

Quantum disentanglement eraser: A cavity QED implementation

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A possible experimental scheme of the Garisto-Hardy disentanglement eraser based on a cavity QED system is presented. This scheme can be used for a *delayed choice* quantum eraser. It also allows us to acquire single-qubit control over teleportation of an arbitrary binary state.

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Complementarity lies at the heart of quantum mechanics [1]. A classic example is the Young's double-slit experiment in which a "particle" exhibits *wavelike* behavior by displaying interference if it goes through both slits. The interference disappears and a *particlelike* behavior is exhibited if the which-path information becomes available [2]. The question then is: What would happen if we *erase* the which-path information after the particle has passed through the slits? Would such a *quantum eraser* process restore the interference fringes? The answer is "yes" [3], and this has been verified experimentally [4,5]. This erasure and fringe retrieval can be achieved even *after* the atoms hit the screen [6].

There is a close link between the notions of quantum eraser and entanglement. For example, in any setup for quantum eraser, the which-path information, and therefore the disappearance of the fringes, is achieved by entangling the state of the particle with another controlling *qubit* that contains the which-path information. One can, however, restore the fringes by erasing the which-path information contained in the controlling qubit.

Garisto and Hardy [7] described an interesting new class of quantum erasers, called *disentanglement eraser*. These consist of at least three subsystems: *A*, *B*, and *T*. The *AB* subsystem is prepared in an entangled state,

$$|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|0_A, 1_B\rangle + |1_A, 0_B\rangle). \quad (1)$$

If the pieces of this entangled state are tagged with *T*, such that the state of the whole system is

$$|\psi_{ABT}\rangle = (|0_A, 1_B\rangle|0_T\rangle + |1_A, 0_B\rangle|1_T\rangle)/\sqrt{2}, \quad (2)$$

then the purity of the entanglement of the subsystem *AB* is lost as the state of the *AB* subsystem is described by the statistical mixture,

$$\rho_{AB} = (|0_A, 1_B\rangle\langle 0_A, 1_B| + |1_A, 0_B\rangle\langle 1_A, 0_B|)/2. \quad (3)$$

However, if the tagged information is erased, then the entanglement of the *AB* subsystem is restored. In order to see this we define the superposition states of the tagged state $|\pm_T\rangle = (|0_T\rangle \pm |1_T\rangle)/\sqrt{2}$. The state of the combined *ABT* system (2) can then be rewritten as

$$|\psi_{ABT}\rangle = \frac{1}{\sqrt{2}} \left\{ |+_T\rangle \left[\frac{1}{\sqrt{2}}(|0_A, 1_B\rangle + |1_A, 0_B\rangle) \right] + |-_T\rangle \left[\frac{1}{\sqrt{2}}(|0_A, 1_B\rangle - |1_A, 0_B\rangle) \right] \right\}. \quad (4)$$

An outcome $|+_T\rangle$ for the tagged state therefore restores the original state (1) for the *AB* subsystem, whereas the outcome $|-_T\rangle$ yields $(|0_A, 1_B\rangle - |1_A, 0_B\rangle)/\sqrt{2}$. A phase shift then restores the original state. Thus a measurement of tagging qubit restores the entangled state. This *disentanglement eraser* of Garisto and Hardy shows that "the entanglement of any two particles that do not interact (directly or indirectly), never disappears but rather is encoded in the ancilla of the system." An implementation of such eraser has been demonstrated in NMR systems [8].

In this paper, we propose the usage of two high quality cavities to realize a new type of quantum eraser as proposed by Garisto and Hardy [7]. In our work, we use an auxiliary atom's passage through the first cavity to control the degree of entanglement between the two cavities. We further discuss how these changes can be studied by using a probe atom and post measurement on the auxiliary atom.

