MODELING AND EVALUATION OF STATISTICALLY AND ECONOMICALLY DESIGNED NARROW LIMIT GAGING (NLG) PROCESS CONTROL PLANS

By

SHAWN SHIH-CHUN YU

Bachelor of Science
Tunghai University
Taichung, Taiwan, R.O.C.
1973

Master of Science
University of Illinois at Chicago Circle
Chicago, Illinois
1979

Submitted to the Faculty of the Graduate College
of the Oklahoma State University
in partial fulfillment of the requirements
for the Degree of
DOCTOR OF PHILOSOPHY
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Thesis Approved:

Kenneth E Case

Thesis Adviser

Joe H. Mize

M. Palmer Terrell

L. D. Broemeling

Philip M. Wolfe

Norman N Durham

Dean of the Graduate College

PREFACE

This study is concerned with the modeling and evaluation of the easy-to-use powerful process control scheme--Narrow Limit Gaging (NLG). The primary objective is to provide systematic methodologies and an interactive computer program to help Quality Control practitioners in understanding, designing, evaluating, and implementing statistically- and economically-based NLG plans. Also, NLG is compared with the alternative $\bar{X}$-chart plan, both statistically and economically, to help users in choosing the control scheme which better suits their individual needs.

I wish to express my sincere appreciation to my major adviser, Dr. Kenneth E. Case, for his constant encouragement, guidance, and assistance throughout this research and during my doctoral program. Thanks also to my committee members, Dr. Joe H. Mize, Dr. M. Palmer Terrell, Dr. Phillip M. Wolfe, and Dr. Lyle D. Broemeling, for their interest and assistance.

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CHAPTER I

THE RESEARCH PROBLEM

Purpose

Process control is one of the major areas of statistical quality control, in which several techniques can be employed to estimate process characteristics and capability, to establish control, and to monitor the process. This study will focus on one of the easiest to use techniques--Narrow Limit Gaging (NLG). The major interest of this research is to help practitioners in understanding, designing, evaluating, and implementing the most appropriate NLG process control scheme by providing the following:

1. a clear taxonomy and recommended standardization of NLG control schemes,

2. comprehensive methodology for statistical and economic design and evaluation of NLG plans,

3. comparison of NLG to the most popular process control alternative, and

4. a user-oriented interactive computer program to accomplish a wide range of design and analysis tasks.

The Need

The implementation of a process control procedure in a production context involves two stages. First, a state of statistical control must be described and achieved; and second, the output can then be monitored in a reasonable fashion. During the monitoring stage, the process begins "in control" but eventually shifts out of control, at the occurrence of an assignable cause which is desired to be detected as early as possible.

Two types of control schemes can be employed to monitor the process, namely, variable plans (such as $\bar{X}$- and R-charts, and the cusum chart) and attribute plans (such as the p-chart and c-chart). Generally, variable plans require a longer time to measure individual items, while attribute plans require larger sample sizes to detect the same degree of process shift. Both the variables measurement of small samples and the attributes gaging of large samples can be quite time consuming and, for some cases, may impede the rapid detection of a process shift.

To solve this problem, a combination of the advantages of both control schemes is strongly desired. A quick-and-easy gaging method, together with a fairly small sample size, is sought. Among all traditional approaches, NLG process control plans seem to be the only ones to fulfill this need.

Introduction

Suppose the measurements of the product characteristic are normally distributed, and the process capability (6σ) is less than the specification tolerance (USL - LSL) (see Figure 1.1). In addition, the process dispersion σ (standard deviation) is assumed to remain unchanged while the process mean may shift.[a] To guide manufacturing, go/no-go gages are

[a]These assumptions are made only to facilitate illustration. In practice, none of them is required.

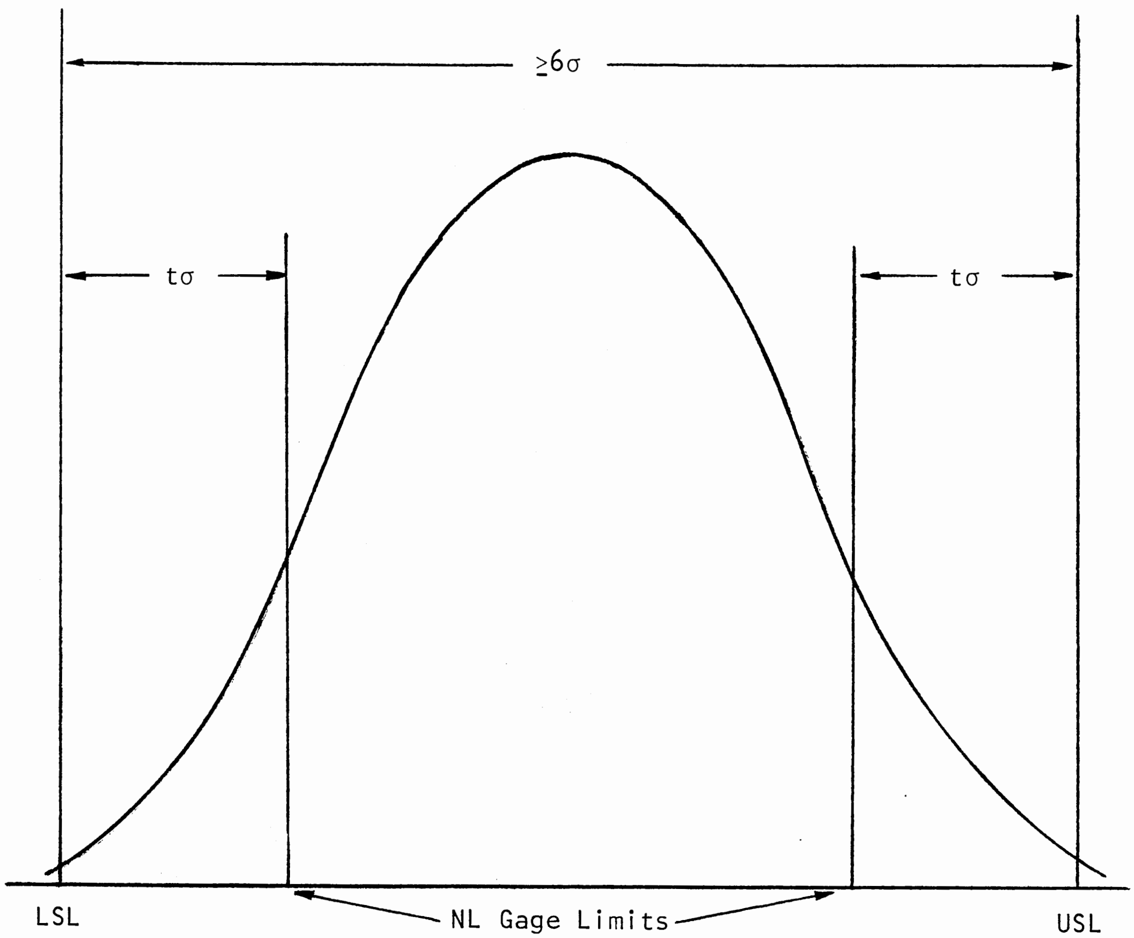

Figure 1.1. Specification Limits and Narrow Gage Limits

prepared which are stricter than specifications by an amount $t\sigma$ and hence are called Narrow Limit Gages. Then small samples are taken and gaged at regular intervals of time, which may be called frequency gaging. Finally, decisions about actions are made according to some predetermined rules.

Two examples follow:

1. Simple rule [33]: In a sample of size n, if the number of units which do not pass the NL gage, is greater than a specified number c, then the process is stopped and investigated for assignable causes. Otherwise, the process keeps going.

2. Complex rule [38]: A sample of three is drawn and two are gaged. The third is gaged only when necessary. Possible outcomes and actions follow:

a. No action required

(1) Both within NLG limits.

(2) One in and one out of NLG limits (but within specification limits) and the third inside NLG limits.

b. Readjust/correct machine

(1) Any one out of specification limits.

(2) Both out on the same side of NLG limits.

(3) One in and one out of NLG limits (but within specification limits) and the third out on the same side of NLG limits.

c. Machine capability questionable

(1) When two out of three (or two out of two) are both out of NLG limits, but on opposite sides, the operation is suspected of having too much variation. A machine

capability study should be made with machine maintenance as necessary.

In addition to the above frequency gaging rules, decisions about <u>sampling frequency</u> and the <u>qualification</u> to begin frequency gaging after each machine setup and reset may also be needed. An example follows [19]:

1. To qualify for frequency checking, make 100 percent inspection until five successive pieces fall between NLG limits. While waiting for five, the process may require a reset as necessary.

2. For sampling frequency, seek an <u>average</u> of 25 checks to a reset. If, on the average, an operator checks more than 25 times without having to reset the process, gaging frequency may be reduced so that more pieces are made between checks. If the process must be reset before 25 checks on the average are made, the gaging frequency may be increased.

Taxonomy and Development of a Standard Formulation

Although NLG is easy to use, there exists a variety of rules in practice. Different people can always make up different rules. The current sets of individual rules for use of NLG seem so arbitrary that they lack a common basis for evaluation and comparison. Furthermore, people always describe NLG rules in their own lengthy words rather than in common terminology and concise notation. These descriptions can easily amount to 20 sentences. This makes the essential structure of NLG even more obscure.

In all, a clarified structure is needed to generalize the NLG rules, to simplify the descriptions, to give appropriate evaluations, and to provide comparisons. This research fulfills this need by developing a clear, notation-stated, comprehensive, and exhaustive NLG statement.

Also, a "standard" NLG scheme is developed on which all of the numerical evaluations of this study are based. This will considerably reduce the total number of possible rules and facilitate evaluation.

Statistical Evaluation

In order to statistically compare different NLG plans on the same basis, proper "performance measures" are first established. For individual samples, the following are investigated:

1. P_a--Probability of acceptance

2. E_n--Expected number of items inspected in each sample

3. OC (Operating Characteristic) curve--P_a as a function of either process mean shift or dispersion change.

For the process as a whole, the following performance measures are considered [19]:

1. APQ and APQL--Average produced quality and its limit

2. AOQ and AOQL--Average outgoing quality and its limit when 100 percent retroactive inspection is performed to remove defective items.

The formulations of all these performance measures are developed as functions of the process fraction defective.

The general effect of each NLG parameter (e.g., sample size, control limit inset, truncation rule, acceptance/rejection rule, . . ., etc.) is analyzed to help in understanding NLG characteristics. Based upon this understanding, flexible procedures are constructed for designing NLG plans. To provide greater flexibility for the user in choosing a preferred plan under certain specified conditions, all qualified plans are listed together with related performance measures provided.

Finally, a performance comparison between the most popular process control plan, the $\bar{X}$-chart, and NLG is analyzed to see if NLG is comparable or even superior to the $\bar{X}$-chart.

Economic Formulation

Traditionally, process control schemes are designed statistically and produce acceptable results. However, in recent years, there has been an increasing emphasis on economic performance since it is intuitively more appealing to design plans with direct consideration of quality costs [31]. In reality, economic performance is the ultimate criterion for evaluating control plans, in which one is balancing the costs associated with sampling, testing, and process surveillance against internal and external failure costs. Since the design of the procedure affects these costs, it is logical to consider this design from an economic viewpoint.

Based upon the maximum income criterion, Duncan [6] has formulated a model which measures the average net income of a process under the surveillance of an $\bar{X}$-chart. The process starts in-control and is subject to random shifts in the process mean (out-of-control). Once out of control, this process remains there until the trouble is removed. Given (1) cost parameters of in-control income, out-of-control income, false alarm cost, real alarm cost, and control chart costs; and (2) time parameters of process shifting, inspection and plotting, and searching for assignable causes, the best values of the decision variables sample size (n), sampling interval (h), and control limit spread (k) are determined using optimization techniques.

This study follows Duncan's approach in formulating an economic NLG scheme in which the decision variables consist of sample size (n), sampling interval (h), control limit inset (t), a truncation rule, and acceptance/rejection rules. For both models, the underlying assumptions are closely matched to ensure the highest degree of formulation similarity for comparison purposes. The significance of possible NLG improvements over $\bar{X}$-charts, resulting from the reduction of control chart costs and plotting delay, is evaluated.

Economic Optimization

In optimizing the values of the decision variables of the economically-based $\bar{X}$-chart model, Duncan [6] uses a complicated and involved search technique after making certain assumptions and approximations about his model. To improve accuracy and speed, Goel et al. [12] develop an algorithm, also employing a search technique, which consists of solving an implicit equation in all decision variables. Both authors utilize the differentiability of the loss-cost function with respect to decision variables n, h, and k to considerably simplify the effort of direct search.

In the economically-based NLG model, the probability of acceptance is a complicated function of decision variables n, h, t, truncation rule, and acceptance/rejection rules. The desirable property of differentiability no longer exists. Therefore, multidimensional direct search techniques represent the most promising optimization approach. Furthermore, since the decision variables sample size n is not continuous, and the truncation rule and acceptance/rejection rules are not even measurable, the general optimization strategy adopts an appropriate direct search algorithm to optimize sampling interval h and control limit inset t simultaneously under every possible set of combinations of n and both rules.

The combination of decision variables n, h, t, truncation rule, and acceptance/rejection rules yielding a minimum loss-cost is the optimal scheme.

Economic Comparison of NLG Plan and $\bar{X}$-Chart

To assess the best conditions for the application of NLG and $\bar{X}$-charts, both models are evaluated under the same environments. This evaluation is performed under each of a number of examples. For each example, in addition to the $\bar{X}$-chart and standard NLG, two more variations of NLG are investigated to reveal the effects of the truncation rule and the reductions in control chart costs and plotting delays.

Based upon the results of these comparisons, in addition to intuitive theoretical interpretation, practical general guidelines are developed to help practitioners in choosing between economic $\bar{X}$-charts and NLG plans under specified environments.

Interactive Computer Program

To help practitioners in the design, evaluation, and implementation of NLG process control plans, all previous developments and analyses are summarized into a comprehensive and flexible interactive computer program. This program has both statistical and economic analysis and design capability. In addition, both design and evaluation, either statistically or economically, of a specified $\bar{X}$-chart are also provided upon the user's request for comparison purposes.

Summary of Research Objectives

Based upon the above discussions, the primary objective of this research is stated:

Objective:

> To provide a systematic methodology and a practical interactive computer program to help Quality Control practitioners in understanding, designing, evaluating, and implementing statistically- and economically-based Narrow Limit Gaging process control plans.

In order to accomplish this objective, several specific subobjectives are included:

Subobjectives:

1. To develop a clearly, symbolically stated, comprehensive NLG taxonomy to generalize and simplify the descriptions of varieties of NLG rules.
2. To propose a "standard" NLG scheme to reduce the total number of possible rules and to facilitate easy numerical evaluation.
3. To provide a methodology for designing and evaluating NLG plans statistically. A comparison with the $\bar{X}$-chart will also be provided.
4. To formulate the economically-based model for evaluating NLG process control plans.
5. To develop a general strategy, together with a direct search technique, to optimize the economically-based NLG model.
6. To economically compare NLG and $\bar{X}$-chart plans under a variety of situations.

7. To develop a comprehensive and flexible interactive computer program to provide
   (a) design and evaluation of statistically-based NLG plans,
   (b) design and evaluation of statistically-based $\bar{X}$-chart plans,
   (c) design and evaluation of economically-based NLG plans, and
   (d) design and evaluation of economically-based $\bar{X}$-chart plans.

Contribution

The successful completion of this research will provide benefits to both theoreticians and practitioners. This study will become the first of its kind in providing (1) a unified taxonomy and a standardization of NLG, (2) thorough statistical analyses of NLG, (3) considerable economic treatment of NLG, and (4) appropriate comparisons, both statistically and economically, between NLG and $\bar{X}$-charts. Most of these results (except a small portion of (2)) are not presented in any textbooks or papers on statistical quality control, although NLG has had considerable application and, even more, is of growing interest in the quality control area.

Practitioners will benefit from this research because it will provide them with practical procedures for designing and evaluating appropriate NLG plans. The flexibility of either statistical or economic comparisons among qualified NLG plans and $\bar{X}$-control chart schemes will improve the user's decision-making capabilities. The fast execution of an interactive computer program will make the design and evaluation of NLG plans considerably easier. Consequently, this will encourage a broader range of NLG applications and therefore result in increased productivity.

CHAPTER II

LITERATURE REVIEW

Introduction

This chapter reviews developments in the literature relevant to the objectives of this research. Support for this specific research is elaborated upon. In addition, other sources which communicate the general concepts relating to this study are also presented.

This chapter is divided into five areas:

1. Process Control Techniques and Their Comparisons
2. Development of NLG
3. Variety of NLG Rules and Applications
4. NLG Statistical Evaluation
5. Economic Modeling, Optimization, and Comparison of Process Control Schemes.

Process Control Techniques and Their Comparisons

Since Shewhart [43] first introduced the concept of statistical quality control a half century ago, many new techniques have been proposed in both the process control and acceptance sampling areas. In process control, important developments include [11, 21]:

1. Shewhart control charts and their ramifications--$\bar{X}$, $\bar{X}$-R, p, c, u, tests for runs, $\bar{X}$-chart

2. Modifications of Shewhart control charts--moving average and range, median and midrange, geometric moving average

3. Cumulative sum control charts

4. Acceptance control charts

5. Multi-characteristic control charts--Hotelling T^2, Q-chart

6. Narrow limit gaging.

In order to select the most appropriate method for a given situation, proper comparisons among all alternatives are needed. However, few authors have compared the different schemes. Among them, Page [35] discusses the general comparison approach of process inspection schemes. Freund [10] compares the cumulative sum, geometric moving average, and acceptance control charts. Roberts [39] compares the moving average, geometric moving average, cumulative sum, Girshick-Rubin, and run sum charts. Unfortunately, NLG has never been compared to other methods, although it has the general advantages of simplicity and speed over all other control schemes.

According to a survey conducted by Sanija and Shirland [40], the $\bar{X}$-control chart remains the most popular process control scheme in industry. Naturally, it becomes the alternative chosen to compare with NLG in this research.

Development of NLG

In the literature, Narrow Limit Gaging [9, 33] has a variety of synonyms. It is also known as Compressed-Limit Gaging [7], Increased Severity Testing [7], Pre-Control[a] [19], and Target Area Control [4]. Some even

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[a]Pre-Control is so named because when the specification interval is

refer to it without giving it a name, such as "Patrol Inspection (np Chart) with special gages" [15]. Among all of these, most often it goes by the names of Narrow Limit Gaging and Pre-Control.

For controlling a current production process and in comparison to variable control schemes, attribute control charts have many advantages. For example, they (1) can accommodate numerous variables in a single chart, (2) are more economical and easier to use because they can use go/no-go gages, and (3) are better for destructive and time consuming testing. However, attribute control charts require larger sample sizes to achieve the same sensitivity as that of variable schemes.

To improve the usefulness of attribute control charts, attempts have been made to devise attribute charts that require a lower than usual sample size. In the last four decades, several suggestions have been made to use gages with limits stricter than product specifications (i.e., NLG) for decision making purposes, either applied to control charts or to acceptance sampling, and in this way to reduce the sample size required for making a decision. Chronologically, this development is divided into three periods: (1) Simple Rule period, (2) Complex Rule period, and (3) Statistical Optimization and Economic Design period.

In the Simple Rule period, all NLG plans require that each of a sample of size n items be compared to narrow gaging limits and that c or fewer be within these limits for process acceptance. These Simple Rule plans do not involve the concept of Qualification and Gaging Frequency. NLG concepts first emerged in Britain in the 1940's [5, 30] and were

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large enough to tolerate some degree of process shifting, it permits a decision for corrective action to be made long before the process has deteriorated to the point that tolerances are exceeded and rejects made.

claimed to be as promising as $\bar{X}$-charts. Mace [27], in 1952, actually designs two NLG plans having similar OC curves as a comparable $\bar{X}$-chart. Ott and Mundel [33], in 1954, systematically investigate the effect of each NLG element (n, c, t) on OC curves and provide some general guidelines in designing NLG plans. As a ramification of NLG, Stevens [46], in 1948, designs (C - A) and (C + A)[b] charts to substitute for $\bar{X}$- and R-charts, respectively. Stevens' charts application is illustrated by Aroian [1] in 1959.

In the Complex Rule period, the Jones and Lamson Machine Co., in 1954, develop an important milestone. In its Quality PRE-Control brochure [19], frequency gaging rules evolve from the Simple Rule into the Complex Rule. Moreover, the concepts of Qualification (to begin frequency gaging), Sampling Frequency, and Average Produced Quality and Its Limit are all integrated into NLG design. Four different plans are provided for typical applications which require very little statistical knowledge. The idea and practicality of NLG is greatly popularized by Juran's [20] Quality Control Handbook in 1962. However, no flexibility is provided to adjust control limit spread t, no evaluation is given to the Qualification rule, no clear methodology for evaluating P_a of each sample is given, and the computation of APQ is questionable. Still, the contribution to the realization and application of NLG schemes in industry by both references is undoubtedly significant.

The Statistical Optimization and Economic Design period broke a 20-year drought of little progress in NLG since Jones and Lamson's [19] innovation in 1954. In 1974, Beja and Ladany [2] present a procedure to

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[b] C is the number of pieces to fall below the lower NLG limit, and A is that number to fall above the upper NLG limit.

optimize (in the sense of minimizing sample size) the NLG Simple Rule under specified acceptable and rejectable quality levels, and their associated α, β risks. They also discuss the interesting and revealing conceptual comparison of attribute and variable measurements, and herein design and optimize an intermediate double-limit per single specification NLG scheme. In 1975, Ladany [24] presents the first economic NLG model by incorporating the above-mentioned optimal statistical Simple Rule NLG plan [2] into an economically-based p chart [23], resulting in a "narrow-limit gaging fraction defective" control chart. However, the optimization of such a combination only results in a suboptimum rather than an overall optimum since the overall costs in using NLG are not considered.

The above discussion indicates some voids to be filled in order to complete the development of NLG to a satisfactory degree. These voids include (1) comprehensive statistical analyses of NLG, (2) accurate economic modeling and true optimization of NLG, and (3) appropriate comparison between NLG and $\bar{X}$-charts, both statistically and economically.

Variety of NLG Rules and Applications

There exists such a variety of rules in practice that there is no standard approach to NLG design and use. But in the less involved Simple Rule NLG plans [5, 9, 27, 30, 33], the design procedure is somewhat standardized. Due to its simplicity and consistency, optimum design is sought by Beja and Ladany [2] and some ramifications are extended. A double NLG limit per single specification limit scheme is proposed and optimized by the same authors. Also, a combined sequential implementation of two NLG plans is demonstrated by Ott [33, 34].

In Complex Rule NLG plans, a great diversity of methods exist. For sample size, n = 2 (Plan A in [19]), [20, 29, 37], and n = 3 [38] are quite popular, but n = 5 [17], n = 6 (Plan B in [19]), and n = 7, 8, 10 [17] are also used in practice. The variation of truncation (i.e., the curtailment of items inspected in each sample) rules depend upon the corresponding sample sizes. For inspection frequency, Jones and Lamson Co. [19] and Juran [20] propose a guideline of 25 or 50 inspections on the average for each process correction, while Whittingham [49], in 1981, suggests three fixed checking intervals for different process classifications. Very little work has been done on Qualification (to start frequency gaging) rules which are employed to ensure the process is under control immediately after every setup and reset. There is currently only one Qualification rule in practice [19].

NLG has a large variety of applications in practice. Harding [16], in 1957, uses--for incoming material acceptance sampling--NLG plans which are comparable to (and more economic than) MIL-STD 105A double sampling plans. Beja and Ladany [2], in 1974, also design NLG plans for use as an acceptance sampling scheme which is compared with single attribute sampling plans and variable sampling plans. When used as a process control tool, in addition to the major function of maintaining control of a process, NLG can also be used to control a trend in process mean [45],[c] to detect either mean or dispersion shifts, or both [42], and as a set-up plan [19]. Finally, after incorporating it with the "feed back" concept [26], NLG can easily be adopted in automatic process control [25, 44, 45].

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[c]Also see footnote b on page 15.

The above discussion reveals a strong need for summarizing, simplifying, and standardizing NLG plans to meet the following general requirements [19]:

1. Protect against unwanted shifts in process mean and/or process spread, yet accommodate the tolerable process trend.

2. Serve both as a set-up plan and a monitor plan, and economically adjust inspection frequency to guarantee a specified level of produced quality.

3. Provide ease of use, require no paperwork, permit use of go/no-go gages, and be easily learned by operators.

4. Be competitive in efficiency with alternative plans, but cost less to administer.

NLG Statistical Evaluation

The statistical evaluation of the NLG process control scheme can be done either with respect to the sample only, or with the process as a whole. When considering the sample only, for a two-point design (i.e., under specified acceptable and rejectable quality levels and their associated α,β risks), Beja and Ladany [2] propose using the sample size n as a performance measure in choosing qualified Simple Rule NLG plans. Similarly, the average sample number E_n [14] resulting from the truncation of sampling inspection under the Complex Rule can be used instead of n. However, if the user specifies only one point, either OC curves or ARL curves [48] incorporated with E_n can be employed to evaluate qualified plans. Furthermore, if the detection of both process mean shift and process dispersion change are considered,[d] ISO-OC or ISO-ARL graphs [48] may be used.

When considering the process as a whole, under specified conditions, Jones and Lamson Co. [19] suggests using the Average Produced Quality Limit (APQL) to evaluate alternative plans. However, under certain conditions, the APQ calculation becomes questionable. This shortcoming should be improved. Also, more information can be provided by supplying the whole APQ curve. Furthermore, the same article [19] indicates that Average Outgoing Quality (AOQ) and its limit (AOQL) can be obtained when the implementation of Retroactive Inspection (100% inspection of recently passed product) is added.

To investigate the general effect of individual NLG decision variables, the work of Ott and Mundel [33] on the Simple Rule can be extended and applied to the Complex Rule. In investigating the rule of Qualification for frequency gaging, Weiler's [47] discussion about the ARL (Average Run Length) of Runs is also useful.

In summary, all the above-discussed ideas and methods are evaluated, improved, and finally integrated into a comprehensive statistical evaluation package which is intended to give practitioners maximum assistance.

Economic Modeling, Optimization, and Comparison of Process Control Schemes

Designing process control schemes using economic instead of statistical

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[d]Almost all of the NLG schemes consider only the process mean shift which Shainin [42] claims happens much more often than process dispersion changes in industry. However, there exist situations where the process dispersion may change.

criteria has received more and more attention in the quality control literature in recent years. Most of the modern work in this area has concentrated on the $\bar{X}$-chart, due to its flexibility, simplicity of administration, and the information content of plotted point pattern. Extensions to the p-chart, cumulative sum charts, control charts with warning limits, joint design of $\bar{X}$- and R-charts, and multivariate quality control procedures have also been reported [31]. In many variations of economically-based $\bar{X}$-control chart models [31], Duncan's [6] fundamental approach is still the most popular one. Therefore, it is used in this research as an alternative to the economically-based NLG model for comparison purposes.

The only related work on the economic design of NLG process control plans is done by Ladany [24]. He combines the optimal Simple Rule NLG plan with the economically-based p-chart and results in a suboptimal solution. To avoid this shortcoming, this research develops a model which combines the "standard" NLG scheme with Duncan's $\bar{X}$-control chart model, and then employs a direct search technique to find the overall optimum.

Himmelblau [18], and Kuester and Mize [22] provide many useful methods for direct search techniques. Among them, the method proposed by Nelder and Mead [32] is quite straightforward, efficient, and easy to use. However, its non-constrained optimization algorithm requires some modification before it can be applied to optimize the economic NLG schemes in which constraints exist on sampling interval h and control limit spread t.

Goel [13] and McFadden [28] perform several comparisons on economically-designed process control schemes. These complement the previously mentioned statistical comparisons done by Page [35], Freund [10], and Roberts [39]. However, there has been no work toward economically comparing NLG and the $\bar{X}$-chart.

Summary

This chapter presents a survey of the literature on the problems, contributions, and needs relative to the objectives of this research on Narrow Limit Gaging for process control. This survey indicates that NLG process control plans have had considerable application in industry due to their inherent advantages. However, NLG plans lack standardization and appropriate design and evaluation procedures.

This survey also demonstrates the increasing interest in economic design of process control models. Unfortunately, there has been very little work done toward developing and optimizing a general economically-based NLG model.

This survey indicates a clear need for the following:

1. To provide a clear taxonomy and standardization for NLG process control schemes.

2. To develop a methodology for statistical design and evaluation of NLG plans.

3. To develop a methodology for economic modeling and optimization of NLG plans.

4. To compare NLG to alternative process control plans.

5. To develop a user-oriented interactive computer program to facilitate the wide range implementation of NLG schemes.

This research accomplishes a significant improvement in the theoretical and applied development of Narrow Limit Gaging process control schemes. Due to this contribution, NLG plans can be used more correctly, more easily, with broader application, and with increasing popularity. Also, their use will eventually result in increased productivity.

CHAPTER III

TAXONOMY AND STANDARDIZATION OF NLG

Introduction

This chapter analyzes the composition of NLG and investigates its complexity and possible variation to provide an overall understanding of its general structure. Based on this understanding, a simplification and standardization of NLG schemes is then developed. Concise notation is presented to effectively describe NLG plans. Pertinent examples are provided.

Notation

To facilitate the comprehensive description of a complicated NLG scheme, the following notation is introduced and will be continuously used throughout the entire research.

USL, LSL--Upper and lower specification limits, respectively (see Figure 3.1)

σ_o--Process standard deviation (before shifting) of the characteristic measurement (x) of the product

USLLSL--Specification interval (in multiples of σ_o) = (USL - LSL)/σ_o (see Figure 3.1)

UNGL, LNGL--Upper and lower narrow gage limits, respectively (see Figure 3.1)

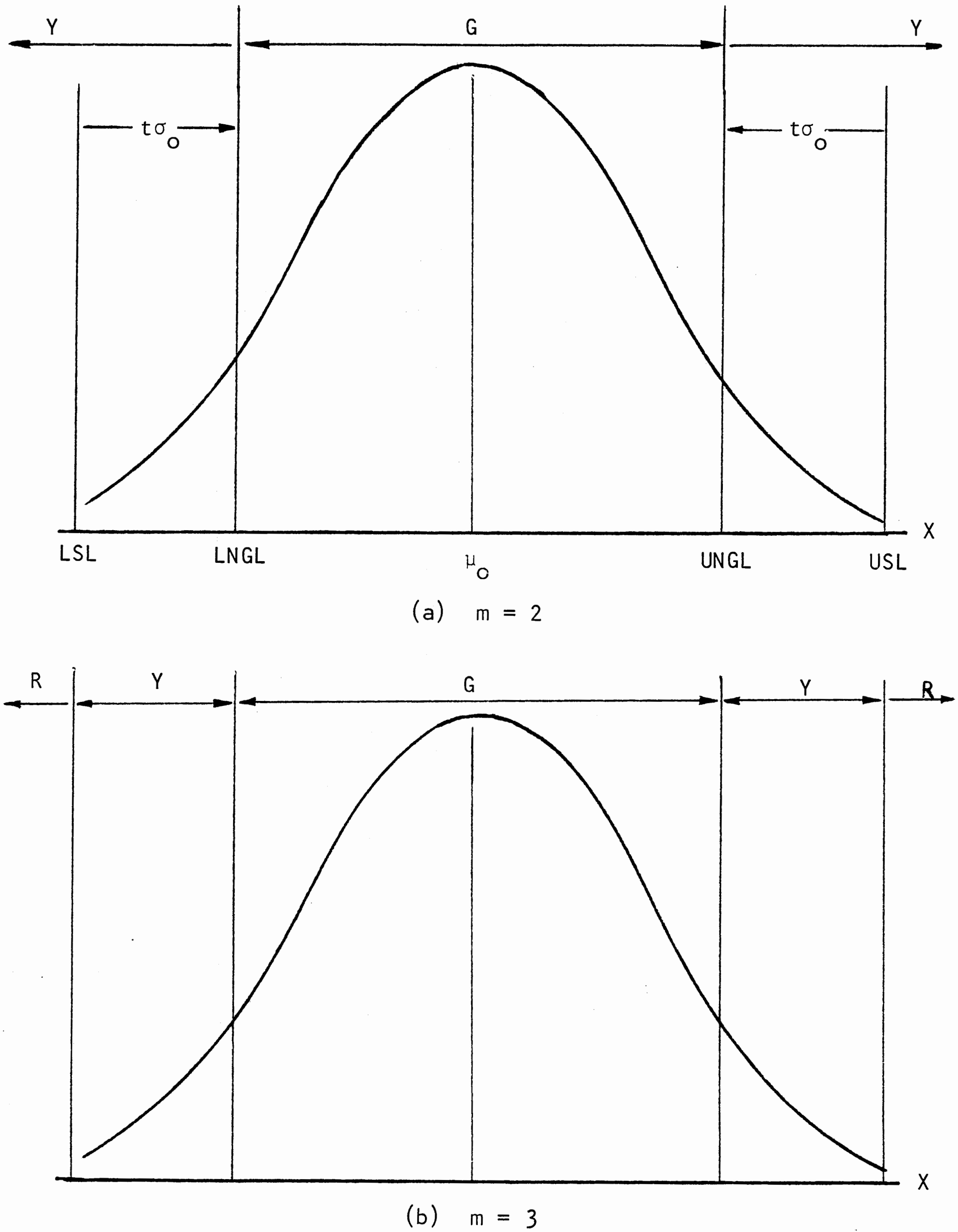


Figure 3.1. Illustration of NLG Notation

t--Control limit inset of NLG. This is the number of standard deviations ($t\sigma_o$) that the narrow gage limits are set in from both USL and LSL. That is, $UNGL = USL - t\sigma_o$; $LNGL = LSL + t\sigma_o$ (see Figure 3.1)

n--Sample size

m--Number of NLG classifications; $m = 2$: Green, Yellow; $m = 3$: Green, Yellow, Red (see Figure 3.1)

G--Green. It denotes any measurement falling between two narrow gage limits; that is, $LNGL \leq x \leq UNGL$ (see Figure 3.1)

Y--Yellow. When $m = 2$, it denotes a non-G measurement; that is, $x < LNGL$ or $x > UNGL$. When $m = 3$, it denotes any measurement falling between the specification limit and the narrow gage limit on the same side; that is, $LSL \leq x < LNGL$ or $UNGL < x \leq USL$ (see Figure 3.1)

R--Red. It denotes any measurement falling beyond USL or LSL; that is, $x < LSL$ or $x > USL$. This classification exists only for $m = 3$ and not for $m = 2$ (see Figure 3.1)

g--Acceptance truncation number. Whenever the first g items of a sample are green, the sample is accepted and the remaining inspection is truncated

y--Maximum acceptance number of items designated as Y. Whenever the number of Y in a sample is $>y$, the sample is rejected and inspection is truncated

r--Maximum acceptance number of items designated as R. Whenever the number of R in a sample is $>r$, the sample is rejected and inspection is truncated

QL--An abbreviation referring to Qualification for starting Frequency Gaging. It is a procedure to ensure that the process has been adjusted to the desired in-control level before starting Frequency Gaging

FG--An abbreviation for Frequency Gaging. It is a procedure to monitor proper operation of the process. Periodically, a sample of size n is taken and inspected for early detection of a process shift

SF--An abbreviation for Sampling Frequency. This is the frequency of taking and inspecting samples in the FG step

RI--An abbreviation for Retroactive Inspection. To improve the average produced quality, items between the final out-of-control sample and the last previous in-control sample are 100% inspected for the removal of defectives

OC curve--Operating Characteristic curve. This curve describes the probability of acceptance as a function of process quality

APQ--Process Average Produced Quality. It is the long term average fraction defective produced by the process

IC--an abbreviation for in-control or "in control"

OOC--An abbreviation for out-of-control or "out of control."

Taxonomy of NLG

General Structure

Theoretically, a complete Narrow Limit Gaging process control scheme consists of four basic elements: Qualification (QL), Frequency Gaging

(FG), Sampling Frequency (SF), and Retroactive Inspection (RI). These elements comprise a complete control cycle as shown in Figure 3.2.

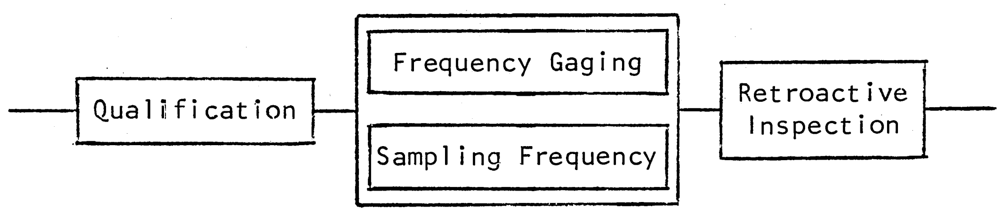


Figure 3.2 NLG Scheme Structure

At the beginning of each control cycle, if necessary, QL is implemented to ensure that the process has been adjusted to the desired in-control (IC) level. In the second step, a sample of size n is taken periodically, according to the SF specification, and inspected to infer whether the process is in or out of control. If in control, FG continues. An out-of-control (OOC) indication necessitates adjustment of the process back to an IC level. This would usually conclude the control cycle. However, if further improvement on the average produced quality is desired without altering the control scheme, RI can be performed. All items produced in the last sampling interval are therefore 100 percent screened for the removal of every defective.

In practice, not all of the above three steps are implemented. While FG and SF are mandatory, QL and RI can be optional depending upon individual situations. Their definitions, functional objectives, ingredients, and variations will be delineated in the following sections.

Frequency Gaging

Generally, each process control cycle starts out in control (which, if desired, can be ensured by QL), remains in control for a certain period of fime, and then eventually shifts out of control due to the occurrence of an assignable cause. To detect this shift as early as possible, a sample of size n is taken from the process periodically. Each item of this sample is then gaged by a pair of Narrow Limit Gages which has a control limit inset t, and is classified into one of m resulting classifications (for example, if m = 3, the classifications will be G, Y, and R). Comparing the gaging results of the sample (or part of the sample) to a set of predetermined rules, a decision is then made to either let the process continue or to take necessary corrective actions.

Unfortunately, the number of "possible" sample acceptance/rejection decision rules is formidable due to the number of variations of acceptance/rejection criterion. Theoretically, the number of all possible NLG outcome permutations can be as large as m^n. For example, if n = 4, m = 3, there will be $3^4 = \underline{81}$ possible criteria. If outcomes are expressed in combinations of (G, Y, R), the number of criteria can be reduced to $\binom{n+m-1}{n}$[a] which is considerably smaller than m^n. For example, when n = 4, m = 3, there will be $\binom{4+3-1}{4} = \binom{6}{4} = \underline{15}$ possible criteria, namely, (G,Y,R) = (4,0,0), (3,0,1), (2,0,2), (1,0,3), (0,0,4), (3,1,0), (2,1,1), (1,1,2), (0,1,3), (2,2,0), (1,2,1), (0,2,2), (1,3,0), (0,3,1), or (0,4,0).

Further reduction to the number of criteria can be achieved by the adoption of acceptance/rejection truncation rules. That is, as soon as

---

[a]This is equivalent to the problem of finding the number of possible ways to put n indistinguishable objects into m distinguishable cells (see [36], p. 74, Exercise 5.3).

the acceptance/rejection criteria are satisfied, the sample is either accepted or rejected without inspecting the rest of the items. For example, when we specify $g = 1$, the sample will be accepted right away if the first item is classified G. When we specify $r = 0$, the sample will be rejected as soon as a R appears. When we specify $y = 1$, the sample will be rejected as soon as the number of Y is 2. Thus, in the previous example of $n = 4$, $m = 3$, if $g = 1$, $y = 1$ and $r = 0$ are imposed, the total number of criteria can be expressed in only 4 sets which is much smaller than either 15 combinations or 81 permutations. These four criteria are: acceptance on first G; rejection on any R; acceptance on one or fewer Y when there is no R; and rejection on two or more Y when there is no R.

In practice, two acceptance/rejection truncation rules are commonly used. First is the most widely used rejection truncation rule, $r = 0$. Since R indicates a real defective and its chance is relatively small as long as the process stays in control, it is quite reasonable to reject the sample whenever R is encountered.

The other commonly used truncation rule is G acceptance truncation (e.g., $0 < g < n$). The reasoning for this rule is based on the concerns for effectiveness and efficiency in inspection timing. Ideally, the best timing for inspection is to make no measurements on the process except immediately following a process shift. But in practice, a process is subject to unknown spontaneous shifts occurring at unpredictable times. Therefore, the efficient control plan calls for a periodic small number of checks with additional gaging (up to the full sample size) whenever the initial gaging results hint that a process shift may have occurred. This tends to concentrate the gaging at times when a process shift has actually occurred. Thus, the control plans with acceptance truncation rules seem to be more efficient than those regular non-truncation plans with an equal number of measurements taken periodically.

Although the adoption of acceptance/rejection truncation rules can certainly reduce the total number of inspections, they may not result in fewer or simpler Frequency Gaging rules as illustrated previously. For example, if $n = 4$, $m = 3$, $r = 0$, and acceptance/rejection decisions are made based on the combinations of G, Y, R, there will be as many as 16 possible truncation rules which are tabulated in Table 3.1. Obviously, further simplification on acceptance/rejection truncation rules is desirable.

Sampling Frequency

Given a set of FG rules, the Average Produced Quality (APQ) of the process can be improved merely by more frequently checking samples, since the shifts can be detected earlier. However, this quality improvement results in higher inspection costs. Thus the essential purpose for proper adjustment of the Sampling Frequency (SF) is to achieve an economic balance between high inspection cost resulting from overly frequent sampling, and high defective cost resulting from less frequent sampling.

In practice, there are two types of SF, namely, fixed SF and self-adjusting SF. The first kind takes samples for a fixed period of time or quantity of production. For example, take a sample of size 3 every production hour or every 1000 items produced. This method is easy to implement, but it lacks the flexibility to properly respond to the gradual deterioration or improvement of the process level.

The second approach self-adjusts SF in accordance with the frequency of OOC indications. It seeks to keep constant the average number of

TABLE 3.1

POSSIBLE TRUNCATION RULES FOR $n = 4$, $m = 3$, $r = 0$ WITH ACCEPTANCE/REJECTION DECISIONS BASED ON THE COMBINATIONS OF G, Y, R

i	Rule No.	Acceptance Truncation		Rejection Truncation	
		Possible acceptance/rejection truncations occur in the first i items of the sample			
		(a) The Main Table			
1	1	≥1G (≤0Y)	and 0R	---	≥1R
	2	---	---	≥1Y or	≥1R
2	3	≥2G (≤0Y)	and 0R	≥2Y or	≥1R
	4	≥2G (≤0Y)	and 0R	---	≥1R
	5	≥1G (≤1Y)	and 0R	---	≥1R
	6	---	---	≥2Y or	≥1R
	7	---	---	≥1Y or	≥1R
3	8	≥3G (≤0Y)	and 0R	≥3Y or	≥1R
	9	≥3G (≤0Y)	and 0R	≥2Y or	≥1R
	10	≥3G (≤0Y)	and 0R	---	≥1R
	11*	≥2G (≤1Y)	and 0R	≥3Y or	≥1R
	12	≥2G (≤1Y)	and 0R	---	≥1R
	13	≥1G (≤2Y)	and 0R	---	≥1R
	14	---	---	≥3Y or	≥1R
	15	---	---	≥2Y or	≥1R
	16	---	---	≥1Y or	≥1R

*(b) An Illustration of Rule 11 in (a)

	1st	2nd	3rd	Trunc. at
Acceptance Truncation	G	G	---	2nd
	G	Y	G	3rd
	Y	G	G	3rd
Rejection Truncation	Y	Y	Y	3rd
	R	---	---	1st
	---	R	---	2nd
	---	---	R	3rd
Continuation	G	Y	Y	none
	Y	G	Y	
	Y	Y	G	

inspected samples per OOC indication. Thus an increase in process shift frequency (with a consequent proportional increase in the number of defectives) is almost exactly counteracted by an increase in SF which proportionally reduces the time required to detect the process shift (and therefore the number of defectives produced before such detection). This approach can give a proper guarantee to the process APQ but it is more difficult to implement.

Qualification

There are times when the accuracy of each process setup or reset is suspect. The assurance that the process has indeed been adjusted to the targeted IC level before starting Frequency Gaging is desired. To achieve this purpose, Qualification (QL) rules are employed to reject all unsatisfied setups and resets, and to properly ensure that the process is in control before beginning FG.

Although the gages used in QL may not necessarily be the same as those used in FG, in practice it is more cost-effective to use the same set of gages in both QL and FG. Theoretically, any control plan which possesses a satisfactory capability to discriminate between good and bad process levels can serve as a QL rule. However, there is only one kind of QL rule ever seen in practice. This QL rule requires 100 percent inspection until a predetermined number of successive pieces, say 5, fall within the same NLG limits used in FG.

