EFFECT OF DISTURBANCE DYNAMICS ON OPTIMUM CONTROL OF SECOND ORDER PLUS DEAD TIME PROCESS

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CHAPTER I

INTRODUCTION

The research work described in this paper deals with a study of the effect of disturbance dynamics on the optimum feedback control systems whose transfer function can be expressed by a second order plus dead time (SOPDT). The disturbances considered in this research are infrequent disturbances where it is expected that the control system will complete its response prior to the entry of another disturbance. The effect of disturbance dynamic was considered by using separate transfer functions for the response to the disturbance variable and the manipulated variable. Only critically damped or overdamped SOPDT processes are considered. Control systems presented are those that involve a single manipulated variable and a single controlled variable (SISO). The controller involved in this research is the conventional three mode proportional-integral-derivative (PID) controller.

Digital computer simulation is used to find the optimum controller tuning constants, controller gain, integral time and derivative time, for the SOPDT systems. The feedback control loop includes the process model (SOPDT), sensor, PID controller, and valve. A simulated control system is used as the object function for an optimization program based on the

"Rosenbrock Technique". An integral criteria, the "Integral of Absolute value of of Error (IAE)", is introduced to compare the performance and to obtain the optimum tuning constants (giving lowest IAE) from the different set of controller tuning constants.

The optimum tuning constants are dependent upon the parameters of the control loop dynamic model and disturbance dynamic model. Previous workers have dealt with this problem using disturbance models based on the simple step changes in set-point and load variables, and a sequence of random step changes in load variable. The unique feature of this research is that the disturbance is modeled as first order and enters the loop at the process output. As the two time constants of the SOPDT process are varied, the time constant of the first order disturbance is varied to examined the effect of disturbance dynamics to a controller design.

For a particular set of conditions optimum tuning constants will be found using the control system model as the objective function of an optimization program suitable for a multiple variable search involving a nonlinear function. Controller actions will be investigated by the closed loop response for the PID controller based on the optimum controller tuning constants obtained by this research.

In this study the manipulated variable is constrained to the limits corresponding to a fully closed or a fully opened valve position. Together with these limits, the lowest instaneous valve signal, will be used to investigate the range of magnitude of disturbance which the optimum controller tuning constants obtained in this research can be applicable to.

The results of this research will be applicable to practical control problems such as a heat exchanger control and a distillation column control. In these applications it is known that the controlled variable responds with different dynamics to changes in the disturbance variable and the manipulated variable.Control system performance based on the conroller tuning constants obtained by the present study should be better than the the performance based on the controller tuning constants found by the previous workers who considered only the dynamic response of the controlled variable to the changes in manipulated variable, when the disturbance comes into the control loop after the process and it has different dynamics from the process. Also, control system performance based on the process approximation by second order plus dead time, instead of first order plus dead time, should be improved.

CHAPTER II

LITERATURE REVIEW

Controller tuning is still a black art in spite of all the technical articles dealing with the subject that have been published in the last four decades. Today, computer simulation is used to extensively analyze the dynamics of chemical processes or aid in the design of controllers and study their effectiveness in controlling a given processes. Analog and digital computers have been used for this purpose, with emphasis having shifted almost entirely in favor of digital computers.

Selection of tuning constants for a control system may be accomplished by a trial-error procedure when a digital computer is available for a simulation of process response. This research describes a study of the effect of disturbance dynamics on the optimum PID control of SOPDT process using digital com-puter to evaluate the performance criteria of IAE.

A first step in the application of the feedback control technology is to model the system mathematically by investi-gating the dynamic response of the controlled variable to a change in some manipulated variable. Latour [1] showed that many processes are effectively represented by a second order with dead time model. Stern [2] developed a fast graphical method to evaluate the two time constants and the dead time of the damped second order process based on the step response curve of the process.

The process response of a second order or a higher order system with delay can be approximated by the following second order plus dead time model in the transfer function notation:

$$\text{SOPDT} \quad G_p(s) = \frac{K_p\, e^{-\theta_d s}}{(\tau_1 s + 1)(\tau_2 s + 1)} \qquad (1)$$

The three mode proportional-integral-derivative controller which first obtained acceptance after World War II is still the most frequently applied controller up to now. Its mathematical description is given below in the time domain and the Laplce domain.

Time domain:

$$V(t) = K_C \left[e(t) + \frac{1}{\tau_I}\int_0^t e(t)\, dt + \tau_D \frac{de(t)}{dt} \right] + V_S \qquad (2)$$

where V_s = controller's bias signal (i.e., its actuating signal when error = 0).

Laplace domain:

$$\frac{V(s)}{E(s)} = G_C(s) = K_C \left[1 + \frac{1}{\tau_I s} + \tau_D\, s \right] \qquad (3)$$

As we see in equation (2) and (3), use of the PID controller involves the specification of the three tuning parameters: K_C-proportional gain, τ_I-integral time and τ_D-derivative time constants. If the derivative time tuning constant is set up to zero, then the PID controller will be reduced to two mode proportional-integral (PI) controller.

Several PID controller tuning methods to evaluate the three tuning constants (K_C, τ_I and τ_D) have been developed for the control of SOPDT process. The Ultimate-cycle method originally proposed by Ziegler and Nichols [3] is based on the frequency response analysis with the feedback loop closed. These early methods were semi-empirical in nature and related the stability considerations found in the linear control theory. More recent correlation have been developed with the aid of the digital computer. Lopez [4] developed correlation to find the optimum tuning constants for systems responding to step changes in load. He used same dynamic model as process for a disturbance and developed the graphs by which the controller tuning constants can be related to the characteristics of a popular process model based on three error integral performance criterion: Integral of the Square Error (ISE), Integral of the Absolute value of Error (IAE), and Integral of the Time-weighted Absolute Error (ITAE). Rovira [5] performed a similar study for FOPDT processes where the tuning relationship can also be derived for set point changes based on the performance criteria of minimum error integrals.

The control systems used by Lopez and Rovira can be described by the block diagram shown in figure 1 given in the next chapter. This diagram shows that they performed their study under the assumption that the controlled variable C(s) responds with the same dynamics represented as $G_p(s)$ to a change in load L(s) or manipulated variable V(s). The optimization program used for their computer simulation is considered as a formalized trial and error procedure. These workers used an optimization program such as the technique derived by Rosenbrock [6] and obtained the tuning constants which produced the minimum value of integral performance criterion by means of the computer simulation.

Smith et al. [7] approached controller tuning from the simple algebraic synthesis. They developed a tuning method which required only a knowledge of the two dominant poles of a process. According to this method, PID controller is "synthesized" to give approximately first order plus dead time (FOPDT) closed loop response to step change in set point.

It is possible to approximate a SOPDT process by a FOPDT (Cohen & Coon [8]), then the tuning constants can be obtained based on the approximate model. Weigand et al. [9] performed a study comparing these methods to the tuning methods based on the full SOPDT model. These workers found out that the tuning techniques based on the full SOPDT model gave much superior results compared to the approximated FOPDT model.

Sood and Huddleston [10,11] performed the digital simulation to obtain the tuning constants for a critically damped SOPDT system exposed to a sequence of step load changes of the random magnitude based on the IAE performance criteria. Disturbances were introduced at random and were filtered by a first order lag model, in which a new one occurred before the effects of last disturbance subsided. In their study, they discovered that different optimum tuning constants existed for frequent disturbances and for infrequent disturbances. In the latter cases the control system response is substantially complete and back at a set point prior to the entry of another disturbance.

An another interesting point indicated by these workers was the presence of local minima in the IAE for the tuning constant values outside of the range predicted by previously developed tuning correlations. In some case these unexpected local minimums proved to be global minimums. This fact tells us that several different starting values of the tuning constants should be considered when a unimodal optimization technique is used in digital simulation.

Hill, Kosinsani, and Basore [18] studied the effect of disturbance dynamics on optimum tuning of FOPDT processes. In this study we consider the effect of first order disturbances on second order plus dead time processes. The disturbance is considered to enter the control loop infrequently being expected that the control system will complete its response prior to the entry of a new disturbance. Also, four different starting values of tuning constants are used in the digital simulation to check the convergence to the global minimum.

CHAPTER III

DIGITAL COMPUTER SIMULATION

Research Objectives

In Chapter II, Literature Review, several methods were introduced to find the optimum PID controller tuning constants. Previous workers investigated the effect of disturbances performing digital computer simulation but they were limited to narrowly defined disturbances. The objective of this research is to study the effect of disturbance dynamics in determining the optimum PID controller tuning constants stants for SOPDT processes, where the simulation is designed using separate transfer functions for the response to the disturbance variable and the manipulated variable.

The disturbance is modeled as first order and enters the control loop at the process output. The control loop which includes the process model, PID controller, valve and sensor, is simulated with a digital computer to calculate the minimum IAE values for the different sets of characteristics of the process and the disturbance model.

Previous workers developed correlations which related the optimum tuning constants and the process dynamic parameters. As previous workers did, in this work the optimum tuning constants will be described as a function of both process and disturbance dynamic parameters. Also, such correlations for the integral of absolute value of error will be provided to illustrate the effect of process dead time and disturbance dynamics on the control system performance.

Another objective of this research will be to investigate the range of disturbance magnitude which obtained controller tuning constants can be applicable to. If there are no constraints in the manipulated variable and the feedback control system is modeled as a system of linear equations, the magnitude of the disturbance will linearly affect the IAE value. Therefore, the values of optimum controller tuning constants will be independent of the change of disturbance magnitude. However, because the manipulated variable is constrained in the limits corresponding to a fully closed or a fully open actual valve, disturbances with magnitude large enough to saturate the valve will affect the calculation of optimum controller tuning constants. In this research optimum tuning constants were determined for load magnitudes small enough to avoid valve saturation during the response.The lowest and highest instaneous valve signal, will be invstigated. Those extremes will be used to calculate the range of magnitude of disturbance which the optimum controller tuning constants obtained in this research can be applicable to.

Digital Simulation Approach

The generalized feedback control loop can be described as a block diagram shown in figure 2. It has an output C(s), a potential disturbance L(s), and an available manipulated variable V(s). The disturbance, L(s), changes in an unpredictable manner and our control objective is to keep the value of output at the desired reference value, R(s). A feedback control takes the following generalized steps:

1. measure the value of output using appropriate measuring device.

2. compare the indicated value, C(s), to the desired set point value, R(S), then let the deviation variable ,e, be error: e = R(s) - C(s)

3. The value of deviation,e, is supplied to the main controller. The controller in turn changes the value of the manipulated variable in such a way as to reduce the magnitude of error deviation, e. Usually, the controller does not affect the manipulated variable directly but through the final control element like a valve.

The control system used by previous workers is described in figure 1. The figure 2 represents the block diagram of the control system used for this research. The main difference between the two systems would be in the way the load disturbance, L(s), enters the control loop. According to the system described by figure 2 the transfer function of disturbance may have the different dynamics from the transfer function for the response to the manipulated variable.

In figure 2, $G_{P_2}(s)$ represents the transfer function of load variable L(s) on the response of the controlled variable C(s) and $G_{P_1}(s)$ represents the transfer function of manipulated

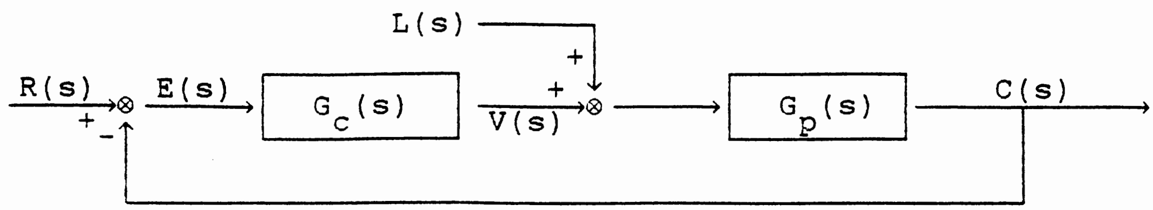


Figure 1. Block diagram used by Lopez and Rovira

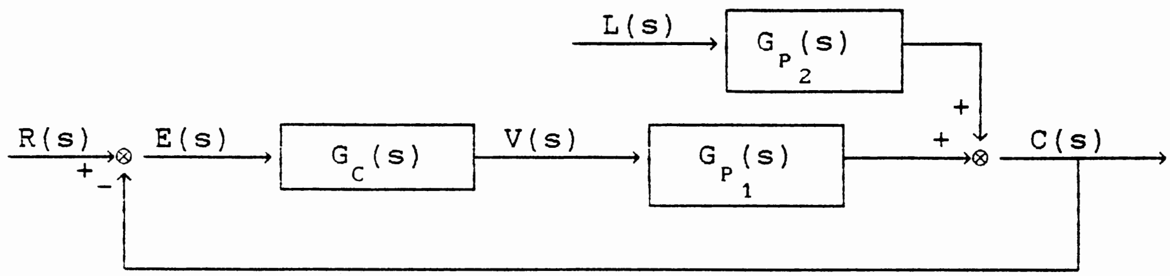


Figure 2. Block diagram used in present study

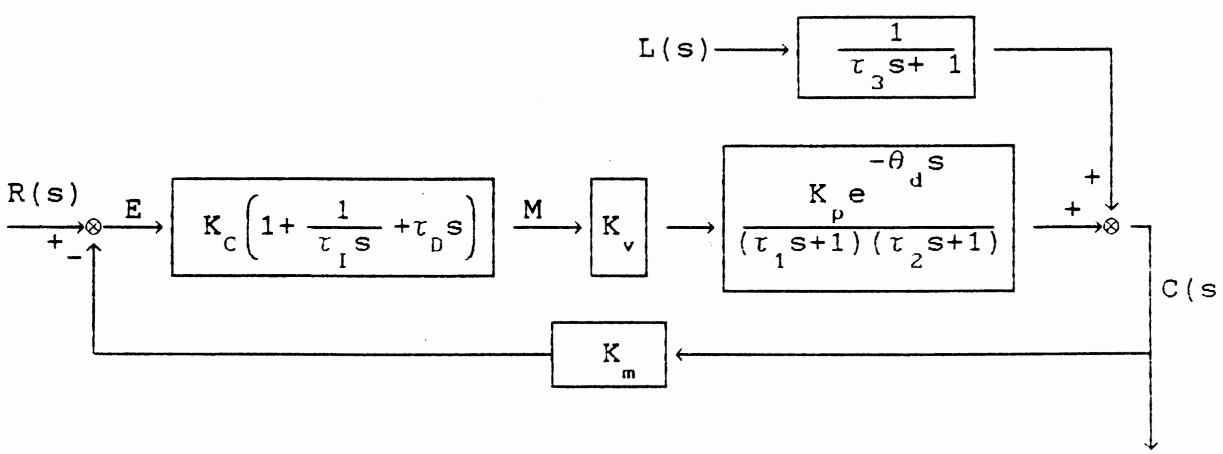


Figure 3. Control loop with a SOPDT process and a First order disturbance.

variable V(s) on the response of the controlled variable. In the control system studied by previous workers (figure 1) a single process transfer function $G_p(s)$ is provided to represent the dynamic effect of not only disturbance but also manipulated variable. The system shown in figure 2 describes the more general cases. It will become the same as the case used by previous workers if the transfer function $G_{p_1}(s)$ and $G_{p_2}(s)$ have the same form.

The control system which will be used in simulation for this research is given in figure 3, giving detailed specificationto a general control loop described in figure 2. In this control system the process model is given by a second order plus dead time with a process gain K_p and the transfer fucton of controller represents a conventional three mode proportional-integral-derivative (PID) controller. It has the three tuning constants: controller gain, K_C, integral time, τ_I,and derivative time constant, τ_D. Also, the measuring element gain K_m and the gain of valve K_v is involved in the control loop.

Physical interpretation of the control system presented in this study is detailed in figure 4. The mixing process depicted by figure 4 are composed by two tanks in series which maintain constant liquid levels and flow rates. Two entering streams are being mixed and stirred by an agitator in the first tank. This well-mixed liquid with the outlet concentration X_2 goes through the second tank, then produce the final product, stream 4, with outlet concentration X_4.

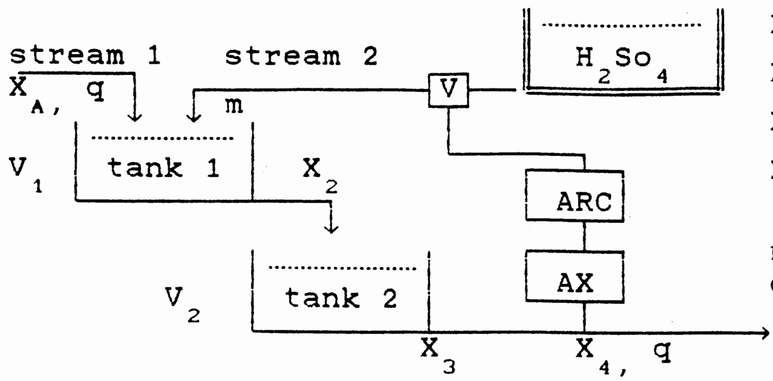
Inlet Stream 1 has a nominal concentration of 500 mg/l of sulfuric acid (H_2So_4) with volumetric flow rate, 1 liter/min, and inlet stream 2 has concentrated sulfuric acid with nominal flow rate m equal to 1000mg/min.

The flow m is manipulated by a feedback controller in order to maintain the desired acid concentration at a 1500 mg/l in the exit liquid line where a concentration analyzer is positioned. Since the mixing tanks are mixed by agitaor, it is assumed that the concentration in the tanks are homogeneous, therefore, its concentration is the same as the exit concentration. The volume of the first tank is equal to one liter providing a tank detention time of one minute . The volume of the second tank is variable in the range 0.1 to 1.0. The liquid flow model in the exit line is assumed to be ideal. The volume of exit liquid line preceding the analyzer is a plug flow allowed to vary, giving transportation lags in the range of 0.1 to 1 minute for the investigation of the effects of dead time on controller tuning.

In figure 4, we see that three different types of disturbance can intrude into the control loop : a set point disturbance R(s), a load disturbance X_A, and another load disturbance X_B. The effect of set point change to the control loop was studied by Rovira et al. [5] and the effect of load disturbance X_A, physically interpreted as a step change in entering liquid concentration, is the type of disturbance studied by Lopez et al. [4].

Finally, the disturbance X_B, studied in this research,

Process diagram

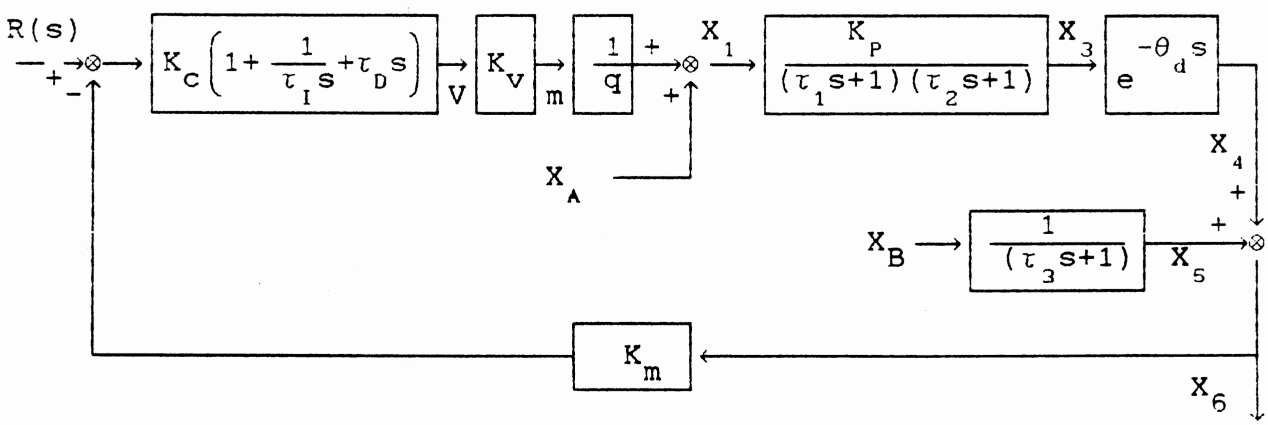


Legend

X_A = entering conc., mg/l
X_2 = conc. of tank 1, mg/l
X_3 = conc. of tank 2, mg/l
X_4 = conc. of X_3 after transportation lag
m = acid flow
q = flow rate, 1 l/min
V_1, V_2 = tank volume
V = valve
ARC = controller
AX = analyzer

not shown : X_1, X_B, X_5, X_6

Control loop block diagram



X_B = step change in the disturbance
X_5 = disturbance after a first order lag

Figure 4. Physical interpretation of a control system

is not so easy to provide a physical interpretation in this system. Perhaps it could be considered as the sudden placement of corrodable piece of metal in the second tank that eventually consumes a steady supply of H_2So_4. This disturbance would be similar to applying a negative value of X_B. However, in this study a positive value of X_B was always applied during the optimization runs. In this research the transfer function of load disturbance X_B is given as a first order with a time constant τ_3 and a unit gain. In the simulation the time constant τ_3 is varied to investigate the effect of disturbance dynamics on the controller tuning constants. The range of variation for τ_3 extends from 0.05 to 7.

Because of the low concentration involve in the process of figure 5 the mass flow of acid, m, is assumed to be much less than the entering liquid. If the entering liquid is to be water, the actual ratio of mass flows of the two stream is less than 1/1000 at normal operating conditions. With the above assumptions the hypothetical concentration X_1 of the two entering stream can be expressed by the following approximated equation:

$$X_1 = X_A + \frac{m}{q} \qquad (4)$$

where q is the constant volumetric liquid flow rate equal to 1 liter per minute. In the above equation and all the following equations , the variables will be defined to be in the deviation (perturbation) form which describes directly the magnitude of dislocation of a system from the desired level of operation (steady state). Therefore, before the introduction of a disturbance, the system is considered to be at a steady state and all the variables would have values which are equal to zero.

Now, mathematical modeling of a process can be derived by setting up a mass balance on the mixing tanks. The principle of conservation of mass states that :

$$\frac{\begin{bmatrix}\text{accumulation of mass} \\ \text{within a system}\end{bmatrix}}{\text{time period}} = \frac{\begin{bmatrix}\text{flow of mass} \\ \text{in the system}\end{bmatrix}}{\text{time period}} - \frac{\begin{bmatrix}\text{flows of mass} \\ \text{out of system}\end{bmatrix}}{\text{time period}} + \frac{\begin{bmatrix}\text{amount of mass} \\ \text{generation within} \\ \text{the system}\end{bmatrix}}{\text{time period}} - \frac{\begin{bmatrix}\text{amount of mass} \\ \text{consumption in} \\ \text{system}\end{bmatrix}}{\text{time period}} \tag{5}$$

Using above principle provides the mass balances of the two tanks :

For a first tank :

$$q\ X_1(t) - q\ X_2(t) = V_1 \frac{dX_2(t)}{dt} \tag{6}$$

For a second tank :

$$q\ X_2(t) - q\ X_3(t) = V_2 \frac{dX_3(t)}{dt} \tag{7}$$

Rearranging equation (6) and (7) produces,

$$\frac{dX_2(t)}{dt} = \frac{X_1(t) - X_2(t)}{\tau_1} \tag{8}$$

and

$$\frac{dX_3(t)}{dt} = \frac{X_2(t) - X_3(t)}{\tau_2} \tag{9}$$

where $\tau_1 = V_1/q$ and $\tau_2 = V_2/q$.

In Laplace domain equations (8) and (9) become,

$$\frac{X_2(s)}{X_1(s)} = \frac{1}{\tau_1 s + 1} \qquad (10)$$

and

$$\frac{X_3(s)}{X_2(s)} = \frac{1}{\tau_2 s + 1} \qquad (11).$$

The transportation lag of liquid passing from the second tank to the analyzer can be obtained by the following calculation:

$$\theta_d = V_L/q \qquad (12)$$

where θ_d = transportation lag (dead time)

V_L = liquid volume of the exit liquid line preceding the analyzer

q = liquid flow rate .

Combining equation (10), (11) and dead time equation (12) produces the following SOPDT model for the two mixing tanks in series :

$$G_p(s) = \frac{X_3(s)}{X_2(s)} = \frac{e^{-\theta_d s}}{(\tau_1 s + 1)\ (\tau_2 s + 1)} \qquad (13)$$

where $G_p(s)$ means the transfer function of process.

Two gain elements were introduced in the control loop of figure 4 : K_v for final control element (valve) and K_m for the measuring device. The most common final element is the pneumatic valve, which receives the output of the controller (actuating signal) and accordingly adjusts the value of manipulated variable. In the control system shown in figure 4, the signals between the analyzer and controller, also the signals between the controller and valve are depicted as pneumatic signals. The pneumatic signal of most devices varies with the range from 3 to 15 psig. Considering the above fact the gains associated with measuring element and vale are calculated as follows :

$$K_v = \frac{\Delta m}{\Delta V} = \frac{2000 \text{ g/min}}{12 \text{ psi}} = 166.667 \text{ (g/min-psi)} \quad (14)$$

where Δm = maximum acid flow at the maximum valve signal, and ΔV = range of the controller output signal.

$$K_m = \frac{\Delta C}{\Delta X} = \frac{12 \text{ psi}}{3000 \text{ mg/l}} = 0.004 \text{ (psi-mg/l)} \quad (15)$$

where ΔC = range of the analyzer output signal, and ΔX = maximum concentration measured by analyzer.

The general form of conventional continuous three mode proportional-integral-derivative controller is given :

$$V(t) = K_C \left[e(t) + \tau_I \int e(t)\, dt + \tau_D \frac{de(t)}{dt} \right] + V_0 \quad (15)$$

where V_0 = controller's bias signal (i.e. its actuating signal when e = 0). Its transfer function can be expressed as a following equation:

$$G_C(s) = K_C \left[1 + \frac{1}{\tau_I s} + \tau_D s \right] \quad (16)$$

In digital simulation this continuous PID algorithm needs to be modified to the digital approximation. Two forms of discrete time approximation are often used for a PID controller. The one is a position form and the other is a velocity form [12]. The position form is,

$$V_n = V_0 + K_c \left[e_n + \frac{T}{\tau_I} \sum_{i=0}^{n} e_i + \frac{\tau_D}{T} (e_n - e_{n-1}) \right] \quad (17)$$

and the velocity form is,

$$V_n = V_{n-1} + K_c \left[(1 + \frac{T}{\tau_I} + \frac{\tau_D}{T}) e_n - (1 + \frac{2\tau_D}{T}) e_{n-1} + \frac{\tau_D}{T} e_{n-2} \right] \quad (18).$$

In the above equation n refers to the nth sampling instant and T refers to the sampling interval. Both equation used rectangular integration to approximate the integral control mode and use first order difference to approximate the derivative mode. The velocity form is derived by subtracting $V_n - V_{n-1}$. In this research the position form is used.

In the digital computer simulation the control system described in figure 4 is programed to be used as the objective function for the optimization program which is based on the Rosenbrock technique [6]. Then, the optimum tuning constants to step changes in X_B are calculated for various sets of $\tau_2/\tau_1, \tau_3/\tau_1$ and θ_d/τ_1.

Objective Function

In figure 5 the program of the objective function which describes the simulated control system is given in a portion of pascal code. Function dtx2dot and dtx3dot in the program are defined to solve the first order differential equaton (8) and (9), modeling of two mixing tanks. Two first order differential equations are solved numerically by application of a fourth order Runge-Kutta method. The object function calculates the integral of absolute value of error (IAE)which accumulates as the system responds to a disturbance. The followings are the list of constants and variables assigned globally with respect to the function definition:

K_{mm} =measuring element gain

K_v = valve gain

r = step change in set point

X_A = step change in entering liquid concentration

X_B = step change in measurement error

delta = time step size for Rnuge-Kutta integration

ttt = total simulation time

thetad = dead time

tau1 = first time constant of the process

tau2 = 2nd time constant of the process

tau3 = first order time constant of load variable, X_B

taudd = derivative time tuning constant

tauii = integral time tuning constant

kcc = proportional gain tuning constant

er = error of current value

erint = time integral of error

epast = error of previous step

absie = time integral of absolute value of error

s = integral number of time steps included in the dead time, thetad

DT = dead time array

kmm,kvv = measurement and valve gain

vv,va = controller output

G,Q = pointers of dead time array

vamax = maximum valve position

```
function object(cx1,cx2:real):real;
     begin
        dtx2dot:=-delta*(cx1-cx2)/tau1;
     end;

  function dtx3dot(cx2,cx3:real):real;
     begin
        dtx3dot:=-delta*(cx2-cx3)/tau2;
     end;

function object(kcc,tauii,taudd:real):real;

var
   DT : array [1..5001] of real {array for dead time}
   i: integer;
   begin
     c:=0.0;epast:=0.0;absie:=0.0;
     vamin:=0.0;vamax:=0.0;
     erint:=0.0;time:=0.0;
     x1:=0.0;x2:=0.0;x3:=0.0;x4:=0.0;
     x5:=0.0;x6:=0.0;
     for i:=1 to s do DT[i]:=0.0; G:=s; Q:=1;
        while time < ttt do
           begin
              c:=kmm*x6;
              er:=r-c;
              va:=kcc*(er+erint/tauii+(er-epast)*taudd);
              vv:=va;
              if va<vamin then vamin:=va;
              if va>vamax then vamax:=va;
              if va<= -6.0 then va:=6.0;
              if va>= 6.0 then va:=6.0;
              x1:=xa+va*kvv
              time:=time+delta;
              if (vv<6.1) and (vv>-6.1) then
              erint:=erint+er*delta;
              absie:=absie+abs(er*delta);
              epast:=er;
              rk11:=dtx2dot(x1,x2);
              rk12:=dtx3dot(x2,x3);
              rk21:=dtx2dot(x1,x2+0.5*rk11);
              rk22:=dtx3dot(x2+0.5*rk11,x3+0.5*rk12);
              rk31:=dtx2dot(x1,x2+0.5*rk21);
              rk32:=dtx3dot(x2+0.5*rk21,x3+0.5*rk22);
              rk41:=dtx2dot(x1,x2+rk31);
              rk42:=dtx3dot(x2+rk31,x3+rk32);
              x2:=x2+(rk11+2.0*rk21+2.0*rk31+rk41)/6.0;
              x3:=x3+(rk12+2.0*rk22+2.0*rk32+rk42)/6.0;
              DT[G]:=x3;
```

Figure 5. Pascal program illustrating the objective function

```
            x4:=DT[Q];
            G:=G+1;
            Q:=Q+1;
              if Q>s then Q:=1;
              x5:=x5*du31+du32xb;
              x6:=x4+x5;
           end;
        object:=absie;
end;
```

Figure 5. (Continued)

vamin = minimum valve posion

x1 = hypothetical inlet concentration

x2 = concentration in the first tank

x3 = concentration in the second tank

x4 = concentration of x3 after the transportation lag

x5 = disturbance after first order lag, tau3

x6 = measured concentration passing the analyzer after imposition of the disturbance

The first part of the code, function object, shows the initialization of several variables setting them to zero corresponding to an initial steady state condition prior to a disturbance, because they were expressed in a deviation form. The next part of code is a computation loop to repeat the several calculations for each step in time until the running value of time is greater than the total simulation time, ttt. The computation steps in the loop are as follows:

The concentration, X_6, measured by an analyzer is changed into a pneumatic signal with the multiplication of measurement gain kmm,then this pneumatic signal, c, is compared to the desired set point value, r, to evaluate the error, er. The controller output, va, is calculated based on the three tuning constants transferred from the optimization procedure. This controller output va is compared to the limits of pneumatic signal corresponding to the valve position of a fully open or a fully closed state. If a valve limit is exceeded more than a small amount integration of the error is stopped to prevent windup. Hypothetical inlet concentration is calculated based on the controller output and valve gain, kvv, then the concentrations of x2 and x3 are calculated with the fourth order Runge-Kutta method. Time integral of the error, erint, used in the controller output and time integral of absolute value of the error, absie, are calculated in next steps. The concentration of x3 is saved in the dead time array. After the time has elapsed corresponding to the dead time the values of concentration x3 saved in the dead time array ,DT,are taken out and designated as variable x4. The disturbance after the first order lag tau3, x5, is computed and added to x4 to produce the controlled output x6 which is measured by the analyzer.

The computation steps stated above are repeated in the loop until the accumulation of time reaches total time, ttt. In the computation loop the function, object, is assigned a value equivalent to the last accumulated value of absie, the integral of absolute value of error. The PID controller equation used in the simulation is written in position form. Valve signal (controller output) was constrained to a range of +6 to -6 (making use of deviation variables and assuming the valve is half open at the start of simulation). The value of erint is allowed to accumulate as long as computed valve position does not exceed the valve constraints by more than a small margin. This is to prevent the integral wind up caused by the integral model of a controller when errors can not be eliminated quickly, and therefore, produce larger and larger values for the integral term which in turns keeps increasing the control action until it is saturated and remains saturated even if the error returns to zero.

The dead time is an important element in the mathematical modeling and has a serious impact on the design of effective controllers. A major advantage of digital simulation is in the fact that the dead time can be handled with ease. The dead time array, DT, includes enough elements to hold a process variable for an integral number of iterations equal to the dead time. The outlet concentration x3 is saved in the array DT sequentially, held in array DT for s-1 number of time increments which is equal to dead time, then taken out to be applied to the computation loop after s number of time increments. Dead time array pointers G and Q were used to keep track of the positions of concentration x3 until it would have entered the pipe line leaving the tank and pass through the analyzer.

The iteration step, delta, was set equal to 0.001. The total time of simulation, ttt, varied from 20 to 50 minutes as the time constant of disturbance tau3 increases. In all the cases, the total time was greater than six times the ultimate period, $2\pi/\omega_{co}$ (ω_{co}: cross over frequency rad/min), found according to the frequency response analysis [12].

The performance criteria, IAE, is accumulated as a variable, absie, and returned as the result of objective function to the main program that successively compares IAE for a wide variety of controller tuning constants to choose the optimum values of tuning constants.

Computer computations were performed on an IBM 3090-200 main frame computer located at the University of Southwestern Louisiana. Floating point calculation was performed in IBM double precision format. For a run initial inputs were introduced with the following values: r=0.0, xa=0.0, xb=10.0, delta=0.001, tt=0.0, and a variety of sets of tau1, tau2, and thetad values. During the run the range of tau3 was extended from 0.05 to 7.0 with 19 different data values to check the effect of speed of disturbance. Those 19 different values of tau3 were kept in the array t3 in the main program and specified as follows: 0.05, 0.1, 0.3, 0.5, 0.7, 1.0, 1.2, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0.

In the optimization results tuning constants are normalized to be applicable to other SOPDT processes. Proportional gain K_C is given as the product of K_C and K_L, where K_L is equivalent to the product of measuring element gain, K_m, process gain, K_P, and valve gain K_v. The integral time is reported as τ_1/τ_I and the derivative is reported as τ_D/τ_1. The integral of absolute value of the error, IAE IAE observed by the controller is divided by the measuring element gain K_m to the IAE in terms of the controlled variable's units. This value is then expressed as dimensionless form in the figures by dividing by the load magnitude X_B and time constant τ_1 to get the normalized value, $IAE/(X_B*\tau_1)$.

Main Program and Optimization Results

The main program intends to find a minimum of a multivariable, unconstrained, non-linear function. The procedure is based on the direct search method which is proposed by H. H. Rosenbrock [6]. Since the procedure assumes a unimodal function several sets of starting values for the independent variable should be used to check if the minimum is global, if the slope surface is unknown. The version of this procedure employed in this study was adapted from the Fortran source code originally developed by A. I. Johnson [13]. The Fortran code was converted to Pascal code for the purpose of readability and structuring. The function object given in figure 5 was employed as a subroutine to be used to calculate and return IAE to the main program. The Pascal code of main program and function object is given in Appendix A.

For the initial input values, three controller tuning constants (K_C, τ_I, τ_D), desired set point value r, magnitude of disturbance X_A and X_B, integration time interval delta, and total simulation time ttt are provided by a user. Since unstable starting value of three controller tuning constants may lead to the unstable optimum controller tuning constants as a result of simulation, simple algebraic synthesis method [7] was employed to get controller tuning constants which can be used as starting values. Also, three different additional starting values of controller tuning constants are employed to check if the minimum value of IAE is global.

sample results of optimum controller tuning constants are graphically described in figure 6 to 9. Complete results are given in appendix A. These curves give normalized three controller tuning constants, $K_C K_L$, τ_1/τ_I, τ_D/τ_1, and

IAE values as the function of θ_d/τ_1 and τ_3/τ_1. The results of optimization program were interpolated to draw continuous smooth curve using the plotting package, Statgraphics [14]. In each of these plots the ratio τ_3/τ_1 is described as the abscissa. The normalized controller tuning constant or IAE value is indicated as the ordinate. A family of curves is given for τ_3/τ_1 in the rage, $0.05 \le \tau_3/\tau_1 \le 7.0$ and the ratio of dead time θ_d to major time constant τ_1 is in the range, $0.1 \le \theta_d/\tau_1 \le 1.0$. A separate line is drawn for each of 10 different values of the parameter θ_d/τ_1 at interval of 0.1.

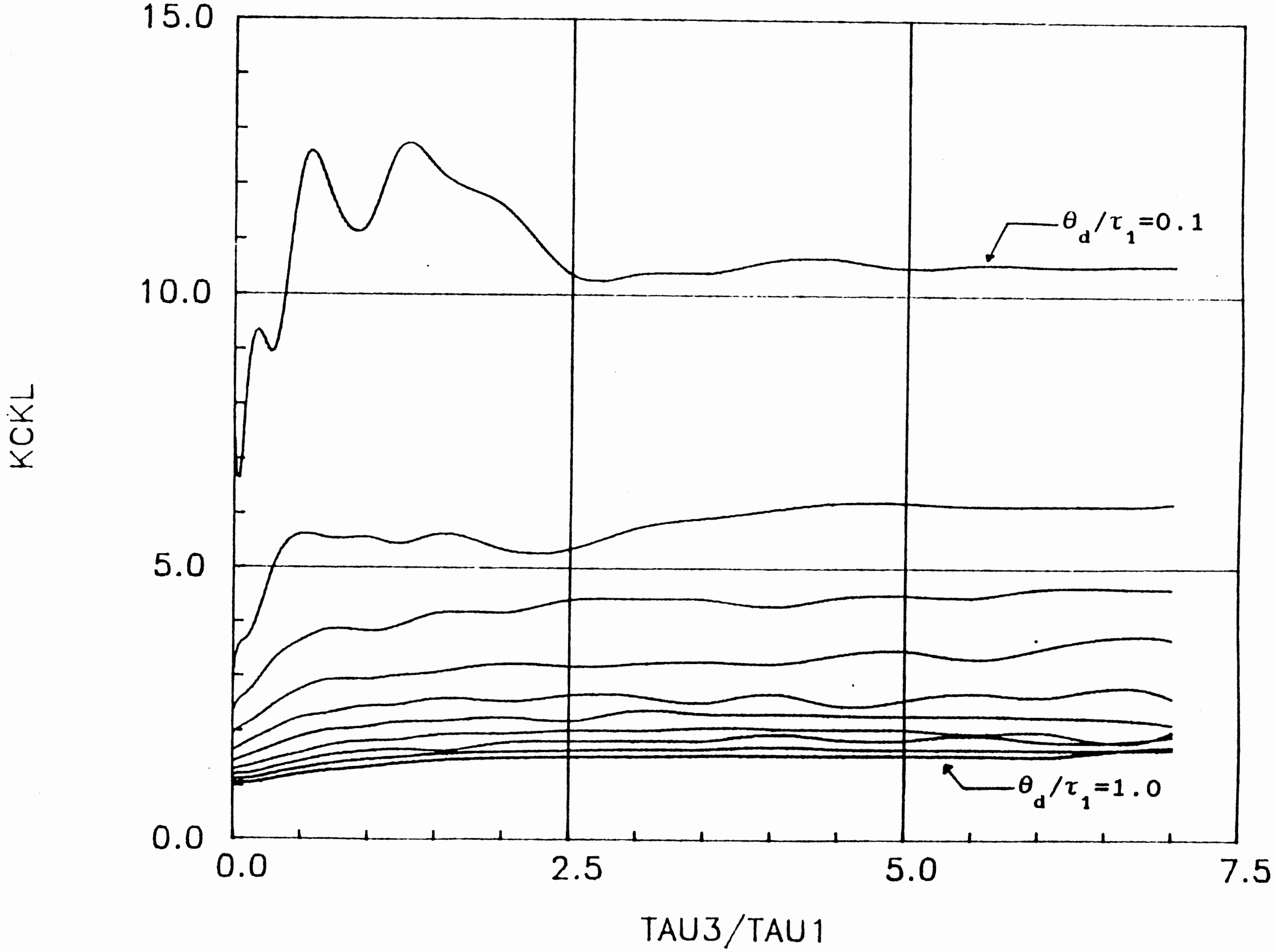


Figure 6. Optimum PID Proportional gain constant (τ_2/τ_1=0.1)

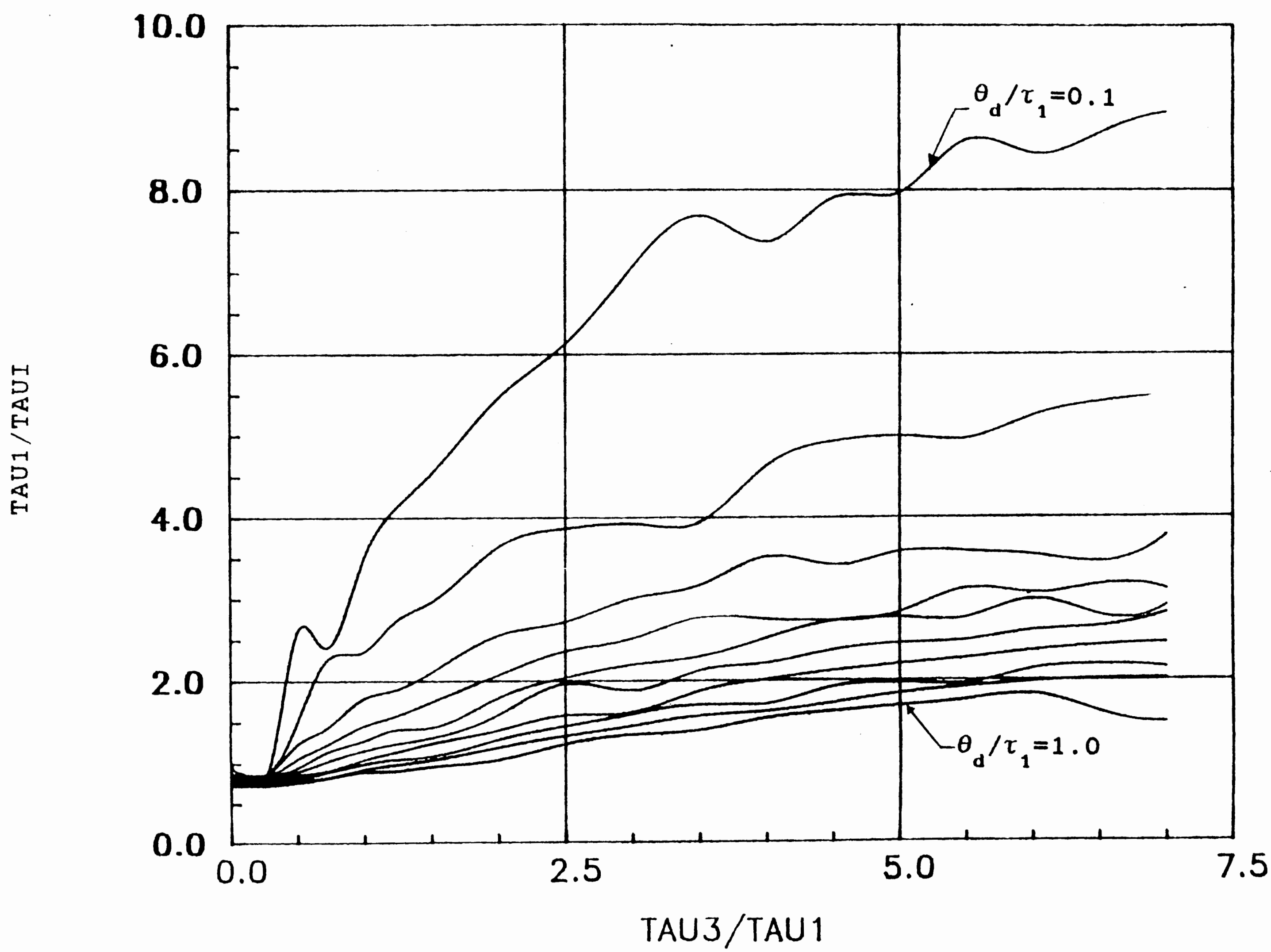
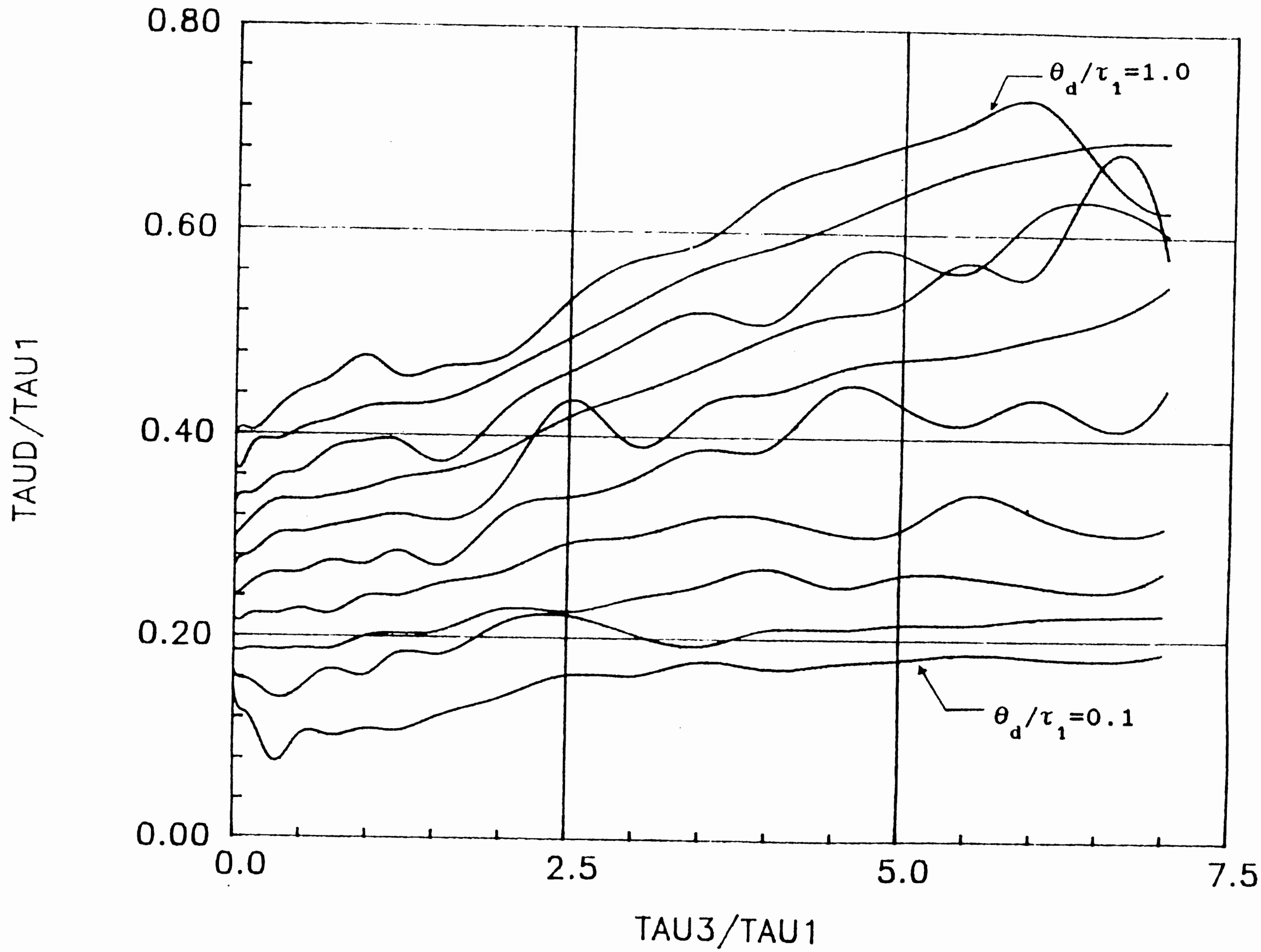


figure 7. Optimum PID Integral Time constant (τ_2/τ_1=0.1)



Figur 8. Optimum PID Derivative Time Constant (τ_2/τ_1=0.1)

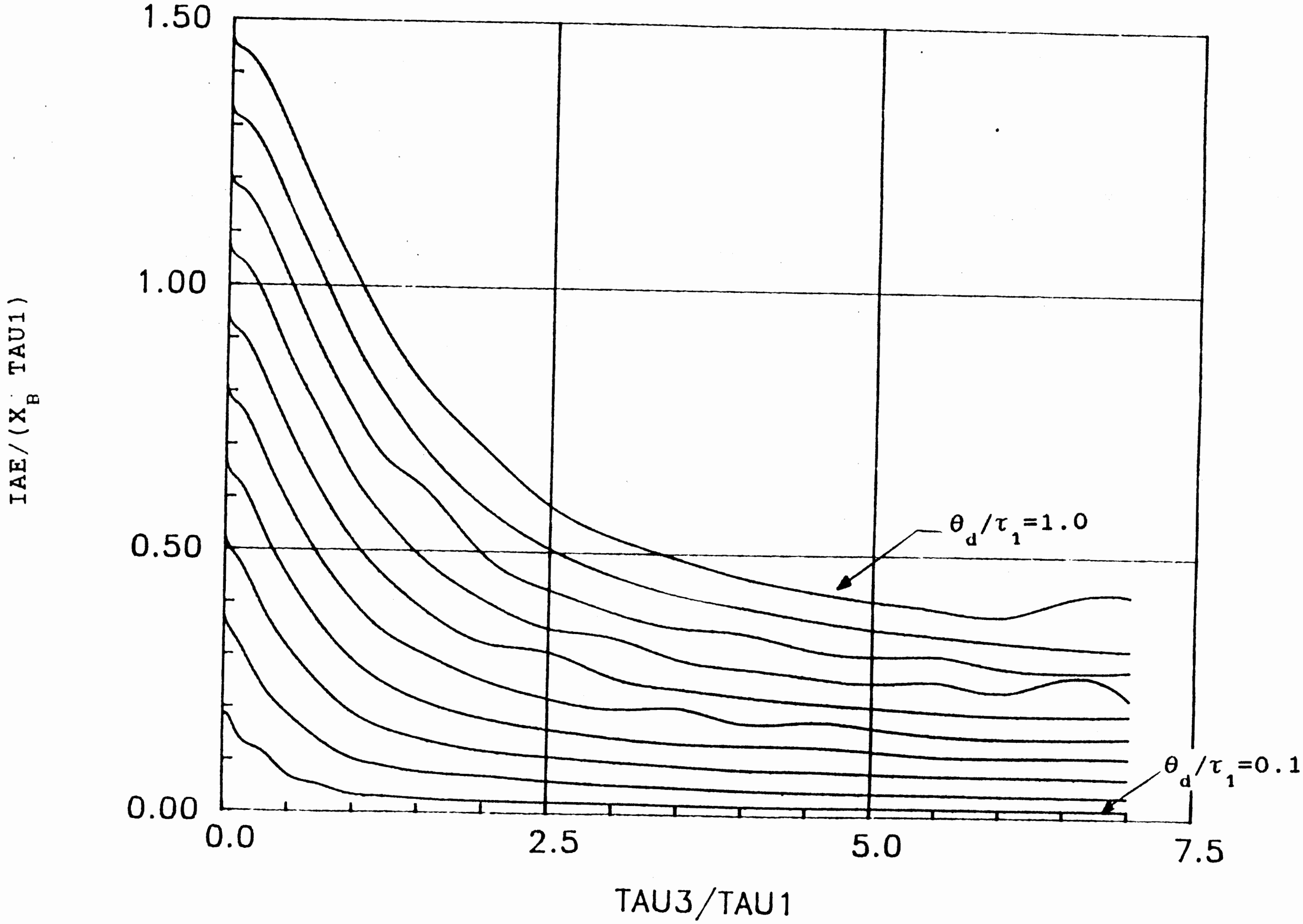


Figure 9. IAE based on Optimum PID Tuning Constants(τ_2/τ_1=0.1)

CHAPTER IV.

CORRELATION OF CONTROLLER TUNING CONSTANTS

The results of computer simulation were presented in a graphical form in the previous chapter and Appendix B. It may be difficult and tedious to use those graphs to obtain the correct controller tuning constants, because the data points are not read accurately with ease. This tedious job can be avoided if we can find the mathematical expression to fit the data very closely. Fitting the functions to the data is frequent task in science wherever we need to superimpose the complex mathematical models on the data. Nedler et al. [15] introduced a simplex method of function minimization for several variables and the curve fitting program based on a simplex procedure was developed by Caceci et al. [16].

The applied method to fit the candidate functions to the available data was based on a least squares criterion. The basic idea may be explained as follows:

If you have n data points, label each value of the independent variable as x_1, x_2, ..., x_n and each value of the dependent variable as y_1, y_2, ..., y_n. Also, label your n predicted values (as calculated by the equation using certain value for unknown parameters) for the dependent variable y_1^*, y_2^*, ..., y_n^*. The sum of $(y_1-y_1^*)^2+(y_2-y_2^*)^2+ \ldots +(y_n-y_n^*)^2$ is called the sum of squared residuals. The lower this sum is, the better the curve fits to the data. The optimization program used for curve fitting in this study was a slight modification of the original pascal program developed by Caceci and Cacheris [16]. a listing of the curve fitting program is given in appendix C in pascal code.

