MODELING AND EVALUATION OF STATISTICALLY AND
ECONOMICALLY DESIGNED NARROW LIMIT GAG-
ING (NLG) PROCESS CONTROL PLANS

By

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MODELING AND EVALUATION OF STATISTICALLY AND
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PREFACE

This study is concerned with the modeling and evaluation of the easy-to-use powerful process control scheme—Narrow Limit Gaging (NLG). The primary objective is to provide systematic methodologies and an interactive computer program to help Quality Control practitioners in understanding, designing, evaluating, and implementing statistically- and economically-based NLG plans. Also, NLG is compared with the alternative $\bar{X}$-chart plan, both statistically and economically, to help users in choosing the control scheme which better suits their individual needs.

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CHAPTER I

THE RESEARCH PROBLEM

Purpose

Process control is one of the major areas of statistical quality control, in which several techniques can be employed to estimate process characteristics and capability, to establish control, and to monitor the process. This study will focus on one of the easiest to use techniques—Narrow Limit Gaging (NLG). The major interest of this research is to help practitioners in understanding, designing, evaluating, and implementing the most appropriate NLG process control scheme by providing the following:

1. a clear taxonomy and recommended standardization of NLG control schemes,
2. comprehensive methodology for statistical and economic design and evaluation of NLG plans,
3. comparison of NLG to the most popular process control alternative, and
4. a user-oriented interactive computer program to accomplish a wide range of design and analysis tasks.

The Need

The implementation of a process control procedure in a production context involves two stages. First, a state of statistical control must
be described and achieved; and second, the output can then be monitored in a reasonable fashion. During the monitoring stage, the process begins "in control" but eventually shifts out of control, at the occurrence of an assignable cause which is desired to be detected as early as possible.

Two types of control schemes can be employed to monitor the process, namely, variable plans (such as $\bar{X}$- and R-charts, and the cusum chart) and attribute plans (such as the p-chart and c-chart). Generally, variable plans require a longer time to measure individual items, while attribute plans require larger sample sizes to detect the same degree of process shift. Both the variables measurement of small samples and the attributes gaging of large samples can be quite time consuming and, for some cases, may impede the rapid detection of a process shift.

To solve this problem, a combination of the advantages of both control schemes is strongly desired. A quick-and-easy gaging method, together with a fairly small sample size, is sought. Among all traditional approaches, NLG process control plans seem to be the only ones to fulfill this need.

Introduction

Suppose the measurements of the product characteristic are normally distributed, and the process capability ($6\sigma$) is less than the specification tolerance (USL - LSL) (see Figure 1.1). In addition, the process dispersion $\sigma$ (standard deviation) is assumed to remain unchanged while the process mean may shift.\(^a\) To guide manufacturing, go/no-go gages are

\(^a\)These assumptions are made only to facilitate illustration. In practice, none of them is required.
Figure 1.1. Specification Limits and Narrow Gage Limits
prepared which are stricter than specifications by an amount $\sigma$ and hence are called Narrow Limit Gages. Then small samples are taken and gaged at regular intervals of time, which may be called frequency gaging. Finally, decisions about actions are made according to some predetermined rules.

Two examples follow:

1. Simple rule [33]: In a sample of size $n$, if the number of units which do not pass the NL gage, is greater than a specified number $c$, then the process is stopped and investigated for assignable causes. Otherwise, the process keeps going.

2. Complex rule [38]: A sample of three is drawn and two are gaged. The third is gaged only when necessary. Possible outcomes and actions follow:
   
a. **No action required**
   
   (1) Both within NLG limits.
   
   (2) One in and one out of NLG limits (but within specification limits) and the third inside NLG limits.

b. **Readjust/correct machine**

   (1) Any one out of specification limits.
   
   (2) Both out on the same side of NLG limits.
   
   (3) One in and one out of NLG limits (but within specification limits) and the third out on the same side of NLG limits.

c. **Machine capability questionable**

   (1) When two out of three (or two out of two) are both out of NLG limits, but on opposite sides, the operation is suspected of having too much variation. A machine
capability study should be made with machine maintenance as necessary.

In addition to the above frequency gaging rules, decisions about sampling frequency and the qualification to begin frequency gaging after each machine setup and reset may also be needed. An example follows [19]:

1. To qualify for frequency checking, make 100 percent inspection until five successive pieces fall between NLG limits. While waiting for five, the process may require a reset as necessary.

2. For sampling frequency, seek an average of 25 checks to a reset. If, on the average, an operator checks more than 25 times without having to reset the process, gaging frequency may be reduced so that more pieces are made between checks. If the process must be reset before 25 checks on the average are made, the gaging frequency may be increased.

Taxonomy and Development of a Standard Formulation

Although NLG is easy to use, there exists a variety of rules in practice. Different people can always make up different rules. The current sets of individual rules for use of NLG seem so arbitrary that they lack a common basis for evaluation and comparison. Furthermore, people always describe NLG rules in their own lengthy words rather than in common terminology and concise notation. These descriptions can easily amount to 20 sentences. This makes the essential structure of NLG even more obscure.

In all, a clarified structure is needed to generalize the NLG rules, to simplify the descriptions, to give appropriate evaluations, and to provide comparisons. This research fulfills this need by developing a clear, notation-stated, comprehensive, and exhaustive NLG statement.
Also, a "standard" NLG scheme is developed on which all of the numerical evaluations of this study are based. This will considerably reduce the total number of possible rules and facilitate evaluation.

Statistical Evaluation

In order to statistically compare different NLG plans on the same basis, proper "performance measures" are first established. For individual samples, the following are investigated:

1. \( P_a \) -- Probability of acceptance
2. \( E_n \) -- Expected number of items inspected in each sample
3. OC (Operating Characteristic) curve -- \( P_a \) as a function of either process mean shift or dispersion change.

For the process as a whole, the following performance measures are considered [19]:

1. APQ and APQL -- Average produced quality and its limit
2. AOQ and AOQL -- Average outgoing quality and its limit when 100 percent retroactive inspection is performed to remove defective items.

The formulations of all these performance measures are developed as functions of the process fraction defective.

The general effect of each NLG parameter (e.g., sample size, control limit inset, truncation rule, acceptance/rejection rule, etc.) is analyzed to help in understanding NLG characteristics. Based upon this understanding, flexible procedures are constructed for designing NLG plans. To provide greater flexibility for the user in choosing a preferred plan under certain specified conditions, all qualified plans are listed together with related performance measures provided.
Finally, a performance comparison between the most popular process control plan, the \( \bar{X} \)-chart, and NLG is analyzed to see if NLG is comparable or even superior to the \( \bar{X} \)-chart.

**Economic Formulation**

Traditionally, process control schemes are designed statistically and produce acceptable results. However, in recent years, there has been an increasing emphasis on economic performance since it is intuitively more appealing to design plans with direct consideration of quality costs [31]. In reality, economic performance is the ultimate criterion for evaluating control plans, in which one is balancing the costs associated with sampling, testing, and process surveillance against internal and external failure costs. Since the design of the procedure affects these costs, it is logical to consider this design from an economic viewpoint.

Based upon the maximum income criterion, Duncan [6] has formulated a model which measures the average net income of a process under the surveillance of an \( \bar{X} \)-chart. The process starts in-control and is subject to random shifts in the process mean (out-of-control). Once out of control, this process remains there until the trouble is removed. Given (1) cost parameters of in-control income, out-of-control income, false alarm cost, real alarm cost, and control chart costs; and (2) time parameters of process shifting, inspection and plotting, and searching for assignable causes, the best values of the decision variables sample size (\( n \)), sampling interval (\( h \)), and control limit spread (\( k \)) are determined using optimization techniques.

This study follows Duncan's approach in formulating an economic NLG scheme in which the decision variables consist of sample size (\( n \)),
sampling interval (h), control limit inset (t), a truncation rule, and acceptance/rejection rules. For both models, the underlying assumptions are closely matched to ensure the highest degree of formulation similarity for comparison purposes. The significance of possible NLG improvements over $\bar{X}$-charts, resulting from the reduction of control chart costs and plotting delay, is evaluated.

**Economic Optimization**

In optimizing the values of the decision variables of the economically-based $\bar{X}$-chart model, Duncan [6] uses a complicated and involved search technique after making certain assumptions and approximations about his model. To improve accuracy and speed, Goel et al. [12] develop an algorithm, also employing a search technique, which consists of solving an implicit equation in all decision variables. Both authors utilize the differentiability of the loss-cost function with respect to decision variables $n, h,$ and $k$ to considerably simplify the effort of direct search.

In the economically-based NLG model, the probability of acceptance is a complicated function of decision variables $n, h, t,$ truncation rule, and acceptance/rejection rules. The desirable property of differentiability no longer exists. Therefore, multidimensional direct search techniques represent the most promising optimization approach. Furthermore, since the decision variables sample size $n$ is not continuous, and the truncation rule and acceptance/rejection rules are not even measurable, the general optimization strategy adopts an appropriate direct search algorithm to optimize sampling interval $h$ and control limit inset $t$ simultaneously under every possible set of combinations of $n$ and both rules.
The combination of decision variables \( n, h, t, \) truncation rule, and acceptance/rejection rules yielding a minimum loss-cost is the optimal scheme.

**Economic Comparison of NLG Plan and \( \bar{X} \)-Chart**

To assess the best conditions for the application of NLG and \( \bar{X} \)-charts, both models are evaluated under the same environments. This evaluation is performed under each of a number of examples. For each example, in addition to the \( \bar{X} \)-chart and standard NLG, two more variations of NLG are investigated to reveal the effects of the truncation rule and the reductions in control chart costs and plotting delays.

Based upon the results of these comparisons, in addition to intuitive theoretical interpretation, practical general guidelines are developed to help practitioners in choosing between economic \( \bar{X} \)-charts and NLG plans under specified environments.

**Interactive Computer Program**

To help practitioners in the design, evaluation, and implementation of NLG process control plans, all previous developments and analyses are summarized into a comprehensive and flexible interactive computer program. This program has both statistical and economic analysis and design capability. In addition, both design and evaluation, either statistically or economically, of a specified \( \bar{X} \)-chart are also provided upon the user's request for comparison purposes.
Summary of Research Objectives

Based upon the above discussions, the primary objective of this research is stated:

Objective:
To provide a systematic methodology and a practical interactive computer program to help Quality Control practitioners in understanding, designing, evaluating, and implementing statistically- and economically-based Narrow Limit Gaging process control plans.

In order to accomplish this objective, several specific subobjectives are included:

Subobjectives:
1. To develop a clearly, symbolically stated, comprehensive NLG taxonomy to generalize and simplify the descriptions of varieties of NLG rules.
2. To propose a "standard" NLG scheme to reduce the total number of possible rules and to facilitate easy numerical evaluation.
3. To provide a methodology for designing and evaluating NLG plans statistically. A comparison with the $\bar{x}$-chart will also be provided.
4. To formulate the economically-based model for evaluating NLG process control plans.
5. To develop a general strategy, together with a direct search technique, to optimize the economically-based NLG model.
6. To economically compare NLG and $\bar{x}$-chart plans under a variety of situations.
7. To develop a comprehensive and flexible interactive computer program to provide
(a) design and evaluation of statistically-based NLG plans,
(b) design and evaluation of statistically-based $\bar{X}$-chart plans,
(c) design and evaluation of economically-based NLG plans, and
(d) design and evaluation of economically-based $\bar{X}$-chart plans.

Contribution

The successful completion of this research will provide benefits to both theoreticians and practitioners. This study will become the first of its kind in providing (1) a unified taxonomy and a standardization of NLG, (2) thorough statistical analyses of NLG, (3) considerable economic treatment of NLG, and (4) appropriate comparisons, both statistically and economically, between NLG and $\bar{X}$-charts. Most of these results (except a small portion of (2)) are not presented in any textbooks or papers on statistical quality control, although NLG has had considerable application and, even more, is of growing interest in the quality control area.

Practitioners will benefit from this research because it will provide them with practical procedures for designing and evaluating appropriate NLG plans. The flexibility of either statistical or economic comparisons among qualified NLG plans and $\bar{X}$-control chart schemes will improve the user's decision-making capabilities. The fast execution of an interactive computer program will make the design and evaluation of NLG plans considerably easier. Consequently, this will encourage a broader range of NLG applications and therefore result in increased productivity.
CHAPTER II

LITERATURE REVIEW

Introduction

This chapter reviews developments in the literature relevant to the objectives of this research. Support for this specific research is elaborated upon. In addition, other sources which communicate the general concepts relating to this study are also presented.

This chapter is divided into five areas:

1. Process Control Techniques and Their Comparisons
2. Development of NLG
3. Variety of NLG Rules and Applications
4. NLG Statistical Evaluation

Process Control Techniques and Their Comparisons

Since Shewhart [43] first introduced the concept of statistical quality control a half century ago, many new techniques have been proposed in both the process control and acceptance sampling areas. In process control, important developments include [11, 21]:

1. Shewhart control charts and their ramifications—\( \bar{X} \), \( \bar{X}-R \), \( p \), \( c \), \( u \), tests for runs, \( \bar{X} \)-chart
2. Modifications of Shewhart control charts -- moving average and range, median and midrange, geometric moving average

3. Cumulative sum control charts

4. Acceptance control charts

5. Multi-characteristic control charts -- Hotelling $T^2$, Q-chart

6. Narrow limit gaging.

In order to select the most appropriate method for a given situation, proper comparisons among all alternatives are needed. However, few authors have compared the different schemes. Among them, Page [35] discusses the general comparison approach of process inspection schemes. Freund [10] compares the cumulative sum, geometric moving average, and acceptance control charts. Roberts [39] compares the moving average, geometric moving average, cumulative sum, Girshick-Rubin, and run sum charts. Unfortunately, NLG has never been compared to other methods, although it has the general advantages of simplicity and speed over all other control schemes.

According to a survey conducted by Sanija and Shirland [40], the $\bar{X}$-control chart remains the most popular process control scheme in industry. Naturally, it becomes the alternative chosen to compare with NLG in this research.

Development of NLG

In the literature, Narrow Limit Gaging [9, 33] has a variety of synonyms. It is also known as Compressed-Limit Gaging [7], Increased Severity Testing [7], Pre-Control\textsuperscript{a} [19], and Target Area Control [4]. Some even

\textsuperscript{a}Pre-Control is so named because when the specification interval is
refer to it without giving it a name, such as "Patrol Inspection (np Chart) with special gages" [15]. Among all of these, most often it goes by the names of Narrow Limit Gaging and Pre-Control.

For controlling a current production process and in comparison to variable control schemes, attribute control charts have many advantages. For example, they (1) can accommodate numerous variables in a single chart, (2) are more economical and easier to use because they can use go/no-go gages, and (3) are better for destructive and time consuming testing. However, attribute control charts require larger sample sizes to achieve the same sensitivity as that of variable schemes.

To improve the usefulness of attribute control charts, attempts have been made to devise attribute charts that require a lower than usual sample size. In the last four decades, several suggestions have been made to use gages with limits stricter than product specifications (i.e., NLG) for decision making purposes, either applied to control charts or to acceptance sampling, and in this way to reduce the sample size required for making a decision. Chronologically, this development is divided into three periods: (1) Simple Rule period, (2) Complex Rule period, and (3) Statistical Optimization and Economic Design period.

In the Simple Rule period, all NLG plans require that each of a sample of size n items be compared to narrow gaging limits and that c or fewer be within these limits for process acceptance. These Simple Rule plans do not involve the concept of Qualification and Gaging Frequency. NLG concepts first emerged in Britain in the 1940's [5, 30] and were

large enough to tolerate some degree of process shifting, it permits a decision for corrective action to be made long before the process has deteriorated to the point that tolerances are exceeded and rejects made.
claimed to be as promising as \( \bar{x} \)-charts. Mace [27], in 1952, actually designs two NLG plans having similar OC curves as a comparable \( \bar{x} \)-chart. Ott and Mundel [33], in 1954, systematically investigate the effect of each NLG element \((n, c, t)\) on OC curves and provide some general guidelines in designing NLG plans. As a ramification of NLG, Stevens [46], in 1948, designs \((C - A)\) and \((C + A)^b\) charts to substitute for \( \bar{x} \)- and R-charts, respectively. Stevens' charts application is illustrated by Aroian [1] in 1959.

In the Complex Rule period, the Jones and Lamson Machine Co., in 1954, develop an important milestone. In its Quality PRE-Control brochure [19], frequency gaging rules evolve from the Simple Rule into the Complex Rule. Moreover, the concepts of Qualification (to begin frequency gaging), Sampling Frequency, and Average Produced Quality and Its Limit are all integrated into NLG design. Four different plans are provided for typical applications which require very little statistical knowledge. The idea and practicality of NLG is greatly popularized by Juran's [20] Quality Control Handbook in 1962. However, no flexibility is provided to adjust control limit spread \( t \), no evaluation is given to the Qualification rule, no clear methodology for evaluating \( P_a \) of each sample is given, and the computation of APQ is questionable. Still, the contribution to the realization and application of NLG schemes in industry by both references is undoubtedly significant.


\[ bC \text{ is the number of pieces to fall below the lower NLG limit, and } A \text{ is that number to fall above the upper NLG limit.} \]
optimize (in the sense of minimizing sample size) the NLG Simple Rule under specified acceptable and rejectable quality levels, and their associated $\alpha, \beta$ risks. They also discuss the interesting and revealing conceptual comparison of attribute and variable measurements, and herein design and optimize an intermediate double-limit per single specification NLG scheme. In 1975, Ladany [24] presents the first economic NLG model by incorporating the above-mentioned optimal statistical Simple Rule NLG plan [2] into an economically-based $p$ chart [23], resulting in a "narrow-limit gaging fraction defective" control chart. However, the optimization of such a combination only results in a suboptimum rather than an overall optimum since the overall costs in using NLG are not considered.

The above discussion indicates some voids to be filled in order to complete the development of NLG to a satisfactory degree. These voids include (1) comprehensive statistical analyses of NLG, (2) accurate economic modeling and true optimization of NLG, and (3) appropriate comparison between NLG and $\bar{X}$-charts, both statistically and economically.

Variety of NLG Rules and Applications

There exists such a variety of rules in practice that there is no standard approach to NLG design and use. But in the less involved Simple Rule NLG plans [5, 9, 27, 30, 33], the design procedure is somewhat standardized. Due to its simplicity and consistency, optimum design is sought by Beja and Ladany [2] and some ramifications are extended. A double NLG limit per single specification limit scheme is proposed and optimized by the same authors. Also, a combined sequential implementation of two NLG plans is demonstrated by Ott [33, 34].
In Complex Rule NLG plans, a great diversity of methods exist. For sample size, \( n = 2 \) (Plan A in \([19]\)), \([20, 29, 37]\), and \( n = 3 \) \([38]\) are quite popular, but \( n = 5 \) \([17]\), \( n = 6 \) (Plan B in \([19]\)), and \( n = 7, 8, 10 \) \([17]\) are also used in practice. The variation of truncation (i.e., the curtailment of items inspected in each sample) rules depend upon the corresponding sample sizes. For inspection frequency, Jones and Lamson Co. \([19]\) and Juran \([20]\) propose a guideline of 25 or 50 inspections on the average for each process correction, while Whittingham \([49]\), in 1981, suggests three fixed checking intervals for different process classifications. Very little work has been done on Qualification (to start frequency gaging) rules which are employed to ensure the process is under control immediately after every setup and reset. There is currently only one Qualification rule in practice \([19]\).

NLG has a large variety of applications in practice. Harding \([16]\), in 1957, uses--for incoming material acceptance sampling--NLG plans which are comparable to (and more economic than) MIL-STD 105A double sampling plans. Beja and Ladany \([2]\), in 1974, also design NLG plans for use as an acceptance sampling scheme which is compared with single attribute sampling plans and variable sampling plans. When used as a process control tool, in addition to the major function of maintaining control of a process, NLG can also be used to control a trend in process mean \([45]\),\(^c\) to detect either mean or dispersion shifts, or both \([42]\), and as a set-up plan \([19]\). Finally, after incorporating it with the "feed back" concept \([26]\), NLG can easily be adopted in automatic process control \([25, 44, 45]\).

\(^c\)Also see footnote b on page 15.
The above discussion reveals a strong need for summarizing, simplifying, and standardizing NLG plans to meet the following general requirements [19]:

1. Protect against unwanted shifts in process mean and/or process spread, yet accommodate the tolerable process trend.

2. Serve both as a set-up plan and a monitor plan, and economically adjust inspection frequency to guarantee a specified level of produced quality.

3. Provide ease of use, require no paperwork, permit use of go/no-go gages, and be easily learned by operators.

4. Be competitive in efficiency with alternative plans, but cost less to administer.

NLG Statistical Evaluation

The statistical evaluation of the NLG process control scheme can be done either with respect to the sample only, or with the process as a whole. When considering the sample only, for a two-point design (i.e., under specified acceptable and rejectable quality levels and their associated $\alpha, \beta$ risks), Beja and Ladany [2] propose using the sample size $n$ as a performance measure in choosing qualified Simple Rule NLG plans. Similarly, the average sample number $E_n$ [14] resulting from the truncation of sampling inspection under the Complex Rule can be used instead of $n$. However, if the user specifies only one point, either OC curves or ARL curves [48] incorporated with $E_n$ can be employed to evaluate qualified plans. Furthermore, if the detection of both process mean shift and process
dispersion change are considered,\(^d\) ISO-OC or ISO-ARL graphs [48] may be used.

When considering the process as a whole, under specified conditions, Jones and Lamson Co. [19] suggests using the Average Produced Quality Limit (APQL) to evaluate alternative plans. However, under certain conditions, the APQ calculation becomes questionable. This shortcoming should be improved. Also, more information can be provided by supplying the whole APQ curve. Furthermore, the same article [19] indicates that Average Outgoing Quality (AOQ) and its limit (AOQL) can be obtained when the implementation of Retroactive Inspection (100% inspection of recently passed product) is added.

To investigate the general effect of individual NLG decision variables, the work of Ott and Mundel [33] on the Simple Rule can be extended and applied to the Complex Rule. In investigating the rule of Qualification for frequency gaging, Weiler's [47] discussion about the ARL (Average Run Length) of Runs is also useful.

In summary, all the above-discussed ideas and methods are evaluated, improved, and finally integrated into a comprehensive statistical evaluation package which is intended to give practitioners maximum assistance.

Economic Modeling, Optimization, and Comparison of Process Control Schemes

Designing process control schemes using economic instead of statistical

\(^d\)Almost all of the NLG schemes consider only the process mean shift which Shainin [42] claims happens much more often than process dispersion changes in industry. However, there exist situations where the process dispersion may change.
criteria has received more and more attention in the quality control literature in recent years. Most of the modern work in this area has concentrated on the \( \bar{X} \)-chart, due to its flexibility, simplicity of administration, and the information content of plotted point pattern. Extensions to the \( p \)-chart, cumulative sum charts, control charts with warning limits, joint design of \( \bar{X} \)- and \( R \)-charts, and multivariate quality control procedures have also been reported [31]. In many variations of economically-based \( \bar{X} \)-control chart models [31], Duncan's [6] fundamental approach is still the most popular one. Therefore, it is used in this research as an alternative to the economically-based NLG model for comparison purposes.

The only related work on the economic design of NLG process control plans is done by Ladany [24]. He combines the optimal Simple Rule NLG plan with the economically-based \( p \)-chart and results in a suboptimal solution. To avoid this shortcoming, this research develops a model which combines the "standard" NLG scheme with Duncan's \( \bar{X} \)-control chart model, and then employs a direct search technique to find the overall optimum.

Himmelblau [18], and Kuester and Mize [22] provide many useful methods for direct search techniques. Among them, the method proposed by Nelder and Mead [32] is quite straightforward, efficient, and easy to use. However, its non-constrained optimization algorithm requires some modification before it can be applied to optimize the economic NLG schemes in which constraints exist on sampling interval \( h \) and control limit spread \( t \).

Goel [13] and McFadden [28] perform several comparisons on economically-designed process control schemes. These complement the previously mentioned statistical comparisons done by Page [35], Freund [10], and Roberts [39]. However, there has been no work toward economically comparing NLG and the \( \bar{X} \)-chart.
Summary

This chapter presents a survey of the literature on the problems, contributions, and needs relative to the objectives of this research on Narrow Limit Gaging for process control. This survey indicates that NLG process control plans have had considerable application in industry due to their inherent advantages. However, NLG plans lack standardization and appropriate design and evaluation procedures.

This survey also demonstrates the increasing interest in economic design of process control models. Unfortunately, there has been very little work done toward developing and optimizing a general economically-based NLG model.

This survey indicates a clear need for the following:

1. To provide a clear taxonomy and standardization for NLG process control schemes.

2. To develop a methodology for statistical design and evaluation of NLG plans.

3. To develop a methodology for economic modeling and optimization of NLG plans.

4. To compare NLG to alternative process control plans.

5. To develop a user-oriented interactive computer program to facilitate the wide range implementation of NLG schemes.

This research accomplishes a significant improvement in the theoretical and applied development of Narrow Limit Gaging process control schemes. Due to this contribution, NLG plans can be used more correctly, more easily, with broader application, and with increasing popularity. Also, their use will eventually result in increased productivity.
CHAPTER III
TAXONOMY AND STANDARDIZATION OF NLG

Introduction

This chapter analyzes the composition of NLG and investigates its complexity and possible variation to provide an overall understanding of its general structure. Based on this understanding, a simplification and standardization of NLG schemes is then developed. Concise notation is presented to effectively describe NLG plans. Pertinent examples are provided.

Notation

To facilitate the comprehensive description of a complicated NLG scheme, the following notation is introduced and will be continuously used throughout the entire research.

USL, LSL--Upper and lower specification limits, respectively (see Figure 3.1)

σ --Process standard deviation (before shifting) of the characteristic measurement (x) of the product

USLLSL--Specification interval (in multiples of σ ) = (USL - LSL)/σ (see Figure 3.1)

UNGL, LNGL--Upper and lower narrow gage limits, respectively (see Figure 3.1)
Figure 3.1. Illustration of NLG Notation
t--Control limit inset of NLG. This is the number of standard deviations (tσ) that the narrow gage limits are set in from both USL and LSL. That is, UNGL = USL - tσ; LNGL = LSL + tσ (see Figure 3.1)
n--Sample size
m--Number of NLG classifications; m = 2: Green, Yellow;
m = 3: Green, Yellow, Red (see Figure 3.1)
G--Green. It denotes any measurement falling between two narrow gage limits; that is, LNGL ≤ x ≤ UNGL (see Figure 3.1)
Y--Yellow. When m = 2, it denotes a non-G measurement; that is, x < LNGL or x > UNGL. When m = 3, it denotes any measurement falling between the specification limit and the narrow gage limit on the same side; that is, LSL ≤ x < LNGL or UNGL < x ≤ USL (see Figure 3.1)
R--Red. It denotes any measurement falling beyond USL or LSL; that is, x < LSL or x > USL. This classification exists only for m = 3 and not for m = 2 (see Figure 3.1)
g--Acceptance truncation number. Whenever the first g items of a sample are green, the sample is accepted and the remaining inspection is truncated
y--Maximum acceptance number of items designated as Y. Whenever the number of Y in a sample is >y, the sample is rejected and inspection is truncated
r--Maximum acceptance number of items designated as R. Whenever the number of R in a sample is >r, the sample is rejected and inspection is truncated
QL--An abbreviation referring to Qualification for starting Frequency Gaging. It is a procedure to ensure that the process has been adjusted to the desired in-control level before starting Frequency Gaging.

FG--An abbreviation for Frequency Gaging. It is a procedure to monitor proper operation of the process. Periodically, a sample of size n is taken and inspected for early detection of a process shift.

SF--An abbreviation for Sampling Frequency. This is the frequency of taking and inspecting samples in the FG step.

RI--An abbreviation for Retroactive Inspection. To improve the average produced quality, items between the final out-of-control sample and the last previous in-control sample are 100% inspected for the removal of defectives.

OC curve--Operating Characteristic curve. This curve describes the probability of acceptance as a function of process quality.

APQ--Process Average Produced Quality. It is the long term average fraction defective produced by the process.

IC--an abbreviation for in-control or "in control".

OOC--An abbreviation for out-of-control or "out of control."

Taxonomy of NLG

General Structure

Theoretically, a complete Narrow Limit Gaging process control scheme consists of four basic elements: Qualification (QL), Frequency Gaging.
(FG), Sampling Frequency (SF), and Retroactive Inspection (RI). These elements comprise a complete control cycle as shown in Figure 3.2.

At the beginning of each control cycle, if necessary, QL is implemented to ensure that the process has been adjusted to the desired in-control (IC) level. In the second step, a sample of size n is taken periodically, according to the SF specification, and inspected to infer whether the process is in or out of control. If in control, FG continues. An out-of-control (OOC) indication necessitates adjustment of the process back to an IC level. This would usually conclude the control cycle. However, if further improvement on the average produced quality is desired without altering the control scheme, RI can be performed. All items produced in the last sampling interval are therefore 100 percent screened for the removal of every defective.

In practice, not all of the above three steps are implemented. While FG and SF are mandatory, QL and RI can be optional depending upon individual situations. Their definitions, functional objectives, ingredients, and variations will be delineated in the following sections.
Frequency Gaging

Generally, each process control cycle starts out in control (which, if desired, can be ensured by QL), remains in control for a certain period of time, and then eventually shifts out of control due to the occurrence of an assignable cause. To detect this shift as early as possible, a sample of size \( n \) is taken from the process periodically. Each item of this sample is then gaged by a pair of Narrow Limit Gages which has a control limit inset \( t \), and is classified into one of \( m \) resulting classifications (for example, if \( m = 3 \), the classifications will be G, Y, and R). Comparing the gaging results of the sample (or part of the sample) to a set of predetermined rules, a decision is then made to either let the process continue or to take necessary corrective actions.

Unfortunately, the number of possible sample acceptance/rejection decision rules is formidable due to the number of variations of acceptance/rejection criterion. Theoretically, the number of all possible NLG outcome permutations can be as large as \( m^n \). For example, if \( n = 4 \), \( m = 3 \), there will be \( 3^4 = 81 \) possible criteria. If outcomes are expressed in combinations of (G, Y, R), the number of criteria can be reduced to \( \binom{n+m-1}{n} \) which is considerably smaller than \( m^n \). For example, when \( n = 4 \), \( m = 3 \), there will be \( \binom{4+3-1}{4} = \binom{6}{4} = 15 \) possible criteria, namely, (G,Y,R) = (4,0,0), (3,0,1), (2,0,2), (1,0,3), (0,0,4), (3,1,0), (2,1,1), (1,1,2), (0,1,3), (2,2,0), (1,2,1), (0,2,2), (1,3,0), (0,3,1), or (0,4,0).

Further reduction to the number of criteria can be achieved by the adoption of acceptance/rejection truncation rules. That is, as soon as

\[ ^a \text{This is equivalent to the problem of finding the number of possible ways to put } n \text{ indistinguishable objects into } m \text{ distinguishable cells (see [36], p. 74, Exercise 5.3).} \]
the acceptance/rejection criteria are satisfied, the sample is either accepted or rejected without inspecting the rest of the items. For example, when we specify \( g = 1 \), the sample will be accepted right away if the first item is classified \( G \). When we specify \( r = 0 \), the sample will be rejected as soon as a \( R \) appears. When we specify \( y = 1 \), the sample will be rejected as soon as the number of \( Y \) is 2. Thus, in the previous example of \( n = 4 \), \( m = 3 \), if \( g = 1 \), \( y = 1 \) and \( r = 0 \) are imposed, the total number of criteria can be expressed in only 4 sets which is much smaller than either 15 combinations or 81 permutations. These four criteria are: acceptance on first \( G \); rejection on any \( R \); acceptance on one or fewer \( Y \) when there is no \( R \); and rejection on two or more \( Y \) when there is no \( R \).

In practice, two acceptance/rejection truncation rules are commonly used. First is the most widely used rejection truncation rule, \( r = 0 \). Since \( R \) indicates a real defective and its chance is relatively small as long as the process stays in control, it is quite reasonable to reject the sample whenever \( R \) is encountered.

The other commonly used truncation rule is \( G \) acceptance truncation (e.g., \( 0 < g < n \)). The reasoning for this rule is based on the concerns for effectiveness and efficiency in inspection timing. Ideally, the best timing for inspection is to make no measurements on the process except immediately following a process shift. But in practice, a process is subject to unknown spontaneous shifts occurring at unpredictable times. Therefore, the efficient control plan calls for a periodic small number of checks with additional gaging (up to the full sample size) whenever the initial gaging results hint that a process shift may have occurred. This tends to concentrate the gaging at times when a process shift has actually occurred. Thus, the control plans with acceptance truncation
rules seem to be more efficient than those regular non-truncation plans with an equal number of measurements taken periodically.

Although the adoption of acceptance/rejection truncation rules can certainly reduce the total number of inspections, they may not result in fewer or simpler Frequency Gaging rules as illustrated previously. For example, if \( n = 4, m = 3, r = 0 \), and acceptance/rejection decisions are made based on the combinations of G, Y, R, there will be as many as 16 possible truncation rules which are tabulated in Table 3.1. Obviously, further simplification on acceptance/rejection truncation rules is desirable.

**Sampling Frequency**

Given a set of FG rules, the Average Produced Quality (APQ) of the process can be improved merely by more frequently checking samples, since the shifts can be detected earlier. However, this quality improvement results in higher inspection costs. Thus the essential purpose for proper adjustment of the Sampling Frequency (SF) is to achieve an economic balance between high inspection cost resulting from overly frequent sampling, and high defective cost resulting from less frequent sampling.

In practice, there are two types of SF, namely, fixed SF and self-adjusting SF. The first kind takes samples for a fixed period of time or quantity of production. For example, take a sample of size 3 every production hour or every 1000 items produced. This method is easy to implement, but it lacks the flexibility to properly respond to the gradual deterioration or improvement of the process level.

The second approach self-adjusts SF in accordance with the frequency of OOC indications. It seeks to keep constant the average number of
### TABLE 3.1

POSSIBLE TRUNCATION RULES FOR n = 4, m = 3, r = 0
WITH ACCEPTANCE/REJECTION DECISIONS BASED
ON THE COMBINATIONS OF G, Y, R

Possible acceptance/rejection truncations occur
in the first i items of the sample

<table>
<thead>
<tr>
<th>i</th>
<th>Rule No.</th>
<th>Acceptance Truncation</th>
<th>Rejection Truncation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(a) The Main Table</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>≥1G (≤0Y) and OR</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>---</td>
<td>≥1Y or ≥1R</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>≥2G (≤0Y) and OR</td>
<td>≥2Y or ≥1R</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>≥2G (≤0Y) and OR</td>
<td>---</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>≥1G (≤1Y) and OR</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>---</td>
<td>≥2Y or ≥1R</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>---</td>
<td>≥1Y or ≥1R</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>≥3G (≤0Y) and OR</td>
<td>≥3Y or ≥1R</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>≥3G (≤0Y) and OR</td>
<td>≥2Y or ≥1R</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>≥3G (≤0Y) and OR</td>
<td>---</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>≥2G (≤1Y) and OR</td>
<td>≥3Y or ≥1R</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>≥2G (≤1Y) and OR</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>≥1G (≤2Y) and OR</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>---</td>
<td>≥3Y or ≥1R</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>---</td>
<td>≥2Y or ≥1R</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>---</td>
<td>≥1Y or ≥1R</td>
</tr>
</tbody>
</table>

*(b) An Illustration of Rule 11 in (a)*

<table>
<thead>
<tr>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>Trunc. at</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceptance Truncation</td>
<td>G</td>
<td>G</td>
<td>---</td>
</tr>
<tr>
<td>Rejection Truncation</td>
<td>---</td>
<td>R</td>
<td>---</td>
</tr>
</tbody>
</table>

Continuation: G Y Y none
inspected samples per OOC indication. Thus an increase in process shift frequency (with a consequent proportional increase in the number of defectives) is almost exactly counteracted by an increase in SF which proportionally reduces the time required to detect the process shift (and therefore the number of defectives produced before such detection). This approach can give a proper guarantee to the process APQ but it is more difficult to implement.

