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AN EXACT PROBABILITY TEST FOR INDEPENDENCE IN 2^k

CONTINGENCY TABLES: A MULTIVARIATE ANALOGUE

OF FISHER'S EXACT PROBABILITY TEST

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CHAPTER I

INTRODUCTION

General Considerations

Frequently medical and epidemiological investigations involve the analyses of samples from populations whose members are classified according to the presence or absence of given characteristics. For example, one might record such variables as sex, race, previous admittance, and diagnosis, measuring each variable as either present or absent from a sample of patients in a mental hospital in order to predict length of stay. Similarly, a clinician might be interested in testing the simultaneous occurrence of a disease and two symptoms in a preliminary investigation of the mechanisms underlying the disease. The first example is typical of problems of prediction where one has to choose among numerous available variables for those which are most efficient in the prediction process. In order to minimize the effort required to perform an on-going analysis, one might wish to restrict himself to measuring only mutually independent variables. The second example represents a purely exploratory study but it also requires knowledge of the dependence, or lack of it, between variables. It is easily seen that these examples are part of the class of statistical problems involving 2^k contingency tables.

Statement of the Problem

Suppose there is given a population whose elements may be classified according to the presence or absence of each of k distinct characteristics. The use of the term "characteristic" in this paper will be quite general in that the absence of a specified condition or attribute could be denoted by the presence of the corresponding characteristic. This usage is necessary to avoid possible confusion when the absence of a condition or attribute is to be measured as the occurrence of an event. Suppose further that one is interested in deciding whether the specified characteristics occur independently within elements of the population. The purpose of this paper is to develop a test statistic based on the joint occurrence of characteristics for deciding whether it is reasonable to assume independence among the characteristics. In terms of a 2^k contingency table, the problem is to develop the probability density function for the cell of the table whose entry is the number of observed elements in which all k of the characteristics of interest occurred simultaneously. This approach is different from the usual treatment of contingency tables in that one is concerned with the contents of one cell only, whereas normally tests for "independence" refer to the contingency table as a whole. This distinction will become clear in the derivation of the appropriate density function.

A distribution for testing the null hypothesis of independence which avoids the use of proportions will be developed in Chapter II. Properties of the distribution and approximations will be investigated in Chapters III and IV. Chapter V will be devoted to worked examples.

The appendix contains tables of critical values for use whenever significance levels are acceptable and it is not desired to compute exact probabilities. For sampling situations not covered by the tables of critical values, a Fortran program for the IBM 1620 Computer is presented which will compute exact probabilities for any allowable range of the test statistic. Also, the program may be used to compute and print out the complete cumulative distribution for a given density function.

Detailed explanations for use of the tables and computing program are also given.

CHAPTER II

DERIVATION OF THE DISTRIBUTION OF THE TEST STATISTIC

Preliminary Discussion

Suppose that there exists an event space E and a probability measure P defined on E. Let $E_1, E_2, \cdots, E_k$ be k sub-events of E. Then the E_i are said to be independent if the joint probability of $E_1, E_2, \cdots, E_k$ can be written as the product of their respective marginal probabilities. That is, the E_i are independent if

$$P(E_1 E_2 \cdots E_k) = P(E_1)P(E_2)\cdots P(E_k). \qquad (2.1)$$

Let us write equation (2.1) in the form

$$D = P(E_1)(P(E_2)\cdots P(E_k) - P(E_1 E_2 \cdots E_k). \qquad (2.2)$$

If each of the $P(E_i) > 0$, $i=1,2,\cdots,k$, then clearly $D = 0$ only if the E_i are mutually independent. We must note carefully that the pair-wise independence of the E_i is not enough to insure that D be zero (Cramér, 1946), although this appears to be no problem for practical events (Feller, 1957). Consider now a random sample of size M from a population defined by the event space E, where a one is recorded if the event E_i occurred in the j^{th} observation and a zero if it did not. We could then build the following matrix of observational column vectors.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1M} \\ a_{21} & a_{22} & \cdots & a_{2M} \\ \cdots & \cdots & \cdots & \cdots \\ a_{k1} & a_{k2} & \cdots & a_{kM} \end{bmatrix} \qquad (2.3)$$

The vector

$$\begin{bmatrix} a_{ij} \\ a_{2j} \\ \cdot \\ \cdot \\ \cdot \\ a_{kj} \end{bmatrix}$$

represents the j^{th} observation where $a_{ij} = 1$ if the event E_i occurred and 0 otherwise.

Let

$$\sum_{j=1}^{M} a_{ij} = N_i ,$$

$$\sum_{i=1}^{k} a_{ij} = w(j),$$

$$d_{ij} = 1, \; i=j,$$
$$= 0, \; i \neq j.$$

Note that $w(j) = k$ only if the j^{th} column of the observational matrix is completely filled with ones. Then $d_{kw(j)} = 1$ only if $w(j) = k$. Thus, D can be estimated from the matrix of observations by

$$D = P(E_1)(P(E_2)\cdots P(E_k) - P(E_1E_2\cdots E_k),$$

$$= \frac{N_1N_2\cdots N_k}{M^k} - \frac{1}{M}\sum_{j=1}^{M} d_{kw(j)}. \qquad (2.4)$$

The quantity

$$n = \sum_{j=1}^{M} d_{kw(j)}$$

can be interpreted to mean the number of times that columns in the matrix (2.3) were "matched" by being completely filled with ones. Multiplying (2.4) on both sides by M and letting $MD= D^*$, we get

$$MD = \frac{N_1 N_2 \cdots N_k}{M^{k-1}} - n, \qquad (2.5)$$

$$= D^* .$$

It will be shown later that the expected number of matches for fixed N_1, N_2, $\cdots$, N_k, and M is given by the first term on the right side of equation (2.5). Thus our proposed test criterion becomes

D^* = Expected number of matches - observed number of matches.

Furthermore, if N_1, N_2, $\cdots$, N_k and M are fixed, then for a given constant D_o^* there are constants N_o and N_o^* such that

$$P(D^*=D_o) = P(\frac{N_1 N_2 \cdots N_k}{M^{k-1}} - n = n_o),$$

$$= P(n = n_o^*),$$

and the distribution of D^* is known if we know that of n. This leads naturally to the use of the observed frequency of simultaneous occurrences of all events within observations as a measure of the independence among events.

Derivation of the Distribution of n

Consider a 2 x M matrix of zeros and ones with N_1 ones in the first row and N_2 ones in the second row. We may assume without loss of generality that $N_1 \leq N_2$.

$$\begin{bmatrix} 1\ 0\ 1\ 1\ 0\ 1 \cdots 0\ 1\ 0 \\ 0\ 1\ 1\ 1\ 1\ 0 \cdots 1\ 0\ 1 \end{bmatrix} \tag{2.6}$$

The problem is to find the probability that exactly n of the columns of (2.6) are composed entirely of ones. For fixed N_1, N_2, and M under the null hypothesis of independence of rows this is clearly the hypergeometric density. That is

$$P(n \mid N_1, N_2) = \frac{\binom{N_1}{n}\binom{M-N_1}{N_2-n}}{\binom{M}{N_2}}, \quad 0 \leq n \leq N_1. \tag{2.7}$$

The matrix (2.6) can be summarized in tabular form to yield

	-	+	
-	$M-N_1-N_2+n$	N_2-n	$M-N_1$
+	N_1-n	n	N_1
	$M-N_2$	N_2	M

which is seen to be a slightly rearranged form of the usual 2 x 2 contingency table associated with Fisher's exact probability test (Siegel, 1956).

Suppose now we add one category to our classification scheme and form a new observational matrix by joining to (2.6) a new row containing $N_3 \geqq N_1$ ones.

$$\begin{bmatrix} 1\,0\,1\,1\,0\,1 \cdots 0\,1\,0 \\ 0\,1\,1\,1\,1\,0 \cdots 1\,0\,1 \\ 1\,1\,0\,0\,1\,1 \cdots 1\,1\,0 \end{bmatrix} \tag{2.8}$$

The problem now is to evaluate $P(n|N_1,N_2,N_3)$.

If we are to have exactly n columns of the matrix (2.8) filled with ones then we must have at least n of the first two rows so constructed. Consider first the case where exactly n columns of the first two rows are matched. Then

$$P(n|N_1,N_2,N_3) = P(n|N_1,N_2)\, P(n|n,N_3)$$

since the rows are independent by hypothesis. Thus

$$P(n|N_1,N_2,N_3) = \frac{\binom{N_1}{n}\binom{M-N_1}{N_2-n}}{\binom{M}{N_2}} \frac{\binom{n}{n}\binom{M-n}{N_3-n}}{\binom{M}{N_3}} .$$

If n+1 columns in the first two rows are matched then

$$P(n|N_1,N_2,N_3) = P(n+1|N_1,N_2)P(n|n+1,N_3)+P(n|N_1,N_2)P(n|n,N_3),$$

$$= \sum_{R_1=0}^{1} P(n+R_1|N_1,N_2)P(n|n+R_1,N_3).$$

In general we find, for three categories,

$$P(n|N_1,N_2,N_3) = \sum_{R_1=0}^{N_1-n} \frac{\binom{N_1}{n+R_1}\binom{M-N_1}{N_2-n-R_1}}{\binom{M}{N_2}} \frac{\binom{n+R_1}{n}\binom{M-n-R_1}{N_3-n}}{\binom{M}{N_3}} \tag{2.9}$$

As before, the matrix of observations can be represented in the form of a contingency table

<table>
<tr><td colspan="2">CATEGORY 1</td><td colspan="2">+</td><td colspan="2">-</td></tr>
<tr><td colspan="2">CATEGORY 2</td><td>+</td><td>-</td><td>+</td><td>-</td></tr>
<tr><td rowspan="2">CATEGORY 3</td><td>+</td><td>A</td><td>B</td><td>C</td><td>D</td></tr>
<tr><td>-</td><td>E</td><td>F</td><td>G</td><td>H</td></tr>
</table>

where

$$N_1 = A+B+E+F,$$

$$N_2 = A+E+C+G,$$

$$N_3 = A+B+C+D,$$

$$n = A.$$

The generalization to the case of k categories is straightforward and yields, for a 2^k contingency table,

$$P(n|N_1,N_2,\cdots,N_k) = \sum_{R_1=0}^{N_1-n} \sum_{R_2=0}^{R_1} \cdots \sum_{R_{k-2}=0}^{R_{k-3}} \left[\frac{\binom{N_1}{n+R_1}\binom{M-N_1}{N_2-n-R_1}}{\binom{M}{N_2}} \frac{\binom{n+R_1}{n+R_2}\binom{M-n-R_1}{N_3-n-R_2}}{\binom{M}{N_3}} \cdots \frac{\binom{n+R_{k-2}}{n}\binom{M-n-R_{k-2}}{N_k-n}}{\binom{M}{N_k}} \right] . \tag{2.10}$$

The proof that (2.10) represents a density function, that it is independent of the order in which the categories appear and the derivation of its mean and variance will be taken up in Chapter III after suitable notation has been developed.

CHAPTER III

PROPERTIES OF THE DISTRIBUTION OF THE TEST STATISTIC

It will be recalled that an expression suitable for testing the null hypothesis of independence among categories in a 2^k contingency table with fixed marginal totals was developed in the preceding chapter. This expression, equation (2.10), is very unwieldy in its present form and subsequent developments require a refinement in notation. The definitions needed will be listed in the order in which they appear in the following arguments so that reference to the list will be facilitated.

Definitions and Notation

Let k = the number of categories,

M = the number of observations,

N_i = the number of occurrences in category i,

S_k = the set $N_1, N_2, \cdots, N_k$,

N_1 = $\min_k S_k$,

n = the number of observations with matches across all categories.

The manipulations required will become greatly simplified if the summation notation used in Chapter II is replaced by vector and matrix notation. To this end we make the following definitions.

<u>Definition 1</u>. The $(N_1+1) \times 1$ probability vector $\vec{P}(S_k)$ will be written as

$$\vec{P}(S_k) = \vec{P}(N_1, N_2, \cdots, N_k)$$

$$= \begin{bmatrix} P(0 \mid N_1, N_2, \cdots, N_k) \\ P(1 \mid N_1, N_2, \cdots, N_k) \\ \cdot \\ \cdot \\ \cdot \\ P(N_1 \mid N_1, N_2 \cdots, N_k) \end{bmatrix} .$$

In particular we reserve $\vec{P}(S_2)$ to mean

$$\vec{P}(S_2) = \vec{P}(N_1, N_2)$$

$$= \frac{1}{\binom{M}{N_2}} \begin{bmatrix} \binom{N_1}{0} \binom{M-N_1}{N_2} \\ \binom{N_1}{1} \binom{M-N_1}{N_2-1} \\ \cdot \\ \cdot \\ \cdot \\ \binom{N_1}{N_1} \binom{M-N_1}{N_2-N_1} \end{bmatrix}$$

<u>Definition 2</u>. Let A_m be a $(N_1+1)x(N_1+1)$ matrix whose columns are probability densities of the form $\vec{P}(X, N_m)$, $0 \leqq x \leqq N_1$. That is

$$A_m = \left[a_{ij}\right], i, j = 1, 2, \cdots, N_1+1; m = 3, 4, \cdots, k,$$

where

$$a_{ij} = \frac{\binom{j-1}{i-1}\binom{M-j+1}{N_m-i+1}}{\binom{M}{N_m}}, \quad i \leqq j,$$

$$a_{ij} = 0 \qquad , \quad i > j.$$

Note that we may write

$$A_m = \left[\vec{P}(0,N_m),\vec{P}(1,N_m),\cdots,\vec{P}(X,N_m),\cdots,\vec{P}(N_1,N_m)\right] ,$$

and that

$$\sum_{i=1}^{j} a_{ij} = 1 \quad , \quad 1\leq j\leq N_1+1$$

since each column of A_m is a hypergeometric density function.

Definition 3. Let the vector operator $\bar{E}(n^r)$ be defined such that

$$\bar{E}(n^r)\vec{P}(S_k) = \left[0,1,2^r, \quad ,N_1^r\right]\vec{P}(S_k)$$

$$= \sum_{n=0}^{N_1} n^r \, P(n|S_k), \qquad r>0.$$

This will be recognized as the vector analogue of the usual expectation operator E.

Lemma 1. $\vec{P}(S_k) = A_k A_{k-1}\cdots A_3\vec{P}(S_2)$.

Proof: (By Induction)

Consider the case for k = 3. From equation (2.9) we have

$$P(n|N_1,N_2,N_3) = \sum_{R_1=0}^{N_1-n} \frac{\binom{N_1}{n+R_1}\binom{M-N_1}{N_2-n-R_1}}{\binom{M}{N_2}} \frac{\binom{n+R_1}{n}\binom{M-n-R_1}{N_3-n}}{\binom{M}{N_3}} .$$

Let n = 0.

$$P(0|N_1,N_2,N_3) = \sum_{R_1=0}^{N_1} \frac{\binom{N_1}{R_1}\binom{M-N_1}{N_2-R_1}}{\binom{M}{N_2}} \frac{\binom{M-R_1}{N_3}}{\binom{M}{N_3}} \quad ,$$

$$= \frac{\binom{N_1}{0}\binom{M-N_1}{N_2}}{\binom{M}{N_2}} \frac{\binom{M}{N_3}}{\binom{M}{N_3}} + \frac{\binom{N_1}{1}\binom{M-N_1}{N_2-1}}{\binom{M}{N_2}} \frac{\binom{M-1}{N_3}}{\binom{M}{N_3}} + \cdots$$

$$+ \frac{\binom{N_1}{N_1}\binom{M-N_1}{N_2-N_1}}{\binom{M}{N_2}} \frac{\binom{M-N_1}{N_3}}{\binom{M}{N_3}} \quad ,$$

$$= \frac{1}{\binom{M}{N_3}} \left[\binom{0}{0}\binom{M}{N_3}, \binom{1}{0}\binom{M-1}{N_3}, \ldots, \binom{N_1}{0}\binom{M-N_1}{N_3}\right] \vec{P}(N_1,N_2).$$

Let $n = 1$.

$$P(1|N_1,N_2,N_3) = \sum_{R_1=0}^{N_1-1} \frac{\binom{N_1}{R_1+1}\binom{M-N_1}{N_2-R_1-1}}{\binom{M}{N_2}} \frac{\binom{R_1+1}{1}\binom{M-R_1-1}{N_3-1}}{\binom{M}{N_3}}$$

$$= \frac{\binom{N_1}{1}\binom{M-N_1}{N_2-1}}{\binom{M}{N_2}} \frac{\binom{1}{1}\binom{M-1}{N_3-1}}{\binom{M}{N_3}} + \frac{\binom{N_1}{2}\binom{M-N_1}{N_2-2}}{\binom{M}{N_2}} \frac{\binom{2}{1}\binom{M-2}{N_3-1}}{\binom{M}{N_3}}$$

$$+ \cdots + \frac{\binom{N_1}{N_1}\binom{M-N_1}{N_2-N_1}}{\binom{M}{N_2}} \frac{\binom{N_1}{1}\binom{M-N_1}{N_3-1}}{\binom{M}{N_3}}$$

$$= \frac{1}{\binom{M}{N_3}} \left[\binom{1}{1}\binom{M-1}{N_3-1}, \binom{2}{1}\binom{M-2}{N_3-1}, \ldots, \binom{N_1}{1}\binom{M-N_1}{N_3-1}\right] \vec{P}(N_1,N_2) \; .$$

Continuing in this fashion we find, by Definition 1,

$$\vec{P}(N_1,N_2,N_3) = \begin{bmatrix} \binom{0}{0}\binom{M}{N_3} & \binom{1}{0}\binom{M-1}{N_3} & \binom{2}{0}\binom{M-2}{N_3} & \cdots & \binom{N_1}{0}\binom{M-N_1}{N_3} \\ 0 & \binom{1}{1}\binom{M-1}{N_3-1} & \binom{2}{1}\binom{M-2}{N_3-1} & \cdots & \binom{N_1}{1}\binom{M-N_1}{N_3-1} \\ 0 & 0 & \binom{2}{2}\binom{M-2}{N_3-2} & \cdots & \binom{N_1}{2}\binom{M-N_1}{N_3-2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \binom{N_1}{N_1}\binom{M-N_1}{N_3-N_1} \end{bmatrix} \frac{\vec{P}(N_1,N_2)}{\binom{M}{N_3}} ,$$

and, by Definition 2, $\vec{P}(N_1,N_2,N_3) = A_3\vec{P}(S_2)$.

Let the induction assumption be that the Lemma is true for the integers k such that $3 \leqq k \leqq m$. Writing equation (2.10) with the last summation sign distributed across terms not involving its lower index, we get

$$P(n|N_1,N_2,\cdots,N_{m+1}) = \sum_{R_1=0}^{N_1-1}\sum_{R_2=0}^{R_1}\cdots\sum_{R_{m-2}=0}^{R_{m-3}}\left[\left(\frac{\binom{N_1}{n+R_1}\binom{M-N_1}{N_2-n-R_1}}{\binom{M}{N_2}}\cdots\right.\right.$$

$$\left.\frac{\binom{n+R_{m-3}}{n+R_{m-2}}\binom{M-n-R_{m-3}}{N_{m-1}-n-R_{m-2}}}{\binom{M}{N_{m-1}}}\right)\sum_{R_{m-1}=0}^{R_{m-2}}\left(\frac{\binom{n+R_{m-2}}{n+R_{m-1}}\binom{M-n-R_{m-2}}{N_m-n-R_{m-1}}}{\binom{M}{N_m}}\right.$$

$$\left.\left.\frac{\binom{n+R_{m-1}}{n}\binom{M-n-R_{m-1}}{N_{m+1}-n}}{\binom{M}{N_{m+1}}}\right)\right] .$$

For the last summation we have, by the first part of the Lemma, letting $J = R_{m-2} + n$ be fixed,

$$\sum_{R_{m-1}=0}^{R_{m-2}} \frac{\binom{n+R_{m-2}}{n+R_{m-1}}\binom{M-n-R_{m-2}}{N_{m-1}-n-R_{m-1}}}{\binom{M}{N_m}} \frac{\binom{n+R_{m-1}}{n}\binom{M-n-R_{m-1}}{N_m-n}}{\binom{M}{N_{m+1}}}$$

$$= \sum_{R_{m-1}=0}^{J-n} \frac{\binom{J}{n+R_{m-1}}\binom{M-J}{N_{m-1}-n-R_{m-1}}}{\binom{M}{N_m}} \frac{\binom{n+R_{m-1}}{n}\binom{M-n-R_{m-1}}{N_m-n}}{\binom{M}{N_{m+1}}},$$

$$= A_{m+1}\vec{P}(J, N_m).$$

Now we must let J range over its possible values $0 \leqq J \leqq N_1$ to get

$$\left[A_{m+1}\vec{P}(0,N_m), A_{m+1}\vec{P}(1,N_m), \cdots, A_{m+1}\vec{P}(N_1,N_m)\right]$$
$$= A_{m+1}\left[\vec{P}(0,N_m), \vec{P}(1,N_m), \cdots, \vec{P}(N_1,N_m)\right],$$
$$= A_{m+1}A_m,$$

whence

$$\vec{P}(N_1, N_2, \cdots, N_{m+1}) = A_{m+1}A_m \cdots A_3\vec{P}(S_2)$$

so that by the Induction Property the Lemma holds for all positive integers k.

<u>Theorem 1</u>: $\vec{P}(S_k)$ is a density function.

Proof: By Lemma 1 we may write

$$\vec{P}(S_k) = A_k A_{k-1} \cdots A_3 \vec{P}(S_2),$$

so that

$$[1,1,\cdots,1]\,\vec{P}(S_k) = \sum_{n=0}^{N_1} P(n|S_k),$$

$$= [1,1,\cdots,1]\,A_k A_{k-1} \cdots A_3 \vec{P}(S_2).$$

However, by Definition 2

$$\left[1,1,\cdots,1\right] A_j = \left[1,1,\cdots,1\right],$$

whence,

$$\begin{aligned}\left[1,1,\cdots,1\right]\vec{P}(S_k) &= \left[1,1,\cdots,1\right] A_k A_{k-1}\cdots A_3\vec{P}(S_2),\\ &= \left[1,1,\cdots,1\right] A_{k-1}\cdots A_3\vec{P}(S_2),\\ &\ \ \vdots\\ &= \left[1,1,\cdots,1\right] \vec{P}(S_2),\\ &= 1,\end{aligned}$$

by virtue of the fact that $\vec{P}(S_2)$ is the ordinary hypergeometric density with parameters N_1, N_2, and M.

<u>Lemma 2</u>. $\quad \vec{E}(n)A_m = \frac{N_m}{M}\vec{E}(n)$

Proof:

$$\vec{E}(n)A_m = \left[0,1,2,\cdots,N_1\right]\left[\vec{P}(0,N_m),\vec{P}(1,N_m),\cdots,\vec{P}(N_1,N_m)\right],$$

$$= \left[\sum_{n=0}^{0} nP(n\,|\,0,N_m), \sum_{n=0}^{1} nP(n\,|\,1,N_m),\cdots,\sum_{n=0}^{N_1} nP(n\,|\,N_1,N_m)\right].$$

Now

$$P(n\,|\,N_j,N_m) = \frac{\binom{N_j}{n}\binom{M-N_j}{N_m-n}}{\binom{M}{N_m}},$$

but the usual form for the hypergeometric density is

$$P(x) = \frac{\binom{Np}{x}\binom{Nq}{r-x}}{\binom{N}{n}}, \quad x = 0, 1, \cdots, r,$$

which has as its mean rp. If we make the identifications $N=M$, $p=\frac{N_j}{M}$, $r=N_m$, we find that the mean of $\vec{P}(N_j,N_m)$ is $\frac{N_jN_m}{M}$.

