OPTIMIZATION OF A HEAT EXCHANGE SYSTEM
IN A PROCESS PLANT

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1970

Submitted to the Faculty of the Graduate College
of the Oklahoma State University
in partial fulfillment of the requirements
for the Degree of
MASTER OF SCIENCE
May, 1975

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PREFACE

The purpose of this study is to propose an optimization procedure which is applicable to the optimization of the heat exchange system. Optimization of a heat exchange system is studied as a three stage problem: optimization of the heat exchangers, optimization of a heat exchange system for a fixed system configuration, and optimal synthesis of a heat exchange system. The Fibonacci search technique is used for the optimum design of a water cooler. The modified simplex method is used for the optimization of a heat exchanger system for a fixed system configuration. Optimal synthesis of the heat exchange system is developed by graphical analysis of a temperature-enthalpy flow rate diagram and a temperature-heat capacity flow rate diagram.

I wish to express my sincere thanks to Dr. Kenneth J. Bell for his invaluable advice and guidance throughout this study and to Dr. John H. Erbar for his assistance in using the OSU PAS system to calculate thermodynamic property data necessary to perform the desired calculations. Appreciation is also expressed to all the faculty of the School of Chemical Engineering for their assistantship throughout this study. A special note of thanks to Mrs. Dolores Behrens is in order for her excellent typing of the final copy.

Finally, I would like to express my gratitude to my parents, wife, Younghae, and brother, Youngwon, for their understnading, encouragement, support and many sacrifices.

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NOMENCLATURE

A, A_i, A_o	- Area of heat exchanger, inside the tube, outside the tube; ft^2
a, b	- Cost constants
a_K, b_K	- Constraint boundaries of K^{th} interval in Fibonacci search
$\dot{C}$, $\dot{C}_a$, $\dot{C}_{av}$	- Heat capacity flow rate, artificial heat capacity flow rate, average heat capacity flow rate; Btu/hr^{o}F
C_s, C_w	- Unit cost of steam, water, \$/lb
E_f	- Fin efficiency
(EXCO), $(EXCO)_s$	- Capital cost of a heat exchanger, a heat exchanger system, \$
F	- Configuration correction factor of a heat exchanger
$\dot{H}$	- Enthalpy flow rate, Btu/hr
$\bar{H}$	- Specific enthalpy of a stream, Btu/lb, or Btu/lb-mole
$\Delta\dot{H}_\ell$	- Latent heat rate, Btu/hr
h_i, h_o	- Heat transfer coefficient inside the tube, outside the tube, Btu/ft^{2}hr^{o}F
k_w	- Thermal conductivity of wall, Btu/ft hr^{o}F

L_K	- Interval of uncertainty after K^{th} trial in Fibonacci search
LMTD	- Log mean temperature difference, ^oF
m	- Mass flow rate of stream, lb/hr or lb-mole/hr
m_s, m_w	- Mass flow rate of steam, water, lb/hr
(MTD), $(MTD)_c$, $(MTD)_s$	- Mean temperature difference, condensing, subcooling; ^oF
N	- Number of shells in series in a shell and tube heat exchanger
n	- Total number of variables
Q, Q_c, Q_s	- Heat duty of a heat exchanger, condensing, subcooling; Btu/hr
R_{fi}, R_{fo}	- Fouling factor, inside the tube, outside the tube; $ft^{2o}F$ hr/Btu
SCOST	- Annual cost of steam, \$/yr
ΔT_a	- Artificial temperature difference, ^oF
T_i, T_o	- Shell side temperature, inlet, outlet; ^oF
t_i, t_o	- Tube side temperature, inlet, outlet; ^oF
T_c, T_h	- Temperature of cold stream, hot stream; ^oF
(TACO), $(TACO)_s$	- Total annual cost of a heat exchanger, a heat exchanger system; \$/yr
U	- Overall heat transfer coefficient, Btu/hr $ft^{2o}F$

WCOST	- Annual cost of water, \$/yr
X_K^M	- Mis point in K^{th} interval of uncertainty in Fibonacci search
X_{cen}, X_{con}, X_{exp}, X_{ref}, X_{worst}	- Centroid point, contracted point, expanded point, reflected point, worst point

Greek Letters

δ	- Annual rate of depreciation
θ	- Operating hours, hr/yr
λ	- Latent heat of condensation, Btu/lb
τ	- Preselected tolerance of solution

Subscripts

c	- Cold stream
h	- Hot stream
s	- Steam
w	- Water
F	- Feed
T	- Top product
B	- Bottom product
H	- Heater
C1, C2	- Cooler 1, Cooler 2
i	- Initial
f	- Final

CHAPTER I

INTRODUCTION

Large amounts of heat are both used and given off in chemical process plants. It often happens that a large amount of heat is required at the beginning of a process to bring the process streams to reaction conditions and that a large amount of heat must be removed from the reactor or from the product streams to keep the products stable or subcool them for storage, shipping or further processing. In distillations, a great deal of heat is needed to provide the vapor phase and almost as much heat must be removed in the condensers. In the past, it has been common practice to provide the heat required by a furnace, either directly or indirectly through the use of steam or hot oil from the furnace, and to remove the heat by water or air cooling in heat exchangers.

However, with the increasing cost of energy in all forms, it is of great interest to try to recover as much of the heat as possible from the heat rejection steps of the process and recycle it into the process at the heat addition points. There are a number of alternatives to consider for recovering heat. The specific method chosen will depend on the particular requirements of the process under consideration, pressure and/or temperature level of the available heat source, and economic considerations.

In this study, the chosen method is to provide a heat exchanger in which a hot stream to be cooled and a cold stream to be heated are allowed to exchange heat.

The cost of a heat exchanger system depends not only on the pairing and sequencing of streams exchanging heat, but also on the amounts of supplemental heating and cooling used. Capital cost is roughly a function of total heat exchanger area while the operating costs are primarily a function of supplemental utilities costs. Exchanger area increases with increased recovery of heat and with decreased utilities requirements, so optimum design of a heat exchanger system involves an economic optimization. Optimization of a heat exchanger system includes the synthesis of the heat exchange system, i.e. optimizing the network structure of heat exchangers as well as the allocation of heat duty to each exchanger in the structure.

Therefore, optimization of the heat exchange system is essentially a three stage problem: optimization of heat exchangers, optimization of heat exchange system for a fixed system configuration, and optimal synthesis of heat exchange system.

In this study, methods which are applicable to each stage of optimization are introduced. The objective function for optimization of a heat exchange system is formulated in Chapter III. Optimization of heat exchangers is discussed and the Fibonacci search technique is introduced for the optimum design of a water cooler in Chapter IV. Optimization of a heat exchanger system for a fixed system configuration is discussed and the modified simplex method (modified form of Nelder and Mead algorithm) is introduced in Chapter V. Optimal synthesis of heat exchange system is developed by graphical analysis of a temperature-enthalpy flow rate diagram and a temperature-heat capacity flow rate diagram in Chapter VI. Finally, a heuristic computer-oriented methodology for optimizing heat recovery system in a typical distillation process is demonstrated in detail by applying the suggested optimization procedures.

CHAPTER II

LITERATURE SURVEY

A heat exchange system is one of the most common engineering systems employed in industrial processes. Several authors have used the methods of calculus to optimize heat exchanger trains. McAdams (23) considered the problems of determining the optimum amount of water for condensers and coolers and of determining the optimum amount of heating in the system where two hot streams and one cold stream are involved. TenBroeck (36) considered the situation in detail for a stream being preheated in a battery of exchangers by setting up a series of simultaneous partial differential equations and solving by trial and error for the optimum area for each exchanger. Whistler (38) discussed the use of "heat pictures" for facilitating such calculations. Plots of temperature versus heat content of various streams enable possible combinations to be set up consistent with a heat balance. Happel (12) gave a comprehensive derivation of the equations and tabulated the solution of the problem to optimize the outlet temperature of a waste heat exchanger followed by water cooling.

In 1961, Westbrook (37) applied dynamic programming to the optimization of a train of five exchangers and a furnace used to preheat the feed to a pipe still. Fan and Wang (9) applied the discrete maximum principle to the same problem.

In 1965, Hwa (14) discussed the synthesis of a heat exchange system, optimizing the network structure of a heat exchanger system as well as the allocation of heat duty to each exchanger, introducing separable programming. Bragin (4) used both the maximum principle and dynamic programming to optimize the heat allocation and the sequencing of the hot streams in feed preheat trains similar to the one studied by Westbrook.

Kesler and Parker (18) proposed a method of finding the optimal network of heat exchangers using an assignment algorithm to maintain the feasibility and a modified linear programming algorithm in which stream heat loads were divided into discrete heat elements in order to linearize the objective function.

The synthesis of heat exchange systems has also been studied in the work of Rudd and his coworkers (21,22) on theoretical lines using the synthesis of the system structure which they have developed to handle general synthesizing problems. Masso and Rudd (22) presented a heuristic approach in which the network was structured exchanger by exchanger and new stream matches were assigned by rules of thumb or "heuristics." Weighting functions were associated with each heuristic at each stage to build up experience on heuristic selection and thus move towards an optimal solution. Lee et al. (21) used the branch and bound theory for the systematic synthesis of heat exchanger systems under the condition that a stream cannot be used in more than one place at the same time. Kobayashi et al. (19) proposed a systematic way of synthesizing an optimal heat exchange system in which they formulate the problem as an optimal assignment problem in linear programming, and carry out the optimal design of the synthesized system by the complex method (3). Nishida et al. (26) discussed necessary conditions for the optimal structure of a heat exchanger system with the mimimum heat transfer area employed as a criterion to express efficiency of the system; on the basis of the necessary conditions obtained they proposed an algorithm to synthesize heat exchange systems with auxiliary heating and cooling equipment.

Hohman (13) tried to use temperature-enthalpy diagram in synthesizing heat exchange networks to avoid guideless combinatorial synthesis problems of network designs.

Most recently Pho and Lapidus (29) derived a compact matrix representation of an acyclic exchanger network. Based on this matrix, a decision tree diagram is constructed whose nodes will encompass all the feasible networks of acyclic and nonsplit streams. This reduces the synthesis problem into a tree searching problem where one seeks to locate a node with minimum cost.

Most of the above authors simplified the problem to avoid the complexity of the synthesis of heat exchanger systems and to enable their method to work, and these methods are still far from being applicable to real world problems.

CHAPTER III

OBJECTIVE FUNCTION FOR OPTIMIZATION

The objective function is a criterion function which the optimization technique seeks to maximize or minimize. Since the ultimate objective concerns economics the objective function must represent the true economic incentives. In the optimization of a heat exchange system, the minimization of the total annual cost of heat exchange system will be our objective.

Cost Calculation for a Heat Exchange System

The main cost items to be considered are the following:

A. Investment Cost of Heat Exchangers

A number of factors influence the initial cost of heat exchangers. Exchanger area, material of construction, pressure, and type of exchanger will be the main factors affecting exchanger cost.

In order to be reasonably accurate, heat exchanger cost estimation must be separated into costs of the rough component parts, and their manufacturing and assembly costs. Palen et al. (27) present an equation for the total cost of shell and tube heat exchangers composed of the costs of the component parts, and their manufacturing and assembly costs.

For a rough cost estimation, heat exchanger cost can be reasonably expressed as a function of heat transfer area if the material of construction, type of exchanger and operating pressure are known. Peters and Timmerhaus (28) present graphical correlations for purchased cost vs. exchanger area. More recently Guthrie (11) presented a "module technique" for making fast, reasonably accurate and consistent capital cost estimates. In his article, he presented a graphical correlation for FOB equipment cost vs. surface area for floating head carbon steel heat exchanger designed for 150 psi, and developed the capital cost calculation method with adjustment factors for the exchanger type, operating pressure, material and escalation and module factors.

Therefore, total capital cost of a heat exchanger EXCO is expressed roughly as

$$EXCO = aA^{b} \tag{3-1}$$

where a and b are constants to be determined by the exchanger type, exchanger material, design pressure, module factor and current cost index. This relation may not give extremely accurate cost calculations but it is very useful for a fast and consistent capital cost estimate of heat exchangers for comparison purposes.

B. Operating Cost

Operating cost includes the cost of the utility heating or cooling streams. For the cases of water cooling and steam heating, the cost of water and steam can be calculated on the basis of unit cost and total amount of utilities required. Annual cooling water cost (WCOST) and heating steam cost (SCOST) can be calculated as

$$WCOST = C_w . m_w . \theta \quad (3\text{-}2)$$

$$SCOST = C_s . m_s . \theta \quad (3\text{-}3)$$

Pumping power cost will be proportional to the pressure drop of streams and to the amounts of fluid that must be pumped.

C. Maintenance Cost

The primary maintenance cost in heat exchangers is cleaning of fouling deposits. Maintenance cost also includes the cost of replacing any corroded components in the exchanger.

Heat exchangers must be designed either to minimize the build up of fouling or at least to withstand the mechanical effects of fouling as it does develop and to have excess area to keep working at fouled condition as long as process conditions are met.

The tube material used will in some cases not only influence fouling but also, in corrosive services, determine the life of the exchangers.

Maintenance cost should include the cost of lost production during the shutdown period necessary for maintenance.

D. Investment Cost of Interconnections

Interconnections include piping, pumps, valving, insulation, and instrumentation to control stream temperatures or rates.

The design and cost importance of these interconnection elements can be a significant part of heat exchange systems in which different and sometimes widely separated units are highly integrated with regard to process stream heat exchange for heat recovery. The cost associated with the interconnection elements is difficult to determine but some consideration must be given to it if different heat exchange system configurations are to be equitably compared. Generally, piping and other costs will be relatively higher for system configurations involving more heat exchangers, but operational flexibility will be greatly improved.

Assumptions and Simplifications

Generating a meaningful cost function as a basis for heat exchange system optimization is extremely difficult, as some of the above important cost factors have too much uncertainty to be expressed in reasonable mathematical terms. Specifications of uncertain accuracy in details can be worse than having none as they can easily lead to a false and misleading pseudo-optimum.

On the other hand, too many assumptions and simplifications will cause the final solution to be far from the real world problem and be useless. Hence reasonable assumptions and simplifications of the cost function of a heat exchange system have to be made to enable the cost calculations to be made.

Since fouling and plugging cause the pressure drop to rise rapidly and possibly cause a premature shutdown, designers usually sacrifice the effect of pressure drop on economics. Usually, the allowable pressure drop is rigorously specified and is not a design variable. The dimensions of the tube are also usually specified so that the designer may vary the number of passes to approach the desired pressure drops. Usually process cooling water is supplied under sufficient pressure so that no pumping power cost calculations are necessary for water. Thus power cost will not be included in the objective function.

Maintenance cost of heat exchangers and capital cost of interconnections have so much uncertainty that it would be very difficult to include them in our objective function formulation. But it is desirable to include an estimate when the data are available, or they should be considered in final design stage.

Therefore, for this problem, the objective function for a heat exchanger system will be the total annual cost including amortized annual capital cost and heating and cooling utility cost. If steam and water are used as heating and cooling utilities respectively, the total annual cost of the system (TACO)s can be expressed as

$$(\mathrm{TACO})s = \delta \cdot \sum_i (\mathrm{EXCO})_i + \sum_j \{ \delta \cdot (\mathrm{EXCO})_j + (\mathrm{WCOST})_j \} + \sum_k \{ \delta \cdot (\mathrm{EXCO})_k + (\mathrm{SCOST})_k \} \tag{3-4}$$

where i, j, and k denote the numbers of i^{th} exchanger of process streams, j^{th} cooler, and k^{th} heater respectively.

The following assumptions will be made to simplify the optimization problem.

1. Shell and tube heat exchangers are to be used in the systems to be considered.

2. Cost of the heat exchangers will be related to the number of shells and exchanger area as

$$EXCO = N \cdot a \cdot (A/N)^b \qquad (3\text{-}5)$$

3. Overall heat transfer coefficient U is assumed to be constant, not changing with the temperature and design during the optimization calculation.

CHAPTER IV

OPTIMIZATION OF HEAT EXCHANGERS

Optimization of heat exchangers has been a very popular topic in the chemical engineering field. Many authors have proposed optimization techniques.

Recently Tarer et al. (35) developed a computer program for the optimum design of heat exchangers without phase changes, considering the objective function as the total cost (including amortized exchanger cost based on heat exchanger area) and operating cost based on pumping cost and utility cost. First, optimization was begun by using the Lagrange multiplier technique, which was originally used by Cichelli and Brinn (6) and the final optimization was performed for the discrete standard equipment sizes surrounding the continuous optimum.

A most notable work was done by J. W. Palen et al. (27). They used the Box Complex Method (3) in designing optimum shell and tube exchangers, considering the objective function as the minimum initial cost of a heat exchanger based on detailed design variables for a given fixed process condition and allowable pressure drops. The optimization procedure for shell and tube exchangers can consider details like shell diameter, tube diameter, tube length, tube pitch, baffle spacing, baffle cut and number of tube passes.

In the present study of optimization of heat exchanger systems, sophisticated optimization of individual heat exchangers is not performed. The simplifying assumptions in formulating the objective function in Chapter III eliminate the necessity of optimization of individual heat exchangers except in the case of water coolers. As the capital cost of a heat exchange system is assumed to be only a function of the number and the areas of the heat exchangers, the heat exchangers are uniquely designed rather than optimized by the process specifications except in the case of water coolers. For water coolers, the total annual cost is the sum of the annual cost of water and the capital cost and an optimum has to be formed between the two limiting conditions: much water and small surface area or little water and large surface area.

In this chapter, the procedures for designing shell and tube heat exchangers and optimum water coolers will be discussed.

Shell and Tube Heat Exchanger Design

The shell and tube exchanger is selected here because of its universality and because of the availability of design procedures. Bell (1) describes the design procedure in detail. The design procedure will be briefly outlined here.

A. Selection of the Basic Configuration

1. In a shell and tube unit in sensible heat transfer service, the first important decision is which fluid goes to which side. The fluid which is corrosive, or fouling, or at high pressure goes to the tube side. In conflicting cases, for example, one fluid is fouling and the other is corrosive, no hard and fast rules can be set; the decision must then be made by a cost comparison between the two cases.

2. Another decision which must be made is whether extended surface is to be employed or not. Extended surface in a shell and tube exchanger means low fin tubes to give a few-fold increase in the shell-side area, the goal being to make

$$h_o A_o E_f \approx h_i A_i \tag{4-1}$$

3. Multishell arrangements are frequently necessary in large scale process applications. The purely series arrangement is mainly useful when (a) the single shell with multiple tube passes gives too low a value of F, the configuration correction factor on the LMTD, or (b) there are limitations on shell length and/or diameter, requiring the total area to be disposed in more than one shell. The purely parallel flow arrangements are mainly used when pressure drop limitations (coupled with diameter and baffle spacing limits) force a reduction in shell side velocity.

B. Estimation of Required Area

Once Q, MTD, and U are known, the area can be easily calculated from

$$A = \frac{Q}{U(MTD)} \tag{4-2}$$

In the case of a condenser with subcooling, to avoid uncertainties of the two-phase behavior it is preferable to design separate exchangers for the separate cooling functions.

$$A_c = \frac{Q_c}{U_c(MTD)_c} \tag{4-3}$$

$$A_s = \frac{Q_s}{U_s (MTD)_s} \tag{4-4}$$

1. Calculation of Q

The calculation of the duty is relatively straightforward if the thermodynamic data are available. For sensible heat transfer, the specific heat at process condition is required; if not known, it may be estimated with sufficient precision for all but the most extreme conditions or unusual compositions. Then:

$$Q = m\ C_p (T_o - T_i) \tag{4-5}$$

For boiling and condensation, the latent heat is required:

$$Q = m\ \lambda \tag{4-6}$$

This is less easy to estimate than the specific heat, especially when mixtures are involved. The most complex case of all is condensation of a mixture because then vapor-liquid equilibrium data and vapor and liquid enthalpies as function of composition are required.

In an exchanger with more than one mode of heat transfer occurring (e.g., a condenser with subcooling), the heat duty for each process should be separately computed to give some feel for the nature of the problem; whether or not the separate duties are actually required for the design calculation is dependent upon the service, the configuration, and the particular design procedure.

$$Q_c = m\ \lambda \tag{4-7}$$

$$Q_s = m\ C_p (T_c - T_o) \tag{4-8}$$

For accurate thermodynamic property calculation OSU PAS system (7) can be used conveniently.

2. Calculation of MTD

The LMTD can be readily be calculated from the terminal temperatures for the countercurrent case

$$LMTD = \frac{(T_i - t_o) - (T_o - t_i)}{\ell n \left(\frac{T_i - t_o}{T_o - t_i} \right)} \tag{4-9}$$

The value of the configuration correction factor for N shells in series and 2N or more tube passes can be obtained from the terminal temperatures if a chart for the configuration is available, or it can be calculated by the following relations by Bowman et al. (2).

If $R \neq 1$

$$F = \frac{\frac{(R^2+1)^{\frac{1}{2}}}{R-1} \ell n \left(\frac{1-P_x}{1-RP_x} \right)}{\ell n \left(\frac{2/P_x - 1 - R + (R^2+1)^{\frac{1}{2}}}{2/P_x - 1 - R - (R^2+1)^{\frac{1}{2}}} \right)} \tag{4-10}$$

where

$$P_x = \frac{1 - \left(\frac{RP-1}{P-1}\right)^{1/N}}{R - \left(\frac{RP-1}{P-1}\right)^{1/N}} \tag{4-11}$$

$$P = \frac{t_o - t_i}{T_i - t_i} \qquad (4\text{-}12)$$

$$R = \frac{T_i - T_o}{t_o - t_i} \qquad (4\text{-}13)$$

N is the total number of shells in series.

If R = 1

$$F = \frac{\dfrac{P_x (R^2+1)^{1/2}}{1-P_x}}{\ln\left(\dfrac{2/P_x - 1 - R + (R^2+1)^{1/2}}{2/P_x - 1 - R - (R^2+1)^{1/2}}\right)} \qquad (4\text{-}14)$$

where

$$P_x = \frac{P}{N - NP + P} \qquad (4\text{-}15)$$

At the design stage, the number of shells in series, N, is unknown and the above equations must be solved for such N for which $F \geq 0.80$ by trial and error calculation.

The mean temperature difference MTD is

$$MTD = F(LMTD) \qquad (4\text{-}16)$$

3. Estimation of U

The step with the greatest uncertainty in the preliminary calculations is estimating the overall heat transfer coefficient. The value of U can be built up from the individual h values, wall resistance and dirt resistances:

$$U_o = \frac{1}{\frac{A_o}{h_i A_i} + \frac{R_{fi} A_o}{A_i} + \frac{A_o \ln(r_o/r_i)}{2\ Lk_w} + \frac{R_{fo}}{E_f} + \frac{1}{h_o E_f}} \qquad (4\text{-}17)$$

Determination of Optimum Outlet-Water Temperature

In using water as the cooling medium for a given duty, it is possible to circulate a large quantity of water with a small temperature range or a smaller quantity with a larger range. The temperature range of the water naturally affects the LMTD. If a large quantity is used, the outlet water temperature will be far from the process stream inlet temperature and less surface area is required as a result of the large LMTD and F. Although this will reduce the original investment and fixed charges, since depreciation and maintenance will also ordinarily be smaller, the operating cost will be increased owing to the greater quantity of water. It is apparent that there must be an optimum between the two conditions: much water and small surface area or little water and large surface. In the following it is assumed that the line pressure on the water is sufficient to overcome the pressure drop in the exchanger and that the cost of water is related only to the amount used. The total annual cost of the cooler will be the sum of the annual cost of water and fixed charges, which include maintenance and depreciation. Therefore the total annual cost will be

$$(TACO) = C_w . m_w . \theta + \delta . N . a . (A/N)^b \qquad (4\text{-}18)$$

where

$$m_w = \frac{Q}{C_{p,w}(t_{w,o}-t_{w,i})} \qquad (4-19)$$

$$A = \frac{Q}{U \cdot F(LMTD)} \qquad (4-20)$$

θ : annual operating hours

C_w : water cost per lb_m

δ : annual rate of depreciation (e.g. 0.2)

N : number of shells

a,b : cost constants

The calculation of U, F, LMTD were shown in Equations (4-17), (4-10), and (4-9).

Assuming U does not change and keeping all temperatures constant except $t_{w,o}$ the outlet water temperature, then LMTD, F, and N are functions of $t_{w,o}$ only, and consequently the heat exchanger area, A is only a function of $t_{w,o}$. Finally the total annual cost function expressed as Equation (17) depends only on $t_{w,o}$, outlet water temperature.

The optimum condition will occur when the total annual cost is a minimum; thus this problem is the minimization problem of a single variable, nonlinear function subject to constraints.

When water is employed as a heat transfer medium, fouling from water of average mineral and air content tends to become excessive at water temperatures higher than 120°F. Consequently, an outlet water temperature above 120°F is avoided and 140°F is usually quoted maximum.

Minimize TACO($t_{w,o}$)

subject to $t_{w,i} \leq t_{w,o} \leq 120^{o}F$

The optimum outlet water temperature and minimum annual cost of a water cooler can be found by a computer program using the Fibonacci search technique.