This scheme seems quite simple and easy to use. Unfortunately, it is very difficult to properly assess its Operating Characteristic (OC) curve which depicts the probability of acceptance as a function of the degree of process shift.

A practical QL rule would require an easy assessment of its OC curve as well as its easy implementation. It should utilize the same set of FG limit gages and its acceptance/rejection decision should be based upon combinations of G, Y, R, outcomes.

Retroactive Inspection

The APQ guaranteed by a specific SF used in conjunction with a specific FG rule may not be satisfactory. The APQ may be improved to some extent without changing the NLG plan by employing Retroactive Inspection (RI). Retroactive Inspection requires 100 percent inspection of all pieces produced since the most recently inspected sample whenever an OOC indication is obtained. Removal of any defectives found during the RI gives, for larger process shifts, an average outgoing fraction defective (AOQ) that will be substantially better than the APQ without RI.. However, this improvement should be carefully evaluated against the consequent increase in inspection cost.

Examples

Following are two examples of NLG actually used in industry, which illustrate the contrast between lengthy wording and the concise notation introduced earlier in this chapter. Also, the relative importance of each NLG component (FG, SF, QL, and RI).

Example 1. The following set of NLG rules was created and first used by Jones and Lamson Machine Company [19] and then greatly popularized by Juran's [20] Quality Control Handbook (2nd edition, section 19). The rules read as follows:

1. Divide the tolerance band with NLG lines at 1/4 and 3/4 of the tolerance (which exceeds six standard deviations of the process).

2. Start job.

3. If piece is outside specification limits, reset.

4. If one piece is inside specification limits but outside a NLG line, check next piece.

5. If second piece is also outside same NLG line, reset.

6. If second piece is inside NLG line, continue process and reset only when two pieces in a row are outside a given NLG line.

7. If two successive pieces show one to be outside the high NLG line and one below the low NLG line, action must be taken immediately to reduce variation.

8. When five successive pieces fall between the NLG lines, frequency gaging may start. While waiting for five, if one piece goes over a NLG line, start count over again.

9. When frequency gaging, let process alone until a piece exceeds a NLG line. Check the very next piece and proceed as in 6 above.

10. When machine is reset, five successive pieces inside the NLG lines must again be realized before returning to frequency gaging.

11. If the operator checks more than 25 times without having to reset his process, his gaging frequency may be reduced so that more pieces are made between checks. If, on the other hand, he must reset before 25 checks are made, increase the gaging frequency. An average of 25 checks to a reset is indication that the gaging frequency is correct.

Now, this same set of rules can be described by using the proposed notation as follows:

FG: USLLSL > 6, t = USLLSL/4, n = 2, m = 3, y = 1, g = 1, r = 0

QL: 100% inspection until 5 consecutive G obtained

SF: 25 samples per OOC indication

RI: none.

Note that the proposed notation and procedure does not distinguish between Y values which fall below the low NLG line and Y values which fall above the high NLG line.

<u>Example 2</u>. The following NLG plan is used by a different major manufacturer [38]. Their description reads as follows: Suppose the work limit spread is equal to, or greater than, seven standard deviations, and NLG limits are established 1.5 standard deviations inside the work limits. A two-out-of-three NLG sampling plan is described herein:

> A sample of three consecutive components is drawn and two of the components are gaged. The third is gaged only when necessary as per below:
>
> <u>IN--NO ACTION REQUIRED</u>
>
> (1) Both components in NLG limits.
> (2) One in and one out of NLG limits (but within work limits) and the third component is in NLG limits.
>
> <u>OUT--READJUST/CORRECT MACHINE</u>
>
> (1) Any component out of work limits.
> (2) Both components out on the same side of NLG limits.
> (3) One in and one out of NLG limits (but within work limits) and the third component out on same side of NLG limits.
>
> <u>OUT--MACHINE CAPABILITY QUESTIONABLE</u>
>
> (1) When two components out of three (or two out of two) are both out of NLG limits, one high and one low, the operation is suspected of having too much variation. A machine capability study should be made with machine maintenance as necessary.

Now, this same set of rules can be described by using the proposed notation as follows:

FG: USLLSL $\geq$ 7, $t = 1.5$, $n = 3$, $m = 3$, $y = 1$, $g = 2$, $r = 0$

QL: none

SF: not specified

RI: none.

Comments

The above analysis, discussion, and illustration of NLG taxonomy make clear the general structure of NLG, and demonstrate the potentially hazardous diversity of possible NLG rules. Without adequate simplification and standardization, the implementation, evaluation, design, and comparison of NLG plans will remain very difficult or even impossible. Among all four NLG components, FG is the most important and most complicated, and therefore needs to be substantially improved. The other three components, SF, QL, and RI, are relatively not as important and are less controversial. In practice, it is quite possible that QL and RI may not even be required.

Simplification and Standardization of NLG

To facilitate easy implementation, accurate numeric evaluation, concise expression, and convenient comparison for NLG plans, a simplified "standard" NLG is proposed in the following sections.

Frequency Gaging

It is recommended that in FG the parameters be constrained, and thereby simplified. Only $m = 2$ or $m = 3$ should be considered, since $m > 3$ will result in complicated NLG gages and cumbersome gaging procedures. The NLG control inset t should always be measured inward from the specification limits rather than measured outward from the center of the specification interval. This puts more emphasis on "defective control" rather than "shift control." In other words, as long as the process keeps producing satisfactory products, the process level is allowed to shift. Finally, when $m = 3$, a R should represent a real defective and the process should always be rejected.

Acceptance/rejection criteria may also be simplified. Acceptance/rejection decisions should be based on combinations (rather than permutations) of G, Y, R such that truncation possibilities are maximized. By letting $r = 0$, and therefore tolerating no R, maximum rejection truncation can be achieved. Field implementation and numeric evaluation will also be made much easier if $r = 0$. Rejection truncation should also be applied to Y. Whenever the cumulative number of Y in a sample exceeds y, the sample should be rejected and inspection truncated. Even acceptance truncation can be allowed. This should be allowed to occur only when g straight Gs are obtained from the beginning of the sample. The rule "g straight Gs from the beginning" is more advantageous than the rule "g Gs out of first x pieces" in terms of easy implementation and evaluation.

Based upon the above discussion, simplified standard NLG FG rules are summarized as below:

n--should be kept small (often in the range from 2 to 6)

m--only $m = 2$ or $m = 3$ are considered

t--$0 < t <$ USLLSL/2 and is always measured inward from USL and LSL

r--$r = 0$ and the sample is rejected and inspection truncated as soon as a R is encountered

y--$0 \leq y \leq n$ (usually in the range $0 \leq y \leq$ INTEGER $(n/2+\cdot 5)$). Whenever the cumulative number of Y in a sample exceeds y, the sample is rejected and inspection truncated

g--$0 \le g \le n - 1$ (usually in the range $0 \le g \le$ INTEGER $(n/2+.5)$). As soon as g consecutive Gs from the beginning of the sample are obtained, acceptance occurs and inspection is truncated.

Sampling Frequency

No rigid SF rule is proposed; rather, the SF depends upon a user's individual need. If the user is concerned with having proper assurance of APQ of the process, a self-adjusting SF is suggested. That is, keep constant the average number of inspected samples per OOC indication (approximately 25 to 50 samples per OOC indication is recommended in Reference [20]). On the other hand, if the user is not concerned about the APQ, any other SF scheme may be selected.

Qualification

To simplify the evaluation, design, and implementation of the QL rule, the concepts underlying single acceptance sampling are adopted. It is recommended that QL make use of the same m, t, r values from FG and also that g = 0. Thus only n and y are allowed to vary. By proper manipulation of n and y, QL's OC curve can be adjusted to the user's desired shape. Standardized QL is summarized as follows:

n--free to vary

m--same as that used in FG

t--same as that used in FG

r--same as that used in FG (i.e., r = 0) y--$0 \leq y \leq n$, free to vary

g--$g = 0$.

Retroactive Inspection

It is recommended in RI that all pieces produced since the most recent acceptable sample be 100 percent inspected whenever an OOC indication is obtained.

Comments

After adequate simplification and standardization, this easy-to-implement, precise-to-evaluate, and concise-to-express version of standardized NLG scheme will certainly have broader application in industry. All later chapters are based upon the standard NLG version as proposed above.

For practical purposes, the implementation of NLG does not require all four of the components discussed above. Except for the mandatory FG, selection of SF, QL, and RI essentially depends upon the user's individual needs. For example, if the user does not care about the assurance of APQ, a simple SF rule may be specified rather than a self-adjusting SF rule as discussed above, which is harder to implement. If the user has no reason to suspect problems in process setup, and resets, there is no need to include the QL rule in a NLG plan. Similarly, if it is desired to improve the APQ by any means other than screening inspection, or if the 100 percent inspection is relatively costly, RI will never be needed.

In all, to better suit individual needs, the user must always carefully evaluate the particular situation before deciding exactly which components to be included in the NLG plan.

CHAPTER IV

STATISTICAL EVALUATION AND DESIGN OF STANDARD (STD) NLG PLANS; COMPARISONS WITH $\bar{X}$-CHARTS

Introduction

This chapter first discusses the statistical evaluation of Standard (STD) NLG plans. The calculation methods for both samplewise and process-wise performance measures are derived. Then, the statistical design of STD NLG is developed. Greater details are provided for the design procedures of both FG and QL, while a more general approach is given to the processwise design. Finally, after the derivation of methodologies for evaluating and designing $\bar{X}$-charts, a comparison between STD NLG and $\bar{X}$-charts is provided through an example.

Notation

In addition to the notation introduced in Chapter III, the following terms are employed to facilitate this chapter's discussion:

STD NLG--Standard NLG plan which is described in Chapter III

P_g, P_y, P_r--probability of an inspected item being classified as Green, Yellow, Red, respectively

Φ, Φ^{-1}--Φ is the cumulative probability function of the standard normal distribution; Φ^{-1} is the inverse function of Φ

μ, μ_o--μ is the process mean which has the value μ_o before any shifting occurs

σ, σ_o--σ is the process standard deviation which has the value of σ_o before shifting

δ--the distance (in multiples of σ_o) between shifted μ and μ_o

p, p_o--p is the process fraction defective which is also called the process level; it has the value of p_o before shifting. $0 \le p$ (or p_o) ≤ 1

P_a (p or δ)--the probability of acceptance of a sample, which is a function of p or δ

E_n (p or δ)--average number of pieces inspected in a sample of size n, which is a function of p or δ; it is also known as average sample number or average inspection number

ARL (p or δ)--average run length; average number of samples inspected before deciding to reset. $ARL(p) = 1/(1 - P_a(p))$. Likewise, $ARL(\delta) = 1/(1 - P_a(\delta))$

PBAPQ--probability bound on average produced quality

PBAOQ--probability bound on average outgoing quality resulting from employing RI

F--average number of samples per OOC indication; it is known as self-adjusting sampling frequency

APL--acceptable process level which is a satisfactorily small p or δ value; the process is considered functioning well at this quality level

RPL--rejectable process level which is an undesirably large p or δ value; the process is considered functioning poorly at this quality level

TLAPL, TLRPL--user-specified lower tolerable limit of P_a (APL) and upper tolerable limit of P_a (RPL), respectively; in other words, values of P_a (APL) $\geq$ TLAPL and P_a (RPL) $\leq$ TLRPL are desired

v--in the modified $\bar{X}$-chart, v is the distance in multiples of σ_o between a specification limit and the corresponding boundary for an acceptable process mean. For both traditional and designed $\bar{X}$-charts, v = USLLSL/2 (see section entitled "Evaluation and Design of $\bar{X}$-Charts")

k--control limit spread in multiples of $\sigma_o/\sqrt{n}$ for $\bar{X}$-charts. In both traditional and designed $\bar{X}$-charts, control limits are $k\sigma_o/\sqrt{n}$ outward from μ_o. In modified $\bar{X}$-charts, control limits are $k\sigma_o/\sqrt{n}$ outward from the boundary of the acceptable process mean on each side (see section entitled "Evaluation and Design of $\bar{X}$-Charts")

UCL, LCL--upper and lower control limits of $\bar{X}$-charts, respectively.

Statistical Evaluation of STD NLG Plans

Assumptions

In order to present exact formulations of numerical evaluations, several assumptions concerning STD NLG parameters are explicitly stated here:

1. The process characteristic of interest is normally distributed with mean μ and standard deviation σ. Before shifting occurs, $\mu = \mu_o$ and $\sigma = \sigma_o$.

2. The specification tolerance is $(USL - LSL) \geq 6\sigma_o$ (or USLLSL ≥ 6).

3. The process may shift in either one (but not both) of the following two forms:

a. Process mean may shift away from μ_o in either direction.

b. Process dispersion may increase and become greater than σ_o.

These assumptions will be maintained throughout this research. Possible relaxations and their effects will be discussed later.

Formulation of Probabilities of G, Y, R

Under the above assumptions, and given values of m, t, USL, LSL, and σ_o, the probabilities of G, Y, R can be obtained. The formulations are derived for three different cases, namely (1) before any process shift, (2) after a process mean shift, and (3) after a process dispersion change. First, m = 3 is considered for each of the three cases.

Case 1: Before any shift occurs, the process has a normal distribution with mean μ_o and standard deviation σ_o. Its probabilities of G, Y, R, namely, P_g, P_y, P_r, respectively, can be derived as follows (see Figure 4.1(a)): Let

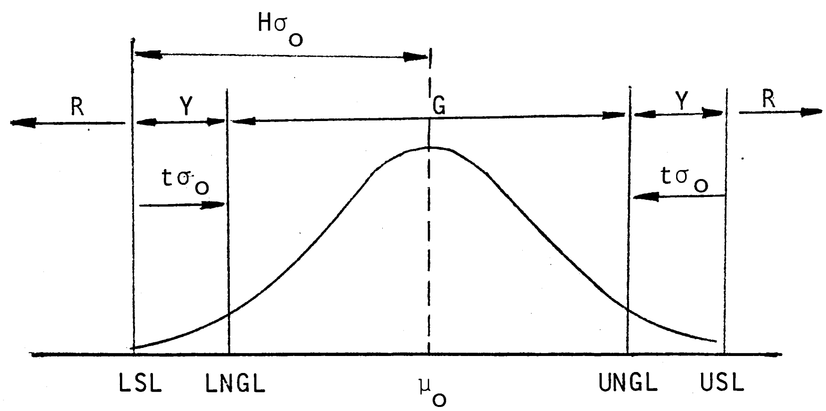
$$H = USLLSL/2 = (USL - LSL)/2\sigma_o$$

$$P_r = \Phi(-H) + [1 - \Phi(H)] = 2\Phi(-H)$$

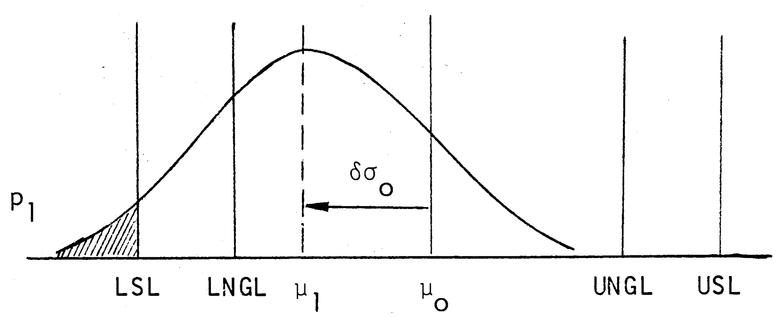
$$P_g = \Phi(H - t) - \Phi[-(H - t)]$$

$$P_y = 1 - P_g - P_r$$

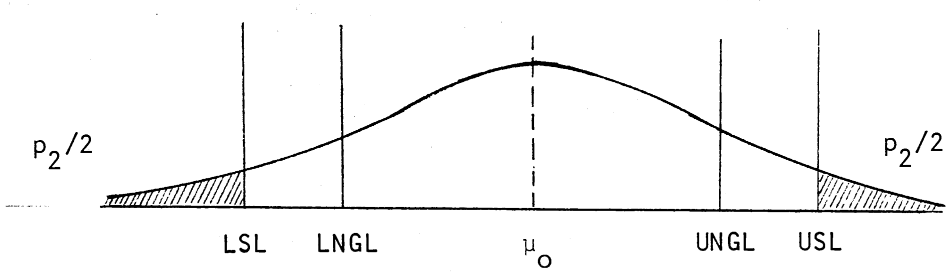
Case 2: While the process standard deviation remains constant, the process mean shifts $\delta\sigma_o$ from μ_o and results in a fraction defective p_1. The calculation of P_g, P_y, and P_r can be derived as follows (see Figure 4.1(b)):



(a) Case 1: Both μ and σ Remain Unchanged ($\mu = \mu_o$, $\sigma = \sigma_o$)



(b) Case 2: μ Shifts While σ Remains Unchanged ($\mu = \mu_1$, $\sigma = \sigma_o$)



(c) Case 3: σ Increases While μ Remains Unchanged ($\mu = \mu_o$, $\sigma = \sigma_2$)

Figure 4.1. Three Cases of Process Shifts Under the Surveillance of an NLG Plan

If δ is given, p_1 can be obtained as:

$$p_1 = 1 - \Phi(H + \delta) + \Phi(-H + \delta)$$

If p_1 is given, δ can be approximately calculated as:

$$\delta = \Phi^{-1}(p_1) + H$$

where $p_1 > p_o$ and USLLSL ≥ 6 are assumed. The greater the differences in both equalities, the better the approximation.

For both situations,

$$P_r = p_1$$

$$P_g = \Phi(H - t + \delta) - \Phi[-(H - t) + \delta]$$

$$P_y = 1 - P_g - P_r$$

Case 3: While the process mean stays at μ_o, the process standard deviation increases to σ_2 and results in a fraction defective p_2. The calculation can be derived as follows (see Figure 4.1(c)):

If σ_2 is given, p_2 can be obtained as

$$p_2 = 2\Phi(-H\sigma_o/\sigma_2)$$

If p_2 is given, σ_2 can be calculated as

$$\sigma_2 = -H\sigma_o/\Phi^{-1}(p_2/2)$$

For both situations,

$$P_r = p_2$$

$$P_g = \Phi[(H - t)\ \sigma_o/\sigma_2] - \Phi[-(H - t)\ \sigma_o/\sigma_2]$$

$$= 2\{0.5 - \Phi[(-H + t)\ \sigma_o/\sigma_2]\} = 1 - 2\Phi[(-H + t)\ \sigma_o/\sigma_2]$$

$$P_y = 1 - P_g - P_r$$

When $m = 2$, the formulations for the above three cases still apply, where P_g remains the same, but $P_y = 1 - P_g$ and P_r no longer exists.

Formulation of Performance Measures for Frequency Gaging

Probability of acceptance (P_a), Average Run Length (ARL), and average number of inspections in a sample (E_n) are the three most important performance measures in FG. The ARL is a function of P_a, namely ARL = $1/(1 - P_a)$. Therefore, it suffices to consider only the formulations of P_a and E_n. Also, since the derivations of P_g, P_y, and P_r have been developed in the last section, it is convenient to express P_a and E_n in terms of P_g, P_y, and P_r instead of the original NLG parameters.

Probability of Acceptance (P_a). In the derivation of P_a, the simpler case without G acceptance truncation is first considered. That is, only Y and R rejection truncations are considered. Then the formulation is advanced to accommodate G acceptance truncation. Finally, all formulas are summarized into a single general equation which suits both situations.

1. For $g = 0$, without G acceptance truncation:

For $m = 2$, the sample is accepted if and only if the total number of Y is no greater than y. This number is binomially distributed. Similarly, for $m = 3$, in addition to the above condition, no R can be tolerated. Now, the combinations of numbers of G, Y, R become multinomially distributed. But since the number of R is restricted to 0, this multinomial distribution actually reduces to the binomial. Thus, when m = 2, g = 0:

$$P_a = \sum_{i=0}^{y} \binom{n}{i} P_y^i P_g^{n-i}$$

where $P_y = 1 - P_g$;

when m = 3, g = 0:

$$P_a = \sum_{\substack{b=0 \\ a+b=n}}^{y} \frac{n!}{a!b!0!} P_g^a P_y^b P_r^0 = \sum_{i=0}^{y} \binom{n}{i} P_y^i P_g^{n-i}$$

where $P_y = 1 - P_g - P_r$.

2. <u>For $0 < g \leq n - 1$ (and hence $y > 0$),[a] G acceptance truncation allowed</u>:

When acceptance truncation is allowed, P_a may become larger than that with no truncation. This is due to the acceptance of the whole acceptance-truncated "branch" (of the probability tree) in which there might be some "paths" which would be rejected should no acceptance truncation be allowed. This additional probability of acceptance is therefore added to the previous formulas in (1) to account for the increase in P_a.

For both m = 2 and m = 3, the value of P_a is:

$$P_a = \sum_{i=0}^{y} \binom{n}{i} P_y^i P_g^{n-i} + P_g^g \left[1 - \sum_{j=0}^{s} \binom{n-g}{j} P_y^j P_g^{n-g-j}\right]$$

where s = min (y, n - g). In this formula, the first term represents the

---

[a]The condition $g > 0$ implies that $y > 0$. If $g > 0$ and $y = 0$, inspection will <u>always</u> be truncated and never reach its full sample size.

P_a with no acceptance truncation. The second term calculates the addition to P_a made possible by acceptance truncation.

3. In general, for both g = 0 and g > 0:

The value of Pa can now be expressed in the following summarized single equation which suits both situations:

$$P_a = \sum_{i=0}^{y} \binom{n}{i} P_y^i P_g^{n-i} + I_g P_g^g \left[1 - \sum_{j=0}^{s} \binom{n-g}{j} P_y^j P_g^{n-g-j}\right]$$

where s is min (y, n - g); and I_g is an indicator function: $I_g = 1$ if $g > 0$ (hence $y > 0$), = 0 otherwise.

Average Number of Inspections (E_n). Similar to the derivation of P_a, the average number of inspected pieces in a sample (E_n) is first derived for the simpler no G acceptance truncation case. Then the formulation is advanced to take into account the effect of G acceptance truncation. Finally, a summarized formula is developed to suit both situations.

In the following derivation of E_n, m = 2 and m = 3 are treated separately. Since n = 1 results in $E_n = 1$, only $n \geq 2$ are considered.

1. For g = 0, m = 2, n ≥ 2:

Three cases are considered: $y = 0$, $0 < y \leq n - 2$, $y \geq n - 1$.

a. y = 0: Whenever a Y is encountered, the sample is rejected and inspection truncated. This truncation can occur anywhere between the first and next to last item. Summing up the product of the numbers of items inspected and their corresponding probabilities of truncation at those numbers results in E_n. Thus,

$$E_n = \sum_{i=1}^{n-1} iP_g^{i-1} P_y + nP_g^{n-1}$$

$$= \sum_{i=1}^{n-1} iP_g^{i-1} (1 - P_g) + nP_g^{n-1}$$

b. $\underline{0 < y \leq n - 2}$: Truncation can only occur on or after the y+1st item. As soon as the number of Y reaches y+1, the inspection is truncated. Therefore, if truncation occurs at the ith item (i > y), the ith item must be classified as Y, and the rest of y Y's can be scattered among the previous i-1 items, which results in $\binom{i-1}{y}$ combinations. Thus,

$$E_n = \sum_{i=y+1}^{n-1} i\binom{i-1}{y} P_y^{y+1} P_g^{i-1-y} + n[1 - \sum_{i=y+1}^{n-1} \binom{i-1}{y} P_y^{y+1} P_g^{i-1-y}]$$

c. $\underline{y \geq n - 1}$: No truncation occurs in this case. Thus,

$$E_n = n.$$

2. <u>For g = 0, m = 3, n ≥ 2</u>

For m = 3, in addition to Y rejection truncation (i.e., the number of Y is greater than y), the sample is also rejected whenever a R is encountered. Based upon similar reasoning, the formulations in (1) above are now modified to accommodate the R rejection effect.

a. <u>y = 0</u>:

$$E_n = \sum_{i=1}^{n-1} iP_g^{i-1} (P_y + P_r) + nP_g^{n-1}$$

$$= \sum_{i=1}^{n-1} iP_g^{i-1} (1 - P_g) + nP_g^{n-1}$$

b. <u>$0 < y \leq n-2$</u>: On or before the yth item, only R truncation can occur. On or after the y+1st item, both Y truncation and R truncation can occur. Thus,

$$E_n = \sum_{i=1}^{y} i(1 - S_{i-1})\ P_r + \sum_{i=y+1}^{n-1} i[(1 - S_{i-1})\ P_r$$

$$+ \binom{i-1}{y}\ P_y^{y+1}\ P_g^{i-1-y}] + n(1 - S_{n-1})$$

$$= \sum_{i=1}^{n} iU_i$$

where

$S_o = 0$

$S_i = S_{i-1} + U_i$ for $0 < i \leq n - 1$

$U_i = (1 - S_{i-1})\ P_r$ for $1 \leq i \leq y$

$= (1 - S_{i-1})\ P_r + \binom{i-1}{y}\ P_y^{y+1}\ P_g^{i-1-y}$ for $y < i \leq n - 1$

$= 1 - S_{n-1}$ for $i = n$

For example, if $n = 5$, $m = 3$, $y = 2$, $g = 0$, $r = 0$

$U_1 = P_r$

$U_2 = (1 - U_1)\ P_r$

$U_3 = (1 - U_1 - U_2)\ P_r + \binom{2}{2}\ P_y^3\ P_g^0$

$U_4 = (1 - U_1 - U_2 - U_3)\ P_r + \binom{3}{2}\ P_y^3\ P_g^1$

$U_5 = 1 - U_1 - U_2 - U_3 - U_4$

$$E_n = \sum_{i=1}^{5} iU_i$$

c. $\underline{y \geq n-1}$: Only R truncations can occur in this case. Thus,

$$E_n = \sum_{i=1}^{n-1} i(1-S_{i-1})\, P_r + n(1-S_{n-1}) = \sum_{i=1}^{n} iU_i$$

where

$$S_o = 0$$

$$S_i = S_{i-1} + U_i \qquad \text{for } 0 < i \leq n-1$$

$$U_i = (1-S_{i-1})\, P_r \qquad \text{for } 1 \leq i \leq n-1$$

$$= 1 - S_{n-1} \qquad \text{for } i = n$$

3. For $0 < g \leq n-1$, $m = 2$, $n \geq 2$

Acceptance truncation $g > 0$ also implies that $y > 0$; otherwise, the process will always be truncated before reaching the full sample size. Therefore, only two cases are considered: $0 < y \leq n-2$ and $y \geq n-1$. In both cases, the acceptance truncation effect is added to the formulas in (1) above.

a. $\underline{0 < y \leq n-2}$:

$$E_n = \sum_{i=y+1}^{n-1} i\binom{i-1}{y} P_y^{y+1} P_g^{i-1-y} + gP_g^g$$

$$+ n[1 - \sum_{i=y+1}^{n-1} \binom{i-1}{y} P_y^{y+1} P_g^{i-1-y} - P_g^g]$$

b. $\underline{y \geq n-1}$:

$$E_n = gP_g^g + n[1 - P_g^g]$$

4. For $0 < g \leq n-1$, $m = 3$, $n \geq 2$

Similar to (3) above, the formulas in (2) above are revised to account for the G acceptance truncation effect for the $0 < y \leq n - 2$ and $y \geq n - 1$ cases.

a. <u>$0 < y \leq n - 2$:</u>

$$E_n = \sum_{i=1}^{n} iU_i$$

where

$$S_o = 0$$

$$S_i = S_{i-1} + U_i \qquad \text{for } 0<i\leq n-1$$

$$U_i = (1 - S_{i-1})\ P_r \qquad \text{for } 1\leq i\leq y \text{ and } g\neq i$$

$$= (1 - S_{i-1})\ P_r + P_g^g \qquad \text{for } 1\leq i\leq y \text{ and } g=i$$

$$= (1 - S_{i-1})\ P_r + \binom{i-1}{y} P_y^{y+1}\ P_g^{i-1-y} \qquad \text{for } y<i\leq n-1 \text{ and } g\neq i$$

$$= (1 - S_{i-1})\ P_r + \binom{i-1}{y} P_y^{y+1}\ P_g^{i-1-y} + P_g^g \qquad \text{for } y<i\leq n-1 \text{ and } g=i$$

$$= 1 - S_{n-1} \qquad \text{for } i=n$$

b. <u>$y \geq n - 1$:</u>

$$E_n = \sum_{i=1}^{n} iU_i$$

where

$$S_o = 0$$

$$S_i = S_{i-1} + U_i \qquad \text{for } 0 < i \leq n - 1$$

$$U_i = (1 - S_{i-1})\ P_r \qquad \text{for } 1 \leq i \leq n - 1 \text{ and } g \neq i$$

$$= (1 - S_{i-1})\ P_r + P_g^g \qquad \text{for } 1 \leq i \leq n - 1 \text{ and } g = i$$

$$= 1 - S_{n-1} \qquad \text{for } i = n$$

5. Summary for $m = 2,\ 0 \le g \le n-1,\ n \ge 2$

a. For $y = 0$ and $g = 0$:

$$E_n = \sum_{i=1}^{n-1} iP_g^{i-1} P_y + nP_g^{n-1}$$

b. For $0 < y \le n-2$ and $0 \le g \le n-1$:

$$E_n = \sum_{i=y+1}^{n-1} i\binom{i-1}{y} P_y^{y+1} P_g^{i-1-y} + I_g\, gP_g^g$$

$$+ n[1 - \sum_{i=y+1}^{n-1} \binom{i-1}{y} P_y^{y+1} P_g^{i-1-y} - I_g P_g^g]$$

where the indicator function

$$I_g = 1 \quad \text{if } g > 0$$
$$= 0 \quad \text{if } g = 0.$$

c. For $y \ge n-1$ and $0 \le g \le n-1$:

$$E_n = I_g\, gP_g^g + n[1 - I_g P_g^g]$$

where the indicator function I_g is defined as above.

6. Summary for $m = 3,\ 0 \le g \le n-1,\ n \ge 2$

a. For $y = 0$ and $g = 0$:

$$E_n = \sum_{i=1}^{n-1} iP_g^{i-1} (i - P_g) + nP_g^{n-1}$$

b. For $0 < y \le n-2$ and $0 \le g \le n-1$:

$$E_n = \sum_{i=1}^{n} iU_i$$

where

$$S_o = 0$$

$$S_i = S_{i-1} + U_i \qquad \text{for} \quad 0 < i \leq n - 1$$

$$U_i = (1 - S_{i-1})P_r + I_i\binom{i-1}{y}P_y^{y+1}\,P_g^{i-1-y} + J_i\,P_g^g \qquad \text{for} \quad 1 \leq i \leq n - 1$$

$$= 1 - S_{n-1} \qquad \text{for } i = n$$

where the indicator functions

$$I_i = 1 \quad \text{for } y < i \leq n - 1 \qquad J_i = 1 \quad \text{for } i = g$$

$$= 0 \quad \text{for } 1 \leq i \leq y \qquad = 0 \quad \text{for } i \neq g$$

c. For $y \geq n - 1$ and $0 \leq g \leq n - 1$:

$$E_n = \sum_{i=1}^{n} iU_i$$

where

$$S_o = 0$$

$$S_i = S_{i-1} + U_i \qquad \text{for} \quad 0 < i \leq n - 1$$

$$U_i = (1 - S_{i-1})\ P_r + J_i\ P_g^g \qquad \text{for} \quad 1 \leq i \leq n - 1$$

$$= 1 - S_{n-1} \qquad \text{for} \quad i = n$$

where

$$J_i = 1 \quad \text{for } i = g$$

$$= 0 \quad \text{for } i \neq g.$$

Formulation of Performance Measures for Qualification

The performance measures for QL are exactly the same as those for FG.

Given values of n and y, letting g = 0, and keeping the same m, t, r values determined for FG, P_a, and E_n can readily be evaluated by the same set of formulas derived in the previous section for FG.

Formulation of Performance Measures for the Process as a Whole

In evaluating the performance of the whole process, Average Produced Quality (APQ) and Average Outgoing Quality (AOQ) are the two performance measures to be investigated. Considering the process as a whole, APQ indicates the long term average of the quality produced by the process, while AOQ represents the long term average of the improved quality after RI.

Probability Bound of APQ (PBAPQ). In order to obtain the exact APQ value, the mean of the time-to-shift distribution of the process must be known. However, this mean may not be easy to estimate. Fortunately, the self-adjusting SF rule can help provide a somewhat conservative estimation of APQ, namely the Probability Bound of APQ (PBAPQ) without knowledge of the mean time-to-shift. This PBAPQ provides a guarantee on the limit of the APQ. In other words, in the long term, the process APQ should be no worse than the PBAPQ.

Following are assumptions needed for the formulation of PBAPQ:

1. The probability of a false alarm is relatively small compared to that of a true alarm.

2. The inspection time, the assignable cause searching time, and the time to reset the process are relatively negligible.

3. The number of pieces inspected is relatively small compared to the number of pieces produced.

4. A second process shift does not occur until the first is detected.

5. Qualification (if needed) takes a relatively short period of time compared to that for FG.

Based on these assumptions, the formula for the PBAPQ can be approximated as follows (see Figure 4.2):

$$PBAPQ(p) = \frac{1}{F} [p (\frac{1}{1 - P_a(p)} - 0.5) + p_o (F - \frac{1}{1 - P_a(p)} + 0.5)]$$

where

p = fraction defective produced by the shifted process;

p_o = fraction defective produced by an unshifted process;

F = average number of samples per OOC indication; and

$1 - P_a(p)$ = probability of an alarm (i.e., an OOC indication) for a process having the fraction defective p.

Here $1/[1 - P_a(p)]$ is the average number of samples required to detect the shifted process and $1/[1 - P_a(p)] - 0.5$ is the average number of inspection intervals between the process shift and its detection, which must be confined in the range of 0 and F to be meaningful. The factors p and p_o are weighted by the expected length of the OOC and IC intervals, and division by F spreads these defectives over the entire period since the previous OOC indication. Finally, without including the mean time-to-shift, the above formulation can therefore only represent an upper bound of the true APQ.

For a specified F and SF, a small value of p can make the OOC indication occur very infrequently in F samples no matter how large the

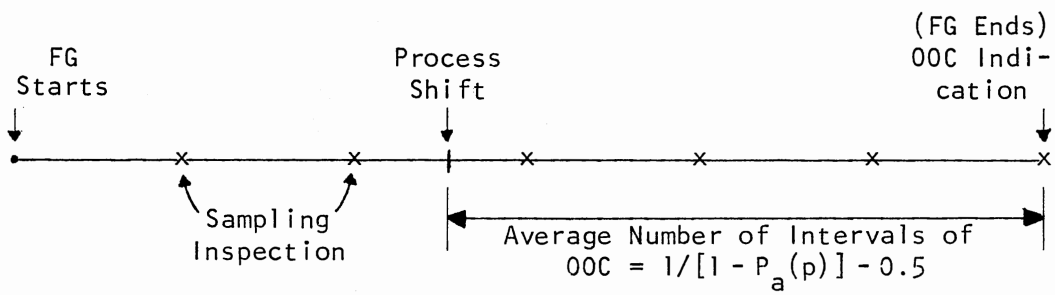


Figure 4.2. NLG Frequency Gaging Cycle

intervals are, and hence impede implementation of the SF rule. This follows because $1/[1 - P_a(p)] - 0.5$ cannot exceed F. In other words, $1 - P_a(p)$ must be greater than $1/(F + 0.5)$ to some extent to make the implementation of F samples per OOC indication possible. If this does not occur, either F can be increased or stricter FG rules can be employed to overcome this difficulty.

The closeness of the PBAPQ to the true APQ depends upon the difference between $1 - P_a(p)$ and $1/(F+0.5)$. The larger the difference (i.e., $1 - P_a(p) << 1/(F+0.5)$), the closer the PBAPQ to APQ. Furthermore, the length of the mean time-to-shift will also affect this accuracy. In all cases, PBAPQ(p) can never exceed p.

Probability Bound of AOQ (PBAOQ). RI calls for inspection of all pieces since the last inspection whenever an OOC indication is obtained. Therefore, no defectives are left in the lot if the control plan picks up the process shift on the first sample after the process shift occurs. But the plan does not always pick it up on the first inspection. Rather, RI can eliminate the defectives of only one interval per F samples. Therefore, the upper bound of the AOQ becomes

$$\mathrm{PBAOQ}(p) = \frac{1}{F}\left[p\left(\frac{1}{1 - P_a(p)} - 0.5 - 1\right) + p_o\left(F - \frac{1}{1 - P_a(p)} + 0.5\right)\right]$$

where $1/[1 - P_a(p)] - 1.5$ must be confined in the range of 0 and F to be meaningful.

Comments

All of the above formulations (P_g, P_y, P_r, P_a, E_n, PBAPQ, and PBAOQ) are based upon the normality assumption which can now be relaxed. For any other distribution, after replacing Φ and Φ^{-1} by the corresponding cumulative and inverse cumulative distribution functions, all of these formulations still apply.

The assumption that USLLSL ≥ 6 can also be relaxed. This assumption facilitates a better P_g, P_y approximation when an unknown δ is derived from a given p under the process mean shift condition. For a smaller USLLSL value, δ can still be obtained to any desirable accuracy from a given p value by employing an iterative procedure. This procedure first evaluates the sum of the p areas under both tails as a function of a trial δ value and then repeatedly adjusts δ until its corresponding p value is close enough to the given p.

When evaluating the process as a whole, PBAPQ and PBAOQ can only be used as conservative approximations of real APQ and AOQ values. However, if in implementation the mean time-to-shift and the assignable cause searching time have been acquired, APQ and AOQ can be more accurately evaluated based on similar reasoning to that used in the PBAPQ and PBAOQ derivation.

Statistical Design of STD NLG Plans

Introduction

Traditionally, the commonly used statistically based process control plans such as the $\bar{X}$-chart, p chart, and c chart are implemented without any design consideration. Their performances are rarely adequately understood by the user and may well not fit the user's own particular need. Consequently, these plans may result in misuse.

In order to help one understand the performance of multi-parameter NLG plans, the statistical design procedure of STD NLG is derived in this section. The general effects of NLG parameters on P_a and E_n are first presented. These measures are critical in understanding NLG's performance and can facilitate its design in each step. Then, detailed design procedures of FG and QL follow. Finally, this section is concluded by a discussion of the general strategy for process-wise NLG design.

General Effects of STD NLG Parameters on P_a and E_n

The general effects of each of the parameters n, t, y, g on FG performance measures P_a and E_n are investigated for both the m = 2 and m = 3 cases under either mean shift or dispersion change conditions. Beginning with a base plan (USLLSL = 7, n = 3, t = 1, y = 1, g = 1, r = 0), each parameter is freed to vary one at a time while the rest remain fixed. Table 4.1 shows the range of variation for each individual parameter. It also identifies the figures which depict the effects of parameter variations on performance measures P_a and E_n. Each figure contains four graphs: (1) m = 2 with mean shift, (2) m = 3 with mean shift, (3) m = 2 with dispersion change, and (4) m = 3 with dispersion change. In the y effect example, the reason for specifying g = 0 instead of g = 1 as used in the base case is to show the effect of y = 0, since g = 1 implies y > 0 as explained previously.

Effects on P_a. In the following discussion, conclusions are based on the mean shift assumption; however, the effects due to dispersion

TABLE 4.1

PARAMETER RANGE AND RELEVANT FIGURE NUMBER FOR INDIVIDUAL NLG PARAMETER EFFECT ON P_a AND E_n

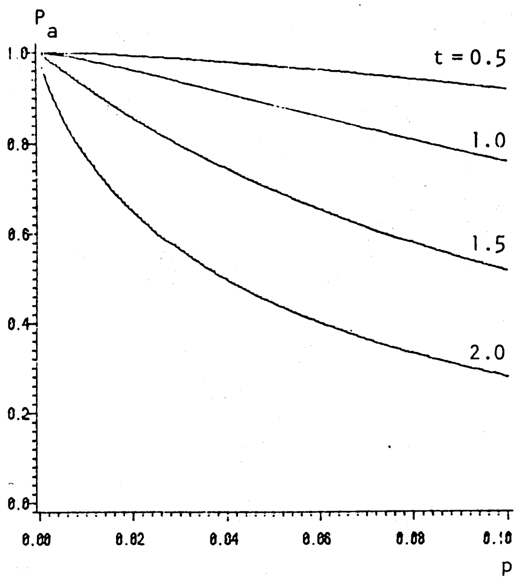
	t	y	g	n	Relevant Figure P_a	Relevant Figure E_n
Base	1	1	1	3	---	---
t Effect	0.5 1 1.5 2	1	1	3	Fig. 4.3	Fig. 4.7
y Effect	1	0 1 2 3	0	3	Fig. 4.4	Fig. 4.8
g Effect	1	1	0 1 2 3	3	Fig. 4.5	Fig. 4.9
n Effect	1	1	1	2 3 5 8	Fig. 4.6	Fig. 4.10

changes are quite similar. Also, in general, m = 2 and m = 3 have similar results. Therefore, their differences are discussed only when necessary. For all graphs, P_a is usually decreasing (and always nonincreasing) as the process fraction defective P increases.

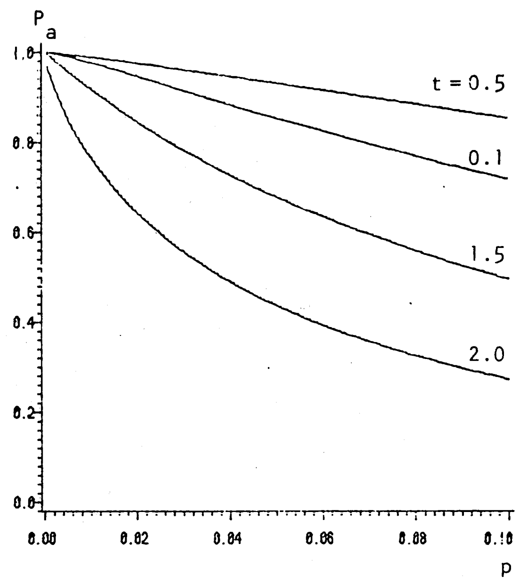
The effect of t is shown in Figure 4.3. For a given process level p, as t increases, P_a decreases. This is because larger t values cause smaller P_g and larger P_y (while P_r remains the same), which consequently yield more Ys and fewer Gs.

The effect of y is shown in Figure 4.4. Under the same process level p, as y increases, P_a also increases. This is because when y increases, more Ys are tolerable. In other words, larger y means a more lenient acceptance criterion. Among y = 0. 1, 2, 3, y = 0 has a very severe impact on the reduction of P_a. It should be noted that when m = 2, y = 3, acceptance always occurs regardless of process levels. On the other hand, due to R rejection, the P_a of m = 3 and y = 3 yields the usual declining OC curve. Finally, the OC curve of y = 2 and y = 3 are very close to each other.

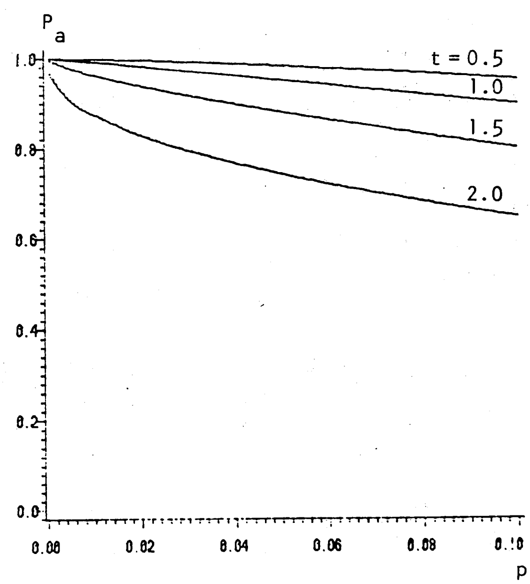
The effect of g is shown in Figure 4.5. There, g = 0 and g = 3 are essentially the same plan. They are just two different expressions for the same situation. Generally, P_a decreases as g increases (from 1 to n), given the same process level. This is because smaller g (excluding g = 0) causes earlier acceptance truncation, which converts more original rejection paths (those which should be rejected if no G acceptance truncation is allowed) into acceptance paths. In this example, g = 2 and g = 3 have the same OC curve when m = 2.