Thus,
$$\bar{E}(n)A_m = \left[0, \frac{N_m}{M}, \frac{2N_m}{M}, \dots, \frac{N_1N_m}{M}\right],$$
$$= \frac{N_m}{M}\left[0,1,2,\cdots,N_1\right],$$
$$= \frac{N_m}{M}\ \bar{E}(n).$$

<u>Theorem 2</u>. The mean of $\vec{P}(S_k)$ is given by
$$\mu = \bar{E}(n)\ \vec{P}(S_k)$$
$$= \frac{N_1N_2\cdots N_k}{M^{k-1}}.$$

Proof: By Lemma 2,
$$\bar{E}(n)\vec{P}(S_k) = \bar{E}(n)A_kA_{k-1}\cdots A_3\vec{P}(S_2),$$
$$= \frac{N_kN_{k-1}\cdots N_3}{M^{k-2}}\ \bar{E}(n)\vec{P}(N_1,N_2),$$
$$= \frac{N_kN_{k-1}\cdots N_3}{M^{k-2}}\ \frac{N_1N_2}{M},$$
$$= \frac{N_1N_2\cdots N_k}{M^{k-1}}.$$

<u>Lemma 3</u>. $\bar{E}(n^2)\vec{P}(S_k) = \frac{N_1N_2\cdots N_k}{M^{k-1}}\left[1+\frac{(N_1-1)(N_2-1)\cdots(N_k-1)}{(M-1)^{k-1}}\right].$

Proof: (By Induction)

By making the identifications with the usual form of the hypergeometric density as shown in Lemma 2, one finds that
$$\bar{E}(n^2)\vec{P}(N_j,N_m) = \frac{N_jN_m}{M}\left[1+\frac{(N_j-1)(N_m-1)}{(M-1)}\right],$$
$$= \frac{N_j}{M(M-1)}(N_j-1)N_m^2 + \frac{N_j(M-N_j)}{M(M-1)}N_m.$$

Let

$$A(N_j) = \frac{N_j(N_j-1)}{M(M-1)} ,$$

$$B(N_j) = \frac{N_j(M-N_j)}{M(M-1)} .$$

Then

$$\overline{\overline{E}}(n^2)\vec{P}(N_j,N_m) = A(N_j)N_m^2 + B(N_j)N_m . \tag{3.1}$$

Now

$$\begin{aligned}\vec{P}(N_1,N_2,N_3) &= A_3\vec{P}(N_1,N_2),\\ &= \left[\vec{P}(0,N_3),\vec{P}(1,N_3),\cdots,\vec{P}(N_1,N_3)\right]\vec{P}(N_1,N_2),\end{aligned}$$

so that

$$\begin{aligned}\overline{\overline{E}}(n^2)\vec{P}(N_1,N_2,N_3) &= \overline{\overline{E}}(n^2)\left[\vec{P}(0,N_3),\vec{P}(1,N_3)\cdots,\vec{P}(N_1,N_3)\right]\vec{P}(N_1,N_2),\\ &= \left[\overline{\overline{E}}(n^2)\vec{P}(0,N_3),\overline{\overline{E}}(n^2)\vec{P}(1,N_3),\cdots,\right.\\ &\qquad\qquad \left.\overline{\overline{E}}(n^2)\vec{P}(N_1,N_3)\right]\vec{P}(N_1,N_2),\\ &= \left[\,0,A(N_3)+B(N_3),4A(N_3)+2B(N_3),\cdots,\right.\\ &\qquad\qquad \left.N_1^2A(N_3)+N_1B(N_3)\right]\vec{P}(N_1,N_2),\\ &= A(N_3)\left[0,1,4,\cdots,N_1^2\right]\vec{P}(N_1,N_2)\\ &\qquad +B(N_3)\left[0,1,2,\cdots,N_1\right]\vec{P}(N_1,N_2),\\ &= A(N_3)A(N_2)N_1^2+A(N_3)B(N_2)N_1+B(N_3)\,\frac{N_1N_2}{M} ,\\ &= \frac{N_3(N_3-1)N_2(N_2-1)N_1^2}{M^2(M-1)^2} + \frac{N_3(N_3-1)N_2(M-N_2)N_1}{M^2(M-1)^2}\\ &\qquad + \frac{N_1N_2N_3(M-N_3)}{M^2(M-1)} ,\\ &= \frac{N_1N_2N_3}{M^2}\left[1+\frac{(N_1-1)(N_2-1)(N_3-1)}{(M-1)^2}\right]\end{aligned}$$

and the Lemma holds for k = 3.

Let the induction assumption be that the Lemma holds for all integers k such that $3 \leq k \leq m$. Now, recalling that $\vec{P}(N_1,N_2,\cdots,N_{m+1}) = \vec{P}(S_{m+1})$, we have

$$\begin{aligned}\vec{P}(S_{m+1}) &= A_{m+1}\vec{P}(S_m), \\ &= \left[\vec{P}(0,N_{m+1}),\vec{P}(1,N_{m+1}),\cdots,\vec{P}(N_1,N_{m+1})\right]\vec{P}(S_m).\end{aligned}$$

Thus, by the first part of the Lemma and by assumption,

$$\begin{aligned}\overline{E}(n^2)\vec{P}(S_{m+1}) &= A(N_{m+1})\overline{E}(n^2)\vec{P}(S_m)+B(N_{m+1})\overline{E}(n)\vec{P}(S_m) \quad , \\ &= A(N_{m+1})\frac{N_1N_2\cdots N_m}{M^{m-1}}\left[1+\frac{(N_1-1)(N_2-1)\ldots(N_m-1)}{(n-1)^{m-1}}\right] \\ &\qquad + B(N_{m+1})\frac{N_1N_2\cdots N_m}{M^{m-1}} \quad , \\ &= \frac{N_{m+1}(N_{m+1}-1)}{M(M-1)}\frac{N_1N_2\cdots N_m}{M^{m-1}}\left[1+\frac{(N_1-1)(N_2-1)\cdots(N_m-1)}{(M-1)^{m-1}}\right] \\ &\qquad + \frac{N_{m+1}(M-N_{m+1})}{M(M-1)}\frac{N_1N_2\cdots N_m}{M^{m-1}} \quad , \\ &= \frac{N_1N_2\cdots N_{m+1}}{M^m}\left[1+\frac{(N_1-1)(N_2-1)\cdots(N_{m+1}-1)}{(M-1)^m}\right].\end{aligned}$$

Therefore, by the Induction Property, the Lemma is true for all positive integers k.

<u>Theorem 3</u>. The variance of $\vec{P}(S_k)$ is given by

$$\sigma^2 = \mu\left[1-\mu+\frac{(N_1-1)(N_2-1)\cdots(N_k-1)}{(M-1)^2}\right].$$

Proof:

$$\begin{aligned}\sigma^2 &= E(n^2) - E(n)^2 \\ &= E(n^2) - \mu^2.\end{aligned}$$

From Lemma 3 we have

$$E(n^2) = \frac{N_1N_2\cdots N_k}{M^{k-1}}\left[1+\frac{(N_1-1)(N_2-1)\cdots(N_k-1)}{(M-1)^{k-1}}\right],$$

$$= \mu\left[1+\frac{(N_1-1)(N_2-1)\cdots(N_k-1)}{(M-1)^{k-1}}\right].$$

Then

$$\sigma^2 = \mu\left[1-\mu+\frac{(N_1-1)(N_2-1)\cdots N_k-1)}{(M-1)^{k-1}}\right].$$

Up to this point the only assumption on the set of occurrences $S_k = N_1,N_2,\cdots,N_k$ has been that $N_1 \leqq N_i$, $i = 2,3,\cdots,k$. No mention has been made of the order in which these parameters are to enter equation (2.10) in calculating $\vec{P}(S_k)$. Thus the question arises whether a permutation among the members of S_k will change the density function of n. It will be shown that the probabilities computed from $P(n|S_k)$, that is equation (2.10), are independent of the order in which the parameters $N_1,N_2,\cdots,N_k$ enter into the equation.

<u>Lemma 4</u>. Let A, B_i, R_j, and m be non-negative integers, where $i = 1,2,\cdots,k-1$ and $j = 1,2,\cdots,k-2$.

Let

$$S_m(k) = \sum_{R_1=0}^{m}\sum_{R_2=0}^{R_1}\cdots\sum_{R_{k-2}=0}^{R_{k-3}}\left[\binom{m}{R_1}\binom{A-m}{A-B_1-R_1}\binom{R_1}{R_2}\binom{A-R_1}{A-B_2-R_2}\cdots\binom{R_{k-2}}{R_{k-1}}\binom{A-R_{k-2}}{A-B_{k-1}-R_{k-1}}\right], \quad (3.1)$$

$$S_m^*(k) = \sum_{i=0}^{m} (-1)^m \binom{m}{i}\binom{A-i}{B_1}\binom{A-i}{B_2}\cdots\binom{A-i}{B_{k-1}} \quad , \tag{3.2}$$

where

$k \geq 3,$

$R_{k-1} = 0.$

Then

$$S_m(k) = S_m^*(k).$$

Proof: (By Induction). Equations (3.1) and (3.2) are in the form necessary for later use but the proof of the lemma will be easier if a different notation is used. Let

$$V_k(R_1,R_2,\cdots,R_{k-2}) = \binom{R}{R_2}\binom{A-R_1}{A-B_2-R_2}\cdots\binom{R_{k-2}}{R_{k-1}}\binom{A-R_{k-2}}{A-B_{k-1}-R_{k-1}} \quad ,$$

$$U(R_1) = \binom{m}{R_1}\binom{A-m}{A-B_1-R_1} \quad ,$$

$$W_i(k) = \binom{A-i}{B_1}\binom{A-i}{B_2}\cdots\binom{A-i}{B_{k-1}} \quad .$$

Then

$$S_m(k) = \sum_{R_1=0}^{m}\sum_{R_2=0}^{R_1}\cdots\sum_{R_{k-2}=0}^{R_{k-3}} U(R_1)V_k(R_1,R_2,\cdots,R_{k-2}),$$

$$S_m^*(k) = \sum_{i=0}^{m} (-1)^i\binom{m}{i}W_i(k).$$

Consider the case for k = 3:

$$S_m^*(3) = \sum_{i=0}^{m} (-1)^i \binom{m}{i} W_i(3),$$

$$S_m(3) = \sum_{R_1=0}^{m} U(R_1)V_3(R_1) \quad ,$$

$$= U(0)V_3(0)+U(1)V_3(1)+\cdots+U(m)V_3(m),$$

$$= \left[U(0),U(1),\cdots,U(m)\right]\begin{bmatrix} V_3(0) \\ V_3(1) \\ \cdot \\ \cdot \\ \cdot \\ V_3(m) \end{bmatrix} \quad . \tag{3.3}$$

The following identities (Feller, 1957) will be useful.

For integral a, b, and m,

$$\binom{a-m}{b-m} = \sum_{j=0}^{m} (-1)^m \binom{m}{j}\binom{a-j}{b} \tag{3.4}$$

and

$$\binom{a-m}{b} = \sum_{j=0}^{m} \binom{m}{j}\binom{a}{b-j} \quad . \tag{3.5}$$

Now

$$V_3(j) = \binom{A-j}{A-B_2} \quad ,$$

$$= \binom{A-j}{B_2-j} \quad ,$$

or

$$V_3(j) = \sum_{i=0}^{j} (-1)^i \binom{j}{i}\binom{A-i}{B_2} \quad .$$

by equation (3.4). Thus we may write equation (3.3) as

$$S_m(3) = \left[U(0),U(1),\cdots,U(m)\right]\begin{bmatrix} \binom{A}{B_2} \\ \binom{A}{B_2} - \binom{A-1}{B_2} \\ \cdots\cdots\cdots\cdots \\ \binom{A}{B_2}-\binom{m}{1}\binom{A-1}{B_2}+\binom{m}{2}\binom{A-2}{B_2}-\cdots+(-1)^m\binom{m}{m}\binom{A-m}{B_2} \end{bmatrix}$$

$$= \binom{A}{B_2}\sum_{i=0}^{m} U(i)-\binom{A-1}{B_2}\sum_{i=1}^{m}\binom{i}{1}U(i)+\cdots+(-1)^j\binom{A-j}{B_2}\sum_{i=j}^{m}\binom{i}{j}U(i)+$$

$$\cdots+(-1)^m\binom{A-m}{B_2}U(m).$$

But

$$\sum_{i=j}^{m}\binom{i}{j}U(i) = \sum_{i=j}^{m}\binom{i}{j}\binom{m}{i}\binom{A-m}{A-B_1-i} \quad ,$$

$$= \binom{m}{j}\sum_{i=j}^{m}\binom{m-j}{i-j}\binom{A-m}{A-B_1-i} \quad ,$$

or

$$\sum_{i=j}^{m} \binom{i}{j} U(i) = \binom{m}{j} \sum_{k=0}^{m-j} \binom{m-j}{k} \binom{A-m}{A-B_1-j-k} \quad ,$$

$$= \binom{m}{j} \binom{A-j}{A-B_1-j} \quad ,$$

$$= \binom{m}{j} \binom{A-j}{B_1} \quad ,$$

using equation (3.5). Therefore,

$$S_m(3) = \binom{A}{B_1}\binom{A}{B_2} - m\binom{A-1}{B_1}\binom{A-1}{B_2} + \binom{m}{2}\binom{A-2}{B_1}\binom{A-2}{B_2} - \cdots + (-1)^m \binom{A-m}{B_1}\binom{A-m}{B_2} \quad ,$$

$$= \sum_{i=0}^{m} (-1)^i \binom{m}{i} W_i(3),$$

$$= S_m^*(3).$$

Thus the lemma holds for $k = 3$.

Let the induction assumption be that the lemma is true for all integers k such that $3 \leqq k \leqq n$.

Now

$$S_m(n+1) = \sum_{R_1=0}^{m} \sum_{R_2=0}^{R_1} \cdots \sum_{R_{n-1}=0}^{R_{n-2}} U(R_1) V_{n+1}(R_1, R_2, \cdots, R_{n-1}),$$

$$= \sum_{R_1=0}^{m} U(R_1) \sum_{R_2=0}^{R_1} \cdots \sum_{R_{n-1}=0}^{R_{n-2}} V_{n+1}(R_1, R_2, \cdots, R_{n-1}),$$

or

$$S_m(n+1) = U(0)V_{n+1}(0,0,\cdots,0) + U(1)\left[V_{n+1}(1,0,\cdots,0)+V_{n+1}(1,1,0,\cdots,0)+\cdots+V_{n+1}(1,1,\cdots,1)\right] + \cdots + U(m)\left[V_{n+1}(m,0,\cdots,0)+\cdots+V_{n+1}(m,m,\cdots,m)\right] .$$

Clearly,

$$V_{n+1}(j,0,\cdots 0)+\cdots+V_{n+1}(j,j,\cdots,j) = \sum_{R_2=0}^{j}\sum_{R_3=0}^{R_2}\cdots\sum_{R_{n-1}=0}^{R_{n-2}} U(j)V_n(R_2,R_3,\cdots,R_{n-1}) ,$$

$$= S_j(n).$$

Therefore

$$S_m(n+1) = \sum_{j=0}^{m} U(j)S_j(n).$$

Thus, by the induction hypothesis,

$$S_m(n+1) = \sum_{j=0}^{m} U(j)S_j^*(n),$$

$$= \left[U(0),U(1),\cdots,U(m)\right]\begin{bmatrix} W_0(n) \\ W_0(n)-W_1(n) \\ W_0(n)-2W_1(n)+W_2(n) \\ \cdots\cdots\cdots \\ W_0(n)-mW_1(n)+\cdots\pm\binom{m}{n}W_m(n) \end{bmatrix} ,$$

or

$$S_m(n+1) = W_0(n) \sum_{i=0}^{m} U(i) - W_1(n) \sum_{i=1}^{m} \binom{i}{1} U(i) +$$

$$\cdots + (-1)^j W_j(n) \sum_{i=j}^{m} \binom{i}{j} U(i) +$$

$$\cdots + (-1)^m W_m(n) U(m).$$

As before,

$$\sum_{i=j}^{m} \binom{i}{j} U(i) = \binom{m}{j}\binom{A-j}{B_1} \quad .$$

Then

$$S_m(n+1) = \binom{A}{B_1}\binom{m}{0} W_0(n) - \binom{A-1}{B_1}\binom{m}{1} W_1(n) + \cdots + (-1)^m \binom{m}{m}\binom{A-m}{B_1} W_m(n),$$

$$= \binom{m}{0} W_0(n+1) - \binom{m}{1} W_1(n+1) + \cdots + (-1)^m \binom{m}{m} W_m(n+1) \ ,$$

$$= \sum_{i=0}^{m} (-1)^i \binom{m}{i} W_i(n+1),$$

$$= S_m^*(n+1).$$

Thus, by the Induction Property, the Lemma is true for all positive integers k such that $3 \geqq k$.

Theorem 4. The density function $P(n|N_1,N_2,\cdots,N_k)$ given by equation (2.10) is independent of the order in which the parameters

$N_2, N_3, \cdots, N_k$ are taken.

Proof: Using the equivalent expression

$$\binom{N_1}{n+R_1}\binom{n+R_1}{n+R_2}\cdots\binom{n+R_{k-2}}{n} = \binom{N_1}{n}\binom{N_1-n}{R_1}\binom{R_1}{R_2}\cdots\binom{R_{k-3}}{R_{k-2}}$$

and letting

$$N_1 - n = m,$$

$$M - n = A,$$

$$M - N_i = B_i,$$

we may rewrite equation (2.10) as

$$P(n|N_1, N_2, \cdots, N_k) = \frac{\binom{N_1}{n}}{\prod_{i=2}^{k}\binom{M}{N_i}} \sum_{R_1=0}^{N_1-n} \cdots \sum_{R_{k-2}=0}^{R_{k-3}} U(R_1)V_k(R_1, R_2, \cdots, R_{k-2}).$$

Then by Lemma 4,

$$P(n|N_1, N_2, \cdots, N_k) = \frac{\binom{N_1}{n}}{\prod_{i=2}^{k}\binom{M}{N_i}} \sum_{i=0}^{m} (-1)^i \binom{m}{i} W_i(k), \qquad (3.6)$$

which is independent of the order of the N_i, $i = 2, 3, \cdots, k$.

Equation (3.6) is a slightly rearranged form of the density function of n derived by Mielke and Siddiqui (1965).

The last theorem completes the present theoretical development of the problem. The question of approximations and the use of the significance tables will be taken up in the next chapter.

CHAPTER IV

THE NORMAL APPROXIMATION TO THE DISTRIBUTION OF THE TEST STATISTIC

When either the number of categories or the number of occurrences in any category is large, the computation of exact probabilities by equation (2.10) becomes tedious and time-consuming. Fortunately, it appears that for reasonable conditions on the parameters the cumulative distribution of n can adequately be represented by the cumulative normal distribution. Although an intensive investigation into the theoretical nature of the approximation has not been carried out as yet, some preliminary empirical results have been obtained which yield a conservative "rule-of-thumb" for deciding when the normal approximation is applicable.

In Chapter III the mean and variance of the distribution of n were found to be, respectively,

$$\mu = \frac{N_1 N_2 \cdots N_k}{M^{k-1}}$$

$$\sigma^2 = \mu\left[1-\mu+\frac{(N_1-1)\cdots(N_{k-1})}{(M-1)^{k-1}}\right].$$

Empirical investigation has shown that when the normal approximation is applicable, the agreement between approximate and exact values is considerably improved if a correction of $\frac{1}{2}$ is added to the mean. In the following paragraphs we will use μ^* for the corrected mean, where

$\mu^* = \mu + \frac{1}{2}$. Also we will let

$$z = \frac{n - \mu^*}{\sigma},$$

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}z^2),$$

$$F(z) = \int_{-\infty}^{z} f(t)dt .$$

There are three conditions which we would expect to affect the shape of the density function of the test statistic. These are the proportion of occurrences within categories, the number of categories, and the total sample size. We shall consider changes in sample size first.

Figure 1 shows the effect of increasing sample size when the proportions per category are all held constant at a value of .5 and the number of categories is three. Smooth curves have been drawn through the discrete data points in order to emphasize the shape of the density function. It appears from Figure 1 that under the given conditions the approach to normality is fairly rapid. Table 1 gives the exact and approximate values of the cumulatives for a sample size of 20. The approximation should be adequate under ordinary circumstances.

When sample size and the number of categories are held constant, the proportions of occurrences per category may be varied in two different ways. One way is to hold a sub-set of S_k, say $N_1, N_2, \cdots, N_j$, constant and let $N_{j+1}, \cdots, N_k$ increase. The other is to let all of the N_i increase together. Figure 2 shows the effect of holding N_1 fixed and letting N_2 and N_3 vary for the case of 3 categories. Figure 3 shows

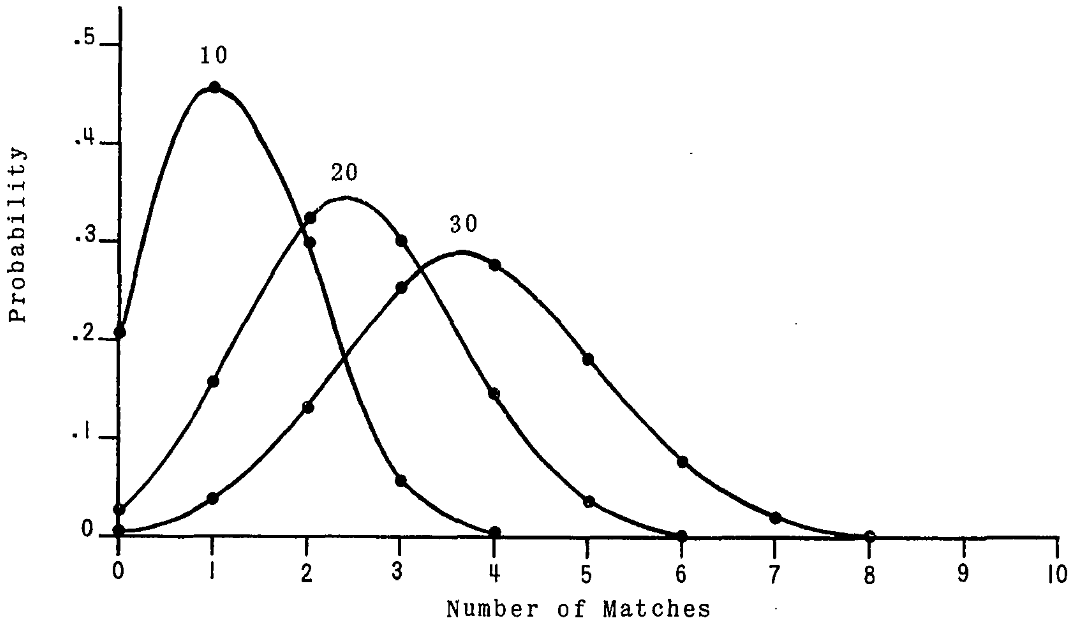

Figure 1. The effect on the shape of the density function of increasing sample size. Sample size is shown above each curve.

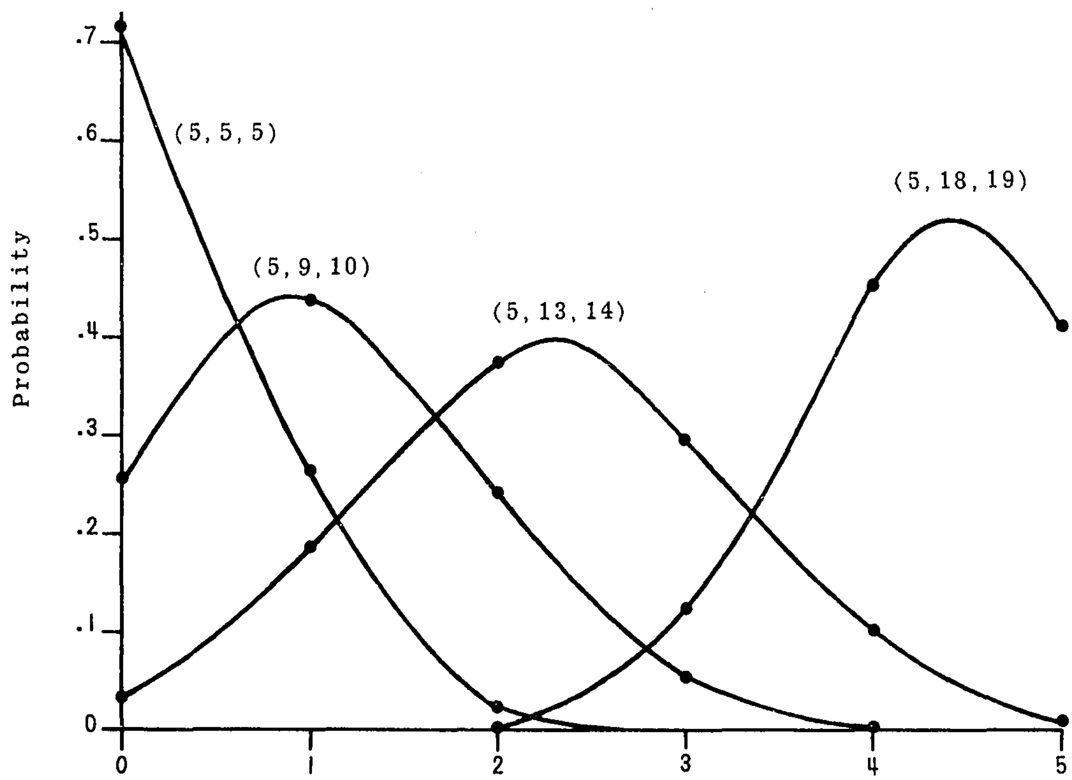

Figure 2. The effect on the shape of the density function of increasing proportions in a subset of the categories. The elements of S_k are given above each curve.