Fibonacci Search Technique

This search technique finds the minimum of a single variable, nonlinear function subject to constraints:

Minimize F(x)

subject to $a_1 \leq x \leq b_1$,

where a_1 and b_1 are constraints

Procedure

The procedure is an interval elimination search method. Thus, starting with the original boundaries on the independent variable, the interval in which the optimum value of the function occurs is reduced to some final value, the magnitude of which depends on the desired accuracy. The location of points for function evaluations is based on the use of positive integers known as the Fibonacci numbers (Table I). No derivatives are required. A specification of the desired accuracy will determine the number of function evaluations required. The number of function evaluations and the interval of uncertainty are shown in Table II. A unimodal function is assumed. Thus the use of multiple starting points is recommended if a multimodal function is suspected. The successive interval elimination steps in the search are shown in Figures 1 and 2, and the logic

TABLE I

FIBONACCI NUMBERS

K	F_K
0	1
1	1
2	2
3	3
4	5
5	8
6	13
7	21
8	34
9	55
10	89
11	144
12	233

TABLE II

INTERVAL OF UNCERTAINTY IN
FIBONACCI SEARCH

Number of Trials	Interval of Uncertainty
0	L
1	L
2	0.500L
3	0.333L
4	0.200L
5	0.125L
6	0.077L
7	0.048L
8	0.0294L
9	0.0182L
10	0.0112L
11	0.0069L
12	0.0043L

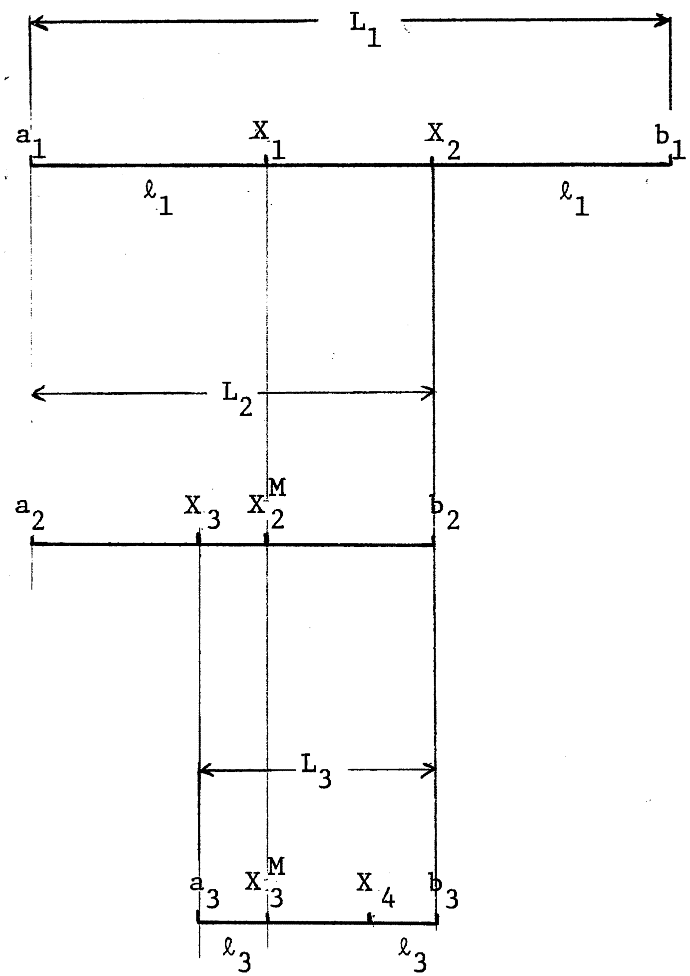


Figure 1. Initial Interval Elimination

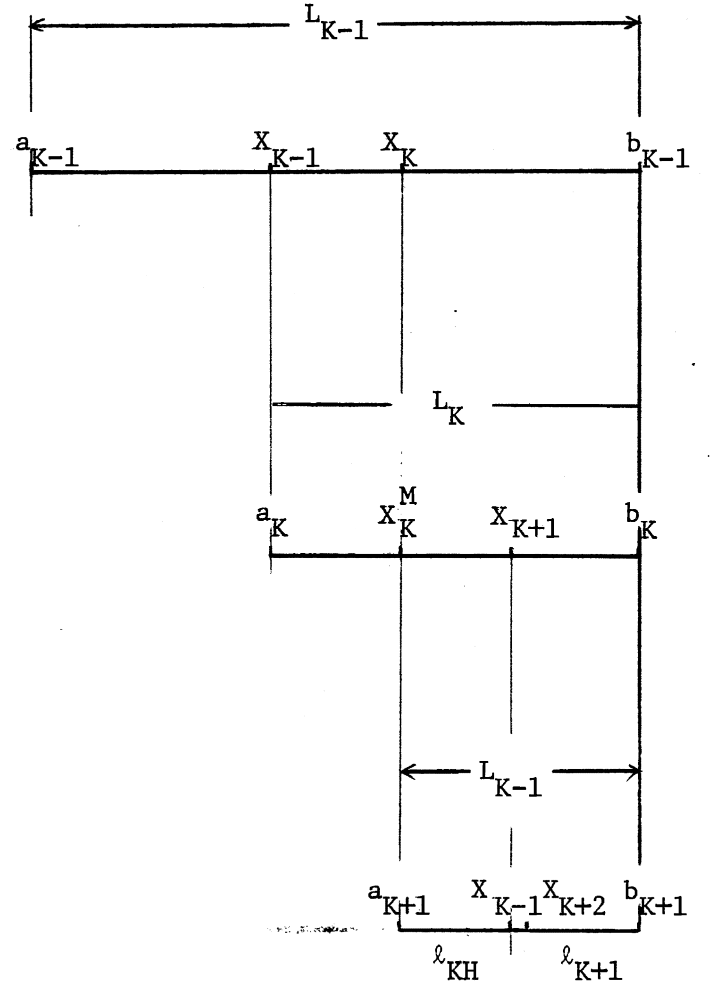


Figure 2. Interval Elimination at $K-1^{th}$ and K^{th} Step

diagram for Fibonacci search is shown in Figure 4 on page 28. The algorithm proceeds are as follows:

1. Designate the original search interval as L_1 with boundaries a_1 and b_1.

2. Predetermine the desired accuracy α which is the ratio of the final interval to the original search interval. Determine the number, N, of the required Fibonacci numbers (equals number of required function evaluations)

$$\alpha = 1/F_N \tag{4-21}$$

$$F_o = F_1 = 1 \tag{4-22}$$

$$F_N = F_{N-1} + F_{N-2} \text{ , } N \geq 2 \tag{4-23}$$

where F_N is called a Fibonacci number.

3. Place the first two points, X_1 and X_2 ($X_1 < X_2$), within L_1 at a distance ℓ_1 from each boundary as Figure 1,

$$\ell_1 = \frac{F_{N-2}}{F_N} L_1 \tag{4-24}$$

$$X_1 = a_1 + \ell_1 \tag{4-25}$$

$$X_2 = b_1 - \ell_1 \tag{4-26}$$

4. Evaluate the objective function at X_1 and X_2. Designate the function as $F(X_1)$ and $F(X_2)$. Narrow the search interval as follows:

if $\quad F(X_1) < F(X_2)$

$$a_1 \le X^* \le X_2 \text{ , and } X_2^M = X_1 \tag{4-27}$$

if $F(X_1) > F(X_2)$

$$X_1 \le X^* \le b_1 \text{ , and } X_2^M = X_2 \tag{4-28}$$

where

X^* is the location of the optimum

X_2^M is mid point in the new interval at which the value of function is known

5. The new search interval is given by

$$L_2 = \frac{F_{N-1}}{F_N} L_1 = L_1 - \ell_1 \text{ with boundaries } a_2 \text{ and } b_2 \tag{4-29}$$

$$a_2 \le X^* \le b_2 \tag{4-30}$$

where

$$a_2 = a_1 \text{ , } b_2 = X_2 \quad \text{for} \quad F_1 (X_1) < F (X_2) \tag{4-31}$$

$$a_2 = X_1 \text{ , } b_2 = b_1 \quad \text{for} \quad F (X_1) > F (X_2) \tag{4-32}$$

6. Place the third point in the new L_2 subinterval symmetric about the remaining points,

$$\ell_2 = \frac{F_{N-3}}{F_{N-1}} L_2 \tag{4-33}$$

$$X_3 = A_2 + \ell_2 \quad \text{or} \quad b_2 - \ell_2 \tag{4-34}$$

7. Evaluate the objective function $F(X_3)$, compare with the function for the point remaining (X_2^M) in the interval and reduce the interval to

$$L_3 = \frac{F_{N-2}}{F_N} L_1 = L_2 - \ell_2 \qquad (4\text{-}35)$$

8. The process is continued per the preceding rules for H iterations. The general equations are

$$\ell_K = \frac{F_{N-(K+1)}}{F_{N-(K-1)}} L_K \qquad (4\text{-}36)$$

$$X_{KH} = A_K + \ell_K \quad \text{or} \quad b_K - \ell_K \qquad (4\text{-}37)$$

(symmetric about mid point)

$$L_K = \frac{F_{N-(K-1)}}{F_N} L_1 = L_{K-1} - \ell_{K-1} \qquad (4\text{-}38)$$

Evaluate the objective function at X_K^M and X_{K+1} and designate the functions as $F(X_K^M)$ and $F(X_{K+1})$. Narrow the search interval as Figure 4. The new interval is

$$a_{K+1} \leq X^* \leq b_{K+1} \qquad (4\text{-}39)$$

where

$$a_{K+1} = a_K,\ b_{K+1} = X_K \quad \text{for} \quad F(X_K^M) < F(X_{K+1}) \qquad (4\text{-}40)$$

$$a_{K+1} = X_K,\ b_{K+1} = b_K \quad \text{for} \quad F(X_K^M) > F(X_{K+1}) \qquad (4\text{-}41)$$

X_K^M is the mid point in K^{th} interval

9. After N-1 evaluation and discarding the appropriate interval at each step, the remaining point (X_{N-1}^{M}) will be precisely in the center of the remaining interval (Figure 3). Thus point X_N is also at the mid point and is replaced by a point perturbed some small distance ε to one side or the other of the mid point. The objective function is then evaluated and the final interval where the optimum is located is thus determined

$$a_N < X^* < b_N$$

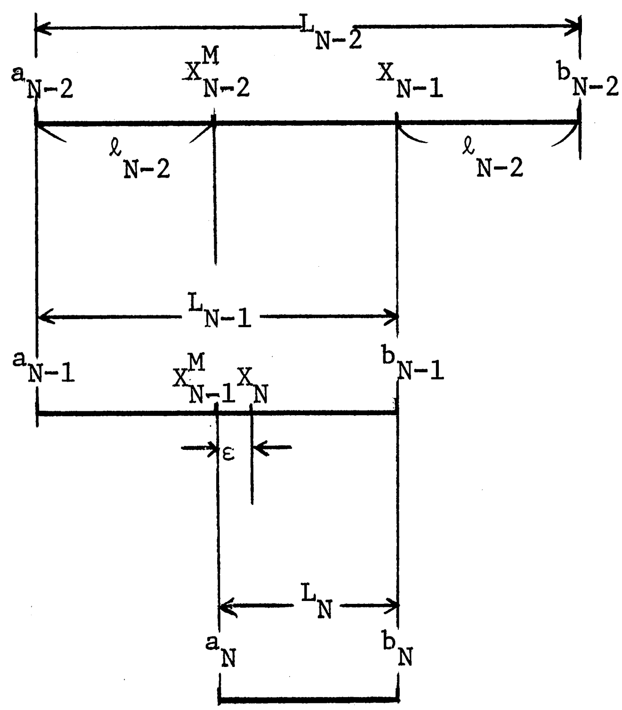


Figure 3. Final Interval Determination

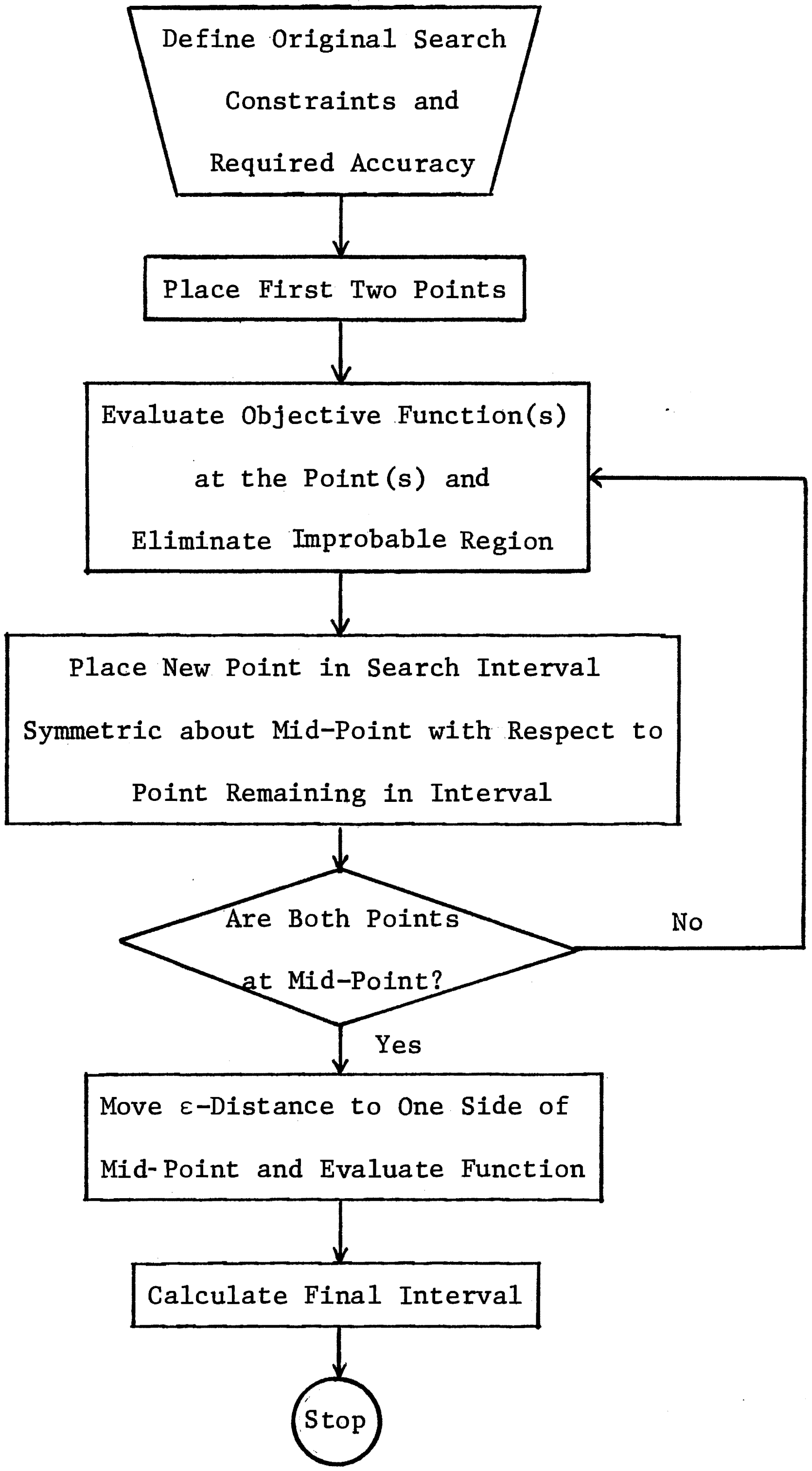


Figure 4. Logic Diagram for Fibonacci Search

CHAPTER V

OPTIMIZATION OF A HEAT EXCHANGER SYSTEM FOR A FIXED SYSTEM CONFIGURATION

For a heat exchanger system with a given process configuration, the optimization is based on the determination of the optimum terminal temperatures (and consequent amount of heat transferred and heat exchanger areas) that will result in minimum total annual cost.

Optimization of this kind is not easy, because most heat exchange systems are composed of many heat exchangers connected in a complex fashion. The size and the complexity of the problem make it very difficult to attain the optimum design by the conventional techniques. Many kinds of optimization techniques have been applied by several authors, but most of them simplified the problem too much to be applicable to design of actual heat exchange systems.

The modified simplex technique (a form of the Nelder and Mead algorithm (25), modified by this author) is used here successfully to minimize the total annual cost, which is a multivariable, nonlinear function subject to nonlinear inequality constraints.

Formulation of Optimization Problem

In the typical distillation system like Figure 5, the feed stream is to be heated to its bubble point before it goes into the distillation

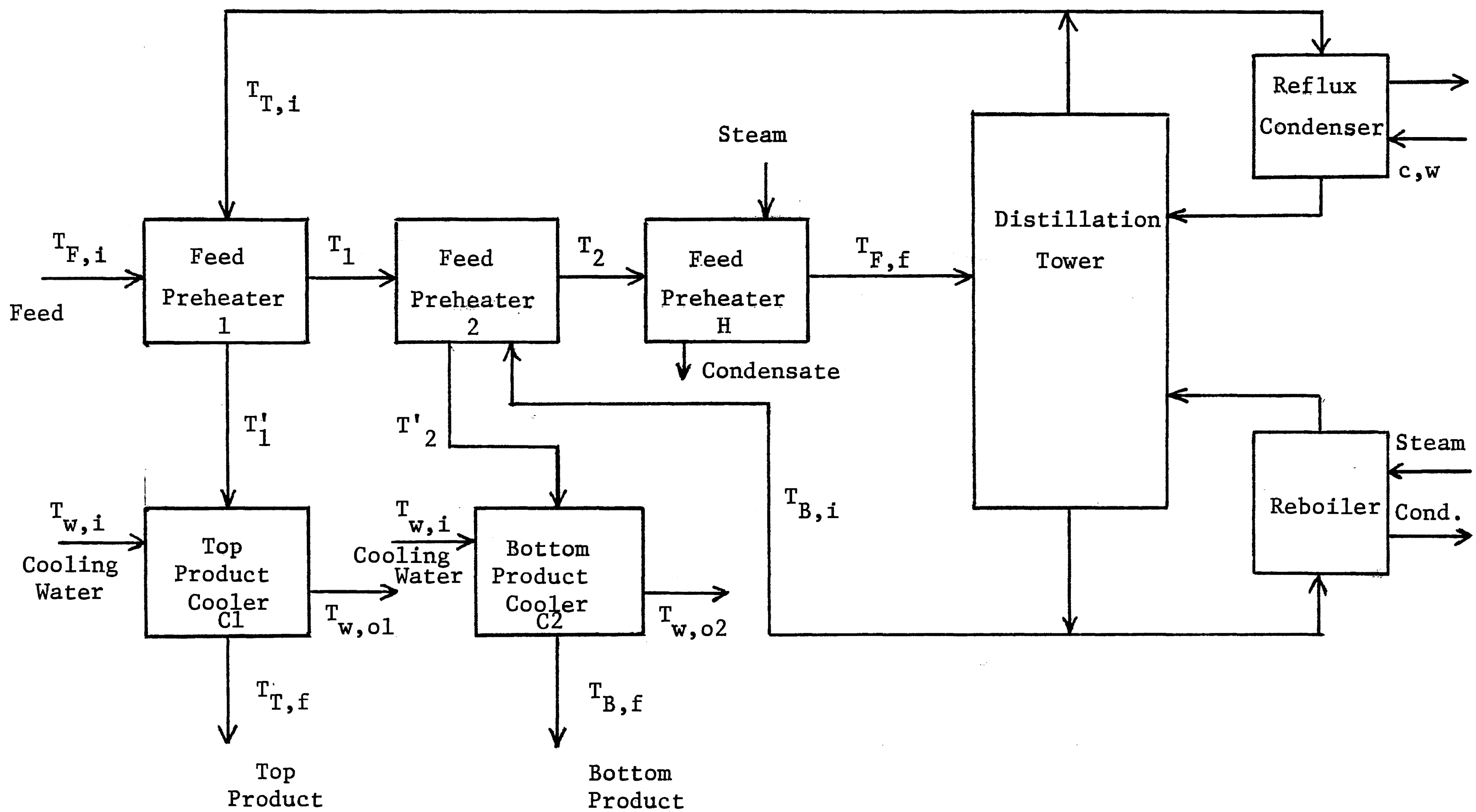


Figure 5. Heat Recovery System in a Distillation Process

column while top product and bottom product streams are to be cooled to specified temperatures for storage. For a given configuration of the heat exchanger system, our object is to minimize the total annual cost by designing the sizes of the exchangers and allocating the heat duty of each exchanger optimally.

By the objective function formulation in Chapter III the total annual cost of this heat recovery system will be the sum of the amortized annual cost of the heat exchangers including coolers and steam heater and the cost of water and steam consumed for a year. The total annual cost of this system, $(TACO)_s$, can be expressed as follows:

$$(TACO)_s = \delta\,.(EXCO, + EXCO_2 + EXCO_{C1} + EXCO_{C2} + EXCO_H) \\ + (C_w\cdot(m_{w,C1} + m_{w,C2} + C_s\cdot m_s)\cdot\theta \qquad (5\text{-}1)$$

Heat exchanger cost was assumed to be a function of surface area only in Chapter III. It is assumed that shell and tube heat exchangers can be designed for the given flow rates of streams, inlet and outlet temperatures both in shell side and in tube side, as described in Chapter IV. For a given heat duty and inlet temperature of process stream, water-cooled exchangers can be designed optimally as explained in Chapter IV.

If all the flowrates of feed, top product and bottom product streams are known and the temperatures $T_{F,i}$, $T_{F,f}$, $T_{T,i}$, $T_{T,f}$, $T_{B,i}$, $T_{B,f}$, $T_{w,i}$, and T_s are given, the unknown variables are T_1, T_2, T_1', T_2', $T_{w,o1}$, and $T_{w,o2}$ (Figure 5). If T_1 and T_2 are determined, T_1 and T_2 will be found by heat balance in exchanger 1 and exchanger

2 respectively. For the calculated values of T_1' and T_2', $T_{w,o1}$ and $T_{w,o2}$ will be determined by the optimum outlet-water temperature calculation procedure described in Chapter IV. Therefore, all the exchanger areas and the amount of utilities can be calculated. Consequently, the total annual cost, the objective function to be minimized, can be determined. In other words, the independent variables in this system are T_1 and T_2, or T_1' and T_2', which determine the total annual cost. Therefore, optimization of heat recovery system shown in Figure 7 is reduced to the problem of finding values of T_1 and T_2 which give the minimum value of TACO.

This problem is the minimization of a multivariable, nonlinear function subject to constraints. The optimum temperatures, T_1 and T_2, and minimum total annual cost of this system can be obtained by a computer program using the modified simplex search technique. Using the same procedure more complicated heat exchanger systems with many heat exchangers can be optimized.

Modified Simplex Method

This method is to find the optimum of a multivariable constrained nonlinear function. The Nelder and Mead algorithm of the simplex method is to accelerate the simplex method of Spendly et al. (32), and adapt itself to the local landscape, using reflected, expanded, and contracted points (Figure 6) to locate the minimum of a nonconstrained multivariable nonlinear function. The algorithm adopted in this work is an extension of the Nelder and Mead algorithm into the multivariable "constraint" nonlinear function problem by the

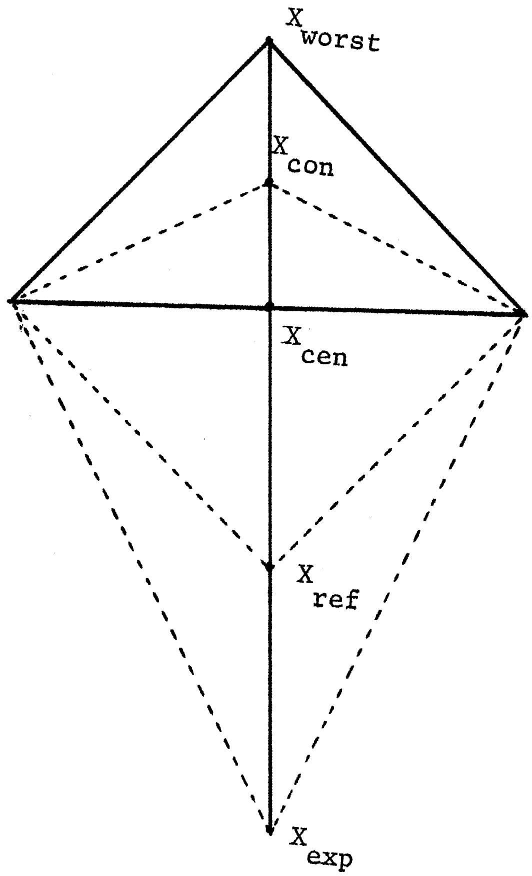


Figure 6. Reflection, Expansion, Contraction Operations in the Nelder and Mead Algorithm (Two Independent Variable Case)

author. Unimodality is assumed and thus several sets of starting points should be considered when the objective function is expected to be multimodal. The simplex method has a wide application since it does not make any assumptions about the objective function except that the function should be continuous. Derivatives are not required. The logic diagram is shown in Figure 7. The algorithm proceeds as follows:

1. A feasible starting point $\underline{X}_1$ is selected.

$$\underline{X}_1 = (X_{1,1}, X_{1,2}, X_{1,3}, \cdots, X_{1,n}) \qquad (5\text{-}2)$$

"Feasible" means satisfying all the constraints.