Qualification

There are times when the accuracy of each process setup or reset is suspect. The assurance that the process has indeed been adjusted to the targeted IC level before starting Frequency Gaging is desired. To achieve this purpose, Qualification (QL) rules are employed to reject all unsatisfied setups and resets, and to properly ensure that the process is in control before beginning FG.

Although the gages used in QL may not necessarily be the same as those used in FG, in practice it is more cost-effective to use the same set of gages in both QL and FG. Theoretically, any control plan which possesses a satisfactory capability to discriminate between good and bad process levels can serve as a QL rule. However, there is only one kind of QL rule ever seen in practice. This QL rule requires 100 percent inspection until a predetermined number of successive pieces, say 5, fall within the same NLG limits used in FG.

This scheme seems quite simple and easy to use. Unfortunately, it is very difficult to properly assess its Operating Characteristic (OC) curve which depicts the probability of acceptance as a function of the degree of process shift.
A practical QL rule would require an easy assessment of its OC curve as well as its easy implementation. It should utilize the same set of FG limit gages and its acceptance/rejection decision should be based upon combinations of G, Y, R, outcomes.

Retroactive Inspection

The APQ guaranteed by a specific SF used in conjunction with a specific FG rule may not be satisfactory. The APQ may be improved to some extent without changing the NLG plan by employing Retroactive Inspection (RI). Retroactive Inspection requires 100 percent inspection of all pieces produced since the most recently inspected sample whenever an OOC indication is obtained. Removal of any defectives found during the RI gives, for larger process shifts, an average outgoing fraction defective (AOQ) that will be substantially better than the APQ without RI. However, this improvement should be carefully evaluated against the consequent increase in inspection cost.

Examples

Following are two examples of NLG actually used in industry, which illustrate the contrast between lengthy wording and the concise notation introduced earlier in this chapter. Also, the relative importance of each NLG component (FG, SF, QL, and RI).

Example 1. The following set of NLG rules was created and first used by Jones and Lamson Machine Company [19] and then greatly popularized by Juran's [20] Quality Control Handbook (2nd edition, section 19). The rules read as follows:
1. Divide the tolerance band with NLG lines at 1/4 and 3/4 of the tolerance (which exceeds six standard deviations of the process).

2. Start job.

3. If piece is outside specification limits, reset.

4. If one piece is inside specification limits but outside a NLG line, check next piece.

5. If second piece is also outside same NLG line, reset.

6. If second piece is inside NLG line, continue process and reset only when two pieces in a row are outside a given NLG line.

7. If two successive pieces show one to be outside the high NLG line and one below the low NLG line, action must be taken immediately to reduce variation.

8. When five successive pieces fall between the NLG lines, frequency gaging may start. While waiting for five, if one piece goes over a NLG line, start count over again.

9. When frequency gaging, let process alone until a piece exceeds a NLG line. Check the very next piece and proceed as in 6 above.

10. When machine is reset, five successive pieces inside the NLG lines must again be realized before returning to frequency gaging.

11. If the operator checks more than 25 times without having to reset his process, his gaging frequency may be reduced so that more pieces are made between checks. If, on the other hand, he must reset before 25 checks are made, increase the gaging frequency. An average of 25 checks to a reset is indication that the gaging frequency is correct.

Now, this same set of rules can be described by using the proposed notation as follows:
FG: $USLLSL > 6$, $t = USLLSL/4$, $n = 2$, $m = 3$, $y = 1$, $g = 1$, $r = 0$

QL: 100% inspection until 5 consecutive G obtained

SF: 25 samples per OOC indication

RI: none.

Note that the proposed notation and procedure does not distinguish between $Y$ values which fall below the low NLG line and $Y$ values which fall above the high NLG line.

**Example 2.** The following NLG plan is used by a different major manufacturer [38]. Their description reads as follows: Suppose the work limit spread is equal to, or greater than, seven standard deviations, and NLG limits are established 1.5 standard deviations inside the work limits. A two-out-of-three NLG sampling plan is described herein:

A sample of three consecutive components is drawn and two of the components are gaged. The third is gaged only when necessary as per below:

**IN--NO ACTION REQUIRED**

(1) Both components in NLG limits.
(2) One in and one out of NLG limits (but within work limits) and the third component is in NLG limits.

**OUT--READJUST/CORRECT MACHINE**

(1) Any component out of work limits.
(2) Both components out on the same side of NLG limits.
(3) One in and one out of NLG limits (but within work limits) and the third component out on same side of NLG limits.

**OUT--MACHINE CAPABILITY QUESTIONABLE**

(1) When two components out of three (or two out of two) are both out of NLG limits, one high and one low, the operation is suspected of having too much variation. A machine capability study should be made with machine maintenance as necessary.

Now, this same set of rules can be described by using the proposed notation as follows:
The above analysis, discussion, and illustration of NLG taxonomy make clear the general structure of NLG, and demonstrate the potentially hazardous diversity of possible NLG rules. Without adequate simplification and standardization, the implementation, evaluation, design, and comparison of NLG plans will remain very difficult or even impossible. Among all four NLG components, FG is the most important and most complicated, and therefore needs to be substantially improved. The other three components, SF, QL, and RI, are relatively not as important and are less controversial. In practice, it is quite possible that QL and RI may not even be required.

Simplification and Standardization of NLG

To facilitate easy implementation, accurate numeric evaluation, concise expression, and convenient comparison for NLG plans, a simplified "standard" NLG is proposed in the following sections.

Frequency Gaging

It is recommended that in FG the parameters be constrained, and thereby simplified. Only $m = 2$ or $m = 3$ should be considered, since $m > 3$ will result in complicated NLG gages and cumbersome gaging procedures. The NLG control inset $t$ should always be measured inward from the...
specification limits rather than measured outward from the center of the specification interval. This puts more emphasis on "defective control" rather than "shift control." In other words, as long as the process keeps producing satisfactory products, the process level is allowed to shift. Finally, when \( m = 3 \), a \( R \) should represent a real defective and the process should always be rejected.

Acceptance/rejection criteria may also be simplified. Acceptance/rejection decisions should be based on combinations (rather than permutations) of \( G, Y, R \) such that truncation possibilities are maximized. By letting \( r = 0 \), and therefore tolerating no \( R \), maximum rejection truncation can be achieved. Field implementation and numeric evaluation will also be made much easier if \( r = 0 \). Rejection truncation should also be applied to \( Y \). Whenever the cumulative number of \( Y \) in a sample exceeds \( y \), the sample should be rejected and inspection truncated. Even acceptance truncation can be allowed. This should be allowed to occur only when \( g \) straight \( Gs \) are obtained from the beginning of the sample. The rule "\( g \) straight \( Gs \) from the beginning" is more advantageous than the rule "\( g \) \( Gs \) out of first \( x \) pieces" in terms of easy implementation and evaluation.

Based upon the above discussion, simplified standard NLG FG rules are summarized as below:

- \( n \)--should be kept small (often in the range from 2 to 6)
- \( m \)--only \( m = 2 \) or \( m = 3 \) are considered
- \( t \)--\( 0 < t < USL/2 \) and is always measured inward from USL and LSL
- \( r \)--\( r = 0 \) and the sample is rejected and inspection truncated as soon as a \( R \) is encountered
- \( y \)--\( 0 \leq y \leq n \) (usually in the range \( 0 \leq y \leq \text{INTEGER} \,(n/2+5) \)). Whenever
the cumulative number of $Y$ in a sample exceeds $y$, the sample is rejected and inspection truncated.

$g = 0 \leq g \leq n - 1$ (usually in the range $0 \leq g \leq \text{INTEGER} \left(\frac{n}{2+\cdot 5}\right)$). As soon as $g$ consecutive $G$s from the beginning of the sample are obtained, acceptance occurs and inspection is truncated.

**Sampling Frequency**

No rigid $SF$ rule is proposed; rather, the $SF$ depends upon a user's individual need. If the user is concerned with having proper assurance of $APQ$ of the process, a self-adjusting $SF$ is suggested. That is, keep constant the average number of inspected samples per $OOC$ indication (approximately 25 to 50 samples per $OOC$ indication is recommended in Reference [20]). On the other hand, if the user is not concerned about the $APQ$, any other $SF$ scheme may be selected.

**Qualification**

To simplify the evaluation, design, and implementation of the QL rule, the concepts underlying single acceptance sampling are adopted. It is recommended that QL make use of the same $m$, $t$, $r$ values from FG and also that $g = 0$. Thus only $n$ and $y$ are allowed to vary. By proper manipulation of $n$ and $y$, QL's OC curve can be adjusted to the user's desired shape. Standardized QL is summarized as follows:

- $n$--free to vary
- $m$--same as that used in FG
- $t$--same as that used in FG
- $r$--same as that used in FG (i.e., $r = 0$)
\[ y - n \leq y \leq n, \text{ free to vary} \]
\[ g = 0. \]

**Retroactive Inspection**

It is recommended in RI that all pieces produced since the most recent acceptable sample be 100 percent inspected whenever an OOC indication is obtained.

**Comments**

After adequate simplification and standardization, this easy-to-implement, precise-to-evaluate, and concise-to-express version of standardized NLG scheme will certainly have broader application in industry. All later chapters are based upon the standard NLG version as proposed above.

For practical purposes, the implementation of NLG does not require all four of the components discussed above. Except for the mandatory FG, selection of SF, QL, and RI essentially depends upon the user's individual needs. For example, if the user does not care about the assurance of APQ, a simple SF rule may be specified rather than a self-adjusting SF rule as discussed above, which is harder to implement. If the user has no reason to suspect problems in process setup, and resets, there is no need to include the QL rule in a NLG plan. Similarly, if it is desired to improve the APQ by any means other than screening inspection, or if the 100 percent inspection is relatively costly, RI will never be needed.

In all, to better suit individual needs, the user must always carefully evaluate the particular situation before deciding exactly which components to be included in the NLG plan.
CHAPTER IV

STATISTICAL EVALUATION AND DESIGN OF STANDARD (STD) NLG PLANS; COMPARISONS WITH $\bar{X}$-CHARTS

Introduction

This chapter first discusses the statistical evaluation of Standard (STD) NLG plans. The calculation methods for both samplewise and processwise performance measures are derived. Then, the statistical design of STD NLG is developed. Greater details are provided for the design procedures of both FG and QL, while a more general approach is given to the processwise design. Finally, after the derivation of methodologies for evaluating and designing $\bar{X}$-charts, a comparison between STD NLG and $\bar{X}$-charts is provided through an example.

Notation

In addition to the notation introduced in Chapter III, the following terms are employed to facilitate this chapter's discussion:

STD NLG--Standard NLG plan which is described in Chapter III

$P_g$, $P_y$, $P_r$--probability of an inspected item being classified as Green, Yellow, Red, respectively

$\phi$, $\phi^{-1}$--$\phi$ is the cumulative probability function of the standard normal distribution; $\phi^{-1}$ is the inverse function of $\phi$

$\mu$, $\mu_o$--$\mu$ is the process mean which has the value $\mu_o$ before any shifting occurs
\[\sigma, \sigma_0 \text{-- } \sigma \text{ is the process standard deviation which has the value of } \sigma_0 \text{ before shifting.}\]

\[\delta \text{-- the distance (in multiples of } \sigma_0 \text{) between shifted } \mu \text{ and } \mu_0\]

\[p, p_o \text{-- } p \text{ is the process fraction defective which is also called the process level; it has the value of } p_o \text{ before shifting. } 0 \leq p \text{ (or } p_o) \leq 1\]

\[P_a (p \text{ or } \delta) \text{-- the probability of acceptance of a sample, which is a function of } p \text{ or } \delta\]

\[E_n (p \text{ or } \delta) \text{-- average number of pieces inspected in a sample of size } n, \text{ which is a function of } p \text{ or } \delta; \text{ it is also known as average sample number or average inspection number}\]

\[\text{ARL} (p \text{ or } \delta) \text{-- average run length; average number of samples inspected before deciding to reset. } \text{ARL}(p) = 1/(1 - P_a(p)). \text{ Likewise, } \text{ARL}(\delta) = 1/(1 - P_a(\delta))\]

\[\text{PBAPQ} \text{-- probability bound on average produced quality}\]

\[\text{PBAOQ} \text{-- probability bound on average outgoing quality resulting from employing RI}\]

\[F \text{-- average number of samples per OOC indication; it is known as self-adjusting sampling frequency}\]

\[\text{APL} \text{-- acceptable process level which is a satisfactorily small } p \text{ or } \delta \text{ value; the process is considered functioning well at this quality level}\]

\[\text{RPL} \text{-- rejectable process level which is an undesirably large } p \text{ or } \delta \text{ value; the process is considered functioning poorly at this quality level}\]
TLAPL, TLRPL—user-specified lower tolerable limit of $P_a$ (APL) and upper tolerable limit of $P_a$ (RPL), respectively; in other words, values of $P_a$ (APL) $\geq$ TLAPL and $P_a$ (RPL) $\leq$ TLRPL are desired.

$v$—in the modified $\bar{X}$-chart, $v$ is the distance in multiples of $\sigma_0$ between a specification limit and the corresponding boundary for an acceptable process mean.

For both traditional and designed $\bar{X}$-charts, $v = \frac{USLLSL}{2}$ (see section entitled "Evaluation and Design of $\bar{X}$-Charts").

$k$—control limit spread in multiples of $\sigma_0/\sqrt{n}$ for $\bar{X}$-charts. In both traditional and designed $\bar{X}$-charts, control limits are $k\sigma_0/\sqrt{n}$ outward from $\mu_0$. In modified $\bar{X}$-charts, control limits are $k\sigma_0/\sqrt{n}$ outward from the boundary of the acceptable process mean on each side (see section entitled "Evaluation and Design of $\bar{X}$-Charts").

UCL, LCL—upper and lower control limits of $\bar{X}$-charts, respectively.

Statistical Evaluation of STD NLG Plans

Assumptions

In order to present exact formulations of numerical evaluations, several assumptions concerning STD NLG parameters are explicitly stated here:

1. The process characteristic of interest is normally distributed with mean $\mu$ and standard deviation $\sigma$. Before shifting occurs, $\mu = \mu_0$ and $\sigma = \sigma_0$. 
2. The specification tolerance is $(USL - LSL) \geq 6\sigma_0$ (or $USL - LSL \geq 6$).

3. The process may shift in either one (but not both) of the following two forms:
   
   a. Process mean may shift away from $\mu_0$ in either direction.
   
   b. Process dispersion may increase and become greater than $\sigma_0$.

   These assumptions will be maintained throughout this research. Possible relaxations and their effects will be discussed later.

**Formulation of Probabilities of $G$, $Y$, $R$**

Under the above assumptions, and given values of $m, t, USL, LSL,$ and $\sigma_0$, the probabilities of $G$, $Y$, $R$ can be obtained. The formulations are derived for three different cases, namely (1) before any process shift, (2) after a process mean shift, and (3) after a process dispersion change. First, $m = 3$ is considered for each of the three cases.

**Case 1:** Before any shift occurs, the process has a normal distribution with mean $\mu_0$ and standard deviation $\sigma_0$. Its probabilities of $G$, $Y$, $R$, namely, $P_g$, $P_y$, $P_r$, respectively, can be derived as follows (see Figure 4.1(a)): Let

$$H = \frac{USL - LSL}{2} = \frac{USL - LSL}{2\sigma_0}$$

$$P_r = \phi(-H) + [1 - \phi(H)] = 2\phi(-H)$$

$$P_g = \phi(H - t) - \phi[(-H - t)]$$

$$P_y = 1 - P_g - P_r$$

**Case 2:** While the process standard deviation remains constant, the process mean shifts $\delta\sigma_0$ from $\mu_0$ and results in a fraction defective $p_1$. The calculation of $P_g$, $P_y$, and $P_r$ can be derived as follows (see Figure 4.1(b)): 
(a) Case 1: Both $\mu$ and $\sigma$ Remain Unchanged

$(\mu = \mu_0, \sigma = \sigma_0)$

(b) Case 2: $\mu$ Shifts While $\sigma$ Remains Unchanged

$(\mu = \mu_1, \sigma = \sigma_0)$

(c) Case 3: $\sigma$ Increases While $\mu$ Remains Unchanged

$(\mu = \mu_0, \sigma = \sigma_2)$

Figure 4.1. Three Cases of Process Shifts Under the Surveillance of an NLG Plan
If $\delta$ is given, $p_1$ can be obtained as:

$$p_1 = 1 - \phi(H + \delta) + \phi(-H + \delta)$$

If $p_1$ is given, $\delta$ can be approximately calculated as:

$$\delta = \phi^{-1}(p_1) + H$$

where $p_1 > p_0$ and USLLSL > 6 are assumed. The greater the differences in both equalities, the better the approximation.

For both situations,

$$P_r = p_1$$

$$P_g = \phi(H - t + \delta) - \phi[-(H - t) + \delta]$$

$$P_y = 1 - P_g - P_r$$

Case 3: While the process mean stays at $\mu_0$, the process standard deviation increases to $\sigma_2$ and results in a fraction defective $p_2$. The calculation can be derived as follows (see Figure 4.1(c)):

If $\sigma_2$ is given, $p_2$ can be obtained as

$$p_2 = 2\phi(-H\sigma_0 / \sigma_2)$$

If $p_2$ is given, $\sigma_2$ can be calculated as

$$\sigma_2 = -H\sigma_0 / \phi^{-1}(p_2/2)$$

For both situations,

$$P_r = p_2$$

$$P_g = \phi[(H - t) \sigma_0 / \sigma_2] - \phi[-(H - t) \sigma_0 / \sigma_2]$$

$$= 2(0.5 - \phi[-(H + t) \sigma_0 / \sigma_2]) = 1 - 2\phi[(-H + t) \sigma_0 / \sigma_2]$$

$$P_y = 1 - P_g - P_r$$
When $m = 2$, the formulations for the above three cases still apply, where $P_g$ remains the same, but $P_y = 1 - P_g$ and $P_r$ no longer exists.

**Formulation of Performance Measures**

for Frequency Gaging

Probability of acceptance ($P_a$), Average Run Length (ARL), and average number of inspections in a sample ($E_n$) are the three most important performance measures in FG. The ARL is a function of $P_a$, namely $ARL = 1/(1 - P_a)$. Therefore, it suffices to consider only the formulations of $P_a$ and $E_n$. Also, since the derivations of $P_g$, $P_y$, and $P_r$ have been developed in the last section, it is convenient to express $P_a$ and $E_n$ in terms of $P_g$, $P_y$, and $P_r$ instead of the original NLG parameters.

**Probability of Acceptance ($P_a$).** In the derivation of $P_a$, the simpler case without G acceptance truncation is first considered. That is, only Y and R rejection truncations are considered. Then the formulation is advanced to accommodate G acceptance truncation. Finally, all formulas are summarized into a single general equation which suits both situations.

1. For $g = 0$, without G acceptance truncation:

For $m = 2$, the sample is accepted if and only if the total number of $Y$ is no greater than $y$. This number is binomially distributed. Similarly, for $m = 3$, in addition to the above condition, no R can be tolerated. Now, the combinations of numbers of G, Y, R become multinomially distributed. But since the number of R is restricted to 0, this multinomial distribution actually reduces to the binomial. Thus,
when \( m = 2, \ g = 0 \):

\[
p_a = \sum_{i=0}^{\gamma} \binom{\gamma}{i} \ p^i \ p^{\gamma-i}
\]

where \( P_y = 1 - P_g \).

when \( m = 3, \ g = 0 \):

\[
p_a = \sum_{b=0}^{n} \frac{n!}{a!b!(n-a-b)!} \ p^a \ p^b \ p^{n-a-b} = \sum_{i=0}^{\gamma} \binom{\gamma}{i} \ p^i \ p^{\gamma-i}
\]

where \( P_y = 1 - P_g - P_r \).

2. For \( 0 < g \leq n - 1 \) (and hence \( y > 0 \)), \( ^a \) G acceptance truncation allowed:

When acceptance truncation is allowed, \( P_a \) may become larger than that with no truncation. This is due to the acceptance of the whole acceptance-truncated "branch" (of the probability tree) in which there might be some "paths" which would be rejected should no acceptance truncation be allowed. This additional probability of acceptance is therefore added to the previous formulas in (1) to account for the increase in \( P_a \).

For both \( m = 2 \) and \( m = 3 \), the value of \( P_a \) is:

\[
p_a = \sum_{i=0}^{\gamma} \binom{\gamma}{i} \ p^i \ p^{\gamma-i} + p_g \ [1 - \sum_{j=0}^{s} \binom{n-g}{j} \ p^j \ p^{n-g-j}]
\]

where \( s = \min (y, n - g) \). In this formula, the first term represents the

\(^a\)The condition \( g > 0 \) implies that \( y > 0 \). If \( g > 0 \) and \( y = 0 \), inspection will always be truncated and never reach its full sample size.
with no acceptance truncation. The second term calculates the addition to $P_a$ made possible by acceptance truncation.

3. In general, for both $g = 0$ and $g > 0$:

The value of $P_a$ can now be expressed in the following summarized single equation which suits both situations:

$$P_a = \sum_{i=0}^{y} \binom{n}{i} p^i g^{n-i} + \sum_{j=0}^{s} \binom{n-g}{j} p^j g^{n-g-j}$$

where $s$ is $\min(y, n-g)$; and $I_g$ is an indicator function: $I_g = 1$ if $g > 0$ (hence $y > 0$), = 0 otherwise.

Average Number of Inspections ($E_n$). Similar to the derivation of $P_a$, the average number of inspected pieces in a sample ($E_n$) is first derived for the simpler no $G$ acceptance truncation case. Then the formulation is advanced to take into account the effect of $G$ acceptance truncation. Finally, a summarized formula is developed to suit both situations.

In the following derivation of $E_n$, $m = 2$ and $m = 3$ are treated separately. Since $n = 1$ results in $E_n = 1$, only $n \geq 2$ are considered.

1. For $g = 0, m = 2, n \geq 2$:

Three cases are considered: $y = 0, 0 < y \leq n - 2, y \geq n - 1$.

a. $y = 0$: Whenever a $Y$ is encountered, the sample is rejected and inspection truncated. This truncation can occur anywhere between the first and next to last item. Summing up the product of the numbers of items inspected and their corresponding probabilities of truncation at those numbers results in $E_n$. Thus,
\[
E_n = \sum_{i=1}^{n-1} ip_i^{i-1} \left( p + p \right) + np^{n-1}
\]

\[
= \sum_{i=1}^{n-1} ip_i^{i-1} \left( 1 - p \right) + np^{n-1}
\]

b. \(0 < y \leq n - 2\): Truncation can only occur on or after the \(y+1\)st item. As soon as the number of \(Y\) reaches \(y+1\), the inspection is truncated. Therefore, if truncation occurs at the \(i\)th item \((i > y)\), the \(i\)th item must be classified as \(Y\), and the rest of \(y\) \(Y\)'s can be scattered among the previous \(i-1\) items, which results in \(\binom{i-1}{y}\) combinations. Thus,

\[
E_n = \sum_{i=y+1}^{n-1} i\binom{i-1}{y} p_Y + \sum_{i=y+1}^{n-1} n[i - \sum_{i=y+1}^{n-1} i\binom{i-1}{y} p_Y]
\]

c. \(y > n - 1\): No truncation occurs in this case. Thus,

\[
E_n = n.
\]

2. For \(g = 0, m = 3, n \geq 2\)

For \(m = 3\), in addition to \(Y\) rejection truncation (i.e., the number of \(Y\) is greater than \(y\)), the sample is also rejected whenever a \(R\) is encountered. Based upon similar reasoning, the formulations in (1) above are now modified to accommodate the \(R\) rejection effect.

a. \(y = 0\):

\[
E_n = \sum_{i=1}^{n-1} ip_i^{i-1} \left( p_y + p_r \right) + np^{n-1}
\]

\[
= \sum_{i=1}^{n-1} ip_i^{i-1} \left( 1 - p \right) + np^{n-1}
\]
b. $0 < y \leq n-2$: On or before the $y$th item, only $R$ truncation can occur. On or after the $(y+1)$st item, both $Y$ truncation and $R$ truncation can occur. Thus,

$$E_n = \sum_{i=1}^{y} i(1-S_{i-1}) P_r + \sum_{i=y+1}^{n-1} i[(1-S_{i-1}) P_r + (1-y)_y^y p^{y+1} g^{i-1-y}] + n(1-S_{n-1})$$

$$= \sum_{i=1}^{n} iU_i$$

where

$$S_0 = 0$$

$$S_i = S_{i-1} + U_i$$  \hspace{1cm} \text{for } 0 < i \leq n - 1$$

$$U_i = (1-S_{i-1}) P_r$$  \hspace{1cm} \text{for } 1 \leq i \leq y$$

$$= (1-S_{i-1}) P_r + (1-y)_y^y p^{y+1} g^{i-1-y}$$  \hspace{1cm} \text{for } y < i \leq n - 1$$

$$= 1-S_{n-1}$$  \hspace{1cm} \text{for } i = n$$

For example, if $n = 5$, $m = 3$, $y = 2$, $g = 0$, $r = 0$

$$U_1 = P_r$$

$$U_2 = (1-U_1) P_r$$

$$U_3 = (1-U_1-U_2) P_r + \binom{2}{2} p^3 \quad g^0$$

$$U_4 = (1-U_1-U_2-U_3) P_r + \binom{3}{2} p^3 \quad g^1$$

$$U_5 = 1-U_1-U_2-U_3-U_4$$

$$E_n = \sum_{i=1}^{5} iU_i$$
c. $y \geq n - 1$: Only $R$ truncations can occur in this case. Thus,

$$E_n = \sum_{i=1}^{n-1} i(1-S_{i-1})P_r + n(1-S_{n-1}) = \sum_{i=1}^{n} iU_i$$

where

$$S_0 = 0$$

$$S_i = S_{i-1} + U_i \quad \text{for} \quad 0 < i \leq n - 1$$

$$U_i = (1-S_{i-1})P_r \quad \text{for} \quad 1 \leq i \leq n - 1$$

$$= 1 - S_{n-1} \quad \text{for} \quad i = n$$

3. For $0 < g \leq n - 1$, $m = 2, n \geq 2$

Acceptance truncation $g > 0$ also implies that $y > 0$; otherwise, the process will always be truncated before reaching the full sample size. Therefore, only two cases are considered: $0 < y \leq n - 2$ and $y \geq n - 1$. In both cases, the acceptance truncation effect is added to the formulas in (1) above.

a. $0 < y \leq n - 2$:

$$E_n = \sum_{i=y+1}^{n-1} i(i-1)y^{y+1}p^{i-1-y}p^g + g^p^g \quad + n[1 - \sum_{i=y+1}^{n-1} (i-1)y^{y+1}p^{i-1-y}p^g]$$

b. $y \geq n - 1$:

$$E_n = g^p^g + n[1 - p^g]$$

4. For $0 < g \leq n - 1$, $m = 3, n \geq 2$

Similar to (3) above, the formulas in (2) above are revised to
account for the G acceptance truncation effect for the $0 < y \leq n - 2$ and $y \geq n - 1$ cases.

a. $0 < y \leq n - 2$:

$$E_n = \sum_{i=1}^{n} iU_i$$

where

$$S_0 = 0$$

$$S_i = S_{i-1} + U_i \quad \text{for } 0 < i \leq n - 1$$

$$U_i = (1 - S_{i-1}) P_r \quad \text{for } 1 \leq i \leq y \text{ and } g \neq i$$

$$= (1 - S_{i-1}) P_r + p^g \quad \text{for } 1 \leq i \leq y \text{ and } g = i$$

$$= (1 - S_{i-1}) P_r + (i-1) p^{y+1} p^{i-1-y} y \quad \text{for } y < i \leq n - 1 \text{ and } g \neq i$$

$$= (1 - S_{i-1}) P_r + (i-1) p^{y+1} p^{i-1-y} + p^g \quad \text{for } y < i \leq n - 1 \text{ and } g = i$$

$$= 1 - S_{n-1} \quad \text{for } i = n$$

b. $y \geq n - 1$:

$$E_n = \sum_{i=1}^{n} iU_i$$

where

$$S_0 = 0$$

$$S_i = S_{i-1} + U_i \quad \text{for } 0 < i \leq n - 1$$

$$U_i = (1 - S_{i-1}) P_r \quad \text{for } 1 \leq i \leq n - 1 \text{ and } g \neq i$$

$$= (1 - S_{i-1}) P_r + p^g \quad \text{for } 1 \leq i \leq n - 1 \text{ and } g = i$$

$$= 1 - S_{n-1} \quad \text{for } i = n$$
5. Summary for \( m = 2, 0 \leq g \leq n - 1, n \geq 2 \)

a. For \( y = 0 \) and \( g = 0 \):

\[
E_n = \sum_{i=1}^{n-1} i p^{i-1} p^y + n p^{n-1} g
\]

b. For \( 0 < y \leq n - 2 \) and \( 0 \leq g \leq n - 1 \):

\[
E_n = \sum_{i=y+1}^{n-1} i(i-1) p^{y+1} p^{i-y} + l_g p^g
\]

\[
+ n[l - \sum_{i=y+1}^{n-1} (i-1) p^{y+1} p^{i-y} - l_g p^g]
\]

where the indicator function

\[
l_g = 1 \text{ if } g > 0
\]

\[
= 0 \text{ if } g = 0.
\]

c. For \( y \geq n - 1 \) and \( 0 \leq g \leq n - 1 \):

\[
E_n = l_g p^g + n[l - l_g p^g]
\]

where the indicator function \( l_g \) is defined as above.

6. Summary for \( m = 3, 0 \leq g \leq n - 1, n \geq 2 \)

a. For \( y = 0 \) and \( g = 0 \):

\[
E_n = \sum_{i=1}^{n-1} i p^{i-1} (i - p) + n p^{n-1} g
\]

b. For \( 0 < y \leq n - 2 \) and \( 0 \leq g \leq n - 1 \):

\[
E_n = \sum_{i=1}^{n} i U_i
\]

where
\[ S_0 = 0 \]
\[ S_i = S_{i-1} + U_i \quad \text{for } 0 < i \leq n - 1 \]
\[ U_i = (1 - S_{i-1}) P_r + I_i (i-1) p_y^1 p_y^{i-1} g + J_i p_g \quad \text{for } 1 \leq i \leq n - 1 \]
\[ = 1 - S_{n-1} \quad \text{for } i = n \]

where the indicator functions
\[ I_i = 1 \quad \text{for } y < i \leq n - 1 \]
\[ J_i = 1 \quad \text{for } i = g \]
\[ = 0 \quad \text{for } 1 \leq i \leq y \]
\[ = 0 \quad \text{for } i \neq g \]

\[ c. \quad \text{For } y \geq n - 1 \text{ and } 0 \leq g \leq n - 1: \]
\[ E_n = \sum_{i=1}^{n} i U_i \]

where
\[ S_0 = 0 \]
\[ S_i = S_{i-1} + U_i \quad \text{for } 0 < i \leq n - 1 \]
\[ U_i = (1 - S_{i-1}) P_r + J_i p_g \quad \text{for } 1 \leq i \leq n - 1 \]
\[ = 1 - S_{n-1} \quad \text{for } i = n \]

where
\[ J_i = 1 \quad \text{for } i = g \]
\[ = 0 \quad \text{for } i \neq g. \]

**Formulation of Performance Measures for Qualification**

The performance measures for QL are exactly the same as those for FG.
Given values of n and y, letting g = 0, and keeping the same m, t, r values determined for FG, \( P_a \), and \( E_n \), can readily be evaluated by the same set of formulas derived in the previous section for FG.

**Formulation of Performance Measures for the Process as a Whole**

In evaluating the performance of the whole process, Average Produced Quality (APQ) and Average Outgoing Quality (AOQ) are the two performance measures to be investigated. Considering the process as a whole, APQ indicates the long term average of the quality produced by the process, while AOQ represents the long term average of the improved quality after RI.

**Probability Bound of APQ (PBAPQ).** In order to obtain the exact APQ value, the mean of the time-to-shift distribution of the process must be known. However, this mean may not be easy to estimate. Fortunately, the self-adjusting SF rule can help provide a somewhat conservative estimation of APQ, namely the Probability Bound of APQ (PBAPQ) without knowledge of the mean time-to-shift. This PBAPQ provides a guarantee on the limit of the APQ. In other words, in the long term, the process APQ should be no worse than the PBAPQ.

Following are assumptions needed for the formulation of PBAPQ:

1. The probability of a false alarm is relatively small compared to that of a true alarm.

2. The inspection time, the assignable cause searching time, and the time to reset the process are relatively negligible.
3. The number of pieces inspected is relatively small compared to the number of pieces produced.

4. A second process shift does not occur until the first is detected.

5. Qualification (if needed) takes a relatively short period of time compared to that for FG.

Based on these assumptions, the formula for the PBAPQ can be approximated as follows (see Figure 4.2):

$$PBAPQ(p) = \frac{1}{F} \left[ p \left( \frac{1}{1-P_d(p)} - 0.5 \right) + P_o \left( F - \frac{1}{1-P_d(p)} + 0.5 \right) \right]$$

where

- $p$ = fraction defective produced by the shifted process;
- $P_o$ = fraction defective produced by an unshifted process;
- $F$ = average number of samples per OOC indication; and
- $1-P_d(p)$ = probability of an alarm (i.e., an OOC indication) for a process having the fraction defective $p$.

Here $1/[1-P_d(p)]$ is the average number of samples required to detect the shifted process and $1/[1-P_d(p)] - 0.5$ is the average number of inspection intervals between the process shift and its detection, which must be confined in the range of 0 and $F$ to be meaningful. The factors $p$ and $P_o$ are weighted by the expected length of the OOC and IC intervals, and division by $F$ spreads these defectives over the entire period since the previous OOC indication. Finally, without including the mean time-to-shift, the above formulation can therefore only represent an upper bound of the true APQ.

For a specified $F$ and $SF$, a small value of $p$ can make the OOC indication occur very infrequently in $F$ samples no matter how large the
Figure 4.2. NLG Frequency Gaging Cycle
intervals are, and hence impede implementation of the SF rule. This follows because \(1/[1 - P_a(p)] - 0.5\) cannot exceed \(F\). In other words, \(1 - P_a(p)\) must be greater than \(1/(F + 0.5)\) to some extent to make the implementation of \(F\) samples per OOC indication possible. If this does not occur, either \(F\) can be increased or stricter FG rules can be employed to overcome this difficulty.

The closeness of the PBAPQ to the true APQ depends upon the difference between \(1 - P_a(p)\) and \(1/(F + 0.5)\). The larger the difference (i.e., \(1 - P_a(p) << 1/(F + 0.5)\)), the closer the PBAPQ to APQ. Furthermore, the length of the mean time-to-shift will also affect this accuracy. In all cases, PBAPQ(p) can never exceed \(p\).

**Probability Bound of AOQ (PBAOQ).** RI calls for inspection of all pieces since the last inspection whenever an OOC indication is obtained. Therefore, no defectives are left in the lot if the control plan picks up the process shift on the first sample after the process shift occurs. But the plan does not always pick it up on the first inspection. Rather, RI can eliminate the defectives of only one interval per \(F\) samples. Therefore, the upper bound of the AOQ becomes

\[
PBAOQ(p) = \frac{1}{F} \left[ p \left( \frac{1}{1 - P_a(p)} - 0.5 - 1 \right) + P_o \left( F - \frac{1}{1 - P_a(p)} + 0.5 \right) \right]
\]

where \(1/[1 - P_a(p)] - 1.5\) must be confined in the range of 0 and \(F\) to be meaningful.
Comments

All of the above formulations ($P_g, P_y, P_r, P_a, E_n, PBAPQ,$ and $PBAOQ$) are based upon the normality assumption which can now be relaxed. For any other distribution, after replacing $\phi$ and $\phi^{-1}$ by the corresponding cumulative and inverse cumulative distribution functions, all of these formulations still apply.

