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Entangling pairs of nano-cantilevers, Cooper-pair boxes and mesoscopic teleportation

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Abstract. We propose two schemes to establish entanglement between two mesoscopic quantum systems through a third mesoscopic quantum system. The first scheme entangles two nano-mechanical oscillators in a non-Gaussian entangled state through a Cooper-pair box (CPB). Entanglement detection of the nano-mechanical oscillators is equivalent to a teleportation experiment in a mesoscopic setting. The second scheme can entangle two CPB qubits through a nano-mechanical oscillator in the presence of arbitrarily strong decoherence.

Contents

Probing quantum superpositions and entanglement with mesoscopic mechanical systems has recently developed into an area of substantial interest [1]–[9]. The most striking experimental demonstrations are the interferometry of mesoscopic free particles (molecules) [1] and the entangling of mesoscopic atomic ensembles [2]. Proposals for the generation of entanglement between Bose–Einstein condensates [3, 10] and coherence between states of mesoscopic atomic ensembles have been made [4]. Some early proposals involving harmonically bound mesoscopic systems were based on opto-mechanical effects where schemes for observing coherent superpositions of states of the movable mirror [5] and entanglement between two such mirrors [6] were proposed. Soon, however, a canonical system of a Cooper-pair box coupled to a mesoscopic cantilever was introduced [7]. It offered an optics-free, fully nano-technological alternative, with switchable couplings for such schemes. Accordingly, a scheme to observe coherent superpositions between states of a mesoscopic cantilever, as well as its entanglement with a Cooper-pair box (CPB) was proposed [7]. Recently, interferometric proposals to probe superpositions of states of movable mirrors have also been proposed [8]. Recently, a proposal to entangle two well-separated nano-electromechanical oscillators through a harmonic chain has also been made [9]. A host of other quantum effects are expected to be seen in mesoscopic mechanical systems [11]–[17]. Even quantum computation has been proposed with such systems [18]. These theoretical proposals are fuelled by the rapid technological progress in the fabrication of nano-mechanical systems and experiments approaching the quantum regime [19, 20].

The Hamiltonian which generates entanglement between a CPB and a cantilever in [7] offers many more exciting entangling possibilities even with *minimal* additions to the number of systems, such as just one extra CPB *or* just one extra cantilever. In this paper, we show that with the above minimal addition, one can entangle two mesoscopic systems of the *same* dimension: two discrete variable systems (two CPBs) or two continuous variable systems (two nano-mechanical cantilevers (NCs)). One can also verify their entanglement with an entanglement witness or teleportation with higher than classically achievable fidelity. An interesting feature of the entangling of the cantilevers is that they are placed in a *non-Gaussian* continuous variable entangled state as a result of our scheme. To date, only Gaussian entangled states have been used in continuous variable implementations of quantum information processing [21], and the scheme we suggest might enable one to realize a non-Gaussian entangled state. The scheme we suggest for detection of the non-Gaussian entanglement is equivalent to possibly the simplest realization of a quantum teleportation experiment with entangled NCs. Positive features of the entangling scheme for the CPBs are its applicability in entangling non-neighbouring (not directly interacting) boxes in an array and its robustness to the thermal nature as well as decoherence of the states of the mediating cantilever. Most importantly, our schemes seek to extend the domain of quantum behaviour by entangling two mesoscopic systems through a third mesoscopic system.

1. Entangling two nano-cantilevers

A CPB is an example of a qubit with states $|0\rangle$ and $|1\rangle$ representing n or $n+1$ Cooper pairs in the box [7, 22]. It can be made to evolve under a Hamiltonian $-\frac{E_J}{2}\sigma_x$ by the application of an appropriate voltage pulse [7, 22], where σ_x is the Pauli-X operator and the parameter E_J is called the Josephson coupling. This gives rise to coherent oscillations between the $|0\rangle$ and $|1\rangle$ states as observed in [22]. A NC, on the other hand is a simple example of a quantum harmonic oscillator. We now proceed to the proposal for entangling two cantilevers based on their interaction with a