2. A starting simplex is constructed with n+1 points consisting of a starting feasible point and n additional feasible points. They are generated by the values of the feasible starting point and initial side length of the starting simplex as follows:

$$\underline{X}_j = (X_{j,1}, X_{j,2}, X_{j,3}, \cdots, X_{j,n}) \qquad (5\text{-}3)$$

where

$j = 2,3, \ldots, n+1$

and

$$X_{j,i} = X_{1,i} + \xi_{j,i}, \quad i = 1,2, \ldots, n; \quad j = 2,3, \ldots, n+1 \qquad (5\text{-}4)$$

j	$\xi_{j,1}$,	$\xi_{j,2}$,	$\xi_{j,3}$	$\cdots\ \xi_{j,n-1}$,	$\xi_{j,n}$
2	p ,	q ,	q ,	... q ,	q
3	q ,	p ,	q ,	... q ,	q

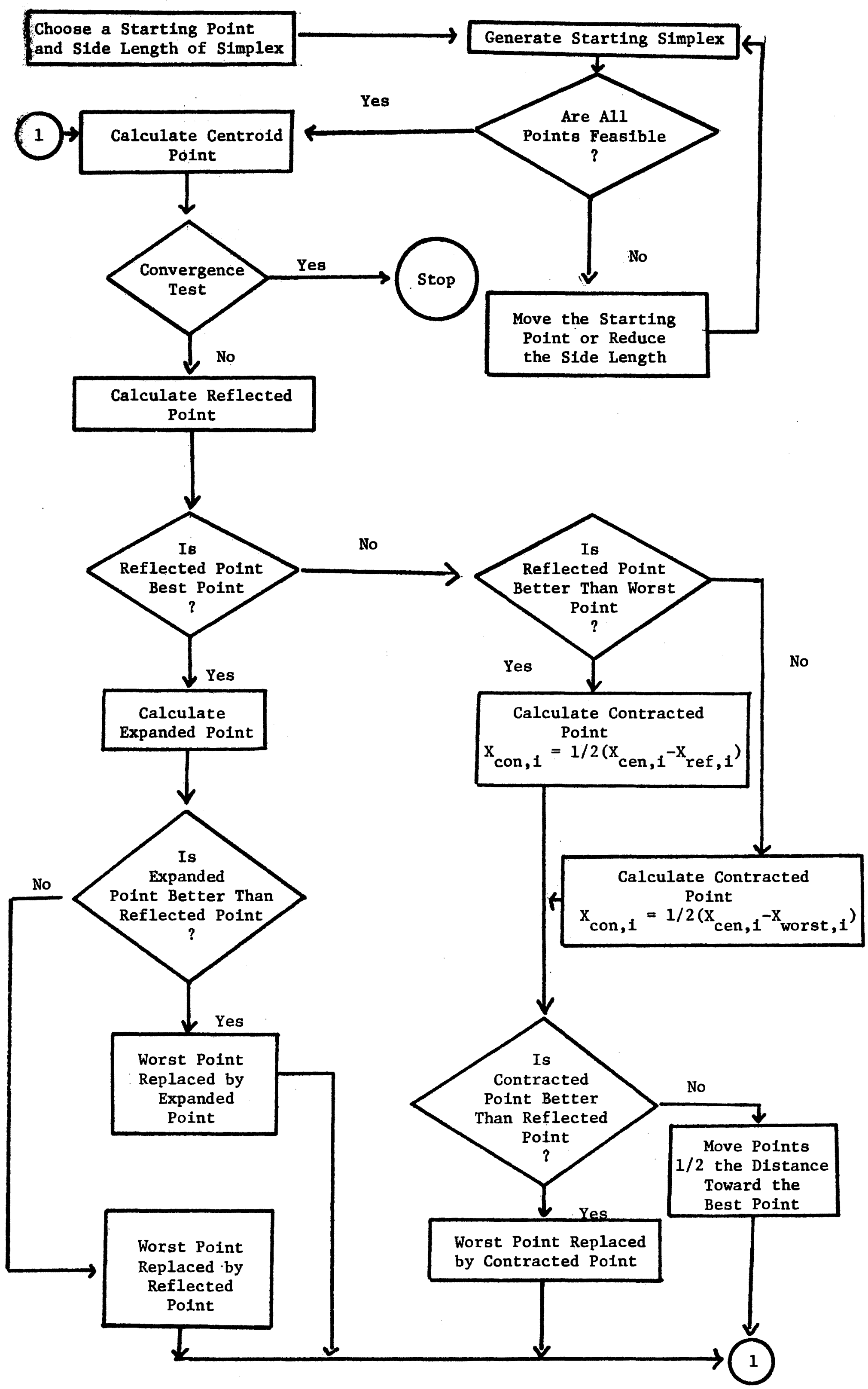


Figure 7. Logic Diagram of Modified Simplex Algorithm

4 q , q , p , ..., q , q

. . . .

. . . .

. . . .

n q , q , q , ..., q , q

n+1 q , q , q , ..., q , p

$$p = \frac{a}{\sqrt{2}\,n} \left(\sqrt{n+1} + n-1 \right) \qquad (5\text{-}5)$$

$$q = \frac{a}{\sqrt{2}\,n} \left(\sqrt{n+1} - 1 \right) \qquad (5\text{-}6)$$

where

n: total number of variables

a: side length of starting simplex

3. Once the starting simplex is formed, the feasibility is tested at each point. The feasibility test checks whether the point satisfies all the explicit and implicit constraints. If there is any infeasible point among the starting simplex points, the starting feasible point is moved or the side length of the starting simplex is decreased to yield only the feasible points.
4. The objective function is evaluated at each point and the worst and best points are selected.
5. The centroid point of all the points (excluding the point having the worst value) is calculated from the following (see Figure 6):

$$X_{cen,i} = \frac{1}{n} \left[\sum_{j=1}^{n+1} X_{j,i} - X_{worst,i} \right], \qquad (5\text{-}7)$$

where

$$i = 1,2,3, \ldots , n$$

6. The objective function is evaluated at the centroid point and a convergence test is performed,

$$\sum_{j=1}^{n+1} [\{ F(X_{j,i}) - F(X_{cen,i}) \} / (n+1)]^{\frac{1}{2}} \leq \tau \qquad (5\text{-}8)$$

where τ is the preselected tolerance of the solution.

7. A reflected point is located as follows:

$$X_{ref,i} = 2\, X_{cen,i} - X_{worst,\, i} \qquad (5\text{-}9)$$

where

$$i = 1,2, \ldots , n$$

8. The feasibility of the reflected point is tested and if it is not a feasible point, a new reflected point is calculated,

$$(X_{ref,i})_{new} = \tfrac{1}{2}\, [(X_{ref,i})_{old} + X_{cen,i}] \qquad (5\text{-}10)$$

where

$$i = 1,2, \ldots , n$$

9. The objective function is evaluated at this reflected point, and if the reflected point has the worst objective function value of the current set of points, a contracted point is located as follows:

$$X_{cen,i} = \tfrac{1}{2}\, [X_{cen,i} - X_{worst,i}] \qquad (5\text{-}11)$$

where

$i = 1,2, \ldots , n$

10. The objective function is now evaluated at the contracted point. If an improvement over the current values is achieved, the worst point is replaced by the contracted point and the process is restarted from step (5).

11. If an improvement is not achieved, the current simplex is shrunk about the best point

$$(X_{j,i})_{new} = [X_{best,i} + (X_{j,i})_{old}] /2 \qquad (5\text{-}12)$$

where

$j = 1,2, \ldots , n+1$

$i = 1,2, \ldots , n$

and the process is restarted from step (5).

12. At step (9), if the reflected point is better than the worst point but is not the best point, a contracted point is located as follows:

$$X_{con,i} = \tfrac{1}{2} [X_{cen,i} - X_{ref,i}] \qquad (5\text{-}13)$$

13. If the objective function at this contracted point is an improvement over that at the reflected point, the worst point is replaced by the contracted point; if not, the worst point is replaced by the reflected point and the process is restarted from step (5).

14. At step (9), if the reflected point is the best point, an expanded point is calculated as follows:

$$X_{exp,i} = 2\, X_{ref,i} - X_{cen,i} \qquad (5\text{-}14)$$

where

$i = 1,2, \ldots , n$

15. If the expanded point calculated in step (14) is infeasible, a new expanded point is calculated as follows:

$$(X_{exp,i})_{new} = \tfrac{1}{2} \left[(X_{exp,i})_{old} + X_{ref,i} \right] \qquad (5\text{-}15)$$

16. If the expanded point is an improvement over the reflected point, the worst point is replaced by the expanded point and the process is restarted from step (5).
17. If the expanded point is not an improvement over the refelcted point, the worst point is replaced by the reflected point and the process is restarted from step (5).
18. The procedure is terminated when the convergence criterion is satisfied or a specified number of iterations have been exceeded.

CHAPTER VI

OPTIMAL SYNTHESIS OF A HEAT EXCHANGE SYSTEM

The most important facet of the optimization of a heat exchange system is the synthesis of the system configuration. One can calculate the optimum parameters for a given system configuration by the optimization technique developed already in the previous chapter. However, different system configurations give different optimum values of the objective function. What one wants to find is the optimum among the many optimum values of all the different system configuration. The direct way of synthesizing an optimal heat exchange system is to generate all the feasible system configurations, to optimize all the systems generated by that procedure and to select the system configuration which gives the final optimum. There are so many feasible system configurations that just generating the system configuration can be an enormous problem if recycle information flow streams and stream splitting are considered for more efficient heat exchange.

Pho (29) attempted the decision tree approach to the synthesis problem using a compact matrix representation of an acyclic exchanger network. Based on this matrix, a decision tree diagram whose nodes encompass all the feasible networks of acyclic and nonsplittable streams is constructed. Then, optimization of all the feasible systems is necessary to determine the best system configuration. Therefore, Pho's attempt cannot eliminate the basic combinatorial difficulty of the synthesis problem and cannot handle the system configurations with recycle information streams and split streams into branch streams.

Hohman (13) suggested using temperature-enthalpy flow rate diagrams to visualize the effect of individual design decisions or changes, and he attempted to develop a synthesis method for a heat exchanger network.

Kobayashi et al. (19) used temperature-heat capacity flow rate diagrams, and linear programming for the synthesis of the heat exchanger network for the crude preheater train problem. Therefore they could avoid the conbinatorial difficulty of the synthesis problem, but to make their method work they sacrificed the nonlinearity of the heat exchange system problems.

Graphical Representation of a Stream System

Graphical analysis is very useful in synthesizing heat exchange systems as well as in evaluating a given heat exchange system. Through graphical visualization, complex and intangible problems can be reduced to simple and concrete ones.

For the graphical analysis, temperature-enthalpy flow rate diagram and temperature-heat capacity flow rate (or thermal capacitance) diagrams are very useful. Enthalpy flow rate is defined as the product of the enthalpy of unit mass of stream and the mass flow rate:

$$\dot{H}(\text{Btu/hr}) = \bar{H}(\text{Btu/lb}) \times \dot{m}\ (\text{lb/hr}) \tag{6-1}$$

or

$$\dot{H}(\text{Btu/hr}) = \overline{H}(\text{Btu/lb mole}) \times \dot{m}\ (\text{lb mole/hr}) \tag{6-2}$$

Heat capacity flow rate (thermal capacitance) is defined as the product of the specific heat and the mass flow rate.

$$\dot{C}(Btu/hr^{o}F) = C_{p}(Btu/lb\ mole^{o}F) \times \dot{m}\ (lb\ mole/hr) \qquad (6\text{-}3)$$

$$\dot{C}(Btu/hr^{o}F) = C_{p}(Btu/lb^{o}F) \times \dot{m}\ (lb/hr) \qquad (6\text{-}4)$$

On the temperature-enthalpy flow rate diagram (Figure 8), the vertical coordinate represents temperature, and the horizontal coordinate is enthalpy flow rate.

On the temperature-heat capacity flow rate diagram (Figure 9), the vertical coordinate is temperature, and the horizontal coordinate is the heat capacity flow rate scale. Heat capacity flow rate is a line parallel to the horizontal coordinate, expressing the magnitude as the absolute length of the line between the two end points arbitrarily chosen for convenience. If the flow rate, temperature, and heat capacity flow rate or enthalpy flow rate of a stream are given, the state of a stream can be expressed as a point on the temperature-enthalpy flow rate diagram (Figure 8) and as a line on the temperature-heat capacity flow rate diagram (Figure 9) Typical example streams are given in Table III and they are represented both on the temperature-enthalpy flow rate diagram (Figure 8) and on the temperature-heat capacity flow rate diagram (Figure 9).

If one plots the stream conditions at all the temperatures between inlet and outlet temperatures, one obtains a line connecting the inlet and outlet conditions on the temperature-enthalpy flow rate diagram (Figure 10), and one obtains a block on temperature-heat capacity flow rate diagram (Figure 11). If the heat capacity is known on the range, the path the stream follows during the heat exchange process can be

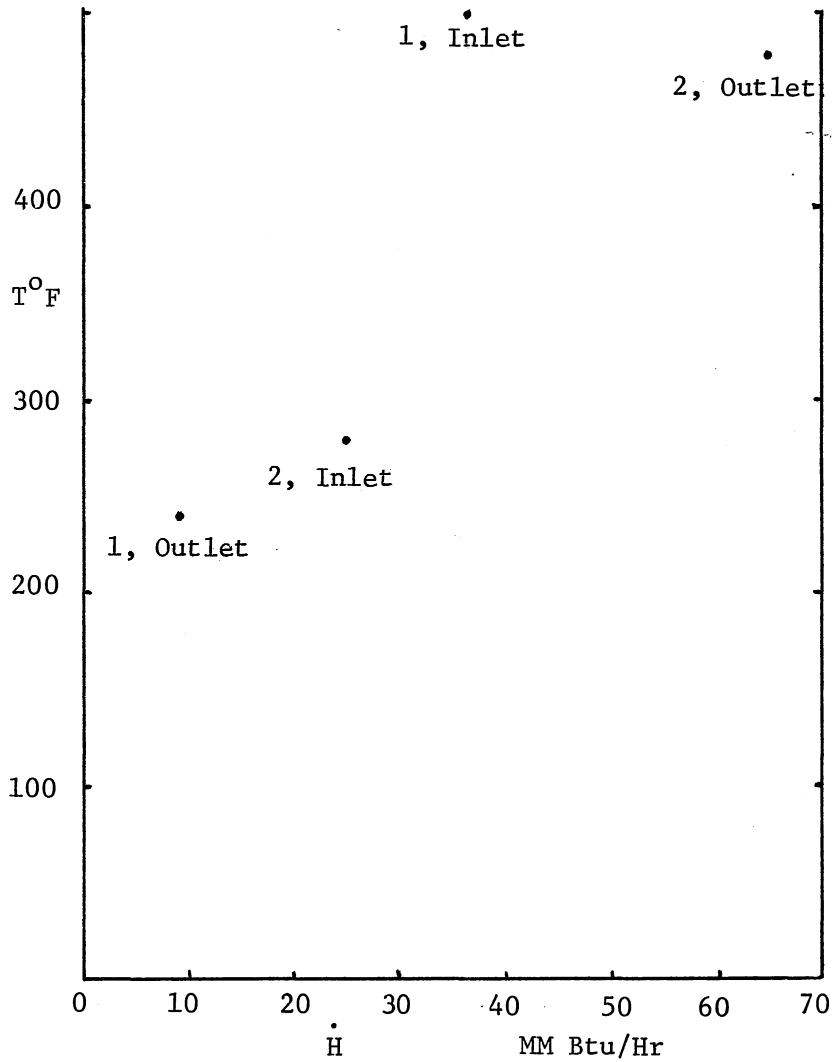


Figure 8. Temperature Enthalpy Flow Rate Diagram (T-Ḣ Diagram)

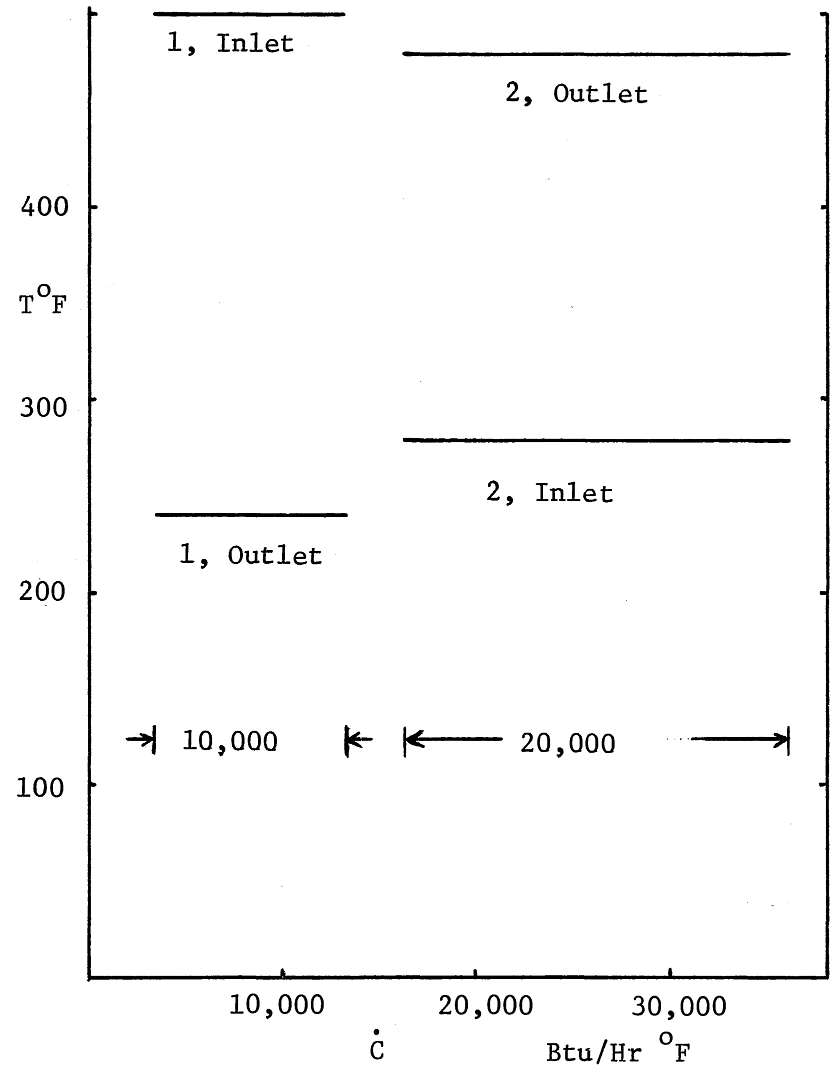
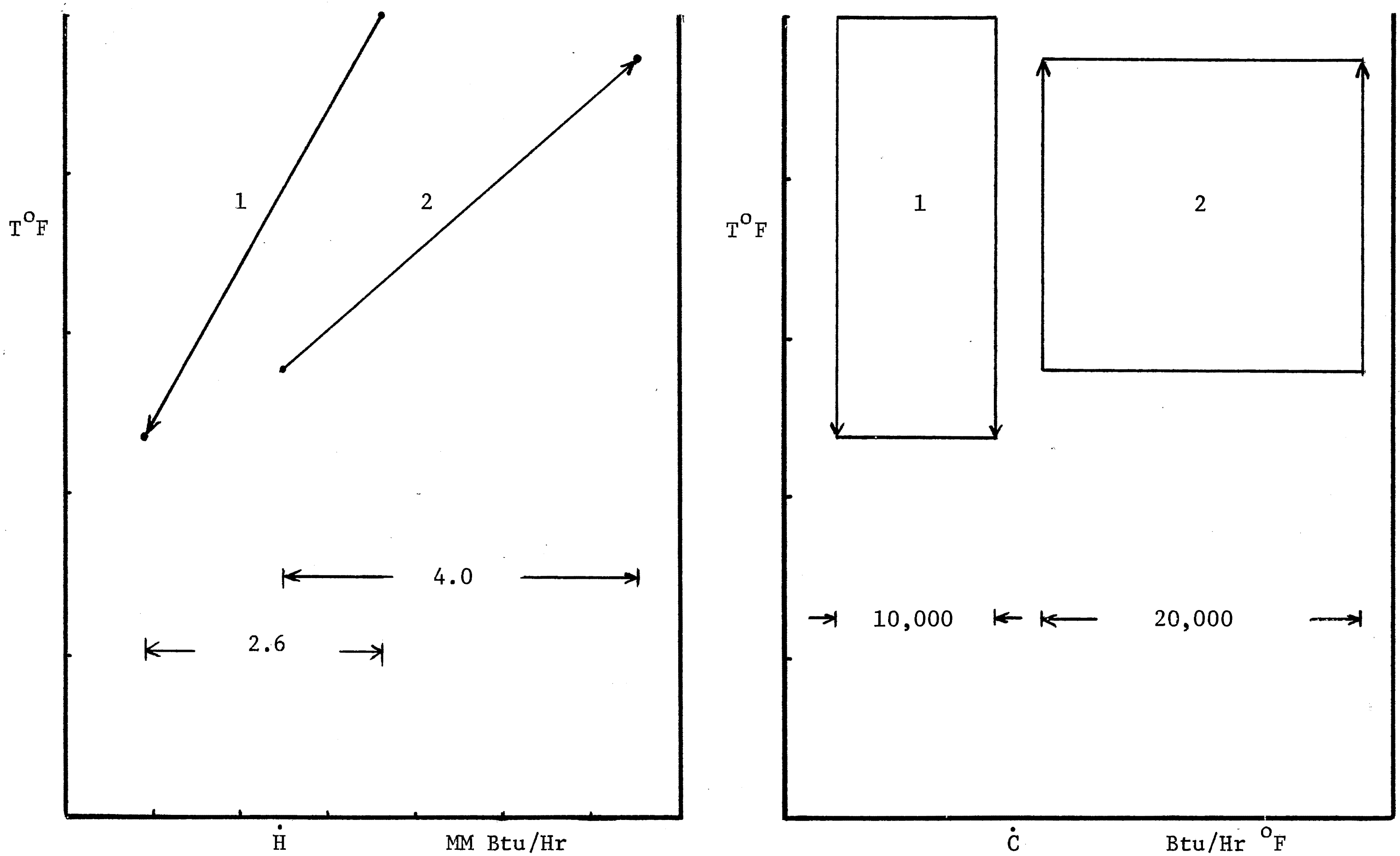


Figure 9. Temperature Heat Capacity Flow Rate Diagram (T-Ċ Diagram)



Figures 10 and 11. Representation of the Change of Stream Condition During the Process drawn by connecting the inlet and outlet condition points with a line on temperature-enthalpy flow rate diagram, or by connecting the inlet and outlet condition lines with two side lines on the temperature-heat capacity flow rate diagram. For example, if the heat capacity is constant in that process range, the line on the temperature-enthalpy flow rate diagram will be a straight line connecting inlet and outlet points (Figure 10) and the block on the temperature-heat capacity flow rate diagram will be a rectangle (Figure 11).

TABLE III

DATA FOR TWO EXAMPLE STREAMS

Stream	Mass Flow Rate lb moles/hr	Average Specific Heat Btu/lb mole ^{o}F	Average Heat Capacity Flow Rate Btu/hr ^{o}F	Temperature In ^{o}F	Temperature Out ^{o}F	Enthalpy Flow Rate In MMBtu/hr	Enthalpy Flow Rate Out MMBtu/hr	Enthalpy Flow Rate Change MMBtu/hr
1	1000	10	10,000	500	240	3.60	1.00	-2.60
2	800	25	20,000	280	480	2.50	6.50	4.00

To specify the process direction of a stream arrow heads are drawn in Figures 10 and 11. Enthalpy flow rate changes of stream for process specification are expressed as the projected lengths of the lines on the horizontal coordinate on the temperature-enthalpy flow rate diagram and as the areas of the blocks on the temperature-heat capacity flow rate diagram because of the following relation

$$\dot{\Delta H} = \dot{C}_{av}(T_{in}-T_{out}) \qquad (6\text{-}5)$$

where $\dot{C}_{av}$ is the average heat capacity flow rate in the temperature range.

Usually the specific heat of a stream in the process increases with temperature increase as given in Table IV. In this case, the change of stream conditions between inlet and outlet can be expressed as a curved line (Figure 12) or as a trapezoid (Figure 13).

TABLE IV

EXAMPLE STREAM DATA

	Tempera-ture ^{o}F	Mass Flow Rate lb/hr	Specific Heat Btu/lb ^{o}F	Heat Capacity Flow Rate But/hr ^{o}F	Enthalpy Flow Rate MMBtu/hr	Enthalpy Flow Rate Change MMBtu/hr
Inlet	50	10,000	0.555	5,550	1.206	1.304
Outlet	250	10,000	0.749	7,490	2.510	

Since the enthalpy scale is only relative, the absolute position is not important. The lines representing the streams may be shifted horizontally or broken into parts for convenience. Therefore one can

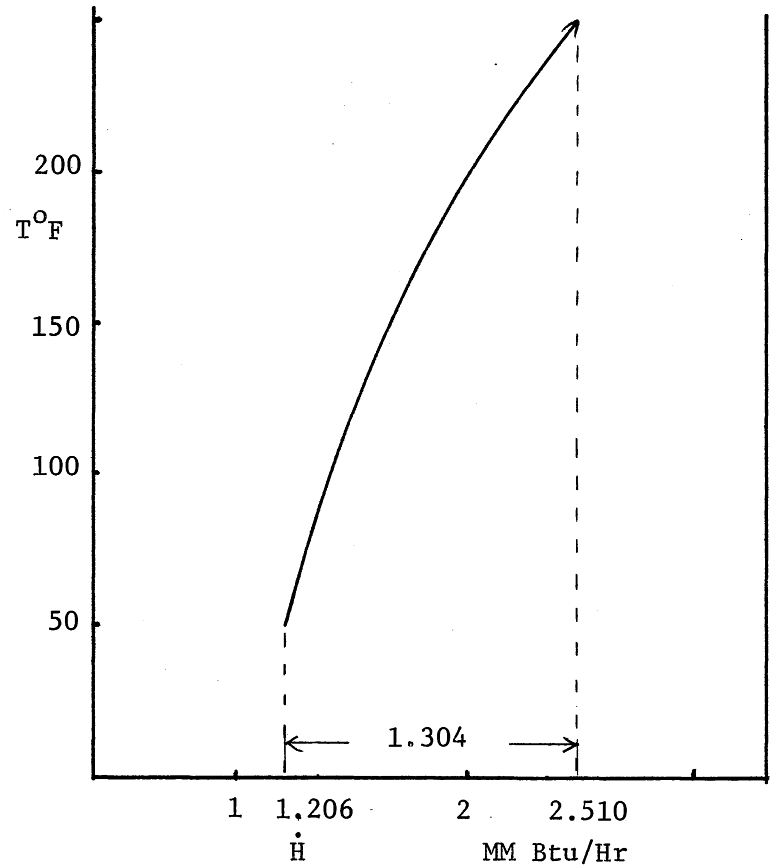


Figure 12. T-Ḣ Diagram

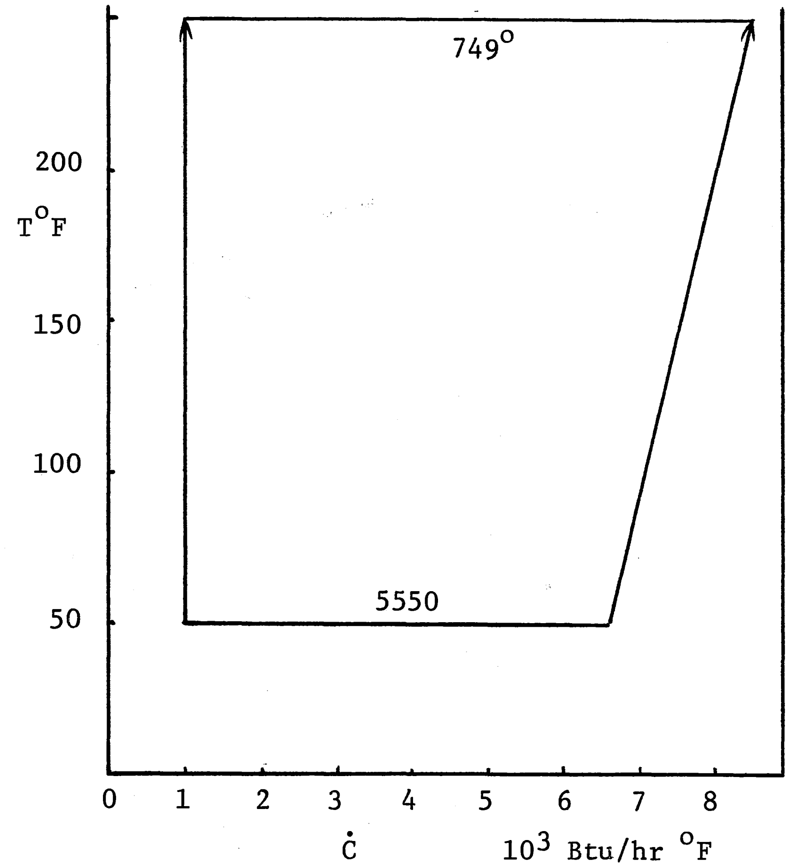


Figure 13. T-Ċ Diagram

locate the stream regardless of the absolute value of enthalpy, maintaining the inlet and outlet temperatures and the enthalpy flow rate change fixed.

