COMPUTER METHODS FOR PLAYING QUBIC: AN ANALYSIS AND STRATEGY IMPLEMENTATION

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COMPUTER METHODS FOR PLAYING QUBIC: AN
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PREFACE

This thesis presents an exhaustive approach to solving a game of QUBIC in the sense of devising a perfect strategy. Various mathematical mappings of the QUBIC game playing board onto itself are described. An implementation of a game playing strategy using the results of analysis is also described.

Further mathematical analysis is required to prove or disprove the conjecture of being able to play according to a perfect strategy.

I would like to thank my major adviser, Dr. James VanDoren for his probing questions and useful suggestions concerning the content of this thesis.

Thanks are due to Dr. Donald Grace and Dr. George Hedrick as committee members who took time away from their regular activities to provide guidance and assistance in the writing of this thesis.

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CHAPTER I

INTRODUCTION

Description of Thesis

This thesis is a description of the author's approach to solving the game of QUBIC (three-dimensional tic-tac-toe) on a computing machine. The approach developed is shown to be an effective tool for solving a given class of situations. This class is a significant subset of the total number of solutions of QUBIC. Included are the definitions related to QUBIC, the development and proofs of various schemes whereby the total number of situations to be considered is reduced, and the implementation of such schemes.

The purpose of this thesis is twofold: 1) to develop and implement a strategy for QUBIC based on four-by-four planes, and 2) to show that, assuming that the machine moves first, this strategy enables the machine to detect winning combinations immediately after the opponent's third move.

The main areas of concentration have been the reduction of "unique situations" and the development of strategies focusing on a plane subset of the cube. The number of unique situations that need be considered is reduced using all known reduction techniques. The strategy developed partially solves the problem of detecting winning combinations, and empirical evidence suggests that exhaustive search of a tree of forces can be used to solve the remaining situations. In conclusion, the author believes that it is reasonable to assume that the game of QUBIC is indeed a forced win for the first player. This conjecture, however, has not been proved or disproved.

Description of Appendixes

Appendix A lists the 48 rotations and reflections of the cube. Appendix B lists all (to the author's knowledge) of the unique situations given X has a marker in cell 1 and a marker in cell 22 and O has two markers anywhere else in the cube. Appendix C gives a listing of all logical games that may result given the situation 1-2,22-4. Appendix C was used to prove that, for this situation, the game is a win for X. Appendix D describes, in some detail, the programs written by the author to find the number of unique combinations with three markers in one plane and to list these situations. Appendix E describes in some detail the programs written by the author to find how many unique combinations there are with four markers in the cube and X having markers in cell 1 and 22.

CHAPTER II

HISTORY OF THE PROBLEM AND SURVEY OF VARIOUS STRATEGIES

Since the advent of computing machines, considerable work has been done in the area of game playing. Computers have been programmed to play chess, checkers, halma, life, nim and numerous other games (1) (6) (7).

Since Parker Brothers released their version of three-dimensional tic-tac-toe, entitled "QUBIC," numerous 3-D tic-tac-toe programs have been written by various people in their quest for a "perfect" strategy. Louden (4) produced a program for QUBIC that plays three moves ahead by examining the cells located at the intersections of rows with only one or two markers of the same color. Louden's program can beat most beginners, but fails to beat more experienced players. A program written by Daly (1) exhibits various levels of playing depth, depending on the declared skill of the opponent. This program uses exhaustive search of known forces (i.e., forcing moves) and is restricted to playing a maximum of 10 moves ahead.

There are two types of look-ahead discussed in this paper: 1) pattern recognition as employed by Louden (4) and 2) exhaustive search of the force tree as used by Daly. The author defines look-ahead as any method which allows a player to predict a certain win whether it uses pattern recognition, exhaustive search, or some other method. Gammil (3) devised a scheme for dynamically evaluating each cell (in terms of resources) and incorporated this idea into a qubic playing program which he claims has beaten many good players. These programs, unfortunately, cannot guarantee a win and, in fact, sometimes lose.

Although QUBIC is simple to play (the object is to get four markers in a row, on any of 76 rows), casual observation leads one to believe there is an upper limit of 64! possible sequences of moves that lead to a conclusion of the games because there are 4x4x4 = 64 playing positions on the board. Usually a conclusion is reached in fewer steps; however, extensive reduction is necessary in order to perform machine analysis in a reasonable period of time.

Methods which do not provide for this reduction have a major weakness in terms of computing time. The author's approach relies on an "a priori" analysis of QUBIC in order to eliminate a significant number of possible situations that might occur during the course of a game.

The author conjectures that one could analyze a subset of the possible opening situations (i.e. with 4 markers on the board) exhaustively and by so doing show that, given enough time and resources, QUBIC is indeed a win for the first player. It was with this idea in mind that the analysis and implementation work described in this thesis was undertaken.

CHAPTER III

ANALYSIS

Definitions

Three-dimensional tic-tac-toe is one of a class of games referred to by Citrenbaum (2) as N^k tic-tac-toe, and is the simplest in this class which remains unsolved. (3) (N^k refers to a board with each side of length N and with k dimensions.) Letting N = 3 and k = 2 results in the 3 by 3 tic-tac-toe familiar to most school children. Letting N = 4 and k = 3 results in the game of QUBIC, to which this entire paper is devoted.

The QUBIC "board" is a three-dimensional cube that measures four-by-four-by-four, with a total of 64 squares, or cells. Figure 1 shows such a cube. With the risk of losing perspective, the board is

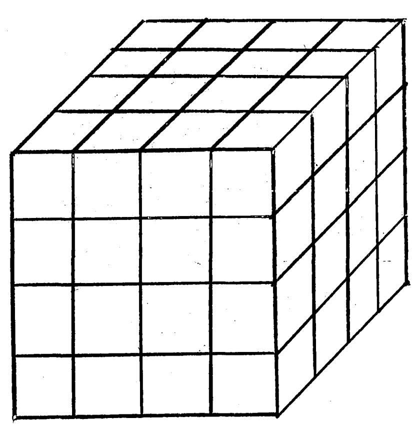

Figure 1. A Three-Dimensional Cube

then peeled apart, level by level, and each level is placed side by side as shown in Figure 2. Each level is then "straightened up" and

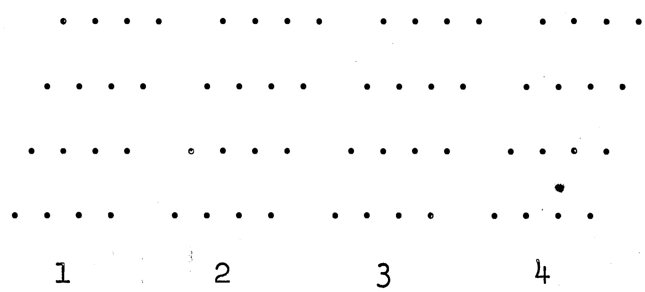

Figure 2. The Four Three-Dimensional Tic-Tac-Toe Levels

the edges are removed. Figure 3 shows the board with four cells containing markers. Each level (and also each plane, as will be shown

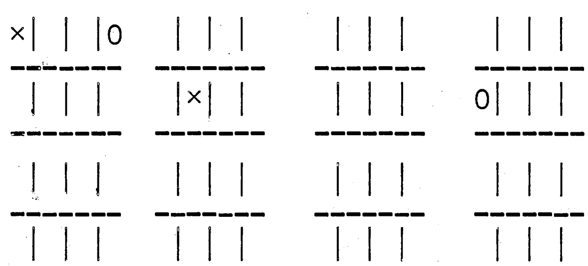

Figure 3. Usual Representation of QUBIC Board

later) can be represented using the scheme shown in Figure 3. The Figure below displays one plane with three markers. For ease in representation, the lines are removed and empty cells are replaced by periods. The situation shown in Figure 4 can be represented by the notation shown in Figure 5. The notation of Figure 5 will be used throughout this thesis.

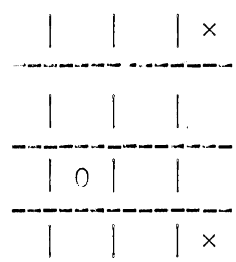

Figure 4. One Level of Three-Dimensional Tic-Tac-Toe

```
. . . X
. . . .
. O . .
. . . .
```

Figure 5. One Level of QUBIC Without the Lines

Each cell within the cube can be referenced by one of two methods. Method I: each cell is assigned a number from one to 64 (see Figure 6). When using this notation, the moves are strung out, beadlike, with

X's and O's moves alternating. X's and O's refer to opposing players' moves. The above representation may be extended to allow for alternative moves by O by placing dashes in front of all the alternatives. The situation 1-2,22-3-4-5 lists, for example, three alternative moves for O for his second move. (The author's representation sometimes differs as in Appendix C.) The situation shown in Figure 3 can be represented by the following sequence of pairs of numbers: 1-4,22-53. Each pair in the sequence represents the move made by each player at the stage of the game.

1	2	3	4	17	18	19	20	33	34	35	36	49	50	51	52
5	6	7	8	21	22	23	24	37	38	39	40	53	54	55	56
9	10	11	12	25	26	27	28	41	42	43	44	57	58	59	60
13	14	15	16	29	30	31	32	45	46	47	48	61	62	63	64

Figure 6. Representing Cells with One and Two Digit Numbers

Method II: each cell is represented by its coordinates within the cube. The order of the coordinates is always the same, i.e., the level followed by the row followed by the column. The levels are numbered as shown in Figure 2. The rows (within a level) are numbered from top to bottom (some authors number from bottom to top), and the columns (within a row) are numbered from left to right. Using this notation cell number 53 would be referred to by 421, indicating the fourth level, the second row, and the first column.

For reasons of programming convenience, Method I is used more frequently.

A plane consists of a four by four grid of 16 cells. The qubic board contains 18 such planes. Since the concept of the plane is used throughout, the 18 planes are listed in Table I and are described in the following paragraph.

There are four horizontal planes parallel to the base of the cube. Four vertical planes parallel to any vertical face four vertical planes perpendicular to the same face, and six diagonal planes, each containing two parallel and diagonally opposite edges of the cube.

A row contains four distinct points within the cube that lie in a straight line. For an N^k board, the number of rows is given by:

$$R = ((N+2) \times k - N \times k)/2. \qquad (1$$

There are 76 such rows in qubic. Different cells have different numbers of rows passing through them. Cells with the most number of rows through them are, intuitively, more "powerful" (3), and will be referred to as prime cells (4). There are 16 prime cells located at cell numbers 1,4,13,16,22,23,26,27,38,39,42,43,49,52,61, and 64. They make up the four corner cells and the four center cells of the board marked with a P in Figure 7. (Silver (5) refers to the primes and non-primes as rich points and poor points, respectively.) Prime cells are located in 2^{k-1} rows in general and in particular, when k equals three, prime cells have seven rows that intersect them, while non-primes have only four.

TABLE I

THE 18 PLANES WITHIN THE QUBIC BOARD

1.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2.	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
3.	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
4.	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64
5.	1	2	3	4	17	18	19	20	33	34	35	36	49	50	51	52
6.	5	6	7	8	21	22	23	24	37	38	39	40	53	54	55	56
7.	9	10	11	12	25	26	27	28	41	42	43	44	57	58	59	60
8.	13	14	15	16	29	30	31	32	45	46	47	48	61	62	63	64
9.	1	5	9	13	17	21	25	29	33	37	41	45	49	53	57	61
10.	2	6	10	14	18	22	26	30	34	38	42	46	50	54	58	62
11.	3	7	11	15	19	23	27	31	35	39	43	47	51	55	59	63
12.	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64
13.	1	5	9	13	18	22	26	30	35	39	43	47	52	56	60	64
14.	1	2	3	4	21	22	23	24	41	42	43	44	61	62	63	64
15.	4	8	12	16	19	23	27	31	34	38	42	46	49	53	57	61
16.	13	14	15	16	25	26	27	28	37	38	39	40	49	50	51	52
17.	1	6	11	16	17	22	27	32	33	38	43	48	49	54	59	64
18.	4	7	10	13	20	23	26	29	36	39	42	45	52	55	58	61

```
P . . P   . . . .   . . . .   P . . P
. . . .   . P P .   . P P .   . . . .
. . . .   . P P .   . P P .   . . . .
P . . P   . . . .   . . . .   P . . P
```

Figure 7. The 16 Prime Cells of the Tic-Tac-Toe Cube

The notion of a prime cell is also used within a plane. Within a plane, there are prime cells (1,4,6,7,10,11,13, and 16 of the left-most plane of Figure 6) and non-prime cells, and are called, by the author, planar primes and planar non-primes. In view of the fact that planes and plane strategies are discussed in some detail, the distinction between cubic primes and planar primes must be made clear, hence, the terminology. Planar primes reside on three intersecting rows, while planar non-primes reside on only two. Figure 8 graphically shows the planar primes in any given plane. The planar prime cells have been marked with the letter P.

```
P . . P
. P P .
. P P .
P . . P
```

Figure 8. Prime Cells Within a Plane

While Silver (5) refers to the cubic non-primes as poor points, things are not so bad as the terminology suggests. Every cell within the cube is a prime cell within some plane. Cubic primes are planar primes in all six planes in which they reside, and cubic non-primes are planar primes in one of the four planes in which they reside.

The rules of the game are simple. The players alternate, placing different colored markers in unoccupied cells of the board. The first player to get four markers in any row horizontally, vertically, or diagonally is declared the winner. By convention, the player who plays first is referred to as X and the second player is referred to as O.

According to Gammil (3), QUBIC, like other sophisticated games, can be divided up into three phases of play: the opening game, the middle game, and the end game. The opening consists of the first five to seven moves. Neither player has three markers in any given row and no direct defensive play (blocking three in a row) is necessary.

The initial objective for both players is to place markers in cubic primes since cubic prime cells are more powerful, and wait for the opponent to fail to block a winning combination. Some situations lend themselves to an early win for X. The sequence 1-2, 22-52, 27 . . . contains a trap for the second player. Unless he plays in plane number 17 (see Table I) and specifically in cell number 6,16,17,32,38,43,49,54, or 64 he will lose on or before the ninth move. The winning sequence for X can be seen by isolating plane number 17 (see Figure 9) then considering this subset of the cube. The reader will note that O does not occupy any cell of plane

```
X . . .
. X X .
. . . .
. . . .
```

Figure 9. Plane Number 17 After X's Third Move

number 17, nor will he be able to get three markers in a row with the given solution (except, perhaps, on his last move). Figure 10 shows plane number 17 with three markers for X and a solution. The numbers represent moves by X (forces), the O's represent blocks (forced) by O and the dashes represent the two options available for O's final move. After O's last move, X will place a marker in the remaining cell and win. The entire sequence is as follows: 1-2, 22-52,27-4,32-17,64-43,16-48,6-11,38-49,54.

```
X . . .     X 4 O 3
. X X .     O X X 1
. . . .     . 5 O O
. . . .     - - . 2
```

Figure 10. Plane 17 with Three Markers and a Solution

The middle game begins after the first force (or threat). The endgame is initiated when one player begins a forcing sequence to an eventual win. In the previous example, the end game begins on X's fourth move.

Mappings

In order to reduce the total number of situations that must be considered for a complete analysis of the game, several automorphisms are discussed and are shown to reduce the number of unique situations with four markers on the board (and X occupying cells 1 and 22) to 198.

Silver (5) defines an automorphism A as a one to one mapping of the board onto itself which preserves lines; i.e. given four collinear cells within the cube (P1,P2,P3,P4), the four points (P1A, P2A, P3A, P4A) are also collinear. Automorphisms of interest are rotations and reflections. One face of the cube can be turned 0, 90, 180, or 270 degrees and the result is an automorphism. Since there are six faces, 24 unique rotations of the board exist. Given the midpoint of the cube, each cell can be mapped to a cell equidistant and "on the other side" of this point. This is termed a reflection about the midpoint. Rotations and reflections account for 48 automorphisms of the cube onto itself.

There are two other automorphisms of interest, the evisceration and scramble mappings (5). The evisceration and scramble mappings are described and defined by the following procedure. Given the coordinates of a cell (refer to Method II, page 8) the coordinates of the eviscerated cell can be determined by substituting 2,1,4 or 3 for 1,2,3 or 4 respectively. Cell 213 transforms into cell 124 in this manner. Given the coordinates of a cell, the coordinates of the scrambled cell can be determined by substituting 1,3,2 or 4 for 1,2,3 or 4 respectively. Figure 11 shows, within a plane subset, the two transformations described using Method I referencing. These transformations play havoc with the layout of the cube, but since all lines are preserved the game remains essentially unchanged.

6	5	8	7		1	3	2	4
2	1	4	3		9	11	10	12
14	13	16	15		5	7	6	8
10	9	12	11		13	15	14	16

Figure 11. Evisceration and Scramble Mappings on a 4 x 4 Board

Figure 12 shows the location of each cell in Figure 6 after the evisceration mapping has been performed and Figure 13 shows the location of each cell after the scramble mapping has been performed.

22 21 24 23	6 5 8 7	54 53 56 55	38 37 40 39
18 17 20 19	2 1 4 3	50 49 52 51	34 33 36 35
30 29 32 31	14 13 16 15	62 61 64 63	46 45 48 47
26 25 28 27	10 9 12 11	58 57 60 59	42 41 44 43

Figure 12. The Board after Evisceration

1 3 2 4	33 35 34 36	17 19 18 20	49 51 50 52
9 11 10 12	41 43 42 44	25 27 26 28	57 59 58 60
5 7 6 8	37 39 38 40	21 23 22 24	53 55 54 56
13 15 14 16	45 47 46 48	29 31 30 32	61 63 62 64

Figure 13. The Board after Scrambling

Given the solution in Figure 10, rotation by 0, 90, 180, and 270 degrees give the four plane positions as shown in Figure 14. and mapping first by evisceration then by scramble then by a combination of both give the positions shown in Figure 15.

