ANALYSIS AND SYNTHESIS OF A GEARED
SPHERICAL CYCLOIDAL
CRANK MECHANISM

By

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ANALYSIS AND SYNTHESIS OF A GEARED

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PREFACE

The synthesis procedures presented in this thesis are intended to bridge a gap in the spherical synthesis procedures. Attaching the rigid body to the rotating gear of a mechanism has not been solved before in closed form.

Very special thanks is extended to my major advisor, Dr. A. H. Soni, for his encouragement, help, and great faith that this endeavor would be completed. The encouragement and counsel of my fellow students of mechanisms, Mr. Tain Chunsiripong, Mr. Jack Lee, Mr. John Vadasz, Mr. Syed Azeez, and Mr. Siddhanty has been greatly appreciated during my stay at Oklahoma State University. My employer, Vought Corporation, Systems Division, contributed the use of their computer for developing the computer programs used in this thesis.

Nontechnical assistance, encouragement and perseverance were supplied in large quantities by my wife Sarah. My parents provided valuable financial assistance and an impetus to finish. A special thanks must also be extended to the many typists who helped in preparing the final copy of this thesis: Patsy Allen, Wilma Brewer, and Kay Coursey.

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CHAPTER I

INTRODUCTION

A large amount of the work on mechanisms done in this country has been done in the area of planar mechanisms. Mechanisms consist of links, gears, chains and springs which are connected by joints. These joints include ball joints (spherical pairs), hinge joints (revolute pairs), screw (threaded) joints and others. Planar mechanisms are the simplest class and are distinguished by all parts of the mechanism translating and rotating in the same plane or parallel planes. This restriction is what makes planar mechanisms the simplest type.

In the past few years interest has been increasing in the two other basic types of linkages, spherical and spatial. Spherical mechanisms are the class of mechanisms in which all the elements of the mechanism have only rotational motion about axes which intersect in a common point. That is, the elements all move on spheres which have a common center. The last category, spatial mechanisms, includes all the mechanisms which do not fall into one of the other categories. The motions of these mechanisms can be resolved into rotations about and translations along three orthogonal axes; the elements have general motion in space. Spatial mechanisms have received the least amount of attention since they are in general difficult both to analyze and to build.

The subject of this thesis is a spherical mechanism, specifically a geared spherical cycloidal crank mechanism. As shown in Figure 1, this mechanism consists of two conical gears and attaching hardware. All of the work of this thesis deals with points on the unit sphere, a sphere located at the origin with a radius of one unit. So any reference to a point refers to the intersection of the unit sphere and the vector from the origin, O, to that point. The center of the fixed sun gear is at M, i.e., the axis of the fixed gear, gear 1, intersects the unit sphere at point M. Similarly the center of the revolving gear, gear 2, is at A. These two gears are connected by a rigid link MA which restrains the moving gear to always be in mesh with the stationary gear. This allows angles of motion measured through the centers of the gears to be related by the gear ratio, GR, which is the radius of the sun gear divided by the radius of the revolving planet gear.

To completely define a geared spherical cycloidal crank mechanism only a few parameters are required. One possibility for defining a given linkage is to give the location of M, the location of A in one position, and the gear ratio. Another set of sufficient parameters is the location of M, the gear ratio, and the included angle between the axes of the gears. In this work, the first of these two methods is used.

In conclusion, this thesis examines one of the large class of spherical mechanisms. The particular mechanism under study is a geared spherical cycloidal crank mechanism. This is a physically simple mechanism consisting of two conical gears and a connecting link.

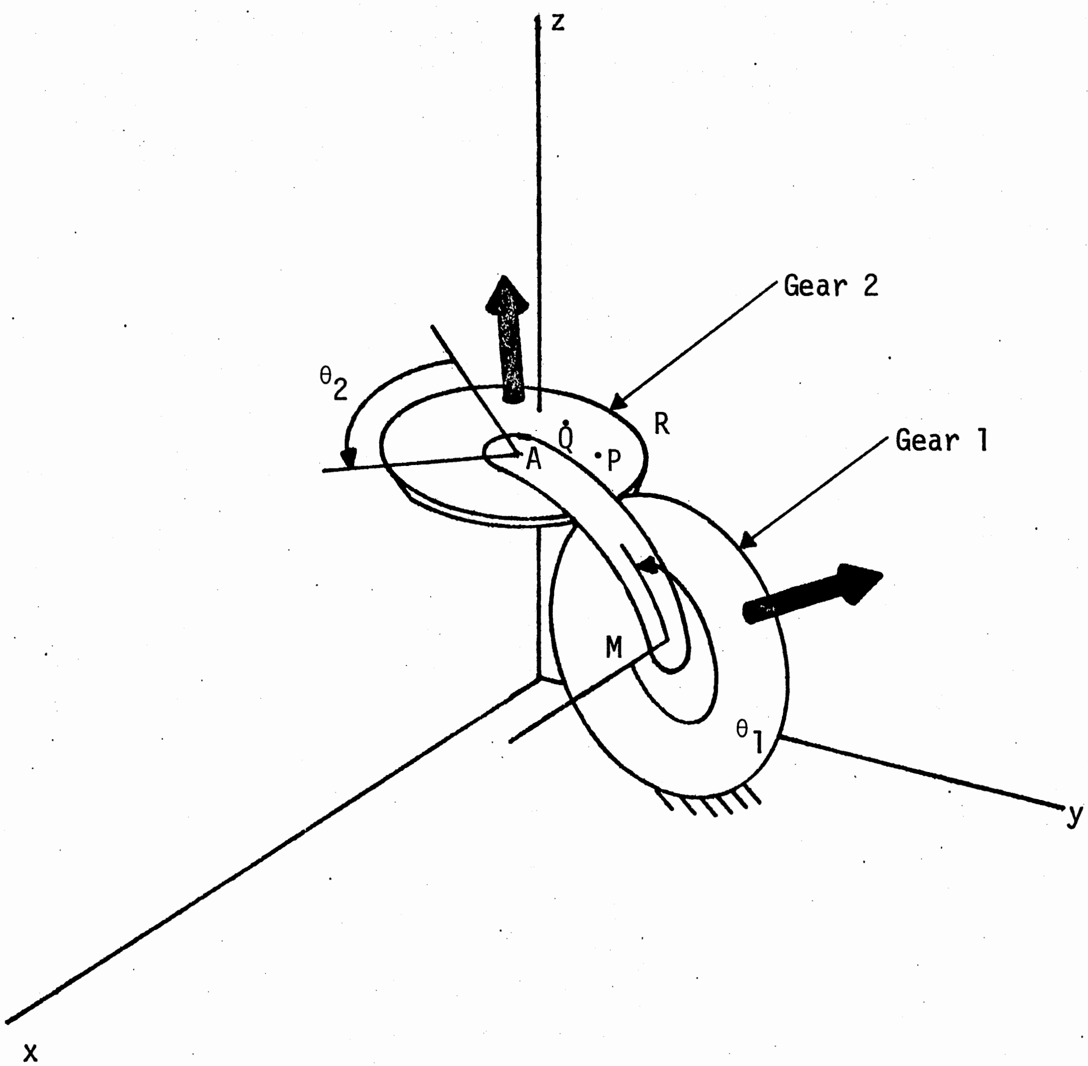

Figure 1. Schematic of Geared Spherical Cycloidal Crank Mechanism

CHAPTER II

KINEMATIC ANALYSIS

2.1 Introduction

Taking a given mechanism with defined proportions and finding the motions of all the various parts for a given motion of one part is the basis of kinematic analysis. This analysis has been done by graphical methods which are fast for some simple cases but have limited accuracy. An example of this type of analysis is given in Chapter 5 of a text on machine design by Martin [5].[1] Chapter 5 of the same text also explains how to analyze plane mechanisms using the instant center technique. A number of analytical methods have been developed to handle more complicated mechanisms quickly and accurately. Dr. A. T. Yang uses dual vectors for analysis as in a paper on spatial four bar mechanisms [2].

Successive screw rotations are used for analysis by Mike McKee in his thesis on a geared spherical five link mechanism [3]. C. H. Suh and C. W. Radcliffe use rotation matrices in their paper on synthesis of spherical linkages both to synthesize and to analyze a spherical four-bar linkage [4]. The basic approach to analysis used in this thesis has the same basis as the one used in the above paper by Radcliffe and Suh.

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[1]Numbers in brackets designate references in the bibliography.

2.2 Development of Equations for Analysis Procedures

This approach uses 3 x 3 rotation matrices which transform the coordinates of a point to yield the coordinates of that point when it is rotated about a given axis through a given angle. If the direction cosines of the rotation axis are U_x, U_y, and U_z and the rotation angle is ϕ, then the rotation matrix has the following form as presented by Suh and Radcliffe [4]:

$$[R]_{\overline{U},\phi} = \begin{bmatrix} U_x^2 \text{ vers } \phi + \cos\phi & U_xU_y \text{ vers } \phi - U_z \sin\phi & U_xU_z \text{ vers } \phi + U_y \sin\phi \\ U_xU_y \text{ vers } \phi + U_z \sin\phi & U_y^2 \text{ vers } \phi + \cos\phi & U_yU_z \text{ vers } \phi - U_x \sin\phi \\ U_xU_z \text{ vers } \phi - U_y \sin\phi & U_yU_z \text{ vers } \phi + U_x \sin\phi & U_z^2 \text{ vers } \phi + \cos\phi \end{bmatrix} \tag{2.1}$$

where:

$\text{vers } \phi = 1 - \cos\phi.$

To find P_2 the coordinates of a given point P_1 after being rotated about an axis $\overline{U}$ through an angle ϕ, the rotation matrix of Equation (2.1) can be used as follows:

$$\begin{Bmatrix} P_{2x} \\ P_{2y} \\ P_{2z} \end{Bmatrix} = [R]_{\overline{U},\phi} \begin{Bmatrix} P_{1x} \\ P_{1y} \\ P_{1z} \end{Bmatrix} \tag{2.2}$$

Equation (2.2) is the basis for this analysis. Through a series of rotations about the axes of the various gears, the entire analysis procedure is carried out.

As explained in Chapter I and shown in Figure 1 the intersection of the axis of the fixed gear and the unit sphere is point M. Similarly point A is the intersection of the unit sphere and the axis of the rotating gear. As shown in Figure 1, θ_1 is the rotation of the arm and is measured CCW from a plane through OM and the x axis, about the axis OM. θ_2 is measured CCW about OA relative to the link MA; by definition θ_2 is zero when θ_1 is zero. For this work only changes in θ_1 and θ_2 will be considered. The change in θ_1 when the mechanism goes from position one to position two is θ_{12}. Similarly the change in θ_2 when the mechanism goes from position one to position two is θ_{22}. For the doubly subscripted θ's the first subscript denotes the θ under discussion, and the second subscript denotes which position is taken as the second position. Finally, to define the motions of the planet gear three arbitrary points on the planet gear are necessary. These three points are labeled P, Q, and R and are on the unit sphere.

Before plunging into the analysis, a brief section to orient the reader will be included. This section defines the given input parameters and the necessary output parameters for this analysis. The input parameters chosen are the location of points M, A, P and Q in their first positions; the gear ratio GR, and θ_{12} the desired increment in θ_1. The first position of R is found by rotating Q about P through an angle of ninety degrees. Output consists of the second positions of points A, P, Q, and R. This defines the goal of kinematic analysis of a geared spherical cycloidal crank mechanism.

The first step is to find the first position of point R. The following equation is used to accomplish this:

$$\{R_1\} = [R]_{\overline{OP}_1, 90^o}\{Q_1\} \tag{2.4}$$

To find the second position of point A a single rotation is required.

$$\{A_2\} = [R]_{\overline{OM}, \theta_{1n}} \{A_1\}$$

The change in θ_2 is the gear ratio times the change in θ_1.

$$\theta_{2n} = GR\ \theta_{1n} \tag{2.5}$$

The second positions of points P, Q, and R can be found by a total of two rotations. The first of these rotations is about point A_1 through an angle of θ_{2n}. The resulting points are then rotated about point M through an angle of θ_{1n}. The equations for these transformations are:

$$\{P_2\} = [R]_{\overline{OM}, \theta_{1n}} [R]_{\overline{OA}, \theta_{2n}} \{P_1\} \tag{2.6}$$

$$\{Q_2\} = [R]_{\overline{OM}, \theta_{1n}} [R]_{\overline{OA}, \theta_{2n}} \{Q_1\} \tag{2.7}$$

$$\{R_2\} = [R]_{\overline{OM}, \theta_{1n}} [R]_{\overline{OA}, \theta_{2n}} \{R_1\} \tag{2.8}$$

At this point all the information required for analysis of the mechanism has been found.

2.3 Computer Program

A computer program was written to perform the above analysis and is included in Appendix A. The input of data into the program is explained in the comment cards at the first of the program. The output is set up to be self-explanatory.

The following set of input data was used for a demonstration of the analysis program:

POINT	COORDINATES		
	x	y	z
M	.1	.5	+
A	.2	.6	+
P	-.2	.3	+
Q	-.3	.4	-

GR = 2.0, $\theta_{12} = 15^0$

The resulting initial positions were:

POINT	COORDINATES		
	x	y	z
M	.1	.5	.860233
A	.2	.6	.774597
P	-.2	.3	.932738
Q	-.3	.4	-.866025
R	-.507348	-.641359	-.575549

The program computed the following second positions:

POINT	COORDINATES		
	x	y	z
A	.163199	.620841	.766761
P	.108740	.146311	.983244
Q	-.814562	.123531	-.566770
R	-.272650	-.831133	-.484643

CHAPTER III

DEVELOPMENT OF GENERAL SYNTHESIS EQUATION

3.1 Introduction

As explained in the introduction of Chapter II a wide range of methods have been used in the analysis of mechanisms. All of these methods have also been used for synthesis. The procedures developed in this thesis are based on the use of the rotation matrix developed by Suh and Radcliffe [4]. The remainder of this chapter will be devoted to explaining the procedures used to take the basic rotation matrix of Equation (2.1) and develop the desired design equation for the problem at hand. This equation is used as the heart of the analysis of the two, three, four, and five position rigid body synthesis problems.

3.2 The Displacement Matrix

The displacement matrix is a (3 x 3) matrix which, when multiplied times any point on the planet gear in its first position, yields the coordinates of that point after the mechanism has gone through a finite rotation. Then, if three points P, Q, and R are given, which move from a given position one to a given position n, the following equations can be written:

$$\{P_{1n}\} = [D_{1n}]\{P_1\} \tag{3.1}$$

$$\{Q_{1n}\} = [D_{1n}]\{Q_1\} \tag{3.2}$$

$$\{R_{1n}\} = [D_{1n}]\{Q_1\} \tag{3.3}$$

where $[D_{1n}]$ is the rotation matrix from position one to position n. These equations can be expanded to give nine equations in terms of the nine unknown elements of the rotation matrix. The three equations containing the elements of the first row are as follows where the lower case d's are the elements of D_{1n} :

$$P_{1x}d_{11} + P_{1y}d_{12} + P_{1z}d_{13} = P_{nz} \tag{3.4}$$

$$Q_{1x}d_{11} + Q_{1y}d_{12} + Q_{1z}d_{13} = Q_{nz} \tag{3.5}$$

$$R_{1x}d_{11} + R_{1y}d_{12} + R_{1z}d_{13} = R_{nz} \tag{3.6}$$

For Equations (3.4) through (3.6) to have a unique solution for d_{11}, d_{12}, and d_{13}, the matrix

$$[PQR]_1^T = \begin{bmatrix} P_{1x} P_{1y} P_{1z} \\ Q_{1x} Q_{1y} Q_{1z} \\ R_{1x} R_{1y} R_{1z} \end{bmatrix} \tag{3.7}$$

must be nonsingular, since at least one of the right hand sides of Equations (3.4) through (3.6) is nonzero. This is guaranteed by making point R the result of rotating Q about P through an angle of ninety degrees, which forces P, Q, and R not to be in one plane.

Equations (3.1) through (3.3) can be rewritten as:

$$[PQR]_n = [D_{1n}]\,[PQR]_1 \tag{3.8}$$

This equation can be solved for the rotation matrix.

$$[D_{1n}] = [PQR]_n\,[PQR_1]^{-1} \tag{3.9}$$

It is seen that the requirements for Equations (3.4) through (3.6) to have a unique solution and for Equation (3.9) to be solvable are the same. So, if the rotation matrix can be found from Equation (3.9), then from Equations (3.4) through (3.6), and from similar equations for the other two rows of the rotation matrix, the elements of the rotation matrix are uniquely determined. This step is critical to the remaining discussion of this chapter.

Since the displacement matrix has uniquely determined elements, any matrix which satisfies Equations (3.1) through (3.6) can be equated to D_{1n} element by element. Equation (2.6) contains a matrix which meets this criterion. Therefore, the following equality holds.

$$[D_{1n}] = [R]_{\overline{OM},\theta_{1n}}\,[R]_{\overline{OA},\theta_{2n}} \tag{3.10}$$

The elements of the matrices on each side of Equation (3.10) can be equated to yield:

$$\begin{aligned} d_{11} = {} & \text{vers}\,\theta_{1n}\,\text{vers}\,\theta_{2n}\,(M_x^2A_x^2 + M_xM_yA_xA_y + M_xM_zA_xA_z) \\ & + \text{vers}\,\theta_{1n}\cos\theta_{2n}M_x^2 + \text{vers}\,\theta_{1n}\sin\theta_{2n}\,(M_xM_yA_z \\ & - M_xM_zA_y) + \cos\theta_{1n}\cos\theta_{2n} + \cos\theta_{1n}\,\text{vers}\,\theta_{2n}A_x^2 \\ & + \sin\theta_{1n}\,\text{vers}\,\theta_{2n}\,(M_yA_xA_z - M_zA_xA_y) \\ & + \sin\theta_{1n}\sin\theta_{2n}\,(-M_yA_y - M_zA_z) \end{aligned} \tag{3.11}$$

$$d_{12} = \text{vers}\ \theta_{1n}\ \text{vers}\ \theta_{2n}\ (M_x^2 A_x A_y + M_x M_y A_y^2 + M_x M_z A_y A_z$$
$$+ \text{vers}\ \theta_{1n}\ \cos\ \theta_{2n} M_x M_y)$$
$$+ \text{vers}\ \theta_{1n}\ \sin\ \theta_{2n}\ (M_x M_z A_x - M_x^2 A_z)$$
$$+ \cos\ \theta_{1n}\ \text{vers}\ \theta_{2n} A_x A_y$$
$$+ \sin\ \theta_{1n}\ \text{vers}\ \theta_{2n}\ (M_y A_y A_z - M_z A_y^2)$$
$$+ \sin\ \theta_{1n}\ \sin\ \theta_{2n} M_y A_x - \sin\ \theta_{1n}\ \cos\ \theta_{2n} M_z$$
$$- \cos\ \theta_{1n}\ \sin\ \theta_{2n} A_z \tag{3.12}$$

$$d_{13} = \text{vers}\ \theta_{1n}\ \text{vers}\ \theta_{2n}\ (M_x^2 A_x A_z + M_x M_y A_y A_z + M_x M_z A_z^2)$$
$$+ \text{vers}\ \theta_{1n}\ \cos\ \theta_{2n} M_x M_z + \text{vers}\ \theta_1\ \sin\ \theta_2\ (M_x^2 A_y - M_x M_y A_x)$$
$$+ \cos\ \theta_{1n}\ \text{vers}\ \theta_{2n} A_x A_z + \sin\ \theta_{1n}\ \text{vers}\ \theta_{2n}\ (M_y A_z^2$$
$$- M_z A_y A_z) + \sin\ \theta_{1n}\ \sin\ \theta_{2n} M_z A_x + \sin\ \theta_{1n}\ \cos\ \theta_{2n} M_y$$
$$+ \cos\ \theta_{1n}\ \sin\ \theta_{2n} A_y \tag{3.13}$$

$$d_{21} = \text{vers}\ \theta_{1n}\ \text{vers}\ \theta_{2n}\ (M_x M_y A_x^2 + M_y^2 A_x A_y + M_y M_z A_x A_z)$$
$$+ \text{vers}\ \theta_{1n}\ \cos\ \theta_{2n} M_x M_y + \text{vers}\ \theta_{1n}\ \sin\ \theta_{2n}\ (M_y^2 A_z$$
$$- M_y M_z A_y) + \cos\ \theta_{1n}\ \text{vers}\ \theta_{2n} A_x A_y$$
$$+ \sin\ \theta_{1n}\ \text{vers}\ \theta_{2n}\ (M_z A_x^2 - M_x A_x A_z)$$
$$+ \sin\ \theta_{1n}\ \sin\ \theta_{2n} M_x A_y + \cos\ \theta_{1n}\ \sin\ \theta_{2n} A_z$$
$$+ \sin\ \theta_{1n}\ \cos\ \theta_{2n} M_z \tag{3.14}$$

$$d_{22} = \text{vers}\ \theta_{1n}\ \text{vers}\ \theta_{2n}\ (M_x M_y A_x A_y + M_y^2 A_y^2 + M_y M_z A_y A_z)$$
$$+ \text{vers}\ \theta_{1n}\ \cos\ \theta_{2n} M_y^2 + \text{vers}\ \theta_{1n}\ \sin\ \theta_{2n}\ (M_y M_z A_x$$
$$- M_x M_y A_z) + \cos\ \theta_{1n}\ \cos\theta_{2n} + \cos\ \theta_{1n}\ \text{vers}\ \theta_{2n} A_y^2$$

$$+ \sin\theta_{1n} \text{ vers } \theta_{2n} (M_z A_x A_y - M_x A_y A_z)$$

$$+ \sin\theta_{1n} \sin\theta_{2n} (-M_z A_z - M_x A_x) \quad (3.15)$$

$$d_{23} = \text{vers } \theta_{1n} \text{ vers } \theta_{2n} (M_x M_y A_x A_z + M_y^2 A_y A_z + M_y M_z A_z^2)$$

$$+ \text{vers } \theta_{1n} \cos\theta_{2n} M_y M_z$$

$$+ \text{vers } \theta_{1n} \sin\theta_{2n} (M_x M_y A_y - M_y^2 A_x)$$

$$+ \cos\theta_{1n} \text{ vers } \theta_{2n} A_y A_z$$

$$+ \sin\theta_{1n} \text{ vers } \theta_{2n} (M_z A_x A_z - M_x A_z^2)$$

$$+ \sin\theta_{1n} \sin\theta_{2n} M_z A_y - \sin\theta_{1n} \cos\theta_{2n} M_x$$

$$- \cos\theta_{1n} \sin\theta_{2n} A_x \quad (3.16)$$

$$d_{31} = \text{vers } \theta_{1n} \text{ vers } \theta_{2n} (M_x M_z A_x^2 + M_y M_z A_x A_y + M_z^2 A_x A_z)$$

$$+ \text{vers } \theta_{1n} \cos\theta_{2n} M_x M_z + \text{vers } \theta_{1n} \sin\theta_{2n} (M_y M_z A_z$$

$$- M_x^2 A_y) + \cos\theta_{1n} \text{ vers } \theta_{2n} A_x A_z$$

$$+ \sin\theta_{1n} \text{ vers } \theta_{2n} (M_x A_x A_y - M_y A_x^2)$$

$$+ \sin\theta_{1n} \sin\theta_{2n} M_x A_z - \cos\theta_{1n} \sin\theta_{2n} A_y$$

$$- \sin\theta_{1n} \cos\theta_{2n} M_y \quad (3.17)$$

$$d_{32} = \text{vers } \theta_{1n} \text{ vers } \theta_{2n} (M_x M_z A_x A_y + M_y M_z A_y^2 + M_z A_y A_z)$$

$$+ \text{vers } \theta_{1n} \cos\theta_{2n} M_y M_z + \text{vers } \theta_{1n} \sin\theta_{2n} (M_z^2 A_x$$

$$- M_x M_z A_z) + \cos\theta_{1n} \text{ vers } \theta_{2n} A_y A_z$$

$$+ \sin\theta_{1n} \text{ vers } \theta_{2n} (M_x A_y^2 - M_y A_x A_y)$$

$$+ \sin\theta_{1n} \sin\theta_{2n} M_y A_z + \cos\theta_{1n} \sin\theta_{2n} A_x$$

$$+ \sin\theta_{1n} \cos\theta_{2n} M_x \quad (3.18)$$

$$d_{33} = \text{vers}\ \theta_{1n}\ \text{vers}\ \theta_{2n}\ (M_xM_zA_xA_z$$
$$+ M_yM_zA_yA_z + M_z^2A_z^2) + \text{vers}\ \theta_{1n}\cos\theta_{2n}M_z^2$$
$$+ \text{vers}\ \theta_{1n}\sin\theta_{2n}\ (M_xM_zA_y - M_yM_zA_x$$
$$+ \cos\theta_{1n}\cos\theta_{2n} + \cos\theta_{1n}\ \text{vers}\ \theta_{2n}A_z^2$$
$$+ \sin\theta_{1n}\ \text{vers}\ \theta_{2n}\ (M_xA_yA_z - M_yA_xA_z)$$
$$+ \sin\theta_{1n}\sin\theta_{2n}\ (-M_yA_y - A_xM_x) \qquad (3.19)$$

where:

$$\text{vers}\ \theta = 1 - \cos\theta.$$

The nine Equations (3.11) through (3.19) are the equations which form the basic building blocks used in the derivative of the design equation.

The nine equations formed from the elements of the rotation matrices are obviously dependent since there are only six unknowns, the coordinates of points M and A. These nine dependent equations will be reduced to one independent equation which is linear in the coordinates of points M and A. Three intermediate equations will be derived first. Adding together the three equations formed from main diagonal terms, Equations (3.11), (3.15), and (3.19), yields the first of the three intermediate equations. After simplification and using the fact that the sum of the squares of the coordinates of points M and A is one yields:

$$\text{vers}\ \theta_{1n}\ \text{vers}\ \theta_{2n}K_1 + 2\sin\theta_{1n}\sin\theta_{2n}K_2$$
$$- 2\cos\theta_{1n}\cos\theta_{2n} - 1 + d_{11} + d_{22} + d_{33} = 0 \qquad (3.20)$$

where, $K_1 = |\overline{OM} \times \overline{OA}|^2$

and, $K_2 = \overline{OM} \cdot \overline{OA}$

To eliminate K_1 and K_2, which are highly nonlinear terms, the other two intermediate equations are needed. The next intermediate equation is found by adding A_x times the result of subtracting Equation (3.18) from Equation (3.16), A_y times the result of subtracting Equation (3.13) from Equation (3.17), and A_z times the result of subtracting Equation (3.12) from Equation (3.14). After simplification this yields

$$\text{vers } \theta_{1n} \sin \theta_{2n} K_1 + 2 \sin \theta_{1n} \cos \theta_{2n} K_2 + 2 \cos \theta_{1n} \sin \theta_{2n} + A_x (d_{23} - d_{32}) + A_y (d_{31} - d_{13}) + A_z (d_{12} - d_{21}) = 0 \quad (3.21)$$

The third and last intermediate equation is similar to the second except that the differences above are multiplied by the coordinates of M instead of the coordinates of A. This yields the following equation.

$$\sin \theta_{1n} \text{ vers } \theta_{2n} K_1 + 2 \cos \theta_{1n} \sin \theta_{2n} K_2 + 2 \sin \theta_{1n} \cos \theta_{2n} + M_x (d_{23} - d_{32}) + M_y (d_{31} - d_{13}) + M_z (d_{12} - d_{21}) = 0 \quad (3.22)$$

With the three Equations (3.20) through (3.22), the unknowns K_1 and K_2 can be eliminated. Solving Equation (3.20) for K_2 and substituting in Equations (3.21) and (3.22) yields the following two equations:

$$\text{vers}\ \theta_{1n}\ \text{vers}\ \theta_{2n}K_1 + 2\cos\theta_{1n}$$
$$+ \cos\theta_{2n}\,(1 - d_{11} - d_{22} - d_{33})$$
$$+ A_x \sin\theta_{2n}\,(d_{23} - d_{32}) + A_y \sin\theta_{2n}\,(d_{31} - d_{13})$$
$$+ A_z \sin\theta_{2n}\,(d_{12} - d_{21}) = 0 \qquad (3.23)$$

and

$$\text{vers}\ \theta_{1n}\ \text{vers}\ \theta_{2n}K_1 + 2\cos\theta_{2n}$$
$$+ \cos\theta_{1n}\,(1 - d_{11} - d_{22} - d_{33}) + M_x \sin\theta_{1n}\,(d_{23}$$
$$- d_{32}) + M_y \sin\theta_{1n}\,(d_{31} - d_{13}) + M_z \sin\theta_{1n}\,(d_{12}$$
$$- d_{21}) \qquad (3.24)$$

By subtracting Equation (3.24) from Equation (3.23), the final design equation is found, which is:

$$A_x \sin\theta_{2n}\,(d_{23} - d_{32}) + A_y \sin\theta_{2n}\,(d_{31} - d_{13})$$
$$+ A_z \sin\theta_{2n}\,(d_{12} - d_{21}) - M_x \sin\theta_{1n}\,(d_{23} - d_{32})$$
$$- M_y \sin\theta_{1n}\,(d_{31} - d_{13}) - M_z \sin\theta_{1n}\,(d_{12} - d_{21})$$
$$+ (\cos\theta_{1n} - \cos\theta_{2n})\,(1 + d_{11} + d_{22} + d_{33}) = 0$$
$$(3.25)$$

This is the design, or synthesis, equation which is used in Chapter IV to design geared spherical cycloidal crank mechanisms which will move a rigid body through given positions.

In conclusion, the nine equations derived from the basic rotation matrix have been combined to yield one equation which is linear in the coordinates of points M and A. This design equation is sufficient when combined with a constraint to keep the points A and M on the unit sphere, to constrain points M and A to satisfy the design goals. That is, the points M and A define axes which yield a mechanism which has the desired properties.

CHAPTER IV

SYNTHESIS PROCEDURES

4.1 Introduction

As referred to before, synthesis is the process used to find the proportions of a mechanism which will satisfy given criteria. In this thesis the design criteria is a series of positions of two points which are located on the surface of the unit sphere. The series of positions are transformed into rotation matrices, one matrix for the rotation from position one to each of the other positions, as explained in Chapter III. These rotation matrices are then used to define design equations; one design equation for each rotation matrix. Besides the design equations, the points M and A must be constrained to lie on the unit sphere. The methods used to accomplish this are the subject of the remainder of this chapter.

4.2 Determination of Rotation Angles

After the rotation matrix D_{1n} has been found for each n from two to the total number of positions specified, the desired rotation angles can be determined. The rotation matrix is explained in Chapter III. The sum of the main diagonal elements yields an equation which contains only known rotation matrix elements and the cosine of the total rotation angle. Solving this equation yields the total rotation angle of the planet gear, when the mechanism goes from position one to position n,

$$\phi_{1n} = \cos^{-1} [\tfrac{1}{2} (d_{11} + d_{22} + d_{33} - 1)] \quad (4.1)$$

The angles θ_1 and θ_2 can be determined from the following two equations.

$$\theta_{1n} = \frac{1}{1 + GR} \phi_{1n} \quad (4.2)$$

$$\theta_{2n} = \frac{GR}{1 + GR} \phi_{1n} \quad (4.3)$$

The rationale for these two equations is actually rather simple. Obviously for a change in θ_1 of θ_{12} the second gear rotates through an angle of θ_{12} times the gear ratio, relative to the arm MA. However, the planet gear rotates through a total angle, ϕ_{1n}, of θ_{12}, plus θ_{22}. This can be written as an equation:

$$\phi_{1n} = \theta_{11} + \theta_{22} = (1 + GR)\ \theta_{11} \quad (4.4)$$

It can be seen that Equations (4.2) and (4.3) follow immediately from Equation (4.4) and the preceding discussion.

