# RADIATION EFFECTS IN CRYSTALLINE $\mathrm{SiO}_{2}$ : STRUCTURE 

OF THE E" CENTERS

## By

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DEDICATION
to

MY PARENTS, RAHUL, PRAGNA, SHAILESH
AND MANDAKINI

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Thesis Approved:


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## INTRODUCTION

## Background

Radiation-induced defects in insulating (i.e., wide band gap) materials have been widely studied in the last fifty years. Much of the interest has arisen because of their fundamental properties and also because of their technological potential. Optical and magnetic resonance techniques have been primarily used in these investigations and a number of sophisticated combinations of these techniques have evolved. Point defects in insulators are generally known as color centers because they usually have two or more electronic states lying within the band gap between which optical transitions are possible. A vast amount of research has been done on color centers in alkali halides and much of this work has been described in depth by Fowler (1). A similar large amount of work on defects in alkaline-earth oxides has been reviewed by Henderson and Wertz (2).

In more recent years, researchers have focused their attention on piezoelectric materials such as quartz, $\mathrm{LiNbO}_{3}, \mathrm{LiTaO}_{3}$, and berlinite $\left(\mathrm{AlPO}_{4}\right)$ because of the many applications in precision frequency control and surface acoustic wave (SAW) devices. Since many of these latter materials have complex crystal structures, a much wider variety of defect configurations are possible and, as a result, it is very difficult
to determine precise models for defects or to disentangle the observed kinetic behavior of defects. However, because of the vast amount of work previously done on point defects in the simpler materials such as alkali halides and alkaline-earth oxides, researchers today have a base of information which allows progress to be made more rapidly in the studies of the more complex, but more useful, materials.

Since its discovery by E. Zavoisky in 1945 (3), the electron spin resonance (ESR) technique has been widely used in different branches of the physical and biological sciences. In solid state physics, ESR is a very powerful technique making it possible to obtain an exact knowledge of the electronic structure and the local environment of a defect. Also, it has been used to study the influence of external stress and the effects of electric fields upon the properties of color centers and their interactions, and it can measure concentrations of these defects.

Magnetic resonance is an experimental technique that can be applied to the study of materials containing permanent magnetic dipole moments. Both electrons and nuclei have permanent magnetic moments. Let's consider a spin system consisting of non-interacting electron spins. The magnetic moment $\vec{\mu}$ of an electron is related to the spin angular momentum $\vec{S}$ as follows

$$
\vec{\mu}=-g_{e} \beta \vec{S}
$$

where $\beta$ is the Bohr magneton $\left(\beta=\frac{e \hbar}{2 m c}=9.2741 \times 10^{-21} \mathrm{erg} /\right.$ gauss $)$ and $g_{e}$ is the free spin $g$ factor. The energy of a magnetic dipole of moment $\vec{\mu}$ in a magnetic field $\vec{H}$ is

$$
W=-\vec{\mu} \cdot \overrightarrow{\mathrm{H}}
$$

From the above two equations, the Hamiltonian can be written as

$$
\partial r=g_{e} \beta \vec{S} \cdot \vec{H} .
$$

If the electron spin is quantized along the direction of the magnetic field (taken to be $Z$ ) then

$$
\hat{H}=g_{e} \beta H \hat{S}_{z}
$$

where $\hat{S}_{z}$ is a spin operator.
The energy is given by

$$
E=\left\langle M_{S}\right| \mathcal{A}\left|M_{S}\right\rangle
$$

which reduces to the following

$$
E=g_{e} \beta \mathrm{HM}_{s}
$$

There are two values for $M_{s},+1 / 2$ and $-1 / 2$. This gives us two energy levels


$$
\Delta E=E_{+1 / 2}-E_{-1 / 2}=g_{e}^{\beta H}
$$

Transitions between the two energy levels can be induced by an electromagnetic field of the appropriate frequency $v$, if the photon energy $h \nu$ matches the energy-level separation $\Delta E$. Then

$$
h \nu=g \beta H_{r}
$$

where $H_{r}$ is the magnetic field at which the resonance condition is met. Further details of the magnetic resonance techniques have been well reviewed in the literature (4-7).

Defects in Quartz

Structural and radiation-induced defects in quartz have been studied extensively over the last twenty-five years because of the wide application of this material in precision frequency control devices; one example being in crystal-controlled oscillators in communication satellites that are continuously bombarded by high-energy radiation. It is now well-known that defects in the quartz crystals play an important role in determining the performance of these oscillators. One of the first radiation-induced defects in quartz to be studied by ESR was reported by Weeks (8) in 1956. It is an oxygen-vacancy-associated trapped electron center and is known as the $E_{1}^{\prime}$ center. Since the mid1950s, many people have reported studies of defects in quartz. Some of the more pertinent of these studies are discussed briefly in the following paragraphs.

Weeks and Nelson (9) investigated radiation-induced defects in quartz and fused silica using optical and magnetic resonance techniques.

Evidence was presented showing a correlation of the observed optical absorption band at 210 nm with the $\mathrm{E}_{\mathrm{l}}^{\prime}$ center and a correlation of the 230 nm band with the $\mathrm{E}_{2}^{\prime}$ center, in both quartz and fused silica. For crystalline quartz it was observed that the $\mathrm{E}_{2}^{\prime}$ band was more easily produced than the $E_{1}^{\prime}$ band, and the $E_{2}^{\prime}$ band could be bleached easily at 78 K using an ultraviolet lamp. Heating treatments between $250-300^{\circ} \mathrm{C}$ revealed an increase in the $E_{1}^{\prime}$ optical absorption band and the ESR absorption associated with this center. In contrast to the behaviour of crystalline quartz, the $E_{1}^{\prime}$ optical absorption band in high purity fused silica seemed to decrease in intensity upon heating above the room temperature. Another optical absorption band at 260 nm was observed in irradiated fused silica, which annealed in the same manner as the $E_{1}^{\prime}$ optical band. This band was not associated with any of the defects observed by ESR. Models for the $E_{1}^{\prime}, E_{2}^{\prime}$, and $E_{1}^{\prime \prime}$ centers were proposed. [The E" notation used in this dissertation is different from their notation.] These workers have suggested that the $E_{1}^{\prime}$ center is an oxygen vacancy with one trapped electron at the vacancy and the $E_{I}^{\prime \prime}$ center is an oxygen vacancy with two electrons $(S=0)$. They proposed that the $E_{2}^{\prime}$ center is an oxygen vacancy with a trapped electron in an unsaturated silicon bond.

Silsbee (10) reported a detailed ESR study of the $E_{1}^{\prime}$ center which followed the initial work by Weeks (8). Neutron irradiation of the quartz crystals at room temperature was used to produce the $E_{1}^{\prime}$ centers. The optical rotation and etch patterns indicated that the crystal was left-handed quartz. The observed ESR spectrum was interpreted as being due to a single unpaired electron interacting strongly with one ${ }^{29}$ Si nucleus and weakly with two other ${ }^{29}$ Si nuclei. The $\vec{g}^{\prime} \overrightarrow{\mathrm{A}}_{29}$
(strong hfs) and $\vec{A}_{29_{S i}}(I, I I)$ (weak hfs) matrix parameters were obtained. On the basis of experimental results and calculations, it was suggested that possibly the $E_{1}^{\prime}$ center is an oxygen vacancy with the electron in a nonbonding $\mathrm{sp}^{3}$ hybrid orbital localized primarily on one silicon. But this model could not explain the observed weak hyperfine ESR spectrum. Further work on the $E_{l}^{\prime}$ center has been reported by Feigl et al. (11) and Yip and Fowler (12). A linear combination of localized orbitalmolecular orbital (LCLO-MO) cluster method was used to calculate the electronic structure of the $E_{1}^{\prime}$ center in $\alpha$-quartz. As a result of their calculations, it was suggested that the $E_{1}^{\prime}$ center is an $O^{-}$vacancy with a highly asymetric relaxation of the two silicons adjacent to the $0^{-}$ vacancy. The unpaired electron is strongly localized in the nonbonding sp ${ }^{3}$ hybrid orbital centered on the closest silicon and oriented along the Si-O short bond direction toward the oxygen vacancy as shown in Figure 1.

The $E_{2}^{\prime}$ center in $\alpha$-quartz was studied in detail by Weeks (13) using ESR. The observed ESR spectrum when the magnetic field is along the c-axis consisted of two primary lines separated by 0.4G at 300K. From the ESR measurements at 24 GHz it was decided that the two-line spectrum was due to an unpaired electron $(S=1 / 2)$ at the defect interacting with a single 100\% abundant nuclear spin $I=1 / 2$. In addition to the primary lines, additional pairs of weak lines, one pair at high field and one pair at low field, separated by 412.0 G were observed in the c-axis spectrum. The separation within these pairs of lines was the same as the primary doublet, 0.4 G . The ratio of the intensities of each of these lines to each component of the primary lines was 0.03 . No additional weak pairs of lines with $7 G$ and $8 G$ spiittings were observed for the $E_{2}^{\prime}$


Silicon


Figure 1. Proposed Model for the $E_{1}^{\prime}$ Center (Yip and Fowler Ref). The Arrows Indicate the Assymetric Relaxation of the Two Silicon Positions. The Unpaired Electron is Located on Si(O) (i.e., the Short-Bond Side)
center, in contrast to the $E_{1}^{\prime}$ center (10). From the comparison of the crystals grown in $\mathrm{H}_{2} \mathrm{O}$ with one grown in $\mathrm{D}_{2} \mathrm{O}$, it was determined that the doublet structure of the $E_{2}^{\prime}$ center is due to a hyperfine interaction of the unpaired electron with a nearby proton. The weak (0.03 intensity), but widely separated, pairs of hyperfine interactions were interpreted as arising from an interaction of the defect electron with a ${ }^{29}$ Si nucleus. Since only one pair of such lines and no additional weak interactions were observed, Weeks proposed that the $E_{2}^{\prime}$ center is an electron trapped in a nonbonding $\mathrm{sp}^{3}$ hybrid orbital on a silicon with an oxygen and the next neighbor silicon missing, as shown in Figure 2. The excess charge due to the remaining oxygen ions around the silicon vacancy are compensated by the monovalent and divalent cation impurities present in the crystal. The nearby proton associated with the $E_{2}^{\prime}$ center is one of these impurities. Considering the weak hyperfine interaction of the $E_{1}^{\prime}$ center with two nearby ${ }^{29}$ Si, the proposed model by Weeks is an oxygen divacancy shown in Figure 2. In this model the weak hyperfine interaction arises from the two nearest-neighbor silicons through the oxygen vacancies. Such a defect could trap four or two electrons in a nonparamagnetic state, and such a nonparamagnetic state has been observed indirectly according to Weeks and Nelson (9).

In support of the models proposed by Weeks (13) for the $E_{1}^{\prime}$ and $E_{2}^{\prime}$ center, Castle et al. (14), measured the spin-lattice relaxation time $\mathrm{T}_{1}$ by the inversion-recovery technique for these centers over a wide temperature range. The data were interpreted in terms of cross relaxation, direct processes, and Raman processes. The theory of spinlattice relaxation was extended to account for the modification of strain at a defect site due to a lattice wave. A detailed comparison


Proposed model for the $E$ ' center. Location of nearby proton is not specified. The unpaired electron is located on Si(4).

Figure 2. Models for the $E_{i}^{\prime}$ and $E_{2}^{\prime}$ Center as Proposed by R. A. Weeks (13)


Figure 2 (Continued)
with theory of the relaxation rates observed for the two $E$ ' centers suggested that each center had two characteristic frequencies or temperatures $\theta_{i}$. For the $E_{2}^{\prime}$ center, one of these was ascribed to the vibration of a neighboring impurity ion, probably a proton. The authors proposed that the $E_{i}^{\prime}$ center is an electron trapped at a silicon ion located in an oxygen divacancy, which is similar to the model proposed by Weeks (13).

Feigl and Anderson (15) used ESR to study paramagnetic defects produced by low energy ionizing radiation in crystalline quartz doped with germanium. Their analysis indicated that these defects are similar to the $E_{1}^{\prime}$ and $E_{2}^{\prime}$ centers in pure crystalline quartz with a germanium ion substituted for the central silicon ion in the E'-defect structures. Two distinct but similar spectra of the $E_{1}^{\prime}$-type associated with Ge impurities were observed, each characteristic of a specific irradiation and heating treatment. In contrast to the proposed divacancy model of Weeks (8), a single-oxygen-vacancy model for the $E_{1}^{\prime}(G e)$ center was proposed where the unpaired electron occupies a non-bonding sp ${ }^{3}$ orbital strongly localized on the Ge impurity.

Another oxygen-vacancy center, the $E_{4}^{\prime}$ center, first reported by Weeks and Nelson (9), has been studied in more detail by Isoya et al. (16) using ESR. Weeks and Nelson (5) suggested that the ESR spectrum of this center, which consisted of four equally spaced and equally lines, was a result of an electron ( $S=1 / 2$ ) interacting with an alkali ion ( $I=3 / 2$ ) present in the crystal as a charge compensating impurity. Isoya et al. (16) observed that for low microwave power levels, the intensity ratio of the outer pair of lines to the inner pair is approximately 1.2:1 at 9.85 GHz . This is not consistent with an $S=1 / 2$ and
$I=3 / 2$ system. In a detailed angular dependence study of the $E_{4}^{\prime}$ spectrum at $X$-band, it was observed that as the separation of the inner pairs of lines decreases, the intensities of these inner lines increases and remain equal while the intensities of the outer pair of lines correspondingly decrease and remain equal. Further, as the splitting between the inner pair of lines approached zero, the outer pairs of lines were separated by 10.5 G , which is very nearly twice the proton Larmor frequency. Therefore, they concluded that the $E_{4}^{\prime}$ center has $S=1 / 2$ and the hyperfine structure arises from interaction with a proton (I = $1 / 2$ ) in a situation permitting observation of all $2 S(2 I+1)^{2}$ possible ESR transitions. The near equality of the amplitudes and spacings of the four primary absorption lines observed at X -band were not found when the spectrum was taken at 20.4 GHz (K-band) with the magnetic field parallel to the c-axis. Also, to a first-order approximation, the separation of the outer pair of lines scaled with the magnetic field and thus with the proton Larmor frequency. This additional work at K-band confirmed their conclusion that the $E_{4}^{\prime}$ center is an $S=1 / 2$, $I=1 / 2$ spin system. The spin Hamiltonian parameter matrices $\vec{g}, \vec{A}_{1 H^{\prime}}$, $\vec{A}_{29_{S i}}(1,2)$ have been measured. A proposed model of the $E_{4}^{\prime}$ center, consisting of an $\mathrm{O}^{-}$vacancy between $\mathrm{Si}(1)$ and $\mathrm{Si}(2)$ with a hydride ion bonded to $\mathrm{Si}(1)$, is shown in Figure 3. Ab initio SCF-MO calculations for a 15-atom cluster using the Gaussian 70 quantum chemistry computer program allowed them to obtain the minimum energy configuration for this cluster. The results of this calculation supported the model and gave good agreement with the experimental ESR data.


Silicon


- H

Figure 3. Model for the $E_{4}^{\prime}$ Center as Proposed by Isoya
et al. (16). The Unpaired Electron is Unequally Shared Between Si(O) and Si(1)

## Present Study

Additional new oxygen-vacancy centers, other than the $E_{1}^{\prime}, E_{2}^{\prime}$ and $E_{4}^{\prime}$ centers, have been observed in $\alpha$-quartz following electron irradiation at room temperature. In the past, lengthy electron irradiations at room temperature had raised the sample temperature during the process and thus thermally annealed these new defects as soon as they were formed. Reduction of radiation times minimized the sample heating effects and allowed the new defects to be monitored in subsequent ESR measurements. After recognizing the heating problem during irradiations, the samples were irradiated at $0^{\circ} \mathrm{C}$ by immersing them in ice water.

These three new centers, labeled as $E_{1}^{\prime \prime}, E_{2}^{\prime \prime}$, and $E_{3}^{\prime \prime}$ centers, were observed along with the $E_{2}^{\prime}$ and $E_{4}^{\prime}$ centers. The ESR linewidths for these centers are 0.05 G . The c-axis doublet separations for the $E_{1}^{\prime \prime}$, $E_{2}^{\prime \prime}$ and $E_{3}^{\prime \prime}$ centers are $5.00 \mathrm{G}, 11.00 \mathrm{G}$ and 17.87 G , respectively. Initial efforts were concentrated on the most intense $E_{1}^{\prime \prime}$ center spectrum. Preliminary angular dependence showed that the doublet separation varied from approximately 5.00 G to 192.00 G . Such a large angular dependence of the splitting effectively eliminated the proton as a cause of the splitting. A half-field line, characteristic of $s=1$ spin systems, was observed for this center. The double primes represent the number of unpaired electrons in the centers, in contrast to the double primes denoting a diamagnetic center as proposed by weeks and Nelson (9). Centers similar to the present $E$ " defects have been reported by Weeks and Abraham $(17,18)$ and Solntsev et al. (19). However, complete analyses were not presented in the above threereferenced papers.

In this study, an analysis of the main line spectra for the $E_{1}^{\prime \prime}$
and $E_{3}^{\prime \prime}$ centers has been presented along with an analysis of the hyperfine interactions for the $E_{1}^{\prime \prime}$ center. From the parameters obtained for the $\vec{g}, \vec{D}$ and $\vec{A}$ matrices, models will be proposed for the $E_{1}^{\prime \prime}$ and $E_{3}^{\prime \prime}$ centers.

## CHAPTER II

## EXPERIMENTAL PROCEDURE

All the ESR spectra reported in this dissertation were obtained using a Varian 4502 ESR spectrometer. The block diagram of this homodyne $X$-band ( $9-10 \mathrm{GHz}$ ) spectrometer is shown in Figure 4.

Microwave power was supplied by a VAl53C klystron and the microwave frequency was stabilized by an automatic frequency control system in which the klystron was locked to the sample cavity. This AFC used 10 kHz frequency modulation of the klystron, via the reflector voltage, to create an error signal. A home-built microwave bridge having both a bias and sample arm was substituted for the less-versatile Varian bridge. The sample arm contained a precision attenuator ( $0-60 \mathrm{db}$ ) to control the level of microwave power incident on the sample. The bias arm contained a phase shifter and an attenuator to vary the detector bias. A self-locking HP 5340A frequency counter was used to measure the microwave frequency.

A home-made $T_{102}$ rectangular cavity was used to measure the angular dependence of the main-line spectra, whereas a V-4531 rectangular cavity was used for the hyperfine spectra measurements. The static magnetic field was amplitude modulated at 100 kHz , with the modulation coils mounted on the outside of the $\mathrm{TE}_{102}$ home-made rectangular cavity and within the wall of the Varian cavity. A GaAs FET solid state microwave amplifier (Narda N62445-37) was incorporated in the sample arm to


Figure 4. Block Diagram of the ESR Spectrometer
increase sensitivity especially at lower microwave power. The output from the amplifier was detected using a properly-biased low noise Schottky barrier diode (MA40075). This resulting signal from the diode was then amplified and fed to a phase-sensitive detector. The output from the phase-sensitive detector was fed to an oscilloscope and to a Houston Instruments (Model 2000) $x-y$ recorder.

A Varian 12-inch fieldial-regulated electromagnet was used to produce the static magnetic field. Stabilization of the magnetic field was achieved by use of a Hall probe, mounted on one of the pole caps, and its associated error-feedback circuitry. The Hall probe supplied a signal proportional to the magnetic field, which was compared to a reference in order to obtain an error signal. This error signal was then used to adjust the magnet current and maintain a fixed magnetic field. All the magnetic field values reported in this dissertation were measured using a self-tracking Varian E-500 digital Gaussmeter.

Samples used in this investigation were obtained from Toyo Communications Company and from western Electric and all were unswept. The angular dependence of the main $E S R$ spectra for the $E_{1}^{\prime \prime}$ and $E_{3}^{\prime \prime}$ centers were measured using sample SQA-10 (2.1x8.7x25.0 mm ${ }^{3}$ ), an X-plate cut from a pure $z$ growth bar of Toyo Supreme 2 quartz. The ${ }^{29}$ Si hyperfine angular dependence data for the $E_{1}^{\prime \prime}$ center were obtained from Sample WEA-1, a $Z$-plate ( $2.72 \times 10 \times 13 \mathrm{~mm}^{3}$ ) cut from the X -growth region of a western Electric synthetic quartz stone.

Crystal Structure

There are three well-known polymorphic crystalline forms of silica: Quartz, Tridymite and Cristobalite (20). Quartz is the most common and
important of these three forms of silica. The structure of all forms of crystalline silica consists of $\mathrm{SiO}_{4}$ tetrahedra sharing each of their corners with other tetrahedra in a continuous three-dimensional network. Quartz occurs in two distinct forms, a low-temperature form called low-quartz (or $\alpha$-quartz) and a high temperature form called high-quartz (or $\beta$-quartz). The transition from one form to the other occurs at about 846 K and can take place reversibly; although cooling through the $\alpha-\beta$ transition temperature nearly always results in an electrically twinned crystal. Both high and low quartz can exist in two enantiamorphous forms, right-handed and left-handed crystals, which are mirror images of one another.

Quartz has a hexagonal unit cell which consists of 3 formulas with $a_{0}=4.9157 \AA$ and $C o=5.406 \AA$ (21). The space group for righthanded quartz is P 3 21. Symmetry elements for quartz consist of a threefold axis $(\vec{c})$ and three twofold axes $\left(\vec{a}_{1}, \vec{a}_{2}\right.$ and $\left.\vec{a}_{3}\right)$. A projection of right-handed $\alpha-q u a r t z$ in a plane perpendicular to the $c$-axis is shown in Figure 5. The crystallographic coordinate system is formed from two axes $\vec{a}_{1}$ and $\vec{a}_{2}$, making an angle of $120^{\circ}$ with each other, and a third axis $\vec{c}$.

