in} + T_{out}$

Figure 3.2. Diagrammatric Explanation of the Cost Model Derivation

mean of the distribution governing the process failure (mean shift) mechanism. Hence the expected length of time, T_{in}, for which the process is in-control at level μ_0 is given by

$$T_{in} = \theta\Gamma(1+1/S). \tag{3.2}$$

(2.a) If subgroups are taken at intervals of h hours, then given the occurrence of the assignable cause in the interval between the i^{th} and $(i+1)^{th}$ subgroup (see Figure 3.3), the average time of occurrence within that interval is given by

$$ATOWI = \frac{\int_{ih}^{(i+1)h} f(t)(t-ih)dt}{\int_{ih}^{(i+1)h} f(t)dt}.$$

This can be simplified as follows :

$$ATOWI = \frac{\int_{ih}^{(i+1)h} f(t)tdt - \int_{ih}^{(i+1)h} f(t)ihdt}{\int_{ih}^{(i+1)h} f(t)dt}$$

$$= \frac{\int_{ih}^{(i+1)h} f(t)tdt}{\int_{ih}^{(i+1)h} f(t)dt} - ih. \tag{3.3}$$

When t is Weibull distributed, from Eq. (3.1) ATOWI is as follows:

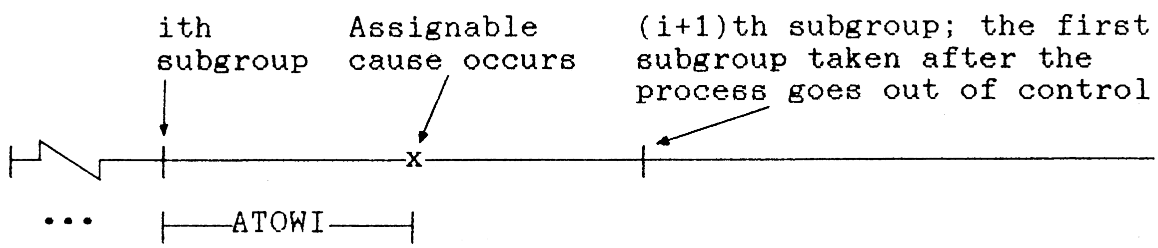


Figure 3.3. Average Time of Occurrence of Assignable Cause Within an Interval Between Subgroups

$$\text{ATOWI} = \frac{\int_{ih}^{(i+1)h} (S/\theta)(t/\theta)^{S-1} \exp(-(t/\theta)^{S})\,t\,dt}{\int_{ih}^{(i+1)h} (S/\theta)(t/\theta)^{S-1} \exp(-(t/\theta)^{S})\,dt} - ih$$

Letting $(t/\theta)^{S} = u$, then $(S/\theta)(t/\theta)^{S-1}\,dt = du$. Also, $(t/\theta)^{S} = u$ implies that $t/\theta = u^{1/S}$ or $t = \theta u^{1/S}$.

Therefore,

$$\text{ATOWI} = \frac{\int_{(ih/\theta)^{S}}^{((i+1)h/\theta)^{S}} u^{1/S} \exp(-u)\,du}{\int_{(ih/\theta)^{S}}^{((i+1)h/\theta)^{S}} \exp(-u)\,du} - ih$$

$$= \frac{\theta(\gamma((1+1/S),((i+1)h/\theta)^{S}) - \gamma((1+1/S),(ih/\theta)^{S}))}{\exp(-(ih/\theta)^{S}) - \exp(-((i+1)h/\theta)^{S})}$$

$$- ih, \qquad (3.4)$$

where

$\gamma(a,x)$ represents the incomplete Gamma integral;

$$\gamma(a,x) = \int_{0}^{x} \exp(-t)t^{a-1}\,dt.$$

(2.b) Given the average time of the occurrence of the assignable cause between subgroups i and i+1 (ATOWI) above, in Eq. 3.3, the expected time of occurrence of the assignable causes within an interval is given by

$$\text{ETOPS} = \sum_{i=0}^{\infty} \text{ATOWI} \int_{ih}^{(i+1)h} f(t)dt$$

$$= \sum_{i=0}^{\infty} \int_{ih}^{(i+1)h} f(t)(t-ih)dt$$

$$= \sum_{i=0}^{\infty} \left(\int_{ih}^{(i+1)h} f(t)tdt - ih \int_{ih}^{(i+1)h} f(t)dt \right).$$

(2.c) When the process mean shifts from μ_0 to $\mu_0 + \delta_U\sigma$, then the average number of subgroups taken before the shift in the process will be caught by virtue of the cumulative sum exceeding decision interval d_U is $ARLA_U(\delta_U)$, and by virtue of exceeding d_L, $ARLA_L(\delta_U)$. Kemp (1961) shows that the average number of subgroups taken before this upward shift in the process will be caught is $ARLA_1(\delta_U)$, where

$$\frac{1}{ARLA_1(\delta_U)} = \frac{1}{ARLA_U(\delta_U)} + \frac{1}{ARLA_L(\delta_U)}.$$

(2.d) When the process mean shifts from μ_0 to $\mu_0 - \delta_L\sigma$, then the average number of subgroups taken before the shift in the process will be caught by virtue of the cumulative sum exceeding decision interval d_U is $ARLA_U(\delta_L)$, and by virtue of exceeding d_L, $ARLA_L(\delta_L)$. Therefore, the average number of subgroups taken before this downward shift in the process will be caught is $ARLA1(\delta_L)$, where

$$\frac{1}{ARLA_1(\delta_L)} = \frac{1}{ARLA_U(\delta_L)} + \frac{1}{ARLA_L(\delta_L)}.$$

(2.e) The average number of subgroups taken before a shift in the process mean is caught is noted as ARL_1. That is, ARL_1 is the ARL of an asymmetric two-sided Cusum control chart when the process is out of control and is given by

$$ARL_1 = \alpha * ARLA_1(\delta_U) + (1-\alpha) * ARLA_1(\delta_L).$$

Therefore, the average time elapsed for which the process mean will be at the rejectable quality level before the detecting subgroup is given by

$$T_1 = ARL_1 * h - ETOPS.$$

(3) The time required to sample, measure, compute, and plot a point is proportional to the subgroup size n. That is, delay until a point is plotted is en hours.

(4) An average time of D hours is required to find an assignable cause after its detection. Thus, the process will continue at the rejectable quality level for an additional $T_2 = en + D$ hours since the process is not shut down while searching for the assignable cause. Therefore, the total expected time the process is out of control, T_{out}, is given by

$$T_{out} = T_1 + T_2$$
$$= ARL_1 * h - ETOPS + en + D. \quad (3.4)$$

Combining expressions in Eqs. (3.2) and (3.4), the average time T_{cycle} for one in-control, out of control cycle is given by

$$T_{cycle} = T_{in} + T_{out}$$
$$= \theta\Gamma(1+1/S) + ARL_1 * h - ETOPS + en + D.$$

Cost Formulation

The components of this model are (1) loss due to defective products being produced, (2) cost of searching for an assignable cause when none exists, (3) cost of searching for an assignable cause when one exists, and (4) cost of sampling, measuring, computing and plotting the control chart.

Based upon the average in-control, out of control and cycle time, the hourly net income from the process is developed as follows:

		(1)		(2)
Process average hourly net income	=	Average hourly in-control income	+	Average hourly out of control income
		(3)		(4)
	−	Average hourly false alarm cost	−	Average hourly real alarm cost
		(5)		
	−	Average hourly cost for sampling, measuring, computing and plotting the control chart		

(1) The proportion of time a process is in-control is

$$\Gamma_o = \frac{\theta\Gamma(1+1/S)}{T_{cycle}}.$$

Therefore, the average hourly income due to the process being in-control is $V_o\Gamma_o$.

(2.a) The proportion of the time a process will be out of control due to an upward shift in the process is

$$\Gamma_U = \frac{\alpha * (ARLA_1(\delta_U) * h - ETOPS + en + D)}{T_{cycle}}.$$

Thus, the average hourly income due to the process being out of control in the upward direction is $V_U \Gamma_U$.

(2.b) The proportion of the time a process will be out of control due to a downward shift in the process is

$$\Gamma_L = \frac{(1-\alpha) * (ARLA_1(\delta_L) * h - ETOPS + en + D)}{T_{cycle}}.$$

Thus, the average hourly income due to the process being out of control in the downward direction is $V_L \Gamma_L$.

(3.a) A false alarm occurs when the cumulative sum value of a subgroup reaches either the upper or lower decision interval, while the process is actually in-control. The false alarm demands a search for the nonexistent assignable cause. The average number of subgroups, taken from an in-control process, between false alarms is ARL_0. Hence the proportion of time a subgroup point will fall outside the decision interval when the process is in-control is $1/ARL_0$.

(3.b) If the process goes out of control in the i^{th} interval, the expected number of subgroups taken during the period in which the process is in-control is given by

$$ENSIN = \sum_{i=0}^{\infty} \int_{ih}^{(i+1)h} i f(t) dt$$

Using Eq. (3.1), ENSIN is as follows :

$$ENSIN = \sum_{i=0}^{\infty} \int_{ih}^{(i+1)h} i (S/\theta)(t/\theta)^{S-1} \exp(-(t/\theta)^{S}) dt$$

Letting $(t/\theta)^{S} = u$, then $(S/\theta)(t/\theta)^{S-1} dt = du$. Also,

$(t/\theta)^S = u$ implies that $t/\theta = u^{1/S}$ or $t = \theta u^{1/S}$.

$$ENSIN = \sum_{i=0}^{\infty} i(\exp(-(ih/\theta)^S) - \exp(-((i+1)h/\theta)^S))$$

$$= \exp(-(h/\theta)^S)+2\exp(-(2h/\theta)^S)+\cdots+ n \exp(-(nh/\theta)^S) +\cdots$$

$$-1\exp(-(2h/\theta)^S)-\cdots-(n-1)\exp(-(nh/\theta)^S)-\cdots$$

$$= \exp(-(h/\theta)^S)+\exp(-(2h/\theta)^S)+\exp(-(3h/\theta)^S)+\cdots$$

$$= \sum_{i=1}^{\infty} \exp(-(ih/\theta)^S)$$

(3.c) The average hourly false alarm cost is therefore

$$\frac{\frac{1}{ARL_0} * T * ENSIN}{T_{cycle}}.$$

(4) The process is truly out of control once every T_{cycle} hours. Therefore, the average number of times per hour that the process actually goes out of control is $1/T_{cycle}$. If the average cost of finding the assignable cause when it occurs is W, the average cost per hour for finding as actual alarm will be W/T_{cycle}.

(5) The average hourly cost for sampling, measuring, computing and plotting charts is

$$\frac{b+cn}{h}.$$

The process hourly net income is therefore:

$$I = V_O\Gamma_O + V_U\Gamma_U + V_L\Gamma_L - \frac{\frac{1}{ARL_O} * T * ENSIN}{T_{cycle}} - \frac{W}{T_{cycle}} - \frac{b+cn}{h}$$

Since $M_U = V_O - V_U$, $M_L = V_O - V_L$ and $\Gamma_O + \Gamma_U + \Gamma_L = 1$,

$$\begin{aligned} V_O\Gamma_O + V_U\Gamma_U + V_L\Gamma_L &= V_O\Gamma_O + (V_O - M_U)\Gamma_U + (V_O - M_L)\Gamma_L \\ &= V_O\Gamma_O + V_O\Gamma_U + V_O\Gamma_L - M_U\Gamma_U - M_L\Gamma_L \\ &= V_O - M_U\Gamma_U - M_L\Gamma_L \end{aligned}$$

Thus,

$$I = V_O - M_U\Gamma_U - M_L\Gamma_L - \frac{\frac{1}{ARL_O} * T * ENSIN}{T_{cycle}} - \frac{W}{T_{cycle}} - \frac{b+cn}{h}$$

$$= V_O - L.$$

where

L = Loss-cost

$$= M_U\Gamma_U + M_L\Gamma_L + \frac{\frac{1}{ARL_O} * T * ENSIN}{T_{cycle}} + \frac{W}{T_{cycle}} + \frac{b+cn}{h}$$

$$= M_U\Gamma_U + M_L\Gamma_L + \frac{T * ENSIN + W * ARL_O}{ARL_O * T_{cycle}} + \frac{b+cn}{h}.$$

According to the formulation above, to maximize average hourly net income is equivalent to minimizing the loss-cost L. This observation corresponds to that of Duncan.

Optimal-Seeking Methods

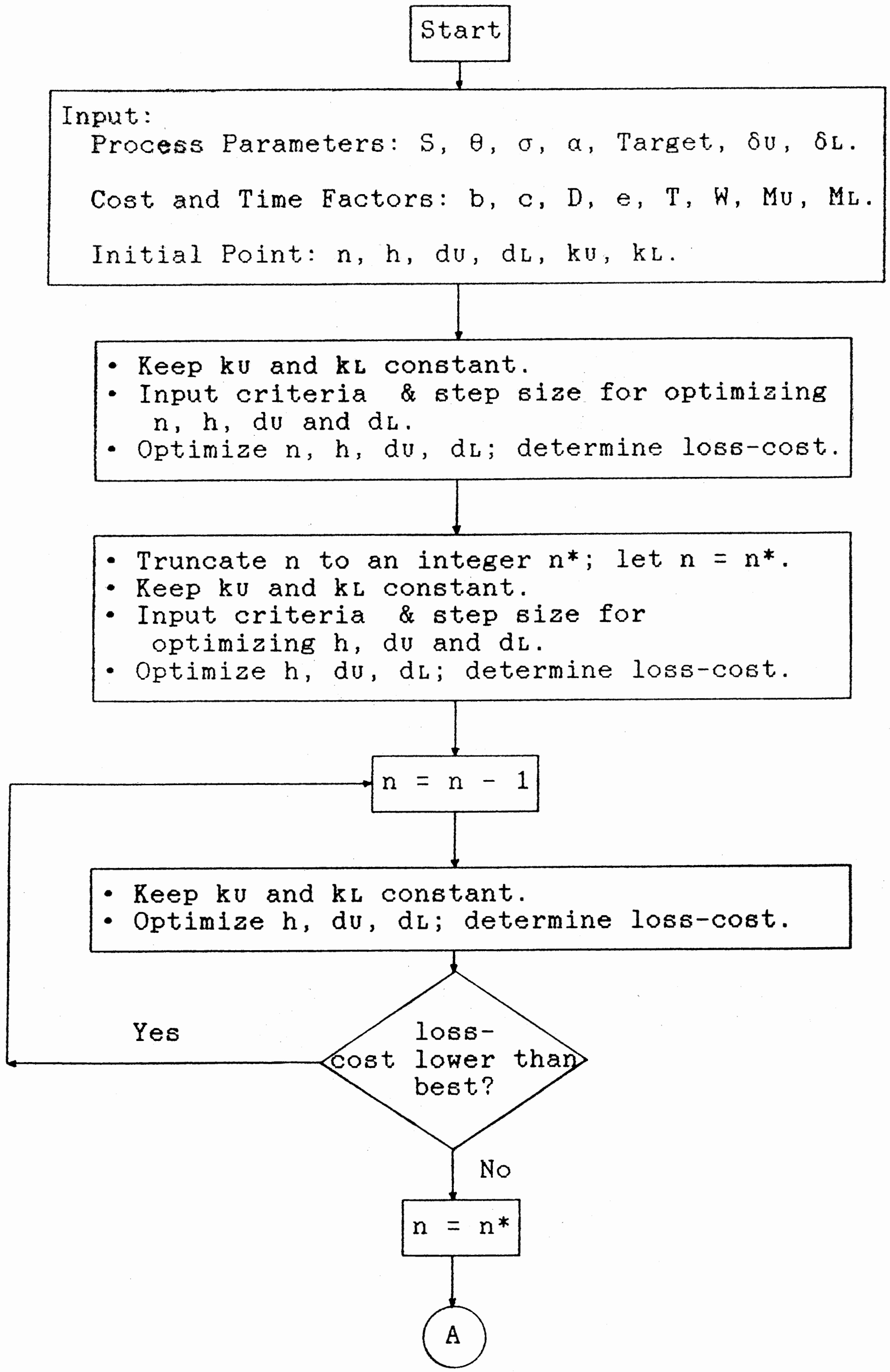
The economically-based Cusum chart model is now used to find an optimal or near-optimal combination of values of the decision variables, minimizing the loss-cost L and thereby maximizing the average hourly net income of the process. An analytically definite optimal solution has not been determined for the value of L as a loss-cost function of the decision variables n, h, k_U, k_L, d_U, and d_L. A multi-dimensional direct search technique is used for near-optimizing the loss-cost function.

The Nelder-Mead simplex procedure (Nelder and Mead, 1965) (O'Neill, 1971) is utilized as the search algorithm. Olsson and Nelson (1975) show the generality of the Nelder-Mead simplex method, its accuracy, and the simplicity of the information required for the computer input statement. This method is described for the minimization of a multivariable function without constraints. The simplex procedure derives its name from the geometric figure which is moved along the response surface in search of the minimum. No derivatives of the objective function are required, which is a so-called "direct" procedure.

The simplex procedure approaches the minimum by moving away from the highest values of the objective function rather than by trying to move in a line toward the minimum. The procedure is operated by reflection, extension, contraction or shrinkage so as to conform to the characteristics of the response surface. The operation continues until either a specified number of evaluations has been reached or the function values differ from themselves by less than a specified amount. Based on empirical evidence, multiple starting points are required in order to lend confidence that an optimal or near-optimal solution of the loss-cost function has been reached.

In this research, the subgroup size n is the only decision variable which must be an integer. A brief schematic description of the search procedure is given in Figure 3.4. Following is a more detailed description of the search procedure.

1. Fix k_U and k_L at the middle of the desired process mean and upper rejectable quality level and lower rejectable quality level, respectively. Apply the Nelder-Mead algorithm with the other four variables to find the near-optimal point of real values of n, h, d_U and d_L.
2. With k_U and k_L remaining at the same values as they were in step 1, the real value of subgroup size n is truncated to an integer and treated as a constant. The values of h, d_U and d_L, obtained from the preceding step, serve as a new starting point in the direct search which is then performed on decision variables h, d_U and d_L. The result of h, d_U and d_L with this integer value n and fixed k_U and k_L is treated as an intermediate best solution.
3. Repeat step 2 by doing a line search along integer values of n to find the minimum loss-cost.
4. Let the best result realized in step 3 be a new starting point and, with n fixed, do a five variables direct search to optimize values of h, d_U, d_L,


Start
Input:
Process Parameters: S, θ, σ, α, Target, δU, δL.
Cost and Time Factors: b, c, D, e, T, W, MU, ML.
Initial Point: n, h, dU, dL, kU, kL.
• Keep kU and kL constant.
• Input criteria & step size for optimizing n, h, dU and dL.
• Optimize n, h, dU, dL; determine loss-cost.
• Truncate n to an integer n*; let n = n*.
• Keep kU and kL constant.
• Input criteria & step size for optimizing h, dU and dL.
• Optimize h, dU, dL; determine loss-cost.
n = n - 1
• Keep kU and kL constant.
• Optimize h, dU, dL; determine loss-cost.
loss-cost lower than best?
Yes
No
n = n*
A

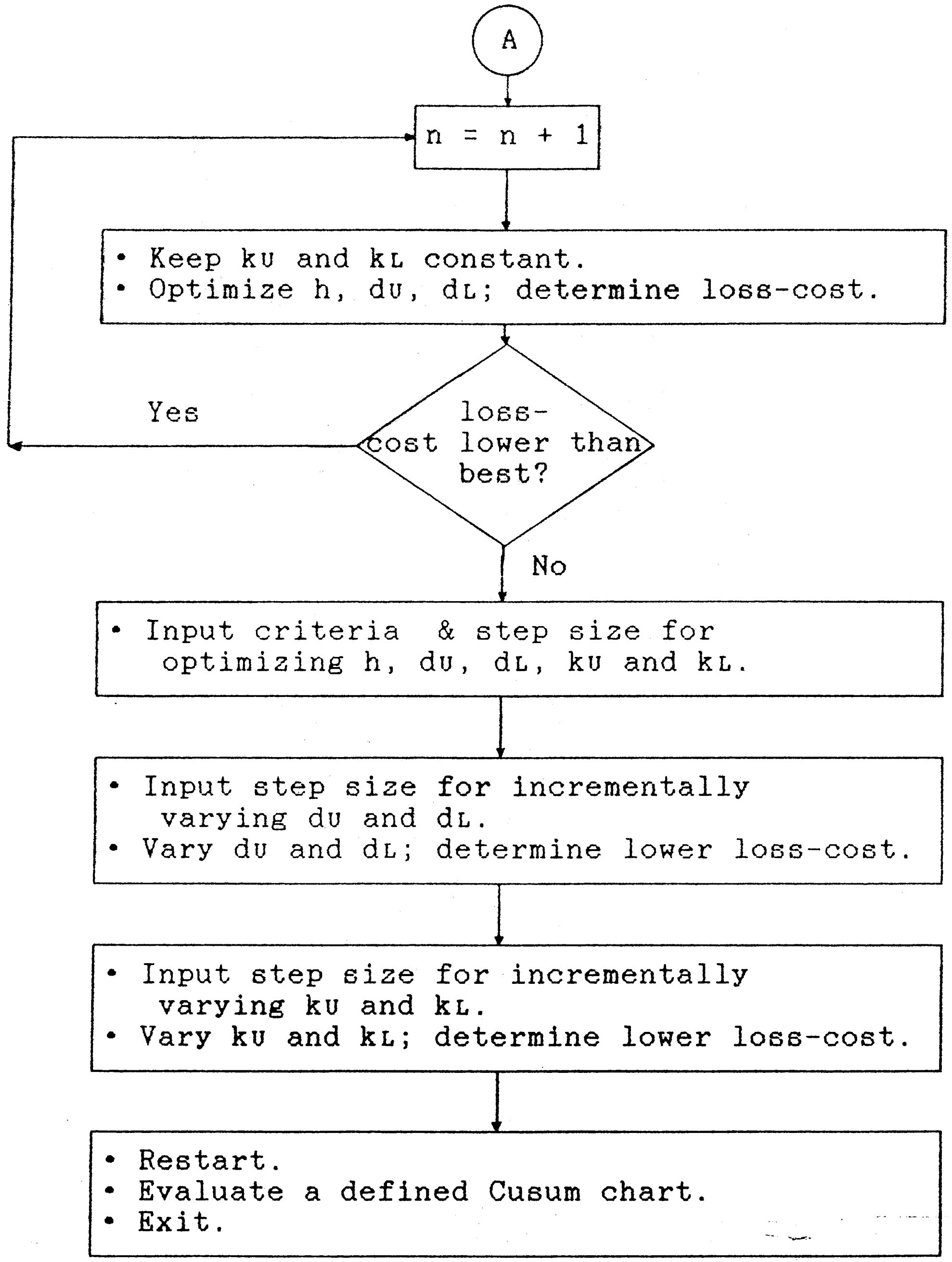


Figure 3.4. Schematic Description of the Search Procedure

k_U and k_L.

5. Incrementally vary d_U and d_L as well as k_U and k_L on the result of step 4. The final outcome is then the best or near-best decision variable set (n, h, k_U, k_L, d_U, d_L) for the economically-based Cusum chart.

Incrementally varying k_U, k_L, d_U and d_L on the result of the search procedure will lead to a slightly better outcome in most of the cases.

In many cases, search methods do not require continuity of the objective function and the existence of derivatives. However, in general, in solving unconstrained nonlinear programming problems, gradient and second-derivative methods converge faster than direct search methods.

Summary

An economically-based model is developed to describe the use of a generalized Cusum chart. This model is developed using Duncan's approach to the economical design of control charts. The mathematical development and derivation of the hourly net income for this generalized Cusum chart is discussed. The model developed in this chapter has the characteristic of representing various process failure mechanisms while Duncan's model only deals with the exponential time to failure mechanism. In addition, this model has the added capability of dealing directly with asymmetrical upper and lower decision intervals, dead bands and costs.

An optimization procedure is used to find the decision variables m, h, k_U, k_L, d_U, and d_L required to construct the control chart and minimize the loss-cost function. The minimum loss-cost design is equivalent to the design which maximizes hourly net income of a process. The Nelder-Mead direct search algorithm is utilized in this optimization procedure.

CHAPTER IV

RESULTS, COMPARISON AND ANALYSIS

Introduction

This chapter first discusses results achieved on Cusum charts of symmetric design. Results of the economically-based model are compared with those of Goel (1968) based on his data sets numbered 1, 16 and 21. Then the asymmetric design is presented through Goel's number 1 data set.

Factors which produce asymmetry of the model are: (1) α, the conditional probability that if there is a shift in the mean, the shift will be in the upward direction, (2) δ, the magnitude of the shift in the process mean in either the upward direction, δ_U, or downward direction, δ_L, (3) M, the diminution of hourly income that attributes to the occurrence of the assignable cause in either the upward direction, M_U, or downward direction, M_L. A $3^2 5^1$ factorial experiment is conducted to verify the validity of the asymmetric design. Different initial points are employed to confirm the validity of the model and its associated search procedure.

Comparison of Results for the Symmetric Design

In order to validate the economically-based asymmetric model developed in Chapter III and the search procedure associated with the model, three representative examples from Goel's research (1968) are optimized. The costs and other relevant parameters for these three examples are given in Table 4.1.

TABLE 4.1

COST AND RISK FACTORS AND PARAMETERS FOR THREE EXAMPLES

Example No.	λ	δ	M	e	D	T	W	b	c
1	0.01	2.0	100.00	0.05	2	50	25	0.5	0.1
16	0.01	1.0	12.87	0.05	2	50	25	0.5	0.1
21	0.01	0.5	2.25	0.05	2	50	25	0.5	0.1

Goel presents his results based on a minimum cost criterion for a two-sided symmetric Cusum chart. Subgroup size n, time interval between subgroups h, decision intervals d_U and d_L, and loss-cost values for these examples are reevaluated and are listed in Table 4.2. These results for the economically-based design are computed under the conditions: (a) α = 0.5, (b) $M_U = M_L$, and (c) $\delta_U = \delta_L$. This is the only circumstance in which the asymmetric model developed herein is used to optimize a symmetric two-sided Cusum chart. Based on the results listed in Table 4.2, it can be noted that the asymmetric model developed has results very close to those of Goel's Cusum chart.

TABLE 4.2

RESULTS FOR GOEL'S CUSUM CHART AND ECONOMICALLY-BASED DESIGN

Example No.	Goel's CUSUM chart as evaluated by Goel				Goel's CUSUM chart as evaluated by model developed					CUSUM chart as optimized and evaluated by asymmetric model				
	n	h	d	Cost	n	h	d_U	d_L	Cost	n	h	d_U	d_L	Cost
1	5	1.4	0.39	4.0093	5	1.40	0.39	0.39	4.0088	5	1.40	0.3893	0.3895	4.0088
16	14	5.4	0.23	1.4128	14	5.40	0.23	0.23	1.4113	14	5.40	0.2371	0.2472	1.4113
21	37	22.29	0.123	0.8339	37	22.29	0.123	0.123	0.8289	38	24.22	0.1069	0.1063	0.8291

A further comparison is to calculate the loss-costs for varying subgroup sizes of these 3 examples. The results are listed in Table 4.3. These loss-costs provide a measure of the performance of the control chart. From Table 4.3, the validity of the economically-based design and its associated search procedure can be confirmed.