4.3 Input to Synthesis Programs

The requirements which input points P and Q must fulfill are fairly restrictive. A computer program to help accurately specify the input points is included in Appendix B. This program takes a given P_1, P_2, Q_1, and Q_{2x} and finds the other coordinates of Q_2. The computed Q_2 must be such that the length of $\overline{P_1Q_1}$ is equal to the length of $\overline{P_2Q_2}$ and Q_2 must be on the unit sphere. First the length of $\overline{P_1Q_1}$ is

$$|\overline{P_1Q_1}| = [(P_{1x} - Q_{1x})^2 + (P_{1y} - Q_{1y})^2 + (P_{1z} - Q_{1z})^2]^{1/2} \quad (4.5)$$

A similar equation holds for the second positions of P and Q. This equation and the equation

$$Q_{2x}^2 + Q_{2y}^2 + Q_{2z}^2 = 1 \tag{4.6}$$

can be solved to yield:

$$c_1 Q_{2z}^2 + c_2 Q_{2z} + c_3 = 0 \tag{4.7}$$

where:

$$c_1 = \frac{P_{2z}^2}{P_{2y}^2} + 1$$

$$c_2 = \frac{P_{2z}}{2P_{2y}} (|\overline{P_1 Q_1}|^2 + 2Q_{2x} P_{2x} - 2)$$

and

$$c_3 = \frac{1}{4P_{2y}^2} (|\overline{P_1 Q_1}|^2 + 2Q_{2x} P_{2x} - 2)^2 + Q_{2x}^2 - 1$$

This equation is then solved to yield two possibilities for Q_{2z}. Q_{2y} is found for each of these Q_{2z} from

$$Q_{2y} = (1 - Q_{2x}^2 - Q_{2x}^2)^{1/2} \tag{4.8}$$

Another feature of the computer program in Appendix B is the option of not inputting Q_{2x}. Under this option, as explained in the comment cards of the program, the maximum possible range for the coordinates of Q_2 is given as output.

4.4 Two Position Synthesis

In two position synthesis only two rigid body positions are given. The rotation matrix from position one to two is found as explained in Chapter III. Then θ_{12} and θ_{22} are found using the equations of paragraph

4.2. The equations which must be satisfied are:

$$M_x^2 + M_y^2 + M_z^2 = 1 \tag{4.9}$$

$$A_{1x}^2 + A_{1y}^2 + A_{1z}^2 = 1 \tag{4.10}$$

and,

$$c_{21}A_x + c_{22}A_y + c_{23} = 0 \tag{4.11}$$

where:

$$c_{21} = \sin \theta_{22} (d_{23} - d_{32})$$

$$c_{22} = \sin \theta_{22} (d_{31} - d_{13})$$

$$\begin{aligned} c_{23} = {} & A_{1z} \sin \theta_{22} (d_{12} - d_{21}) - M_x \sin \theta_{12} (d_{23} - d_{32}) \\ & - M_y \sin \theta_{12} (d_{31} - d_{13}) - M_z \sin \theta_{12}(d_{12} - d_{21}) \\ & + (\cos \theta_{12} - \cos \theta_{22}) (1 + d_{11} + d_{22} + d_{33}) \end{aligned}$$

At this point there are three equations, Equations (4.9) through (4.11), which must be satisfied. There are also six unknowns, the coordinates of points M and A. The gear ratio, M_x, M_y, A_{1z}, and the sign of M_z are assumed. M_z is found from Equation (4.9) and is

$$M_z = \pm (1 - M_x^2 - M_y^2) \tag{4.12}$$

where the sign takes the assumed value. The coefficients c_{21}, c_{22}, and c_{23} can now be calculated. Equation (4.11) is solved for A_{1y} to give:

$$A_{1y} = h_{11}A_{1x} + h_{12} \tag{4.13}$$

where:

$$h_{11} = -c_{21}/c_{22}$$

and,

$$h_{12} = -c_{23}/c_{22}$$

which can be substituted in Equation (4.10) to give the final equation.

$$e_1 A_{1x}^2 + e_2 A_{1x} + e_3 = 0 \qquad (4.14)$$

where:

$$e_1 = h_{11}^2 + 1$$

$$e_2 = 2h_{11} h_{12}$$

and,

$$e_3 = A_{1z}^2 + h_{12}^2 - 1$$

This equation is solved to give two roots for A_{1x}. If the two roots are real and equal, then one solution has been found. If the roots are real and unequal, then one or two solutions have been found. If the roots are imaginary, then no solutions are possible and either the rigid body positions or one of the assumed values must be changed to reach a solution. Solutions for A_{1x} are substituted in Equation (4.13) to find the corresponding value or values for A_{1y}. The sets of possible M and A values are then used in the analysis program to find which values are good values and which are extraneous values introduced by squaring Equation (4.13).

A computer program to perform the above analysis, but not to substitute back in the analysis program, is included in Appendix C. The input procedures for the program are explained in the comment cards at the first of the program and the output is self-explanatory. An example problem is solved using the computer. The results of this example are given in Table I.

TABLE I

TWO POSITION SYNTHESIS, EXAMPLE PROBLEM, WITH A GEAR RATIO OF TWO

Point	Coordinates x	y	z
		Input	
M	.6	.31	.737496
A			.552449
P_1	-.2	.3	.932738
P_2	-.134274	.102227	.985657
Q_1	-.3	.4	-.866025
Q_2	-.454178	.482660	-.748840
		Output	
First A_1	.761222	.339620	.552449
Second A_1	.675930	.487768	.552449

4.5 Three Position Synthesis

The synthesis procedure,when three rigid body positions are given, is similar to the procedure when two rigid body positions are given. Two rotation matrices are found using the steps outlined in Chapter III. One rotation matrix from position one to two, and one matrix from position one to three must be found. Using the equations of paragraph 4.2, the angles θ_{12}, θ_{22}, θ_{13}, and θ_{23} are found. The equations which will be used in this analysis are given below. The first subscript on the elements of the rotation matrices refers to which rigid body position is taken as the second position.

$$M_x^2 + M_y^2 + M_z^2 = 1 \tag{4.15}$$

$$A_{1x}^2 + A_{1y}^2 + A_{1z}^2 = 1 \tag{4.16}$$

$$c_{21} A_{1x} + c_{22} A_{1y} + c_{23} A_{1z} + c_{24} = 0 \tag{4.17}$$

$$c_{31} A_{1x} + c_{32} A_{1y} + c_{33} A_{1z} + c_{34} = 0 \tag{4.18}$$

where:

$$c_{n1} = \sin \theta_{2n} (d_{n23} - d_{n32}) \qquad \text{for } n = 2, 3$$

$$c_{n2} = \sin \theta_{2n} (d_{n31} - d_{n13}) \qquad \text{for } n = 2, 3$$

$$c_{n3} = \sin \theta_{2n} (d_{n12} - d_{n21}) \qquad \text{for } n = 2, 3$$

$$c_{n4} = -M_x \sin \theta_{1n} (d_{n23} - d_{n32}) \qquad \text{for } n = 2, 3$$

$$- M_y \sin \theta_{1n} (d_{n31} - d_{n13}) - M_z \sin \theta_{1n} (d_{n12} - d_{n21})$$

$$+ (\cos \theta_{1n} - \cos \theta_{2n}) (1 + d_{n11} + d_{n22} + d_{n33})$$

At this point there are four equations and six unknowns. Therefore, two of the unknowns can take on assumed values. The values to be assumed are M_x, M_y, and the gear ratio. M_z is calculated from Equation (4.12). This leaves Equations (4.16) through (4.18) to be solved. Equation (4.17) is first solved for A_{1z}, which gives:

$$A_{1z} = g_{11} A_{1x} + g_{12} A_{1y} + g_{13} \tag{4.19}$$

where:

$$g_{11} = -c_{21}/c_{23}$$

$$g_{12} = -c_{22}/c_{23}$$

$$g_{13} = -c_{24}/c_{23}$$

Substituting Equation (4.19) in Equations (4.16) and (4.17) yields two equations:

$$h_{21} A_{1x} + h_{22} A_{1y} + h_{23} = 0 \tag{4.20}$$

where:

$$h_{21} = g_{31} + g_{33} g_{11}$$

$$h_{22} = g_{32} + g_{33} g_{12}$$

$$h_{23} = g_{34} + g_{33} g_{13}$$

and,

$$a_1 A_{1x}^2 + a_2 A_{1y}^2 + a_3 A_{1x} A_{1y} + a_4 A_{1x} + a_5 A_{1y} + a_6 = 0 \tag{4.21}$$

where:

$$a_1 = g_{11}^2 + 1$$

$$a_2 = g_{12}^2 + 1$$

$$a_3 = 2 g_{11} g_{12}$$

$$a_4 = 2 g_{11} g_{13}$$

$$a_5 = 2\, g_{12}\, g_{13}$$

$$a_6 = g_{13}^2 - 1$$

The next step is to solve Equation (4.20) for A_{1y} and substitute in Equation (4.21).

$$A_{1y} = c_{11}\, A_{1x} + c_{12} \qquad (4.22)$$

where:

$$k_{11} = \frac{h_{21}}{h_{22}}$$

and,

$$k_{12} = - \frac{h_{23}}{h_{22}}$$

Substituting this in Equation (4.20) yields the final design quadratic.

$$b_1\, A_{1x}^2 + b_2\, A_{1x} + b_3 = 0 \qquad (4.23)$$

where:

$$b_1 = a_1 + k_{11}^2\, a_2 + k_{11}\, a_3$$

$$b_2 = a_4 + 2k_{11}\, k_{12}\, a_2 + k_{12}\, a_3 + c_{11}\, a_5$$

$$b_3 = a_6 + k_{12}^2\, a_2 + k_{12}\, a_5$$

The quadratic equation is then solved for A_{1x}. If the imaginary parts of the two roots of Equation (4.23) are zero, then at least one solution has been found. For each A_{1x} the corresponding A_{1y} and A_{1z} are found from:

$$A_{1y} = k_{11}\, A_{1x} + k_{12} \qquad (4.24)$$

and,

$$A_{1z} = \frac{-1}{c_{33}} (c_{31} A_{1x} + c_{32} A_{1y} + c_{34}) \qquad (4.25)$$

The sets of values for point A must be checked for extraneous roots by substituting in the analysis program. This way, extraneous roots can be eliminated. If no roots with zero imaginary parts are found by using this method, then one of the input parameters must be changed to reach a solution.

A computer program is presented in Appendix D which solves the above equations but does not substitute in the analysis program to check for extraneous roots. The input to this program is explained in the comment cards at the first of the program. The output from the program is self-explanatory. An example problem has been solved on the computer. The results of this example are given in Table II.

4.6 Four Position Synthesis

Four rigid body positions increase the complexity of the problem considerably over the three position problem. The reason for the greater complexity is that after the linear substitutions, there are two second order equations which must be solved. Proceeding with the derivation, three rotation matrices are found using the procedures of Chapter III. The three rotation matrices are for rotations from position one to positions two, three, and four. The subscripts on the θ's and the elements of the rotation matrices are the same as that used in paragraph 4.5, three position synthesis. The equations necessary for synthesis are equations to insure M and A are on the unit sphere, and equations derived from Equation (3.25). Equation (3.25) yields one equation for each rotation matrix.

TABLE II

THREE POSITION SYNTHESIS, EXAMPLE PROBLEM, WITH A GEAR RATIO OF TWO

Point	Coordinates x	y	z
		Input	
M	.1	.5	.860233
P_1	-.2	.3	.932738
P_2	-.113094	.226181	.967498
P_3	-.00757381	.173646	.984779
Q_1	-.3	.4	-.866025
Q_2	-.503378	.350705	-.789694
Q_3	-.679041	.256321	-.687897
		Output	
First A_1	.122874	.518829	.846001
Second A_1	.220877	.605974	.764204

$$M_x^2 + M_y^2 + M_z^2 = 1 \tag{4.26}$$

$$A_{1x}^2 + A_{1y}^2 + A_{1z}^2 = 1 \tag{4.27}$$

$$c_{21} A_{1x} + c_{22} A_{1y} + c_{23} A_{1z} + c_{24} M_x + c_{25} M_y + c_{26} = 0 \tag{4.28}$$

$$c_{31} A_{1x} + c_{32} A_{1y} + c_{33} A_{1z} + c_{34} M_x + c_{35} M_y + c_{26} = 0 \tag{4.29}$$

$$c_{41} A_{1x} + c_{42} A_{1y} + c_{43} A_{1z} + c_{44} M_x + c_{45} M_y + c_{26} = 0 \tag{4.30}$$

where:

$$c_{n1} = \sin \theta_{2n} (d_{n23} - d_{n32}) \qquad \text{for } n = 2, 3, 4$$

$$c_{n2} = \sin \theta_{2n} (d_{n31} - d_{n13}) \qquad \text{for } n = 2, 3, 4$$

$$c_{n3} = \sin \theta_{2n} (d_{n12} - d_{n21}) \qquad \text{for } n = 2, 3, 4$$

$$c_{n4} = -\sin \theta_{1n} (d_{n23} - d_{n32}) \qquad \text{for } n = 2, 3, 4$$

$$c_{n5} = -\sin \theta_{1n} (d_{n31} - d_{n13}) \qquad \text{for } n = 2, 3, 4$$

$$c_{n6} = -M_z \sin \theta_{1n} (d_{n12} - d_{n21}) + (\cos \theta_{1n} - \cos \theta_{2n}) (1 + d_{n11} + d_{n22} + d_{n33}) \qquad \text{for } n = 2, 3, 4$$

The above equations, Equations (4.26) through (4.30), are the equations which must be satisfied. These five equations have six unknowns, the coordinates of points M and A. Therefore, the z component of M can be assumed. Equation (4.28) is solved for M_y, yielding

$$M_y = h_{11} A_{1x} + h_{12} A_{1y} + h_{13} A_{1z} + h_{14} M_x + h_{15} \tag{4.31}$$

where:

$$h_{11} = -c_{21}/c_{25}$$

$$h_{12} = -c_{22}/c_{25}$$

$$h_{13} = -c_{23}/c_{25}$$

$$h_{14} = -c_{24}/c_{25}$$

$$h_{15} = -c_{26}/c_{25}$$

This equation is then substituted in Equations (4.26), (4.29) and (4.30), giving the next set of equations.

$$\begin{aligned} & e_1 A_{1x}^2 + e_2 A_{1y}^2 + e_3 A_{1z}^2 + e_4 M_x^2 + e_5 A_{1x} A_{1y} \\ & \quad + e_6 A_{1x} A_{1z} + e_7 A_{1x} M_x + e_8 A_{1x} + e_9 A_{1y} A_{1z} \\ & \quad + e_{10} A_{1y} M_x + e_{11} A_{1y} + e_{12} A_{1z} M_x + e_{13} A_{1z} \\ & \quad + e_{14} M_x + e_{15} = 0 \end{aligned} \tag{4.32}$$

where:

$$e_1 = h_{11}^2$$

$$e_2 = h_{12}^2$$

$$e_3 = h_{13}^2$$

$$e_4 = h_{14}^2 + 1$$

$$e_5 = 2\, h_{11}\, h_{12}$$

$$e_6 = 2\, h_{11}\, h_{13}$$

$$e_7 = 2\, h_{11}\, h_{14}$$

$$e_8 = 2\, h_{11}\, h_{15}$$

$$e_9 = 2\, h_{12}\, h_{13}$$

$$e_{10} = 2\ h_{12}\ h_{14}$$

$$e_{11} = 2\ h_{12}\ h_{15}$$

$$e_{12} = 2\ h_{13}\ h_{14}$$

$$e_{13} = 2\ h_{13}\ h_{15}$$

$$e_{14} = 2\ h_{14}\ h_{15}$$

$$e_{15} = M_z^2 - 1 + h_{15}^2$$

$$h_{21}\ A_{1x} + h_{22}\ A_{1y} + h_{23}\ A_{1z} + h_{24}\ M_x + h_{25} = 0 \qquad (4.33)$$

where:

$$h_{21} = c_{31} + c_{35}\ c_{11}$$

$$h_{22} = c_{32} + c_{35}\ c_{12}$$

$$h_{23} = c_{33} + c_{35}\ c_{13}$$

$$h_{24} = c_{34} + c_{35}\ c_{14}$$

$$h_{25} = c_{36} + c_{35}\ c_{15}$$

$$h_{31}\ A_{1x} + h_{32}\ A_{1y} + h_{33}\ A_{1z} + h_{34}\ M_x + h_{35} = 0 \qquad (4.34)$$

where:

$$h_{31} = c_{41} + c_{45}\ c_{11}$$

$$h_{32} = c_{42} + c_{45}\ c_{12}$$

$$h_{33} = c_{43} + c_{45}\ c_{13}$$

$$h_{34} = c_{44} + c_{45}\ c_{14}$$

$$h_{35} = c_{46} + c_{45}\ c_{15}$$

Next, Equation (4.33) is solved for M_x.

$$M_x = k_{11}\ A_{1x} + k_{12}\ A_{1y} + k_{13}\ A_{1z} + k_{14} \qquad (4.35)$$

where:

$$k_{11} = -h_{21}/h_{24}$$

$$k_{12} = -h_{22}/h_{24}$$

$$k_{13} = h_{23}/h_{24}$$

$$k_{14} = h_{25}/h_{24}$$

Then this equation is substituted in Equations (4.32) and (4.34) with the following results.

$$f_1 A_{1x}^2 + f_2 A_{1y}^2 + f_3 A_{1z}^2 + f_4 A_{1x} A_{1y} + f_5 A_{1x} A_{1z} + f_6 A_{1x} + f_7 A_{1y} A_{1z} + f_8 A_{1y} + f_9 A_{1z} + f_{10} = 0 \tag{4.36}$$

where:

$$f_1 = e_1 + k_{11}^2 e_4 + k_{11} e_7$$

$$f_2 = e_2 + k_{12}^2 e_4 + k_{12} e_{10}$$

$$f_3 = e_3 + k_{13}^2 e_4 + k_{13} e_{12}$$

$$f_4 = e_5 + 2 k_{11} k_{12} e_4 + k_{11} e_{10} + k_{12} e_7$$

$$f_5 = e_6 + 2 k_{11} k_{13} e_4 + k_{11} e_{12} + k_{13} e_7$$

$$f_6 = e_8 + 2 k_{11} k_{14} e_4 + k_{11} e_{14} + k_{14} e_7$$

$$f_7 = e_9 + 2 k_{12} k_{13} e_4 + k_{12} e_{12} + k_{13} e_{10}$$

$$f_8 = e_{11} + 2 k_{12} k_{14} e_4 + k_{12} e_{14} + k_{14} e_{10}$$

$$f_9 = e_{13} + 2 k_{13} k_{14} e_4 + k_{13} e_{14} + k_{14} e_{12}$$

$$f_{10} = e_{15} + k_{14}^2 e_4 + k_{14} e_{14}$$

$$k_{21} A_{1x} + k_{22} A_{1y} + k_{23} A_{1z} + k_{24} = 0 \tag{4.37}$$

where:

$$k_{21} = h_{31} + h_{34} h_{11}$$

$$k_{22} = h_{32} + h_{34} h_{12}$$

$$k_{23} = h_{33} + h_{34} h_{13}$$

$$k_{24} = h_{35} + h_{34} h_{14}$$

The next substitution eliminates A_{1z}. Solving Equation (4.37) for A_z yields the following equation.

$$A_{1z} = s_{11} A_{1x} + s_{12} A_{1y} + s_{13} \qquad (4.38)$$

where:

$$s_{11} = -k_{21}/k_{23}$$

$$s_{12} = -k_{22}/k_{23}$$

$$s_{13} = -k_{24}/k_{23}$$

Substituting Equation (4.38) in Equations (4.36) and (4.27) yields the two second order equations in A_{1x} and A_{1y} which must be solved to find a solution for this case.

$$a_1 A_{1x}^2 + a_2 A_{1y}^2 + a_3 A_{1x} A_{1y} + a_4 A_{1x} + a_5 A_{1y} + a_6 = 0 \qquad (4.39)$$

where:

$$a_1 = f_1 + s_{11}^2 f_3 + s_{11} f_5$$

$$a_2 = f_2 + s_{12}^2 f_3 + s_{12} f_7$$

$$a_3 = f_4 + 2 s_{11} f_3 + s_{11} f_7 + s_{12} f_5$$

$$a_4 = f_6 + 2 s_{11} s_{13} f_3 + s_{11} f_9 + s_{13} f_5$$

$$a_5 = f_8 + 2\, s_{12}\, s_{13}\, f_3 + s_{12}\, f_9 + s_{13}\, f_7$$

$$a_6 = f_{10} + s_{13}^2\, f_3 + s_{13}\, f_9$$

$$b_1\, A_{1x}^2 + b_2\, A_{1y}^2 + b_3\, A_{1x}\, A_{1y} + b_4\, A_{1x} + b_5\, A_{1y} + b_6 = 0 \qquad (4.40)$$

where:

$$b_1 = s_{11}^2 + 1$$

$$b_2 = s_{12}^2 + 1$$

$$b_3 = 2\, s_{11}\, s_{12}$$

$$b_4 = 2\, s_{11}\, s_{13}$$

$$b_5 = 2\, s_{12}\, s_{13}$$

$$b_6 = s_{13}^2 - 1$$

Equations (4.39) and (4.40) can be combined into one fourth order equation in only one unknown. The first step is to combine the two equations to eliminate the $A_{1x}\, A_{1y}$ terms.

$$c_1\, A_{1x}^2 + c_2\, A_{1y}^2 + c_3\, A_{1x} + c_4\, A_{1y} + c_5 = 0 \qquad (4.41)$$

where:

$$c_1 = a_1\, b_3 - a_3\, b_1$$

$$c_2 = a_2\, b_3 - a_3\, b_2$$

$$c_3 = a_4\, b_3 - a_3\, b_4$$

$$c_4 = a_5\, b_3 - a_3\, b_5$$

$$c_5 = a_6\, b_3 - a_3\, b_6$$

Next, "complete the square" on Equation (4.41), which yields:

$$\left(\sqrt{c_1}\, A_{1x} + \frac{c_3}{2\sqrt{c_1}}\right)^2 + \left(\sqrt{c_2}\, A_{1y} + \frac{c_4}{2\sqrt{c_1}}\right)^2 + c_5 - \frac{c_3^2}{4c_1}$$

$$- \frac{c_4^2}{4c_2} = 0 \qquad (4.42)$$

To get this in a simpler form, the following transformation is made.

$$A'_x = \sqrt{c_1}\, A_{1x} + \frac{c_3}{2\sqrt{c_1}} \qquad (4.43)$$

and,

$$A'_y = \sqrt{c_2}\, A_{1y} + \frac{c_4}{2\sqrt{c_1}} \qquad (4.44)$$

These can be rearranged as:

$$A_{1x} = \frac{A'_x}{\sqrt{c_1}} - \frac{c_3}{2c_1} \qquad (4.45)$$

$$A_{1y} = \frac{A'_y}{\sqrt{c_2}} - \frac{c_4}{2c_2} \qquad (4.46)$$

Equation (4.42) can be rewritten as follows:

$$A'^2_x + A'^2_y + R_1 = 0 \qquad (4.47)$$

where:

$$R_1 = c_5 - \frac{c_3^2}{4c_1} - \frac{c_4^2}{4c_2}$$

Substituting Equations (4.45) and (4.46) in Equation (4.39) yields the following equation:

$$d_1\, A'^2_x + d_2\, A'^2_y + d_3\, A'_x\, A'_y + d_4\, A'_x + d_5\, A'_y + d_6 = 0 \qquad (4.48)$$

where:

where:

$$d_1 = a_1/c_1$$

$$d_2 = a_2/c_2$$

$$d_3 = a_3/\sqrt{c_1 c_2}$$

$$d_4 = \frac{1}{\sqrt{c_1}} (a_4 - \frac{a_1 c_3}{c_1} - \frac{a_3 c_4}{2c_2})$$

$$d_5 = \frac{1}{\sqrt{c_2}} (a_5 - \frac{a_2 c_4}{c_2} - \frac{a_3 c_3}{2c_1})$$

$$d_6 = - \frac{a_4 c_3}{2c_1} - \frac{a_5 c_4}{2c_2} + \frac{a_1 c_3^2}{4c_1^2} + \frac{a_2 c_4^2}{4c_2^2} + \frac{a_3 c_3 c_4}{4c_1 c_2} + a_6$$

To combine Equations (4.47) and (4.48), the latter must be rearranged and squared.

$$d_1 A_x'^2 + d_2 A_y'^2 + d_4 A_x' + d_6 - d_3 A_x' A_y' - d_5 A_y' \qquad (4.49)$$

Squaring this gives an equation which contains only even powers of A_y'. This allows substitution for $A_y'^2$ from Equation (4.47) which results in the following fourth order equation.

$$e_1 A_x'^4 + e_2 A_x'^3 + e_3 A_x'^2 + e_4 A_x' + e_5 = 0 \qquad (4.50)$$

where:

$$e_1 = d_1^2 + d_2^2 - 2 d_1 d_2 + d_3^2$$

$$e_2 = 2 d_1 d_4 - 2 d_2 d_4 + 2 d_3 d_5$$

$$e_3 = 2 d_2^2 R_1 + d_4^2 - 2 d_1 d_2 R_1 + 2 d_1 d_6 - 2 d_2 d_6$$

$$+ d_3^2 R_1 + d_5^2$$

TABLE III

FOUR POSITION SYNTHESIS, EXAMPLE PROBLEM, WITH A GEAR RATIO OF TWO

Point	Coordinates x	y	z
		Input	
M	--	--	.672512
P_1	-.2	.3	.932738
P_2	-.134274	.102227	.985657
P_3	-.0312716	-.0852917	.995866
P_4	.101886	-.249175	.963084
Q_1	-.3	.4	-.866025
Q_2	-.454178	.482660	-.748840
Q_3	-.606622	.515393	-.605293
Q_4	-.745965	.495813	-.444641
		Output	
First M	.5446801	.498735	.672512
First A_1	.584604	.409250	.700537
Second M	.632716	.383925	.672512
Second A_1	.697172	.420892	.580345
Third M	.619970	.404184	.672512
Third A_1	.692129	.431593	.578520
Fourth M	.580250	.464841	.672512
Fourth A_1	.667463	.455398	.587922

$$e_4 = -2\, d_2\, d_4\, R_1 + 2\, d_4\, d_6 + 2\, d_3\, d_5\, R_1$$

$$e_5 = d_2^2\, R_1^2 + d_6^2 - 2\, d_2\, d_6\, R_1 + d_5^2\, R_1$$

This polynomial is then solved using a standard polynomial solving routine. Any of the roots of the polynomial which have zero imaginary parts are possible solutions. For each of these possible solutions, the following equations are used to find the other coordinates of points A and M.

$$A'_y = (-A'^2_x - R_1)^{\frac{1}{2}} \tag{4.51}$$

$$A_{1x} = \frac{A'_x}{\sqrt{c_1}} - \frac{c_3}{2c_2} \tag{4.52}$$

$$A_{1y} = \frac{A'_y}{\sqrt{c_2}} - \frac{c_4}{2c_2} \tag{4.53}$$

$$A_{1z} = -\frac{1}{c_{23}}\,(c_{21}\, A_{1x} + c_{22}\, A_{1y} + c_{24}) \tag{4.54}$$

$$M_x = \frac{-1}{c_{34}}\,(c_{31}\, A_{1x} + c_{32}\, A_{1y} + c_{33}\, A_{1z} + c_{35}) \tag{4.55}$$

$$M_y = \frac{-1}{c_{45}}\,(c_{41}\, A_{1x} + c_{42}\, A_{1y} + c_{43}\, A_{1z} + c_{44}\, M_x + c_{46}) \tag{4.56}$$

These coordinates are then used as inputs into the analysis program to sort extraneous roots from the solutions. A program to carry out the above computations is included as Appendix E. This program is used to work an example problem, with the results given in Table III.

