PROGRAM EVALUATION AND REVIEW TECHNIQUE: AN INNOVATIVE APPROACH TO PROJECT COMPLETION TIME

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PREFACE

This study presents a new approach to find an accurate and realistic estimate of the completion time of PERT type network projects. It compares the project completion times calculated by the new method with those of the standard textbook PERT method against bench mark values supplied by a simulation program.

Beta distribution with four parameters is assumed for each activity. The four moments of the paths are calculated and Schmeiser-Deutsch distribution is fitted to each path. The project completion time is computed as the maximum of n stochastic variables represented by each path length.

The project completion times found by this new technique are more accurate and much closer to the simulated values than those of PERT values. The significance is more prominent in cases where there are multiple paths with near equal lengths. The results may have practical applications in project management and overcome the perennial problem of "late projects."

I wish to express my sincere gratitude to all who made my stay at Oklahoma State University a highly productive and rewarding experience. I am immensely indebted to my major advisor and the Ph.D. committee chairperson, Dr. Hon-Shiang

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NOMENCLATURE

t_i	stochastic time of activity i
$\mu(t_i)$	expected time of activity i
$\mu(x_i)$	expected value of stochastic variable x_i
$\mu_m(x_i)$	mth central moment of x_i
μ_i	ith central moment
μ_i'	ith raw moment
$\sigma(t_i)$	standard deviation of activity i
P_i	path i
$\mu(p_i)$	expected value of P_i
$\sigma(p_i)$	standard deviation of $P(_i)$
$\mu(T)$	expected time of T
$\sigma(T)$	standard deviation of T
m	mode
T	project completion time
α_3	μ_3/σ^3 } skewness measures
β_1	$= (\alpha_3)^2$ } skewness measures
α_4	$= \beta_2 = \mu_4/\sigma^4$, kurtosis measure
Θ_1	first parameter of Schmeiser-Deutsch distribution
Θ_2	second parameter of Schmeiser-Deutsch distribution
Θ_3	third parameter of Schmeiser-Deutsch distribution
Θ_4	fourth parameter of Schmeiser-Deutsch distribution

$\beta(0,1,p,q)$	Standard Beta distribution over the range [0,1]
$\beta(a,b,p,q)$	four parameter Beta distribution
a,b	abscissa intercepts of the Beta distribution
p,q	shape parameters of the Beta distribution
p.d.f.	probability density function
c.d.f.	cumulative density function
S-D	Schmeiser-Deutsch

CHAPTER I

INTRODUCTION

Program Evaluation and Review Technique (PERT) is a network model widely employed to aid management in planning and controlling large projects. Malcolm et al. (1959) developed PERT in the late 1950's in an effort by the U.S. Navy Special Projects Office to speed up the Polaris missile project. PERT stresses probabilistic activity time estimates and is suitable for an environment typified by high uncertainty.

1.1 Representing Projects as Networks

A project is represented by a network or by a precedence diagram to depict major project activities and their sequential relationships. The diagram is composed of arrows, representing project activities, and nodes, named events, representing points in time when the activities represented by incoming arrows are completed and the activities represented by outgoing arrows can be started. There is only one starting (origin) node and one ending (terminal) node. Figure 1 is an example. A path is a continuous chain of activities from the starting node to the ending node, and each path will be identified by this chain of nodes. A network typically has many paths.

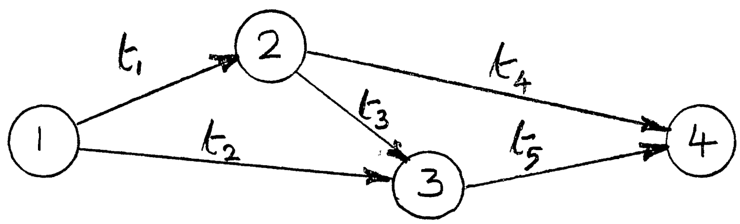

Figure 1. An Illustrative Network

The three paths in the given example are 1-2-4, 1-2-3-4 and 1-3-4, represented by P_1, P_2 and P_3, respectively. The critical path is the path with the longest duration and the critical activities are the activities on the critical path. The critical activities are the ones that need the maximum attention from the manager.

Let t_i be the stochastic time of the activity i. $\mu(t_i)$ and $\sigma^2(t_i)$ are the expected value and variance of t_i. Let p_i represent the stochastic path time of P_i, $\mu(p_i)$ its expected value and $\sigma^2(p_i)$ the variance. Then $p_1 = t_1 + t_4$, $p_2 = t_1 + t_3 + t_5$ and $p_3 = t_2 + t_5$. Let T represent the project completion time, $\mu(T)$ the expected time of T and $\sigma^2(T)$ its variance. The project completion time $T = \text{Max}[p_1, p_2, p_3] = \text{Max}[(t_1 + t_4), (t_1 + t_3 + t_5), (t_2 + t_5)]$.

The basic OR problems in PERT are: (1) to determine the probability distributions of the individual activities; and (2) given the distributions of the activities, find the distribution of the project completion time, its expected value and the variance.

1.2 Basic PERT Methodology

When a project manager consults an expert regarding the time span of a future activity, often the first time estimate that comes to mind is an approximation of the mode of distribution of possible time. This is the "most likely" time estimate "m". After the most likely time, the next information most experts can give with some confidence is an idea of the extreme times that would be required in cases of favorable and unfavorable situations. These are the optimistic time estimate "a", and the pessimistic time estimate "b" (Clark, 1962).

PERT handles stochasticity of individual activities by assuming the probable duration of an activity to be Beta distributed. A Beta distribution can assume different forms like "U", "J" ,"reversed J" or "inverted U". Malcolm et al. (1959), the authors of PERT, chose the "inverted U" Beta distribution to represent the activities. This type of Beta distribution is unimodal and has two positive abscissa intercepts. The mode and the two abscissa intercepts are equated to, the most likely, the optimistic and the pessimistic time estimates, respectively.

To take care of the fourth parameter of the Beta distribution the authors of PERT decided to choose a restricted Beta distribution (PERT Beta) that has a standard deviation equal to 1/6 of its range. The justification is that the Normal distribution truncated at ±2.66 has its standard deviation equal 1/6 of the range. According to PERT, the activity's expected time of completion is

$$\mu = (a + 4m + b)/6 \ . \tag{1.1}$$

The standard deviation and variance of completion time are:

$$\sigma = (b - a)/6 \ , \tag{1.2}$$

$$\sigma^2 = (b - a)^2/36 \ . \tag{1.3}$$

To obtain the distribution of the project completion time "T", PERT considers only the critical path, say P_1, whose expected time $\mu(p_1)$ is not less than any of the expected values of other paths. PERT calculates the expected time of the critical path as the sum of the expected times of the critical activities. $\mu(p_1)$ and $\sigma^2(p_1)$ are the project mean and the variance. The project duration is assumed to be normally distributed, drawing support from the central limit theorem.

1.3 Numerical Illustration

Consider an activity whose stochastic time t has a most likely estimate m = 12 days, optimistic estimate a = 9 days and pessimistic estimate b = 16 days. Equations (1.1) and (1.3) give the expected time and variance as:

$$\mu(t) = (a + 4m + b)/6$$

$$= (9 + 4*12 + 16)/6 = 12.17 \text{ days},$$

$$\sigma^2 = (b - a)^2/36$$

$= (16 - 9)^2/36 = 1.36.$

Consider now the network in Figure 2. The first and second numbers above each activity are the expected duration time of that activity in days and the variance, respectively.

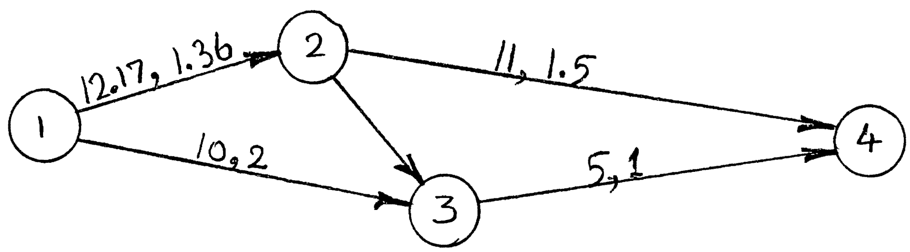

Figure 2. A PERT Network

The expected path times are:

$\mu(p_1) = 12.17 + 11 = 23.17$ days,

$\mu(p_2) = 12.17 + 9 + 5 = 26.17$ days and

$\mu(p_3) = 10 + 5 = 15$ days.

The critical path is P_2. The expected project completion time is then taken to be

$\mu(T) = \mu(p_2) = 26.17$ days,

and the variance of the critical path is

$\sigma^2(T) = \sigma^2(p_2) = 1.36 + 2 + 1 = 4.36.$

1.4 Directions for Improvement

Among others, MacCrimmon and Ryavec (1964) have shown that the preceding procedure of using only three parameters of the Beta distribution and considering only one critical path for project completion time is unsound. These assumptions can involve errors up to 25% in the activity time estimates and project completion time parameters.

There is an abundance of literature on the subject of PERT project completion times, but none of them has addressed the problem of finding the project completion time from the standpoint of using four parameter distributions for the activities and path times. The problem of multiple paths in calculating the project completion times is also not adequately dealt with. Advantages of using four parameter probability distributions for the activities and the path times are explained in chapter III. We address the above in this study.

Our objectives are: (i) to study the suitability of Schmeiser-Deustch distribution (1977) to represent a stochastic path time as the sum of several four-parameter Beta variables representing the activity-times in an individual path; and (ii) to compute the project completion time considering all the paths. PERT considers only one path, the critical path. We have developed methods to use Schmeiser-Deutsch distributions for estimating the maximum of several path times and to compute the expected project completion time, its variance, the third and the fourth central moments. Results of our method will be compared to the results of standard PERT and benchmarks results obtained with simulation.

CHAPTER II

LITERATURE REVIEW

2.1 Introduction

From among the hundreds of published papers on PERT and related network problems, this review will concentrate on those relating to distributional issues of stochastic activity, path and project completion times.

2.2 Problem Assumption

The first assumption in this study concerns the use of moments to represent the probability distributions. In any project we do not know the true underlying distributions of activity durations.

Kendall and Stuart (1969) have shown that we can approximate an unknown distribution by finding another distribution of known form using three or four moments. Statisticians usually accept moments as convenient summaries of a probability distribution.

2.3 Errors Due to PERT Assumptions

MacCrimmon and Ryavec (1962) pointed out that many errors can accrue due to the basic PERT assumptions. These errors are summarized in 2.3.1 and 2.3.2.

2.3.1 Errors at the Activity Level

The follwing sections explain the errors introduced at the individual activities level.

2.3.1.1 Errors Due to Using Beta Distribution.

Beta distribution seems appropriate for approximating the unknown actual distribution having the three properties of unimodality, continuity and two finite abscissa intercepts. See Appendix E for details and formulas for a four parameter Beta distribution. However, MacCrimmon and Ryavec (1964) compared three distributions having the range [0,1], modes at m and $0 \leq m < 1/2$. They are the Beta distribution, a quasi-uniform distribution and a quasi-delta function represented by D_1, D_2 and D_3 respectively in Figure 3.

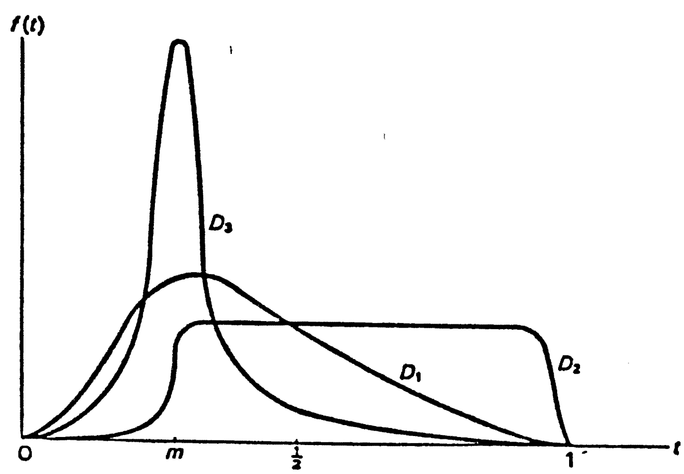

Figure 3. Three Illustrative Distributions

MacCrimmon and Ryavec showed that the possible error in the calculation of the mean depends on the position of the mode and skewness of the distribution. The worst absolute error in the mean is given by

$$\max\{|(4m+1)/6 - (m+1)/2|, |(4m+1)/6 - m|\}$$
$$= (2-m)/6. \qquad (2.1)$$

If the mode is allowed to vary between zero and one, the PERT-calculated error in the mean could be as much as 33.3 percent of the range. Even if the mode is more centralized, the maximum possible error would not drop below 25 percent.

Using a similar approach, MacCrimmon and Ryavec also showed that the worst absolute error in standard deviation is about 17 percent and does not depend on the mode.

2.3.1.2 Errors Caused by Using Equations (1,2) and (1.3). The possible errors from using the approximations $\mu(t) = (a + 4m + b)/6$ (equation 1.1) and $\sigma(t) = (b - a)/6$ (equation 1.2) can be analyzed by comparing them with the actual values for the mean and standard deviation of a Beta distribution (on [0,1]). The expressions for the actual mode, mean and standard deviation of the Standard Beta distribution (x: 0, 1, p, q) are:

Mode: $m = p/(p + q)$ (2.2)

Mean: $\mu = (p + 1)/(p + q + 2)$ (2.3)

Standard deviation:

$\sigma = \sqrt{[(p + 1)(q + 1)/(p + q + 2)^2 (p + q + 3)]}$ (2.4)

The mean and the standard deviation may be rewritten as functions of p and m, and these appear as the second terms in the error expressions below. The worst absolute error in the mean is

$$|1/6(4m + 1) - m(p + 1)(p + 2m))| \qquad (2.5)$$

The worst absolute error in the standard deviation is

$$|[(1/6) - \sqrt{\{m^2(p + 1)(p - pm + m)\}/\{(p + 2m)^2(p + 3m)\}}]| \qquad (2.6)$$

The worst absolute error in the mean can be 33 percent, and in the standard deviation 17 percent. This occurs for extreme values of p and m. If we assume $1 \le p \le 6$ and $|(1/2) - m| \le 1/6$, then the errors in the mean and standard deviation reduce to 4 percent and 7 percent respectively (MacCrimmon and Ryavec, 1962).

2.3.2 Errors at the Network Level

The longest path, P_1, is called the critical path. Following the notation we used in the previous paragraphs, we mentioned that PERT uses $\mu(p_1)$ and $\sigma(p_1)$ as the project mean and standard deviation, and assumes the project completion time is normally distributed. If there is more than one path with the largest expected value, PERT labels them as the critical paths and uses the path with the largest variance as P_1. But the correct distribution for the project is given by $F(t) = Pr(\max_i p_i \le t)$. The expected value of the random variable $T = \max_i p_i$ is greater than the expected value of any one of the p_i's. Therefore the PERT calculated mean is generally less than and never greater than the true project mean. The PERT calculated standard deviation can be greater or less than the actual standard deviation. If the distribution is symmetric and has finite tails, the actual standard deviation will be less than any of the σ_i. However, if the distributions are considerably skewed to the right, the reverse may be true (MacCrimmon and Ryavec, 1964).

If there is one path through a network that is significantly larger than any other path, then the PERT procedure for calculating the project mean and the standard deviation will give approximately the correct results. But, if there are a large number paths having approximately the same length and having few activities in common, substantial errors will be introduced in the PERT-calculated project mean and standard deviation. The more parallel paths a network has, the larger the error.

2.4 The Literature on Activity Distributions

The research literature on the problem of finding a suitable probability distribution to closely approximate the actual activity distribution is meager. Clark (1962) originally used end points or extremes for computing the standard deviation for the individual activities. Subsequently Pearson and Tukey (1965) found that using the 5th (p_5) and 95th (p_{95}) percentiles leads to better estimates of the variance. Moder and Rodgers (1968) suggested using the 5th and 95th percentiles to find the standard deviation by the relatively distribution-free approximation formula, $\sigma = (p_{95} - p_5)/3.20$. One interesting study was conducted by Perry and Greig (1975). They suggested using the 5th and 95th percentiles and a heuristic formula,

$$\sigma = (p_{95} - p_5)/3.25, \qquad (2.8)$$

which they claim is more accurate than Moder and Rodgers'. They also claimed that the formula for the mean, i.e., $\mu = (p_5 + 0.95m + p_{95})/2.95$, is distribution free and is very accurate.

2.5 A Classification of the Literature on Project Completion Times

Most of the PERT research published in the last three decades deals with the problem of finding the completion time distribution of a PERT network. Works on this problem can be classified into four main groups (Scully and Wong 1985).

2.5.1 The First Group

The first group suggested methods to find a lower bound approximation to the distribution function of the total completion time and derive bounds for the expected value and variance of the total completion time. Examples of the first group's work to find close lower and upper bounds to the completion time distribution include Robillard and Trahan (1976) and Kleindorfer (1971).

Robillard and Trahan computed a lower bound approximation for the total duration of the PERT network and used this approximation to propose an upper bound for the expected value of the total project duration and a lower and upper bound for its variance. They also assumed that all activities were normally distributed, which may not be appropriate since the normal distribution has only two parameters.

Kleindorfer (1971) obtained upper and lower bounding distributions for the activity starting- and finishing-time probability distributions, as well as upper and lower bounds for the expected starting and finishing time for each network activity, and for expected network flows. But they did not find the exact completion-time probability distribution of the network or of any activity in it, rather, they showed only how to find distributions that bound the activity- and completion-time probability distributions from above and below.

2.5.2 The Second Group

The second group of research literature consists of approximations in which the distributions of the individual activities are assumed to be of the discrete type. This approximation involved the manipulation of a fixed number of time values and the corresponding probabilities. They were usually suitable for implementations on digital computers.

However, using discrete random variables to represent activity times does not seem to be a sound idea. Two examples are Fulkerson (1962) and Elmaghraby (1967). Fulkerson suggested a method of obtaining a fairly good lower approximation to the expected duration time of a project whose individual activity times are discrete random variables. He assumed independence among activities. Elmaghraby's method was an extension of Fulkerson's results in two different directions to obtain closer approximations, but his improved result was obtained at the expense of extra computing effort.

2.5.3 The Third Group

The third group's approach consists of approximations, where the computations involve the manipulation of distribution parameters. Representative works are by Clark (1961), Greer and LaCava (1979) and Sculli (1985). Normal distribution was assumed for individual activities. Given multivariate-normally distributed t_i's (i = 1 to n) with arbitrary means and variance-covariance matrix, Clark (1961) derived approximate formulas for the first four moments of max(t_i, i = 1 to n). Thus Clark's paper provided formulas to find the expected value and variance of the greatest of a finite set of normally distributed variables. Greer and LaCava applied Clark's formulas to find normal approximations for the greater of two random variables. Sculli proposed a simpler approximation for the completion time, mean and variance of PERT networks in which the durations of individual activities were normally and independently distributed.

2.5.4 The Fourth Group

The fourth group used simulation to estimate the completion time distribution. Van Slyke (1963) developed the idea of using crude simulation as a tool for finding the cumulative density function (c.d.f.) of a PERT network completion time. He also suggested two methods for potentially reducing simulation computation times. Klingel (1966) used a crude simulation approach to study the direction and magnitude of the errors of PERT methodology when parallel paths are present in a network.

Stratification, control variates and regression were suggested by Burt and Garman (1971, 1971) as ways to reduce the computational effort required in crude simulation. They also developed a new simulation procedure called Conditional Monte Carlo Simulation in which certain activity times were fixed at their original sampled value thus reducing computational effort and variance. But simulation is a very costly and time consuming process.

2.6 Where do We Stand To-day?

A direct and general solution appears to have escaped the researchers so far. Difficulties arise because of the need to find the maximum of a set of stochastic path times. As such, as of date, satisfactory solutions or techniques to the problem of finding a suitable approximation for activity distribution and to deal with multiple parallel paths without extensive and costly simulation techniques in PERT networks are not available. In the next chapter we propose some approaches to deal with the problem of multiple parallel paths in PERT networks and to find a more realistic and accurate project completion time than that of PERT.

CHAPTER III

RESEARCH METHODOLOGY

3.1 Introduction

Modern day project managers and the supporting staff can afford to use more sophisticated formulas and statistical techniques to determine the activity distributions and project completion times with the help of computers. The procedures in this thesis are developed with the above prospect into consideration.

3.2 Research Problems

Two distinct problems we consider are:

1) To determine the underlying statistical distribution of individual activities.
2) To determine the network completion time distribution when the distributions of individual activities are known. To do this we go through two steps:
    a) Determine the distribution of individual paths.
    b) Compute the maximum of two or more random variables representing different parallel paths in the network.

3.3 Using Four Parameter Distributions

In chapter II we briefly mentioned the conditions under which Malcolm et al. (1959) chose the Beta distribution and tailored it to suit the prevailing conditions, the existing computing power and facilities available to the managers in the late fifties.

From basic statistics the four most observable characteristics of a distribution are : 1. Location, 2. Dispersion, 3. Skewness and 4. Kurtosis. Karl Pearson (1895) pointed out that to fit adequately the four independently varying characteristics, a mathematical distribution must have at least four free parameters. Although the Beta distribution has four parameters, PERT effectively uses only three parameters when the approximate formulas (1.1) to (1.3) are used. In this study four-parameter distributions will be used to model the stochastic times of activities and projects.

3.4 A Set of Useful formulas

Hahn and Shapiro (1968) have given exact formulas for calculating the first four central moments of the sum of n independent stochastic variables with known first four central moments.

Let $y = (x_1 + x_2 + . . . + x_n)$, where the x_i's are independent stochastic variables whose first four moments are known. Define $\mu(x_i) = E(x_i) =$ expected value of x_i; and

$\mu_m(x_i) = (x_i - \mu(x_i))^m$ = the mth central moment of x_i. Then

$$\mu(y) = \sum_{i=1}^{n} \mu(x_i) \tag{3.1}$$

$$\mu_2(y) = \sum_{i=1}^{n} \mu_2(x_i) \tag{3.2}$$

$$\mu_3(y) = \sum_{i=1}^{n} \mu_3(x_i) \tag{3.3}$$

$$\mu_4(y) = \sum_{i=1}^{n} \mu_4(x_i) + 6\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \mu_2(x_i)\mu_2(x_j) \tag{3.4}$$

The above formulas are exact and valid for all distribution forms. These formulas will be used to find the four moments of the path times and the project completion times.

3.5 β_1 - β_2 Diagram

One way to study the versatility of a distribution is through a β_1-β_2 diagram. Figure 4 is adapted from the standard β_1-β_2 diagram in Pearson and Hartley (1970) and Hahn and Shapiro (1968).

β_1 and β_2 are respectively the square of the standardized measure of skewness and the standardized measure of peakedness (kurtosis).

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} \tag{3.5}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} \tag{3.6}$$

Each point in the diagram represents a (β_1-β_2) combination. (β_1-β_2) combinations above the line AAA in Figure 4 are

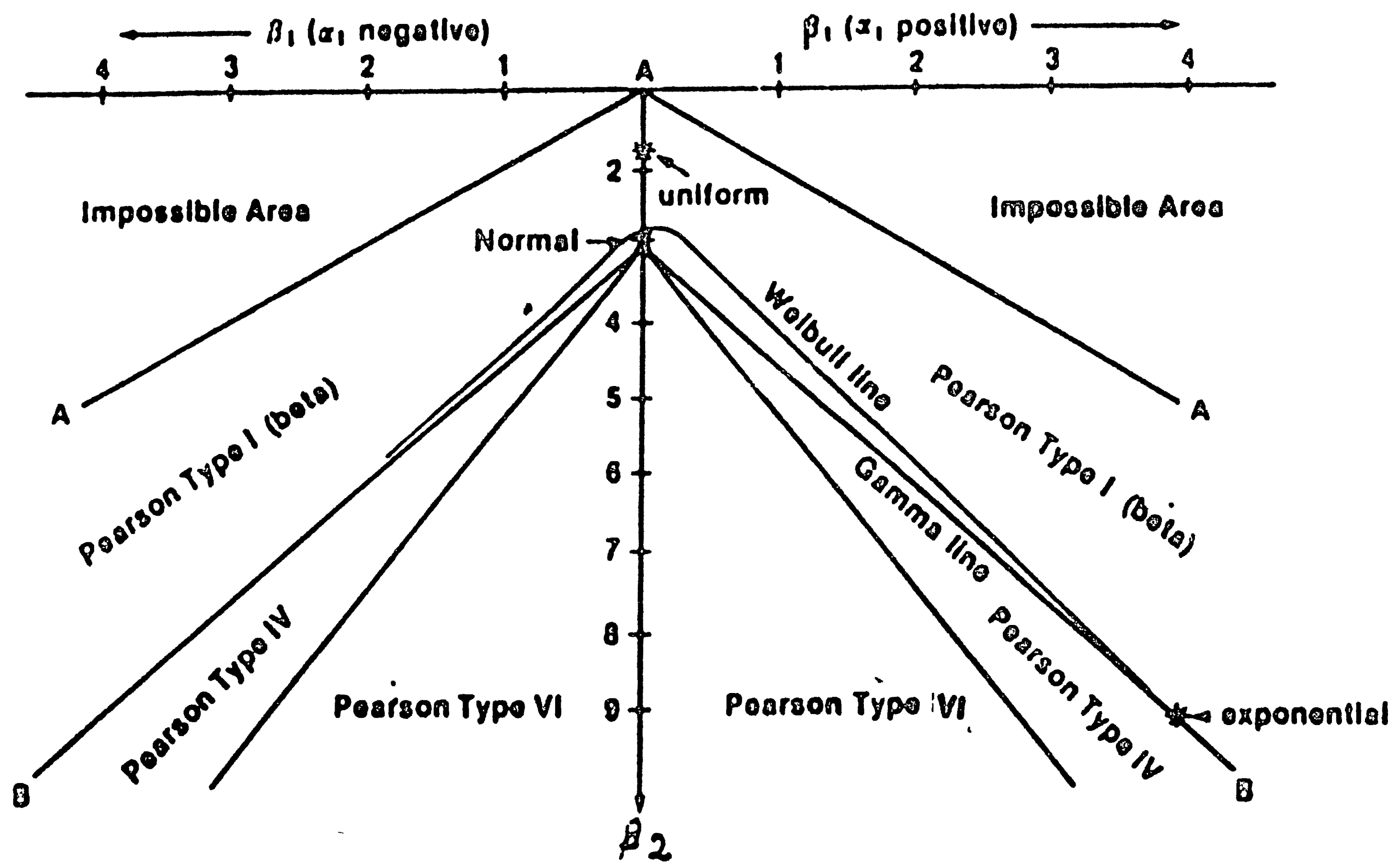

Figure 4. $\beta_1 - \beta_2$ Diagram

mathematically impossible, but empirical distributions can have any (β_1-β_2) combinations below the line.

We need a system of density functions to represent different path times and the project completion time. Ideally this density function should cover the entire area under AAA. For the Normal distribution, $\beta_1 = 0$ and $\beta_2 = 3$. This is represented by one point in Figure 4. Exponential and Uniform distributions are also represented by single points. Gamma, Weibull, Lognormal and t-distribution are represented by straight lines. The four-parameter Beta distribution covers all β_1-β_2 combinations between AAA and BBB, but it cannot cover the large area under BBB. Examples of four parameter distributions that cover the entire area under AAA are the Pearson's, Johnson's and Schmeiser-Deutsch's systems. We limit our study to the Pearson's and Schmeiser-Deutsch's systems.