Finding a correlation to fit each of the curves shown in figure 6 to 9 was based on the optimization runs for a total of ten curves, corresponding to ten different ratios of θ_d/τ_1 values, was generated for three normalized controller tuning constants $K_C\ K_L$, τ_1/τ_I, τ_D/τ_1 and IAE. In the majority of cases the form of these curves were suggested by the fact that a non-linear function would provide the best fit. The functions used to fit the data and the values of its parameters obtained by the curve fitting runs are given in the tables I to IV for the case of τ_3/τ_1=0.1, θ_d/τ_1=0.1 and τ_2/τ_1=0.1. Other cases are given in Appendix D. Each table corresponds to a graph given in Chapter III. The form of equation used as a correlation function is given at the top of each table and the parameter values obtained by the curve fitting method for each curve of ten different value of θ_d/τ_1 are listed below the correlation function in the table. The number of parameter used for the curve fitting was varied depending on three controller tuning constants and IAE. From three to seven variables were introduced to be used as function parameters. The standard deviation of experimental data points from the fitted function is given in the last column of each table along with the function parameter values for

TABLE I

CURVE FITTING RESULTS OF
PROPORTIONAL GAIN
AT $\tau_2/\tau_1 = 0.1$

$$K_C K_L = A - e^{-B\tau_3/\tau_1}[C\cos(D\,\tau_3/\tau_1) + E\sin(D\,\tau_3/\tau_1)]$$

θ_d/τ_1	A	B	C	D	E	Standard Deviation
0.1	10.541223	1.180019	4.081162	1.272492	-7.398963	0.449385
0.2	5.890894	3.425357	2.846238	7.24×10^{-6}	2.403632	0.290349
0.3	4.484194	1.267431	1.954393	9.16×10^{-7}	1.268995	0.119251
0.4	3.446715	0.985048	1.384971	2.95×10^{-7}	0.649919	0.129946
0.5	2.622772	1.773274	1.016053	-9.26×10^{-7}	2.696408	0.067870
0.6	2.172058	0.625983	0.718355	0.003308	-164.5421	0.045793
0.7	1.839872	0.496145	0.550092	0.005831	-84.99422	0.041626
0.8	1.873584	0.712189	0.696749	0.022343	-7.329004	0.040928
0.9	1.670054	0.729667	0.591636	0.427691	-0.352738	0.011888
1.0	1.609807	0.879666	0.606288	-4.44×10^{-7}	-2.840634	0.036769

TABLE II

CURVE FITTING RESULTS OF
INTEGRAL TIME
AT $\tau_2/\tau_1 = 0.1$

$$\tau_1/\tau_I = A + B(1 - e^{C\,\tau_3/\tau_1}) + D\,e^{E\,\tau_2/\tau_1}\sin(F\,\tau_3/\tau_1 + G)$$

θ_d/τ_1	A	B	C	D	E	F	G	Standard Deviation
0.1	-3.30267	9.01153	-0.41464	5.79647	-0.00625	-0.000027	2.43275	0.2424
0.2	5.78255	-0.09001	-12.2736	5.13833	-0.39375	-0.00058	-1.78762	0.18450
0.3	2.77685	0.58238	-6.76347	2.38840	-0.34719	0.46117	-1.79252	0.11018
0.4	3.67113	-0.39645	-5.85407	4.71432	-0.47918	-0.04031	-0.63706	0.06248
0.5	-1.94808	0.40691	0.00948	3.90218	0.03892	0.16394	0.75193	0.05624
0.6	1.58023	0.37603	-0.04395	2.92361	-0.08631	0.14973	-0.30795	0.06160
0.7	2.85940	-0.31889	-1.36371	2.22301	-0.21063	0.15687	-1.24698	0.02911
0.8	1.35737	1.33223	-0.13991	1.96826	-0.66789	0.28259	-2.82426	0.03606
0.9	2.13835	0.02338	-0.03980	2.51418	-0.41486	0.23493	-2.54317	0.01802
1.0	0.37534	0.00740	0.71613	0.56146	0.20946	0.20258	0.60896	0.02968

TABLE III

CURVE FITTING RESULTS OF DERIVATIVE TIME AT $\tau_2/\tau_1 = 0.1$

$$\tau_D/\tau_1 = A + B\, e^{C\, \tau_3/\tau_1}$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	-43360.4670	43360.5758	3.193977×10^{-7}	0.0153100
0.2	-44636.8244	44636.9557	2.434525×10^{-7}	0.0136737
0.3	-53461.0341	53461.2240	2.401277×10^{-7}	0.0121607
0.4	-83746.1773	83746.4041	2.044911×10^{-7}	0.0182938
0.5	-77234.5607	77234.8133	4.055537×10^{-7}	0.0210251
0.6	-32407.2158	32407.4991	1.176143×10^{-6}	0.0161665
0.7	-3811.73677	3812.04278	1.216128×10^{-5}	0.0205976
0.8	-55195.8537	55196.1993	7.840349×10^{-7}	0.0190948
0.9	-31467.2744	32467.6501	1.582992×10^{-6}	0.0185181
1.0	-134661.5610	134661.9795	3.354662×10^{-7}	0.0374493

TABLE 4.

CURVE FITTING RESULT OF IAE
AT $\tau_2/\tau_1 = 0.1$

$$IAE = A + B/(C + \tau_3/\tau_1)$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	0.004136	0.049875	0.228442	0.005886
0.2	0.015031	0.138218	0.352544	0.007893
0.3	0.029530	0.260434	0.483797	0.013004
0.4	0.043210	0.418941	0.619501	0.017866
0.5	0.051111	0.638021	0.785571	0.020877
0.6	0.051936	0.912282	0.956557	0.024283
0.7	0.058688	1.207813	1.109474	0.031217
0.8	0.045950	1.638504	1.325902	0.032701
0.9	0.034825	2.079027	1.501410	0.036342
1.0	0.058584	2.415581	1.610953	0.045892

each curve.

Using the parameters obtained by the curve fitting runs new graphs similar to those given in the previous chapter were generated. Figure 10 to 13 describes those generated curves. The graphs prepared using the fitted equation may be compared to those based on the optimization runs described in Chapter III. Those generated graphs should be well fitted to the graphs based on real experimental data. The correlation between the controller tuning constants and the parameters of process and disturbance dynamics would lead to a simple calculation in obtaining the optimum PID controller tuning constants rather than reading graphs.

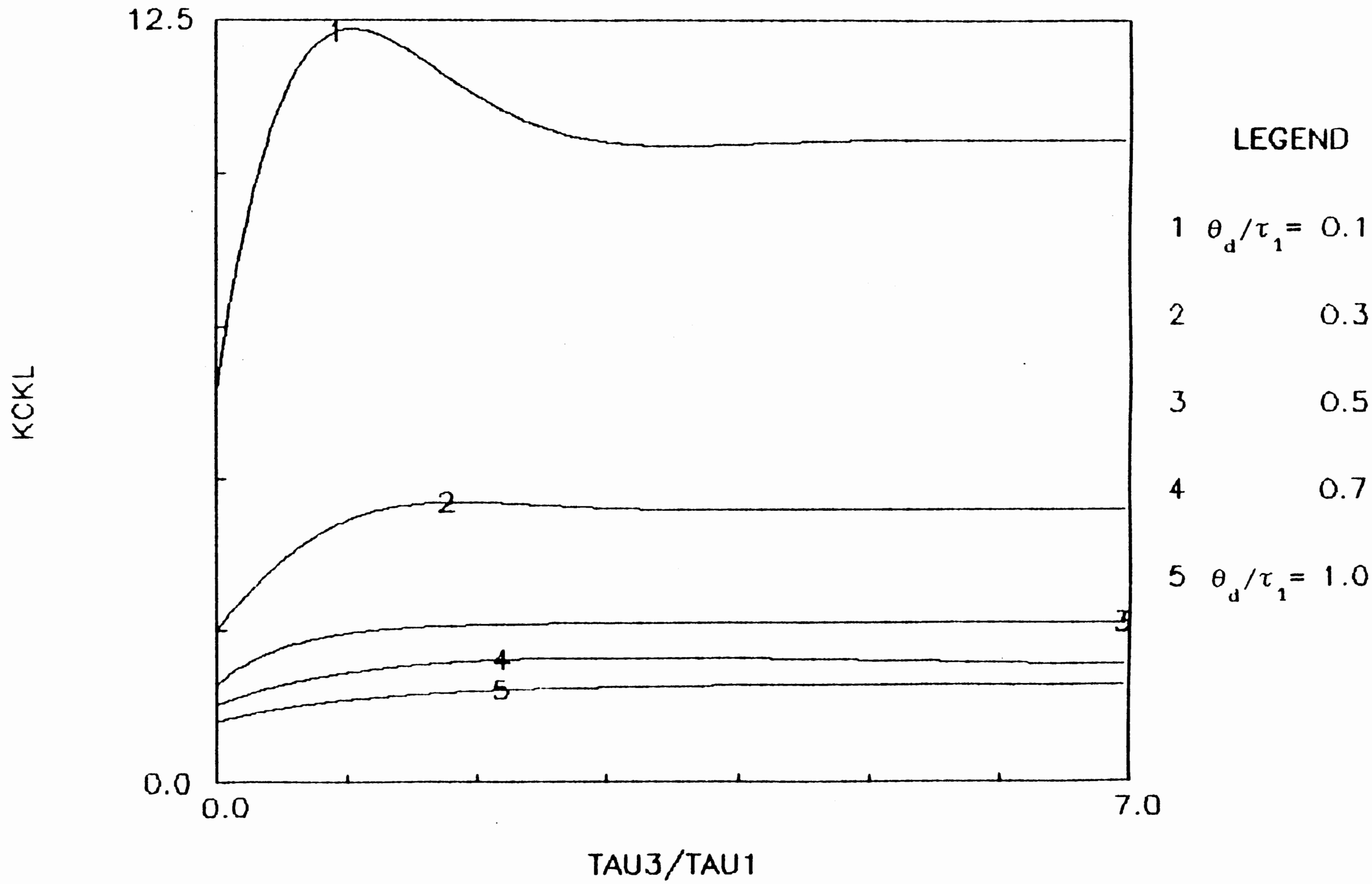


Figure 10. PID Proportioal Gain Based on Curve Fitting Results at τ_2/τ_1=0.1

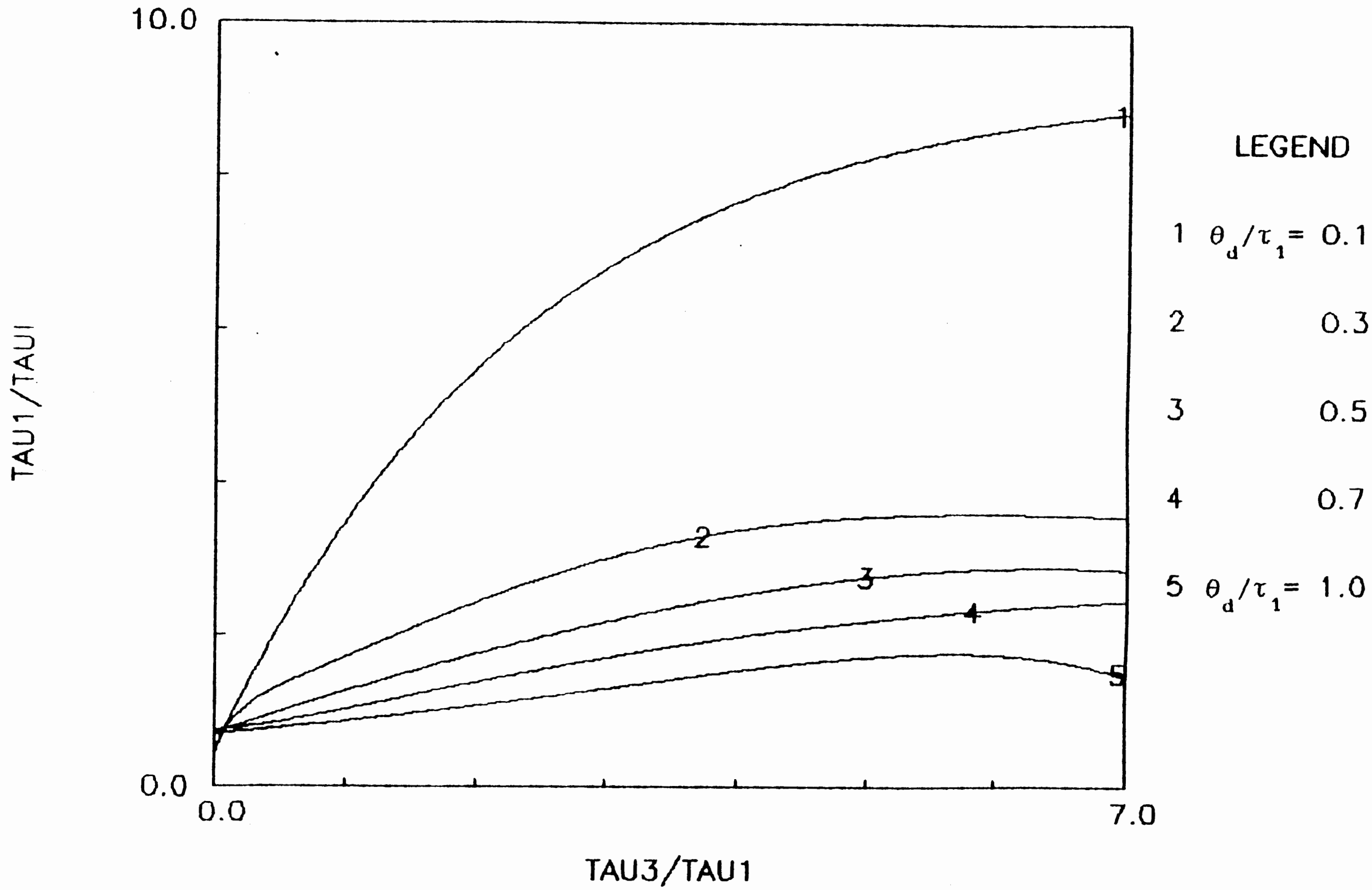


Figure 11. PID Integral Time Based on Curve Fitting Results at τ_2/τ_1=0.1

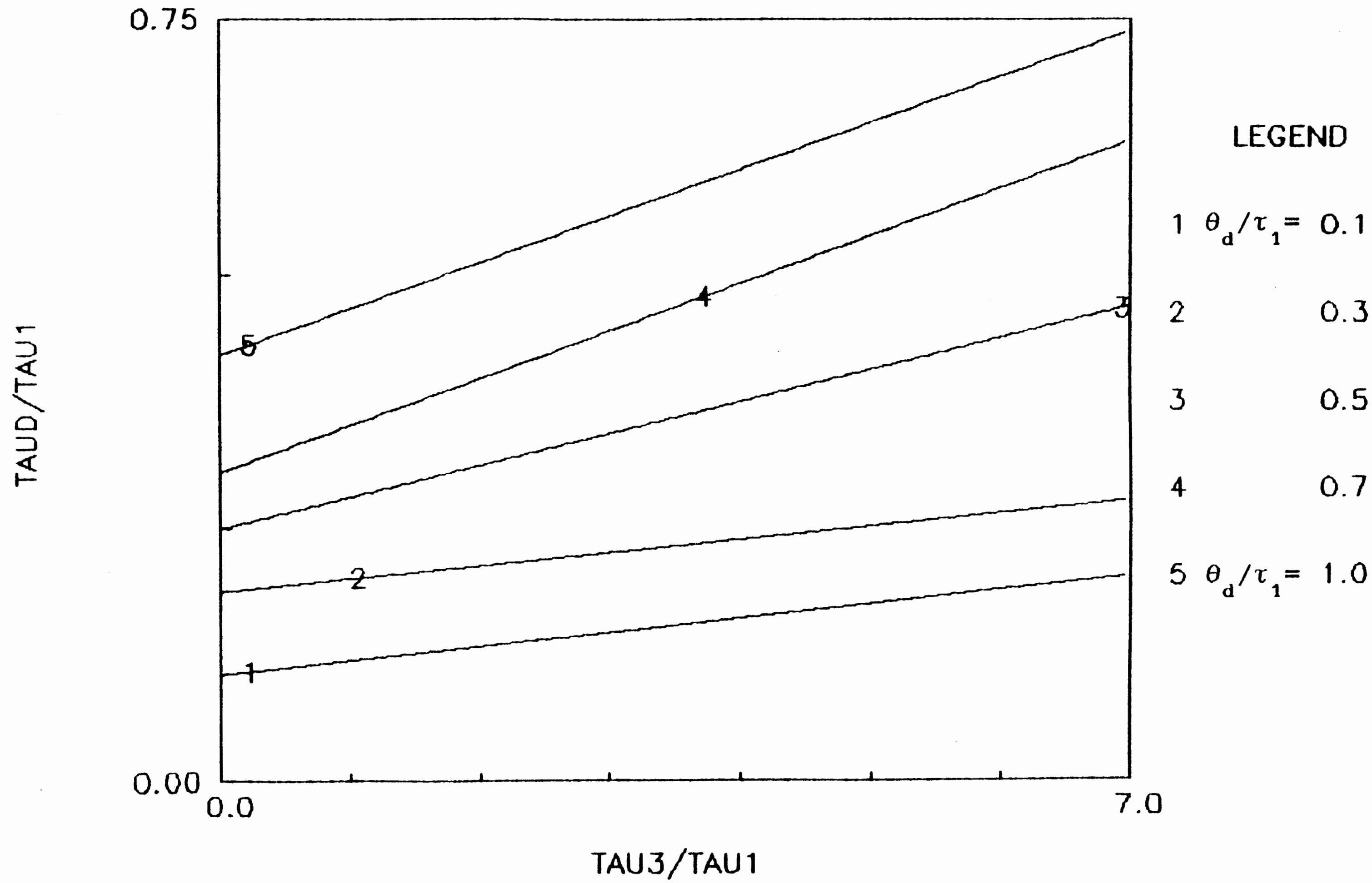


Figure 12. PID Derivative Time Based on Curve Fitting Results at τ_2/τ_1=0.1

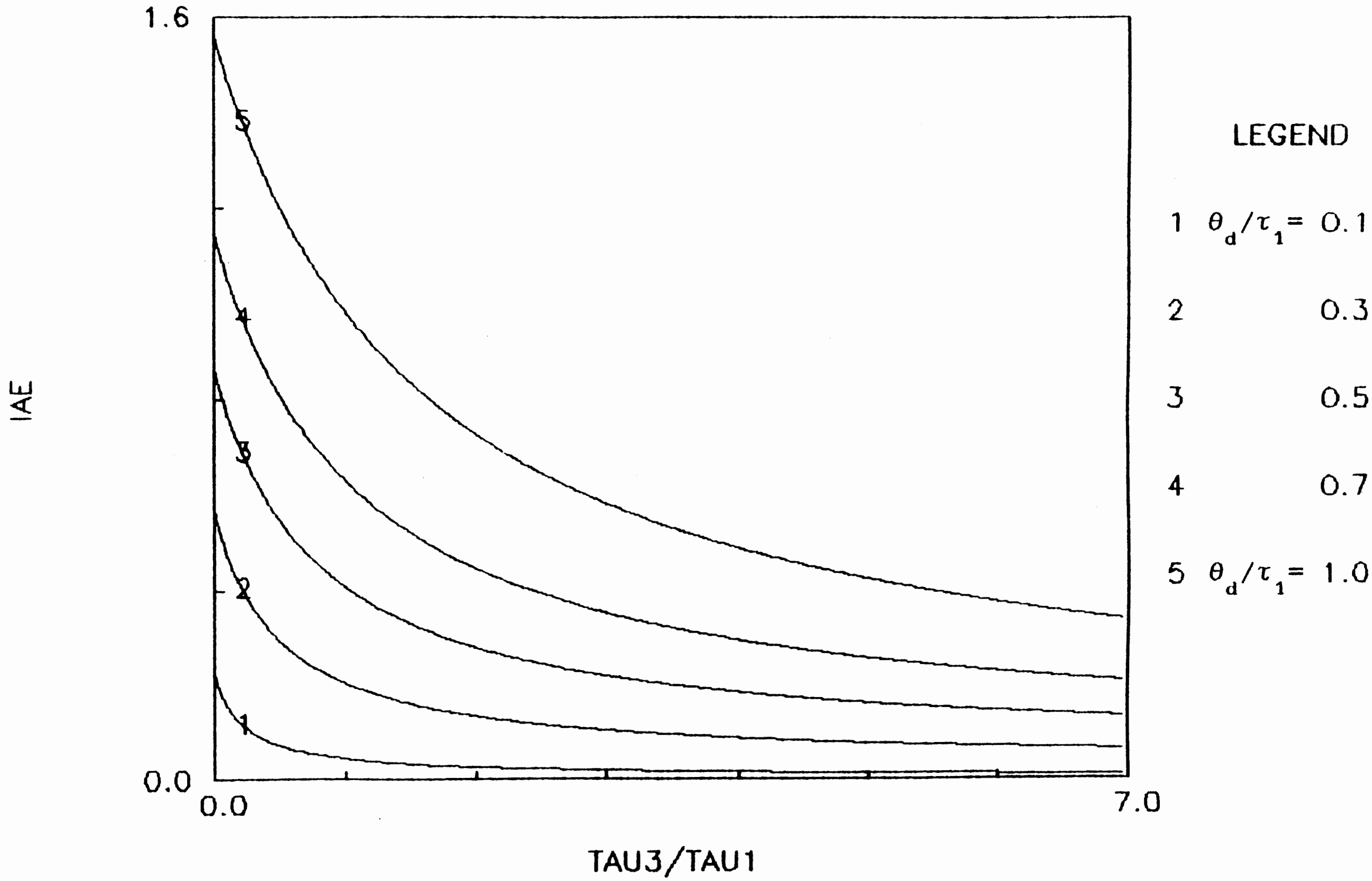


Figure 13. IAE Based on Optimum PID Controller Tuning Constants at $\tau_2/\tau_1=0.1$

CHAPTER V

DISCUSSION

Optimum Controller Tuning Constants

The optimum controller tuning constants were obtained by computer simulation and illustrated graphically in the figure 6 to 9 and Appendix B. The controller tuning constants have been normalized as follows:

Proportional gain K_C is reported as the product $K_C K_L$ where K_L is the product of measuring element K_m, process gain K_P, and valve gain K_v. The integral time is reported as τ_1/τ_I. The derivative time is reported as τ_D/τ_1. The integral of absol- lute value of the error observed by controller is divided by the measuring element gain K_m to express IAE in terms of con- trolled variable's units. Then, this value is expressed in a dimensionless form for the figures by dividing by the load magnitude X_B and the time constant τ_1 to produce the normal- lized value IAE/(X_B τ_1). A total of 40 graphs expressing the relationship of normalized tuning constants and IAE values ($K_C K_L$, τ_1/τ_I, τ_D/τ_1) versus the parameters of the process and disturbance dynamics (τ_3/τ_1, θ_d/τ_1) were made in Chapter III and Appendix B. For each family of curves, the ratios of dis- turbance time constant of the first order lag process to the principal time constant of SOPDT process, τ_3/τ_1, is given in the range, $0.05 \le \tau_3/\tau_1 \le 7.0$, and the ratios of dead time θ_d to principal process time constant τ_1 is given in the range, $0.1 \le \theta_d/\tau_1 \le 1.0$. The ratio of τ_2/τ_1 was in the range $0.1 \le \tau_2/\tau_1 \le 1.0$.

According to the prepared curves the following important observations could be made by taking consideration of the results of the optimum tuning constants calculation. The effect of disturbance dynamics on the proportional gain K_c is most pronounced when the ratios of τ_3/τ_1 are less than 5.0. As expected the proportional gain K_c is decreased with the increase of dead time. The frequency analysis [12] describes that the dead time causes phase shift and can lead system to the instability, therefore, dead time becomes the principal source of destabilizing effects in the control systems. Since most of the chemical system process exhibit an open loop response which can be approximated by a first order or a second order with dead time, it becomes clear that the possibility of instability for the closed loop will be present almost always. Therefore, time delay in the response forces the use of lower gain to maintain the stability in the feedback controller design. When the τ_3/τ_1 becomes greater than 5.0, K_cK_L values become almost constant, while integral time and derivative time continue to vary. This means that the controller gain term does not have much effect on slow disturbance compared to two other terms, integral time and derivative time constants, when a slow disturbance enters the control loop.

The effect of disturbance dynamics on the integral time tuning constant τ_I of the optimum PID controller was illustrated in figure 7 and Appendix B. Those results show that the integral action expressed as the ratio, τ_1/τ_I, should be increased as the ratio of τ_3/τ_1 increases throughout most of the range of τ_3/τ_1. This fact can be interpreted as indicating larger reset rates, $1/\tau_I$, can be used to control the slower disturbances. The smaller reset rates should be used to control the system when dead time becomes larger, because the system becomes closer to the instability when the dead time becomes larger.

The effects of disturbance dynamics on the derivative time constant of the optimum PID controller was illustrated in figure 8 and Appendix B. According to those graphs τ_D/τ_1 values increase as the ratio of τ_3/τ_1 increase. With the presence of the derivative term, $\tau_D \frac{de(t)}{dt}$, of the PID controller equation in the time domain, the PID controller anticipates what the deviation will be in immediate future and applies the control action which is proportional to the current rate of change in deviation. This anticipatory control action can be explained by figure 8. It shows that the larger value of derivative time constant τ_D can be used to control the slower disturbance (large τ_3), while the smaller value of derivative time constant, τ_D, is used for the faster disturbance (small τ_3). According to figure 8 derivative time constant τ_D tends to increases with the increase of dead time, which is contrary to the case of integral time constant.

The normalized IAE values of the closed loop with a PID controller were presented in figure 9 and Appendix B. Those results show that IAE values increase as the ratio of θ_d/τ_1 increases and it decreases as the ratio of τ/τ increases.

Application of Optimum Tuning Constants

In Chapter III, the optimum tuning constants for the PID feedback controller were obtained using the digital computer simulation and optimization procedure. In this chapter we will examine the dynamic behavior of a process that is controlled by a feedback controller with the optimum tuning constants determined in this work. To check the response of the control system, the value of disturbance, X_B, is applied as a step input to the control loop. The closed loop response of con-trolled variable, X_6, must be investigated to check the control system performance when the desired tuning constants are applied to the system disturbed by a step change in load. For this purpose, several sample applications are executed. A computer program is employed to see the response of controlled system and the Fortran code of program is given in Appendix E.

The examples of the result from a single sample application are presented in figure 14 and 15. These examples of the sample application uses the optimum tuning constants of PID controller which is applied to a control system with the following specifications:
θ_d/τ_1=0.1, τ_3/τ_1=0.1, τ_2/τ_1=0.1, and, X_B=10. Figure 14 plots the response of X_5, load disturbance after the first order

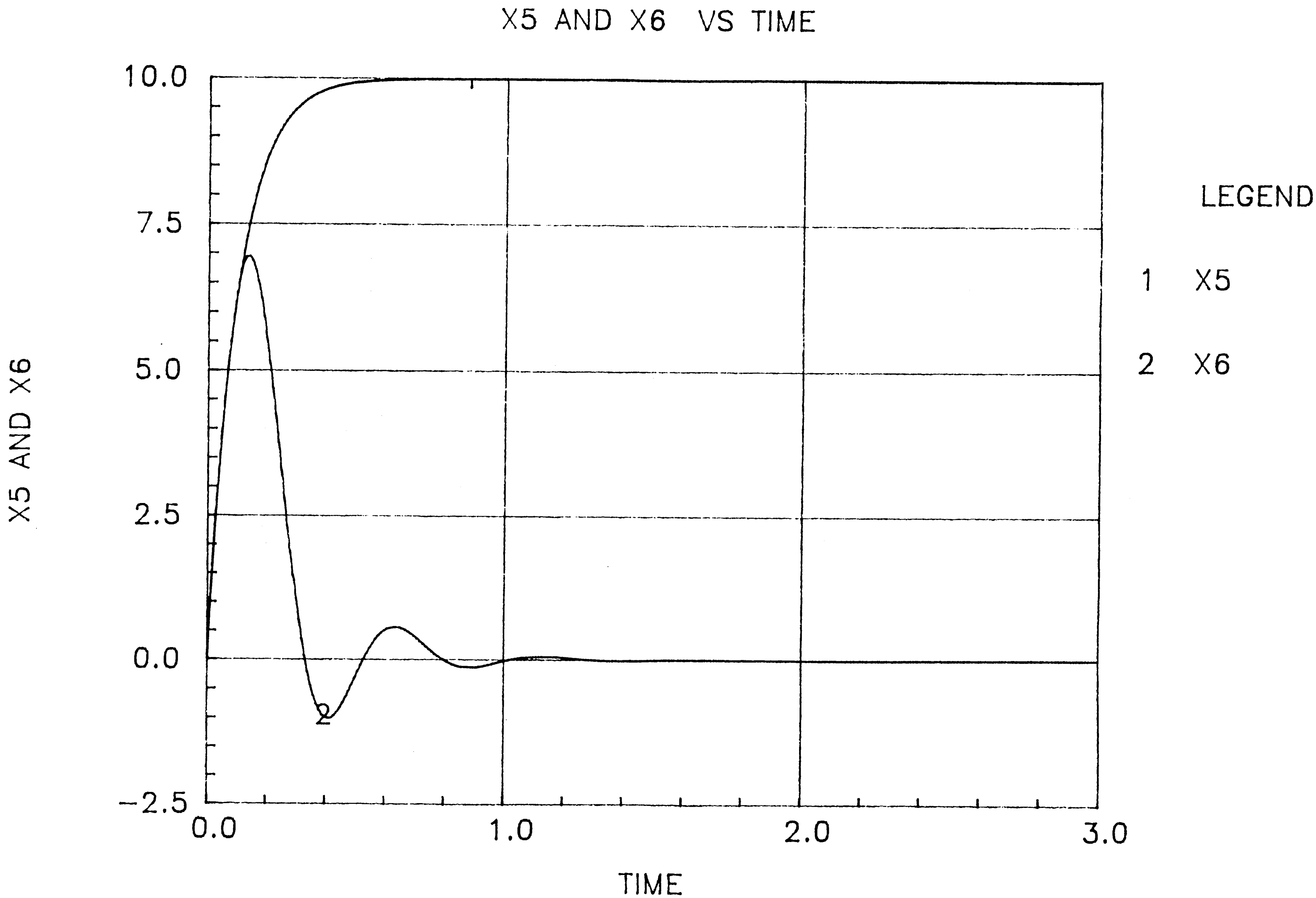


Figure 14. Response, X_6, of a Sample Application at θ_d/τ_1=0.1, τ_3/τ_1=0.1, τ_2/τ_1=0.1, and X_B=10.0

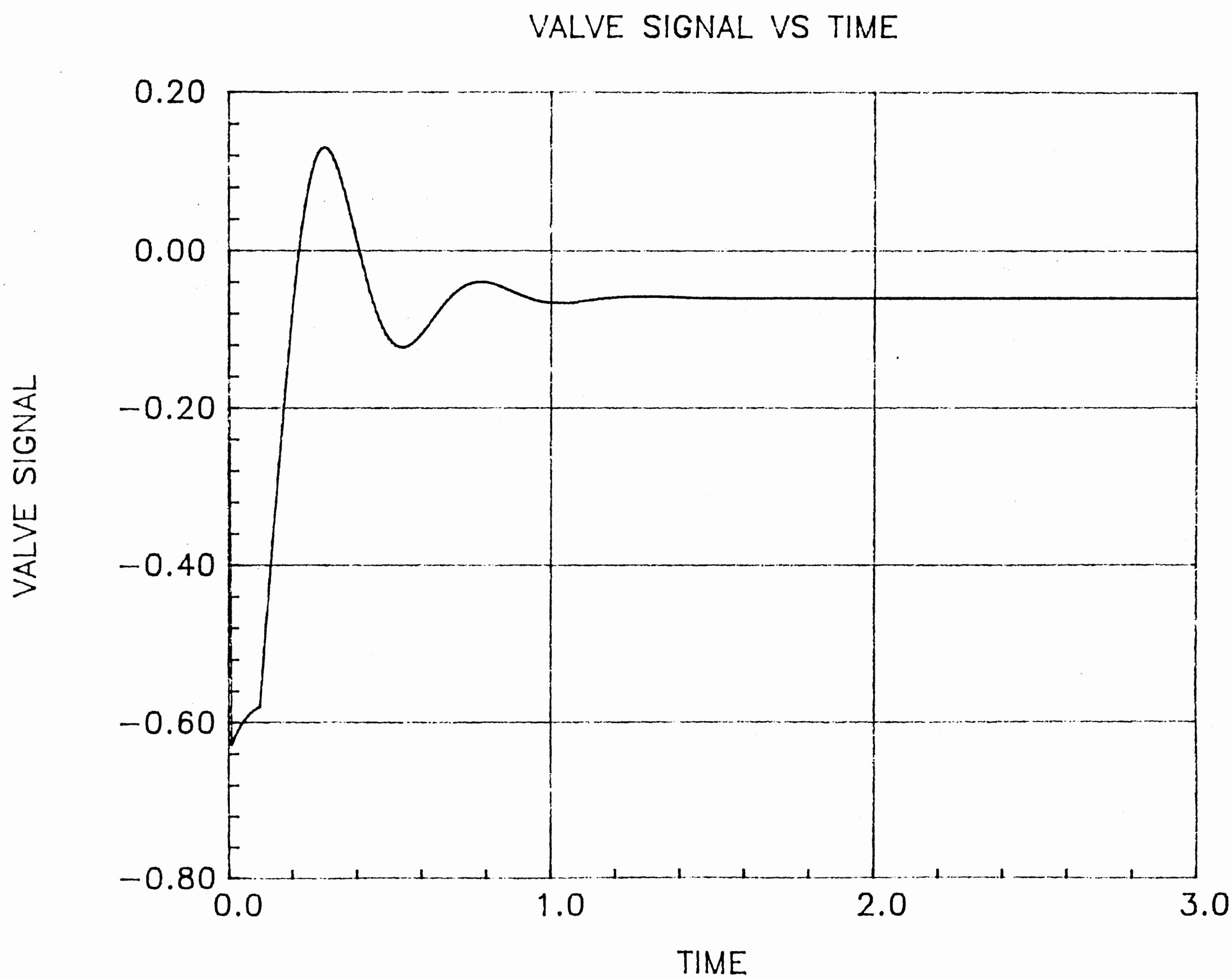


Figure 15. Valve Signal of a Sample Application at θ_d/τ_1=0.1, τ_3/τ_1=0.1, τ_2/τ_1=0.1, and X_B=10.0

lag, τ_3, and X_6 which is measured concentration passing the analyzer after the imposition of the load disturbance. Figure 15 expresses the time response of the valve signal of sample application of the above case.

More examples of sample application runs are shown in Appendix F. Runs were performed for the several sets of combination of θ_d/τ_1 (0.1, 0.5, 1.0), τ_3/τ_1 (0.1, 2.0,4.0), and τ_2/τ_1 (0.1, 0.5, 0.8) by applying a step input of disturbance, X_B=10. Figures prepared for the sample application show that the applied tuning constants lead the disturbed system to a desired steady state very effectively. According to thesis of Kosinsani [18] instability of the control loop was observed in certain cases of sample applications. According to his results the points of optimum tuning constants given in the graphs were not applicable to the disturbed control system, when the value of derivative time constant τ_D is greater than the value of integral time constant τ_I. But such instability was not observed in any case of sample application using the controller tuning constants obtained in this research.

Comparison of Tuning Methods

In order to evaluate the relative improvement attainable with the optimum tuning constants developed by this research, the responses of controlled variable, X_6, would be investigated after the application of tuning constants based on both this new study and previous worker's method. Here, the performance of tuning constants based on the new method of this

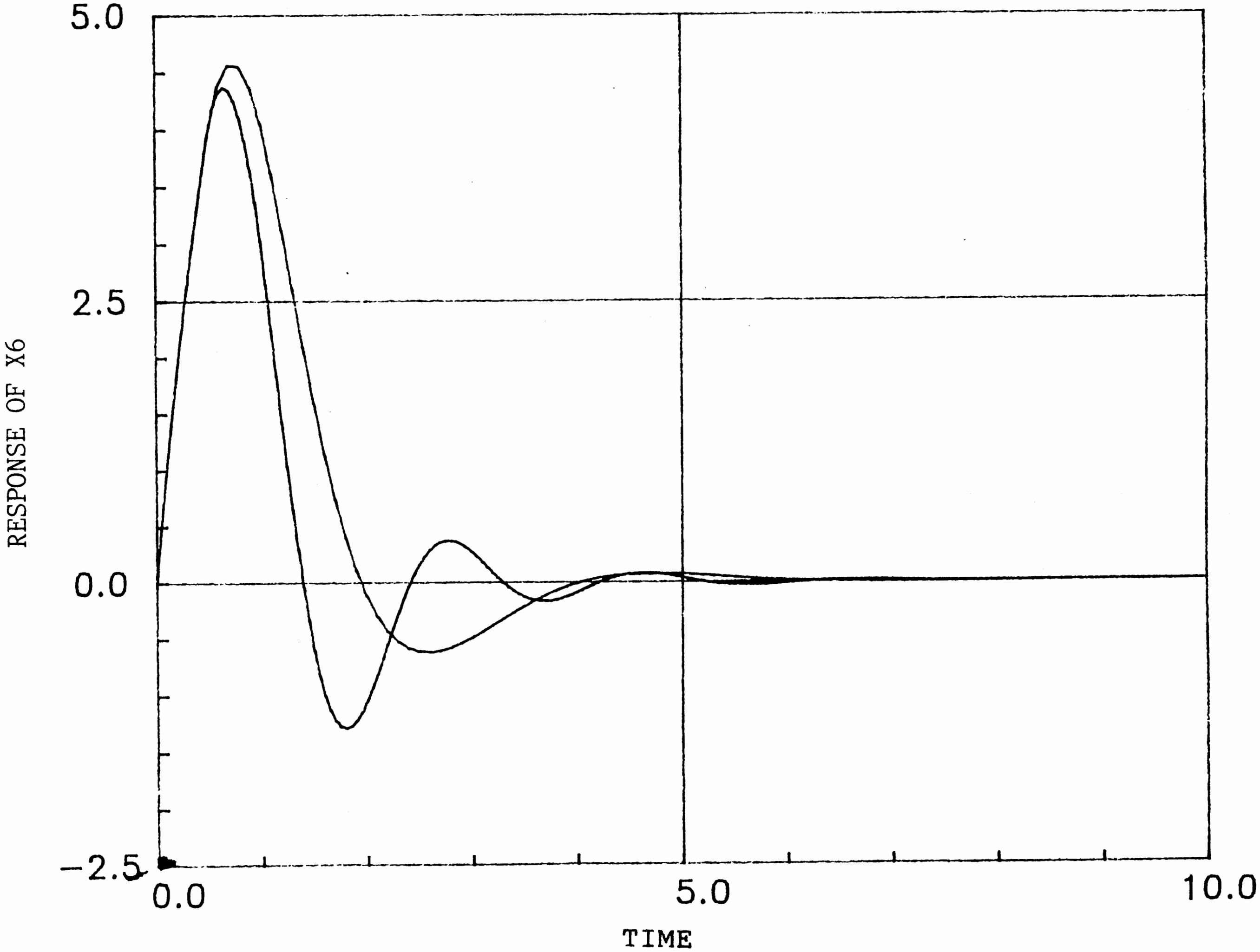


Figure 16. Comparison of Tuning Methods, Lopez and New Method, for τ_3/τ_1=1.0, τ_2/τ_1=0.1, and θ_d/τ_1=0.5

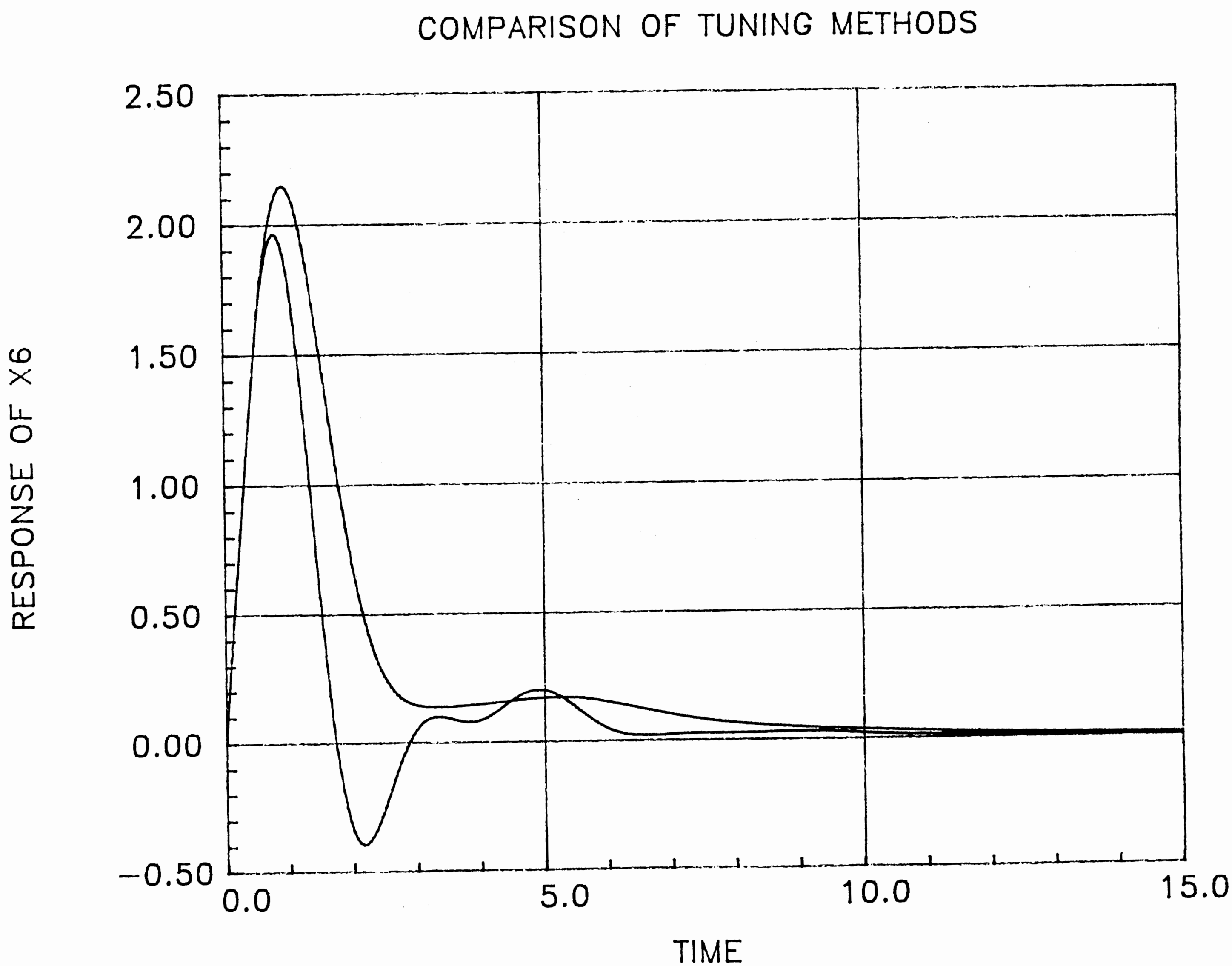


Figure 17. Comaprison of Tuning Methods, Lopez nad New Method, for τ_3/τ_1=3.0, τ_2/τ_1=0.5, and θ_d/τ_1=0.5

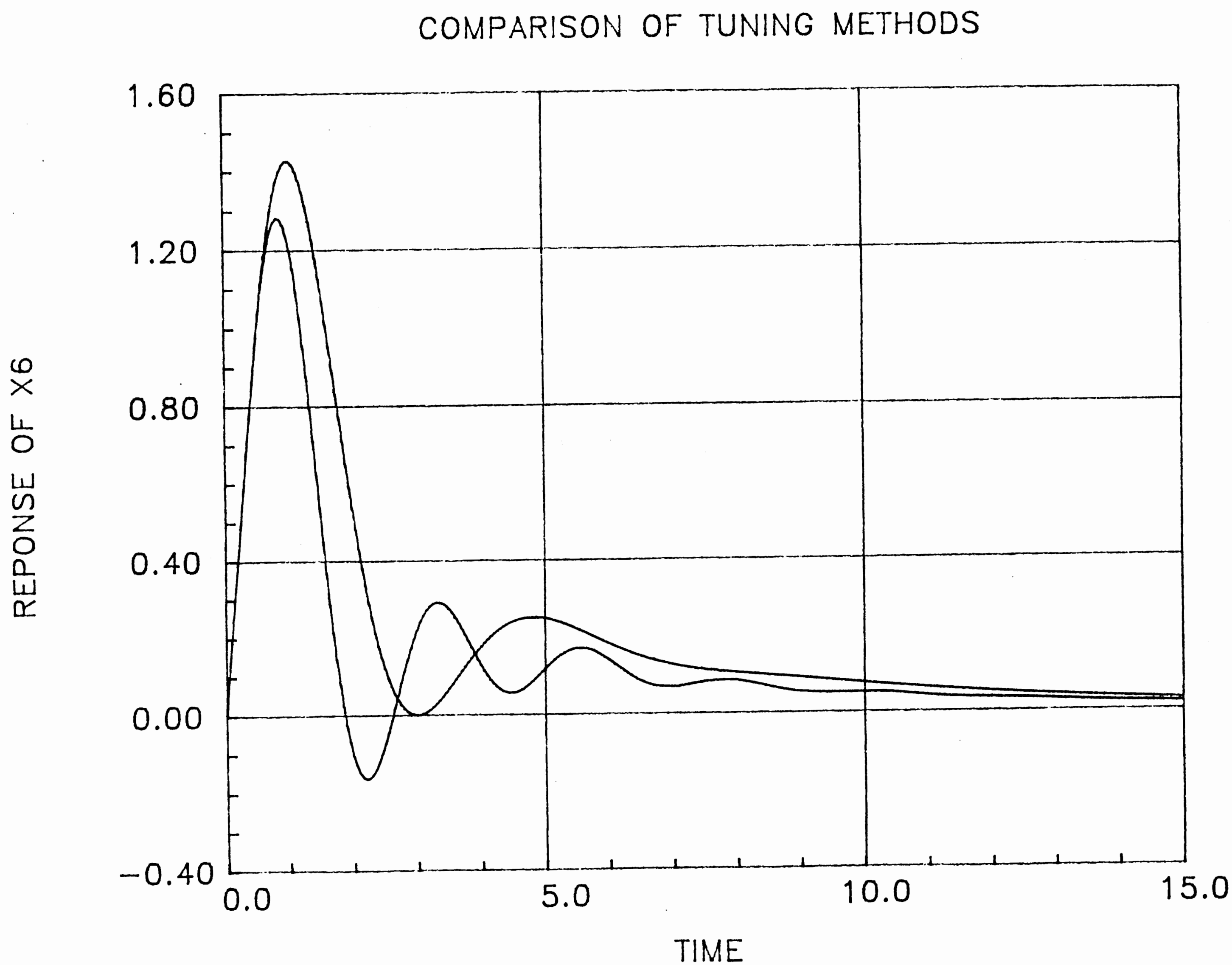


Figure 18. Comparison of Tuning Methods, Lopez and New Method, for τ_3/τ_1=5.0, τ_2/τ_1=0.8, and θ_d/τ_1=0.5

work and Lopez's (ITAE criterion) method [17] are compared.

Figure 16, 17, and 18 provide the graphical comparison of responses of controlled variable, X_6, that resulted from the computer simulation with the controller tuning constants obtained by Lopez and this study. Each of the three graphs represents the responses of controlled variable, X_6, for a specific set of three tuning constants. Figure 16 describes the responses of X_6 resulted from the Lopez method and this method, when the parameters of the process and disturbance dynamics have the following specifications: τ_1=1.0, τ_2=0.1, τ_3=1.0, and θ_d=0.5. According to figure 16 Lopez's and the new method showed very close performance even if new method looks slightly better than Lopez's method. This can be recognized as a special case because in Lopez's and this study, two control loops becomes almost identical at that particular set of parameters of process and disturbance dynamics. When the sets of parameters of process and disturbance dynamics are evaluated far away from that particular case (figure 17 and 18), significant improvements can be observed in the new tuning method compared to Lopez's tuning method. According figures 17 and 18, the new tuning method gives smaller overshoot, faster settling time, and finally much smaller value of IAE than that of the Lopez method.

Comparison of First Order and Second Order Tuning

The relative difficulties involved in obtaining the experimental information required to either a first order or a second order tuning is generally the same. Both method are based on open loop models, and hence, they are classified as open loop methods. The popularity of methods based on first order approximations (FOPDT models) to the process reaction curve is related to the relatively simplicity. This process reaction curve was developed by Cohen and Coon [8] and its approximation (FOPDT) can be described as a following equation:

$$G_p(s) = \frac{K e^{-\theta_d s}}{\tau s + 1} \tag{19}$$

Its equation has three parameters: static gain K, dead time θ_d, and time constant τ. However, since the approximation as second order model has more parameters than the approximation of first order model, it should express the process dynamics more accurately.

Sten [2] developed a method to approximate a process reaction curve as a second order model (second order lag with dead time). Its transfer function can be described with the following equation for a overdamped system:

$$G_p(s) = \frac{K e^{-\theta_d s}}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{20}$$

which has the four parameters: two time constants τ_1 and τ_2, static gain K, and dead time θ_d. To obtain a quantitative measure of improvement in the control action presented by the method based on second order tuning, one process was tuned by two techniques and responses were compared. The transfer function of process was given by:

$$G_p(s) = e^{-1.0s} /\{(s+1)(0.5s+1)\} \quad (21)$$

When the above transfer function is approximated as a first order model with time delay, its equation becomes:

$$G_p(s) = e^{-1.19s}/(1.39s+1) \quad (22)$$

Figure 19 is the graph to compare two tuning techniques for PID controller based on IAE criteria. Figure 19 describes the responses of controlled variable X_6 resulted from the two tuning techniques. In a comparison based on the response of controlled variable X_6, second order tuning did give improvement against first order tuning as well as the comparison of of Lopez and new method did. As we can see in figure 19, controller tuning based on the second order approximation gave faster settling time, smaller overshoot, and smaller IAE values. Also, in most cases the valve signals showed big differences in the two tuning methods. With second order tuning the valve response was relatively gradual and continuous as it moved to correct a load disturbance, while the valve response by first order tuning showed rapid changes. In order to use tuning constants obtained by the first order approximation it may be necessary to install a high performance valve capable of quick and accurate responses to the valve signal.

Load Fraction

In this research the manipulated variable, va, was cons-

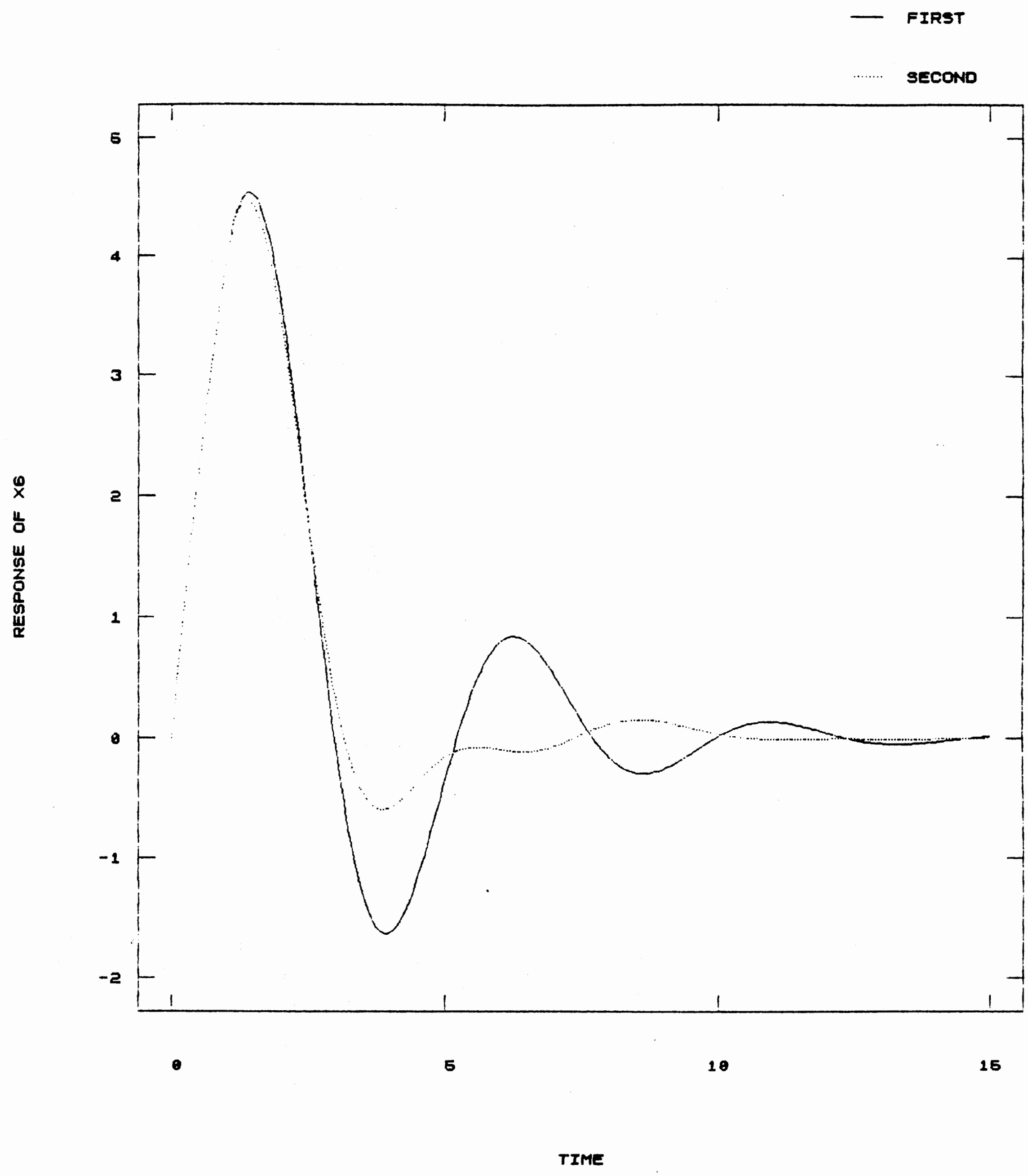


Figure 19. Comparison of Tuning methods between First Order and Second Order Approximation

trained to the limits corresponding to a fully closed or a fully open state. The valve was considered to operate using a pneumatic signal in the range 3 to 15 psig. The maximum acid flow available was 2000 mg/l when the valve was fully open. At normal operating conditions the valve would be half open supplying 1000 mg/l of acid flow to the mixing tank. In terms of deviation variables valve signal could increase +6 psi before reaching its upper constraints and decrease -6 psi before reaching its lower constraints. The valve signal of a fully open or a fully closed state is equivalent at the value of 6 and -6. Use of these limits made it possible to determine the effect of load magnitude. Examination of the objective function in figure 5 will show that the lowest and the highest controller signals (prior to application of constraints) were retained as variables ,called vamin and vamax, respectively. During the optimization runs the load magnitude, X_B=10 mg/l, was used. It ended to a final steady state with the valve signal, va= -0.06 psi, when the controlled variable X_6 returned to zero. The available movement of valve signal in this direction was -6.0.