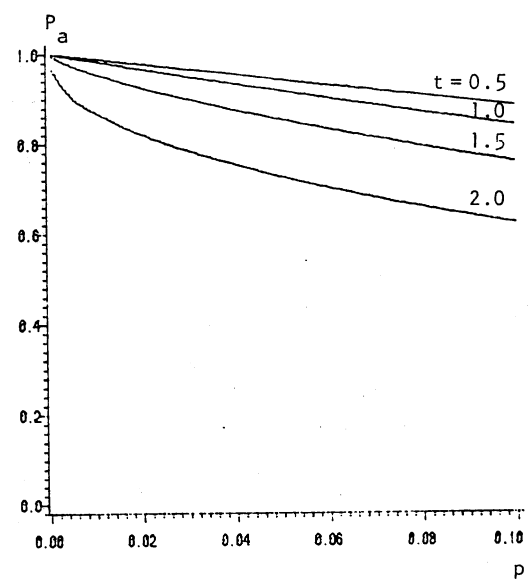
(a) m = 2; Mean Shift



(b) m = 3; Mean Shift



(c) m = 2; Dispersion Change



(d) m = 3; Dispersion Change

Figure 4.3. The Effect of t on P_a

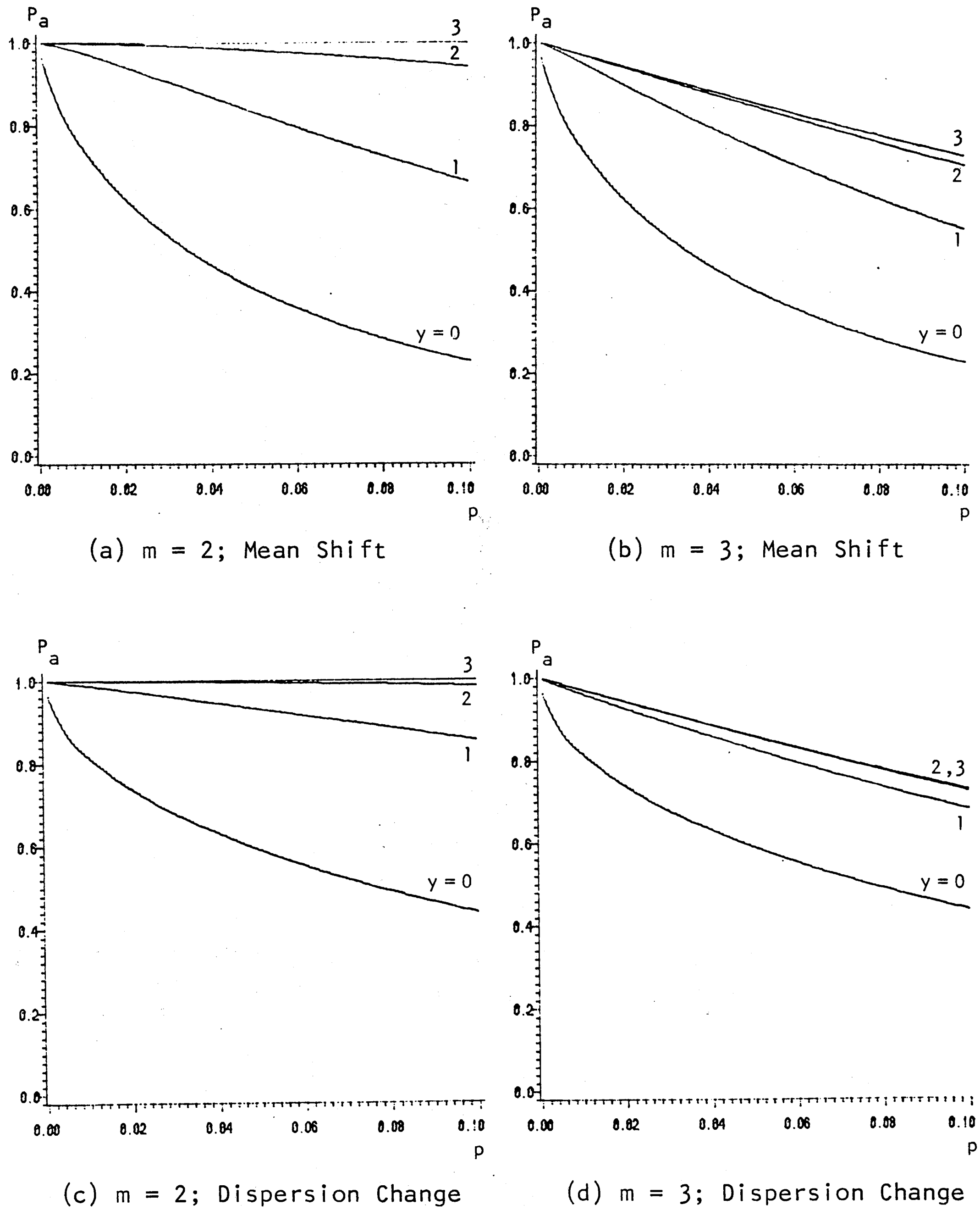
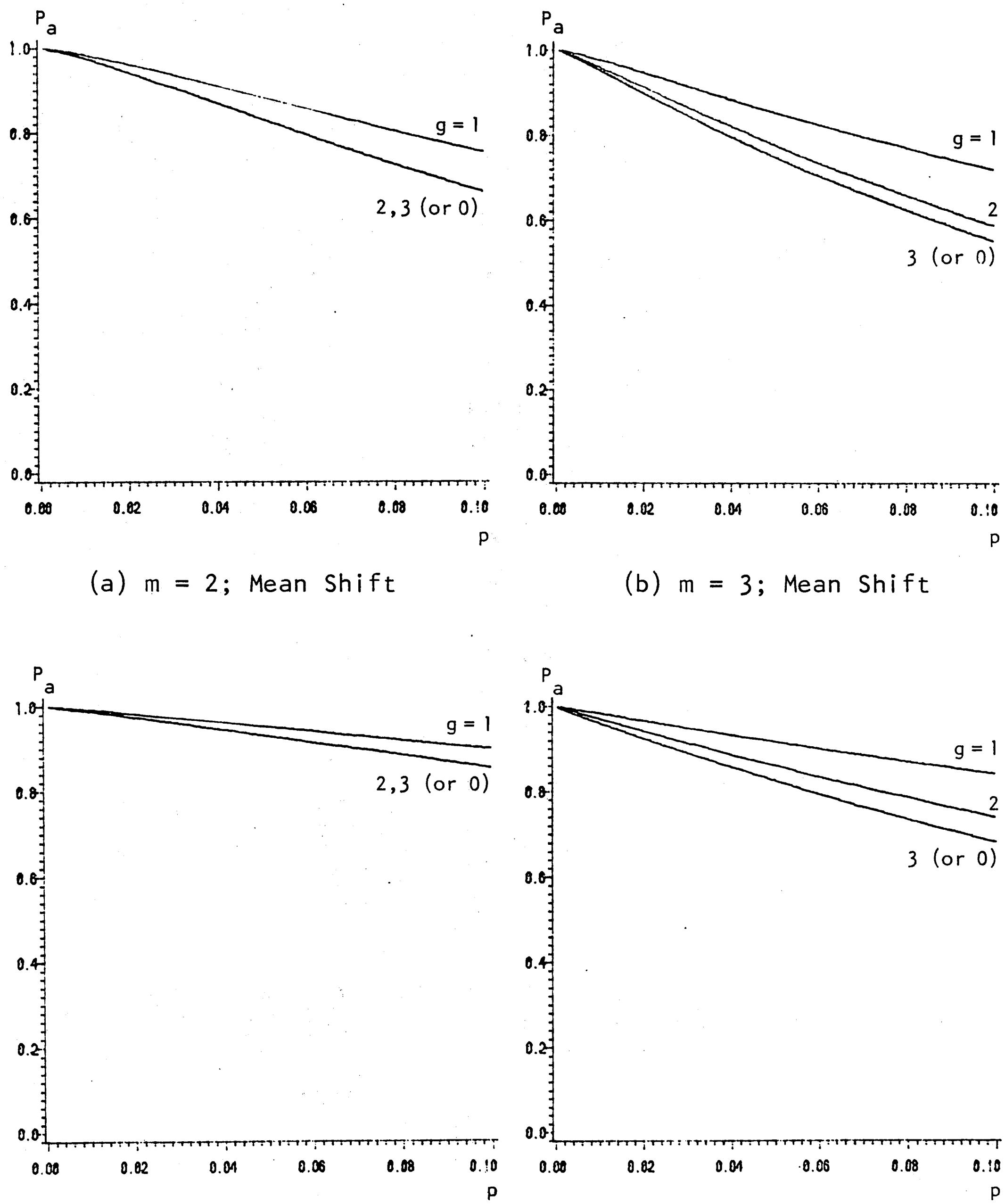


Figure 4.4. The Effect of y on P_a



(a) m = 2; Mean Shift

(b) m = 3; Mean Shift

(c) m = 2; Dispersion Change

(d) m = 3; Dispersion Change

Figure 4.5. The Effect of g on P_a

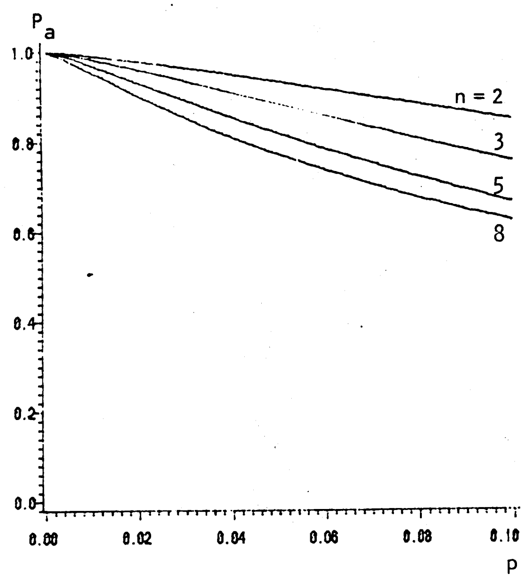
The effect of n is shown in Figure 4.6. Under the same process level, P_a decreases as n increases. This is because for the same process fraction defective, the average number of Y in a sample should increase proportionally as the sample size n increases. Consequently, to increase n without increasing y accordingly will certainly result in a stricter NLG plan and hence smaller P_a.

In short, the increases of t, g, or n, or the decrease of y, all result in steeper OC curves which provide better discrimination between good and bad process levels, but at the price of a higher false alarm rate. Among t, y, g, and n, the value of P_a (and hence the OC curve) is more sensitive to the adjustment of t and y, but less sensitive to that of g and n.

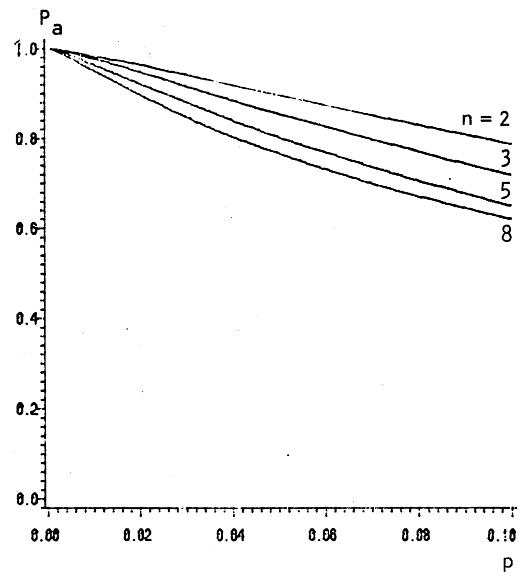
Effects on E_n. Similar to the previous section, the following discussions are based only on the mean shift assumption. Effects of dispersion changes are quite similar. Also, in general, m = 2 and m = 3 have similar results. Their differences are pointed out only when necessary.

The effect of t is shown in Figure 4.7. For all t values, E_n increases over low values of p. Under the same process level, E_n decreases as t decreases. This is because smaller t values result in larger P_g which causes more G acceptance truncation. Although larger P_g also causes less Y rejection truncation, the effect of Y rejection truncation is dominated by G acceptance truncation in this example.

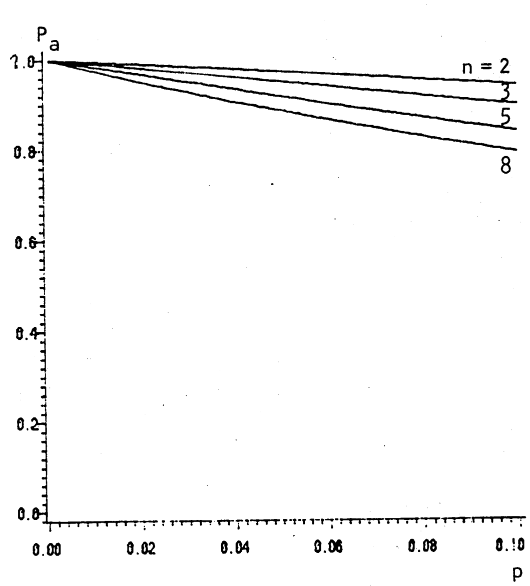
The effect of y is shown in Figure 4.8. For all y values, E_n is usually decreasing (and always non-increasing) over low values of p. Under the same process level, as y increases, E_n also increases. This is



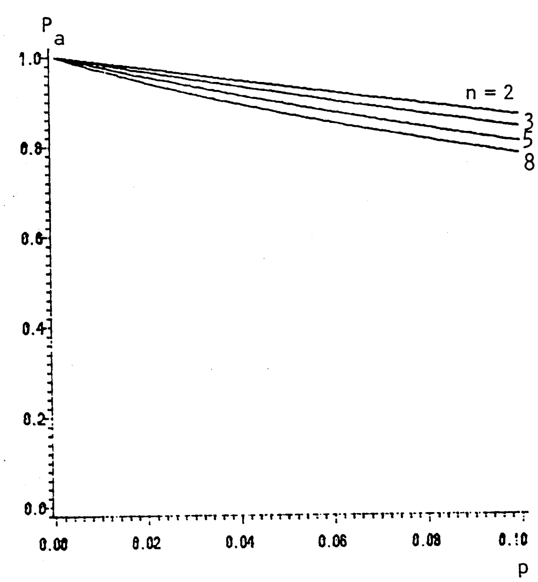
(a) m = 2; Mean Shift



(b) m = 3; Mean Shift



(c) m = 2; Dispersion Change



(d) m = 3; Dispersion Change

Figure 4.6. The Effect of n on P_a

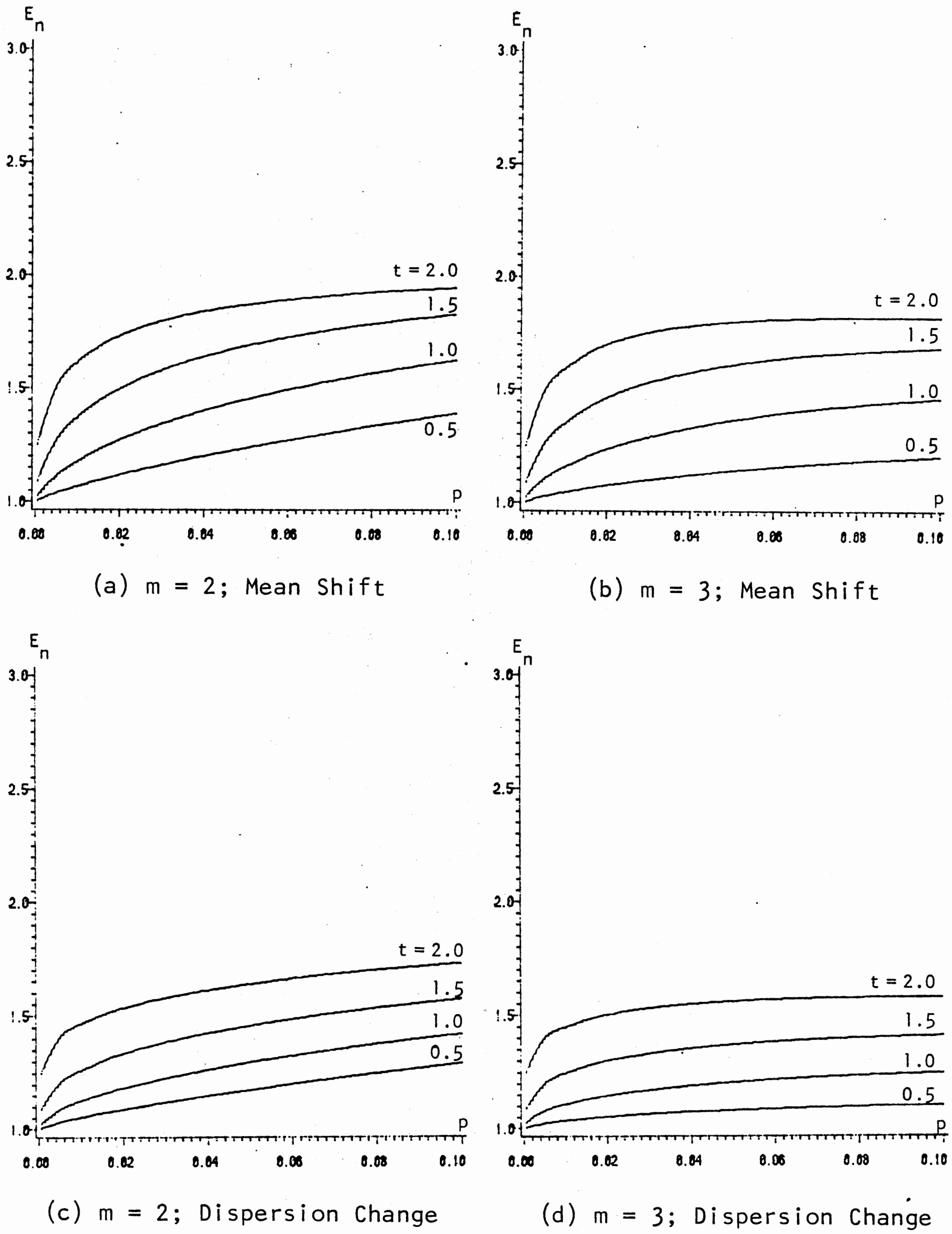
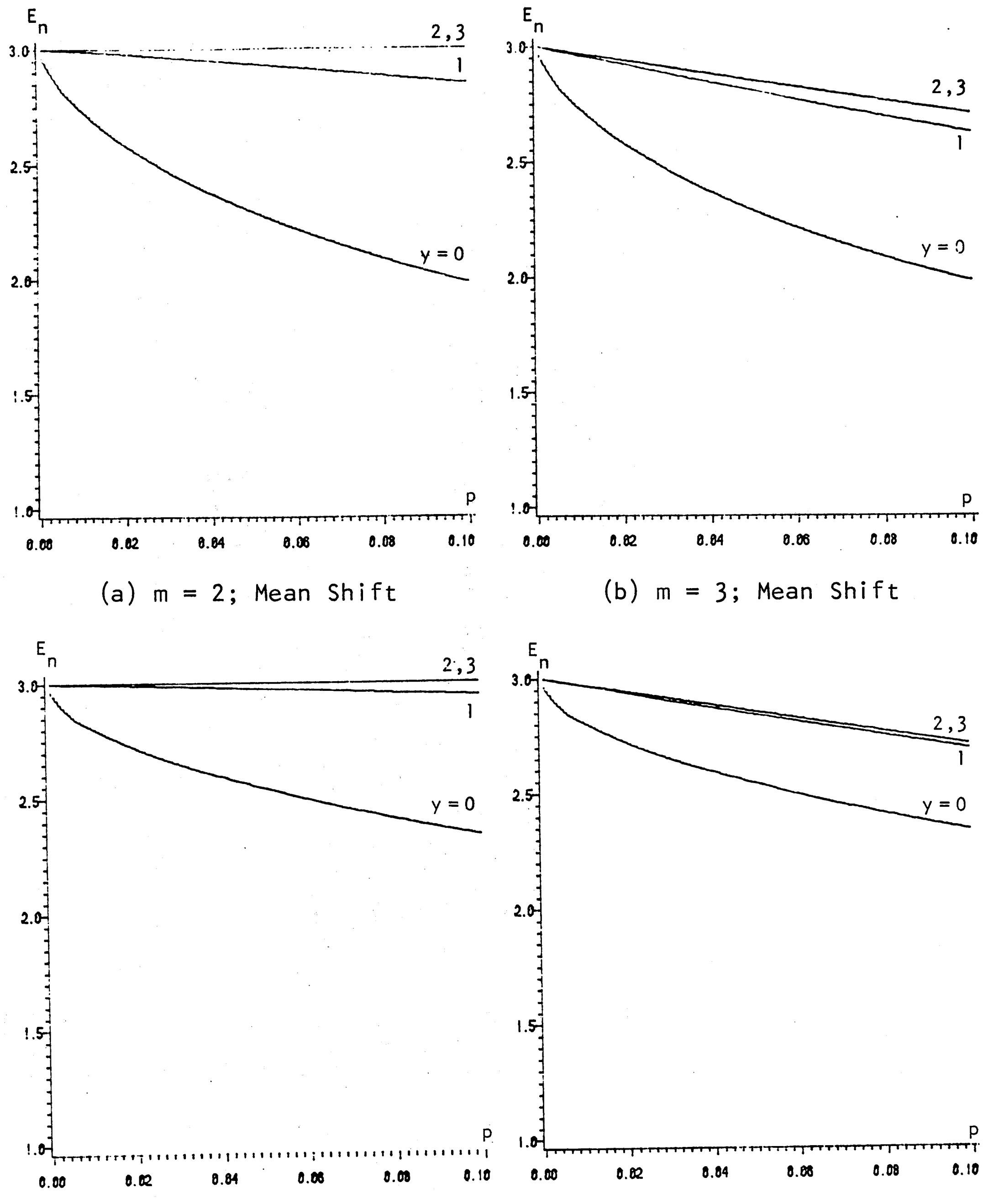


Figure 4.7. The Effect of t on E_n



(a) m = 2; Mean Shift

(b) m = 3; Mean Shift

(c) m = 2; Dispersion Change

(d) m = 3; Dispersion Change

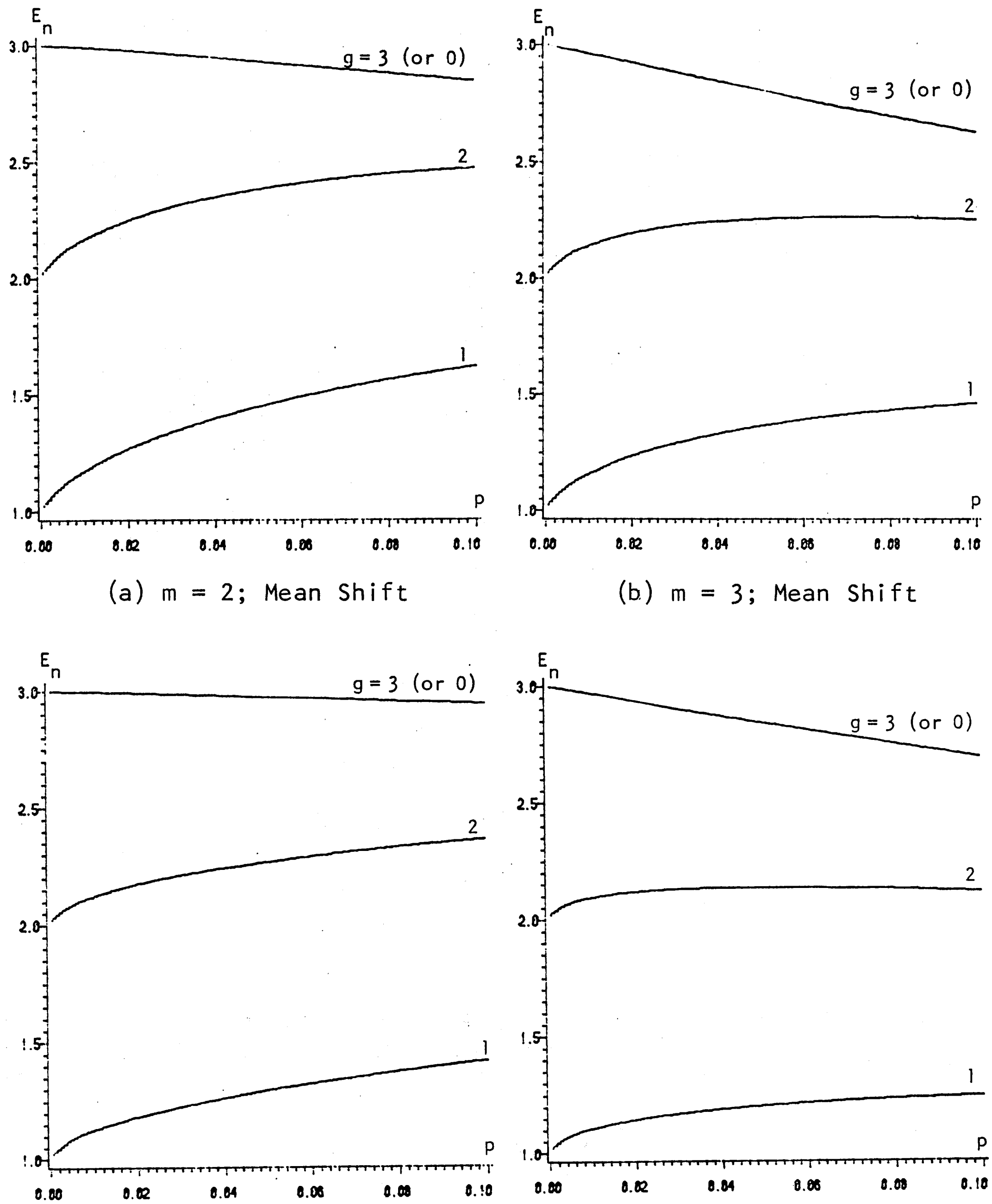
Figure 4.8. The Effect of y on E_n

because larger y means more Ys are tolerable, which in turn reduces the probability of Y rejection truncation. Among the values, y = 0, 1, 2, and 3, $y = 0$ has a dramatic impact on the reduction of E_n. For both m = 2 and m = 3 when n = 3, y = 2 has the same E_n curve as y = 3. In fact, it is always true that $y = n - 1$ has the same E_n curve as that of $y = n$. Since for $y = n - 1$, truncation can only occur at nth item (which is no truncation at all), $y = n$ and $y = n - 1$ are essentially equivalent in terms of the E_n calculation. When m = 2, it is also always true that the E_n for $y = n - 1$ or n remains $E_n = n$ regardless of process level as indicated by this example. Finally, the E_n curve for $y = 1$ and $y = 2,3$ are relatively close together.

The effect of g is shown in Figure 4.9. Here, $g = 0$ and $g = 3$ are equivalent as explained earlier. For $g = 3$ (or 0), E_n decreases over low values of p. But for $g = 1$ or 2, E_n increases over low values of p. Generally, as g increases (from 1 to 3), E_n increases significantly. This is because larger values of g cause reduced probability of G acceptance truncation.

The effect of n is shown in Figure 4.10. For all n values, E_n increases over low values of p. Under the same process level, E_n increases as n increases. As n increases from 2 to 8, E_n increases only about 50 percent. This is due to the combined effectiveness of all the acceptance/rejection truncation measures which are $g = 1$, $y = 1$, and $r = 0$.

In short, E_n is most sensitive to the adjustment of g, moderately sensitive to y and t, and least sensitive to n for these examples. However, the effects of y and n depend on the power of the acceptance/rejection truncation measures specified.



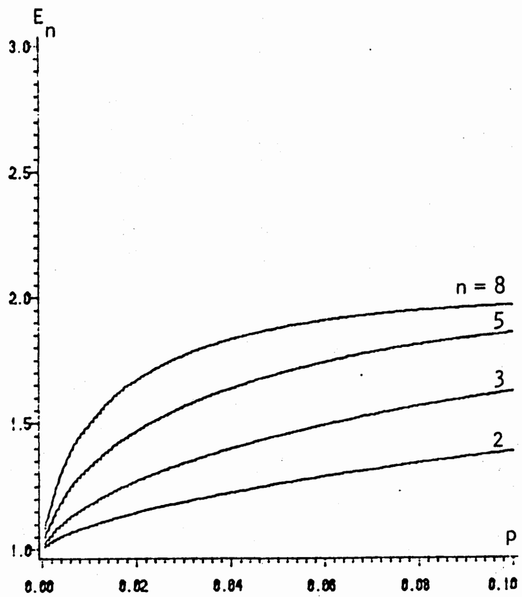
(a) m = 2; Mean Shift

(b) m = 3; Mean Shift

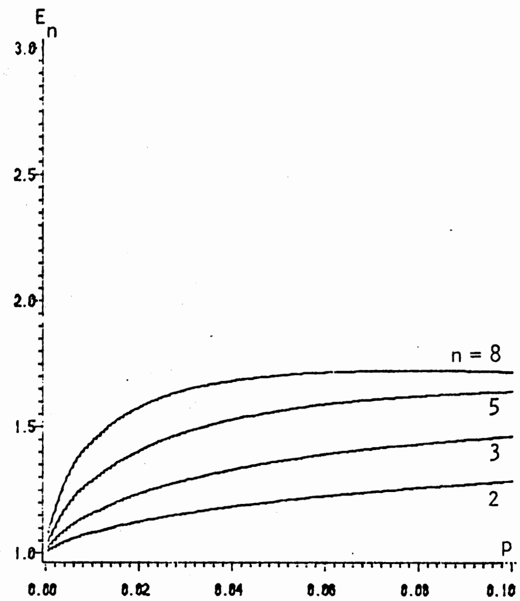
(c) m = 2; Dispersion Change

(d) m = 3; Dispersion Change

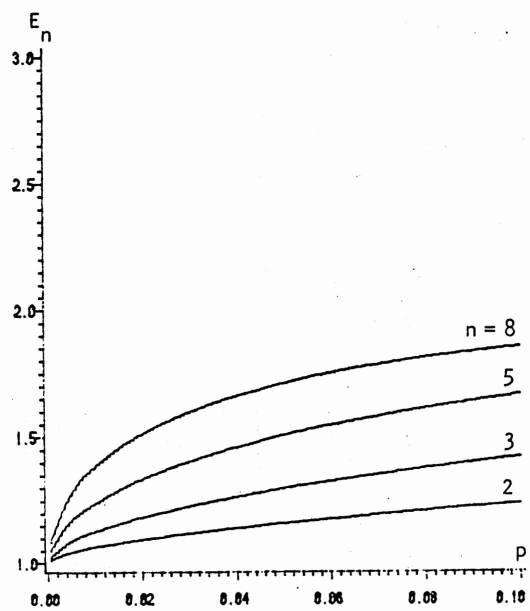
Figure 4.9. The Effect of g on E_n



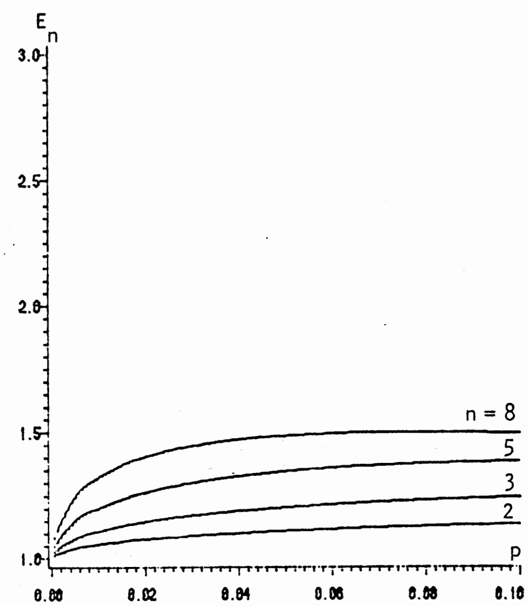
(a) m = 2; Mean Shift



(b) m = 3; Mean Shift



(c) m = 2; Dispersion Change



(d) m = 3; Dispersion Change

Figure 4.10. The Effect of n on E_n

Design of Frequency Gaging Rule

Ideally, every user would like to have a FG rule with absolute discriminative power to detect a process shift on the first sample after it occurs. Also, it is desired that the FG rule not signal any false alarms when there are no shifts at all. However, due to randomness, two types of errors may occur: (1) when the process is at the desirable Acceptable Process Level (APL), its samples may be erroneously rejected; (2) when the process is at the undesirable Rejectable Process Level (RPL), its samples may be erroneously accepted. Hence, in practice, we can specify the tolerable limits for either one or both of these two wrong decision cases. For convenience, these are called "one point" or "two point" designs.

If the defective cost is very significant and setup and reset costs are relatively negligible, one may adopt a one point design by specifying the Tolerable Limit of P_a (RPL)--TLRPL. In this case, any STD NLG rule which satisfies P_a (RPL) $\leq$ TLRPL will be considered as a qualified candidate. On the other hand, if setup and reset costs are also significant, one then should adopt a two point design by specifying the Tolerable Limits of both P_a (APL) and P_a (RPL)--TLAPL, TLRPL. In this case, all the qualified candidate plans must satisfy both P_a (APL) $\geq$ TLAPL and P_a (RPL) $\leq$ TLRPL. These strategies are similar to the design strategies of Attribute Single Sampling Plans, in which the counterparts of APL, TLAPL, RPL, and TLRPL are AQL (Acceptable Quality Level), $1 - \alpha$ (where α is Type I Error), LTPD (Lot Tolerance Percent Defective) and β (Type II Error), respectively.

To select the most appropriate plan from all of the candidates requires proper statistical comparison. Unfortunately, there is no ultimate objective criterion for statistical comparison like the "total cost" used in economic comparisons. Different users may emphasize different performance measures, and eventually the final decision must resort to individual subjective judgment.

Among P_a and E_n, generally, P_a is used as a primary criterion and E_n is secondary. Except when unit inspection cost is very high, the user prefers a plan with a better OC curve (in the sense that it fits better to those user-designated design points) but with a slightly worse E_n curve, rather than the opposite situation. However, if two qualified plans have quite similar OC curves, the user surely prefers the one with a better E_n curve, thus resulting in lower inspection cost. For those cases with non-comparable OC and E_n curves, the decision of selection will rely heavily on individual needs and the user's subjective judgment.

Theoretically, the design procedure for FG is quite straightforward. After specifying the design points for the OC curve, the user proceeds to separate out all qualified plans from the complete set of possible plans. Finally, proper comparisons among those candidates lead to the selection of a most desired FG rule. However, in practice, due to the large number of possible variations of multiple FG parameters, the number of qualified candidates becomes formidable and hence makes the comparisons and final selection very difficult or even impossible.

To alleviate this problem, proper restrictions can first be imposed on the variations of n, t, y, and g to considerably reduce the number of possible plans considered. This number can be further reduced by evaluating each at the APL and RPL and eliminating all but the qualified plans. For example, for USLLSL = 7, mean shift assumed, and m = 2, we may confine the variations as follows: $2 \le n \le 5$; $0 \le y \le$ INTEGER $(n/2 + 0.5)$; $1 \le g \le n - y$ (but $g = 0$ if $y = 0$); $t = 1, 1.5, 2$; which results in 66 plans.

Then the P_a and E_n of each plan are evaluated at the APL and RPL. Suppose APL = 0.01, TLAPL = 0.90, RPL = 0.10, and TLRPL = 0.20. Among these 66 plans, only 9 plans are qualified. After proper comparisons, the final decision may be subjectively reached. However, if further improvement on the selected plan is still desired, it may be modified in the direction of the user's interest by properly adjusting individual parameters (mainly t, or if necessary, n, y, and even g). This adjustment may utilize the general properties of the effects of individual parameters on P_a and E_n as revealed previously.

Design of Qualification Rule

Based upon similar reasoning as that used for FG, the QL rule can be designed using a one- or two-point approach depending on the user's need. Recall that in STD NLG QL, m, t, and r have the same values as those used in FG; g is set equal to 0; and only n and y are allowed to vary.

For specified values of TLAPL and TLRPL of QL, any qualified QL rule should have an OC curve satisfying the following:

$$P_a(APL) = \sum_{i=0}^{y} \binom{n}{i} P_y^i\ (APL)\ P_g^{n-i}\ (APL) \geq TLAPL$$

and

$$P_a(RPL) = \sum_{i=0}^{y} \binom{n}{i} P_y^i\ (RPL)\ P_g^{n-i}\ (RPL) \leq TLRPL$$

In QL, since only n and y are allowed to vary, and both are integers, the number of possible QL plans is quite limited for typical values of n. Hence, searching for the most desirable QL rule is much easier, with no trial and error needed. For the same example used in the FG design section (i.e., USLLSL = 7, mean shift assumed, m = 2), suppose the final t chosen is 1.7. Now, for APL = 0.2σ, TLAPL = 0.90, RPL = 2σ, TLRPL = 0.10, and $2 \leq n \leq 8$, among 35 possible plans, only 3 are qualified. Consequently, the final selection can easily be made.

General Procedure to Satisfy a Designated PBAPQ

If assurance is desired for the APQ being less than a designated value, the following general procedure may be followed. The user should first evaluate the PBAPQ of the currently used FG and SF rules to see if it is satisfactory. If not, the user may increase the SF to reduce PBAPQ to the desired level. If for some reason SF should not be changed, the user may modify the FG rule to achieve the same purpose. Finally, RI can also be employed to temporarily improve the PBAPQ.

Comments

The effects of NLG parameters on P_a and E_n have been demonstrated only for one typical example. Some of the properties revealed may change somewhat for different cases. Thus, more examples covering a wider range of NLG applications may be found worthwhile.

Since the flexible general procedures for designing FG and QL are quite cumbersome and time consuming, an alternative might be considered for real world practice. To provide a convenient application, standard tabulation of already-designed FG and QL plans suitable for a wide range of typical conditions can be developed for use. These may include typical values of n and t under typical sets of APL, TLAPL, RPL, TLRPL, and typical USLLSL intervals. Thus, users can just look up the table and select the plans which match best with their particular needs.

Evaluation and Design of $\bar{X}$-Charts

Introduction

It is desirable to compare NLG to the most popular process control scheme, the $\bar{X}$-chart. In order to do this properly, methodologies for designing and evaluating an $\bar{X}$-chart are presented. The $\bar{X}$-chart is the counterpart of only one phase of STD NLG, namely NLG FG.

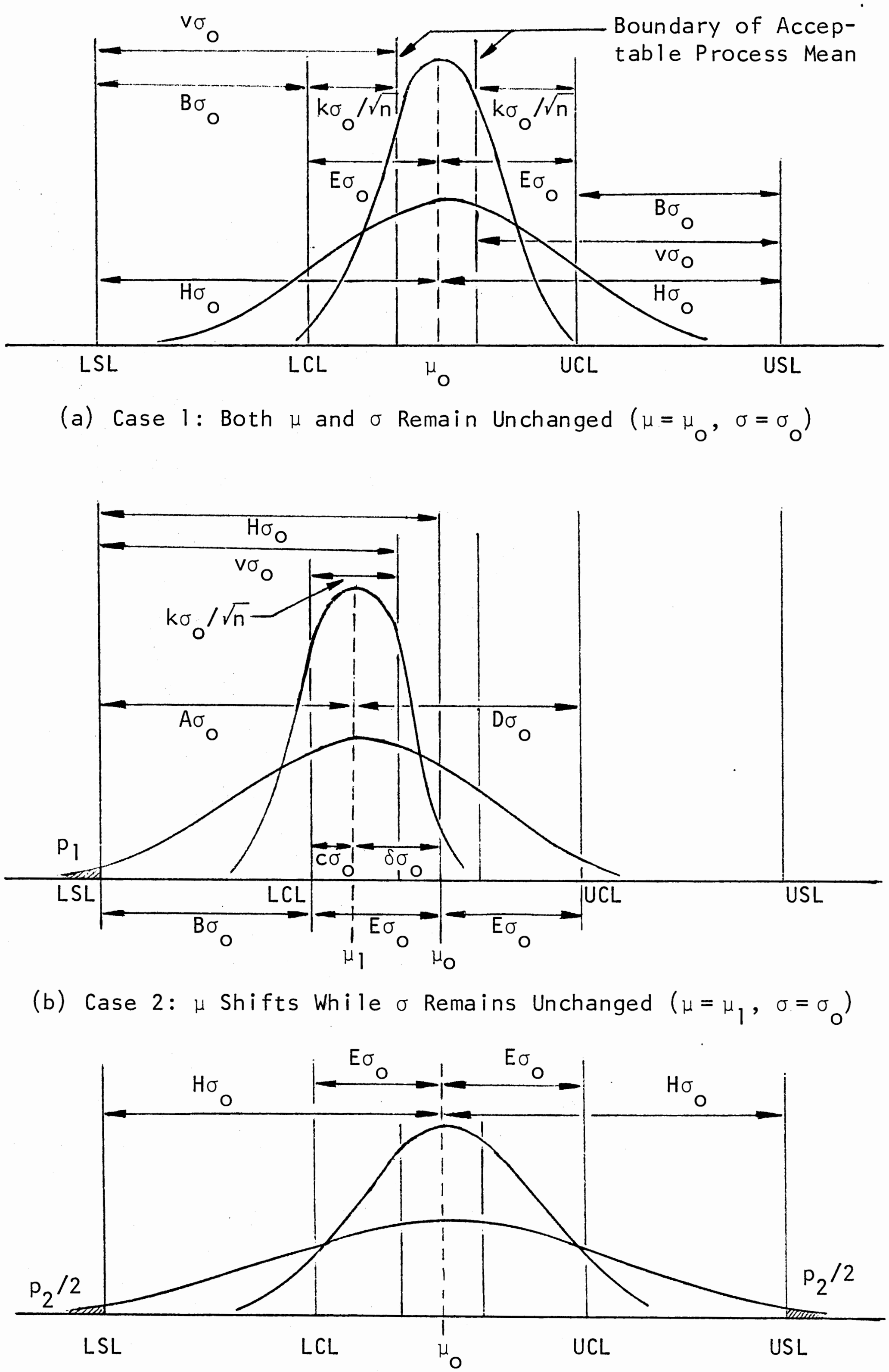
In an $\bar{X}$-chart control scheme, a sample of size n is taken regularly with its average value calculated and compared to the predetermined upper and lower control limits, UCL and LCL. Whenever a sample average falls beyond the control limits, the process is reset accordingly. Otherwise, it continues. There are three major variations used in specifying UCL and LCL, which in turn yield three versions of $\bar{X}$-charts.

1. Traditional $\bar{X}$-chart: The sample size n and control limits UCL and LCL are always fixed. No design is required. The sample size is usually set equal to 4 or 5, while UCL and LCL are often $3\sigma_o/\sqrt{n}$ away from μ_o.

2. Designed $\bar{X}$-chart: Both n and the control spread k are design variables. In this case, UCL and LCL are $k\sigma_o/\sqrt{n}$ away from μ_o.

3. Modified $\bar{X}$-chart: Both n and k are design variables. Both UCL and LCL are $k\sigma_o/\sqrt{n}$ outward from the boundaries of acceptable values of process mean. These boundaries themselves are $v\sigma_o$ inward from USL and LSL (see Figure 4.11(a)).

Among these three versions, only the modified $\bar{X}$-chart is comparable to NLG since its control limits are measured from specification limits and thus control the defectives rather than the shifts. Furthermore, both the traditional and designed $\bar{X}$-charts are just special cases of the modified $\bar{X}$-chart. Therefore, only the modified $\bar{X}$-chart will be considered



(a) Case 1: Both μ and σ Remain Unchanged ($\mu = \mu_o$, $\sigma = \sigma_o$)

(b) Case 2: μ Shifts While σ Remains Unchanged ($\mu = \mu_1$, $\sigma = \sigma_o$)

(c) Case 3: σ Increases While μ Remains Unchanged ($\mu = \mu_o$, $\sigma = \sigma_2$)

Figure 4.11. Three Cases of Process Shifts Under the Surveillance of the Modified $\bar{X}$-Chart

in the following sections which describe its evaluation and design methodologies.

Evaluation

For all versions of the $\bar{X}$-chart, no inspection truncation is allowed. Hence, $E_n = n$, the sample size. As to the evaluation of P_a, three different cases are considered for formula derivation: (1) before any shifts occur, (2) μ shifts while σ remains unchanged, and (3) σ increases while μ remains unchanged.

Case 1: Before any shifts occur, the process is normally distributed with mean μ_o, standard deviation σ_o, and fraction defective p_o. Its $P_a\ (p_o)$ can be derived as follows (see Figure 4.11(a)): Let

$$H = (USL - LSL)/2\sigma_o$$

$$LCL = LSL + B\sigma_o = LSL + (v\sigma_o - k\sigma_o/\sqrt{n}) = LSL + (v - k/\sqrt{n})\sigma_o$$

$$UCL = USL - B\sigma_o = USL - (v - k/\sqrt{n})\sigma_o$$

Since

$$E\sigma_o = E\sqrt{n}\ (\sigma_o/\sqrt{n}),$$

$$\begin{aligned} P_a(p_o) &= \Phi(E\sqrt{n}) - \Phi(-E\sqrt{n}) = \Phi[(H - B)\sqrt{n}] - \Phi[-(H - B)\sqrt{n}] \\ &= \Phi[(H - v + k/\sqrt{n})\sqrt{n}] - \Phi[(-H - v - k/\sqrt{n})\sqrt{n}] \end{aligned}$$

where

$$p_o = 2\Phi(-H)$$

Case 2: While the process dispersion stays constant, the process mean shifts $\delta\sigma_o$ away from μ_o and results in a fraction defective p_1. Its $P_a\ (p_1)$ can be derived as follows (see Figure 4.11(b)):

If σ is given, p_1 can be obtained as:

$$p_1 = 1 - \Phi(H + \delta) + \Phi(-H + \delta)$$

If p_i is given, δ can be approximated by:

$$\delta = \Phi^{-1}(p_1) + H$$

with $p_1 > p_0$ and USLLSL ≥ 6 assumed. The greater the differences in both inequalities, the better the approximation.