4.7 Five Position Synthesis

When five rigid body positions are specified, the problem is an extension of the four position synthesis problem. Four rotation matrices and linear design equations must be found using the methods of Chapter III, one design equation for each of the rotations from position one to position two, three, four, and five. In addition, the two equations to constrain points M and A to be on the unit sphere are necessary. The six design equations are listed below:

$$M_x^2 + M_y^2 + M_z^2 = 1 \tag{4.57}$$

$$A_{1x}^2 + A_{1y}^2 + A_{1z}^2 = 1 \tag{4.58}$$

$$c_{21} A_{1x} + c_{22} A_{1y} + c_{23} A_{1z} + c_{24} M_x + c_{25} M_y + c_{26} M_z + c_{27} = 0 \tag{4.59}$$

$$c_{31} A_{1x} + c_{32} A_{1y} + c_{33} A_{1z} + c_{34} M_x + c_{35} M_y + c_{36} M_z + c_{37} = 0 \tag{4.60}$$

$$c_{41} A_{1x} + c_{42} A_{1y} + c_{43} A_{1z} + c_{44} M_x + c_{45} M_y + c_{46} M_z + c_{47} = 0 \tag{4.61}$$

$$c_{51} A_{1x} + c_{52} A_{1y} + c_{53} A_{1z} + c_{54} M_x + c_{55} M_y + c_{56} M_z + c_{57} = 0 \tag{4.62}$$

where:

$c_{n1} = \sin \theta_{2n} (d_{n23} - d_{n32})$ for n = 2, 3, 4, 5

$c_{n2} = \sin \theta_{2n} (d_{n31} - d_{n13})$ for n = 2, 3, 4, 5

$c_{n3} = \sin \theta_{2n} (d_{n12} - d_{n21})$ for n = 2, 3, 4, 5

$c_{n4} = -\sin \theta_{1n} (d_{n23} - d_{n32})$ for n = 2, 3, 4, 5

$c_{n5} = -\sin \theta_{1n} (d_{n31} - d_{n13})$ for n = 2, 3, 4, 5

$c_{n6} = -\sin \theta_{1n} (d_{n12} - d_{n21})$ for n = 2, 3, 4, 5

$$c_{n7} = (\cos \theta_{1n} - \cos \theta_{2n})(1 + d_{n11} + d_{n22} + d_{n33})$$

for n = 2, 3, 4, 5

The solution of these equations for the coordinates of points M and A is similar to the procedure used for four positions. First solve Equation (4.59) for M_z and substitute in Equations (4.57) and (4.60) through (4.62).

$$M_z = g_{11} A_{1x} + g_{12} A_{1y} + g_{13} A_{1z} + g_{14} M_x + g_{15} M_y + g_{16} \tag{4.63}$$

where:

$$g_{11} = -c_{21}/c_{26}$$

$$g_{12} = -c_{22}/c_{26}$$

$$g_{13} = -c_{23}/c_{26}$$

$$g_{14} = -c_{24}/c_{26}$$

$$g_{15} = -c_{25}/c_{26}$$

$$g_{16} = -c_{27}/c_{26}$$

After substitution, the following equations are generated:

$$g_{21} A_{1x} + g_{22} A_{1y} + g_{23} A_{1z} + g_{24} M_x + g_{25} M_y + g_{26} = 0 \tag{4.64}$$

where:

$$g_{21} = c_{31} + g_{11} c_{36}$$

$$g_{22} = c_{32} + g_{12} c_{36}$$

$$g_{23} = c_{33} + g_{13} c_{36}$$

$$g_{24} = c_{34} + g_{14} c_{36}$$

$$g_{25} = c_{35} + g_{15}\, c_{36}$$

$$g_{26} = c_{37} + g_{16}\, c_{36}$$

$$g_{31}\, A_{1x} + g_{32}\, A_{1y} + g_{33}\, A_{1z} + g_{34}\, M_x + g_{35}\, M_y + g_{36} = 0 \tag{4.65}$$

where:

$$g_{31} = c_{41} + g_{11}\, c_{46}$$

$$g_{32} = c_{42} + g_{12}\, c_{46}$$

$$g_{33} = c_{43} + g_{13}\, c_{46}$$

$$g_{34} = c_{44} + g_{14}\, c_{46}$$

$$g_{35} = c_{45} + g_{15}\, c_{46}$$

$$g_{36} = c_{47} + g_{16}\, c_{46}$$

$$g_{41}\, A_{1x} + g_{42}\, A_{1y} + g_{43}\, A_{1z} + g_{44}\, M_x + g_{45}\, M_y + g_{46} = 0 \tag{4.66}$$

where:

$$g_{41} = c_{51} + g_{11}\, c_{56}$$

$$g_{42} = c_{52} + g_{12}\, c_{56}$$

$$g_{43} = c_{53} + g_{13}\, c_{56}$$

$$g_{44} = c_{54} + g_{14}\, c_{56}$$

$$g_{45} = c_{55} + g_{15}\, c_{56}$$

$$g_{46} = c_{57} + g_{16}\, c_{56}$$

$$d_1\, A_{1x}^2 + d_2\, A_{1y}^2 + d_3\, A_{1z}^2 + d_4\, M_x^2 + d_5\, M_y^2 + d_6$$

$$+ d_7\, A_{1x}\, A_{1y} + d_8\, A_{1x}\, A_{1z} + d_9\, A_{1x}\, M_x + d_{10}\, A_{1x}\, M_y$$

$$+ d_{11} A_{1x} + d_{12} A_{1y} A_{1z} + d_{13} A_{1y} M_x + d_{14} A_{1y} M_y$$
$$+ d_{15} A_{1y} + d_{16} A_{1z} M_x + d_{17} A_{1z} M_y + d_{18} A_{1z}$$
$$+ d_{19} M_x M_y + d_{20} M_x + d_{21} M_y = 0 \qquad (4.67)$$

where:

$$d_1 = g_{11}^2$$
$$d_2 = g_{12}^2$$
$$d_3 = g_{13}^2$$
$$d_4 = g_{14}^2 + 1$$
$$d_5 = g_{15}^2 + 1$$
$$d_6 = g_{16}^2 - 1$$
$$d_7 = 2\ g_{11}\ g_{12}$$
$$d_8 = 2\ g_{11}\ g_{13}$$
$$d_9 = 2\ g_{11}\ g_{14}$$
$$d_{10} = 2\ g_{11}\ g_{15}$$
$$d_{11} = 2\ g_{11}\ g_{16}$$
$$d_{12} = 2\ g_{12}\ g_{13}$$
$$d_{13} = 2\ g_{12}\ g_{14}$$
$$d_{14} = 2\ g_{12}\ g_{15}$$
$$d_{15} = 2\ g_{12}\ g_{16}$$
$$d_{16} = 2\ g_{13}\ g_{14}$$
$$d_{17} = 2\ g_{13}\ g_{15}$$

$$d_{18} = 2\, g_{13}\, g_{16}$$

$$d_{19} = 2\, g_{14}\, g_{15}$$

$$d_{20} = 2\, g_{14}\, g_{16}$$

$$d_{21} = 2\, g_{15}\, g_{16}$$

The next step in the synthesis procedure is to solve Equation (4.64) for M_y, yielding Equation (4.68). Equation (4.68) is then substituted in Equations (4.65) and (4.66) yielding Equations (4.69) and (4.70).

$$M_y = h_{11}\, A_{1x} + h_{12}\, A_{1y} + h_{13}\, A_{1z} + h_{14}\, M_x + h_{15} \tag{4.68}$$

where:

$$h_{11} = -g_{21}/g_{25}$$

$$h_{12} = -g_{22}/g_{25}$$

$$h_{13} = -g_{23}/g_{25}$$

$$h_{14} = -g_{24}/g_{25}$$

$$h_{15} = -g_{26}/g_{25}$$

$$h_{21}\, A_{1x} + h_{22}\, A_{1y} + h_{23}\, A_{1z} + h_{24}\, M_x + h_{25} = 0 \tag{4.69}$$

where:

$$h_{21} = g_{31} + h_{11}\, g_{35}$$

$$h_{22} = g_{32} + h_{12}\, g_{35}$$

$$h_{23} = g_{33} + h_{13}\, g_{35}$$

$$h_{24} = g_{34} + h_{14}\, g_{35}$$

$$h_{25} = g_{36} + h_{15}\, g_{35}$$

$$h_{31}\, A_{1x} + h_{32}\, A_{1y} + h_{33}\, A_{1z} + h_{34}\, M_x + h_{35} = 0 \tag{4.70}$$

where:

$$h_{31} = g_{41} + h_{11}\, g_{45}$$

$$h_{32} = g_{42} + h_{12}\, g_{45}$$

$$h_{33} = g_{43} + h_{13}\, g_{45}$$

$$h_{34} = g_{44} + h_{14}\, g_{45}$$

$$h_{35} = g_{45} + h_{15}\, g_{45}$$

Equation (4.68) is also substituted in Equation (4.67) with the following result.

$$\begin{aligned} e_1\, A_{1x}^2 &+ e_2\, A_{1y}^2 + e_3\, A_{1z}^2 + e_4\, M_x^2 \\ &+ e_5\, A_{1x}\, A_{1y} + e_6\, A_{1x}\, A_{1z} + e_7\, A_{1x}\, M_{1x} + e_8\, A_{1x} \\ &+ e_9\, A_{1y}\, A_{1z} + e_{10}\, A_{1y}\, M_x + e_{11}\, A_{1y} + e_{12}\, A_{1z}\, M_x \\ &+ e_{13}\, A_{1z} + e_{14}\, M_x + e_{15} = 0 \end{aligned} \quad (4.71)$$

where:

$$e_1 = d_1 + h_{11}^2\, d_5 + h_{11}\, d_{10}$$

$$e_2 = d_2 + h_{12}^2\, d_5 + h_{12}\, d_{14}$$

$$e_3 = d_3 + h_{13}^2\, d_5 + h_{13}\, d_{17}$$

$$e_4 = d_4 + h_{14}^2\, d_5 + h_{14}\, d_{19}$$

$$e_5 = d_7 + 2\, h_{11}\, h_{12}\, d_5 + h_{11}\, d_{14} + h_{12}\, d_{10}$$

$$e_6 = d_8 + 2\, h_{11}\, h_{13}\, d_5 + h_{11}\, d_{17} + h_{13}\, d_{10}$$

$$e_7 = d_9 + 2\, h_{11}\, h_{14}\, d_5 + h_{11}\, d_{19} + h_{14}\, d_{10}$$

$$e_8 = d_{11} + 2\, h_{11}\, h_{15}\, d_5 + h_{11}\, d_{21} + h_{15}\, d_{10}$$

$$e_9 = d_{12} + 2\, h_{12}\, h_{13}\, d_5 + h_{12}\, d_{17} + h_{13}\, d_{14}$$

$$e_{10} = d_{13} + 2\,h_{12}\,h_{14}\,d_5 + h_{12}\,d_{19} + h_{14}\,d_{14}$$

$$e_{11} = d_{15} + 2\,h_{12}\,h_{15}\,d_5 + h_{12}\,d_{21} + h_{15}\,d_{14}$$

$$e_{12} = d_{16} + 2\,h_{13}\,h_{14}\,d_5 + h_{13}\,d_{19} + h_{14}\,d_{17}$$

$$e_{13} = d_{18} + 2\,h_{13}\,h_{15}\,d_5 + h_{13}\,d_{21} + h_{15}\,d_{17}$$

$$e_{14} = d_{20} + 2\,h_{14}\,h_{15}\,d_5 + h_{14}\,d_{21} + h_{15}\,d_{19}$$

$$e_{15} = d_6 + h_{15}^2\,d_5 + h_{15}\,d_{21}$$

Next, Equation (4.72) is solved for M_x to yield Equation (4.35). This equation is substituted in Equations (4.70) and (4.71), yielding Equations (4.37) and (4.36), respectively. Then Equation (4.37) is solved for A_{1z} which yields Equation (4.38). This value of A_{1z} is substituted in Equations (4.36) and (4.58), resulting in Equations (4.39) and (4.40), respectively. These two equations are then solved exactly as in the four position synthesis. Equations (4.41) through (4.56) are valid. One additional equation is necessary to define M_z in terms of the other variables.

$$M_z = \frac{-1}{c_{56}}\,(c_{51}\,A_{1x} + c_{52}\,A_{1y} + c_{53}\,A_{1z} + c_{54}\,M_x + c_{55}\,M_y + c_{57}) \qquad (4.72)$$

As before, the analysis program must be employed at this point to sort out the extraneous roots. Appendix F contains a computer program to perform the above synthesis. This computer program is used to work an example problem, with the results given in Table IV.

TABLE IV

FIVE POSITION SYNTHESIS, EXAMPLE PROBLEM, WITH A GEAR RATIO OF TWO

Point	Coordinates x	y	z
		Input	
P_1	-.2	.3	.932738
P_2	-.113094	.226181	.967498
P_3	-.00757381	.173646	.984779
P_4	.108740	.146311	.093244
P_5	.227249	.146282	.962787
Q_1	-.3	.4	.866025
Q_2	-.503378	.350705	-.789694
Q_3	-.679041	.256321	-.687897
Q_4	-.814562	.123531	-.566771
Q_5	-.900225	-.0383110	-.433736
		Output	
First M	.150548	.693181	.704866
First A_1	.0655505	.533533	.843235
Second M	.0371080	.601662	.797888
Second A_1	.0763731	.553933	.829051
Third M	.100053	.637487	.764233
Third A_1	.162990	.626260	.762386
Fourth M	.100053	.637487	.764233
Fourth A_1	.162990	.626260	.762386

CHAPTER V

CONCLUSION

This thesis presents a method for simplifying the nonlinear equations derived from rotation matrices. This procedure is illustrated by applying it to a specific spherical mechanism. The mechanism chosen as an example is a spherical cycloidal crank mechanism as defined in Chapter I. The kinematic analysis of the selected mechanism is explained in Chapter II. Chapter III is devoted to development of the major synthesis equation. This design, or synthesis, equation is then used for designing mechanisms, given various sets of design criteria. The design procedures for two, three, four and five rigid body positions are given in Chapter IV. Computer programs to perform the analysis and synthesis described above are included in the Appendices. The advantage of this method is that it yields a closed form solution, eliminating the convergence problems associated with numerical techniques.

The use of rotation matricies for synthesis may be extended to three rotation matrices by using the methods outlined in this thesis. Using three rotation matrices would allow the synthesis of more complex mechanisms. However, several problems must first be confronted. First, the nonlinearity of the equations which result from multiplying three rotation matrices together is at least sixth order. Also, there are nine unknowns, which greatly increase the size of the equations.

Determining the proper sequence of operations to manipulate the original nonlinear equations into the desired number of linear equations is a large and rather time-consuming task. With three rotation matrices there will be three equations necessary to insure that the rotation vectors remain on the unit sphere. While this would be a useful extension of the present problem, the difference mentioned would be formidable.

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APPENDIX A

ANALYSIS PROGRAM

```
0000000001111111111222222222233333333334444444444555555555566666666667777777778
1234567890123456789012345678901234567890123456789012345678901234567890123456789
CARD
C**      THIS PROGRAM IS DESIGNED TO PERFORM A DISPLACEMENT ANALYSIS OF A       **
C**      SPHERICAL CYCLOIDAL CRANK MECHANISM WHICH IS LOCATED ON A SPHERE       **
C**      WHOSE CENTER IS AT THE ORIGIN. THE VARIABLES ARE AS FOLLOWS.           **
C**      IC - INPUT DATA CHOICE PARAMETER; (FIRST DATA CARD WITH AN             **
C**             I1 FORMAT)                                                      **
C**         IC=1, COORDINATES GIVEN ARE ASSUMED TO LIE ON A UNIT SPHERE.        **
C**         IC=2, THE X AND Y COORDINATES OF EACH POINT ARE GIVEN IN THE        **
C**               FIRST 20 COLUMNS WITH A +1. OR A -1. IN THE NEXT 10           **
C**               COLUMNS TO GIVE THE SIGN OF THE Z COORDINATE.                 **
C**      GR - THE GEAR RATIO N = (THE RADIUS OF THE SUN GEAR)/(THE RADIUS       **
C**             OF THE PLANET GEAR); (SECOND DATA CARD WITH AN F10.0 FORMAT)    **
C**      M - A VECTOR CONTAINING THE CARTESIAN COORDINATES OF THE AXIS OF THE   **
C**             FIXED SUN GEAR; (THIRD DATA CARD WITH A 3F10.0 FORMAT)          **
C**      A - A VECTOR CONTAINING THE CARTESIAN COORDINATES OF THE AXIS OF       **
C**             THE PLANET GEAR IN ITS ORIGINAL POSITION IN COLUMN 1. THE       **
C**             SUBSEQUENT POSITIONS OF A ARE IN THE SECOND COLUMN OF A:        **
C**             (FOURTH DATA CARD WITH A 3F10.0 FORMAT FOR POSITION 1)          **
C**      P,Q - VECTORS CONTAINING THE CARTESIAN COORDINATES OF TWO POINTS       **
C**             ON THE PLANET GEAR. THE FIRST COLUMN CONTAINS THE FIRST         **
C**             POSITIONS OF THE POINTS AND SUBSEQUENT POSITIONS ARE IN         **
C**             COLUMN THREE. (P1 AND Q1 ARE ON THE FIFTH DATA CARD WITH A      **
C**             6F10.0 FORMAT)                                                  **
C**      R - A VECTOR CONTAINING THE CARTESIAN COORDINATES OF THE               **
C**             RESULT OF ROTATING Q ABOUT P THROUGH AN ANGLE OF                **
C**             NINETY DEGREES.                                                 **
C**      PQR - A MATRIX WHOSE COLUMNS ARE THE VECTORS P, Q, AND R.              **
C**      PQR2 - A MATRIX WHOSE COLUMNS ARE THE CARTESIAN COORDINATES OF THE     **
C**             SECOND POSITIONS OF P,Q, AND R.                                 **
C**      ROT - THE ROTATION MATRIX FROM POSITION ONE TO POSITION TWO.           **
C**      N - THE NUMBER OF SECOND POSITIONS OF THE MECHANISM; (SIXTH DATA       **
C**             CARD WITH AN I2 FORMAT)                                         **
C**      PHI - THE TOTAL ROTATION ANGLE REQUIRED TO GO FROM POSITION ONE TO     **
C**             POSITION TWO.                                                   **
C**      TH1 - THE ROTATION OF A ABOUT M WHEN THE MECHANISM GOES FROM           **
C**             POSITION ONE TO EACH SECOND POSITION. (STARTS ON SEVENTH        **
C**             DATA CARD WITH AN  F10.0 FORMAT; IN DEGREES)                    **
C**      TH2 - THE ROTATION OF P ABOUT A WHEN THE MECHANISM GOES FROM           **
C**             POSITION ONE TO EACH SECOND POSITION.                           **
C*******************************************************************************
C*******************************************************************************
      REAL M(3)
      DIMENSION A(3,2),P(3,3),Q(3,3),R(3,3),PQR(3,3),PQR2(3,3),ROTA(3,3)
     1,ROTM(3,3),ROTT(3,3)
C**     READ IN DATA                                                            **
      KCOUNT = 0
      READ 100, IC
  100 FORMAT(I1)
      PRINT 101, IC
  101 FORMAT(1H1,37H    THE INPUT DATA CHOICE PARAMETER =   ,I2)
      READ 102, GR
  102 FORMAT(8F10.0)
      PRINT 103, GR
  103 FORMAT(///,28H    THE INPUT GEAR RATIO IS: ,/,G15.6)
```

```
$JOB LIST                                                                    14:59:36.610  76/03/28  V000L00006
                                               80/80 LIST                         PAGE  001

          00000000011111111112222222222333333333344444444445555555555666666666677777777778
          12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
0001            PROGRAM LYZE(INPUT,OUTPUT)
0002      C*******************************************************************************
0003      C*******************************************************************************
0004      $IBSYS
```

```
 00000000011111111112222222222333333333344444444445555555555666666666677777777778
 12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
          READ 102, (M(I),I=1,3)
          PRINT 104, (M(I),I=1,3)
      104 FORMAT(///,31H    THE INPUT SUN GEAR AXIS IS: ,/,3G15.6)
          READ 102, (A(I,1),I=1,3)
          PRINT 105, (A(I,1),I=1,3)
      105 FORMAT(///,34H    THE INPUT PLANET GEAR AXIS IS: ,/,3G15.6)
          READ 106, (P(I,1),I=1,3),(Q(I,1),I=1,3)
      106 FORMAT(6F10.0)
          PRINT 107, (P(I,1),I=1,3),(Q(I,1),I=1,3)
      107 FORMAT(///,50H    THE FIRST RIGID BODY POSITIONS ARE AS FOLLOWS: ,
         1/,12H    POINT P: ,3X,3G15.6,/,12H    POINT Q: ,3X,3G15.6)
          PI = 355./(113.*180.)
          GO TO (5,10), IC
        5 CP = RAD(P(1,1),P(2,1),P(3,1)) - 1.
          CA = RAD(A(1,1),A(2,1),A(3,1)) - 1.
          CM = RAD(M(1),M(2),M(3)) - 1.
          CQ = RAD(Q(1,1),Q(2,1),Q(3,1)) - 1.
          CHECK = ABS(CP) + ABS(CA) + ABS(CM) + ABS(CQ)
          IF(CHECK.LT.0.01) GO TO 20
          PRINT 108, CP,CA,CM,CQ
      108 FORMAT(///,64H    $$$$THIS JOB ABORTED, THE INPUT VALUES ARE NOT O
         1N A UNIT$$$$ ,/,75H    $$$$SPHERE. THE AMOUNTS EACH POINT (P,A,M,A
         2ND Q) ARE TOO LARGE ARE:$$$$ ,/,4G15.6)
          GO TO 2000
       10 CONTINUE
C**       CALCULATE THE Z COORDINATE OF M,A,P, AND Q                       **
          CP = CHK(P(1,1),P(2,1))
          CA = CHK(A(1,1),A(2,1))
          CQ = CHK(Q(1,1),Q(2,1))
          CM = CHK(M(1),M(2))
          IF(CP.GT.0.) GO TO 15
          IF(CA.GT.0.) GO TO 15
          IF(CQ.GT.0.) GO TO 15
          IF(CM.GT.0.) GO TO 15
          P(3,1) = RAD2(P(1,1),P(2,1),P(3,1))
          Q(3,1) = RAD2(Q(1,1),Q(2,1),Q(3,1))
          A(3,1) = RAD2(A(1,1),A(2,1),A(3,1))
          M(3) = RAD2(M(1),M(2),M(3))
          GO TO 20
       15 CONTINUE
          PRINT 109, CP,CA,CM,CQ
      109 FORMAT(///,68H    $$$$THIS JOB ABORTED, THE INPUT VALUES X AND Y A
         1RE TOO LARGE$$$$ ,/,87H    $$$$TO BE ON THE UNIT SPHERE. THE AMOUN
         2T THEY (P,A,M, AND Q) ARE TOO LARGE ARE:$$$$ ,/,4G15.6)
          GO TO 2000
       20 CONTINUE
          R(1,1) = P(1,1)*P(1,1)*Q(1,1) + (P(1,1)*P(2,1)-P(3,1))*Q(2,1)
         1  + (P(1,1)*P(3,1)+P(2,1))*Q(3,1)
          R(2,1) = (P(1,1)*P(2,1)+P(3,1))*Q(1,1) + P(2,1)*P(2,1)*Q(2,1)
         1  + (P(2,1)*P(3,1)-P(1,1))*Q(3,1)
          R(3,1) = (P(1,1)*P(3,1)-P(2,1))*Q(1,1)
         1  + (P(2,1)*P(3,1)+P(1,1))*Q(2,1) + P(3,1)*P(3,1)*Q(3,1)
C**       READ IN NUMBER OF SECOND POSITIONS                               **
          PRINT 116,(M(I),I=1,3),(A(I,1),I=1,3),(P(I,1),I=1,3),
         1      (Q(I,1),I=1,3),(R(I,1),I=1,3)
```

```
00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
  116 FORMAT(///,37H    THE ORIGINAL M,A,P,Q, AND R, ARE: ,/,
     16H    M: ,3G15.6,/,6H    A: ,3G15.6,/,
     26H    P: ,3G15.6,/,6H    Q: ,3G15.6,/,
     36H    R: ,3G15.6)
      READ 110, N
  110 FORMAT(I2)
      PRINT 117,N
  117 FORMAT(///,35H    THE NUMBER OF SECOND POSITIONS: ,I5)
   25 CONTINUE
      READ 102, TH1D
      TH1 = TH1D*PI
      TH2 = TH1*GR
C**      NEXT FIND THE MATRIX FOR ROTATIONS ABOUT A                           **
      CALL ROT(A(1,1),A(2,1),A(3,1),TH2,ROTA)
      TH2D = TH2/PI
C        THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE ROTATION
C        MATRIX ABOUT A FOR EACH POSITION
C     PRINT 111,((ROTA(I,J),J=1,3),I=1,3),TH2D
C 111 FORMAT(///,35H    THE ROTATION MATRIX ABOUT A IS: ,/, 3(3G15.6,/),
C    128H    FOR A ROTATION ANGLE OF: ,G15.6,8H DEGREES )
C**      NEXT FIND THE NEW POSITIONS OF P,Q, AND R AFTER A ROTATION ABOUT A   **
      DO 30 I = 1,3
      P(I,2) = 0.0
      Q(I,2) = 0.0
      R(I,2) = 0.0
   30 CONTINUE
      DO 35 I = 1,3
      DO 35 J = 1,3
      P(I,2) = ROTA(I,J)*P(J,1) + P(I,2)
      Q(I,2) = ROTA(I,J)*Q(J,1) + Q(I,2)
      R(I,2) = ROTA(I,J)*R(J,1) + R(I,2)
   35 CONTINUE
C        THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE INTERMEDIATE
C        POSITIONS OF ALL THE POINTS; AFTER THE ROTATION ABOUT POINTA
C     PRINT 112,(P(I,2),I=1,3),(Q(I,2),I=1,3),(R(I,2),I=1,3)
C 112 FORMAT(///,*    AFTER THE FIRST ROTATION ABOUT A THE RIGID BODY PO
C    1POINTS ARE: ,/,6H    P: , 3G15.6/,  6H    Q: , 3G15.6,/,6H    R:
C    23G15.6)
C**      FIND THE ROTATION MATRIX ABOUT M                                     **
      CALL ROT(M(1),M(2),M(3),TH1,ROTM)
      DO 40 I = 1,3
      P(I,3) = 0.0
      Q(I,3) = 0.0
      R(I,3) = 0.0
      A(I,2) = 0.0
   40 CONTINUE
C        THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE ROTATION
C        MATRIX ABOUT POINT M
C     PRINT 113,((ROTM(I,J),J=1,3),I=1,3),TH1D
C 113 FORMAT(///,35H    THE ROTATION MATRIX ABOUT M IS: ,/, 3(3G15.6,/),
C    128H    FOR A ROTATION ANGLE OF: ,G15.6,8H DEGREES )
      DO 45 I = 1,3
      DO 45 J = 1,3
      P(I,3) = ROTM(I,J)*P(J,2) + P(I,3)
      Q(I,3) = ROTM(I,J)*Q(J,2) + Q(I,3)
```

```
00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
      R(I,3) = ROTM(I,J)*R(J,2) + R(I,3)
      A(I,2) = ROTM(I,J)*A(J,1) + A(I,2)
   45 CONTINUE
      PRINT 114,(A(I,2),I=1,3),(P(I,3),I=1,3),(Q(I,3),I=1,3),(P(I,3),I=1
     1,3), THID
  114 FORMAT(///,58H    AFTER THE TOTAL ROTATION THE RIGID BODY POSITION
     1S ARE: ,/,6H    A: , 3G15.6,/,6H    P: ,3G15.6 ,/,6H    Q: ,3G15.6
     2,/,6H    R: ,3G15.6,/,28H    FOR A ROTATION ANGLE OF: ,G15.6,
     38HDEGREES  )
      DO 50 I = 1,3
      DO 50 J = 1,3
   50 ROTT(I,J) = 0.
      DO 55 I = 1,3
      DO 55 J = 1,3
      DO 55 K = 1,3
   55 ROTT(I,J) = ROTM(I,K)*ROTA(K,J) + ROTT(I,J)
      PRINT 115,((ROTT(I,J),J=1,3),I=1,3)
  115 FORMAT(///58H    THE TOTAL ROTATION MATRIX FROM POSITION ONE TO TW
     1O IS: ,/, 3(3G15.6,/))
      KCOUNT = KCOUNT + 1
      IF(KCOUNT.LT.N) GO TO 25
 2000 CONTINUE
      STOP
      END
      SUBROUTINE ROT(UX,UY,UZ,PHI,D)
      DIMENSION D(3,3)
      C = COS(PHI)
      V = 1. - C
      S = SIN(PHI)
      D(1,1) = UX*UX*V + C
      D(2,2) = UY*UY*V + C
      D(3,3) = UZ*UZ*V + C
      DXY = UX*UY*V
      DXZ = UX*UZ*V
      DYZ = UY*UZ*V
      D(1,2) = DXY - UZ*S
      D(1,3) = DXZ + UY*S
      D(2,1) = DXY + UZ*S
      D(2,3) = DYZ - UX*S
      D(3,1) = DXZ - UY*S
      D(3,2) = DYZ + UX*S
      RETURN
      END
      FUNCTION RAD2(A,B,C)
      D = 1. - A*A - B*B
      RAD2 = C*SQRT(D)
      RETURN
      END
      FUNCTION RAD(A,B,C)
      D = A*A + B*B + C*C
      RAD = SQRT(D)
      RETURN
      END
      FUNCTION CHK(A,B)
      CHK = A*A + B*B - 1.
```

```
00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
         RETURN
         END
*00
2
2.0
.6              .31             1.
.67593          .487768         1.
-.2             .3              1.            -.3            .4            -1.
  1
4.9661
$IBSYS
```

APPENDIX B

INPUT DATA PROGRAM

```
 000000000111111111122222222223333333333444444444455555555556666666666777777777778
 12345678901234567890123456789012345678901234567890123456789012345678901234567890
      PROGRAM VALU(INPUT,OUTPUT)
C*******************************************************************************
C*******************************************************************************
C**     THIS ROUTINE IS DESIGNED TO TAKE TWO POSITICNS OF POINT P, ONE         **
C**     THIS ROUTINE IS DESIGNED TO TAKE N POSITIONS OF POINT P, ONE         **
C**     THIS ROUTINE IS DESIGNED TO TAKE N POSITICNS OF POINT P, ONE           **
C**     POSITION OF POINT Q WHICH CORRESPONDS TO ONE POSITION OF P AND         **
C**     FIND THE SECOND POSITION OF POINT Q. THE PROGRAM IS DESIGNED TO        **
C**     EITHER GIVE THE TOTAL RANGE OF THE X,Y, AND Z COMPONENTS OF THE        **
C**     SECOND POSITION OF Q FOR EACH SECOND POSITION OF P, OR WITH THE        **
C**     SECOND POSITION OF Q,X COORDINATE GIVEN, TO FIND THE OTHER             **
C**     COORDINATES OF Q. THE FIRST POSITIONS OF P AND Q ARE TAKEN AS THE      **
C**     REFERENCE POSITION FOR ALL OF THE OTHER INPUT P'S AND Q'S.             **
C**     THE VARIABLES USED ARE AS FOLLOWS:                                     **
C**     N - THE TOTAL NUMBER OF P,Q CARDS, NOT COUNTING THE INITIAL            **
C**     IC - CHOICE OF OPTIONS PARAMETER                                       **
C**          IC = 1: THE RANGES OF POSIBLE Q VALUES WILL BE CALCULATED.        **
C**          IC = 2: WITH THE GIVEN SECOND Q(X), THE OTHER POSSIBLE VALUES     **
C**               FOR Q(Y) AND Q(Z) ARE CALCULATED                             **
C**             POSITION P1 AND Q1                                             **
C**     P1 - A VECTOR WHICH CONTAINS THE CARTESIAN COORDINATES OF THE FIRST    **
C**             POINT P.                                                       **
C**     Q1 - A VECTOR WHICH CONTAINS THE CARTESIAN COORDINATES OF THE FIRST    **
C**             POINT Q                                                        **
C**     P - A VECTOR WHICH CONTAINS CONSECUTIVELY THE CARTESIAN COORDINATES    **
C**             OF EACH OF THE SECOND POINTS P                                 **
C**     Q - A VECTOR WHICH CONTAINS CONSECUTIVELY THE CARTESIAN COORDINATES    **
C**             OF EACH OF THE SECOND POINTS Q                                 **
C******           DATA INPUT INSTRUCTIONS                                 *******
C**     DATA CARD     VARIABLE             FORMAT                              **
C**         1              N                 I2                                **
C**         2             IC                 I2                                **
C**         3            P1,Q1               6F10.0                            **
C**         4            P,Q                 4F10.0                            **
C**         5            P,Q                 4F10.0                            **
C**     THE P'S AND Q'S ARE ENTERED WITH THE X AND Y COMPONENTS ENTERED IN     **
C**     TWENTY COLUMNS AND THE SIGN OF THE Z COMPONENT GIVEN BY A +1. OR A     **
C**     -1. IN THE NEXT TEN COLUMNS.                                           **
C*******************************************************************************
C*******************************************************************************
      DIMENSION P(3),Q(3),P1(3),Q1(3),Q2(3)
      READ 100, N
  100 FORMAT(I2)
      READ 100, IC
      READ 101,(P1(I),I=1,3),(Q1(I),I=1,3)
  101 FCRMAT(6F10.0)
      PRINT 102,N,IC,(P1(I),I=1,3),(Q1(I),I=1,3)
  102 FORMAT(1H1,///,38H    THE NUMBER OF SECOND PCSITIONS IS: ,I2,//
     125H    THE CHOICE OPTION IS: ,I2,/,
     237H    THE FIRST POSITION IS AS FOLLOWS: ,/,7H    P1: ,3G15.6,/,
     37H    Q1: ,3G15.6)
C**     COMPUTE THE LENGTH CF P1-Q1                                            **
C**     FIND THE Z COORDINATES OF P1 AND Q1
```

```
00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
      P1(3) = RAD2(P1(1),P1(2),P1(3))
      Q1(3) = RAD2(Q1(1),Q1(2),Q1(3))
      XL = P1(1) - Q1(1)
      YL = P1(2) - Q1(2)
      ZL = P1(3) - Q1(3)
      FLS = SEG(XL,YL,ZL)
      PRINT 107,(P1(I),I=1,3),(Q1(I),I=1,3),FLS
  107 FORMAT(///,48H     THE COMPUTED FIRST POSITIONS ARE AS FOLLOWS
     1,/,7H     P1: ,3G15.6,/,  7H     Q1: ,3G15.6,/,27H     THE LENGTH OF
     2P1-Q1 IS:  ,G15.6)
      ICOUNT = 1
      IF(IC.EQ.2) GO TO 10
C**     FIND THE POSSIBLE RANGE FOR THE COMPONENTS OF THE SECOND Q FOR A       **
C**     GIVEN SECOND P.                                                         **
    5 CONTINUE
      READ 101, (P(I),I=1,3)
C**     FIRST FIND P(3)                                                         **
      P(3) = RAD2(P(1),P(2),P(3))
      RX1 = P(1) - FLS
      RX2 = P(1) + FLS
      RY1 = P(2) - FLS
      RY2 = P(2) + FLS
      RZ1 = P(3) - FLS
      RZ2 = P(3) + FLS
      PRINT 103,(P(I),I=1,3),RX1,RX2,RY1,RY2,RZ1,RZ2
  103 FORMAT(///,40H     FOR THE ABOVE P1 AND Q1, AND FOR P2: ,/, 3G15.6,
     1/38H     THE X COORDINATE OF Q2 IS BETWEEN  ,/,G15.6,3HAND,G15.6,
     2/38H     THE Y COORDINATE OF Q2 IS BETWEEN  ,/,G15.6,3HAND,G15.6,
     3/38H     THE Z COORDINATE OF Q2 IS BETWEEN  ,/,G15.6,3HAND,G15.6)
      ICOUNT = ICOUNT + 1
      IF(ICOUNT.LE.N) GO TO 5
      GO TO 35
   10 CONTINUE
      READ 101, (P(I),I=1,3),Q(1)
      P(3) = RAD2(P(1),P(2),P(3))
C**     NOW FIND THE OTHER COORDINATES OF Q                                     **
      C1 = -2.*P(2)
      C2 = -2.*P(3)
      FLS = FLS*FLS
      C3 = -FLS + (P(1)-Q(1))*(P(1)-Q(1)) + P(2)*P(2) + P(3)*P(3)
      C4 = 1. - Q(1)*Q(1)
      C5 = C3 + C4
      CHK = ABS(C2)
      CHK2 = ABS(C1)
      IF(CHK.GE.1.E-6) GO TO 20
      IF(CHK2.GE.1.E-4) GO TO 15
      PRINT 104,(P(I),I=1,3),C1,C2,C3,C4,C5
  104 FORMAT(///,11H     FOR P2=  ,/, 3G15.6,/,31H     THE COEFFICIENTS C1
     1-C5 ARE:   ,/,3G15.6,48H     THESE COEFFICIENTS DO NOT YIELD A SOLU
     2TION         )
      ICOUNT = ICOUNT + 1
      IF(ICOUNT.LE.N) GO TO 10
      GO TO 35
   15 CONTINUE
      Q(2) = -C5/C1
```

```
0000000001111111111222222222233333333334444444444555555555566666666667777777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
      Q(3) = 1.
      Q(3) = RAD2(Q(1),Q(2),Q(3))
      Q2(1) = Q(1)
      Q2(2) = Q(2)
      Q2(3) = -Q(3)
      PRINT 105,(P(I),I=1,3),(Q(I),I=1,3),(Q2(I),I=1,3)
  105 FORMAT(///,40H    FOR THE ABOVE P1 AND Q1, AND FOR P2: ,/, 3G15.6,
     1/,31H    THE CORRESPONDING Q2'S ARE:  /,(3G15.6))
      ICOUNT = ICOUNT + 1
      IF(ICOUNT.LE.N) GO TO 10
      GO TO 35
   20 CONTINUE
C**     SET UP COEFFICIENTS FOR QUADRATIC                                    **
      C2S = C2*C2
      C6 = 1. + (C1*C1)/C2S
      C7 = 2.*C5*C1/C2S
      C8 = C5*C5/C2S - C4
      C9 = C7*C7 - 4.*C6*C8
      IF(C9.GT.0.) GO TO 25
      PRINT 106,(P(I),I=1,3),Q(1),C1,C2,C3,C4,C5,C6,C7,C8,C9
  106 FORMAT(///,40H    FOR THE ABOVE P1 AND Q1, AND FOR P2: ,/, 3G15.6,
     1/,35H    THERE IS NO SOLUTION WITH Q(X)= ,/,G15.6,/,
     231H    THE COEFFICIENTS C1-C9 ARE: ,/,(3G15.6))
      ICOUNT = ICOUNT + 1
      IF(ICOUNT.LE.N) GO TO 10
      GO TO 35
   25 CONTINUE
      C9 = SQRT(C9)
      CHK = ABS(C6)
      IF(CHK.GE.1.E-6) GO TO 30
      Q(3) = -C5/C2
      Q(2) = 1.
      Q(2) = RAD2(Q(1),Q(3),Q(2))
      Q2(1) = Q(1)
      Q2(2) = -Q(2)
      Q2(1) = Q(1)
      Q2(2) = -Q(2)
      Q2(3) = Q(3)
      ICOUNT = ICOUNT + 1
      IF(ICOUNT.LE.N) GO TO 10
      GO TO 35
   30 CONTINUE
      Q(2) = (-C7 + C9)/(2.*C6)
      Q2(1) = Q(1)
      Q2(2) = (-C7 - C9)/(2.*C6)
      Q2(3) = 1.
      Q(3) = 1.
      Q2(3) = RAD2(Q2(1),Q2(2),Q2(3))
      Q(3) = RAD2(Q(1),Q(2),Q(3))
      PRINT 105,(P(I),I=1,3),(Q(I),I=1,3),(Q2(I),I=1,3)
      ICOUNT = ICOUNT +1
      IF(ICOUNT.LE.N) GO TO 10
   35 CONTINUE
      STOP
      END
```

```
     00000000011111111112222222222333333333344444444445555555555666666666677777777778
     12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
           FUNCTION RAD2(A,B,C)
           D = 1. - A*A - B*B
           RAD2 = C * SQRT(D)
           RETURN
           END
           FUNCTION SEG(A,B,C)
           D = A*A + B*B + C*C
           SEG = SQRT(D)
           RETURN
           END
     "00
      2
      2
         .2        .61       -1.        .12       .52       1.
         .36       .32       -1.        .26
         .48       .27        1.        .302
     $IBSYS
```