3.6 Pearson Distribution

The family of distributions defined by

$$\frac{df(x)}{dx} = \frac{(x - a)\ f(x)}{b_0 + b_1 x + b_2 x^2} \tag{3.7}$$

are known as the Pearson system of distributions (Kendall & Stuart, 1969); The solution to (3.7) gives density functions labeled Type I through Type XII.

Of the above Types I, IV and VI cover areas of (β_1-β_2)

combinations as shown in Figure 4, and are known as the "main types." Others are "transition types" and cover only lines or points of (β_1-β_2) combinations. Together, they cover the entire feasible β_1-β_2 area under AAA. Although K. Pearson (1895) obtained closed-form density functions for all Types of the system, closed-form cumulative distribution function and inverse cumulative distribution function do not exist for most Types.

The solution of equation 3.7 leads to large number of distribution families, including the Normal, Beta (Pearson Type I), and Gamma (Pearson Type III) distributions.

3.7 Schmeiser-Deutsch Distribution

The cumulative distribution function and the distribution function of the Schmeiser-Deutsch distribution are:

$$F(x) = p = \begin{cases} \theta_4 - [(\theta_1 - x)/\theta_2]^{1/\theta_3} & \text{if } \theta_1 - \theta_2\theta_4^{\theta_3} \le x \le \theta_1 \\ \theta_4 + [(x - \theta_1)/\theta_2]^{1/\theta_3} & \text{if } \theta_1 \le x \le \theta_1 + \theta_2(1-\theta_4)^{\theta_3} \end{cases} \tag{3.8}$$

$$f(x) = [1/\theta_2\theta_3] \; |(\theta_1 - x)/\theta_2|^{[(1-\theta_3)/\theta_3]} \tag{3.9}$$

$$\text{for all } x \in [\; \theta_1 - \theta_2\theta_4^{\theta_3} \;,\; \theta_1 + \theta_2(1-\theta_4)^{\theta_3} \;]$$

where θ_1 and θ_2 are the location and scaling parameters

and Θ_3 and Θ_4 are the shape parameters.
Straightforward procedures are available for determining the parameters Θ_1, Θ_2, Θ_3 and Θ_4 for a given empirical distribution. Symmetric distributions correspond to $\Theta_4 = 0.5$. For $\Theta_3 > 1$, skew is to the right for $\Theta_4 < 0.5$ and to the left for $\Theta_4 > 0.5$. For $\Theta_3 < 1$, the direction of skew is reversed. For $\Theta_3 > 1$, the unique mode is at $x = \Theta_1$. $\Theta_3 = 1$ gives a uniform distribution. The exponential distribution is a limiting distribution of this family.

The inverse cumulative distribution function is

$$x = F^{-1}(p) = \begin{cases} \Theta_1 - \Theta_2(\Theta_4 - p)^{\Theta_3} & \text{if } 0 \le p \le \Theta_4 \\ \Theta_1 + \Theta_2(p - \Theta_4)^{\Theta_3} & \text{if } \Theta_4 \le p \le 1 \end{cases} \qquad (3.10)$$

This closed form inverse cumulative distribution function enables Schmeiser-Deutsch distributed variates to be generated easily with the inverse transformation technique.

3.8 Research Objective I

Research objective I is to determine the first four central moments of the distribution for the individual path times when the first four moments of the distributions for the individual activities comprising the path are known. In the past although many authors have studied the suitability of distributions like Beta or Normal to represent path times, it seems that none has investigated methods to compute path times as the sum of more than two activities represented by four-parameter distributions.

Once we find the first four moments of the variables for the individual path times, the next step is to compute the network completion time.

3.9 Research Objective II

Research objective II is to determine a suitable distribution for the network completion time and its first four central moments. To achieve this objective it is necessary to compute the maximum of two or more four-parameter random variables, representing different parallel path times in the network.

In chapter IV we explore the suitability of using the Pearson distribution to find the maximum of n stochastic variables represented by Uniform, Normal, Beta and Gamma distributions. In chapter V we fit the Schmeiser-Deutsch distribution to the path times and then find the maximum of all the path times.

CHAPTER IV

USING PEARSON DISTRIBUTIONS TO DERIVE THE DISTRIBUTION OF PROJECT COMPLETION TIME

4.1 The Basic Approach

The technique described below has been successfully used (e.g., Lau 1986) to estimate the distribution of the maximum of several random variables.

For a random variable X, define

$$X^+ = \text{Max}(X, 0) = \begin{cases} X \text{ if } X > 0 \\ 0 \text{ if } X = 0 \end{cases} \tag{4.1}$$

That is, the random variable X^+ takes the value of the random variable X when X is positive but the value of 0 when X is negative. If y is the maximum of random variables t_1 and t_2, equation (4.1) enables y to be stated as

$$y = t_2 + (t_1 - t_2)^+ \tag{4.2}$$

$$= t_2 + t_0^+ \tag{4.3}$$

where $t_0 = t_1 - t_2$. (4.4)

Therefore,

$$E(y) = E(t_2) + E[(t_1 - t_2)^+]$$

$$= \mu_2 + E(t_0^+) \tag{4.5}$$

Given the empirical distributions or distribution functions of t_1 and t_2, the following steps can now be used to find y's moments:

Step 1: Use equations (3.1) to (3.4) to compute the first four moments of t_o.

Step 2: Use t_o's first four moments to fit a Pearson distribution to t_o.

Step 3: Use the fitted Pearson distribution and Winkler et al.'s (1972) method to find the moments of t_o.

Step 4: Use equations (3.1) to (3.4) with equation (4.2) to find the moments of y.

Steps 2 and 3 are explained in the following sections. If y is the maximum of more than 2 variables, the above procedure can be reiterated, handling two variables at a time, to determine y.

4.2 Step 2: Fitting a Pearson Distribution

Out of the twelve types of Pearson density functions, types I, IV, VI, III and V collectively cover the entire feasible continuum in the $(\beta_1 - \beta_2)$ diagram. They are the only types considered below.

To fit a variable with given first four moments to a Pearsn distribution, first compute the value of the "criterion" K:

$$K = \frac{\beta_1(\beta_2 + 3)^2}{4(2\beta_2 - 3\beta_1 - 6)(4\beta_2 - 3\beta_1)} \tag{4.6}$$

K may have any value from $-\infty$ to ∞ and the following diagram shows how the different Pearson types cover all the possible values of the criterion and do not overlap.

$K = -\infty$		$K = 0$		$K = 1$		$K = -\infty$
	K negative		$K>0$ & $K<1$		$K > 1$	
	Type I		Type IV		Type VI	
Type III		Normal curve when $\beta_2 = 3$; Type II (VII) when $\beta_2<3$ (>3)		Type V		Type III

A given K-value enables one to identify the appropriate type of Pearson distribution to fit. Detailed below are procedures for fitting Types I, IV, VI, III and V of the Pearson distributions.

4.3 Fitting Pearson Type I Distribution

When $K < 0$ a Type I distribution is the suitable one.

$$y = y_0(1 + x/a_1)^{m_1} (1 - x/a_2)^{m_2} \quad , \quad (-a_1<x<a_2)$$

where $m_1/a_1 = m_2/a_2$

Origin at mode (antimode)

The values to be calculated are

$$r = 6(\beta_2 - \beta_1 -1)/(6 + 3\beta_1 -2\beta_2)$$

$$a_1 + a_2 = \tfrac{1}{2} \sqrt{\mu_2} \sqrt{\{\beta_1(r + 2)^2 + 16(r + 1)\}}$$

The m's are given by

$$\frac{1}{2}\left[r - 2 \pm r(r + 2) \frac{\sqrt{\beta_1}}{\sqrt{[\beta_1(r + 2)^2 + 16(r + 1)]}} \right]$$

(when u_3 is positive, m_2 is the positive root)

$$y_0 = \frac{1}{a_1 + a_2} - \frac{{m_1}^{m_1}{m_2}^{m_2}}{(m_1 + m_2)^{m_1+m_2}} * \frac{\Gamma(m_1 + m_2 + 2)}{\Gamma(m_1 + 1)\ \Gamma(m_2 + 1)}$$

$$\text{mode} = \text{mean} - \frac{1}{2} * \frac{\mu_3}{\mu_2} * \frac{r + 2}{r - 2}$$

Expressing the curve with origin at mean:

$$A_1 + A_2 = a_1 + a_2$$

$$(m_1 + 1)/A_1 = (m_2 + 1)/A_2$$

$$y_e = \frac{1}{A_1 + A_2} \frac{(m_1 + 1)^{m_1}(m_2 + 1)^{m_2}}{(m_1 + m_2 + 2)^{m_1+m_2}} \frac{\Gamma(m_1 + m_2 + 2)}{\Gamma(m_1 + 1)\ \Gamma(m_2 + 1)}$$

4.4 Fitting Pearson Type IV Distribution

When $K > 1$, Type IV distribution is suitable.

$$y = y_0\ [1 + (x^2/a^2)^{-m}]\ e^{-\tau \tan^{-1}(x/a)}$$

Origin is $\tau a/r$ above mean.

The values to be calculated are

$$r = \frac{6(\beta_2 - \beta_1 - 1)}{2\beta_2 - 3\beta_1 - 6}$$

$$m = 0.5(\ r + 2\ )$$

$$\tau = \frac{-r(r - 2)\sqrt{\beta_1}}{\sqrt{\{16(r - 1) - \beta_1(r - 2)^2\}}}$$

$$a = \sqrt{(\mu_2/16)}\ \sqrt{\{16(r - 1) - \beta_1(r - 2)^2\}}$$

$$y_0 = 1 - [\{aF(r,\tau\}]$$

$$\text{mode} = \text{mean} - \frac{1}{2} * \frac{\mu_3(r - 2)}{\mu_2(r + 2)}$$

(Since $(2\beta_2 - 3\beta_1 - 6) > 0$ for Type IV, r must be greater than 3)

With the origin at the mean the equation becomes

$$y = y_0 \left[1 + \left[\frac{x}{a} - \frac{\tau}{r}\right]^2\right]^{-m} e^{-\tau \tan^{-1}[(x/a)-(\tau/r)]}$$

$$F(r,v) = \int_0^{\pi} \sin^r\phi\ e^{\tau\phi} d\phi$$

4.5 Fitting Pearson Type VI Distribution

When K > 1 Type VI distribution is the appropriate one.

$$y = y_0(x - a)^{q_2} x^{-q_1}$$

Origin at a before start of curve.

The values to be calculated are

$$r = \frac{6(\beta_2 - \beta_1 - 1)}{6 + 3\beta_1 - 2\beta_2}$$

$$a = \tfrac{1}{2} \sqrt{\mu_2}\ \sqrt{\{\beta_1 (r + 2)^2 + 16(r + 1)\}}$$

q_2 and $-q_1$ are given by

$$\frac{r - 2}{2} \pm \frac{r(r + 2)}{2} \sqrt{\frac{\beta_1}{\beta_1(r + 2)^2 + 16(r + 1)}}$$

$$y_0 = \frac{a^{q_1-q_2-1}\ \Gamma(q_1)}{\Gamma(q_1 - q_2 - 1)\ \Gamma(q_2 + 1)}$$

$$\text{origin} = \text{mean} - \frac{a(q_1 - 1)}{q_1 - q_2 - 2}$$

$$\text{mode} = \text{mean} - \frac{1}{2} * \frac{\mu_3}{\mu_2} * \frac{r + 2}{r - 2}$$

Expressing the curve with origin at mean:

$$A_1 = \frac{a(q_1 - 1)}{(q_1 - 1) - (q_2 + 1)}, \qquad A_2 = \frac{a(q_2 + 1)}{(q_1 - 1) - (q_2 + 1)}$$

$$y_e = \frac{(q_2 + 1)^{q_2}(q_1 - q_2 - 2)^{q_1 - q_2}\Gamma(q_1)}{a(q_1 - 1)^{q_1}\Gamma(q_1 - q_2 - 1)\ \Gamma(q_2 + 1)}$$

4.6 Fitting Pearson Type III Distribution

When $K = \infty$ or $-\infty$ Type III distribution is suitable.

$$y = y_0\ [1 + (x/a)]^{\tau a} e^{-\tau x} \qquad (x>a)$$

Origin at mode (= mean)

$$\tau = \frac{2\mu_2}{\mu_3}$$

$$p = \tau a = \frac{4}{\beta_1} - 1$$

$$a = \frac{2\mu_2^2}{\mu_3} - \frac{\mu_3}{2\mu_2}$$

$$y_0 = \frac{1}{2} * \frac{p^{p+1}}{e^p\Gamma(p + 1)}$$

$$\text{Mode} = \text{Mean} - \frac{\mu_3}{2\mu_2}$$

Expressing the curve with origin at mean:

$$y_e = \tau \frac{(p + 1)^p}{e^{p+1}\ \Gamma(p + 1)}$$

4.7 Fitting Pearson Type V Distribution

When K = 1 Type V distribution is to be fitted.

$$y = y_0 x^{-p} e^{-r/x} \qquad (x>0) \qquad (p>1)$$

Origin at start of curve.

$$p = \frac{4 + 8 + 4\surd(4 + \beta_1)}{\beta_1} \qquad (p>4)$$

$$\tau = (p - 2) \surd\{\mu_2(p - 3)\} \qquad (p>3)$$

$$y_0 = \frac{\tau^{p-1}}{\Gamma(p - 1)}$$

$$\text{origin} = \text{mean} - \frac{\tau}{p - 2}$$

$$\text{mode} = \text{mean} - \frac{2\tau}{p(p - 2)}$$

The sign of Γ is the same as that of μ_3.

Expressing the curve with origin at mean:

$$A = \tau/(p - 2)$$

$$y_e = \frac{(p - 2)^p}{\tau e^{p-2}\Gamma(p - 1)}$$

4.8 Step 3: Using Winkler et al.'s Procedure to Compute the Moments of t_o^+

Determining the moments of a quantity such as t_o^+ is the statistical problem of determining "partial moments." Winkler et al. (1972) suggested a method for determining the partial moments of a number of frequently encountered distributions. In the case of the Pearson family of distributions, multiplying both sides of (3.7) by x^{n-1} and integrating by parts over $(-\infty, z)$ gives

$$[x^{n-1}(b_0 + b_1 x + b_2 x^2)\ f(x)]_{\infty}^{z} - (n-1)\ b_0\ E_{\infty}^{z}\ (x^{n-2}) - nb_1 E_{\infty}^{z}\ (x^{n-1}) - (n+1)\ b_2 E_{\infty}^{z}\ (x^n) = E_{\infty}^{z}\ (x^n) - aE_{\infty}^{z}\ (x^{n-1}) \tag{4.8}$$

Assuming that the expression in square brackets vanishes at the extremities of the distribution,

$$E_{-\infty}^{z}(x^n) = \frac{z^{n-1}(b_0 + b_1 z + b_2 z^2)\ f(z) - (n-1) b_0 E_{-\infty}^{z}(x^{n-2})}{(n+1) b_2 + 1} - \frac{(nb_1 - a) E_{-\infty}^{z}(x^{n-1})}{(n+1) b_2 + 1} \tag{4.9}$$

Thus we have a simple recursive relationship for the partial moments of members of the Pearson family. For any member of the family, the four parameters a, b_0, b_1 and b_2 can be expressed in terms of the first four complete moments about the origin.

Note that Winkler et al.'s formulas give the partial moments from $-\infty$ to z, which are simply the complements of the moments of t_o^+.

4.9 The Computer Program

A FORTRAN computer program written by the author finds the maximum of n stochastic variables (Beta, Normal, Gamma or Uniform) representing individual paths (see Appendix F for output from one of the test runs). The program has successfully run and the results were favorable when compared with the simulation results generated by a simulation program, which is part of the FORTRAN program. The following paragraphs give very brief explanations of the program variables, IMSL (IMSL 1982) routines used and the program subroutines.

Main Program: All the variables are double precision type. The following IMSL routines are called.

1. GGBTR to generate Beta random variables
2. GGUBS to generate uniform random variables
3. GGAMR to generate random variables
4. DCADRE for numerical integration

The Variables:

X	input stochastic variables
y	computer variables
IPOP	input variable population type
NSIZ	sample size (number of observations generated)
NPR	print switch for debugging
NUMVAR	number of variables
P, Q	distribution parameters for normal distribution - the mean and the standard deviation
PRM's	population raw moments
PCM's	population central moments

AM's raw moments

CM's central moments.

Read the first set of input values namely,

IPOP - Population type

p, q - location and scale parameters

ab and bb - shape parameters

Calculate the population raw moments (PRM's) for the given distribution. (see Appendix A, B, C & D for the derivations.)

Call subroutine POPCEN (population central moments) to compute the central moments from the raw moments.

$$\mu_2 = \mu_2' - (\mu_1') ** 2 \qquad (4.10)$$

$$\mu_3 = \mu_3' - (3\mu_2'\mu_1') + 2(\mu_1')^3 \qquad (4.11)$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4 \qquad (4.12)$$

(see Kendall and Stuart (1969))

Convert the single precision deviates generated by the IMSL subroutines to double precision deviates.

Call subroutine MOMENT to compute the four raw moments (AM1, AM2, AM3, AM4) and the central moments (CM1, CM2, CM3, CM4). The input is Z(NSIZ). The subroutine MOMENT in turn calls POPCEN to calculate the corresponding central moments. Calculate the Population raw moments and the central moments.

After computing the raw and central moments the next step is to compute the maximum of all the samples and calculate the four raw and central moments for the sample made of maximum values using the DMAX built-in function. Later these values will be compared with the values generated by the latter part of the program. Now we go to the Pearson curve fitting part of the program.

To find Yn = MAX(X1, X2, . . ., Xn)

NM1 = NUMVAR - 1 = number of iterations to find the MAX(X1, X2, . . ., Xn)

$$Y1 = MAX(X1, X2)$$

$$= X1 + (X2 - X1)^{+}$$

To find the four moments of Y1 = MAX(X1, X2) we find the four moments of $X1 + (X2 - X1)^{+}$. We use Winkler's method to find the partial moments for the distribution Z = X2 - X1. But to use the Winkler's formula we need the value f(z) when z = 0. So we fit the Pearson type distribution to Z = X2 - X1 to find f(z) at z = 0. We need the partial moments for Z = (X2 - X1) from z = 0 to upper limit. Subtract from the moments of the whole distribution the results from the Winkler's to get it. Add up the moments of X1 and the above to get the four moments of Y1. Read the values of raw moments from the sample generation part of the program. Compute central moments B1, B2 and K. Depending on the values of K, branch off to subroutine Type I, subroutine Type IV, subroutine Type VI, subroutine Type III and subroutine

Type V. Subroutine Type I fits a Pearson Type I distribution. Subroutine Type IV fits a Pearson Type IV distribution and so on.

Call subroutine WINKLR to find the partial moments

$$E_{-\infty}^{z}(X^n) = \int_{-\infty}^{z} x^n df(x)$$

$$\int_{-\infty}^{\infty} X^n f(x)\ dx = \int_{-\infty}^{z} x^m\ f(x)\ dx + \int_{z}^{\infty} x^n f(x)\ dx$$

$$=> \int_{z}^{\infty} x^n f(x)\ dx = \underbrace{\int_{-\infty}^{\infty} x^n f(x)\ dx}_{\text{from integration}} - \underbrace{\int_{-\infty}^{z} x^n f(x)\ dx}_{\text{by Winkler's method}}$$

E1, E2, E3 and E4 are the output values from the Winkler's subroutine, which are the four partial moments evaluated from the lower limit of the distribution to 0. The raw moments of our truncated distribution are the differences between the moments of the old distribution and the moments from the Winkler subroutine.

Call POPCEN to compute the central moments. The output from the Pearson curve fitting subroutines are ET1, CMS2, CMS3 and CMS4 that are the four central moments of Yn, n = 1,2 , . . . , n the value of n depending upon the iteration number. The iterations are continued till the data are exhausted.

4.10 The Study

We have run the program for several different data sets comprising of Normal and Beta variables and combinations of them. The results are given in table I. The table gives the summary of results for the following ninteen sets of variables:

1. five sets of two Normal variables
2. three sets of three Normal variables
3. three sets of five normal variables
4. one set of two Beta variables
5. two sets of three Beta variables
6. two sets of five Beta variables
7. two sets of one Beta and one Normal variable
8. one set of four variables, consisting of two Beta variables and two Normal variables.

Column 2 in the table I gives the number of variables in the set. Column 3 gives the type of the variable, columns 4 through 7 denote the parameter values (p, q, a and b). Column 8 denotes the appropriate Pearson type selected for each iteration. Column 9 gives the expected value and the variance of the simulated values. Column 10 gives the expected values and the variances derived from the method at each iteration and the final values. The number of simulation runs is 1000.

4.11 Analysis of the Results

It can be seen from the summary of output (Table I) that in all cases the mean of the iteratively fitted Pearson distributions for Max(X1, X2, . . . , Xn) converge towards the simulated mean of the maximum of n variables as n increases. Since we have confined our study to a maximum of five variables, and because of computer limitations the convergence is not very obvious in the variance, third and fourth central moments. This method assumes dependency between all the variables and it may not be the case always in PERT type networks. This needs further research. In the next chapter we study another approach by using the Schmeiser-Deutsch distribution. We use Schmeiser-Deustch variables to calculate the four parameters of the paths, given the four Beta parameters of the activities and then compute the project completion time of the network.

TABLE I

RESULTS FOR THE PEARSON DISTRIBUTION METHOD
AND THE SIMULATION RESULTS

1	2	3		4	5	6	7	8	9		10	
Sl. No.	# Var.	Var. Type		p	q	aa	bb	Psn. Type	Simul. Values E(Y) V(Y)		Max(X1,...Xn) E(T) V(T)	
1	2	N	X1	10.00	5.00	0	1	I	E	13.4553	E	12.5755
		N	X2	12.00	3.50	0	1		V	11.2994	V	39.6254
2	2	N	X1	10.00	5.00	0	1	I	E	13.5439	E	12.6891
		N	X2	12.00	3.989	0	1		V	13.1348	V	40.8181
3	2	N	X1	10.00	15.00	0	1	I	E	18.4170	E	18.0510
		N	X2	12.00	11.90	0	1		V	122.8164	V	366.8427
4	2	N	X1	10.00	5.00	0	1	I	E	13.2189	E	12.1924
		N	X2	12.00	1.00	0	1		V	5.4097	V	35.9960
5	2	N	X1	12.00	3.989	0	1	I	E	13.6083	E	14.9918
		N	X2	10.00	5.00	0	1		V	14.7911	V	34.2046
											Iteration 1	
6	3	N	X1	10.00	3.989	0	1	I	E	16.1138	E	12.6076
		N	X2	12.00	5.00	0	1		V	11.5236	V	31.6468
		N	X3	14.00	4.00	0	1				Iteration 2	
											E	15.4249
											V	25.4393

TABLE I (Continued)

Sl No.	# Var.	Var.. Type	p	q	aa	bb	Psn. Type	Simul. Values E(Y) V(Y)	Max(X1,...Xn) E(T) V(T)
									Iteration 1
7	3	N X1	16.75	8.00	0	1	I	E 19.5079	E 20.8487
		N X2	13.75	5.5	0	1		V 35.0533	V 101.0802
		N X3	9.00	3.25	0	1	IV		Iteration 2
									E 25.0152
									V 140.6789
									Iteration 1
8	3	N X1	-16.25	8.00	0	1	I	E -13.4890	E -12.6513
		N X2	13.75	5.5	0	1		V 29.9996	V 101.0802
		N X3	-9.00	3.25	0	1	IV		Iteration 2
									E -8.4848
									V 140.6789
									Iteration 1
9	5	N X1	-16.75	8.00	0	1	I	E 21.6825	E 5.3879
		N X2	13.75	5.50	0	1		V 66.4993	V 373.0618
		N X3	-9.00	3.25	0	1	IV		Iteration 2
		N X4	11.00	9.00	0	1			E -12.6513
		N X5	15.00	13.00	0	1			V 10.0802
							IV		Iteration 3
									E -8.4848
									V 140.6789
							IV		Iteration 4
									E 5.8717
									V 80.4605
									Iteration 1
10	5	N X1	150.00	10.00	0	1	I	E 168.1619	E 154.9409
		N X2	145.00	6.00	0	1		V 15.2446	V 154.2819
		N X3	97.00	11.00	0	1	IV		Iteration 2
		N X4	168.00	4.00	0	1			E 161.5241
		N X5	115.00	12.30	0	1			V 249.8384
							IV		Iteration 3
									E 167.9442
									V 354.8033
							IV		Iteration 4
									E 176.9742
									V 556.2303

TABLE I (Continued)

Sl. No.	# Var.	Var. Type	p	q	aa	bb	Psn. Type	Simul. Values E(Y) V(Y)	Max(X1,...Xn) E(T) V(T)
									Iteration 1
11	5	N X1	1.00	10.00	0	1	I	E 5.9409	E .4560
		N X2	0.45	6.00	0	1		V 2.8795	V 154.2819
		N X3	15.00	1.00	0	1	IV		Iteration 2
		N X4	-12.00	1.5	0	1			E 10.8492
		N X5	5.00	2.3	0	1			V 208.9991
							IV		Iteration 3
									E 16.5709
									V 289.2714
							IV		Iteration 4
									E 23.3358
									V 407.8499
12	2	B X1	1.00	2.00	0	1	I	E 0.4987	E 0.4576
		B X2	2.00	3.00	0	1		V 0.0403	V 0.0979
									Iteration 1
13	3	B X1	4.5	6.5	0	1	I	E 0.7313	E 0.7182
		B X2	2.5	7.0	0	1		V 0.0478	V 0.0343
		B X3	1.0	0.5	0	1	I		Iteration 2
									E 0.6378
									V 0.0949
									Iteration 1
14	3	B X1	0.50	0.50	0	1	I	E 0.8067	E 0.7182
		B X2	0.35	0.17	0	1		V 0.0730	V 0.2440
		B X3	0.05	0.95	0	1	IV		Iteration 2
									E 0.9197
									V 0.3496
									Iteration 1
15	5	B X1	0.05	0.95	0	1	I	E 0.8620	E 0.3031
		B X2	4.00	5.00	0	1		V 0.0330	V 0.1228
		B X3	0.50	0.50	0	1	IV		Iteration 2
		B X4	0.35	0.17	0	1			E 0.1333
		B X5	5.00	6.00	0	1			V 0.0455
							IV		Iteration 3
									E 0.5042

TABLE I (Continued)

Sl. No.	# Var.	Var. Type		p	q	aa	bb	Psn. Type		Simul. Values E(Y) V(Y)	Max(X1,...Xn) E(T) V(T)
											V 0.2227
								IV			Iteration 4
											E 0.6927
											V 0.3217
											Iteration 1
16	5	B	X1	2.00	2.00	0	1	I	E	0.7024	E 0.6140
		B	X2	3.00	3.00	0	1		V	0.0127	V 0.0796
		B	X3	4.00	4.00	0	1	I			Iteration 2
		B	X4	5.00	5.00	0	1				E 0.7447
		B	X5	6.07	6.00	0	1				V 0.1217
								IV			Iteration 3
											E 0.8930
											V 0.1730
								IV			Iteration 4
											E 1.0648
											V 0.2473
17	2	B	X1	2.00	3.00	-3	3	I	E	0.3985	E 0.0149
		N	X2	0.00	1.00	0	1		V	0.7715	V 2.1941
18	2	B	X1	1.50	2.50	1	10	I	E	5.8950	E 5.5320
		N	X2	5.00	2.00	0	1		V	2.7532	V 6.5140
											Iteration 1
19	4	B	X1	1.50	2.50	1	10	I	E	6.8789	E 5.4090
		N	X2	5.50	1.50	0	1		V	1.4454	V 5.9471
		B	X3	2.00	3.00	1	10	IV			Iteration 2
		N	X4	5.00	2.00	0	1				E 6.6114
											V 9.1175
								IV			Iteration 3
											E 8.0405
											V 14.0006

CHAPTER V

USING A SCHMEISER-DEUTSCH DISTRIBUTION TO DERIVE THE DISTRIBUTION OF PROJECT COMPLETION TIME

5.1 Overview

The procedure described in this chapter has the following components:

1. Obtain the first four moments of each activity time.
2. For each path, obtain the first four moments of the path time by applying equations (3.1) to (3.4) to the first four moments of the activity times.
3. Fit a Schmeiser-Deutsch (1977) distribution to each path-time's first four moments.
4. Use the Schmeiser-Deutsch cumulative distribution functions of the path-times to obtain the completion-time-distributions of parallel paths, and hence the distribution of the project completion time.