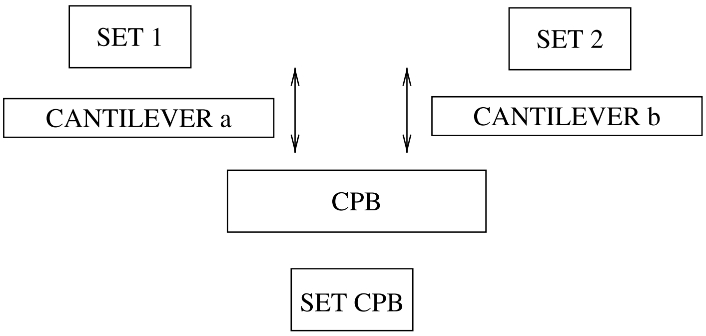

Figure 1. The figure shows a schematic diagram of the setup for entangling two cantilevers, denoted as cantilever 1 and cantilever 2 respectively, through a CPB. For the entangling, measurements are only needed to be performed on the CPB, which is done with the help of the single electron transistor SET CPB. For verification of the entanglement of cantilevers 1 and 2 by a mesoscopic teleportation, measurements need to be performed on them through SET1 and SET2 respectively.

single CPB. The setup is shown in figure 1. The Hamiltonian required for the scheme is given by

$$H = -2E_C\sigma_z + \hbar\omega_m a^\dagger a + \hbar\omega_m b^\dagger b + \lambda\{(a + a^\dagger) + (b + b^\dagger)\}\sigma_z, \tag{1}$$

where the parameter E_C is called the charging energy of the CPB, σ_z is the Pauli-Z operator for the CPB, operators a, $a^\dagger$ and b, $b^\dagger$ are the creation/annihilation operators for two oscillators and λ is a coupling strength. We assume that the NCs are prepared initially in their ground state (this is quite realistic for the GHz oscillators available now [20] by cooling, as suggested in [18]). Accordingly, we start with the cantilevers in the initial state $|0\rangle_a|0\rangle_b$, where subscripts a and b denote the two cantilevers, and the CPB in the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. (This state can be prepared by using a voltage pulse to accomplish a $\pi/2$ rotation about the x-axis through $-\frac{E_J}{2}\sigma_x$ followed by local phase adjustments). The evolution induced by the Hamiltonian H has the feature that corresponding to the $|0\rangle$ state of the CPB the cantilevers undergo an oscillation given by the state $|\beta(\mathrm{e}^{-\mathrm{i}\omega_m t} - 1)\rangle$ and corresponding to the $|1\rangle$ state of the CPB the cantilevers undergo an oscillation given by the state $|-\beta(\mathrm{e}^{-\mathrm{i}\omega_m t} - 1)\rangle$. The evolution that takes place in a time $T = \pi/\omega_m$ is

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle_a|0\rangle_b \rightarrow \frac{1}{\sqrt{2}}(\mathrm{e}^{-\mathrm{i}\frac{2E_C T}{\hbar}}|0\rangle|{-2\beta}\rangle_a|{-2\beta}\rangle_b + \mathrm{e}^{\mathrm{i}\frac{2E_C T}{\hbar}}|1\rangle|2\beta\rangle_a|2\beta\rangle_b), \tag{2}$$

where $\beta = \lambda/\hbar\omega_m$ is a dimensionless coupling and $|\pm 2\beta\rangle$ are coherent states. For simplicity, we will assume that $\frac{2E_C T}{\hbar}$ is an integral multiple of 2π. We now measure the CPB in the basis $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ to obtain the state

$$|\psi(\pm)\rangle_{ab} = \frac{1}{\sqrt{2}}(|{-2\beta}\rangle_a|{-2\beta}\rangle_b \pm |2\beta\rangle_a|2\beta\rangle_b), \tag{3}$$

where the upper and lower signs stand for the $|+\rangle$ and $|-\rangle$ outcomes respectively. If $\beta \sim 1$, as will happen, for example, if one takes the parameters of [7] (even if one took the higher frequency oscillators of [20], which decreases λ by about a factor of 6, one can reduce the mass of the oscillators by the same factor to keep $\beta \sim 1$) then $|\langle -2\beta|2\beta\rangle|^2 = \mathrm{e}^{-16\beta^2} \sim O(10^{-7})$. This means that the states $|-2\beta\rangle$ and $|2\beta\rangle$ involved in $|\psi(\pm)\rangle_{ab}$ are nearly orthogonal and thus $|\psi(\pm)\rangle_{ab}$ has nearly one ebit of entanglement. The above also implies that each outcome has a probability of nearly 1/2 to occur. $|\psi(\pm)\rangle_{ab}$ are a class of non-Gaussian continuous variable entangled states known as *entangled coherent states*, proposed originally in the optical context [23]. An analogous calculation will show that the scheme also works if the cantilevers started in coherent states of nonzero amplitude.