X	4	0	3	-	.	0	X	2	.	-	-	3	1	0	2
0	X	X	1	-	5	X	4	0	0	5	.	0	X	0	.
.	5	0	0	9	0	X	0	1	X	X	0	4	X	5	-
-	-	.	2	2	0	1	3	3	0	4	X	X	0	.	-

Figure 14. The Four Rotations of 1,6,7

Original				Evisceration				Scramble				Both			
X	4	0	3	X	0	1	X	X	0	4	3	X	1	0	X
0	X	X	1	4	X	3	0	.	0	5	0	-	2	-	.
.	5	0	0	-	-	2	.	0	X	X	1	4	3	X	0
-	-	.	2	5	.	0	0	-	.	-	2	5	0	.	0

Figure 15. Evisceration and Scramble Mappings of 1,6,7

Taking the evisceration and scramble mappings into consideration, the automorphisms of the cube onto itself total 198. Silver has given a formal proof of this and has shown that no other automorphisms exist (5).

Practical Restrictions

It will be shown that the first move of the game can appear in only two unique cells (cell 1 or cell 2). It is easily seen that all of the corner cells are equivalent to each other by rotation and reflection, and all of the center cells are equivalent also by rotation and reflection. Cell 1, a corner cell, is equivalent to cell 22, a center cell, by evisceration; therefore, the corner cells and the center cells, otherwise known as prime cells are equivalent (by the various automorphisms). Cell 2 is equivalent to cell 3,5,8, 9,12,14,15,17,20,29,32,33,36,45,48,50,51,53,56,57,60,62 and 63 by rotation and/or reflection. Cell 2 is equivalent to cell 21 (by evisceration) which is also equivalent to cells 18,19,24,25,28,30,31, 34,35,37,40,41,44,46, and 47 by rotation and reflection. Cell 17 is equivalent to cell 6 (by evisceration) which is equivalent to cells 7, 10,11,54,55,58, and 59 by rotation and reflection. Therefore, every cubic non-prime cell is equivalent (by the various automorphisms), and as a first move in the game of QUBIC, one can place a marker in one of two unique cells, cell 1 (prime) or cell 2 (non-prime).

It has been shown that the first move can appear in two unique cells. Cell 1 was chosen for two reasons: 1) because it lies on the intersection of seven rows, while cell two lies on the intersection of four rows, and 2) using evisceration and scramble mappings cell 1 maps to either itself or to cell 22.

CHAPTER IV

GAME PLAYING STRATEGY

This Chapter details the author's version of a QUBIC playing program. Empirical evidence suggests that some situations require more than a nominal amount of machine time to arrive at their respective solutions. It was decided, therefore, to place these situations along with the respective solutions on disk so that when any of these situations arose in actual play, the solution would be found in a reasonably short period of time. The first part of this Chapter is devoted to describing the situations that need be stored. The next part of the Chapter describes the author's defensive strategy. Planar traps are discussed in some detail. All unique planar traps are listed, and the author's defense to these traps is described.

The last part describes the programs that incorporate these offensive and defensive strategies into their logic and these methods are compared with other qubic playing programs.

The reader is advised that although this Chapter is divided into two parts (offense and defense) this was done for convenience only. Any offensive or defensive moves by X can also be employed by O. While the strategies are interchangeable, the terminology was chosen mainly for purposes of illustration.

Approach

Moves which block direct threats (three in a row), which build traps and which force the opponent are the only moves considered. With 64! possible games these restrictions are reasonable. The sooner the machine can control the game, the fewer possibilities need be considered.

Offense

The first two moves for the author's QUBIC playing program are fixed. The machine (playing first for X) will play in cell 1. (Any other of the prime cells could have been chosen, the cells being equivalent.) The opponent may play in cell 22 or some other cell. If he chooses cell 22, the machine will play in cell 13; otherwise, the machine will play in cell 22. At this point the opponent has 61 cells in which to play. After the opponent has moved, there are four markers on the board; and this position is called a situation. It has been shown that if X has markers in cells 1 and 22, then after evisceration, although O's markers will have been moved, X will still have markers in cells 1 and 22. This was the prime motivating force behind choosing cells 1 and 22 as the first and second moves for X. The first requirement, in an approach of this type, is to find the number of unique situations with four pieces on the board also with X occupying cells 1 and 22 or 13. Considering all automorphisms described in Chapter III, there are 198 unique situations that must be considered to accomplish an exhaustive analysis of the game. These situations can be found in Appendix B. Due to limited resources, it was decided to look rather closely at about 15 percent of the situations starting at the top, and derive the conclusions from the cases studied. Also, one other situation 1-22,13-49 was examined in detail, this being one exception to the rule that X have markers in cells 1 and 22. The situations examined are listed: 1-2,22-3-4-5-6-7-8-9-10-11-12-13-14-15-16-21-23-24-26-27-28-29-30-31-42-44-45-46-47-48-61-62-63 and 1-2, 13-49. These situations total 33 and out of 198 possible, a little over 16 percent form the basis of the conclusions.

It was determined in all cases studied that, if X played according to a prescribed strategy, X would win every game.

Attacking the problem involved making an assumption that after a given number of moves X can force a win. Without this assumption, one must rely on an objective function for a good strategy, a method the author finds lacking in this particular game. Objective functions are usually formulas that examine a set of possible alternative moves and evaluate the "goodness" of these moves in terms of the state of the board. Using an objective function the next move can be selected from this set of alternatives in terms of the one with the best "goodness" measure. An objective function selects a good move for a "good" strategy, but it does not, necessarily, select the best move for a winning strategy. For a method that uses an objective function the reader is referred to Gammil (3).

Some of the winning sequences for the different situations involve only one plane and the forced win is relatively straightforward. However, when a suitable plane cannot be found, then the board is treated as 76 intersecting rows, and several planes are involved. The author defines the term "solution" to mean a sequence of forcing moves that result in a win. At a certain point within a game there may exist a situation which, if pursued, will yield a win for one player. This sequence of moves is called a solution.

The situations are divided into two groups: those with "simple" solutions and those with "complex" solutions. Simple solutions are for those situations for which an algorithm that determines the solution within a reasonable period of time is known to exist, and usually includes only 1 plane. A complex solution is one in which no known algorithm, which will select the perfect cell, is known to exist, or an algorithm exists but cannot be used in a practical sense. For these situations a method of kibitzing by the author is allowed. The solution found is punched into cards and never again considered (except, perhaps, in actual play). In a sense, the machine is developing solution strategies as it is exposed to new situations. Representation of solutions is discussed later in this Chapter.

All complex solutions and many simple ones are stored on magnetic disk for the benefit of the game playing program. In actual play the disk is searched for a solution to the game in progress. If no solution is found, it is assumed the solution is a simple solution and treated accordingly.

Defense

Defensive play is an important part of any QUBIC strategy. Recognizing a potentially dangerous situation is, at best, difficult. Other than exhaustive analysis, no perfect defensive strategy is known. However, most traps set by the opponent can be discovered by the method described in the following pages. The algorithm used depends heavily upon recognition, within a plane, of a planar trap that is a potentially dangerous situation. A planar trap has the property that 0 has three markers occupying selected cells within a plane and with these markers 0 can ultimately force a win. The following paragraphs describe some examples of planar traps, describe the solutions to traps in general, list all the unique planar traps, show that they are unique, and present a method, or scheme, for blocking these traps.

In Figure 16A 0 has three markers in prime cell numbers 1,4 and 13. Unless X blocks (by placing a marker in cell 2,3,5,6,7,9,10,11 or 16) or "gets the move" by obtaining three in a row utilizing the rest of the cube, 0 can force a win.

```
0 . . 0     0 . . 0     0 . . 0
. . . .     . . . X     . . . .
. . . .     . . . .     . . . X
0 . . .     0 . . .     0 . . .
   A           B           C
```

Figure 16. Traps Formed by Player 2

The first player (X) having a marker somewhere in the plane (see Figure 16B and 16C) will not necessarily stop the forced win. The solutions for the three traps shown in Figure 16 can be seen in Figure 17. The three 0's in the corner cells (and the X in the rightmost column) represent the initial positions. The numbers represent forces by 0, the asterisks represent blocks by X and the dashes show the trap. The final state of each plane of Figure 17 yield 0 with two rows with three in a row.

```
0 1 * 0     0 1 * 0     0 * 1 0
* 4 * .     * 4 * X     2 5 3 -
2 3 5 -     2 3 5 -     * * 4 X
0 * . -     0 * . -     0 . * -
```

Figure 17. Solutions to Traps Formed by Player 2

There are several variations on this theme. If 0 has three markers on any three planar primes and the rest of the plane is clear, 0 can probably force a win. With eight planar primes, there are three unique situations. These situations are shown in Figure 18. All other combinations with X having three markers in prime cells make up the set of rotations, reflections and/or mappings of the three shown in Figure 18. For completeness all possible combinations are given in Table II along with the equivalent (unique) situations. If a situation has "three in a row," then the situation is considered trivial.

```
0 . . 0    0 . . 0    0 . . .
. . . .    . 0 . .    . . 0 .
. . . .    . . . .    . 0 . .
0 . . .    . . . .    . . . .
```

Figure 18. The Three Unique Planar Situations

The reader should note that X may or may not have a marker in one of the two cells that are empty (in Figure 17). However, the solutions shown still hold and are shown in Figure 19. (The reader will note: Figure 19C is equivalent to the rightmost plane of Figure 18 after evisceration.)

The order of the moves may vary, but the result is the same, i.e., a loss for X. The solutions to the three basic situations given in Figure 18 are equivalent as will be shown by the following argument. Referring to Figure 19 and disregarding the order of the moves (only the final state is relevant), A and B are equivalent by reflection and A and C are identically equivalent.

TABLE II

ALL COMBINATIONS OF THREE MARKERS IN PLANAR PRIME CELLS

1 4 6	Unique
1 4 7	1 4 6
1 4 10	1 4 6
1 4 11	1 4 10
1 4 13	Unique
1 4 16	1 4 13
1 6 7	1 4 6
1 6 10	1 4 6
1 6 11	Three in a row
1 6 13	1 4 6
1 6 16	Three in a row
1 7 10	Unique
1 7 11	1 4 10
1 7 13	1 4 10
1 7 16	1 7 10
1 10 11	1 4 10
1 10 13	1 4 6
1 10 16	1 7 10
1 11 13	1 4 10
1 11 16	Three in a row
1 13 16	1 4 13
4 6 7	1 4 6
4 6 10	1 4 10
4 6 11	1 7 10
4 6 13	1 7 10
4 6 16	1 4 10
4 7 10	1 6 11
4 7 11	1 4 6

TABLE II (Continued)

4	7	13	1	6	16
4	7	16	1	4	6
4	10	11	1	4	10
4	10	13	1	6	16
4	10	16	1	4	10
4	11	13	1	7	10
4	11	16	1	4	6
4	13	16	1	4	13

```
0 * 4 0     0 4 * 0     3 * 4 0
1 3 2 *     * 0 3 2     2 0 1 *
* * 5 .     . 5 * *     * * 5 .
0 . - -     - - . 1     0 . - -
   A           B           C
```

Figure 19. The Solutions to the Three Planar Situations

The traps previously described obviously should be discovered and blocked. Rather than use a separate algorithm for each situation, two basic patterns can be used to discover all traps formed by the opponent within the context of a plane. The two solutions are shown in Figure 20 and these patterns can be used to find any dangerous situation given that 0 has three markers in a plane and X has zero or one marker in the same plane. The situation shown in Figure 16A with 0 having markers in cell numbers 1,4 and 13 is called a 1,4,13 trap. The sequence of moves resulting in a win for 0 in a 1,4,13 trap can be played one of two ways. Both solutions are given in Figure 20. The positions of the two cells (in A and B) marked with dashes (-) are X's choice for the next to the final move and they determine the uniqueness of these solutions. In A the two dashes are in the same row, while in B the dashes are not in the same row. Since rotations, reflections and evisceration and scramble mappings preserve two markers in a row, the solutions are unique.

0	1	*	0		0	1	*	0
*	4	*	.		-	3	*	.
2	3	5	-		5	2	4	-
0	*	.	-		0	*	.	*

Figure 20. The Two Solutions to the 1,4,13 Trap

Table III represents a table of forcing moves. An exhaustive search was made of the table for the situation shown in Figure 16A. By referring to Table III it can be said that no other unique solutions to the 1,4,13 trap exist other than the two shown in Figure 20.

TABLE III

POSSIBLE VARIATIONS OF THE 1,4,13 TRAP

2-3,5-9,7-10,6-8,11-16.
2-3,5-9,7-10,6-8,16-11,12.
2-3,5-9,7-10,8-6,12-16.
2-3,5-9,7-10,8-6,16-12,11.
2-3,5-9,10-7,6-14,11-16.
2-3,5-9,10-7,6-14,16-11,15.
2-3,5-9,10-7,14-6,15-16.
2-3,5-9,10-7,14-6,16-15,11.
2-3,7-10,5-9,6-8,11-16.
2-3,7-10,5-9,6-8,16-11,12.
2-3,7-10,5-9,8-6,12-16.
2-3,7-10,5-9,8-6,16-12,11.
2-3,7-10,9-5.
2-3,9-5,7-10.
2-3,9-5,10-7,6-14,11-12,16. Solution
2-3,9-5,10-7,6-14,11-16,12. Solution
2-3,9-5,10-7,6-14,12-11,15.
2-3,9-5,10-7,6-14,16-11,15.
2-3,9-5,10-7,14-6,8.
2-3,10-7,5-9, See 2-3,5-9,10-7.
2-3,10-7,6-14,5-9.
2-3,10-7,6-14,9-5, See 2-3,9-5,10-7,6-14.
2-3,10-7,6-14,11-16,5-9.
2-3,10-7,6-14,11-16,9-5,12. Solution
2-3,10-7,6-14,11-16,9-12,5. Solution
2-3,10-7,6-14,11-16,12-9.
2-3,10-7,6-14,16-11,15.
2-3,10-7,9-5, See 2-3,9-5,10-7.
2-3,10-7,14-6,5-9,15-16.

TABLE III (Continued)

2-3,10-7,14-6,5-9,16-15,11.
2-3,10-7,14-6,9-5,8.
2-3,10-7,14-6,15-16,5-9.
2-3,10-7,14-6,15-16,9-5,8.
2-3,10-7,14-6,16-15,11.
3-2 Same as 2-3 with scramble mapping.
5-9,2-3 Same as 2-3,5-9.
5-9,3-2 Same as 3-2,5-9.
5-9,7-10,2-3 Same as 2-3,5-9,7-10.
5-9,7-10,3-2 Same as 3-2,5-9,7-10.
5-9,7-10,6-8,2-3 Same as 2-3,5-9,7-10,6-8.
5-9,7-10,8-6,2-3,12-16.
5-9,7-10,8-6,2-3,16-12,11.
5-9,7-10,8-6,3-2,14.
5-9,7-10,8-6,12-16,2-3.
5-9,7-10,8-6,16-12,11.
5-9,10-7,2-3 Same as 2-3,5-9,10-7.
5-9,10-7,3-2 Same as 3-2,5-9,10-7.
7-10,2-3 Same as 2-3,7-10.
7-10,3-2 Same as 3-2,7-10.
7-10,5-9, Same as 5-9,7-10.
7-10,9-5,2-3.
7-10,9-5,3-2,11-15.
7-10,9-5,3-2,15-11,14-16.
7-10,9-5,3-2,15-11,16-14,6.
9-5 Same as 5-9 with scramble mapping
10-7 Same as 7-10 with scramble mapping

These two schemes also discover winning combinations for 0 when 0 has one or more markers in planar non-primes. Given the situation shown in Figure 21, 0 has only one marker in a prime cell, but can force a win, none the less. The solution is shown.

```
. 0 . .     3 0 - 5
. . . .     * 2 - .
0 . 0 .     0 1 0 *
. . . .     4 * . *
```

Figure 21. A Winning Combination With 0 Occupying One Planar Prime

There are 28 unique situations with either player having three markers in a plane and these are given in Table IV. The list was compiled using the program described in Appendix D. This program (UNQPL) lists all the possible situations and follows each situation with an equivalent one, if there is one. The output of UNQPL shows that there are 28 different situations with three markers of the same color in a plane and also that these 28 are unique. The first three situations in Table IV give 0 three in a row and obviously would result in a win for 0 if not blocked by X. In situations four through eight 0 does not have any rows with two in a row, therefore a force will not be immediately forthcoming. Situations nine through seventeen are such that 0 cannot force a win within the context of the given plane. This was determined by exhaustively

TABLE IV

THE 28 UNIQUE SITUATIONS WITH 0 HAVING THREE MARKERS IN A PLANE

Three in a Row	No Two in a Row	No Solution	Forced Wins
1. 1 2 3	4. 1 7 12	9. 1 2 5	18. 1 2 6
2. 1 2 4	5. 1 8 14	10. 1 2 12	19. 1 2 7
3. 1 6 11	6. 1 8 15	11. 1 2 14	20. 1 2 8
	7. 2 5 12	12. 1 2 15	21. 1 2 9
	8. 2 8 9	13. 1 4 14	22. 1 2 10
		14. 1 6 12	23. 1 2 11
		15. 1 8 12	24. 1 2 13
		16. 2 3 5	25. 1 2 16
		17. 2 3 14	26. 1 4 6
			27. 1 4 13
			28. 1 7 10

searching the table of forces and considering every possible force by 0 and block by X. Situations 18 through 28 are traps for X. Situations 26 through 28 refer to positions given 0 has three markers in planar primes and have been discussed previously. Situations 18 through 25 make up the eight unique traps with 0 having at least one marker in a planar non-prime. These eight traps, along with the first three moves of the solutions, are shown in

```
0  0  *  1    0  0  *  1    0  0  *  1    0  0  *  1
.  0  .  3    3  .  0  .    .  3  .  0    *  .  *  .
.  .  *  *    *  *  .  .    .  .  *  *    0  3  .  .
.  .  .  2    2  .  .  .    .  .  .  2    2  .  .  .

0  0  *  1    0  0  *  1    0  0  *  1    0  0  *  1
*  .  *  .    .  *  .  *    *  .  *  .    .  3  .  2
3  0  .  .    .  .  0  3    2  3  .  .    .  .  *  *
2  .  .  .    .  .  .  2    0  .  .  .    .  .  .  0
```

Figure 22. Solution to the Eight Traps Described in the Text

Figure 22. It can be shown that the eight positions are equivalent and that they make up a subset of the first solution shown in Figure 20, i.e. they are equivalent to the solution for the 1,4,13 trap given in Figure 20.