Components 3 and 4 are explained in section 5.2 and 5.3 respectively. Section 5.4 presents numerical illustrations of this procedure; these illustrations assume that the activity times are defined initially with four-parameter Beta distributions. Properties of the four-parameter Beta distribution are summarized in Appendix E.

5.2 Component 3: Fitting Schmeiser-Deutsch Functions to Path Times

The Schmeiser-Deutsch distribution was described in section 3.6. It is chosen for our procedure because:

1. It can cover empirical distributions of all possible (β_1,β_2) combinations with a single distribution function.
2. It has a closed form cumulative density function. The relevance of this second factor will be apparent in section 5.3.

A three-step procedure is used to compute the four parameters (Θ_1, Θ_2, Θ_3 and Θ_4 in equation 3.8) of the Schmeiser-Deutsch distribution from the path-time's four central moments.

5.2.1 Step 1

We get an initial estimate of Θ_3 and Θ_4 by interpolation from an "Initial Table" we have borrowed from Lau and Martin (1986). This "Initial Table" gives the values of Θ_3 and Θ_4 for (i) α_3 from -2 to 2 in steps of 0.4 and (ii) α_4 from 2 to 8 in unit increments. A two dimensional bicubic spline interpolation procedure is performed by the IMSL subroutine IBCIEU (IMSL 1982). Given this pair of trial (Θ_3^*,Θ_4^*) values, the corresponding (α_3^*,α_4^*) values can be computed by the following procedure:

5.2.2 (α_3^*,α_4^*) Computation Method:

Given a pair of (Θ_3,Θ_4), the moments about point zero for a "standardized" Schmeiser-Deutsch variable x (with $\Theta_1 = 0$, $\Theta_2 = 1$) can be computed as:

$$E(x^k|\Theta_1 = 0,\ \Theta_2 = 1) = \frac{(-1)^k\ \Theta_4^{k\Theta_3+1} + (1-\Theta_4)^{k\Theta_3+1}}{\Theta_2^{-k}\ (k\Theta_3 + 1)} \tag{5.1}$$

The central moments of x can then be computed from these moments about zero using formulas (4.10) to (4.12). With x's central moments, (α_3^*,α_4^*) can be computed according to the definitions in equations (3.5) and (3.6).

5.2.3 Step 2

The values of Θ_3 and Θ_4 are adjusted until α_3^* and α_4^* become equal to α_3 and α_4. This is achieved by solving the non-linear programming problem

$$\text{Min } Z = (\alpha_3 - \alpha_3^*)^2 + (\alpha_4 - \alpha_4^*)^2 \tag{5.2}$$

$$\text{subject to } 0 \le \Theta_4 =< 1$$

In the above non-linear programming problem α_3 and α_4 are the "constants"; and Θ_3 and Θ_4 are the "variables." α_3^* and α_4^* are functions of Θ_3 and Θ_4 as defined in Step-1's "(α_3^*,α_4^*) Computation Method." Here we have to minimize the sum of the squares of two non-linear functions. The subroutine ZXSSQ from IMSL (1982), which is based on the Brown-Dennis (1972) modification of the Levenberg-Marquardt algorithm, is used.

5.2.4 Step 3

After Θ_3 and Θ_4 are determined accurately with Step 2, the values of Θ_1 and Θ_2 can be computed as: (Schmeiser and Deutsch 1977):

$$\Theta_2 = \sigma\left[\frac{(2\Theta_3+1)(\Theta_3+1)^2}{(\Theta_3+1)^2\left[\Theta_4^{2\Theta_3+1}+(1-\Theta_4)^{2\Theta_3+1}\right]-(2\Theta_3+1)\left[(1-\Theta_4)^{\Theta_3+1}-\Theta_4^{\Theta_3+1}\right]^2}\right]^{\frac{1}{2}} \qquad (5.3)$$

$$\Theta_1 = \mu - \left[\frac{(1-\Theta_4)^{\Theta_3+1}-\Theta_4^{\Theta_3+1}}{\Theta_3+1}\right]\Theta_2 \qquad (5.4)$$

5.3 Component 4: Finding the Maximum of n Paths

This component is based on the following statistical principle: if X, Y, Z are stochastic variables and Z = Max(X, Y) then

$$f_Z = f_Y . F_X + f_X . F_Y \qquad (5.5)$$

and the four raw moments are:

$$\mu_r' = \int_l^u z^r f_z \, dz \qquad r = 1, 2, 3, 4 \qquad (5.6)$$

For two parallel paths with stochastic times X and Y, since their closed-form Schmeiser-Deutsch f_X, F_X, f_Y and F_Y are now known, equations (5.5) and (5.6) can be used to compute the raw moments of the completion time of the two parallel paths. Equations (4.10) to (4.12) then provide the central moments. The IMSL subroutine DCADRE (IMSL 1982) is used to perform the numerical integration in equation (5.6).

The preceding procedure is reiterated to obtain the moments of completion times of more than two paths.

5.4 A Numerical Example

Consider the network shown below:

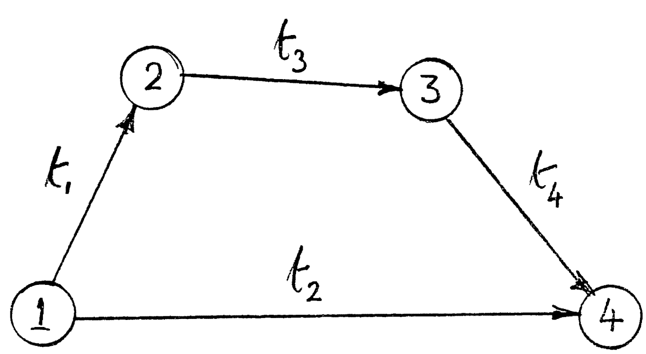

Figure 5. Network for a Numerical Example

There are four activities, 1, 2, 3 and 4. Let t_i {i = 1, 2, 3, 4} be the stochastic time of the activity i. There are two paths 1-2-3-4 and 1-4 represented by P1 and P2. Let p1 and p2 represent the stochastic path times of P1 and P2 respectively.

$p1 = t_1 + t_3 + t_4$

$p2 = t_2$

Let us assume that the activity durations follow a four-parameter generalized Beta distribution (see Appendix E). The parameters are a, b, p and q respectively. The parameter values for our example are:

$t_1(x;\ 0.5,\ 5.5,\ 2.0,\ 4.0)$

$t_2(x;\ 3.0,\ 24.0,\ 2.0,\ 3.5)$

$t_3(x;\ 1.0,\ 9.0,\ 2.0,\ 3.0)$

$t_4(x;\ 2.0,\ 10.0,\ 3.0,\ 2.0)$

Now, let us do this problem by the textbook PERT method.

$m(t_1)= 1.750$; $\mu(t_1)= 2.167$; $\sigma^2(t_1)= 0.694$; $cv(t_1)= 0.385$

$m(t_2)= 9.000$; $\mu(t_2)= 10.500$; $\sigma^2(t_2)= 12.250$; $cv(t_2)= 0.333$

$m(t_3)= 3.667$; $\mu(t_3)= 4.111$; $\sigma^2(t_3)= 1.778$; $cv(t_3)= 0.324$

$m(t_4)= 7.333$; $\mu(t_4)= 6.889$; $\sigma^2(t_4)= 1.778$; $cv(t_4)= 0.194$

The critical path is: 1-2-3

μ of PERT critical path = 13.166

Variance = 4.250

Standard Deviation = 2.062

Coeff. of variation (cv) = 0.156

See page 49 for the computed values for the new approach, the four moments, α_3 and α_4 for activities (see Appendix E).

Act.	μ_1	μ_2	μ_3	μ_4	α_3	α_4
1	2.167	0.794	0.331	1.653	0.468	2.625
2	10.636	15.700	23.978	613.942	0.385	2.491
3	4.200	2.560	1.170	15.448	0.286	2.357
4	6.800	2.560	-1.170	15.448	-0.286	2.357

The parameters of the paths (see equations (3.1) to (3.4)):

Parameters	Path 1	Path 2
Mean	13.167	10.636
Variance	5.414	15.700
μ_3	0.331	23.978
μ_4	96.252	613.942
α_3	0.023	0.385
α_4	2.752	2.441
cv	0.185	0.373

Schmeiser-Deutsch distribution parameters for the paths:

(see section 5.3 for formulas and procedure)

Parameters	Path 1	Path 2
θ_1	13.138	9.523
θ_2	21.252	26.679
θ_3	1.974	1.783
θ_4	0.497	0.431

The four central moments of the project completion time computed as maximum of all the path lengths are: (see section 5.4 for formulas and procedures)

μ_1 = 14.099 μ_2 = 6.496

μ_3 = 0.922 μ_4 = 104.389

α_3 = 0.056 α_4 = 2.474

With the sample size of 10,000, five simulation runs were made and the means of the simulated central moments are: (see Appendix G for the simulation program)

μ_1 = 14.057 μ_2 = 6.362

μ_3 = 2.612 μ_4 = 116.700

α_3 = 0.163 α_4 = 2.883

Using the simulated values as bench marks, the errors are:

PERT error in the mean = -6.335 %

New approach error in the mean = 0.298 %

PERT error in the standard deviation = -18.270 %

New approach error in the standard deviation = 1.044 %

From the above values we can clearly see that the new approach provides us with more accurate values for the project completion time when compared with the PERT values.

CHAPTER VI

COMPUTER PROGRAMS FOR THE NEW APPROACH

6.1 Introduction

There are two computer programs for the new approach presented in Chapter V, one for simulation and the other for fitting the Schmeiser-Deutsch distribution to different path times and to find the project completion time as the maximum of all the path times in the network. Both the programs are written in FORTRAN to make use of efficient subroutines available from major library packages (e.g., IMSL 1982); to run them interactively when need arises; and to change conveniently the number of activities, paths and the activities in each path, depending on the network.

6.2 The Simulation Program

The next two sections explain briefly the simulation program, its components and the notation.

6.2.1 A Brief Summary

Input to the simulation program consists of the number of activities, the number of paths and the activity numbers for each path in sequence. The input data is read from an external file. Each activity has four parameters corresponding to a generalized Beta distribution. The first part of the program reads and echo prints the input data. Part two computes the mean, variance, the third and the fourth moments for each activity. Part three generates for each activity their Beta-distributed completion times using the IMSL procedure GGBTR (IMSL 1982) and appropriate transformations. The path lengths are calculated as the sum of the activity durations and the maximum of all the paths is computed for each iteration, with the help of FORTRAN library function AMAX1. Then the mean path lengths, their raw and central moments are calculated.

The program also finds the mode, the standard deviation and the variance for standard PERT. The PERT critical path length and its standard deviation are also calculated.

Finally, the simulation program uses the simulated values as the bench mark values and calculates the percentage of errors in the PERT mean and the PERT standard deviation.

The simulation is run five times, each for a sample size of 10,000. The final results are the mean values of all the five runs. The output is given in Appendix G.

6.2.2 Selected Notation

The following is a list of selected notation :

a, b: abscissa intercepts of the Beta distributions for the activities.

p, q: shape parameters for the above

The data are entered in single precision.

NUMVAR: number of variables

NSIZ: units per simulation run

DSEED: seed values for the simulation

DRD: Beta random deviate

ACTBRV: Beta random variable for activity

NACT: Number of activities in the network

NP: number of paths in the network

NAP: array of number of activities in each path

Z: array for the maximum of all paths for NSIZ units per simulation run

PL1: array for simulated path lengths

SUMPL: sum of path lengths for NSIZ units per simulation run

PLENGTH: array for mean path lengths for NSIZ units per simulation run

CLM1, CLM2, CLM3 AND CLM4 are central moments of the simulated paths. CM(I, J), J = 1,4 are the simulated moments of the completion time for each run. Subroutine MOMENT calculates raw and central moments.

6.3 The New Approach Program

The new approach program, its components and the notation are briefly explained in the next two sections.

6.3.1 A Brief Summary

The FORTRAN program output for the new approach is given in Appendix H. The input data is the same for both the programs and is read from an external file. Part one reads and echo prints the given data. Part two calculates the mean, variance, the third and the fourth moments for each activity. Part three finds the four moments, α_3 and α_4 for each path time. In part four we fit Schmeiser-Deutsch distribution functions to each path time and find the four parameters, Θ_1, Θ_2, Θ_3 and Θ_4. Part five calculates the maximum of all the paths, the four central moments, α_3 and α_4 and the four central moments from the four parameters of the Schmeiser-Deutsch distribution.

6.3.2 Selected Notation

NP: number of paths in the network

NNP: array for the number of activities in each path

NAP: array for the number of activities of path(i)

IP: array for the activities in different paths

ALPHA3: α_3

ALPHA4: α_4

AMU's: moments for the activities

CMU's: central moments for the activities

AMUP's: moments for the paths

CMUP's: central moments for the paths

PL's: lambdas for the paths

RMZ's: moments for the joint distributions

CMZ's: central moments of the joint distributions

XZ's: project completion lambdas

ZM's: central moments of the project completion time

ZA3: α_3 of the project completion time

ZA4: α_4 of the project completion time

IMSL procedure DCADRE (IMSL 1982) is used for numerical integration and ZXSSQ (IMSL 1982) for non-linear programming. Functions FM2Z, FM2ZC and FM1ZC are called by DCADRE subroutine. Subroutine CFUN calculates the central moments from the raw moments. Subroutine LAMBDA transforms the four central moments of each path to mean, coefficient of variation, α_3 and α_4 for interpolation and non-linear programming. The output are the four parameters for the Schmeiser-Deutsch distribution. Subroutine ALTOMU finds the four central moments from the given four parameters of the Schmeiser-Deutsch function.

The IMSL subroutines used are: UERTST, UERSET, ZXSSQ, USPKD, UGETIO, LEQT1P, LUDECP, LUELMP, IBCIEU, ICSEVU, ICSCCU and DCADRE.

The program utilizes double precision throughout. The selected notations and the abbreviations given above will help to understand the computer outputs.

CHAPTER VII

TEST RESULTS AND ANALYSIS

7.1 Introduction

In this chapter we use ten test problems to compare the performance of: (i) our method presented in chapter 5; (ii) the "textbook" PERT method; and (iii) simulation, which is used as a bench mark method.

7.2 Source of Test Problems

We have taken from the existing literature the following PERT networks. The only change is: each activity time is assumed to be four-parameter-Beta distributed with nearly the same mean and variance, but arbitrary values of skewness and kurtosis are assumed. This enables us to test the performance of alternative methods under a wider range of activity-time distributed forms.

Network 1: This is the four activities-two path network used to show the computations in the previous chapter.

Network 2: This network is from Production/ Operations Management by Stevenson, William J., Irwin, 1986 - p: 642. This is a nine activities-four paths network.

Network 3: This network has ten activities and three paths. The source is Activity Networks: Project Planning and Control by Network Models by Elmaghraby, S.E., John Wiley, 1977 - p: 273.

Network 4: This six activities-four paths network is from MacCrimmon, K.R., and Ryavec, "An Analytical Study of PERT Assumptions," Operations Research, 12 (1964), 404-421.

Network 5: There are nine activities and four paths in this network. It is taken from Kleindorfer, G.B., "Bounding Distributions for a Stochastic Acyclic Networks," Operational Research, 19 (1971), 1586-1601.

Network 6: The source for this network is: Project Management, A Managerial Approach by Meredith, Jack R., and Samuel J. Mantel, Jr., John Wiley, 1989, 274. This network has ten activities and five paths.

Network 7: There are eleven activities and five paths in this network and is taken from page 54 of the book by Elmaghraby quoted in network 3.

Network 8: This eight activities and six paths network is from page 294 of Elmaghraby's book.

Network 9: There are thirteen activities and six paths in this network. This network is due to Pritsker and Kiviat, quoted by Elmaghraby on page 275 of his book.

Network 10: This network is from Project Management with CPM and PERT by Moder, Joseph J., and Cecil R. Phillips, Van Nostrand Reinhold Company, Second Edition, 1970, 307.

7.3 The comparison Tables

The comparison tables for the above ten networks are given in the following pages.

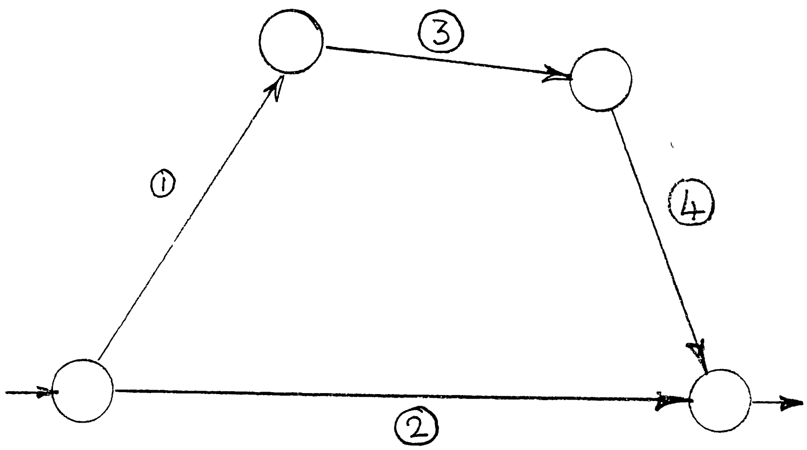

Figure 6. Network # 1

TABLE II

SUMMARY OF TEST RUNS FOR NETWORK # 1

Activities = 4 ; # Paths = 2 ; Source: Test

Activity Parameters					PERT Parameters				
Act#	a	b	p	q	mode	μ	σ^2	σ	CV
1	0.5	5.5	2.0	4.0	1.75	2.17	0.69	0.83	0.3846
2	3.0	24.0	2.0	3.5	9.00	10.50	12.25	3.50	0.3333
3	1.0	9.0	2.0	3.0	3.67	4.11	1.78	1.33	0.3243
4	2.0	10.0	3.0	2.0	7.33	6.89	1.78	1.33	0.1935

TABLE II (Continued)

Simulation Results:

runs = 5: sample size = 10,000

run	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	SB1	B2	CV
1	14.070	6.325	1.904	114.763	0.120	2.868	0.1787
2	14.055	6.467	2.740	119.025	0.167	2.846	0.1809
3	14.042	6.327	2.684	117.200	0.169	2.927	0.1791
4	14.054	6.374	2.911	115.275	0.181	2.838	0.1796
5	14.064	6.318	2.820	117.238	0.178	2.937	0.1787

Mean of 5 simulation runs, New Approach and PERT values:

Sim.	14.057	6.362	2.612	116.700	0.163	2.883	0.1794
pgm**	14.099	6.496	0.922	104.389	0.056	2.474	0.1808
PERT	13.166	4.250					0.1560

Results:

PERT error = -6.335 %

Program error = 0.298 %

Ratio of PERT error to Program error = -21.258

Note:

PERT error = ((PERT Mean - Simulation Mean) /Simulation Mean) * 100

Program error = ((New Approach Mean - Simulation Mean) / Simulation Mean) * 100

Ratio of PERT error to Program error = (PERT error ÷ New Approach error)

** Values generated by the New approach program for the project completion time.

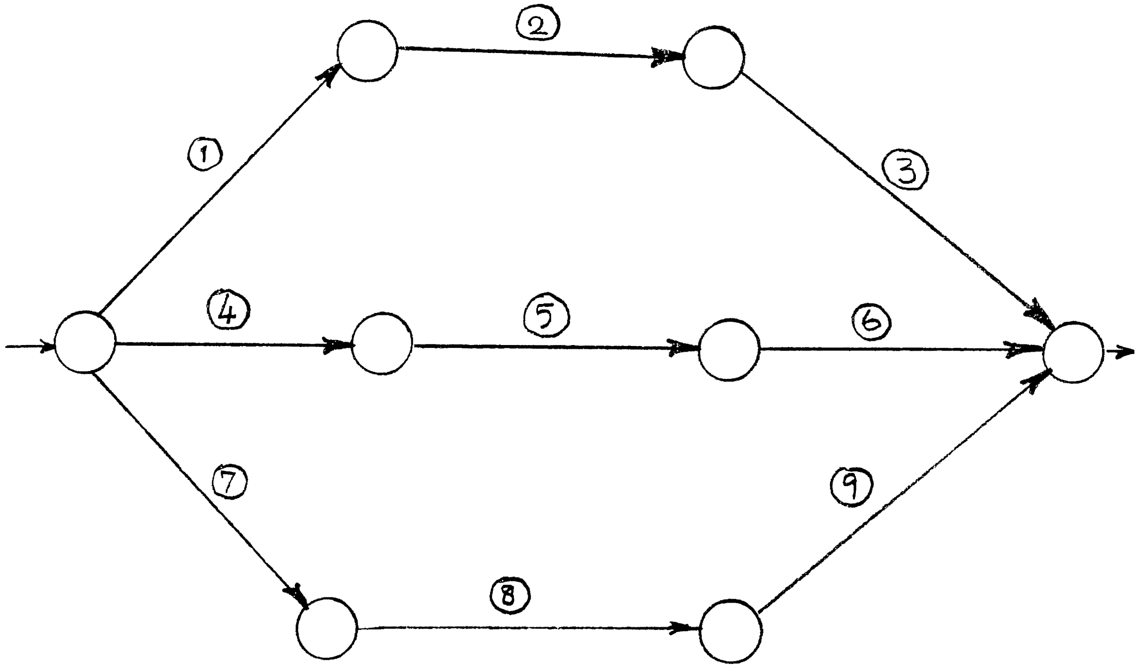

Figure 7. Network #2

TABLE III

SUMMARY OF TEST RUNS FOR NETWORK # 2

Activities = 9 ; # Paths = 3 ; Source: Stevenson

	Activity Parameters				PERT Parameters			
Act#	a	b	p	q	mode	μ	σ^2	SD	CV
1	1.5	9.5	2.0	2.5	4.70	4.97	1.78	1.33	0.269
2	0.5	7.5	3.0	3.0	4.00	4.00	1.36	1.17	0.292
3	1.0	12.0	2.0	3.0	4.66	5.28	3.36	1.83	0.347
4	1.0	7.0	2.0	3.0	3.00	3.33	1.00	1.00	0.300
5	1.5	8.5	4.0	4.0	5.00	5.00	1.36	1.17	0.233

TABLE III(Continued)

Act#	a	b	p	q	mode	μ	σ^2	SD	CV
	Activity Parameters				PERT Parameters				
6	2.0	12.0	1.5	2.5	4.50	5.33	2.78	1.67	0.313
7	1.0	11.0	2.0	4.0	3.50	4.33	2.78	1.67	0.385
8	2.0	10.0	3.0	3.0	6.00	6.00	1.78	1.33	0.222
9	0.5	8.5	2.0	5.0	2.10	2.90	1.78	1.33	0.460

Simulation Results:

runs = 5: sample size = 10,000

run	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	SB1	B2	CV
1	16.429	5.291	2.945	81.617	0.242	2.916	0.140
2	16.405	5.037	2.435	74.544	0.215	2.938	0.137
3	16.425	5.182	2.405	78.569	0.204	2.926	0.139
4	16.423	5.071	2.795	74.549	0.245	2.899	0.137
5	16.434	5.188	2.537	79.243	0.215	2.944	0.139

Mean of 5 simulation runs, New Approach and PERT values:

Sim.	16.423	5.154	2.624	77.704	0.224	2.925	0.138
pgm	16.414	5.373	1.880	68.363	0.151	2.368	0.1412
PERT	14.244	6.500					0.1790

Results:

Program error = -0.05 %

PERT error = -13.267 %

Ratio of PERT error to Program error = 265

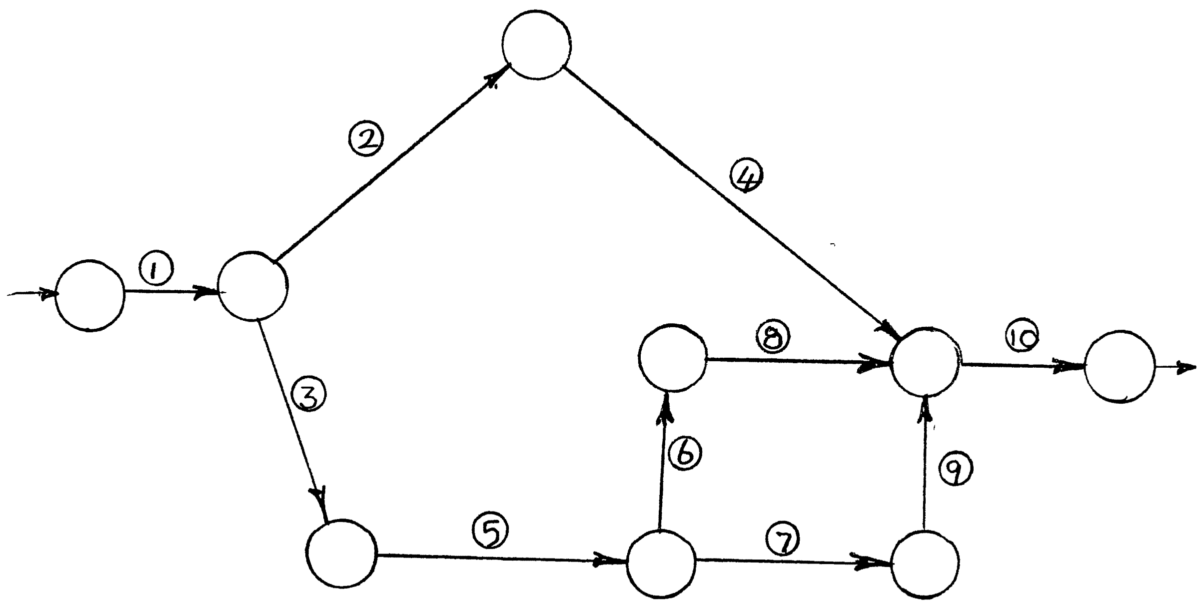

Figure 8. Network # 3

TABLE IV

SUMMARY OF TEST RUNS FOR NETWORK # 3

Activities = 10; # Paths = 3;