Different initial points are employed to further validate the model and its associated search procedure. Each example is performed starting from two subgroup sizes to search for the optimal or near-optimal plan. As shown in

TABLE 4.3

LOSS-COSTS FOR VARIOUS SUBGROUP SIZES
FOR THREE EXAMPLES

		LOSS-COST		
Example No.	Subgroup size	Goel's CUSUM chart as evaluated by Goel	Goel's CUSUM chart as evaluated by model developed	CUSUM chart as optimized and evaluated by asymmetric model
	3	4.1265	4.1257	4.1264
1	4	4.0232	4.0225	4.0227
	5	4.0093	4.0088	4.0088
	6	4.0464	4.0461	4.0462
	13	1.4138	1.4122	1.4191
16	14	1.4128	1.4113	1.4113
	15	1.4145	1.4130	1.4152
	16	1.4184	1.4173	1.4183
	37	0.8339	0.8289	0.8292
21	38	0.8340	0.8291	0.8291
	39	0.8342	0.8294	0.8293
	40	0.8346	0.8299	0.8296

Table 4.4, for examples 1 and 16, results of the asymmetric model are very close to those of Goel. An interpretation of example number 21, in which the decision variables do not match well, is that the surfaces of the loss-cost become flatter as δ decreases, as declared by Goel and Wu (1973).

In order to explore the slope of the loss-cost surfaces, loss-costs are investigated by increasing and decreasing the subgroup size n from its optimum value. For each value of n, the model is optimized using the Nelder-Mead technique, holding only n constant, with the other five decision variables initially set to their original optimum

TABLE 4.4

OPTIMUM RESULTS OF ECONOMICALLY-BASED DESIGN FOR DIFFERENT INITIAL POINTS

Example No.	From small subgroup size (n=1)					From large subgroup size (n=10)				
	n	h	d_U	d_L	Cost	n	h	d_U	d_L	Cost
1 (δ=2.0)	5	1.39	0.3991	0.3926	4.0089	5	1.40	0.3893	0.3895	4.0088
16 (δ=1.0)	14	5.31	0.2325	0.2316	1.4114	14	5.40	0.2371	0.2472	1.4113
21 (δ=0.5)	35	20.86	0.1291	0.1330	0.8292	38	24.22	0.1069	0.1063	0.8291

value. The deviations in loss-cost with subgroup size n, as shown in Table 4.5, are the largest for $\delta = 2$ (example #1) and are the smallest for $\delta = 0.5$ (example #21) in either increasing or decreasing subgroup sizes from optimum.

Analysis of the Asymmetric Design

Factors, α, δ, and M reflect the asymmetry of the model. The optimal results for asymmetric Cusum charts are analyzed by evaluating a $3^2 5^1$ factorial design. For factor α, there are five levels of interest, which are levels 0.00, 0.25, 0.50, 0.75, and 1.00. For factor δ, there are two different values, 4 and 2, for each of δ_U and δ_L which are used to form three pairwise combinations of δ_U and δ_L. Those are:

(1) $\delta_U > \delta_L$ where $\delta_U = 4$, $\delta_L = 2$.

(2) $\delta_U = \delta_L$ where $\delta_U = 2$, $\delta_L = 2$.

(3) $\delta_U < \delta_L$ where $\delta_U = 2$, $\delta_L = 4$.

TABLE 4.5

DEVIATIONS IN LOSS-COST WITH SUBGROUP SIZE n

Example No.	Subgroup size	Loss-cost	Deviation
	3	4.1264	
	4	4.0227	0.1037
1	5	4.0088	0.0139
	6	4.0462	0.0374
	7	4.1123	0.0661
	12	1.4180	
	13	1.4127	0.0053
16	14	1.4113	0.0014
	15	1.4138	0.0025
	16	1.4188	0.0050
	36	0.8294	
	37	0.8291	0.0003
21	38	0.8291	0.0000
	39	0.8293	0.0002
	40	0.8298	0.0005

Likewise, for factor M, three combinations of M_U and M_L are :

(1) $M_U > M_L$ where $M_U = 1000$, $M_L = 100$.

(2) $M_U = M_L$ where $M_U = 100$, $M_L = 100$.

(3) $M_U < M_L$ where $M_U = 100$, $M_L = 1000$.

<u>Decision Variables and Loss-costs</u>

To study the nature of the asymmetry, consider the design of a two-sided Cusum chart based on Goel's example number 1 with the following cost and risk factors:

b = \$ 0.50 D = 2.00

c = \$ 0.10	e = 0.05
T = \$50.00	σ = 1.00
W = \$25.00	Target = 100.00

The results are obtained using the optimization procedure described in Chapter III, and are summarized in Tables 4.6-4.9.

It can be seen that each cell of each table has its mirror image through the centroid. Based on the results listed in the tables, conclusions within each table can be generated as follows:

1. Each subgroup size (n) has its mirror image through the centroid.
2. Two cells the same distance from and mirrored through the centroid have the same or nearly the same values of the time intervals between subgroups (h) and loss-costs (Cost).
3. Two cells the same distance from and mirrored through the centroid have the upper and lower decision intervals (d_U and d_L) very close to the lower and upper decision intervals (d_U and d_L), respectively, in the other cell.
4. The value of the upper decision interval (d_U) at α = 0.00 tends to be a relatively large number. This results in a very small possibility of a false alarm in the upward direction. Likewise, the value of lower decision interval (d_L) at α = 1.00 tends to be a relatively large number. This results in a

TABLE 4.6

OPTIMUM DECISION VARIABLES AND LOSS-COSTS FOR THE WEIBULL PROCESS FAILURE MECHANISM WITH SHAPE PARAMETER S=1, SCALE PARAMETER θ=100 AND INITIAL POINT AS FOLLOWS: DECISION INTERVAL-UPPER d_U AND LOWER d_L, TIME INTERVAL BETWEEN SUBGROUPS h=0.1, DEAD BAND-UPPER k_U AND LOWER k_L, SUBGROUP SIZE n=1:

		$d_U=4, d_L=2, k_U=102, k_L=99$			$d_U=d_L=2, k_U=101, k_L=99$			$d_U=2, d_L=4, k_U=101, k_L=98$		
		$\delta_U > \delta_L$ (4) (2)			$\delta_U = \delta_L$ (2) (2)			$\delta_U < \delta_L$ (2) (4)		
α		$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000	$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000	$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000
	h	1.31	1.31	0.36	1.31	1.31	0.36	1.19	1.19	0.33
	d_U	4.1125	4.1125	3.2116	3.5875	3.5875	2.1692	5.1201	5.1201	6.4357
	d_L	0.4270	0.4270	0.5782	0.4126	0.4126	0.5824	0.4700	0.4700	0.9420
0.00	n	4	4	3	4	4	3	2	2	1
	k_U	102.0524	102.0524	102.0579	101.0000	101.0000	101.0866	101.0269	101.0269	101.0000
	k_L	99.0116	99.0116	98.9967	99.0000	99.0000	99.0010	98.0217	98.0217	98.0000
	Cost	3.9556	3.9556	26.6099	3.9556	3.9556	26.6099	3.4840	3.4840	24.4790
	h	0.59	1.33	0.41	0.71	1.40	0.40	0.72	1.23	0.42
	d_U	0.4893	0.6300	0.8324	0.4673	0.4821	1.2271	0.4366	0.7863	1.7336
	d_L	1.2523	0.4464	0.6059	0.6343	0.3587	0.6039	0.4467	0.4581	0.4838
0.25	n	2	4	3	4	5	3	4	3	2
	k_U	102.0154	102.0000	101.9203	100.9858	100.9844	100.9772	101.0082	101.0030	100.9913
	k_L	98.9971	99.0000	99.0159	99.0192	99.0012	99.0091	98.0619	98.1103	98.0169
	Cost	9.5369	3.9146	21.2164	10.1529	4.0024	21.2944	9.9925	3.7559	19.6238
	h	0.48	1.19	0.49	0.55	1.39	0.55	0.49	1.19	0.48
	d_U	0.4893	0.4474	0.5277	0.4263	0.3991	0.7511	0.6051	0.6952	1.4529
	d_L	1.4695	0.6969	0.6082	0.7591	0.3926	0.4353	0.5419	0.4001	0.4985
0.50	n	2	3	3	4	5	4	3	3	2
	k_U	101.9835	101.9185	102.0211	101.0061	101.0000	100.9862	100.9916	100.9916	101.0066
	k_L	99.0029	99.0098	99.0046	99.0158	99.0000	99.0021	98.0253	98.0432	98.0164
	Cost	14.6227	3.8619	15.7083	15.8229	4.0089	15.8229	15.7081	3.8619	14.6229
	h	0.42	1.23	0.72	0.40	1.41	0.71	0.41	1.31	0.59
	d_U	0.4675	0.5088	0.6110	0.6008	0.3514	0.6076	0.5929	0.4770	1.2344
	d_L	1.7287	0.7989	0.4425	1.2260	0.4601	0.4604	0.7594	0.4703	0.5269
0.75	n	2	3	4	3	5	4	3	4	2
	k_U	102.0000	101.8491	101.8429	100.9933	100.9975	101.0137	100.9938	100.9758	101.0044
	k_L	99.0000	99.0085	98.9962	99.0216	98.9920	99.0087	98.0549	98.0904	97.9945
	Cost	19.6238	3.7560	9.9926	21.2944	4.0023	10.1530	21.2161	3.9139	9.5370
	h	0.33	1.23	1.23	0.36	1.31	1.31	0.36	1.29	1.29
	d_U	0.9420	0.4719	0.4719	0.5796	0.4126	0.4126	0.5753	0.4324	0.4324
	d_L	6.4357	5.1151	5.1151	2.2578	3.5875	3.5875	3.2093	4.2934	4.2934
1.00	n	1	2	2	3	4	4	3	4	4
	k_U	102.0000	101.9910	101.9910	101.0018	101.0000	101.0000	101.0067	101.0000	101.0000
	k_L	99.0000	99.0219	99.0219	99.0586	99.0000	99.0000	98.0556	98.0000	98.0000
	Cost	24.4790	3.4840	3.4840	26.6099	3.9556	3.9556	26.6099	3.9563	3.9563

TABLE 4.7

OPTIMUM DECISION VARIABLES AND LOSS-COSTS FOR THE WEIBULL PROCESS FAILURE MECHANISM WITH SHAPE PARAMETER S=2, SCALE PARAMETER θ=100 AND INITIAL POINT AS FOLLOWS: DECISION INTERVAL-UPPER d_U AND LOWER d_L, TIME INTERVAL BETWEEN SUBGROUPS h=0.1, DEAD BAND-UPPER k_U AND LOWER k_L, SUBGROUP SIZE n=1:

		$d_U=4, d_L=2, k_U=102, k_L=99$			$d_U=d_L=2, k_U=101, k_L=99$			$d_U=2, d_L=4, k_U=101, k_L=98$		
		$\delta_U > \delta_L$ (4) (2)			$\delta_U = \delta_L$ (2) (2)			$\delta_U < \delta_L$ (2) (4)		
α		$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000	$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000	$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000
	h	1.24	1.24	0.34	1.21	1.21	0.34	1.13	1.13	0.31
	d_U	3.9248	3.9248	3.6906	3.7403	3.7403	2.6252	4.0213	4.0213	5.6487
	d_L	0.4102	0.4102	0.5803	0.4268	0.4268	0.5876	0.4329	0.4329	0.9453
0.00	n	4	4	3	4	4	3	2	2	1
	k_U	102.0581	102.0581	102.0000	101.0000	101.0000	101.0667	101.0284	101.0284	101.1000
	k_L	98.9961	98.9961	99.0000	99.0000	99.0000	99.0035	97.9826	97.9826	98.0000
	Cost	4.3459	4.3459	29.5913	4.3464	4.3464	29.5914	3.8412	3.8412	27.2751
	h	0.56	1.24	0.39	0.67	1.32	0.39	0.61	1.16	0.39
	d_U	0.5120	0.3332	0.7787	0.4617	0.4550	1.2198	0.6118	0.7855	1.7341
	d_L	1.2543	0.4492	0.5870	0.6345	0.3714	0.6000	0.4950	0.3541	0.4768
0.25	n	2	4	3	4	5	3	3	3	2
	k_U	101.9957	102.0266	101.8932	100.9908	101.0050	100.9832	101.0049	101.0098	101.0073
	k_L	99.0009	99.0005	99.0005	99.0192	99.0198	99.0102	98.0637	98.0248	98.0184
	Cost	10.5562	4.3017	23.5656	11.2297	4.3981	23.6468	11.0552	4.1319	21.8538
	h	0.46	1.13	0.46	0.52	1.33	0.52	0.47	1.14	0.45
	d_U	0.4784	0.4318	0.5162	0.4371	0.3988	0.7328	0.6006	0.6777	1.4834
	d_L	1.4966	0.6875	0.5981	0.7351	0.4094	0.4258	0.5416	0.3724	0.4916
0.50	n	2	3	3	4	5	4	3	3	2
	k_U	101.9955	101.9240	102.0000	100.9914	100.9951	101.0000	100.9925	101.0000	100.9891
	k_L	99.0173	98.9990	99.0000	98.9998	99.0187	99.0000	98.0151	98.0000	98.0206
	Cost	16.2491	4.2441	17.4187	17.5549	4.4051	17.5549	17.4187	4.2442	16.2490
	h	0.40	1.16	0.61	0.38	1.34	0.67	0.39	1.25	0.56
	d_U	0.4658	0.3446	0.4588	0.5962	0.3517	0.6280	0.5950	0.4464	1.2321
	d_L	1.7525	0.8014	0.6339	1.2742	0.4817	0.4677	0.6866	0.3066	0.5157
0.75	n	2	3	3	3	5	4	3	4	2
	k_U	101.9879	101.9743	101.9771	100.9964	100.9961	100.9917	100.9918	101.0000	101.0152
	k_L	99.0145	99.0089	99.0143	99.0467	99.0156	99.0172	98.0586	98.0000	98.0123
	Cost	21.8534	4.1319	11.0552	23.6471	4.3982	11.2298	23.5655	4.3017	10.5564
	h	0.31	1.14	1.14	0.34	1.24	1.24	0.34	1.24	1.24
	d_U	0.9453	0.4372	0.4372	0.5858	0.4280	0.4280	0.5803	0.4057	0.4057
	d_L	6.2307	3.9068	3.9068	2.5529	3.5632	3.5632	3.6906	3.9232	3.9232
1.00	n	1	2	2	3	4	4	3	4	4
	k_U	102.0000	102.0262	102.0262	100.9967	100.9871	100.9871	101.0000	101.0082	101.0082
	k_L	99.0000	99.0473	99.0473	99.0600	99.0562	99.0562	98.0000	98.0565	98.0565
	Cost	27.2751	3.8412	3.8412	29.5913	4.3459	4.3459	29.5913	4.3459	4.3459

TABLE 4.8

OPTIMUM DECISION VARIABLES AND LOSS-COSTS FOR THE WEIBULL PROCESS FAILURE MECHANISM WITH SHAPE PARAMETER S=1, SCALE PARAMETER θ=50 AND INITIAL POINT AS FOLLOWS: DECISION INTERVAL-UPPER d_U AND LOWER d_L, TIME INTERVAL BETWEEN SUBGROUPS h=0.1, DEAD BAND-UPPER k_U AND LOWER k_L, SUBGROUP SIZE n=1:

		$d_U=4, d_L=2, k_U=102, k_L=99$			$d_U=d_L=2, k_U=101, k_L=99$			$d_U=2, d_L=4, k_U=101, k_L=98$		
		$\delta_U > \delta_L$ (4) (2)			$\delta_U = \delta_L$ (2) (2)			$\delta_U < \delta_L$ (2) (4)		
α		$\mu_U > \mu_L$ 1000 100	$\mu_U = \mu_L$ 100 100	$\mu_U < \mu_L$ 100 1000	$\mu_U > \mu_L$ 1000 100	$\mu_U = \mu_L$ 100 100	$\mu_U < \mu_L$ 100 1000	$\mu_U > \mu_L$ 1000 100	$\mu_U = \mu_L$ 100 100	$\mu_U < \mu_L$ 100 1000
	h	0.95	0.95	0.26	0.95	0.95	0.26	0.88	0.88	0.24
	d_U	3.6791	3.6791	5.1222	2.6959	2.6959	2.8175	3.5393	3.5393	5.6492
	d_L	0.4188	0.4188	0.5798	0.4209	0.4209	0.5769	0.4387	0.4387	0.9370
0.00	n	4	4	3	4	4	3	2	2	1
	k_U	102.0257	102.0257	102.0260	101.0198	101.0198	101.0248	101.0241	101.0241	101.1000
	k_L	99.0064	99.0064	99.0020	99.0040	99.0040	98.9982	97.9758	97.9758	98.0000
	Cost	6.8420	6.8420	49.0532	6.8421	6.8421	49.0532	6.1450	6.1450	45.6028
	h	0.43	0.94	0.30	0.51	0.94	0.30	0.46	0.77	0.26
	d_U	0.5510	0.4908	1.5469	0.4616	0.5869	1.2079	0.6235	1.2349	3.4403
	d_L	1.2676	0.4540	0.5770	0.6292	0.4539	0.5939	0.6978	0.5491	0.9692
0.25	n	2	4	3	4	4	3	3	2	1
	k_U	101.9493	101.9075	101.1452	100.9907	101.0000	101.0104	100.9906	100.9878	100.9967
	k_L	99.0113	99.0054	98.9949	99.0099	99.0000	99.0089	98.2012	98.0191	98.0013
	Cost	17.1569	6.7852	38.8731	18.1935	6.9188	38.9634	17.9080	6.5421	36.3442
	h	0.35	0.87	0.35	0.35	0.93	0.35	0.36	0.87	0.35
	d_U	0.4796	0.4517	0.5680	0.5953	0.5106	1.0492	0.5851	0.7039	1.4869
	d_L	1.4896	0.6959	0.5890	1.0374	0.5110	0.5987	0.5786	0.4007	0.4825
0.50	n	2	3	3	3	4	3	3	3	2
	k_U	101.9854	101.9167	101.9993	100.9984	100.9926	100.9859	101.0067	100.9813	100.9985
	k_L	99.0012	99.0141	98.9963	99.0099	99.0029	98.9996	98.0536	98.0461	98.0133
	Cost	26.8545	6.6886	28.5330	28.7306	6.9338	28.7305	28.5329	6.6886	26.8545
	h	0.26	0.76	0.47	0.29	0.94	0.51	0.30	0.96	0.43
	d_U	0.9488	0.5254	0.7645	0.5918	0.4539	0.6221	0.4824	0.4286	1.2493
	d_L	3.4419	1.2440	0.6395	1.2184	0.5869	0.4605	2.6494	0.6159	0.5286
0.75	n	1	2	3	3	4	4	3	4	2
	k_U	102.0235	102.0020	101.6681	100.9946	101.0000	100.9936	101.0832	101.0122	100.9978
	k_L	99.0030	99.0183	99.0252	98.9989	99.0000	99.0141	99.3830	98.1202	98.0225
	Cost	36.3444	6.5423	17.9082	38.9634	6.9188	18.1933	38.8890	6.7854	17.1568
	h	0.24	0.87	0.87	0.26	0.95	0.95	0.26	0.96	0.96
	d_U	0.9490	0.4294	0.4294	0.5739	0.4201	0.4201	0.5837	0.4151	0.4151
	d_L	5.5102	3.5300	3.5300	2.8168	2.7750	2.7750	5.1236	3.7534	3.7534
1.00	n	1	2	2	3	4	4	3	4	4
	k_U	101.9854	102.0265	102.0265	101.0046	100.9944	100.9944	100.9929	101.0000	101.0000
	k_L	98.7947	99.0148	99.0148	99.0242	99.0219	99.0219	98.0273	98.0000	98.0000
	Cost	45.6028	6.1450	6.1450	49.0533	6.8421	6.8421	49.0533	6.8422	6.8422

TABLE 4.9

OPTIMUM DECISION VARIABLES AND LOSS-COSTS FOR THE WEIBULL PROCESS FAILURE MECHANISM WITH SHAPE PARAMETER S=2, SCALE PARAMETER θ=50 AND INITIAL POINT AS FOLLOWS: DECISION INTERVAL-UPPER d_U AND LOWER d_L, TIME INTERVAL BETWEEN SUBGROUPS h=0.1, DEAD BAND-UPPER k_U AND LOWER k_L, SUBGROUP SIZE n=1:

		$d_U=4, d_L=2, k_U=102, k_L=99$			$d_U=d_L=2, k_U=101, k_L=99$			$d_U=2, d_L=4, k_U=101, k_L=98$		
		$\delta_U > \delta_L$ (4) (2)			$\delta_U = \delta_L$ (2) (2)			$\delta_U < \delta_L$ (2) (4)		
		$M_U > M_L$	$M_U = M_L$	$M_U < M_L$	$M_U > M_L$	$M_U = M_L$	$M_U < M_L$	$M_U > M_L$	$M_U = M_L$	$M_U < M_L$
α		1000 100	100 100	100 1000	1000 100	100 100	100 1000	1000 100	100 100	100 1000
0.00	h	0.90	0.90	0.25	0.91	0.91	0.25	0.83	0.83	0.22
	d_U	3.7567	3.7567	4.1468	2.7168	2.7168	2.5665	4.6371	4.6371	5.6355
	d_L	0.4024	0.4024	0.5860	0.4123	0.4123	0.5808	0.4413	0.4413	0.9564
	n	4	4	3	4	4	3	2	2	1
	k_U	102.0290	102.0290	102.0223	101.0192	101.0192	101.0574	101.0270	101.0270	101.1222
	k_L	98.9893	98.9893	99.0074	99.0039	99.0039	99.0048	97.9786	97.9786	98.0205
	Cost	7.5323	7.5323	54.5689	7.5323	7.5323	54.5688	6.7872	6.7872	50.8164
0.25	h	0.41	0.91	0.28	0.49	0.87	0.28	0.44	0.72	0.25
	d_U	0.5091	0.4967	1.1555	0.4439	0.5870	1.2263	0.6283	1.2012	3.4225
	d_L	1.2459	0.4704	0.5817	0.6197	0.4543	0.5892	0.5730	0.5187	0.9591
	n	2	4	3	4	4	3	3	2	1
	k_U	102.0000	101.9568	101.5405	101.0000	101.0048	100.9990	100.9870	101.0106	101.0057
	k_L	99.0000	99.0217	99.0033	99.0000	98.9948	99.0071	98.1367	97.9933	97.9996
	Cost	19.0130	7.4726	43.2033	20.1476	7.6134	43.2942	19.8328	7.2064	40.4336
0.50	h	0.33	0.82	0.34	0.33	0.89	0.33	0.34	0.82	0.33
	d_U	0.4788	0.6829	0.6494	0.5955	0.4993	1.0315	0.5857	0.6771	1.5187
	d_L	1.5265	0.7075	0.5919	1.0315	0.5002	0.5955	0.5730	0.4821	0.4803
	n	2	3	3	3	4	3	3	3	2
	k_U	101.9998	101.6723	101.9237	101.0000	101.0000	101.0000	100.9977	101.0003	100.9851
	k_L	99.0141	99.0220	99.0057	99.0000	99.0000	99.0000	98.0411	98.1151	98.0099
	Cost	29.8561	7.3658	31.6689	31.8717	7.6290	31.8717	31.6689	7.3656	29.8558
0.75	h	0.24	0.73	0.44	0.28	0.89	0.49	0.28	0.92	0.41
	d_U	0.9608	0.5412	0.8308	0.5508	0.4535	0.6197	0.5852	0.4458	1.2492
	d_L	3.4336	1.2230	0.6230	1.6897	0.6031	0.4439	0.9255	0.6316	0.5111
	n	1	2	3	3	4	4	3	4	2
	k_U	102.0001	101.9668	101.6060	101.0343	100.9940	101.0000	100.9948	100.9949	101.0032
	k_L	98.9978	99.0121	99.0099	99.2136	99.0123	99.0000	98.2969	98.2051	98.0134
	Cost	40.4336	7.2062	19.8326	43.3037	7.6134	20.1476	43.2032	7.4724	19.0129
1.00	h	0.23	0.84	0.84	0.25	0.91	0.91	0.25	0.90	0.90
	d_U	0.9516	0.4352	0.4352	0.5822	0.4132	0.4132	0.5843	0.4027	0.4027
	d_L	5.7737	4.7101	4.7101	2.6684	2.7183	2.7183	4.1504	3.7530	3.7530
	n	1	2	2	3	4	4	3	4	4
	k_U	101.9902	102.0000	102.0000	100.9935	100.9955	100.9955	100.9922	101.0088	101.0088
	k_L	98.9231	99.0000	99.0000	99.0259	99.0207	99.0207	98.0259	98.0253	98.0253
	Cost	50.8165	6.7871	6.7871	54.5688	7.5323	7.5323	54.5689	7.5323	7.5323

very small possibility of a false alarm in the downward direction.

5. The upper dead band value (k_U) is in the vicinity of $\mu_0 + \frac{1}{2}\delta_U\sigma$. Similarly to the lower dead band value (k_U) is in the vicinity of $\mu_0 - \frac{1}{2}\delta_L\sigma$.

<u>Effect of Probability of Upward Shift, α</u>

Figure 4.1.a shows that there is no major change in loss-cost as factor α is varied, when the magnitude of a shift in the process mean is equal in either direction, $\delta_U = \delta_L$. However, the curve of $\delta_U = \delta_L$ shows that whenever $\alpha = 0.00$ or $\alpha = 1.00$ there is a slightly lower average loss-cost. This is because a two-sided asymmetric Cusum control chart becomes a pure one-sided Cusum control chart whenever $\alpha = 0.00$ or $\alpha = 1.00$. Only when $\alpha = 0.50$ is the two-sided asymmetric chart considered to be a two-sided symmetric chart. Yet, when α is at an extreme value of 0.00 or 1.00, the Cusum chart can be made more efficient for detecting an out of control condition. This leads to a slightly lower average loss-cost. When $\alpha = 0.50$, however, the Cusum chart must be able to detect an out of control condition in either direction, causing it to be slightly less efficient, resulting in a higher average loss-cost.

