NUMERICAL SIMULATION OF SALTWATER UPCONING
IN INLAND AQUIFERS

By

MD MOZZAMMEL HOQUE

Bachelor of Science
Bangladesh Agricultural University
Mymensingh, Bangladesh
1972

Master of Science
University of Southampton
Southampton, England
1979

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of the Oklahoma State University
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Thesis Approved:

Ardhesh K. Tyagi
Thesis Adviser

Richard N. DeVries

Douglas C. Kent

Don F Kincannon

Norman N Durham
Dean of the Graduate College

Dedicated to my Parents and Parents-in-law, and to all those whose contributions to the numerical methods made it possible today to solve complex Engineering problems.

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CHAPTER I

INTRODUCTION

Background Statement

Increasing use of aquifers as a source of water supply for agricultural, municipal, industrial, and recreational purposes is severely hampered in many regions of the world by the encroachment of unusable salt water. Although saltwater encroachment is frequent in coastal aquifers, it often presents a problem in inland aquifers as well, where the fresh water is underlain by a body of salt water. To some extent this situation is similar to the typical petroleum reservoir where the oil is underlain by water.

A sharp interface does not exist between the fresh and salt waters. This is because they are miscible and, therefore, the zone of contact between them occurs as a transition zone due to hydrodynamic dispersion and molecular diffusion (Bear, 1979). In field problems, however, this transition zone is relatively small compared to the thickness of the aquifer, and it is possible to consider the transition zone as a sharp interface.

The freshwater-saltwater interface is not static but dynamic because of the movement of freshwater and saltwater bodies with time. When the fresh water is withdrawn, the reduced head causes the interface to move upward. This upward movement of the interface below a pumping well can be significant and is known as local upconing. The upconing phenomenon, either local or regional, is very complex, and even numerical methods fail to give satisfactory solutions without significant approximation (Bear, 1979). Only in recent years has work concentrated significantly on local upconing.

Digital models based on numerical techniques are tools for managing large-scale complex groundwater systems. Parallel to the advancement of digital computer-technology, in many parts of the world efforts have been focused on deriving numerical solutions of partial differential equations that describe the flow of water in aquifer systems of various types and boundary conditions.

The simulation of groundwater reservoirs where fresh water is underlain by salt water forms an important and difficult problem in water resources. Appearing in the literature are several analytical solutions for local upconing in the case of steady flow in a vertical plane (Bear and Dagan, 1964; Yih, 1964; Schmorak and Mercado, 1969; Bear, 1972), and some numerical solutions for local upconing (Sahni, 1973; Chandler and McWhorter, 1975) and movement of saltwater fronts in coastal aquifers (Tyagi, 1971; Pinder and Page, 1976; Mercer et al., 1980). However, the regional simulation of the movement of freshwater-saltwater interface in an inland aquifer has drawn less attention. For economic and efficient management of aquifers, and to maximize freshwater development, the location of the transient freshwater-saltwater interface is very important. Thus, this study concentrates on predicting the regional interface and the approximate shape of the interface near the pumping wells in inland aquifers.

Study Objectives

This study simulates an inland aquifer system where the freshwater body is underlain by a saltwater body and where pumping wells exist, and evaluates the followings:

(a) The transient position of the piezometric level/water table of the fresh water.

(b) The transient position of saltwater potentials.

(c) The transient position of freshwater-saltwater interface.

A numerical model is developed using the block-centered finite difference method to approximate the governing partial differential equations in a two-dimensional horizontal physical plane. The line successive overrelaxation (LSOR) technique in conjunction with the bitridiagonal algorithm is used to solve the generated algebraic equations. The solution algorithm has been developed both in the x- and the y-directions so that the model can be applied in either direction to meet the requirements of the field conditions. The model has options to simulate confined/unconfined, homogeneous/heterogeneous, and isotropic/anisotropic conditions in a horizontal plane. The model has options to simulate the discharge or recharge in both saltwater and freshwater regions in the field situation. The model also has an option to terminate execution in the time step in which the interface moves to the critical position from where a sudden rise of interface occurs.

The performance of the numerical model developed in this study is compared with an analytical solution, taking the ratio of the horizontal to vertical permeability as unity for various depths of penetration of the well (Schmorak and Mercado, 1969). The model is applied to the

Yukon municipal well field in the Garber-Wellington aquifer of Oklahoma. The Garber-Wellington aquifer is considered confined, heteregeneous, and isotropic. But in the field situation, because of the presence of intermittent shale, the ratio of the horizontal to the vertical permeability is not unity. This can be handled by the present model with further modifications and calibrating the model for the aquifer system to be simulated.

CHAPTER II

REVIEW OF LITERATURE

Analytical Methods

The approximate solution to the problem of upconing beneath pumping wells was first derived by Muskat and Wyckoff (1935). They analyzed an idealized problem of brine upconing that resulted from pumping in the overlying oil zone in an homogeneous saturated formation; the water was considered to be in a static equilibrium condition. They obtained a steady-state distribution of hydraulic head for flow toward a well that partially penetrated a confined aquifer. They presented the basic physical principles of water upconing for individual wells and then concluded that the water-oil interface becomes unstable when it has reached a point about 1/2 to 3/4 of the height from the bottom of the well. At the upper portion of the region of stability the cone is observed to be very sensitive to small changes in pressure difference. Thus, for a given average pressure differential a steadily flowing well induces a lower cone height than one in which the flow is intermittent.

Meyer and Garder (1954) reported their investigation on the mechanics of two immiscible fluids in porous media. The flow of two and three fluids is considered in which the various fluids are separated into different zones by gravity. They considered three cases,--the production of oil or gas from a reservoir that has an underlying water table; the production of oil from a reservoir that has an underlying gas sands; and production from a reservoir with both an underlying water stratum and an overlying gas cap. For a fluid, a flow equation was derived and solved to get that fluid's maximum rate of flow into a well from a radially symmetrical porous medium without producing the other fluids present in the formation.

To compute a sharp freshwater-saltwater interface in a coastal aquifer, Henry (1959) developed analytical equations that are mathematically identical to the solution of a seepage problem developed by Kozeny (Harr, 1962). The hodograph theory was employed to locate the interface.

Bear and Dagan (1964) presented analytical solutions and nomographs of sharp interfaces using the hodograph method. They extended the work of Henry (1959) on intercepting the freshwater flow in a coastal collector in the form of a continuous drain operating above the interface. They considered four cases,--interface in a confined aquifer, upconing toward a sink in an unbounded aquifer, upconing toward a sink in a confined aquifer of infinite thickness, and a drain above the interface in an aquifer of infinite depth. They concluded that in the first and the third cases the approximate solution based on Dupuit assumptions is sufficiently accurate.

Wang (1965) reported an approximate theoretical analysis for a partially penetrating well used for skimming fresh water overlying salt water, and specified the interrelation between well spacings, well depth; rate of pumping, thickness of the aquifer, and freshwater and saltwater densities. He assumed that the rising saltwater mound beneath the pumping well does not affect the discharge. His theory is similar to that of Muskat and Wyckoff (1935), but it makes no use of a detailed distribution of hydraulic head for locating the position of the interface. Wang used the formula presented by Muskat (1937), which relates the well discharge to drawdown and incorporates the Ghyben-Herzberg relation (Bear, 1979). Wang neglected the vertical component of flow and did not predict the instability phenomenon in upconing.

Using the method of small perturbations, Dagan and Bear (1967) solved the problems of determining the shape and position of a rising interface caused by the withdrawal of fresh water from shallow wells that operate a short distance above the freshwater-saltwater interface in a coastal aquifer. They developed the analytical equation required for solving the problem of the moving interface. Because of the nonlinear nature of the boundary condition along the interface, a linearized approximate solution based on the method of small perturbations was presented. They considered both the two- and three-dimensional cases and the validity of the analytical solution was verified by experiments in a sand-box model. The solution was valid up to a deviation at which the crest of upconing interface reaches the value of one-third the critical distance between the interface and the well bottom.

Schmorak and Mercado (1969) compared the results of a field investigation with the analytical solution given by Dagan and Bear (1967). They developed the design criteria for skimming fresh water above the saline water. The theoretical formulas developed by Dagan and Bear (1967) and Muskat (1935) agree well with the field experiment result of Schmorak and Mercado (1969) up to a critical rise of the interface that seems to be approximately half the distance between the bottom of the well and the undisturbed interface. They also used linear approximations in computing the transition zone. They have developed the design criteria based both on theoretical formulas and field investigations in the form of nomographs which were constructed on the assumption that the mixing mechanism for other geometries is similar to the observed one.

Collins and Gelhar (1971) investigated water intrusion in layered aquifers by formulating a steady-state Dupuit model to predict the Piezometric head variation landward and above the intruding sea water. They assumed that the flow zones are homogeneous and that the flow obeys Darcy's law. The Dupuit assumption they employed in this study is that the pressure is hydrostatically distributed in a vertical direction in the flow zone. The mathematical model was compared to experimental results of a Hele-Shaw model. The analysis reasonably predicts the interface shape between the fresh water and salt water, but the location of the interface toe is inadequately described.

Tyagi (1971) applied analytical solutions for recharge and overdraft conditions in coastal aquifers. Stark (1972) reported his investigation on the shape and position of the interface in a coastal aquifer, where the fresh water flows from the land toward the sea. He solved this problem by conformal mapping and the hodograph theory. He investigated the upconing under a drain and performed a test in a parallel-plate model to verify the formulas he derived for upconing.

Numerical Methods

Pinder and Cooper (1970) developed a numerical technique for determining the transient position of the freshwater-saltwater interface and the pattern of flow under the effects of dispersion involving irregular boundaries and nonuniform permeabilities. They solved the flow equation for velocity and pressure by the iterative alternating direction implicit procedure (ADIP) and the solute transport equation by the method of characteristics. They assumed that the release of water from storage has a negligible effect on the movement of the interface, porosity, and dynamic viscosity. Porosity, dynamic viscosity, and dispersion coefficients were assumed constant in time and space. The finite difference results were compared with the analytical solution.

Letkeman and Ridings (1970) discussed different numerical techniques for the upconing in a water-oil phase. They considered for simplicity an oil-water system and discussed the successive overrelaxation (SOR) method, the alternating direction implicit procedure (ADIP), and the strongly implicit procedure (SIP) numerical techniques to solve upconing problems. The ADIP was found to solve a larger class of upconing problems than the SOR. Furthermore, the SIP would converge better than the ADIP for many problems.

Spivak and Coats (1970) simulated a multiphase, multidimensional mathematical model to predict two- or three-phase coning behavior. They concluded that instabilities in the numerical simulation of oil coning result from explicit handling of saturation dependent quantities in the finite difference equations. They showed that the use of implicit production terms alone in finite difference equations for coning simulation can result in a five-fold increase in the permissible time step for a stable solution with no increase in computing time per time step. They discussed gas and water coning in an oil-well and the numerical method that gives a stable solution. They demonstrated two examples--one for gas coning and the other for water upconing--using an implicit finite difference method.

McDonald and Coats (1970) developed three models of gas and water upconing in an oil well. The first method employs the implicit pressure-explicit saturation analysis with the production terms treated implicitly. The second model is similar to the first model except that the interblock transmissivities are also treated implicitly in the saturation equation. The third model is fully implicit with respect to all variables. They compared these techniques with respect to computing, rate of convergence, and the length of time step. These techniques were found stable in saturation production behavior during formation and after breakthrough.

Shamir and Dagan (1971) solved three one-dimensional partial differential equations numerically. These equations describe the motion of a shallow interface and the free surface in an unconfined coastal aquifer or the freshwater head in a confined case. These equations were based on Dupuit approximation and considered the geometry of the vertical section through the aquifer. They used an implicit finite difference scheme to approximate the equations. It employs a grid with one spacing over the intrusion length and a different spacing in the remaining flow field. The numerical results are in good agreement with an exact solution of a simple case and in fair agreement with the experimental results of a Hele-Shaw model.

Fetter (1972) developed an equation describing the two-dimensional steady-state position of an interface in an unconfined aquifer beneath an oceanic island. The equation can be solved analytically for simple boundary problems and numerically for an oceanic island of any shape. The multilayered aquifers can be treated by assuming an average conductivity of all aquifer systems. This model has been successfully used to generate the known position of a saltwater interface beneath the South

Fork of Long Island, New York.

Streltsova and Kashef (1974) studied the critical state of freshwater-saltwater interface upconing beneath an artesian discharge well. The initial potentiometric surface was horizontal. They discussed the transition stage from pumping fresh water to pumping salt water at the critical condition and an expansion of the transition zone caused by well discharge for coastal aquifers. The shape of upconing boundaries was determined by an electric resistance network model or by a finite difference method. Their findings in the steady-state case were applied to transient case and used by analogy to find the upconing curve at the critical condition.

Kashef and Smith (1975) extended the work of Streltsova and Kashef (1974) to evaluate the effect of well discharge in the expansion of the transition zone for various conditions of aquifer properties, pump capacities, natural flows, time effects, and well locations. The initially deformed potentiometric surface resulting from saltwater intrusion was considered. The effect of various rates of pumping and various durations were studied for certain conditions. The boundaries between the totally freshwater zone and the intruded zone were determined for the selected cases. They concluded that the technique developed may be applied to recharge wells.

Using the finite element method and the hydraulic approach method, Kashef and Safar (1975) made a comparative study of freshwater-saltwater interface. The freshwater-saltwater interface in artesian aquifers has been investigated; however, the hydraulic forces technique has not been checked thoroughly because of the lack of a wide range of coverage by the exact solutions. Both methods, however, proved to be in close agreement. Moreover, the hydraulic heads along the upper boundary of the artesian aquifer were found to be in close agreement with Dupuit assumptions.

Segol et al. (1975) applied the finite element technique for calculating the transient position of the saltwater front in a coastal aquifer. They coupled the equations of groundwater flow and mass transport. They indicated that their solution can be readily applied for layered nonhomogeneous media.

Tyagi (1975) used the finite element method to predict the transition zone between fresh and salt waters in coastal aquifers. Analytical solutions were employed to locate the sharp interface at different times. Then, the dispersion effect was imposed on the interface to determine the transition zone.

Investigating the discharge of fresh water to the sea, Gupta (1976) obtained a finite element solution for the shape and location of two unknown boundaries--the free surface and the interface. He assumed that there is a steady, two-dimensional flow in a homogeneous and isotropic water table aquifer, with the static salt water underlying the fresh water. He studied the upconing of the interface in the presence of an infiltration gallery system.

Pinder and Page (1976) employed the Galerkin finite element technique to solve the saltwater intrusion in a coastal aquifer. They solved a pair of partial differential equations in vertical dimensions. They considered a sharp interface between seawater and freshwater regions. The model worked satisfactorily to simulate the movement of the saltwater front in a coastal aquifer in North Haven, Long Island, New York.

Rubin and Pinder (1977) presented the effect of salinity dispersion on the dynamics of flow as well as the salinity distribution in coastal aquifers. They considered pumpage, both from an infinite strip of wells and from a single well, with the assumption that a sharp interface between fresh water and salt water initially exists within a finite distance from the pumpage location.

Mercer et al. (1980) presented the solution of a pair of partial differential equations that describe the motion of a saltwater and freshwater front in a coastal aquifer. The areal equations are based on Dupuit approximations and are obtained from partial integration over the vertical dimension. They concluded that the most efficient matrix solution scheme is the block successive overrelaxation method. The model can handle the time dependent problems and can treat water table or confined conditions having a steady leakage of fresh water.

Panigrahi (1980) studied the upconing phenomenon and, using the finite difference approximation, solved a two-dimensional groundwater flow equation in a horizontal plane. He used the iterative alternating direction implicit procedure (ADIP) to solve the system of algebraic equations. His model, which is similar to the U.S.G.S. groundwater flow model (Prickett and Lonnquist, 1971), calculates the transient position of freshwater potentials in a confined aquifer system and, using these freshwater potentials in the Ghyben-Herzberg formula, calculates the upconing. This is an approximate solution because the saltwater potential is considered static.

Bear and Kapular (1981), using an implicit finite difference technique, derived a solution for the movement of the freshwater-saltwater interface in a multilayered coastal aquifer. The aquifers are divided into a number of subaquifers by impervious and relatively thin layers. Because the freshwater discharge to the sea varies, seawater intrusion occurs as a separate interface in each subaquifer. The flow is normal to the coastline and essentially horizontal in both the freshwater and saltwater zones. They formulated three mathematical equations, similar to those of Shamir and Dagan (1971), to represent the complete physical systems and used three types of cells, each a different size. The equations used by Bear and Kapular are nonlinear; however, they solved these equations after linearization in the same way Shamir and Dagan (1971) did in their work. To solve the resulting system of algebraic equations they adopted the iterative alternating direction implicit procedure (ADIP). In their conclusions they mentioned that although they have given the solution for a one-dimensional and two-layer case, their work can be extended to a two-dimensional horizontal flow in coastal aquifers having more layers.

Wilson and Costa (1982), using the Galerkin finite element technique, studied the movement of an interface. They solved the problem in one dimension and this solution includes the involvement of one or two moving boundaries and the toe points in which layers become absent. The method avoids the use of a moving grid or grid generation in favor of an indirect, fixed grid toe tracking algorithm. This algorithm makes special use of the spatial integrations of the finite element method and, across these elements, employs a nonlinear variation of layer saturation that contains a moving boundary. The linear elements perform as accurately as the quadratic elements. The time step and grid spacing need to be selected carefully, however, so that the toe does not move too rapidly through an individual element.

Kishi et al. (1982) reported their investigations of the freshwater-saltwater interface in a coastal aquifer; they gave a numerical solution to the Poisson equation in a confined, isotropic and homogeneous condition and applied the model to the estuary of the Naka River in the Tokushima Prefecture, Japan. They concluded that the saline region advances rapidly when the rate of seawater intrusion approaches the rate of pumping.

