COMPUTER AIDED AERODYNAMIC DESIGN OF LOW TIP SPEED RATIO WIND TURBINES

By

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NOMENCLATURE

Theory	FORTRAN	Description
A		area of actuator disk
a	A	axial interference factor
a'	AP	angular interference factor
	AMAX	maximum a
	AMIN	minimum a
	AMOD	axial interference model
	AAT(I)	tabulated angle of attack
B	B	number of blades
c	C	chord at element
C_D	C_D	drag coefficient
C_L	C_L	lift coefficient
C_p	CP	power coefficient
C_{pL}	CPL	blade element local C_p
C_x	CXX	coefficient of force in x-direction
C_y	CYY	coefficient of force in y-direction
	CI(I)	tabulated local chord
	CDT(I)	tabulated C_D
	CLT(I)	tabulated C_L
D	TY	drag on actuator disk
dr	DR	blade element width
f	FWI	far wake interference factor

Theory	FORTRAN	Description
F_x	FXF	force in x-direction
F_y	FYF	force in y-direction
	H	altitude of turbine
	HB	hub radius
	HH	height of hub above ground
L/D		lift over drag ratio
$\dot{m}$		mass flow rate
	NF	number of tabulated inputs for blade geometry
	NFS	number of tabulated inputs for airfoil characteristics
	NK	number of tip speed ratios
P	PY	power
p_∞		freestream static pressure
p^+		static pressure in front of turbine
p^-		static pressure behind turbine
Q	QY	torque on actuator disk
R	R	radius of turbine
r_L	RL	local blade element radius
r_2		far wake radius
	RR(I)	tabulated blade radius
	SI	coning angle
	THETI(I)	tabulated blade twist
U		velocity at actuator disk
U_θ		tangential velocity behind blades
V_∞	V	freestream velocity
V_2		velocity of far wake

Theory	FORTRAN	Description
$V_{2\theta}$		tangential velocity of far wake
W	W	velocity relative to the blades
X	X	tip speed ratio
x	XL	local tip speed ratio
	XIC	tip speed ratio increment
α	ALPHA	angle of attack
γ	WIR	axial interference factor parameter
η	RF	angular interference factor parameter
θ	THET	twist of the blades
ρ	RHO	air density
σ_L	SIG	local solidity
ϕ	PHI	angle of the relative velocity
Ω	OMEGA	angular velocity of blades
ω		angular velocity of wake behind blades
ω_2		angular velocity of far wake

CHAPTER I

INTRODUCTION

In recent years an upsurge of interest in wind energy systems has occurred due to the depletion of conventional energy supplies. For the most part this interest has centered on the development of very large turbines (30 m in diameter and greater) which produce power in the 100 kw-1 Mw range. However, there has been growing interest in the smaller turbines suitable for individual homes or farm work. Extensive research in this area of small turbines has been conducted at Oklahoma State University since the early 1970s. Recent effort has centered on the development of a practical wind turbine and electrical generator coupling. This wind turbine application is an offshoot of the Field Modulated Generator (FMG) research at Oklahoma State University. The unique feature of this generator is that it produces constant frequency a-c power although the input is of variable RPM. Hence, this generator is well suited for operation with simple wind turbines which do not have control of the RPM. Details of the FMG design and operation can be found in publications by Allison, Ramakumar, and Hughes (1) (2).

The wind turbines used in the Oklahoma State University research effort are of the "spoked wheel" type designed and built by the American Wind Turbine Company (Figure 1). As the name suggests, these turbines are constructed in a way very similar to a bicycle wheel. Stainless steel wires serve as the spokes which support the aluminum airfoils on one side of the 15' diameter wheel while providing structural support on the other side. The turbine has proven to be fairly simple and inexpensive to build while studies are now being conducted at Oklahoma State University to determine (and improve upon) its operational reliability. If its dependability is proven, the spoked wheel wind turbine has the potential of being a substantial improvement over the old American farm windmill. Furthermore, the spoked wheel wind turbine may prove to be an efficient producer of small amounts of electrical power for domestic uses.

The spoked wheel wind turbine was originally designed by Tom Chalk of the American Wind Turbine Company. Several models with variations in the number of blades and in the blade pitch were tested by Chalk. The resulting design was marketed by the American Wind Turbine Company and several were used by Oklahoma State University in the wind energy field facility. This "Chalk" turbine seemed to operate with surprisingly good efficiency. Because of this, interest in a complete aerodynamic analysis of the turbine developed.

Further inducement for an aerodynamic analysis of the Chalk wind turbine came from a preliminary study of the major literature on wind turbines. Work by Glauert (3) and Hutter (4) (5) indicated that a low efficiency should be expected from a wind turbine with low tip speed ratio (the ratio of the tip velocity of the turbine to the wind velocity). Consequently most, if not all, significant research has concentrated on high tip speed ratio wind turbines. Therefore, if the Chalk turbine, which inherently operates at low tip speed ratio, was indeed achieving high efficiency, it was desirable to find the reason why.

The first significant work on wind turbine theory was performed by two specialists in propeller theory. Betz (6) (7) and Glauert (3) developed a basic analysis of wind turbines along lines very similar to propeller theory in the late 1920s. Glauert established design parameters for an ideal turbine operating at optimum conditions. Although his technique for an ideal wind turbine were of limited use for designing an actual turbine, the basic analysis he developed is the foundation upon which the following work has been built.

In the early 1960s Hutter (4) presented a procedure for designing the airfoil twist on high tip speed ratio wind turbines. Although the technique is somewhat complicated and time consuming, it represents the first practical method of designing wind turbines for optimal performance. Recently Wilson and Lissaman (8) (12) have developed a computer program which solves the complex system of equations associated with the wind turbine model. The program incorporates an extension of Glauert's basic mathematical model and uses an iterative solution technique well suited for the computer. Also included are corrections for tip losses, hub losses, and blade coning. Using the basic physical parameters (number and width of blades, twist, airfoil characteristics, RPM, and windspeed) the program determines the operational characteristics of both the turbine and the air flowing through the turbine. Wilson's program has formed the basis for the computer programs to be presented in this study.

When initially studying the Chalk turbine the validity of the theory contained in Wilson's computer program was questioned at the low tip speed ratios being examined. In Glauert's basic theory for axial flow wind turbines it is assumed that the rotation of the wake behind the turbine does not significantly influence the final translational velocity behind the turbine. This problem has not concerned most researchers in wind energy since they have been trying to develop high speed turbines. However, when working with low speed wind turbines, this assumption must be examined in greater detail.

At the outset of performing an aerodynamic analysis of "spoked wheel" type wind turbines there were three major objectives.

1. Obtain experimental data on the operational characteristics of the existing Chalk turbine for comparison with the theoretical prediction. More specifically, measure the power coefficient, C_p, of the Chalk turbine over a range of tip speed ratios and compare these with values predicted by the Wilson computer program.

2. Use modified versions of Wilson's program to investigate the various parameters affecting wind turbine operation. The effect of each parameter on the wind turbine performance can be evaluated more or less independent of the effects of other parameters. From this examination a realization of the parameters critical for efficient turbine operation would be obtained. Although optimum values for many of these parameters might be found from a theoretical standpoint, in designing a turbine it is important to know how deviations from these optimum values will affect turbine efficiency. Furthermore, the study would give additional support to theoretically determined optimum values for design parameters.

3. Design a new "spoked wheel" type wind turbine using the information obtained in the first part of this study. A 15' diameter turbine of this new design would then be built and tested at the Oklahoma State University wind energy field facility. The tests on the new turbine

would not only prove out the design procedure used, but would provide further support for the mathematical model used in the Wilson computer program.

CHAPTER II

AERODYNAMIC MODEL

As discussed in the Introduction the basic theory for the axial flow wind turbine was developed by Glauert (3) and has been followed by several other investigators including Wilson (8) (12). A summary of the essential details of this analysis is presented here together with the extension worked out at Oklahoma State University to more accurately model the low speed wind turbine.

The analysis of the wind turbine uses both a control volume approach, for determining the momentum flux of the air passing through the turbine, and aerodynamically derived lift and drag forces on the blades of the turbine. The momentum equations are applied to the control volume as shown in Figure 2. As is suggested by this figure, it is assumed that there is a well defined slipstream passing through the turbine disc which has negligible influence on the air passing around the turbine. In the case of the Chalk turbine it is also assumed that the slipstream through the blades does not significantly affect the flow of the air through the center hole of the turbine. The validity of these assumptions will be discussed in more detail later.

It is further assumed that the static pressure of the slipstream far in front of and far behind the turbine is equal to the free-stream static pressure. In reality there is a centrifugal force in the far wake due to the rotation of the flow. This centrifugal force must be balanced by a pressure gradient which, however, is small in the far wake. The momentum equation applied to the control volume of Figure 2 is simply:

$$D = \dot{m} \, (V_\infty - V_2) = \rho \, A \, U \, (V_\infty - V_2). \tag{2.1}$$

In analyzing the wind turbine further it is necessary to introduce the blade element or strip theory. The blade element theory is frequently used in theoretical studies of propellers and helicopter rotors as discussed in papers by Glauert (3) and Wiesner (9). Wilson (8) (12) incorporated the blade element technique in his wind turbine analysis, a brief explanation of which will be presented here.

The blade element concept is depicted in Figure 3. Instead of an analysis being performed on the full disc area of the turbine, a small strip of width Δr is studied. Each strip is characterized by a distinct radius, local tip speed ratio, blade twist, etc. It is assumed that each strip is independent of the other strips. This assumption is well founded if there is little change in the thrust loading along the radius of the wind turbine disc. Experimental support for this assumption when applied to propellers was demonstrated in wind tunnel tests performed by Lock (10).

An aerodynamic analysis is performed on an element of blade of length Δr within each strip. The aerodynamic forces thus found are multiplied by the number of blades in the turbine for the total aerodynamic force acting on the strip in question. These forces are then equivalent with the forces due to the momentum flux as derived from the control volume analysis. The performance of the entire turbine is found as a summation of the values obtained for the strips from the root to the tip of the blades.

The aerodynamic analysis of a blade element has been well presented by Wilson (8), Glauert (3), and McCormick (11) and will not be repeated in great detail here. The basic concept can be visualized with the help of Figure 4. In this diagram the axial interference factor "a" and the angular interference factor "a'" are defined as:

$$a \equiv 1 - \frac{U}{V_\infty} . \qquad (2.2)$$

$$a' \equiv \frac{\omega}{2\Omega} . \qquad (2.3)$$

The axial interference factor is a measure of how much the air flow normal to the turbine disc is slowed down relative to the free stream velocity. As the axial interference factor increases the air velocity at the blades decreases. Similarly the angular interference factor is a measure of the amount of angular velocity, or rotation, imparted to the wake relative to the angular velocity of the turbine. As the angular interference factor increases the amount of rotation in the wake increases.

Using trigonometry and basic equations for lift and drag on an airfoil, the coefficients of force in the x and y directions are:

$$C_x = C_L \sin \phi - C_D \cos \phi. \qquad (2.4)$$

$$C_y = C_L \cos \phi + C_D \sin \phi. \qquad (2.5)$$

From these coefficients the differential equations for drag and torque on the wind turbine are determined to be:

$$dD = \frac{1}{2} B c \rho W^2 C_y dr_L. \tag{2.6}$$

$$dQ = (\frac{1}{2} B c \rho W^2 C_x) r_L dr_L. \tag{2.7}$$

In the above equations W is the air velocity relative to the blades of the turbine. From Figure 4 it can be seen that the magnitude and direction of W is dependent on a and a'. The angle of attack α is determined by the direction of W and therefore α, C_D and C_L are also dependent on a and a'. Thus, Equations (2.6) and (2.7) alone are insoluble since the axial and angular interference factors are unknown.

Two more equations for drag and torque can be found from the momentum analysis. The differential drag quantity is derived from Equation (2.1) for a circular strip of width dr (see Figure 3).

$$dD = 2\pi r_L \rho U (V_\infty - V_2) dr_L. \tag{2.8}$$

In Glauert's basic analysis the rotation of the wake is now neglected in order to obtain an expression for V_2 in terms of V_∞ and a. At this point the conventional analysis will be extended in an attempt to more accurately model the low speed turbine. If one assumes no significant rotation in the flow until the turbine blades are encountered, Bernoulli's equation from far in front of the turbine to a point immediately in front of the blades becomes:

$$p_\infty + \frac{1}{2} \rho V_\infty^2 = p^+ + \frac{1}{2} \rho U^2. \tag{2.9}$$

Bernoulli's equation from a point directly behind the blades to a position in the far wake of the turbine is:

$$p^- + \frac{1}{2} \rho \, (U^2 + U_\theta^2) = p_\infty + \frac{1}{2} \rho \, (V_2^2 + V_{2\theta}^2). \tag{2.10}$$

Combining the two above equations:

$$p^+ - p^- = \frac{1}{2} \rho \, V_\infty^2 - \frac{1}{2} \rho \, V_2^2 + (\frac{1}{2} \rho \, U_\theta^2 - \frac{1}{2} \quad V_{2\theta}^2). \tag{2.11}$$

However, the drag on the turbine can also be expressed in terms of this pressure difference across the blades, i.e.:

$$dD = (p^+ - p^-) \, 2\pi \, r_L \, dr_L. \tag{2.12}$$

Substituting Equation (2.11) for the pressure difference term in Equation (2.12), the differential drag is found to be:

$$dD = \{\frac{1}{2} \rho \, V_\infty^2 - \frac{1}{2} \rho \, V_2^2 + (\frac{1}{2} \rho \, U_\theta^2 - \frac{1}{2} \rho \, V_{2\theta}^2)\} \, (2\pi \, r_L \, dr_L). \tag{2.13}$$

In the above equation V_2, U_θ, and $V_{2\theta}$ are dependent on a and a'. It is desirable to express the differential drag of the turbine as a more concise function of a and a'. This will facilitate the combining of equations for the turbine drag derived from the momentum analysis and from the aerodynamic analysis. This combination will yield a soluble equation for the axial interference factor a.

The following manipulations with equations is performed in order to reduce Equation (2.8) into a more concise function of a and a'. Combining Equations (2.8) and (2.13):

$$\rho U \, (V_\infty - V_2) = \frac{1}{2} \rho \, [V_\infty^2 - V_2^2 + (U_\theta^2 - V_{2\theta}^2)]. \tag{2.14}$$

Rearranging the above:

$$U = \frac{V_\infty + V_2}{2} + \frac{U_\theta^2 - V_{2\theta}^2}{2(V_\infty - V_2)} . \tag{2.15}$$

The tangential velocities V_θ and $V_{2\theta}$ in the above equation can be expressed in terms of the angular velocities ω_L, ω_2:

$$U_\theta = r_L \omega_L . \tag{2.16}$$

$$V_{2\theta} = r_2 \omega_2 . \tag{2.17}$$

Because of conservation of angular momentum the angular velocity of the air behind the blades, ω_L, and the angular velocity in the far wake, ω_2, are related by the equation:

$$r_2^2 \omega_2 = r_L^2 \omega_L . \tag{2.18}$$

Using Equations (2.16), (2.17) and (2.18) the tangential velocities in Equation (2.15) can be expressed in terms of the angular velocity ω_L and the radial lengths r_L and r_2:

$$U = \frac{V_\infty + V_2}{2} + \frac{(r_L \omega_L)^2 [1 - (\frac{r_L}{r_2})^2]}{2(V_\infty - V_2)} . \tag{2.19}$$

Now a new axial interference factor for the far wake will be defined as follows:

$$f \equiv V_2/V_\infty . \tag{2.20}$$

Expressing Equation (2.19) in terms of the axial interference factors:

$$2(1-a) = (1+f) + \left(\frac{r_L \omega_L}{V_\infty}\right) \frac{\left[1 - \left(\frac{r_L}{r_2}\right)^2\right]}{(1-f)} . \tag{2.21}$$

From continuity it can be shown that:

$$\left(\frac{r_L}{r_2}\right)^2 = \frac{V_2 \ V_\infty}{V_\infty \ U} = f/(1-a). \tag{2.22}$$

Also, defining the local tip speed ratio as:

$$x \equiv \frac{r_L \ \Omega}{V_\infty} . \tag{2.23}$$

Substituting Equations (2.3) and (2.23) into Equation (2.21):

$$2(1-a)\ (1-f) = 1-f^2 + 2\eta\ [1-f/(1-a)] \tag{2.24}$$

where

$$\eta \equiv 2(a'\ x)^2 \tag{2.25}$$

Equation (2.24) is a quadratic with respect to the far wake interference factor "f" and its solution is found to be:

$$f = [(1-a) - \eta/(1-a)] - \left[\left(\frac{\eta}{1-a}\right)^2 + a^2\right]^{1/2} \tag{2.26}$$

A simplification can be accomplished by introducing the variable γ defined as:

$$\gamma \equiv \frac{(1-a) - f}{(1-a)(1-f)} \tag{2.27}$$

Substituting into Equation (2.24):

$$2(1-a) = 1 + f + 2\,\eta\,\gamma \tag{2.28}$$

and therefore:

$$f = 1 - 2a - 2\,\eta\,\gamma. \tag{2.29}$$

It should be noted that Equation (2.29) is an implicit equation since γ is a function of the far wake interference factor f. However, this is not particularly troublesome since the equation can easily be solved by an iterative process on the computer.

Returning to the momentum equation for drag, Equation (2.8), it is now possible to express the differential drag quantity as a function of the axial interference factor a. Equation (2.8) expressed in terms of the axial interference factors a and f is found to be:

$$dD = 2\pi\, r_L\, \rho\, (1-a)\, (1-f)\, V_\infty^2\, dr_L \tag{2.30}$$

or, substituting Equation (2.29) for f:

$$dD = 4\pi\, r_L\, \rho\, (1-a)\, (a+\eta\gamma)\, V_\infty^2\, dr_L. \tag{2.31}$$

It is interesting to compare this with the momentum equation for drag as developed by Wilson while neglecting rotation in the wake. The Wilson equation for drag is:

$$dD = 4\pi\, r_L\, \rho\, (1-a)\, (a)\, V_\infty^2\, dr_L. \tag{2.32}$$

The two equations differ only in the "$\eta\gamma$" term which is present in the analysis in which rotational effects were taken into account. If one assumes that a' is equal to zero, analogous to neglecting the rotation in the wake, the quantity "$\eta\gamma$" becomes zero and Equation (2.31) becomes identical to that derived by Wilson. In effect, the quantity "$\eta\gamma$" is a measure of how much the rotation of the wake effects the axial velocity of the air. Consequently, the "$\eta\gamma$" term is a function of not only a but of a'.

