ADAPTIVE POLE PLACEMENT CONTROL OF NONLINEAR SYSTEMS

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NOMENCLATURE

A - matrix

A - area of piston

$\underline{a}$ - vector of nonlinear functions

B - matrix of nonlinear functions

b - viscous damping

C_t - coefficient of leakage

$\underline{C}$ - vector of nonlinear functions

$\underline{c}$ - vector of Coriolis forces

$D(d)$ - diagonal matrix

d - differential operator, $\frac{d}{dt}$

E - matrix

$\underline{e}$ - error vector

F_1 - nonlinear function

$\underline{f}$ - vector of nonlinear functions

g - acceleration due to gravity

$\underline{g}$ - vector of gravitational forces

$\underline{h}$ - scalar field

I - inertia of links

i - servovalve current

J - inertia matrix

J_N - cost function

$L_{[\cdot,\cdot]}h$ - Lie derivative of h

l - length of links

M - mass of piston

N - number of inputs

m - mass of links

m_L - mass of payload

n_i - degree of subsystem i

P_s - supply pressure

P_1 - load pressure drop across cylinder

P - covariance matrix

p - pole of first order filter

Q - flow rate

$\underline{q}_1$ - sampled value of $D(d)\underline{y}$

s - Laplace transform operator

$\underline{s}$ - vector function

T - transformation matrix

t_s - sampling time

$\underline{t}$ - vector function

$\underline{u}$ - input vector

V - cost function

V_t - volume of fluid under compression

$\underline{v}$ - reference input

$\underline{W}$ - vector of $d^{n-1}\underline{y}$

$\underline{x}$ - state vector

$\underline{x}^*$ - desired state vector

x_1 - conversion of reactant 1 to reactant 2

x_2 - dimensionless reactor temperature

$\dot{x}$ - velocity

$\underline{y}$ - output vector

$\underline{y}_d$ - desired output vector

z - z-transform operator

β - bulk modulus

Γ_i - matrices of state feedback gains

γ - parameter

Δ - parameter

$\underline{\phi}$ - regressor vector

$\underline{\theta}$ - manipulator position vector

$\underline{\theta}_d$ - desired position vector

Θ - parameter vector

λ - forgetting factor

Λ - parameter

Γ_o - exponential time decay constant

ζ - damping factor

ρ - parameter

ω_n - natural frequency

$\hat{}$ - denotes estimated values

CHAPTER I

INTRODUCTION AND SUMMARY

The very high performance required over wide operating ranges in many control applications has, in recent years, led to the development of nonlinear controllers for systems with known dynamics and adaptive controllers for linear systems with time varying parameters and constant, unknown parameters. The adaptive control of nonlinear systems with unknown dynamics is an area of current research activity.

Statement of the Problem

The adaptive feedback control of multi-input, multi-output, linear-in-control, nonlinear systems with unknown dynamics is the subject of this study. Such systems occur naturally in many applications such as robotics and aircraft controls.

The emphasis was on two very important aspects of adaptive control of nonlinear systems: minimization of the number of estimated parameters and the tracking of rapidly varying quantities.

The problem considered involves the adaptive pole-placement control of nonlinear systems of the form

$$J(\underline{y})\ D(d)\ \underline{y} + \underline{f}(\Phi(d)\underline{y}) = \underline{u} \tag{1.1}$$

where

$\underline{f} \in R^m$, an unknown nonlinear function

$J \in R^{m\times m}$, an unknown nonlinear matrix function

and

d = differential operator, d/dt

$D(d) = \text{diag.} \quad \{d^{ni}\}\ i = 1,2,\ldots,m$

ni = degree of the i-th diagonal element of D

m = number of inputs = number of outputs

$\underline{u} \in R^m$, input vector

$\underline{y} \in R^m$, output vector

$$\Phi(d) = \begin{bmatrix} I \\ \hline dI \\ \hline \vdots \\ \hline d^{n-1}I \end{bmatrix}$$

The dynamic system described by Eq. (1.1) is a multi-input, multi-output system. Since the nonlinear functions $\underline{f}$ and J contain unknown quantities, the need for adaptive control arises.

The following assumptions were made regarding the system (1.1):

1. the degrees (ni's) of the sub-systems are known;
2. the functions $\underline{f}$ and J are analytic; and
3. $J(\underline{y}) \neq 0$ for all $\underline{y}$

A knowledge of the degrees of the sub-systems is essential for the pole-placement compensation scheme. A study of the uncertainty in the order of the sub-systems is beyond the scope of this study.

The assumption regarding the analycity of the functions is essential from the view of the pole-placement compensation of nonlinear systems. The function J has to be inverted to obtain the input to the nonlinear system. Hence, $J(\underline{y})$ is assumed to be non-singular.

Objective of this Study

The objective of this study was to develop an adaptive pole-placement control algorithm for nonlinear systems governed by Eq. (1.1). To meet this objective, it was necessary to develop on-line estimation methods for the unknown nonlinear functions $\underline{f}$ and J which minimize the number of estimated parameters. The criterion of performance for the estimation methods was established to be the ability of the adaptive pole-placement algorithm to overcome poor estimates of $\underline{f}$ and J and still produce good steady-state and dynamic performance.

Exact Feedback Linearization

A pole-placement scheme can be devised for nonlinear systems of the form (1.1) by using a method known as exact feedback linearization (Somer, 1980). The input for such a scheme is given by:

$$\underline{u} = \underline{f} + J\underline{v} \tag{1.2}$$

where

$$\underline{v} = \Gamma_{n-1}\, d^{n-1}\,(\underline{y}_d - \underline{y}) + \ . \ . \ . \ + \Gamma_o(\underline{y}_d - \underline{y}) \tag{1.3}$$

The Γ's are chosen to yield the location of the closed-loop poles according to the characteristic equation

$$|D(d) + \Gamma_{n-1}\, d^{n-1} + \ . \ . \ . \ + \Gamma_o| = 0 \tag{1.4}$$

The method depends on complete knowledge of the nonlinear functions. If some parameters in the function $\underline{f}$ are unknown, the feedback linearization approach fails since $\underline{f}$ cannot be calculated explicitly. Further, when there is a mismatch between the actual and estimated values of $\underline{f}$, a steady-state error is introduced which may not be overcome by the location of the poles alone. Illustrative examples are given in Chapter V.

Adaptive Pole-Placement Algorithm

The problem resulting from mismatch may be overcome by using an adaptive pole-placement scheme, which is achieved by replacing the actual values of $\underline{f}$ and J in (1.2) with their estimated values $\hat{\underline{f}}$ and $\hat{J}$. The input to the system is modified as:

$$\underline{u} = \hat{\underline{f}} + \hat{J}\underline{v} \tag{1.5}$$

The values of $\hat{f}$ and $\hat{J}$ are estimated on-line. A brief description of the estimation schemes evaluated in this study is given in this chapter and a detailed discussion is provided in later chapters.

In this study, the function $\underline{f}$ was considered unknown. Recursive estimation schemes based on expanding $\underline{f}$ in a Taylor's series in terms of the state variables $\Phi(d)\underline{y}$ and estimation of the coefficients of the series may result in a large number of coefficients being estimated. The number of coefficients depends on the number of state variables as well as on the order of the truncated Taylor's series.

The first goal of this study was to reduce the number of parameters that must be estimated. To accomplish this goal in the estimation of $\underline{f}$, the following differential equation model for $\underline{f}$ was used

$$\dot{\underline{f}} = \underline{w}(t) \tag{1.6}$$

where $\underline{w}(t)$ is a vector of random variables. An on-line integral feedback estimation method was devised based on this model, which eliminates the steady-state error due to a mismatch in the estimation of $\underline{f}$. The function $\underline{f}$ can be estimated using an integral feedback approach as follows:

$$\dot{\hat{\underline{f}}} = \hat{J}^{-1} \, \Gamma_{-1} \, (\underline{y}_d - \underline{y}) \tag{1.7}$$

Integration of (1.7) provides the estimate of $\hat{\underline{f}}$. The characteristic equation (1.4) is modified to accommodate the estimated value as:

$$|d \, (D(d) + \Gamma \, (d)) + \Gamma_{-1}| = 0 \tag{1.8}$$

This method is referred to as the modified pole-placement method. For very quick error reduction between the actual and desired output, Γ_{-1} should be as high as possible. The highest value of Γ_{-1} is constrained by equation (1.8). This method of estimating $\underline{f}$ works very well for step inputs as well as for smooth inputs. When square wave inputs are used, the performance is poor.

The second goal of this study was to develop a control algorithm which provides for tracking of rapidly varying quantities; that is, a high performance estimator is needed. A steepest gradient algorithm should perform better than the integral feedback algorithm of (1.7) and (1.8). In this case, equation (1.7) is replaced by:

$$\dot{\hat{\underline{f}}} = -k \, \hat{J}^{-1}\underline{e} \tag{1.9}$$

where

$$\underline{e} = D(d)\underline{y} - \hat{J}^{-1}[\underline{u} - \hat{\underline{f}}] \tag{1.10}$$

The steepest gradient algorithm remains alert to changes in $\underline{f}$ over long periods of time and is also easy to implement. $D(d)\underline{y}$ is estimated using numerical differentiation.

The matrix $\hat{J}^{-1}$ in (1.10) is estimated using the recursive least squares (RLS) algorithm or its variant, the exponentially weighted least squares (ELS) algorithm. The recursive least squares algorithm converges fast but does not track time-varying quantities. The exponentially weighted least squares algorithm, though somewhat slower in convergence, tracks time-varying quantities well.

The theory of nonlinear transformations used to obtain the input (1.2) is discussed in Chapter III. The use of nonlinear transformations highlights the reasons why $\underline{f}$ and J should be analytic.

The methods described in this section were applied to two example problems: a two-link manipulator with revolute joints and an electrohydraulic velocity control system with a nonlinear load. The two-link manipulator has two inputs and two outputs and hence is considered to be a multi-input, multi-output system. The electro-hydraulic velocity control system is a single-input, single-output system. The dynamic equations for these systems are of the form (1.1). Studies were conducted through computer simulations.

Example Problem: Manipulator

A two-link manipulator in which the links are of equal length and geometry was considered. The load at the end of the manipulator was assumed to be unknown. The formulation of this problem is presented in Chapter V.

Pole-placement compensation using the exact linearization approach of Somer (1980) performed very well when all the functions and parameters were known. When some of the parameters were not known, this approach performed poorly. To illustrate the disadvantages of exact linearization when some parameters were unknown, the load at the end of the manipulator was assumed to be zero when it actually was not. The presence of this uncertainty in the load introduced a steady-state error when a step-input was applied to the second joint with the first joint fixed.

To improve the steady-state performance, the modified pole-placement method (equations (1.7) and (1.8)) developed in this study was used. The steady-state error was eliminated. This method performed very well for smooth inputs. When square wave inputs were applied to both joints, the performance of the modified pole-placement scheme was poor because the nonlinear functions $\underline{f}$ were not estimated quickly.

The use of the steepest gradient algorithm alleviated the problem associated with slow estimation of $\underline{f}$. The steepest gradient algorithm performed very well for both smooth and square-wave inputs.

The matrix J, which corresponds to the inertia matrix for the manipulator, varies with the angle of the second link. The recursive least squares (RLS) algorithm did not track this variation in the values of the inertia. The use of the ELS algorithm mitigated this difficulty.

The values of $D(d)\underline{y}$ in equation (1.10) were obtained by numerical differentiation from the next lower derivatives. The performance of the numerical differentiation in the presence of noise was very poor. The use of a filter reduced the effects of the noisy measurements. The trade-offs involved in filter selection are discussed in Chapter V.

Example Problem: Electrohydraulic Velocity Control System

The electrohydraulic velocity control system considered is modeled as second-order in the velocity with nonlinear coefficients that are dependent on the velocity and the fluid bulk modulus. The load on the system is nonlinear in the velocity. Air entrapment in the system and air entrainment in the hydraulic fluid result in considerable uncertainty in the effective value of the bulk modulus of the fluid. Typical values of bulk modulus encountered in practice vary between 1.372 X 10^8 N/m^2 (20,000 psi) and 1,715 X 10^3 N/m^2 (250,000 psi). In this example, the value of the bulk modulus was considered unknown. The formulation of the problem is described in Chapter V.

A conventional PID controller was designed for a bulk modulus value of 6.86 X 10^8 N/m^2 (100,000 psi) to satisfy the ITAE criterion (D'Azzo and Houpis). This bulk modulus value is approximately in the middle of the range of variations of the bulk modulus. The performance of this PID controller is good at the design value of 6.86 X 10^8 N/m^2 (25,000 psi) but is not good at bulk modulus values of 1.715 X 10^8 N/m^2 and 13.12 X 10^8 N/m^2. Since it is not possible to predict the value of the bulk modulus exactly, the need for adaptive control arises.

An adaptive controller using the steepest gradient algorithm for estimation of $\underline{f}$ and the recursive least squares algorithm for estimation of J^{-1} was developed for this system. The adaptive controller performed well over a range of values of bulk modulus. Detailed results are presented in Chapter V.

CHAPTER II

BASIS FOR STUDY

Background

There are many approaches for designing controllers for systems with nonlinear dynamics. The most common approach is to linearize the system about a nominal operating point and design a linear controller for the linearized system. This approach provides a design that has a limited operating range. That is, if the operating point strays too far from the nominal point, the system performance could be adversely affected.

A less common approach is to approximate the input-output characteristics of the nonlinear system with sinusoidal input describing functions and to design a linear controller for the approximate characteristics. While this approach produces a design with a somewhat expanded operating range, the result often is not sufficient.

Motivated by these concerns, two different approaches have been attempted in recent years. One approach used the simultaneous stabilization theory (Vidyasagar, 1982). Taylor (1983) and Nassirharand, et al. (1988) have used this theory and describing functions to find a class of linear compensators that satisfy a particular set of performance criteria at multiple nominal operating ranges. The approach has the drawback that it produces compensators with orders very much higher than that of the plant. Model reduction techniques have to be used to approximate the compensator. The design method is carried out off-line.

Another approach is to find nonlinear transforms that globally linearize the nonlinear systems for any given operating point; a nonlinear controller that satisfies linear control performance specifications is synthesized. Somer (1980) proposed a method that transforms a nonlinear system of the form

$$\dot{\underline{x}} = \underline{a}(\underline{x}) + B(\underline{x})\underline{u} \tag{2.1}$$

to a phase-variable canonical form.
The terms in (2.1) are defined as

$\underline{x} \in R^n$ is the state vector

$\underline{u} \in R^m$ is the input vector

$\underline{a} \in R^n$ is a vector of nonlinear functions

$\underline{B} \in R^{nxm}$ is a matrix of nonlinear functions

A pole-placement feedback controller is designed in the transformed domain and transformed back to the original domain. Certain integrability conditions have to be satisfied for the inverse transform to exist.

Hunt, et al. (1983) proposed a global linearization procedure about the origin that transforms a nonlinear system into a linear one over the whole state space. They obtained conditions for the existence of the transform. This method was generalized by Reboulet and Champetier (1984) whose "pseudolinearization" procedure transforms the nonlinear system into a linear one over the whole state space for any given operating point. The method consists of linearizing the system about a general operating point and transforming the system into a linear controller form as in Kailath (1980). The linear transform is then integrated to obtain a nonlinear transform. The transformed canonical system has the same characteristic equation as the locally linearized system.

A similar idea has been used by Baumann and Rugh (1986) to develop an "extended linearization" procedure which finds the feedback gains using Ackermann's formula and then integrates the gains to find the nonlinear input to the system. These ideas of global linearization are summarized in Zak and MacCarley (1986), who also proposed a non-unique partial linearization procedure which improves the performance of the system.

The global linearization methods require exact knowledge of the nonlinear functions and parameters. When unknown parameters are considered, constructing the linearizing transforms becomes a very difficult task (Nam and Arapostathis, 1987). Further, the computation of the nonlinear functions may turn out to be too complicated for on-line implementation. These considerations form the basis for the development of an adaptive pole-placement feedback controller. The motivation for considering nonlinear systems with unknown functions or parameters is provided later in this chapter.

The general type of nonlinear system for which the methods of this study are applicable is described by Eq. (2.1). This type of nonlinear system occurs naturally in many situations. They arise in aircraft control (Su and Hunt, 1984), open loop robotic manipulators (Nam and Arapostathis, 1987), as well as in radar tracking (Bowles and Cartreli, 1983).

The specific type of nonlinear system considered in this study is a sub-class of (2.1) and is described by:

$$J(\underline{y})\ D(d)\ \underline{y} + \underline{f}(\Phi(d)\underline{y}) = \underline{u} \qquad (2.2)$$

where

$\underline{f}$ is an unknown nonlinear function

and J is an unknown matrix function.

Available Methods

For the case of unknown $\underline{f}$ and J, the "pseudo-linearization" procedure of Reboulet and Chempetier (1984) and the "extended linearization" procedure of Baumann and Rugh (1986) provide possible ways of finding the unknown transform. The unknown functions could be assumed to be in Taylor's series form and the unknown coefficients of the Taylor's series could be estimated on-line. In the development of an adaptive control procedure for multi-input, multi-output systems, it is desirable to minimize the number of parameters that must be estimated on-line (Elliott and Wolovich, 1983). The drawback of trying to apply the "pseudo-linearization" and "extended linearization" procedures to estimate the unknown f and J is that a large number of parameters must estimated on-line. For example, if Baumann and Rugh's "extended linearization" procedure is used, then approximately

$$\sum_{i=1}^{m} (n^{2i} (i^2+1))$$

parameters must be estimated on-line.

Another characteristic of the type of nonlinear system considered is the rapid change in the quantities $\underline{f}$ and J. Chen and Norton (1987) describe various techniques for tracking rapid parameter changes. The methods are basically of the recursive least squares (RLS) type with modifications to detect parameter changes. Fortescu et al. (1981) proposed a "variable forgetting factor" scheme in which the weighting in the Recursive Weighted Least Squares (RWLS) method is adjusted so that old data is discarded in an orderly manner. This method does not distinguish between errors caused by large variations in parameters and a large noise level.

Anderson (1985) has proposed a Bayesian update method for "adaptive forgetting through multiple models" (AFMM) in which several parameter models with different probability densities are averaged to provide a single model parameter estimate. The number of estimated parameters increases linearly with the number of models used.

Isaksson (1987) proposed an adaptive Kalman filtering technique in which knowledge about likely variations of the parameters is assumed. Zheng (1987) and Xianya and Evans (1984) proposed methods based on Taylor's series expansion of the time-varying terms and co-variance resetting. The likely variations in parameters are characterized by the second moment or co-variance matrix.

Motivation

In practical situations, it is not possible to obtain the values of every relevant parameter exactly. In such a case, the linearizing transforms and describing function methods fail to provide a satisfactory solution. It may also not be possible to measure the values of the parameters on-line.

As an example, consider the two-link manipulator shown in Figure 1 (Appendix). It consists of two links of similar geometry having rotary joints about the same axis and carrying an unknown payload at its end. The dynamics of the manipulator is dependent on the payload. Hence, for control of the manipulator using any of the global linearization methods, an accurate estimate of the payload is essential. A method for measuring the mass of the payload on-line has been suggested by Paul (1981, pp. 225-9) under the assumption that the center of mass of the payload coincides with the center of the end-effector. For a two-link manipulator, this is equivalent to the center of mass of the payload being at the tip of the second link. This assumption may not be valid under all circumstances. Since the mass of the payload enters the dynamic equations in a nonlinear manner, the assumption that the functions are linear in the unknown parameters is also invalid. Youcef-Toumi, et al. (1987) discuss a method to adaptively control manipulators having unknown dynamics.

Another example arises in the modeling of a continuous stirred tank reactor (CSTR). Reactants produce an exothermic reaction. The heat of reaction is removed by a coolant flowing in the surrounding jacket (Figure 2, Appendix). On the assumption that the flow rate in is equal to the flow rate out, the mathematical model could be stated as follows (Ray, p. 7 and Stephanopoulos, pp. 59-64):

$$\dot{x}_1 = -x_1+\Delta(1-x_1)\exp(\Lambda x_2/(\Lambda+x_2))+d_1 \tag{2.3}$$

$$\dot{x}_2 = -(1+\gamma x_2+\rho\Delta(1-x_1 \exp(\Lambda x_2/\Lambda+x_2))+\gamma u+d_2 \tag{2.4}$$

where

x_1 = conversion of product 1 to product 2

x_2 = dimensionless reactor temperature

Δ, Λ, ρ and γ are parameters

u = jacket coolant temperature (input)

d_1 = disturbance in feed reactant concentration

d_2 = disturbance in feed temperature

The parameter Λ is dependent on the activation energy and cannot be determined accurately. The margin for error in estimating the value of Λ is about 25 percent and its value affects the stability of the equations. Thus, one is faced with the situation of having to estimate uncertain nonlinear parameters.

A third example is the electrohydraulic velocity control system subjected to unknown disturbances (Yun and Cho, 1988). A schematic of the control system consisting of a servovalve and a cylinder driving a load is shown in Figure 3 (Appendix). The load F_1 is a nonlinear function of the velocity $\dot{x}$ and the control objective is to make the velocity follow a desired trajectory.

The mathematical model for this system is:

$$MD^2\dot{x} + BD\dot{x} + \partial f/\partial\dot{x}\ D\dot{x} + \frac{4\beta}{V_t} A^2\dot{x} = \frac{4\beta}{V_t} A[Ki\sqrt{(P_s-P_1)}-C_tP_1] \tag{2.5}$$

Equation (2.5) is a second-order differential equation in the velocity $\dot{x}$ and the coefficients are nonlinear functions of $\dot{x}$ and its derivatives. The parameter β is seldom known accurately.

These three examples illustrate cases of nonlinear systems in which the models have coefficients which are rapidly varying and cannot be determined accurately beforehand. In such cases, an adaptive control approach is likely to be preferable over a non-adaptive approach.

CHAPTER III

THEORETICAL DEVELOPMENT

Very high performance control systems are developed using the exact linearization method on the assumption that all functions and parameters are known. Exact linearization is achieved through the use of linearizing transforms and output feedback. It may not be possible to determine the linearizing transforms if some parameters of the nonlinear functions are not known. However, the input determined through the use of linearizing transforms is useful in adaptive control. The theory of the linearizing transforms as well as effect of unmodeled dynamics on the exact linearization method are discussed in this chapter. The adaptive pole-placement control algorithm developed in this study is outlined.

Linearizing Transforms

In this section, a method for global feedback linearization which is directly applicable to the system under consideration is presented. The theory given by Zak and MacCarley (1986) for the global linearization of nonlinear systems is summarized. The linearization aspect leads to the pole-placement control law. A nonlinear system is considered which is described by:

$$\dot{\underline{x}} = \underline{a}(\underline{x}) + B(\underline{x})\underline{u}(t) \tag{3.1a}$$

$$\underline{y} = \underline{c}(\underline{x}) \tag{3.1b}$$

where

$\underline{x} \in R^n$, a state vector

$\underline{a}(\underline{x}) \in R^n$, smooth functions of $\underline{x}$

$B(\underline{x}) \in R^{nxm}$, smooth matrix functions of $\underline{x}$

$\underline{c}(\underline{x}) \in R^m$, smooth vector functions of $\underline{x}$

$\underline{u} \in R^m$, the input vector

$\underline{y} \in R^m$, the output vector

It is desired to transform this system (3.1), to a globally state equivalent linear system

$$\dot{\underline{x}}^* = A\underline{x}^* + D\underline{v} \tag{3.2}$$

where

$\underline{x}^* \in R^n$, the new state vector

(A,E), a controllable pair

$\underline{v} \in R^m$, the new reference input

Notation:

For two vector functions $\underline{s}$ and $\underline{t} \in R^n$, the Lie bracket $[\underline{s},\underline{t}]$ is defined by:

$$[\underline{s},\underline{t}] = \frac{\partial \underline{t}}{\partial x}\,\underline{s} - \frac{\partial \underline{s}}{\partial x}\,\underline{t}$$

The Lie bracket is also denoted by:

$$[\underline{s},\underline{t}] = [ad^1\underline{s},\underline{t}]$$

$$(ad^k\underline{s},t) = [\underline{s},(ad^{k-1}\underline{s},\underline{t})]$$

$$(ad^o\underline{s},\underline{t}) = \underline{t}$$

The Lie derivative of a scalar field h with respect to a vector field $\underline{s}$ is:

$$<dh,\underline{s}> = \partial h/\partial x_1 s_1 + \ . \ . \ . + \partial h/\partial x_n\ s_n$$

The Lie derivative is also denoted as:

$$L_s h = <dh,s>$$

A useful result is:

$$L_{[\underline{s},\underline{t}]}h = L_s L_t h - L_t L_s h$$

or

$$<dh,[\underline{s},\underline{t}]> = <d<dh,\underline{t}>,\underline{s}> - <d<dh,\underline{s}>,\underline{t}>$$

Under the assumption that

$$L_{bi}L_a^k c = 0,\ k = 0,1,...,ni -1$$

$$L_{bi}L_f^{ni}c \neq 0,$$

the transformation,

$$T(\underline{x}) = [\underline{c}, L_a\underline{c}, \ldots, L_a^{n-1}\underline{c}](\underline{x}) \qquad (3.3)$$

and the input,

$$u_k = 1/L_{gk}L_f^{n-k}L \; (V_k - \sum_{j=k+1}^{m} u_j L_{gj} L_f^{n-k} h), \; 1 \leq k \leq m \qquad (3.4)$$

applied to the system (3.1) leads to (3.2).

As pointed out in Chapter II, the system of equations considered in this study are of the form:

$$J(\underline{y})D(d)\underline{y} + \underline{f}(\Phi(d)\underline{y}) = \underline{u}(t) \qquad (3.5)$$

The orders (ni) of all subsystems are assumed to be equal. This implies that all the assumptions are satisfied and

$$L_a^{ni}c = f_i \qquad (3.6)$$

The input to the system is:

$$\underline{u} = \underline{f} + J\underline{v} \qquad (3.7)$$

The reference input $\underline{v}$ is chosen as:

$$\underline{v} = \Gamma_{n-1}(\underline{y}_d^{(n-1)}(t) - \underline{y}^{(n-1)}(t)) + \ldots + \Gamma_1(\dot{\underline{y}}_d(t) - \dot{\underline{y}}(t))$$

$$+ \Gamma_0(\underline{y}_d(t) - \underline{y}(t)) = \Gamma(d)(\underline{y}_d - \underline{y}) \qquad (3.8)$$

where y_d is the desired output vector. The closed loop characteristic equation is:

$$|S(d)| = |D(d) + \Gamma(d)| = 0 \qquad (3.9)$$

With the control law (3.7), the closed loop system is globally feedback equivalent to:

$$S(d)\underline{y}(t) = \Gamma(d)\ \underline{y}_d(t) \tag{3.10}$$

Effects of Unmodeled Dynamics

In the previous section, the nonlinear functions $\underline{f}$ and J in (3.5) are completely compensated by the nonlinear feedback (3.7). No effort is made to account for the difference between the estimated values and the actual values of the nonlinear functions. The actual values could include unmodeled dynamics and such dynamics could seriously affect the performance of the system.