CHAPTER III

MATHEMATICAL STATEMENT

The development of a mathematical model begins with a conceptual understanding of the physical system. Once these concepts are formulated they can be transferred into a mathematical framework resulting in an equation or a system of equations that describes the physical process.

The equations that describe the freshwater and saltwater potentials and the transient condition of the freshwater-saltwater interface in an inland aquifer system are developed from the basic equations that describe the flow of a fluid through a porous medium. There are three major phases of development of the required equations.

In the first phase, two partial differential equations developed from the principle of momentum and mass balance to represent the hydraulics of groundwater flow in the freshwater subdomain and the saltwater subdomain are described. For details of the development of these equations reader is referred to texts by Bear (1972, 1979).

In the second phase, two-dimension areal flow equations are derived by vertical integration of the flow equations described in phase one.

In the third phase, the final equations are developed; for an inland aquifer, they describe the regional distribution of freshwater and saltwater potentials and the interface elevation evaluating the surface conditions of the dynamic interface.

Governing Flow Equations

The present problem involves two liquids separated by an abrupt interface in such a way that each liquid occupies a separate part of the entire flow domain. Figure 1 represents a flow domain (an inland aquifer system) where the freshwater subdomain, Ω_f, is underlain by the saltwater subdomain, Ω_s. The transient three-dimensional equation that describes the hydraulics of flow in the freshwater subdomain may be written as

$$\frac{\partial}{\partial x} (K_{fx} \frac{\partial \phi_f}{\partial x}) + \frac{\partial}{\partial y} (K_{fy} \frac{\partial \phi_f}{\partial y}) + \frac{\partial}{\partial z} (K_{fz} \frac{\partial \phi_f}{\partial z}) = S_f \frac{\partial \phi_f}{\partial t} \qquad (3.1)$$

where

ϕ_f = Z + P/γ_f (assuming water is incompressible), L;

Z = elevation head, L;

p = hydrostatic pressure, FL^{-2};

γ_f = density of fresh water, ML^{-3};

K_{fx}, K_{fy}, K_{fz} = coefficients of permeability in the freshwater region in the directions of principal axes x, y, and z respectively, LT^{-1}; and

S_f = specific storage of the freshwater region, L^{-1}.

A similar equation can be written to describe the hydraulics of flow in the saltwater subdomain by replacing the freshwater subscript f in Equation (3.1) by the saltwater subscript s as follows:

$$\frac{\partial}{\partial x} (K_{sx} \frac{\partial \phi_s}{\partial x}) + \frac{\partial}{\partial y} (K_{sy} \frac{\partial \phi_s}{\partial y}) + \frac{\partial}{\partial z} (K_{sz} \frac{\partial \phi_s}{\partial z}) = S_s \frac{\partial \phi_s}{\partial t} \qquad (3.2)$$

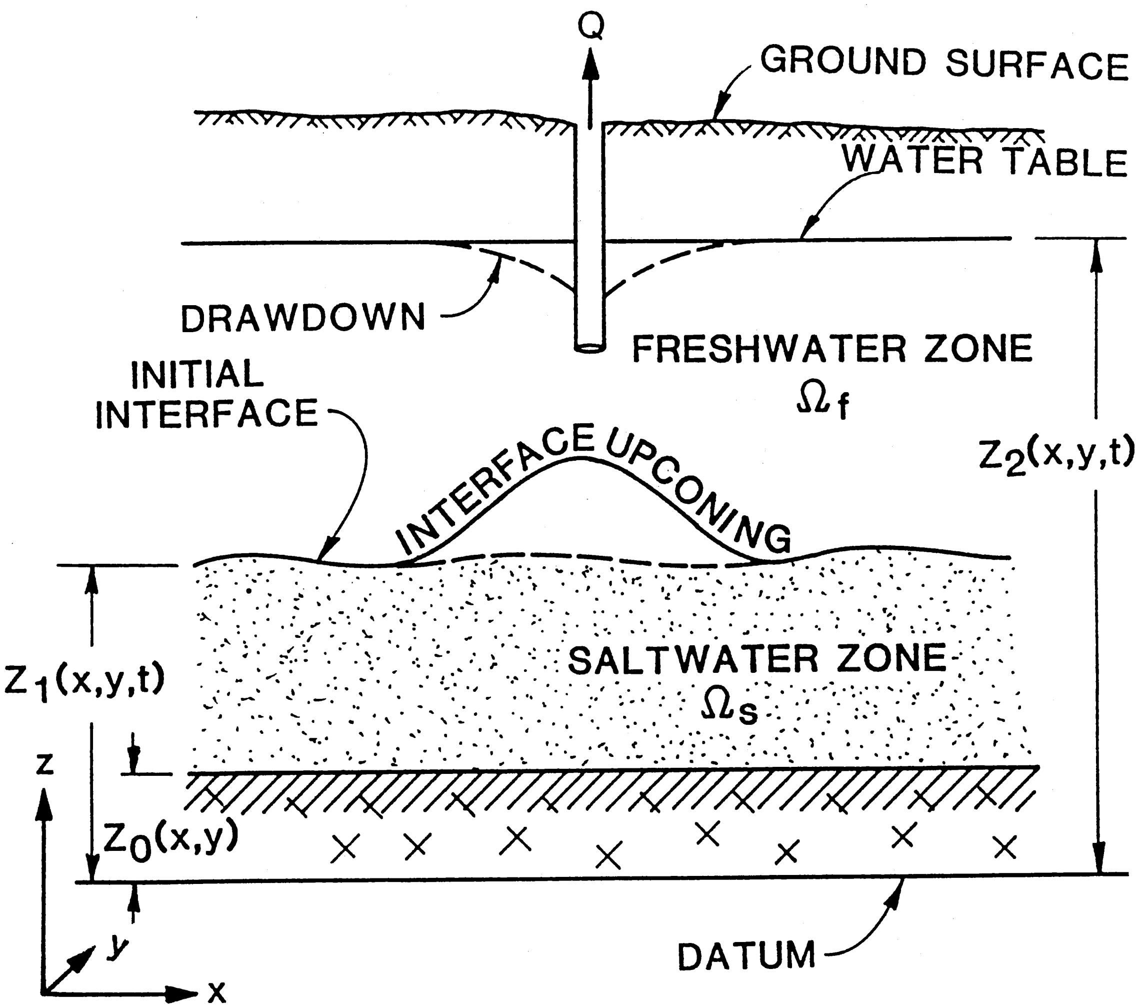

Figure 1. An Aquifer System With the Fresh Water is Underlain by Salt Water

Equations (3.1) and (3.2) must be continuous and have a continuous first derivative everywhere in the respective flow subdomain. Sources or sinks of liquid (pumping and artificial recharge) may exist in both regions. Adding a source or sink term and introducing the vector operator ∇, Equations (3.1) and (3.2) can be written in compact form respectively as

$$\nabla(\vec{K}_f \nabla \phi_f) + q_f = S_f \frac{\partial \phi_f}{\partial t} \tag{3.3}$$

and

$$\nabla(\vec{K}_s \nabla \phi_s) + q_s = S_s \frac{\partial \phi_s}{\partial t} \tag{3.4}$$

where

q_f, q_s = the source/sink term (positive for sink and negative for source), T^{-1}; and

$$\nabla = \frac{\partial}{\partial x} \bar{i} + \frac{\partial}{\partial y} \bar{j} + \frac{\partial}{\partial z} \bar{k}$$

$\bar{i}, \bar{j}, \bar{k}$ = the unit vectors in the x, y and z directions, respectively.

Two-Dimensional Areal Flow Equations

The shape and position of a transient interface in an inland aquifer can be derived by solving two equations in a three-dimensional space. Employing a hydraulic approach and averaging three-dimensional equations for each region in the vertical plane, two-dimensional areal flow equations can be developed. To derive two-dimensional equations, Equations

(3.3) and (3.4) are integrated in the vertical plane. In Figure 1, the freshwater subdomain, Ω_f, is bounded by an interface at $Z_1(x,y,t)$ and by the phreatic surface for unconfined aquifer or by the piezometric surface for confined aquifer at $Z_2(x,y,t)$. The saltwater subdomain, Ω_s, is bounded by the interface at $Z_1(x,y,t)$ and by an impervious bottom at $Z_0(x,y)$. Integration of Equation (3.3) from depth $Z_1(x,y,t)$ to $Z_2(x,y,t)$ gives

$$\int_{Z_1}^{Z_2} [\nabla(\vec{K}_f \cdot \nabla \cdot \phi_f) + q_f - S_f \frac{\partial \phi_f}{\partial t}] \, dz = 0 \qquad (3.5)$$

Similarly, integration of Equation (3.4) from depth $Z_0(x,y)$ to $Z_1(x,y,t)$ results in

$$\int_{Z_0}^{Z_1} [\nabla(\vec{K}_s \cdot \nabla \cdot \phi_s) + q_s - S_s \frac{\partial \phi_s}{\partial t}] \, dz = 0 \qquad (3.6)$$

The following assumptions are made for evaluating the integrals of Equations (3.5) and (3.6):

1. For a small inclination of the line of seepage, equipotential lines approach the vertical and the hydraulic gradient is equal to the slope of the free surface and is invarient with depth (i.e., the Dupuit approximations are valid).

2. The coefficient of permeability is colinear with the coordinate axis.

3. The coefficient of permeability is invarient with the depth.

Evaluating the integrals by the Leibnitz rule (Korn and Korn, 1968), the vertically integrated equations for saltwater and freshwater regions are developed. These equations for freshwater and saltwater regions are respectively

$$\frac{\partial}{\partial x}\{b_f K_{fx}(\frac{\partial \phi_f}{\partial x})_{aver}\} + \frac{\partial}{\partial y}\{b_f K_{fy}(\frac{\partial \phi_f}{\partial y})_{aver}\}$$

$$+ b_f \tilde{q}_f - S_f b_f \frac{\partial \tilde{\phi}_f}{\partial t} - q_{fx}\Big|_{Z_1} \frac{\partial z}{\partial x} + q_{fx}\Big|_{Z_2} \frac{\partial z}{\partial x}$$

$$-q_{fy}\Big|_{Z_1} \frac{\partial z}{\partial y} + q_{fy}\Big|_{Z_2} \frac{\partial z}{\partial y} + q_{fz}\Big|_{Z_1} - q_{fz}\Big|_{Z_2} = 0 \qquad (3.7)$$

and

$$\frac{\partial}{\partial x}\{b_s K_{sx}(\frac{\partial \phi_s}{\partial x})_{aver}\} + \frac{\partial}{\partial y}\{b_s K_{sy}(\frac{\partial \phi_s}{\partial y})_{aver}\} + b_s \tilde{q}_s$$

$$-S_s b_s \frac{\partial \tilde{\phi}_s}{\partial t} - q_{sx}\Big|_{Z_0} \frac{\partial z}{\partial x} + q_{sx}\Big|_{Z_1} \frac{\partial z}{\partial x} - q_{sy}\Big|_{Z_0} \frac{\partial z}{\partial y} + q_{sy}\Big|_{Z_1} \frac{\partial z}{\partial y}$$

$$+ q_{sz}\Big|_{Z_0} - q_{sz}\Big|_{Z_1} = 0 \qquad (3.8)$$

where

b_f = thickness of freshwater subdomain, L;

b_s = thickness of saltwater subdomain, L;

q_{fx}, q_{fy}, q_{fz} = discharge velocity in x, y and z directions in the freshwater subdomain, LT^{-1}; and

q_{sx}, q_{sy}, q_{sz} = discharge velocity in x, y and z directions in the saltwater subdomain LT^{-1}.

For an impermeable base (Hantush, 1964)

$$q_{sx}\Big|_{Z_0} \frac{\partial z}{\partial x} + q_{sy}\Big|_{Z_0} \frac{\partial z}{\partial y} - q_{sz}\Big|_{Z_0} = 0 \qquad (3.9)$$

The base in Figure 1 is considered impermeable, so the terms in Equation (3.9) can be eliminated from Equation (3.8). So, Equation (3.8) can be written as

$$\frac{\partial}{\partial x}\{b_s K_{sx}(\frac{\partial \phi_s}{\partial x})_{aver}\} + \frac{\partial}{\partial y}\{b_s K_{sy}(\frac{\partial \phi_s}{\partial y})_{aver} + b_s \tilde{q}_s$$

$$- S_s b_s \frac{\partial \tilde{\phi}_s}{\partial t} + q_{sx}\Big|_{Z_1} \frac{\partial z}{\partial x} + q_{sy}\Big|_{Z_1} \frac{\partial z}{\partial y} - q_{sz}\Big|_{Z_1} = 0 \qquad (3.10)$$

The terms evaluated at $Z_1(x,y,t)$ and $Z_2(x,y,t)$ can be replaced by evaluating the conditions of the material surface of the interface on the piezometric surface or water table.

Evaluation of Surface Conditions

The relationship necessary to evaluate the location and shape of an interface and the movement of the water table or piezometric surface in an aquifer at time t may be mathematically represented by (Muskat, 1937; Bear, 1979).

$$F(x,y,z,t) = 0 \qquad (3.11)$$

After a time period Δt, a fluid particle at a point (x,y,z) will move to a new position $[(x + \Delta x), (y + \Delta y), (z + \Delta z)]$. If V_x, V_y and V_z are the Darcian velocity components of the particle at (x,y,z) on the interface, the new position of the particle is $[(x + V_x\Delta t), (y + V_y\Delta t), (z + V_z\Delta t)]$.

The new interface position is given by

$$F(x + V_x\Delta t,\ y + V_y\Delta t,\ z + V_z\Delta t) - F(x,y,z,t) = 0 \qquad (3.12)$$

The boundary conditions on the interface (F) are

1. Same specific discharge on both sides;

$$(q_n)_f = (q_n)_s$$

2. Same pressure on both sides;

$$\gamma_f(\phi_f - z_1) = \gamma_s(\phi_s - z_1)$$

The interface is a material surface with fluid particles remaining always on it.

So in the freshwater zone,

$$\frac{dF}{dt} \equiv \frac{\partial F}{\partial t} + \vec{V}_f \nabla F = 0 \qquad (3.13)$$

and in the saltwater zone,

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \vec{V}_s \nabla F = 0 \qquad (3.14)$$

where

$\vec{V}_f$ = Darcian velocity vector in the freshwater subdomain, LT^{-1};

$\vec{V}_s$ = Darcian velocity vector in the saltwater subdomain, LT^{-1};

and

$\frac{d}{dt}$ = Material derivative following motion.

Either of the Equations (3.13) and (3.14) is the Kelvin equation

(Muskat, 1937) governing the motion of a liquid on a surface containing a given set of particles. Denoting the elevation of points on the interface by $Z = Z_1(x,y,t)$, the relationship for F becomes

$$F \equiv Z - Z_1(x,y,t) = 0 \qquad (3.15)$$

Substituting Equation (3.15) into Equation (3.13) and rearranging the terms, yields

$$\frac{\partial Z_1}{\partial t} = \left(V_{fz} - V_{fx}\frac{\partial Z_1}{\partial x} - V_{fy}\frac{\partial Z_1}{\partial y}\right)\Bigg|_{Z_1} \qquad (3.16)$$

Multiplying Equation (3.16) by the porosity n of the aquifer, the equation obtained in terms of discharge velocity, is

$$n\frac{\partial Z_1}{\partial t} = q_{fz}\Big|_{Z_1} - q_{fx}\Big|_{Z_1}\frac{\partial Z_1}{\partial x} - q_{fy}\Big|_{Z_1}\frac{\partial Z_1}{\partial y} \qquad (3.17)$$

The top surface of the aquifer defined as $\phi(x,y,z,t)$, from Figure 1, $\phi = Z_2(x,y)$ for a confined aquifer and $\phi = \phi_f(x,y,z,t)$ for an unconfined aquifer. The surface is then

$$F \equiv Z - \phi_f(x,y,z,t) \qquad (3.18)$$

With the material derivative,

$$\frac{DF}{Dt} = \frac{DZ}{Dt} - \frac{D\phi}{Dt} = 0$$

or,

$$\frac{dz}{dt} = \frac{\partial\phi_f}{\partial t} + \frac{\partial\phi_f}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial\phi_f}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial\phi_f}{\partial t}\cdot\frac{dz}{dt}$$

or

$$V_{fz}(1 - \frac{\partial \phi_f}{\partial z}) = \frac{\partial \phi_f}{\partial t} + V_{fx} \frac{\partial \phi_f}{\partial x} + V_{fy} \frac{\partial \phi_f}{\partial y} \tag{3.19}$$

With the introduction of a new term α to distinguish between confined and unconfined aquifers and multiplication by porosity n, Equation (3.19) when rearranged gives,

$$nV_{fz}(1 - \frac{\partial \phi_f}{\partial z})\Big|_{\phi} - \alpha n \frac{\partial \phi_f}{\partial t}\Big|_{\phi} = q_{fx} \frac{\partial \phi_f}{\partial x}\Big|_{\phi} + q_{fy} \frac{\partial \phi_f}{\partial y}\Big|_{\phi} \tag{3.20}$$

Because Z_2 does not vary with time for a confined aquifer, $\alpha = 0$ and for an unconfined aquifer, $\alpha = 1$. With the Dupuit approximation, $\frac{\partial \phi_f}{\partial z}$ is zero in the case of a confined aquifer and is very small for an unconfined aquifer. Thus, Equation (3.20) becomes,

$$nV_{fz}\Big|_{\phi} - \alpha n \frac{\partial \phi_f}{\partial t}\Big|_{\phi} = q_{fx}\Big|_{\phi} \frac{\partial \phi_f}{\partial x} + q_{fy}\Big|_{\phi} \frac{\partial \phi_f}{\partial y} \tag{3.21}$$

In Equation (3.21) the term nV_{fz} represents the discharge velocity of a particle at the surface ϕ and composed of $q_{fz}\Big|_{\phi}$ and recharge or leakage if they exist in the system. For the freshwater region,

$$nV_{fz} = q_{fz}\Big|_{\phi} - \alpha R + (1 - \alpha) \frac{K'}{b'} (\tilde{\phi}_f - \phi') \tag{3.22}$$

where

R = the rate of recharge to the system, LT^{-1};

K' = the coefficient of permeability of the overlying confining bed, LT^{-1};

b' = the thickness of the confining bed, L; and

ϕ' = the head in adjoining aquifer, L.