The two equations for turbine drag as obtained from the momentum analysis and from the aerodynamic analysis will yield a single equation for the axial interference factor a. Expressing the aerodynamic drag, Equation (2.6), in terms of the axial interference factor a:

$$dD = \frac{1}{2} B c \rho \frac{(1-a)}{\sin^2 \phi} V_\infty^2 C_y \, dr_L \tag{2.33}$$

where from Figure 4:

$$W = (1-a) V_\infty / \sin \phi . \tag{2.34}$$

Combining the aerodynamic (2.33) and the momentum (2.31) equations for the drag on the wind turbine, it is found that:

$$4\pi r_L (a + \eta\gamma) = \frac{1}{2} B c \frac{(1-a)}{\sin^2 \phi} C_y \tag{2.35}$$

this can be reduced to:

$$a = (\sigma_L C_y - 8\eta\gamma \sin^2 \phi)/(8 \sin^2 \phi + \sigma_L C_y) \tag{2.36}$$

where

$$\sigma_L = \frac{B c}{\pi r_L} . \tag{2.37}$$

An equation for determining a' is now required to complete the mathematical model. This equation can be obtained in much the same way as was done for a. In this case two equations for the torque produced by the turbine are developed from the momentum and from the aerodynamic analyses. When these equations are combined, a single equation for the angular interference factor a' will be attained. The torque equation as developed from momentum principles is found to be:

$$dQ = d\dot{m}\ (U_\theta\ r_L) = 2\pi\ r_L^3\ \rho\ U\ \omega\ dr_L. \tag{2.38}$$

Wilson has shown that combining this with the aerodynamic equation for torque, Equation (2.7), leads to an expression for the angular interference factor "a'":

$$a' = \sigma_L\ C_x/(8\ \sin\phi\ \cos\phi - \sigma_L\ C_x). \tag{2.39}$$

Equations (2.36) and (2.39) for the axial and angular interference factors are implicit since the values of C_x, C_y, η, γ, and ϕ are all dependent on "a" and "a'". Still these equations are easily solved using a computer iterative process which will be described shortly. Once the interference factors are known, all other physical parameters can easily be calculated. Specifically, the torque and drag on the turbine disc can be found from Equations (2.25), (2.27), (2.31), and (2.38).

It should be noted that blade coning and tip loss effects have not been taken into account in the foregoing derivation. This was mainly because these effects can be shown to be negligible for the low speed "spoked wheel" wind turbine. Wilson has done extensive work on these coning and tip loss corrections and should be referred to in cases where such corrections might be significant. It should also be mentioned that an analysis including the effect of wake rotation on the axial velocity was performed by Nilberg (13) in the early 1950s. In his analysis, however, Nilberg has assumed that the tangential velocity is constant as the air moves from the blades to the far wake. This differs from the assumption of conservation of angular momentum used in the analysis presented in this study (see Equation (2.16)). If the angular momentum is relatively constant as the flow moves downstream of the turbine, the tangential velocity of the flow must necessarily decrease substantially.

The computer iteration technique used to solve Equations (2.36) and (2.39) for the axial and angular interference factors, respectively, is as follows:

1. Assume a and a'.
2. Calculate ϕ:

$$\phi = \tan^{-1} [(1-a)/(1+a')\, x].$$

3. Calculate η:

$$\eta = 2\,(a'\, x)^2.$$

4. Calculate f:

$$f = [(1-a) - \frac{\eta}{1-a}] - [(\frac{\eta}{1-a})^2 + a^2]^{1/2}.$$

5. Calculate γ:

$$\gamma = \frac{1-a-f}{(1-a)(1-f)} .$$

6. Calculate α:

$$\alpha = \phi - \theta$$

7. Determine C_L, C_D.

8. Calculate C_x, C_y.

9. Calculate a:

$$a = (\sigma_L\ C_y - 8\eta\gamma\ \sin^2\phi)/(8\ \sin^2\phi + \sigma_L\ C_y).$$

10. Calculate a':

$$a' = \sigma_L\ C_x/(8\sin\phi\ \cos\phi - \sigma_L\ C_x).$$

11. Compare a and a' values with assumed values; if not within a specified tolerance, repeat with a and a' values just calculated.

CHAPTER III

DESIGN LIMITS ON OPERATIONAL PARAMETERS

One of the most significant revelations from Glauert's basic analysis of the ideal wind turbine is the optimum value for the axial interference factor a. Glauert (3) demonstrated that the ideal turbine (i.e., a turbine with no aerodynamic losses) has a maximum power output when the axial interference factor is equal to 1/3. In his analysis of high speed wind turbines, Hutter (4) came to essentially the same conclusion. For tip speed ratios greater than 2.5, Hutter found that the optimum value of ξ ($\xi = V_2/V_\infty$) was approximately 1/3. It can be shown that this corresponds to a value of 1/3 for a.

Both Glauert and Hutter have suggested that high tip speed ratio wind turbines should be more efficient than low tip speed ratio turbines. This was generally accepted since it was felt that the large amount of rotation induced into the wake at low speeds constituted a large loss. Experimental support for the theory was provided by the American farm windmill which generally operates at about a tip speed ratio of 1. However, the data from this turbine are misleading since, as will be shown later, the inefficiency of the American windmill is due to inefficient airfoils.

Hutter has also stated that greatest efficiency can be expected from airfoils operating at the maximum lift over drag ratio (L/D). Using blade element theory Glauert (3) presented a proof of this in the case of propellers. In his analysis Glauert defines the efficiency of the propeller as the work done by the blades in providing thrust divided by the power input to the blades. However, this definition has little meaning for wind turbines where maximum power output is important regardless of the drag applied by the turbine on the air flow.

Determining Optimum Parameters

As a preliminary step in this study, a mathematical derivation of optimum airfoil operation was performed. In the following analysis the optimum angle of attack (α) for an airfoil on a wind turbine will be determined. Following the format of Figure 4, the force in the y direction (F_y) is the drag on the turbine and the force in the x direction (F_x) is responsible for the torque on the turbine. These forces can be found from the equations:

$$F_y = \frac{1}{2} B \, c \, \rho \, W^2 \, C_y \tag{3.1}$$

$$F_x = \frac{1}{2} B \, c \, \rho \, W^2 \, C_x \tag{3.2}$$

where C_x and C_y are found from Equations (2.4) and (2.5). For any given tip speed ratio (X) and axial interference factor, the optimum α is that for which the power output of the turbine is maximum. The power output is found as the product of the torque produced by the turbine and the angular velocity of the turbine. At a given X, the angular velocity of the turbine is constant for a given wind speed. Therefore, to maximize the power output, the torque force (F_x) must be as large as possible.

For any given value of "a" there is a corresponding value for F_y. Hence, when "a" is constant, F_y is constant and maximum F_x occurs at the same point as the maximum ratio F_x/F_y. This ratio is found by combining Equations (3.1) and (3.2):

$$F_x/F_y = C_x/C_y. \tag{3.3}$$

The angle α for which F_x/F_y is maximum can be determined by finding α for maximum C_x/C_y. From Equations (2.4) and (2.5) the ratio C_x/C_y is found to be:

$$\frac{C_x}{C_y} = \frac{(\sin\phi)\ C_L - (\cos\phi)\ C_D}{(\cos\phi)\ C_L + (\sin\phi)\ C_D}\ . \tag{3.4}$$

In the above equation ϕ is constant with respect to α since it is a function of X and a only. Differentiating Equation (3.4) with respect to α and setting it equal to zero, the following relationship is found for maximum C_x/C_y:

$$\frac{d\ C_D}{d\ C_L} = \frac{C_D}{C_L}\ . \tag{3.5}$$

This can easily be shown to occur at maximum C_L/C_D. Thus the optimum angle of attack α occurs at the maximum lift over drag ratio. This confirms what Hutter (4) had indicated in his work on design of high tip speed ratio wind turbines.

It should be noted that, when determining the maximum L/D, the drag on supporting structures on the turbine wheel (such as the wire spokes on a "spoked wheel" wind turbine) should be taken into account. The importance of this can be seen in Figure 5 where a turbine with

Clark Y airfoils has maximum efficiency at α equal to 1°. When wire spokes are added, the optimum α is at the angle for maximum L/D for the airfoil and wire combination.

Although theoretical optimum values have been determined for the axial interference factor and the angle of attack of the airfoils, for design purposes it is important to know how much one can deviate from these values and still have acceptable performance. To answer this question a systematic evaluation of the effect of α, a, and X on the power output of the wind turbine was performed using variations of Wilson's computer program. The effect of each of these parameters on the power output was determined while holding the remaining parameters constant. In order to obtain specified conditions for axial interference factor, tip speed ratio, and L/D, the solidity of the turbine was allowed to vary.

The basic computer program used in this study is a simplified form of a program written by Wilson (12). The major simplification involved the removal of tip loss and hub loss correction techniques. In doing this much computer time was saved while only a small error was introduced, since tip and hub losses are minimal on a "spoked wheel" wind turbine. A block diagram of the program is shown in Table I and a listing is given in Table II (see Appendix B). The only other major change in the program is the calculation which uses the rotational analysis as mentioned earlier.

The program is written in FORTRAN and was used on the Oklahoma State University IBM 360/65 computer in this study. The inputs to the program are shown in Table III; the output format of the program, shown in Tables IV and V, lists important parameters for each blade element at a given tip speed ratio (see Appendix B). The tables are arranged so that a complete picture of the operating characteristics of the entire blade can be seen. The power coefficient (C_p) in Table V is defined as the power output of the turbine divided by the total power in the wind. That is:

$$C_p = P/\frac{1}{2} \rho A V_\infty^3. \tag{3.6}$$

The program is ordinarily used to determine the operational characteristics of a wind turbine of a given design. However, by modifying the subroutine "CALC" (Table VI), the program can be made to alter the design of the turbine until specified operational characteristics are achieved. Design parameters that are altered in this process are blade number, chord width, or angle of attack. Since the optimum angle of attack is usually set at the maximum L/D, the optimum axial interference factor is obtained by varying either the number of blades or the chord width. The product of these latter two parameters determine the solidity of the turbine.

In order to find the optimum axial interference factor the program was used to study a single blade element under varying conditions. The local coefficient of power (C_{pL}) found for this blade element is then applicable to any part of the blade so long as it has the same local tip speed ratio and L/D. In the first analysis the L/D was set at 7 which approximated the maximum L/D for the Chalk airfoils plus the wire spokes. The local power coefficient was determined over a range of "a" values for tip speed ratios from 0.8 to 2.8. The results from this analysis can be seen in Figure 6. As this figure indicates the optimum "a" is found to vary from a value of 0.31 to 0.25 as the tip speed ratio goes from 0.8 to 2.8. Before concluding that the optimum "a" is dependent on tip speed ratio, one must determine the effect of changing the airfoil characteristics.

The same analysis described above was performed for L/D ratios of 25 and 50 with the results shown in Figures 7 and 8. Comparing these figures with Figure 6, it can be seen that as the airfoil characteristics become more efficient the optimum "a" comes closer to the ideal wind turbine optimum of 0.33. When aerodynamic losses occur, the optimum "a" becomes smaller than the "ideal a" and progressively decreases as the losses become greater. Apparently as the tip speed ratio increases the aerodynamic losses become greater and the optimum "a" decreases.

In studying Figures 6, 7, and 8 it is apparent that, although a definite optimum "a" occurs, the C_{pL} does not usually vary much in the range of "a" values from 0.25 to 0.40. The exception to this rule is for very poor airfoils at relatively high tip speed ratio in which case "a" should be kept around 0.25 or lower. It can be concluded from this analysis that, when designing a wind turbine, the axial interference factor should be kept around 0.3 for optimum performance. However, there is a considerable range from 0.25 to 0.4 within which the power coefficient is not highly affected. Hence, this allows for some freedom of choosing an "a" value and possibly permitting operation of the airfoils at more efficient angles of attack.

The next computer analysis was performed to verify the theoretical optimum α value and determine how strongly the C_{pL} is influenced by changing α. Figure 5 shows that the computer analysis does agree with the theoretical optimum α at the maximum L/D. It can also be seen that, for the airfoil examined, the C_{pL} is not highly affected if α is kept within about 4° of the optimum α.

The last parameter to be examined with the computer analysis was the tip speed ratio X. Tip speed ratios were studied from 1 to 5 over a full range of "a" values and with L/D ratios from ∞ to 7. Tip speed ratios much less than 1 could not be studied since the flow in the wake of the turbine would reverse and the momentum analysis used would become invalid. Operation at tip speed ratios smaller than 1 also required prohibitively large solidities in which case losses from airfoil cascade effects would predominate.

The analysis of an airfoil with L/D equal to ∞ corresponded to the analysis of an ideal wind turbine. The computer program was run using both the new "model" which includes effects of wake rotation on the axial velocity and the standard Wilson method. From Figure 9 it can be seen that there is little difference between the two methods at tip speed ratios above 2. This is to be expected since rotation in the wake becomes smaller at higher tip speed ratios due to a decrease in the torque on the blades. The analysis using both methods asymptotes toward a C_{pL} equal to 0.593 which is the theoretical maximum when ignoring the rotation of the wake. However, when using the new method, the C_{pL} tends to increase at lower tip speed ratios while the Wilson method indicates a decreasing C_{pL}. The validity of both methods is in doubt due to the unknown influence of a pressure gradient on the stream tube going through the turbine disc. Thss problem, which becomes more acute at very low tip speed ratios, was first pointed out by Goorjian (14). At tip speed ratios above 1, the two methods differ only slightly in predicted power output. As can be seen in Figure 10 for the Chalk turbine, the predictions for turbine performance by the two methods are very close. Since the Chalk turbine is representative of low tip speed ratio wind turbines and since predictions by the two methods become even closer at higher tip speed ratios, one can conclude that the rotation of the wake induced by the wind turbine does not significantly affect the turbine performance.

Figure 9 also shows the characteristics of non-ideal wind turbines with L/D ratios from 50 to 7. It is seen that turbines with poor airfoils ($L/D < 20$) can gain a significant increase in maximum C_{pL} if they are designed to run at lower tip speed ratios. It should be remembered that the design should avoid local tip speed ratios less than 1. If the airfoils have a L/D over 50 (which can easily be achieved in most cases, see Figure 11), then the drop in C_{pL} becomes very small with increasing tip speed ratio. Hence, the fact that there is a drastic decrease in the required solidity of the turbine at higher tip speed ratios becomes an important factor.

Design of the Wind Turbine

Having completed the above analysis of the important operational parameters for wind turbines, the next step was to design a turbine to operate at maximum efficiency. The design of the new "spoked wheel" wind turbine was undertaken with two constraints due to economic and structural considerations. These constraints were that the turbine must still have 48 blades and that the blades must be of the same profile as previously used. However, although the aerodynamic characteristics of the blades could not be altered, the overall aerodynamics of the turbine could be improved by changing the wire spokes.

The 1/16" wire spokes which lie opposite to the blades on the "spoked wheel" turbine are a source of considerable aerodynamic drag. Emphasis is brought to this point when it is realized that the maximum L/D of the airfoils is reduced from 29 to 7 when the wires are taken into account. Referring to Figure 9 it can be seen that the maximum possible efficiency is seriously reduced by the presence of the wires. In order to reduce drag from the wires the number of wires was reduced from 96 to 24 and the diameter of each wire was increased to approximately 1/8". Accordingly the drag from the wires was cut in half while the total strength remained about the same. The maximum L/D ratio for the blade and wire combination was thus increased to 14.

With the number of blades and the airfoil characteristics set, the next step was to determine the optimum tip speed ratio at which to operate the turbine. This was done by running the version of the computer program in which the axial interference factor is kept constant by adjusting the angle of attack of the airfoils. The axial interference factor was specified to be about 0.3 and the program was run to determine the operational characteristics at various tip speed ratios. From this analysis it was found that the wind turbine could be run at a tip speed ratio of about 2.2 with "a" at optimum and with α at optimum near the tip of the blades. Running at a lower tip speed ratio resulted in highly inefficient values of α for inner portions of the blade. Hence, the optimum tip speed ratio for the 48-blade configuration was set at 2.2.

The twist distribution for the new turbine was calculated from the values of α and ϕ at the optimum tip speed ratio in the above program.

From Figure 4 the equation for twist is found to be:

$$\theta = \phi - \alpha. \tag{3.7}$$

Since the structural design of the "spoked wheel" turbine constrains the twist to be linear along the blade, the optimum values of θ were approximated by 14.5° at the tip increasing linearly to a twist of 31° at the base of the blade.

With the design complete, the configuration of the new turbine was analyzed using the main computer program to determine its characteristics over a range of tip speed ratios. The result of this analysis is shown in Figure 12. Also shown in this figure is the performance curve for the original Chalk turbine. The maximum power coefficient for the new design is over 39% as compared with 23% for the Chalk turbine. This represents an increase of 70% in power output from the wind turbine.

The reason for this drastic increase in power can be found in studying the operational characteristics of the two turbines shown in Tables IV and V. The Chalk turbine, when operating at its optimum tip speed ratio of about 2.0, has values of "a" far above the optimum value of 0.3. This is caused by the small twist (7° at the tip of 18° at the base of the blades) incorporated in the Chalk turbine which, in turn, causes a large value for α and thus high lift on the blades. The large lift on the blades drastically slows down the wind resulting in very inefficient operation. The new turbine, however, is seen to be operating within acceptable ranges of "a" and α all along the blades. It should be noted that larger values of α are needed as one moves toward the base of the blades so that optimum "a" can be maintained. This is found to be beneficial from an operational standpoint since the whole blade will not stall at the same time if there is sudden change in wind velocity.