Source: Elmaghraby (p.273); Martin's Problem

activity parameters						PERT Parameters			
Act#	a	b	p	q	mode	μ	σ^2	σ	CV
1	3.0	20.0	1.5	4.0	5.429	7.452	8.028	2.833	0.3802
2	10.0	46.0	2.0	3.5	20.288	22.857	36.000	6.000	0.2625
3	3.0	21.0	2.0	4.0	7.500	9.000	9.000	3.000	0.3333
4	15.0	55.0	1.5	4.5	20.000	25.000	44.444	6.667	0.2667

TABLE IV (continued)

Act#	a	b	p	q	mode	μ	σ^2	σ	CV
Activity parameters						PERT Parameters			
5	7.0	18.0	4.5	4.0	10.667	11.278	3.361	1.833	0.1626
6	4.0	26.0	3.0	4.0	12.800	13.533	13.444	3.667	0.2709
7	5.0	29.0	2.0	4.5	10.333	12.556	16.000	4.000	0.3186
8	3.0	26.0	3.0	3.5	13.222	13.648	14.694	3.833	0.2809
9	7.0	22.0	3.0	4.5	12.455	13.136	6.250	2.500	0.1903
10	2.0	28.0	1.5	3.5	6.333	9.222	18.778	4.333	0.4699

Simulation Results:

runs = 5: sample size = 10,000

run	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	SB1	B2	CV
1	70.823	77.379	287.078	19341	0.422	3.230	0.1242
2	70.637	77.696	287.366	20110	0.420	3.331	0.1248
3	70.769	79.028	287.977	19877	0.410	3.183	0.1256
4	70.779	75.516	253.377	18485	0.386	3.242	0.1228
5	70.898	77.946	263.392	19896	0.383	3.275	0.1245

Mean of 5 simulation runs, New Approach and PERT values:

Sim.	70.781	77.513	275.838	19542	0.404	3.253	0.1244
pgm	72.606	65.810	203.370	10516	0.381	2.428	0.1117
PERT	64.532	107.250					0.1605

Results:

Program error = 2.57 %

PERT error = -8.829 %

Ratio of PERT error to Program error = 3.5

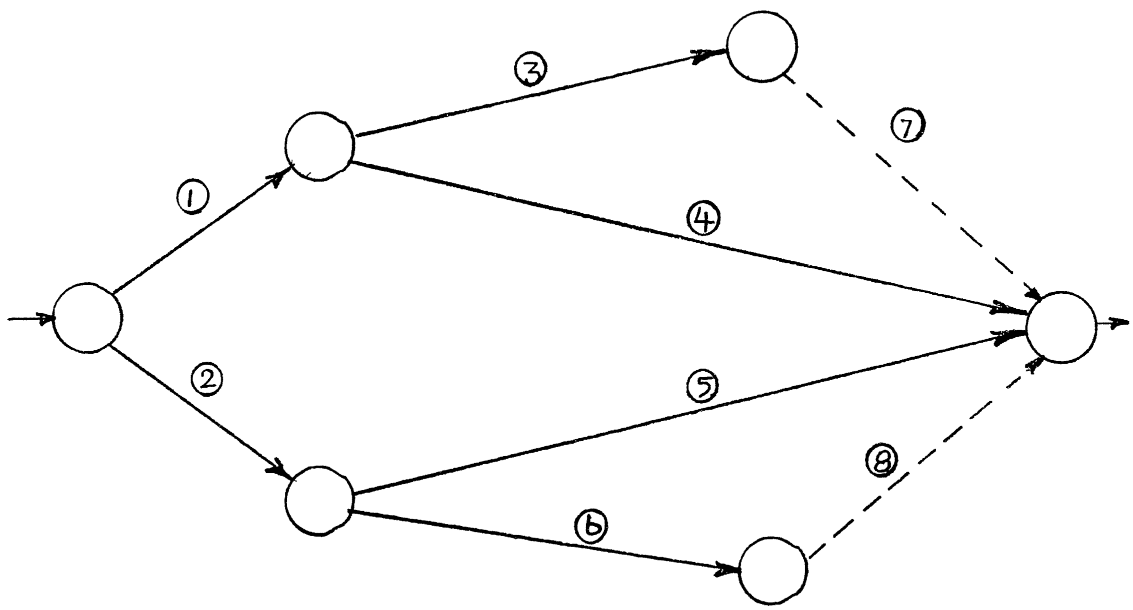

Figure 9. Network # 4

TABLE V

SUMMARY OF TEST RUNS FOR NETWORK # 10

Activities = 6 ; # Paths = 4 ; Source: MacCrimmon & Ryavec

Activity Parameters					PERT Parameters			
Act# a	b	p	q	mode	μ	σ^2	σ	CV
1 2.0	16.0	2.0	2.0	9.00	9.00	5.44	2.33	0.2593
2 2.0	15.0	4.0	3.0	9.80	9.37	4.69	2.17	0.2313
3 2.0	6.0	2.0	2.0	4.00	4.00	0.44	0.67	0.1667
4 1.0	10.0	1.5	4.5	2.13	3.25	2.25	1.50	0.4615

TABLE V (continued)

Activity parameters					PERT Parameters			
Act#	a	b	p	q	mode	μ	σ^2	σ	CV
5	2.5	8.5	1.5	4.5	3.25	4.00	1.00	1.00	0.2500
6	0.9	8.9	1.5	4.0	2.04	3.00	1.78	1.33	0.4452

<u>Simulation Results:</u>

runs = 5: sample size = 10,000

run	μ1	μ2	μ3	μ4	SB1	B2	CV
1	15.266	5.472	-1.412	81.468	-0.110	2.721	0.1532
2	15.250	5.433	-1.545	80.951	-0.122	2.742	0.1528
3	15.263	5.488	-1.456	84.240	-0.113	2.797	0.1534
4	15.262	5.569	-0.978	84.500	-0.074	2.725	0.1546
5	15.270	5.456	-1.444	80.542	-0.113	2.706	0.1530

<u>Mean of 5 simulation runs, New Approach and PERT values:</u>

Sim.	15.262	5.484	-1.367	82.340	-0.106	2.738	0.1534
pgm	15.933	3.885	-0.774	41.875	-0.101	2.775	0.1237
PERT	13.667	5.694					0.1785

Results:

Program error = <u>4.3 %</u>

PERT error = <u>-12.420 %</u>

Ratio of PERT error to Program error = <u>2.89</u>

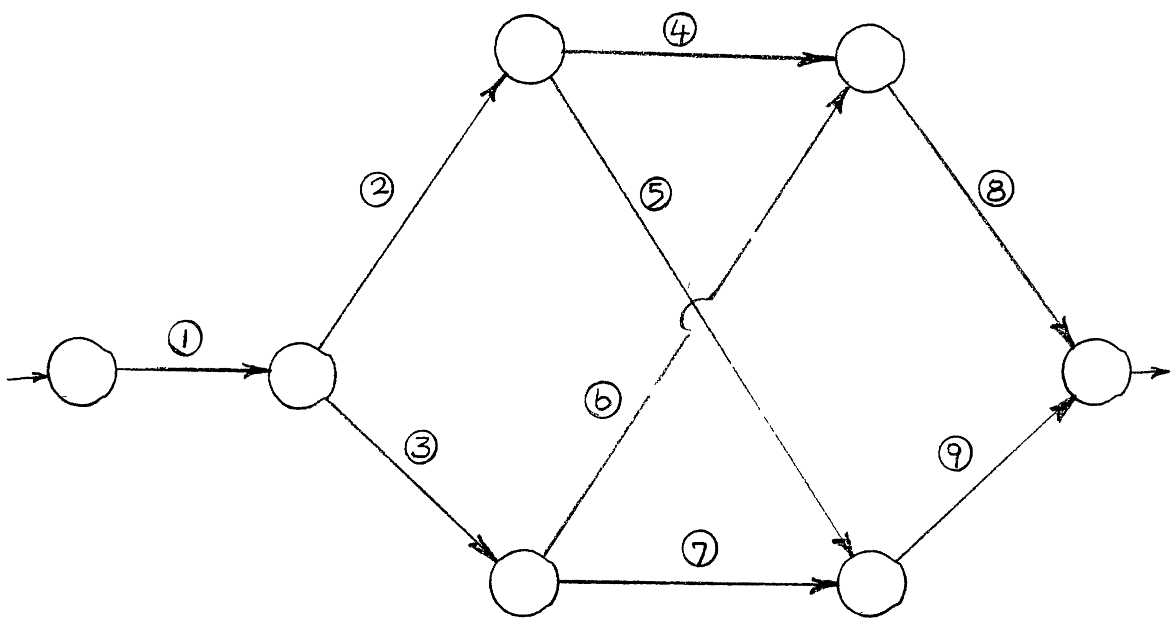

Figure 10. Network # 5

TABLE VI

SUMMARY OF TEST RESULTS FOR NETWORK # 5

Activities = 9 ; # Paths = 4 ; Source: Kleindorfer

Activity Parameters					PERT Parameters			
Act#	a	b	p	q	mode	μ	σ^2	σ	CV
1	5.0	22.0	1.5	4.0	7.43	9.45	8.03	2.83	0.2997
2	2.5	17.5	2.0	3.0	7.50	8.33	6.25	2.50	0.3000
3	1.5	21.0	3.0	3.0	11.25	11.25	10.56	3.25	0.2889
4	2.0	29.0	2.5	3.5	12.13	13.25	20.25	4.50	0.3396

TABLE VI (continued)

Activity parameters						PERT Parameters			
Act#	a	b	p	q	mode	μ	σ^2	σ	CV
5	5.0	47.0	3.0	5.0	19.00	21.33	49.00	7.00	0.3281
6	8.3	25.3	1.5	3.5	11.13	13.02	8.03	2.83	0.2176
7	3.0	28.0	2.5	1.5	21.75	19.67	17.36	4.17	0.2119
8	8.0	37.0	3.0	3.0	22.50	22.50	23.36	4.83	0.2148
9	7.0	25.0	2.0	4.0	11.50	13.00	9.00	3.00	0.2308

Simulation Results:

runs = 5: sample size = 10,000

run	$\mu1$	$\mu2$	$\mu3$	$\mu4$	SB1	B2	CV
1	61.057	44.376	39.892	5875	0.135	2.983	0.1091
2	61.212	45.161	23.370	5946	0.077	2.916	0.1099
3	61.114	44.468	28.427	5734	0.096	2.899	0.1091
4	61.111	44.906	31.374	5917	0.104	2.935	0.1097
5	61.239	45.218	33.670	5871	0.111	2.872	0.1098

Mean of 5 simulation runs, New Approach and PERT values:

Sim.	61.147	44.826	31.346	5868	0.105	2.921	0.1095
pgm	62.623	36.833	14.865	3157	0.067	2.327	0.0969
PERT	56.225	49.979					0.1257

Results:

Program error = 2.4 %

PERT error = -8.050 %

Ratio of PERT error to Program error = 3.35

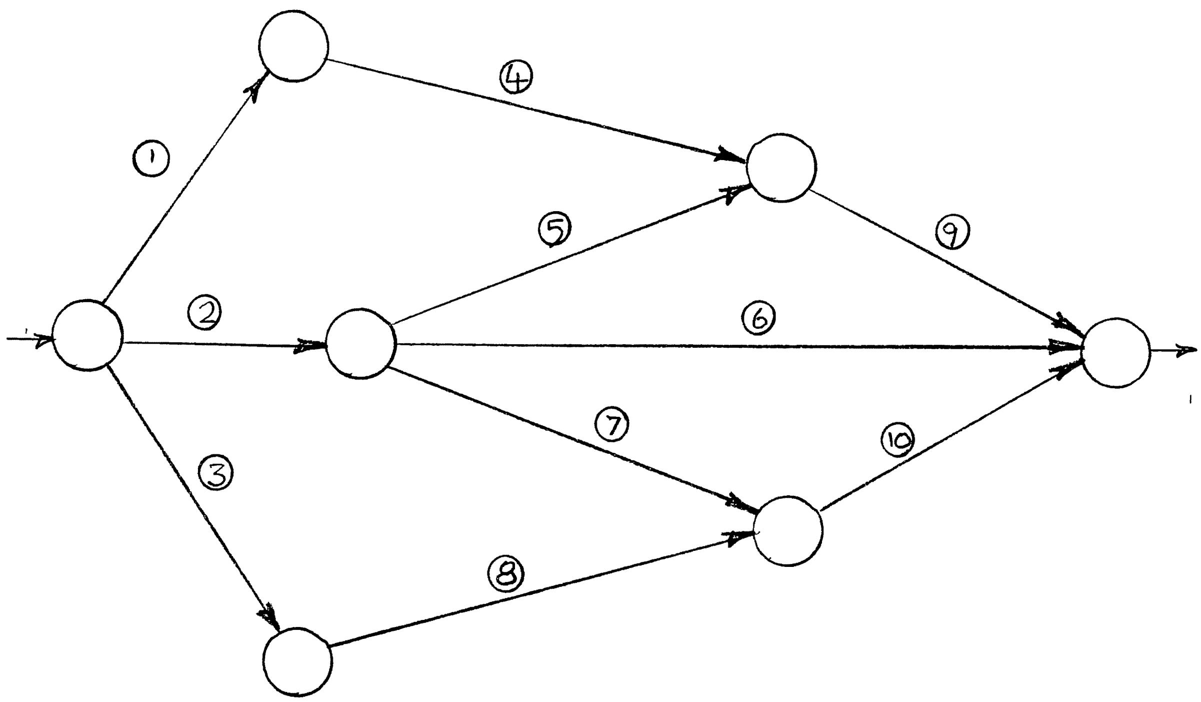

Figure 11. Network # 6

TABLE VII

SUMMARUY OF TEST RESULTS FOR NETWORK # 6

Activities = 10 ; # Paths = 5 ; Source: Meredith (Project Management)

Activities Parameters					PERT Parameters			
Act# a	b	p	q	mode	μ	σ^2	σ	CV
1 5.0	27.0	2.0	1.0	27.00	23.33	13.44	3.67	30.1571
2 12.0	28.0	2.0	2.0	20.00	20.00	7.11	2.67	0.1333
3 10.0	16.0	3.0	3.0	13.00	13.00	1.00	1.00	0.0769
4 7.0	37.0	1.5	3.5	12.00	15.33	25.00	5.00	0.3261

TABLE VII (continued)

Activity parameters						PERT Parameters			
Act#	a	b	p	q	mode	μ	σ^2	σ	CV
5	3.0	25.0	2.0	2.0	14.00	14.00	13.44	3.67	0.2619
6	22.0	34.0	2.5	2.5	28.00	28.00	4.00	2.00	0.0714
7	4.0	4.0	2.0	2.0	4.00	4.00	0.00	0.00	0.0000
8	4.0	18.0	2.0	2.0	11.00	11.00	5.44	2.33	0.2121
9	8.0	16.0	2.0	2.0	12.00	12.00	1.78	1.33	0.1111
10	13.0	45.0	1.45	2.0	22.93	24.95	28.44	5.33	0.2137

Simulation Results:

runs = 5: sample size = 10,000

run	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	SB1	B2	CV
1	55.982	35.089	60.135	3302	0.289	2.682	0.1058
2	56.081	36.160	66.888	3419	0.308	2.615	0.1072
3	56.026	34.629	62.005	3295	0.304	2.748	0.1050
4	55.974	35.673	63.464	3409	0.298	2.679	0.1067
5	56.104	35.435	56.057	3407	0.266	2.714	0.1061

Mean of 5 simulation runs, New Approach and PERT values:

Sim.	56.033	35.397	61.710	3366	0.293	2.687	0.1062
pgm	57.511	31.549	14.168	1965	0.080	1.975	0.0977
PERT	50.667	40.222					0.1252

Results:

Program error = 2.6 %

PERT error = - 9.577 %

Ratio of PERT error to Program error = 3.7

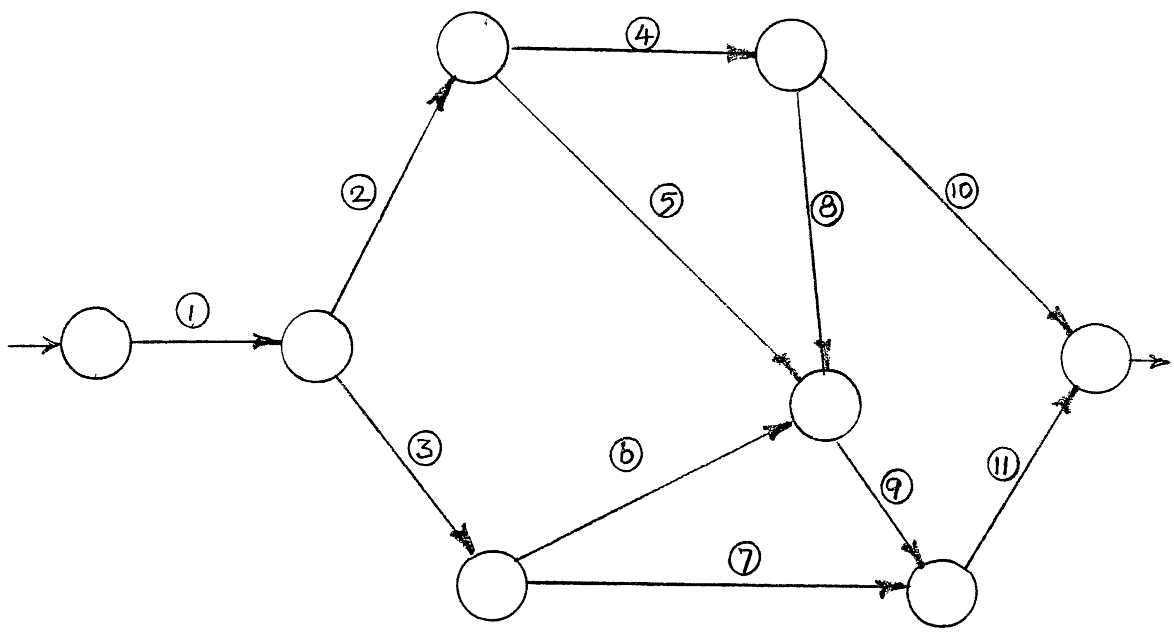

Figure 12. Network # 7

TABLE VIII

SUMMARY OF TEST RESULTS FOR NETWORK # 7

Activities = 11 ; # Paths = 5 ; Source: Elmaghraby (p.54)

Activities Parameters					PERT Parameters			
Act#	a	b	p	q	mode	μ	σ^2	σ	CV
1	2.0	17.0	2.5	4.0	7.00	7.83	6.25	2.50	0.3191
2	4.0	26.0	3.0	3.0	15.00	15.00	13.44	3.67	0.2444
3	1.5	13.5	2.0	4.0	4.50	5.50	4.00	2.00	0.3636
4	1.0	36.0	3.5	3.5	18.50	18.50	34.03	5.83	0.3153

TABLE VIII (continued)

Act#	a	b	p	q	mode	PERT μ	σ^2	σ	CV
5	10.0	50.0	3.0	3.0	30.00	30.00	44.44	6.67	0.2222
6	5.0	72.0	2.5	3.5	30.13	32.92	124.69	11.17	0.3392
7	33.0	72.0	2.0	4.0	42.75	46.00	42.25	6.50	0.1413
8	3.0	18.0	4.0	2.0	14.25	13.00	6.25	2.50	0.1923
9	2.0	10.0	2.5	2.5	56.00	6.00	1.78	1.33	0.2222
10	5.0	55.0	3.0	3.0	30.00	30.00	69.44	8.33	0.2778
11	2.0	27.0	2.0	3.5	9.14	10.93	17.36	4.17	0.3813

(Activity parameters: Act#, a, b, p, q, mode; PERT Parameters: μ, σ^2, σ, CV)

Simulation Results:

\# runs = 5: sample size = 10,000

run	μ_1	μ_2	μ_3	μ_4	SB1	B2	CV
1	80.704	71.193	71.205	14883	0.119	2.936	0.1045
2	80.775	72.275	60.335	15133	0.099	2.897	0.1052
3	80.758	71.934	60.369	15095	0.099	2.917	0.1050
4	80.828	70.622	46.693	13958	0.079	2.799	0.1040
5	80.684	71.843	63.053	15165	0.104	2.939	0.1050

Mean of 5 simulation runs, New Approach and PERT values:

Sim.	80.749	71.573	60.331	14847	0.100	2.898	0.1048
pgm	82.702	57.009	-13.243	7869	-0.030	2.421	0.0913
PERT	71.333	123.167					0.1556

Results:

Program error = 2.4 %

PERT error = -11.661 %

Ratio of PERT error to Program error = 4.83

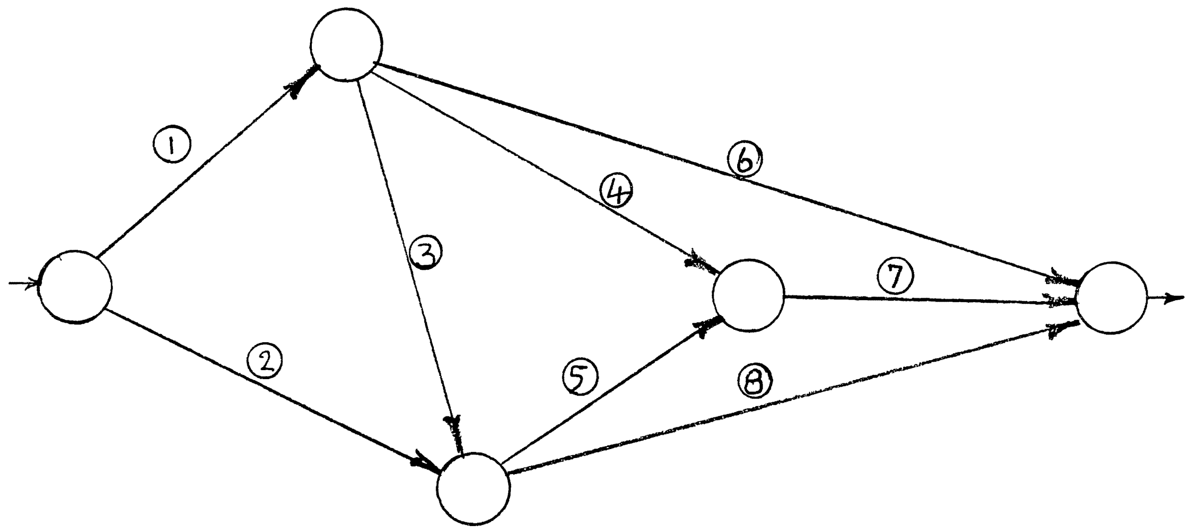

Figure 13. Network # 8

TABLE IX

SUMMARY OF TEST RUNS FOR NETRWORK # 8

Activities = 8 ; # Paths = 6 ; Source: Elmaghraby (p.294)

Activities Parameters					PERT Parameters			
Act#	a	b	p	q	mode	μ	σ^2	σ	CV
1	1.5	9.5	2.0	4.0	3.50	4.17	1.78	1.33	0.3200
2	1.0	19.0	3.0	3.0	10.00	10.00	9.00	3.00	0.3000
3	1.5	7.5	3.0	2.0	5.50	5.17	1.00	1.00	0.1935
4	4.0	15.0	3.0	3.0	9.50	9.50	3.36	1.83	0.1930

TABLE IX (continued)

Activity parameters						PERT Parameters			
Act#	a	b	p	q	mode	μ	σ^2	σ	CV
5	1.0	7.0	2.0	2.0	4.00	4.00	1.00	1.00	0.2500
6	12.0	35.0	1.5	2.5	17.75	19.67	14.69	3.83	0.1949
7	1.5	15.5	2.0	4.0	5.00	6.17	5.44	2.33	0.3784
8	3.0	21.0	4.0	2.0	16.50	15.00	9.00	3.00	0.2000

<u>Simulation Results:</u>

runs = 5: sample size = 10,000

run	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	SB1	B2	CV
1	28.354	13.646	6.330	529.354	0.126	2.843	0.1303
2	28.437	13.544	4.101	513.671	0.082	2.800	0.1294
3	28.367	13.802	4.692	524.280	0.092	2.752	0.1309
4	28.377	13.613	5.680	514.446	0.113	2.776	0.1300
5	28.387	13.628	4.938	530.577	0.098	2.857	0.1300

<u>Mean of 5 simulation runs, New Approach and PERT values:</u>

Sim.	28.384	13.647	5.148	522.465	0.102	2.805	0.1301
pgm	28.868	11.013	9.868	277.818	0.270	2.290	0.1150
PERT	25.000	18.000					0.1697

Results:

Program error = <u>1.7 %</u>

PERT error = <u>-11.924%</u>

Ratio of PERT error to Program error = 7.01

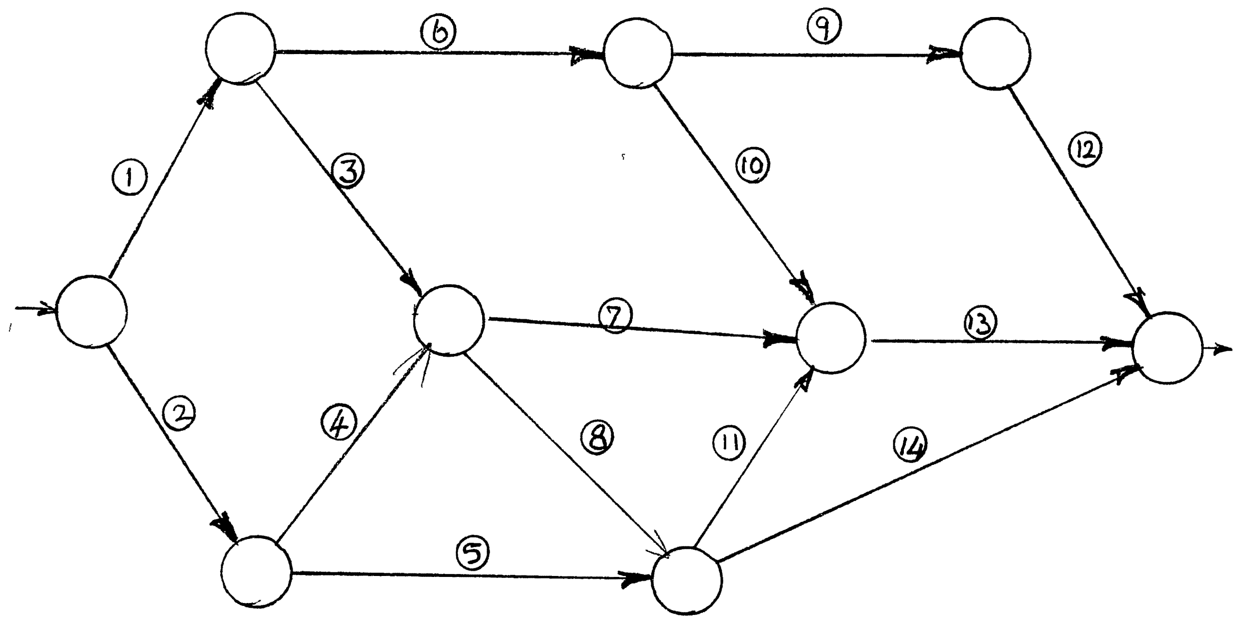

Figure 13. Network # 9

TABLE X

SUMMARY OF TEST RUNS FOR NETWORK # 9

Activities = 13 ; # Paths = 6 ;