The assumption that $USLLSL \geq 6$ can also be relaxed. This assumption facilitates a better $P_g, P_y$ approximation when an unknown $\delta$ is derived from a given $p$ under the process mean shift condition. For a smaller $USLLSL$ value, $\delta$ can still be obtained to any desirable accuracy from a given $p$ value by employing an iterative procedure. This procedure first evaluates the sum of the $p$ areas under both tails as a function of a trial $\delta$ value and then repeatedly adjusts $\delta$ until its corresponding $p$ value is close enough to the given $p$.

When evaluating the process as a whole, PBAPQ and PBAOQ can only be used as conservative approximations of real APQ and AOQ values. However, if in implementation the mean time-to-shift and the assignable cause searching time have been acquired, APQ and AOQ can be more accurately evaluated based on similar reasoning to that used in the PBAPQ and PBAOQ derivation.

Statistical Design of STD NLG Plans

Introduction

Traditionally, the commonly used statistically based process control plans such as the $\bar{X}$-chart, $p$ chart, and $c$ chart are implemented without any design consideration. Their performances are rarely adequately
understood by the user and may well not fit the user's own particular need. Consequently, these plans may result in misuse.

In order to help one understand the performance of multi-parameter NLG plans, the statistical design procedure of STD NLG is derived in this section. The general effects of NLG parameters on $P_a$ and $E_n$ are first presented. These measures are critical in understanding NLG's performance and can facilitate its design in each step. Then, detailed design procedures of FG and QL follow. Finally, this section is concluded by a discussion of the general strategy for process-wise NLG design.

General Effects of STD NLG Parameters on $P_a$ and $E_n$

The general effects of each of the parameters $n$, $t$, $y$, $g$ on FG performance measures $P_a$ and $E_n$ are investigated for both the $m = 2$ and $m = 3$ cases under either mean shift or dispersion change conditions. Beginning with a base plan ($USL\downarrow SL = 7, \ n = 3, \ t = 1, \ y = 1, \ g = 1, \ r = 0$), each parameter is freed to vary one at a time while the rest remain fixed. Table 4.1 shows the range of variation for each individual parameter. It also identifies the figures which depict the effects of parameter variations on performance measures $P_a$ and $E_n$. Each figure contains four graphs: (1) $m = 2$ with mean shift, (2) $m = 3$ with mean shift, (3) $m = 2$ with dispersion change, and (4) $m = 3$ with dispersion change. In the $y$ effect example, the reason for specifying $g = 0$ instead of $g = 1$ as used in the base case is to show the effect of $y = 0$, since $g = 1$ implies $y > 0$ as explained previously.

Effects on $P_a$. In the following discussion, conclusions are based on the mean shift assumption; however, the effects due to dispersion
TABLE 4.1
PARAMETER RANGE AND RELEVANT FIGURE NUMBER FOR INDIVIDUAL NLG PARAMETER EFFECT ON $P_a$ AND $E_n$

| Parameter Effect | t | y | g | n | Relevant Figure | $P_a$ | $E_n$
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>t Effect</td>
<td>0.5, 1, 1.5, 2</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>Fig. 4.3, Fig. 4.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y Effect</td>
<td>1</td>
<td>0, 1, 2, 3</td>
<td>0</td>
<td>3</td>
<td>Fig. 4.4, Fig. 4.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>g Effect</td>
<td>1</td>
<td>1</td>
<td>0, 1, 2, 3</td>
<td>3</td>
<td>Fig. 4.5, Fig. 4.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n Effect</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2, 3, 5, 8</td>
<td>Fig. 4.6, Fig. 4.10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
changes are quite similar. Also, in general, \( m = 2 \) and \( m = 3 \) have similar results. Therefore, their differences are discussed only when necessary. For all graphs, \( P_a \) is usually decreasing (and always nonincreasing) as the process fraction defective \( P \) increases.

The effect of \( t \) is shown in Figure 4.3. For a given process level \( p \), as \( t \) increases, \( P_a \) decreases. This is because larger \( t \) values cause smaller \( P_g \) and larger \( P_y \) (while \( P_r \) remains the same), which consequently yield more Ys and fewer Gs.

The effect of \( y \) is shown in Figure 4.4. Under the same process level \( p \), as \( y \) increases, \( P_a \) also increases. This is because when \( y \) increases, more Ys are tolerable. In other words, larger \( y \) means a more lenient acceptance criterion. Among \( y = 0, 1, 2, 3 \), \( y = 0 \) has a very severe impact on the reduction of \( P_a \). It should be noted that when \( m = 2, y = 3 \), acceptance always occurs regardless of process levels. On the other hand, due to R rejection, the \( P_a \) of \( m = 3 \) and \( y = 3 \) yields the usual declining OC curve. Finally, the OC curve of \( y = 2 \) and \( y = 3 \) are very close to each other.

The effect of \( g \) is shown in Figure 4.5. There, \( g = 0 \) and \( g = 3 \) are essentially the same plan. They are just two different expressions for the same situation. Generally, \( P_a \) decreases as \( g \) increases (from \( 1 \) to \( n \)), given the same process level. This is because smaller \( g \) (excluding \( g = 0 \)) causes earlier acceptance truncation, which converts more original rejection paths (those which should be rejected if no G acceptance truncation is allowed) into acceptance paths. In this example, \( g = 2 \) and \( g = 3 \) have the same OC curve when \( m = 2 \).
(a) $m = 2$; Mean Shift

(b) $m = 3$; Mean Shift

(c) $m = 2$; Dispersion Change

(d) $m = 3$; Dispersion Change

Figure 4.3. The Effect of $t$ on $P_a$
Figure 4.4. The Effect of $y$ on $P_a$
Figure 4.5. The Effect of \( g \) on \( P_a \)
The effect of $n$ is shown in Figure 4.6. Under the same process level, $P_a$ decreases as $n$ increases. This is because for the same process fraction defective, the average number of $Y$ in a sample should increase proportionally as the sample size $n$ increases. Consequently, to increase $n$ without increasing $y$ accordingly will certainly result in a stricter NLG plan and hence smaller $P_a$.

In short, the increases of $t$, $g$, or $n$, or the decrease of $y$, all result in steeper OC curves which provide better discrimination between good and bad process levels, but at the price of a higher false alarm rate. Among $t$, $y$, $g$, and $n$, the value of $P_a$ (and hence the OC curve) is more sensitive to the adjustment of $t$ and $y$, but less sensitive to that of $g$ and $n$.

Effects on $E_n$. Similar to the previous section, the following discussions are based only on the mean shift assumption. Effects of dispersion changes are quite similar. Also, in general, $m = 2$ and $m = 3$ have similar results. Their differences are pointed out only when necessary.

The effect of $t$ is shown in Figure 4.7. For all $t$ values, $E_n$ increases over low values of $p$. Under the same process level, $E_n$ decreases as $t$ decreases. This is because smaller $t$ values result in larger $P_g$ which causes more $G$ acceptance truncation. Although larger $P_g$ also causes less $Y$ rejection truncation, the effect of $Y$ rejection truncation is dominated by $G$ acceptance truncation in this example.

The effect of $y$ is shown in Figure 4.8. For all $y$ values, $E_n$ is usually decreasing (and always non-increasing) over low values of $p$. Under the same process level, as $y$ increases, $E_n$ also increases. This is
Figure 4.6. The Effect of $n$ on $P_a$
Figure 4.7. The Effect of $t$ on $E_n$
Figure 4.8. The Effect of \( y \) on \( E_n \)

(a) \( m = 2; \) Mean Shift

(b) \( m = 3; \) Mean Shift

(c) \( m = 2; \) Dispersion Change

(d) \( m = 3; \) Dispersion Change
because larger $y$ means more $Y$s are tolerable, which in turn reduces the probability of $Y$ rejection truncation. Among the values, $y = 0, 1, 2,$ and $3$, $y = 0$ has a dramatic impact on the reduction of $E_n$. For both $m = 2$ and $m = 3$ when $n = 3$, $y = 2$ has the same $E_n$ curve as $y = 3$. In fact, it is always true that $y = n - 1$ has the same $E_n$ curve as that of $y = n$. Since for $y = n - 1$, truncation can only occur at $n$th item (which is no truncation at all), $y = n$ and $y = n - 1$ are essentially equivalent in terms of the $E_n$ calculation. When $m = 2$, it is also always true that the $E_n$ for $y = n - 1$ or $n$ remains $E_n = n$ regardless of process level as indicated by this example. Finally, the $E_n$ curve for $y = 1$ and $y = 2, 3$ are relatively close together.

The effect of $g$ is shown in Figure 4.9. Here, $g = 0$ and $g = 3$ are equivalent as explained earlier. For $g = 3$ (or $0$), $E_n$ decreases over low values of $p$. But for $g = 1$ or $2$, $E_n$ increases over low values of $p$. Generally, as $g$ increases (from $1$ to $3$), $E_n$ increases significantly. This is because larger values of $g$ cause reduced probability of $G$ acceptance truncation.

The effect of $n$ is shown in Figure 4.10. For all $n$ values, $E_n$ increases over low values of $p$. Under the same process level, $E_n$ increases as $n$ increases. As $n$ increases from $2$ to $8$, $E_n$ increases only about 50 percent. This is due to the combined effectiveness of all the acceptance/rejection truncation measures which are $g = 1$, $y = 1$, and $r = 0$.

In short, $E_n$ is most sensitive to the adjustment of $g$, moderately sensitive to $y$ and $t$, and least sensitive to $n$ for these examples. However, the effects of $y$ and $n$ depend on the power of the acceptance/rejection truncation measures specified.
Figure 4.9. The Effect of \( g \) on \( E_n \)
Figure 4.10. The Effect of \( n \) on \( E_n \)
Design of Frequency Gaging Rule

Ideally, every user would like to have a FG rule with absolute discriminative power to detect a process shift on the first sample after it occurs. Also, it is desired that the FG rule not signal any false alarms when there are no shifts at all. However, due to randomness, two types of errors may occur: (1) when the process is at the desirable Acceptable Process Level (APL), its samples may be erroneously rejected; (2) when the process is at the undesirable Rejectable Process Level (RPL), its samples may be erroneously accepted. Hence, in practice, we can specify the tolerable limits for either one or both of these two wrong decision cases. For convenience, these are called "one point" or "two point" designs.

If the defective cost is very significant and setup and reset costs are relatively negligible, one may adopt a one point design by specifying the Tolerable Limit of $P_a (\text{RPL}) \leq \text{TLRPL}$. In this case, any STD NLG rule which satisfies $P_a (\text{RPL}) \leq \text{TLRPL}$ will be considered as a qualified candidate. On the other hand, if setup and reset costs are also significant, one then should adopt a two point design by specifying the Tolerable Limits of both $P_a (\text{APL})$ and $P_a (\text{RPL}) \leq \text{TLAPL, TLRPL}$. In this case, all the qualified candidate plans must satisfy both $P_a (\text{APL}) \geq \text{TLAPL}$ and $P_a (\text{RPL}) \leq \text{TLRPL}$. These strategies are similar to the design strategies of Attribute Single Sampling Plans, in which the counterparts of APL, TLAPL, RPL, and TLRPL are AQL (Acceptable Quality Level), $1 - \alpha$ (where $\alpha$ is Type I Error), LTPD (Lot Tolerance Percent Defective) and $\beta$ (Type II Error), respectively.

To select the most appropriate plan from all of the candidates requires proper statistical comparison. Unfortunately, there is no ultimate objective criterion for statistical comparison like the "total cost"
used in economic comparisons. Different users may emphasize different performance measures, and eventually the final decision must resort to individual subjective judgment.

Among $P_a$ and $E_n$, generally, $P_a$ is used as a primary criterion and $E_n$ is secondary. Except when unit inspection cost is very high, the user prefers a plan with a better OC curve (in the sense that it fits better to those user-designated design points) but with a slightly worse $E_n$ curve, rather than the opposite situation. However, if two qualified plans have quite similar OC curves, the user surely prefers the one with a better $E_n$ curve, thus resulting in lower inspection cost. For those cases with non-comparable OC and $E_n$ curves, the decision of selection will rely heavily on individual needs and the user's subjective judgment.

Theoretically, the design procedure for FG is quite straightforward. After specifying the design points for the OC curve, the user proceeds to separate out all qualified plans from the complete set of possible plans. Finally, proper comparisons among those candidates lead to the selection of a most desired FG rule. However, in practice, due to the large number of possible variations of multiple FG parameters, the number of qualified candidates becomes formidable and hence makes the comparisons and final selection very difficult or even impossible.

To alleviate this problem, proper restrictions can first be imposed on the variations of $n, t, y,$ and $g$ to considerably reduce the number of possible plans considered. This number can be further reduced by evaluating each at the APL and RPL and eliminating all but the qualified plans. For example, for $USLLSL = 7$, mean shift assumed, and $m = 2$, we may confine the variations as follows: $2 \leq n \leq 5$; $0 \leq y \leq \text{INTEGER} \left( n/2 + 0.5 \right)$; $1 \leq g \leq n - y$ (but $g = 0$ if $y = 0$); $t = 1, 1.5, 2$; which results in 66 plans.
Then the $P_a$ and $E_n$ of each plan are evaluated at the APL and RPL. Suppose APL = 0.01, TLAPL = 0.90, RPL = 0.10, and TLRPL = 0.20. Among these 66 plans, only 9 plans are qualified. After proper comparisons, the final decision may be subjectively reached. However, if further improvement on the selected plan is still desired, it may be modified in the direction of the user's interest by properly adjusting individual parameters (mainly $t$, or if necessary, $n$, $y$, and even $g$). This adjustment may utilize the general properties of the effects of individual parameters on $P_a$ and $E_n$ as revealed previously.

**Design of Qualification Rule**

Based upon similar reasoning as that used for FG, the QL rule can be designed using a one- or two-point approach depending on the user's need. Recall that in STD NLG QL, $m$, $t$, and $r$ have the same values as those used in FG; $g$ is set equal to 0; and only $n$ and $y$ are allowed to vary.

For specified values of TLAPL and TLRPL of QL, any qualified QL rule should have an OC curve satisfying the following:

\[
P_a(\text{APL}) = \sum_{i=0}^{\infty} \binom{n}{i} P^i(\text{APL}) P^{n-i}(\text{APL}) \geq \text{TLAPL}
\]

and

\[
P_a(\text{RPL}) = \sum_{i=0}^{\infty} \binom{n}{i} P^i(\text{RPL}) P^{n-i}(\text{RPL}) \leq \text{TLRPL}
\]

In QL, since only $n$ and $y$ are allowed to vary, and both are integers, the number of possible QL plans is quite limited for typical values of $n$. Hence, searching for the most desirable QL rule is much easier, with no trial and error needed. For the same example used in the FG design
section (i.e., USLLSL = 7, mean shift assumed, m = 2), suppose the final t chosen is 1.7. Now, for APL = 0.2σ, TLAPL = 0.90, RPL = 2σ, TLRPL = 0.10, and 2 ≤ n ≤ 8, among 35 possible plans, only 3 are qualified. Consequently, the final selection can easily be made.

General Procedure to Satisfy a Designated PBAPQ

If assurance is desired for the APQ being less than a designated value, the following general procedure may be followed. The user should first evaluate the PBAPQ of the currently used FG and SF rules to see if it is satisfactory. If not, the user may increase the SF to reduce PBAPQ to the desired level. If for some reason SF should not be changed, the user may modify the FG rule to achieve the same purpose. Finally, RI can also be employed to temporarily improve the PBAPQ.

Comments

The effects of NLG parameters on $P_a$ and $E_n$ have been demonstrated only for one typical example. Some of the properties revealed may change somewhat for different cases. Thus, more examples covering a wider range of NLG applications may be found worthwhile.

Since the flexible general procedures for designing FG and QL are quite cumbersome and time consuming, an alternative might be considered for real world practice. To provide a convenient application, standard tabulation of already-designed FG and QL plans suitable for a wide range of typical conditions can be developed for use. These may include typical values of $n$ and $t$ under typical sets of APL, TLAPL, RPL, TLRPL, and typical USLLSL intervals. Thus, users can just look up the table and select the plans which match best with their particular needs.
Evaluation and Design of \( \bar{X} \)-Charts

Introduction

It is desirable to compare NLG to the most popular process control scheme, the \( \bar{X} \)-chart. In order to do this properly, methodologies for designing and evaluating an \( \bar{X} \)-chart are presented. The \( \bar{X} \)-chart is the counterpart of only one phase of STD NLG, namely NLG FG.

In an \( \bar{X} \)-chart control scheme, a sample of size \( n \) is taken regularly with its average value calculated and compared to the predetermined upper and lower control limits, UCL and LCL. Whenever a sample average falls beyond the control limits, the process is reset accordingly. Otherwise, it continues. There are three major variations used in specifying UCL and LCL, which in turn yield three versions of \( \bar{X} \)-charts.

1. Traditional \( \bar{X} \)-chart: The sample size \( n \) and control limits UCL and LCL are always fixed. No design is required. The sample size is usually set equal to 4 or 5, while UCL and LCL are often \( 3\sigma_0/\sqrt{n} \) away from \( \mu_0 \).

2. Designed \( \bar{X} \)-chart: Both \( n \) and the control spread \( k \) are design variables. In this case, UCL and LCL are \( k\sigma_0/\sqrt{n} \) away from \( \mu_0 \).

3. Modified \( \bar{X} \)-chart: Both \( n \) and \( k \) are design variables. Both UCL and LCL are \( k\sigma_0/\sqrt{n} \) outward from the boundaries of acceptable values of process mean. These boundaries themselves are \( v\sigma_0 \) inward from USL and LSL (see Figure 4.11(a)).

Among these three versions, only the modified \( \bar{X} \)-chart is comparable to NLG since its control limits are measured from specification limits and thus control the defectives rather than the shifts. Furthermore, both the traditional and designed \( \bar{X} \)-charts are just special cases of the modified \( \bar{X} \)-chart. Therefore, only the modified \( \bar{X} \)-chart will be considered.
(a) Case 1: Both $\mu$ and $\sigma$ Remain Unchanged ($\mu = \mu_0$, $\sigma = \sigma_0$)

(b) Case 2: $\mu$ Shifts While $\sigma$ Remains Unchanged ($\mu = \mu_1$, $\sigma = \sigma_0$)

(c) Case 3: $\sigma$ Increases While $\mu$ Remains Unchanged ($\mu = \mu_0$, $\sigma = \sigma_2$)

Figure 4.11. Three Cases of Process Shifts Under the Surveillance of the Modified X-Chart
in the following sections which describe its evaluation and design methodologies.

Evaluation

For all versions of the $\bar{X}$-chart, no inspection truncation is allowed. Hence, $n = n$, the sample size. As to the evaluation of $P_a$, three different cases are considered for formula derivation: (1) before any shifts occur, (2) $\mu$ shifts while $\sigma$ remains unchanged, and (3) $\sigma$ increases while $\mu$ remains unchanged.

Case 1: Before any shifts occur, the process is normally distributed with mean $\mu_0$, standard deviation $\sigma_0$, and fraction defective $p_0$. Its $P_a(p_0)$ can be derived as follows (see Figure 4.11(a)): Let

$$ H = (USL - LSL)/2\sigma_0 $$
$$ LCL = LSL + B\sigma_0 = LSL + (v\sigma_0 - k\sigma_0/\sqrt{n}) = LSL + (v - k/\sqrt{n})\sigma_0 $$
$$ UCL = USL - B\sigma_0 = USL - (v - k/\sqrt{n})\sigma_0 $$

Since

$$ E\sigma_0 = E\sqrt{n} (\sigma_0/\sqrt{n}) $$

$$ P_a(p_0) = \phi(E\sqrt{n}) - \phi(-E\sqrt{n}) = \phi[(H - B)/\sqrt{n}] - \phi[-(H - B)/\sqrt{n}] $$

$$ = \phi[(H - \nu + k/\sqrt{n})/\sqrt{n}] - \phi[(-H - \nu - k/\sqrt{n})/\sqrt{n}] $$

where

$$ p_0 = 2\phi(-H) $$

Case 2: While the process dispersion stays constant, the process mean shifts $\delta \sigma_0$ away from $\mu_0$ and results in a fraction defective $p_1$. Its $P_a(p_1)$ can be derived as follows (see Figure 4.11(b)): 
If \( \sigma \) is given, \( p_1 \) can be obtained as:

\[
p_1 = 1 - \phi(H + \delta) + \phi(-H + \delta)
\]

If \( p_1 \) is given, \( \delta \) can be approximated by:

\[
\delta = \phi^{-1}(p_1) + H
\]

with \( p_1 > p_0 \) and USLLSL \( \geq \delta \) assumed. The greater the differences in both inequalities, the better the approximation.

Since

\[
C\sigma = C\sqrt{n} \left( \sigma / \sqrt{n} \right)
\]

and

\[
D\sigma = D\sqrt{n} \left( \sigma / \sqrt{n} \right),
\]

\[
Pa(p_1) = \phi(D\sqrt{n}) - \phi(C\sqrt{n})
\]

But

\[
D = \delta + E = \delta + (H - B) = \delta + H - (v - k/\sqrt{n})
\]

\[
C = -A + B = \delta - H + (v - k/\sqrt{n})
\]

Hence

\[
Pa(p_1) = \phi[\delta + H - v + k/\sqrt{n}] - \phi[\delta - H + v - k/\sqrt{n}]
\]

Case 3: While the process mean stays at \( \mu_0 \), the process standard deviation increases to \( \sigma_2 \) and results in a fraction defection \( p_2 \). Its \( Pa(p_2) \) can be derived as follows (see Figure 4.11(c)):

If \( \sigma_2 \) is given, \( p_2 \) can be obtained as

\[
p_2 = 2\phi(-H0 / \sigma_2)
\]
If \( p_2 \) is given, \( \sigma_2 \) can be calculated as

\[
\sigma_2 = -\frac{H \sigma_o}{\Phi^{-1}(p_2/2)}
\]

Since

\[
E \sigma_o = (E \sqrt{n} \sigma_o/\sigma_2)(\sigma_2/\sqrt{n}),
\]

\[
P_a(p_2) = \Phi(E \sqrt{n} \sigma_o/\sigma_2) - \Phi(-E \sqrt{n} \sigma_o/\sigma_2) = 2[0.5 - \Phi(-E \sqrt{n} \sigma_o/\sigma_2)]
\]

\[
= 1 - 2\Phi([-H + v - k/\sqrt{n}] \sqrt{n} \sigma_o/\sigma_2)
\]

**Design**

Among the three variables \((n, v, k)\) involved in a modified \( \bar{X} \)-chart, \( v \) is usually subjectively designated by the user and often assumes a value of 3 or 3.5. When \( v = (USL - LSL)/2\sigma_o \), the modified \( \bar{X} \)-chart reduces to the Traditional and Designed \( \bar{X} \)-charts. Thus, the only two design variables of the Modified \( \bar{X} \)-charts are sample size \( n \) and control spread \( k \).

In designing a Modified \( \bar{X} \)-chart, the same STD NLG one point or two point design strategy used for FG applies. By imposing similar variation restrictions on \( n \) and \( k \), followed by similar searching and modification procedures, the most desirable control plan can be more easily located for \( \bar{X} \)-charts than for STD NLG FG.

**Comments**

Usually \( \bar{X} \)-charts are used only as the counterpart of FG in NLG. For the entire \( \bar{X} \)-chart process control scheme, if qualification of process setup and reset is needed, a similar \( \bar{X} \)-chart control mechanism (which may have different \( n, v, k \) values) can be adopted as its QL plan. The evaluation and design of this QL plan uses the same evaluation formulation and
design procedure previously developed for Modified $\bar{X}$-charts. Furthermore, the evaluation of performance measures such as PBAPQ and PBAOQ for the whole process, under the surveillance of $\bar{X}$-charts, are exactly the same as that of NLG if similar SF and RI (as needed) rules are incorporated into the entire control scheme.

Comparison of STD NLG With the $\bar{X}$-Chart

Based on the understanding of methodologies for evaluating and designing both NLG plans and $\bar{X}$-charts, the user is now able to properly compare NLG with $\bar{X}$-charts. That is, based on the same set of user-designated APL, TLAPL, RPL, and TLRPL criteria, both NLG and the Modified $\bar{X}$-chart can be properly designed to qualify this same set of criteria and can then be compared to each other by their $P_a$ and $E_n$ curves. Finally, a decision on choosing either NLG or the $\bar{X}$-chart can be reached with proper justification.

An example comparing NLG, an $\bar{X}$-chart, and a traditional attribute gaging plan (i.e., attribute single sampling plan) is illustrated in Figure 4.12. Under mean shift assumption, given USLLSL = 7, APL = 0.01, TLAPL = 0.95, RPL = 0.10, and TLRPL = 0.33, three different types of process plans are considered for use. In the traditional attribute gaging control scheme (i.e., specification gages instead of narrow limit gages are used), the qualified plan with minimum sample size is $n = 23$, $c = 1$ (i.e., >1 defective is not acceptable). On the other hand, in the Modified $\bar{X}$-chart control scheme, a plan with $n = 4$, $v = 3$, and $k = 3$ satisfies the same set of criteria. Obviously, this variable scheme $\bar{X}$-chart requires a much smaller sample size, while it is relatively more difficult to implement when compared to an attributes scheme.
Figure 4.12. A Comparison Among Three Types of Process Control Schemes (Comparison Basis: USLLSL = 7, Mean Shift Assumed, APL = 0.01, TLAPL = 0.95, RPL = 0.10, TLRPL = 0.33)
However, if the traditional specification gages are replaced by narrow limit gages, a significant improvement on the attribute scheme can be achieved by an NLG plan with $n = 6$, $m = 2$, $t = 1.7$, $y = 3$, and $g = 3$. In this plan, all $E_n(p)$ are no greater than 5.4 for $p \leq 0.10$ and the average $E_n$ will be less than 4.5 if the process is assumed to be IC for more than 50 percent of the time. Thus, in a typical application, this plan's $E_n$ is very close to that of the $\bar{X}$-chart.

In this example, based on similar go/no-go gaging methods, apparently NLG is much better than traditional attribute gaging due to its much smaller average inspection number. Compared to the $\bar{X}$-chart, NLG seems equally competitive since its average inspection number is as small as that of the $\bar{X}$-chart. In fact, NLG should be administratively and economically superior to the $\bar{X}$-chart due to its easier-to-use go/no-go gaging method and no-calculation-required control scheme. In short, the statistical performance of NLG plans seems at least comparable and in some respects better than that of $\bar{X}$-charts.

Summary

In the preceding NLG statistical evaluation, the formulations of $P_g$, $P_y$, and $P_r$ are first developed for either mean shift or dispersion change conditions. Based on these formulas, $P_a$ and $E_n$ are derived to evaluate the performance of FG or QL. All of these evaluations can be adapted to accommodate different distributions and narrower USLLSL intervals. For the entire process, $PBAPQ$ and $PBAOQ$ are developed to provide conservative upper bounds of APQ and AOQ. With the additional knowledge of mean time-to-shift and/or assignable cause searching time, the estimation of APQ and AOQ can be improved accordingly.
In NLG statistical design, the general effects of \( t, y, g, \) and \( n \) on \( P_a \) and \( E_n \) are investigated based on a typical example. Some general properties have been revealed to help design FG and QL rules. Then a flexible general procedure is constructed for designing the FG rule. This procedure starts with enumerating all possible rules followed by eliminating all those unqualified within a restricted parameter space, and finally concludes with trial and error modifications to eventually locate the most desirable plan. A similar but simpler procedure is also provided for QL. As to the design of an entire NLG plan, a very general strategy is discussed. Finally, to alleviate the design burden on users, a standard tabulation of FG and QL designs for a wide range of typical conditions is suggested.

To properly compare NLG with the most popular alternative, the \( \bar{X} \)-chart, methodologies for evaluating and designing a Modified \( \bar{X} \)-chart have been presented. Among all versions, only the Modified \( \bar{X} \)-chart is comparable to NLG and both the Traditional and Designed \( \bar{X} \)-charts are special cases of it.

Finally, this chapter is concluded by an example comparing NLG, the \( \bar{X} \)-chart, and a traditional attribute gaging plan. This example reveals that NLG can significantly improve the sensitivity of an attribute scheme and become as good as the most popular variable scheme--the \( \bar{X} \)-chart in terms of sample size. Furthermore, with the additional administrative and economic advantages, NLG has the potential to become superior to the \( \bar{X} \)-chart.
CHAPTER V

ECONOMIC FORMULATION AND OPTIMIZATION OF STD NLG;
ECONOMIC COMPARISONS WITH THE $\bar{X}$-CHART

Introduction

This chapter provides a good alternative to statistically-based NLG and $\bar{X}$-chart control schemes--economically-based NLG and $\bar{X}$-charts. Economic schemes are more appealing in two aspects: (1) they do not require the user to supply subjective design points (such as APL, TLAPL, RPL, and TLRPL), and (2) they use "total cost" as the only performance measure, which in fact is the ultimate criterion in evaluating all control plans. In order to provide an economic comparison between NLG and the $\bar{X}$-chart, both the formulation and design of NLG plans must be considered from an economic viewpoint. The economic formulation of $\bar{X}$-charts has previously been treated in the literature.

This chapter follows Duncan's [6] $\bar{X}$-chart model (the Designed $\bar{X}$-chart) and its assumptions to formulate an economic NLG scheme. Then, an optimization algorithm utilizing a direct search technique is developed and improved to optimize the five decision variables of the economic NLG model. Finally, based on several representative examples, both models are optimized and extensively compared. General guidelines are eventually developed for the better application of both models.
Notation

In addition to notation introduced in previous chapters, the following terms are employed to facilitate this chapter's discussion:

- \( h \) -- the sampling interval; samples of size \( n \) are taken from the process every \( h \) hours

- \( \lambda \) -- the parameter related to the probability of occurrence of the assignable cause. The distribution of IC time is exponentially distributed with mean \( 1/\lambda \)

- \( e \) -- the rate at which the average sampling, gaging, and evaluation time for a sample increases with the average sample number (\( E_n \) for NLG or \( n \) for \( \bar{X} \)-chart)

- \( D \) -- the average search time for an assignable cause

- \( V_0 \) -- the hourly income from operation of an IC process

- \( V_1 \) -- the hourly income from operation of an OOC process for which the mean has shifted by \( \delta \sigma_0 \).

- \( M \) -- the reduction in process hourly income that is attributed to the occurrence of the assignable cause; \( M = V_0 - V_1 \)

- \( T \) -- the average cost per occasion of looking for an assignable cause when none exists

- \( W \) -- the average cost per occasion of finding the assignable cause when it exists

- \( b \) -- the cost per sample of sampling, gaging, and acceptance/rejection decision making that is independent of the sample size

- \( c \) -- the unit cost of sampling, gaging, and evaluation that is related to the sample size; this relationship is assumed to be linear
p_δ -- the fraction defective resulting from an OOC process whose mean has shifted by δσ_0

α -- the probability of a false alarm (i.e., the control scheme indicates an OOC indication when the process is still IC); α = 1 - P_a (p_0)

P -- the probability of a real alarm (i.e., the control scheme indicates an OOC indication when the process is actually OOC);

P = 1 - P_a (p_δ)

β -- the average proportion of time a process is IC

E_1' -- the average number of pieces inspected per sample from an IC process; E_1' = E_n (p_0)

E_1'' -- the average number of pieces inspected per sample from an OOC process for which the mean has shifted by δσ_0; E_1'' = E_n (p_δ)

E_1* -- the overall average number of pieces inspected per sample for the entire process; E_1* = βE_1' + (1 - β) E_1''

L -- the loss-cost; the minimization of L will result in the maximization of process hourly net income.

Economic NLG Formulation

General Structure

Among economically designed process control schemes, Duncan's [6] fundamental economic X-chart (the Designed X-chart) is the most popular one due to its flexibility, simplicity of administration, and the information content of the plotted point pattern. Hence, it is used in this research as the basis against which the economic NLG model is compared.
In order to ensure proper comparison between both models, the general structure of Duncan's economic $\bar{X}$-chart is adopted for the economic NLG formulation in this research. That is, based upon the maximum income criterion, the economic model (either NLG or Duncan's $\bar{X}$-chart) measures the average net income of a process under the surveillance of its control scheme. The process starts IC and is subject to random shifts in the process mean (OOC). Once OOC, the process remains there until corrected. Given associated cost and time parameters, the optimal values of decision variables for each model are then determined using optimization techniques.

Assumptions

The economic NLG formulation is based on the same set of assumptions as used for Duncan's economic $\bar{X}$-chart. These assumptions are stated as follows:

1. Due to an assignable cause, the process mean may randomly shift to $\mu_0 \pm \delta_0$ and stay there until corrected while $\sigma$ remains unchanged.
2. The process is not shut down while the search for the assignable cause is in progress.
3. Neither the cost of adjustment or repair, nor the cost of bringing the process back into a state of IC after the assignable cause is discovered, is considered in the economic model.

Formula Derivation

Control Cycle. A complete economic NLG control cycle consists of four time intervals as follows (see Figure 5.1):
Figure 5.1. Economic NLG Control Cycle
Control cycle length = (a) + (b) + (c) + (d)

(a) Since the average time for the occurrence of an assignable cause is $1/\lambda$, so is the process average IC time.

(b) Given the occurrence of an assignable cause in the interval between the $n$th and $(n+1)$st sample, the average time of occurrence within an interval between samples will be

$$
\int_{nh}^{(n+1)h} e^{-\lambda x} \frac{\lambda (x - nh) dx}{\int_{nh}^{(n+1)h} e^{-\lambda x} \lambda dx} = \frac{e^{-\lambda nh} \int_0^h e^{-\lambda z} \lambda zdz}{e^{-\lambda nh} \int_0^h e^{-\lambda z} \lambda dz}
$$

$$= \frac{1 - (1 + \lambda h) e^{-\lambda h}}{\lambda (1 - e^{-\lambda h})}
$$

$$\approx \frac{h - \frac{\lambda h^2}{2}}{12}$$

The average number of samples taken before the shift in the process is caught is $1/P$, where $P$ is the probability of a real alarm ($P = 1 - P_a (p_\delta)$). Hence, $h/P - (h/2 - \lambda h^2/12)$ is approximately the average time the process will be OOC before the sample destined to detect the process shift is taken.

(c) The average sampling and evaluation time for each sample is $eE'_n$, where $e$ is average sampling, gaging, and evaluation time for each piece; $E'_n = E_n (p_\delta)$.