CHAPTER IV

EXPERIMENTAL EVALUATION OF WIND TURBINE PERFORMANCE

Experiments were performed on both the Chalk turbine and the Oklahoma State University designed wind turbine to determine the power coefficient at various tip speed ratios. The tests were conducted on 15' diameter models at the Oklahoma State University wind energy field facility. The main purpose of the experiments was to evaluate the theoretical aerodynamic model developed by Wilson and used in this study. Unfortunately the experimental data obtained was not as definitive as hoped. The main reason for this, as will be explained later, was the difficulty encountered in testing the wind turbines in an atmospheric environment.

Determination of the experimental power coefficient of any wind turbine requires the simultaneous measurement of wind velocity, turbine (or dynamometer) RPM and dynamometer torque.

The experimental procedure used on the Chalk turbine is listed below. This procedure was improved before testing the Oklahoma State University turbine in an effort to obtain more accurate data.

1. Wind Velocity. Ball-cup anemometers with a calibrated d-c voltage output linearly proportional to the wind velocity were used. One anemometer was placed approximately 100 ft directly upwind of the turbine and at the same height as the center of the turbine. This

anemometer indicated the free stream velocity of the wind acting on the wind turbine.

2. Dynamometer RPM. The transducer pickup of a magnet imbedded in the dynamometer shaft was fed into a digital counter. The turbine RPM, used in tip speed ratio calculations, is directly proportional to the dynamometer RPM.

3. Dynamometer Torque. A hydraulic piston connected to a pressure gauge was calibrated to measure the torque on the dynamometer housing. This was used in conjunction with the dynamometer RPM to determine the power output of the wind turbine.

Simultaneous measurements of the three noted quantities were made at five second intervals for periods of about ten minutes. From these measurements the power coefficient (Equation (3.6)) and the corresponding tip speed ratio can be determined. The power output of the turbine is simply the product of the angular velocity and the torque applied to the dynamometer.

In analyzing this data it was found that the turbine was rarely in steady state operation. This was due to the large fluctuation in wind speeds that are typically encountered. Since the turbine efficiency is a function of the wind power which, in turn, is a function of the wind velocity cubed, any discrepancy between the turbine power output and the corresponding wind velocity can cause large errors. A criterion was therefore established in an attempt to distinguish data points occurring during steady state operation. This criterion was that the wind velocity should not fluctuate by more than 15% ten seconds before and five seconds after the measurements. Also, the tip speed ratio was required to fluctuate less than 10% five seconds before and after the measurement. Only data meeting these conditions were considered valid.

Before any comparison between experiment and computer prediction can be made, accurate input data for the computer analysis is needed. For the most part simple measurements of the physical structure of the turbine sufficed for this data. However, the aerodynamic characteristics of the airfoils used on the Chalk wind turbine constituted an unknown factor since the profile of the blades did not exactly conform to any standard airfoil profile. Since the exact aerodynamic characteristics of the blades are of prime importance in the computer analysis, wind tunnel tests on an airfoil section were undertaken. The tests were run at a Reynolds number of about 75,000 which corresponds to the average value at which the wind turbine operates. Corrections were applied to the data according to procedures described by Pope and Harper (15). The resulting sectional coefficients of life and drag are shown in Figure 13. These data were entered into the computer program in tabular form.

The experimental data from the Chalk turbine and the theoretical performance curve are shown in Figure 10. Each individual experimental point on this graph represents the average of several measurements made in the individual ranges of tip speed ratio. A plot of the raw data from which these averages are taken is shown in Figure 14. There is a large amount of scatter in the data despite efforts to distinguish measurements taken during steady state operation. However, the scatter of experimental measurements seems to be distributed about the curve predicted by the theoretical analysis. Some support for the analysis is therefore given by the experimental data, although the margin of error on the peak power output is somewhat large. Note should be made that there are four data points derived from a brief experiment made using a prony brake instead of the dynamometer. The prony brake system proved to be highly unstable and therefore was not used extensively.

Before tests were conducted on the new Oklahoma State University wind turbine, the experimental procedure used was completely automated. Continuous analog signals for the wind velocity, dynamometer RPM, and dynamometer torque were simultaneously recorded on a strip chart recorder. It was hoped that a better determination of when the turbine was in steady state operation could be obtained from the analysis of a continuous chart recording.

In order to obtain an analog voltage output for the dynamometer RPM, the signal from the magnetic transducer was fed into a frequency to voltage converter. A strain gauge transducer connected to the dynamometer housing (Figure 15) provided a voltage output proportional to the torque on the housing. The voltage output from the ball-cup anemometer was fed directly into the strip chart recorder. Figure 16 is a typical tracing of two minutes of actual data obtained from the Oklahoma State University wind turbine. The resulting traces were analyzed and portions of the record were marked where the wind velocity and dynamometer RPM were reasonably steady. The same criterion for steady state operation that was used on the data from the Chalk turbine was applied to the strip chart data.

Figure 17 shows the resulting experimental data and the theoretical performance curve for the new Oklahoma State University wind turbine. As can be seen, the experimental data did not agree with the predicted performance curve and further investigation was warranted. Since the airfoils used on the second turbine were slightly different in shape than those on the original Chalk turbine, the lift and drag performance of the new airfoils was measured in the same way as described earlier. The resulting airfoil characteristics for the new turbine are shown in Figure 18. Although the shape of the new airfoils was little changed, the angle of attack corresponding to zero lift shifted by almost 4°. When the new airfoil data were entered into the computer program, a very different power production curve was predicted. This revised curve is also shown in Figure 17.

Although the testing of the new turbine was performed under better controlled conditions than used for the Chalk turbine, the experimental data still contained an excessive amount of scatter. The predicted performance curve shown in Figure 17 is well above the averaged experimental data at tip speed ratios greater than 2.0. However, the averaged unloaded tip speed ratio is quite close to the theoretically predicted value. This is significant since the turbine shaft bearings are the only source of friction loss when making this measurement and therefore the experimental error is minimized. Whether the discrepancy at high tip speed ratio is due to the experiment or the theory is unresolved.

CHAPTER V

CONCLUSIONS

During the initial part of this study of "spoked wheel" wind turbines it was found that the existing aerodynamic theory was geared for high tip speed ratio wind turbines. In the analysis of high tip speed ratio wind turbines it was assumed that the rotation of the wake behind the turbine has negligible effect on the axial velocity of the air. Since this assumption was questionable at the low tip speed ratios characteristic of the "spoked wheel" wind turbine, an analysis which includes the effects due to wake rotation was developed. However, when this new analysis was applied to the Chalk turbine, the performance curve predicted was not substantially different from that predicted by the Wilson analysis (Figure 10). The effects due to the rotation of the wake can therefore be considered negligible at tip speed ratios as low as 1.

In developing a design procedure for wind turbines it was found that five basic parameters can be optimized. In brief these are:

1. Axial Interference Factor "a". This was found to have an optimum value around 0.3 in most cases. The range of acceptable values, however, is relatively large; i.e., $0.25 < a < 0.40$.

2. Angle of Attack α. The optimum value is found to be at the angle of maximum L/D. A range of acceptable values is plus or minus 4° from the optimum value for α.

3. Airfoil Characteristics. Airfoils should be selected to have highest L/D ratio under given operating conditions.

4. Tip Speed Ratio X. This parameter is not crucial except for turbines with poor airfoil characteristics. In this case the turbine should be designed for low tip speed ratios in the range from 1 to 2.

5. Solidity. This value is set so that the above optimum parameters can be achieved. However, the solidity may often be determined by structural or economic considerations. In this case the tip speed ratio can be adjusted to maintain optimum operation.

One of the most interesting results from the analysis of optimum parameters is that an increase in efficiency can be expected by operating at lower tip speed ratios. This is especially the case when operating with poor airfoils where the tip speed ratio has a significant effect. However, when operating with reasonably good airfoils, the tip speed ratio does not have much effect on the maximum power coefficient for the wind turbine. Thus, the inefficiency of the American farm windmill can be attributed to the poor airfoil characteristics rather than the low tip speed ratio. It should be pointed out, however, that the American farm windmill has a very high starting torque and thus may be well suited for some applications such as deep well mechanical pumping.

There is also a close coupling between the tip speed ratio of the turbine and the solidity. At low tip speed ratios a high solidity is required while conversely at high tip speed ratios a low solidity is required. Thus the popular two- and three-bladed modern wind turbines must necessarily operate at high tip speed ratios due to the low solidity. However, a high solidity wind turbine, such as "spoked wheel" turbines, should be able to operate very efficiently at low tip speed ratios when built with reasonably efficient airfoils.

The design of the new Oklahoma State University wind turbine followed basically a trial and error procedure. With the solidity and airfoil characteristics set, the "constant a" version of the computer program was run to determine at which tip speed ratio the axial interference factor and the angle of attack of the airfoils are both at optimum. Once each section of the blade is brought within the optimum ranges as mentioned earlier, very little improvement can be accomplished by further adjustments.

Difficulties still exist in modeling wind turbine operation at local tip speed ratios less than 1. The basic theory used in the wind turbine analysis becomes suspect at very low tip speed ratios due to the large amount of rotation induced into the wake. The physics of this situation should probably be described from wind tunnel experiments so that an accurate model can be developed. The necessity for this, however, may prove minimal since preliminary experiments by Sweeney et al. (16) have shown that greater efficiency can be achieved by replacing the low speed inner portion of the blades with a large hub. The physics of this flow remain to be mathematically described so that the effect of the hub can be incorporated in the basic wind turbine theory.

The experimental measurements performed on the wind turbines in order to give support for the theoretical model were not as conclusive as hoped. However, qualitative support for the theory was gained, although a discrepancy occurred at the high tip speed ratios on the new Oklahoma State University wind turbine. The source of this discrepancy is unknown due to uncertainty in the wind tunnel tests on the airfoils, the friction losses in the dynamometer at high speeds, and the effects of highly turbulent atmospheric conditions.

At this point it seems that the use of experimentally derived C_p versus X curves for comparison with theoretical predictions remains questionable. There are far too many intermediate quantities, such as the velocity of the air as it flows through the blades, the velocity of the air in the far wake, and the drag on the turbine disc, which remain unknown. Measurement of these intermediate quantities would provide much stronger support for theoretical predictions while the data would be invaluable for developing a new aerodynamic model if necessary. Although many of these measurements might be made in wind tunnel tests, measurements in a true field situation would be particularly useful in determining the effects of turbulent atmospheric conditions.