Let $\hat{J}$ and $\hat{\underline{f}}$ denote the estimated values of J and $\underline{f}$ respectively. The input to the system is:

$$\underline{u} = \hat{\underline{f}} + \hat{J}\underline{v} \tag{3.11}$$

Substituting (3.11) into (3.5), the actual dynamics of the system under this control law is:

$$D(d)\underline{y} = J^{-1}(\underline{y})[\hat{\underline{f}}+\hat{J}(\underline{y})\ \Gamma(d)\ (\underline{y}_d - \underline{y})]$$

$$- J^{-1}(\underline{y})\ \underline{f}(\Phi(d)\underline{y})$$

$$= J^{-1}(\underline{y})\ \tilde{\underline{f}}(\Phi(d)\underline{y})+J^{-1}\ \hat{J}\ \Gamma(d)(\underline{y}_d - \underline{y}) \tag{3.12}$$

where

$$\tilde{\underline{f}} = \hat{\underline{f}} - \underline{f} \tag{3.13}$$

An analysis of the transients is very difficult. In the steady-state, the error is dependent on $\tilde{\underline{f}}$, the difference between the actual and computed values of $\underline{f}$. Since this method is based on exact compensation of $\underline{f}$, either Γ_o has to be very high or the nonlinear functions have to be estimated on-line. Some estimation schemes for $\underline{f}$ and J are discussed in Chapter IV.

Modeling

In case some parameters of the function $\underline{f}(\Phi(d)\underline{y})$ are unknown, the feedback linearization approach fails, since the linearizing functions cannot be calculated explicitly. If the functions are linear in the unknown parameters, then it is possible to devise a method to estimate the unknown parameters on-line. Such a scheme for estimation and control is developed by Nam and Arapostathis (1987). But, as explained in Chapter II, it may not be possible to express the nonlinear functions as linear in the unknown parameters. In such a case, the entire nonlinear function may have to be treated as unknown.

Consider the case when $\underline{f}$ in (3.5) is unknown. Following Baumann and Rugh (1986), the functions $\underline{f}$ may be expanded in Taylor's series in terms of the state variables $(\Phi(d)\underline{y})$. The coefficients of the expansion could be considered unknown and a recursive estimation scheme devised to estimate the unknown coefficients. Depending on the order of the truncated Taylor's series, a large number of coefficients may have to be estimated.

A convenient method of reducing the number of parameters, that is commonly used for linear time-varying systems, is to assume differential equation models for the unknown parameters or functions. The differential equation model in this case is of the form:

$$\underline{\dot{f}} = \underline{Gf} + \underline{w}(t) \tag{3.14}$$

where $\underline{w}(t)$ is a white noise process. Often G is taken to be zero and the noise w(t) introduces uncertainty about the consistency of f (Mendel, 1987). When G = 0, (3.14) is of the form:

$$\underline{\dot{f}}(t) = \underline{w}(t) \tag{3.15}$$

and only a few parameters are estimated on-line.

A Modified Pole-Placement Scheme

An evaluation of equation (3.12) shows that any mismatch in the computation of $\underline{f}$ leads to steady-state errors. An on-line estimation scheme to minimize the steady-state errors in the presence of such a mismatch is detailed in this section. A modified pole-placement scheme based on this estimation along with its limitations is also discussed in this section.

Again looking at equation (3.12), if the estimation of $\underline{f}$ is tied to the error between the desired and actual output values, then the steady-state error due to the mismatch in $\underline{f}$ can be eliminated. Such an estimator for $\underline{f}$ is given by:

$$\dot{\hat{\underline{f}}} = \hat{J}^{-1} \, \Gamma_{-1} \, (\underline{y}_d - \underline{y}) \tag{3.16}$$

The estimate $\hat{\underline{f}}$ obtained from this equation is then used as part of the control input to the system. The nonlinear functions $\underline{f}$ are not explicitly calculated on-line. Instead, integration of equation (3.16) provides the estimates of $\hat{\underline{f}}$.

In order to establish the selection of Γ_{-1} and its effects on the speed of convergence, let $\underline{x} = \Phi(d)\underline{y}$ denote the states of the system (3.5) under the influence of the control input (3.11). Then

$$\dot{\hat{\underline{x}}} = A\hat{\underline{x}} + B_1 \, \hat{\underline{f}}(\hat{\underline{x}}) + B_2 \, \underline{y} + B_1 \, \underline{f} \, (\hat{\underline{x}}) \tag{3.17}$$

where

$$A = \text{diag } \{A_i\} \; i = 1,2,\ldots,m$$

$$A_i = \begin{bmatrix} 0 \; 1 \; 0 \; . \; . \; . & 0 \\ 0 \; 0 \; 1 \; 0 \; . \; . & 0 \\ & . \\ & . \\ & 1 \\ 0 . \; . \; . \; . \; . \; . & 0 \end{bmatrix} \in R^{nixni}$$

$$B_1 = \text{col } [B_{1i}] \; , \; i = 1, 2, \ldots ,m$$

$$B_{1i} = 0_{nixm} \quad , \; i = 1,2,\ldots,m-1$$

$$B_{1m} = J^{-1}$$

$$B_2 = \text{col}\ [B_{2i}]\ ,\ i = 1,2,\ .\ .\ .,m$$

$$B_{2i} = 0_{ni\times m}\qquad ,\ i = 1,2,\ .\ .\ .,m-1$$

$$B_{2m} = J^{-1}\ \hat{J}^{-1}$$

Augmenting (3.16) with (3.17), the combined system of equations is given by:

$$\begin{bmatrix} \dot{\hat{\underline{x}}} \\ \dot{\hat{\underline{f}}} \end{bmatrix} = \begin{bmatrix} A_f & B_1 \\ E & 0 \end{bmatrix} \begin{bmatrix} \hat{\underline{x}} \\ \hat{\underline{f}} \end{bmatrix} + \begin{bmatrix} B_2 \\ \hline 0 \end{bmatrix} \Gamma\ (d)\underline{y}_d + \begin{bmatrix} B_1 \\ \hline 0 \end{bmatrix} \underline{f} \quad (3.18)$$

where $A_f = A+\hat{J}^{-1}\Gamma(d)$

and $E = [J^{-1}\Gamma_{-1}\ 0.\ .\ .\ 0]\ \epsilon\ R^{m\times n}$

Ideally, Γ_{-1} should be as high as possible in order to quickly follow any mismatch between the desired output and the actual output. The highest value of Γ_{-1} is constrained by (3.18). The characteristic equation for the augmented system (3.18) is given by

$$|d(D(d) + \Gamma(d))+ \Gamma_{-1}| = 0 \qquad (3.19)$$

The gain Γ_{-1} is also included in the characteristic equation. This leads to a constraint on Γ_{-1} that is similar to the stability problem using integral feedback for disturbance rejection in linear systems (Kailath, 1980) Equation (3.19) is the modified pole-placement scheme.

The effect of a mismatch in the estimated and the actual values of J affects only the transient performance of the system. A more elaborate scheme than the simple integral feedback scheme that is used for $\underline{f}$ is needed. The least squares scheme discussed in the next chapter is used for the estimation of J.

Adaptive Pole-Placement Control

The use of on-line estimation schemes for $\underline{f}$ and J and the application of these estimated values in the control law (3.11) along with the pole-placement equations (3.9) and (3.19) lead to an adaptive pole-placement controller. When the integral feedback is used as the estimator for $\underline{f}$, (3.19) is used as the pole-placement equation to ensure the stability of the system.

When faster convergence of $\hat{\underline{f}}$ to the true value is desired, an estimation scheme like the gradient algorithm described in the next chapter could be used. In such a case, (3.9) is used for pole-placement.

In the integral feedback scheme, the output converges only when the estimated value $\hat{\underline{f}}$ converges to the true value. In (3.12), when $\hat{\underline{f}} \to \underline{f}$, $\tilde{\underline{f}} \to 0$ and $\underline{y}_d \to \underline{y}$ since $\tilde{D}(d)\underline{y} \to 0$. The rate of convergence of $\tilde{f} \to 0$ may not be satisfactory in the case of varying $\underline{f}$.

The gradient algorithm and the recursive least squares algorithm and their properties and application to equation (3.5) are described in the next chapter.

CHAPTER IV

ESTIMATION METHODS

The various parameter estimation methods used to estimate the functions $\hat{f}$ and $\hat{J}$ in equation (3.11) are presented in this chapter. The steepest gradient algorithm is used for estimating $\hat{f}$ and the recursive least squares (RLS) and exponentially weighted least squares (ELS) algorithms are used for estimating $\hat{J}^{-1}$. $\hat{J}$ is obtained by the inversion of $\hat{J}^{-1}$. The selection criteria for the gains of these algorithms are also provided.

Steepest Gradient

Suppose the parameters to be estimated are related to the system output as:

$$\underline{y} = \Theta^T(t)\ \underline{\phi}(t) \tag{4.1}$$

where Θ represents the parameters, $\underline{\phi}(t)$ represents the inputs and $\underline{y}$ represents the outputs. The parameter estimates $\hat{\Theta}(t)$ must minimize a cost function of the form:

$$V = [\underline{y} - \hat{\Theta}^T\underline{\phi}]^T[\underline{y} - \hat{\Theta}^T\underline{\phi}] \tag{4.2}$$

Let $\underline{e}$ denote the error vector introduced by not using the actual parameters, that is:

$$\underline{e} = \underline{y} - \hat{\Theta}\,\underline{\phi} \tag{4.3}$$

Then, (4.2) can be written as:

$$V = \underline{e}^T \underline{e} \tag{4.4}$$

Minimizing (4.4) with respect to Θ, $\hat{\Theta}$ must satisfy:

$$\frac{\partial V}{\partial \Theta} + [\hat{\Theta} - \Theta]^T \left[\frac{\partial^2 V}{\partial \Theta^2} \right] = 0 \tag{4.5}$$

Solving (4.5) for Θ,

$$\hat{\Theta} = \Theta - \left[\frac{\partial^2 V}{\partial \Theta^2} \right]^{-1} \frac{\partial V}{\partial \Theta} \tag{4.6}$$

Θ is the old estimate and $\hat{\Theta}$ is the new estimate.

For the steepest gradual algorithm, the Hessian matrix

$$\left[\frac{\partial^2 V}{\partial \Theta^2} \right]$$

is replaced by a constant 2/k. From (4.4) and (4.3),

$$\frac{\partial V}{\partial \Theta} = k\underline{e} \frac{\partial e}{\partial \Theta}$$

$$= k\underline{e\phi}^T \qquad (4.7)$$

For continuous time systems, equation (4.6) can be rewritten as:

$$\dot{\hat{\Theta}} = k\underline{e\phi}^T \qquad (4.8)$$

A selection criterion for k is:

$$0 < k < \frac{2}{\phi^T \phi}$$

The steepest gradient algorithm exhibits superior performance in the sense $\hat{\Theta}$ has better global convergence than other algorithms. However, it converges very slowly in the vicinity of the minimum (Sorenson, p. 66). The algorithm is alert to variations in Θ with time as any error introduced is due to the wrong estimate of Θ and k is a constant. This property is useful for estimating the nonlinear function $\underline{f}$ in (3.5). The nonlinear functions vary with time and the algorithm needs to estimate $\underline{f}$ for any value.

Recursive Least Squares

Another parameter estimation method that is commonly used for systems of the type (4.1) is the recursive least squares algorithm. The algorithm involves updating the parameters as well as a related covariance matrix. Since the estimation method is carried out on-line and it is not desirable to integrate too many quantities on-line, a discretized version of the method is presented.

For a multi-output system, the output $\underline{y}$ is an m-vector, the regressor $\underline{\phi}$ is an n-vector and the parameter matrix Θ is of order n x m. The parameters are estimated by minimizing the following cost function: (Goodwin and Sin, p. 94)

$$J_N(\Theta) \overset{\Delta}{=} 1/2 \sum_{i=1}^{N} (\underline{y} - \Theta^T \underline{\phi})^T (\underline{y} - \Theta^T \underline{\phi}) \tag{4.9}$$

The estimate $\hat{\Theta}(N)$ is obtained by differentiating (4.9) with respect to Θ, setting the derivative equal to zero and solving the resulting equation.

$$\frac{\partial J_N(\Theta)}{\partial \Theta} = -\sum_{i=1}^{N} (\underline{y} - \Theta^T \underline{\phi}) \underline{\phi}^T = 0 \tag{4.10}$$

$$\hat{\Theta}(N) = [\sum_{i=1}^{N} \underline{\phi}(i)\underline{\phi}^T(i)]^{-1} \sum_{i=1}^{N} \underline{\phi}(i) \underline{y}^T(i) \tag{4.11}$$

To obtain a recursive solution, consider $\hat{\Theta}(N-1)$

$$\hat{\Theta}(N-1) = [\sum_{i=1}^{N-1} \underline{\phi}(i)\underline{\phi}^T(i)]^{-1} \sum_{i=1}^{N-1} \underline{\phi}(i) \underline{y}^T(i) \tag{4.12}$$

Defining,

$$P^{-1}(N) = \sum_{i=1}^{N} \underline{\phi}(i) \underline{\phi}^T(i) \tag{4.13}$$

equation (4.11) can be rewritten as:

$$\hat{\Theta}(N) = [P^{-1}(N-1) + \underline{\phi}(N)\,\underline{\phi}^T(N)]^{-1}\,[\sum_{i=1}^{N-1} \underline{\phi}(i)\underline{y}^T(i) + \underline{\phi}(N)\underline{\phi}^T(N)] \qquad (4.14)$$

and $P^{-1}(N) = P^{-1}(N-1) + \underline{\phi}(N)\,\underline{\phi}^T(N)$ (4.15)

Using the matrix inversion lemma,

$$P(N) = [P^{-1}(N-1) + \underline{\phi}(N)\,\underline{\phi}^T(N)]^{-1}$$

$$= P(N-1) - \frac{P(N-1)\,\underline{\phi}(N)\underline{\phi}^T(N)\,P(N-1)}{1 + \underline{\phi}^T(N)\,P(N-1)\,\underline{\phi}(N)} \qquad (4.16)$$

$$\hat{\Theta}(N) = P^{-1}(N)\,[\sum_{i=1}^{N-1} \underline{\phi}(i)\,\underline{y}^T(i) + \underline{\phi}(N)\,\underline{\phi}^T(N)]$$

$$= \hat{\Theta}(N-1) + \frac{P(N-1)\,\underline{\phi}(N)\,\underline{e}^T(N)}{1 + \underline{\phi}^T(N)\,P(N-1)\,\underline{\phi}(N)} \qquad (4.17)$$

Equations (4.16) and (4.17) form the recursive least squares algorithm. (4.17) updates the parameter estimates and (4.16) updates the associated covariance matrix based on the error between the current output measurement $\underline{y}(N)$ and the predicted output $\Theta^T(N-1)\,\underline{\phi}(N)$.

The following key properties can be established for the least squares algorithm (Goodwin and Sin, pp. 60-61):

1. $||\hat{\Theta}(t) - \Theta||^2 \leq k_1 ||\hat{\Theta}(o) - \Theta||^2$ (4.18)

where k_1 = condition number of $P^{-1}(0)$

2. $$\lim_{n \to 1} \sum_{i=1}^{N} \frac{\underline{e}^T(t)\underline{e}(t)}{1 + \underline{\phi}^T(t)\, P(t-1)\, \underline{\phi}(t)} < 1 \qquad (4.19)$$

3. $$\lim_{t \to 1} \frac{\underline{e}(t)}{[1 + \underline{\phi}(t-1)\, \underline{\phi}^T(t-1)]^{1/2}} = 0 \qquad (4.20)$$

4. $$\lim_{t \to 1} ||\hat{\Theta}(t) - \hat{\Theta}(t-k)|| = 0 \text{ for any finite } k \qquad (4.21)$$

These properties imply that $\hat{\Theta}(N)$ converges to some value, but this value need not be the true value. The matrix P(N) in (4.16) tends to zero as $N \to \infty$. This poses a problem in the estimation of time-varying parameters as P(N) should be non-zero for the parameters to be updated.

Exponentially Weighted Least Squares

For time varying systems, the recursive least squares algorithm is not suitable as the parameter estimates are not updated after a long period of time. To alleviate this problem, a weighted least squares algorithm in which the old data are weighted less as compared to the more recent data, is used.

The cost function to be minimized is (Ljung and Soderstrom, 1983):

$$J_N(\Theta) = \frac{1}{2} \sum_{i=1}^{N} \lambda^{N-i}[y(i)-\Theta^T\phi]^2 \qquad (4.22)$$

where λ is chosen to be less than 1. As N becomes very large, $\lambda^{N-i} \rightarrow 0$, and the old data are weighted less compared to the new data. By proper selection of λ, the old data can be effectively discarded. The parameter λ is called the forgetting factor. Repeating the calculations of the previous section, the following parameter and covariance update equations are obtained:

$$\hat{\Theta}(N) = \hat{\Theta}(N-1) + \frac{P(N-1)\underline{\phi}(N)\ \underline{e}^T(N)}{\lambda + \underline{\phi}^T(N)P(N-1)\underline{\phi}(N)} \qquad (4.23)$$

and $$P(N) = \{P(N-1) - \frac{P(N-1)\underline{\phi}(N)\ \underline{\phi}^T(N)P(N-1)}{\lambda + \underline{\phi}^T(N)P(N-1)\underline{\phi}(N)}\}/\lambda \qquad (4.24)$$

A potential disadvantage of exponential forgetting is that old data are discounted even if there is no information in the new data. The algorithm does not behave very well where there are long periods with no excitation. The estimator will forget the proper values of the parameters and the uncertainties will grow. This is called estimator wind up (Astrom and Wittenmark, 1988) and can be understood from (4.24). If there is no information in the last measurement, then $P(N-1)\underline{\phi}(N)$ will be zero and (4.24) reduces to $P(N) = (P(N-1)/\lambda$. Since $\lambda < 1$, P(t) will grow exponentially until $\underline{\phi}$ changes. As P(N-1) also influences the parameter updates in (4.23), there may be large changes in the estimated parameters leading to a burst in the output. The parameter λ is usually chosen close to 1, giving an exponential decay time constant of

$$\tau_o = \frac{1}{1-\lambda} \qquad (4.25)$$

Hence a prediction error older than τ_o time units has a weight less than 36% of the most recent data.

Application to Nonlinear Systems

The methods of the three previous sections can be applied to nonlinear systems of the form (3.5). Since the nonlinear functions $\underline{f}(\Phi(d)\underline{y})$ in (3.5) are time-varying and the algorithm has to be alert to these time variations, the steepest gradient algorithm is used to estimate them. The values of J are not critical for the steady state convergence of the outputs. Hence, the recursive least squares algorithm is used for the estimation of $J(\underline{y})$. The algorithm for estimating f requires that $J(\underline{y})$ be known and the estimation for $J(y)$requires $\underline{f}(\Phi(d)\underline{y})$. Since these values are not known, the two algorithms are boot-strapped by using their estimated values in the algorithms.

The estimation of $\underline{f}(\Phi(d)\underline{y})$ is carried out using the steepest gradient algorithm. Rewriting (3.5) as:

$$D(d)\underline{y}\ (t) = J^{-1}(\underline{y})[\underline{u} - \underline{f}(\Phi(d)\ \underline{y})] \qquad (4.26)$$

the steepest gradient algorithm can be developed. The error vector is defined as:

$$\underline{e} = D(d)\underline{y} - \hat{J}^{-1}(\underline{y})\,[u - \underline{f}(\Phi(d)\underline{y}] \tag{4.27}$$

and the cost function to be minimized is:

$$V = [1/2]\underline{e}^T\,\underline{e}$$

The steepest gradient algorithm is:

$$\dot{\hat{\underline{f}}} = \frac{k\partial\underline{e}}{\partial\hat{\underline{f}}}\,\underline{e} \tag{4.28}$$

The partial derivative $\frac{\partial \underline{e}}{\partial \hat{\underline{f}}}$ from (4.27) is:

$$\partial\underline{e}/\partial\hat{\underline{f}} = \hat{J}^{-1}(y) \tag{4.29}$$

Substituting (4.29) into (4.28),

$$\dot{\hat{\underline{f}}} = -k\,\hat{J}^{-1}(y)\,\underline{e} \tag{4.30}$$

This equation is the steepest gradient algorithm for estimating $f(\Phi(d)\underline{y})$.

The estimation of $J^{-1}(y)$ is carried out using the recursive least squares algorithm. The variable t in (4.26) could represent discrete instants of time. The error vector $\underline{e}$ is defined as in (4.27). The following equivalences could be made between (4.27) and (4.3).

$$\hat{\Theta}^T = \hat{J}^{-1}$$

$\underline{\phi} = [\underline{u} - \hat{\underline{f}}]$

Thus, the recursive least squares algorithm can be used to estimate $J^{-1}(\underline{y})$ after the necessary modifications have been made.

Estimation of Highest Derivatives

The steepest gradient and the recursive least squares estimation algorithms use the quantity $D(d)\underline{y}$, the highest derivative of the outputs, in estimating the parameters. Often, the highest derivatives of the outputs are not available for measurement. In this section, two methods of estimating $D(d)\underline{y}$ are outlined. The first one is numerical differentiation in which the next lower derivative is differentiated to obtain $D(d)\underline{y}$. The second is a filtering technique in which the next lower derivative of the output is passed through a filter to obtain $D(d)\underline{y}$

Numerical Differentiation

Let

$$\underline{q} = D(d)\underline{y} \tag{4.31}$$

denote the vector of the highest derivative of the outputs $\underline{y}$. This vector has to be estimated from the measured states. Let t_s be the sampling time and let

$$\underline{w} = d^{n-1}\underline{y}(t) \tag{4.32}$$

denote the vector of the next lower derivative that is measured. The vector $\underline{q}$ is obtained through numerical differentiation as:

$$\underline{q} = (\dot{\underline{w}}(t) - \underline{w}(t-1))/t_s \qquad (4.33)$$

The numerical differentiation scheme is very easy to implement on a digital computer. In practice, measurement noise in the vector $\underline{w}(t)$ could affect the estimate of $\underline{q}(t)$ and hence all the other estimated parameters. If the measurement noise is very high some filtering technique would have to be used.

Filtering

The vector $\underline{q}(t)$ is obtained by filtering the vector $\underline{w}(t)$. A first order filter with a pole at -p is used. Thus, $\underline{q}(t)$ is given by:

$$\underline{q}(t) = \frac{1}{s + p} \underline{w}(t) \qquad (4.34)$$

where s is the Laplace operator. Again, for purposes of digital computer implementation, this filter is discretized. In order to discretize the filter, Tustin's approximation (Astrom and Wittenmark, p. 176) is used. The Tustin's approximation is:

$$s = \frac{2}{t_s} \quad \frac{(z-1)}{(z+1)} \tag{4.35}$$

where z is the z-transform operator. Substituting (4.35) in (4.34),

$$\underline{q}(t) = \frac{1}{\frac{2}{t_s} \frac{(z-1)}{(z+1)} + p} \underline{w}(t)$$

from which $\underline{q}(t)$ is given by:

$$\underline{q}(t) = \{[\underline{w}(t) + \underline{w}(t-1)] - (p - 2/t_s)\underline{z}\} / (p + 2/t_s) \tag{4.36}$$

The highest order derivative thus obtained could then be used in the estimation schemes of (4.30) and (4.3).

The selection of the pole is dependent upon the frequency content of $\underline{w}(t)$. The rule of thumb is that p should be 10 times the bandwidth of $\underline{w}(t)$. The use of this filter helps in attenuating the effects of the measurement noise $\underline{w}(t)$ for estimation of $\underline{q}(t)$. It does not affect the feedback to the system. The filter would enhance the performance of the estimation methods in the presence of noise while diminishing the performance in the absence of measurement noise.

CHAPTER V

APPLICATIONS OF THE METHOD

The dynamic equations of articulated robotic manipulators and electrohydraulic velocity control systems have the same structure as equation (3.5). A two-link manipulator whose dynamic equations are well documented (Brady, et. al, 1982) is used to demonstrate the applicability of the control algorithm developed. In addition, an electrohydraulic velocity control system whose dynamic equations were developed in Chapter 2, is used to demonstrate the versatility of the algorithm.

Example Problem: Two-link manipulator

Figure 1 shows a schematic of the two-link mainpulator. Assuming that the manipulator links are rigid, excluding the dynamics of the control devices, and neglecting friction and backlash, the equations of motion of a manipulator are a set of coupled, second-order, nonlinear differential equations. The equations include inertia loading, coriolis and centrifugal coupling forces between joints and gravity loading effects. The torques/forces to be applied depend on the joint position, velocity and acceleration, as well as on the payload.

For the two-link manipulator shown in Figure 1, let the vector $\underline{\theta} = [\theta_1, \theta_2]^T$ denote the instantaneous position of the joints. Let $\dot{\underline{\theta}}$ and $\underline{\theta}^{(2)}$ denote the vectors of instantaneous joint velocities and joint accelerations respectively. The two links are assumed to be of equal length and to have the same geometry. The dynamic equations are given by:

$$J(\underline{\theta})\ \underline{\theta}^{(2)} + \underline{c}(\underline{\theta}, \dot{\underline{\theta}}) + g(\underline{\theta}) = \underline{u} \qquad (5.1)$$

where $J(\underline{\theta})$ is the inertia matrix whose elements are given by:

$$J_{11}(\underline{\theta}) = 2I + ml^2(\tfrac{3}{2} + \cos\theta_2) + m_L l^2(2 + \cos\theta_2) \qquad (5.2)$$

$$J_{12}(\underline{\theta}) = I + ml^2(\tfrac{1}{4} + \tfrac{1}{2}\cos\theta_2) + m_L l^2(2 + \cos\theta_2) = J_{21}(\underline{\theta}) \qquad (5.3)$$

$$J_{22}(\theta) = I + \tfrac{1}{4}ml^2 + m_L l^2 \qquad (5.4)$$

The term $\underline{c}(\underline{\theta}, \dot{\underline{\theta}})$ is the vector of coriolis and centrifugal forces and is given by:

$$c_1(\underline{\theta}, \dot{\underline{\theta}}) = -ml^2\ \sin\theta_2(\tfrac{1}{2}\dot{\theta}_2{}^2 + \dot{\theta}_1\dot{\theta}_2) - m_L l^2 \sin\theta_2(\dot{\theta}_2{}^2 + \dot{\theta}_1\ \dot{\theta}_2) \qquad (5.5)$$

$$c_2(\underline{\theta}, \dot{\underline{\theta}}) = -\tfrac{1}{2}ml^2\ \dot{\theta}_1{}^2\sin\theta_2 - m_L l\dot{\theta}_1{}^2\sin\theta_2 \qquad (5.6)$$

The term $\underline{g}(\underline{\theta})$ is the vector of gravitational forces and is given by:

$$g_1(\underline{\theta})=\frac{1}{2}mgl\ (\cos\ (\theta_1+\theta_2)+3\cos\theta_1)$$

$$+m_Lgl(\cos(\theta_1+\theta_2)\ +\ \cos\theta_2) \qquad (5.7)$$

$$g_2(\underline{\theta})=\frac{1}{2}mgl\cos(\theta_1+\theta_2)+m_Lgl\cos(\theta_1+\theta_2) \qquad (5.8)$$

The term $\underline{u}$ is the vector of torque inputs to the manipulator.

The various parameters are:

I = inertia of each link

m = mass of each link

g = acceleration due gravity (9.8/m/sec.)

l = length of each link

m_L = mass of payload

In equation (5.1), the vector of Coriolis and centrifugal forces, $\underline{c}(\underline{\theta}, \underline{\theta})$ is highly nonlinear and coupled. The gravitational loading effects, given by $\underline{g}(\theta)$, are dependent only on the instantaneous joint positions. The inertia loading matrix $J(\underline{\theta})$ is also dependent only on the instantaneous joint positions.