(d) The average time taken to locate an assignable cause is $D$. Therefore,
Control cycle length = \(\frac{1}{\lambda} + (\frac{1}{P} - \frac{1}{2} + \frac{\lambda h}{12})h + eE_n + D\)

\[= \frac{1}{\lambda} + B\]

where

\[B = (\frac{1}{P} - \frac{1}{2} + \frac{\lambda h}{12})h + eE_n + D\]

Thus, the proportion of the time a piece is IC is

\[\beta = \frac{1/\lambda}{1/\lambda + B} = \frac{1}{1 + \lambda B}\]

Cost Formulation. Based upon the above derivation of a control cycle, formulation of the process average hourly net income is now developed as follows:

\[
\begin{align*}
\text{(Process average)} & = \text{(a) Weighted hourly IC income) + (b) Weighted hourly OOC income} - \text{(c) (Hourly false alarm cost)} \\
& \text{(d) (Hourly real alarm cost) - (e) (Hourly FG cost)}
\end{align*}
\]

(a) Weighted hourly IC income = \(\text{(Hourly income from IC process)} \times (\text{Fraction of the time the process is IC)}\)

\[= V_o \times \beta\]

(b) Weighted hourly OOC income = \(\text{(Hourly income from OOC process)} \times (\text{Fraction of the time the process is OOC)}\)

\[= V_1 \times (1 - \beta)\]
(c) \( \text{(Average hourly false alarm cost)} = \left( \frac{\text{Expected number of false alarms per hour}}{\text{Average cost of searching for an assignable cause when a false alarm is encountered}} \right) \)

The expected number of false alarms before the process goes OOC will be the probability of false alarm (\( \alpha \)) times the expected number of samples taken in the period. This is

\[
\alpha \sum_{i=0}^{\infty} \int_{ih}^{(i+1)h} i e^{-\lambda t} dt = \alpha \sum_{i=0}^{\infty} i [e^{-ih\lambda} - e^{-(i+1)h\lambda}]
\]

\[
= \alpha (1 - e^{-\lambda h}) \sum_{i=0}^{\infty} i e^{-ih\lambda}
\]

\[
= -\alpha (1 - e^{-\lambda h}) \frac{1}{\lambda} \sum_{i=0}^{\infty} e^{-ih\lambda}
\]

\[
= \frac{ae^{-\lambda h}}{\lambda} \approx \frac{\alpha}{\lambda h}
\]

Thus, the average hourly false alarm cost = \( \frac{\alpha/\lambda h}{\text{Control cycle length}} \times T \)

\[
= \frac{T\alpha/\lambda h}{1/\lambda + B} = \frac{\beta T}{h}
\]

(d) \( \text{(Average hourly real alarm cost)} = \left( \frac{\text{Expected number of real alarms per hour}}{\text{Average cost of searching for an assignable cause when a real alarm is encountered}} \right) \)

\[
= \frac{1}{\text{Control cycle length}} \times W
\]

\[
= \frac{W}{1/\lambda + B} = \frac{\lambda W}{1 + \lambda B}
\]
Average hourly cost for FG cost sampling, gaging and evaluation

\[
= \frac{b/h + c \left[ \beta E_n^I + (1 - \beta) E_n^J \right]}{h} = \frac{(b + cE_n^H)}{h}
\]

Therefore,

\[
\text{(Process hourly net income)} = \beta V_0 + (1 - \beta)V_1 - \beta aT/h - \frac{\lambda W}{1 + \lambda B} - \frac{(b + cE_n^H)}{h}
\]

\[
= V_0 - \frac{\lambda MB + aT/h + \lambda W}{1 + \lambda B} - \frac{b + cE_n^H}{h}
\]

where

\[
M = V_0 - V_1 = V_0 - L
\]

where

\[
L = \frac{\lambda MB + aT/h + \lambda W}{1 + \lambda B} - \frac{b + cE_n^H}{h}
\]

In this formulation, to maximize average hourly net income is equivalent to minimizing the loss-cost L.

Summary of Parameters and Decision Variables

In the above economic NLG formulation, all the involved parameters and variables can be classified into three categories according to their nature:
1. Time parameters: $\delta, \lambda, e, D$
2. Cost parameters: $M$ (or $V_o$ and $V_1$), $T, W, b, c$
3. Decision variables: $n, m, h, t, y, g$.

**Differences Between Economic NLG and the Economic $\bar{X}$-Chart**

The major difference between these two process control methods is the number of decision variables: $n, h,$ and $k$ for the $\bar{X}$-chart; $n, m, h, t, y,$ and $g$ for NLG. As to the average inspection number, $n$ is used throughout the entire $\bar{X}$-chart plan, while $E_n^1, E_n^1$, or $E_n^*$ is adopted depending upon the individual stage in the NLG control scheme. Finally, while all the time and cost parameters assume the same values for both models to ensure the highest degree of resemblance, the real world values of $e$ and $c$ for NLG may be much smaller than those for the $\bar{X}$-chart due to the simple gaging methods and evaluation procedures for NLG.

**Comments**

In the first assumption, a single OOC state caused by a single assignable cause is assumed. Although the multiplicity of assignable causes is more realistic in the real world, the much simpler single cause has been demonstrated by Duncan [8] to be a satisfactory approximation, and hence is somewhat preferred for use. The single OOC state is traditionally justified as representing the threshold beyond which process deterioration is intolerable and which thus represents the most difficult such OOC state to detect.

Under the second assumption, the process is not shut down during the search for an assignable cause. This is quite typical in practice.
However, there are situations when shutdown is preferred or required. In this case, the previous model no longer applies and a different model must be constructed. An example model considering shutdown has been shown by Baker [3].

Under the third assumption, the cost of resetting the process is not included in the model. In fact, the inclusion of this cost item will only add a constant term to the total cost formula, and thus has no effect on the optimal solution.

Economic NLG Optimization

General Optimization Strategy

The ultimate goal in optimizing an economic NLG model is to find the optimal combination of values of the decision variables, in order to minimize the loss-cost \( L \) and hence maximize the average hourly net income of the process under surveillance. Since \( L \) is a very complicated function of the decision variables \( n, m, y, g, t, \) and \( h \), there exists no analytically explicit optimal solution. Therefore, multidimensional direct search techniques become the only means for optimization.

However, all six control variables cannot be simultaneously optimized using direct search, since \( n, m, y, \) and \( g \) are integers and \( m, y, g \) scatter unevenly in integer space. Therefore, the only feasible optimization strategy for economic NLG is as follows:

1. Simultaneously optimize \( (h,t) \) under each specified set of \( (n,m, y, g) \) values, resulting in a local optimum set.

2. Compare all local optimums and locate the overall optimum.
Direct Search Technique

The direct search technique employed in this research is the Nelder and Mead algorithm [32], which is straightforward, efficient, and easy to use. This method finds the minimum of a multivariable \(n_v\) unconstrained, nonlinear function. The minimization is achieved by the comparison of function values at the \((n_v + 1)\) vertices of a general simplex, followed by replacement of the vertex having the highest value by another point. This simplex method efficiently adapts itself to the local landscape by using reflected, expanded, and contracted points; it finally contracts onto the final minimum. Derivatives are not required.

Since this algorithm is intended only for unconstrained variables, a minor modification is needed before it can be applied to NLG optimization. In NLG, the feasible ranges for \(h\) and \(t\) are: \(h > 0\) and \(0 \leq t \leq \text{USLLSL}/2\). This modification is thus achieved by confining all the reflected and expanded points (and hence contracted points) to the above feasible region.

About 100 different combinations of \((n, m, y, g)\) for several examples with different sets of parameter values have been investigated to reveal the general shape of the cost surface of \(L\). Each cost surface of \(L\) is tabulated in a rectangular table with 25 \(h\) rows \((0 < h \leq 100)\) and 11 \(t\) columns \((0.01 \leq t \leq 2.99)\). The results have shown that \(L\) surfaces are shallow and convex shaped with a minimum located a substantial distance from both ends of the feasible range of \(t\). Only a few occasions have shown a mild ridge close to the high end border of \(t\) (i.e., \(t \to 3\)). In this case, once in a while the minimum lies right on the high \(t\) border. In summary, none

\[a\] In actual computer programming, \(h > 0.001\) and \(0.001 \leq t \leq \text{USLLSL}/2 - 0.001\) are used to avoid intermediate underflow and overflow problems.
of the \( L \) surfaces investigated has ever indicated shapes other than the above two types.

**NLG Optimization Algorithm**

To find the overall optimum, all the possible combinations of \((n, m, y, g)\) must be investigated. If \( n \) is not restricted, the number of combinations becomes infinite. Even if \( n \) is restricted to a moderate number, say 6, still there will be about 130 possible combinations, requiring extensive computational effort. Consequently, an efficient search algorithm other than the above enumeration approach is strongly desired, if there exist some favorable properties in the relations among different combinations of \((n, m, y, g)\) which can be utilized to make such an algorithm possible.

Based on this motivation, an investigation of several examples, each with a different set of parameter values, has been performed. The results have revealed that a nice relation does exist among \( n, y, \) and \( g \) for \( m = 2 \) or \( m = 3 \), respectively. This relation can be described as follows:

1. The value of \( m \) is first specified. That is, either \( m = 2 \) or \( m = 3 \).

2. Under each set of \((n, y)\) values, the local optimums of loss-cost (one \( L^*_g \) for each \( g \)) for \( g \) values from \( g = 1 \) to \( g = n \) form either a convex curve or strictly increasing curve. The optimum of this curve is labeled \( L^*_g \).

3. Under each \( n \) value, the local loss-cost optimums (one \( L^*_g \) for each \( y \)) for \( y \) values from \( y = 0 \) to \( y = n \) form either a convex curve or a strictly increasing curve. The optimum of this curve is labeled \( L^*_y \).

4. The local loss-cost optimums (one \( L^*_n \) for each \( n \)) for \( n \) values
from \( n = 1 \) and above form either a convex curve or a strictly increasing curve. This overall optimum is labeled \( L^*_n \).

All of these cases have shown either convex or strictly increasing values of local optimum within each of the \((n,y,g)\) levels. In fact, in addition to all the above preliminary examples, generally all production cases investigated support this property without exception. However, in practice, the possibilities of a strictly decreasing (or non-increasing) or a very flat "generally convex" curve with a few very small bumps (due to the approximation of formulation and the cumulative inaccuracy of calculation) must be considered.

Based on this convex property, the efficient NLG optimization algorithm can now be constructed as follows:

**A. General Structure of the NLG Optimization Algorithm**

**Notation:**

\[
L^*_g, L^*_y, L^*_n = \text{local optimal } L \text{ values within each of the } (g,y,n) \text{ levels, respectively, as explained previously.}
\]

\( n_s, n_e; y_s, y_e; g_s, g_e \) = starting and ending values for \( n, y, \) and \( g \), respectively.

1. Specify \( m \) value (\( m = 2 \) or \( 3 \)).
2. Start with \( n_s, y_s \).
3. Under specified \( n,y \) values, optimize \( L \) for each \( g \) (resulting in \( L^*_g \)) from \( g_s \) to \( g_e \); compare all \( L^*_g \) and locate their minimum as \( L^*_g \).
4. Under specified \( n \), repeat step 3 for each \( y \) from \( y_s \) to \( y_e \); compare all \( L^*_g \) and locate their minimum as \( L^*_y \).
5. Repeat step 4 for each \( n \) from \( n_s \) to \( n_e \); compare all \( L^*_y \) and locate their minimum as \( L^*_n \).
6. Optimal NLG plan = the plan associated with \( L^*_n \).
After some experience in implementing the above algorithm, further improvement in optimization efficiency can be achieved by effectively dynamically adjusting $n_s$, $n_e$, $y_s$, $y_e$, and $g_s$, $g_e$ values as follows:

B. Efficiency Improvement on General NLG Optimization Structure

1. In A-3:
   a. For $y_s \geq 1$, $g_s(y_s) = 1$. For $y_s = 0$, $g_s(y_s) = 0$.
   b. Under the same $n$, $g_s(y_{i+1}) = \min [1, g_s^*(y_i) - \varepsilon_g]$; where $i \geq s$, $g_s^*(y_i) = \text{optimal } g \text{ under } y_i$, and $\varepsilon_g = \text{a user specified allowance}$.
   c. When searching for $L_s^\star$, $g_e$ can be dynamically determined as the $g$ having its $L_s^\star \geq L_s^\prime + \varepsilon_L$; where $L_s^\prime$ is the minimal $L_s^\star$ from $g_s$ up to the current $g$, and $\varepsilon_L$ is a user specified allowance to overcome those small bumps (if there are any) in a fairly flat curve.

2. Similarly, in A-4:
   a. $y_s(n_s) = 0$.
   b. $y_s(n_{i+1}) = \min [0, y_s^*(n_i) - \varepsilon_y]$; where $i \geq s$, $y_s^*(n_i) = \text{optimal } y \text{ under } n_i$; and $\varepsilon_y = \text{a user specified allowance}$.
   c. When searching for $L_y^\star$, $y_e$ can be dynamically determined as the $y$ having its $L_y^\star \geq L_y^\prime + \varepsilon_L$, where $L_y^\prime$ is the minimal $L_y^\star$ from $y_s$ up to the current $y$.

3. $n_e = \text{the } n \text{ having its } L_y^\star \geq L_n^\prime + \varepsilon_L$, where $L_n^\prime$ is the minimal $L_y^\star$ from $n_s$ up to the current $n$.

Comments

In direct search for the optimum $(h,t)$ under specified $(n,m,y,g)$, sometimes the result may deviate as the starting point changes due to the
existence of multiple local minima or special shapes of the loss-cost surface. Therefore, whenever the optimum \((h,t)\) and its associated \(L^*\) found by the direct search algorithm are suspect, either an investigation on the tabulation of the loss-cost surface or a rerun on several starting points should be performed to ensure the location of the real optimum.

Similarly, if the final result obtained by the improved version of the NLG optimization algorithm is suspect, a complete enumeration of all \(n, y,\) and \(g\) should be performed to help locate the real overall optimal plan.

Economic Comparison Between NLG and the \(\bar{X}\)-Chart

Examples for Comparison

To assess the best conditions for the application of NLG and the \(\bar{X}\)-chart, both control schemes are compared. Both schemes are based upon the same assumptions and evaluated under the same environments. Twelve representative examples are chosen from Duncan's [6] paper as shown in Table 5.1. The values assigned to the cost and time factors in this table cover a wide range of variations. Under each example, both control schemes are compared for their optimal loss-costs.

These 12 examples are divided into two groups: 1 to 13 and 16 to 26. In group 1 \((δ = 2)\), example 1 is the base case, and the rest are its variations. In group 2 \((δ = 1)\), example 16 is the base case, and the rest are its variations. Example 26 is the only exception not from Duncan's paper. It is newly created and added into group 2 to show the effect of e variation.
### TABLE 5.1

**EXAMPLES CHOSEN FOR ECONOMIC COMPARISON BETWEEN NLG AND $\bar{x}$-CHART**

<table>
<thead>
<tr>
<th>No.</th>
<th>$\delta$</th>
<th>$\lambda$</th>
<th>$M$</th>
<th>$e$</th>
<th>$D$</th>
<th>$T$</th>
<th>$W$</th>
<th>$b$</th>
<th>$c$</th>
<th>Characteristics</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>.01</td>
<td>100</td>
<td>.05</td>
<td>2</td>
<td>50</td>
<td>25</td>
<td>.50</td>
<td>.10</td>
<td>Basis for 1 to 13</td>
<td>$\delta = 2$ base</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\lambda$ increases 3 times</td>
<td>$\lambda + 3$</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td>1000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$M$ increases 10 times</td>
<td>$M + 10$</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td>.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$e$ increases 10 times</td>
<td>$e + 10$</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$D$ increases 10 times</td>
<td>$D + 10$</td>
</tr>
<tr>
<td>9</td>
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<td></td>
<td></td>
<td></td>
<td>5</td>
<td>2.5</td>
<td></td>
<td></td>
<td></td>
<td>$T$ and $W$ decrease 10 times</td>
<td>$T$ and $W + 10$</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>500</td>
<td>250</td>
<td></td>
<td></td>
<td></td>
<td>$T$ and $W$ increase 10 times</td>
<td>$T$ and $W + 10$</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td></td>
<td></td>
<td>$b$ increases 10 times</td>
<td>$b + 10$</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td>$c$ increases 10 times</td>
<td>$c + 10$</td>
</tr>
<tr>
<td>16</td>
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<td>12.87</td>
<td>.05</td>
<td>2</td>
<td>50</td>
<td>25</td>
<td>.50</td>
<td>.10</td>
<td>Basis for 16, 26, and 20</td>
<td>$\delta = 1$ base</td>
</tr>
<tr>
<td>26</td>
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<td></td>
<td>$e$ increases 10 times</td>
<td>$e + 10$</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td>$c$ increases 10 times</td>
<td>$c + 10$</td>
</tr>
</tbody>
</table>

*All example numbers are the same as those used in Duncan's paper, with the exception of example 26 which is newly created.*
Explanation and Analysis

Within each of these examples, four cases are investigated under both $m = 2$ and $m = 3$ situations:

1. Duncan's model (abbreviated as DC)
2. NLG without G acceptance truncation, i.e., $g = 0$ (NC)
3. STD NLG (with G acceptance truncation, i.e., $g > 0$) (TC)
4. STD NLG with both $e, c$ values reduced by half (RC).

All of the optimal results of all these cases are shown in Table 5.2. This table also provides comparisons among the above four cases and between $m = 2$ and $m = 3$.

In Table 5.2, for Duncan's model, optimal solutions are either provided by Goel et al. [12] (examples 1, 3, 5, 7, 8, 10, 12, and 16) or by a $\bar{x}$-chart optimization subroutine developed in this research (examples 9, 13, 26, and 20). For NLG plans, the investigation of both NC and RC in addition to standard TC is to illustrate the effects of (1) G acceptance truncation, and (2) the NLG reduction of sample inspection and evaluation costs, respectively.

To provide proper comparison, both Duncan's model (DC) and STD NLG (TC) adopt exactly the same set of parameter values. In actual implementation, however, the NLG parameters $e$ and $c$ should assume much smaller values than their DC counterparts. For example, in DC, $e$ (the time of sampling, measuring, and evaluating each piece) can be decomposed into several steps: sampling; measuring and recording; and calculating and plotting. But in NLG, for the same parameter $e$, the calculating and plotting step can be totally eliminated; and the measuring and recording step requires much less time. Therefore, for the same process under surveillance, the $e$ value in NLG should be much smaller than that of the counter-
### TABLE 5.2

**OPTIMAL ECONOMIC DESIGNS OF \( \bar{x} \)-CHART AND THEIR COMPARISONS**

<table>
<thead>
<tr>
<th>Ex. No.</th>
<th>Desc.</th>
<th>( m = 2 )</th>
<th>( \text{100L}^\alpha )</th>
<th>( m = 3 )</th>
<th>( \text{100L}^\beta )</th>
<th>( \text{100L}^\gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td></td>
<td></td>
<td>(B)</td>
<td></td>
<td>(C)</td>
<td>(D)</td>
</tr>
<tr>
<td>1</td>
<td>6 = 2</td>
<td>DC</td>
<td>5 1.41 3.08</td>
<td>401.38</td>
<td>5 1.41 3.08</td>
<td>401.38</td>
</tr>
<tr>
<td></td>
<td>Base</td>
<td>NC</td>
<td>8 1.59 1.342</td>
<td>441.480</td>
<td>5 2.0 1.657 1.272</td>
<td>463.424</td>
</tr>
<tr>
<td></td>
<td>TC</td>
<td>11 1.84 1.329</td>
<td>413.173*</td>
<td>2.9 -6.4</td>
<td>9 4 2 1.422 1.382</td>
<td>426.619</td>
</tr>
<tr>
<td></td>
<td>RC</td>
<td>13 1.74 1.194</td>
<td>377.595</td>
<td>-5.9 -8.6</td>
<td>9 4 2 1.328 1.388</td>
<td>404.783</td>
</tr>
<tr>
<td>3</td>
<td>1+3</td>
<td>DC</td>
<td>4 0.78 2.94</td>
<td>962.39</td>
<td>4 0.78 2.94</td>
<td>962.39</td>
</tr>
<tr>
<td></td>
<td>NC</td>
<td>7 1.28 1.121</td>
<td>1026.662</td>
<td>6.7</td>
<td>5 2 0.998 1.274</td>
<td>1050.602</td>
</tr>
<tr>
<td></td>
<td>TC</td>
<td>9 3 2 0.691 1.246</td>
<td>984.525</td>
<td>2.3 -4.1</td>
<td>8 3 2 0.803 1.250</td>
<td>994.566</td>
</tr>
<tr>
<td></td>
<td>RC</td>
<td>12 4 3 0.722 1.253</td>
<td>917.534</td>
<td>-4.7 -6.8</td>
<td>9 4 2 0.801 1.397</td>
<td>949.851</td>
</tr>
<tr>
<td>5</td>
<td>M+10</td>
<td>DC</td>
<td>4 0.41 2.95</td>
<td>2697.63</td>
<td>4 0.41 2.95</td>
<td>2697.63</td>
</tr>
<tr>
<td></td>
<td>NC</td>
<td>6 2 0 0.448 1.216</td>
<td>2850.739</td>
<td>5.7</td>
<td>5 2 0 0.525 1.293</td>
<td>2868.689</td>
</tr>
<tr>
<td></td>
<td>TC</td>
<td>7 2 2 0.330 1.094</td>
<td>2762.063</td>
<td>2.4 -3.1</td>
<td>6 3 1 0.299 1.462</td>
<td>2757.345</td>
</tr>
<tr>
<td></td>
<td>RC</td>
<td>10 3 3 0.358 1.152</td>
<td>2598.059</td>
<td>-3.7 -5.9</td>
<td>9 4 2 0.415 1.397</td>
<td>2637.541</td>
</tr>
<tr>
<td>7</td>
<td>et10</td>
<td>DC</td>
<td>2 0.94 2.69</td>
<td>541.16</td>
<td>2 0.94 2.69</td>
<td>541.16</td>
</tr>
<tr>
<td></td>
<td>NC</td>
<td>3 1 0 1.037 1.099</td>
<td>592.644</td>
<td>9.5</td>
<td>2 1 0 0.902 1.214</td>
<td>576.269</td>
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<tr>
<td></td>
<td>TC</td>
<td>4 1 1 0.712 0.371</td>
<td>553.922</td>
<td>2.4 -6.5</td>
<td>5 2 1 0.850 1.232</td>
<td>538.946</td>
</tr>
<tr>
<td></td>
<td>RC</td>
<td>6 2 1 0.732 1.190</td>
<td>485.958</td>
<td>-10.2 -12.3</td>
<td>6 3 1 0.900 1.442</td>
<td>476.928</td>
</tr>
<tr>
<td>8</td>
<td>D+10</td>
<td>DC</td>
<td>5 1.62 3.05</td>
<td>1837.28</td>
<td>5 1.62 3.05</td>
<td>1837.28</td>
</tr>
<tr>
<td></td>
<td>NC</td>
<td>8 3 0 1.658 1.360</td>
<td>1868.284</td>
<td>1.7</td>
<td>5 2 0 1.877 1.280</td>
<td>1883.827</td>
</tr>
<tr>
<td></td>
<td>TC</td>
<td>11 3 3 1.558 1.129</td>
<td>1848.401</td>
<td>0.6 -1.1</td>
<td>9 4 2 1.663 1.405</td>
<td>1856.424</td>
</tr>
<tr>
<td></td>
<td>RC</td>
<td>13 4 3 1.421 1.211</td>
<td>1819.459</td>
<td>-1.0 -1.6</td>
<td>9 4 2 1.537 1.406</td>
<td>1838.454</td>
</tr>
<tr>
<td>9</td>
<td>T&amp;6</td>
<td>DC</td>
<td>3 1.273 2.220</td>
<td>360.952</td>
<td>3 1.273 2.220</td>
<td>360.952</td>
</tr>
<tr>
<td></td>
<td>NC</td>
<td>4 1 0 1.361 1.351</td>
<td>382.016</td>
<td>5.8</td>
<td>4 1 0 1.361 1.230</td>
<td>377.520</td>
</tr>
<tr>
<td></td>
<td>TC</td>
<td>6 2 2 1.201 1.477</td>
<td>370.308</td>
<td>2.6 -3.1</td>
<td>6 2 2 1.203 1.430</td>
<td>365.383</td>
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<tr>
<td></td>
<td>RC</td>
<td>8 2 3 1.163 1.241</td>
<td>364.642</td>
<td>-4.5 -6.9</td>
<td>9 3 3 1.216 1.343</td>
<td>341.945</td>
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<tr>
<td>10</td>
<td>T&amp;6</td>
<td>DC</td>
<td>6 1.45 3.67</td>
<td>637.05</td>
<td>6 1.45 3.67</td>
<td>637.05</td>
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<tr>
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<td>NC</td>
<td>11 4 0 1.753 1.185</td>
<td>691.607</td>
<td>8.6</td>
<td>5 2 0 3.449 1.140</td>
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<tr>
<td></td>
<td>TC</td>
<td>14 5 2 1.146 1.192</td>
<td>647.701</td>
<td>1.7 -6.3</td>
<td>8 4 1 1.685 1.365</td>
<td>815.687</td>
</tr>
<tr>
<td></td>
<td>RC</td>
<td>17 6 3 1.210 1.203</td>
<td>606.482</td>
<td>-4.8 -6.4</td>
<td>8 4 1 1.666 1.365</td>
<td>803.909</td>
</tr>
<tr>
<td>Ex. No.</td>
<td>Desc.</td>
<td>n</td>
<td>y g</td>
<td>h</td>
<td>tork</td>
<td>100L^*</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td>---</td>
<td>-----</td>
<td>---</td>
<td>------</td>
<td>--------</td>
</tr>
<tr>
<td>12 b+10</td>
<td>DC</td>
<td>6</td>
<td>3.47</td>
<td>2.88</td>
<td>586.95</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>NC</td>
<td>11.30</td>
<td>3.640</td>
<td>1.248</td>
<td>612.218</td>
<td>11.30</td>
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<tr>
<td></td>
<td>TC</td>
<td>13.35</td>
<td>3.486</td>
<td>1.136</td>
<td>601.634</td>
<td>14.4</td>
</tr>
<tr>
<td>13 c+10</td>
<td>DC</td>
<td>3</td>
<td>2.601</td>
<td>2.426</td>
<td>563.497</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>NC</td>
<td>4100</td>
<td>2.553</td>
<td>1.218</td>
<td>640.423</td>
<td>310</td>
</tr>
<tr>
<td></td>
<td>TC</td>
<td>6.21</td>
<td>1.447</td>
<td>1.324</td>
<td>561.326</td>
<td>631</td>
</tr>
<tr>
<td></td>
<td>RC</td>
<td>9.32</td>
<td>1.796</td>
<td>1.281</td>
<td>487.563</td>
<td>631</td>
</tr>
<tr>
<td>16 d=1</td>
<td>DC</td>
<td>14</td>
<td>5.47</td>
<td>2.68</td>
<td>141.80</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>Base</td>
<td>30.70</td>
<td>7.508</td>
<td>1.480</td>
<td>200.345</td>
<td>21.60</td>
</tr>
<tr>
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<td>TC</td>
<td>36.74</td>
<td>4.286</td>
<td>1.334</td>
<td>185.132</td>
<td>26.64</td>
</tr>
<tr>
<td></td>
<td>RC</td>
<td>49.105</td>
<td>4.292</td>
<td>1.369</td>
<td>156.668</td>
<td>30.74</td>
</tr>
<tr>
<td>26 e+10</td>
<td>DC</td>
<td>8</td>
<td>4.080</td>
<td>2.486</td>
<td>190.183</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>NC</td>
<td>9.20</td>
<td>4.052</td>
<td>1.304</td>
<td>261.819</td>
<td>7.20</td>
</tr>
<tr>
<td></td>
<td>TC</td>
<td>21.51</td>
<td>1.670</td>
<td>1.423</td>
<td>232.340</td>
<td>17.51</td>
</tr>
<tr>
<td>20 c+10</td>
<td>DC</td>
<td>8</td>
<td>12.159</td>
<td>1.898</td>
<td>243.362</td>
<td>8</td>
</tr>
<tr>
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<td>NC</td>
<td>710</td>
<td>13.596</td>
<td>1.466</td>
<td>315.654</td>
<td>510</td>
</tr>
<tr>
<td></td>
<td>TC</td>
<td>10.23</td>
<td>8.936</td>
<td>1.501</td>
<td>301.953</td>
<td>10.23</td>
</tr>
<tr>
<td></td>
<td>RC</td>
<td>19.43</td>
<td>6.774</td>
<td>1.446</td>
<td>258.344</td>
<td>17.43</td>
</tr>
</tbody>
</table>

In column (A): DC = Duncan's model; NC = NLG without G acceptance truncation; TC = STD NLG (with G acceptance truncation); RC = STD NLG with both e,c values reduced by half.

In column 100L^*: The evaluation of L is based on the assumptions that (1) the process characteristic of interest is normally distributed, and (2) USLLSL = 6.

In column (B): Each of NC, TC, and RC is compared to DC to obtain the percent change with respect to 100L^*.

In column (C): Percent difference of 100L^* for the TC row is obtained from comparing TC to NC; similarly, that for the RC row is obtained from comparing RC to TC.

In column (E): Shows the percent difference of 100L^* between m = 3 and m = 2 for each case.
part of the $\bar{X}$-chart. Likewise, TC's c value should also be much smaller than that of its DC counterpart. However, the degree of the reduction of e and c values for NLG depends upon the particular situation. Therefore, on the safe side, a conservative value of 50 percent reduction for both e and c are adopted for this research.

The economic comparisons in Table 5.2 are further summarized in Tables 5.3 and 5.4 for $m = 2$ and $m = 3$, respectively. Based upon these three tables, analyses are first provided for the $m = 2$ situation. Then $m = 2$ and $m = 3$ are compared. Finally, this section is concluded by a discussion of the $m = 3$ case.

First, $m = 2$ is considered. Although the nominal NLG plans (TC--which assumes the same e,c values as those of the $\bar{X}$-chart) always perform worse than the $\bar{X}$-chart (DC) does, the more realistic NLG plans (RC--which assumes reduced e,c values) do become superior under certain conditions. That is, when $\delta$, e, or c is relatively large, RC becomes better than DC. On the other hand, when $\delta$ is relatively small, RC is always worse. However, with a large D value, the performances of RC and DC show almost no difference.

Table 5.3 also suggests that the NLG plan with G acceptance truncation is always better than that without it. Similarly, the NLG plan with e,c reductions is always better than that without them. However, the degree of both the effects of G acceptance truncation and e,c reductions may vary depending upon individual situations. When e or c is relatively large, or $\delta$ is relatively small, these effects are most significant. On the other hand, when D is relatively large, these effects are least significant.
### TABLE 5.3

A SUMMARY TABLE FOR THE ECONOMIC COMPARISON OF X-CHART AND NLG PLANS WHEN m = 2

<table>
<thead>
<tr>
<th>Comparison</th>
<th>Condition</th>
<th>Result Description</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC → DC</td>
<td>$\delta = 2$; $D^+, c^+$</td>
<td>Almost the same</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td>The rest</td>
<td>TC slightly worse</td>
<td>2-3</td>
</tr>
<tr>
<td></td>
<td>$\delta = 1$; Base case</td>
<td>TC much worse</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>$e^+, c^+$</td>
<td>TC much worse</td>
<td>23-24</td>
</tr>
<tr>
<td>RC → DC</td>
<td>$\delta = 2$; $e^+, c^+$</td>
<td>RC moderately better</td>
<td>10-14</td>
</tr>
<tr>
<td></td>
<td>$D^+$</td>
<td>Almost the same</td>
<td>&lt;1</td>
</tr>
<tr>
<td></td>
<td>The rest</td>
<td>RC slightly better</td>
<td>3-6</td>
</tr>
<tr>
<td></td>
<td>$\delta = 1$; Base case</td>
<td>RC moderately worse</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>$e^+, c^+$</td>
<td>RC slightly worse</td>
<td>4-6</td>
</tr>
<tr>
<td>TC → NC</td>
<td>$\delta = 2$; $D^+, b^+$</td>
<td>Almost the same</td>
<td>&lt;2</td>
</tr>
<tr>
<td></td>
<td>$c^+$</td>
<td>TC moderately better</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>The rest</td>
<td>TC slightly better</td>
<td>3-7</td>
</tr>
<tr>
<td></td>
<td>$\delta = 1$; $e^+$</td>
<td>TC moderately better</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>The rest</td>
<td>TC slightly better</td>
<td>4-8</td>
</tr>
<tr>
<td>RC → TC</td>
<td>$\delta = 2$; $e^+, c^+$</td>
<td>RC moderately better</td>
<td>12-13</td>
</tr>
<tr>
<td></td>
<td>$D^+$</td>
<td>Almost the same</td>
<td>&lt;2</td>
</tr>
<tr>
<td></td>
<td>The rest</td>
<td>RC slightly better</td>
<td>5-9</td>
</tr>
<tr>
<td></td>
<td>$\delta = 1$; All cases</td>
<td>RC moderately better</td>
<td>14-15</td>
</tr>
</tbody>
</table>

*"+$" means "compared to."

†"+$ means "relatively large;" "-$ means "relatively small."

**"Almost the same" means "<2% difference;" "slight" means "3-10% difference;" "moderate" means "11-20% difference;" and "much" means ">20% difference."
### TABLE 5.4

A SUMMARY TABLE\textsuperscript{*} FOR THE ECONOMIC COMPARISON OF X-CHART AND NLG PLANS WHEN \( m = 3 \)

<table>
<thead>
<tr>
<th>Comparison</th>
<th>Condition</th>
<th>Result Description</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC (\rightarrow) DC</td>
<td>(\delta = 2; ) e(\dagger), D(\dagger), T&amp;W(\dagger), c(\dagger)</td>
<td>Almost the same</td>
<td>&lt;2</td>
</tr>
<tr>
<td></td>
<td>T&amp;W(\dagger)</td>
<td>TC much worse</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>The rest</td>
<td>TC slightly worse</td>
<td>2-6</td>
</tr>
<tr>
<td></td>
<td>(\delta = 1; ) Base case</td>
<td>TC much worse</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>e(\dagger), c(\dagger)</td>
<td>TC much worse</td>
<td>23-24</td>
</tr>
<tr>
<td>RC (\rightarrow) DC</td>
<td>(\delta = 2; ) e(\dagger), c(\dagger)</td>
<td>RC moderately better</td>
<td>12-13</td>
</tr>
<tr>
<td></td>
<td>T&amp;W(\dagger)</td>
<td>RC slightly better</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>T&amp;W(\dagger)</td>
<td>RC much worse</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>The rest</td>
<td>Almost the same</td>
<td>&lt;2</td>
</tr>
<tr>
<td></td>
<td>(\delta = 1; ) Base case</td>
<td>RC much worse</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>e(\dagger), c(\dagger)</td>
<td>RC slightly worse</td>
<td>5-10</td>
</tr>
<tr>
<td>TC (\rightarrow) NC</td>
<td>(\delta = 2; ) T&amp;W(\dagger), c(\dagger)</td>
<td>TC moderately better</td>
<td>11-14</td>
</tr>
<tr>
<td></td>
<td>D(\dagger)</td>
<td>Almost the same</td>
<td>&lt;2</td>
</tr>
<tr>
<td></td>
<td>The rest</td>
<td>TC slightly better</td>
<td>3-8</td>
</tr>
<tr>
<td></td>
<td>(\delta = 1; ) All cases</td>
<td>TC slightly better</td>
<td>5-10</td>
</tr>
<tr>
<td>RC (\rightarrow) TC</td>
<td>(\delta = 2; ) e(\dagger), c(\dagger)</td>
<td>RC moderately better</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>D(\dagger), T&amp;W(\dagger)</td>
<td>Almost the same</td>
<td>&lt;2</td>
</tr>
<tr>
<td></td>
<td>The rest</td>
<td>RC slightly better</td>
<td>3-6</td>
</tr>
<tr>
<td></td>
<td>(\delta = 1; ) Base case</td>
<td>RC slightly better</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>e(\dagger), c(\dagger)</td>
<td>RC moderately better</td>
<td>11-14</td>
</tr>
</tbody>
</table>

\textsuperscript{*}Notation is explained in Table 5.3.
Now, consider the comparison between m = 2 and m = 3. Column (E) of Table 5.2 suggests that "on the average" m = 3 is worse than m = 2. Especially when T and W are relatively large, m = 3 is much worse. With a relatively small δ value (but together with average e,c values), m = 3 is also considerably worse. The only exception is that when e or c is relatively large (together with a relatively large δ value), m = 3 becomes slightly better.