APPENDIX C

TWO POSITION SYNTHESIS PROGRAM

```
0000000001111111111222222222233333333334444444444555555555566666666667777777778
1234567890123456789012345678901234567890123456789012345678901234567890123456789

      PROGRAM SC2P(INPUT,OUTPUT)
C*****************************************************************************
C*****************************************************************************
C**     THIS PROGRAM IS DESIGNED TO TAKE TWO GIVEN RIGID BODY POSITIONS      **
C**     WHICH LIE ON A SPHERE AND COMPUTE A GEARED SPHERICAL CYCLOIDAL       **
C**     CRANK MECHANISM WHICH WILL GUIDE A RIGID BODY CONNECTED TO THE       **
C**     PLANET GEAR, THROUGH THE TWO GIVEN POSITIONS.                        **
C**     THE VARIABLES USED ARE AS FOLLOWS :                                  **
C**     IC - INPUT DATA CHOICE PARAMETER;  (FIRST DATA CARD WITH AN          **
C**            I1 FORMAT)                                                     **
C**        IC=1, COORDINATES GIVEN ARE ASUMED TO LIE ON A UNIT SPHERE.       **
C**        IC=2, THE X AND Y COORDINATES OF EACH POINT ARE GIVEN IN THE      **
C**              FIRST 20 COLUMNS  WITH  A +1. OR A -1. IN THE NEXT 10       **
C**              COLUMNS TO GIVE THE SIGN OF THE Z COORDINATE.               **
C**     GR - THE GEAR RATIO N = (THE RADIUS OF THE SUN GEAR)/(THE            **
C**            RADIUS OF THE PLANET GEAR);  (SECOND DATA CARD WITH AN        **
C**            F10.0 FORMAT)                                                  **
C**     M - A VECTOR CONTAINING THE CARTESIAN COORDINATES OF THE AXIS        **
C**            OF THE FIXED SUN GEAR;  (SECOND DATA CARD WITH  A             **
C**            3F10.0 FORMAT)                                                 **
C**     A - A VECTOR CONTAINING THE CARTESIAN COORDINATES OF THE AXIS OF     **
C**            THE PLANET GEAR IN ITS ORIGINAL POSITION;  (THIRD DATA CARD   **
C**            WITH AN F10.0 FORMAT, ONLY THE THIRD OR Z COORDINATE IS       **
C**            INPUT IN THIS CASE)                                            **
C**     P,Q - VECTORS CONTAINING THE CARTESIAN COORDINATES OF TWO            **
C**            POINTS ON THE PLANET GEAR;  FIFTH DATA CARD--P1;              **
C**            SIXTH DATA CARD--P2                                            **
C**            SEVENTH DATA CARD--Q1                                          **
C**            EIGHTH DATA CARD--Q2;  ALL WITH A 3F10.0 FORMAT AS            **
C**            EXPLAINED UNDER IC.                                            **
C**     R - A VECTOR CONTAINING THE CARTESIAN COORDINATES OF THE             **
C**            RESULT OF ROTATING Q ABOUT P THROUGH AN ANGLE OF              **
C**            NINETY DEGREES.                                                **
C**     PQR - A MATRIX WHOSE COLUMNS ARE THE VECTORS P, Q, AND R.            **
C**     PINV - A MATRIX WHICH IS THE INVERSE OF PQR.                         **
C**     PQR2 - A MATRIX WHOSE COLUMNS ARE THE CARTESIAN COORDINATES OF THE   **
C**            SECOND POSITIONS OF P,Q, AND R.                                **
C**     ROT12 - THE ROTATION MATRIX FROM POSITION ONE TO POSITION TWO. IT    **
C**            IS THE PRODUCT OF PQR2 AND PINV                                **
C**     PHI - THE TOTAL ROTATION ANGLE REQUIRED TO GO FROM POSITION ONE TO   **
C**            POSITION TWO.                                                  **
C**     TH1 - THE ROTATION OF A ABOUT M WHEN THE MECHANISM GOES FROM         **
C**            POSITION ONE TO POSITION TWO; THETA ONE.                       **
C**     TH2 - THE ROTATION OF P ABOUT A WHEN THE MECHANISM GOES FROM         **
C**            POSITION ONE TO POSITION TWO; THETA TWO.                       **
C*****************************************************************************
C*****************************************************************************
      REAL M(3)
      DIMENSION A(3),P(3,2),Q(3,2),R(3,2),PQR(3,3),PINV(3,3),
     1          PQR2(3,3),ROT12(3,3),CHKL(2)
C**     READ IN DATA                                                          **
      KCOUNT = 1
      READ 100,IC
```

```
00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
  100 FORMAT(I1)
      PRINT 101,IC
  101 FORMAT(1H1,37H    THE INPUT DATA CHOICE PARAMETER =    I2)
      READ 102 , GR
      PRINT 103,GR
  103 FORMAT(///,28H    THE INPUT GEAR RATIO IS: ,/,G15.6)
      READ 102, M(1),M(2),M(3)
      PRINT 104,M(1),M(2),M(3)
  104 FORMAT(/// ,31H    THE INPUT SUN GEAR AXIS IS: ,/,3G15.6)
      READ 102,A(3)
      PRINT 105,A(3)
  105 FORMAT(///,47H    THE INPUT PLANET GEAR AXIS Z COORDINATE IS: ,/,G
     115.6)
      READ 102,((P(I,J),I=1,3),J=1,2),((Q(I,J),I=1,3),J=1,2)
  102 FORMAT(3F10.0)
      PRINT 106,((P(I,J),J=1,2),I=1,3),((Q(I,J),J=1,2),I=1,3)
  106 FORMAT(///,50H    THE INPUT RIGID BODY POSITIONS ARE AS FOLLOWS:,/
     146H      POINTS P; WHERE EACH COLUMN IS ONE POINT. ,/,3(2G15.6,/),
     2//   45H    POINTS Q; WHERE EACH COLUMN IS ONE POINT.  ,/,
     3 3(2G15.6,/))
C**      CALCULATE THE Z COORDINATE OF P, Q, AND M                           **
      PI = 355./(113.*180.)
      GO TO (5,20), IC
    5 CPS = 0.0
      CQS = 0.0
      DO 10 J = 1,2
      CP = RAD(P(1,J),P(2,J),P(3,J)) - 1.
      CQ = RAD(Q(1,J),Q(2,J),Q(3,J)) - 1.
      CPS = CPS + ABS(CP)
      CQS = CQS + ABS(CQ)
   10 CONTINUE
      CM = RAD(M(1),M(2),M(3)) - 1.
      CHECK = ABS(CPS) + ABS(CQS) + ABS(CM)
      IF (CHECK.GT. 0.01) GO TO 180
      GO TO 30
   20 DO 25  J = 1,2
      CP = CHK(P(1,J),P(2,J))
      CQ = CHK(Q(1,J),Q(2,J))
      IF (CP.GT.0.) GO TO 185
      IF (CQ.GT.0.) GO TO 185
      P(3,J) = RAD2(P(1,J),P(2,J) ,P(3,J))
      Q(3,J) = RAD2(Q(1,J),Q(2,J) ,Q(3,J))
   25 CONTINUE
      PRINT 106,((P(I,J),J=1,2),I=1,3),((Q(I,J),J=1,2),I=1,3)
      CM = CHK(M(1),M(2))
      IF (CM.GT.0.) GO TO 185
      M(3) = RAD2(M(1),M(2),M(3))
   30 CONTINUE
C**      NEXT FIND THE COORDINATES OF POINT R                                **
      DO 35 I = 1,2
      R(1,I) = P(1,I)*P(1,I)*Q(1,I) +  (P(1,I)*P(2,I)-P(3,I))*Q(2,I)
     1  + (P(1,I)*P(3,I)+P(2,I))*Q(3,I)
      R(2,I) = (P(1,I)*P(2,I)+P(3,I))*Q(1,I) + P(2,I)*P(2,I)*Q(2,I)
     1  + (P(2,I)*P(3,I)-P(1,I))*Q(3,I)
   35 R(3,I) = (P(1,I)*P(3,I)-P(2,I))*Q(1,I)
```

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12345678901234567890123456789012345678901234567890123456789012345678901234567890
     1   + (P(2,I)*P(3,I)+P(1,I))*Q(2,I) + P(3,I)*P(3,I)*Q(3,I)
      PRINT 107,((R(I,J),J=1,2),I=1,3)
  107 FORMAT(///,68H    THE COORDINATES OF THE COMPUTED R'S ARE AS FOLLO
     1WS. EACH COLUMN  ,/,17H    IS ONE POINT.  ,/,(2G15.6))
C**     NEXT SET UP THE FIRST PQR MATRIX                                     **
      DO 40 I = 1,3
      PQR(I,1) = P(I,1)
      PQR(I,2) = Q(I,1)
   40 PQR(I,3) = R(I,1)
C**     CHECK THE LENGTH OF P(I)Q(I)                                         **
      DO 41 I = 1,2
      DX = P(1,I) - Q(1,I)
      DY = P(2,I) - Q(2,I)
      DZ = P(3,I) - Q(3,I)
      CHKL(I) = RAD(DX,DY,DZ)
   41 CONTINUE
      CHK2 = CHKL(2) - CHKL(1)
      CHK2 = ABS(CHK2)
      IF(CHK2.GT.1.E-3) GO TO 186
C**     NEXT FIND THE INVERSE OF PQR                                         **
      IS = 2
      CALL AIN(PQR,PINV,PMAG,IS)
C       THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE MAGNITUDE
C       OF THE MATRIX PQR, AND ITS INVERSE
C     PRINT 1001,PMAG,((PINV(I,J),J=1,3),I=1,3)
C1001 FORMAT(///,28H    THE MAGNITUDE OF PQR IS:  ,/,G15.6,
C    1/,26H    THE INVERSE OF PQR IS:    ,/,3(3G15.6,/))
      IF(IS.EQ.101) GO TO 187
C**     NEXT FIND THE ROTATION MATRIX FROM POSITION ONE TO TWO               **
C**     NEXT SET UP THE SECOND PQR MATRIX.                                   **
      DO 45 I = 1,3
      PQR2(I,1) = P(I,2)
      PQR2(I,2) = Q(I,2)
   45 PQR2(I,3) = R(I,2)
C**     FIRST ZERO THE ROTATION MATRIX                                       **
      DO 50 I = 1,3
      DO 50 J = 1,3
   50 ROT12(I,J) = 0.0
      DO 55 J = 1,3
      DO 55 K = 1,3
      DO 55 L = 1,3
   55 ROT12(K,J) = PQR2(K,L) * PINV(L,J) + ROT12(K,J)
      PRINT 1002,((ROT12(I,J),J=1,3),I=1,3)
 1002 FORMAT(///,63H    THE COMPUTED ROTATIONAL MATRIX FROM POSITION ONE
     1 TO TWO IS:    ,/,3(3G15.6,/))
C**     NEXT FIND THE ROTATION ANGLE PHI                                     **
      PHI = .5*(ROT12(1,1) + ROT12(2,2) + ROT12(3,3) - 1.)
      PHI = ACOS(PHI)
      PHID = PHI / PI
      PRINT 108, PHID
  108 FORMAT(///,72H    THE ROTATION ANGLE FROM POSITION ONE TO POSITION
     1 TWO, IN DEGREES IS:  ,/,G15.6)
C**     NEXT FIND THE OTHER ANGLES                                           **
      TH1 = PHI / (1. + GR)
      TH2 = TH1 * GR
```

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     12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
     C**     NEXT DEFINE THE DESIGN EQUATION PARAMETERS                               **
           C1 = SIN(TH1)
           C2 = SIN(TH2)
           C3 = COS(TH1)
           C4 = COS(TH2)
           C5 = ROT12(2,3) - ROT12(3,2)
           C6 = ROT12(3,1) - ROT12(1,3)
           C7 = ROT12(1,2) - ROT12(2,1)
           C8 = C2 * C5
           C9 = C2 * C6
           C10 = A(3)*C2*C7 - M(1)*C1*C5 - M(2)*C1*C6 - M(3)*C1*C7
          1      + C3 - C4 + (ROT12(1,1)+ROT12(2,2)+ROT12(3,3))*(C3-C4)
     C       THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE INTERMEDIATE
     C        COEFFICIENTS C1 THROUGH C10
     C     PRINT 1003, C1,C2,C3,C4,C5,C6,C7,C8,C9,C10
     C1003 FORMAT(///,43H    THE FIRST TEN COEFFICIENTS, C1-C10 ARE: ,/,(3G15
     C    1.6))
           IF(ABS(C9).GT..001) GO TO 70
           C11 = C8 * C10 * 0.001
           IF(ABS(C9).GT.C11) GO TO 70
           IF(ABS(C8).GT.1.E-8) GO TO 65
        60 CONTINUE
           PRINT 109,C8,C9,C10
           GO TO 170
        65 A(1) = C10 / C8
           KCOUNT = KCOUNT + 1
           GO TO 150
        70 CONTINUE
     C**     AT THIS POINT THERE ARE TWO EQUATIONS:                                   **
     C**         C8*A(1) + C9*A(2) + C10 = 0     AND                                  **
     C**         A(1)**2 + A(2)**2 + A(3)**2 = 1                                      **
     C**     NEXT FIND THE COEFFICIENTS OF THE QUADRATIC TO FIND A(1)                 **
           IF(ABS(C9).LT.1.E-8) GO TO 60
           C9S = C9*C9
           C12 = 1. + C8*C8/C9S
           C13 = 2.*C8*C10/C9S
           C14 = C10*C10/C9S + A(3)*A(3) - 1.
       109 FORMAT(///,66H    THIS JOB ABORTED, THE SOLUTION MAY BE FOUND FROM
          1 THE EQUATION:  ,/,  55H    C8 A(1)+C9 A(2)+C10 = 0. SOLVING THIS
          2EQUATION AND:   /39H    A(1)  2 + A(2)  (2) + A(3)  2 = 1 . ,/,
          351H    THE COEFFICIENTS C8,C9, AND C10 ARE AS FOLLOWS:  ,/, 3G15.6
          4)
           IF(ABS(C12).GT.1.E-6) GO TO 75
           PRINT 110, C12,C13,C14
       110 FORMAT(///,66H    THIS JOB ABORTED, THE SOLUTION MAY BE FOUND FROM
          1 THE EQUATION:  ,/,36H    C12 A(1)  2 + C13 A(1) + C14 = 0  ,/,
          253H    THE COEFFICIENTS C12,C13, AND C14 ARE AS FOLLOWS: ,/, 3G15.
          36)
        75 CONTINUE
     C**     NEXT FIND THE DISCRIMINANT FOR THE A(1) QUADRATIC                        **
           C15 = C13*C13 - 4.*C12*C14
           C15 = SQRT(C15)
     C      THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE COEFFICIENTS
     C       OF THE FINAL QUADRATIC EQUATION
     C     PRINT 1004, C12,C13,C14,C15
```

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12345678901234567890123456789012345678901234567890123456789012345678901234567890
C1004 FORMAT(///,52H    THE COEFFICIENTS OF THE QUADRATIC (C12-C15) ARE:
C    1  ,/,(3G15.6))
C**      TEST DISCRIMINANT TO SEE IF THERE ARE ANY SOLUTIONS                    **
      IF(C15.GE.0.) GO TO 80
      PRINT 111
  111 FORMAT(///,54H    THERE IS NO SOLUTION FOR THE GIVEN INPUT CONDITI
     1ONS ,/,83H    THE GEAR RATIO OR INPUT ASSUMED COORDINATES MUST BE
     2CHANGED TO YIELD A SOLUTION  )
      GO TO 170
   80 CONTINUE
      PRINT 117
  117 FORMAT(1H1)
      IF(C15.EQ.0.0) GO TO 160
      IF(ABS(C12).GT.1.E-6) GO TO 90
      IF(ABS(C13).GT.1.E-6) GO TO 85
      PRINT 110,C12,C13,C14
      GO TO 170
   85 CONTINUE
      A(1) = -C14/C13
      KCOUNT = KCOUNT + 1
      GO TO 150
   90 CONTINUE
C**      SOLVE FOR A(1)                                                         **
      A(1) = (-C13 + C15)/(2.*C12)
      GO TO 150
   95 CONTINUE
      A(1) = (-C13 - C15)/(2.*C12)
  150 CONTINUE
      A(2) =(-C10 - C8*A(1))/C9
C**      FIND THE INCLUDED ANGLE BETWEEN M AND A                                **
      TH = A(1)*M(1) + A(2)*M(2) + A(3)*M(3)
      IF(ABS(TH).GT.1.) GO TO 155
      TH = ACOS(TH)
      THD = TH/PI
      TH1D = TH1/PI
      PRINT 112,(M(I),I=1,3),(A(I),I=1,3),GR,TH1D,THD
      GO TO 160
  112 FORMAT(1H1,//,66H    THE FOLLOWING ARE THE RESULTS FOR THE INPUT D
     1ATA LISTED ABOVE.  ,/,56H    THE COORDINATES OF THE SUN GEAR AXIS
     2ARE AS FOLLOWS:  ,/, 3G15.6,//,
     365H   THE COORDINATES OF THE AXIS OF THE PLANET GEAR ARE AS FOLLOW
     4S: ,/,3G15.6,//,      39H    THE GEAR RATIO OF THE MECHANISM IS: ,
     5G15.6,//,78H    THE ROTATION OF THE CONNECTING ARM IN GOING FROM P
     63SITION ONE TO TWO IS:   ,/,G15.6,7HDEGREES ,//, 55H    THE ANGLE
     7BETWEEN THE AXIS OF THE SUN GEAR AND THE  ,/,31H    AXIS OF THE PL
     8ANET GEAR IS:  ,/, G15.6, 8H DEGREES  )
  155 CONTINUE
      PRINT 112,(M(I),I=1,3),(A(I),I=1,3),GR,TH1
  160 CONTINUE
      KCOUNT =  KCOUNT + 1
      IF(KCOUNT.EQ.2) GO TO 95
      GO TO 190
  186 PRINT 115,(CHKL(I),I=1,2)
  115 FORMAT(///,70H    $$$$THIS JOB ABORTED, THE LENGTHS OF THE INPUT P
     1Q ARCS ARE NOT$$$$ ,/,42H    $$$$EQUAL. THE LENGTHS ARE AS FOLLOWS
```

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12345678901234567890123456789012345678901234567890123456789012345678901234567890
      2: ,/,2G15.6)
       GO TO 190
  187 PRINT 116
  116 FORMAT(///,66H    $$$$THE INITIAL POINTS  PAND Q ARE SUCH THAT THE
     1 MAGNITUDE$$$$   /,51H    $$$$OF THE MATRIX PQP IS ESSENTIALLY ZER
     2O.$$$$  )
  170 CONTINUE
      GO TO 190
  175 CONTINUE
      KCOUNT = KCOUNT + 1
      GO TO 150
  180 PRINT 113
  113 FORMAT(///,51H    $$$$ INPUT DATA POINTS NOT ON UNIT SPHERE. $$$$
     1/,42H    $$$$ INPUT DATA UNDER OPTION TWO. $$$$ )
      GO TO 190
  185 PRINT 114
  114 FORMAT(///,83H   $$$$ THE INPUT DATA POINT COORDINATES ARE TOO LAR
     1GE TO BE ON A UNIT SPHERE. $$$$ )
  150 CONTINUE
      STOP
      END
      FUNCTION RAD2(A,B,C)
      D = 1. - A*A - B*B
      RAD2 = C*SQRT(D)
      RETURN
      END
      FUNCTION RAD(A,B,C)
      D = A*A + B*B + C*C
      RAD = SQRT(D)
      RETURN
      END
      FUNCTION CHK(A,B)
      CHK = A*A + B*B - 1.
      RETURN
      END
      SUBROUTINE AIN(A,AINV,AMAG,IS)
C**   FIRST FIND THE MAGNITUDE OF THE MATRIX                                  **
C**   THIS ROUTINE IS DESIGNED TO FIND THE INVERSE OF A (3,3) MATRIX A,       **
C**   THE INVERSE IS RETURNED IN AINV AND THE MAGNITUDE IN AMAG. IF ONLY      **
C**     THE MAGNITUDE OF THE MATRIX A IS DESIRED, IS SHOULD BE SET TO 1. IF   **
C**     BOTH THE MAGNITUDE AND THE INVERSE ARE DESIRED, IS SHOULD BE SET TO 2.**
      DIMENSION A(3,3),AINV(3,3)
      AMAG = A(1,1)*A(2,2)*A(3,3) + A(1,2)*A(2,3)*A(3,1)
     1     + A(1,3)*A(2,1)*A(3,2) - A(3,1)*A(2,2)*A(1,3)
     2     - A(2,1)*A(1,2)*A(3,3) - A(1,1)*A(3,2)*A(2,3)
      IF(IS.EQ.1) GO TO 150
      A1 = 1./AMAG
      AINV(1,1) = (A(2,2)*A(3,3) - A(3,2)*A(2,3))*A1
      AINV(1,2) = (A(3,2)*A(1,3) - A(1,2)*A(3,3))*A1
      AINV(1,3) = (A(1,2)*A(2,3) - A(2,2)*A(1,3))*A1
      AINV(2,1) = (A(3,1)*A(2,3) - A(2,1)*A(3,3))*A1
      AINV(2,2) = (A(1,1)*A(3,3) - A(3,1)*A(1,3))*A1
      AINV(2,3) = (A(2,1)*A(1,3) - A(1,1)*A(2,3))*A1
      AINV(3,1) = (A(2,1)*A(3,2) - A(3,1)*A(2,2))*A1
      AINV(3,2) = (A(3,1)*A(1,2) - A(1,1)*A(3,2))*A1
```

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     12345678901234567890123456789012345678901234567890123456789012345678901234567890
           AINV(3,3) = (A(1,1)*A(2,2) - A(2,1)*A(1,2))*A1
       150 CONTINUE
           RETURN
           END
     "00
     2
      2.0
     .6         .31         1.
     .552449
     -.2         .3         1.
     -.134274   .102227     1.
     -.3         .4          -1.
     -.454178   .48266       -1.
     $IBSYS
```

APPENDIX D

THREE POSITION SYNTHESIS PROGRAM

```
00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
      PROGRAM SC3P(INPUT,OUTPUT)
C*******************************************************************************
C*******************************************************************************
C**     THIS PROGRAM IS DESIGNED TO TAKE THREE GIVEN RIGID BODY POSITIONS      **
C**     WHICH LIE ON A SPHERE AND COMPUTE A GEARED SPHERICAL CYCLOIDAL         **
C**     CRANK MECHANISM WHICH WILL GUIDE A RIGID BODY, CONNECTED TO THE        **
C**     PLANET GEAR, THROUGH THE THREE GIVEN POSITIONS.                        **
C**     THE VARIABLES USED ARE AS FOLLOWS :                                    **
C**     IC - INPUT DATA CHOICE PARAMETER;  (FIRST DATA CARD WITH AN            **
C**             I1 FORMAT)                                                     **
C**        IC=1, COORDINATES GIVEN ARE ASSUMED TO LIE ON A UNIT SPHERE.        **
C**        IC=2, THE X AND Y COORDINATES OF EACH POINT ARE GIVEN IN THE        **
C**              FIRST 20 COLUMNS  WITH  A +1. OR A -1. IN THE NEXT 10         **
C**              COLUMNS TO GIVE THE SIGN OF THE Z COORDINATE.                 **
C**     GR - THE GEAR RATIO N = (THE RADIUS OF THE SUN GEAR)/(THE              **
C**             RADIUS OF THE PLANET GEAR);  (SECOND DATA CARD WITH AN         **
C**             F10.0 FORMAT)                                                  **
C**     M - A VECTOR CONTAINING THE CARTESIAN COORDINATES OF THE AXIS          **
C**             OF THE FIXED SUN GEAR; (NINTH DATA CARD WITH A 3F10.0 FORMAT;  **
C**             ONLY THE X AND Y COORDINATES OF M ARE INPUT)                   **
C**     A - A VECTOR CONTAINING THE CARTESIAN COORDINATES OF THE AXIS OF       **
C**             THE PLANET GEAR IN ITS ORIGINAL POSITION.                      **
C**     P.Q - VECTORS CONTAINING THE CARTESIAN COORDINATES OF TWO              **
C**             POINTS ON THE PLANET GEAR;  THIRD DATA CARD--P1;               **
C**             FOURTH DATA CARD--P2                                           **
C**             FIFTH DATA CARD--P3                                            **
C**             SIXTH DATA CARD--Q1                                            **
C**             SEVENTH DATA CARD--Q2                                          **
C**             EIGHTH DATA CARD--Q3                                           **
C**             FORMATTED AS EXPLAINED UNDER VARIABLE IC ABOVE.                **
C**     R - A VECTOR CONTAINING THE CARTESIAN COORDINATES OF THE               **
C**             RESULT OF ROTATING Q ABOUT P THROUGH AN ANGLE OF               **
C**             NINETY DEGREES.                                                **
C**     PQR - A MATRIX WHOSE COLUMNS ARE THE VECTORS P, Q, AND R.              **
C**     PINV - A MATRIX WHICH IS THE INVERSE OF PQR.                           **
C**     PQR2 - A MATRIX WHOSE COLUMNS ARE THE CARTESIAN COORDINATES OF THE     **
C**             SECOND POSITIONS OF P,Q, AND R.                                **
C**     ROT12 - THE ROTATION MATRIX FROM POSITION ONE TO POSITION TWO. IT      **
C**             IS THE PRODUCT OF PQR2 AND PINV                                **
C**     PHI - THE TOTAL ROTATION ANGLE REQUIRED TO GO FROM POSITION ONE TO     **
C**             POSITION TWO.                                                  **
C**     TH1(I) - THE ROTATION OF A ABOUT M WHEN THE MECHANISM GOES FROM        **
C**             POSITION ONE TO POSITION I, THETA ONE.                         **
C**     TH2(I) - THE ROTATION OF P ABOUT A WHEN THE MECHANISM GOES FROM        **
C**             POSITION ONE TO POSITION I, THETA TWO.                         **
C*******************************************************************************
C*******************************************************************************
      REAL M(3)                                                                **
      DIMENSION A(3),P(3,3),Q(3,3),R(3,3),PQR(3,3),PINV(3,3),PQR2(3,3)
     1         ,ROT12(3,3,3),TH1(3),TH2(3),CHKL(3),PHI(3),PHID(3)
     2         ,F(10),C(3,7)
C**     READ IN DATA                                                           **
      READ 100,IC
```

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12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
  100 FORMAT(I1)
      PRINT 101,IC
  101 FORMAT(1H1,37H    THE INPUT DATA CHOICE PARAMETER =   ,I2)
      READ 102 , GR
      PRINT 103,GR
  103 FORMAT(///,28H    THE INPUT GEAR RATIO IS: ,/,G15.6)
      READ 102,((P(I,J),I=1,3),J=1,3),((Q(I,J),I=1,3),J=1,3)
      PRINT 104,((P(I,J),J=1,3),I=1,3),((Q(I,J),J=1,3),I=1,3)
  102 FORMAT(3F10.0)
  104 FORMAT(///,50H    THE INPUT RIGID BODY POSITIONS ARE AS FOLLOWS:
     1,/,45H    POINTS P; WHERE EACH COLUMN IS ONE POINT.,/,3(3G15.6,/),
     2//,45H    POINTS Q; WHERE EACH COLUMN IS ONE POINT.,/,3(3G15.6,/))
      PI = 355./(113.*180.)
      READ 102,(M(I),I=1,3)
      PRINT 105,(M(I),I=1,3)
  105 FORMAT(///,41H    THE INPUT SUN GEAR AXIS (POINT M) IS: ,/,3G15.6)
      GO TO (5,15), IC
    5 CPS = 0.0
      DO 10 J = 1,3
      CP = RAD(P(1,J),P(2,J),P(3,J)) - 1.
      CQ = RAD(Q(1,J),Q(2,J),Q(3,J)) - 1.
      CPS = CPS + ABS(CP) + ABS(CQ)
   10 CONTINUE
      CM = RAD(M(1),M(2),M(3)) - 1.
      CPS = CPS + ABS(CM)
      IF(CPS.LT.0.03) GO TO 30
      PRINT 106
  106 FORMAT(///,51H    $$$$ INPUT DATA POINTS NOT ON UNIT SPHERE. $$$$
     1 ,/,60H    $$$$ INPUT DATA USING INPUT DATA OPTION NUMBER TWO. $$$
     2$  )
      GO TO 2000
C**     CALCULATE THE Z COORDINATES OF THE P AND Q POINTS
   15 CONTINUE
      DO 25 J = 1,3
      CP = CHK(P(1,J),P(2,J))
      CQ = CHK(Q(1,J),Q(2,J))
      IF(CP.LE.0.) GO TO 20
      IF(CQ.LE.0.) GO TO 20
      PRINT 115,J,CP,CQ
  115 FORMAT(///, 73H    $$$$ THE INPUT P,AND Q ARE TOO LARGE TO BE ON T
     1HE UNIT SPHERE. $$$$     ,/,4X,2HJ= ,I2,/,4X, 3HCP=,G15.6,
     2 /,4X,3HCQ=,G15.6)
      GO TO 2000
   20 CONTINUE
      P(3,J) = RAD2(P(1,J),P(2,J),P(3,J))
      Q(3,J) = RAD2(Q(1,J),Q(2,J),Q(3,J))
   25 CONTINUE
      M(3) = RAD2(M(1),M(2),M(3))
   30 CONTINUE
      PRINT 105,(M(I),I=1,3)
      PRINT 104,((P(I,J),J=1,3),I=1,3),((Q(I,J),J=1,3),I=1,3)
C**     NEXT FIND THE COORDINATES OF POINT R                                **
      DO 35 I = 1,3
      R(1,I) = P(1,I)*P(1,I)*Q(1,I) +  (P(1,I)*P(2,I)-P(3,I))*Q(2,I)
     1 + (P(1,I)*P(3,I)+P(2,I))*Q(3,I)
```