The condition where $\delta_U > \delta_L$ indicates a shift in the upward direction, if it occurs, will be larger and more easily detected than a downward shift. The average loss-cost with a small value of factor α is higher and the

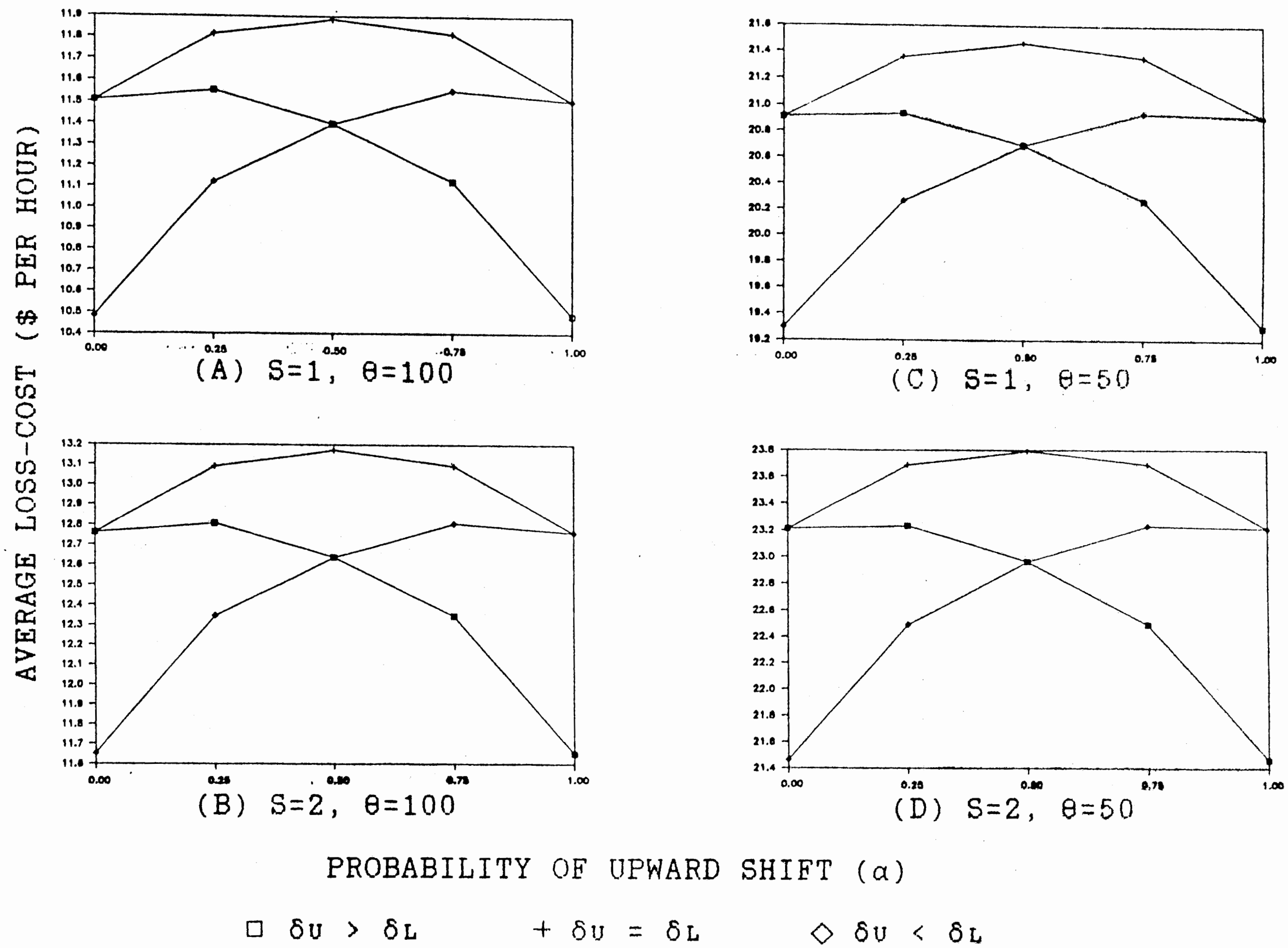


Figure 4.1. Average Loss-Cost Vs. Probability of Upward Shift (α) for Overall Factor M, Average Over All Combinations of Factor δ

average loss-cost with a high value of α is lower. When α is low, there will more likely be a downward shift in the process mean, which is less easily detected, resulting in a higher average loss-cost. On the contrary, when α is high, there is more likely an upward shift in the process mean which is more easily detected, resulting in a lower average loss-cost.

The condition where $\delta_U < \delta_L$ indicates a shift in the downward direction, if it occurs, will be larger and more easily detected than an upward shift. The average loss-cost with a small value of factor α is lower and the average loss-cost with high value of α is higher. When α is low, there will more likely be a downward shift in the process mean, which is more easily detected, resulting in a lower average loss-cost. On the contrary, when α is high, there is more likely an upward shift in the process mean, which is less easily detected, resulting in a higher average loss-cost.

Figure 4.2.a shows that there is virtually no change in average loss-cost as factor α is varied, when the magnitude of the diminution of hourly income is equal in either direction, $M_U = M_L$. This is because the proportion of time the process is out of control is the same regardless of the value of α, and there is no differential cost effect in either direction.

The condition where $M_U > M_L$ indicates that a shift in the upward direction, if it occurs, will be extremely

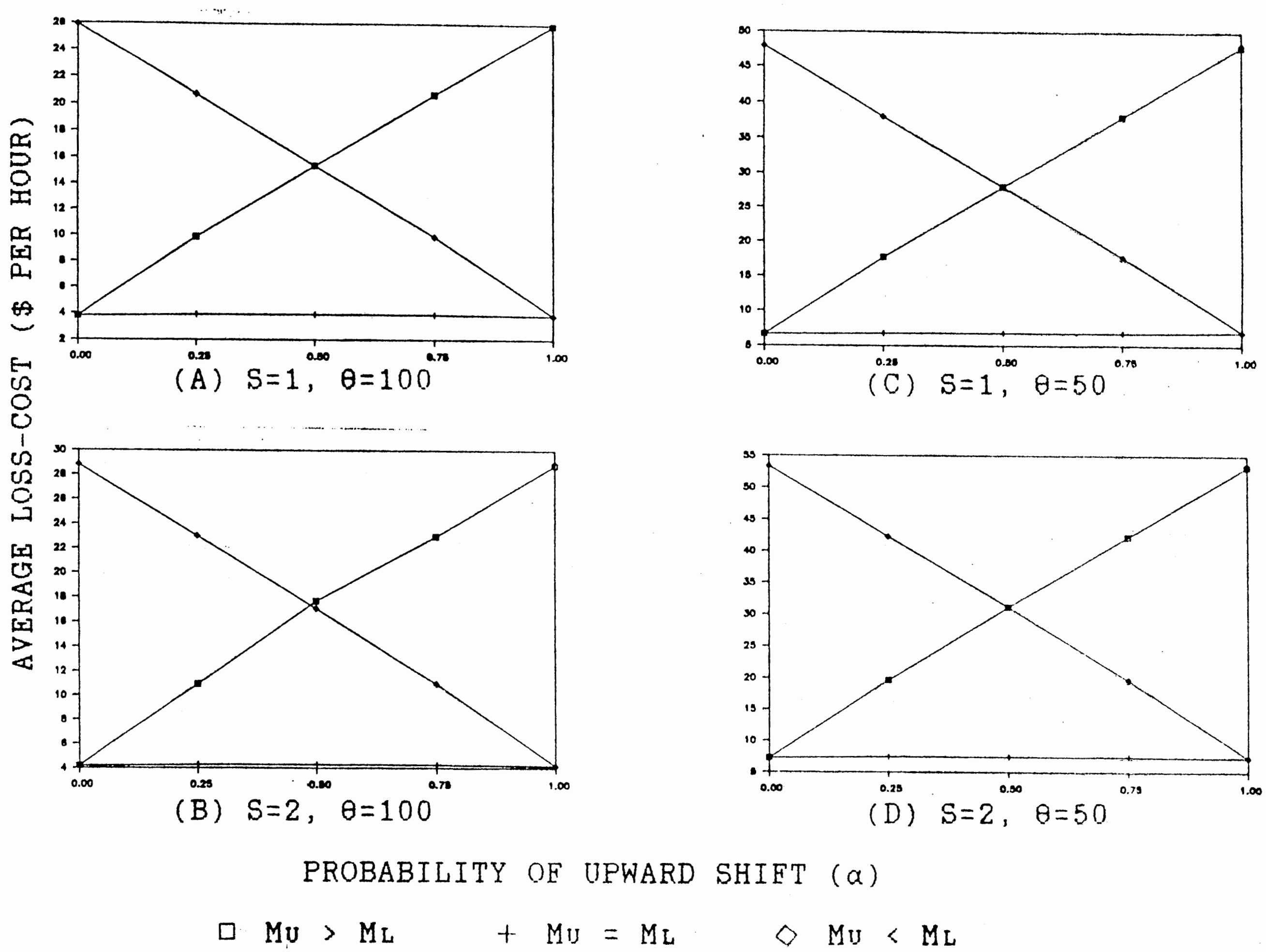


Figure 4.2. Average Loss-Cost Vs. Probability of Upward Shift (α) for Overall Factor δ, Average Over All Combinations of Factor M

costly, \$1000 per hour. On the contrary, a downward shift is not so costly, \$100 per hour, when it occurs. The average loss-cost with a low value of factor α is lower and the average loss-cost with a high value of α is higher. This is because when α is small, it is more likely a shift in the process mean will be in the downward direction, which and is not so costly.

The condition where $M_U < M_L$ indicates that a shift in the downward direction, if it occurs, will be extremely costly, \$1000 per hour. On the contrary, an upward shift is not so costly, \$100 per hour, when it occurs. The average loss-cost with a low value of factor α is higher and the average loss-cost with a high value of α is lower. This is because when α is small, it is more likely a shift in the process mean will be in the downward direction, which is extremely costly.

Effect of Risk Parameter, M

Figure 4.3.a shows that there is no major change in average loss-cost when the diminution of hourly income $M_U = M_L$, whether $\delta_U > \delta_L$, $\delta_U = \delta_L$ or $\delta_U < \delta_L$. When $\delta_U > \delta_L$, however, a shift in the upward direction is more easily detected and results in a slightly lower average loss-cost than that of $\delta_U = \delta_L$. Likewise, when $\delta_U < \delta_L$, a shift in the downward direction is more easily detected and results in a slightly lower average loss-cost than that of $\delta_U = \delta_L$.

The condition in which $M_U > M_L$ causes a strong upward

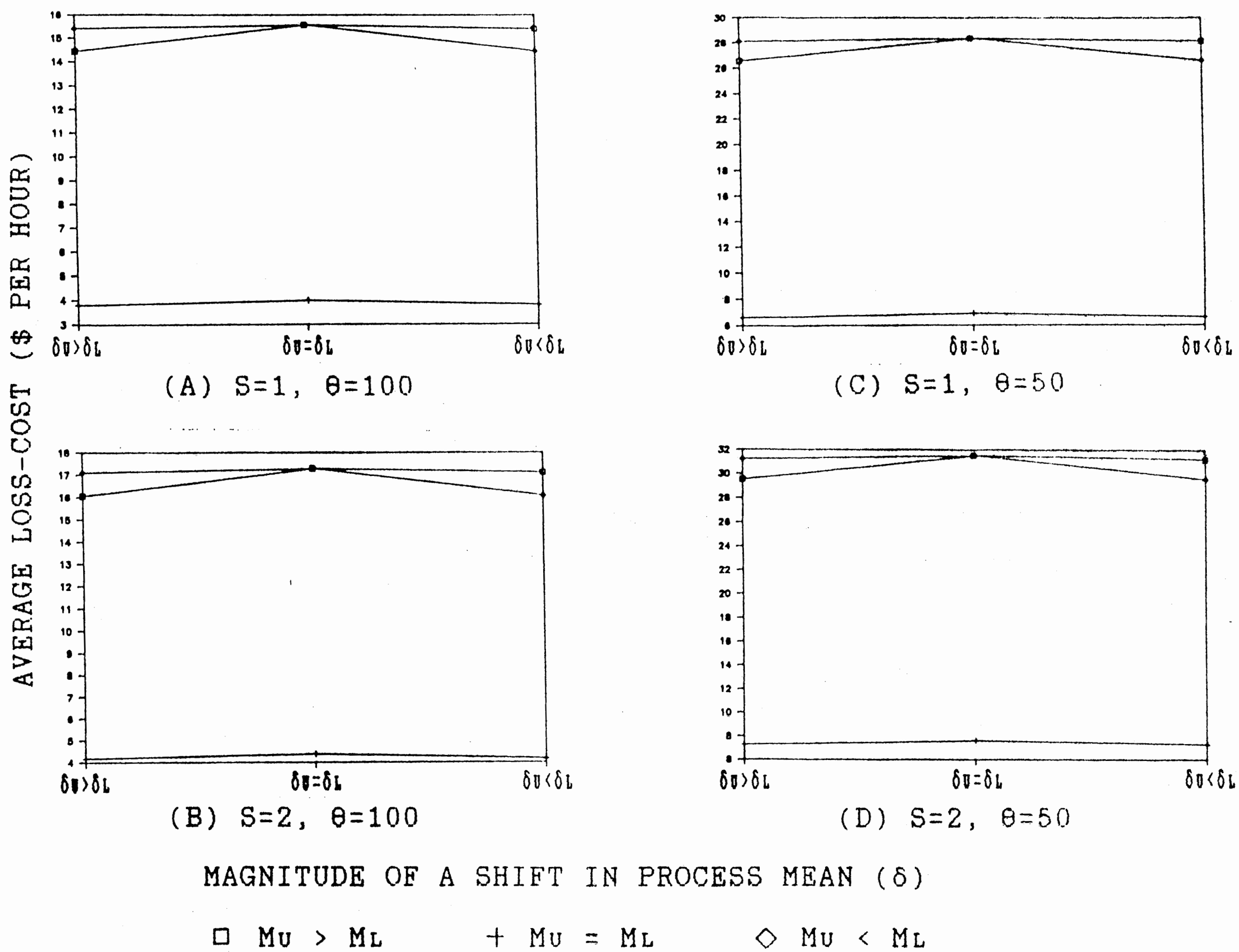


Figure 4.3. Average Loss-Cost Vs. Magnitude of a Shift in Process Mean (δ) for Overall Factor α, Average Over All Combinations of Factor M

shift in the average loss-cost, due primarily to the large increase of M_U to $1000 per hour. When $\delta_U > \delta_L$, the average loss-cost is lower than in the situation where $\delta_U < \delta_L$. This is because the magnitude of the upward shift makes it easier to detect. Likewise, when $M_U < M_L$, there is again a strong upward shift in the average loss-cost. When $\delta_U < \delta_L$, the average loss-cost is lower than in the situation where $\delta_U > \delta_L$. This is because the magnitude of the downward shift makes it easier to detect.

Effect of Weibull Shape Parameter, S

The shape parameter, S, governs the shape of the process failure distribution. When S = 1, the Weibull distribution reduces to an exponential distribution. From Figure 4.1.a to 4.1.b and 4.2.a to 4.2.b and 4.3.a to 4.3.b where the scale parameter $\theta = 100$, the shape parameter increases from 1 to 2. It can be seen that shapes of figures do not change, but the average loss-cost increases as S increases. Similarly, from Figure 4.1.c to 4.1.d and 4.2.c to 4.2.d and 4.3.c to 4.3.d, where the scale parameter $\theta = 50$, the observation above continues to hold.

In addition, from Table 4.6 to 4.7 where $\theta = 100$, in all cases the time interval between subgroups (h) decreases as S increases. Similarly, in Tables 4.8 and 4.9, where $\theta = 50$, the observation continues to hold.

From Table 4.10 to 4.11, again S increases from 1 to 2 while holding constant the scale parameter $\theta = 100$. It can be seen that in all cases: (a) the total proportion of time the process is out of control ($\Gamma_U + \Gamma_L$) increases as S increases and (b) the cycle time (T_{cycle}) decreases as S increases. Likewise, in Tables 4.12 and 4.13 for $\theta = 50$, observations (a) and (b) also hold.

Figures 4.4, 4.5 and 4.6 show the overall effect of α, δ and M, respectively, on the average loss-cost. Again the average loss-cost increases as S increases from 1 to 2.

Effect of Weibull Scale Parameter, θ

The scale parameter, θ, also has relevance to the change in the process failure mechanism. When the Weibull distribution reduces to an exponential distribution, the reciprocal of θ is equal to the average number of assignable causes per unit time. A decrease in θ is equivalent to an increase in the frequency of assignable causes.

From Figures 4.1.a to 4.1.c, 4.2.a to 4.2.c, 4.3.a to 4.3.c, where the shape parameter S = 1, the scale parameter decreases from 100 to 50. It can be seen that shapes of figures do not change, but the average loss-cost increases as θ decreases. Similarly, from Figures 4.1.b to 4.1.d and 4.2.b to 4.2.d and 4.3.b to 4.3.d, where the shape parameter S = 2, the observation above continues to hold.

In addition, from Tables 4.6 and 4.8 where S = 1, in all cases the time interval between subgroups (h) decreases as θ decreases. Similarly, in Tables 4.7 and 4.9, where S = 2, the same observation continues to hold.

TABLE 4.10

OPTIMUM VALUES OF Γ_U, Γ_L, ARL_0, ARL_1, h*ENSIN AND CYCLE TIME FOR THE WEIBULL PROCESS FAILURE MECHANISM WITH SHAPE PARAMETER S=1, SCALE PARAMETER θ=100 AND INITIAL POINT AS FOLLOWS: n=1, h=.1, d_U, d_L, k_U AND k_L:

		$d_U=4, d_L=2, k_U=102, k_L=99$			$d_U=d_L=2, k_U=101, k_L=99$			$d_U=2, d_L=4, k_U=101, k_L=98$		
		$\delta_U > \delta_L$ (4) (2)			$\delta_U = \delta_L$ (2) (2)			$\delta_U < \delta_L$ (2) (4)		
		$M_U > M_L$	$M_U = M_L$	$M_U < M_L$	$M_U > M_L$	$M_U = M_L$	$M_U < M_L$	$M_U > M_L$	$M_U = M_L$	$M_U < M_L$
α		1000 100	100 100	100 1000	1000 100	100 100	100 1000	1000 100	100 100	100 1000
	$ARLA_1(\delta_U)$	2.644	2.644	2.196	4.203	4.203	3.041	5.956	5.956	7.183
	Γ_U	0	0	0	0	0	0	0	0	0
	$ARLA_1(\delta_L)$	1.129	1.129	1.267	1.128	1.128	1.267	1.014	1.014	1.156
0.00	Γ_L	0.0294	0.0294	0.0237	0.0294	0.0294	0.0237	0.0264	0.0264	0.0222
	ARL_1	1.129	1.129	1.267	1.128	1.128	1.267	1.014	1.014	1.156
	HENSIN	99.35	99.35	99.82	99.35	99.35	99.82	99.41	99.41	99.84
	ARL_0	413.2	413.2	297.7	408.0	408.0	297.0	3720	3720	584.7
	T_{cycle}	103.0262	103.0262	102.4286	103.0260	103.0260	102.4291	102.7119	102.7119	102.2655
	$ARLA_1(\delta_U)$	1.017	1.003	1.015	1.147	1.122	1.817	1.143	1.423	2.401
	Γ_U	0.0059	0.0070	0.0058	0.0065	0.0076	0.0065	0.0065	0.0080	0.0071
	$ARLA_1(\delta_L)$	1.924	1.144	1.272	1.240	1.078	1.275	1.001	1.002	1.015
0.25	Γ_L	0.0215	0.0222	0.0181	0.0199	0.0223	0.0180	0.0187	0.0202	0.0170
	ARL_1	1.698	1.109	1.208	1.217	1.089	1.411	1.036	1.107	1.362
	HENSIN	99.70	99.34	99.80	99.65	99.30	99.80	99.64	99.39	99.79
	ARL_0	731.8	501.6	307.1	384.3	565.1	304.9	498.1	856.1	2346
	T_{cycle}	102.8118	103.0082	102.4403	102.7064	103.0794	102.5188	102.5862	102.8967	102.4621
	$ARLA_1(\delta_U)$	1.015	1.002	1.006	1.137	1.093	1.332	1.277	1.340	2.138
	Γ_U	0.0114	0.0134	0.0117	0.0124	0.0149	0.0129	0.0123	0.0153	0.0140
	$ARLA_1(\delta_L)$	2.139	1.340	1.281	1.337	1.091	1.138	1.005	1.002	1.016
0.50	Γ_L	0.0141	0.0153	0.0124	0.0129	0.0149	0.0124	0.0117	0.0134	0.0114
	ARL_1	1.577	1.171	1.144	1.237	1.092	1.235	1.141	1.171	1.577
	HENSIN	99.76	99.40	99.76	99.73	99.31	99.73	99.76	99.40	99.76
	ARL_0	1343	498.8	330.4	408.4	540.6	407.3	318.3	498.2	1360
	T_{cycle}	102.6161	102.9516	102.4651	102.6025	103.0739	102.6025	102.4632	102.9527	102.6137
	$ARLA_1(\delta_U)$	1.015	1.002	1.001	1.275	1.075	1.247	1.270	1.146	1.908
	Γ_U	0.0170	0.0202	0.0187	0.0180	0.0223	0.0199	0.0181	0.0222	0.0214
	$ARLA_1(\delta_L)$	2.413	1.422	1.144	1.818	1.123	1.146	1.012	1.001	1.019
0.75	Γ_L	0.0071	0.0080	0.0065	0.0065	0.0076	0.0065	0.0058	0.0069	0.0059
	ARL_1	1.365	1.107	1.037	1.411	1.087	1.221	1.206	1.110	1.686
	HENSIN	99.79	99.39	99.64	99.80	99.30	99.65	99.79	99.35	99.70
	ARL_0	2398	852.6	501.9	304.4	544.0	388.5	303.7	517.0	707.4
	T_{cycle}	102.4634	102.8993	102.5869	102.5189	103.0806	102.7091	102.4410	103.0001	102.8047
	$ARLA_1(\delta_U)$	1.156	1.015	1.015	1.267	1.128	1.128	1.268	1.137	1.137
	Γ_U	0.0222	0.0266	0.0266	0.0237	0.0294	0.0294	0.0237	0.0294	0.0294
	$ARLA_1(\delta_L)$	7.183	5.689	5.689	2.769	4.203	4.203	2.113	2.685	2.685
1.00	Γ_L	0	0	0	0	0	0	0	0	0
	ARL_1	1.156	1.015	1.015	1.267	1.128	1.128	1.268	1.137	1.137
	HENSIN	99.84	99.39	99.39	99.82	99.35	99.35	99.82	99.36	99.36
	ARL_0	584.7	4019	4019	297.5	408.0	408.0	299.1	460.4	460.4
	T_{cycle}	102.2655	102.7328	102.7328	102.4288	103.0260	103.0260	102.4290	103.0242	103.0242

TABLE 4.11

OPTIMUM VALUES OF Γ_U, Γ_L, ARL_0, ARL_1, h*ENSIN AND CYCLE TIME FOR THE WEIBULL PROCESS FAILURE MECHANISM WITH SHAPE PARAMETER S=2, SCALE PARAMETER θ=100 AND INITIAL POINT AS FOLLOWS: n=1, h=.1, d_U, d_L, k_U AND k_L:

		$d_U=4, d_L=2, k_U=102, k_L=99$			$d_U=d_L=2, k_U=101, k_L=99$			$d_U=2, d_L=4, k_U=101, k_L=98$		
		$\delta_U > \delta_L$ (4) (2)			$\delta_U = \delta_L$ (2) (2)			$\delta_U < \delta_L$ (2) (4)		
		$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000	$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000	$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000
α										
0.00	$ARLA_1(\delta_U)$	2.537	2.537	2.361	4.356	4.356	3.472	4.834	4.834	7.028
	Γ_U	0	0	0	0	0	0	0	0	0
	$ARLA_1(\delta_L)$	1.129	1.129	1.266	1.135	1.135	1.269	1.014	1.014	1.157
	Γ_L	0.0325	0.0325	0.0265	0.0324	0.0324	0.0265	0.0293	0.0293	0.0248
	ARL_1	1.129	1.129	1.266	1.135	1.135	1.269	1.014	1.014	1.157
	HENSIN	88.00	88.00	88.45	88.02	88.02	88.45	88.06	88.06	88.47
	ARL_0	412.1	412.1	295.4	444.9	444.9	300.5	3764	3764	590.7
	Tcycle	91.6027	91.6027	91.0349	91.5928	91.5928	91.0359	91.3019	91.3019	90.8778
0.25	$ARLA_1(\delta_U)$	1.018	1.001	1.011	1.147	1.119	1.818	1.291	1.429	2.433
	Γ_U	0.0065	0.0077	0.0064	0.0072	0.0084	0.0073	0.0072	0.0088	0.0078
	$ARLA_1(\delta_L)$	1.921	1.145	1.270	1.241	1.076	1.272	1.003	1.002	1.015
	Γ_L	0.0238	0.0246	0.0202	0.0221	0.0246	0.0201	0.0202	0.0224	0.0190
	ARL_1	1.695	1.109	1.206	1.217	1.087	1.408	1.075	1.109	1.369
	HENSIN	88.34	88.00	88.43	88.29	87.96	88.43	88.32	88.05	88.43
	ARL_0	724.9	508.5	304.4	383.9	539.5	297.6	353.8	885.8	2405
	Tcycle	91.3915	91.5782	91.0460	91.3022	91.6461	91.1231	91.1222	91.4759	91.0628
0.50	$ARLA_1(\delta_U)$	1.016	1.002	1.005	1.135	1.091	1.330	1.274	1.334	2.139
	Γ_U	0.0128	0.0148	0.0131	0.0139	0.0166	0.0144	0.0138	0.0170	0.0156
	$ARLA_1(\delta_L)$	2.142	1.342	1.278	1.332	1.090	1.134	1.005	1.002	1.015
	Γ_L	0.0156	0.0170	0.0138	0.0144	0.0166	0.0139	0.0131	0.0149	0.0128
	ARL_1	1.579	1.172	1.142	1.233	1.091	1.232	1.140	1.168	1.577
	HENSIN	88.40	88.06	88.39	88.36	87.96	88.36	88.39	88.05	88.40
	ARL_0	1357	506.1	322.2	396.3	525.0	390.6	313.0	479.0	1341
	Tcycle	91.2140	91.5337	91.0701	91.2024	91.6575	91.2030	91.0720	91.5365	91.2124
0.75	$ARLA_1(\delta_U)$	1.014	1.002	1.003	1.274	1.075	1.244	1.270	1.144	1.922
	Γ_U	0.0190	0.0224	0.0202	0.0201	0.0247	0.0222	0.0202	0.0246	0.0238
	$ARLA_1(\delta_L)$	2.409	1.424	1.291	1.832	1.122	1.145	1.009	1.000	1.017
	Γ_L	0.0079	0.0088	0.0072	0.0073	0.0084	0.0072	0.0064	0.0077	0.0065
	ARL_1	1.363	1.107	1.075	1.414	1.087	1.219	1.205	1.108	1.696
	HENSIN	88.42	88.04	88.32	88.43	87.95	88.29	88.43	88.00	88.34
	ARL_0	2282	856.6	354.7	303.3	537.0	383.2	303.4	501.1	725.7
	Tcycle	91.0661	91.4755	91.1223	91.1223	91.6611	91.3063	91.0480	91.5854	91.3942
1.00	$ARLA_1(\delta_U)$	1.157	1.015	1.015	1.268	1.129	1.129	1.266	1.129	1.129
	Γ_U	0.0248	0.0294	0.0294	0.0265	0.0325	0.0325	0.0265	0.0325	0.0325
	$ARLA_1(\delta_L)$	6.978	4.409	4.409	3.040	3.980	3.980	2.361	2.399	2.399
	Γ_L	0	0	0	0	0	0	0	0	0
	ARL_1	1.157	1.015	1.015	1.268	1.129	1.129	1.266	1.129	1.129
	HENSIN	88.47	88.05	88.05	88.45	88.00	88.00	88.45	88.00	88.00
	ARL_0	590.7	4033	4033	298.2	412.1	412.1	295.4	412.1	412.1
	Tcycle	90.8778	91.3079	91.3079	91.0357	91.6000	91.6000	91.0349	91.6020	91.6020