Furthermore, in actual implementation, m = 3 results in higher e,c values than that of m = 2, due to its longer measuring and recording time. This may well counteract the above described exception (i.e., with a relatively large δ value, the relatively large e or c results in a slightly better performance for m = 3) and make m = 2 always superior to m = 3.

Finally, m = 3 is considered. The general observations for m = 2 follow quite well for m = 3. The only significant exception is that relatively large T and W values make RC much worse than DC.

General Guidelines for Improved Application of NLG and the X-Chart

Based on the analyses of the 12 representative examples, general guidelines can now be provided for better application of both NLG and X-chart control plans.

1. For improved NLG application:
   a. The value m = 2 (instead of m = 3) should always be used whenever possible, especially when either T and W are relatively large or δ is relatively small (≤1).
   b. G acceptance should always be considered.
2. Possible situations for NLG to perform better than the \(\bar{X}\)-chart:
   a. The value of \(\delta\) is relatively large \((\geq 2)\)
   b. Either \(e\) or \(c\) is relatively large
   c. The relative difference of the actual values of \(e\) and \(c\) between the \(\bar{X}\)-chart and NLG is significant.

3. Possible situations for the \(\bar{X}\)-chart to perform better than NLG:
   a. The value of \(\delta\) is relatively small \((\leq 1)\)
   b. Both \(e\) and \(c\) are relatively small
   c. The relative difference of the actual values of \(e\) and \(c\) between the \(\bar{X}\)-chart and NLG is not significant.

4. Possible situations for equivalent performance between the \(\bar{X}\)-chart and NLG plan:
   a. \(D\) is relatively large
   b. The value of \(\delta\) is moderate \((1 < \delta < 2)\).

Comments

The properties revealed in the foregoing discussion match quite well with one's intuition. Since the parameter space of a variable scheme is continuous and that of an attribute scheme is discrete, it is believed that the \(\bar{X}\)-chart is more sensitive to changes than NLG. Thus, for a small process shift, the \(\bar{X}\)-chart should perform better. Due to its much simpler gaging requirements and lack of charting, NLG likely becomes superior whenever either the values of \(e\) and \(c\) of the \(\bar{X}\)-chart are relatively large or the NLG reduction on the \(e\) and \(c\) is significant enough. Finally, the bigger the portion of a control cycle which is occupied by the assignable cause search time \(D\) (which is independent of either control scheme), the smaller effect the control scheme will contribute to the total cost. In
other words, the adoption of either NLG or an $\bar{X}$-chart will make no significant difference on total cost whenever $D$ is big enough.

Although $m = 2$ is on the average more cost-effective than $m = 3$, in practice the latter seems to be psychologically more appealing. This is because $m = 2$ indiscriminately classifies both $Y$ items and $R$ items as "defectives" while $m = 3$ differentiates between the two. Hence, $m = 3$ may be preferred by on-line workers and even inspectors. For better implementation of $m = 2$, more explanation and training must be provided to soften the possible psychological resistance from workers.

In short, both NLG and the $\bar{X}$-chart have their own advantages and disadvantages. A thorough understanding of the environment and one's own needs is crucial in choosing the better-suited model.

Summary

In order to properly compare NLG and the $\bar{X}$-chart, the assumptions and general structure of Duncan's economically-based $\bar{X}$-chart are followed in developing the economic NLG model to ensure the highest degree of similarity and comparability. In the model development, their differences are pointed out and the effects and justifications of assumptions are discussed.

In economic NLG optimization, a general strategy of optimizing $(h,t)$ under each specified set of $(n,m,y,g)$ is followed. To simultaneously optimize $(h,t)$, the loss-cost surface is investigated and the slightly modified Nelder and Mead direct search algorithm is employed. To optimize $(n,m,y,g)$, an appealing convexity property of local optimums among each level of $(n,y,g)$ under specified $m$ has been revealed and is utilized to
construct an efficient NLG optimization algorithm. With adequate experience, this algorithm can be further improved by dynamically adjusting the searching range for each of \((n,y,g)\).

To economically compare NLG with the \(\bar{X}\)-chart, 12 representative examples covering a wide range of variations are selected from Duncan's paper. For each example, the \(\bar{X}\)-chart and three variations of NLG are optimized and compared to each other under \(m = 2\) and \(m = 3\) situations. All of these results are tabulated in Table 5.2 and are further summarized in Tables 5.3 and 5.4. After proper interpretations and analyses, general guidelines are provided for better applications of both models.
CHAPTER VI

USING THE INTERACTIVE COMPUTER PROGRAM

Introduction

Overview

This chapter illustrates the use of an interactive computer program which permits easy utilization of the design and evaluation methodology presented in previous chapters. The actual FORTRAN program is well documented and appears in the Appendix. It has been implemented on an IBM 3081D using various time share terminals.

The user is prompted for all necessary inputs by the computer. All these values together with some preprogrammed parameter values are presented to the user for verification or change. Only when a set of inputs has been verified does the program continue.

When several values are to be entered, they only need be separated by a space or a comma. Integer numbers are usually entered without a decimal point; however, a decimal may be included. The input mechanism is virtually self-explanatory, as long as the user understands the terms being input as well as their mathematically feasible range.

In the remainder of this chapter, actual interactive output is interspersed with comments and explanations. All computer output to follow is automatically generated except for the terminal input which follows a question mark (?).
General Structure and Input Requirements

The general structure and input requirements of this interactive computer program are shown in Figure 6.1. Twelve major functions perform: (1) statistical design and evaluation of NLG, (2) statistical design and evaluation of the $\bar{X}$-chart, (3) economic design, evaluation, and loss-cost surface investigation of NLG, and (4) economic design, evaluation, and loss-cost surface investigation of the $\bar{X}$-chart. Both common input and individual input requirements for each function module are listed.

Getting Started

The program begins by prompting option menu (M.1). The selection of "1" indicates the statistically based scheme is to be pursued.

### ENTER OPTION NUMBER
1 = STATISTICALLY BASED PROCESS CONTROL
2 = ECONOMICALLY BASED PROCESS CONTROL
5 = EXIT SYSTEM

(M.1)

Statistical NLG FG Design

After the statistically-based scheme is selected, values for the common statistical parameters USLLSL and assignable cause are entered and verified. Then, the major statistical option menu (M.2) is presented. A selection of "1" from this menu leads to the statistical NLG FG design.

IN STATISTICALLY BASED PROCESS CONTROL
### ENTER VALUES:
USLLSL, ASSIGNABLE CAUSE (1= MEAN SHIFT; 2= DISPERSION CHANGE)

7
USLLSL = 7.00 (STD) 1= MEAN SHIFT ASSUMED.
CORRECT ? 1=YES 2=NO 3=RETURN

1
Figure 6.1. General Structure and Input Requirements for the Interactive Computer Program
In statistical NLG FG design, the user is sequentially prompted for the input values of three sets of design parameters. After proper verification, all possible plans within the user-specified range are then listed. Each plan is evaluated at four process levels: exact setup for 1-P_a (labeled by PRO); APL, midpoint, and RPL for P_a. The value of PRO represents the probability of a false alarm for each sample. In addition to P_a and 1-P_a, E is also provided for exact setup and RPL. The qualified plans are labeled by "**". To save space, only the results of t=1 are illustrated, since t=2 has a similar output format. At this point, program control returns to menu (M.2) for the next option.
Statistical NLG FG Evaluation

A selection of "2" from menu (M.2) leads to statistical NLG FG evaluation. There are three options for FG evaluation, namely, FG only, FG + PBAPQ, and FG + PBAPQ + PBAOQ. In order to evaluate either PBAPQ or PBAOQ, the value of sampling frequency F (number of samples per OOC indication) must be provided. The procedure for entering the required parameter values and verifying them is the same as that in the last section. In the final evaluation listing, DEL = δ, the degree of mean shift measured in multiples of the standard deviation. Upon completing the evaluation, program control again returns to menu (M.2) for the next option.
A selection of "3" from menu (M.2) leads to statistical NLG QL design. The interactive procedure and the input parameters are almost the same as those of statistical NLG FG design. The only difference is that APL and RPL are now measured in multiples of $\sigma$ (labeled by STD) instead of probability. The format of the resulting listing is very similar to
that of FG design. Note in the following example that \( n \) and \( y \) for QL may differ from the \( n \) and \( y \) values used in FG.

**FOR STAT NLG QL DESIGN**

**ENTER VALUES:** 
\( M = 2 \)  
\( NHIN = 2 \)  
\( NMAX = 6 \)  
\( AFL = 0.200(\text{STD}) \)  
\( TLAF = 0.800 \)  
\( RPL = 2,000(\text{STD}) \)  
\( TLRPL = 0.300 \)  
\( T = 1.700 \)

**VALUES ENTERED:** 
\( M = 2 \)  
\( NHIN = 2 \)  
\( NMAX = 6 \)  
\( AFL = 0.200(\text{STD}) \)  
\( TLAF = 0.800 \)  
\( RPL = 2,000(\text{STD}) \)  
\( TLRPL = 0.300 \)  
\( T = 1.700 \)

**CORRECT?**  
1 = YES  
2 = NO  
3 = RETURN FOR OTHER STAT OPTIONS

**STATISTICALLY BASED NLG QL DESIGN**

\( USL\_SL = 7.00 \) (STD)  
\( \text{DEL MEAN SHIFT: ASSUMED (MULTIPLES OF STD)} \)  
\( M = 2 \)  
\( NHIN = 2 \)  
\( NMAX = 6 \)  
\( AFL = 0.200(\text{STD}) \)  
\( TLAF = 0.800 \)  
\( RPL = 2,000(\text{STD}) \)  
\( TLRPL = 0.300 \)  
\( T = 1.700 \)

**EN3**

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**Statistical NLG QL Evaluation**

A selection of "4" from menu (M.2) leads to statistical NLG QL evaluation. Following the standard interactive procedure, \( P_a \) and \( E_n \) are provided as functions of \( \delta \) (DEL) which ranges from 0 to 5.
Statistical X-Chart Design

A selection of "5" from menu (M.2) leads to the statistical X-chart design. The interactive procedure and input requirements generally follow those in the statistical NLG FG design section.
In the output listing, for each \((n,k)\) combination, four process levels are evaluated: exact setup for \(1-P\), APL, midpoint, and RPL for \(P\). The value of \(1-P\) (labeled by PRO) represents the probability of a false alarm for each sample.

**** STATISTICALLY BASED MODIFIED X-BAR CHART DESIGN ****

\[
\begin{array}{cccccccccccc}
\text{USL} & = & 7.00 & (\text{STD}) & \text{MEAN SHIFT ASSUMED} & (\text{MULTIPLES OF STD}) \\
\text{V} & = & 3.000 & \text{MMIN} & = & 2 & \text{MMAX} & = & 6 \\
\text{APL} & = & 0.010 & \text{TLAPL} & = & 0.900 & \text{RPL} & = & 0.100 & \text{TLRPL} & = & 0.400 \\
\text{INVESTIGATED K VALUES} & = & 1.500 & 2.000 & 2.500 & 2.750 & 3.000 & 3.250 & 3.500 & 4.000 \\
\end{array}
\]

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Statistical \(\bar{x}\)-Chart Evaluation

A selection of "6" from menu (M.2) leads to the statistical \(\bar{x}\)-chart evaluation. The interactive procedure and evaluation results follow.
Economically based process schemes can be accessed by either selecting '3' from menu (M.2) or selecting '2' from menu (M.1). Once accessed, menu (M.3) is listed. Then a selection of '1' from this menu leads to the economic NLG scheme.
Once in the economic NLG scheme, the user is prompted for the values of common economic NLG parameters. After proper verification, menu (M.4) is presented. A selection of "1" from this menu finally results in economic NLG design.

*** FOR ECON NLG, ENTER VALUES:

\[
\begin{align*}
\text{USLLSL} & = 6.00 \\
\text{DELTA} & = 2.00 \\
\text{LAMBDAM} & = 0.01 \\
\text{M} & = 100.00 \\
\text{E} & = 0.05 \\
\text{D} & = 2.00 \\
\text{T} & = 50.00 \\
\text{W} & = 25.00 \\
\text{B} & = 0.50 \\
\text{C} & = 1.00
\end{align*}
\]

VALUES ENTERED:

\[
\begin{align*}
\text{USLLSL} & = 6.00 \\
\text{MM} & = 2 \\
\text{DELTA} & = 2.00 \\
\text{LAMBDAM} & = 0.01 \\
\text{M} & = 100.00 \\
\text{E} & = 0.05 \\
\text{D} & = 2.00 \\
\text{T} & = 50.00 \\
\text{W} & = 25.00 \\
\text{B} & = 0.50 \\
\text{C} & = 1.00
\end{align*}
\]

CORRECT? 1=YES 2=NO 3=RETURN

Y

*** ENTER OPTION NUMBER

1= ECON NLG DESIGN (OPTIMIZATION)
2= ECON NLG EVALUATION
3= ECON NLG LOSS-COST SURFACE INVESTIGATION
4= SWITCH TO ECON X-BAR CHART
5= RETURN TO REVISE USLLSL, MM, AND TIME AND COST PARAMETERS
6= EXIT SYSTEM

The user is then prompted for the values of design parameters. Pre-programmed values of optimization parameters are listed for the user's examination. If desired, these values can be changed to those of the user's preference. In (h,t) optimization, YACC and XACC are quitting criteria; STEP = step size; ITRMAX = maximum iteration number; HO = h_0 and TO = t_0 are starting h,t values; IRESET = 1 requires that each optimization start with the user-specified h_0 and t_0 values; and IRESET = 0 requires that each optimization start with the optimal (h,t) point of the last optimization. In overall optimization, EY = \epsilon_y, EG = \epsilon_g, and EL = \epsilon_L, which are explained in Chapter V, the section entitled "Economic NLG Optimization." For more detail, users are referred to Reference [32] and the subroutines NECOPT, XECOPT, and HTOPT in the Appendix.
optimization output follows. The local optimal solution is first listed for each (n,m,y,g) combination. Each n then has its own suboptimum indicated. Finally, the overall optimum is printed. In the output notation, MM = m; 100L = loss-cost per 100 hours; STDY = standard deviation of 100L for the three vertices of the final simplex; and STDX = standard deviation of the distances among the three vertices of the final simplex. For normal termination of (h,t) optimization (rather than maximum iteration termination), either STDY < YACC or STDX < XACC must be satisfied. The total iteration number TITR must not exceed the specified maximum iteration number ITRMAX; MAXITR indicates whether ITRMAX has been reached or not (if reached, iteration stops and a '***' is printed).

***** ECONOMICALLY BASED NLG DESIGN *****

| USLSEL | MM=2 | MEAN SHIFT ASSUMED |
| DELTA | 2.00 | | | |
| T= 50.00 | W= 25.00 | B= 0.50 | E= 1.00 |

(H,T) OPTIMIZATION: YACC= 0.003 XACC= 0.002 STEP= 1.000 ITRMAX= 60
OVERALL OPTIMIZATION: EY= 2 EG= 3 EL= 0.0 NMIN= 4 NMAX= 10

<table>
<thead>
<tr>
<th>N</th>
<th>MM</th>
<th>Y</th>
<th>G</th>
<th>H</th>
<th>T</th>
<th>100L</th>
<th>STDY</th>
<th>STDX</th>
<th>TITR</th>
<th>MAXITR</th>
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<td>0</td>
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<td>0.532</td>
<td>679.441</td>
<td>0.0014</td>
<td>0.0067</td>
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<tr>
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<td>1</td>
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<td>581.852</td>
<td>0.0014</td>
<td>0.0038</td>
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<tr>
<td>4</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1.479</td>
<td>1.577</td>
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<td>0.0051</td>
<td>17</td>
<td></td>
</tr>
<tr>
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<td>1.609</td>
<td>604.015</td>
<td>0.0019</td>
<td>0.0076</td>
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<td></td>
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<tr>
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<td>3</td>
<td>1</td>
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<td>2.016</td>
<td>645.742</td>
<td>0.0014</td>
<td>0.0069</td>
<td>18</td>
<td></td>
</tr>
</tbody>
</table>

FOR N= 41 MIN 100L = 572.771
<table>
<thead>
<tr>
<th>N</th>
<th>Y</th>
<th>G</th>
<th>H</th>
<th>T</th>
<th>Loss-Cost PER 100 HOURS</th>
<th>Overall Optimal 100L</th>
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<tr>
<td>6</td>
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<td>1.331</td>
<td>561.336</td>
<td>561.336</td>
<td></td>
</tr>
<tr>
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<td>2</td>
<td>1.462</td>
<td>1.331</td>
<td>561.336</td>
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<tr>
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<td>2</td>
<td>1.462</td>
<td>1.331</td>
<td>561.336</td>
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<td></td>
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<tr>
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<td>2</td>
<td>1.462</td>
<td>1.331</td>
<td>561.336</td>
<td>561.336</td>
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<td>2</td>
<td>1.462</td>
<td>1.331</td>
<td>561.336</td>
<td>561.336</td>
<td></td>
</tr>
</tbody>
</table>

**Economic NLG Evaluation**

A selection of "2" from menu (M.4) leads to economic NLG evaluation.

The interactive procedure and output are illustrated below.

FOR ECON NLG EVALUATION, ENTER VALUES: N,Y,G,H,T

VALUES ENTERED: N= 6, Y= 2, G= 1, H= 1.462, T= 1.331

CORRECT 1=YES 2=NO 3=RETURN FOR OTHER ECON NLG OPTIONS

**** ECONOMICALLY BASED NLG EVALUATION ****

USL=SL= 6.00 (STD) H=2 MEAN SHIFT ASSUMED

DELTA= 2.00 LAMBDA= 0.01 \(\lambda= 100.00 \) \(E= 0.05 \) \(D= 2.00 \)

\(T= 50.00 \) \(\mu= 25.00 \) \(B= 0.50 \) \(C= 1.00 \)

\(N= 6 \) \(Y= 2 \) \(G= 1 \) \(H= 1.462 \) \(T= 1.331 \)

LOSS-COST PER 100 HOURS = 561.336 (HOURLY LOSS-COST = 5.613)
Economic NLG Loss-Cost Surface Investigation

A selection of "3" from menu (M.4) leads to the economic NLG loss-cost surface investigation. Loss-cost is evaluated at each (h,t) combination of the user's specified h and t values. Among them, the optimal combination is identified. For each t value, the probability of a false alarm (ALPHA), the probability of a true alarm (P), the in-control average sample number (EN IC), and the out-of-control average sample number (EN OOC) are also provided for the user's reference. A wider terminal width (132) is required for a better loss-cost tabulation. The standard interactive procedure and the final output are illustrated below.

### FOR ECON NLG COST SURFACE INVESTIGATION; ENTER VALUES: N Y G

6 2 1
ENTER VALUES:
NUMH (NUMBER OF H; <= 10), FOLLOWED BY ALL H VALUES TO BE INVESTIGATED

14 .1 .5 .75 1 1.25 1.5 2 2.5 3 5 10 25 50 100
ENTER VALUES:
NUMT (NUMBER OF T; <= 11), FOLLOWED BY ALL T VALUES TO BE INVESTIGATED

11 .1 .5 .75 1 1.25 1.5 1.75 2 2.25 2.5 2.9
VALUES ENTERED: N= 6  T= 2  G= 1
14 H VALUES = 0.100 0.500 0.750 1.000 1.250 1.500
2.000 2.500 3.000 5.000 10.000 25.000
50.000 100.000
11 T VALUES = 0.100 0.500 0.750 1.000 1.250 1.500
1.750 2.000 2.250 2.500 2.900

### ENTER OPTION NUMBER:
1= ALL ON; NO REVISION NEEDED
2= NEED TO REVISE (N Y G) VALUES
3= NEED TO REVISE NUMH AND H VALUES
4= NEED TO REVISE NUMT AND T VALUES
5= RETURN FOR OTHER ECON NLG OPTIONS
?

1
| **USLLSL** | 6.00 STD | **H** = 2 | **DELTA** | 2.00 | **LAMBDA** | 0.01 | **E** | 0.05 | **D** | 2.00 | **T** | 50.00 | **B** | 0.50 | **C** | 1.00 |
|------------|----------|-----------|-----------|-------|------------|------|-----|-----|------|------|-----|-------|------|------|-----|
| **T**      | 0.100    | 0.500     | 0.750     | 1.000 | 1.250      | 1.500 | 1.750 | 2.000 | 2.250 | 2.500 | 2.900 |
| **ALPHA**  | 0.000    | 0.000     | 0.000     | 0.000 | 0.000      | 0.004 | 0.018 | 0.060 | 0.161 | 0.339 | 0.571 |
| **F**      | 0.042    | 0.151     | 0.267     | 0.406 | 0.546      | 0.668 | 0.766 | 0.840 | 0.897 | 0.939 | 0.989 |
| **EN IC**  | 1.019    | 1.042     | 1.122     | 1.227 | 1.396      | 1.647 | 1.981 | 2.359 | 2.700 | 2.917 | 3.000 |
| **EN GCC** | 1.869    | 2.331     | 2.588     | 2.781 | 2.902      | 2.989 | 2.997 | 2.999 | 3.000 | 3.000 |        |
| **H**      |          |           |           |       |            |       |       |       |       |       |       |       |       |       |      |
| 0.100      | 2000.021 | 1886.679  | 1927.934  | 2056.677 | 2390.076  | 3305.394 | 5701.195 | 10984.695 | 20045.992 | 31595.848 | 46757.227 |
| 0.500      | 1545.786 | 641.241   | 722.133   | 489.854  | 727.598   | 894.734  | 1363.772 | 2412.747  | 4217.469  | 4520.695  | 9944.036  |
| 0.750      | 1866.720 | 871.277   | 486.093   | 615.561  | 616.170   | 714.242  | 1018.863 | 1712.390 | 2911.094  | 4442.734  | 6721.016  |
| 1.000      | 2199.844 | 955.451   | 704.229   | 600.885  | 576.925   | 635.670  | 855.895  | 1370.456 | 2265.182  | 3410.451  | 5115.172  |
| 1.250      | 2519.586 | 1047.137  | 743.379   | 609.811  | 564.040   | 597.803  | 765.664  | 1171.784 | 2911.094  | 4442.734  | 6721.016  |
| 1.500      | 2819.993 | 1150.891  | 792.503   | 630.333  | 566.242   | 580.239  | 711.770  | 1044.721 | 1633.763  | 2391.499  | 3521.431  |
| 2.000      | 3361.250 | 1364.276  | 904.176   | 688.182  | 591.499   | 575.412  | 658.391  | 897.942  | 1332.456  | 1895.286  | 2736.603  |
| 2.500      | 3830.891 | 1575.771  | 1022.844  | 756.463  | 630.298   | 590.586  | 641.160  | 822.640  | 1363.115  | 1608.108  | 2275.299  |
| 3.000      | 4240.328 | 1781.027  | 1143.147  | 829.214  | 675.514   | 615.565  | 641.877  | 782.989  | 1059.691  | 1425.393  | 1975.711  |
| 5.000      | 5453.273 | 2523.636  | 1610.708  | 1129.684 | 877.977   | 752.409  | 715.085  | 765.960  | 926.863   | 1111.752  | 1423.761  |
| 10.000     | 7025.105 | 3917.419  | 2612.947  | 1833.098 | 1309.456  | 1140.441 | 1009.068 | 942.512  | 985.308   | 1052.443  | 1170.973  |
| 25.000     | 8541.031 | 6108.883  | 4581.746  | 3433.282 | 2672.205  | 2194.272 | 1898.953 | 1720.441 | 1620.335  | 1570.188  | 1532.942  |
| 50.000     | 9211.668 | 7574.598  | 6218.969  | 5073.910 | 4164.559  | 3571.825 | 3108.308 | 2627.229 | 2441.847  | 2519.523  | 2391.927  |
| 100.000    | 9589.816 | 8622.711  | 7709.730  | 6749.344 | 5907.844  | 5256.594 | 4764.398 | 4449.543 | 4213.156  | 4045.380  | 2858.577  |

| **H**      |          |           |           |       |            |       |       |       | H = 1.250 | **T** = 1.250 | **LOSS-COST** = 564.040 (PER 100 HOURS) |
Economic $\bar{X}$-Chart Design (Optimization)

The economic $\bar{X}$-chart scheme can be accessed by either selecting "4" from menu (M.4) or selecting "2" from menu (M.3). Once accessed, the user is first prompted for the values of common economic $\bar{X}$-chart parameters. After proper verification, menu (M.5) is presented. And a selection of "1" from this menu leads to the economic $\bar{X}$-chart design.

### FOR ECON $\bar{X}$-BAR CHART, ENTER VALUES:
- USL, SL, DELTA, LAMBDA, M, E, V, T, W, B, C

VALUES ENTERED:
- USL = 6.00
- DELTA = 2.00
- LAMBDA = 0.01
- M = 100.00
- E = 0.05
- D = 20.00
- T = 50.00
- W = 25.00
- B = 0.50
- C = 0.10

CORRECT? 1=YES 2=NO 3=RETURN

Then the user is prompted for the values of design parameters. The pre-programmed values of optimization parameters are listed for the user's examination. These values can be changed upon the user's request. After proper verification, the optimization subroutine is executed and optimal results printed. The interactive procedure, notation, and output format are similar to those for economic NLG design.

### FOR ECON $\bar{X}$-BAR CHART DESIGN, ENTER VALUES:  NMIN, NMAX

VALUES ENTERED:
- NMIN = 2
- NMAX = 10

PARAMETER VALUES FOR:
- $H(T)$ OPTIMIZATION
- OVERALL OPTIMIZATION

DEFAULT:
- YACC = 0.003
- XACC = 0.002
- STEP = 1.00
- ITMAX = 60
- HO = 1.000
- TO IRESET
- EL = 1.000

CURRENT:
- YACC = 0.003
- XACC = 0.002
- STEP = 1.00
- ITMAX = 60
- HO = 1.000
- TO IRESET
- EL = 1.000

### ENTER OPTION NUMBER:
1= ALL OK, NO REVISION NEEDED
2= NEED TO REVISE (NMIN, NMAX) VALUES
3= NEED TO REVISE ($H(T)$ OPTIMIZATION PARAMETER VALUES
4= NEED TO REVISE OVERALL OPTIMIZATION PARAMETER VALUE
5= RETURN FOR OTHER ECON $\bar{X}$-BAR CHART OPTIONS

4
ENTER VALUE: EL

VALUES ENTERED: NMIN = 2  NMAX = 10

PARAMETER VALUES FOR:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Default</th>
<th>Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>YACC</td>
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<td>0.003</td>
</tr>
<tr>
<td>XACC</td>
<td>0.002</td>
<td>0.002</td>
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<tr>
<td>STEP</td>
<td>1.00</td>
<td>1.00</td>
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<tr>
<td>ITR</td>
<td>60</td>
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<tr>
<td>TRMAX</td>
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<td>1</td>
</tr>
<tr>
<td>EL</td>
<td>20.00</td>
<td>20.00</td>
</tr>
</tbody>
</table>

*** ENTER OPTION NUMBER:
1 = ALL OK, NO REVISION NEEDED
2 = NEED TO REVISE (NMIN + NMAX) VALUES
3 = NEED TO REVISE (H,T) OPTIMIZATION PARAMETER VALUES
4 = NEED TO REVISE OVERALL OPTIMIZATION PARAMETER VALUE
5 = RETURN FOR OTHER ECON X-BAR CHART OPTIONS

1

****** ECONOMICALLY BASED X-BAR CHART DESIGN ******

USL = 6.00  MEAN SHIFT ASSUMED
USL = 6.00  DELTA = 2.00  LAMBDA = 0.01  M = 100.00  .E = 0.05  D = 20.00
T = 50.00  W = 25.00  B = 0.50  C = 0.10

(H,T) OPTIMIZATION:

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<thead>
<tr>
<th>N</th>
<th>H</th>
<th>K</th>
<th>100L</th>
<th>STDY</th>
<th>STDX</th>
<th>TITR</th>
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<td>5</td>
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****** ECONOMICALLY BASED X-BAR CHART DESIGN ******

OVERALL OPTIMAL 100L = 1837.204

Economic X-Chart Evaluation

A selection of "1" from menu (M.5) leads to the economic X-chart evaluation. The interactive procedure and evaluation output are very similar to those in economic NLG evaluation and are illustrated below.
FOR ECON X-BAR CHART EVALUATION, ENTER VALUES: N, H, K
?

VALUES ENTERED: N = 5, H = 1.669, K = 3.046
CORRECT? 1= YES  2= NO  3= RETURN FOR OTHER ECON X-BAR CHART OPTIONS
?

ERROR!! OUT OF RANGE!! DO IT OVER AGAIN
CORRECT? 1= YES  2= NO  3= RETURN FOR OTHER ECON X-BAR CHART OPTIONS
?

**** ECONOMICALLY BASED X-BAR CHART EVALUATION ****
USLLSL = 6.00 (STD)  MEAN SHIFT ASSUMED

DELTA = 2.00  LAMBDA = 0.01  M = 100.00  E = 0.05  D = 20.00
T = 50.00  W = 25.00  B = 0.50  C = 0.10

N = 5  H = 1.669  K = 3.046

LOSS-COST FOR 100 HOURS = 1837.204  (HOURLY LOSS-COST = 18.372)

Economic \( \bar{X} \)-Chart Loss-Cost Surface Investigation

A selection of "3" from menu (M.5) leads to the economic \( \bar{X} \)-chart loss-cost surface investigation. The interactive procedure, notation, and explanation are very similar to those in the economic NLG loss-cost surface investigation. They are illustrated below.

**** FOR ECON X-BAR CHART COST SURFACE INVESTIGATION, ENTER VALUE: N
?

5
ENTER VALUES:
NUMBER OF H <= 30, FOLLOWED BY ALL H VALUES TO BE INVESTIGATED
14 1.5 1.75 1.25 1.5 1.75 2 2.25 2.5 3 5 10 50
ENTER VALUES:
NUMBER OF K <= 31, FOLLOWED BY ALL K VALUES TO BE INVESTIGATED
11 1.5 1.75 2 2.25 2.5 2.75 3 3.25 3.5 3.75
VALUES ENTERED:
N = 5
H VALUES = 0.100 0.500 0.750 1.000 1.250 1.500
1.750 2.000 2.250 2.500 3.000 5.000
10.000 50.000
K VALUES = 1.500 1.750 2.000 2.250 2.500 2.750
3.000 3.250 3.500 3.750 4.000
### ECONOMICALLY BASED X-BAR CHART LOSS-COST SURFACE INVESTIGATION

**USL/LSL**: 6.00 STD  
**MEAN SHIFT**: ASSUMED  
**N**: 5  
**DELTA**: 2.00  
**LAMBDA**: 0.01  
**M**: 100,000  
**E**: 0.05  
**D**: 20.00  
**T**: 50,000  
**W**: 25.00  
**B**:  
**ALPHA**:  

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<th>1.750</th>
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<th>2.250</th>
<th>2.500</th>
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<td>1999.730</td>
<td>1933.842</td>
<td>1899.144</td>
<td>1882.745</td>
<td>1876.712</td>
<td>1876.686</td>
<td>1880.682</td>
<td>1888.232</td>
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<tr>
<td>1.500</td>
<td>2191.214</td>
<td>2044.023</td>
<td>1949.014</td>
<td>1891.691</td>
<td>1859.735</td>
<td>1843.933</td>
<td>1838.164</td>
<td>1838.089</td>
<td>1844.485</td>
<td>1854.767</td>
<td>1870.630</td>
</tr>
<tr>
<td>3.000</td>
<td>2024.097</td>
<td>1951.135</td>
<td>1904.454</td>
<td>1876.959</td>
<td>1862.031</td>
<td>1857.922</td>
<td>1859.785</td>
<td>1867.476</td>
<td>1881.287</td>
<td>1902.637</td>
<td>1934.132</td>
</tr>
</tbody>
</table>

**MINIMUM**: H = 1.750  
**T = 3.000**  
**LOSS-COST = 1837.396 (PER 100 HOURS)**
Summary

Nearly every feature of the interactive computer program of this research has been illustrated in this chapter. The interactive feature and its convenience, flexibility and comprehensiveness make this computer program a powerful process control tool. The implementation of this program can substantially help practitioners in designing and evaluating NLG process control plans both statistically and economically. Through its additional statistical and economic $\bar{X}$-chart design and evaluation capability, NLG can also be properly compared to the $\bar{X}$-chart. As such, this interactive computer program will greatly help with better assessment, easier implementation, and broader application of the NLG process control scheme.
CHAPTER VII

SUMMARY AND CONCLUSION

To fulfill the objective and subobjectives of this research stated in Chapter I, the following have been accomplished:

1. The general structure of NLG has been made clear by a comprehensive analysis, discussion, and illustration of NLG taxonomy. The undesirable diversity of possible NLG rules has been demonstrated.

2. A symbolically stated standard NLG scheme has been developed to standardize and simplify the design and evaluation of NLG. The relative importance and applicability of its individual basic elements have been examined.

3. The formulations for statistically evaluating both sample-wise and process-wise NLG performance have been derived, wherein either the mean shift or dispersion change is considered as an assignable cause.

4. General procedures have been constructed for statistically designing FG, QL, and the entire NLG plan. The general effects of individual NLG parameters on $P_a$ and $E_n$ have been investigated to help design FG and QL rules.

5. Methodologies for statistically evaluating and designing an $\bar{X}$-chart have been presented. An example comparing NLG, the $\bar{X}$-chart, and a traditional attribute gaging plan has been presented.

6. An economically-based NLG model has been formulated by following
the general structure of Duncan's fundamental economic $\bar{X}$-chart. Assumptions, similarities, and differences of both models have been investigated.

7. A general strategy together with a direct search technique has been developed to optimize the economic NLG model. For each $m$, this strategy optimizes $(h, t)$ under each specified set of $(n, y, g)$. This strategy is further improved by utilizing the convexity property of local optima among each level of $(n, y, g)$ and by dynamically adjusting the searching range for each value of $n, y,$ and $g$.

8. Economic NLG and the economic $\bar{X}$-chart have been compared under a variety of situations. From this analysis, general guidelines have been developed for better application of both models.

9. A convenient, flexible, and comprehensive interactive computer program has been constructed and demonstrated to facilitate the design and evaluation of (1) statistically-based NLG plans, (2) statistically-based $\bar{X}$-chart plans, (3) economically-based NLG plans, and (4) economically-based $\bar{X}$-chart plans.

Based on the results obtained in this research, the NLG process control scheme has proved to have combined the advantages of both variable and attribute control schemes. Therefore, it becomes potentially very suitable for the rapid detection of a process shift. In comparison to $\bar{X}$-charts both statistically and economically, NLG plans have been shown to be at least equally competitive, and in several aspects quite better than $\bar{X}$-charts, due to their easier-to-use go/no-go gaging method and no-calculation-required control scheme.

The following are major recommendations for future research on the
same subject to facilitate NLG implementation and to cover a wider range of NLG applications:

1. For statistically-based control schemes, comprehensive standard tabulations of already-designed plans can be provided for FG, QL, entire NLG, and the $\bar{X}$-chart under a wide range of APL, TLAPL, RPL, and TLRPL design criteria. This can significantly reduce the cumbersome design procedures to a simple table-lookup for both NLG and $\bar{X}$-chart plans. It can also provide an alternative selection between NLG and $\bar{X}$-chart plans to better suit the user's individual needs.

2. The economically-based formulations of both NLG and the $\bar{X}$-chart can be extended to include dispersion change as an alternative assignable cause.