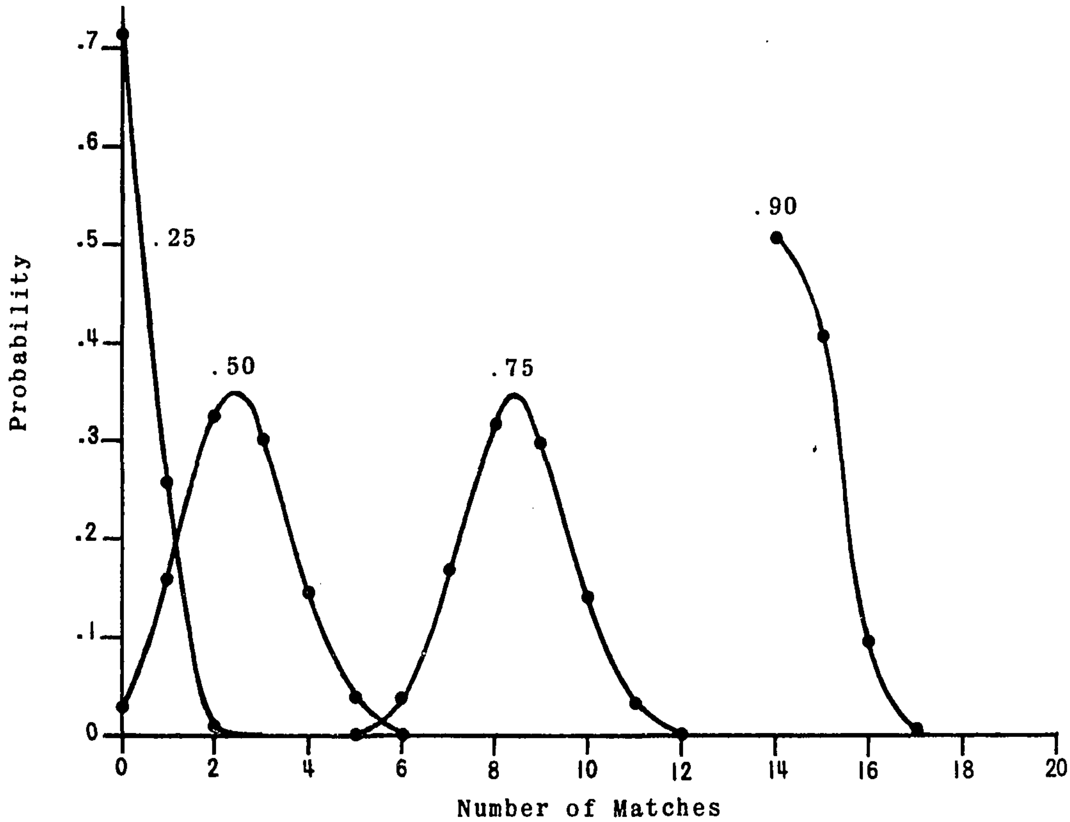

Figure 3. The effect on the shape of the density function of increasing proportions simultaneously in all categories. The common proportion for all categories is given above each curve (see text).

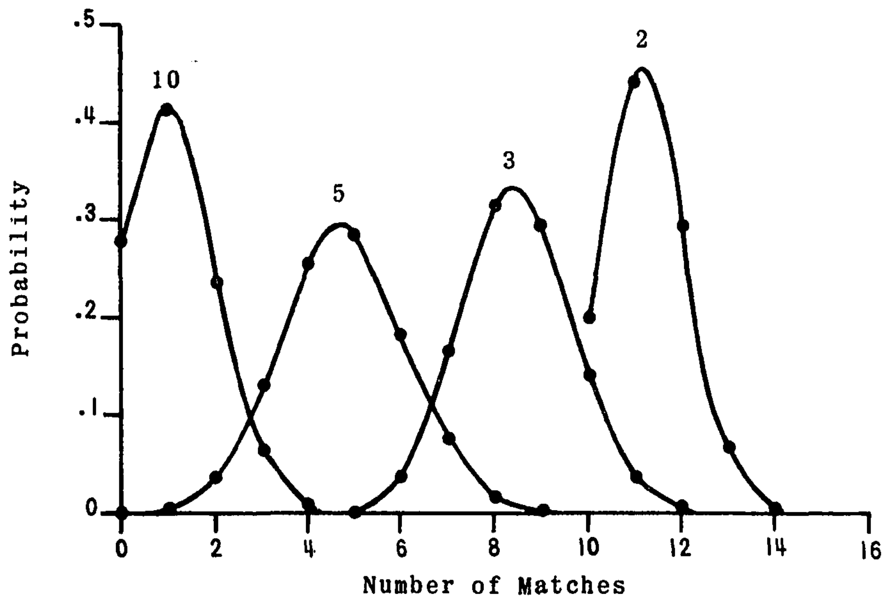

Figure 4. The effect on the shape of the density function of increasing the number of categories. The number of categories is given above each curve.

the effect of increasing N_1, N_2, and N_3 simultaneously. There are two points to be noted about these figures. The first is that even for a small proportion in one of the categories, in this case $p_1 = .25$, large p values in the other categories can make the normal approximation applicable. This is shown in Table 2.

TABLE 1

EXACT AND APPROXIMATE VALUES FOR A SAMPLE SIZE OF 20 WHEN N=3 AND $N_1=N_2=N_3=10$.

	Cumulative		
n	Exact	F(z)	Difference
0	0.030	.040	-0.010
1	.189	.189	0.000
2	.512	.500	.012
3	.816	.811	.005
4	.960	.960	.000
5	.995	.996	-0.001
6	1.000	1.000	0.000

The other point is that when any of the categories has a "large" number of occurrences then the density function becomes skewed and the normal approximation is not valid. Obviously, the ranges of the proportions and sample size interact strongly in determining the shape of the density.

The effect of increasing the number of categories while holding

TABLE 2

EXACT AND APPROXIMATE VALUES FOR A SAMPLE SIZE OF 20 WHEN $N=3, N_1=5, N_2=13, N_3=14$.

	Cumulative		
n	Exact	F(z)	Difference
0	0.032	0.040	-0.008
1	.221	.221	0.000
2	.592	.587	.005
3	.890	.885	.005
4	.989	.986	.003
5	1.000	.999	.001

sample size and proportions fixed is shown in Figure 4. For convenience the proportions were held constant at .75 for each category. The results resemble somewhat those shown in Figures 2 and 3 in that the distributions become skewed at each end of the horizontal scale while in the middle of the range the densities can usefully be approximated by the appropriate normal density.

From the foregoing it is clear that there is a bounded region in the k+2 dimensional hyperspace defined by the set $(N_1, N_2, \cdots, N_k, M, N)$ where the appropriate normal approximation is applicable. At this point in the investigation it is suggested that the following rule be used: when $0 < \mu - 2\sigma$ and $\mu + 2\sigma < N_1$, then use the normal approximation $z = (n - \mu^*)/\sigma$. Otherwise, do not. This rule has the effect of insuring that the density function will be approximately symmetric and that both tails will be depressed. The theoretical basis for approximation, as well as empirical investigations, will be left for further study at a later date.

CHAPTER V

SPECIFIC EXAMPLES OF THE USE OF THE TEST STATISTIC

Coding of the Observational Vector

As was pointed out in the introduction, the test statistic refers to independence among events as measured relative to fixed marginal totals and the contents of the cell of the contingency table specified by the number of joint occurrences observed. Thus the null hypothesis of independence concerns only the mutual occurrence of events and, in a sense, ignores such questions as mutual non-occurrence and specific associations between subsets of the observations. The restrictive form of the hypothesis requires that careful attention be paid to the coding of the characteristics which are to be tested for independence. Suppose for instance that one is interested in testing three characteristics, say the presence of A, the absence of B, and the presence of C, for mutual independence. Then, in the proper rows of the observational vector, one would record a one if A is present, a one if B is absent, and a one if C is present. This coding scheme allows one to consider joint occurrences and to make suitable hypotheses in a straightforward manner.

Examples Using the Tables of Critical Values

Table 7 of the Appendix may be used to test for independence between two categories for a sample size of 30 and any marginal totals compatible with this total. The following tables will illustrate the use of Table 7.

Suppose that a physician is interested in investigating the possibility that a recent outbreak of viral infections among his patients represents the effects of two different viruses in terms of the symptoms of headache and nausea. That is, one virus predominantly causes headaches while the other predominantly causes nausea. The null hypothesis of two sources of infection would be that nausea and headaches occur jointly no more than one would expect by chance. This is a one-tailed hypothesis. The data (artificial) are shown in Table 3.

TABLE 3

NAUSEA AND HEADACHE IN VIRAL INFECTIONS
(ARTIFICIAL DATA)

		Headache -	Headache +	
Nausea	-	6	2	8
	+	1	<u>6</u>	<u>7</u>
		7	<u>8</u>	<u>15</u>

The relevant values for testing the null hypothesis are those which are underlined in Table 3, namely,

$$n = 6,$$

$$N_1 = 7,$$

$$N_2 = 8,$$

$$M = 15.$$

Reference to Table 7 shows that $P(n \geq 6) \leq .05$ so we reject the hypothesis that nausea and headaches are occurring independently.

In order to be sure of the results, suppose that new data is collected from a larger group of people as shown in Table 4.

TABLE 4

REPEATED STUDY OF NAUSEA AND HEADACHE IN VIRAL INFECTIONS (ARTIFICIAL DATA).

		Headache -	Headache +	
Nausea	-	9	3	12
	+	1	15	16
		10	18	28

The entries in Table 4 which are appropriate for testing the null hypothesis are

$$n = 15,$$

$$N_1 = 16,$$

$$N_2 = 18,$$

$$M = 28.$$

However, the table of critical values for two categories (Table 7) was constructed such that $N_1 \leqq \frac{1}{2} M$ and $N_1 + N_2 \leqq M$ in order to keep the table as small as possible. Thus we must use the symmetry properties of the hypergeometric density to test the hypothesis. The relevant values from Table 4 are

$$n' = 9,$$

$$N_1' = 10,$$

$$N_2' = 12,$$

$$M = 28,$$

where primes have been used to indicate that we are considering the complements of our original test parameters. This is equivalent to a complete relabeling of the contingency table. Reference to Table 7 shows that $P(n' \geqq 9) < .01$ so that once again we reject the null hypothesis of independence of the symptoms of nausea and headache.

Unfortunately, the density function $\vec{P}(S_k)$ is not symmetric in general for $k > 2$ and the effort necessary to compute critical values rapidly becomes prohibitive. Table 8 gives critical values for three categories up to a sample size of 15. The following artificial data illustrate the use of this table. The null hypothesis is that the three factors A, B, and C occur independently.

The relevant parameters for the test are

$$n = 4,$$

$$N_1 = 8,$$

$$N_2 = 9,$$

$$N_3 = 9,$$

$$M = 14.$$

TABLE 5

ARTIFICIAL DATA FOR ILLUSTRATING THE USE OF THE TABLE OF CRITICAL VALUES FOR THREE CATEGORIES

A		+		-		
B		+	-	+	-	TOTAL
C	+	4	2	3	0	9
	-	1	1	1	2	5
TOTAL		5	3	4	2	14

Suppose that we wish to test the hypothesis at the 10% level, noting that a two-tailed test is implied. Table 8 shows the left and right critical values at the 10% level (5% each) to be 1 and 6 respectively. Since $1 < n < 6$ we accept the null hypothesis of independence of the joint occurrence of the factors A, B, and C.

An Example Using the Computer Program

Data from a study designed to determine the factors which influence perinatal deaths was used to test the computing program in the case of very large sample sizes. This data was collected from birth certificates corresponding to perinatal deaths in Oklahoma over the two-year period 1961-1962.

TABLE 6

FACTORS INFLUENCING PERINATAL DEATHS; OKLAHOMA, 1961-1962.

Length of Gestation		$\leqslant$ 36 Weeks				>36 Weeks			
Race		Negro		White		Negro		White	
Previous Loss		yes	no	yes	no	yes	no	yes	no
Age	> 30 years	40	43	195	202	45	39	143	244
	≦ 30 years	107	114	343	659	40	62	184	506

It is felt that a short gestation period, a history of previous child loss, and an age greater than thirty among Negro mothers gives rise to a higher risk of perinatal death, but the question arises as to whether these factors are indeed independent among themselves when measured on the sub-set of deaths only. Table 6 is the appropriate data extracted from the overall study which included all births during the specified period. The relevant parameters from these data are

$$n = 40,$$
$$N_1 = 490,$$
$$N_2 = 951,$$
$$N_3 = 1097,$$
$$N_4 = 1703,$$
$$M = 2966.$$

Using these parameters in the computer program it was found that $P(n \geqq 40) = .097$ so that there is a reasonable basis for considering the factors to be independent if one considers the usual 5% rejection region. The computing time for these data was approximately two hours. The expected value of n was found to be 33.4 and a standard deviation of 5.45. Using the normal approximation one finds $P(n \geqq 40) = .097$. In fact, the exact distribution function and its normal approximation are almost coincident throughout. This points up the drastic saving of time and expense when the normal approximation is valid.

CHAPTER VI

SUMMARY

Given a population whose elements may be classified according to the presence or absence of k distinct characteristics, the probability density function for the number of elements in a sample which display a simultaneous occurrence of all k characteristics has been developed, subject to the constraints of fixed marginal totals and a fixed sample size. The mean and variance of the distribution are given as well as a broad general rule for deciding when it is appropriate to use the normal density function as an approximation.

For $k = 2$, two-tailed tables of critical values at the .01, .025, and .05 levels are given up to a sample size of 30. For $k = 3$, two-tailed tables of critical values at the .01 and .05 levels are given up to a sample size of 15. Examples demonstrating the use of these tables are presented. A Fortran program for the IBM 1620 for computing the complete probability density function is also given.

The probability density function is based on the assumption of independence among the k characteristics within elements, so that it may be used to test hypotheses concerning such independence. Although the data layout falls naturally into the form of a 2^k contingency table, not all such tables are amenable to analysis by this method. For example, a 2^2 factorial experiment which could result in only two measurements, say alive and dead, could be considered as a 2^3 contingency table. The design, however, requires that the four treatment groups, say A^+B^+, A^+B^-, A^-B^+, A^-B^-, representing the possible combinations of the levels of factors A and B, each be applied to an equal number of experimental units. Thus the joint occurrence of two of the three characteristics represented in the contingency table is determined by the design, which violates the assumption of independence among characteristics. In general one might say that the proposed technique should be applied only to situations where it is known that the design does not imply dependence among subsets of the characteristics in order to avoid bias or misinterpretation of the results.

There are still several areas of the problem to be investigated. Some of these are the possibility of testing sub-sets of the characteristics for independence, investigating the power functions of the distribution, establishing more stringent rules for the use of the normal approximation based on theoretical considerations, and performing an empirical study of the effect of allowing the marginal totals to vary. Hopefully, some of these questions may be answered in the near future.

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APPENDIX A

TABLES OF CRITICAL VALUES

The following tables give rejection regions, or critical values, at the probability levels specified at the top of the table for the sample size and number of occurrences listed at the left of the body of the table. For a particular entry the probability level is for the tabulated value or one more extreme. The left-most columns of critical values refer to the probability of observing the tabular value or smaller values while the right-most columns refer to the probability of observing the tabular value or larger values. Thus the tables can be used for one or two-tailed hypotheses.

If a particular set of observed parameters does not occur in the table then there are no critical values at the specified probability levels for that set. A dash in the body of the table indicates that there is no critical value for the specified probability level.

TABLE 7

CRITICAL VALUES BY SAMPLE SIZE
TWO CATEGORIES

M	N1	N2	.01	.025	.05	.05	.025	.01	M	N1	N2	.01	.025	.05	.05	.025	.01
6	3	3	-	-	0	3	-	-	10	4	4	-	-	-	4	4	4
7	2	2	-	-	-	2	-	-		4	5	-	0	0	4	4	-
	2	5	-	-	0	-	-	-		4	6	0	0	0	-	-	-
	3	3	-	-	-	3	-	-		5	5	0	0	0	5	5	5
	3	4	-	-	0	-	-	-	11	2	2	-	-	-	2	2	-
8	2	2	-	-	-	2	-	-		2	9	-	0	0	-	-	-
	2	6	-	-	0	-	-	-		3	3	-	-	-	3	3	3
	3	3	-	-	-	3	3	-		3	4	-	-	-	3	3	-
	3	5	-	0	0	-	-	-		3	7	-	0	0	-	-	-
	4	4	-	0	0	4	4	-		3	8	0	0	0	-	-	-
9	2	2	-	-	-	2	-	-		4	4	-	-	-	4	4	4
	2	7	-	-	0	-	-	-		4	5	-	-	0	4	4	-
	3	3	-	-	-	3	3	-		4	6	-	0	0	4	-	-
	3	4	-	-	-	3	-	-		4	7	0	0	0	-	-	-
	3	5	-	-	0	-	-	-		5	5	-	0	0	5	5	5
	3	6	-	0	0	-	-	-		5	6	0	0	0	5	5	-
	4	4	-	-	0	4	4	4	12	2	2	-	-	-	2	2	-
	4	5	0	0	0	4	-	-		2	3	-	-	-	2	-	-
10	2	2	-	-	-	2	2	-		2	9	-	-	0	-	-	-
	2	8	-	0	0	-	-	-		2	10	-	0	0	-	-	-
	3	3	-	-	-	3	3	3		3	3	-	-	-	3	3	3
	3	4	-	-	-	3	-	-		3	4	-	-	-	3	3	-
	3	6	-	-	0	-	-	-		3	5	-	-	-	3	-	-
	3	7	0	0	0	-	-	-		3	7	-	-	0	-	-	-

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01	M	N1	N2	.01	.025	.05	.05	.025	.01
12	3	8	–	0	0	–	–	–	13	5	8	0	0	1	5	–	–
	3	9	0	0	0	–	–	–		6	6	0	0	0	5	6	6
	4	4	–	–	–	4	4	4		6	7	0	0	1	6	6	6
	4	5	–	–	–	4	4	–	14	2	2	–	–	–	2	2	–
	4	6	–	–	0	4	–	–		2	3	–	–	–	2	–	–
	4	7	–	0	0	–	–	–		2	11	–	–	0	–	–	–
	4	8	0	0	0	–	–	–		2	12	–	0	0	–	–	–
	5	5	–	–	0	4	5	5		3	3	–	–	–	3	3	3
	5	6	0	0	0	5	5	5		3	4	–	–	–	3	3	–
	5	7	0	0	1	5	–	–		3	5	–	–	–	3	–	–
	6	6	0	0	1	5	6	6		3	9	–	–	0	–	–	–
13	2	2	–	–	–	2	2	–		3	10	–	0	0	–	–	–
	2	3	–	–	–	2	–	–		3	11	0	0	0	–	–	–
	2	10	–	–	0	–	–	–		4	4	–	–	–	3	4	4
	2	11	–	0	0	–	–	–		4	5	–	–	–	4	4	4
	3	3	–	–	–	3	3	3		4	6	–	–	–	4	4	–
	3	4	–	–	–	3	3	–		4	7	–	–	0	4	–	–
	3	5	–	–	–	3	–	–		4	8	–	0	0	–	–	–
	3	8	–	–	0	–	–	–		4	9	0	0	0	–	–	–
	3	9	–	0	0	–	–	–		4	10	0	0	1	–	–	–
	3	10	0	0	0	–	–	–		5	5	–	–	–	4	4	5
	4	4	–	–	–	4	4	4		5	6	–	–	0	5	5	5
	4	5	–	–	–	4	4	4		5	7	–	0	0	5	5	–
	4	6	–	–	0	4	4	–		5	8	0	0	0	5	–	–
	4	7	–	0	0	4	–	–		5	9	0	1	1	–	–	–
	4	8	0	0	0	–	–	–		6	6	0	0	0	5	5	6
	4	9	0	0	0	–	–	–		6	7	0	0	0	6	6	6
	5	5	–	–	0	4	5	5		6	8	0	1	1	6	6	6
	5	6	–	0	0	5	5	5		7	7	0	1	1	6	6	7
	5	7	0	0	0	5	5	–	15	2	2	–	–	–	2	2	2

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01	M	N1	N2	.01	.025	.05	.05	.025	.01
15	2	3	-	-	-	2	-	-	15	7	8	1	1	1	6	7	7
	2	12	-	-	0	-	-	-	16	2	2	-	-	-	2	2	2
	2	13	0	0	0	-	-	-		2	3	-	-	-	2	2	-
	3	3	-	-	-	3	3	3		2	4	-	-	-	2	-	-
	3	4	-	-	-	3	3	3		2	12	-	-	0	-	-	-
	3	5	-	-	-	3	3	-		2	13	-	0	0	-	-	-
	3	6	-	-	-	3	-	-		2	14	0	0	0	-	-	-
	3	9	-	-	0	-	-	-		3	3	-	-	-	3	3	3
	3	10	-	0	0	-	-	-		3	4	-	-	-	3	3	3
	3	11	0	0	0	-	-	-		3	5	-	-	-	3	3	-
	3	12	0	0	0	-	-	-		3	6	-	-	-	3	-	-
	4	4	-	-	-	3	4	4		3	10	-	-	0	-	-	-
	4	5	-	-	-	4	4	4		3	11	-	0	0	-	-	-
	4	6	-	-	-	4	4	-		3	12	0	0	0	-	-	-
	4	7	-	-	-	4	-	-		3	13	0	0	0	-	-	-
	4	8	-	-	0	-	-	-		4	4	-	-	-	3	4	4
	4	9	-	0	0	-	-	-		4	5	-	-	-	4	4	4
	4	10	0	0	0	-	-	-		4	6	-	-	-	4	4	4
	4	11	0	0	1	-	-	-		4	7	-	-	-	4	4	-
	5	5	-	-	-	4	4	5		4	8	-	-	0	4	-	-
	5	6	-	-	0	4	5	5		4	9	-	0	0	-	-	-
	5	7	-	0	0	5	5	5		4	10	0	0	0	-	-	-
	5	8	0	0	0	5	5	-		4	11	0	0	0	-	-	-
	5	9	0	0	1	5	-	-		4	12	0	0	1	-	-	-
	5	10	0	1	1	-	-	-		5	5	-	-	-	4	4	5
	6	6	-	0	0	5	5	6		5	6	-	-	-	4	5	5
	6	7	0	0	0	5	6	6		5	7	-	-	0	5	5	5
	6	8	0	0	1	6	6	6		5	8	-	0	0	5	5	-
	6	9	0	1	1	6	6	-		5	9	0	0	0	5	-	-
	7	7	0	0	1	6	6	6		5	10	0	0	1	-	-	-

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01	M	N1	N2	.01	.025	.05	.05	.025	.01
16	5	11	0	1	1	-	-	-	17	4	10	-	0	0	-	-	-
	6	6	-	-	0	5	5	5		4	11	0	0	0	-	-	-
	6	7	-	0	0	5	5	6		4	12	0	0	0	-	-	-
	6	8	0	0	0	6	6	6		4	13	0	1	1	-	-	-
	6	9	0	1	1	6	6	-		5	5	-	-	-	4	4	4
	6	10	1	1	1	6	-	-		5	6	-	-	-	4	5	5
	7	7	0	0	0	6	6	6		5	7	-	-	0	5	5	5
	7	8	0	1	1	6	6	7		5	8	-	0	0	5	5	5
	7	9	1	1	1	7	7	7		5	9	0	0	0	5	5	-
	8	8	1	1	1	7	7	7		5	10	0	0	0	5	-	-
17	2	2	-	-	-	2	2	2		5	11	0	0	1	-	-	-
	2	3	-	-	-	2	2	-		5	12	1	1	1	-	-	-
	2	4	-	-	-	2	-	-		6	6	-	-	0	5	5	5
	2	13	-	-	0	-	-	-		6	7	-	0	0	5	5	6
	2	14	-	0	0	-	-	-		6	8	0	0	0	5	6	6
	2	15	0	0	0	-	-	-		6	9	0	0	1	6	6	6
	3	3	-	-	-	3	3	3		6	10	0	1	1	6	6	-
	3	4	-	-	-	3	3	3		6	11	1	1	1	6	-	-
	3	5	-	-	-	3	3	-		7	7	0	0	0	6	6	6
	3	6	-	-	-	3	-	-		7	8	0	0	1	6	6	7
	3	11	-	-	0	-	-	-		7	9	0	1	1	6	7	7
	3	12	-	0	0	-	-	-		7	10	1	1	1	7	7	7
	3	13	0	0	0	-	-	-		8	8	0	1	1	6	7	7
	3	14	0	0	0	-	-	-		8	9	1	1	2	7	7	8
	4	4	-	-	-	3	3	4	18	2	2	-	-	-	2	2	2
	4	5	-	-	-	4	4	4		2	3	-	-	-	2	2	-
	4	6	-	-	-	4	4	4		2	4	-	-	-	2	-	-
	4	7	-	-	-	4	4	-		2	14	-	-	0	-	-	-
	4	8	-	-	-	4	-	-		2	15	-	0	0	-	-	-
	4	9	-	-	0	-	-	-		2	16	0	0	0	-	-	-