TABLE 4.12

OPTIMUM VALUES OF Γ_U, Γ_L, ARL_0, ARL_1, h*ENSIN AND CYCLE TIME FOR THE WEIBULL PROCESS FAILURE MECHANISM WITH SHAPE PARAMETER S=1, SCALE PARAMETER θ=50 AND INITIAL POINT AS FOLLOWS: n=1, h=.1, d_U, d_L, k_U AND k_L:

		d_U=4, d_L=2, k_U=102, k_L=99			d_U=d_L=2, k_U=101, k_L=99			d_U=2, d_L=4, k_U=101, k_L=98		
		$\delta_U > \delta_L$ (4) (2)			$\delta_U = \delta_L$ (2) (2)			$\delta_U < \delta_L$ (2) (4)		
α		$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000	$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000	$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000
	$ARLA_1(\delta_U)$	2.356	2.356	3.145	3.369	3.369	3.538	4.320	4.320	7.028
	Γ_U	0	0	0	0	0	0	0	0	0
	$ARLA_1(\delta_L)$	1.128	1.128	1.265	1.130	1.130	1.266	1.015	1.015	1.155
0.00	Γ_L	0.0529	0.0529	0.0449	0.0530	0.0530	0.0449	0.0486	0.0486	0.0423
	ARL_1	1.128	1.128	1.265	1.130	1.130	1.266	1.015	1.015	1.155
	HENSIN	49.53	49.53	49.87	49.53	49.53	49.87	49.56	49.56	49.88
	ARL_0	406.6	406.6	291.6	418.1	418.1	293.5	4023	4023	575.7
	Tcycle	52.7951	52.7951	52.3508	52.8003	52.8003	52.3512	52.5520	52.5520	52.2063
	$ARLA_1(\delta_U)$	1.017	1.001	1.012	1.146	1.223	1.846	1.288	1.885	4.177
	Γ_U	0.0110	0.0127	0.0110	0.0120	0.0136	0.0122	0.0120	0.0150	0.0143
	$ARLA_1(\delta_L)$	1.919	1.145	1.268	1.244	1.147	1.269	1.005	1.019	1.163
0.25	Γ_L	0.0386	0.0399	0.0341	0.0368	0.0399	0.0340	0.0341	0.0356	0.0318
	ARL_1	1.693	1.109	1.204	1.219	1.166	1.413	1.076	1.235	1.916
	HENSIN	49.79	49.53	49.85	49.75	49.53	49.85	49.77	49.62	49.87
	ARL_0	715.1	507.3	298.8	386.8	368.5	292.6	346.9	650.1	557.7
	Tcycle	52.6132	52.7760	52.3606	52.5675	52.8262	52.4205	52.4179	52.6665	52.4161
	$ARLA_1(\delta_U)$	1.015	1.002	1.007	1.275	1.173	1.647	1.275	1.338	2.159
	Γ_U	0.0217	0.0245	0.0222	0.0231	0.0268	0.0243	0.0232	0.0273	0.0256
	$ARLA_1(\delta_L)$	2.162	1.336	1.275	1.641	1.176	1.279	1.005	1.002	1.015
0.50	Γ_L	0.0255	0.0273	0.0231	0.0243	0.0268	0.0230	0.0222	0.0245	0.0217
	ARL_1	1.589	1.169	1.141	1.458	1.174	1.463	1.140	1.170	1.587
	HENSIN	49.83	49.57	49.82	49.83	49.53	49.83	49.82	49.57	49.82
	ARL_0	1409	483.9	314.5	287.5	363.0	296.0	313.5	489.8	1408
	Tcycle	52.4811	52.7308	52.3768	52.4855	52.8305	52.4828	52.3774	52.7311	52.4830
	$ARLA_1(\delta_U)$	1.164	1.019	1.003	1.270	1.147	1.241	1.263	1.142	1.914
	Γ_U	0.0318	0.0355	0.0342	0.0340	0.0399	0.0368	0.0341	0.0401	0.0386
	$ARLA_1(\delta_L)$	4.179	1.885	1.287	1.843	1.223	1.143	1.102	1.001	1.017
0.75	Γ_L	0.0143	0.0149	0.0120	0.0121	0.0136	0.0120	0.0111	0.0127	0.0110
	ARL_1	1.918	1.235	1.074	1.413	1.166	1.217	1.223	1.106	1.690
	HENSIN	49.87	49.62	49.76	49.85	49.53	49.74	49.85	49.52	49.78
	ARL_0	567.1	649.4	342.8	294.9	368.5	373.8	283.9	486.8	706.8
	Tcycle	52.4154	52.6567	52.4211	52.4185	52.8262	52.5687	52.3699	52.7865	52.6137
	$ARLA_1(\delta_U)$	1.154	1.015	1.015	1.266	1.129	1.129	1.264	1.129	1.129
	Γ_U	0.0423	0.0485	0.0485	0.0449	0.0530	0.0530	0.0449	0.0531	0.0531
	$ARLA_1(\delta_L)$	7.673	4.164	4.164	3.389	3.327	3.327	3.079	2.368	2.368
1.00	Γ_L	0	0	0	0	0	0	0	0	0
	ARL_1	1.154	1.015	1.015	1.266	1.129	1.129	1.264	1.129	1.129
	HENSIN	49.88	49.57	49.57	49.87	49.53	49.53	49.87	49.52	49.52
	ARL_0	569.4	3877	3877	293.6	411.8	411.8	289.1	414.1	414.1
	Tcycle	52.2066	52.5495	52.5495	52.3507	52.8007	52.8007	52.3517	52.8045	52.8045

TABLE 4.13

OPTIMUM VALUES OF Γ_U, Γ_L, ARL_0, ARL_1, h*ENSIN AND CYCLE TIME FOR THE WEIBULL PROCESS FAILURE MECHANISM WITH SHAPE PARAMETER S=2, SCALE PARAMETER θ=50 AND INITIAL POINT AS FOLLOWS: n=1, h=.1, d_U, d_L, k_U AND k_L:

		$d_U=4, d_L=2, k_U=102, k_L=99$			$d_U=d_L=2, k_U=101, k_L=99$			$d_U=2, d_L=4, k_U=101, k_L=98$		
		$\delta_U > \delta_L$ (4) (2)			$\delta_U = \delta_L$ (2) (2)			$\delta_U < \delta_L$ (2) (4)		
α		$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000	$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000	$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000
	$ARLA_1(\delta_U)$	2.403	2.403	2.630	3.389	3.389	3.379	5.460	5.460	7.171
	Γ_U	0	0	0	0	0	0	0	0	0
	$ARLA_1(\delta_L)$	1.129	1.129	1.265	1.126	1.126	1.263	1.015	1.015	1.154
0.00	Γ_L	0.0588	0.0588	0.0502	0.0588	0.0588	0.0502	0.0540	0.0540	0.0472
	ARL_1	1.129	1.129	1.265	1.126	1.126	1.263	1.015	1.015	1.154
	HENSIN	43.86	43.86	44.19	43.86	43.86	44.19	43.90	43.90	44.20
	ARL_0	410.5	410.5	291.7	397.1	397.1	288.5	4016	4016	571.5
	T_{cycle}	47.0795	47.0795	46.6511	47.0821	47.0821	46.6513	46.8398	46.8398	46.5083
	$ARLA_1(\delta_U)$	1.018	1.001	1.012	1.143	1.226	1.848	1.289	1.884	4.191
	Γ_U	0.0123	0.0141	0.0123	0.0134	0.0150	0.0135	0.0134	0.0165	0.0158
	$ARLA_1(\delta_L)$	1.914	1.144	1.265	1.245	1.150	1.267	1.003	1.019	1.160
0.25	Γ_L	0.0429	0.0444	0.0380	0.0410	0.0441	0.0380	0.0381	0.0395	0.0355
	ARL_1	1.690	1.109	1.202	1.219	1.169	1.412	1.075	1.235	1.918
	HENSIN	44.11	43.86	44.17	44.07	43.87	44.17	44.09	43.95	44.19
	ARL_0	708.3	504.8	292.0	373.9	382.3	288.9	347.4	647.6	546.7
	T_{cycle}	46.8994	47.0643	46.6606	46.8630	47.0966	46.7184	46.7126	46.9421	46.7089
	$ARLA_1(\delta_U)$	1.016	1.002	1.007	1.276	1.171	1.648	1.268	1.334	2.169
	Γ_U	0.0243	0.0272	0.0249	0.0257	0.0297	0.0270	0.0258	0.0301	0.0284
	$ARLA_1(\delta_L)$	2.178	1.338	1.270	1.648	1.171	1.276	1.006	1.002	1.015
0.50	Γ_L	0.0284	0.0301	0.0258	0.0270	0.0297	0.0257	0.0249	0.0272	0.0243
	ARL_1	1.597	1.170	1.138	1.462	1.171	1.462	1.137	1.168	1.592
	HENSIN	44.14	43.90	44.14	44.15	43.87	44.15	44.14	43.90	44.15
	ARL_0	1503	489.4	302.8	291.2	350.4	291.2	299.3	478.2	1446
	T_{cycle}	46.7764	47.0076	46.6768	46.7758	47.1060	46.7758	46.6781	47.0092	46.7740
	$ARLA_1(\delta_U)$	1.161	1.018	1.003	1.272	1.144	1.245	1.266	1.141	1.922
	Γ_U	0.0355	0.0396	0.0381	0.0379	0.0442	0.0410	0.0380	0.0444	0.0429
	$ARLA_1(\delta_L)$	4.190	1.873	1.287	2.001	1.225	1.143	1.009	1.001	1.017
0.75	Γ_L	0.0158	0.0165	0.0134	0.0137	0.0151	0.0134	0.0123	0.0141	0.0123
	ARL_1	1.918	1.231	1.074	1.454	1.164	1.219	1.202	1.106	1.695
	HENSIN	44.19	43.95	44.09	44.17	43.86	44.07	44.17	43.85	44.11
	ARL_0	548.7	615.8	344.4	302.3	360.0	373.9	294.0	483.4	721.2
	T_{cycle}	46.7088	46.9445	46.7164	46.7246	47.1058	46.8630	46.6600	47.0670	46.8993
	$ARLA_1(\delta_U)$	1.156	1.014	1.014	1.263	1.126	1.126	1.263	1.128	1.128
	Γ_U	0.0473	0.0540	0.0540	0.0502	0.0589	0.0589	0.0502	0.0587	0.0587
	$ARLA_1(\delta_L)$	7.006	5.399	5.399	3.239	3.274	3.274	2.575	2.340	2.340
1.00	Γ_L	0	0	0	0	0	0	0	0	0
	ARL_1	1.156	1.014	1.014	1.263	1.126	1.126	1.263	1.128	1.128
	HENSIN	44.20	43.89	43.89	44.19	43.85	43.85	44.19	43.86	43.86
	ARL_0	583.1	3475	3475	287.8	397.9	397.9	288.7	406.2	406.2
	T_{cycle}	46.5090	46.8431	46.8431	46.6515	47.0832	47.0832	46.6513	47.0757	47.0757

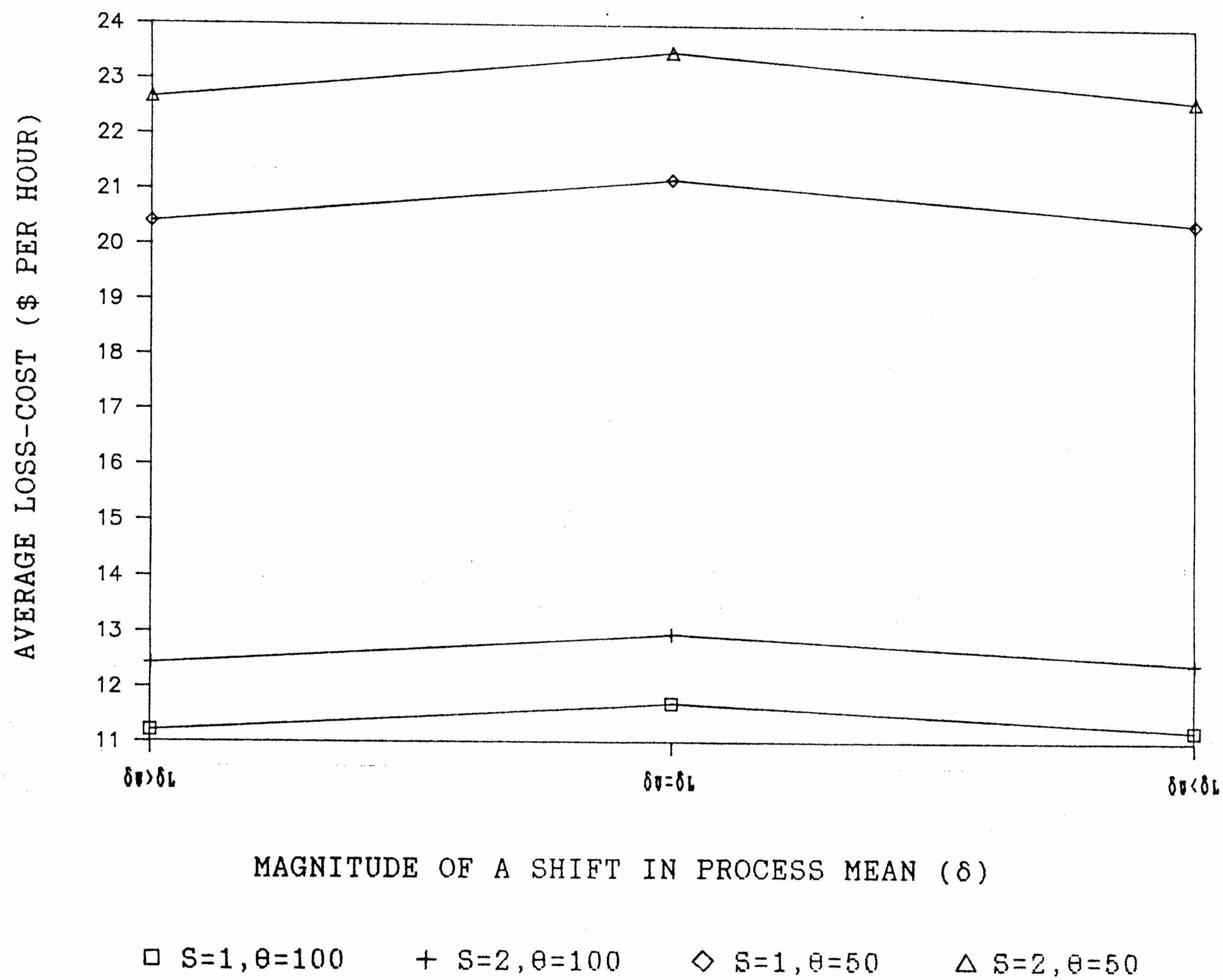


Figure 4.4. Average Loss-Cost Vs. Magnitude of a Shift in Process Mean (δ) for Overall Factors M and α

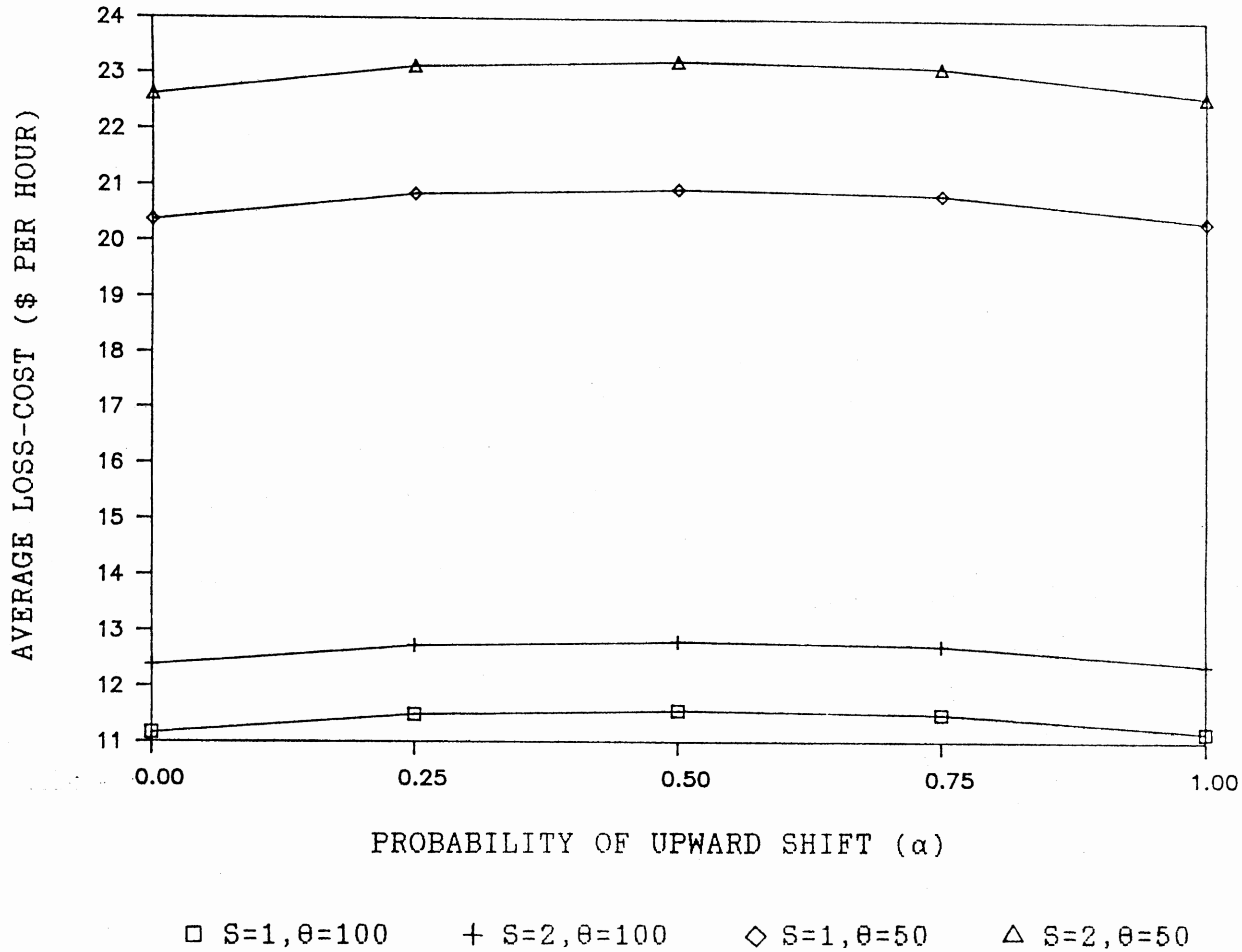


Figure 4.5. Average Loss-Cost Vs. Probability of Upward Shift (α) for Overall Factors δ and M

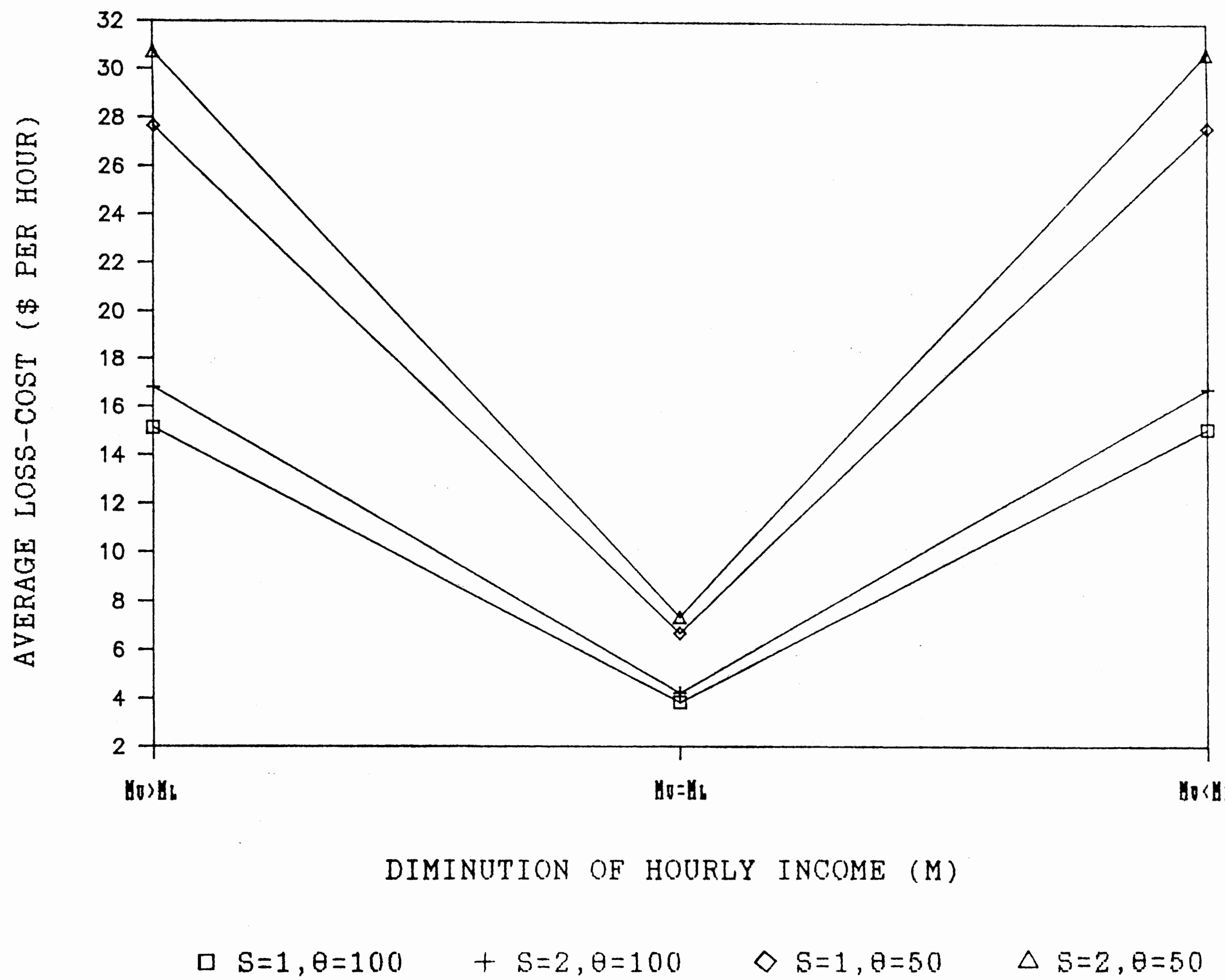


Figure 4.6. Average Loss-Cost Vs. Diminution of Hourly Income (M) for Overall Factors δ and α

From Table 4.10 to 4.12, again θ decreases from 100 to 50 while holding constant the shape parameter S = 1. It can be seen that in all cases: (a) the total proportion of time the process is out of control ($\Gamma_U + \Gamma_L$) increases as θ decreases, and (b) the cycle time (T_{cycle}) decreases to about half as θ decreases by a 2:1 ratio. Likewise, in Tables 4.11 and 4.13 for S = 2, observations (a) and (b) also hold.

Figures 4.4, 4.5 and 4.6 show the overall effect of α, δ and M, respectively, on the average loss-cost. Again the average loss-cost increases as θ decreases from 100 to 50. Furthermore, from these figures, it can be seen that the scale parameter has more effect on the variation in average loss-cost than does the shape parameter. Also, Tables 4.10, 4.11, 4.12 and 4.13 show that the scale parameter has more effect on the variation in cycle time than does the shape parameter.

Effect of Shift Parameter, δ

The shift parameter δ specifies the degree of change in the process mean, $\delta_U\sigma$ or $\delta_L\sigma$, which a Cusum chart is designed to detect. Table 4.6 is chosen as representative for investigating its effect on n, h and loss-cost. Table 4.14 is a summary of selected data from Table 4.6 where $M_U > M_L$. It can be seen that in all cases subgroup sizes and loss-costs for $\delta_U = \delta_L$ are no smaller than those for $\delta_U > \delta_L$. Likewise, the optimum time intervals between subgroups for $\delta_U = \delta_L$ is no smaller than those for $\delta_U > \delta_L$, with one exception which is probably due to the imperfection of the search algorithm. In other words, as the shift to be detected increases, small subgroup sizes should be taken more often, and less expense is expected.