Equations (5.1 through 5.8) are used for simulation. When the applied torques $\underline{u}(t)$ are given, (5.1) is solved to obtain $\underline{\theta}^{(2)}$ and then the equations are integrated simultaneously to obtain the actual motion $\underline{\theta}(t)$ of the joints. From the viewpoint of controller design, equation (5.1) is of the form of equation (3.5). The sum of $\underline{c}(\underline{\theta}, \dot{\underline{\theta}})$ and $\underline{g}(\underline{\theta})$ is equivalent to $\underline{f}$ in equation (3.5). The nonlinear effects can be compensated exactly when all the parameters are known.

The input to the manipulator is generated according to equation (3.7). Such a controller for manipulators was proposed by Freund (1982). In this study, the nonlinear forces $\underline{f}$ and the inertia matrix J are assumed to be unknown and are estimated on-line. The estimated values are then used to generate the input to the manipulator according to equation (3.11).

The dynamics of open-loop robotic manipulators are representative of the complexity of the nonlinear systems considered in this study. Examples are presented in this section to illustrate the problems associated with feedback control through exact linearization and the advantages provided by adaptive control in overcoming such difficulties. The drawbacks of the adaptive control method when noisy measurements are used for estimation are also discussed.

For the examples, the manipulator links were assumed to be of length 1 meter and mass 1 kg. The term $\Gamma(d)$ in (3.8) is of the form:

$$\Gamma(d) = \Gamma_1 d + \Gamma_0 \tag{5.9}$$

where

$$\Gamma_1 = \text{diag}\ \{2\zeta_i \omega_{ni}\}, \quad i=1,2 \tag{5.10}$$

$$\Gamma_0 = \text{diag}\ \{\omega_{ni}^2\}, \quad i=1,2 \tag{5.11}$$

The values of ζ_i and ω_{ni} used were: $\zeta_1=\zeta_2=1.0$, $\omega_{n1}=\omega_{n2}=10.0$.

Various types of input trajectories $\underline{\theta}_d$ are used. The step input is used to illustrate that a steady-state error occurs when the exact linearization technique (3.7) is used and there is a mismatch between the estimated payload and its actual value. A continuous input is used to study how the modified pole-placement scheme tracks time-varying inputs. A square wave input is used to illustrate the deficiency of the integral feedback scheme.

The use of the steepest gradient algorithm is shown to minimize the problem that occurs when $\underline{f}$ is time-varying. The effect of noisy measurements on the estimation of the accelerations and hence the overall performance is studied for the case of the integral feedback scheme. The use of a filter improves the performance in the presence of noise.

Case 1. The Need for Adaptive Control

The performance of the exact linearization method when all parameters are known as well as when some parameters are unknown is studied in this example. The system of equations (5.1) through (5.8) was simulated with the input being generated according to equation (3.7). The payload (m_L) is 0.5 kg. The command is a step input to the second joint with the first joint in a fixed position. The payload and all other parameters are assumed to be known exactly. The terms $J(\underline{\theta})$, $\underline{c}(\underline{\theta},\underline{\theta})$ and $\underline{g}(\underline{\theta})$ in equations (5.2) through (5.8) are calculated and then the input (equation (3.7) is calculated. The result is shown in Figure 4. The steady-state error is zero and there is no overshoot. The exact linearization approach works very well when the estimated and actual values of the nonlinear functions match.

To illustrate the drawbacks of exact linearization and to demonstrate the need for adaptive control, a mismatch between the actual and estimated payload is considered. The terms $J(\underline{\theta})$, $\underline{c}(\underline{\theta},\underline{\theta})$ and $\underline{g}(\underline{\theta})$ in equation (5.2) through (5.8) are calculated based on an estimated payload of 0 kg and used for obtaining the input (equation (3.7)). The actual payload is 0.5 kg. The command is a step input input to the second joint with the first joint in a fixed position. The response is shown in Figure 5. There is a steady-state error of about 12 percent. Clearly, the nonlinear functions $J(\underline{\theta})$ and $\underline{f}$ need to be estimated on-line.

<u>Case 2. Adaptive Control Using Integral Feedback and RLS</u>

The performance of the adaptive control scheme using integral feedback and the recursive least squares (RLS) method is studied in this example. The values of $\underline{f}$ and J are considered unknown and are estimated on-line. $\hat{f}$ is obtained by integrating equation (3.17) and $\hat{J}$ is obtained using equations (4.16) and (4.17). The acceleration $\underline{\theta}^{(2)}$ is obtained by numerical differentiation.

The response of the second joint when a step input is applied to it with the first joint in a fixed position is shown in Figure 6. The steady-state error is zero even for different payloads. The estimates of f_2 using equation (3.17) and the actual values of f_2 are shown in Figure 7. The transient adaptation is very poor. As the steady-state value is reached, the estimated value converges to the actual value.

The response of both the joints to square-wave inputs is shown in Figures 8 and 9. The input to joint 1 varies from 0 to -1 with a period of 4 seconds. The input to joint 2 varies from 0 to 1 with a period of 5 seconds. The results indicate that the response of one joint is influenced by sudden changes in the input to the other joint. An examination of the actual and the estimated forces $\underline{f}$ at the two joints, shown in Figures 10 and 11, indicates that the force estimates do not track the actual forces. The gain Γ_{-1} used to estimate $\underline{f}$ is limited by equation (3.3). The estimated inertia values are shown in Figures 12 through 14. The estimated inertia values do not track the actual values. Since the estimates of both $\underline{f}$ and J do not converge, the overall system exhibits poor performance for square-wave inputs.

Case 3. Adaptive Control Using the Steepest Gradient Algorithm and RLS

The performance of the adaptive pole placement scheme using the steepest gradient algorithm for estimation of $\underline{f}$ (Equation (4.30)) and the recursive least squares algorithm for the estimation of J, is studied in this example. The values of $\underline{f}$ and J are considered to be completely unknown. The acceleration $\underline{\theta}^{(2)}$ is obtained by numerical differentiation.

The inputs are smooth and given by:

$$\theta_{1d}(t) = \begin{cases} -90 + 52.5\ (1-\cos(1.26t)), & 0 \leq t \leq 2.5 \\ 15.0 & t > 2.5 \end{cases} \tag{5.12}$$

$$\theta_{2d}(t) = \begin{cases} 170 - 60\ (1-\cos(1.26t)), & 0 \leq t \leq 2.5 \\ 50.0 & t > 2.5 \end{cases} \tag{5.13}$$

The output response for the smooth commands are exhibited in Figures 15 and 16. The algorithm performed very well in tracking for different loads. It is also able to overcome errors in the estimates of the initial output. The actual and estimated forces $\underline{f}$ are shown in Figures 17 and 18. The estimates follow the actual forces well except in the initial period. These results demonstrate that the adaptive control scheme using the steepest gradient algorithm is able to track time-varying quantities reasonably well. The actual and estimated inertias are shown in Figures 19 through 21. The inertia estimates do not converge.

The response of the joints to square wave inputs is shown in Figures 22 and 23. The input to joint 1 was varied from 0 to -1 with a period of 4 seconds and the input to joint 2 was varied from 0 to 1 with a period of 5 seconds. The response of either joint is not affected by sudden changes in the input of the other joint. The actual and estimated values of the nonlinear forces $\underline{f}$ are compared in Figures 24 and 25. The estimates of $\underline{f}$ track the actual values very closely, thus providing superior performance when compared with Case 2. The actual and estimated inertia values for J_{11} are compared in Figure 26. The estimate converges to a mean value of the time-varying inertia, but does not track the time-varying nature of the inertias. This lack of tracking is a property of the recursive least squares algorithm.

Case 4. Adaptive Control Using Steepest Gradient and ELS

The performance of the adaptive pole-placement scheme using steepest gradient method for estimation of $\underline{f}$ and the exponentially weighted least squares (ELS) algorithm (equations (4.23) and 4.24)) is studied in this case. The responses of the joints to square-wave inputs of Case 3 are shown in Figures 27 and 28. There is no difference in the outputs as compared to the recursive least squares shown in Figures 19 through 21. The inertia estimates do not converge.

The response of the joints to square wave inputs is shown in Figures 22 and 23. The input to joint 1 was varied from 0 to -1 with a period of 4 seconds and the input to joint 2 was varied from 0 to 1 with a period of 5 seconds. The response of either joint is not affected by sudden changes in the input of the other joint. The actual and estimated values of the nonlinear forces $\underline{f}$ are compared in Figures 24 and 25. The estimates of $\underline{f}$ track the actual values very closely, thus providing superior performance when compared with Case 2. The actual and estimated inertia values for J_{11} are compared in Figure 26. The estimate converges to a mean value of the time-varying inertia, but does not track the time-varying nature of the inertias. This lack of tracking is a property of the recursive least squares algorithm.

<u>Case 4. Adaptive Control Using Steep-
est Gradient and ELS</u>

The performance of the adaptive pole-placement scheme using steepest gradient method for estimation of $\underline{f}$ and the exponentially weighted least squares (ELS) algorithm (equations (4.23) and 4.24)) is studied in this case. The responses of the joints to square-wave inputs of Case 3 are shown in Figures 27 and 28. There is no difference in the outputs as compared to the recursive least squares using a constant estimate for $\hat{J}$ is quite high, the need for adaptive control is clear.

Case 5. Effect of Noise

The performance of the adaptive control methods when measurement noise is present is studied in this example. The integral feedback method is used for estimating $\underline{f}$ and the RLS algorithm is used for estimating J. The desired inputs are the smooth inputs (equations (5.12) and (5.13)) given in Case 3. The noise is added to the velocity measurements. The response of joint 1 is shown in Figure 35; the numerical differentiation method is used for obtaining the acceleration. Even at a signal-to-noise ratio (SNR) of about 33 to 1, the dynamic performance is very poor.

A filter with a cut-off frequency of 100 rad/sec also was used for estimation of the acceleration. The response of joint 1 with this filter is shown in Figure 36. The SNR is 10 to 1. The effects of noise are diminished and the system performance is improved.

The response of joint 1 when the filter is used in the absence of noise is compared with the response obtained when numerical differentiation is used, in Figure 37. In the absence of noise, the performance when the filter is used is inferior to the performance when numerical differentiation is used. Thus, the selection of a method to estimate the acceleration is dependent on the strength of the noise.

The integral feedback scheme to estimate the nonlinear functions $\underline{f}$ used in conjunction with the recursive least squares scheme to estimate J works very well for set point regulation as well as for tracking smooth inputs. The speed of adaptation of $\hat{\underline{f}}$ in this scheme is constrained by the selection of Γ_{-1}. This limitation leads to problems in rejecting sudden disturbances at the other joints as evidenced by the performance of the method to square-wave inputs at both joints.

The use of the steepest gradient algorithm alleviates the disturbance rejection problem. This algorithm also improves the estimates of $\hat{\underline{f}}$ and stays alert to changes in $\underline{f}$.

The use of numerical differentiation to estimate accelerations leads to poor performance in the presence of noise. The use of a first order filter in the estimation of the accelerations mitigates the effect of measurement noise. The use of the filter results in a sluggish response in the absence of noise. Thus, there is a trade-off in using the filter to estimate the acceleration.

Example Problem: Electrohydraulic Velocity Control System

A schematic of an electrohydraulic velocity control system is shown in Figure 3. For a servovalve current, i, supply pressure, P_s, and a load pressure across the cylinder, P_1, the flow rate, Q_1, through the servovalve is:

$$Q_1 = Ki\sqrt{(P_s - |P_1|)} \qquad (5.14)$$

The continuity equation yields:

$$Q_1 = A\dot{x} + C_t P_1 + (V_t/(4(\beta))\dot{P}_1 \qquad (5.15)$$

and the equation of motion of the piston is:

$$AP_1 = MD\dot{x} + B\dot{x} + F_1 \qquad (5.16)$$

where

A = Area of the piston

M = Mass of the piston

B = viscous damping

C_t = coefficient of leakage

V_t = Volume of fluid under compression

β = bulk modulus of the fluid

D = d/dt.

The load is a nonlinear function of the velocity $\dot{x}$ and contains many uncertain parameters

$$F_1 = f(\dot{x}) \qquad (5.17)$$

By combining equation (5.14) through (5.17) yields:

$$MD^2\dot{x} + BD\dot{x} + \frac{\partial f}{\partial \dot{x}} D\dot{x} + \frac{4\beta A^2}{V_t} \dot{x} = \frac{4\beta A}{V_t}[Ki \quad (P_s - P_1) - C_t P_1] \qquad (5.18)$$

Yun and Cho (1988) used this system to develop an adaptive model following control system for uncertain loads F_l. In this study, uncertain values of the bulk modulus β are considered.

The values for the different parameters used in this example are (Yun and Cho, 1986):

Mass, M 53.4 kg;

Damping constant, B, 882 n-sec/m;

Volume, v_t, 1.79 x 10^{-3} m^3;

Area, A, 1.52 x 10^{-3} m^3;

Amplifier gain, K, 1.62 x 10^{-9} m^4/sec/(mA · N);

Supply pressure, P_s, 6.86 x 10^6 N/m^2 (≈ 1,000 psi);

Leakage coefficient, C_t 2·24 x 10^{-12} m^5/sec/.

The load F_l is nonlinear and is given by

$$F_l = f(\dot{x}) = c\dot{x}^2 \tag{5.19}$$

where C is 4.9 x 10^6 n-sec^2/m^2

Equation (5.18) can be cast in the form of equation (3.5) by setting

$$f(\dot{x},\ D\dot{x}) = \frac{v_t}{4\beta AK\sqrt{P_s - P_l}}\left[BD\dot{x} + \frac{\partial f}{\partial \dot{x}}D\dot{x} + \frac{4\beta}{v_t}A^2\dot{x} + \frac{4\beta A}{v_t}C_tP_l\right] \tag{5.20}$$

and
$$J(\dot{x},\ D\dot{x}) = \frac{M\ v_t}{4\beta AK\sqrt{P_s - P_l}} \tag{5.21}$$

An adaptive pole-placement was designed using the methods in this study and the results were compared to a linearized PID controller which was designed based on (5.18).

A PID controller was designed for the linearized system (based on linearized form of Eq. (5.18) using the ITAE criterion. The fluid bulk modulus of 6.86 x 10^8 N/m^2 (100,000 psi) and the natural frequency was 234 radian/sec. Since the control was applied to a nonlinear system, the gains obtained from the linearized equations were adjusted to satisfy the overshoot and settling time of the third order ITAE criterion. The proportional gain constant was 1892, the derivative gain was 7.29, and the integral gain was 109820.

An adaptive pole-placement controller was designed for a natural frequency of 234 radians/sec. and a damping ratio of 0.707. The steepest gradient algorithm was used for estimation of the nonlinear functions f. The gain for the steepest gradient algorithm was 5.0. The recursive least squares (RLS) estimation scheme was used for estimating J^{-1}. The performance of the PID controller and the adaptive controller for a step input is compared in Figures 38-40.

Figure 38 shows the response of the PID and adaptive controllers for a bulk modulus of 6.86 x 10^8 N/m^2. There is not much difference in the two responses since the PID controller was designed for this value. Figure 39 shows the response of the two controllers for a bulk modulus of 13.72×10^8 N/m^2. The adaptive controller has a slower response time but a faster settling time than the PID controller. Figure 40 shows the response of the PID and adaptive controllers for a bulk modulus of 1.715×10^8 N/m^2. Even though the rise time seems the same, the adaptive controller has a faster settling time. The PID controller has a very large settling time and a very high overshoot. The difference in the performance of the PID and adaptive controller is dramatic for the conditions considered.

The performance of the PID controller over wide ranges of the bulk modulus is poor. The performance of the adaptive controller is good over the same range of the bulk modulus. This shows the versatility of the applicability of the adaptive control algorithm and its superior performance compared to conventional PID control.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The objective of this study was the development of an adaptive feedback controller for multi-input, multi-output, linear-in-control, nonlinear systems with unknown functions. Specifically, the systems considered were assumed to be modeled by equations of the form:

$$J(y)D(d)\underline{y} + \underline{f}(\Phi(d)\underline{y}) = \underline{u} \qquad (6.1)$$

The emphasis was on two very important aspects of adaptive control of nonlinear systems: minimization of the number of estimated and the tracking of rapidly varying quantities.

The input to equation (6.1) was obtained as:

$$\underline{u} = \underline{\hat{f}} + \underline{\hat{J}}\ \underline{v} \qquad (6.2)$$

Various estimation schemes were used for the on-line estimation of $\hat{f}$ and $\hat{J}$. The number of parameters estimated was reduced by estimating $\underline{f}$ on-line using a differential equation of the form:

$$\underline{\dot{f}} = \underline{w}\ (t) \qquad (6.3)$$

An integral feedback scheme that eliminated steady-state errors and the steepest gradient algorithm were used for estimation of $\underline{f}$. The integral feedback scheme (equation 1.7) was constrained by the selection of its gain. It eliminated steady-state errors but provided poor transient performance. The performance of the estimator was improved using the steepest gradient algorithm (equation (1.9)).

The matrix J, whose values are dependent on the output, was estimated on-line using the recursive least squares (RLS) and exponentially weighted least squares (ELS) algorithms. When the RLS algorithm was used for estimation, the estimated values did not track the actual values. The use of the ELS algorithm mitigated this difficulty.

In this study, the nature of the nonlinear functions has been assumed to be unknown except for the order of the sub-systems. However, certain types of prior knowledge can be used. For example, if the nonlinear function $\underline{f}$ in (2.2) were composed of two components, one of which is known and the other unknown, i.e.

$$\underline{f} = \underline{f}_{known} + \underline{f}_{unknown} \tag{6.4}$$

then, the algorithm can be applied by using

$$\underline{u}' = \underline{u} - \underline{f}_{known} \tag{6.5}$$

and utilizing $\underline{u}'$ in the estimation of $\hat{\underline{f}}$ and $\hat{J}$, as well as by redefining the input to the system as

$$\underline{u} = \underline{f}_{known} + \hat{\underline{f}} + \hat{J}\underline{v} \qquad (6.6)$$

$\hat{\underline{f}}$ is the estimate of the unknown portion of $\underline{f}$.

Recommendations for Future Work

In this study, the primary emphasis was on systems with known order and with no hard nonlinearities such as saturation. Hard nonlinearities violate the analycity assumption and hence the exact linearization procedure will fail. The nonlinearities cannot be cancelled using input feedback as in equation (3.7). However, the performance of the overall scheme could be studied from the bounded-input, bounded-output stability viewpoint. The input could be calculated according to equation (3.11) and the effect of saturation on the output need to be studied.

The effects of uncertainties regarding the orders of the sub-systems were not considered in this study. The order of the characteristic equation (3.9) is dependent on the order of the sub-systems and the selection of the poles is affected by any uncertainty in the system order. The effects of uncertainties in the system order on the adaptive pole-placement control method require further study.

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APPENDIX

FIGURES

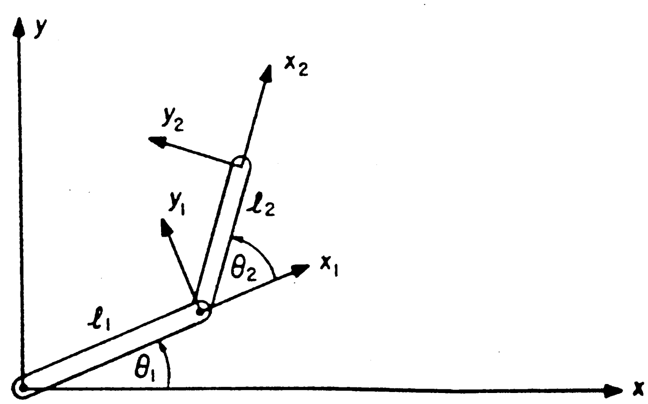

Figure 1. Schematic of a two-link manipulator

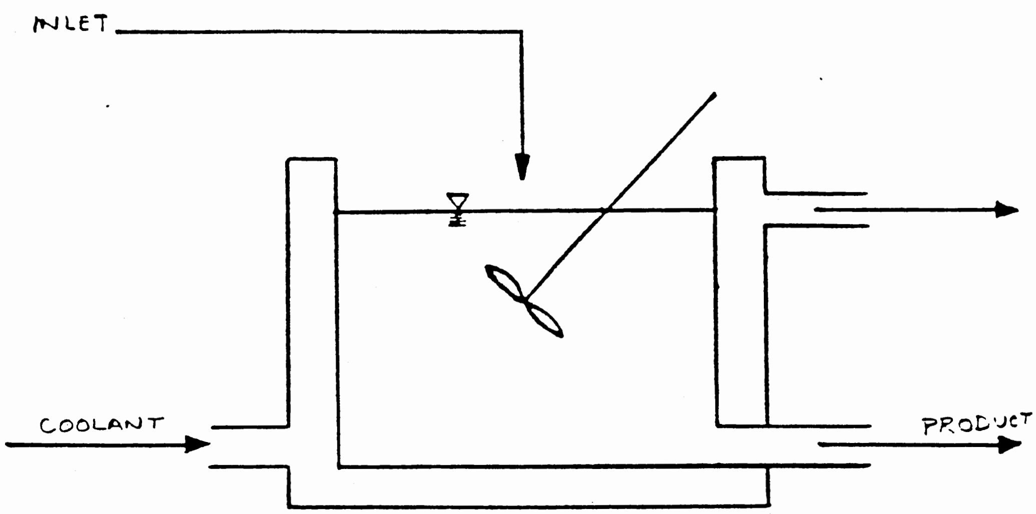

Figure 2. A continuous stirred tank reactor

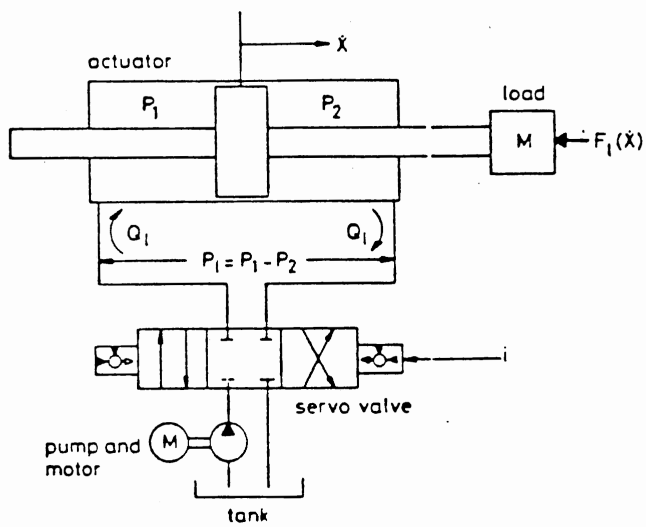

Figure 3. Schematic of an electro-hydraulic velocity control system

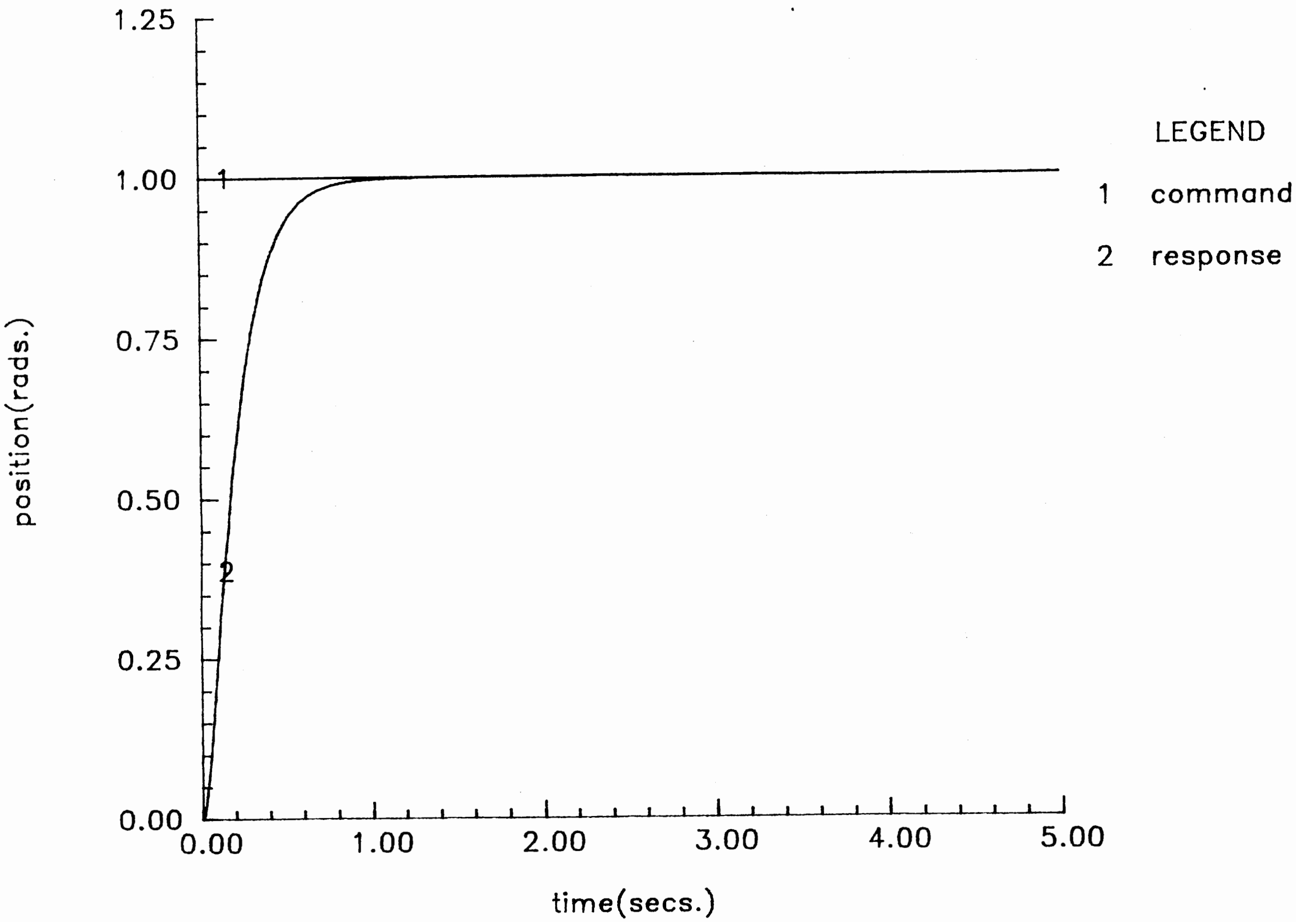

Figure 4. Step response of joint 2 using exact linearization; no mismatch in actual and computed values