3. Different economically-based models of both NLG and the $\bar{X}$-chart requiring process shutdown during the search for an assignable cause can be considered.

4. More present-time examples containing realistic time and cost parameter values can be adopted for comparing economic NLG and $\bar{X}$-chart performance. This comparison should include the extended and the new economic control schemes proposed in items 2 and 3.

5. The economic portion of the interactive computer program should be extended accordingly.
REFERENCES


C***********************************************************************
C THIS INTERACTIVE PROGRAM PERFORMS
C (1) STATISTICAL DESIGN AND EVALUATION OF NLG
C (2) STATISTICAL DESIGN AND EVALUATION OF X-BAR CHART
C (3) ECONOMIC DESIGN AND EVALUATION OF NLG
C (4) ECONOMIC DESIGN AND EVALUATION OF X-BAR CHART
C BY SHAWN S. YU, SCHOOL OF INDUSTRIAL ENGINEERING AND MANAGEMENT
C OKLAHOMA STATE UNIVERSITY
C DISSERTATION ADVISOR: DR. KENNETH E. CASE
C VERSION 1 -- JULY, 1983
C
C***********************************************************************

C*** GENERAL STRUCTURE AND INPUT REQUIREMENTS:
C ( MAIN PROGRAM DRIVES SUBROUTINES STAT AND ECON. )
C ( STAT DRIVES S1 THROUGH S6; ECON DRIVES E1 THROUGH E6 )

<table>
<thead>
<tr>
<th>COMMON INPUT</th>
<th>MAJOR FUNCTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>STAT --------</td>
<td>USLLSL ; NLG ; X-BAR</td>
</tr>
<tr>
<td>ECON --&gt; NLG</td>
<td>USLLSL,M ; X-BAR</td>
</tr>
<tr>
<td>--&gt; X-BAR</td>
<td>USLLSL ; ASSIGNABLE CAUSE</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SUBROUTINE</th>
<th>FUNCTION</th>
<th>MODULE INPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1: FGENE</td>
<td>NLG FQ DESIGN</td>
<td>M; NMIN,NMAX;</td>
</tr>
<tr>
<td>S2: GFEVAL</td>
<td>NLG FQ EVALU.</td>
<td>APL,TLAPL; T VALUES</td>
</tr>
<tr>
<td>S3: OGENE</td>
<td>NLG QL DESIGN</td>
<td>N,Y,G; T VALUES;</td>
</tr>
<tr>
<td>S4: QLEVAL</td>
<td>NLG QL EVALU.</td>
<td>M; NMIN,NMAX;</td>
</tr>
<tr>
<td>S5: XSTGE</td>
<td>X-BAR DESIGN</td>
<td>APL,TLAPL; T</td>
</tr>
<tr>
<td>S6: XSTEV</td>
<td>X-BAR EVALU.</td>
<td>N,V,K</td>
</tr>
<tr>
<td>E1: NECOPT</td>
<td>NLG DESIGN</td>
<td>NMIN,NMAX;</td>
</tr>
<tr>
<td>E2: NECFV</td>
<td>NLG EVALUATION</td>
<td>N,Y,G; OPTIMIZATION PARAMETERS (OPTIONAL)</td>
</tr>
<tr>
<td>E3: NCOF</td>
<td>NLG COST SURF.</td>
<td>N,Y,G; H VALUES; T VALUES</td>
</tr>
<tr>
<td>E4: XCOF</td>
<td>X-BAR DESIGN</td>
<td>NMIN,NMAX;</td>
</tr>
<tr>
<td>E5: XCEV</td>
<td>X-BAR EVALU.</td>
<td>N,H,K</td>
</tr>
<tr>
<td>E6: XDSF</td>
<td>X-BAR COST SURF.</td>
<td>N; H VALUES; K VALUES</td>
</tr>
</tbody>
</table>

C*** EXTERNAL FUNCTIONS REQUIRED:
C (1) REGULAR SYSTEM SUPPLIED FORTRAN FUNCTIONS
C (2) TWO IMSL SUBROUTINES:
C MDNOR -- CUMULATIVE PROBABILITY FUNCTION OF STANDARD NORMAL
C MDNRIS -- INVERSE FUNCTION OF MDNOR

C*** COMMON BLOCK VARIABLE DEFINITIONS:
C NN -- SMALL N, SAMPLE SIZE
C MM -- SMALL M, NUMBER OF NLG CLASSIFICATIONS
C NG -- SMALL G, GREEN ACCEPTANCE TRUNCATION NUMBER
C NY -- SMALL Y, MAXIMUM YELLOW ACCEPTANCE NUMBER
C /C1/ ---- NLG PARAMETERS

C***********************************************************************
NY1 = NY + 1
TLNG = SMALL T, NLG CONTROL SPREAD
------------------------ FOR STATISTICAL BASED SCHEMES ------------------------
/S1/
/WSTD -- ASSIGNABLE CAUSE (1 = MEAN SHIFT; 2 = DISPERSION CHANGE)
NHL,NNH -- RANGE OF SAMPLE SIZE
APL,TLAP,RPL,TLRPL -- USER SPECIFIED QC CURVE DESIGN POINTS,
ACCEPTABLE AND REJECTABLE PROCESS LEVELS AND THEIR ASSOCIATED
TOLERABLE LIMITS
NUM.T,AT(10) -- NUMBER OF T VALUES. THESE T VALUES ARE STORED IN
ARRAY AT
/S2/
PG,PY,PR -- PROBABILITY OF GREEN, YELLOW AND RED
/S3/
DELMU -- DEGREE OF PROCESS MEAN SHIFT (IN MULTIPLES OF STD)
STD10 -- DEGREE OF DISPERSION CHANGE (THE RATIO OF NEW OLD)
/S4/
IFG = 1=FG 2 = FG + PBAPQ , 3 = FG + PBAPQ + PBAPQ
NF = CAPITAL F, THE SELF-ADJUST SAMPLING FREQUENCY, THE NUMBER
OF SAMPLES PER OUT-OF-CONTROL INDICATION
/S5/
RY -- RELATIVE LOCATION OF THE LOWER SPECIFICATION LIMIT MEASURED
FROM THE PROCESS MEAN (IN MULTIPLES OF STD)
DEL -- DEGREE OF MEAN SHIFT (IN MULTIPLES OF STD)
STD10 -- DEGREE OF DISPERSION CHANGE (NEW TO OLD RATIO)
/S6/
IFG -- PARAMETERS FOR X-BAR CHART PLANS
/S7/
VX -- SMALL V, THE DISTANCE BETWEEN A SPECIFICATION LIMIT AND ITS
CORRESPONDING BOUNDARY FOR AN ACCEPTABLE PROCESS MEAN (IN
MULTIPLES OF STD)
RKX -- SMALL K, X-BAR CHART CONTROL LIMIT SPREAD
NX -- SMALL N, SAMPLE SIZE OF X-BAR CHART PLAN
NXL,NXH -- RANGE OF NX
NUMK,AK(10) -- NUMBER OF K VALUES. THESE VALUES ARE STORED IN
ARRAY AK
C /E2/
PG,PY,PR -- PROBABILITY OF GREEN, YELLOW AND RED
PR1,PR2 -- FRACTION DEFECTIVES BEFORE AND AFTER PROCESS MEAN SHIFT
/E3/ ------ COST AND TIME PARAMETERS FOR NLG OR X-BAR CHART SCHEME
/E4/ ------ (H,T) DIRECT SEARCH OPTIMIZATION PARAMETERS
/XSTART(2) -- THE ADOPTED STARTING VALUES OF H AND T
/X(3.2) -- THREE VERTICES OF A ITERATION SIMPLEX
/Y(3) -- FUNCTION VALUES (LOSS-COST) OF X(3.2)
/ITRFLG -- 1 = MAXIMUM ITERATION NUMBER REACHED AND ITERATION
TERMINATED
/IRESET -- 1 = EACH (H,T) OPTIMIZATION STARTS WITH THE USER SPECIFIED
(H,T) STARTING VALUES
O = EACH (H,T) OPTIMIZATION STARTS WITH THE OPTIMAL (H,T)
VALUES FROM LAST OPTIMIZATION
/STDX -- STANDARD DEVIATION OF THE DISTANCES AMONG ALL VERTICES OF
A SIMPLEX
/STDY -- STANDARD DEVIATION OF THE FUNCTION VALUES OF ALL VERTICES
OF A SIMPLEX
/XACC,YACC -- USER SPECIFIED QUITTING CRITERIA. (H,T) OPTIMIZATION
TERMINATES WHENEVER STDX < XACC OR STDY < YACC
/STEP -- STEP SIZE
/ITRMAX -- USER SPECIFIED MAXIMUM ITERATION NUMBER
NLGX = 1 = NLG SCHEME 2 = X-BAR CHART SCHEME
/E5/ ------ NLG OVERALL OPTIMIZATION PARAMETERS
NYBACK -- EPSILON SUB SMALL Y, THE VALUE TO DYNAMICALLY DETERMINE
NEXT STARTING Y VALUE
NYBACK -- EPSILON SUB SMALL G, THE VALUE TO DYNAMICALLY DETERMINE
NEXT STARTING G VALUE
YIMPV -- EPSILON SUB L, THE VALUE TO OVERCOME BUMPS IN A CONVEX
CURVE
/NMIN,NMAX -- RANGE OF SAMPLE SIZE
/E6/ ------ PARAMETERS FOR LOSS-COST SURFACE INVESTIGATION
HNLG = SMALL H, THE SAMPLING INTERVAL FOR NLG PLAN
HX = SMALL H, THE SAMPLING INTERVAL FOR X-BAR CHART PLAN
```
C RKX: -- SMALL K, THE CONTROL SPREAD, FOR X-BAR CHART PLAN
C /E7/ 
C NH,AH(30) -- NUMBER OF K VALUES. THESE VALUES ARE STORED IN ARRAY
C AH 
C MT,AT(11); NK,AK(11) -- SIMILAR FOR T AND K
C
C*******************************************************************************
C MAIN PROGRAM -- THE PRIMARY DRIVER PROGRAM
C
C *** THE MAIN PROGRAM DRIVES SUBROUTINES STAT AND ECON
C
COMMON /C1/ USLLSL, NN, MM, NG, NY, NY1, TNLG, HALF, IR, IW
IR=5
IW=6
10 WR !TE ( IW, 11 )
11 FORMAT(/ 
** ENTER OPTION NUMBER'/
* T6,'1 =STATISTICALLY-BASED 
PROCESS CONTROL'/
* T6,'2 =ECONOMICALLY-BASED PROCESS CONTROL'/
* T6, '3 =EXIT 
SYSTEM')
READ(IR,* ) N13
GOTO(100,200,300),N13
WRITE(IW,20)
20 FORMAT(' 
ERROR 
OUT OF 
RANGE DO 
IT OVER AGAIN')
GOTO 10
100 CALL STAT
GOTO 10
200 CALL ECON
GOTO 10
300 STOP
END
C
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C *** THIS BLOCK DATA SUBPROGRAM INITIALIZE VARIABLES IN COMMON /S8/
C
COMMON /S8/ ASSCOZ(10,2), BLANK, STAR2, DELSTD(2)
DATA ASSCOZ/'MEAN', 'SHI', 'FT A', 'SSUM', 'ED (', 'MULT', 'IPLE',
* 'S OF', 'STD')','
* 'DISP', 'ERSI', 'ON C', 'HANG', 'E AS', 'SUME', 'D (S',
* 'TD R', 'AT I')'/
DATA BLANK/ '/, STAR2/* ,DELSTD/'DEL', 'STD'/
END
C
C*******************************************************************************
C*******************************************************************************
C*******************************************************************************
C *** THIS SUBROUTINE SERVICES AS THE PROMPTER PROGRAM AND DRIVES THE 
C FOLLOWING SIX SUBROUTINES FOR THE STATISTICALLY BASED 
C PROCESS CONTROL SCHEMES: 
C
SUBROUTINE STAT
C
FGGENE -- STAT NLG FG DESIGN
FGEVAL -- STAT NLG FG EVALUATION
QLGENE -- STAT NLG QL DESIGN
QLEVAL -- STAT NLG QL EVALUATION
XSTGE -- STAT X-BAR CHART DESIGN
XSTEV -- STAT X-BAR CHART EVALUATION
```
COMMON /C1/ USLLSL, NN, MM, NG, NY, NV1, TNLG, HALF, IR, IW 00027000
COMMON /S2/MUSTD, NNL, NNH, APL, TLAPL, RPL, TLRPL, NUMT, AT(10) 00021800
COMMON /S5/IFG, NF 00021900
COMMON /S7/VX, NXL, NXH, NUMK, AK(10), NX, RKX 000220000
20 FORMAT(‘!! ERROR !! OUT OF RANGE !! DO IT OVER AGAIN’) 00022100
C------------------------------------ STAT OPTION MENU ----------------------------------- 00022300
100 WRITE(IW,101) 00022400
101 FORMAT(‘IN STATISTICALLY BASED PROCESS CONTROL’/’*** ENTER’, 00022500
*’ VALUES:/T2,’USLLSL, ASSIGNABLE CAUSE (1 = MEAN SHIFT’, 00022600
*’ 2 = DISPERSION CHANGE’) 00022700
READ(IR,*) USLLSL, MUSTD 00022800
IF(MUSTD.EQ.1) WRITE(IW,103) USLLSL 00022900
103 FORMAT(‘USLLSL=’,F5.2,’(STD),’; ’ MEAN SHIFT ASSUMED.’) 00023000
IF(MUSTD.EQ.0) WRITE(IW,104) USLLSL 00023100
104 FORMAT(‘USLLSL=’,F5.2,’(STD),’; ’ DISPERSION CHANGE ASSUMED.’) 00023200
102 WRITE(IW,107) 00023300
107 FORMAT(‘CORRECT? ’; ‘1=YES 2=NO 3=RETURN’) 00023400
READ(IR,*) IYN 00023500
GOTO (105,100,250),IYN 00023600
WRITE(IW,20) 00023700
GOTO 102 00023800
C 00023900
105 WRITE(IW,106) 00024000
106 FORMAT(‘*** ENTER OPTION NUMBER’/) 00024100
*’ T6,’1= STAT NLG FG DESIGN’/ 00024200
*’ T6,’2= STAT NLG FG EVALUATION (+ OPTIONAL PBAPQ AND PBADQ’)/ 00024300
*’ T6,’3= STAT NLG OI DESIGN’/ 00024400
*’ T6,’4= STAT NLG OI EVALUATION’/ 00024500
*’ T6,’5= STAT XBAR CHART DESIGN’/ 00024600
*’ T6,’6= STAT XBAR CHART EVALUATION’/ 00024700
*’ T6,’7= RETURN TO REVISE USLLSL AND ASSIGNABLE CAUSE’/ 00024800
*’ T6,’8= SWITCH TO ECN PROCESS CONTROL SCHEME’/ 00024900
*’ T6,’9= EXIT SYSTEM’/ 00025000
READ(IR,*) NSTAT 00025100
GOTO (110,120,130,140,150,160,100,250,300),NSTAT 00025200
WRITE(IW,20) 00025300
GOTO 105 00025400
C 00025500
C------------------------------------- STAT NLG FG DESIGN ------------------------------------- 00025600
110 WRITE(IW,111) 00025700
111 FORMAT(‘FOR STAT NLG FG DESIGN’/)’ 00025800
*’ *** ENTER VALUES: M,NMIN,NMAX’/) 00025900
READ(IR,*) MM, NNL, NNH 00026000
WRITE(IW,112) 00026100
112 FORMAT(‘*** ENTER VALUES: APL,TLAPL,RPL,TLRPL’/) 00026200
READ(IR,*) APL, TLAPL, RPL, TLRPL 00026300
WRITE(IW,113) 00026400
113 FORMAT(‘*** ENTER VALUES:/T2,’NUMT (NUMBER OF T; <= 10),’/ 00026500
*’ FOLLOWED BY T VALUES TO BE INVESTIGATED’/) 00026600
READ(IR,*) NUMT,(AT(I),I=1,NUMT) 00026700
WRITE(IW,114) MM, NNL, NNH, APL, TLAPL, RPL, TLRPL, NUMT, (AT(I), I=1, NUMT) 00026800
114 FORMAT(‘VALUES ENTERED: M’,F5.2,’(NXL, NXL=12.4X,’/ 00026900
*’ NMIN=F5.3,4X,’RPL=F5.3,4X,’/ 00027000
*’ TLAPL=F5.3,4X,’RPL=F5.3,4X,’/ 00027100
*’ T=10/(F5.3,1X))’/) 00027200
117 WRITE(IW,115) 00027300
115 FORMAT(‘CORRECT? ’; ‘1=YES 2=NO 3= RETURN FOR OTHER’/) 00027400
*’ STAT OPTIONS’/) 00027500
READ(IR,*) IYN 00027600
GOTO (116,110,105),IYN 00027700
WRITE(IW,20) 00027800
GOTO 117 00027900
116 CALL FGGENG 00028000
GOTO 105 00028100
C 00028200
C------------------------------------- STAT NLG FG EVALUATION ------------------------------------- 00028300
120 WRITE(IW,121) 00028400
121 FORMAT(‘*** FOR STAT NLG FG EVALUATION, ENTER OPTION NUMBER’/) 00028500
*’ T5,’1= FG ONLY 2= FG + PBAPQ 3= FG + PBAPQ + PBADQ’/) 00028600
READ(IR,*) IFG 00028700
WRITE(IW,122) 00028800
122 FORMAT(‘*** FDR FG, ENTER VALUES: N,M,Y,G’/) 00028900
READ(IR,*) NN, NM, NY, NG
WRITE(IW, 113)
READ(IR,*) NUMT, (AT(I), I=1, NUMT)
GOTO (128, 127, 127), IFG
127 WRITE(IW, 123)
123 FORMAT(' *** FOR PBAPQ, ENTER VALUE OF F'/T13, '(NUMBER OF', 'SAMPLES PER OCR INDICATION)')
READ(IR,*) NF
WRITE(IW, 124)
GOTO (129, 1124, 1124), IFG
1124 WRITE(IW, 125) NF
125 FORMAT(' SAMPLING FREQUENCY F=' ,I3, 'SAMPLES PER DOC ')..INDICATION')
WRITE(IW, 115)
READ(IR,*) IYN
GOTO (126, 120, 105), IYN
WRITE(IW, 20)
GOTO 129
126 CALL FGEVAL
GOTO 105
C------------------------- STAT NLG QL DESIGN ------------------------
130 WRITE(IW, 131)
131 FORMAT(' FOR STAT NLG QL DESIGN/' *** ENTER VALUES: ', M, NMIN, NMAX')
READ(IR,*) MM, NMINL, NMAXH
WRITE(IW, 132)
132 FORMAT(' *** ENTER VALUES OF APL, TLAPL, RPL, TLRPL'/T6, 'HERE', APL, RPL MUST BE IN MULTIPLES OF STD')
READ(IR,*) APL, TLAPL, RPL, TLRPL
WRITE(IW, 133)
133 FORMAT(' *** ENTER T VALUE')
READ(IR,*) TNLG
WRITE(IW, 134) MM, NMINL, NMAXH, APL, TLAPL, RPL, TLRPL, TNLG
WRITE(IW, 135)
135 Format(' *** ENTER VALUES: V, NMIN, NMAX, NUMK (NUMBER OF K; <= 10), ', V, NXL, NXH)
READ(IR,*) VX, NXL, NXH
WRITE(IW, 112)
APL, TLAPL, RPL, TLRPL
WRITE(IW, 153)
153 FORMAT(' *** ENTER VALUES: '/T6, 'NUMK (NUMBER OF K; <= 10), ', VX, NXL, NXH)
WRITE(IW, 141)
141 FORMAT(' FOR STAT NLG QL EVALUATION/' *** ENTER VALUES: N, M, Y, T')
READ(IR,*) NN, MM, NY, TNLG
WRITE(IW, 144) NN, MM, NY, TNLG
WRITE(IW, 145)
145 WRITE(IW, 115) READ(IR,*) IYN
GOTO (146, 140, 105), IYN
WRITE(IW, 20)
GOTO 145
146 CALL QLEVAL
GOTO 105
C--------------------- STAT MODIFIED X-BAR CHART DESIGN ----------
150 WRITE(IW, 151)
151 FORMAT(' FOR STAT MODIFIED X-BAR CHART DESIGN/' *** ENTER VALUES: V, NMIN, NMAX')
READ(IR,*) VX, NXL, NXH
WRITE(IW, 122)
APL, TLAPL, RPL, TLRPL
WRITE(IW, 153)
153 FORMAT(' *** ENTER VALUES: '/T6, 'NUMK (NUMBER OF K; <= 10), ', VX, NXL, NXH)
* 'FOLLOWED BY K VALUES TO BE INVESTIGATED'
READ (IR,*) NUMK,(AK(I),I=1,NUMK)
WRITE(IW,154) VX,NXL,NXH,APL,TLAPL,RPL,TLRPL,NUMK,(AK(I),I=1,NUMK)
* T3,'APL=',F5.3,'TLAPL=',F5.3,'RPL=',F5.3,'TLRPL='
READ(IR,*) VX,NXL,NXH,APL,TLAPL,RPL,TLRPL,NUMK,(AK(I),I=1,NUMK)
WRITE(IW,155)
READ(IR,*) IYN
GOTO (156,150,105),IYN
WRITE(IW,20)
GOTO 155
156 CALL XSTGE
GOTO 105

C----------------------- STAT MODIFIED X-BAR CHART EVALUATION

WRITE(IW,161)
WRITE(IW,60) TNLG
FORMAT(//,T2,10('*'),' T =',F6.3)
WRITE(IW,70)
FORMAT(//,T19,(APL=','F5.3,') (RPL=','F5.3,')',/T4,'N M Y G',T20,'END','4X,
PRO',T6,T4,'PA1',T4,'PA2',T4,'PA3',T4,'PA4',T4,'PA5',T4,'PA6',T4,'PA7',T4,'PA8',T4,'PA9',T4,'PA10',/T4,'F5.3,')
* T5,'INVESTIGATED T VALUES=',9(F6.3,1X))
DO 130 I=1,NUMT
TNLG=AT(I)
WRITE(IW,60) TNLG
FORMAT(//,T2,10('*'),' T =',F6.3)
WRITE(IW,70)
FORMAT(//,T19,(APL=','F5.3,') (RPL=','F5.3,')',/T4,'N M Y G',T20,'END','4X,
PRO',T6,T4,'PA1',T4,'PA2',T4,'PA3',T4,'PA4',T4,'PA5',T4,'PA6',T4,'PA7',T4,'PA8',T4,'PA9',T4,'PA10',/T4,'F5.3,')
* T5,'INVESTIGATED T VALUES=',9(F6.3,1X))

C------------------------------- N LOOP

C------------------------------------------------------------------------
SUBROUTINE FGGENE
C *** THIS SUBROUTINE STATISTICALLY DESIGN NLG FREQUENCY GAGING RULES
C COMMON /C1/ USLLSL,NN,MM,NG,NY,NY1,TNLG,HALF,IR,IW
COMMON /S2/MUSTD,NNL,NNH,APL,TLAPL,RPL,TLRPL,NUMT,AT(10)
COMMON /S3/ PG,PY,PR
COMMON /S6/ RY,DEL,STD10
COMMON /SB/ ASSCOZ(10,2),BLANK,STAR2,DELSTD(2)

PMID=(APL+RPL)/2.
HALF=.5*USLLSL
CALL MDNOR(-HALF,PPO)
PP2=PPO*2.
WRITE(IW,50) USLLSL,(ASSCOZ(I,MUSTD),1=1,10),MM,NNL,NNH,
APL,TLAPL,RPL,TLRPL,(AT(I),I=1,NUMT)
FORMAT(//,T5,'USLLSL=',F5.2,' (STD)',5X,10A4/T5,'M=',I2,4X,
 'NMIN=',I2,'NAMX=',I2/T5,'APL=',F5.3,'TLAPL=',F5.3,'RPL=',F5.3,'TLRPL=',F5.3/
T5,'INVESTIGATED T VALUES=',9(F6.3,1X))
C----------------------------- T LOOP

DO 130 I=1,NUMT
TNLG=AT(I)
WRITE(IW,60) TNLG
FORMAT(//,T2,10('*'),' T =',F6.3)
WRITE(IW,70)
FORMAT(//,T19,(APL=','F5.3,') (RPL=','F5.3,')',/T4,'N M Y G',T20,'END','4X,
PRO',T6,T4,'PA1',T4,'PA2',T4,'PA3',T4,'PA4',T4,'PA5',T4,'PA6',T4,'PA7',T4,'PA8',T4,'PA9',T4,'PA10',/T4,'F5.3,')
* T5,'INVESTIGATED T VALUES=',9(F6.3,1X))

C------------------------------- N LOOP

C++------- PRINT TITLE AND PARAMETER VALUES
C
WRITE(IW,50) USLLSL,(ASSCOZ(I,MUSTD),1=1,10),MM,NNL,NNH,
APL,TLAPL,RPL,TLRPL,(AT(I),I=1,NUMT)
FORMAT(//,T5,'USLLSL=',F5.2,' (STD)',5X,10A4/T5,'M=',I2,4X,
 'NMIN=',I2,'NAMX=',I2/T5,'APL=',F5.3,'TLAPL=',F5.3,'RPL=',F5.3,'TLRPL=',F5.3/
T5,'INVESTIGATED T VALUES=',9(F6.3,1X))
C----------------------------- T LOOP

DO 130 I=1,NUMT
TNLG=AT(I)
WRITE(IW,60) TNLG
FORMAT(//,T2,10('*'),' T =',F6.3)
WRITE(IW,70)
FORMAT(//,T19,(APL=','F5.3,') (RPL=','F5.3,')',/T4,'N M Y G',T20,'END','4X,
PRO',T6,T4,'PA1',T4,'PA2',T4,'PA3',T4,'PA4',T4,'PA5',T4,'PA6',T4,'PA7',T4,'PA8',T4,'PA9',T4,'PA10',/T4,'F5.3,')
* T5,'INVESTIGATED T VALUES=',9(F6.3,1X))

C------------------------------- N LOOP

C------------------------------------------------------------------------
SUBROUTINE FGGENE
C *** THIS SUBROUTINE STATISTI...
DO 120 NN=NNL,NNH
WRITE(IW,80)
FORMAT(’’)
NYH1=INT(NN/2.+6)+1
GOTO (131,22,33),MM
22 NGH=NN-NY+1
GOTO 83
33 NGH=NN+1
C------------------------------- Y LOOP
DD 110 J=1,NYH1
NY=J-1
NY1=NY+1
NFLAG=0
GOTO (131,22,33),MM
NGH=NN-NY+1
GOTO 83
C------------------------------- G LOOP
83 DD 100 K=2,NGH
IF(NFLAG.EQ.1)GO TO 110
NG*K=1
IF(NY.EQ.0)GO TO 90
CALL EQN(ENO)
CALL PA8(PAO)
PRO=1.-PAO
CALL GYR(APL)
CALL PA8(PA1)
CALL GYR(PMID)
CALL PA8(PA2)
CALL GYR(RPL)
CALL PA8(PA3)
CALL EQN(EN3)
STAR=BLANK
GOTO 90
CONTINUE
CONTINUE
CONTINUE
CONTINUE
RETURN
END
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++00048500
SUBROUTINE FGEVAL
C
C *** THIS SUBROUTINE STATISTICALLY EVALUATES NLG FREQUENCY GAGING RULES00048800
C ( EVALUATED PERFORMANCE MEASURES: PA,EN, PBAPQ,PB&QQ )
C
COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,MHALF, IR, IW
COMMON /S2/MUSTD, NNL,NNH, APL,TLAPL,RPL,TLRPL, NUMT,AT(10)
COMMON /S3/ PG,PY,PR
COMMON /S5/IFG,NF
COMMON /S6/ RY,DEL,STD10
COMMON /S8/ ASSCOZ(10,2),BLANK,STAR2,DELSTD(2)
DIMENSION APP(27)
C------------------------- SPECIFY FRACTION DEFECTIVE VALUES
NY1=NY+1
HALF=.5*USLLSL
CALL MNDOR(-HALF,PRO)
APP(I)=2.*PRO
DO 10 I=2,21
10 APP(I)=(I-1)*.005
C----- FOR NY=0, NG MUST BE 0, OR INSPECTION WILL ALWAYS BE TRUNCATED00046700
C + REMATURELY
90 NG=0
NFLAG=1
GO TO 85
95 WRITE(IW,96)NN,MM,NY,NG,STAR, ENO,PRO, PA1,STAR,
PA2, PA3,STAR, EN3
FORMAT(T2,4I3,1X,A2,T18,F6.2,1X,F7.4,T34,F6.3,1X,A2,2X,F6.2)
CONTINUE
CONTINUE
CONTINUE
CONTINUE
C++----------------------------- LABEL QUALIFIED PLAN BY ’**’
IF(PA1.GE.TLAPL .AND. PA3.LE.TLRPL) STAR=STAR2
GO TO 95
C----- FDR
NY=0, NG MUST BE 0, OR INSPECTION WILL ALWAYS BE TRUNCATED00046700
C PREMATURELY
DO 12 I=22,26
 12 APP(I)=+(I-21)*.02+.1
APP(27)=.40
C------------------------ PRINT TITLE AND PARAMETER VALUES
C WRITE(IW,50) USLLSL.(ASSCOZ(I,MUSTD),I=1,10), NN,MM,NY,NG,
  * (AT(I),I=1,NUMT)
50 FORMAT('***** STATISTICALLY BASED NLG FG EVALUATION *****'/
  * TS, 'USLLSL=',F5.2,' (STD)' ,5X,10A4,/T5, 'N=',I2,4X, 'M=',I2,4X,
  * 'Y=',I2,4X,'G=',I2,'INVESTIGATED T VALUES =',10(F6.3,1X))
C-------------------------- T LOOP
DO 200 IU=1,NUMT
  TNLG=AT(IU)
  WRITE(IW,85) TNLG
  FORMAT(/T2' 10(,T',' FG. 3)
85 FORMAT(/T2,Y10,A4)C---------- CHECK OPTION NUMBER AND PRINT APPROPRIATE LABELS
C (1=FG 2= FG + PBAPQ 3= FG + PBAPQ + PBAOQ)
  IF(MUSTD.EQ.0.1.AND.I.EQ.1) GOTO 107
  CALL GYR(APP(I))
  CALL PAFG(PA)
  CALL EOFN(EN)
  IF(IFG.EQ.2) GOTO 96
  IF(01.LT.1.) 01=1.
  IF(Q1 .GT.NF) Q1=NF
  Q2=APP(I)*(NF-Q1)
  PBAPQ=(APP(I)*Q1 + 02)/NF
  IF(PBAPQ.GT.APP(I)) PBAPQ=APP(I)
  IF(IFG.EQ.2) GOTO 96
  GOTO 110
C------------------------ PROCESS BEFORE SHIFTING
107 CALL GYR(PPQ)
   GOTO 95
C------------- CALCULATION FOR PBAPQ AND PBAOQ--------------
C----- (O <= PA <= 1) ==> (.5 <= Q1 <= INFINITY)AND (-.5 <= Q1-1 <= INF)
C----- BUT IN REALITY, IT IS REQUIRED THAT
C----- (O <= Q1 <= NF) FOR PBAPQ AND (O <= Q1-1 <= NF) FOR PBAOQ
C----- Q1=1/(1.-PA)-.5
  IF(Q1.GT.NF) Q1=NF
  Q2=APP(I)*(NF-Q1)
  PBAPQ=(APP(I)*Q1 + Q2)/NF
  IF(PBAPQ.GT.APP(I)) PBAPQ=APP(I)
  IF(IFG.EQ.2) GOTO 96
  IF(Q1.LT.1.) Q1=1.
  C=}
SUBROUTINE QLGENE

C THIS SUBROUTINE STATISTICALLY DESIGNS NLG QUALIFICATION RULES

COMMON USLLSL, NN, MM, NG, NY, NY1, TNLG, HALF, IR, IW
COMMON MUSTD, NNL, NNH, APL, TLAPL, RPL, TLRPL, NUMT, AT(10)
COMMON PG, PY, PR
COMMON ASSCOZ(10,2), BLANK, STAR2, DELSTD(2)

C IN QL DESIGN, APL AND RPL ARE EXPRESSED IN MULTIPLES OF STD

IR, IW
NUMT

C-------------------

WRITE(IW,50)

50 FORMAT(//' ***** STATISTICALLY BASED NLG QL DESIGN *****'/
  * T5, 'USLLSL=' ,F5.2,' (STD)' ,5X, 10A4/T5, 'M=' ,I2,4X,
  * 'NMIN=' ,I2,4X, 'NMAX=' ,I2/
  * 4X, 'TLRPL=','F6.3/T5, 'T=' ,F6.3)

WRITE(IW,70)

70 FORMAT(/ T18, '(EXACT SETUP)'; , '(APL=' ,F5.3, ') (MID=' ,F5.3, '
  * (RPL=','F5.3,')/T25, 'O.O STD',T40, 'STD',T52, 'STD',
  * T64, 'STD'/,T74, 'N Y ',T20, 'ENO ',4X, 
  * 'PRO',T36, 'PA1', T48, 'PA2', T58, 'PA3', T69, 'EN3' /)

C---------------------- N LOOP

DD 120 NN=NNL, NNH

C---------------------- Y LOOP

DD 110 J=1, NN

NY=J-1

IF(MUSTD.EQ.1) CALL GYRC(0.)