As the "heat capacity" of phase change is infinite, phase change can be expressed as a line parallel to the enthalpy flow rate coordinate in temperature-enthalpy diagram (Figure 14). On the other hand, it is impossible to represent the phase change on the temperature-heat capacity flow diagram without modification.

What one wants to do with graphical representation is to visualize the stream temperature levels and enthalpy flow rate changes during the process. Therefore one can draw a line with a convenient arbitrary length to express the condensing temperature, parallel to the horizontal coordinate and draw a block with dotted lines on the temperature level line of which the area corresponds to the amount of the latent heat of the stream as in Figure 15.

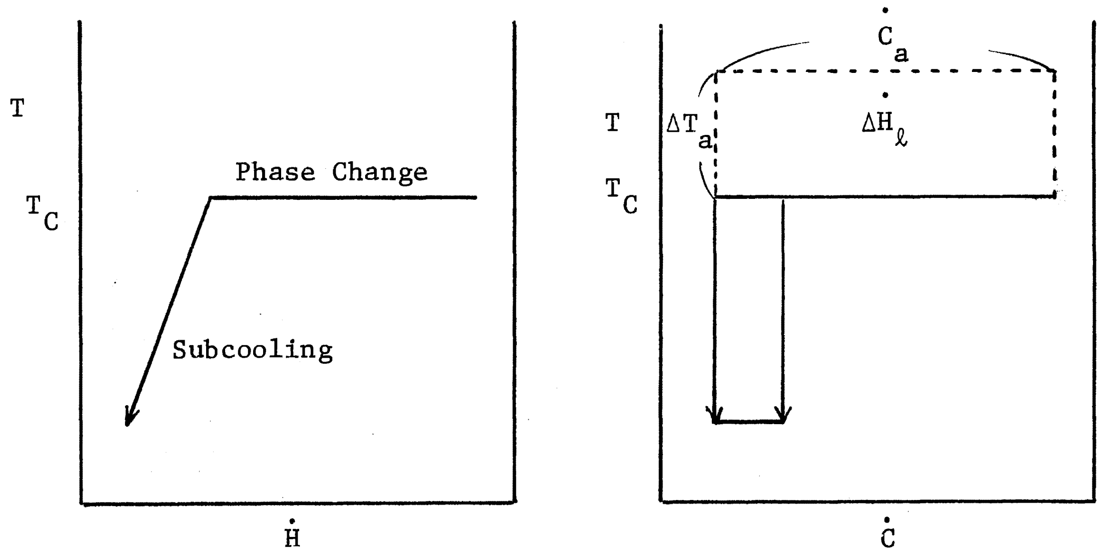
$$\Delta T_a = \frac{\Delta \dot{H}_\ell}{\dot{C}_a} \qquad (6\text{-}6)$$

ΔT_a = artificial temperature change

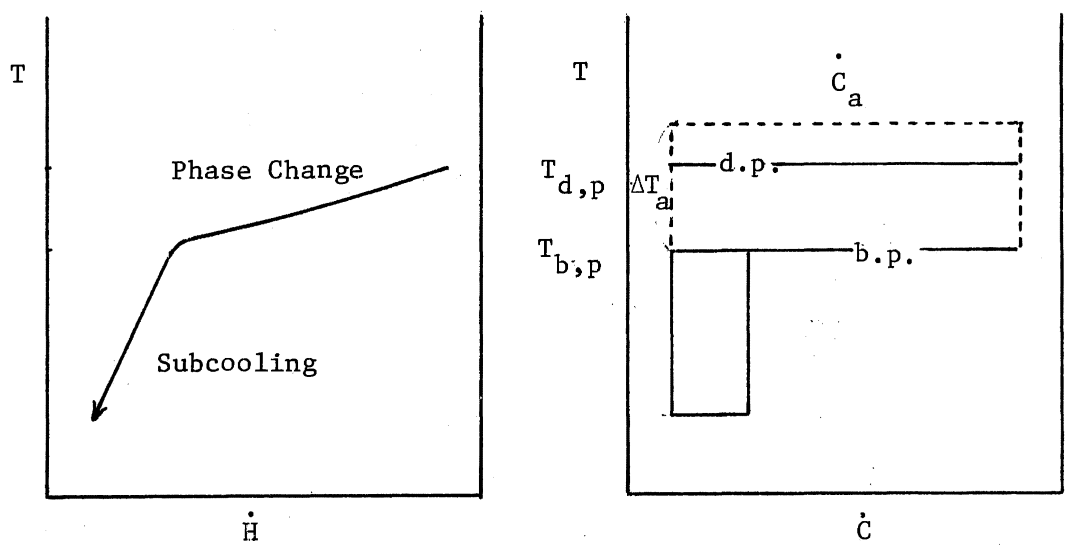
$\dot{C}_a$ = artificial heat capacity flow rate (arbitrarily determined for graphical representation)

$\Delta \dot{H}_\ell$ = latent heat rate

When the problem concerns phase changes of multicomponent streams, the graphical representation can be done on temperature-enthalpy flow rate diagram as Figure 16, but it is a little awkward on the temperature-heat capacity flow rate diagram. In this case one has to



Figures 14 and 15. Phase Change of Single Component Stream



Figures 16 and 17. Phase Change of Multicomponent Stream specify the bubble point and dew point temperature levels as in Figure 17.

The "stream system" is defined as the combination of the given process streams. The number of streams to be cooled or heated, flow rates, heat capacities or enthalpies, and initial and final temperatures of the streams define and describe the stream system. A "hot stream" is defined as a stream to be cooled and a "cold stream" is defined as a stream to be heated regardless of their initial temperatures. The stream system in Table V can be visualized on temperature-enthalpy flow rate and temperature-heat capacity flow rate diagrams, Figures 18 and 19. To simplify graphical representation, the average specific heat of each stream is used.

On temperature-heat capacity flow rate diagrams of stream systems, hot streams to be cooled are shown on the left side of the vertical axis and cold streams to be heated on the right side to make it convenient to compare the temperature levels of those two classes of streams.

Synthesis of Heat Exchange System by Graphical Analysis

In the synthesis of heat exchange systems, one tries to minimize the thermodynamic irreversibility of the process (by keeping the temperature difference as small as possible) and to maximize the amount of heat recovered. On the other hand, one would like to keep the temperature difference as large as possible in order to minimize the size and cost of the heat exchangers. Obviously these are contradictory goals and one has to seek the economic optimum between them.

TABLE V

EXAMPLE STREAM SYSTEM I

Stream Classification	State	Mass Flow Rate in(lb/hr)	Temperature T(^{o}F)	Average Specific Heat $\bar{C}_p$ (Btu/lb^{o}F)	Average Heat Capacity Flow Rate $\dot{C}_{av}$(Btu/hr^{o}F)	Enthalpy Flow Rate Change $\Delta\dot{H}$(MMBtu/hr)
A (Hot)	Initial	300,000	200	0.5	150,000	-15.0
	Final		100			
B (Hot)	Initial	100,000	350	0.8	80,000	-12.0
	Final		200			
C (Hot)	Initial	267,000	500	0.75	200,000	-28.0
	Final		360			
D (Cold)	Initial	500,000	70	0.6	300,000	15.0
	Final		120			

TABLE V (Continued)

Stream Classification	State	Mass Flow Rate in(lb/hr)	Temperature $T(^oF)$	Average Specific Heat $\bar{C}_p$ (Btu/lb^{o}F)	Average Heat Capacity Flow Rate $\dot{C}_{av}$ (Btu/hr^{o}F)	Enthalpy Flow Rate Change $\Delta\dot{H}$(MMBtu/hr)
E (Cold)	Initial	120,000	130	1.0	120,000	12.0
	Final		230			
F (Cold)	Initial	312,000	250	0.6	187,000	28.0
	Final		400			

By using graphical representation one tries to construct all feasible heat exchangers with the largest driving forces possible. One tries not to design some exchangers with excessive driving forces while making others have small driving forces. If there are several hot and cold streams, perhaps the best policy of matching the stream for heat exchange is to start with the highest temperature hot and cold streams in order to minimize the irreversibility of the process.

One feasibility criterion of heat exchange between a hot and a cold stream is that the amount of heat to be gained in the cold stream and the amount to be removed from the hot stream must be identical.

$$\dot{C}_{av,c}(T_{c,o}-T_{c,i}) = \dot{C}_{av,h}(T_{h,i}-T_{h,o}) \qquad (6\text{-}7)$$

Heat exchange between a hot and cold stream is indicated by assigning the same numbers to the hot and cold lines or blocks (Figures 20, 21). The areas in the hot and cold blocks have to be same to permit their matching on the temperature-heat capacity flow rate diagram. On the temperature-enthalpy flow rate diagram, cold stream, or the hot stream moved over the cold stream or the hot stream to show the heat exchange matching. The projected lengths of the lines on the enthalpy flow rate coordinate of the two matching streams have to be the same. The example stream system shown in Figures 18 and 19 can be matched as shown in Figures 20 and 21.

Another feasibility criterion of heat exchange between cold and hot streams is that the temperature of hot stream has to remain higher than the temperature of cold stream at all points during the heat exchange process. Usually, if the inlet temperature of hot stream is

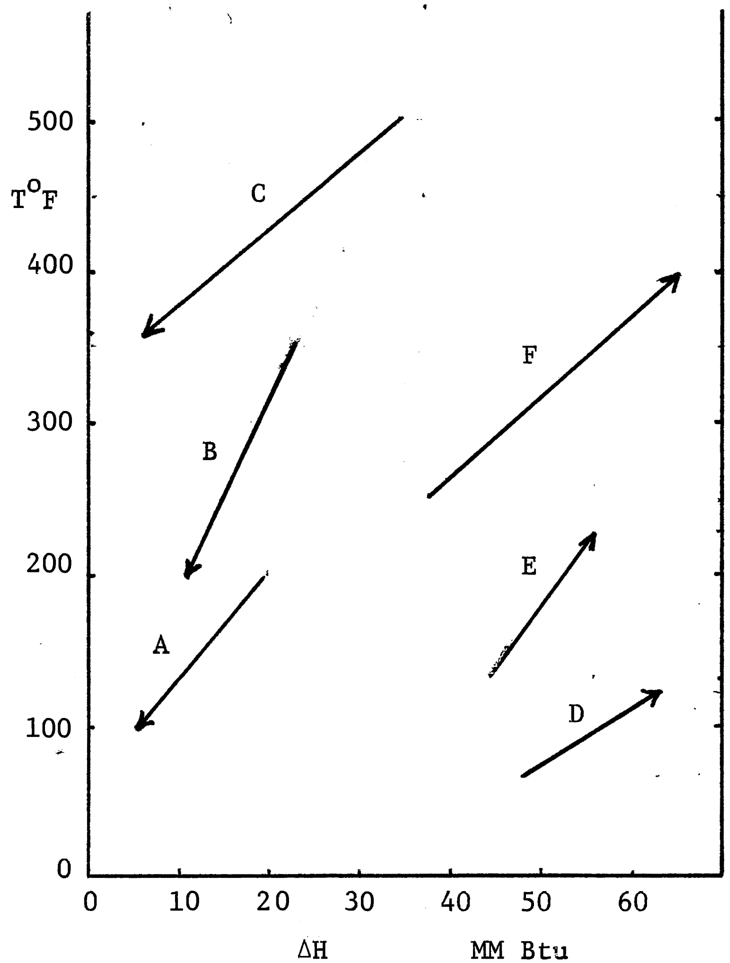


Figure 18. T-$\dot{H}$ Diagram

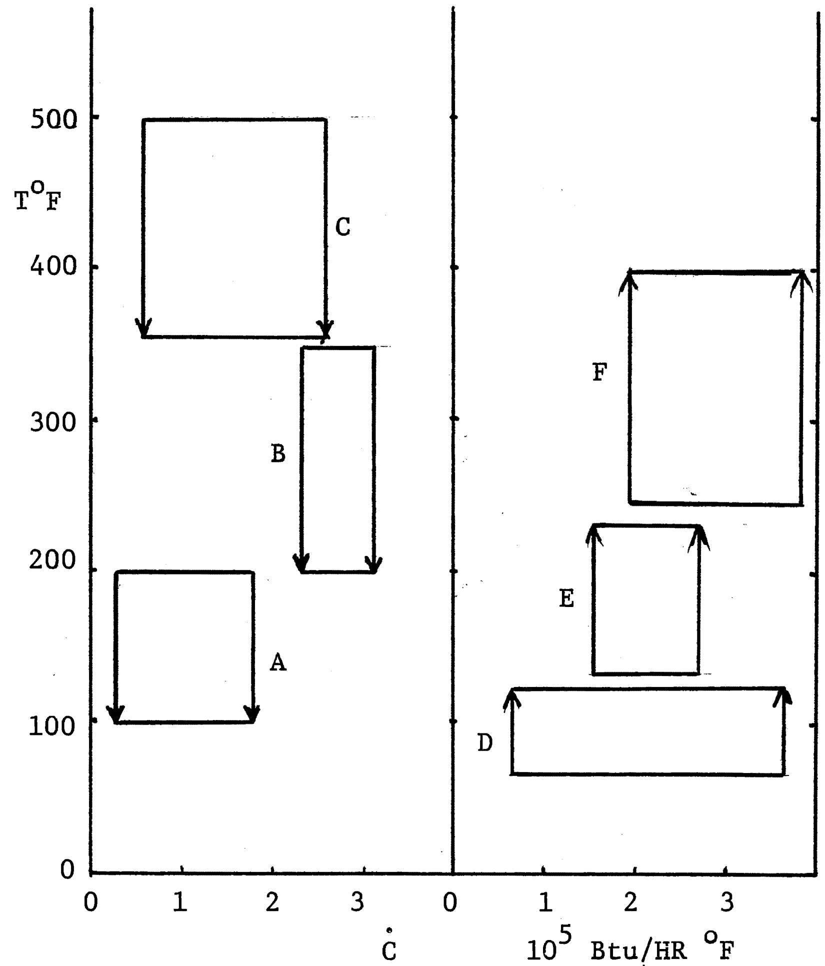


Figure 19. T-$\dot{C}$ Diagram

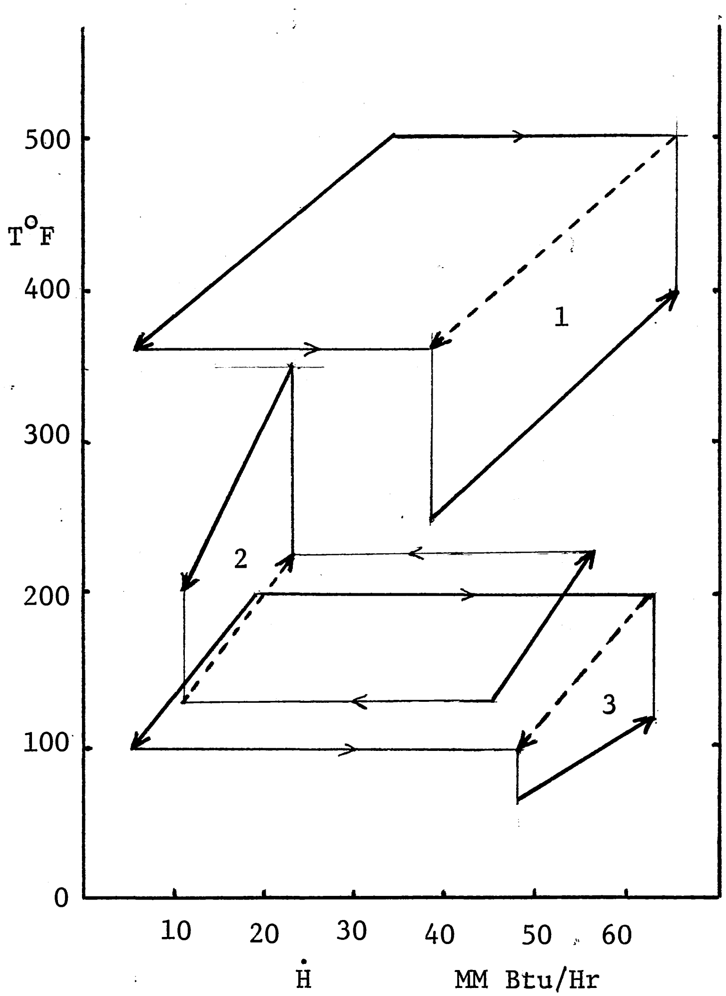


Figure 20. Synthesis of a Heat Exchange System by T-Ḣ Diagram

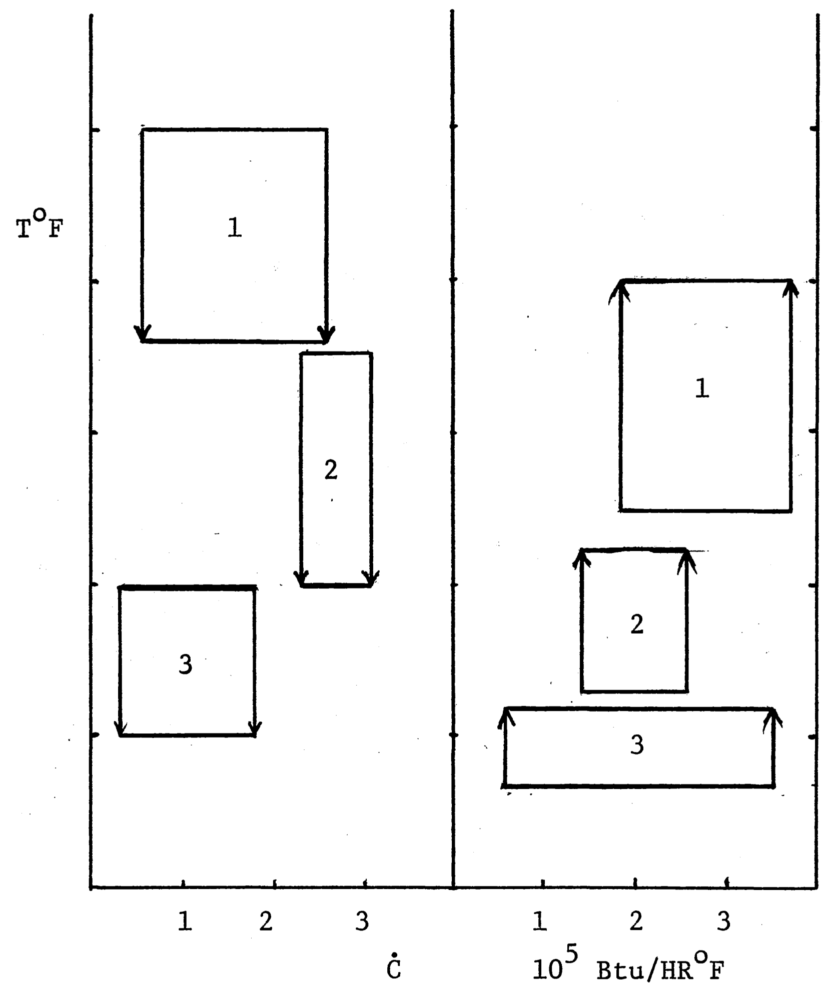
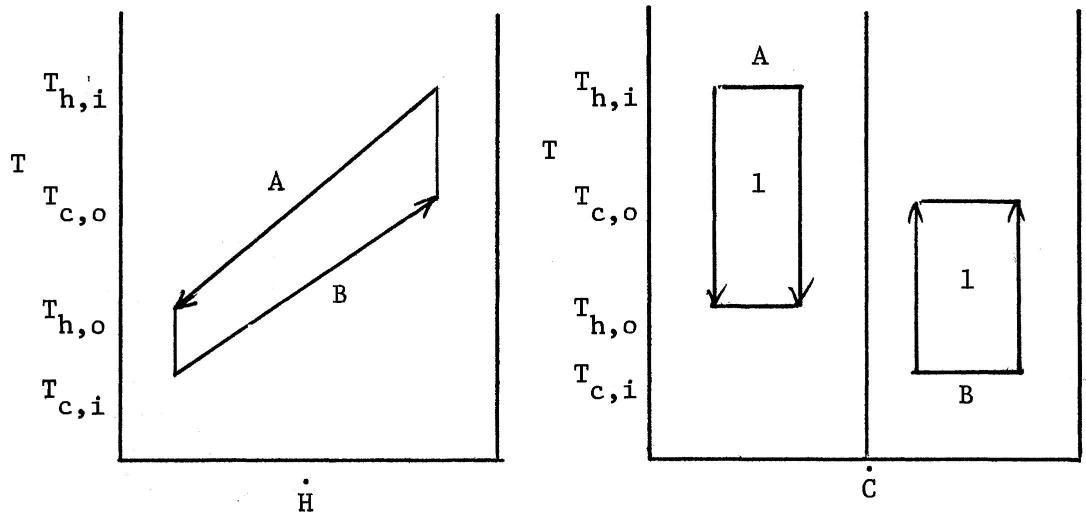
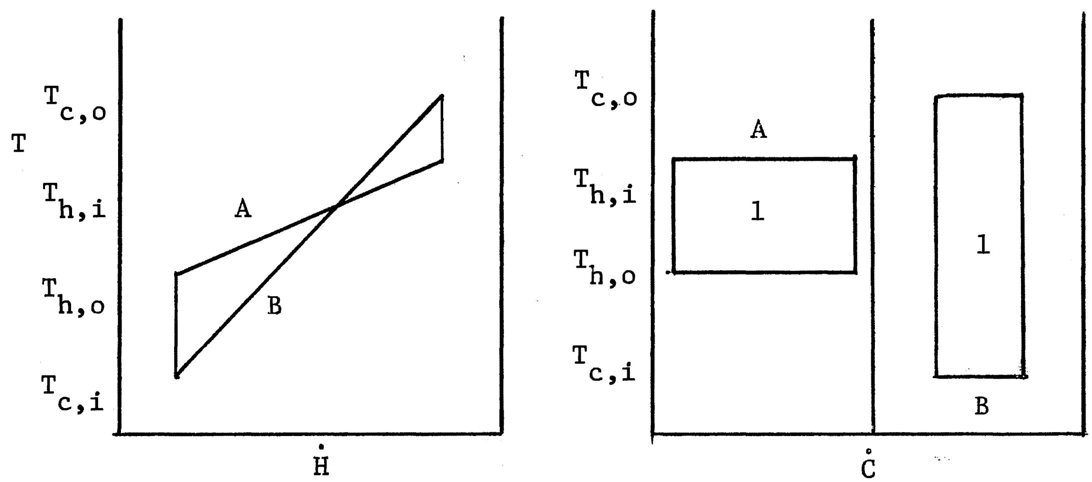


Figure 21. Synthesis of a Heat Exchange System by T-Ċ Diagram



Figures 22 and 23. Feasible Matching



Figures 24 and 25. Infeasible Matching

higher than the outlet temperature of cold stream,

$$T_{h,i} > T_{c,o} \qquad (6\text{-}8)$$

and the outlet temperature of the hot stream is higher than the inlet temperature of the cold stream,

$$T_{h,o} > T_{c,i} \qquad (6\text{-}9)$$

the criterion will be satisfied.

The previous two feasibility criteria can be tested easily by the graphical analysis. Each of the two diagrams has its own characteristic conveniences. For example, for a multicomponent phase change over a large temperature range, the temperature-enthalpy flow rate diagram can be drawn reasonably, but the temperature-heat capacity flow rate diagram has some difficulty expressing the amount of heat available at temperature levels between bubble point and dew point temperatures. But the latter is preferable to the former in visualizing the amount of heat duty which is shown as the area, and is convenient for the synthesis of more complex heat exchanger systems.

On the temperature-heat capacity flow rate diagram, one can divide a block vertically or horizontally. Vertical dividing means stream splitting into branch streams, and horizontal dividing corresponds to multiple heat exchange of a stream with other streams in series.

If two or more streams of same kind are at the same temperature level, the streams are said to be in "temperature contention." That is, any one of them can deliver heat to the cold stream or absorb heat from the hot stream by the same temperature difference driving force. For the ranges in which temperature contention is present, all streams in contention are required to transfer heat with the same temperature driving force to minimize thermodynamic irreversibility.

An example of streams in temperature contention is shown in Figures 26 and 27. Synthesis of a heat exchange system for this example is shown in Figures 28 and 29 and Figures 30 and 31. In Figures 28 and 30 the cold stream is split into two branch streams and the system configuration is shown in Figure 32. In Figures 29 and 31 the heat exchanger system is synthesized without splitting the cold stream and the system configuration is shown in Figure 33. In this example, it is shown that the system structure synthesized by splitting the cold stream into two branch streams does not require an additional cooler or heater while the other one needs two additional exchangers to satisfy the system specification. It is obvious from this example that the system synthesized by splitting the cold stream into two branch streams is better from a thermodynamic efficiency standpoint than the one synthesized without splitting the cold stream.

Temperature contention of streams is easily found by graphical visualization of stream system. In cases where temperature contention exists, the possibility of stream splitting has to be considered in the synthesis of heat exchange system.