Since

$$C\sigma_o = C\sqrt{n}\ (\sigma_o/\sqrt{n})$$

and

$$D\sigma_o = D\sqrt{n}\ (\sigma_o/\sqrt{n}),$$

$$P_a(p_1) = \Phi(D\sqrt{n}) - \Phi(C\sqrt{n})$$

But

$$D = \delta + E = \delta + (H - B) = \delta + H - (v - k/\sqrt{n})$$

$$C = -A + B = \delta - H + (v - k/\sqrt{n})$$

Hence

$$P_a(p_1) = \Phi[(\delta + H - v + k/\sqrt{n})\sqrt{n}] - \Phi[(\delta - H + v - k/\sqrt{n})\sqrt{n}]$$

Case 3: While the process mean stays at μ_o, the process standard deviation increases to σ_2 and results in a fraction defection p_2. Its $P_a(p_2)$ can be derived as follows (see Figure 4.11(c)):

If σ_2 is given, p_2 can be obtained as

$$p_2 = 2\Phi(-H\sigma_o/\sigma_2)$$

If p_2 is given, σ_2 can be calculated as

$$\sigma_2 = -H\sigma_o/\Phi^{-1}(p_2/2)$$

Since

$$E\sigma_o = (E\sqrt{n}\ \sigma_o/\sigma_2)(\sigma_2/\sqrt{n}),$$

$$P_a(p_2) = \Phi(E\sqrt{n}\ \sigma_o/\sigma_2) - \Phi(-E\sqrt{n}\ \sigma_o/\sigma_2) = 2[0.5 - \Phi(-E\sqrt{n}\ \sigma_o/\sigma_2)]$$

$$= 1 - 2\Phi[(-H + v - k/\sqrt{n})\sqrt{n}\ \sigma_o/\sigma_2]$$

Design

Among the three variables (n, v, k) involved in a modified $\bar{X}$-chart, v is usually subjectively designated by the user and often assumes a value of 3 or 3.5. When $v = (USL - LSL)/2\sigma_o$, the modified $\bar{X}$-chart reduces to the Traditional and Designed $\bar{X}$-charts. Thus, the only two design variables of the Modified $\bar{X}$-chart are sample size n and control spread k.

In designing a Modified $\bar{X}$-chart, the same STD NLG one point or two point design strategy used for FG applies. By imposing similar variation restrictions on n and k, followed by similar searching and modification procedures, the most desirable control plan can be more easily located for $\bar{X}$-charts than for STD NLG FG.

Comments

Usually $\bar{X}$-charts are used only as the counterpart of FG in NLG. For the entire $\bar{X}$-chart process control scheme, if qualification of process setup and reset is needed, a similar $\bar{X}$-chart control mechanism (which may have different n, v, k values) can be adopted as its QL plan. The evaluation and design of this QL plan uses the same evaluation formulation and design procedure previously developed for Modified $\bar{X}$-charts. Furthermore, the evaluation of performance measures such as PBAPQ and PBAOQ for the whole process, under the surveillance of $\bar{X}$-charts, are exactly the same as that of NLG if similar SF and RI (as needed) rules are incorporated into the entire control scheme.

Comparison of STD NLG With the $\bar{X}$-Chart

Based on the understanding of methodologies for evaluating and designing both NLG plans and $\bar{X}$-charts, the user is now able to properly compare NLG with $\bar{X}$-charts. That is, based on the same set of user-designated APL, TLAPL, RPL, and TLRPL criteria, both NLG and the Modified $\bar{X}$-chart can be properly designed to qualify this same set of criteria and can then be compared to each other by their P_a and E_n curves. Finally, a decision on choosing either NLG or the $\bar{X}$-chart can be reached with proper justification.

An example comparing NLG, an $\bar{X}$-chart, and a traditional attribute gaging plan (i.e., attribute single sampling plan) is illustrated in Figure 4.12. Under mean shift assumption, given USLLSL = 7, APL = 0.01, TLAPL = 0.95, RPL = 0.10, and TLRPL = 0.33, three different types of process plans are considered for use. In the traditional attribute gaging control scheme (i.e., specification gages instead of narrow limit gages are used), the qualified plan with minimum sample size is n = 23, c = 1 (i.e., >1 defective is not acceptable). On the other hand, in the Modified $\bar{X}$-chart control scheme, a plan with $n=4$, $v=3$, and $k=3$ satisfies the same set of criteria. Obviously, this variable scheme $\bar{X}$-chart requires a much smaller sample size, while it is relatively more difficult to implement when compared to an attributes scheme.

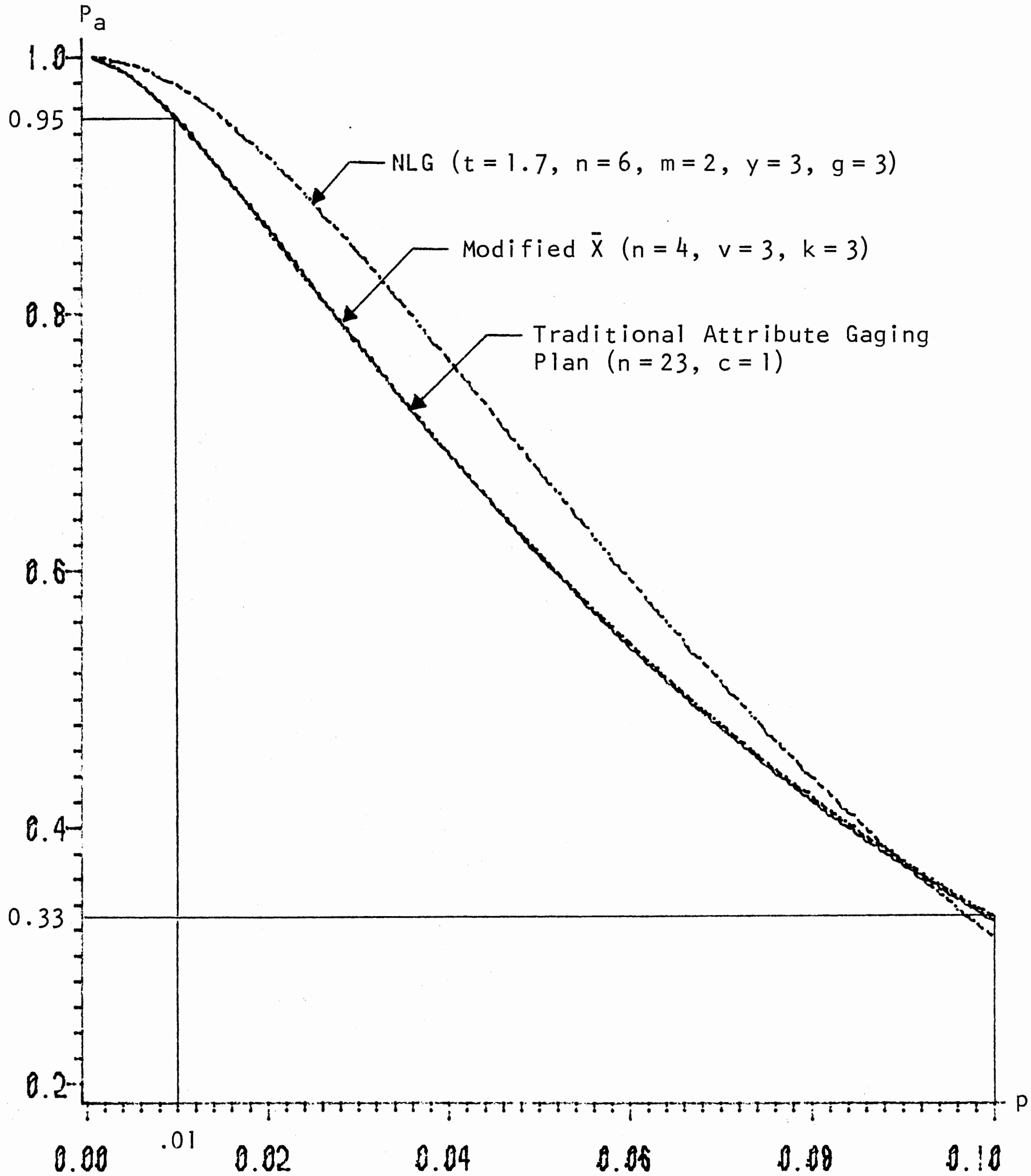


Figure 4.12. A Comparison Among Three Types of Process Control Schemes (Comparison Basis: USLLSL = 7, Mean Shift Assumed, APL = 0.01, TLAPL = 0.95, RPL = 0.10, TLRPL = 0.33)

However, if the traditional specification gages are replaced by narrow limit gages, a significant improvement on the attribute scheme can be achieved by an NLG plan with $n = 6$, $m = 2$, $t = 1.7$, $y = 3$, and $g = 3$. In this plan, all $E_n(p)$ are no greater than 5.4 for $p \leq 0.10$ and the average E_n will be less than 4.5 if the process is assumed to be IC for more than 50 percent of the time. Thus, in a typical application, this plan's E_n is very close to that of the $\bar{X}$-chart.

In this example, based on similar go/no-go gaging methods, apparently NLG is much better than traditional attribute gaging due to its much smaller average inspection number. Compared to the $\bar{X}$-chart, NLG seems equally competitive since its average inspection number is as small as that of the $\bar{X}$-chart. In fact, NLG should be administratively and economically superior to the $\bar{X}$-chart due to its easier-to-use go/no-go gaging method and no-calculation-required control scheme. In short, the statistical performance of NLG plans seems at least comparable and in some respects better than that of $\bar{X}$-charts.

Summary

In the preceding NLG statistical evaluation, the formulations of P_g, P_y, and P_r are first developed for either mean shift or dispersion change conditions. Based on these formulas, P_a and E_n are derived to evaluate the performance of FG or QL. All of these evaluations can be adapted to accommodate different distributions and narrower USLLSL intervals. For the entire process, PBAPQ and PBAOQ are developed to provide conservative upper bounds of APQ and AOQ. With the additional knowledge of mean time-to-shift and/or assignable cause searching time, the estimation of APQ and AOQ can be improved accordingly.

In NLG statistical design, the general effects of t, y, g, and n on P_a and E_n are investigated based on a typical example. Some general properties have been revealed to help design FG and QL rules. Then a flexible general procedure is constructed for designing the FG rule. This procedure starts with enumerating all possible rules followed by eliminating all those unqualified within a restricted parameter space, and finally concludes with trial and error modifications to eventually locate the most desirable plan. A similar but simpler procedure is also provided for QL. As to the design of an entire NLG plan, a very general strategy is discussed. Finally, to alleviate the design burden on users, a standard tabulation of FG and QL designs for a wide range of typical conditions is suggested.

To properly compare NLG with the most popular alternative, the $\bar{X}$-chart, methodologies for evaluating and designing a Modified $\bar{X}$-chart have been presented. Among all versions, only the Modified $\bar{X}$-chart is comparable to NLG and both the Traditional and Designed $\bar{X}$-charts are special cases of it.

Finally, this chapter is concluded by an example comparing NLG, the $\bar{X}$-chart, and a traditional attribute gaging plan. This example reveals that NLG can significantly improve the sensitivity of an attribute scheme and become as good as the most popular variable scheme--the $\bar{X}$-chart in terms of sample size. Furthermore, with the additional administrative and economic advantages, NLG has the potential to become superior to the $\bar{X}$-chart.

CHAPTER V

ECONOMIC FORMULATION AND OPTIMIZATION OF STD NLG; ECONOMIC COMPARISONS WITH THE $\bar{X}$-CHART

Introduction

This chapter provides a good alternative to statistically-based NLG and $\bar{X}$-chart control schemes--economically-based NLG and $\bar{X}$-charts. Economic schemes are more appealing in two aspects: (1) they do not require the user to supply subjective design points (such as APL, TLAPL, RPL, and TLRPL), and (2) they use "total cost" as the only performance measure, which in fact is the ultimate criterion in evaluating all control plans. In order to provide an economic comparison between NLG and the $\bar{X}$-chart, both the formulation and design of NLG plans must be considered from an economic viewpoint. The economic formulation of $\bar{X}$-charts has previously been treated in the literature.

This chapter follows Duncan's [6] $\bar{X}$-chart model (the Designed $\bar{X}$-chart) and its assumptions to formulate an economic NLG scheme. Then, an optimization algorithm utilizing a direct search technique is developed and improved to optimize the five decision variables of the economic NLG model. Finally, based on several representative examples, both models are optimized and extensively compared. General guidelines are eventually developed for the better application of both models.

Notation

In addition to notation introduced in previous chapters, the following terms are employed to facilitate this chapter's discussion:

h--the sampling interval; samples of size n are taken from the process every h hours

λ--the parameter related to the probability of occurrence of the assignable cause. The distribution of IC time is exponentially distributed with mean $1/\lambda$

e--the rate at which the average sampling, gaging, and evaluation time for a sample increases with the average sample number (E_n for NLG or n for $\bar{X}$-chart)

D--the average search time for an assignable cause

V_o--the hourly income from operation of an IC process

V_1--the hourly income from operation of an OOC process for which the mean has shifted by $\delta\sigma_o$.

M--the reduction in process hourly income that is attributed to the occurrence of the assignable cause; $M = V_o - V_1$

T--the average cost per occasion of looking for an assignable cause when none exists

W--the average cost per occasion of finding the assignable cause when it exists

b--the cost per sample of sampling, gaging, and acceptance/rejection decision making that is independent of the sample size

c--the unit cost of sampling, gaging, and evaluation that is related to the sample size; this relationship is assumed to be linear

p_δ--the fraction defective resulting from an OOC process whose mean has shifted by $\delta\sigma_o$

α--the probability of a false alarm (i.e., the control scheme indicates an OOC indication when the process is still IC); $\alpha = 1 - P_a\ (p_o)$

P--the probability of a real alarm (i.e., the control scheme indicates an OOC indication when the process is actually OOC); $P = 1 - P_a\ (p_\delta)$

β--the average proportion of time a process is IC

E_n'--the average number of pieces inspected per sample from an IC process; $E_n' = E_n\ (p_o)$

E_n''--the average number of pieces inspected per sample from an OOC process for which the mean has shifted by $\delta\sigma_o$; $E_n'' = E_n\ (p_\delta)$

E_n^*--the overall average number of pieces inspected per sample for the entire process; $E_n^* = \beta E_n' + (1 - \beta)\ E_n''$

L--the loss-cost; the minimization of L will result in the maximization of process hourly net income.

Economic NLG Formulation

General Structure

Among economically designed process control schemes, Duncan's [6] fundamental eco-omic $\bar{X}$-chart (the Designed $\bar{X}$-chart) is the most popular one due to its flexibility, simplicity of administration, and the information content of the plotted point pattern. Hence, it is used in this research as the basis against which the economic NLG model is compared.

In order to ensure proper comparison between both models, the general structure of Duncan's economic $\bar{X}$-chart is adopted for the economic NLG formulation in this research. That is, based upon the maximum income criterion, the economic model (either NLG or Duncan's $\bar{X}$-chart) measures the average net income of a process under the surveillance of its control scheme. The process starts IC and is subject to random shifts in the process mean (OOC). Once OOC, the process remains there until corrected. Given associated cost and time parameters, the optimal values of decision variables for each model are then determined using optimization techniques.

Assumptions

The economic NLG formulation is based on the same set of assumptions as used for Duncan's economic $\bar{X}$-chart. These assumptions are stated as follows:

1. Due to an assignable cause, the process mean may randomly shift to $\mu_o \pm \delta\sigma_o$ and stay there until corrected while σ remains unchanged.

2. The process is not shut down while the search for the assignable cause is in progress.

3. Neither the cost of adjustment or repair, nor the cost of bringing the process back into a state of IC after the assignable cause is discovered, is considered in the economic model.

Formula Derivation

Control Cycle. A complete economic NLG control cycle consists of four time intervals as follows (see Figure 5.1):

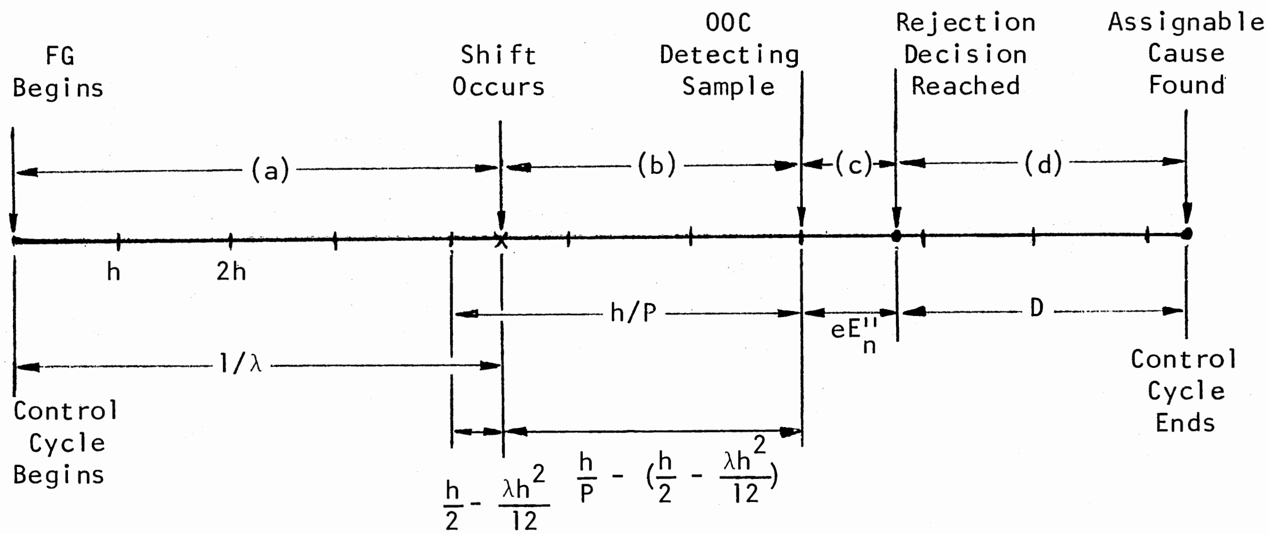


Figure 5.1. Economic NLG Control Cycle

$$\text{Control cycle length} = \overset{(a)}{(\text{IC})} + \overset{(b)}{(\text{OOC before the detecting sample})}$$
$$+ \overset{(c)}{(\text{sample inspection and evaluation})}$$
$$+ \overset{(d)}{(\text{search for assignable cause})}$$

(a) Since the average time for the occurrence of an assignable cause is $1/\lambda$, so is the process average IC time.

(b) Given the occurrence of an assignable cause in the interval between the nth and n+1st sample, the average time of occurrence within an interval between samples will be

$$\frac{\int_{nh}^{(n+1)h} e^{-\lambda x} \lambda(x - nh)\,dx}{\int_{nh}^{(n+1)h} e^{-\lambda x} \lambda dx} = \frac{e^{-\lambda nh}\int_{o}^{h} e^{-\lambda z} \lambda z dz}{e^{-\lambda nh}\int_{o}^{h} e^{-\lambda z} \lambda dz}$$

$$= \frac{1 - (1+\lambda h)\ e^{-\lambda h}}{\lambda(1 - e^{-\lambda h})}$$

$$\doteq \frac{h}{2} - \frac{\lambda h^2}{12} \quad \text{approximately.}$$

The average number of samples taken before the shift in the process is caught is $1/P$, where P is the probability of a real alarm $(P = 1 - P_a\ (p_\delta))$. Hence, $h/P - (h/2 - \lambda h^2/12)$ is approximately the average time the process will be OOC before the sample destined to detect the process shift is taken.

(c) The average sampling and evaluation time for each sample is eE_n'', where e is average sampling, gaging, and evaluation time for each piece; $E_n'' = E_n(p_\delta)$.

(d) The average time taken to locate an assignable cause is D.

Therefore,

$$\text{Control cycle length} = 1/\lambda + (1/P - 1/2 + \lambda h/12)h + eE''_n + D$$
$$= 1/\lambda + B$$

where

$$B = (1/P - 1/2 + \lambda h/12)h + eE''_n + D$$

Thus, the proportion of the time a piece is IC is

$$\beta = \frac{1/\lambda}{1/\lambda + B} = \frac{1}{1 + \lambda B}$$

Cost Formulation. Based upon the above derivation of a control cycle, formulation of the process average hourly net income is now developed as follows:

$$\begin{pmatrix}\text{Process average}\\ \text{hourly}\\ \text{net income}\end{pmatrix} = \overset{(a)}{\begin{pmatrix}\text{Weighted}\\ \text{hourly IC}\\ \text{income}\end{pmatrix}} + \overset{(b)}{\begin{pmatrix}\text{Weighted}\\ \text{hourly OOC}\\ \text{income}\end{pmatrix}} - \overset{(c)}{\begin{pmatrix}\text{Hourly false}\\ \text{alarm cost}\end{pmatrix}}$$
$$- \overset{(d)}{\begin{pmatrix}\text{Hourly real}\\ \text{alarm cost}\end{pmatrix}} - \overset{(e)}{\begin{pmatrix}\text{Hourly FG}\\ \text{cost}\end{pmatrix}}$$

(a) $\text{Weighted hourly IC income} = \begin{pmatrix}\text{Hourly income}\\ \text{from IC process}\end{pmatrix}$

$$\times \begin{pmatrix}\text{Fraction of the time}\\ \text{the process is IC}\end{pmatrix}$$
$$= V_o \times \beta$$

(b) $\text{Weighted hourly OOC income} = \begin{pmatrix}\text{Hourly income}\\ \text{from OOC process}\end{pmatrix}$

$$\times \begin{pmatrix}\text{Fraction of the time}\\ \text{the process is OOC}\end{pmatrix}$$
$$= V_1 \times (1 - \beta)$$

(c) $\begin{pmatrix}\text{Average hourly}\\ \text{false alarm cost}\end{pmatrix} = \begin{pmatrix}\text{Expected number of}\\ \text{false alarms per hour}\end{pmatrix}$

$$\times \begin{pmatrix}\text{Average cost of searching for}\\ \text{an assignable cause when a}\\ \text{false alarm is encountered}\end{pmatrix}$$

The expected number of false alarms before the process goes OOC will be the probability of false alarm (α) times the expected number of samples taken in the period. This is

$$\alpha \sum_{i=o}^{\infty} \int_{ih}^{(i+1)h} i\lambda e^{-\lambda t} dt = \alpha \sum_{i=o}^{\infty} i[e^{-ih\lambda} - e^{-(i+1)h\lambda}]$$

$$= \alpha(1 - e^{-\lambda h}) \sum_{i=o}^{\infty} ie^{-ih\lambda}$$

$$= -\alpha(1 - e^{-\lambda h}) \frac{\partial}{\partial\lambda} \frac{1}{h} \sum_{i=o}^{\infty} e^{-ih\lambda}$$

$$= \frac{\alpha e^{-\lambda h}}{1 - e^{-\lambda h}}$$

$$\doteq \frac{\alpha}{\lambda h} \quad \text{approximately.}$$

Thus, the average hourly false alarm cost $= \dfrac{\alpha/\lambda h}{\text{Control cycle length}} \times T$

$$= \frac{T\alpha/\lambda h}{1/\lambda + B} = \frac{\beta\alpha T}{h}$$

(d) $\begin{pmatrix}\text{Average hourly}\\ \text{real alarm cost}\end{pmatrix} = \begin{pmatrix}\text{Expected number of}\\ \text{real alarms per hour}\end{pmatrix}$

$$\times \begin{pmatrix}\text{Average cost of searching for}\\ \text{an assignable cause when}\\ \text{a real alarm is encountered}\end{pmatrix}$$

$$= \frac{1}{\text{Control cycle length}} \times W$$

$$= \frac{W}{1/\lambda + B} = \frac{\lambda W}{1 + \lambda B}$$

(e) $$\begin{pmatrix}\text{Average hourly}\\ \text{FG cost}\end{pmatrix} = \begin{pmatrix}\text{Hourly fixed cost per sample for}\\ \text{sampling, gaging and evaluation}\end{pmatrix}$$

$$+\begin{pmatrix}\text{Hourly variable cost per}\\ \text{piece for sampling, gag-}\\ \text{ing and evaluation}\end{pmatrix}$$

$$= b/h + c[\beta E'_n + (1 - \beta)E''_n]/h$$

$$= (b + cE^*_n)/h$$

Therefore,

$$\begin{pmatrix}\text{Process hourly}\\ \text{net income}\end{pmatrix} = \beta V_o + (1 - \beta)V_1 - \beta\alpha T/h$$

$$- \lambda W/(1 + \lambda B) - (b + cE^*_n)/h$$

$$= V_o - \frac{\lambda MB + \alpha T/h + \lambda W}{1 + \lambda B} - \frac{b + cE^*_n}{h}$$

where

$$M = V_o - V_1$$

$$= V_o - L$$

where

$$L = \frac{\lambda MB + \alpha T/h + \lambda W}{1 + \lambda B} - \frac{b + cE^*_n}{h}$$

In this formulation, to maximize average hourly net income is equivalent to minimizing the loss-cost L.

Summary of Parameters and Decision Variables

In the above economic NLG formulation, all the involved parameters and variables can be classified into three categories according to their nature:

1. Time parameters: δ, λ, e, D
2. Cost parameters: M (or V_o and V_1), T, W, b, c
3. Decision variables: n, m, h, t, y, g.

Differences Between Economic NLG and the Economic $\bar{X}$-Chart

The major difference between these two process control methods is the number of decision variables: n, h, and k for the $\bar{X}$-chart; n, m, h, t, y, and g for NLG. As to the average inspection number, n is used throughout the entire $\bar{X}$-chart plan, while E'_n, E''_n, or E^*_n is adopted depending upon the individual stage in the NLG control scheme. Finally, while all the time and cost parameters assume the same values for both models to ensure the highest degree of resemblance, the real world values of e and c for NLG may be much smaller than those for the $\bar{X}$-chart due to the simple gaging methods and evaluation procedures for NLG.

Comments

In the first assumption, a single OOC state caused by a single assignable cause is assumed. Although the multiplicity of assignable causes is more realistic in the real world, the much simpler single cause has been demonstrated by Duncan [8] to be a satisfactory approximation, and hence is somewhat preferred for use. The single OOC state is traditionally justified as representing the threshold beyond which process deterioration is intolerable and which thus represents the most difficult such OOC state to detect.

Under the second assumption, the process is not shut down during the search for an assignable cause. This is quite typical in practice.

However, there are situations when shutdown is preferred or required. In this case, the previous model no longer applies and a different model must be constructed. An example model considering shutdown has been shown by Baker [3].

Under the third assumption, the cost of resetting the process is not included in the model. In fact, the inclusion of this cost item will only add a constant term to the total cost formula, and thus has no effect on the optimal solution.

Economic NLG Optimization

General Optimization Strategy

The ultimate goal in optimizing an economic NLG model is to find the optimal combination of values of the decision variables, in order to minimize the loss-cost L and hence maximize the average hourly net income of the process under surveillance. Since L is a very complicated function of the decision variables n, m, y, g, t, and h, there exists no analytically explicit optimal solution. Therefore, multidimensional direct search techniques become the only means for optimization.

However, all six control variables cannot be simultaneously optimized using direct search, since n, m, y, and g are integers and m, y, g scatter unevenly in integer space. Therefore, the only feasible optimization strategy for economic NLG is as follows:

1. Simultaneously optimize (h,t) under each specified set of (n, m, y, g) values, resulting in a local optimum set.

2. Compare all local optimums and locate the overall optimum.

Direct Search Technique

The direct search technique employed in this research is the Nelder and Mead algorithm [32], which is straightforward, efficient, and easy to use. This method finds the minimum of a multivariable (n_v) unconstrained, nonlinear function. The minimization is achieved by the comparison of function values at the ($n_v + 1$) vertices of a general simplex, followed by replacement of the vertex having the highest value by another point. This simplex method efficiently adapts itself to the local landscape by using reflected, expanded, and contracted points; it finally contracts onto the final minimum. Derivatives are not required.

Since this algorithm is intended only for unconstrained variables, a minor modification is needed before it can be applied to NLG optimization. In NLG, the feasible ranges for h and t are: $h > 0$ and $0 \le t \le USLLSL/2$.[a] This modification is thus achieved by confining all the reflected and expanded points (and hence contracted points) to the above feasible region.

About 100 different combinations of (n, m, y, g) for several examples with different sets of parameter values have been investigated to reveal the general shape of the cost surface of L. Each cost surface of L is tabulated in a rectangular table with 25 h rows ($0 < h \le 100$) and 11 t columns ($0.01 \le t \le 2.99$). The results have shown that L surfaces are shallow and convex shaped with a minimum located a substantial distance from both ends of the feasible range of t. Only a few occasions have shown a mild ridge close to the high end border of t (i.e., $t \to 3$). In this case, once in a while the minimum lies right on the high t border. In summary, none

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[a] In actual computer programming, $h > 0.001$ and $0.001 \le t \le USLLSL/2 - 0.001$ are used to avoid intermediate underflow and overflow problems.

of the L surfaces investigated has ever indicated shapes other than the above two types.

NLG Optimization Algorithm

To find the overall optimum, all the possible combinations of (n, m, y, g) must be investigated. If n is not restricted, the number of combinations becomes infinite. Even if n is restricted to a moderate number, say 6, still there will be about 130 possible combinations, requiring extensive computational effort. Consequently, an efficient search algorithm other than the above enumeration approach is strongly desired, if there exist some favorable properties in the relations among different combinations of (n, m, y, g) which can be utilized to make such an algorithm possible.

Based on this motivation, an investigation of several examples, each with a different set of parameter values, has been performed. The results have revealed that a nice relation does exist among n, y, and g for m = 2 or m = 3, respectively. This relation can be described as follows:

1. The value of m is first specified. That is, either m = 2 or m = 3.

2. Under each set of (n,y) values, the local optimums of loss-cost (one L* for each g) for g values from g = 1 to g = n form either a convex curve or strictly increasing curve. The optimum of this curve is labeled L^*_g.

3. Under each n value, the local loss-cost optimums (one L^*_g for each y) for y values from y = 0 to y = n form either a convex curve or a strictly increasing curve. The optimum of this curve is labeled L^*_y.

4. The local loss-cost optimums (one L^*_y for each n) for n values

from n = 1 and above form either a convex curve or a strictly increasing curve. This overall optimum is labeled L^*_n.

All of these cases have shown either convex or strictly increasing values of local optimum within each of the (n, y, g) levels. In fact, in addition to all the above preliminary examples, generally all production cases investigated support this property without exception. However, in practice, the possibilities of a strictly decreasing (or non-increasing) or a very flat "generally convex" curve with a few very small bumps (due to the approximation of formulation and the cumulative inaccuracy of calculation) must be considered.

Based on this convex property, the efficient NLG optimization algorithm can now be constructed as follows:

A. General Structure of the NLG Optimization Algorithm

Notation:

L^*_g, L^*_y, L^*_n = local optimal L values within each of the (g,y,n) levels, respectively, as explained previously.

n_s, n_e; y_s, y_e; g_s, g_e = starting and ending values for n, y, and g, respectively.

1. Specify m value (m = 2 or 3).

2. Start with n_s, y_s.

3. Under specified n,y values, optimize L for each g (resulting in L^*) from g_s to g_e; compare all L^* and locate their minimum as L^*_g.

4. Under specified n, repeat step 3 for each y from y_s to y_e; compare all L^*_g and locate their minimum as L^*_y.

5. Repeat step 4 for each n from n_s to n_e; compare all L^*_y and locate their minimum as L^*_n.

6. Optimal NLG plan = the plan associated with L^*_n.

After some experience in implementing the above algorithm, further improvement in optimization efficiency can be achieved by effectively dynamically adjusting n_s, n_e, y_s, y_e, and g_s, g_e values as follows:

B. Efficiency Improvement on General NLG Optimization Structure

1. In A-3:

   a. For $y_s \geq 1$, $g_s(y_s) = 1$. For $y_s = 0$, $g_s(y_s) = 0$.

   b. Under the same n, $g_s(y_{i+1})$ = minimum $[1, g^*(y_i) - \varepsilon_g]$; where $i \geq s$, $g^*(y_i)$ = optimal g under y_i, and ε_g = a user specified allowance.

   c. When searching for L_g^*, g_e can be dynamically determined as the g having its $L^* \geq L_g' + \varepsilon_L$; where L_g' is the minimal L^* from g_s up to the current g, and ε_L is a user specified allowance to overcome those small bumps (if there are any) in a fairly flat curve.

2. Similarly, in A-4:

   a. $y_s(n_s) = 0$.

   b. $y_s(n_{i+1})$ = minimum $[0, y^*(n_i) - \varepsilon_y]$; where $i \geq s$, $y^*(n_i)$ = optimal y under n_i; and ε_y = a user specified allowance.

   c. When searching for L_y^*, y_e can be dynamically determined as the y having its $L_g^* \geq L_y' + \varepsilon_L$, where L_y' is the minimal L_g^* from y_s up to the current y.

3. n_e = the n having its $L_y^* \geq L_n' + \varepsilon_L$, where L_n' is the minimal L_y^* from n_s up to the current n.

Comments

In direct search for the optimum (h,t) under specified (n, m, y, g), sometimes the result may deviate as the starting point changes due to the existence of multiple local minima or special shapes of the loss-cost surface. Therefore, whenever the optimum (h,t) and its associated L* found by the direct search algorithm are suspect, either an investigation on the tabulation of the loss-cost surface or a rerun on several starting points should be performed to ensure the location of the real optimum.

Similarly, if the final result obtained by the improved version of the NLG optimization algorithm is suspect, a complete enumeration of all n, y, and g should be performed to help locate the real overall optimal plan.

Economic Comparison Between NLG and the $\bar{X}$-Chart

Examples for Comparison

To assess the best conditions for the application of NLG and the $\bar{X}$-chart, both control schemes are compared. Both schemes are based upon the same assumptions and evaluated under the same environments. Twelve representative examples are chosen from Duncan's [6] paper as shown in Table 5.1. The values assigned to the cost and time factors in this table cover a wide range of variations. Under each example, both control schemes are compared for their optimal loss-costs.

These 12 examples are divided into two groups: 1 to 13 and 16 to 26. In group 1 ($\delta = 2$), example 1 is the base case, and the rest are its variations. In group 2 ($\delta = 1$), example 16 is the base case, and the rest are its variations. Example 26 is the only exception not from Duncan's paper. It is newly created and added into group 2 to show the effect of e variation.

TABLE 5.1

EXAMPLES CHOSEN FOR ECONOMIC COMPARISON BETWEEN NLG AND $\bar{X}$-CHART

No.*	δ	λ	M	e	D	T	W	b	c	Characteristics	Abbreviation
1	2	.01	100	.05	2	50	25	.50	.10	Basis for 1 to 13	δ = 2 base
3		.03								λ increases 3 times	$\lambda \uparrow 3$
5			1000							M increases 10 times	$M \uparrow 10$
7				.50						e increases 10 times	$e \uparrow 10$
8					20					D increases 10 times	$D \uparrow 10$
9						5	2.5			T and W decrease 10 times	T and $W \downarrow 10$
10						500	250			T and W increase 10 times	T and $W \uparrow 10$
12								5		b increases 10 times	$b \uparrow 10$
13									1	c increases 10 times	$c \uparrow 10$
16	1	.01	12.87	.05	2	50	25	.50	.10	Basis for 16, 26, and 20	δ = 1 base
26				.50						e increases 10 times	$e \uparrow 10$
20									1	c increases 10 times	$c \uparrow 10$

*All example numbers are the same as those used in Duncan's paper, with the exception of example 26 which is newly created.

Explanation and Analysis

Within each of these examples, four cases are investigated under both m = 2 and m = 3 situations:

1. Duncan's model (abbreviated as DC)
2. NLG without G acceptance truncation, i.e., g = 0 (NC)
3. STD NLG (with G acceptance truncation, i.e., $g \geq 0$)(TC)
4. STD NLG with both e,c values reduced by half (RC).

All of the optimal results of all these cases are shown in Table 5.2. This table also provides comparisons among the above four cases and between m = 2 and m = 3.

In Table 5.2, for Duncan's model, optimal solutions are either provided by Goel et al. [12] (examples 1, 3, 5, 7, 8, 10, 12, and 16) or by a $\bar{X}$-chart optimization subroutine developed in this research (examples 9, 13, 26, and 20). For NLG plans, the investigation of both NC and RC in addition to standard TC is to illustrate the effects of (1) G acceptance truncation, and (2) the NLG reduction of sample inspection and evaluation costs, respectively.

To provide proper comparison, both Duncan's model (DC) and STD NLG (TC) adopt exactly the same set of parameter values. In actual implementation, however, the NLG parameters e and c should assume much smaller values than their DC counterparts. For example, in DC, e (the time of sampling, measuring, and evaluating each piece) can be decomposed into several steps: sampling; measuring and recording; and calculating and plotting. But in NLG, for the same parameter e, the calculating and plotting step can be totally eliminated; and the measuring and recording step requires much less time. Therefore, for the same process under surveillance, the e value in NLG should be much smaller than that of the counter-

TABLE 5.2

OPTIMAL ECONOMIC DESIGNS OF $\bar{X}$-CHART AND THEIR COMPARISONS

Ex. No.	Desc.	(A)	m = 2: n y g	h	t or k	100L*	(B)	(C)	m = 3: n y g	h	t or k	100L*	(B)	(C)	(E)
1	$\delta = 2$	DC	5	1.41	3.08	401.38			5	1.41	3.08	401.38			
	Base	NC	8 3 0	1.591	1.342	441.480	10.0		5 2 0	1.657	1.272	463.424	15.5		5.0
		TC	11 4 2	1.184	1.329	413.173	2.9	-6.4	9 4 2	1.422	1.382	426.619	6.3	-7.9	3.3
		RC	13 4 3	1.174	1.194	377.595	-5.9	-8.6	9 4 2	1.328	1.388	404.783	0.8	-5.1	7.2
3	$\lambda\uparrow 3$	DC	4	0.78	2.94	962.39			4	0.78	2.94	962.39			
		NC	7 2 0	0.928	1.121	1026.662	6.7		5 2 0	0.998	1.274	1050.602	9.2		2.3
		TC	9 3 2	0.691	1.246	984.525	2.3	-4.1	8 3 2	0.803	1.260	994.566	3.3	-5.3	1.0
		RC	12 4 3	0.722	1.253	917.534	-4.7	-6.8	9 4 2	0.801	1.387	949.851	-1.3	-4.5	3.5
5	$M\uparrow 10$	DC	4	0.41	2.95	2697.63			4	0.41	2.95	2697.63			
		NC	6 2 0	0.448	1.216	2850.739	5.7		5 2 0	0.525	1.283	2868.689	6.3		0.6
		TC	7 2 2	0.330	1.094	2762.063	2.4	-3.1	6 3 1	0.299	1.462	2757.345	2.2	-3.9	-0.2
		RC	10 3 3	0.358	1.152	2598.059	-3.7	-5.9	9 4 2	0.415	1.381	2637.541	-2.2	-4.3	1.5
7	$e\uparrow 10$	DC	2	0.94	2.69	541.16			2	0.94	2.69	541.16			
		NC	3 1 0	1.037	1.099	592.644	9.5		2 1 0	0.902	1.214	576.269	6.5		-2.8
		TC	4 1 1	0.712	0.971	553.922	2.4	-6.5	5 2 1	0.850	1.232	538.946	-0.4	-6.5	-2.7
		RC	6 2 1	0.732	1.190	485.958	-10.2	-12.3	6 3 1	0.900	1.442	476.928	-11.9	-11.5	-1.9
8	$D\uparrow 10$	DC	5	1.62	3.05	1837.28			5	1.62	3.05	1837.28			
		NC	8 3 0	1.858	1.360	1868.284	1.7		5 2 0	1.877	1.280	1883.827	2.5		0.8
		TC	11 3 3	1.558	1.129	1848.401	0.6	-1.1	9 4 2	1.663	1.405	1856.424	1.0	-1.5	0.4
		RC	13 4 3	1.421	1.211	1819.458	-1.0	-1.6	9 4 2	1.537	1.406	1838.454	0.1	-1.0	1.0
9	T & W	DC	3	1.273	2.220	360.952			3	1.273	2.220	360.952			
	$\downarrow 10$	NC	4 1 0	1.361	1.351	382.016	5.8		4 1 0	1.361	1.290	377.520	4.6		-1.2
		TC	6 2 2	1.201	1.477	370.308	2.6	-3.1	6 2 2	1.203	1.430	365.383	1.2	-3.2	-1.3
		RC	8 2 3	1.163	1.241	344.642	-4.5	-6.9	9 3 3	1.216	1.343	341.945	-5.3	-6.4	-0.8
10	T & W	DC	6	1.45	3.67	637.05			6	1.45	3.67	637.05			
	$\uparrow 10$	NC	11 4 0	1.753	1.185	691.607	8.6		5 2 0	3.449	1.140	951.679	49.4		37.6
		TC	14 5 2	1.146	1.192	647.701	1.7	-6.3	8 4 1	1.685	1.365	815.687	28.0	-14.3	25.9
		RC	17 6 3	1.210	1.203	606.482	-4.8	-6.4	8 4 1	1.666	1.365	803.909	26.2	-1.4	32.6

TABLE 5.2 (Continued)

Ex. No.	Desc.	(A)	m = 2: n	y	g	h	t or k	100L*	(B)	(C)	m = 3: n	y	g	h	t or k	100L*	(B)	(C)	(E)
12	$b\uparrow 10$	DC	6			3.47	2.88	586.95			6			3.47	2.88	586.95			
		NC	11	3	0	3.640	1.248	612.218	4.3		6	2	0	3.589	1.339	631.363	7.6		3.1
		TC	13	3	5	3.486	1.136	601.634	2.5	-1.7	11	4	4	3.652	1.368	606.514	3.3	-3.9	0.8
		RC	16	4	6	3.406	1.166	572.050	-2.5	-4.9	12	4	4	3.562	1.307	586.853	-0.0	-3.2	2.6
13	$c\uparrow 10$	DC	3			2.601	2.426	563.497			3			2.601	2.426	563.497			
		NC	4	1	0	2.953	1.218	640.423	13.7		3	1	0	2.506	1.297	624.603	10.8		-2.5
		TC	6	2	1	1.447	1.324	561.326	-0.4	-12.4	6	3	1	1.649	1.541	553.132	-1.8	-11.4	-1.5
		RC	9	3	2	1.796	1.281	487.563	-13.5	-13.1	6	3	1	1.306	1.516	488.423	-13.3	-11.7	0.2
16	$\delta = 1$ Base	DC	14			5.47	2.68	141.80			14			5.47	2.68	141.80			
		NC	30	7	0	7.508	1.480	200.345	41.3		21	6	0	8.528	1.580	216.288	52.5		8.0
		TC	36	7	4	4.286	1.334	185.132	30.6	-7.6	26	6	4	5.409	1.398	199.885	41.0	-7.6	8.0
		RC	49	10	5	4.292	1.369	156.668	10.5	-15.4	30	7	4	5.122	1.406	184.625	30.2	-7.6	17.8
26	$e\uparrow 10$	DC	8			4.080	2.486	190.183			8			4.080	2.486	190.183			
		NC	9	2	0	4.052	1.304	261.819	37.7		7	2	0	4.052	1.396	260.503	37.0		-0.5
		TC	21	5	1	1.670	1.423	232.940	22.5	-11.0	17	5	1	1.978	1.503	235.302	23.7	-9.7	1.0
		RC	28	6	2	2.119	1.341	198.633	4.4	-14.7	20	5	2	2.724	1.392	209.387	10.1	-11.0	5.4
20	$c\uparrow 10$	DC	8			12.159	1.898	243.362			8			12.159	1.898	243.362			
		NC	7	1	0	13.596	1.466	315.654	29.7		5	1	0	10.632	1.563	314.601	29.3		-0.3
		TC	10	2	3	8.936	1.501	301.953	24.1	-4.3	10	2	3	8.681	1.468	298.752	22.8	-5.0	-1.1
		RC	19	4	3	6.774	1.446	258.344	6.2	-14.4	17	4	3	6.965	1.473	256.494	5.4	-14.1	-0.7

In column (A): DC = Duncan's model; NC = NLG without G acceptance truncation; TC = STD NLG (with G acceptance truncation); RC = STD NLG with both e,c values reduced by half.