The optimum tuning constants determined by this study were based on load magnitudes that are small enough such that the control valve does not saturate to a constraint during a response to a disturbance. We define load fraction to be the steady state change in valve position created by a load disturbance divided by the available movement of the valve in that direction:

$$\text{Load fraction} = \frac{\text{steady state change in valve signal}}{\text{available change in valve signal}} \quad (23)$$

In the case of positive change in X_B the control system will respond with a negative change in valve signal and a negative change in X_B will cause a positive change in valve signal. Load fraction can be taken as a normalized measure of load magnitude. During the optimization runs the final steady state value signal was -0.06. Therefore, load magnitude may be expressed as load fraction = -0.06/-6.0 = 0.01. During each run the lowest instaneous valve signal was recorded in a variable vamin. We will use this value to determine the load fraction that would have caused the valve to reach instaneously lower constraints. We term this load the "Max. Load Fraction".

The maximum allowable load fraction (with a valve signal remaining in the range of $-6.0 \leq va \leq 6.0$) may be determined by the following equation:

$$\text{Maximum Load Fraction} = \text{applied load fraction} * \frac{6.0}{\text{vamin}} \quad (24)$$

If a magnitude of disturbance larger than the Maximum load fraction enters into the control loop, the controller tuning constants obtained by this research can not be considered as optimum. In this case looser tuning would be better, such as the controller synthesis described in reference 7. The maximum allowable load fraction (preventing valve saturation) has been calculated for each of the runs with the application of equation (13).

Figures (20), (21), and (22) are presented to describe the relationship of maximum load fraction versus τ_3/τ_1 for the case of τ_2/τ_1=0.1, 0.5, and 1.0. The rest of the results for the description of maximum load fraction versus τ_2/τ_1 are given in Appendix G. The important observation from the prepared graphs indicates that the obtained optimum controller tuning constants can be applicable only for the small disturbance magnitudes, if the disturbance is very fast (small τ_3). This trend becomes more prominent as the value of τ_2/τ_1 increases. But even the large magnitude of disturbance can be controlled with new tuning constants when the disturbance is slow (large τ_3). The concept of allowable load fraction will address this practical problem in an effective manner.

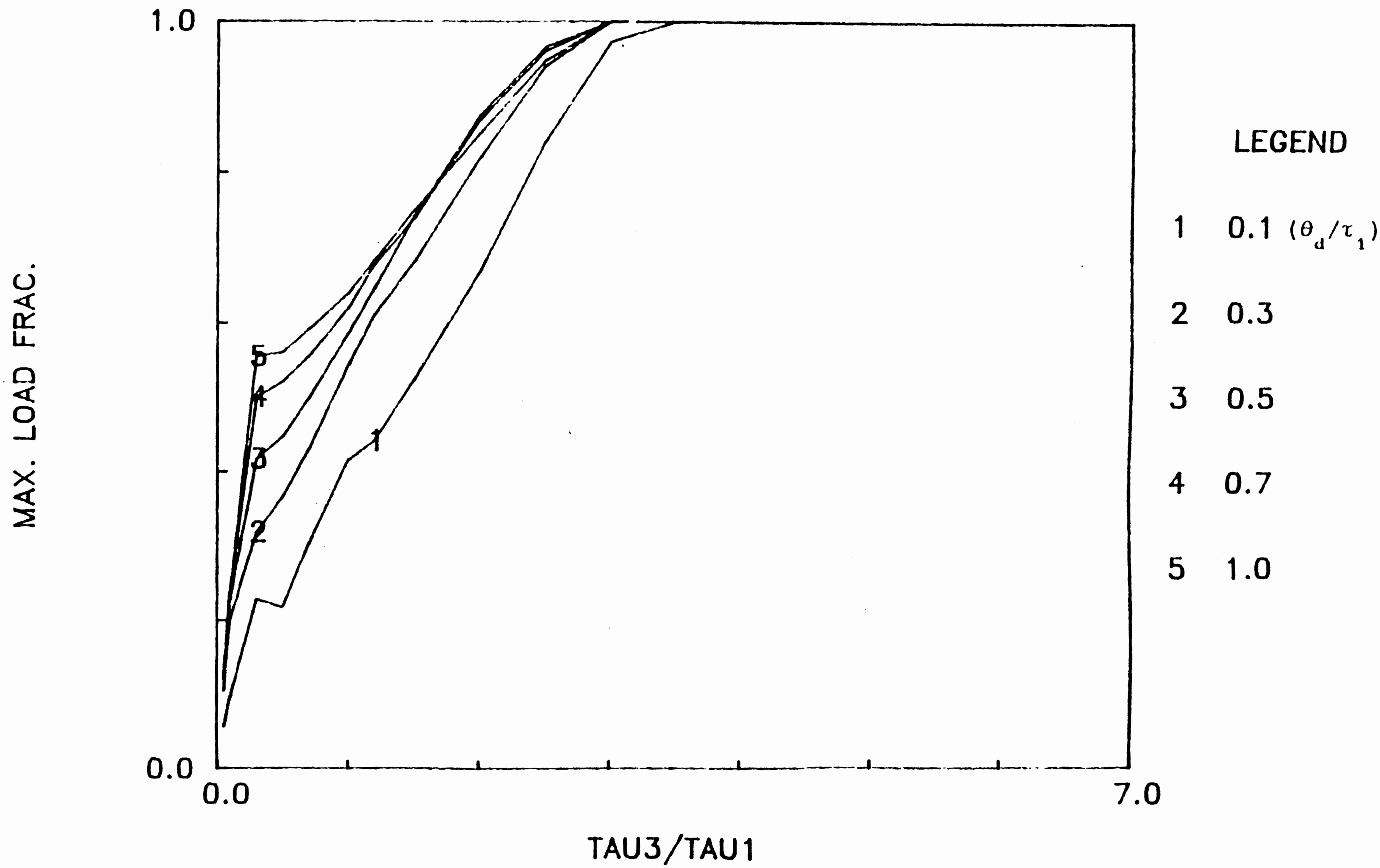


Figure 20. Max. Load Fraction for τ_2/τ_1=0.1

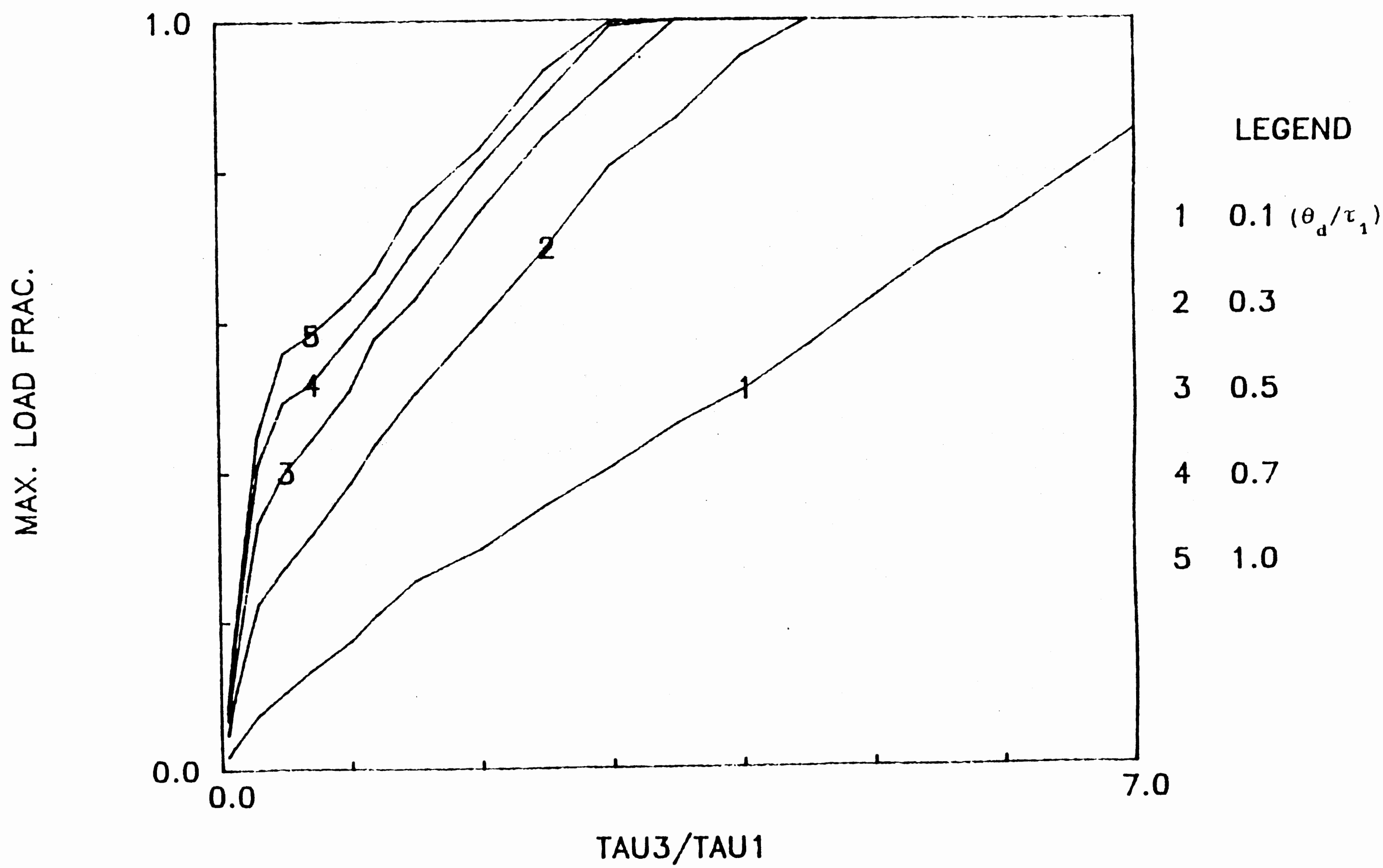


Figure 21. Max. Load Fraction for τ_2/τ_1=0.5

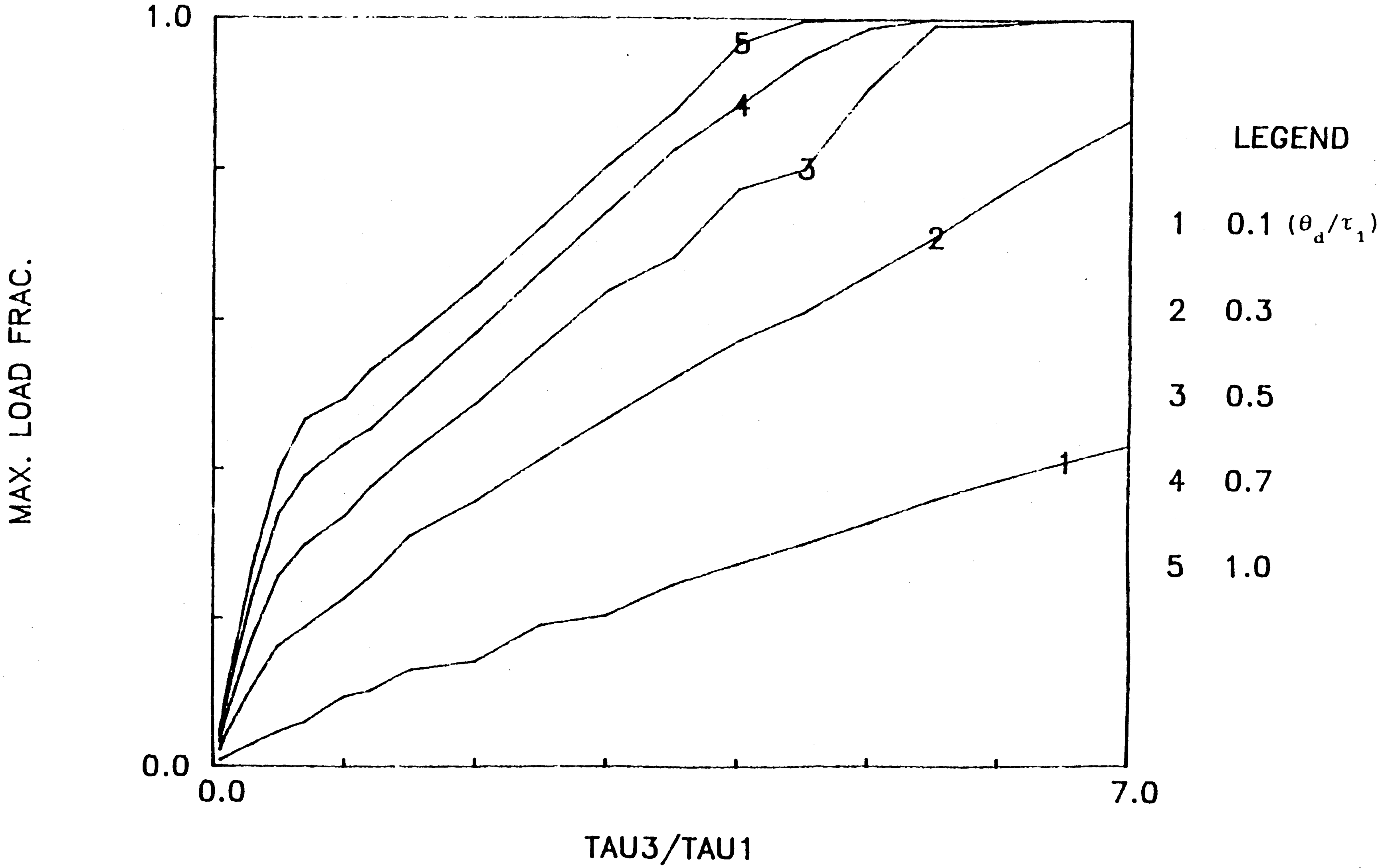


Figure 22. Max. Load Fraction for τ_2/τ_1=1.0

CHAPTER VI

CONCLUSIONS

Throughout this research work five important tasks were carried out to investigate the effect of disturbance dynamics on a control system:

1. calculation of optimum tuning constants
2. correlation of optimum tuning constants
3. sample application
4. comparison of tuning methods
5. consideration of disturbance magnitude

Taking observation of the results of the above tasks lead to the following conclusions:

The figures provided in this study show that optimum tuning constants are dependent upon the effect of disturbance dynamics. Optimum controller tuning constants (K_C, τ_I, and τ_D) could be described as a function of the parameters of the disturbance and process dynamics (τ_3/τ_1 and θ_d/τ_1). Those optimum tuning constants can be applicable to systems that allow the process to be modeled as a second order plus dead time (with gain K_P, two time constants τ_1 and τ_2, and dead time θ_d) and disturbance to be modeled as a first order with time constant τ_3. Also, the range of ratios of θ_d/τ_1, τ_3/τ_1, and τ_2/τ_1 should be in the range of $0.05 \le \theta_d/\tau_1 \le 1.0$, $0.1 \le \tau_2/\tau_1$

≤ 1.0, and $0 \leq \tau_3/\tau_1 \leq 7.0$. Examination of the IAE charts indicates that the value of IAE decrease as the disturbance slows down (increase of τ_3/τ_1) and as the dead time decreases (decrease of θ_d/τ_1).

Sample applications show that optimum tuning constants obtained by this research lead the disturbed system to a desired steady state very effectively. Cases of instability of the control loop were observed in some sample applications with Kosinsani's tuning constants. But, no such cases of the instability were found in the sample application of tuning constants obtained from this work.

In the comparison of two tuning methods, the new tuning method provided by this study gives significant improvements when it is compared to Lopez tuning method by producing much smaller overshoot, faster settling time, and smaller IAE value. The controller tuning based on the approximation of secod order with delay showed much improvement against the controller tuning based on first order approximation, when the responses of controlled variable X_6 were compared. Also, controller tuning based on second order approximation gave much better valve reaction than controller tuning based on first order approximation. With controller tuning based on second order approximation the valve signal showed more gradual and flexible response compared to the valve signal based on first order approximation as it moved to correct a load disturbance.

The disturbance magnitude, for which calculated optimum tuning constants can be applicable, were dependent upon the speed of disturbance dynamics. When the disturbance was fast (small τ_3), optimum tuning constants could be available only for small magnitudes of disturbance and this situation became more prominent as τ_2/τ_1 is increased. Even the large magnitude of disturbance can be controlled by calculated tuning constants when disturbance is slow (large τ_3). The concept of allowable load fraction will address this practical problem in an effective manner.

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APPENDIX A

DIGITAL SIMULATION PROGRAM FOR OPTIMUM TUNING CONSTANTS

```
program rbonce;

LABEL 1,2,3,4,5,6,7;

CONST

    KM=3;MAXK=5000;MKAT=800;MCYC=500;NSTEP=1;
    EPSY=1.0E-06;ALPHA=3.00;BETA=5.0E-01;
    KMM=4.0E-03;KV=1.6666666667E02;

VAR

    k:array [1..km] of real; (*INITIAL GUESSES*)
    v:array [1..km,1..km] of real; (*DIRECTIONAL VECTORS*)
    b:array [1..km,1..km] of real;
    d:array [1..km] of real; (*SUM OF SUCCCESSFUL STEPS*)
    n:array [1..km] of real;
    p:array [1..km] of real; (*STEP SIZE*)
    j:array [1..km] of real; (*SUCCESS/FAIL INDEX*)
    e:array [1..km] of real;
    l:array [1..km,1..km] of real;
    t3:array [1..40] of real; (*time const. of
                                 disturb.=tau3*);
    kkcc:array [1..4] of real;
    kat,i,ii,m,kk1,iii,kl,z,mm,mmm,kkk,iiii,kkkk:integer;
    sumo,sumn,fbest,sumdif,sumavv,sumav:real;
    c, (*TRANSMITTED VARIABLE,PSI*)
    er, (*ERROR, CURRENT VALUE*)
    erint, (*ERROR INTERGRAL*)
    epast, (*ERROR, PREVIOUS ITERATION*)
    absie:real; (*TIME INTEGRAL OF ABSOLUTE VALUE OF THE
                  ERROR*)
    G,Q,S:integer;(*dead time array pointers,elements*)
    thetad, (* deadtime *)
    du31,du32,
    du32xb, (*CONSTANTS IN DISCRETE EQUIVALENT OF 1ST
              ORDER*)
    x1, (*xa+v*kv/1*)
    x2,x3,x4,(*INTERMEDIATE VALUES*)
    x5, (*PROCESS RESPONSE TO xb*)
    x6, (*THE CONTROLLED VARIABLE,MG/L*)
    rk11,rk21,rk31,rk41,
    rk12,rk22,rk32,rk42,
    time,
    vv, (*CONTROLLER OUTPUT BEFORE CONSTRAINTS,PSI*)
    va:real; (*CONTROLLER OUTPUT AFTER CONSTRAINTS,PSI*)
    vamin,vamax, (*MINIMUM AND MAXIMUM VALUES OF VA*)
    kc,taui,taud, (*TUNING CONSTANTS*)
    r, (*STEP CHANGE IN SET POINT,PSI*)
    xa, (*STEP CHANGE IN LOAD VARIABLE NO.1,MG/L*)
    xb, (*STEP CHANGE IN LOAD VARIABLE NO.2,MG/L
        (FOLLOWED BY 1ST ORDER DELAY TAU5*)
    delta, (*ITERATION TIME INTERVAL*)
```

```
    tau1,tau2, (*PROCESS TIME CONSTANTS*)
    tau3,(*1ST ORDER TIME CONSTANT FOR RESPONSE TO xb*)
    lambda,(* CMS TUNING CONSTANT*)
    tt,ttt:real; (*TOTAL TIME OF SIMULATION*)

PROCEDURE DATA;

    begin
       readln(r,xa,xb);
       readln(delta,tt);
       readln(tau1,tau2,thetad);
       t3[1]:=0.05;t3[2]:=0.1;t3[3]:=0.3;t3[4]:=0.5;
       t3[5]:=0.7;t3[6]:=1.0; t3[7]:=1.2;t3[8]:=1.5;
       t3[9]:=2.0;t3[10]:=2.5;t3[11]:=3.0;t3[12]:=3.5;
       t3[13]:=4.0;t3[14]:=4.5;t3[15]:=5.0;t3[16]:=5.5;
       t3[17]:=6.0;t3[18]:=6.5;t3[19]:=7.0;
    end;

procedure answer  (a,bb:integer;cc,dd,eee,f:real);
    begin
       writeln(kc,taui,taud);
       writeln(r,xa,xb,delta,tt);
       writeln(tau1,tau2,tau3,thetad);
       writeln;
       writeln('TRIAL NO.',kkkk);
       writeln('NO. OF STAGES= ',a:3);
       writeln('NO. OF FUNCTION EVALUATIONS= ',bb:3);
       writeln('IAE/(xb*taui= ',cc/(kmm*(xb+r/kmm)*tau1):
                                  16:8);
       writeln('kckl= ',dd*kmm*kv:16:8, 'tau1/taui=',
                tau1/eee:16:8,'tau1*taud= ',tau1*f:16:8);
       writeln('vamax= ',vamax:16:8,'vamin= ',vamin:16:8);
       kc:=kkcc[kkkk];
       k[1]:=kc; k[2]:=taui; k[3]:=taud;
       kkkk:=kkkk+1;
    end;

function dtx2dot(cx1,cx2:real):real;
    begin
       dtx2dot:=delta*(cx1-cx2)/tau1;
    end;

function dtx3dot(cx2,cx3:real):real;
    begin
       dtx3dot:=delta*(cx2-cx3)/tau2;
    end;

function object(kcc,tauii,taudd:real):real;

var
   DT: array [1..5001] of real; (*array for the dead time*)
   i:integer;
    begin
       c:=0.0;epast:=0.0;absie:=0.0;vamin:=0.0;vamax:=0.0;
```

```
      erint:=0.0;time:=0.0;x1:=0.0;
        x2:=0.0;x3:=0.0;x4:=0.0;x5:=0.0;x5:=0.0;x6:=0.0;
        for i:=1 to s do DT[i]:=0.0; G:=S; Q:=1;
           while time<ttt do
              begin
                 c:=kmm*x6;
                 er:=r-c;
                 va:=kcc*(er+erint/tauii+(er-epast)*
                     taudd/delta);
                 vv:=va;
                 if va<vamin then vamin:=va;
                 if va>vamax then vamax:=va;
                 if va <= -6.0 then va:= -6.0;
                 if va >= 6.0 then va:=  6.0;
                 x1:= xa + va*kv;
                 time:= time + delta;
                 if (vv < 6.1) and (vv > -6.1) then
                     erint:= erint + er * delta;
                 absie:= absie + abs(er*delta);
                 epast:= er;
                 rk11:=dtx2dot(x1,x2);
                 rk12:=dtx3dot(x2,x3);

                 rk21:=dtx2dot(x1,x2+0.5*rk11);
                 rk22:=dtx3dot(x2+0.5*rk11,x3+0.5*rk12);

                 rk31:=dtx2dot(x1,x2+0.5*rk21);
                 rk32:=dtx3dot(x2+0.5*rk21,x3+0.5*rk22);

                 rk41:=dtx2dot(x1,x2+rk31);
                 rk42:=dtx3dot(x2+rk31,x3+rk32);

                 x2:=x2+(rk11+2.0*rk21+2.0*rk31+rk41)/6.0;
                 x3:=x3+(rk12+2.0*rk22+2.0*rk32+rk42)/6.0;

                 DT[G]:=x3;
                 x4:=DT[Q];
                 G:=G+1; Q:=Q+1;
                 if G>S then G:=1; if Q>S then Q:=1;
                 x5:=x5 * du31 + du32xb;
                 x6:=x4 + x5;
              end;
           object:=absie;
           writeln(kcc:8:3,tauii:8:3,taudd:8:3, absie);
  END;

 begin  (*MAIN PROGRAM*)
     TERMIN(INPUT);
     TERMOUT(OUTPUT);
     data;

     for mmm:=1 to 10 do
     begin
     thetad:=0.1*mmm;
```

```
    ttt:=tt;
    for iiii:= 1 to 19 do
    begin
    tau3:=t3[iiii];
    ttt:=ttt+0.20*iiii;
    lambda:=1.0/(tau1+tau2);
    kc:=(tau1*tau2)*lambda/(kv*kmm*(lambda*thetad+1.0));
    taui:=tau1+tau2;
    taud:=tau1*tau2/(tau1+tau2);

    kkcc[1]:=kc;kkcc[2]:=kc/1.25;kkcc[3]:=kc*1.10;

    k[1]:=kkcc[1];
    k[2]:=taui;
    K[3]:=taud;

    kkkk:=1;
5:  if kkkk>4 then goto 6;

    P[1]:=0.10;
    P[2]:=0.02;
    P[3]:=0.005;
    S:=round(thetad/delta)+1;

    DU31:=EXP(-DELTA/TAU3);DU32:=1.0-DU31;
    DU32XB:=XB * DU32;

    KAT:=1;   (*FUNCTION EVALUATIONS*)

    FOR I:=1 TO KM DO (*INITIALIZE DIRECTIONAL VECTORS*)
       BEGIN
          FOR M:=1 TO KM DO
             BEGIN
                v[i,m]:=0.0;
                if (i-m)=0 then
                   v[i,m]:=1.0;
             end;
       end;

    sumn:=object(k[1],k[2],k[3]);
    writeln('initial IAE/(xb*tau1)=',sumn/(kmm*(xb+r/kmm)*
            tau1));
    sumo:=sumn;
    kk1:=1; (*STAGES*)

    if nstep<>1 then
       begin
          for i:=1 to km do
             e[i]:=p[i];
       end;
 1: for i:=1 to km do
       begin
          fbest:=sumn;
          j[i]:=2.0;
```

```
         if nstep=1 then
         e[i]:=p[i];
         d[i]:=0.0;
  end;
   iii:=0;
2: iii:=iii+1;
3: i:=1;
4: for m:=1 to km do
      k[m]:=k[m]+e[i]*v[i,m];
   sumn:=object(k[1],k[2],k[3]);
   kat:=kat+1;
   sumdif:=fbest-sumn;

  if (abs(sumdif)-epsy)<=0.0 then
     begin
      answer(kk1,kat,sumo,k[1],k[2],k[3]);
      goto 5;
     end;

   if (kat-maxk)>=0.0 then
      begin
         writeln('maxk exceeded');
         answer(kk1,kat,sumo,k[1],k[2],k[3]);
         goto 5;
      end;

   if (sumn-sumo)>0.0 then
      begin (*FAILURE*)
         for m:=1 to km do
            k[m]:=k[m]-e[i]*v[i,m];
         e[i]:=-beta*e[i];
         if abs(e[i])< 1.0E-20. then goto 7;
         if (j[i]-1.5)<0.0 then j[i]:=0.0;
      end

   else
      begin (*SUCCESS*)
         d[i]:=d[i]+e[i];
         e[i]:=alpha*e[i];
         if abs(e[i])< 1.0E-20. then goto 7;
         sumo:=sumn;
         if (j[i]-1.5)>0 then j[i]:=1.0;
      end;

   for m:=1 to km do (*CHECK END OF STAGE*)
      if (j[m]-0.5)>0 then (*STAGE NOT ENDED*)
         begin
            if (i-km)<>0 then (*CHECK END OF CYCLE*)
               begin
                  i:=i+1;
                  goto 4;
               end
            else
                begin
```

```
                  for z:=1 to km do
                  if (j[z]-2.0)<0 then goto 3;
                  if (iii-mcyc)<0 then goto 2
                  else
                     begin
                       writeln('mcyc exceeded');
                       answer(kk1,kat,sumo,k[1],k[2],k[3]);
                       goto 5;
                     end;
                  end;
            end;
      for i:=1 to km do
         for m:=1 to km do
            l[i,m]:=0.0;
      writeln(output,'STAGE NO.=',kk1:3);
      writeln(output,'FUNCTION=',sumo:16:8);
      for i:=1 to km do
         writeln(output,'x(',i,')=',k[i]:16:8);

      (*ROTATE AXES*)

      for i:=1 to km do
         begin
            kl:=i;
            for m:=1 to km do
               begin
                  for z:=kl to km do
                     l[i,m]:=d[z]*v[z,m]+l[i,m];
                  b[i,m]:=l[i,m];
               end;
         end;

      n[1]:=0.0;
      for z:=1 to km do
         n[1]:=n[1]+b[1,z]*b[1,z];
      n[1]:=sqrt(n[1]);
      for m:=1 to km do
         v[1,m]:=b[1,m]/n[1];
      for i:=2 to km do
         begin
            ii:=i-1;
            for m:=1 to km do
               begin
                  sumavv:=0.0;
                  for z:=1 to ii do
                     begin
                        sumav:=0.0;
                        for mm:=1 to km do
                           sumav:=sumav+l[i,mm]*v[z,mm];
                           sumavv:=sumav*v[z,m]+sumavv;
                     end;
                  b[i,m]:=l[i,m]-sumavv;
               end;
         end;
```

```
        for i:=2 to km do
           begin
              n[i]:=0.0;
              for z:=1 to km do
                 n[i]:=n[i]+b[i,z]*b[i,z];
                 n[i]:=sqrt(n[i]);
                 for m:=1 to km do
                    v[i,m]:=b[i,m]/n[i];
           end;
7;      kk1:=kk1+1;
        if (kk1-mkat)<0 then goto 1;
        writeln('maxk exceeded');
        answer(kk1,kat,sumo,k[1],k[2],k[3]);
        goto 5;
6:   end;
     end;
     end.
```