TABLE 4.14

VALUES OF SUBGROUP SIZE, TIME INTERVAL BETWEEN SUBGROUPS, DECISION INTERVALS AND LOSS-COST FOR $M_U > M_L$

		$\delta_U > \delta_L$					$\delta_U = \delta_L$			
			(4)	(2)				(2)	(2)	
α	n	h	d_U	d_L	Cost	n	h	d_U	d_L	Cost
0.00	4	1.31	4.1125	0.4270	3.9556	4	1.31	3.5875	0.4126	3.9556
0.25	2	0.59	0.4893	1.2523	9.5369	4	0.71	0.4673	0.6343	10.1529
0.50	2	0.48	0.4893	1.4695	14.6227	4	0.55	0.4263	0.7591	15.8229
0.75	2	0.42	0.4675	1.7287	19.6238	3	0.40	0.6008	1.2260	21.2944
1.00	1	0.33	0.9420	6.4357	24.4790	3	0.36	0.5796	2.2578	26.6099

<u>Effect of Initial Point for Search Procedure</u>

Results which are listed in Tables 4.15-4.18 are obtained by the optimization methods described in Chapter III with a significantly different initial point from that discussed in the earlier presentation on asymmetric design. It is noted that results in Table 4.15 are very close to

TABLE 4.15

OPTIMUM DECISION VARIABLES AND LOSS-COSTS FOR THE WEIBULL PROCESS FAILURE MECHANISM WITH SHAPE PARAMETER S=1, SCALE PARAMETER θ=100 AND INITIAL POINT AS FOLLOWS: DECISION INTERVAL-UPPER d_U AND LOWER d_L, TIME INTERVAL BETWEEN SUBGROUPS h=3.0, DEAD BAND-UPPER k_U AND LOWER k_L, SUBGROUP SIZE n=10:

		$d_U=0.4, d_L=0.2, k_U=102, k_L=99$			$d_U=d_L=0.2, k_U=101, k_L=99$			$d_U=0.2, d_L=0.4, k_U=101, k_L=98$		
		$\delta_U > \delta_L$ (4) (2)			$\delta_U = \delta_L$ (2) (2)			$\delta_U < \delta_L$ (2) (4)		
α		$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000	$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000	$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000
	h	1.31	1.31	0.36	1.31	1.31	0.36	1.21	1.21	0.33
	d_U	1.9345	1.9345	2.1013	1.7515	1.7515	2.5293	2.7384	2.7384	6.3187
	d_L	0.4162	0.4162	0.5858	0.4162	0.4162	0.5858	0.4418	0.4418	0.9570
0.00	n	4	4	3	4	4	3	2	2	1
	k_U	102.0187	102.0187	102.0237	101.0187	101.0187	101.0237	101.1000	101.1000	101.0193
	k_L	98.9966	98.9966	99.0041	98.9966	98.9966	99.0041	98.0000	98.0000	98.0148
	Cost	3.9557	3.9557	26.6099	3.9557	3.9557	26.6099	3.4838	3.4838	24.4790
	h	0.59	1.33	0.42	0.71	1.39	0.40	0.73	1.23	0.42
	d_U	0.5246	0.3641	0.5848	0.4708	0.4603	1.2192	0.4430	0.7788	1.7568
	d_L	1.2724	0.4462	0.5919	0.6339	0.3521	0.5887	0.4320	0.3620	0.4824
0.25	n	2	4	3	4	5	3	4	3	2
	k_U	101.9873	102.0000	102.0204	100.9816	101.0000	100.9821	101.0042	101.0084	100.9818
	k_L	99.0152	99.0000	99.0052	99.0185	99.0000	98.9979	98.0527	98.0187	98.0253
	Cost	9.5369	3.9139	21.2161	10.1529	4.0023	21.2943	9.9925	3.7559	19.6237
	h	0.48	1.19	0.49	0.55	1.40	0.55	0.49	1.19	0.48
	d_U	0.4797	0.4834	0.5137	0.4399	0.3893	0.7525	0.6025	0.6959	1.4481
	d_L	1.4745	0.6933	0.6007	0.7600	0.3895	0.4345	0.5355	0.4522	0.5040
0.50	n	2	3	3	4	5	4	3	3	2
	k_U	101.9916	101.9052	102.0205	100.9898	101.0023	100.9941	100.9949	100.9922	101.0064
	k_L	99.0043	99.0021	99.0045	99.0179	98.9988	99.0056	98.0223	98.0946	98.0100
	Cost	14.6227	3.8620	15.7081	15.8230	4.0088	15.8229	15.7081	3.8619	14.6229
	h	0.42	1.23	0.73	0.41	1.39	0.71	0.41	1.31	0.59
	d_U	0.4779	0.3700	0.4909	0.5905	0.3521	0.6394	0.5928	0.4581	1.2600
	d_L	1.7463	0.7870	0.4429	1.1931	0.4603	0.4659	0.6131	0.3941	0.5533
0.75	n	2	3	4	3	5	4	3	4	2
	k_U	101.9887	101.9935	101.9436	101.0013	101.0000	100.9750	100.9944	100.9925	100.9903
	k_L	99.0114	98.9970	99.0010	99.0028	99.0000	99.0121	98.0056	98.0300	98.0475
	Cost	19.6240	3.7560	9.9925	21.2943	4.0023	10.1529	21.2160	3.9138	9.5368
	h	0.33	1.20	1.20	0.36	1.31	1.31	0.36	1.31	1.31
	d_U	0.9451	0.4399	0.4399	0.5825	0.4109	0.4109	0.5825	0.4109	0.4109
	d_L	6.3994	2.7864	2.7864	2.6056	1.7284	1.7284	2.1775	1.9114	1.9114
1.00	n	1	2	2	3	4	4	3	4	4
	k_U	102.0000	102.0219	102.0219	101.0000	101.0040	101.0040	101.0000	101.0040	101.0040
	k_L	99.0000	98.9167	98.9167	99.0000	99.0136	99.0136	98.0000	98.0136	98.0136
	Cost	24.4792	3.4839	3.4839	26.6099	3.9556	3.9556	26.6099	3.9556	3.9556

TABLE 4.16

OPTIMUM DECISION VARIABLES AND LOSS-COSTS FOR THE WEIBULL PROCESS FAILURE MECHANISM WITH SHAPE PARAMETER S=2, SCALE PARAMETER θ=100 AND INITIAL POINT AS FOLLOWS: DECISION INTERVAL-UPPER d_U AND LOWER d_L, TIME INTERVAL BETWEEN SUBGROUPS h=3.0, DEAD BAND-UPPER k_U AND LOWER k_L, SUBGROUP SIZE n=10:

		d_U=0.4, d_L=0.2, k_U=102, k_L=99			d_U=d_L=0.2, k_U=101, k_L=99			d_U=0.2, d_L=0.4, k_U=101, k_L=98		
		$\delta_U > \delta_L$ (4) (2)			$\delta_U = \delta_L$ (2) (2)			$\delta_U < \delta_L$ (2) (4)		
α		$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000	$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000	$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000
0.00	h	1.24	1.24	0.34	1.24	1.24	0.34	1.14	1.14	0.31
	d_U	2.1988	2.1988	2.7718	1.9988	1.9988	2.5718	2.7384	2.7384	5.6501
	d_L	0.4232	0.4232	0.5820	0.4232	0.4232	0.5820	0.4442	0.4442	0.9471
	n	4	4	3	4	4	3	2	2	1
	k_U	102.0596	102.0596	102.0257	101.0596	101.0596	101.0257	101.1000	101.1000	101.1000
	k_L	99.0096	99.0096	99.0004	99.0096	99.0096	99.0004	98.0000	98.0000	98.0000
	Cost	4.3459	4.3459	29.5913	4.3459	4.3459	29.5913	3.8411	3.8411	27.2754
0.25	h	0.56	1.26	0.39	0.67	1.34	0.39	0.61	1.13	0.40
	d_U	0.5174	0.4341	0.6336	0.4637	0.4700	1.1833	0.6150	0.8107	1.7419
	d_L	1.2498	0.4458	0.5887	0.6323	0.3578	0.5966	0.4394	0.4289	0.4639
	n	2	4	3	4	5	3	3	3	2
	k_U	102.0000	102.0000	102.0211	100.9925	100.9887	101.0145	101.0000	100.9909	101.0015
	k_L	99.0000	99.0000	98.9989	99.0122	99.0084	99.0071	98.0000	98.1058	98.0062
	Cost	10.5563	4.3018	23.5658	11.2299	4.3981	23.6468	11.0551	4.1322	21.8535
0.50	h	0.46	1.13	0.47	0.52	1.32	0.52	0.46	1.13	0.46
	d_U	0.4583	0.3566	0.5976	0.4290	0.4112	0.7456	0.5968	0.6793	1.4371
	d_L	1.4500	0.6836	0.5964	0.7456	0.4013	0.4290	0.6207	0.3950	0.4632
	n	2	3	3	4	5	4	3	3	2
	k_U	102.0095	102.0004	101.9914	101.0000	100.9812	101.0000	101.0000	101.0018	101.0194
	k_L	98.9946	98.9986	98.9977	99.0000	99.9956	99.0000	98.1000	98.0026	97.9943
	Cost	16.2490	4.2440	17.4191	17.5551	4.4054	17.5551	17.4189	4.2441	16.2491
0.75	h	0.40	1.16	0.61	0.38	1.32	0.66	0.39	1.24	0.56
	d_U	0.4610	0.3847	0.4394	0.5913	0.3509	0.6176	0.5894	0.4501	1.2563
	d_L	1.7140	0.8196	0.6150	1.1852	0.4665	0.4526	0.6334	0.3597	0.5280
	n	2	3	3	3	5	4	3	4	2
	k_U	101.9933	101.9642	102.0000	101.0003	101.0000	100.9988	100.9976	100.9995	100.9935
	k_L	98.9991	99.0239	99.0000	98.9964	99.0000	99.0019	98.0210	98.0589	98.0233
	Cost	21.8534	4.1320	11.0551	23.6467	4.3982	11.2297	23.5656	4.3017	10.5561
1.00	h	0.31	1.15	1.15	0.34	1.24	1.24	0.34	1.24	1.24
	d_U	0.9648	0.4471	0.4471	0.5824	0.4255	0.4255	0.5824	0.4255	0.4255
	d_L	5.9974	2.7648	2.7648	2.6461	2.1049	2.1049	2.8461	2.3049	2.3049
	n	1	2	2	3	4	4	3	4	4
	k_U	101.9765	102.0213	102.0213	101.0000	100.9893	100.9893	101.0000	100.9893	100.9893
	k_L	98.9621	98.9090	98.9090	99.0000	99.0323	99.0323	98.0000	98.0323	98.0323
	Cost	27.2751	3.8413	3.8413	29.5913	4.3459	4.3459	29.5913	4.3459	4.3459

TABLE 4.17

OPTIMUM DECISION VARIABLES AND LOSS-COSTS FOR THE WEIBULL PROCESS FAILURE MECHANISM WITH SHAPE PARAMETER S=1, SCALE PARAMETER θ=50 AND INITIAL POINT AS FOLLOWS: DECISION INTERVAL-UPPER d_U AND LOWER d_L, TIME INTERVAL BETWEEN SUBGROUPS h=3.0, DEAD BAND-UPPER k_U AND LOWER k_L, SUBGROUP SIZE n=10:

		$d_U=0.4, d_L=0.2, k_U=102, k_L=99$			$d_U=d_L=0.2, k_U=101, k_L=99$			$d_U=0.2, d_L=0.4, k_U=101, k_L=98$		
		$\delta_U > \delta_L$ (4) (2)			$\delta_U = \delta_L$ (2) (2)			$\delta_U < \delta_L$ (2) (4)		
α		$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000	$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000	$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000
	h	0.96	0.96	0.26	0.96	0.96	0.26	0.87	0.87	0.24
	d_U	2.4522	2.4522	2.0882	2.2522	2.2522	2.1213	3.1544	3.1544	5.9036
	d_L	0.4143	0.4143	0.5794	0.4143	0.4143	0.5877	0.4568	0.4568	0.9262
0.00	n	4	4	3	4	4	3	2	2	1
	k_U	102.0194	102.0194	102.0000	101.0194	101.0194	101.0558	101.0000	101.0000	101.0582
	k_L	99.0034	99.0034	99.0000	99.0034	99.0034	99.0086	98.0000	98.0000	97.9926
	Cost	6.8420	6.8420	49.0533	6.8420	6.8420	49.0533	6.1450	6.1450	45.6028
	h	0.43	0.95	0.30	0.51	0.93	0.29	0.47	0.78	0.26
	d_U	0.5032	0.4023	0.6535	0.4428	0.6013	1.2193	0.6295	1.2098	3.4776
	d_L	1.2614	0.4504	0.5962	0.6189	0.4691	0.6155	0.6064	0.5071	0.9731
0.25	n	2	4	3	4	4	3	3	2	1
	k_U	101.9977	102.0000	101.9934	101.0044	100.9890	101.0033	100.9820	101.0000	101.0056
	k_L	99.0086	99.0000	99.0121	99.0018	99.0150	99.0232	98.1746	98.0000	98.0048
	Cost	17.1569	6.7852	38.8729	18.1933	6.9187	38.9641	17.9080	6.5421	36.3447
	h	0.35	0.87	0.36	0.35	0.92	0.35	0.36	0.87	0.35
	d_U	0.5031	0.4963	0.4641	0.5974	0.5109	1.0264	0.5860	0.7254	1.4932
	d_L	1.4947	0.7155	0.5858	1.0318	0.5245	0.5979	0.5719	0.4645	0.4876
0.50	n	2	3	3	3	4	3	3	3	2
	k_U	101.9631	101.9154	102.0585	100.9993	100.9950	101.0000	101.0014	100.9613	100.9926
	k_L	98.9981	99.0304	98.9970	99.0055	99.0188	99.0000	98.0101	98.0720	98.0232
	Cost	26.8546	6.6890	28.5328	28.7305	6.9337	28.7304	28.5328	6.6891	26.8545
	h	0.26	0.77	0.47	0.30	0.93	0.51	0.29	0.95	0.43
	d_U	0.9595	0.5207	0.7187	0.5760	0.4678	0.6169	0.5875	0.4504	1.2665
	d_L	3.4828	1.2309	0.6348	1.3903	0.5960	0.4566	0.7257	0.4023	0.5131
0.75	n	1	2	3	3	4	4	3	4	2
	k_U	102.0072	102.0031	101.7546	101.0055	100.9910	100.9982	101.0000	101.0000	100.9897
	k_L	99.0138	99.0149	99.0177	99.0949	99.0064	99.0105	98.1000	98.1000	98.0130
	Cost	36.3444	6.5420	17.9080	38.9648	6.9188	18.1933	38.8735	6.7851	17.1569
	h	0.24	0.87	0.87	0.26	0.96	0.96	0.26	0.96	0.96
	d_U	0.9314	0.4568	0.4568	0.5842	0.4154	0.4154	0.5794	0.4154	0.4154
	d_L	6.2294	3.1544	3.1544	2.1960	2.2536	2.2536	2.0882	2.4536	2.4536
1.00	n	1	2	2	3	4	4	3	4	4
	k_U	102.0000	102.0000	102.0000	100.9933	100.9955	100.9955	101.0000	100.9955	100.9955
	k_L	99.0000	99.0000	99.0000	98.9642	99.0207	99.0207	98.0000	98.0207	98.0207
	Cost	45.6028	6.1450	6.1450	49.0534	6.8420	6.8420	49.0533	6.8420	6.8420

TABLE 4.18

OPTIMUM DECISION VARIABLES AND LOSS-COSTS FOR THE WEIBULL PROCESS FAILURE MECHANISM WITH SHAPE PARAMETER S=2, SCALE PARAMETER θ=50 AND INITIAL POINT AS FOLLOWS: DECISION INTERVAL-UPPER d_U AND LOWER d_L, TIME INTERVAL BETWEEN SUBGROUPS h=3.0, DEAD BAND-UPPER k_U AND LOWER k_L, SUBGROUP SIZE n=10:

		$d_U=0.4, d_L=0.2, k_U=102, k_L=99$			$d_U=d_L=0.2, k_U=101, k_L=99$			$d_U=0.2, d_L=0.4, k_U=101, k_L=98$		
		$\delta_U > \delta_L$ (4) (2)			$\delta_U = \delta_L$ (2) (2)			$\delta_U < \delta_L$ (2) (4)		
α		$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000	$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000	$M_U > M_L$ 1000 100	$M_U = M_L$ 100 100	$M_U < M_L$ 100 1000
0.00	h	0.91	0.91	0.25	0.91	0.91	0.25	0.82	0.82	0.23
	d_U	2.0541	2.0541	2.6609	1.9945	1.9945	2.9275	2.6262	2.6262	4.8534
	d_L	0.4222	0.4222	0.5800	0.4222	0.4222	0.5827	0.4410	0.4410	0.9429
	n	4	4	3	4	4	3	2	2	1
	k_U	102.0207	102.0207	102.0575	101.0207	101.0207	101.0573	101.1400	101.1400	101.3826
	k_L	99.0139	99.0139	99.0047	99.0139	99.0139	99.0053	97.9971	97.9971	98.0110
	Cost	7.5323	7.5323	54.5688	7.5323	7.5323	54.5689	6.7871	6.7871	50.8163
0.25	h	0.41	0.91	0.29	0.48	0.88	0.28	0.44	0.72	0.24
	d_U	0.5469	0.5815	1.2824	0.4441	0.6035	1.2287	0.6168	1.2128	3.4461
	d_L	1.2621	0.4422	0.5869	0.6180	0.4592	0.5843	0.4947	0.5369	0.9703
	n	2	4	3	4	4	3	3	2	1
	k_U	101.9521	101.9251	101.4416	101.0000	100.9928	100.9972	100.9946	101.0000	101.0027
	k_L	99.0066	98.9954	99.0059	99.0000	99.0076	99.0040	98.0604	98.0000	98.0085
	Cost	19.0129	7.4728	43.2039	20.1475	7.6133	43.2942	19.8325	7.2064	40.4336
0.50	h	0.33	0.82	0.34	0.33	0.88	0.33	0.33	0.82	0.33
	d_U	0.4838	0.4306	0.9276	0.5966	0.5273	1.0517	0.5798	0.6728	1.4749
	d_L	1.5016	0.6799	0.5883	1.0334	0.5050	0.5961	0.6876	0.3774	0.4930
	n	2	3	3	3	4	3	3	3	2
	k_U	101.9810	101.9236	101.5994	101.0000	100.9769	100.9852	101.0091	101.0109	101.0060
	k_L	98.9999	98.9990	99.0013	99.0000	99.0038	98.9982	98.1610	98.0237	98.0311
	Cost	29.8558	7.3656	31.6688	31.8717	7.6291	31.8718	31.6690	7.3657	29.8557
0.75	h	0.25	0.73	0.44	0.28	0.89	0.48	0.28	0.91	0.41
	d_U	0.9614	0.5389	0.5394	0.6597	0.4561	0.6180	0.5848	0.4397	1.2485
	d_L	3.4373	1.2196	0.6109	1.4484	0.5950	0.4441	0.9697	0.4468	0.5316
	n	1	2	3	3	4	4	3	4	2
	k_U	102.0000	101.9676	101.8961	100.9404	100.9953	101.0000	100.9990	101.0055	100.9991
	k_L	99.0000	99.0117	99.0010	99.1154	99.0094	99.0000	98.3130	98.0571	98.0350
	Cost	40.4337	7.2062	19.8325	43.3009	7.6134	20.1475	43.2034	7.4723	19.0129
1.00	h	0.23	0.82	0.82	0.25	0.90	0.90	0.25	0.90	0.90
	d_U	0.9519	0.4317	0.4317	0.5777	0.4201	0.4201	0.5803	0.4201	0.4201
	d_L	5.0082	3.0551	3.0551	3.0306	2.0030	2.0030	2.7624	2.0627	2.0627
	n	1	2	2	3	4	4	3	4	4
	k_U	101.9822	102.0093	102.0093	100.9985	100.9913	100.9913	100.9983	100.9913	100.9913
	k_L	98.7450	99.0012	99.00112	99.0271	99.0292	99.0292	98.0257	98.0292	98.0292
	Cost	50.8163	6.7870	6.7870	54.5688	7.5323	7.5323	54.5689	7.5323	7.5323

those in Table 4.6. A similar statement applies to Tables 4.16 and 4.7, Tables 4.17 and 4.8 and Tables 4.18 and 4.9. This lends confidence that the asymmetric economically-based design and search procedure are valid. As mentioned previously, due to the flatness of some loss-cost functions, there are several combinations of time interval between subgroups (h), decision intervals (d_U and d_L), and dead band values (k_U and k_L) which yield close to the same loss-cost.

Loss-costs listed in Tables 4.6 to 4.9 and 4.15 to 4.18 are outcomes when the process is the steady state. A simulation technique might be applied to obtain the variation of the loss-cost over a particular duration during which the process is operated. Performing this analysis is beyond the scope of this research.

Summary

The economically-based asymmetric Cusum model and the optimization procedure are analyzed and validated using two approaches: (1) evaluate symmetric Cusum examples with known solutions using the asymmetric model and compare solutions with Goel's data sets, (2) perform a $3^2 5^1$ factorial design using asymmetric examples and the asymmetric model to obtain near-optimal results, and (3) again perform the optimization of (2) using different initial points for the search.

CHAPTER V

USING THE INTERACTIVE COMPUTER PROGRAM

Introduction

This chapter demonstrates the use of an interactive computer program which allows utilization of the design and evaluation methodology presented in previous chapters. The actual FORTRAN program is documented and appears in the Appendix. It has been performed on an IBM 3081D using various time share terminals and an IBM PC.

The user is prompted for all necessary inputs by the computer. The entire program is interactive and values of all the parameters are presented to the user for verification. Only when a set of inputs has been confirmed does the program continue.

When several values are to be entered, a space or a comma is used to separate them. Integer numbers should be entered without decimal points. If a decimal point is included, an error message is issued and the user is prompted to reenter values. The input mechanism is virtually self-explanatory, as long as the user understands the terms being input and their mathematically feasible range.

In the remainder of this chapter, actual interactive outputs are interspersed with comments and explanations.

All computer outputs illustrated are generated automatically by the computer except for the terminal inputs which follow a question mark (?).

The interactive computer program provides the capability to do two activities: (1) design an economically-based asymmetric Cusum control chart and (2) evaluate a user-defined Cusum control chart. The program begins by prompting option menu (M.1). The selection of "1" indicates the design of an economically-based asymmetric Cusum control chart is to be performed.

```
*************************
*      MAIN  MENU       *
*************************

WHAT WOULD YOU LIKE TO DO ?
  1. DESIGN AN ECONOMICALLY-BASED CUSUM CONTROL CHART
  2. EVALUATE A CUSUM CONTROL CHART
  3. EXIT.

ENTER THE OPTION NUMBER PLEASE!
?
1
```

(M.1)

Design of an Economically-Based Asymmetric Cusum Control Chart

After the economically-based chart design is chosen, input of the following values are sequentially prompted by the program:

(1) The process parameters,

(2) The cost and time factors,

(3) The initial point for the search procedure,

(4) The criteria and step sizes for optimization of n, h, d_U and d_L,

(5) The criteria and step sizes for optimization of h, d_U and d_L,

(6) The criteria and step sizes for optimization of h, d_U, d_L, k_U and k_L,

(7) The step size for varing incrementally the values of d_U and d_L,

(8) The step size for varing incrementally the values of k_U and k_L.

The program prints these input data each time for verification by the user. Only after the user confirms the validity of the input does the program continue.

```
  PLEASE ENTER PROCESS PARAMETERS, INPUT VALUES OF:
    SHAPE, SCALE, SIGMA, ALPHA, TARGET, DELTA(UP), DELTA(LOW)

?
2.0,100.0,1.0,0.25,100.0,2.0,4.0

  THE FOLLOWING VALUES HAVE BEEN INPUTTED:
    SHAPE     =  2.00     SCALE     = 100.00     SIGMA    =  1.00
    ALPHA     =  0.25     TARGET    = 100.00
    DELTA(UP)=  2.00      DELTA(LOW)=   4.00

  ARE THESE DATA RIGHT?
  PLEASE ENTER 1 FOR YES, 2 FOR NO.

?
1

  PLEASE ENTER COST AND TIME FACTORS, INPUT VALUES OF:
    B, C, D, E, T, W, MU, ML

?
0.5,0.1,2.0,0.05,50.0,25.0,100.0,100.0

  THE FOLLOWING VALUES HAVE BEEN INPUTTED:
    B=  0.50      C=  0.10      D =   2.00      E =   0.05
    T=  50.00     W=  25.00     MU= 100.00      ML= 100.00
```

```
ARE THESE DATA RIGHT?
PLEASE ENTER 1 FOR YES, 2 FOR NO.

?
1

THE FOLLOWING INITIAL POINT IS SUGGESTED:
  SUBGROUP SIZE N        =  10       SAMPLING INTERVAL H      :   3.00
  DECISION INTERVAL(UP) DU=   0.2000   DECISION INTERVAL(LOW) DL=   0.4000
  DEAD BAND VALUE(UP) KU  = 101.0000   DEAD BAND VALUE(LOW) KL  =  98.0000

DO YOU ACCEPT THIS POINT?
PLEASE ENTER 1 FOR YES, 2 FOR NO.

?
1

THE FOLLOWING VALUES ARE SUGGESTED FOR OPTIMIZATION:
  TERMINATION LIMIT=   0.100D-03
  MAX. EVALUATIONS = 200
  STEP FOR N = 1.000     STEP FOR H = 0.200
  STEP FOR DU= 0.200     STEP FOR DL= 0.200

DO YOU ACCEPT THIS SUGGESTION?
PLEASE ENTER 1 FOR YES, 2 FOR NO.

?
1
```

The Nelder and Mead direct search method is performed after the criteria and step sizes for n, h, d_U and d_L have been verified. The optimal point values and their associated hourly loss-cost are printed.

```
** OPTIMIZATION IS PROCESSING **

**************************************************
AFTER OPTIMIZATION THE DESIGN IS
  N=  2.46     DU= 1.0751     KU=101.0000
  H=  1.02     DL= 0.5247     KL= 98.0000
  LOSS-COST=    4.1325
**************************************************
```

Thereafter, the subgroup size is automatically truncated to an integer and the intermediate values of n, d_U, d_L, k_U and k_L are used. The next phase of the optimization is then run after the criteria and step sizes for h, d_U and d_L have been inputted and verified. The search for an integer n, the optimal decision variable values and their associated hourly loss-cost are then printed.

```
THE FOLLOWING VALUES ARE SUGGESTED:
  TERMINATION LIMIT=   0.100D-05
  MAX. EVALUATIONS = 300
  STEP FOR H = 0.150     STEP FOR DU= 0.150     STEP FOR DL= 0.150

DO YOU ACCEPT THIS SUGGESTION?
PLEASE ENTER 1 FOR YES, 2 FOR NO.

?
1

              ***  OPTIMIZATION ITERATION  ***

    N     H      DU       DL        KU        KL        LOSS-COST

    2.   1.01   1.1943   0.5077  101.0000   98.0000    4.1476

    1.   0.75   2.4269   1.1677  101.0000   98.0000    4.4586

    3.   1.19   0.7889   0.4371  101.0000   98.0000    4.1332

    4.   1.27   0.5635   0.3789  101.0000   98.0000    4.1879

**************************************************
 AFTER OPTIMIZATION THE DESIGN IS
   N=  3.00     DU= 0.7889     KU=101.0000
   H=  1.19     DL= 0.4371     KL= 98.0000
   LOSS-COST=    4.1332
**************************************************
```

The direct search is again applied, automatically using a fixed subgroup size n and the new intermediate values of h, d_U, d_L, k_U and k_L as an initial point for another iteration. Again, new criteria and step sizes must be inputted and verified.

```
  THE FOLLOWING VALUES ARE SUGGESTED:
    TERMINATION LIMIT=   0.100D-06
    MAX. EVALUATIONS = 300
    STEP FOR H = 0.100
    STEP FOR DU= 0.100      STEP FOR DL= 0.100
    STEP FOR KU= 0.100      STEP FOR KL= 0.100

  DO YOU ACCEPT THIS SUGGESTION?
  PLEASE ENTER 1 FOR YES, 2 FOR NO.

?
1

************************************************
  AFTER OPTIMIZATION THE DESIGN IS
   N=  3.00     DU= 0.8027     KU=100.9889
   H=  1.13     DL= 0.4029     KL= 98.1078
   LOSS-COST=    4.1323
************************************************
```