Source: Elmaghraby (p.275): Pritsker & Kiviat

Activities Parameters					PERT Parameters				
Act#	a	b	p	q	mode	μ	σ^2	σ	CV
1	2.0	9.0	2.0	3.0	4.33	4.72	1.36	1.17	0.2471
2	10.5	36.5	3.0	3.0	23.50	23.50	18.78	4.33	0.1844
3	7.0	32.0	1.5	3.5	11.17	13.94	17.36	4.17	0.2988
4	1.0	10.0	2.0	2.5	4.60	4.90	2.25	1.50	0.3061

TABLE X (continued)

Act#	a	b	p	q	mode	μ	σ^2	σ	CV
Activity parameters						PERT Parameters			
5	20.0	54.0	2.0	4.0	28.50	31.33	32.11	5.67	0.1809
6	5.0	22.0	1.5	3.0	8.40	10.10	8.03	2.33	0.2805
7	1.5	10.5	3.0	3.0	6.00	6.00	2.25	1.50	0.2500
8	9.0	31.0	2.5	2.5	20.00	20.00	13.44	3.67	0.1833
9	3.0	18.0	2.0	2.5	9.00	9.50	6.25	2.50	0.2632
10	2.5	12.5	3.0	3.0	7.50	7.50	2.78	1.67	0.2222
11	1.0	5.0	1.5	2.5	2.00	2.33	0.44	0.67	0.2857
12	1.5	7.5	1.5	4.5	2.25	3.00	1.00	1.00	0.3333
13	6.0	26.0	2.0	4.0	11.00	12.67	11.11	3.33	0.2632

Simulation Results:

runs = 5: sample size = 10,000

run	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	SB1	B2	CV
1	43.730	18.734	19.234	1075	0.237	3.064	0.0990
2	43.765	18.686	13.663	1079	0.69	3.091	0.0988
3	43.740	19.328	19.226	1129	0.226	3.023	0.1005
4	43.794	19.432	17.542	1114	0.205	2.951	0.1006
5	43.719	18.889	17.804	1092	0.217	3.063	0.0994

Mean of 5 simulation runs, New Approach and PERT values:

Sim.	43.750	19.014	17.496	1098	0.211	3.038	0.0997
pgm	44.231	16.978	-0.901	708	-0.013	2.459	0.0932
PERT	38.389	33.917					0.1517

Results:

Program error = 1.09 %

PERT error = -12.253 %

Ratio of PERT error to Program error = 11.2

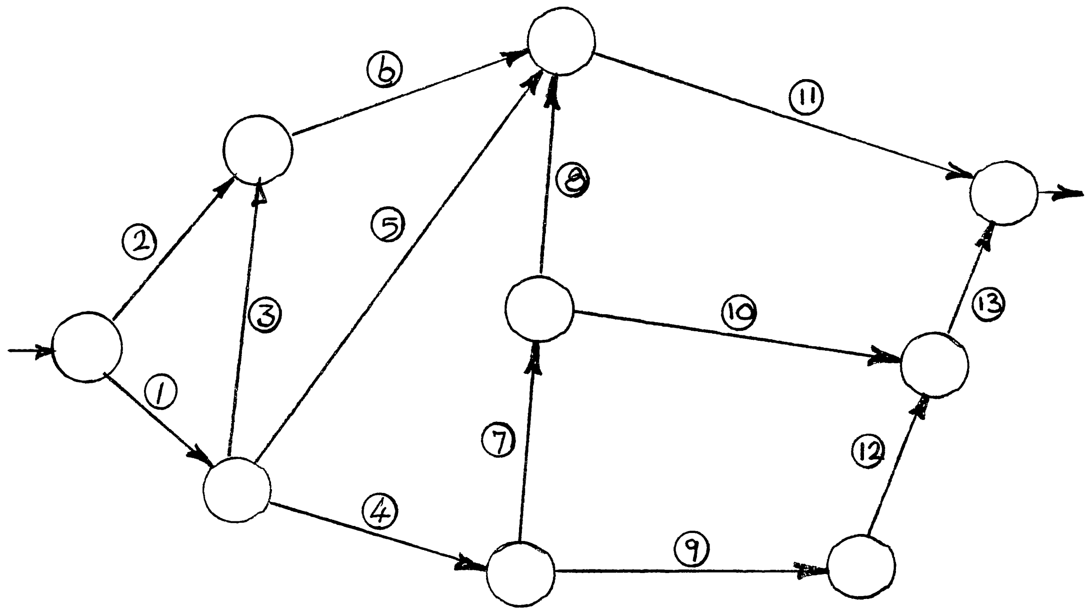

Figure 15. Network # 10

TABLE XI

SUMMARY OF TEST RUNS FOR NETWORK # 10

Activities = 14 ; # Paths = 10 ; Source: Moder & Phillips

Activities Parameters					PERT Parameters				
Act#	a	b	p	q	mode	μ	σ^2	σ	CV
1	4.0	23.0	3.0	2.0	16.67	15.61	10.03	3.17	0.2028
2	3.0	28.0	2.0	3.0	11.33	12.72	17.36	4.17	0.3275
3	8.0	34.0	1.5	2.5	14.50	16.67	18.78	4.33	0.2600
4	7.0	32.0	1.5	3.5	11.17	13.94	17.36	4.17	0.2988

TABLE XI (continued)

Activity parameters					PERT Parameters			
Act#	a	b	p	q	mode	μ	σ^2	σ	CV
5	5.0	35.0	3.0	3.5	18.33	18.89	25.00	5.00	0.2647
6	10.0	37.0	2.0	2.5	20.80	21.70	20.25	4.50	0.2074
7	2.0	22.0	3.0	3.0	12.00	12.00	11.11	3.33	0.2778
8	1.0	2.0	2.0	2.0	1.50	1.50	0.03	0.17	0.1111
9	1.0	1.0	2.5	2.5	1.00	1.00	0.00	0.00	0.0000
10	3.0	20.0	2.0	3.0	8.67	9.61	8.03	2.83	0.2948
11	5.0	24.0	1.5	2.5	9.75	11.33	10.03	3.17	0.2794
12	11.0	14.0	3.0	2.0	13.00	12.83	0.25	0.50	0.0390
13	4.0	33.0	2.0	3.0	13.67	15.28	23.36	4.83	0.3164
14	12.0	48.0	3.0	3.0	30.00	30.00	36.00	6.00	0.2000

<u>Simulation Results:</u>

runs = 5: sample size = 10,000

run	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	SB1	B2	CV
1	72.393	59.991	17.040	10108	0.037	2.809	0.1070
2	72.603	61.082	18.989	10589	0.040	2.838	0.1077
3	72.539	62.232	17.668	10726	0.036	2.770	0.1088
4	72.574	60.862	8.506	10539	0.018	2.845	0.1075
5	72.515	60.681	2.355	10451	0.005	2.838	0.1074

<u>Mean of 5 simulation runs, New Approach and PERT values:</u>

Sim.	72.525	60.969	12.911	10482	0.027	2.820	0.1077
pgm	75.986	33.789	-77.247	2980	-0.393	2.611	0.0765
PERT	63.778	64.833					0.1262

Results:

Program error = <u>4.7 %</u>

PERT error = <u>-12.060 %</u>

Ratio of PERT error to Program error = <u>2.73</u>

TABLE XII

SUMMARY OF TEN TEST CASES

NETWORK #	ERROR IN THE MEAN PERT	ERROR IN THE MEAN NEW METHOD	RATIO OF PERT ERROR TO NEW METHOD ERROR
1	- 6.335%	0.29%	-21.258
2	-13.267%	-0.05%	265.00
3	- 8.829%	2.57%	-3.50
4	-12.420%	4.3%	-2.89
5	- 8.050%	2.4%	-3.35%
6	- 9.577%	2.6%	-3.70
7	-11.661%	2.4%	-4.83
8	-11.924%	1.7%	-7.01%
9	-12.253%	1.09%	-11.20
10	-12.060%	4.7%	2.73
MEAN VALUE	-10.638%	2.2%	

7.4 Summary and Contributions

Though PERT has come to stay as one of the most powerful tools of OR/MS there are many potential advantages and insights to be gained by empirically analyzing the basic assumptions of PERT. There is substantial scope for research in the search for a suitable approximation for the actual distribution of individual activities, paths and project completion time in PERT networks using four parameter distributions. This study contributes to a greater understanding and precision of the path distributions and project completion time in a PERT network.

There has been considerable research in the last three decades regarding the PERT completion time. Most of the published research has dealt with only some minor aspects of the problem or other. The research efforts have been sporadic. This study has investigated the suitability of the highly versatile but often overlooked Pearson system of distribution and the new Schmeiser-Deutsch distribution to represent the completion time of projects, considering the stochasticity of multiple parallel paths in a PERT network. We have come to the conclusion that the new Schmeiser-Deutsch distribution can be used to overcome the drawbacks of the PERT and provide more precise and accurate project completion time with four parameters. Our study has clearly shown that PERT underestimates the project completion time(Table XII).

When compared to the accuracy of results and the elimination of errors, a little extra computer time is of no consequence to the modern day managers. The CPU time for running the largest among the test jobs, i.e., network #10 having 14 activities and 10 paths network, was only 4.2 seconds on an IBM 3090 computer with a WATFIV compiler. The compile time was 0.14 seconds. The program can be rewritten to run on any personal computer. The object code needs about 75K bytes only. Also the running time can be reduced by rewriting portions of the program to reduce the number of calls to the IMSL subroutines. IMSL library is available for personal computers.

The results of the study are expected to have far reaching implications for the management scientist and practicing project managers. "It has almost become axiomatic . . .that planned project schedule . . .will be optimistic. The only question is: By how much?" (Schonberger, 1977; Feiler, 1972). The projects are "always" late because the current PERT understates the likely project completion time. This study may help to overcome the perennial problem of "late" projects. The cost savings may run into millions of dollars, when we think of about the billions of dollars being spent for projects by corporations, civilian and defence oriented, big and small, and regional and multinational.

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APPENDIX A

DERIVATIONS FOR THE MOMENTS OF THE BETA DISTRIBUTION

$$\mu_n' = \frac{\beta(r+a,b)}{\beta(a,b)}$$

$$= \frac{\Gamma(r+a)\,\Gamma(b)}{\Gamma(r+a+b)} \quad \frac{1}{\beta(a,b)}$$

$$= \frac{(r+a-1)\,\Gamma(r+a-1)\,\Gamma(b)}{(a+b+r-1)\,\Gamma(r+\ +b-1)} \quad \frac{1}{\beta(a,b)} \qquad \mu_1' = \frac{a}{(a+b)}$$

$$\mu_n' = \frac{(a+r-1)}{(a+b+r-1)} \quad \frac{\beta(a+r-1,b)}{B(a,b)}$$

r = 2,

$$\mu_2' = \frac{(a+2-1)}{(a+b+2-1)} \quad \frac{\beta(a+2-1,b)}{\beta(a,b)}$$

$$= \frac{(a+1)}{(a+b+1)} \quad \frac{\beta(a+1,b)}{\beta(a,b)}$$

$$= \frac{(a+1)}{(a+b+1)} \quad \frac{\Gamma(a+1)\,\Gamma(b)}{\Gamma(a+b+1)} \quad \frac{\Gamma(a+b)}{\Gamma(a)\,\Gamma(b)}$$

$$= \frac{(a+1)}{(a+b+1)} \quad \frac{a\Gamma(a)\,\Gamma(b)}{(a+b)\,\Gamma(a+b)} \quad \frac{\Gamma(a+b)}{\Gamma(a)\,\Gamma(b)}$$

$$= \frac{(a+1)}{(a+b+1)} \quad \frac{a}{(a+b)}$$

$$= \frac{(a+1)}{(a+b+1)} \quad \mu_1'$$

$$\mu_3' = \frac{(a+2)}{(a+b+2)} \quad \frac{\beta(a+2,b)}{\beta(a,b)}$$

$$= \frac{(a+2)}{(a+b+2)} \quad \frac{\Gamma(a+2)\,\Gamma(b)}{\Gamma(a+b+2)} \quad \frac{\Gamma(a+b)}{\Gamma(a)\,\Gamma(b)}$$

$$= \frac{(a+2)}{(a+b+2)} \quad \frac{a\Gamma(a+1)\;\Gamma(b)}{(a+b+1)\;\Gamma\;(a+b+1)} \quad \frac{\Gamma(a+b)}{\Gamma(a)\;\Gamma(b)}$$

$$= \frac{(a+2)}{(a+b+2)} \quad \frac{(a+1)}{(a+b+1)} \quad \frac{a\Gamma(a)\;\Gamma(b)}{(a+b)\;\Gamma(a+b)} \quad \frac{\Gamma(a+b)}{\Gamma(a)\;\Gamma(b)}$$

$$= \frac{(a+2)}{(a+b+2)} \quad \frac{(a+1)}{(a+b+1)} \quad \frac{a}{(a+b)}$$

$$= \frac{(a+2)}{(a+b+2)} \; \mu_2'$$

$$\mu_4' = \frac{(a+3)}{(a+b+3)} \quad \frac{\beta(a+3,b)}{\beta(a,b)}$$

$$= \frac{(a+3)}{(a+b+3)} \quad \frac{\Gamma(a+3)\;\Gamma(b)}{\Gamma(a+b+3)} \quad \frac{1}{\beta(a,b)}$$

$$= \frac{(a+3)}{(a+b+3)} \quad \frac{(a+2)\;\Gamma(a+2)\;\Gamma(b)}{(a+b+2)\;\Gamma(a+b+2)} \quad \frac{1}{\beta(a,b)}$$

$$= \frac{(a+3)}{(a+b+3)} \quad \frac{(a+2)}{(a+b+2)} \quad \frac{(a+1)\;\Gamma(a+1)}{(a+b+1)\;\Gamma(a+b+1)} \quad \Gamma(b) \quad \frac{1}{B(a,b)}$$

$$= \frac{(a+3)}{(a+b+3)} \quad \frac{(a+2)}{(a+b+2)} \quad \frac{(a+1)}{(a+b+1)} \quad \frac{a\;\Gamma(a)\;\Gamma(b)}{(a+b)\;\Gamma(a+b)} \quad \frac{\Gamma(a+b)}{\Gamma(a)\;\Gamma(b)}$$

$$= \frac{(a+3)}{(a+b+3)} \; \mu_3'$$

APPENDIX B

DERIVATION OF MOMENTS FOR THE UNIFORM DISTRIBUTION

$$f(x) = \begin{cases} 1/(b-a) & ; a<x<b \\ 0 & ; \text{elsewhere} \end{cases}$$

$$E(x^r) = \int_a^b \frac{x^r}{(b-a)} \quad f(x)\ dx$$

$$\mu_n' = \frac{b^{r+1} - a^{r+1}}{(b-a)(r+1)}$$

$$\mu_1' = \frac{b^2 - a^2}{2(b-a)}$$

$$\mu_2' = \frac{b^3 - a^3}{3(b-a)}$$

$$\mu_3' = \frac{b^4 - a^4}{4(b-a)}$$

$$\mu_4' = \frac{b^5 - a^5}{5(b-a)}$$

Central Moments:

$$E(x-(a+b)/2)^r = \frac{1}{b-a} \int_a^b (x-(a+b)/2)^r \, dx$$

$$= \frac{1}{b-a} \frac{1}{2^r} \int_a^b (2x-a-b)^r \, dx$$

$$= \frac{1}{b-a} \frac{1}{2^r} \left[\frac{(2x-a-b)^{r+1}}{2(r+1)} \right]_a^b$$

$$= \frac{1}{2^{r+1}} \frac{1}{(b-a)(r+1)} [(b-a)^{r+1} (a-b)^{r+1}]$$

$$= \frac{1}{2^{r+1} (b-a)(r+1)} [2(b-a)^{r+1}] \qquad r, \text{ even}$$

$$\text{Therefore } \mu^r = \frac{(b-a)^r}{2^r (r+1)}$$

$$\mu_2 = \frac{(b-a)^2}{12}$$

$$\mu_3 = 0$$

$$\mu_4 = \frac{(b-a)^4}{2^4 (5)}$$

$$= \frac{(b-a)^2}{12} \; \frac{(b-a)^2}{12} \; \frac{144}{80}$$

$$= \mu_2^2 \, (144/80)$$

APPENDIX C

DERIVATION OF MOMENTS FOR THE

GAMMA DISTRIBUTION

$$f(x) = \frac{\alpha}{\Gamma(r)} x^{r-1} e^{-\alpha x} I_{(0,\infty)}(x) \qquad \alpha > 0,\ r > 0 \qquad p = r,\ q = \alpha$$

$$\mu_j' = \frac{\Gamma(r+j)}{\alpha^j \Gamma(r)}$$

$$\mu_1' = \frac{r}{\alpha}$$

$$\mu_2' = \frac{r+1}{\alpha} \frac{r}{\alpha} = \frac{r+1}{\alpha} \mu_1'$$

$$\mu_3' = \frac{r+2}{\alpha} \frac{r+1}{\alpha} \frac{r}{\alpha} = \frac{r+2}{\alpha} \mu_2'$$

$$\mu_4' = \frac{r+3}{\alpha} \frac{r+2}{\alpha} \frac{r+1}{\alpha} \frac{r}{\alpha} = \frac{r+3}{\alpha} \mu_3'$$

$$\mu_1 = \frac{r}{\alpha}$$

$$\mu_2 = \frac{r}{\alpha^2}$$

APPENDIX D

DERIVATIONS FOR THE MOMENTS OF THE NORMAL DISTRIBUTION

$$f(x) = \frac{1}{2\pi\sigma}\, e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$$\mu_1{}' = \mu$$

$$\mu_2{}' = \sigma^2 + \mu^2$$

$$\mu_3{}' = 3\mu_1{}'\mu_2{}' - 2\mu_1^{3}$$

$$\mu_4{}' = 3\sigma^4 + 4\mu_1{}'\mu_3{}' - 6\mu_1{}'^{2}\mu_2{}' + 3\mu_1{}'^{4}$$

$$\mu_r = 0 \quad , \quad r,\ \text{odd}$$

$$\mu_r = \frac{r!}{(r/2)} \quad \frac{\sigma^r}{2^{r/2}} \quad r,\ \text{even}$$

$$\mu_2 = \frac{2!\ \sigma^2}{(2/2)\,2^{2/2}} = \frac{2}{1x2}\,\sigma^2 = \sigma^2$$

$$\mu_4 = \frac{4!}{4/2} \quad \frac{\sigma^4}{2^{4/2}} = \frac{4x3x2}{4} \quad \frac{\sigma^4}{\;} = 3\sigma^4$$

APPENDIX E

BETA DISTRIBUTION

Let a, b, p, and q represent the four-parameters of a generalized Beta distribution. The first two parameters, a and b, represent positive abscissa intercepts. The parameters p and q are the shape parameters. The pdf of the generalized Beta is given by

$$f(X;\ a,b,p,q) = \begin{cases} \dfrac{1}{b-a}\ \dfrac{\Gamma(p+q)}{\Gamma(p)\Gamma(y)} \left[\dfrac{x-a}{b-a}\right]^{(p-1)} \left[1 - \dfrac{x-a}{b-a}\right]^{(q-1)} & a \le x \le b,\ 0 < p,\ 0 < q \\ 0 & \text{elsewhere} \end{cases} \qquad \text{(A.E.1)}$$

The mean, the mode, variance and α_3 & α_4, the standardized third and fourth moments are calculated by the following formulas: (See Johnson and Kotz 1986)

If X has the standard Beta distribution, its rth moment about zero is:

$$\mu'_r = \frac{B(p + r,q)}{B(p,q)}$$

$$= \frac{\Gamma(p + r)\ \Gamma(p + q)}{\Gamma(p)\ \Gamma(p + q + r)}$$

$$= \frac{p^{[r]}}{(p + q)^{[r]}} \qquad \text{(if r is an integer)} \qquad (A.E.2)$$

where $y^{[r]} = y(y + 1), \ldots, (y + r - 1)$ is the ascending factorial.

$$\text{Mode, } m = a + (b-a) \frac{p-1}{p + q - 2} ; \qquad (A.E.3)$$

$$\alpha_3(X) = \sqrt{\{\beta_1(X)\}} = \frac{\mu_3(X)}{\mu_2(X)^{1.5}} ; \qquad (A.E.4)$$

$$\alpha_4(X) = \beta_2(X) = \frac{\mu_4(X)}{\mu_2(X)^2} ; \qquad (A.E.5)$$

$$E(X) = \mu_1 = a + (b - a) \frac{p}{p + q} ; \qquad (A.E.6)$$

$$\sigma^2(X) = \mu_2(X) = (b - a)^2 \left[\frac{pq}{(p + q)^2 (p + q + 1)} \right] \qquad (A.E.7)$$

$$\alpha_3(X) = \frac{2(q - p) \sqrt{p^{-1} + q^{-1} + (pq)^{-1}}}{(p + q + 2)} \qquad (A.E.8)$$

$$\alpha_4(X) = \frac{3(p + q + 1) \{ 2(p + q)^2 + pq (p + q - 6)\}}{pq (p + q + 2) (p + q + 3)} \qquad (A.E.9)$$

If μ_3 and μ_4 are the third and fourth moments,

$$\mu_3(X) = [\sigma^2(X)]^{1.5} \left[\frac{2(q - p) \sqrt{p^{-1} + q^{-1} + (pq)^{-1}}}{(p + q + 2)} \right] \qquad (A.E.10)$$

$$\mu_4(X) = [\sigma^2(X)] \left[\frac{3(p + q + 1) \{2(p + q)^2 + pq (p + q - 6)\}}{pq (p + q + 2) (p + q + 3)} \right] \qquad (A.E.11)$$

APPENDIX F

COMPUTER OUTPUT FROM A PEARSON CURVE FITTING PROGRAM TO FIND THE MAXIMUM OF SEVERAL STOCHASTIC VARIABLES

```
        $ENTRY

 NO OF VARIABLES =   4      SAMPLE SIZE =  1000
 THE INPUT PARAMATER VALUES ARE :
  #  IPOP       P          Q          A          B
  1    1     1.5000     2.5000     1.0000    10.0000
  2    4     5.5000     1.5000     0.0000     1.0000
  3    1     2.0000     3.0000     1.0000    10.0000
  4    4     5.0000     2.0000     0.0000     1.0000

 SAMPLE NUMBER =   1

 THE SAMPLE IS GENERATED FROM A BETA POP.
IPOP =    1 P =    1.5000 Q =    2.5000

 THE MOMENTS OF THE POPULATION ARE :
RAW MOMENTS :
PRM1 =        0.37500000    PRM2 =        0.18750000    PRM3 =
0.10937500
THE CENTRAL MOMENTS     ARE :
PCM2 =        0.04687500    PCM3 =        0.00390625    PCM4 =
0.00512695

 THE MOMENTS OF THE GENERATED SAMPLE ARE :
RAW MOMENTS :
RM1 =      4.43300912 RM2 =     23.33595776 RM3 =    138.40129186
RM4 =
CENTRAL MOMENTS :
CM2 =      3.68438788 CM3 =      2.28692806 CM4 =     30.99500746

                   ******* RUN OVER ******

 SAMPLE NUMBER =   2

 THIS SAMPLE IS GENERATED FROM A NORMAL POP.
IPOP =    4 MEAN =    5.5000 STD. DEV =   1.5000
```

THE MOMENTS OF THE POPULATION ARE :
RAW MOMENTS :
PRM1 = 5.50000000 PRM2 = 32.50000000 PRM3 =
203.50000000
THE CENTRAL MOMENTS ARE :
PCM2 = 2.25000000 PCM3 = 0.00000000 PCM4 =
15.18750000

THE MOMENTS OF THE GENERATED SAMPLE ARE :
RAW MOMENTS :
RM1 = 5.58279687 RM2 = 33.30266571 RM3 = 209.48479231
RM4 =
CENTRAL MOMENTS :
CM2 = 2.13504484 CM3 = -0.27626879 CM4 = 14.20890923

******* RUN OVER ******

SAMPLE NUMBER = 3

THE SAMPLE IS GENERATED FROM A BETA POP.
IPOP = 1 P = 2.0000 Q = 3.0000

THE MOMENTS OF THE POPULATION ARE :
RAW MOMENTS :
PRM1 = 0.40000000 PRM2 = 0.20000000 PRM3 =
0.11428571
THE CENTRAL MOMENTS ARE :
PCM2 = 0.04000000 PCM3 = 0.00228571 PCM4 =
0.00377143

THE MOMENTS OF THE GENERATED SAMPLE ARE :
RAW MOMENTS :
RM1 = 4.67509154 RM2 = 25.10171229 RM3 = 148.97465735
RM4 =
CENTRAL MOMENTS :
CM2 = 3.24523137 CM3 = 1.27834706 CM4 = 23.34305992