APPENDIX B

GRAPHS FOR PID CONTROLLER TUNING CONSTANTS

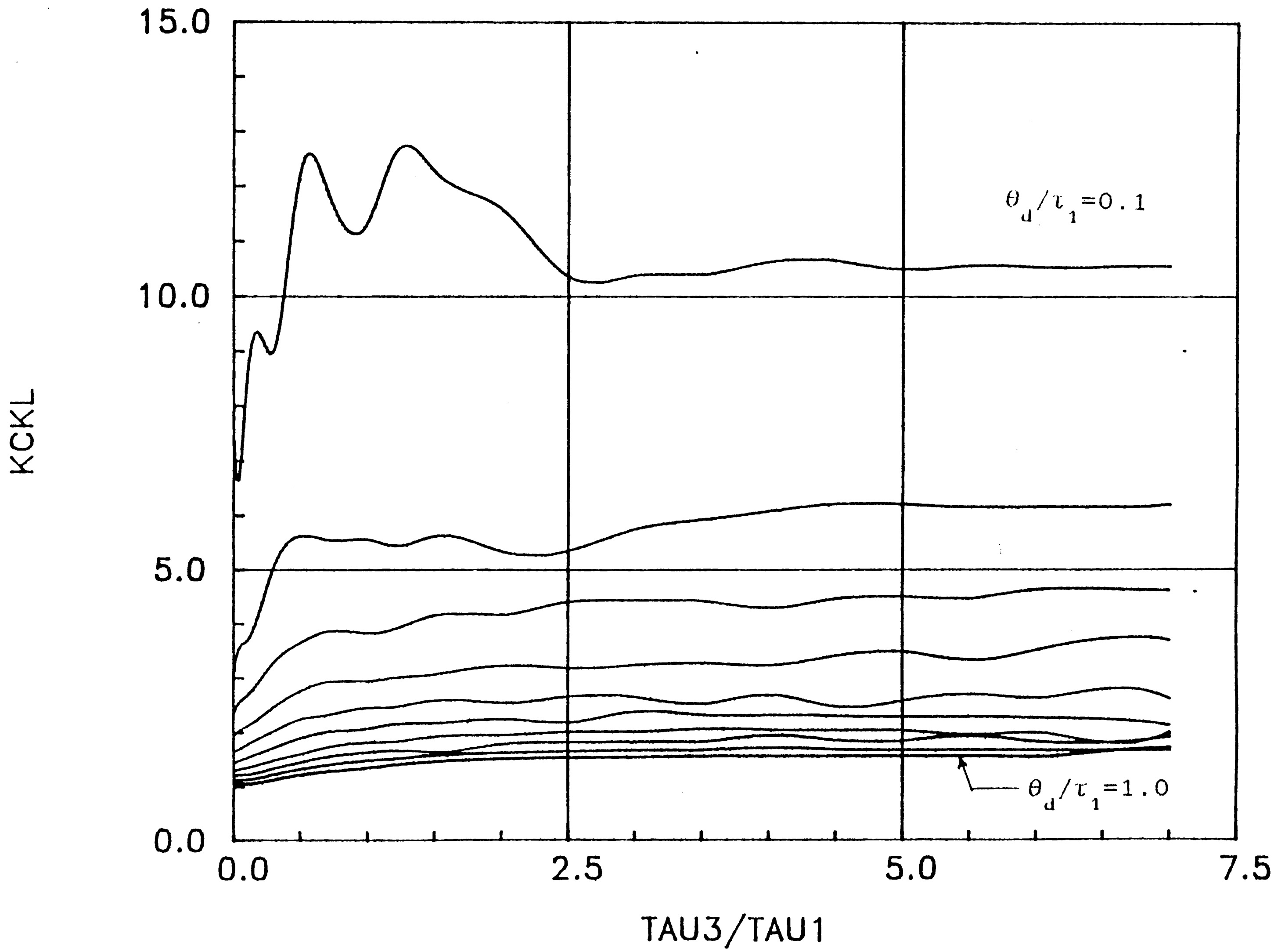

Figure 23. Optimu PID Proportional Gain at $\tau_2/\tau_1=0.1$

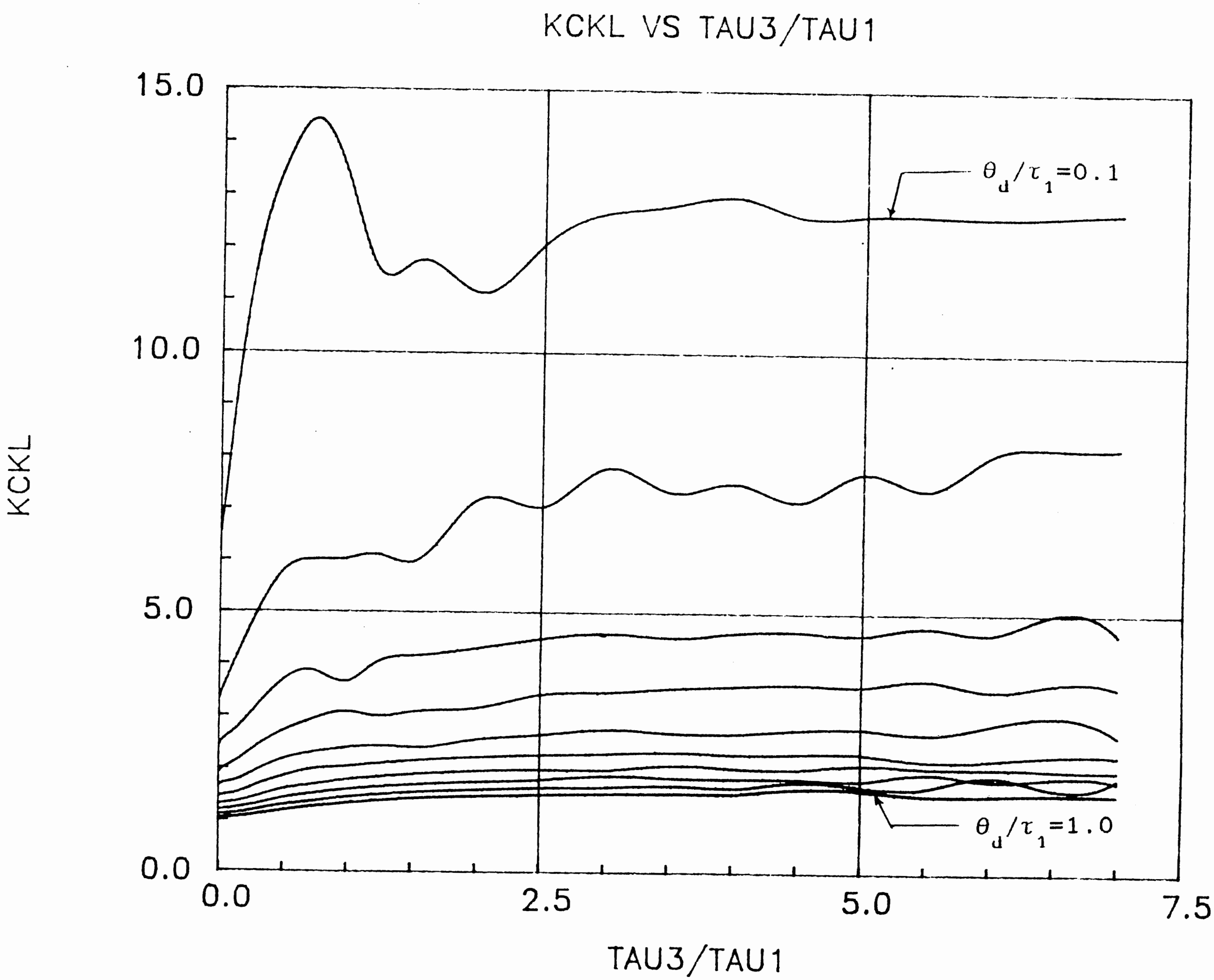

Figure.24. Optimu PID Proportional Gain at $\tau_2/\tau_1=0.2$

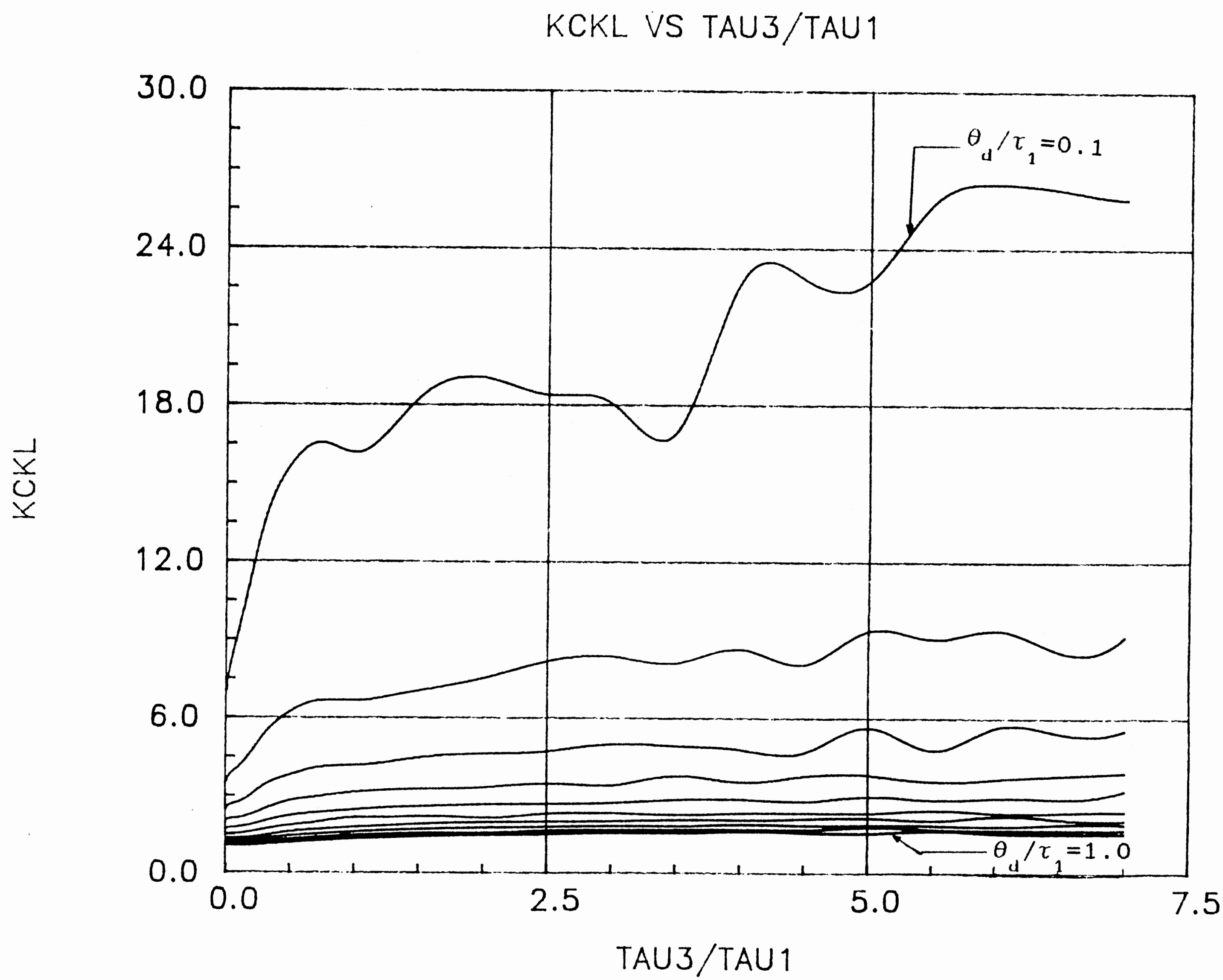

Figure.25. Optimu PID Proportional Gain at τ_2/τ_1=0.3

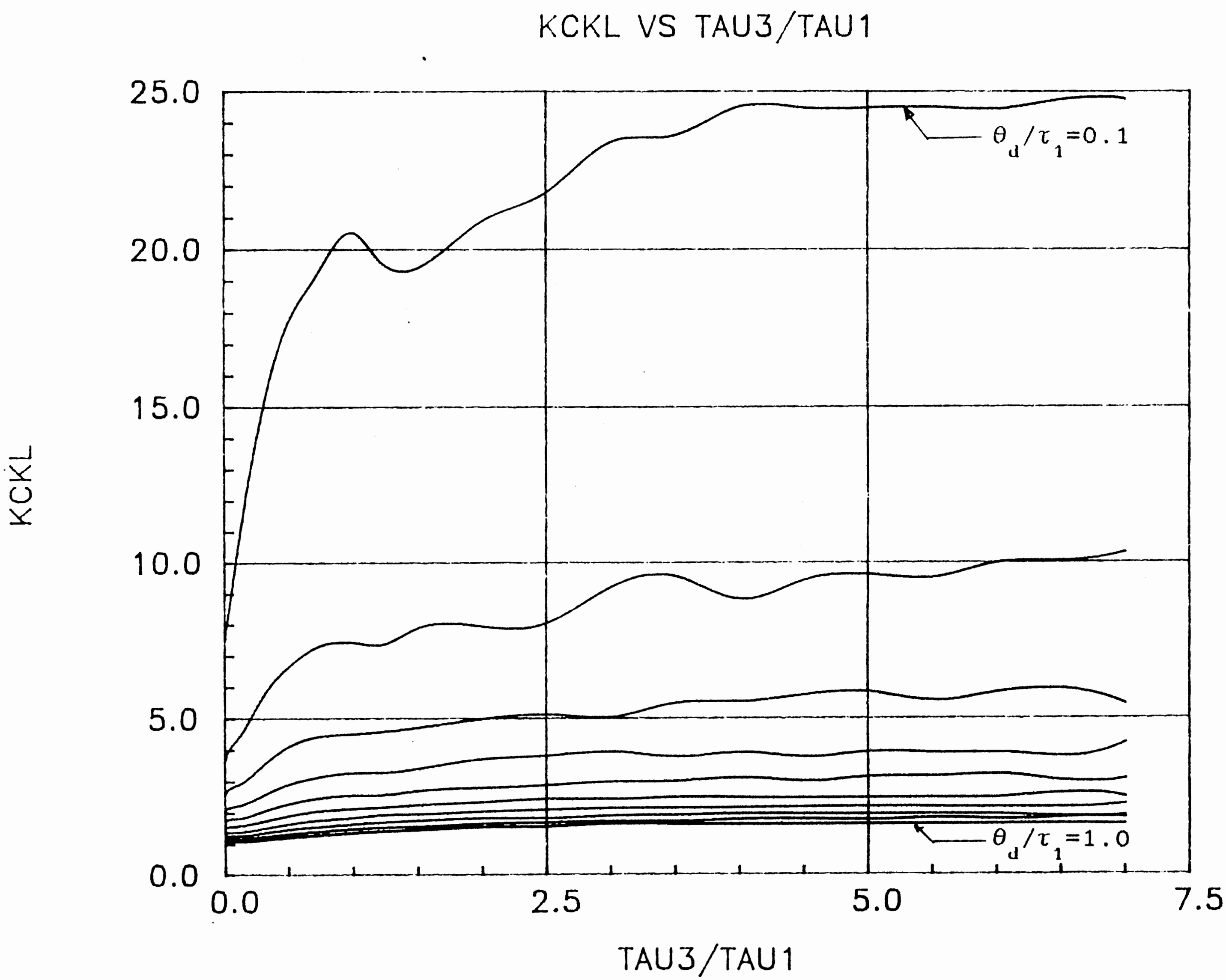

Figure 26. Optimu PID Proportional Gain at τ_2/τ_1=0.4

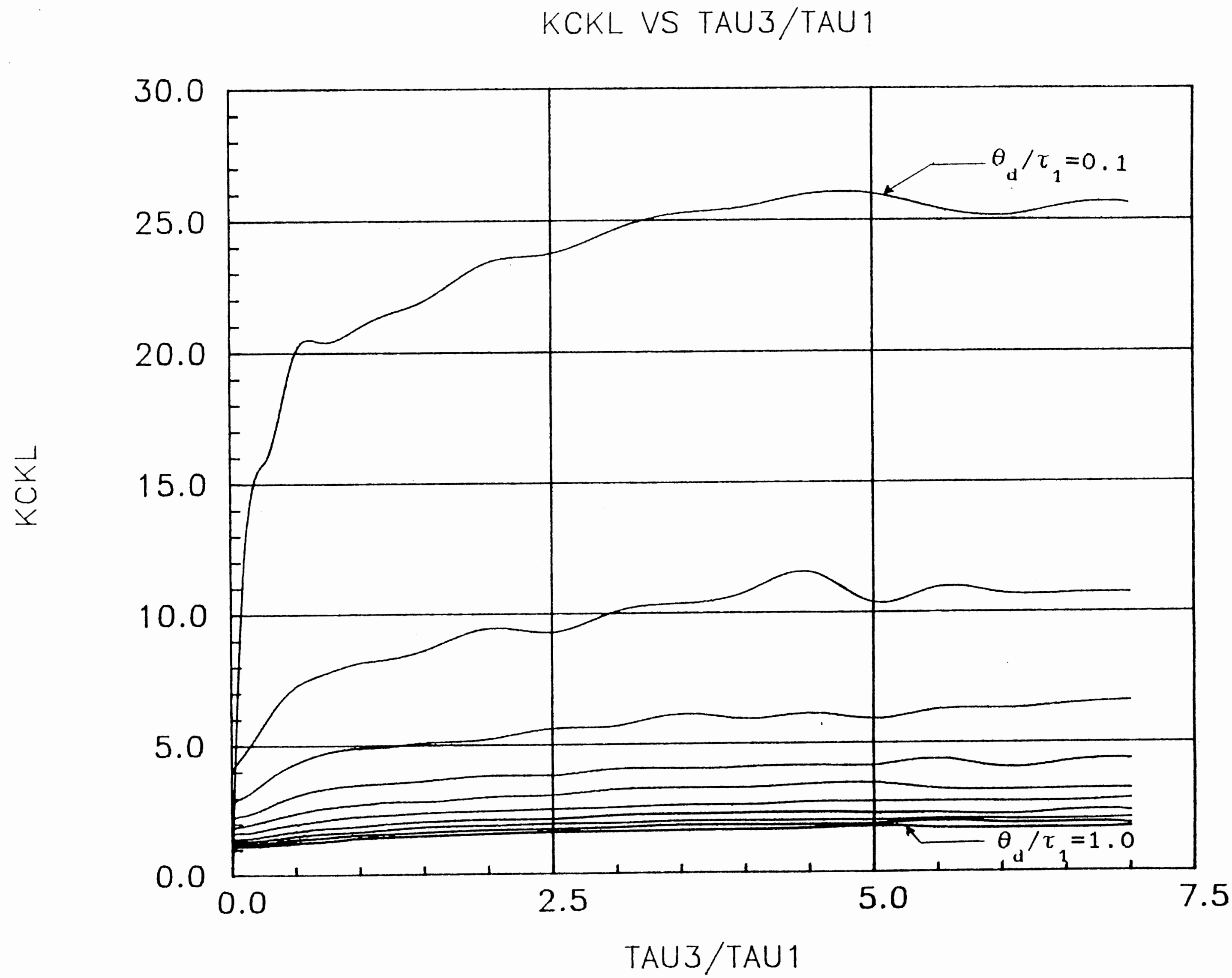

Figure 27. Optimu PID Proportional Gain at τ_2/τ_1=0.5

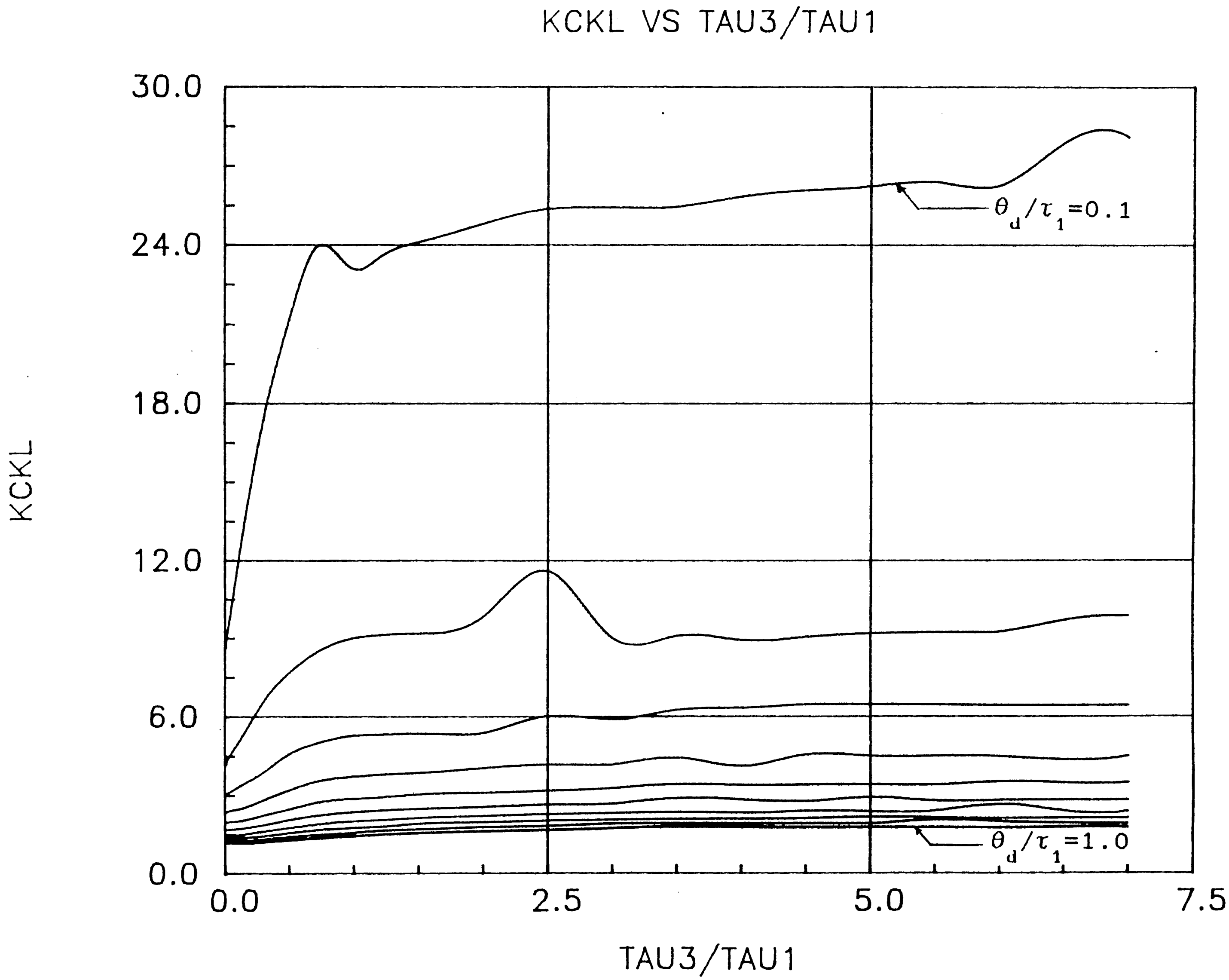

Figure 28 Optimu PID Proportional Gain at τ_2/τ_1=0.6

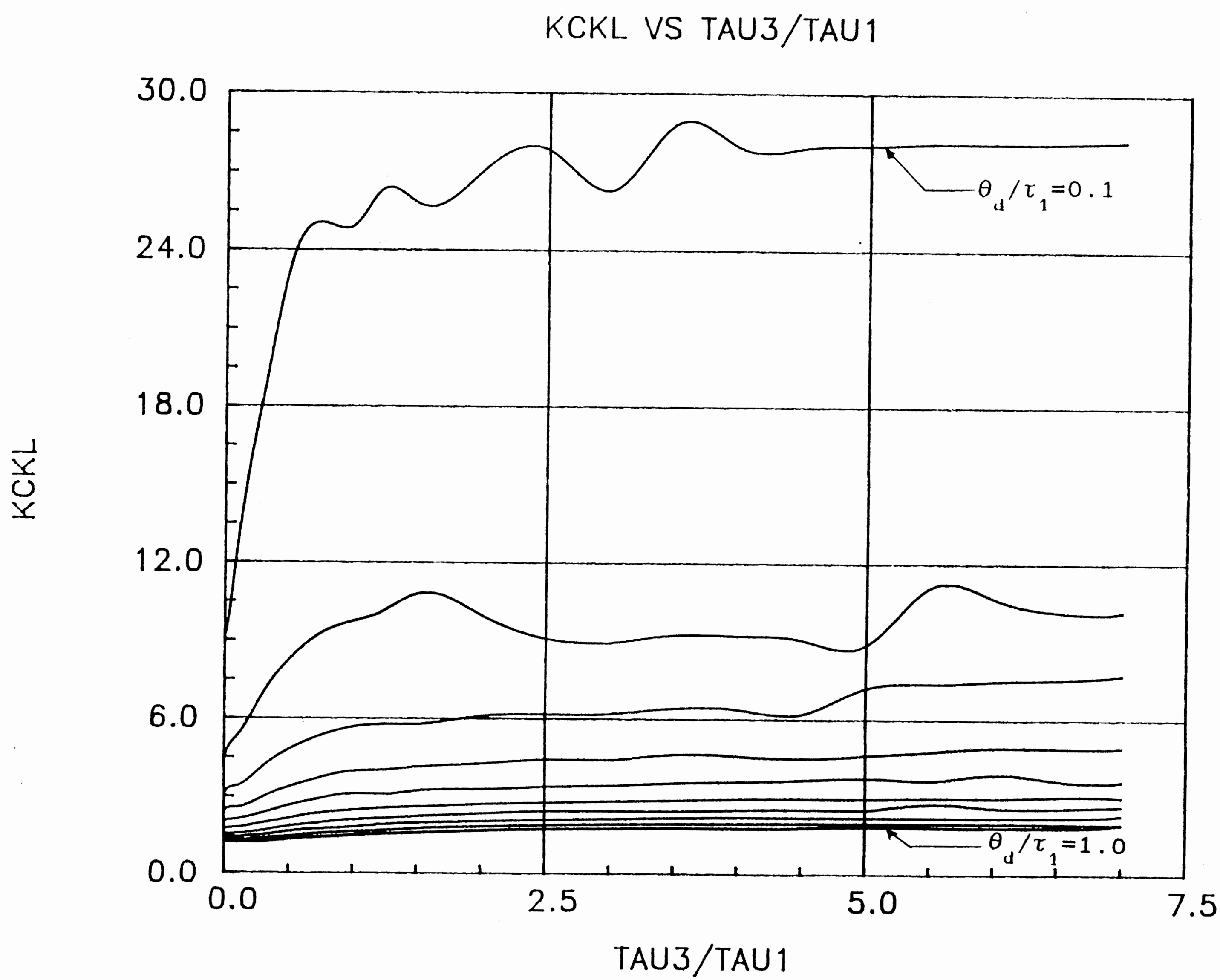

Figure 29. Optimu PID Proportional Gain at τ_2/τ_1=0.7

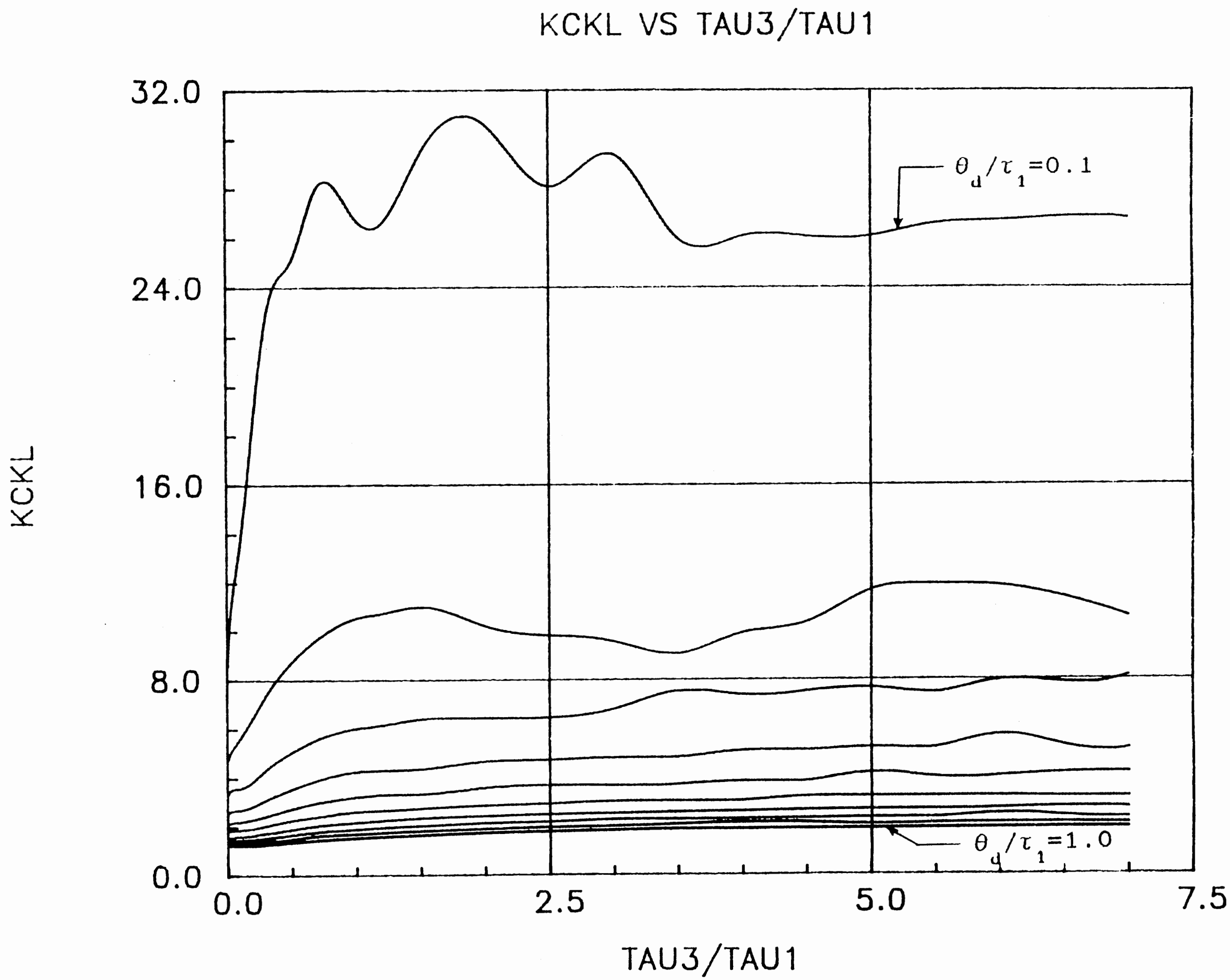

Figure 30. Optimu PID Proportional Gain at τ_2/τ_1=0.8

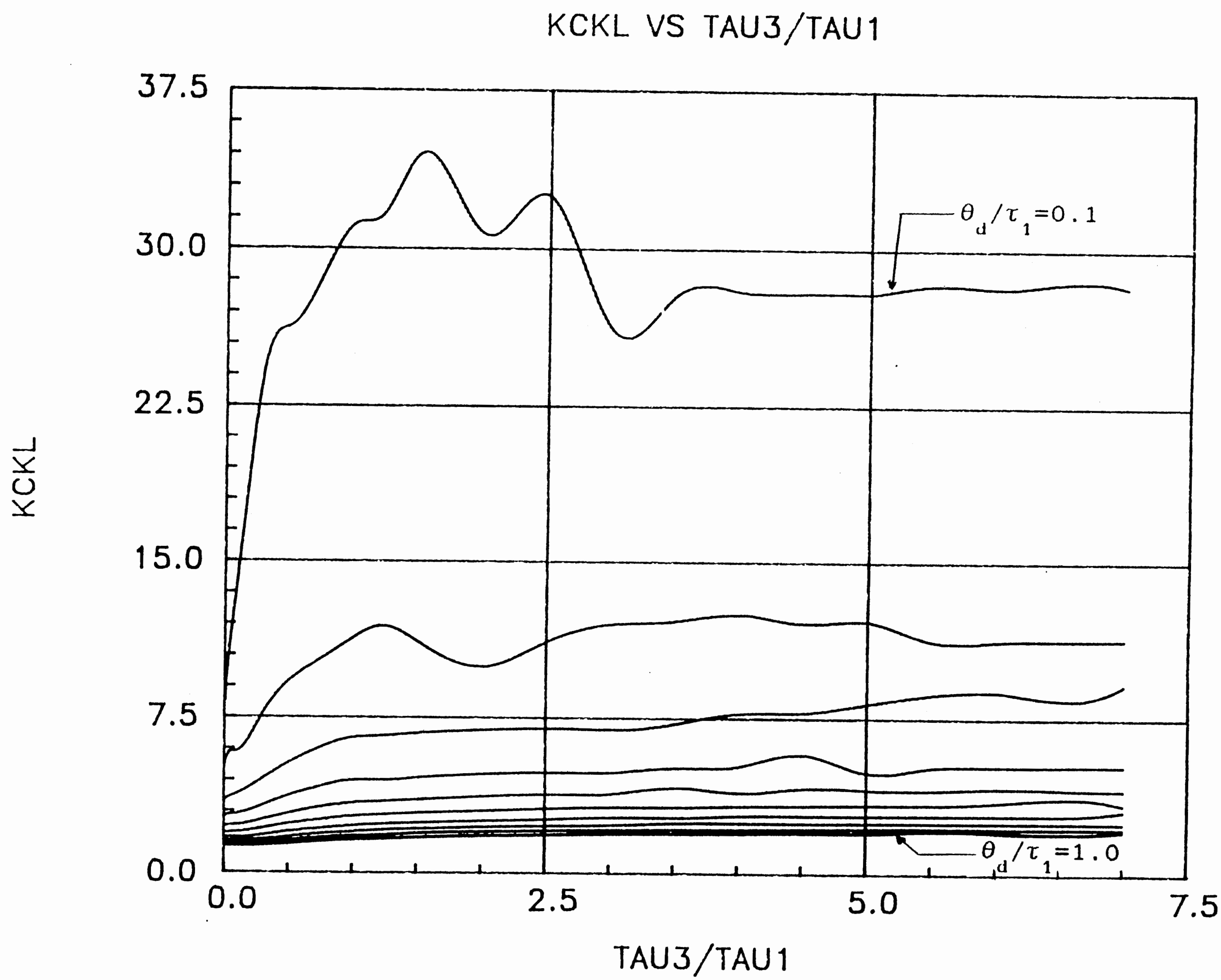

Figure 31. Optimu PID Proportional Gain at τ_2/τ_1=0.9

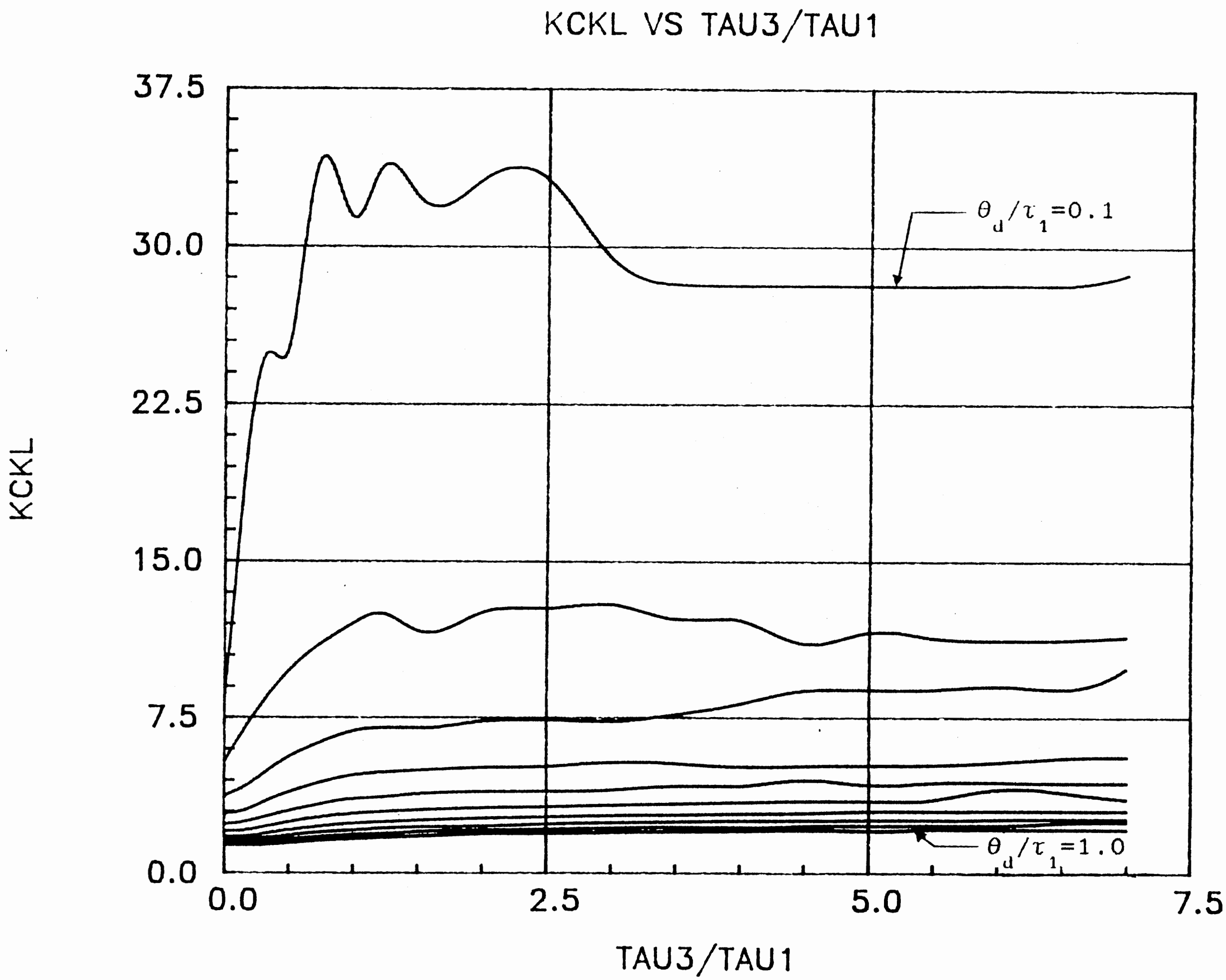

Figure 32. Optimu PID Proportional Gain at τ_2/τ_1=1.0

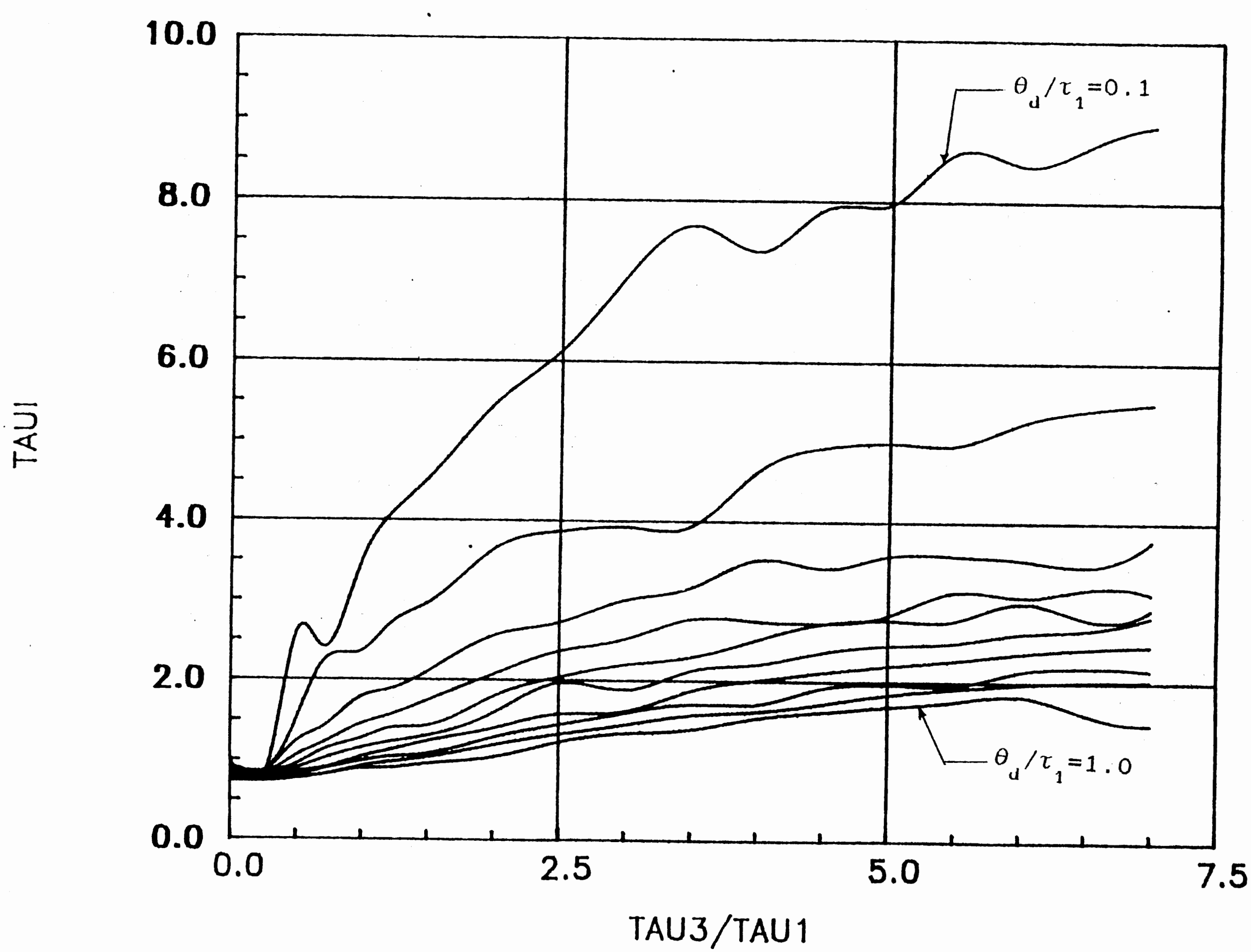

Figure 33.. Optimum PID integral Time at τ_2/τ_1=0.1

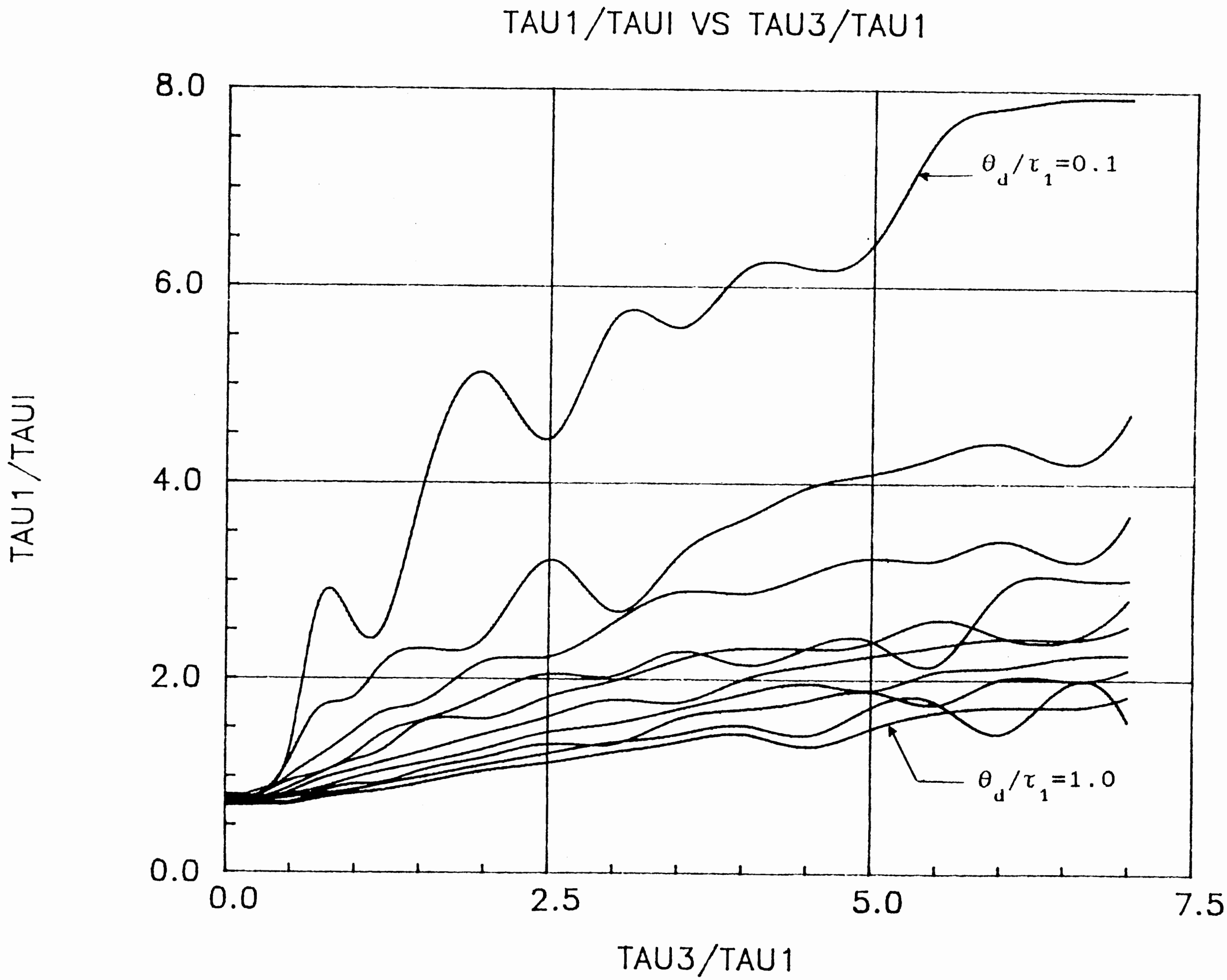

Figure 34. Optimum PID integral Time at τ_2/τ_1=0.2

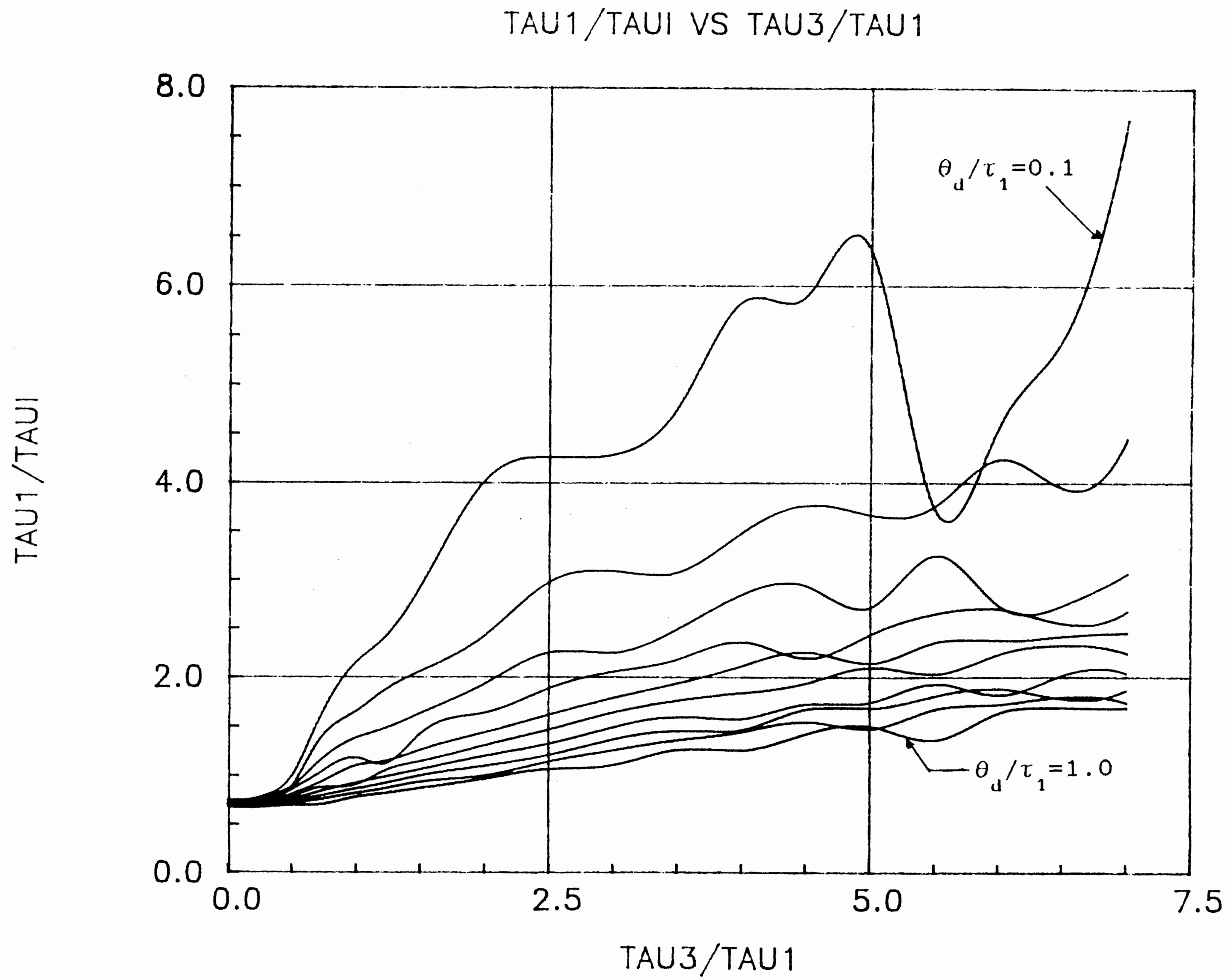

Figure 35. Optimum PID integral Time at τ_2/τ_1=0.3

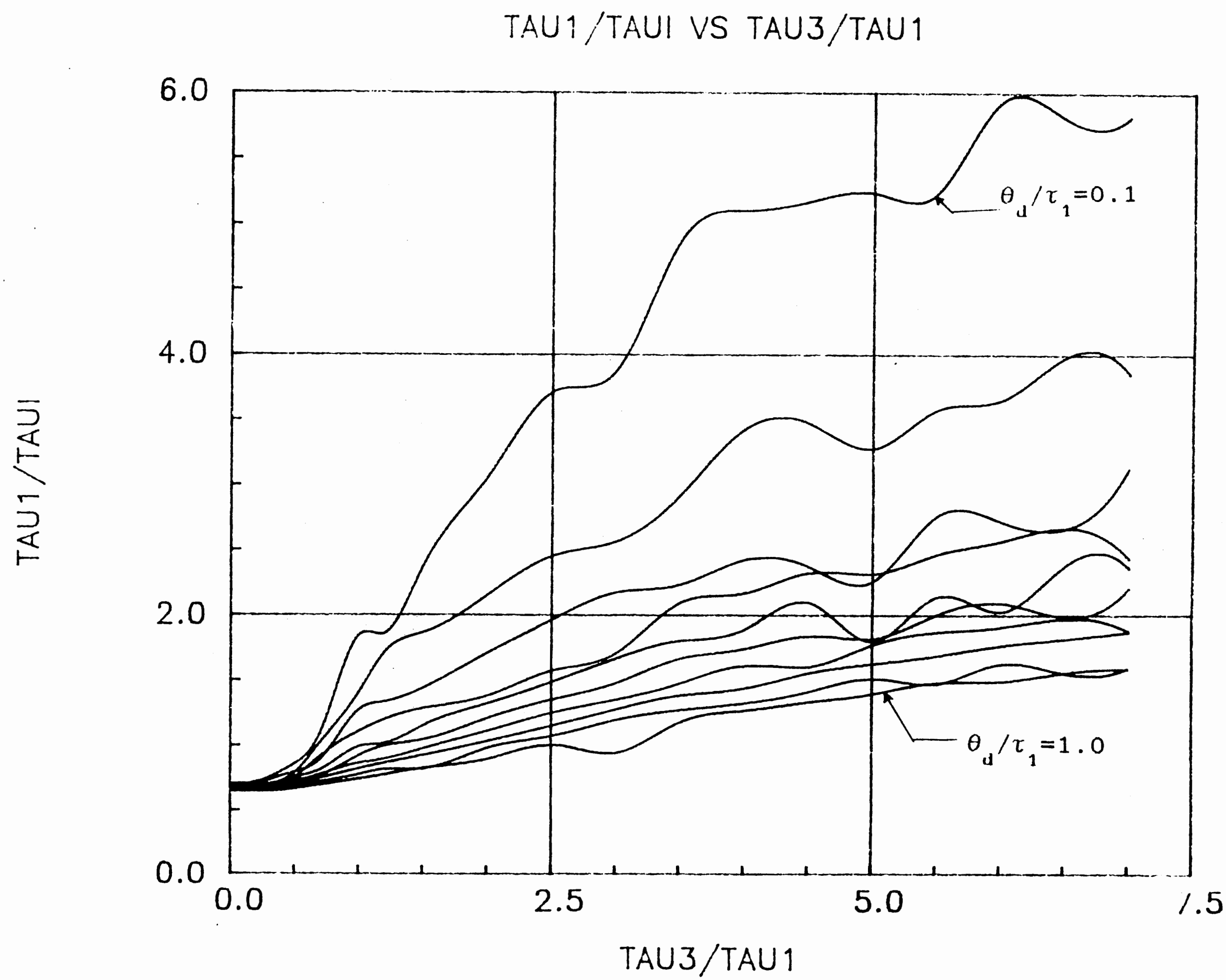

Figure 36. Optimum PID integral Time at $\tau_2/\tau_1=0.4$

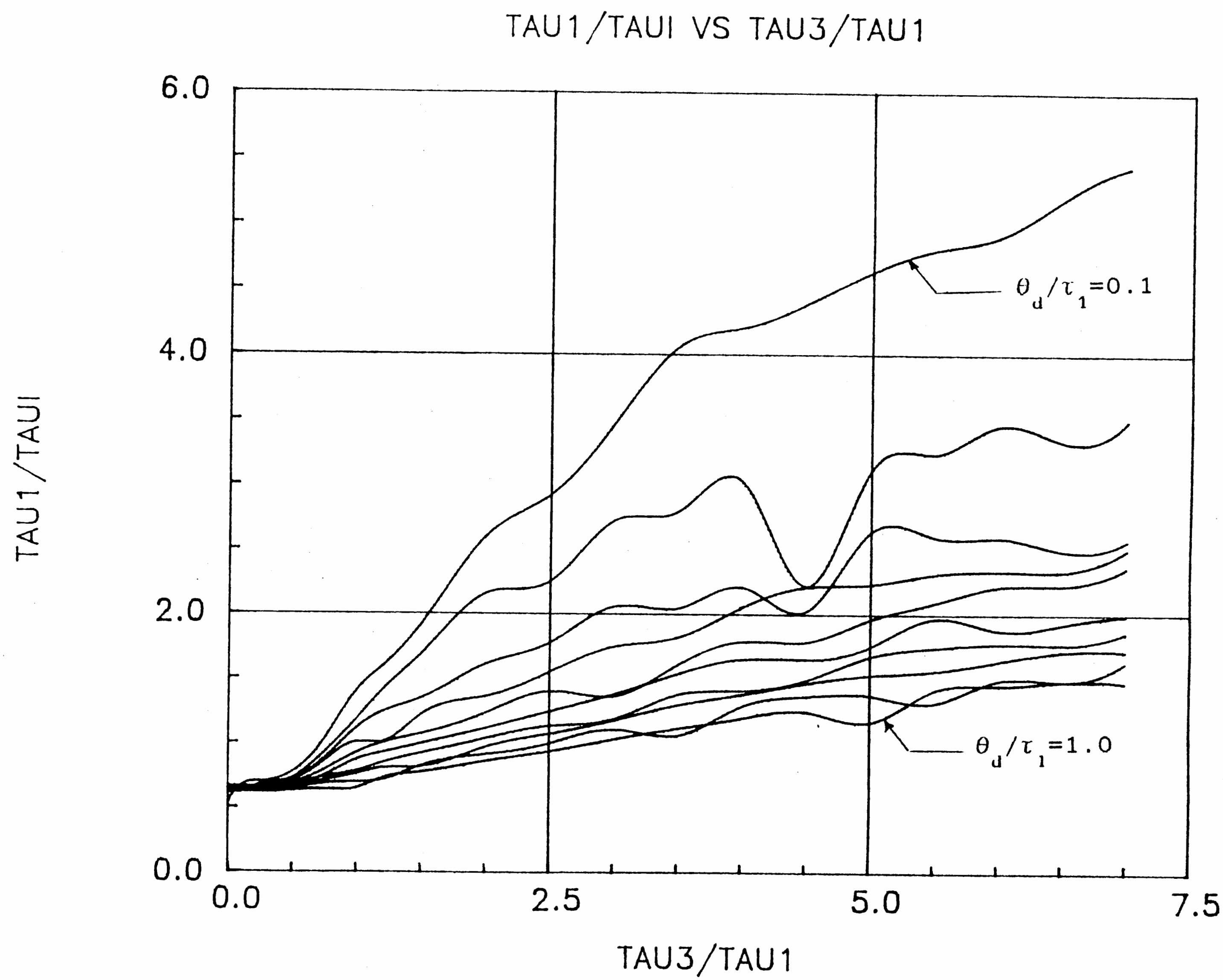

Figure 37. Optimum PID integral Time at τ_2/τ_1=0.5

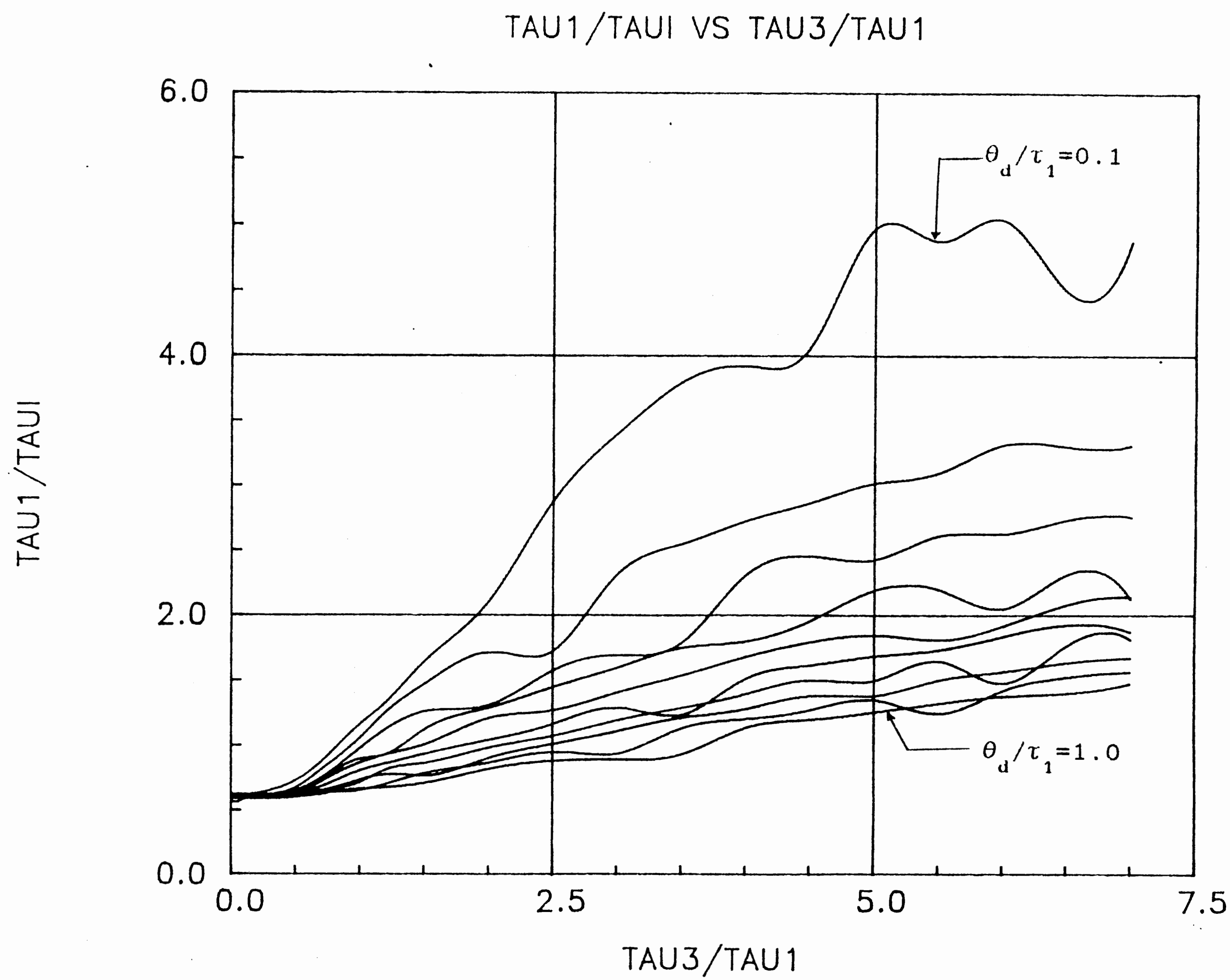

Figure 38. Optimum PID integral Time at $\tau_2/\tau_1=0.6$

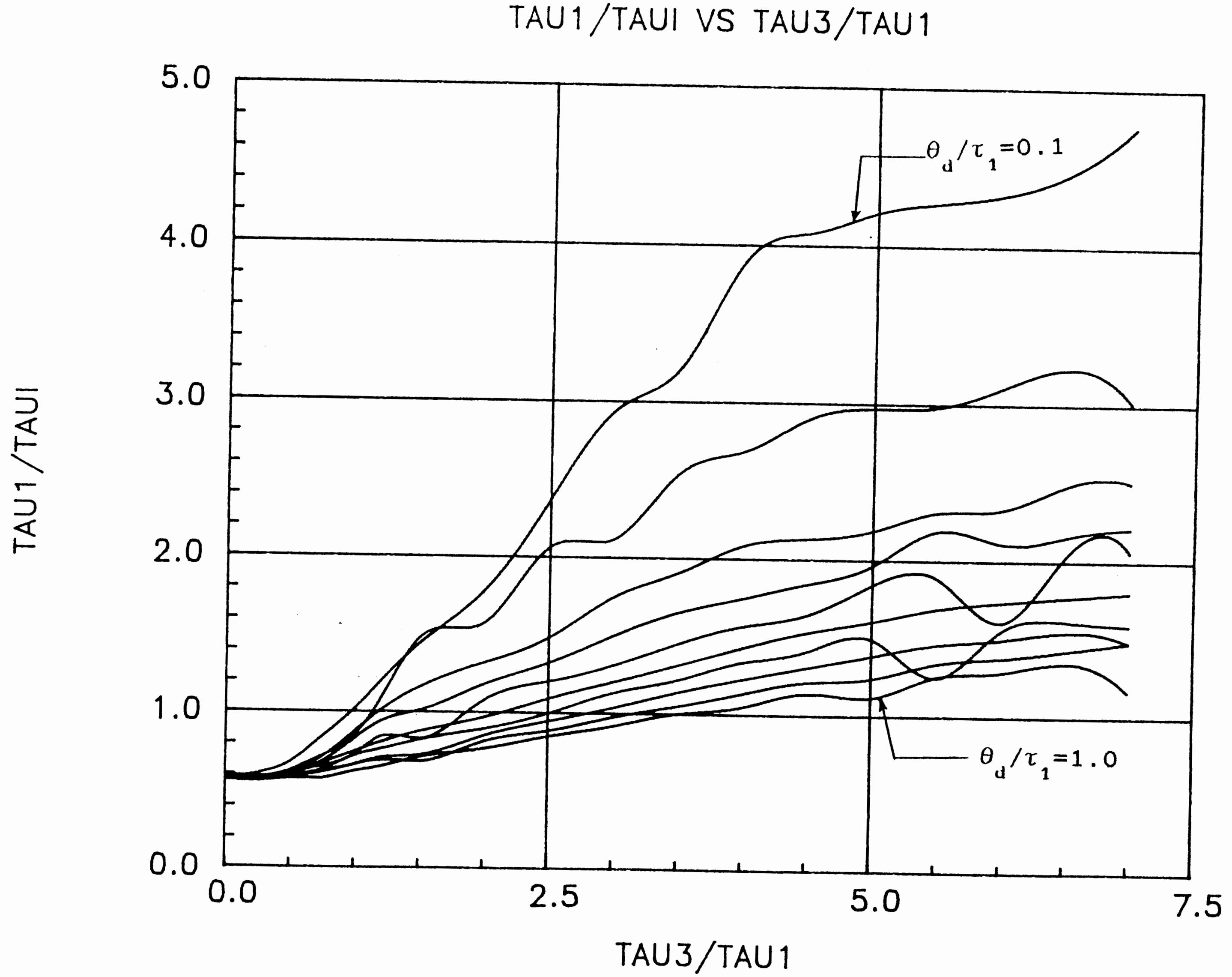

Figure 39. Optimum PID integral Time at τ_2/τ_1=0.7

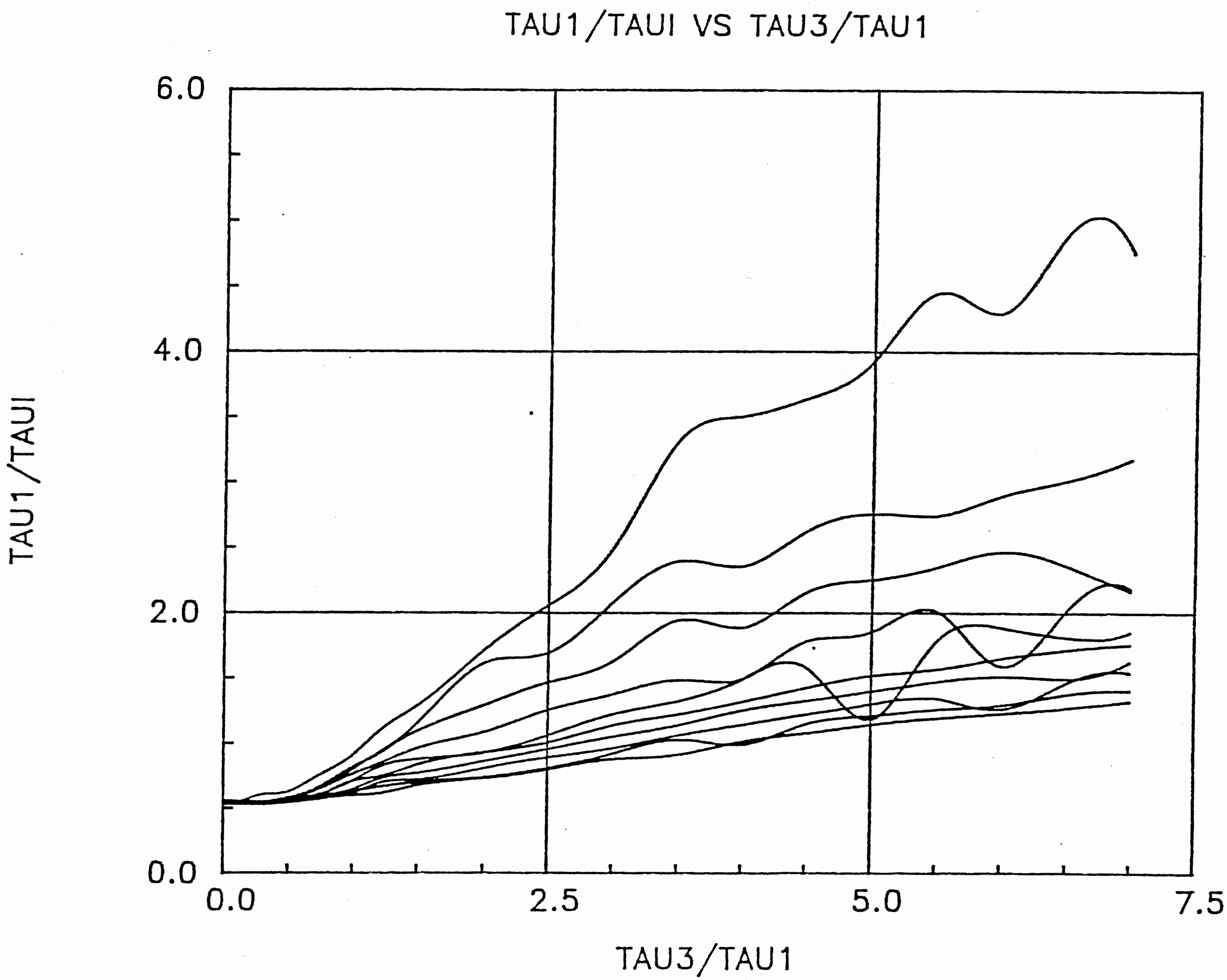

Figure 40. Optimum PID integral Time at τ_2/τ_1=0.8

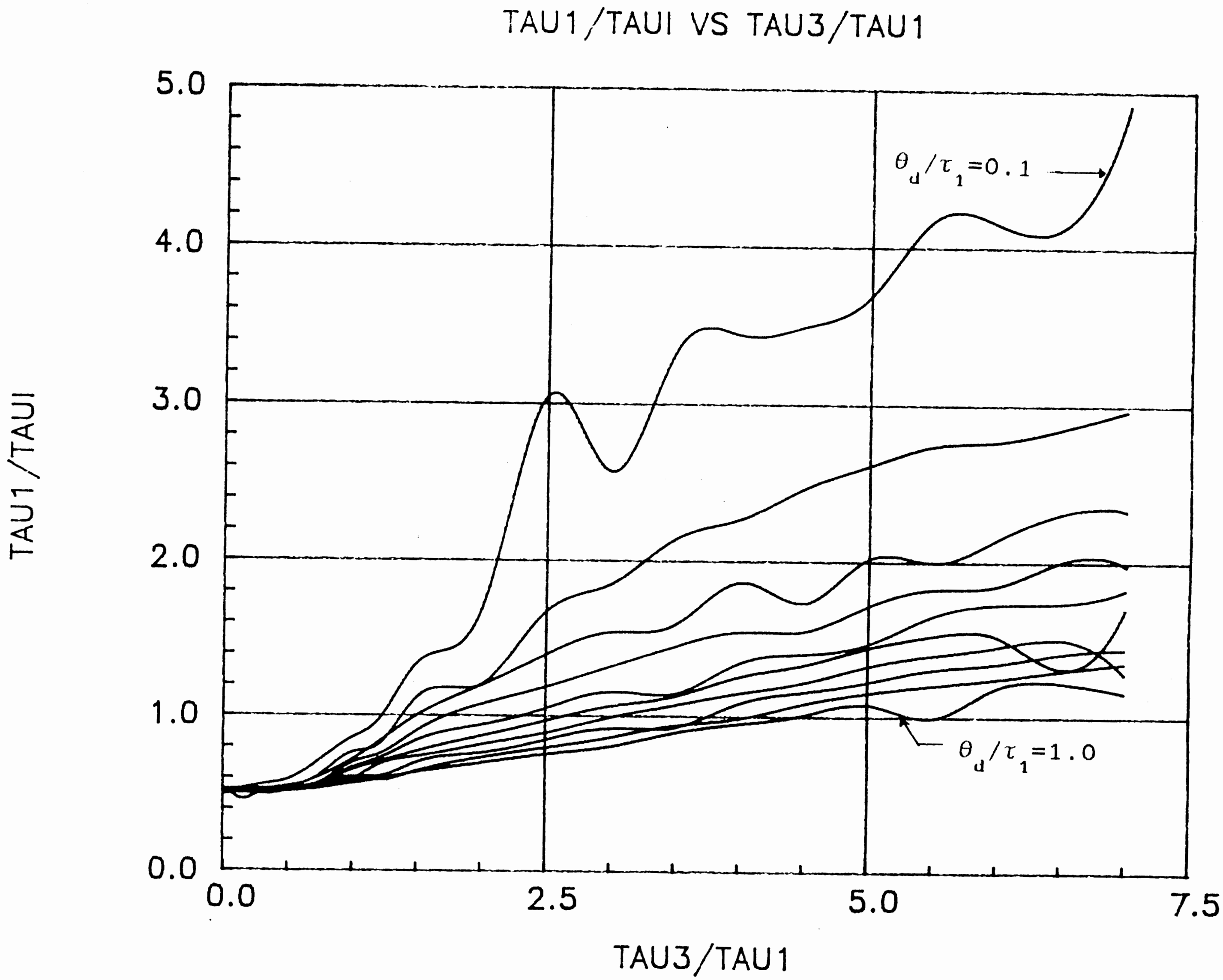

Figure 41. Optimum PID integral Time at τ_2/τ_1=0.9

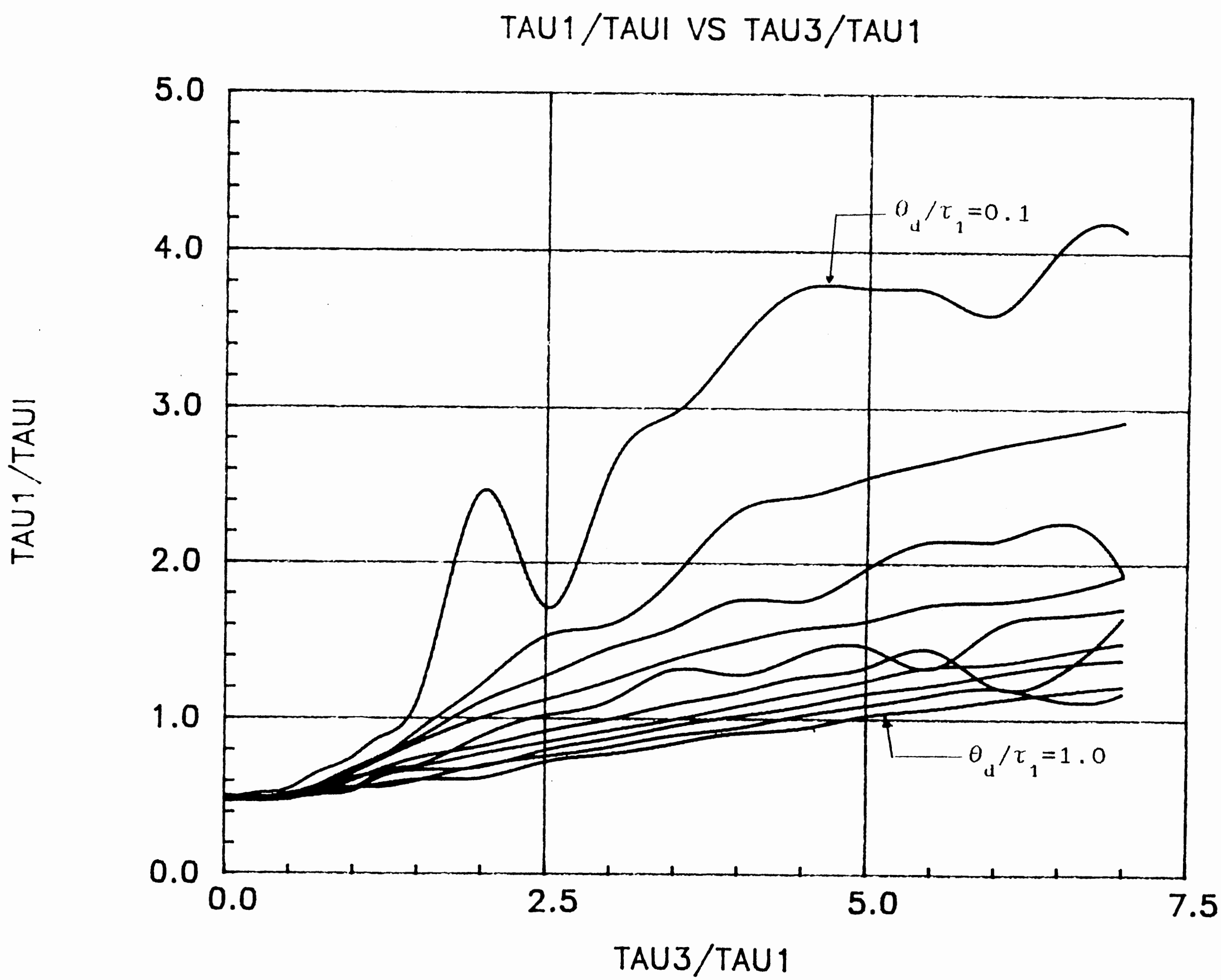

Figure 42. Optimum PID integral Time at $\tau_2/\tau_1=1.0$

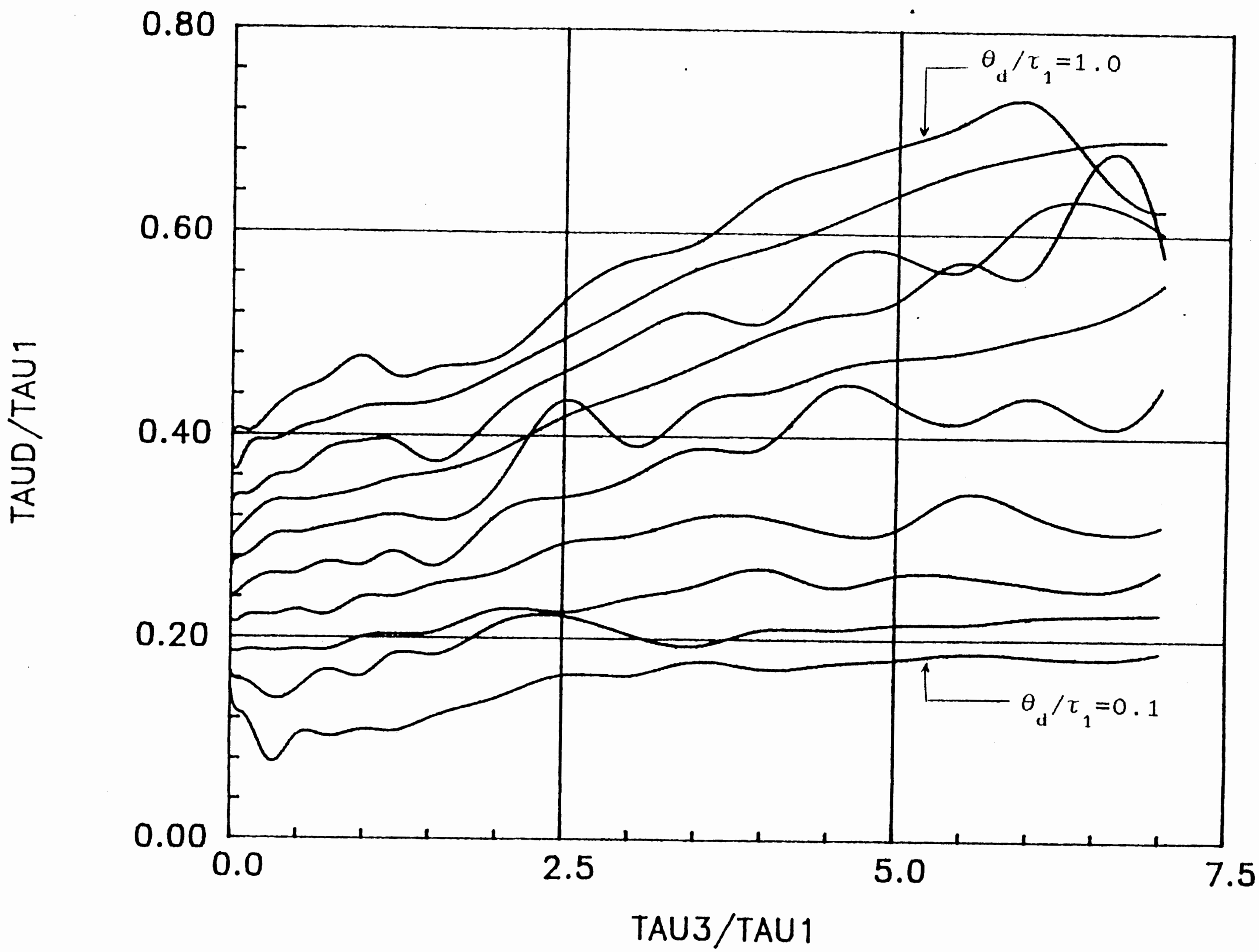

Figure 43. Optimum PID Derivative Time at $\tau_2/\tau_1=0.1$

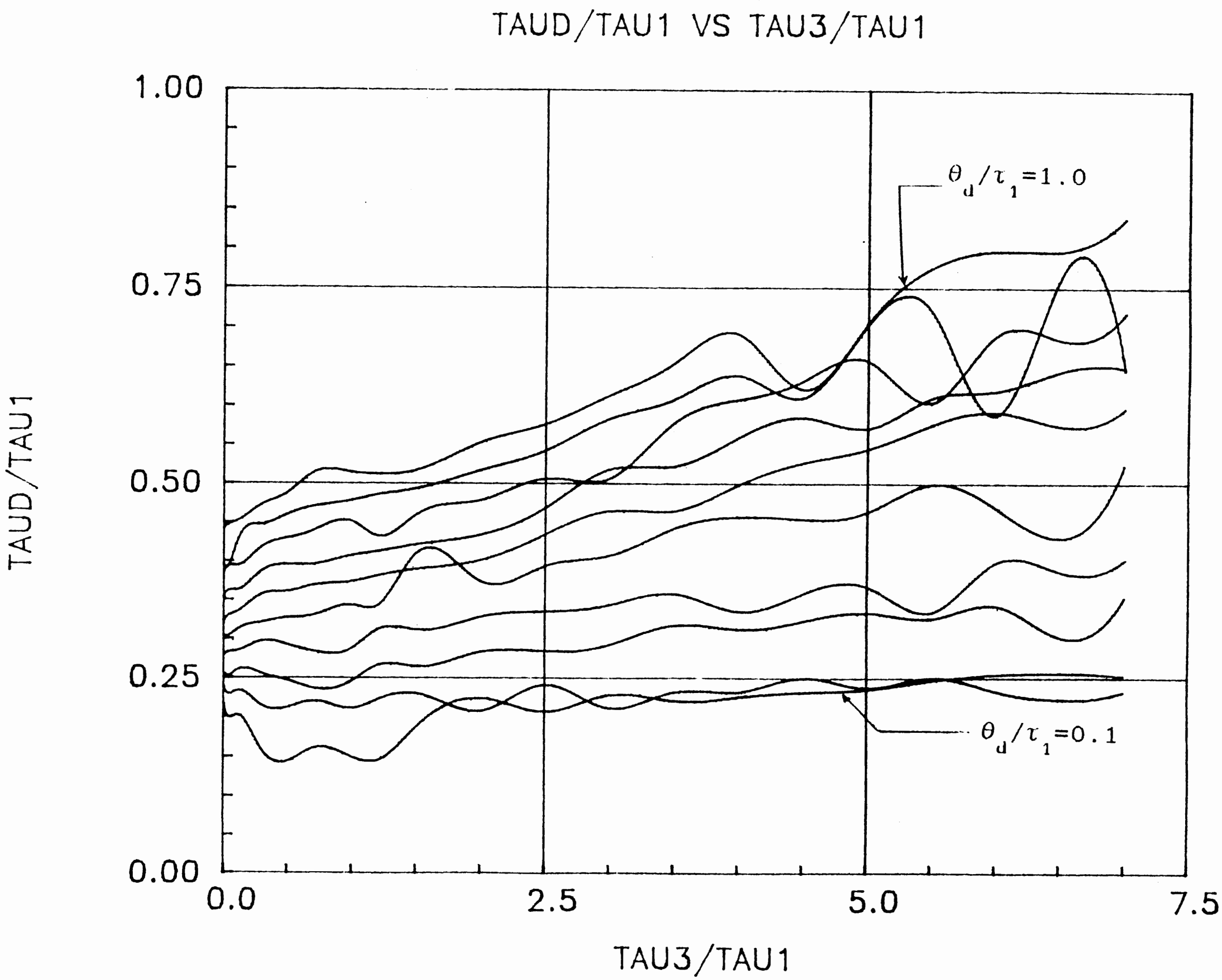

Figure 44. Optimum PID Derivative Time at $\tau_2/\tau_1=0.2$

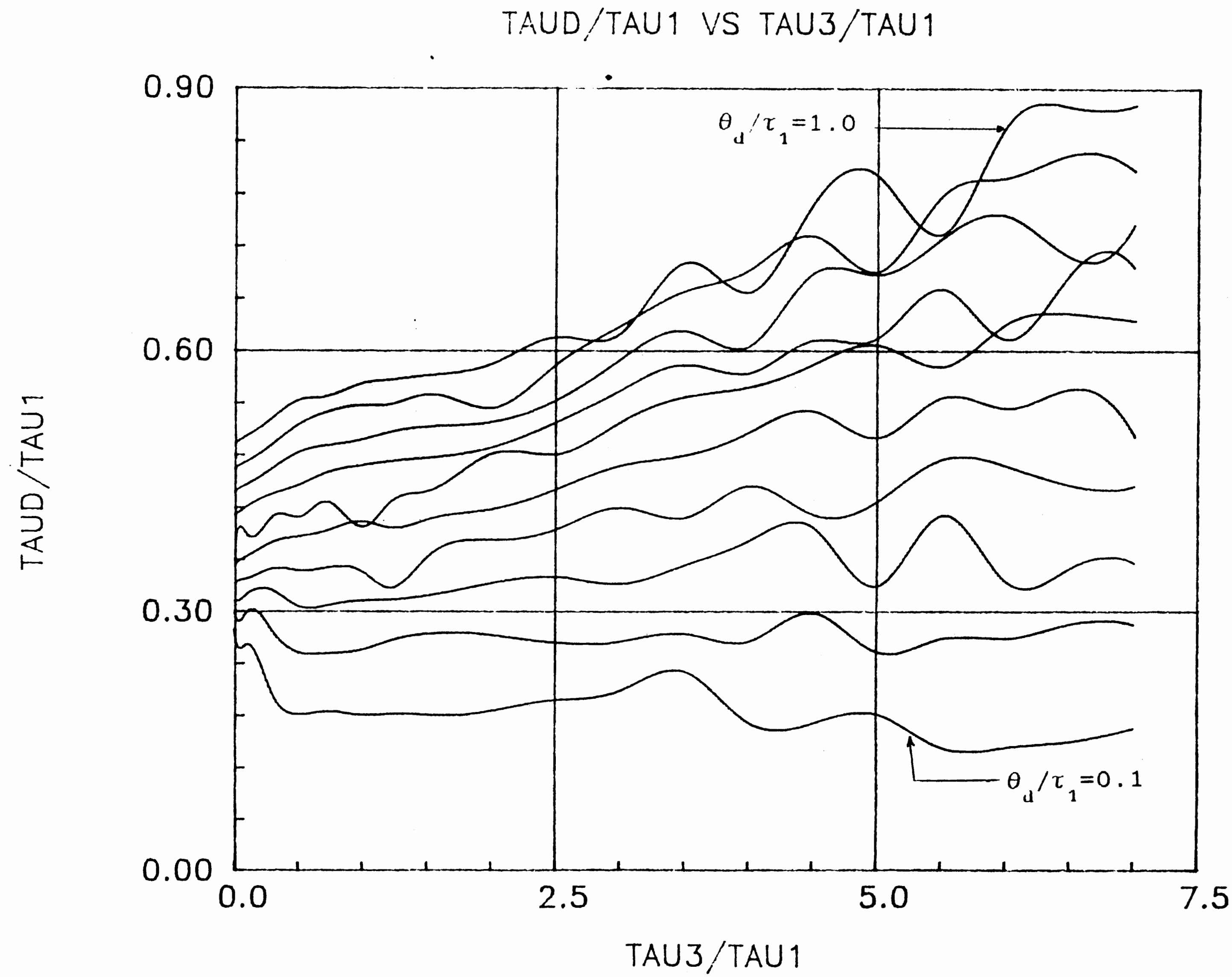

Figure 45. Optimum PID Derivative Time at $\tau_2/\tau_1=0.3$

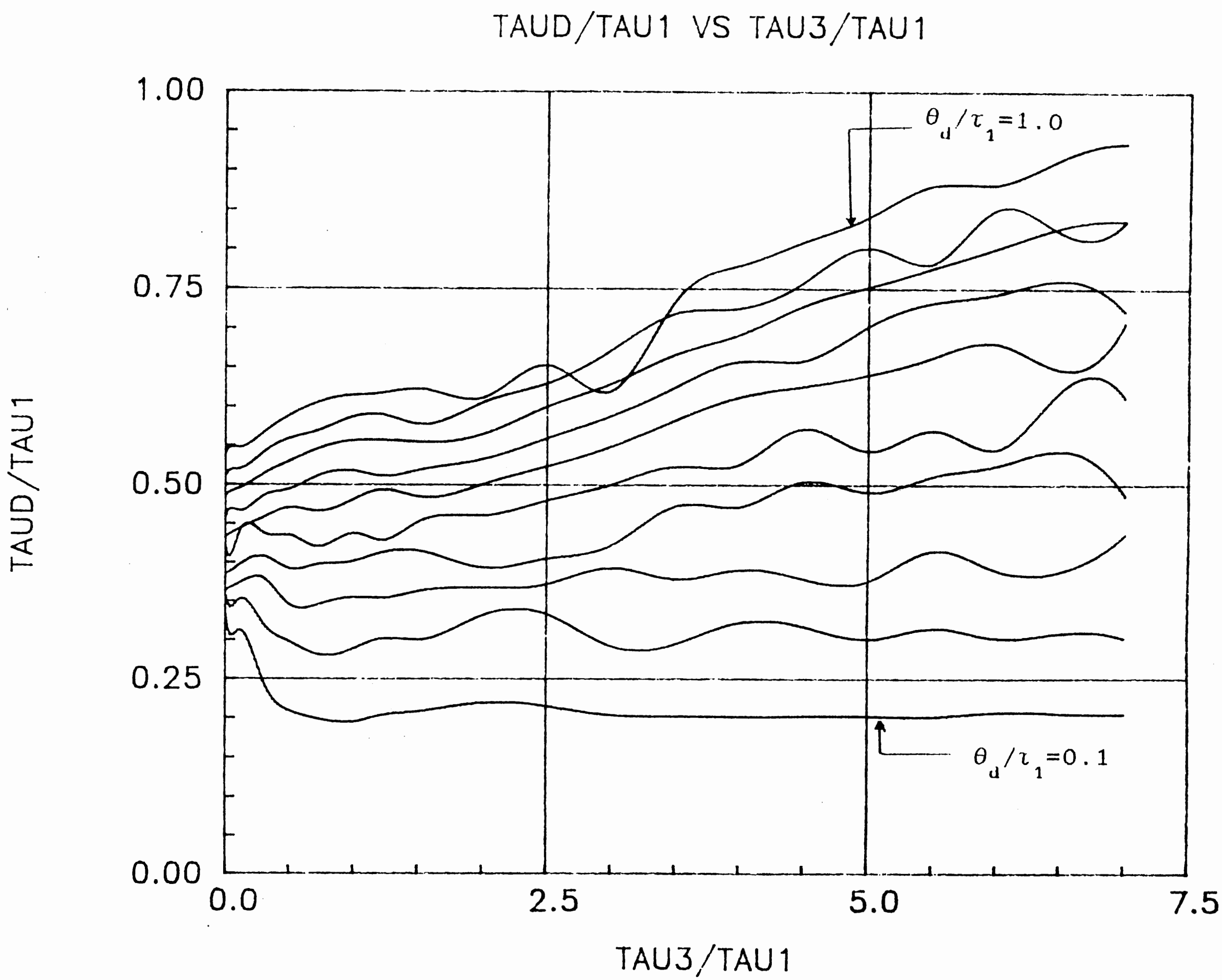

Figure 46. Optimum PID Derivative Time at τ_2/τ_1=0.4

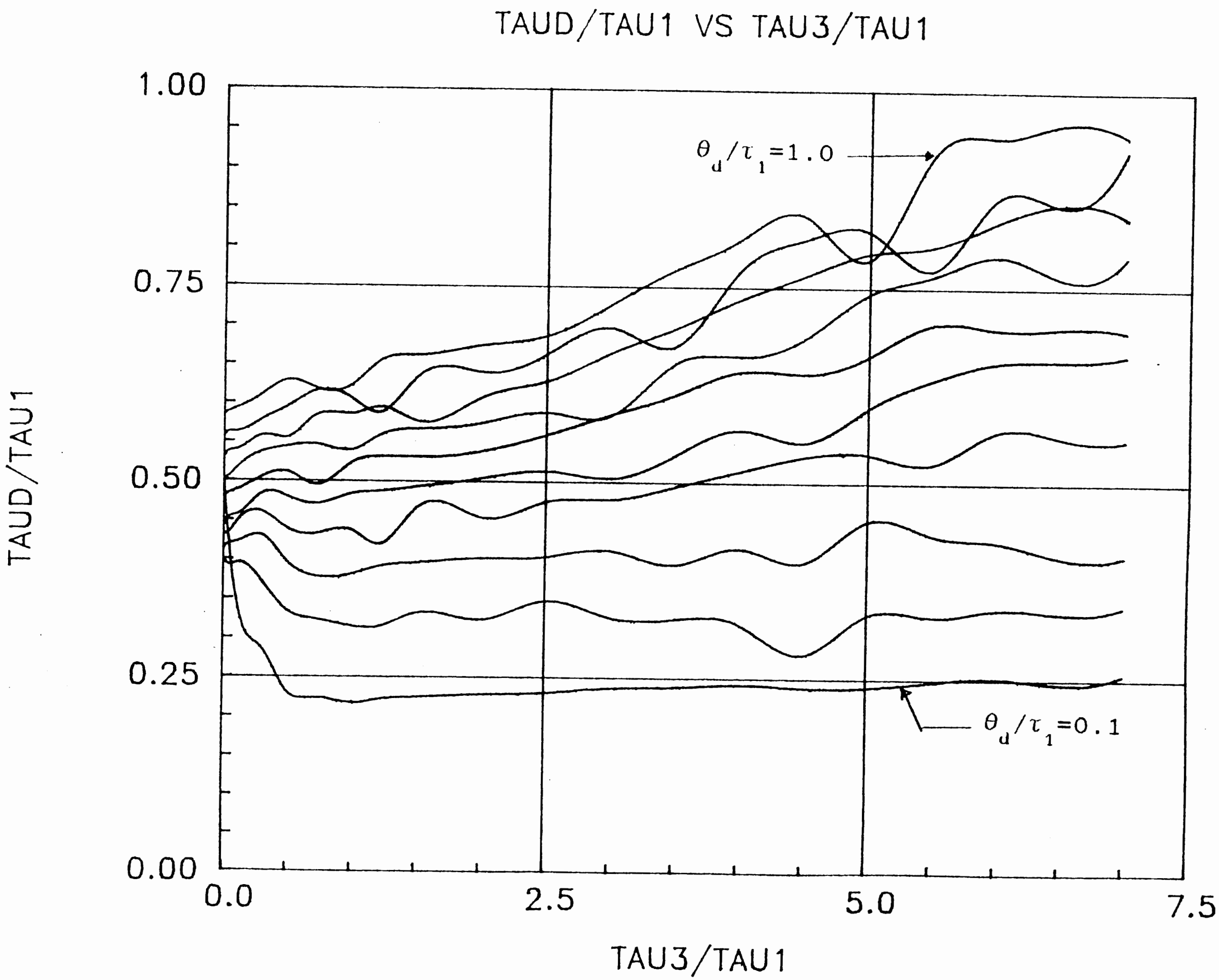

Figure 47. Optimum PID Derivative Time at τ_2/τ_1=0.5

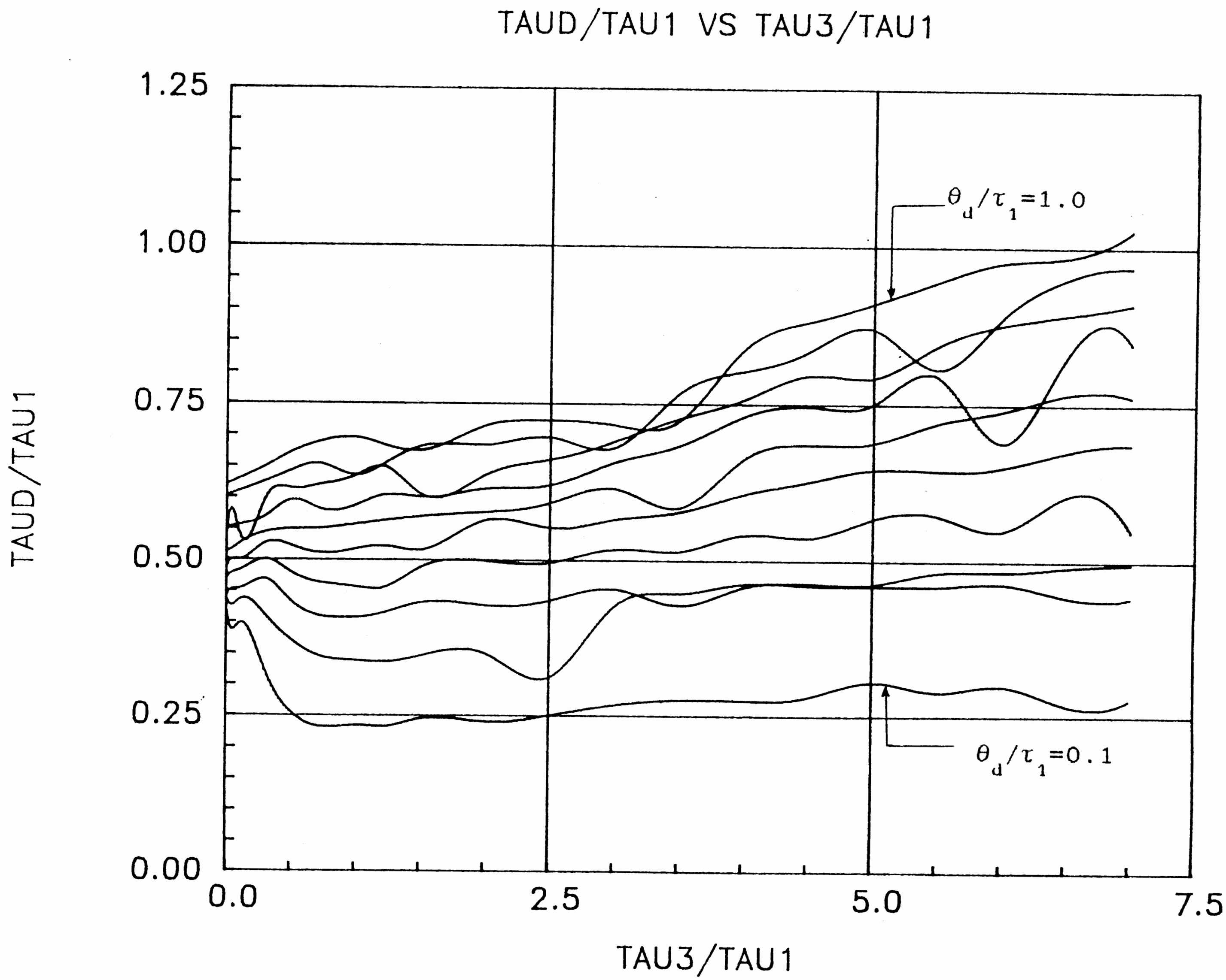

Figure 48. Optimum PID Derivative Time at τ_2/τ_1=0.6

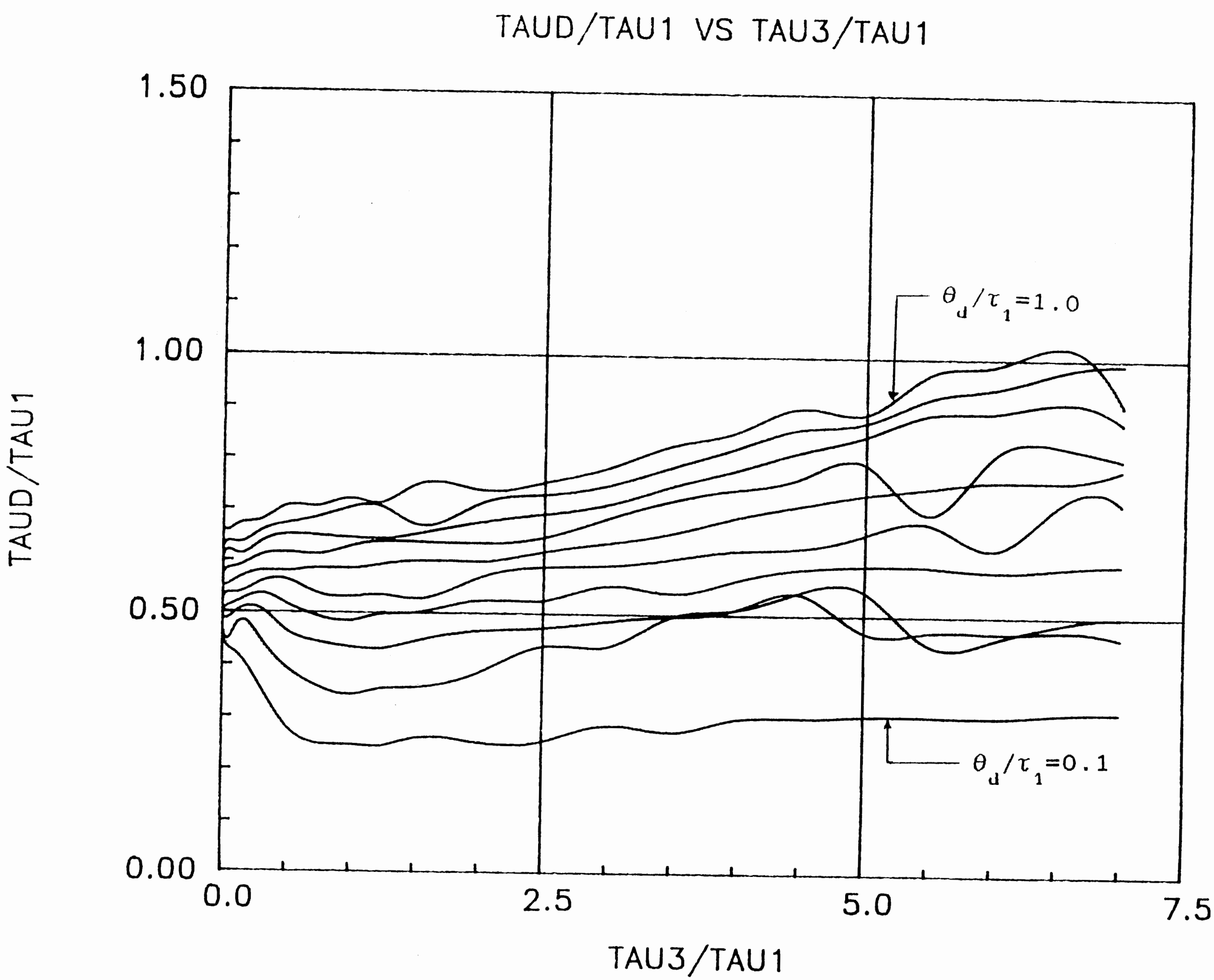

Figure 49. Optimum PID Derivative Time at τ_2/τ_1=0.7

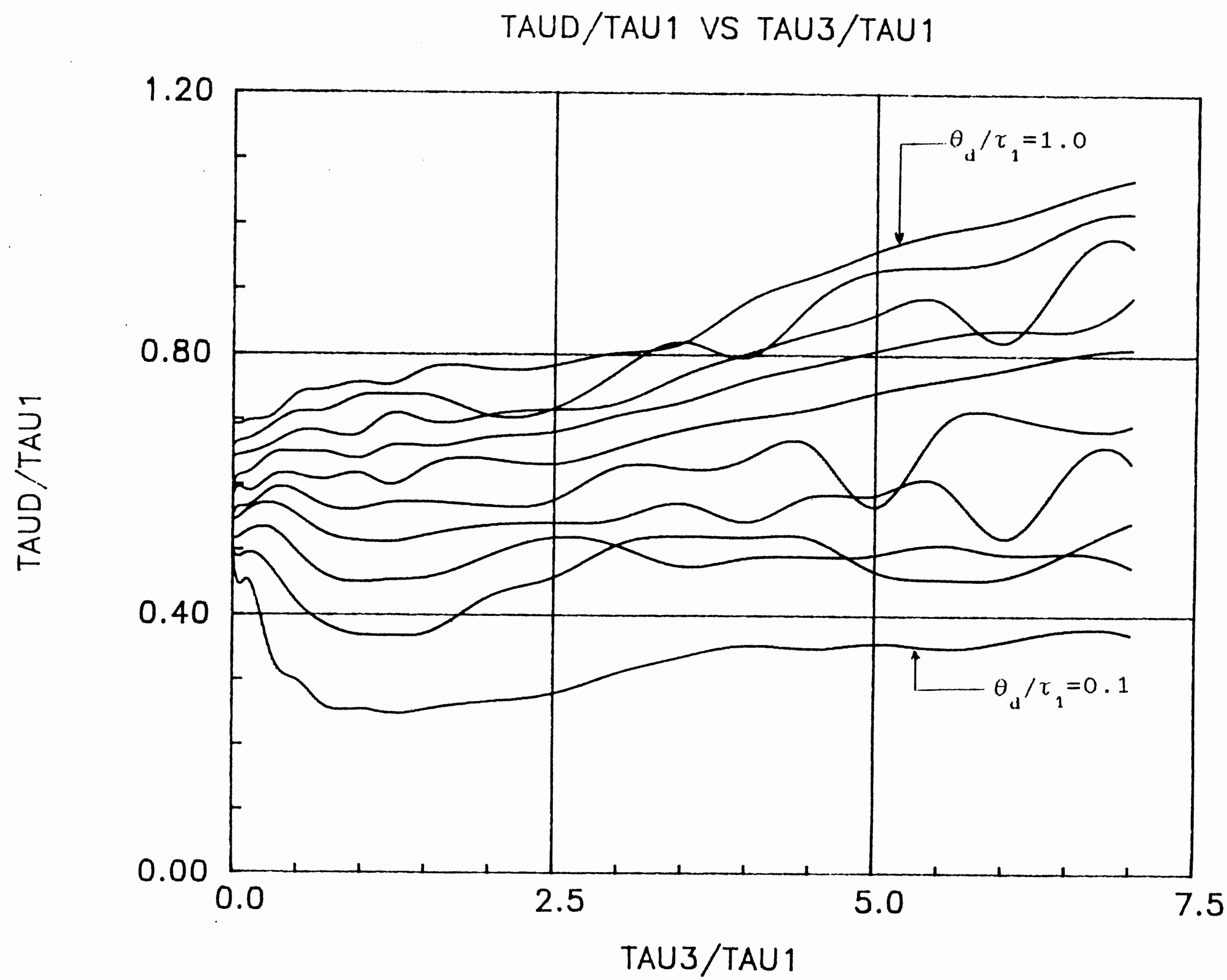

Figure 50. Optimum PID Derivative Time at $\tau_2/\tau_1=0.8$

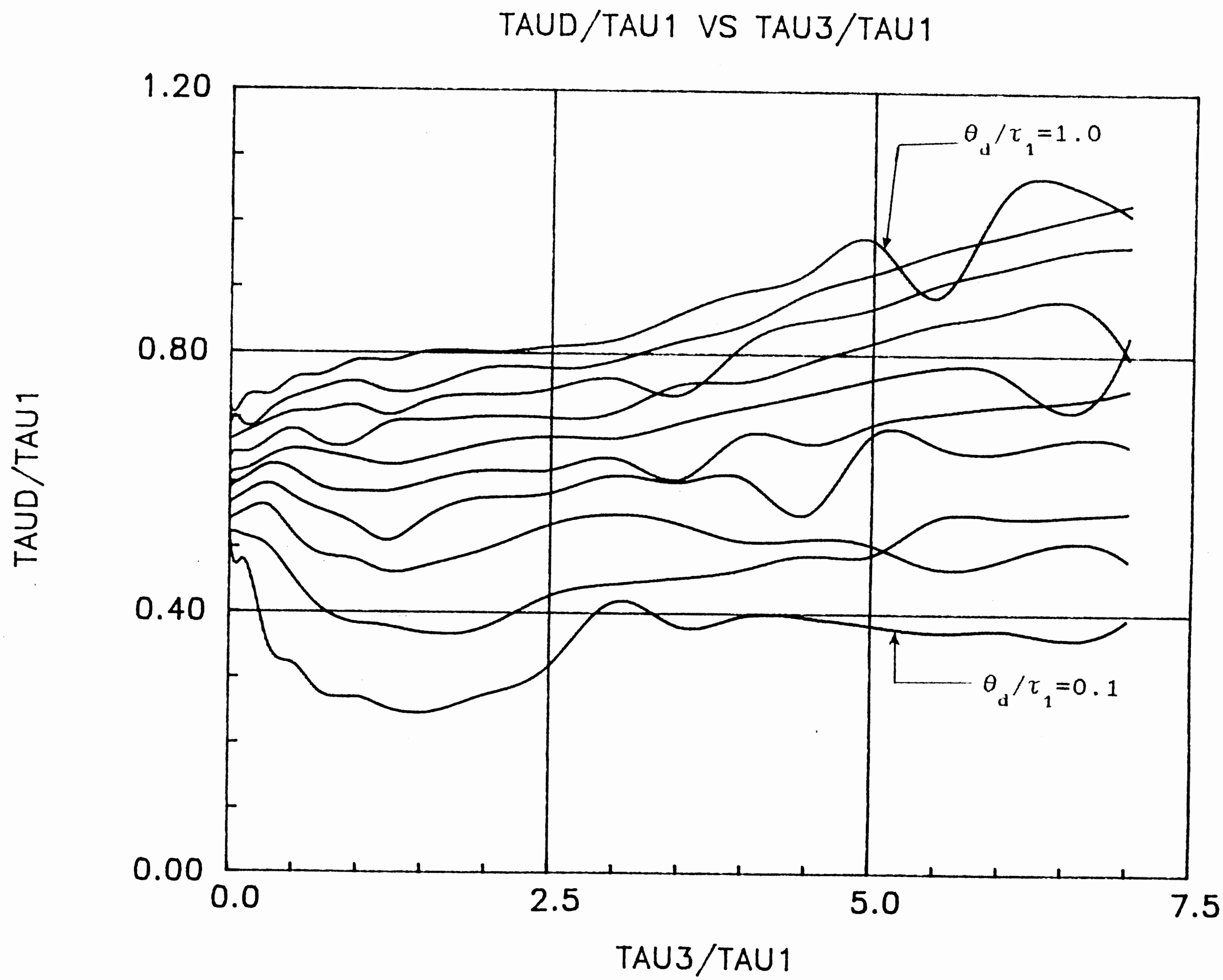

Figure 51. Optimum PID Derivative Time at τ_2/τ_1=0.9

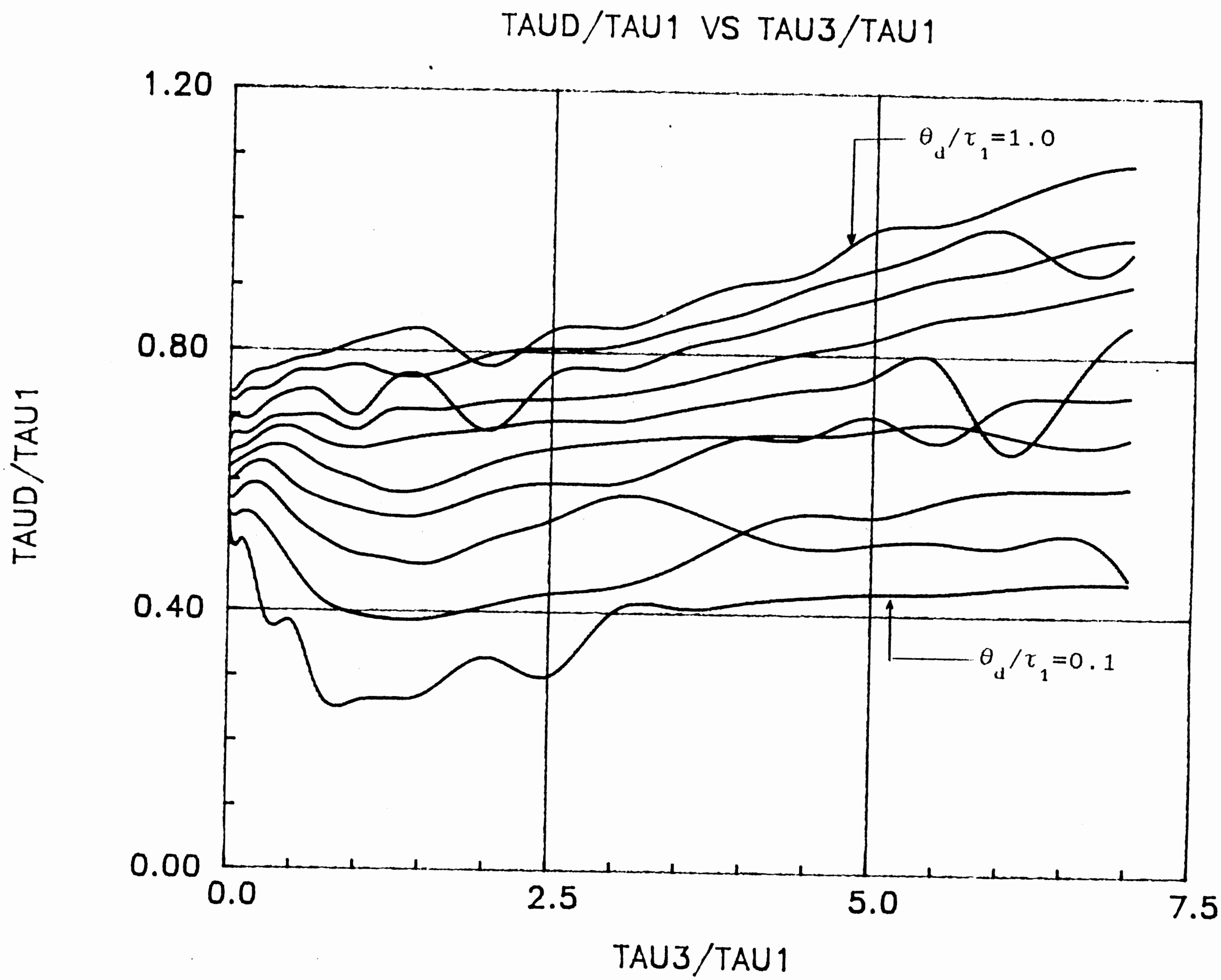

Figure 52. Optimum PID Derivative Time at $\tau_2/\tau_1=1.0$

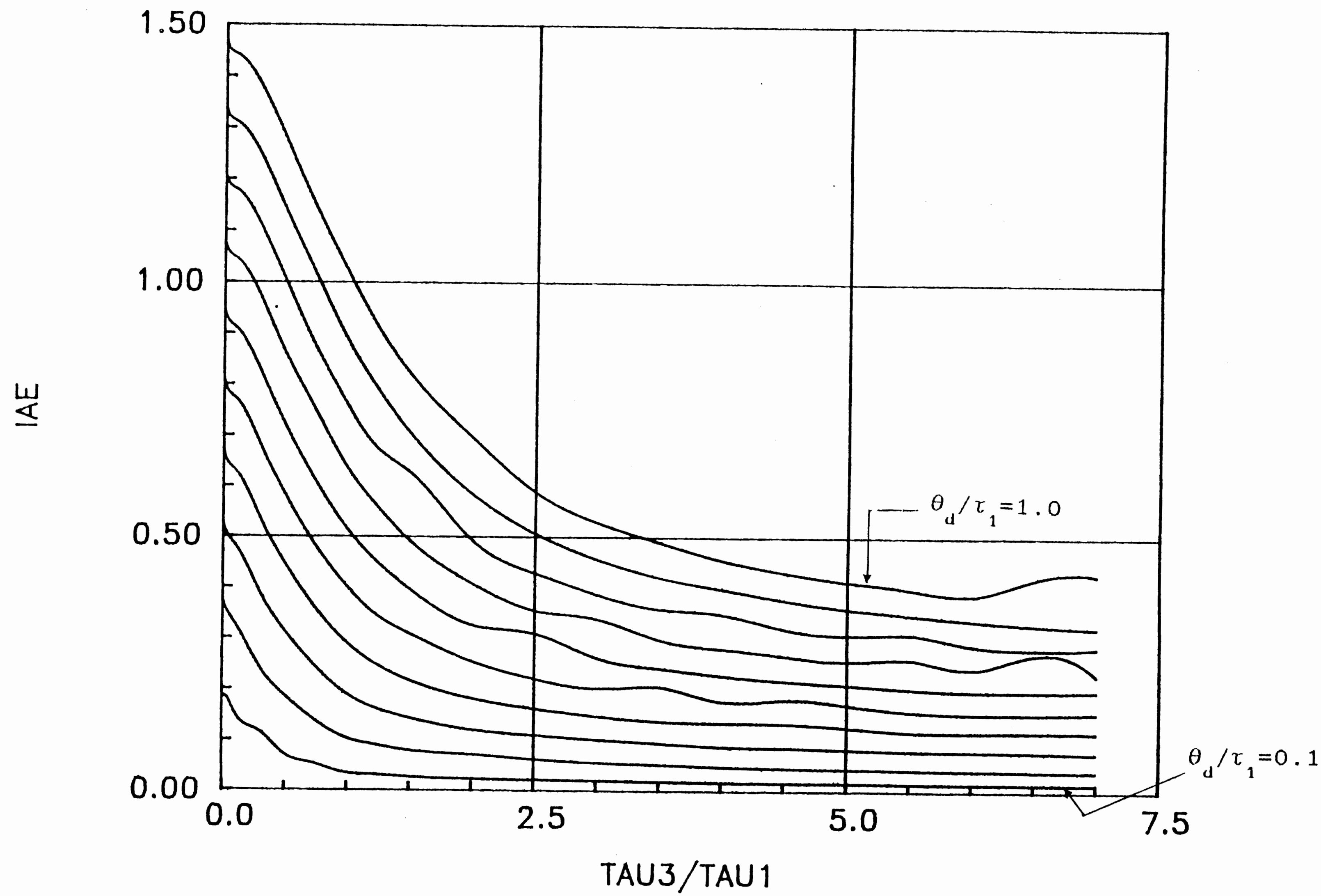

Figure 53. IAE Based on Optimum Tuning Constants at τ_2/τ_1=0.1

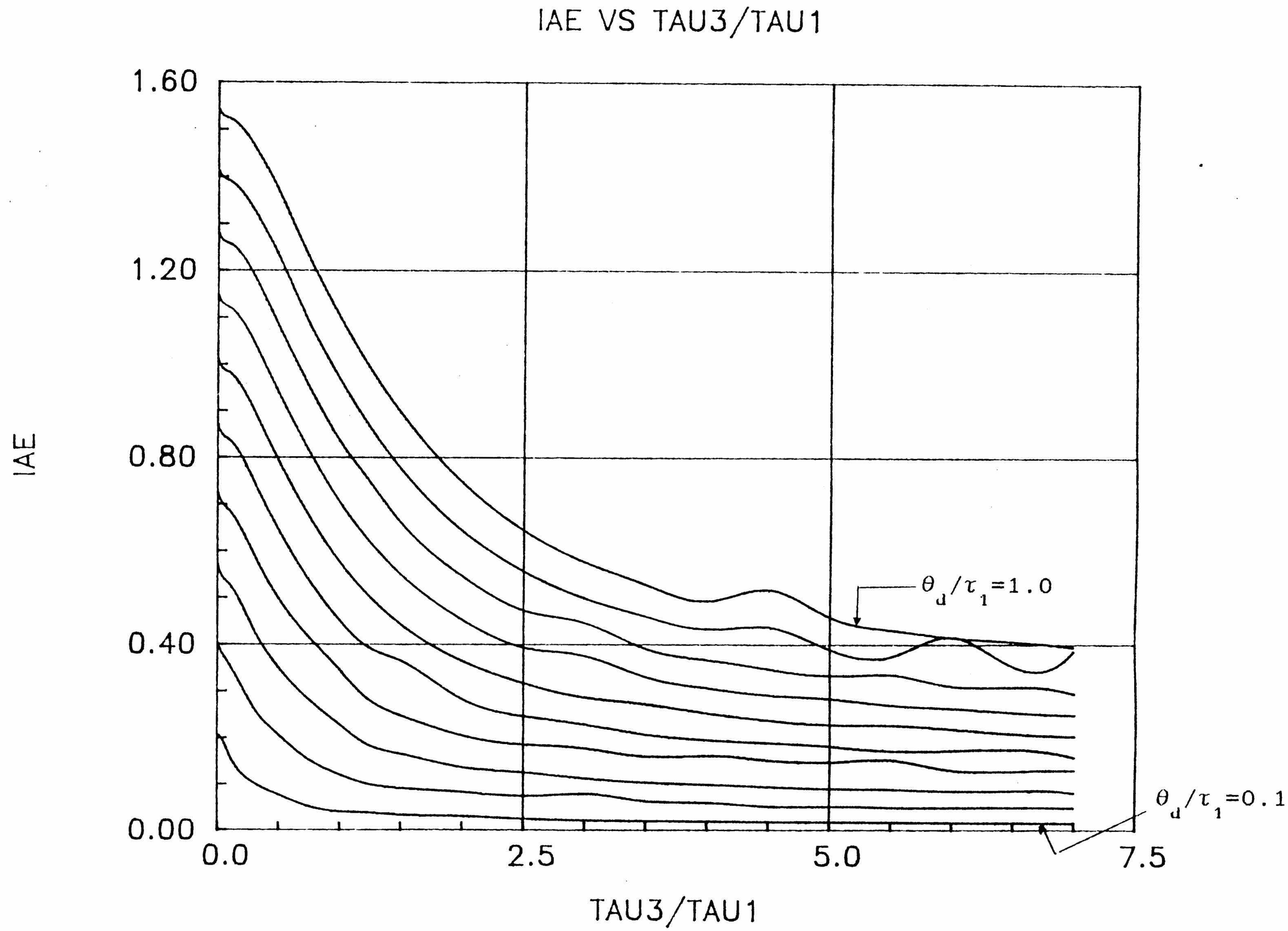

Figure 54. IAE Based on Optimum Tuning Constants at τ_2/τ_1=0.2

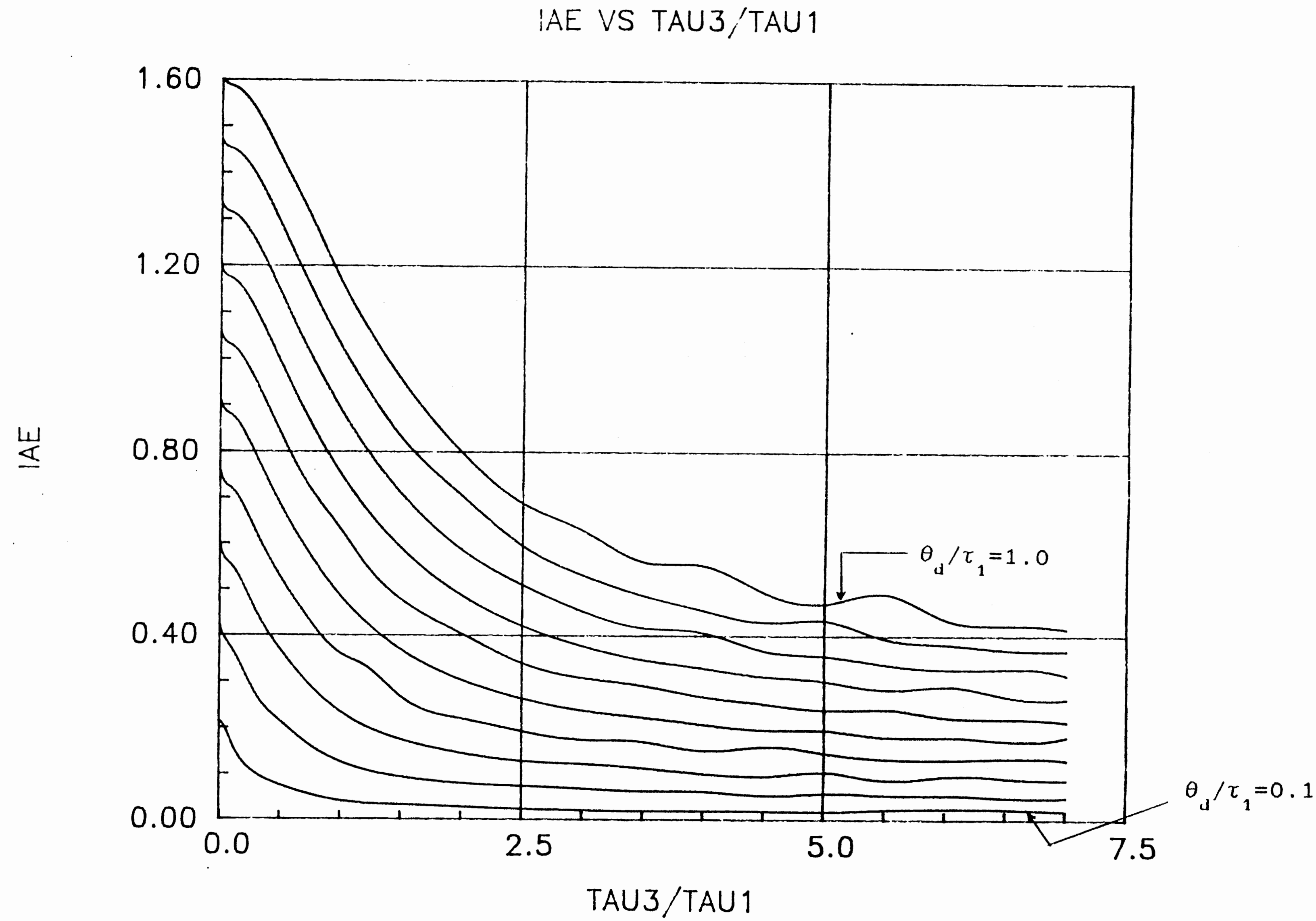

Figure 55. IAE Based on Optimum Tuning Constants at τ_2/τ_1=0.3

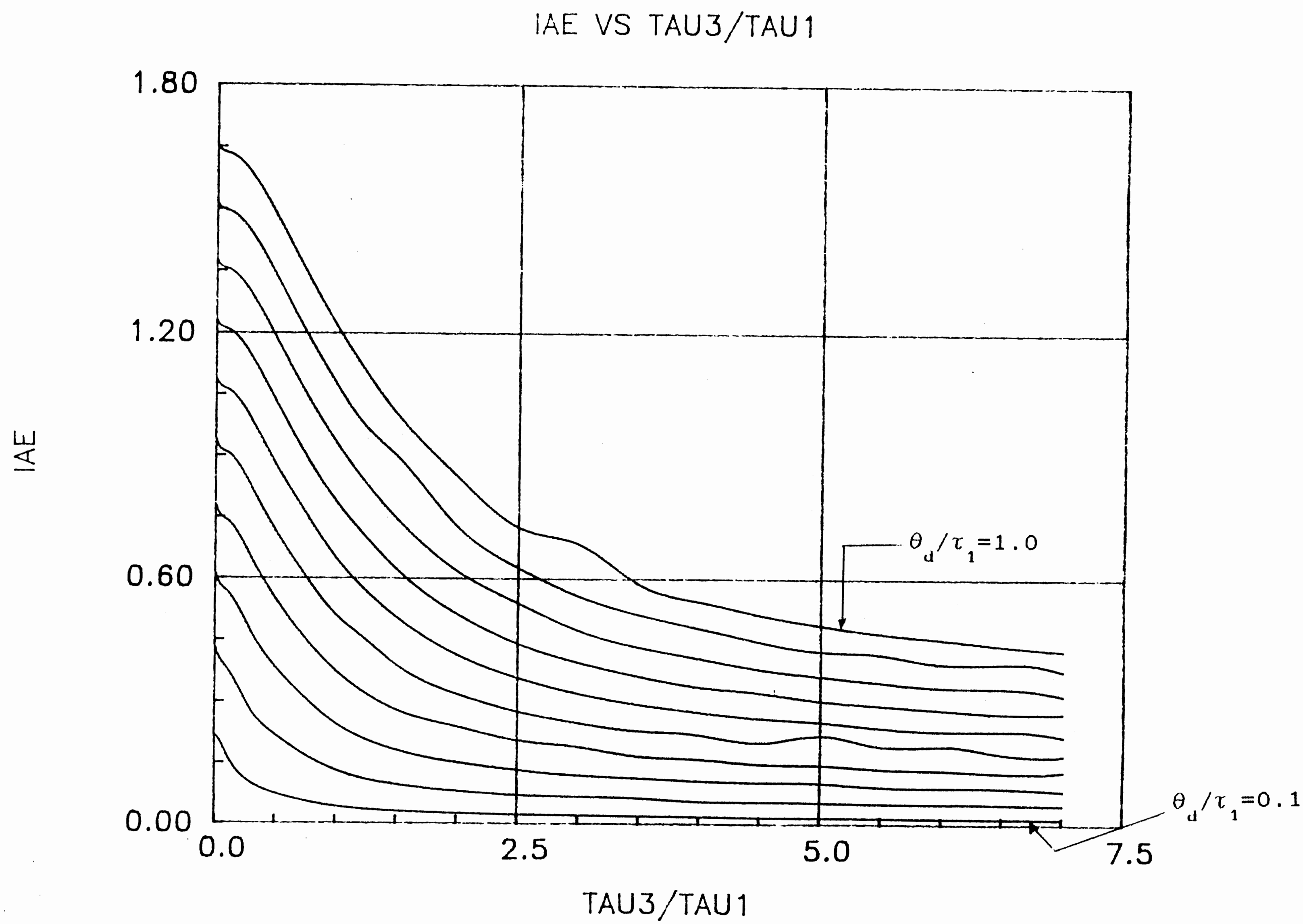

Figure 56. IAE Based on Optimum Tuning Constants at τ_2/τ_1=0.4

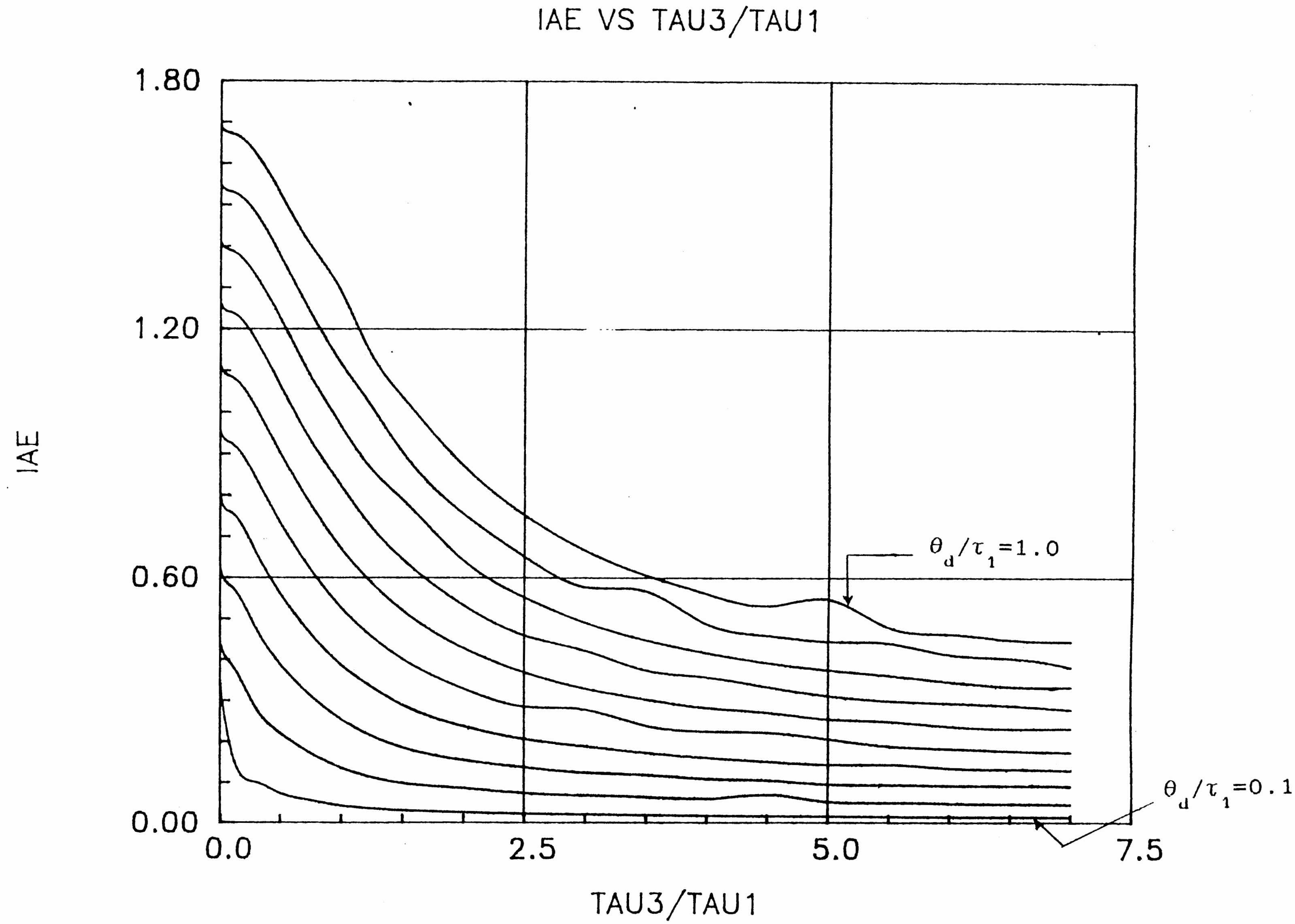

Figure 57. IAE Based on Optimum Tuning Constants at τ_2/τ_1=0.5

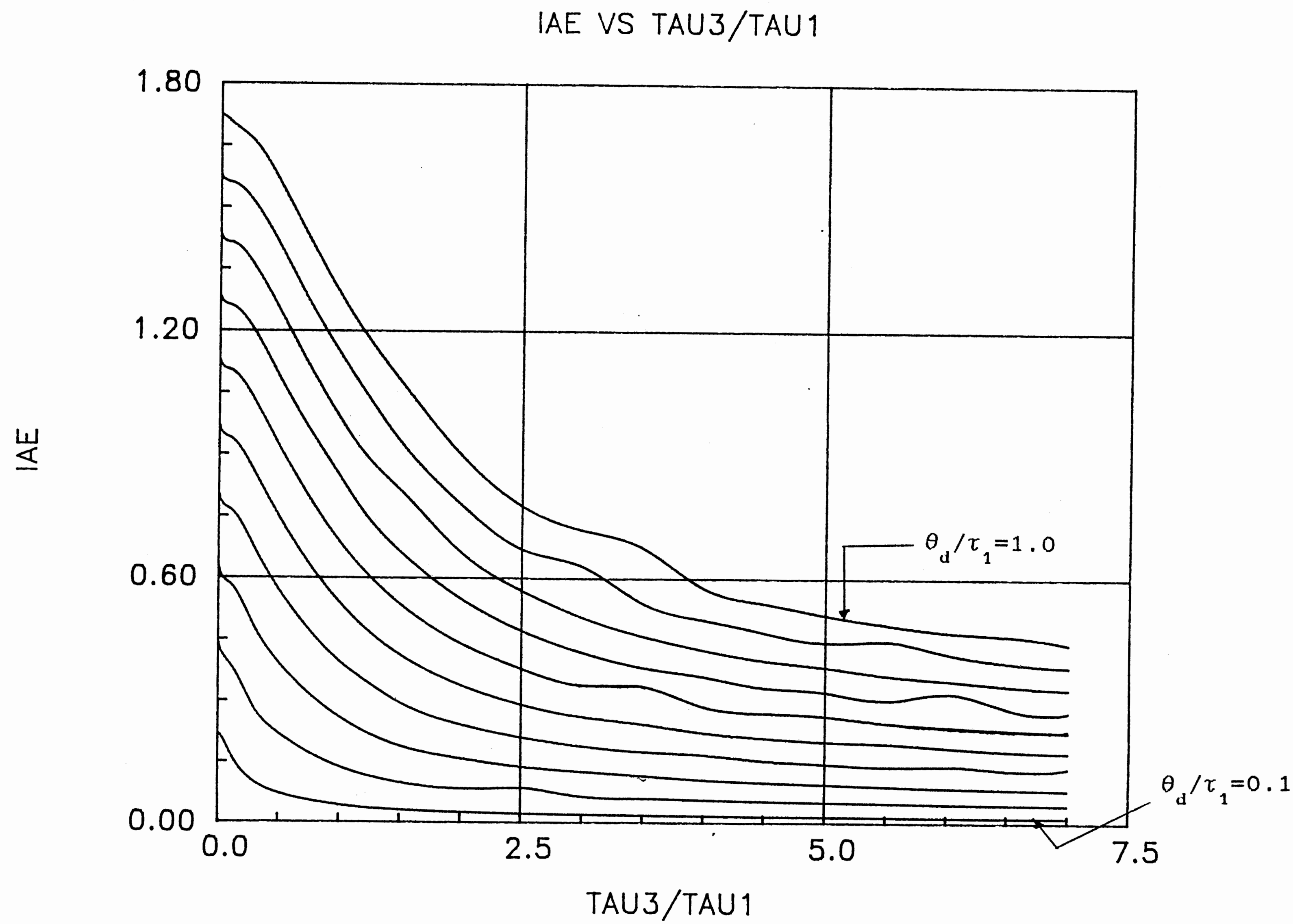

Figure 58. IAE Based on Optimum Tuning Constants at τ_2/τ_1=0.6

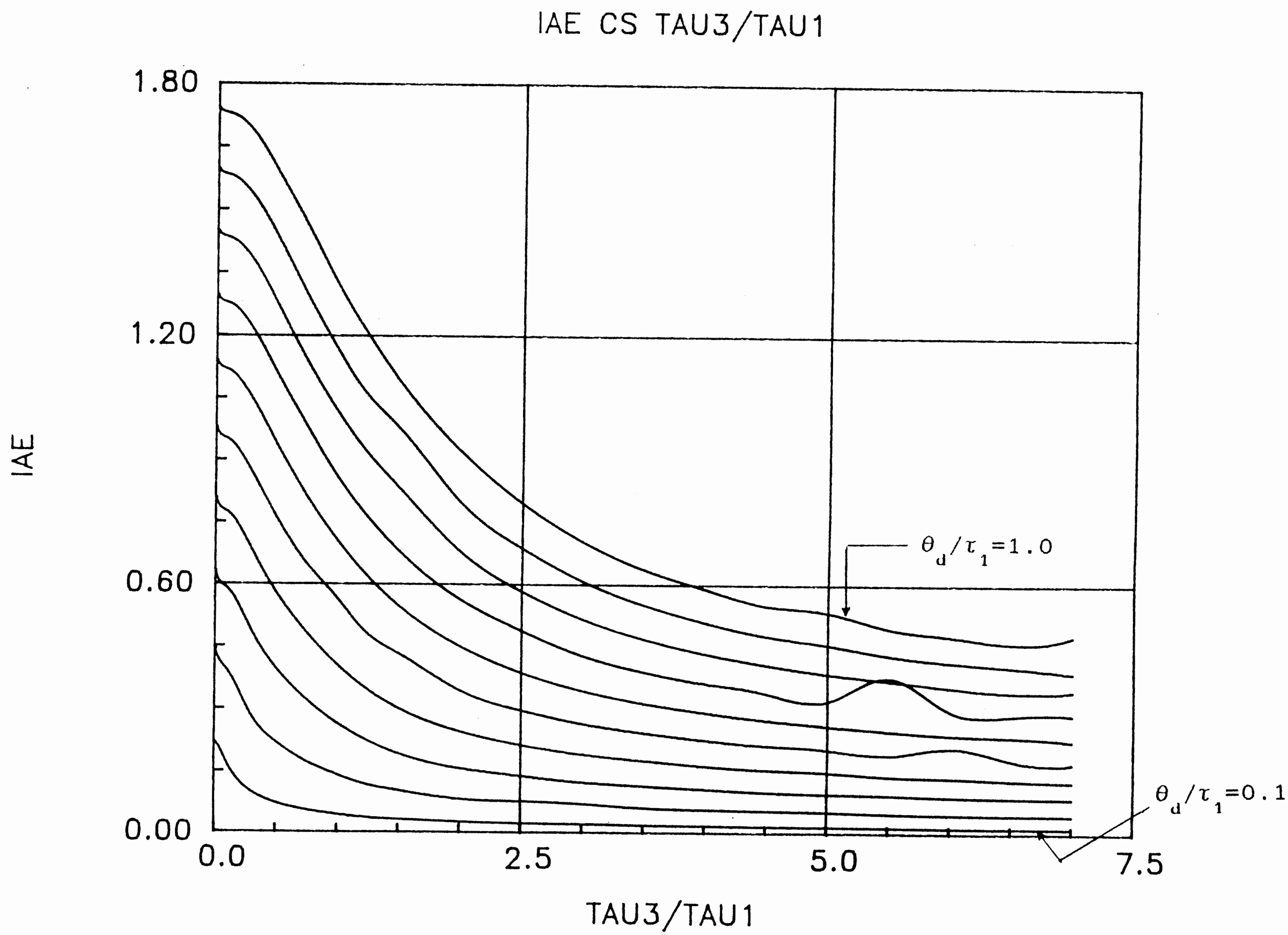

Figure 59. IAE Based on Optimum Tuning Constants at τ_2/τ_1=0.7

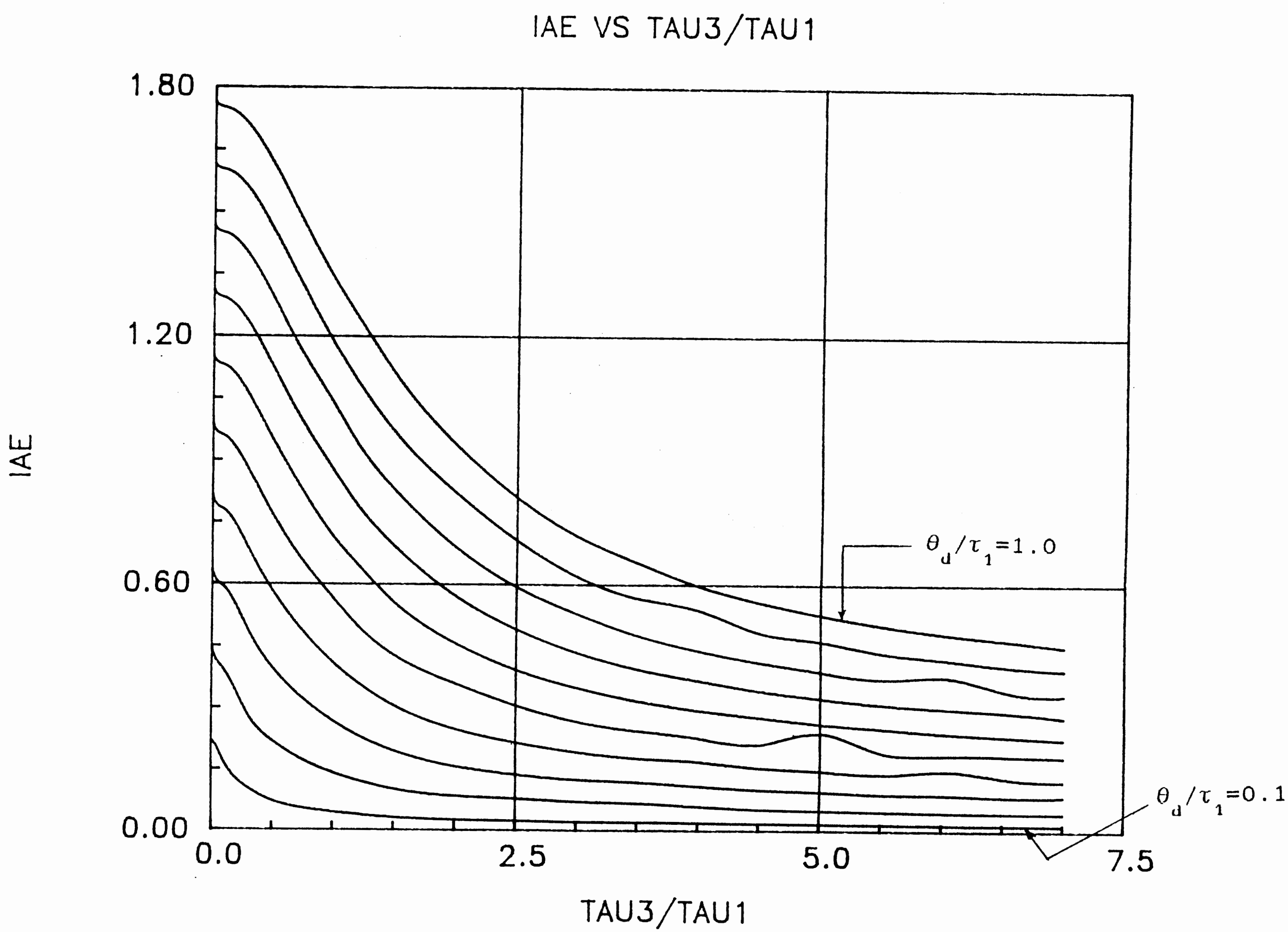

Figure 60. IAE Based on Optimum Tuning Constants at τ_2/τ_1=0.8

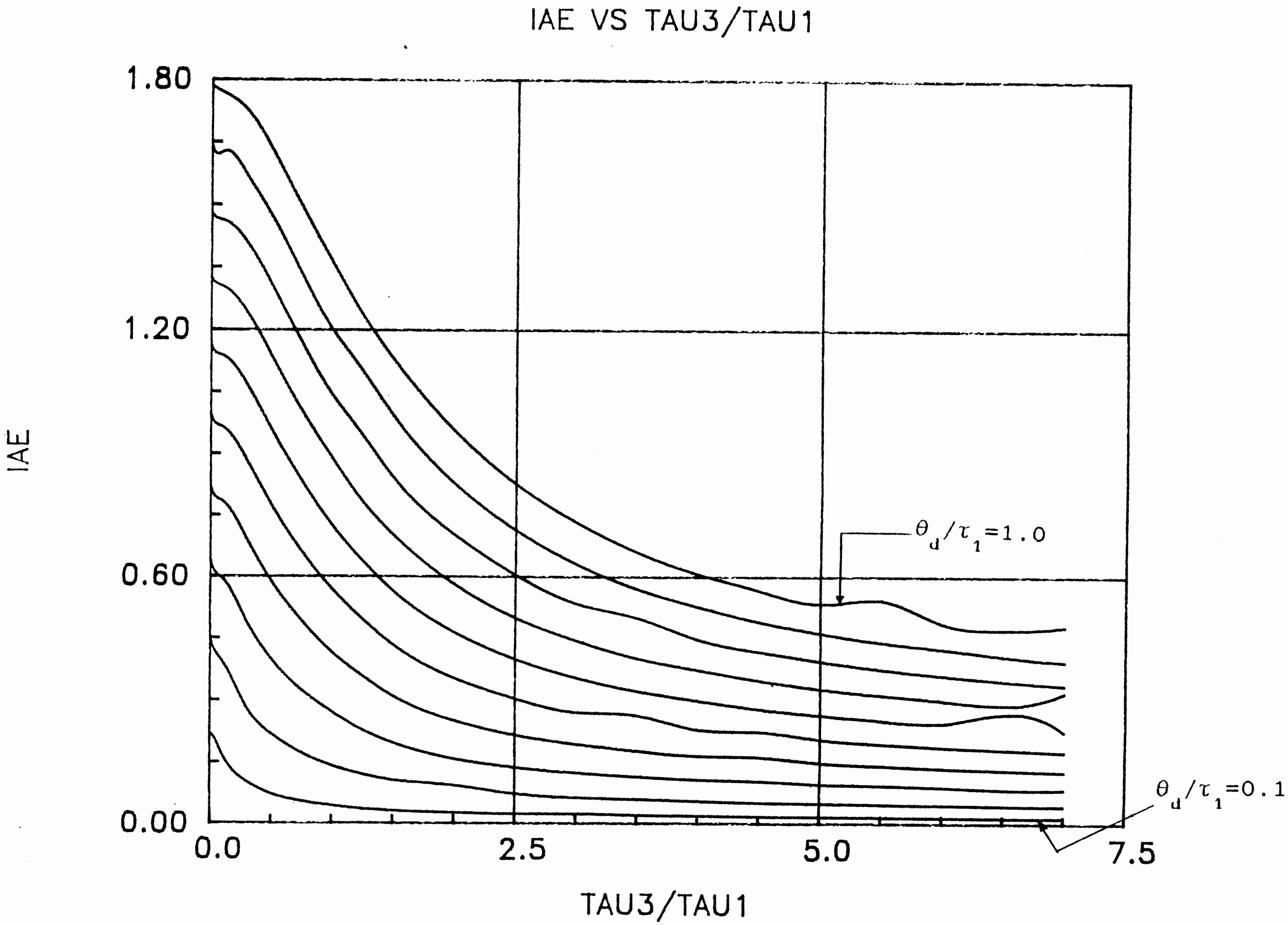

Figure 61. IAE Based on Optimum Tuning Constants at τ_2/τ_1=0.9

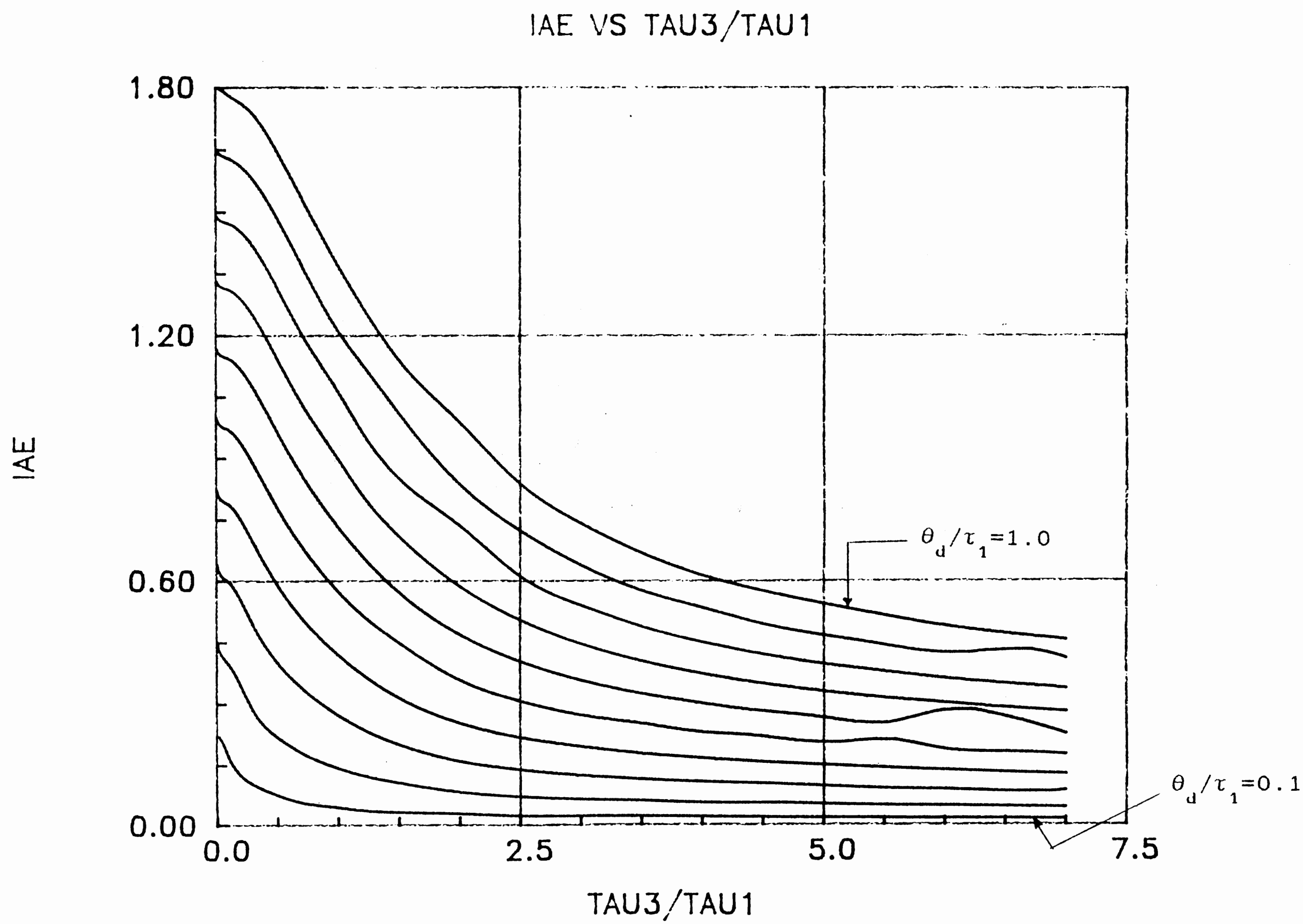

Figure 62. IAE Based on Optimum Tuning Constants at $\tau_2/\tau_1=1.0$

APPENDIX C

CURVE FITTING PROGRAM

```
program simp (din, dout);

 const m      =  5; (* number of parameters to fit *)
       nvpp = 2; (* total number of vars. per data point *)
       n      =  6; (* m+1 dimension *)
       mnp    = 200; (* max. number of data point *)
       alpha = 1.0; (* reflection coefficient *)
       beta   = 0.5; (* contraction coefficient *)
       gamma = 2.0; (* expansion coefficient *)
       lw     = 7; (* width of line in data fields + 1 *)
       root2  = 1.414214;
 type  vector  =  array(.1..n.) of real;
       datarow =  array(.1..nvpp.) of real;
       index   =  0..255;
 var   done     :  boolean;  (* convergence *)
       i,j,k    :  index;
       h,l      :  array(.1..n.) of index;
       np,                    (* number of data points *)
       maxiter,             (* max. number of iteration *)
       niter    :  integer;   (* number of iteration *)
       next,                  (* new vertex to be tested *)
       center,                (* center of hyperplane *)
       mean,error,
       maxerr,                (* max error accepted *)
       p,q,                   (* to compute first simplex *)
       step   :  vector;    (* input starting step *)
       simp   : array(.1..n.) of vector;  (* the simplex *)
       data   : array(.1..mnp.) of datarow;   (* data *)
       odat     :  array(.1..200,1..10.) of real;
       din,dout:  text;

function f (x : vector; d : datarow) : real;
 begin
   f:=x(.1.)-exp(-x(.2.)*d(.1.))*(x(.3.)*cos(x(.4.)*d(.1.))
      +x(.5.)*sin(x(.4.)*d(.1.)));
 end;

procedure sum_of_residuals (var x : vector);
  var  i : index;
   begin
    x(.n.) := 0.0;
     for i:=1 to 19 do
      begin
       x(.n.):=x(.n.)+sqr(f(x,data(.i.))-data(.i,2.))
      end
   end;

procedure enter;
  var i,j,np : integer;
   begin
   writeln (dout, 'to find kckl case of PID, thetad=1.0');
   writeln (dout, 'model is kckl=a-exp(-bx)*(c*cos(dx)+
            e*sin(dx))');
   read (din, maxiter);
```

```
    writeln (dout, 'max. number of iteration is :=' ,
              maxiter:5);
    writeln (dout, 'start coord.:');
    for i:=1 to m do
     begin
      read (din, simp(.1,i.));
         if (i mod lw) = 0 then writeln (dout);
      write (dout, simp(.1,i.))
     end;
    writeln (dout);
    write (dout, 'start steps: ');
    for i:=1 to m do
     begin
      read (din, step(.i.));
         if (i mod lw) = 0 then writeln (dout);
      write (dout, step(.i.))
     end;
    writeln (dout);
    write (dout, 'max errors: ');
     for i:=1 to n do
      begin
       read (din, maxerr(.i.));
          if (i mod lw) = 0 then writeln (dout);
       write (dout,maxerr(.i.))
      end;
     writeln (dout);
     writeln (dout, 'data: ');
     writeln (dout, 'x':14, 'kckl':14);
     for np:=1 to 19 do
      begin
       write (dout, ' #',np:3);
        for j:=1 to nvpp do
         begin
            read (din, data(.np,j.));
            write (dout, data(.np,j.))
         end;
        writeln (dout);
       end
      end;

procedure report ;
 var kckl,dkckl,
     sigma       :  real;
     i,j         :  integer;
  begin
   writeln (dout, 'program exited after',niter:5,
             'iterations');
   writeln (dout, ' the final simplex is');
   for j:=1 to n do
    begin
     for i:=1 to n do
      begin
         if (i mod lw) = 0 then writeln (dout);
         write (dout,simp(.j,i.):10);
```

```
      end;
     writeln (dout);
    end;
   writeln (dout, ' the mean is');
   for i:=1 to n do
    begin
       if (i mod lw) = 0 then writeln (dout);
       write (dout, mean(.i.))
    end;
   writeln (dout);
   writeln (dout,' the estimated fractional error is ');
   for i:=1 to n do
    begin
       if (i mod lw) = 0 then writeln (dout);
       write (dout, error(.i.))
    end;
   writeln (dout);
   writeln (dout,' #':4,'x':10,'kckl':15,'kckl"':15,
            'dkckl':15);
   sigma:=0.0;
   for i:=1 to 19 do
    begin
     kckl:=f(mean,data(.i.));
     dkckl:=data(.i,2.)-kckl;
     sigma:=sigma+sqr(dkckl);
     writeln(dout,i:4,'   ',data(.i,1.):13,'   ',
    data(.i,2.):13,'   ',kckl:13,'   ',dkckl:13);
    end;
   sigma:=sqrt(sigma/19);
   writeln (dout,' the standard deviation is',sigma);
   sigma:=sigma/sqrt(19-m);
   write(dout,' the estimated error of the');
   writeln (dout,' function is',sigma);
  end;

procedure first;
  var i,j : integer;
    begin
     writeln (dout,' starting simplex');
     for j:=1 to n do
      begin
       write (dout,' simp(',j:1,')');
        for i:=1 to n do
         begin
          if (i mod lw) = 0 then writeln (dout);
          write (dout,simp(.j,i.))