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12345678901234567890123456789012345678901234567890123456789012345678901234567890
      R(2,I) = (P(1,I)*P(2,I)+P(3,I))*Q(1,I) + P(2,I)*P(2,I)*Q(2,I)
     1  + (P(2,I)*P(3,I)-P(1,I))*Q(3,I)
   35 R(3,I) = (P(1,I)*P(3,I)-P(2,I))*Q(1,I)
     1  + (P(2,I)*P(3,I)+P(1,I))*Q(2,I) + P(3,I)*P(3,I)*Q(3,I)
      PRINT 107,((R(I,J),J=1,3),I=1,3)
  107 FORMAT(///,68H    THE COORDINATES OF THE COMPUTED R S ARE AS FOLLO
C**     CHECK THE LENGTH OF P(I)Q(I)                                         **
     1WS. EACH COLUMN  ,/,17H     IS ONE POINT. ,/,(3G15.6))
      DO 40 I = 1,3
      DX = P(1,I) - Q(1,I)
      DY = P(2,I) - Q(2,I)
      DZ = P(3,I) - Q(3,I)
      CHKL(I) = RAD(DX,DY,DZ)
   40 CONTINUE
      DO 50 I = 2,3
      CHK2 = CHKL(I) - CHKL(1)
      CHK2 = ABS(CHK2)
      IF(CHK2.LT.1.E-3) GO TO 45
      PRINT 108,(CHKL(L),L=1,3)
  108 FORMAT(///,72H     $$$$ THIS JOB ABORTED, THE LENGTHS OF THE INPUT
     1PG ARCS ARE NOT $$$$ ,/,48H     $$$$ EQUAL. THE LENGTHS ARE AS FOLL
     2OWS: $$$$ ,/, 5G15.6)
      GO TO 2000
   45 CONTINUE
   50 CONTINUE
C**     THE FOLLOWING DO LOOPS FIND THE ROTATION MATRICES                    **
C**     NEXT SET UP THE FIRST POSITION PQR MATRIX                            **
      DO 55 I = 1,3
      PQR(I,1) = P(I,1)
      PQR(I,2) = Q(I,1)
   55 PQR(I,3) = R(I,1)
C**     NEXT FIND THE INVERSE OF PQR                                         **
      IS = 2
      CALL AIN(PQR,PINV,PMAG,IS)
C       THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE MAGNITUDE
C       AND INVERSE OF THE MATRIX PQR
C     PRINT 1000,PMAG,((PINV(I,J),J=1,3),I=1,3)
C1000 FORMAT(///,28H     THE MAGNITUDE OF PQR IS: ,/,G15.6,/,
C    126H     THE INVERSE OF PQR IS:  ,/,3(3G15.6,/))
      IF(IS.NE.101) GO TO 60
      PRINT 109
  109 FORMAT(///,68H     $$$$ THE INITIAL POINTS P AND Q ARE SUCH THAT TH
     1E MAGNITUDE $$$$ ,/,52H     $$$$ OF THE MATRIX PQR IS ESSENTIALLY Z
     2ERO. $$$$  )
      GO TO 2000
   60 CONTINUE
C**     NEXT FIND THE ROTATION MATRICES
      DO 65 I = 1,3
      DO 65 J = 1,3
      DO 65 K = 1,3
   65 ROT12(K,J,I) = 0.0
      DO 80 I = 2,3
      DO 70 J = 1,3
      PQR2(J,1) = P(J,I)
      PQR2(J,2) = Q(J,I)
```

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12345678901234567890123456789012345678901234567890123456789012345678901234567890
      PQR2(J,3) = R(J,I)
   70 CONTINUE
      DO 75 J = 1,3
      DO 75 K = 1,3
      DO 75 L = 1,3
   75 ROT12(I,K,J) = PQR2(K,L) * PINV(L,J) + ROT12(I,K,J)
      PRINT 1001, I,((ROT12(I,J,K),K=1,3),J=1,3)
 1001 FORMAT(///,64H    THE COMPUTED ROTATIONAL MATRIX FROM POSITION ONE
     1 TO POSITION,I2,  4H IS: ,/,(3G15.6))
C**     NEXT FIND THE ROTATION ANGLE PHI                                      **
      PHI(I) = .5*(ROT12(I,1,1) + ROT12(I,2,2) + ROT12(I,3,3) - 1.)
      PHI(I) = ACOS(PHI(I))
      PHID(I) = PHI(I)/PI
      PRINT 110,I,PHID(I)
  110 FORMAT(///,52H    THE ROTATION ANGLE FROM POSITION ONE TO POSITION
     1 , I2,16H, IN DEGREES IS: ,/,G15.6)
C**     NEXT FIND THE OTHER ANGLES                                            **
      TH1(I) = PHI(I) / (1. + GR)
      TH2(I) = TH1(I) * GR
C**     NEXT DEFINE THE DESIGN EQUATION PARAMETERS                            **
      C1 = SIN(TH2(I))
      C2 = SIN(TH1(I))
      C3 = ROT12(I,2,3) - ROT12(I,3,2)
      C4 = ROT12(I,3,1) - ROT12(I,1,3)
      C5 = ROT12(I,1,2) - ROT12(I,2,1)
      C(I,1) = C1*C3
      C(I,2) = C1*C4
      C(I,3) = C1*C5
      C(I,4) = -M(1)*C2*C3 - M(2)*C2*C4 - M(3)*C2*C5
     1          + (1. + ROT12(I,1,1) + ROT12(I,2,2) + ROT12(I,3,3))
     2          *(COS(TH1(I)) - COS(TH2(I)))
C       THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE INTERMEDIATE
C       COEFFICIENTS FOR EACH ROTATION, AND THE DESIGN COEFFICIENTS.
C     PRINT 1002,I,C1,C2,C3,C4,C5,(C(I,J),J=1,4)
C1002 FORMAT(///,50H    FOR THE ROTATION FROM POSITION ONE TO POSITION ,
C    1I2,18H, THE COEFFICIENTS  ,/,14H    C1-C5 ARE:  ,/, 5G15.6,/
C    241H    AND THE FOUR DESIGN COEFFICIENTS ARE: ,/, 4G15.6)
   80 CONTINUE
      C1 = -1./C(2,3)
      C(1,1) = C(2,1)*C1
      C(1,2) = C(2,2)*C1
      C(1,3) = C(2,4)*C1
      C(2,1) = C(3,1) + C(3,3)*C(1,1)
      C(2,2) = C(3,2) + C(3,3)*C(1,2)
      C(2,3) = C(3,4) + C(3,3)*C(1,3)
      A1 = C(1,1)*C(1,1) + 1.                                               AX2
      A2 = C(1,2)*C(1,2) + 1.                                               AY2
      A3 = 2.*C(1,1)*C(1,2)                                                AXAY
      A4 = 2.*C(1,1)*C(1,3)                                                 AX
      A5 = 2.*C(1,2)*C(1,3)                                                 AY
      A6 = C(1,3)*C(1,3) - 1.                                              ----
C       THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE COEFFICIENT
C       MATRIX AND THE A COEFFICIENTS AFTER THE FIRST SUBSTITUTION.
C     PRINT 112, C1,((C(I,J),J=1,3),I=1,2),A1,A2,A3,A4,A5,A6
C 112 FORMAT(///, 9H    C1 = ,G15.6,/,59H    THE COEFFICIENT MATRIX AFTE
```

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         00000000011111111112222222222333333333344444444445555555555666666666677777777778
         12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
      C     1R THE FIRST SUBSTITUTION IS:/, 2(3G15.6,/), 27H    THE A COEFFICIE
      C     2NTS ARE: , 2(3G15.6,/))
      C**      THIS STARTS THE FINAL SUBSTITUTION (ELIMINATES AY)                      **
            C(1,1) = -C(2,1)/C(2,2)
            C(1,2) = -C(2,3)/C(2,2)
            B1 = A1 + C(1,1)*(C(1,1)*A2 + A3)
            B2 = A4 + C(1,1)*(2.*C(1,2)*A2 + A5) + C(1,2)*A3
            B3 = A6 + C(1,2)*(C(1,2)*A2 + A5)
      C        THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE COEFFICIENT
      C        MATRIX AND B COEFFICIENTS AFTER THE SECOND SUBSTITUTION.
      C     PRINT 113,C(1,1),C(1,2),B1,B2,B3
      C 113 FORMAT(///,56H    AFTER THE LAST SUBSTITION THE COEFFICIENT MATRIX
      C    1 IS: ,/,2G15.6,/, 45H    AND THE FINAL QUADRATIC COEFFICIENTS ARE:
      C    2,/, 3G15.6)
            DIS = B2*B2 - 4.*B1*B3
            IF(DIS.GT.0.) GO TO 89
            PRINT 2006,DIS
       2006 FORMAT(///, 14H    DIS IS        , G15.6)
            COMP = B2/(2.*B1)
            COMP = DIS/COMP
            COMP = ABS(COMP)
            IF(COMP.LT.035) DIS = 0.
            IF(DIS.EQ.0.) GO TO 89
            GO TO 2000
         89 CONTINUE
            DIS = SQRT(DIS)
            ICOUNT =1
         90 CONTINUE
            IF(ICOUNT.EQ.2) GO TO 95
            A(1) = (-B2 - DIS)/(2.*B1)
            GO TO 190
         95 CONTINUE
            A(1) = (-B2 + DIS)/(2.*B1)
        190 CONTINUE
            IF(ICOUNT.GE.3) GO TO 2000
            A(2) = C(1,1)*A(1) + C(1,2)
            A(3) = -(C(3,1)*A(1) + C(3,2)*A(2) + C(3,4))/C(3,3)
            G1 = RAD(A(1),A(2),A(3))
            A(1) = A(1)/G1
            A(2) = A(2)/G1
            A(3) = A(3)/G1
            ICOUNT = ICOUNT + 1
            PRINT 114,(M(L),L=1,3),(A(L),L=1,3)
        114 FORMAT(///,43H    THE AXIS OF THE SUN GEAR (POINT M) IS:
           1      ,/,3G15.6,/,56H    THE COMPUTED AXIS OF THE PLANET GEAR (POIN
           2T A) IS:      ,/,3G15.6 )
            GO TO 90
       2000 CONTINUE
            STOP
            END
            FUNCTION RAD2(A,B,C)
            D = 1. - A*A - B*B
            RAD2 = C*SQRT(D)
            RETURN
            END
```

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12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
      FUNCTION RAD(A,B,C)
      D = A*A + B*B + C*C
      RAD = SQRT(D)
      RETURN
      END
      FUNCTION CHK(A,B)
      CHK = A*A + B*B - 1.
      RETURN
      END
      SUBROUTINE AIN(A,AINV,AMAG,IS)
C**   FIRST FIND THE MAGNITUDE OF THE MATRIX                                  **
C**   THIS ROUTINE IS DESIGNED TO FIND THE INVERSE OF A (3,3) MATRIX A,       **
C**   THE INVERSE IS RETURNED IN AINV AND THE MAGNITUDE IN AMAG. IF ONLY      **
C**     THE MAGNITUDE OF THE MATRIX A IS DESIRED, IS SHOULD BE SET TO 1. IF   **
C**     BOTH THE MAGNITUDE AND THE INVERSE ARE DESIRED, IS SHOULD BE SET TO 2.**
      DIMENSION A(3,3),AINV(3,3)
      AMAG = A(1,1)*A(2,2)*A(3,3) + A(1,2)*A(2,3)*A(3,1)
     1     + A(1,3)*A(2,1)*A(3,2) - A(3,1)*A(2,2)*A(1,3)
     2     - A(2,1)*A(1,2)*A(3,3) - A(1,1)*A(3,2)*A(2,3)
      IF(IS.EQ.1) GO TO 150
      A1 = 1./AMAG
      AINV(1,1) = (A(2,2)*A(3,3) - A(3,2)*A(2,3))*A1
      AINV(1,2) = (A(3,2)*A(1,3) - A(1,2)*A(3,3))*A1
      AINV(1,3) = (A(1,2)*A(2,3) - A(2,2)*A(1,3))*A1
      AINV(2,1) = (A(3,1)*A(2,3) - A(2,1)*A(3,3))*A1
      AINV(2,2) = (A(1,1)*A(3,3) - A(3,1)*A(1,3))*A1
      AINV(2,3) = (A(2,1)*A(1,3) - A(1,1)*A(2,3))*A1
      AINV(3,1) = (A(2,1)*A(3,2) - A(3,1)*A(2,2))*A1
      AINV(3,2) = (A(3,1)*A(1,2) - A(1,1)*A(3,2))*A1
      AINV(3,3) = (A(1,1)*A(2,2) - A(2,1)*A(1,2))*A1
  150 CONTINUE
      RETURN
      END
"00
2
2.
-.2       .3        1.
-.113094  .226181    1.
-.00757381.173646   1.
-.3       .4       -1.
-.503378  .350705    -1.
-.679041  .256321  -1.
 .1       .5        1.
$IBSYS
```

APPENDIX E

FOUR POSITION SYNTHESIS PROGRAM

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0C00C00000111111111122222222223333333333444444444455555555556666666666777777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
      PROGRAM SC4P(INPUT,OUTPUT)
C*******************************************************************************
C*******************************************************************************
C**     THIS PROGRAM IS DESIGNED TO TAKE FOUR GIVEN RIGID BODY POSITIONS       **
C**     WHICH LIE ON A SPHERE AND COMPUTE A GEARED SPHERICAL CYCLOIDAL         **
C**     CRANK MECHANISM WHICH WILL GUIDE A RIGID BODY, CONNECTED TO THE        **
C**     PLANET GEAR, THROUGH THE FOUR GIVEN POSITIONS.                         **
C**     THE VARIABLES USED ARE AS FOLLOWS :                                    **
C**     IC - INPUT DATA CHOICE PARAMETER;  (FIRST DATA CARD WITH AN            **
C**            I1 FORMAT)                                                      **
C**        IC=1, COORDINATES GIVEN ARE ASSUMED TO LIE ON A UNIT SPHERE.        **
C**        IC=2, THE X AND Y COORDINATES OF EACH POINT ARE GIVEN IN THE        **
C**              FIRST 20 COLUMNS  WITH  A +1. OR A -1. IN THE NEXT 10         **
C**              COLUMNS TO GIVE THE SIGN OF THE Z COORDINATE.                 **
C**     GR - THE GEAR RATIO N = (THE RADIUS OF THE SUN GEAR)/(THE              **
C**            RADIUS OF THE PLANET GEAR);  (SECOND DATA CARD WITH AN          **
C**            F10.0 FORMAT)                                                   **
C**     M - A VECTOR CONTAINING THE CARTESIAN COORDINATES OF THE AXIS          **
C**            OF THE FIXED SUN GEAR; (ELEVENTH DATA CARD WITH AN F10.0        **
C**            FORMAT; ONLY THE Z COORDINATE OF M IS INPUT)                    **
C**     A - A VECTOR CONTAINING THE CARTESIAN COORDINATES OF THE AXIS OF       **
C**            THE PLANET GEAR IN ITS ORIGINAL POSITION.                       **
C**     P,Q - VECTORS CONTAINING THE CARTESIAN COORDINATES OF TWO              **
C**            POINTS ON THE PLANET GEAR;  THIRD DATA CARD--P1;                **
C**            FOURTH DATA CARD--P2                                            **
C**            FIFTH DATA CARD--P3                                             **
C**            SIXTH DATA CARD--P4                                             **
C**            SEVENTH DATA CARD--Q1                                           **
C**            EIGHTH DATA CARD--Q2                                            **
C**            NINTH DATA CARD--Q3                                             **
C**            TENTH DATA CARD--Q4                                             **
C**            FORMATTED AS EXPLAINED UNDER VARIABLE IC ABOVE.                 **
C**     R - A VECTOR CONTAINING THE CARTESIAN COORDINATES OF THE               **
C**            RESULT OF ROTATING Q ABOUT P THROUGH AN ANGLE OF                **
C**            NINETY DEGREES.                                                 **
C**     PQR - A MATRIX WHOSE COLUMNS ARE THE VECTORS P, Q, AND R.              **
C**     PINV - A MATRIX WHICH IS THE INVERSE OF PQR.                           **
C**     PQR2 - A MATRIX WHOSE COLUMNS ARE THE CARTESIAN COORDINATES OF THE     **
C**            SECOND POSITIONS OF P,Q, AND R.                                 **
C**     ROT12 - THE ROTATION MATRIX FROM POSITION ONE TO POSITION TWO. IT      **
C**            IS THE PRODUCT OF PQR2 AND PINV                                 **
C**     PHI - THE TOTAL ROTATION ANGLE REQUIRED TO GO FROM POSITION ONE TO     **
C**            POSITION TWO.                                                   **
C**     TH1(I) - THE ROTATION OF A ABOUT M WHEN THE MECHANISM GOES FROM        **
C**            POSITION ONE TO POSITION I, THETA ONE.                          **
C**     TH2(I) - THE ROTATION OF P ABOUT A WHEN THE MECHANISM GOES FROM        **
C**            POSITION ONE TO POSITION I, THETA TWO.                          **
C*******************************************************************************
C*******************************************************************************
      COMPLEX Z(5)
      REAL M(3)                                                                **
      DIMENSION A(3),P(3,4),Q(3,4),R(3,4),PQR(3,3),PINV(3,3),PQR2(3,3)
     1         ,ROT12(4,3,3),TH1(4),TH2(4),CHKL(4),PHI(4),PHID(4),D(5)
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12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
     2         ,F(10),C(4,7)
C**     READ IN DATA                                                           **
      READ 100,IC
  100 FORMAT(I1)
      PRINT 101,IC
  101 FORMAT(1H1,37H    THE INPUT DATA CHOICE PARAMETER =  ,I2)
      READ 102 , GR
      PRINT 103,GR
  103 FORMAT(///,28H    THE INPUT GEAR RATIO IS: ,/,G15.6)
      READ 102,((P(I,J),I=1,3),J=1,4),((Q(I,J),I=1,3),J=1,4)
      PRINT 104,((P(I,J),J=1,4),I=1,3),((Q(I,J),J=1,4),I=1,3)
  102 FORMAT(3F10.0)
  104 FORMAT(///,50H    THE INPUT RIGID BODY POSITIONS ARE AS FOLLOWS:
     1,/,45H    POINTS P; WHERE EACH COLUMN IS ONE POINT.,/,3(4G15.6,/),
     2//,45H    POINTS Q; WHERE EACH COLUMN IS ONE POINT.,/,3(4G15.6,/))
      PI = 355./(113.*180.)
      READ 102,M(3)
      PRINT 105,M(3)
  105 FORMAT(///,61H    THE INPUT Z COORDINATE OF THE SUN GEAR AXIS (POI
     1NT M) IS: ,/,G15.6)
      GO TO (5,15), IC
    5 CPS = 0.0
      DO 10 J = 1,4
      CP = RAD(P(1,J),P(2,J),P(3,J)) - 1.
      CQ = RAD(Q(1,J),Q(2,J),Q(3,J)) - 1.
      CPS = CPS + ABS(CP) + ABS(CQ)
   10 CONTINUE
      IF(CPS.LT.0.03) GO TO 30
      PRINT 106
  106 FORMAT(///,51H    $$$$ INPUT DATA POINTS NOT ON UNIT SPHERE. $$$$
     1 ,/,60H    $$$$ INPUT DATA USING INPUT DATA OPTION NUMBER TWO. $$$
     2$  )
      GO TO 2000
C**     CALCULATE THE Z COORDINATES OF THE P AND Q POINTS
   15 CONTINUE
      DO 25 J = 1,4
      CP = CHK(P(1,J),P(2,J))
      CQ = CHK(Q(1,J),Q(2,J))
      IF(CP.LE.0.) GO TO 20
      IF(CQ.LE.0.) GO TO 20
      PRINT 115,J,CP,CQ
  115 FORMAT(///, 73H    $$$$ THE INPUT P,AND Q ARE TOO LARGE TO BE ON T
     1HE UNIT SPHERE. $$$$     ,/,4X,2HJ= ,I2,/,4X, 3HCP=,G15.6,
     2 /,4X,3HCQ=,G15.6)
      GO TO 2000
   20 CONTINUE
      P(3,J) = RAD2(P(1,J),P(2,J),P(3,J))
      Q(3,J) = RAD2(Q(1,J),Q(2,J),Q(3,J))
   25 CONTINUE
   30 CONTINUE
      PRINT 104,((P(I,J),J=1,4),I=1,3),((Q(I,J),J=1,4),I=1,3)
C**     NEXT FIND THE COORDINATES OF POINT R                                   **
      DO 35 I = 1,4
      R(1,I) = P(1,I)*P(1,I)*Q(1,I) +  (P(1,I)*P(2,I)-P(3,I))*Q(2,I)
     1  + (P(1,I)*P(3,I)+P(2,I))*Q(3,I)
```

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00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
      R(2,I) = (P(1,I)*P(2,I)+P(3,I))*Q(1,I) + P(2,I)*P(2,I)*Q(2,I)
     1  + (P(2,I)*P(3,I)-P(1,I))*Q(3,I)
   35 R(3,I) = (P(1,I)*P(3,I)-P(2,I))*Q(1,I)
     1  + (P(2,I)*P(3,I)+P(1,I))*Q(2,I) + P(3,I)*P(3,I)*Q(3,I)
      PRINT 107,((R(I,J),J=1,4),I=1,3)
  107 FORMAT(///,68H    THE COORDINATES OF THE CCMPUTED R S ARE AS FOLLO
     1WS. EACH COLUMN  ,/,17H    IS ONE POINT. ,/,(4G15.6))
C**     CHECK THE LENGTH OF P(I)Q(I)                                          **
      DO 40 I = 1,4
      DX = P(1,I) - Q(1,I)
      DY = P(2,I) - Q(2,I)
      DZ = P(3,I) - Q(3,I)
      CHKL(I) = RAD(DX,DY,DZ)
   40 CONTINUE
      DO 50 I = 2,4
      CHK2 = CHKL(I) - CHKL(1)
      CHK2 = ABS(CHK2)
      IF(CHK2.LT.1.E-3) GO TO 45
      PRINT 108,(CHKL(L),L=1,4)
  108 FORMAT(///,72H    $$$$ THIS JOB ABORTED, THE LENGTHS OF THE INPUT
     1PG ARCS ARE NOT $$$$ ,/,48H    $$$$ EQUAL. THE LENGTHS ARE AS FOLL
     2OWS: $$$$ ,/, 5G15.6)
      GO TO 2000
   45 CONTINUE
   50 CONTINUE
C**     THE FOLLOWING DO LOOPS FIND THE ROTATION MATRICES                     **
C**     NEXT SET UP THE FIRST POSITION PQR MATRIX                             **
      DO 55 I = 1,3
      PQR(I,1) = P(I,1)
      PQR(I,2) = Q(I,1)
   55 PQR(I,3) = R(I,1)
C**     NEXT FIND THE INVERSE OF PQR                                          **
      IS = 2
      CALL AIN(PQR,PINV,PMAG,IS)
C      THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE MAGNITUDE
C       AND THE INVERSE OF THE PQR MATRIX.
C     PRINT 1000,PMAG,((PINV(I,J),J=1,3),I=1,3)
C1000 FORMAT(///,28H    THE MAGNITUDE OF PQR IS: ,/,G15.6,/,
C    126H    THE INVERSE OF PQR IS:  ,/,3(3G15.6,/))
      IF(IS.NE.101) GO TO 60
      PRINT 109
  109 FORMAT(///,68H    $$$$ THE INITIAL POINTS P AND Q ARE SUCH THAT TH
     1E MAGNITUDE $$$$ ,/,52H    $$$$ OF THE MATRIX PQR IS ESSENTIALLY Z
     2ERO. $$$$  )
      GO TO 2000
   60 CONTINUE
C**     NEXT FIND THE ROTATION MATRICES
      DO 65 I = 1,3
      DC 65 J = 1,3
      DO 65 K = 1,4
   65 ROT12(K,J,I) = 0.0
      DO 80 I = 2,4
      DO 70 J = 1,3
      PQR2(J,1) = P(J,I)
      PQR2(J,2) = Q(J,I)
```

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12345678901234567890123456789012345678901234567890123456789012345678901234567890
      PQR2(J,3) = R(J,I)
   70 CONTINUE
      DO 75 J = 1,3
      DO 75 K = 1,3
      DO 75 L = 1,3
   75 ROT12(I,K,J) = PQR2(K,L) * PINV(L,J) + ROT12(I,K,J)
      PRINT 1001, I,((ROT12(I,J,K),K=1,3),J=1,3)
 1001 FORMAT(///,64H    THE COMPUTED ROTATIONAL MATRIX FROM POSITION ONE
     1 TO POSITION,I2,  4H IS: ,/,(3G15.6))
C**      NEXT FIND THE ROTATION ANGLE PHI                                   **
      PHI(I) = .5*(ROT12(I,1,1) + ROT12(I,2,2) + ROT12(I,3,3) - 1.)
      PHI(I) = ACOS(PHI(I))
      PHID(I) = PHI(I)/PI
      PRINT 110,I,PHID(I)
  110 FORMAT(///,52H    THE ROTATION ANGLE FROM POSITION ONE TO POSITION
     1 , I2,16H, IN DEGREES IS: ,/,G15.6)
C**      NEXT FIND THE OTHER ANGLES                                         **
      TH1(I) = PHI(I) / (1. + GR)
      TH2(I) = TH1(I) * GR
C**      NEXT DEFINE THE DESIGN EQUATION PARAMETERS                         **
      C1 = SIN(TH2(I))
      C2 = SIN(TH1(I))
      C3 = ROT12(I,2,3) - ROT12(I,3,2)
      C4 = ROT12(I,3,1) - ROT12(I,1,3)
      C5 = ROT12(I,1,2) - ROT12(I,2,1)
      C(I,1) = C1*C3
      C(I,2) = C1*C4
      C(I,3) = C1*C5
      C(I,4) = -C2*C3
      C(I,5) = -C2*C4
      C(I,6) = (1. + ROT12(I,1,1) + ROT12(I,2,2) + ROT12(I,3,3))
     1         *(COS(TH1(I)) - COS(TH2(I))) - C2*C5*M(3)
C        THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE COEFFICIENT
C        MATRIX AND DESIGN COEFFICIENTS FOR EACH ROTATION.
C     PRINT 1002, I,C1,C2,C3,C4,C5,(C(I,J),J=1,6)
C1002 FORMAT(///,50H    FOR THE ROTATION FROM POSITION ONE TO POSITION ,
C    1I2,18H, THE COEFFICIENTS ,/,14H    C1-C5 ARE: ,/,5G15.6,/,42H    A
C    2ND THE SEVEN DESIGN COEFFICIENTS ARE:  ,/, 2(4G15.6,/))
   80 CONTINUE
C**      AT THIS POINT WE HAVE FIVE EQUATIONS.                              **
C**      THREE OF THE FORM:                                                 **
C**      C21AX + C22AY + C23AZ + C24MX + C25MY + C26MZ + C27 = 0            **
C**      AND TWO: AX**2 + AY**2 + AZ**2 - 1 = 0                             **
C**               MX**2 + MY**2 + MZ**2 - 1 = 0                             **
C**      THIS STARTS THE FIRST SUBSTITUTION (ELIMINATES MY)
      C1 = - 1./C(2,5)
      C(1,1) = C(2,1)*C1
      C(1,2) = C(2,2)*C1
      C(1,3) = C(2,3)*C1
      C(1,4) = C(2,4)*C1
      C(1,5) = C(2,6)*C1
      C(2,1) = C(3,1) + C(3,5)*C(1,1)
      C(2,2) = C(3,2) + C(3,5)*C(1,2)
      C(2,3) = C(3,3) + C(3,5)*C(1,3)
      C(2,4) = C(3,4) + C(3,5)*C(1,4)
```