We consider a system of two high-*Q* microwave cavities *A* and *B* initially in the vacuum states. The cavities can be prepared in the entangled state (1) by first passing an atom (atom 1) in excited state $|a\rangle$ through the two cavities (Fig. 1). The interaction times between the atom and the cavities are chosen such that the interaction time with cavity *A* corresponds to a $\pi/2$ pulse, with the resulting state,

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|0_A\rangle|a_1\rangle + |1_A\rangle|b_1\rangle)|0_B\rangle; \quad (5)$$

and the interaction time with cavity *B* corresponds to a π pulse, yielding the final state,

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0_A\rangle|1_B\rangle + |1_A\rangle|0_B\rangle)|b_1\rangle. \quad (6)$$

Thus the photons in the cavities *A* and *B* are entangled and atom 1 is decoupled from the *AB* system.

In the second step we *tag* a qubit with the entangled state (6). The tagging qubit in our system is a three-level atom

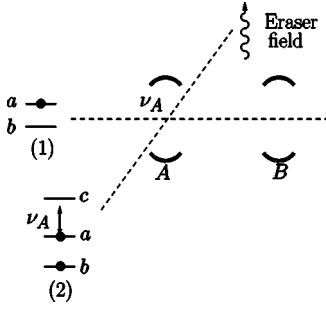


FIG. 1. The preparation of entangled states. Cavities A and B , each resonant with the $|a\rangle-|b\rangle$ transition, are prepared in an entangled state [Eq. (6)] by passing the two-level atom 1 initially in the excited state $|a\rangle$ with appropriate passage times through the two cavities. The entangled state given by Eq. (8) is prepared via the passage of the three-level atom 2 through the cavity A .

(atom 2) that passes through cavity A only (Fig. 1). The level structure of atom 2 is such that the field inside cavity A is nonresonant with the $|a\rangle \rightarrow |b\rangle$ transition, but is dispersively coupled with the $|c\rangle \rightarrow |a\rangle$ transition with $\omega_{ca} = \nu_A + \Delta$. Here the atomic levels $|a\rangle$ and $|b\rangle$ represent the states $|0\rangle$ and $|1\rangle$ for the tagging qubit, respectively. The effective Hamiltonian for the atom and the cavity field is given by $\mathcal{H}_{eff} = (\hbar g^2 / \Delta)(aa^\dagger |c\rangle\langle c| - a^\dagger a |a\rangle\langle a|)$, where g is a coupling coefficient, and a and a^\dagger are destruction and creation operators, respectively, for the field state inside the cavity [9]. Atom 2 is initially prepared in a superposition state $(|a\rangle + |b\rangle) / \sqrt{2}$. After passage through cavity A , a quantum phase gate is made with a phase shift $\eta = g^2 \tau / \Delta$. Here τ is the interaction time between the atom and the cavity. Such a quantum phase gate has been discussed and experimentally implemented in [10].

The resulting state after the passage of the atom through the cavity is

$$|\psi_3\rangle = e^{-i\mathcal{H}_{eff}/\hbar \tau} |\psi_2\rangle = \frac{1}{\sqrt{2}} \left\{ |b_2\rangle \left[\frac{1}{\sqrt{2}} (|0_A, 1_B\rangle + |1_A, 0_B\rangle) \right] + |a_2\rangle \left[\frac{1}{\sqrt{2}} (|0_A, 1_B\rangle + e^{i\eta} |1_A, 0_B\rangle) \right] \right\}. \quad (7)$$

We chose the interaction time τ such that $\eta = \pi$. Then $|\psi_3\rangle$ is of the form (4). If atom 2 then interacts with a classical Ramsey field, such that $(|a\rangle + |b\rangle) / \sqrt{2} \rightarrow |a\rangle$ and $(-|a\rangle + |b\rangle) / \sqrt{2} \rightarrow |b\rangle$, we obtain

$$|\psi_4\rangle = \frac{1}{\sqrt{2}} (|0_A\rangle |1_B\rangle |a_2\rangle + |1_A\rangle |0_B\rangle |b_2\rangle) |b_1\rangle. \quad (8)$$

We have thus obtained an entangled state of the form (2) and the entanglement between photons inside the cavities A and B is controlled by atom 2.