Finally, incrementally varying the value of d_U and d_L as well as k_U and k_L brings about the optimal or near-optimal design of an economically-based asymmetric Cusum control scheme.

```
   STEP=  0.0020 IS SUGGESTED FOR INCREMENTALLY VARYING DU AND DL.

  DO YOU ACCEPT IT?   PLEASE ENTER 1 FOR YES, 2 FOR NO.

?
1

************************************************
  AFTER VARYING DU AND DL THE DESIGN IS
   N=  3.       DU= 0.8107     KU= 100.9889
   H=  1.13     DL= 0.4289     KL=  98.1078
   LOSS-COST=    4.1322
************************************************
```

```
   STEP=  0.0020 IS SUGGESTED FOR INCREMENTALLY VARYING KU AND KL.

  DO YOU ACCEPT IT?   PLEASE ENTER 1 FOR YES, 2 FOR NO.

?
1

************************************************
   AFTER VARYING KU AND KL THE DESIGN IS
   N=  3.        DU= 0.8107     KU= 100.9909
   H=  1.13      DL= 0.4289     KL=  98.1058
   LOSS-COST=    4.1322
************************************************

************************************************************************
         THE ECONOMICALLY-BASED CUSUM CHART IS EVALUATED AS:
SUBGROUP SIZE N          =  3.       SAMPLING INTERVAL H      =  1.13 HRS
DECISION INTERVAL(UP) DU=   0.8107  DECISION INTERVAL(LOW) DL=  0.4289
DEAD BAND VALUE(UP) KU  = 100.9909  DEAD BAND VALUE(LOW) KL  = 98.1058
  GAMMA(U)= 0.0088      ARL1=       1.11       ENSIN      =  74.09
  GAMMA(L)= 0.0223      ARL0=     898.63       CYCLE TIME=  91.46 HRS
  GAMMA(0)= 0.9690      THE HOURLY LOSS-COST IS $    4.1322
************************************************************************
```

Evaluation of A Cusum Control Chart

A selection of "2" from menu (M.1) leads to the evaluation of a specified Cusum control chart. The interactive procedure and the input data follow the first three steps in designing an economically-based asymmetric Cusum control chart. The format of the resulting listing is very similar to that of economically-based design.

```
          ************************
          *      MAIN MENU       *
          ************************

  WHAT WOULD YOU LIKE TO DO ?
    1. DESIGN AN ECONOMICALLY-BASED CUSUM CONTROL CHART
    2. EVALUATE A CUSUM CONTROL CHART
    3. EXIT.

  ENTER THE OPTION NUMBER PLEASE!
?
2
```

```
   PLEASE ENTER PROCESS PARAMETERS, INPUT VALUES OF:
     SHAPE, SCALE, SIGMA, ALPHA, TARGET, DELTA(UP), DELTA(LOW)

 ?
 2.0,100.0,1.0,0.25,100.0,100.0,2.0

   THE FOLLOWING VALUES HAVE BEEN INPUTTED:
     SHAPE     =  2.00      SCALE      = 100.00      SIGMA     =  1.00
     ALPHA     =  0.25      TARGET     = 100.00
     DELTA(UP)=100.00       DELTA(LOW)=   2.00

   ARE THESE DATA RIGHT?
   PLEASE ENTER 1 FOR YES, 2 FOR NO.

 ?
 2

   PLEASE ENTER PROCESS PARAMETERS, INPUT VALUES OF:
     SHAPE, SCALE, SIGMA, ALPHA, TARGET, DELTA(UP), DELTA(LOW)

 ?
 1.0,100.0,1.0,0.25,100.0,2.0,2.0

   THE FOLLOWING VALUES HAVE BEEN INPUTTED:
     SHAPE     =  1.00      SCALE      = 100.00      SIGMA     =  1.00
     ALPHA     =  0.25      TARGET     = 100.00
     DELTA(UP)=  2.00       DELTA(LOW)=   2.00

   ARE THESE DATA RIGHT?
   PLEASE ENTER 1 FOR YES, 2 FOR NO.

 ?
 1

   PLEASE ENTER COST AND TIME FACTORS, INPUT VALUES OF:
     B, C, D, E, T, W, MU, ML

 ?
 0.5,0.1,2.0,0.05,50.0,25.0,100.0,100.0

   THE FOLLOWING VALUES HAVE BEEN INPUTTED:
     B=   0.50      C=   0.10      D =   2.00      E =   0.05
     T=  50.00      W=  25.00      MU= 100.00      ML= 100.00

   ARE THESE DATA RIGHT?
   PLEASE ENTER 1 FOR YES, 2 FOR NO.

 ?
 1

   PLEASE ENTER INITIAL POINT, INPUT VALUES OF:
     N, H, DU, DL, KU, KL

 ?
 5,1.40,0.4821,0.3587,100.9844,99.0012
```

```
  THE FOLLOWING VALUES HAVE BEEN INPUTTED:
    SUBGROUP SIZE N          =  5        SAMPLING INTERVAL H       :  1.40
    DECISION INTERVAL(UP) DU=   0.4821   DECISION INTERVAL(LOW) DL=   0.3587
    DEAD BAND VALUE(UP) KU  = 100.9844   DEAD BAND VALUE(LOW) KL  :  99.0012

  ARE THESE DATA RIGHT?
  PLEASE ENTER 1 FOR YES, 2 FOR NO.

?
1

*************************************************************************
                    THE CUSUM CHART IS EVALUATED AS:
SUBGROUP SIZE N          =  5.      SAMPLING INTERVAL H       =  1.40 HRS
DECISION INTERVAL(UP) DU=   0.4821  DECISION INTERVAL(LOW) DL=  0.3587
DEAD BAND VALUE(UP) KU  = 100.9844  DEAD BAND VALUE(LOW) KL  = 99.0012
  GAMMA(U)= 0.0076      ARL1=      1.09      ENSIN     =  70.93
  GAMMA(L)= 0.0223      ARL0=    565.05      CYCLE TIME= 103.08 HRS
  GAMMA(0)= 0.9702      THE HOURLY LOSS-COST IS $    4.0024
*************************************************************************
```

In the main menu, a selection of "3" terminates the execution of the interactive computer program.

```
           ************************
           *      MAIN  MENU      *
           ************************

  WHAT WOULD YOU LIKE TO DO ?
    1. DESIGN AN ECONOMICALLY-BASED CUSUM CONTROL CHART
    2. EVALUATE A CUSUM CONTROL CHART
    3. EXIT.

  ENTER THE OPTION NUMBER PLEASE!
?
3
READY
```

Summary

According to the numerical results in Chapter IV, as shown in Tables 4.6 to 4.9 and 4.15 to 4.18, there is an average of 1.8251 minutes CPU time with a standard deviation of 0.5833 minutes for a single run. The minimum CPU time is

0.8688 minutes and the maximum CPU time is 3.3565 minutes. It has been observed that the major effect in the variation of CPU time is the quality of the initial point for the search procedure.

Nearly every feature of the interactive computer program of this research has been demonstrated in this chapter. The interactive feature and its flexibility make this computer program a useful tool for designing and evaluating Cusum control schemes economically. Through its additional design and evaluation capability, this interactive computer program will help with better design and assessment and broader application of Cusum control schemes.

CHAPTER VI

SUMMARY AND CONCLUSION

This research extends the state of the art in quality control charting by fulfilling the objective and subobjectives stated in Chapter I. It provides an operational tool which will permit the Cusum control chart to be used in an economically optimum manner as an alternative to Shewhart control charts for monitoring a process in a realistic environment. This has been achieved by accomplishing the following:

1. An asymmetric Cusum control chart methodology has been developed in which shifts in process mean, probabilities of shift direction and the associated costs of process shifts are asymmetric.
2. A Weibull process failure mechanism has been assumed and incorporated into the asymmetric Cusum control chart model.
3. An economically-based Cusum model has been formulated by following the same cost structure as in Duncan's classic economically-based $\overline{X}$-chart model.
4. Methodologies for statistically evaluating and designing an asymmetric Cusum control chart have been presented.

5. Economical design of the asymmetric Cusum control chart has been compared under a variety of conditions. The effect of the Weibull process failure mechanism has been examined.
6. A versatile interactive computer program has been developed and demonstrated to facilitate the design and evaluation of (1) economically-based asymmetric Cusum control chart, and (2) user defined Cusum control charts.

Based on the results obtained in this research:

1. The Weibull scale parameter affects more the variation in loss-cost and cycle time than does the Weibull shape parameter.
2. It is observed that smaller subgroup sizes should be taken more often when the magnitude of shift in the process mean, which is to be detected, increases.
3. A symmetric Cusum control chart is a special case of the asymmetric Cusum control scheme.
4. Based on the loss-costs obtained, a symmetric Cusum control chart seems slightly less efficient than does a one-sided asymmetric Cusum control chart.
5. In order to have more confidence in the near-optimal solution, multiple starting points are used in the optimal-seeking search procedure.
6. In this study, the upper dead band value k_U is about $\mu_0 + \frac{1}{2}\delta_U\sigma$ and the lower dead band value k_L is about $\mu_0 - \frac{1}{2}\delta_L\sigma$.

The following are recommendations for future research on the same subject to facilitate implementation of Cusum control charts:

1. Multiple assignable causes may be considered in an extension to this research. In this study, a single assignable cause is assumed.
2. The economically-based formulations of Cusum control charts can be extended to have a process failure mechanism which follows the rich Weibull distribution.
3. Step sizes for the decision variables in optimization procedures do affect the final result. Optimal step sizes should be a consideration in improving the computer program and obtaining a better solution.