Combination of Equations (3.21) and (3.22) and rearrangment of the terms gives

$$q_{fz}\Big|_{\phi} - q_{fx}\Big|_{\phi} \frac{\partial \phi_f}{\partial x} - q_{fy}\Big|_{\phi} \frac{\partial \phi_f}{\partial y} = \alpha R - (1-\alpha) \frac{K'}{b'} (\tilde{\phi}_f - \phi') + \alpha n \frac{\partial \tilde{\phi}_f}{\partial t} \tag{3.23}$$

where

$\tilde{\phi}_f \simeq \phi_f\Big|_{\phi}$ using Dupuit approximation.

Combination of Equations (3.7), (3.17) and (3.23) gives,

$$\frac{\partial}{\partial x} \{b_f K_{fx} (\frac{\partial \phi_f}{\partial x})_{aver}\} + \frac{\partial}{\partial y} \{b_f K_{fy} (\frac{\partial \phi_f}{\partial y})\}_{aver} + b_f \tilde{q}_f - S_f b_f \frac{\partial \tilde{\phi}_f}{\partial t}$$

$$+ n \frac{\partial Z_1}{\partial t} - \alpha R + (1-\alpha) K'/b' \; (\tilde{\phi}_f - \phi') - \alpha n \frac{\partial \tilde{\phi}_f}{\partial t} = 0 \tag{3.24}$$

Equation (3.17) for salt water combined with Equation (3.8), results in the following equation:

$$\frac{\partial}{\partial x} \{b_s K_{sx} (\frac{\partial \phi_s}{\partial x})_{aver}\} + \frac{\partial}{\partial y} \{b_s K_{xy} (\frac{\partial \phi_s}{\partial y})_{aver}\} + b_s \tilde{q}_s$$

$$- S_s b_s \frac{\partial \tilde{\phi}_s}{\partial t} - n \frac{\partial Z_1}{\partial t} = 0 \tag{3.25}$$

The interface time derivative $\frac{\partial Z_1}{\partial t}$ from Equations (3.24) and (3.25) is evaluated to obtain the final form of the equations. This can be done because of the principle that the same pressure on both sides of the interface defines the boundary conditions on the interface. This principle and the assumption that the head is uniform with depth yield the interface equation

$$Z_1(x,y,t) = \tilde{\phi}_s(1 + \sigma) - \tilde{\phi}_f\sigma \tag{3.26}$$

where

$\sigma = \gamma_f/(\gamma_s-\gamma_f)$

Once the distributions of $\tilde{\phi}_f = \phi_f(x,y,z,t)$ and $\tilde{\phi}_s = \phi_s(x,y,z,t)$ are known, the equation for F(x,y,z,t) is obtained as

$$F \equiv Z - \tilde{\phi}_s(1+\sigma) + \tilde{\phi}_f\sigma = 0 \tag{3.27}$$

The time derivative of Equation (3.26) is

$$\frac{\partial Z_1}{\partial t} = (1+\sigma)\frac{\partial \tilde{\phi}_s}{\partial t} - \sigma\frac{\partial \tilde{\phi}_f}{\partial t} \tag{3.28}$$

Equation (3.28) shows that the rate of change of the interface elevation is proportional to the rate of change in the head. Replacing the time derivative of Equation (3.24) with the right side of Equation (3.28) and neglecting recharge and leakage terms, the equation for the freshwater subdomain, Ω_f, is

$$\frac{\partial}{\partial x}\{b_f K_{fx}(\frac{\partial \phi_f}{\partial x})_{aver}\} + \frac{\partial}{\partial y}\{b_f K_{fy}(\frac{\partial \phi_f}{\partial y})_{aver}\} + W_f\sigma'\ (x-x_i,\ y-y_i)$$

$$- (S_f b_f + n\sigma + \alpha n)\frac{\partial \tilde{\phi}_f}{\partial t} + n(1+\sigma)\frac{\partial \tilde{\phi}_s}{\partial t} = 0 \tag{3.29}$$

Similarly, the equation for the saltwater subdomain, Ω_s, is

$$\frac{\partial}{\partial x} \{b_s K_{sx} (\frac{\partial \phi_s}{\partial x})_{aver}\} + \frac{\partial}{\partial y} \{b_s K_{sy} (\frac{\partial \phi_s}{\partial y})_{aver}\} + W_s \sigma'(x-x_i, y-y_i)$$

$$- (S_s b_s + n(1+\sigma)) \frac{\partial \tilde{\phi}_s}{\partial t} + n\sigma \frac{\partial \phi_f}{\partial t}) = 0 \qquad (3.30)$$

where

$W_f = \tilde{q}_f b_f$ = the volume flux per unit area (positive sign for inflow and negative sign for outflow) in the freshwater zone, LT^{-1};

$W_s = \tilde{q}_s b_s$ = the volume flux per unit area (positive sign for inflow and negative sign for outflow) in the saltwater zone, LT^{-1}; and

σ' = Dirac-delta Function

Equations (3.29) and (3.30) are the vertically integrated two-dimensional areal partial differential equations developed for describing regional distributions of freshwater and saltwater potentials in an inland aquifer system, where fresh water is underlain by salt water. These equations are to be solved for the freshwater potential $\phi_f(x,y,t)$ and the saltwater potential $\phi_s(x,y,t)$. The interface elevation can then be determined with Equation (3.26).

Initial and Boundary Conditions

In order to obtain a unique solution of a partial differential equation or a system of partial differential equations corresponding to a given physical system, additional information about the physical state of the process is required. This information is described by initial and boundary conditions. For steady-state problems only boundary conditions are required, but for unsteady-state problems both initial and boundary conditions are required. Mathematically the boundary conditions include the geometry of the boundary and the values of the dependent variables or the derivatives normal to the boundary. An initial condition of unsteady-state represents the variables in the system at the beginning of simulation.

The initial condition in aquifer simulation is the head distributions over the region at time equal to zero. Here, the initial freshwater potential is specified as $\phi_f = \phi_f(x,y,o) \equiv Z_2(x,y,o)$, and the initial saltwater potential as $\phi_s = \phi_s(x,y,o)$. The initial freshwater-saltwater interface is $Z_1 = Z_1(x,y,o)$. The initial freshwater potential and freshwater-saltwater interface are obtained from field observations. The initial saltwater potential $\phi_s(x,y,o)$ is then obtained from Equation (3.31)

$$\phi_s = \left(\frac{\sigma}{1+\sigma}\right)\phi_f + \left(\frac{1}{1+\sigma}\right)Z_1 \qquad (3.31)$$

The boundary conditions in aquifer simulation refer to the head distributions to the boundary nodes of the domain of interest at all times of the simulation. An aquifer system is normally larger than the project area. Nevertheless, the physical boundary of the problems is included in the model if it is feasible. Where it is impractical to include physical boundaries, the finite difference grid can be expanded and the boundaries located far enough from the project area, so that they will have a negligible effect on the area of interest at the simulation period. Generally, there are two types of boundaries (Trescott et al., 1975) in groundwater flow simulation--constant head boundaries and constant flux boundaries. Because Equations (3.29) and (3.30) are formulated in terms of potentials ϕ_f and ϕ_s, the use of constant head boundaries will be facilitated.

CHAPTER IV

NUMERICAL FORMULATION

Computer based numerical techniques are the major tools for solving complex groundwater problems that are described in the form of partial differential equations. The finite difference and the finite element methods are the available powerful numerical techniques that can be used to solve these equations. Because the finite difference method requires relatively fewer operations and consequently less computational time and storage, it is used in this study.

Finite Difference Approximations

One approach to the solution of Equations (3.29) and (3.30) by the finite difference approximations for an aquifer with irregular boundaries, is to subdivide the domain of interest into rectangular grids, where the aquifer properties are assumed to be uniform. Figure 2 shows a block-centered finite difference grid. In the finite difference formulation i, j and k are used as the indices of x-direction, y-direction and time step, respectively.

In the following steps the derivatives of Equations (3.29) and (3.30) are replaced one by one and the resulting finite difference equations are assembled to obtain the final form of the finite difference equation at each node for the governing partial differential equations.

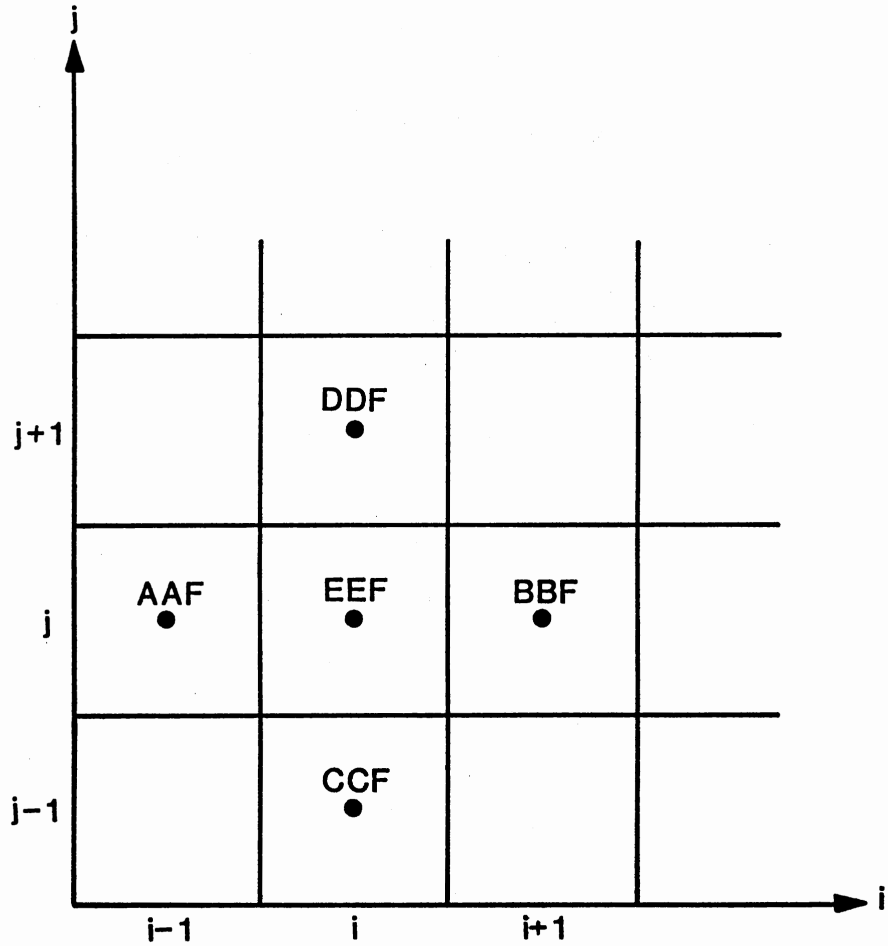

Figure 2. Index Scheme for Finite Difference Grid and Coefficients of Finite Difference Equation Written for Node (i,j)

$$\frac{\partial \phi_f}{\partial t} \equiv \left[\frac{\phi_{f(i,j,k)} - \phi_{f(i,j,k-1)}}{\Delta t}\right] \qquad (4.1.a)$$

Equation (4.1.a) is the backward difference approximation to the time derivative of the freshwater potential. The time derivative of the saltwater potential is replaced as

$$\frac{\partial \phi_s}{\partial t} = \left[\frac{\phi_{s(i,j,k)} - \phi_{s(i,j,k-1)}}{\Delta t}\right] \qquad (4.1.b)$$

The spatial derivative for the freshwater potential is approximated as

In the x-direction,

$$\frac{\partial}{\partial x}\left(b_f K_{fx} \frac{\partial \phi_f}{\partial x}\right) = \frac{1}{\Delta x_i}\left[\left(b_f K_{fx} \frac{\partial \phi_f}{\partial x}\right)_{i+\frac{1}{2},j,k} - \left(b_f K_{fx} \frac{\partial \phi_f}{\partial x}\right)_{i-\frac{1}{2},j,k}\right]$$

$$= \frac{1}{\Delta x_i}\left[(b_f K_{fx})_{i+\frac{1}{2},j,k}\left\{\frac{\phi_{f(i+1,j,k)} - \phi_{f(i,j,k)}}{\Delta x_{i+\frac{1}{2}}}\right\}\right.$$

$$\left. - (b_f K_{fx})_{i-\frac{1}{2},j,k}\left\{\frac{\phi_{f(i,j,k)} - \phi_{f(i-1,j,k)}}{\Delta x_{i-\frac{1}{2}}}\right\}\right] \qquad (4.1.c)$$

In the y-direction,

$$\frac{\partial}{\partial y}\left(b_f K_{fy} \frac{\partial \phi_f}{\partial y}\right) = \frac{1}{\Delta y_j}\left[(b_f K_{fy})_{i,j+\frac{1}{2},k}\left\{\frac{\phi_{f(i,j+1,k)} - \phi_{f(i,j,k)}}{\Delta y_{j+\frac{1}{2}}}\right\}\right.$$

$$\left. - (b_f K_{fy})_{i,j-\frac{1}{2},k}\left\{\frac{\phi_{f(i,j,k)} - \phi_{f(i,j-1,k)}}{\Delta y_{j-\frac{1}{2}}}\right\}\right] \qquad (4.1.d)$$

where

$(b_f K_{fx})_{i+\frac{1}{2},j,k}$ = the product of the depth and the hydraulic conductivity of the freshwater zone between the nodes (i,j) and (i+1,j), and time k, L^2T^{-1};

$(b_f K_{fx})_{i-\frac{1}{2},j,k}$ = the product of the depth and the hydraulic conductivity of the freshwater zone between the nodes (i,j) and (i-1,j), and time k, L^2T^{-1};

$(b_f K_{fy})_{i,j+\frac{1}{2},k}$ = the product of the depth and the hydraulic conductivity of the freshwater zone between the nodes (i,j) and (i,j+1), and time k, L^2T^{-1};

$(b_f K_{fy})_{i,j-\frac{1}{2},k}$ = the product of the depth and the hydraulic conductivity of the freshwater zone between the nodes (i,j) and (i,j-1), and time k, L^2T^{-1};

$\Delta x_{i+\frac{1}{2}}$ = the distance between the nodes (i,j) and (i+1,j), L;

$\Delta x_{i-\frac{1}{2}}$ = the distance between the nodes (i,j) and (i-1,j), L;

$\Delta y_{j+\frac{1}{2}}$ = the distance between the nodes (i,j) and (i,j+1), L;

$\Delta y_{j-\frac{1}{2}}$ = the distance between the nodes (i,j) and (i,j-1), L; and

Δt = the time increment, T.

The spatial derivative for the saltwater potential can be approximated in the same way as the freshwater potential and may be written as In the x-direction,

$$\frac{\partial}{\partial x}\left(b_s K_{sx} \frac{\partial \phi_s}{\partial x}\right) = \frac{1}{\Delta x_i}\left[\left(b_s K_{sx} \frac{\partial \phi_s}{\partial x}\right)_{i+\frac{1}{2},j,k} - \left(b_s K_{sx} \frac{\partial \phi_s}{\partial x}\right)_{i-\frac{1}{2},j,k}\right]$$

$$= \frac{1}{\Delta x_i} \left[(b_s K_{sx})_{i+\frac{1}{2},j,k} \left\{ \frac{\phi_{s(i+1,j,k)} - \phi_{s(i,j,k)}}{\Delta x_{i+\frac{1}{2}}} \right\} \right.$$

$$\left. - \; (b_s K_{sx})_{i-\frac{1}{2},j,k} \left\{ \frac{\phi_{s(i,j,k)} - \phi_{s(i-1,j,k)}}{\Delta x_{i-\frac{1}{2}}} \right\} \right] \qquad (4.1.e)$$

In the y-direction,

$$\frac{\partial}{\partial y} \left(b_s K_{sy} \frac{\partial \phi_s}{\partial y} \right) = \frac{1}{\Delta y_j} \left[(b_s K_{sy})_{i,j+\frac{1}{2},k} \left\{ \frac{\phi_{s(i,j+1,k)} - \phi_{s(i,j,k)}}{\Delta y_{j+\frac{1}{2}}} \right\} \right.$$

$$\left. - \; (b_s K_{sy})_{i-\frac{1}{2},j,k} \left\{ \frac{\phi_{s(i,j,k)} - \phi_{s(i,j-1,k)}}{\Delta y_{j-\frac{1}{2}}} \right\} \right] \qquad (4.1.f)$$

The source or sink terms for freshwater and saltwater subdomains may be written as

$$W_{f_{i,j}} = \frac{Q_{f_{i,j}}}{\Delta x_i \; \Delta y_j} \qquad (4.1.g)$$

$$W_{s_{i,j}} = \frac{Q_{s_{i,j}}}{\Delta x_i \; \Delta y_j} \qquad (4.1.h)$$

where

$Q_{f_{i,j}}$ = the discharge or recharge in the freshwater subdomain at node (i,j), L^3T^{-1}; and

$Q_{s_{i,j}}$ = the discharge or recharge in the saltwater subdomain at node (i,j), L^3T^{-1}.