******* RUN OVER ******

SAMPLE NUMBER = 4

THIS SAMPLE IS GENERATED FROM A NORMAL POP.
IPOP = 4 MEAN = 5.0000 STD. DEV = 2.0000

THE MOMENTS OF THE POPULATION ARE :
RAW MOMENTS :
PRM1 = 5.00000000 PRM2 = 29.00000000 PRM3 =
185.00000000
THE CENTRAL MOMENTS ARE :
PCM2 = 4.00000000 PCM3 = 0.00000000 PCM4 =
48.00000000

```
 THE MOMENTS OF THE GENERATED SAMPLE ARE :
RAW MOMENTS :
RM1 =      5.00012798 RM2 =     28.93202305 RM3 =    184.15208375
RM4 =
CENTRAL MOMENTS :
CM2 =      3.93074323 CM3 =      0.17982741 CM4 =     42.69472951

                   ******* RUN OVER ******

 THE MOMENTS OF Y ARE :
RAW MOMENTS :
YRM1  =         6.87888238  YRM2  =        48.76442949  YRM3  =
355.57733117 YRM
CENTRAL MOMENTS :

YCM2  =        1.44540671  YCM3  =        0.24699092  YCM4  =
5.95974032

                    ***** Y = MAX(X1,X2,...,XN) ******


***********************************
*   NOTATION :                    *
*      Y1 = X1                    *
*      Y2 = MAX(X1,X2)            *
*      Y3 = MAX(X1,X2,X3)         *
*           .                     *
*           .                     *
*           .                     *
*      YN = MAX(X1,X2,X3,...,XN)  *
***********************************


************************************************
*            CM1 = E(XN+1) - E(YN)             *
*            CM2 = M(XN+1**2)+M(YN**2)         *
*            CM3 = M(XN+1**3)-M(YN**3)         *
*            CM4 = M(XN+1**4)+M(YN**4)         *
*                 +6*M(XN+1**2)*M(YN**2)       *
*       WHERE M(X**R) DENOTES RTH CENTRAL      *
*           MOMENT OF X                        *
*            B1 = CM3**2 / CM2**3              *
*            B2 = CM4 / CM2**2                 *
*            KAPPA = CRITERION WHICH DECIDES   *
*                    THE PEARSON TYPE TO BE    *
*                    FITTED.                   *
************************************************


CM1 =        1.1497877 CM2 =        5.8194327 CM3 =       -2.5631969
```

```
CM4 =
B1 =        0.0333367 B2 =        2.7284731
KAPPA =       -0.0393282


 ********************************************
 *   THE PROGRAM WILL TRY TO FIT A TYPE I   *
 *   PEARSON DISTRIBUTION TO THE VARIABLE   *
 *   XN+1-YN :(X2-X1) OR (X3-Y2),...        *
 ********************************************

 THE PEARSON TYPE I FUNCTION IS :
     0.1577386 ( 1 + Z/     12.0469130)**      8.4464184 ( 1 - Z/
    8
THIS FUNCTION WHEN INTEGRATED FROM -A1T1 =     -12.04691301 TO
     A


*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE DCADRE
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE DCADRE

 THE RAW MOMENTS BY INTEGRATION ARE :
    RM1 =      -0.00000044 RM2 =      5.81943261
    RM3 =      -2.56319685 RM4 =     92.40191680



 THE FOUR PARTIAL MOMENTS CALCULATED FROM THE
LOWER LIMIT OF DISTRIBUTION TO 0 , USING WINKLER METHOD ARE :
PART. MOMENT 1 =       -0.9759877 PART. MOMENT 2 =       2.6041440

PART. MOMENT 3 =       -9.5747936 PART. MOMENT 4 =      34.6417957




THE 4 PARTIAL MOMENTS OF THE ABOVE TYPE I PEARSON FROM :
0 TO        8.1233000 ARE :
MOMENT 1 =        0.9759872 MOMENT 2 =      3.2152886
MOMENT 3 =        7.0115967 MOMENT 4 =     57.7601211



  THE CENTRAL MOMENTS OF (X 2-Y 1)+ ARE :
CMU1 =        0.9759872
CMU2 =        2.2627375
CMU3 =       -0.5432898
CMU4 =       46.0415048

  THE MOMENTS OF Y = Y 1 + (X 2-Y 1)+    ARE :
MU1Y =        5.4089964
MU2Y =        5.9471254
```

```
MU3Y =       1.7436383
MU4Y =     127.0573289

                   ***ITERATION  1 STOPS HERE***


************************************************
*            CM1 = E(XN+1) - E(YN)             *
*            CM2 = M(XN+1**2)+M(YN**2)         *
*            CM3 = M(XN+1**3)-M(YN**3)         *
*            CM4 = M(XN+1**4)+M(YN**4)         *
*                  +6*M(XN+1**2)*M(YN**2)      *
*       WHERE M(X**R) DENOTES RTH CENTRAL      *
*           MOMENT OF X                        *
*            B1 = CM3**2 / CM2**3              *
*            B2 = CM4 / CM2**2                 *
*            KAPPA = CRITERION WHICH DECIDES   *
*                    THE PEARSON TYPE TO BE    *
*                    FITTED.                   *
************************************************


CM1 =      -0.7339048 CM2 =       9.1923568 CM3 =      -0.4652912
CM4 =
B1 =        0.0002787 B2 =       3.1503076
KAPPA =       0.0006978


 ********************************************
 *   THE PROGRAM WILL TRY TO FIT A TYPE IV  *
 *   PEARSON DISTRIBUTION TO THE VARIABLE   *
 *   XN+1-YN :(X2-X1) OR (X3-Y2),...        *
 ********************************************
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE DCADRE
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE DCADRE
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE DCADRE

  THE PEARSON TYPE 4 FUNCTION IS :
     0.1320085 (1 + (X/      19.6495805 -      -0.0264247)**2)**
 -22.5
*EXP(          1.1371153  ARCTAN(X  /        19.6495805  -
-0.0264247))

THIS FUNCTION WHEN INTEGRATED FROM -INFINITY(-1.D6) TO INFINITY
(1.D6) G
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE DCADRE
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE DCADRE

*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE DCADRE
```

```
 THE RAW MOMENTS BY INTEGRATION ARE :
RM1 =        0.0000001 RM2 =        9.1923568
RM3 =        0.4652913 RM4 =      266.1991781


 THE FOUR PARTIAL MOMENTS CALCULATED FROM THE
LOWER LIMIT OF DISTRIBUTION TO 0 , USING WINKLER METHOD ARE :
PART. MOMENT 1 =       -1.2023705 PART. MOMENT 2 =        4.5762812

PART. MOMENT 3 =      -22.4123263 PART. MOMENT 4 =      130.7611843


THE 4 PARTIAL MOMENTS OF THE ABOVE TYPE IV PEARSON FROM  :
0 TO INFINITY ARE :
MOMENT 1 =        1.2023705 MOMENT 2 =        4.6160756
MOMENT 3 =       22.8776175 MOMENT 4 =      135.4379938


  THE CENTRAL MOMENTS OF (X 3-Y 2)+ ARE :

CMU1 =        1.2023705
CMU2 =        3.1703807
CMU3 =        9.7034396
CMU4 =       59.1790212

  THE MOMENTS OF Y = Y 2 + (X 3-Y 2)+    ARE :
MU1Y =        6.6113669
MU2Y =        9.1175061
MU3Y =       11.4470778
MU4Y =      299.3642596

                    ***ITERATION  2 STOPS HERE***


************************************************
*            CM1 = E(XN+1) - E(YN)             *
*            CM2 = M(XN+1**2)+M(YN**2)         *
*            CM3 = M(XN+1**3)-M(YN**3)         *
*            CM4 = M(XN+1**4)+M(YN**4)         *
*                  +6*M(XN+1**2)*M(YN**2)      *
*       WHERE M(X**R) DENOTES RTH CENTRAL      *
*           MOMENT OF X                        *
*            B1 = CM3**2 / CM2**3              *
*            B2 = CM4 / CM2**2                 *
*            KAPPA = CRITERION WHICH DECIDES   *
*                    THE PEARSON TYPE TO BE    *
*                    FITTED.                   *
************************************************
```

```
CM1 =       -1.6112389 CM2 =      13.0482493 CM3 =     -11.2672504
CM4 =
B1 =        0.0571451 B2 =       3.2720597
KAPPA =        0.1167465


 ********************************************
 *   THE PROGRAM WILL TRY TO FIT A TYPE IV  *
 *   PEARSON DISTRIBUTION TO THE VARIABLE   *
 *   XN+1-YN :(X2-X1) OR (X3-Y2),...        *
 ********************************************
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE DCADRE
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE DCADRE
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE DCADRE

  THE PEARSON TYPE 4 FUNCTION IS :
     0.0126983 (1 + (X/     19.9860049 -      -0.3635626)**2)**
 -18.8
*EXP(         12.9642304  ARCTAN(X  /         19.9860049  -
-0.3635626))

THIS  FUNCTION  WHEN  INTEGRATED  FROM  -INFINITY(-1.D6)  TO  INFINITY
(1.D6) G
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE DCADRE
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE DCADRE
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE DCADRE
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE DCADRE

 THE RAW MOMENTS BY INTEGRATION ARE :
RM1 =        0.0000001 RM2 =      13.0482515
RM3 =       11.2672476 RM4 =     557.0903387



 THE FOUR PARTIAL MOMENTS CALCULATED FROM THE
LOWER LIMIT OF DISTRIBUTION TO 0 , USING WINKLER METHOD ARE :
PART. MOMENT 1 =       -1.4291584 PART. MOMENT 2 =        6.1226103

PART. MOMENT 3 =     -33.1171739 PART. MOMENT 4 =     210.1363093


THE 4 PARTIAL MOMENTS OF THE ABOVE TYPE IV PEARSON FROM   :
0 TO INFINITY ARE :
MOMENT 1 =        1.4291584 MOMENT 2 =      6.9256412
MOMENT 3 =       44.3844216 MOMENT 4 =     346.9540293
```

```
  THE CENTRAL MOMENTS OF (X 4-Y 3)+ ARE :
CMU1 =       1.4291584
CMU2 =       4.8831473
CMU3 =      20.5290006
CMU4 =     165.5826802

  THE MOMENTS OF Y = Y 3 + (X 4-Y 3)+    ARE :
MU1Y =       8.0405253

MU2Y =      14.0006534
MU3Y =      31.9760784
MU4Y =     732.0796933

                    ***ITERATION  3 STOPS HERE***


STATEMENTS EXECUTED=  306405

CORE USAGE        OBJECT CODE=   90328 BYTES,ARRAY AREA=  213594
BYTES,TO

DIAGNOSTICS        NUMBER OF ERRORS=       0, NUMBER OF WARNINGS=


COMPILE  TIME=       0.20  SEC,EXECUTION  TIME=       0.77  SEC,
10.37.19
         C$STOP

++++ BOTTOM OF DATA SET +++
==>
```

APPENDIX G

COMPUTER OUTPUT FROM THE

SIMULATION PROGRAM FOR

NETWORK #10

```
20E740 BYTES USED
EXECUTION BEGINS

              ** PGM: SIMF10K.CNTL **SEPT.4,1989 ***

                  *********** SIMULATION  ***********

    SIMULATION RUNS = 10000     DSEED =  0.3250170D+06

    + + + + +  PART I + + + + + + + + +
    * * * READ & ECHO PRINT THE GIVEN DATA * * *
    NUMBER OF ACTIVITIES =  14

    A,B,P & Q OF ACTIVITY   1  ARE:        12.0000      15.0000
   3.0000       2.0000

    A,B,P & Q OF ACTIVITY   2  ARE:        14.0000      17.0000
   2.0000       3.0000

    A,B,P & Q OF ACTIVITY   3  ARE:        18.0000      24.0000
   1.5000       2.5000

    A,B,P & Q OF ACTIVITY   4  ARE:        17.0000      22.0000
   1.5000       3.5000

    A,B,P & Q OF ACTIVITY   5  ARE:        17.0000      23.0000
   3.0000       3.5000

    A,B,P & Q OF ACTIVITY   6  ARE:        22.0000      25.0000
   2.0000       2.5000

    A,B,P & Q OF ACTIVITY   7  ARE:        10.0000      14.0000
   3.0000       3.0000

    A,B,P & Q OF ACTIVITY   8  ARE:         1.0000       2.0000
   2.0000       2.0000
```

```
 A,B,P & Q OF ACTIVITY   9  ARE:       1.0000       1.0000
2.5000      2.5000

 A,B,P & Q OF ACTIVITY  10  ARE:       8.0000      15.0000
2.0000      3.0000

 A,B,P & Q OF ACTIVITY  11  ARE:      11.0000      18.0000
1.5000      2.5000

 A,B,P & Q OF ACTIVITY  12  ARE:      21.0000      24.0000
3.0000      2.0000

 A,B,P & Q OF ACTIVITY  13  ARE:      17.0000      20.0000
2.0000      3.0000

 A,B,P & Q OF ACTIVITY  14  ARE:      24.0000      36.0000
3.0000      3.0000

 NUMBER OF PATHS ARE:   10
 ACTIVITIES ON PATH   1 ARE    1    6    9   12
 ACTIVITIES ON PATH   2 ARE    1    6   10   13
 ACTIVITIES ON PATH   3 ARE    1    3    7   13
 ACTIVITIES ON PATH   4 ARE    1    3    8   11   13
 ACTIVITIES ON PATH   5 ARE    1    3    8   14
 ACTIVITIES ON PATH   6 ARE    2    4    7   13
 ACTIVITIES ON PATH   7 ARE    2    4    8   11   13
 ACTIVITIES ON PATH   8 ARE    2    4    8   14
 ACTIVITIES ON PATH   9 ARE    2    5   11   13
 ACTIVITIES ON PATH  10 ARE    2    5   14

 + + END PART I & BEGIN PART II + + + +

 ** FIND MU,VAR,MU3 & MU4 OF EACH ACTIVITY**

 ACTIVITY   1:
 EXP =     13.8000   VAR =       0.3600   ALPHA3 =     -0.2857
ALPHA4=     2.3571
   MU3 =    -0.0617  MU4 =      0.3055
  PERT MODE=     14.0000   PERT MU =    13.8333   PERT VAR=
0.2500   PERT SD=     0.5000  PERT CV= 0.0361

 ACTIVITY   2:
 EXP =     15.2000   VAR =       0.3600   ALPHA3 =      0.2857
ALPHA4=     2.3571
   MU3 =     0.0617  MU4 =      0.3055
  PERT MODE=     15.0000   PERT MU =    15.1667   PERT VAR=
0.2500   PERT SD=     0.5000  PERT CV= 0.0330

 ACTIVITY   3:
 EXP =     20.2500   VAR =       1.6875   ALPHA3 =      0.3849
ALPHA4=     2.3333
```

MU3 = 0.8437 MU4 = 6.6445
PERT MODE= 19.5000 PERT MU = 20.0000 PERT VAR=
1.0000 PERT SD= 1.0000 PERT CV= 0.0500

ACTIVITY 4:
EXP = 18.5000 VAR = 0.8750 ALPHA3 = 0.6109
ALPHA4= 2.7398
MU3 = 0.5000 MU4 = 2.0977
PERT MODE= 17.8333 PERT MU = 18.3889 PERT VAR=
0.6944 PERT SD= 0.8333 PERT CV= 0.0453

ACTIVITY 5:
EXP = 19.7692 VAR = 1.1929 ALPHA3 = 0.0994
ALPHA4= 2.3817
MU3 = 0.1295 MU4 = 3.3892
PERT MODE= 19.6667 PERT MU = 19.7778 PERT VAR=
1.0000 PERT SD= 1.0000 PERT CV= 0.0506

ACTIVITY 6:
EXP = 23.3333 VAR = 0.4040 ALPHA3 = 0.1614
ALPHA4= 2.2338
MU3 = 0.0414 MU4 = 0.3647
PERT MODE= 23.2000 PERT MU = 23.3000 PERT VAR=
0.2500 PERT SD= 0.5000 PERT CV= 0.0215

ACTIVITY 7:
EXP = 12.0000 VAR = 0.5714 ALPHA3 = 0.0
ALPHA4= 2.3333
MU3 = 0.0 MU4 = 0.7619
PERT MODE= 12.0000 PERT MU = 12.0000 PERT VAR=
0.4444 PERT SD= 0.6667 PERT CV= 0.0556

ACTIVITY 8:
EXP = 1.5000 VAR = 0.0500 ALPHA3 = 0.0
ALPHA4= 2.1429
MU3 = 0.0 MU4 = 0.0054
PERT MODE= 1.5000 PERT MU = 1.5000 PERT VAR=
0.0278 PERT SD= 0.1667 PERT CV= 0.1111

ACTIVITY 9:
EXP = 1.0000 VAR = 0.0 ALPHA3 = 0.0
ALPHA4= 2.2500
MU3 = 0.0 MU4 = 0.0
PERT MODE= 1.0000 PERT MU = 1.0000 PERT VAR= 0.0
PERT SD= 0.0 PERT CV= 0.0

ACTIVITY 10:
EXP = 10.8000 VAR = 1.9600 ALPHA3 = 0.2857
ALPHA4= 2.3571
MU3 = 0.7840 MU4 = 9.0552
PERT MODE= 10.3333 PERT MU = 10.7222 PERT VAR=
1.3611 PERT SD= 1.1667 PERT CV= 0.1088

ACTIVITY 11:
EXP = 13.6250 VAR = 2.2969 ALPHA3 = 0.3849
ALPHA4= 2.3333
MU3 = 1.3398 MU4 = 12.3098
PERT MODE= 12.7500 PERT MU = 13.3333 PERT VAR=
1.3611 PERT SD= 1.1667 PERT CV= 0.0875

ACTIVITY 12:
EXP = 22.8000 VAR = 0.3600 ALPHA3 = -0.2857
ALPHA4= 2.3571
MU3 = -0.0617 MU4 = 0.3055
PERT MODE= 23.0000 PERT MU = 22.8333 PERT VAR=
0.2500 PERT SD= 0.5000 PERT CV= 0.0219

ACTIVITY 13:
EXP = 18.2000 VAR = 0.3600 ALPHA3 = 0.2857
ALPHA4= 2.3571
MU3 = 0.0617 MU4 = 0.3055
PERT MODE= 18.0000 PERT MU = 18.1667 PERT VAR=
0.2500 PERT SD= 0.5000 PERT CV= 0.0275

ACTIVITY 14:
EXP = 30.0000 VAR = 5.1429 ALPHA3 = 0.0
ALPHA4= 2.3333
MU3 = 0.0 MU4 = 61.7142
PERT MODE= 30.0000 PERT MU = 30.0000 PERT VAR=
4.0000 PERT SD= 2.0000 PERT CV= 0.0667

*** END PART II & BEGIN PART III *****

***** SIMULATION RUN NO = 1********

** NO OF SIMULATION RUNS = 10000 ***

PATH LENGTH 1 = 60.88924

SIMULATED MOMENTS OF PATH NO : 1 ARE :
RAW MOMENTS :
RM1 = 60.91716 RM2 = 3712.04834 RM3 = 226267.187 RM4
= 13796321.0
CENTRAL MOMENTS:
CM1= 60.91716 CM2 = 1.14722 CM3 = -0.05736 CM4
= 3.68228
STD DEV= 1.07108 COEFF.VAR.= 0.0176

********* RUN OVER ************

PATH LENGTH 2 = 66.09187

SIMULATED MOMENTS OF PATH NO : 2 ARE :
RAW MOMENTS :

RM1 = 66.12029 RM2 = 4374.92969 RM3 = 289674.000 RM4 = 19193328.0
CENTRAL MOMENTS:
CM1= 66.12029 CM2 = 3.03716 CM3 = 0.75432 CM4 = 25.12547
STD DEV= 1.74274 COEFF.VAR.= 0.0264

********* RUN OVER ************

PATH LENGTH 3 = 64.21664

SIMULATED MOMENTS OF PATH NO : 3 ARE :
RAW MOMENTS :
RM1 = 64.24474 RM2 = 4130.34766 RM3 = 265734.937 RM4 = 17108960.0
CENTRAL MOMENTS:
CM1= 64.24474 CM2 = 2.96348 CM3 = 0.87471 CM4 = 23.91571
STD DEV= 1.72148 COEFF.VAR.= 0.0268

********* RUN OVER ************

PATH LENGTH 4 = 67.35942

SIMULATED MOMENTS OF PATH NO : 4 ARE :
RAW MOMENTS :
RM1 = 67.38762 RM2 = 4545.76172 RM3 = 306959.687 RM4 = 20749392.0
CENTRAL MOMENTS:
CM1= 67.38762 CM2 = 4.66986 CM3 = 2.14714 CM4 = 59.85732
STD DEV= 2.16099 COEFF.VAR.= 0.0321

********* RUN OVER ************

PATH LENGTH 5 = 65.47922

SIMULATED MOMENTS OF PATH NO : 5 ARE :
RAW MOMENTS :
RM1 = 65.50716 RM2 = 4298.37109 RM3 = 282516.562 RM4 = 18599680.0
CENTRAL MOMENTS:
CM1= 65.50716 CM2 = 7.18511 CM3 = 1.00362 CM4 = 135.13179
STD DEV= 2.68051 COEFF.VAR.= 0.0409

********* RUN OVER ************

PATH LENGTH 6 = 63.86578

SIMULATED MOMENTS OF PATH NO : 6 ARE :
RAW MOMENTS :

RM1 = 63.89380 RM2 = 4084.54468 RM3 = 261249.187 RM4 = 16718347.0
CENTRAL MOMENTS:
CM1= 63.89380 CM2 = 2.12578 CM3 = 0.46871 CM4 = 12.74008
STD DEV= 1.45801 COEFF.VAR.= 0.0228

********* RUN OVER ************

PATH LENGTH 7 = 67.00832

SIMULATED MOMENTS OF PATH NO : 7 ARE :
RAW MOMENTS :
RM1 = 67.03670 RM2 = 4497.68359 RM3 = 302016.937 RM4 = 20297392.0
CENTRAL MOMENTS:
CM1= 67.03670 CM2 = 3.76830 CM3 = 1.62973 CM4 = 38.84732
STD DEV= 1.94121 COEFF.VAR.= 0.0290

********* RUN OVER ************

PATH LENGTH 8 = 65.12796

SIMULATED MOMENTS OF PATH NO : 8 ARE :
RAW MOMENTS :
RM1 = 65.15622 RM2 = 4251.68359 RM3 = 277852.562 RM4 = 18185024.0
CENTRAL MOMENTS:
CM1= 65.15622 CM2 = 6.35215 CM3 = 1.02263 CM4 = 105.04929
STD DEV= 2.52035 COEFF.VAR.= 0.0387

********* RUN OVER ************

PATH LENGTH 9 = 66.78877

SIMULATED MOMENTS OF PATH NO : 9 ARE :
RAW MOMENTS :
RM1 = 66.81731 RM2 = 4468.67187 RM3 = 299136.437 RM4 = 20042976.0
CENTRAL MOMENTS:
CM1= 66.81731 CM2 = 4.11774 CM3 = 1.47192 CM4 = 46.73332
STD DEV= 2.02922 COEFF.VAR.= 0.0304

********* RUN OVER ************

PATH LENGTH 10 = 64.90855

SIMULATED MOMENTS OF PATH NO :10 ARE :
RAW MOMENTS :

```
    RM1 =     64.93684 RM2 =   4223.41016 RM3 =   275115.250 RM4
= 17949056.0
    CENTRAL MOMENTS:
    CM1=    64.93684  CM2 =     6.61872  CM3 =     0.54677   CM4
=   113.96292
   STD DEV=     2.57269   COEFF.VAR.= 0.0396

                   ********* RUN OVER   ************

   SIMULATED MOMENTS OF COMP.TIME FOR RUN NO: 1
    RAW MOMENTS :
    RM1 =     68.77054 RM2 =   4732.37500 RM3 =   325859.625 RM4
= 22452112.0
    CENTRAL MOMENTS:
    CM1=    68.77054  CM2 =     2.98652  CM3 =     0.85764   CM4
=    25.18864
   ** SB1 =       0.16617 **B2 =       2.82407
     STD DEV =      1.72815  COEFF. OF VAR =      0.02513

                   ********* RUN OVER   ************

              ***** SIMULATION RUN NO =  2********

    ** NO OF SIMULATION RUNS = 10000   ***

   PATH LENGTH  1 =    60.90431

   SIMULATED MOMENTS OF PATH NO : 1  ARE :
    RAW MOMENTS :
    RM1 =     60.93213 RM2 =   3713.84692 RM3 =   226429.125 RM4
= 13809285.0
    CENTRAL MOMENTS:
    CM1=    60.93213  CM2 =     1.12103  CM3 =    -0.10394   CM4
=     3.45502
   STD DEV=     1.05878   COEFF.VAR.= 0.0174

                   ********* RUN OVER   ************

   PATH LENGTH  2 =    66.09833

   SIMULATED MOMENTS OF PATH NO : 2  ARE :
    RAW MOMENTS :
    RM1 =     66.12657 RM2 =   4375.80859 RM3 =   289766.000 RM4
= 19201888.0
    CENTRAL MOMENTS:
    CM1=    66.12657  CM2 =     3.08360  CM3 =     0.92266   CM4
=    25.66837
   STD DEV=     1.75602   COEFF.VAR.= 0.0266

                   ********* RUN OVER   ************

   PATH LENGTH  3 =    64.21498
```

```
   SIMULATED MOMENTS OF PATH NO : 3  ARE :
    RAW MOMENTS :
    RM1 =      64.24304 RM2 =   4130.13672 RM3 =   265715.000 RM4
= 17107280.0
    CENTRAL MOMENTS:
    CM1=    64.24304  CM2 =     2.96887  CM3 =     0.82976   CM4
=    24.40315
   STD DEV=     1.72304   COEFF.VAR.= 0.0268

               ********* RUN OVER   ************

   PATH LENGTH  4 =    67.35641

   SIMULATED MOMENTS OF PATH NO : 4  ARE :
    RAW MOMENTS :
    RM1 =      67.38478 RM2 =   4545.41406 RM3 =   306928.750 RM4
= 20746992.0
    CENTRAL MOMENTS:
    CM1=    67.38478  CM2 =     4.70724  CM3 =     2.39745   CM4
=    62.17242
   STD DEV=     2.16962   COEFF.VAR.= 0.0322

               ********* RUN OVER   ************

   PATH LENGTH  5 =    65.50627

   SIMULATED MOMENTS OF PATH NO : 5  ARE :
    RAW MOMENTS :
    RM1 =      65.53456 RM2 =   4302.09375 RM3 =   282895.750 RM4
= 18634000.0
    CENTRAL MOMENTS:
    CM1=    65.53456  CM2 =     7.31751  CM3 =     0.67252   CM4
=   141.84810
   STD DEV=     2.70509   COEFF.VAR.= 0.0413

               ********* RUN OVER   ************

   PATH LENGTH  6 =    63.87080

   SIMULATED MOMENTS OF PATH NO : 6  ARE :
    RAW MOMENTS :
    RM1 =      63.89877 RM2 =   4085.20435 RM3 =   261314.875 RM4
= 16724162.0
    CENTRAL MOMENTS:
    CM1=    63.89877  CM2 =     2.14985  CM3 =     0.57023   CM4
=    13.30443
   STD DEV=     1.46624   COEFF.VAR.= 0.0229

               ********* RUN OVER   ************

   PATH LENGTH  7 =    67.01241
```

```
SIMULATED MOMENTS OF PATH NO : 7  ARE :
 RAW MOMENTS :
 RM1 =     67.04051 RM2 =   4498.33594 RM3 =   302096.812 RM4
= 20305856.0
 CENTRAL MOMENTS:
 CM1=    67.04051  CM2 =     3.90711  CM3 =     2.03197   CM4
=    41.71822
STD DEV=     1.97664   COEFF.VAR.= 0.0295

               ********* RUN OVER   ************

PATH LENGTH  8 =    65.16199

SIMULATED MOMENTS OF PATH NO : 8  ARE :
 RAW MOMENTS :
 RM1 =     65.19029 RM2 =   4256.32812 RM3 =   278326.937 RM4
= 18228048.0
 CENTRAL MOMENTS:
 CM1=    65.19029  CM2 =     6.55570  CM3 =     0.77850   CM4
=    108.55299
STD DEV=     2.56041   COEFF.VAR.= 0.0393

               ********* RUN OVER   ************

PATH LENGTH  9 =    66.77362

SIMULATED MOMENTS OF PATH NO : 9  ARE :
 RAW MOMENTS :
 RM1 =     66.80214 RM2 =   4466.67969 RM3 =   298940.125 RM4
= 20025760.0
 CENTRAL MOMENTS:
 CM1=    66.80214  CM2 =     4.15352  CM3 =     1.38953   CM4
=    47.55359
STD DEV=     2.03802   COEFF.VAR.= 0.0305

               ********* RUN OVER   ************

PATH LENGTH 10 =    64.92371

SIMULATED MOMENTS OF PATH NO :10  ARE :
 RAW MOMENTS :
 RM1 =     64.95192 RM2 =   4225.55078 RM3 =   275341.937 RM4
= 17970288.0
 CENTRAL MOMENTS:
 CM1=    64.95192  CM2 =     6.80160  CM3 =     0.56618   CM4
=    118.66930
STD DEV=     2.60799   COEFF.VAR.= 0.0402

               ********* RUN OVER   ************

SIMULATED MOMENTS OF COMP.TIME FOR RUN NO: 2
```

```
     RAW MOMENTS :
     RM1 =      68.81271 RM2 =   4738.16406 RM3 =   326456.687 RM4
= 22506880.0
     CENTRAL MOMENTS:
     CM1=    68.81271  CM2 =     2.97629  CM3 =     1.00834   CM4
=    25.61478
    ** SB1 =       0.19638 **B2 =      2.89162
      STD DEV =     1.72519  COEFF. OF VAR =      0.02507

                    ********* RUN OVER   ************

               ***** SIMULATION RUN NO =  3********

     ** NO OF SIMULATION RUNS = 10000  ***

    PATH LENGTH  1 =    60.89664

    SIMULATED MOMENTS OF PATH NO : 1  ARE :
     RAW MOMENTS :
     RM1 =      60.92455 RM2 =   3712.93286 RM3 =   226346.625 RM4
= 13802666.0
     CENTRAL MOMENTS:
     CM1=    60.92455  CM2 =     1.13208  CM3 =    -0.06212   CM4
=      3.53448
    STD DEV=     1.06399   COEFF.VAR.= 0.0175

                    ********* RUN OVER   ************

    PATH LENGTH  2 =    66.07272

    SIMULATED MOMENTS OF PATH NO : 2  ARE :
     RAW MOMENTS :
     RM1 =      66.10094 RM2 =   4372.39453 RM3 =   289425.125 RM4
= 19171600.0
     CENTRAL MOMENTS:
     CM1=    66.10094  CM2 =     3.06149  CM3 =     0.93727   CM4
=     25.69403
    STD DEV=     1.74971   COEFF.VAR.= 0.0265

                    ********* RUN OVER   ************

    PATH LENGTH  3 =    64.20488

    SIMULATED MOMENTS OF PATH NO : 3  ARE :
     RAW MOMENTS :
     RM1 =      64.23311 RM2 =   4128.83594 RM3 =   265587.187 RM4
= 17096096.0
     CENTRAL MOMENTS:
     CM1=    64.23311  CM2 =     2.94497  CM3 =     0.69531   CM4
=     23.85249
    STD DEV=     1.71609   COEFF.VAR.= 0.0267
```

```
                    ********* RUN OVER   ************

    PATH LENGTH  4 =    67.32942

    SIMULATED MOMENTS OF PATH NO : 4  ARE :
     RAW MOMENTS :
     RM1 =      67.35800 RM2 =   4541.76172 RM3 =   306553.812 RM4
 = 20712704.0
     CENTRAL MOMENTS:
     CM1=    67.35800  CM2 =     4.66228  CM3 =     1.70960   CM4
 =    58.65518
    STD DEV=     2.15923   COEFF.VAR.= 0.0321

                    ********* RUN OVER   ************

    PATH LENGTH  5 =    65.54680

    SIMULATED MOMENTS OF PATH NO : 5  ARE :
     RAW MOMENTS :
     RM1 =      65.57544 RM2 =   4307.58594 RM3 =   283450.125 RM4
 = 18683856.0
     CENTRAL MOMENTS:
     CM1=    65.57544  CM2 =     7.44791  CM3 =     1.33834   CM4
 =   146.51619
    STD DEV=     2.72908   COEFF.VAR.= 0.0416

                    ********* RUN OVER   ************

    PATH LENGTH  6 =    63.86259

    SIMULATED MOMENTS OF PATH NO : 6  ARE :
     RAW MOMENTS :
     RM1 =      63.89093 RM2 =   4084.21216 RM3 =   261220.750 RM4
 = 16716226.0
     CENTRAL MOMENTS:
     CM1=    63.89093  CM2 =     2.15949  CM3 =     0.65579   CM4
 =    13.55317
    STD DEV=     1.46952   COEFF.VAR.= 0.0230

                    ********* RUN OVER   ************

    PATH LENGTH  7 =    66.98750

    SIMULATED MOMENTS OF PATH NO : 7  ARE :
     RAW MOMENTS :
     RM1 =      67.01582 RM2 =   4494.98047 RM3 =   301754.375 RM4
 = 20274784.0
     CENTRAL MOMENTS:
     CM1=    67.01582  CM2 =     3.86094  CM3 =     1.98878   CM4
 =    41.91035
    STD DEV=     1.96493   COEFF.VAR.= 0.0293
```