For $\eta = \pi$, a slight change in the proposal would make it easier to have it implemented experimentally. Instead of assuming a dispersive interaction between levels $|a\rangle$ and $|c\rangle$ and the cavity mode, we may assume a resonant interaction, such that if the atom is in state $|a\rangle$ and there is one photon in the cavity, the atom undergoes a 2π Rabi rotation, thus changing its phase by a minus sign. A resonant 2π transition

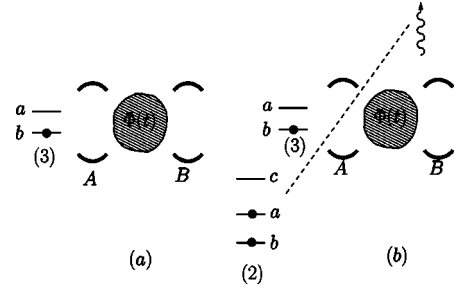


FIG. 2. (a) A two-level atom initially in its ground state $|b\rangle$ passes through the entangled cavities A and B . The atom in state $|a\rangle$ acquires a phase shift while passing between the two cavities. The probability of finding the atom in the excited or ground state exhibits interference fringes. (b) The system is the same as in (a), but a tagging qubit can partially or completely erase the interference fringes via dispersive coupling of a three-level atom with cavity A .

requires a smaller interaction time than a dispersive interaction, thus making the proposal more resistant to decoherence. This 2π -phase change was demonstrated experimentally by Noguez *et al.* [11].

We now consider two examples where the entanglement-based phenomena can be controlled by the tagging qubit. In the first example, we consider an atomic double cavity Ramsey interferometry, where the appearance or disappearance of interference fringes in the final atomic state probability depends on the presence or absence of entanglement between the photons of the two cavities. In the second example, we consider the control of quantum teleportation via the tagging qubit.

We proceed to analyze the passage of a two-level atom 3 initially in its ground state $|b\rangle$ through two cavities that are initially in an entangled state (6). First, we discuss the system in the absence of a tagging qubit [Fig. 2(a)]. A theoretical study and an experimental demonstration of a similar setup are discussed in [12–14]. The initial atom-field state is

$$|\psi_1\rangle = \frac{1}{\sqrt{2}} (|0_A, 1_B\rangle + |1_A, 0_B\rangle) |b_3\rangle. \quad (9)$$

After the passage through cavity A with a passage time corresponding to a π pulse, we have

$$|\psi_2\rangle = \frac{1}{\sqrt{2}} (|b_3, 1_B\rangle - |a_3, 0_B\rangle) |0_A\rangle. \quad (10)$$

If the atom interacts with a classical field dispersively, such that a phase $\Phi(t)$ is acquired when the atom is in the excited state $|a\rangle$ and no phase when it is in ground state $|b\rangle$, then the resulting state is

$$|\psi_3\rangle = \frac{1}{\sqrt{2}} [|b_3, 1_B\rangle - e^{i\Phi(t)} |a_3, 0_B\rangle] |0_A\rangle. \quad (11)$$

Finally the atomic passage through cavity B with interaction time corresponding to a $\pi/2$ pulse yields

$$|\psi_4\rangle = \frac{1}{2} [|b_3, 1_B\rangle (1 - e^{i\Phi(t)}) + |a_3, 0_B\rangle (1 + e^{i\Phi(t)})] |0_A\rangle. \quad (12)$$

The probability of finding atom 3 in the excited and ground states exhibits antisymmetric interference fringes, i.e.,

$$P_{a_3} = 1 - P_{b_3} = \frac{1}{2} [1 + \cos \Phi(t)]. \quad (13)$$

The appearance of the fringes (say in P_{a_3}) is due to the interference between two paths: The atom absorbs a photon in cavity A or in cavity B.