From Equation (4.1), the finite difference approximations to all the terms of Equation (3.29) are assembled resulting the finite difference equation for the freshwater subdomain at node (i,j) and time k.

$$\frac{1}{\Delta x_i}\,[(b_f K_{fx})_{i+\frac{1}{2},j,k}\,\{\frac{\phi_{f(i+1,j,k)} - \phi_{f(i,j,k)}}{\Delta x_{i+\frac{1}{2}}}\}$$

$$-\ (b_f K_{fx})_{i-\frac{1}{2},j,k}\,\{\frac{\phi_{f(i,j,k)} - \phi_{f(i-1,j,k)}}{\Delta x_{i-\frac{1}{2}}}\}]$$

$$+\ \frac{1}{\Delta y_j}\,[(b_f K_{fy})_{i,j+\frac{1}{2},k}\,\{\frac{\phi_{f(i,j+1,k)} - \phi_{f(i,j,k)}}{\Delta y_{j+\frac{1}{2}}}\}$$

$$-\ (b_f K_{fy})_{i,j-\frac{1}{2},k}\,\{\frac{\phi_{f(i,j,k)} - \phi_{f(i,j-1,k)}}{\Delta y_{j-\frac{1}{2}}}\}]$$

$$+\ (-(S_{f_{i,j}} b_{f_{i,j,k}} + n\sigma + \alpha n))\ [\frac{\phi_{f(i,j,k)} - \phi_{f(i,j,k-1)}}{\Delta t}]$$

$$+\ n(1+\sigma)\ [\frac{\phi_{s(i,j,k)} - \phi_{s(i,j,k-1)}}{\Delta t}$$

$$=\ Q_{f_{i,j}}/\Delta x_i\ \Delta y_j \qquad (4.2)$$

The harmonic average of the transmissivity terms taken to ensure continuity across cell boundaries and to make the appropriate coefficients zero at no flow boundaries, Equation (4.2) can be written as

$$AAF_{i,j,k}\ \phi_{f(i-1,j,k)} + BBF_{i,j,k}\ \phi_{f(i+1,j,k)} + EEF_{i,j,k}\ \phi_{f(i,j,k)}$$

$$+\ CCF_{i,j,k}\ \phi_{f(i,j-1,k)} + DDF_{i,j,k}\ \phi_{f(i,j+1,k)} + CS1_{i,j,k}\ \phi_{s(i,j,k)}$$

$$=\ Q_{f_{i,j}}/\Delta x_i \Delta y_j + CS1_{i,j,k}\ \phi_{s(i,j+1,k-1)} - CF1_{i,j,k}\ \phi_{f(i,j,k-1)} \qquad (4.3)$$

where

$$BBF_{i,j,k} = \left[\frac{2(b_f K_{fx})_{i,j,k} \cdot (b_f K_{fx})_{i+1,j,k}}{(b_f K_{fx})_{i,j,k} \cdot \Delta x_{i+1} + (b_f K_{fx})_{i+1,j,k} \cdot \Delta x_i}\right] / \Delta x_i$$

$$AAF_{i,j,k} = \left[\frac{2(b_f K_{fx})_{i,j,k} \, (b_f K_{fx})_{i-1,j,k}}{(b_f K_{fx})_{i,j,k} \, \Delta x_{i-1} + (b_f K_{fx})_{i-1,j,k} \, \Delta x_i}\right] / \Delta x_i$$

$$DDF_{i,j,k} = \left[\frac{2(b_f K_{fy})_{i,j,k} \, (b_f K_{fy})_{i,j+1,k}}{(b_f K_{fy})_{i,j,k} \, \Delta y_{j+1} + (b_f K_{fy})_{i,j+1,k} \, \Delta y_i}\right] / \Delta y_j$$

$$CCF_{i,j,k} = \left[\frac{2(b_f K_{fy})_{i,j,k} \, (b_f K_{fy})_{i,j-1,k}}{(b_f K_{fy})_{i,j,k} \, \Delta y_{j-1} + (b_f K_{fy})_{i,j-1,k} \, \Delta y_j}\right] / \Delta y_j$$

$$CF1_{i,j,k} = (n\sigma + S_{f_{i,j}} \, b_{f_{i,j,k}} + \alpha n)/\Delta t$$

$$EEF_{i,j,k} = -(AAF_{i,j,k} + BBF_{i,j,k} + CCF_{i,j,k} + DDF_{i,j,k} + CF1_{i,j,k})$$

$$CS1_{i,j,k} = n(1+\sigma)/\Delta t$$

Similarly, the finite difference approximation of the saltwater subdomain represented by Equation (3.30) at node (i,j) and time k may be written as

$$AAS_{i,j,k} \, \phi_{s(i-1,j,k)} + EES_{i,j,k} \, \phi_{s(i,j,k)} + BBS_{i,j,k} \, \phi_{s(i+1,j,k)}$$

$$+ DDS_{i,j,k}\, \phi_{s(i,j+1,k)} + CCS_{i,j,k}\, \phi_{s(i,j-1,k)} + CF2_{i,j,k}$$

$$x\ \phi_{f(i,j,k)} = Q_{s_{i,j}}/\Delta x_i \Delta y_j + CF2_{i,j,k}\, \phi_{f(i,j,k-1)}$$

$$- CS2_{i,j,k}\, \phi_{s(i,j,k-1)} \qquad (4.4)$$

where

$$BBS_{i,j,k} = \left[\frac{2(b_s K_{sx})_{i,j,k}\,(b_s K_{sx})_{i+1,j,k}}{(b_s K_{sx})_{i,j,k}\,\Delta x_{i+1} + (b_s K_{sx})_{i+1,j,k}\,\Delta x_i}\right]/\Delta x_i$$

$$AAS_{i,j,k} = \left[\frac{2(b_s K_{sx})_{i,j,k}\,(b_s K_{sx})_{i-1,j,k}}{(b_s K_{sx})_{i,j,k}\,\Delta x_{i-1} + (b_s K_{sx})_{i-1,j,k}\,\Delta x_i}\right]/\Delta x_i$$

$$DDS_{i,j,k} = \left[\frac{2(b_s K_{sy})_{i,j,k}\,(b_s K_{sy})_{i,j+1,k}}{(b_s K_{sy})_{i,j,k}\,\Delta y_{i+1} + (b_s K_{sy})_{i,j+1,k}\,\Delta y_j}\right]/\Delta y_j$$

$$CCS_{i,j,k} = \left[\frac{2(b_s K_{sy})_{i,j,k}\,(b_s K_{sy})_{i,j-1,k}}{(b_s K_{sy})_{i,j,k}\,\Delta y_{j-1} + (b_s K_{sy})_{i,j-1,k}\,\Delta y_j}\right]/\Delta y_j$$

$$CF2_{i,j,k} = n\sigma/\Delta t$$

$$CS2_{i,j,k} = [n(1+\sigma) + S_{s_{i,j}} b_{s_{i,j,k}}]/\Delta t$$

$$EES_{i,j,k} = -\,(AAS_{i,j,k} + BBS_{i,j,k} + CCS_{i,j,k} + DDS_{i,j,k} + CS2_{i,J,k})$$

From Equations (4.3) and (4.4), N number of algebraic equations are generated for each subdomain to cover the entire aquifer having N nodes.

Solution of Difference Equations

This section deals with the computer solution of a system of algebraic equations. The equations are solved using an indirect method. There are three iterative techniques--namely, the alternating direction implicit procedure (ADIP) (Peaceman and Rachford, 1955; Remson et al., 1971; Trescott et al., 1976; Konikow et al., 1978), the line successive overrelaxation (LSOR) (Varga, 1962; Remson et al., 1971; Aziz and Settari, 1972; Cooley, 1974; Trescott et al., 1976) and the strongly implicit procedure (SIP) (Stone 1968; Remson et al., 1971; Cooley, 1974; Trescott et al., 1976). These methods have been successfully used for solving linear system of algebraic equations. The line successive overrelaxation technique is used in this study.

Line Successive Overrelaxation Technique

The line successive overrelaxation (LSOR) is a particular type of block iterative method in which the asymptotic rate of convergence is improved. LSOR is rigorously applicable and numerically stable with respect to rounding errors (Varga, 1962). LSOR improves the values of the variables (ϕ_f and ϕ_s), one row or column at a time. For isotropic conditions it is not important whether the solution is oriented along rows or columns, but a solution oriented along rows or columns has a significant effect (Trescott et al., 1976) on the convergence rate in anisotropic aquifers. For faster convergence the solution should be oriented in the direction of larger coefficients. The difference in the magnitude of coefficients may result either from anisotropic transmissivity or from a large difference in the grid spacing between x- and y-directions. In this study, to generalize the solution, the algorithms are developed separately for x- and y-directions. The appropriate algorithm can be used to meet the requirement for the field condition that exists.

For the solution in the x-direction, the elements in Equations (4.3) and (4.4) in the x-direction are implicit and the elements in the y-direction are explicit. Therefore, these equations can be written by taking the terms in the y-direction known from the previous time step and from the previous iteration as

$$AAF_{i,j,k}\, \phi^{n}_{f(i-1,j,k)} + EEF_{i,j,k}\, \phi^{n}_{f(i,j,k)} + CS1_{i,j,k}\, \phi^{n}_{s(i,j,k)} + BBF_{i,j,k}\, \phi^{n}_{f(i+1,j,k)} = MMF_{i,j,k} \tag{4.5}$$

and

$$AAS_{i,j,k}\, \phi^{n}_{s(i-1,j,k)} + EES_{i,j,k}\, \phi^{n}_{s(i,j,k)} + CF2_{i,j,k}\, \phi^{n}_{f(i,j,k)} + BBS_{i,j,k}\, \phi^{n}_{s(i+1,j,k)} = MMS_{i,j,k} \tag{4.6}$$

where

$$MMF_{i,j,k} = Q_{f_{i,j}}/\Delta x_i\, \Delta y_i + CS1_{i,j,k}\, \phi^{n}_{s(i,j,k-1)} - CF1_{1,j,k} \times \phi^{n}_{f(i,j,k-1)} - DDF_{i,j,k}\, \phi^{n-1}_{f(i,j+1,k)} - CCF_{i,j,k} \times \phi^{n}_{f(i,j-1,k)};$$

$$MMS_{i,j,k} = Q_{s_{i,j}}/\Delta x_i \Delta y_j + CF2_{i,j,k}\, \phi^{n}_{f(i,j,k-1)} - CS2_{i,j,k}\, \phi^{n}_{s(i,j,k-1)} - DDS_{i,j,k}\, \phi^{n-1}_{s(i,j+1,k)} - CCS_{i,j,k}\, \phi^{n}_{s(i,j-1,k)};$$

and

n = the iteration level.

Equations (4.5) and (4.6) are the LSOR equations to be solved row by row through the domain.

In solution of the matrix equations, the round-off error sometimes may be large. To reduce it in the matrix solution, Wilkinson (1963) suggested a formulation which is called "residual form". This formulation was later successfully used by Bjordamman and Coats (1969); Weinstein et al. (1969); Trescott et al. (1976). In this residual form, Equations (4.5) and (4.6) can be written, respectively, as

$$AAF_{i,j,k}\ \xi^n_{f(i-1,j,k)} + EEF_{i,j,k}\ \xi^n_{f(i,j,k)} + CS1_{i,j,k}\ \xi^n_{s(i,j,k)}$$

$$+ BBF_{i,j,k}\ \xi^n_{f(i+1,j,k)} = RF_{i,j,k} \tag{4.7}$$

and

$$AAS_{i,j,k}\ \xi^n_{s(i-1,j,k)} + EES_{i,j,k}\ \xi^n_{s(i,j,k)} + CF2_{i,j,k}\ \xi^n_{f(i,j,k)}$$

$$+ BBS_{i,j,k}\ \xi^n_{s(i+1,j,k)} = RS_{i,j,k} \tag{4.8}$$

where

$$RF_{i,j,k} = MMF_{i,j,k} - AAF_{i,j,k}\ \phi^{n-1}_{f(i-1,j,k)} - EEF_{i,j,k}\ \phi^{n-1}_{f(i,j,k)}$$

$$- CS1_{i,j,k}\ \phi^{n-1}_{s(i,j,k)} - BBF_{i,j,k}\ \phi^{n-1}_{f(i+1,j,k)}$$

$$RS_{i,j,k} = MMS_{i,j,k} - AAS_{i,j,k}\ \phi^{n-1}_{s(i-1,j,k)} - EES_{i,j,k}\ \phi^{n-1}_{s(i,j,k)}$$

$$- CF2_{i,j,k}\ \phi^{n-1}_{f(i,j,k)} - BBS_{i,j,k}\ \phi^{n-1}_{s(i+1,j,k)}$$

$$\xi^n_{f(i,j,k)} = \phi^n_{f(i,j,k)} - \phi^{n-1}_{f(i,j,k)}$$

$$\xi^n_{s(i,j,k)} = \phi^n_{s(i,j,k)} - \phi^{n-1}_{s(i,j,k)}$$

To use a standard and an efficient solution algorithm, and introduce new terms that are eventually zero, these Equations (4.7) and (4.8) can be written respectively, as

$$AAF_{i,j,k}\,\xi^n_{f(i-1,j,k)} + AAS^1_{i,j,k}\,\xi^n_{s(i-1,j,k)} + EEF_{i,j,k}\,\xi^n_{f(i,j,k)}$$

$$CS1_{i,j,k}\,\xi^n_{s(i,j,k)} + BBF_{i,j,k}\,\xi^n_{f(i+1,j,k)} + BBS^1_{i,j,k}\,\xi^n_{s(i+1,j,k)}$$

$$= RF_{i,j,k} \qquad (4.9)$$

and

$$AAF^1_{i,j,k}\,\xi^n_{f(i-1,j,k)} + AAS_{i,j,k}\,\xi^n_{s(i-1,j,k)} + EES_{i,j,k}\,\xi^n_{s(i,j,k)}$$

$$+ CF2_{i,j,k}\,\xi^n_{f(i,j,k)} + BBF^1_{i,j,k}\,\xi^n_{f(i+1,j,k)} + BBS_{i,j,k}\,\xi^n_{s(i+1,j,k)}$$

$$= RS_{i,j,k} \qquad (4.10)$$

where

$$AAS^1_{i,j,k} = BBS^1_{i,j,k} = AAF^1_{i,j,k} = BBF^1_{i,j,k} = 0$$

In matrix form, Equations (4.9) and (4.10) can be written for each row and for every iteration as

$$
\begin{bmatrix}
EEF_1 & CS1_1 & BBF_1 & BBS_1^1 & & \\
EES_1 & CF2_1 & BBF_1^1 & BBS_1 & & \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
AAF_{i-1} & AAS_{i-1}^1 & EEF_i & CS1_i & BBF_{i+1} & BBS_{i+1}^1 \\
AAF_{i-1}^1 & AAS_{i-1} & EES_i & CF1_i & BBF_{i+1}^1 & BBS_{i+1} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
 & & AAF_{NC-1} & AAS_{NC-1}^1 & EEF_{NC} & CS1_{NC} \\
 & & AAF_{NC-1}^1 & AAS_{NC-1} & EES_{NC} & CF1_{NC}
\end{bmatrix}
\begin{Bmatrix}
\xi_{f1} \\ \xi_{s1} \\ \xi_{f2} \\ \xi_{s2} \\ \cdots \\ \cdots \\ \xi_{f(i-1)} \\ \xi_{s(i-1)} \\ \xi_{fi} \\ \xi_{si} \\ \xi_{f(i+1)} \\ \xi_{s(i+1)} \\ \cdots \\ \cdots \\ \xi_{f(NC-1)} \\ \xi_{s(NC-1)} \\ \xi_{fNC} \\ \xi_{sNC}
\end{Bmatrix}
=
\begin{bmatrix}
RF_1 \\ RS_1 \\ RF_2 \\ RS_2 \\ \cdots \\ \cdots \\ RF_{i-1} \\ RS_{i-1} \\ RF_i \\ RS_i \\ RF_{i+1} \\ RS_{i+1} \\ \cdots \\ \cdots \\ RF_{NC-1} \\ RS_{NC-1} \\ RF_{NC} \\ RS_{NC}
\end{bmatrix}
\tag{4.11}
$$

where NC is the number of columns in the solution domain. For the solution in the y-direction, the elements in Equations (4.3) and (4.4) in the y-direction are implicit and the elements in the x-direction are explicit. In this case, these equations can be written as

$$
CCF_{i,j,k}\,\phi^n_{f(i,j-1,k)} + EEF_{i,j,k}\,\phi^n_{f(i,j,k)} + CS1_{i,j,k}\,\phi^n_{s(i,j,k)} + DDF_{i,j,k}\,\phi^n_{f(i+1,j,k)} = MMF_{i,j,k} \tag{4.12}
$$

and

$$
CCS_{i,j,k}\,\phi^n_{s(i,j-1,k)} + EES_{i,j,k}\,\phi^n_{s(i,j,k)} + CF2_{i,j,k}\,\phi^n_{f(i,j,k)} + DDS_{i,j,k}\,\phi^n_{s(i+1,j,k)} = MMS_{i,j,k} \tag{4.13}
$$

where

$$MMF_{i,j,k} = Q_{f_{i,j}}/\Delta x_i \Delta y_j + CS1_{i,j,k}\, \phi^{n}_{s(i,j,k-1)}$$

$$- \; CF1_{i,j,k}\, \phi^{n}_{f(i,j,k-1)} - AAF_{i,j,k}\, \phi^{n}_{f(i-1,j,k)}$$

$$- \; BBF_{i,j,k}\, \phi^{n-1}_{f(i+1,j,k)}$$

$$MMS_{i,j,k} = Q_{s_{i,j}}/\Delta x_i \Delta y_j + CF2_{i,j,k}\, \phi^{n}_{f(i,j,k-1)}$$

$$- \; CS2_{i,j,k}\, \phi^{n}_{s(i,j,k-1)} - AAS_{i,j,k}\, \phi^{n}_{s(i-1,j,k)}$$