2. Verifying the entanglement of the cantilevers by teleportation

An interesting question now is how to verify the entanglement of the states $|\psi(\pm)\rangle_{ab}$. The non-local character can be ascertained *in principle* from Bell's inequality experiments [24]. However, these involve measurements in a highly non-classical (Schrödinger Cat-like) basis [24], and could be rather difficult for an NC. For an NC, position/momentum measurements seem natural. Unfortunately, from joint uncertainties in position and momentum of the two NCs, the entangled nature of the state $|\psi(\pm)\rangle_{ab}$ cannot be inferred. We will thus use quantum teleportation through $|\psi(\pm)\rangle_{ab}$ to demonstrate its entangled nature. Note that the possibility of teleportation of Schrödinger Cat states of a third oscillator through the entangled coherent state of two oscillators has already been pointed out by van Enk and Hirota [25] in the quantum optical context. However, for NCs, preparing a third NC in a highly non-classical state such as a Schrödinger Cat is challenging, making it directly interact with one of the entangled NCs is difficult and, moreover, we do not want to increase the complexity of the system by adding an extra NC. We will thus concentrate on the teleportation of the state of a qubit through $|\psi(\pm)\rangle_{ab}$ with better than classically achievable (2/3) fidelity. This will prove the entangled nature of the state $|\psi(\pm)\rangle_{ab}$.

For the teleportation protocol, first assume that the NCs were prepared in $|\psi(+)\rangle_{ab}$ as a result of the measurement of the CPB in the $|\pm\rangle$ basis. The CPB is now, of course, disentangled from the state of the NCs. It is thus now prepared in the arbitrary state $\cos\theta/2|0\rangle + \mathrm{e}^{\mathrm{i}\delta}\sin\theta/2|1\rangle$ which we want to teleport through $|\psi(+)\rangle_{ab}$. The CPB interacts with cantilever a for a time T and the resulting evolution is:

$$(\cos\theta/2|0\rangle + \mathrm{e}^{\mathrm{i}\delta}\sin\theta/2|1\rangle)|\psi(+)\rangle_{ab} \to \frac{1}{\sqrt{2}}(\cos\theta/2|0\rangle|0\rangle_a|-2\beta\rangle_b + \mathrm{e}^{\mathrm{i}\delta}\sin\theta/2|1\rangle|4\beta\rangle_a|-2\beta\rangle_b$$
$$+\cos\theta/2|0\rangle|-4\beta\rangle_a|2\beta\rangle_b + \mathrm{e}^{\mathrm{i}\delta}\sin\theta/2|1\rangle|0\rangle_a|2\beta\rangle_b). \quad (4)$$

The position of the cantilever a and the state of the CPB in the $|\pm\rangle$ basis are now measured. All the above corresponds to the Bell state measurement part of the teleportation procedure. As $\mathrm{e}^{-8\beta^2} \ll 1$, there is a probability $\sim 1/2$ that the cantilever is projected to the state $|0\rangle_a$. Let us, for the moment, concentrate on this outcome. Contingent on this outcome, the state of the CPB is projected to $|+\rangle$ and $|-\rangle$ with 1/2 probability each, corresponding to which the state of cantilever b goes to $\cos\theta/2|-2\beta\rangle_b + \mathrm{e}^{\mathrm{i}\delta}\sin\theta/2|2\beta\rangle_b$ and $\cos\theta/2|-2\beta\rangle_b - \mathrm{e}^{\mathrm{i}\delta}\sin\theta/2|2\beta\rangle_b$. Let us assume the state to be $\cos\theta/2|-2\beta\rangle_b + \mathrm{e}^{\mathrm{i}\delta}\sin\theta/2|2\beta\rangle_b$ for the moment. In some sense the above state of cantilever b already contains the teleported quantum information from the original state of the CPB. However, it is difficult to verify this information while it resides in the state of cantilever b. So we map it back from cantilever b to the CPB (which is now disentangled as a result of the previous measurement) by preparing the CPB in the state $|+\rangle$, allowing for the evolution

$$|+\rangle(\cos\theta/2|-2\beta\rangle_b + \mathrm{e}^{\mathrm{i}\delta}\sin\theta/2|2\beta\rangle_b) \to \frac{1}{\sqrt{2}}(\cos\theta/2|0\rangle|0\rangle_b + \mathrm{e}^{\mathrm{i}\delta}\sin\theta/2|1\rangle|4\beta\rangle_b + \cos\theta/2|0\rangle|-4\beta\rangle_b + \mathrm{e}^{\mathrm{i}\delta}\sin\theta/2|1\rangle|0\rangle_b), \quad (5)$$