********* RUN OVER ************

PATH LENGTH 8 = 65.20509

SIMULATED MOMENTS OF PATH NO : 8 ARE :
RAW MOMENTS :
RM1 = 65.23328 RM2 = 4261.87500 RM3 = 278864.625 RM4 = 18274432.0
CENTRAL MOMENTS:
CM1= 65.23328 CM2 = 6.49748 CM3 = 0.67931 CM4 = 107.23639
STD DEV= 2.54902 COEFF.VAR.= 0.0391

********* RUN OVER ************

PATH LENGTH 9 = 66.75760

SIMULATED MOMENTS OF PATH NO : 9 ARE :
RAW MOMENTS :
RM1 = 66.78612 RM2 = 4464.45703 RM3 = 298709.125 RM4 = 20004464.0
CENTRAL MOMENTS:
CM1= 66.78612 CM2 = 4.07143 CM3 = 1.57257 CM4 = 45.99821
STD DEV= 2.01778 COEFF.VAR.= 0.0302

********* RUN OVER ************

PATH LENGTH 10 = 64.97528

SIMULATED MOMENTS OF PATH NO :10 ARE :
RAW MOMENTS :
RM1 = 65.00356 RM2 = 4232.28125 RM3 = 275999.875 RM4 = 18027552.0
CENTRAL MOMENTS:
CM1= 65.00356 CM2 = 6.81745 CM3 = 0.19684 CM4 = 120.30133
STD DEV= 2.61102 COEFF.VAR.= 0.0402

********* RUN OVER ************

SIMULATED MOMENTS OF COMP.TIME FOR RUN NO: 3
RAW MOMENTS :
RM1 = 68.77953 RM2 = 4733.64844 RM3 = 325995.125 RM4 = 22464912.0
CENTRAL MOMENTS:
CM1= 68.77953 CM2 = 3.02565 CM3 = 0.76301 CM4 = 25.50966
** SB1 = 0.14498 **B2 = 2.78655
STD DEV = 1.73944 COEFF. OF VAR = 0.02529

********* RUN OVER ************

```
                 ***** SIMULATION RUN NO =  4********

     ** NO OF SIMULATION RUNS = 10000   ***

    PATH LENGTH  1 =    60.89824

    SIMULATED MOMENTS OF PATH NO : 1  ARE :
     RAW MOMENTS :
     RM1 =       60.92616 RM2 =   3713.14111 RM3 =   226366.625 RM4
= 13804371.0
     CENTRAL MOMENTS:
     CM1=    60.92616  CM2 =     1.14238  CM3 =    -0.03954   CM4
=      3.59259
    STD DEV=     1.06882   COEFF.VAR.= 0.0175

                      ********* RUN OVER   ************

    PATH LENGTH  2 =    66.13383

    SIMULATED MOMENTS OF PATH NO : 2  ARE :
     RAW MOMENTS :
     RM1 =       66.16225 RM2 =   4380.53906 RM3 =   290236.750 RM4
= 19243520.0
     CENTRAL MOMENTS:
     CM1=    66.16225  CM2 =     3.09635  CM3 =     0.58405   CM4
=     25.43146
    STD DEV=     1.75965   COEFF.VAR.= 0.0266

                      ********* RUN OVER   ************

    PATH LENGTH  3 =    64.22672

    SIMULATED MOMENTS OF PATH NO : 3  ARE :
     RAW MOMENTS :
     RM1 =       64.25497 RM2 =   4131.70312 RM3 =   265869.125 RM4
= 17120784.0
     CENTRAL MOMENTS:
     CM1=    64.25497  CM2 =     3.00118  CM3 =     0.92592   CM4
=     25.10934
    STD DEV=     1.73239   COEFF.VAR.= 0.0270

                      ********* RUN OVER   ************

    PATH LENGTH  4 =    67.37845

    SIMULATED MOMENTS OF PATH NO : 4  ARE :
     RAW MOMENTS :
     RM1 =       67.40654 RM2 =   4548.41797 RM3 =   307239.625 RM4
= 20775568.0
     CENTRAL MOMENTS:
     CM1=    67.40654  CM2 =     4.77927  CM3 =     1.99383   CM4
```

= 62.40570
STD DEV= 2.18615 COEFF.VAR.= 0.0324

********* RUN OVER ************

PATH LENGTH 5 = 65.51790

SIMULATED MOMENTS OF PATH NO : 5 ARE :
RAW MOMENTS :
RM1 = 65.54610 RM2 = 4303.64453 RM3 = 283052.500 RM4 = 18648144.0
CENTRAL MOMENTS:
CM1= 65.54610 CM2 = 7.35448 CM3 = 1.12264 CM4 = 139.34448
STD DEV= 2.71191 COEFF.VAR.= 0.0414

********* RUN OVER ************

PATH LENGTH 6 = 63.87749

SIMULATED MOMENTS OF PATH NO : 6 ARE :
RAW MOMENTS :
RM1 = 63.90561 RM2 = 4086.06763 RM3 = 261396.750 RM4 = 16731070.0
CENTRAL MOMENTS:
CM1= 63.90561 CM2 = 2.14006 CM3 = 0.60064 CM4 = 13.09506
STD DEV= 1.46290 COEFF.VAR.= 0.0229

********* RUN OVER ************

PATH LENGTH 7 = 67.02895

SIMULATED MOMENTS OF PATH NO : 7 ARE :
RAW MOMENTS :
RM1 = 67.05716 RM2 = 4500.64062 RM3 = 302335.937 RM4 = 20327888.0
CENTRAL MOMENTS:
CM1= 67.05716 CM2 = 3.97908 CM3 = 1.88350 CM4 = 43.09061
STD DEV= 1.99476 COEFF.VAR.= 0.0297

********* RUN OVER ************

PATH LENGTH 8 = 65.16850

SIMULATED MOMENTS OF PATH NO : 8 ARE :
RAW MOMENTS :
RM1 = 65.19673 RM2 = 4257.04687 RM3 = 278385.312 RM4 = 18232096.0
CENTRAL MOMENTS:
CM1= 65.19673 CM2 = 6.43381 CM3 = 0.72573 CM4

```
=    105.33801
    STD DEV=      2.53650   COEFF.VAR.= 0.0389

                    ********* RUN OVER   ************

    PATH LENGTH  9 =    66.79892

    SIMULATED MOMENTS OF PATH NO : 9  ARE :
     RAW MOMENTS :
     RM1 =      66.82715 RM2 =   4470.05078 RM3 =   299281.750 RM4
= 20056544.0
     CENTRAL MOMENTS:
     CM1=    66.82715  CM2 =     4.18503  CM3 =     1.38783   CM4
=     47.91531
    STD DEV=      2.04573   COEFF.VAR.= 0.0306

                    ********* RUN OVER   ************

    PATH LENGTH 10 =    64.93808

    SIMULATED MOMENTS OF PATH NO :10  ARE :
     RAW MOMENTS :
     RM1 =      64.96672 RM2 =   4227.47656 RM3 =   275529.125 RM4
= 17986480.0
     CENTRAL MOMENTS:
     CM1=    64.96672  CM2 =     6.80090  CM3 =     0.16613   CM4
=    117.06027
    STD DEV=      2.60785   COEFF.VAR.= 0.0401

                    ********* RUN OVER   ************

    SIMULATED MOMENTS OF COMP.TIME FOR RUN NO: 4
     RAW MOMENTS :
     RM1 =      68.81490 RM2 =   4738.48047 RM3 =   326490.562 RM4
= 22510096.0
     CENTRAL MOMENTS:
     CM1=    68.81490  CM2 =     2.99017  CM3 =     0.85822   CM4
=     25.70125
    ** SB1 =       0.16598 **B2 =       2.87451
      STD DEV =     1.72921  COEFF. OF VAR =      0.02513

                    ********* RUN OVER   ************

               ***** SIMULATION RUN NO =  5********

     ** NO OF SIMULATION RUNS = 10000   ***

    PATH LENGTH  1 =    60.89954

    SIMULATED MOMENTS OF PATH NO : 1  ARE :
     RAW MOMENTS :
     RM1 =      60.92790 RM2 =   3713.35107 RM3 =   226385.687 RM4
```

```
= 13805910.0
     CENTRAL MOMENTS:
     CM1=    60.92790  CM2 =     1.14141  CM3 =    -0.06993   CM4
=      3.63389
    STD DEV=      1.06837   COEFF.VAR.= 0.0175

                    ********* RUN OVER   ************

    PATH LENGTH  2 =    66.10176

    SIMULATED MOMENTS OF PATH NO : 2  ARE :
     RAW MOMENTS :
     RM1 =      66.12997 RM2 =   4376.19922 RM3 =   289799.187 RM4
= 19204320.0
     CENTRAL MOMENTS:
     CM1=    66.12997  CM2 =     3.02646  CM3 =     0.91752   CM4
=     25.22762
    STD DEV=      1.73967   COEFF.VAR.= 0.0263

                    ********* RUN OVER   ************

    PATH LENGTH  3 =    64.21068

    SIMULATED MOMENTS OF PATH NO : 3  ARE :
     RAW MOMENTS :
     RM1 =      64.23900 RM2 =   4129.59766 RM3 =   265661.187 RM4
= 17102496.0
     CENTRAL MOMENTS:
     CM1=    64.23900  CM2 =     2.95012  CM3 =     0.76433   CM4
=     23.84547
    STD DEV=      1.71759   COEFF.VAR.= 0.0267

                    ********* RUN OVER   ************

    PATH LENGTH  4 =    67.31926

    SIMULATED MOMENTS OF PATH NO : 4  ARE :
     RAW MOMENTS :
     RM1 =      67.34763 RM2 =   4540.42187 RM3 =   306424.625 RM4
= 20701664.0
     CENTRAL MOMENTS:
     CM1=    67.34763  CM2 =     4.71984  CM3 =     2.02719   CM4
=     61.39009
    STD DEV=      2.17252   COEFF.VAR.= 0.0323

                    ********* RUN OVER   ************

    PATH LENGTH  5 =    65.54277

    SIMULATED MOMENTS OF PATH NO : 5  ARE :
     RAW MOMENTS :
     RM1 =      65.57082 RM2 =   4306.79297 RM3 =   283353.375 RM4
```

```
= 18673664.0
     CENTRAL MOMENTS:
     CM1=    65.57082  CM2 =     7.26433  CM3 =     0.57629   CM4
=    141.22313
    STD DEV=     2.69524   COEFF.VAR.= 0.0411

                   ********* RUN OVER   ************

    PATH LENGTH  6 =    63.87564

    SIMULATED MOMENTS OF PATH NO : 6  ARE :
     RAW MOMENTS :
     RM1 =      63.90370 RM2 =   4085.85034 RM3 =   261378.500 RM4
= 16729738.0
     CENTRAL MOMENTS:
     CM1=    63.90370  CM2 =     2.16678  CM3 =     0.63474   CM4
=     13.54492
    STD DEV=     1.47200   COEFF.VAR.= 0.0230

                   ********* RUN OVER   ************

    PATH LENGTH  7 =    66.98430

    SIMULATED MOMENTS OF PATH NO : 7  ARE :
     RAW MOMENTS :
     RM1 =      67.01233 RM2 =   4494.64062 RM3 =   301733.250 RM4
= 20274048.0
     CENTRAL MOMENTS:
     CM1=    67.01233  CM2 =     3.98956  CM3 =     2.04070   CM4
=     43.46088
    STD DEV=     1.99739   COEFF.VAR.= 0.0298

                   ********* RUN OVER   ************

    PATH LENGTH  8 =    65.20717

    SIMULATED MOMENTS OF PATH NO : 8  ARE :
     RAW MOMENTS :
     RM1 =      65.23550 RM2 =   4262.14062 RM3 =   278887.687 RM4
= 18276160.0
     CENTRAL MOMENTS:
     CM1=    65.23550  CM2 =     6.47151  CM3 =     0.26325   CM4
=    106.52518
    STD DEV=     2.54392   COEFF.VAR.= 0.0390

                   ********* RUN OVER   ************

    PATH LENGTH  9 =    66.72624

    SIMULATED MOMENTS OF PATH NO : 9  ARE :
     RAW MOMENTS :
     RM1 =      66.75446 RM2 =   4460.33203 RM3 =   298306.312 RM4
```

= 19969504.0
CENTRAL MOMENTS:
CM1= 66.75446 CM2 = 4.17426 CM3 = 1.85446 CM4
= 48.93257
STD DEV= 2.04310 COEFF.VAR.= 0.0306

********* RUN OVER ************

PATH LENGTH 10 = 64.94939

SIMULATED MOMENTS OF PATH NO :10 ARE :
RAW MOMENTS :
RM1 = 64.97765 RM2 = 4228.71484 RM3 = 275632.437 RM4
= 17993888.0
CENTRAL MOMENTS:
CM1= 64.97765 CM2 = 6.62281 CM3 = -0.29979 CM4
= 112.71318
STD DEV= 2.57348 COEFF.VAR.= 0.0396

********* RUN OVER ************

SIMULATED MOMENTS OF COMP.TIME FOR RUN NO: 5
RAW MOMENTS :
RM1 = 68.77196 RM2 = 4732.62500 RM3 = 325891.375 RM4
= 22455568.0
CENTRAL MOMENTS:
CM1= 68.77196 CM2 = 3.04352 CM3 = 0.82584 CM4
= 26.73695
** SB1 = 0.15554 **B2 = 2.88642
STD DEV = 1.74457 COEFF. OF VAR = 0.02537

********* RUN OVER ************

MEAN OF SIMULATED VALUES FOR 5 RUNS & SAMPLE SIZE=10000

CM1= 68.78990 CM2= 3.00443 CM3= 0.86261 CM4=
25.75024

SB1= 0.16564 SB2= 2.85271 CV= 0.0252

PERT PATH NO: 1 LENGTH = 60.9666 VAR= 0.7500 CV =
0.0142

PERT PATH NO: 2 LENGTH = 66.0222 VAR= 2.1111 CV =
0.0220

PERT PATH NO: 3 LENGTH = 64.0000 VAR= 1.9444 CV =
0.0218

PERT PATH NO: 4 LENGTH = 66.8333 VAR= 2.8889 CV =
0.0254

```
    PERT PATH NO: 5   LENGTH =    65.3333   VAR=       5.2778   CV =
0.0352

    PERT PATH NO: 6   LENGTH =    63.7222   VAR=       1.6389   CV =
0.0201

    PERT PATH NO: 7   LENGTH =    66.5555   VAR=       2.5833   CV =
0.0241

    PERT PATH NO: 8   LENGTH =    65.0555   VAR=       4.9722   CV =
0.0343

    PERT PATH NO: 9   LENGTH =    66.4444   VAR=       2.8611   CV =
0.0255

    PERT PATH NO:10   LENGTH =    64.9444   VAR=       5.2500   CV =
0.0353

      PERT C.PATH IS =   66.83331   CP VAR =    2.88889   CP CV=
0.0254

    ** PERT ERROR OVER SIM MU =  -2.844%
STOP         0
*End
*Go
```