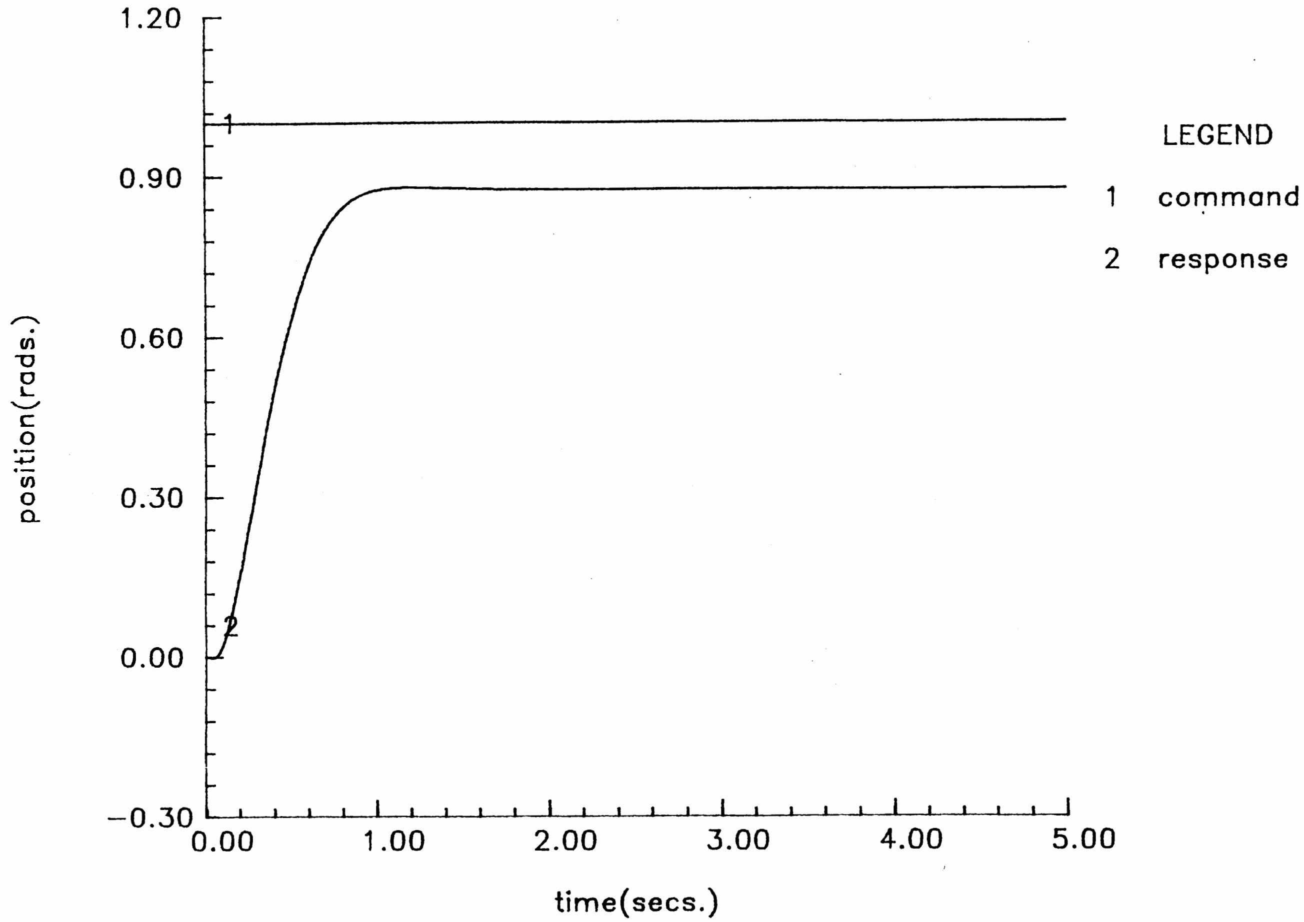

Figure 5. Step response of joint 2 using exact linearization; actual payload = 0.5 kg; assumed payload = 0.0 kg

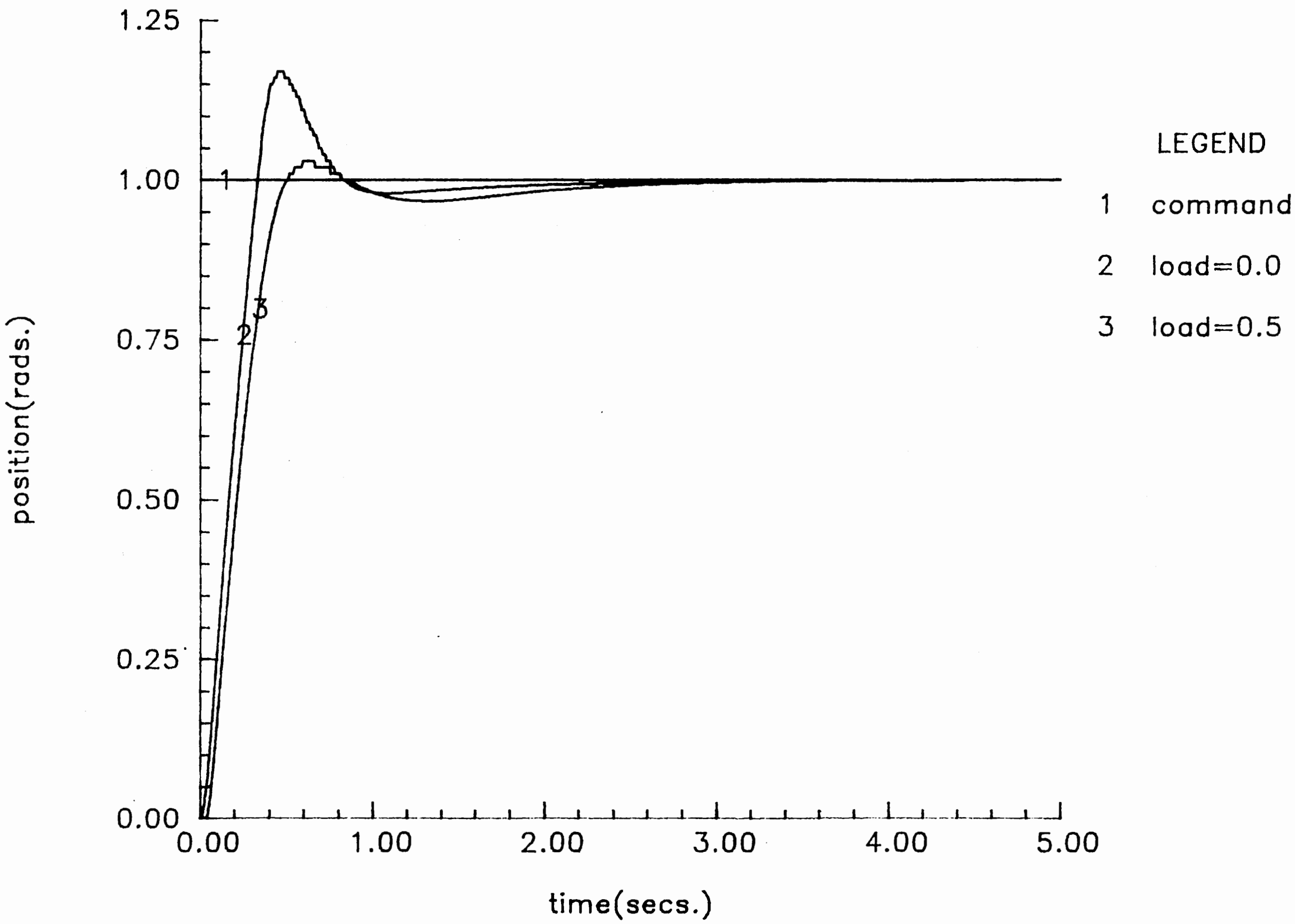

Figure 6. Step response of joint 2 when using integral feedback; payloads of 0.0 and 0.5 kg

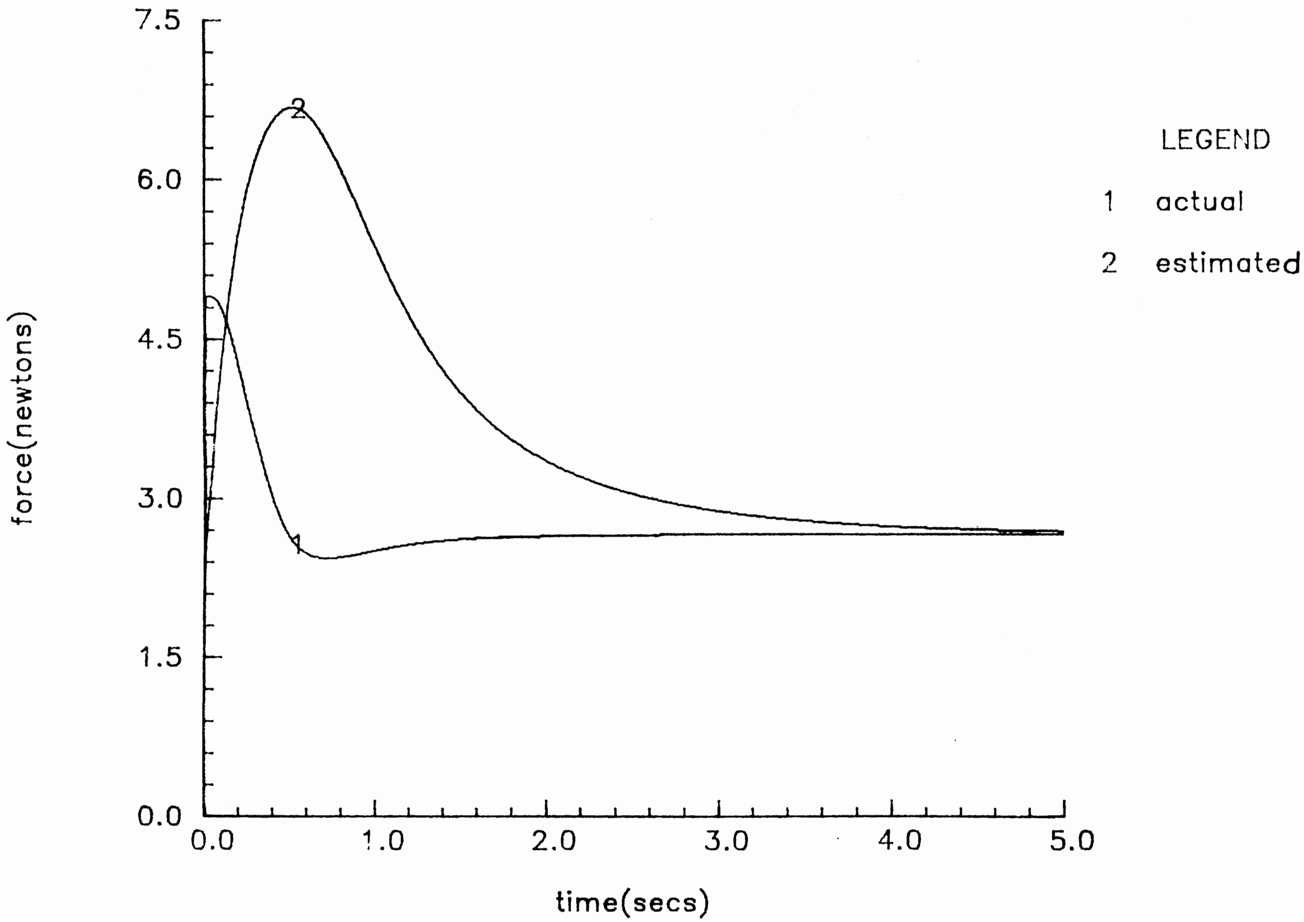

Figure 7. Comparison of actual and estimated values of f_2 using integral feedback

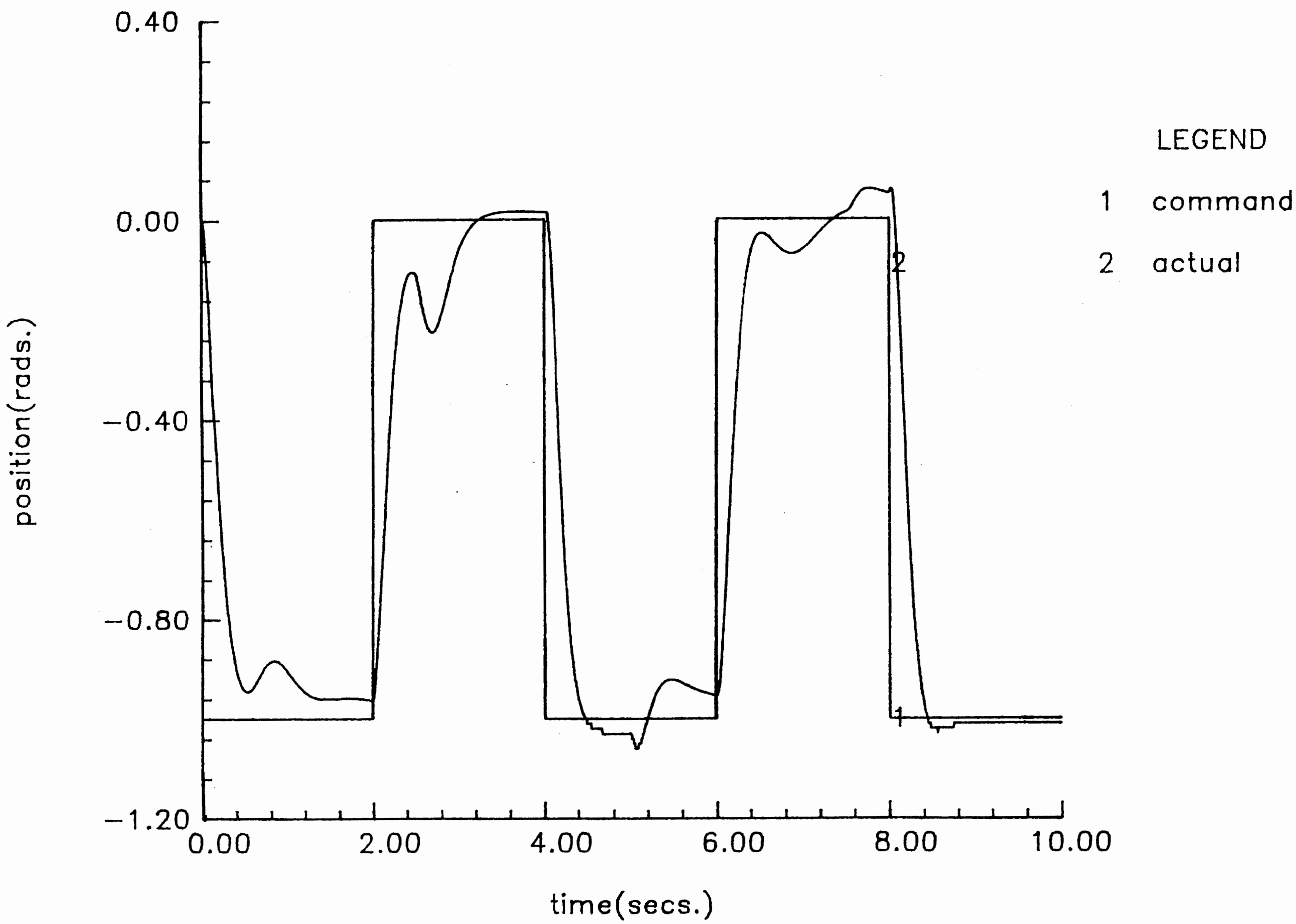

Figure 8. Response of joint 1 to square wave input; integral feedback method

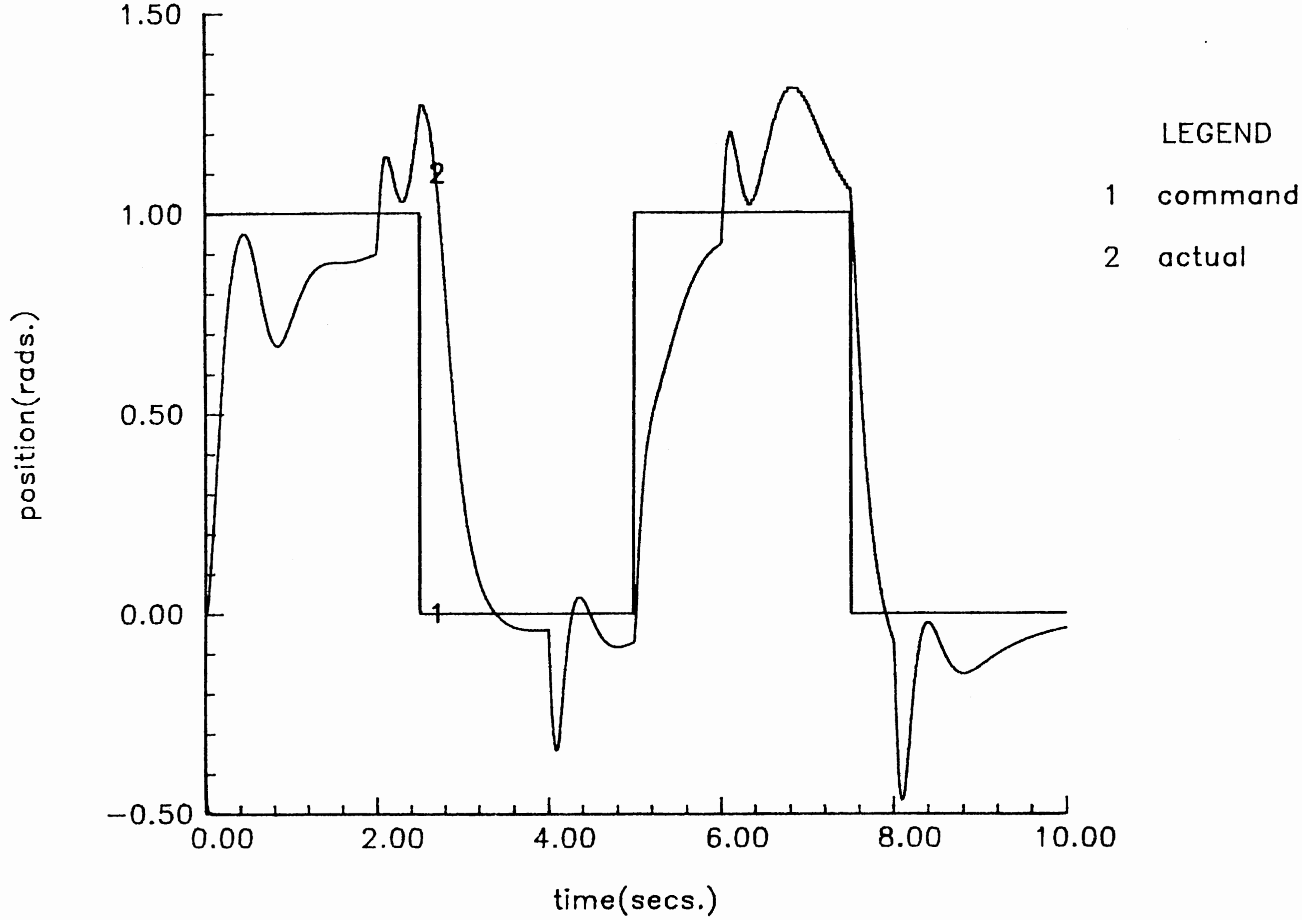

Figure 9. Response of joint 2 to square wave input; integral feedback method

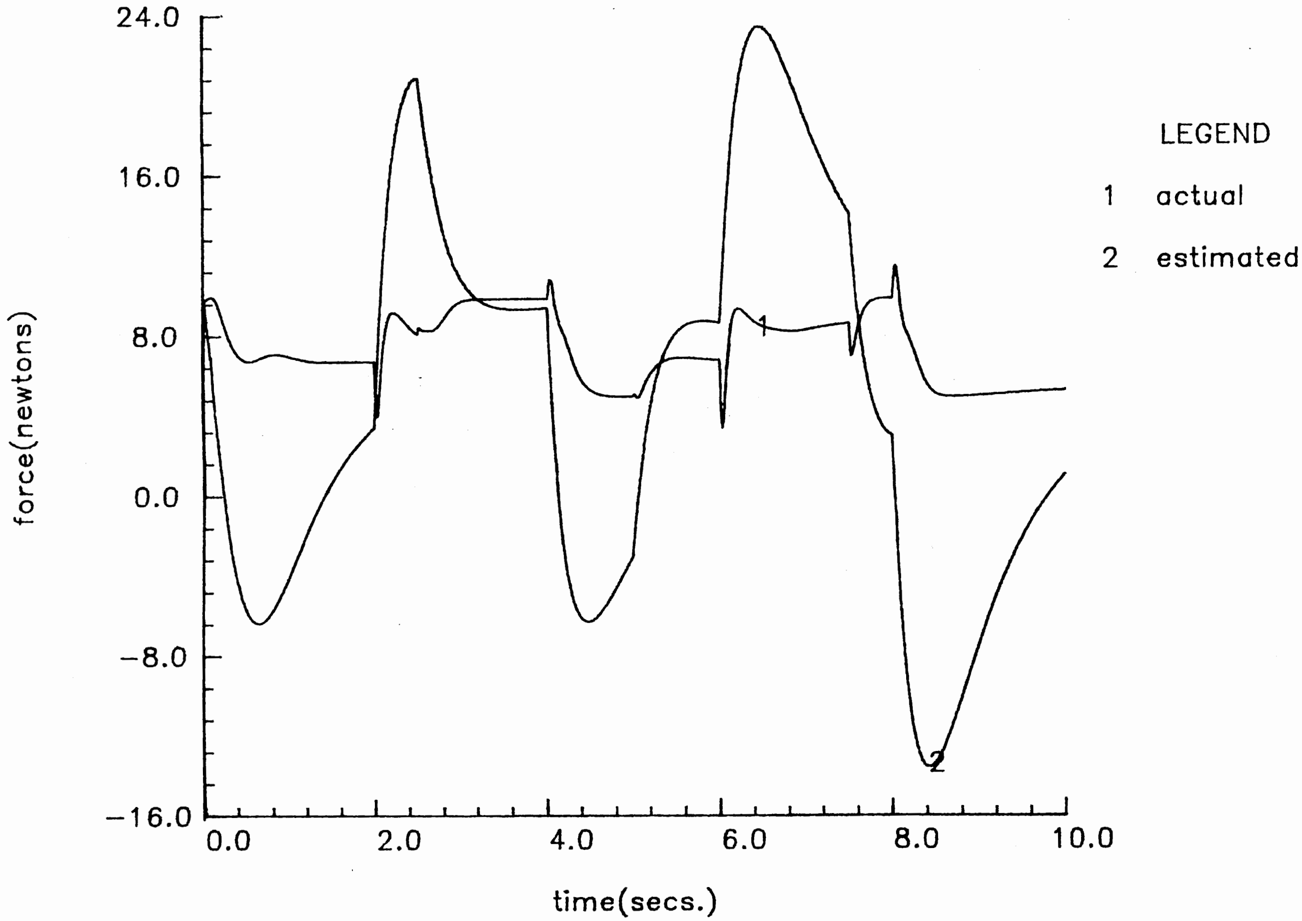

Figure 10. Comparison of actual and estimated forces at joint 1; integral feedback method

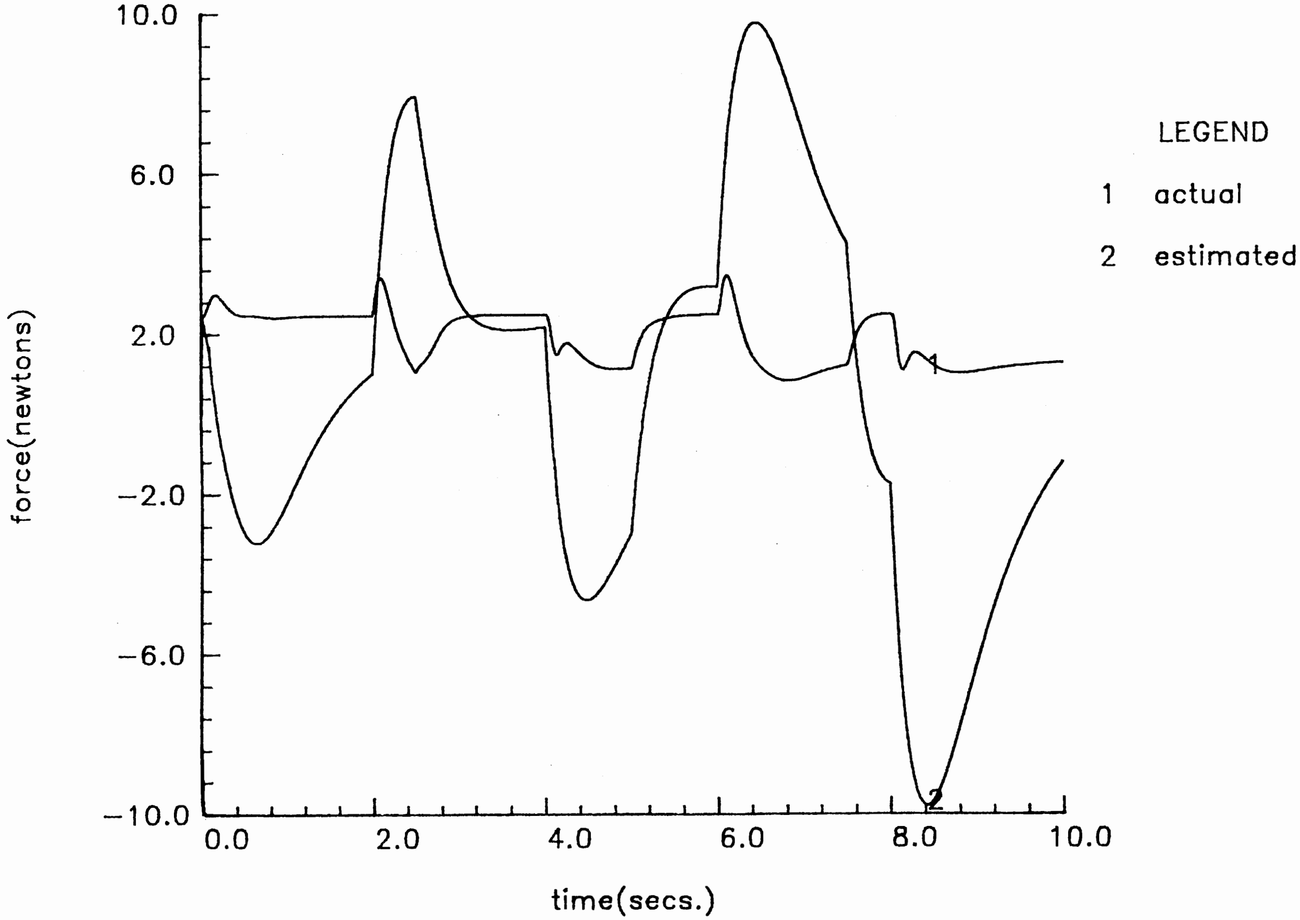

Figure 11. Comparison of actual and estimated forces at joint 2; integral feedback method