and then measuring the position of cantilever b. With a probability $1/2$ it is $|0\rangle_b$, for which the CPB is projected to the state $\cos\theta/2|0\rangle + \mathrm{e}^{\mathrm{i}\delta}\sin\theta/2|1\rangle$, thereby concluding a chain of operations leading to teleportation with unit fidelity. In the case when the outcome $|-\rangle|0\rangle_a$ is obtained during the Bell measurement procedure, a teleportation with unit fidelity can also be performed on obtaining $|0\rangle_b$ in the mapping back stage followed by the correction of a known phase factor. For the outcomes $|\pm\rangle|-4\beta\rangle_a$ and $|\pm\rangle|4\beta\rangle_a$ in the Bell state measurement, the CPB is prepared in states $|0\rangle$ and $|1\rangle$ respectively, while for $|-4\beta\rangle_b$ and $|4\beta\rangle_b$ in the mapping back stage, it is prepared in states $|0\rangle$ and $|1\rangle$ respectively. This completes our teleportation protocol. The fidelity of the procedure is thus unity with probability $1/4$, $\cos^2\theta/2$ with probability $(3/8)\cos^2\theta/2$ and $\sin^2\theta/2$ with probability $(3/8)\sin^2\theta/2$. Averaging over all possible initial states one then gets an average fidelity of $3/4$, which is greater than the classical teleportation fidelity of $2/3$.

Let us clarify the sense in which the above is a bona fide teleportation procedure despite the systems being adjacent and the same CPB being *reused*. The CPB interacts with only cantilever a during the Bell state measurement procedure and hence this can be considered as a local action by a party holding cantilever a. The CPB is automatically reset in the process as a fresh qubit not bearing any memory of its initial state. In the mapping back stage it can thus be regarded as a local device used by the party holding cantilever b for extraction of the state.

Decoherence of the cantilever, if significant, will of course affect both the generation of the state $|\psi(+)\rangle_{ab}$, as well as the teleportation. However, decoherence of a cantilever is in the coherent state basis and it will simply multiply the off diagonal term $|-2\beta\rangle_a|-2\beta\rangle_b\langle 2\beta|_a\langle 2\beta|_b$ (and its conjugate) in $|\psi(+)\rangle_{ab}$ by a factor of the form $\mathrm{e}^{-\Gamma}$ where $\mathrm{e}^{-\Gamma}\sim \mathrm{e}^{-8\beta^2\pi/Q}$ in which Q is the quality factor of the cantilevers [7] (note that as physically expected, higher the quality factor lower the decoherence). Similarly, in evolutions given by equations (4) and (5), the off diagonal terms $|0\rangle|0\rangle_a|2\beta\rangle_b\langle 1|\langle 0|_a\langle -2\beta|_b$ and $|0\rangle|0\rangle_b\langle 1|\langle 0|_b$ (and their conjugates) are multiplied by $\mathrm{e}^{-5\Gamma/2}$ and $\mathrm{e}^{-\Gamma/2}$ respectively. The net effect of decoherence at the end of the teleportation will then be a reduction of fidelity corresponding to the $|\pm\rangle|0\rangle_a$ outcome of the Bell state measurement to $(2+\mathrm{e}^{-4\Gamma})/3$, while the fidelity corresponding to other outcomes will remain unchanged. Thus unless *all* coherence is destroyed by decoherence i.e., $\mathrm{e}^{-4\Gamma}\sim 0$, we have an average teleportation fidelity $2/3+\mathrm{e}^{-4\Gamma}/12$, which is better than $2/3$. For example, for $Q\sim 1000$ [7], we have $\mathrm{e}^{-\Gamma}\sim 0.975$ (for $\beta\sim 1$ [7]) and average teleportation fidelity is 0.74. In this paper, we assume that the CPB hardly decoheres over the *ns* timescale of experiments with a GHz NC [7].

3. Entangling two CPBs

The setting of our scheme of entangling two CPBs as depicted in figure 2 is two CPBs coupled to a single NC. The Hamiltonian for this system, in the absence of the voltage pulse giving rise to $-\frac{E_J}{2}\sigma_x$, is well approximated (by straightforward extrapolation of [7]) as

$$H = -2E_C(\sigma_z^{(1)}+\sigma_z^{(2)}) + \hbar\omega_m a^\dagger a + \lambda(a+a^\dagger)(\sigma_z^{(1)}+\sigma_z^{(2)}), \quad (6)$$