APPENDIX H

COMPUTER OUTPUT FROM THE

NEW APPROACH PROGRAM

FOR NETWORK #10

```
017400 BYTES USED
EXECUTION BEGINS
    + + + + +  PART I + + + + + + + + +
    * * * READ & ECHO PRINT THE GIVEN DATA * * *
    NUMBER OF ACTIVITIES =  14

    A,B,P & Q OF ACTIVITY    1   ARE:          12.0000
15.0000       3.0000       2.0000

    A,B,P & Q OF ACTIVITY    2   ARE:          14.0000
17.0000       2.0000       3.0000

    A,B,P & Q OF ACTIVITY    3   ARE:          18.0000
24.0000       1.5000       2.5000

    A,B,P & Q OF ACTIVITY    4   ARE:          17.0000
22.0000       1.5000       3.5000

    A,B,P & Q OF ACTIVITY    5   ARE:          17.0000
23.0000       3.0000       3.5000

    A,B,P & Q OF ACTIVITY    6   ARE:          22.0000
25.0000       2.0000       2.5000

    A,B,P & Q OF ACTIVITY    7   ARE:          10.0000
14.0000       3.0000       3.0000

    A,B,P & Q OF ACTIVITY    8   ARE:           1.0000
2.0000       2.0000       2.0000

    A,B,P & Q OF ACTIVITY    9   ARE:           1.0000
1.0000       2.5000       2.5000

    A,B,P & Q OF ACTIVITY   10   ARE:           8.0000
15.0000       2.0000       3.0000
```

```
   A,B,P & Q OF ACTIVITY  11  ARE:          11.0000
18.0000       1.5000       2.5000

   A,B,P & Q OF ACTIVITY  12  ARE:          21.0000
24.0000       3.0000       2.0000

   A,B,P & Q OF ACTIVITY  13  ARE:          17.0000
20.0000       2.0000       3.0000

   A,B,P & Q OF ACTIVITY  14  ARE:          24.0000
36.0000       3.0000       3.0000

    NUMBER OF PATHS ARE:   10
    ACTIVITIES ON PATH   1 ARE    1    6    9   12
    ACTIVITIES ON PATH   2 ARE    1    6   10   13
    ACTIVITIES ON PATH   3 ARE    1    3    7   13
    ACTIVITIES ON PATH   4 ARE    1    3    8   11   13
    ACTIVITIES ON PATH   5 ARE    1    3    8   14
    ACTIVITIES ON PATH   6 ARE    2    4    7   13
    ACTIVITIES ON PATH   7 ARE    2    4    8   11   13
    ACTIVITIES ON PATH   8 ARE    2    4    8   14
    ACTIVITIES ON PATH   9 ARE    2    5   11   13
    ACTIVITIES ON PATH  10 ARE    2    5   14

    + + END PART I & BEGIN PART II + + + +

    ** FIND MU,VAR,MU3 & MU4 OF EACH ACTIVITY**

    ACTIVITY   1:
    EXP =    13.8000  VAR =     0.3600  ALPHA3 =    -0.2857
ALPHA4=     2.3571
      MU3 =    -0.0617  MU4 =     0.3055

    ACTIVITY   2:
    EXP =    15.2000  VAR =     0.3600  ALPHA3 =     0.2857
ALPHA4=     2.3571
      MU3 =     0.0617  MU4 =     0.3055

    ACTIVITY   3:
    EXP =    20.2500  VAR =     1.6875  ALPHA3 =     0.3849
ALPHA4=     2.3333
      MU3 =     0.8437  MU4 =     6.6445

    ACTIVITY   4:
    EXP =    18.5000  VAR =     0.8750  ALPHA3 =     0.6109
ALPHA4=     2.7398
      MU3 =     0.5000  MU4 =     2.0977

    ACTIVITY   5:
    EXP =    19.7692  VAR =     1.1929  ALPHA3 =     0.0994
ALPHA4=     2.3817
      MU3 =     0.1295  MU4 =     3.3892
```

```
    ACTIVITY   6:
    EXP =     23.3333   VAR =      0.4040  ALPHA3 =      0.1614
ALPHA4=     2.2338
     MU3 =      0.0414  MU4 =      0.3647

    ACTIVITY   7:
    EXP =     12.0000   VAR =      0.5714  ALPHA3 =      0.0
ALPHA4=     2.3333
     MU3 =      0.0     MU4 =      0.7619

    ACTIVITY   8:
    EXP =      1.5000   VAR =      0.0500  ALPHA3 =      0.0
ALPHA4=     2.1429
     MU3 =      0.0     MU4 =      0.0054

    ACTIVITY   9:
    EXP =      1.0000   VAR =      0.0     ALPHA3 =      0.0
ALPHA4=     2.2500
     MU3 =      0.0     MU4 =      0.0

    ACTIVITY  10:
    EXP =     10.8000   VAR =      1.9600  ALPHA3 =      0.2857
ALPHA4=     2.3571
     MU3 =      0.7840  MU4 =      9.0552

    ACTIVITY  11:
    EXP =     13.6250   VAR =      2.2969  ALPHA3 =      0.3849
ALPHA4=     2.3333
     MU3 =      1.3398  MU4 =     12.3098

    ACTIVITY  12:
    EXP =     22.8000   VAR =      0.3600  ALPHA3 =     -0.2857
ALPHA4=     2.3571
     MU3 =     -0.0617  MU4 =      0.3055

    ACTIVITY  13:
    EXP =     18.2000   VAR =      0.3600  ALPHA3 =      0.2857
ALPHA4=     2.3571
     MU3 =      0.0617  MU4 =      0.3055

    ACTIVITY  14:
    EXP =     30.0000   VAR =      5.1429  ALPHA3 =      0.0
ALPHA4=     2.3333
     MU3 =      0.0     MU4 =     61.7143

    *** END PART II & BEGIN PART III  *****

    *FIND MU,CVP,VAR,MU3,MU4,ALPHA3 & ALPHA4 OF PATHS*

    *** MEAN    ,VARIANCE,        MU3        MU4       ALFA3P,
ALFA4P  &   CVP :  ****
```

```
 PATH  1:
     60.933        1.124      -0.082       3.499      -0.069
2.769        0.017

 PATH  2:
     66.133        3.084       0.825      25.773       0.152
2.710        0.027

 PATH  3:
     64.250        2.979       0.844      24.339       0.164
2.743        0.027

 PATH  4:
     67.375        4.754       2.184      62.228       0.211
2.753        0.032

 PATH  5:
     65.550        7.240       0.782     137.652       0.040
2.626        0.041

 PATH  6:
     63.900        2.166       0.623      13.497       0.196
2.876        0.023

 PATH  7:
     67.025        3.942       1.963      42.730       0.251
2.750        0.030

 PATH  8:
     65.200        6.428       0.562     106.035       0.034
2.566        0.039

 PATH  9:
     66.794        4.210       1.593      48.603       0.184
2.742        0.031

 PATH 10:
     64.969        6.696       0.191     115.904       0.011
2.585        0.040

    + + + END PART III & BEGIN PART IV + + + +

    ** FIT S-D DISTRNS TO PATHS & FIND 4 LAMBDAS *

    *******  FOR PATH NO = 1*******

    ********** IN SUBROUTINE LAMBDA************
    *** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
    XL(1) =      0.6880D-01     YL(1) =      0.2769D+01

    FROM IBCIEU: FC(1,1)=   1.83690  FD(1,1)=    0.54026
```

```
   CD(1) =       0.1837D+01     CD(2) =      0.5403D+00

   ****** OUTPUT FROM ZXSSQ  *******
   X3,  X4  =      1.99073      0.49221

   ****** OUTPUT FROM ZXSSQ  *******
      AL3,   AL4 =      1.99073      0.50779

   PATH :  1 THE 4 LAMBDAS ARE :
          60.9701726        9.3989110        1.9907266
0.5077882

   *******  FOR PATH NO = 2*******

   ********** IN SUBROUTINE LAMBDA************
   *** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
   XL(1) =       0.1524D+00     YL(1) =      0.2710D+01

   FROM IBCIEU: FC(1,1)=   1.71328  FD(1,1)=    0.54048

   CD(1) =       0.1713D+01     CD(2) =      0.5405D+00

   ****** OUTPUT FROM ZXSSQ  *******
   X3,  X4  =      1.92915      0.48126

   ****** OUTPUT FROM ZXSSQ  *******
      AL3,   AL4 =      1.92915      0.48126

   PATH :  2 THE 4 LAMBDAS ARE :
          65.9886727       14.6946781        1.9291465
0.4812626

   *******  FOR PATH NO = 3*******

   ********** IN SUBROUTINE LAMBDA************
   *** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
   XL(1) =       0.1641D+00     YL(1) =      0.2743D+01

   FROM IBCIEU: FC(1,1)=   1.74060  FD(1,1)=    0.54536

   CD(1) =       0.1741D+01     CD(2) =      0.5454D+00

   ****** OUTPUT FROM ZXSSQ  *******
   X3,  X4  =      1.96196      0.48063

   ****** OUTPUT FROM ZXSSQ  *******
      AL3,   AL4 =      1.96196      0.48063

   PATH :  3 THE 4 LAMBDAS ARE :
          64.1020700       14.8681454        1.9619642
0.4806280
```

```
*******  FOR PATH NO = 4*******

********** IN SUBROUTINE LAMBDA************
*** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
XL(1) =       0.2106D+00     YL(1) =      0.2753D+01

FROM IBCIEU: FC(1,1)=   1.76051  FD(1,1)=    0.53667

CD(1) =       0.1761D+01     CD(2) =      0.5367D+00

****** OUTPUT FROM ZXSSQ  *******
X3,  X4  =       1.97049      0.47527

****** OUTPUT FROM ZXSSQ  *******
   AL3,   AL4 =      1.97049      0.47527

PATH :  4 THE 4 LAMBDAS ARE :
       67.1365031       18.8841123        1.9704917
0.4752717

*******  FOR PATH NO = 5*******

********** IN SUBROUTINE LAMBDA************
*** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
XL(1) =       0.4014D-01     YL(1) =      0.2626D+01

FROM IBCIEU: FC(1,1)=   1.75379  FD(1,1)=    0.51544

CD(1) =       0.1754D+01     CD(2) =      0.5154D+00

****** OUTPUT FROM ZXSSQ  *******
X3,  X4  =       1.84592      0.49452

****** OUTPUT FROM ZXSSQ  *******
   AL3,   AL4 =      1.84592      0.49452

PATH :  5 THE 4 LAMBDAS ARE :
       65.4861356       20.9471404        1.8459184
0.4945202

*******  FOR PATH NO = 6*******

********** IN SUBROUTINE LAMBDA************
*** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
XL(1) =       0.1955D+00     YL(1) =      0.2876D+01

FROM IBCIEU: FC(1,1)=   1.87191  FD(1,1)=    0.55805

CD(1) =       0.1872D+01     CD(2) =      0.5580D+00

****** OUTPUT FROM ZXSSQ  *******
```

```
X3,  X4  =      2.09350      0.48019

****** OUTPUT FROM ZXSSQ  *******
   AL3,   AL4 =      2.09350      0.48019

PATH :  6 THE 4 LAMBDAS ARE :
       63.7676730       14.2433139        2.0935030
0.4801868

*******  FOR PATH NO = 7*******

********** IN SUBROUTINE LAMBDA************
*** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
XL(1) =      0.2509D+00     YL(1) =      0.2750D+01

FROM IBCIEU: FC(1,1)=   1.78532  FD(1,1)=    0.52128

CD(1) =      0.1785D+01     CD(2) =      0.5213D+00

****** OUTPUT FROM ZXSSQ  *******
X3,  X4  =      1.96582      0.47021

****** OUTPUT FROM ZXSSQ  *******
   AL3,   AL4 =      1.96582      0.47021

PATH :  7 THE 4 LAMBDAS ARE :
       66.7642120       17.0794818        1.9658238
0.4702102

*******  FOR PATH NO = 8*******

********** IN SUBROUTINE LAMBDA************
*** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
XL(1) =      0.3447D-01     YL(1) =      0.2566D+01

FROM IBCIEU: FC(1,1)=   1.71124  FD(1,1)=    0.50765

CD(1) =      0.1711D+01     CD(2) =      0.5077D+00

****** OUTPUT FROM ZXSSQ  *******
X3,  X4  =      1.78564      0.49487

****** OUTPUT FROM ZXSSQ  *******
   AL3,   AL4 =      1.78564      0.49487

PATH :  8 THE 4 LAMBDAS ARE :
       65.1443687       18.6853778        1.7856421
0.4948677

*******  FOR PATH NO = 9*******

********** IN SUBROUTINE LAMBDA************
```

```
*** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
XL(1) =      0.1844D+00     YL(1) =      0.2742D+01

FROM IBCIEU: FC(1,1)=   1.74208  FD(1,1)=    0.54165

CD(1) =      0.1742D+01     CD(2) =      0.5417D+00

****** OUTPUT FROM ZXSSQ  *******
X3,  X4  =      1.96104      0.47816

****** OUTPUT FROM ZXSSQ  *******
   AL3,   AL4 =      1.96104      0.47816

PATH :  9 THE 4 LAMBDAS ARE :
       66.5961749        17.6434997         1.9610430
0.4781604

*******  FOR PATH NO =10*******

********** IN SUBROUTINE LAMBDA************
*** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
XL(1) =      0.1104D-01     YL(1) =      0.2585D+01

FROM IBCIEU: FC(1,1)=   1.77744  FD(1,1)=    0.50366

CD(1) =      0.1777D+01     CD(2) =      0.5037D+00

****** OUTPUT FROM ZXSSQ  *******
X3,  X4  =      1.80480      0.49840

****** OUTPUT FROM ZXSSQ  *******
   AL3,   AL4 =      1.80480      0.49840

PATH : 10 THE 4 LAMBDAS ARE :
       64.9514846        19.4099044         1.8048031
0.4984028

*** END OF PART4 & START PART5 ****

*** PART 5: FIND THE MAXIMUM OF ALL PATHS ****


    VALUE OF XD(=YD) IS    0.10000D-12


%%% PART 5: START MAX FOR 1ST & 2ND PATHS %%%

** E1X =  58.53140  X1 =  60.97017  E2X =  63.26229

** E1Y =  62.40417 Y1 =  65.98867  E2Y =  70.13107
```

```
    * COMPUTE MU1(Z1); Z1=Y*PDF(Y)*CDF(X) ***
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    YTA=  65.93200  YTB=   0.00770 UZY=  65.93970

    ** COMPUTE MU1(Z2); Z2=X*PDF(X)*CDF(Y) **

    XTA=   0.19449  XTB=   0.0     UZX=   0.19449

    ** UZ =  66.13419  UZX =   0.19449  UZY =  65.93970

     **START THE MOMENTS OF Z1 ** NN = 1
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
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*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
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*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    2ND,3RD & 4TH MOMENTS OF Z1 ARE:
          3.04724      0.94493     25.32682

    ** START: THE MOMENTS OF Z2 ** NN = 2

    XTA=   0.03108 XTB=   0.0      NN= 2

    ** START: THE MOMENTS OF Z2 ** NN = 3
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE

    XTA=  -0.09921 XTB=   0.0      NN= 3

    ** START: THE MOMENTS OF Z2 ** NN = 4
```

```
XTA=   0.31809 XTB=   0.0      NN= 4

2,3 & 4TH  MOMENTS OF Z2 ARE:
      0.03108     -0.09921      0.31809

** 4 CENTRAL MU"S OF JOINT DISTN OF  2 PATHS:
      66.1342       3.0783       0.8457       25.6449

ALPHA3, ALPHA4=       0.15659      2.70629

********** IN SUBROUTINE LAMBDA************
*** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
XL(1) =       0.1566D+00     YL(1) =       0.2706D+01

FROM IBCIEU: FC(1,1)=   1.70979  FD(1,1)=    0.53931

CD(1) =       0.1710D+01     CD(2) =       0.5393D+00

****** OUTPUT FROM ZXSSQ  *******
X3,  X4  =       1.92565     0.48065

****** OUTPUT FROM ZXSSQ  *******
   AL3,   AL4 =       1.92565      0.48065

Z =   1 THE 4 LAMBDAS ARE :
       65.9850933       14.6321424        1.9256486
 0.4806535

START MAX OF MAX OF FIRST  2PATHS & PATH 3

** E1X =  62.41542  X1 =  65.98509  E2X =  70.12870

** E1Y =  60.57042 Y1 =  64.10207  E2Y =  68.21391

* COMPUTE MU1(Z1); Z1=Y*PDF(Y)*CDF(X) ***
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

YTA=  14.87658  YTB=   0.00119 UZY=  14.87777

** COMPUTE MU1(Z2); Z2=X*PDF(X)*CDF(Y)  **
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
```

```
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE

    XTA=  51.57093  XTB=   0.00629 UZX=  51.57722

    ** UZ =  66.45499  UZX =  51.57722  UZY =  14.87777

     **START THE MOMENTS OF Z1 ** NN = 1
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
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*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE

    2ND,3RD & 4TH MOMENTS OF Z1 ARE:
          0.50975      -0.83064       2.61334

    ** START: THE MOMENTS OF Z2 ** NN = 2
*** WARNING WITH FIX ERROR  (IER =   66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE

    XTA=   1.95561 XTB=   0.00002  NN= 2

    ** START: THE MOMENTS OF Z2 ** NN = 3
*** WARNING WITH FIX ERROR  (IER =   66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
```

```
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=   1.64456 XTB=  -0.00001  NN= 3

    ** START: THE MOMENTS OF Z2 ** NN = 4
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=  13.96576 XTB=   0.00001  NN= 4

    2,3 & 4TH  MOMENTS OF Z2 ARE:
          1.95563      1.64455     13.96577

    ** 4 CENTRAL MU"S OF JOINT DISTN OF  3 PATHS:
          66.4550       2.4654       0.8139      16.5791

    ALPHA3, ALPHA4=        0.21026      2.72768

    ********** IN SUBROUTINE LAMBDA************
    *** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
    XL(1) =       0.2103D+00     YL(1) =       0.2728D+01

    FROM IBCIEU: FC(1,1)=   1.73847  FD(1,1)=    0.53189

    CD(1) =       0.1738D+01     CD(2) =       0.5319D+00

    ****** OUTPUT FROM ZXSSQ  *******
    X3,  X4  =       1.94540      0.47452

    ****** OUTPUT FROM ZXSSQ  *******
       AL3,   AL4 =       1.94540      0.47452

    Z =   2 THE 4 LAMBDAS ARE :
           66.2789503        13.2941849        1.9453994
 0.4745205

    START MAX OF MAX OF FIRST  3PATHS & PATH 4

    ** E1X =  63.16115  X1 =  66.27895  E2X =  70.08112

    ** E1Y =  62.77623 Y1 =  67.13650  E2Y =  72.43594
```

```
    * COMPUTE MU1(Z1); Z1=Y*PDF(Y)*CDF(X) ***
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE

    YTA=  43.84342  YTB=   0.00608 UZY=  43.84950

    ** COMPUTE MU1(Z2); Z2=X*PDF(X)*CDF(Y) **
*** WARNING WITH FIX ERROR  (IER =   66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE

    XTA=  24.19231  XTB=   0.00236 UZX=  24.19466

    ** UZ =  68.04417  UZX =  24.19466  UZY =  43.84950

     **START THE MOMENTS OF Z1 ** NN = 1
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
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*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
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*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
```

```
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    2ND,3RD & 4TH MOMENTS OF Z1 ARE:
          2.20390      3.90344      21.17216

    ** START: THE MOMENTS OF Z2 ** NN = 2
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=   0.87360 XTB=   0.00011  NN= 2

    ** START: THE MOMENTS OF Z2 ** NN = 3
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=  -1.46222 XTB=  -0.00020  NN= 3

    ** START: THE MOMENTS OF Z2 ** NN = 4
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=   5.43732 XTB=   0.00035  NN= 4

    2,3 & 4TH  MOMENTS OF Z2 ARE:
          0.87371     -1.46242      5.43766

    ** 4 CENTRAL MU"S OF JOINT DISTN OF  4 PATHS:
          68.0442       3.0776       2.4410       26.6098

    ALPHA3, ALPHA4=       0.45212      2.80942

    ********** IN SUBROUTINE LAMBDA************
    *** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
    XL(1) =       0.4521D+00     YL(1) =       0.2809D+01

    FROM IBCIEU: FC(1,1)=   2.09168  FD(1,1)=    0.41504

    CD(1) =       0.2092D+01     CD(2) =       0.4150D+00
```

```
****** OUTPUT FROM ZXSSQ  *******
X3,  X4  =      2.01001      0.44692

****** OUTPUT FROM ZXSSQ  *******
   AL3,   AL4 =      2.01001      0.44692

Z =   3 THE 4 LAMBDAS ARE :
       67.6368022       15.3964710        2.0100148
 0.4469174

START MAX OF MAX OF FIRST  4PATHS & PATH 5

** E1X =  64.58629  X1 =  67.63680  E2X =  72.31874

** E1Y =  59.77643 Y1 =  65.48614  E2Y =  71.43161

* COMPUTE MU1(Z1); Z1=Y*PDF(Y)*CDF(X) ***
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

YTA=  14.94718  YTB=   0.00029 UZY=  14.94747

** COMPUTE MU1(Z2); Z2=X*PDF(X)*CDF(Y) **
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

XTA=  53.49289  XTB=   0.00897 UZX=  53.50186

** UZ =  68.44933  UZX =  53.50186  UZY =  14.94747

 **START THE MOMENTS OF Z1 ** NN = 1
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
```

```
DCADRE
*** WARNING WITH FIX ERROR  (IER =   66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
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*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
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*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE

    2ND,3RD & 4TH MOMENTS OF Z1 ARE:
          0.65486      0.18663      3.90620

    ** START: THE MOMENTS OF Z2 ** NN = 2
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE

    XTA=   2.33273 XTB=   0.00009  NN= 2

    ** START: THE MOMENTS OF Z2 ** NN = 3
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE

    XTA=   1.15654 XTB=  -0.00007  NN= 3

    ** START: THE MOMENTS OF Z2 ** NN = 4
*** WARNING WITH FIX ERROR  (IER =   66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =   65) FROM IMSL ROUTINE
DCADRE
```

```
XTA=  17.58841 XTB=   0.00006  NN= 4

2,3 & 4TH  MOMENTS OF Z2 ARE:
      2.33282      1.15647     17.58847

** 4 CENTRAL MU"S OF JOINT DISTN OF  5 PATHS:
      68.4493       2.9877       1.3431       21.4947

ALPHA3, ALPHA4=        0.26008      2.40803

********** IN SUBROUTINE LAMBDA************
*** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
XL(1) =       0.2601D+00     YL(1) =      0.2408D+01

FROM IBCIEU: FC(1,1)=   1.57038  FD(1,1)=    0.41131

CD(1) =       0.1570D+01     CD(2) =      0.4113D+00

****** OUTPUT FROM ZXSSQ  *******
X3,  X4  =       1.63363      0.44847

****** OUTPUT FROM ZXSSQ  *******
   AL3,   AL4 =       1.63363      0.44847

Z =    4 THE 4 LAMBDAS ARE :
       68.0858429       10.9234255        1.6336274
 0.4484682

START MAX OF MAX OF FIRST  5PATHS & PATH 6

** E1X =  65.13858  X1 =  68.08584  E2X =  72.21804

** E1Y =  60.70118 Y1 =  63.76767  E2Y =  67.38791

* COMPUTE MU1(Z1); Z1=Y*PDF(Y)*CDF(X) ***
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

YTA=   1.40234  YTB=   0.0     UZY=   1.40234

** COMPUTE MU1(Z2); Z2=X*PDF(X)*CDF(Y) **
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

XTA=  67.05997  XTB=   0.00163 UZX=  67.06160

** UZ =  68.46394  UZX =  67.06160  UZY =   1.40234
```

```
 **START THE MOMENTS OF Z1 ** NN = 1
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE

    2ND,3RD & 4TH MOMENTS OF Z1 ARE:
          0.07844     -0.17003      0.39268

    ** START: THE MOMENTS OF Z2 ** NN = 2
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=   2.84492 XTB=   0.00001  NN= 2

    ** START: THE MOMENTS OF Z2 ** NN = 3
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=   1.60596 XTB=  -0.00001  NN= 3

    ** START: THE MOMENTS OF Z2 ** NN = 4
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=  20.30474 XTB=   0.00001  NN= 4

    2,3 & 4TH  MOMENTS OF Z2 ARE:
          2.84492      1.60596      20.30475

    ** 4 CENTRAL MU"S OF JOINT DISTN OF  6 PATHS:
          68.4639       2.9234       1.4359       20.6974

    ALPHA3, ALPHA4=       0.28728       2.42186
```

```
********** IN SUBROUTINE LAMBDA************
*** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
XL(1) =      0.2873D+00     YL(1) =      0.2422D+01

FROM IBCIEU: FC(1,1)=   1.59588  FD(1,1)=    0.41118

CD(1) =      0.1596D+01     CD(2) =      0.4112D+00

****** OUTPUT FROM ZXSSQ  *******
X3,  X4  =      1.64861      0.44419

****** OUTPUT FROM ZXSSQ  *******
   AL3,   AL4 =      1.64861      0.44419

Z =   5 THE 4 LAMBDAS ARE :
       68.0741696       10.9243569        1.6486076
 0.4441913

START MAX OF MAX OF FIRST  6PATHS & PATH 7

** E1X =  65.20750  X1 =  68.07417  E2X =  72.22254

** E1Y =  62.88933 Y1 =  66.76421  E2Y =  71.66325

* COMPUTE MU1(Z1); Z1=Y*PDF(Y)*CDF(X) ***
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

YTA=  19.76500  YTB=   0.00145 UZY=  19.76646

** COMPUTE MU1(Z2); Z2=X*PDF(X)*CDF(Y) **
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

XTA=  49.18317  XTB=   0.00134 UZX=  49.18452

** UZ =  68.95097  UZX =  49.18452  UZY =  19.76646

 **START THE MOMENTS OF Z1 ** NN = 1
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
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DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
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*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
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*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    2ND,3RD & 4TH MOMENTS OF Z1 ARE:
          0.74645     -0.14644      3.55422

    ** START: THE MOMENTS OF Z2 ** NN = 2
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=   1.82135 XTB=   0.00002  NN= 2

    ** START: THE MOMENTS OF Z2 ** NN = 3
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=   0.74004 XTB=  -0.00001  NN= 3

    ** START: THE MOMENTS OF Z2 ** NN = 4
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
```

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*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=  10.75826 XTB=   0.00001  NN= 4

    2,3 & 4TH  MOMENTS OF Z2 ARE:
          1.82136      0.74002      10.75827

    ** 4 CENTRAL MU"S OF JOINT DISTN OF  7 PATHS:
          68.9510       2.5678       0.5936       14.3125

    ALPHA3, ALPHA4=        0.14426      2.17065

    ********** IN SUBROUTINE LAMBDA************
    *** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
    XL(1) =       0.1443D+00     YL(1) =      0.2171D+01

    FROM IBCIEU: FC(1,1)=   1.48875  FD(1,1)=    0.30597

    CD(1) =       0.1489D+01     CD(2) =      0.3060D+00

    ****** OUTPUT FROM ZXSSQ  *******
    X3,  X4  =       1.39336      0.45182

    ****** OUTPUT FROM ZXSSQ  *******
       AL3,   AL4 =       1.39336      0.45182

    Z =   6 THE 4 LAMBDAS ARE :
           68.6523221        8.1353058         1.3933570
 0.4518240

    START MAX OF MAX OF FIRST  7PATHS & PATH 8

    ** E1X =  65.96312  X1 =  68.65232  E2X =  72.17276

    ** E1Y =  59.82367 Y1 =  65.14437  E2Y =  70.66374

    * COMPUTE MU1(Z1); Z1=Y*PDF(Y)*CDF(X) ***
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE

    YTA=   7.69293  YTB=   0.0     UZY=   7.69293

    ** COMPUTE MU1(Z2); Z2=X*PDF(X)*CDF(Y) **
```

```
*** WARNING WITH FIX ERROR   (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR   (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR   (IER =   65) FROM IMSL ROUTINE
DCADRE

    XTA=  61.41206  XTB=   0.00026 UZX=  61.41232

    ** UZ =  69.10524  UZX =  61.41232  UZY =   7.69293

     **START THE MOMENTS OF Z1 ** NN = 1
*** WARNING WITH FIX ERROR   (IER =   66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR   (IER =   66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR   (IER =   66) FROM IMSL ROUTINE
DCADRE

    2ND,3RD & 4TH MOMENTS OF Z1 ARE:
          0.13781     -0.10756      0.43475

    ** START: THE MOMENTS OF Z2 ** NN = 2
*** WARNING WITH FIX ERROR   (IER =   66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR   (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR   (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR   (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR   (IER =   65) FROM IMSL ROUTINE
DCADRE

    XTA=   2.23292 XTB=   0.00000  NN= 2

    ** START: THE MOMENTS OF Z2 ** NN = 3
*** WARNING WITH FIX ERROR   (IER =   66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR   (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR   (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR   (IER =   65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR   (IER =   65) FROM IMSL ROUTINE
DCADRE

    XTA=   0.25408 XTB=  -0.00000  NN= 3

    ** START: THE MOMENTS OF Z2 ** NN = 4
*** WARNING WITH FIX ERROR   (IER =   65) FROM IMSL ROUTINE
```

```
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=  12.11871 XTB=   0.00000  NN= 4

    2,3 & 4TH  MOMENTS OF Z2 ARE:
          2.23292       0.25408      12.11871

    ** 4 CENTRAL MU"S OF JOINT DISTN OF  8 PATHS:
          69.1052       2.3707       0.1465       12.5535

    ALPHA3, ALPHA4=       0.04014      2.23358

    ********** IN SUBROUTINE LAMBDA************
    *** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
    XL(1) =      0.4014D-01     YL(1) =      0.2234D+01

    FROM IBCIEU: FC(1,1)=   1.45426  FD(1,1)=    0.44327

    CD(1) =      0.1454D+01     CD(2) =      0.4433D+00

    ****** OUTPUT FROM ZXSSQ  *******
    X3,  X4  =      1.44737      0.48862

    ****** OUTPUT FROM ZXSSQ  *******
       AL3,   AL4 =      1.44737      0.48862

    Z =   7 THE 4 LAMBDAS ARE :
           69.0361023        8.2829565        1.4473696
 0.4886184

    START MAX OF MAX OF FIRST  8PATHS & PATH 9

    ** E1X =  66.09839  X1 =  69.03610  E2X =  72.17395

    ** E1Y =  62.44458 Y1 =  66.59617  E2Y =  71.52408

    * COMPUTE MU1(Z1); Z1=Y*PDF(Y)*CDF(X) ***
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    YTA=  12.87437  YTB=   0.00049 UZY=  12.87485
```

```
    ** COMPUTE MU1(Z2); Z2=X*PDF(X)*CDF(Y) **
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=  56.50085  XTB=   0.00039 UZX=  56.50125

    ** UZ =  69.37610  UZX =  56.50125  UZY =  12.87485

     **START THE MOMENTS OF Z1 ** NN = 1
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
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*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    2ND,3RD & 4TH MOMENTS OF Z1 ARE:
          0.39168     -0.26505      1.74043

    ** START: THE MOMENTS OF Z2 ** NN = 2
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=   1.75257 XTB=   0.00000  NN= 2

    ** START: THE MOMENTS OF Z2 ** NN = 3
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
```

```
    XTA=  -0.05679 XTB=  -0.00000  NN= 3

    ** START: THE MOMENTS OF Z2 ** NN = 4
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=   8.60270 XTB=   0.00000  NN= 4

    2,3 & 4TH  MOMENTS OF Z2 ARE:
          1.75257     -0.05679      8.60270

    ** 4 CENTRAL MU"S OF JOINT DISTN OF  9 PATHS:
          69.3761        2.1443      -0.3218       10.3431

    ALPHA3, ALPHA4=      -0.10250      2.24956

    ********** IN SUBROUTINE LAMBDA************
    *** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
    XL(1) =       0.1025D+00     YL(1) =      0.2250D+01

    FROM IBCIEU: FC(1,1)=   1.47593  FD(1,1)=    0.38675

    CD(1) =       0.1476D+01     CD(2) =      0.3868D+00

    ****** OUTPUT FROM ZXSSQ  *******
    X3,  X4  =      1.46680      0.47209

    ****** OUTPUT FROM ZXSSQ  *******
       AL3,   AL4 =       1.46680      0.52791

    Z =   8 THE 4 LAMBDAS ARE :
           69.5378362        8.0046800        1.4667967
 0.5279144

    START MAX OF MAX OF FIRST  9PATHS & PATH10

    ** E1X =  66.40166  X1 =  69.53784  E2X =  72.19978

    ** E1Y =  59.42797 Y1 =  64.95148  E2Y =  70.53906

    * COMPUTE MU1(Z1); Z1=Y*PDF(Y)*CDF(X) ***
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE

    YTA=   5.26629  YTB=   0.0     UZY=   5.26629
```

```
    ** COMPUTE MU1(Z2); Z2=X*PDF(X)*CDF(Y)  **
*** WARNING WITH FIX ERROR  (IER =  66) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=  64.19968  XTB=   0.00054 UZX=  64.20022

    ** UZ =  69.46651  UZX =  64.20022  UZY =   5.26629

     **START THE MOMENTS OF Z1 ** NN = 1
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    2ND,3RD & 4TH MOMENTS OF Z1 ARE:
          0.07902     -0.10749      0.25904

    ** START: THE MOMENTS OF Z2 ** NN = 2
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=   1.90700 XTB=   0.00000  NN= 2

    ** START: THE MOMENTS OF Z2 ** NN = 3
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=  -0.32392 XTB=   0.00000  NN= 3

    ** START: THE MOMENTS OF Z2 ** NN = 4
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE
*** WARNING WITH FIX ERROR  (IER =  65) FROM IMSL ROUTINE
DCADRE

    XTA=   9.10696 XTB=   0.00000  NN= 4

    2,3 & 4TH  MOMENTS OF Z2 ARE:
          1.90701     -0.32392      9.10696

    ** 4 CENTRAL MU"S OF JOINT DISTN OF 10 PATHS:
```

```
        69.4665       1.9860      -0.4314       9.3660

  ALPHA3, ALPHA4=      -0.15414      2.37456

  ********** IN SUBROUTINE LAMBDA************
  *** FIT S-D DISTN TO PATH MU"S & FIND 4 LAMBDAS ***
  XL(1) =      0.1541D+00     YL(1) =      0.2375D+01

  FROM IBCIEU: FC(1,1)=   1.51445  FD(1,1)=    0.42493

  CD(1) =      0.1514D+01     CD(2) =      0.4249D+00

  ****** OUTPUT FROM ZXSSQ  *******
  X3,  X4  =      1.59457      0.46790

  ****** OUTPUT FROM ZXSSQ  *******
     AL3,   AL4 =      1.59457      0.53210

  Z =   9 THE 4 LAMBDAS ARE :
        69.6508594         8.6666413         1.5945708
 0.5320989

  ** 4 RMU"S OF PROJ. COMP ARE:          -0.18435
2.02001         -1.53604         10.09025

  #@#@#@#@#@#@#@#@#@#@#@#@#@#@#@#@#@#@#@#@#@#@#@#@

  THE 4 MUS FOR PROJ COMP TIME ARE :
          69.46651           1.98603          -0.43141
   9.36600
    SB1 AND B2 ARE :    -0.15414     2.37456 & CV= 0.0203
STOP          0
*End
```

2

VITA

Chellappan Somarajan

Candidate for the Degree of

Doctor of Philosophy

Thesis: PROGRAM EVALUATION AND REVIEW TECHNIQUE: AN INNOVATIVE APPROACH TO PROJECT COMPLETION TIME

Major Field: Business Administration

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