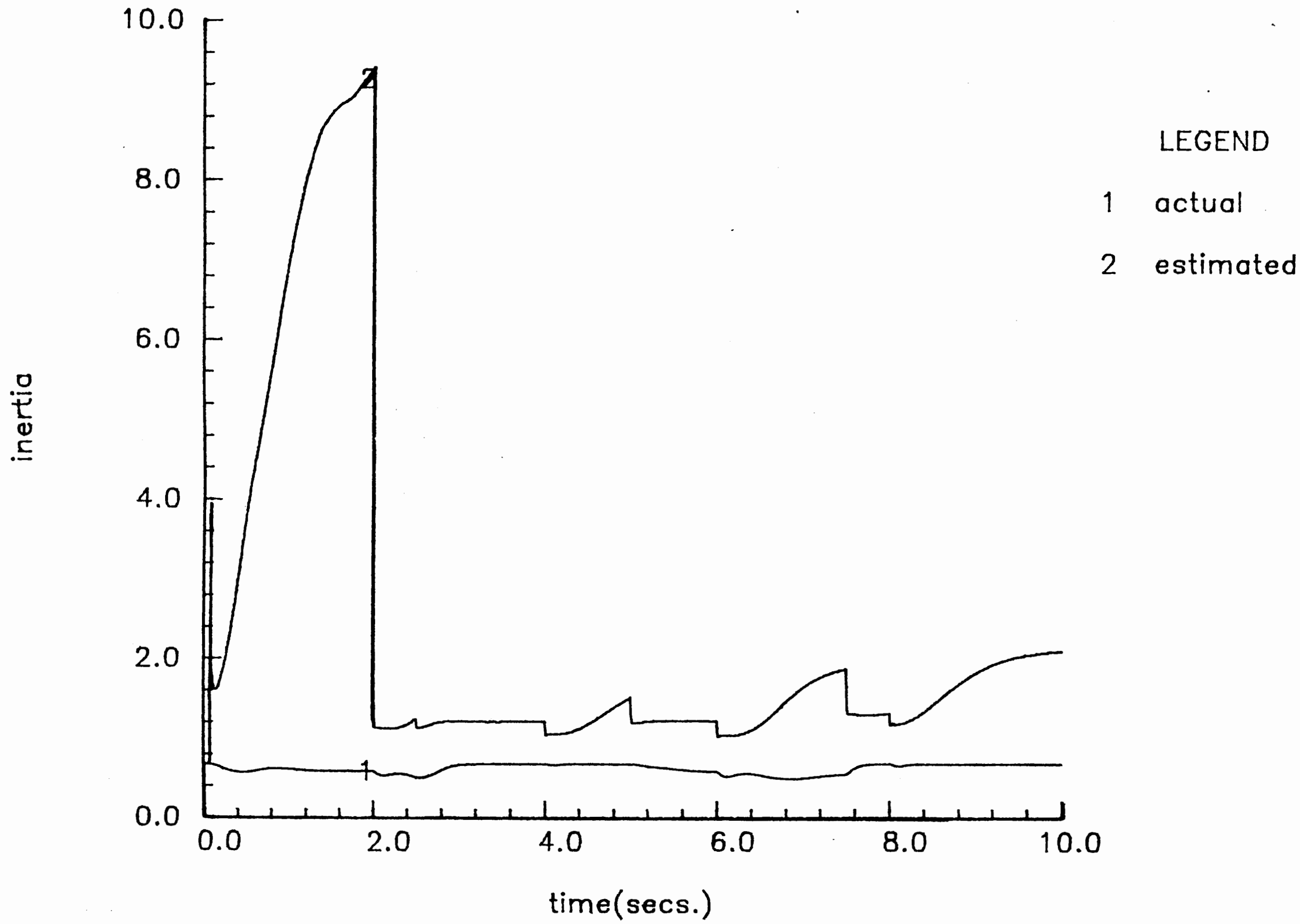

Figure 12. Comparison of actual and estimated inertias J_{11}; integral feedback method

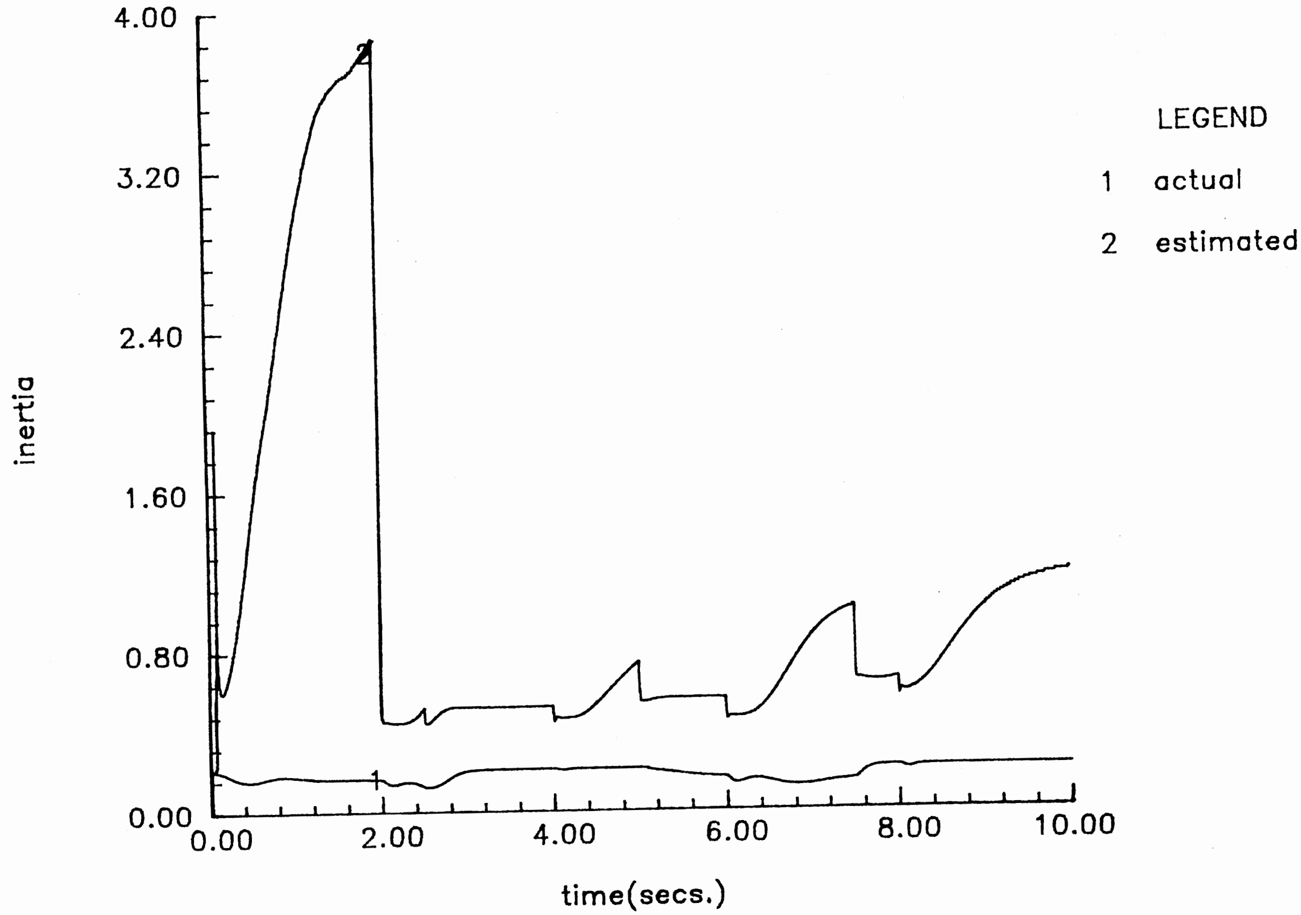

Figure 13. Comparison of actual and estimated inertias J_{12}; integral feedback method

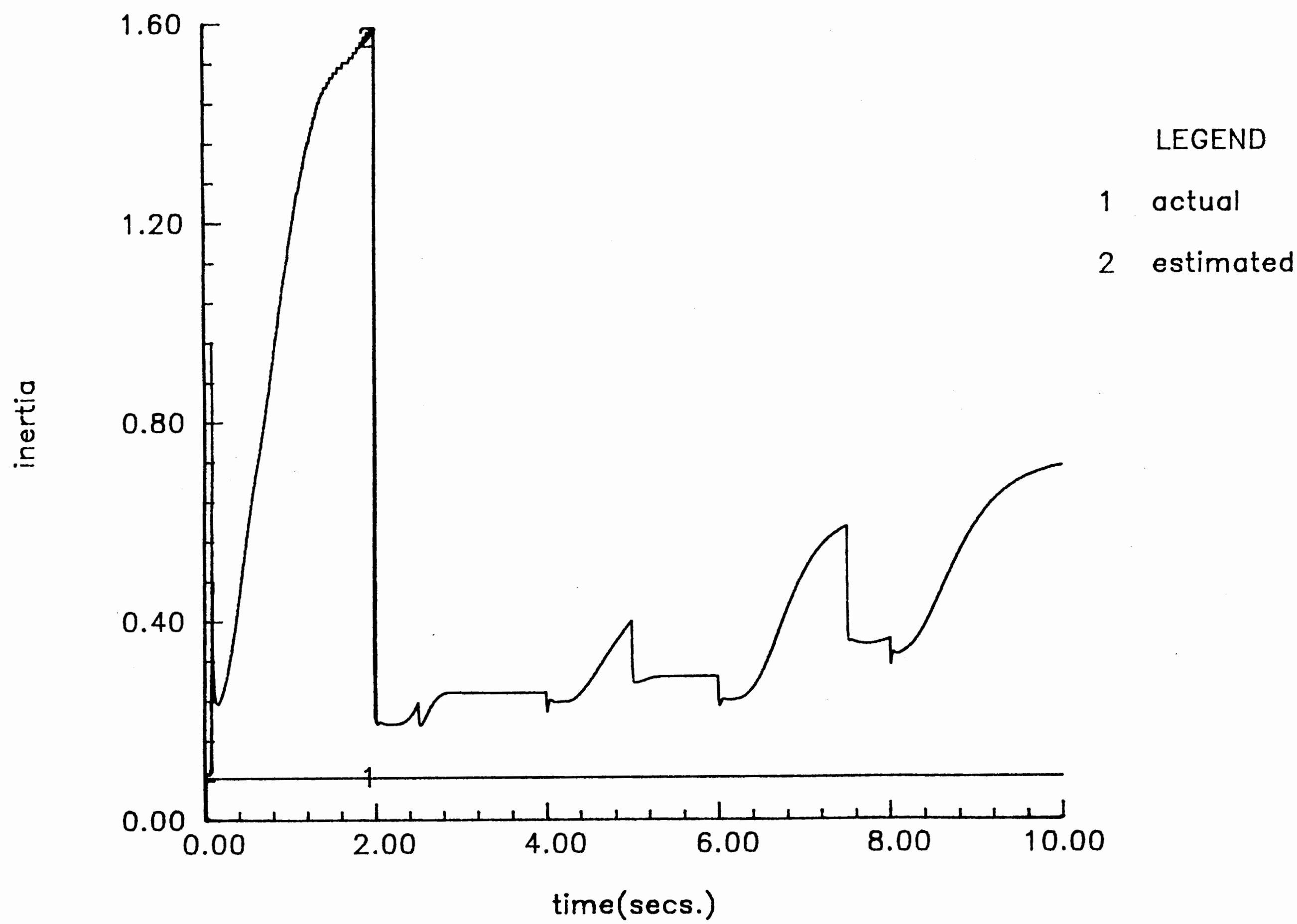

Figure 14. Comparison of actual and estimated inertias J_{22}; integral feedback method

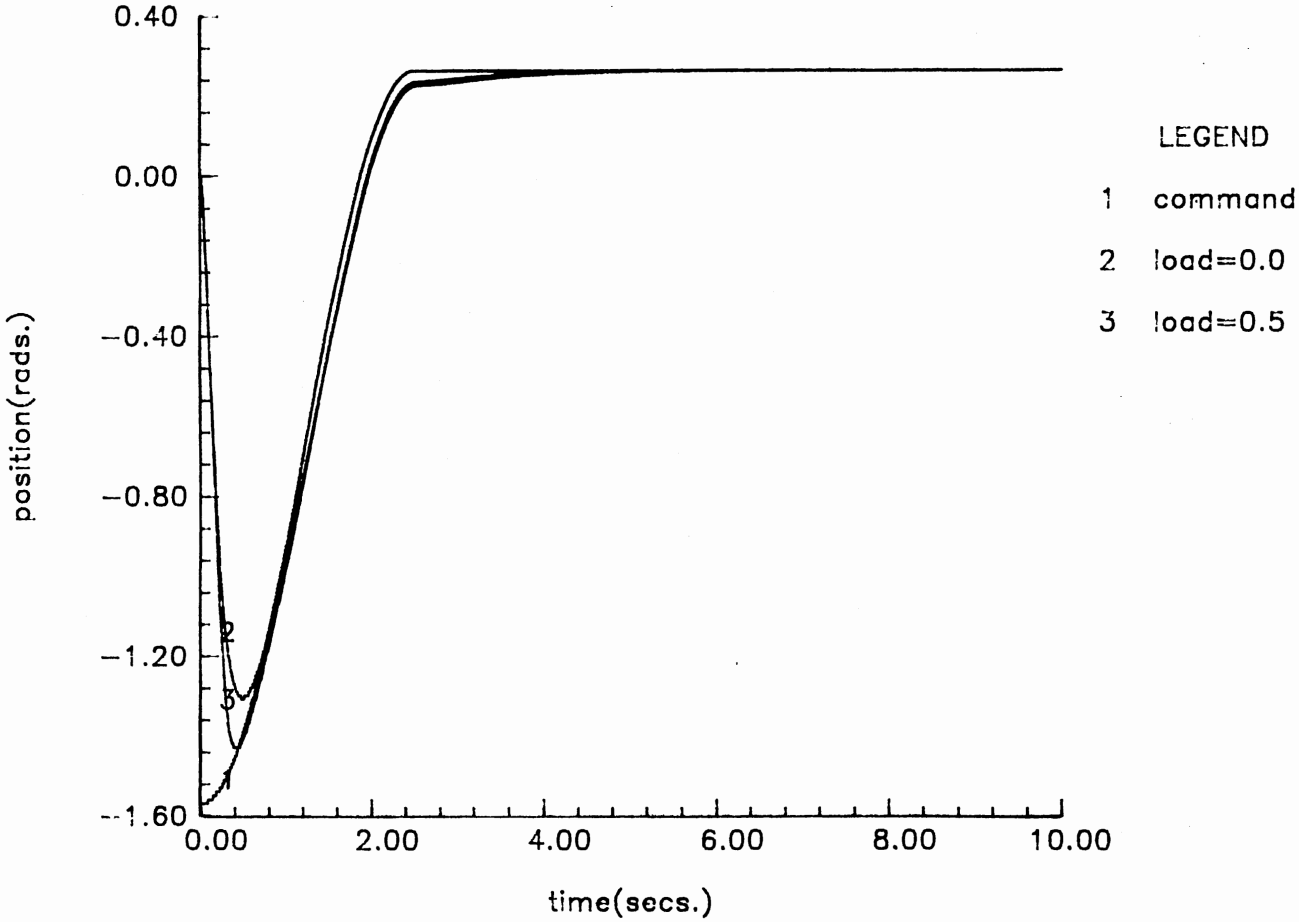

Figure 15. Response of joint 1 to smooth input; steepest gradient algorithm

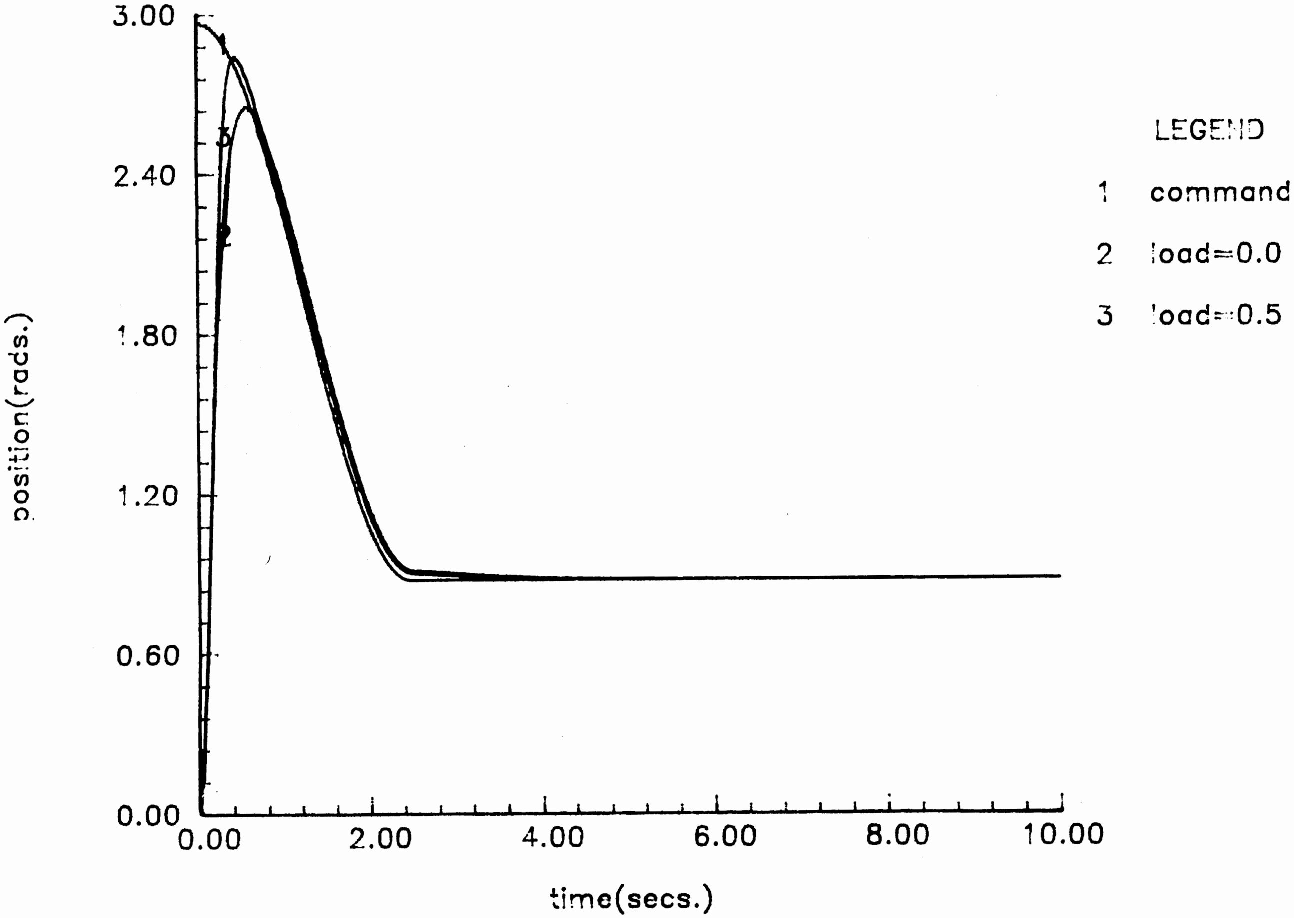

Figure 16. Response of joint 2 to smooth input; steepest gradient algorithm

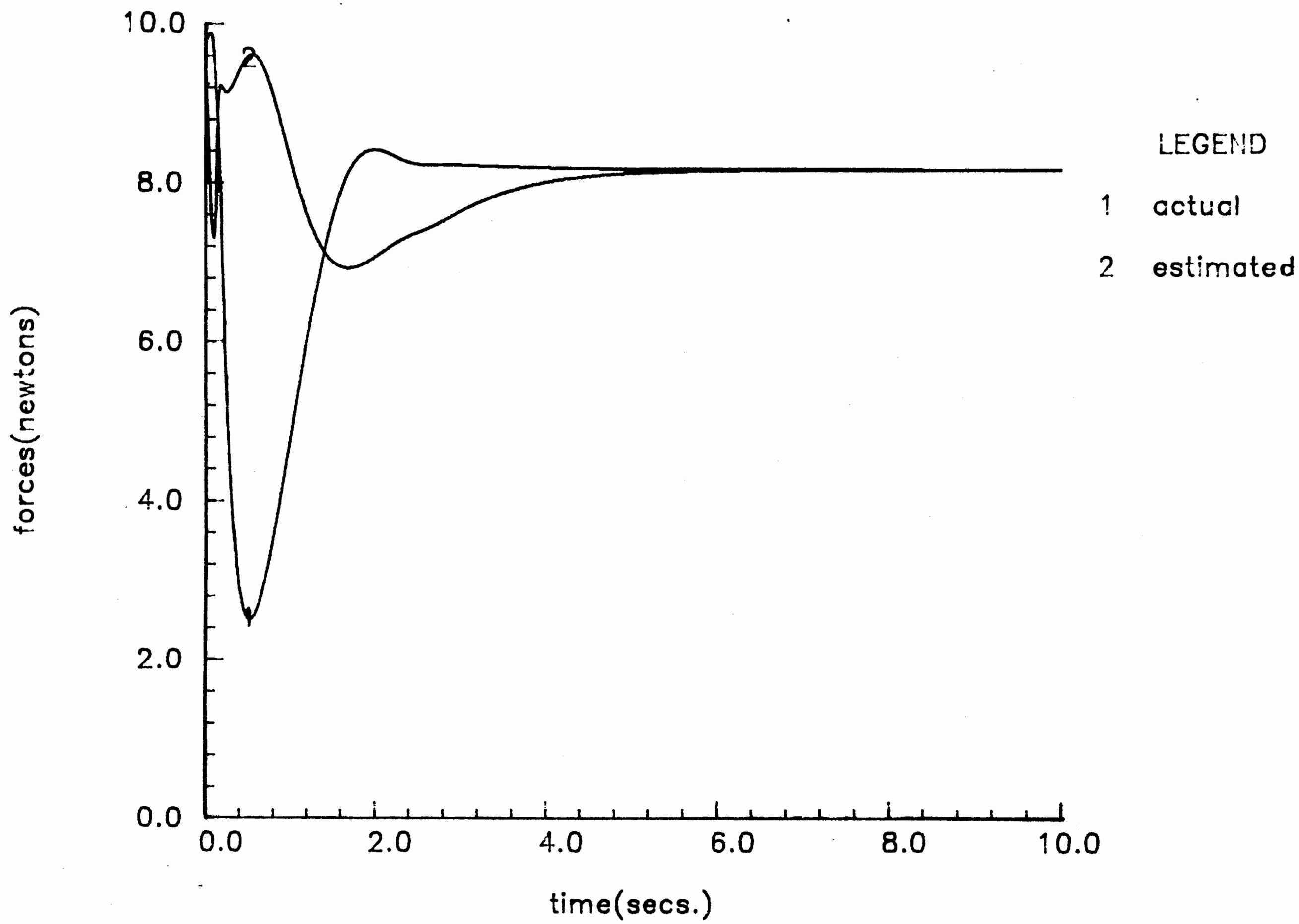

Figure 17. Comparison of actual and estimated forces at joint 1; steepest gradient algorithm

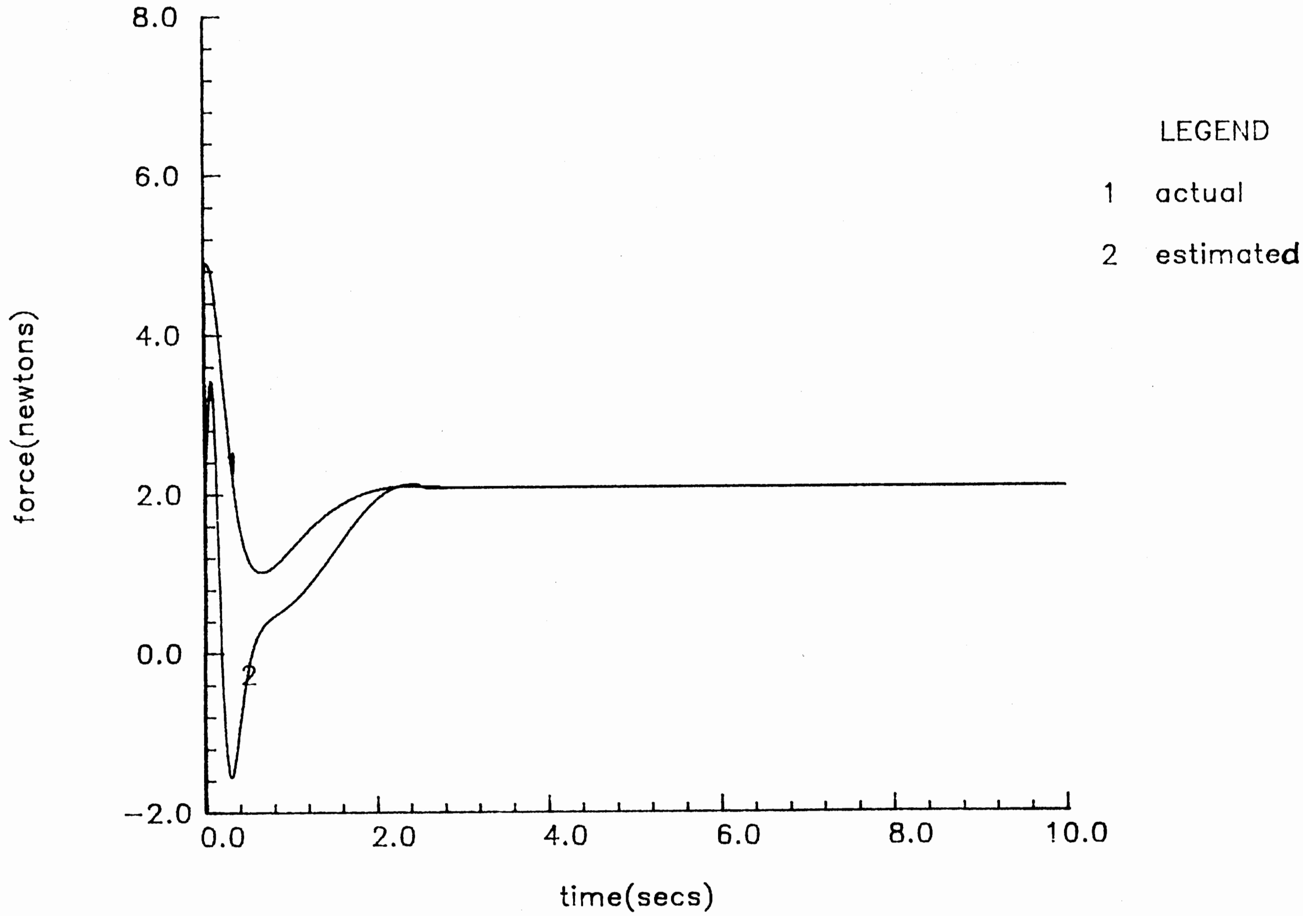

Figure 18. Comparison of actual and estimated forces at joint 2; steepest gradient algorithm

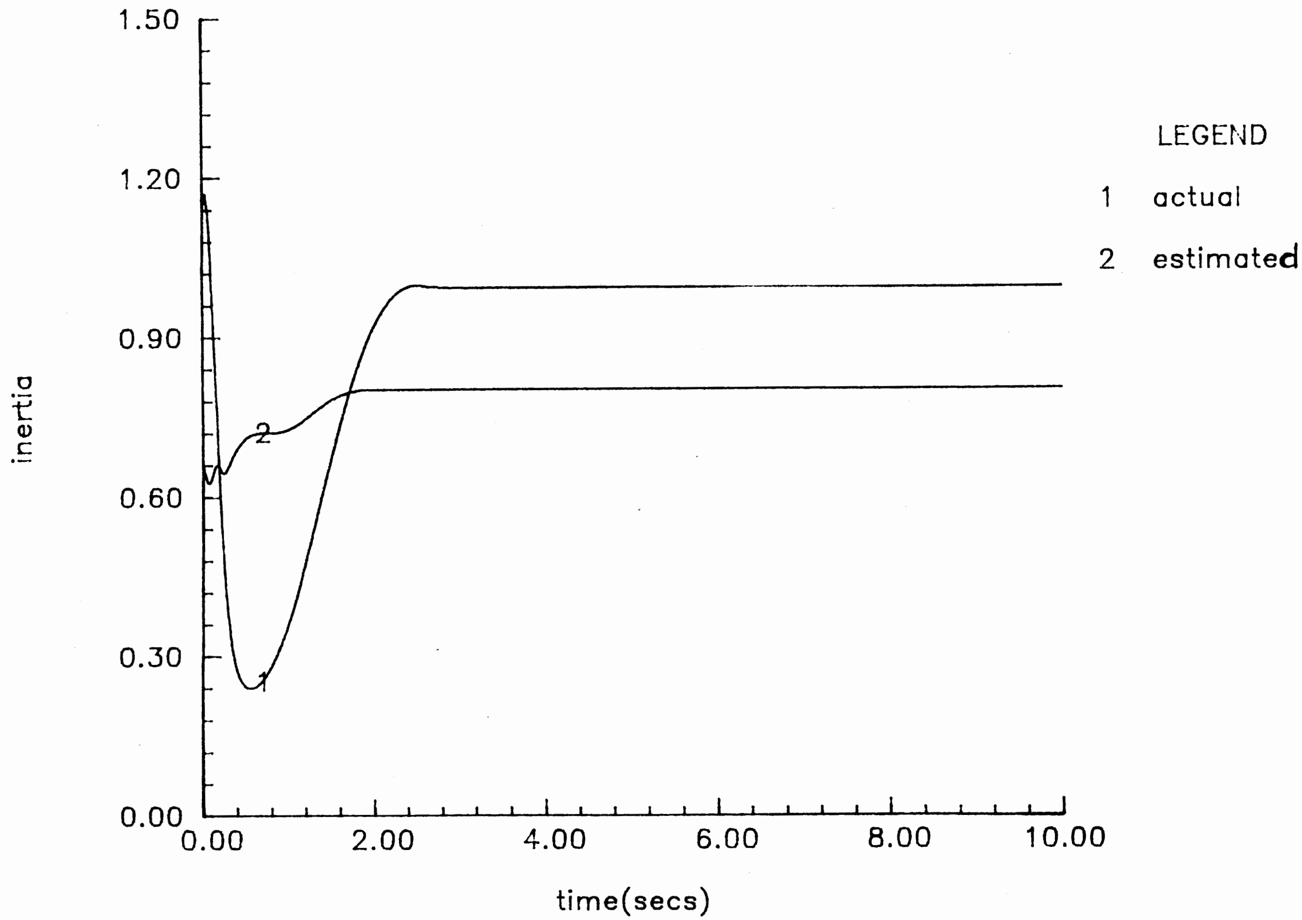

Figure 19. Comparison of actual and estimated inertias J_{11}; steepest gradient algorithm