The system is, however, more complicated when the tagging atom (atom 2) passes through cavity A [Fig. 2(b)]. In this case, the initial state is

$$|\psi'_1\rangle = \frac{1}{\sqrt{2}} [|0_A\rangle |1_B\rangle |a_2\rangle + |1_A\rangle |0_B\rangle |b_2\rangle] |b_3\rangle. \quad (14)$$

Atom 3 now passes through the cavities A and B with a phase shift $\Phi(t)$ acquired by the level $|a\rangle$, while the atom passes in between the two cavities. The final state after the passage of atom 3 can be determined as above, and is given by

$$|\psi'_4\rangle = \frac{1}{2} \{ |b_3, 1_B\rangle [|a_2\rangle - |b_2\rangle e^{i\Phi(t)}] - |a_3, 0_B\rangle [|a_2\rangle + |b_2\rangle e^{i\Phi(t)}] \} |0_A\rangle. \quad (15)$$

The probabilities for atom 3 to be in the excited and ground states are

$$P_{a_3} = P_{b_3} = \frac{1}{2}, \quad (16)$$

and the fringes disappear. The disappearance of the fringes is due to the availability of which-path information. The state of the tagged atom controls the which-path information. We assume that the atom is found in the ground state. Thus the atom must have followed the path absorbing a photon in cavity A and emitting the photon in cavity B if the tag atom 2 is in state $|b_2\rangle$. Similarly, if it followed the path with no interaction with either cavity A or B, the tag atom 2 must be in state $|a_2\rangle$.

How do we restore the interference fringes? In a ‘‘delayed choice’’ quantum eraser we might like to do so even after atom 3 has been detected. A quantum eraser would require erasing the which-path information.

Let the tag atom 2 interact with a classical Ramsey field, such that

$$\begin{aligned} |a_2\rangle &\rightarrow \cos \theta |a_2\rangle + \sin \theta |b_2\rangle \\ |b_2\rangle &\rightarrow -\sin \theta |a_2\rangle + \cos \theta |b_2\rangle. \end{aligned} \quad (17)$$

Here the angle θ depends on the Rabi frequency of the field and the interaction time.

A subsequent measurement of tag atom in state $|a_2\rangle$ yields the following expressions for the probabilities:

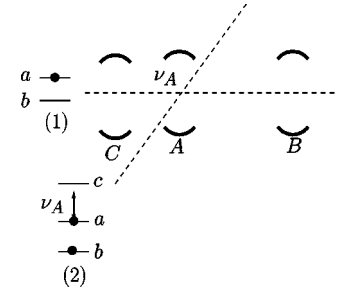


FIG. 3. Cavities A and B are prepared in an entangled state (1) via passage of atom 1 initially in its excited state. The field inside cavity C is teleported to cavity B by making a Bell-state measurement of AC system. The tagging qubit is atom 2 that interacts with the field in cavity A dispersively.

$$P_{a_3} = 1 - P_{b_3} = \frac{1}{2} [1 - \sin 2\theta \cos \Phi(t)] \quad (18)$$

Similar expressions are obtained if the tag atom is detected in the ground state $|b_2\rangle$.

The visibility of the fringes is given by $\mathcal{V} = \sin 2\theta$. It is clear that the interference fringes are fully restored (with unit visibility) for $\theta = \pi/4$. We then obtain

$$P_{a_3} = \frac{1}{2} [1 - \cos \Phi(t)], \quad (19)$$

if atom 2 is found in state $|a_2\rangle$ and

$$P_{a_3} = \frac{1}{2} [1 + \cos \Phi(t)], \quad (20)$$

if atom 2 is found in state $|b_2\rangle$.