Every possible planar trap has been identified and all the planar primes are used in the solutions; therefore, by placing a marker in any available planar prime, X can block the win.

Methods Used

Implementing the strategies described involved writing several programs.

The programs are divided into two groups, the move-generating programs (ML64, GENEX and associated subprograms) and the game-playing programs and associated subprograms. Each group will be discussed in detail using, as an example, 1-2,22-4, one of the 198 situations listed in Appendix B. Refer to Figure 23. The given situation is punched on a card and this card is input to GENEX. GENEX determines, within the context of a plane, the best move for X using the defensive strategy described above. It does this by first checking to see if 0 commands a plane from which he can force a win given the chance. If 0 commands a plane that contains a potentially dangerous situation then this plane is blocked by GENEX (defensive). If 0 does not command a plane then a "stay away" strategy is invoked and DENEM (a subroutine called by GENEX) chooses a cell which resides in a plane that contains other markers for X and as few markers for 0 as is possible (offensive). For the situation 1-2,22-4 DENEM chose 27 as X's next move, giving X three markers in prime cells in plane 17 (refer to Table I), a situation shown to be dangerous unless blocked by 0 in cells 6,11,17,27,38, 43,48,49,54 or 64. Whatever the choice (offensive or defensive), the move generated by GENEX is punched onto a card and this new situation is used as input to ML64.

ML64 and associated subroutines constitute the bulk of the a priori analysis programs. (Most of the programming effort has been directed toward one of ML64's subroutines, WHIZ.) ML64 accepts as input a situation of the form

X 0 X 0 ... X 1.

All the possible moves for 0 are then examined and the result of each possibility (whether it be a win or draw for X or a force by 0) is punched into cards. The final three states for each of the 64 possibilities are: 1) X has a forced win; 2) X does not have a force win; and 3) 0 has three in a row. If 1), the solution is punched to cards. If 2), the situation is punched to cards and input to GENEX at a later date. If 3), the situation along with the force, the block by X and the number 1 are punched to cards and input to ML 64 at a later date. This process continues until all possibilities have been considered and all possible moves by 0 have been accounted for.

These programs merely punch solutions into cards (to be stored on disk). When the game playing program is challenged by a human opponent, the disk is searched for the current game in progress and if this game is found on the disk then the program has little to do except play the moves as specified.

A number of improvements were incorporated into ML64 to minimize the number of cards to be punched. When a sequence of moves unlike any other sequence in this run is found, this "solution" is kept and used again (if it will yield a win for X) on O's other 63 options. If O moves in cell 3, X can force a win in plane 17 by the following sequence of moves: 43-64,11-59,17-32,49-33,16-38,6. The same sequence of moves will result in a win for X if O moves in any of the following cells: 3,5,7,8,9,10,12,13,14,15,18,19,20,21,23,25, 26,28,29,30,31,34,35,36,37,39,40,41,42,45,46,47,50,51,52,53,55,56, 57,58,60,61,62 or 63. Rather than punch a separate set of cards for each move by O, the moves are combined and the entire sequence (including O's moves) is punched on three cards as shown in Figure 24. The result of using this scheme is that 44 separate solutions can be contained on three cards. Not all situations lend themselves to this form of compaction. Had O placed a marker in cell 16, the result would have been quite different. Cell 16 is a planar prime and need be empty if X is to force a win in plane 17. After trying all possible moves for X, ML64 reports "no solution", i.e. failure to find a forced win for this situation and punches 1-2,22-4,27-16 onto a card. This card is input to GENEX, the next move for X (cell number 26) is generated and punched and the entire process is repeated until all possibilities, i.e. all moves for O, have been exhausted.

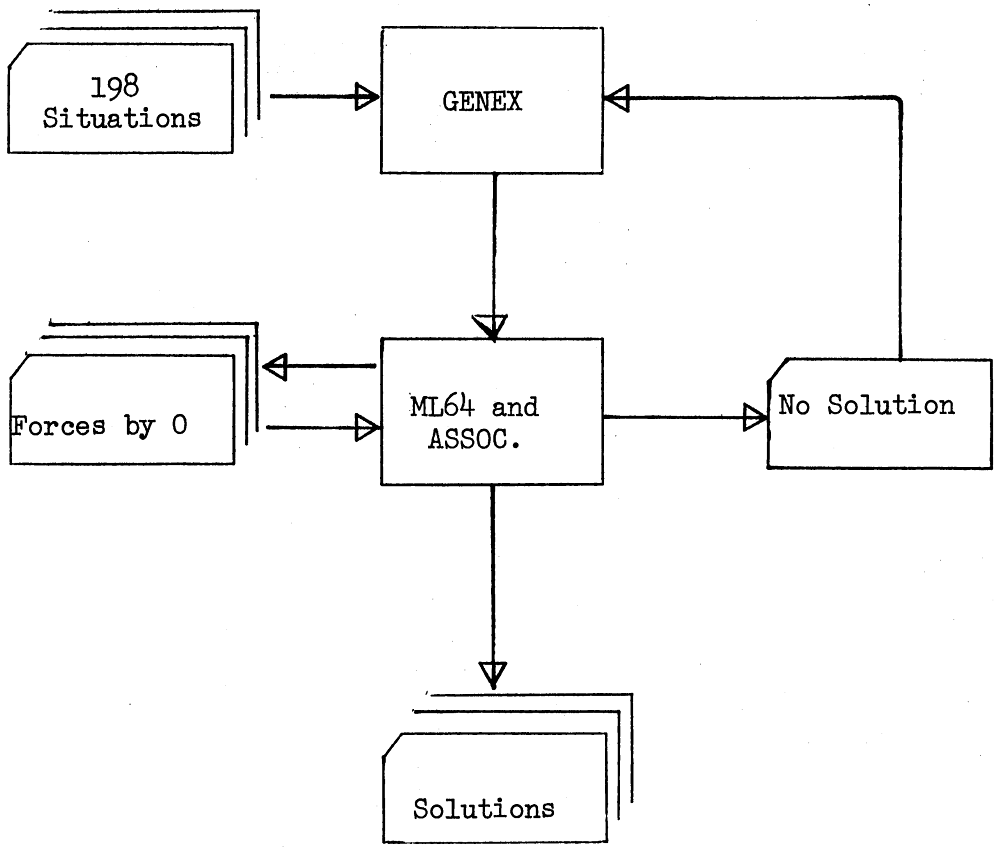

Figure 23. The Move Generating Programs

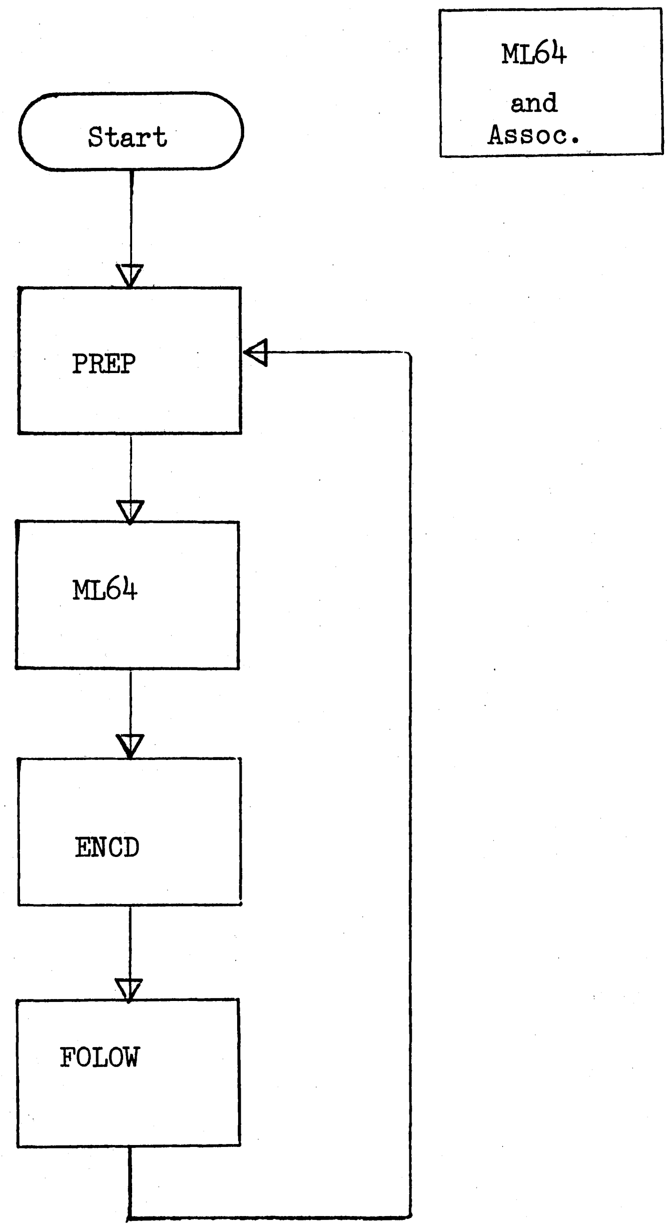

Figure 23. (Continued)

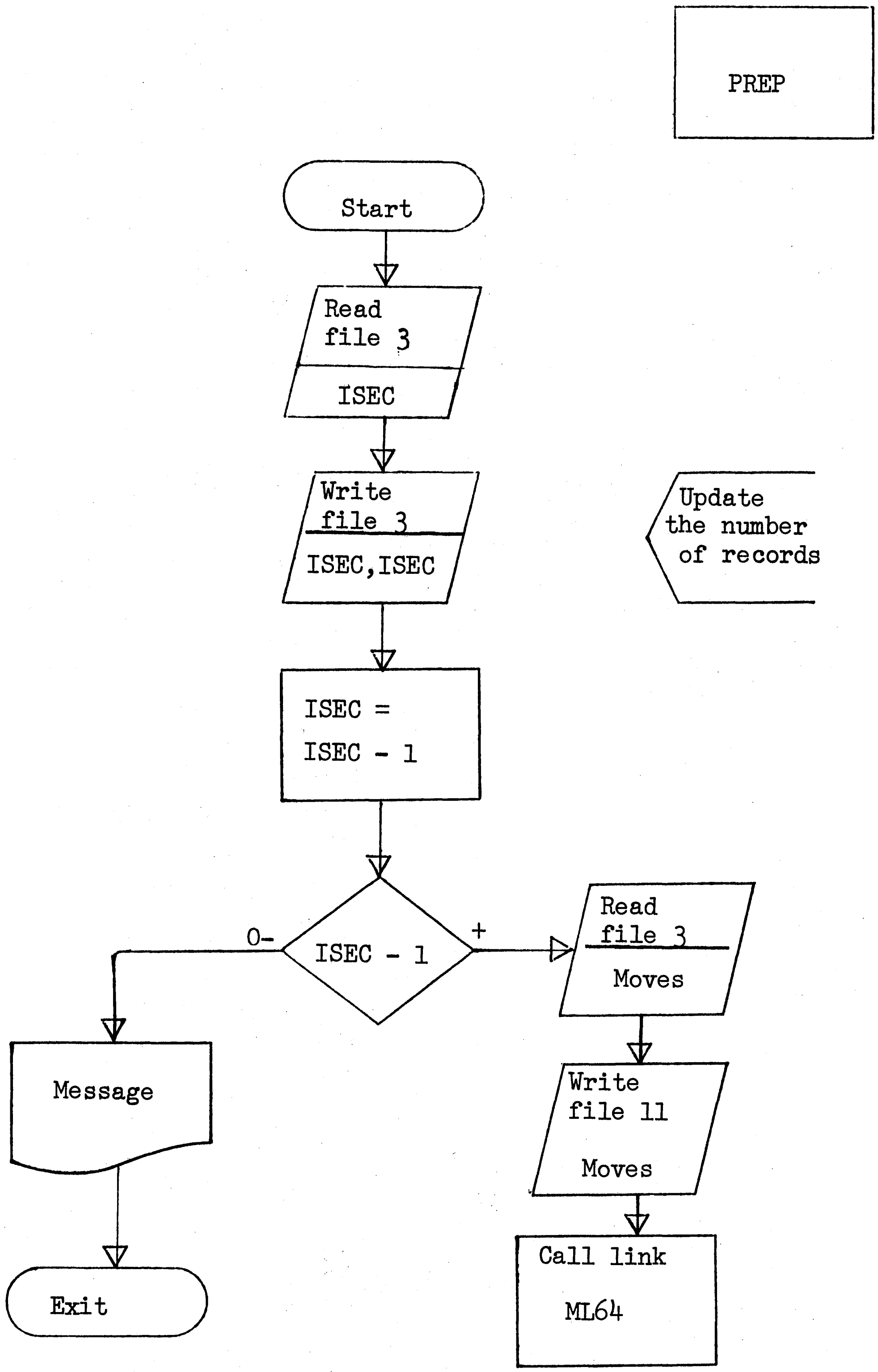

Figure 23. (Continued)

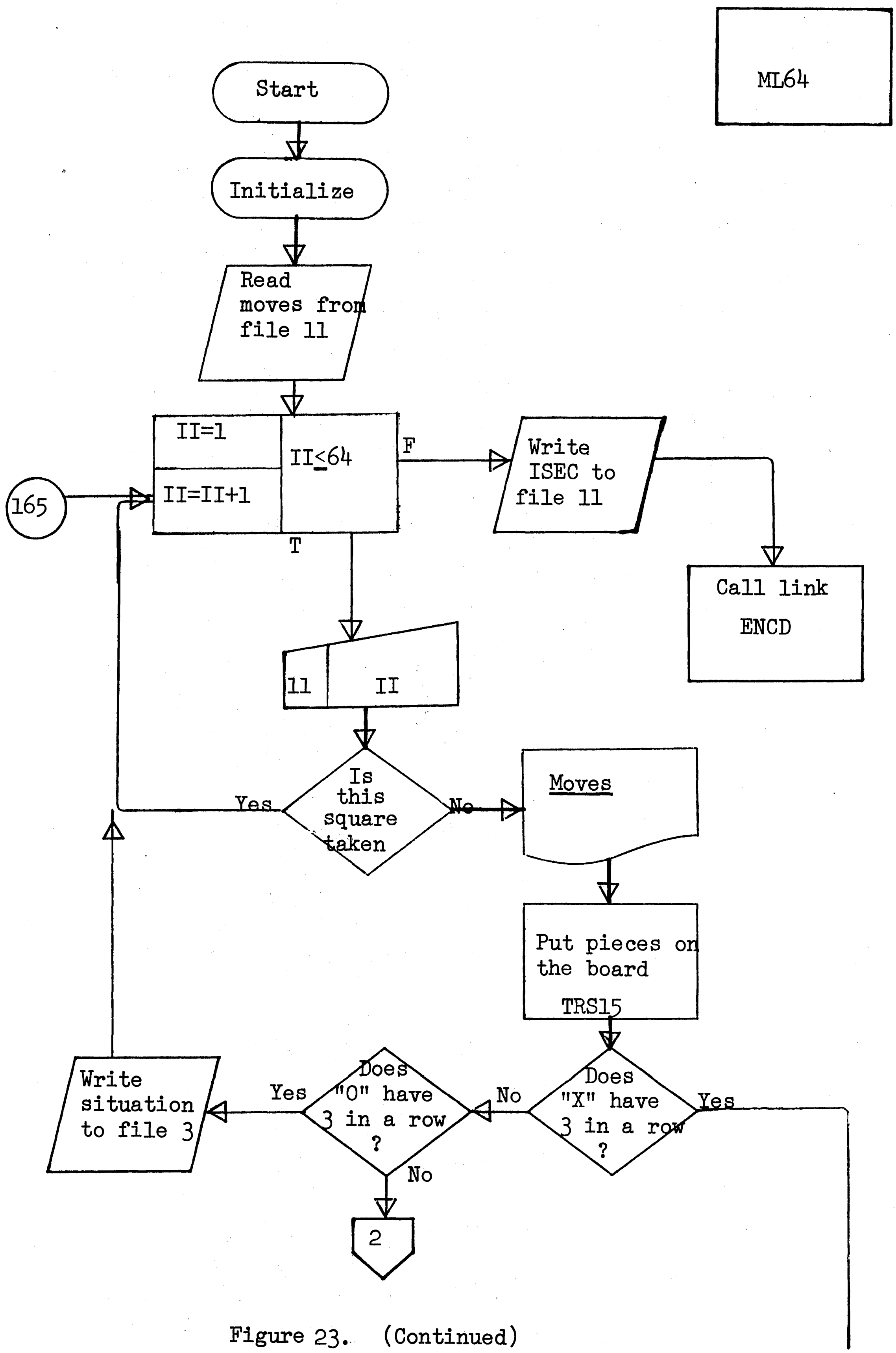

Figure 23. (Continued)