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01	M	N1	N2	.01	.025	.05	.05	.025	.01
18	3	3	-	-	-	3	3	3	18	6	6	-	-	0	5	5	5
	3	4	-	-	-	3	3	3		6	7	-	0	0	5	5	6
	3	5	-	-	-	3	3	-		6	8	-	0	0	5	6	6
	3	6	-	-	-	3	3	-		6	9	0	0	0	6	6	6
	3	7	-	-	-	3	-	-		6	10	0	0	1	6	6	-
	3	11	-	-	0	-	-	-		6	11	0	1	1	6	6	-
	3	12	-	0	0	-	-	-		6	12	1	1	1	6	-	-
	3	13	-	0	0	-	-	-		7	7	-	0	0	5	6	6
	3	14	0	0	0	-	-	-		7	8	0	0	0	6	6	6
	3	15	0	0	0	-	-	-		7	9	0	1	1	6	6	7
	4	4	-	-	-	3	3	4		7	10	1	1	1	7	7	7
	4	5	-	-	-	3	4	4		7	11	1	1	2	7	7	-
	4	6	-	-	-	4	4	4		8	8	0	1	1	6	7	7
	4	7	-	-	-	4	4	-		8	9	1	1	1	7	7	7
	4	8	-	-	-	4	4	-		8	10	1	1	2	7	7	8
	4	9	-	-	0	4	-	-		9	9	1	1	2	7	8	8
	4	10	-	0	0	-	-	-	19	2	2	-	-	-	2	2	2
	4	11	-	0	0	-	-	-		2	3	-	-	-	2	2	-
	4	12	0	0	0	-	-	-		2	4	-	-	-	2	-	-
	4	13	0	0	1	-	-	-		2	15	-	-	0	-	-	-
	4	14	0	1	1	-	-	-		2	16	-	0	0	-	-	-
	5	5	-	-	-	4	4	4		2	17	0	0	0	-	-	-
	5	6	-	-	-	4	4	5		3	3	-	-	-	3	3	3
	5	7	-	-	-	4	5	5		3	4	-	-	-	3	3	3
	5	8	-	-	0	5	5	5		3	5	-	-	-	3	3	-
	5	9	-	0	0	5	5	-		3	6	-	-	-	3	3	-
	5	10	0	0	0	5	-	-		3	7	-	-	-	3	-	-
	5	11	0	0	1	-	-	-		3	12	-	-	0	-	-	-
	5	12	0	1	1	-	-	-		3	13	-	0	0	-	-	-
	5	13	1	1	1	-	-	-		3	14	-	0	0	-	-	-

TABLE 7--<u>CONTINUED</u>

M	N1	N2	.01	.025	.05	.05	.025	.01	M	N1	N2	.01	.025	.05	.05	.025	.01
19	3	15	0	0	0	-	-	-	19	6	12	1	1	1	6	-	-
	3	16	0	0	0	-	-	-		6	13	1	1	2	-	-	-
	4	4	-	-	-	3	3	4		7	7	-	0	0	5	6	6
	4	5	-	-	-	3	4	4		7	8	0	0	0	6	6	6
	4	6	-	-	-	4	4	4		7	9	0	0	1	6	6	7
	4	7	-	-	-	4	4	4		7	10	0	1	1	6	7	7
	4	8	-	-	-	4	4	-		7	11	1	1	1	7	7	7
	4	9	-	-	-	4	-	-		7	12	1	1	2	7	7	-
	4	10	-	-	0	-	-	-		8	8	0	0	1	6	6	7
	4	11	-	0	0	-	-	-		8	9	0	1	1	7	7	7
	4	12	0	0	0	-	-	-		8	10	1	1	1	7	7	8
	4	13	0	0	0	-	-	-		8	11	1	2	2	7	8	8
	4	14	0	0	1	-	-	-		9	9	1	1	1	7	7	8
	4	15	0	1	1	-	-	-		9	10	1	2	2	8	8	8
	5	5	-	-	-	4	4	4	20	1	1	-	-	-	1	-	-
	5	6	-	-	-	4	4	5		1	19	-	-	0	-	-	-
	5	7	-	-	-	4	5	5		2	2	-	-	-	2	2	2
	5	8	-	-	0	5	5	5		2	3	-	-	-	2	2	-
	5	9	-	0	0	5	5	-		2	4	-	-	-	2	-	-
	5	10	-	0	0	5	5	-		2	16	-	-	0	-	-	-
	5	11	0	0	0	5	-	-		2	17	-	0	0	-	-	-
	5	12	0	0	1	-	-	-		2	18	0	0	0	-	-	-
	5	13	0	1	1	-	-	-		3	3	-	-	-	2	3	3
	5	14	1	1	1	-	-	-		3	4	-	-	-	3	3	3
	6	6	-	-	-	4	5	5		3	5	-	-	-	3	3	3
	6	7	-	-	0	5	5	5		3	6	-	-	-	3	3	-
	6	8	-	0	0	5	5	6		3	7	-	-	-	3	-	-
	6	9	0	0	0	5	6	6		3	8	-	-	-	3	-	-
	6	10	0	0	1	6	6	6		3	12	-	-	0	-	-	-
	6	11	0	1	1	6	6	-		3	13	-	-	0	-	-	-

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01	M	N1	N2	.01	.025	.05	.05	.025	.01
20	3	14	–	0	0	–	–	–	20	6	8	–	0	0	5	5	6
	3	15	0	0	0	–	–	–		6	9	–	0	0	5	6	6
	3	16	0	0	0	–	–	–		6	10	0	0	0	6	6	6
	3	17	0	0	1	–	–	–		6	11	0	0	1	6	6	–
	4	4	–	–	–	3	3	4		6	12	0	1	1	6	6	–
	4	5	–	–	–	3	4	4		6	13	1	1	1	6	–	–
	4	6	–	–	–	4	4	4		6	14	1	1	2	–	–	–
	4	7	–	–	–	4	4	4		7	7	–	0	0	5	5	6
	4	8	–	–	–	4	4	–		7	8	–	0	0	6	6	6
	4	9	–	–	–	4	–	–		7	9	0	0	0	6	6	7
	4	10	–	–	0	4	–	–		7	10	0	0	1	6	7	7
	4	11	–	–	0	–	–	–		7	11	0	1	1	7	7	7
	4	12	–	0	0	–	–	–		7	12	1	1	1	7	7	–
	4	13	0	0	0	–	–	–		7	13	1	2	2	7	7	–
	4	14	0	0	0	–	–	–		8	8	0	0	0	6	6	7
	4	15	0	0	1	–	–	–		8	9	0	1	1	6	7	7
	4	16	0	1	1	–	–	–		8	10	1	1	1	7	7	7
	5	5	–	–	–	4	4	4		8	11	1	1	2	7	7	8
	5	6	–	–	–	4	4	5		8	12	1	2	2	8	8	8
	5	7	–	–	–	4	5	5		9	9	1	1	1	7	7	8
	5	8	–	–	–	5	5	5		9	10	1	1	2	7	8	8
	5	9	–	–	0	5	5	5		9	11	1	2	2	8	8	8
	5	10	–	0	0	5	5	–		10	10	1	2	2	8	8	9
	5	11	0	0	0	5	–	–	21	1	1	–	–	–	1	–	–
	5	12	0	0	0	–	–	–		1	20	–	–	0	–	–	–
	5	13	0	0	1	–	–	–		2	2	–	–	–	2	2	2
	5	14	0	1	1	–	–	–		2	3	–	–	–	2	2	–
	5	15	1	1	1	–	–	–		2	4	–	–	–	2	–	–
	6	6	–	–	–	4	5	5		2	5	–	–	–	2	–	–
	6	7	–	–	0	5	5	5		2	16	–	–	0	–	–	–

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01	M	N1	N2	.01	.025	.05	.05	.025	.01
21	2	17	-	-	0	-	-	-	21	5	6	-	-	-	4	4	5
	2	18	-	0	0	-	-	-		5	7	-	-	-	4	5	5
	2	19	0	0	0	-	-	-		5	8	-	-	-	4	5	5
	3	3	-	-	-	2	3	3		5	9	-	-	0	5	5	5
	3	4	-	-	-	3	3	3		5	10	-	0	0	5	5	-
	3	5	-	-	-	3	3	3		5	11	-	0	0	5	5	-
	3	6	-	-	-	3	3	-		5	12	0	0	0	5	-	-
	3	7	-	-	-	3	-	-		5	13	0	0	1	-	-	-
	3	8	-	-	-	3	-	-		5	14	0	0	1	-	-	-
	3	13	-	-	0	-	-	-		5	15	0	1	1	-	-	-
	3	14	-	-	0	-	-	-		5	16	1	1	1	-	-	-
	3	15	-	0	0	-	-	-		6	6	-	-	-	4	5	5
	3	16	0	0	0	-	-	-		6	7	-	-	-	5	5	5
	3	17	0	0	0	-	-	-		6	8	-	-	0	5	5	6
	3	18	0	0	1	-	-	-		6	9	-	0	0	5	6	6
	4	4	-	-	-	3	3	4		6	10	0	0	0	6	6	6
	4	5	-	-	-	3	4	4		6	11	0	0	0	6	6	6
	4	6	-	-	-	4	4	4		6	12	0	0	1	6	6	-
	4	7	-	-	-	4	4	4		6	13	0	1	1	6	-	-
	4	8	-	-	-	4	4	-		6	14	1	1	1	-	-	-
	4	9	-	-	-	4	4	-		6	15	1	1	2	-	-	-
	4	10	-	-	-	4	-	-		7	7	-	-	0	5	5	6
	4	11	-	-	0	-	-	-		7	8	-	0	0	5	6	6
	4	12	-	0	0	-	-	-		7	9	0	0	0	6	6	6
	4	13	-	0	0	-	-	-		7	10	0	0	1	6	6	7
	4	14	0	0	0	-	-	-		7	11	0	1	1	6	7	7
	4	15	0	0	0	-	-	-		7	12	1	1	1	7	7	7
	4	16	0	0	1	-	-	-		7	13	1	1	2	7	7	-
	4	17	0	1	1	-	-	-		7	14	1	2	2	7	-	-
	5	5	-	-	-	4	4	4		8	8	0	0	0	6	6	7

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01	M	N1	N2	.01	.025	.05	.05	.025	.01
21	8	9	0	0	1	6	7	7	22	3	17	0	0	0	-	-	-
	8	10	0	1	1	7	7	7		3	18	0	0	0	-	-	-
	8	11	1	1	1	7	7	8		3	19	0	0	1	-	-	-
	8	12	1	1	2	7	8	8		4	4	-	-	-	3	3	3
	8	13	1	2	2	8	8	8		4	5	-	-	-	3	3	4
	9	9	0	1	1	7	7	7		4	6	-	-	-	3	4	4
	9	10	1	1	1	7	7	8		4	7	-	-	-	4	4	4
	9	11	1	2	2	8	8	8		4	8	-	-	-	4	4	4
	9	12	2	2	2	8	8	9		4	9	-	-	-	4	4	-
	10	10	1	2	2	8	8	8		4	10	-	-	-	4	-	-
	10	11	2	2	2	8	8	9		4	11	-	-	0	4	-	-
22	1	1	-	-	-	1	-	-		4	12	-	-	0	-	-	-
	1	21	-	-	0	-	-	-		4	13	-	0	0	-	-	-
	2	2	-	-	-	2	2	2		4	14	0	0	0	-	-	-
	2	3	-	-	-	2	2	-		4	15	0	0	0	-	-	-
	2	4	-	-	-	2	-	-		4	16	0	0	1	-	-	-
	2	5	-	-	-	2	-	-		4	17	0	1	1	-	-	-
	2	17	-	-	0	-	-	-		4	18	1	1	1	-	-	-
	2	18	-	-	0	-	-	-		5	5	-	-	-	4	4	4
	2	19	-	0	0	-	-	-		5	6	-	-	-	4	4	4
	2	20	0	0	0	-	-	-		5	7	-	-	-	4	4	5
	3	3	-	-	-	2	3	3		5	8	-	-	-	4	5	5
	3	4	-	-	-	3	3	3		5	9	-	-	0	5	5	5
	3	5	-	-	-	3	3	3		5	10	-	-	0	5	5	5
	3	6	-	-	-	3	3	-		5	11	-	0	0	5	5	-
	3	7	-	-	-	3	3	-		5	12	0	0	0	5	-	-
	3	8	-	-	-	3	-	-		5	13	0	0	0	5	-	-
	3	14	-	-	0	-	-	-		5	14	0	0	1	-	-	-
	3	15	-	0	0	-	-	-		5	15	0	1	1	-	-	-
	3	16	-	0	0	-	-	-		5	16	1	1	1	-	-	-

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01
22	5	17	1	1	1	-	-	-
	6	6	-	-	-	4	5	5
	6	7	-	-	-	5	5	5
	6	8	-	-	0	5	5	6
	6	9	-	0	0	5	5	6
	6	10	-	0	0	5	6	6
	6	11	0	0	0	6	6	6
	6	12	0	0	1	6	6	-
	6	13	0	1	1	6	6	-
	6	14	0	1	1	6	-	-
	6	15	1	1	1	-	-	-
	6	16	1	1	2	-	-	-
	7	7	-	-	0	5	5	6
	7	8	-	0	0	5	6	6
	7	9	-	0	0	6	6	6
	7	10	0	0	0	6	6	7
	7	11	0	0	1	6	7	7
	7	12	0	1	1	7	7	7
	7	13	1	1	1	7	7	-
	7	14	1	1	2	7	7	-
	7	15	1	2	2	7	-	-
	8	8	0	0	0	6	6	6
	8	9	0	0	0	6	6	7
	8	10	0	0	1	6	7	7
	8	11	0	1	1	7	7	8
	8	12	1	1	2	7	8	8
	8	13	1	2	2	8	8	8
	8	14	2	2	2	8	8	8
	9	9	0	1	1	7	7	7
	9	10	0	1	1	7	7	8
22	9	11	1	1	2	7	8	8
	9	12	1	2	2	8	8	9
	9	13	2	2	2	8	8	9
	10	10	1	1	2	7	8	8
	10	11	1	2	2	8	8	9
	10	12	2	2	3	8	9	9
	11	11	2	2	3	8	9	9
23	1	1	-	-	-	1	-	-
	1	22	-	-	0	-	-	-
	2	2	-	-	-	2	2	2
	2	3	-	-	-	2	2	-
	2	4	-	-	-	2	2	-
	2	5	-	-	-	2	-	-
	2	18	-	-	0	-	-	-
	2	19	-	0	0	-	-	-
	2	20	-	0	0	-	-	-
	2	21	0	0	0	-	-	-
	3	3	-	-	-	2	3	3
	3	4	-	-	-	3	3	3
	3	5	-	-	-	3	3	3
	3	6	-	-	-	3	3	-
	3	7	-	-	-	3	3	-
	3	8	-	-	-	3	-	-
	3	9	-	-	-	3	-	-
	3	14	-	-	0	-	-	-
	3	15	-	-	0	-	-	-
	3	16	-	0	0	-	-	-
	3	17	-	0	0	-	-	-
	3	18	0	0	0	-	-	-
	3	19	0	0	0	-	-	-

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01
23	3	20	0	0	1	-	-	-
	4	4	-	-	-	3	3	3
	4	5	-	-	-	3	3	4
	4	6	-	-	-	3	4	4
	4	7	-	-	-	4	4	4
	4	8	-	-	-	4	4	4
	4	9	-	-	-	4	4	-
	4	10	-	-	-	4	4	-
	4	11	-	-	-	4	-	-
	4	12	-	-	0	-	-	-
	4	13	-	0	0	-	-	-
	4	14	-	0	0	-	-	-
	4	15	0	0	0	-	-	-
	4	16	0	0	0	-	-	-
	4	17	0	0	1	-	-	-
	4	18	0	1	1	-	-	-
	4	19	1	1	1	-	-	-
	5	5	-	-	-	3	4	4
	5	6	-	-	-	4	4	4
	5	7	-	-	-	4	4	5
	5	8	-	-	-	4	5	5
	5	9	-	-	-	5	5	5
	5	10	-	-	0	5	5	5
	5	11	-	0	0	5	5	-
	5	12	-	0	0	5	5	-
	5	13	0	0	0	5	-	-
	5	14	0	0	0	-	-	-
	5	15	0	0	1	-	-	-
	5	16	0	1	1	-	-	-
	5	17	1	1	1	-	-	-
23	5	18	1	1	2	-	-	-
	6	6	-	-	-	4	4	5
	6	7	-	-	-	4	5	5
	6	8	-	-	0	5	5	5
	6	9	-	-	0	5	5	6
	6	10	-	0	0	5	6	6
	6	11	0	0	0	6	6	6
	6	12	0	0	0	6	6	6
	6	13	0	0	1	6	6	-
	6	14	0	1	1	6	-	-
	6	15	1	1	1	6	-	-
	6	16	1	1	2	-	-	-
	6	17	1	2	2	-	-	-
	7	7	-	-	0	5	5	6
	7	8	-	-	0	5	6	6
	7	9	-	0	0	6	6	6
	7	10	0	0	0	6	6	7
	7	11	0	0	1	6	6	7
	7	12	0	1	1	6	7	7
	7	13	0	1	1	7	7	7
	7	14	1	1	1	7	7	-
	7	15	1	1	2	7	-	-
	7	16	1	2	2	7	-	-
	8	8	-	0	0	6	6	6
	8	9	0	0	0	6	6	7
	8	10	0	0	1	6	7	7
	8	11	0	1	1	7	7	7
	8	12	1	1	1	7	7	8
	8	13	1	1	2	7	8	8
	8	14	1	2	2	8	8	8

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01
23	8	15	2	2	2	8	8	-
	9	9	0	0	1	6	7	7
	9	10	0	1	1	7	7	8
	9	11	1	1	1	7	8	8
	9	12	1	1	2	8	8	8
	9	13	1	2	2	8	8	9
	9	14	2	2	3	8	9	9
	10	10	1	1	1	7	8	8
	10	11	1	1	2	8	8	9
	10	12	1	2	2	8	9	9
	10	13	2	2	3	9	9	9
	11	11	2	2	2	8	9	9
	11	12	2	2	3	9	9	9
24	1	1	-	-	-	1	-	-
	1	23	-	-	0	-	-	-
	2	2	-	-	-	2	2	2
	2	3	-	-	-	2	2	-
	2	4	-	-	-	2	2	-
	2	5	-	-	-	2	-	-
	2	19	-	-	0	-	-	-
	2	20	-	0	0	-	-	-
	2	21	-	0	0	-	-	-
	2	22	0	0	0	-	-	-
	3	3	-	-	-	2	3	3
	3	4	-	-	-	3	3	3
	3	5	-	-	-	3	3	3
	3	6	-	-	-	3	3	3
	3	7	-	-	-	3	3	-
	3	8	-	-	-	3	-	-
	3	9	-	-	-	3	-	-
24	3	15	-	-	0	-	-	-
	3	16	-	-	0	-	-	-
	3	17	-	0	0	-	-	-
	3	18	0	0	0	-	-	-
	3	19	0	0	0	-	-	-
	3	20	0	0	0	-	-	-
	3	21	0	0	1	-	-	-
	4	4	-	-	-	3	3	3
	4	5	-	-	-	3	3	4
	4	6	-	-	-	3	4	4
	4	7	-	-	-	4	4	4
	4	8	-	-	-	4	4	4
	4	9	-	-	-	4	4	-
	4	10	-	-	-	4	4	-
	4	11	-	-	-	4	-	-
	4	12	-	-	0	4	-	-
	4	13	-	-	0	-	-	-
	4	14	-	0	0	-	-	-
	4	15	-	0	0	-	-	-
	4	16	0	0	0	-	-	-
	4	17	0	0	0	-	-	-
	4	18	0	0	1	-	-	-
	4	19	0	1	1	-	-	-
	4	20	1	1	1	-	-	-
	5	5	-	-	-	3	4	4
	5	6	-	-	-	4	4	4
	5	7	-	-	-	4	4	5
	5	8	-	-	-	4	5	5
	5	9	-	-	-	4	5	5
	5	10	-	-	0	5	5	5

TABLE 7--<u>CONTINUED</u>

M	N1	N2	.01	.025	.05	.05	.025	.01
24	5	11	–	–	0	5	5	–
	5	12	–	0	0	5	5	–
	5	13	–	0	0	5	–	–
	5	14	0	0	0	5	–	–
	5	15	0	0	1	–	–	–
	5	16	0	0	1	–	–	–
	5	17	0	1	1	–	–	–
	5	18	1	1	1	–	–	–
	5	19	1	1	2	–	–	–
	6	6	–	–	–	4	4	5
	6	7	–	–	–	4	5	5
	6	8	–	–	–	5	5	5
	6	9	–	–	0	5	5	6
	6	10	–	0	0	5	6	6
	6	11	–	0	0	5	6	6
	6	12	0	0	0	6	6	6
	6	13	0	0	1	6	6	–
	6	14	0	0	1	6	6	–
	6	15	0	1	1	6	–	–
	6	16	1	1	1	–	–	–
	6	17	1	1	2	–	–	–
	6	18	1	2	2	–	–	–
	7	7	–	–	–	5	5	5
	7	8	–	–	0	5	5	6
	7	9	–	0	0	5	6	6
	7	10	0	0	0	6	6	6
	7	11	0	0	0	6	6	7
	7	12	0	0	1	6	7	7
	7	13	0	1	1	7	7	7
	7	14	1	1	1	7	7	7
24	7	15	1	1	2	7	7	–
	7	16	1	2	2	7	–	–
	7	17	2	2	2	–	–	–
	8	8	–	0	0	5	6	6
	8	9	0	0	0	6	6	7
	8	10	0	0	0	6	7	7
	8	11	0	0	1	7	7	7
	8	12	0	1	1	7	7	8
	8	13	1	1	1	7	8	8
	8	14	1	1	2	8	8	8
	8	15	1	2	2	8	8	8
	8	16	2	2	3	8	8	–
	9	9	0	0	1	6	7	7
	9	10	0	1	1	7	7	7
	9	11	0	1	1	7	7	8
	9	12	1	1	2	7	8	8
	9	13	1	2	2	8	8	9
	9	14	2	2	2	8	8	9
	9	15	2	2	3	8	9	9
	10	10	0	1	1	7	7	8
	10	11	1	1	2	8	8	8
	10	12	1	2	2	8	8	9
	10	13	2	2	2	8	9	9
	10	14	2	3	3	9	9	10
	11	11	1	2	2	8	8	9
	11	12	2	2	3	8	9	9
	11	13	2	3	3	9	9	10
	12	12	2	3	3	9	9	10
25	1	1	–	–	–	1	–	–
	1	24	–	–	0	–	–	–

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01
25	2	2	–	–	–	2	2	2
	2	3	–	–	–	2	2	2
	2	4	–	–	–	2	2	–
	2	5	–	–	–	2	–	–
	2	6	–	–	–	2	–	–
	2	19	–	–	0	–	–	–
	2	20	–	–	0	–	–	–
	2	21	–	0	0	–	–	–
	2	22	0	0	0	–	–	–
	2	23	0	0	0	–	–	–
	3	3	–	–	–	2	3	3
	3	4	–	–	–	3	3	3
	3	5	–	–	–	3	3	3
	3	6	–	–	–	3	3	3
	3	7	–	–	–	3	3	–
	3	8	–	–	–	3	3	–
	3	9	–	–	–	3	–	–
	3	16	–	–	0	–	–	–
	3	17	–	0	0	–	–	–
	3	18	–	0	0	–	–	–
	3	19	0	0	0	–	–	–
	3	20	0	0	0	–	–	–
	3	21	0	0	0	–	–	–
	3	22	0	0	1	–	–	–
	4	4	–	–	–	3	3	3
	4	5	–	–	–	3	3	4
	4	6	–	–	–	3	4	4
	4	7	–	–	–	4	4	4
	4	8	–	–	–	4	4	4
	4	9	–	–	–	4	4	4