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APPENDIX A

FIGURES

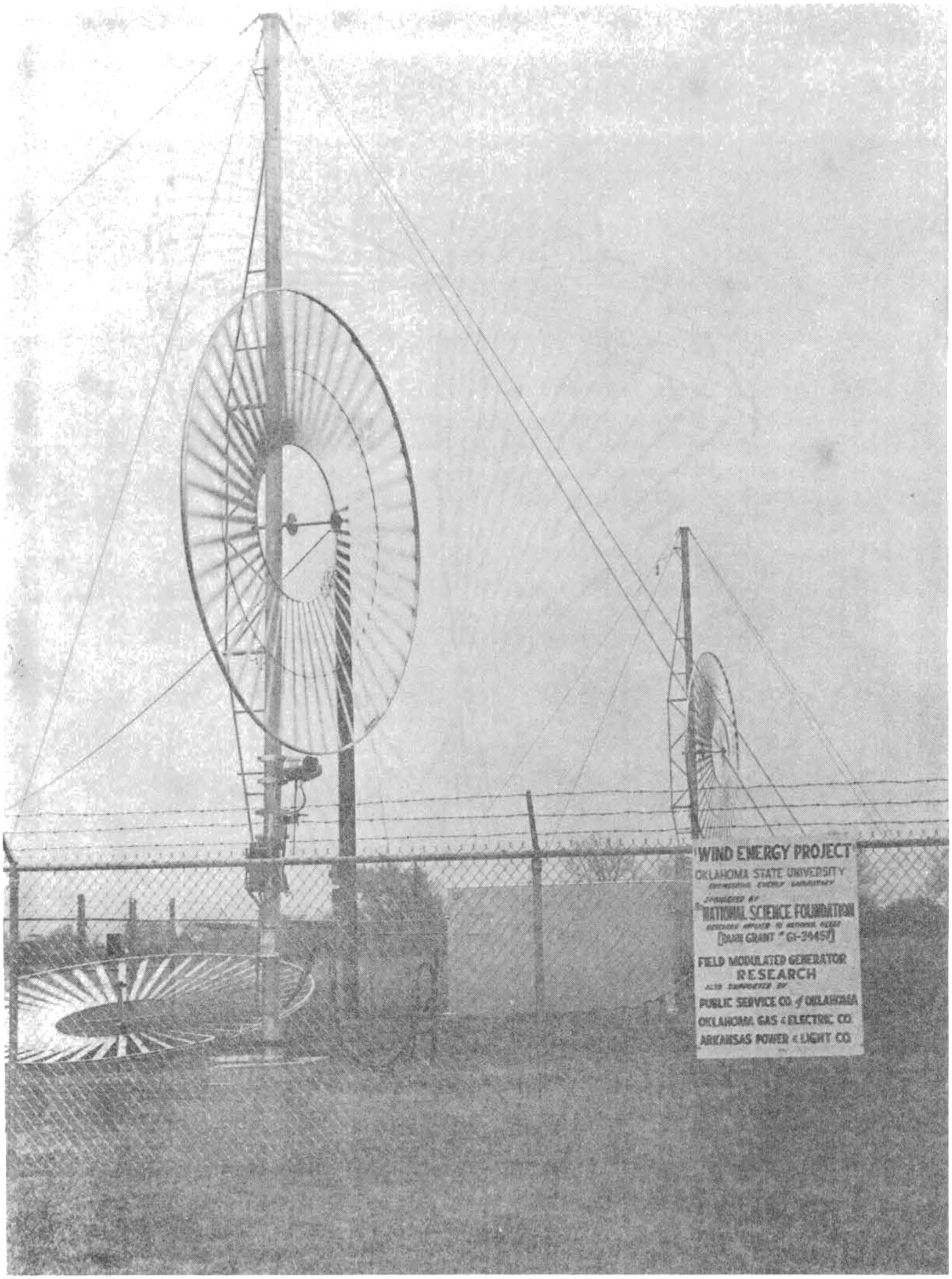

Figure 1. "Spoked Wheel" Wind Turbines

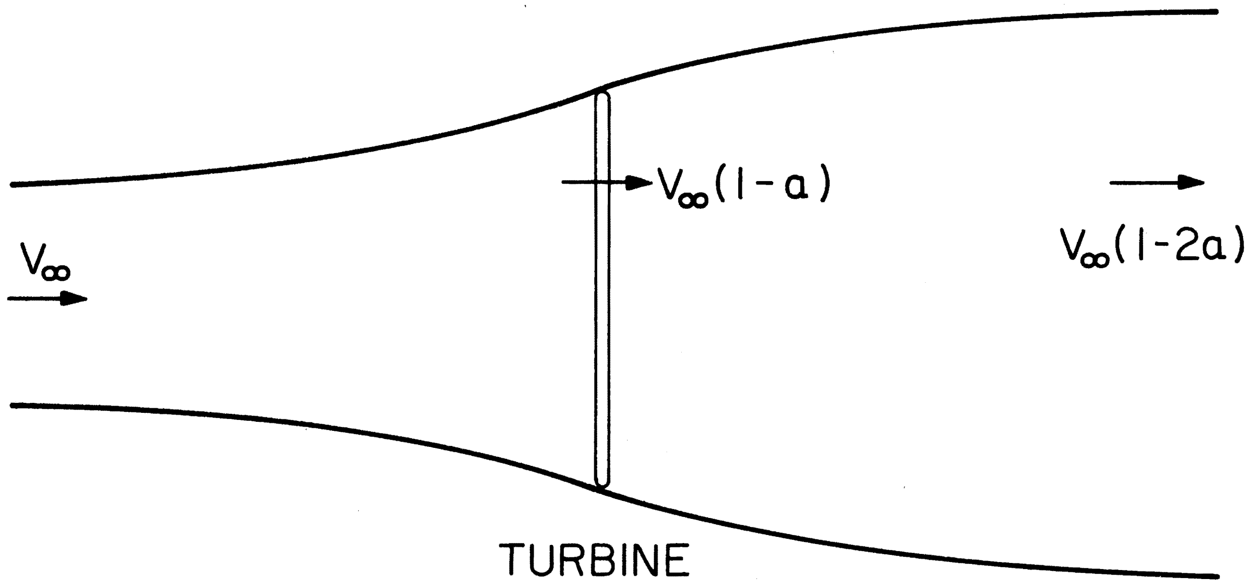

Figure 2. Wind Turbine Control Volume

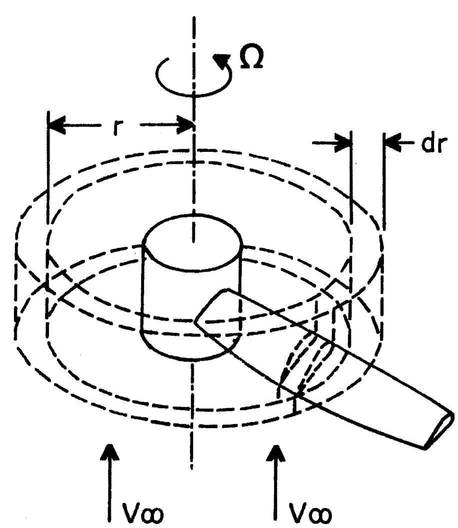

Figure 3. Blade Element Diagram

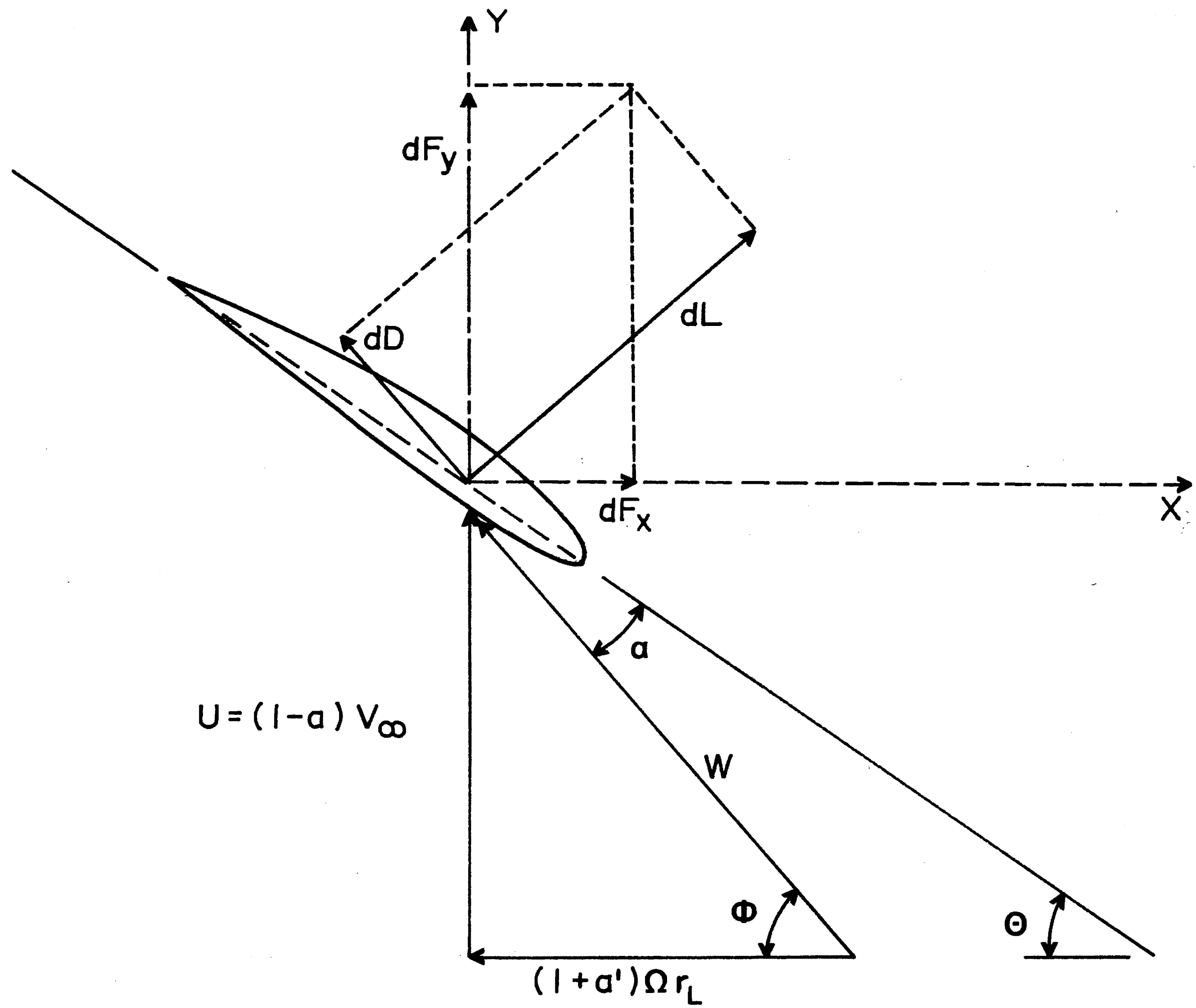

Figure 4. Aerodynamic Vector Diagram

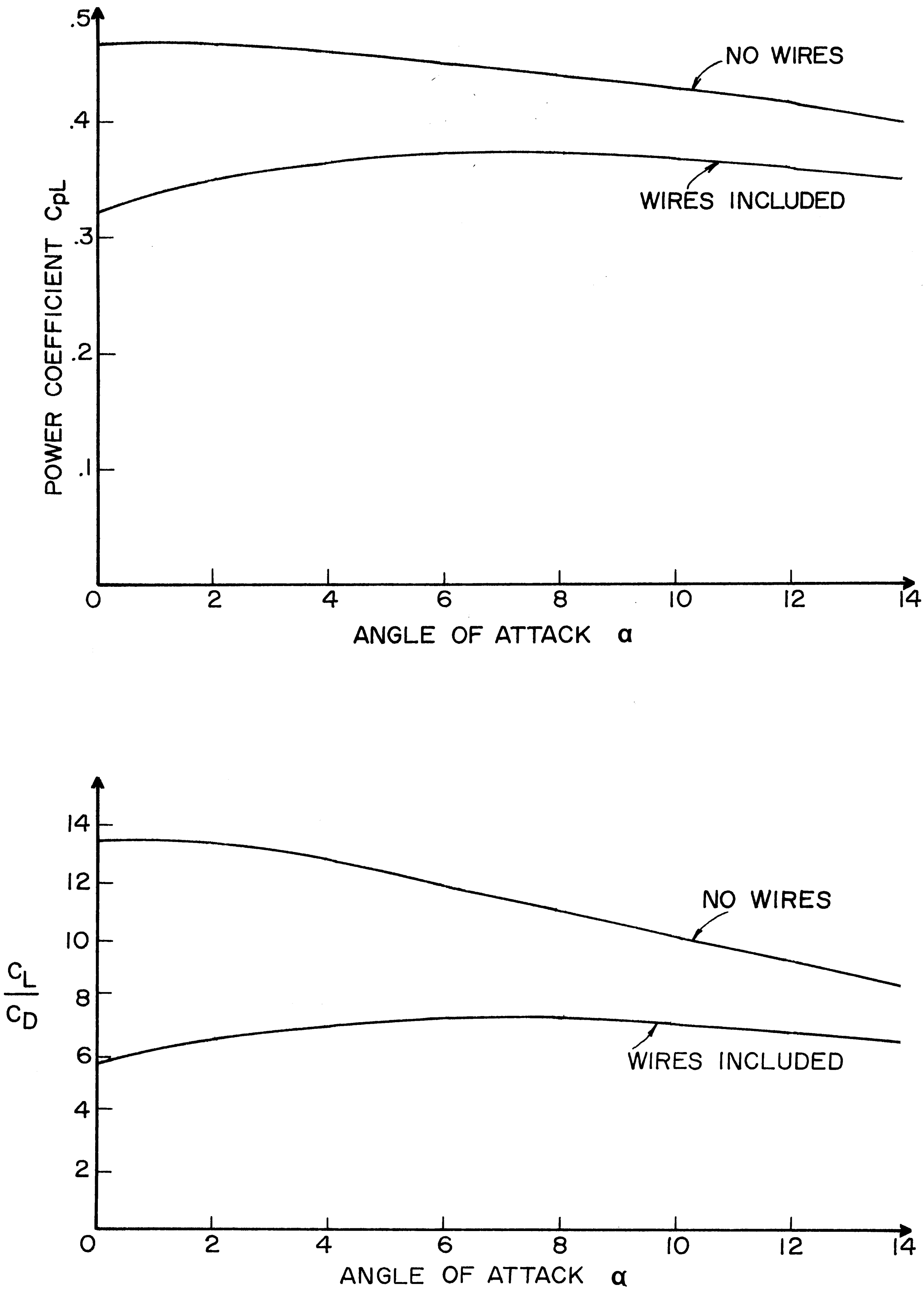

Figure 5. Optimum Angle of Attack

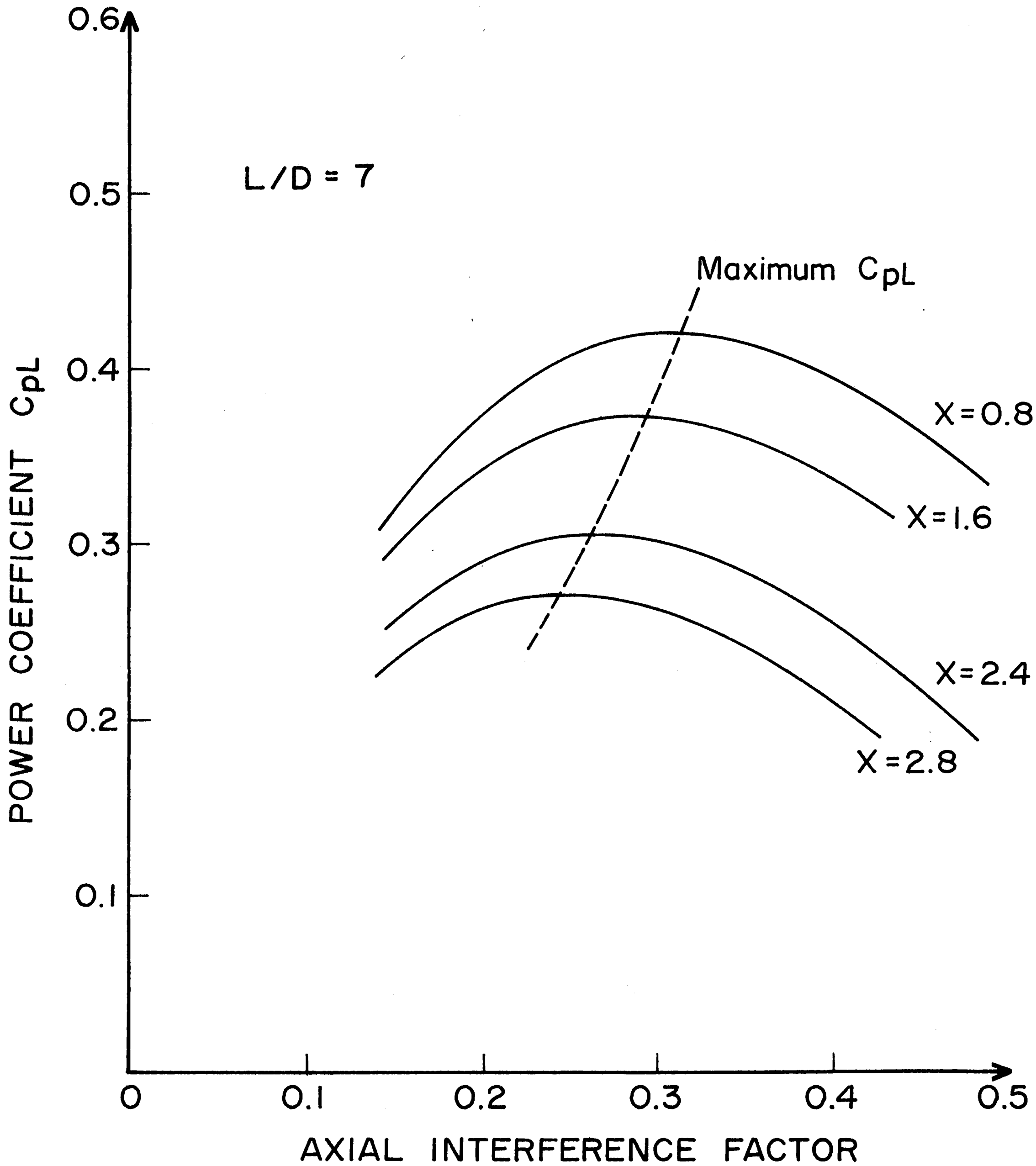

Figure 6. Optimum a, L/D = 7

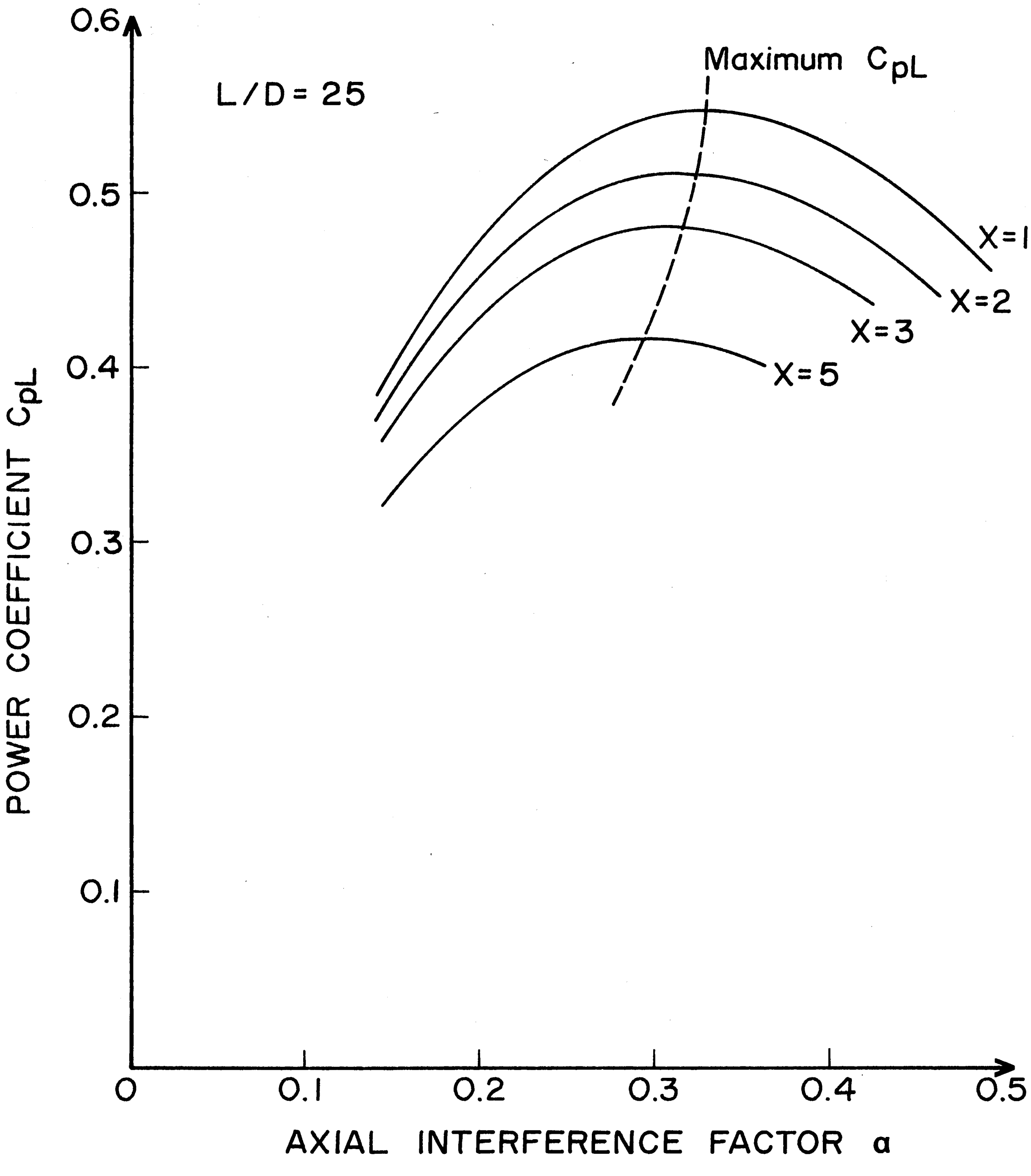