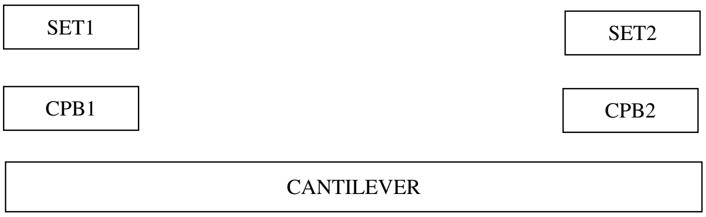

Figure 2. The figure shows a schematic diagram of the setup for entangling two CPBs, denoted as CPB1 and CPB2 respectively, through a cantilever. For the entangling procedure, no measurements are required.

where $\sigma_z^{(i)}$ is a Pauli-Z operator of the ith CPB, a, $a^\dagger$ are the annihilation–creation operators of the nano-cantilever. We initially consider the NC to be starting in the coherent state $|\alpha\rangle$ (we shall generalize later to a thermal state) and the CPB's to be initialized in the state $|0\rangle_1|0\rangle_2$, where labels 1 and 2 stand for the two CPBs. At first, the Hamiltonian $-\frac{E_J}{2}\sigma_x$ is applied to each CPB to rotate their states from $|0\rangle$ to $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. Then evolution according to the Hamiltonian H kicks in and in a time $T = \pi/\omega_m$ the evolution of the state can be calculated from [11] to be

$$\frac{1}{\sqrt{2}}(|0\rangle_1 + |1\rangle_1)\frac{1}{\sqrt{2}}(|0\rangle_2 + |1\rangle_2)|\alpha\rangle \rightarrow \tfrac{1}{2}\{\mathrm{e}^{-\mathrm{i}(E_C T+\phi(T,\beta,\alpha))}|0\rangle_1|0\rangle_2|-\alpha-4\beta\rangle + (|0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2)|-\alpha\rangle + \mathrm{e}^{\mathrm{i}(E_C T-\phi(T,\beta,\alpha))}|1\rangle_1|1\rangle_2|-\alpha+4\beta\rangle\}, \tag{7}$$

where $\phi(T, \beta, \alpha) = 2\beta\mathrm{Im}\alpha$ is a phase factor and $|-\alpha\rangle$, $|-\alpha-4\beta\rangle$ and $|-\alpha+4\beta\rangle$ are coherent states. The sign flip from $\alpha \rightarrow -\alpha$ in the above evolution occurs due to the oscillator evolution for half a time period. The production of states of the above type has been noted earlier in the context of cavity-QED [26] and very recently in the context of measurement-based quantum computation [27]. In [27], it has been pointed out that for a large β, a measurement of the oscillator (NC in our case) will project the two qubits (CPBs in our case) probabilistically to the maximally entangled state $|\psi^+\rangle_{12} = \frac{1}{\sqrt{2}}(|0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2)$. Such an entangled state can, of course, be verified through Bell's inequalities by measurements on the CPBs.

Note that when decoherence of the states of the cantilever is taken into account, as occurs in the coherent state basis [7], we can, without loss of generality, replace $|-\alpha\rangle$, $|-\alpha-4\beta\rangle$ and $|-\alpha+4\beta\rangle$ in equation (7) by $|-\alpha\rangle|\xi_{-\alpha}\rangle$, $|-\alpha-4\beta\rangle|\xi_{-\alpha-4\beta}\rangle$ and $|-\alpha+4\beta\rangle|\xi_{-\alpha+4\beta}\rangle$, where $|\xi_{-\alpha}\rangle$, $|\xi_{-\alpha-4\beta}\rangle$ and $|\xi_{-\alpha+4\beta}\rangle$ are three distinct environmental states with pair-wise mutual overlap tending to zero in the limit of strong decoherence. Thereby, for $\beta \sim 1$, the projected state $|\psi^+\rangle_{12}$ of the two CPBs for a state $|-\alpha\rangle$ of the cantilever is unaffected by decoherence.

We have thus proposed a way of entangling two CPBs through a cantilever in the presence of decoherence. This is an useful alternative to entangling the CPBs by direct interaction, as it will work even when the CPBs fall outside the range of each other's interaction. We have also proposed a method to verify their entanglement through local measurements on each of the CPBs. Of course, if the CPBs were allowed to resonantly exchange energy with a mode of the cantilever in analogy with [18], then not only entanglement, but any quantum computation would be possible in low decoherence [18]. The presence of arbitrarily strong decoherence will, however, affect such a method. What we have shown is that even given the Hamiltonian of [7], arbitrarily strong decoherence, entanglement between the CPBs is still possible.

4. Conclusions

In this paper, we have proposed a scheme to entangle two mesoscopic systems of the same type through a third mesoscopic system. In this context, we have also proposed a teleportation experiment in the mesoscopic setting using continuous variable entanglement for discrete variable teleportation.

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