```
000000000111111111122222222223333333333444444444455555555556666666666777777777778
1234567890123456789012345678901234567890123456789012345678901234567890123456789 0
      C(2,5) = C(3,6) + C(3,5)*C(1,5)
      C(3,1) = C(4,1) + C(4,5)*C(1,1)
      C(3,2) = C(4,2) + C(4,5)*C(1,2)
      C(3,3) = C(4,3) + C(4,5)*C(1,3)
      C(3,4) = C(4,4) + C(4,5)*C(1,4)
      C(3,5) = C(4,6) + C(4,5)*C(1,5)
      E1 = C(1,1)*C(1,1)
      E2 = C(1,2)*C(1,2)
      E3 = C(1,3)*C(1,3)
      E4 = C(1,4)*C(1,4) + 1.
      E5 = 2.*C(1,1)*C(1,2)
      E6 = 2.*C(1,1)*C(1,3)
      E7 = 2.*C(1,1)*C(1,4)
      E8 = 2.*C(1,1)*C(1,5)
      E9 = 2.*C(1,2)*C(1,3)
      E10 = 2.*C(1,2)*C(1,4)
      E11 = 2.*C(1,2)*C(1,5)
      E12 = 2.*C(1,3)*C(1,4)
      E13 = 2.*C(1,3)*C(1,5)
      E14 = 2.*C(1,4)*C(1,5)
      E15 = M(3)*M(3) - 1. + C(1,5)*C(1,5)
      CALL SCA(E1,E2,E3,E4,E5,E6,E7,E8,E9,E10,E11,E12,E13,E14,E15,0.,
     1         0.,0.,0.,0.,0.,15)
C       THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE COEFFICIENT
C       MATRIX AND THE E COEFFICIENTS AFTER THE FIRST SUBSTITUTION.
C     PRINT 1005,((C(I,J),J=1,5),I=1,3),E1,E2,E3,E4,E5,E6,E7,E8,E9,E10,
C    1E11,E12,E13,E14,E15
C1005 FORMAT(///,61H    AFTER THE FIRST SUBSTITUTION, THE COEFFICIENT MA
C    1TRIX IS:  ,/,3(5G15.6,/),/,31H     AND THE E COEFFICIENTS ARE: ,/,
C    23(5G15.6,/))
C**     THIS STARTS THE SECOND SUBSTITUTION   (ELIMINATES MX)               **
      C1 = - 1./C(2,4)
      C(1,1) = C(2,1)*C1
      C(1,2) = C(2,2)*C1
      C(1,3) = C(2,3)*C1
      C(1,4) = C(2,5)*C1
      C(2,1) = C(3,1) + C(3,4)*C(1,1)
      C(2,2) = C(3,2) + C(3,4)*C(1,2)
      C(2,3) = C(3,3) + C(3,4)*C(1,3)
      C(2,4) = C(3,5) + C(3,4)*C(1,4)
      F1 = E1 + C(1,1)*(C(1,1)*E4 + E7)                                 AX2
      F2 = E2 + C(1,2)*(C(1,2)*E4 + E10)                                AY2
      F3 = E3 + C(1,3)*(C(1,3)*E4 + E12)                                AZ2
      F4 = E5 + C(1,1)*(2.*C(1,2)*E4 + E10) + C(1,2)*E7                AXAY
      F5 = E6 + C(1,1)*(2.*C(1,3)*E4 + E12) + C(1,3)*E7                AXAZ
      F6 = E8 + C(1,1)*(2.*C(1,4)*E4 + E14) + C(1,4)*E7                AX
      F7 = E9 + C(1,2)*(2.*C(1,3)*E4 + E12) + C(1,3)*E10               AYAZ
      F8 = E11 + C(1,2)*(2.*C(1,4)*E4 + E14) + C(1,4)*E10              AY
      F9 = E13 + C(1,3)*(2.*C(1,4)*E4 + E14) + C(1,4)*E12              AZ
      F10 = E15 + C(1,4)*(C(1,4)*E4 + E14)                              ----
      CALL SCA(F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,0.,0.,0.,0.,0.,0.,0.,0.,
     1          0.,0.,0.,10)
C       THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE COEFFICIENT
C       MATRIX AND THE F COEFFICIENTS AFTER THE SECOND SUBSTITUTION.
C     PRINT 1006,((C(I,J),J=1,4),I=1,2),F1,F2,F3,F4,F5,F6,F7,F8,F9,F10
```

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12345678901234567890123456789012345678901234567890123456789012345678901234567890
C1006 FORMAT(///,61H    AFTER THE SECOND SUBSTITUTION, THE COEFFICIENT M
C     1ATRIX IS:,/,2(4G15.6,/),/,31H    AND THE F COEFFICIENTS ARE: ,/,
C     23(4G15.6,/))
C**      THIS STARTS THE THIRD SUBSTITUTION    (ELIMINATES  AZ)                 **
      C1 = - 1./C(2,3)
      C(1,1) = C(2,1)*C1
      C(1,2) = C(2,2)*C1
      C(1,3) = C(2,4)*C1
      A1 = F1 + C(1,1)*(C(1,1)*F3 + F5)                                      AX2
      A2 = F2 + C(1,2)*(C(1,2)*F3 + F7)                                      AY2
      A3 = F4 + C(1,1)*(2.*C(1,2)*F3 + F7) + C(1,2)*F5                      AXAY
      A4 = F6 + C(1,1)*(2.*C(1,3)*F3 + F9) + C(1,3)*F5                      AX
      A5 = F8 + C(1,2)*(2.*C(1,3)*F3 + F9) + C(1,3)*F7                      AY
      A6 = F10 + C(1,3)*(C(1,3)*F3 + F9)                                    ----
      B1 = C(1,1)*C(1,1) + 1.                                                AX2
      B2 = C(1,2)*C(1,2) + 1.                                                AY2
      B3 = 2.*C(1,1)*C(1,2)                                                 AXAY
      B4 = 2.*C(1,1)*C(1,3)                                                 AX
      B5 = 2.*C(1,2)*C(1,3)                                                 AY
      B6 = C(1,3)*C(1,3) - 1.                                               ----
C     CALL SCA(A1,A2,A3,A4,A5,A6,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,
C    1           0.,0.,0.,0.,0.,6)
C     CALL SCA(B1,B2,B3,B4,B5,B6,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,
C    1           0.,0.,0.,0.,0.,6)
C        THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE COEFFICIENT
C        MATRIX AND THE A AND B COEFFICIENTS AFTER THE THIRD SUBTITUTION.
C     PRINT 1007,(C(1,I),I=1,3),A1,A2,A3,A4,A5,A6,B1,B2,B3,B4,B5,B6
C1007 FORMAT(///,67H    AFTER ALL THE LINEAR SUBSTITUTIONS, THE FINAL CO
C     1EFFICIENTS ARE: ,/,3G15.6,/,59H    THE COEFFICIENTS OF THE TWO SEC
C     2OND ORDER EQUATIONS ARE: ,/,2(3G15.6,/),/,19H    AND THE SECOND: ,
C     3/,2(3G15.6,/))
C**      AT THIS POINT THERE ARE TWO EQUATIONS                                  **
C**      A1*AX**2+A2*AY**2+A3*AX*AY+A4*AX+A5*AY+A6=0      (EQN 1)              **
C**      B1*AX**2+B2*AY**2+B3+AX*AY+B4*AX+B5*AY+B6=0      (EQN 2)              **
C**      ELIMINATE THE AX*AY TERM                                               **
      C1 = B3*A1 - A3*B1                                                     AX2
      C2 = B3*A2 - A3*B2                                                     AY2
      C3 = B3*A4 - A3*B4                                                     AX
      C4 = B3*A5 - A3*B5                                                     AY
      C5 = B3*A6 - A3*B6                                                     ---
C        THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE COEFFICIENTS
C        OF THE EQUATION WITHOUT THE CROSS PRODUCT TERM.
C     PRINT 1008,C1,C2,C3,C4,C5
C1008 FORMAT(///,31H    THE COEFFICIENTS C1-C5 ARE: ,/,2(3G15.6,/))
C**      TAKE THIS EQUATION AND COMPLETE THE SQUARE TO GET EQN 3                **
      IF(C1*C2.LE.0.) GO TO 2000
      IF(C1.GT.0.) GO TO 85
      C1 = -C1
      C2 = -C2
      C3 = -C3
      C4 = -C4
      C5 = -C5
   85 CONTINUE
      R1 = C5 - C3*C3/(4.*C1) - C4*C4/(4.*C2)
C**      SUBSTITUTE AX = AX'/SQRT(C1) - .5*C3/C1                                **
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 12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
 C**          AND  AY = AY'/SQRT(C2) - .5*C4/C2 INTO EQN 3, AND EQN 1           **
 C**       WHICH GIVES AX'**2 + AY'**2 + R1 = 0  AND ANOTHER EQUATION OF        **
 C**       THE FORM D1*AX'**2+D2*AY'**2+D3*AX'*AY'+D4*AX'+D5*AY'+D6 = 0         **
          OC1 = 1./C1
          OC2 = 1./C2
          SC1 = SQRT(OC1)
          SC2 = SQRT(OC2)
          D1 = OC1*A1                                                          AX2
          D2 = OC2*A2                                                          AY2
          D3 = SC1*SC2*A3                                                     AXAY
          D4 = SC1*(A4 - C3*OC1*A1 - .5*C4*OC2*A3)                            AX
          D5 = SC2*(A5 - C4*OC2*A2 - .5*C3*OC1*A3)                            AY
          D6 = -.5*C3*OC1*A4 - .5*C4*OC2*A5 + .25*C3*C3*OC1*OC1*A1            ----
         1     +.25*C4*C4*OC2*OC2*A2 + .25*C3*C4*OC1*OC2*A3 + A6
 C        CALL SCA(D1,D2,D3,D4,D5,D6,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,
 C       1        0.,0.,0.,0.,6)
 C         THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE COEFFICIENTS
 C         OF THE TRANSFORMED SECOND DEGREE EQUATION.
 C        PRINT 1009, D1,D2,D3,D4,D5,D6
 C1009 FORMAT(///,28H     THE D COEFFICIENTS ARE: ,/,(3G15.6))
 C**       TAKE THIS LAST EQUATION AND REWRITE AS:                               **
 C**       D1*AX**2+D2*AY**2+D4*AX+D6 = D3*AX*AY+D5*AY                           **
 C**       SQUARING BOTH SIDES YIELDS AN EQUATION IN AX**2,AX,AND AY**2          **
 C**       SO SUBSTITUTE FOR AY**2, YIELDING THE FOLLOWING COEFFICIENTS          **
          D(1) = D1*D1 + D2*(D2 - 2.*D1) + D3*D3                               AX4
          D(2) = 2.*(D1*D4 - D2*D4 + D3*D5)                                    AX3
          D(3) = 2.*D2*D2*R1 + D4*D4 - 2.*D1*D2*R1 + 2.*D1*D6 - 2.*D2*D6       AX2
         1       + D3*D3*R1 + D5*D5
          D(4) = 2.*(D4*D6 - D2*D4*R1 + D3*D5*R1)                              AX
          D(5) = D2*D2*R1*R1 + D6*D6 - 2.*D2*D6*R1 + D5*D5*R1                  --
          CALL ZPOLR(D,4,Z,IER)
          PRINT 1011,(D(I),I=1,5),(Z(I),I=1,4),IER
  1011 FORMAT(///,42H     THE INPUT POLYNOMIAL COEFFICIENTS ARE: , 3G15.6
         1,/, 2G15.6,/,28H     THE RESULTING ROOTS ARE: ,/, 4(2G15.6,1HI,/),
         2            /,39H     THE ERROR PARAMETER FROM ZPOLR IS: , I3)
          DO 90 I = 1,4
          J = 2*I - 1
          K = 2*I
          F(J) = REAL(Z(I))
          F(K) = AIMAG(Z(I))
     90 CONTINUE
          DO 200 I = 1,4
          J = 2*I - 1
          K = 2*I
          CHECK = ABS(F(K))
          IF(CHECK.GT.1.E-5) GO TO 95
 C**       CALCULATE CORRESPONDING AY'                                          **
          AYS = -F(J)*F(J) - R1
          A(2) = SQRT(AYS)
 C**       NOW BACKTRANSFORM TO THE ORIGINAL COORDINATE SYSTEM                  **
          A(1) = F(J)*SC1 - .5*C3/C1
          A(2) = A(2)*SC2 - .5*C4/C2
          A(3) = -(C(2,1)*A(1) + C(2,2)*A(2) + C(2,4))/C(2,3)
          M(1) = -(C(3,1)*A(1) + C(3,2)*A(2) + C(3,3)*A(3) + C(3,5))/C(3,4)
          M(2) = -(C(4,1)*A(1) + C(4,2)*A(2) + C(4,3)*A(3) + C(4,4)*M(1)
```

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00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
     1          + C(4,6))/C(4,5)
      PRINT 111,(M(L),L=1,3),(A(L),L=1,3)
  111 FORMAT(///,53H    THE COMPUTED AXIS OF THE SUN GEAR (POINT M) IS
     1     ,/,3G15.6,/,56H    THE COMPUTED AXIS OF THE PLANET GEAR (POIN
     2T A) IS:      ,/,3G15.6 )
   95 CONTINUE
  200 CONTINUE
 2000 CONTINUE
      STOP
      END
      FUNCTION RAD2(A,B,C)
      D = 1. - A*A - B*B
      RAD2 = C*SQRT(D)
      RETURN
      END
      FUNCTION RAD(A,B,C)
      D = A*A + B*B + C*C
      RAD = SQRT(D)
      RETURN
      END
      FUNCTION CHK(A,B)
      CHK = A*A + B*B - 1.
      RETURN
      END
      SUBROUTINE AIN(A,AINV,AMAG,IS)
C**   FIRST FIND THE MAGNITUDE OF THE MATRIX                                   **
C**   THIS ROUTINE IS DESIGNED TO FIND THE INVERSE OF A (3,3) MATRIX A,        **
C**   THE INVERSE IS RETURNED IN AINV AND THE MAGNITUDE IN AMAG. IF ONLY       **
C**     THE MAGNITUDE OF THE MATRIX A IS DESIRED, IS SHOULD BE SET TO 1. IF    **
C**     BOTH THE MAGNITUDE AND THE INVERSE ARE DESIRED, IS SHOULD BE SET TO 2.**
      DIMENSION A(3,3),AINV(3,3)
      AMAG = A(1,1)*A(2,2)*A(3,3) + A(1,2)*A(2,3)*A(3,1)
     1     + A(1,3)*A(2,1)*A(3,2) - A(3,1)*A(2,2)*A(1,3)
     2     - A(2,1)*A(1,2)*A(3,3) - A(1,1)*A(3,2)*A(2,3)
      IF(IS.EQ.1) GO TO 150
      A1 = 1./AMAG
      AINV(1,1) = (A(2,2)*A(3,3) - A(3,2)*A(2,3))*A1
      AINV(1,2) = (A(3,2)*A(1,3) - A(1,2)*A(3,3))*A1
      AINV(1,3) = (A(1,2)*A(2,3) - A(2,2)*A(1,3))*A1
      AINV(2,1) = (A(3,1)*A(2,3) - A(2,1)*A(3,3))*A1
      AINV(2,2) = (A(1,1)*A(3,3) - A(3,1)*A(1,3))*A1
      AINV(2,3) = (A(2,1)*A(1,3) - A(1,1)*A(2,3))*A1
      AINV(3,1) = (A(2,1)*A(3,2) - A(3,1)*A(2,2))*A1
      AINV(3,2) = (A(3,1)*A(1,2) - A(1,1)*A(3,2))*A1
      AINV(3,3) = (A(1,1)*A(2,2) - A(2,1)*A(1,2))*A1
  150 CONTINUE
      RETURN
      END
      SUBROUTINE SCA(A1,A2,A3,A4,A5,A6,A7,A8,A9,A10,A11,A12,A13,A14,A15,
     1          A16,A17,A18,A19,A20,A21,N)
      DIMENSION A(21)
      A(1) = A1
      A(2) = A2
      A(3) = A3
      A(4) = A4
```

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00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
      A(5) = A5
      A(6) = A6
      A(7) = A7
      A(8) = A8
      A(9) = A9
      A(10) = A10
      A(11) = A11
      A(12) = A12
      A(13) = A13
      A(14) = A14
      A(15) = A15
      A(16) = A16
      A(17) = A17
      A(18) = A18
      A(19) = A19
      A(20) = A20
      A(21) = A21
      AMAX = ABS(A(1))
      DO 20 I = 2,N
      A(I) = ABS(A(I))
      IF(A(I).GT.AMAX) AMAX = A(I)
   20 CONTINUE
      ADIV = 2./AMAX
      A1 = A1*ADIV
      A2 = A2*ADIV
      A3 = A3*ADIV
      A4 = A4*ADIV
      A5 = A5*ADIV
      A6 = A6*ADIV
      A7 = A7*ADIV
      A8 = A8*ADIV
      A9 = A9*ADIV
      A10 = A10*ADIV
      A11 = A11*ADIV
      A12 = A12*ADIV
      A13 = A13*ADIV
      A14 = A14*ADIV
      A15 = A15*ADIV
      A16 = A16*ADIV
      A17 = A17*ADIV
      A18 = A18*ADIV
      A19 = A19*ADIV
      A20 = A20*ADIV
      A21 = A21*ADIV
      RETURN
      END
"00
2
2.
 -.2        .3          1.
 -.134274 .102227   1.
-.0312716 -.0852817 1.
.101886   -.249175   1.
 -.3        .4          -1.
 -.454178 .482660     -1.
```

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     00000000011111111112222222222333333333344444444445555555555666666666677777777778
     12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
0494 -.606622  .515393    -1.
0495 -.745965  .495813    -1.
0496 .672512
0497 $IBSYS
```

APPENDIX F

FIVE POSITION SYNTHESIS PROGRAM

```
 0000000001111111111222222222233333333334444444444555555555566666666667777777777 8
 1234567890123456789012345678901234567890123456789012345678901234567890123456789 0
      PROGRAM SC5P(INPUT,OUTPUT)
C********************************************************************************
C********************************************************************************
C**     THIS PROGRAM IS DESIGNED TO TAKE FIVE GIVEN RIGID BODY POSITIONS       **
C**     WHICH LIE ON A SPHERE AND COMPUTE A GEARED SPHERICAL CYCLOIDAL         **
C**     CRANK MECHANISM WHICH WILL GUIDE A RIGID BODY, CONNECTED TO THE        **
C**     PLANET GEAR, THROUGH THE FIVE GIVEN POSITIONS.                         **
C**     THE VARIABLES USED ARE AS FOLLOWS :                                    **
C**     IC - INPUT DATA CHOICE PARAMETER;  (FIRST DATA CARD WITH AN            **
C**            I1 FORMAT)                                                      **
C**        IC=1, COORDINATES GIVEN ARE ASSUMED TO LIE ON A UNIT SPHERE.        **
C**        IC=2, THE X AND Y COORDINATES OF EACH POINT ARE GIVEN IN THE        **
C**              FIRST 20 COLUMNS  WITH  A +1. OR A -1. IN THE NEXT 10         **
C**              COLUMNS TO GIVE THE SIGN OF THE Z COORDINATE.                 **
C**     GR - THE GEAR RATIO N = (THE RADIUS OF THE SUN GEAR)/(THE              **
C**            RADIUS OF THE PLANET GEAR);  (SECOND DATA CARD WITH AN          **
C**            F10.0 FORMAT)                                                   **
C**     M - A VECTOR CONTAINING THE CARTESIAN COORDINATES OF THE AXIS          **
C**            OF THE FIXED SUN GEAR.                                          **
C**     A - A VECTOR CONTAINING THE CARTESIAN COORDINATES OF THE AXIS OF       **
C**            THE PLANET GEAR IN ITS ORIGINAL POSITION.                       **
C**     P.Q - VECTORS CONTAINING THE CARTESIAN COORDINATES OF TWO              **
C**            POINTS ON THE PLANET GEAR;  THIRD DATA CARD--P1;                **
C**            FOURTH DATA CARD--P2                                            **
C**            FIFTH DATA CARD--P3                                             **
C**            SIXTH DATA CARD--P4                                             **
C**            SEVENTH DATA CARD--P5                                           **
C**            EIGHTH DATA CARD--Q1                                            **
C**            NINTH DATA CARD--Q2                                             **
C**            TENTH DATA CARD--Q3                                             **
C**            ELEVENTH DATA CARD--Q4                                          **
C**            TWELFTH DATA CARD--Q5                                           **
C**            FORMATTED AS EXPLAINED UNDER VARIABLE IC ABOVE.                 **
C**     R - A VECTOR CONTAINING THE CARTESIAN COORDINATES OF THE               **
C**            RESULT OF ROTATING Q ABOUT P THROUGH AN ANGLE OF                **
C**            NINETY DEGREES.                                                 **
C**     PQR - A MATRIX WHOSE COLUMNS ARE THE VECTORS P, Q, AND R.              **
C**     PINV - A MATRIX WHICH IS THE INVERSE OF PQR.                           **
C**     PQR2 - A MATRIX WHOSE COLUMNS ARE THE CARTESIAN COORDINATES OF THE     **
C**            SECOND POSITIONS OF P,Q, AND R.                                 **
C**     ROT12 - THE ROTATION MATRIX FROM POSITION ONE TO POSITION TWO. IT      **
C**            IS THE PRODUCT OF PQR2 AND PINV                                 **
C**     PHI - THE TOTAL ROTATION ANGLE REQUIRED TO GO FROM POSITION ONE TO     **
C**            POSITION TWO.                                                   **
C**     TH1(I) - THE ROTATION OF A ABOUT M WHEN THE MECHANISM GOES FROM        **
C**            POSITION ONE TO POSITION I, THETA ONE.                          **
C**     TH2(I) - THE ROTATION OF P ABOUT A WHEN THE MECHANISM GOES FROM        **
C**            POSITION ONE TO POSITION I, THETA TWO.                          **
C********************************************************************************
C********************************************************************************
      COMPLEX Z(5)
      REAL M(3)                                                                **
      DIMENSION A(3),P(3,5),Q(3,5),R(3,5),PQR(3,3),PINV(3,3),PQR2(3,3)
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CARD
     1        ,ROT12(5,3,3),TH1(5),TH2(5),CHKL(5),PHI(5),PHID(5),D(5)
     2         ,F(10),C(5,7),CH(3,3)
C**     READ IN DATA                                                          **
      READ 100,IC
  100 FORMAT(I1)
      PRINT 101,IC
  101 FORMAT(1H1,37H    THE INPUT DATA CHOICE PARAMETER =  ,I2)
      READ 102 , GR
      PRINT 103,GR
  103 FORMAT(///,28H    THE INPUT GEAR RATIO IS: ,/,G15.6)
      READ 102,((P(I,J),I=1,3),J=1,5),((Q(I,J),I=1,3),J=1,5)
  102 FORMAT(3F10.0)
      PRINT 104,((P(I,J),J=1,5),I=1,3),((Q(I,J),J=1,5),I=1,3)
  104 FORMAT(///,50H    THE INPUT RIGID BODY POSITIONS ARE AS FOLLOWS:
     1,/,45H    POINTS P; WHERE EACH COLUMN IS ONE POINT.,/,3(5G15.6,/),
     2//,45H    POINTS Q; WHERE EACH COLUMN IS ONE POINT.,/,3(5G15.6,/))
      PI = 355./(113.*180.)
      GO TO (5,15), IC
    5 CPS = 0.0
      DO 10 J = 1,2
      CP = RAD(P(1,J),P(2,J),P(3,J)) - 1.
      CQ = RAD(Q(1,J),Q(2,J),Q(3,J)) - 1.
      CPS = CPS + ABS(CP) + ABS(CQ)
   10 CONTINUE
      IF(CPS.LT.0.03) GO TO 30
      PRINT 105
  105 FORMAT(///,51H    $$$$ INPUT DATA POINTS NOT ON UNIT SPHERE. $$$$
     1 ,/,60H    $$$$ INPUT DATA USING INPUT DATA OPTION NUMBER TWO. $$$
     2$  )
      GO TO 2000
C**     CALCULATE THE Z COORDINATES OF THE P AND Q POINTS
   15 CONTINUE
      DO 25 J = 1,5
      CP = CHK(P(1,J),P(2,J))
      CQ = CHK(Q(1,J),Q(2,J))
      IF(CP.LE.0.) GO TO 20
      IF(CQ.LE.0.) GO TO 20
      PRINT 115,J,CP,CQ
  115 FORMAT(///, 73H    $$$$ THE INPUT P,AND Q ARE TOO LARGE TO BE ON T
     1HE UNIT SPHERE. $$$$      ,/,4X,2HJ= ,I2,/,4X, 3HCP=,G15.6,
     2 /,4X,3HCQ=,G15.6)
      GO TO 2000
   20 CONTINUE
      P(3,J) = RAD2(P(1,J),P(2,J),P(3,J))
      Q(3,J) = RAD2(Q(1,J),Q(2,J),Q(3,J))
   25 CONTINUE
   30 CONTINUE
      PRINT 104,((P(I,J),J=1,5),I=1,3),((Q(I,J),J=1,5),I=1,3)
C**     NEXT FIND THE COORDINATES OF POINT R                                  **
      DO 35 I = 1,5
      R(1,I) = P(1,I)*P(1,I)*Q(1,I) +  (P(1,I)*P(2,I)-P(3,I))*Q(2,I)
     1 + (P(1,I)*P(3,I)+P(2,I))*Q(3,I)
      R(2,I) = (P(1,I)*P(2,I)+P(3,I))*Q(1,I) + P(2,I)*P(2,I)*Q(2,I)
     1 + (P(2,I)*P(3,I)-P(1,I))*Q(3,I)
   35 R(3,I) = (P(1,I)*P(3,I)-P(2,I))*Q(1,I)
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CARD
     1  + (P(2,I)*P(3,I)+P(1,I))*Q(2,I) + P(3,I)*P(3,I)*Q(3,I)
      PRINT 106,((R(I,J),J=1,5),I=1,3)
  106 FORMAT(///,68H    THE COORDINATES OF THE COMPUTED R S ARE AS FOLLO
     1WS. EACH COLUMN  ,/,17H    IS ONE POINT. ,/,(5G15.6))
C**     CHECK THE LENGTH OF P(I)Q(I)                                   **
      DO 40 I = 1,5
      DX = P(1,I) - Q(1,I)
      DY = P(2,I) - Q(2,I)
      DZ = P(3,I) - Q(3,I)
      CHKL(I) = RAD(DX,DY,DZ)
   40 CONTINUE
      DO 50 I = 2,5
      CHK2 = CHKL(I) - CHKL(1)
      CHK2 = ABS(CHK2)
      IF(CHK2.LT.1.E-3) GO TO 45
      PRINT 107,(CHKL(L),L=1,5)
  107 FORMAT(///,72H     $$$$ THIS JOB ABORTED, THE LENGTHS OF THE INPUT
     1PG ARCS ARE NOT $$$$ ,/,48H    $$$$ EQUAL. THE LENGTHS ARE AS FOLL
     2OWS: $$$$ ,/, 5G15.6)
      GC TO 2000
   45 CONTINUE
   50 CONTINUE
C**     THE FOLLOWING DO LOOPS FIND THE ROTATION MATRICES              **
C**     NEXT SET UP THE FIRST POSITION PQR MATRIX                      **
      DO 55 I = 1,3
      PQR(I,1) = P(I,1)
      PQR(I,2) = Q(I,1)
   55 PQR(I,3) = R(I,1)
C**     NEXT FIND THE INVERSE OF PQR                                   **
      IS = 2
      CALL AIN(PQR,PINV,PMAG,IS)
C       THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE MAGNITUDE
C       AND THE INVERSE OF THE PQR MATRIX.
C     PRINT 1000,PMAG,((PINV(I,J),J=1,3),I=1,3)
C1000 FORMAT(///,28H    THE MAGNITUDE OF PQR IS: ,/,G15.6,/,
C    126H    THE INVERSE OF PQR IS:  ,/,3(3G15.6,/))
      IF(IS.NE.101) GO TO 60
      PRINT 108
  108 FORMAT(///,68H    $$$$ THE INITIAL POINTS P AND Q ARE SUCH THAT TH
     1E MAGNITUDE $$$$ ,/,52H    $$$$ OF THE MATRIX PQR IS ESSENTIALLY Z
     2ERO. $$$$  )
      GO TO 2000
   60 CONTINUE
C**     NEXT FIND THE ROTATION MATRICES
      DO 65 I = 1,3
      DO 65 J = 1,3
      DO 65 K = 1,5
   65 ROT12(K,J,I) = 0.0
      DO 80 I = 2,5
      DO 70 J = 1,3
      PQR2(J,1) = P(J,I)
      PQR2(J,2) = Q(J,I)
      PQR2(J,3) = R(J,I)
   70 CONTINUE
      DO 75 J = 1,3
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CARD
      DO 75 K = 1,3
      DC 75 L = 1,3
   75 ROT12(I,K,J) = PQR2(K,L) * PINV(L,J) + ROT12(I,K,J)
      PRINT 1001, I,((ROT12(I,J,K),K=1,3),J=1,3)
 1001 FORMAT(///,64H    THE COMPUTED ROTATIONAL MATRIX FROM POSITION ONE
     1 TO POSITION,I2,  4H IS: ,/,(3G15.6))
C**     NEXT FIND THE ROTATION ANGLE PHI                                      **
      PHI(I) = .5*(ROT12(I,1,1) + ROT12(I,2,2) + ROT12(I,3,3) - 1.)
      PHI(I) = ACOS(PHI(I))
      PHID(I) = PHI(I)/PI
      PRINT 109,I,PHID(I)
  109 FORMAT(///,52H    THE ROTATION ANGLE FROM POSITION ONE TO POSITION
     1 , I2,16H, IN DEGREES IS: ,/,G15.6)
C**     NEXT FIND THE OTHER ANGLES                                            **
      TH1(I) = PHI(I) / (1. + GR)
      TH2(I) = TH1(I) * GR
C**     NEXT DEFINE THE DESIGN EQUATION PARAMETERS
      C1 = SIN(TH2(I))
      C2 = SIN(TH1(I))
      C3 = ROT12(I,2,3) - ROT12(I,3,2)
      C4 = ROT12(I,3,1) - ROT12(I,1,3)
      C5 = ROT12(I,1,2) - ROT12(I,2,1)
      C(I,1) = C1*C3
      C(I,2) = C1*C4
      C(I,3) = C1*C5
      C(I,4) = -C2*C3
      C(I,5) = -C2*C4
      C(I,6) = -C2*C5
      C(I,7) = (1. + ROT12(I,1,1) + ROT12(I,2,2) + ROT12(I,3,3))
     1         *(COS(TH1(I)) - COS(TH2(I)))
C       THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE
C       THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE DESIGN
C       EQUATION PARAMETERS FOR EACH ROTATION.
C     PRINT 1002, I,C1,C2,C3,C4,C5,(C(I,J),J=1,7)
C1002 FORMAT(///,50H    FOR THE ROTATION FROM POSITION ONE TO POSITION ,
C    1I2,18H, THE COEFFICIENTS ,/,14H    C1-C5 ARE: ,/,5G15.6,/,42H    A
C    2ND THE SEVEN DESIGN COEFFICIENTS ARE:  ,/, 2(4G15.6,/))
   80 CONTINUE
C**     AT THIS POINT WE HAVE SIX EQUATIONS                                   **
C**     FOUR OF THE FORM:                                                     **
C**     C21AX + C22AY + C23AZ + C24MX + C25MY + C26MZ + C27 = 0               **
C**     AND TWO: AX**2 + AY**2 + AZ**2 - 1 = 0                                **
C**              MX**2 + MY**2 + MZ**2 - 1 = 0                                **
C**     SOLVE THE FIRST EQUATION FOR MZ AND SUBSTITUTE IN ALL OTHERS          **
      C1 = - 1./C(2,6)
      C(1,1) = C(2,1)*C1
      C(1,2) = C(2,2)*C1
      C(1,3) = C(2,3)*C1
      C(1,4) = C(2,4)*C1
      C(1,5) = C(2,5)*C1
      C(1,6) = C(2,7)*C1
C       THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE COEFFICIENT
C       MATRIX FOR THE FIRST SUBSTITUTION.
C     PRINT 1003,C1,(C(1,I),I=1,6)
C1003 FORMAT(///,80H    FOR THE FIRST SUBSTITUTION, THE COEFFICIENTS C1,
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CARD
     C     1 AND C(1,1) THRU C(1,6) ARE: ,/,2(4G15.6,/))
     C**      COMPUTE THE COEFFICIENTS AFTER SUBSTITUTING                             **
            C(2,1) = C(3,1) + C(3,6)*C(1,1)
            C(2,2) = C(3,2) + C(3,6)*C(1,2)
            C(2,3) = C(3,3) + C(3,6)*C(1,3)
            C(2,4) = C(3,4) + C(3,6)*C(1,4)
            C(2,5) = C(3,5) + C(3,6)*C(1,5)
            C(2,6) = C(3,7) + C(3,6)*C(1,6)
            C(3,1) = C(4,1) + C(4,6)*C(1,1)
            C(3,2) = C(4,2) + C(4,6)*C(1,2)
            C(3,3) = C(4,3) + C(4,6)*C(1,3)
            C(3,4) = C(4,4) + C(4,6)*C(1,4)
            C(3,5) = C(4,5) + C(4,6)*C(1,5)
            C(3,6) = C(4,7) + C(4,6)*C(1,6)
            C(4,1) = C(5,1) + C(5,6)*C(1,1)
            C(4,2) = C(5,2) + C(5,6)*C(1,2)
            C(4,3) = C(5,3) + C(5,6)*C(1,3)
            C(4,4) = C(5,4) + C(5,6)*C(1,4)
            C(4,5) = C(5,5) + C(5,6)*C(1,5)
            C(4,6) = C(5,7) + C(5,6)*C(1,6)
            D1 = C(1,1)*C(1,1)                                                    AX2
            D2 = C(1,2)*C(1,2)                                                    AY2
            D3 = C(1,3)*C(1,3)                                                    AZ2
            D4 = C(1,4)*C(1,4) + 1.                                               MX2
            D5 = C(1,5)*C(1,5) + 1.                                               MY2
            D6 = C(1,6)*C(1,6) - 1.                                               ---
            D7 = 2.*C(1,1)*C(1,2)                                                 AXAY
            D8 = 2.*C(1,1)*C(1,3)                                                 AXAZ
            D9 = 2.*C(1,1)*C(1,4)                                                 AXMX
            D10 = 2.*C(1,1)*C(1,5)                                                AXMY
            D11 = 2.*C(1,1)*C(1,6)                                                AX
            D12 = 2.*C(1,2)*C(1,3)                                                AYAZ
            D13 = 2.*C(1,2)*C(1,4)                                                AYMX
            D14 = 2.*C(1,2)*C(1,5)                                                AYMY
            D15 = 2.*C(1,2)*C(1,6)                                                AY
            D16 = 2.*C(1,3)*C(1,4)                                                AZMX
            D17 = 2.*C(1,3)*C(1,5)                                                AZMY
            D18 = 2.*C(1,3)*C(1,6)                                                AZ
            D19 = 2.*C(1,4)*C(1,5)                                                MXMY
            D20 = 2.*C(1,4)*C(1,6)                                                MX
            D21 = 2.*C(1,5)*C(1,6)                                                MY
     C        THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE COEFFICIENT
     C        MATRIX AND THE D COEFFICIENTS AFTER THE FIRST SUBSTITUTION.
     C      PRINT 1004,((C(I,J),J=1,6),I=1,4),D1,D2,D3,D4,D5,D6,D7,D8,D9,D10,
     C     1D11,D12,D13,D14,D15,D16,D17,D18,D19,D20,D21
     C1004 FORMAT(///,60H    AFTER THE FIRST SUBSTITUTION, THE COEFFICIENT MA
     C     1TRIX IS: ,/,4(6G15.6,/),/,31H    AND THE D COEFFICIENTS ARE: ,/,
     C     24(6G15.6,/))
     C**      THIS STARTS THE SECOND SUBSTITUTION (ELIMINATES MY)
            C1 = - 1./C(2,5)
            C(1,1) = C(2,1)*C1
            C(1,2) = C(2,2)*C1
            C(1,3) = C(2,3)*C1
            C(1,4) = C(2,4)*C1
            C(1,5) = C(2,6)*C1
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CARD
      C(2,1) = C(3,1) + C(3,5)*C(1,1)
      C(2,2) = C(3,2) + C(3,5)*C(1,2)
      C(2,3) = C(3,3) + C(3,5)*C(1,3)
      C(2,4) = C(3,4) + C(3,5)*C(1,4)
      C(2,5) = C(3,6) + C(3,5)*C(1,5)
      C(3,1) = C(4,1) + C(4,5)*C(1,1)
      C(3,2) = C(4,2) + C(4,5)*C(1,2)
      C(3,3) = C(4,3) + C(4,5)*C(1,3)
      C(3,4) = C(4,4) + C(4,5)*C(1,4)
      C(3,5) = C(4,6) + C(4,5)*C(1,5)
      E1 = D1 + C(1,1)*(C(1,1)*D5 + D10)                                AX2
      E2 = D2 + C(1,2)*(C(1,2)*D5 + D14)                                AY2
      E3 = D3 + C(1,3)*(C(1,3)*D5 + D17)                                AZ2
      E4 = D4 + C(1,4)*(C(1,4)*D5 + D19)                                MX2
      E5 = D7 + C(1,1)*(2.*C(1,2)*D5 + D14) + C(1,2)*D10               AXAY
      E6 = D8 + C(1,1)*(2.*C(1,3)*D5 + D17) + C(1,3)*D10               AXAZ
      E7 = D9 + C(1,1)*(2.*C(1,4)*D5 + D19) + C(1,4)*D10               AXMX
      E8 = D11 + C(1,1)*(2.*C(1,5)*D5 + D21) + C(1,5)*D10              AX
      E9 = D12 + C(1,2)*(2.*C(1,3)*D5 + D17) + C(1,3)*D14              AYAZ
      E10 = D13 + C(1,2)*(2.*C(1,4)*D5 + D19) + C(1,4)*D14             AYMX
      E11 = D15 + C(1,2)*(2.*C(1,5)*D5 + D21) + C(1,5)*D14             AY
      E12 = D16 + C(1,3)*(2.*C(1,4)*D5 + D19) + C(1,4)*D17             AZMX
      E13 = D18 + C(1,3)*(2.*C(1,5)*D5 + D21) + C(1,5)*D17             AZ
      E14 = D20 + C(1,4)*(2.*C(1,5)*D5 + D21) + C(1,5)*D19             MX
      E15 = D6 + C(1,5)*(C(1,5)*D5 + D21)                              ----
      CALL SCA(E1,E2,E3,E4,E5,E6,E7,E8,E9,E10,E11,E12,E13,E14,E15,0.,
     1         0.,0.,0.,0.,0.,15)
C         THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE COEFFICIENT
C         MATRIX AND THE E COEFFICIENTS AFTER THE SECOND SUBSTITUTION.
C     PRINT 1005,((C(I,J),J=1,5),I=1,3),E/,E2,E3,E4,E5,E6,E7,E8,E9,E10,
C    1E11,E12,E13,E14,E15
C1005 FORMAT(///,61H    AFTER THE SECOND SUBSTITUTION, THE COEFFICIENT M
C    1ATRIX IS: ,/,3(5G15.6,/),/,31H    AND THE E COEFFICIENTS ARE: ,/,
C    23(5G15.6,/))
C**       THIS STARTS THE THIRD SUBSTITUTION    (ELIMINATES MX)          **
      C1 = - 1./C(2,4)
      C(1,1) = C(2,1)*C1
      C(1,2) = C(2,2)*C1
      C(1,3) = C(2,3)*C1
      C(1,4) = C(2,5)*C1
      C(2,1) = C(3,1) + C(3,4)*C(1,1)
      C(2,2) = C(3,2) + C(3,4)*C(1,2)
      C(2,3) = C(3,3) + C(3,4)*C(1,3)
      C(2,4) = C(3,5) + C(3,4)*C(1,4)
      F1 = E1 + C(1,1)*(C(1,1)*E4 + E7)                                 AX2
      F2 = E2 + C(1,2)*(C(1,2)*E4 + E10)                                AY2
      F3 = E3 + C(1,3)*(C(1,3)*E4 + E12)                                AZ2
      F4 = E5 + C(1,1)*(2.*C(1,2)*E4 + E10) + C(1,2)*E7                AXAY
      F5 = E6 + C(1,1)*(2.*C(1,3)*E4 + E12) + C(1,3)*E7                AXAZ
      F6 = E8 + C(1,1)*(2.*C(1,4)*E4 + E14) + C(1,4)*E7                AX
      F7 = E9 + C(1,2)*(2.*C(1,3)*E4 + E12) + C(1,3)*E10               AYAZ
      F8 = E11 + C(1,2)*(2.*C(1,4)*E4 + E14) + C(1,4)*E10              AY
      F9 = E13 + C(1,3)*(2.*C(1,4)*E4 + E14) + C(1,4)*E12              AZ
      F10 = E15 + C(1,4)*(C(1,4)*E4 + E14)                             ----
      CALL SCA(F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,0.,0.,0.,0.,0.,0.,0.,0.,
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     1          0.,0.,C.,10)
C      THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE COEFFICIENT
C      MATRIX AND THE F COEFFICIENTS AFTER THE THIRD SUBSTITUTION.
C     PRINT 1006,((C(I,J),J=1,4),I=1,2),F1,F2,F3,F4,F5,F6,F7,F8,F9,F10
C1006 FORMAT(///,60H    AFTER THE THIRD SUBSTITUTION, THE COEFFICIENT MA
C    1TRIX IS: ,/,2(4G15.6,/),/,31H    AND THE F COEFFICIENTS ARE: ,/,
C    23(4G15.6,/))
C**    THIS STARTS THE FOURTH SUBSTITUTION
      C1 = - 1./C(2,3)
      C(1,1) = C(2,1)*C1
      C(1,2) = C(2,2)*C1
      C(1,3) = C(2,4)*C1
      A1 = F1 + C(1,1)*(C(1,1)*F3 + F5)                                 AX2
      A2 = F2 + C(1,2)*(C(1,2)*F3 + F7)                                 AY2
      A3 = F4 + C(1,1)*(2.*C(1,2)*F3 + F7) + C(1,2)*F5                  AXAY
      A4 = F6 + C(1,1)*(2.*C(1,3)*F3 + F9) + C(1,3)*F5                  AX
      A5 = F8 + C(1,2)*(2.*C(1,3)*F3 + F9) + C(1,3)*F7                  AY
      A6 = F10 + C(1,3)*(C(1,3)*F3 + F9)                                ----
      CALL SCA(A1,A2,A3,A4,A5,A6,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,
     1          0.,0.,C.,0.,0.,6)
      B1 = C(1,1)*C(1,1) + 1.                                           AX2
      B2 = C(1,2)*C(1,2) + 1.                                           AY2
      B3 = 2.*C(1,1)*C(1,2)                                             AXAY
      B4 = 2.*C(1,1)*C(1,3)                                             AX
      B5 = 2.*C(1,2)*C(1,3)                                             AY
      B6 = C(1,3)*C(1,3) - 1.                                           ----
      CALL SCA(B1,B2,B3,B4,B5,B6,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,
     1          0.,0.,C.,0.,0.,6)
C      THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE COEFFICIENT
C      MATRIX AND THE A AND B COEFFICIENTS AFTER THE FOURTH
C      SUBSTITUTION.
C     PRINT 1007,(C(1,I),I=1,3),A1,A2,A3,A4,A5,A6,B1,B2,B3,B4,B5,B6
C1007 FORMAT(///,67H    AFTER ALL THE LINEAR SUBSTITUTIONS, THE FINAL CO
C    1EFFICIENTS ARE: ,/,3G15.6,/,59H    THE COEFFICIENTS OF THE TWO SEC
C    2OND ORDER EQUATIONS ARE: ,/,2(3G15.6,/),/,19H    AND THE SECOND: ,
C    3/,2(3G15.6,/))
C**    AT THIS POINT THERE ARE TWO EQUATIONS                             **
C**    A1*AX**2+A2*AY**2+A3*AX*AY+A4*AX+A5*AY+A6=0     (EQN 1)           **
C**    B1*AX**2+B2*AY**2+B3+AX*AY+B4*AX+B5*AY+B6=0     (EQN 2)           **
C**    ELIMINATE THE AX*AY TERM                                          **
      C1 = B3*A1 - A3*B1                                                AX2
      C2 = B3*A2 - A3*B2                                                AY2
      C3 = B3*A4 - A3*B4                                                AX
      C4 = B3*A5 - A3*B5                                                AY
      C5 = B3*A6 - A3*B6                                                ---
      CALL SCA(C1,C2,C3,C4,C5,0.,0.,0.,0.,0.,0.,0.,0.,C.,0.,0.,
     1          0.,0.,0.,0.,0.,5)
C**    TAKE THIS EQUATION AND COMPLETE THE SQUARE TO GET ECN 3           **
C      THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE COEFFICIENTS
C     OF THE EQUATION WITHOUT A CROSS PRODUCT TERM.
C     PRINT 1008,C1,C2,C3,C4,C5
C1008 FORMAT(///,31H    THE COEFFICIENTS C1-C5 ARE: ,/,2(3G15.6,/))
      IF(C1*C2.LE.0.) GO TO 2000
      IF(C1.GT.0.) GO TO 85
      C1 = -C1
```

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12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
      C2 = -C2
      C3 = -C3
      C4 = -C4
      C5 = -C5
   85 CONTINUE
      R1 = C5 - C3*C3/(4.*C1) - C4*C4/(4.*C2)
C**     SUBSTITUTE AX = AX'/SQRT(C1) - .5*C3/C1                              **
C**            AND  AY = AY'/SQRT(C2) - .5*C4/C2 INTO EQN 3, AND EQN 1       **
C**     WHICH GIVES AX'**2 + AY'**2 + R1 = 0  AND ANOTHER EQUATION OF        **
C**     THE FORM D1*AX'**2+D2*AY'**2+D3*AX'*AY'+D4*AX'+D5*AY'+D6 = 0         **
      OC1 = 1./C1
      OC2 = 1./C2
      SC1 = SQRT(OC1)
      SC2 = SQRT(OC2)
      D1 = OC1*A1                                                         AX2
      D2 = OC2*A2                                                         AY2
      D3 = SC1*SC2*A3                                                     AXAY
      D4 = SC1*(A4 - C3*OC1*A1 - .5*C4*OC2*A3)                            AX
      D5 = SC2*(A5 - C4*OC2*A2 - .5*C3*OC1*A3)                            AY
      D6 = -.5*C3*OC1*A4 - .5*C4*OC2*A5 + .25*C3*C3*OC1*OC1*A1            ----
     1     +.25*C4*C4*OC2*OC2*A2 + .25*C3*C4*OC1*OC2*A3 + A6
      CALL SCA(D1,D2,D3,D4,D5,D6,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,
     1           0.,0.,0.,0.,6)
C       THE FOLLOWING CARDS CAN BE USED TO PRINT OUT THE COEFFICIENTS
C       OF THE SECOND TRANSFORMED EQUATION.
C     PRINT 1009, D1,D2,D3,D4,D5,D6
C1009 FORMAT(///,28H    THE D COEFFICIENTS ARE: ,/,(3G15.6))
C**     TAKE THIS LAST EQUATION AND REWRITE AS:                              **
C**     D1*AX**2+D2*AY**2+D4*AX+D6 = D3*AX*AY+D5*AY                          **
C**     SQUARING BOTH SIDES YIELDS AN EQUATION IN AX**2,AX,AND AY**2         **
C**     SO SUBSTITUTE FOR AY**2, YIELDING THE FOLLOWING COEFFICIENTS         **
      D(1) = D1*D1 + D2*(D2 - 2.*D1) + D3*D3                              AX4
      D(2) = 2.*(D1*D4 - D2*D4 + D3*D5)                                   AX3
      D(3) = 2.*D2*D2*R1 + D4*D4 - 2.*D1*D2*R1 + 2.*D1*D6 - 2.*D2*D6      AX2
     1        + D3*D3*R1 + D5*D5
      D(4) = 2.*(D4*D6 - D2*D4*R1 + D3*D5*R1)                             AX
      D(5) = D2*D2*R1*R1 + D6*D6 - 2.*D2*D6*R1 + D5*D5*R1                 --
      CALL ZPOLR(D,4,Z,IER)
      PRINT 1011,(D(I),I=1,5),(Z(I),I=1,4),IER
 1011 FORMAT(///,42H    THE INPUT POLYNOMIAL COEFFICIENTS ARE: , 3G15.6
     1,/, 2G15.6,/,28H    THE RESULTING ROOTS ARE: ,/, 4(2G15.6,1HI,/),
     2            /,39H    THE ERROR PARAMETER FROM ZPOLR IS: , I3)
      DO 90 I = 1,4
      J = 2*I - 1
      K = 2*I
      F(J) = REAL(Z(I))
      F(K) = AIMAG(Z(I))
   90 CONTINUE
      DO 200 I = 1,4
      J = 2*I - 1
      K = 2*I
      CHECK = ABS(F(K))
      IF(CHECK.LT.1.E-5) GO TO 91
      CHECK = F(K)/F(J)
      CHECK = ABS(CHECK)
```