In column 100L*: The evaluation of L is based on the assumptions that (1) the process characteristic of interest is normally distributed, and (2) USLLSL = 6.

In column (B): Each of NC, TC, and RC is compared to DC to obtain the percent change with respect to 100L*.

In column (C): Percent difference of 100L* for the TC row is obtained from comparing TC to NC; similarly, that for the RC row is obtained from comparing RC to TC.

In column (E): Shows the percent difference of 100L* between m = 3 and m = 2 for each case.

part of the $\bar{X}$-chart. Likewise, TC's c value should also be much smaller than that of its DC counterpart. However, the degree of the reduction of e and c values for NLG depends upon the particular situation. Therefore, on the safe side, a conservative value of 50 percent reduction for both e and c are adopted for this research.

The economic comparisons in Table 5.2 are further summarized in Tables 5.3 and 5.4 for m = 2 and m = 3, respectively. Based upon these three tables, analyses are first provided for the m = 2 situation. Then m = 2 and m = 3 are compared. Finally, this section is concluded by a discussion of the m = 3 case.

First, m = 2 is considered. Although the nominal NLG plans (TC--which assumes the same e,c values as those of the $\bar{X}$-chart) always perform worse than the $\bar{X}$-chart (DC) does, the more realistic NLG plans (RC--which assumes reduced e,c values) do become superior under certain conditions. That is, when δ, e, or c is relatively large, RC becomes better than DC. On the other hand, when δ is relatively small, RC is always worse. However, with a large D value, the performances of RC and DC show almost no difference.

Table 5.3 also suggests that the NLG plan with G acceptance truncation is always better than that without it. Similarly, the NLG plan with e,c reductions is always better than that without them. However, the degree of both the effects of G acceptance truncation and e,c reductions may vary depending upon individual situations. When e or c is relatively large, or δ is relatively small, these effects are most significant. On the other hand, when D is relatively large, these effects are least significant.

TABLE 5.3

A SUMMARY TABLE FOR THE ECONOMIC COMPARISON OF $\bar{X}$-CHART AND NLG PLANS WHEN m = 2

Comparison*	Condition†	Result Description**	Percent Difference
TC → DC	δ = 2; D↑, c↑	Almost the same	< 1
	The rest	TC slightly worse	2~3
	δ = 1; Base case	TC much worse	31
	e↑, c↑	TC much worse	23~24
RC → DC	δ = 2; e↑, c↑	RC moderately better	10~14
	D↑	Almost the same	< 1
	The rest	RC slightly better	3~6
	δ = 1; Base case	RC moderately worse	11
	e↑, c↑	RC slightly worse	4~6
TC → NC	δ = 2; D↑, b↑	Almost the same	< 2
	c↑	TC moderately better	12
	The rest	TC slightly better	3~7
	δ = 1; e↑	TC moderately better	11
	The rest	TC slightly better	4~8
RC → TC	δ = 2; e↑, c↑	RC moderately better	12~13
	D↑	Almost the same	<2
	The rest	RC slightly better	5~9
	δ = 1; All cases	RC moderately better	14~15

*"→" means "compared to."

†"↑" means "relatively large;" "↓" means "relatively small."

**"Almost the same" means "<2% difference;" "slight" means "3~10% difference;" "moderate" means "11~20% difference;" and "much" means ">20% difference."

TABLE 5.4

A SUMMARY TABLE* FOR THE ECONOMIC COMPARISON OF $\bar{X}$-CHART AND NLG PLANS WHEN m = 3

Comparison	Condition	Result Description	Percent Difference
TC → DC	$\delta = 2$; e↑, D↑, T&W↓, c↑	Almost the same	<2
	T&W↑	TC much worse	28
	The rest	TC slightly worse	2~6
	$\delta = 1$; Base case	TC much worse	41
	e↑, c↑	TC much worse	23~24
RC → DC	$\delta = 2$; e↑, c↑	RC moderately better	12~13
	T&W↓	RC slightly better	5
	T&W↑	RC much worse	26
	The rest	Almost the same	<2
	$\delta = 1$; Base case	RC much worse	30
	e↑, c↑	RC slightly worse	5~10
TC → NC	$\delta = 2$; T&W↑, c↑	TC moderately better	11~14
	D↑	Almost the same	<2
	The rest	TC slightly better	3~8
	$\delta = 1$; All cases	TC slightly better	5~10
RC → TC	$\delta = 2$; e↑, c↑	RC moderately better	12
	D↑, T&W↑	Almost the same	<2
	The rest	RC slightly better	3~6
	$\delta = 1$; Base case	RC slightly better	8
	e↑, c↑	RC moderately better	11~14

*Notation is explained in Table 5.3.

Now, consider the comparison between $m = 2$ and $m = 3$. Column (E) of Table 5.2 suggests that "on the average" $m = 3$ is worse than $m = 2$. Especially when T and W are relatively large, $m = 3$ is much worse. With a relatively small δ value (but together with average e,c values), $m = 3$ is also considerably worse. The only exception is that when e or c is relatively large (together with a relatively large δ value), $m = 3$ becomes slightly better.

Furthermore, in actual implementation, $m = 3$ results in higher e,c values than that of $m = 2$, due to its longer measuring and recording time. This may well counteract the above described exception (i.e., with a relatively large δ value, the relatively large e or c results in a slightly better performance for $m = 3$) and make $m = 2$ always superior to $m = 3$.

Finally, $m = 3$ is considered. The general observations for $m = 2$ follow quite well for $m = 3$. The only significant exception is that relatively large T and W values make RC much worse than DC.

General Guidelines for Improved Application of NLG and the $\bar{X}$-Chart

Based on the analyses of the 12 representative examples, general guidelines can now be provided for better application of both NLG and $\bar{X}$-chart control plans.

1. For improved NLG application:
   a. The value $m = 2$ (instead of $m = 3$) should always be used whenever possible, especially when either T and W are relatively large or δ is relatively small (≤ 1).
   b. G acceptance should always be considered.

2. Possible situations for NLG to perform better than the $\bar{X}$-chart:

   a. The value of δ is relatively large (≥ 2)

   b. Either e or c is relatively large

   c. The relative difference of the actual values of e and c between the $\bar{X}$-chart and NLG is significant.

3. Possible situations for the $\bar{X}$-chart to perform better than NLG:

   a. The value of δ is relatively small (≤ 1)

   b. Both e and c are relatively small

   c. The relative difference of the actual values of e and c between the $\bar{X}$-chart and NLG is not significant.

4. Possible situations for equivalent performance between the $\bar{X}$-chart and NLG plan:

   a. D is relatively large

   b. The value of δ is moderate ($1 < \delta < 2$).

Comments

The properties revealed in the foregoing discussion match quite well with one's intuition. Since the parameter space of a variable scheme is continuous and that of an attribute scheme is discrete, it is believed that the $\bar{X}$-chart is more sensitive to changes than NLG. Thus, for a small process shift, the $\bar{X}$-chart should perform better. Due to its much simpler gaging requirements and lack of charting, NLG likely becomes superior whenever either the values of e and c of the $\bar{X}$-chart are relatively large or the NLG reduction on the e and c is significant enough. Finally, the bigger the portion of a control cycle which is occupied by the assignable cause search time D (which is independent of either control scheme), the smaller effect the control scheme will contribute to the total cost. In other words, the adoption of either NLG or an $\bar{X}$-chart will make no significant difference on total cost whenever D is big enough.

Although m = 2 is on the average more cost-effective than m = 3, in practice the latter seems to be psychologically more appealing. This is because m = 2 indiscriminately classifies both Y items and R items as "defectives" while m = 3 differentiates between the two. Hence, m = 3 may be preferred by on-line workers and even inspectors. For better implementation of m = 2, more explanation and training must be provided to soften the possible psychological resistance from workers.

In short, both NLG and the $\bar{X}$-chart have their own advantages and disadvantages. A thorough understanding of the environment and one's own needs is crucial in choosing the better-suited model.

Summary

In order to properly compare NLG and the $\bar{X}$-chart, the assumptions and general structure of Duncan's economically-based $\bar{X}$-chart are followed in developing the economic NLG model to ensure the highest degree of similarity and comparability. In the model development, their differences are pointed out and the effects and justifications of assumptions are discussed.

In economic NLG optimization, a general strategy of optimizing (h,t) under each specified set of (n, m, y, g) is followed. To simultaneously optimize (h,t), the loss-cost surface is investigated and the slightly modified Nelder and Mead direct search algorithm is employed. To optimize (n, m, y, g), an appealing convexity property of local optimums among each level of (n, y, g) under specified m has been revealed and is utilized to construct an efficient NLG optimization algorithm. With adequate experience, this algorithm can be further improved by dynamically adjusting the searching range for each of (n, y, g).

To economically compare NLG with the $\bar{X}$-chart, 12 representative examples covering a wide range of variations are selected from Duncan's paper. For each example, the $\bar{X}$-chart and three variations of NLG are optimized and compared to each other under m = 2 and m = 3 situations. All of these results are tabulated in Table 5.2 and are further summarized in Tables 5.3 and 5.4. After proper interpretations and analyses, general guidelines are provided for better applications of both models.

CHAPTER VI

USING THE INTERACTIVE COMPUTER PROGRAM

Introduction

Overview

This chapter illustrates the use of an interactive computer program which permits easy utilization of the design and evaluation methodology presented in previous chapters. The actual FORTRAN program is well documented and appears in the Appendix. It has been implemented on an IBM 3081D using various time share terminals.

The user is prompted for all necessary inputs by the computer. All these values together with some preprogrammed parameter values are presented to the user for verification or change. Only when a set of inputs has been verified does the program continue.

When several values are to be entered, they only need be separated by a space or a comma. Integer numbers are usually entered without a decimal point; however, a decimal may be included. The input mechanism is virtually self-explanatory, as long as the user understands the terms being input as well as their mathematically feasible range.

In the remainder of this chapter, actual interactive output is interspersed with comments and explanations. All computer output to follow is automatically generated except for the terminal input which follows a question mark (?).

General Structure and Input Requirements

The general structure and input requirements of this interactive computer program are shown in Figure 6.1. Twelve major functions perform: (1) statistical design and evaluation of NLG, (2) statistical design and evaluation of the $\bar{X}$-chart, (3) economic design, evaluation, and loss-cost surface investigation of NLG, and (4) economic design, evaluation, and loss-cost surface investigation of the $\bar{X}$-chart. Both common input and individual input requirements for each function module are listed.

Getting Started

The program begins by prompting option menu (M.1). The selection of "1" indicates the statistically based scheme is to be pursued.

```
*** ENTER OPTION NUMBER
    1 = STATISTICALLY BASED PROCESS CONTROL
    2 = ECONOMICALLY BASED PROCESS CONTROL
    3 = EXIT SYSTEM
?
1
```

(M.1)

Statistical NLG FG Design

After the statistically-based scheme is selected, values for the common statistical parameters USLLSL and assignable cause are entered and verified. Then, the major statistical option menu (M.2) is presented. A selection of "1" from this menu leads to the statistical NLG FG design.

```
IN STATISTICALLY BASED PROCESS CONTROL
*** ENTER VALUES:
USLLSL, ASSIGNABLE CAUSE (1= MEAN SHIFT;  2= DISPERSION CHANGE)
?
7  1
USLLSL= 7.00 (STD);  MEAN SHIFT ASSUMED.
CORRECT ?   1=YES   2=NO   3=RETURN
?
1
```

Common Input	Major Function	Module Input

- STAT — USLLSL, Assignable Cause
  - NLG FG Design--m, range of n; APL, TLAPL, RPL, TLRPL; t values
  - NLG FG Eval.--n, m, y, g; t values; F (for PBAQP and PBAOQ eval.)
  - NLG QL Design--m, range of n; APL, TLAPL, RPL, TLRPL; t
  - NLG QL Eval.--n, m, y, t
  - $\bar{X}$-Chart Design--v, range of n; APL, TLAPL, RPL, TLRPL: k values
  - $\bar{X}$-Chart Eval.--n, v, k
- ECON
  - NLG — USLLSL, m; Time and Cost Parameters
    - NLG Design--range of n; optimization parameters (optional)
    - NLG Evaluation--n, y, g, h, t
    - NLG Cost Surf.--n, y, g; h values; t values
  - $\bar{X}$-Chart — USLLSL; Time and Cost Parameters
    - $\bar{X}$-Chart Design--range of n; optimization parameters (optional)
    - $\bar{X}$-Chart Eval.--n, h, k
    - $\bar{X}$-Chart Cost Surf.--n; h values; k values

Figure 6.1. General Structure and Input Requirements for the Interactive Computer Program

```
*** ENTER OPTION NUMBER
    1= STAT NLG FG DESIGN
    2= STAT NLG FG EVALUATION ( + OPTIONAL PBAFQ AND PBAOQ )
    3= STAT NLG QL DESIGN
    4= STAT NLG QL EVALUATION
    5= STAT X-BAR CHART DESIGN
    6= STAT X-BAR CHART EVALUATION
    7= RETURN TO REVISE USLLSL AND ASSIGNABLE CAUSE                    (M.2)
    8= SWITCH TO ECON PROCESS CONTROL SCHEME
    9= EXIT SYSTEM
?
1
```

In statistical NLG FG design, the user is sequentially prompted for the input values of three sets of design parameters. After proper verification, all possible plans within the user-specified range are then listed. Each plan is evaluated at four process levels: exact setup for $1-P_a$ (labeled by PR0); APL, midpoint, and RPL for P_a. The value of PR0 represents the probability of a false alarm for each sample. In addition to P_a and $1-P_a$, E_n is also provided for exact setup and RPL. The qualified plans are labeled by "**". To save space, only the results of t = 1 are illustrated, since t = 2 has a similar output format. At this point, program control returns to menu (M.2) for the next option.

```
FOR STAT NLG FG DESIGN
*** ENTER VALUES:  M,NMIN,NMAX
?
2  2  6
*** ENTER VALUES:  APL,TLAPL,RPL,TLRPL
?
.01  .90  .10  .40
*** ENTER VALUES:
NUMT (NUMBER OF T; <= 10), FOLLOWED BY T VALUES TO BE INVESTIGATED
?
2    1  2
VALUES ENTERED:  M= 2    NMIN= 2    NMAX= 6
 APL=0.010    TLAPL=0.900    RPL=0.100    TLRPL=0.400
  2 T VALUES =  1.000  2.000
CORRECT ?    1=YES    2=NO    3= RETURN FOR OTHER STAT OPTIONS
?
1


 ***** STATISTICALLY BASED NLG FG DESIGN *****
   USLLSL= 7.00 (STD)      MEAN SHIFT ASSUMED (MULTIPLES OF STD)
   M= 2    NMIN= 2    NMAX= 6
   APL=0.010    TLAPL=0.900    RPL=0.100    TLRPL=0.400
   INVESTIGATED T VALUES = 1.000  2.000
```

********** T = 1.000

					(P0=0.0005)		(APL=0.010)		(MID=0.055)	(RPL=0.100)		
N	M	Y	G		ENC	PRO	PA1		PA2	PA3		EN3
2	2	0	0		1.99	0.0247	0.824		0.526	0.373		1.61
2	2	1	1		1.01	0.0002	0.991		0.924	0.849		1.39
3	2	0	0		2.96	0.0368	0.747		0.381	0.228		1.98
3	2	1	1		1.02	0.0003	0.984		0.870	0.756		1.63
3	2	1	2		2.02	0.0005	0.976		0.815	0.664		2.48
3	2	2	1		1.02	0.0000	0.999		0.979	0.941		1.78
4	2	0	0		3.93	0.0488	0.678		0.276	0.139		2.21
4	2	1	1		1.04	0.0005	0.977		0.830	0.700		1.77
4	2	1	2		2.05	0.0008	0.962		0.735	0.551		2.77
4	2	1	3		3.04	0.0009	0.955		0.696	0.494		3.28
4	2	2	1		1.04	0.0000	0.998		0.949	0.869		2.11
4	2	2	2		2.05	0.0000	0.997		0.934	0.833		3.19
5	2	0	0		4.88	0.0606	0.616		0.200	0.085		2.35
5	2	1	1		1.05	0.0006	0.970		0.801	0.665		1.86
5	2	1	2		2.07	0.0011	0.949		0.678	0.482		2.94
5	2	1	3	**	3.07	0.0014	0.936	**	0.609	0.390	**	3.55
5	2	1	4	**	4.05	0.0015	0.929	**	0.580	0.356	**	3.87
5	2	2	1		1.05	0.0000	0.996		0.916	0.803		2.37
5	2	2	2		2.07	0.0000	0.994		0.879	0.723		3.65
5	2	2	3		3.07	0.0000	0.993		0.869	0.701		4.32
5	2	3	1		1.05	0.0	1.000		0.982	0.935		2.53
5	2	3	2		2.07	0.0000	1.000		0.978	0.921		3.86
6	2	0	0		5.82	0.0722	0.559		0.145	0.052		2.44
6	2	1	1		1.06	0.0008	0.964		0.780	0.644		1.91
6	2	1	2		2.10	0.0014	0.937		0.636	0.439		3.05
6	2	1	3	**	3.11	0.0018	0.918	**	0.547	0.327	**	3.71
6	2	1	4	**	4.09	0.0021	0.906	**	0.497	0.272	**	4.08
6	2	1	5	**	5.06	0.0022	0.900	**	0.476	0.251	**	4.28
6	2	2	1		1.06	0.0000	0.993		0.885	0.749		2.56
6	2	2	2		2.10	0.0000	0.990		0.824	0.629		4.00
6	2	2	3		3.11	0.0000	0.988		0.797	0.580		4.79
6	2	2	4		4.10	0.0000	0.987		0.789	0.567		5.20
6	2	3	1		1.06	0.0000	0.999		0.964	0.884		2.86
6	2	3	2		2.10	0.0000	0.999		0.951	0.844		4.41
6	2	3	3		3.11	0.0000	0.999		0.948	0.835		5.21

Statistical NLG FG Evaluation

A selection of "2" from menu (M.2) leads to statistical NLG FG evaluation. There are three options for FG evaluation, namely, FG only, FG + PBAPQ, and FG + PBAPQ + PBAOQ. In order to evaluate either PBAPQ or PBAOQ, the value of sampling frequency F (number of samples per OOC indication) must be provided. The procedure for entering the required parameter values and verifying them is the same as that in the last section. In the final evaluation listing, DEL = δ, the degree of mean shift measured in multiples of the standard deviation. Upon completing the evaluation, program control again returns to menu (M.2) for the next option.

```
*** FOR STAT NLG FG EVALUATION,  ENTER OPTION NUMBER
   1= FG ONLY      2= FG + PBAPQ      3= FG + PBAPQ + PBAOQ
?
3
 *** FOR FG, ENTER VALUES:  N,M,Y,G
?
6  2  3  3
*** ENTER VALUES:
NUMT (NUMBER OF T; <= 10), FOLLOWED BY T VALUES TO BE INVESTIGATED
?
1    1.7
*** FOR PBAPQ, ENTER VALUE OF F
          (NUMBER OF SAMPLES PER OOC INDICATION)
?
25
 VALUES ENTERED:  N= 6    M= 2    Y= 3    G= 3
  1 T VALUES =  1.700
 SAMPLING FREQUENCY F = 25 SAMPLES PER OOC INDICATION
CORRECT ?    1=YES   2=NO   3= RETURN FOR OTHER STAT OPTIONS
?
1


 ***** STATISTICALLY BASED NLG FG EVALUATION *****
   USLLSL= 7.00 (STD)      MEAN SHIFT ASSUMED (MULTIPLES OF STD)
   N= 6    M= 2    Y= 3    G= 3
   INVESTIGATED T VALUES = 1.700

**********  T= 1.700
```

P	DEL	PA	EN	PBAPQ	PBAOQ
0.0005	-0.000	1.000	3.60	0.0005	0.0004
0.0050	0.924	0.985	4.42	0.0050	0.0048
0.0100	1.174	0.953	4.79	0.0083	0.0079
0.0150	1.330	0.912	5.01	0.0068	0.0062
0.0200	1.446	0.869	5.14	0.0060	0.0052
0.0250	1.540	0.824	5.23	0.0055	0.0045
0.0300	1.619	0.779	5.30	0.0052	0.0040
0.0350	1.688	0.735	5.34	0.0050	0.0036
0.0400	1.749	0.693	5.36	0.0048	0.0032
0.0450	1.805	0.653	5.38	0.0047	0.0029
0.0500	1.855	0.614	5.38	0.0046	0.0026
0.0550	1.902	0.577	5.38	0.0045	0.0023
0.0600	1.945	0.543	5.38	0.0045	0.0021
0.0650	1.986	0.510	5.37	0.0044	0.0018
0.0700	2.024	0.479	5.35	0.0044	0.0016
0.0750	2.060	0.450	5.34	0.0044	0.0014
0.0800	2.095	0.422	5.32	0.0044	0.0012
0.0850	2.128	0.397	5.30	0.0044	0.0010
0.0900	2.159	0.373	5.28	0.0044	0.0008
0.0950	2.189	0.350	5.26	0.0044	0.0006
0.1000	2.218	0.328	5.24	0.0044	0.0004
0.1200	2.325	0.255	5.15	0.0045	0.0004
0.1400	2.420	0.198	5.06	0.0046	0.0004
0.1600	2.506	0.154	4.97	0.0048	0.0004
0.1800	2.585	0.120	4.89	0.0050	0.0004
0.2000	2.658	0.093	4.81	0.0053	0.0004
0.4000	3.247	0.007	4.31	0.0086	0.0004

Statistical NLG QL Design

A selection of "3" from menu (M.2) leads to statistical NLG QL design. The interactive procedure and the input parameters are almost the same as those of statistical NLG FG design. The only difference is that APL and RPL are now measured in multiples of σ (labeled by STD) instead of probability. The format of the resulting listing is very similar to that of FG design. Note in the following example that n and y for QL may differ from the n and y values used in FG.

```
 FOR STAT NLG QL DESIGN
 *** ENTER VALUES:  M,NMIN,NMAX
?
2  2  6
 *** ENTER VALUES OF APL,TLAPL,RPL,TLRPL
    (HERE APL, RPL MUST BE IN MULTIPLES OF STD)
?
.2  .8   2.   .3
 *** ENTER T VALUE
?
1.7
VALUES ENTERED:  M= 2    NMIN= 2     NAMX= 6
 APL= 0.200(STD)     TLAPL=0.800     RPL= 2.000(STD)     TLRPL=0.300
 T= 1.700
CORRECT ?    1=YES    2=NO    3= RETURN FOR OTHER STAT OPTIONS
?
1

 ***** STATISTICALLY BASED NLG QL DESIGN *****
   USLLSL= 7.00 (STD)       MEAN SHIFT ASSUMED (MULTIPLES OF STD)
   M= 2     NMIN= 2     NMAX= 6
   APL= 0.200(STD)     TLAPL= 0.800     RPL= 2.000(STD)     TLRPL= 0.300
   T= 1.700

                 (EXACT SETUP)  (APL=0.200) (MID=1.100) (RPL=2.000)
                        0.0 STD         STD         STD         STD
  N     Y         ENO    PRO       PA1          PA2        PA3          EN3

  2     0    **   1.93  0.1386    0.851 **     0.572      0.177 **     1.42
  2     1         2.00  0.0052    0.994        0.941      0.664        2.00

  3     0         2.79  0.2005    0.785        0.432      0.074        1.60
  3     1         2.99  0.0147    0.983        0.851      0.382        2.66
  3     2         3.00  0.0004    1.000        0.986      0.806        3.00

  4     0         3.59  0.2579    0.724        0.327      0.031        1.67
  4     1    **   3.98  0.0281    0.968 **     0.749      0.204 **     3.05
  4     2         4.00  0.0014    0.998        0.953      0.560        3.81
  4     3         4.00  0.0000    1.000        0.996      0.887        4.00

  5     0         4.33  0.3112    0.668        0.247      0.013        1.70
  5     1    **   4.95  0.0446    0.949 **     0.648      0.104 **     3.25
  5     2         5.00  0.0033    0.996        0.903      0.354        4.37
  5     3         5.00  0.0001    1.000        0.986      0.698        4.89
  5     4         5.00  0.0000    1.000        0.999      0.935        5.00

  6     0         5.02  0.3607    0.616        0.187      0.006        1.72
  6     1    **   5.91  0.0638    0.927 **     0.549      0.051 **     3.35
  6     2    **   5.99  0.0063    0.992 **     0.840      0.209 **     4.72
  6     3         6.00  0.0004    1.000        0.966      0.498        5.59
  6     4         6.00  0.0000    1.000        0.996      0.797        5.93
  6     5         6.00  0.0000    1.000        1.000      0.962        6.00
```

Statistical NLG QL Evaluation

A selection of "4" from menu (M.2) leads to statistical NLG QL evaluation. Following the standard interactive procedure, P_a and E_n are provided as functions of δ (DEL) which ranges from 0 to 5.

```
 FOR STAT NLG QL EVALUATION
*** ENTER VALURES:  N,M,Y,T
?
6  2  1    1.7
 VALUES ENTERED:  N= 6    M= 2    Y= 1    T= 1.700
CORRECT ?    1=YES   2=NO   3= RETURN FOR OTHER STAT OPTIONS
?
1


 ***** STATISTICALLY BASED NLG QL EVALUATION *****

   USLLSL= 7.00 (STD)      MEAN SHIFT ASSUMED (MULTIPLES OF STD)
   N= 6    M= 2    Y= 1    G= 0       T= 1.700

              DEL          PA        EN

             0.0          0.936     5.91
             0.100        0.934     5.90
             0.200        0.927     5.89
             0.300        0.915     5.87
             0.400        0.896     5.84
             0.500        0.871     5.80
             0.600        0.837     5.75
             0.700        0.795     5.68
             0.800        0.745     5.58
             0.900        0.686     5.47
             1.000        0.620     5.34
             1.200        0.474     5.01
             1.400        0.328     4.61
             1.600        0.202     4.18
             1.800        0.109     3.75
             2.000        0.051     3.35
             2.500        0.004     2.63
             3.000        0.000     2.26
             4.000        0.000     2.03
             5.000        0.000     2.00
```

Statistical $\bar{X}$-Chart Design

A selection of "5" from menu (M.2) leads to the statistical $\bar{X}$-chart design. The interactive procedure and input requirements generally follow those in the statistical NLG FG design section.

```
FOR STAT MODIFIED X-BAR CHART DESIGN
*** ENTER VALUES:  V,NMIN,NMAX
?
3   2  6
*** ENTER VALUES:  APL,TLAPL,RPL,TLRPL
?
.01  .9   .10  .4
*** ENTER VALUES:
    NUMK (NUMBER OF K; <= 10), FOLLOWED BY K VALUES TO BE INVESTIGATED
?
8    1.5  2  2.5  2.75  3  3.25  3.5  4
VALUES ENTERED:  V= 3.000     NMIN= 2    NAMX= 6
 APL=0.010    TLAPL=0.900    RPL=0.100    TLRPL=0.400
  8 K VALUES =  1.500  2.000  2.500  2.750  3.000  3.250  3.500  4.000
CORRECT ?    1=YES   2=NO   3= RETURN FOR OTHER STAT OPTIONS
?
1
```

In the output listing, for each (n,k) combination, four process levels are evaluated: exact setup for $1-P_a$, APL, midpoint, and RPL for P_a. The value of $1-P_a$ (labeled by PRO) represents the probability of a false alarm for each sample.

```
***** STATISTICALLY BASED MODIFIED X-BAR CHART DESIGN *****

 USLLSL= 7.00 (STD)      MEAN SHIFT ASSUMED (MULTIPLES OF STD)
 V= 3.000      NMIN= 2    NMAX= 6
 APL=0.010     TLAPL=0.900    RPL=0.100    TLRPL=0.400
 INVESTIGATED K VALUES = 1.500  2.000  2.500  2.750  3.000  3.250  3.500  4.000

      LCL = LSL + (V - K/SQRT(N))*STD      UCL = USL - (V - K/SQRT(N))*STD
```

	N	K	(EXACT SETUP) PRO	(APL=0.010) PA1	(MID=0.055) PA2	(RPL=0.100) PA3
	2	1.50	0.0273	0.708	0.315	0.176
	2	2.00	0.0068	0.853	0.507	0.334
	2	2.50	0.0013	0.939	0.698	0.528
	2	2.75	0.0005	0.964	0.779	0.625
	2	3.00	0.0002	0.980	0.846	0.716
	2	3.25	0.0001	0.989	0.898	0.794
	2	3.50	0.0000	0.995	0.935	0.858
	2	4.00	0.0000	0.999	0.978	0.942
	3	1.50	0.0180	0.631	0.177	0.070
	3	2.00	0.0042	0.798	0.334	0.164
**	3	2.50	0.0008	0.909 **	0.529	0.317 **
	3	2.75	0.0003	0.943	0.626	0.410
	3	3.00	0.0001	0.967	0.716	0.509
	3	3.25	0.0000	0.981	0.794	0.608
	3	3.50	0.0000	0.990	0.858	0.700
	3	4.00	0.0000	0.998	0.942	0.847
	4	1.50	0.0124	0.561	0.096	0.026
	4	2.00	0.0027	0.743	0.211	0.075
	4	2.50	0.0005	0.875	0.381	0.174
**	4	2.75	0.0002	0.920 **	0.479	0.246 **
**	4	3.00	0.0001	0.951 **	0.578	0.331 **
	4	3.25	0.0000	0.971	0.672	0.426
	4	3.50	0.0000	0.984	0.757	0.525
	4	4.00	0.0000	0.996	0.884	0.713
	5	1.50	0.0088	0.497	0.051	0.010
	5	2.00	0.0018	0.689	0.128	0.033
	5	2.50	0.0003	0.840	0.263	0.090
	5	2.75	0.0001	0.893	0.350	0.137
**	5	3.00	0.0000	0.932 **	0.446	0.200 **
**	5	3.25	0.0000	0.959 **	0.546	0.277 **
**	5	3.50	0.0000	0.977 **	0.643	0.366 **
	5	4.00	0.0000	0.994	0.807	0.563
	6	1.50	0.0064	0.440	0.027	0.003
	6	2.00	0.0013	0.637	0.076	0.014
	6	2.50	0.0002	0.802	0.175	0.044
	6	2.75	0.0001	0.864	0.247	0.072
**	6	3.00	0.0000	0.911 **	0.332	0.113 **
**	6	3.25	0.0000	0.945 **	0.427	0.169 **
**	6	3.50	0.0000	0.968 **	0.526	0.239 **
	6	4.00	0.0000	0.991	0.714	0.417

Statistical $\bar{X}$-Chart Evaluation

A selection of "6" from menu (M.2) leads to the statistical $\bar{X}$-chart evaluation. The interactive procedure and evaluation results follow.

```
 FOR STAT MODIFIED X-BAR CHART EVALUATION
*** ENTER VALURES:  N,V,K
?
5   3  3
 VALUES ENTERED:  N= 5    V= 3.000     K= 3.000
CORRECT ?     1=YES    2=NO    3= RETURN FOR OTHER STAT OPTIONS
?
1

 ***** STATISTICALLY BASED MODIFIED X-BAR CHART EVALUATION *****

   USLLSL= 7.00 (STD)      MEAN SHIFT ASSUMED (MULTIPLES OF STD)
   N= 5     V= 3.00     K= 3.000

     LCL= LSL + (V-K/SQRT(N))*STD = LSL +  1.658 STD
     UCL= USL - (V-K/SQRT(N))*STD = USL -  1.658 STD

      P         DEL          PA
   0.0005    -0.000        1.000
   0.0050     0.924        0.980
   0.0100     1.174        0.932
   0.0150     1.330        0.874
   0.0200     1.446        0.812
   0.0250     1.540        0.750
   0.0300     1.619        0.691
   0.0350     1.688        0.634
   0.0400     1.749        0.582
   0.0450     1.805        0.533
   0.0500     1.855        0.488
   0.0550     1.902        0.446
   0.0600     1.945        0.408
   0.0650     1.986        0.374
   0.0700     2.024        0.342
   0.0750     2.060        0.312
   0.0800     2.095        0.286
   0.0850     2.128        0.261
   0.0900     2.159        0.239
   0.0950     2.189        0.218
   0.1000     2.218        0.200
   0.1200     2.325        0.140
   0.1400     2.420        0.098
   0.1600     2.506        0.069
   0.1800     2.585        0.048
   0.2000     2.658        0.034
   0.4000     3.247        0.001
```

Economic NLG Design (Optimization)

Economically based process schemes can be accessed by either selecting "8" from menu (M.2) or selecting "2" from menu (M.1). Once accessed, menu (M.3) is listed. Then a selection of "1" from this menu leads to the economic NLG scheme.

```
*** ENTER OPTION NUMBER
       1 = ECONOMICALLY BASED NLG (MEAN SHIFT ASSUMED)
       2 = ECONOMICALLY BASED X-BAR CHART (MEAN SHIFT ASSUMED)
       3 = SWITCH TO STATISTICALLY BASED SCHEME                      (M.3)
       4 = EXIT SYSTEM
?
1
```

Once in the economic NLG scheme, the user is prompted for the values of common economic NLG parameters. After proper verification, menu (M.4) is presented. A selection of "1" from this menu finally results in economic NLG design.

```
*** FOR ECON NLG, ENTER VALUES:
    USLLSL, MM;  DELTA, LAMBDA, M, E, D, T, W, B, C
?
6 2    2 .01  100  .05  2  50  25  .5  1
VALUES ENTERED:    USLLSL= 6.00     MM=2
 DELTA=    2.00   LAMBDA=    0.01    M= 100.00    E=    0.05    D=    2.00
     T=  50.00          W=  25.00    B=    0.50   C=    1.00
CORRECT ?   1=YES   2=NO    3=RETURN
?
1
*** ENTER OPTION NUMBER
    1= ECON NLG DESIGN (OPTIMIZATION)
    2= ECON NLG EVALUATION
    3= ECON NLG LOSS-COST SURFACE INVESTIGATION
    4= SWITCH TO ECON X-BAR CHART
    5= RETURN TO REVISE USLLSL, MM, AND TIME AND COST PARAMETERS      (M.4)
    6= EXIT SYSTEM
?
1
```

The user is then prompted for the values of design parameters. Preprogrammed values of optimization parameters are listed for the user's examination. If desired, these values can be changed to those of the user's preference. In (h,t) optimization, YACC and XACC are quitting criteria; STEP = step size; ITRMAX = maximum iteration number; HO = h_o and TO = t_o are starting h,t values; IRESET = 1 requires that each optimization start with the user-specified h_o and t_o values; and IRESET = 0 requires that each optimization start with the optimal (h,t) point of the last optimization. In overall optimization, EY = ε_y, EG = ε_g, and EL = ε_L, which are explained in Chapter V, the section entitled "Economic NLG Optimization." For more detail, users are referred to Reference [32] and the subroutines NECOPT, XECOPT, and HTOPT in the Appendix.

```
*** FOR ECON NLG DESIGN, ENTER VALUES:  NMIN, NMAX
?
4  10
VALUES ENTERED:  NMIN= 4    NMAX=10

PARAMETER VALUES FOR:          (H,T) OPTIMIZATION              OVERALL OPTIMIZATION
              YACC    XACC    STEP ITRMAX    HO      TO IRESET   EY  EG      EL
  DEFAULT:    0.003   0.002   1.00   60    1.000  1.000  1        2   3     0.0
  CURRENT:    0.003   0.002   1.00   60    1.000  1.000  1        2   3     0.0

*** ENTER OPTION NUMBER:
1= ALL OK, NO REVISION NEEDED
2= NEED TO REVISE (NMIN,NMAX) VALUES
3= NEED TO REVISE (H,T) OPTIMIZATION PARAMETER VALUES
4= NEED TO REVISE OVERALL OPTIMIZATION PARAMETER VALUES
5= RETURN FOR OTHER ECON NLG OPTIONS
?
1
```

Optimization output follows. The local optimal solution is first listed for each (n,m,y,g) combination. Each n then has its own suboptimum indicated. Finally, the overall optimum is printed. In the output notation, MM = m; 100L = loss-cost per 100 hours; STDY = standard deviation of 100L for the three vertices of the final simplex; and STDX = standard deviation of the distances among the three vertices of the final simplex. For normal termination of (h,t) optimization (rather than maximum iteration termination), either STDY < YACC or STDX < XACC must be satisfied. The total iteration number TITR must not exceed the specified maximum iteration number ITRMAX; MAXITR indicates whether ITRMAX has been reached or not (if reached, iteration stops and a '**' is printed).

```
***** ECONOMICALLY BASED NLG DESIGN *****

USLLSL=  6.00    MM=2       MEAN SHIFT ASSUMED
 DELTA=   2.00   LAMBDA=    0.01    M= 100.00   E=    0.05    D=    2.00
     T=  50.00        W=   25.00    B=    0.50  C=    1.00

(H,T) OPTIMIZATION:  YACC=  0.003   XACC=  0.002    STEP=  1.000   ITRMAX= 60
                     STARTING POINT:  HO=  1.000    TO=  1.000    IRESET=1
OVERALL OPTIMIZATION:  EY=2   EG=3   EL=   0.0         NMIN= 4    NMAX=10

 N  MM  Y  G        H          T         100L        STDY       STDX     TITR  MAXITR

 4  2  0  0       3.141      0.532     679.441     0.0014     0.0067     19
-------------------------------------------------------------------------------------
 4  2  1  1       1.314      1.158     581.852     0.0014     0.0038     15
 4  2  1  2       2.075      1.142     587.536     0.0002     0.0068     16
-------------------------------------------------------------------------------------
 4  2  2  1       1.479      1.577     572.771     0.0017     0.0051     17
 4  2  2  2       2.138      1.609     604.015     0.0019     0.0076     18
-------------------------------------------------------------------------------------
 4  2  3  1       1.514      2.016     645.742     0.0014     0.0069     18
-------------------------------------------------------------------------------------
                                                  FOR N=  4:  MIN 100L =    572.771
```

5	2	0	0	3.564	0.475	692.718	0.0018	0.0184	18
5	2	1	1	1.279	1.076	584.274	0.0005	0.0096	14
5	2	1	2	2.082	1.049	583.656	0.0024	0.0065	16
5	2	1	3	2.633	1.046	601.438	0.0028	0.0107	17
5	2	2	1	1.482	1.432	561.982	0.0024	0.0089	17
5	2	2	2	2.234	1.435	579.733	0.0023	0.0081	17
5	2	3	1	1.657	1.777	583.748	0.0008	0.0041	18
5	2	3	2	2.330	1.819	623.661	0.0019	0.0106	19
							FOR N= 5:	MIN 100L =	561.982
6	2	0	0	3.883	0.423	707.010	0.0009	0.0137	19
6	2	1	1	1.260	1.024	588.880	0.0014	0.0069	13
6	2	1	2	2.059	0.981	584.730	0.0029	0.0060	16
6	2	1	3	2.650	0.968	598.525	0.0013	0.0104	18
6	2	2	1	1.462	1.331	561.336	0.0021	0.0111	14
6	2	2	2	2.251	1.315	571.498	0.0021	0.0111	15
6	2	3	1	1.663	1.623	566.511	0.0004	0.0049	17
6	2	3	2	2.454	1.639	593.708	0.0015	0.0082	18
							FOR N= 6:	MIN 100L =	561.336
7	2	0	0	4.219	0.382	721.774	0.0026	0.0157	18
7	2	1	1	1.234	0.974	594.191	0.0010	0.0036	16
7	2	1	2	2.033	0.930	587.871	0.0003	0.0073	17
7	2	1	3	2.659	0.910	599.151	0.0022	0.0117	18
7	2	2	1	1.412	1.245	564.487	0.0004	0.0057	15
7	2	2	2	2.257	1.223	569.964	0.0017	0.0096	15
7	2	3	1	1.624	1.507	562.377	0.0008	0.0062	17
7	2	3	2	2.477	1.504	581.691	0.0021	0.0073	18
7	2	4	1	1.865	1.765	581.979	0.0026	0.0115	18
7	2	4	2	2.710	1.789	616.956	0.0003	0.0102	20
							FOR N= 7:	MIN 100L =	562.377

******************************** OVERALL OPTIMAL 100L = 561.336

Economic NLG Evaluation

A selection of "2" from menu (M.4) leads to economic NLG evaluation. The interactive procedure and output are illustrated below.

```
FOR ECON NLG EVALUATION, ENTER VALUES:  N,Y,G,H,T
?
6  2  1   1.462  1.331
VALUES ENTERED:  N= 6  Y= 2  G= 1    H=   1.462    T= 1.331
CORRECT ?   1=YES    2=NO    3= RETURN FOR OTHER ECON NLG OPTIONS
?
1

 ***** ECONOMICALLY BASED NLG EVALUATION *****

 USLLSL=  6.00 (STD)     MM=2      MEAN SHIFT ASSUMED

 DELTA=   2.00   LAMBDA=    0.01    M= 100.00   E=   0.05   D=    2.00
     T=  50.00          W=  25.00   B=    0.50   C=   1.00

 N=  6     Y=  2     G=  1          H=   1.462        T=   1.331


 LOSS-COST PER 100 HOURS =   561.337  (HOURLY LOSS-COST =     5.613)
```

Economic NLG Loss-Cost Surface Investigation

A selection of "3" from menu (M.4) leads to the economic NLG loss-cost surface investigation. Loss-cost is evaluated at each (h,t) combination of the user's specified h and t values. Among them, the optimal combination is identified. For each t value, the probability of a false alarm (ALPHA), the probability of a true alarm (P), the in-control average sample number (EN IC), and the out-of-control average sample number (EN OOC) are also provided for the user's reference. A wider terminal width (132) is required for a better loss-cost tabulation. The standard interactive procedure and the final output are illustrated below.

```
*** FOR ECON NLG COST SURFACE INVESTIGATION, ENTER VALUES:  N,Y,G
?
6  2  1
ENTER VALUES:
NUMH (NUMBER OF H; <= 30), FOLLOWED BY ALL H VALUES TO BE INVESTIGATED
?
14   .1 .5 .75 1  1.25 1.5 2 2.5 3 5 10 25 50 100
ENTER VALUES:
NUMT (NUMBER OF T; <= 11), FOLLOWED BY ALL T VALUES TO BE INVESTIGATED
?
11   .1 .5 .75 1 1.25 1.5 1.75 2 2.25 2.5 2.9
VALUES ENTERED:  N= 6    Y= 2    G= 1
14 H VALUES =     0.100     0.500     0.750     1.000     1.250     1.500
               2.000     2.500     3.000     5.000    10.000    25.000
              50.000   100.000
11 T VALUES =   0.100     0.500     0.750     1.000     1.250     1.500
               1.750     2.000     2.250     2.500     2.900

*** ENTER OPTION NUMBER:
1= ALL OK, NO REVISION NEEDED
2= NEED TO REVISE (N,Y,G) VALUES
3= NEED TO REVISE NUMH AND H VALUES
4= NEED TO REVISE NUMT AND T VALUES
5= RETURN FOR OTHER ECON NLG OPTIONS
?
1
```

***** ECONOMICALLY BASED NLG LOSS-COST SURFACE INVESTIGATION *****

USLLSL= 6.00 STD MM=2 MEAN SHIFT ASSUMED N= 6 Y= 2 G= 1
DELTA= 2.00 LAMBDA= 0.01 M= 100.00 E= 0.05 D= 2.00 T= 50.00 W= 25.00 B= 0.50 C= 1.00

T	0.100	0.500	0.750	1.000	1.250	1.500	1.750	2.000	2.250	2.500	2.900
ALPHA	0.000	0.000	0.000	0.001	0.004	0.018	0.060	0.161	0.339	0.571	0.920
P	0.042	0.151	0.267	0.406	0.546	0.668	0.766	0.840	0.897	0.939	0.989
EN IC	1.019	1.062	1.122	1.227	1.396	1.647	1.981	2.359	2.700	2.917	3.000
EN OOC	1.869	2.331	2.588	2.781	2.902	2.963	2.989	2.997	2.999	3.000	3.000
H											
0.100	2000.021	1886.679	1927.934	2056.677	2390.076	3305.394	5701.195	10984.695	20045.992	31595.848	48757.227
0.500	1545.786	841.241	722.133	689.854	727.598	894.734	1363.772	2412.547	4217.469	6520.695	9944.836
0.750	1866.720	871.277	686.093	615.561	616.170	714.242	1018.863	1712.390	2911.094	4442.734	6721.016
1.000	2199.844	950.451	704.229	600.885	576.925	635.670	855.895	1370.456	2265.182	3410.451	5115.172
1.250	2519.586	1047.137	743.379	609.811	564.040	597.803	765.664	1171.784	1883.439	2796.420	4156.508
1.500	2819.993	1150.891	792.503	630.333	566.242	580.239	711.770	1044.721	1633.763	2391.498	3521.431
2.000	3361.250	1364.276	904.178	688.182	591.499	575.412	658.391	897.942	1332.456	1895.286	2736.603
2.500	3830.891	1575.771	1022.844	756.463	630.298	590.586	641.160	822.640	1163.115	1608.108	2275.299
3.000	4240.328	1781.027	1143.147	829.214	675.514	615.565	641.877	782.989	1059.691	1425.393	1975.711
5.000	5453.273	2523.836	1610.708	1129.684	877.977	752.409	715.085	765.960	908.883	1111.752	1423.761
10.000	7025.105	3917.419	2612.947	1833.098	1389.456	1140.441	1009.068	962.512	985.308	1052.443	1170.973
25.000	8541.031	6108.883	4581.746	3433.282	2672.205	2194.272	1898.953	1720.441	1620.335	1570.188	1532.942
50.000	9211.668	7574.508	6261.969	5073.910	4164.559	3531.825	3108.308	2827.229	2641.847	2519.523	2391.927
100.000	9589.816	8622.711	7709.730	6749.344	5907.844	5256.594	4784.398	4449.543	4213.156	4045.380	3858.577