Figure 7. Optimum a, L/D = 25

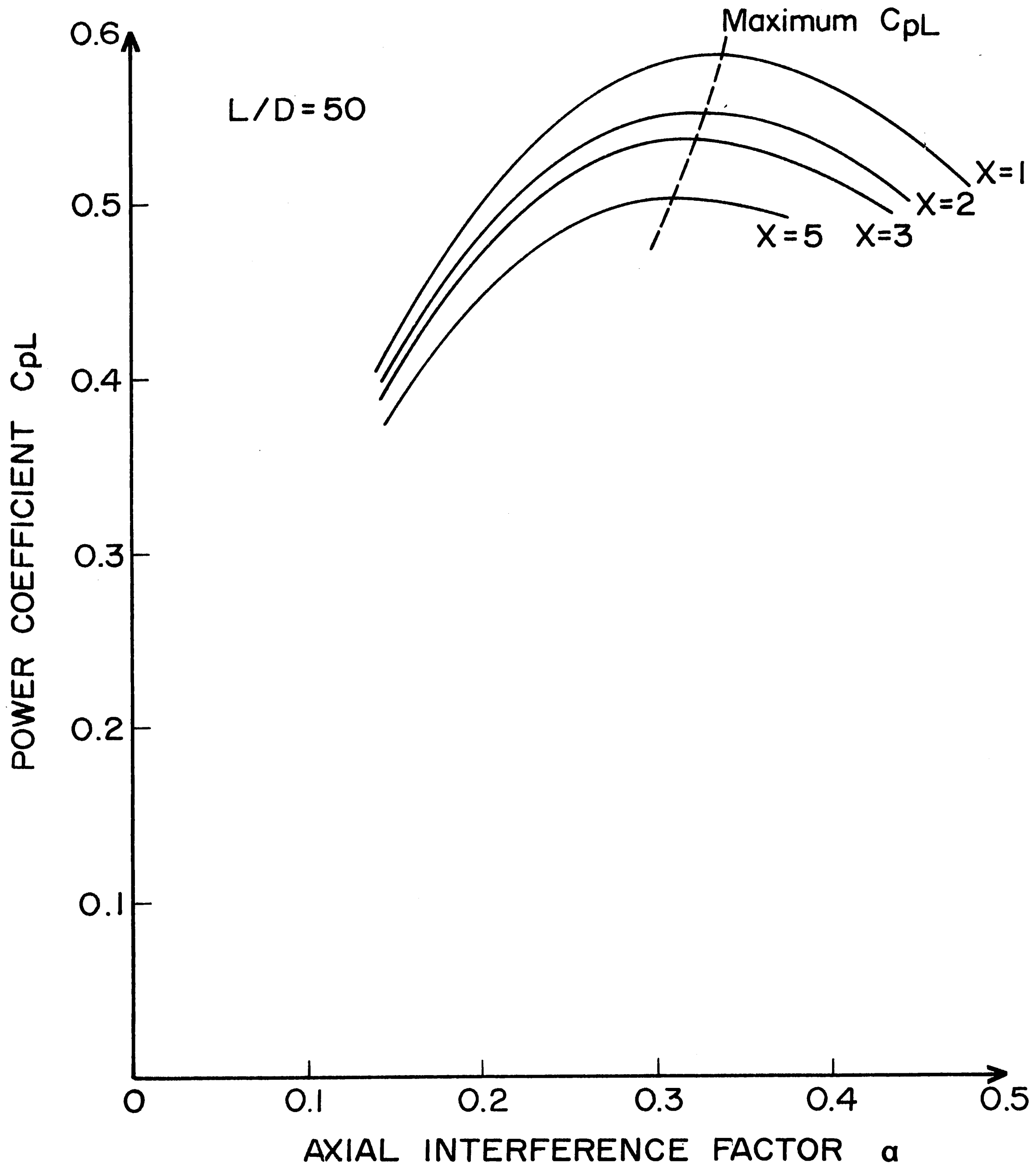

Figure 8. Optimum a, L/D = 50

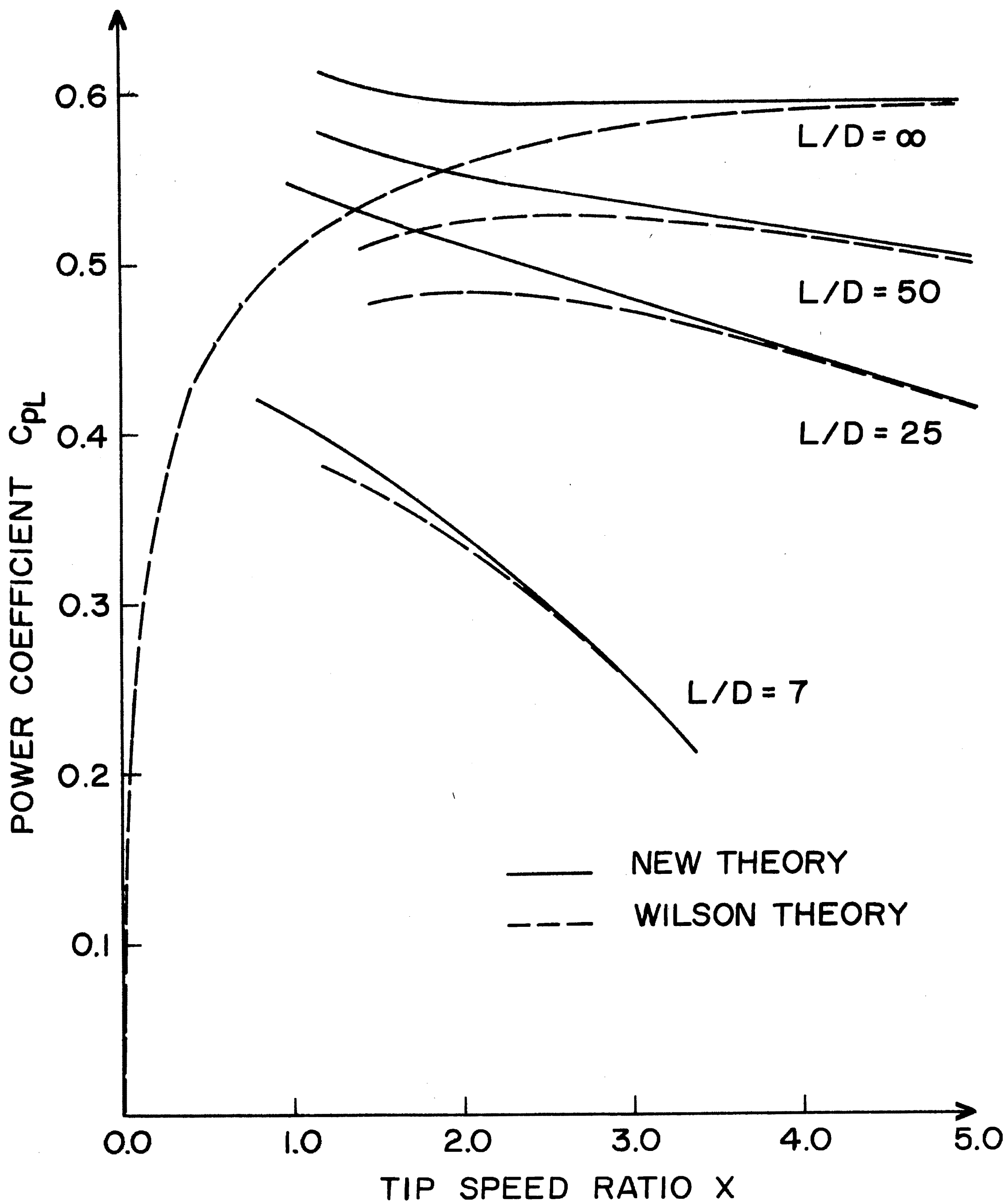

Figure 9. Optimum Tip Speed Ratio

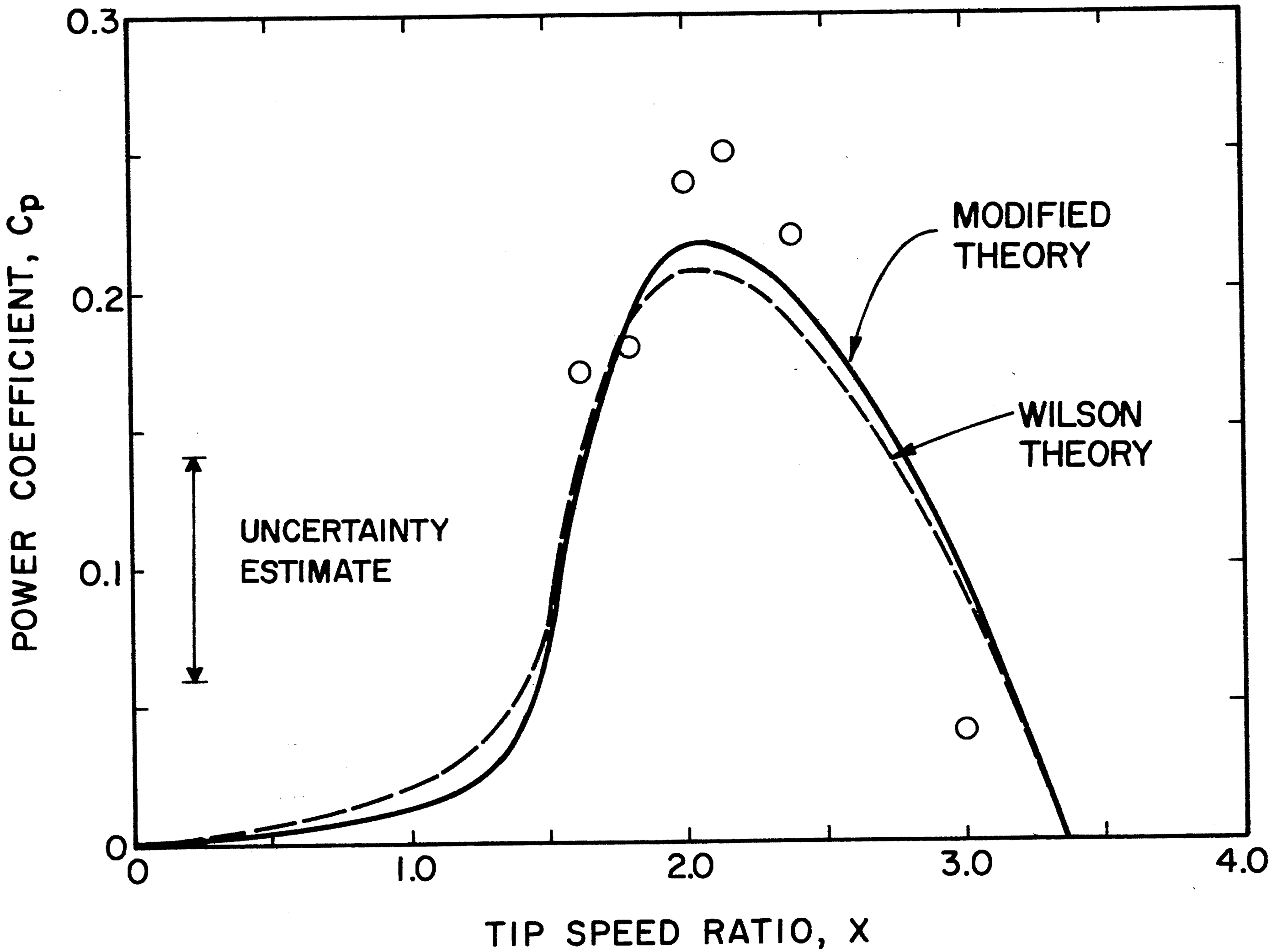

Figure 10. Chalk Turbine; Averaged Experimental Data Compared With Theoretical Models

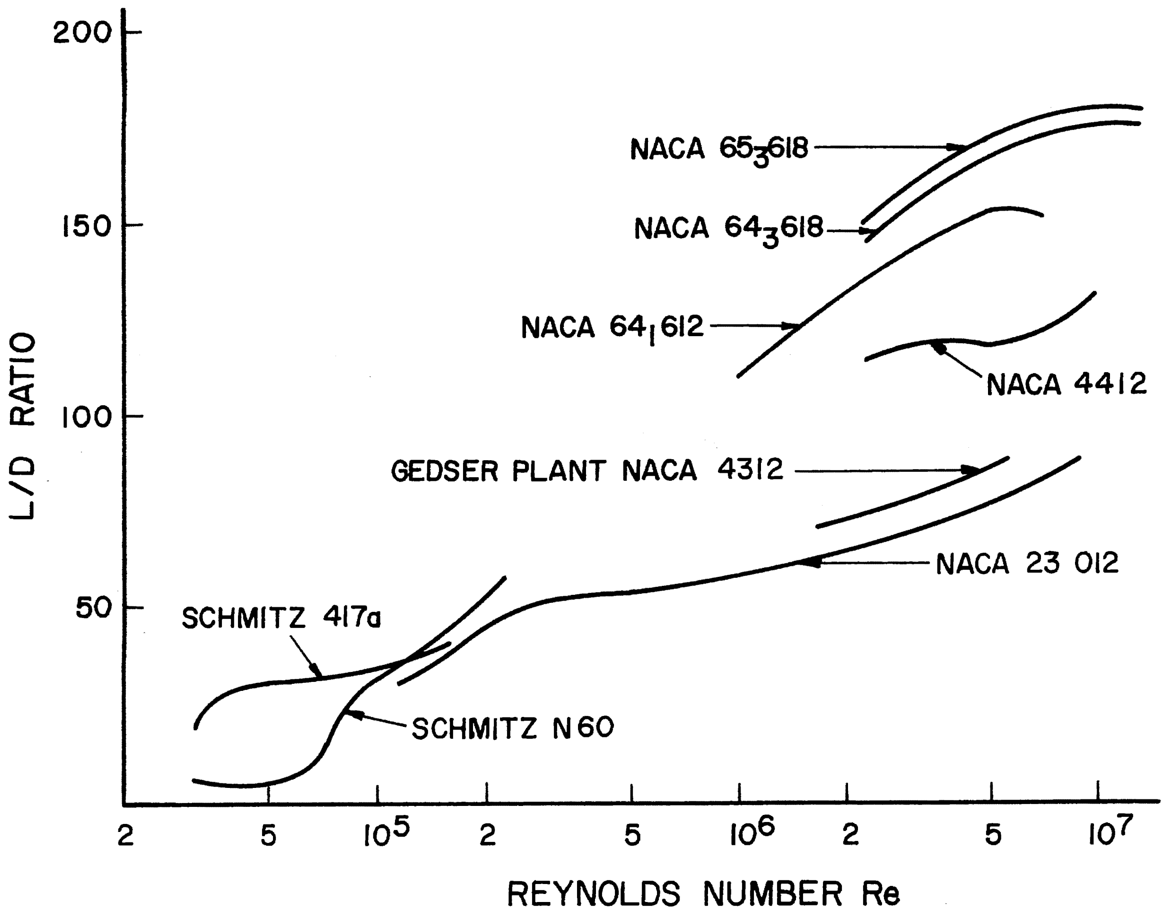

Figure 11. Maximum L/D Ratios Over Range of Reynolds Number

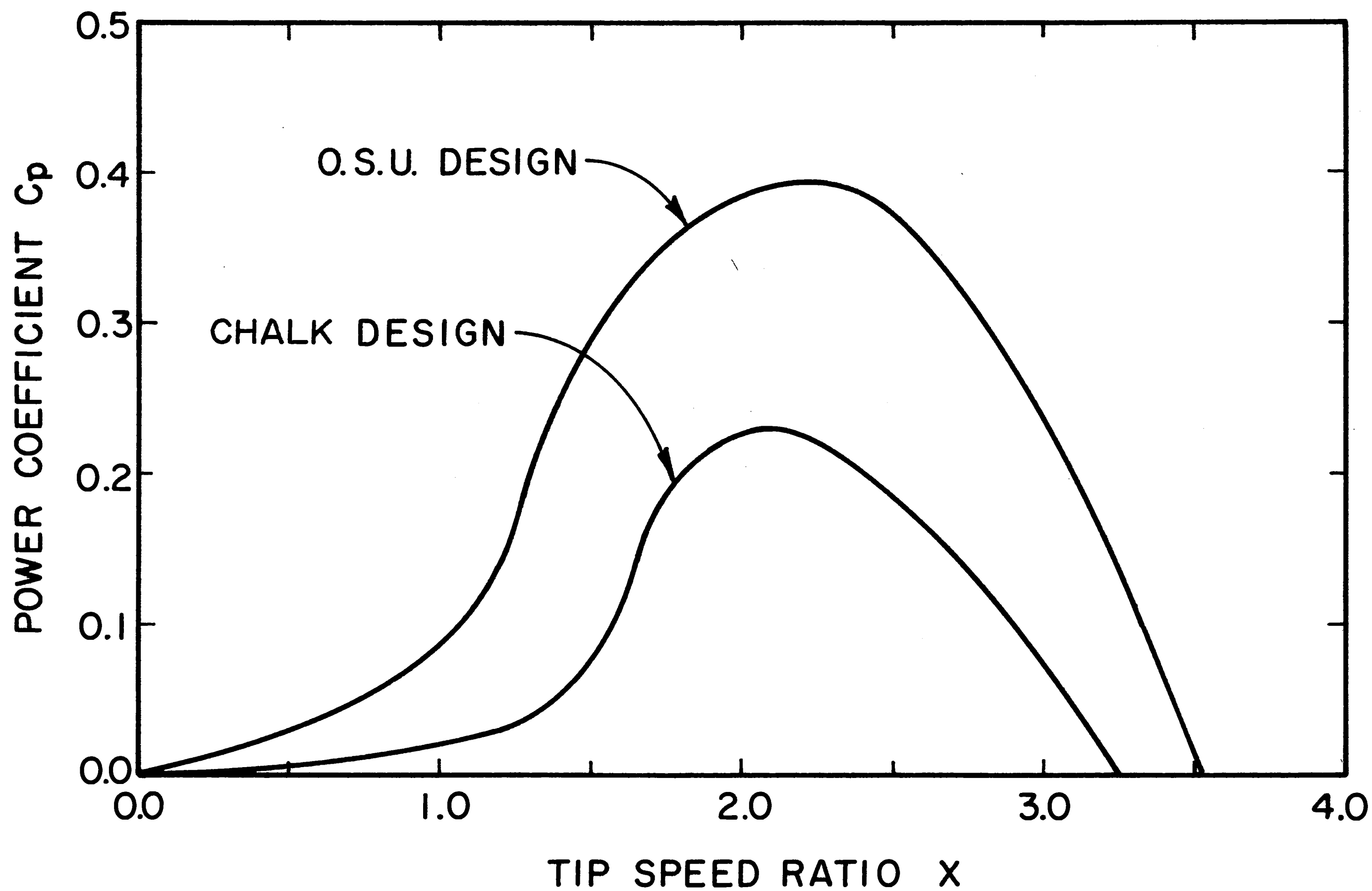

Figure 12. Comparison of Theoretical Power Coefficient of Chalk Turbine With Redesigned Version