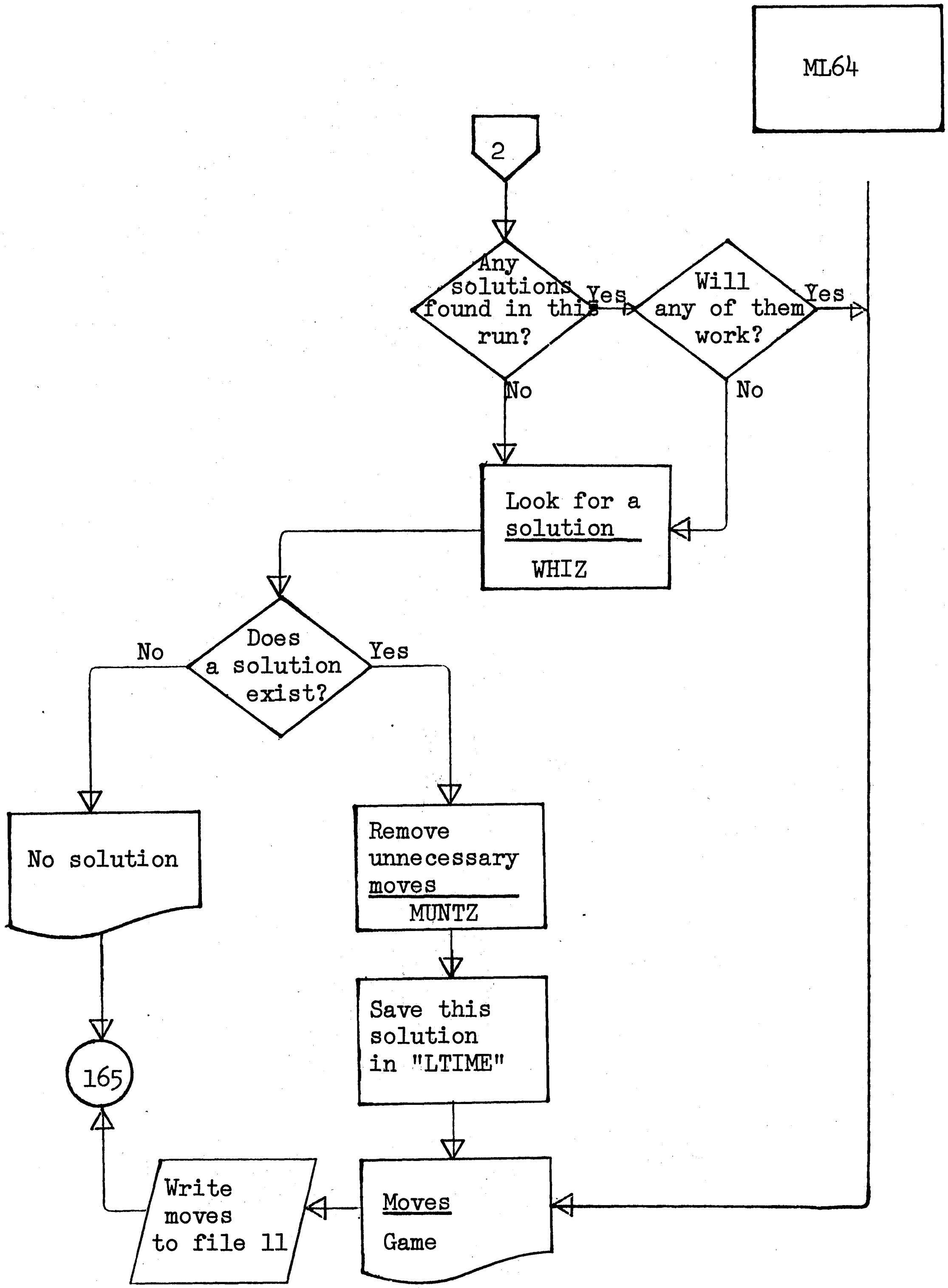

Figure 23. (Continued)

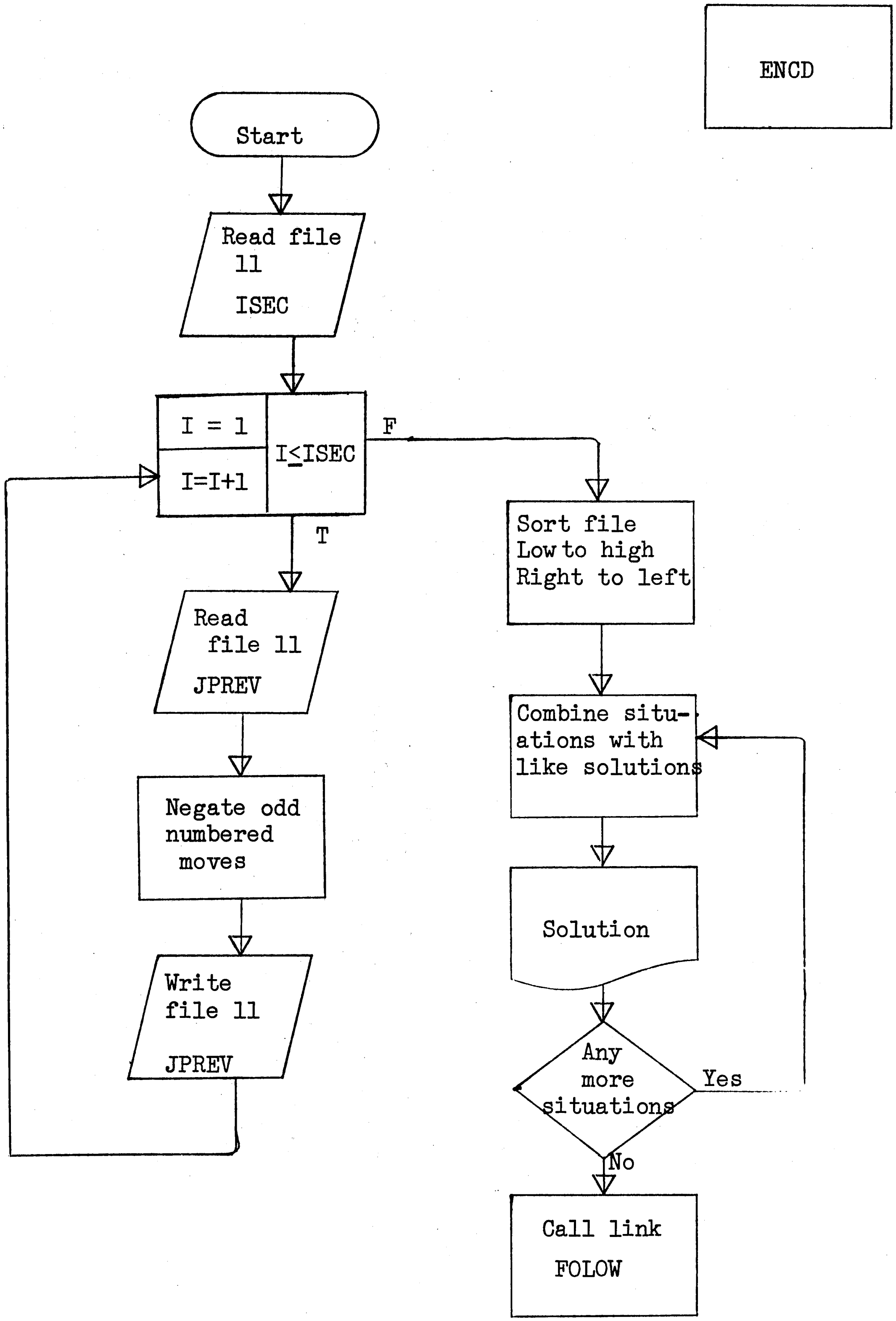

Figure 23. (Continued)

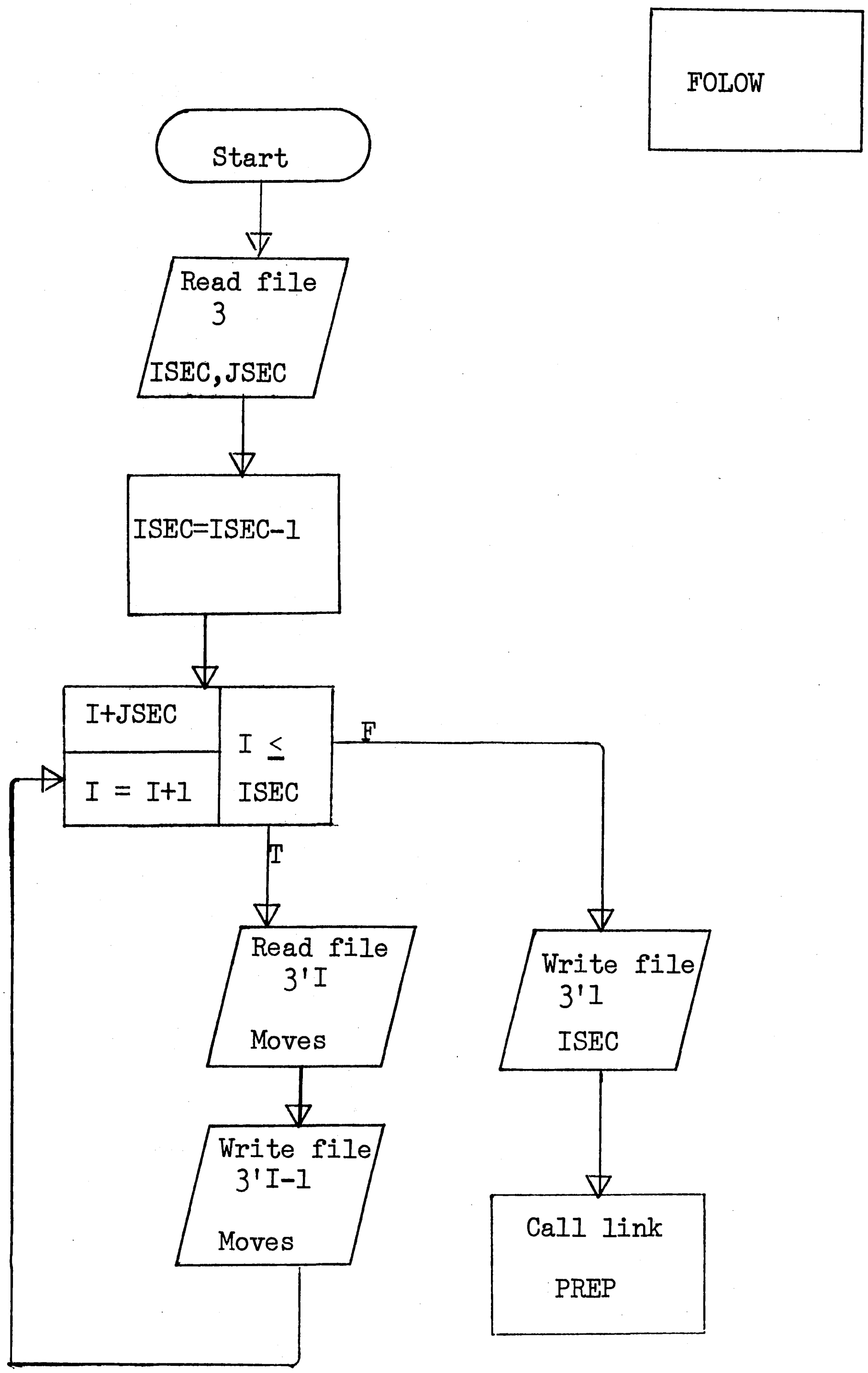

Figure 23. (Continued)

1 -2 22 -4 27 -3 -5 -7 -8 -9-10-12-13-14-15-18-19-20-21-23-25
-26-28-29-30-31-34-35-36-37-39-40-41-42-45-46-47-50-51-52-53-55
-56-57-58-60-61-62-63 43-64 11-59 17-32 49-33 16-38 6.

Figure 24. Winning Sequence for Several of O's Moves of a Situation Described in the Text

Appendix C contains the results for the entire sequence of moves for the situation 1-2,22-4. By referring to Appendix C, it can be said that if O places markers in cells 2 and 4 on his first two moves, he will surely lose. Since all possibilities have been considered, the outcome is entirely predictable.

Previous mention has been made of subroutine WHIZ. WHIZ is the longest and the most complex of the tic-tac-toe programs, and is shown, in flowchart form, in Figure 25. The flowchart is not a finely detailed account of WHIZ, but rather it displays the relationship between the logic of WHIZ and the strategy described in Chapter IV. The primary concern within WHIZ is keeping attention focused upon a plane that contains a possible win for X (i.e. X has three markers in prime cells and the rest of the plane is clear). WHIZ receives from ML64 a situation, determines if this situation is a win for X, and if so returns the solution to ML64. A method of exhaustively searching for all forces is used and if a solution exists, WHIZ will find it. WHIZ builds a dynamic table of forces by X and blocks by O. For purposes of illustration

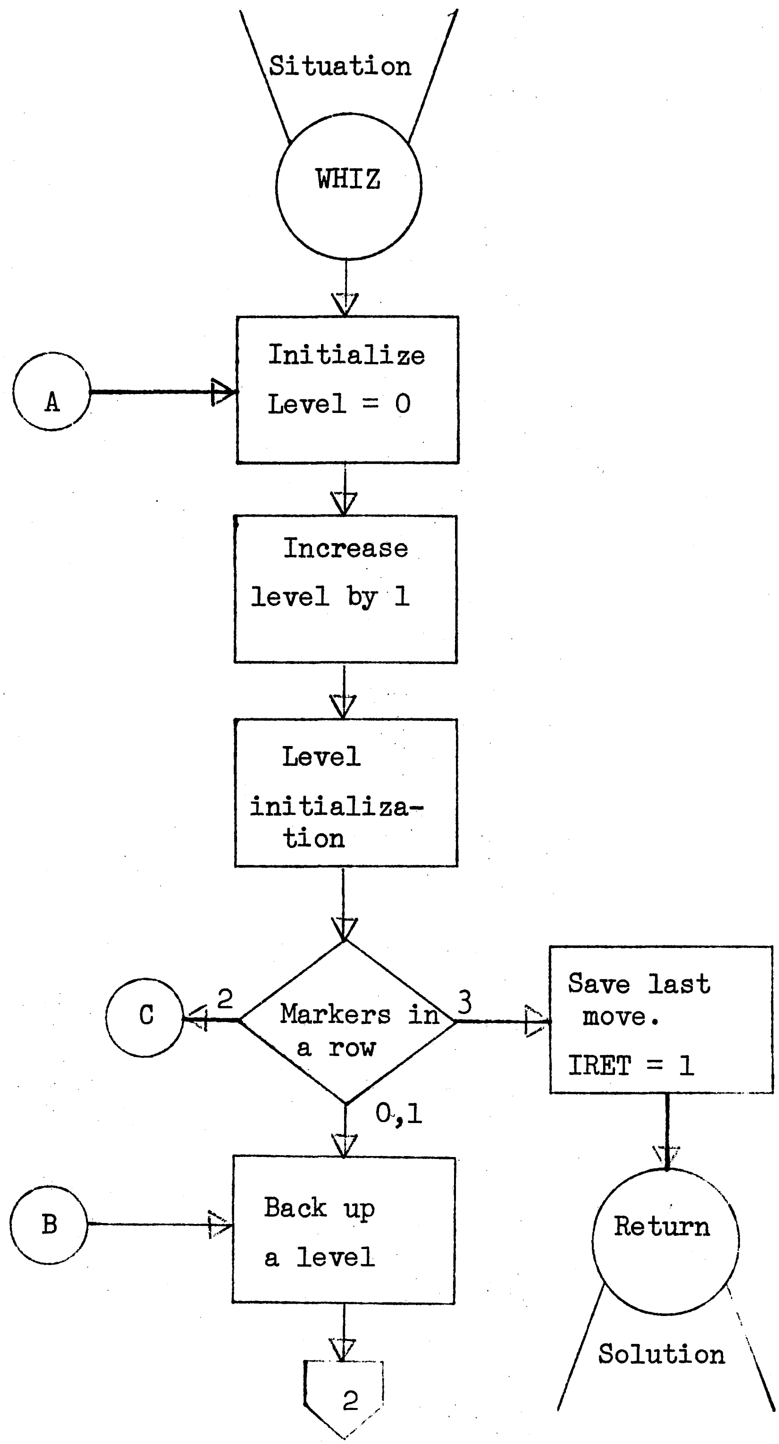

Figure 25. Logic of WHIZ

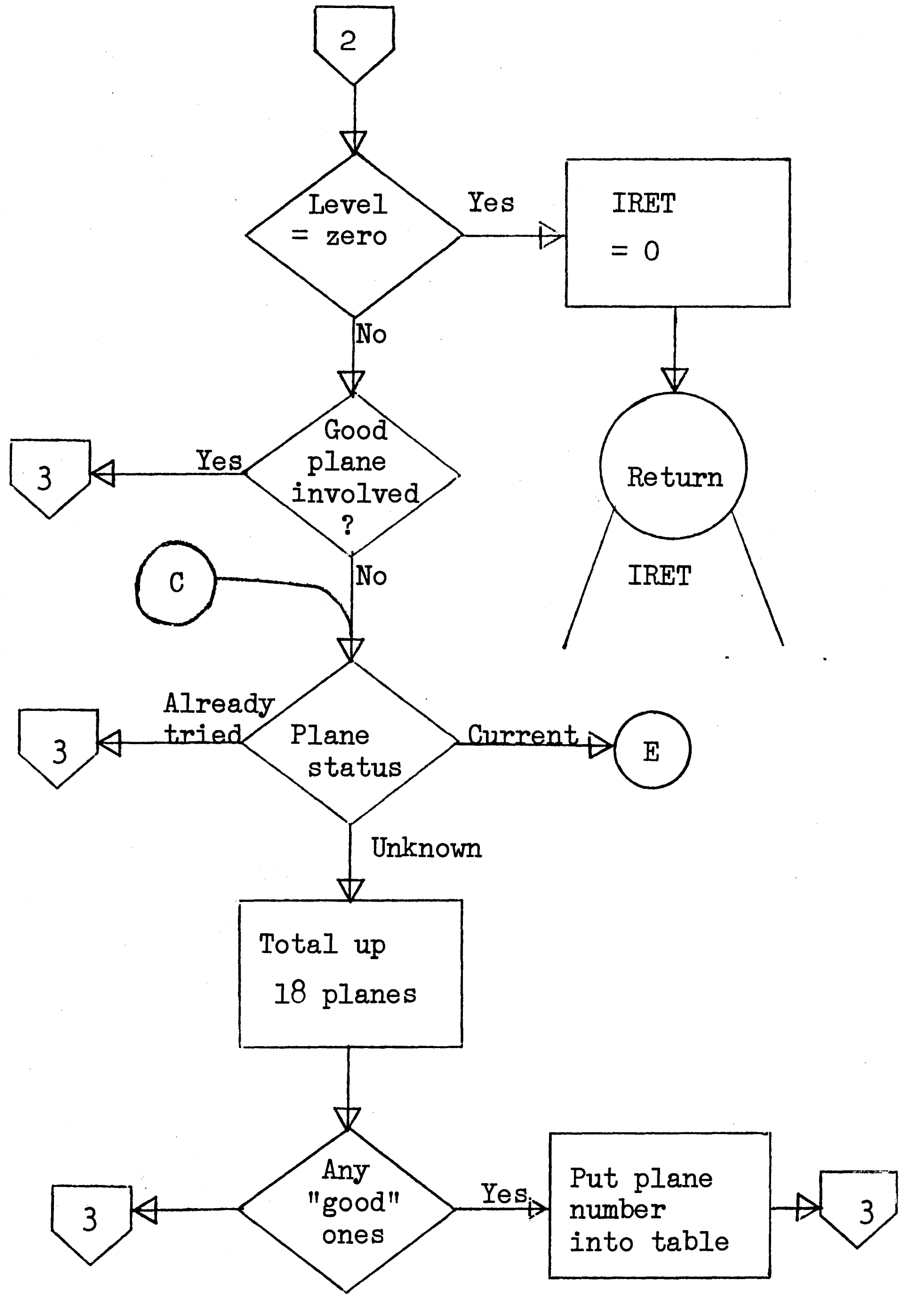

Figure 25. (Continued)