The atomic position coordinates are evaluated in terms of these three axes $\left(\vec{a}_{1}, \vec{a}_{2}, \vec{c}\right)$ by Wyckoff (22) using fractions of the unit cell dimensions.

$$
\begin{array}{lll}
\text { Sil: }(\bar{u}, \bar{u}, 1 / 3) & \text { Si2: }(u, 0,0) & \text { Si3 }:(0, u, 2 / 3) \quad \text { Ol: }(x, y, z) \\
02:(y-x, \bar{x}, z+1 / 3) & 03:(\bar{y}, x-y, z+2 / 3) & 04:(x-y, \bar{y}, \bar{z}) \\
05:(y, x, 2 / 3-z) & 06:(\bar{x}, y-x, 1 / 3-z)
\end{array}
$$

The position parameters $x, y, z$ and $u$ at room temperature have been reported by Lepage and Donnay (23) using cell dimensions measured at 298 K


Figure 5. Projection of Atomic Positions in Right Quartz Onto [001] Plane, Showing Co-ordinate System Conventions Used in This Dissertation. Wyckoff's Atomic Positions are Labeled
by Cohen and Summer (2l). These parameters are as follows.

$$
x=0.41372 \AA \quad y=0.26769 \AA, \quad z=0.11880 \AA
$$

and

$$
u=0.46981{ }^{\circ} \mathrm{A}
$$

Quartz exhibits a piezoelectric effect and the positive end of the x-axis (or a-axis) is defined as the side developing a negative charge during compression of the crystal along that axis (24). A projection of quartz on the plane perpendicular to the $x$-axis is shown in Figure 6. One can generate every atomic position in the quartz lattice by lattice translations along $\vec{a}_{1}, \vec{a}_{2}$ and $\vec{c}$ from one of the nine positions specified by Wyckoff. The nomeclature used to specify any atom position in the lattice is $A(i, j, k)$, where $A$ represents one of the Wyckoff atoms and $i, j$ and $k$ are integers representing the numbers of lattice translation along $\vec{a}_{1}, \vec{a}_{2}$ and $\vec{c}$, respectively.

The $\mathrm{SiO}_{4}$ groups which form the quartz lattice are not perfect tetrahedra and local symmetry at a silicon site is $\mathrm{C}_{2}$. Thus, there are two types of Si-O bonds, slightly different in bond lengths from each other. The equilibrium bond lengths for these two types are $1.611 \AA$ and $1.607 \AA$ at room temperature (23) . Consequently, when considering the environment of a particular silicon the neighboring oxygens are in general referred to as long-bonded and short-bonded oxygens. For our discussion, a shorter nomenclature for atoms surrounding $\operatorname{si} 2(0,0,0)$ is introduced in Table $I$ and Figure 6. As an example, $0 x[0,2]$ represents the oxygen bonded between $S i(0)$ and $S i(2)$. The $x, y, z$ coordinates, separation distances, and bond directions for the set of atoms in Table I are listed in Tables II and III.


Figure 6. A Thirty-three Atom Projection of Right Quartz on the Plane Perpendicular to the X-axis. The Six Outer Silicons Actually Represent Two Silicon Sites Each, One Above the Other

## TABLE I

REIATION BETWEEN FULL SPECIFICATION AND SHORT FORM NOMENCLATURE

| $\begin{gathered} \text { Full } \\ \text { Specifications } \end{gathered}$ | Short <br> Form | $\begin{gathered} \text { Full } \\ \text { Specifications } \end{gathered}$ | Short Form |
| :---: | :---: | :---: | :---: |
| Si2 (0,0,0) | Si(0) | $04(0,0,0)$ | $0[1,0]$ |
| Si3 $(0, \overline{1}, \overline{1})$ | Si ${ }^{(1)}$ | $03(0, \overline{1}, \overline{1})$ | $0[1,5]$ |
| Sil (1,1,0) | Si(2) | $02(0,0, \overline{1})$ | $0[1,6]$ |
| Si3 (1,0, 1 ) | Si (3) | $05(0, \overline{1}, \overline{1})$ | $0[1,7]$ |
| Sil (1,0,0) | Si (4) | $01(0,0,0)$ | $0[2,0]$ |
| Si2 ( $\overline{1}, \overline{1}, 0)$ | Si (5) | $05(0,0,0)$ | $0[2,8]$ |
| $\operatorname{Sil}(0,0, \overline{1})$ | Si ${ }^{(6)}$ | $02(1,1,0)$ | $0[2,9]$ |
| Sil (1,0,1) | Si ${ }^{(7)}$ | $06(1,1,0)$ | $\bigcirc[2,10]$ |
| Si3 (0,0,0) | Si (8) | $03(1,0, \overline{1})$ | $0[3,0]$ |
| Si3(1,0,0) | Si ${ }^{(9)}$ | $05(1,0, \overline{1})$ | $0[3,11]$ |
| Si2 (0, 1, 0) | Si(10) | $04(1,1,0)$ | $0[3,12]$ |
| $\operatorname{Sil}(2,1, \overline{1})$ | Si(ll) | $02(1,1, \overline{1})$ | $0[3,13]$ |
| Si2 (1,1,0) | Si(12) | $06(1,0,0)$ | $0[4,0]$ |
| $\operatorname{Sil}(1,1, \overline{1})$ | Si(13) | $05(0, \overline{1}, 0)$ | $0[4,14]$ |
| Si3 (0, $\overline{1}, 0)$ | Si ${ }^{(14)}$ | $02(1,0,0)$ | $0[4,15]$ |
| Si3 $(1, \overline{1}, 0)$ | Si (15) | $01(0, \overline{1}, 0)$ | $0[4,16]$ |
| Si2 (0, $\overline{1}, 0)$ | Si ${ }^{(16)}$ |  |  |

TABLE II
ATOMIC POSITION CO-ORDINATES OF ATOMS IN QUARTZ AT 300K

| Short Form | x | Y | z |
| :---: | :---: | :---: | :---: |
| Si(0) | 2.30944 | 0.0 | 0.0 |
| Si(I) | 1.30313 | -2.25708 | -1.80206 |
| Si(2) | 1.30313 | 2.25708 | 1.80206 |
| Si (3) | 3.76097 | 2.00004 | -1.80206 |
| Si(4) | 3.76097 | -2.00004 | 1.80206 |
| Si (5) | -0.14840 | $-4.25712$ | 0.0 |
| Si(6) | -1.15472 | -2.00004 | $-3.60413$ |
| Si(7) | 3.76097 | -2.00004 | $-3.60413$ |
| Si (8) | -1. 15472 | 2.00004 | 3.60413 |
| Si (9) | 3.76097 | 2.00004 | 3.60413 |
| Si(10) | -0.14840 | 4.25712 | 0.0 |
| Si(11) | 6.21882 | 2.25708 | -3.60413 |
| Si(12) | 4.76729 | 4.25712 | 0.0 |
| Si(13) | 1.30313 | 2.25708 | -3.60413 |
| Si (14) | 1.30313 | -2.25708 | 3.60413 |
| Si(15) | 6.21883 | -2.25708 | 3.60413 |
| Si(16) | 4.76729 | -4.25712 | 0.0 |
| $0[1,0]$ | 1.37578 | -1.13958 | -0.64226 |
| $0[1,5]$ | 0.78305 | $-3.63545$ | -1.15981 |
| $0[1,6]$ | 0.29902 | -1.76125 | -2.06187 |
| $0[1,7]$ | 2.75687 | -2.49586 | -2.44432 |
| $0[2,0]$ | 1.37578 | 1.13958 | 0.64226 |
| $0[2,8]$ | 0.29902 | 1.76125 | 2.96187 |

TABLE II (Continued)

| Short Form | $x$ | $y$ | $z$ |
| :--- | :---: | :---: | :---: |
| $0[2,9]$ | 2.75687 | 2.49586 | 2.44432 |
| $0[2,10]$ | 0.78305 | 3.63545 | 1.15981 |
| $0[3,0]$ | 3.24089 | 0.62166 | -1.15981 |
| $0[3,11]$ | 5.21472 | 1.76125 | -2.44432 |
| $0[3,12]$ | 3.83363 | 3.11753 | -0.64226 |
| $0[3,13]$ | 2.75687 | 2.49586 | -2.96187 |
| $0[4,0]$ | 3.24089 | -0.62166 | 1.15981 |
| $0[4,14]$ | 2.75687 | -2.49586 | 2.96187 |
| $0[4,15]$ | 5.21472 | -1.76125 | 2.44432 |
| $0[4,16]$ | 3.83363 |  | 0.117529 |

## TABLE III

INTERATOMIC DISTANCES AND BOND DIRECTIONS BETWEEN ATOMS IN QUARTZ AT 300K

| Atoms | Distance in $\AA$ | Direction |  |
| :---: | :---: | :---: | :---: |
|  |  | $\theta$ | $\phi$ |
| Si(0) - Si(l) | 3.0585 | $126.1^{\circ}$ | $245.0^{\circ}$ |
| Si(0) - Si(2) | 3.0585 | $53.9{ }^{\circ}$ | $114.0^{\circ}$ |
| Si(0) - Si(3) | 3.0585 | $126.1^{\circ}$ | $54.0^{\circ}$ |
| Si(0) - Si(4) | 3.0585 | $53.9{ }^{\circ}$ | $306.0^{\circ}$ |
| Si(1) - Si(2) | 5.7764 | $51.39^{\circ}$ | $90.0^{\circ}$ |
| Si(1) - Si(O) | 3.0585 | $53.9{ }^{\circ}$ | $65.97^{\circ}$ |
| Si(8) - Si(7) | 9.598 | $138.7^{\circ}$ | $320.9^{\circ}$ |
| Si(0) - Ox[ 1,0$]$ | 1.6071 | $113.5^{\circ}$ | $230.7^{\circ}$ |
| Si(0) - Ox[2,0] | 1.6071 | $66.4{ }^{\circ}$ | $129.3^{\circ}$ |
| Si(0) - Ox[ 3,0$]$ | 1.6122 | $136.0^{\circ}$ | $33.7{ }^{\circ}$ |
| Si(0) - Ox[4,0] | 1.6122 | $44.0^{\circ}$ | $326.3^{\circ}$ |
| Si(1) - Ox[1,0] | 1.6122 | $49.0^{\circ}$ | $86.3^{\circ}$ |
| Si(8) - Ox[ 1,0$]$ | 5.8559 | $136.5^{\circ}$ | $308.9^{\circ}$ |
| Si(8) - Ox[2,0] | 3.9895 | $137.9^{\circ}$ | $341.2^{\circ}$ |
| $\mathrm{Ox}[2,0]-\mathrm{Si}(2)$ | 1.6122 | $44.0^{\circ}$ | $93.7^{\circ}$ |
| $\mathrm{Ox}[1,0]-\mathrm{Si}(2)$ | 4.1853 | $54.3{ }^{\circ}$ | $91.2{ }^{\circ}$ |

All the results reported in this work are for right-handed $\alpha$-quartz using a right-handed coordinate system. The $\bar{x}$ axis is parallel to a twofold axis having the same sense, as determined by the piezoelectric response on compression of the crystal. The proper point group governing the symmetry-related properties of most defects is $D_{3}$. There are six equivalent symmetry sites for defects containing an oxygen site, all of which can be obtained from a particular site by using the proper transformation matrices for point group $D_{3}$.

The spin Hamiltonian describing an $S=1$ system (two unpaired electrons) interacting with one magnetic nucleus is given, in general by

$$
H=\beta \vec{H} \cdot \cdot \vec{g} \cdot \vec{S}+\vec{S} \cdot \vec{D} \cdot \vec{S}+\vec{I} \cdot \vec{A}_{A} \cdot \vec{S}-g_{N} \beta_{N} \vec{H} \cdot \vec{I}
$$

where the first term represents the electron Zeeman interaction, the second term represents the dipolar and/or exchange interaction (fine structure) between the two unpaired electrons, and the third term represensts the hyperfine interaction due to the magnetic nucleus. The last term is the nuclear zeeman interaction.

The following coordinate systems are used in converting this spin Hamiltonian to a suitable form for computer manipulation.
$x, y, z: \quad$ magnetic coordinate system chosen such that the magnetic field is along the $z$ direction.
$x_{C}, y_{c}, z_{C}: \quad$ Crystal coordinate system.
$x_{g}, y_{g}, z_{g}: \quad$ Principal axes of the $g$-matrix.
$x_{1}, Y_{1}, Z_{1}$ : Principal axes of the D-matrix.
$x_{2}, y_{2}, z_{2}$ : Principal axes of the hyperfine matrix $\vec{A}$.

Rewriting the Hamiltonian in terms of these different coordinate systems, we have

$$
\begin{aligned}
H & =\beta\left[S_{x_{g}} g_{x} H_{x_{g}}+S_{y_{g}} g_{y} H_{y_{g}}+S_{z_{g}} g_{z} H_{z}\right] \\
& +S_{x_{1}} D_{x} S_{x_{1}}+S_{y_{1}} D_{y} S_{y_{1}}+S_{z_{1}} D_{z} S_{z_{1}} \\
& +S_{x_{2}} A_{x} I_{x_{2}}+S_{y_{2}} A_{y} I_{y_{2}}+S_{z_{2}} A_{z} I_{z_{2}} \\
& -g_{N} \beta_{N}{ }^{H I} z_{2}
\end{aligned}
$$

The relationships between the different coordinate systems are

$$
\begin{aligned}
& {\left[\begin{array}{l}
x_{g} \\
y_{g} \\
z_{g}
\end{array}\right]=[\mathrm{TG}]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right],} \\
& {\left[\begin{array}{l}
x_{1} \\
y_{1} \\
z_{1}
\end{array}\right]=[\mathrm{TH}]\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right],}
\end{aligned}
$$

and

$$
\left[\begin{array}{l}
\mathrm{x}_{2} \\
\mathrm{y}_{2} \\
\mathrm{z}_{2}
\end{array}\right]=[\mathrm{TZ}]\left[\begin{array}{l}
\mathrm{x} \\
\mathrm{y} \\
\mathrm{z}
\end{array}\right]
$$

where $[T G],[T H]$, and $[T Z]$ are rotation matrices transforming the principal axes coordinate systems of the $\overleftrightarrow{g}, \overleftrightarrow{D}$, and $\overleftrightarrow{A}$ matrices to the magnetic field coordinate system. The spin vectors and magnetic field transform in the same manner, i.e.,

$$
\left[\begin{array}{c}
s_{x_{g}} \\
s_{y_{g}} \\
s_{z}
\end{array}\right]=[T G]\left[\begin{array}{c}
s_{x} \\
s_{y} \\
s_{z}
\end{array}\right],\left[\begin{array}{c}
H_{x_{g}} \\
H_{y_{g}} \\
H_{z_{g}}
\end{array}\right]=[T G]\left[\begin{array}{c}
0 \\
0 \\
H
\end{array}\right],
$$

etc. Similar expressions are used for the $\left(x_{1}, y_{1}, z_{1}\right)$ and $\left(x_{2}, y_{2}, z_{2}\right)$ coordinate systems when [TH] and [TZ], respectively, replace [TG].

Using the above transformations, the Hamiltonian is written only in terms of the magnetic field coordinate system ( $x, y, z$ ), as follows:

$$
\begin{aligned}
& H=W 1 S_{x}+W 2 S_{y}+W 3 S_{z}+W 4 S_{x}{ }^{2}+W 5 S_{Y}{ }^{2}+W 6 S_{z}{ }^{2} \\
& +W 7 S_{y} S_{x}+W 7 S_{x} S_{y}+W 8 S_{x} S_{z}+W 8 S_{z} S_{x} \\
& + \text { w9 } S_{z} S_{y}+W 9 S_{y} S_{z}+W 10 S_{x} I_{x}+W 11 S_{x} I_{y}+W 11 S_{y} I_{x} \\
& + \text { W12 } S_{x} I_{z}+W 12 S_{z} I_{x}+W 13 S_{y} I_{y}+W 14 S_{z} I_{y}+W 14 S_{y} I_{z} \\
& +\mathrm{W} 15 \mathrm{~S}_{\mathrm{z}} \mathrm{I}_{\mathrm{z}}-\left(\mathrm{g}_{\mathrm{N}} \mathrm{~B}_{\mathrm{N}}\right) \mathrm{HI} \mathrm{z}_{\mathrm{z}}
\end{aligned}
$$

where the w's are:

$$
\begin{aligned}
& \mathrm{WI}=\mathrm{HB}\left[\mathrm{TG}(1,1) \mathrm{TG}(1,3) \mathrm{g}_{\mathrm{x}}+\mathrm{TG}(2,1) \mathrm{TG}(2,3) \mathrm{g}_{\mathrm{y}}+\mathrm{TG}(3,1) \mathrm{TG}(3,3) \mathrm{g}_{2}\right] \\
& \mathrm{W} 2=\mathrm{HB}\left[\mathrm{TG}(1,2) \mathrm{TG}(1,3) \mathrm{g}_{\mathrm{x}}+\mathrm{TG}(2,2) \mathrm{TG}(2,3) \mathrm{g}_{\mathrm{y}}+\mathrm{TG}(3,2) \mathrm{TG}(3,3) \mathrm{g}_{\mathrm{z}}\right] \\
& \mathrm{W} 3=\mathrm{HB}\left[\mathrm{TG}(1,3) \mathrm{TG}(1,3) \mathrm{g}_{\mathrm{x}}+\mathrm{TG}(2,3) \mathrm{TG}(2,3) \mathrm{g}_{\mathrm{y}}+\mathrm{TG}(3,3) \mathrm{TG}(3,3) \mathrm{g}_{z}\right] \\
& \mathrm{W} 4=\mathrm{TH}(1,1) \mathrm{TH}(1,1) \mathrm{D}_{\mathrm{x}}+\mathrm{TH}(2,1) \mathrm{TH}(2,1) \mathrm{D}_{\mathrm{y}}+\mathrm{TH}(3,1) \mathrm{TH}(3,1) \mathrm{D}_{\mathrm{z}} \\
& \mathrm{~W} 5=\mathrm{TH}(1,2) \mathrm{TH}(1,2) \mathrm{D}_{\mathrm{x}}+\mathrm{TH}(2,2) \mathrm{TH}(2,2) \mathrm{D}_{\mathrm{y}}+\mathrm{TH}(3,2) \mathrm{TH}(3,2) \mathrm{D}_{z} \\
& \mathrm{~W} 6=\mathrm{TH}(1,3) \mathrm{TH}(1,3) \mathrm{D}_{\mathrm{x}}+\mathrm{TH}(2,3) \mathrm{TH}(2,3) \mathrm{D}_{\mathrm{y}}+\mathrm{TH}(3,3) \mathrm{TH}(3,3) \mathrm{D}_{z} \\
& \mathrm{~W} 7=\mathrm{TH}(1,1) \mathrm{TH}(1,2) \mathrm{D}_{\mathrm{x}}+\mathrm{TH}(2,1) \mathrm{TH}(2,2) \mathrm{D}_{\mathrm{y}}+\mathrm{TH}(3,1) \mathrm{TH}(3,2) \mathrm{D}_{z} \\
& \mathrm{~W} 8=\mathrm{TH}(1,1) \mathrm{TH}(1,3) \mathrm{D}_{\mathrm{x}}+\mathrm{TH}(2,1) \mathrm{TH}(2,3) \mathrm{D}_{\mathrm{y}}+\mathrm{TH}(3,1) \mathrm{TH}(3,3) \mathrm{D}_{\mathrm{z}} \\
& \mathrm{~W} 9=\mathrm{TH}(1,2) \mathrm{TH}(1,3) \mathrm{D}_{\mathrm{x}}+\mathrm{TH}(2,2) \mathrm{TH}(2,3) \mathrm{D}_{\mathrm{y}}+\mathrm{TH}(3,2) \mathrm{TH}(3,3) \mathrm{D}_{z}
\end{aligned}
$$

```
W10=TZ(1,1)TZ(1,1)A 
```



```
Wl2 = TZ (1,1)TZ (1,3)A A
W13 = TZ (1,2)TZ (1,2)A A
Wl4 = TZ (1, 2)TZ (1,3)A A
```



Using the raising and the lowering operators

$$
\begin{aligned}
& S_{+}=S_{x}+i S_{y}, S_{-}=S_{x}-i S_{y^{\prime}} \\
& I_{+}=I_{x}+i I_{y^{\prime}} \text { and } I_{-}=I_{x}-i I_{y^{\prime}}
\end{aligned}
$$

we can write the Hamiltonian in the following form,

$$
\begin{aligned}
H & =W 3 S_{z}+21 * S_{+}+21 S_{-}+Q_{2}{ }^{*} S_{Z}^{2}+22 S_{-}^{2} \\
& +W 6 S_{z}^{2}+23 S_{+} S_{-}+23 S_{-} S_{+} \\
& +24 * S_{+} S_{Z}+24 S_{-} S_{Z}+24 * S_{Z} S_{+} \\
& +24 S_{Z} S_{-}+25 * I_{+} S_{+}+26 I_{+} S_{-} \\
& +26 I_{-} S_{+}+25 I_{+} S_{-}+Q 7 I_{Z} S_{-} \\
& +27 I_{-} S_{Z}+Q 7 * I_{Z} S_{+}+Q 7 * I_{+} S_{Z} \\
& +W 15 I_{Z} S_{Z}-g_{N} \beta_{N} H I_{Z} .
\end{aligned}
$$

where

$$
\begin{aligned}
& Q 1=1 / 2(W 1+i W 2) \\
& Q 2=1 / 4(W 4-W 5+2 i W 7) \\
& Q 3=1 / 4(W 4+W 5) \\
& Q 4=1 / 2(W 8+i W 9)
\end{aligned}
$$

```
Q5 = 1/4(WlO+2iW11-W13)
Q6 = 1/4(W10+W13)
Q7 = l/2(Wl2+iWl4).
```