         end;
        writeln (dout)
       end;
      writeln (dout)
     end;
procedure new_vertex;
  var i : integer;
   begin
```

```
    for i:=1 to n do
     begin
      simp(.h(.n.),i.) := next(.i.);
     end;
    end;

procedure order;
  var i,j : index;
    begin
     for j:=1 to n do
      begin
       for i:=1 to n do
        begin
         if simp(.i,j.) < simp(.l(.j.),j.) then l(.j.) :=i;
         if simp(.i,j.) > simp(.h(.j.),j.) then h(.j.) :=i;
        end
      end
    end;

begin
  assign(din,'mmm.dat');
  assign(dout,'func.dat');
  reset (din);
  rewrite(dout);
  (* enter; *)
  for i:=1 to 190 do
    begin
     for j:=1 to nvpp do
      begin
       read(din,odat(.i,j.));
      end;
    end;
  for k:=1 to 10 do
  begin
  maxiter:=10000;
  simp(.1,1.):=0.1;simp(.1,2.):=0.2;simp(.1,3.):=0.1;
  simp(.1,4.):=0.2;simp(.1,5.):=0.1;
  step(.1.):=0.2;step(.2.):=0.1;step(.3.):=0.2;
  step(.4.):=0.1;step(.5.):=0.2;
  maxerr(.1.):=1E-5; maxerr(.2.):=1E-5;maxerr(.3.):=1E-5;
  maxerr(.4.):=1E-5;maxerr(.5.):=1E-5;maxerr(.6.):=1E-5;
  for np:=1 to 19 do
    begin
     for j:=1 to nvpp do
      begin
       data(.np,j.):=odat(.(np+(k-1)*19),j.);
      end;
     end;
  sum_of_residuals(simp(.1.));
  for i:=1 to m do
    begin
     p(.i.) := step(.i.)*(sqrt(n)+m-1)/(m*root2);
     q(.i.) := step(.i.)*(sqrt(n)-1)/(m*root2);
    end;
```

```
for i:=2 to n do
 begin
  for j:=1 to m do
   simp(.i,j.) := simp(.1,j.)+q(.j.);
   simp(.i,(i-1).) := simp(.1,(i-1).)+p(.(i-1).);
   sum_of_residuals(simp(.i.))
  end;
 for i:=1 to n do
  begin
   l(.i.) :=1; h(.i.) :=1
  end;
 order;
 (* first; *)
 niter :=0;
 repeat;
 done := true;
 niter := succ(niter);
 for i:=1 to n do
  center(.i.) := 0.0;
  for i:=1 to n do
   if i <> h(.n.) then
    for j:=1 to m do
     center(.j.):=center(.j.)+simp(.i,j.);
  for i:=1 to n do
   begin
    center(.i.) := center(.i.)/m;
    next(.i.):=(1.0+alpha)*center(.i.)
                -alpha*simp(.h(.n.),i.)
   end;
  sum_of_residuals(next);
  if next(.n.) <= simp(.l(.n.),n.) then
   begin
    new_vertex;
     for i:=1 to m do
       next(.i.) := gamma*simp(.h(.n.),i.)+(1.0-gamma)*
                    center(.i.);
       sum_of_residuals(next);
      if next(.n.) <= simp(.l(.n.),n.) then new_vertex
     end
     else
     begin
      if next(.n.) <= simp(.h(.n.),n.) then new_vertex
       else
        begin
         for i:=1 to m do
          next(.i.) := beta*simp(.h(.n.),i.)+(1.0-beta)
                       *center(.i.);
          sum_of_residuals(next);
        if next(.n.) <= simp(.h(.n.),n.) then new_vertex
        else
         begin
          for i:=1 to n do
           begin
            for j:=1 to m do
```

```
              simp(.i,j.) := (simp(.i,j.)+simp(.l(.n.),j.))
                             *beta;
             sum_of_residuals(simp(.i.))
            end
           end
          end
        end;
        order;
        for j:=1 to n do
         begin
          error(.j.):=(simp(.h(.j.),j.)-simp(.l(.j.),j.))/
                      simp(.h(.j.),j.);
              if done then
            if error(.j.) > maxerr(.j.) then
           done := false
          end
         until (done or (niter = maxiter));
         for i:=1 to n do
          begin
           mean(.i.) := 0.0;
             for j:=1 to n do
             mean(.i.):=mean(.i.)+simp(.j,i.);
            mean(.i.):=mean(.i.)/n
           end;
           report;
  end;
  close(din);
  close(dout);
  end.
```

APPENDIX D

TABLES OF CURVE FITTING RESULTS

TABLE I

CURVE FITTING RESULTS OF PROPORTIONAL GAIN AT $\tau_2/\tau_1 = 0.1$

$$K_c K_L = A - e^{-B\tau_3/\tau_1}[C\cos(D\,\tau_3/\tau_1) + E\sin(D\,\tau_3/\tau_1)]$$

θ_d/τ_1	A	B	C	D	E	Standard Deviation
0.1	10.541223	1.180019	4.081162	1.272492	-7.398963	0.449385
0.2	5.890894	3.425357	2.846238	7.24×10^{-6}	2.403632	0.290349
0.3	4.484194	1.267431	1.954393	9.16×10^{-7}	1.268995	0.119251
0.4	3.446715	0.985048	1.384971	2.95×10^{-7}	0.649919	0.129946
0.5	2.622772	1.773274	1.016053	-9.26×10^{-7}	2.696408	0.067870
0.6	2.172058	0.625983	0.718355	0.003308	-164.5421	0.045793
0.7	1.839872	0.496145	0.550092	0.005831	-84.99422	0.041626
0.8	1.873584	0.712189	0.696749	0.022343	-7.329004	0.040928
0.9	1.670054	0.729667	0.591636	0.427691	-0.352738	0.011888
1.0	1.609807	0.879666	0.606288	-4.44×10^{-7}	-2.840634	0.036769

TABLE II

CURVE FITTING RESULTS OF
INTEGRAL TIME
AT $\tau_2/\tau_1 = 0.1$

$$\tau_1/\tau_I = A + B(1 - e^{C\,\tau_3/\tau_1}) + D\,e^{E\,\tau_2/\tau_1} \sin(F\,\tau_3/\tau_1 + G)$$

θ_d/τ_1	A	B	C	D	E	F	G	Standard Deviation
0.1	-3.30267	9.01153	-0.41464	5.79647	-0.00625	-0.000027	2.43275	0.2424
0.2	5.78255	-0.09001	-12.2736	5.13833	-0.39375	-0.00058	-1.78762	0.18450
0.3	2.77685	0.58238	-6.76347	2.38840	-0.34719	0.46117	-1.79252	0.11018
0.4	3.67113	-0.39645	-5.85407	4.71432	-0.47918	-0.04031	-0.63706	0.06248
0.5	-1.94808	0.40691	0.00948	3.90218	0.03892	0.16394	0.75193	0.05624
0.6	1.58023	0.37603	-0.04395	2.92361	-0.08631	0.14973	-0.30795	0.06160
0.7	2.85940	-0.31889	-1.36371	2.22301	-0.21063	0.15687	-1.24698	0.02911
0.8	1.35737	1.33223	-0.13991	1.96826	-0.66789	0.28259	-2.82426	0.03606
0.9	2.13835	0.02338	-0.03980	2.51418	-0.41486	0.23493	-2.54317	0.01802
1.0	0.37534	0.00740	0.71613	0.56146	0.20946	0.20258	0.60896	0.02968

TABLE III

CURVE FITTING RESULTS OF DERIVATIVE TIME AT $\tau_2/\tau_1 = 0.1$

$$\tau_D/\tau_1 = A + B\, e^{C\, \tau_3/\tau_1}$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	-43360.4670	43360.5758	3.193977×10^{-7}	0.0153100
0.2	-44636.8244	44636.9557	2.434525×10^{-7}	0.0136737
0.3	-53461.0341	53461.2240	2.401277×10^{-7}	0.0121607
0.4	-83746.1773	83746.4041	2.044911×10^{-7}	0.0182938
0.5	-77234.5607	77234.8133	4.055537×10^{-7}	0.0210251
0.6	-32407.2158	32407.4991	1.176143×10^{-6}	0.0161665
0.7	-3811.73677	3812.04278	1.216128×10^{-5}	0.0205976
0.8	-55195.8537	55196.1993	7.840349×10^{-7}	0.0190948
0.9	-31467.2744	32467.6501	1.582992×10^{-6}	0.0185181
1.0	-134661.5610	134661.9795	3.354662×10^{-7}	0.0374493

TABLE V

CURVE FITTING RESULTS OF
PROPORTIONAL GAIN
AT $\tau_2/\tau_1 = 0.2$

$$K_C K_L = A - e^{-B\,\tau_3/\tau_1}\left[C\cos(D\,\tau_3/\tau_1) + E\sin(D\,\tau_3/\tau_1)\right]$$

θ_d/τ_1	A	B	C	D	E	Standrad Deviation
0.1	12.399046	1.627010	5.681818	3.7080941194	-1.9898339	0.4224554545
0.2	7.803883	0.797572	3.929099	0.0000000032	-5.5804481	0.3650978605
0.3	4.676351	1.007973	2.111299	-0.0000000052	0.1850836	0.1419999611
0.4	3.608588	0.907358	1.606882	0.0000000072	1.0609761	0.0882600788
0.5	2.784449	0.922551	1.098633	0.0000000057	1.7570548	0.0857822829
0.6	2.233905	0.715294	0.812053	0.0072544976	-59.2209331	0.0343512919
0.7	1.999058	0.610323	0.718012	0.0054062788	-56.2213620	0.0248629267
0.8	1.831384	0.730353	0.668455	-0.3595292639	0.3607367	0.0332465735
0.9	1.754170	0.748779	0.674030	0.0000000085	-1.7062681	0.0558489975
1.0	1.4136881	0.367970	0.4056009	0.0041167373	-67.2205867	0.0255370225

TABLE IV

CURVE FITTING RESULT OF IAE
AT $\tau_2/\tau_1 = 0.1$

$$IAE = A + B/(C + \tau_3/\tau_1)$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	0.004136	0.049875	0.228442	0.005886
0.2	0.015031	0.138218	0.352544	0.007893
0.3	0.029530	0.260434	0.483797	0.013004
0.4	0.043210	0.418941	0.619501	0.017866
0.5	0.051111	0.638021	0.785571	0.020877
0.6	0.051936	0.912282	0.956557	0.024283
0.7	0.058688	1.207813	1.109474	0.031217
0.8	0.045950	1.638504	1.325902	0.032701
0.9	0.034825	2.079027	1.501410	0.036342
1.0	0.058584	2.415581	1.610953	0.045892

TABLE VI

CURVE FITTING RESULTS OF
INTEGRAL TIME
AT $\tau_2/\tau_1 = 0.2$

$$\tau_1/\tau_I = A + B(1 - e^{C\,\tau_3/\tau_1}) + D\,e^{E\,\tau_3/\tau_1}\sin(F\,\tau_3/\tau_1 + G)$$

θ_d/τ_1	A	B	C	D	E	F	G	Standard Deviation
0.1	1.46941	-0.92782	0.30091	0.50210	0.46893	0.26288	1.34227	0.77867
0.2	-3.53221	0.99416	-1.19698	4.43834	0.08304	0.11933	1.19243	0.17850
0.3	4.20245	-0.35109	-9.20077	10.11599	-0.32324	-2.0×10^{-7}	-2.8052	0.08696
0.4	4.32900	-0.72499	-0.19990	4.78175	-0.19990	-0.00000	-0.8526	0.16347
0.5	2.42577	-0.00088	0.83076	5.58883	-0.82066	0.23450	-2.8328	0.07965
0.6	1.35715	0.72435	-0.04618	1.51784	-0.04764	0.22003	-0.44845	0.04102
0.7	0.57396	0.20299	-0.00311	1.56583	0.01040	0.19324	0.07189	0.03552
0.8	0.26774	0.54056	0.01712	1.26326	0.05622	0.19165	0.33249	0.05483
0.9	4.32036	2.41582	0.14200	5.92115	0.01260	0.12715	-0.66533	0.09485
1.0	-1.35707	0.17442	-0.02570	3.11022	0.01128	0.07160	0.70241	0.04796

TABLE VII

CURVE FITTING RESULTS OF
DERIVATIVE TIME
AT $\tau_2/\tau_1 = 0.2$

$$\tau_D/\tau_1 = A + B\, e^{C\,\tau_3/\tau_1}$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	-12340.39508	12340.56067	0.000001165	0.019849436
0.2	-8253.00838	8253.22817	0.000000321	0.010916347
0.3	-24521.82030	24522.06750	0.000000598	0.013354271