With the above graphical analysis, one can easily synthesize the probable optimum system configuration by minimizing thermodynamic irreversibility and consequently maximizing the amount of heat

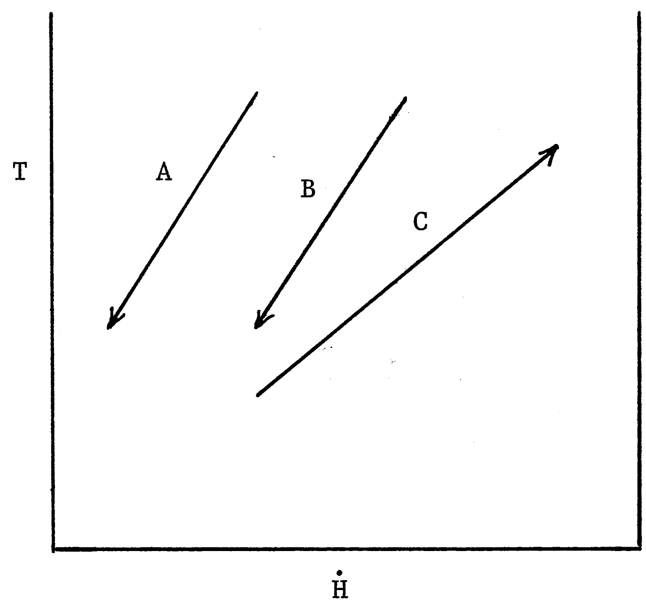


Figure 26. T-Ḣ Diagram of Stream System (A and B, are in Temperature Contention)

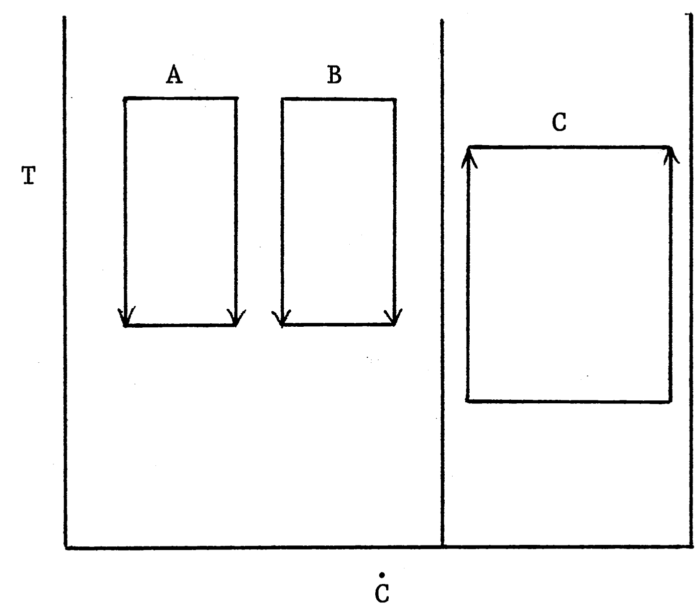


Figure 27. T-Ċ Diagram of Stream System (A and B, are in Temperature Contention)

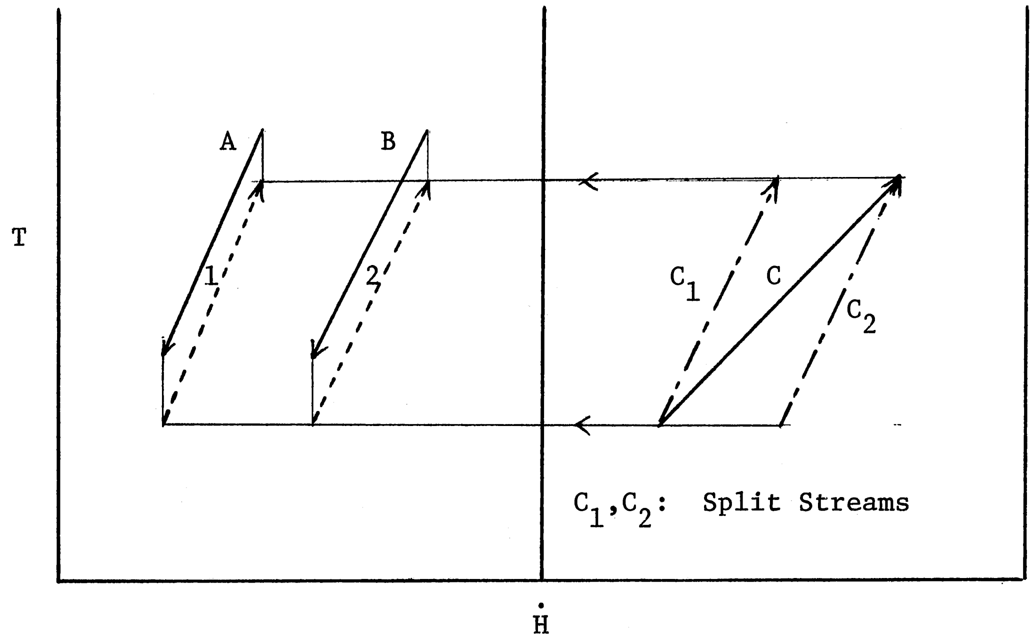


Figure 28. T-$\dot{H}$ Diagram for a Heat Exchanger System with Split Stream

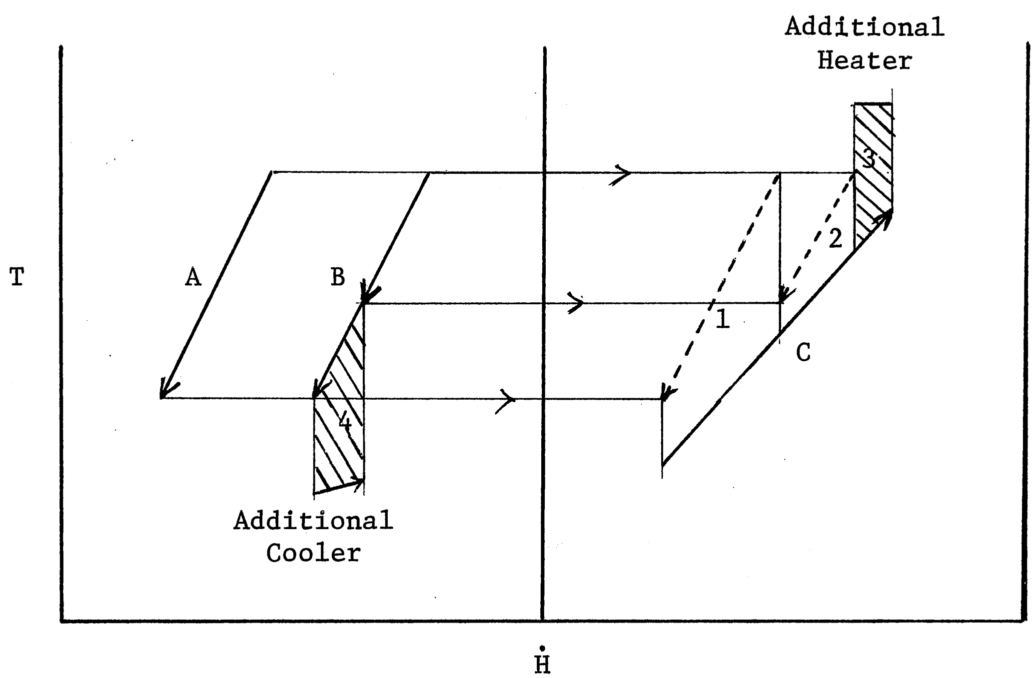


Figure 29. T-$\dot{H}$ Diagram for a Heat Exchanger System Without Split Stream

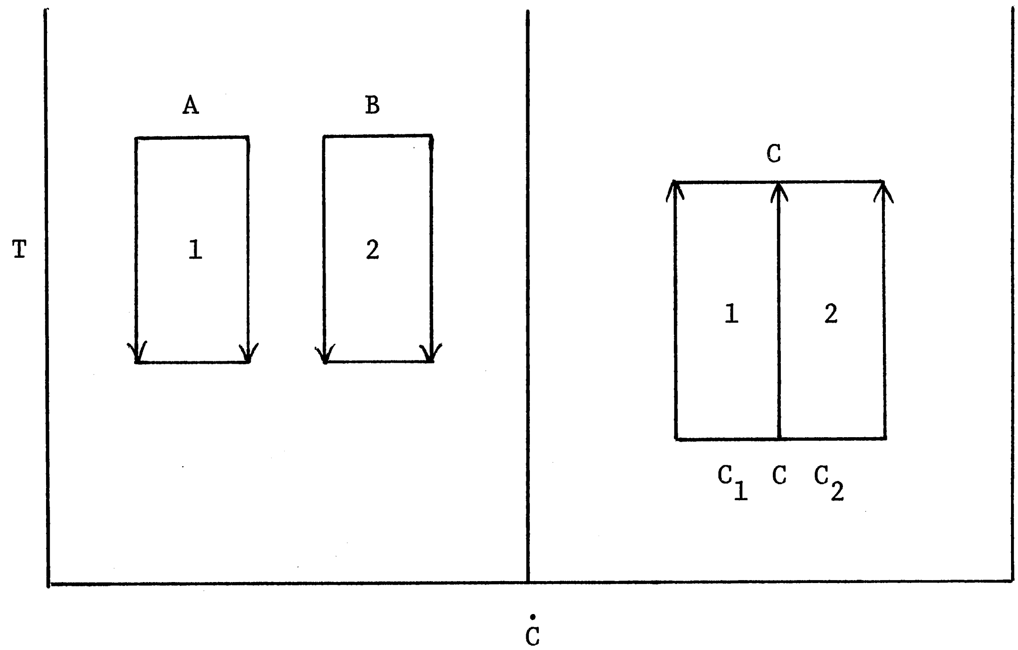


Figure 30. T-$\dot{C}$ Diagram for a Heat Exchanger System with Split Stream

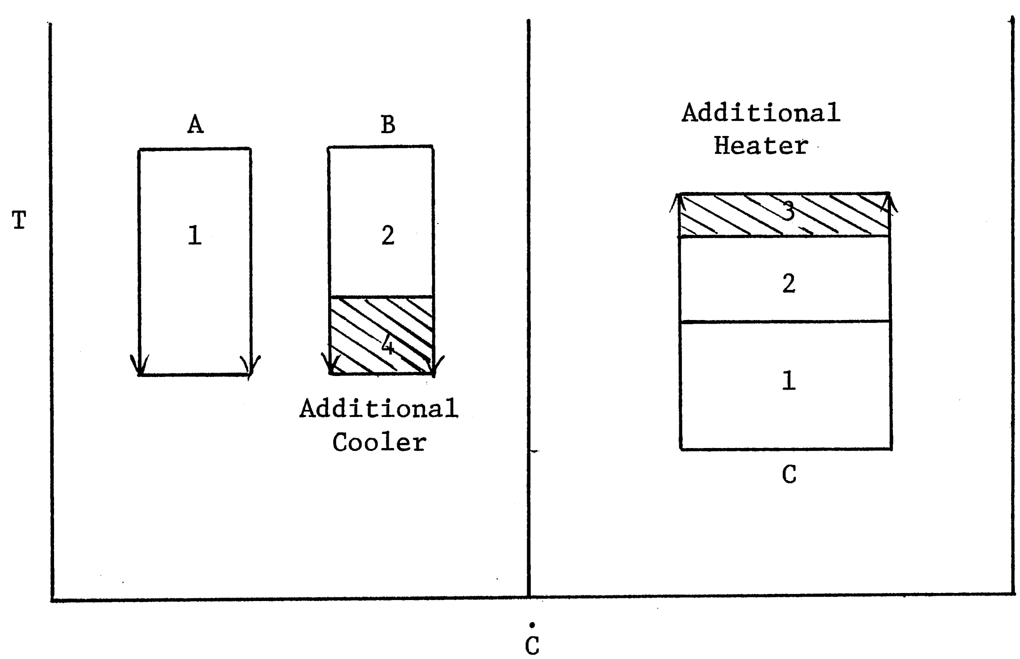


Figure 31. T-$\dot{C}$ Diagram for a Heat Exchanger System Without Split Stream

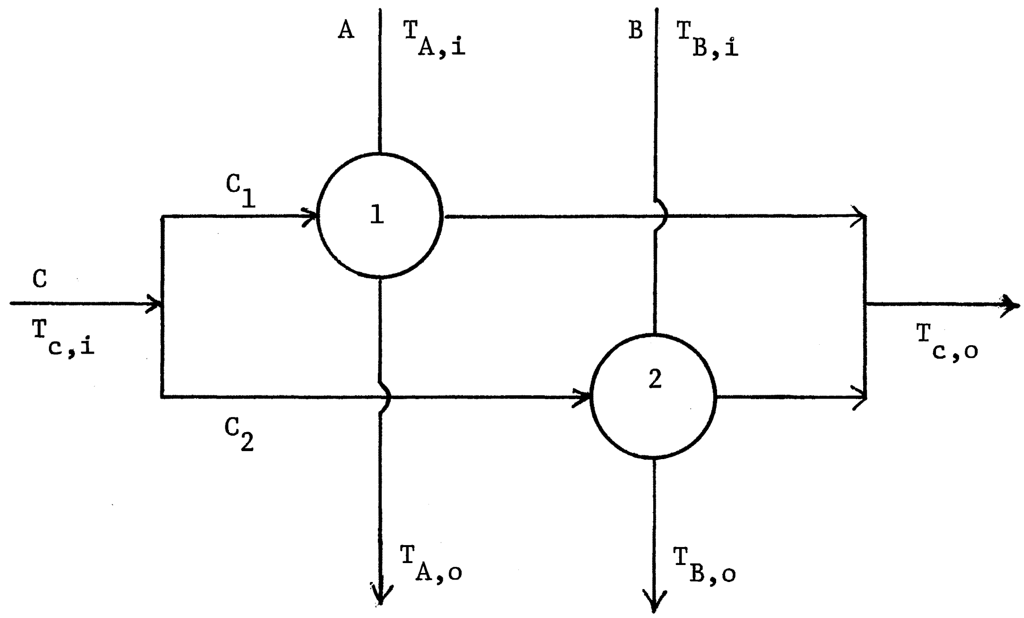


Figure 32. Flow Sheet of a Heat Exchanger System with Split Stream

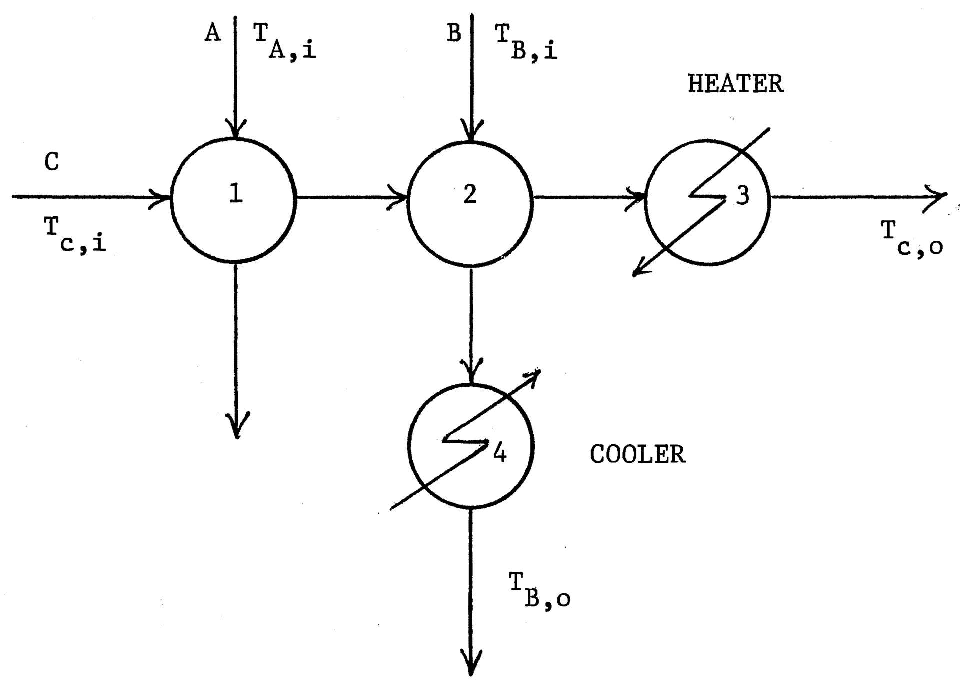


Figure 33. Flow Sheet of a Heat Exchanger System Without Split Stream

recovered. But the final determination can only be made by the total cost calculations of the probable optimum system configurations. One can choose a few possible optimum configurations by the graphical analysis.

Optimal Synthesis of Heat Exchange System

The detailed procedure of synthesizing an optimal heat exchange configuration will be explained by solving the following example problem, using temperature-heat capacity flow rate diagram.

Procedure

1. Draw the temperature-heat capacity flow rate diagram of stream system.
2. Determine the limits of heating and cooling temperatures without additional heaters and coolers. The lower limit temperature is defined as the lowest temperature the hot stream can attain without additional cooling and the upper limit temperature is defined as the highest temperature the cold stream can attain without additional heating.
   a. The lowest inlet temperature of the cold streams is the preliminary lower limit temperature of the hot streams and the highest inlet temperature of the hot streams is considered as the preliminary upper limit temperature of the cold streams to be heated.
   b. Calculate total heat duties required for the hot and cold streams between the preliminary upper and lower

TABLE VI

EXAMPLE STREAM SYSTEM II

Stream Classification	State	Mass Flow Rate m (lb/hr)	Temperature T(°F)	Average Specific Heat Btu/lb°F	Average Heat Capacity Flow Rate Btu/hr°F	Enthalpy Flow Rate Change MMBtu/hr
A (Cold)	Initial	1.6×10^4	150	0.9	1.44×10^4	2.728
	Final		330			
B (Cold)	Initial	2.3×10^4	240	0.5	1.15×10^4	2.990
	Final		500			
C (Hot)	Initial	2.8×10^4	290	0.6	1.68×10^4	-2.016
	Final		170			
D (Hot)	Initial	2.0×10^4	480	1.0	2.00×10^4	-4.000
	Final		280			

limit temperatures.

$$\sum_i \Delta H_{hot_i} = \sum_i C_{hi} \Delta T_i \qquad (6\text{-}10)$$

$$\sum_j \Delta H_{cold_j} = \sum C_{cj} \Delta T_j \qquad (6\text{-}11)$$

c. If $\sum_i \Delta H_{hot_i} < \sum \Delta H_{cold_j}$, the upper limiting temperature of the cold streams that can be attained without additional heating is determined to make $\sum_i \Delta H_{hot_i} = \sum_j \Delta H_{cold_j}$ (between new temperature limits).

d. If $\sum_i \Delta H_{hot_i} > \sum_j \Delta H_{cold_j}$, the lower limiting temperature of hot streams to be attained without additional cooling is determined to make $\sum_i \Delta H_{cold_j} = \sum_i \Delta H_{hot_i}$ (between new temperature limits). In this example, preliminary lower limit temperature is 140^{o}F and preliminary upper limit temperature is 480^{o}F. Within these limit temperatures,

$$\sum_i \Delta H_{hot_i} = 2.00 \times 10^4 \ (480-280) + 1.68 \times 10^4 \ (290-170)$$

$$= 4.00 \times 10^6 + 2.016 \times 10^6 = 6.016 \times 10^6 \ (\text{Btu/hr})$$

$$\sum_i \Delta H_{cold_i} = 1.44 \times 10^4 \ (330-150) + 1.15 \times 10^4 \ (480-240)$$

$$= 2.592 \times 10^6 + 2.96 \times 10^6 = 5.352 \times 10^6 \ (\text{Btu/hr})$$

Therefore,

$$\sum_i \Delta H_{hot_i} > \sum_j \Delta H_{cold_j}$$

$$\sum_i \Delta H_{hot_i} - \sum_j \Delta H_{cold_j} = (6.016-5.352) \times 10^6 =$$

6.64×10^5 (Btu/hr)

The lower limit temperature in this example can be found as

$$T_{\ell,\ell} = 170 + \frac{6.64 \times 10^5}{1.68 \times 10^4} = 209.5 \quad (^oF)$$

and the upper limit temperature is same as the preliminary upper limit temperature. These limit temperatures decide the temperature range to be considered and the minimum additional cooling requirement and minimum additional heating requirement of the heat exchange system.

3. Find the temperature contentions of the stream system and divide the blocks in temperature contention horizontally at the point where the horizontal edges of the other blocks are located (Figure 34).
4. Make matches of hot and cold blocks one by one starting from the highest temperature level. The feasibility criteria should be observed for every matching. Usually, if the areas are different between hot and cold blocks to be matched, the higher temperature portion of the larger block is taken to exchange heat with the smaller block, and the residual of the block is left for the next match. This procedure is repeated until all the blocks are used up (Figure 35). Temperature contention over a narrow temperature range should be ignored to avoid constructing an excessively complicated system structure. An additional heat exchanger can be used to reduce the small amount of thermodynamic irreversibility.

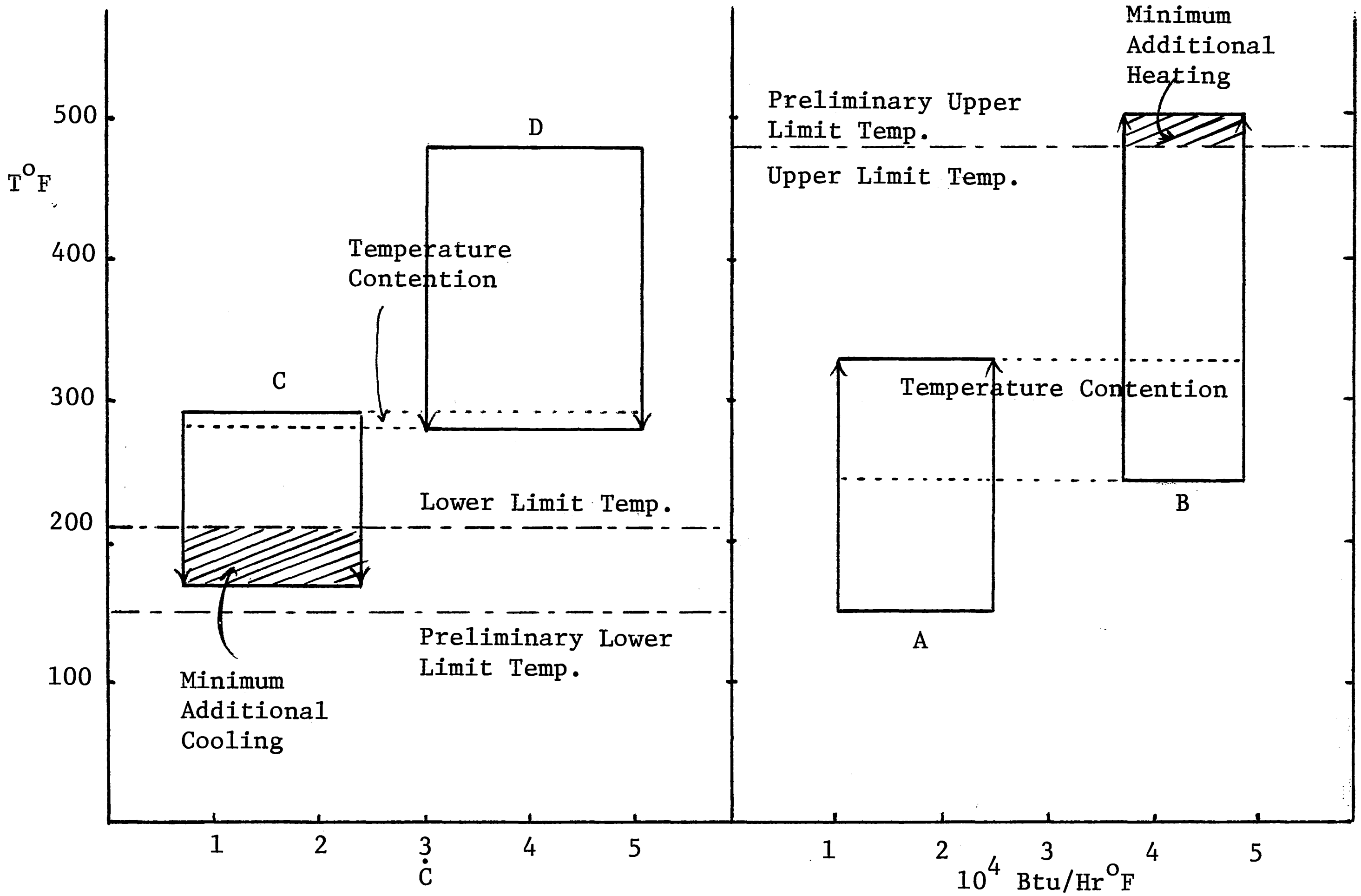


Figure 34. T-$\dot{C}$ Diagram for Determination of Limit Temperatures and Temperature Contention

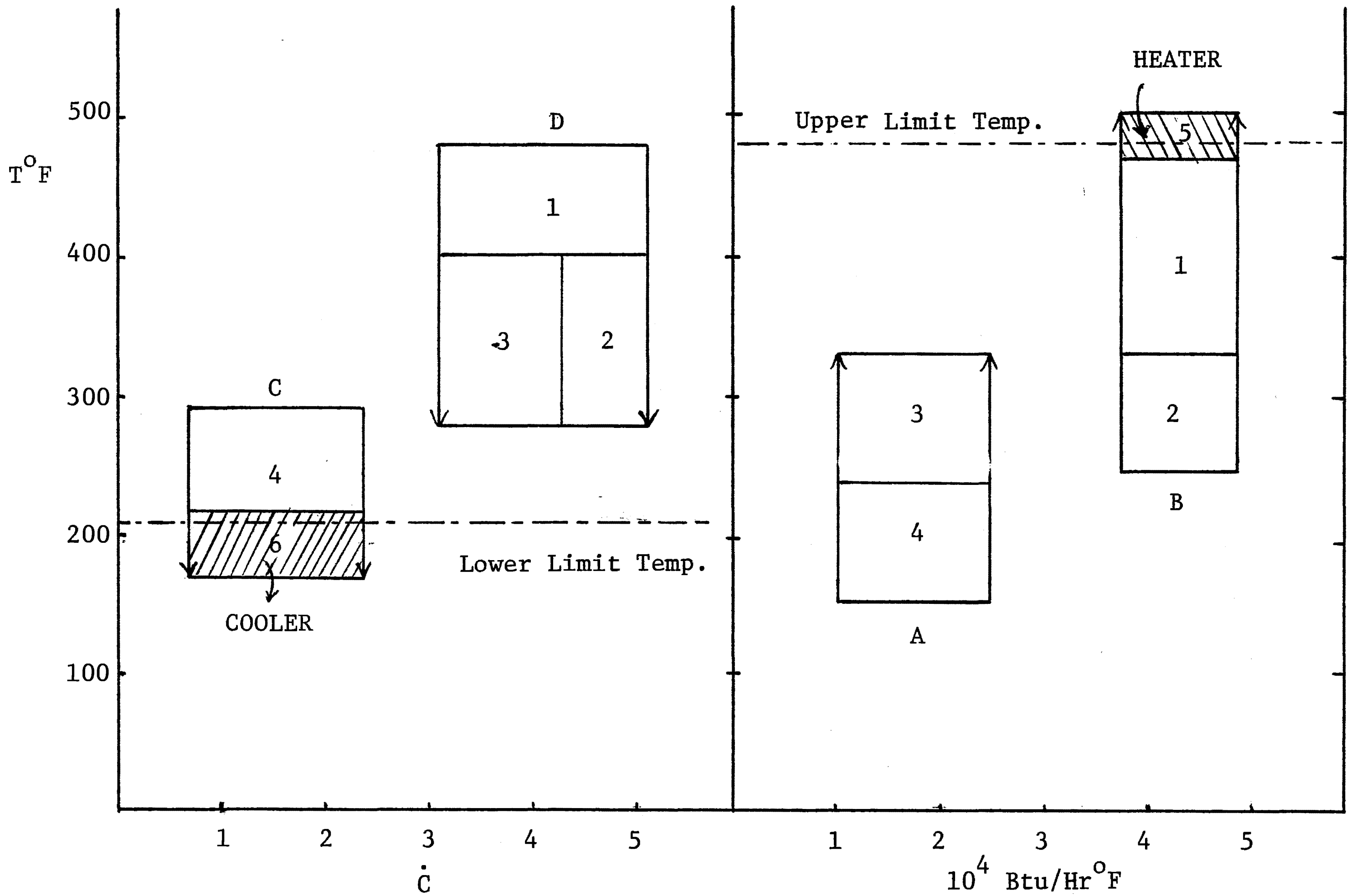


Figure 35. Synthesis of Heat Exchanger System by T-Ċ Diagram Analysis

5. Draw a flow sheet of the synthesized system, as Figure 36.

From the above procedure, heat exchange systems can be synthesized based on the temperature levels in order to avoid infeasible matching and to minimize thermodynamic irreversibilities, consequently maximizing the total amount of heat recovered.