******* MINIMUM: H= 1.250 T= 1.250 LOSS-COST= 564.040 (PER 100 HOURS)

Economic $\bar{X}$-Chart Design (Optimization)

The economic $\bar{X}$-chart scheme can be accessed by either selecting "4" from menu (M.4) or selecting "2" from menu (M.3). Once accessed, the user is first prompted for the values of common economic $\bar{X}$-chart parameters. After proper verification, menu (M.5) is presented. And a selection of "1" from this menu leads to the economic $\bar{X}$-chart design.

```
*** FOR ECON X-BAR CHART, ENTER VALUES:
    USLLSL,  DELTA, LAMBDA, M, E, D, T, W, B, C
?
6  2    .01  100  .05  20  50  25  .5  .1
VALUES ENTERED:   USLLSL= 6.00
 DELTA=    2.00   LAMBDA=    0.01    M= 100.00    E=    0.05    D=  20.00
      T=  50.00          W=  25.00   B=    0.50   C=    0.10
CORRECT ?   1=YES    2=NO    3=RETURN
?
1
*** ENTER OPTION NUMBER
    1= ECON X-BAR CHART DESIGN (OPTIMIZATION)
    2= ECON X-BAR CHART EVALUATION
    3= ECON X-BAR CHART LOSS-COST SURFACE INVESTIGATION                    (M.5)
    4= SWITCH TO ECON NLG
    5= RETURN TO REVISE USLLSL, AND TIME AND COST PARAMETERS
    6= EXIT SYSTEM
?
1
```

Then the user is prompted for the values of design parameters. The pre-programmed values of optimization parameters are listed for the user's examination. These values can be changed upon the user's request. After proper verification, the optimization subroutine is executed and optimal results printed. The interactive procedure, notation, and output format are similar to those for economic NLG design.

```
*** FOR ECON X-BAR CHART DESIGN, ENTER VALUES:  NMIN,NMAX
?
2  10
VALUES ENTERED:  NMIN= 2     NMAX=10

PARAMETER VALUES FOR:       (H,T) OPTIMIZATION                  OVERALL OPTIMIZATION
                  YACC    XACC    STEP ITRMAX    HO      TO IRESET        EL
   DEFAULT:      0.003   0.002    1.00    60    1.000   1.000  1          0.0
   CURRENT:      0.003   0.002    1.00    60    1.000   1.000  1          0.0

*** ENTER OPTION NUMBER:
1= ALL OK, NO REVISION NEEDED
2= NEED TO REVISE (NMIN,NMAX) VALUES
3= NEED TO REVISE (H,T) OPTIMIZATION PARAMETER VALUES
4= NEED TO REVISE OVERALL OPTIMIZATION PARAMETER VALUE
5= RETURN FOR OTHER ECON X-BAR CHART OPTIONS
?
4
```

```
ENTER VALUE: EL
?
20
VALUES ENTERED:  NMIN= 2     NMAX=10

PARAMETER VALUES FOR:            (H,T) OPTIMIZATION                OVERALL OPTIMIZATION
                 YACC    XACC    STEP ITRMAX   HO      TO IRESET          EL
   DEFAULT:     0.003   0.002    1.00   60   1.000   1.000  1             0.0
   CURRENT:     0.003   0.002    1.00   60   1.000   1.000  1            20.00

*** ENTER OPTION NUMBER:
1= ALL OK, NO REVISION NEEDED
2= NEED TO REVISE (NMIN,NMAX) VALUES
3= NEED TO REVISE (H,T) OPTIMIZATION PARAMETER VALUES
4= NEED TO REVISE OVERALL OPTIMIZATION PARAMETER VALUE
5= RETURN FOR OTHER ECON X-BAR CHART OPTIONS
?
1


***** ECONOMICALLY BASED X-BAR CHART DESIGN *****

USLLSL=  6.00       MEAN SHIFT ASSUMED
 DELTA=   2.00   LAMBDA=    0.01    M= 100.00   E=    0.05    D=  20.00
     T=  50.00          W=  25.00   B=    0.50   C=    0.10

(H,T) OPTIMIZATION:  YACC=  0.003   XACC=  0.002   STEP=  1.000   ITRMAX= 60
                     STARTING POINT:  HO=  1.000   TO=  1.000   IRESET=1
OVERALL OPTIMIZATION:  EL=  20.000                      NMIN= 2   NMAX=10

  N                  H         K        100L        STDY       STDX    TITR  MAXITR

  2              1.115     2.644   1882.393     0.0019     0.0060     19
  3              1.343     2.782   1850.358     0.0027     0.0053     20
  4              1.534     2.913   1839.157     0.0017     0.0091     19
  5              1.669     3.046   1837.204     0.0009     0.0085     21
  6              1.778     3.201   1839.927     0.0029     0.0256     14
  7              1.859     3.312   1845.077     0.0017     0.0065     18
  8              1.934     3.432   1851.561     0.0021     0.0141     19
  9              2.006     3.558   1858.726     0.0025     0.0256     14

        *********************************   OVERALL OPTIMAL 100L =   1837.204
```

Economic $\bar{X}$-Chart Evaluation

A selection of "2" from menu (M.5) leads to the economic $\bar{X}$-chart evaluation. The interactive procedure and evaluation output are very similar to those in economic NLG evaluation and are illustrated below.

```
FOR ECON X-BAR CHART EVALUATION, ENTER VALUES:  N,H,K
?
5   1.669  3.046
VALUES ENTERED:  N= 5    H=   1.669    K= 3.046
CORRECT ?   1=YES   2=NO   3= RETURN FOR OTHER ECON X-BAR CHART OPTIONS
?
±00
!! ERROR !! OUT OF RANGE !! DO IT OVER AGAIN
CORRECT ?   1=YES   2=NO   3= RETURN FOR OTHER ECON X-BAR CHART OPTIONS
?
1

 ***** ECONOMICALLY BASED X-BAR CHART EVALUATION *****
 USLLSL=  6.00 (STD)      MEAN SHIFT ASSUMED

 DELTA=   2.00   LAMBDA=    0.01   M= 100.00   E=   0.05   D=  20.00
     T=  50.00         W=  25.00   B=    0.50  C=   0.10

   N=  5          H=    1.669          K=    3.046

 LOSS-COST PER 100 HOURS =  1837.204  (HOURLY LOSS-COST =     18.372)
```

Economic $\bar{X}$-Chart Loss-Cost Surface Investigation

A selection of "3" from menu (M.5) leads to the economic $\bar{X}$-chart loss-cost surface investigation. The interactive procedure, notation, and explanation are very similar to those in the economic NLG loss-cost surface investigation. They are illustrated below.

```
*** FOR ECON X-BAR CHART COST SURFACE INVESTIGATION, ENTER VALUE:  N
?
5
ENTER VALUES:
NUMH (NUMBER OF H; <= 30), FOLLOWED BY ALL H VALUES TO BE INVESTIGATED
?
14   .1 .5 .75 1 1.25 1.5 1.75 2 2.25 2.5 3 5 10 50
ENTER VALUES:
NUMK (NUMBER OF K; <= 11), FOLLOWED BY ALL K VALUES TO BE INVESTIGATED
?
11   1.5 1.75 2 2.25 2.5 2.75 3 3.25 3.5 3.75 4
VALUES ENTERED:  N= 5
14 H VALUES =    0.100     0.500     0.750     1.000     1.250     1.500
                1.750     2.000     2.250     2.500     3.000     5.000
               10.000    50.000
11 K VALUES =  1.500     1.750     2.000     2.250     2.500     2.750
              3.000     3.250     3.500     3.750     4.000
```

```
*** ENTER OPTION NUMBER:
1= ALL OK, NO REVISION NEEDED
2= NEED TO REVISE N VALUE
3= NEED TO REVISE NUMH AND H VALUES
4= NEED TO REVISE NUMK AND K VALUES
5= RETURN FOR OTHER ECON X-BAR CHART OPTIONS'
?
1
```

***** ECONOMICALLY BASED X-BAR CHART LOSS-COST SURFACE INVESTIGATION *****

USLLSL= 6.00 STD MEAN SHIFT ASSUMED N= 5
DELTA= 2.00 LAMBDA= 0.01 M= 100.00 E= 0.05 D= 20.00 T= 50.00 W= 25.00 B= 0.50 C= 0.10

K	1.500	1.750	2.000	2.250	2.500	2.750	3.000	3.250	3.500	3.750	4.000
ALPHA	0.134	0.080	0.046	0.024	0.012	0.006	0.003	0.001	0.000	0.000	0.000
P	0.999	0.997	0.993	0.987	0.976	0.957	0.930	0.889	0.835	0.765	0.682
H											
0.100	8261.625	6038.180	4599.379	3724.473	3224.571	2956.219	2820.955	2757.046	2728.927	2717.692	2714.078
0.500	3030.882	2586.990	2299.821	2125.346	2025.914	1972.978	1947.008	1935.856	1932.669	1934.015	1938.545
0.750	2602.458	2306.886	2115.739	1999.730	1933.842	1899.144	1882.745	1876.712	1876.686	1880.682	1888.232
1.000	2392.548	2171.150	2028.042	1941.322	1892.303	1866.893	1855.542	1852.478	1854.655	1860.890	1871.284
1.250	2270.032	2093.153	1978.894	1909.790	1870.972	1851.260	1843.153	1842.197	1846.184	1854.490	1867.644
1.500	2191.214	2044.023	1949.014	1891.691	1859.735	1843.933	1838.164	1838.884	1844.485	1854.767	1870.630
1.750	2137.358	2011.381	1930.141	1881.262	1854.266	1841.348	1837.396	1839.542	1846.643	1858.844	1877.379
2.000	2099.101	1989.041	1918.140	1875.625	1852.394	1841.722	1839.258	1842.673	1851.198	1865.280	1886.456
2.250	2071.239	1973.564	1910.719	1873.175	1852.917	1844.060	1842.869	1847.449	1857.344	1873.277	1897.069
2.500	2050.648	1962.891	1906.499	1872.954	1855.110	1847.769	1847.700	1853.367	1864.598	1882.355	1908.739
3.000	2024.007	1951.135	1904.454	1876.959	1862.831	1857.922	1859.785	1867.476	1881.287	1902.637	1934.132
5.000	2004.419	1961.430	1934.393	1919.411	1913.441	1914.619	1922.314	1937.074	1960.574	1995.697	2046.852
10.000	2105.458	2085.240	2073.653	2069.407	2071.947	2081.691	2100.160	2130.048	2175.371	2241.798	2337.240
50.000	3244.098	3244.534	3250.215	3263.736	3289.207	3332.558	3401.692	3506.468	3658.578	3871.162	4158.129

******* MINIMUM: H= 1.750 T= 3.000 LOSS-COST= 1837.396 (PER 100 HOURS)

Summary

Nearly every feature of the interactive computer program of this research has been illustrated in this chapter. The interactive feature and its convenience, flexibility and comprehensiveness make this computer program a powerful process control tool. The implementation of this program can substantially help practitioners in designing and evaluating NLG process control plans both statistically and economically. Through its additional statistical and economic $\bar{X}$-chart design and evaluation capability, NLG can also be properly compared to the $\bar{X}$-chart. As such, this interactive computer program will greatly help with better assessment, easier implementation, and broader application of the NLG process control scheme.

CHAPTER VII

SUMMARY AND CONCLUSION

To fulfill the objective and subobjectives of this research stated in Chapter I, the following have been accomplished:

1. The general structure of NLG has been made clear by a comprehensive analysis, discussion, and illustration of NLG taxonomy. The undesirable diversity of possible NLG rules has been demonstrated.

2. A symbolically stated standard NLG scheme has been developed to standardize and simplify the design and evaluation of NLG. The relative importance and applicability of its individual basic elements have been examined.

3. The formulations for statistically evaluating both sample-wise and process-wise NLG performance have been derived, wherein either the mean shift or dispersion change is considered as an assignable cause.

4. General procedures have been constructed for statistically designing FG, QL, and the entire NLG plan. The general effects of individual NLG parameters on P_a and E_n have been investigated to help design FG and QL rules.

5. Methodologies for statistically evaluating and designing an $\bar{X}$-chart have been presented. An example comparing NLG, the $\bar{X}$-chart, and a traditional attribute gaging plan has been presented.

6. An economically-based NLG model has been formulated by following

the general structure of Duncan's fundamental economic $\bar{X}$-chart. Assumptions, similarities, and differences of both models have been investigated.

7. A general strategy together with a direct search technique has been developed to optimize the economic NLG model. For each m, this strategy optimizes (h,t) under each specified set of (n, y, g). This strategy is further improved by utilizing the convexity property of local optima among each level of (n, y, g) and by dynamically adjusting the searching range for each value of n, y, and g.

8. Economic NLG and the economic $\bar{X}$-chart have been compared under a variety of situations. From this analysis, general guidelines have been developed for better application of both models.

9. A convenient, flexible, and comprehensive interactive computer program has been constructed and demonstrated to facilitate the design and evaluation of (1) statistically-based NLG plans, (2) statistically-based $\bar{X}$-chart plans, (3) economically-based NLG plans, and (4) economically-based $\bar{X}$-chart plans.

Based on the results obtained in this research, the NLG process control scheme has proved to have combined the advantages of both variable and attribute control schemes. Therefore, it becomes potentially very suitable for the rapid detection of a process shift. In comparison to $\bar{X}$-charts both statistically and economically, NLG plans have been shown to be at least equally competitive, and in several aspects quite better than $\bar{X}$-charts, due to their easier-to-use go/no-go gaging method and no-calculation-required control scheme.

The following are major recommendations for future research on the same subject to facilitate NLG implementation and to cover a wider range of NLG applications:

1. For statistically-based control schemes, comprehensive standard tabulations of already-designed plans can be provided for FG, QL, entire NLG, and the $\bar{X}$-chart under a wide range of APL, TLAPL, RPL, and TLRPL design criteria. This can significantly reduce the cumbersome design procedures to a simple table-lookup for both NLG and $\bar{X}$-chart plans. It can also provide an alternative selection between NLG and $\bar{X}$-chart plans to better suit the user's individual needs.

2. The economically-based formulations of both NLG and the $\bar{X}$-chart can be extended to include dispersion change as an alternative assignable cause.

3. Different economically-based models of both NLG and the $\bar{X}$-chart requiring process shutdown during the search for an assignable cause can be considered.

4. More present-time examples containing realistic time and cost parameter values can be adopted for comparing economic NLG and $\bar{X}$-chart performance. This comparison should include the extended and the new economic control schemes proposed in items 2 and 3.