0.4	-9326.23961	9326.52523	0.000001714	0.012946171
0.5	-65910.78431	65911.10552	0.000000416	0.023743855
0.6	-18164.86311	18165.19897	0.000002201	0.011322792
0.7	-32116.07804	32116.44446	0.000001345	0.012369514
0.8	-14644.24234	14644.63868	0.000003178	0.019443755
0.9	-104185.86456	104186.29961	0.000000404	0.040395520
1.0	-0.68734	1.14425	0.041199001	0.021419328

TABLE VIII

CURVE FITTING RESULTS OF IAE
AT $\tau_2/\tau_1 = 0.2$

$$IAE = A + B/(C - \tau_3/\tau_1)$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	0.824358	-25.971456	39.306470	0.0197789
0.2	0.288611	-1.346790	19.356491	0.0108858
0.3	0.558740	-4.686823	14.802296	0.0127967
0.4	0.842183	-15.854805	28.336644	0.0128158
0.5	0.661527	-2.219593	6.150730	0.0202570
0.6	2.612603	-114.962580	50.399479	0.0108605
0.7	2.045952	-54.620496	32.384603	0.0112515
0.8	4.379741	-315.149436	79.051963	0.0192967
0.9	1.014596	-4.526766	7.465166	0.0365062
1.0	40.280229	-29052.053	729.40595	0.0219471

TABLE IX

CURVE FITTING RESULTS OF PROPORTIONAL GAIN AT $\tau_2/\tau_1=0.3$

$$K_C K_L = A - e^{-B\,\tau_3/\tau_1}\left[C \cos(D\,\tau_3/\tau_1) + E \sin(D\,\tau_3/\tau_1)\right]$$

θ_d/τ_1	A	B	C	D	E	Standard Deviation
0.1	31.05310	0.19782	19.56376	0.000000004	50.84995	1.88491
0.2	8.89738	0.75203	4.81710	0.000000002	10.05059	0.38503
0.3	5.22663	0.80957	2.50109	-0.000000019	0.91970	0.26836
0.4	3.72221	0.94057	1.66788	0.000000017	1.50663	0.11305
0.5	2.94594	0.85623	1.22571	0.000000001	1.96095	0.08496
0.6	2.36947	1.15698	0.93260	0.000000008	1.76138	0.05174
0.7	2.13794	0.89757	0.84751	0.000000005	-0.54728	0.04122
0.8	1.89036	1.26391	0.70382	0.005179852	61.74040	0.02345
0.9	1.65255	0.44314	0.55708	0.002812078	-81.86672	0.02972
1.0	1.38061	0.31864	0.35699	0.009400342	-30.63374	0.02745

TABLE X

CURVE FITTING RESULTS OF
INTEGRAL TIME AT
$\tau_2/\tau_1=0.3$

$$\tau_1/\tau_I = A + B(1-e^{-C\,\tau_3/\tau_1}) + D\,e^{E\,\tau_3/\tau_1}\sin(F\,\tau_3/\tau_1 + G)$$

θ_d/τ_1	A	B	C	D	E	F	G	Stanard Deviation
0.1	-5.1642	-7.8589	-59.7046	17.5459	0.01126	0.1232	0.8587	0.1876
0.2	10.6290	3.9623	-0.2532	28.8933	-3.14079	0.1020	-3.1486	0.1098
0.3	3.2314	0.1559	-0.0595	2.6706	-0.26734	0.0029	-1.5791	0.1201
0.4	-6.2692	2.8106	0.2276	7.1376	0.15085	-0.00079	1.3085	0.0670
0.5	-7.0245	3.0109	-0.1622	8.0350	-0.00574	0.0000039	1.9000	0.0898
0.6	1.1447	2.1274	-0.0896	0.9688	-0.61380	0.45525	-2.6079	0.0424
0.7	-1.1754	-0.0923	-1.5271	1.8399	0.15697	0.12023	1.6856	0.0286
0.8	1.9433	0.1943	-0.0972	2.3422	-0.35892	0.19242	-2.5540	0.0112
0.9	1.0655	1.1216	-0.0728	0.5016	-0.27034	0.54268	-2.1566	0.0261
1.0	0.6310	-0.3735	0.2301	0.0093	0.58052	0.68567	-0.0951	0.0304

TABLE XI

CURVE FITTING RESULTS OF
DERIVATIVE TIME
AT $\tau_2/\tau_1=0.3$

$$\tau_D/\tau_1 = A + B e^{C \tau_3/\tau_1}$$

θ_d/τ_1	A	B	C	
0.1	0.177104453	0.109699478	-4.297265244	0.020561542
0.2	0.269897596	0.000000172	1.631470727	0.013501095
0.3	-348123.0021	348123.3156	0.000000025	0.021621470
0.4	-3858.253878	3858.591291	0.000005015	0.017036656
0.5	-567873.5989	567873.9715	0.000000048	0.019590287
0.6	-259458.9424	259459.3331	0.000000150	0.015666708
0.7	-129648.4667	129648.8903	0.000000303	0.015564386
0.8	-186693.1187	186693.5675	0.000000244	0.019806701
0.9	-9423.673701	9424.148398	0.000005448	0.019937965
1.0	-0.007643597	0.515286580	0.080544207	0.024677633

TABLE XII

CURVE FITTING RESULT OF IAE
AT $\tau_2/\tau_1=0.3$

$$IAE = A + B/(C + \tau_3/\tau_1)$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	0.011098297	0.039852765	0.159983862	0.004071687
0.2	0.022284462	0.155769205	0.359096495	0.008687723
0.3	0.031262811	0.333140279	0.549406093	0.014580333
0.4	0.036351793	0.582699668	0.756589117	0.019120960
0.5	0.039090780	0.863105465	0.924645422	0.024678190
0.6	0.030801374	1.267542201	1.158723119	0.027224668
0.7	0.032954641	1.642926223	1.320254546	0.033354320
0.8	0.021547219	2.154040924	1.537903348	0.036632620
0.9	0.002971299	2.761776087	1.769306412	0.040343378
1.0	-0.010395189	3.416304568	1.992986833	0.044437867

TABLE XIII

CURVE FITTING RESULTS OF PROPORTIONAL GAIN AT $\tau_2/\tau_1=0.4$

$$K_C K_L = A - e^{-B\,\tau_3/\tau_1}\,[C\cos(D\,\tau_3/\tau_1) + E\sin(D\,\tau_3/\tau_1)]$$

θ_d/τ_1	A	B	C	D	E	Standrad Deviation
0.1	24.07574	1.19830	14.85883	0.000000001	45.63781	1.09814
0.2	9.81443	0.68014	5.36092	-0.000000000	57.39144	0.43386
0.3	5.66194	0.83228	2.81486	-0.000000007	0.06172	0.19475
0.4	3.92689	1.02022	1.82713	0.000000008	1.09482	0.10374
0.5	3.10710	0.85978	1.34758	-0.000001985	0.74023	0.06016
0.6	2.50924	0.99475	1.01303	0.000000003	2.34726	0.03568
0.7	2.19370	1.26302	0.86206	0.003496145	96.44209	0.02763
0.8	1.77722	0.40619	0.55816	0.018986802	-18.09822	0.02183
0.9	1.80703	0.73340	0.68875	-0.000000000	1.74749	0.02049
0.9	1.61055	0.90646	0.54870	0.715797497	0.31955	0.01497

TABLE XIV

CURVE FITTING RESULTS OF
INTEGRAL TIME AT
$\tau_2/\tau_1=0.4$

$$\tau_1/\tau_I = A - B(1-e^{-C\,\tau_3/\tau_1}) + D\,e^{E\,\tau_3/\tau_1}\sin(F\,\tau_3/\tau_1 + G)$$

θ_d/τ_1	A	B	C	D	E	F	G	Stanard Deviation
0.1	-5.1642	-7.8589	-59.7046	17.5459	0.01126	0.1232	0.8587	0.1876
0.2	10.6290	3.9623	-0.2532	28.8933	-3.14079	0.1020	-3.1486	0.1098
0.3	3.2314	0.1559	-0.0595	2.6706	-0.26734	0.0029	-1.5791	0.1201
0.4	-6.2692	2.8106	0.2276	7.1376	0.15085	-0.00079	1.3085	0.0670
0.5	-7.0245	3.0109	-0.1622	8.0350	-0.00574	0.0000039	1.9000	0.0898
0.6	1.1447	2.1274	-0.0896	0.9688	-0.61380	0.45525	-2.6079	0.0424
0.7	-1.1754	-0.0923	-1.5271	1.8399	0.15697	0.12023	1.6856	0.0286
0.8	1.9433	0.1943	-0.0972	2.3422	-0.35892	0.19242	-2.5540	0.0112
0.9	1.0655	1.1216	-0.0728	0.5016	-0.27034	0.54268	-2.1566	0.0261
1.0	0.6310	-0.3735	0.2301	0.0093	0.58052	0.68567	-0.0951	0.0304

TABLE XV

CURVE FITTING RESULTS OF
DERIVATIVE TIME
AT $\tau_2/\tau_1=0.4$

$$\tau_D/\tau_1 = A + B\, e^{-C\, \tau_3/\tau_1}$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	0.203951502	0.143325232	-4.523662773	0.008137682
0.2	0.305494815	0.061180854	-6.043138525	0.013870909
0.3	0.352407234	0.008231174	0.298011046	0.013364184
0.4	-0.893109110	1.277234464	0.015614548	0.019103968
0.5	-1.104583444	1.519742181	0.017479511	0.015388633
0.6	-142415.6809	142416.1219	0.000000267	0.013857436
0.7	-130671.1526	130671.6194	0.000000333	0.017136710
0.8	-0.914422981	1.410955608	0.032167417	0.011268156
0.9	-276263.3034	276263.8291	0.000000177	0.017907724
1.0	-0.013144635	0.564361466	0.077458722	0.025319861

TABLE XVI

CURVE FITTING RESULTS OF IAE
AT $\tau_2/\tau_1=0.4$

$$IAE = A + B/(C + \tau_3/\tau_1)$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	0.005782802	0.045065168	0.179496464	0.001653484
0.2	0.018056607	0.167232310	0.375805020	0.007691563
0.3	0.025735342	0.364772915	0.583748522	0.013838110
0.4	0.028893103	0.632073323	0.791338632	0.018100325
0.5	0.035312179	0.939957544	0.974898306	0.025382438
0.6	0.022067090	1.366699285	1.201006359	0.029144753
0.7	0.013504055	1.840951373	1.414650333	0.033536074
0.8	-0.002212593	2.412063964	1.644878052	0.036983885
0.9	-0.017475754	3.066857857	1.884221320	0.041987247
1.0	-0.054314429	3.908748928	2.165438948	0.042671194

TABLE XVII

CURVE FITTING RESULTS OF
PROPORTIONAL GAIN
AT $\tau_2/\tau_1 = 0.5$

$$K_C K_L = A - e^{-B\,\tau_3/\tau_1}\,[C \cos (D\,\tau_3/\tau_1) + E \sin (D\,\tau_3/\tau_1)]$$

θ_d/τ_1	A	B	C	D	E	Standard Deviation
0.1	25.17440	1.49331	15.76741	-0.000009944	17.16323	0.903045
0.2	10.79241	0.82521	6.319318	-0.000000032	27.54787	0.371841
0.3	6.278828	0.72285	3.233786	-0.000000002	0.86228	0.201368
0.4	4.166211	0.95431	1.944290	0.000000010	1.25545	0.098547
0.5	2.820382	0.35227	0.960844	0.031869143	-23.11652	0.071758
0.6	2.741136	0.76753	1.156651	-0.000000015	1.02499	0.039524
0.7	2.291224	0.59669	0.908106	0.236511173	-0.81194	0.037319
0.8	2.032331	1.40030	0.760692	0.005506539	109.76785	0.021349
0.9	1.804311	0.40025	0.643524	0.009895981	-22.87312	0.032148
1.0	1.698388	1.31130	0.608902	0.004621596	111.80662	0.027326

TABLE XVII

CURVE FITTING RESULTS OF INTEGRAL TIME AT $\tau_2/\tau_1=0.5$

$$\tau_1/\tau_I = A + B(1-e^{-C\,\tau_3/\tau_1}) + D\,e^{-E\,\tau_3/\tau_1}\sin(F\,\tau_3/\tau_1 + G)$$

θ_d/τ_1	A	B	C	D	E	F	G	Standard Deviation
0.1	6.3876	-0.85602	-4.72198	5.5783	-0.28236	0.18526	-1.60655	0.08551
0.2	3.4175	-0.00102	-8.97281	9.1401	-0.67431	0.16805	-2.82084	0.21438
0.3	-1.9108	0.94591	-0.28356	2.8334	0.05311	0.14335	1.03930	0.10599
0.4	2.2159	-0.35525	-1.52783	3.1687	-0.15305	0.15706	-0.53215	0.04187
0.5	0.0392	0.78840	-0.32169	1.1374	0.28811	0.04156	2.66150	0.04644
0.6	-0.6232	-0.16651	-3.63481	3.3930	-0.02129	0.12380	0.38373	0.03751
0.7	0.6530	0.05971	-0.08695	0.6226	0.09346	0.27191	-0.06911	0.03021
0.8	1.8876	0.43637	0.00880	2.57028	-0.40365	0.20558	-2.62029	0.01472
0.9	1.3828	0.60787	-0.08742	2.56107	-0.45037	0.14807	-2.83373	0.04585
1.0	1.5831	-0.00933	0.22935	2.04456	-0.38331	0.20054	-2.64220	0.03871