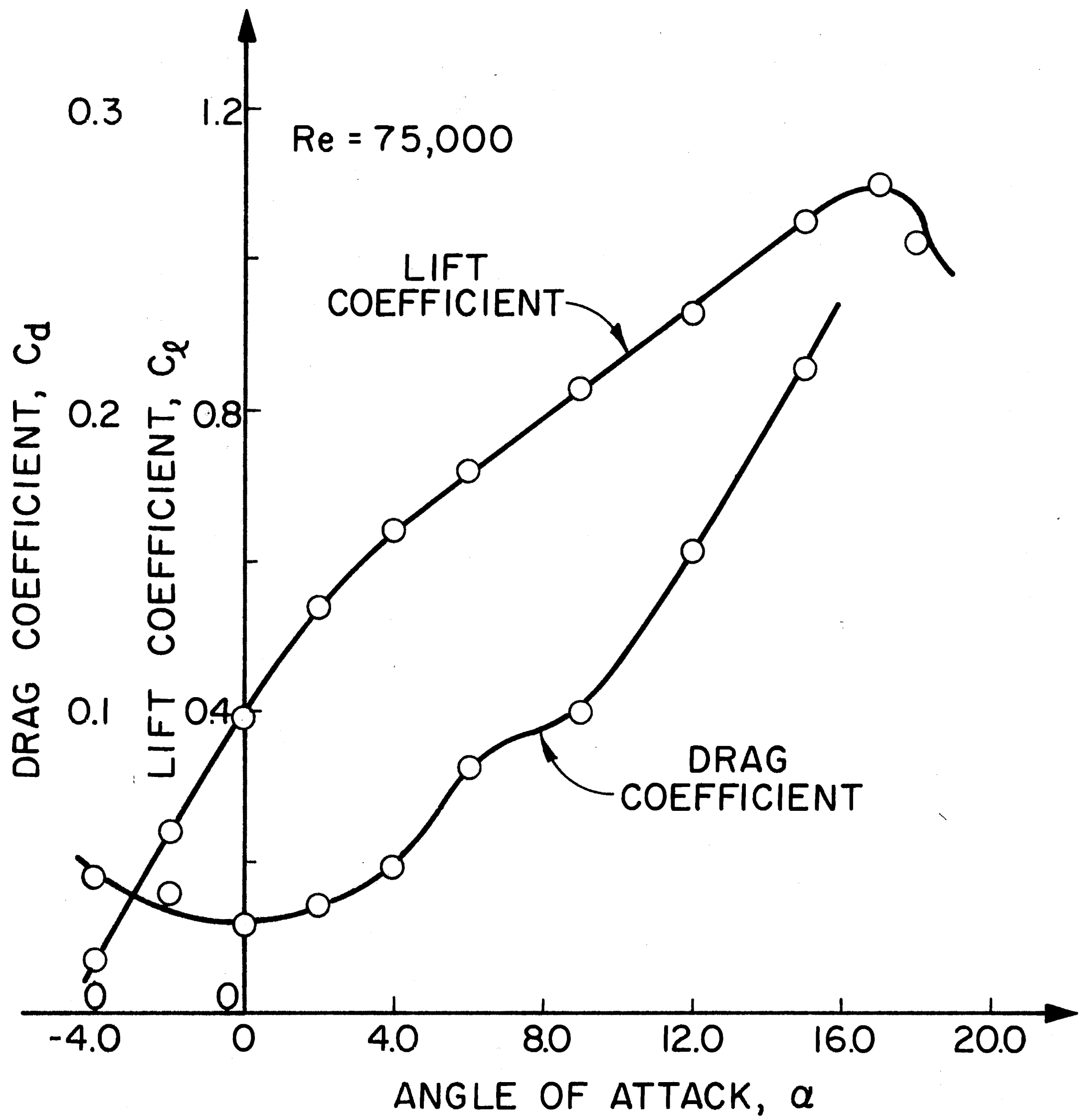

Figure 13. Airfoil Data, Chalk I

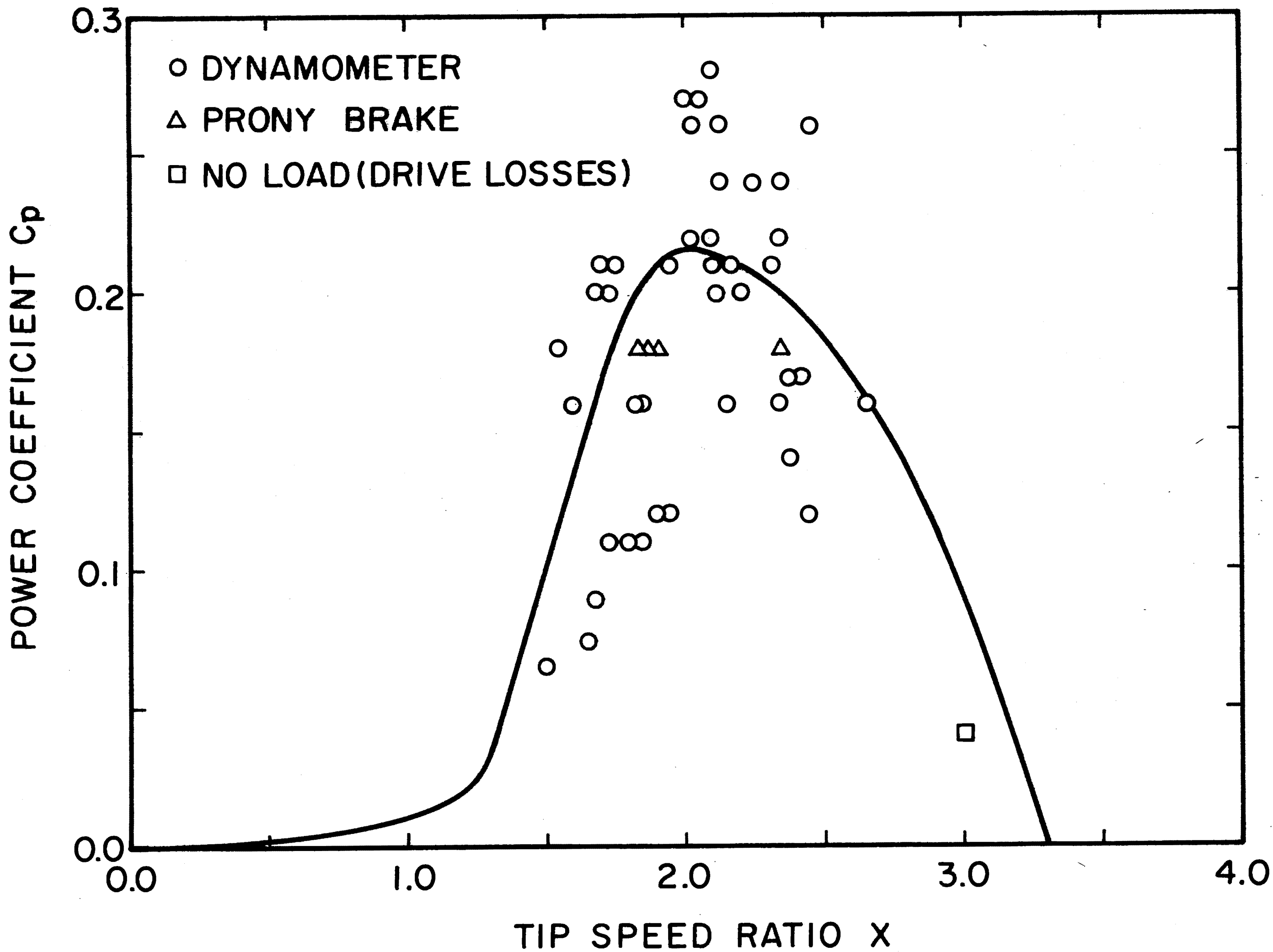

Figure 14. Experimental Power Coefficient Values Compared With the Theoretical Prediction

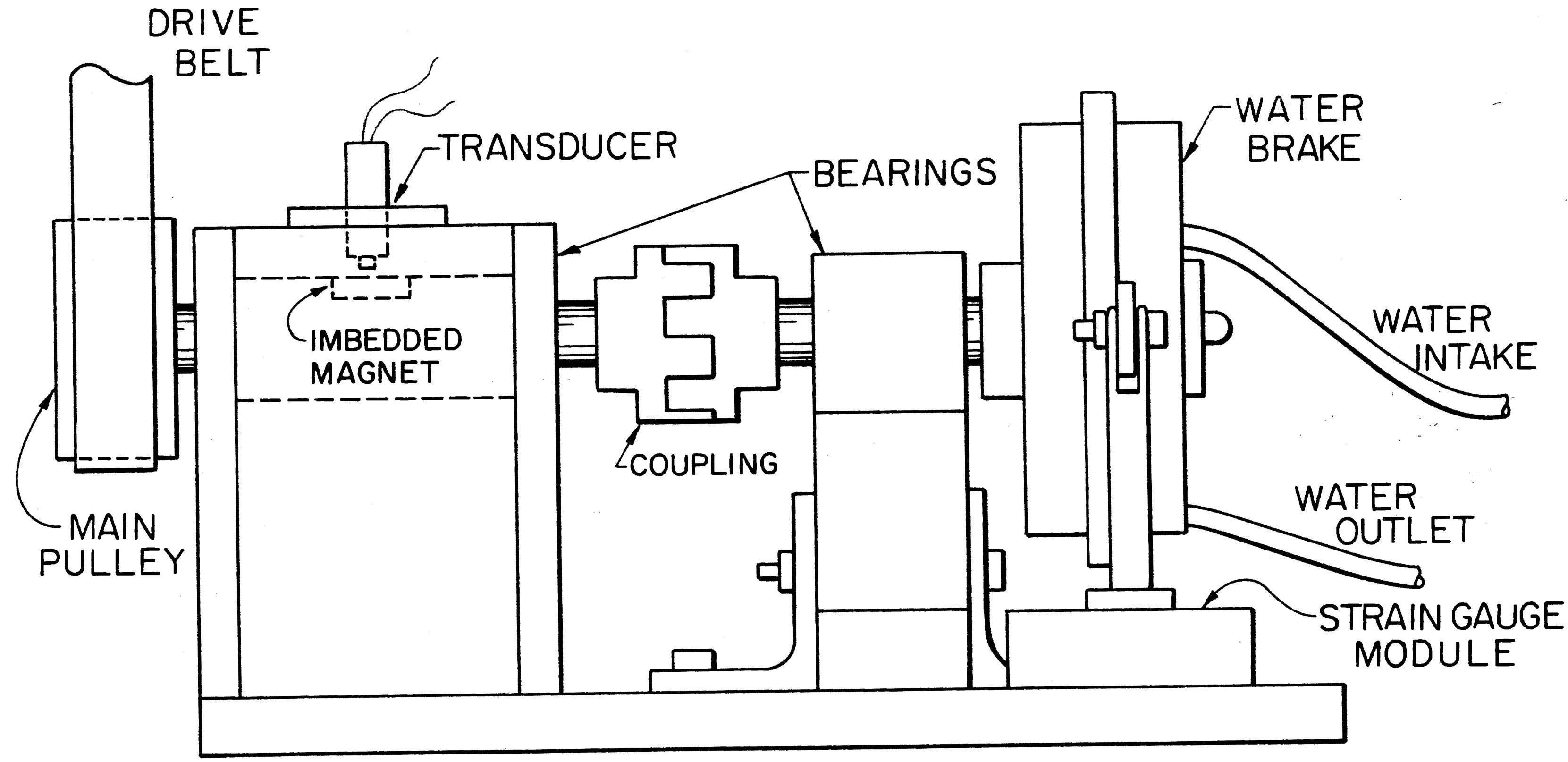

Figure 15. Schematic of Dynamometer

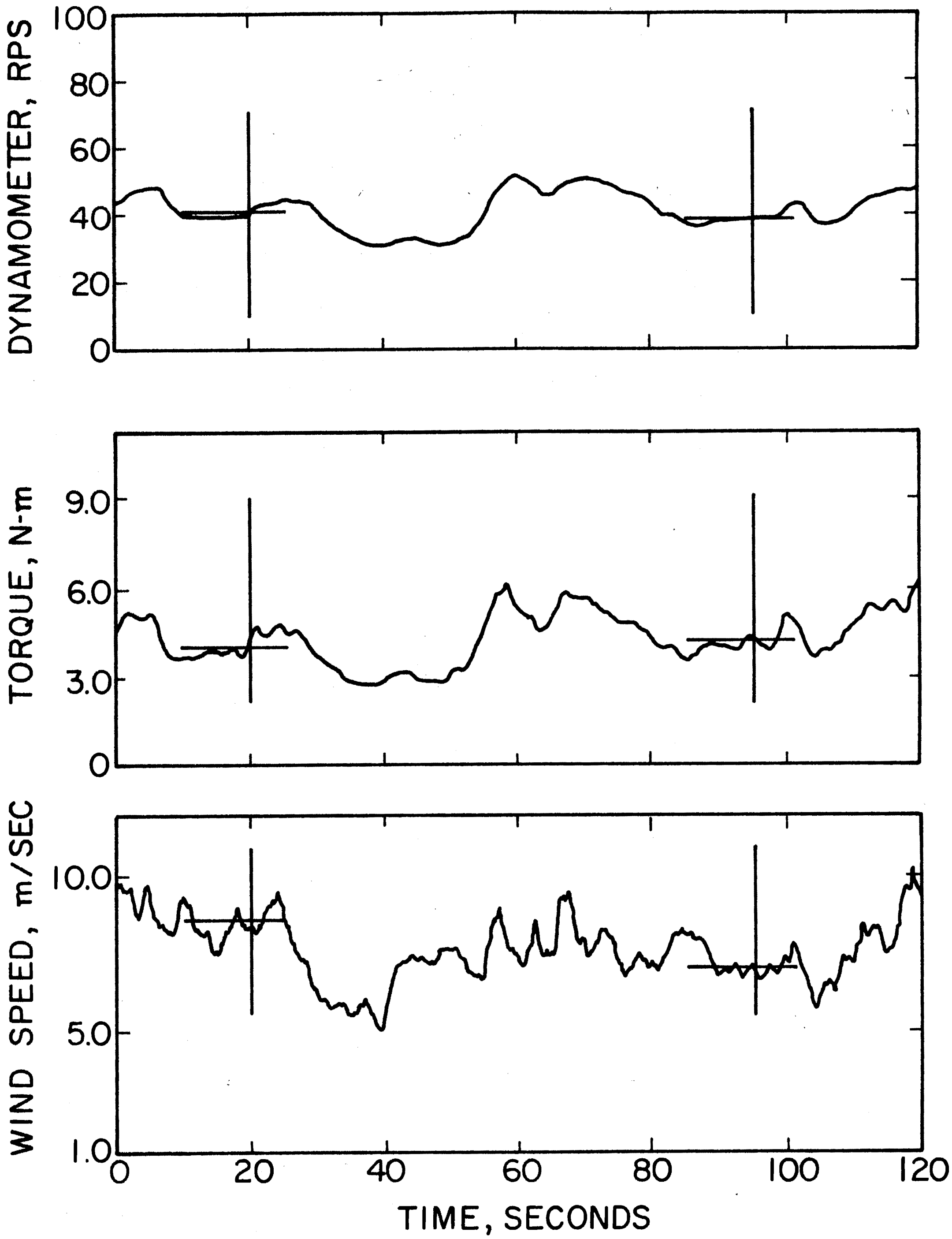

Figure 16. Strip-Chart Readings of Wind Velocity, Torque, and Dynamometer RPS

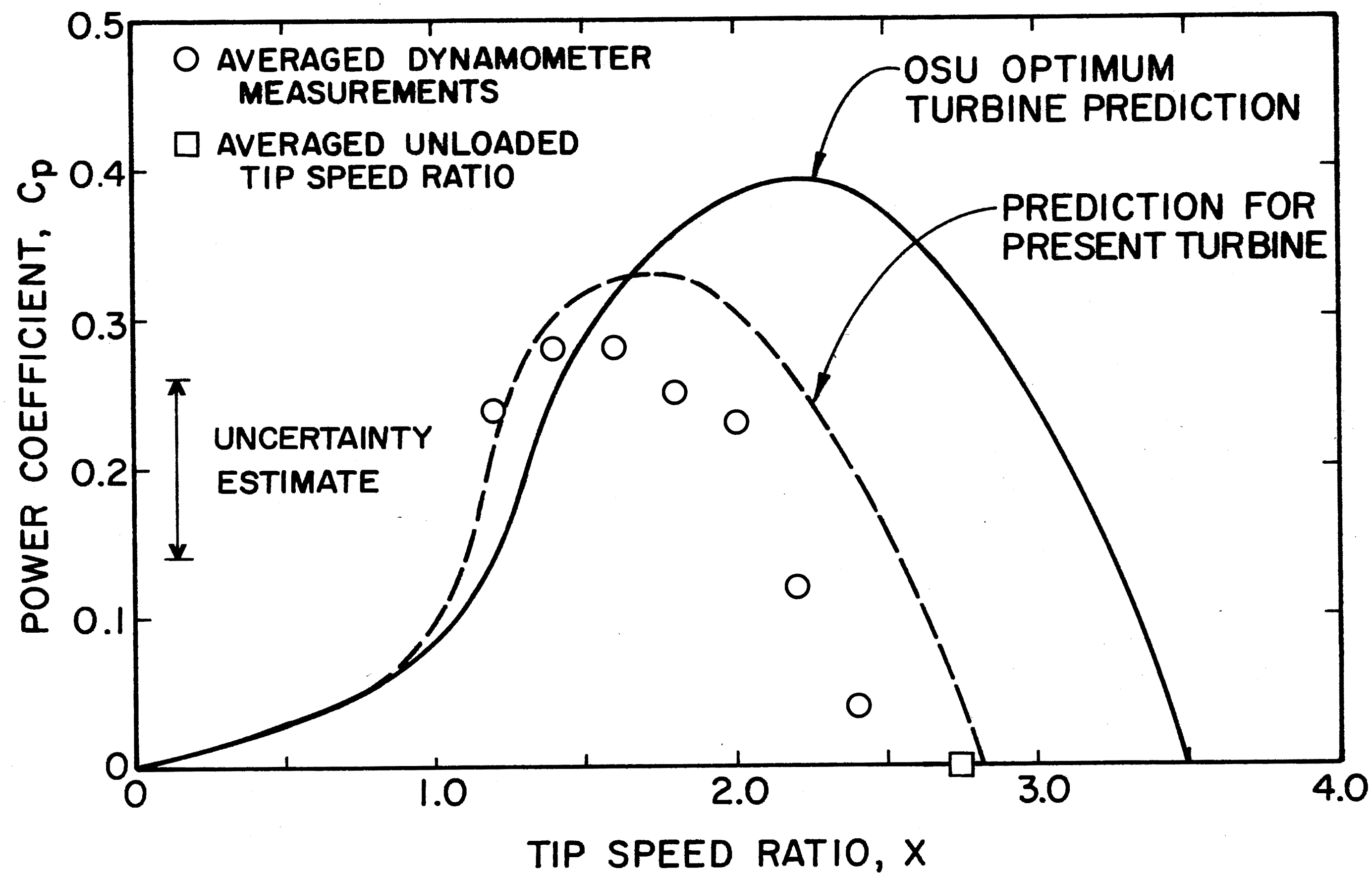

Figure 17. Averaged Experimental Data From the New Oklahoma State University Turbine Compared With Theoretical Predictions