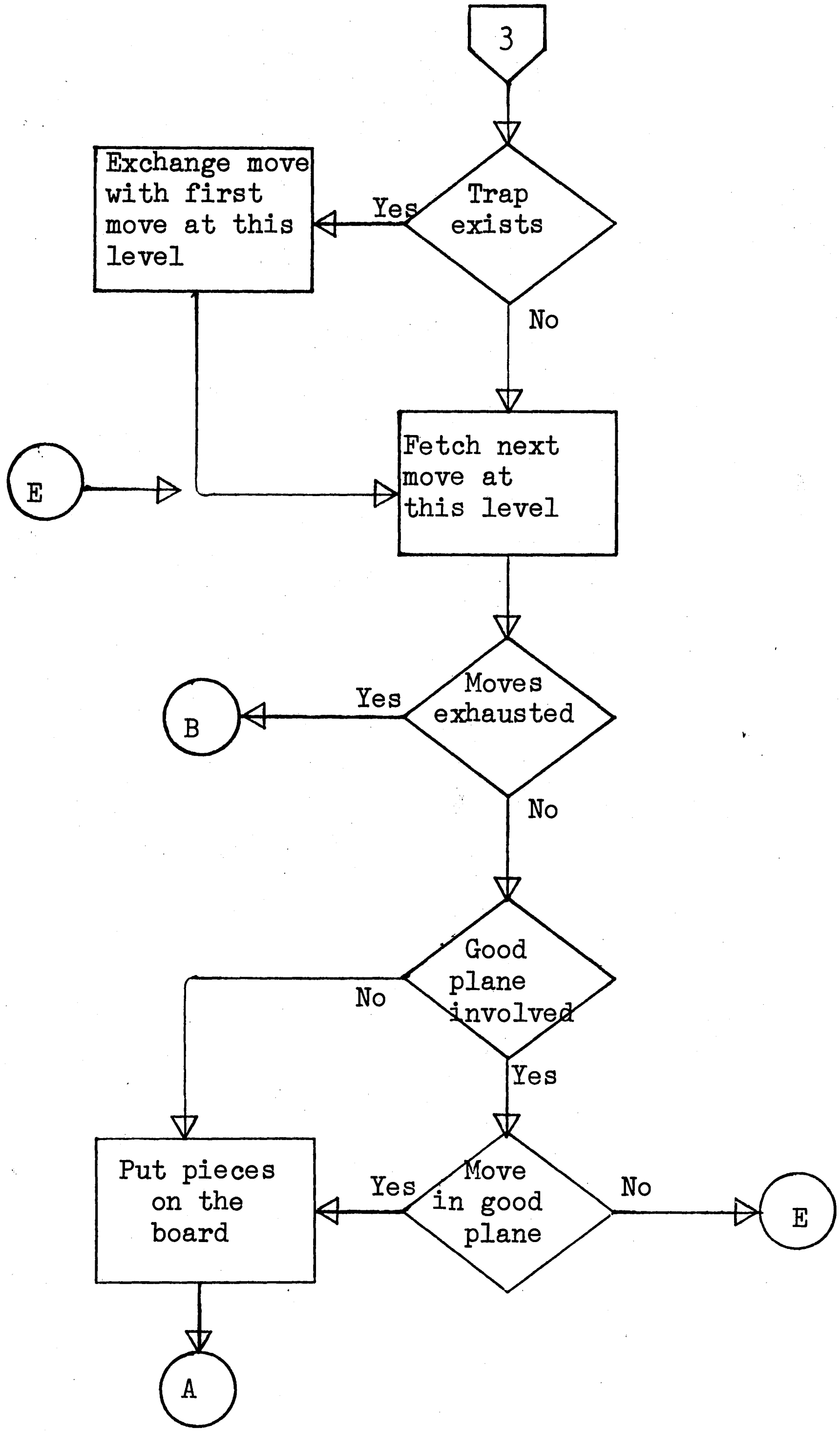

Figure 25. (Continued)

one situation, 1-2,22-4,27-3, is examined in detail. During the first three moves of the game, X has been careful to place all of his markers in plane number 17. Figure 26 displays the cubic cells that constitute plane 17 and also the position formed by X (for 1-2,22-4,27-3) within plane 17. The reader will note that X has three markers in planar primes and 0 does not have any markers within plane 17. The defensive strategy described in the section on Approach becomes an offensive strategy that should result in a win for X. A variable "level" is set to 1. This variable keeps track of the level of play. Figure 26 shows that X can force at cells 43,64,17 or 32. These four forces are placed into a table with a "current level pointer" array pointing to the first option at this level. A level counter is set to 4 (reflecting the number of forces at this level). The first move and corresponding block are fetched and played, i.e. 64-43. The level goes to 2. The forces available at this level are 32 and 17 (left over from level 1), therefore these two moves are placed into the table and the current level pointer array is set accordingly. By placing a marker at cell 32, X opens up a row and has forces at 16 and 48. WHIZ detects these possibilities and places 16 and 48 into the table.

1	6	11	16		X	.	.	.
17	22	27	32		.	X	X	.
33	38	43	48		.	.	.	.
49	54	59	64		.	.	.	.

Figure 26. Cubic Cells of Plane 17 and X's Position in Plane 17 for the Situation 1-2, 22-4,27-3

The status of the various tables after X's third move can be seen in Figure 27. TKNT is a table of level counters, i.e. TKNT contains the number of forces per level. There are four forces at the first level, two at the second and two at the third. TPONT is a table of pointers that point to the first move (within TABLE) of the set of forces for each level. The forces for level 1 start in TABLE (1). For level two in TABLE (5), and for level three in TABLE (7). TABLE is the dynamic array of forces for all levels up to and including the current level, and GAME contains the moves currently pending.

	TKNT	TPONT	TABLE	GAME
1	4	1	64	64
2	2	5	43	43
3	2	7	32	32
4	.	.	17	17
5	.	.	32	16
6			17	48
7			16	
8			48	
			..	
			..	
			..	

Figure 27. The Dynamic Tables of WHIZ

The process of locating forces, placing them in the table, and choosing the first one is continued until a solution is found or until no more forces exist. Some situations require two complete passes through the available moves at each level. The first pass selects only those cells that contain possible traps for X as described earlier in this Chapter. The second pass selects each force in turn, until no more forces are available. When this situation occurs, WHIZ "backs up a level," takes the next force available at the previous level and continues. Each force is tried, in turn, until all possible forces have been tried, in which case WHIZ backs up another level and repeats the process. If the level backs up to zero, no solution exists and an appropriate flag is set for interrogation by ML64. If a winning combination is found for X (i.e., X has three markers in a row and it is his turn to play) this solution is passed to ML64 and eventually punched to cards.

Occasionally, WHIZ chooses a poor alternative and spends a great deal of computing time looking for a nonexistent solution on that alternative. This is one of the weaknesses of the brute force method. This situation led to the introduction of kibitzing by the author.

The IBM 1130 (on which this version of QUBIC is implemented) contains 16 console entry switches (numbered zero through 15) on the front of the console typewriter (hereafter called switches). These switches can be interrogated by a program in progress. By setting switch 13 to on (up), WHIZ will halt the tree search and back up, taking the next move available on level two then stop, allowing the author to set switch 13 off then press program start. This procedure, in effect, allows the author to force WHIZ to select another alternative where a solution can be found within a reasonably short period of time. (The current status of the moves, i.e., the contents of "game," can be printed by turning on switch 15, and the table can be printed by turning on switch 14.) Although the program will run on almost any machine, only on the IBM 1130 are the switcher used.

The preceding paragraphs have been limited to a discussion of a set of programs that accept an initial situation, solve this situation and punch the result to cards. These solutions are then stored on magnetic disk.

Included, as a part of the package, is a set of programs (tailored for the IBM 1130) designed to pit the author's planar strategies against a human opponent. These programs rely heavily upon routines described previously, i.e., WHIZ, GENEX and accompanying subroutines. The essential logic of the game playing strategy is shown in Figure 28. A brief exposition of the flowchart follows.

Starting at circle A, the board is cleared and the program (always playing first) places a marker in cell 1. The opponent's first move is accepted. If the opponent places a marker in cell 22, the program will place a marker in cell 13, else the program will select cell 22. At circle B the opponent's second move is accepted. At this point in time each player has two markers on the cubic board. The situation, as it exists, may have already been solved during the a priori analysis; and, if so, the solution is stored on disk. The program searches the disk using a hashing method combined with a linear search for the current game in progress. If the game is found, the program has little to do except play the moves as determined in the a priori analysis. If the game is not on the disk, control is given to subroutine WHIZ. This subroutine determines if any solution exists, and if so, the solution is returned to the game playing program and the game will proceed accordingly.

(At this point one might ask: "Why not place the new solution to disk, since it wasn't there in the first place?" The primary reason each new solution is not automatically stored on the disk immediately upon detection is one of space. It would take far more disk storage than is usually available on an IBM 1130 to save literally all solutions. Since most solutions, to situations that have solutions, take less than 30 seconds to be solved, it was decided <u>not</u> to try to

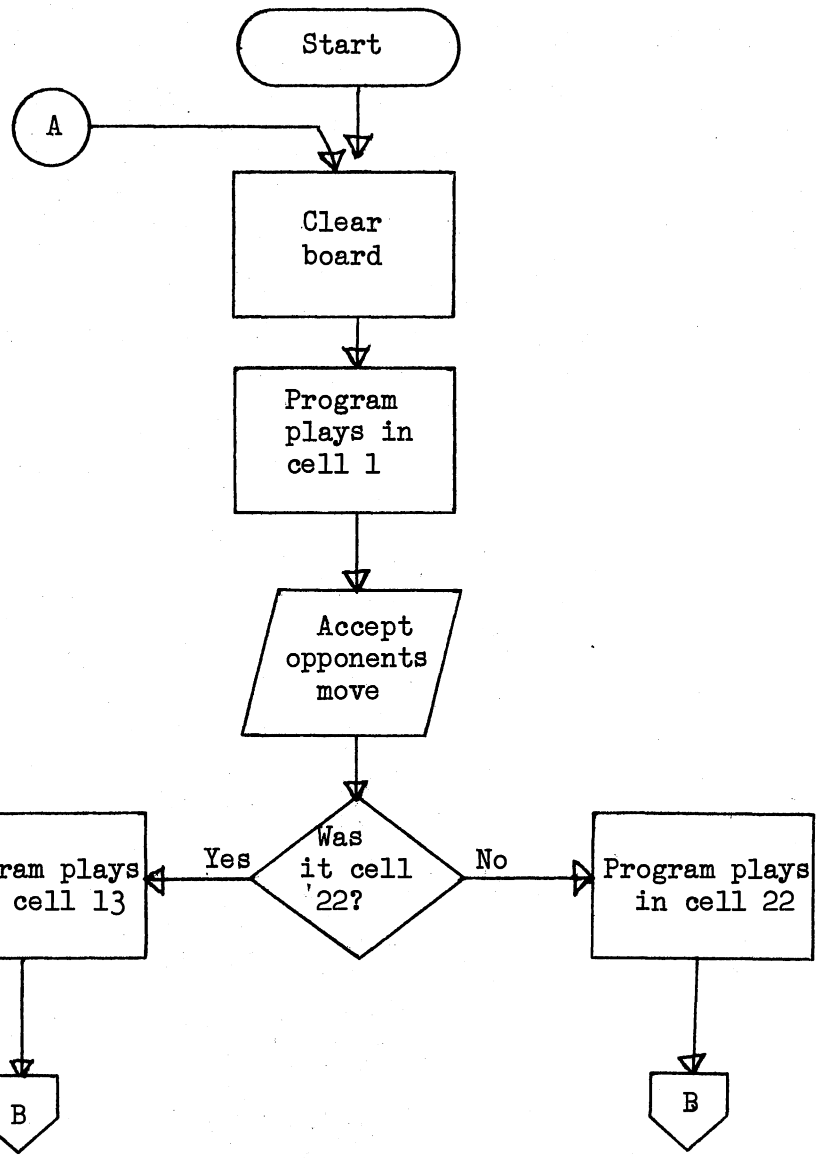

Figure 28. The Game Playing Programs

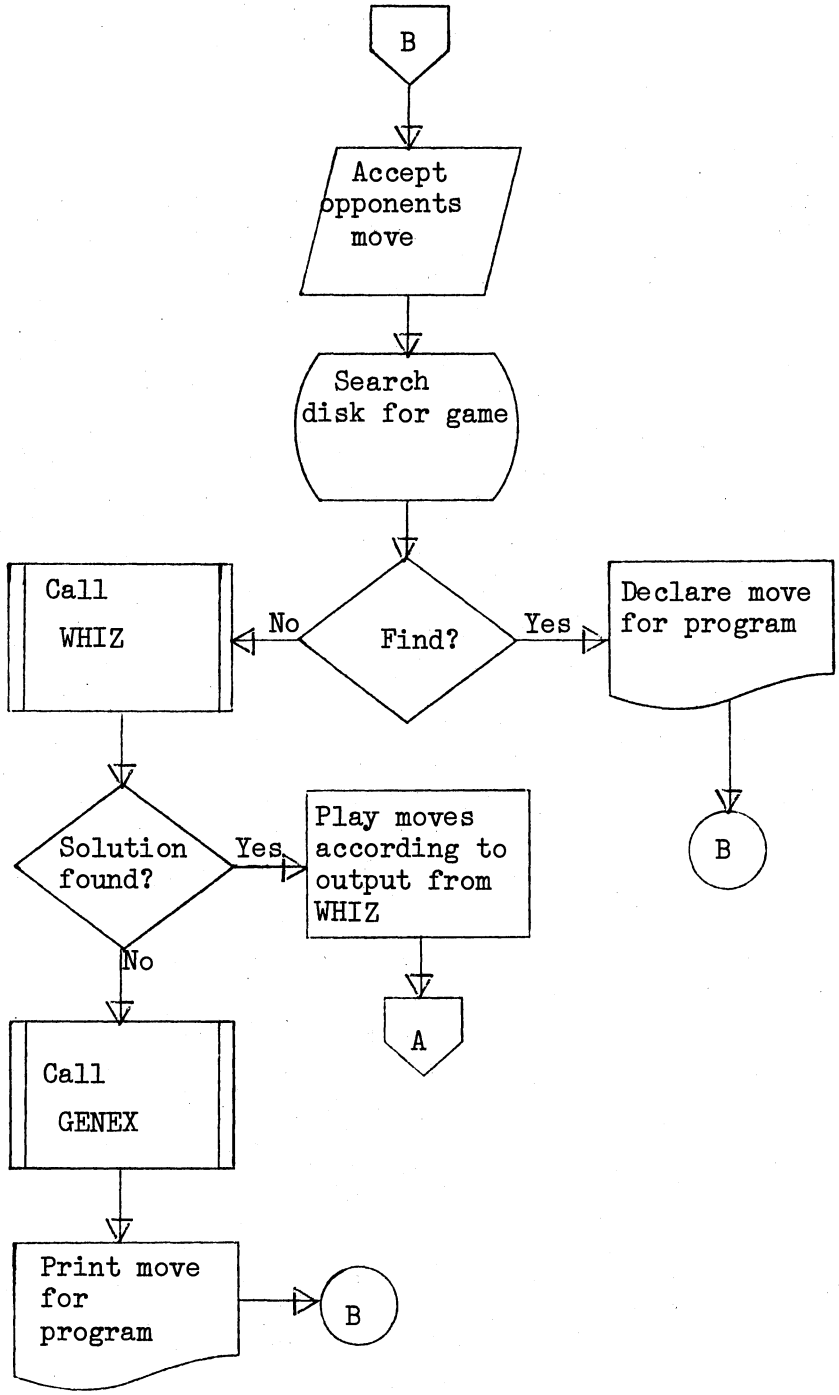

Figure 28. (Continued)

keep all solutions on the disk but, rather, to solve the "easy" ones each time they were encountered. It was felt that the savings in disk space far outweighed the timing considerations and also that the human opponent would not mind waiting a few seconds, each game, while the program "thought it over.")