$$- \; BBS_{i,j,k}\, \phi^{n-1}_{s(i+1,j,k)}$$

In residual form, Equations (4.12) and (4.13) are written as

$$CCF_{i,j,k}\, \xi^{n}_{f(i,j-1,k)} + EEF_{i,j,k}\, \xi^{n}_{f(i,j,k)} + CS1_{i,j,k}\, \xi^{n}_{s(i,j,k)}$$

$$+ \; DDF_{i,j,k}\, \xi^{n}_{f(i,j+1,k)} = RF_{i,j,k} \qquad (4.14)$$

and

$$CCS_{i,j,k}\, \xi^{n}_{s(i,j-1,k)} + EES_{i,j,k}\, \xi^{n}_{s(i,j,k)} + CF2_{i,J,k}\, \xi^{n}_{f(i,j,k)}$$

$$+ \; DDS_{i,j,k}\, \xi^{n}_{s(i,j+1,k)} = RS_{i,j,k} \qquad (4.15)$$

where

$$RF_{i,j,k} = MMF_{i,j,k} - CCF_{i,j,k}\, \phi^{n-1}_{f(i,j-1,k)} - EEF_{i,j,k}\, \phi^{n-1}_{f(i,j,k)}$$

$$- \; CS1_{i,j,k}\, \phi^{n-1}_{s(i,j,k)} - DDF_{i,j,k}\, \phi^{n-1}_{f(i,j+1,k)}$$

$$RS_{i,j,k} = MMS_{i,j,k} - CCS_{i,j,k}\,\phi^{n-1}_{s(i,j-1,k)} - EES_{i,j,k}\,\phi^{n-1}_{s(i,j,k)}$$

$$- CF2_{i,j,k}\,\phi^{n-1}_{f(i,j,k)} - DDS_{i,j,k}\,\phi^{n-1}_{s(i,j+1,k)}$$

$$\xi^{n}_{f(i,j,k)} = \phi^{n}_{f(i,j,k)} - \phi^{n-1}_{f(i,j,k)}$$

and

$$\xi^{n}_{s(i,j,k)} = \phi^{n}_{s(i,j,k)} - \phi^{n-1}_{s(i,j,k)}$$

Similar to Equations (4.9) and (4.10) to use a standard and efficient direct solution algorithm that introduces new terms, Equations (4.14) and (4.15) are written as

$$CCF_{i,j,k}\,\xi_{f(i,j-1,k)} + CCS^{1}_{i,j,k}\,\xi^{n}_{s(i,j-1,k)} + EEF_{i,j,k}\,\xi^{n}_{f(i,j,k)}$$

$$+ CS1_{i,j,k}\,\xi^{n}_{s(i,j,k)} + DDF_{i,j,k}\,\xi^{n}_{f(i,j+1,k)} + DDS^{1}_{i,j,k}\,\xi^{n}_{s(i,j+1,k)}$$

$$= RF_{i,j,k} \tag{4.16}$$

and

$$CCF^{1}_{i,j,k}\,\xi^{n}_{f(i,j-1,k)} + CCS_{i,j,k}\,\xi^{n}_{s(i,j-1,k)} + EES_{i,j,k}\,\xi^{n}_{s(i,j,k)}$$

$$+ CF2_{i,j,k}\,\xi^{n}_{f(i,j,k)} + DDF^{1}_{i,j,k}\,\xi^{n}_{f(i,j+1,k)} + DDS_{i,j,k}$$

$$\times\ \xi^{n}_{s(i,j+1,k)} = RS_{i,j,k} \tag{4.17}$$

where

$$CCS^{1}_{i,j,k} = DDS^{1}_{i,j,k} = CCF^{1}_{i,j,k} = DDF^{1}_{i,j,k} = 0$$

Similar to Equation (4.11), the Equations (4.16) and (4.17) can be written in matrix form for each column and for every iteration as follows:

$$
\begin{bmatrix}
EEF_1 & CS1_1 & DDF_1 & DDS_1^1 & & \\
EES_1 & CF2_1 & DDF_1^1 & DDS_1 & & \\
\hdashline
 & & & & & \\
\hdashline
CCF_{j-1} & CCS_{j-1}^1 & EEF_j & CS1_j & DDF_{j+1} & DDS_{j+1}^1 \\
CCF_{j-1}^1 & CCS_{j-1} & EES_j & CF1_j & DDF_{j+1}^1 & DDS_{j+1} \\
\hdashline
 & & & & & \\
\hdashline
 & & CCF_{NR-1} & CCS_{NR-1}^1 & EEF_{NR} & CS1_{NR} \\
 & & CCF_{NR-1}^1 & CCS_{NR-1} & EES_{NR} & CF1_{NR}
\end{bmatrix}
\begin{Bmatrix}
\xi_{f1} \\ \xi_{s1} \\ \xi_{f2} \\ \xi_{s2} \\
\hdashline
\\
\hdashline
\xi_{f(j-1)} \\ \xi_{s(j-1)} \\ \xi_{fj} \\ \xi_{sj} \\ \xi_{f(j+1)} \\ \xi_{s(j+1)} \\
\hdashline
\\
\hdashline
\xi_{fNR-1} \\ \xi_{sNR-1} \\ \xi_{fNR} \\ \xi_{sNR}
\end{Bmatrix}
=
\begin{Bmatrix}
RF_1 \\ RS_1 \\ RF_2 \\ RS_2 \\
\hdashline
\\
\hdashline
RF_{(j-1)} \\ RS_{(j-1)} \\ RF_j \\ RS_j \\ RF_{(j+1)} \\ RS_{(j+1)} \\
\hdashline
\\
\hdashline
RF_{(NR-1)} \\ RS_{(NR-1)} \\ RF_{NR} \\ RS_{NR}
\end{Bmatrix}
\tag{4.18}
$$

where NR is the number of rows in the solution domain. The solution is continued column by column until the whole domain is covered. In Equation (4.11) or Equation (4.18), the coefficient matrix is a banded bitridiagonal matrix that can be solved for solution vectors with the bitridiagonal algorithm (as presented in Appendix A). When the solution is in the x-direction, Equation (4.11) is solved by rows until all the rows in the solution domain are completed. If the solution is in the y-direction, Equation (4.18) is solved by columns until all the columns in the solution domain are covered. As soon as the solution vectors $\xi_{f(i,j)}$ and $\xi_{s(i,j)}$ are obtained from either Equation (4.11) or Equation (4.18), the freshwater and saltwater potentials for that particular time step are calculated from

$$\phi^{n}_{f(i,j,k)} = \phi^{n-1}_{f(i,j,k)} + \xi^{n}_{f(i,j,k)}$$

and

$$\phi^{n}_{s(i,j,k)} = \phi^{n-1}_{s(i,j,k)} + \xi^{n}_{s(i,j,k)} \tag{4.19}$$

For faster convergence, in practice, a parameter ω (known as the LSOR overrelaxation factor) is introduced and Equation (4.19) is written as

$$\phi^{n}_{f(i,j,k)} = \phi^{n-1}_{f(i,j,k)} + \omega\xi^{n}_{f(i,j,k)}$$

$$\phi^{n}_{s(i,j,k)} = \phi^{n-1}_{s(i,j,k)} + \omega\xi^{n}_{s(i,j,k)} \tag{4.20}$$

The theory of LSOR overrelaxation factor ω, is discussed briefly in the following section.

LSOR Overrelaxation Factor

Varga (1962) showed that if the coefficient matrix is Harmitian, then LSOR converges if and only if $0 < \omega < 2$ and the coefficient matrix is positive definite. If only a few runs are made in a problem, it is probably best to choose an overrelaxation factor ω based on individual experience. But, if many runs are made, it is worthwhile to choose an overrelaxation factor ω close to the maximum limiting value (Trescott et al., 1976). For relatively simple problems, theoretically, the optimum value of ω is given by

$$\omega_{opt} = \frac{2}{1 + \sqrt{1 - \rho(\sigma)}} \tag{4.21}$$

where

$\rho(\sigma)$ = the spectral radius (dominant eigenvalue) of Gauss-Seidel iteration matrix

$\rho(\sigma)$ can be calculated with the equation

$$\rho(\sigma) = |\lambda_1| = \frac{\max_i |\phi^{n+1}_{f(i,j,k)} - \phi^{n}_{f(i,j,k)}|}{\max_i |\phi^{n}_{f(i,j,k)} - \phi^{n-1}_{f(i,j,k)}|} \tag{4.22}$$

Equation (4.22) is for a rectangular or square domain. For other types of domain, the ω_{opt} can be computed (Remson et al., 1971) using

$$\omega_{opt} = \frac{2}{1 + 1.701 \frac{\Delta x}{r_o}} \tag{4.23}$$

where

Δx = the node spacing, L; and

r_o = the radius of the circle having same area as the domain, L.

For Equation (4.22), finding the spectral radius is difficult. Fortunately, the convergence of LSOR does not seem to be very sensitive to the value of ω. The success of these methods, however, is dependent mainly on the problem type (Remson et al., 1971).

Convergence of Iterative Methods

The iteration process stops when convergence is achieved. The convergence test can be performed in three ways: (a) the absolute test, (b) the relative test, and (c) the natural machine-independent convergence test. The first test requires that the absolute difference between the present and previous iterations be less than or equal to a pre-specified value of error; here it is called the steady-state error criterion. The absolute test is expressed as

$$
\begin{aligned}
|\xi_f^n| &= |\phi_f^n - \phi_f^{n-1}| \le \xi_1 \\
|\xi_s| &= |\phi_s^n - \phi_s^{n-1}| \le \xi_1
\end{aligned}
\qquad (4.24)
$$

where, ξ_1 = pre-specified error criterion. When there is no idea of the magnitude of the final ϕ values to be tested, this test is poor. Suppose the pre-specified error criterion chosen is 1.E-8, and the value to be tested happens to be 1.E-10, then the error is very large. Performance of this test is good, however, when the value to be tested is large, not a fraction.

For the situation where there is no idea about the value to be tested it is wise to use the relative convergence test as it naturally adapts the size. The relative test is expressed as

$$
\frac{|\phi_f^n - \phi_f^{n-1}|}{\mathrm{Max}(|\phi_f^n|, |\phi_f^{n-1}|)} \le \varepsilon_2
$$

$$\frac{|\phi_s^n - \phi_s^{n-1}|}{\mathrm{Max}(|\phi_s^n|,|\phi_s^{n-1}|)} \le \varepsilon_2 \qquad (4.25)$$

where ε_2 = pre-specified error criterion for relative test. This test takes more operations than the absolute test.

The natural machine-independent convergence test adapts naturally to the accuracy of the computer used. Its chief drawback is that if full accuracy is not needed, a great amount of computer time may be wasted in the run.

In this study the values to be tested are the freshwater and saltwater potentials, which are large. So, among the three tests described above, the absolute convergence test is used because less computer time is required to perform the test. The convergence test accounts for the total system by controlling the largest absolute change in saltwater and freshwater heads during the iteration process over the entire domain. If the greatest difference (ξ_f or ξ_s) between the potential values calculated in a particular iteration from that of the previous iteration is less than or equal to the given tolerance limit (ε_1), the iteration process is terminated. If Equation (4.24) is satisfied, then the values of freshwater and saltwater potentials are obtained for that time period with Equation (4.20).

CHAPTER V

MODEL VERIFICATION

Verification of the capacity of the model to simulate field problems is necessary; the model can be evaluated by comparing it with existing numerical and analytical models. No comparable numerical model exists to test the validity of the model developed. There is, however, the analytical solution developed by Dagan and Bear (1967) and used by Schmorak and Mercado (1969) and by Ayers and Vacher (1980) to calculate the upconing of an interface in a two-dimensional vertical plane below a pumping well. This solution is adopted to compare with the numerical model developed for a two-dimensional horizontal plane.

Model Description

The model developed is written in ANSI Standard FORTRAN and run on an IBM-370 digital computer, Model 75. The program is portable and, with minor changes, can be run on other computers. The model has a main program and four subprograms. The main program controls the time loop of simulation and coordinates the function of the subprograms. The main program also performs the initial calculation for saltwater heads.

Subroutine DATA is the first subroutine to be called by the main program. This subroutine reads all the input data in the model and performs the echo-check of them. This subroutine calls subroutine ARRAY to read the two-dimensional data and to perform their echo-check.

Subroutine SOLV is the most important subprogram in the model. This program calculates the elements of the coefficients matrix, controls the direction of the LSOR solution, and generates the vectors of the bitridiagonal algorithm. SOLV also controls the iteration loop and updates and calculates the model variables for each iteration until the desired result is obtained.

Subroutine PRINT prints the output at each time step in terms of freshwater potentials, saltwater potentials, freshwater-saltwater interface and the upconing height. The complete model is presented in Appendix B and the input format is presented in Appendix C. The units of the input data are given in the model variables and parameters description.

Analytical Solution

The rate of rise of the freshwater-saltwater interface is an important aspect of groundwater development from aquifers that contain a saline zone. The analytical technique to calculate the interface movement in a vertical plane reported by Dagan and Bear (1967) was developed with the assumptions that the porous medium is homogeneous and nondeformable, that the two fluids are incompressible, are separated by an abrupt interface, and that the groundwater follows Darcy's law. For a partially penetrating pumping well and a relatively thick aquifer as shown in Figure 3, this equation is written as

$$Z(r,t) = \frac{Q_f \gamma_f}{2\pi(\Delta\gamma)K_x d}\left[\frac{1}{(1+R'^2)^{\frac{1}{2}}} - \frac{1}{[(1+\gamma')^2 + R'^2]^{\frac{1}{2}}}\right] \qquad (5.1)$$

where

R' = dimensionless distance parameter; $= \frac{r}{d}(K_z/K_x)^{\frac{1}{2}}$

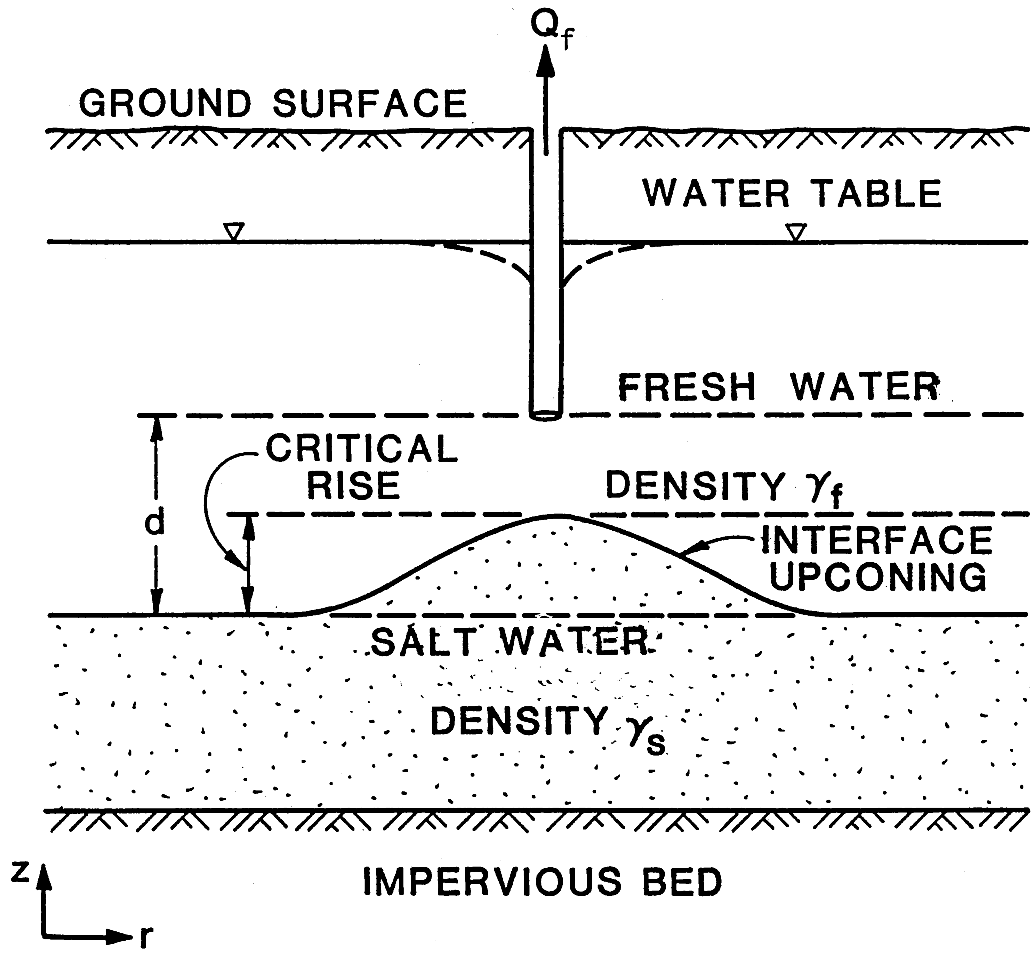

Figure 3. Diagram Defining Variables in Analytical Solution of Freshwater-Saltwater Interface Upconing

γ' = dimensionless time parameter; = $\frac{(\Delta\gamma/\gamma)}{2nd} K_z t$

Z = the rise of interface above its original position, L;

Q_f = the pumping rate of the well, L^3T;

d = the distance between the well bottom and the interface at $t = 0$, L;

r = the distance from the well, L;

n = porosity of the aquifer;

$\Delta\gamma$ = $(\gamma_s - \gamma_f)$, ML^{-3};

γ_s = saltwater density, ML^{-3};

γ_f = freshwater density, ML^{-3};

K_z = vertical permeability, LT^{-1};

K_x = horizontal permeability, LT^{-1}; and

t = time elapsed since pumping started, T.