Since we are concerned with an $S=1$ electron spin system and a ${ }^{29}$ Si nucleus which has $I=1 / 2$, the basis set chosen is $\left|M_{S} M_{I}\right\rangle$ where $M_{S}$ can be $+1,0$, or -1 and $M_{I}$ can be $+1 / 2$ or $-1 / 2$. This basis set consists of six eigen vectors, which allows one to write the Hamiltonian in a $6 \times 6$ matrix form. The Hamiltonian must be hermitian and the lower half of the matrix elements are sufficient to calculate the energy eigenvalues. The notation for the lower half of the matrix elements is presented in Table IV and the resulting energy level scheme, upon diagonalization of the $6 \times 6$ matrix, is shown in Figure 7. The non-zero elements of the energy matrix are given as follows:

$$
\begin{aligned}
& A(1,1)=W 3+W 6+W 15 / 2+2 Q 3-\frac{g_{N} \beta_{N} H}{2} \\
& A(2,1)=Q 7 \\
& A(2,2)=W 3+W 6+2 Q 3-\frac{W 15}{2}+\frac{g_{N} \beta_{N} H}{2} \\
& A(3,1)=\sqrt{2}\left(Q 1+Q 4+\frac{Q 7}{2}\right) \\
& A(3,2)=\sqrt{2} Q 6 \\
& A(3,3)=4 Q 3-\frac{g_{N} \beta_{N} H}{2} \\
& A(4,1)=\sqrt{2} Q^{5} \\
& A(4,2)=\sqrt{2}\left(Q 1+Q 4-\frac{27}{2}\right) \\
& A(4,4)=4 Q 3+\frac{g_{N} \beta_{N} H}{2} \\
& A(5,1)=2 Q 2 \\
& A(5,3)=\sqrt{2}\left(Q 1-Q 4+\frac{Q 7}{2}\right) \\
& A(5,4)=\sqrt{2} Q 6
\end{aligned}
$$

TABLE IV
LOWER HALF OF THE SPIN HAMILTONIAN MATRIX FOR AN $S=1, ~ I=1 / 2$ SPIN SYS'TEM

|  | $\|+1,+1 / 2\rangle$ | $\|+1-1 / 2\rangle$ | $\|0,+1 / 2\rangle$ | $\|0,-1 / 2\rangle$ | $\|-1,+1 / 2\rangle$ | $\|-1,-1 / 2\rangle$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| < $+1,+1 / 21$ | A (1, 1) |  |  |  |  |  |
| < $+1,-1 / 2$ \| | A $(2,1)$ | A (2, 2) |  |  |  |  |
| <0, +1/2\| | A $(3,1)$ | A ( 3,2$)$ | A $(3,3)$ |  |  |  |
| $\triangleleft 0,-1 / 2 \mid$ | A (4, 1) | A (4, 2) | A (4, 3) | A $(4,4)$ |  |  |
| <-1, +1/2\| | A ( 5,1 ) | A $(5,2)$ | A $(5,3)$ | A $(5,4)$ | $A(5,5)$ |  |
| $\langle-1,-1 / 2\|$ | A $(6,1)$ | A $(6,2)$ | A $(6,3)$ | A $(6,4)$ | A ( 6,5 ) | A $(6,6)$ |



Figure 7. Energy Level Diagram for an $S=1, I=\frac{1}{2}$ Spin System
$A(5,5)=W 6-W 3-\frac{W 15}{2}+2 Q 3-\frac{g_{N} \beta_{N} H}{2}$
$A(6,2)=2 Q^{2}$
$A(6,3)=\sqrt{2} Q 5$
$A(6,4)=\sqrt{2}(Q 1-Q 4-Q 7 / 2)$
$A(6,5)=-27$
$A(6,6)=W 6+\frac{W 15}{2}+2 Q 3-W 3+\frac{g_{N} \beta_{N} H}{2}$

For the analysis of only the $\overleftrightarrow{g}$ and $\stackrel{\leftrightarrow}{D}$ tensors, the Hamiltonian can be simplified by setting the $\overleftrightarrow{A}$ tensor and the nuclear zeeman term equal to zero. In this case, the basis set of vectors chosen is $\left|M_{S}\right\rangle$, where $M_{S}$ can be $+1,0$, or -1 . This allows one to obtain the $3 \times 3$ energy matrix shown in Table V. Three energy eigenvalues are calculated for this matrix and the corresponding energy level diagram is shown in Figure 8.

## TABLE V

LOWER HALF OF THE SPIN HAMILTONIAN MATRIX FOR AN S=1 SPIN SYSTEM

|  | $\|+1\rangle$ | $\|0\rangle$ | $\|-1\rangle$ |
| :--- | :--- | :--- | :--- |
| $<+1 \mid$ | $A(1,1)$ |  |  |
| $<0 \mid$ | $A(2,1)$ | $A(2,2)$ |  |
| $<-I \mid$ | $A(3,1)$ | $A(3,2)$ | $A(3,3)$ |



Figure 8. Energy Level Diagram for an $S=1$ Spin System. The Two Allowed $\Delta M_{S}= \pm 1$ Transitions are Shown Along With the Semi-forbidden $\Delta M_{S}= \pm 2$ (HalfField) Transition

## EXPERIMENTAL RESULTS

When high quality synthetic quartz was irradiated at 300 K , followed by a short 77 K electron irradiation, the $E S R$ spectrum shown in Figure 9 was observed at 300 K for the magnetic field H parallel to the [001] direction. This c-axis primary-line spectrum consists of three doublets with $5.00 \mathrm{G}, 11.00 \mathrm{G}$, and 17.87 G separations and intensity ratios of approximately 3:2:1, respectively.

The doublet with ll.00G separation annealed in about 6 hours at the room temperature and the doublet with 5.00 G separation annealed in a few days. The ESR spectrum taken after the sample had been sitting at room temperature for a few days consi-ted of only the doublet with 17.87G. Hence it was concluded that the three doublets were due to three different centers. These doublets have been assigned the labels of $E_{1}^{\prime \prime}, E_{2}^{\prime \prime}$, and $E_{3}^{\prime \prime}$ centers, respectively. Since the ESR linewidths were extremely narrow $(=0.05 \mathrm{G})$ and the lines were easily power saturated because of long spin-lattice relaxation times, the phase of the 100 kHz modulation unit was adjusted out-of-phase to optimize the intensity of the lines and to prevent modulation "sidebands" from appearing and giving distorted ESR line shapes. All the ESR spectra presented in this chapter (except for the half-field line discussed in the next paragraph) were recorded with the phase adjusted in this manner.

In principle, a doublet ESR spectrum can either result from an interaction between an unpaired electron $(S=1 / 2)$ and a nuclear spin $I=1 / 2$ or


Figure 9. ESR Trace Showing the E" Centers at 300 K When $\vec{H}$ is Along the C-Axis. Many of the Weak Lines are Unidentified. The Microwave Frequency is 9.31412 GHz
from a triplet system ( $S=1$ ) with no hyperfine as shown in Figure 8 in Chapter III. The definitive evidence for the triplet nature of a center is the presence of the semi-forbidden transition ( $M_{S}=+1 \leftrightarrow M_{S}=-1$ ) at half the magnetic field of the allowed transitions. Such a half-field line was observed for the $E$ " centers and it is shown in Figure lo. Since the three centers have nearly identical $g_{C}$ values, the three half-field lines overlap and only one line was observed when the magnetic field was along the c-axis. This spectrum was recorded at higher microwave power, higher gain, and in-phase relative to the primary lines.

An angular dependence study of the $E_{1}^{\prime \prime}$ center was carried out using sample SQA-10. This allowed the $\stackrel{\rightharpoonup}{g}$ and $\overleftrightarrow{D}$ matrix principal values and directions to be determined. The magnetic field was rotated about a twofold crystal axis (i.e., the $x$ axis) and ESR spectra were recorded at $20^{\circ}, 40^{\circ}, 60^{\circ}$, and $70^{\circ}$ on each side of the [001] direction. Spectra beyond the $70^{\circ}$ angle were not obtained because the signal intensity was dropping rapidly. This is because the microwave field component perpendicular to the static magnetic field is proportional to $\cos ^{2} \beta$ for the orientation of the rectangular cavity being used where $\beta$ is the angle between the static field and the crystal's c-axis. The magnetic field values and corresponding microwave frequencies are listed in Table VI for each measured ESR line. Since the gaussmeter probe was not located at exactly the same place as the sample, a standard sample $\mathrm{MgO}: \mathrm{Cr}^{3+}$ (having a known $g$ value of l.9799) was placed in the cavity and correction factors determined for the different orientations of the magnetic field. These are listed in Table VII.

The best sets of parameters for the $\vec{g}$ and $\vec{D}$ matrices were obtained using the two computer programs listed in Appendix A and B. The first


Figure 10. Half-Field Transition of the E" Centers When $\vec{H}$ is Along the C-Axis. The Microwave Frequency is 9.31412 GHZ , and the Temperature is 300K

## TABLE VI

ORIGINAL DATA FOR THE PRIMARY LINES OF THE E" CENTER

| Angle | Uncorrected Field in Gauss | $\begin{aligned} & \text { Corrected } \\ & \text { Field } \\ & \text { in Gauss } \end{aligned}$ | $\begin{aligned} & \text { Calculated } \\ & \text { Field } \\ & \text { in Gauss } \end{aligned}$ | Microwave Frequency in GHz |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & -70^{\circ} \\ & \left(-69.70^{\circ}\right) \end{aligned}$ | 3185.1 | 3185.51 | 3185.44 | 9.1151497 |
|  | 3202.46 | 3202.90 | 3202.84 | 9.1151451 |
|  | 3239.82 | 3240.26 | 3240.21 | 9.1151460 |
|  | 3268.23 | 3268.68 | 3268.61 | 9.1151423 |
|  | 3306.04 | 3306.50 | 3306.45 | 9.1151389 |
|  | 3321.89 | 3322.34 | 3322.30 | 9.1151377 |
| $\begin{aligned} & -60^{\circ} \\ & \left(-59.76^{\circ}\right) \end{aligned}$ | 3173.42 | 3173.94 | 3173.81 | 9.1145077 |
|  | 3211.31 | 3211.84 | 3211.72 | 9.1145028 |
|  | 3217.71 | 3218.24 | 3218.13 | 9.1145014 |
|  | 3290.34 | 3290.88 | 3290.76 | 9.1145014 |
|  | 3296.54 | 3297.09 | 3296.97 | 9.1145016 |
|  | 3333.04 | 3333.59 | 3333.49 | 9.1144987 |
| $\begin{aligned} & -40^{\circ} \\ & \left(-39.86^{\circ}\right) \end{aligned}$ | 3175.36 | 3175.73 | 3175.74 | 9.1145658 |
|  | 3195.38 | 3195.75 | 3195.75 | 9.1145661 |
|  | 3232.00 | 3232.38 | 3232.38 | 9.1145609 |
|  | 3275.75 | 3276.12 | 3276.13 | 9.1145617 |
|  | 3313.48 | 3313.86 | 3313.89 | 9.1145598 |
|  | 3331.36 | 3331.74 | 3331.76 | 9.1145571 |
| $\begin{aligned} & -20^{\circ} \\ & \left(-19.92^{\circ}\right) \end{aligned}$ | 3207.98 | 3208.27 | 3208.38 | 9.1146018 |
|  | 3208.58 | 3208.88 | 3208.96 | 9.1146014 |
|  | 3247.84 | 3248.14 | 3248.19 | 9.1145992 |
|  | 3259.91 | 3260.21 | 3260.26 | 9.1145946 |
|  | 3299.12 | 3299.42 | 3299.42 | 9.1145944 |
|  | 3300.08 | 3300.39 | 3300.42 | 9.1145959 |
| $0^{\circ}$ | 3251.38 | 3251.58 | 3251.72 | 9.1146036 |
|  | 3256.38 | 3256.68 | 3256.75 | 9.1146013 |
| $\begin{aligned} & +20^{\circ} \\ & \left(19.94^{\circ}\right) \end{aligned}$ | 3203.26 | 3203.60 | 3203.59 | 9.1152273 |
|  | 3210.71 | 3211.05 | 3211.04 | 9.1152218 |
|  | 3241.24 | 3241.58 | 3241.56 | 9.1152224 |
|  | 3267.12 | 3267.46 | 3267.43 | 9.1152201 |
|  | 3298.42 | 3298.77 | 3298.73 | 9.1152169 |
|  | 3304.35 | 3304.70 | 3304.65 | 9.1152175 |
| $\begin{aligned} & +40^{\circ} \\ & \left(39.88^{\circ}\right) \end{aligned}$ | 3164.94 | 3165.32 | 3165.27 | 9.1151215 |
|  | 3195.46 | 3195.84 | 3195.82 | 9.1151183 |
|  | 3221.75 | 3222.13 | 3222.08 | 9.1151099 |
|  | 3286.64 | 3287.03 | 3287.02 | 9.1151012 |
|  | 3313.85 | 3314.24 | 3314.18 | 9.1151003 |
|  | 3342.35 | 3342.74 | 3342.70 | 9.115097 |

TABLE VI (Continued)

|  | Uncorrected <br> Field <br> in Gauss | Corrected <br> Field <br> in Gauss | Calculated <br> Field <br> in Gauss | Microwave <br> Frequency <br> in GHz |
| :---: | :---: | :---: | :---: | :---: |
| $+60^{\circ}$ |  |  |  |  |
| $\left(59.76^{\circ}\right)$ | 3159.21 | 3159.53 | 3159.59 | 9.1150115 |
|  | 3202.26 | 3202.59 | 3202.64 | 9.1150053 |
|  | 3212.81 | 3213.13 | 3212.17 | 9.1150000 |
|  | 3295.72 | 3296.06 | 3296.10 | 9.1149937 |
| $+70^{\circ}$ | 3306.23 | 3306.57 | 3306.60 | 9.1149901 |
| $\left(69.76^{\circ}\right)$ | 3347.85 | 3348.19 | 3348.22 | 9.1149850 |
|  |  |  |  |  |
|  | 3170.09 | 3170.36 | 3170.47 | 9.1149567 |
|  | 3195.43 | 3195.70 | 3195.83 | 9.1149560 |
|  | 3231.83 | 3232.11 | 3232.23 | 9.1149549 |
|  | 3313.10 | 3276.40 | 3276.51 | 9.1149545 |
|  | 3336.85 | 3313.39 | 3313.50 | 9.1149530 |
|  |  | 3337.14 | 3337.27 | 9.1149541 |

TABLE VII
FIELD-CORRECTION FACTORS AS DETERMINED FROM
STANDARD MgO: $\mathrm{Cr}^{3+}$ SAMPLE

| Degrees | $\left.\mathrm{Cr}^{3+}\right)^{\text {) }}$ | Microwave Frequency (GHZ) | $\Delta \mathrm{H}$ |
| :---: | :---: | :---: | :---: |
| $0^{\circ}$ | 3300.87 | 9.1478600 | 0.308 |
| $-20^{\circ}$ | 3300.88 | 9.1478843 | 0.307 |
| $-40^{\circ}$ | 3300.80 | 9.1478717 | 0.381 |
| $-60^{\circ}$ | 3300.61 | 9.1478134 | 0.543 |
| $-70^{\circ}$ | 3300.69 | 9.1477849 | 0.453 |
| $+20^{\circ}$ | 3300.79 | 9.1477551 | 0.349 |
| $+40^{\circ}$ | 3300.75 | 9.1477739 | 0.391 |
| $+60^{\circ}$ | 3300.82 | 9.1477996 | 0.337 |
| $+70^{\circ}$ | 3300.87 | 9.1477980 | 0.284 |
| $\Delta \mathrm{H}=\left(\mathrm{H}_{\text {sample }}\right) \mathrm{Cr}^{3+}-\left(\mathrm{Hr}^{3+}{ }^{3+}\right. \text { Gaussmeter }$ |  |  |  |
| $\frac{6.6262 \times 10^{-27} \times v_{\mathrm{Cr}^{3+}}}{9.2741 \times 10^{-21} \times 1.9799}$ |  |  |  |

program, listed in Appendix A, was used to predict line positions (i.e., magnetic field values) for different orientations of the magnetic field when given a set of spin Hamiltonian parameters and a microwave frequency. The second program calculates the final set of parameters when the microwave frequencies and experimental ESR line positions are provided as input data.

In the first program, it was assumed that the parameters for the $\vec{g}$ and $\vec{D}$ matrices were known. Then the magnetic field values associated with different ESR resonance lines were predicted by an iteration scheme. The direction of the magnetic field relative to the crystal axes was specified by the angles alpha( $\alpha$ ) and beta $(\beta)$ as shown in Figure ll. For each set of these angles there were two transitions per inequivalent defect site according to the spin selection rules $\Delta M_{S}= \pm 1$. These possible transitions for one site are shown in Figure 8 in Chapter III and were determined by the following procedure. First, an initial value of magnetic field, $H$, was assigned. Then the $3 x 3$ matrix shown in Table $V$ was diagonalized by the computer and three energy eigenvalues $D(I)$ given in order of ascending values were obtained. The microwave frequency necessary for a transition is the difference between the appropriate eigenvalues. The assigned magnetic field was then varied systematically and the microwave frequency corresponding to a particular transition was recalculated and compared with the experimental microwave frequency for each assigned magnetic field value. The iteration continues until the calculated microwave frequency lies within 0.1 MHz of the experimental frequency, the last value of the magnetic field was taken to be the particular line position.

The values of the different spin Hamiltonian parameters were systematically varied in the second program, listed in Appendix B, until


Figure 11. Definition of Angles Alpha ( $\alpha$ ) and Beta ( $\beta$ )
good agreement between the calculated and experimental microwave frequency for each experimental transition was obtained. The program essentially uses a least-squares-fit method. An initial set of parameters was assumed and the magnetic field positions for different orientation of the magnetic field were provided as experimental data. Using this information the energy eigenvalues were obtained as in the first program and the microwave frequency associated with each resonance was calculated.

Since the assumed set of parameters was most likely not the correct one, the calculated microwave frequency was not the same as the experimental value. Hence a quantity called SUM

$$
\operatorname{sUM}={ }_{i=1}\left[v_{i}^{\exp }-v_{i}^{c a l}\right]^{2}
$$

was calculated where the summation is over all measured ESR lines. In each iteration, one of the parameters was increased by a pre-determined amount and a new set of microwave frequencies were obtained. The new value of SUM thus obtained was compared against the previous value of SUM. If the new value of SUM was greater than the previous value, then the value of that parameter (which had just been increased) was decreased by twice the specified increment. All the microwave frequencies were recalculated and the new value of SUM was obtained. This SUM was again compared with the initial value of SUM and the value of this parameter which gave the smallest SUM was retained. This procedure was repeated for all the other parameters. The final set of parameters correspond to the case when any variation in any of the parameters failed to lower the value of SUM.

The above discussed procedure was used to fit the experimental
field values and microwave frequencies for different transitions at various magnetic field directions of the $E_{1}^{\prime \prime}$ center in the $y-z$ plane. The final set of parameters as determined from these computations is listed in Table VIII. This set of parameters was used in the first (i.e., line position) program to predict the angular dependence for the main line spectrum, shown in Figure 12 . The O's represent the magnetic field values used as experimental data in the fitting program. The calculated magnetic field values for different lines are listed in Table VI for comparison with the measured field values. Angles listed in the first column of Table VI were also varied as parameters in the fitting program since the angular scale on the magnet seemed to be very slightly in error. The best values of the angles obtained from the program are listed in parenthesis (in the first column of Table VI). Similar analysis was done for the $E_{3}^{\prime}$ center using the two programs discussed above. The measured values of the magnetic field and microwave frequency for different line positions are listed in Table IX. The best set of parameters for the $E_{3}^{\prime \prime}$ center is presented in Table $X$. The predicted angular dependence for the $E_{3}^{\prime \prime}$ center is shown in Figure 13. The O's represent the magnetic field values used as experimental data in the fitting program.