M	N1	N2	.01	.025	.05	.05	.025	.01
25	4	10	–	–	–	4	4	–
	4	11	–	–	–	4	–	–
	4	12	–	–	–	4	–	–
	4	13	–	–	0	–	–	–
	4	14	–	–	0	–	–	–
	4	15	–	0	0	–	–	–
	4	16	0	0	0	–	–	–
	4	17	0	0	0	–	–	–
	4	18	0	0	0	–	–	–
	4	19	0	0	1	–	–	–
	4	20	0	1	1	–	–	–
	4	21	1	1	1	–	–	–
	5	5	–	–	–	3	4	4
	5	6	–	–	–	4	4	4
	5	7	–	–	–	4	4	5
	5	8	–	–	–	4	4	5
	5	9	–	–	–	4	5	5
	5	10	–	–	–	5	5	5
	5	11	–	–	0	5	5	5
	5	12	–	0	0	5	5	–
	5	13	–	0	0	5	5	–
	5	14	0	0	0	5	–	–
	5	15	0	0	0	–	–	–
	5	16	0	0	1	–	–	–
	5	17	0	1	1	–	–	–
	5	18	0	1	1	–	–	–
	5	19	1	1	1	–	–	–
	5	20	1	1	2	–	–	–
	6	6	–	–	–	4	4	5
	6	7	–	–	–	4	5	5

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01
25	6	8	–	–	–	5	5	5
	6	9	–	–	0	5	5	6
	6	10	–	–	0	5	5	6
	6	11	–	0	0	5	6	6
	6	12	0	0	0	6	6	6
	6	13	0	0	0	6	6	6
	6	14	0	0	1	6	6	–
	6	15	0	1	1	6	–	–
	6	16	0	1	1	6	–	–
	6	17	1	1	1	–	–	–
	6	18	1	1	2	–	–	–
	6	19	1	2	2	–	–	–
	7	7	–	–	–	5	5	5
	7	8	–	–	0	5	5	6
	7	9	–	0	0	5	6	6
	7	10	–	0	0	6	6	6
	7	11	0	0	0	6	6	7
	7	12	0	0	1	6	7	7
	7	13	0	0	1	6	7	7
	7	14	0	1	1	7	7	7
	7	15	1	1	1	7	7	–
	7	16	1	1	2	7	7	–
	7	17	1	2	2	7	–	–
	7	18	2	2	2	–	–	–
	8	8	–	0	0	5	6	6
	8	9	–	0	0	6	6	6
	8	10	0	0	0	6	6	7
	8	11	0	0	1	6	7	7
	8	12	0	1	1	7	7	7
	8	13	1	1	1	7	7	8
25	8	14	1	1	2	7	8	8
	8	15	1	2	2	8	8	8
	8	16	2	2	2	8	8	–
	8	17	2	2	3	8	8	–
	9	9	0	0	0	6	7	7
	9	10	0	0	1	7	7	7
	9	11	0	1	1	7	7	8
	9	12	1	1	1	7	8	8
	9	13	1	1	2	8	8	8
	9	14	1	2	2	8	8	9
	9	15	2	2	2	8	9	9
	9	16	2	2	3	9	9	9
	10	10	0	1	1	7	7	8
	10	11	1	1	1	7	8	8
	10	12	1	1	2	8	8	9
	0	13	1	2	2	8	9	9
	10	14	2	2	3	9	9	9
	10	15	2	3	3	9	9	10
	11	11	1	1	2	8	8	9
	11	12	1	2	2	8	9	9
	11	13	2	2	3	9	9	10
	11	14	2	3	3	9	10	10
	12	12	2	2	3	9	9	10
	12	13	2	3	3	9	10	10
26	1	1	–	–	–	1	–	–
	1	25	–	–	0	–	–	–
	2	2	–	–	–	2	2	2
	2	3	–	–	–	2	2	2
	2	4	–	–	–	2	2	–
	2	5	–	–	–	2	–	–

TABLE 7--<u>CONTINUED</u>

M	N1	N2	.01	.025	.05	.05	.025	.01
26	2	6	-	-	-	2	-	-
	2	20	-	-	0	-	-	-
	2	21	-	-	0	-	-	-
	2	22	-	0	0	-	-	-
	2	23	0	0	0	-	-	-
	2	24	0	0	0	-	-	-
	3	3	-	-	-	2	3	3
	3	4	-	-	-	3	3	3
	3	5	-	-	-	3	3	3
	3	6	-	-	-	3	3	3
	3	7	-	-	-	3	3	-
	3	8	-	-	-	3	3	-
	3	9	-	-	-	3	-	-
	3	10	-	-	-	3	-	-
	3	16	-	-	0	-	-	-
	3	17	-	-	0	-	-	-
	3	18	-	0	0	-	-	-
	3	19	-	0	0	-	-	-
	3	20	0	0	0	-	-	-
	3	21	0	0	0	-	-	-
	3	22	0	0	0	-	-	-
	3	23	0	0	1	-	-	-
	4	4	-	-	-	3	3	3
	4	5	-	-	-	3	3	4
	4	6	-	-	-	3	4	4
	4	7	-	-	-	3	4	4
	4	8	-	-	-	4	4	4
	4	9	-	-	-	4	4	4
	4	10	-	-	-	4	4	-
	4	11	-	-	-	4	4	-

M	N1	N2	.01	.025	.05	.05	.025	.01
26	4	12	-	-	-	4	-	-
	4	13	-	-	0	4	-	-
	4	14	-	-	0	-	-	-
	4	15	-	0	0	-	-	-
	4	16	-	0	0	-	-	-
	4	17	0	0	0	-	-	-
	4	18	0	0	0	-	-	-
	4	19	0	0	1	-	-	-
	4	20	0	0	1	-	-	-
	4	21	0	1	1	-	-	-
	4	22	1	1	1	-	-	-
	5	5	-	-	-	3	4	4
	5	6	-	-	-	4	4	4
	5	7	-	-	-	4	4	5
	5	8	-	-	-	4	4	5
	5	9	-	-	-	4	5	5
	5	10	-	-	-	5	5	5
	5	11	-	-	0	5	5	5
	5	12	-	-	0	5	5	-
	5	13	-	0	0	5	5	-
	5	14	-	0	0	5	-	-
	5	15	0	0	0	5	-	-
	5	16	0	0	0	-	-	-
	5	17	0	0	1	-	-	-
	5	18	0	1	1	-	-	-
	5	19	0	1	1	-	-	-
	5	20	1	1	1	-	-	-
	5	21	1	1	2	-	-	-
	6	6	-	-	-	4	4	5
	6	7	-	-	-	4	5	5

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01	M	N1	N2	.01	.025	.05	.05	.025	.01
26	6	8	-	-	-	5	5	5	26	8	12	0	0	1	7	7	7
	6	9	-	-	-	5	5	5		8	13	0	1	1	7	7	8
	6	10	-	-	0	5	5	6		8	14	1	1	1	7	8	8
	6	11	-	0	0	5	6	6		8	15	1	1	2	7	8	8
	6	12	-	0	0	6	6	6		8	16	1	2	2	8	8	8
	6	13	0	0	0	6	6	6		8	17	2	2	2	8	8	-
	6	14	0	0	0	6	6	-		8	18	2	2	3	8	-	-
	6	15	0	0	1	6	6	-		9	9	0	0	0	6	6	7
	6	16	0	1	1	6	-	-		9	10	0	0	1	6	7	7
	6	17	1	1	1	-	-	-		9	11	0	1	1	7	7	8
	6	18	1	1	1	-	-	-		9	12	0	1	1	7	8	8
	6	19	1	1	2	-	-	-		9	13	1	1	2	7	8	8
	6	20	1	2	2	-	-	-		9	14	1	1	2	8	8	9
	7	7	-	-	-	5	5	5		9	15	1	2	2	8	8	9
	7	8	-	-	0	5	5	6		9	16	2	2	3	8	9	9
	7	9	-	-	0	5	6	6		9	17	2	3	3	9	9	9
	7	10	-	0	0	6	6	6		10	10	0	1	1	7	7	8
	7	11	0	0	0	6	6	7		10	11	0	1	1	7	8	8
	7	12	0	0	0	6	6	7		10	12	1	1	2	8	8	8
	7	13	0	0	1	6	7	7		10	13	1	2	2	8	8	9
	7	14	0	1	1	7	7	7		0	14	2	2	2	8	9	9
	7	15	0	1	1	7	7	7		0	15	2	2	3	9	9	10
	7	16	1	1	1	7	7	-		10	16	2	3	3	9	9	10
	7	17	1	1	2	7	-	-		11	11	1	1	2	8	8	9
	7	18	1	2	2	7	-	-		11	12	1	2	2	8	9	9
	7	19	2	2	2	-	-	-		11	13	2	2	2	9	9	9
	8	8	-	-	0	5	6	6		1	14	2	2	3	9	9	10
	8	9	-	0	0	6	6	6		11	15	2	3	3	9	10	10
	8	10	0	0	0	6	6	7		2	12	2	2	2	9	9	9
	8	11	0	0	1	6	7	7		12	13	2	3	3	9	9	10

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01	M	N1	N2	.01	.025	.05	.05	.025	.01
26	12	14	3	3	3	10	10	10	27	4	4	-	-	-	3	3	3
	13	13	3	3	3	10	10	10		4	5	-	-	-	3	3	4
27	1	1	-	-	-	1	-	-		4	6	-	-	-	3	3	4
	1	26	-	-	0	-	-	-		4	7	-	-	-	3	4	4
	2	2	-	-	-	2	2	2		4	8	-	-	-	4	4	4
	2	3	-	-	-	2	2	2		4	9	-	-	-	4	4	4
	2	4	-	-	-	2	2	-		4	10	-	-	-	4	4	-
	2	5	-	-	-	2	-	-		4	11	-	-	-	4	4	-
	2	6	-	-	-	2	-	-		4	12	-	-	-	4	-	-
	2	21	-	-	0	-	-	-		4	13	-	-	-	4	-	-
	2	22	-	-	0	-	-	-		4	14	-	-	0	-	-	-
	2	23	-	0	0	-	-	-		4	15	-	-	0	-	-	-
	2	24	0	0	0	-	-	-		4	16	-	0	0	-	-	-
	2	25	0	0	0	-	-	-		4	17	-	0	0	-	-	-
	3	3	-	-	-	2	2	3		4	18	0	0	0	-	-	-
	3	4	-	-	-	2	3	3		4	19	0	0	0	-	-	-
	3	5	-	-	-	3	3	3		4	20	0	0	1	-	-	-
	3	6	-	-	-	3	3	3		4	21	0	1	1	-	-	-
	3	7	-	-	-	3	3	-		4	22	0	1	1	-	-	-
	3	8	-	-	-	3	3	-		4	23	1	1	1	-	-	-
	3	9	-	-	-	3	-	-		5	5	-	-	-	3	4	4
	3	10	-	-	-	3	-	-		5	6	-	-	-	4	4	4
	3	17	-	-	0	-	-	-		5	7	-	-	-	4	4	4
	3	18	-	-	0	-	-	-		5	8	-	-	-	4	4	5
	3	19	-	0	0	-	-	-		5	9	-	-	-	4	5	5
	3	20	-	0	0	-	-	-		5	10	-	-	-	4	5	5
	3	21	0	0	0	-	-	-		5	11	-	-	-	5	5	5
	3	22	0	0	0	-	-	-		5	12	-	-	0	5	5	5
	3	23	0	0	1	-	-	-		5	13	-	0	0	5	5	-
	3	24	0	1	1	-	-	-		5	14	-	0	0	5	5	-

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01	M	N1	N2	.01	.025	.05	.05	.025	.01
27	5	15	0	0	0	5	-	-	27	7	13	0	0	1	6	7	7
	5	16	0	0	0	-	-	-		7	14	0	0	1	6	7	7
	5	17	0	0	1	-	-	-		7	15	0	1	1	7	7	7
	5	18	0	0	1	-	-	-		7	16	1	1	1	7	7	-
	5	19	0	1	1	-	-	-		7	17	1	1	2	7	7	-
	5	20	1	1	1	-	-	-		7	18	1	2	2	7	-	-
	5	21	1	1	1	-	-	-		7	19	1	2	2	-	-	-
	5	22	1	1	2	-	-	-		7	20	2	2	3	-	-	-
	6	6	-	-	-	4	4	5		8	8	-	-	0	5	6	6
	6	7	-	-	-	4	4	5		8	9	-	0	0	6	6	6
	6	8	-	-	-	4	5	5		8	10	-	0	0	6	6	7
	6	9	-	-	-	5	5	5		8	11	0	0	0	6	7	7
	6	10	-	-	0	5	5	6		8	12	0	0	1	6	7	7
	6	11	-	-	0	5	6	6		8	13	0	1	1	7	7	8
	6	12	-	0	0	5	6	6		8	14	0	1	1	7	7	8
	6	13	-	0	0	6	6	6		8	15	1	1	2	7	8	8
	6	14	0	0	0	6	6	-		8	16	1	1	2	8	8	8
	6	15	0	0	1	6	6	-		8	17	1	2	2	8	8	-
	6	16	0	0	1	6	-	-		8	18	2	2	2	8	8	-
	6	17	0	1	1	6	-	-		8	19	2	2	3	8	-	-
	6	18	1	1	1	-	-	-		9	9	-	0	0	6	6	7
	6	19	1	1	2	-	-	-		9	10	0	0	0	6	7	7
	6	20	1	2	2	-	-	-		9	11	0	0	1	7	7	7
	6	21	1	2	2	-	-	-		9	12	0	1	1	7	7	8
	7	7	-	-	-	4	5	5		9	13	1	1	1	7	8	8
	7	8	-	-	-	5	5	6		9	14	1	1	2	8	8	8
	7	9	-	-	0	5	5	6		9	15	1	2	2	8	8	9
	7	10	-	0	0	5	6	6		9	16	2	2	2	8	9	9
	7	11	-	0	0	6	6	6		9	17	2	2	3	9	9	9
	7	12	0	0	0	6	6	7		9	18	2	3	3	9	9	-

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01	M	N1	N2	.01	.025	.05	.05	.025	.01
27	10	10	0	0	1	7	7	8	28	2	25	0	0	0	-	-	-
	10	11	0	1	1	7	7	8		2	26	0	0	0	-	-	-
	10	12	1	1	1	7	8	8		3	3	-	-	-	2	2	3
	10	13	1	1	2	8	8	9		3	4	-	-	-	2	3	3
	10	14	1	2	2	8	9	9		3	5	-	-	-	3	3	3
	10	15	2	2	3	9	9	9		3	6	-	-	-	3	3	3
	10	16	2	3	3	9	9	10		3	7	-	-	-	3	3	-
	10	17	2	3	3	9	10	10		3	8	-	-	-	3	3	-
	11	11	1	1	1	8	8	8		3	9	-	-	-	3	-	-
	11	12	1	1	2	8	8	9		3	10	-	-	-	3	-	-
	11	13	1	2	2	8	9	9		3	18	-	-	0	-	-	-
	11	14	2	2	3	9	9	10		3	19	-	-	0	-	-	-
	11	15	2	3	3	9	10	10		3	20	-	0	0	-	-	-
	11	16	3	3	3	10	10	10		3	21	-	0	0	-	-	-
	12	12	1	2	2	8	9	9		3	22	0	0	0	-	-	-
	12	13	2	2	3	9	9	10		3	23	0	0	0	-	-	-
	12	14	2	3	3	9	10	10		3	24	0	0	1	-	-	-
	12	15	3	3	4	10	10	11		3	25	0	1	1	-	-	-
	13	13	2	3	3	9	10	10		4	4	-	-	-	3	3	3
	13	14	3	3	4	10	10	11		4	5	-	-	-	3	3	4
28	1	1	-	-	-	1	-	-		4	6	-	-	-	3	3	4
	1	27	-	-	0	-	-	-		4	7	-	-	-	3	4	4
	2	2	-	-	-	2	2	2		4	8	-	-	-	4	4	4
	2	3	-	-	-	2	2	2		4	9	-	-	-	4	4	4
	2	4	-	-	-	2	2	-		4	10	-	-	-	4	4	-
	2	5	-	-	-	2	-	-		4	11	-	-	-	4	4	-
	2	6	-	-	-	2	-	-		4	12	-	-	-	4	4	-
	2	22	-	-	0	-	-	-		4	13	-	-	-	4	-	-
	2	23	-	-	0	-	-	-		4	14	-	-	0	4	-	-
	2	24	-	0	0	-	-	-		4	15	-	-	0	-	-	-

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01	M	N1	N2	.01	.025	.05	.05	.025	.01
28	4	16	-	0	0	-	-	-	28	6	8	-	-	-	4	5	5
	4	17	-	0	0	-	-	-		6	9	-	-	-	5	5	5
	4	18	-	0	0	-	-	-		6	10	-	-	0	5	5	6
	4	19	0	0	0	-	-	-		6	11	-	-	0	5	5	6
	4	20	0	0	0	-	-	-		6	12	-	0	0	5	6	6
	4	21	0	0	1	-	-	-		6	13	-	0	0	6	6	6
	4	22	0	1	1	-	-	-		6	14	0	0	0	6	6	6
	4	23	0	1	1	-	-	-		6	15	0	0	0	6	6	-
	4	24	1	1	1	-	-	-		6	16	0	0	1	6	6	-
	5	5	-	-	-	3	4	4		6	17	0	1	1	6	-	-
	5	6	-	-	-	4	4	4		6	18	0	1	1	6	-	-
	5	7	-	-	-	4	4	4		6	19	1	1	1	-	-	-
	5	8	-	-	-	4	4	5		6	20	1	1	2	-	-	-
	5	9	-	-	-	4	5	5		6	21	1	2	2	-	-	-
	5	10	-	-	-	4	5	5		6	22	2	2	2	-	-	-
	5	11	-	-	-	5	5	5		7	7	-	-	-	4	5	5
	5	12	-	-	0	5	5	5		7	8	-	-	-	5	5	5
	5	13	-	-	0	5	5	-		7	9	-	-	0	5	5	6
	5	14	-	0	0	5	5	-		7	10	-	-	0	5	6	6
	5	15	-	0	0	5	-	-		7	11	-	0	0	6	6	6
	5	16	0	0	0	5	-	-		7	12	0	0	0	6	6	7
	5	17	0	0	0	-	-	-		7	13	0	0	0	6	6	7
	5	18	0	0	1	-	-	-		7	14	0	0	1	6	7	7
	5	19	0	0	1	-	-	-		7	15	0	1	1	7	7	7
	5	20	0	1	1	-	-	-		7	16	0	1	1	7	7	7
	5	21	1	1	1	-	-	-		7	17	1	1	1	7	7	-
	5	22	1	1	1	-	-	-		7	18	1	1	2	7	-	-
	5	23	1	1	2	-	-	-		7	19	1	2	2	7	-	-
	6	6	-	-	-	4	4	4		7	20	2	2	2	-	-	-
	6	7	-	-	-	4	4	5		7	21	2	2	3	-	-	-

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01
28	8	8	–	–	0	5	5	6
	8	9	–	0	0	5	6	6
	8	10	–	0	0	6	6	7
	8	11	0	0	0	6	6	7
	8	12	0	0	1	6	7	7
	8	13	0	0	1	7	7	7
	8	14	0	1	1	7	7	8
	8	15	1	1	1	7	8	8
	8	16	1	1	2	7	8	8
	8	17	1	2	2	8	8	8
	8	18	1	2	2	8	8	–
	8	19	2	2	3	8	8	–
	8	20	2	3	3	8	–	–
	9	9	–	0	0	6	6	7
	9	10	0	0	0	6	7	7
	9	11	0	0	1	7	7	7
	9	12	0	1	1	7	7	8
	9	13	0	1	1	7	8	8
	9	14	1	1	1	8	8	8
	9	15	1	1	2	8	8	9
	9	16	1	2	2	8	8	9
	9	17	2	2	2	8	9	9
	9	18	2	2	3	9	9	9
	9	19	2	3	3	9	9	–
	10	10	0	0	1	7	7	7
	10	11	0	1	1	7	7	8
	10	12	0	1	1	7	8	8
	10	13	1	1	2	8	8	9
	10	14	1	2	2	8	8	9
	10	15	1	2	2	8	9	9
28	10	16	2	2	3	9	9	10
	10	17	2	3	3	9	9	10
	10	18	3	3	3	9	10	10
	11	11	0	1	1	7	8	8
	11	12	1	1	2	8	8	9
	11	13	1	2	2	8	9	9
	11	14	2	2	2	9	9	9
	11	15	2	2	3	9	9	10
	11	16	2	3	3	9	10	10
	11	17	3	3	4	10	10	11
	12	12	1	2	2	8	9	9
	2	13	2	2	2	9	9	10
	12	14	2	2	3	9	10	10
	12	15	2	3	3	10	10	10
	12	16	3	3	4	10	10	11
	13	13	2	2	3	9	10	10
	3	14	2	3	3	10	10	11
	13	15	3	3	4	10	11	11
	14	14	3	3	4	10	11	11
29	1	1	–	–	–	1	–	–
	1	28	–	–	0	–	–	–
	2	2	–	–	–	2	2	2
	2	3	–	–	–	2	2	2
	2	4	–	–	–	2	2	–
	2	5	–	–	–	2	2	–
	2	6	–	–	–	2	–	–
	2	23	–	–	0	–	–	–
	2	24	–	0	0	–	–	–
	2	25	–	0	0	–	–	–
	2	26	0	0	0	–	–	–

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01
29	2	27	0	0	0	–	–	–
	3	3	–	–	–	2	2	3
	3	4	–	–	–	2	3	3
	3	5	–	–	–	3	3	3
	3	6	–	–	–	3	3	3
	3	7	–	–	–	3	3	3
	3	8	–	–	–	3	3	–
	3	9	–	–	–	3	3	–
	3	10	–	–	–	3	–	–
	3	11	–	–	–	3	–	–
	3	18	–	–	0	–	–	–
	3	19	–	–	0	–	–	–
	3	20	–	0	0	–	–	–
	3	21	–	0	0	–	–	–
	3	22	0	0	0	–	–	–
	3	23	0	0	0	–	–	–
	3	24	0	0	0	–	–	–
	3	25	0	0	1	–	–	–
	3	26	0	1	1	–	–	–
	4	4	–	–	–	3	3	3
	4	5	–	–	–	3	3	4
	4	6	–	–	–	3	3	4
	4	7	–	–	–	3	4	4
	4	8	–	–	–	4	4	4
	4	9	–	–	–	4	4	4
	4	10	–	–	–	4	4	4
	4	11	–	–	–	4	4	–
	4	12	–	–	–	4	4	–
	4	13	–	–	–	4	–	–
	4	14	–	–	–	4	–	–
29	4	15	–	–	0	–	–	–
	4	16	–	–	0	–	–	–
	4	17	–	0	0	–	–	–
	4	18	–	0	0	–	–	–
	4	19	0	0	0	–	–	–
	4	20	0	0	0	–	–	–
	4	21	0	0	0	–	–	–
	4	22	0	0	1	–	–	–
	4	23	0	1	1	–	–	–
	4	24	0	1	1	–	–	–
	4	25	1	1	1	–	–	–
	5	5	–	–	–	3	3	4
	5	6	–	–	–	3	4	4
	5	7	–	–	–	4	4	4
	5	8	–	–	–	4	4	5
	5	9	–	–	–	4	4	5
	5	10	–	–	–	4	5	5
	5	11	–	–	–	5	5	5
	5	12	–	–	–	5	5	5
	5	13	–	–	0	5	5	–
	5	14	–	–	0	5	5	–
	5	15	–	0	0	5	–	–
	5	16	–	0	0	5	–	–
	5	17	0	0	0	–	–	–
	5	18	0	0	0	–	–	–
	5	19	0	0	1	–	–	–
	5	20	0	1	1	–	–	–
	5	21	0	1	1	–	–	–
	5	22	1	1	1	–	–	–
	5	23	1	1	2	–	–	–

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01
29	5	24	1	2	2	-	-	-
	6	6	-	-	-	4	4	4
	6	7	-	-	-	4	4	5
	6	8	-	-	-	4	5	5
	6	9	-	-	-	5	5	5
	6	10	-	-	-	5	5	6
	6	11	-	-	0	5	5	6
	6	12	-	-	0	5	6	6
	6	13	-	0	0	5	6	6
	6	14	-	0	0	6	6	6
	6	15	0	0	0	6	6	-
	6	16	0	0	1	6	6	-
	6	17	0	0	1	6	-	-
	6	18	0	1	1	6	-	-
	6	19	0	1	1	-	-	-
	6	20	1	1	1	-	-	-
	6	21	1	1	2	-	-	-
	6	22	1	2	2	-	-	-
	6	23	2	2	2	-	-	-
	7	7	-	-	-	4	5	5
	7	8	-	-	-	5	5	5
	7	9	-	-	0	5	5	6
	7	10	-	-	0	5	6	6
	7	11	-	0	0	6	6	6
	7	12	-	0	0	6	6	7
	7	13	0	0	0	6	6	7
	7	14	0	0	1	6	7	7
	7	15	0	0	1	6	7	7
	7	16	0	1	1	7	7	7
	7	17	0	1	1	7	7	-
29	7	18	1	1	1	7	7	-
	7	19	1	1	2	7	-	-
	7	20	1	2	2	7	-	-
	7	21	2	2	2	-	-	-
	7	22	2	2	3	-	-	-
	8	8	-	-	0	5	5	6
	8	9	-	-	0	5	6	6
	8	10	-	0	0	6	6	6
	8	11	-	0	0	6	6	7
	8	12	0	0	0	6	7	7
	8	13	0	0	1	7	7	7
	8	14	0	1	1	7	7	8
	8	15	0	1	1	7	7	8
	8	16	1	1	1	7	8	8
	8	17	1	1	2	8	8	8
	8	18	1	2	2	8	8	-
	8	19	2	2	2	8	8	-
	8	20	2	2	3	8	-	-
	8	21	2	3	3	8	-	-
	9	9	-	0	0	6	6	7
	9	10	0	0	0	6	6	7
	9	11	0	0	0	6	7	7
	9	12	0	0	1	7	7	8
	9	13	0	1	1	7	7	8
	9	14	1	1	1	7	8	8
	9	15	1	1	2	8	8	8
	9	16	1	2	2	8	8	9
	9	17	1	2	2	8	9	9
	9	18	2	2	3	9	9	9
	9	19	2	3	3	9	9	9