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APPENDIX

```
C*******************************************************************
C                                                                  *
C  THIS INTERACTIVE PROGRAM PERFORMS AN ASYMMETRICAL ECONOMICALLY- *
C  BASED DESIGN OF CUMULATIVE SUM CONTROL CHART.                   *
C                                                                  *
C  BY CHUNG-YU PAN,  SCHOOL OF INDUSTRIAL ENGINEERING              *
C                    AND MANAGEMENT                                *
C                    OKLAHOMA STATE UNIVERSITY                     *
C                                                                  *
C  DISSERTATION ADVISOR:  DR. KENNETH E. CASE                      *
C                                                                  *
C*******************************************************************
C                                                                  *
C  DEFINITION OF SUBROUTINES:                                      *
C                                                                  *
C    DESIGN : PERFORM THE DESIGN OF ECONOMICALLY-BASED CUMULATIVE  *
C             SUM CONTROL CHARTS.                                  *
C                                                                  *
C    EVALUE : PERFORM THE EVALUATION OF A CUSUM CONTROL CHART.     *
C                                                                  *
C    NELM1  : PERFORM THE NELDER AND MEAD DIRECT SEARCH ALGORITHM  *
C             WITH THREE OR FOUR VARIABLES TO FIND THE OPTIMAL OR  *
C             NEAR-OPTIMAL.                                        *
C                                                                  *
C    NELM2  : PERFORM THE NELDER AND MEAD DIRECT SEARCH ALGORITHM  *
C             WITH FIVE VARIABLES TO FIND THE OPTIMAL OR NEAR-     *
C             OPTIMAL.                                             *
C                                                                  *
C    LOSS   : PERFORM THE EVALUATION OF LOSS-COST.                 *
C                                                                  *
C    CYCLE  : PERFORM THE EVALUATION OF CYCLE TIME.                *
C                                                                  *
C    LENGTH : PERFORM THE EVALUATION OF AVERAGE RUN LENGTH (ARL).  *
C                                                                  *
C    SCALH  : PERFORM HAMMING'S METHOD TO SCALE A SQUARE MATRIX.   *
C                                                                  *
C    RESCAL : PERFORM THE OPERATION OF RESCALING A SQUARE MATRIX.  *
C                                                                  *
C    LSOLV  : PERFORM GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING   *
C             TO SOLVE A SYSTEM OF LINEAR EQUATION.                *
C                                                                  *
C    INCRED : PERFORM THE LINEAR ADJUSTMENT OF DECISION INTERVALS  *
C             TO FIND AN OPTIMAL.                                  *
C                                                                  *
C    INCRED : PERFORM THE LINEAR ADJUSTMENT OF DEAD BAND VALUES    *
C             TO FIND AN OPTIMAL.                                  *
C                                                                  *
C                                                                  *
C  DEFINITION OF FUNCTIONS:                                        *
C                                                                  *
C    DPHI   : PERFORM THE CUMULATIVE DISTRIBUTION FUNCTION OF      *
C             STANDARD NORMAL VARIABLE.                            *
C                                                                  *
C    ENSIN  : PERFORM THE EVALUATION OF THE EXPECTED NUMBER OF     *
```

```
C          SUBGROUPS TAKEN IN THE PERIOD OF THE PROCESS IN-     *
C          CONTROL.                                             *
C                                                               *
C                                                               *
C DEFINITION OF VARIABLES:                                      *
C                                                               *
C   N      : THE NUMBER OF INDIVIDUAL MEASUREMENTS OR SAMPLES   *
C            THAT COMPRISE A SUBGROUP.                          *
C   H      : THE TIME INTERVAL BETWEEN SUBGROUPS.               *
C   DU     : THE UPPER DECISION INTERVAL.                       *
C   DL     : THE LOWER DECISION INTERVAL.                       *
C   KU     : THE UPPER DEAD BAND VALUE.                         *
C   KL     : THE LOWER DEAD BAND VALUE.                         *
C   TARGET : THE DESIRED PROCESS MEAN.                          *
C   SIGMA  : THE STANDARD OR DESIRED PROCESS STANDARD DEVIATION. *
C   DELTAU : THE MAGNITUDE OF AN UPPER SHIFT IN THE PROCESS MEAN. *
C   DELTAL : THE MAGNITUDE OF A DOWNWARD SHIFT IN THE PROCESS   *
C            MEAN.                                              *
C   XU     : THE LOCATION OF THE PROCESS MEAN WITH AN UPPER     *
C            SHIFT, THAT IS, XU = TARGET + DELTAU * SIGMA.      *
C   XL     : THE LOCATION OF THE PROCESS MEAN WITH A DOWNWARD   *
C            SHIFT, THAT IS, XL = TARGET - DELTAL * SIGMA.      *
C   ALPHA  : THE CONDITIONAL PROBABILITY THAT IF THERE IS A     *
C            SHIFT IN THE MEAN, THE SHIFT WILL BE IN THE UPPER  *
C            DIRECTION.                                         *
C   SHAPE  : THE SHAPE PARAMETER OF THE PROCESS FAILURE MECHANISM.*
C   SCALE  : THE SCALE PARAMETER OF THE PROCESS FAILURE MECHANISM.*
C   A1DELU : THE AVERAGE NUMBER OF SUBGROUPS TAKEN BEFORE AN    *
C            UPPER SHIFT WITH A MAGNITUDE OF DELTAU WILL BE     *
C            DETECTED BY VIRTUE OF EXCEEDING EITHER UPPER       *
C            DECISION INTERVAL OR LOWER DECISION INTERVAL.      *
C   A1DELL : THE AVERAGE NUMBER OF SUBGROUPS TAKEN BEFORE A     *
C            DOWNWARD SHIFT WITH A MAGNITUDE OF DELTAL WILL BE  *
C            DETECTED BY VIRTUE OF EXCEEDING EITHER UPPER       *
C            DECISION INTERVAL OR LOWER DECISION INTERVAL.      *
C   ARL0   : THE AVERAGE NUMBER OF SUBGROUPS TAKEN WHEN A PROCESS *
C            IS IN-CONTROL AT ACCEPTABLE LEVEL.                 *
C   ARL1   : THE AVERAGE NUMBER OF SUBGROUPS TAKEN BEFORE A SHIFT *
C            IN THE PROCESS MEAN IS DETECTED BY VIRTUE OF       *
C            EXCEEDING EITHER UPPER OR LOWER DECISION INTERVALS. *
C   GAM0   : THE PROPORTION OF TIME THE PROCESS IS IN-CONTROL.  *
C   GAMU   : THE PROPORTION OF TIME THE PROCESS IS OUT-OF-CONTROL *
C            IN UPWARD DIRECTION.                               *
C   GAML   : THE PROPORTION OF TIME THE PROCESS IS OUT-OF-CONTROL *
C            IN DOWNWARD DIRECTION.                             *
C   CYC    : THE AVERAGE TIME FOR ONE IN-CONTROL, OUT-OF-CONTROL *
C            CYCLE.                                             *
C   B      : THE COST PER SUBGROUP OF SAMPLING, PLOTTING AND    *
C            MAKING THE ACCEPTANCE/REJECTION DECISION.          *
C   C      : THE PER UNIT COST OF SAMPLING, MEASURING, COMPUTING *
C            AND PLOTTING.                                      *
C   D      : THE AVERAGE TIME TAKEN TO FIND THE ASSIGNABLE CAUSE. *
C   E      : THE PER UNIT AVERAGE TIME SAMPLING, MEASURING,     *
C            COMPUTING AND PLOTTING.                            *
```

```
C    T      : THE AVERAGE COST PER EVENT OF SEARCHING FOR AN        *
C             ASSIGNABLE CAUSE WHEN NONE EXISTS.                    *
C    W      : THE AVERAGE COST PER EVENT OF SEARCHING FOR AN        *
C             ASSIGNABLE CAUSE WHEN ONE DOES EXIST.                 *
C    MU     : THE DIMINUTION OF HOURLY INCOME ATTRIBUTED TO THE     *
C             OCCURRENCE OF AN UPPER MEAN SHIFT FROM TARGET TO XU.  *
C    ML     : THE DIMINUTION OF HOURLY INCOME ATTRIBUTED TO THE     *
C             OCCURRENCE OF A DOWNWARD MEAN SHIFT FROM TARGET TO    *
C             XL.                                                   *
C    COST   : THE VALUE OF LOSS-COST.                               *
C                                                                   *
C********************************************************************
C
C  MAIN PROGRAM
C
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 XU,XL,MU,ML,X(6),MIN(6),CONS(8),STEP(6),Y(6),YTEMP(6)
      COMMON SHAPE,SCALE,SIGMA,ALPHA,CONS,DELTAU,DELTAL,TARGET,XU,XL
      COMMON GAMMA,A1DELU,GAMU,A1DELL,GAML,ARL1,HENSIN,ARL0,CYC,GAM0
C
C  PROMT MAIN MENU
C
   10 WRITE(6,200)
C
      READ(5,*)MENU
      GO TO (30,30,300) MENU
   20 WRITE(6,210)
C
      READ(5,*)IENTER
      GO TO (10,300) IENTER
      GO TO 20
C
C  INPUT PROCESS PARAMETERS
C
   30 WRITE(6,220)
      READ(5,*)SHAPE,SCALE,SIGMA,ALPHA,TARGET,DELTAU,DELTAL
      GAMMA=DGAMMA(1.D0+1.D0/SHAPE)
C
C  ECHO PROCESS PARAMETERS
C
   40 WRITE(6,230)SHAPE,SCALE,SIGMA,ALPHA,TARGET,DELTAU,DELTAL
      READ(5,*)ICHECK
      GO TO (50,30) ICHECK
      GO TO 40
C
C  INPUT COST AND TIME FACTORS
C
   50 WRITE(6,240)
      READ(5,*)B,C,D,E,T,W,MU,ML
C
C  ECHO COST AND TIME FACTORS
C
   60 WRITE(6,250)B,C,D,E,T,W,MU,ML
      READ(5,*)ICHECK
```

```
      GO TO (70,50) ICHECK
      GO TO 60
C
   70 CONS(1)=B
      CONS(2)=C
      CONS(3)=D
      CONS(4)=E
      CONS(5)=T
      CONS(6)=W
      CONS(7)=MU
      CONS(8)=ML
      XU=TARGET+DELTAU*SIGMA
      XL=TARGET-DELTAL*SIGMA
      GO TO (80,90) MENU
   80 CALL DESIGN
      GO TO 10
   90 CALL EVALUE
      GO TO 10
C
  200 FORMAT(1H1,12X,24(1H*),/,
     &       13X,'*      MAIN  MENU      *',/,13X,24(1H*),//,
     &         3X,'WHAT WOULD YOU LIKE TO DO ?'
     &      ,/,5X,'1. DESIGN AN ECONOMICALLY-BASED CUSUM CONTROL CHART'
     &      ,/,5X,'2. EVALUATE A CUSUM CONTROL CHART'
     &      ,/,5X,'3. EXIT.'
     &     ,//,3X,'ENTER THE OPTION NUMBER PLEASE!')
  210 FORMAT(///,5X,'ENTERED NUMBER ERROR!',//,
     &           5X,'1. REENTER OPTION NUMBER,',/,
     &           5X,'2. EXIT.')
  220 FORMAT(/,3X,'PLEASE ENTER PROCESS PARAMETERS,',
     &             ' INPUT VALUES OF:',/,5X,
     &  'SHAPE, SCALE, SIGMA, ALPHA, TARGET, DELTA(UP), DELTA(LOW)',/)
  230 FORMAT(/,3X,'THE FOLLOWING VALUES HAVE BEEN INPUTTED:',/,
     &         5X,'SHAPE    =',F6.2,5X,'SCALE     =',F7.2,
     &         5X,'SIGMA    =',F6.2,/,
     &         5X,'ALPHA    =',F6.2,5X,'TARGET    =',F7.2,/,
     &         5X,'DELTA(UP)=',F6.2,5X,'DELTA(LOW)=',F7.2,//,
     &         3X,'ARE THESE DATA RIGHT?',/,
     &         3X,'PLEASE ENTER 1 FOR YES, 2 FOR NO.',/)
  240 FORMAT(/,3X,'PLEASE ENTER COST AND TIME FACTORS, INPUT VALUES OF:'
     ,/,5X,'B, C, D, E, T, W, MU, ML',/)
  250 FORMAT(/,3X,'THE FOLLOWING VALUES HAVE BEEN INPUTTED:',/,
     &         5X,'B=',F7.2,5X,'C=',F7.2,5X,'D =',F7.2,5X,'E =',F7.2,
     &      /,5X,'T=',F7.2,5X,'W=',F7.2,5X,'MU=',F7.2,5X,'ML=',F7.2,
     &     //,3X,'ARE THESE DATA RIGHT?',/,
     &         3X,'PLEASE ENTER 1 FOR YES, 2 FOR NO.',/)
  300 STOP
      END
C
C***************************************************************************
      SUBROUTINE DESIGN
C***************************************************************************
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 KU,KL,MU,ML,X(6),MIN(6),CONS(8),STEP(6),Y(6),YTEMP(6)
```

```
      COMMON SHAPE,SCALE,SIGMA,ALPHA,CONS,DELTAU,DELTAL,TARGET,XU,XL
      COMMON GAMMA,A1DELU,GAMU,A1DELL,GAML,ARL1,HENSIN,ARL0,CYC,GAM0
      DATA N/10/,H/3.0/
      DU=SIGMA*DELTAU/10.0
      DL=SIGMA*DELTAL/10.0
      KU=TARGET+0.5*SIGMA*DELTAU
      KL=TARGET-0.5*SIGMA*DELTAL
C
C  INPUT INITIAL POINT OF CUSUM CHARTS
C
      WRITE(6,300)N,H,DU,DL,KU,KL
      READ(5,*)ICHECK
      GO TO (6,2) ICHECK
    2 WRITE(6,305)
      READ(5,*)N,H,DU,DL,KU,KL
C
C  ECHO THE INITIAL POINT
C
    4 WRITE(6,310)N,H,DU,DL,KU,KL
      READ(5,*)ICHECK
      GO TO (6,2) ICHECK
      GO TO 4
    6 X(1)=H
      X(2)=DU
      X(3)=DL
      X(4)=FLOAT(N)
      X(5)=KU
      X(6)=KL
C
C  INPUT CRITERIA AND STEP SIZES FOR NELDER-MEAD OPTIMIZATION
C  PROCEDURE WITH FOUR VARIABLES
C
      REQ=0.0001
      ICOUNT=200
      STEP(1)=0.2
      STEP(2)=0.2
      STEP(3)=0.2
      STEP(4)=1.0
      WRITE(6,315)REQ,ICOUNT,STEP(4),(STEP(I),I=1,3)
      READ(5,*)ICHECK
      GO TO (30,10) ICHECK
   10 WRITE(6,400)
      READ(5,*)REQ,ICOUNT,STEP(4),(STEP(I),I=1,3)
C
C  ECHO INPUT DATA
C
   20 WRITE(6,410)REQ,ICOUNT,STEP(4),(STEP(I),I=1,3)
      READ(5,*)ICHECK
      GO TO (30,10) ICHECK
      GO TO 20
C
C  PERFORM OPTIMIZATION PROCEDURE
C
   30 WRITE(6,415)
```

```
      CALL NELM1(X,4,STEP,REQ,MIN,ZMIN,ICOUNT)
      WRITE(6,420)MIN(4),MIN(2),MIN(5),MIN(1),MIN(3),MIN(6),ZMIN
C
C  TRUNCATE SUBGROUP SIZE AN INTEGER AND OPTIMIZE H, DU AND DL
C
      X(4)=AINT(MIN(4))
      IF (X(4) .EQ. 0.0) X(4)=1.0
      OPER=1.0
      DO 40 M1=1,6
   40 Y(M1)=MIN(M1)
      Y(4)=X(4)
      ZMIN=1.D10
      REQ=0.000001
      ICOUNT=300
      STEP(1)=0.15
      STEP(2)=0.15
      STEP(3)=0.15
      WRITE(6,425)REQ,ICOUNT,(STEP(I),I=1,3)
      READ(5,*)ICHECK
      GO TO (70,50) ICHECK
   50 WRITE(6,430)
      READ(5,*)REQ,ICOUNT,(STEP(I),I=1,3)
C
C  ECHO INPUT DATA
C
   60 WRITE(6,440)REQ,ICOUNT,(STEP(I),I=1,3)
      READ(5,*)ICHECK
      GO TO (70,50) ICHECK
      GO TO 60
C
C  PERFORM OPTIMATION PROCEDURE
C
   70 MCOUNT=ICOUNT
      WRITE(6,450)
   80 DO 90 M2=1,3
   90 X(M2)=MIN(M2)
      CALL NELM1(X,3,STEP,REQ,MIN,Z,ICOUNT)
      WRITE(6,460)MIN(4),MIN(1),MIN(2),MIN(3),MIN(5),MIN(6),Z
      IF (Z .LT. ZMIN) GO TO 100
      GO TO 120
  100 DO 110 II=1,6
  110 Y(II)=MIN(II)
      OPER=OPER+1.0
      X(4)=X(4)-1.0
      ZMIN=Z
      ICOUNT=MCOUNT
      IF (X(4) .NE. 0.0) GO TO 80
  120 X(4)=X(4)+OPER
  130 ICOUNT=MCOUNT
      CALL NELM1(X,3,STEP,REQ,MIN,Z,ICOUNT)
      WRITE(6,460)MIN(4),MIN(1),MIN(2),MIN(3),MIN(5),MIN(6),Z
      IF (Z .GE. ZMIN) GO TO 160
      DO 140 M3=1,3
  140 X(M3)=MIN(M3)
```

```
      DO 150 M4=1,6
  150 Y(M4)=MIN(M4)
      ZMIN=Z
      X(4)=X(4)+1.0
      GO TO 130
  160 WRITE(6,420)Y(4),Y(2),Y(5),Y(1),Y(3),Y(6),ZMIN
C
C  FIX SUBGROUP SIZE AND OPTIMIZE H, DU, DL, KU AND KL
C
      DO 170 M5=1,6
  170 X(M5)=Y(M5)
      REQ=0.0000001
      ICOUNT=300
      STEP(1)=0.1
      STEP(2)=0.1
      STEP(3)=0.1
      STEP(4)=0.0
      STEP(5)=0.1
      STEP(6)=0.1
      WRITE(6,465)REQ,ICOUNT,(STEP(I),I=1,3),(STEP(J),J=5,6)
      READ(5,*)ICHECK
      GO TO (200,180) ICHECK
  180 WRITE(6,470)
      READ(5,*)REQ,ICOUNT,(STEP(I),I=1,3),(STEP(J),J=5,6)
C
C  ECHO INPUT DATA
C
  190 WRITE(6,480)REQ,ICOUNT,(STEP(I),I=1,3),(STEP(J),J=5,6)
      READ(5,*)ICHECK
      GO TO (200,180) ICHECK
      GO TO 190
C
C  PERFORM OPTIMIZATION PROCEDURE
C
  200 CALL NELM2(X,6,STEP,REQ,MIN,Z,ICOUNT)
      WRITE(6,420)MIN(4),MIN(2),MIN(5),MIN(1),MIN(3),MIN(6),Z
C
C  INCREMENTALLY VARY DU AND DL
C
      DATA STEPD/0.002/
      WRITE(6,485)STEPD
      READ(5,*)ICHECK
      GO TO (230,210) ICHECK
  210 WRITE(6,490)
      READ(5,*)STEPD
C
C  ECHO INPUT DATA
C
  220 WRITE(6,500)STEPD
      READ(5,*)ICHECK
      GO TO (230,210) ICHECK
      GO TO 220
  230 CALL INCRED(MIN,Z,STEPD)
      WRITE(6,510)MIN(4),MIN(2),MIN(5),MIN(1),MIN(3),MIN(6),Z
```

```
C
C  INCREMENTALLY VARY KU AND KL
C
      DATA STEPK/0.002/
      WRITE(6,515)STEPK
      READ(5,*)ICHECK
      GO TO (260,240) ICHECK
  240 WRITE(6,520)
      READ(5,*)STEPK
C
C  ECHO INPUT DATA
C
  250 WRITE(6,500)STEPK
      READ(5,*)ICHECK
      GO TO (260,240) ICHECK
      GO TO 250
  260 CALL INCREK(MIN,Z,TARGET,STEPK)
      WRITE(6,530)MIN(4),MIN(2),MIN(5),MIN(1),MIN(3),MIN(6),Z
      ENSIN=HENSIN/X(1)
      WRITE(6,540)MIN(4),(MIN(I),I=1,3),(MIN(J),J=5,6)
      WRITE(6,550)GAMU,ARL1,ENSIN,GAML,ARL0,CYC,GAM0,Z
C
  300 FORMAT(/,3X,'THE FOLLOWING INITIAL POINT IS SUGGESTED:',/,
     &         5X,'SUBGROUP SIZE N         =',I4,10X,
     &            'SAMPLING INTERVAL H      =',F7.2,/,
     &         5X,'DECISION INTERVAL(UP) DU=',F9.4,5X,
     &            'DECISION INTERVAL(LOW) DL=',F9.4,/,
     &         5X,'DEAD BAND VALUE(UP) KU  =',F9.4,5X,
     &            'DEAD BAND VALUE(LOW) KL  =',F9.4,//,
     &         3X,'DO YOU ACCEPT THIS POINT?',/,
     &         3X,'PLEASE ENTER 1 FOR YES, 2 FOR NO.',/)
  305 FORMAT(/,3X,'PLEASE ENTER INITIAL POINT, INPUT VALUES OF:',/,
     &         5X,'N, H, DU, DL, KU, KL',/)
  310 FORMAT(/,3X,'THE FOLLOWING VALUES HAVE BEEN INPUTTED:',/,
     &         5X,'SUBGROUP SIZE N         =',I4,10X,
     &            'SAMPLING INTERVAL H      =',F7.2,/,
     &         5X,'DECISION INTERVAL(UP) DU=',F9.4,5X,
     &            'DECISION INTERVAL(LOW) DL=',F9.4,/,
     &         5X,'DEAD BAND VALUE(UP) KU  =',F9.4,5X,
     &            'DEAD BAND VALUE(LOW) KL  =',F9.4,//,
     &         3X,'ARE THESE DATA RIGHT?',/,
     &         3X,'PLEASE ENTER 1 FOR YES, 2 FOR NO.',/)
  315 FORMAT(/,3X,'THE FOLLOWING VALUES ARE SUGGESTED FOR OPTIMIZATION:'
     &      ,/,5X,'TERMINATION LIMIT=',D12.3,
     &       /,5X,'MAX. EVALUATIONS =',I4,
     &       /,5X,'STEP FOR N =',F6.3,5X,'STEP FOR H =',F6.3,
     &       /,5X,'STEP FOR DU=',F6.3,5X,'STEP FOR DL=',F6.3,
     &      //,3X,'DO YOU ACCEPT THIS SUGGESTION?',/,
     &         3X,'PLEASE ENTER 1 FOR YES, 2 FOR NO.',/)
  400 FORMAT(/,3X,'PLEASE ENTER CRITERIA AND STEP SIZES FOR',
     &  ' OPTIMIZATION,',/,3X,'INPUT VALUES OF:',/,5X,
     & '1. TERMINATING LIMIT FOR VARIANCE OF FUNCTION VALUES.',/,5X,
     & '2. MAXIMUM NUMBER OF FUNCTION EVALUATIONS.',/,5X,
     & '3. STEP SIZES FOR N, H, DU AND DL, RESPECTIVELY.',/)
```

```
410 FORMAT(/,3X,'THE FOLLOWING VALUES HAVE BEEN INPUTTED:',
   &        /,5X,'TERMINATION LIMIT=',D12.3,
   &        /,5X,'MAX. EVALUATIONS =',I4,
   &        /,5X,'STEP FOR N =',F6.3,5X,'STEP FOR H =',F6.3,
   &        /,5X,'STEP FOR DU=',F6.3,5X,'STEP FOR DL=',F6.3,
   &       //,3X,'ARE THESE DATA RIGHT?',/,
   &          3X,'PLEASE ENTER 1 FOR YES, 2 FOR NO.',/)
415 FORMAT(/,3X,'** OPTIMIZATION IS PROCESSING **',/)
420 FORMAT(/,1X,47(1H*),/,3X,'AFTER OPTIMIZATION THE DESIGN IS',/,
   &       5X,'N=',F6.2,5X,'DU=',F7.4,5X,'KU=',F8.4,/,
   &       5X,'H=',F6.2,5X,'DL=',F7.4,5X,'KL=',F8.4,/,
   &       5X,'LOSS-COST=',F10.4,/,1X,47(1H*))
425 FORMAT(/,3X,'THE FOLLOWING VALUES ARE SUGGESTED:',
   &        /,5X,'TERMINATION LIMIT=',D12.3,
   &        /,5X,'MAX. EVALUATIONS =',I4,
   &        /,5X,'STEP FOR H =',F6.3,5X,'STEP FOR DU=',F6.3,5X,
   &            'STEP FOR DL=',F6.3,
   &       //,3X,'DO YOU ACCEPT THIS SUGGESTION?',/,
   &          3X,'PLEASE ENTER 1 FOR YES, 2 FOR NO.',/)
430 FORMAT(/,3X,'PLEASE INPUT VALUES OF:',/,5X,
   & '1. TERMINATING LIMIT FOR VARIANCE OF FUNCTION VALUES.',/,5X,
   & '2. MAXIMUM NUMBER OF FUNCTION EVALUATIONS.',/,5X,
   & '3. STEP SIZES FOR H, DU AND DL, RESPECTIVELY.',/)
440 FORMAT(/,3X,'THE FOLLOWING VALUES HAVE BEEN INPUTTED:',
   &        /,5X,'TERMINATION LIMIT=',D12.3,
   &        /,5X,'MAX. EVALUATIONS =',I4,
   &        /,5X,'STEP FOR H =',F6.3,5X,'STEP FOR DU=',F6.3,5X,
   &            'STEP FOR DL=',F6.3,
   &       //,3X,'ARE THESE DATA RIGHT?',/,
   &          3X,'PLEASE ENTER 1 FOR YES, 2 FOR NO.',/)
450 FORMAT(/,1X,18X,'***  OPTIMIZATION ITERATION  ***',//,
   & 6X,'N',5X,'H',6X,'DU',7X,'DL',8X,'KU',8X,'KL',8X,'LOSS-COST')
460 FORMAT(/,5X,F3.0,2X,F6.2,2(2X,F7.4),2(2X,F8.4),2X,F9.4)
465 FORMAT(/,3X,'THE FOLLOWING VALUES ARE SUGGESTED:',
   &        /,5X,'TERMINATION LIMIT=',D12.3,
   &        /,5X,'MAX. EVALUATIONS =',I4,
   &        /,5X,'STEP FOR H =',F6.3,
   &        /,5X,'STEP FOR DU=',F6.3,5X,'STEP FOR DL=',F6.3,
   &        /,5X,'STEP FOR KU=',F6.3,5X,'STEP FOR KL=',F6.3,
   &       //,3X,'DO YOU ACCEPT THIS SUGGESTION?',/,
   &          3X,'PLEASE ENTER 1 FOR YES, 2 FOR NO.',/)
470 FORMAT(/,3X,'PLEASE INPUT VALUES OF:',/,5X,
   & '1. TERMINATING LIMIT FOR VARIANCE OF FUNCTION VALUES.',/,5X,
   & '2. MAXIMUM NUMBER OF FUNCTION EVALUATIONS.',/,5X,
   & '3. STEP SIZES FOR H, DU, DL, KU AND KL, RESPECTIVELY.',/)
480 FORMAT(/,3X,'THE FOLLOWING VALUES HAVE BEEN INPUTTED:',
   &        /,5X,'TERMINATION LIMIT=',D12.3,
   &        /,5X,'MAX. EVALUATIONS =',I4,
   &        /,5X,'STEP FOR H =',F6.3,
   &        /,5X,'STEP FOR DU=',F6.3,5X,'STEP FOR DL=',F6.3,
   &        /,5X,'STEP FOR KU=',F6.3,5X,'STEP FOR KL=',F6.3,
   &       //,3X,'ARE THESE DATA RIGHT?',/,
   &          3X,'PLEASE ENTER 1 FOR YES, 2 FOR NO.',/)
485 FORMAT(/,5X,'STEP=',F8.4,' IS SUGGESTED FOR INCREMENTALLY VARYING'
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```
     & ,' DU AND DL.',//,3X,'DO YOU ACCEPT IT?   PLEASE ENTER 1 FOR YES'
     & ,', 2 FOR NO.',/)
  490 FORMAT(/,3X,'PLEASE ENTER STEP SIZE FOR INCREMENTALLY VARYING',
     &   ' DU AND DL.',/)
  500 FORMAT(/,5X,'STEP=',F8.4,' HAS BEEN INPUTTED.',//
     &        ,3X,'IS IT RIGHT?   PLEASE ENTER 1 FOR YES, 2 FOR NO.',/)
  510 FORMAT(/,1X,47(1H*),/,3X,'AFTER VARYING DU AND DL THE DESIGN IS',
     &       /,5X,'N=',F4.0,7X,'DU=',F7.4,5X,'KU=',F9.4,/,
     &         5X,'H=',F6.2,5X,'DL=',F7.4,5X,'KL=',F9.4,/,
     &         5X,'LOSS-COST=',F10.4,/,1X,47(1H*))
  515 FORMAT(/,5X,'STEP=',F8.4,' IS SUGGESTED FOR INCREMENTALLY VARYING'
     & ,' KU AND KL.',//,3X,'DO YOU ACCEPT IT?   PLEASE ENTER 1 FOR YES'
     & ,', 2 FOR NO.',/)
  520 FORMAT(/,3X,'PLEASE ENTER STEP SIZE FOR INCREMENTALLY VARYING',
     &    ' KU AND KL.',/)
  530 FORMAT(/,1X,47(1H*),/,5X,'AFTER VARYING KU AND KL THE DESIGN IS',
     &       /,5X,'N=',F4.0,7X,'DU=',F7.4,5X,'KU=',F9.4,/,
     &         5X,'H=',F6.2,5X,'DL=',F7.4,5X,'KL=',F9.4,/,
     &         5X,'LOSS-COST=',F10.4,/,1X,47(1H*))
  540 FORMAT(/,1X,72(1H*),/,12X,
     &        'THE ECONOMICALLY-BASED CUSUM CHART IS EVALUATED AS:',
     &       /,1X,'SUBGROUP SIZE N          =',F5.0,6X,
     &           'SAMPLING INTERVAL H      =',F6.2,' HRS',/,
     &         1X,'DECISION INTERVAL(UP) DU=',F9.4,2X,
     &           'DECISION INTERVAL(LOW) DL=',F8.4,/,
     &         1X,'DEAD BAND VALUE(UP) KU  =',F9.4,2X,
     &           'DEAD BAND VALUE(LOW) KL  =',F8.4)
  550 FORMAT(3X,'GAMMA(U)=',F7.4,6X,'ARL1=',F10.2,6X,'ENSIN     =',F7.2,
     &     /,3X,'GAMMA(L)=',F7.4,6X,'ARL0=',F10.2,6X,'CYCLE TIME=',F7.2,
     &' HRS',/,3X,'GAMMA(0)=',F7.4,6X,'THE HOURLY LOSS-COST IS $',F10.4,
     & /,72(1H*),//)
      RETURN
      END
C
C************************************************************************
      SUBROUTINE EVALUE
C************************************************************************
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 X(6),CONS(8),KU,KL,MU,ML
      COMMON SHAPE,SCALE,SIGMA,ALPHA,CONS,DELTAU,DELTAL,TARGET,XU,XL
      COMMON GAMMA,A1DELU,GAMU,A1DELL,GAML,ARL1,HENSIN,ARL0,CYC,GAM0
C
C  INPUT INITIAL POINT
C
   10 WRITE(6,100)
      READ(5,*)N,H,DU,DL,KU,KL
C
C  ECHO THE INITIAL POINT
C
   20 WRITE(6,110)N,H,DU,DL,KU,KL
      READ(5,*)ICHECK
      GO TO (30,10) ICHECK
      GO TO 20
   30 X(1)=H
```

```
      X(2)=DU
      X(3)=DL
      X(4)=FLOAT(N)
      X(5)=KU
      X(6)=KL
C
C  EVALUE A CUSUM CONTROL CHART
C
      CALL LOSS(X,COST)
      ENSIN=HENSIN/X(1)
      WRITE(6,120)X(4),(X(I),I=1,3),(X(J),J=5,6)
      WRITE(6,130)GAMU,ARL1,ENSIN,GAML,ARL0,CYC,GAM0,COST
C
  100 FORMAT(/,3X,'PLEASE ENTER INITIAL POINT, INPUT VALUES OF:',/,
     &          5X,'N, H, DU, DL, KU, KL',/)
  110 FORMAT(/,3X,'THE FOLLOWING VALUES HAVE BEEN INPUTTED:',/,
     &          5X,'SUBGROUP SIZE N          =',I4,10X,
     &             'SAMPLING INTERVAL H       =',F7.2,/,
     &          5X,'DECISION INTERVAL(UP) DU=',F9.4,5X,
     &             'DECISION INTERVAL(LOW) DL=',F9.4,/,
     &          5X,'DEAD BAND VALUE(UP) KU  =',F9.4,5X,
     &             'DEAD BAND VALUE(LOW) KL  =',F9.4,//,
     &          3X,'ARE THESE DATA RIGHT?',/,