TABLE VIII

SPIN-HAMILTONIAN PARAMETERS FOR THE E CENTER AS DETERMINED AT ROOM TEMPERATURE

|  |  | $\theta$ | $\phi$ |
| :---: | :---: | :---: | :---: |
| $g_{x}$ | 2.0004 | $32.3^{\circ}$ | $246.0^{\circ}$ |
| $g_{y}$ | 2.0006 | $91.8^{\circ}$ | $333.1^{\circ}$ |
| $g_{z}$ | 2.0014 | $57.8^{\circ}$ | $61.9^{\circ}$ |
| $D_{x}$ | 110.0 MHZ | $34.2^{\circ}$ | $234.9^{\circ}$ |
| $D_{y}$ | 117.6 MHZ | $86.0^{\circ}$ | $330.8^{\circ}$ |
| $D_{z}$ | -227.6 MHZ | $56.0^{\circ}$ | $63.5^{\circ}$ |



Figure 12. The Angular Variation for the $E_{1}^{\prime \prime}$ Center (Primary Lines)

TABLE IX

ORIGINAL DATA FOR THE PRIMARY LINES OF THE E" CENTER

| Angle | ```Uncorrected Field in Gauss``` | ```Corrected Field in Gauss``` | ```Calculated Field in Gauss``` | Microwave Frequency in GHZ |
| :---: | :---: | :---: | :---: | :---: |
| $-70^{\circ}$ | 3237.64 | 3238.08 | 3238.07 | 9.1151368 |
| $\left(-69.70^{\circ}\right.$ ) | 3241.92 | 3242.36 | 3242.35 | 9.1151351 |
|  | 3243.01 | 3243.45 | 3243.46 | 9.1151352 |
|  | 3267.06 | 3267.51 | 3267.49 | 9.1151350 |
|  | 3267.96 | 3268.41 | 3268.00 | 9.1151314 |
|  | 3270.63 | 3271.08 | 3271.04 | 9.1151312 |
| $-60^{\circ}$ | 3232.27 | 3232.80 | 3232.71 | 9.1144975 |
| $\left(-59.76^{\circ}\right.$ ) | 3241.27 | 3241.80 | 3241.73 | 9.1144960 |
|  | 3243.45 | 3243.98 | 3243.95 | 9.1144937 |
|  | 3265.97 | 3266.51 | 3266.48 | 9.1144928 |
|  | 3268.04 | 3268.58 | 3268.55 | 9.1144930 |
|  | 3275.15 | 3275.69 | 3275.64 | 9.1144890 |
| $-40^{\circ}$ | 3228.14 | 3228.51 | 3228.52 | 9.1145589 |
| (-39.86) | 3243.89 | 3244.26 | 3244.28 | 9.1145555 |
|  | 3250.14 | 3250.51 | 3250.51 | 9.1145517 |
|  | 3259.47 | 3259.85 | 3259.85 | 9.1145490 |
|  | 3265.12 | 3265.50 | 3265.50 | 9.1145455 |
|  | 3279.24 | 3279.62 | 3279.63 | 9.1145440 |
| $-20^{\circ}$ | 3233.23 | 3233.53 | 3233.60 | 9.1145947 |
| $(-19.92)$ | 3251.27 | 3251.57 | 3251.61 | 9.1145912 |
|  | 3252.49 | 3252.79 | 3252.85 | 9.1145895 |
|  | 3256.10 | 3256.40 | 3256.44 | 9.1145866 |
|  | 3257.87 | 3258.17 | 3258.21 | 9.1145856 |
|  | 3274.48 | 3274.78 | 3274.82 | 9.1145860 |
| $0^{\circ}$ | 3245.26 | 3245.56 | 3245.61 | 9.1146003 |
|  | 3263.13 | 3263.43 | 3263.47 | 9.1145986 |
| $+20^{\circ}$ | 3237.92 | 3238.26 | 3238.24 | 9.1152095 |
| (19.94 ${ }^{\circ}$ ) | 3244.52 | 3244.86 | 3244.85 | 9.1152064 |
|  | 3250.75 | 3251.09 | 3251.05 | 9.1152061 |
|  | 3258.93 | 3259.27 | 3259.23 | 9.1152086 |
|  | 3263.74 | 3264.08 | 3264.05 | 9.1152097 |
|  | 3271.13 | 3271.48 | 3271.44 | 9.1152085 |
| $+40^{\circ}$ | 3237.02 | 3237.40 | 3237.40 | 9.1150955 |
| (39.88 ${ }^{\circ}$ ) | 3242.66 | 3243.04 | 3242.98 | 9.1150984 |
|  | 3248.93 | 3249.31 | 3249.31 | 9.1150985 |
|  | 3258.98 | 3259.37 | 3259.33 | 9.1151017 |
|  | 3267.41 | 3267.80 | 3267.73 | 9.1151006 |
|  | 3272.31 | 3272.70 | 3272.67 | 9.1151078 |

TABLE IX (Continued)

|  | Uncorrected <br> Field <br> in Gauss | Corrected <br> Field <br> in Gauss | Calculated <br> Field <br> in Gauss | Microwave <br> Frequency <br> in GHZ |
| :---: | :---: | :---: | :---: | :---: |
| $+60^{\circ}$ | 3242.88 | 3243.21 |  |  |
| $\left(59.76^{\circ}\right)$ | 3243.27 | 3243.60 | 3243.24 | 9.1149813 |
|  | 3251.34 | 3251.67 | 3251.76 | 9.1149801 |
|  | 3256.73 | 3257.06 | 3257.14 | 9.1149804 |
|  | 3266.42 | 3266.75 | 3266.85 | 9.1149771 |
| $+70^{\circ}$ | 3267.03 | 3267.36 | 3267.40 | 9.1149781 |
| $\left(69.76^{\circ}\right)$ | 3246.24 | 3246.52 |  | 9246.63 |
|  | 3247.53 | 3247.81 | 3247.97 | 9.1149604 |
|  | 3248.29 | 3248.57 | 3248.72 | 9.1149618 |
|  | 3260.80 | 3261.08 | 3261.24 | 9.1149609 |
|  | 3261.55 | 3261.83 | 3262.01 | 9.1149582 |
|  | 3263.40 | 3263.68 | 3263.79 | 9.1149617 |
|  |  |  |  | 9.1149601 |

TABLE X


|  |  | $\theta$ | $\phi$ |
| :---: | :---: | :---: | :---: |
| $g_{x}$ | 2.0004 | $83.6^{\circ}$ | $78.9^{\circ}$ |
| $g_{y}$ | 2.0005 | $47.4^{\circ}$ | $343.0^{\circ}$ |
| $g_{z}$ | 2.0013 | $136.7^{\circ}$ | $355.8^{\circ}$ |
| $D_{x}$ | -25.0 MHZ | $50.0^{\circ}$ | $335.4^{\circ}$ |
| $D_{y}$ | -25.2 MHZ | $79.3^{\circ}$ | $236.3^{\circ}$ |
| $D_{z}$ | +50.2 MHZ | $138.0^{\circ}$ | $314.3^{\circ}$ |



Figure 13. The Angular Variation for the $E_{3}^{\prime \prime}$ Center (Primary Lines)

## CHAPTER V

## HYPERFINE RESULTS FOR THE E" CENTER

When samples WEA-1 or SQA-10 were electron irradiated at 300 K and then irradiated at low temperature (77K), additional weak ESR lines were observed, four on the low field side and four on the high field side of the main line spectrum for each of the $E$ " centers when the magnetic field is along the c-axis of the crystal. These lines are shown in Figures 14 and 15. This means that there are four pairs of these smaller-intensity lines for each defect and they are approximately centered on the two primary lines for each center. These widely-split pairs of lines have been assigned to hyperfine interactions of each triplet center with two inequivalent ${ }^{29}$ Si nuclei ( $I=1 / 2$ and $4.7 \%$ natural abundance).

The magnetic field values for the hyperfine lines corresponding to the $E_{1}^{\prime \prime}$ center when the field is parallel to the c-axis are given in Table XI along with the assignments to specific nuciei. The fine structure line splittings of the two hyperfine doublets on the low field side of the main spectrum are 4.90 G and 4.30 G , whereas the similar splittings of the two hyperfine doublets on the high field side are 13.12G and 12.75G. The separation between the doublets on the low and high field side are 202.23G and 195.82G for nucleus 1 and 2 , respectively. All the ESR spectra were recorded with the 100 kHz modulation "in-phase" and with higher microwave power and gain, which is quite different from the conditions found to optimize the primary lines.


Figure 14. Low-field ${ }^{29}$ Si Hyperfine Spectrum for the E" Centers
at 300 K When $\overrightarrow{\mathrm{H}}$ is Along the C-axis. The Microwave
Frequency is 9.31412 GHZ


## TABLE XI

MAGNETIC FIELD POSITIONS OF THE PRIMARY DOUBLET AND THE EIGHT
${ }^{29}$ Si HYPERFINE LINES FOR THE E" CENTER. THE MAGNETIC FIELD VALUES ARE IN GAUSS AND THE MICROWAVE

FREQUENCY IS 9.3165 GHZ

| Primary Lines: | 3323.74 | 3328.75 |
| :---: | :---: | :---: |
| Hyperfine Lines: |  |  |
| Nucleus 1 | $3219.43,3224.25$ | $3417.56,3430.75$ |
| Nucleus 2 | $3223.18,3227.44$ | $3414.75,3427.54$ |

An angular-dependence study of the $E_{1}^{\prime \prime}$ center ${ }^{29}$ Si hyperfine lines for rotation in the plane perpendicular to the $X($ or $\vec{a}$ ) axis was done at 300K. The interactions with the two nuclei proved to be similar and it was very difficult to determine to which nucleus a particular hyperfine line should be assigned. In order to distinguish between lines resulting from the two different nuclei, data was taken for every one-tenth of a degree of rotation of the magnetic field up to fifteen degrees on each side of the c-axis. Although very tedious and time-consuming, this helped considerably in making proper line assignments and for later use in the computer program to determine hyperfine matrix principal values and directions for two ${ }^{29}$ Si nuclei. The angular dependence for $-15^{\circ}$ to $+15^{\circ}$ rotation of the magnetic field are shown in Figures 16 and 17 for the low and high field sides. The solid and dashed lines represent the calculated angular dependence for nucleus 1 and 2 , respectively, using parameters determined later in this chapter, whereas the O's represent the measured field values at five degree intervals.

ESR spectra for the angles beyond fifteen degrees on each side of the c-axis were obtained by rotating the sample in the Varian Cavity (instead of rotating the magnetic field and keeping the sample fixed as in the $-15^{\circ}$ to $+15^{\circ}$ case). Normally, the samples were not cut with perfect $x$-plane ends and the rotating mechanism could not maintain the sample properly in the plane at all angles; therefore, the following procedure was used. First, the sample was placed at an approximate angle setting, making sure that the magnetic field was perpendicular to the two-fold (a) axis. [The approximate settings of the angles were $-30^{\circ},-48^{\circ}$, and $-70^{\circ}$ on one side and $30^{\circ}, 40^{\circ}, 60^{\circ}$ and $75^{\circ}$ on the other side of the c-axis.] Then, at each approximate angle setting, the values of the magnetic field and the microwave frequency were recorded


Figure 16. Calculated Low-field ${ }^{29}$ Si Hyperfine Angular Dependence of the E" Center for Rotation of the Magnetic Field From $-15^{\circ}$ to $+15^{\circ}$ in the X-plane. The O's Represent Experimentally Measured Field Values. The Solid Curve Corresponds to Nucleus 1 and the Dashed Curve Corresponds to Nucleus 2


Figure 17. Calculated High-field ${ }^{29}$ Si Hyperfine Angular Dependence of the $E_{1}^{\prime \prime}$ Center for Rotation of the Magnetic Field From $-15^{\circ}$ to $+15^{\circ}$ in the X-plane. The O's Represent Experimentally Measured Field Values. The Solid Curve Corresponds to Nucleus 1 and the Dashed Curve Corresponds to Nucleus 2
recorded for each of the main line and hyperfine line positions. This set of raw data for the various approximate angles is listed in Appendix C. Also, ESR spectra were recorded at several in-between angles to follow the different hyperfine lines as they shifted positions.

Since the angles at which the hyperfine data were taken were known to be slightly in error, a separate computer program making use of the previously determined $\vec{g}$ and $\vec{D}$ matrix parameters was designed to correct the values of these angles. The best set of values for the $\vec{g}$ and $\vec{D}$ matrices for the $E_{1}^{\prime}$ center, listed in Table VIII, was given to the line position program. The separations within pairs of fine-structure-split lines in the primary spectrum were used as experimental data. For a particular angular setting, the experimental fine-structure separations (DF) were compared to the calculated separations (DFC) and the value of SUM

$$
\operatorname{SUM}=\sum_{k=1}^{3}[\operatorname{DFC}(k)-\operatorname{DF}(k)]^{2}
$$

was evaluated. Then the value of the angle was increased by a specified increment and the value of Sum was calculated again. The new value of SUM was compared with the previous value and the smaller of the two was retained along with the corresponding angle. If the new value of SUM was greater than the previous value, then the angle was decreased by twice the specified increment and another new value of SUM was calculated and compared to the previous value. Using this iteration scheme, the minimum value of SUM and the corresponding angle were obtained. This process was repeated for each of the other approximate angles. The best values of the angles obtained from this program were used in analyzing the hyperfine data, i.e., to evaluate the $\vec{A}_{29}{ }_{S i_{I}}$ and $\stackrel{\rightharpoonup}{A}_{29_{S i}}$ matrices.

Two programs, quite similar to the ones discussed in Chapter IV, were written to analyze the hyperfine data. These are listed in Appendix $D$ and E. The first program predicts the line positions given a set of parameters and a microwave frequency. The fitting program (second program) was modified to allow analyzing the hyperfine data in a slightly different fashion, as discussed below.

Best values of the $\vec{g}$ and $\vec{D}$ matrices were provided as non-varying parameters and initial parameters for the $\vec{A}$ matrix were chosen. Instead of individual line-position field values, experimental differences in the field values for each particular site were provided along with the average frequency for a particular angle. The magnetic field values associated with different ESR resonance lines were predicted by an iteration scheme. An initial value of the magnetic field, $H$, was assigned. For each angle, there were four transitions according to the spin selection rules $\Delta M_{S}= \pm 1, \Delta M_{I}=0$. These possible four transitions are shown in Figure 7. The $6 \times 6$ spin-Hamiltonian matrix shown in Table IV was diagonalized and six energy eigenvalues $D(I)$ given in order of ascending value were obtained. The four transitions are given by

$$
\begin{aligned}
& h \nu_{1}=D(4)-D(1) \\
& h \nu_{2}=D(6)-D(4) \\
& h \nu_{3}=D(5)-D(3) \\
& h \nu_{4}=D(3)-D(2)
\end{aligned}
$$

The assigned field value was then varied and the microwave frequency corresponding to a particular transition was recalculated and compared with the experimental microwave frequency. If the calculated microwave frequency was within 0.1 MHz of the experimental frequency, then the microwave frequency for the other 3 transitions were calculated by a similar iteration. The differences were calculated for each angle for the dif-
ferent sites as follows:

$$
\begin{aligned}
& \operatorname{DFC}(\mathrm{MM}, 1)=\mathrm{HF}(3)-\mathrm{HF}(2) \\
& \operatorname{DFC}(\mathrm{MM}, 2)=\mathrm{HF}(4)-\mathrm{HF}(1)
\end{aligned}
$$

Since the initial parameters for the A tensor were not necessarily the correct ones, the calculated differences were not the same as the experimental values $D F(M M, 1)$ and $D F(M M, 2)$. Therefore, a quantity called SUM

$$
\operatorname{SUM}=\sum_{M M=1}^{62}\left[(\operatorname{DFC}(M M, 1)-D F(M M, 1))^{2}+(\operatorname{DFC}(M M, 2)-D F(M M, 2))^{2}\right]
$$ was calculated. Then the value of SUM was minimized by the iteration technique discussed in Chapter IV until any variation in parameters failed to lower the value of the SUM. It was necessary for us to use the differences in field values, instead of individual line positions, since the corrections to the magnetic field were not measured because of the obstacles involved in placing the standard $\mathrm{MgO}: \mathrm{Cr}^{3+}$ sample in the cavity at the same position as the quartz sample. This procedure of minimizing the SUM in the above-discussed program does not in anyway hinder us in obtaining the best set of parameters since each individual line would have to be corrected by an almost identical amount for a particular angle, and this correction is not necessary if differences are used.

Tables XII and XIII lists the experimentally measured and calculated differences for nucleus 1 and 2 , respectively. A fairly good agreement is obtained between the measured and calculated differences using the best set of parameters listed in Table XIV. The calculated angular dependence for the low and high field side are shown in Figures

TABLE XII
hYPERFINE DATA FOR NUCLEUS 1 IN THE E ${ }_{1}$ CENTER

| Angle | Measured Differences in Gauss | Calculated Differences <br> in Gauss |
| :---: | :---: | :---: |
| $-70^{\circ}$ | 199.22 | 199.02 |
| (-70.27 ${ }^{\circ}$ ) | 217.02 | 216.80 |
|  | 201.4 | 202.30 |
|  | 218.57 | 219.48 |
|  | 196.71 | 196.71 |
|  | 211.32 | 211.35 |
| -48 ${ }^{\circ}$ | 202.08 | 201.92 |
| $\left(-49^{\circ}\right.$ ) |  | 218.87 |
|  | 192.14 | 192.69 |
|  | 206.42 | 206.98 |
|  | 198.63 | 198.69 |
|  | 214.98 | 215.05 |
| $-30^{\circ}$ | 200.43 | 200.39 |
| ( $-30.45^{\circ}$ ) | 218.14 | 218.17 |
|  | 188.05 | 188.29 |
|  | 201.71 | 201.95 |
|  | 198.14 | 198.20 |
|  | 215.49 | 215.61 |
| $-15^{\circ}$ | 197.01 | 196.94 |
| $\left(-14.92{ }^{\circ}\right.$ ) | 215.53 | 215.55 |
|  | 188.68 | 188.67 |
|  | 204.54 | 204.51 |
|  | 196.2 | 196.19 |
|  | 214.02 | 214.05 |
| $-10^{\circ}$ | 195.8 | 195.70 |
| (-9.95 ${ }^{\circ}$ ) | 214.3 | 214.3 |
|  | 189.93 | 189.72 |
|  | 206.53 | 206.45 |
|  | 195.35 | 195.30 |
|  | 213.27 | 213.24 |
| $-5^{\circ}$ | 191.37 | 191.28 |
| $\left(-4.96{ }^{\circ}\right.$ ) | 208.82 | 208.75 |

TABLE XII (Continued)

| Angle | Measured Differences in Gauss | Calculated Differences in Gauss |
| :---: | :---: | :---: |
| $0^{\circ}$ | 193.3 | 193.28 |
|  | 211.37 | 211.30 |
| $\begin{gathered} +5^{\circ} \\ \left(4.98^{\circ}\right) \end{gathered}$ | 192.26 | 192.17 |
|  | 209.72 | 209.67 |
|  | 195.66 | 195.66 |
|  | 214.02 | 214.04 |
|  | 192.3 | 192.21 |
|  | 210.3 | 210.23 |
| $\begin{gathered} +10^{\circ} \\ \left(9.93^{\circ}\right) \end{gathered}$ | 191.28 | 191.19 |
|  | 208.08 | 208.02 |
|  | 198.32 | 198.34 |
|  | 216.7 | 216.79 |
|  | 191.27 | 191.16 |
|  | 209.2 | 209.13 |
| $\begin{aligned} & +15^{\circ} \\ & (14.95) \end{aligned}$ | 190.42 | 190.37 |
|  | 206.45 | 206.39 |
|  | 201.21 | 201. 30 |
|  | 219.35 | 219.53 |
|  | 190.45 | 190.15 |
|  | 208.09 | 208.02 |
| $\begin{gathered} +30^{\circ} \\ \left(29.71^{\circ}\right) \end{gathered}$ | 188.78 | 188.67 |
|  | 202.64 | 202.55 |
|  | 209.99 | 210.38 |
|  | 226.26 | 226.67 |
|  | 187.99 | 187.84 |
|  | 205.06 | 204.94 |
| $\begin{gathered} +43^{\circ} \\ \left(43.09^{\circ}\right) \end{gathered}$ | 188.03 | 187.93 |
|  | 201.55 | 201.46 |
|  | 187.08 | 186.96 |
|  | 203.14 | 203.02 |
| $\begin{gathered} +75^{\circ} \\ \left(75.02^{\circ}\right) \end{gathered}$ | 190.48 | 190.19 |
|  | 208.2 | 207.99 |

TABLE XII (Continued)

| Angle | Measured Differences <br> in Gauss | Calculated Differences <br> in Gauss |
| :---: | :---: | :---: |
| 190.36 | 190.29 |  |
| 203.3 | 203.69 |  |

TABLE XIII
HYPERFINE DATA FOR NUCLEUS 2 IN THE E" CENTER

| Angle | Measured Differences in Gauss | Calculated Differences in Gauss |
| :---: | :---: | :---: |
| $\begin{gathered} -70^{\circ} \\ \left(-70.27^{\circ}\right) \end{gathered}$ | 212.02 | 211.76 |
|  | 226.73 | 226.54 |
|  | 182.93 | 183.26 |
|  | 199.64 | 199.97 |
|  | 181.86 | 181.91 |
|  | 195.35 | 195.39 |
| $\begin{aligned} & -48^{\circ} \\ & \left(-49^{\circ}\right) \end{aligned}$ | 211.9 | 211.71 |
|  | 221.91 | 225.77 |
|  | 181.05 | 181.25 |
|  | 194.41 | 194.61 |
|  | 179.92 | 181.01 |
|  | 195.43 | 195.53 |
| $\begin{gathered} -30^{\circ} \\ \left(-30.45^{\circ}\right) \end{gathered}$ | 204.45 | 204.42 |
|  | 220.1 | 220.07 |
|  | 181.98 | 182.08 |
|  | 194.8 | 194.89 |
|  | 181.22 | 181.34 |
|  | 197.96 | 198.10 |
| $\begin{gathered} -15^{\circ} \\ \left(-14.92^{\circ}\right) \end{gathered}$ | 194.59 | 195.38 |
|  | 212.62 | 212.60 |
|  | 184.06 | 184.04 |
|  | 199.88 | 199.00 |
|  | 184.07 | 184.11 |
|  | 201.2 | 201. 24 |
| $\begin{aligned} & -10^{\circ} \\ & \left(-9.95^{\circ}\right) \end{aligned}$ | 192.41 | 192.48 |
|  | 209.9 | 209.88 |
|  | 185.03 | 184.95 |
|  | 200.94 | 200.75 |
|  | 185.17 | 185.16 |
|  | 202.31 | 202.30 |