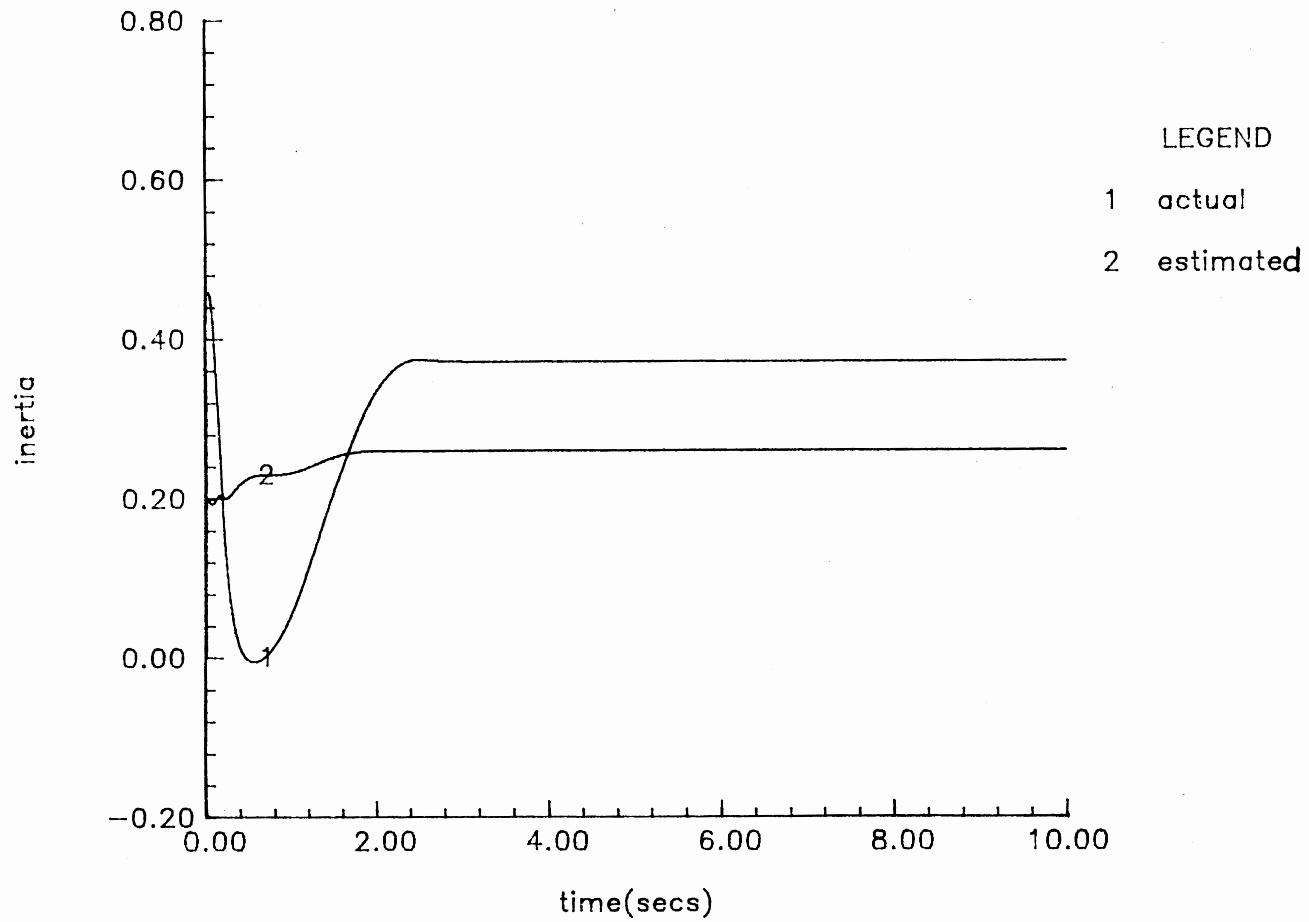

Figure 20. Comparison of actual and estimated inertias J_{12}; steepest gradient algorithm

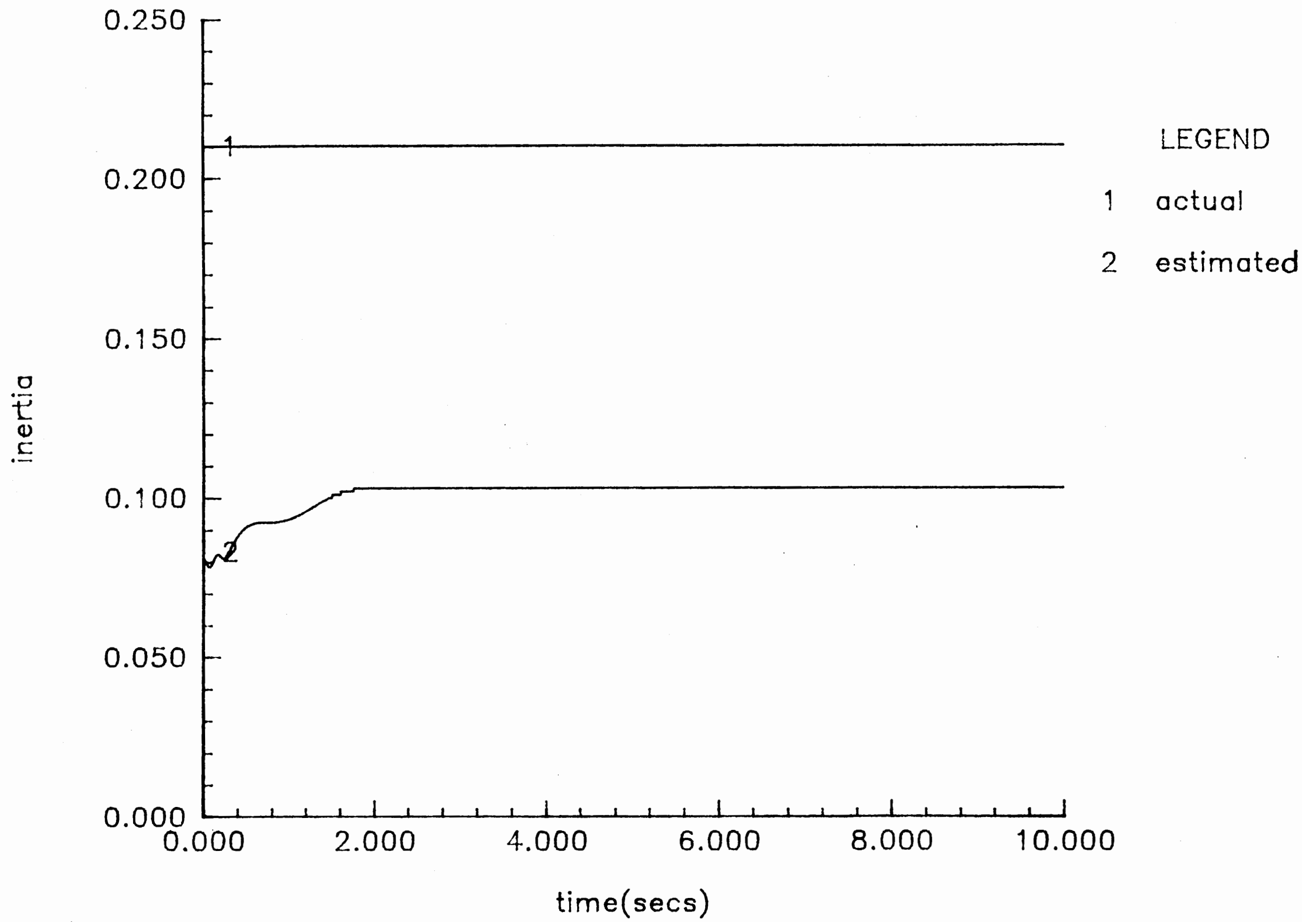

Figure 21. Comparison of actual and estimated inertias J_{22}; steepest gradient algorithm

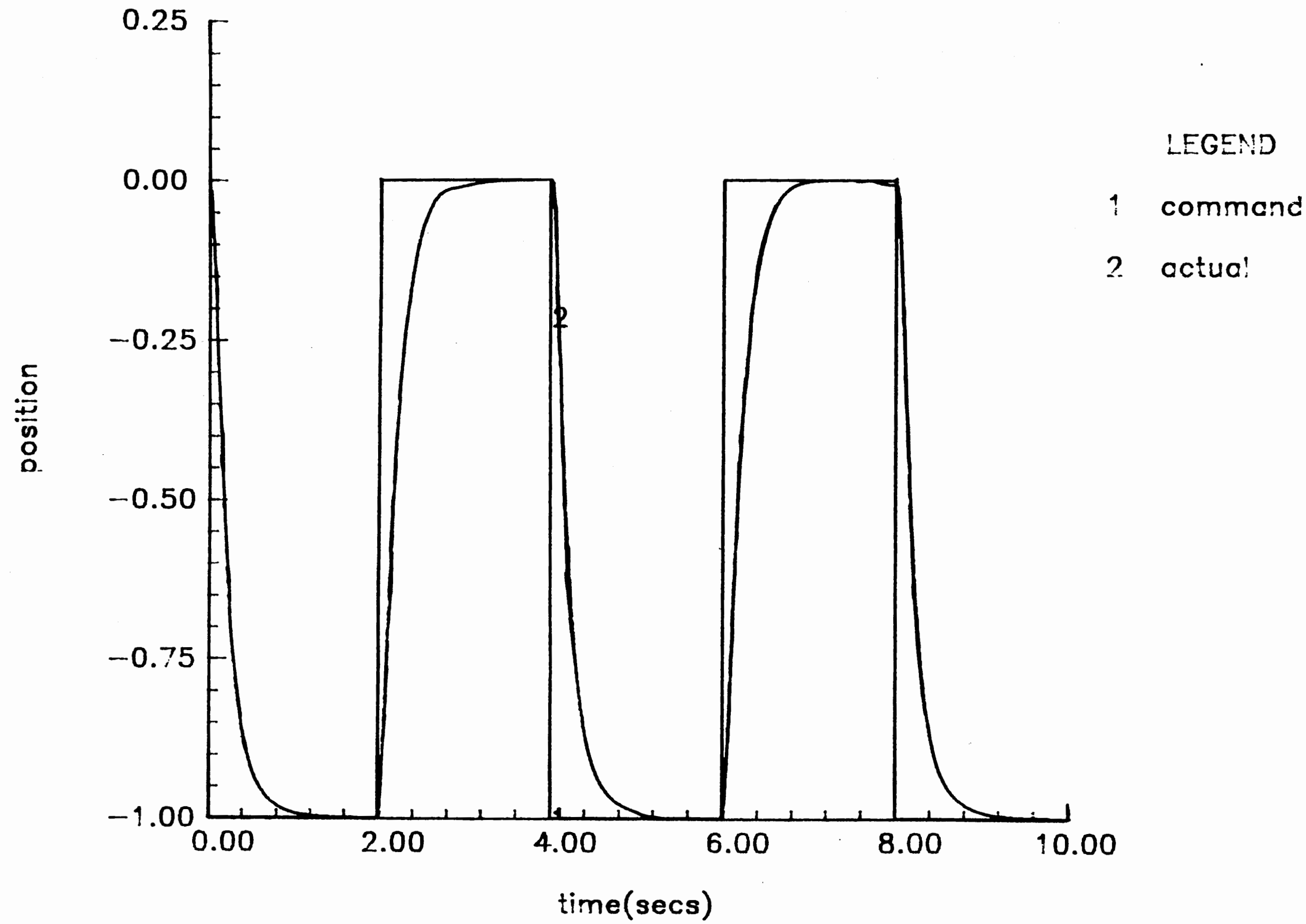

Figure 22. Response of joint 1 to square wave input; steepest gradient algorithm

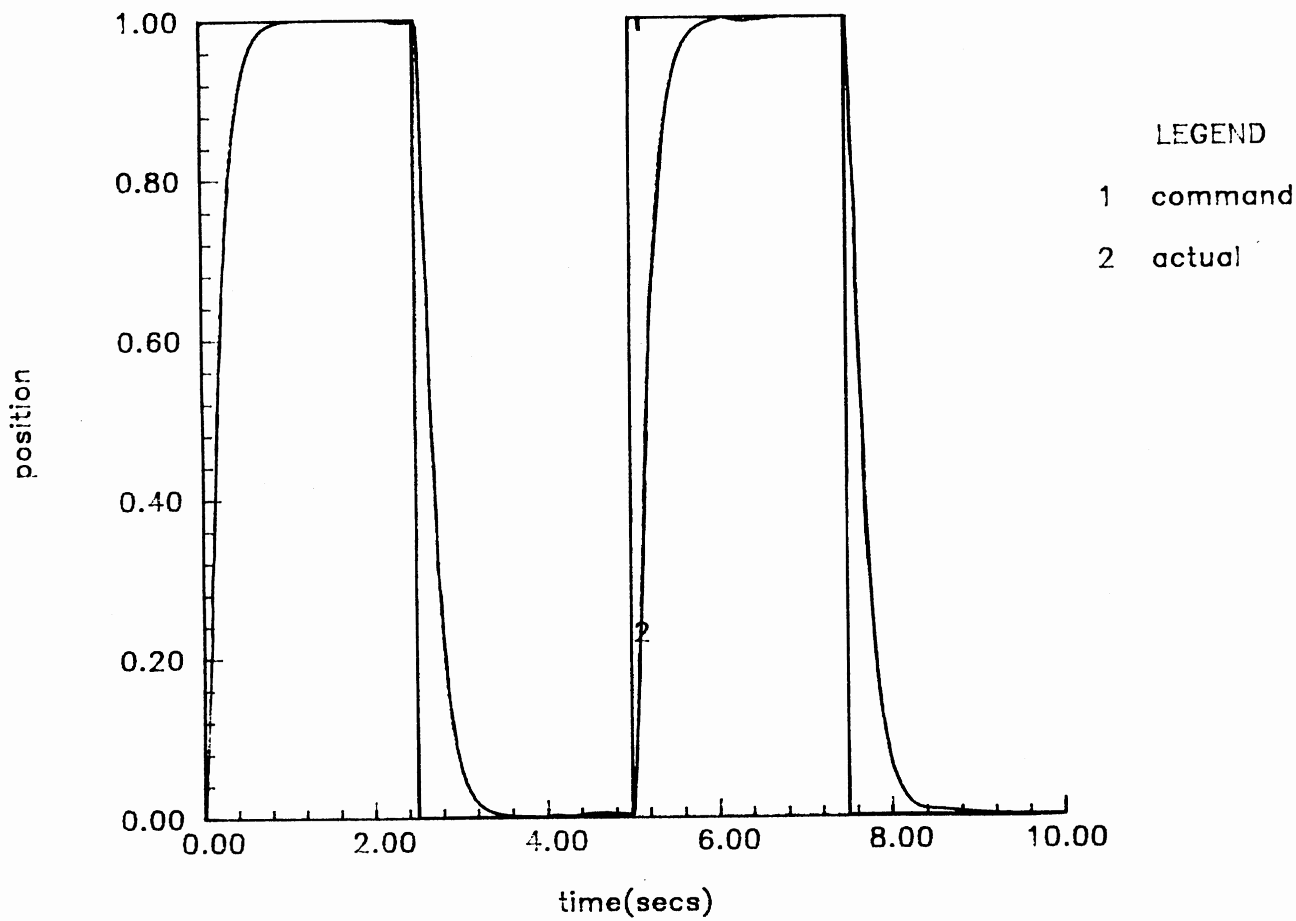

Figure 23. Response of joint 2 to square wave input; steepest gradient algorithm

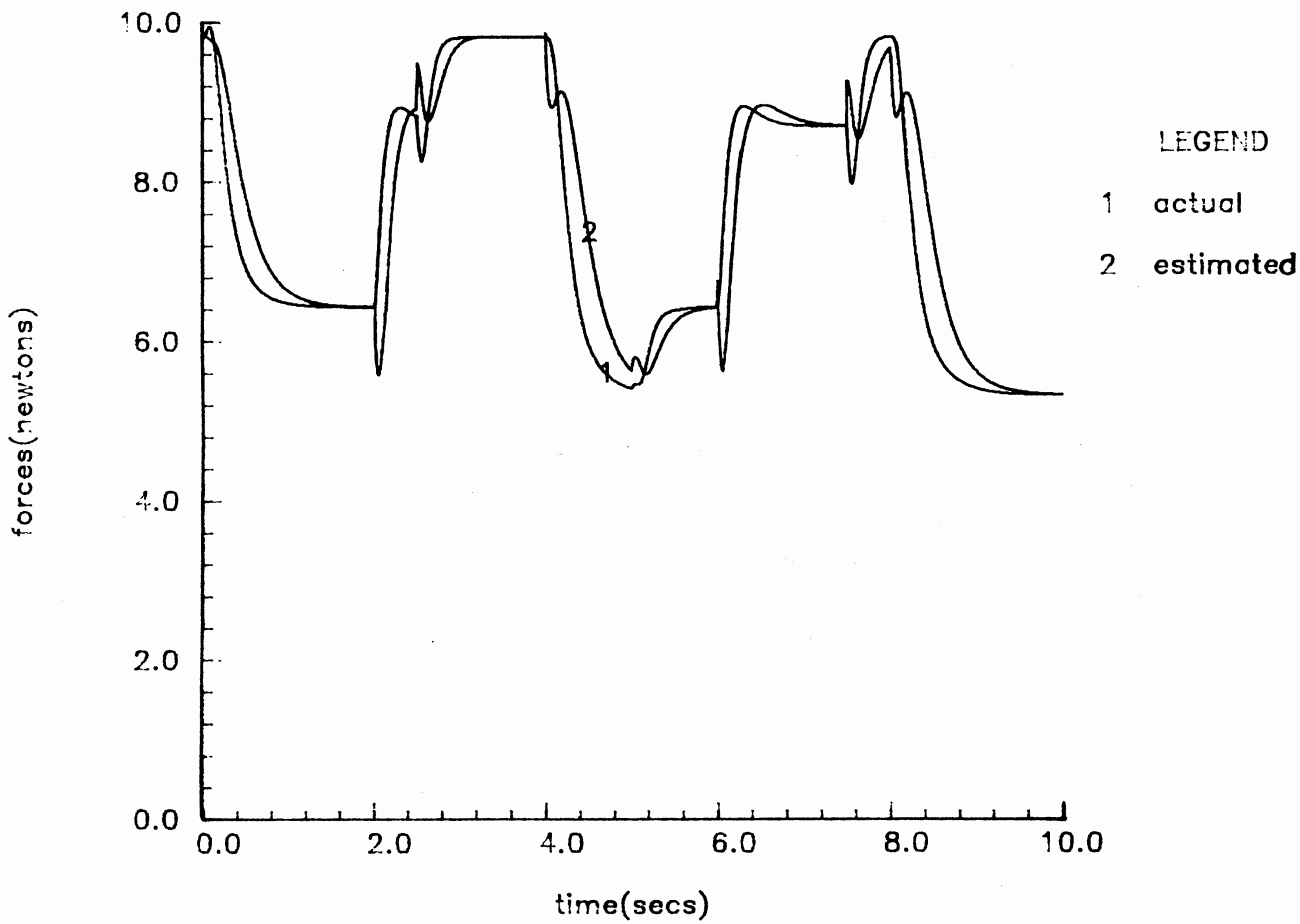

Figure 24. Comparison of actual and estimated forces at joint 1; steepest gradient algorithm

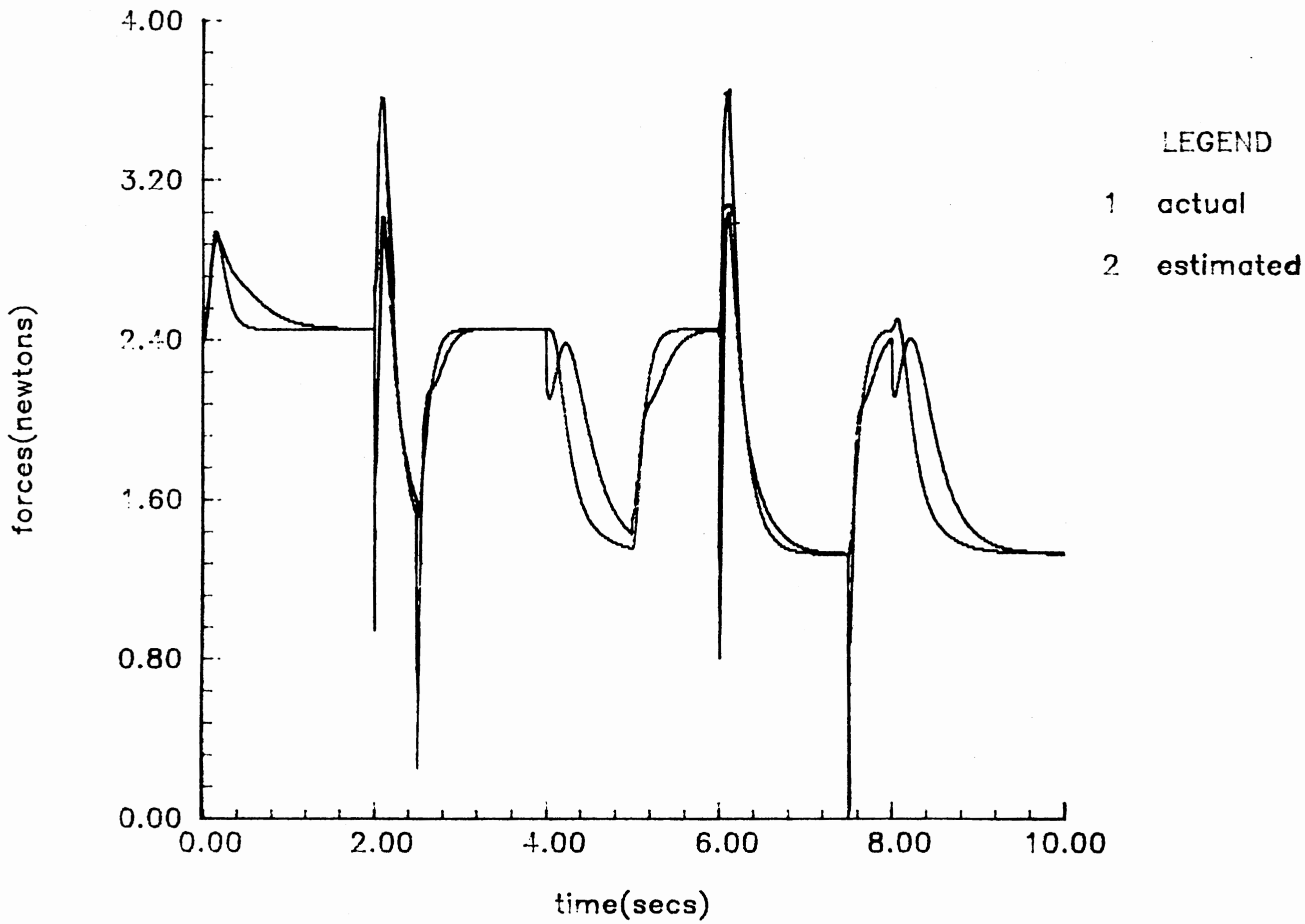

Figure 25. Comparison of actual and estimated forces at joint 2; steepest gradient algorithm

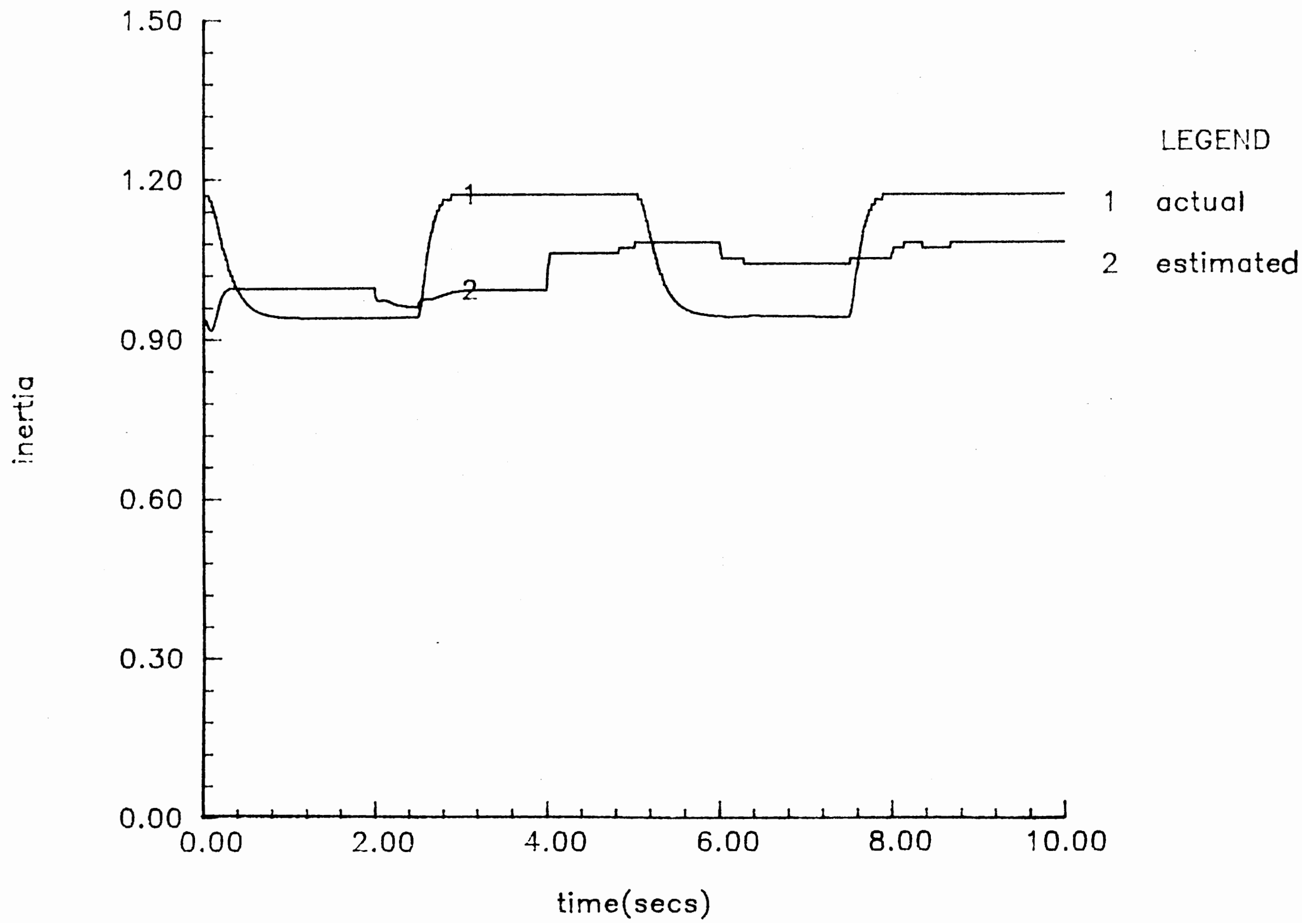

Figure 26. Comparison of actual and estimated inertias J_{11}; steepest gradient algorithm