5. The economic portion of the interactive computer program should be extended accordingly.

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APPENDIX

```
C***********************************************************************
C
C  THIS INTERACTIVE PROGRAM PERFORMS
C     (1) STATISTICAL DESIGN AND EVALUATION OF NLG
C     (2) STATISTICAL DESIGN AND EVALUATION OF X-BAR CHART
C     (3) ECONOMIC    DESIGN AND EVALUATION OF NLG
C     (4) ECONOMIC    DESIGN AND EVALUATION OF X-BAR CHART
C
C  BY SHAWN S. YU,  SCHOOL OF INDUSTRIAL ENGINEERING AND MANAGEMENT
C                   OKLAHOMA STATE UNIVERSITY
C  DISSERTATION ADVISOR:  DR. KENNETH E. CASE
C
C  VERSION 1 -- JULY, 1983
C
C
C***********************************************************************
C
C *** GENERAL STRUCTURE AND INPUT REQUIREMENTS:
C
C     ( MAIN PROGRAM DRIVES SUBROUTINES STAT AND ECON.          )
C     ( STAT DRIVES S1 THROUGH S6;  ECON DRIVES E1 THROUGH E6 )
C
C                             COMMON INPUT           MAJOR FUNCTIONS
C                          --------------------      ---------------
C    STAT -------------> USLLSL  ; ASSIGNABLE CAUSE ---> S1 THROUGH S6
C    ECON --> NLG -----> USLLSL,M; ASSIGNABLE CAUSE ---> E1 THROUGH E3
C         --> X-BAR ---> USLLSL  ; ASSIGNABLE CAUSE ---> E4 THROUGH E6
C
C
C      SUBROUTINE    FUNCTION            MODULE INPUT
C      ---------    -----------    ------------------------------
C    S1:  FGGENE   NLG FG DESIGN       M;  NMIN,NMAX;
C                                      APL,TLAPL,RPL,TLRPL;  T VALUES
C    S2:  FGEVAL   NLG FG EVALU.       N,M,Y,G;  T VALUES;
C                                      F (FOR PBAPQ AND PBAOQ EVALU.)
C    S3:  QLGENE   NLG QL DESIGN       M;  NMIN,NMAX;
C                                      APL,TLAPL,RPL,TLRPL;  T
C    S4:  QLEVAL   NLG QL EVALU.       N,M,Y,T
C    S5:  XSTGE    X-BAR  DESIGN       V;  NMIN,NMAX;
C                                      APL,TLAPL,RPL,TLRPL;  K VALUES
C    S6:  XSTEV    X-BAR  EVALU.       N,V,K
C
C
C    E1:  NECOPT   NLG DESIGN          NMIN,NMAX;
C                                      OPTIMIZATION PARAMETERS (OPTIONAL)
C    E2:  NECEV    NLG EVALUATION      N,Y,G,H,T
C    E3:  NCOSF    NLG COST SURF.      N,Y,G;  H VALUES;  T VALUES
C    E4:  XECOPT   X-BAR DESIGN        NMIN,NMAX;
C                                      OPTIMIZATION PARAMETERS (OPTIONAL)
C    E5:  XECEV    X-BAR EVALU.        N,H,K
C    E6:  XCOSF    X-BAR COST SURF.    N;  H VALUES;  K VALUES
C
C
C***********************************************************************
C
C *** EXTERNAL FUNCTIONS REQUIRED:
C     (1) REGULAR SYSTEM SUPPLIED FORTRAN FUNCTIONS
C     (2) TWO IMSL SUBROUTINES:
C         MDNOR  -- CUMULATIVE PROBABILITY FUNCTION OF STANDARD NORMAL
C         MDNRIS -- INVERSE FUNCTION OF MDNOR
C
C
C***********************************************************************
C
C *** COMMON BLOCK VARIABLE DEFINITIONS:
C
C------------- FOR BOTH STATISTICALLY AND ECONOMICALLY BASED SCHEMES ---
C /C1/ ----- NLG PARAMETERS
C    NN -- SMALL N,  SAMPLE SIZE
C    MM -- SMALL M,  NUMBER OF NLG CLASSIFICATIONS
C    NG -- SMALL G,  GREEN ACCEPANCE TRUNCATION NUMBER
C    NY -- SMALL Y,  MAXIMUM YELLOW ACCEPTANCE NUMBER
```

```
C     NY1 -- NY + 1
C     TNLG -- SMALL T,  NLG CONTROL SPREAD
C
C-------------- FOR STATISTICALLY BASED SCHEMES ------------------------
C /S1/
C     MUSTD -- ASSIGNABLE CAUSE (1= MEAN SHIFT;  2= DISPERSION CHANGE)
C     NNL,NNH -- RANGE OF SAMPLE SIZE
C     APL,TLAPL,RPL,TLRPL -- USER SPECIFIED OC CURVE DESIGN POINTS,
C        ACCEPTABLE AND REJECTABLE PROCESS LEVELS AND THEIR ASSOCIATED
C        TOLERABLE LIMITS
C     NUMT,AT(10) -- NUMBER OF T VALUES.  THESE T VALUES ARE STORED IN
C        ARRAY AT
C /S2/
C     PG,PY,PR -- PROBABILITY OF GREEN, YELLOW AND RED
C /S3/
C     DELMU -- DEGREE OF PROCESS MEAN SHIFT (IN MULTIPLES OF STD)
C     STD10 -- DEGREE OF DISPERSION CHANGE (THE RATIO OF NEW OVER OLD)
C /S4/
C     IFG -- 1=FG   2= FG + PBAPQ ,  3= FG + PBAPQ + PBAOQ
C     NF -- CAPITAL F,  THE SELF-ADJUST SAMPLING FREQUENCY,  THE NUMBER
C           OF SAMPLES PER OUT-OF-CONTROL INDICATION
C /S5/
C     RY -- RELATIVE LOCATION OF THE LOWER SPECIFICATION LIMIT MEASURED
C           FROM THE PROCESS MEAN (IN MULTIPLES OF STD)
C     DEL -- DEGREE OF MEAN SHIFT (IN MULTIPLES OF STD)
C     STD10 -- DEGREE OF DISPERSION CHANGE (NEW TO OLD RATIO)
C /S6/ ----- PARAMETERS FOR X-BAR CHART PLANS
C     VX -- SMALL V,  THE DISTANCE BETWEEN A SPECIFICATION LIMIT AND ITS
C           CORESPONDING BOUNDARY FOR AN ACCEPTABLE PROCESS MEAN (IN
C           MULTIPLES OF STD)
C     RKX -- SMALL K,  X-BAR CHART CONTROL LIMIT SPREAD
C     NX -- SMALL N,  SAMPLE SIZE OF X-BAR CHART PLAN
C     NXL,NXH -- RANGE OF NX
C     NUMK,AK(10) -- NUMBER OF K VALUES.  THESE VALUES ARE STORED IN
C                    ARRAY AK
C /S7/ ----- CHRACTER STRINGS
C
C------------------ FOR ECONOMICALLY BASED SCHEMES ---------------------
C /E2/
C     PG,PY,PR -- PROBABILITY OF GREEN, YELLOW AND RED
C     PR1,PR2 -- FRACTION DEFECTIVES BEFORE AND AFTER PROCESS MEAN SHIFT
C /E3/ ----- COST AND TIME PARAMETERS FOR NLG OR X-BAR CHART SCHEME
C /E4/ ----- (H,T) DIRETC SEARCH OPTIMIZATION PARAMETERS
C     XSTART(2) -- THE ADOPTED STARTING VALUES OF H AND T
C     X(3,2) -- THREE VERTICES OF A ITERATION SIMPLEX
C     Y(3) -- FUNCTION VALUES (LOSS-COST) OF X(3,2)
C     ITRFLG -- 1= MAXIMUM ITERATION NUMBER REACHED AND ITERATION
C                  TERMINATED
C     IRESET -- 1= EACH (H,T) OPTIMIZATION STARTS WITH THE USER SPECIFIED
C                  (H,T) STARTING VALUES
C               O= EACH (H,T) OPTIMIZATION STARTS WITH THE OPTIMAL (H,T)
C                  VALUES FROM LAST OPTIMIZATION
C     STDX -- STANDARD DEVIATION OF THE DISTANCES AMONG ALL VERTICES OF
C             A SIMPLEX
C     STDY -- STANDARD DEVIATION OF THE FUNCTION VALUES OF ALL VERTICES
C             OF A SIMPLEX
C     XACC,YACC -- USER SPECIFIED QUITTING CRITERIA.  (H,T) OPTIMIZATION
C                  TERMINATES WHENEVER  STDX < XACC  OR  STDY < YACC
C     STEP -- STEP SIZE
C     ITRMAX -- USER SPECIFIED MAXIMUM ITERATION NUMBER
C     NLGXB -- 1= NLG SCHEME    2= X-BAR CHART SCHEME
C /E5/ ----- NLG OVERALL OPTIMIZATION PARAMETERS
C     NYBACK -- EPSILON SUB SMALL Y,  THE VALUE TO DYNAMICALLY DETERMINE
C               NEXT STARTING Y VALUE
C     NYBACK -- EPSILON SUB SMALL G,  THE VALUE TO DYNAMICALLY DETERMINE
C               NEXT STARTING G VALUE
C     YIMPRV -- EPSILON SUB L,  THE VALUE TO OVERCOME BUMPS IN A CONVEX
C               CURVE
C     NNMIN,NNMAX -- RANGE OF SAMPLE SIZE
C /E6/ ----- PARAMETERS FOR LOSS-COST SURFACE INVESTIGATION
C     HNLG -- SMALL H,  THE SAMPLING INTERVAL FOR NLG PLAN
C     HX -- SMALL H,  THE SAMPLING INTERVAL FOR X-BAR CHART PLAN
```

```
C    RKX -- SMALL K,  THE CONTROL SPREAD FOR X-BAR CHART PLAN
C /E7/
C    NH,AH(30) -- NUMBER OF K VALUES.  THESE VALUES ARE STORED IN ARRAY
C                 AH
C    NT,AT(11); NK,AK(11) -- SIMILAR FOR T AND K
C
C***********************************************************************
C
C
C
C
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
C     MAIN PROGRAM -- THE PRIMARY DRIVER PROGRAM
C
C *** THE MAIN PROGRAM DRIVES SUBROUTINES STAT AND ECON
C
      COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      IR=5
      IW=6
   10 WRITE(IW,11)
   11    FORMAT(/' *** ENTER OPTION NUMBER'/
     *   T6,'1 = STATISTICALLY-BASED PROCESS CONTROL'/
     *   T6,'2 = ECONOMICALLY-BASED PROCESS CONTROL'/
     *   T6,'3 = EXIT SYSTEM')
         READ(IR,*) N13
         GOTO(100,200,300),N13
         WRITE(IW,20)
   20       FORMAT(' !! ERROR !! OUT OF RANGE !! DO IT OVER AGAIN')
         GOTO 10
  100 CALL STAT
         GOTO 10
  200 CALL ECON
         GOTO 10
  300 STOP
      END
C
C
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      BLOCK DATA
C
C *** THIS BLOCK DATA SUBPROGRAM INITIALIZE VARIABLES IN COMMON /S8/
C
      COMMON /S8/ ASSCOZ(10,2),BLANK,STAR2,DELSTD(2)
      DATA ASSCOZ/'MEAN',' SHI','FT A','SSUM','ED (','MULT','IPLE',
     *            'S OF',' STD',')',
     *            'DISP','ERSI','ON C','HANG','E AS','SUME','D (S',
     *            'TD R','ATIO',')'/
      DATA BLANK/' '/, STAR2/'**'/,DELSTD/'DEL','STDR'/
      END
C
C
C
C
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE STAT
C
C *** THIS SUBROUTINE SERVES AS THE PROMPTER PROGRAM AND DRIVES THE
C     FOLLOWING SIX SUBROUTINES FOR THE STATISTICALLY BASED
C     PROCESS CONTROL SCHEMES:
C
C          FGGENE -- STAT NLG FG DESIGN
C          FGEVAL -- STAT NLG FG EVALUATION
C          QLGENE -- STAT NLG QL DESIGN
C          QLEVAL -- STAT NLG QL EVALUATION
C          XSTGE -- STAT X-BAR CHART DESIGN
C          XSTEV -- STAT X-BAR CHART EVALUATION
C
```

```
      COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /S2/MUSTD, NNL,NNH, APL,TLAPL,RPL,TLRPL, NUMT,AT(10)
      COMMON /S5/IFG,NF
      COMMON /S7/VX,NXL,NXH, NUMK,AK(10), NX,RKX
   20 FORMAT(' !! ERROR !! OUT OF RANGE !! DO IT OVER AGAIN')
C
C--------------------------- STAT OPTION MENU -------------------------
  100 WRITE(IW,101)
  101    FORMAT(' IN STATISTICALLY BASED PROCESS CONTROL'/' *** ENTER',
     *   ' VALUES:'/T2,'USLLSL, ASSIGNABLE CAUSE (1= MEAN SHIFT;',
     *   '  2= DISPERSION CHANGE)')
         READ(IR,*) USLLSL,MUSTD
      IF(MUSTD.EQ.1) WRITE(IW,103) USLLSL
  103    FORMAT(' USLLSL=',F5.2,' (STD)',';  MEAN SHIFT ASSUMED.')
      IF(MUSTD.EQ.2) WRITE(IW,104) USLLSL
  104   FORMAT(' USLLSL=',F5.2,' (STD)',';  DISPERSION CHANGE ASSUMED.')
  102    WRITE(IW,107)
  107       FORMAT(' CORRECT ?   1=YES   2=NO   3=RETURN')
         READ (IR,*) IYN
         GOTO (105,100,250),IYN
         WRITE(IW,20)
         GOTO 102
C
  105 WRITE(IW,106)
  106    FORMAT(/' *** ENTER OPTION NUMBER'/
     *   T6,'1= STAT NLG FG DESIGN'/
     *   T6,'2= STAT NLG FG EVALUATION ( + OPTIONAL PBAPQ AND PBAOQ )'/
     *   T6,'3= STAT NLG QL DESIGN'/
     *   T6,'4= STAT NLG QL EVALUATION'/
     *   T6,'5= STAT X-BAR CHART DESIGN'/
     *   T6,'6= STAT X-BAR CHART EVALUATION'/
     *   T6,'7= RETURN TO REVISE USLLSL AND ASSIGNABLE CAUSE'/
     *   T6,'8= SWITCH TO ECON PROCESS CONTROL SCHEME'/
     *   T6,'9= EXIT SYSTEM')
         READ(IR,*) NSTAT
         GOTO (110,120,130,140,150,160,100,250,300),NSTAT
            WRITE(IW,20)
            GOTO 105
C
C---------------------------- STAT NLG FG DESIGN ------------------------
  110 WRITE(IW,111)
  111    FORMAT(' FOR STAT NLG FG DESIGN'/
     *   ' *** ENTER VALUES:  M,NMIN,NMAX')
         READ(IR,*) MM,NNL,NNH
      WRITE(IW,112)
  112    FORMAT(' *** ENTER VALUES:  APL,TLAPL,RPL,TLRPL')
         READ(IR,*) APL,TLAPL,RPL,TLRPL
      WRITE(IW,113)
  113    FORMAT(' *** ENTER VALUES:'/T2,'NUMT (NUMBER OF T; <= 10), ',
     *   'FOLLOWED BY T VALUES TO BE INVESTIGATED')
         READ (IR,*) NUMT,(AT(I),I=1,NUMT)
      WRITE(IW,114)MM,NNL,NNH,APL,TLAPL,RPL,TLRPL,NUMT,(AT(I),I=1,NUMT)
  114    FORMAT(' VALUES ENTERED:  M=',I2,4X,'NMIN=',I2,4X,'NAMX=',I2/
     *   T3,'APL=',F5.3,4X,'TLAPL=',F5.3,4X,'RPL=',F5.3,4X,'TLRPL=',
     *   F5.3/ T3,I2,' T VALUES = ',10(F6.3,1X))
  117 WRITE(IW,115)
  115    FORMAT(' CORRECT ?   1=YES   2=NO   3= RETURN FOR OTHER',
     *   ' STAT OPTIONS')
         READ(IR,*) IYN
         GOTO (116,110,105),IYN
         WRITE(IW,20)
         GOTO 117
  116 CALL FGGENE
         GOTO 105
C
C---------------------------- STAT NLG FG EVALUATION --------------------
  120 WRITE(IW,121)
  121    FORMAT(' *** FOR STAT NLG FG EVALUATION,  ENTER OPTION NUMBER'/
     *   T5,'1= FG ONLY     2= FG + PBAPQ     3= FG + PBAPQ + PBAOQ')
         READ(IR,*) IFG
         WRITE(IW,122)
  122       FORMAT('  *** FOR FG, ENTER VALUES:  N,M,Y,G')
```

```
         READ(IR,*) NN,MM,NY,NG
      WRITE(IW,113)
         READ(IR,*) NUMT,(AT(I),I=1,NUMT)
         GOTO (128,127,127),IFG
  127    WRITE(IW,123)
  123       FORMAT(' *** FOR PBAPQ, ENTER VALUE OF F'/T13,'(NUMBER OF',
     *      ' SAMPLES PER OOC INDICATION)')
         READ(IR,*) NF
  128    WRITE(IW,124) NN,MM,NY,NG,NUMT,(AT(I),I=1,NUMT)
  124       FORMAT('  VALUES ENTERED:  N=',I2,4X,'M=',I2,4X,'Y=',I2,4X,
     *      'G=',I2/T3,I2,' T VALUES = ',10(F6.3,1X))
         GOTO (129,1124,1124),IFG
 1124    WRITE(IW,125) NF
  125       FORMAT('  SAMPLING FREQUENCY F =',I3,' SAMPLES PER OOC ',
     *      'INDICATION')
  129 WRITE(IW,115)
      READ(IR,*) IYN
      GOTO (126,120,105),IYN
      WRITE(IW,20)
      GOTO 129
  126 CALL FGEVAL
      GOTO 105
C
C---------------------------- STAT NLG QL DESIGN -----------------------
  130 WRITE(IW,131)
  131    FORMAT('  FOR STAT NLG QL DESIGN'/'  *** ENTER VALUES:  ',
     *   'M,NMIN,NMAX')
         READ(IR,*) MM,NNL,NNH
      WRITE(IW,132)
  132    FORMAT('  *** ENTER VALUES OF APL,TLAPL,RPL,TLRPL'/T6,'(HERE ',
     *   'APL, RPL MUST BE IN MULTIPLES OF STD)')
         READ(IR,*) APL,TLAPL,RPL,TLRPL
      WRITE(IW,133)
  133    FORMAT('  *** ENTER T VALUE')
         READ(IR,*) TNLG
      WRITE(IW,134) MM,NNL,NNH,APL,TLAPL,RPL,TLRPL,TNLG
  134    FORMAT(' VALUES ENTERED:  M=',I2,4X,'NMIN=',I2,4X,'NMAX=',I2/
     *   T3,'APL=',F6.3,'(STD)',4X,'TLAPL=',F5.3,4X,'RPL=',F6.3,'(STD)',
     *   4X,'TLRPL=',F5.3/ T3,'T=',F6.3)
  135 WRITE(IW,115)
         READ(IR,*) IYN
         GOTO (136,130,105),IYN
         WRITE(IW,20)
         GOTO 135
  136 CALL QLGENE
         GOTO 105
C
C---------------------------- STAT NLG QL EVALUATION -------------------
  140 WRITE(IW,141)
  141    FORMAT('  FOR STAT NLG QL EVALUATION'/
     *   ' *** ENTER VALURES:  N,M,Y,T')
         READ(IR,*) NN,MM,NY,TNLG
      WRITE(IW,144) NN,MM,NY,TNLG
  144    FORMAT('  VALUES ENTERED:  N=',I2,4X,'M=',I2,4X,'Y=',I2,4X,
     *   'T=',F6.3)
  145 WRITE(IW,115)
      READ(IR,*) IYN
      GOTO (146,140,105),IYN
      WRITE(IW,20)
      GOTO 145
  146 CALL QLEVAL
      GOTO 105
C
C-------------------------- STAT MODIFIED X-BAR CHART DESIGN ----------
  150 WRITE(IW,151)
  151    FORMAT(' FOR STAT MODIFIED X-BAR CHART DESIGN'/
     *   ' *** ENTER VALUES:  V,NMIN,NMAX')
         READ(IR,*) VX,NXL,NXH
      WRITE(IW,112)
         READ(IR,*) APL,TLAPL,RPL,TLRPL
      WRITE(IW,153)
  153    FORMAT(' *** ENTER VALUES:'/T6,'NUMK (NUMBER OF K; <= 10), ',
```

```
     *    'FOLLOWED BY K VALUES TO BE INVESTIGATED')
          READ (IR,*) NUMK,(AK(I),I=1,NUMK)
      WRITE(IW,154)VX,NXL,NXH,APL,TLAPL,RPL,TLRPL,NUMK,(AK(I),I=1,NUMK)
  154     FORMAT(' VALUES ENTERED:  V=',F6.3,4X,'NMIN=',I2,4X,'NAMX=',I2/
     *    T3,'APL=',F5.3,4X,'TLAPL=',F5.3,4X,'RPL=',F5.3,4X,'TLRPL=',
     *    F5.3/ T3,I2,' K VALUES = ',10(F6.3,1X))
  155 WRITE(IW,115)
          READ(IR,*) IYN
          GOTO (156,150,105),IYN
          WRITE(IW,20)
          GOTO 155
  156 CALL XSTGE
          GOTO 105
C
C----------------------- STAT MODIFIED X-BAR CHART EVALUATION ----------
  160 WRITE(IW,161)
  161     FORMAT('  FOR STAT MODIFIED X-BAR CHART EVALUATION'/
     *    ' *** ENTER VALURES:  N,V,K')
          READ(IR,*) NX,VX,RKX
      WRITE(IW,164) NX,VX,RKX
  164     FORMAT('  VALUES ENTERED:  N=',I2,4X,'V=',F6.3,4X,'K=',F6.3)
  165 WRITE(IW,115)
      READ(IR,*) IYN
      GOTO (166,160,105),IYN
      WRITE(IW,20)
      GOTO 165
  166 CALL XSTEV
      GOTO 105
C
  250 RETURN
  300 STOP
      END
C
C
C
C
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE FGGENE
C
C *** THIS SUBROUTINE STATISTICALLY DESIGN NLG FREQUENCY GAGING RULES
C
      COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /S2/MUSTD, NNL,NNH, APL,TLAPL,RPL,TLRPL, NUMT,AT(10)
      COMMON /S3/ PG,PY,PR
      COMMON /S6/ RY,DEL,STD10
      COMMON /S8/ ASSCOZ(10,2),BLANK,STAR2,DELSTD(2)
C
          PMID=(APL+RPL)/2.
          HALF=.5*USLLSL
          CALL MDNOR(-HALF,PPO)
          PP2=PPO*2.
C
C------------------- PRINT TITLE AND PARAMETER VALUES
C
      WRITE(IW,50) USLLSL,(ASSCOZ(I,MUSTD),I=1,10), MM,NNL,NNH,
     *                 APL,TLAPL,RPL,TLRPL, (AT(I),I=1,NUMT)
   50 FORMAT( //'  ***** STATISTICALLY BASED NLG FG DESIGN *****'/
     *T5,'USLLSL=',F5.2,' (STD)',5X,10A4/T5,'M=',I2,4X,
     *'NMIN=',I2,4X,'NMAX=',I2/
     * T5,'APL=',F5.3,4X,'TLAPL=',F5.3,4X,'RPL=',F5.3,4X,'TLRPL=',F5.3/
     * T5,'INVESTIGATED T VALUES =',9(F6.3,1X))
C------------------------------- T LOOP
      DO 130 I=1,NUMT
          TNLG=AT(I)
          WRITE(IW,60) TNLG
   60      FORMAT(// T2,10('*'),'    T =',F6.3)
          WRITE(IW,70) PP2,APL,PMID,RPL
   70        FORMAT(//,T19,'(PO=',F6.4,')  (APL=',F5.3,') (MID=',F5.3,
     *               ') (RPL=',F5.3,')',/,T4,'N  M  Y  G',T20,'ENO',4X,
     *               'PRO',T36,'PA1',T48,'PA2',T58,'PA3',T69,'EN3')
C------------------------------- N LOOP
```

```
         DO 120 NN=NNL,NNH
            WRITE(IW,80)
   80          FORMAT(' ')
            NYH1=INT(NN/2.+.6)+1
C------------------------------- Y LOOP
            DO 110 J=1,NYH1
               NY=J-1
               NY1=NY+1
               NFLAG=0
                  GOTO (131,22,33),MM
   22                NGH=NN-NY+1
                     GOTO 83
   33                NGH=NN+1
C------------------------------- G LOOP
   83          DO 100 K=2,NGH
                  IF(NFLAG.EQ.1)GO TO 110
                  NG=K-1
                     IF(NY.EQ.0) GO TO 90
   85             IF(MUSTD.EQ.1) CALL GYR(PP0)
                  IF(MUSTD.EQ.2) CALL GYR(PP2)
                  CALL EOFN(EN0)
                  CALL PAFG(PA0)
                     PR0=1.-PA0
                  CALL GYR(APL)
                  CALL PAFG(PA1)
                  CALL GYR(PMID)
                  CALL PAFG(PA2)
                  CALL GYR(RPL)
                  CALL PAFG(PA3)
                  CALL EOFN(EN3)
                     STAR=BLANK
C---------------------- LABEL QUILIFIED PLAN BY '**'
                     IF(PA1.GE.TLAPL .AND. PA3.LE.TLRPL) STAR=STAR2
                  GO TO 95
C----- FOR NY=0, NG MUST BE 0, OR INSPECTION WILL ALWAYS BE TRUNCATED
C      PREMATURELY
   90                NG=0
                     NFLAG=1
                     GO TO 85
   95             WRITE(IW,96)NN,MM,NY,NG,STAR, EN0,PR0, PA1,STAR,
     *                     PA2, PA3,STAR, EN3
   96                FORMAT(T2,4I3,1X,A2,T18,F6.2,1X,F7.4,T34,F6.3,1X,A2,
     *                     T46,F6.3,T56,F6.3,1X,A2,2X,F6.2)
  100          CONTINUE
  110       CONTINUE
  120    CONTINUE
  130 CONTINUE
  131 RETURN
      END
C
C
C
C++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE FGEVAL
C
C *** THIS SUBROUTINE STATISTICALLY EVALUATES NLG FREQUENCY GAGING RULES
C        ( EVALUATED PERFORMANCE MEASURES:  PA,EN, PBAPQ,PBAOQ )
C
      COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /S2/MUSTD, NNL,NNH, APL,TLAPL,RPL,TLRPL, NUMT,AT(10)
      COMMON /S3/ PG,PY,PR
      COMMON /S5/IFG,NF
      COMMON /S6/ RY,DEL,STD10
      COMMON /S8/ ASSCOZ(10,2),BLANK,STAR2,DELSTD(2)
      DIMENSION APP(27)
C------------------------ SPECIFY FRACTION DEFECTIVE VALUES
      NY1=NY+1
      HALF=.5*USLLSL
      CALL MDNOR(-HALF,PP0)
      APP(1)=2.*PP0
      DO 10 I=2,21
   10 APP(I)=(I-1)*.005
```

```
      DO 12 I=22,26
   12 APP(I)=(I-21)*.02+.1
      APP(27)=.40
C
C------------------- PRINT TITLE AND PARAMETER VALUES
C
      WRITE(IW,50) USLLSL,(ASSCOZ(I,MUSTD),I=1,10), NN,MM,NY,NG,
     *             (AT(I),I=1,NUMT)
   50 FORMAT(  //'  ***** STATISTICALLY BASED NLG FG EVALUATION *****'/
     * T5,'USLLSL=',F5.2,' (STD)',5X,10A4,/T5,'N=',I2,4X,'M=',I2,4X,
     * 'Y=',I2,4X,'G=',I2,/T5,'INVESTIGATED T VALUES =',10(F6.3,1X))
C
C------------------------- T LOOP
      DO 200 IJ=1,NUMT
         TNLG=AT(IJ)
         WRITE(IW,85) TNLG
   85       FORMAT(/T2,10('*'),'  T=',F6.3)
C
C---------- CHECK OPTION NUMBER AND PRINT APPROPRIATE LABELS
C           (1=FG   2= FG + PBAPQ   3= FG + PBAPQ + PBAOQ)
         GOTO (89,91,93),IFG
C
   89    WRITE(IW,90) DELSTD(MUSTD)
   90    FORMAT(/  T7,'P',T15,A4,T27,'PA',T35,' EN'/)
         GOTO 94
   91    WRITE(IW,92) DELSTD(MUSTD)
   92    FORMAT(/  T7,'P',T15,A4,T27,'PA',T35,' EN',T44,'PBAPQ'/)
         GOTO 94
   93    WRITE(IW,88) DELSTD(MUSTD)
   88    FORMAT(/  T7,'P',T15,A4,T27,'PA',T35,' EN',T44,'PBAPQ',T54,
     *           'PBAOQ'/)
C
   94    DO 110 I=1,27
C
C------------- PROCESS BEFORE SHIFTING IS EVALUATED "EXACTLY".
C              OTHERWISE, EVALUATED APPROXIMATELY
            IF(MUSTD.EQ.1.AND.I.EQ.1) GOTO 107
C
            CALL GYR(APP(I))
   95       CALL PAFG(PA)
            CALL EOFN(EN)
               GOTO (96,108,108),IFG
   96          GOTO (97,99,101),IFG
   97       IF(MUSTD.EQ.1) WRITE(IW,105) APP(I),DEL ,PA,EN
            IF(MUSTD.EQ.2) WRITE(IW,105) APP(I),STD10,PA,EN
  105                      FORMAT(T4,F7.4,2X,F7.3,4X,F7.3,2X,F6.2)
            GOTO 110
   99       IF(MUSTD.EQ.1) WRITE(IW,100) APP(I),DEL ,PA,EN,PBAPQ
            IF(MUSTD.EQ.2) WRITE(IW,100) APP(I),STD10,PA,EN,PBAPQ
  100          FORMAT(T4,F7.4,2X,F7.3,4X,F7.3,2X,F6.2,T42,F7.4)
            GOTO 110
  101       IF(MUSTD.EQ.1) WRITE(IW,102) APP(I),DEL ,PA,EN,PBAPQ,PBAOQ
            IF(MUSTD.EQ.2) WRITE(IW,102)APP(I),STD10,PA,EN,PBAPQ,PBAOQ
  102          FORMAT(T4,F7.4,2X,F7.3,4X,F7.3,2X,F6.2,T42,F7.4,T52,F7.4)
            GOTO 110
C---------------- PROCESS BEFORE SHIFTING
  107          CALL GYR(PPO)
               GOTO 95
C
C ----------- CALCULATION FOR PBAPQ AND PBAOQ -----------
C
C---- (0 <= PA <= 1) ==> (.5 <= Q1 <= INFINITY)AND (-.5 <= Q1-1 <= INF)
C---- BUT IN REALITY, IT IS REQUIRED THAT
C----     (0 <= Q1 <=NF) FOR PBAPQ   AND   (0 <= Q1-1 <= NF) FOR PBAOQ
C
  108          Q1=1./(1.-PA)-.5
                  IF(Q1.GT.NF) Q1=NF
               Q2=APP(1)*(NF-Q1)
            PBAPQ=(APP(I)*Q1 + Q2)/NF
            IF(PBAPQ.GT.APP(I)) PBAPQ=APP(I)
               IF(IFG.EQ.2) GOTO 96
               IF(Q1.LT.1.) Q1=1.
```

```
              NF1=NF+1
            IF(Q1.GT.NF1) Q1=NF1
            Q2=APP(1)*(NF-Q1)
          PBAOQ=(APP(I)*(Q1-1.) + Q2)/NF
          IF(PBAOQ.GT.APP(I)) PBAOQ=APP(I)
          GOTO 96
  110   CONTINUE
  200 CONTINUE
  210 RETURN
      END
C
C
C
C++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE QLGENE
C
C *** THIS SUBROUTINE STATISTICALLY DESIGNS NLG QUALIFICATION RULES
C
      COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /S2/MUSTD, NNL,NNH, APL,TLAPL,RPL,TLRPL, NUMT,AT(10)
      COMMON /S3/ PG,PY,PR
      COMMON /S6/ RY,DEL,STD10
      COMMON /S8/ ASSCOZ(10,2),BLANK,STAR2,DELSTD(2)
      NG=0
C--- IN QL DESIGN, APL AND RPL ARE EXPRESSED IN MULTIPLES OF STD
C
        PMID=(APL+RPL)/2.
        HALF=.5*USLLSL
C
C------------------- PRINT TITLE AND PARAMETER VALUES
C
      WRITE(IW,50) USLLSL,(ASSCOZ(I,MUSTD),I=1,10), MM,NNL,NNH,
     *             APL,TLAPL,RPL,TLRPL, TNLG
   50 FORMAT(  //'  ***** STATISTICALLY BASED NLG QL DESIGN *****'/
     * T5,'USLLSL=',F5.2,' (STD)',5X,10A4/T5,'M=',I2,4X,
     * 'NMIN=',I2,4X,'NMAX=',I2/
     * T5,'APL=',F6.3,'(STD)',4X,'TLAPL=',F6.3,4X,'RPL=',F6.3,'(STD)',
     * 4X,'TLRPL=',F6.3/T5,'T=',F6.3)
      WRITE(IW,70) APL,PMID,RPL
   70   FORMAT(/  T18,'(EXACT SETUP)','  (APL=',F5.3,') (MID=',F5.3,
     *        ') (RPL=',F5.3,')'/T25,'0.0 STD',T40,'STD',T52,'STD',
     *        T64,'STD'/,T4,'N     Y    ',T20,'ENO',4X,
     *        'PRO',T36,'PA1',T48,'PA2',T58,'PA3',T69,'EN3'/)
C-------------------------- N LOOP
      DO 120 NN=NNL,NNH
C-------------------------- Y LOOP
        DO 110 J=1,NN
          NY=J-1
          NY1=NY+1
            IF(MUSTD.EQ.1) CALL GYRC(0.)
            IF(MUSTD.EQ.2) CALL GYRC(1.)
          CALL EOFN(ENO)
          CALL PAQL(PAO)
            PRO=1.-PAO
          CALL GYRC(APL)
          CALL PAQL(PA1)
          CALL GYRC(PMID)
          CALL PAQL(PA2)
          CALL GYRC(RPL)
          CALL PAQL(PA3)
          CALL EOFN(EN3)
            STAR=BLANK
C--------------------- LABEL QUILIFIED PLAN BY '**'
            IF(PA1.GE.TLAPL .AND. PA3.LE.TLRPL) STAR=STAR2
   95     WRITE(IW,96)NN,NY,STAR, ENO,PRO, PA1,STAR,PA2, PA3,STAR, EN3
   96       FORMAT(T2,I3,3X,I3,3X,1X,A2,T18,F6.2,1X,F7.4,T34,F6.3,1X,
     *               A2,T46,F6.3,T56,F6.3,1X,A2,2X,F6.2)
  110   CONTINUE
  120 WRITE(IW,121)
  121   FORMAT(' ')
  131 RETURN
      END
```

```
C
C
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE QLEVAL
C
C *** THIS SUBROUTINE STATISTICALLY EVALUATES NLG QUALIFICATION RULES
C
      DIMENSION ACHG(20,2)
      COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /S2/MUSTD, NNL,NNH, APL,TLAPL,RPL,TLRPL, NUMT,AT(10)
      COMMON /S3/ PG,PY,PR
      COMMON /S6/ RY,DEL,STD10
      COMMON /S8/ ASSCOZ(10,2),BLANK,STAR2,DELSTD(2)
C
C-- PREDETERMINE 20 PROCESS LEVELS (IN MULTIPLES OF STANDARD DEVIATION)
      DATA ACHG/0.,.1,.2,.3,.4,.5,.6,.7,.8,.9,1., 1.2,1.4,1.6,1.8,2.,
     *          2.5,3.,4.,5.,   1.,1.1,1.2,1.3,1.4,1.5,1.6,1.7,1.8,
     *          1.9,2., 2.2,2.4,2.6,2.8,3., 3.5,4.,5.,6./
      NG=0
C
      NY1=NY+1
      HALF=.5*USLLSL
C
C------------------- PRINT TITLE AND PARAMETER VALUES
C
      WRITE(IW,50) USLLSL,(ASSCOZ(I,MUSTD),I=1,10), NN,MM,NY,NG, TNLG
   50 FORMAT(// '  ***** STATISTICALLY BASED NLG QL EVALUATION *****'//
     * T5,'USLLSL=',F5.2,' (STD)',5X,10A4,/
     * T5,'N=',I2,4X,'M=',I2,4X,'Y=',I2,4X,'G=',I2,6X,'T=',F6.3)
         WRITE(IW,90) DELSTD(MUSTD)
   90    FORMAT(/T16,A4,T27,'PA',T35,' EN'/)
C
         DO 110 I=1,20
            CALL GYRC(ACHG(I,MUSTD))
   95       CALL PAQL(PA)
            CALL EOFN(EN)
            WRITE(IW,105) ACHG(I,MUSTD),PA,EN
  105          FORMAT(T13,F7.3,4X,F7.3,2X,F6.2)
  110    CONTINUE
  210 RETURN
      END
C
C
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE XSTGE
C
C *** THIS SUBROUTINE STATISTICALLY DESIGNS MODIFIED X-BAR CHARTS
C
      COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /S2/MUSTD, NNL,NNH, APL,TLAPL,RPL,TLRPL, NUMT,AT(10)
      COMMON /S4/ DELMU,STD10, SQN,B1,B2
      COMMON /S7/VX,NXL,NXH, NUMK,AK(10), NX,RKX
      COMMON /S8/ ASSCOZ(10,2),BLANK,STAR2,DELSTD(2)
C
         PMID=(APL+RPL)/2.
         HALF=USLLSL/2.
         CALL MDNOR(-HALF,POH)
         PO=2.*POH
C
C------------------- PRINT TITLE AND PARAMETER VALUES
C
      WRITE(IW,50) USLLSL,(ASSCOZ(I,MUSTD),I=1,10), VX,NXL,NXH,
     *             APL,TLAPL,RPL,TLRPL, (AK(I),I=1,NUMK)
   50 FORMAT(//  '  ***** STATISTICALLY BASED MODIFIED X-BAR CHART',
     * ' DESIGN *****'//T5,'USLLSL=',F5.2,' (STD)',5X,10A4,/
     * T5,'V=',F6.3,4X,'NMIN=',I2,4X,'NMAX=',I2/
     * T5,'APL=',F5.3,4X,'TLAPL=',F5.3,4X,'RPL=',F5.3,4X,'TLRPL=',F5.3/
     * T5,'INVESTIGATED K VALUES =',10(F6.3,1X))
      WRITE(IW,60)
   60      FORMAT(/T10,'LCL = LSL + (V - K/SQRT(N))*STD',5X,
```

```
     *      'UCL = USL - (V - K/SQRT(N))*STD')
      WRITE(IW,70) APL,PMID,RPL
   70    FORMAT(/T10,'N',T16,'K',T23,'(EXACT SETUP)',T40,'(APL=',F5.3,
     *          ')',T54,'(MID=',F5.3,')',T68,'(RPL=',F5.3,')'/,T28,
     *          'PRO',T45,'PA1',T59,'PA2',T73,'PA3'/)
C------------------------------- N LOOP
      DO 120 NX=NXL,NXH
            RNX=FLOAT(NX)
         SQN=SQRT(RNX)
C------------------------------- K LOOP
        DO 110 J=1,NUMK
            RKX=AK(J)
                CLK=VX-RKX/SQN
              B1=CLK*SQN
              B2=-HALF*SQN+B1
            IF(MUSTD.EQ.1) CALL PAXB(1,POH, PAO)
            IF(MUSTD.EQ.2) CALL PAXB(2,PO,  PAO)
              PRO=1.-PAO
          CALL PAXB(MUSTD,APL, PA1)
          CALL PAXB(MUSTD,PMID,PA2)
          CALL PAXB(MUSTD,RPL, PA3)
            STAR=BLANK
C
C---------------------- LABEL QUILIFIED PLAN BY '**'
            IF(PA1.GE.TLAPL .AND. PA3.LE.TLRPL) STAR=STAR2
          WRITE(IW,96) STAR,NX,RKX, PRO,PA1,STAR, PA2, PA3,STAR
   96       FORMAT(T5,A2,1X,I3,3X,F5.2, T27,F7.4,T44,F6.3,1X,A2,T58,
     *              F6.3,T72,F6.3,1X,A2)
  110   CONTINUE
  120 WRITE(IW,121)
  121   FORMAT(' ')
  131 RETURN
      END
C
C
C
C++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE XSTEV
C
C *** THIS SUBROUTINE STATISTICALLY EVALUATES MODIFIED X-BAR CHART
C
      COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /S2/MUSTD, NNL,NNH, APL,TLAPL,RPL,TLRPL, NUMT,AT(10)
      COMMON /S4/ DELMU,STD10, SQN,B1,B2
      COMMON /S7/VX,NXL,NXH, NUMK,AK(10), NX,RKX
      COMMON /S8/ ASSCOZ(10,2),BLANK,STAR2,DELSTD(2)
      DIMENSION APP(27)
C
        HALF= USLLSL/2.
          RNX=FLOAT(NX)
        SQN=SQRT(RNX)
        CLK=VX-RKX/SQN
        B1=CLK*SQN
        B2=-HALF*SQN+B1
      CALL MDNOR(-HALF,POH)
C------------------------ SPECIFY FRACTION DEFECTIVE VALUES
      APP(1)=2.*POH
      DO 1 I=2,21
    1 APP(I)=(I-1)*.005
      DO 2 I=22,26
    2 APP(I)=(I-21)*.02+.1
      APP(27)=.40
C
C------------------- PRINT TITLE AND PARAMETER VALUES
C
      WRITE(IW,50) USLLSL,(ASSCOZ(I,MUSTD),I=1,10), NX,VX,RKX,CLK,CLK
   50 FORMAT(//  '   ***** STATISTICALLY BASED MODIFIED X-BAR CHART',
     * ' EVALUATION *****'//T5,'USLLSL=',F5.2,' (STD)',5X,10A4,/
     * T5,'N=',I2,4X,'V=',F5.2,4X,'K=',F6.3//T5,
     * '  LCL= LSL + (V-K/SQRT(N))*STD = LSL + ',F6.3,' STD',/T5,
     * '  UCL= USL - (V-K/SQRT(N))*STD = USL - ',F6.3,' STD'/)
        WRITE(IW,12) DELSTD(MUSTD)
```

```
   12     FORMAT( T7,'P',T15,A4,T27,'PA' )
          DO 20 I=1,27
C
C--------------- PROCESS BEFORE SHIFTING IS EVALUATED "EXACTLY".
C                 OTHERWISE, EVALUATED APPROXIMATELY
                IF(MUSTD.EQ.1.AND.I.EQ.1) GOTO 16
C
             CALL PAXB(MUSTD,APP(I),PA)
   13           IF(MUSTD.EQ.1) WRITE(IW,14) APP(I),DELMU,PA
   14                         FORMAT(T4,F7.4,2X,F7.3,4X,F7.3)
                IF(MUSTD.EQ.2) WRITE(IW,14) APP(I),STD10,PA
             GOTO 20
C---------------- PROCESS BEFORE SHIFTING
   16           CALL PAXB(1,POH,PA)
                GOTO 13
   20     CONTINUE
   32 RETURN
      END
C
C
C
C
C
C++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE PAFG (PACC)
C
C *** THIS SUBROUTINE CALCULATES THE PROBABILITY OF ACCEPTANCE (PACC)
C     FOR NLG FREQUENCY GAGING RULE
C
      COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /S3/ PG,PY,PR
      PSUM=0.
   20 DO 22 I=1,NY1
         IL1=I-1
         CALL BINOML(NN,IL1,PAC)
   22 PSUM=PSUM+PAC
      PACC=PSUM
      IF(NG.EQ.0) RETURN
      PSUM2=0.
      IN=NY1
      NNLNG=NN-NG
      IF(NY.GT.NNLNG) IN=NNLNG+1
      DO 24 I=1,IN
         IL1=I-1
         CALL BINOML(NNLNG,IL1,PAC)
   24 PSUM2=PSUM2+PAC
      PACC=PSUM+(1.-PSUM2)*(PG**NG)
      RETURN
      END
C
C
C
C++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE PAQL (PA)
C
C *** THIS SUBROUTINE CALCULATES THE PROBABILITY OF ACCEPTANCE (PA) FOR
C     NLG QUALIFICATION RULE
C
      COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      PSUM=0.
   20 DO 22 I=1,NY1
         IL1=I-1
         CALL BINOML(NN,IL1,PAC)
   22 PSUM=PSUM+PAC
      PA=PSUM
      RETURN
      END
C
C
C
C++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE BINOML (N,IX, PROB)
```

```
C
C *** THIS SUBROUTINE CALCULATES BINOMIAL PROBABILITY AND ITS SIMILARS
C
      COMMON /S3/ PG,PY,PR
      DOUBLE PRECISION DY,DG,DLGPB
      DY=PY
      DG=PG
      DLGPB=DLGAMA(N+1.DO)-DLGAMA(IX+1.DO)-DLGAMA(N-IX+1.DO)
     *       +IX*DLOG(DY)+(N-IX)*DLOG(DG)
      IF (DLGPB.LE.-180.DO) DLGPB=-180.DO
      PROB=DEXP(DLGPB)
      RETURN
      END
C
C
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE GYR(PP)
C
C *** THIS SUBROUTINE CALCULATES THE PROBABILITY OF GREEN, YELLOW AND
C     RED (PG,PY,PR) AS FUNCTIONS OF PROCESS FRACTION DEFECTIVE
C
C *** TWO IMSL SUBROUTINES ARE REQUIRED:
C
C     MDNOR(XIN,XOUT) -- MDNOR = THE CUMULATIVE PROBABILITY FUNCTION
C        (PHI) OF STANDARD NORMAL DISTRIBUTION.  XOUT= PHI(XIN).
C
C     MDNRIS(YIN,YOUT,IERR) -- MDNRIS = THE INVERSE FUNCTION OF MDNOR.
C        YIN= PHI(YOUT).  IERR= ERROR FLAG.
C
      COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /S2/MUSTD, NNL,NNH, APL,TLAPL,RPL,TLRPL, NUMT,AT(10)
      COMMON /S3/ PG,PY,PR
      COMMON /S6/ RY,DEL,STD10
C
      IF(MUSTD.EQ.2) GOTO 10
C----------------------------- MEAN SHIFT
      CALL MDNRIS(PP,RY,IERR)
      DEL=RY+HALF
      HTD1=HALF-TNLG+DEL
      HTD2=-HALF+TNLG+DEL
      CALL MDNOR(HTD1,PHI1)
      CALL MDNOR(HTD2,PHI2)
      PG=PHI1-PHI2
      GOTO 15
C------------------------- DISPERSION CHANGE
   10 PP2=PP/2.
      CALL MDNRIS(PP2,Q1,IERR)
      STD10=-HALF/Q1
      Q2=(HALF-TNLG)/STD10
      CALL MDNOR(Q2,Q3)
      PG=2.*(Q3-.5)
C
   15 GO TO (99,20,30),MM
   20 PY=1.-PG
      RETURN
   30 PR=PP
      PY=1.-PG-PR
   99 RETURN
      END
C
C
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE GYRC(CHANGE)
C
C *** THIS SUBROUTINE CALCULATES THE PROBABILITY OF GREEN, YELLOW AND
C     RED (PG, PY, PR) AS FUNCTIONS OF (1) DEGREE OF MEAN SHIFT, OR (2)
C     DEGREE OF DISPERSION CHANGE.
C
C ----- MUSTD=1  ==> CHANGE= DEL OF MU = DEGREE OF MEAN SHIFT
C ----- MUSTD=2  ==> CHANGE= RATIO OF STD = DEGREE OF DISPERSION CHANGE
```

```
C
      COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /S2/MUSTD, NNL,NNH, APL,TLAPL,RPL,TLRPL, NUMT,AT(10)
      COMMON /S3/ PG,PY,PR
      COMMON /S6/ RY,DEL,STD10
      IF(MUSTD.EQ.2) GOTO 10
C---------------------------- MEAN SHIFT
      HTD1=HALF-TNLG+CHANGE
      HTD2=-HALF+TNLG+CHANGE
      CALL MDNOR(HTD1,PHI1)
      CALL MDNOR(HTD2,PHI2)
      PG=PHI1-PHI2
      GOTO 15
C -------------------------- DISPERSION CHANGE
   10 Q2=(HALF-TNLG)/CHANGE
      CALL MDNOR(Q2,Q3)
      PG=2.*(Q3-.5)
C
   15 GO TO (99,20,30),MM
   20 PY=1.-PG
      RETURN
   30 PR=PP
      PY=1.-PG-PR
   99 RETURN
      END
C
C
C
C++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE EOFN(REN)
C
C *** THIS SUBROUTINE CALCULATES AVERAGE INSPECTION NUMBER (ALSO KNOWN
C     AS AVERAGE SAMPLE NUMBER)
C
      COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /S3/ PG,PY,PR
      DOUBLE PRECISION ABC,SABC,EN, G,Y,R,YGF,GC
      G=PG
      Y=PY
      IF(MM.EQ.3) R=PR
      ABC=0.D0
      SABC=0.D0
      EN=0.D0
      NNL1=NN-1
      IF(NN.GT.1) GO TO 10
C---------------------------- NN = 1 ----
      REN=1.
      RETURN
C---------------------------- NN > 1 ----
   10 GO TO (900,200,300,900,900),MM
C
C--------------------------------------------- MM=2 ----------------
  200 IF(NY.EQ.0) GO TO 201
      IF(NY.LT.NNL1) GO TO 221
      GO TO 251
C----------------- MM=2;  NY=0 (NG=0)  ---------
  201 IF(NG.GE.1) GO TO 212
      DO 210 I=1,NNL1
  210 EN=EN+ I*(G**(I-1))*Y
      REN=EN+NN*G**NNL1
      RETURN
C
  212 WRITE(IW,214)
  214 FORMAT(//,T2,10('-'),'  NLG ERROR:  M=2  Y=0   G>0;',
     *        '   EXECUTION INTERRUPTED IN SUBROUTINE EOFN (LABEL 212)')
      RETURN
C----------------- MM=2;  0<NY<(NN-1)  ---------
  221 IF(NG.EQ.0 .OR. NG.GT.NY) GO TO 225
      ABC=G**NG
      EN=EN+NG*ABC
      SABC=SABC+ABC
  225 DO 240 J=NY1,NNL1
```

```
         JL1=J-1
         IF(J.EQ.NG) GO TO 229
         ABC=YGF(JL1,NY,G,Y)
         EN=EN+J*ABC
         GO TO 240
  229    ABC=YGF(JL1,NY,G,Y)+G**NG
         EN=EN+J*ABC
  240 SABC=SABC+ABC
      REN=EN+ NN*(1.DO-SABC)
      RETURN
C-----------------  MM=2;  NY>O & NY>=(NN-1)  ---
  251 IF(NG.GE.1) GO TO 254
      REN=NN
      RETURN
  254 REN=NG*(G**NG)+NN*(1.DO-G**NG)
      RETURN
C
C------------------------------------------------ MM=3 ---------------
  300 IF(NY.EQ.O) GO TO 301
      IF(NY.LT.NNL1) GO TO 321
      GO TO 351
C------------------  MM=3;  NY=O (NG=O)  --------
  301 IF(NG.GE.1) GO TO 312
      GC=1.DO-G
      DO 310 I=1,NNL1
  310 EN=EN+I*(G**(I-1))*GC
      REN=EN+NN*G**NNL1
      RETURN
C
  312 WRITE(IW,314)
  314 FORMAT(//,T2,10('-'),'  NLG ERROR:  M=3  Y=O  G>O;',
     * '  EXECUTION INTERRUPTED IN SUBROUTINE EOFN (LABEL 312)')
      RETURN
C---------------------  MM=3;  O<NY< NN-1   ----
  321 DO 330 I=1,NY
         IF(I.EQ.NG) GO TO 329
         ABC=(1.DO-SABC)*R
         EN=EN+I*ABC
         GO TO 330
  329    ABC=(1.DO-SABC)*R+G**NG
         EN=EN+I*ABC
  330 SABC=SABC+ABC
      DO 340 J=NY1,NNL1
         JL1=J-1
         IF(J.EQ.NG) GO TO 339
         ABC=(1.DO-SABC)*R + YGF(JL1,NY, G,Y)
         EN=EN+J*ABC
         GO TO 340
  339    ABC=(1.DO-SABC)*R + YGF(JL1,NY, G,Y) + G**NG
         EN=EN+J*ABC
  340 SABC=SABC+ABC
      REN=EN+ NN*(1.DO-SABC)
      RETURN
C---------------------  MM=3;  NY>O & NY>=(NN-1)  --
  351 DO 360 I=1,NNL1
         IF(I.EQ.NG) GO TO 359
         ABC=(1.DO-SABC)*R
         EN=EN+I*ABC
         GO TO 360
  359    ABC=(1.DO-SABC)*R + G**NG
         EN=EN+I*ABC
  360 SABC=SABC+ABC
      REN=EN+NN*(1.DO-SABC)
      RETURN
C
  900 WRITE(IW,901) MM
  901 FORMAT(/// T3,10('-'),'ERROR:  IN SUBROUTINE EOFN, M=',I2,
     *  ' .NE. 2 OR 3;  EXECUTION INTERRUPTED (LABEL 900)')
      RETURN
      END
C
C
```

```
C
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
      FUNCTION YGF(N,K, G,Y)
C
C *** THIS FUNCTION SUBPROGRAM EVALUATES THE TERM ASSOCIATED WITH
C    BINOMIAL COEFFICENT IN THE CALCULATION OF AVERAGE INSPECTION NUMBER
C
      COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      DOUBLE PRECISION BINCOE, G,Y, YGF
      IF(K.GT.N) GO TO 90
      NLNG=N-NG
      IF(NG.EQ.O.OR.NLNG.LT.K) GO TO 10
C------------------------------- NG>O AND (N-NG)>=K --------------
      YGF=(BINCOE(N,K)-BINCOE(NLNG,K))*(Y**(K+1))*(G**(N-K))
      RETURN
C------------------------------- NG=O OR (N-NG)<K ----------------
   10 YGF=BINCOE(N,K)*(Y**(K+1))*(G**(N-K))
      RETURN
C
   90 WRITE (IW,91) K,N
   91 FORMAT(/// 10('-'),' NLG ERROR:  IN FUNCTION SUBPROGRAM YGF, K=',
     * I2, ' > N=',I2,';  EXECUTION INTERRUPTED (LABEL 90)')
      RETURN
      END
C
C
C
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
      FUNCTION BINCOE(N,K)
C
C *** THIS FUNCTION SUBPROGRAM EVALUATES BINOMIAL COEFFICIENT USED IN
C     FUNCTION SUBPROGRAM YGF
C
      DOUBLE PRECISION COEF,DNUM,BINCOE
      IF(K.EQ.O.OR.K.EQ.N) GO TO 20
      NL1=N-1
      IF(K.EQ.1.OR.K.EQ.NL1) GO TO 30
C--------------  1 < K < (N-1)  -----------
      COEF=1.DO
      HN=N/2.
      KK=K
      IF(K.GT.HN) KK=N-K
      DNUM=N
      DO 10 I=1,KK
         COEF=COEF*(DNUM/I)
   10 DNUM=DNUM-1.DO
      BINCOE=COEF
      RETURN
C--------------  K=O  OR  K=N  ------------
   20 BINCOE=1.
      RETURN
C--------------  K=1  OR  K=N-1  ----------
   30 BINCOE=N
      RETURN
      END
C
C
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE PAXB(I12,P, PA)
C
C *** THIS SUBROUTINE CALCULATES THE PROBABILITY OF ACCEPTANCE OF
C     MODIFIED X-BAR CHART, WHERE  I12=1  ==> MEAN SHIFT
C                                  I12=2  ==> DISPERSION CHANGE
C
      COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /S4/ DELMU,STD10, SQN,B1,B2
      COMMON /S7/VX,NXL,NXH, NUMK,AK(10), NX,RKX
      IF(I12.EQ.2) GOTO 20
C---------------------------------------- MEAN SHIFT
      CALL MDNRIS(P,XP, IERR)
      DELMU=XP+HALF
```

```
         A=(DELMU+HALF)*SQN-B1
         B=XP*SQN+B1
      CALL MDNOR(A,PHIA)
      CALL MDNOR(B,PHIB)
      PA=PHIA-PHIB
      RETURN
C------------------------------------- DISPERSION CHANGE
   20    PH=P/2.
      CALL MDNRIS (PH,XPH, IERR)
      STD10= -HALF/XPH
      .  C=B2/STD10
      CALL MDNOR(C,PHIC)
      PA=1.-2.*PHIC
      RETURN
      END
C
C
C
C
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE ECON
C
C *** THIS SUBROUTINE SERVES AS THE PROMPTER PROGRAM AND DRIVES THE
C     FOLLOWING SIX SUBROUTINES FOR THE ECONOMICALLY BASED PROCESS
C     CONTROL SCHEMES
C
C        NECOPT -- ECON NLG OPTIMIZATION (DESIGN)
C        NECEV  -- ECON NLG EVALUATION
C        NCOSF  -- ECON NLG LOSS-COST SURFACE INVESTIGATION
C        XECOPT -- ECON X-BAR CHART OPTIMIZATION (DESIGN)
C        XECEV  -- ECON X-BAR CHART EVALUATION
C        XCOSF  -- ECON X-BAR CHART LOSS-COST SURFACE INVESTIGATION
C
      COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /E2/ PG,PY,PR, PR1,PR2
      COMMON /E3/ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
      COMMON /E4/ XSTART(2),X(3,2),Y(3),        ITRFLG,IRESET,
     *            STDX,STDY,KPP, NVAR,N1,YACC,XACC,STEP,ITRMAX,NLGXB
      COMMON /E5/ NYBACK,NGBACK,YIMPRV, NNMIN,NNMAX
      COMMON /E6/ HNLG,HX,RKX
      COMMON /E7/NH,AH(30), NT,AT(11), NK,AK(11)
C
C-------------- SELECTION FOR ECON NLG OR ECON X-BAR CHART -------------
    5 WRITE(IW,10)
   10    FORMAT(  ' *** ENTER OPTION NUMBER'/
     *   T8,'1 = ECONOMICALLY BASED NLG (MEAN SHIFT ASSUMED)'/
     *   T8,'2 = ECONOMICALLY BASED X-BAR CHART (MEAN SHIFT ASSUMED)'/
     *   T8,'3 = SWITCH TO STATISTICALLY BASED SCHEME'/
     *   T8,'4 = EXIT SYSTEM')
      READ(IR,*) N123
      GOTO (100,200,250,300),N123
      WRITE(IW,20)
   20    FORMAT(' !! ERROR !! OUT OF RANGE !! DO IT OVER AGAIN')
      GOTO 5
C
C--------------------------- ECON NLG OPTION MENU ----------------------
  100 WRITE(IW,101)
  101    FORMAT(' *** FOR ECON NLG, ENTER VALUES:'/
     *   T5,' USLLSL, MM;  DELTA, LAMBDA, M, E, D, T, W, B, C')
         READ(IR,*) USLLSL,MM, ZDEL,ZLAM,ZM,ZE,ZD,ZT,ZW,ZB,ZC
      WRITE(IW,102) USLLSL,MM, ZDEL,ZLAM,ZM,ZE,ZD,ZT,ZW,ZB,ZC
  102    FORMAT(' VALUES ENTERED:   USLLSL=',F5.2,4X,'MM=',I1/
     *   '  DELTA=',F7.2,3X,'LAMBDA=',F7.2,3X,'M=',F7.2,3X,'E=',
     *  F7.2,3X,'D=',F7.2/T7,'T=',F7.2,T24,'W=',F7.2,T36,'B=',F7.2,T48,
     *  'C=',F7.2)
  103 WRITE(IW,104)
  104    FORMAT(' CORRECT ?   1=YES   2=NO   3=RETURN')
         READ(IR,*) IYN
         GOTO (105,100,5),IYN
```

```
         WRITE(IW,20)
         GOTO 103
C
C------------- CALCULATES FRACTION DEFECTIVES:  PR1, PR2 --------------
C                 PR1= BEFORE SHIFTING;    PR2= AFTER  SHIFTING
  105    HALF=.5*USLLSL
         CALL MDNOR(-HALF,PRHALF)
      PR1=2.*PRHALF
            H2L=-HALF+ZDEL
            H2R=HALF+ZDEL
         CALL MDNOR (H2L,PH2L)
         CALL MDNOR (H2R,PH2R)
      PR2=PH2L+(1.-PH2R)
C
  106 WRITE(IW,107)
  107    FORMAT(    ' *** ENTER OPTION NUMBER'/
     *   T6,'1= ECON NLG DESIGN (OPTIMIZATION)'/
     *   T6,'2= ECON NLG EVALUATION'/
     *   T6,'3= ECON NLG LOSS-COST SURFACE INVESTIGATION'/
     *   T6,'4= SWITCH TO ECON X-BAR CHART'/
     *T6,'5= RETURN TO REVISE USLLSL, MM, AND TIME AND COST PARAMETERS'/
     *   T6,'6= EXIT SYSTEM')
         READ(IR,*) N16
         GOTO (110,120,130,200,100,300),N16
            WRITE(IW,20)
            GOTO 106
C
C-------------------------- ECON NLG DESIGN (OPTIMIZATION ) -----------
C
C-- INITIALIZATION OF DEFAULT VALUES FOR OPTIMIZATION PARAMETERS
  110       YACC=.003
            XACC=.002
            STEP=1.
            ITRMAX=60
            XSTART(1)=1.
            XSTART(2)=1.
            IRESET=1
            NYBACK=2
            NGBACK=3
            YIMPRV=0.
      WRITE(IW,111)
  111    FORMAT(' *** FOR ECON NLG DESIGN, ENTER VALUES:  NMIN, NMAX')
         READ(IR,*) NNMIN,NNMAX
 1111 WRITE(IW,112) NNMIN,NNMAX, YACC,XACC,STEP,ITRMAX,
     *              (XSTART(I),I=1,2),IRESET, NYBACK,NGBACK,YIMPRV
  112    FORMAT(' VALUES ENTERED:  NMIN=',I2,4X,'NMAX=',I2//
     *   ' PARAMETER VALUES FOR:',T30,'(H,T) OPTIMIZATION',T61,
     *   'OVERALL OPTIMIZATION'/T15,'YACC     XACC    STEP ITRMAX    HO',
     *   T51,'TO IRESET',T63,'EY  EG      EL'/T4,'DEFAULT:',T15,
     *   '0.003  0.002',T30,'1.00    60    1.000  1.000  1',T64,
     *   '2    3     0.0'/T4,'CURRENT: ',2(1X,F6.3),1X,F6.2,1X,I4,1X,F7.3,
     *   1X,F6.3,2X,I1,T63,2(I2,2X),F6.2)
  113 WRITE(IW,114)
  114    FORMAT(/' *** ENTER OPTION NUMBER:'/
     *   ' 1= ALL OK, NO REVISION NEEDED'/
     *   ' 2= NEED TO REVISE (NMIN,NMAX) VALUES'/
     *   ' 3= NEED TO REVISE (H,T) OPTIMIZATION PARAMETER VALUES'/
     *   ' 4= NEED TO REVISE OVERALL OPTIMIZATION PARAMETER VALUES'/
     *   ' 5= RETURN FOR OTHER ECON NLG OPTIONS')
         READ(IR,*) N15
         GOTO (2119,115,117,119,106),N15
         WRITE(IW,20)
         GOTO 113
  115 WRITE(IW,116)
  116    FORMAT(' ENTER VALUES:  NMIN,NMAX')
         READ(IR,*) NNMIN,NNMAX
         GOTO 1111
  117 WRITE(IW,118)
  118    FORMAT(' ENTER VALUES:  YACC,XACC,STEP,ITRMAX,HO,TO,IRESET')
         READ(IR,*) YACC,XACC,STEP,ITRMAX,(XSTART(I),I=1,2),IRESET
         GOTO 1111
  119 WRITE(IW,1119)
```

```
 1119     FORMAT(' ENTER VALUES:  EY,EG,EL')
          READ(IR,*) NYBACK,NGBACK,YIMPRV
          GOTO 1111
 2119 CALL NECOPT
          GOTO 106
C
C-------------------------- ECON NLG EVALUATION ------------------------
  120 WRITE(IW,121)
  121     FORMAT(' FOR ECON NLG EVALUATION, ENTER VALUES:  N,Y,G,H,T')
          READ(IR,*) NN,NY,NG, HNLG,TNLG
      WRITE(IW,122) NN,NY,NG, HNLG,TNLG
  122     FORMAT(' VALUES ENTERED:  N=',I2,2X,'Y=',I2,2X,'G=',I2,4X,
     *    'H=',F8.3,4X,'T=',F6.3)
  123 WRITE(IW,124)
  124     FORMAT(' CORRECT ?   1=YES   2=NO   3= RETURN FOR OTHER',
     *    ' ECON NLG OPTIONS')
          READ(IR,*) IYN
          GOTO (126,120,106),IYN
          WRITE(IW,20)
          GOTO 123
  126 CALL NECEV
          GOTO 106
C
C--------------------- ECON NLG COST SURFACE INVESTIGATION -------------
  130 WRITE(IW,131)
  131     FORMAT(' *** FOR ECON NLG COST SURFACE INVESTIGATION, ENTER',
     *    ' VALUES:  N,Y,G')
          READ(IR,*) NN,NY,NG
      WRITE(IW,132)
  132     FORMAT(' ENTER VALUES:'/' NUMH (NUMBER OF H; <= 30), FOLLOWED',
     *    ' BY ALL H VALUES TO BE INVESTIGATED')
          READ(IR,*) NH,(AH(I),I=1,NH)
      WRITE(IW,133)
  133     FORMAT(' ENTER VALUES:'/' NUMT (NUMBER OF T; <= 11), FOLLOWED',
     *    ' BY ALL T VALUES TO BE INVESTIGATED')
          READ(IR,*) NT,(AT(I),I=1,NT)
 1133 WRITE(IW,134) NN,NY,NG, NH,(AH(I),I=1,NH)
  134     FORMAT(' VALUES ENTERED:  N=',I2,4X,'Y=',I2,4X,'G=',I2/
     *    T2,I2,' H VALUES = ',6(F8.3,1X)/4(T16,6(F8.3,1X)))
      WRITE(IW,135) NT,(AT(I),I=1,NT)
  135     FORMAT(T2,I2,' T VALUES = ',6(F6.3,3X)/T16,5(F6.3,3X))
 1135 WRITE(IW,136)
  136     FORMAT(/' *** ENTER OPTION NUMBER:'/
     *    ' 1= ALL OK, NO REVISION NEEDED'/
     *    ' 2= NEED TO REVISE (N,Y,G) VALUES'/
     *    ' 3= NEED TO REVISE NUMH AND H VALUES'/
     *    ' 4= NEED TO REVISE NUMT AND T VALUES'/
     *    ' 5= RETURN FOR OTHER ECON NLG OPTIONS')
          READ(IR,*) N15
          GOTO (143,137,139,141,106),N15
          WRITE(IW,20)
          GOTO 1135
C
  137 WRITE(IW,138)
  138     FORMAT(' ENTER VALUES:  N,Y,G')
          READ(IR,*) NN,NY,NG
          GOTO 1133
  139 WRITE(IW,140)
  140     FORMAT(' ENTER VALUES:  NUMH AND H VALUES')
          READ(IR,*) NH,(AH(I),I=1,NH)
          GOTO 1133
  141 WRITE(IW,142)
  142     FORMAT(' ENTER VALUES:  NUMT AND T VALUES')
          READ(IR,*) NT,(AT(I),I=1,NT)
          GOTO 1133
C
  143 CALL NCOSF
          GOTO 106
C
C--------------------------- ECON X-BAR OPTION MENU --------------------
  200 WRITE(IW,201)
  201     FORMAT(' *** FOR ECON X-BAR CHART, ENTER VALUES:'/
```

```
     *    T5,' USLLSL,  DELTA, LAMBDA, M, E, D, T, W, B, C')
          READ(IR,*) USLLSL, ZDEL,ZLAM,ZM,ZE,ZD,ZT,ZW,ZB,ZC
      WRITE(IW,202) USLLSL, ZDEL,ZLAM,ZM,ZE,ZD,ZT,ZW,ZB,ZC
  202     FORMAT(' VALUES ENTERED:   USLLSL=',F5.2/
     *    '  DELTA=',F7.2,3X,'LAMBDA=',F7.2,3X,'M=',F7.2,3X,'E=',
     *   F7.2,3X,'D=',F7.2/T7,'T=',F7.2,T24,'W=',F7.2,T36,'B=',F7.2,T48,
     *   'C=',F7.2)
  203 WRITE(IW,104)
          READ(IR,*) IYN
          GOTO (206,200,5),IYN
          WRITE(IW,20)
          GOTO 203
  206 WRITE(IW,207)
  207     FORMAT(    ' *** ENTER OPTION NUMBER'/
     *    T6,'1= ECON X-BAR CHART DESIGN (OPTIMIZATION)'/
     *    T6,'2= ECON X-BAR CHART EVALUATION'/
     *    T6,'3= ECON X-BAR CHART LOSS-COST SURFACE INVESTIGATION'/
     *    T6,'4= SWITCH TO ECON NLG'/
     *    T6,'5= RETURN TO REVISE USLLSL, AND TIME AND COST PARAMETERS'/
     *    T6,'6= EXIT SYSTEM')
          READ(IR,*) N16
          GOTO (210,220,230,100,200,300),N16
             WRITE(IW,20)
             GOTO 206
C
C------------------------ ECON X-BAR CHART DESIGN (OPTIMIZATION) ------
C
C-- INITIALIZATION OF DEFAULT VALUES FOR OPTIMIZATION PARAMETERS
  210        YACC=.003
             XACC=.002
             STEP=1.
             ITRMAX=60
             XSTART(1)=1.
             XSTART(2)=1.
             IRESET=1
             NYBACK=2
             NGBACK=3
             YIMPRV=0.
      WRITE(IW,211)
  211     FORMAT(' *** FOR ECON X-BAR CHART DESIGN, ENTER VALUES:  ',
     *    'NMIN,NMAX')
          READ(IR,*) NNMIN,NNMAX
 1211 WRITE(IW,212) NNMIN,NNMAX, YACC,XACC,STEP,ITRMAX,
     *               (XSTART(I),I=1,2),IRESET, YIMPRV
  212     FORMAT(' VALUES ENTERED:  NMIN=',I2,4X,'NMAX=',I2//
     *    ' PARAMETER VALUES FOR:',T30,'(H,T) OPTIMIZATION',T61,
     *    'OVERALL OPTIMIZATION'/T15,'YACC    XACC     STEP ITRMAX    HO',
     *    T51,'TO IRESET',T68,'EL'/T4,'DEFAULT:',T15,
     *    '0.003  0.002',T30,'1.00    60    1.000  1.000   1',T67,
     *    '0.0'/T4,'CURRENT: ',2(1X,F6.3),1X,F6.2,1X,I4,1X,F7.3,
     *    1X,F6.3,2X,I1,T65,F6.2)
  213 WRITE(IW,214)
  214     FORMAT(/' *** ENTER OPTION NUMBER:'/
     *    ' 1= ALL OK, NO REVISION NEEDED'/
     *    ' 2= NEED TO REVISE (NMIN,NMAX) VALUES'/
     *    ' 3= NEED TO REVISE (H,T) OPTIMIZATION PARAMETER VALUES'/
     *    ' 4= NEED TO REVISE OVERALL OPTIMIZATION PARAMETER VALUE'/
     *    ' 5= RETURN FOR OTHER ECON X-BAR CHART OPTIONS')
          READ(IR,*) N15
          GOTO (2219,215,217,219,206),N15
          WRITE(IW,20)
          GOTO 213
  215 WRITE(IW,116)
          READ(IR,*) NNMIN,NNMAX
          GOTO 1211
  217 WRITE(IW,118)
          READ(IR,*) YACC,XACC,STEP,ITRMAX,(XSTART(I),I=1,2),IRESET
          GOTO 1211
  219 WRITE(IW,1219)
 1219     FORMAT(' ENTER VALUE: EL')
          READ(IR,*) YIMPRV
          GOTO 1211
```

```
 2219 CALL XECOPT
         GOTO 206
C
C----------------------- ECON X-BAR CHART EVALUATION ------------------
  220 WRITE(IW,221)
  221    FORMAT(' FOR ECON X-BAR CHART EVALUATION, ENTER VALUES:',
     *   '   N,H,K')
         READ(IR,*) NN, HX,RKX
      WRITE(IW,222) NN, HX,RKX
  222    FORMAT(' VALUES ENTERED:  N=',I2,4X,'H=',F8.3,4X,'K=',F6.3)
  223 WRITE(IW,224)
  224    FORMAT(' CORRECT ?   1=YES   2=NO   3= RETURN FOR OTHER',
     *   ' ECON X-BAR CHART OPTIONS')
         READ(IR,*) IYN
         GOTO (226,220,206),IYN
         WRITE(IW,20)
         GOTO 223
  226 CALL XECEV
         GOTO 206
C
C---------------- ECON X-BAR CHART COST SURFACE INVESTIGATION ----------
  230 WRITE(IW,231)
  231    FORMAT(' *** FOR ECON X-BAR CHART COST SURFACE INVESTIGATION,',
     *   ' ENTER VALUE:  N')
         READ(IR,*) NN
      WRITE(IW,132)
         READ(IR,*) NH,(AH(I),I=1,NH)
      WRITE(IW,233)
  233    FORMAT(' ENTER VALUES:'/' NUMK (NUMBER OF K; <= 11), FOLLOWED',
     *   ' BY ALL K VALUES TO BE INVESTIGATED')
         READ(IR,*) NK,(AK(I),I=1,NK)
 1233 WRITE(IW,234) NN, NH,(AH(I),I=1,NH)
  234    FORMAT(' VALUES ENTERED:  N=',I2/
     *   T2,I2,' H VALUES = ',6(F8.3,1X)/4(T16,6(F8.3,1X)))
      WRITE(IW,235) NK,(AK(I),I=1,NK)
  235    FORMAT(T2,I2,' K VALUES = ',6(F6.3,3X)/T16,5(F6.3,3X))
 1235 WRITE(IW,236)
  236    FORMAT(/' *** ENTER OPTION NUMBER:'/
     *   ' 1= ALL OK, NO REVISION NEEDED'/
     *   ' 2= NEED TO REVISE N VALUE'/
     *   ' 3= NEED TO REVISE NUMH AND H VALUES'/
     *   ' 4= NEED TO REVISE NUMK AND K VALUES'/
     *   ' 5= RETURN FOR OTHER ECON X-BAR CHART OPTIONS')
         READ(IR,*) N15
         GOTO (243,237,239,241,206),N15
         WRITE(IW,20)
         GOTO 1235
  237 WRITE(IW,238)
  238    FORMAT(' ENTER VALUE:  N')
         READ(IR,*) NN
         GOTO 1233
  239 WRITE(IW,140)
         READ(IR,*) NH,(AH(I),I=1,NH)
         GOTO 1233
  241 WRITE(IW,242)
  242    FORMAT(' ENTER VALUES:  NUMK AND K VALUES')
         READ(IR,*) NK,(AK(I),I=1,NK)
         GOTO 1233
  243 CALL XCOSF
         GOTO 206
C
  250 RETURN
  300 STOP
      END
C
C
C
C
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE NECOPT
C
```

```
C *** THIS SUBROUTINE ECONOMICALLY OPTIMIZE NLG MODEL
C
      COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /E2/ PG,PY,PR, PR1,PR2
      COMMON /E3/ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
      COMMON /E4/ XSTART(2),X(3,2),Y(3),        ITRFLG,IRESET,
     *            STDX,STDY,KPP, NVAR,N1,YACC,XACC,STEP,ITRMAX,NLGXB
      COMMON /E5/ NYBACK,NGBACK,YIMPRV, NNMIN,NNMAX
      DATA STAR2/'**'/, BLANK/' '/
         N1=3
         NLGXB=1
C--------------- PRINT TITLE AND PARAMETER VALUES ---------------------
C
      WRITE(IW,11) USLLSL,MM
   11    FORMAT(/  ' ***** ECONOMICALLY BASED NLG DESIGN *****'//
     *   ' USLLSL=',F6.2,4X,'MM=',I1,6X,'MEAN SHIFT ASSUMED')
      WRITE (IW,113) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
  113   FORMAT(  '  DELTA=',F7.2,3X,'LAMBDA=',F7.2,3X,'M=',F7.2,3X,'E=',
     *  F7.2,3X,'D=',F7.2/T7,'T=',F7.2,T24,'W=',F7.2,T36,'B=',F7.2,T48,
     *  'C=',F7.2 )
      WRITE(IW,12) YACC,XACC,STEP,ITRMAX,(XSTART(I),I=1,2),IRESET
   12    FORMAT(/' (H,T) OPTIMIZATION:  YACC=',F7.3,3X,'XACC=',F7.3,3X,
     *   'STEP=',F7.3,3X,'ITRMAX=',I3/T23,'STARTING POINT:  HO=',
     *   F7.3,T53,'TO=',F7.3,T66,'IRESET=',I1)
      WRITE(IW,14) NYBACK,NGBACK,YIMPRV,NNMIN,NNMAX
   14    FORMAT(' OVERALL OPTIMIZATION:  EY=',I1,3X,'EG=',I1,3X,'EL=',
     *   F8.3,T56,'NMIN=',I2,3X,'NMAX=',I2)
      WRITE(IW,13)
   13 FORMAT(// T4,'N  MM  Y  G',T23,'H',T33,'T',T41,'100L',T52,'STDY',
     *  T62,'STDX',T69,'TITR  MAXITR'/)
C
C------------------------------NN,MM,NY INCREMENT --------------------
C--------YMN=YMIN AMONG ALL NN,  YMY=YMIN AMONG ALL NY,
C--------YMG=YMIN AMONG ALL NG
         NYMIN=0
         NGMIN=1
         YMN=100000000.
C
C ------------------------------------- N LOOP
      DO 200 NN=NNMIN,NNMAX
         NN1=NN+1
         J3U=NN
         IF(MM.EQ.3) J3U=NN1
         YMY=10000000.
C------------- DINAMICALLY DETERMINE THE STARTING VALUE OF Y
            NYMIN2=NYMIN-NYBACK+1
            J3L=MAX0(1,NYMIN2)
            IF(NYMIN.EQ.0) J3L=1
C
C ------------------------------------- Y LOOP
         DO 170 J3=J3L,J3U
            NY=J3-1
            NY1=NY+1
            NGJU=NN-NY
            IF(MM.EQ.3) NGJU=NN
            NYFLG=0
            YMG=1000000.
            IYMGF=0
C---------------- DINAMICALLY DETERMINE THE STARTING VALUE OF G
               NGMIN2=NGMIN-NGBACK
               NGJL=MAX0(1,NGMIN2)
C
C ------------------------------------- G LOOP
            DO 160 NGJ=NGJL,NGJU
               NG=NGJ
                  IF(NYFLG.EQ.1.OR. IYMGF.EQ.1) GO TO 161
                  IF(NY.EQ.0) GO TO 155
C
C----- (H,T) OPTIMIZATION USING DIRECT SEARCH TECHNIQUE
  152          CALL HTOPT
                  IF(IRESET.EQ.0) GOTO 159
C----- CHECK TO SEE IF THE LOSS-COST L IS BIG ENOUGH TO QUIT G LOOP
```

```
  154             IF(Y(N1).GT.(YMG+YIMPRV)) GO TO 158
 1153             IF(Y(N1).GT. YMG        ) GO TO 153
               NGMIN=NG
               YMG=Y(N1)
  153          STAR=BLANK
               IF(ITRFLG.EQ.1) STAR=STAR2
               WRITE(IW,20) NN,MM,NY,NG,(X(N1,J),J=1,NVAR),Y(N1),
     *                   STDY,STDX,KPP,STAR
   20             FORMAT(T2,4I3,T17,3F10.3,2F10.4,I6,2X,A2)
               GO TO 160
  155             NG=0
                  NYFLG=1
                  GO TO 152
  158          IYMGF=1
               GO TO 1153
C--- ADOPT THE OPTIMAL POINT AS THE STARTING POINT FOR NEXT OPTIMIZATION
  159             DO 1159 JJ=1,NVAR
 1159             XSTART(JJ)=X(N1,JJ)
               GOTO 154
  160       CONTINUE
  161       WRITE(IW,163)
  163          FORMAT('+',T2,77('_'))
C----- CHECK TO SEE IF THE LOSS-COST L IS BIG ENOUGH TO QUIT Y LOOP
               IF(YMG.GT.(YMY+YIMPRV)) GO TO 171
               IF(YMG.GT. YMY        ) GO TO 170
            NYMIN=NY
            YMY=YMG
  170    CONTINUE
  171    WRITE(IW,172) NN,YMY
  172       FORMAT(     T48,'FOR N=',I3,':  MIN 100L =',F10.3)
         WRITE(IW,162)
  162       FORMAT('0')
C----- CHECK TO SEE IF THE LOSS-COST L IS BIG ENOUGH TO QUIT N LOOP
            IF(YMY.GT.(YMN+YIMPRV)) GO TO 201
            IF(YMY.GT. YMN        ) GO TO 200
         YMN=YMY
  200 CONTINUE
  201 WRITE(IW,202) YMN
  202 FORMAT(/ T15,32('*'),3X,'OVERALL OPTIMAL 100L =',F10.3)
  888 RETURN
      END
C
C
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE NECEV
C
C *** THIS SUBROUTINE ECONOMICALLY EVALUATES A NLG PLAN
C
      COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /E2/ PG,PY,PR, PR1,PR2
      COMMON /E3/ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
      COMMON /E6/ HNLG,HX,RKX
      DIMENSION AHT(2)
C----------------------------------- EVALUATION ----------------------
         NY1=NY+1
         AHT(1)=HNLG
         AHT(2)=TNLG
C        -------------
      ZL100=VYNLG(AHT)
C        -------------
      ZL=ZL100/100.
C----------------------------------- OUTPUT SECTION ------------------
      WRITE (IW,9) USLLSL,MM
    9    FORMAT( / T2,' ***** ECONOMICALLY BASED NLG EVALUATION *****'//
     *   '  USLLSL=',F6.2,' (STD)',4X,'MM=',I1,5X,'MEAN SHIFT ASSUMED')
      WRITE (IW,113) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
  113   FORMAT(/ '  DELTA=',F7.2,3X,'LAMBDA=',F7.2,3X,'M=',F7.2,3X,'E=',
     *  F7.2,3X,'D=',F7.2/T7,'T=',F7.2,T24,'W=',F7.2,T36,'B=',F7.2,T48,
     *  'C=',F7.2)
      WRITE(IW,114) NN,NY,NG,HNLG,TNLG
  114    FORMAT(/T3,'N=',I3,4X,'Y=',I3,4X,'G=',I3,10X,
```

```
     *   'H=',F8.3,7X,'T=',F8.3)
      WRITE (IW,115) ZL100,ZL
  115    FORMAT(//    '  LOSS-COST PER 100 HOURS =',F10.3,2X,
     *   '(HOURLY LOSS-COST =',F10.3,')')
   99 RETURN
      END
C
C
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE NCOSF
C
C *** THIS SUBROUTINE INVESTIGATES THE LOSS-COST SURFACE OF A NLG PLAN
C
      COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /E2/ PG,PY,PR, PR1,PR2
      COMMON /E3/ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
      COMMON /E7/NH,AH(30), NT,AT(11), NK,AK(11)
      DIMENSION ACOST(30,11),AALFAP(2,11),LABEL(2),AASN(2,11)
      DATA LABEL/'ALFA','P   '/
         NN1=NN+1
         NY1=NY+1
C------------------------- LOSS-COST SURFACE EVALUATION -------------
      DO 20 I=1,NT
            TNLG=AT(I)
            CALL GYRMU(O.)
         CALL PAFG2(ZALFA)
         CALL EOFN2(ZNIC)
            CALL GYRMU(ZDEL)
         CALL PAFG2(ZP)
         CALL EOFN2(ZNOOC)
            IF(ZP.LT. .0000001) ZP=.0000001
C
         AALFAP(1,I)=ZALFA
         AALFAP(2,I)=ZP
         AASN(1,I)=ZNIC
         AASN(2,I)=ZNOOC
C
         DO 10 J=1,NH
                  ZH=AH(J)
               ZBB=(1./ZP-.5+ ZLAM*ZH/12.)*ZH + ZE*ZNOOC + ZD
               ZBETA=1./(1.+ZLAM*ZBB)
            ZNAVE=ZBETA*ZNIC+(1.-ZBETA)*ZNOOC
            VY=(ZLAM*ZM*ZBB + ZALFA*ZT/ZH +ZLAM*ZW)*ZBETA
     *                     + (ZB+ZC*ZNAVE)/ZH
            ACOST(J,I)=VY*100.
   10    CONTINUE
   20 CONTINUE
C------------------------------------ LOCATE MINUM COST -------------
         AMIN=9999999.
         IX=0
         JX=0
      DO 50 I=1,NH
         DO 40 J=1,NT
               IF (ACOST(I,J).GE.AMIN) GO TO 40
            AMIN=ACOST(I,J)
            IX=I
            JX=J
   40    CONTINUE
   50 CONTINUE
C----------------------------------- OUTPUT SECTION ------------------
      WRITE (IW,9)
    9    FORMAT('1',T5,5('*'),'  ECONOMICALLY BASED NLG LOSS-COST ',
     *   'SURFACE INVESTIGATION   ',5('*'))
      WRITE(IW,112) USLLSL,MM,NN,NY,NG
  112    FORMAT ( /T3,'USLLSL=',F6.2,' STD',5X,'MM=',I1,5X,'MEAN SHIFT',
     *   ' ASSUMED',10X,'N=',I3,4X,'Y=',I3,4X,'G=',I3)
      WRITE (IW,111) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
  111   FORMAT(  '  DELTA=',F7.2,3X,'LAMBDA=',F7.2,3X,'M=',F7.2,3X,'E=',
     *  F7.2,3X,'D=',F7.2,3X,'T=',F7.2,3X,'W=',F7.2,3X,'B=',F7.2,3X,'C='
     *  ,F7.2)
      WRITE (IW,114) (AT(I),I=1,NT)
```

```
 114      FORMAT ( //,T5,'T',T10,11F11.3/)
       DO 30 I=1,2
  30   WRITE (IW,115) LABEL(I),(AALFAP(I,J),J=1,NT)
 115      FORMAT (  T5,A4,T10,11F11.3)
       WRITE (IW,121) (AASN(1,I),I=1,NT)
  121     FORMAT(T4,'EN IC',T10,11F11.3)
       WRITE (IW,122) (AASN(2,I),I=1,NT)
  122     FORMAT(T4,'EN OOC',T10,11F11.3)
       WRITE (IW,117)
 117      FORMAT (  T2,129('-')/T7,'H')
       DO 35 I=1,NH
  35   WRITE (IW,116) AH(I), (ACOST(I,J),J=1,NT)
 116      FORMAT (/,T3,F7.3,T10,11F11.3)
       WRITE (IW,118) AH(IX),AT(JX), AMIN
 118      FORMAT (//,T3,7('*'),'  MINIMUM:  H=',F7.3,'    T=',F8.3,
     *             '  .   LOSS-COST=',F11.3,2X,'(PER 100 HOURS)')
   99  RETURN
       END
C
C
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
       SUBROUTINE XECOPT
C
C *** THIS SUBROUTINE ECONOMICALLY OPTIMIZE X-BAR CHART MODEL
C
       COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
       COMMON /E3/ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
       COMMON /E4/ XSTART(2),X(3,2),Y(3),        ITRFLG,IRESET,
     *             STDX,STDY,KPP, NVAR,N1,YACC,XACC,STEP,ITRMAX,NLGXB
       COMMON /E5/ NYBACK,NGBACK,YIMPRV, NNMIN,NNMAX
       DATA STAR2/'**'/, BLANK/' '/
          N1=3
          NLGXB=2
C----------------------- PRINT TITLE AND PARAMETER VALUES --------------
C
       WRITE(IW,11) USLLSL
   11   FORMAT(/' ***** ECONOMICALLY BASED X-BAR CHART DESIGN *****'//
     *    ' USLLSL=',F6.2,6X,'MEAN SHIFT ASSUMED')
       WRITE (IW,113) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
 113     FORMAT(   '  DELTA=',F7.2,3X,'LAMBDA=',F7.2,3X,'M=',F7.2,3X,'E=',
     *   F7.2,3X,'D=',F7.2/T7,'T=',F7.2,T24,'W=',F7.2,T36,'B=',F7.2,T48,
     *   'C=',F7.2 )
       WRITE(IW,12) YACC,XACC,STEP,ITRMAX,(XSTART(I),I=1,2),IRESET
   12     FORMAT(/' (H,T) OPTIMIZATION:  YACC=',F7.3,3X,'XACC=',F7.3,3X,
     *    'STEP=',F7.3,3X,'ITRMAX=',I3/T23,'STARTING POINT:  HO=',
     *    F7.3,T53,'TO=',F7.3,T66,'IRESET=',I1)
       WRITE(IW,14) YIMPRV,NNMIN,NNMAX
   14     FORMAT(' OVERALL OPTIMIZATION:  EL=',
     *    F8.3,T56,'NMIN=',I2,3X,'NMAX=',I2)
       WRITE(IW,13)
   13 FORMAT(// T4,'N'          ,T23,'H',T33,'K',T41,'100L',T52,'STDY',
     *   T62,'STDX',T69,'TITR  MAXITR'/)
C
C--------------------NN INCREMENT (YMN=YMIN AMONG ALL NN) -------------
          YMN=100000000.
          IOPTF=0
       DO 200 NN=NNMIN,NNMAX
          NN1=NN+1
                IF(IOPTF.EQ.1) GOTO 201
C---- (H,T) OPTIMIZATION USING DIRECT SEARCH TECHNIQUE
          CALL HTOPT
             IF(IRESET.EQ.0) GOTO 159
C----- CHECK TO SEE IF THE LOSS-COST L IS BIG ENOUGH TO QUIT LOOP
  154        IF(Y(N1).GT.(YMN+YIMPRV)) GO TO 170
  153        IF(Y(N1).GT.YMN) GO TO 155
          YMN=Y(N1)
  155        STAR=BLANK
          IF(ITRFLG.EQ.1) STAR=STAR2
          WRITE(IW,156)NN,(X(N1,J),J=1,2),Y(N1),STDY,STDX,KPP,STAR
  156        FORMAT(T2, I3,T17,3F10.3,2F10.4,I6,2X,A2/' ')
          GO TO 200
```

```
  159       DO 160 JJ=1,NVAR
  160       XSTART(JJ)=X(N1,JJ)
            GOTO 154
  170          IOPTF=1
               GOTO 153
  200 CONTINUE
  201 WRITE(IW,202) YMN
  202 FORMAT(/T11,32('*'),3X,'OVERALL OPTIMAL 100L =',F10.3)
  888 RETURN
 999  WRITE (IW,114)
 114  FORMAT (//  10('*'),'  NELDER ERROR:  THIS PROGRAM IS NOT APPLICAB
     *LE WHEN THE NUMBER OF VARIABLES NVAR=',I1,'  .LT.2')
      RETURN
      END
C
C
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE XECEV
C
C *** THIS SUBROUTINE ECONOMICALLY EVALUATES AN X-BAR CHART PLAN
C
      COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /E3/ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
      COMMON /E6/ HNLG,HX,RKX
      DIMENSION AHT(2)
C
C------- PRINT TITLE AND PARAMETERS -----------------
C
      WRITE (IW,9) USLLSL
    9  FORMAT(/T2,' ***** ECONOMICALLY BASED X-BAR CHART EVALUATION ',
     *    '*****'/'  USLLSL=',F6.2,' (STD)',5X,'MEAN SHIFT ASSUMED')
      WRITE (IW,113) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
 113    FORMAT(/ '  DELTA=',F7.2,3X,'LAMBDA=',F7.2,3X,'M=',F7.2,3X,'E=',
     *  F7.2,3X,'D=',F7.2/T7,'T=',F7.2,T24,'W=',F7.2,T36,'B=',F7.2,T48,
     *  'C=',F7.2)
      WRITE(IW,114) NN,HX,RKX
  114    FORMAT (/ T5,'N=',I3,10X,'H=',F8.3,10X,'K=',F8.3)
C
C------------------------- EVALUATION ------------
         AHT(1)=HX
         AHT(2)=RKX
C     --------------
      ZL100=VYXBAR(AHT)
C     --------------
      ZL=ZL100/100.
      WRITE (IW,115) ZL100,ZL
  115    FORMAT(/ '  LOSS-COST PER 100 HOURS =',F10.3,2X,
     *   '(HOURLY LOSS-COST =',F10.3,')')
   99 RETURN
      END
C
C
C
C++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE XCOSF
C
C *** THIS SUBROUTINE INVESTIGATES THE LOSS-COST SURFACE OF AN X-BAR
C     CHART PLAN
C
      DIMENSION ACOST(30,11),AALFAP(2,11),LABEL(2),AASN(2,11)
      COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /E3/ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
      COMMON /E7/NH,AH(30), NT,AT(11), NK,AK(11)
      DATA LABEL/'ALFA','P   '/
         ZN=NN
C----------------------------- COST SURFACE EVALUATION -------------
      DO 20 I=1,NK
            ZK=AK(I)
                 DN=ZDEL*SQRT(ZN)
              Y1= -ZK -DN
              Y2= ZK -DN
```

```
            Y3= -ZK
         CALL MDNOR (Y1,P1)
         CALL MDNOR (Y2,P2)
         CALL MDNOR (Y3,P3)
      ZP=P1+1.-P2
         IF(ZP.LT. .0000001) ZP=.0000001
      ZALFA=2.*P3
C
      AALFAP(1,I)=ZALFA
      AALFAP(2,I)=ZP
C
      DO 10 J=1,NH
               ZH=AH(J)
            ZBB=(1./ZP-.5+ ZLAM*ZH/12.)*ZH + ZE*ZN +ZD
         VY=(ZLAM*ZM*ZBB + ZALFA*ZT/ZH +ZLAM*ZW)/(1.+ZLAM*ZBB)
     *                + (ZB+ZC*ZN)/ZH
         ACOST(J,I)=VY*100.
   10 CONTINUE
   20 CONTINUE
C------------------------------------- LOCATE MINUM COST -------------
      AMIN=9999999.
      IX=0
      JX=0
    DO 50 I=1,NH
      DO 40 J=1,NK
          IF (ACOST(I,J).GE.AMIN) GO TO 40
        AMIN=ACOST(I,J)
        IX=I
        JX=J
  40  CONTINUE
  50  CONTINUE
C------------------------------------- OUTPUT SECTION -----------------
      WRITE (IW,9)
    9   FORMAT('1',T5,5('*'),'  ECONOMICALLY BASED X-BAR CHART ',
     *   'LOSS-COST SURFACE INVESTIGATION  ',5('*'))
      WRITE(IW,112) USLLSL,NN
  112   FORMAT ( /T3,'USLLSL=',F6.2,' STD',5X,'MEAN SHIFT',
     *    ' ASSUMED',10X,'N=',I3)
      WRITE (IW,111) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
  111  FORMAT(  '  DELTA=',F7.2,3X,'LAMBDA=',F7.2,3X,'M=',F7.2,3X,'E=',
     *  F7.2,3X,'D=',F7.2,3X,'T=',F7.2,3X,'W=',F7.2,3X,'B=',F7.2,3X,'C='
     *  ,F7.2)
      WRITE (IW,114) (AK(I),I=1,NK)
 114    FORMAT ( //,T5,'K',T10,11F11.3/)
      DO 30 I=1,2
  30  WRITE (IW,115) LABEL(I),(AALFAP(I,J),J=1,NK)
 115    FORMAT (  T5,A4,T10,11F11.3)
      WRITE (IW,117)
 117    FORMAT (  T2,129('-')/T7,'H')
      DO 35 I=1,NH
  35  WRITE (IW,116) AH(I), (ACOST(I,J),J=1,NK)
 116    FORMAT (/,T3,F7.3,T10,11F11.3)
      WRITE (IW,118) AH(IX),AK(JX), AMIN
 118    FORMAT (//,T3,7('*'),' MINIMUM:  H=',F7.3,'    T=',F8.3,
     *          '       LOSS-COST=',F11.3,2X,'(PER 100 HOURS)')
   99 RETURN
      END
C
C
C
C
C
C++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE HTOPT
C
C *** THIS SUBROUTINE OPTIMIZE (H,T) FOR BOTH NLG AND X-BAR CHART
C     CONTROL SCHEMES BY NELDER AND MEAD DIRECT SEARCH TECHNIQUE
C
C *** REFERENCE:  NELDER, J.A., AND R. MEAD. "A SIMPLEX METHOD FOR
C     FUNCTION MINIMIZATION." THE COMPUTER JOURNAL, 7(1965),308-313
C
      COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
```

```
      COMMON /E2/ PG,PY,PR, PR1,PR2
      COMMON /E3/ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
      COMMON /E4/ XSTART(2),X(3,2),Y(3),         ITRFLG,IRESET,
     *            STDX,STDY,K  , N   ,N1,YACC,XACC,STEP,ITRMAX, NLGXB
      DIMENSION XR(2),XB(2),XF(2),XH(2),XE(2),XC(2),XL(2),
     *          XT(2),NTYPE(6)
C     DATA NTYPE/'EXPE','REFL','CONI','SHRI','CONO','STAR'/
      DATA ALP,BET,GAM/1.0, .50, 2.0/
      N=2
C
C-------  INITIAL SIMPLEX AND PARAMETER INITIALIZATION -----------------
      DO 5 J=1,N
    5 X(N1,J)=XSTART(J)
         P=(STEP/(N*SQRT(2.)))*(SQRT(N+1.)+N-1.)
         Q=(STEP/(N*SQRT(2.)))*(SQRT(N+1.)-1.)
      DO    8  I=1,N
         DO    7 J=1,N
               IF(J .EQ. I) GO TO    6
            X(I,J)=X(N1,J)+STEP*Q
            GO TO    7
    6       X(I,J)=X(N1,J)+STEP*P
    7    CONTINUE
    8 CONTINUE
C
         K=0
         KR=0
C        NTP=6
         ITRFLG=0
         STDY=0.
         STDX=0.
C
C----------- EVALUATE ALL VERTICES AND RANKS THEM PROPERLY -------------
C
C-- FUNCTION EVALUATION (Y) FOR ALL POINTS (X) ---
      DO    11   J=1,N
   11 XF(J)=X(N1,J)
         IF(NLGXB.EQ.1) Y(N1)=VYNLG (XF)
         IF(NLGXB.EQ.2) Y(N1)=VYXBAR(XF)
            DO   12 J=1,N
   12       X(N1,J)=XF(J)
C     NFC=1
   13 DO   17   I=1,N
         DO    14   J=1,N
   14    XF(J)=X(I,J)
            IF(NLGXB.EQ.1) Y(I)=VYNLG (XF)
            IF(NLGXB.EQ.2) Y(I)=VYXBAR(XF)
               DO   16 J=1,N
   16          X(I,J)=XF(J)
C     NFC=NFC+1
   17 CONTINUE
C-----------  FIND BEST PT --> (N+1)TH POINT ----
   19 YL=Y(N1)
      NL=N1
      DO  21   I=1,N
         IF(Y(I) .GE. YL ) GO TO 21
         YL=Y(I)
         NL=I
   21 CONTINUE
      DO  22   J=1,N
         XL(J)=X(NL,J)
         X(NL,J)=X(N1,J)
   22 X(N1,J)=XL(J)
      Y(NL)=Y(N1)
      Y(N1)=YL
C-----------  FIND WORST PT --> 1ST POINT -------
      YH=Y(1)
      NH=1
      DO  23   I=2,N
         IF(Y(I) .LT. YH) GO TO 23
         YH=Y(I)
         NH=I
   23 CONTINUE
```

```
      DO   24  J=1,N
         XH(J)=X(NH,J)
         X(NH,J)=X(1,J)
   24 X(1,J)=XH(J)
      Y(NH)=Y(1)
      Y(1)=YH
C-----------------  FIND 2ND WORST POINT ---------
      YSH=Y(2)
      IF(N .LT. 3) GO TO  27
      DO   26  I=3,N
         IF(Y(I) .LE. YSH) GO TO  26
         YSH=Y(I)
   26 CONTINUE
C
C------------------ CHECK TO SEE IF IT IS TIME TO QUIT -----------------
C
C------ CHECK TO SEE IF MAX ITERATION REACHED --------
   27 IF(K .LT. ITRMAX) GO TO 127
C---- TURN ON FLAG OF MAX ITERATION, AND QUIT
      ITRFLG=1
      RETURN
C---- CALCULATE MEANS OF X (W/O & W/ WORST PT) & Y ---
  127 DO   29  J=1,N
         XB(J)=0.0
         DO 28 I=2,N1
   28    XB(J)=XB(J)+X(I,J)
         XT(J)=XB(J)+XH(J)
         XB(J)=XB(J)/N
   29 XT(J)=XT(J)/N1
      YB=0.0
      DO   31  I=1,N1
   31 YB=YB+Y(I)
      YB=YB/N1
C-------- CALCULATE STANDARD DEVIATION OF Y ----------
      STDY=0.0
      DO  32  I=1,N1
   32 STDY=STDY+(Y(I)-YB)**2
      STDY=STDY/N
      STDY=SQRT(STDY)
C-------- CALCULATE STANDARD DEVIATION OF X ----------
      STDX=0.0
      DO  34  I=1,N1
         SZ=0.0
         DO  33 J=1,N
   33    SZ=SZ+(X(I,J)-XT(J))**2
         SZ=SQRT(SZ)
   34 STDX=STDX+SZ
      STDX=STDX/N1
C
C---- CHECK TO SEE IF QUITTING CRITERIA  SATISFIED
      IF(STDY .LT. YACC .OR. STDX.LT.XACC) RETURN
C
C--------- REFLECTION, EXPANSION, CONTRACTION AND SHRINKAGE ------------
C
C---------------------------------- REFLECTION --------
      DO  37  J=1,N
   37 XR(J)=XB(J)+ALP*(XB(J)-XH(J))
         IF(NLGXB.EQ.1) YR=VYNLG (XR)
         IF(NLGXB.EQ.2) YR=VYXBAR(XR)
C     NFC=NFC+1
      K=K+1
      IF(YR .LT. YL) GO TO 52
      IF(YSH .LT. YR) GO TO 39
C---  WORST REPLACED BY REFLECTION PT  ----
      DO   38  J=1,N
   38 X(1,J)=XR(J)
      Y(1)=YR
C     NTP=2
      GO TO  19
   39 IF(YH .LE. YR) GO TO 43
C--------------------------------- CONTRACTION --------
C------------- CONTRACTION OUTWARD -----
```

```
      DO   41  J=1,N
   41 XC(J)=XB(J)+BET*(XR(J)- XB(J))
C     NTP=5
      IF(NLGXB.EQ.1) YC=VYNLG (XC)
      IF(NLGXB.EQ.2) YC=VYXBAR(XC)
C     NFC=NFC+1
      IF(YC.LT.YR) GO TO  47
      DO  42 J=1,N
   42 X(1,J)=XR(J)
      GO TO 49
C-------  CONTRACTION INWARD  -----------
   43 DO   44  J=1,N
   44 XC(J)=XB(J)+BET*(XH(J)- XB(J))
C     NTP=3
         IF(NLGXB.EQ.1) YC=VYNLG (XC)
         IF(NLGXB.EQ.2) YC=VYXBAR(XC)
C     NFC=NFC+1
      IF(YC .GE. YH ) GO TO 49
C---- WORST REPLACED BY CONTRACTION PT ---
   47 DO   48  J=1,N
   48 X(1,J)=XC(J)
      Y(1)=YC
      GO TO  19
C------------------------------- SHRINKAGE  -----------
   49 DO   51  I=1,N
      DO   51  J=1,N
   51 X(I,J)=X(I,J)+.50*(XL(J)-X(I,J))
C     NTP=4
      GO TO 13
C------------------------------- EXPANSION ------------
   52 DO   53  J=1,N
   53 XE(J)=XB(J)+GAM*(XR(J)-XB(J))
         IF(NLGXB.EQ.1) YE=VYNLG (XE)
         IF(NLGXB.EQ.2) YE=VYXBAR(XE)
C     NFC=NFC+1
      IF(YE .LT. YR) GO TO 56
C----- WORST REPLACED BY REFLECTION PT ----
      DO   54  J=1,N
   54 X(1,J)=XR(J)
      Y(1)=YR
C     NTP=2
      GO TO  19
C---- WORST REPLACED BY EXPANSION PT ----
   56 DO   57  J=1,N
   57 X(1,J)=XE(J)
      Y(1)=YE
C     NTP=1
      GO TO  19
      END
C
C
C
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
      FUNCTION VYXBAR(XF)
C
C *** THIS FUNCTION SUBPROGRAM EVALUATES THE LOSS-COST (PER 100 HOURS)
C     FOR AN X-BAR CHART PLAN
C
      COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /E3/ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
      DIMENSION XF(2)
C
C---------- MEASURES ARE TAKEN TO PREVENT UNDERFLOW (OVERFLOW) PROBLEM
      ZN=NN
         IF(XF(1).LT.0.001) XF(1)=.001
      ZH=XF(1)
         IF(XF(2).LT..001) XF(2)=.001
      ZK=XF(2)
                  DN=ZDEL*SQRT(ZN)
              Y1= -ZK -DN
              Y2= ZK -DN
              Y3= -ZK
```

```
            CALL MDNOR (Y1,P1)
            CALL MDNOR (Y2,P2)
            CALL MDNOR (Y3,P3)
         ZP=P1+1.-P2
            IF(ZP.LT. .0000001) ZP=.0000001
         ZALFA=2.*P3
C
      ZBB=(1./ZP-.5+ ZLAM*ZH/12.)*ZH + ZE*ZN +ZD
      VY=(ZLAM*ZM*ZBB + ZALFA*ZT/ZH +ZLAM*ZW)/(1.+ZLAM*ZBB)
     *  + (ZB+ZC*ZN)/ZH
      VYXBAR=VY*100.
      RETURN
      END
C
C
C
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
      FUNCTION VYNLG(XF)
C
C *** THIS FUNCTION SUBPROGRAM EVALUATES THE LOSS-COST (PER 100 HOURS)
C     FOR AN NLG PLAN
C
      COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /E3/ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
      DIMENSION XF(2)
C
C---------- MEASURES ARE TAKEN TO PREVENT UNDERFLOW (OVERFLOW) PROBLEM
      ZN=NN
         IF(XF(1).LT.0.001) XF(1)=.001
      ZH=XF(1)
         IF(XF(2).GT. HALF) XF(2)= HALF-.001
         IF(XF(2).LT..001) XF(2)=.001
      TNLG=XF(2)
         CALL GYRMU(0.)
      CALL PAFG2(ZALFA)
      CALL EOFN2(ZNIC)
         CALL GYRMU(ZDEL)
      CALL PAFG2(ZP)
      CALL EOFN2(ZNOOC)
         IF(ZP.LT. .0000001) ZP=.0000001
C
      ZBB=(1./ZP-.5+ ZLAM*ZH/12.)*ZH + ZE*ZNOOC + ZD
      ZBETA=1./(1.+ZLAM*ZBB)
      ZNAVE=ZBETA*ZNIC+(1.-ZBETA)*ZNOOC
      VY=(ZLAM*ZM*ZBB + ZALFA*ZT/ZH +ZLAM*ZW)*ZBETA
     *  + (ZB+ZC*ZNAVE)/ZH
      VYNLG=VY*100.
      RETURN
      END
C
C
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE GYRMU(DEL)
C
C *** THIS SUBROUTINE CALCULATES THE PROBABILITY OF GREEN, YELLOW AND
C     RED AS FUNCTIONS OF MEAN SHIFT
C
C *** SAME AS THE FIRST PART OF SUBROUTINE GYRC
C
      COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /E2/ PG,PY,PR, PR1,PR2
         HTD1=HALF-TNLG+DEL
         HTD2=-HALF+TNLG+DEL
      CALL MDNOR(HTD1,PHI1)
      CALL MDNOR(HTD2,PHI2)
      PG=PHI1-PHI2
      GO TO (99,20,30),MM
   20 PY=1.-PG
      RETURN
   30 PR=PR1
      IF(DEL.GT.0.) PR=PR2
```

```
      PY=1.-PG-PR
  99  RETURN
      END
C
C
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE PAFG2 (PREJ)
C
C *** THE UNDERFLOW-PROOF VERSION OF SUBROUTINE PAFG
C
      COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /E2/ PG,PY,PR, PR1,PR2
      PSUM=O.
 20   DO 22 I=1,NY1
         IL1=I-1
 22   PSUM=PSUM+BINOM2(NN,IL1)
      PREJ=1.-PSUM
      IF(NG.EQ.O) RETURN
      PSUM2=O.
      IN=NY1
      NNLNG=NN-NG
      IF(NY.GT.NNLNG) IN=NNLNG+1
      DO 24 I=1,IN
         IL1=I-1
 24   PSUM2=PSUM2+BINOM2(NNLNG,IL1)
         EE=NG*ALOG(PG)
         IF(EE.LT.-170.) EE=-170.
      PREJ=1.-(PSUM+(1.-PSUM2)*EXP(EE))
      RETURN
      END
C
C
C
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
      FUNCTION BINOM2 (N,IX)
C
C *** THE UNDERFLOW-PROOF VERSION OF FUNCTION SUBPROGRAM BINOML
C
      COMMON /E2/ PG,PY,PR, PR1,PR2
      DOUBLE PRECISION DY,DG,DLGPB
C --- THIS ROUTINE CALCULATES BINOMIAL AND ITS SIMILARS
      DY=PY
      DG=PG
      DLGPB=DLGAMA(N+1.DO)-DLGAMA(IX+1.DO)-DLGAMA(N-IX+1.DO)
     *      +IX*DLOG(DY)+(N-IX)*DLOG(DG)
      IF (DLGPB.LT.-170.DO) DLGPB=-170.DO
      BINOM2=DEXP(DLGPB)
      RETURN
      END
C
C
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
      SUBROUTINE EOFN2(REN)
C
C *** THE UNDERFLOW-PROOF VERSION OF SUBROUTINE EOFN
C
      COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      COMMON /E2/ PG,PY,PR, PR1,PR2
      DOUBLE PRECISION ABC,SABC,EN, G,Y,R,YGF2,GC,EE,E2,DEXPEE
      G=PG
      Y=PY
      IF(MM.EQ.3) R=PR
      ABC=O.DO
      SABC=O.DO
      EN=O.DO
      NNL1=NN-1
      IF(NN.GT.1) GO TO 10
C---------------------------- NN = 1 ----
      REN=1.
      RETURN
```

```
C---------------------------- NN > 1 ----
   10 GO TO (900,200,300,900,900),MM
C-----------------------------------------MM=2  ---------------------
  200 IF(NY.EQ.0) GO TO 201
      IF(NY.LT.NNL1) GO TO 221
      GO TO 251
C-----------------  MM=2;  NY=0 (NG=0)  ---------
  201 IF(NG.GE.1) GO TO 212
      DO 210 I=1,NNL1
         EE=(I-1)*DLOG(G)
         IF(EE.LT.-170.D0) EE=-170.D0
  210 EN=EN+ I*DEXP(EE)*Y
         E2=NNL1*DLOG(G)
         IF(E2.LT.-170.D0) E2=-170.D0
      REN=EN+NN*DEXP(E2)
      RETURN
C
  212 WRITE(IW,214)
  214 FORMAT(//,T2,10('-'),'  NLG ERROR:  MM=2  Y=0   G>0;',
     *  '    EXECUTION INTERRUPTED IN SUBROUTINE EOFN2 (LABEL 212)')
      RETURN
C-----------------  MM=2;  0<NY<(NN-1)  ---------
  221 IF(NG.EQ.0 .OR. NG.GT.NY) GO TO 225
         EE=NG*DLOG(G)
         IF(EE.LT.-170.D0) EE=-170.D0
      ABC=DEXP(EE)
      EN=EN+NG*ABC
      SABC=SABC+ABC
  225 DO 240 J=NY1,NNL1
         JL1=J-1
         IF(J.EQ.NG) GO TO 229
         ABC=YGF2(JL1,NY,G,Y)
         EN=EN+J*ABC
         GO TO 240
  229       EE=NG*DLOG(G)
            IF(EE.LT.-170.D0) EE=-170.D0
         ABC=YGF2(JL1,NY,G,Y)+DEXP(EE)
         EN=EN+J*ABC
  240 SABC=SABC+ABC
      REN=EN+ NN*(1.D0-SABC)
      RETURN
C-----------------  MM=2;  NY>0 & NY>=(NN-1)  ---
  251 IF(NG.GE.1) GO TO 254
      REN=NN
      RETURN
  254    EE=NG*DLOG(G)
         IF(EE.LT.-170.D0) EE=-170.D0
         DEXPEE=DEXP(EE)
      REN=NG*DEXPEE +NN*(1.D0-DEXPEE)
      RETURN
C--------------------------------------- MM=3  ---------------------
  300 IF(NY.EQ.0) GO TO 301
      IF(NY.LT.NNL1) GO TO 321
      GO TO 351
C------------------  MM=3;  NY=0 (NG=0)  --------
  301 IF(NG.GE.1) GO TO 312
      GC=1.D0-G
      DO 310 I=1,NNL1
         EE=(I-1)*DLOG(G)
         IF(EE.LT.-170.D0) EE=-170.D0
  310 EN=EN+I*DEXP(EE)*GC
         E2=NNL1*DLOG(G)
         IF(E2.LT.-170.D0) E2=-170.D0
      REN=EN+NN*DEXP(E2)
      RETURN
C
  312 WRITE(IW,314)
  314 FORMAT(//,T2,10('-'),'  NLG ERROR:  MM=3  Y=0  G>0;',
     * '  EXECUTION INTERRUPTED IN SUBROUTINE EOFN2 (LABEL 312)')
      RETURN
C---------------------  MM=3;  0<NY< NN-1   ----
  321 DO 330 I=1,NY
```

```
          IF(I.EQ.NG) GO TO 329
          ABC=(1.DO-SABC)*R
          EN=EN+I*ABC
          GO TO 330
  329        EE=NG*DLOG(G)
             IF(EE.LT.-170.DO) EE=-170.DO
          ABC=(1.DO-SABC)*R+DEXP(EE)
          EN=EN+I*ABC
  330 SABC=SABC+ABC
      DO 340 J=NY1,NNL1
          JL1=J-1
          IF(J.EQ.NG) GO TO 339
          ABC=(1.DO-SABC)*R + YGF2(JL1,NY, G,Y)
          EN=EN+J*ABC
          GO TO 340
  339        EE=NG*DLOG(G)
             IF(EE.LT.-170.DO) EE=-170.DO
          ABC=(1.DO-SABC)*R + YGF2(JL1,NY, G,Y) + DEXP(EE)
          EN=EN+J*ABC
  340 SABC=SABC+ABC
      REN=EN+ NN*(1.DO-SABC)
      RETURN
C--------------------- MM=3;  NY>O & NY>=(NN-1)  --
  351 DO 360 I=1,NNL1
          IF(I.EQ.NG) GO TO 359
          ABC=(1.DO-SABC)*R
          EN=EN+I*ABC
          GO TO 360
  359        EE=NG*DLOG(G)
             IF(EE.LT.-170.DO) EE=-170.DO
          ABC=(1.DO-SABC)*R + DEXP(EE)
          EN=EN+I*ABC
  360 SABC=SABC+ABC
      REN=EN+NN*(1.DO-SABC)
      RETURN
C
  900 WRITE(IW,901) MM
  901 FORMAT(/// T3,10('-'),'ERROR:  IN SUBROUTINE EOFN2, MM=',I2,
     *  ' .NE. 2 OR 3;  EXECUTION INTERRUPTED (LABEL 900)')
      RETURN
      END
C
C
C
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
      FUNCTION YGF2(N,K, G,Y)
C
C *** THE UNDERFLOW-PROOF VERSION OF FUNCTION SUBPROGRAM YGF
C
      COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
      DOUBLE PRECISION BINCOE, G,Y, YGF2, EE,E2
      IF(K.GT.N) GO TO 90
      NLNG=N-NG
         EE=(K+1)*DLOG(Y)
         IF(EE.LT.-170.DO) EE=-170.DO
         E2=(N-K)*DLOG(G)
         IF(E2.LT.-170.DO) E2=-170.DO
      IF(NG.EQ.O.OR.NLNG.LT.K) GO TO 10
C------------------------------- NG>O AND (N-NG)>=K --------------
      YGF2=(BINCOE(N,K)-BINCOE(NLNG,K))*DEXP(EE)*DEXP(E2)
      RETURN
C------------------------------- NG=O OR (N-NG)<K ----------------
   10 YGF2=BINCOE(N,K)*DEXP(EE)*DEXP(E2)
      RETURN
C
   90 WRITE (IW,91) K,N
   91 FORMAT(///10('-'),'  NLG ERROR:  IN FUNCTION SUBPROGRAM YGF2, K=',
     * I2, ' > N=',I2,';  EXECUTION INTERRUPTED (LABEL 90)')
      RETURN
      END
```