If no solution exists at this point, then GENEX is given the situation. GENEX tries to place a marker into such a cell so as to strengthen the program's position. GENEX is divided into two parts. The first part is defensive, the second, offensive. Any planar traps that exist are detected and blocked. If the opponent does not have an attack pending, GENEX will attack and place a marker into a prime cell for the express purpose of developing a planar trap. After GENEX has selected the next move for X, the move is printed, control returns to circle B, and the game continues. After accepting the opponent's next move, the program will check to see if the opponent has won (not shown in Figure 28) and, if so, will concede.

Comparisons with Other Methods

Brief mention was made in Chapter I of a QUBIC playing program designed by Louden (4). This program treats the board as 76 interlocking rows. Each row is checked, in turn, for the opponent having two markers in a row or the machine having two markers in a row. When a row with one of these conditions is found, tests are then made to see if the row intersects another row with one marker. If so, the program concludes that a trap (for one player or the other) will be forthcoming and either blocks it or forces it. The most complex trap recognized by Louden's program is given in Figure 29 and is shown to be a forced win for X in three moves. The initial state reveals some important facts about this program: 1) the machine fails to play in cubic primes early in the game; 2) there are two planar non-primes occupied by X's markers; and 3) four markers within the plane were required before the forced win was recognized. These facts lead one to believe that a human opponent (or another program), drawing upon the planar strategy described earlier in this chapter, would be able to force a winning sequence every time.

```
X . X .      X - X 2
X . . .      X . - .
. X . .      0 X . .
. . . .      1 . . .
```

Figure 29. Louden's Three Level Forced Win

Gammil (3) describes a draw strategy which he attributes to unpublished work by Erdos and Selfridge. This strategy determines, dynamically, whether 0 can achieve a draw. The draw strategy relies heavily upon the Erdos-Selfridge values of a given cell. The Erdos-Selfridge values are used to evaluate dynamically each cell in the cube in terms of resources, and Gammil uses these values to generate moves in his QUBIC playing program. The scheme used is described in the following paragraphs.

The Erdos-Selfridge value of a line that contains one or more of

O's markers is zero. The value of a line that contains no O's and contains zero or more X's is 2_N where N is the number of markers X has in any given row. Quoting Gammil,

> The value of a board state V(B) is the sum of the values of all lines on the board. The value of a move V(M) is the sum of the values of the lines passing through the point, before a piece has been played there.

Gammil further states that if V(B) dips below 2^N, where N is the number of cells in one row, "the Erdos-Selfridge criterion declares that draw has been reached," i.e., if V(B) is less than 16, then O has all the rows blocked.

Within a plane, the Erdos-Selfridge strategy tends to have some limitations. Using Gammil's notation and development, the value of a plane V(P) is the sum of the values of all lines within the plane. The Erdos-Selfridge values of the planes for the 28 unique planar situations found in Table IV were determined and it was concluded that the strategy does not find all draws possible. The Erdos-Selfridge values fail to detect draws in situations four through eight of Table IV. These situations (repeated for the convenience of the reader in Table V) are certain draws for X, since O does not have two markers in a row with which to force a win. If a plane has an Erdos-Selfridge value less than 16, the draw criterion is satisfied; but the converse is true only in special cases.

TABLE V

THE FIVE UNIQUE "DRAW" SITUATIONS

4.	1	7	12
5.	1	8	14
6.	1	8	15
7.	2	5	12
8.	2	8	9

The defensive strategy implemented does not use the Erdos-Selfridge formula but rather a method of masking moves onto or out of a plane. Unlike the Erdos-Selfridge strategy, this scheme recognizes all planar situations that can force a win, and can be used for offensive or defensive play.

CHAPTER V

CONCLUSIONS, EVALUATION AND FURTHER STUDY

4^3 tic-tac-toe (QUBIC) has been examined with an eye toward the use of planar strategies. A program to find QUBIC solutions, written by the author, has been described. Given favorable conditions this program can detect winning combinations immediately after the opponent's third move.

The planar strategies described in this paper build, or find and block, traps that involve only one plane. However, the methods described fail when pitted against strategies that involve two or more planes (interplanar traps). Empirical evidence suggests that it may be possible to play a perfect game of QUBIC, but to do so, a method that can detect interplaner traps must be developed. Following the lead described in Chapter IV, several hundred complete games were punched into cards to test the author's notion that a perfect strategy may exist. All the evidence gathered indicates that theoretically QUBIC is indeed a win for the first player.

The author punched to 22,614 cards the solutions to 33 of the 198 different situations. The cost was $2,530.50 at approximately $5.00 per computing minute. Extrapolating, the total number of cards needed to solve all 198 situations should total 135,680 and the total cost, at a comparable rate, would be in the neighborhood of $15,183.00.

When all logical games either have been punched to cards or proven to be a win for X will the game of QUBIC be considered solved. Much work is left to be done before the game of QUBIC will be considered solved. As Gammil points out, brute force analysis is out of the question; however, it is the author's belief, based entirely on empirical evidence, that a select subset of situations will yield the secrets of their solutions by no other means.

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APPENDIX A

48 ROTATIONS AND REFLECTIONS OF THE CUBE

111 112 113 114	211 212 213 214	311 312 313 314	411 412 413 414
121 122 123 124	221 222 223 224	321 322 323 324	421 422 423 424
131 132 133 134	231 232 233 234	331 332 333 334	431 432 433 434
141 142 143 144	241 242 243 244	341 342 343 344	441 442 443 444
111 121 131 141	211 221 231 241	311 321 331 341	411 421 431 441
112 122 132 142	212 222 232 243	312 322 332 342	412 422 432 442
113 123 133 143	213 223 233 243	313 323 333 343	413 423 433 443
114 124 134 144	214 224 234 244	314 324 334 344	414 424 434 444
111 112 113 114	121 122 123 124	131 132 133 134	141 142 143 144
211 212 213 214	221 222 223 224	231 232 233 234	241 242 243 244
311 312 313 314	321 322 323 324	331 332 333 334	341 342 343 344
411 412 413 414	421 422 423 424	431 432 433 434	441 442 443 444
111 121 131 141	112 122 132 142	113 123 133 143	114 124 134 144
211 221 231 241	212 222 232 242	213 223 233 243	214 224 234 244
311 321 331 341	312 322 332 342	313 323 333 343	314 325 334 344
411 421 431 441	412 422 432 442	413 423 433 443	414 424 434 444
111 211 311 411	121 221 321 421	131 231 331 431	141 241 341 441
112 212 312 412	122 222 322 422	132 232 332 432	142 242 342 442
113 213 313 413	123 223 323 423	133 233 333 433	143 243 343 443
114 214 314 414	124 224 323 424	134 234 334 434	144 244 344 444
111 211 311 411	112 212 312 412	113 213 313 413	114 214 314 414
121 221 321 421	122 222 322 422	123 223 323 423	124 224 324 424
131 231 331 431	132 232 332 432	133 233 333 433	134 234 334 434
141 241 341 441	142 242 342 442	143 243 343 443	144 244 344 444
114 113 112 111	214 213 212 211	314 313 312 311	414 413 412 411
124 123 122 121	224 223 222 221	324 323 322 321	424 423 422 421
134 133 132 131	234 233 232 231	334 333 332 331	434 433 432 431
144 143 142 141	244 243 242 241	344 343 342 341	444 443 442 441
114 124 134 144	324 224 234 244	314 324 334 344	414 424 434 444
113 123 133 143	213 223 233 243	313 323 333 343	413 423 433 443
112 122 132 142	212 222 232 242	312 322 332 342	412 422 432 442
111 121 131 141	211 221 231 241	311 321 331 341	411 421 431 441

114 113 112 111   124 123 122 121   134 133 132 131   144 143 142 141
214 213 212 211   224 223 222 221   234 233 232 231   244 243 242 241
314 313 312 311   324 323 322 321   334 333 332 331   344 343 342 341
414 413 412 411   424 423 422 421   434 433 432 431   444 443 442 441

114 124 134 144   113 123 133 143   112 122 132 142   111 121 131 141
214 224 234 244   213 223 233 243   213 222 232 242   211 221 231 241
314 324 334 344   313 323 333 343   312 322 332 342   311 321 331 341
414 424 434 444   413 423 433 443   412 422 432 442   411 421 431 441

114 214 314 414   124 224 324 424   134 234 334 434   144 244 344 444
113 213 313 413   123 223 323 423   133 233 333 433   143 243 343 443
112 212 312 412   122 222 322 422   132 232 332 432   142 242 342 442
111 211 311 411   121 221 321 421   131 231 331 431   141 241 341 441

114 214 314 414   113 213 313 413   112 212 312 412   111 211 311 411
124 224 324 424   123 223 323 423   122 222 322 422   121 221 321 421
134 234 334 434   133 233 333 433   132 232 332 432   131 231 331 431
144 244 344 444   143 243 343 443   142 242 342 442   141 241 341 441

141 142 143 144   241 242 243 244   341 342 343 344   441 442 443 444
131 132 133 134   231 232 233 234   331 332 333 334   431 432 433 434
121 122 123 124   221 222 223 224   321 322 323 324   421 422 423 424
111 112 113 114   211 212 213 214   311 312 313 314   411 412 413 414

141 131 121 111   241 231 221 211   341 331 321 311   441 431 421 411
142 132 122 112   242 232 222 212   342 332 322 312   442 432 422 412
143 133 123 113   243 233 223 213   343 333 323 313   443 433 423 413
144 134 124 114   244 234 224 214   344 334 324 314   444 434 424 414

141 142 143 144   131 132 133 134   121 122 123 124   111 112 113 114
241 242 243 244   231 232 233 234   221 222 223 224   211 212 213 214
341 342 343 344   331 332 333 334   321 322 323 324   311 312 313 314
441 442 443 444   431 432 433 434   421 422 423 424   411 412 413 414

141 131 121 111   142 132 122 112   143 133 123 113   144 134 124 114
241 231 221 211   242 232 222 212   243 233 223 213   244 234 224 214
341 331 321 311   342 332 322 312   343 333 323 313   344 334 324 314
441 431 421 411   442 432 422 412   443 433 423 413   444 434 424 414

141 241 341 441   131 231 331 431   121 221 321 421   111 211 311 411
142 242 342 442   132 232 332 432   122 222 322 422   112 212 312 412
143 243 343 443   133 233 333 433   123 223 323 423   113 213 313 413
144 244 344 444   134 234 334 434   124 224 324 424   114 214 314 414

141 241 341 441   142 242 342 442   143 243 343 443   144 244 344 444
131 231 331 431   132 232 332 432   133 233 333 433   134 234 334 434
121 221 321 421   122 222 322 422   123 223 323 423   124 224 324 424
111 211 311 411   112 212 312 412   113 213 313 413   114 214 314 414

144 143 142 141   244 243 242 241   344 343 342 341   444 443 442 441
134 133 132 131   234 233 232 231   334 333 332 331   434 433 432 431
124 123 122 121   224 223 222 221   324 323 322 321   424 423 422 421
114 113 112 111   214 213 212 211   314 313 312 311   414 413 412 411

144	134	124	114	244	234	224	214	344	334	324	314	444	434	424	414
143	133	123	113	243	233	223	213	343	333	323	313	443	433	423	413
142	132	122	112	242	232	222	212	342	332	322	312	442	432	422	412
141	131	121	111	241	231	221	211	341	331	321	311	441	431	421	411
144	143	142	141	134	133	132	131	124	123	122	121	114	113	112	111
244	243	242	241	234	233	232	231	224	223	222	221	214	213	212	211
344	343	342	341	334	333	332	331	324	323	322	321	314	313	312	311
444	443	442	441	434	433	432	431	424	423	422	421	414	413	412	411
144	134	124	114	143	133	123	113	142	132	122	112	141	131	121	111
244	234	224	214	243	233	223	213	242	232	222	212	241	231	221	211
344	334	324	314	343	333	323	313	342	332	322	312	341	331	321	311
444	434	424	414	443	433	423	413	442	432	422	412	441	431	421	411
144	244	344	444	134	234	334	434	124	224	324	424	114	214	314	414
143	243	343	443	133	233	333	433	123	223	323	423	113	213	313	413
142	242	342	442	132	232	332	432	122	222	322	422	112	212	312	412
141	241	341	441	131	231	331	431	121	221	321	421	111	211	311	411
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124	224	324	424	123	223	323	423	122	222	322	422	121	221	321	421
114	214	314	414	113	213	313	413	112	212	312	412	111	211	311	411
411	412	413	414	311	312	313	314	211	212	213	214	111	112	113	114
421	422	423	424	321	322	323	324	221	222	223	224	121	122	123	124
431	432	433	434	331	332	333	334	231	232	233	234	131	132	133	134
441	442	443	444	341	342	343	344	241	242	243	244	141	142	143	144
411	421	431	441	311	321	331	341	211	221	231	241	111	121	131	141
412	422	432	442	312	322	332	342	212	222	232	242	112	122	132	142
413	423	433	443	313	323	333	343	212	223	233	243	113	123	133	143
414	424	434	444	314	324	334	344	214	224	234	244	114	124	134	144
411	412	413	414	421	422	423	424	431	432	433	434	441	442	443	444
311	312	313	314	321	322	323	324	331	332	333	334	341	342	343	344
211	212	213	214	221	222	223	224	231	232	233	234	241	242	423	244
111	112	113	114	121	122	123	124	131	132	133	134	141	142	143	144
411	421	431	441	412	422	432	442	413	423	433	443	414	424	434	444
311	321	331	341	312	322	332	342	313	323	333	343	314	324	334	344
211	221	231	241	212	222	232	242	213	223	233	243	214	224	234	244
111	121	131	141	112	122	132	142	113	123	133	143	114	124	134	144
411	311	211	111	421	321	221	121	431	331	231	131	441	341	241	141
412	312	212	112	422	322	222	122	432	332	232	132	442	342	242	142
413	313	213	113	423	323	223	123	433	333	233	133	443	343	243	143
414	314	214	114	424	324	224	124	434	334	234	134	444	344	244	144
411	311	211	111	412	312	212	112	413	313	213	113	414	314	214	114
421	321	221	121	422	322	222	122	423	323	223	123	424	324	224	124
431	331	231	131	432	332	232	132	433	333	233	133	434	334	234	134
441	341	241	141	442	342	242	142	443	343	243	143	444	344	244	144

414	413	412	411	314	313	312	311	214	213	212	211	114	113	112	111
424	423	422	421	324	323	322	321	224	223	222	221	124	123	122	121
434	433	432	431	334	333	332	331	234	233	232	231	134	133	132	131
444	443	442	441	344	343	342	341	244	243	242	241	144	143	142	141

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412	422	432	442	312	322	332	342	212	222	232	242	112	122	132	142
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314	313	312	311	324	323	322	321	334	333	332	331	344	343	342	341
214	213	212	211	224	223	222	221	234	233	232	231	244	243	242	241
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414	424	434	444	413	423	433	443	412	422	432	442	411	421	431	441
314	324	334	344	313	323	333	343	312	322	332	342	311	321	331	341
214	224	234	244	213	223	233	243	212	222	232	242	211	221	231	241
114	124	134	144	113	123	133	143	112	122	132	142	111	121	131	141

414	314	214	114	424	324	224	124	434	334	234	134	444	344	244	144
413	313	213	113	423	323	223	123	433	333	233	133	443	343	243	143
412	312	212	112	422	322	222	122	432	332	232	132	442	342	242	142
411	311	211	111	421	321	221	121	431	331	231	131	441	341	241	141

414	314	214	114	413	313	213	113	412	312	212	112	411	311	211	111
424	324	224	124	423	323	223	123	422	322	222	122	421	321	221	121
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431	432	433	434	331	332	333	334	231	232	233	234	131	132	133	134
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241	242	443	244	231	232	233	234	221	222	223	224	211	212	213	214
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141	131	121	111	142	132	122	112	143	133	123	113	144	134	124	114

441	341	241	141	431	331	231	131	421	321	221	121	411	311	211	111
442	342	242	142	432	332	232	132	422	322	222	122	412	312	212	112
443	343	243	143	433	333	233	133	423	323	223	123	413	313	213	113
444	344	244	144	434	334	234	134	424	324	224	124	414	314	214	114

441	341	241	141	442	342	242	142	443	343	243	143	444	344	244	144
431	331	231	131	432	332	232	132	433	333	233	133	434	334	234	134
421	321	221	121	422	322	222	122	423	323	223	123	424	324	224	124
411	311	211	111	412	312	212	112	413	313	213	113	414	314	214	114

444	443	442	441	344	343	342	341	244	243	242	241	144	143	142	141
434	433	432	431	334	333	332	331	234	233	232	231	134	133	132	131
424	423	422	421	324	323	322	321	224	223	222	221	124	123	122	121
414	413	412	411	314	313	312	311	214	213	212	211	114	113	112	111

444	434	424	414	344	334	324	314	244	234	224	214	144	134	124	114
443	433	423	413	343	333	323	313	243	233	223	213	143	133	123	113
442	432	422	412	342	332	322	312	242	232	222	212	142	132	122	112
441	431	421	411	341	331	321	311	241	231	221	211	141	131	121	111

444	443	442	441	434	433	432	431	424	423	422	421	414	413	412	411
344	343	342	341	334	333	332	331	324	323	322	321	314	313	312	311
244	243	242	241	234	233	232	231	224	223	222	221	214	213	212	211
144	143	142	141	134	133	132	131	124	123	122	121	114	113	112	111

444	434	424	414	443	433	423	413	442	432	422	412	411	431	421	411
344	334	324	314	343	333	323	313	342	332	322	312	341	331	321	311
244	234	224	214	243	233	223	213	242	232	222	212	241	231	221	211
144	134	124	114	143	133	123	113	142	132	122	112	141	131	121	111