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12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
      IF(CHECK.GT.0.052) GC TC 95
   91 CONTINUE
C**     CALCULATE CORRESPONDING AY'                                           **
      AYS = -F(J)*F(J) - R1
      A(2) = SQRT(AYS)
C**     NOW BACKTRANSFORM TO THE ORIGINAL COORDINATE SYSTEM                   **
      A(1) = F(J)*SC1 - .5*C3/C1
      A(2) = A(2)*SC2 - .5*C4/C2
      A(3) = -(C(2,1)*A(1) + C(2,2)*A(2) + C(2,4))/C(2,3)
      M(1) = -(C(3,1)*A(1) + C(3,2)*A(2) + C(3,3)*A(3) + C(3,5))/C(3,4)
      M(2) = -(C(4,1)*A(1) + C(4,2)*A(2) + C(4,3)*A(3) + C(4,4)*M(1)
     1         + C(4,6))/C(4,5)
      M(3) = -(C(5,1)*A(1) + C(5,2)*A(2) + C(5,3)*A(3) + C(5,4)*M(1)
     1         + C(5,5)*M(2) + C(5,7))/C(5,6)
      PRINT 110,(M(L),L=1,3),(A(L),L=1,3)
  110 FORMAT(///,53H     THE COMPUTED AXIS OF THE SUN GEAR (POINT M) IS:
     1     ,/,3G15.6,/,56H     THE COMPUTED AXIS OF THE PLANET GEAR (POIN
     2T A) IS:       ,/,3G15.6 )
      DO 92 L=2,5
      CO1 = COS(TH1(L))
      CO2 = COS(TH2(L))
      S1 = SIN(TH1(L))
      S2 = SIN(TH2(L))
      V1 = 1. - CO1
      V2 = 1. - CO2
      CH(1,1) = V1*V1*(M(1)*A(1))*(M(1)*A(1)+M(2)*A(2)+M(3)*A(3))
     1        + V1*CO2*M(1)*M(1) + V1*S2*(M(1)*M(2)*A(3)-M(1)*M(3)*A(2))
     2        + CO1*CO2 + CO1*V2*A(1)*A(1)
     3        + S1*V2*(M(2)*A(1)*A(3)-M(3)*A(1)*A(2))
     4        + S1*S2*(-M(2)* A(2)-M(3)*A(3))
      PRINT 120,CH(1,1)
  120 FORMAT(26H     THE ELEMENTS ARE        ,G15.6)
   92 CONTINUE
   95 CONTINUE
  200 CONTINUE
 2000 CONTINUE
      STOP
      END
      FUNCTION RAD2(A,B,C)
      D = 1. - A*A - B*B
      RAD2 = C*SQRT(D)
      RETURN
      END
      FUNCTION RAD(A,B,C)
      D = A*A + B*B + C*C
      RAD = SQRT(D)
      RETURN
      END
      FUNCTION CHK(A,B)
      CHK = A*A + B*B - 1.
      RETURN
      END
      SUBROUTINE AIN(A,AINV,AMAG,IS)
C**   FIRST FIND THE MAGNITUDE OF THE MATRIX                                  **
C**   THIS ROUTINE IS DESIGNED TO FIND THE INVERSE OF A (3,3) MATRIX A,       **
```

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000000000111111111122222222223333333333444444444455555555556666666666777777777778
123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890
C**   THE INVERSE IS RETURNED IN AINV AND THE MAGNITUDE IN AMAG. IF ONLY      **
C**     THE MAGNITUDE OF THE MATRIX A IS DESIRED, IS SHOULD BE SET TO 1. IF    **
C**     BOTH THE MAGNITUDE AND THE INVERSE ARE DESIRED, IS SHOULD BE SET TO 2.**
      DIMENSION A(3,3),AINV(3,3)
      AMAG = A(1,1)*A(2,2)*A(3,3) + A(1,2)*A(2,3)*A(3,1)
     1     + A(1,3)*A(2,1)*A(3,2) - A(3,1)*A(2,2)*A(1,3)
     2     - A(2,1)*A(1,2)*A(3,3) - A(1,1)*A(3,2)*A(2,3)
      IF(IS.EQ.1) GO TO 150
      A1 = 1./AMAG
      AINV(1,1) = (A(2,2)*A(3,3) - A(3,2)*A(2,3))*A1
      AINV(1,2) = (A(3,2)*A(1,3) - A(1,2)*A(3,3))*A1
      AINV(1,3) = (A(1,2)*A(2,3) - A(2,2)*A(1,3))*A1
      AINV(2,1) = (A(3,1)*A(2,3) - A(2,1)*A(3,3))*A1
      AINV(2,2) = (A(1,1)*A(3,3) - A(3,1)*A(1,3))*A1
      AINV(2,3) = (A(2,1)*A(1,3) - A(1,1)*A(2,3))*A1
      AINV(3,1) = (A(2,1)*A(3,2) - A(3,1)*A(2,2))*A1
      AINV(3,2) = (A(3,1)*A(1,2) - A(1,1)*A(3,2))*A1
      AINV(3,3) = (A(1,1)*A(2,2) - A(2,1)*A(1,2))*A1
  150 CONTINUE
      RETURN
      END
      SUBROUTINE SCA(A1,A2,A3,A4,A5,A6,A7,A8,A9,A10,A11,A12,A13,A14,A15,
     1            A16,A17,A18,A19,A20,A21,N)
      DIMENSION A(21)
      A(1) = A1
      A(2) = A2
      A(3) = A3
      A(4) = A4
      A(5) = A5
      A(6) = A6
      A(7) = A7
      A(8) = A8
      A(9) = A9
      A(10) = A10
      A(11) = A11
      A(12) = A12
      A(13) = A13
      A(14) = A14
      A(15) = A15
      A(16) = A16
      A(17) = A17
      A(18) = A18
      A(19) = A19
      A(20) = A20
      A(21) = A21
      AMAX = A(1)
      DO 20 I = 2,N
      A(I) = ABS(A(I))
      IF(A(I).GT.AMAX) AMAX = A(I)
   20 CONTINUE
      ADIV = 2./AMAX
      A1 = A1*ADIV
      A2 = A2*ADIV
      A3 = A3*ADIV
      A4 = A4*ADIV
```

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       00000000011111111112222222222333333333344444444445555555555666666666677777777778
       12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
              A5 = A5*ADIV
              A6 = A6*ADIV
              A7 = A7*ADIV
              A8 = A8*ADIV
              A9 = A9*ADIV
              A10 = A10*ADIV
              A11 = A11*ADIV
              A12 = A12*ADIV
              A13 = A13*ADIV
              A14 = A14*ADIV
              A15 = A15*ADIV
              A16 = A16*ADIV
              A17 = A17*ADIV
              A18 = A18*ADIV
              A19 = A19*ADIV
              A20 = A20*ADIV
              A21 = A21*ADIV
              RETURN
              END
       "00
       2
       2.
       -.2       .3        1.
       -.113094  .226181     1.
       -.00757381.173646   1.
       .108740   .146311   1.
       .227249   .146282   1.
       -.3       .4        -1.
       -.503378  .350705     -1.
       -.679041  .256321   -1.
       -.814562  .123531   -1.
       -.900225  -.038311   -1.
```

APPENDIX G

COMPUTER ROUTINES USED TO FIND THE ROOTS OF THE FOURTH ORDER POLYNOMIAL

```
C***********************************************************************
C***********************************************************************
C**     THE FOLLOWING ROUTINES ARE FROM THE INTERNATIONAL MATHEMATICAL       **
C**     AND STATISTICAL LIBRARY (IMSL). THESE ROUTINES ARE THE ONES USED     **
C**     TO SOLVE THE FOURTH ORDER DESIGN POLYNOMIAL.                         **
C***********************************************************************
C***********************************************************************
      SUBROUTINE UERTST (IER,NAME)
C
C-UERTST----------------LIBRARY 3-------------------------------------
C
C   FUNCTION            - ERROR MESSAGE GENERATION
C   USAGE               - CALL UERTST(IER,NAME)
C   PARAMETERS   IER    - ERROR PARAMETER. TYPE + N  WHERE
C                           TYPE= 128 IMPLIES TERMINAL ERROR
C                                  64 IMPLIES WARNING WITH FIX
C                                  32 IMPLIES WARNING
C                           N    = ERROR CODE RELEVANT TO CALLING ROUTINE
C                NAME   - INPUT SCALAR CONTAINING THE NAME OF THE
C                           CALLING ROUTINE AS A 6-CHARACTER LITERAL
C                           STRING.
C   LANGUAGE            - FORTRAN
C--------------------------------------------------------------------
C   LATEST REVISION     - AUGUST 1, 1973
C
      DIMENSION          ITYP(2,4),IBIT(4)
      INTEGER            WARN,WARF,TERM,PRINTR
      EQUIVALENCE        (IBIT(1),WARN),(IBIT(2),WARF),(IBIT(3),TERM)
      DATA      ITYP      /10HWARNING    ,10H          ,
     *                     10HWARNING(WI,10HTH FIX)    ,
     *                     10HTERMINAL  ,10H           ,
     *                     10HNON-DEFINE,10HD          /,
     *          IBIT       / 32,64,128,0/
      DATA               PRINTR/6LOUTPUT/
      IER2=IER
      IF (IER2 .GE. WARN) GO TO 5
C                                  NON-DEFINED
      IER1=4
      GO TO 20
    5 IF (IER2 .LT. TERM) GO TO 10
C                                  TERMINAL
      IER1=3
      GO TO 20
   10 IF (IER2 .LT. WARF) GO TO 15
C                                  WARNING(WITH FIX)
      IER1=2
      GO TO 20
C                                  WARNING
   15 IER1=1
C                                  EXTRACT *N*
   20 IER2=IER2-IBIT(IER1)
C                                  PRINT ERROR MESSAGE
      WRITE (PRINTR,25) (ITYP(I,IER1),I=1,2),NAME,IER2,IER
   25 FORMAT(26H *** I M S L(UERTST) ***  ,2A10,4X,A6,4X,I2,
     1   8H (IER = ,I3,1H))
      RETURN
      END
```

```
      SUBROUTINE ZPOLR  (A,NDEG,Z,IER)
C
C-ZFOLR---------S--------LIBRARY 3--------------------------------------
C
C   FUNCTION            - ZEROS OF A POLYNOMIAL WITH REAL
C                           COEFFICIENTS (LAGUERRE).
C   USAGE               - CALL ZPOLR(A,NDEG,Z,IER)
C   PARAMETERS   A      - REAL VECTOR OF LENGTH NDEG+1 CONTAINING THE
C                           COEFFICIENTS IN ORDER OF DECREASING
C                           POWERS OF THE VARIABLE (INPUT).
C                NDEG   - INTEGER DEGREE OF THE POLYNOMIAL (INPUT).
C                Z      - COMPLEX VECTOR OF LENGTH NDEG CONTAINING
C                           THE COMPUTED ROOTS OF THE POLYNOMIAL
C                           (OUTPUT).
C                IER    - ERROR PARAMETER (OUTPUT)
C                         TERMINAL ERROR
C                           IER = 129, INDICATES THAT THE DEGREE OF THE
C                             POLYNOMIAL IS GREATER THAN 100 OR LESS
C                             THAN 1.
C                           IER = 130, INDICATES THAT THE LEADING
C                             COEFFICIENT IS ZERO. THIS RESULTS IN AT
C                             LEAST ONE ROOT, Z(NDEG), BEING SET TO
C                             POSITIVE MACHINE INFINITY.
C                           IER = 131, INDICATES THAT ZPOLR FOUND
C                             FEWER THAN NDEG ZEROS. IF ONLY M ZEROS
C                             ARE FOUND Z(J),J=M+1,...,NDEG ARE SET TO
C                             POSITIVE MACHINE INFINITY.
C   PRECISION           - SINGLE
C   REQD. IMSL ROUTINES - UERTST,ZQADC,ZQADR
C   LANGUAGE            - FORTRAN
C-----------------------------------------------------------------------
C   LATEST REVISION     - FEBRUARY 7, 1975
C
      DIMENSION          A(101),Z(NDEG),DA(101),ACF1(2),DZ(100),
     1                   ACF2(2),ACF(2),ACDIR(2),AC(2),ACL(2)
      DOUBLE PRECISION   DA,DZNR,DZNI,DZOR,DZOI,DXT,DZ,
     1                   DX,DR,DSC,DY,DX2,DV,DT,DT1,DZERO,DTWO
      COMPLEX            Z
      COMPLEX            CF1,CF2,CF,CDIRO,CSPIR,CDIR,C,CL
      LOGICAL            STARTD,SPIRAL
      EQUIVALENCE        (CF1,ACF1(1)),(CF2,ACF2(1)),(CF,ACF(1)),
     1                   (CDIR,ACDIR(1)),(C,AC(1)),(CL,ACL(1))
      DATA               RADIX/2.0/,
     1                   SINF/36404000000000000000B/,
     2                   SDEPS/15614000000000000000B/,
     3                   TWOD3/.66666666666667/,
     4                   RNLGRX/.69314718055995/
      DATA               FINITY/37767777777777777777B/
      DATA               F0/0.0/,GAMA/0.5/,THETA/1.0/,PHI/0.2/,
     1                   ZERO/0.0/,ONE/1.0/,TWO/2.0/,TENM3/1.0E-3/,
     2                   DZERO/0.0D0/,THREE/3.0/,RNINE/9.0/,DTWO/2.0D0/,
     3                   HALF/0.5/,OPTFM/-1.25/
C                                  INITIALIZE CONSTANTS
      SINFSQ = SQRT(SINF)
      SINFT = SINFSQ
      SETASQ = ONE/SINFSQ
      IER = 0
      N = NDEG
      NP1 = N+1
      IF (N.GT.0) GO TO 5
      IER = 129
      GO TO 9000
    5 IF (N.LE.100) GO TO 10
      IER = 129
      GO TO 9000
   10 CONTINUE
C                                  MOVE THE COEFFICIENTS A(I) TO
C                                    DA(I).
      DO 15 I=1,NP1
         DA(I) = A(I)
```

```
   15 CONTINUE
      IF (N.LE.2) GO TO 65
C                                  SCALING (WHEN N .GT. 2)
      ASSIGN 85 TO LSW
      SC = ZERO
      DO 20 I=1,NP1
         SC = AMAX1(SC,ABS(SNGL(DA(I))))
   20 CONTINUE
      IF (SC.EQ.ZERO) GO TO 30
      ISC = ALOG(SC)/RNLGRX
      SC = RADIX**ISC
      DSC = SC
      IF (SC.GE.SINFT) GO TO 30
      SC = SINFT/SC
      DSC = SC
C                                  SCALE BY SC TO HAVE MAX(DA(I),I=1,
C                                    NP1) APPROACH SINFT
      DO 25 I=1,NP1
         DA(I) = DSC*DA(I)
   25 CONTINUE
C                                  FIND NUMBER I OF CONSECUTIVE LEADING
C                                    COEFFICIENTS EQUAL TO ZERO.
   30 DO 35 I=1,NP1
         IF (SNGL(DA(I)).NE.ZERO) GO TO 40
C                                  EACH VANISHED LEADING COEFFICIENT
C                                    YIELDS AN INFINITE ZERO.
         J = NP1-I
         Z(J) = CMPLX(FINITY,ZERO)
   35 CONTINUE
      GO TO 9000
   40 IF (I.EQ.1) GO TO 65
C                                  SLIDE COEFFICIENTS BACK
      IER = 130
      DO 45 K=I,NP1
         J = K-I
         DA(J+1) = DA(K)
   45 CONTINUE
      N = N-I
      GO TO 60
C                                  RE-ENTRIES FOR CURRENT (REDUCED)
C                                    POLYNOMIAL.
   50 N = N1
   55 N = N-1
   60 NP1 = N+1
      ASSIGN 85 TO LSW
   65 CONTINUE
      ITER = 0
      IF (N-2) 70,75,80
   70 Z(1) = CMPLX(SNGL(-DA(2)/DA(1)),ZERO)
      GO TO 9000
   75 CALL ZQADR (SNGL(DA(1)),SNGL(DA(2)),SNGL(DA(3)),Z(2),Z(1),IIER)
      IF (IIER.NE.0) IER = 130
      GO TO 9000
C                                  CHECK FOR ZEROS = (0.,0.)
   80 IF (SNGL(DA(NP1)).NE.ZERO) GO TO LSW, (85,100)
      Z(N) = CMPLX(ZERO,ZERO)
      GO TO 55
C                                  HENCEFORTH N .GT. 2, DA(1) .NE. 0.0
C                                    AND DA(NP1) .NE. 0.0. INITIALIZE
C                                    SOME USEFUL CONSTANTS.
   85 CONTINUE
      XN = N
      XN1 = XN-ONE
      XN2 = XN1-ONE
      X2N = TWO/XN
      X2N1 = X2N/XN1
```