TABLE XIII (Continued)

| Angle | Measured Differences in Gauss | Calculated Differences in Gauss |
| :---: | :---: | :---: |
| $\begin{gathered} -5^{0} \\ \left(4.96^{\circ}\right) \end{gathered}$ | 186.11 | 186.05 |
|  | 202.6 | 202.55 |
| $0^{\circ}$ | 187.3 | 187.27 |
|  | 204.35 | 204.33 |
|  | 185.17 | 185.13 |
| $\left(4.98{ }^{+5}\right.$ | 201.78 | 201.71 |
|  | --- | --- |
|  | 206.05 | 206.02 |
|  | 188.33 | 188.37 |
|  | 205.3 | 205.25 |
| $\begin{gathered} +10^{\circ} \\ \left(9.93^{\circ}\right) \end{gathered}$ | 183.43 | 183.40 |
|  | 199.34 | 199.32 |
|  | 189.99 | 190.00 |
|  | 207.56 | 207.60 |
|  | 189.26 | 189.19 |
|  | 206.12 | 206.05 |
| $\begin{gathered} +15^{\circ} \\ \left(14.95^{\circ}\right) \end{gathered}$ | 182.12 | 182.13 |
|  | 196.99 | 197.23 |
|  | 191.65 | 191.45 |
|  | 208.89 | 208.98 |
|  | 189.91 | 190.02 |
|  | 206.81 | 206.73 |
| $\begin{gathered} +30^{\circ} \\ \left(29.71^{\circ}\right) \end{gathered}$ | 181.09 | 181.10 |
|  | 194.01 | 193.99 |
|  | 195.21 | 195.41 |
|  | 211.49 | 211.68 |
|  | ---- | 191.76 |
|  | 207.84 | 207.72 |
| $\begin{gathered} +43^{\circ} \\ \left(43.09^{\circ}\right) \end{gathered}$ | 183.23 | 183.22 |
|  | 195.87 | 195.84 |
|  | 192.22 | 192.14 |
|  | 207.35 | 207.24 |

TABIE XIII (Continued)

| Angle | Measured Differences <br> in Gauss | Calculated Differences <br> in Gauss |
| :---: | :---: | :---: |
| $+75^{\circ}$ | 196.87 | 196.64 |
| $\left(75.02^{\circ}\right)$ | 213.15 | 212.93 |
|  | 188.87 | 188.53 |
|  | 202.96 | 201.41 |



|  | $\theta$ | $\phi$ |  |
| :---: | :---: | :---: | :---: |
| $\mathrm{A}_{\mathrm{X}_{1}}$ | -544.6 MHZ | $95.9^{\circ}$ | $359.5^{\circ}$ |
| $\mathrm{A}_{\mathrm{y}_{1}}$ | -546.6 MHZ | $152.2^{\circ}$ | $100.1^{\circ}$ |
| $\mathrm{A}_{\mathrm{Z}_{1}}$ | -637.1 MHZ | $63.0^{\circ}$ | $86.5^{\circ}$ |


| ${ }^{A} \mathrm{X}_{2}$ | -526.6 MHZ | $79.2^{\circ}$ | $300.5^{\circ}$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{A}_{2}$ | -525.6 MHZ | $148.7^{\circ}$ | $12.2^{\circ}$ |
| $\mathrm{Y}_{2}$ |  |  |  |
| $\mathrm{Z}_{2}$ | -616.1 MHZ | $61.0^{\circ}$ | $36.5^{\circ}$ |



Figure 18. Calculated Low-field ${ }^{29}$ Si Hyperfine Angular Dependence of the $E_{1}^{\prime \prime}$ Center for Rotation of the Magnetic Field in the X-plane. The Solid Curve Corresponds to Nucleus 1 and the Dashed Curve Corresponds to Nucleus 2. The Microwave Frequency is


Figure 19. Calculated High-field ${ }^{29}$ Si Hyperfine Angular Dependence of the $E_{1}^{\prime \prime}$ Center for Rotation of the Magnetic Field in the X-Plane. The Solid Curve Corresponds to Nucleus 1 and the Dashed Curve Corresponds to Nucleus 2. The Microwave Frequency is

18 and 19, respectively. The solid lines represent nucleus 1 and the dashed lines represent nucleus 2.

It is concluded from the experimental results that the E" centers are $S=1$ spin systems, interacting with two almost equivalent ${ }^{29}$ Si nuclei with nuclear spin $I=1 / 2$. The $\vec{g}, \vec{D}$ and $\vec{A}$ matrices have been determined for the $E_{1}^{\prime \prime}$ center and the $\vec{g}$ and $\vec{D}$ matrices for the $E_{3}^{\prime \prime}$ centers. Since the ESR spectra of the E" centers could be observed only if the sample had been irradiated at 300 K prior to 77 K irradiation, further detailed growth and pulse anneal studies not reported in this dissertation were performed by Dr. Robert B. Bossoli and the present author. It was possible to produce only very small concentrations of these $E$ " centers in hydrogen-swept quartz crystals.

A variable-temperature electron irradiation between 77 K and 300 K of an as-grown Premium Q quartz sample was performed where each irradiation was followed by a short 77 K irradiation. It was observed that the 77 K irradiation by itself will not produce the $E$ " centers unless the sample has previously received an electron irradiation above 200k. This is the temperature at which the alkali interstitial ions become mobile under electron irradiation, as reported by Markes and Halliburton in the ESR studies of the $\left[\mathrm{Al} \mathrm{e}^{+}\right]^{0}$ centers in quartz (25). Since the $E$ " centers have not been observed in Sawyer-swept Premium $Q$ samples, it suggests that alkalis ( $\mathrm{Na}^{+}, \mathrm{Li}^{+}$) must be present in the quartz samples and must be removed from the $A l^{3+}$ substitutional ions
(by the above-200K irradiation) in order for the $E$ " centers to be formed. No hyperfine ESR lines due to alkalis ( Na is $100 \%$ abundant and Li is $93 \%$ abundant, both with nuclear $\operatorname{spin} I=3 / 2$ ) were observed for the $E$ " centers even though the ESR linewidth is less than 0.05 gauss. Thus far, it has been impossible to determine the precise role of the alkalis in the formation or stabilization of the $E$ " centers.

It has also been observed that if we keep a sample, which has $E$ " centers, at 300 K for a few days then the $E_{1}^{\prime \prime}$ and $E_{3}^{\prime \prime}$ centers slowly convert into $E_{1}^{\prime}$ centers, whereas the $E_{2}^{\prime}$ centers convert into the $E_{2}^{\prime}$ and the $E_{4}^{\prime}$ centers. This conversion suggest a correlation between the $S=1$ spin systems and the $S=1 / 2$ spin systems. But, during pulse anneal studies, it was observed that the $E{ }_{2}^{\prime \prime}$ centers decay at $40^{\circ} \mathrm{C}$ while the $E_{1}^{\prime \prime}$ and $E_{3}^{\prime \prime}$ centers decay at $90^{\circ} \mathrm{C}$ and none of the $E^{\prime}(S=1 / 2)$ centers seem to grow in while the $E$ " centers are decaying. The concentration of the $E$ " centers is approximately the same as the concentration of the $E_{1}^{\prime}$ center obtained after a $350^{\circ} \mathrm{C}$ anneal. The conversion of E " to $E^{\prime}$ centers seems to be a complicated process. And the dynamics of this process are not yet understood well.

A simple way to look at the $S=1$ centers is that there are two $S=1 / 2$ centers at a distance $r$ from each other. An approximation to the value of $r$ can be made by examining the $D$ matrix. The electron dipole-electron dipole contribution to $\vec{D}$ is given by

$$
\vec{D}=\left(g^{2} \beta^{2} / 2\right)\left[\begin{array}{ccc}
\left\langle\frac{r^{2}-3 x^{2}}{5}\right. & \left.<-\frac{3 x y}{5}\right\rangle & \left.<-\frac{3 x z}{5}\right\rangle \\
<-\frac{3 x y}{5}> & \left.<\frac{r^{2}-3 y^{2}}{5}\right\rangle & \left.<-\frac{3 y z}{5}\right\rangle \\
\left.<-\frac{3 x z}{5}\right\rangle & \left.<-\frac{i y z}{5}\right\rangle & <\frac{r^{2}-3 z^{2}}{r^{5}}
\end{array}\right]
$$

where $x, y$, and $z$ are the components of the position vector $r$ between the two electrons. The integrals are over the spatial distribution of the electron. In the point dipole approximation we have

$$
D=-(3 / 2) D_{z z}=(3 / 4) g^{2} \beta^{2}\left(3 \cos ^{2} \theta-1\right) \stackrel{1}{R^{3}}
$$

from which we get the value $R$, the separation of the electrons: by using the value of $D$ obtained from fitting the spin-Hamiltonian:

$$
\begin{aligned}
& R=4.85 \AA \text { at room temperature for the } E_{1}^{\prime \prime} \text { center } \\
& R=8.03 \AA \text { at room temperature for the } E_{3}^{\prime \prime} \text { center }
\end{aligned}
$$

Taking into account the value of $R$ and the principal directions of $\vec{g}, \vec{D}, \vec{A}_{29}{ }_{S i_{1}}$, and $\vec{A}_{29_{S_{2}}}$, a plausible model for the $E_{1}^{\prime \prime}$ center is shown in Figure 20. It is suggested that two oxygens and a silicon are missing from the cluster and the two electrons are in the sp ${ }^{3}$ hybrid orbitals extending from $\mathrm{Si}(1)$ and $\mathrm{Si}(2)$. The separation between $\mathrm{Si}(1)$ and Si(2) is $5.78 \AA$ as reported in Table III. Each hybrid orbital extends approximately $0.6 \AA$ out from the silicon. Taking this extension of $\mathrm{sp}^{3}$ hybrid orbitals into account, the separation between the two unpaired electrons is 4.58 . This separation is about $5 \%$ from the predicted separation $4.85 \AA$. This discrepancy could be explained by taking into consideration the relaxation effects associated with the lattice. The possible model proposed for the $E_{1}^{\prime}$ center has a net charge of -2 . Numerical values of the different principle axes directions and Si-O and Si-Si directions are presented in Table XV for comparison.

The precursor of this center could possibly consist of the two oxygen and one silicon vacancies with charge compensating impurities


TABLE XV

SUMMARY OF DISTANCES AND DIRECTIONS FOR VARIOUS SILICON-SILICON AND SILICON-OXYGEN PAIRS IN QUARTZ, AND COMPARISON WITH Z-COMPONENT DIRECTIONS OF THE SPIN HAMILTONIAN MATRICES OF THE E" CENTER

| Atoms | Distance | $\theta$ | $\phi$ |
| :---: | :---: | :---: | :---: |
| Si(1)-Si(2) | $5.7764 \AA$ | 51.39 | $90.0^{\circ}$ |
| Si(1)-Si(0) | $3.0585 \AA$ | $53.9{ }^{\circ}$ | $65.97{ }^{\circ}$ |
| Si(1)-Ox[1,0] | 1.61221 ® | $43.99{ }^{\circ}$ | $86.28^{\circ}$ |
| Ox[2,0]-Si(2) | $1.61221 \AA$ | $43.99{ }^{\circ}$ | $93.71{ }^{\circ}$ |
| Ox[1,0]-Si(2) | 4.1853 Å | $54.3{ }^{\circ}$ | $91.2{ }^{\circ}$ |
| Si(1)-Ox[2,0] | $4.185 \AA$ | $54.26^{\circ}$ | $88.77^{\circ}$ |
| $\theta$ |  |  | $\phi$ |
| $g_{z}$ | $57.77^{\circ}$ |  | $61.9^{\circ}$ |
| $\mathrm{D}_{\mathrm{z}}$ | 56.05 |  | $63.5{ }^{\circ}$ |
| $63.0^{\circ}$ |  |  | $86.5^{\circ}$ |
| $61.0^{\circ}$ |  |  | $36.5^{\circ}$ |

nearby. It has been observed by Sibley, et al. (26) that two infrared bands, at $3367 \mathrm{~cm}^{-1}$ and $3306 \mathrm{~cm}^{-1}$, appear with irradiation at temperatures above l80K, which is the same temperature reported by Markes and Halliburton (20) for movement of alkalis away from $A l^{3+}$ ions. Kats (27) has postulated that these two infrared bands are due to $\mathrm{OH}^{-}$ions in the vicinity of $A 1^{3+}$ ions, with no alkali ions nearby. This model can be explained in the following manner. Alkali ions near $\mathrm{Al}^{3+}$ ions are moved away by irradiation above 180 K and are replaced by $\mathrm{H}^{+}$ions or holes trapped in the non-bonding orbitals of neighboring oxygen ions. We have observed that it is very difficult to form the $E$ " centers in hydrogen-swept quartz crystals. Thus it is not possible to form the E" centers unless one is able to move hydrogen in the crystal. This fact along with the appearance of the two $\mathrm{Al}-\mathrm{OH}^{-}$infrared bands for irradiations above 180 K suggests that the protons may be somehow associated with the precursors of the $E$ " center. A variety of such models are conceivable.

A similar type of model is reasonable for the $E_{3}^{\prime \prime}$ center except for the fact that the two unpaired electrons are separated by $8.033 \%$. For this center also there are two slightly inequivalent ${ }^{29}$ Si nuclei giving rise to hyperfine ESR spectra. Since the concentration of this center is almost one third that of the $E_{1}^{\prime \prime}$ center it was not possible to collect experimental data for evaluating the $\vec{A}_{29}$ matrices. A plausible model for the $E_{3}^{\prime \prime}$ center is shown in Figure 21 taking into account the separation between the two dipoles and the z-components directions for the $\vec{g}$ and $\vec{D}$ matrices.

It has also been observed in our study that the separation of the c-axis primary lines of the $\mathrm{E}_{1}^{\prime \prime}$ center increases from 5.00 G at room


Figure 21. Proposed Model for the $E_{3}^{\prime}$ Center
temperature to approximately 7.40G at about 90K. Hence, calculations of the principal values and directions for the $\vec{g}, \vec{D}$ and $\vec{A}$ matrices at 90K will enable one to shed more light on the defect structure and associated relaxation effects of the lattice. The correlation of the ESR results for the $E_{1}^{\prime \prime}$ centers with known results for the $E^{\prime}(S=1 / 2)$ and $\left[\mathrm{Al}_{\mathrm{e}^{+}}\right]$centers will help to determine the role of the $E$ " centers in the overall response of high-quality quartz to radiation.

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APPENDIX A

LISTING OF THE LINE POSITION PROGRAM FOR AN S=1 SPIN SYSTEM

```
//U1<1)55A دC3 (77?7?.448-70-37E1). (JANI',CLASS=K,TIME=(2,00).
// M\UこL~ASS=X,NCTIFY=Ul 2105A
/#PASSMOÑ? 
// EXEC FUFTGCLG
/JFEAT.SYSIN DO #
            IMPLICIT REAL * E{AH,ODZ)
```



```
        C3(3,3),H(3,3),FM(3,3), R2{3,3),R3(3,3),FT(3,3),TG(3,3),TH(3,3).
        CR(3.3)
            THE FAFAMETERS FCR THE G TENSOR A FE 1-GX, z-GY, z-GZ. a- THETA.,
            J-2M&, -OSI. THE PARAMETERS FGR THE FIVE-ST OUCTURE TEVSOR ARE
            7-ux, E-OY, 9-0Z,10-THETA, !1-PHI, 12-ESI.
            P(i)=2.3 20410+00
            P(2)=2.000E20 400
            O(2)=<..101387=+00
            2(4)=57.770+40
            P(亏)=11.925+00
            つ(د)=92.130+00
            P(7)=609.5EED+C0
            ~(0)=1:7.t010+00
            つ(j)=-227.557E+00
            ~(10)=55.05c+C0
            P(il)=33.480+00
            P(12)=25.1\in0+60
            N^!TE(E.!O) (F(I),I=1,12)
        lJ FOR:AAT (12F10.S)
            |=5.27410+0J/6.62\leqslant20+\infty0
            FinEO@=5.1:460+03
            v=3
            NM=3
            NM=1
            ALPRMA=,.0O+NO
            JETA= -90.00+00
c
c
                    GrANGE ANGLES IN JEGREES TJ FADIANS.
            OU Fi L=i,3
            P(_-3)=P(L+3)*(3.1:!5S0+00/!. -5+02)
        31 ~(L+S)=ص(L+9)*(3.1+1ミSC+00/1.ヨC+02)
            SET UP FCTATION MATEICES G(3,3), H(3,3),
            २*(3,3)。
    A O=OSIN{P(a,)
    AAv=OC[S(2(A))
    Gう=OS&N(P(!))
    ここ=UCCS(ص(5))
    F6=0SlM(2(&))
    FFG=uccs(P(E))
    AM=US\N(P(1.J))
    ムAM=こここミ{つ(!0))
    GH=OS\N(P(11))
    ECn=OCCS(F(11))
    F-mosin(P(12))
    FFr=こCCS(P(12))
    こ0=0CこS(2.0543950+40)
    SI=0SIN(2.054:9ED+00)
    心(1,1) =FFG*CCG-AAC*CC*FG
    G(b,2)=FFG*CG+AAG*CG*FG
```



```
    S(&,1)=-FG*CCC-AدG*CE*FFG
    *(L.(i) =-FG*(G+AAG*CCG*FFG
    う(<.3)=FFG*AG
```

```
    U(j,1)=AG*CG
    G(3.2)=-AG*CCG
    G(j,3)=AAG
    H(b; D)=FFH*CCH-AAH*CH*F*
    H(1, 2) =FFH*CH+AAH*CCH*F%
    H(1,3) =FH*AM
    H(&.b)=ーFH*(CF-AAr*CH*FFM
    H(2,2)=-FH=CH+AAH*CCHFFFH
    H(2,3) = FFF官AF
    H(3,1) =AM*CH
    H{j. ふ}=-Ar*CEj
    H(3,3)=AAH
2J ALこHAR=AL?HA*(3.141550+00/:.30+13)
```





```
C
6
        SET UP MAGNETIC FIELD FETATIGN MATP{X حU(I.I).
    ##(1, 1)=CCES(, 组PAR)
    Fa(1:2)=-DSIN(ALFMA=) #CSIN(3ETAR)
    \tilde{*W(L.E)=OSIN(ALPMAF) #OCCS{GETAF)}
    FM(2.1)=0.00 +00
    RM(<.2)=CCCS(EETAE)
    ₹㴘(2.3)=0 S[N(EETAE)
    NM(3,l)=-DS IN(ALPFAR)
    FM(3.2)=-DCCS(ALFHAD)*OSTN(\ThetaETAF)
    FM(3.3)=0COS(ALPHAF)*CCOS(日ETAR)
    K=1
```



```
ju =21b,1 \=1.CC+C0
    R<61.2.j=0.0C+0!
    =2(1.3)=0.20+00
    ₹ 2(2.1)=0.00+00
    R<(2.2)=1.00+00
    2 2(2,3)=0.CD+CO
    22(3.1)=0.00+00
    2 ( ( 3.2) =3.00+05
    ₹ 2(3.3)=1.00+60
    心に r6 (70,80.90).K
60 ₹ a(d.1)=1.30+60
    22(1.2)=0.0C+00
    ₹ 21.1.3)=0.00+00
    2.2(2.1)=0.00+C0
    <<2(2,2)=-1.0E+00
    2<(2,J)=0.0C+C0
    ~2(3,1)=0.0C+00
    ミ<(3.2)=0.0C+60
    <<<63.3)=-!.030+00
    N^= <-3
    SC I= (70,80,90),kK
7J ミ3(1.!)=1.00+EJ
    -3(1.2)=0.00+00
    2 3(1,3)=0.00+C0
    2j(2. }1)=0.00+6
    R 三(2,2)=1.00+00
    F-(2.3)=0.00+60
    Na(3.1)=0.00+C0
    23(3.2)=0.00+60
    R J(3,3)=1.00+00
    Gu TU 100
3v 2*(1.1)=(0
    i ((1,2)=S!
    #3(1, 土) =0.00+00
    ₹ د(2.1) =-SI
    23(2,2)=C0
```

```
    N3(2,3)=0.00+00
    23(3.1)=0.0C+60
    ~3(3.2)=0.0c+00
    R3(3.3)=1.00+00
    Gu TO 100
90 E3(1.1)=<
    Rj(1,2)=-5I
    F 3( (, 3)=0.00+60
    *3(2,b)=5 1
    2 36 2.2)=C C
    ₹3{2. ड)=0.00+00
    R 3(3,1)=0.00+00
    ? 3(3,2)=0.00+6 5
    #3(3,3)=1.00+00
    buU vO liJ L=:,3
    ) \1 2 k={.3
```



```
    OO 1 20 L= = .3
    OU 12J M=1.3
```



```
    2U 13U L=1.3
    u 1 ذ人 M=1.\Xi
    TG(L,M)=G(L,1)*E(1,M)+G(L, 2) #R(2,M)*G(L, 3) *Q(3.M)
```



```
    b=!
    14v MM=4-vOC+03
```



```
    C「G(3.3))
```



```
    OTG(3.3))
```



```
        GTご{.3})
```



```
            \bulletj=P(7)*TH(1,2)*TH(1,2)+P(3)*TH(2,2)*TH(2.2)+O(5)*TH(2.2)*TH(3.2)
```






```
            GTr(3,2)
            こうに=06/2.DC+00
            د11= 2/2. 50 + C0
            ヒヒミ=(*4-*3)/4.00+00
            うこ!=*7/2.CD+JJ
            ~N=6*4+\infty E)/4.00+CO
            い4ッ=w3/2.00+!30
            4i=* 5/2.00+00
            u0 lov L=1,3
            Ju lov n=1,Z
            AR(L.*)= 0.00+00
    ij」 A(lL:M)=0.00+00
C
C
```