The objective function for optimization of the heat exchange system is the total annual cost function of the system as established in Chapter III. This can be expressed as a nonlinear function of the temperatures of interconnecting streams of the heat exchangers if the other design variables are given. Therefore, the temperatures of interconnecting streams, which are the levels of horizontal lines dividing the blocks on the temperature-heat capacity flow rate diagram, have to be redecided to include the associated nonlinearities of the objective function by the optimization procedure in Chapter V.

As shown in Figure 36, there are eight known temperatures, t_1, t_2, t_3, t_5, T_1, T_2, T_4, and $T_{w,o}$ to be determined. For the design of the system one also needs to determine the amount of cooling water and heating steam required and the split of stream D. One can set seven energy balance equations for the six exchangers and the summation point of the branch streams. Therefore if the three unknowns T_1, T_2, t_2, out of above eleven unknowns are assumed, the seven energy balance equations, and the optimization procedure for the optimum outlet water temperature determination, will determine all the other unknown variables and specify the system.

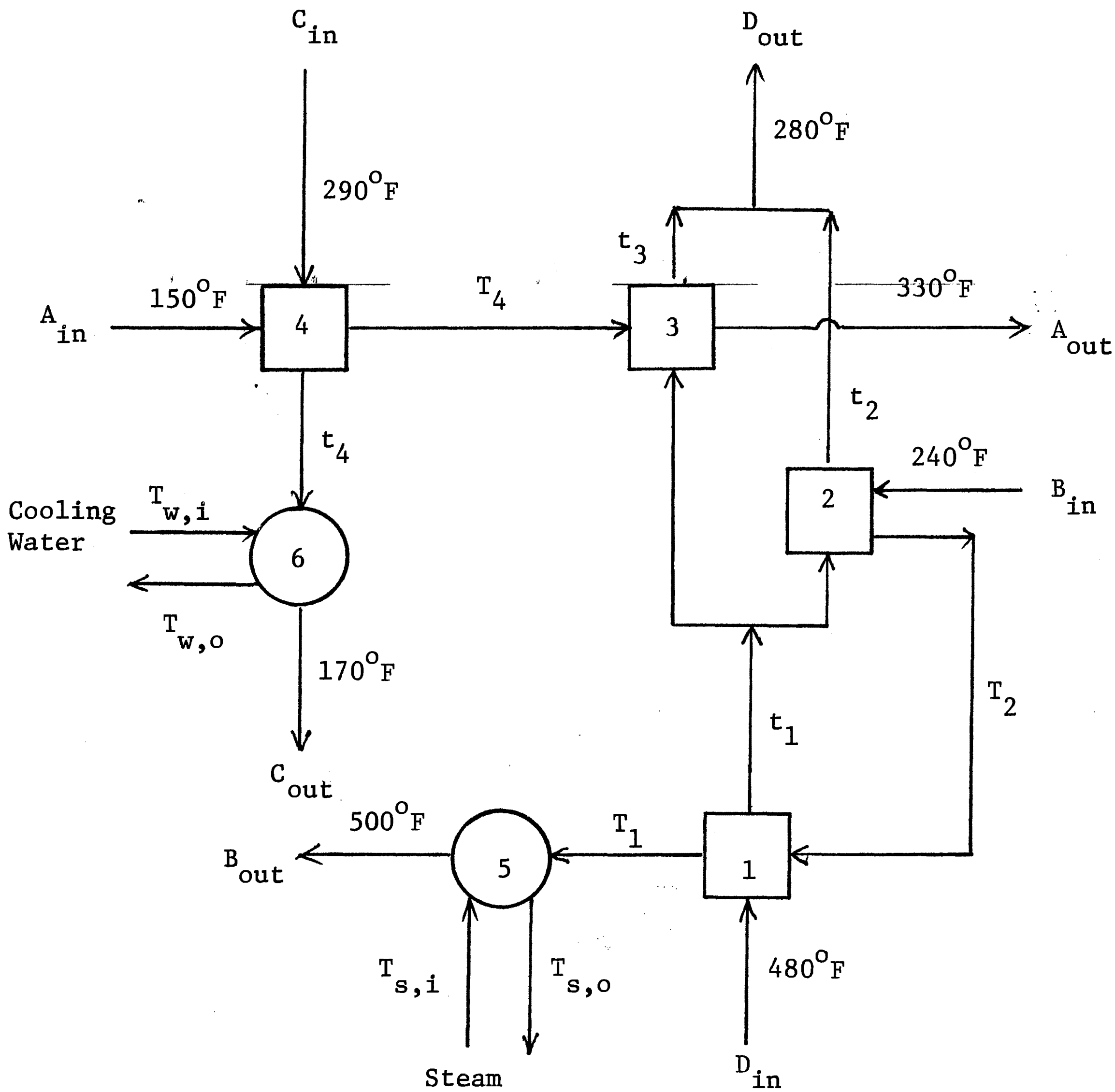


Figure 36. Heat Exchange System Configuration of Example Problem

Therefore, this problem reduces to a three variable, nonlinear optimization problem subject to constraints. This problem can be solved by the same procedure in Chapter V.

CHAPTER VII

OPTIMIZATION OF A HEAT RECOVERY SYSTEM IN A DISTILLATION PROCESS

Distillation is one of the most common processes in the chemical process industry. In this chapter, optimization of a heat recovery system for a distillation process is conducted by means of the proposed optimization procedure.

Problem Statement

Optimimum design of a heat recovery system is desired for a distillation column which is to split a butane feed into normal butane and iso butane (Figure 37). Properties of the feed stream and other available numerical data for the design are summarized in Table VII and VIII.

The feed stream is initially at 70°F and it is desired to be heated up to bubble point temperature before it goes into the distillation column. The temperatures of the distillate and the bottom products from the column are 180.15°F and 207.20°F respectively and are required to be cooled to 100°F. It is desired to recover heat from the product streams and use it to preheat the feed stream. The GPA program (8) is used for the bubble point calculation and the OSU PAS system (7) is used for material and energy balance calculation.

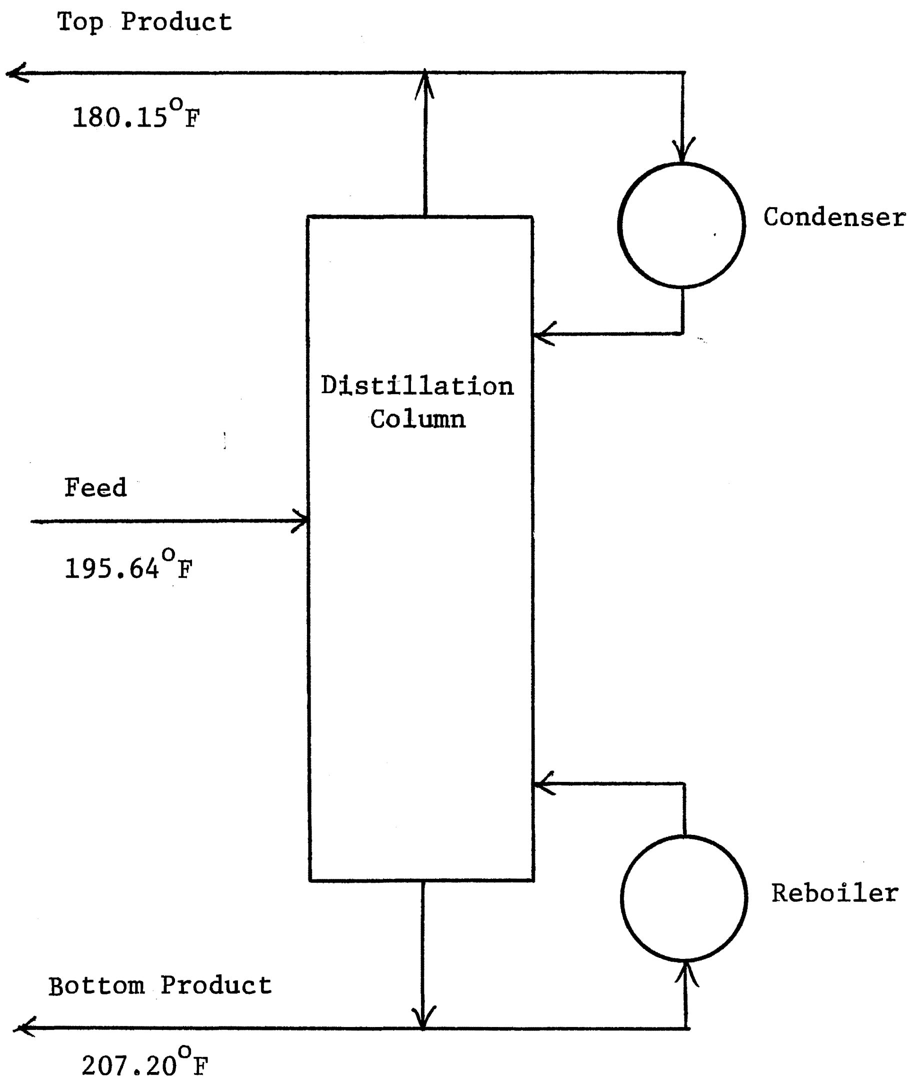


Figure 37. Distillation Column

TABLE VII

STREAM DATA 1

Component	Feed	Distillate Product	Bottom Product
Propane	60.00	60.00	0.00
I-Butane (lb moles/hr)	2260.00	2248.00	22.38
N-Butane	3530.00	35.30	3434.70
I-Pentane	90.00	0.00	90.00
N-Pentane	60.00	0.00	60.00
Total (lb moles/hr)	6000.00	2344.00	3667.08
Temperature, ^{o}F	195.64	180.15	207.20
Pressure, psia	200.00	200.00	200.00
Enthalpy, MM Btu/hr	19.5224	22.0279	13.5948
Fraction of Vapor	0.00	1.00	0.00

TABLE VIII

STREAM DATA 2

Stream	State	Temperature ^oF	Enthalpy Flow Rate MM Btu/hr	$\Delta\dot{H}$ MM Btu/hr	Heat Capacity Flow Rate MM Btu/hr^oF
Feed	Initial	70	-5.3670	24.8894	1.98
	Final	195.64	19.5224		
Top Product	Initial	180.15	22.0278	-16.2553	Infinity
	Bubble Point	175.22	5.7727	- 5.6408	0.75
	Final	100.00	0.1319		
Bottom Product	Initial	207.20	13.5948	-13.2255	1.23
	Final	100.00	0.3683		

Cooling water is available at 80^{o}F and 15 psig steam (250.33^{o}F) is chosen for heating. Film heat transfer coefficients and fouling factors of the streams are given in Table IX.

Specified economic conditions for this problem are given in Table X.

TABLE IX

HEAT TRANSFER COEFFICIENT AND FOULING FACTORS

Stream	Film Heat Transfer Coefficient, Btu/Hr Ft^2 ^oF	Fouling Factor Ft^2 Hr ^oF/Btu
Cooling Water	1000	0.001
Hydrocarbon (sensible heat transfer)	300	0.001
Condensing Steam	1500	0.005
Condensing Hydrocarbon	450	0.005

TABLE X

DATA FOR COST CALCULATION

Capital Cost of Heat Exchanger	$C = aA^b$	a = 350 x 1.35 b = 0.65
δ in Qu. (4-17)		0.2
Unit Cost of Cooling Water		6×10^{-6} \$/1b
Unit Cost of Steam		1.5×10^{-3} \$/1b
Annual Operating Days		350 days

Synthesis of System Configuration by Graphical Analysis

The temperature-heat capacity flow rate diagram can be drawn with the data in Table VIII, resulting in Figure 38.

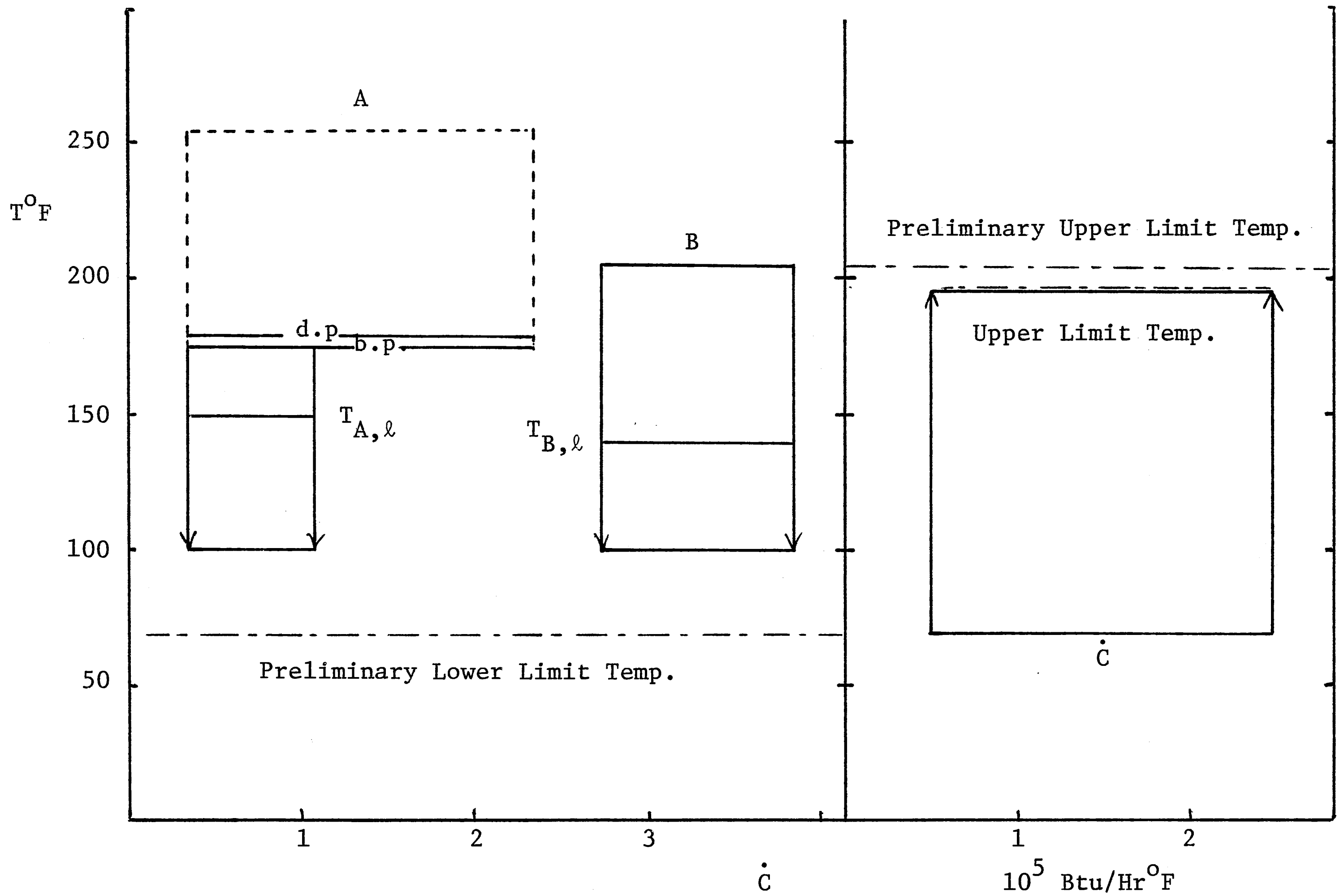


Figure 38. T-$\dot{C}$ Diagram of Heat Recovery System

Preliminary temperature limits are $70^{o}F$ and $207.20^{o}F$ and between these limits, $\sum_i \Delta H_{hot,i}$ is obviously larger than $\sum_j \Delta H_{cold,j}$. Therefore, lower limit of the temperature of hot streams has to be redetermined satisfying

$$\Sigma\Delta H_{hot} = \Sigma\Delta H_{cold}. \tag{7-1}$$

But in this problem, temperature contention exists at the lower end of the hot stream temperatures. If one of the two temperatures, $T_{s,\ell}$ and $T_{B,\ell}$ (Figure 39) is fixed, the other one will be determined. In this case, the lower limit temperature cannot be uniquely defined as in Chapter VI.

It appears possible in the diagram (Figure 38) to heat the cold stream up to the final specified temperature. But while matching the hot and cold streams by the procedure in Chapter VI one finds there is such a close temperature approach that one must accept the necessity of an additional heater for the final heating of the cold stream. Therefore, one can synthesize a possible optimum configuration of the heat exchange system of this problem as Figure 39 (Configuration 1).

For the configuration determined, one can draw the flow sheets of the synthesized system as Figure 40.

Formulation of Optimization Problem

From Figures 39 and 40, one can find that if T_4 becomes equal to $T_{T,f}$ exchangers No. 3 and No. 7 are not necessary as in Figures 41 and 43 (Configuration 2). If T_4 is equal to T_2 in Figures 39 and 40, this configuration becomes the one in Figures 42 and 44 (Configuration 3) in which exchanger No. 4 is not necessary.

The variables in Figure 39 or Figure 40 are the interconnecting stream temperatures, t_1, t_2, t_3, t_4, $\overline{t}_{3,4}$, $t_{w,6}$, $t_{w,7}$, T_1, T_3, and T_4,

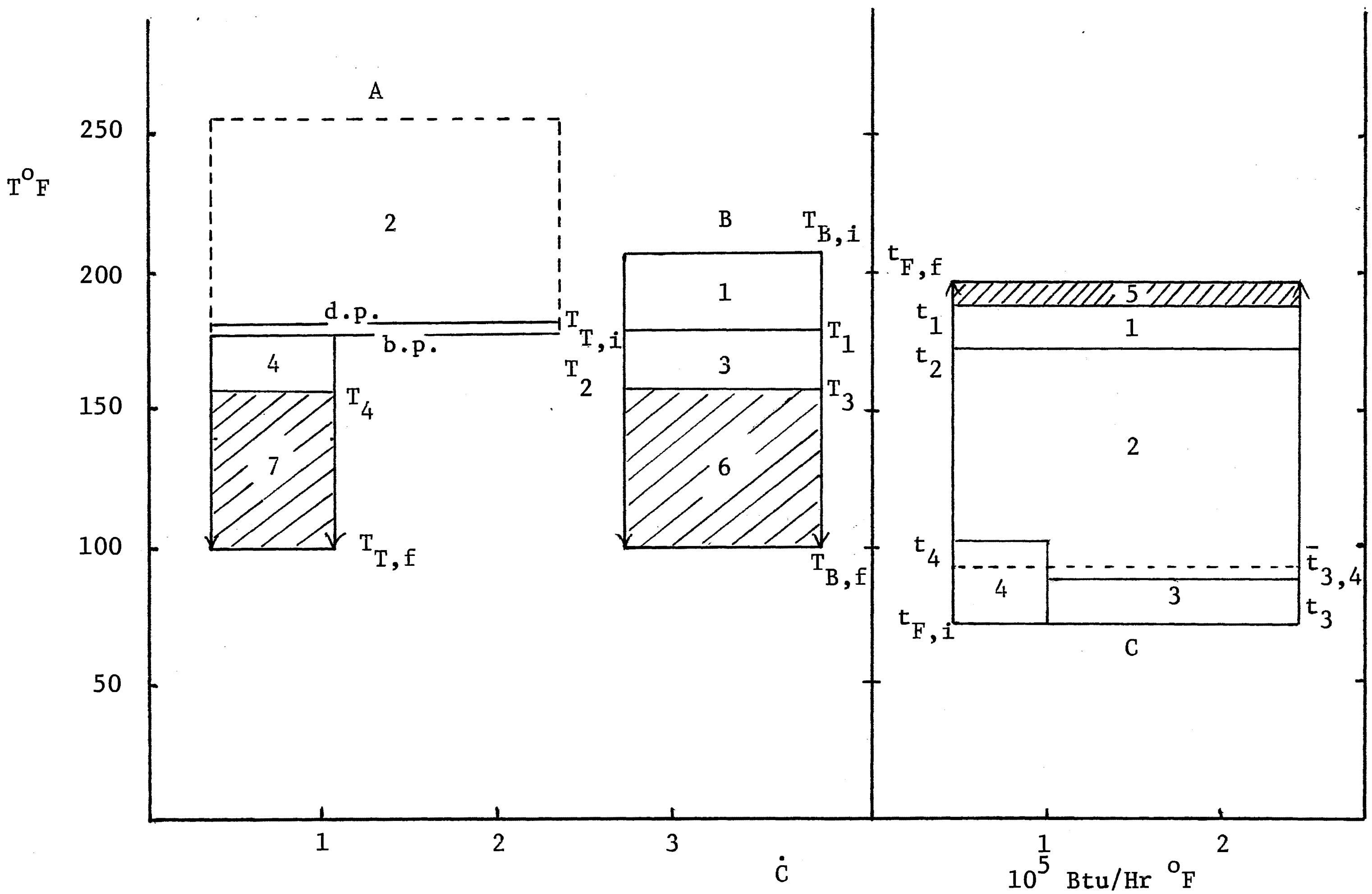


Figure 39. Synthesis of Heat Recovery System (Configuration 1)

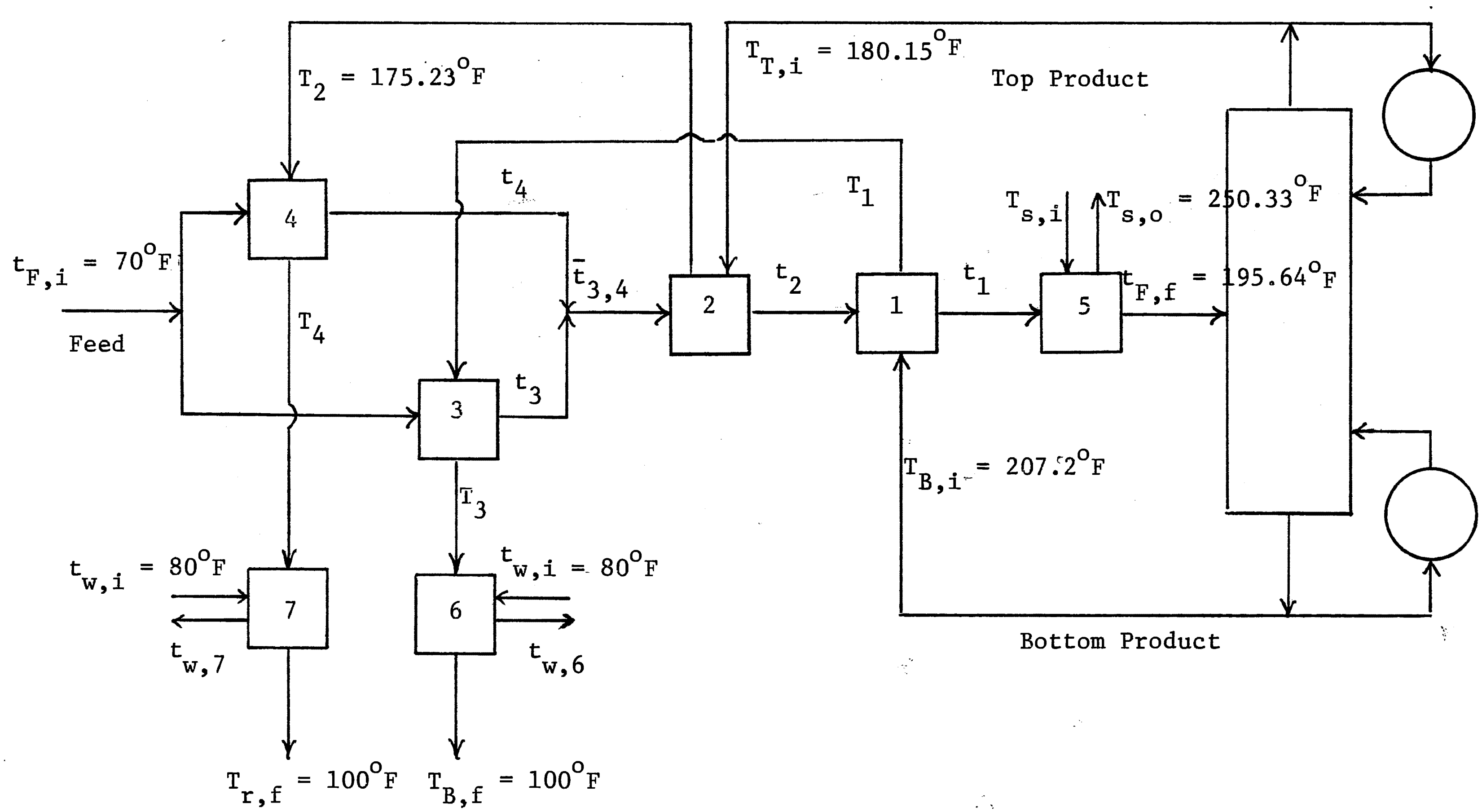


Figure 40. Flow Sheet of Synthesized Heat Recovery System (Configuration 1)