For $r = 0$ (just below a pumping well), Equation (5.1) reduces to

$$Z(t) = \frac{Q_f \gamma_f}{2\pi(\Delta\gamma)K_x d} \left(1 - \frac{1}{1+\gamma'}\right) \tag{5.2}$$

When time becomes very large ($t \to \infty$), Equation (5.1) reduces to

$$Z(r,\infty) = \frac{Q_f \gamma_f}{2d\pi(\Delta\gamma)K_x} \left[\frac{1}{\left(1 + \left(\frac{r}{d}\right)^2 \frac{K_x}{K_z}\right)^{\frac{1}{2}}}\right] \tag{5.3}$$

Verification

A homogeneous and isotropic aquifer was used to compare the numerical model with the analytical solution.

The permeability of the aquifer equals 16.5 gpd/ft^2. One discharge well at the center of the aquifer pumped at a rate of 225 gpm. The ratio of the density of fresh water to that of salt water is used as 1.025. The porosity of the aquifer is assumed to be 30 percent.

The numerical model contains 19 columns and 14 rows, respectively. The grids in both x- and y-directions are assumed equal and 960 feet long. The storage coefficient is 5 x 10^{-4} indicating a confined aquifer. The thickness of the aquifer is 915 feet and the thickness of freshwater zone is 555 feet. The initial freshwater head and the initial interface elevation are 955 and 360 feet respectively.

The analytical solution was run for 120 days of simulation and three penetration depths of the well (25, 27, and 29 percent of the depth of the freshwater zone). The numerical model was run for both x- and y-directions for the same duration. Figure 4 compares upconing of the interface at the bottom of the well for analytical and numerical models. Note that the numerical model gives close results with the analytical solution for a depth of penetration of 162 feet (29 percent). As the depth of penetration decreases, the deviation between numerical and analytical solutions gets larger. Sahni (1972) concluded that the optimum depth of penetration is about 30 percent of the depth of freshwater zone in an aquifer. In Figure 4, the numerical model agrees with the analytical solution for 29 percent of the penetration depth, which is close to the 30 percent criterion of Sahni (1972).

Figures 5, 6 and 7 compare the upconing of the interface computed for penetration of 162 feet (29 percent) in the analytical solution and the upconing computed by the numerical model at various distances from the center of the well for simulation periods of 200, 600 and 1000 days,

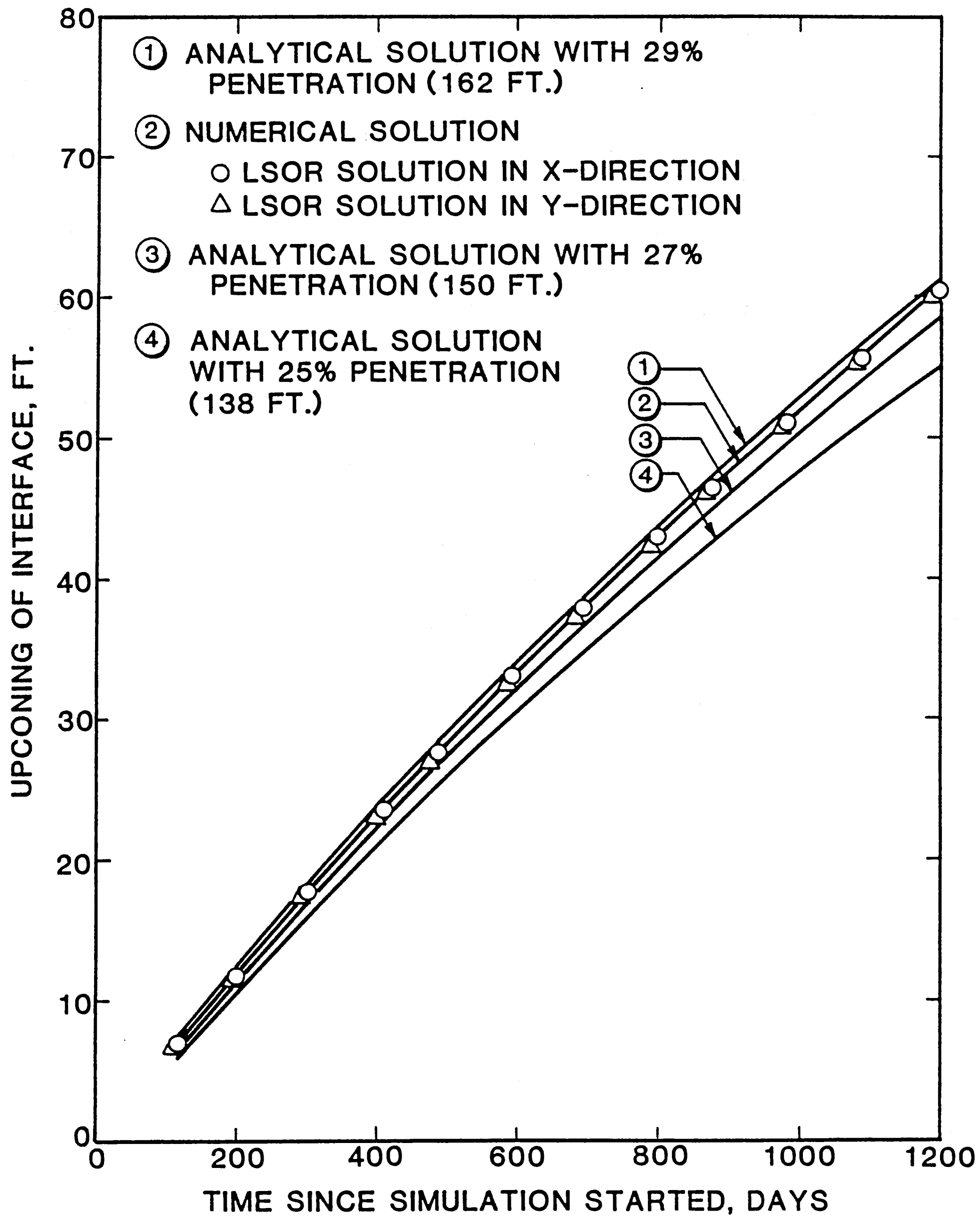

Figure 4. Comparison of Upconing Below a Pumping Well Computed by the Numerical Model and the Analytical Method With Rate of Pumping 225 gpm

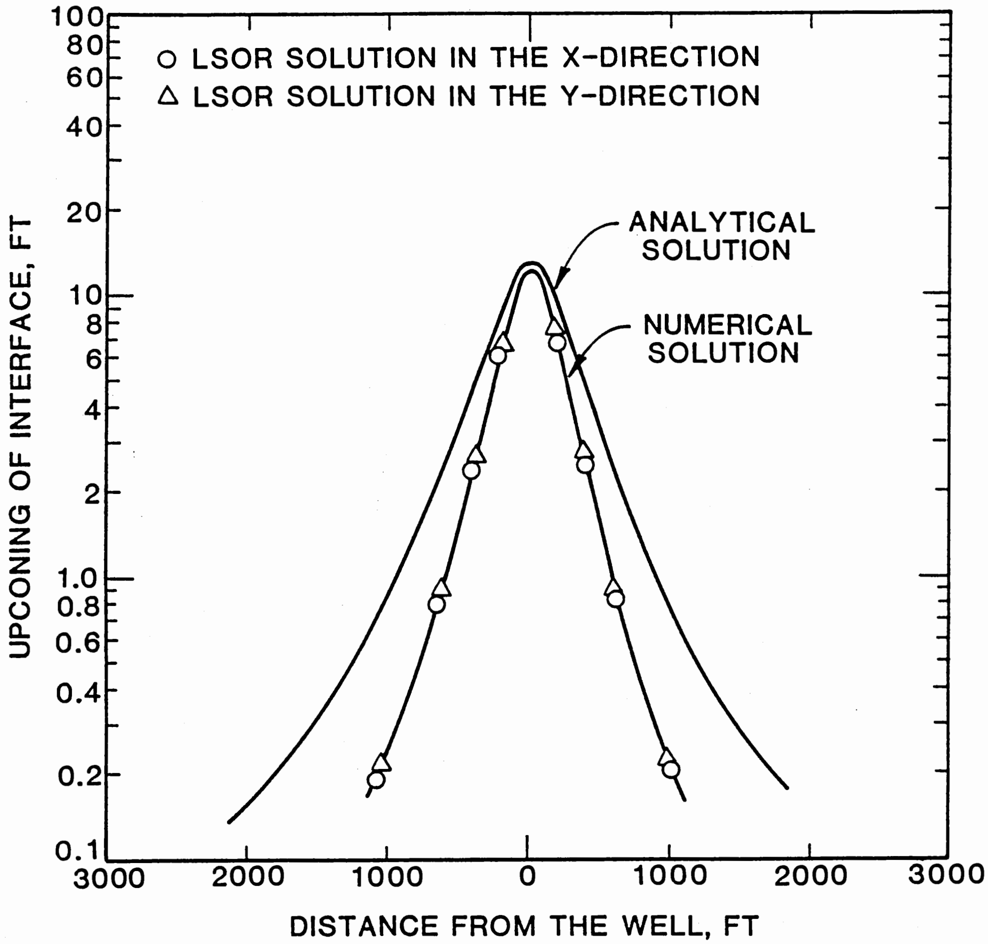

Figure 5. Comparison of Upconing Computed by the Numerical Model and the Analytical Method With Depth of Penetration 162 Ft (29 Percent), Time of Simulation 200 Days and Rate of Pumping 225 gpm

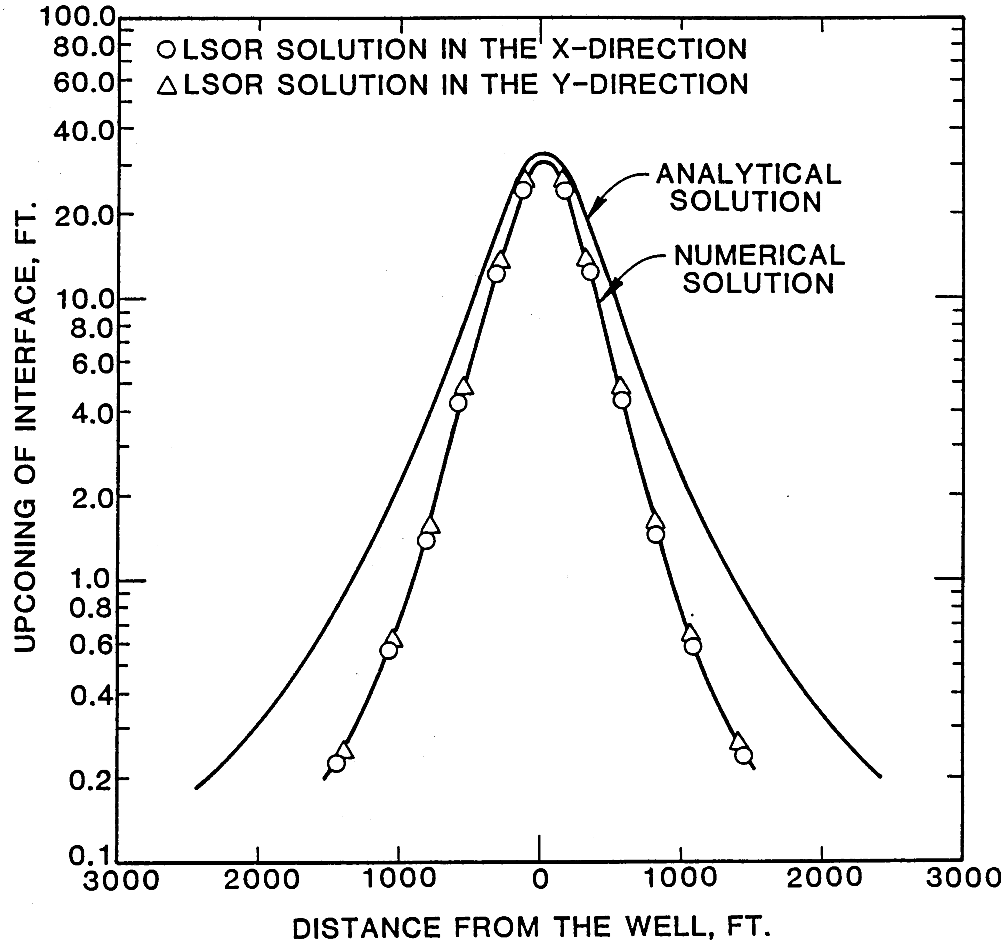

Figure 6. Comparison of Upconing Computed by the Numerical Model and the Analytical Method With Depth of Penetration 162 Ft (29 Percent), Time of Simulation 600 Days and Rate of Pumping 225 gpm

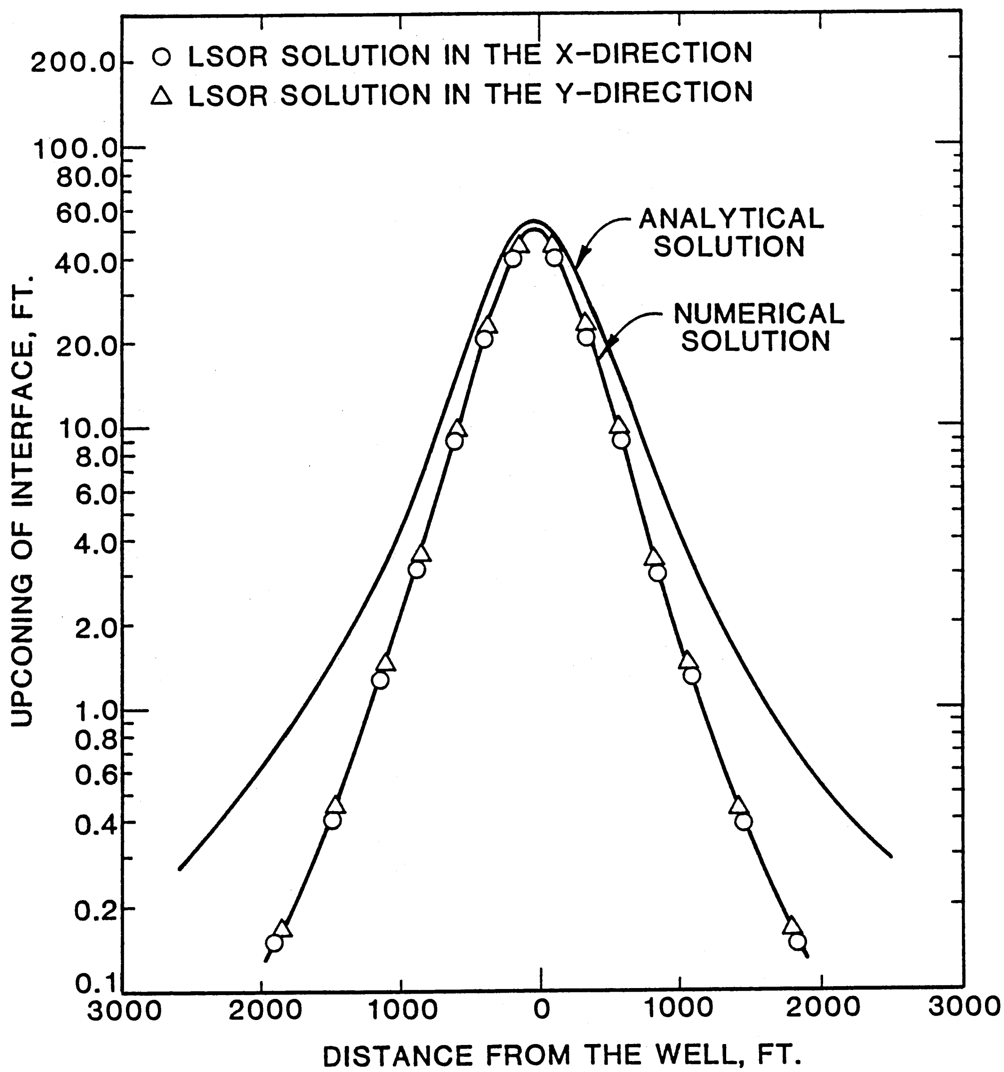

Figure 7. Comparison of Upconing Computed by the Numerical Model and the Analytical Method With Depth of Penetration 162 Ft (29 Percent), Time of Simulation 1000 Days and Rate of Pumping 225 gpm

respectively. The difference between the analytical and numerical models varies with the distance from the well. The analytical solution used here has been developed to calculate the upconing at the well bottom. It is interesting to note that in Figures 5, 6, and 7 the maximum difference in upconing height between the analytical and numerical models at the well is about 2 feet compared to the maximum upconing of about 52 feet. The maximum error is about 4 percent between the two models. Thus, the developed model has been verified and found capable of simulating the upconing height in an aquifer.

CHAPTER VI

FIELD APPLICATION

After verification of the numerical model, the present chapter deals with its application to a field aquifer. The capacity and restrictions of the model in relation to its application and performance have already been mentioned. Application of the model has been made to a field situation in the Yukon well field, Garber-Wellington aquifer of Oklahoma.

Garber-Wellington Aquifer

The Garber-Wellington aquifer which dips westward at 30 to 40 feet per mile, consists of about 900 feet of interbedded sandstone, shale, and siltstone. Sandstone composes 35 to 75 percent of the aquifer. The Garber-Wellington aquifer is exposed at the land surface in eastern Oklahoma, and it starts downward to the west and at the western edge the top of the formation is several hundred feet down below the land surface as shown in Figure 8. The details of the aquifer are given by Carr and Marcher (1977) and Wickersham (1979).