    CALL HTEIOI (YM,N.AN.AI,O,E,E2,TAUS
    CALL IMTELI (N,C,E,IERR)
    GO TC (170.1ec).I
    170 FFEO=0 (2)-E(1)
    60 TC 210
    130 FiEQ=0(3)-D(2)
    2\U !F(EAES(FREQQ-FREQ)-1.00-03)220.22J.ここ0
    220 HF(I)=HH
    02 T0 240
    230 HHFHHW(FREGQ/FREQ)
    GO TC 150
    ```

```

    2うう 1=& +1
    G0 10 !.60
    zov vIFF=.دASS(HF(i)-MF(2))
    #NitE{G.270) (rF(I).I=!.2).0IFF
    270 FここMAT(3F20.4)
    {F(K-3) &80, < ¢0, 2sc
    200 }x=x+
    GこT0 A.J
    ZナU ЭETA=EETA+10.GD+00
    1F{EETA-91.00400120,300,30%
    ЈuJ बहITE{E.3:O) (P(1),I=1.12)
    310 FUNMAT('0.,12F10.E)
    STこP
    ENJ
    //LKEv•SSP OO OSNAME=OSU.ACT:2105.ESNLI3.J: SP=SHQ
/1

```

\section*{APPENDIX B}

LISTING OF THE FITTING PROGRAM FOR AN S=I SPIN SYSTEM
```

/(U1LdJJA JCB (???7?,442-70-3751).'JANI',CLASS=K.T:ME=(4,0.)),
// NSUCLASS=x,NOTIFY=L1210SA
/FFASSMCO゙O?
// EXEC FUFTGCLG
//FGET.SYSIN OO
c
C THIS PRQGRAMFITS SPIN-HAMILTENIMA
TO ESR CATA FOR S=1 DEFSCT.
IMPLICIT REAL * G (A-H,OLZ)
REAL * A AR(3,3),A{(3.3),E(3),E2(3),TAU(こ, こ),J(3),HF(3),P(20),

```

```

        Cर(3.3),FEEO1(46),FEEGO(46)
            THE PAFAMETEFS FOQ THE G TENSOR ARE 1-GX, E-GY, z-GZ, &-THETA.
    ラ-コHI, G-OSI. THE EAGAMETETS FCR THE FIVE-STRUCTURE TEVSOR ARE
    7-., نーニ. GTHETA, !0-PHI, ll-PSI, 12-19 EEPEESENT ANGLES
    BET|EEA TrE MAGNETIC FIEL= AND THE C-AXIS GF GכYSTAL.
    2(i)=2.00G410+10
    P(2)=2.000620+00
    P(3)=2.2013E5C+0.J
        P(+)= ミ7.750+00
        N(三) =32.00+00
        O(0)=92.10+00
        P(7)=341.5550+00
        P(j)=3.30+00
        P(;)=je.05D+N0
        2(:0)=33. 30+00
        p(1, )=95.1c+00
        2(12)=65.7 00+C0
        P(13)=55.7 6C+00
        2(14)=39.880+0)
        P(i5)=19.G4C+CO
        P(1G)=-19.94C+00
        -(17)=-39.EE0+00
        P(13)=-59.79C+00
        P(. Э)=-69.720+00
    ```

```

        iv=3
        vi=3
        HLPHA= J.JO+CC
    C
GHANGE ANGLES IN DEGREES TL FADIANS.
c
J0 :1 L=1,3
2(L+3)=0(L+3)=(3.171390+00/1.g0+02)
j; 3(-+S)=N(L+S)*(3.17:590+00/1. 30+02)
2u 11 t=:1.8
(1 P(u+12)=F(n+12) =(3.141590+1)/1.30+32)
*F(TE(E.!J) (F(I).I=1.20)
!J FC.EMAT (BF!O.E)
y< jC 7 ミ - L=1,1%
>><!=!
うう - -4=1

```



```

        ころ54.きヲ太,396).mM
    C
j=j HM=327E.4OC+10
\XiETAR=P{:2}
K=1
l=!
Fマミコこ(:)=5.114455450+03

```
```

        G0 T0 57
    352 HH=3232.1060+00
        l=2
        FFGGQ(2)=9.1:4C54 ¢D +03
        3` TC S7
    3s> HH=3176.3 60+00
        k=2
        l=1
        FREQO(I)=9.1149EヒTכ +03
        Sむ T6 57
    354 tr=3337.140+00
        I=2
        FAEQQ(4)=9.1149 $4 10+03
        ひこ TC ¢7
    دうこ +r=3こ12.こSO+OC
    < =3
    i=1
    ```

```

    Gu TC 57
    jう0 mH=315E.710+00
        i =2
        FFEOQ(E)=9.11495\inC+0?
        Gu TC 57
    c
6
j57 HH=32;6.0.60+00
BETAK=0(:Z)
K=l
l=1
F{゙E0667)=9.1:499370+03
うこ TL 57
300 Hm=3213.130+20
i=2
r^EO6(9)=9.1:1500+03
ن\ 50 57
35; HH=3153.53C+00
K=2
i=l
F\tilde{NCOG(G)=9.115011 co+03}
うO T心 ミ7
ジう HH= ミ34\&.1 SO+OC
I=2
F2三006:)J=5.1149@50+03
ju Ta ;7
j01 mM=33vE.570+00
*=0
i=l
F๙EこG(11)=9.1:49901 J\$03
iu TC ¢7
30< mr=32vz.ESO+CC
1=<
=マEつC(:2)=5.1:500530+03
」こ「こ!く
`
joj Hn=3こ14.24D+00
OETAR=F(14)
x=1
!=1
F\tilde{EGG(13)=9.11510030+03}
30 Tじ¢7
3\leqslant4 th= इ19 S. A4C+OC
1 =2
=^E\0(14)= %.1:151: 230+03
G二 TO ¢T
so\ HH=3125.320+00

```
```

        <=2
        i=1
        FREG((15)=9.11512150+0J
        j0 T0 57
    200 HM=3342.740+00
        I =2
        =RE00(16)=9.11509870 +03
        | }105
    367,tn=3237.030+00
        < =3
        \ = \
        FREOQ(17)=S.11510120+03
        GU TC ST
    303 nH=3222.130+00
        I=<
        FREQG(:&)=5.1:1510530+03
        ju TE 57
    C
365 HM=32;E.770+00
DETAR=F(15)
< =1
\=1
FRE00619)=9.1:1521690+03
iv r0 57
370 HH=3211.050+00
[=<
F\tilde{NOO( 20)=5.11E22130+0]}
む二 TG 57
371 HH=32N3.60+30
k=2
\ =\
FREQG(E1)=5.1:1522730+03
Gu TO 57
372 MH=3304.700+00
1 }2
FпEaG(z2)=5.!l:E ! iED + C?
心5 56 97
د7う 4mx 32E7.4ED+OG
<=3
l=1
=RE0Q6 23)=5.11522010+03
GU TG 5T
S7* AH=3241.530+00
I=2
F%EOQ(2%)=S.11522240+0こ
ju TG ST
C
-
j75 MH=3<j6.690+00
0ETAरि=0. 3つ+0.)
<=2
l=!
Fスシ60(25)=9.114\in0130+0ミ
Gu TG 57
37= Hm=325l.580+00
\ =2
FFEOQ(26)=9.11460330+03
GO 10 57
G
-
377 HH=3260.210+C0
GET AK=F(:6)
K=3
I=1
FREG6(27)=9.11459460+03

```
50 TO 97
\(375 \mathrm{HH}=3242.140+00\) \(t=2\)
FREGUイ \(23:=5.11455520+0=\)
FRE TC 57

SOj HH=3173.940+00




TC
\(=3237.090+00\)
EGC(39) \(=9.11450150+03\)
TC 57

\(l=\mathrm{F}\)
\(F R=Q 0(43)=5.114 \leq 0290+03\)
\(G U T G 97\) \(\infty\)
0
17
\begin{tabular}{l}
\(\boldsymbol{n}\) \\
\(\mathbf{0}\) \\
in \\
\\
\hline
\end{tabular}
M
```

\vartheta
351 HM=313{.510+0C
GETAF=F(1%)
K=1
i=1
FfEOG(41)=9.11514570 +03
ju TO 57
39く Hh=3こ2E.34C+OC
l=L
F₹EOQ142)=5.11513770+03
GO TU 57
35ड Mr=320E.ESC+00
<2<
l=!
=रEGG{43)=9.11514230+03
j0 T0 !?
354 חn= 324G.2\leqslant0+00
l=<
FRE20(44)=5.1151460 +03
iv rc gT
3)5 MH=3306.300+00
k=3
! =1
=शE0Q(25)=5.11513890+03
jJ 「E 57
シシ0 mH=32U2.90C*00
1 =6
Fミミ@a(at)=5.11514\leqslant10+0こ
6
C
SET UP ECTATICN MATRICES C(3.3). F(3.3). FV(3.З).
\#7 Aज=OS{A(P(4))
AA\dot{=UCES(D(4))}
Gu=0SIN(P(E))
CGi=0CES(P(E))
Fu=u SiM{F(6))
FFGacces(P(E))
AH=OS(N(P(:O))
A AM=UCCS(口(10))
LHOS{N(P(!l))
CEr=cces(P(11))
Fn=0ミ\&A(P(12))
FFH=OCCS(D(12))
Gこ=OCこ5{2.094 295C+.00)
Si=US{N(2.CS4ミ5ED+00)
U4b.1 i=FFG*CCE-AAC*CE*FG
j, 1,2J=FFGG*CG+AAG*CCG*FG
*(d,3)=FG\#\#GG
S(E,1)=-FG*CGC-AAG*CG*FFG
(2,2)=-FG*CG+AAG*CCG*FFG
i(2.3)=FFG*AG
U(3.1)=AG*CC
心(J.<)=-AG*CCG
O(2, ב)=AAG
M(!,:)=FFr*CCR-AAF*CR*F%
H(1, 己) =FFH*CH+AAH*CCH*FH
H(1,03)=FH*AM
H(2.1) =-fH*CCr-AAH*CR\#FFH
H(2,2)=-FH\#CH+AAM*CCH*FFH
H(2,3)=FFH*AH
H(3,1) =AH*CH
H(J.2)=ーAH*CCH
H(\Sigma.3)=AAM
20 ALPMAR=ALFHA*(3.14155C+0J/1.35+.)2)
G
C
SET UF MAGNETIC FIELD FOTATICN WATFIX FM(3.3).

```
```

    ₹M(1.1)=OCCS(ALPHAR)
    FM(1.,2)=-DS[N(ALPFAR)*OSIN(BETAR)
    RM(1,J)=0SIM(ALFHAR)*CCCS(EETAR)
    RM(2,1)=0.00+00
    RM(2,2)=CCES(EETAR)
    FM(2,\Xi)=OSIN(EETAF)
    Fm(3.b)=-0SIN(ALPRAR)
    AM(3.2) =-CCCS(ALPYAR)*OSIN(BETAR)
    RM(3.3)=OCCS(ALPHAR)末CCCS(3ETAR)
    4J G0 TU (50,50,50,60,60,60),K
50 ₹2(1,1)=1.00+00
२2(t.2)=0.00+60
R2(1., J)=0.0C+00
2 <(2.1)=0. 20+00
22(2.玉)=1.00+60
R2(2,3)=0.00+40
₹ 2( 3, 1)=0.50+00
N2(3,2)=0.00+00
2 21 3,3)=1.00+\infty0
iv T0 (70,80,90).K
00 त2(1.1)={.00+00
₹ 2(1.2)=0.00+C0
M2(1.,3)=0.00+00
22(2.1)=0.00+00
\hat{L}<(2.2)=-1.00+00
; 2(2,3)=0.00+00
2 2(3.1)=0.00+00
₹ 2(3.2)=0.c0+C0
k2(3.3)=-1.00+00
KN = к-3
GC TU (70.80, 80).KK
70 न3(1.1)=1.OC+00
23(1, 2)=0. 20+60
Fa(1., J)=0.0C+C0
Fj(2.b)=0.0c+00
R3(2.2)=1.00+00
\&こ(2.3)=0.00+00
23(3,1)=0.00+00
2 3(3.2)=0.00+00
23(3.3)=1.0C+00
うこ TC iJつ
ou \tilde{n}(1,1)=C0
n3(1,2)=5:
ミコ(1.3)=J.C00+60
₹3(2.1)=-s:
R3(2.2)=C5
23(2.3)=0. 30+0:)
ij(3,!)=0.00+00
₹う(3,2)=0.0.0+.00
₹ j( 3.3)=1.\omega+00
iu Tこ 100
ju 2こ(1.1)=C0
\# - (1, < )=-51
N3(1,3)=0.00+00
23(2.1)=51
M 3(2,2)=CO
R3(2,3)=0.0c+00
N-(3,1)=0.00+00
₹ 3(3.2)=0.00+00
23(3,3)=1.00+30
1JJ วO 11J L=1.3
0C 110 M=1.3

```

```

    20 12.J L=1,3
    UC 120 4=1.3
    ```

```

    DO 1 3U L= 1,3
    0 !JU val:3
    TG(L.,A)=G(L,1)=R(1,M)+G(L,2) =R(2,N)+G(L,3) #R(3,*)
    13u TH(L,M)=r(L,1)*R(1,M)+H(L, 2)*R(2,M)+H(L,3)*R(3,N)
        O X=P(7)/3.OC+00-P(3)
        2YユP(7)/3.CD+00+e(8)
        DZ=-2.00+00*F(7)/?.00+00
    ```

```

        CTG{3.3)\
    ```

```

    こTG(3.3))
    ```

```

    :TG(3.3))
        * 4=Ux* TH(1, b)*TH(1,1)+0Y*TH(2,:)*TH(2,1)+C2*TH(3,1)*TH(3,1)
    ```


```

        * T=コ X*TH(1, 2)*TH(1,1)+OY#TH(2,2)*TH(2,1) + O 2*TH(3,2) FTH(3.1)
    ```


```

        \1R=W1/2.00+00
        w1j= 2/2.00+00
        d 2F=(*4-w5)/4.N0+00
        02i=a7/Z.00+00
        ゝJミ=(44+w5)/4.00+00
        24ri=wd/Z.00+0c
        v*i=* 9/2.00+00
        UU LEU L=1.3
    Jこ lo0 M=1.3
    Aヘ̄(L.M)z 3.0C+00
    C
150 4((L.:W)=0.00+00
AR(1, 1) =w 3 +w6 +2事绿

```

```

    AI(2.1)=(01I+G41)*SSORT(2.00+00)
    Am(2,2) =& # G3F
    Aп(3.1)=2*629
    A[(3.b)=2*Q21
    A^(3.2)=(61\approx-6.E)*0S CgT(2...6+00)
    A l(3,2)=(01t-64!) =OSQRT(2. SO+CO)
    \lambdaत(3,3)=w 6-w 3+(2#63R)
    =
6
EALG FTOIDI (AN,N,AE,AI,C,E,EE,TAU)
こALL IMTQL! (N,D.E,IERR)
*こ TE (170.180),1
17う = ミEQ=0(2)-E(1)
Gu TO \&10
bju F=Eजज=c(3)-C(2)
2b」 =\#E21(MM)=FREG
MM=MMA+1
1FGMM-46;96,F6,399
J5; SUM=0.00+00
Dこ 400 4M=1,4\epsilon
4JU SUM=SUN+(FEEGI(*M)-FEEGG(MM))*\#2
GO TC (401,40\&,410).KI
4いし うUm2=sUM
1F(LL-1)402:402,403
*J๕゙ SN|l=SUM
405 5i re (404.404.404.405.405.405,456.457.405.405.4.05.
= 406.406.406.406.406.436.430.4J6).LL
404 PP=1.UC-05

```
```

    SC TG 407
    ```

```

    G0 50 407
    400 PP=0.U10+00
    GO TO 407
    457 P0=0.0010+00
    Gu TO 407
    +vo PP=0.0 D + OO*{\Xi. 1419 50+00/130.00+00)
    +U7 P(LL)=F(LL)+PR
    x1=xi+1
    GO TOGS
    400 &F(SUM-SUMZ)93.409.409
    4UG コ(-L)=P(LL)-2.00+00*P0
        K1=K1+1
        Gu TC 55
    4dJ [F(SU:A-SUM1)9E.411.411
    41b P(LL)=F(LL)+FF
    gJ GENTINLE
        1F(SUM-SUM2)412.413.413
    +12 SUNM2=SUM
    4|J CLNTINLE
    #RITE{E.420)SLMZ
    42U FORMAT(' SUMZ EOUALS',F!2.3)
    #R(TE(E.430) (O(I).t =1.20)
    +JJ F=\tilde{mat(*O!,EFI2.E)}
    (F(SUM1-SU:W2)450,450,52
    45O GUNTINLE
    0し 45! L=1.3
    P(_+3)=O(L+3) (1 (120. 50+0.3/3.141590+)0)
    ```

```

    OO 4Eこ N=1,\varepsilon
    ->E3 (M+11)=P(M+1 1) #(13).00+30/3.1&1550+00)
    * F1TEí0,452)
    ->E FENTAT(1HO,Z2H THE FINAL SET CF EAOANETEFS IS )
    *N(ITE{G.430) {F(I),I=1.19)
    Jx=ص(7)/3.0&+00-0(9)
    ว Y=0(7)/3. 30+00+E{9)
    O<=-2.OC+00*O{(7)/E.00+00
    #RITE{E,500)0X,0Y,0Z
    ```