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01
29	9	20	2	3	3	9	9	-
	10	10	0	0	0	6	7	7
	10	11	0	0	1	7	7	8
	10	12	0	1	1	7	8	8
	10	13	1	1	1	8	8	8
	10	14	1	1	2	8	8	9
	10	15	1	2	2	8	9	9
	10	16	2	2	2	9	9	9
	10	17	2	2	3	9	9	10
	10	18	2	3	3	9	10	10
	10	19	3	3	4	10	10	10
	11	11	0	1	1	7	8	8
	11	12	1	1	1	8	8	9
	11	13	1	1	2	8	8	9
	11	14	1	2	2	8	9	9
	11	15	2	2	3	9	9	10
	11	16	2	3	3	9	10	10
	11	17	2	3	3	10	10	10
	11	18	3	3	4	10	10	11
	12	12	1	1	2	8	9	9
	12	13	1	2	2	9	9	9
	12	14	2	2	3	9	9	10
	12	15	2	3	3	9	10	10
	12	16	3	3	3	10	10	11
	12	17	3	3	4	10	11	11
	13	13	2	2	3	9	9	10
	13	14	2	3	3	9	10	10
	13	15	3	3	4	10	10	11
	13	16	3	4	4	10	11	11
	14	14	3	3	4	10	10	11
29	14	15	3	4	4	10	11	11
30	1	1	-	-	-	1	-	-
	1	29	-	-	0	-	-	-
	2	2	-	-	-	2	2	2
	2	3	-	-	-	2	2	2
	2	4	-	-	-	2	2	-
	2	5	-	-	-	2	2	-
	2	6	-	-	-	2	-	-
	2	7	-	-	-	2	-	-
	2	23	-	-	0	-	-	-
	2	24	-	-	0	-	-	-
	2	25	-	0	0	-	-	-
	2	26	-	0	0	-	-	-
	2	27	0	0	0	-	-	-
	2	28	0	0	0	-	-	-
	3	3	-	-	-	2	2	3
	3	4	-	-	-	2	3	3
	3	5	-	-	-	3	3	3
	3	6	-	-	-	3	3	3
	3	7	-	-	-	3	3	3
	3	8	-	-	-	3	3	-
	3	9	-	-	-	3	3	-
	3	10	-	-	-	3	-	-
	3	11	-	-	-	3	-	-
	3	19	-	-	0	-	-	-
	3	20	-	-	0	-	-	-
	3	21	-	0	0	-	-	-
	3	22	-	0	0	-	-	-
	3	23	0	0	0	-	-	-
	3	24	0	0	0	-	-	-

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01	M	N1	N2	.01	.025	.05	.05	.025	.01
30	3	25	0	0	0	-	-	-	30	5	9	-	-	-	4	4	5
	3	26	0	0	1	-	-	-		5	10	-	-	-	4	5	5
	3	27	0	1	1	-	-	-		5	11	-	-	-	4	5	5
	4	4	-	-	-	3	3	3		5	12	-	-	-	5	5	5
	4	5	-	-	-	3	3	3		5	13	-	-	0	5	5	5
	4	6	-	-	-	3	3	4		5	14	-	-	0	5	5	-
	4	7	-	-	-	3	4	4		5	15	-	0	0	5	5	-
	4	8	-	-	-	3	4	4		5	16	-	0	0	5	-	-
	4	9	-	-	-	4	4	4		5	17	0	0	0	5	-	-
	4	10	-	-	-	4	4	4		5	18	0	0	0	-	-	-
	4	11	-	-	-	4	4	-		5	19	0	0	1	-	-	-
	4	12	-	-	-	4	4	-		5	20	0	0	1	-	-	-
	4	13	-	-	-	4	-	-		5	21	0	1	1	-	-	-
	4	14	-	-	-	4	-	-		5	22	0	1	1	-	-	-
	4	15	-	-	0	4	-	-		5	23	1	1	1	-	-	-
	4	16	-	-	0	-	-	-		5	24	1	1	2	-	-	-
	4	17	-	-	0	-	-	-		5	25	1	2	2	-	-	-
	4	18	-	0	0	-	-	-		6	6	-	-	-	4	4	4
	4	19	-	0	0	-	-	-		6	7	-	-	-	4	4	5
	4	20	0	0	0	-	-	-		6	8	-	-	-	4	5	5
	4	21	0	0	0	-	-	-		6	9	-	-	-	4	5	5
	4	22	0	0	1	-	-	-		6	10	-	-	-	5	5	5
	4	23	0	0	1	-	-	-		6	11	-	-	0	5	5	6
	4	24	0	1	1	-	-	-		6	12	-	-	0	5	6	6
	4	25	1	1	1	-	-	-		6	13	-	0	0	5	6	6
	4	26	1	1	1	-	-	-		6	14	-	0	0	6	6	6
	5	5	-	-	-	3	3	4		6	15	0	0	0	6	6	6
	5	6	-	-	-	3	4	4		6	16	0	0	0	6	6	-
	5	7	-	-	-	4	4	4		6	17	0	0	1	6	6	-
	5	8	-	-	-	4	4	5		6	18	0	0	1	6	-	-

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01
30	6	19	0	1	1	6	-	-
	6	20	1	1	1	-	-	-
	6	21	1	1	2	-	-	-
	6	22	1	1	2	-	-	-
	6	23	1	2	2	-	-	-
	6	24	2	2	2	-	-	-
	7	7	-	-	-	4	5	5
	7	8	-	-	-	5	5	5
	7	9	-	-	-	5	5	6
	7	10	-	-	0	5	6	6
	7	11	-	0	0	5	6	6
	7	12	-	0	0	6	6	6
	7	13	0	0	0	6	6	7
	7	14	0	0	0	6	7	7
	7	15	0	0	1	6	7	7
	7	16	0	0	1	7	7	7
	7	17	0	1	1	7	7	7
	7	18	1	1	1	7	7	-
	7	19	1	1	2	7	7	-
	7	20	1	1	2	7	-	-
	7	21	1	2	2	-	-	-
	7	22	2	2	2	-	-	-
	7	23	2	2	3	-	-	-
	8	8	-	-	-	5	5	6
	8	9	-	-	0	5	6	6
	8	10	-	0	0	6	6	6
	8	11	-	0	0	6	6	7
	8	12	0	0	0	6	7	7
	8	13	0	0	1	6	7	7
	8	14	0	0	1	7	7	7
30	8	15	0	1	1	7	7	8
	8	16	1	1	1	7	8	8
	8	17	1	1	2	7	8	8
	8	18	1	1	2	8	8	8
	8	19	1	2	2	8	8	-
	8	20	2	2	2	8	8	-
	8	21	2	2	3	8	-	-
	8	22	2	3	3	-	-	-
	9	9	-	0	0	6	6	6
	9	10	-	0	0	6	6	7
	9	11	0	0	0	6	7	7
	9	12	0	0	1	7	7	7
	9	13	0	1	1	7	7	8
	9	14	0	1	1	7	8	8
	9	15	1	1	1	8	8	8
	9	16	1	1	2	8	8	9
	9	17	1	2	2	8	8	9
	9	18	2	2	2	8	9	9
	9	19	2	2	3	9	9	9
	9	20	2	3	3	9	9	-
	9	21	3	3	3	9	9	-
	10	10	0	0	0	6	7	7
	10	11	0	0	1	7	7	8
	10	12	0	1	1	7	7	8
	10	13	0	1	1	7	8	8
	10	14	1	1	2	8	8	9
	10	15	1	1	2	8	9	9
	10	16	1	2	2	8	9	9
	10	17	2	2	3	9	9	10
	10	18	2	3	3	9	9	10

TABLE 7--CONTINUED

M	N1	N2	.01	.025	.05	.05	.025	.01
30	10	19	2	3	3	9	10	10
	10	20	3	3	4	10	10	10
	11	11	0	1	1	7	8	8
	11	12	0	1	1	8	8	8
	11	13	1	1	2	8	8	9
	11	14	1	2	2	8	9	9
	11	15	1	2	2	9	9	10
	11	16	2	2	3	9	9	10
	11	17	2	3	3	9	10	10
	11	18	3	3	3	10	10	11
	11	19	3	3	4	10	10	11
	12	12	1	1	2	8	8	9
	12	13	1	2	2	8	9	9
	12	14	2	2	2	9	9	10
	12	15	2	2	3	9	10	10
	12	16	2	3	3	10	10	10
	12	17	3	3	4	10	10	11
	12	18	3	4	4	10	11	11
	13	13	2	2	2	9	9	10
	13	14	2	2	3	9	10	10
	13	15	2	3	3	10	10	11
	13	16	3	3	4	10	11	11
	13	17	3	4	4	11	11	11
	14	14	2	3	3	10	10	11
	14	15	3	3	4	10	11	11
	14	16	3	4	4	11	11	12
	15	15	3	4	4	11	11	12

TABLE 8

CRITICAL VALUES BY SAMPLE SIZE
THREE CATEGORIES

M	N1	N2	N3	.01	.05	.05	.01	M	N1	N2	N2	.01	.05	.05	.01
5	1	1	1	-	-	1	-	7	3	6	6	0	0	-	-
	2	2	2	-	-	2	2		4	4	4	-	-	3	4
	2	2	3	-	-	2	-		4	4	5	-	-	4	4
	3	3	3	-	-	3	3		4	4	6	-	0	4	-
	3	3	4	-	-	3	-		4	5	5	-	0	4	-
	3	4	4	0	0	-	-		4	5	6	0	0	-	-
	4	4	4	1	1	4	-		4	6	6	1	1	-	-
6	1	1	1	-	-	1	-		5	5	5	0	0	5	5
	2	2	2	-	-	2	2		5	5	6	1	1	5	-
	2	2	3	-	-	2	-		5	6	6	2	2	-	-
	2	2	4	-	-	2	-		6	6	6	3	3	6	-
	2	2	5	-	-	2	-	8	1	1	1	-	-	1	-
	2	3	3	-	-	2	-		1	1	2	-	-	1	-
	3	3	3	-	-	3	3		1	1	3	-	-	1	-
	3	3	4	-	-	3	3		2	2	2	-	-	2	2
	3	3	5	-	-	3	-		2	2	3	-	-	2	2
	3	4	4	-	-	3	-		2	2	4	-	-	2	2
	3	4	5	-	0	-	-		2	2	5	-	-	2	-
	3	5	5	0	0	-	-		2	2	6	-	-	2	-
	4	4	4	-	0	4	4		2	2	7	-	-	2	-
	4	4	5	0	0	4	-		2	3	3	-	-	2	-
	4	5	5	1	1	-	-		2	3	4	-	-	2	-
	5	5	5	2	2	5	-		2	3	5	-	-	2	-
7	1	1	1	-	-	1	-		2	4	4	-	-	2	-
	1	1	2	-	-	1	-		2	7	7	-	0	-	-
	2	2	2	-	-	2	2		3	3	3	-	-	2	3
	2	2	3	-	-	2	2		3	3	4	-	-	3	3
	2	2	4	-	-	2	-		3	3	5	-	-	3	3
	2	2	5	-	-	2	-		3	3	6	-	-	3	3
	2	2	6	-	-	2	-		3	3	7	-	-	3	-
	2	3	3	-	-	2	-		3	4	4	-	-	3	3
	2	3	4	-	-	2	-		3	4	5	-	-	3	-
	2	6	6	-	0	-	-		3	4	6	-	-	3	-
	3	3	3	-	-	3	3		3	4	7	-	-	3	-
	3	3	4	-	-	3	3		3	5	5	-	-	3	-
	3	3	5	-	-	3	3		3	6	6	-	0	-	-
	3	3	6	-	-	3	-		3	6	7	-	0	-	-
	3	4	4	-	-	3	-		3	7	7	0	0	-	-
	3	4	5	-	-	3	-		4	4	4	-	-	3	4
	3	5	6	-	0	-	-		4	4	5	-	-	3	4

TABLE 8--CONTINUED

M	N1	N2	N3	.01	.05	.05	.01	M	N1	N2	N2	.01	.05	.05	.01
8	4	4	6	-	-	4	4	9	3	4	6	-	-	3	-
	4	4	7	-	0	4	4		3	4	7	-	-	3	-
	4	5	5	-	-	4	4		3	4	8	-	-	3	-
	4	5	6	-	0	4	-		3	5	5	-	-	3	-
	4	5	7	0	0	4	-		3	5	6	-	-	3	-
	4	6	6	0	0	4	-		3	5	7	-	-	3	-
	4	6	7	0	0	-	-		3	6	8	-	0	-	-
	4	7	7	1	1	-	-		3	7	7	-	0	-	-
	5	5	5	-	0	4	5		3	7	8	0	0	-	-
	5	5	6	0	0	5	5		3	8	8	0	0	-	-
	5	5	7	0	1	5	5		4	4	4	-	-	3	3
	5	6	6	0	1	5	-		4	4	5	-	-	3	4
	5	6	7	1	1	5	-		4	4	6	-	-	3	4
	5	7	7	2	2	-	-		4	4	7	-	-	4	4
	6	6	6	1	1	6	6		4	4	8	-	-	4	4
	6	6	7	2	2	6	6		4	5	5	-	-	4	4
	6	7	7	3	3	-	-		4	5	6	-	-	4	4
	7	7	7	4	4	7	-		4	5	7	-	-	4	-
9	1	1	1	-	-	1	-		4	5	8	-	0	4	-
	1	1	2	-	-	1	-		4	6	6	-	-	4	-
	1	1	3	-	-	1	-		4	6	7	-	0	4	-
	1	1	4	-	-	1	-		4	6	8	0	0	-	-
	1	2	2	-	-	1	-		4	7	7	0	0	-	-
	2	2	2	-	-	2	2		4	7	8	0	1	-	-
	2	2	3	-	-	2	2		4	8	8	1	1	-	-
	2	2	4	-	-	2	2		5	5	5	-	-	4	4
	2	2	5	-	-	2	2		5	5	6	-	0	4	5
	2	2	6	-	-	2	-		5	5	7	-	0	4	5
	2	2	7	-	-	2	-		5	5	8	0	0	5	5
	2	2	8	-	-	2	-		5	6	6	-	0	5	5
	2	3	3	-	-	2	2		5	6	7	0	0	5	5
	2	3	4	-	-	2	-		5	6	8	0	1	5	-
	2	3	5	-	-	2	-		5	7	7	0	1	5	-
	2	3	6	-	-	2	-		5	7	8	1	1	-	-
	2	3	7	-	-	2	-		5	8	8	2	2	-	-
	2	4	4	-	-	2	-		6	6	6	0	0	5	6
	2	4	5	-	-	2	-		6	6	7	0	1	5	6
	2	8	8	-	0	-	-		6	6	8	1	1	6	6
	3	3	3	-	-	2	3		6	7	7	1	1	6	6
	3	3	4	-	-	2	3		6	7	8	2	2	6	-
	3	3	5	-	-	3	3		6	8	8	3	3	-	-
	3	3	6	-	-	3	3		7	7	7	2	2	6	7
	3	3	7	-	-	3	3		7	7	8	3	3	7	7
	3	3	8	-	-	3	3		7	8	8	4	4	7	-
	3	4	4	-	-	3	3		8	8	8	5	5	8	-
	3	4	5	-	-	3	3	10	1	1	1	-	-	1	1

TABLE 8--CONTINUED

M	N1	N2	N3	.01	.05	.05	.01	M	N1	N2	N2	.01	.05	.05	.01
10	1	1	2	-	-	1	-	10	3	9	9	0	0	-	-
	1	1	3	-	-	1	-		4	4	4	-	-	3	3
	1	1	4	-	-	1	-		4	4	5	-	-	3	4
	1	1	5	-	-	1	-		4	4	6	-	-	3	4
	1	2	2	-	-	1	-		4	4	7	-	-	3	4
	2	2	2	-	-	2	2		4	4	8	-	-	4	4
	2	2	3	-	-	2	2		4	4	9	-	-	4	4
	2	2	4	-	-	2	2		4	5	5	-	-	3	4
	2	2	5	-	-	2	2		4	5	6	-	-	3	4
	2	2	6	-	-	2	2		4	5	7	-	-	4	4
	2	2	7	-	-	2	-		4	5	8	-	-	4	4
	2	2	8	-	-	2	-		4	5	9	-	0	4	-
	2	2	9	-	-	2	-		4	6	6	-	-	4	4
	2	3	3	-	-	2	2		4	6	7	-	-	4	-
	2	3	4	-	-	2	2		4	6	8	-	0	4	-
	2	3	5	-	-	2	-		4	6	9	-	0	4	-
	2	3	6	-	-	2	-		4	7	7	-	0	4	-
	2	3	7	-	-	2	-		4	7	8	-	0	-	-
	2	3	8	-	-	2	-		4	7	9	0	0	-	-
	2	4	4	-	-	2	-		4	8	8	0	0	-	-
	2	4	5	-	-	2	-		4	8	9	0	1	-	-
	2	4	6	-	-	2	-		4	9	9	1	1	-	-
	2	5	5	-	-	2	-		5	5	5	-	-	4	4
	2	9	9	-	0	-	-		5	5	6	-	-	4	4
	3	3	3	-	-	2	3		5	5	7	-	-	4	5
	3	3	4	-	-	2	3		5	5	8	-	0	4	5
	3	3	5	-	-	2	3		5	5	9	-	0	5	5
	3	3	6	-	-	3	3		5	6	6	-	-	4	5
	3	3	7	-	-	3	3		5	6	7	-	0	4	5
	3	3	8	-	-	3	3		5	6	8	-	0	5	5
	3	3	9	-	-	3	3		5	6	9	0	0	5	-
	3	4	4	-	-	2	3		5	7	7	0	0	5	5
	3	4	5	-	-	3	3		5	7	8	0	0	5	-
	3	4	6	-	-	3	3		5	7	9	0	1	5	-
	3	4	7	-	-	3	3		5	8	8	0	1	5	-
	3	4	8	-	-	3	-		5	8	9	1	1	-	-
	3	4	9	-	-	3	-		5	9	9	2	2	-	-
	3	5	5	-	-	3	3		6	6	6	-	0	5	5
	3	5	6	-	-	3	-		6	6	7	0	0	5	6
	3	5	7	-	-	3	-		6	6	8	0	0	5	6
	3	5	8	-	-	3	-		6	6	9	0	1	6	6
	3	6	6	-	-	3	-		6	7	7	0	1	5	6
	3	6	7	-	-	3	-		6	7	8	0	1	6	6
	3	7	9	-	0	-	-		6	7	9	1	2	6	-
	3	8	8	-	0	-	-		6	8	8	1	2	6	-
	3	8	9	0	0	-	-		6	8	9	2	2	-	-

TABLE 8--CONTINUED

M	N1	N2	N3	.01	.05	.05	.01	M	N1	N2	N2	.01	.05	.05	.01
10	6	9	9	3	3	-	-	11	3	3	5	-	-	2	3
	7	7	7	1	1	6	6		3	3	6	-	-	2	3
	7	7	8	1	2	6	7		3	3	7	-	-	3	3
	7	7	9	2	2	7	7		3	3	8	-	-	3	3
	7	8	8	2	2	7	7		3	3	9	-	-	3	3
	7	8	9	3	3	7	-		3	3	10	-	-	3	3
	7	9	9	4	4	-	-		3	4	4	-	-	2	3
	8	8	8	3	3	7	8		3	4	5	-	-	3	3
	8	8	9	4	4	8	8		3	4	6	-	-	3	3
	8	9	9	5	5	8	-		3	4	7	-	-	3	3
	9	9	9	6	6	9	9		3	4	8	-	-	3	3
11	1	1	1	-	-	1	1		3	4	9	-	-	3	-
	1	1	2	-	-	1	-		3	4	10	-	-	3	-
	1	1	3	-	-	1	-		3	5	5	-	-	3	3
	1	1	4	-	-	1	-		3	5	6	-	-	3	3
	1	1	5	-	-	1	-		3	5	7	-	-	3	-
	1	1	6	-	-	1	-		3	5	8	-	-	3	-
	1	2	2	-	-	1	-		3	5	9	-	-	3	-
	1	2	3	-	-	1	-		3	5	10	-	-	3	-
	2	2	2	-	-	2	2		3	6	6	-	-	3	-
	2	2	3	-	-	2	2		3	6	7	-	-	3	-
	2	2	4	-	-	2	2		3	6	8	-	-	3	-
	2	2	5	-	-	2	2		3	7	7	-	-	3	-
	2	2	6	-	-	2	2		3	7	10	-	0	-	-
	2	2	7	-	-	2	2		3	8	9	-	0	-	-
	2	2	8	-	-	2	2		3	8	10	-	0	-	-
	2	2	9	-	-	2	-		3	9	9	-	0	-	-
	2	2	10	-	-	2	-		3	9	10	0	0	-	-
	2	3	3	-	-	2	2		3	10	10	0	1	-	-
	2	3	4	-	-	2	2		4	4	4	-	-	3	3
	2	3	5	-	-	2	2		4	4	5	-	-	3	3
	2	3	6	-	-	2	-		4	4	6	-	-	3	4
	2	3	7	-	-	2	-		4	4	7	-	-	3	4
	2	3	8	-	-	2	-		4	4	8	-	-	3	4
	2	3	9	-	-	2	-		4	4	9	-	-	3	4
	2	3	10	-	-	2	-		4	4	10	-	-	4	4
	2	4	4	-	-	2	-		4	5	5	-	-	3	4
	2	4	5	-	-	2	-		4	5	6	-	-	3	4
	2	4	6	-	-	2	-		4	5	7	-	-	3	4
	2	4	7	-	-	2	-		4	5	8	-	-	4	4
	2	5	5	-	-	2	-		4	5	9	-	-	4	4
	2	5	6	-	-	2	-		4	5	10	-	-	4	4
	2	9	10	-	0	-	-		4	6	6	-	-	4	4
	2	10	10	-	0	-	-		4	6	7	-	-	4	4
	3	3	3	-	-	2	2		4	6	8	-	-	4	4
	3	3	4	-	-	2	3		4	6	9	-	-	4	-