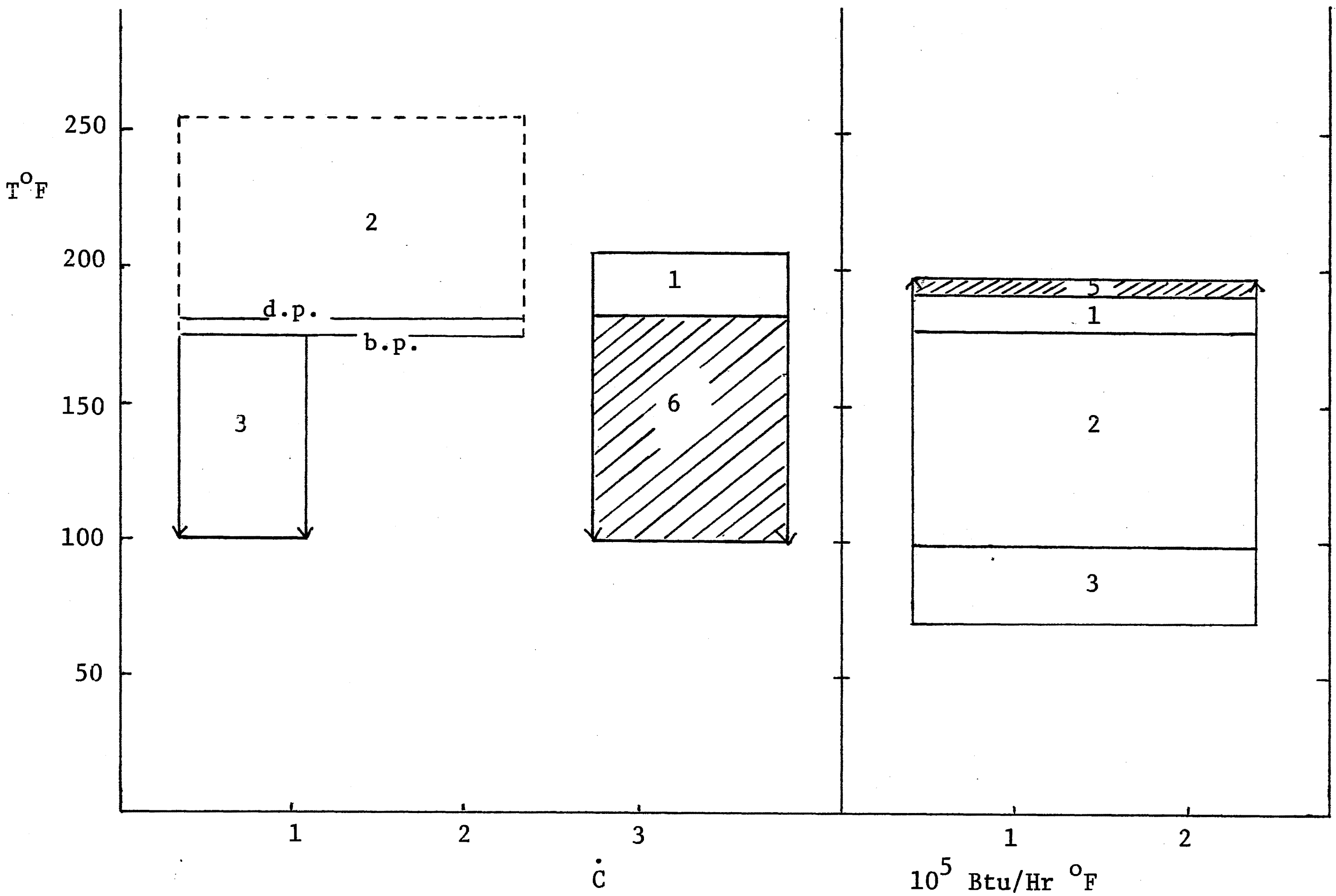


Figure 41. Synthesis of Heat Recovery System (Configuration 2)

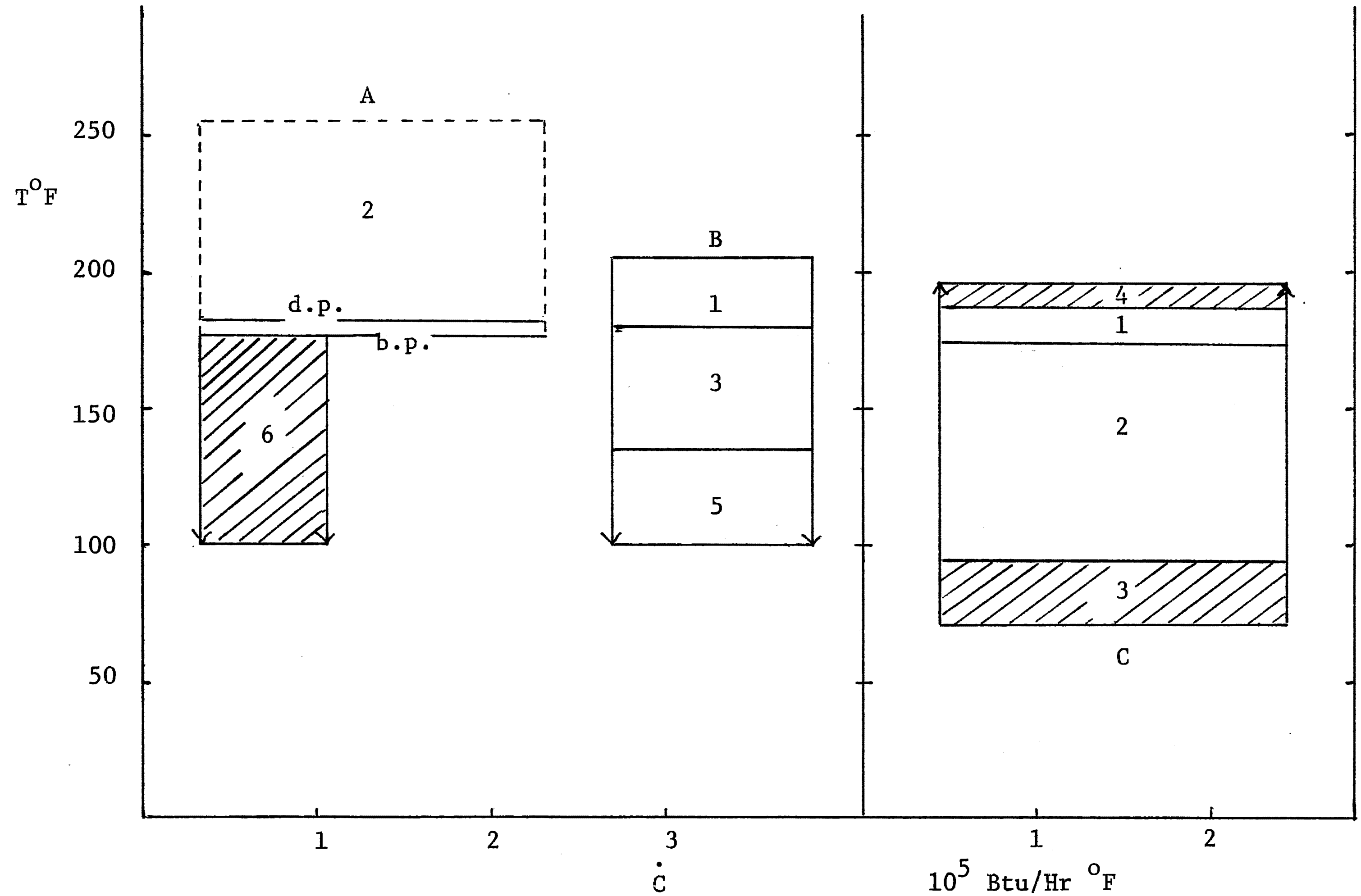


Figure 42. Synthesis of Heat Recovery System (Configuration 3).

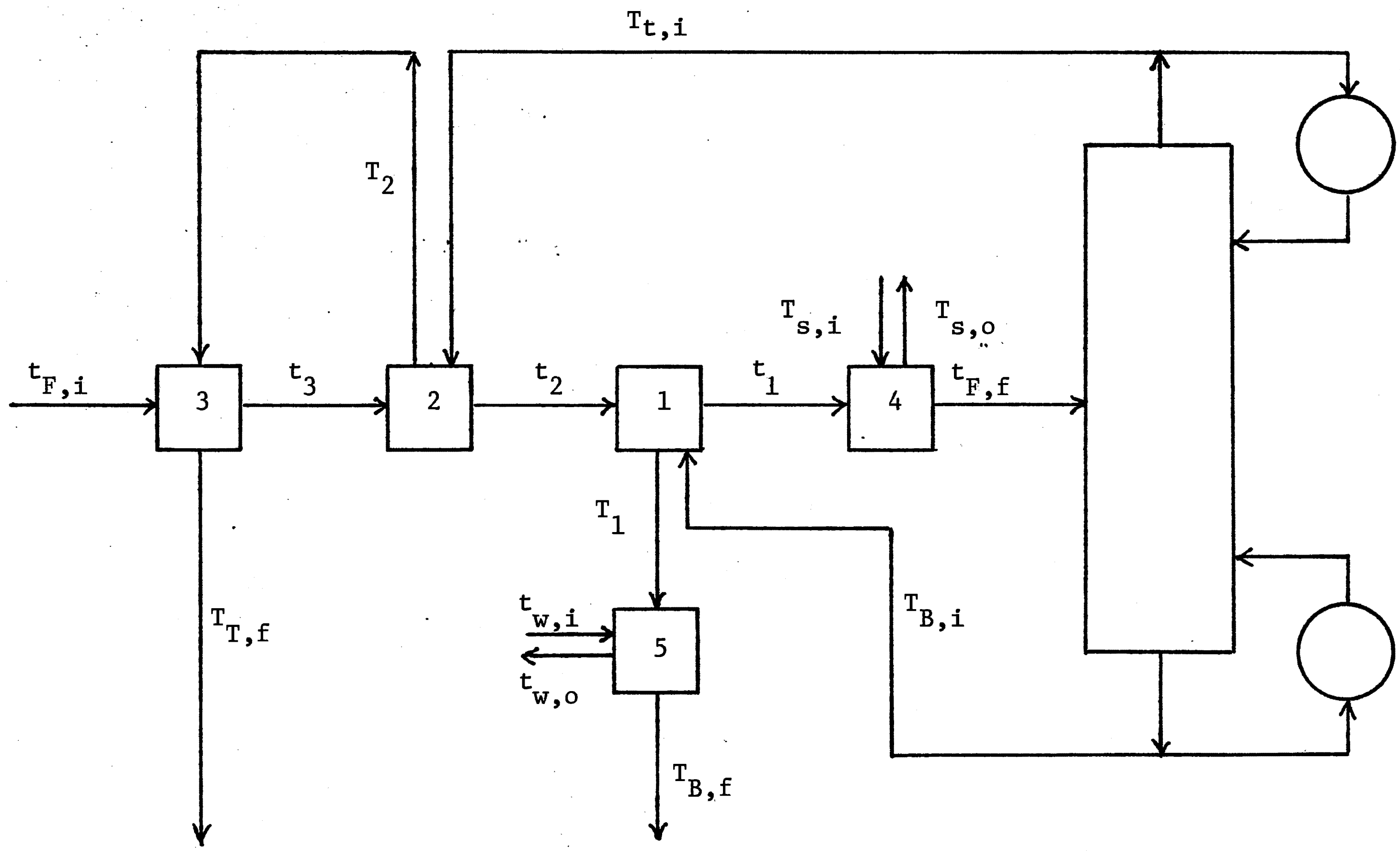


Figure 43. Flow Sheet of Heat Recovery System (Configuration 2)

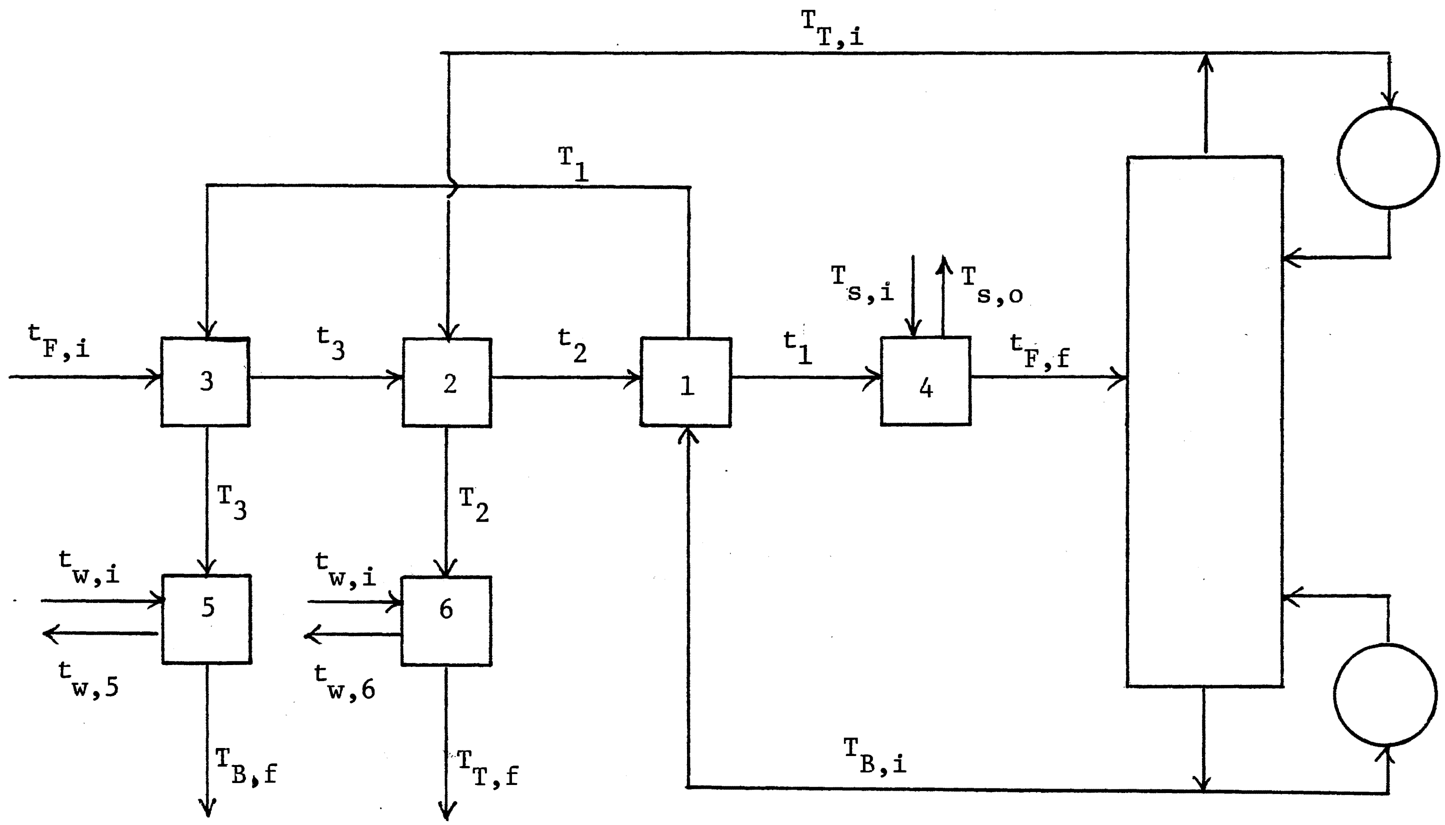


Figure 44. Flow Sheet of Heat Recovery System (Configuration 3)

split of feed stream, the amounts of water required in the coolers, and the amount of steam required in the heater. The total number of variables to be determined is fourteen.

There are eight energy balance equations. Two optimum outlet water temperature determination calculations can determine two unknown variables.

Therefore, this problem becomes the minimization of a four variable, nonlinear function subject to constraints. If one chooses t_1, $\bar{t}_{3,4}$, t_4, and T_4 as independent variables, one can design the optimum heat exchanger system.

For the designs of individual heat exchangers, data tables as Table XI can be helpful.

Considering the startup condition, when no or reduced hot stream flow is available, the steam heater has to be designed to have the capacity to heat the initial feed stream up to the final feed stream temperature. (But this need not be done at full flow rate.)

A. Energy Balance Equations

Overall energy balance equation can be expressed as follows:

$$\dot{C}_F \cdot (t_1 - T_{F,i}^{\,o}) = \dot{C}_B \cdot (T_{B,i}^{\,o} - T_3') + Q_C + \dot{C}_T \cdot (T_2^{\,o} - T_4) \qquad (7\text{-}2)$$

T_3' can be determined rearranging Equation (7-2)

$$T_3' = (\dot{C}_T \cdot (T_2^{\,o} - T_4) + Q_C - \dot{C}_F \cdot (t_1 - t_{F,i}^{\,o}))/\dot{C}_B + T_{B,i}^{\,o} \qquad (7\text{-}3)$$

For Exchanger No. 2 following energy balance equation can be obtained

$$Q_C = \dot{C}_F (t_2' - t_{3,4}) \qquad (7\text{-}6)$$

TABLE XI

DATA FOR INDIVIDUAL HEAT EXCHANGER DESIGN

Exchanger No.	1	2	3	4	5	6	7	(8)
Tube Side	Feed	Top Product	Feed	Feed	Steam	Water	Water	Water
Shell Side	Bottom Product	Feed	Bottom Product	Top Product	Feed	Bottom Product	Bottom Product	Top Product
TIT	t_2'	$\bar{t}_{3,4}$	$t^o_{F,i}$	$t^o_{F,i}$	T_s^o	$t^o_{w,i}$	$t^o_{w,i}$	T^o_s
TOT	t_1	t_2'	t_3'	t_4	T^o_s	$t''_{w,6}$	$t''_{w,7}$	T^o_s
SIT	$T^o_{B,i}$	$T^o_{T,i}$	T_1'	T^o_2	t_1	T_3'	T_4	$t^o_{F,i}$
SOT	T_1'	T^o_2	T_3'	T_4	$t^o_{F,f}$	$T^o_{B,f}$	$T^o_{T,f}$	$t^o_{F,f}$
h_i	300	300	300	300	1500	1000	1000	1500
h_o	300	450	300	300	300	300	300	300
$R_{f,i}$	0.001	0.001	0.001	0.001	0.0005	0.001	0.001	0.0005
$R_{f,o}$	0.001	0.0005	0.001	0.001	0.001	0.001	0.001	0.001

( ) means the design for the start up condition

"o" denotes given value by process specification

"'" denotes independent variables to be determined by energy balance

"''" denotes independent variables to be determined by optimum calculation by Fibonacci search

"no superscript" denotes chosen independent variable

t_2' can be determined rearranging Equation (7-4)

$$t_2' = Q_C/\dot{C}_F + \bar{t}_{3,4} \qquad (7\text{-}5)$$

From Exchanger No. 1

$$\dot{C}_F.(t_1 - t_2') = \dot{C}_B.(T_{B,i}{}^o - T_1') \qquad (7\text{-}6)$$

T_1' can be determined rearranging Equation (7-6)

$$T_1' = T_{B,i}{}^o - (t_1 - t_2')\ \dot{C}_F/\dot{C}_B \qquad (7\text{-}7)$$

From Exchanger No. 4

$$\dot{C}_{F,1}.(t_4 - t^o_{F,i}) = \dot{C}_T.(T_2^o - T_4) \qquad (7\text{-}8)$$

$C_{F,1}$ and $C_{F,2}$ can be determined as follows:

$$\dot{C}_{F,1} = \dot{C}_T.(T_2{}^o - T_4)/(t_4 - t_{F,i}{}^o) \qquad (7\text{-}9)$$

$$\dot{C}_{F,2} = \dot{C}_F - \dot{C}_{F,1} \qquad (7\text{-}10)$$

From Exchanger No. 3

$$\dot{C}_{F,2}.(t_3' - t^o_{F,i}) = \dot{C}_F.(\bar{t}_{3,4} - t^o_{F,i}) - \dot{C}_{F,1}.(t_4 - t^o_{F,i}) \qquad (7\text{-}11)$$

t_3' can be determined by rearranging Equation (7-11)

$$t_3' = (\dot{C}_F.(\bar{t}_{3,4} - t^o_{F,i}) - \dot{C}_{F,1}.(t_4 - t^o_{F,i}))/\dot{C}_{F,2} + t^o_{F,i} \qquad (7\text{-}12)$$

From Exchanger No. 5

$$\dot{C}_F.(t_{F,f}{}^o - T_1) = \dot{m}_s.\lambda \qquad (7\text{-}13)$$

Rearranging gives

$$\dot{m}_s = \dot{C}_F \cdot (t^o_{F,f} - t_1)/\lambda \tag{7-14}$$

From Exchanger No. 6

$$\dot{C}_3 \cdot (T'_3 - T^o_{B,f}) = \dot{C}'_{w,6} \cdot (t''_{w,6} - t^o_{w,i}) \tag{7-15}$$

Rearranging gives

$$t''_{w,6} = \dot{C}_B (T'_3 - T^o_{B,f})/\dot{C}'_{w,6} + t^o_{w,i} \tag{7-16}$$

From Exchanger No. 7

$$\dot{C}_7 \cdot (T_4 - T^o_{T,f}) = \dot{C}'_{w,7} \cdot (t''_{w,7} - t^o_{w,i}) \tag{7-17}$$

Rearranging gives

$$t''_{w,7} = \dot{C}_T \cdot (T_4 - T^o_{T,f})/\dot{C}'_{w,7} + t^o_{w,i} \tag{7-18}$$

B. Constraints

The constraints for a counter-current heat exchanger design is that the inlet temperature of hot stream should be higher than the outlet temperature of cold stream and the outlet temperature of hot stream should be higher than the inlet temperature of cold stream. The outlet temperature of a hot stream must be lower than its inlet temperature and the outlet temperature of the cold stream should be higher than the inlet temperature of the cold stream. All of the heat exchangers have to satisfy the above constraints.
For shell and tube heat exchangers, the above constraints can be expressed as follows:

1. When cold stream goes to shell side and hot stream to tube side (Figure 45)

$$TIT > SOT \text{ and } TOT > SIT \tag{7-19}$$

and

$$TIT > TOT \text{ and } SOT > SIT \tag{7-20}$$

2. When hot stream goes to shell side and cold stream to tube side (Figure 46)

$$SIT > TOT \text{ and } SOT > TIT \tag{7-21}$$

and

$$SIT > SOT \text{ and } TOT > TIT \tag{7-22}$$

C. Total Annual Cost

Total annual cost of the heat exchanger system, $(TACO)_s$ can be expressed as follows:

$$(TACO)_s = \delta \cdot (EXCO_1 + EXCO_2 + EXCO_3 + EXCO_4 + EXCO_8 + EXCO_6 + EXCO_7) + SCOST_5 + WCOST_6 + WCOST_7$$

The steam heater is to be designed to have the capacity to heat the feed stream to the bubble point for the startup condition.

The Computer Program

The computer program for the optimization of the heat recovery system in a distillation process using the modified simplex method (for the optimization of a given system configuration) and the Fibonacci search technique (for the optimum water outlet temperature calculation) is written in FORTRAN IV for use on the IBM 360 model 65

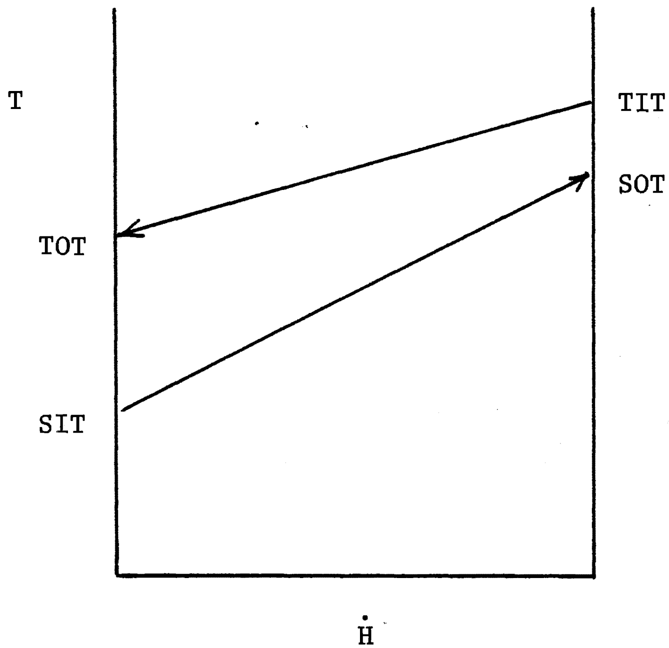


Figure 45. T-Ḣ Diagram for a Shell and Tube Exchanger (Cold Stream to Shell Side and Hot Stream to Tube Side)

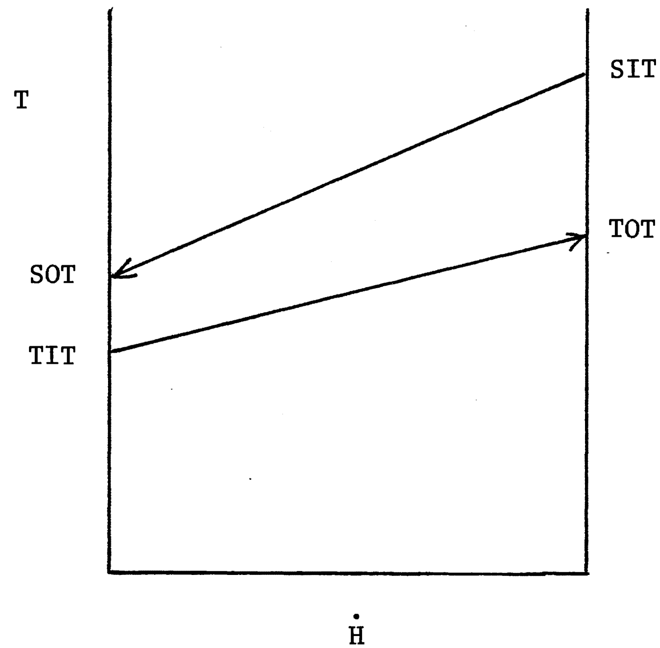


Figure 46. T-Ḣ Diagram for a Shell and Tube Exchanger (Cold Stream to Tube Side and Hot Stream to Shell Side)

digital computer. A description of the main program and the subroutines is presented in the following sections.