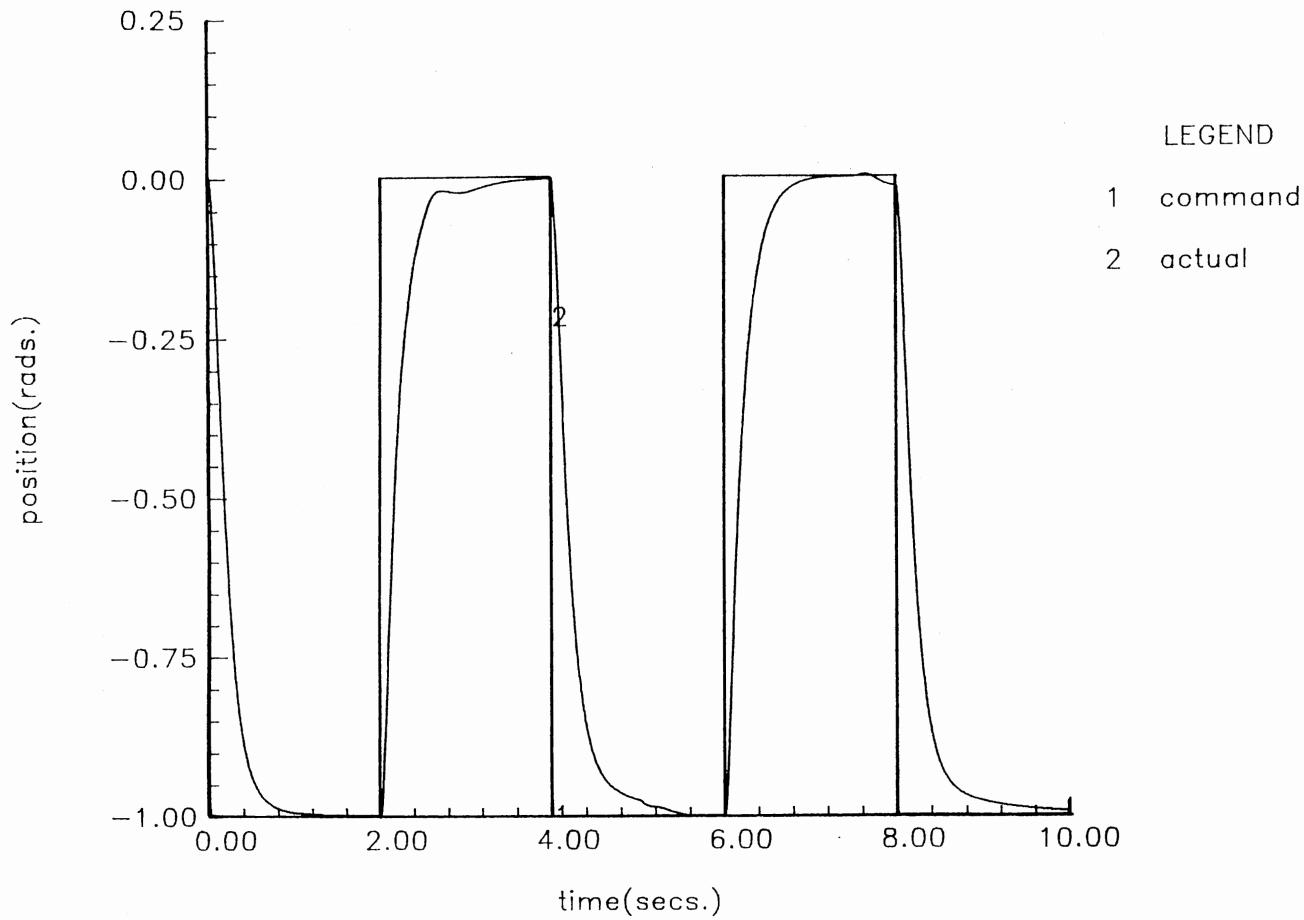

Figure 27. Response of joint 1 to square wave input; steepest gradient for forces; ELS for inertias

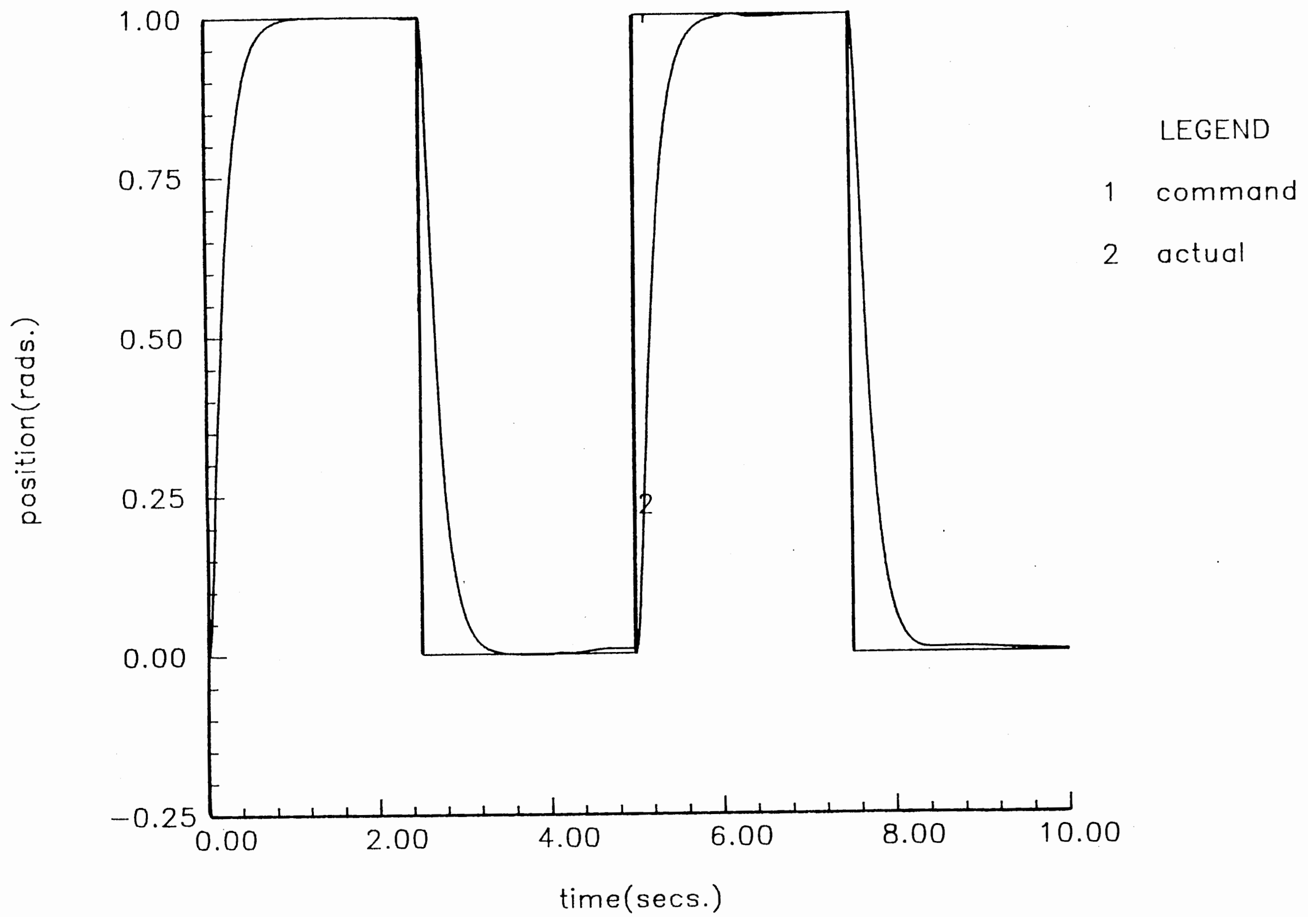

Figure 28. Response of joint 2 to square wave input; steepest gradient for forces; ELS for inertias

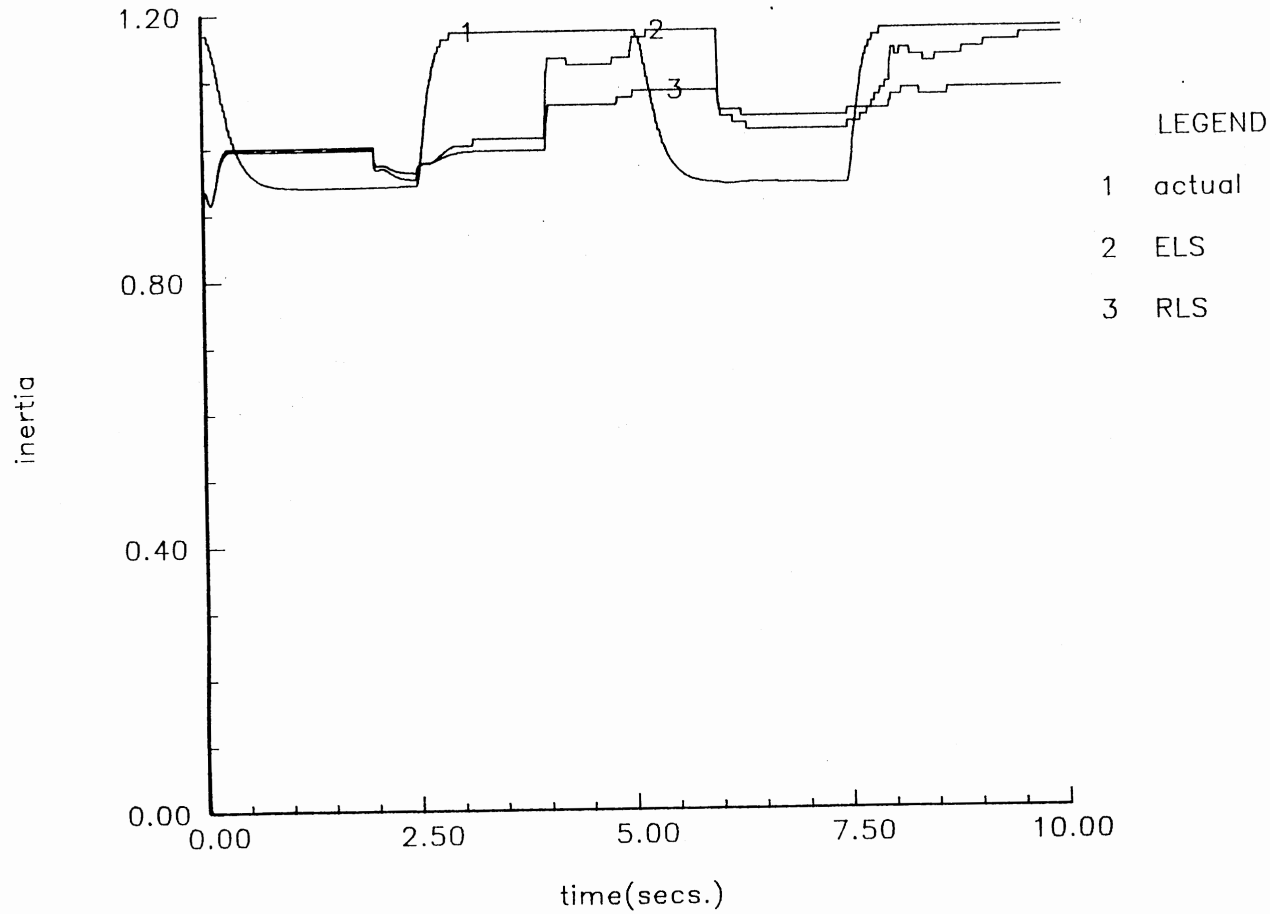

Figure 29. Comparison of actual and estimated inertias; recursive and exponentially weighted least squares

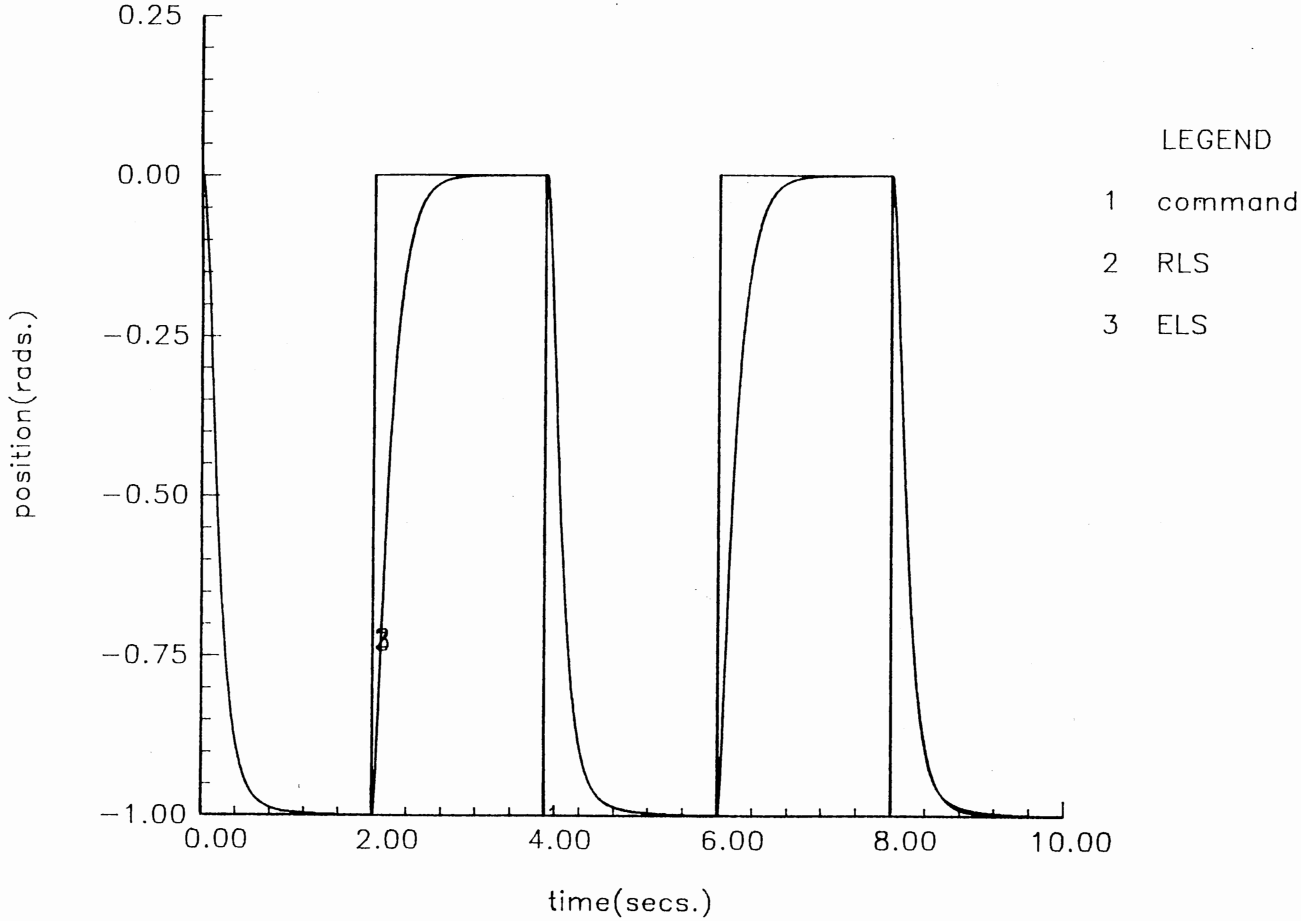

Figure 30. Response of joint 1 to square wave input of period 4 secs; Payload variation 0 to 0.5 kg; Recursive and exponentially weighted least squares

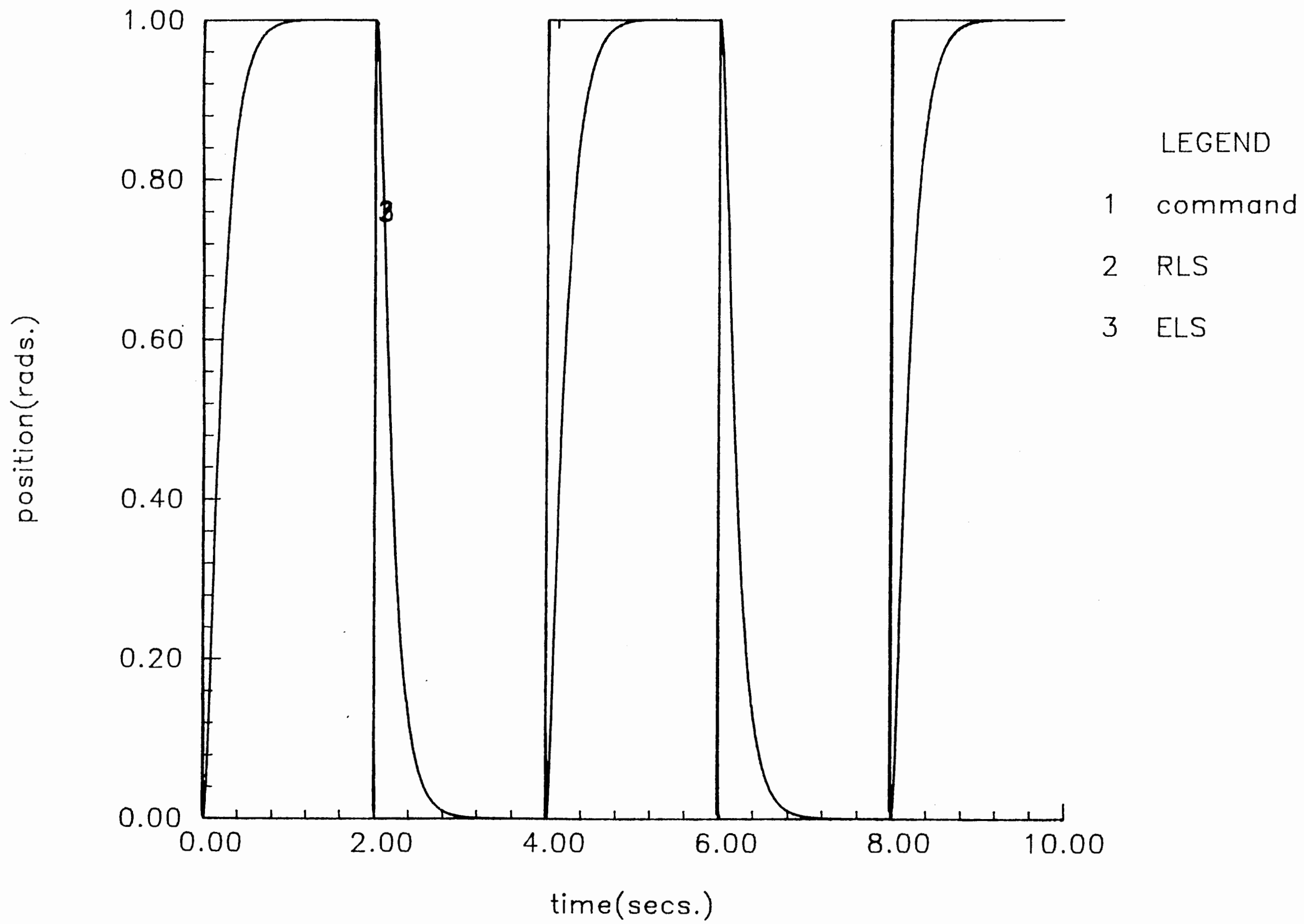

Figure 31. Response of joint 2 to square wave input of period 4 secs; Payload variation 0 to 0.5 kg; Recursive and exponentially weighted least squares

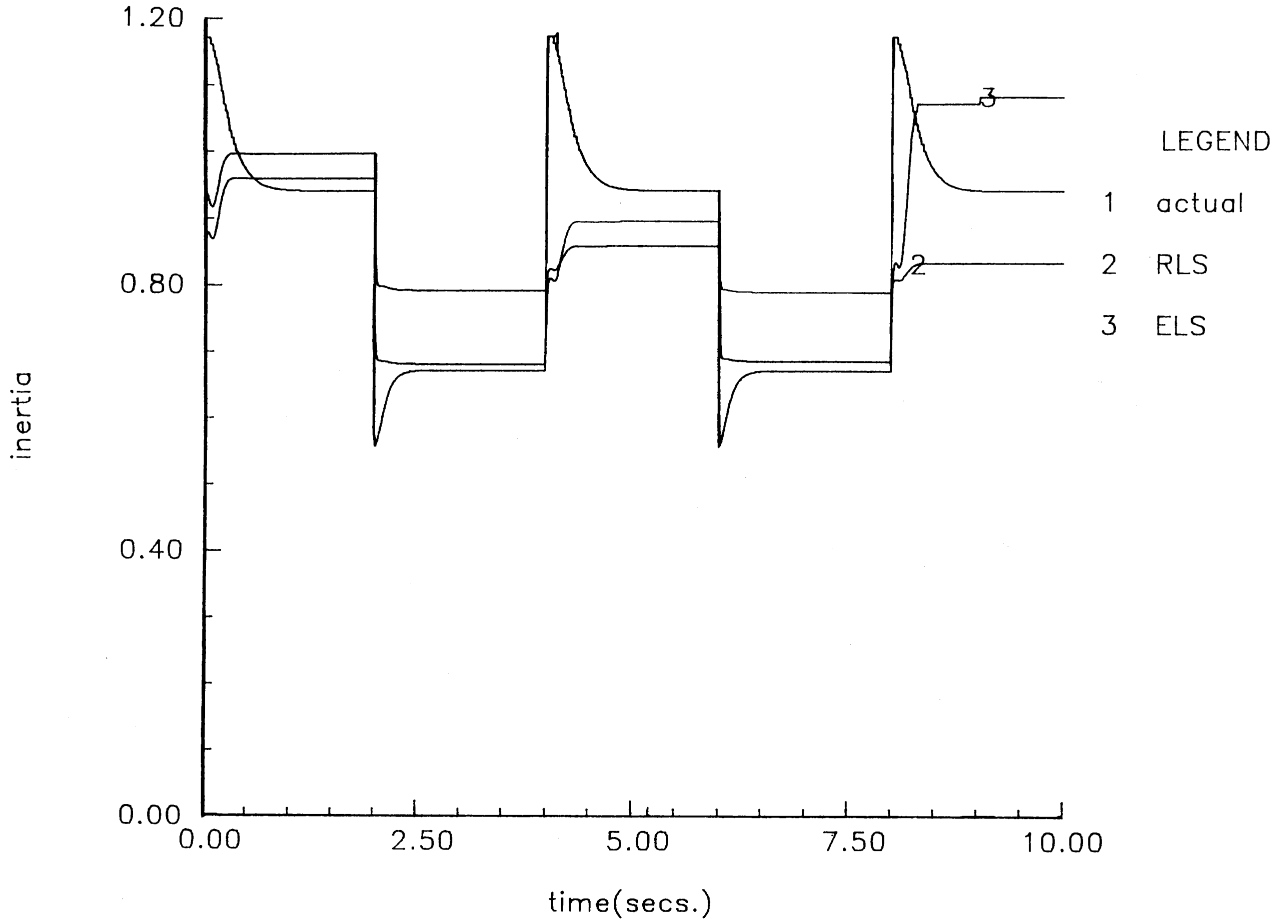

Figure 32. Comparison of actual and estimated inertias; Recursive and exponentially weighted least squares

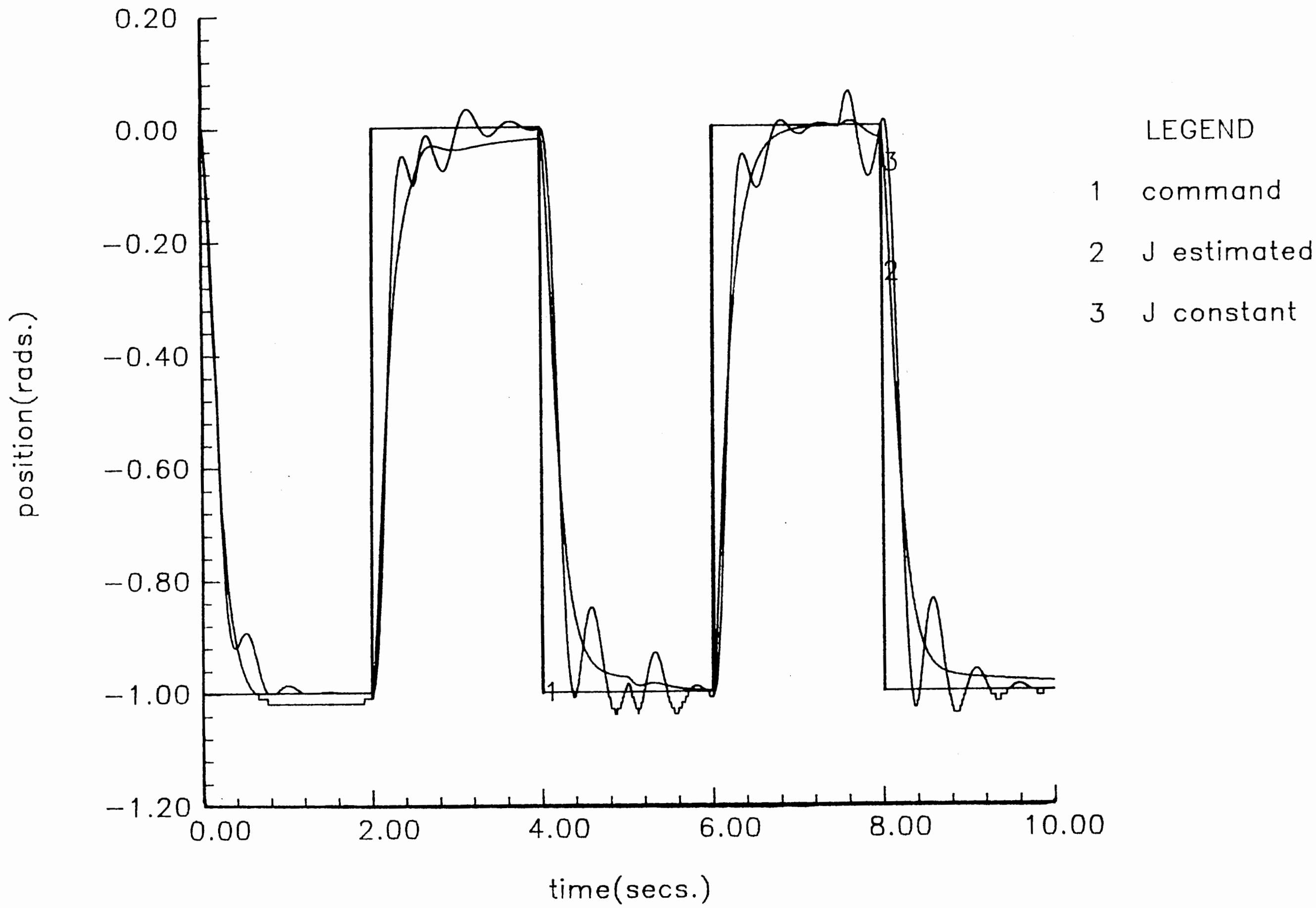

Figure 33. Comparison of response of joint 1 with constant and estimated inertias; m_L=1.5 kg