Vertical and lateral variations in the lithology of the Garber-Wellington aquifer result in groundwater occurring under unconfined, semi-artesian and artesian conditions. Unconfined conditions generally exist at depths of less than 200 feet, where the aquifer is exposed at the surface. Artesian conditions exist below 200 feet and in most of

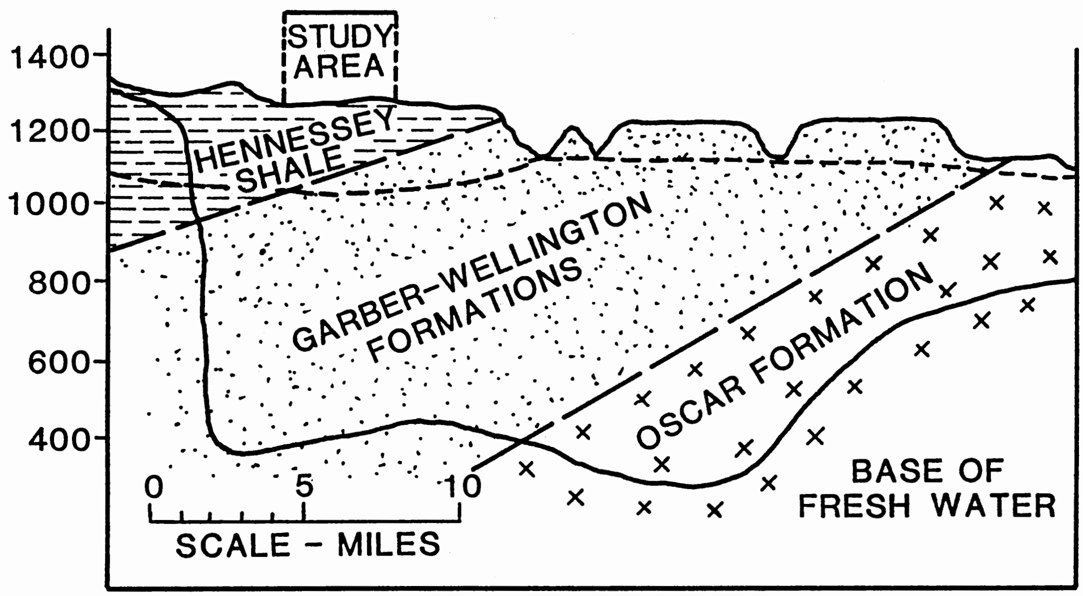

Figure 8. Sketch of the Garber-Wellington Aquifer (Panigrahi, 1980)

the area the aquifer is overlain by the Hennessey group. Vertical variations in hydraulic characteristics of the aquifer present a significant problem in defining the hydrologic system. The average transmissivity of the Garber-Wellington formation is estimated to be 3,000 gallons per day per foot (Wood and Barton, 1968).

Water in the upper part of the aquifer has two components of movement. The principal component is essentially lateral from areas of recharge to points of discharge. A secondary component of movement is vertically downward.

The total volume of water available from storage in the freshwater zone may be estimated by multiplying the area, one-half of the thickness of the freshwater zone, and the porosity of the sandstone. One-half of the thickness of the freshwater zone is used because the aquifer consists of about equal amounts of sandstone and shale. Although porosity determines the amount of water the aquifer can hold, the amount of water that the rocks will yield is less because some of the water is retained in the pore spaces. Thus, a better estimate of water available from storage is based on specific yield rather than porosity.

Recharge to the Garber-Wellington aquifer is derived primarily from rainfall on the outcrop area in the northeastern portion of the basin. Wood and Burton (1968) estimated recharge to the Garber-Wellington aquifer to be 5 percent of the average annual precipitation. Carr and Marcher (1977) used actual field data to determine the recharge for the Garber-Wellington aquifer and found that it was at least 10 percent of the average annual rainfall.

Generally, the good quality water in the Garber-Wellington aquifer has a higher piezometric level. Well-yield varies widely from a higher value in the east to the lower value in the west.

Chemical analysis of water from the aquifer indicates that the hardness is greater in the upper part of the aquifer than in the lower part, and that sulphate, chloride, and dissolved solids increase with depth. Intrusion of saline water into the freshwater zone is a potential threat to water quality in the aquifer if the pressure head in the freshwater zone is reduced sufficiently to allow upward movement of saline water. Salt water underlies the fresh water at depths greater than 1,000 feet at the western edge and at a shallower depth in the east. There is a transitional zone of brackish water 100-150 feet in thickness separating the fresh water and salt water. Presence of salt water means that deep-water wells in the Garber-Wellington basin must be drilled with caution. The freshwater-saltwater interface is still 50-100 feet below the bottom perforation of most municipal wells in the basin. The threat of salt-water upconing exists, however, if the basin is not properly managed.

The Study Area

The City of Yukon has its well field located within the alluvium of the North Canadian River. Investigations about renovating this well field, reveal that limitations exist, such as naturally poor water quality, competition among irrigators, and a limited long-term potential of water.

The groundwater reservoir formed by the Garber-Wellington aquifer underlies much of central Oklahoma, however, the part that is supposed to supply Yukon's water requirement, underlies Township 11 North, Range 4, West of the Indian Meridian, Oklahoma County and is bounded by I-40 on the north, Portland Avenue on the east, the Cleveland county line on the south, and the Canadian county line on the west. The area under the study, the municipal production deep well locations, and the piezometric head distribution are shown in Figures 9, 10 and 11, respectively.

The geologic framework in the Yukon well field is very similar to the general trend of the Garber-Wellington formation, that is, the Hennessey group above the aquifer thickens toward west and south. In the study area, the thickness of the Hennessey group ranges between 300 and 450 feet. Within the study area the Garber-Wellington aquifer is slightly artesian (see Figure 12).

The study area is a part of the Prairie Plains Homocline. The surface is a gently eastward sloping plane with westward dipping rocks. Within the study area the surface elevation varies from about 1,250 to 1,300 feet. Drainage consists of eastward flowing streams. Tributary streams generally flow northward or southward. The area has a sub-humid climate with pronounced day-to-day changes and mild seasonal variations. The average annual precipitation is about 32 inches. May is commonly the wettest month. The average annual temperature is 61^{o} F. Within the study area recharge from directly above the aquifer is negligible, however, the study area receives a reasonable amount of recharge from the North Canadian River along its northern portion.

The freshwater-saltwater interface is not a sharp surface in the study area. Therefore, the upper boundary of the transition zone, with the total dissolved solids content of the water less than 1000 mg/ℓ, is considered as the location of the interface. In the study area, the thickness of the aquifer varies from 820-920 feet. The discharge rate of the pumping wells is about 200 gpm, and the area under study is about 10 square miles.

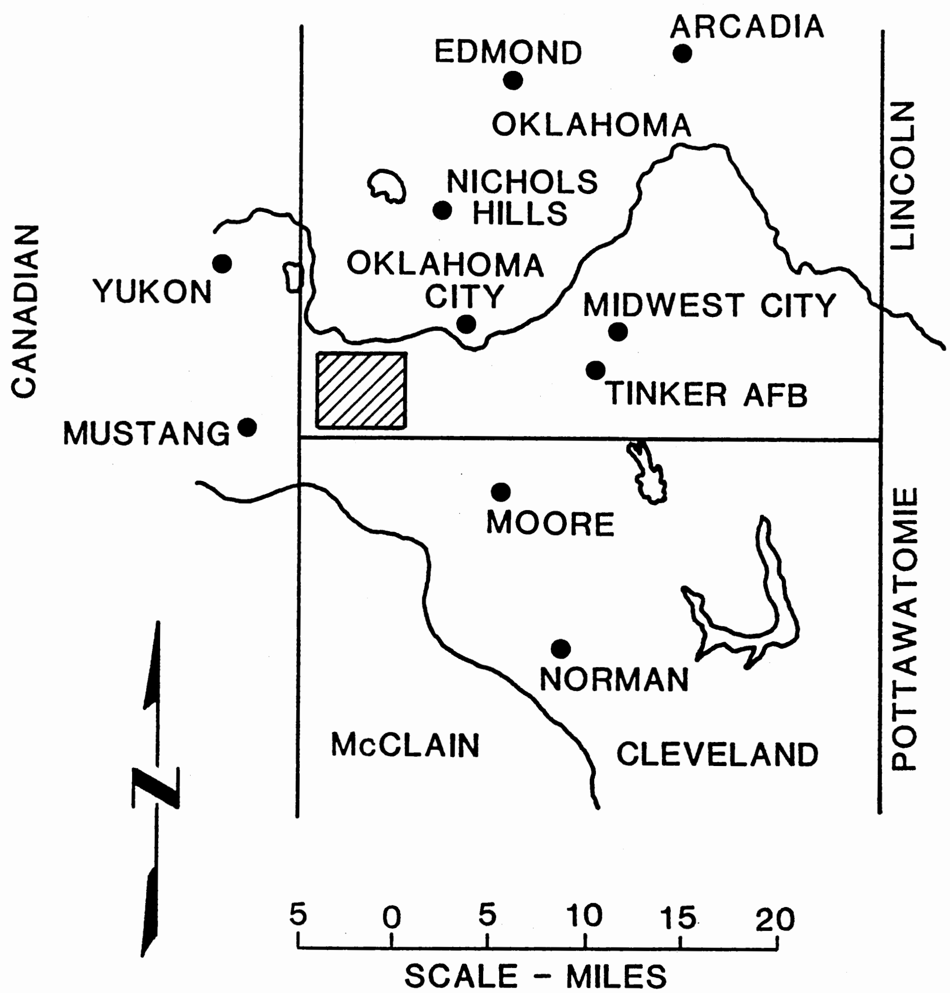

Figure 9. Location of the Study Area (Panigrahi, 1980)

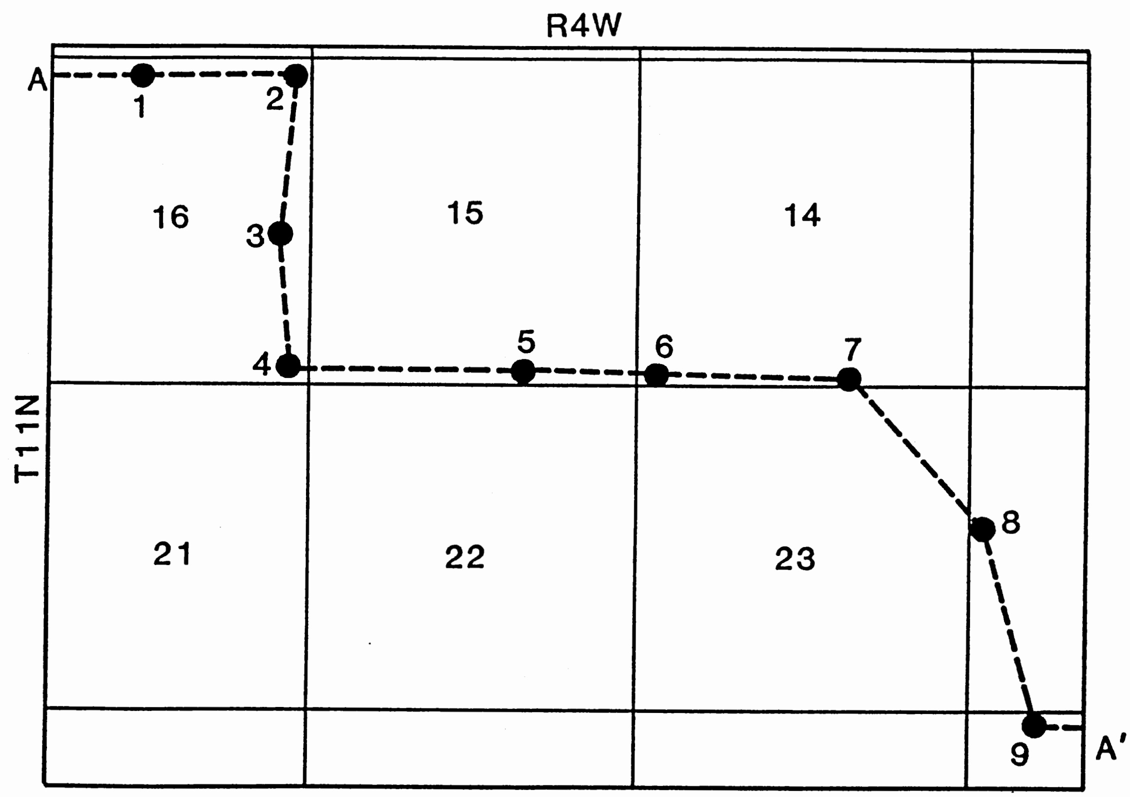

Figure 10. Well Location Map (Panigrahi, 1980)

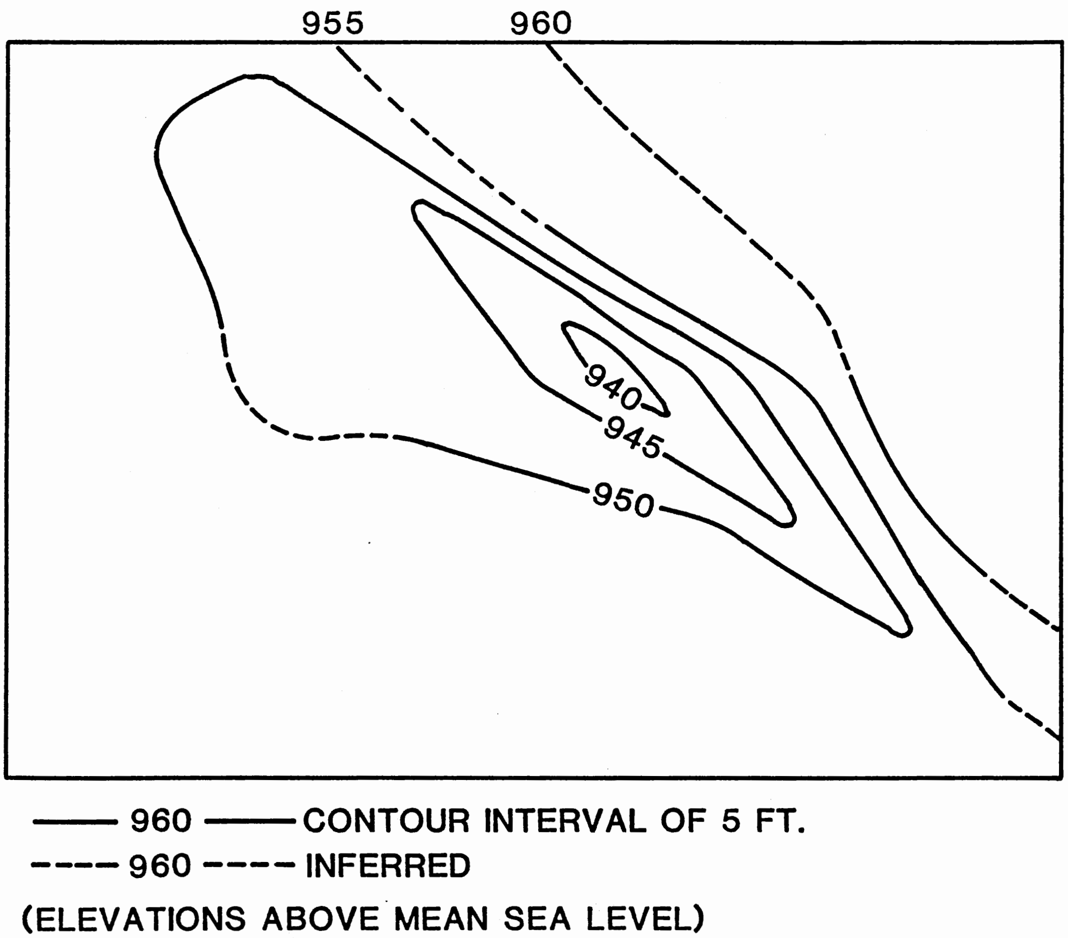

Figure 11. Piezometric Head Distribution Map (Panigrahi, 1980)

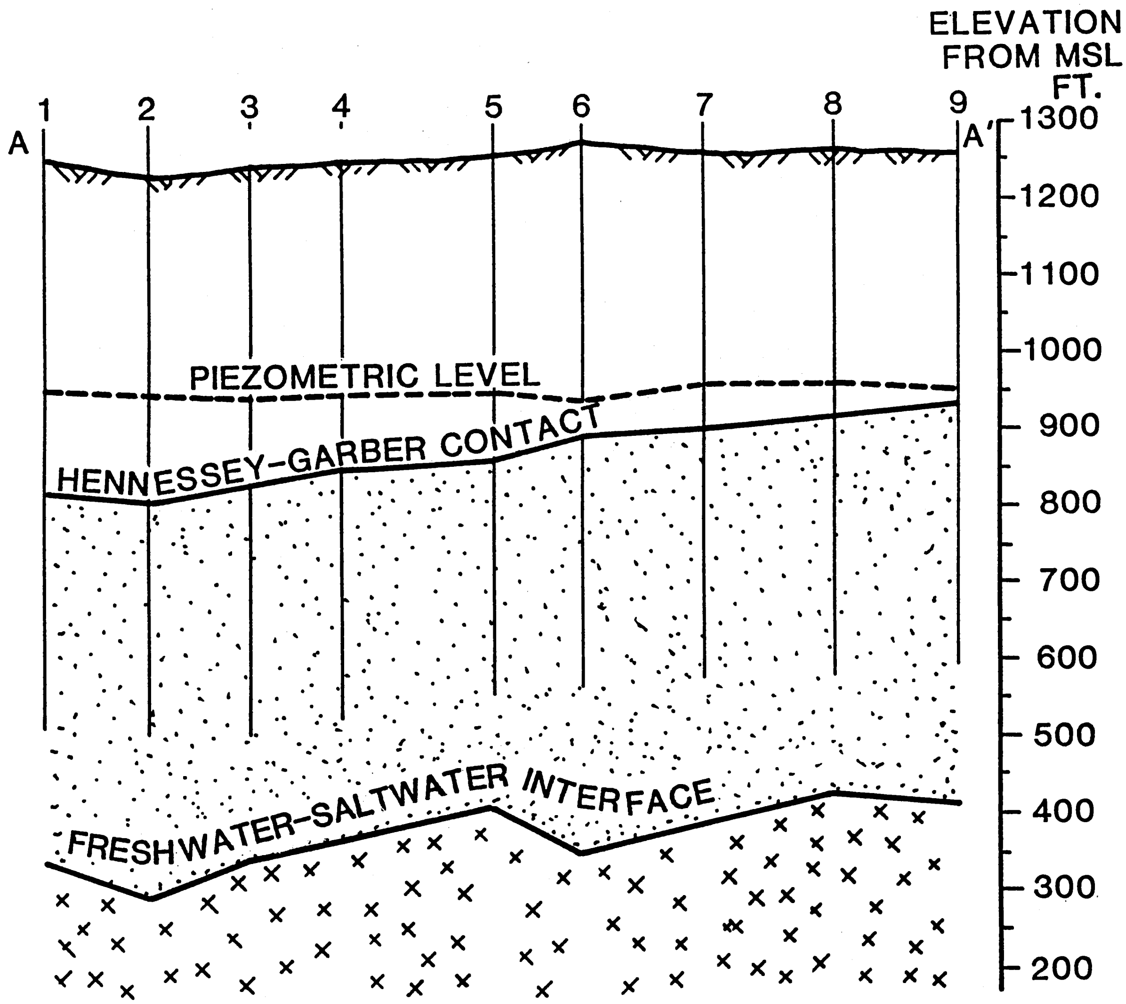

Figure 12. Diagramatic Sketch of Cross-Section Along A-A' (Panigrahi, 1980)

Aquifer Descritization

To design a finite difference grid, consideration should be given to the following requirements.