TABLE 8--CONTINUED

M	N1	N2	N3	.01	.05	.05	.01	M	N1	N2	N2	.01	.05	.05	.01
11	4	6	10	-	0	4	-	11	6	10	10	3	3	-	-
	4	7	7	-	-	4	-		7	7	7	0	0	5	6
	4	7	8	-	0	4	-		7	7	8	0	1	6	6
	4	7	9	-	0	4	-		7	7	9	1	1	6	7
	4	7	10	-	0	-	-		7	7	10	1	2	6	7
	4	8	8	-	0	4	-		7	8	8	1	1	6	7
	4	8	9	0	0	-	-		7	8	9	1	2	7	7
	4	8	10	0	0	-	-		7	8	10	2	2	7	7
	4	9	9	0	0	-	-		7	9	9	2	2	7	-
	4	9	10	0	1	-	-		7	9	10	3	3	7	-
	4	10	10	1	1	-	-		7	10	10	4	4	-	-
	5	5	5	-	-	3	4		8	8	8	1	2	7	7
	5	5	6	-	-	4	4		8	8	9	2	2	7	8
	5	5	7	-	-	4	4		8	8	10	3	3	8	8
	5	5	8	-	-	4	5		8	9	9	3	3	8	8
	5	5	9	-	-	4	5		8	9	10	4	4	8	-
	5	5	10	-	0	4	5		8	10	10	5	5	-	-
	5	6	6	-	-	4	4		9	9	9	4	4	8	9
	5	6	7	-	-	4	5		9	9	10	5	5	9	9
	5	6	8	-	0	4	5		9	10	10	6	6	9	-
	5	6	9	-	0	5	5		10	10	10	7	7	10	10
	5	6	10	0	0	5	5	12	1	1	1	-	-	1	1
	5	7	7	-	0	4	5		1	1	2	-	-	1	-
	5	7	8	-	0	5	5		1	1	3	-	-	1	-
	5	7	9	0	0	5	-		1	1	4	-	-	1	-
	5	7	10	0	1	5	-		1	1	5	-	-	1	-
	5	8	8	0	0	5	-		1	1	6	-	-	1	-
	5	8	9	0	1	5	-		1	1	7	-	-	1	-
	5	8	10	0	1	-	-		1	2	2	-	-	1	-
	5	9	9	1	1	-	-		1	2	3	-	-	1	-
	5	9	10	1	2	-	-		2	2	2	-	-	2	2
	5	10	10	2	2	-	-		2	2	3	-	-	2	2
	6	6	6	-	-	4	5		2	2	4	-	-	2	2
	6	6	7	-	0	5	5		2	2	5	-	-	2	2
	6	6	8	-	0	5	5		2	2	6	-	-	2	2
	6	6	9	0	0	5	6		2	2	7	-	-	2	2
	6	6	10	0	1	5	6		2	2	8	-	-	2	2
	6	7	7	-	0	5	6		2	2	9	-	-	2	2
	6	7	8	0	0	5	6		2	2	10	-	-	2	-
	6	7	9	0	1	6	6		2	2	11	-	-	2	-
	6	7	10	1	1	6	6		2	3	3	-	-	2	2
	6	8	8	0	1	6	6		2	3	4	-	-	2	2
	6	8	9	1	1	6	-		2	3	5	-	-	2	2
	6	8	10	1	2	6	-		2	3	6	-	-	2	-
	6	9	9	1	2	6	-		2	3	7	-	-	2	-
	6	9	10	2	2	-	-		2	3	8	-	-	2	-

TABLE 8--CONTINUED

M	N1	N2	N3	.01	.05	.05	.01	M	N1	N2	N2	.01	.05	.05	.01
12	2	3	9	-	-	2	-	12	3	9	10	-	0	-	-
	2	3	10	-	-	2	-		3	9	11	-	0	-	-
	2	3	11	-	-	2	-		3	10	10	-	0	-	-
	2	4	4	-	-	2	2		3	10	11	0	0	-	-
	2	4	5	-	-	2	-		3	11	11	0	1	-	-
	2	4	6	-	-	2	-		4	4	4	-	-	2	3
	2	4	7	-	-	2	-		4	4	5	-	-	3	3
	2	4	8	-	-	2	-		4	4	6	-	-	3	3
	2	4	9	-	-	2	-		4	4	7	-	-	3	4
	2	5	5	-	-	2	-		4	4	8	-	-	3	4
	2	5	6	-	-	2	-		4	4	9	-	-	3	4
	2	5	7	-	-	2	-		4	4	10	-	-	3	4
	2	10	11	-	0	-	-		4	4	11	-	-	3	4
	2	11	11	-	0	-	-		4	5	5	-	-	3	3
	3	3	3	-	-	2	2		4	5	6	-	-	3	4
	3	3	4	-	-	2	3		4	5	7	-	-	3	4
	3	3	5	-	-	2	3		4	5	8	-	-	3	4
	3	3	6	-	-	2	3		4	5	9	-	-	4	4
	3	3	7	-	-	2	3		4	5	10	-	-	4	4
	3	3	8	-	-	3	3		4	5	11	-	-	4	4
	3	3	9	-	-	3	3		4	6	6	-	-	3	4
	3	3	10	-	-	3	3		4	6	7	-	-	3	4
	3	3	11	-	-	3	3		4	6	8	-	-	4	4
	3	4	4	-	-	2	3		4	6	9	-	-	4	4
	3	4	5	-	-	2	3		4	6	10	-	-	4	-
	3	4	6	-	-	3	3		4	6	11	-	-	4	-
	3	4	7	-	-	3	3		4	7	7	-	-	4	4
	3	4	8	-	-	3	3		4	7	8	-	-	4	4
	3	4	9	-	-	3	3		4	7	9	-	-	4	-
	3	4	10	-	-	3	3		4	7	10	-	0	4	-
	3	4	11	-	-	3	-		4	7	11	-	0	4	-
	3	5	5	-	-	3	3		4	8	8	-	-	4	-
	3	5	6	-	-	3	3		4	8	9	-	0	4	-
	3	5	7	-	-	3	3		4	8	10	-	0	-	-
	3	5	8	-	-	3	-		4	8	11	0	0	-	-
	3	5	9	-	-	3	-		4	9	9	-	0	-	-
	3	5	10	-	-	3	-		4	9	10	0	0	-	-
	3	5	11	-	-	3	-		4	9	11	0	0	-	-
	3	6	6	-	-	3	3		4	10	10	0	1	-	-
	3	6	7	-	-	3	-		4	10	11	0	1	-	-
	3	6	8	-	-	3	-		4	11	11	1	1	-	-
	3	6	9	-	-	3	-		5	5	5	-	-	3	4
	3	6	10	-	-	3	-		5	5	6	-	-	3	4
	3	7	7	-	-	3	-		5	5	7	-	-	4	4
	3	7	8	-	-	3	-		5	5	8	-	-	4	4
	3	8	11	-	0	-	-		5	5	9	-	-	4	5

TABLE 8--CONTINUED

M	N1	N2	N3	.01	.05	.05	.01
12	5	5	10	-	-	4	5
	5	5	11	-	0	4	5
	5	6	6	-	-	4	4
	5	6	7	-	-	4	4
	5	6	8	-	-	4	5
	5	6	9	-	-	4	5
	5	6	10	-	0	5	5
	5	6	11	-	0	5	5
	5	7	7	-	-	4	5
	5	7	8	-	0	4	5
	5	7	9	-	0	5	5
	5	7	10	-	0	5	5
	5	7	11	0	0	5	-
	5	8	8	-	0	5	5
	5	8	9	-	0	5	-
	5	8	10	0	0	5	-
	5	8	11	0	1	5	-
	5	9	9	0	0	5	-
	5	9	10	0	1	-	-
	5	9	11	1	1	-	-
	5	10	10	1	1	-	-
	5	10	11	1	2	-	-
	5	11	11	2	2	-	-
	6	6	6	-	-	4	5
	6	6	7	-	-	4	5
	6	6	8	-	0	4	5
	6	6	9	-	0	5	5
	6	6	10	-	0	5	6
	6	6	11	0	0	5	6
	6	7	7	-	0	5	5
	6	7	8	-	0	5	5
	6	7	9	0	0	5	6
	6	7	10	0	0	5	6
	6	7	11	0	1	6	6
	6	8	8	0	0	5	6
	6	8	9	0	0	6	6
	6	8	10	0	1	6	6
	6	8	11	1	1	6	-
	6	9	9	0	1	6	6
	6	9	10	1	1	6	-
	6	9	11	1	2	6	-
	6	10	10	1	2	-	-
	6	10	11	2	2	-	-
	6	11	11	3	3	-	-
	7	7	7	-	0	5	6
	7	7	8	0	0	5	6

M	N1	N2	N2	.01	.05	.05	.01
12	7	7	9	0	1	6	6
	7	7	10	0	1	6	7
	7	7	11	1	1	6	7
	7	8	8	0	1	6	6
	7	8	9	0	1	6	7
	7	8	10	1	1	6	7
	7	8	11	1	2	7	7
	7	9	9	1	1	6	7
	7	9	10	1	2	7	7
	7	9	11	2	2	7	-
	7	10	10	2	3	7	-
	7	10	11	3	3	-	-
	7	11	11	4	4	-	-
	8	8	8	0	1	6	7
	8	8	9	1	1	7	7
	8	8	10	1	2	7	8
	8	8	11	2	3	7	8
	8	9	9	1	2	7	8
	8	9	10	2	3	7	8
	8	9	11	3	3	8	8
	8	10	10	3	3	8	8
	8	10	11	4	4	8	-
	8	11	11	5	5	-	-
	9	9	9	2	3	8	8
	9	9	10	3	3	8	9
	9	9	11	4	4	8	9
	9	10	10	4	4	9	9
	9	10	11	5	5	9	-
	9	11	11	6	6	-	-
	10	10	10	5	5	9	10
	10	10	11	6	6	10	10
	10	11	11	7	7	10	-
	11	11	11	8	8	11	11
13	1	1	1	-	-	1	1
	1	1	2	-	-	1	-
	1	1	3	-	-	1	-
	1	1	4	-	-	1	-
	1	1	5	-	-	1	-
	1	1	6	-	-	1	-
	1	1	7	-	-	1	-
	1	1	8	-	-	1	-
	1	2	2	-	-	1	-
	1	2	3	-	-	1	-
	1	2	4	-	-	1	-
	2	2	2	-	-	1	2
	2	2	3	-	-	2	2

TABLE 8--CONTINUED

M	N1	N2	N3	.01	.05	.05	.01	M	N1	N2	N2	.01	.05	.05	.01
13	2	2	4	-	-	2	2	13	3	4	7	-	-	3	3
	2	2	5	-	-	2	2		3	4	8	-	-	3	3
	2	2	6	-	-	2	2		3	4	9	-	-	3	3
	2	2	7	-	-	2	2		3	4	10	-	-	3	3
	2	2	8	-	-	2	2		3	4	11	-	-	3	3
	2	2	9	-	-	2	2		3	4	12	-	-	3	-
	2	2	10	-	-	2	2		3	5	5	-	-	2	3
	2	2	11	-	-	2	2		3	5	6	-	-	3	3
	2	2	12	-	-	2	-		3	5	7	-	-	3	3
	2	3	3	-	-	2	2		3	5	8	-	-	3	3
	2	3	4	-	-	2	2		3	5	9	-	-	3	-
	2	3	5	-	-	2	2		3	5	10	-	-	3	-
	2	3	6	-	-	2	2		3	5	11	-	-	3	-
	2	3	7	-	-	2	-		3	5	12	-	-	3	-
	2	3	8	-	-	2	-		3	6	6	-	-	3	3
	2	3	9	-	-	2	-		3	6	7	-	-	3	3
	2	3	10	-	-	2	-		3	6	8	-	-	3	-
	2	3	11	-	-	2	-		3	6	9	-	-	3	-
	2	3	12	-	-	2	-		3	6	10	-	-	3	-
	2	4	4	-	-	2	2		3	6	11	-	-	3	-
	2	4	5	-	-	2	2		3	7	7	-	-	3	-
	2	4	6	-	-	2	-		3	7	8	-	-	3	-
	2	4	7	-	-	2	-		3	7	9	-	-	3	-
	2	4	8	-	-	2	-		3	8	8	-	-	3	-
	2	4	9	-	-	2	-		3	9	11	-	0	-	-
	2	4	10	-	-	2	-		3	9	12	-	0	-	-
	2	5	5	-	-	2	-		3	10	10	-	0	-	-
	2	5	6	-	-	2	-		3	10	11	-	0	-	-
	2	5	7	-	-	2	-		3	10	12	-	0	-	-
	2	5	8	-	-	2	-		3	11	11	-	0	-	-
	2	6	6	-	-	2	-		3	11	12	0	0	-	-
	2	11	12	-	0	-	-		3	12	12	0	1	-	-
	2	12	12	-	0	-	-		4	4	4	-	-	2	3
	3	3	3	-	-	2	2		4	4	5	-	-	3	3
	3	3	4	-	-	2	2		4	4	6	-	-	3	3
	3	3	5	-	-	2	3		4	4	7	-	-	3	3
	3	3	6	-	-	2	3		4	4	8	-	-	3	4
	3	3	7	-	-	2	3		4	4	9	-	-	3	4
	3	3	8	-	-	2	3		4	4	10	-	-	3	4
	3	3	9	-	-	3	3		4	4	11	-	-	3	4
	3	3	10	-	-	3	3		4	4	12	-	-	3	4
	3	3	11	-	-	3	3		4	5	5	-	-	3	3
	3	3	12	-	-	3	3		4	5	6	-	-	3	3
	3	4	4	-	-	2	3		4	5	7	-	-	3	4
	3	4	5	-	-	2	3		4	5	8	-	-	3	4
	3	4	6	-	-	2	3		4	5	9	-	-	3	4

TABLE 8--CONTINUED

M	N1	N2	N3	.01	.05	.05	.01	M	N1	N2	N2	.01	.05	.05	.01
13	4	5	10	-	-	4	4	13	5	7	7	-	-	4	5
	4	5	11	-	-	4	4		5	7	8	-	-	4	5
	4	5	12	-	-	4	4		5	7	9	-	-	4	5
	4	6	6	-	-	3	4		5	7	10	-	0	5	5
	4	6	7	-	-	3	4		5	7	11	-	0	5	5
	4	6	8	-	-	3	4		5	7	12	-	0	5	-
	4	6	9	-	-	4	4		5	8	8	-	-	4	5
	4	6	10	-	-	4	4		5	8	9	-	0	5	5
	4	6	11	-	-	4	4		5	8	10	-	0	5	5
	4	6	12	-	-	4	-		5	8	11	0	0	5	-
	4	7	7	-	-	4	4		5	8	12	0	0	5	-
	4	7	8	-	-	4	4		5	9	9	-	0	5	5
	4	7	9	-	-	4	4		5	9	10	0	0	5	-
	4	7	10	-	-	4	-		5	9	11	0	0	5	-
	4	7	11	-	-	4	-		5	9	12	0	1	-	-
	4	7	12	-	0	4	-		5	10	10	0	1	5	-
	4	8	8	-	-	4	4		5	10	11	0	1	-	-
	4	8	9	-	-	4	-		5	10	12	1	1	-	-
	4	8	10	-	0	4	-		5	11	11	1	1	-	-
	4	8	11	-	0	4	-		5	11	12	1	2	-	-
	4	8	12	-	0	-	-		5	12	12	2	2	-	-
	4	9	9	-	0	4	-		6	6	6	-	-	4	4
	4	9	10	-	0	-	-		6	6	7	-	-	4	5
	4	9	11	-	0	-	-		6	6	8	-	-	4	5
	4	9	12	0	0	-	-		6	6	9	-	-	4	5
	4	10	10	-	0	-	-		6	6	10	-	0	5	5
	4	10	11	0	0	-	-		6	6	11	-	0	5	5
	4	10	12	0	1	-	-		6	6	12	0	0	5	6
	4	11	11	0	1	-	-		6	7	7	-	-	4	5
	4	11	12	0	1	-	-		6	7	8	-	0	5	5
	4	12	12	1	1	-	-		6	7	9	-	0	5	5
	5	5	5	-	-	3	4		6	7	10	-	0	5	6
	5	5	6	-	-	3	4		6	7	11	0	0	5	6
	5	5	7	-	-	3	4		6	7	12	0	0	5	6
	5	5	8	-	-	4	4		6	8	8	-	0	5	5
	5	5	9	-	-	4	4		6	8	9	-	0	5	6
	5	5	10	-	-	4	4		6	8	10	0	0	5	6
	5	5	11	-	-	4	5		6	8	11	0	1	6	6
	5	5	12	-	-	4	5		6	8	12	0	1	6	6
	5	6	6	-	-	3	4		6	9	9	0	0	5	6
	5	6	7	-	-	4	4		6	9	10	0	1	6	6
	5	6	8	-	-	4	4		6	9	11	0	1	6	-
	5	6	9	-	-	4	5		6	9	12	1	1	6	-
	5	6	10	-	-	4	5		6	10	10	0	1	6	-
	5	6	11	-	0	4	5		6	10	11	1	1	6	-
	5	6	12	-	0	5	5		6	10	12	1	2	-	-

TABLE 8--CONTINUED

M	N1	N2	N3	.01	.05	.05	.01	M	N1	N2	N2	.01	.05	.05	.01
13	6	11	11	1	2	-	-	13	9	11	11	4	4	9	9
	6	11	12	2	2	-	-		9	11	12	5	5	9	-
	6	12	12	3	3	-	-		9	12	12	6	6	-	-
	7	7	7	-	0	5	5		10	10	10	3	3	8	9
	7	7	8	-	0	5	6		10	10	11	4	4	9	9
	7	7	9	-	0	5	6		10	10	12	5	5	9	10
	7	7	10	0	0	5	6		10	11	11	5	5	9	0
	7	7	11	0	1	6	6		10	11	12	6	6	10	10
	7	7	12	0	1	6	7		10	12	12	7	7	-	-
	7	8	8	-	0	5	6		11	11	11	6	6	10	11
	7	8	9	0	0	6	6		11	11	12	7	7	11	1
	7	8	10	0	1	6	7		11	12	12	8	8	11	-
	7	8	11	0	1	6	7		12	12	12	9	9	12	12
	7	8	12	1	1	6	7	14	1	1	1	-	-	1	1
	7	9	9	0	1	6	7		1	1	2	-	-	1	-
	7	9	10	1	1	6	7		1	1	3	-	-	1	-
	7	9	11	1	2	7	7		1	1	4	-	-	1	-
	7	9	12	1	2	7	7		1	1	5	-	-	1	-
	7	10	10	1	2	7	7		1	1	6	-	-	1	-
	7	10	11	2	2	7	-		1	1	7	-	-	1	-
	7	10	12	2	3	7	-		1	1	8	-	-	1	-
	7	11	11	2	3	7	-		1	1	9	-	-	1	-
	7	11	12	3	3	-	-		1	2	2	-	-	1	-
	7	12	12	4	4	-	-		1	2	3	-	-	1	-
	8	8	8	0	0	6	6		1	2	4	-	-	1	-
	8	8	9	0	1	6	7		1	3	3	-	-	1	-
	8	8	10	1	1	6	7		2	2	2	-	-	1	2
	8	8	11	1	2	7	7		2	2	3	-	-	2	2
	8	8	12	1	2	7	8		2	2	4	-	-	2	2
	8	9	9	1	1	6	7		2	2	5	-	-	2	2
	8	9	10	1	2	7	7		2	2	6	-	-	2	2
	8	9	11	2	2	7	8		2	2	7	-	-	2	2
	8	9	12	2	3	8	8		2	2	8	-	-	2	2
	8	10	10	2	2	7	8		2	2	9	-	-	2	2
	8	10	11	2	3	8	8		2	2	10	-	-	2	2
	8	10	12	3	3	8	-		2	2	11	-	-	2	2
	8	11	11	3	3	8	-		2	2	12	-	-	2	2
	8	11	12	4	4	8	-		2	2	13	-	-	2	2
	8	12	12	5	5	-	-		2	3	3	-	-	2	2
	9	9	9	1	2	7	8		2	3	4	-	-	2	2
	9	9	10	2	2	7	8		2	3	5	-	-	2	2
	9	9	11	2	3	8	8		2	3	6	-	-	2	2
	9	9	12	3	3	8	9		2	3	7	-	-	2	2
	9	10	10	2	3	8	8		2	3	8	-	-	2	-
	9	10	11	3	3	8	9		2	3	9	-	-	2	-
	9	10	12	4	4	9	9		2	3	10	-	-	2	-

TABLE 8--CONTINUED

M	N1	N2	N3	.01	.05	.05	.01
14	2	3	11	-	-	2	-
	2	3	12	-	-	2	-
	2	3	13	-	-	2	-
	2	4	4	-	-	2	2
	2	4	5	-	-	2	2
	2	4	6	-	-	2	-
	2	4	7	-	-	2	-
	2	4	8	-	-	2	-
	2	4	9	-	-	2	-
	2	4	10	-	-	2	-
	2	4	11	-	-	2	-
	2	4	12	-	-	2	-
	2	5	5	-	-	2	-
	2	5	6	-	-	2	-
	2	5	7	-	-	2	-
	2	5	8	-	-	2	-
	2	5	9	-	-	2	-
	2	6	6	-	-	2	-
	2	6	7	-	-	2	-
	2	12	13	-	0	-	-
	2	13	13	-	0	-	-
	3	3	3	-	-	2	2
	3	3	4	-	-	2	2
	3	3	5	-	-	2	3
	3	3	6	-	-	2	3
	3	3	7	-	-	2	3
	3	3	8	-	-	2	3
	3	3	9	-	-	2	3
	3	3	10	-	-	2	3
	3	3	11	-	-	3	3
	3	3	12	-	-	3	3
	3	3	13	-	-	3	3
	3	4	4	-	-	2	3
	3	4	5	-	-	2	3
	3	4	6	-	-	2	3
	3	4	7	-	-	2	3
	3	4	8	-	-	3	3
	3	4	9	-	-	3	3
	3	4	10	-	-	3	3
	3	4	11	-	-	3	3
	3	4	12	-	-	3	3
	3	4	13	-	-	3	3
	3	5	5	-	-	2	3
	3	5	6	-	-	2	3
	3	5	7	-	-	3	3
	3	5	8	-	-	3	3

M	N1	N2	N2	.01	.05	.05	.01
14	3	5	9	-	-	3	3
	3	5	10	-	-	3	3
	3	5	11	-	-	3	-
	3	5	12	-	-	3	-
	3	5	13	-	-	3	-
	3	6	6	-	-	3	3
	3	6	7	-	-	3	3
	3	6	8	-	-	3	3
	3	6	9	-	-	3	-
	3	6	10	-	-	3	-
	3	6	11	-	-	3	-
	3	6	12	-	-	3	-
	3	6	13	-	-	3	-
	3	7	7	-	-	3	3
	3	7	8	-	-	3	-
	3	7	9	-	-	3	-
	3	7	10	-	-	3	-
	3	7	11	-	-	3	-
	3	8	8	-	-	3	-
	3	8	9	-	-	3	-
	3	9	13	-	0	-	-
	3	10	12	-	0	-	-
	3	10	13	-	0	-	-
	3	11	11	-	0	-	-
	3	11	12	-	0	-	-
	3	11	13	0	0	-	-
	3	12	12	0	0	-	-
	3	12	13	0	0	-	-
	3	13	13	0	1	-	-
	4	4	4	-	-	2	3
	4	4	5	-	-	2	3
	4	4	6	-	-	3	3
	4	4	7	-	-	3	3
	4	4	8	-	-	3	3
	4	4	9	-	-	3	3
	4	4	10	-	-	3	4
	4	4	11	-	-	3	4
	4	4	12	-	-	3	4
	4	4	13	-	-	3	4
	4	5	5	-	-	3	3
	4	5	6	-	-	3	3
	4	5	7	-	-	3	3
	4	5	8	-	-	3	4
	4	5	9	-	-	3	4
	4	5	10	-	-	3	4
	4	5	11	-	-	3	4

TABLE 8--CONTINUED

M	N1	N2	N3	.01	.05	.05	.01	M	N1	N2	N2	.01	.05	.05	.01
14	4	5	12	-	-	4	4	14	5	5	13	-	-	4	5
	4	5	13	-	-	4	4		5	6	6	-	-	3	4
	4	6	6	-	-	3	4		5	6	7	-	-	3	4
	4	6	7	-	-	3	4		5	6	8	-	-	4	4
	4	6	8	-	-	3	4		5	6	9	-	-	4	4
	4	6	9	-	-	3	4		5	6	10	-	-	4	5
	4	6	10	-	-	4	4		5	6	11	-	-	4	5
	4	6	11	-	-	4	4		5	6	12	-	-	4	5
	4	6	12	-	-	4	4		5	6	13	-	0	4	5
	4	6	13	-	-	4	-		5	7	7	-	-	4	4
	4	7	7	-	-	3	4		5	7	8	-	-	4	5
	4	7	8	-	-	3	4		5	7	9	-	-	4	5
	4	7	9	-	-	4	4		5	7	10	-	-	4	5
	4	7	10	-	-	4	4		5	7	11	-	0	4	5
	4	7	11	-	-	4	-		5	7	12	-	0	5	5
	4	7	12	-	-	4	-		5	7	13	-	0	5	5
	4	7	13	-	-	4	-		5	8	8	-	-	4	5
	4	8	8	-	-	4	4		5	8	9	-	-	4	5
	4	8	9	-	-	4	4		5	8	10	-	0	5	5
	4	8	10	-	-	4	-		5	8	11	-	0	5	5
	4	8	11	-	-	4	-		5	8	12	-	0	5	-
	4	8	12	-	0	4	-		5	8	13	0	0	5	-
	4	8	13	-	0	4	-		5	9	9	-	0	5	5
	4	9	9	-	-	4	-		5	9	10	-	0	5	5
	4	9	10	-	-	4	-		5	9	11	-	0	5	-
	4	9	11	-	0	4	-		5	9	12	0	0	5	-
	4	9	12	-	0	-	-		5	9	13	0	1	5	-
	4	9	13	-	0	-	-		5	10	10	-	0	5	-
	4	10	10	-	0	4	-		5	10	11	0	0	5	-
	4	10	11	-	0	-	-		5	10	12	0	1	5	-
	4	10	12	0	0	-	-		5	10	13	0	1	-	-
	4	10	13	0	0	-	-		5	11	11	0	1	-	-
	4	11	11	0	0	-	-		5	11	12	0	1	-	-
	4	11	12	0	0	-	-		5	11	13	1	1	-	-
	4	11	13	0	1	-	-		5	12	12	1	1	-	-
	4	12	12	0	1	-	-		5	12	13	1	2	-	-
	4	12	13	1	1	-	-		5	13	13	2	2	-	-
	4	13	13	1	1	-	-		6	6	6	-	-	3	4
	5	5	5	-	-	3	3		6	6	7	-	-	4	4
	5	5	6	-	-	3	4		6	6	8	-	-	4	5
	5	5	7	-	-	3	4		6	6	9	-	-	4	5
	5	5	8	-	-	3	4		6	6	10	-	-	4	5
	5	5	9	-	-	4	4		6	6	11	-	0	5	5
	5	5	10	-	-	4	4		6	6	12	-	0	5	5
	5	5	11	-	-	4	4		6	6	13	-	0	5	6
	5	5	12	-	-	4	5		6	7	7	-	-	4	5