TABLE XIX

CURVE FITTING RESULTS OF
DERIVATIVE TIME
AT $\tau_2/\tau_1=0.5$

$$\tau_D/\tau_1 = A + B e^{C\, \tau_3/\tau_1}$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	0.235371025	0.162361151	-4.855050578	0.012972427
0.2	0.325336057	0.090122590	-3.534032330	0.014815305
0.3	0.385598433	0.016493207	0.110234093	0.017235546
0.4	0.047299085	0.383584461	0.044678559	0.013902512
0.5	0.357329082	0.104878133	0.161842406	0.012424313
0.6	-173470.5323	173471.0184	0.000000197	0.013089320
0.7	-0.023963949	0.537069378	0.063285998	0.018456049
0.8	-3.893494066	4.426451734	0.010599263	0.015348172
0.9	-0.110807511	0.674499373	0.059193034	0.023345046
1.0	-0.286222108	0.874817830	0.052556535	0.023026721

TABLE XX

CURVE FITTING RESULTS OF IAE
AT $\tau_2/\tau_1=0.5$

$$IAE = A + B/(C + \tau_3/\tau_1)$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	0.005723501	0.044636203	0.175905496	0.001548603
0.2	0.018421233	0.170275133	0.378533305	0.007698625
0.3	0.024982446	0.376934291	0.593279038	0.013576250
0.4	0.023885942	0.667527763	0.814519833	0.018727666
0.5	0.025313246	1.021796746	1.030279770	0.023433154
0.6	0.019418472	1.438737015	1.233900160	0.030232949
0.7	0.006473725	1.972655198	1.476180938	0.033398483
0.8	-0.016637006	2.607974929	1.720485654	0.038292199
0.9	-0.026450639	3.277881598	1.959725166	0.042601618
1.0	-0.064234566	4.149732510	2.232595244	0.049525759

TABLE XXI

CURVE FITTING RESULTS OF PROPORTIONAL GAIN AT τ_2/τ_1=0.6

$$K_C K_L = A - e^{-B\,\tau_3/\tau_1}\,[C\cos(D\,\tau_3/\tau_1) - E\sin(D\,\tau_3/\tau_1)]$$

θ_d/τ_1	A	B	C	E	F	Standard Deviation
0.1	26.01722	2.29125	17.64455	-0.000000004	87.22845	0.963223
0.2	9.30203	0.91006	4.90682	0.956787623	-2.80380	0.517259
0.3	6.39028	0.90794	3.25597	0.000000005	11.65437	0.181501
0.4	4.42277	1.00460	2.10465	0.000000003	2.59171	0.105597
0.5	3.45279	0.89220	1.53625	-0.000000005	1.548603	0.051564
0.6	2.84460	0.87737	1.21646	0.000000007	2.847037	0.049199
0.7	2.43683	0.82642	1.00447	-0.000000002	3.487253	0.061928
0.8	2.12771	1.40966	0.80738	0.002412956	281.778372	0.017395
0.9	1.75061	0.31927	0.55093	0.001532237	-202.951608	0.034652
1.0	1.74287	0.67492	0.61150	0.605624953	0.171433	0.017289

TABLE XXII

CURVE FITTING RESULTS OF
INTEGRAL TIME AT
$\tau_2/\tau_1=0.6$

$$\tau_1/\tau_I = A + B(1-e^{-B\ \tau_3/\tau_1}) + D\ e^{E\ \tau_3/\tau_1}\ \sin\ (F\ \tau_3/\tau_1 + G)$$

θ_d/τ_1	A	B	C	D	E	F	G	Standard Deviation
0.1	4.0331	-0.27636	-9.5196	3.3238	-0.17124	0.47005	-1.66247	0.16281
0.2	2.6040	0.31545	0.0497	2.2010	-0.12636	0.34418	-1.24938	0.08070
0.3	0.0012	0.34348	-0.3461	0.9782	0.16556	0.23971	0.57321	0.08229
0.4	0.7043	0.21007	0.2621	2.1962	0.02892	0.19264	-0.08434	0.06073
0.5	0.9996	1.80525	-0.1337	1.9469	-0.72558	0.19734	-2.90700	0.04186
0.6	0.3608	0.79708	-0.2263	0.1901	0.37855	0.16296	1.66256	0.04203
0.7	-0.0240	1.23100	-0.0137	1.2420	0.04262	0.15688	0.47906	0.05537
0.8	1.4091	0.96715	-0.0684	1.7939	-0.38885	0.19820	-2.66156	0.01989
0.9	1.0035	3.81389	0.0564	3.1341	0.02826	0.11700	-0.14357	0.04391
1.0	0.7807	0.21010	-0.2489	0.2479	0.10057	0.38764	-1.05633	0.02685

TABLE XXIII

CURVE FITTING RESULTS OF
DERIVATIVE TIME
AT $\tau_2/\tau_1=0.6$

$$\tau_D/\tau_1 = A + B\, e^{C\, \tau_3/\tau_1}$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	0.264326516	0.183215578	-5.125286637	0.023506568
0.2	0.280211339	0.089596905	0.109112121	0.040979224
0.3	0.427081197	0.004957846	0.393595557	0.016116829
0.4	0.031352681	0.436013860	0.033684509	0.018470226
0.5	0.077840398	0.422322973	0.053440390	0.010352637
0.6	0.290453325	0.236682115	0.104258247	0.014116076
0.7	-0.199801389	0.756245496	0.044111733	0.027029529
0.8	0.098796511	0.476531864	0.078594691	0.017859875
0.9	0.336517516	0.275309050	0.118747755	0.023278755
1.0	0.414683946	0.220446061	0.150580955	0.025698944

TABLE XXIV

CURVE FITTING RESULTS OF IAE
AT $\tau_2/\tau_1=0.6$

$$IAE = A - B/(C + \tau_3/\tau_1)$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	0.005555820	0.044564179	0.174624427	0.001450922
0.2	0.015771988	0.176596147	0.388040761	0.006596532
0.3	0.018094853	0.402301547	0.621958356	0.012048839
0.4	0.024188710	0.686589086	0.828206710	0.019298545
0.5	0.016343437	1.076817140	1.060067331	0.024344899
0.6	0.009153153	1.542303908	1.293436572	0.029062106
0.7	-0.001269079	2.075340753	1.516690928	0.036023204
0.8	-0.033097292	2.793936890	1.796307874	0.038048556
0.9	-0.052505463	3.547737517	2.054861139	0.042081739
1.0	-0.093276028	4.519497875	2.358059142	0.042202062

TABLE XXV

CURVE FITTING RESULTS OF PROPORTIONAL GAIN AT $\tau_2/\tau_1=0.7$

$$K_C K_L = A - e^{-B\,\tau_3/\tau_1}\,[C\cos(D\,\tau_3/\tau_1) + E\sin(D\,\tau_3/\tau_1)]$$

θ_d/τ_1	A	B	C	D	E	Standard Deviation
0.1	27.69377	2.567600	19.30382	-0.000000371	55.54893	0.740850
0.2	9.62531	1.172778	4.89178	1.782716326	-0.41195	0.567129
0.3	7.28867	0.583158	3.72150	0.000000004	-1.29708	0.383679
0.4	4.76541	0.920671	2.31443	-0.000000001	-0.40174	0.122889
0.5	3.70815	0.850482	1.69000	-0.000000075	1.60395	0.082126
0.6	3.01409	0.821919	1.30946	0.000000005	2.03925	0.031818
0.7	2.65485	0.688195	1.14687	-0.000000010	-0.41448	0.053259
0.8	2.27549	0.764863	0.92822	-0.000000023	0.37670	0.024944
0.9	2.00670	1.354058	0.74655	0.008271015	80.68862	0.012694
1.0	1.89259	0.632914	0.74549	-0.000000006	0.58780	0.033810

TABLE XXVI

CURVE FITTING RESULTS OF INTEGRAL TIME AT $\tau_2/\tau_1=0.7$

$$\tau_1/\tau_I = A + B(1-e^{-C\ \tau_3/\tau_1}) + D\ e^{-E\ \tau_3/\tau_1} \sin(F\ \tau_3/\tau_1 + G)$$

θ_d/τ_1	A	B	C	D	E	F	G	Standard Deviation
0.1	0.3422	8.1236	-0.12236	0.33079	-0.00467	1.24529	-3.56925	0.06339
0.2	0.5405	2.6857	-0.49521	3.16727	-0.74571	0.47416	-3.15364	0.08000
0.3	3.9802	18.4645	0.00680	53.14755	-0.40354	0.01803	-3.07597	0.04594
0.4	1.6645	2.0395	-0.02386	1.15781	-0.19637	0.36312	-1.59532	0.04070
0.5	0.3818	0.0661	-0.12358	1.30656	0.02759	0.20352	0.07950	0.08192
0.6	1.1853	0.2012	-0.43661	0.67629	-0.05883	0.36676	-1.32293	0.01682
0.7	2.1021	-0.4987	-0.76848	1.55725	-0.31115	0.21950	-1.70526	0.06031
0.8	1.5993	0.3686	0.18203	1.13695	-0.00869	0.29869	-1.17113	0.01562
0.9	2.4492	-0.5913	-0.70767	3.27271	-0.22700	-0.00037	-0.61624	0.01358
1.0	0.4056	0.000010	1.47230	0.24969	0.23577	0.22791	0.55064	0.03035

TABLE XXVII

CURVE FITTING RESULTS OF
DERIVATIVE TIME
AT $\tau_2/\tau_1=0.7$

$$\tau_D/\tau_1 = A + B\, e^{C\, \tau_3/\tau_1}$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	0.280322198	0.209153818	-4.847470366	0.026383444
0.2	-3739.472446	3739.867383	0.000004448	0.049294172
0.3	-24821.433674	24821.904083	0.000000059	0.027317077
0.4	0.255335620	0.245344088	0.053786376	0.016589548
0.5	0.420797567	0.112202746	0.138804267	0.017982601
0.6	0.020783587	0.537510957	0.052035615	0.009990302
0.7	-8805.370351	8805.960187	0.000003808	0.024534212
0.8	-0.335221379	0.944753310	0.041538190	0.020990415
0.9	0.293609472	0.348623639	0.102367245	0.014434986
1.0	-12949.157933	12949.820202	0.000003718	0.030028359

TABLE XXVIII

CURVE FITTING RESULTS OF IAE
AT $\tau_2/\tau_1=0.7$

$$IAE = A - B/(C + \tau_3/\tau_1)$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	0.005458164	0.044722977	0.174688724	0.001286274
0.2	0.015306185	0.178202497	0.388704766	0.005902130
0.3	0.019881743	0.402947058	0.618803619	0.012844453
0.4	0.017175085	0.722428287	0.857270011	0.018593623
0.5	0.012731650	1.131855938	1.100017894	0.025234654
0.6	-0.000279755	1.611543034	1.323993878	0.029720495
0.7	0.006147035	2.096322459	1.520547321	0.039123653
0.8	-0.042351118	2.930744175	1.847527993	0.039744570
0.9	-0.077001407	3.781385706	2.130552622	0.042785581
1.0	-0.089333920	4.543064601	2.338592491	0.049491427

TABLE XXIX

CURVE FITTING RESULTS OF
PROPORTIONAL GAIN
AT $\tau_2/\tau_1=0.8$

$$K_C K_L = A - e^{-B\,\tau_3/\tau_1}\,[C\cos(D\,\tau_3/\tau_1) + E\sin(D\,\tau_3/\tau_1)]$$

θ_d/τ_1	A	B	C	D	E	Standard Deviation
0.1	27.32212	8.48587	15.81669	0.063323817	2598.9216	1.268503
0.2	10.556903	2.03212	5.63889	-2.073522846	-2.557892	0.726696
0.3	7.742700	0.70565	4.15667	-0.000000000	59.153361	0.271635
0.4	5.252423	0.79585	2.64103	-0.000000101	3.626108	0.160436
0.5	4.069885	0.68822	1.91817	0.000000006	0.427682	0.101287
0.6	3.208561	0.77745	1.40624	-0.000000000	1.998330	0.041696
0.7	2.740232	0.74861	1.17557	-0.000000323	3.209259	0.029565
0.8	2.406327	0.71258	1.00408	0.000000014	3.562319	0.036050
0.9	1.909436	0.31803	0.61037	0.031267403	-10.644044	0.025920
1.0	1.931674	1.15231	0.72571	0.005389270	90.643926	0.008946

TABLE XXX

CURVE FITTING RESULTS OF INTEGRAL TIME AT $\tau_2/\tau_1=0.8$

$$\tau_1/\tau_3 = A + B(1-e^{-B\,\tau_3/\tau_1}) + D\,e^{-E\,\tau_3/\tau_1}\sin(F\,\tau_3/\tau_1 + G)$$

θ_d/τ_1	A	B	C	D	E	F	G	Standard Deviation
0.1	2.0670	-0.81998	-1.30625	3.7987	-0.00825	0.25857	-0.40637	0.12138
0.2	1.8649	8.29391	-0.02365	16.3023	-0.99412	0.12919	-3.06409	0.05949
0.3	-0.3455	1.95396	-0.10623	1.6928	0.34148	0.05778	2.65552	0.05833
0.4	-9.4900	1.92823	-0.07725	10.3935	0.00510	0.05074	1.27365	0.10184
0.5	0.2705	0.39736	-0.52047	0.2580	0.27548	0.22134	0.84474	0.10977
0.6	1.5124	-0.07469	0.23510	1.6833	-0.43241	0.25697	-2.50386	0.02183
0.7	0.6105	0.38731	0.06515	0.7676	0.06315	0.24581	-0.14247	0.02448
0.8	1.3185	-0.00781	-1.53201	0.7948	-0.12478	0.29997	-1.58384	0.03685
0.9	1.4696	-3.67146	-0.00108	1.0013	-0.16409	0.25501	-1.89497	0.02287
1.0	0.8890	11.58321	-0.00531	0.5443	-0.31748	0.36220	-2.39940	0.00951

TABLE XXXI

CURVE FITTING RESULTS OF DERIVATIVE TIME AT $\tau_2/\tau_1=0.8$

$$\tau_D/\tau_1 = A + B\, e^{C\, \tau_3/\tau_1}$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	0.313054829	0.001651750	0.543880163	0.055739571
0.2	0.336894921	0.090623337	0.098747130	0.046853081
0.3	0.485684608	0.053440674	-3.697428326	0.020059867
0.4	0.538059462	0.001916993	0.549796524	0.024401043
0.5	0.511458711	0.054447330	0.182649419	0.025350400
0.6	0.444115226	0.148721839	0.133209349	0.008802239
0.7	0.294458410	0.327252663	0.084555799	0.011106046
0.8	0.449393430	0.204161976	0.128899226	0.021250243
0.9	0.515912270	0.164062946	0.164331718	0.022915223
1.0	0.435895049	0.267779382	0.125931586	0.015916512

TABLE XXXII

CURVE FITTING RESULTS OF IAE
AT $\tau_2/\tau_1=0.8$

$$IAE = A + B/(C + \tau_3/\tau_1)$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	0.006348029	0.044831013	0.176182026	0.001717612
0.2	0.015369666	0.179195159	0.389618512	0.005693040
0.3	0.018257812	0.411217156	0.626693384	0.012311945
0.4	0.017727088	0.733634934	0.865200204	0.018796204
0.5	0.016334054	1.139001693	1.101646717	0.025642044
0.6	-0.008270732	1.688002244	1.365935558	0.029815431
0.7	-0.025066494	2.294059752	1.613125810	0.034853071
0.8	-0.046120274	3.004847736	1.870020153	0.039979835
0.9	-0.082888729	3.912008646	2.174028711	0.041552235
1.0	-0.124862455	4.895824272	2.451653088	0.046649100

TABLE XXXIII

CURVE FITTING RESULTS OF
PROPORTIONAL GAIN
AT $\tau_2/\tau_1=0.9$

$$K_C K_L = A - e^{-B\,\tau_3/\tau_1}\,[C\cos(D\,\tau_3/\tau_1) + E\sin(D\,\tau_3/\tau_1)]$$

θ_d/τ_1	A	B	C	D	E	Standard Deviation
0.1	29.29959	4.787268	21.13594	-2.332855213	-14.99868	1.943485
0.2	11.54144	4.506433	5.54898	0.062077275	526.60280	0.552682
0.3	8.55324	0.544232	4.59922	-0.000001698	1.87016	0.419526
0.4	5.20808	1.066881	2.57344	-0.000000035	0.03550	0.172926
0.5	4.10794	0.890244	1.93688	0.000000002	0.77670	0.072607
0.6	3.46153	0.690204	1.56889	-0.000000005	3.55701	0.076357
0.7	2.93523	0.646403	1.27028	-0.000000008	2.52906	0.054451
0.8	2.50438	1.044097	1.03577	0.008242687	47.93714	0.020662
0.9	2.21726	1.166595	0.85345	0.003252753	165.66986	0.016029
1.0	2.06582	0.830563	0.81193	0.001656572	141.09397	0.036460

TABLE XXXIV

CURVE FITTING RESULTS OF INTEGRAL TIME AT $\tau_2/\tau_1=0.9$

$$\tau_1/\tau_I = A + B(1-e^{-C\,\tau_3/\tau_1}) + D\,e^{-E\,\tau_3/\tau_1}\sin(F\,\tau_3/\tau_1 + G)$$

θ_d/τ_1	A	B	C	D	E	F	G	Standard Deviation
0.1	-1.4364	-0.04905	0.82025	1.7629	0.40462	0.24180	1.75113	0.22755
0.2	2.6754	-0.01963	0.38434	4.1196	-0.52773	0.37965	-2.59431	0.04312
0.3	3.1788	-0.36885	-2.73077	5.2538	-0.24340	-0.00476	-0.52761	0.05737
0.4	0.6056	-0.22020	-6.18932	3.1404	-0.05451	0.12315	-0.01671	0.03476
0.5	0.2693	0.94777	-0.21846	0.4210	0.39568	0.04207	2.72585	0.03799
0.6	1.5418	0.13500	-0.22997	1.9908	-0.42469	0.23771	-2.57909	0.06498
0.7	0.2143	0.11339	-0.14577	0.3023	0.41054	0.14080	1.96123	0.03174
0.8	0.7409	1.65364	0.00917	0.5187	0.06537	0.28069	-0.49938	0.01995
0.9	0.5230	1.14280	-0.26755	1.5530	-0.33628	0.16877	-3.11774	0.01120
1.0	0.2821	0.38740	0.07375	0.4362	0.15546	0.22257	0.46422	0.03250

TABLE XXXV

CURVE FITTING RESULTS OF DERIVATIVE TIME AT $\tau_2/\tau_1=0.9$

$$\tau_D/\tau_1 = A + B e^{C \tau_3/\tau_1}$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	0.332379419	0.006979067	0.309974587	0.064630189
0.2	0.428541000	0.005660188	0.474990923	0.045914960
0.3	0.501425120	0.068635982	-3.105743341	0.025433727
0.4	0.522229788	0.035209341	0.214729064	0.028571062
0.5	0.561668230	0.033603011	0.252106363	0.015708873
0.6	0.287761997	0.332298163	0.061364028	0.021800547
0.7	-743.1650429	743.8105325	0.000042029	0.023254251
0.8	0.545647517	0.135033859	0.169548694	0.016526330
0.9	0.501469121	0.202292747	0.141093396	0.012866524
1.0	0.229916733	0.499982175	0.070327518	0.028121240

TABLE XXXVI

CURVE FITTING RESULTS OF IAE
AT $\tau_2/\tau_1=0.9$

$$IAE = A + B/(C + \tau_3/\tau_1)$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	0.006812337	0.044324756	0.173071687	0.001680627
0.2	0.013380539	0.183683955	0.396325379	0.004856191
0.3	0.017561009	0.416255734	0.630916309	0.012253302
0.4	0.013005267	0.760306616	0.888018731	0.018483715
0.5	0.005894452	1.190563353	1.133643675	0.024088249
0.6	-0.000092676	1.676314699	1.353200757	0.032476558
0.7	-0.018017842	2.296107912	1.606413832	0.037688033
0.8	-0.063656572	3.174869806	1.940334269	0.038511269
0.9	-0.100018841	4.074139373	2.216964190	0.042882621
1.0	-0.107443107	4.854608499	2.425098081	0.049352346

TABLE XXXVII

CURVE FITTING RESULTS OF
PROPORTIONAL GAIN
AT $\tau_2/\tau_1=1.0$

$$K_C K_L = A - e^{-B\,\tau_3/\tau_1}\left[C\cos(D\,\tau_3/\tau_1) + E\sin(D\,\tau_3/\tau_1)\right]$$

θ_d/τ_1	A	B	C	D	E	Standard Deviation
0.1	27.97665	1.277220	18.71138	-0.013734497	2691.38532	1.243995
0.2	11.03440	0.831614	5.76175	0.017341021	-426.24851	0.325783
0.3	9.11317	0.527492	4.94877	0.000000001	7.61301	0.433618
0.4	5.35217	1.329382	2.69544	-0.000000054	-0.35813	0.123758
0.5	4.36313	0.881278	2.07582	-0.000000001	11.85649	0.081050
0.6	3.73176	0.585626	1.71378	0.000000002	1.64459	0.134795
0.7	3.03594	0.703950	1.32162	0.000000001	2.86447	0.034941
0.8	2.64029	0.674629	1.11481	-0.000000002	2.94392	0.022607
0.9	2.42488	0.538789	1.01055	-0.000000001	1.64608	0.042295
1.0	2.11698	1.093662	0.80927	0.002258698	231.14457	0.023096

TABLE XXXVIII

CURVE FITTING RESULTS OF
INTEGRAL TIME AT
$\tau_2/\tau_1=1.0$

$$K_C K_L = A + B(1-e^{-C\,\tau_3/\tau_1}) + D\,e^{-E\,\tau_3/\tau_1}\sin(F\,\tau_3/\tau_1 + G)$$

θ_d/τ_1	A	B	C	D	E	F	G	Standard Deviation
0.1	4.9788	-317.313	-0.000313	68.4228	-0.66537	0.05120	-3.07697	0.23203
0.2	-0.4640	-0.02649	0.770540	0.83656	0.40341	0.23307	1.68286	0.06642
0.3	-0.3964	0.00359	0.820710	1.55175	0.11662	0.13840	0.55684	0.06111
0.4	1.7666	0.25587	-0.050763	1.56676	-0.31861	0.32394	-2.12808	0.02756
0.5	0.7659	2.35067	-0.070482	3.33389	-1.06694	0.20502	-3.05806	0.05312
0.6	0.9567	0.87363	-0.031840	0.92797	-0.08163	0.19296	-0.62152	0.07621
0.7	0.3342	-0.37382	0.227059	0.13029	0.23911	0.36880	1.01465	0.02007
0.8	0.3319	0.30451	0.069559	0.54881	0.11885	0.21494	0.23286	0.01732
0.9	0.2471	1.33289	0.042734	0.43116	0.20230	0.24918	0.51822	0.01906
1.0	1.1162	-0.01643	0.302313	0.72669	-0.20858	0.27543	-2.03399	0.01076

TABLE XXXIX

CURVE FITTING RESULTS OF
DERIVATIVE TIME
AT $\tau_2/\tau_1=1.0$

$$\tau_D/\tau_1 = A + B\, e^{C\, \tau_3/\tau_1}$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	0.346807412	0.009318093	0.375526558	0.068368258
0.2	0.438213388	0.012208334	0.396096441	0.052524993
0.3	0.513235098	0.089222744	-2.783175980	0.029998423
0.4	0.416640880	0.159690346	0.081117548	0.029461411
0.5	0.591686804	0.026180365	0.255673066	0.021314050
0.6	0.508411250	0.144039475	0.094638039	0.033397436
0.7	0.552430972	0.122364404	0.156130305	0.008693198
0.8	0.568489066	0.130165347	0.170849612	0.021920842
0.9	-2.139914097	2.867235334	0.012439425	0.020982080
1.0	0.614959678	0.145996564	0.174546359	0.019681037

TABLE XXXX

CURVE FITTING RESULTS OF IAE
AT τ_2/τ_1=1.0

$$IAE = A + B/(C + \tau_3/\tau_1)$$

θ_d/τ_1	A	B	C	Standard Deviation
0.1	0.007523122	0.043729788	0.168409667	0.001592648
0.2	0.013544995	0.181414679	0.390095215	0.005703725
0.3	0.015455970	0.424391413	0.639752301	0.012028255
0.4	0.010440535	0.773800926	0.897400310	0.018183932
0.5	0.002680748	1.221853387	1.154808658	0.024398188
0.6	0.006550486	1.668701284	1.345038548	0.034143199
0.7	-0.041439356	2.457885708	1.683735847	0.035092382
0.8	-0.078739700	3.330473942	2.003304313	0.038672251
0.9	-0.094706881	4.116087418	2.231189877	0.045464691
1.0	-0.152692543	5.287633800	2.567201538	0.045625728

APPENDIX E

SAMPLE APPLICATION PROGRAM

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                         C
C                                                         C
C         THIS IS THE PROGRAM TO SEE THE RESPONSE TO A    C
C         STEP CHANGE IN LOAD.                            C
C                                                         C
C         DEFINITON OF VARIABLES;                         C
C            D : DEAD TIME ARRAY                          C
C            TAU1,TAU2 : TIME CONSTANTS                   C
C            RKC : CONTROLLER GAIN                        C
C            TAUI : INTEGRAL TIME                         C
C            TAUD : DERIVATIVE TIME                       C
C            R : SET POINT CHANGE                         C
C            XA,XB,X2,X3,X4 : CONCENTRATIONS              C
C            THETAD : DEAD TIME                           C
C            RKM : MEASUREMENT GAIN                       C
C            RKV : VALVE GAIN                             C
C            E : ERROR                                    C
C            V : VALVE SIGNAL                             C
C            C : MEASUREMENT OUTPUT                       C
C            EPAST : ONE STEP PREVIOUS ERROR              C
C            ERINT : INTEGRAL OF ERROR                    C
C            ABSIE : INTEGRAL OF ABSOLUTE VALUE OF ERROR  C
C            IDT : MAX. LENGTH OF DEAD TIME ARRAY         C
C            TT : TOTAL SIMULATION TIME                   C
C            TPRINT : PRINT INTERVAL                      C
C            K,I : LOOP COUNTER                           C
C            TFLAG : PRINT COUNTER                        C
C                                                         C
C                                                         C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

      EXTERNAL DTX2DOT,DTX3DOT
      DIMENSION D(2000)
C
C     DATA INPUT
C
      DATA TAU1,TAU2,TAU3/1.0, 0.5,1.0/
      DATA RKC,TAUI,TAUD/2.206,1.213,0.6562/
      DATA R,XA,XB/0.,0.,1.0/
      DATA DELTA,THETAD/0.001,1.0/
C
C     DEFINE GAIN OF MEASUREMENT AND VALVE
C
      RKM=0.004
      RKV=166.6667
      OPEN(UNIT=6,FILE='C:\COMPILER\FOR77\RESP2.DAT',
           STATUS='NEW')
      WRITE(6,3)
3     FORMAT(3X,'TIME',7X,'X4',8X,'X6',8X,'V',9X,'ABSIE')
C
C     INITIALIZE VARIABLES
C
      E=0.
```

```
      V=0.
      C=0.
      EPAST=0.
      ERINT=0.
      ABSIE=0.
      IDT=INT(THETAD/DELTA+1.)
      DU31=EXP(-DELTA/TAU3)
      DU32=1.0-DU31
      DU32XB=XB*DU32
      DO 25 I=1,IDT
         D(I)=0.
25    CONTINUE
      TT=25.0*TAU1
      TPRINT=0.1
      K=IDT
      L=1
      TIME=0.
      TFLAG=0.
      X2=0.
      X3=0.
      X4=0.
      x5=0.
      x6=0.
C
C     CONTROL LOOP
C
28    C=RKM*X6
      E=R-C
      V=RKC*(E+ERINT/TAUI+(E-EPAST)*TAUD/DELTA)
      VV=V
      IF (V .GE. 6.)    V=6.
      IF (V .LE.-6.)    V=-6.
      X1=XA+V*RKV/1.
      RK11=DTX2DOT(X1,X2,TAU1)
      RK12=DTX3DOT(X2,X3,TAU2)
C
      RK21=DTX2DOT(X1,X2+0.5*RK11,TAU1)
      RK22=DTX3DOT(X2+0.5*RK11,X3+0.5*RK12,TAU2)
C
      RK31=DTX2DOT(X1,X2+0.5*RK21,TAU1)
      RK32=DTX3DOT(X2+0.5*RK21,X3+0.5*RK22,TAU2)
C
      RK41=DTX2DOT(X1,X2+RK31,TAU1)
      RK42=DTX3DOT(X2+RK31,X2+RK32,TAU2)
C
      X2=X2+(RK11+2.0*RK21+2.0*RK31+RK41)/6.0
      X3=X3+(RK12+2.0*RK22+2.0*RK32+RK42)/6.0
C

      IF (TIME .LT. (TFLAG-DELTA/2.)) GO TO 30
      WRITE(6,40) TIME,X5,X6,V, ABSIE
40    FORMAT(5F10.5)
      TFLAG=TFLAG+TPRINT
```