```

    3丁陆
    ENO
    ノルKEJ.SSP こD OSNAMEZCSU.ACT:2135.ES=LIE,DISF=SHa
//

```

\section*{APPENDIX C}

INITIAL DATA FOR \({ }^{29}\) Si HYPERFINE INTERACTION OF THE E" CENTER

Nucleus 1
\begin{tabular}{|c|c|c|c|c|}
\hline \multicolumn{3}{|c|}{Low Field} & \multicolumn{2}{|c|}{High Field} \\
\hline & Magnetic & Microwave & Magnetic & Microwave \\
\hline & Field & Frequency & Field & Frequency \\
\hline Angle & in Gauss & in GHz & in Gauss & in GHz \\
\hline \multirow[t]{6}{*}{\(-70^{\circ}\)} & 3163.58* & 9.3657049 & 3380.60* & 9.3657625 \\
\hline & 3190.57 & 9.3657064 & 3387.28 & 9.3657957 \\
\hline & 3227.43* & 9.3657087 & 3428.83* & 9.3658104 \\
\hline & 3243.60 & 9.3657107 & 3462.17* & 9.3658222 \\
\hline & 3286.78 & 9.3657127 & 3498.1 & 9.3658355 \\
\hline & 3308.08 & 9.3657149 & 3507.37 & 9.3658410 \\
\hline \multirow[t]{6}{*}{\(-48^{\circ}\)} & 3139.84* & 9.3448948 & 3352.59* & 9.3449090 \\
\hline & 3185.12 & 9.3448952 & 3377.26 & 9.3449114 \\
\hline & 3202.96 & 9.3448960 & 3401.59 & 9.3449126 \\
\hline & 3256.59 & 9.3448976 & 3471.57 & 9.3449143 \\
\hline & 3282.56 & 9.3448987 & 3488.98 & 9.3449157 \\
\hline & 3314.86* & 9.3449003 & 3516.94* & 9.3449164 \\
\hline \multirow[t]{6}{*}{\(-30^{\circ}\)} & 3150.54 & 9.3255709 & 3363.88 & 9.3255487 \\
\hline & 3175.83 & 9.3255934 & 3368.68 & 9.3255472 \\
\hline & 3214.34 & 9.3255884 & 3412.48 & 9.3255418 \\
\hline & 3231.21 & 9.3255844 & 3446.70 & 9.3255368 \\
\hline & 3282.50 & 9.3255818 & 3489.21 & 9.3255340 \\
\hline & 3291.67 & 9.3255771 & 3492.1 & 9.3255325 \\
\hline \multirow[t]{6}{*}{\(-15^{\circ}\)} & 3180.32 & 9.3161709 & 3380.85 & 9.3161746 \\
\hline & 3192.17 & 9.3161707 & 3395.85 & 9.3161746 \\
\hline & 3219.2 & 9.3161712 & 3417.56 & 9.3161743 \\
\hline & 3221.36 & 9.3161715 & 3433.22 & 9.3161740 \\
\hline & 3257.39 & 9.3161712 & 3455.56 & 9.3161740 \\
\hline & 3258.55 & 9.3161726 & 3461.93 & 9.3161742 \\
\hline \multirow[t]{6}{*}{\(-10^{\circ}\)} & 3193.0 & 9.3161886 & 3391.59 & 9.3161842 \\
\hline & 3201.76 & 9.3161890 & 3407.30 & 9.3161827 \\
\hline & 3218.23 & 9.3161896 & 3418.48 & 9.3161819 \\
\hline & 3223.13 & 9.3161899 & 3431.50 & 9.3161821 \\
\hline & 3246.05 & 9.3161905 & 3443.04 & 9.3161811 \\
\hline & 3247.24 & 9.3161907 & 3452.58 & 9.3161800 \\
\hline \multirow[t]{6}{*}{\(-5^{0}\)} & 3206.16* & 9.3162544 & 3403.93 & 9.3162780 \\
\hline & 3212.56 & 9.3162517 & 3418.43* & 9.3162745 \\
\hline & 3218.27* & 9.3162497 & 3419.03* & 9.3162735 \\
\hline & 3224.07* & 9.3162483 & 3430.16* & 9.3162697 \\
\hline & 3233.18 & 9.3162473 & 3430.6* & 9.3162686 \\
\hline & 3235.62* & 9.3162467 & 3442.0 & 9.3162661 \\
\hline \multirow[t]{2}{*}{\(0^{\circ}\)} & 3219.2 & 9.3163091 & 3417.4 & 9.3162963 \\
\hline & 3224.1 & 9.3163066 & 3430.57 & 9.3162945 \\
\hline
\end{tabular}

Nucleus 1 (Continued)
\begin{tabular}{cccccc}
\hline & Low Field & & & & High Field \\
\hline & \begin{tabular}{c} 
Magnetic \\
Field \\
in Gauss
\end{tabular} & \begin{tabular}{c} 
Microwave \\
Frequency \\
Angle
\end{tabular} & & Magnetic & Field
\end{tabular}
*These values were not used in the fitting procedure to determine the hyperfine matrix parameters.

Nucleus 2
\begin{tabular}{|c|c|c|c|c|}
\hline \multicolumn{3}{|c|}{Low Field} & \multicolumn{2}{|c|}{High Field} \\
\hline & Magnetic & Microwave & Magnetic & Microwave \\
\hline & Field & Frequency & Field & Frequency \\
\hline Angle & in Gauss & in GHz & in Gauss & in GHz \\
\hline \multirow[t]{6}{*}{\(-70^{\circ}\)} & 3159.23 & 9.3657035 & 3379.91 & 9.3657729 \\
\hline & 3198.05 & 9.3657077 & 3385.96 & 9.3657839 \\
\hline & 3236.66* & 9.3657096 & 3419.59* & 9.3658035 \\
\hline & 3253.14* & 9.3657120 & 3452.78* & 9.3658156 \\
\hline & 3294.88 & 9.3657133 & 3490.23 & 9.3658156 \\
\hline & 3302.21 & 9.3657143 & 3514.23 & 9.3658464 \\
\hline \multirow[t]{6}{*}{\(-48^{\circ}\)} & 3136.82* & 9.3448946 & 3358.73* & 9.3449086 \\
\hline & 3190.77 & 9.3448958 & 3371.82 & 9.3449107 \\
\hline & 3212.32 & 9.3448965 & 3392.24 & 9.3449118 \\
\hline & 3266.43 & 9.3448984 & 3461.86 & 9.3449136 \\
\hline & 3288.76 & 9.3448991 & 3483.17 & 9.3449152 \\
\hline & 3310.41* & 9.3448997 & 3522.31 & 9.3449170 \\
\hline \multirow[t]{6}{*}{\(-30^{\circ}\)} & 3149.94 & 9.3255721 & 3361.02 & 9.3255509 \\
\hline & 3179.04 & 9.3255909 & 3370.04 & 9.3255453 \\
\hline & 3222.83 & 9.3255863 & 3404.05 & 9.3255434 \\
\hline & 3240.07 & 9.3255831 & 3438.03 & 9.3255401 \\
\hline & 3286.23 & 9.3255806 & 3481.03 & 9.3255353 \\
\hline & 3290.02 & 9.3255784 & 3494.47 & 9.3255311 \\
\hline \multirow[t]{6}{*}{\(-15^{\circ}\)} & 3182.1 & 9.3161706 & 3378.73 & 9.3161748 \\
\hline & 3194.67 & 9.3161709 & 3394.72 & 9.3161746 \\
\hline & 3225.78 & 9.3161717 & 3411.59 & 9.3161746 \\
\hline & 3227.52 & 9.3161712 & 3426.98 & 9.3161743 \\
\hline & 3259.59 & 9.3161727 & 3455.03 & 9.3161739 \\
\hline & 3260.44 & 9.3161722 & 3459.47 & 9.3161741 \\
\hline \multirow[t]{6}{*}{\(-10^{\circ}\)} & 3195.51 & 9.3161888 & 3389.39 & 9.3161847 \\
\hline & 3204.36 & 9.3161893 & 3405.41 & 9.3161837 \\
\hline & 3223.91 & 9.3161901 & 3413.50 & 9.3161823 \\
\hline & 3228.33 & 9.3161902 & 3426.22 & 9.3161814 \\
\hline & 3249.08 & 9.3161916 & 3441.64 & 9.3161819 \\
\hline & 3249.20 & 9.3161917 & 3450.02 & 9.3161807 \\
\hline \multirow[t]{6}{*}{\(-5^{\circ}\)} & 3209.32* & 9.3162530 & 3401.50 & 9.3162795 \\
\hline & 3215.39 & 9.3162504 & 3414.52* & 9.3162770 \\
\hline & 3222.99* & 9.3162490 & 3416.43* & 9.3162757 \\
\hline & 3228.28* & 9.3162476 & 3426.34* & 9.3162723 \\
\hline & 3236.57 & 9.3162463 & 3427.99* & 9.3162707 \\
\hline & 3238.20* & 9.3162459 & 3439.17 & 9.3162670 \\
\hline \multirow[t]{2}{*}{\(0^{\circ}\)} & 3223.00 & 9.31630 .76 & 3414.60 & 9.3162972 \\
\hline & 3227.30 & 9.3163059 & 3427.35 & 9.3162954 \\
\hline
\end{tabular}

\section*{Nucleus 2 (Continued)}
\begin{tabular}{|c|c|c|c|c|}
\hline \multicolumn{3}{|c|}{Low Field} & \multicolumn{2}{|c|}{High Field} \\
\hline & Magnetic & Microwave & Magnetic & Microwave \\
\hline & Field & Frequency & Field & Frequency \\
\hline Angle & in Gauss & in GHz & in Gauss & in GHz \\
\hline \multirow[t]{6}{*}{\(+5^{\circ}\)} & 3206.46 & 9.3096967 & 3399.49 & 9.3096869 \\
\hline & 3214.32 & 9.3096959 & 3411.45 & 9.3096854 \\
\hline & 3221.61 & 9.3096946 & 3412.51 & 9.3096844 \\
\hline & 3223.12 & 9.3096945 & 3426.14 & 9.3096822 \\
\hline & 3233.85 & 9.3096941 & 3426.91 & 9.3096814 \\
\hline & 3237.52 & 9.3096441 & 3435.63 & 9.3096801 \\
\hline \multirow[t]{6}{*}{\(+10^{\circ}\)} & 3192.18 & 9.3096660 & 3387.72 & 9.3096599 \\
\hline & 3204.29 & 9.3096653 & 3399.74 & 9.3096594 \\
\hline & 3220.44 & 9.3096647 & 3409.70 & 9.3096591 \\
\hline & 3223.43 & 9.3096638 & 3409.70 & 9.3096591 \\
\hline & 3246.14 & 9.3096629 & 3440.18 & 9.3096582 \\
\hline & 3250.19 & 9.3096626 & 3445.48 & 9.3096576 \\
\hline \multirow[t]{6}{*}{\(+15^{\circ}\)} & 3178.02 & 9.3096562 & 3377.18 & 9.3096565 \\
\hline & 3195.06 & 9.3096567 & 3386.91 & 9.3096572 \\
\hline & 3216.99 & 9.3096573 & 3406.90 & 9.3096575 \\
\hline & 3226.16 & 9.3096575 & 3432.97 & 9.3096581 \\
\hline & 3257.25 & 9.3096577 & 3454.24 & 9.3096581 \\
\hline & 3262.85 & 9.3096575 & 3454.50 & 9.3096579 \\
\hline \multirow[t]{6}{*}{\(+30^{\circ}\)} & 3164.06 & 9.3715450 & 3375.55 & 9.3715417 \\
\hline & 3197.84 & 9.3715437 & 3378.93 & 9.3715417 \\
\hline & 3225.62* & 9.3715433 & ----- & ------ \\
\hline & 3260.52* & 9.3715426 & 3468.36* & 9.3715426 \\
\hline & 3301.26 & 9.3715418 & 3495.27 & 9.3715426 \\
\hline & 3317.66 & 9.3715415 & 3512.87 & 9.3715431 \\
\hline \multirow[t]{6}{*}{\(+43^{\circ}\)} & 3131.96* & 9.3377253 & 3353.42* & 9.3377391 \\
\hline & 3181.55 & 9.3377262 & 3364.78 & 9.3377400 \\
\hline & 3199.42 & 9.3377268 & 3391.64 & 9.3377400 \\
\hline & 3262.70 & 9.3377274 & 3470.05 & 9.3377398 \\
\hline & 3291.35 & 9.3377284 & 3487.23 & 9.3377394 \\
\hline & 3314.58* & 9.3377293 & 3521.26* & 9.3377397 \\
\hline \multirow[t]{6}{*}{\(+75^{\circ}\)} & 3164.40* & 9.3754088 & 3378.18* & 9.3754042 \\
\hline & 3189.31* & 9.3754088 & 3385.57* & 9.3754029 \\
\hline & 3235.92 & 9.3754088 & 3432.79 & 9.3754008 \\
\hline & 3247.37 & 9.3754088 & 3460.52 & 9.3753999 \\
\hline & 3302.96* & 9.3754085 & 3505.92* & 9.3753992 \\
\hline & 3309.62* & 9.3754069 & 3514.70* & 9.3753972 \\
\hline
\end{tabular}
*These values were not used in the fitting procedure to determine the hyperfine matrix parameters.

APPENDIX D

LISTING OF THE LINE POSITION PROGRAM FOR \(S=1, \quad I=\frac{1}{2}\) SPIN SYSTEM
```

//UL2105A -CS (?77??.443-70-3751).'JANI',CLASS=A.TIME=(0,40),
// MSUCLASS=X,NOTIFY=U1210ミA
/*PMゝう^ごRD ?
| ExEC FGOTGCLG
//FCET.SYSIN OD *
\MPLICIT REAL * S (A-H,O-Z)
FEAL S AR(6,6),A{(5,6),E(6),E2(6),TAU(2,E),C(E),MF(3),P(:3).

```

```

            にR(土.3),Z(3,3),TZ(3.3)
    c
c
C
C
E
E
C
THE PAFAMETEFS FQE THE G TENSOQ ADE i-GX, z-GY, Z-GZ, 4-THETA,
S-JM\&. S-OSI. THE PARAMETERS FCR THE FINE-STVUCTURE TEVSCR ARE

```

```

        THE HYFEQFINE TENSOR ARE 13-AX, IQ-AY, 1J-AZ, IE-THETA, 1T-FHI,
        Hivu d &OSI.
        2(1)=2. 5 04:0+00
        P(2)=2 .000e20+00
        N(3)=2.00138700+00
        (+)=ミ7.770+00
        P(j)=31.920+00
        P(2)=92.1 30+00
        2(7)=105.95E0+00
        P(v)=11:1.6010+00
        つ(シ)=-227.557C+00
        P(bu)=55.050+00
        P(11)=33.4 90+c0
        P(12)=E5.1 LO+00
        O(13)=-К42.\epsilon0+00
        O(14)=-54 5 . Є O + + O
        つ(jミ)=-643.10+00
        P(i0)=E3.0 O+00
        2(17)=56.50+00
        2:13)=175.\in\inD+00
    E
E
OGTPUT GIVEN SET CF OAEAMETEOS.

```

```

    L\nu F:GMAT (:H ,EFIO.5)
        3=y.27410+00/E.6242D+00
        joiv=-0.4530-04
        F\EOO=9.3! <30+J3
        N=j
        .v M=S
        NN=1
        A6.3HA= 0. 30+06
        دミTA=-90.OC+NO
    E
つこ j1 L=1:3
p(-+2)=ص(L+3)*(3.141\leq50+0011. ع\+02)
P(に+9)=尺(L+9) *(3.141气9C+0)/1.90+02)
31 P(-+1E)=0(L+1E)*(3.14:590+90/1.30+.32)
C
G SETUP FCTATIEN MATEICES G(3, J). H(3.3), 2M(3, 3).
C
AG=DS{N{P(A))
AAv=OCES(P(4))
Cu=OS(N(P(S))
ccumucas(P(E))
Fj=0SIN(P(6))

```
```

    FFG=ucos{P(6))
    Am=OS iN{P(:0))
    AAM=CCES(P(10))
    = MxOSIN(P(:l))
    CCH=OCOS(P(1:1))
    FH=OSIM(P(12))
    FFH=UCCS(P(12))
    AL=OS{N(P(16))
    AA<=OCES(D(16))
    CZ=OSIN(P(17))
    CG=0CES(P{17])
    =z=0SIN(P(1e))
    FFZ=CCOS{(\18)}
    ここ=0Cコ S(2.054 5950+00)
    Si=0SIN(2.c543550+00)
    G(L,D)=FFFG*CCG-AAG*CE*FG
    G(1,2) =FFG*CG+AAG*CCG*FG
    G(b, 3)=FG*AG
    J(2,L)=~FG*CCG-AAG*CG*FFG
    *(L.2) ~FG*CG +AAG*CCG*FFG
    6(L.3)=FFFG#AG
    G(3,1) =AG#CG
    G(3,2)=-AG#CCG
    G(j.3)=AAG
    H(1.1)=FFH*CC5-AARFCH#FF
    +(&,2) =FFH*Cr+AAR*CCH*FH
    #(1, 3) = FW%AM
    H(Z,1) =-FF*CGH-AAR*CF#FFH
    H(2,2) =-FH#CH+AAH*CCH#FFH
    H(2,3)=FFH*AM
    H(j,1)=AM*CH
    H(j.2) =ص&H#CCH
    H(3,3)=AAH
    L&&.1)=FFZ*CCZ-A4Z*CZ*FZ
    Z(d.2)=FFZ*CZ+AAZ*CこZ*FZ
    L(b,J)=FZ*A2
    Z(く.d)=-FZ*(CZ-AAZ*CZ*FFZ
    Z(2.2)=-FZ*GZ+AAZ*CCZ*FFZ
    -(く.3)=FFZ#AZ
    <(う.1)=AZ*CZ
    Z(3., ) =-AZ *CCZ
    L(コ,コ) =AAZ
    2) ALJHAR=AL?HA* (3.1715 90 +0./1.30+32)
    ~ETAन}=\mathrm{ EETA*{3.141590+00/1.30+02)
    *तiTE(\epsilon.3D) 4LFRA. EETA
    ```

```

<

- SET LP *AGNETIC FIELD ECTATIEN AATFIX OM(I.I).
O
FM(1, 1)=CCOS(A_OPAZ)
\#N(1,2)=-DSIN(ALPHAO)*CS:N(BETAO)

```

```

    ₹ N(2.i)=0.00+00
    ₹M^2.ट)=OCCS(EETAE)
    २M(2.3)=0 SIN(EETAR)
    ₹m(3.1) =-CSIN(ALPHAR)
    RM(3,2)=-OCCS(ALOHAZ)*OS[N(EETAR)
    R(A(3.Z)=0COS(ALPHAR)*CCDS(9ETAR)
    K =1
    * SO TC (50.50.E0,60.63.6)).K
53 ^2(1,:1)=1.60+60
~2(1,2)=0.00+00
22i 1.3)=0.50+00
22(2,i)=0.00+00
2\angle(2,2)=1.0C+00
२<(2,3)=0.00+00

```
```

    R2(3.1)=0.00+00
    ₹ < ( 3,2)=0.00+00
    <2(3.3)=1.00+60
    GE rG (70.30.90).K
    O) ₹ 2(1, 1)=1.00+60
N2(1,2)=0.0C+00
226 (.3)=0.00+60
2 2(2.1)=0.00+C0
N2(2.2)=-1.00 +00
2 2{2, 3)=0.c0+C0
*2(3,1)=0.0C+00
R2(3.2)=0.00+00
2 2(3.3)=-1.00+00
Kx = x-3
jこ T0 (79.30.50).0k
7) ₹ 3(1,1)=1.00+00
F3(1,2)=0.00+00
2 3(1.,3)=0.00+00
23(2.1)=0.00+60
23(2.2)=1.00+00
736 2.3)=0.0E+60
F3(3.1)=0.0C+00
\# 3(3.2)=0.00+00
२ z(3.3)=1.00+00
ت゙ TC 100
*) 73.1.1)=60
₹ 3(1, < )=5!
F3(1.,3)=0.00+00
E -3(2.1) =-51
2 3(2.2)=c0
R3(2.3)=0.0C+00
₹ う(コ.1)=1.OC+C0
\approx3(3.2)=0.0c+00
23(3.3)=1.0C+00
-0 TC 1J0
ジ N3(1.,1)=<0
ミう(1,2)=-5!
₹ 3( (., 3)=0. 00+60
\#う!2,1)=S!
231 2,21=c0
२a(2, E)=0.CD+C0
N3(3.1)=0.00+00
२ E( 3.2)=0. 50 + C0
₹3(3.3)=:.CO+CO
bu\ uL 1!0 L=1,3
je 11 J v=1.3

```

```

    ว 1こうL=1.3
    2U 12, 4*1.J
    ```

```

    うせ 1こJ L=1.3
    J0 130 *=1.3
    J`(L,M)=G(L,1)*F(1,M)+G(L, 2)*2(2,M)+G(L, 3)*R(3,*)
    TH(L,0&)=H(L,1)*R(1,M)+H(L, 2) #F(2,M)+H(L, 3)*E(3,*)
    ```

```

    I =1
    140 HH=4.OCD+03
15u 1^8*rr*(O(1)*TG(1,1)*TG(1,3)+O(2)*TG(E,1)*TG(2,3)+P(3)*TG(3.1)*
こ「心(3,3)}

```

```

    CTG(3,3))
    ```

```

    CTG(3,3))
    #4=P(7)*TH(1,1)*TH(1,1)+P(3)*TM(2,1)*TH(2,1)+0(5)*TH(3.1)*TH(3.1)
    ```





```

        CTH(3.2)
            *1UユP(13)*TZ(1, b)*TZ(i,1) +つ(14)*TZ(2,1)*TZ(2,1)+P(15)*TZ(3,1)
    GFT:6(3.1)
    ```

```

    CP(15)*TZ{3.1)*TZ(3.2)
    *12=P(13)*TZ(1,1)#TZ(1, Z)+0(1&)#TZ(2.1)*TZ(2,3)+
    6?(15)#TZ(3.1)*TZ(3.3)
    * 13 = F(13)*TZ(1,2)*TZ(1,2)+P(14)*TZ(2,2)*TZ(2,2)+
    ここ(15)*TZ(*.2)*TZ(3.2)
    * (->=~(13)*TZ(1, <)*TZ({, 3)+O(!*)*TZ(2,2)*TZ(2,3)+
    =P(i5)=TZ(3,2)#TZ(3, 工)
    ```

```

    -2(15)*TZ(3.3)*TZ(3, 3)
    \1R=N1/2.30+00
    \1」= (2/2. JD+0.)
    * 2R=(* 4-m S 1/4.00 +00
    G2I=*7/2.00+00
    2 FR = (* 4+w 5)/4.CD + 00
    ~4F=#0/2.00+00
    -4l=#5/2.00+00
    ```

```

    \bullet5\ =.41 1/2.0C+00
    j 0ミ= (-1 J+w(3)14.00+30
    47スニツ12/2.00+10
    O7I=1 14/2.00+00
    うこ léll=1.E
    JC 100 v=1,6
    AR(L,M)=0.00+00
    100 + ((L.N:M)=0.00+00
        SET LP SEIM-HAMILTCNIAN FEAL ANE IMAGINARY MATOIX
        ELEMENTS AF(E.E), A{{(6.6).
    AF(1,1)=w3+w6+w1\leqR.CD+.j0+2.JC+!J*(23R-(GヨN#F+)/E.JO+1))
    ```

```

    AF(J.J)=4.0C+00*63R-(GEN*#H)/2.00+00
    ```



```

    i^(2.1)=07R
    i\(2,1)=07!
    ```

```

    Ai(3.1)={01!+C4!)*SSORT(2.00+00)+QT1/0SGRT(2. 20+00)
    Н\tilde{R}(3.2)=65F#CSERT(2.00+00)
    ~内(4, ) =0 SF*OSGRT(2.00 +0つ)
    \lambdai(4,1)=0SI*0\leq0FT{2.00+00)
    ```


```

    AE(5.1)=2.00+CO*Q2?
    A((5.1)=2.100+0n*&2!
    ```


```

    A*( j. -) =0 SGET(2.J(t+0)) #06R
    ```

```

    A1(6,2)=22.00+00*621
    A\tilde{N}(5.2)=0 SCRT(2.00+00) #05R
    Ai(5.3)=OSQRT(2.00+00)*0EI
    AR(6.4)=(01F-64F)*OSG2T(2.00+00)-G70/CSCRT(2.)040)
    AI(6.4)=(0!I-64!)*mSORT(2.00+00)-07I/0\leq@RT(2.)0+n0)
    AF(5.5)=-67F
    A!(c.ミ)=-07!
    DIAGCNALIZE SPINEENERGY YATRIX A(5,E).
    ```
```

C
CALL HTFIOI (NM,N,AP,AI,D,E,E?,TAL)
CALL IMTGL! (N,G.E.IERR)
i0 TO (170.180.190.200).!
170 FFEG=0(4)-E(1)
SC TG 210
18U FE=Q=0(6)-C(4)
* TO 210
190 FREG=0(5)-0(3)
*0 TO 210
200 FEEQ=0(3)-0(2)
21J 1F(DABS{FEEGGRFEEG)-1.0C-:)1)223,220.23J
220 HF(I)=HH
UC TC 240
2コ\nu TH=NHF(FQEGG/FREQ)
GC TC 150
2%J IF(I-4)250.260.260
えう」 !={+!
GO TO }14
LOU DIFF1=C\lambdaBS(FF(3)-HF(2))
O{FFE=CABS(MF(4)-HF(1))
*RITE(6,270) (HF(!),I=1.4).0!FF!.0!FF2
27U FGRMAT(5Fこ0.4)
|F(K-J)280.E50,250
2dコ K=K+1
jo ic 40
cSu 3ETA=3ETA +10.CD+00
lF(EミT د-今1.00 +00) 80, 300. 300
3u\ MR!TE(E,3!J) (F{!).!=!.18)
31J FGズMAT(1O*,GF10.E)
TTUP
ENU
//-KEA.SSP \O DSNANE=OSL.ACTI210S.ESRLIE.OISP=SKミ
/%