TABLE 8--CONTINUED

M	N1	N2	N3	.01	.05	.05	.01	M	N1	N2	N2	.01	.05	.05	.01
14	6	7	8	-	-	4	5	14	7	10	11	1	1	7	7
	6	7	9	-	-	4	5		7	10	12	1	2	7	7
	6	7	10	-	0	5	5		7	10	13	2	2	7	-
	6	7	11	-	0	5	6		7	11	11	1	2	7	7
	6	7	12	-	0	5	6		7	11	12	2	2	7	-
	6	7	13	0	0	5	6		7	11	13	2	3	7	-
	6	8	8	-	-	5	5		7	12	12	2	3	-	-
	6	8	9	-	0	5	5		7	12	13	3	3	-	-
	6	8	10	-	0	5	6		7	13	13	4	4	-	-
	6	8	11	0	0	5	6		8	8	8	-	0	5	6
	6	8	12	0	0	5	6		8	8	9	0	0	6	6
	6	8	13	0	1	6	6		8	8	10	0	1	6	7
	6	9	9	-	0	5	6		8	8	11	0	1	6	7
	6	9	10	0	0	5	6		8	8	12	1	1	7	7
	6	9	11	0	0	6	6		8	8	13	1	2	7	7
	6	9	12	0	1	6	6		8	9	9	0	1	6	7
	6	9	13	0	1	6	-		8	9	10	0	1	6	7
	6	10	10	0	0	6	6		8	9	11	1	1	7	7
	6	10	11	0	1	6	-		8	9	12	1	2	7	8
	6	10	12	1	1	6	-		8	9	13	2	2	7	8
	6	10	13	1	2	6	-		8	10	10	1	1	7	7
	6	11	11	1	1	6	-		8	10	11	1	2	7	8
	6	11	12	1	2	6	-		8	10	12	2	2	7	8
	6	11	13	1	2	-	-		8	10	13	2	3	8	8
	6	12	12	1	2	-	-		8	11	11	2	2	7	8
	6	12	13	2	3	-	-		8	11	12	2	3	8	8
	6	13	13	3	3	-	-		8	11	13	3	3	8	-
	7	7	7	-	-	4	5		8	12	12	3	3	8	-
	7	7	8	-	-	5	5		8	12	13	4	4	-	-
	7	7	9	-	0	5	6		8	13	13	5	5	-	-
	7	7	10	-	0	5	6		9	9	9	0	1	6	7
	7	7	11	0	0	5	6		9	9	10	1	1	7	8
	7	7	12	0	0	6	6		9	9	11	1	2	7	8
	7	7	13	0	1	6	6		9	9	12	2	2	8	8
	7	8	8	-	0	5	6		9	9	13	2	3	8	8
	7	8	9	-	0	5	6		9	10	10	1	2	7	8
	7	8	10	0	0	5	6		9	10	11	2	2	8	8
	7	8	11	0	1	6	6		9	10	12	2	3	8	9
	7	8	12	0	1	6	7		9	10	13	3	4	8	9
	7	8	13	1	1	6	7		9	11	11	2	3	8	9
	7	9	9	0	0	6	6		9	11	12	3	4	9	9
	7	9	10	0	1	6	7		9	11	13	4	4	9	9
	7	9	11	0	1	6	7		9	12	12	4	4	9	-
	7	9	12	1	1	6	7		9	12	13	5	5	9	-
	7	9	13	1	2	7	7		9	13	13	6	6	-	-
	7	10	10	0	1	6	7		10	10	10	2	2	8	8

TABLE 8--CONTINUED

M	N1	N2	N3	.01	.05	.05	.01	M	N1	N2	N2	.01	.05	.05	.01
14	10	10	11	2	3	8	9	15	2	2	13	-	-	2	2
	10	10	12	3	4	9	9		2	2	14	-	-	2	2
	10	10	13	4	4	9	10		2	3	3	-	-	2	2
	10	11	11	3	4	9	9		2	3	4	-	-	2	2
	10	11	12	4	4	9	10		2	3	5	-	-	2	2
	10	11	13	5	5	10	10		2	3	6	-	-	2	2
	10	12	12	5	5	10	10		2	3	7	-	-	2	2
	10	12	13	6	6	10	-		2	3	8	-	-	2	2
	10	13	13	7	7	-	-		2	3	9	-	-	2	2
	11	11	11	4	4	9	10		2	3	10	-	-	2	-
	11	11	12	5	5	10	10		2	3	11	-	-	2	-
	11	11	13	6	6	10	11		2	3	12	-	-	2	-
	11	12	12	6	6	10	11		2	3	13	-	-	2	-
	11	12	13	7	7	11	11		2	3	14	-	-	2	-
	11	13	13	8	8	11	-		2	4	4	-	-	2	2
	12	12	12	7	7	11	12		2	4	5	-	-	2	2
	12	12	13	8	8	12	12		2	4	6	-	-	2	2
	12	13	13	9	9	12	-		2	4	7	-	-	2	-
	13	13	13	10	10	13	13		2	4	8	-	-	2	-
15	1	1	1	-	-	1	1		2	4	9	-	-	2	-
	1	1	2	-	-	1	1		2	4	10	-	-	2	-
	1	1	3	-	-	1	-		2	4	11	-	-	2	-
	1	1	4	-	-	1	-		2	4	12	-	-	2	-
	1	1	5	-	-	1	-		2	4	13	-	-	2	-
	1	1	6	-	-	1	-		2	4	14	-	-	2	-
	1	1	7	-	-	1	-		2	5	5	-	-	2	2
	1	1	8	-	-	1	-		2	5	6	-	-	2	-
	1	1	9	-	-	1	-		2	5	7	-	-	2	-
	1	1	10	-	-	1	-		2	5	8	-	-	2	-
	1	1	11	-	-	1	-		2	5	9	-	-	2	-
	1	2	2	-	-	1	-		2	5	10	-	-	2	-
	1	2	3	-	-	1	-		2	5	11	-	-	2	-
	1	2	4	-	-	1	-		2	6	6	-	-	2	-
	1	2	5	-	-	1	-		2	6	7	-	-	2	-
	1	3	3	-	-	1	-		2	6	8	-	-	2	-
	2	2	2	-	-	1	2		2	6	9	-	-	2	-
	2	2	3	-	-	2	2		2	7	7	-	-	2	-
	2	2	4	-	-	2	2		2	13	13	-	0	-	-
	2	2	5	-	-	2	2		2	13	14	-	0	-	-
	2	2	6	-	-	2	2		2	14	14	0	0	-	-
	2	2	7	-	-	2	2		3	3	3	-	-	2	2
	2	2	8	-	-	2	2		3	3	4	-	-	2	2
	2	2	9	-	-	2	2		3	3	5	-	-	2	2
	2	2	10	-	-	2	2		3	3	6	-	-	2	3
	2	2	11	-	-	2	2		3	3	7	-	-	2	3
	2	2	12	-	-	2	2		3	3	8	-	-	2	3

TABLE 8--CONTINUED

M	N1	N2	N3	.01	.05	.05	.01	M	N1	N2	N2	.01	.05	.05	.01
15	3	3	9	-	-	2	3	15	3	8	11	-	-	3	-
	3	3	10	-	-	2	3		3	9	9	-	-	3	-
	3	3	11	-	-	2	3		3	9	10	-	-	3	-
	3	3	12	-	-	3	3		3	10	14	-	0	-	-
	3	3	13	-	-	3	3		3	11	13	-	0	-	-
	3	3	14	-	-	3	3		3	11	14	-	0	-	-
	3	4	4	-	-	2	2		3	12	12	-	0	-	-
	3	4	5	-	-	2	3		3	12	13	-	0	-	-
	3	4	6	-	-	2	3		3	12	14	0	0	-	-
	3	4	7	-	-	2	3		3	13	13	0	0	-	-
	3	4	8	-	-	2	3		3	13	14	0	0	-	-
	3	4	9	-	-	3	3		3	14	14	0	1	-	-
	3	4	10	-	-	3	3		4	4	4	-	-	2	3
	3	4	11	-	-	3	3		4	4	5	-	-	2	3
	3	4	12	-	-	3	3		4	4	6	-	-	2	3
	3	4	13	-	-	3	3		4	4	7	-	-	3	3
	3	4	14	-	-	3	3		4	4	8	-	-	3	3
	3	5	5	-	-	2	3		4	4	9	-	-	3	3
	3	5	6	-	-	2	3		4	4	10	-	-	3	3
	3	5	7	-	-	3	3		4	4	11	-	-	3	4
	3	5	8	-	-	3	3		4	4	12	-	-	3	4
	3	5	9	-	-	3	3		4	4	13	-	-	3	4
	3	5	10	-	-	3	3		4	4	14	-	-	3	4
	3	5	11	-	-	3	3		4	5	5	-	-	2	3
	3	5	12	-	-	3	-		4	5	6	-	-	3	3
	3	5	13	-	-	3	-		4	5	7	-	-	3	3
	3	5	14	-	-	3	-		4	5	8	-	-	3	4
	3	6	6	-	-	3	3		4	5	9	-	-	3	4
	3	6	7	-	-	3	3		4	5	10	-	-	3	4
	3	6	8	-	-	3	3		4	5	11	-	-	3	4
	3	6	9	-	-	3	3		4	5	12	-	-	3	4
	3	6	10	-	-	3	-		4	5	13	-	-	3	4
	3	6	11	-	-	3	-		4	5	14	-	-	4	4
	3	6	12	-	-	3	-		4	6	6	-	-	3	3
	3	6	13	-	-	3	-		4	6	7	-	-	3	4
	3	6	14	-	-	3	-		4	6	8	-	-	3	4
	3	7	7	-	-	3	3		4	6	9	-	-	3	4
	3	7	8	-	-	3	3		4	6	10	-	-	3	4
	3	7	9	-	-	3	-		4	6	11	-	-	4	4
	3	7	10	-	-	3	-		4	6	12	-	-	4	4
	3	7	11	-	-	3	-		4	6	13	-	-	4	4
	3	7	12	-	-	3	-		4	6	14	-	-	4	4
	3	7	13	-	-	3	-		4	7	7	-	-	3	4
	3	8	8	-	-	3	-		4	7	8	-	-	3	4
	3	8	9	-	-	3	-		4	7	9	-	-	3	4
	3	8	10	-	-	3	-		4	7	10	-	-	4	4

TABLE 8--CONTINUED

M	N1	N2	N3	.01	.05	.05	.01	M	N1	N2	N2	.01	.05	.05	.01
15	4	7	11	-	-	4	4	15	5	6	10	-	-	4	4
	4	7	12	-	-	4	4		5	6	11	-	-	4	5
	4	7	13	-	-	4	-		5	6	12	-	-	4	5
	4	7	14	-	-	4	-		5	6	13	-	-	4	5
	4	8	8	-	-	4	4		5	6	14	-	-	4	5
	4	8	9	-	-	4	4		5	7	7	-	-	3	4
	4	8	10	-	-	4	4		5	7	8	-	-	4	4
	4	8	11	-	-	4	-		5	7	9	-	-	4	5
	4	8	12	-	-	4	-		5	7	10	-	-	4	5
	4	8	13	-	-	4	-		5	7	11	-	-	4	5
	4	8	14	-	0	4	-		5	7	12	-	-	4	5
	4	9	9	-	-	4	4		5	7	13	-	0	5	5
	4	9	10	-	-	4	-		5	7	14	-	0	5	5
	4	9	11	-	-	4	-		5	8	8	-	-	4	5
	4	9	12	-	0	4	-		5	8	9	-	-	4	5
	4	9	13	-	0	4	-		5	8	10	-	-	4	5
	4	9	14	-	0	-	-		5	8	11	-	-	4	5
	4	10	10	-	-	4	-		5	8	12	-	0	5	5
	4	10	11	-	0	4	-		5	8	13	-	0	5	5
	4	10	12	-	0	-	-		5	8	14	-	0	5	-
	4	10	13	-	0	-	-		5	9	9	-	-	4	5
	4	10	14	0	0	-	-		5	9	10	-	0	5	5
	4	11	11	-	0	-	-		5	9	11	-	0	5	5
	4	11	12	-	0	-	-		5	9	12	-	0	5	-
	4	11	13	0	0	-	-		5	9	13	-	0	5	-
	4	11	14	0	0	-	-		5	9	14	0	0	5	-
	4	12	12	0	0	-	-		5	10	10	-	0	5	5
	4	12	13	0	0	-	-		5	10	11	-	0	5	-
	4	12	14	0	1	-	-		5	10	12	0	0	5	-
	4	13	13	0	1	-	-		5	10	13	0	0	5	-
	4	13	14	1	1	-	-		5	10	14	0	1	-	-
	4	14	14	1	1	-	-		5	11	11	0	0	5	-
	5	5	5	-	-	3	3		5	11	12	0	0	5	-
	5	5	6	-	-	3	3		5	11	13	0	1	-	-
	5	5	7	-	-	3	4		5	11	14	0	1	-	-
	5	5	8	-	-	3	4		5	12	12	0	1	-	-
	5	5	9	-	-	3	4		5	12	13	0	1	-	-
	5	5	10	-	-	3	4		5	12	14	1	1	-	-
	5	5	11	-	-	4	4		5	13	13	1	1	-	-
	5	5	12	-	-	4	4		5	13	14	1	2	-	-
	5	5	13	-	-	4	4		5	14	14	2	2	-	-
	5	5	14	-	-	4	5		6	6	6	-	-	3	4
	5	6	6	-	-	3	4		6	6	7	-	-	4	4
	5	6	7	-	-	3	4		6	6	8	-	-	4	4
	5	6	8	-	-	3	4		6	6	9	-	-	4	5
	5	6	9	-	-	4	4		6	6	10	-	-	4	5

TABLE 8--CONTINUED

M	N1	N2	N3	.01	.05	.05	.01	M	N1	N2	N2	.01	.05	.05	.01
15	6	6	11	-	-	4	5	15	7	7	13	0	0	5	6
	6	6	12	-	-	4	5		7	7	14	0	0	6	6
	6	6	13	-	0	5	5		7	8	8	-	-	5	5
	6	6	14	-	0	5	5		7	8	9	-	0	5	6
	6	7	7	-	-	4	4		7	8	10	-	0	5	6
	6	7	8	-	-	4	5		7	8	11	-	0	5	6
	6	7	9	-	-	4	5		7	8	12	0	0	6	6
	6	7	10	-	-	4	5		7	8	13	0	1	6	7
	6	7	11	-	0	5	5		7	8	14	0	1	6	7
	6	7	12	-	0	5	5		7	9	9	-	0	5	6
	6	7	13	-	0	5	6		7	9	10	0	0	5	6
	6	7	14	-	0	5	6		7	9	11	0	0	6	6
	6	8	8	-	-	4	5		7	9	12	0	1	6	7
	6	8	9	-	-	4	5		7	9	13	0	1	6	7
	6	8	10	-	0	5	5		7	9	14	1	1	6	7
	6	8	11	-	0	5	6		7	10	10	0	0	6	6
	6	8	12	-	0	5	6		7	10	11	0	1	6	7
	6	8	13	0	0	5	6		7	10	12	0	1	6	7
	6	8	14	0	0	6	6		7	10	13	1	1	7	7
	6	9	9	-	0	5	5		7	10	14	1	2	7	7
	6	9	10	-	0	5	6		7	11	11	0	1	6	7
	6	9	11	-	0	5	6		7	11	12	1	1	7	7
	6	9	12	0	0	5	6		7	11	13	1	2	7	-
	6	9	13	0	1	6	6		7	11	14	2	2	7	-
	6	9	14	0	1	6	-		7	12	12	1	2	7	-
	6	10	10	-	0	5	6		7	12	13	2	2	7	-
	6	10	11	0	0	6	6		7	12	14	2	3	-	-
	6	10	12	0	1	6	6		7	13	13	2	3	-	-
	6	10	13	0	1	6	-		7	13	14	3	3	-	-
	6	10	14	1	1	6	-		7	14	14	4	4	-	-
	6	11	11	0	1	6	6		8	8	8	-	0	5	6
	6	11	12	0	1	6	-		8	8	9	-	0	5	6
	6	11	13	1	1	6	-		8	8	10	0	0	6	6
	6	11	14	1	2	-	-		8	8	11	0	0	6	7
	6	12	12	1	1	6	-		8	8	12	0	1	6	7
	6	12	13	1	2	-	-		8	8	13	0	1	6	7
	6	12	14	2	2	-	-		8	8	14	1	1	7	7
	6	13	13	2	2	-	-		8	9	9	0	0	6	6
	6	13	14	2	3	-	-		8	9	10	0	1	6	7
	6	14	14	3	3	-	-		8	9	11	0	1	6	7
	7	7	7	-	-	4	5		8	9	12	0	1	7	7
	7	7	8	-	-	4	5		8	9	13	1	2	7	7
	7	7	9	-	-	5	5		8	9	14	1	2	7	8
	7	7	10	-	0	5	5		8	10	10	0	1	6	7
	7	7	11	-	0	5	6		8	10	11	1	1	7	7
	7	7	12	-	0	5	6		8	10	12	1	2	7	8

TABLE 8--CONTINUED

M	N1	N2	N3	.01	.05	.05	.01
15	8	10	13	1	2	7	8
	8	10	14	2	2	8	8
	8	11	11	1	2	7	8
	8	11	12	1	2	7	8
	8	11	13	2	3	8	8
	8	11	14	2	3	8	-
	8	12	12	2	3	8	8
	8	12	13	2	3	8	-
	8	12	14	3	4	8	-
	8	13	13	3	4	8	-
	8	13	14	4	4	-	-
	8	14	14	5	5	-	-
	9	9	9	0	1	6	7
	9	9	10	0	1	6	7
	9	9	11	1	1	7	7
	9	9	12	1	2	7	8
	9	9	13	1	2	7	8
	9	9	14	2	2	8	8
	9	10	10	1	1	7	7
	9	10	11	1	2	7	8
	9	10	12	1	2	7	8
	9	10	13	2	3	8	9
	9	10	14	2	3	8	9
	9	11	11	1	2	8	8
	9	11	12	2	3	8	9
	9	11	13	3	3	8	9
	9	11	14	3	4	9	9
	9	12	12	3	3	8	9
	9	12	13	3	4	9	9
	9	12	14	4	4	9	-
	9	13	13	4	4	9	-
	9	13	14	5	5	-	-
	9	14	14	6	6	-	-
	10	10	10	1	2	7	8
	10	10	11	2	2	8	8
	10	10	12	2	3	8	9
	10	10	13	3	3	8	9
	10	10	14	3	4	9	9
	10	11	11	2	3	8	9
	10	11	12	3	3	8	9
	10	11	13	3	4	9	10
	10	11	14	4	4	9	10
	10	12	12	3	4	9	10
	10	12	13	4	4	9	10
	10	12	14	5	5	10	10
	10	13	13	5	5	10	10

M	N1	N2	N2	.01	.05	.05	.01
15	10	13	14	6	6	10	-
	10	14	14	7	7	-	-
	11	11	11	3	3	9	9
	11	11	12	3	4	9	10
	11	11	13	4	5	10	10
	11	11	14	5	5	10	1
	11	12	12	4	5	10	0
	11	12	13	5	5	10	1
	11	12	14	6	6	11	1
	11	13	13	6	6	11	1
	11	13	14	7	7	11	-
	11	14	14	8	8	-	-
	12	12	12	5	5	10	11
	12	12	13	6	6	11	11
	12	12	14	7	7	11	12
	12	13	13	7	7	11	12
	12	13	14	8	8	12	12
	12	14	14	9	9	12	-
	13	13	13	8	8	12	12
	13	13	14	9	9	13	13
	13	14	14	10	10	13	-
	14	14	14	11	11	14	14

APPENDIX B

COMPUTING PROGRAM FOR EXACT RROBABILITIES

```
      DIMENSION P(500),Y(500),Z(10)
      ERROR(0)
      READ(5,100) N,M
C     Z(1) MUST BE THE SMALLEST MARGINAL TOTAL.
      READ(5,200) (Z(I),I=1,N)
      INUL=0
      IMAX=Z(1)+1.
      XM=M
      JOB=1
      A=M
      B=Z(2)
      C=Z(1)
      D=INUL
      E=XM-Z(1)
      F=Z(2)-D
      II=0
      KK=IMAX-INUL
      GO TO 40
    1 P(II)=PRD
      X1=C-D
      X2=D+1.
      X3=E-F+1.
      II=II+1
      IF(KK-II)64,63,63
   63 DO 3 I=II,KK
      J=I-1
      S=I-II
      X4=(X1-S)/(X2+S)
      X5=(F-S)/(X3+S)
      P(I)=X4*X5*P(J)
    3 CONTINUE
C     OUTPUT IS GIVEN FOR ALL CATEGORIES.
      J=2
      WRITE(6,100) J,M,INUL,IMAX
      WRITE(6,400) (Z(I),I=1,2)
      X1=0.
      DO 68 I=1,KK
      J=I-1
```

```
      X1=X1+P(I)
   68 WRITE(6,300) J,P(I),X1
      IF(N-2)201,201,65
   65 JOB=2
      DO 9 LOOP=3,N
      WRITE(6,100) LOOP,M,INUL,IMAX
      WRITE(6,400) (Z(I),I=1,LOOP)
      X5=0.
      II=0
      JJ=0
      B=Z(LOOP)
      D=INUL
      C=D
      E=XM-D
      F=Z(LOOP)-D
      IF(D)50,50,40
   50 PRD=1.
      II=1
    2 Y(II)=PRD
      II=II+1
      JJ=JJ+1
      X1=E-F
      IF(KK-II)21,20,20
   20 DO 6 J=II,KK
      I=J-1
      S=J-II
      U=S+1.
      X3=(X1-S)/(E-S)
      X2=(C+U)/U
      Y(J)=X2*X3*Y(I)
    6 CONTINUE
      SUM=0.
      J=II-1
      DO 7 I=J,KK
    7 SUM=SUM+P(I)*Y(I)
      P(JJ)=SUM
      I=JJ-1
      X5=X5+P(JJ)
      IF(I)91,91,92
   92 IF(P(JJ)-P(I))90,91,91
   90 IF(P(JJ)-T)93,91,91
   93 P(JJ)=0.
   91 WRITE(6,300) I,P(JJ),X5
      D=D+1.
      IF(Z(1)-D)9,51,51
   51 X4=1./E
      PRD=F*X4*Y(J)
      II=J+1
      C=D
      F=F-1.
```

```
      E=E-1.
      IF(PRD)40,40,2
    9 CONTINUE
      GO TO 201
   40 PRD=1.
      IF(JOB-1)46,46,48
   46 II=II+1
      IF(E-F)47,48,48
   47 D=D+1.
      F=F-1.
      P(II)=0.
      GO TO 46
   48 L=B
      DO 45 K=1,L
      S=K
      IF(D-S)42,41,41
   41 PRD=PRD*((C-D+S)/S)
   42 IF(F-S)44,43,43
   43 PRD=PRD*((E-F+S)/S)
   44 PRD=PRD*(S/(A-B+S))
      IF(PRD)80,80,45
   45 CONTINUE
      GO TO (1,2),JOB
   80 IF(KK-II)57,57,58
   57 GO TO(1,2),JOB
   58 PRD=1.
      IF(JOB-1)55,55,49
   49 C=C+1.
      E=E-1.
      II=II+1
      GO TO 48
   55 D=D+1.
      F=F-1.
      P(II)=0.
      II=II+1
      GO TO 48
  201 CONTINUE
  100 FORMAT(4I5,2E12.6)
  200 FORMAT(F10.4)
  300 FORMAT(2X,I4,2(3X,E12.6))
  400 FORMAT(1X,I5)
      END
```