444	344	244	144	434	334	234	134	424	324	224	124	414	314	214	114
443	343	243	143	433	333	233	133	423	323	223	123	413	313	213	113
442	342	242	142	432	332	232	132	422	322	222	122	412	312	212	112
441	341	241	141	431	331	231	131	421	321	221	121	411	311	211	111

444	344	244	144	443	343	243	143	442	342	242	142	441	341	241	141
434	334	234	134	433	333	233	133	432	332	232	132	431	331	231	131
424	324	224	124	423	323	223	123	422	322	222	122	421	321	221	121
414	314	214	114	413	313	213	113	412	312	212	112	411	311	211	111

APPENDIX B

THE 198 UNIQUE SITUATIONS

1 2 22 3	1 3 22 10	1 4 22 27	1 7 22 45
1 2 22 4	1 3 22 11	1 4 22 28	1 7 22 46
1 2 22 5	1 3 22 12	1 4 22 29	1 7 22 47
1 2 22 6	1 3 22 13	1 4 22 31	1 7 22 48
1 2 22 7	1 3 22 14	1 4 22 32	1 7 22 50
1 2 22 8	1 3 22 15	1 4 22 41	1 7 22 51
1 2 22 9	1 3 22 16	1 4 22 42	1 7 22 52
1 2 22 10	1 3 22 23	1 4 22 43	1 7 22 55
1 2 22 11	1 3 22 24	1 4 22 44	1 7 22 56
1 2 22 12	1 3 22 25	1 4 22 45	1 7 22 57
1 2 22 13	1 3 22 26	1 4 22 46	1 7 22 58
1 2 22 14	1 3 22 27	1 4 22 47	1 7 22 59
1 2 22 15	1 3 22 28	1 4 22 48	1 7 22 60
1 2 22 16	1 3 22 29	1 4 22 61	1 7 22 61
1 2 22 21	1 3 22 30	1 4 22 62	1 7 22 62
1 2 22 23	1 3 22 31	1 4 22 63	1 7 22 63
1 2 22 24	1 3 22 32	1 4 22 64	1 7 22 64
1 2 22 25	1 3 22 41	1 7 22 8	1 11 22 12
1 2 22 26	1 3 22 42	1 7 22 10	1 11 22 16
1 2 22 27	1 3 22 43	1 7 22 11	1 11 22 27
1 2 22 28	1 3 22 44	1 7 22 12	1 11 22 28
1 2 22 29	1 3 22 45	1 7 22 14	1 11 22 32
1 2 22 30	1 3 22 46	1 7 22 15	1 11 22 35
1 2 22 31	1 3 22 47	1 7 22 16	1 11 22 36
1 2 22 32	1 3 22 48	1 7 22 19	1 11 22 39
1 2 22 41	1 3 22 61	1 7 22 20	1 11 22 40
1 2 22 42	1 3 22 62	1 7 22 25	1 11 22 43
1 2 22 43	1 3 22 63	1 7 22 27	1 11 22 44
1 2 22 44	1 3 22 64	1 7 22 28	1 11 22 48
1 2 22 45	1 4 22 7	1 7 22 29	1 11 22 51
1 2 22 46	1 4 22 8	1 7 22 31	1 11 22 52
1 2 22 47	1 4 22 10	1 7 22 32	1 11 22 55
1 2 22 48	1 4 22 11	1 7 22 35	1 11 22 56
1 2 22 61	1 4 22 12	1 7 22 36	1 11 22 59
1 2 22 62	1 4 22 13	1 7 22 37	1 11 22 60
1 2 22 63	1 4 22 14	1 7 22 39	1 11 22 64
1 2 22 64	1 4 22 15	1 7 22 40	1 12 22 15
1 3 22 4	1 4 22 16	1 7 22 41	1 12 22 16
1 3 22 7	1 4 22 23	1 7 22 42	1 12 22 27
1 3 22 8	1 4 22 25	1 7 22 43	1 12 22 28
1 3 22 9	1 4 22 26	1 7 22 44	1 12 22 31

1 12 22 36
1 12 22 39
1 12 22 40
1 12 22 42
1 12 22 43
1 12 22 44
1 12 22 45
1 12 22 46
1 12 22 47
1 12 22 48
1 12 33 52
1 12 22 55
1 12 22 57
1 12 22 59
1 12 22 60
1 12 22 61
1 12 22 63
1 12 22 64
1 16 22 27
1 16 22 39
1 16 22 43
1 16 22 44
1 16 22 48
1 16 22 52
1 16 22 59
1 16 22 60
1 16 22 64
1 43 22 44
1 43 22 48
1 43 22 64
1 44 22 47
1 44 22 48
1 44 22 63
1 22 13 49

APPENDIX C

ALL LOGICAL GAMES FOR 1-2,22-4

1 -2 22 -4 27 -3 -5 -7 -8 -9-10-12-13-14-15-18-19-20-21-23-25-26-28-29 -30-31-34-35-36-37-39-40-41-42-45-46-47-50-51-52-53-55-56-57-58-60-61 -62-63 43-64 11-59 17-32 49-33 16-38 6

1 -2 22 -4 27-54 43-64 17-32 49-33 16-38 11-59 6

1 -2 22 -4 27-33-59 64-43 32-17 16-48 38-49 6-54 11

1 -2 22 -4 27-48 43-64 11-59 16 -6 49-38 17-32 33

1 -2 22 -4 27-11-24-44 64-43 17-32 49-33 38-16 54 -6 59

1 -2 22 -4 27 -6 26 -3 -5 -7 -8 -9-11-12-13-15-16-33-35-36-37-38-39-40 -41-42-43-44-45-46-47-48-49-51-52-53-54-55-56-57-58-59-60-61-62-63-64 28-25 30-18 32-17 29-31 20-23 24

1 -2 22 -4 27 -6 26-17-19-20-21-23-24-28-29-31-32-34-50 43-64 9-60 18 -30 52-35 13-39 5

1 -2 22 -4 27 -6 26-24 64-43 30-18 13-47 39-52 5-56 9

1 -2 22 -4 27 -6 26-18-30 43-64 9-60 11-59 25-28 57-41 12-42 10

1 -2 22 -4 27 -6 26-10 14 -3 -5 -8 -1-11-12-15-16-33-35-36-37-38-39-40 -41-42-43-44-45-46-47-48-49-51-52-53-54-55-56-57-58-59-60-61-62-63-64 28-25 30-18 32-17 29-31 20-23 24

1 -2 22 -4 27 -6 26-10 14-17-19-20-21-23-25-28-29-31-32-34-50 43-64 9 -60 18-30 52-35 13-39 5

1 -2 22 -4 27 -6 26-10 14-18-24-30 50-38 28-25 17-32 49-33 52-51 13-39 16-15 40

1 -2 22 -4 27 -6 26-10 14 -7 13 -3 -9-11-12-15-16-33-36-37-38-39-40 -41-42-43-44-45-46-47-48-49-51-52-53-54-55-56-57-58-59-60-61-62-63-64 28-25 30-18 32-17 29-31 20-23 24

1 -2 22 -4 27 -6 26-10 14 -7 13-17-18-19-21-23-24-25-28-29-30-31-32-34 -50 39-52 5-56 9

1 -2 22 -4 27 -6 26-10 14 -7 13-20 52-39 18-30 35

1 -2 22 -4 27 -6 26-10 14 -7 13 -5 8 -3 -9-11-12-15-16-33-35-36-37-38
-39-40-41-42-43-44-45-46-47-48-49-51-52-53-54-55-56-57-58-59-60-61-62
-63-64 28-25 30-18 32-17 29-31 20-23 24

1 -2 22 -4 27 -6 26-10 14 -7 13 -5 8-18-25-28-34-50 39-52 15-51 16

1 -2 22 -4 27 -6 26-10 14 -7 13 -5 8-17-19-20-21-23-24-29-30-31-32 38
-50 49-16 25-28 37

1 -2 22 -4 27 -6 26-10 14 -7 13 -8 5 -3-11-12-15-16-17-18-19-20-21-23
-24-25-282-9-30-31-32-33-34-35-36-37-38-39-40-41-42-43-44-45-46-47-48
-49-50-51-52-53-54-55-56-57-58-59-60-61-62-63-64 9

1 -2 22 -4 27 -6 26-10 14 -7 13 -8 5 -9 28-25 30-18 32-17 29-31 20-23 24

1 -2 22 -4 27 -6 26-10 14-13 7 -3 -5 -8 -9-11-12-15-16-33-35-36-38-39
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28-25 30-18 32-17 29-31 20-23 24

1 -2 22 -4 27 -6 26-10 14-13 7 -3 -5 -8 -9-11-12-15-16-33-35-36-38-39
-40-41-42-43-44-45-46-47-48-51-52-53-54-55-56-57-58-59-60-61-62-63-64
28-25 30-18 32-17 29-31 20-23 24

1 -2 22 -4 27 -6 26-10 14-13 7-20-24-25-28-30-37 38-50 49-16 17-32 33

1 -2 22 -4 27 -6 26-10 14-13 7-49 25-28 18-30 17-32 29-21 20-23 19

1 -2 22 -4 27 -6 26-10 14-13 7-34-50 28-25 32-17 20-24 29-23 30-18 31

1 -2 22 -4 27 -6 26-10 14-13 7-17-19-21-23-29-31-32 43-64 60 -9 11-59
38-50 48-33 28-25 44-12 40-36 16-49 52

1 -2 22 -4 27 -6 26-10 14-13 7-18 43-64 60 -9 11-59 3-15 63-23 50-38
28-25 44-12 41-42 61-21 17-32 49-33 52-51 16-40 46-31 58-55 62-30 54

1 -2 22 -4 27 -6 26-14 10 -3 -5 -7 -8 -9-11-12-12-15-16-33-35-36-37-38
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-63-64 28-25 30-18 32-17 29-31 20-23 24

1 -2 22 -4 27 -6 26-14 10-17-19-20-21-24-25-28-29-31-32-34-50 43-64 9
-60 18-30 52-35 13-39 5

1 -2 22 -4 27 -6 26-14 10-24 64-43 30-18 13-47 39-52 5-56 9

1 -2 22 -4 27 -6 26-14 10-18-30 43-64 9-60 42-58 57-12 25-28 41

1 -2 22 -4 27-16 26 -3 -5 -6 -7 -9-10-11-13-14-15-33-35-36-37-38-39-40
-41-42-43-44-45-47-48-49-51-52-53-54-55-56-57-58-59-60-62-63-64 28-25 30
-18 32-17 29-31 20-23 24

1 -2 22 -4 27-16 26-18-30 43-64 9-60 11-59 25-28 57-41 12-42 10

1 -2 22 -4 27-16 26-17-19-20-21-23-25-28-29-31-34-46-50-61 43-64 9-60 18-30 52-35 13-39 5

1 -2 22 -4 27-16 26-24-32 64-43 30-18 13-47 39-52 5-56 9

1 -2 22 -4 27-16 26 -8 12 -3 -5 -6 -7 -9-10-11-13-14-15-33-35-36-37-38 -39-40-41-42-43-44-45-47-48-49-51-52-53-54-55-56-57-58-59-60-62-63-64 28-25 30-18 32-17 29-31 20-23 24

1 -2 22 -4 27-16 26 -8 12-17-18-19-20-21-23-29-30-31-34-46-50-61 43-64 60 -9 28-25 44

1 -2 22 -4 27-16 26 -8 12-24 42-57 43-64 44-41 60 -9 28

1 -2 22 -4 27-16 26 -8 12-32 42-57 28-25 44-60 43-64 41

1 -2 22 -4 27-16 26 -8 12-25-28 43-64 9-60 18-30 52-35 13-39 5

1 -2 22 -4 27-16 26-12 8 -3 -5 -6 -7 -9-10-11-13-14-15-33-35-36-37-38 -39-40-41-42-43-44-45-47-48-49-51-52-53-54-55-56-57-58-59-60-62-63-64 28-25 30-18 32-17 29-31 20-23 24

1 -2 22 -4 27-16 26-12 8-18 43-64 9-60 28-25 32-17 5-13 56-39 24-40 20

1 -2 22 -4 27-16 26-12 8-30 43-64 9-60 11-59 28-25 32-17 24-20 21-23 41-61 42-44 58-10 56-40 5-39 13

1 -2 22 -4 27-16 26-12 8-17-19-20-21-23-25-29-31-34-46-50-61 43-64 9 -60 18-30 52-35 13-39 5

1 -2 22 -4 27-16 26-12 8-28 43-64 18-30 52-35 13-39 9-60 5

1 -2 22 -4 27-16 26-12 8-24-32 64-43 30-18 13-47 39-52 5-56 9

1 -2 22 -4 27-17 26 -3 -6 -7 -8-10-11-12-14-15-16-19-20-21-23-25-28-29 -31-32-33-34-36-37-38-40-41-42-45-46-48-49-50-51-53-54-55-57-58-59-61-62 -63 43-64 9-60 18-30 52-35 13-39 5

1 -2 22 -4 27-17 26-24-44 64-43 30-18 13-47 39-52 5-56 9

1 -2 22 -4 27-17 26 -5-13-18-30-35-39-47 43-64 9-60 11-59 25-28 57-41 12-42 10

1 -2 22 -4 27-17 26-52 43-64 60 -9 59-11 28-25 57-58 12-42 44

1 -2 22 -4 27-17 26-64 28-25 18-30 52-35 13-39 16-40 38-49 14-50 15

1 -2 22 -4 27-17 26 -9-56-60 43-64 11-59 6-16 25-28 18-30 52-35 13-39 49-37 38-54 50-14 51

1 -2 22 -4 27-17 26-43 28-25 18-30 52-35 39-13 15-51 16-40 49-38 56 -5 64-60 11 -6 54-59 55-53 7-23 3

1 -2 22 -4 27-32 26 -3 -6 -7 -8-10-11-12-14-15-17-19-20-21-23-25-28-33 -34-36-37-38-40-41-42-45-46-49-50-51-53-54-55-57-58-59-61-62-63 43-64 9 -60 18-30 42-35 13-39 5

1 -2 22 -4 27-32 26-16-24-44-48 64-43 30-18 13-47 39-52 5-56 9

1 -2 22 -4 27-32 26 -5-13-18-29-30-31-35-39-47 43-64 9-60 11-59 25-28 57-41 12-42 10

1 -2 22 -4 27-32 26-52 43-64 60 -9 59-11 28-25 57-58 12-42 44

1 -2 22 -4 27-32 26-64 28-25 18-30 52-35 13-39 16-40 38-49 14-50 15

1 -2 22 -4 27-32 26 -9-56-60 43-64 11-59 16 -6 49-38 28-25 52-40 39-13 51-15 50

1 -2 22 -4 27-32 26-43 28-25 18-30 52-35 39-13 15-51 16-40 49-38 56 -5 64-60 54-59 55-53 11 -6 7-23 3

1 -2 22 -4 27-38 26 -3 -5 -6 -7 -8 -9-10-11-12-13-14-15-16-33-35-36-37 -39-40-41-42-43-44-45-46-47-48-49-51-52-53-54-55-56-57-58-59-60-61-62-63 -64 28-25 30-18 32-17 29-31 20-23 24

1 -2 22 -4 27-38 26-18-30 43-64 9-60 11-59 25-28 57-41 12-42 10

1 -2 22 -4 27-38 26-17-19-20-21-23-25-28-29-31-32-34-50 43-64 9-60 18 -30 52-35 13-39 5

1 -2 22 -4 27-38 26-24 64-43 30-18 13-47 39-52 5-56 9

1 -2 22 -4 27-43 26 -3 -5 -6 -7 -8 -9-10-11-12-13-14-15-16-33-35-36-37 -38-39-40-41-42-44-45-46-47-48-49-51-52-53-54-55-56-57-58-59-60-61-62-63 -64 28-25 30-18 32-17 29-31 20-23 24

1 -2 22 -4 27-43 26-23 28-25 32-17 18-30 52-35 39-13 16-40 56 -5 64-60 48

1 -2 22 -4 27-43 26-18-19-30 17-32 49-33 38-16 50-14 52-51 13-39 5 -9 6-54 8 -7 23-53 21-24 29-20 25

1 -2 22 -4 27-43 26-17-20-24-29-31-32-34-50 28-25 18-30 52-35 39-13 15 -51 16-40 49-38 56 -5 64-60 11 -6 54-59 55-53 7-23 3

1 -2 22 -4 27-43 26-21 28-25 17-32 18-30 35-53 33-49 36-34 19-20 51 -3 39-15 5-56 42-45 12-57 10-58 9-13 11

1 -2 22 -4 27-43 26-25-28 30-18 17-32 49-33 38-16 50-14 52-51 13-39 5 -9 6-54 8 -7 23-53 24-21 56-40 64-60 47

1 -2 22 -4 27-49 26-19 34-24-44-54-59-60 64-43 30-18 13-47 39-52 5-56 9

1 -2 22 -4 27-49 26-19 34 -3 -6 -7 -8-10-11-12-14-15-16-17-20-21-23-25
-28-29-31-32-33-36-37-38-40-41-42-45-46-48-50-51-53-55-57-58-61-62-63
43-64 9-60 18-30 13 -5 52-39 35

1 -2 22 -4 27-49 26-19 34 -5-13-18-30-35-39-47 43-64 9-60 25-28 57-41
12-42 11-59 10

1 -2 22 -4 27-49 26-19 34-52-56 43-64 60 -9 59-11 28-25 57-58 12-42 44

1 -2 22 -4 27-49 26-19 34-64 28-25 18-30 52-35 13-39 16-40 14-15 38-50
6-54 11

1 -2 22 -4 27-49 26-19 34-43 28-25 32-17 18-30 52-35 16-40 64-48 56-60
39 -5 13

1 -2 22 -4 27-49 26-19 34 -9 43-64 59-11 28-25 32-17 24-20 18-30 52-35
13-39 16-40 14-15 38-50 6-54 21-23 41-61 42-44 46