     &          3X,'PLEASE ENTER 1 FOR YES, 2 FOR NO.',/)
  120 FORMAT(/,1X,72(1H*),/,21X,
     &       'THE CUSUM CHART IS EVALUATED AS:',
     &       /,1X,'SUBGROUP SIZE N          =',F5.0,6X,
     &             'SAMPLING INTERVAL H       =',F6.2,' HRS',/,
     &          1X,'DECISION INTERVAL(UP) DU=',F9.4,2X,
     &             'DECISION INTERVAL(LOW) DL=',F8.4,/,
     &          1X,'DEAD BAND VALUE(UP) KU  =',F9.4,2X,
     &             'DEAD BAND VALUE(LOW) KL  =',F8.4)
  130 FORMAT(3X,'GAMMA(U)=',F7.4,6X,'ARL1=',F10.2,6X,'ENSIN     =',F7.2,
     &     /,3X,'GAMMA(L)=',F7.4,6X,'ARL0=',F10.2,6X,'CYCLE TIME=',F7.2,
     &' HRS',/,3X,'GAMMA(0)=',F7.4,6X,'THE HOURLY LOSS-COST IS $',F10.4,
     &     /,72(1H*),//)
      RETURN
      END
C
C***********************************************************************
      SUBROUTINE LOSS(X,COST)
C***********************************************************************
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 X(6),CONS(8)
      COMMON SHAPE,SCALE,SIGMA,ALPHA,CONS,DELTAU,DELTAL,TARGET,XU,XL
      COMMON GAMMA,A1DELU,GAMU,A1DELL,GAML,ARL1,HENSIN,ARL0,CYC,GAM0
C
C  CALL SUBROUTINE CYCLE.  THOSE DECISION VARIABLES TO BE
C  OPTIMIZED ARE CONTAINED IN X.
C
      CALL CYCLE(X,COEF,STDDU,STDDL)
C
C  COMPUTE DISTANCES BETWEEN TARGET AND UPPER AND LOWER
C  DEAD BANDS, RESPECTIVELY.  THOSE DISTANCES ARE COMPARED
```

```
C  WITH THE UPPER AND LOWER DECISION INTERVALS TO COMPUTE
C  THE ARLO.
C
      DIFFU=(TARGET-X(5))*COEF
      DIFFL=(X(6)-TARGET)*COEF
      CALL LENGTH(STDDU,DIFFU,ARL0U)
      CALL LENGTH(STDDL,DIFFL,ARL0L)
      TEMP=1.D0/ARL0U+1.D0/ARL0L
      ARL0=1.D0/TEMP
C
C  TO EVALUTE THE LOSS COST EQUATION
C
      ELM1=GAMU*CONS(7)+GAML*CONS(8)
      ELM2=(CONS(5)*HENSIN/X(1)+CONS(6)*ARL0)/(ARL0*CYC)
      ELM3=(CONS(1)+CONS(2)*X(4))/X(1)
      COST=ELM1+ELM2+ELM3
      RETURN
      END
C
C**********************************************************************
      SUBROUTINE CYCLE(X,COEF,STDDU,STDDL)
C**********************************************************************
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 X(6),CONS(8)
      COMMON SHAPE,SCALE,SIGMA,ALPHA,CONS,DELTAU,DELTAL,TARGET,XU,XL
      COMMON GAMMA,A1DELU,GAMU,A1DELL,GAML,ARL1,HENSIN,ARL0,CYC,GAM0
      COEF=DSQRT(X(4))/SIGMA
      STDDU=X(2)*COEF
      STDDL=X(3)*COEF
      DIFFU=(XU-X(5))*COEF
      DIFFL=(X(6)-XU)*COEF
      CALL LENGTH(STDDU,DIFFU,AUDELU)
      CALL LENGTH(STDDL,DIFFL,ALDELU)
      TEMP1=1.D0/AUDELU+1.D0/ALDELU
      A1DELU=1.D0/TEMP1
      DIFFU=(XL-X(5))*COEF
      DIFFL=(X(6)-XL)*COEF
      CALL LENGTH(STDDU,DIFFU,AUDELL)
      CALL LENGTH(STDDL,DIFFL,ALDELL)
      TEMP2=1.D0/AUDELL+1.D0/ALDELL
      A1DELL=1.D0/TEMP2
      ARL1=ALPHA*A1DELU+(1.D0-ALPHA)*A1DELL
      HENSIN=ENSIN(SHAPE,SCALE,X(1))*X(1)
      CYC=ARL1*X(1)+HENSIN+CONS(4)*X(4)+CONS(3)
      TIMEIN=SCALE*GAMMA
      GAM0=TIMEIN/CYC
      TEMP3=A1DELU*X(1)-TIMEIN+HENSIN+CONS(4)*X(4)+CONS(3)
      GAMU=ALPHA*TEMP3/CYC
      TEMP4=A1DELL*X(1)-TIMEIN+HENSIN+CONS(4)*X(4)+CONS(3)
      GAML=(1.D0-ALPHA)*TEMP4/CYC
      RETURN
      END
C
C**********************************************************************
```

```
      SUBROUTINE LENGTH(STDH,DIFF,ARL)
C***********************************************************************
      IMPLICIT  REAL*8 (A-H,O-Z)
      REAL*8 AK(24),A(24,24),X(24),ZZ(24),ZK(24),Y(24),Z1(12),A1(12),
     &DA(24),DB(24),C(24,24)
      DIMENSION NA(24),NB(24)
      DATA Z1/-.9951872199970214D0,-.9747285559713095D0,
     2        -.9382745520027328D0,-.8864155270044010D0,
     3        -.8200019859739029D0,-.7401241915785544D0,
     4        -.6480936519369756D0,-.5454214713888395D0,
     5        -.4337935076260451D0,-.3150426796961634D0,
     6        -.1911188674736163D0,-.0640568928626056D0/
      DATA A1/.0123412297999872D0,.0285313886289337D0,
     2        .0442774388174198D0,.0592985849154368D0,
     3        .0733464814440803D0,.0861901615319533D0,
     4        .0976186521041139D0,.1074442701159655D0,
     5        .1155056680537256D0,.1216704729278034D0,
     6        .1258374563468283D0,.1279381953467522D0/
C
C  INITIALIZE PARAMETERS
C
      DATA N/24/,MAX/25/,PI/3.1415926535898D0/
C
      DO 40 L=1,12
      ZZ(L)=Z1(L)
      ZZ(25-L)=-Z1(L)
      AK(L)=DLOG(A1(L))
   40 AK(25-L)=AK(L)
C
C  TRANSFORM ZK FROM THE (-1,1) INTERVAL TO THE (0,STDH) INTERVAL
C  FOR GAUSSIAN ELIMINATION
C
      DO 10 I=1,N
   10 ZK(I)=(ZZ(I)+1.D0)*STDH/2.D0
C
C  SET UP THE A MATRIX AND THE B VECTOR AND I VECTOR
C
      TEMVAL=DLOG(STDH)+DLOG(.5D0)-DLOG(DSQRT(2.D0*PI))
      DO 20 I=1,N
      DO 20 J=1,N
      AD=.5D0*((ZK(J)-ZK(I)-DIFF)**2)
      TEMP=AK(J)+TEMVAL-AD
      IF (TEMP .GT. -1.8D2) GO TO 15
      A(I,J)=0.0D0
      GO TO 18
   15 A(I,J)=-DEXP(TEMP)
   18 IF (I.EQ.J) A(I,J)=A(I,J)+1.D0
   20 CONTINUE
C
C  SCALING A MATRIX
C
      CALL SCALH(A,24,24,24,0,NA,NB,DA,DB)
      CHECK=0.0
      BIGNUM=0.0
```

```
      DO 25 I=1,N
      DO 24 J=1,N
      A(I,J)=DA(I)*A(I,J)*DB(J)
      IF (DABS(A(I,J)) .LT. 1.D-38) CHECK=1.0
      IF (DABS(A(I,J)) .GT. BIGNUM) BIGNUM=DABS(A(I,J))
   24 CONTINUE
      AR=-ZK(I)-DIFF
      P=DPHI(AR)
      X(I)=DA(I)*P
   25 Y(I)=DA(I)
      IF (CHECK .EQ. 0.0) GO TO 26
      CALL RESCAL(A,24,24,24,BIGNUM,X,Y)
   26 DO 30 I=1,N
      DO 30 J=1,N
   30 C(I,J)=A(I,J)
      CALL LSOLV(A,X,24,24)
      CALL LSOLV(C,Y,24,24)
      DO 60 I=1,N
      X(I)=DB(I)*X(I)
   60 Y(I)=DB(I)*Y(I)
      AE=-DIFF
      PR=DPHI(AE)
      XNZERO=0.0D0
      PZERO=0.0D0
      DO 90 I=1,N
      AD1=.5D0*((ZK(I)-DIFF)**2)
      TEMP=AK(I)+TEMVAL-AD1
      IF (X(I) .LE. 0.0) GO TO 50
      TEMP1=TEMP+DLOG(X(I))
      IF (TEMP1 .LT. -1.8D2) GO TO 50
      PZERO=PZERO+DEXP(TEMP1)
   50 IF (Y(I) .LE. 0.0) GO TO 90
      TEMP2=TEMP+DLOG(Y(I))
      IF (TEMP2 .LT. -1.8D2) GO TO 90
      XNZERO=XNZERO+DEXP(TEMP2)
   90 CONTINUE
      PZERO=PZERO+PR
      XNZERO=1.D0+XNZERO
      IF (1.D0-PZERO.LT.1.D-6) GO TO 95
      ARL=XNZERO/(1.D0-PZERO)
      GO TO 100
   95 ARL=1.D9
  100 RETURN
      END
C
C*************************************************************************
      DOUBLE PRECISION FUNCTION DPHI(X)
C*************************************************************************
      IMPLICIT REAL*8(A-H,O-Z)
      DATA B1/.319381530D0/,B2/-.356563782D0/,B3/1.781477937D0/,
     &     B4/-1.821255978D0/,B5/1.330274429D0/,B6/.2316419D0/,
     &     PI/3.1415926535898D0/
      T=1.D0/(1.D0+B6*DABS(X))
      ELM1=DLOG(B1*T+B2*T**2+B3*T**3+B4*T**4+B5*T**5)
```

```
      ELM2=DLOG(DSQRT(2.D0*PI))+X*X/2.D0
      TEMP=ELM1-ELM2
      DPHI=0.0D0
      IF (TEMP .GT. -1.8D2) DPHI=DEXP(TEMP)
      IF (X.GE.0.0D0) DPHI=1.0D0-DPHI
      RETURN
      END
C
C***********************************************************************
      SUBROUTINE SCALH (A,M,N,LADIM,KRENRM,NA,NB,DA,DB)
C***********************************************************************
C
C  THIS PROGRAM IS PROVIDED BY  J. P. CHANDLER
C  COMPUTER SCIENCE DEPT., OKLAHOMA STATE UNIVERSITY
C
C  INPUT:
C
C    A(*,*) : THE MATRIX TO BE SCALED
C    M      : NUMBER OF ROWS IN THE MATRIX A
C    N      : NUMBER OF COLUMNS IN THE MATRIX A
C    LADIM  : THE FIRST DIMENSION OF THE ARRAY A (M.LE.LADIM)
C    KRENRM : =1 TO RENORMALIZE SO THAT THE LARGEST
C                MAGNITUDE IS 1.0,
C             =0 NOT TO RENORMALIZE
C
C  OUTPUT :
C
C    DA(*)  : LEFT DIAGONAL SCALING MATRIX
C    DB(*)  : RIGHT DIAGONAL SCALING MATRIX
C
C  SCRATCH STORAGE: : NA(*),NB(*)
C
C***********************************************************************
      DOUBLE PRECISION A,DA,DB,    QSQRT,ARG,QABS,QLOG,QEXP,    RZERO,
     *  SUM,SUMK,TEMP,HALFAV,AVE,AMX
      DIMENSION A(LADIM,N),NA(M),NB(N),DA(M),DB(N)
      RZERO=0.0D0
C
      IF(M.LT.1 .OR. M.GT.LADIM .OR. N.LT.1) STOP
C
C  INITIALIZE.
C
      DO 10 J=1,M
         DA(J)=RZERO
   10    NA(J)=0
      DO 20 K=1,N
         DB(K)=RZERO
   20    NB(K)=0
      SUM=RZERO
      JKSUM=0
C
C  ACCUMULATE ALL SUMS AND PROCESS A(*,*) BY COLUMNS.
C
      DO 40 K=1,N
```

```
      SUMK=RZERO
      KSUM=0
      DO 30 J=1,M
         TEMP=DABS(A(J,K))
         IF(TEMP.EQ.RZERO) GO TO 30
         TEMP=DLOG(TEMP)
         DA(J)=DA(J)+TEMP
         SUMK=SUMK+TEMP
         SUM=SUM+TEMP
         NA(J)=NA(J)+1
         KSUM=KSUM+1
         JKSUM=JKSUM+1
 30      CONTINUE
      DB(K)=SUMK
 40   NB(K)=KSUM
C
C COMPUTE DA(*) AND DB(*).
C
    IF(JKSUM.EQ.0) GO TO 70
    TEMP=JKSUM+JKSUM
    HALFAV=SUM/TEMP
    DO 50 J=1,M
       IF(NA(J).EQ.0) GO TO 50
       TEMP=NA(J)
       DA(J)=HALFAV-DA(J)/TEMP
 50    CONTINUE
    DO 60 K=1,N
       IF(NB(K).EQ.0) GO TO 60
       TEMP=NB(K)
       DB(K)=HALFAV-DB(K)/TEMP
 60    CONTINUE
C
C TAKE ANTILOGS.
C
 70 DO 80 J=1,M
 80    DA(J)=DEXP(DA(J))
    DO 90 K=1,N
 90    DB(K)=DEXP(DB(K))
C
    IF(KRENRM.NE.1) RETURN
C
C RENORMALIZE SO THAT THE LARGEST MAGNITUDE IS 1.0 .
C
    AMX=RZERO
    DO 100 K=1,N
       DBK=DB(K)
       DO 100 J=1,M
          TEMP=DABS(DA(J)*A(J,K)*DBK)
          IF(TEMP.GT.AMX) AMX=TEMP
100       CONTINUE
    TEMP=DSQRT(AMX)
    IF(TEMP.EQ.RZERO) RETURN
    DO 110 J=1,M
110    DA(J)=DA(J)/TEMP
```

```
      DO 120 K=1,N
  120    DB(K)=DB(K)/TEMP
C
      RETURN
      END
C
C***********************************************************************
      SUBROUTINE LSOLV (A,BX,N,LDIM)
C***********************************************************************
C
C  THIS PROGRAM IS PROVIDED BY  J. P. CHANDLER
C  COMPUTER SCIENCE DEPT., OKLAHOMA STATE UNIVERSITY
C
C  N IS THE NUMBER OF EQUATIONS IN THE LINEAR SYSTEM.
C  ON INPUT, A(*,*) CONTAINS THE MATRIX OF COEFFICIENTS AND BX(*)
C  CONTAINS THE VECTOR OF CONSTANTS (THE RIGHTHAND SIDES).
C  ON OUTPUT, BX(*) CONTAINS THE SOLUTION VECTOR AND A(*,*) CONTAINS
C  GARBAGE.
C  LDIM IS THE VALUE OF THE DIMENSIONS OF THE ARRAYS A AND BX.
C  THE VALUE OF N MUST NOT EXCEED THE VALUE OF LDIM.
C
      DOUBLE PRECISION A,BX,QABS,ARG,BIGA,TEMP,EM,SUM
      DIMENSION A(LDIM,LDIM),BX(LDIM)
C
C  CHECK FOR AN INVALID VALUE OF N OR LDIM.
C
      IF(N)240,240,10
   10 IF(N-LDIM)20,20,240
C
C  TRIANGULARIZE THE MATRIX A.
C
   20 NMU=N-1
      IF(NMU)240,140,30
   30 DO 130 J=1,NMU
C
C  SEARCH COLUMN J FOR THE PIVOT ELEMENT.
C
        BIGA=0.
        DO 50 K=J,N
          TEMP=DABS(A(K,J))
          IF(TEMP-BIGA)50,50,40
   40     BIGA=TEMP
          JPIV=K
   50     CONTINUE
        IF(BIGA)130,130,60
   60   IF(JPIV-J)90,90,70
C
C  INTERCHANGE EQUATIONS J AND JPIV.
C
   70   DO 80 L=J,N
          TEMP=A(J,L)
          A(J,L)=A(JPIV,L)
   80     A(JPIV,L)=TEMP
        TEMP=BX(J)
```

```
      BX(J)=BX(JPIV)
      BX(JPIV)=TEMP
  90  JPU=J+1
      DO 120 K=JPU,N
C
C  PERFORM ELIMINATION ON EQUATION K.
C
        EM=A(K,J)/A(J,J)
        IF (DABS(EM) .LT. 1.D-30) EM=0.0
        IF(EM)100,120,100
 100    DO 110 L=JPU,N
 110      A(K,L)=A(K,L)-EM*A(J,L)
        BX(K)=BX(K)-EM*BX(J)
 120    CONTINUE
 130  CONTINUE
C
C  DO THE BACK SOLUTION.
C
 140 DO 230 JINV=1,N
      J=N+1-JINV
      TEMP=A(J,J)
      IF(TEMP)160,150,160
 150  BX(J)=0.
      GO TO 230
 160  SUM=0.
      IF(J-N)200,220,220
 200  JPU=J+1
      DO 210 K=JPU,N
 210    SUM=SUM+A(J,K)*BX(K)
 220  BX(J)=(BX(J)-SUM)/TEMP
 230  CONTINUE
 240 RETURN
     END
C
C*************************************************************************
      FUNCTION ENSIN(SHAPE,SCALE,H)
C*************************************************************************
      IMPLICIT REAL*8 (A-H,O-Z)
      PTUP=(-DLOG(1.D-10))**(1.D0/SHAPE)
      LIMUP=INT(PTUP*SCALE/H)
      PTLOW=(-DLOG(1.D0-1.D-10))**(1.D0/SHAPE)
      LIMLOW=INT(PTLOW*SCALE/H)
      IF (LIMLOW .LE. 0) LIMLOW=1
      ENSIN=0.D0
      DO 1 I=LIMLOW,LIMUP
      B=(I*H/SCALE)**SHAPE
    1 ENSIN=ENSIN+DEXP(-B)
      RETURN
      END
C
C*************************************************************************
      SUBROUTINE RESCAL(A,M,N,LADIM,BIGNUM,X,Y)
C*************************************************************************
      IMPLICIT REAL*8 (A-H,O-Z)
```

```
      REAL*8 A(24,24),DA(24),DB(24),X(24),Y(24)
      DIMENSION NA(24),NB(24)
      BIGA=BIGNUM*1.D-38
      DO 10 I=1,N
      DO 10 J=1,N
      IF (DABS(A(I,J)) .LT. BIGA) A(I,J)=0.0
   10 CONTINUE
      CALL SCALH(A,M,N,LADIM,0,NA,NB,DA,DB)
      DO 30 I=1,N
      DO 20 J=1,N
   20 A(I,J)=DA(I)*A(I,J)*DB(J)
      X(I)=DA(I)*X(I)
   30 Y(I)=DA(I)*Y(I)
      RETURN
      END
C
C*************************************************************************
      SUBROUTINE NELM1(X,N,STEP,REQ,MIN,ZMIN,ICOUNT)
C*************************************************************************
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 X(6),MIN(6),STEP(6),P(20,21),PX(20),P2X(20),PBAR(20),
     2       Y(20),ZMIN,REQ,DN,DNN,Z,SUM,SUMM,YLO,YX,Y2X,CURMIN,DEL,
     3       RCOEFF,ECOEFF,CCOEFF,CONS(8)
      DOUBLE PRECISION DFLOAT
      COMMON SHAPE,SCALE,SIGMA,ALPHA,CONS,DELTAU,DELTAL,TARGET,XU,XL
      COMMON GAMMA,A1DELU,GAMU,A1DELL,GAML,ARL1,HENSIN,ARLO,CYC
C
C  REFLECTION,EXTENSION AND CONTRACTION COEFFICIENTS
C
      DATA RCOEFF/1.D0/,ECOEFF/2.D0/,CCOEFF/.5D0/
      DATA KONVGE/5/
      CHECK=0.0
      KCOUNT=ICOUNT
      ICOUNT=0
      JCOUNT=KONVGE
      DN=DFLOAT(N)
      NN=N+1
      DNN=DFLOAT(NN)
C
C  CONSTRUCTION OF INITIAL SIMPLEX
C
      DO 20 I=1,6
      P(I,NN)=X(I)
      PX(I)=X(I)
      P2X(I)=X(I)
   20 MIN(I)=X(I)
      CALL LOSS(X,Z)
      ICOUNT=ICOUNT+1
      Y(NN)=Z
      SUM=Z
      SUMM=Z*Z
      DO 40 J=1,N
      X(J)=X(J)+STEP(J)
      DO 30 I=1,N
```

```
   30 P(I,J)=X(I)
      CALL LOSS(X,Z)
      ICOUNT=ICOUNT+1
      Y(J)=Z
      SUM=SUM+Z
      SUMM=SUMM+Z*Z
   40 X(J)=X(J)-STEP(J)
C
C  SIMPLEX CONSTRUCTION COMPLETE
C
C  FIND HIGHTEST AND LOWEST Y VALUES. Z ( =Y(IHI ) INDICATES
C  THE VERTEX OF THE SIMPLEX TO BE REPLACED.
C
   50 YLO=Y(1)
      ZMIN=YLO
      ILO=1
      IHI=1
      DO 70  I=2,NN
      IF (Y(I).GE.YLO) GO TO 60
      YLO=Y(I)
      ILO=I
      GO TO 70
   60 IF (Y(I).LE.ZMIN) GO TO 70
      ZMIN=Y(I)
      IHI=I
   70 CONTINUE
      SUM=SUM-ZMIN
      SUMM=SUMM-ZMIN*ZMIN
C
C  CALCULATE PBAR, THE CENTROID OF THE SIMPLEX VERTICES
C  EXCEPTING THAT WITH Y VALUE ZMIN.
C
      DO 90 I=1,N
      Z=0.D0
      DO 80 J=1,NN
   80 Z=Z+P(I,J)
      Z=Z-P(I,IHI)
   90 PBAR(I)=Z/DN
C
C  REFLECTION THROUGH THE CENTROID
C
      DO 100 I=1,N
      PX(I)=(1.D0+RCOEFF)*PBAR(I)-RCOEFF*P(I,IHI)
      IF (PX(I) .GT. 0.0) GO TO 100
      CHECK=1.0
      GO TO 110
  100 CONTINUE
  110 YX=1.D10
      IF (CHECK .NE. 0.0) GO TO 120
      CALL LOSS(PX,YX)
  120 CHECK=0.0
      ICOUNT=ICOUNT+1
      IF (YX.GE.YLO) GO TO 180
C
```

```
C  SUCCESSFUL REFLECTION, THEN EXTENSION
C
      DO 130 I=1,N
      P2X(I)=ECOEFF*PX(I)+(1.D0-ECOEFF)*PBAR(I)
      IF (P2X(I) .GT. 0.0) GO TO 130
      CHECK=1.0
      GO TO 201
  130 CONTINUE
  201 Y2X=1.D10
      IF (CHECK .NE. 0.0) GO TO 150
      CALL LOSS(P2X,Y2X)
  150 CHECK=0.0
      ICOUNT=ICOUNT+1
C
C  RETAIN EXTENSION OR CONTRACTION
C
      IF (Y2X .GE. YX) GO TO 280
  160 DO 170 I=1,N
  170 P(I,IHI)=P2X(I)
      Y(IHI)=Y2X
      GO TO 300
C
C  NO EXTENSION
C
  180 L=0
      DO 190 I=1,NN
      IF (Y(I).GT.YX) L=L+1
  190 CONTINUE
      IF (L-1) 220,200,280
C
C  CONTRACTION ON THE REFLECTION SIDE OF THE CENTROID
C
  200 DO 210 I=1,N
  210 P(I,IHI)=PX(I)
      Y(IHI)=YX
C
C  CONTRACTION ON THE Y(IHI) SIDE OF THE CENTROID
C
  220 DO 230 I=1,N
      P2X(I)=CCOEFF*P(I,IHI)+(1.D0-CCOEFF)*PBAR(I)
      IF (P2X(I) .GT. 0.0) GO TO 230
      CHECK=1.0
      GO TO 240
  230 CONTINUE
  240 Y2X=1.D10
      IF (CHECK .NE. 0.0) GO TO 250
      CALL LOSS(P2X,Y2X)
  250 CHECK=0.0
      ICOUNT=ICOUNT+1
      IF (Y2X.LE.Y(IHI)) GO TO 160
C
C  CONTRACT WHOLE SIMPLEX
C
      SUM=0.D0
```

```
      SUMM=0.D0
      DO 270 J=1,NN
      DO 260 I=1,N
      P(I,J)=(P(I,J)+P(I,ILO))*.5D0
  260 MIN(I)=P(I,J)
      CALL LOSS(MIN,Y(J))
      SUM=SUM+Y(J)
  270 SUMM=SUMM+Y(J)*Y(J)
      ICOUNT=ICOUNT+NN
      GO TO 310
C
C  RETAIN REFLECTION
C
  280 DO 290 I=1,N
  290 P(I,IHI)=PX(I)
      Y(IHI)=YX
  300 SUM=SUM+Y(IHI)
      SUMM=SUMM+Y(IHI)*Y(IHI)
  310 JCOUNT=JCOUNT-1
      IF (JCOUNT.NE.0) GO TO 50
C
C  CHECK TO SEE IF MINIMUM REACHED
C
      IF (ICOUNT .GE. KCOUNT) GO TO 320
      JCOUNT=KONVGE
      CURMIN=(SUMM-(SUM*SUM)/DNN)/DN
C
C  CURMIN IS THE VARIANCE OF THE N+1 LOSS VALUES AT THE
C  VERTICES
C
      IF (CURMIN.GT.REQ) GO TO 50
  320 YLO=Y(1)
      ILO=1
      DO 330 I=2,NN
      IF (Y(I) .GE. YLO) GO TO 330
      YLO=Y(I)
      ILO=I
  330 CONTINUE
      DO 340 I=1,N
  340 MIN(I)=P(I,ILO)
      ZMIN=YLO
      RETURN
      END
C
C**************************************************************************
      SUBROUTINE NELM2(X,N,STEP,REQ,MIN,ZMIN,ICOUNT)
C**************************************************************************
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 X(6),MIN(6),STEP(6),P(20,21),PX(20),P2X(20),PBAR(20),
     2       Y(20),ZMIN,REQ,DN,DNN,Z,SUM,SUMM,YLO,YX,Y2X,CURMIN,DEL,
     3       RCOEFF,ECOEFF,CCOEFF,CONS(8)
      DOUBLE PRECISION DFLOAT
      COMMON SHAPE,SCALE,SIGMA,ALPHA,CONS,DELTAU,DELTAL,TARGET,XU,XL
      COMMON GAMMA,A1DELU,GAMU,A1DELL,GAML,ARL1,HENSIN,ARL0,CYC
```

```
C
C  REFLECTION,EXTENSION AND CONTRACTION COEFFICIENTS
C
      DATA RCOEFF/1.D0/,ECOEFF/2.D0/,CCOEFF/.5D0/
      DATA KONVGE/5/
      CHECK=0.0
      KCOUNT=ICOUNT
      ICOUNT=0
      JCOUNT=KONVGE
      DN=DFLOAT(N)
      NN=N+1
      DNN=DFLOAT(NN)
C
C  CONSTRUCTION OF INITIAL SIMPLEX
C
      DO 20 I=1,6
   20 P(I,NN)=X(I)
      CALL LOSS(X,Z)
      ICOUNT=ICOUNT+1
      Y(NN)=Z
      SUM=Z
      SUMM=Z*Z
      DO 40 J=1,N
      X(J)=X(J)+STEP(J)
      DO 30 I=1,N
   30 P(I,J)=X(I)
      CALL LOSS(X,Z)
      ICOUNT=ICOUNT+1
      Y(J)=Z
      SUM=SUM+Z
      SUMM=SUMM+Z*Z
   40 X(J)=X(J)-STEP(J)
C
C  SIMPLEX CONSTRUCTION COMPLETE
C
C  FIND HIGHTEST AND LOWEST Y VALUES. Z ( =Y(IHI ) INDICATES
C  THE VERTEX OF THE SIMPLEX TO BE REPLACED.
C
   50 YLO=Y(1)
      ZMIN=YLO
      ILO=1
      IHI=1
      DO 70 I=2,NN
      IF (Y(I).GE.YLO) GO TO 60
      YLO=Y(I)
      ILO=I
      GO TO 70
   60 IF (Y(I).LE.ZMIN) GO TO 70
      ZMIN=Y(I)
      IHI=I
   70 CONTINUE
      SUM=SUM-ZMIN
      SUMM=SUMM-ZMIN*ZMIN
C
```

```
C  CALCULATE PBAR, THE CENTROID OF THE SIMPLEX VERTICES
C  EXCEPTING THAT WITH Y VALUE ZMIN.
C
      DO 90 I=1,N
      Z=0.D0
      DO 80 J=1,NN
   80 Z=Z+P(I,J)
      Z=Z-P(I,IHI)
   90 PBAR(I)=Z/DN
C
C  REFLECTION THROUGH THE CENTROID
C
      DO 100 I=1,N
      PX(I)=(1.D0+RCOEFF)*PBAR(I)-RCOEFF*P(I,IHI)
      IF (PX(I) .GT. 0.0) GO TO 100
      CHECK=1.0
      GO TO 110
  100 CONTINUE
  110 YX=1.D10
      IF (PX(5).LT.TARGET .OR. PX(6).GT.TARGET) CHECK=1.0
      IF (CHECK .NE. 0.0) GO TO 120
      PX(4)=X(4)
      CALL LOSS(PX,YX)
  120 CHECK=0.0
      ICOUNT=ICOUNT+1
      IF (YX.GE.YLO) GO TO 180
C
C  SUCCESSFUL REFLECTION, THEN EXTENSION
C
      DO 130 I=1,N
      P2X(I)=ECOEFF*PX(I)+(1.D0-ECOEFF)*PBAR(I)
      IF (P2X(I) .GT. 0.0) GO TO 130
      CHECK=1.0
      GO TO 140
  130 CONTINUE
  140 Y2X=1.D10
      IF (P2X(5).LT.TARGET .OR. P2X(6).GT.TARGET) CHECK=1.0
      IF (CHECK .NE. 0.0) GO TO 150
      P2X(4)=X(4)
      CALL LOSS(P2X,Y2X)
  150 CHECK=0.0
      ICOUNT=ICOUNT+1
C
C  RETAIN EXTENSION OR CONTRACTION
C
      IF (Y2X .GE. YX) GO TO 280
  160 DO 170 I=1,N
  170 P(I,IHI)=P2X(I)
      Y(IHI)=Y2X
      GO TO 300
C
C  NO EXTENSION
C
  180 L=0
```

```
      DO 190 I=1,NN
      IF (Y(I).GT.YX) L=L+1
  190 CONTINUE
      IF (L-1) 220,200,280
C
C  CONTRACTION ON THE REFLECTION SIDE OF THE CENTROID
C
  200 DO 210 I=1,N
  210 P(I,IHI)=PX(I)
      Y(IHI)=YX
C
C  CONTRACTION ON THE Y(IHI) SIDE OF THE CENTROID
C
  220 DO 230 I=1,N
      P2X(I)=CCOEFF*P(I,IHI)+(1.D0-CCOEFF)*PBAR(I)
      IF (P2X(I) .GT. 0.0) GO TO 230
      CHECK=1.0
      GO TO 240
  230 CONTINUE
  240 Y2X=1.D10
      IF (P2X(5).LT.TARGET .OR. P2X(6).GT.TARGET) CHECK=1.0
      IF (CHECK .NE. 0.0) GO TO 250
      P2X(4)=X(4)
      CALL LOSS(P2X,Y2X)
  250 CHECK=0.0
      ICOUNT=ICOUNT+1
      IF (Y2X.LE.Y(IHI)) GO TO 160
C
C  CONTRACT WHOLE SIMPLEX
C
      SUM=0.D0
      SUMM=0.D0
      DO 270 J=1,NN
      DO 260 I=1,N
      P(I,J)=(P(I,J)+P(I,ILO))*.5D0
  260 MIN(I)=P(I,J)
      MIN(4)=X(4)
      CALL LOSS(MIN,Y(J))
      SUM=SUM+Y(J)
  270 SUMM=SUMM+Y(J)*Y(J)
      ICOUNT=ICOUNT+NN
      GO TO 310
C
C  RETAIN REFLECTION
C
  280 DO 290 I=1,N
  290 P(I,IHI)=PX(I)
      Y(IHI)=YX
  300 SUM=SUM+Y(IHI)
      SUMM=SUMM+Y(IHI)*Y(IHI)
  310 JCOUNT=JCOUNT-1
      IF (JCOUNT.NE.0) GO TO 50
C
C  CHECK TO SEE IF MINIMUM REACHED
```

```
C
      IF (ICOUNT .GE. KCOUNT) GO TO 320
      JCOUNT=KONVGE
      CURMIN=(SUMM-(SUM*SUM)/DNN)/DN
C
C  CURMIN IS THE VARIANCE OF THE N+1 LOSS VALUES AT THE
C  VERTICES
C
      IF (CURMIN.GT.REQ) GO TO 50
  320 YLO=Y(1)
      ILO=1
      DO 330 I=2,NN
      IF (Y(I) .GE. YLO) GO TO 330
      YLO=Y(I)
      ILO=I
  330 CONTINUE
      DO 340 I=1,N
  340 MIN(I)=P(I,ILO)
      ZMIN=YLO
      RETURN
      END
C
C***************************************************************************
      SUBROUTINE INCRED(X,ZMIN,STEPD)
C***************************************************************************
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 X(6),ZMIN,COST
   10 CHECK=0.0
C
C  TWEAK DL
C
      A1=0.0
      A2=0.0
   20 X(3)=X(3)+STEPD
      CALL LOSS(X,COST)
      IF (COST .GE. ZMIN) GO TO 30
      ZMIN=COST
      CHECK=1.0
      A1=A1+1.0
      GO TO 20
   30 X(3)=X(3)-STEPD*(A1+1.0)
   40 IF (X(3) .LT. 0.0) GO TO 50
      CALL LOSS(X,COST)
      IF (COST .GE. ZMIN) GO TO 50
      ZMIN=COST
      CHECK=1.0
      A2=A2+1.0
      X(3)=X(3)-STEPD
      GO TO 40
   50 IF (A2 .NE. 0.0) GO TO 60
      X(3)=X(3)+STEPD*A1
      GO TO 70
   60 X(3)=X(3)+STEPD
C
```

```
C  TWEAK DO
C
   70 A1=0.0
      A2=0.0
   80 X(2)=X(2)+STEPD
      CALL LOSS(X,COST)
      IF (COST .GE. ZMIN) GO TO 90
      ZMIN=COST
      CHECK=1.0
      A1=A1+1.0
      GO TO 80
   90 X(2)=X(2)-STEPD*(A1+1.0)
  100 IF (X(2) .LT. 0.0) GO TO 110
      CALL LOSS(X,COST)
      IF (COST .GE. ZMIN) GO TO 110
      ZMIN=COST
      CHECK=1.0
      A2=A2+1.0
      X(2)=X(2)-STEPD
      GO TO 100
  110 IF (A2 .NE. 0.0) GO TO 120
      X(2)=X(2)+STEPD*A1
      GO TO 130
  120 X(2)=X(2)+STEPD
  130 IF (CHECK .EQ. 0.0) RETURN
      GO TO 10
      END
C
C************************************************************************
      SUBROUTINE INCREK(X,ZMIN,TARGET,STEPK)
C************************************************************************
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 X(6),ZMIN,COST,TARGET
   10 CHECK=0.0
C
C  TWEAK KL
C
      A1=0.0
      A2=0.0
   20 X(6)=X(6)-STEPK
      CALL LOSS(X,COST)
      IF (COST .GE. ZMIN) GO TO 30
      ZMIN=COST
      CHECK=1.0
      A1=A1+1.0
      GO TO 20
   30 X(6)=X(6)+STEPK*(A1+1.0)
   40 IF (X(6) .GT. TARGET) GO TO 50
      CALL LOSS(X,COST)
      IF (COST .GE. ZMIN) GO TO 50
      ZMIN=COST
      CHECK=1.0
      A2=A2+1.0
      X(6)=X(6)+STEPK
```

```
      GO TO 40
   50 IF (A2 .NE. 0.0) GO TO 60
      X(6)=X(6)-STEPK*A1
      GO TO 70
   60 X(6)=X(6)-STEPK
C
C  TWEAK KU
C
   70 A1=0.0
      A2=0.0
   80 X(5)=X(5)+STEPK
      CALL LOSS(X,COST)
      IF (COST .GE. ZMIN) GO TO 90
      ZMIN=COST
      CHECK=1.0
      A1=A1+1.0
      GO TO 80
   90 X(5)=X(5)-STEPK*(A1+1.0)
  100 IF (X(5) .LT. TARGET) GO TO 110
      CALL LOSS(X,COST)
      IF (COST .GE. ZMIN) GO TO 110
      ZMIN=COST
      CHECK=1.0
      A2=A2+1.0
      X(5)=X(5)-STEPK
      GO TO 100
  110 IF (A2 .NE. 0.0) GO TO 120
      X(5)=X(5)+STEPK*A1
      GO TO 130
  120 X(5)=X(5)+STEPK
  130 IF (CHECK .EQ. 0.0) RETURN
      GO TO 10
      END
```

VITA

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Doctor of Philosophy

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