```
30     IF (TIME .GT. TT) GO TO 100
         TIME=TIME+DELTA
         ERINT=ERINT+E*DELTA
         ABSIE=ABSIE+ABS(E*DELTA)
         EPAST=E
         D(K)=X3
         X4=D(L)
         X5=X5*DU31+DU32XB
         X6=X4+X5
         K=K+1
         L=L+1
         IF (K .LE. (IDT+1)) GO TO 50
         K=1
  50     IF (L .LE. (IDT+1)) GO TO 60
         L=1
  60     GO TO 28
  100    STOP
         END
  C
  C
         REAL FUNCTION DTX2DOT(CX1,CX2,TAU1)
         DELTA=0.001
         DTX2DOT=DELTA*(CX1-CX2)/TAU1
         RETURN
         END
  C
  C
         REAL FUNCTION DTX3DOT(CX2,CX3,TAU2)
         DELTA=0.001
         DTX3DOT=DELTA*(CX2-CX3)/TAU2
         RETURN
         END
```

APPENDIX F

GRAPHS OF SAMPLE

APPLICATION

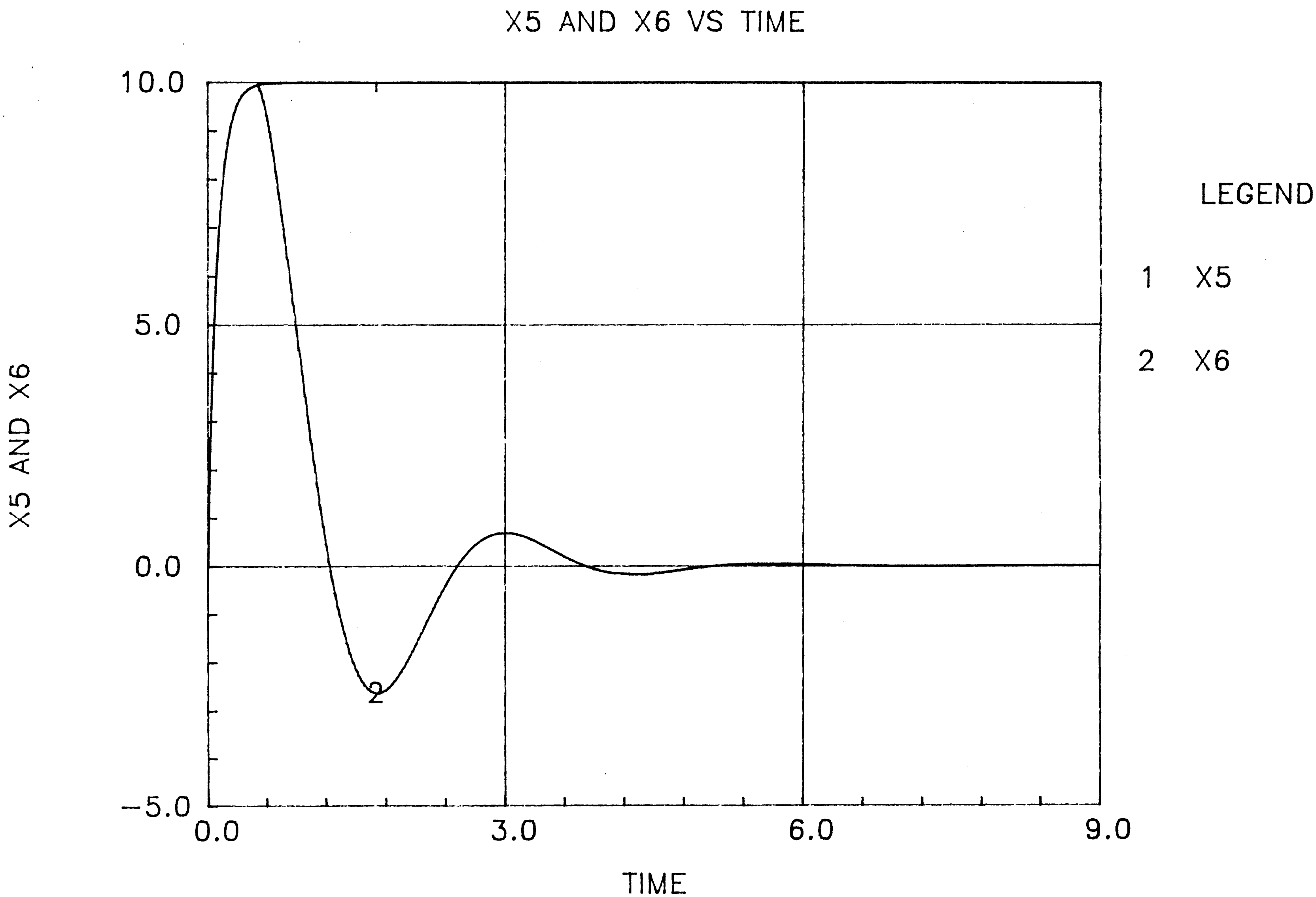

Figure 63. Response of X_5 and X_6 in Sample Application at τ_2/τ_1=0.1, τ_3/τ_1=0.1, and θ_d/τ_1=0.5

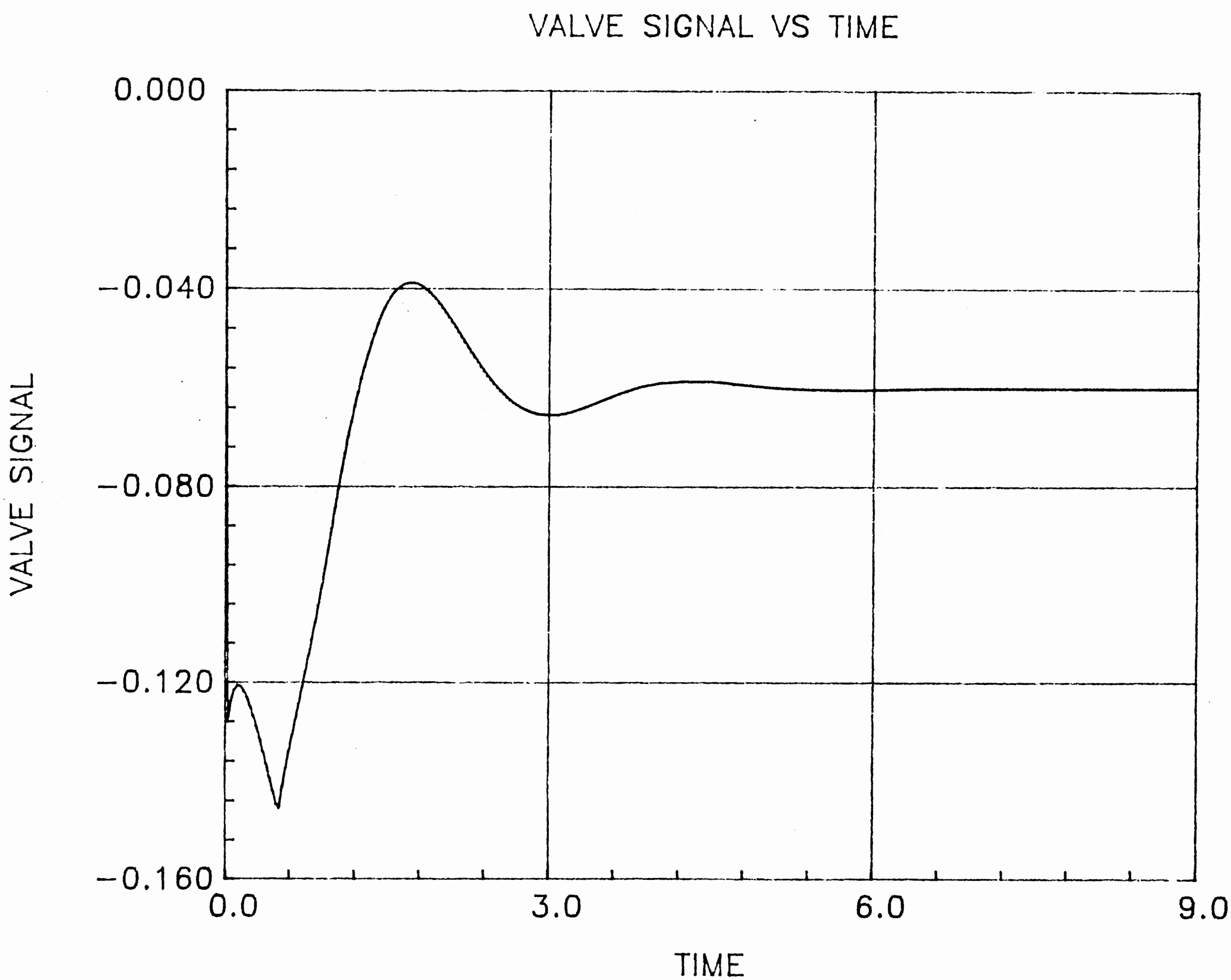

Figure 64. Valve Signal in Sample Application at τ_2/τ_1=0.1, τ_3/τ_1=0.1, θ_d/τ_1=0.5

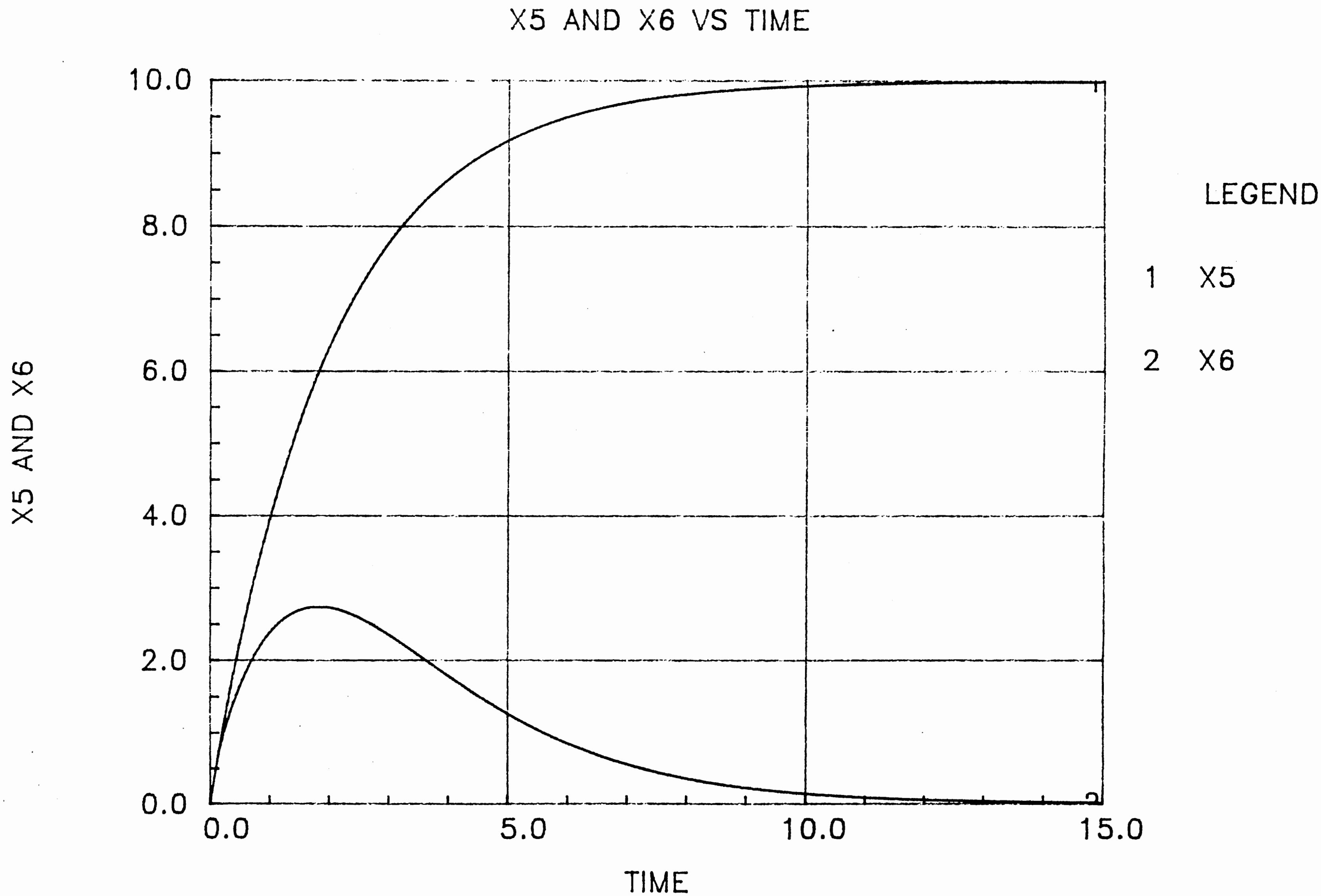

Figure 65. Response of X_5 and X_6 in Sample Application at τ_2/τ_1=0.1, τ_3/τ_1=0.1, and θ_d/τ_1=1.0

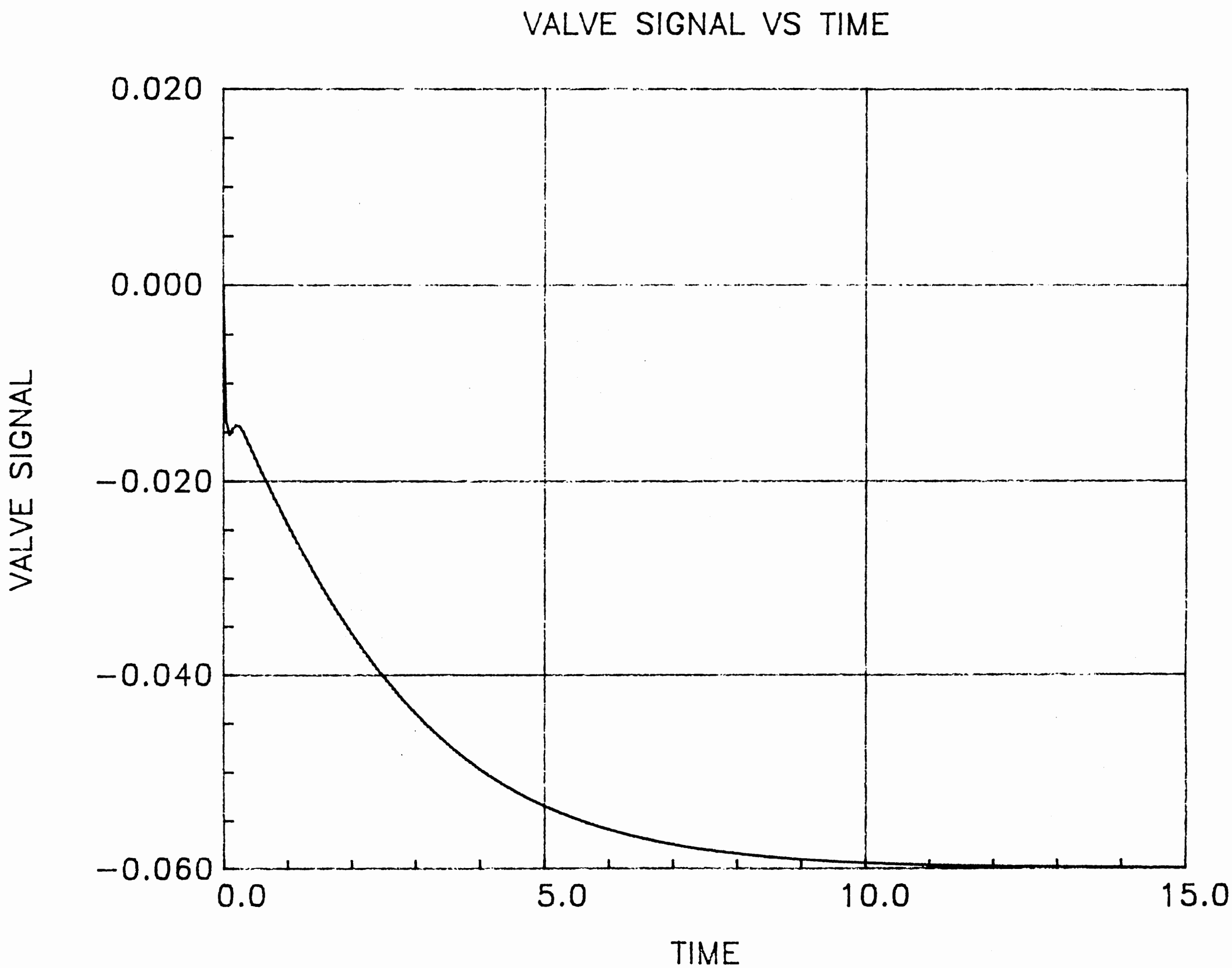

Figure 66. Valve Signal in Sample Application at τ_2/τ_1=0.1 τ_3/τ_1=0.1, and θ_d/τ_1=1.0

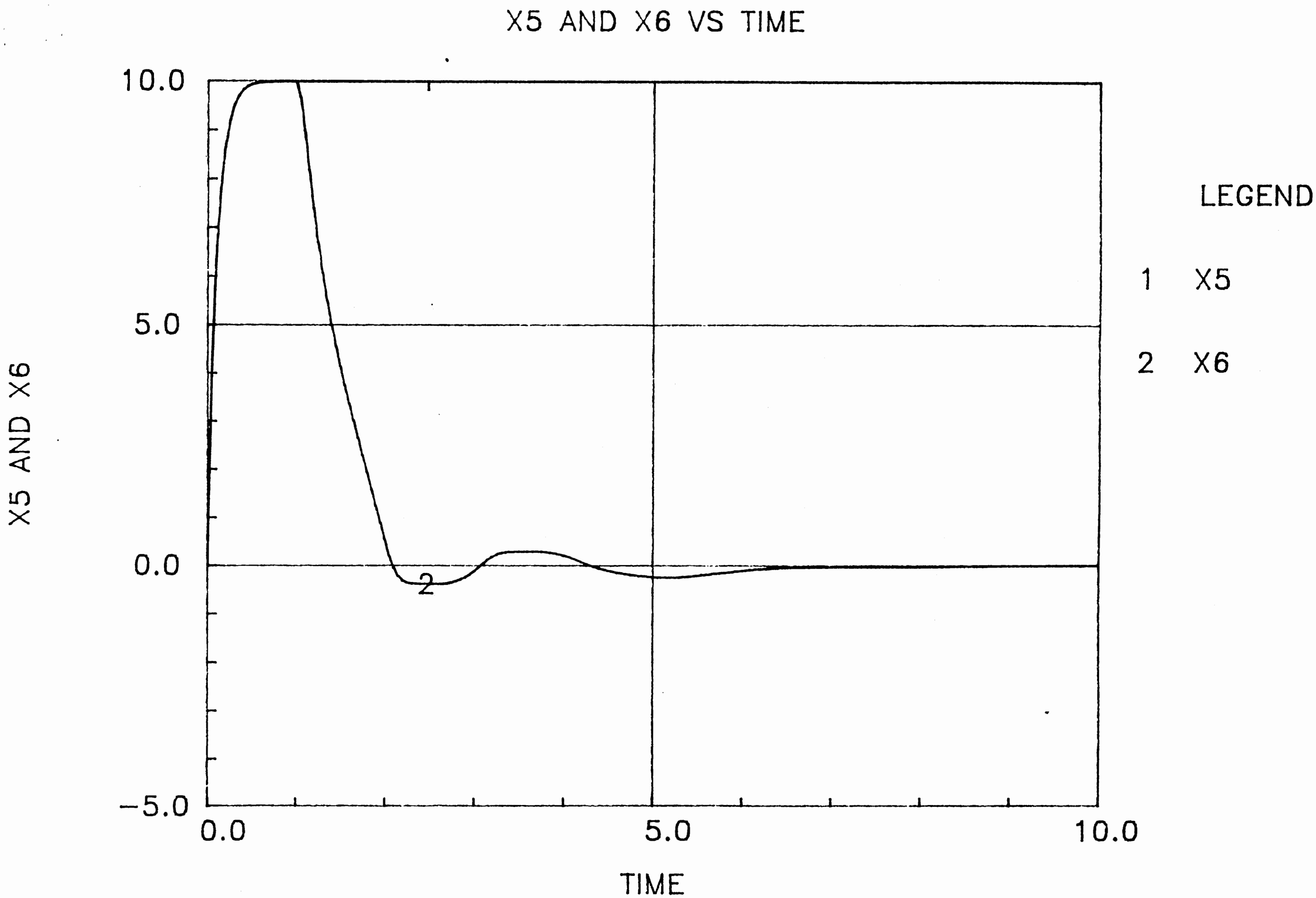

Figure 67. Response of X_5 and X_6 in Sample Application at τ_2/τ_1=0.1, τ_3/τ_1=2.0, and θ_d/τ_1=0.1

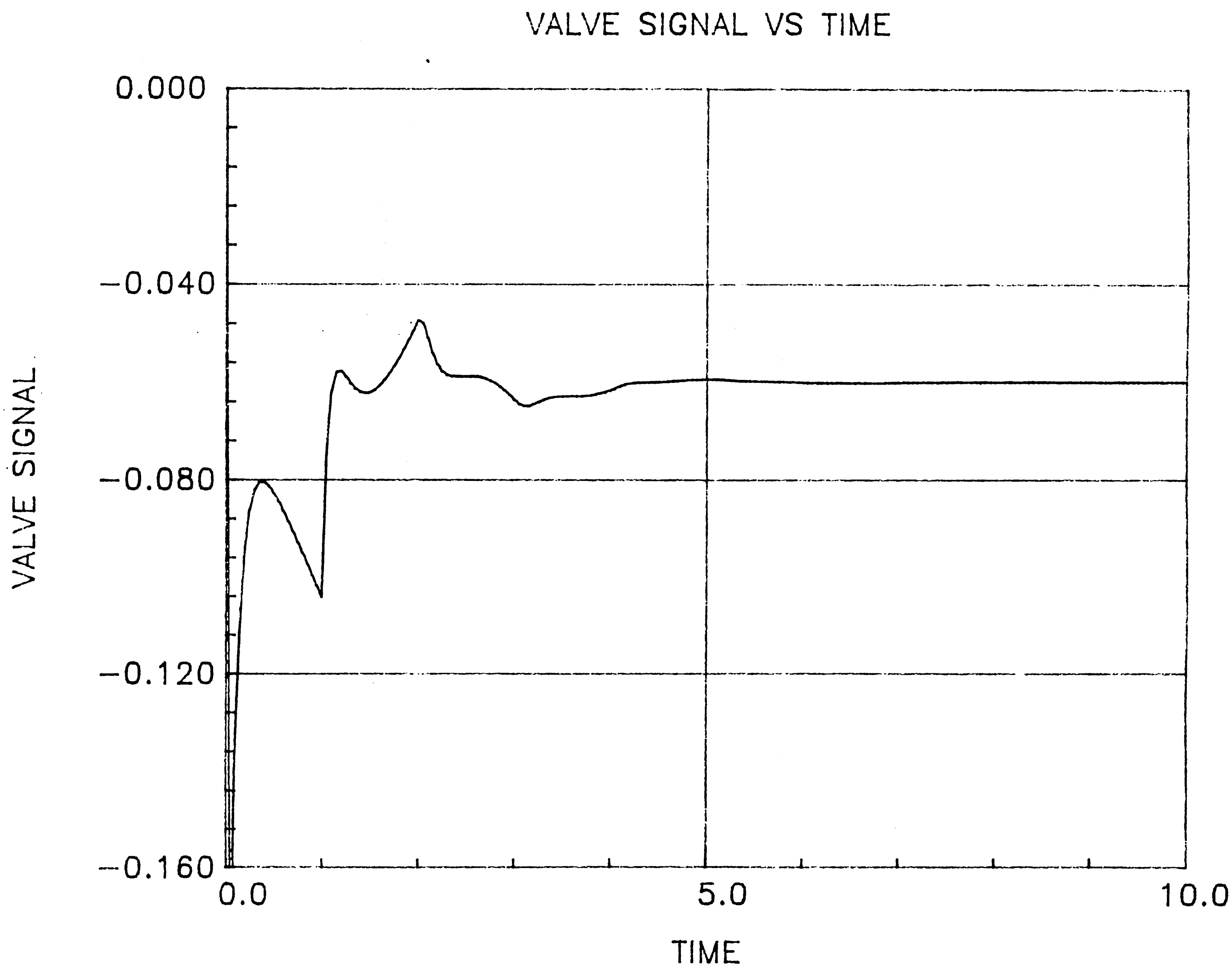

Figure 68. Valve Signal in Sample Application at τ_2/τ_1=0.1 τ_3/τ_1=2.0, and θ_d/τ_1=0.1

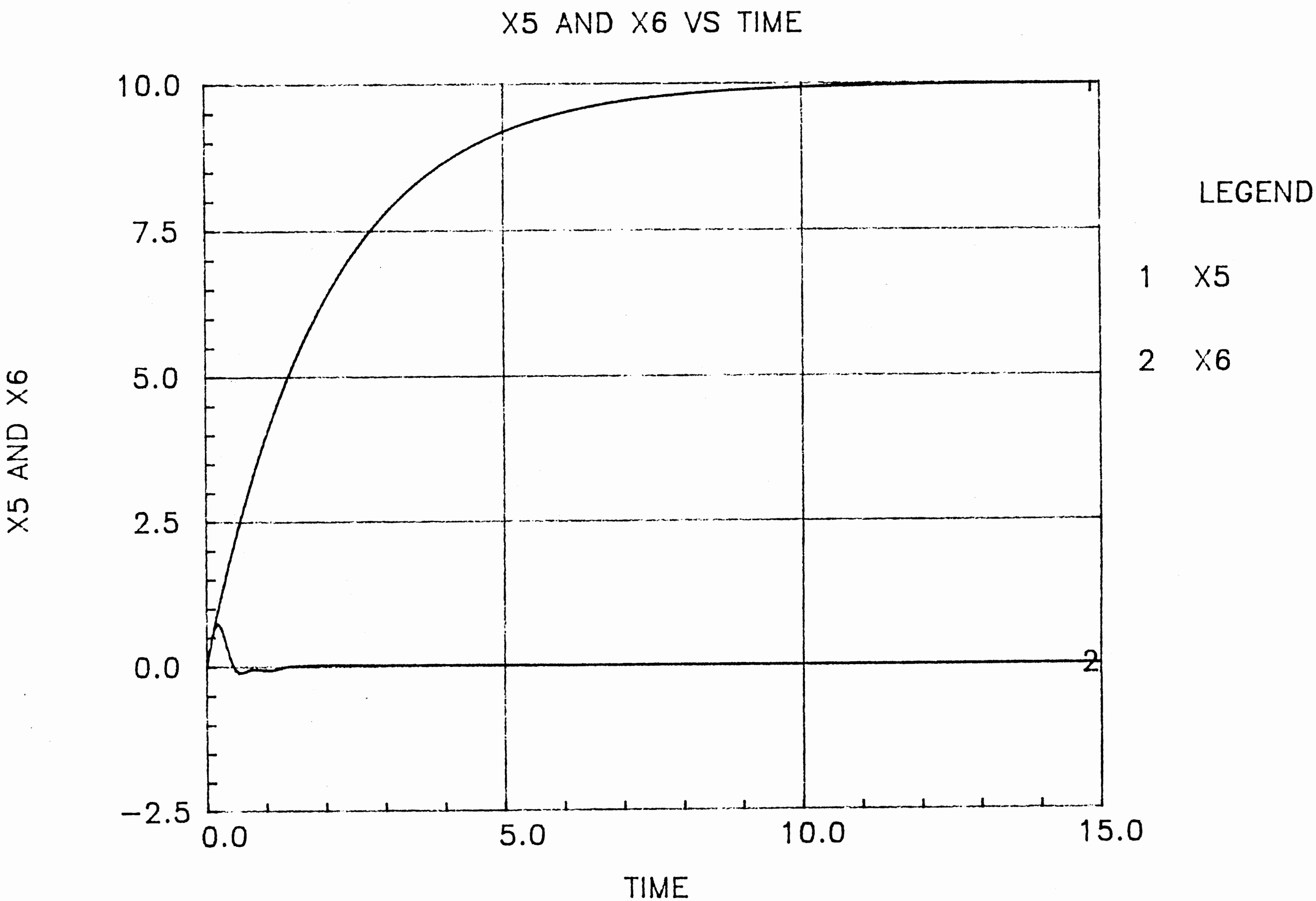

Figure 69. Response of X_5 and X_6 in Sample Application at $\tau_2/\tau_1=0.5$, $\tau_3/\tau_1=2.0$, and $\theta_d/\tau_1=0.1$

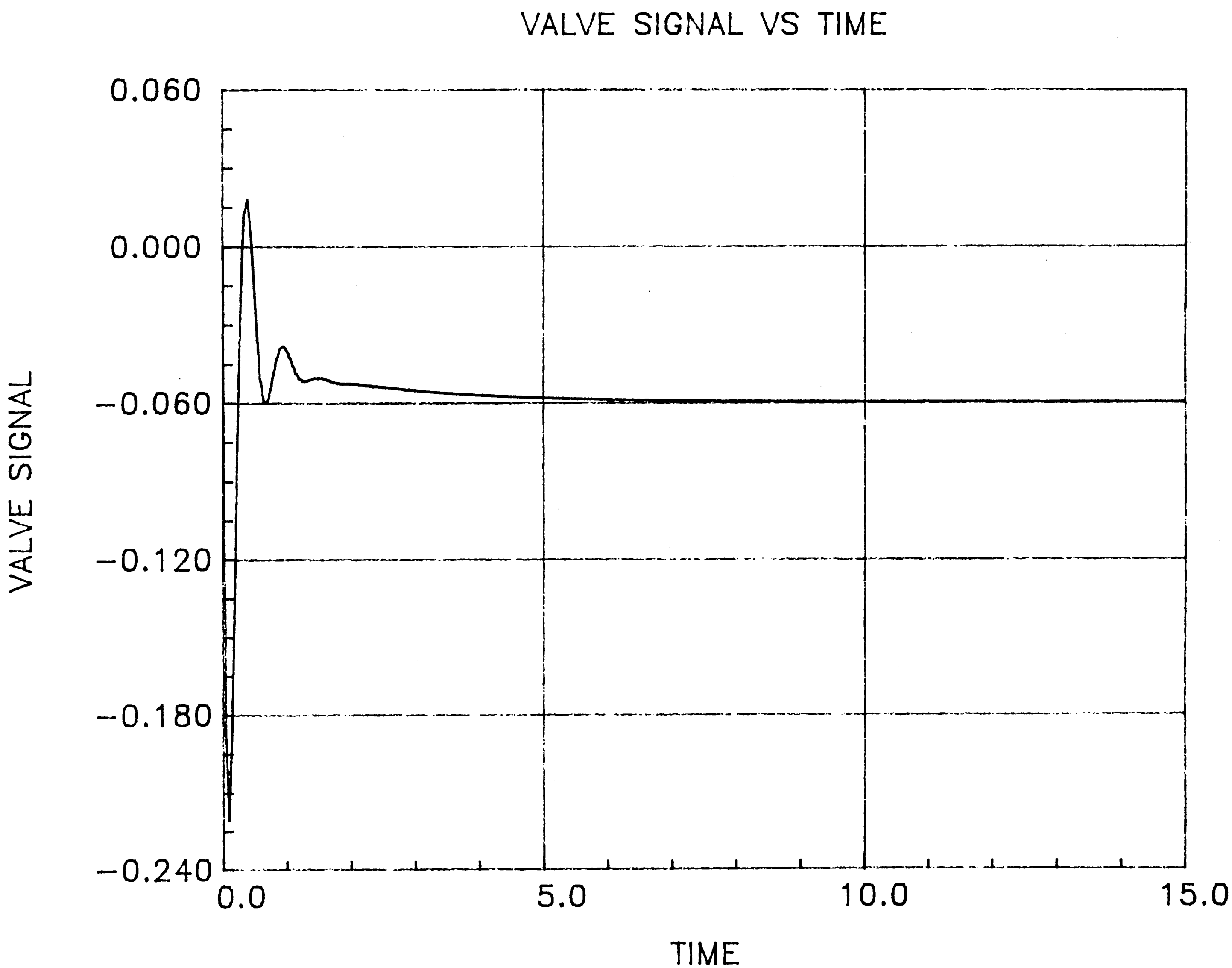

Figure 70. Valve Signal in Sample Application at τ_2/τ_1=0.5, τ_3/τ_1=2.0, and θ_d/τ_1=0.1

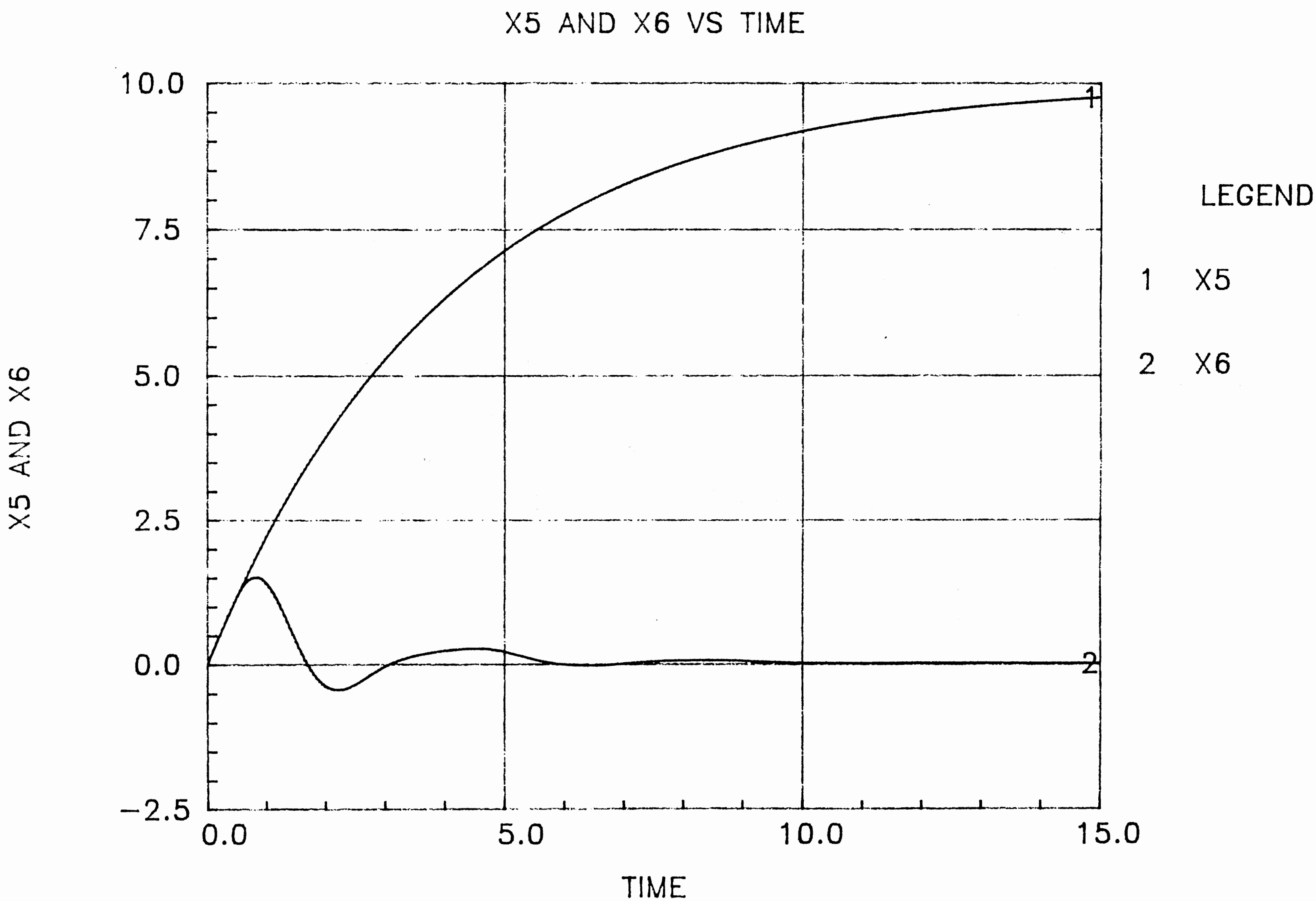

Figure 71. Response of X_5 and X_6 in Sample Application at τ_2/τ_1=0.5, τ_3/τ_1=4.0, and θ_d/τ_1=0.5

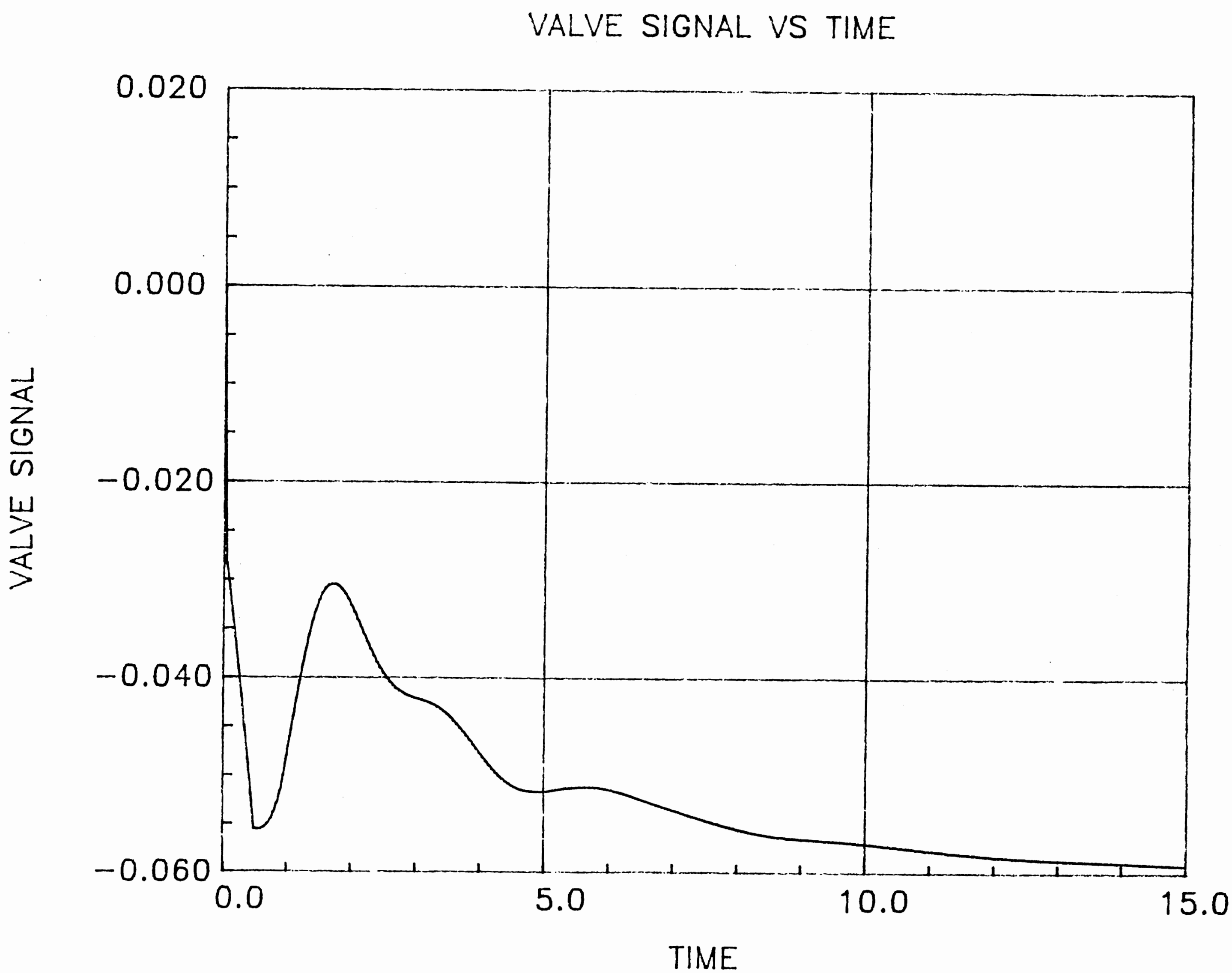

Figure 72. Valve Signal in Sample Application at τ_2/τ_1=0.5 τ_3/τ_1=4.0, and θ_d/τ_1=0.5

APPENDIX G

GRAPHS OF MAXIMUM

LOAD FRACTION

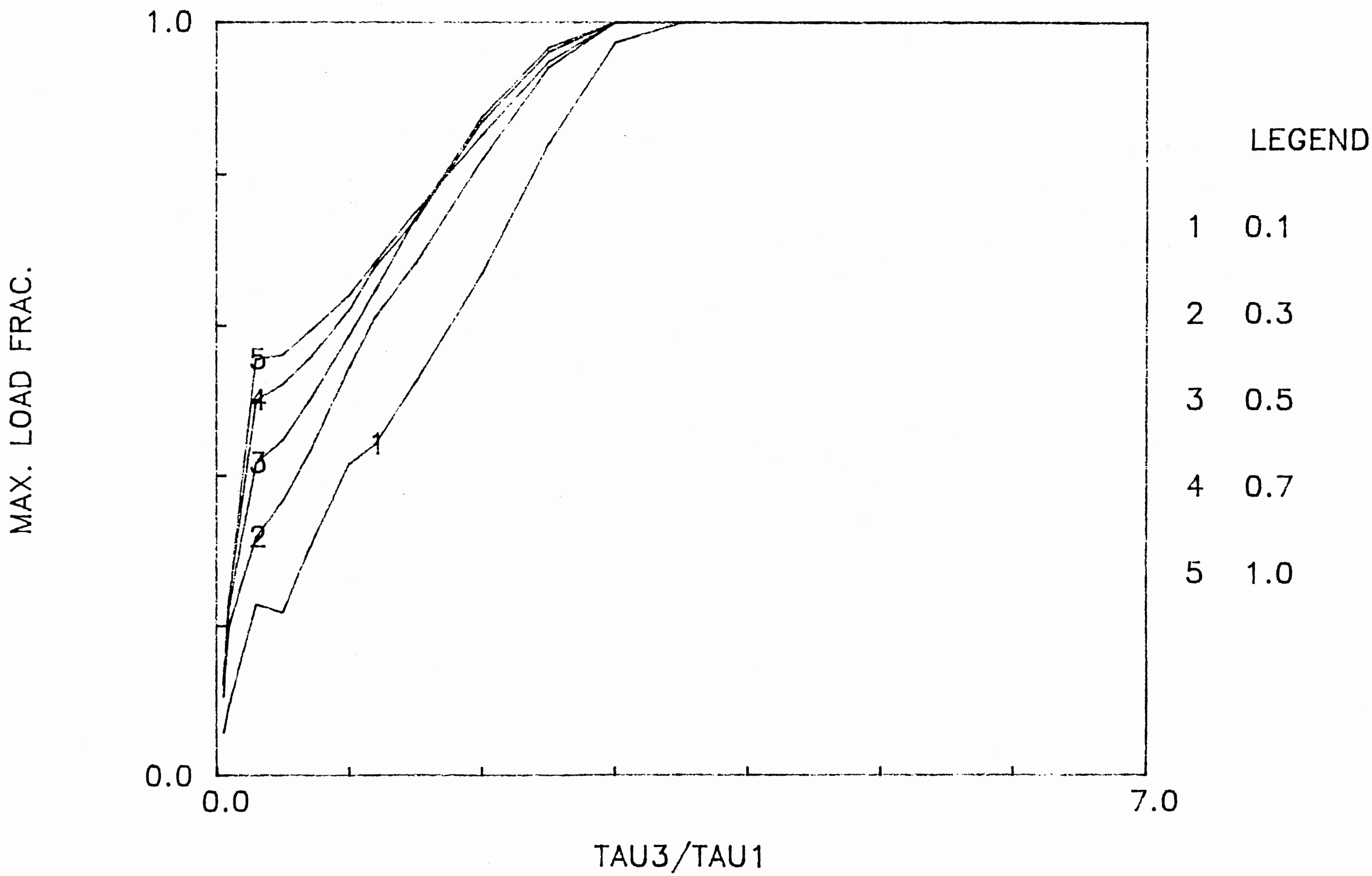

Figure 73. Max. Load Fraction at τ_2/τ_1=0.1

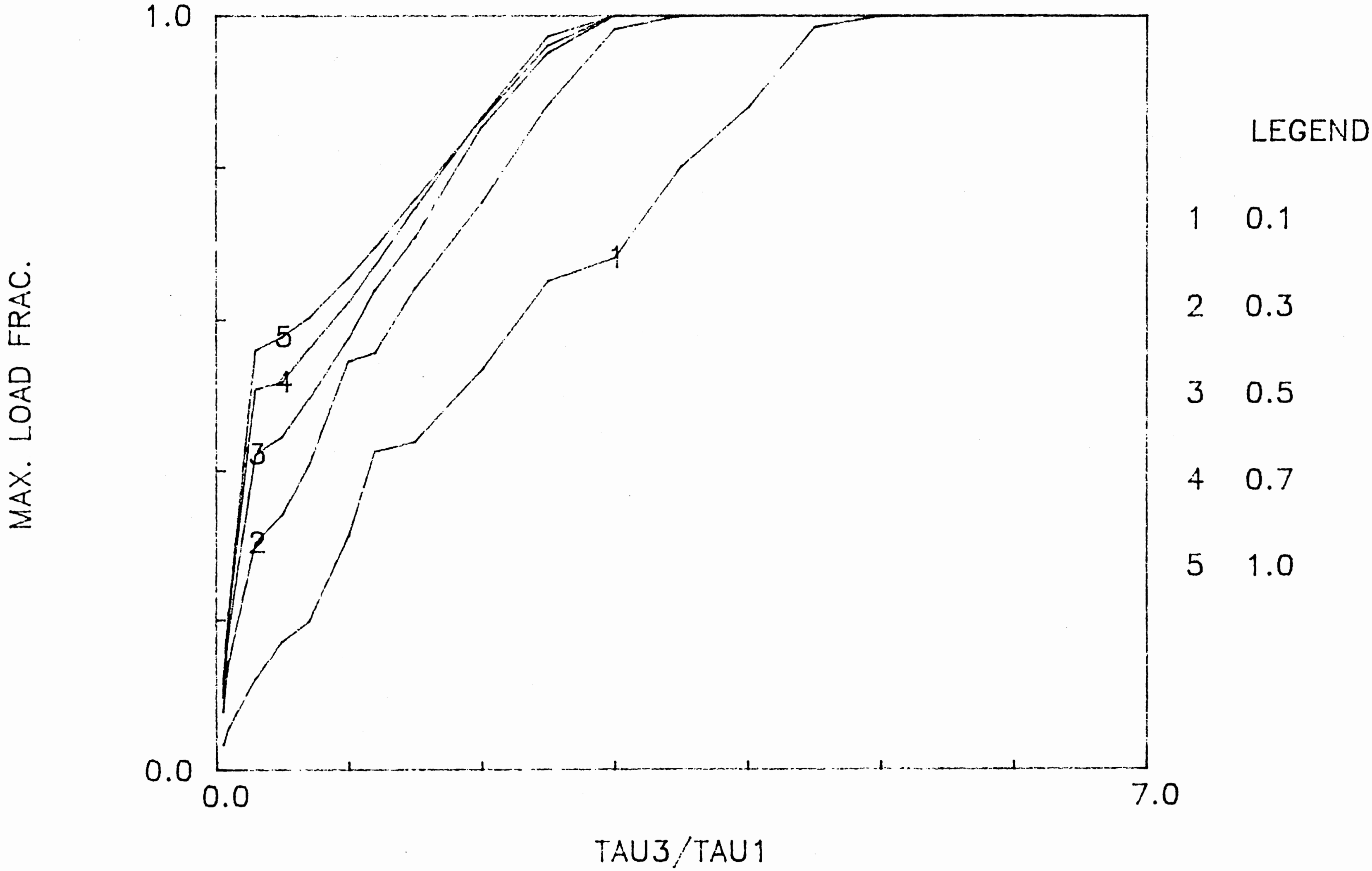

Figure 74. Max. Load Fraction at τ_2/τ_1=0.2

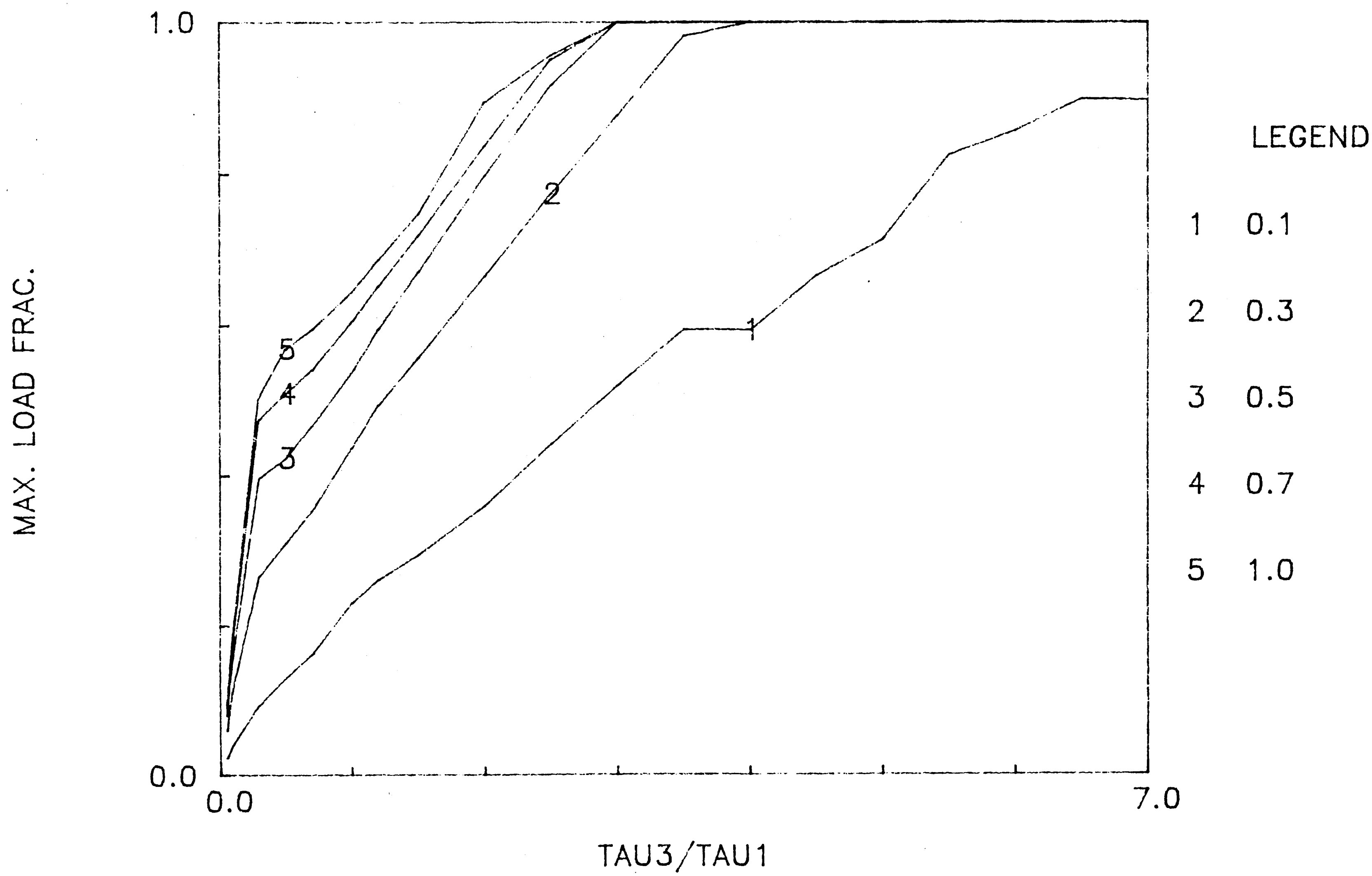

Figure 75. Max. Load Fraction at τ_2/τ_1=0.3

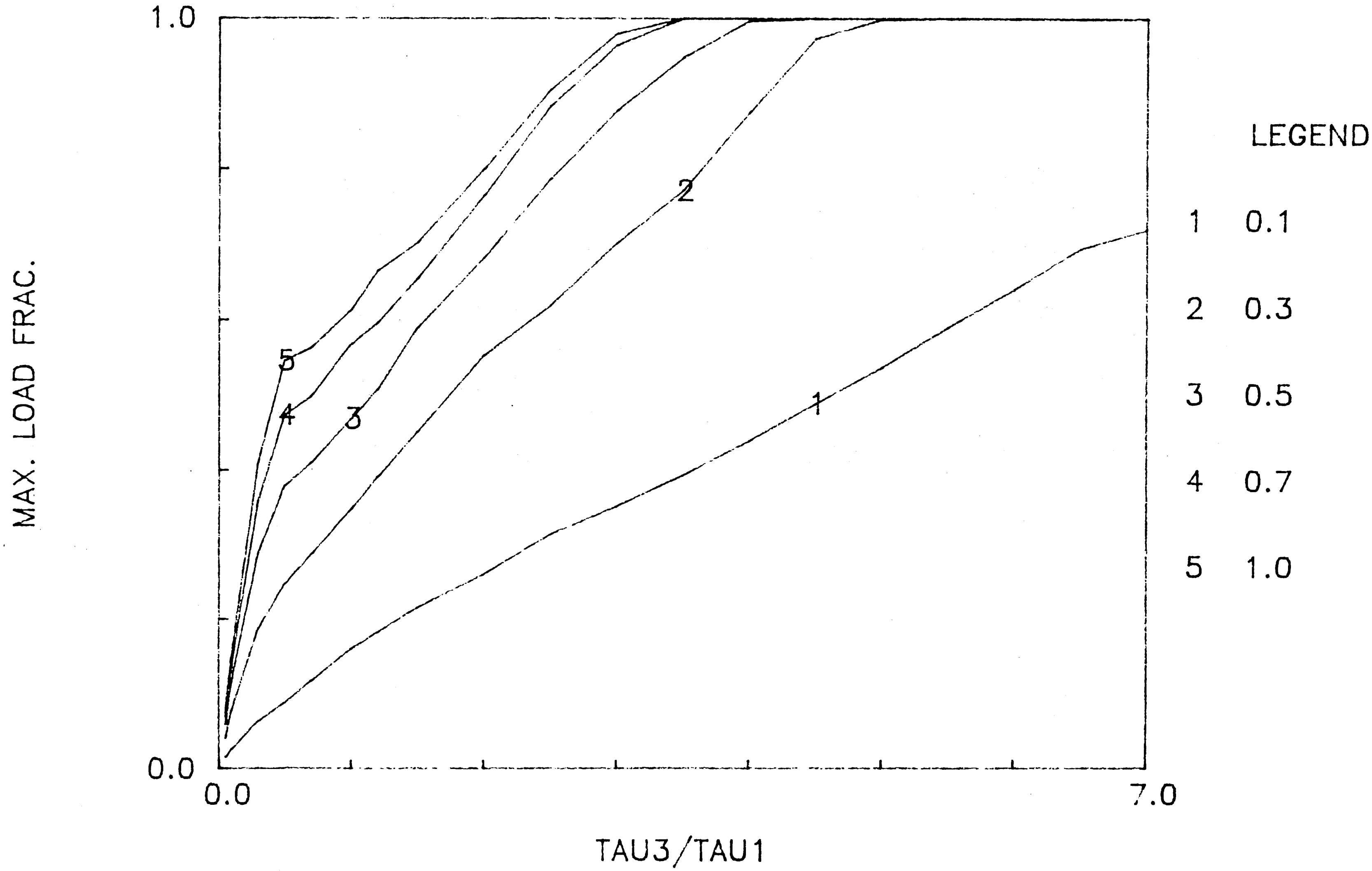

Figure 76. Max. Load Fraction at $\tau_2/\tau_1=0.4$

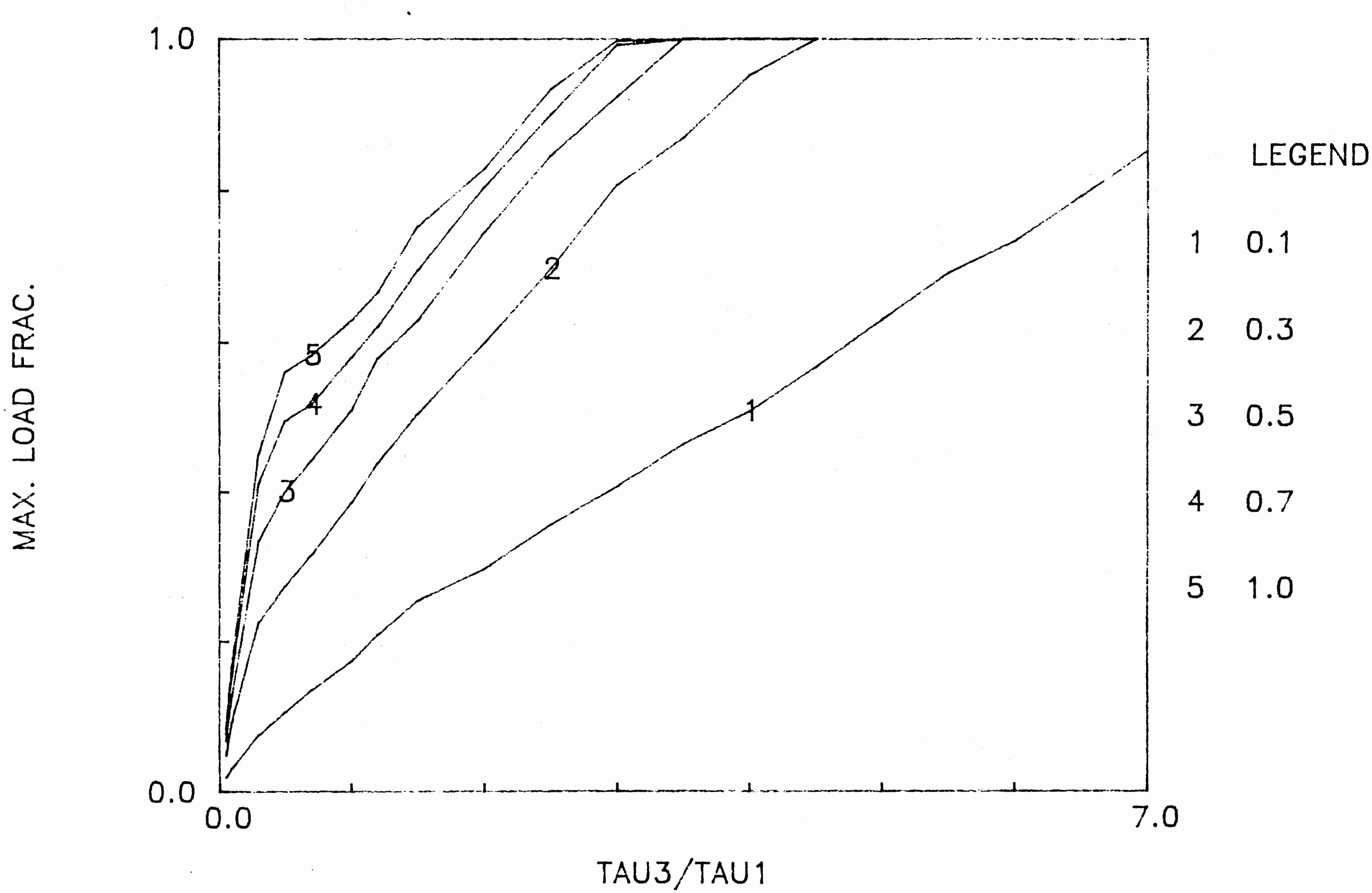

Figure 77. Max. Load Fraction at $\tau_2/\tau_1=0.5$

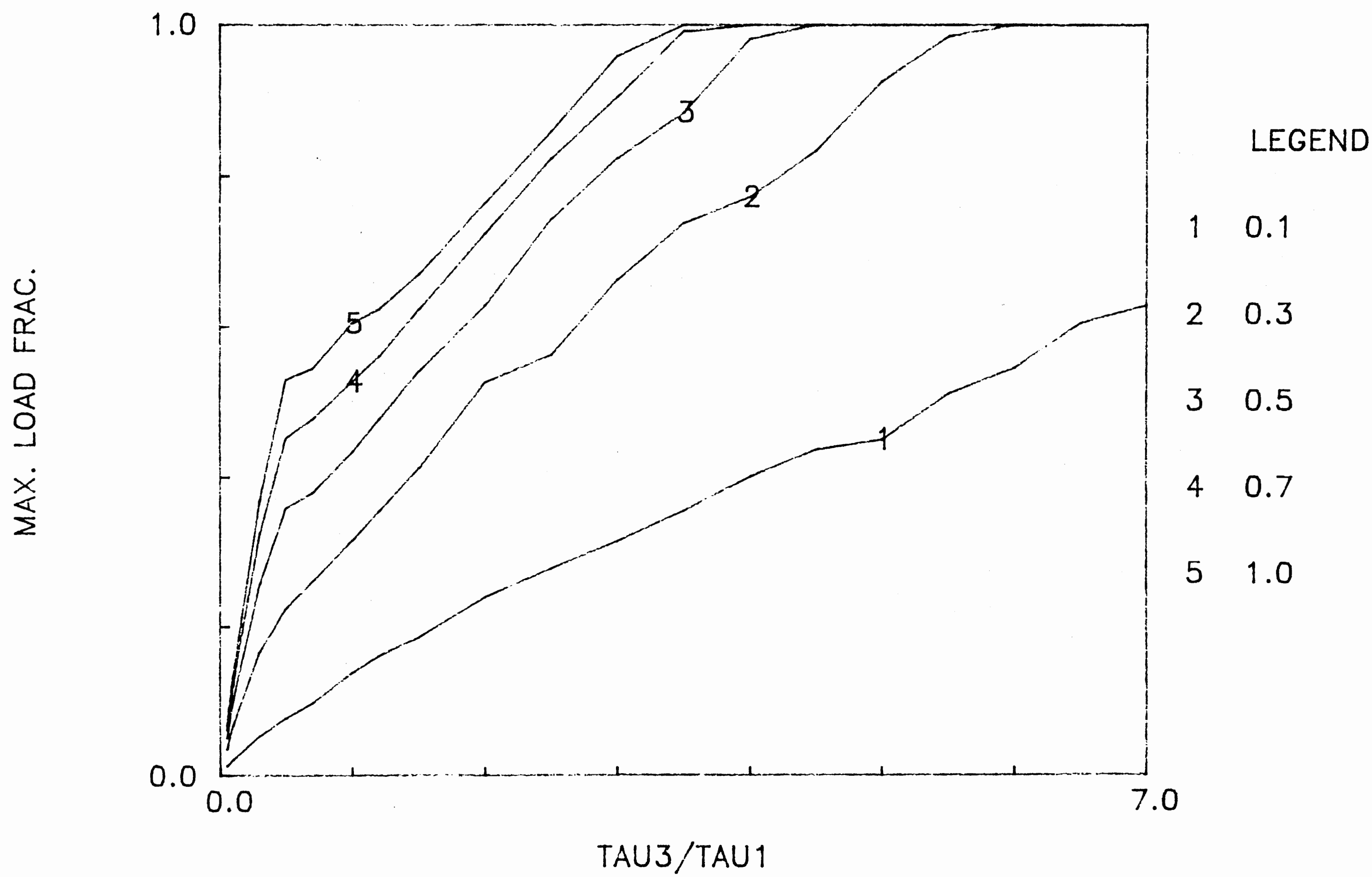

Figure 78. Max. Load Fraction at $\tau_2/\tau_1=0.6$

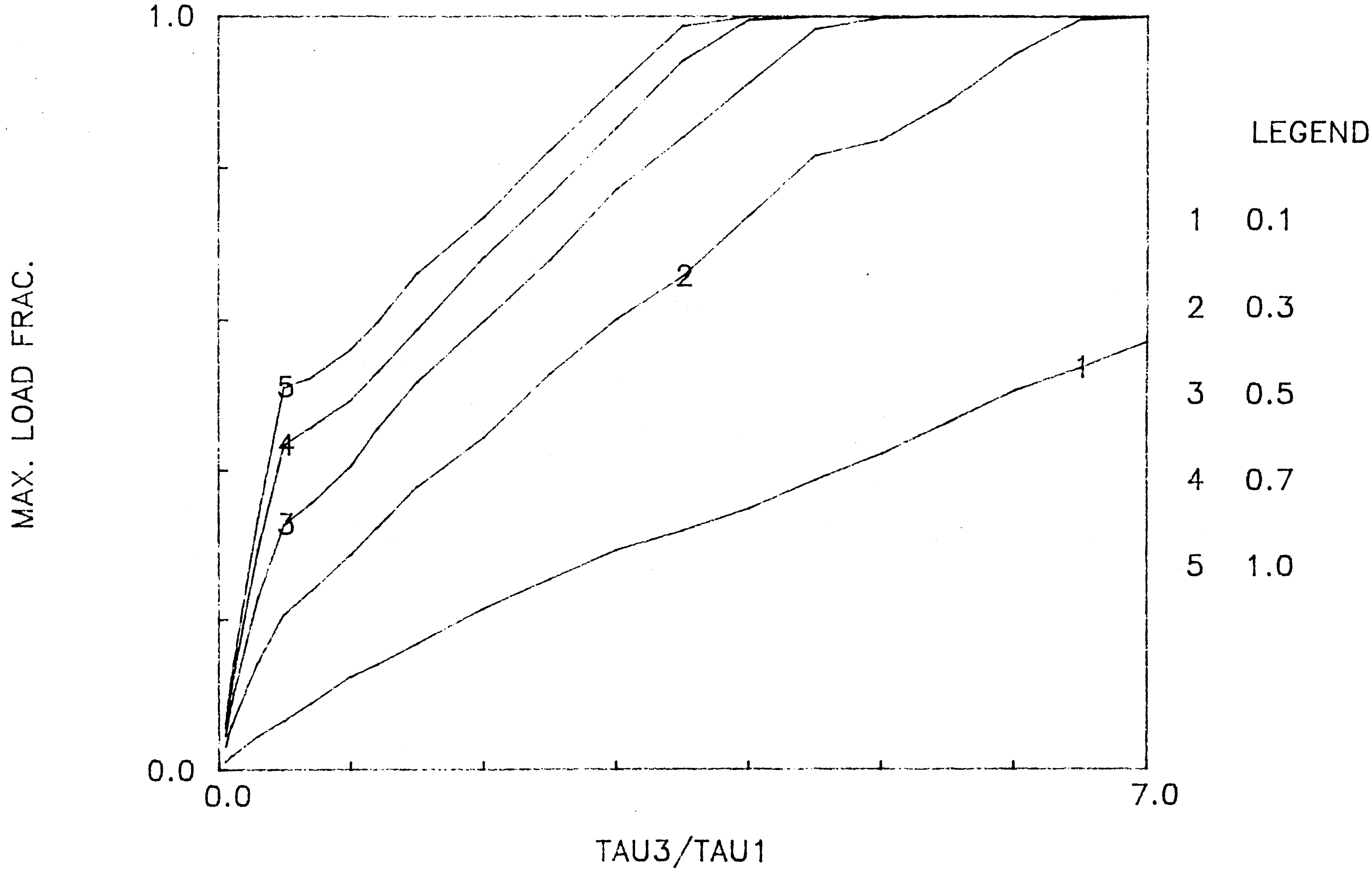

Figure 79. Max. Load Fraction at τ_2/τ_1=0.7

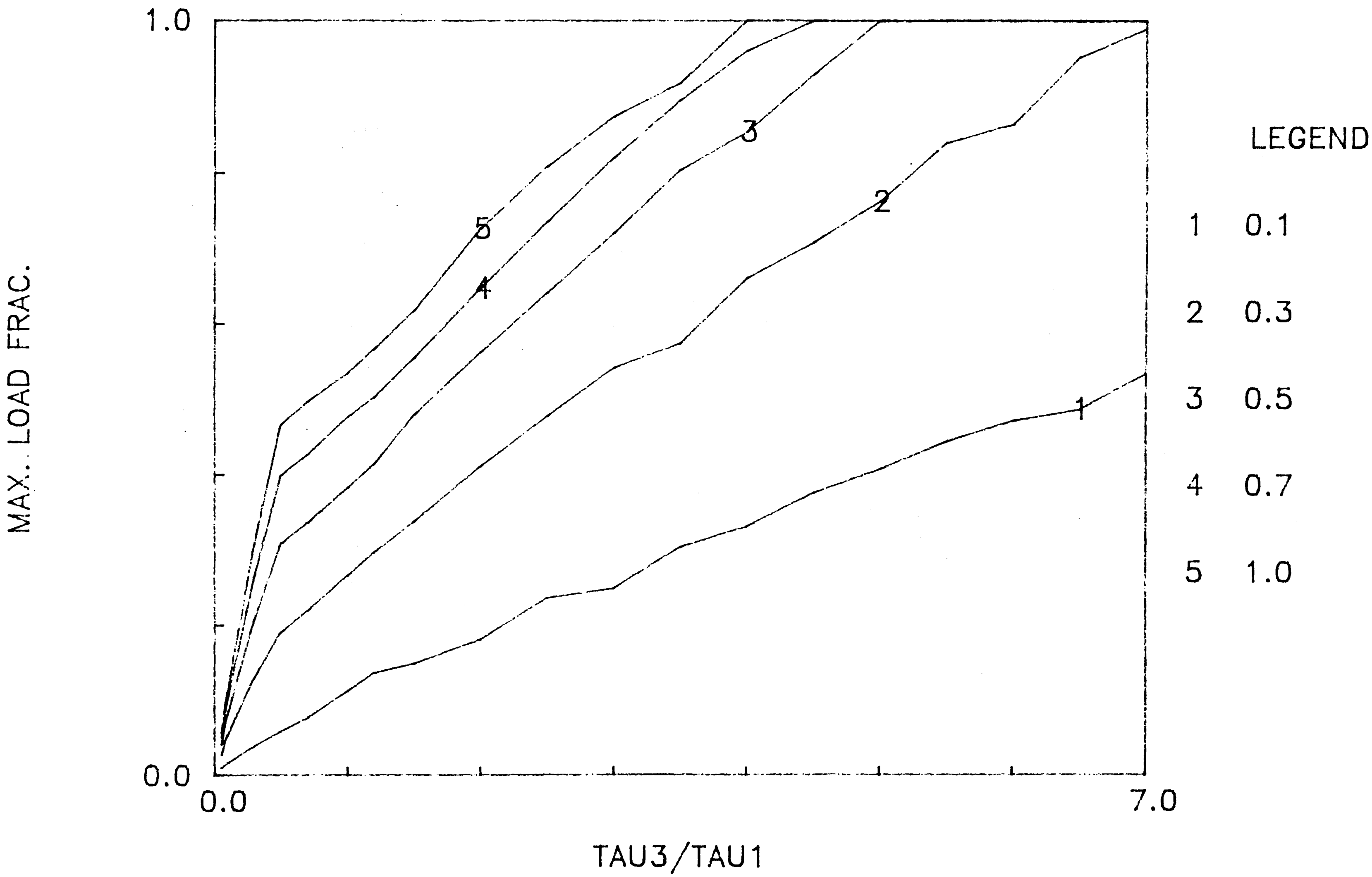

Figure 80. Max. Load Fraction at τ_2/τ_1=0.8

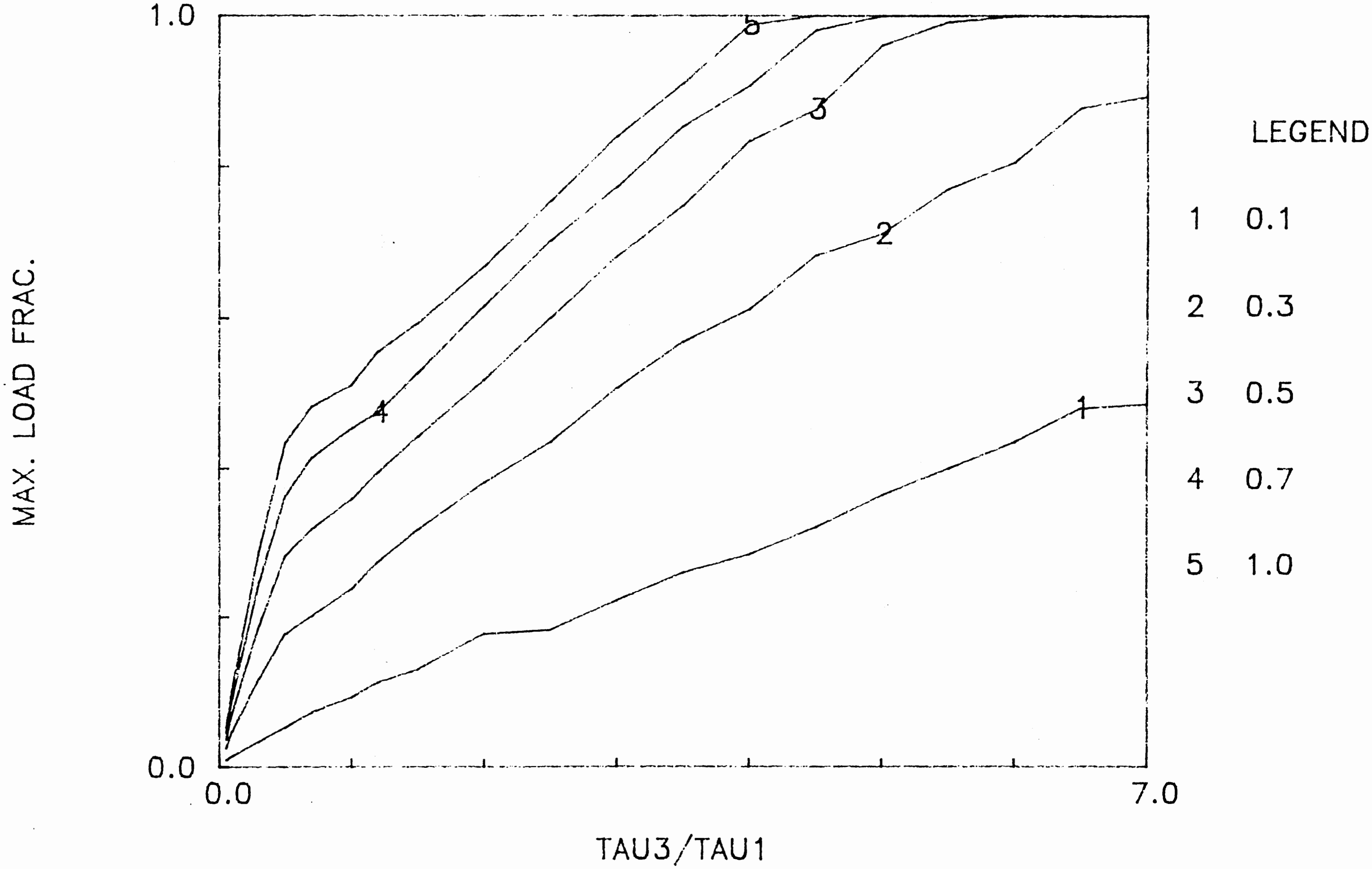

Figure 81. Max. Load Fraction at τ_2/τ_1=0.9

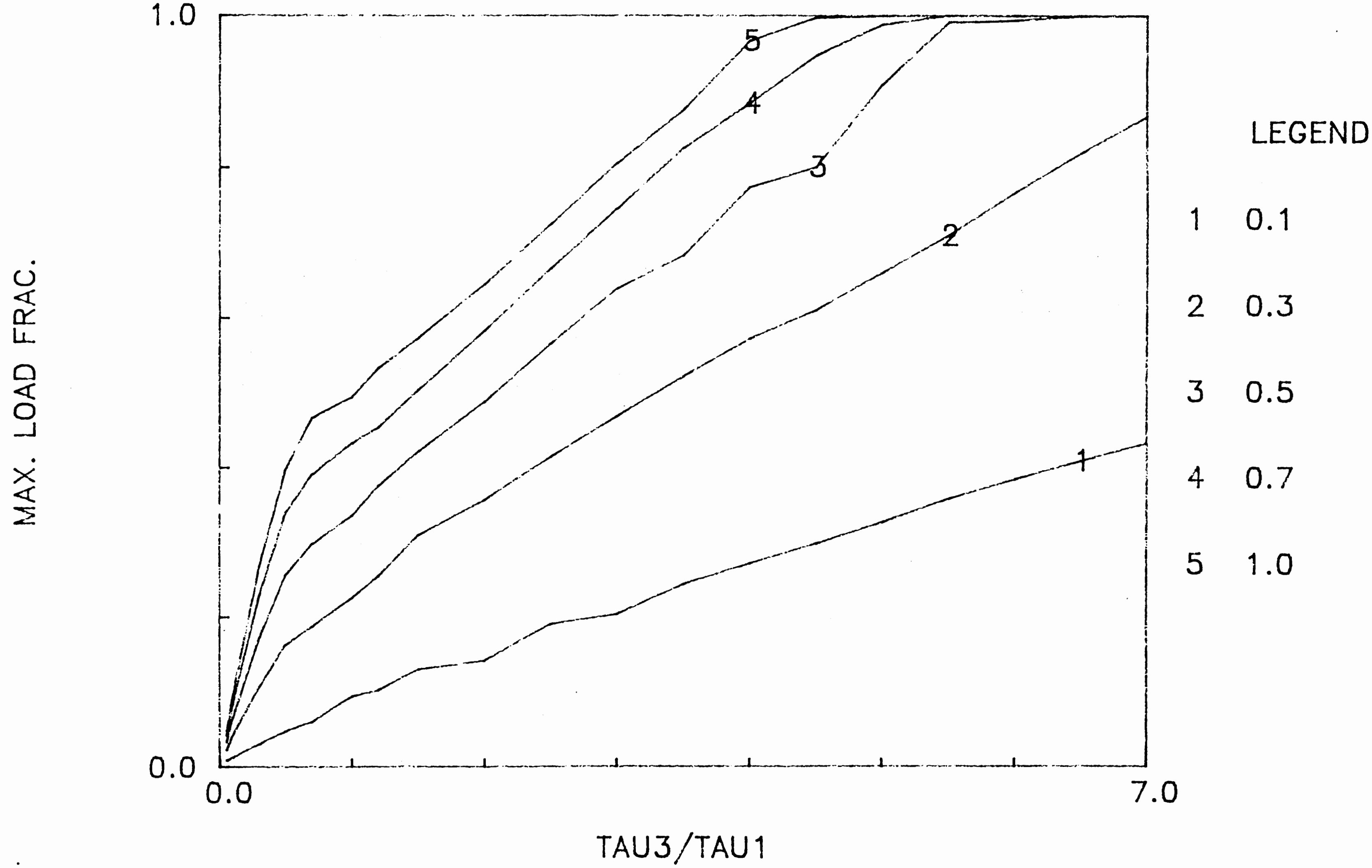

Figure 82. Max. Load Fraction at τ_2/τ_1=1.0

VITA

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