```
      XN2N = XN2/XN
      N1 = N-1
      RTN = SQRT(XN)
C                                 CALCULATE G , AN UPPER BOUND FOR THE
C                                   NEAREST ZERO. INITIALLY G =
C                                   CABS(GEOMETRIC MEAN OF THE ZEFOS).
C
      G = EXP((ALOG(ABS(SNGL(DA(NP1))))-ALOG(ABS(SNGL(DA(1)))))/
     1        XN+TENM3)
C                                 CALCULATE LAGUERRE-STEP CDIR AND
C                                   FEJER-BOUND FOR G.
      R = SNGL(DA(N))/SNGL(DA(NP1))
      CALL ZQADR (X2N1*SNGL(DA(N-1)),X2N*SNGL(DA(N)),SNGL(DA(NP1)),C,
     1           CF1,IKER)
      R = XN2N*R
      IF (IKER .EQ. 65) CDIRO = CMPLX(AC(1)/XN1,ZERO)
      IF (IKER .NE. 65) CDIRO = C/CMPLX(R*AC(1)+XN1,R*AC(2))
      ABDIRO = ABS(REAL(CDIRO))+ABS(AIMAG(CDIRO))
      G = AMIN1(G,1.0001*AMIN1(ABS(AC(1))+ABS(AC(2)),RTN*ABDIRO))
C                                 CALCULATE THE CAUCHY-LOWER BOUND R
C                                   FOR THE SMALLEST ZERO BY SOLVING
C                                   ABS(DA(NP1)) = SUM(ABS(DA(I))
C                                   *R**(NP1-I),I=1,N)
C                                   USING NEWTON*S METHOD.
      R = G
   90 T = ABS(SNGL(DA(1)))
      S = ZERO
      DO 95 I=2,N
         S = R*S+T
         T = R*T+ABS(SNGL(DA(I)))
   95 CONTINUE
      S = R*S+T
      T = (R*T-ABS(SNGL(DA(NP1))))/S
      S = R
      R = SNGL(DBLE(R)-DBLE(T))
      IF (R.LT.S) GO TO 90
C                                 R/(2**(1/N) - 1 ) .LT. 1.445*N*R IS
C                                   ANOTHER UPPER BOUND.
      GO = AMIN1(1.445*XN*R,G)
      RO = 0.99999*S
      ASSIGN 100 TO LSW
C                                 INITIALIZE THE ITERATION TO BEGIN AT
C                                   THE ORIGIN.
  100 CONTINUE
      FEJER = GO
      G = GO
      CDIR = CDIRO
      DZNR = DZERO
      ABDIR = ABDIRO
      DZNI = DZERO
      FN = ABS(SNGL(DA(NP1)))
      SPIRAL = .FALSE.
      STARTD = .FALSE.
C                                 RE-ENTRY POINT TO ACCEPT, MODIFY,
C                                   OR REJECT THE LAGUERRE STEP.
  105 CONTINUE
C                                 ACCEPT CDIR IF CABS(CDIR) .LE.
C                                   GAMA*G.
      IF (ABDIR .LE. G*GAMA) GO TO 110
C                                 REJECT CDIR IF CABS(CDIR) .GT.
C                                   THETA*G.
      IF (ABDIR .GT. G*THETA) GO TO 215
C                                 MODIFY CDIR SO THAT CABS(CDIR) =
C                                   GAMA*G.
      IF (.NOT.(STARTD.OR.SPIRAL).AND.RO.GT.GAMA*G) GO TO 110
      V = GAMA*(G/ABDIR)
      CDIR = CMPLX(V*ACDIR(1),V*ACDIR(2))
      ABDIR = ABDIR*V
```

```
C                                        ACCEPT PREVIOUS ITERATE. SAVE DATA   ZPLR2050
C                                          ASSOCIATED WITH CURRENT ITERATE.   ZPLR2060
  110 CONTINUE                                                                ZPLR2070
      G = FEJER                                                               ZPLR2080
      CL = CDIR                                                               ZPLR2090
      ABSCL = ABDIR                                                           ZPLR2100
      F0 = FN                                                                 ZPLR2110
      DZ0R = DZNR                                                             ZPLR2120
      DZ0I = DZNI                                                             ZPLR2130
C                                        CDIR AT THE ORIGIN IS IN THE         ZPLR2140
C                                          DIRECTION OF DECREASING FUNCTION   ZPLR2150
C                                          VALUE                              ZPLR2160
      STARTD = .TRUE.                                                         ZPLR2170
C                                        NEXT ITERATE IS ZN=CMPLX(DZNR,DZNI). ZPLR2180
  115 DZNR = DZ0R+ACL(1)                                                      ZPLR2190
      DZNI = DZ0I+ACL(2)                                                      ZPLR2200
C                                        IS ZN CLOSE TO THE REAL AXIS         ZPLR2210
C                                          RELATIVE TO STEP SIZE.             ZPLR2220
  120 CONTINUE                                                                ZPLR2230
      ITER = ITER+1                                                           ZPLR2240
      IF (ITER.GT.200*NDEG) GO TO 220                                         ZPLR2250
      IF (ABS(SNGL(DZNI)).LE.PHI*ABSCL) GO TO 175                             ZPLR2260
C                                        ZN IS COMPLEX .                      ZPLR2270
C                                        FACTORIZATION OF THE POLYNOMIAL BY   ZPLR2280
C                                          QUADRATIC FACTOR (Z**2-X2*Z+R)     ZPLR2290
C                                          SUM(DA(I)*Z**(N-I)) =              ZPLR2300
C                                          (Z**2-X2*Z+R)*SUM(Z(I)*Z**(N-I-2)) ZPLR2310
C                                          + Z(N-1)*(Z-X) + Z(N) FOR ALL Z ,  ZPLR2320
C                                          THE VALUE OF THE POLYNOMIAL AT     ZPLR2330
C                                          (X,Y) IS CF , FIRST DERIVATIVE OF  ZPLR2340
C                                          THE POLYNOMIAL AT (X,Y) IS CF1,    ZPLR2350
C                                          AND THE SECOND DERIVATIVE OF THE   ZPLR2360
C                                          POLYNOMIAL AT (X,Y) IS 2.*CF2,     ZPLR2370
C                                          WHERE (X,Y) IS A ZERO OF           ZPLR2380
C                                          Z**2-X2*Z+R.                       ZPLR2390
C                                          E IS ERROR BOUND FOR THE VALUE OF  ZPLR2400
C                                          THE POLYNOMIAL AND DZ(I) ARE THE   ZPLR2410
C                                          COEFFICIENTS OF THE QUOTIENT       ZPLR2420
C                                          POLYNOMIAL.                        ZPLR2430
C                                        INITIALIZATIONS FOR EVALUATION LOOPS ZPLR2440
      S = ZERO                                                                ZPLR2450
      S1 = ZERO                                                               ZPLR2460
      DT1 = DZERO                                                             ZPLR2470
      T1 = ZERO                                                               ZPLR2480
      DT = DA(1)                                                              ZPLR2490
C                                        INDEX J IS USED TO CHANGE DX ON THE  ZPLR2500
C                                          LAST ITERATION.                    ZPLR2510
      J = 3                                                                   ZPLR2520
C                                        SET Z(X,Y) TO ZN(ZNR,ZNI).           ZPLR2530
      DX = DZNR                                                               ZPLR2540
      DY = DZNI                                                               ZPLR2550
      SC = CABS(CMPLX(SNGL(DX),SNGL(DY)))                                     ZPLR2560
      DSC = SC                                                                ZPLR2570
      IF (SC.GE.SINFSQ.OR.SC.LE.SETASQ) GO TO 140                             ZPLR2580
      DX2 = DX+DX                                                             ZPLR2590
      X2 = DX2                                                                ZPLR2600
      DR = DX*DX+DY*DY                                                        ZPLR2610
      R = DR                                                                  ZPLR2620
      DZ(1) = DA(2)+DX2*DA(1)                                                 ZPLR2630
      DZ(2) = DA(3)+(DX2*DZ(1)-DR*DA(1))                                      ZPLR2640
      IF (J.LT.N) GO TO 130                                                   ZPLR2650
  125 DX2 = DX                                                                ZPLR2660
      X2 = DX2                                                                ZPLR2670
      J = N                                                                   ZPLR2680
```

```
  130 NLM = MAX0(N1,J)
      DO 135 I=J,NLM
         V = S1*R
         S1 = S
         S = T1+(X2*S-V)
         DV = DT1*DR
         DT1 = DT
         T1 = DT1
         DT = (DX2*DT-DV)+DZ(I-2)
         DZ(I) = DA(I+1)+(DX2*DZ(I-1)-DR*DZ(I-2))
  135 CONTINUE
      IF (J.LT.N) GO TO 125
      GO TO 160
  140 DX = DX/DSC
      DY = DY/DSC
      DR = (DX*DX+DY*DY)*DSC
      R = DR
      DX2 = DX+DX
      X2 = DX2
      DZ(1) = DA(2)+(DX2*DA(1))*DSC
      DZ(2) = DA(3)+(DX2*DZ(1)-DR*DA(1))*DSC
      IF (J.LT.N) GO TO 150
  145 DX2 = DX
      X2 = DX2
      J = N
  150 NLM = MAX0(N1,J)
      DO 155 I=J,NLM
         V = S1*R
         S1 = S
         S = T1+(X2*S-V)*DSC
         DV = DT1*DR
         DT1 = DT
         T1 = DT1
         DV = DX2*DT-DV
         DT = DZ(I-2)+DV*DSC
         DZ(I) = DA(I+1)+(DX2*DZ(I-1)-DR*DZ(I-2))*DSC
  155 CONTINUE
      IF (J.LT.N) GO TO 145
  160 CF = CMPLX(SNGL(DZ(NLM)),SNGL(DZNI*DZ(NLM-1)))
      FN = CABS(CF)
      E = ABS(SNGL(DA(1)))
      DO 165 I=2,N1
         E = ABS(SNGL(DZ(I-1)))+SC*E
  165 CONTINUE
      E = SDEPS*((RNINE*E*SC+THREE*ABS(SNGL(DZ(N-1))))*SC+
     1             ABS(SNGL(DZ(N))))
C                                   HAS AN ACCEPTABLE ZERO BEEN FOUND
      IF (FN.LE.E) GO TO 195
      IF (FN.GE.F0.AND.STARTD) GO TO 215
      DV = DTWO*DZNI
      V = DV
      T = DT
      CF1 = CMPLX(SNGL(DZ(NLM-1)-(DV*(DT1*DZNI))),SNGL(DV*DT))
      CF2 = CMPLX(T-V*(V*S),SNGL(DZNI)*(3.*T1-V*(V*S1)))
C                                   FIND THE LAGUERRE STEP AT ZN.
C                                   X = AMAX1(ABS(ACF(1)),ABS(ACF(2)))
C                                   C = CMPLX(ACF1(1)/X,ACF1(2)/X) /
C                                     CMPLX(ACF(1)/X,ACF(2)/X)
      IF (CABS(CF).GE.ONE) GO TO 170
C                                   IF CABS(CF1/CF) .GT. SINF, THERE IS
C                                     A ZERO WITHIN N*(1.0/SINF) OF ZN.
      IF (CABS(CF1).GT.CABS(CF)*SINF) GO TO 195
  170 C = CF1/CF
C                                   COMPUTE THE LAGUERRE STEP CDIR AND
C                                     THE BOUND FEJER AT ZN.
      CALL ZQADC (CMPLX(X2N1*ACF2(1),X2N1*ACF2(2)),CMPLX(X2N*ACF1(1),
```

```
     1              X2N*ACF1(2)),CF,CDIR,CF1,IJER)
      FEJER = ABS(ACDIR(1))+ABS(ACDIR(2))
      C = CMPLX(XN2N*AC(1),XN2N*AC(2))
      C = C*CDIR
      C = CMPLX(AC(1)+XN1,AC(2))
      CDIR = CDIR/C
      ABDIR = ABS(ACDIR(1))+ABS(ACDIR(2))
      FEJER = AMIN1(RTN*ABDIR,FEJER)
      DX = DABS(DZNR)+DABS(DZNI)
      DXT = DX+ABDIR
C                                     IS THE STEP SIZE NEGLIGIBLE
      IF (SNGL(DXT-DX).EQ.ZERO) GO TO 195
      GO TO 105
  175 CONTINUE
C                                     ZN IS REAL
C                                     FACTORIZATION OF POLYNOMIAL BY
C                                       LINEAR FACTOR (Z-X) AS FOLLOWS
C                                       SUM(DU(I)*Z**(N-I)) =
C                                       (Z-X)*SUM(Z(I)*Z**(N-I-1)) + Z(N)
C                                       FOR ALL Z ,
C                                       SO Z(N) IS VALUE OF POLYNOMIAL AT
C                                       Z=X , FIRST DERIVATIVE OF
C                                       POLYNOMIAL AT Z=X IS V , AND
C                                       SECOND DERIVATIVE OF POLYNOMIAL AT
C                                       Z=X IS 2*W . E IS ERROR BOUND FOR
C                                       THE VALUE OF POLYNOMIAL AND DZ(I)
C                                       ARE THE COEFFICIENTS OF QUOTIENT
C                                       POLYNOMIAL.
      DX = DZNR
      X = DX
      DZNI = DZERO
      ABX = ABS(X)
      DV = DA(1)
      V = DV
      W = ZERO
      DZ(1) = DA(2)+DX*DA(1)
      DO 180 I=2,N
         W = V+X*W
         DV = DZ(I-1)+DX*DV
         V = DV
         DZ(I) = DA(I+1)+DX*DZ(I-1)
  180 CONTINUE
      F = SNGL(DZ(N))
      FN = ABS(F)
      E = ABS(SNGL(DA(1)))*TWOD3
      DO 185 I=1,N1
         E = ABS(SNGL(DZ(I)))+ABX*E
  185 CONTINUE
      E = SDEPS*(THREE*ABX*E+ABS(SNGL(DZ(N))))
C                                     HAS AN ACCEPTABLE ZERO BEEN FOUND.
      IF (FN.LE.E) GO TO 205
C                                     HAS THE FUNCTION VALUE DECREASED.
      IF (FN.GE.F0.AND.STARTD) GO TO 215
C                                     FIND THE LAGUERRE STEP AT DZNR.
      IF (ABS(F).GE.ONE) GO TO 190
C                                     IF ABS(V/F) .GT. SINF, THERE IS
C                                       A ZERO WITHIN N*(1.0/SINF) OF ZN.
      IF (ABS(V).GT.ABS(F)*SINF) GO TO 205
  190 R = V/F
      CALL ZQADR (X2N1*W,X2N*V,F,C,CF1,IKER)
C                                     CALCULATE THE FEJER BOUND FOR
C                                       SMALLEST ZERO.
      FEJER = ABS(AC(1))+ABS(AC(2))
      R = XN2N*R
      CDIR = C/CMPLX(R*AC(1)+XN1,R*AC(2))
      ABDIR = ABS(ACDIR(1))+ABS(ACDIR(2))
```

```
      FEJER = AMIN1(RTN*ABDIR,FEJER)
      DX = DABS(DZNR)
C                                      IS THE STEP SIZE NEGLIGIBLE.
      DXT = DX+ABDIR
      IF (SNGL(DXT-DX).EQ.ZERO) GO TO 205
      GO TO 105
C                                      CZN IS A COMPLEX ZERO.
C                                      STORE COEFFICIENTS OF QUOTIENT
C                                        POLYNOMIAL IN DA ARRAY.
C                                      DA(1) IS UNCHANGED FOR THE DEFLATED
C                                        POLYNOMIAL.
  195 DO 200 I=3,N
         DA(I-1) = DZ(I-2)
  200 CONTINUE
      Z(N) = CMPLX(SNGL(DZNR),SNGL(DZNI))
      Z(N-1) = CONJG(Z(N))
      GO TO 50
C                                      ZN IS A REAL ZERO.
C                                      STORE COEFFICIENTS OF QUOTIENT
C                                        POLYNOMIAL IN DA ARRAY.
C                                      DA(1) IS UNCHANGED FOR THE DEFLATED
C                                        POLYNOMIAL.
  205 DO 210 I=2,N
         DA(I) = DZ(I-1)
  210 CONTINUE
      Z(N) = CMPLX(SNGL(DZNR),ZERO)
      GO TO 55
C                                      CURRENT LAGUERRE STEP IS
C                                        UNACCEPTABLE.
  215 CONTINUE
      IF (.NOT.STARTD) GO TO 245
C                                      REDUCE PREVIOUS LAGUERRE STEP BY
C                                        HALF.
      ABSCL = HALF*ABSCL
      CL = CMPLX(HALF*ACL(1),HALF*ACL(2))
      DX = DABS(DZNR)+DABS(DZNI)
      DXT = DX+ABSCL
      IF (SNGL(DXT-DX).NE.ZERO) GO TO 115
      IF (FN.LT.E*XN*XN) GO TO 240
  220 CONTINUE
      IF (N.EQ.NDEG) GO TO 230
      DO 225 I=NP1,NDEG
         Z(I-N) = Z(I)
  225 CONTINUE
  230 NTOM = NDEG-N+1
      DO 235 I=NTOM,NDEG
         Z(I) = CMPLX(FINITY,ZERO)
  235 CONTINUE
      IER = 131
      GO TO 9000
  240 IF (DZNI) 195,205,195
  245 CONTINUE
C                                      IF .NOT. STARTD, HAS CZN BEEN ON THE
C                                        INNER CAUCHY RADIUS.
      IF (SPIRAL) GO TO 250
C                                      SET SPIRAL TO .TRUE.. PUT ZN ON THE
C                                        INNER CIRCLE OF THE ANNULUS
C                                        CONTAINING THE SMALLEST ZERO IN
C                                        THE DIRECTION OF THE LAGUERRE STEP.
      SPIRAL = .TRUE.
      CSPIR = CMPLX(OPTFM/XN,ONE)
      ABSCL = RO/(XN*XN)
      C = CMPLX((ACDIR(1)/ABDIR)*RO,(ACDIR(2)/ABDIR)*RO)
      GO TO 255
C                                      SET ZN TO ANOTHER POINT ON THE
C                                        SPIRAL.
  250 C = CSPIR*CMPLX(SNGL(DZNR),SNGL(DZNI))
  255 DZNR = AC(1)
      DZNI = AC(2)
      GO TO 120
 9000 CONTINUE
      IF (IER.GT.0) CALL UERTST (IER,6HZPOLR )
 9005 RETURN
      END
```

```
      SUBROUTINE ZQADC  (A,B,C,ZSM,ZLG,IER)
C
C-ZQADC---------S-------LIBRARY 3--------------------------------------
C
C   FUNCTION            - FIND THE ROOTS OF THE QUADRATIC EQUATION
C                           A*Z**2+B*Z+C = 0.0, WHERE THE
C                           COEFFICIENTS A, B, AND C ARE COMPLEX
C                           NUMBERS.
C   USAGE               - CALL ZQADC(A,B,C,ZSM,ZLG,IER)
C   PARAMETERS   A      - COEFFICIENT OF THE QUADRATIC EQUATION (INPUT).
C                B      - COEFFICIENT OF THE QUADRATIC EQUATION (INPUT).
C                C      - COEFFICIENT OF THE QUADRATIC EQUATION (INPUT).
C                           (NOTE - A, B, AND C MUST BE DECLARED TYPE
C                             COMPLEX.)
C                ZSM    - ROOT OF THE QUADRATIC EQUATION (OUTPUT).
C                ZLG    - ROOT OF THE QUADRATIC EQUATION (OUTPUT).
C                           (NOTE - ZSM AND ZLG MUST BE DECLARED TYPE
C                             COMPLEX.)
C                           FOR THE ROOTS ZSM AND ZLG THE FOLLOWING
C                             CONDITION HOLDS - CABS(ZSM) .LE. CABS(ZLG)
C                IER    - ERROR PARAMETER
C                         WARNING (WITH FIX)
C                           IER = 65, IMPLIES A=B=0.0
C                             IN THIS CASE, THE LARGE ROOT,
C                             ZLG = SIGN(FINITY,-B), AND
C                             THE SMALL ROOT, ZSM = -ZLG, WHERE
C                             FINITY = LARGEST NUMBER WHICH CAN BE
C                             REPRESENTED IN THE MACHINE.
C                           IER = 66, IMPLIES A=0.0
C                             IN THIS CASE, THE LARGE ROOT,
C                             ZLG = SIGN(FINITY,-B), WHERE
C                             FINITY = LARGEST NUMBER WHICH CAN BE
C                             REPRESENTED IN THE MACHINE.
C   PRECISION           - SINGLE
C   REQD. IMSL ROUTINES - UERTST
C   LANGUAGE            - FORTRAN
C-----------------------------------------------------------------------
C   LATEST REVISION     - FEBRUARY 7, 1975
C
      DIMENSION          AA(2),BB(2),CC(2),ZSA(2),ZLA(2)
      DOUBLE PRECISION   DR,DI,D,D1
      COMPLEX            A,B,C,ZSM,ZLG,A0,B0,C0,ZS,ZL
      EQUIVALENCE        (AA(1),A0),(BB(1),B0),(CC(1),C0),(ZSA(1),ZS),
     1                   (ZLA(1),ZL),(AA(1),AR),(BB(1),BR),(CC(1),CR),
     2                   (ZSA(1),ZSR),(ZLA(1),ZLR),(AA(2),AI),
     3                   (BB(2),BI),(CC(2),CI),(ZSA(2),ZSI),(ZLA(2),ZLI)
      DATA               FINITY/377677777777777777777B/
      DATA               RADIX/2.0/,
     1                   RNLGRX/.69314718055995/
      DATA               ZERO/0.0/,HALF/0.5/
      IER = 0
C                                  PUT THE COEFFICIENTS IN TEMPORARY TO
C                                    SAVE EXECUTION TIME.
      A0 = A
      B0 = -B
      C0 = C
C                                  CHECK FOR A=ZERO OR C=ZERO.
      IF(AR .NE. ZERO .OR. AI .NE. ZERO) GO TO 5
      IER = 65
      ZL = CMPLX(FINITY,ZERO)
```

```
      ZS = ZL
      IF(BR .EQ. ZERO .AND. BI .EQ. ZERO) GO TO 35
      IER = 66
      ZS = C0/B0
      GO TO 35
    5 IF(CR .NE. ZERO .OR. CI .NE. ZERO) GO TO 10
      ZS = CMPLX(ZERO,ZERO)
      GO TO 30
C                                  SCALING TO AVOID OVERFLOW OR
C                                    UNDERFLOW. SCALE THE COEFFICIENTS
C                                    SO THAT A*C IS APPROXIMATELY ONE.
C                                    THE SCALE FACTOR CSQRT(A*C) FITS
C                                    THIS REQUIREMENT BUT MAY CAUSE
C                                    OVERFLOW OR UNDERFLOW IN THE
C                                    SCALING PROCEDURE.
C                                    LET AMAX1(ABS(AR),ABS(AI)) BE
C                                    REPRESENTED BY RADIX**IA AND LET
C                                    AMAX1(ABS(CR),ABS(CI)) BE
C                                    REPRESENTED BY RADIX**IC.
C                                    THE SCALE FACTOR, SCALE, IS DEFINED
C                                    BY THE FOLLOWING FORMULAS
C                                    SCALE=RADIX**IS, WHERE
C                                    IS=ENTIER((IA+IC+1)/2) AND
C                                    ENTIER IS THE MATHEMATICAL GREATEST
C                                    INTEGER FUNCTION.
   10 IS = (ALOG(AMAX1(ABS(AR),ABS(AI)))+ALOG(AMAX1(ABS(CR),ABS(CI)))+
     1       RNLGRX)/(RNLGRX+RNLGRX)
      SCALE = RADIX**IS
C                                  IF THE SCALE FACTOR .LE.
C                                    DEPS*MAX(ABS(BR),ABS(BI))
C                                    DO NOT SCALE THE COEFFICIENTS.
      TEMP = AMAX1(ABS(BR),ABS(BI))
      D1 = DBLE(TEMP)
      D = D1+SCALE
      D = D-D1
      IF (SNGL(D) .EQ. ZERO) GO TO 25
C                                  IF MAX(ABS(BR),ABS(BI)) .GE.
C                                    DEPS*SCALE FACTOR THEN SCALE
C                                    B0. OTHERWISE SET B0 = ZERO.
      D = D1+SCALE
      D = D-SCALE
      IF (SNGL(D) .NE. ZERO) GO TO 15
      BR = ZERO
      BI = ZERO
      GO TO 20
   15 BR = (BR/SCALE)*HALF
      BI = (BI/SCALE)*HALF
   20 AR = AR/SCALE
      AI = AI/SCALE
      CR = CR/SCALE
      CI = CI/SCALE
C                                  SOLVE A0*Z**2-2.0*B0*Z+C0=ZERO
      DR = DBLE(BR)**2
      DI = DBLE(BI)*(2.0D0*DBLE(BR))
      ZS = CMPLX(SNGL(((DR-DBLE(BI)**2)-DBLE(AR)*DBLE(CR))+DBLE(AI)*
     1       DBLE(CI)),SNGL((DI-DBLE(AI)*DBLE(CR))-DBLE(AR)*DBLE(CI)))
      ZS = CSQRT(ZS)
C                                  CHOOSE THE SIGN OF ZS SUCH THAT
C                                  CABS(B)=AMAX1(CABS(B+ZS),CABS(B-ZS)).
      IF(DBLE(ZSR)*DBLE(BR)+DBLE(ZSI)*DBLE(BI) .LE. ZERO) ZS = -ZS
      B0 = B0+ZS
C                                  PERFORM THE FINAL COMPLEX OPERATION
C                                    FOR THE ZEROS.
   25 ZS = C0/B0
   30 ZL = B0/A0
   35 ZSM = ZS
      ZLG = ZL
 9000 CONTINUE
      IF(IER .NE. 0) CALL UERTST(IER,6HZQADC )
 9005 RETURN
      END
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      SUBROUTINE ZQADR  (A,B,C,ZSM,ZLG,IER)                             ZQDR0010
C                                                                       ZQDR0020
C-ZQADR---------S-------LIBRARY 3---------------------------------------ZQDR0030
C                                                                       ZQDR0040
C   FUNCTION            - FIND THE ROOTS OF THE QUADRATIC EQUATION      ZQDR0050
C                           A*Z**2+B*Z+C = 0.0, WHERE THE               ZQDR0060
C                           COEFFICIENTS A, B, AND C ARE REAL           ZQDR0070
C                           NUMBERS.                                    ZQDR0080
C   USAGE               - CALL ZQADR(A,B,C,ZSM,ZLG,IER)                 ZQDR0090
C   PARAMETERS   A      - COEFFICIENT OF THE QUADRATIC EQUATION (INPUT).ZQDR0100
C                B      - COEFFICIENT OF THE QUADRATIC EQUATION (INPUT).ZQDR0110
C                C      - COEFFICIENT OF THE QUADRATIC EQUATION (INPUT).ZQDR0120
C                ZSM    - ROOT OF THE QUADRATIC EQUATION (OUTPUT).      ZQDR0130
C                ZLG    - ROOT OF THE QUADRATIC EQUATION (OUTPUT).      ZQDR0140
C                           (NOTE - ZSM AND ZLG MUST BE DECLARED TYPE   ZQDR0150
C                             COMPLEX).                                 ZQDR0160
C                           IF ZSM AND ZLG ARE REAL, THEN               ZQDR0170
C                             ABS(ZSM) .LE. ABS(ZLG).                   ZQDR0180
C                           IF ZSM AND ZLG ARE COMPLEX, THEN            ZQDR0190
C                             ZSM = CONJG(ZLG) AND                      ZQDR0200
C                             AIMAG(ZLG) .GT. 0.0.                      ZQDR0210
C                IER    - ERROR PARAMETER                               ZQDR0220
C                         WARNING (WITH FIX)                            ZQDR0230
C                           IER = 65, IMPLIES A=B=0.0                   ZQDR0240
C                             IN THIS CASE, THE LARGE ROOT,             ZQDR0250
C                             ZLG = SIGN(FINITY,-B), AND                ZQDR0260
C                             THE SMALL ROOT, ZSM = - ZLG , WHERE       ZQDR0270
C                             FINITY = LARGEST NUMBER WHICH CAN BE      ZQDR0280
C                             REPRESENTED IN THE MACHINE.               ZQDR0290
C                           IER = 66, IMPLIES A=0.0                     ZQDR0300
C                             IN THIS CASE, THE LARGE ROOT,             ZQDR0310
C                             ZLG = SIGN(FINITY,-B), WHERE              ZQDR0320
C                             FINITY = LARGEST NUMBER WHICH CAN BE      ZQDR0330
C                             REPRESENTED IN THE MACHINE.               ZQDR0340
C   PRECISION           - SINGLE                                        ZQDR0350
C   REQD. IMSL ROUTINES - UERTST                                        ZQDR0360
C   LANGUAGE            - FORTRAN                                       ZQDR0370
C-----------------------------------------------------------------------ZQDR0380
C   LATEST REVISION     - FEBRUARY, 7, 1975                             ZQDR0390
C                                                                       ZQDR0400
      DOUBLE PRECISION   D,D1                                           ZQDR0410
      COMPLEX            ZSM,ZLG,ZS,ZL                                  ZQDR0420
      DATA               FINITY/37767777777777777777B/                  ZQDR0430
      DATA               RADIX/2.0/,                                    ZQDR0440
     1                   RNLGRX/.69314718055995/                        ZQDR0450
      DATA               ZERO/0.0/,HALF/0.5/                            ZQDR0460
      IER = 0                                                           ZQDR0470
C                                 PUT THE COEFFICIENTS IN TEMPORARY TO  ZQDR0480
C                                   SAVE EXECUTION TIME.                ZQDR0490
      A0 = A                                                            ZQDR0500
      B1 = -B                                                           ZQDR0510
      C0 = C                                                            ZQDR0520
C                                 CHECK FOR A=ZERO OR C=ZERO.           ZQDR0530
      IF(A0 .NE. ZERO) GO TO 5                                          ZQDR0540
      IER = 65                                                          ZQDR0550
      ZL = CMPLX(SIGN(FINITY,B1),ZERO)                                  ZQDR0560
      ZS = -ZL                                                          ZQDR0570
      IF(B1 .EQ. ZERO) GO TO 30                                         ZQDR0580
      IER = 66                                                          ZQDR0590
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      ZS = CMPLX(C0/B1,ZERO)
      GO TO 30
    5 IF(C0 .NE. ZERO) GO TO 10
      ZS = CMPLX(ZERO,ZERO)
      GO TO 25
C                                  SCALING TO AVOID OVERFLOW OR
C                                    UNDERFLOW. SCALE THE COEFFICIENTS
C                                    SO THAT A*C IS APPROXIMATELY ONE.
C                                    THE SCALE FACTOR SQRT(A*C) FITS
C                                    THIS REQUIREMENT BUT MAY CAUSE
C                                    OVERFLOW OR UNDERFLOW IN THE
C                                    SCALING PROCEDURE.
C                                    LET A=RADIX**IA AND C=RADIX**IC.
C                                    THE SCALE FACTOR, SCALE, IS DEFINED
C                                    BY THE FOLLOWING FORMULAS
C                                    SCALE=RADIX**IS, WHERE
C                                    IS=ENTIER((IA+IC+1)/2) AND
C                                    ENTIER IS THE MATHEMATICAL GREATEST
C                                    INTEGER FUNCTION.
   10 IS = (ALOG(ABS(A0))+ALOG(ABS(C0))+RNLGRX)/(RNLGRX+RNLGRX)
      SCALE = RADIX**IS
C                                  IF THE SCALE FACTOR .LE.
C                                    DEPS*ABS(B1) DO NOT SCALE
C                                    THE COEFFICIENTS.
      D1 = DBLE(ABS(B1))
      D = D1+SCALE
      D = D-D1
      IF (SNGL(D) .EQ. ZERO) GO TO 20
C                                  IF ABS(B1) .GE. DEPS*SCALE FACTOR
C                                    THEN SCALE B0. OTHERWISE SET
C                                    B0 = ZERO.
      B0 = ZERO
      D = D1+SCALE
      D = D-SCALE
      IF (SNGL(D) .NE. ZERO) B0 = (B1/SCALE)*HALF
      A0 = A0/SCALE
      C0 = C0/SCALE
C                                  SOLVE A0*Z**2-2.0*B0*Z+C0=ZERO
      DD = DBLE(B0)**2-DBLE(A0)*DBLE(C0)
      S = SQRT(ABS(DD))
      IF(DD .GT. ZEPO) GO TO 15
C                                  COINCIDENT OR COMPLEX ROOTS
C                                    (D .LE. ZERO).
      ZL = CMPLX(B0/A0,ABS(S/A0))
      ZS = CONJG(ZL)
      GO TO 30
C                                  DISTINCT REAL ROOTS (D .GT. ZERO).
   15 B1 = SIGN(S,B0)+B0
   20 ZS = CMPLX(C0/B1,ZERO)
   25 ZL = CMPLX(B1/A0,ZERO)
      IF(ABS(REAL(ZL)) .LT. ABS(REAL(ZS))) ZS = -ZL
   30 ZSM = ZS
      ZLG = ZL
 9000 CONTINUE
      IF(IER .NE. 0) CALL UERTST(IER,6HZQADR )
 9005 RETURN
      END
```

VITA

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Master of Science

Thesis: ANALYSIS AND SYNTHESIS OF A GEARED SPHERICAL CYCLOIDAL CRANK MECHANISM

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