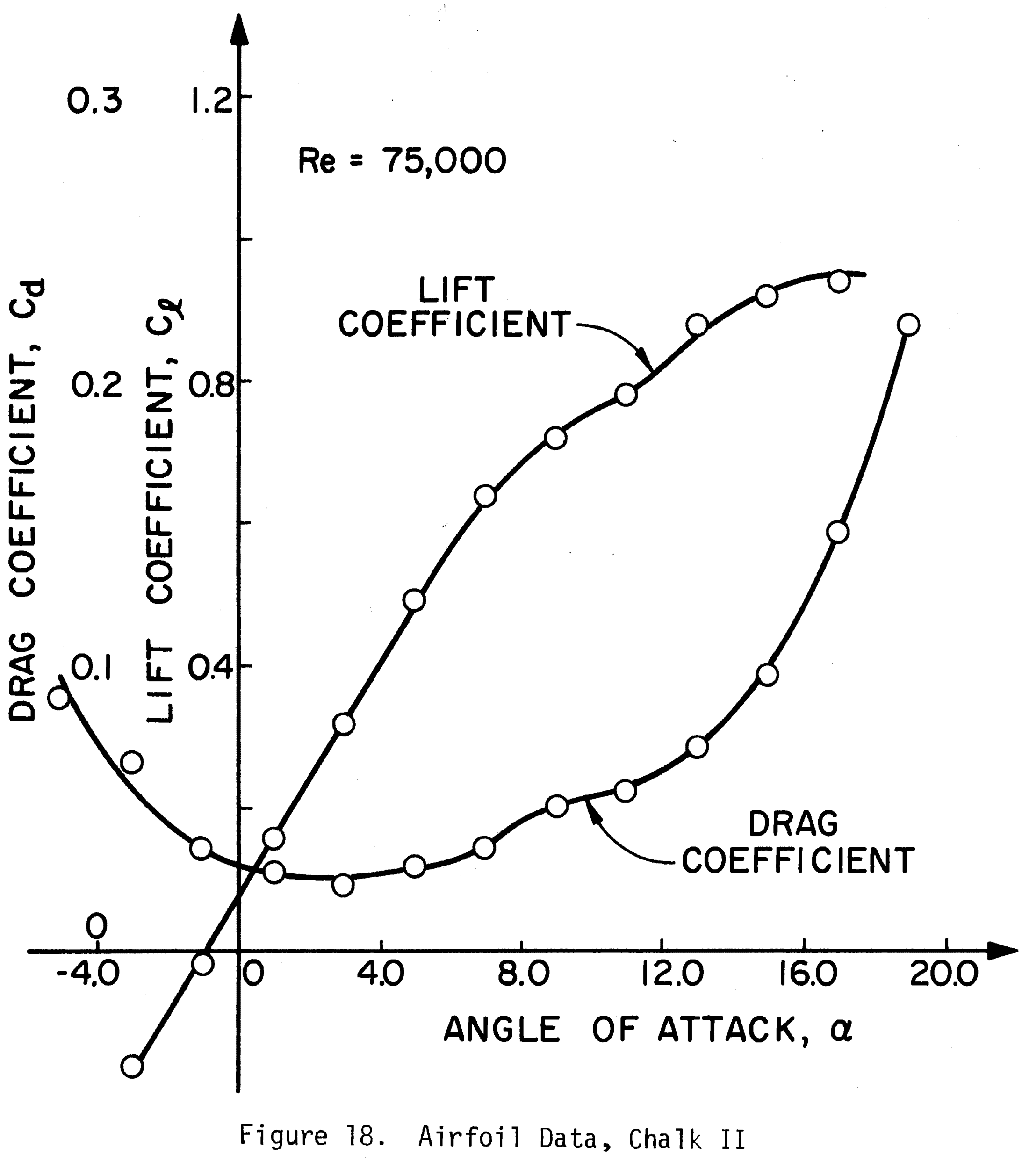

Figure 18. Airfoil Data, Chalk II

APPENDIX B

TABLES

TABLE I

MAIN PROGRAM FLOW DIAGRAM

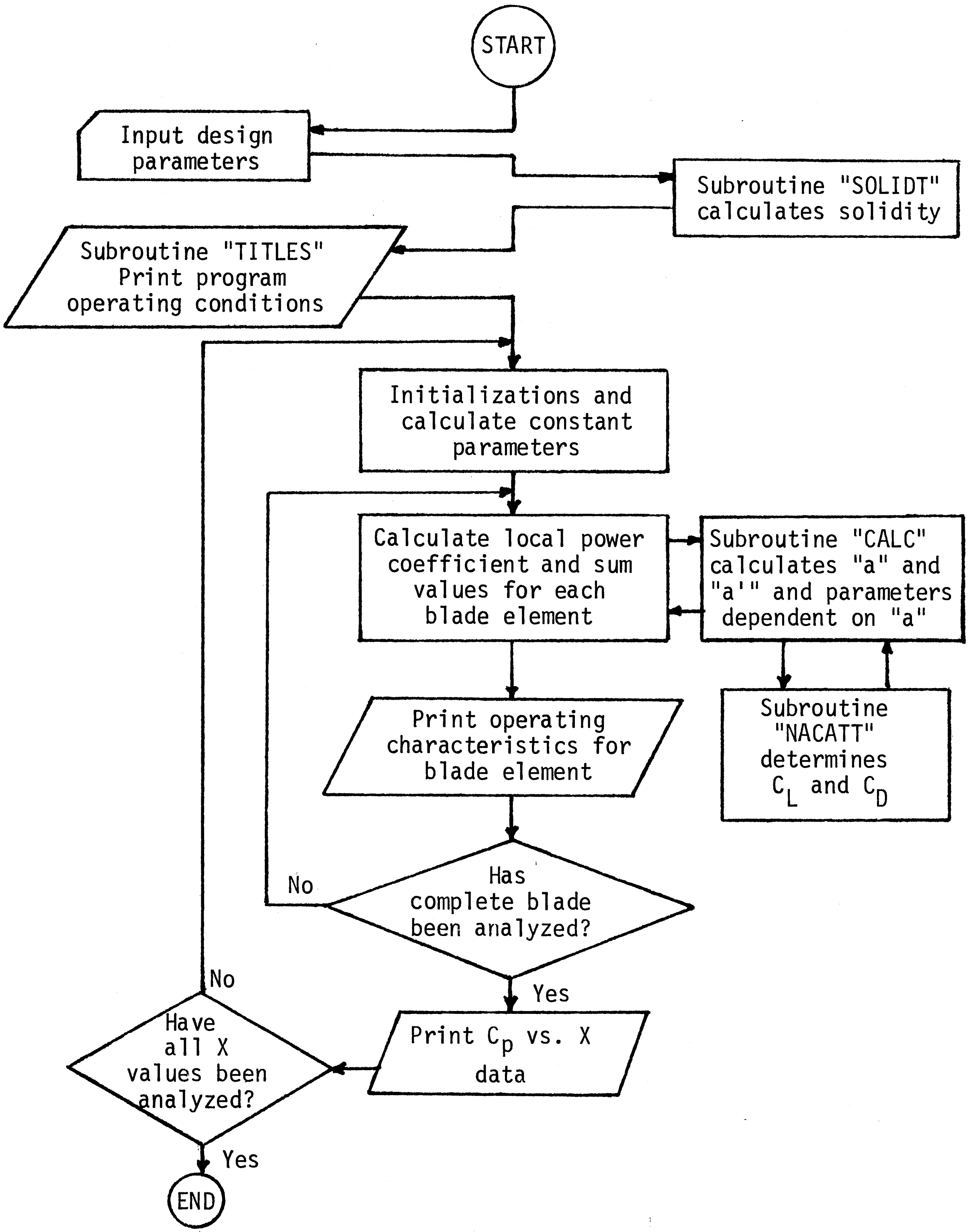

TABLE II

MAIN COMPUTER PROGRAM

```
$JOB TIME=30
C          O.S.U.  REVISED APRIL 1976
C
C       ....... MAIN PROGRAM ......
C
C       INPUT PARAMETERS:
C
C       ....R--RADIUS OF BLADE - FT
C
C       ....DR--INCREMENTAL PERCENTAGE
C
C       ....HB--HUB RADIUS - TF
C
C       ....HH--ALTITUDE OF HUB ABOVE GROUND LEVEL - FT
C
C       ....H--ALTITUDE OF HUB ABOVE SEA LEVEL - FT
C
C       ....B--NUMBER OF BLADES
C
C       ....V--WIND VELOCITY - MPH
C
C       ....X--TIP SPEED RATIO
C
C       ....AMOD--AXIAL INTERFERENCE MODEL CODE
C       ..........0--BOGARD
C       .......... 1 -- WILSON
C
C       ....XIC--TIP SPEED RATIO INCREMENT
C
C       ....AMAX,AMIN--RANGE OF ACCEPTABLE "A" VALUES
C
C       ....ALPHA--ANGLE OF ATTACK
C
C       ....SI--CONING ANGLE - DEGREES
C
C       ....NFS--NUMBER OF ROWS IN TABLE OF CD AND CL VALUES
C
C       ....NF--NUMBER OF INPUTED STATIONS FOR BLADE GEOMETRY
C
C       ....NK--NUMBER OF TIP SPEED RATIOS TO BE ANALYZED
C
C       ....RR(I)--PERCENT RADIUS FOR STATIONS
C
C       ....CI(I)--CHORD FOR STATIONS - FT
C
C       ....THET(I)--TWIST ANGLE FOR STATIONS - DEGREES
C
C       ....CLT(I)--COEF. OF LIFT DATA
C
C       ....CDT(I)--COEF. OF DRAG DATA
C
C       ....AAT(I)--ANGLE OF ATTACK - DEGREES
C
        DIMENSION RR(25), CI(25), THETI(25), AAT(25), CLT(25), CDT(25),
       1XT( 25),TCPY(25),TCTY(25)
        COMMON R,DR,HB,B,V,X,THETP,AMOD,H,SI,GO,OMEGA,RHO,VIS,HL,PI,RX,
       1W,NPROF,APF,T1,T2,T3,T4,T5,T6,T7,T8,TEST,XETA,HH,AMAX,AMIN
C
C       ........READ INPUT DATA.........
C
```

TABLE II (Continued)

```
      READ(5,15)R,DR,HB,HH,H
      READ(5,10)B,V,X,AMOD
      READ(5,15)XIC,AMAX,AMIN,ALPHA,SI
      READ(5,4)NFS,NF,NK
      READ(5 ,20)(RR(I),CI(I),THETI(I),I=1,NF)
      READ(5 ,5) (AAT(I), CLT(I), CDT(I), I=1,NFS)
 4    FORMAT(3I4)
 5    FORMAT(3F10.5)
 10   FORMAT(4F10.3)
 15   FORMAT(5F10.3)
 20   FORMAT(F5.1,5X,F10.5,F10.5)
      PI=3.1415926536
      CALL SOLIDT(RR,CI,NF,B,R,PI,SOLD)
C
C     ....... PRINT INPUT AND TITLES FOR OUTPUT ......
C
      CALL TITLES(RR,CI,THETI,NF,SOLD)
C
C     ....... INITIALIZATION AND CONSTANT PARAMETER CALCULATIONS ......
C
      C=0.2896
      TH=0.124
      TIP=R
      V=V*5280./3600.
      SI=SI*PI/180.
      RHO=0.0023769199*EXP(-0.297*H/10000.)
      VIS=0.0000003719 - 0.00000000204*H/1000.
      NN=(R-HB)/DR +1.
      RX=R
      RLB=(1.-DR)*RX
      DRO=DR
      R=R*COS(SI)
      HB=HB*COS(SI)
      DO 200 K=1,NK
      OMEGA=V*X/R
      WRITE(6,6)X
 6    FORMAT(///,20X,'TIP SPEED RATIO =',F6.3,/)
      WRITE(6,754)
 754  FORMAT(/,11X,'RADIUS',5X,'A',7X,'AP',7X,'FWI',5X,'PHI',5X,'ALPHA',
     14X,'CX',7X,'CY',6X,'CX/CY',5X,'CT',7X,'CP',7X,'XL',7X,'CPL',/)
      DR=(RX-RLB)*COS(SI)
      QX=0.0
      TX=0.0
      FXXP1=0.0
      FYXP1=0.0
      QY=C.0
      TY=C.0
      PY=0.0
      ASTCP=0.0
      A=0.0
      AP=0.0
      DR2=DR/2.0
      RL=R-DR2
      CAT=0.0
      DO 100 L=1,NN
      IF((RL-HB).GE.DR2) GO TO 311
      ASTCP=ASTOP+1.
      IF(ASTOP.GE.2.) GO TO 93
      RL=RL+DR2
      DP=(RL-HB)
```

TABLE II (Continued)

```
          RL=RL-DR/2.0
    311   CONTINUE
          CALL SEARCH(RL,RR,CI,THETI,NF,C,THET)
          CALL CALC(RL,C,THET,FXY,FYY,XMXXP,XMYXP,QXP,TXP,RE,PHIR,CL,
         1CD,CX,CY,A,AP,XL,AK,ALPHA,F,CLF,CAT,AAT,CLT,CDT,NFS,SOLD,TH,FWI)
    C
    C     .......CALCULATION OF TOTAL AND LOCAL POWER COEFFICIENTS
    C
          TYL=TXP*DR
          PYL=OMEGA*QXP*DR
    97    TY=TY+TYL
          PY=PY+PYL
          CTY=TY/(.5*RHO*V**2*PI*RX**2)
          CPY=PY/(.5*RHO*V**3*PI*RX**2)
          CTYL=CTY*TYL/TY
          CPYL=CPY*PYL/PY
          CPYLA=PYL/(.5*RHO*V**3*PI*((RL+DR/2.)**2-(RL-DR/2.)**2))
          TP=PY/737.6
          PHIO=PHIR*180./PI
          ALPHA=ALPHA*180./PI
          PR=RL/(RX*COS(SI))
          RCXCY=CX/CY
    C
    C     .......PRINT OPERATIONAL CHARACTERISTICS OF THE TURBINE
    C
          WRITE(6,755)RL,A,AP,FWI,PHIO,ALPHA,CX,CY,RCXCY,CTYL,CPYL,XL,CPYLA
    755   FORMAT(/,11X,F5.2,4X,F5.3,3X,F6.3,3X,F6.3,4X,F4.1,4X,F4.1,3X,F6.3,
         14X,F5.3,3X,F6.3,4X,F5.3,3X,F6.3,4X,F5.3,3X,F6.3)
          RL=RL-DR
    100   CONTINUE
    93    CONTINUE
          XT(K)=X
          TCPY(K)=CPY
          TCTY(K)=CTY
          X=X+XIC
    200   CONTINUE
    C
    C     ....... PRINT OUTPUT ......
    C
          WRITE(6,750)
    750   FORMAT(//,14X,'TIP SPEED RATIO',4X,'POWER COEFFICIENT',5X,'THRUST
         1COEFFICIENT',/)
          WRITE(6,751)(XT(I),TCPY(I),TCTY(I),I=1,NK)
    751   FORMAT(,19X,F5.2,13X,F7.5,16X,F7.5)
    C
          STOP
          END

          SUBROUTINE TITLES(RR,CI,THETI,NF,SOLD)
    C
    C     ...... TITLES - PRINTS OUT INPUT DATA AND PROGRAM OPERATING
    C     CONDITIONS IN A DESCRIPTIVE FORM.
    C
          DIMENSION RR(25),CI(25),THETI(25)
          COMMON R,DR,HB,B,V,X,THETP,AMOD,H,SI,GO,OMEGA,RHO,VIS,HL,PI,RX,
         1W,NPROF,APF,T1,T2,T3,T4,T5,T6,T7,T8,TEST,XETA,HH,AMAX,AMIN
          WRITE(6 ,25)
    25    FORMAT('1')
          WRITE(6 ,1)
    1     FORMAT(' THEORETICAL PERFORMANCE OF A PROPELLER TYPE WIND TURBINE'
```

TABLE II (Continued)

```
          1)
          WRITE(6 ,9)
     9    FORMAT(///,5X,' BLADE DESIGN:')
          WRITE(6,690)B
   690    FORMAT(/,15X,' NO. OF BLADES =',F5.1)
          WRITE(6 ,11) R
    11    FORMAT(/,15X,' TIP RADIUS - FT = ',F7.4)
          WRITE(6 ,12) HB
    12    FORMAT(/,15X,' HUB RADIUS - FT = ',F7.4)
          WRITE(6,13)
    13    FORMAT(/,16X,'AIRFOIL PROFILE :  CHAULK SPECIAL 1 ')
          WRITE(6 ,15)
    15    FORMAT(//,10X,' CHORD AND TWIST DISTRIBUTION')
          WRITE(6 ,16)
    16    FORMAT(//,16X,' PERCENT RADIUS',5X,'CHORD-FT',10X,'TWIST-DEG')
          WRITE(6 ,17) (RR(I),CI(I),THETI(I),I=1,NF)
    17    FORMAT(/,20X,F5.1,8X,F10.5,10X,F10.5)
          WRITE(6 ,18)
    18    FORMAT(///,5X,' PROGRAM OPERATING CONDITIONS:')
          WRITE(6 ,19)DR
    19    FORMAT(//,15X,' INCREMENTAL PERCENTAGE =',F7.4)
   141    IF(AMOD.EQ.0.0) GO TO 300
          WRITE(6 ,310)
          GO TO 340
   300    WRITE(6 ,320)
   340    CONTINUE
   657    WRITE(6 ,659)
   777    WRITE(6 ,779)
   310    FORMAT(//,15X,' WILSON AXIAL INTERFERENCE METHOD USED')
   320    FORMAT(//,15X,' BOGARD AXIAL INTERFERENCE METHOD USED')
   659    FORMAT(//,15X,' NO TIP LOSS MODEL')
   779    FORMAT(//,15X,' NO HUBLOSS MODEL USED')
          RETURN
          END

          SUBROUTINE CALC(RL,C,THET,FXF,FYF,XMFXF,XMFYF,QF,TF,RE,PHIR,CL,
         1CD,CX,CY,A,AP,XL,AK,ALPHA,F,CLF,CAT,AAT,CLT,CDT,NFS,SOLD,TH,FWI)
     C
     C    ....... CALC - DETERMINES THE AXIAL AND ANGULAR INTERFERENCE
     C    FACTORS AT A GIVEN RADIUS AND DETERMINES FUNCTIONS DEPENDENT
     C    UPON THESE PARAMETERS.
     C
          DIMENSION AAT(25),CLT(25),CDT(25)
          COMMON R,DR,HB,B,V,X,THETP,AMOD,H,SI,GO,OMEGA,RHO,VIS,HL,PI,RX,
         1W,NPROF,APF,T1,T2,T3,T4,T5,T6,T7,T8,TEST,XETA,HH,AMAX,AMIN
          XL=RL*OMEGA/V
          IF(A.GT..5) AP=0.
          IF(A.GT..5) A=0.
          RH=HB
   200    DO 10 J=1,100
          BETA=A
          DELTA=AP
          IF (AP.LT..001)GO TO 12
     C
     C    .......DETERMINATION OF THE BLADE INTERFERENCE FACTOR (EWI) AND THE
     C           FAR WAKE INTERFERENCE FACTOR (FWI)
     C
          EWI=1.0-A
          RF=2.0*(AP*XL)**2.0
          FWI=(EWI-RF/EWI)-((RF/EWI)**2+(1.0-EWI)**2)**0.5
```