As a second example, we consider a control on quantum teleportation via tagging qubit. In a teleportation scheme [15], two systems, A and B, are prepared in an entangled state (1). A quantum state of system C,

$$|\psi_C\rangle = c_0 |0_C\rangle + c_1 |1_C\rangle, \quad (21)$$

can be teleported to system B by making a Bell-basis measurement of the AC system and communicating the outcome to B.

Here we consider the system shown in Fig. 3, where a quantum state of the radiation field of the form (21) is teleported from cavity C to cavity B. As before, we prepare an entangled state between cavities A and B along with the tagged qubit T (atom 2),

$$|\psi_{ABT}\rangle = \frac{1}{\sqrt{2}} [|0_A\rangle |1_B\rangle |a_2\rangle + |1_A\rangle |0_B\rangle |b_2\rangle]. \quad (22)$$

The state of the combined system ABCT can be written as

$$\begin{aligned} |\psi_{ABCT}\rangle &= \frac{1}{2\sqrt{2}} [+_2 \{ \psi_{AC}^+ (c_0 |0_B\rangle + c_1 |1_B\rangle) + \phi_{AC}^+ (c_0 |1_B\rangle \\ &\quad + c_1 |0_B\rangle) - \psi_{AC}^- (c_0 |0_B\rangle - c_1 |1_B\rangle) + \phi_{AC}^- (c_0 |1_B\rangle \\ &\quad - c_1 |0_B\rangle) \}] + \frac{1}{2\sqrt{2}} [-_2 \{ \psi_{AC}^+ (-c_0 |0_B\rangle + c_1 |1_B\rangle) \}] \end{aligned}$$

$$+ \phi_{AC}^+(c_0|1_B\rangle - c_1|0_B\rangle) + \psi_{AC}^-(c_0|0_B\rangle + c_1|1_B\rangle) \\ + \phi_{AC}^-(c_0|1_B\rangle + c_1|0_B\rangle)], \quad (23)$$

where

$$|\psi_{AC}^+\rangle = (|0_A\rangle|1_C\rangle \pm |1_A\rangle|0_C\rangle)/\sqrt{2},$$

$$|\psi_{AC}^-\rangle = (|0_A\rangle|0_C\rangle \pm |1_A\rangle|1_C\rangle)/\sqrt{2}$$

are the states of the AC system in the Bell basis. A measurement of the Bell basis can be carried out along the lines proposed in [16]. A Bell-basis measurement of (say) $|\psi_{AC}^+\rangle$ reduces the BT state to

$$|\psi_{BT}\rangle = \frac{1}{2}[c_0|0_B\rangle(|_{+2}\rangle - |_{-2}\rangle) + c_1|1_B\rangle(|_{+2}\rangle + |_{-2}\rangle)] \\ = \frac{1}{\sqrt{2}}(c_0|0_B1_2\rangle + c_1|1_B0_2\rangle). \quad (24)$$

It follows on taking a trace over the tagging qubit, that the teleportation is not accomplished; the state inside cavity B will be a statistical mixture of states $|0\rangle$ and $|1\rangle$. We however note that the state (21) is teleported if the tag state is measured in the $|\pm\rangle$ basis and the outcome is communicated

through a classical channel to B . Thus the tagging qubit is therefore a controller bit for teleportation.

In conclusion, we have demonstrated how the high quality cavities can be used to realize the new class of quantum erasers referred to as quantum disentanglement erasers. We have further shown the possibility of controlling quantum teleportation by the tagging qubit.

Finally, we also mention that our proposal should be perfectly feasible as we make use of the well-known methods developed in cavity QED [17,18]. The use of resonant π and other type of pulses is perfected by them. The dispersive interaction has been used by them in a number of experiments including the implementation of a quantum phase gate [10]. The only additional complication that we bring in is the usage of multiple cavities. This problem can be overcome by considering the two cavity fields by the two modes of the same cavity. The atomic interaction with these modes can be controlled by shifting the atomic levels via electric (Stark effect) or magnetic (Zeeman shift) fields. Note that, for cavities in use, the decoherence is not a serious issue.

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