```

APPENDIX E

\section*{LISTING OF THE FITTING PROGRAM FOR AN \(S=1\), \(I=\frac{1}{2}\) SPIN SYSTEM}

```

// MSGCLASS=X,NOTIFY=U12105A
/*PASSNORD ?
/f EXEC FORTGCLG
//FORT.SYSIN DO *
SMPLICIT REAL 8 (AOH*O-Z)
REAL * B AR(6:6):AI(6,6),E(G),E2(6),TAU(2,6),0(6),HF(6),P(18),

```

```

        CR(3,3),2&3,3),TZ13:3),FREQQ(34);DF{34,2):DFC(34,2)
    c
C
C
C
C
C
C
C
C
P(1)=2.000410+00
P{2}=2.000620+00
P(3)=2.00138700+00
P{41=57.770+00
P(5)=31.920+00
P(E)=92.18D+00
P(7)=109.9550+00
P{8}=117.6010+00
P(9)=-227.5570+00
P{10)=56.050+00
P(11)=33.480+00
P{12}=85,160+00
p(13)=-545.60+00
P(1+4)=-545.60+00
P{15}=-633.10+00
P{16}=63.00+00
P(17)=55.50*00
P{18)=193.060+00
WRITE(6,10) (P{(\),I=1,18)
10 FORMAT (1H ,6F10.51
B=9.27410+00/6.62620+00
GBN=-8.4580-04
N=6
iNM=6
ALPHA= O.OD\&OO
00 91 L=1,3
P(L+3)=P{L+3)=13.141590+00/1.80+02)
P{L+9)=P(L+9)*{3.141590+00/2.30+02)
91 P(L+15)=P(L+15i*{ 3.141590+00/1.80+02)
9200 93 LL=13,18
94 < 1=1
95Mm=1
96 GOTO 1 351,352,353,354,355,356,357,358,359,360,
C 361,362,363,364,365,366,367,368,369,370,
C 371,372,373,374,375,376,377,378,379,380,
C 3811.MM
C
C
C
C
C
THE PARAMETERS FOR THE G TENSOR ARE 1-GX, 2-GY, 3-GZ, 4-THETA,
S-PHI, GOPSI. TME PARAMETERS FOR THE FINE-STRUCTURE TENSOR APE
7-UX, 8-DY, 9-0Z, 10-THETA, 11-PHI, 12-PSI.THE PARAMETERS
FOR HYPERFIME MATRIX ARE 13-AX, 14-AY, 15-AZ. 16-THETA,
1T-PHI, 18-PSI.
(15)=31.720+00
P(1)
C
INPUT EXPERTMENTAL DATA IN THE fORM OF OIFFERENCES
getmeen different field yalues ratmer than paricular
magnetic field values since the corrections mere
NOT OBTAINED FOR INDIyIDUAL LINE POSITIONS.
351 gETA=14.950+00
DF{1,1)=190.420+00
DF{1,2)=206.450+00
X=1

```

FREQQ\{11=9•30965780+03
GOTD 97
352 DF( 2,1\()=201 \cdot 210+00\) DF\{ \(2,21=219 \cdot 350+00\) \(K=2\) FREOQ121=FREOOS11 GO TO 97
353 DF\{3.1) \(=190.250+00\) OFC 3, \(21=208.090+00\) \(K=3\)
FREQO(3) =FREQQ(1)
GO 1097
354 BETA=9.93D+00
DF\{ 4,1\(\}=191.280+00\) UFi \(4,2 t=208.080+00\)
\[
k=1
\]

FREQQ(4)=9.30966170+03
GOTO 97
355 DFA5.11:198.320+00 DF(5,2)=216.70+00
\(K=2\)
FREQQR 5 ) \(=\) FREQOR4
GO TO 97
356 DF\{ 6,1\()=191.27 D+00\)
\(D F(6 ; 2)=209 \cdot 20+00\)
\(K=3\)
FREQO(6):FREQQ(4)
GO TO 97
357 BETA=4.980+00
DF(7,1)=192.260 +00
DF\{7,21=209.72D+00
\(K=1\)
FRE QQ6 \(71=9.30968890+03\)
\(60 \quad 1097\)
358 DF \((8,1)=195.560+00\) DF \((8,2)=214,020+00\)
\(K=2\)
FRE OQ( 8 ) =FREOQRT)
GO TO 97
359 DF(9.1) \(=192 \cdot 30+00\)
DF\{9:21=210.30+00
\(K=3\)
FREGO(9)=FREQQ(T)
60 TO 97
360 BET \(A=0.00+00\)
DF \((10: 1)=193 \cdot 30+00\)
DF( 10,2\()=211 \cdot 370+00\)
\(K=2\)
FRE QQ 10) \(=9 \cdot 31630160+03\)
GO 1097
361 BETA=-4.960+00
DF(11,1) \(=191 \cdot 370+00\)
DF( 11,2\()=208 \cdot 820+00\)
\(K=2\)
FREQ01111=9.3162607D+03
\(60 \quad 1097\)
362 BETA=-9.950+00
DF( 12,1\()=195.8 D+00\)
DF(12.2) \(=214 \cdot 30+00\)
\(K=1\)
FRE OO\& \(121=9 \cdot 31618590+03\)
GOTO 97
363 DF\{ 13.1\(\}=189.830+00\)
\(D F\{13=21=206 \cdot 530+00\)
\(K=2\)
FREQQR13J=FREOQ\{123

GO 1097
364 DF(14,1) \(=195 \cdot 350+00\) DF \((14,2)=213 \cdot 270+00\) \(K=3\)
FREQQS14 3=FREQQ( 221 GOTO 97
365 BETA \(=-14.920+00\)
DF(15,1) \(=197.010+00\)
OF 15,2\(\}=215.530+00\) \(K=1\)
FREQQ1151=9.31617280+03 GO 1097
366 DF( 16,1\()=188 \cdot 680+00\) DF( 16,2\()=204.540+00\) \(K=2\)
FREGG1 16 JIFFRECOI 151 601097
367 DF\{17.1) \(=196.20+00\) DF\{17,21=214-020+00 \(K=3\)
FREQU117) =FREQQ1 15: \(60 T 097\)
368 日ETA \(=30.45 \mathrm{D}+00\) DF \((18,1\}=188.050+00\) \(D F\{18,2\}=201,710+00\) \(K=2\)
FREQQ(18) \(=9 \cdot 32556140+03\) GO TO 97
369 DF(19, 1) \(=198.140+00\) DF(19:2) \(=215.490+00\) \(K=3\)
FREOOR19) =FRECOR 181 GO TO 97
370 DF \((20,1)=200.430+00\) DF( 20,2\()=218.1+0+00\) \(K=1\)
FRE GQR 20 ) \(x\) FREQQC 28 1 GO TO 97
371 EETA=-49.00+00
DF(21,1): \(192 \cdot 140+00\)
DF\{21,2)=206.420+00
\(k=2\)
FRE 00 (21) \(=0.34490510+03\) GOTO 97
372 טF( 22, 11: \(198.630+00\) UF \(122,2:=214.980+00\) \(K=3\)
FREGOR 22 : FFREQOI 21 : 601097
373 日 OF (23. \(11=199.220+00\) DFi 23, \(21=217,020+00\)
\(K=1\)
FRE CO1 23 : \(=9.36575530+03\)
GO TO 97
374 DF \(124,11=196.710+00\) DFS 24, \(21=211 \cdot 320+00\) FREQQ(24) =FREQQ(23) \(K=3\)
GO \(10 \quad 97\)
375 geTAx29.710+00
DF(25,1) \(=188 \cdot 780+00\)
DF\{ 25,2\(\}=202 \cdot 640+00\)
\(K=1\)
FREOQ(25) \(=9.3715410+03\) GO TO 97
```

376 9ET A=29.71D+00
DF(26,1)=187.990+00
DF{ 26,2i=205•060+00
X=3
FRECQ(26)=9.3715410+03
GO TO 97
377 BETA=43.090*00
DF{27:1:=188.030+00
OF(27,2:=201.550+00
K=1
FREGU{27:=9.3377335D*03
GO TO 97
378 UF(29,1)=187.080+00
DF(26,2)=203=140+00
K=3
FREQG\&28)=FREGQ(27)
GO TO 97
379 BETA=75.02D+00
DF(29,1)=190.480+00
DF{ 29:2i=208.2D+00
K=1
FRE CH429 2=9.37540450 +03
GOTO 97
380 DF{30.11=190.360+00
DF{30,2)=203.30+00
k=3
FREQQ(30) =FREQOE 29)
GO TO 97
381 BETA=29.710+00
DF{ 31,1)=209.990+00
DFi31,2:=226.260+00
K=2
FRECO{31}=9.3715410+03
97 AG=DSIN<P{41:
AAG=DCOS{P{4;:
CG=OSIN\&P{5:3
CCG=DCOS\&P{5:)
FG=OSIN(P(6)!
FFG=CCOS(P{6:)
A H=DSIN{P{10)}
A AH=OCOS{P{10,)
CH=DSIN(P(11))
CCH=OCOS\P(11:1
FH=CSIN{P{12!}
FFH=0COS{P{121)
AZ=OSIN(P(16))
AAZ=DCOS{P{161:
CZ=OSIM(P{17)!
CC2=0COS\&P{171)
F2=0SIM(P{18)!
FF2=0COS{P{181:
CO= DCOS{2.094395D +00)
SI=DSIN\&2.0943950+00:
G{1:1)=FFG*CCG-AAG=CG*FG
G(1:2)=FFG=CG+AAG*CCG*FG
G(1:3)=FG=AG
G(2,1):=-FG\#CCG-AAG\#CG*FFG
G(2,2)=-FG\#CG+AAG \#CCG*FFG
G(2,3)=FFG*AG
G(3.1)=AG*CG
G(3,2)=-AG*CCG
G{3,3:=AAG
H(1,1)=FFH=CCH-AAH*CH*FH
H(1,2)=FFH*CH+AAH*CCH*FH
H(1,3)=FH=AH
H(2,1)=-FH*CCH-AAH*CH*FFH

```

H\{2,2) =-FHFCH+AAH =CCH*FFH
H\{2:3) =FFH:AH
\(H(3,1)=A H \approx C H\)
H\{3,2):
H(3,3)=AAH

Z 11,2 ) =FF Z \(F C Z+A A Z=C C Z * F Z\)
211,3)=FZ*A2
\(Z(2,1)=-F Z=C C Z-A A Z=C Z * F F Z\)
Z(2,2)=-FZ*CZ+AAZ =CCZ*FFZ
Z(2,3)=FF2:AZ
2(3,1) =A2\#CZ
Z(3.2) =-AZ \(=\) CCI
こ(3, 3 ) =AAZ
20 ALPHAR=ALPHA \(=\{3.141590+00 / 1.80+021\)
\(\triangle E T A R=B E T A=\{3.14159 D+00 / 1.80+02)\)
KM(1,1) \(=\) OCOS(ALPHAR)
RM( 1,2 ) \(=-\) OS IN(ALPHAR) \(=0 S I N(B E T A R)\)
RME \(1 ; 3\) ) \(=0\) SIN ALPHARI\#OCOS(BETAR)
RP\{2:1)=0.00+00
RM(2,2):OCOSPBETAR1
RMA 2:3 \(3=0\) IM\& 日ETARI
RM(3,2)=
RM( 3,2\()=-\) OCOS\{ALPHARI\#DSIN(BETAR)
RM( 3, 3)=DCOSGALPHARIFDCOSRBETARI
40 GO TO \(150: 50=50,60,60,601, R\)
50 R2 \(11,11=1.0 D+00\)
R2(1:2) \(=0.00+00\)
\(R 2(1,3)=0.00+00\)
\(R 212,11=0.00+00\)
R212,2s=1.00+00
\(R 212,3,=0.00+00\)
\(R 2(3.1)=0.00+00\)
\(R 2(3,2)=0.00+00\)
R2\{3, \(31=1=00+00\)
GO TO (70,80,90):K
60 R2 1,\(2 ;=1,00+00\)
\(R 2(1,2)=0.00+00\)
\(R 211,31=0.00+00\)
R2(2,1)=0.0D+00
R2 \(2,2:=-1.00+00\)
\(R 2\{2,3\}=0.00+00\)
\(k 2(3,1)=0.00+00\)
\(R 2(3,2)=0.00+00\)
R21 3, 3\()=-1,0 D+00\)
\(K K=K-3\)
GO TO (70, 80,90), KK
\(70 R 3(1,1\}=1,00+00\)
\(R 3\{1,2)=0.00+00\)
R3( 1,\(3 ;=0.00 \rightarrow 00\)
\(R 3(2,1)=0.00+00\)
र \(312,21=1.00+00\)
\(R 3\{2,3)=0.00+00\)
R \(3(3.1)=0.00+00\)
\(R 3\{3.2)=0.00+00\)
R 3: 3,3\()=1.00+00\)
GO TO 100
80 R3(1,1) \(=C 0\)
R311,21=SI
\(R 3(1.3)=0.00+00\)
R3(2,1) \(=\) - SI
R3i 2: \(21=\) C0
R3i2:3:=0.00+00
\(R 3\{3,1 ;=0.00 \div 00\)
\(R 3\{3,2:=0.00+00\)
R313,3i=1.00+00
```

    GO TO 100
    90 R3(1:1)=60
R3\&1,2:=-SI
R3(1:3)=0.00+00
R3(2,1)=5I
R3{2,2)= =0
R3(2.3)=0.00+00
R3{3,1:=0.00+00
k 3( 3,2)=0.00+00
R 3( 3,3:=1.00+00
100 00 110 L=1,3
DO 120 m= 1,3
{10 只T(L,M)=R2(L,1)*R3{1,M)+R2{L, 2)*R3(2,M)+R2(L,3)=R3(3,M)
DC 120 6x1:3
vO 120 m=1:3

```

```

    OO 130 L= 1,3
    UC 130 M=1.3
    ```

```

    TH(L,M)=H(L,1) #R(1,M)+H(L,2)=R(2,M)+H{L, 3)*R(3,M1
    ```

```

    I=1
    140 HM=3.500+03

```

```

    CTG{3,3)!
    ```

```

    CTG&3:311
    ```

```

    CTG(3.3))
    ```




```

    #8=P(7)ETH{1,1)=TM&1,3)+P(8)*TH(2,1)*TH(2,3)+P(9):TH(3,1)*TH& 3,3)
    m9 =P(7)#TH(1,3)*TH{1,2)+P(B)*TH{2,3)*TH(2,2)+P{9)*TH{3,3)=
    CTHi3:2:
    ```

```

    C=r (2 3, 1)
    m11=P(13)*TZ{1.1)#TZ{1021+P(14):TZ&2,1)=TZ:2,2)+
    CP(15)=TZ(3,1)*TZ(3,2)
    ~12=P{13)=TZ{1,11*TZ11,3)+P(1*)*TZ(2,1)*TZ(2,3)+
    CP(15)=T2(3,1)=T2{3,3)
    m13=P{13)*TZ(1,2)*TZ(1,2)+P(14)*TZ(2,2)*TZ12,2)*
    CP{15)=\2{3,2)=TZ{3.2}
    ```

```

    CP(15)*TZ{3,2)क\2{3,3)
    N15=P(13)=TZ(1,3)*TZ(1,3)+P(14)*TZ(2,3)*TZ(2,3)+
    CP!15%*TZ:3,3!*TZ13&3)
    W1R=a1/2.00+00
    *1I=m2/2.00+00
    O2R=(n4-55)/4.00+00
    \2I=w7/2.00+00
    6 3R=(w4+N5)/4.00+00
    0 4R=* 8/2.00 +00
    4II=w9/2.00+00
    H SR={N10-m13|/4.00+00
    0 II=*11/2.00+00
    0 6R={N20+m13)/4.00+00
    |R=W12/2.0D+00
    CTI=N14/2.00+00
    OD 160 L=1,6
    U0 100 M=1,6
    ARIL.MI= 0.00+00
    150 LIIL,M)= 0.OD+00

```

AR(1, 1) \(=m 3+m 6+w 15 / 2.00+00+2.00+00 * 03 R-(G 8 N \neq H H) / 2.00+00\)
 \(A R(3,3)=4.00+00=03 R-1 G 8 N=H H 1 / 2.00+00\)
AR(4;4) \(34.00+00 \approx 03 R+4 G B N * H H / / 2.0 D+00\)

AR( \(5: 6)=(m-m 3+m 15 / 2,00+00+2.00+00 \equiv Q 3 R+(G B N \neq H H) / 2.00+00\)
\(A R(2,1)=Q 7 R\)
A\{(2,1)=071
AR(3,1) \(=(Q 1 R+04 R)\) \#DSORT \(2.00+001+07 R / 0 S Q R T(2.00+00)\)
AI(3,1)=(01I+Q4I)*OSORT(2.00+00)+Q7I/OSQRT(2.00+00)
ART 3.2) = \(06 R\) : OSERT \(\{2.00+001\)
AR( 4.1\()=05 R=0 S Q R T(2.00+001\)
\(A I(4,1)=G 5 I * D S Q R(2.00+00\) )
AR\{4.2 \(2=(Q 1 R+Q 4 R)=0 S Q R T(2.00+00)-Q 7 R / O S Q R T(2.00+00)\)
AIf 4,2\(\}=(01 I+04 I) \approx 0 S Q R T(2.00+00)=Q 7 I / O S Q R T(2.00+00)\)
ARU \(5.11=2.00+00=02 R\)
AI(5,1) \(=2.00+00 * 22 I\)
AR\& 5.3\()=(01 R-04 R)=0 S O R T(2.00+00)+07 R / 0 S Q R T(2.00+00)\)
\(A I(5,3)=\{01 I-Q+I\}=0 S O R T\{2.00+001+07 I / O S Q R T\{2.00+00)\)
AR(5,4)=DSORT\{2.00+001=06R
ARI 6,2\()=2.00 \rightarrow 00 \approx 92 R\)
A116,2)=2.00 \(+00=021\)
AR(6:3) \(=0\) SQRT(2.0.0 000 ) \(=05 R\)
AIf \(6,31=0\) SORT \(2.00+00:=05 I\)

AI( 6.4\()=\{Q 1 I-04 I\} \approx O S Q R T\{2.00+00)=07 I / 0 S O R T(2.00+00\) )
AR(6.5) \(=-07\) R
AIf 6,5 ) \(=-071\)

\section*{DIAGDNALIEE SPIN-ENERGY MATRIX ACG: 6 L.}

CALL HTRIDI INM,NOAR,AI, D,E,EZ,TAUS
CALS IMTQLI (N:D:E:IERR)
60 10 (170,180:190:200).1
170 FRE O=D 4 4 - D (1)
GO TO 210
180 FREQ=O(6)-014)
GO TO 210
190 FRE Q=0 (51-0 (3)
GO TO 210
200 FREQ=0(3)-D:21
210 IFI OABSiFREQQIMM\& -FREQ:-1.OD-011220.220.230
220 HFAI) =HM
GO TO 240
230 HHE HHF (FREQQIMMBTFREOB
GO TO 150
240.IF(I-4) 250, 250,260
\(2501=1+1\)
GO TO 140
250 CONTINUE

DFC\{MMs 2 )=(HF\{4:-HF(1))
\(M M=M M+1\)
IFIMM-31:96:96:399
399 SuM \(=0.0 \mathrm{D}+00\)
DO 400 mm=1.31
 GO 10 (401:408.4101,R1
401 SUM2=SUM
IF (LL- 131402,402,403
402 SUM \(1=\) SUM
403 LLL \(\times\) LL- 12
60 TO \(1404,404,404,405,405,4051, \mathrm{LLL}\)
404
\(P P=1.00+00\)
```

        GO TO 407
    405 PP=1.0D+00={3.141590+00/180.00+00)
    407 P{LL}=P{LL}+PP
    K1=K1+1
    GO 10 95
    408 IF\SUM-SUm2193:409.409
    409 P(LL)=P(LLJ-2.00*00*PP
    K1=K1+1
    GO TO }9
    410 IF\SUM-SUM2:93.411:412
    411P(LL)=P(LL)*PP
    93 CONTINUE
        IF\SUM-SUM2,1412,412,413
    412 SUM2=SUM
    413 CONTINUE
    #RITE\6,420:SUMZ
    420 FORMAT:" SUM2 EQUALS ©F12.3)
    MRITE{6,430) &PII|SI=1:18)
    430 FORMAT:'O*, 6F12.6)
    IF{SUM1-SUM21450.450:92
    450 CONTINUE
        DO 451 L=1:3
        P(L+3)=P(L+3)*{180.00+00/3.1+1590+00)
        P{L+9)=P{L+9:*1180.00+00%3.141590*001
    451P{L+15) =P{L+15)*(180.00+00/3.141590+00}
    MRITE\6:452)
    452 FORMATY IHO," THE FINAL SET OF PARAMETERS IS *)
        WRITE{6,430) (P{IJ:I=1.18:
        STOP
        END
    //LKED.SSP OD DSNAME=OSU.ACT1210S.ESRLIS,OISP=SHR
/1

```
\[
\begin{gathered}
\text { VITA } \\
\text { Mahendrakumar G. Jani } \\
\text { Candidate for the Degree of } \\
\text { Doctor of Philosophy }
\end{gathered}
\]

Thesis: RADIATION EFFECTS IN CRYSTALLINE SiO \(_{2}\) : STRUCTURE OF THE E" CENTERS

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