CALL EOFN(ENO)

CALL EQLN(ENO)

CALL PAQL(PAC)

CALL GYRC(APL)

CALL PAQL(PA1)

CALL GYRC(PMID)

CALL PAQL(PA2)

CALL GYRC(RPL)

CALL PAQL(PA3)

CALL EOFN(EN3)

STAR=BLANK

C---------------------- LABEL QUALIFIED PLAN BY '**'

IF(PA1.GE.TLAPL .AND. PA3.LE.TLRPL) STAR=STAR2

95 WRITE(IW,96)NN, NY, STAR, ENO, PRO, PA1, STAR, PA2, PA3, STAR, ENO

96 FORMAT(T2,I3,X,I3,2X,A2,T18,F6.2,1X,F7.4,T34,F6.3,1X)

* A2,T46,F6.3,T56,F6.3,A2,T46,6.2)

110 CONTINUE

120 WRITE(IW,121)

121 FORMAT('' )

131 RETURN

END
SUBROUTINE QLEVAL

*** THIS SUBROUTINE STATISTICALLY EVALUATES NLG QUALIFICATION RULES

DIMENSION ACHG(20,2)
COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
COMMON /S2/MUSTD, NNL,NNH, APL,TLAPL,RPL,TLRPL, NUMT,AT(10)
COMMON /S3/ PMID,DEL,STD
COMMON /S6/ ASSCOZ(10,2), BLANK,STAR2,DELSTD(2)

C-- PREDETERMINE 20 PROCESS LEVELS
DATA ACHG/0., .1, .2, .3, .4, .5, .6, .7, .8, .9, 1., 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2., 2.1, 2.2, 2.3, 2.4, 2.5, 3., 3.5, 4., 5., 6./
NG=0 NY1=NY+1
HALF=.5*USLLSL

PRINT TITLE AND PARAMETER VALUES
WRITE(IW,50) USLLSL,(ASSCOZ(I,MUSTD),I=1,10), NN,MM,NG,NY,NUMT
50 FORMAT(// ' *****STATISTICALLY BASED NLG QUALIFICATION EVALUATION *****'// TS,'USLLSL=',F5.2,' (STD)' ,5X,10A4,)//
51 FORMAT('NMIN=',I2,4X,'NMAX=' ,I2)
52 FORMAT('APL=',F5.3,4X,'RPL=',FS.3,4X,'TNR=',F5.3,4X)//
53 FORMAT('INVESTIGATED K VALUES=',10(F6.3,1X))
WRITE(IW,60) PMID=(APL+RPL)/2.
HALF=USLLSL/2.
CALL MDNOR(-HALF,POH)
P0=2.*POH

PRINT TITLE AND PARAMETER VALUES
WRITE(IW,50) USLLSL,(ASSCOZ(I,MUSTD),I=1,10), NN,MM,NG,NY,NUMT
50 FORMAT(// ' *****STATISTICALLY BASED MODIFIED X-BAR CHART DESIGN *****'// TS,'USLLSL=',F5.2,' (STD)' ,5X,10A4,)//
51 FORMAT('NMIN=',I2,4X,'NMAX=' ,I2)
52 FORMAT('APL=',F5.3,4X,'RPL=',FS.3,4X,'TNR=',F5.3,4X)//
53 FORMAT('INVESTIGATED K VALUES=',10(F6.3,1X))
WRITE(IW,60) PMID=(APL+RPL)/2.
HALF=USLLSL/2.
CALL MDNOR(-HALF,POH)
P0=2.*POH
'UCL = USL - (V - K/SQRT(N))*STD')
WRITE(IW,70) APL,PMID,RPL
C------------------------------- N LOOP
DO 120 NX=NXL,NXH
   RNX=FLOAT(NX)
   SQN=SQRT(RNX)
C------------------------------- K LOOP
   DO 110 J=1,NUMK
      RKX=AK(J)
      CLK=VX-RKX/SQN
      B1=CLK*SQN
      B2=-HALF*SQN+B1
      IF(MUSTD.EQ.1) CALL PAXB(1,POH, PAO)
      IF(MUSTD.EQ.2) CALL PAXB(2,PO, PAO)
      PRO=1.-PAO
      CALL PAXB(MUSTD,APL, PA1)
      CALL PAXB(MUSTD,PMID,PA2)
      CALL PAXB(MUSTD,RPL, PA3)
      STAR=BLANK
      IF(PA1.GE.TLAPL .AND. PA3.LE.TLRPL) STAR=STAR2
   WRITE(IW,96) STAR,NX,RKX, PRO,PA1,STAR, PA2, PA3,STAR
96 FORMAT(T5,A2,1X,I3,3X,F5.2, T27,F7.4,T44,F6.3,1X,A2.)
110 CONTINUE
120 WRITE(IW,121)
121 FORMAT(')
131 RETURN
END
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++0007S700
SUBROUTINE XSTEV
C *** THIS SUBROUTINE STATISTICALLY EVALUATES MODIFIED X-BAR CHART
C
COMM /C1/ USLLSL, NN,MM,NG, NY , N1, TNLG,HALF, IR, IW
COMM /S2/ MUSTD, NNL,NNH, APL,TLAPL, RPL,TLRPL, NUMT,AT(10)
COMM /S4/ DELMU,STD10, SON,B1,B2
COMM /S7/VX,NX,NXH, NUMK,AK(10), NK, RKX
COMM /S8/ ASSCOZ(10,2),BLANK,STAR2,DELSTD(2)
DIMENSION APP(27)
C
   HALF= USLLSL/2.
   RNX=FLOAT(NX)
   SQN=SQRT(RNX)
   CLK=VX-RKX/SQN
   B1=CLK*SQN
   B2=-HALF*SQN+B1
   CALL MDNOR(-HALF,POH)
C------------------------- SPECIFY FRACTION DEFECTIVE VALUES
   APP(1)=2.*POH
   DO 1 I=2,21
      1 APP(I)=(I-1)*.005
   DO 2 I=22,26
      2 APP(I)=(I-21)*.02+.1
   APP(27)=.40
C------------------- PRINT TITLE AND PARAMETER VALUES
   WRITE(IW,50) USLLSL,(ASSCOZ(I,MUSTD),I=1,10), NX, VX, RKX,CLK,CLK
50 FORMAT(/'*EVALUATION'****''/T5,'USLLSL=' ,F5.2, ' (STD)' ,5X,10A4 ,/)
   '* T5,'N=' ,12,4X, 'V=' ,F5.2,4X, 'K=' ,F6.3,7X, 'F5.3,' '/T5,
   '* LCL= USL - (V-K/SQRT(N))*STD = USL - ',F6.3, ' STD' ,/T5,
   '* UCL= USL - ',F6.3, ' STD' ,/)
   WRITE(IW,12) DELSTD(MUSTD)
DO 20 I=1,27
C---------------
PROCESS BEFORE SHIFTING IS EVALUATED "EXACTLY".
C OTHERWISE, EVALUATED APPROXIMATELY
IF(MUSTD.EQ.1.AND.I.EQ.1) GOTO 16
CALL PAXB(MUSTD,APP(I),PA)
13 IF(MUSTD.EQ.1) WRITE(IW,14) APP(I),DELMU,PA
14 FORMAT(T4,F7.4,2X,F7.3,4X,F7.3)
IF(MUSTD.EQ.2) WRITE(IW, 14) APP(I),STD10,PA
GOTO 20
C----------------
PROCESS BEFORE SHIFTING
C c
16 CALL PAXB(1,POH,PA)
GOTO 13
20 CONTINUE
32 RETURN
END

SUBROUTINE PAFG (PACC)
THIS SUBROUTINE CALCULATES THE PROBABILITY OF ACCEPTANCE (PACC) FOR NLG FREQUENCY GAGING RULE
COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
COMMON /S3/ PG,PY,PR
PSUM=O.
DO 22 I= 1, NY1
   IL1=I-1
   CALL BINOML(NN,IL1,PAC)
   PSUM=PSUM+PAC
22 PSUM2=PSUM
   IF(NG.EQ.0) RETURN
   PSUM2=0.
   IN=NY1
   NNLNG=NN-NG
   IF(NY.GT.NNLNG) IN=NNLNG+1
   DO 24 I=1,IN
      IL1=I-1
      CALL BINOML(NNLNG,IL1,PAC)
      PSUM2=PSUM2+PAC
24 PSUM2=PSUM2-PAC
   PAC=PSUM*(1.-PSUM2)**(PG**NG)
RETURN
END

SUBROUTINE PAQL (PA)
C *** THIS SUBROUTINE CALCULATES THE PROBABILITY OF ACCEPTANCE (PA) FOR NLG QUALIFICATION RULE
C
COMMON /C1/ USLLSL, NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
PSUM=O.
DO 22 I= 1, NY1
   IL1=I-1
   CALL BINOML(NN,IL1,PAC)
   PA=PSUM
22 RETURN
END

SUBROUTINE BINOML (N,IX, PROB)
*** THIS SUBROUTINE CALCULATES BINOMIAL PROBABILITY AND ITS SIMILARS

COMMON /S3/ PG,PY,PR
DOUBLE PRECISION DY,DG,DLG

DY=PG
DG=PY
DLG=DLGAMA(N+1.DO)-DLGAMA(N)-DLGAMA(N-IX+1.DO)
+ IX*DLOG(DY)+(N-IX)*DLOG(DG)
IF (DLG.LT.-180.DO) DLG=-180.DO
PROB=DEXP(DLG)
RETURN
END

SUBROUTINE GYR(PP)
*** THIS SUBROUTINE CALCULATES
THE PROBABILITY OF
GREEN, YELLOW
AND RED
(PG, PY, PR)
AS FUNCTIONS OF PROCESS FRACTION DEFECTIVE
MUSTD=1 ==> CHANGE= DEL OF MU = DEGREE OF MEAN SHIFT
MUSTD=2 ==> CHANGE= RATIO OF STD = DEGREE OF DISPERSION CHANGE

COMMON /C1/ USLLSL, NN, MM, NG, NY, NY1, TNLG, HALF, IR, IW
COMMON /S2/ MUSTD, NN, MM, APL, TLAPL, RPL, TLRPL, NUMT, AT(10)
COMMON /S3/ PG, PY, PR
COMMON /S6/ RY, DEL, STD10

IF(MUSTD.EQ.2) GOTO 10
PP2=PP/2.
CALL MDRNOR(PP2, Q1, IERR)
STD10=-Q1/Q1
Q2=(HALF-TNLG)/STD10
CALL MDRNOR(Q2, Q3)
PG=2.*(Q3-.5)
GOTO 15
10 IF(MUSTD.EQ.2) GOTO 10
15 GO TO (99, 20, 30), MM
20 PY=-PG
RETURN
30 PR=PP
PY=-PG-PR
99 RETURN
END

SUBROUTINE GYR(CHANGE)
*** THIS SUBROUTINE CALCULATES THE PROBABILITY OF GREEN, YELLOW AND
RED (PG, PY, PR) AS FUNCTIONS OF (1) DEGREE OF MEAN SHIFT, OR (2)
DEGREE OF DISPERSION CHANGE.
MUSTD=1 ==> CHANGE= DEL OF MU = DEGREE OF MEAN SHIFT
MUSTD=2 ==> CHANGE= RATIO OF STD = DEGREE OF DISPERSION CHANGE
C COMMON /C1/ USLLSL, NN,MM,NG, NY1, TNLG,HALF, IR, IW
COMMON /S2/ MUSTD, NNL,NNH, APL, TLAPL, RPL, TLRLP, NUMT, AT(10)
COMMON /S3/ PG, PY, PR
COMMON /S6/ RY, DEL, STD10
C---------------------------------- MEAN SHIFT
HTD1=HALF-TNLG+CHANGE
HTD2=-HALF+TNLG+CHANGE
CALL MDNOR(HTD1, PHI1)
CALL MDNOR(HTD2, PHI2)
PG=PHI1-PHI2
GOTO 15
C---------------------------------- DISPERSION CHANGE
10 Q2=(HALF-TNLG)/CHANGE
CALL MDNOR(Q2, Q3)
PG=2.*(Q3-.5)
GOTO (99,20,30), MM
20 PY=1.-PG
RETURN
30 PR=PP
PY=1.-PG-PR
99 RETURN
END

SUBROUTINE EOFN(REN)
C *** THIS SUBROUTINE CALCULATES AVERAGE INSPECTION NUMBER (ALSO KNOWN AS AVERAGE SAMPLE NUMBER)
C
COMMON /C1/ USLLSL, NN,MM,NG, NY1, TNLG,HALF, IR, IW
COMMON /S2/ MUSTD, NNL,NNH, APL, TLAPL, RPL, TLRLP, NUMT, AT(10)
COMMON /S3/ PG, PY, PR
COMMON /S6/ RY, DEL, STD10
IF(MM.EQ.3) R=PR
ABC=O.DO
SABC=O.DO
EN=O.DO
NNL1=NN-1
IF(NN.GT.1) GO TO 10
C---------------------------- NN = 1 ----
REN=1.
RETURN
C---------------------------- NN > 1 ----
10 GO TO (900,200,300,900,900), MM
C---------------------------- MM=2 ----------------
200 IF(NY.EQ.O) GO TO 201
IF(NY.LT.NNL1) GO TO 221
GO TO 251
C---------------------------- MM=2; NY=O (NG=O) ------
201 IF(NG.GE.1) GO TO 212
DD 210 I=1,NNL1
210 EN=EN+I*(G**((I-1))*Y
REN=EN+NN*G**NNL1
RETURN
C 212 WRITE(IW,214)
213 FORMAT(/,T2,10(' -'), 'NLG ERROR: M=2 Y=O G>O:' ,
' * EXECUTION INTERRUPTED IN SUBROUTINE EOFN (LABEL 212)')
RETURN
C---------------------------- MM=2; O<NY<(NN-1) ------
221 IF(NY.EQ.O OR NG.GT.NY) GO TO 225
ABC=G**NG
EN=EN+NG*ABC
SABC=SABC+ABC
225 DD 240 J=NY1,NNL1
JL1 = J-1

IF(J .EQ. NG) GO TO 229

ABC = YGF(JL1, NY, G, Y)
EN = EN + J*ABC
GO TO 240

229

ABC = YGF(JL1, NY, G, Y) + G**NG
EN = EN + J*ABC

GO TO 240

SABC = SABC + ABC

EN = EN + NN*(1.00 - SABC)
RETURN

C------------------------------------- MM = 2; NY > 0 & NY >= (NN - 1) ----

251 IF(NG .GE. 1) GO TO 254

EN = EN + NN
RETURN

254

EN = EN + NN*(1.00 - G**NG)
RETURN

C------------------------------------- MM = 3 ------------------------------------------

300 IF(NY .EQ. 0) GO TO 301
IF(NY .LT. NNL1) GO TO 321
GO TO 351

C------------------------------------- MM = 3; NY > 0 (NG = 0) ------------------

301 IF(NG .GE. 1) GO TO 312
GC = 1.00 - G
DO 310 I = 1, NNL1

310

EN = EN + I*(GC**I) + GC
REN = EN + NN*G**NNL1
RETURN

C------------------------------------- MM = 3; O < NY < NN - 1 ----

321 DO 330 I = 1, NY

330

IF(I .EQ. NG) GO TO 329
ABC = (1.00 - SABC)*R
EN = EN + I*ABC
GO TO 330

329

ABC = (1.00 - SABC)*R + G**NG
EN = EN + I*ABC

GO TO 340

SABC = SABC + ABC

EN = EN + NN*(1.00 - SABC)
RETURN

C------------------------------------- MM = 3; NY > 0 & NY >= (NN - 1) ----

351 DO 360 I = 1, NNL1

360

IF(I .EQ. NG) GO TO 359
ABC = (1.00 - SABC)*R
EN = EN + I*ABC
GO TO 360

359

ABC = (1.00 - SABC)*R + G**NG
EN = EN + I*ABC

SABC = SABC + ABC

EN = EN + NN*(1.00 - SABC)
RETURN

C-----------------------------------------------------

900 WRITE(IW, 901) MM
901 FORMAT(//, T3, 10('-'), 'ERROR: IN SUBROUTINE EDFN, M = ', I2,
* ', NE. 2 OR 3; EXECUTION INTERRUPTED (LABEL 900)')
RETURN
FUNCTION YGF(N, K, G, Y)

*** THIS FUNCTION SUBPROGRAM EVALUATES THE TERM ASSOCIATED WITH BINOMIAL COEFFICIENT IN THE CALCULATION OF AVERAGE INSPECTION NUMBER.

COMMON /C1/ USLLSL, NN, MM, NG, NY, NY1, TNLG, HALF, IR, IW
DOUBLE PRECISION BINCOE, G, Y, YGF
IF(K.GT.N) GO TO 90
NLNG=N-NG
IF(NLNG.EQ.0.OR.NLNG.LT.K) GO TO 10

YGF=(BINCOE(N, K)-BINCOE(NLNG, K))*((Y**((K+1)+1))*(G**((N-K))))
RETURN

C---------- NG=0 OR (N-NG)<K ----------------
10 YGF=BINCOE(N, K)*Y**(K+1)*G**(N-K)
RETURN

C -----------------------------------------------
90 WRITE (IW,91) K,N
91 FORMAT(/// 10('-'),' NLG ERROR: IN FUNCTION SUBPROGRAM YGF, K='', I2, ', N=', !2, '; EXECUTION INTERRUPTED (LABEL 90)')
RETURN

END

FUNCTION BINCOE(N, K)

*** THIS FUNCTION SUBPROGRAM EVALUATES BINOMIAL COEFFICIENT USED IN FUNCTION SUBPROGRAM YGF.

DOUBLE PRECISION COEF, DNUM, BINCOE
IF(K.EQ.0.OR.K.EQ.N) GO TO 20
NL1=N-1
IF(K.EQ.1.OR.K.EQ.NL1) GO TO 30

C-------------- 1 < K < (N-1) ----------------
COEF=1.0
HN=N/2.
KK=K
IF(K.GT.HN) KK=N-K
DNUM=N
DO 10 I= 1, KK
COEF=COEF*(DNUM/I)
10 DNUM=DNUM-1.DO
BINCOE=COEF
RETURN

C------------- K=0 OR K=N ----------------
20 BINCOE=1.
RETURN

C---------- K=1 OR K=N-1 --------
30 BINCOE=N
RETURN
END

SUBROUTINE PAXB(I12, P, PA)

*** THIS SUBROUTINE CALCULATES THE PROBABILITY OF ACCEPTANCE OF MODIFIED X-BAR CHART, WHERE I12=1 == MEAN SHIFT I12=2 == DISPERSION CHANGE.

COMMON /C1/ USLLSL, NN, MM, NG, NY, NY1, TNLG, HALF, IR, IW
COMMON /S4/ DELMU, STD10, SON, B1, B2
COMMON /S7/VX, NXL, NXH, NUMK, AK(10), NX, RKX
IR(I12.EQ.2) GOTO 20

C------------------------------- MEAN SHIFT
CALL MDNRIS(P, XP, IERR)
DELMU=XP+HALF
\[ A = (\text{DELMU} + \text{HALF}) \times \text{SQN} - B_1 \]
\[ B = \text{XP} \times \text{SQN} + B_1 \]
\[ \text{CALL MDNOR}(A, \text{PHIA}) \]
\[ \text{CALL MDNOR}(B, \text{PHIB}) \]
\[ \text{PA} = \text{PHIA} - \text{PHIB} \]
\[ \text{RETURN} \]

---

**DISPERSION CHANGE**

\[ c = \frac{1}{2} \phi \]
\[ \text{CALL MDNRIS}(\phi, \text{XPH}, \text{ERROR}) \]
\[ \text{STD10} = -\frac{1}{2} / \text{XPH} \]
\[ B_2 / \text{STD10} \]
\[ \text{CALL MDNOR}(C, \text{PHIC}) \]
\[ \text{PA} = 1 - 2 \times \text{PHIC} \]
\[ \text{RETURN} \]
\[ \text{END} \]
WRITE(IW,20)
GOTO 103

C------------------- CALCULATES FRACTION DEFECTIVES: PR1, PR2 -------------------

105        HALF= .5*USLLSL
        CALL MDNOR(-HALF,PRHALF)
        PR1=2.*PRHALF
        H2L=HALF+ZDEL
        H2R=HALF+ZDEL
        CALL MDNOR(H2L,PH2L)
        CALL MDNOR(H2R,PH2R)
        PR2=PH2L+(1.-PH2R)

C------------------- ECON NLG DESIGN (OPTIMIZATION) -------------------

106 WRITE(IW,107)
107        FORMAT('/ *** ENTER OPTION NUMBER'/'
* T6,'1= ECON NLG DESIGN (OPTIMIZATION)'/
* T6,'2= ECON NLG EVALUATION'/
* T6,'3= ECON NLG LOSS-COST SURFACE INVESTIGATION'/
* T6,'4= SWITCH TO ECON X-BAR CHART'/
* T6,'5= RETURN TO REVISE USLLSL, MM, AND TIME AND COST PARAMETERS'/
* T6,'6= EXIT SYSTEM')
        READ(IR,*) N16
        GOTO (110,120,130,200,100,300),N16
        WRITE(IW,20)
        GOTO 106

C-- INITIALIZATION OF DEFAULT VALUES FOR OPTIMIZATION PARAMETERS

110        YACC=.003
        XACC=.002
        STEP=1.
        ITRMAX=60
        XSTART(1)=1.
        XSTART(2)=1.
        IRESET=1
        NYBACK=2
        NGBACK=3
        YIMPRV=0.

111 WRITE(IW,111)
112 WRITE(IW,112)

113 WRITE(IW,113)

114 WRITE(IW,114)

115 WRITE(IW,115)

116 WRITE(IW,116)

117 WRITE(IW,117)

118 WRITE(IW,118)

119 WRITE(IW,119)
1119 FORMAT(' ENTER VALUES: EY,EG,EL')
READ(IR,*), NYBACK,NGBACK,YIMPRV
GOTO 1111
2119 CALL NECOPT
GOTO 106
C----------------------------------------- ECON NLG EVALUATION ------------------------------------------
120 WRITE(IW,121)
121 FORMAT(' FOR ECON NLG EVALUATION, ENTER VALUES: N,Y,G,H,T')
READ(IR,*), NN,NY,NG,HNLG,TNLG
WRITE(IW,122)
123 WRITE(IW,124)
124 FORMAT(' CORRECT? 1=YES 2=NO 3= RETURN FOR OTHER')
READ(IR,*) IYN
GOTO (126,120,106),IYN
WRITE(IW,20)
GOTO 123
126 CALL NECEV
GOTO 106
C----------------------------------------- ECON NLG COST SURFACE INVESTIGATION ------------------------------------------
130 WRITE(IW,131)
131 FORMAT(' FOR ECON NLG COST SURFACE INVESTIGATION, ENTER')
* ' VALUES: N,Y,G')
READ(IR,*), NN,NY,NG
WRITE(IW,132)
132 FORMAT(' ENTER VALUES: NUMH (NUMBER OF H VALUES TO BE INVESTIGATED)
* BY ALL H VALUES TO BE INVESTIGATED')
READ(IR,*), NH,(AH(I),I=1,NH)
WRITE(IW,133)
133 FORMAT(' ENTER VALUES: NUMT (NUMBER OF T VALUES TO BE INVESTIGATED)
* BY ALL T VALUES TO BE INVESTIGATED')
READ(IR,*), NT,(AT(I),I=1,NT)
WRITE(IW,134)
134 FORMAT(' ENTER OPTION NUMBER: 1= ALL OK, NO REVISION NEEDED'/' 2= NEED TO REVISE (N,Y,G) VALUES'/' 3= NEED TO REVISE NUMH AND H VALUES'/' 4= NEED TO REVISE NUMT AND T VALUES'/' 5= RETURN FOR OTHER ECON NLG OPTIONS')
READ(IR,*), N15
GOTO (143,137,149,141,106),N15
WRITE(IW,20)
GOTO 1135
137 WRITE(IW,138)
138 FORMAT(' ENTER VALUES: N,Y,G')
READ(IR,*), NN,NY,NG
GOTO 1133
139 WRITE(IW,140)
140 FORMAT(' ENTER VALUES: NUMH AND H VALUES')
READ(IR,*), NH,(AH(I),I=1,NH)
GOTO 1133
141 WRITE(IW,142)
142 FORMAT(' ENTER VALUES: NUMT AND T VALUES')
READ(IR,*), NT,(AT(I),I=1,NT)
GOTO 1133
C
143 CALL NCOSF
GOTO 106
C----------------------------------------- ECON X-BAR OPTION MENU ------------------------------------------
200 WRITE(IW,201)
201 FORMAT(' FOR ECON X-BAR CHART, ENTER VALUES: ')

READ(IR,*) USLLSL, DELTA, LAMBDA, M, E, T, W, B, C
WRITE(IW,202) USLLSL, ZDEL, ZLAM, ZM, ZE, ZD, ZT, ZW, ZB, ZC
202 FORMAT(' VALUES ENTERED: USLLSL=',F5.2/, 
DELT=',F7.2,3X,'LAMBDA=',F7.2,3X,'M=',F7.2,3X,'E=',F7.2,3X,'T=',F7.2,3X,'W=',F7.2,3X,'B=',F7.2,3X,'C=',F7.2)
WRITE(IW,104) READ(IR,*) IYN
GOTO (206,200,5),IYN
WR
IT E (IW,20)
GOTO 203
206 WRITE(IW,207)
207 FORMAT(' *** ENTER OPTION NUMBER'/' 
T6,'1= ECON X-BAR CHART DESIGN (OPTIMIZATION)'/' 
T6,'2= ECON X-BAR CHART EVALUATION'/' 
T6,'3= ECON X-BAR CHART LOSS-COST SURFACE INVESTIGATION'/' 
T6,'4= SWITCH TO ECON NLG'/' 
T6,'5= RETURN TO REVISE USLLSL, AND TIME AND COST PARAMETERS'/' 
T6,'6= EXIT SYSTEM')
READ(IR,*) N16
GOTO (210,220,230,100,200,300), N16
WRITE(IW,20)
GOTO 206
C------------------------
ECON X-BAR CHART DESIGN (OPTIMIZATION) ------

C-- INITIALIZATION OF DEFAULT VALUES FOR OPTIMIZATION PARAMETERS
210 YACC=.003
XACC=.002
STEP=1
ITRMAX=60
XSTART(1)=1.
XSTART(2)=1.
IRESET=1
NYBACK=2
NGBACK=3
YIMPRV=0.
WRITE(IW,211)
211 FORMAT(' *** FOR ECON X-BAR CHART DESIGN, ENTER VALUES: ' ,
'NMIN,NMAX')
READ(IR,*) NNMIN,NNMAX
1211 WRITE(IW,212) NNMIN,NNMAX, YACC,XACC,STEP,ITRMAX.

212 FORMAT(' VALUES ENTERED: NNMIN=',I2,4X,'NMAX=',I2// 
'PARAMETER VALUES FOR: ',T30,'(H,T) OPTIMIZATION',T61, 
'OVERALL OPTIMIZATION'/T15,'YACC XACC STEP ITRMAX HO/', 
'T51,'TO IRESET',T68,'EL/T4,'DEFAULT:',T15, 
'0.003 0.002,730,1.00 60 1.000 1.000 1.767, 
'0.003 0.002 T30,1.00 60 1.000 1.000 1.767, 
'0.003 0.002 T30,1.00 60 1.000 1.000 1.767, 
'0.003 0.002 T30,1.00 60 1.000 1.000 1.767, 
'0.003 0.002 T30,1.00 60 1.000 1.000 1.767, 
'0.003 0.002 T30,1.00 60 1.000 1.000 1.767, 
WRITE(IW,214)
214 FORMAT(' *** ENTER OPTION NUMBER:' ,
'ALL OK, NO REVISION NEEDED'/' 
'2= NEED TO REERVE (NMIN,NMAX) VALUES'/' 
'3= NEED TO REERVE (H,T) OPTIMIZATION PARAMETER VALUES'/' 
'4= NEED TO REERVE OVERALL OPTIMIZATION PARAMETER VALUE'/' 
'5= RETURN FOR OTHER ECON X-BAR CHART OPTIONS')
READ(IR,*) N16
GOTO (2219,218,217,219,208), N16
WRITE(IW,20)
GOTO 213
215 WRITE(IW,116) READ(IR,*) NNMIN,NNMAX
GOTO 1211
217 WRITE(IW,116) READ(IR,*) YACC,XACC,STEP,ITRMAX.(XSTART(I),I=1,2), IRESET
GOTO 1211
219 WRITE(IW,1219) READ(IR,*) YIMPRV
GOTO 1211

C------------ ECON X-BAR CHART DESIGN (OPTIMIZATION) ------
CALL XECOPT
GOTO 206

WRITE(IW,221) FOR ECON X-BAR CHART EVALUATION, ENTER VALUES:

READ(IR.*) NN, HX,RXX WRITE(IW,222) NN, HX,RXX

WRITE(IW,224) NN, HX, RKX FORMAT(' VALUES ENTERED: N=', I2,4X, 'H=' ,F8.3,4X, 'K=' ,F6.3)

WRITE(IW,226) CALL XCEEV GOTO 206

WRITE(IW,231) FOR ECON X-BAR CHART COST SURFACE INVESTIGATION, ENTER VALUE:

READ(IR.*) NN, NH,(AH(I),I=1,NH)

WRITE(IW,233) ENTER VALUES: NUMK (NUMBER BY ALL K VALUES TO BE INVESTIGATED)

READ(IR.*) NK,(AK(I),I=1,NK)

WRITE(IW,235) NK,(AK(I),I=1,NK) FORMAT(T2,I2,' K VALUES = ',6(F6.3,3X)/T16,5(F6.3,3X))

WRITE(IW,236) ENTER VALUE:

READ(IR.*) NN

WRITE(IW,238) ENTER VALUES: NUMK AND K VALUES

READ(IR.*) NK,(AK(I),I=1,NK)

WRITE(IW,240) ENTER VALUE:

READ(IR.*) NN

WRITE(IW,242) ENTER VALUES: NUMK AND K VALUES

READ(IR.*) NK,(AK(I),I=1,NK)

WRITE(IW,243) CALL XCOSF GOTO 206

CALL XECDF GOTO 206

C---------------------------------- ECON X-BAR CHART EVALUATION ---------------------

220 WRITE(IW,221) FOR ECON X-BAR CHART EVALUATION, ENTER VALUES:

* ' N,H,K'

READ(IR.*) NN, HX, RXX WRITE(IW,222) NN, HX, RXX

222 FORMAT(' VALUES ENTERED: N=',I2,4X,'H=',F8.3,'K=',F6.3)

WRITE(IW,224) NN, HX, RKX FORMAT(' VALUES ENTERED: N=', I2,4X, 'H=' ,F8.3,4X, 'K=' ,F6.3)

WRITE(IW,226) CALL XCEEV GOTO 206

WRITE(IW,231) FOR ECON X-BAR CHART COST SURFACE INVESTIGATION, Enter Value:

READ(IR.*) NN, NH,(AH(I),I=1,NH)

WRITE(IW,233) ENTER VALUES: NUMK (NUMBER BY ALL K VALUES TO BE INVESTIGATED)

READ(IR.*) NK,(AK(I),I=1,NK)

WRITE(IW,235) NK,(AK(I),I=1,NK) FORMAT(T2,I2,' K VALUES = ',6(F6.3,3X)/T16,5(F6.3,3X))

WRITE(IW,236) ENTER VALUE:

READ(IR.*) NN

WRITE(IW,238) ENTER VALUES: NUMK AND K VALUES

READ(IR.*) NK,(AK(I),I=1,NK)

WRITE(IW,240) ENTER VALUE:

READ(IR.*) NN

WRITE(IW,242) ENTER VALUES: NUMK AND K VALUES

READ(IR.*) NK,(AK(I),I=1,NK)

WRITE(IW,243) CALL XCOSF GOTO 206

CALL XECDF GOTO 206

C---------------------------------- ECON X-BAR CHART COST SURFACE INVESTIGATION ---------------------

230 WRITE(IW,231) FOR ECON X-BAR CHART COST SURFACE INVESTIGATION, Enter Value:

READ(IR.*) NN, NH,(AH(I),I=1,NH)

WRITE(IW,233) ENTER VALUES: NUMK (NUMBER BY ALL K VALUES TO BE INVESTIGATED)

READ(IR.*) NK,(AK(I),I=1,NK)

WRITE(IW,235) NK,(AK(I),I=1,NK) FORMAT(T2,I2,' K VALUES = ',6(F6.3,3X)/T16,5(F6.3,3X))

WRITE(IW,236) ENTER VALUE:

READ(IR.*) NN

WRITE(IW,238) ENTER VALUES: NUMK AND K VALUES

READ(IR.*) NK,(AK(I),I=1,NK)

WRITE(IW,240) ENTER VALUE:

READ(IR.*) NN

WRITE(IW,242) ENTER VALUES: NUMK AND K VALUES

READ(IR.*) NK,(AK(I),I=1,NK)

WRITE(IW,243) CALL XCOSF GOTO 206

CALL XECDF GOTO 206

C SUBROUTINE NECOPT

SUBROUTINE NECOPT
THIS SUBROUTINE ECONOMICALLY OPTIMIZE NLG MODEL

COMMON /C1/USLLSL,NN,MM,NG,NY,NY1,TLNLG,HALF,IR,IW
COMMON /E2/PQ,PY,PR,P1,P2
COMMON /E3/ZDEL,ZLAM,ZM,ZE,ZD,ZT,ZW,ZB,ZC
COMMON /E4/ XSTART(2),X(3,2),Y(3),ITRFLG,IRESET,
* STDY,STYO,KP,NNVAR,N1,YACC,XACC,STEP,ITRMAX,NLGXB
COMMON /E5/ NYBACK,NGBACK,YIMPRV,NNMIN,NNMAX

DATA STAR2/'**'/, BLANK/' '/

N1=3
NLGXB=1

------------- PRINT TITLE AND PARAMETER VALUES ---------------

WRITE(IW,11) USLLSL,MM
11 FORMAT(/ ' **** ECONOMICALLY BASED NLG DESIGN ****'//
* 'USLLSL=',F6.2,6X,'MM=',I1,6X,'MEAN SHIFT ASSUMED')
WRITE (IW, 113)
113 FORMAT( 'DELTA=' ,F7.2,6X,'LAMBDA=' ,F7.2,6X, 'M=' ,F7.2,6X, 'E='
* F7.2,6X,'D=',F7.2/T7, 'T='
WRITE(IW,12) YACC, XACC, STEP, ITRMAX, (XSTART(I), I=1, 2), IRESET
12 FORMAT(/' (H,T) OPTIMIZATION: YACC='
* 'XACC='
* 'STEP=',F7.3,6X, 'ITRMAX=',I3/T23, 'STARTING
* POINT: HO=', F7.3,T53, 'TO=',F7.3,T66, 'IRESET=' ,I1)
WRITE(IW,14) NYBACK,NGBACK,YIMPRV,NNMIN,NNMAX
14 FORMAT(' OVERALL OPTIMIZATION:
* 'EY=',I1,6X,'EG=',I1,6X,'EL=',F8.3,T56, 'NMIN=' ,I2,6X, 'NMAX=', I2)
WRITE( IW, 13)
13 FORMAT(// T4, 'N MM
G' ,T23, 'H',T33, 'T' ,T41, 'L01L'
* 'STOY', T62, 'STDX'
* T69, 'TITR MAXITR'/)

------------- YMN=YMIN AMONG ALL NN, YMY=YMIN AMONG ALL NY,
------------- YMG=YMIN AMONG ALL NG

NYMIN=O
NGMIN=1
YMN=10000000.

DO 200 NN=NNMIN,NNMAX
200 IF(MM.EQ.3) J3U=NN+1
     J3U=NN
     NYMIN2=NYMIN-NYBACK+1
     J3L=MAX0(1,NYMIN2)
     IF(NYMIN.EQ.O) J3L=1
     YMY=10000000.

---------- DYNAMICALY DETERMINE THE STARTING VALUE OF Y
     NYMIN2=NYMIN-NYBACK+1
     J3L=MAX0(1,NYMIN2)
     IF(NYMIN.EQ.O) J3L=1

---------- DYNAMICALY DETERMINE THE STARTING VALUE OF G
     NGMIN2=NGMIN-NGBACK
     NGJL=MAX0(1,NGMIN2)
     IF(NGMIN.EQ.O) NGJL=1

---------- (H,T) OPTIMIZATION USING DIRECT SEARCH TECHNIQUE
     CALL HTOPT
     IF(IRESET.EQ.O) GOTO 159

-- CHECK TO SEE IF THE LOSS-COST L IS BIG ENOUGH TO QUIT G LOOP

C
154 IF(Y(N1).GT.(YMG+YIMPRV)) GO TO 158
1153 IF(Y(N1).GT. YMG ) GO TO 153

NGMIN=NG
YMG=Y(N1)
STAR=BLANK
IF(ITRFLG.EQ. 1) STAR=STAR2

WRITE(IW,20) NN,MM,NG,Y(N1),STDY,STDX,KPP,STAR

FORMAT(T2,4I3,T17,3F10.3,2F10.4,I6,2X,A2)

GO TO 160

NG=O
NYFLG=1
GO TO 152

IYMGF=1
GO TO 1153

C--- ADOPT THE OPTIMAL POINT AS THE STARTING POINT FOR NEXT OPTIMIZATION

DO 1159 JJ=1,NVAR

XSTART(JJ)=X(N1,JJ)
GO TO 154

CONTINUE

WRITE(IW,163)
FORMAT(/ T15,32('*'),3X, 'OVERALL OPTIMAL 100L =',F10.3)
RETURN

END

C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++C

C *** THIS SUBROUTINE ECONOMICALLY EVALUATES A NLG PLAN

COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
COMMON /E2/ PG,PY,PR, PR1,PR2
COMMON /E3/ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
COMMON /E6/ HNLG,HX,RKX
DIMENSION AHT(2)

C------------------------------------ EVALUATION ----------------------C

NY1=NY+1
AHT(1)=HNLG
AHT(2)=TNLG

C---------------- OUTPUT SECTION -------------------------------C

WRITE (IW,9) USLLSL,MM

FORMAT / T2, ' ECONOMICALLY BASED NLG EVALUATION' //0016400
* ' USLLSL',F6.2, ' (STD)',4X, 'MM=',I1,5X, 'MEAN SHIFT ASSUMED') //0016400
WRITE (IW,113) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC

FORMAT(/ T3,'N=',I3,4X,'Y=',I3,4X,'G=',I3,10X.