VITA

Shawn Shih-Chun Yu

Candidate for the Degree of

Doctor of Philosophy

Thesis: MODELING AND EVALUATION OF STATISTICALLY AND ECONOMICALLY DESIGNED NARROW LIMIT GAGING (NLG) PROCESS CONTROL PLANS

Major Field: Industrial Engineering and Management

Biographical:

Personal Data: Born in Keelung, Taiwan, Republic of China, March 17, 1950, the son of Mr. and Mrs. Tong-tsann Yu.

Education: Graduated from Jan-kuo High School, Taipei, Taiwan, R.O.C., in June, 1968; received the Bachelor of Science degree in Industrial Engineering from Tunghai University, Taichung, Taiwan, in 1973; received the Master of Science degree in Systems Engineering from the University of Illinois at Chicago Circle, Chicago, Illinois, in 1979; completed requirements for the Doctor of Philosophy degree at Oklahoma State University in July, 1983.

Professional Experience: Teaching Assistant and Administration Assistant, Department of Industrial Engineering, Tunghai University, 1975-1977; Teaching Assistant, Department of Systems Engineering, University of Illinois at Chicago Circle, 1978-1979; Teaching Assistant and Research Associate, School of Industrial Engineering and Management, Oklahoma State University, 1979-1983.

Professional Organizations: Institute of Industrial Engineers, Alpha Pi Mu, Omega Rho, Phi Tau Phi.