Main Program

The main program is the optimization calculation of the minimum total annual cost by using the modified simplex method. The subroutine CONST is called to check the feasibility of the starting simplex points, reflected point, and expanded point. The subroutine SYSTM is called to calculate the total annual cost of the heat exchange system. The basic optimization procedure is discussed in Chapter V.

Subroutine SYSTM

This subroutine supplies the main program with the objective function for optimization. Design of all the heat exchangers and all cost calculations are carried in the program. The subroutine FIBON is called to calculate the optimum water outlet temperatures for the water cooler designs. The subroutines HEXCH, COOLER, HEATER are called to design the heat exchangers, coolers, and heaters and to calculate the costs of exchangers, coolers, and heaters. The logic diagram of subroutine SYSTM is shown in Figure 47.

Subroutine CONST

This subroutine supplies the main program with feasibility tests by checking all the constraints. The logic diagram of subroutine CONST is shown in Figure 48.

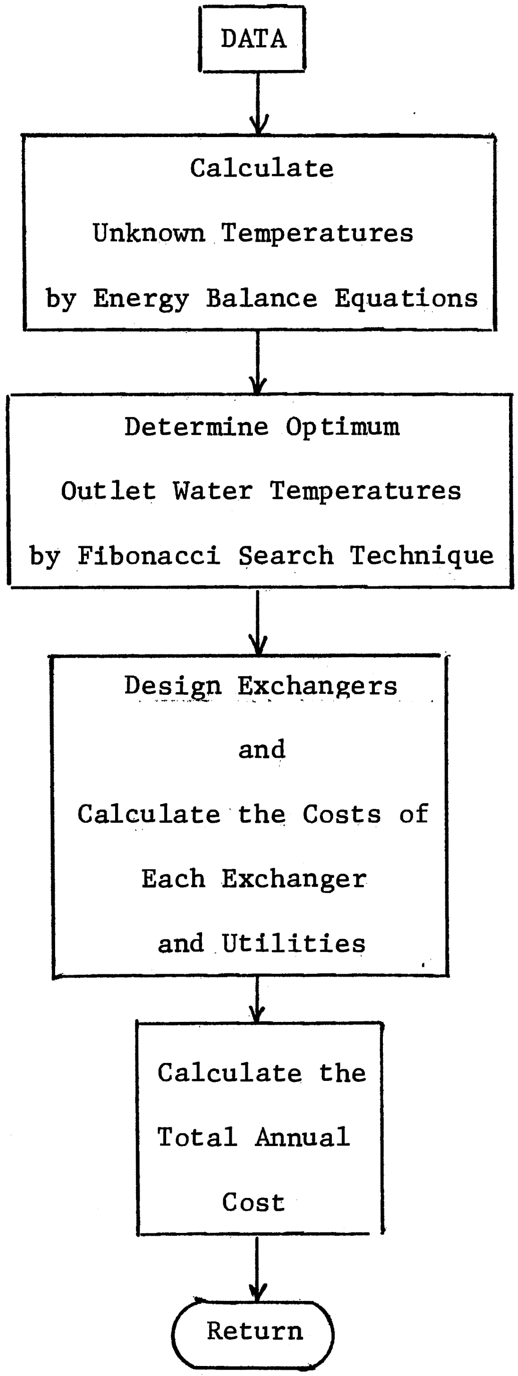


Figure 47. Logic Diagram of Subroutine SYSTM

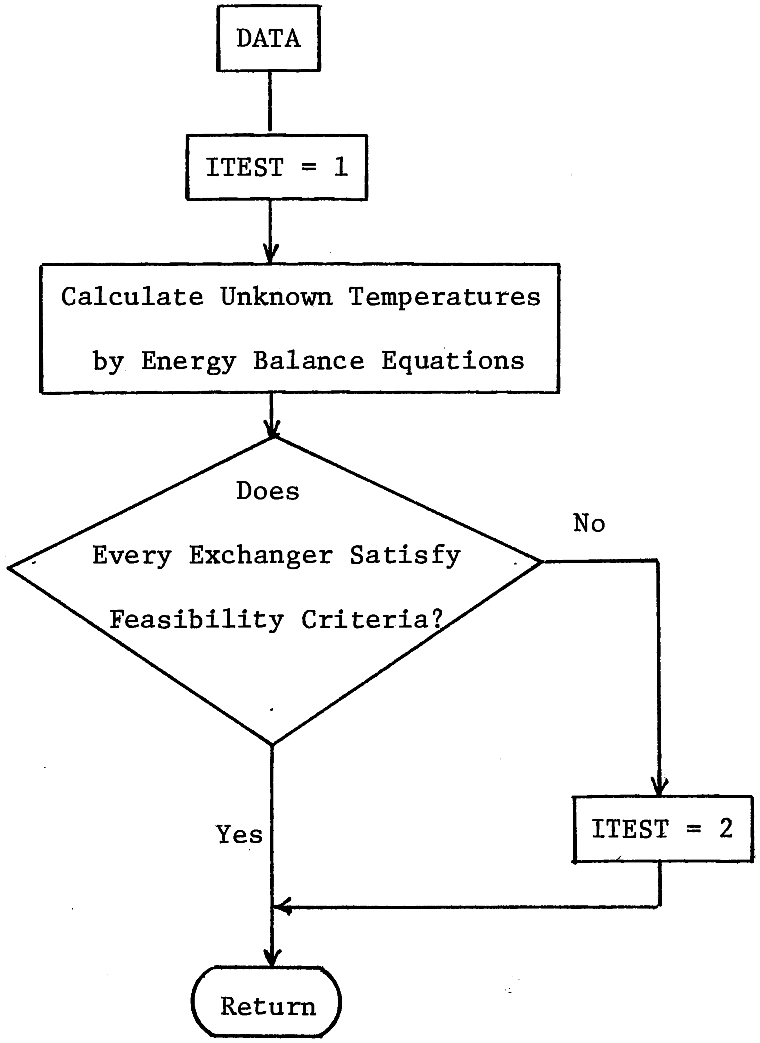


Figure 48. Logic Diagram of Subroutine CONST

Subroutine FIBON

This subroutine supplies the subroutine SYSTM with the calculation of optimum outlet water temperatures. The subroutine FUNC is called to calculate the total annual cost of water cooler. The logic diagram of the subroutine FIBON is given in Figure 49.

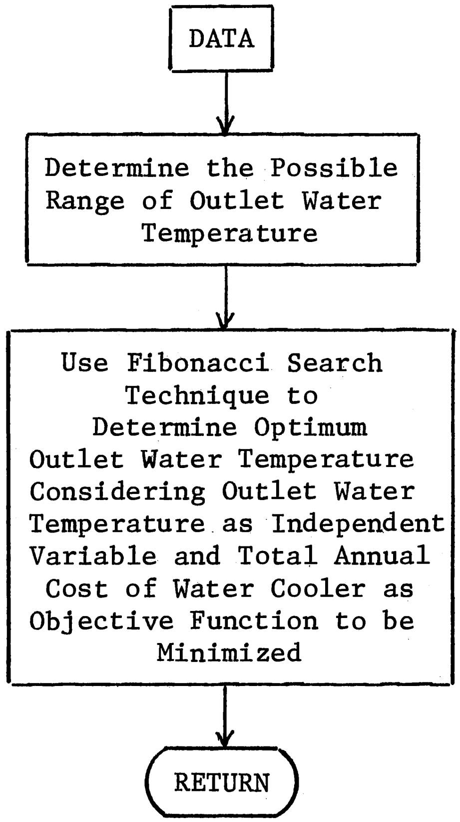


Figure 49. Logic Diagram of Subroutine FIBON

Subroutine FUNC

This subroutine supplies the subroutine FIBON with objective function by computing the annual cost of water cooler including the annual amortized capital cost and water cost. The subroutine COOLER is called to calculate the capital cost and water cost. The logic diagram of subroutine FUNC is given in Figure 50.

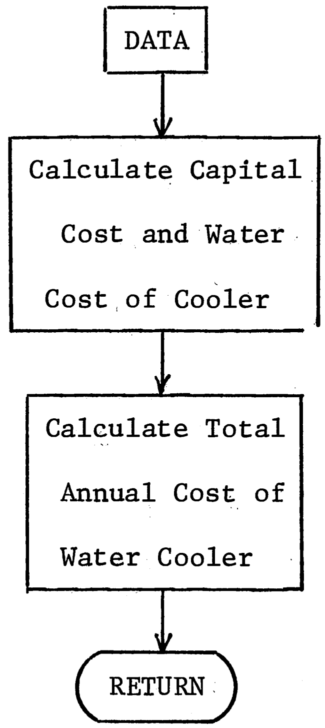


Figure 50. Logic Diagram of Subroutine FUNC

Subroutine HEXCH

This subroutine designs the heat exchangers and calculates the capital cost of the exchangers. The subroutine OHTC is called to calculate the overall heat transfer coefficient, the subroutine TMTD is called to calculate the log mean temperature difference, the subroutine FNSP is called to calculate the configuration correction factor F and the number of shells in series, and the subroutine EXCOS is called to calculate the capital cost of heat exchanger and the annual amortized capital cost. The logic diagram is shwon in Figure 51.

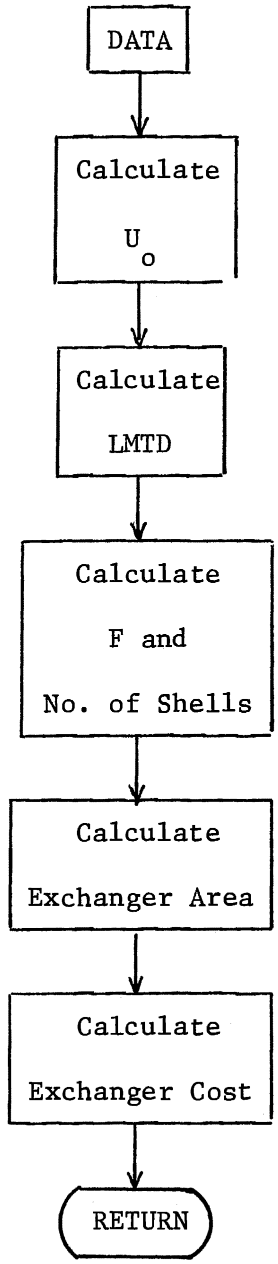


Figure 51. Logic Diagram of Subroutine HEXCH

Subroutine COOLER

This subroutine designs the cooler and calculates the capital cost of the cooler and the annual cooling water cost.

The subroutines OHTC, TMTD, FNSP, EXCOS are called as in the subroutine HEXCH and the subroutine WATER is called to calculate the annual cooling water cost.

The logic diagram is the same as that of the subroutine HEXCH except the additional calculation of annual cooling water cost.

Subroutine HEATER

This subroutine designs the steam heaters and calculates the captial cost of the heaters and annual heating steam cost.

The subroutines OHTC, TMTD, EXCOS, STEAM are called to calculate the overall heat transfer coefficient, LMTD, total and annual amortized capital cost of heat exchanger, and annual steam cost respectively.

The logic diagram is the same as that of the subroutine HEXCH except for the addition of the calculation of annual heating steam cost.

Subroutine TCNDS

This subroutine designs the total condensers without sub-cooling.

The subroutines OHTC, TMTD, EXCOS are called to calculate the overall heat transfer coefficient, LMTD, total and amortized capital cost of heat exchanger respectively.

The logic diagram is the same as that of the subroutine HEXCH.

Subroutine OHTC

This subroutine calculates the overall heat transfer coefficient for all kinds of heat exchangers. Equation (4-17) is used to calculate U_o.

Subroutine TMTD

This subroutine calculates the log mean temperature difference for the countercurrent case for all kinds of heat exchanger design subroutines. Equation (4-9) is used to calculate LMTD.

Subroutine FNSP

This subroutine supplies to the subroutines HEXCH and COOLER the number of shells in series and configuration correction factor for N shells and 2N or more tube passes. Equations 10, 11, 12, 13, 14 and 15 in Chapter IV are used. The logic diagram of the subroutine FNSP is shown in Figure 52.

Subroutine EXCOS

This subroutine supplies the total and the amortized annual capital costs of heat exchangers to all kind of exchanger subroutines.

$$(EXCO) = N \cdot a \left(\frac{A}{N}\right)^b$$

where

EXCO: total capital cost of an exchanger

N: No. of shells in series in an exchanger

$$(AREX) = \delta \cdot (EXCO)$$

where

AREX: amortized annual capital cost

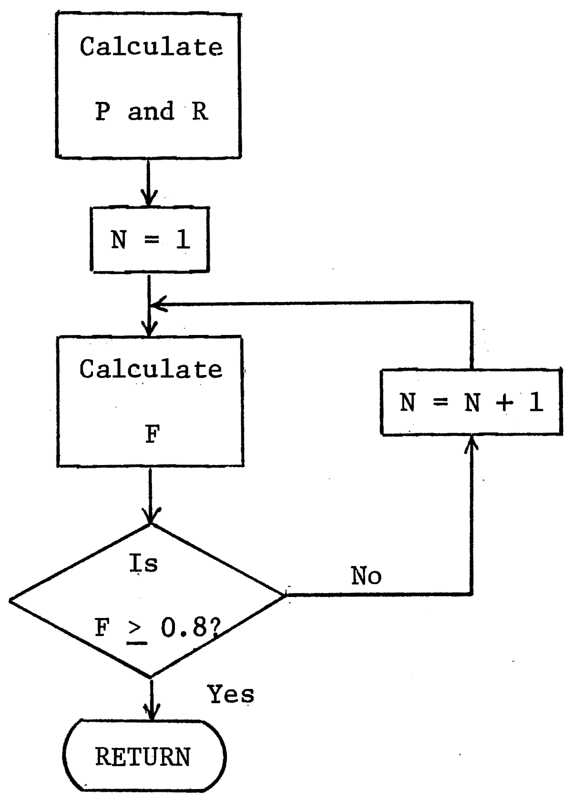


Figure 52. Logic Diagram of Subroutine FNSP

Subroutine WATER

This subroutine calculates the annual cost of cooling water:

$$(WCOST) = C_w \frac{Q}{C_{pw}(WOT-WIT)} \cdot (24 \cdot OPDY)$$

where

(WCOST) : annual water cost (\$/yr)

C_w : unit cost of water (\$/lb)

WOT, WIT: water outlet and (^oF)

inlet temperatures

Q : heat duty of cooler (Btu/hr)

$C_{p,w}$: heat capacity of water (Btu/lb ^oF)

OPDY : annual operating days (days/yr)

Subroutine STEAM

This subroutine claculates the annual cost of heating steam:

$$(SCOST) = C_s \frac{Q}{(HTLA)} \cdot (24.(OPDY))$$

where

SCOST: annual steam cost (\$/yr)

C_s : unit cost of steam (\$/lb)

Q : heat duty of heater (Btu/hr)

HTLA : latent heat of steam (Btu/lb)

OPDY : annual operating days (days/yr)

Results and Discussion

Optimum design of the heat recovery system in Table XII is obtained by the computer program for the system configuration I in Figures 39 and 40. From Table XII, one can see that exchanger No. 3 and No. 7 are too small to include. This means that the exchangers No. 3 and No. 7 are negligible and the optimum heat exchange system configuration is configuration 2 in Figure 41. Therefore, stream splitting is unnecessary in this problem. The reason is that temperature contention exists over the lower end zone of the hot stream temperatures and does not affect the total amount of heat recoverable. Another reason is that the heat exchanger cost is a function of the number as well as the area of the exchangers. In configuration 2 the number of exchangers needed is five and the amount of heat recovered is nearly same as that of configuration 1 in which two more exchangers are used. This can be observed in the graphical analysis.

TABLE XII

OPTIMUM DESIGN OF CONFIGURATION 1

Exchange No.	1	2	3	4	5(OP)	6	7	8(ST)
Tube Side	Feed	Feed	Feed	Feed	Heating Steam	Cooling Water	Cooling Water	Heating Steam
Shell Side	Bottom Product	Condensing Top Product	Bottom Product	Top Product	Feed	Bottom Product	Top Product	Feed
TIT (^{o}F)	176.7	98.57	70.00	70.00	250.33	80.00	80.00	250.00
TOT (^{o}F)	188.3	176.7	70.69	102.3	250.33	120.00	86.07	250.33
SIT (^{o}F)	207.2	180.1	188.4	175.2	188.3	188.3	100.0	70.0
SOT (^{o}F)	188.4	175.2	188.3	100.0	195.6	100.0	100.0	195.6
Q(MMBtu/hr)	2.309	15.46	0.0159	5.641	1.451	10.86	0.00077	24.88
U(Btu/hr^{o}Fft^{2})	95.81	113.3	95.81	95.81	157.0	134.4	134.4	157.0
LMTD(^{o}F)	15.05	23.70	118	48.35	58.28	39.33	16.79	105.3
F	0.8075	1.00	1	0.9546	1.00	0.8939	1.0	1.00
NSP	1	1	1	2	1	2	1	1
A (ft^{2})	1982	5759	1.407	1276	158.6	2299	0.343	1505
EXCO ($)	6.57E4	13.14E4	0.059E4	6.288E4	12.7E4	9.22E4	0.0236E4	5.49E4
AREX ($/Yr)	1.314E4	2.628E4	0.0118E4	1.258E4	0.255E4	1.844E4	0.0047E4	1.099E4
UTILITY ($/Yr)	--	--	--		1.934E4	1.368E4	0.006E4	33.16E4

Annual Amortized Exchanger Cost	$81,590
Steam Cost	$19,340
Water Cost	$13,690
Total Annual Cost	$114,600

CHAPTER VIII

CONCLUSIONS

In this work, optimization of heat exchange systems is studied as a three stage optimization procedure. First, the procedure for designing an optimum water cooler is discussed, applying Fibonacci search technique. Second, optimization of a heat exchange system for a fixed system configuration is discussed applying a modified Nelder and Mead algorithm. Third, optimal synthesis of heat exchange system is studied by graphical analysis of temperature-enthalpy flow rate diagrams and temperature-heat capacity flow rate diagrams.

Thus any heat exchanger system can be synthesized and designed optimally with the proposed optimization procedures in this work. For more sophisticated design of individual exchangers, the results from this optimization procedure can be used to determine the exchanger details. If the thermodynamic property calculation package is available to use as a subroutine to calculate the exact values of enthalpy flow rate and heat capacity flow rate, the final result will be more accurate.

The modified simplex method introduced here needs to be further refined and can be compared with the Box Complex method from the calculation efficiency point of view.

The graphical visualization concept used here can be applied to many other energy recovery problems to maximize the amount of energy to be recovered.

The optimization procedure adapted here requires fewer assumptions and simplifications than the other ones in open literature as far as this author knows.

SELECTED BIBLIOGRAPHY

1. Bell, K. J., "Process Heat Transfer," Class Notes, Oklahoma State University, Spring, 1974.

2. Bowman, R. A., A. C. Mueller and W. M. Nagle, "Mean Temperature Difference in Design," Trans. Am. Soc. Mech. Engrs., 62, 283 (1940).

3. Box, M. J., "A New Method of Constrained Optimization and a Comparison with Other Methods," The Computer Journal, 8, 42-52 (1965).

4. Bragin, M. S., "Optimization of Multistage Heat Exchanger Systems," Doctoral Dissertation, University Microfilm Inc. 1966.

5. Brooke, M., "Process Plant Utilities - Water," Chem. Eng., 77, No. 12, 135-138 (1970).

6. Cichelli, M. T. and M. S. Brinn, "How to Design the Optimum Heat Exchanger," Chem. Eng., 63, No. 5, 196-200 (1956).

7. Erbar, J. H., Process Analysis System, Oklahoma State University, 1974.

8. Erbar, J. H., GPA K & H Computer Program Mod II, Gas Processors Association, 1974.

9. Fan, L. T., C. L. Hwang and C. S. Wang, "Optimization of Multistage Heat Exchanger System by the Discrete Maximum Principle," Chem. Eng. Progr. Symposium Series, 61, No. 59, 243-252 (1965).

10. Frith, J. F., B. M. Berger and M. M. Shreehan, "Optimize Heat Train Design," Hydrocarbon Processing, No. 7, 89-91 (1973).

11. Guthrie, K. M., "Capital Cost Estimating," Chem. Engr., 76, Vol. 3, 114-142 (1969).

12. Happel, J., Chemical Process Economics, John Wiley & Sons, Inc., New York, 1958.

13. Hohman, E. C., "Optimum Networks for Heat Exchange," Doctoral Dissertation, University Microfilm Inc., 1971.

14. Hwa, C. S., "Mathematical Formulation and Optimization of Heat Exchanger Networks Using Separable Programming," A.I.Ch.E. - I. Chem. E. Symposium Series, No. 4, 101-106, (1965).

15. Jacoby, S. L. S., J. S. Kowalik and J. T. Pizzo, Iterative Methods for Nonlinear Optimization Problems, Prentice Hall, 1972.

16. Jenssen, S. K., "Heat Exchanger Optimization," Chem. Eng. Prog., 65, No. 7, 59-66 (1969).

17. Kern, D. A., Process Heat Transfer, McGraw-Hill, New York, 1950.

18. Kesler, M. G. and P. O. Parker, "Optimal Networks of Heat Exchange," Chem. Eng. Progr. Symposium Series, 65, No. 92, 111-120 (1969)

19. Kobayashi, S., T. Umeda and A. Ichikawa, "Synthesis of Optimal Heat Exchange Systems - An Approach by the Optimal Assignment Problem in Linear Programming," Chem. Eng. Sci., 26, 1367-1380 (1971).

20. Kuester, J. L. and J. H. Mize, Optimization Techniques with Fortran, McGraw-Hill, 1973.

21. Lee, K. F., A. H. Masso and D. F. Rudd, "Branch and Bound Synthesis of Integrated Process Designs," I.&E.C. Fundamentals, 9, No. 1, 48-58 (1970.

22. Masso, A. H. and D. F. Rudd, "The Synthesis of System Designs: II Heuristic Structuring," A.I.Ch.E. Journal, 15, No. 1, 10-17 (1969).

23. McAdams, W. H., Heat Transmission, 2nd ed., McGraw-Hill, New York, 1942.

24. Monroe, L. R., "Process Plant Utilities - Steam," Chem. Eng., 77, No. 12, 130-134 (1970).

25. Nelder, J. A. and R. Mead, "A Simplex Method for Function Minimization," The Computer Journal, Vol. 7, 308-313 (1965).

26. Nishida, N. S., S. Kobayashi and A. Ichikawa, "Optimal Synthesis of Heat Exchange Systems - Necessary Conditions for Minimum Heat Transfer Area and their Spplication to System Synthesis," Chem. Eng. Sci., 27, 1841-1855, (1971).

27. Palen, J. W., T. P. Cham and J. Taborek, "Optimization of Shell and Tube Heat Exchangers by Case Study Method," A.I.Ch.E.-A.S.M.E. Fourteenth National Heat Transfer Conference, August 5-8, 1973.

28. Peters, M. S. and K. D. Timmerhaus, Plant Design and Economics for Chemical Engineers, McGraw-Hill, New York, 1968.

29. Pho, T. K. and L. Lapidus, "Topics in Computer-Aided Design: Park II. Synthesis of Optimal Heat Exchanger Networks by Tree Searching Algorithms," A.I.Ch.E. Journal, 19, No. 6, 1182-1189 (1973).

30. Rudd, D. F., "The Synthesis of System Designs: I. Elementary Decomposition Theory," A.I.Ch.E. Journal, 14, No. 2, 343-349 (1968).

31. Rudd, D. F. and C. C. Watson, Strategy of Process Engineering, John Wiley & Sons, Inc., New York, 1968.

32. Spendley, W., G. R. Hext and F. R. Himsworth, "Sequential Application of Simplex Designs in Optimization and Evolutionary Operation," Technometrics, Vol. 4, 441 (1962).

33. Taborek, J. J., "Optimization in Process Equipment Design," Chem. Eng. Pro., 56, No. 2, August 1960, p37.

34. Taborek, J. J., "Organization of Heat Exchanger Programs on Digital Computers," Chem. Eng. Progr. Symposium Series, 56, No. 30, 207-218 (1960).

35. Tarrer, A. R., H. C. Lim and L. B. Koppel, "Finding the Economically Optimum Heat Exchanger," Chem. Eng., 78, No. 10, 79-84 (1971).

36. TenBroeck, H., "Economic Selection of Exchanger Sizes," Ind. Eng. Chem., 36, 64 (1944).

37. Westbrook, G. T., "Use This Method to Size Each Stage for Best Operation," Hydrocarbon Processing & Petroleum Refiner, 40, No. 9, 201-206 (1961).

38. Whistler, A. M., "Heat Exchangers as Money Makers," Petroleum Refiner, 27, No. 1, 83-86 (1948).

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