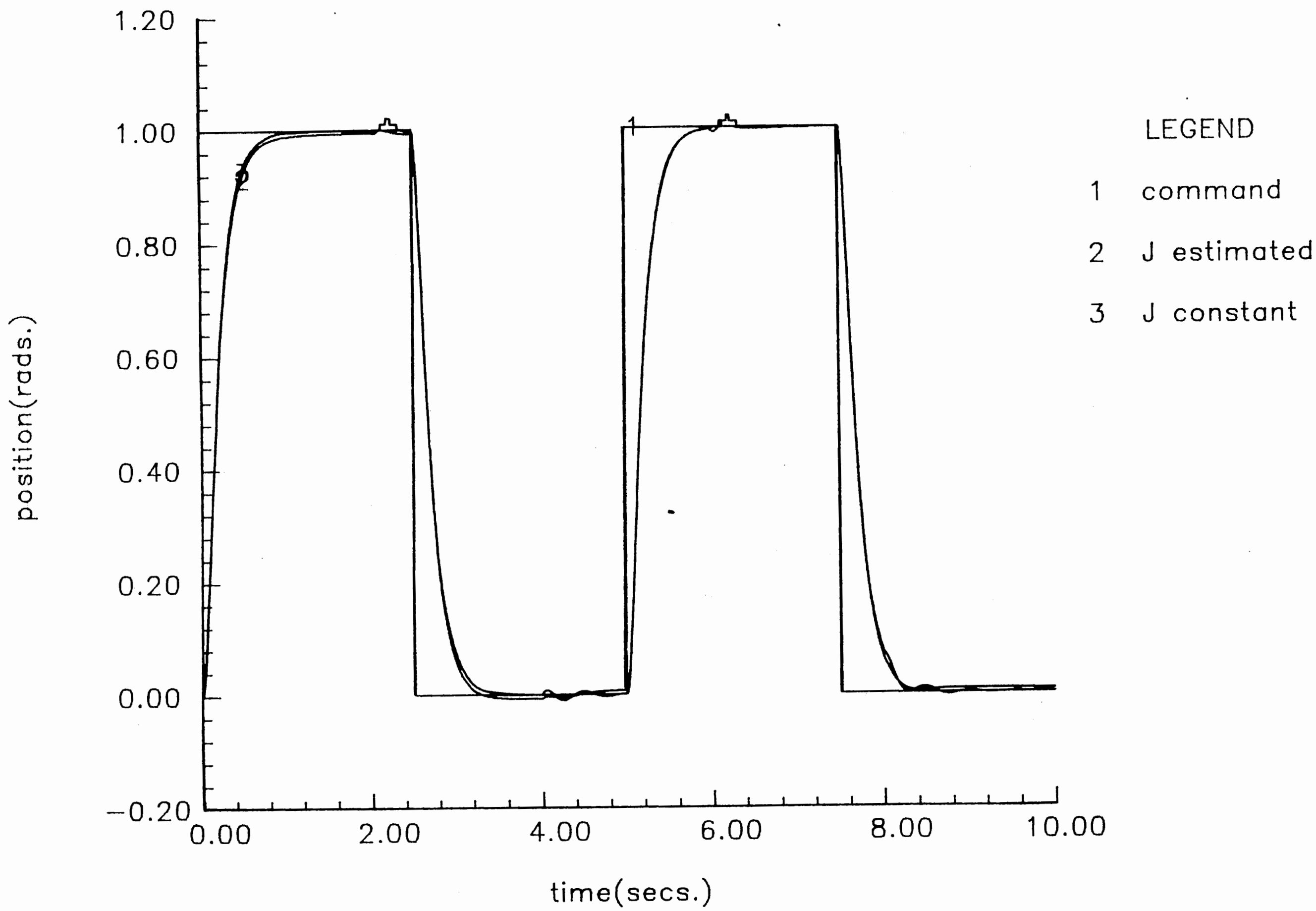

Figure 34. Comparison of response of joint 2 with constant and estimated inertias; m_L=1.5 kg

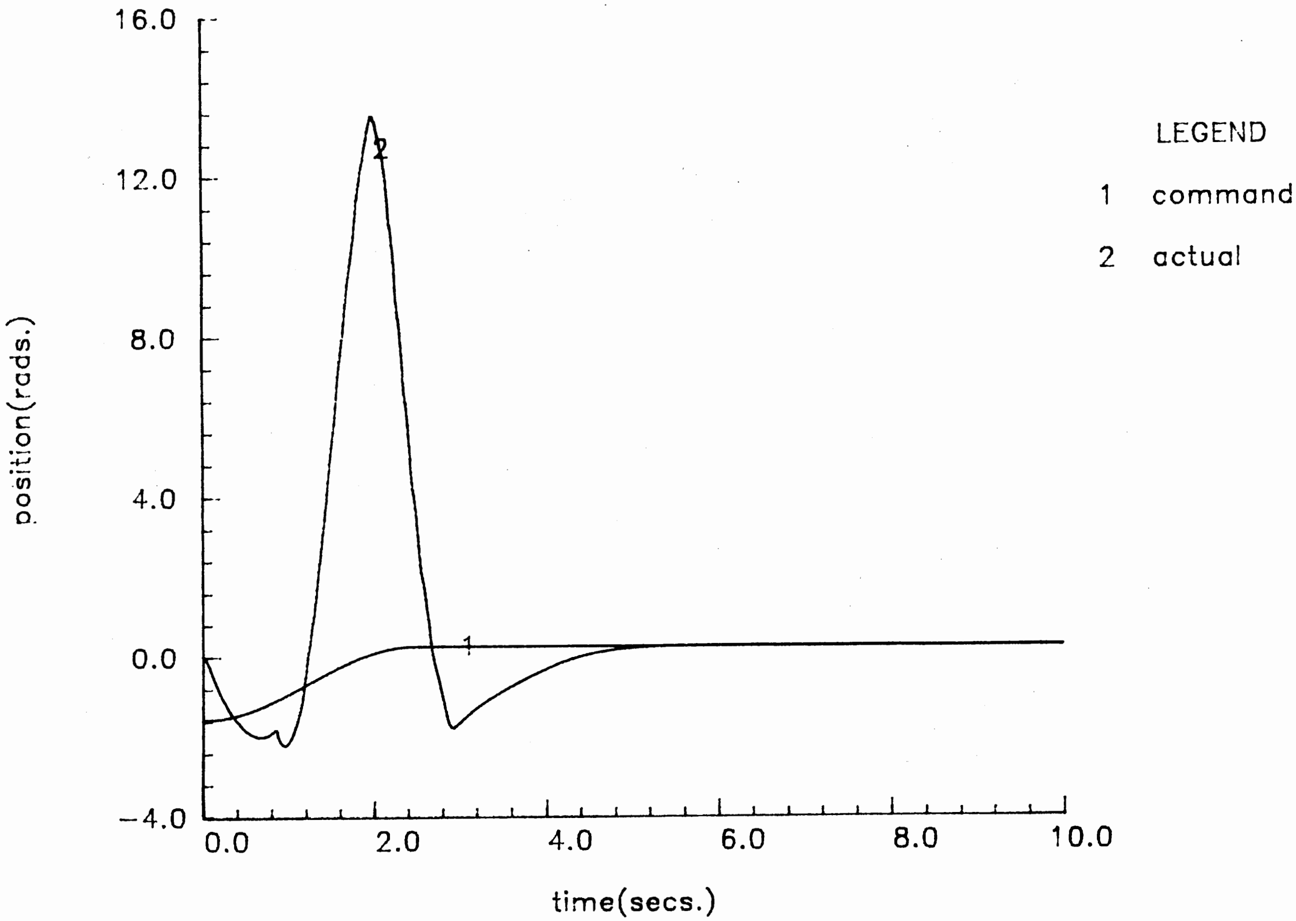

Figure 35. Response of joint 1 with measurement noise; steepest gradient algorithm; numerical differentiation

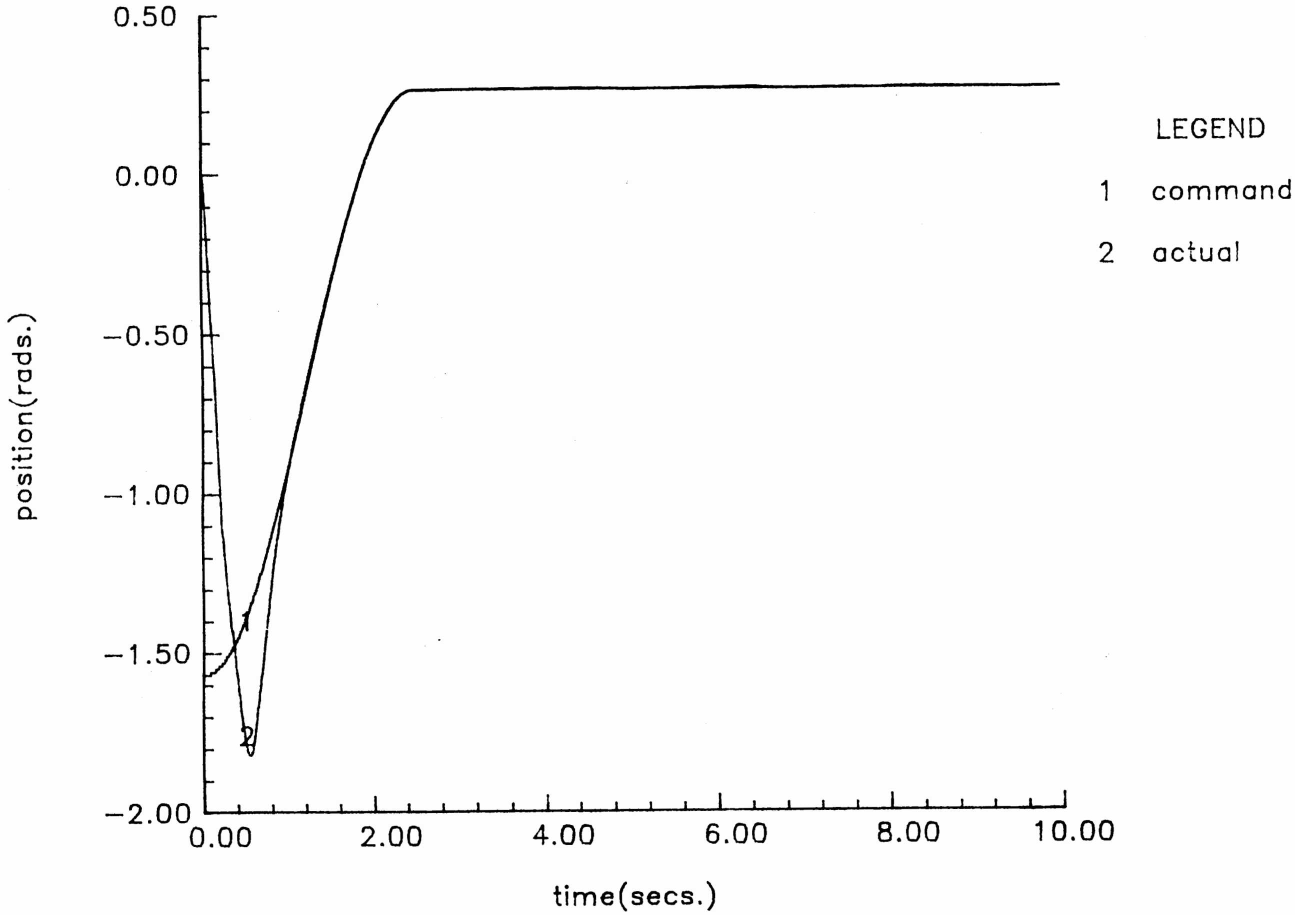

Figure 36. Response of joint 1 with measurement; noise; steepest gradient algorithm filter

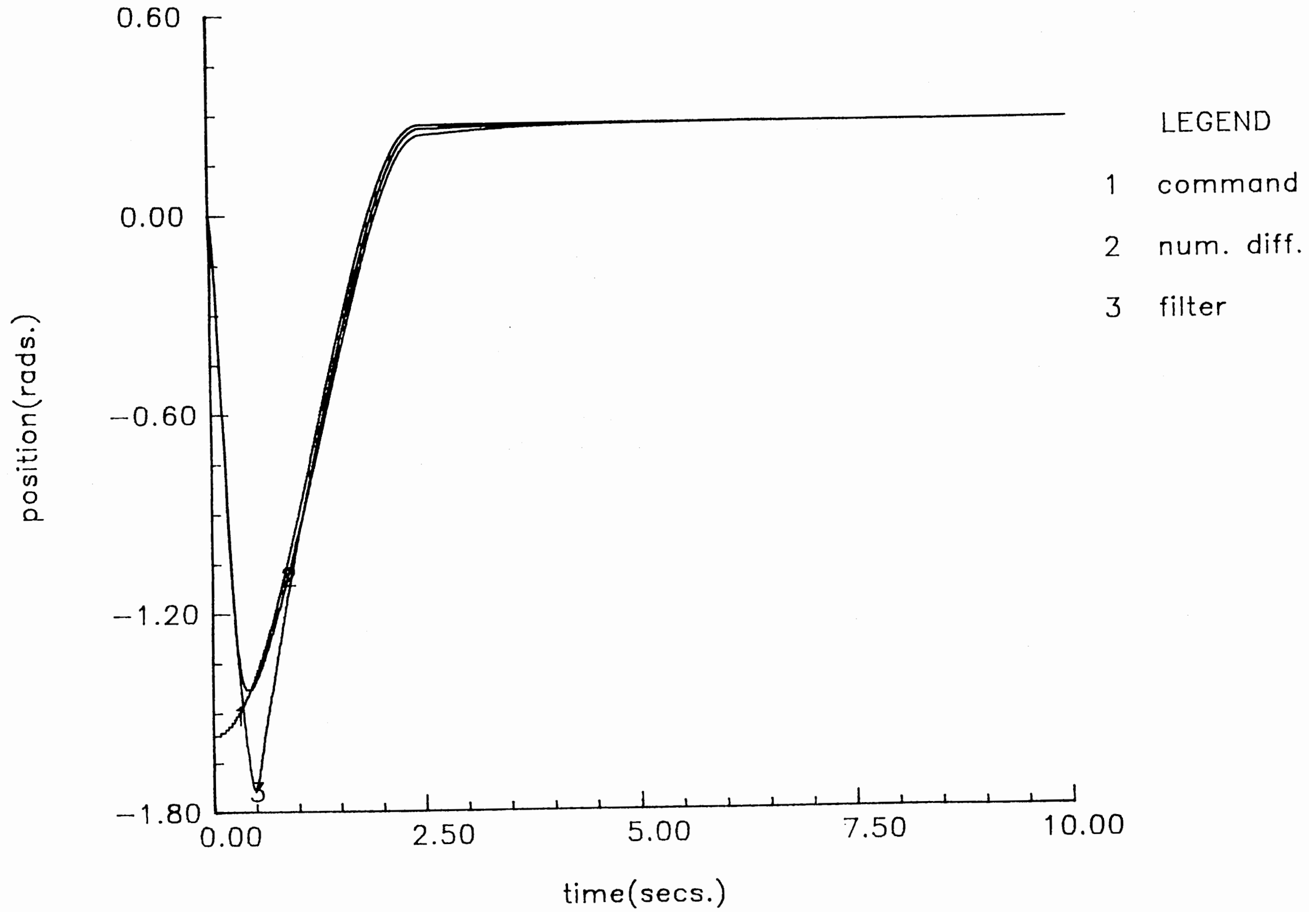

Figure 37. Comparison of response of joint 1 with and without the filter in the absence of noise

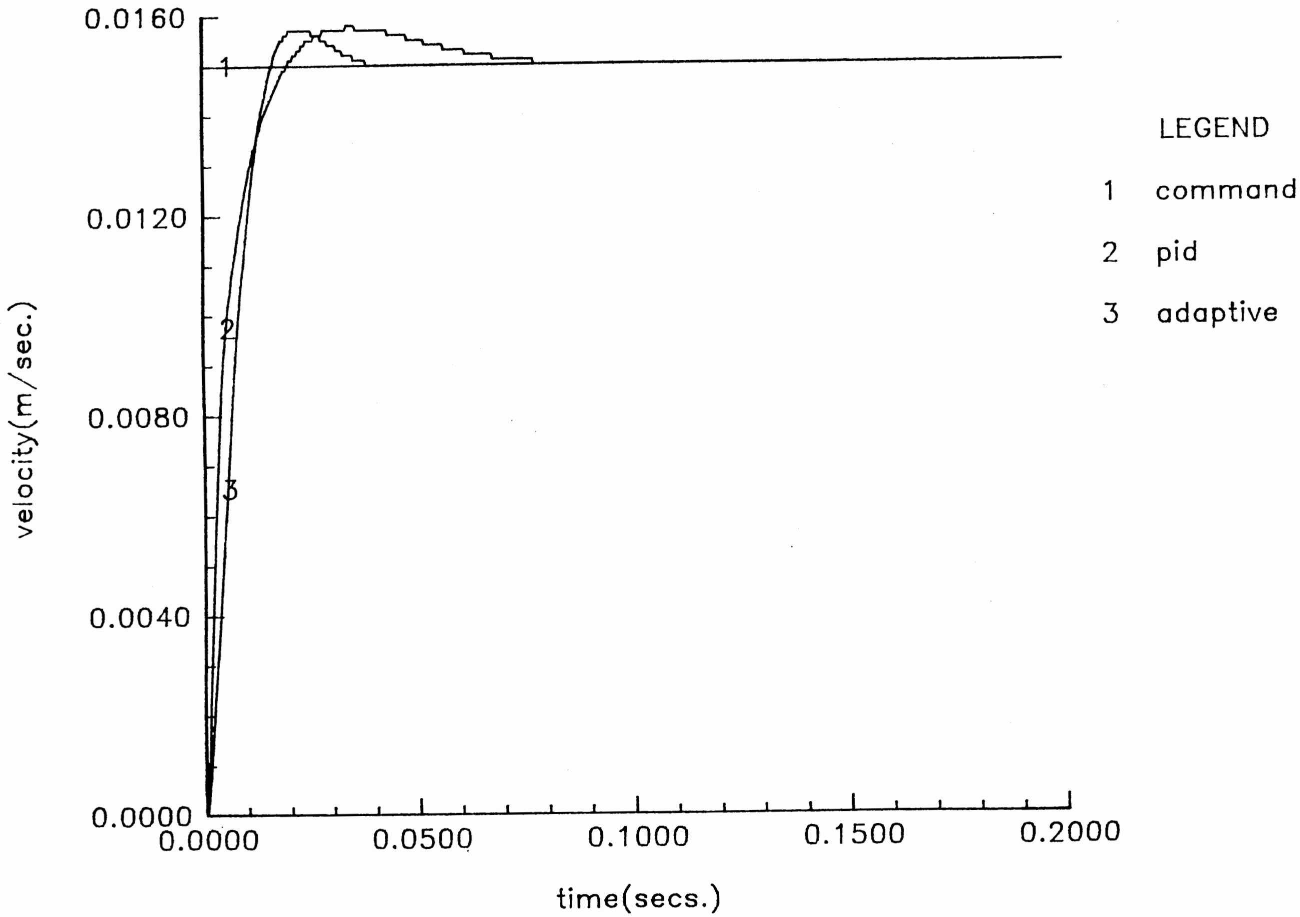

Figure 38. Comparison of response of PID and adaptive controllers; bulk modulus: 6.86×10^8 N/m^2

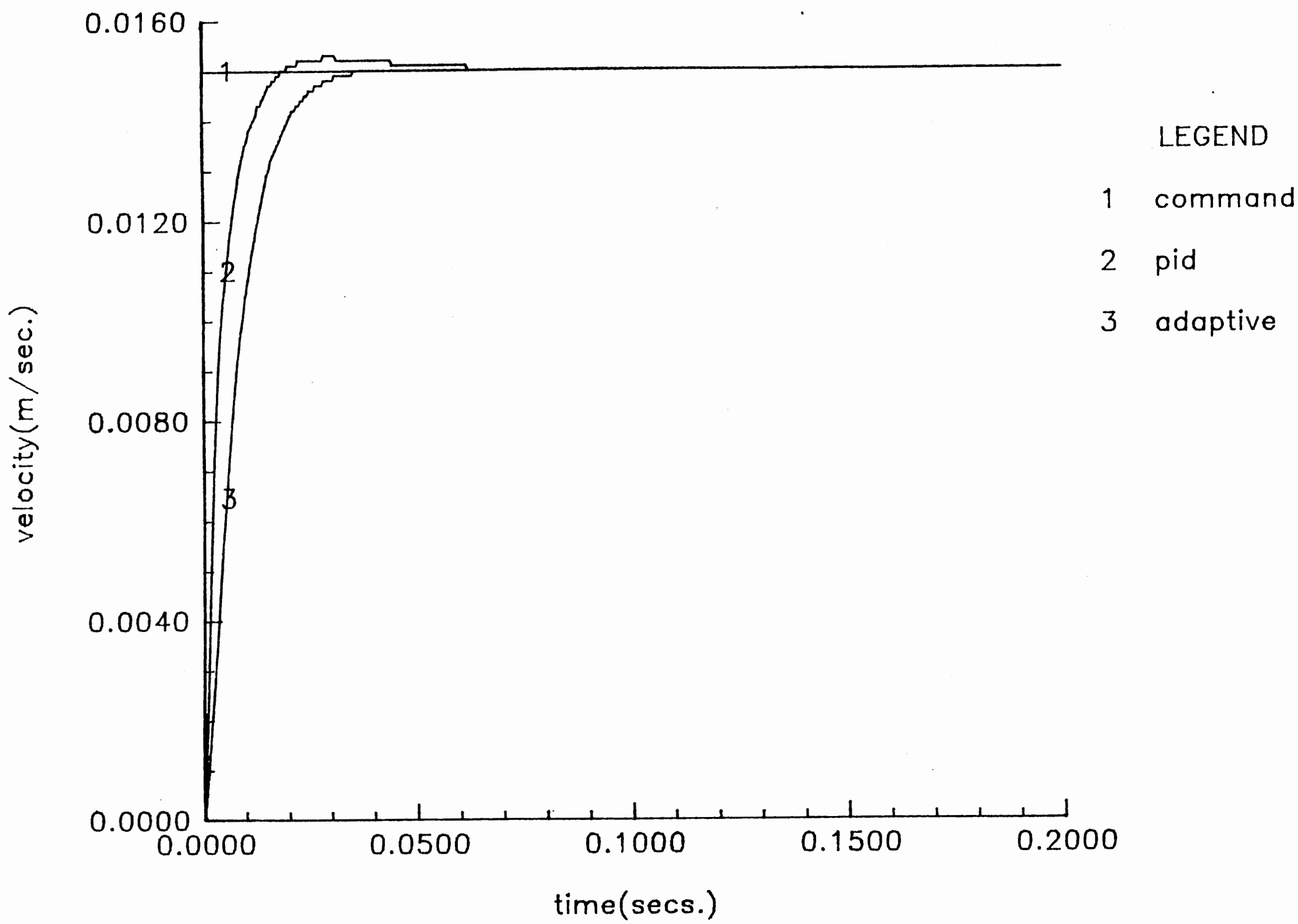

Figure 39. Comparison of response of PID and adaptive controllers; bulk modulus: 13.72×10^8 N/m^2

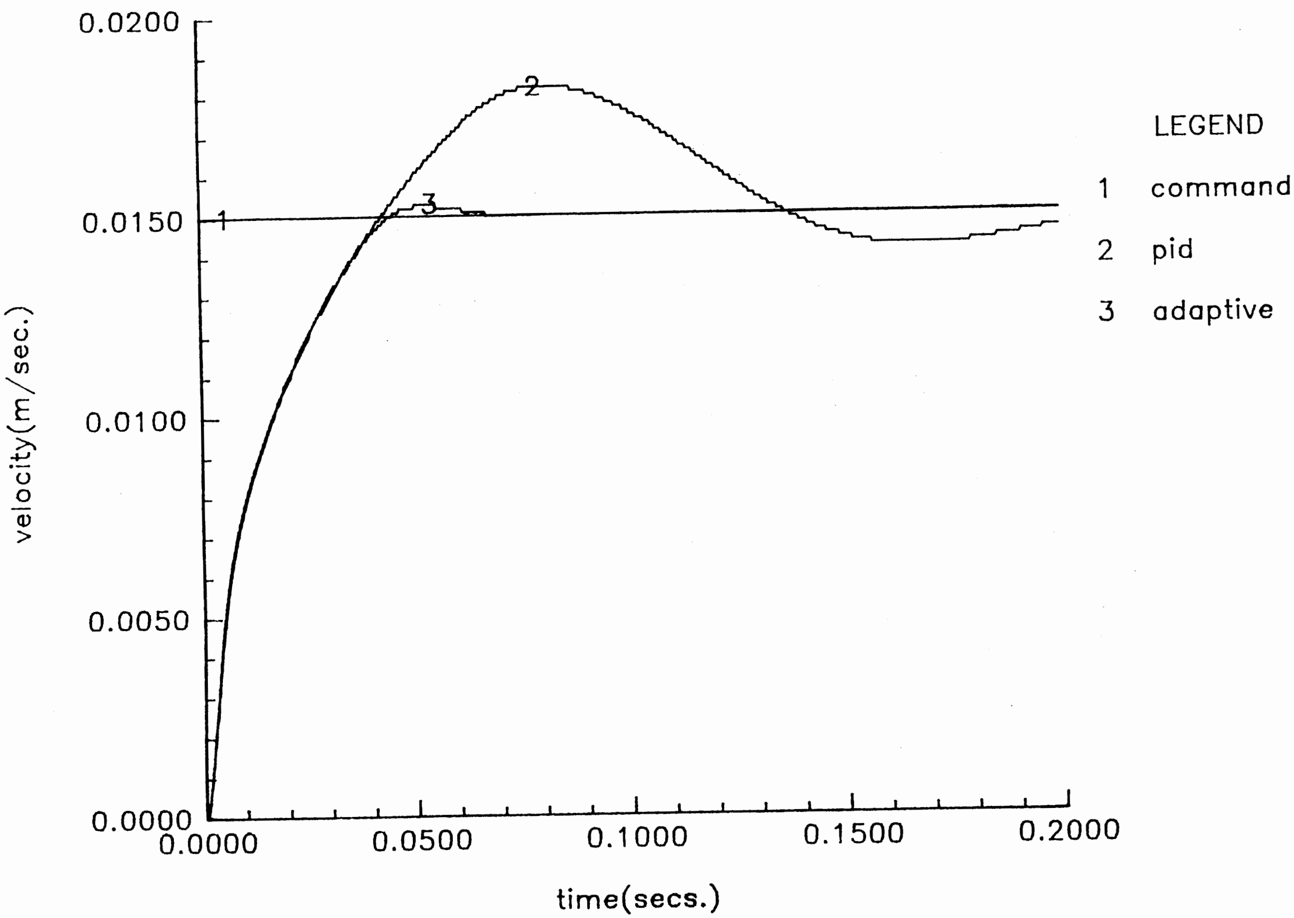

Figure 40. Comparison of response of PID and adaptive controllers; bulk modulus: 1.715×10^8 N/m^2

VITA

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Doctor of Philosophy

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