1 -2 22 -4 27-49 26-17-20-21-23-25-28-29-31-32 43-64 9-60 18-30 52-35
13-39 5

1 -2 22 -4 27-49 26-24 64-43 30-18 13-47 39-52 5-56 9

1 -2 22 -4 27-49 26-13-37-50 25-28 18-30 17-32 29-21 20-23 19

1 -2 22 -4 27-49 26 -3 -5 -6 -7 -8 -9-10-11-12-14-15-16-33-35-36-38-39
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28-25 30-18 32-17 29-31 20-23 24

1 -2 22 -4 27-49 26-18-30 43-64 9-60 25-28 57-41 12-42 11-59 10

1 -2 22 -4 27-49 26-34 19-13-21-23-24-25-28-29-31-37 18-30 17-32 20

1 -2 22 -4 27-49 26-34 19 -3 -5 -6 -7 -8 -9-10-11-12-14-15-16-30-33-35
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-62-63-64 28-25 23-31 24-21 32-17 20

1 -2 22 -4 27-49 26-34 19-17-20-32 43-64 9-60 18-30 52-35 13-39 5

1 -2 22 -4 27-49 26-34 19-18 50 -9-10-11-12-41-42-43-44-54-57-58-59-60
-64 28-25 32-17 23-31 24-21 20

1 -2 22 -4 27-49 26-34 19-18 50-24-25-28 64-43 32-17 16-48 38-14 6-54
11

1 -2 22 -4 27-49 26-34 19-18 50-13-37 43-64 60 -9 59-11 57-58 12-42 28
-25 44

1 -2 22 -4 27-49 26-34 19-18 50 -3 -5 -6 -7 -8-14-15-16-17-20-21-23-29
-30-31-32-33-35-36-38-39-40-45-46-47-48-51-52-53-55-56-61-62-63 43-64
60-9 59-11 28-25 57-58 12-42 44

1 -2 22 -4 27-49 26-34 19-50 18 -3 -5 -6 -7 -8 -9-10-11-12-13-14-15-16
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1 -2 22 -4 27-49 26-34 19-50 18-30 17-32 20

1 -2 22 -4 27-64 26 -3 -5 -6 -7 -8 -9-10-11-12-13-14-15-16-33-35-36-37
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28-25 30-18 32-17 29-31 20-23 24

1 -2 22 -4 27-64 26-50 28-25 18-30 52-35 16-40 38-49 6-54 11

1 -2 22 -4 27-64 26-17-20-21-23-29-31-32 28-25 18-30 52-35 13-39 16-40
38-49 14-50 15

1 -2 22 -4 27-64 26-18-19-34 28-25 17-32 49-33 16-38 52-40 39-13 51-15
50

1 -2 22 -4 27-64 26-30 28-25 17-32 49-33 16-38 52-40 35-18 11 -6 43-59
39-47 13

1 -2 22 -4 27-64 26-28 30-18 17-32 49-33 16-38 11 -6 43-59 9-60 13 -5
14-15 46-62 61-31 41-21 25-37 57

1 -2 22 -4 27-64 26-25 30-18 17-32 49-33 16-38 11 -6 43-59 9-60 13 -5
52-39 40-28 14-15 46-62 61-31 41-21 44-42 58-55 45-29 48-47 36

1 -2 22 -4 27-64 26-24 44 -3 -5-17-18-19-20-21-23-29-31-32-33-34-35-36
28-25 60-12 43 -9 11-59 16 -6 49-38 52-40 39-13 51-15 50

1 -2 22 -4 27-64 26-24 44-12 28-25 18-30 52-35 13-39 16-40 38-49 14-50
15

1 -2 22 -4 27-64 26-24 44 -6 -7-10-11-30 28-25 60-12 9-43 13 -5 52-39
16-40 38-49 14-50 15

1 -2 22 -4 27-64 26-24 44 -8-15-16-37-38 28-25 32-17 30-18 29-31 12-60
42-57 58-10 41-43 61-21 13-45 52-39 55

1 -2 22 -4 27-64 26-24 44 -9-14 28-25 32-17 30-18 29-31 20-23 12-60 42
-57 41-43 61-21 13-45 52-39 58-55 10

1 -2 22 -4 27-64 26-24 44-28 18-30 17-32 20-19 29-23 21-25 41-61 42-43
12-57 10-58 9-11 13 -5 52-39 35
21-13 45-14 15-38 50-49 44-39 28-25 32-17 30-18 29-31 60-12 43 -9 11-59
16 -6 40-52 36-48 41-42 61-21 13-45 14-15 38-50 49

1 -2 22 -4 27-64 26-24 44-25 30-18 17-32 49-33 16-38 11 -6 43-59 9-60
41-42 61-21 46-31 62-14 10-12 13 -5 52-39 50-51 58-55 54

1 -2 22 -4 27-64 26-24 44-13 28-25 32-17 30-18 29-31 60-12 43 -9 11-59
16 -6 49-38 52-40 41-42 61-21 53-57 55-58 56-54 39 -5 51-15 50

1 -2 22 -4 27-64 26-24 44-39-40-43-52-60 25-28 18-30 17-32 20-19 29-23
21

1 -2 22 -4 27-64 26-24 44-41-43-45-46-48-49-50-51-53-54-55-56-57-58-59
-61-62-63 28-25 60-12 9-43 13 -5 52-39 18-30 35

1 -2 22 -4 27-64 26-24 44-47 28-25 60-12 9-43 13 -5 39-52 15-51 16-14
49-38 17-32 33

1 -2 22 -4 27-64 26-44 24 -3 -5 -6 -7 -8 -9-10-11-12-13-14-15-16-18-19
-21-23-29-30-31-33-34-35-36-37-38-39-40-41-42-43-45-46-47-48-49-50-51
-52-53-54-55-56-57-58-59-60-61-62-63 28-25 32-17 20

1 -2 22 -4 27-64 26-44 24-25-28 23-21 31-19 32-17 29-20 30

1 -2 22 -4 27-64 26-44 24-17-20 18-30 25-28 52-34 13-39 49-37 38-16 50
-14 51

1 -2 22 -4 27-64 26-44 24-32 23-21 31-19 18-30 20-29 52-35 13-39 25-28
9 -5 57-41 12-42 43-60 11-59 10

APPENDIX D

UNQPL, RAN16, TRS15, EVSPL, SKRBL PROGRAMS

This appendix describes the programs used to find all unique combinations of three markers of the same color in one plane.

Program Name - UNQPL

Purpose - List all unique combinations of three markers in one plane.

Methods Used - Three markers are placed in plane number one. The board is rotated and reflected all 48 ways. If any rotation or reflection results in markers in cell numbers less than the original, the rotation or reflection is printed and a flag is set. If the flag has not been set, the board is eviscerated then rotated and reflected again and the situation checked. If still no situation is found, then the board is scrambled and the entire process is repeated. This continues until all combinations of three markers on the board have been exhausted.

Program Name - RAN16

Purpose - Rotate and reflect the board all 48 ways. If any rotation or reflection is found to be "less than" the original, then print this situation and return. If any rotation or reflection yields markers in a plane other than plane one, then skip this rotation/reflection.

Methods Used - Each cell is mapped to one other cell for each of the 48 rotations and reflections. There are four nested loops, three having values from 1 to 2 and one loop having values from one to six. The three outer loops map each coordinate onto itself or onto five minus itself. The inner loop interchanges the coordinates through all six permutations. The three outer loops combine with the inner loop to give all 48 rotations and reflections of the cube onto itself.

Program Name - TRS15

Purpose - Put markers on a QUBIC board, X's markers will have a value of five, O's markers will have a value of one.

Methods Used - Input to TRS15 is in the form 1-22,16-25.... with the preceding data, cells 1 and 16 will contain values of 5 and cells 22 and 25 will contain values of 1 upon return.

Program Name - EVSPL

Purpose - Perform the evisceration mapping within the context of a plane.

Methods Used - The second and third coordinates of the eviscerated cell is determined by performing the following operations upon the corresponding coordinates of the original cell:

```
DIMENSION JEVSC(4)
DATA JEVSC/2,1,4,3/
NEW = JEVSC(OLD)
```

Restating, if the old coordinates have a value of 1,2,3 or 4, the new coordinate will have a value of 2,1,4 or 3 respectively. For speed, table loopup was used.

Program Name - SKRBL

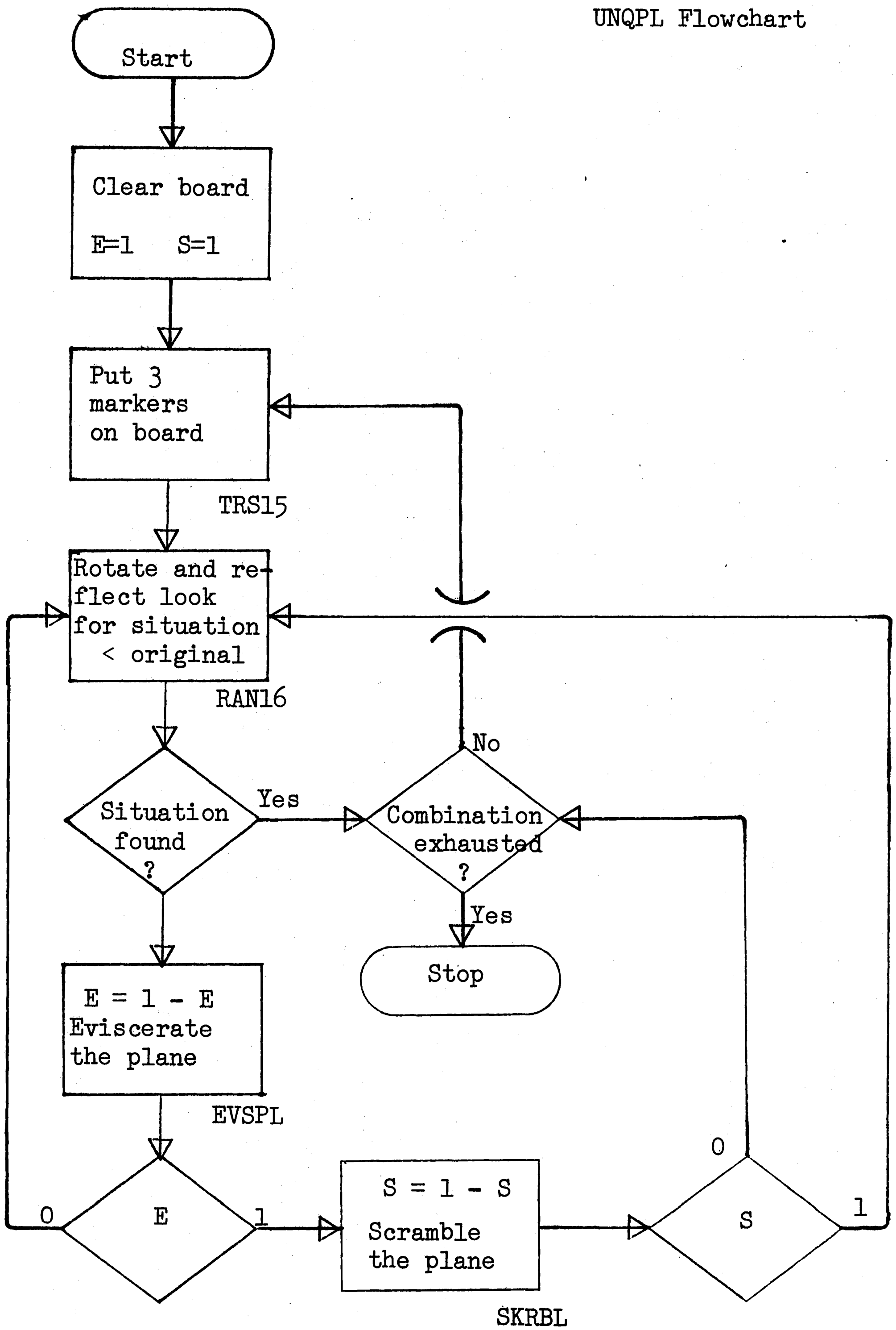
Purpose - Perform the scramble mapping upon the markers on a board.

Methods Used - Each coordinate of the scrambled cell is determined by performing the following operations upon the coordinates of the original cell:

```
DIMENSION JSKBL(4)
DATA JSKBL/1,3,2,4/
NEW = JSKBL(OLD)
```

Restating, if the old coordinates have a value of 1,2,3, or 4, the new coordinates will have a value of 1,3,2 or 4 respectively.

UNQPL Flowchart



APPENDIX E

UNQUB, RANDR, EVISC PROGRAMS

This appendix describes the programs used to find all unique combinations of four markers in a cube and X having markers in cells 1 and 22.

Program Name - UNQUB

Purpose - List all unique combinations of four markers in a cube with X having markers in cells 1 and 22.

Methods Used - Four markers are placed in the cube. X's markers are placed in cells 1 and 22. 9's markers are placed in the remaining cells, two at a time. All permutations are considered. The board is rotated, reflected, eviscerated and scrambled using a method not unlike UNQPL.

Comments - UNQUB output reveals there are 197 unique situations with four markers on the board and X has markers in cells 1 and 22. The author conjectures that there are no more. It remains to be proven rigorously.

Program Name - RANDR

Purpose - Rotate and reflect the board all 48 ways. If any rotation or reflection is found to be "less than" the original, then print this situation and return.

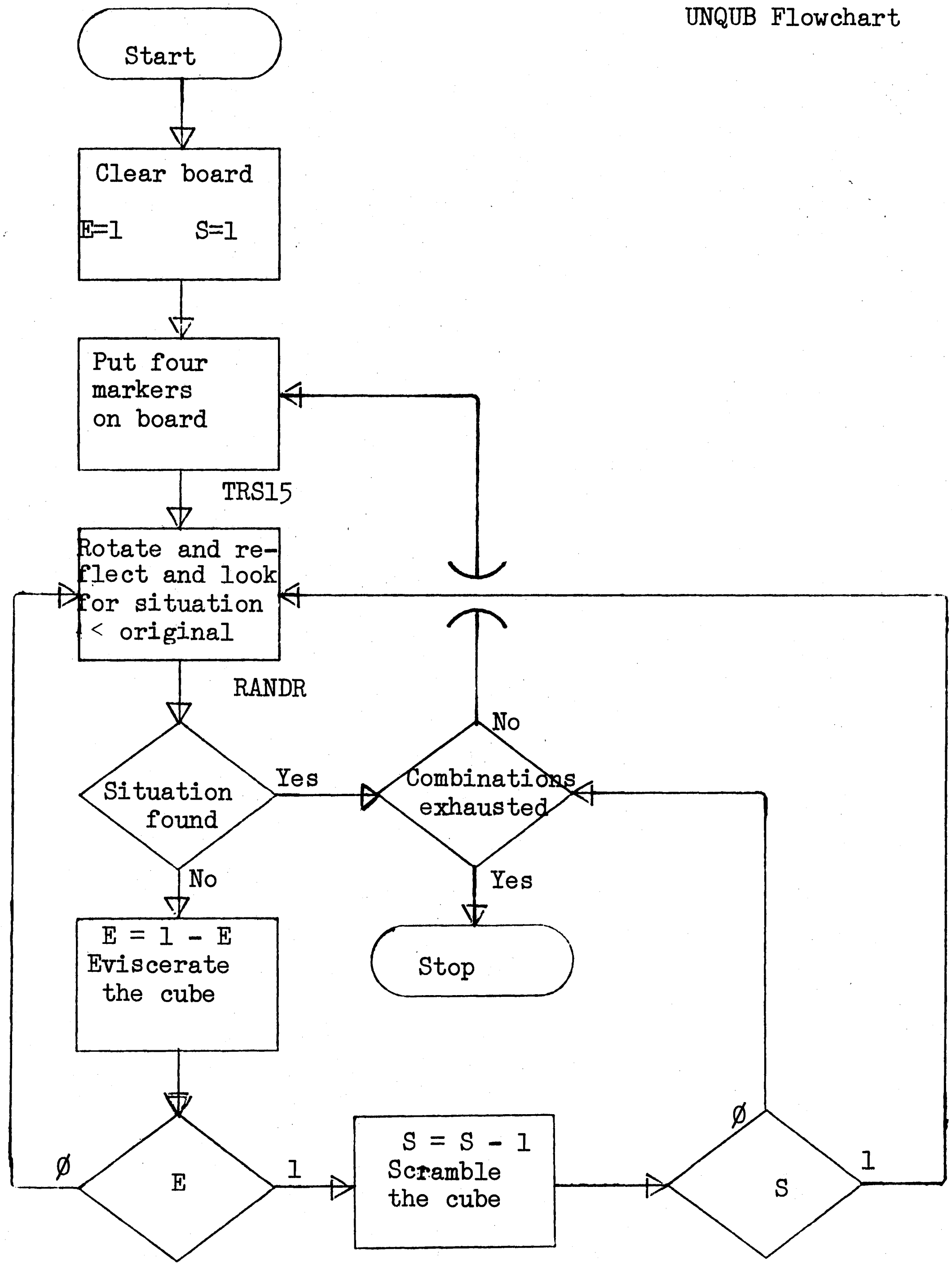
Methods used are identical to the methods used in RAN16.

Program Name - EVISC

Purpose - Perform the evisceration mapping on the markers within a cube.

Methods Used - All three coordinates are eviscerated using the scheme described in EVSPL.

UNQUB Flowchart



VITA

John William McDaniel

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Master of Science

Thesis: COMPUTER METHODS FOR PLAYING QUBIC: AN ANALYSIS AND STRATEGY IMPLEMENTATION

Major Field: Computing and Information Sciences

Biographical:

Personal Data: Born in Bakersfield, California, June 19, 1942, the son of Mrs. John W. Smithson.

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