TABLE II (Continued)

```
           WIR=(EWI-FWI)/(EWI-FWI*EWI)
           GO TO 13
  12       WIR=0.0
           RF=0.0
  13       PHI=ATAN((1.-A)*COS(SI)/((1.+AP)*XL))
           PHIAA=ABS(PHI)
           PHIR=PHI
           ALPHA=PHI-THET
           DDTC=ATAN((1.-A)/(XL*(1.+2.*AP)))-ATAN((1.-A)/XL)
           DA1=DDTC/4.
           DA2=((4./15.)*(SOLD*TH)/X)/((1./X)**2+(RL/R)**2)
           DALPHA=DA1+DA2
           ALPHA=ALPHA-DALPHA
  C
  C        ...... CALCULATION OF SECTIONAL LIFT AND DRAG COEFFICIENTS
  C
  510      CALL NACATT(RL,RX,SI,ALPHA,CL,CD,W,AAT,CLT,CDT,NFS,SOLD)
  C
  670      F=1.0
  667      CX=CL*SIN(PHI)-CD*COS(PHI)
           CY=  CL*COS(PHI)+CD*SIN(PHI)
           CXX=CX
           CYY=CY
           SIG=(B*C)/(PI*RL)
           IF(AMOD.EQ.0.) GO TO 575
  C
  C        .......WILSON AXIAL INTERFERENCE METHOD
  C
           VBR=((.125*SIG*CYY)*(COS(SI)**2))/(SIN(PHI)**2)
           VAR=(0.125*SIG*CXX)/(F*SIN(PHI)*COS(PHI))
           CAN=F*F+4.*VBR*F*(1.-F)
           A=(2.*VBR+F-SQRT(CAN))/(2.*(VBR+F*F))
           AP=VAR/(1.-VAR)
           GO TO 580
  C
  C        .......BOGARD AXIAL INTERFERENCE METHOD
  C
  575      VBR=0.125*SIG*CYY*(COS(SI)**2)
           VAR=0.125*SIG*CXX
           A=(SIG*CY-8.0*RF*WIR*SIN(PHI)**2)/(8.0*SIN(PHI)**2+SIG*CY)
           AP=VAR/(F*SIN(PHI)*COS(PHI)-VAR)
  580      PCR=RL/(RX*COS(SI))
  C        ....... DAMPENING OF AXIAL AND ANGULAR INTERFERENCE FACTOR
  C        ITERATIONS.
  C
           IF(J-4) 30,40,90
  90       IF(J-10)  30,40,110
  110      IF(J-15) 30,40,30
  40       A=(A+BETA)*.5
           AP=(AP+DELTA)*.5
  30       CONTINUE
  C
  C        ....... TEST FOR CONVERGENCE ......
  C
           IF(AP.EQ.0.0) GO TO 70
           IF(ABS((AP-DELTA)/AP).LE..0001) GO TO 70
           GO TO 10
  70       IF(ABS((A-BETA)/A).LE..0001) GO TO 50
  C
  10       CONTINUE
```

TABLE II (Continued)

```
99    WRITE(6,756)RL
756   FORMAT(/,11X,F5.2,10X,'NO CONVERGENCE')
50    CONTINUE
      PCCR=RL/RX
C
C     ....... CALCULATION OF FUNCTIONS DEPENDENT UPON AXIAL AND
C     ANGULAR INTERFERENCE FACTORS.
C
      W=SQRT((((1.-A)*V*COS(SI))**2+((1.+AP)*RL*OMEGA)**2)
      CONST=(0.5*RHO*(W**2)*C)
      FXF=CONST*CX
      FYF=CONST*CY
      CT1=(0.5*RHO*B*C)*(W*W)
      QF=CT1*RL*CX
      TF=CT1*CY*COS(SI)
C
402   RETURN
      END

      SUBROUTINE NACATT(RL,RX,SI,ALPHA,CL,CD,W,AAT,CLT,CDT,NFS,SOLD)
C
C     ...... NACATT - IS AN INTERPOLATING SUBROUTINE TO INTERPOLATE
C     AIRFOIL DATA INPUTED IN TABLE FORM.
C
      DIMENSION AAT(25),CLT(25),CDT(25)
      A=ALPHA*180./3.141593
      DO 20 I=1,NFS
      IF(A.LE.AAT(1)) GO TO 100
      IF(A.LE.AAT(I)) GO TO 10
      IF(I.EQ.NFS) GO TO 30
20    CONTINUE
10    J=I+1
      PER=(A-AAT(J-1))/(AAT(J-2)-AAT(J-1))
      CL=PER*(CLT(J-2)-CLT(J-1))+CLT(J-1)
      CD=PER*(CDT(J-2)-CDT(J-1))+CDT(J-1)
      GO TO 40
30    CL=CLT(NFS)
      CD=CDT(NFS)
      GO TO 40
100   CL=CLT(1)
      CD=CDT(1)
40    CD=(CD*3.475+1.2*.124)/3.475
      RETURN
      END

      SUBROUTINE SEARCH(RL,RR,CI,THETI,NF,C,THET)
      DIMENSION RR(25),CI(25),THETI(25)
      COMMON R,DR,HB,B,V,X,THETP,AMOD,H,SI,GO,OMEGA,RHO,VIS,HL,PI,RX,
     1W,NPROF,APF,T1,T2,T3,T4,T5,T6,T7,T8,TEST,XETA,HH,AMAX,AMIN
      DO 20 I=1,NF
      RRV=RL/(RX*COS(SI))*100.
      IF(RRV.EQ.RR(1)) GO TO 50
      IF(RRV.GE.RR(I)) GO TO 10
      IF(I.EQ.NF) GO TO 30
20    CONTINUE
10    J=I+1
      PER=(RRV-RR(J-1))/(RR(J-2)-RR(J-1))
      C=PER*(CI(J-2)-CI(J-1))+CI(J-1)
      THET=PER*(THETI(J-2)-THETI(J-1))+THETI(J-1)
      GO TO 40
```

TABLE II (Continued)

```
30    C=CI(NF)
      THET=THETI(NF)
      GO TO 40
50    C=CI(1)
      THET=THETI(1)
40    THET=THET*PI/180.
      RETURN
      END

      SUBROUTINE SOLIDT(RR,CI,NF,B,R,PI,SOLD)
C
C     ..... SOLIDTY - DETERMINES THE TOTAL SOLIDITY OF THE WIND
C     TURBINE DESIGN.
C
      DIMENSION RR(25),CI(25)
      NFX=NF-1
      S1=0.
      DO 20 I=1,NFX
      SOL=((CI(I+1)+CI(I))/2.)*(RR(I)-RR(I+1))*R/100.
      S1=S1+SOL
20    CONTINUE
      SOLD=B*S1/(PI*R**2)
      RETURN
      END

$ENTRY
```

TABLE III

PROGRAM INPUTS

<table>
<tr><th colspan="8">Input Format</th></tr>
<tr><th></th><th colspan="7">Columns</th></tr>
<tr><th>Cards</th><th colspan="3">1-10</th><th>11-20</th><th>21-30</th><th>31-40</th><th>41-50</th></tr>
<tr><td>1</td><td colspan="3">R</td><td>DR</td><td>HB</td><td>HH</td><td>H</td></tr>
<tr><td>2</td><td colspan="3">B</td><td>V</td><td>X</td><td>AMOD</td><td>SI</td></tr>
<tr><td>3</td><td colspan="3">XIC</td><td>AMAX</td><td>AMIN</td><td>ALPHA</td><td></td></tr>
<tr><td></td><td>1-4</td><td>5-8</td><td>9-12</td><td></td><td></td><td></td><td></td></tr>
<tr><td>4</td><td>NFS</td><td>NF</td><td>NK</td><td></td><td></td><td></td><td></td></tr>
<tr><td>5</td><td colspan="3">PR(I)</td><td>CI(I)</td><td>THETI(I)</td><td></td><td></td></tr>
<tr><td>4 + NF</td><td colspan="3">↓</td><td>↓</td><td>↓</td><td></td><td></td></tr>
<tr><td>5 + NF</td><td colspan="3">AAT(I)</td><td>CLT(I)</td><td>CDT(I)</td><td></td><td></td></tr>
<tr><td>4 + NF + NFS</td><td colspan="3">↓</td><td>↓</td><td>↓</td><td></td><td></td></tr>
</table>

TABLE IV

CHALK TURBINE

THEORETICAL PERFORMANCE OF A PROPELLER TYPE WIND TURBINE

BLADE DESIGN:

NO. OF BLADES = 48.0

TIP RADIUS - FT = 7.6250

HUB RADIUS - FT = 2.6250

AIRFOIL PROFILE : CHAULK SPECIAL 1

CHORD AND TWIST DISTRIBUTION

PERCENT RADIUS	CHORD-FT	TWIST-DEG
100.0	0.28960	7.00000
34.4	0.28960	18.00000

PROGRAM OPERATING CONDITIONS:

INCREMENTAL PERCENTAGE = 0.0700

BOGARD AXIAL INTERFERENCE METHOD USED

NO TIP LOSS MODEL

NO HUBLOSS MODEL USED

TIP SPEED RATIO = 2.000

RADIUS	A	AP	FWI	PHI	ALPHA	CX	CY	CX/CY	CT	CP	XL	CPL
7.36	0.525	0.030	-0.065	13.4	5.7	0.086	0.805	0.107	0.137	0.028	1.930	0.210
6.82	0.484	0.038	0.015	15.5	6.7	0.115	0.845	0.136	0.127	0.031	1.790	0.248
6.29	0.454	0.046	0.071	17.6	7.6	0.145	0.880	0.165	0.117	0.032	1.650	0.276
5.76	0.431	0.057	0.112	19.6	8.6	0.177	0.917	0.193	0.107	0.031	1.510	0.294
5.22	0.414	0.069	0.139	21.8	9.7	0.212	0.956	0.221	0.097	0.029	1.370	0.306
4.69	0.395	0.083	0.175	24.4	11.2	0.245	0.989	0.248	0.086	0.026	1.230	0.304
4.16	0.391	0.104	0.175	26.9	12.7	0.285	1.039	0.274	0.077	0.023	1.090	0.301
3.62	0.311	0.121	0.338	32.9	17.8	0.322	0.929	0.346	0.061	0.020	0.950	0.300
3.09	0.306	0.071	0.378	38.6	21.7	0.181	0.976	0.186	0.049	0.007	0.810	0.130
2.72	0.305	0.062	0.385	42.5	24.6	0.143	0.995	0.144	0.016	0.002	0.714	0.088

TABLE V

NEW TURBINE DESIGN

THEORETICAL PERFORMANCE OF A PROPELLER TYPE WIND TURBINE

BLADE DESIGN:

NO. OF BLADES = 48.0

TIP RADIUS - FT = 7.6250

HUB RADIUS - FT = 2.6250

AIRFOIL PROFILE : CHAULK SPECIAL 1

CHORD AND TWIST DISTRIBUTION

PERCENT RADIUS	CHORD-FT	TWIST-DEG
100.0	0.28960	14.50000
34.4	0.28960	31.00000

PROGRAM OPERATING CONDITIONS:

INCREMENTAL PERCENTAGE = 0.0700

BOGARD AXIAL INTERFERENCE METHOD USED

NO TIP LOSS MODEL

NO HUBLOSS MODEL USED

TIP SPEED RATIO = 2.000

RADIUS	A	AP	FWI	PHI	ALPHA	CX	CY	CX/CY	CT	CP	XL	CPL
7.36	0.331	0.042	0.317	18.4	3.1	0.162	0.676	0.239	0.123	0.057	1.930	0.422
6.82	0.306	0.048	0.366	20.3	3.3	0.185	0.678	0.273	0.110	0.054	1.790	0.430
6.29	0.286	0.056	0.404	22.3	3.5	0.211	0.683	0.309	0.098	0.050	1.650	0.434
5.76	0.270	0.065	0.432	24.4	4.0	0.240	0.692	0.346	0.088	0.046	1.510	0.434
5.22	0.259	0.077	0.449	26.6	4.5	0.272	0.706	0.385	0.078	0.041	1.370	0.430
4.69	0.253	0.094	0.456	29.0	5.3	0.310	0.727	0.426	0.070	0.037	1.230	0.427
4.16	0.251	0.118	0.451	31.6	6.3	0.355	0.755	0.470	0.063	0.032	1.090	0.422
3.62	0.253	0.154	0.430	34.3	7.6	0.406	0.792	0.513	0.057	0.028	0.950	0.414
3.09	0.260	0.209	0.391	37.1	9.1	0.464	0.835	0.555	0.051	0.023	0.810	0.405
2.72	0.267	0.264	0.353	39.1	10.4	0.503	0.864	0.582	0.017	0.007	0.714	0.394

TABLE VI

MODIFIED "CONSTANT a" SUBROUTINE

```
      SUBROUTINE CALC(RL,C,THET,FXF,FYF,XMFXF,XMFYF,QF,TF,RE,PHIR,CL,
     1CD,CX,CY,A,AP,XL,AK,ALPHA,F,CLF,CAT,AAT,CLT,CDT,NFS,SOLD,TH,FWI)
C
C     ....... CALC - DETERMINES THE AXIAL AND ANGULAR INTERFERENCE
C     FACTORS AT A GIVEN RADIUS AND DETERMINES FUNCTIONS DEPENDENT
C     UPON THESE PARAMETERS.
C
      DIMENSION AAT(25),CLT(25),CDT(25)
      COMMON R,DR,HB,B,V,X,THETP,AMOD,H,SI,GO,OMEGA,RHO,VIS,HL,PI,RX,
     1W,NPROF,APF,T1,T2,T3,T4,T5,T6,T7,T8,TEST,XETA,HH,AMAX,AMIN
      ALPHA=0.0
      XL=RL*OMEGA/V
      IF(A.GT..5) AP=0.
      IF(A.GT..5) A=0.
      RH=HB
  200 DO 10 J=1,100
      BETA=A
      DELTA=AP
      IF (AP.LT..001)GO TO 12
C
C     .......DETERMINATION OF THE BLADE INTERFERENCE FACTOR (EWI) AND THE
C            FAR WAKE INTERFERENCE FACTOR (FWI)
C
      EWI=1.0-A
      RF=2.0*(AP*XL)**2.0
      FWI=(EWI-RF/EWI)-((RF/EWI)**2+(1.0-EWI)**2)**0.5
      WIR=(EWI-FWI)/(EWI-FWI*EWI)
      GO TO 13
   12 WIR=0.0
      RF=0.0
   13 PHI=ATAN((1.-A)*COS(SI)/((1.+AP)*XL))
      PHIAA=ABS(PHI)
      PHIR=PHI
C
C     ...... CALCULATION OF SECTIONAL LIFT AND DRAG COEFFICIENTS
C
  510 CALL NACATT(RL,RX,SI,ALPHA,CL,CD,W,AAT,CLT,CDT,NFS,SOLD)
C
  670 F=1.0
  667 CX=CL*SIN(PHI)-CD*COS(PHI)
      CY=  CL*COS(PHI)+CD*SIN(PHI)
      CXX=CX
      CYY=CY
      SIG=(B*C)/(PI*RL)
      IF(AMOD.EQ.0.) GO TO 575
C
C     .......WILSON AXIAL INTERFERENCE METHOD
C
      VBR=((.125*SIG*CYY)*(COS(SI)**2))/(SIN(PHI)**2)
      VAR=(0.125*SIG*CXX)/(F*SIN(PHI)*COS(PHI))
      CAN=F*F+4.*VBR*F*(1.-F)
      A=(2.*VBR+F-SQRT(CAN))/(2.*(VBR+F*F))
      AP=VAR/(1.-VAR)
      GO TO 580
```

TABLE VI (Continued)

```
      C
      C       .......BOGARD AXIAL INTERFERENCE METHOD
      C
      575     VBR=0.125*SIG*CYY*(COS(SI)**2)
              VAR=0.125*SIG*CXX
              A=(SIG*CY-8.0*RF*WIR*SIN(PHI)**2)/(8.0*SIN(PHI)**2+SIG*CY)
              AP=VAR/(F*SIN(PHI)*COS(PHI)-VAR)
      580     PCR=RL/(RX*COS(SI))
      C       ....... DAMPENING OF AXIAL AND ANGULAR INTERFERENCE FACTOR
      C       ITERATIONS.
      C
              IF(J-4) 30,40,90
      90      IF(J-10)  30,40,110
      110     IF(J-15) 30,40,30
      40      A=(A+BETA)*.5
              AP=(AP+DELTA)*.5
      30      CONTINUE
      C
      C       ....... TEST FOR CONVERGENCE ......
      C
              IF(AP.EQ.0.0) GO TO 70
              IF(ABS((AP-DELTA)/AP).LE..0001) GO TO 70
              GO TO 10
      70      IF(ABS((A-BETA)/A).LE..0001) GO TO 50
      C
      10      CONTINUE
      99      WRITE(6,756)RL
      756     FORMAT(/,11X,F5.2,10X,'NO CONVERGENCE')
      50      IF(ALPHA.GT..28)GO TO 18
      C
      C       .......CHECK THE VALUE OF "A"

      C
              IF(A.LT.AMIN)GO TO 16
              IF(A.GT.AMAX)GO TO 17
              GO TO 18
      C
      C       .......INCREMENT THE NUMBER OF BLADES TO INCREASE SOLIDITY IF "A" IS
      C             TOO SMALL;DECREASE SOLIDITY IF "A" IS TOO LARGE
      C
      16      B=B+1.0
              GO TO 200
      17      B=B-0.8
              GO TO 200
      18      CONTINUE
              PCCR=RL/RX
      C
      C       ....... CALCULATION OF FUNCTIONS DEPENDENT UPON AXIAL AND
      C       ANGULAR INTERFERENCE FACTORS.
      C
              W=SQRT(((1.-A)*V*COS(SI))**2+((1.+AP)*RL*OMEGA)**2)
              CONST=(0.5*RHO*(W**2)*C)
              FXF=CONST*CX
              FYF=CONST*CY
              CT1=(0.5*RHO*B*C)*(W*W)
              QF=CT1*RL*CX
              TF=CT1*CY*COS(SI)
      C
      402     RETURN
              END
```

VITA

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