ZL=ZL100/100.

C------------------------- SUBROUTINE NECEV -------------------------C

C+++ -----------------------------------------------------------------
SUBROUTINE NCOSF

** THIS SUBROUTINE INVESTIGATES THE LOSS-COST SURFACE OF A NLG PLAN **

COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
COMMON /E2/PG,PY,PR, PR1,PR2
COMMON /E3/ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
COMMON /E7/NH,AH(30), NT,AT(11), NK,AK(11)
DIMENSION ACOST(30,11),AALFAP(2,11),LABEL(2),AASN(2,11)
DATA LABEL/'ALFA','P '/

** LOSS-COST SURFACE EVALUATION **

DO 20 I=1,NT
    TNLG=AT(I)
    CALL GYRMU(0.)
    CALL PAFG2(ZALFA)
    CALL EDFN2(ZNIC)
    CALL GYRMU(ZDEL)
    CALL PAFG2(ZP)
    CALL EDFN2(ZNDOC)
    IF(ZP.LT..0000001) ZP=.0000001
    AALFAP(1,I)=ZALFA
    AALFAP(2,I)=ZP
    AASN(1,I)=ZNIC
    AASN(2,I)=ZNDOC

DD 10 J=1,NH
    ZH=AH(J)
    ZBB=(1./ZP-.5+ ZLAM*ZH/12.)*ZH + ZE*ZNDOC + ZD
    ZBETA=1./(1.+ZLAM*ZBB)
    ZNAVE=ZBETA•ZNIC+(1.-ZBETA)*ZNDDC
    VY=(ZLAM*ZM*ZBB
        + ZALFA*ZT/ZH +ZLAM*ZW)*ZBETA
        + (ZB+ZC*ZNAVE)/ZH
    ACOST(J,I)=VY*100.
20 CONTINUE

** LOCATE MINIMUM COST **

AMIN=9999999.
IX=O
JX=O
DD 50 I=1,NH
    DD 40 J=1,NT
    IF (ACOST(I,J).GE.AMIN) GD TO 40
    AMIN=ACOST(I,J)
    IX=I
    JX=J
40 CONTINUE
50 CONTINUE

** OUTPUT SECTION **

WRITE (IW,9)
9 ** ECONOMICALLY BASED NLG LOSS-COST **
    WRITE (IW,116) USLLSL,MM,NN,NY,NG
116 ** SURFACE INVESTIGATION **
    WRITE (IW,118) USLLSL,MM,NN,NY,NG
118 ** ASSUMED **
    WRITE (IW,111) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
111 ** DELTA **
    WRITE (IW,114) (AT(I),I=1,NT)
114 ** WRITE **
    WRITE (IW,115) ZL100,ZL
115 ** WRITE **
    WRITE (IW,112) ZDEL,ZM,ZE,ZD, ZT,ZW,ZB,ZC
112 ** WRITE **
    WRITE (IW,117) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
117 ** WRITE **
    WRITE (IW,113) ZDEL,ZM,ZE,ZD, ZT,ZW,ZB,ZC
113 ** WRITE **
    WRITE (IW,119) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
119 ** WRITE **
    WRITE (IW,120) ZDEL,ZM,ZE,ZD, ZT,ZW,ZB,ZC
120 ** WRITE **
    WRITE (IW,121) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
121 ** WRITE **
    WRITE (IW,122) ZDEL,ZM,ZE,ZD, ZT,ZW,ZB,ZC
122 ** WRITE **
    WRITE (IW,123) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
123 ** WRITE **
    WRITE (IW,124) ZDEL,ZM,ZE,ZD, ZT,ZW,ZB,ZC
124 ** WRITE **
    WRITE (IW,125) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
125 ** WRITE **
    WRITE (IW,126) ZDEL,ZM,ZE,ZD, ZT,ZW,ZB,ZC
126 ** WRITE **
    WRITE (IW,127) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
127 ** WRITE **
    WRITE (IW,128) ZDEL,ZM,ZE,ZD, ZT,ZW,ZB,ZC
128 ** WRITE **
    WRITE (IW,129) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
129 ** WRITE **
    WRITE (IW,130) ZDEL,ZM,ZE,ZD, ZT,ZW,ZB,ZC
130 ** WRITE **
    WRITE (IW,131) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
131 ** WRITE **
    WRITE (IW,132) ZDEL,ZM,ZE,ZD, ZT,ZW,ZB,ZC
132 ** WRITE **
    WRITE (IW,133) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
133 ** WRITE **
    WRITE (IW,134) ZDEL,ZM,ZE,ZD, ZT,ZW,ZB,ZC
134 ** WRITE **
    WRITE (IW,135) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
135 ** WRITE **
    WRITE (IW,136) ZDEL,ZM,ZE,ZD, ZT,ZW,ZB,ZC
136 ** WRITE **
    WRITE (IW,137) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
137 ** WRITE **
    WRITE (IW,138) ZDEL,ZM,ZE,ZD, ZT,ZW,ZB,ZC
138 ** WRITE **
    WRITE (IW,139) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
139 ** WRITE **
    WRITE (IW,140) ZDEL,ZM,ZE,ZD, ZT,ZW,ZB,ZC
140 ** WRITE **
    WRITE (IW,141) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
141 ** WRITE **
    WRITE (IW,142) ZDEL,ZM,ZE,ZD, ZT,ZW,ZB,ZC
142 ** WRITE **
    WRITE (IW,143) ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
SUBROUTINE XECOPT

COMMON /C1/USLLSL, NN, MM, NG, NY, NY1, TNLG, HALF, IR, IW
COMMON /E3/ZDEL, ZLAM, ZM, ZE, ZD, Z7, ZW, ZB, ZC
COMMON /E4/ XSTART(2), X(3,2), Y(3), ITRFLG, IRESET, STDX, STDY, KPP, NVAR, N1, YACC, XACC, STEP, ITRMAX, NLGXB
COMMON /E5/ NYBACK, NGBACK, YIMPRV, NNMIN, NNMAX

DATA STAR2/'**'/, BLANK/' '/

N1=3
NLGXB=2

PRINT TITLE AND PARAMETER VALUES
WRITE(IW,11) USLLSL
11 FORMAT(/'*****ECONOMICALLY BASED X-BAR CHART DESIGN*****'//,T5,'USLLSL=' ,F6.2,6X, 'MEAN SHIFT ASSUMED')
WRITE(IW,113) ZDEL, ZLAM, ZM, ZE, ZD, Z7, ZW, ZB, ZC
WRITE(IW,12) YACC, XACC, STEP, ITRMAX, (XSTART(I),I=1,2), IRESET
WRITE(IW,14) YIMPRV, NNMIN, NNMAX
14 FORMAT(/ (H,T) OVERALL OPTIMIZATION: EL=' ,F8.3,5X, 'MIN=' ,I2,3X, 'MAX=' ,I2)

C----------NN INCREMENT (YMN=YMIN AMONG ALL NN)----------

YMN=10000000.

DO 200 NN=NNMIN, NNMAX
200 NN1=NN+1

C++++++ ECONOMICALLY OPTIMIZE X-BAR CHART MODEL++++++

CALL HTOPT

IF(IRESET.EQ.0) GOTO 159

WRITE(IW,14) YIMPRV, NNMIN, NNMAX


C--------------------------------- PRINT TITLE AND PARAMETER VALUES ---------------
C
WRITE(IW,11) USLLSL
11 FORMAT(/'*****ECONOMICALLY BASED X-BAR CHART DESIGN *****'//,T5,'USLLSL=' ,F6.2,6X, 'MEAN SHIFT ASSUMED')
WRITE(IW,113) ZDEL, ZLAM, ZM, ZE, ZD, Z7, ZW, ZB, ZC
WRITE(IW,12) YACC, XACC, STEP, ITRMAX, (XSTART(I),I=1,2), IRESET
WRITE(IW,14) YIMPRV, NNMIN, NNMAX
14 FORMAT(/ (H,T) OVERALL OPTIMIZATION: EL=' ,F8.3,5X, 'MIN=' ,I2,3X, 'MAX=' ,I2)

C--------------------------------- NN INCREMENT (YMN=YMIN AMONG ALL NN)------------------

YMN=10000000.

DO 200 NN=NNMIN, NNMAX
200 NN1=NN+1

C++++++ ECONOMICALLY OPTIMIZE X-BAR CHART MODEL++++++

CALL HTOPT

IF(IRESET.EQ.0) GOTO 159

WRITE(IW,14) YIMPRV, NNMIN, NNMAX


C--------------------------------- PRINT TITLE AND PARAMETER VALUES ---------------
C
DO 160 JJ=1,NVAR
  XSTART(JJ)=X(N1, JJ)
GOTO 154
170  IOPTF=1
GOTO 153
200 CONTINUE
201 WRITE(IW,202) YMN
DO 153 JJ=1,NVAR
  XSTART(JJ)=X(N1, JJ)
GOTO 154
CONTINUE
IOPTF=1
GOTO 153
WRITE(IW,202) YMN
FORMAT(/T11,32('*'),3X,'OVERALL OPTIMAL 100L =',F10.3)
RETURN
WRITE(IW,114)
FORMAT(//T0('*'),'
  *LE WHEN THE NUMBER OF VARIABLES NVAR=',I1,' .LT.2')
END
C++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
SUBROUTINE XECEV
C *** THIS SUBROUTINE ECONOMICALLY EVALUATES AN X-BAR CHART PLAN
C
COMMON /C1/USLLSL,NN,MM,NG, NY, NY1, TNLG,HALF, IR, IW
COMMON /E3/ZDEL,ZLAM,ZM, ZE, ZD, ZT,ZW, ZB, ZC
COMMON /E6/HNLG,HX,RKX
DIMENSION AHT(2)
C-------
PRINT TITLE AND PARAMETERS ------------
WRITE (IW,9) USLLSL
9  FORMAT(/T2,' *****ECONOMICALLY BASED X-BAR CHART EVALUATION ', "/", USLLSL=': F6.2.' ,STD'),5X,'MEAN SHIFT ASSUMED')
WRITE (IW,113) ZDEL, ZLAM, ZM, ZE, ZD, ZT, ZW, ZB, ZC
113  FORMAT(' DELTA=',F7.2,3X,'LAMBDA=',F7.2,3X,
  'M=',F7.2,3X,'E=',F7.2,3X, '0=',F7.2/T7, 'T=',F7.2,3X,
  'W=',F7.2,3X,'B=',F7.2,3X, 'C=',F7.2)
WRITE(IW,114) NN, HX, RKX
114  FORMAT(/ TS, 'N=', I3,
  'H=', F8.3,
  'K=', F8.3)
C-------------------------------
AHT( 1 )=HX
AHT(2)=RKX
ZL100=VYXBAR(AHT)
ZL=ZL100/100.
WRITE(IW,115) ZL100,ZL
115  FORMAT(// T5, 'N=', I3, 10X, 'H=', F8.3, 10X, 'K=', F8.3)
C+C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
C *** THIS SUBROUTINE INVESTIGATES THE LOSS-COST SURFACE OF AN X-BAR
C CHART PLAN
C
DIMENSION ACOST(30,11),AALFAP(2,11),LABEL(2),AASN(2,11)
COMMON /C1/USLLSL,NN,MM,NG, NY, NY1, TNLG,HALF, IR, IW
COMMON /E3/ZDEL,ZLAM,ZM, ZE, ZD, ZT, ZW, ZB, ZC
COMMON /E7/NH,AH(30), NT, AT(11), NK, AK(11)
DATA LABEL/'ALFA',p '/
ZN=NN
C+C------------------------------- COST SURFACE EVALUATION --------------
DD 20 i=1,NK
  ZK=AK(I)
  DW=ZDEL*SQRZ(ZN)
  Y1=-ZK-DN
  Y2= ZK-DN
C=}159
C=16C
C=170
C=200
C=201
C=202
C=888
C=999
C=114
C=115
CALL MDNOR(Y1,P1)
CALL MDNOR(Y2,P2)
CALL MDNOR(Y3,P3)
ZP=P1+1.-P2
IF(ZP.LT.0.0000001) ZP=0.0000001
ZALFA=2.*P3
AALFAP(1,I)=ZALFA
AALFAP(2,I)=ZP
ZP=.0000001
DO 10 J=1,NH
ZH=AH(J)
ZBB=(1./ZP-.5+ZLAM*ZH/12.)*ZH+ZE*ZN+ZD
VY=(ZLAM*ZM*ZBB+ZALFA*T/ZH+ZLAM*ZW)/(1.+ZLAM*ZBB)+(ZB+ZC*ZN)/ZH
ACOST(J,I)=VY*100.
CONTINUE
10 CONTINUE
20 CONTINUE
C------------------------------------- LOCATE MINUM COST -----------------
AMIN=99999999.
IX=0
JX=0
DO 50 !=1,NH
DO 40 J=1,NK
IF (ACOST(I,J).GE.AMIN) GO TO 40
AMIN=ACOST(I,J)
IX=I
JX=J
40 CONTINUE
50 CONTINUE
C------------------------------------- OUTPUT SECTION ------------------
WRITE (IW,9)
9 FORMAT(' ',T5,'ECONOMICALLY BASED X-BAR CHART ','LOSS-COST SURFACE INVESTIGATION ','F5(''))
WRITE(IW,112) USLLSL,NN
112 FORMAT( /T3, 'USLLSL.=' ,F6.2,' STD' ,5X, 'MEAN SHIFT', F7.2,3X, 'N=',I3)
WRITE(IW,111) ZDEL,ZLAM,ZM,ZE,ZD,ZT,ZW,ZB,ZC
WRITE(IW,114) (AK(I),I=1,NK)
114 FORMAT( //,T5,'K' ,T10,11F11.3/) DO 30 I=1,2
30 WRITE (IW,115) LABEL(I),(AALFAP(I,J),J=1,NK)
115 FORMAT ( T5,A4,T10,11F11.3)
WRITE(IW,117) ( T2,129('')/T7,'H')
117 FORMAT ( //,T5,K,T10,11F11.3)
WRITE(IW,116) AH(I), (ACOST(I,J),J=1,NK)
116 FORMAT ( //,T3,F7.3,T10,11F11.3)
WRITE(IW,118) AH(I), (ACOST(I,J),J=1,NK)
118 FORMAT ( //,T3,F7.3,T10,11F11.3)
WRITE(IW,119) AH(I), (ACOST(I,J),J=1,NK)
119 FORMAT ( //,T3,F7.3,T10,11F11.3)
WRITE(IW,110) AH(I), (ACOST(I,J),J=1,NK)
110 FORMAT ( //,T3,F7.3,T10,11F11.3)
WRITE(IW,111) AH(I), (ACOST(I,J),J=1,NK)
111 FORMAT ( //,T3,F7.3,T10,11F11.3)
WRITE(IW,112) AH(I), (ACOST(I,J),J=1,NK)
112 FORMAT ( //,T3,F7.3,T10,11F11.3)
WRITE(IW,113) AH(I), (ACOST(I,J),J=1,NK)
113 FORMAT ( //,T3,F7.3,T10,11F11.3)
RETURN
END
C*** THIS SUBROUTINE OPTIMIZE (H,T) FOR BOTH NLG AND X-BAR CONTROL SCHEMES BY NELDER AND MEAD DIRECT SEARCH TECHNIQUE
COMMON /C1/USLLSL,MM,NG,NY, NY1, TNLG,HALF, IR, IW
C---------------------------------------------------------------
SUBROUTINE HTOPT
C

Y3= -ZK
CALL MDNOR(Y1,P1)
CALL MDNOR(Y2,P2)
CALL MDNOR(Y3,P3)
ZP=P1+1.-P2
IF(ZP.LT.0.0000001) ZP=0.0000001
ZALFA=2.*P3
AALFAP(1,I)=ZALFA
AALFAP(2,I)=ZP
ZP=.0000001
DO 10 J=1,NH
ZH=AH(J)
ZBB=(1./ZP-.5+ZLAM*ZH/12.)*ZH+ZE*ZN+ZD
VY=(ZLAM*ZM*ZBB+ZALFA*T/ZH+ZLAM*ZW)/(1.+ZLAM*ZBB)+(ZB+ZC*ZN)/ZH
ACOST(J,I)=VY*100.
CONTINUE
10 CONTINUE
20 CONTINUE
C------------------------------------- LOCATE MINUM COST -----------------
AMIN=99999999.
IX=0
JX=0
DO 50 !=1,NH
DO 40 J=1,NK
IF (ACOST(I,J).GE.AMIN) GO TO 40
AMIN=ACOST(I,J)
IX=I
JX=J
40 CONTINUE
50 CONTINUE
C------------------------------------- OUTPUT SECTION ------------------
WRITE (IW,9)
9 FORMAT(' ',T5,'ECONOMICALLY BASED X-BAR CHART ','LOSS-COST SURFACE INVESTIGATION ','F5(''))
WRITE(IW,112) USLLSL,NN
112 FORMAT( /T3, 'USLLSL.=' ,F6.2,' STD' ,5X, 'MEAN SHIFT', F7.2,3X, 'N=',I3)
WRITE (IW,111) ZDEL,ZLAM,ZM,ZE,ZD,ZT,ZW,ZB,ZC
WRITE(IW,114) (AK(I),I=1,NK)
114 FORMAT( //,T5,'K' ,T10,11F11.3/) DO 30 I=1,2
30 WRITE (IW,115) LABEL(I),(AALFAP(I,J),J=1,NK)
115 FORMAT ( T5,A4,T10,11F11.3)
WRITE(IW,117) ( T2,129('')/T7,'H')
117 FORMAT ( //,T5,K,T10,11F11.3)
WRITE(IW,116) AH(I), (ACOST(I,J),J=1,NK)
116 FORMAT ( //,T3,F7.3,T10,11F11.3)
WRITE(IW,118) AH(I), (ACOST(I,J),J=1,NK)
118 FORMAT ( //,T3,F7.3,T10,11F11.3)
WRITE(IW,119) AH(I), (ACOST(I,J),J=1,NK)
119 FORMAT ( //,T3,F7.3,T10,11F11.3)
WRITE(IW,110) AH(I), (ACOST(I,J),J=1,NK)
110 FORMAT ( //,T3,F7.3,T10,11F11.3)
WRITE(IW,111) AH(I), (ACOST(I,J),J=1,NK)
111 FORMAT ( //,T3,F7.3,T10,11F11.3)
WRITE(IW,112) AH(I), (ACOST(I,J),J=1,NK)
112 FORMAT ( //,T3,F7.3,T10,11F11.3)
WRITE(IW,113) AH(I), (ACOST(I,J),J=1,NK)
113 FORMAT ( //,T3,F7.3,T10,11F11.3)
RETURN
END
COMMON /E2/ PG, PY, PR, PR1, PR2
COMMON /E3/ ZDEL, ZLAM, ZM, ZE, ZD, ZT, ZW, ZB, ZC
DIMENSION XR(2), XB(2), XF(2), XH(2), XE(2), XC(2), XL(2),
CT(2), NTYPE(6)

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00194600
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00201500
00201600

C-- FUNCTION EVALUATION (y) FOR ALL POINTS (x) ----

DO 11 J=1,N
11 XF(J)=X(N1,J)
   IF(NLGXB.EQ.1) Y(N1)=VYNLG(XF)
   IF(NLGXB.EQ.2) Y(N1)=VVXBAR(XF)

DO 12 J=1,N
12 X(N1,J)=XF(J)

C-- FIND BEST PT --> (N+1)TH POINT ----

19 YL=Y(N1)
   NL=N1
   DO 21 I=1,N
      IF(Y(I).GE.YL) GO TO 21
      YL=Y(I)
   21 CONTINUE
   X(N1,J)=Y(N1)
   N=I

C-- FIND WORST PT --> 1ST POINT ----

22 YH=Y(1)
   NH=1
   DO 23 I=1,N
      IF(Y(I).LT.YH) GO TO 23
   23 CONTINUE
DO 24 J=1,N
X(H(J))=X(NH,J)
X(NH,J)=X(1,J)
24 X(1,J)=XH(J)
Y(NH)=Y(1)
Y(1)=YH
C----------------- FIND 2ND WORST POINT ----------------
YSH=Y(2)
IF(N .LT. 3) GO TO 27
DO 26 I=3,N
  IF(Y(I) .LE. YSH) GO TO 26
YSH=Y(I)
26 CONTINUE
C---------------- CHECK TO SEE IF IT IS TIME TO QUIT ---------------
C---- CHECK TO SEE IF MAX ITERATION REACHED ---------------
27 IF(K .LT. ITRMAX) GO TO 127
C---- TURN ON FLAG OF MAX ITERATION, AND QUIT
ITRFLG=1
RETURN
C----- CALCULATE MEANS OF X (W/O & W/ WORST PT) & Y ------
127 DD 29 J=1,N
  XBJ(J)=0.0
  DO 28 I=2,N1
    XBJ(J)=XBJ(J)+X(I,J)
  XT(J)=XBJ(J)+XH(J)
  XBJ(J)=XBJ(J)/N1
29 XT(J)=XT(J)/N1
YB=0.0
DO 31 I=1,N1
  YB=YB+Y(I)
31 YB=YB/N1
C-------- CALCULATE STANDARD DEVIATION OF Y ---------
STDY=0.0
DD 32 I=1,N1
  STDY=STDY+(Y(I)-YB)**2
32 STDY=SQRT(STDY)
C-------- CALCULATE STANDARD DEVIATION OF X ---------
STDX=0.0
DD 34 I=1,N1
  SZ=0.0
  DO 33 J=1,N
    SZ=SZ+(X(I,J)-XT(J))**2
  33 SZ=SQRT(SZ)
34 STDX=STDX+SZ
STDX=STDX/N1
C----- CHECK TO SEE IF QUITTING CRITERIA SATISFIED
IF(STDY .LT. YACC .OR. STDX.LT.XACC) RETURN
C----- REFLECTION, EXPANSION, CONTRACTION AND SHRINKAGE -----
C----------------------------- REFLECTION ----------------
37 XR(J)=XBJ(J)+ALP*(XBJ(J)-XH(J))
  IF(NLGXB.EQ.1) YR=VYNLG(XR)
  IF(NLGXB.EQ.2) YR=VYXBAR(XR)
  NFC=NFC+1
  K=K+1
  IF(YR .LT. YL) GO TO 52
  IF(YSH .LT. YR) GO TO 39
C---- WORST REPLACED BY REFLECTION PT ----
38 X(1,J)=XR(J)
  Y(1)=YR
C NTP=2
  GO TO 19
39 IF(YH .LE. YR) GO TO 43
C---------------------- CONTRACTION -----------------
C------------ CONTRACTION OUTWARD ----------
DO 41 J=1,N

169 DO 41 J=1,N
41 XC(J)=XB(J)+BET*(XR(J)-XB(J))

C NTP=5
IF(NLGXB.EQ.1) YC=VYNLG(XC)
IF(NLGXB.EQ.2) YC=VXBAR(XC)

C NFC=NFC+1
IF(YC.LT.YR) GO TO 47
DO 42 J=1,N
42 X(1,J)=XR(J)
GO TO 49

C-------- CONTRACTION INWARD ---------

DO 44 J=1,N
44 XC(J)=XB(J)+BET*(XH(J)-XB(J))

C NTP=3
IF(NLGXB.EQ.1) YC=VYNLG(XC)
IF(NLGXB.EQ.2) YC=VXBAR(XC)

C NFC=NFC+1
IF(YC.LT.YH) GO TO 47
DO 48 J=1,N
48 X(1,J)=XC(J)
Y(1)=YC
GO TO 19

C------------------------ SHRINKAGE -----------------------

DO 49 I=1,N
51 XI(I,J)=X(I,J)+.50*(XL(I,J)-X(I,J))

C NTP=4
GO TO 13

C--------------------- EXPANSION ---------------------

DO 53 J=1,N
53 XE(J)=XB(J)+GAM*(XR(J)-XB(J))

IF(NLGXB.EQ.1) YE=VYNLG(XE)
IF(NLGXB.EQ.2) YE=VXBAR(XE)

C NFC=NFC+1
IF(YE.LT.YH) GO TO 56
DO 54 J=1,N
54 X(1,J)=XR(J)
Y(1)=YE
GO TO 19

C-------- WORST REPLACED BY CONTRACTION PT -----

DO 48 J=1,N
48 X(1,J)=XC(J)
Y(1)=YC
GO TO 19

FUNCTION VXBAR(XF)

C---------- MEASURES ARE TAKEN TO PREVENT UNDERFLOW (OVERFLOW) PROBLEM
ZN=NN
IF(YF(1).LT.0.001) XF(1)=.001
ZD=ZDEL*SQR(ZN)
Y1=-ZD-DN
Y2=ZK-DN
Y3=-ZK

C------- THIS FUNCTION SUBPROGRAM EVALUATES THE LOSS-COST (PER 100 HOURS)
C FOR AN X-BAR CHART PLAN
C
COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
COMMON /E3/ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
DIMENSION XF(2)

C THIS FUNCTION SUBPROGRAM EVALUATES THE LOSS-COST (PER 100 HOURS)
C FOR AN X-BAR CHART PLAN
C
COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
COMMON /E3/ZDEL,ZLAM,ZM,ZE,ZD, ZT,ZW,ZB,ZC
DIMENSION XF(2)

C-------- MEASURES ARE TAKEN TO PREVENT UNDERFLOW (OVERFLOW) PROBLEM
ZN=NN
IF(YF(1).LT.0.001) XF(1)=.001
ZD=ZDEL*SQR(ZN)
Y1=-ZD-DN
Y2=ZK-DN
Y3=-ZK

C---------------------- END ----------------------
CALL MDNOR (Y1,P1)
CALL MDNOR (Y2,P2)
CALL MDNOR (Y3,P3)
ZP=P+1. -P2
IF(ZP.LT. 0000001) ZP=00000001
ZALFA=2.*P3

ZBB=(1./ZP -.5 + ZLAM*ZH/12.)*ZH + ZE*ZN + ZD
VY=(ZLAM*ZM*ZBB + ZALFA*ZT/ZH + ZLAM*ZW)/(1.+ZLAM*ZBB)
* + (ZB+2*ZC)/ZH
VYBAR=VY+100.
RETURN
END

FUNCTION VYNLG(XF)

ZN=NN
IF(XF(1).LT.0.001) XF(1)=.001
ZH=XF(1)
IF(XF(2).GT. HALF) XF(2)=HALF-.001
IF(XF(2).LT..001) XF(2)=.001
TNLG=ZF(2)
CALL GYRMU(O.)
CALL PAFG2(ZALFA)
CALL EOFN2(ZNIC)
CALL GYRMU(ZDEL)
CALL PAFG2(ZP)
CALL EOFN2(ZNOOC)
IF(ZP.LT..0000001) ZP=.0000001
ZBB=(1./ZP -.5 + ZLAM*ZH/12.)*ZH + ZE*ZNOOC + ZD
ZBETA=1./(1.+ZLAM*ZBB)
ZNAVE=ZBETA*ZNIC+(1.-ZBETA)*ZNOOC
VY=(ZLAM*ZM*ZBB + ZALFA*ZT/ZH + ZLAM*ZW)*ZBETA
* + (ZB+ZC*ZNAVE)/ZH
VYNLG=VY*100.
RETURN
END

SUBROUTINE GYRMU(DEL)

HTD1=HALF-TNLG+DEL
HTD2=-HALF+TNLG+DEL
CALL MDNOR(HTD1,PHI1)
CALL MDNOR(HTD2,PHI2)
PG=PHI1-PHI2
GO TO (99,20,30),MM
20  PY=1.+PG
RETURN
30  PG=PR1
IF(DEL.GT.0.) PR=PR2

** THIS SUBROUTINE CALCULATES THE PROBABILITY OF GREEN, YELLOW AND RED AS FUNCTIONS OF MEAN SHIFT **

** SAME AS THE FIRST PART OF SUBROUTINE GYRC **

COMMON /C1/USLLSL,NN,MM,NG,NY,NY1, TNLG,HALF, IR,IW
COMMON /E2/ PG,PY,PR, PR1,PR2
HTD1=HALF-TNLG+DEL
HTD2=-HALF+TNLG+DEL
CALL MDNOR(HTD1,PHI1)
CALL MDNOR(HTD2,PHI2)
PG=PHI1-PHI2
GO TO (99,20,30),MM
20  PY=1.+PG
RETURN
30  PG=PR1
IF(DEL.GT.0.) PR=PR2
SUBROUTINE PAFG (PREJ)

*** THE UNDERFLOW-PROOF VERSION OF SUBROUTINE PAFG

COMMON /C1/USLLSL,NN,MM,NG,NV,NY,NY1, TNLG,HALF, IR, IW
COMMON /E2/ PG,PY,PR, PR1,PR2
PSUM=0.

DO 22 I=1,NY1
    IL1=I-1
22  PSUM=PSUM+BINOM2(NN,IL1)
PREJ=1 .-PSUM
IF(NG.EQ.0) RETURN

PSUM2=0.
IN=NY1
NNLNG=NN-NG
IF(NY.GT.NNLNG) IN=NNLNG+1

DO 24 I=1,IN
    IL1=I-1
24  PSUM2=PSUM2+BINOM2(NNLNG,IL1)
EE=NG*ALOG(PG)
IF(EE.LT.-170.) EE=-170.
PREJ=1 .-(PSUM+(1.-PSUM2)*EXP(EE))
RETURN
END

FUNCTION BINOM2 (N,IX)

*** THE UNDERFLOW-PROOF VERSION OF FUNCTION SUBPROGRAM BINOM1

COMMON /E2/ PG,PY,PR, PR1,PR2
DOUBLE PRECISION DY,DG,DLGPB

C THIS ROUTINE CALCULATES BINOMIAL AND ITS SIMILERS

DY=PY
DG=PG
DLGPB=DLGAMA(N+1.DO)-DLGAMA(IX+1.DO)-DLGAMA(N-IX+1.DO)
* +IX*DLG(DDY)+(N-IX)*DLG(DG)
IF (DLGPB.LT.-170.DO) DLGPB=-170.DO
BINOM2=DEXP(DLGPB)
RETURN
END

SUBROUTINE EOFN2(REN)

*** THE UNDERFLOW-PROOF VERSION OF SUBROUTINE EOFN

COMMON /C1/USLLSL,NN,MM,NG,NV,NY,NY1, TNLG,HALF, IR, IW
COMMON /E2/ PG,PY,PR, PR1,PR2
DOUBLE PRECISION ABC,SABC,EN, G,Y,R,YGF2,GC,EE,E2,DEXPEE
G=PG
Y=PY
IF(MM.EQ.3) R=PR
ABC=0.DO
SABC=0.DO
EN=0.DO
NNL1=NN-1
IF(NY.GT.10) GO TO 10

REN=1.
RETURN
C---------------------------- NN > 1 ------
10 GO TO (900,200,300,900,900),MM
C-----------------------------------------MM=2 ---------------------
200 IF(NY.EQ.O) GO TO 201
IF(NY.LT.NNL1) GO TO 221
GO TO 281
C---------------------------- MM=2; NY=O (NG=O) ----------
201 IF(NG.GE.1) GO TO 212
DO 210 I=1,NNL1
EE=(I-1)*DLOG(G)
IF(EE.LT.-170.DO) EE=-170.DO
210 EN=EN+I*DEXP(EE)*Y
E2=NNL1*DLOG(G)
IF(E2.LT.-170.DO) E2=-170.DO
REn=EN+NN*DEXP(E2)
RETURN
C
212 WRITE(IW,214)
214 FORMAT(//,T2,10(' '),' NLG ERROR: MM=2 Y=O G>G;' ,
* ' EXECUTION INTERRUPTED IN SUBROUTINE EOFN2 (LABEL 212)')
RETURN
C----------------------------------------- MM=2 ---------------------
221 IF(NG.EQ.O .OR. NG.LT.NY) GO TO 225
EE=NG*DLOG(G)
IF(EE.LT.-170.DO) EE=-170.DO
ABC=DEXP(EE)
EN=EN+NG*ABC
SABC=SABC+ABC
GO TO 240
225 DC 240 J=NY+1,NNL1
JL1=J-1
IF(J.EQ.NG) GO TO 229
ABC=YGF2(JL1,NY,G,Y)
EN=EN+J*ABC
SABC=SABC+ABC
GO TO 240
229 EE=NG*DLOG(G)
IF(EE.LT.-170.DO) EE=-170.DO
ABC=YGF2(JL1,NY,G,Y)+DEXP(EE)
EN=EN+J*ABC
SABC=SABC+ABC
RETURN
C---------------------------- MM=3 ---------------------
300 IF(NY.EQ.O) GO TO 301
IF(NY.LT.NNL1) GO TO 321
C---------------------------- MM=3; NY=O (NG=O) ----------
301 IF(NG.GE.1) GO TO 312
GO TO 351
C
312 WRITE(IW,314)
314 FORMAT(//,T2,10(' '),' NLG ERROR: MM=3 Y=G G>G;' ,
* ' EXECUTION INTERRUPTED IN SUBROUTINE EOFN2 (LABEL 312)')
RETURN
C---------------------------- MM=3; O<NY< NN-1 ----
321 DO 330 I=1,NN

C                  MM=3;  NY>O & NY>=(NN-1) --
351 DO 360  I=1,NN1
      IF (I.EQ.NG) GO TO 359
      ABC=(1.DO-SABC)*R
      EN=EN+I*ABC
      GO TO 360
359 EE=NG*DLOG(G)
      IF(EE.LT.-170.DO) EE=-170.DO
      ABC=(1.DO-SABC)*R+YGF2(JL1,NY,G,Y)
      EN=EN+I*ABC
360 SABC=SABC+ABC
      REN=EN+NN*(1.DO-SABC)
      RETURN
C--------------------- MM=3;
C                  NY>O & NY>=(NN-1) --
C-------------------------------
900 WRITE(IW,901) MM
901 FORMAT(/// T3,10('-'), 'ERROR: IN SUBROUTINE EOFN2, MM= ', I2, 'N', 2 OR 3; EXECUTION INTERRUPTED (LABEL 900))
RETURN
END
FUNCTION YGF2(N,K,G,Y)
C-------------------------------
THE UNDERFLOW-PROOF VERSION OF FUNCTION SUBPROGRAM YGF
COMMON /C1/USLLSL,NN,MM,NG,NY,NY1,TNLG,HALF, IR, IW
DOUBLE PRECISION BINCOE, G,Y, YGF2, EE,E2
IF(K.GT.N) GO TO 90
NLNG=N-NG
EE=(K+1)*DLOG(Y)
E2=(N-K)*DLOG(G)
IF(EE.LT.-170.DO) EE=-170.DO
IF(E2.LT.-170.DO) E2=-170.DO
IF(NG.EQ.O.OR.NLNG.LT.K) GO TO 10
YGF2=(BINCOE(N,K)-BINCOE(NLNG,K))*DEXP(EE)*DEXP(E2)
RETURN
C-------------------------------
NG=O OR (N-NG)<K ----------------
YGF2=BINCOE(N,K)*DEXP(EE)*DEXP(E2)
RETURN
C
80 WRITE (IW,91) K,N
91 FORMAT(///10('-'),'NLG ERROR: IN FUNCTION SUBPROGRAM YGF2, K=', I2, 'N=', I2, 'N', 2 OR 3; EXECUTION INTERRUPTED (LABEL 90))
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