1. Nodes representing pumping and observation wells should be close to their respective positions to facilitate simulation. If several pumping wells are clustered together, their discharge should essentially be combined and assigned to the center of the cell.

2. Boundary conditions within the study area should be located accurately. Distant boundaries can be located approximately and with fewer nodes by expanding the grid. In expanding a finite difference grid in the positive x-direction, Trescott et al. (1976) have shown that restricting the ratio $\Delta x_i/\Delta x_{i-1} \leqq 1.5$ will avoid large truncation errors and possible convergence problems.

3. Nodes should be placed together in areas where there are spatial changes in transmissivity. The grid should be oriented so that a minimum number of nodes are outside the aquifer. If the aquifer is anisotropic, the grid should be oriented with its axes parallel to the principal directions of the transmissivity tensor.

In the present example of application of the model, the aquifer system properties are descritized by superimposing a square mesh finite difference grid over maps of the aquifer properties. The number of columns in the model is 19 and the number of rows is 14. The grid spacing is 960 feet in each direction. The aquifer descritization is shown in Figure 13. For all the grids (inner and boundary), the node is placed at the center of the grid. The boundary nodes are assigned constant freshwater-saltwater interface elevation and constant freshwater head

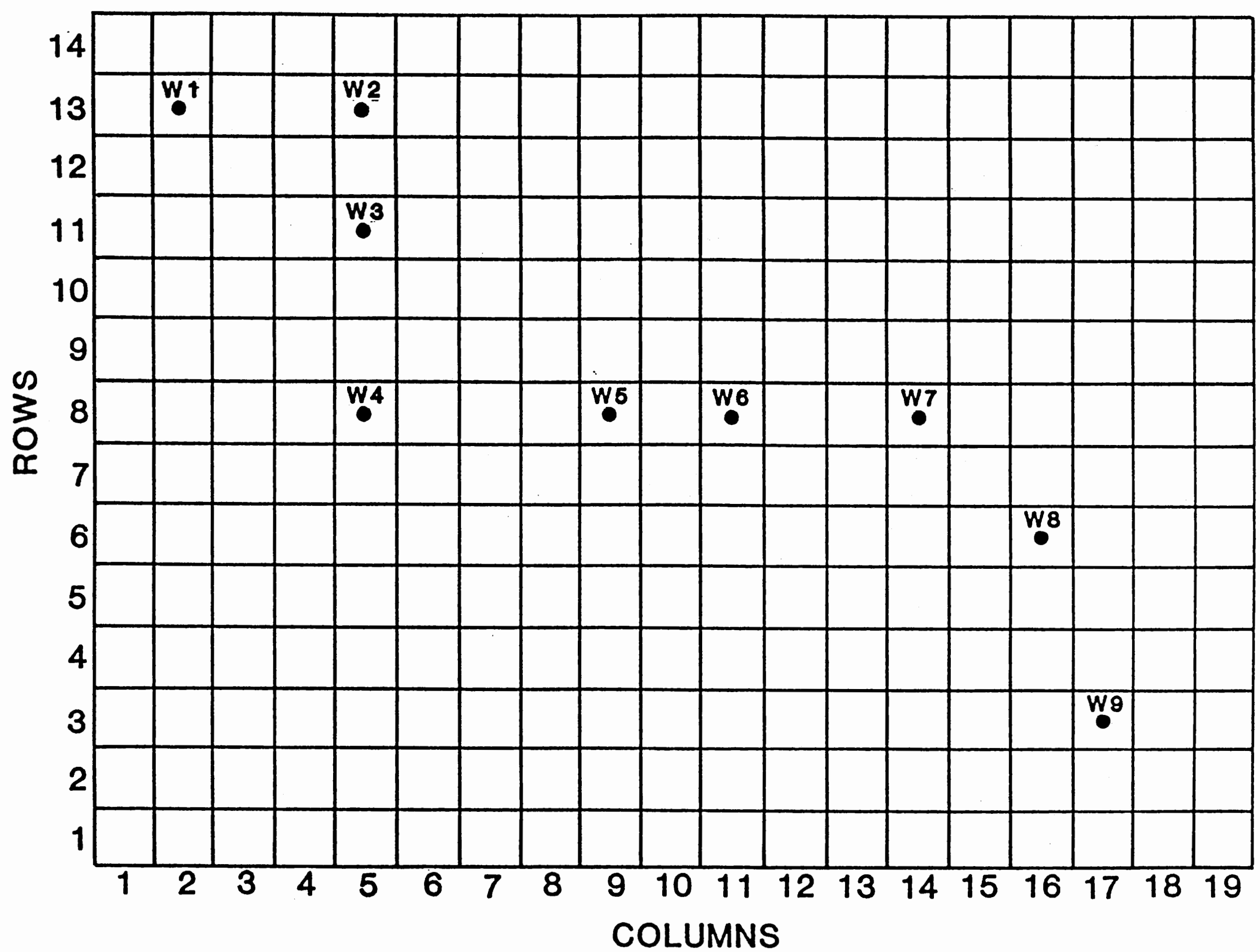

Figure 13. Discritization of the Yukon Well Field

throughout the simulation. Each node is assigned a value of permeability, an initial freshwater head and interface elevation, a thickness of the aquifer (for a confined aquifer) and a discharge rate. The format of input data in the model is given in Appendix C.

Numerical Performance of the Model

The numerical performance of the model, such as rate of convergence, accuracy, and stability, needs to be evaluated through well established mathematical relations. The convergence criterion is a condition in which the solution of the finite difference equation for a finte grid size approaches the true solution of the governing partial differential equations. Using Equation (4.21), the convergence criterion (LSOR over-relaxation factor) is calculated as 1.2 for the problem solved in this study. As shown in Appendix F, the model converges to the solution in every time step and, as the time of simulation progresses, the convergence rate becomes faster. The number of iterations required for every time step is the same in each direction of the LSOR solution, which is ture for the isotropic case.

The dynamic stability of the numerical solution is evaluated from the condition created by the size of the time step. A table in Appendix F also shows that for any size of time step the model converges to the solution. There is, however, an optimum size of time step that requires a minimum number of iterations to converge to the true solution. For the present problem, the range of this optimum time step is approximately 600-2400 days. For a time step below or beyond this optimum range, more iterations are required to converge to the true solution. The model is, however, unconditionally stable. A CPU time of 0.00684 is required to simulate a domain of 19 columns and 14 rows for five time steps (4650 days) either in the x- or in the y-direction.

Upconing in the Yukon Well Field

The numerical model has been applied to the Yukon well field of Oklahoma. There are nine discharge wells in this area, and locations of these wells are shown in Figure 10. The upconing below the pumping wells for different times of simulation is shown in Table I. The minimum and the maximum upconings are observed in well 2 and well 5, respectively. The upconing in wells 2 and 5 for a simulation period of about 4200 days is shown in Figure 14. The upconing in the Yukon well field increases in both the numerical and the analytical solutions, with the same trend as observed during the model verification.

From the concept of the critical rise of interface (Todd, 1980; Ayers, 1980), taking 50 percent of the depth of freshwater subdomains, the limiting rises over which a sudden rise of interface may occur are 173.0 and 161.0 ft. for wells 2 and 5, respectively. It takes about 4200 and 3200 days (approximately 11.5 and 9.0 years) for the interface to reach the critical point for wells 2 and 5, respectively, if pumping is continuous for the simulation period (see Figure 14). For the other seven wells, this time lies between 3200 and 4200 days. The freshwater and saltwater potentials, the interface elevation, and the interface upconed at the end of time steps 1 and 5 for the study area are presented in Appendix E.

In the Yukon well field, most wells are not installed at the optimum depths of penetration, the pumpage is not continuous and there exists interbedded shale and sandstone in the Garber-Wellington aquifer.

TABLE I

UPCONING OF FRESHWATER-SALTWATER INTERFACE AT THE PUMPING WELLS IN THE YUKON WELL FIELD

Length of Time Step (Days)	Simulation Time (Days)	Upconing Above the Initial Interface Position, Ft.																	
		Well 1		Well 2		Well 3		Well 4		Well 5		Well 6		Well 7		Well 8		Well 9	
		X	Y	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y
150	150	8.5	8.5	7.9	7.9	8.8	8.8	8.7	8.7	9.4	9.4	8.7	8.7	8.6	8.6	9.0	9.0	9.0	9.0
300	450	24.9	25.0	23.2	23.3	25.5	25.7	25.4	25.6	27.4	27.6	25.3	25.5	25.2	25.3	26.1	26.6	26.2	26.3
600	1050	55.5	55.6	51.9	52.0	56.6	56.8	56.3	56.4	61.1	61.2	56.5	56.6	56.5	56.5	57.6	57.7	58.3	58.4
1200	2250	108.5	108.6	102.1	102.2	110.2	110.3	109.0	109.1	119.5	119.7	110.9	111.0	111.8	112.0	111.5	111.6	113.7	113.8
2400	4650	191.9	192.0	182.9	182.8	195.3	195.0	191.1	191.2	213.7	214.2	198.5	198.5	202.4	202.4	194.9	195.0	201.2	201.2

X = the solution in the x-direction.

Y = the solution in the y-direction.

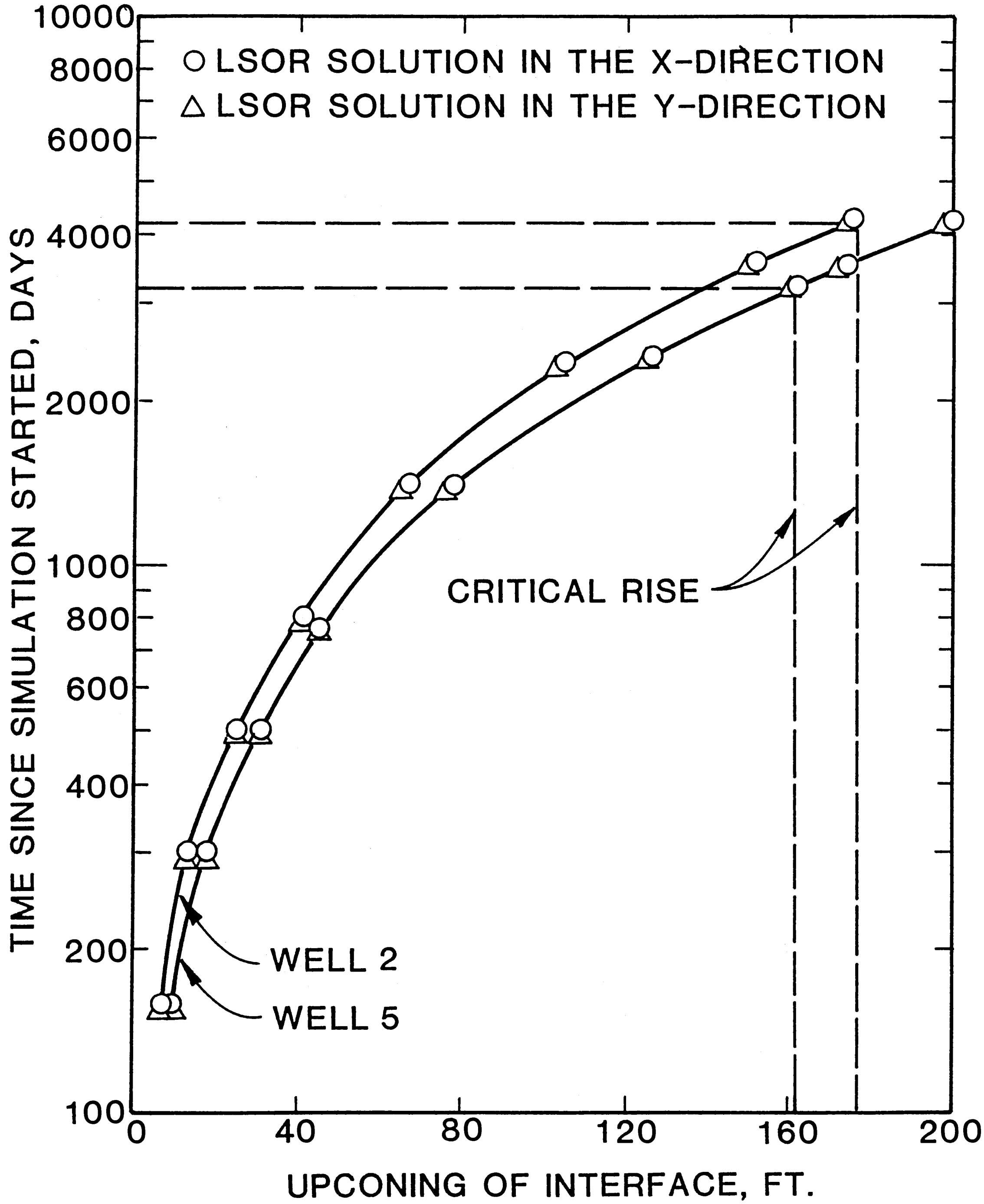

Figure 14. Upconing Below Well 2 and Well 5 in the Yukon Well Field With Rate of Pumping 200 gpm

Therefore, the interface upconing computed by applying the numerical model may not exactly represent the field conditions in the Garber-Wellington aquifer. The results obtained are useful, however, in that they give a detailed insight in the movement of the freshwater-saltwater interface in the study area. The application of the numerical model to the Yukon well field is intended to serve as an example rather than a final solution. The limitations of the existing hydrologic data must be considered seriously before the results of this study are used.

CHAPTER VII

SUMMARY AND CONCLUSIONS

A digital model written in ANSI standard FORTRAN has been developed for simulating the transient position of a freshwater-saltwater interface in an inland aquifer system. The model is capable of simulating the interface in confined/unconfined, isotropic/anisotropic and homogeneous/heterogeneous conditions in the horizontal plane. The model has the option to handle recharge and discharge in both saltwater and freshwater regions.

A finite difference technique to approximate the partial differential equations used to generate a system of algebraic equations. The line successive overrelaxation (LSOR) method is used as an iterative technique to solve the resulting banded matrix in conjunction with the modified form of the bitridiagonal algorithm (Von Rosenberg, 1969). The model has the option to apply the LSOR solution both in the x- and y-directions depending on the direction of anisotropy in the aquifer. This model has four subprograms controlled by a main program.

The performance of the numerical model is evaluated by comparing it with an analytical solution for a hypothetical situation. The model is applied to the Yukon, Oklahoma municipal well field.

Based on the results obtained from verification and application of the model, the following conclusions are made:

1. The LSOR finite difference numerical method solves the partial

differential equations that simulate the freshwater and saltwater potentials and the freshwater-saltwater interface in an inland aquifer.

2. The rate of convergence of LSOR in conjunction with the bitridiagonal algorithm to the solution of the problem is rapid.

3. The model is stable for any size of the time step. There is, however, an optimum size of time step for which the number of iterations is minimum. This time step ranges between 600 and 2400 days, with three iterations required to converge.

4. The optimum value of the overrelaxation factor, ω, obtained for the present problem, is 1.2, which is within the range of the optimum values of the LSOR overrelaxation factor.

5. The optimum penetration depth of a well is approximately 30 percent of the freshwater depth. The results of an analytical solution and the numerical model developed in this study are very close at the well. As the distance from the center of the well increases the difference between the results of the numerical model and the analytical solution increases; a maximum of four percent error develops between the two solutions at the well.

6. The results of the numerical model indicate that well 5 in the Yukon well field will become contaminated by saline water in about 9 years, and well 2 in about 11.5 years given the continuous pumpage of 200 gpm that was used throughout the simulation period. The other seven wells in the well field will be contaminated between 9 and 11.5 years.

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APPENDIX A

BITRIDIAGONAL ALGORITHM

To elaborate the bitridiagonal algorithm used in the text, systems of algebraic Equations (A.1) and (A.2), which are supposed to be generated for each row or for each column by the finite difference approximations of Equations (3.29) and (3.30) are considered. These equations are arranged in the residual form to be solved with the help of the LSOR technique and are analogous to Equations (4.9) and (4.10) for row by row solution or to Equations (4.16) and (4.17) for column by column solution.

$$a_i^{(1)}\xi_{f(i-1)} + a_i^{(2)}\xi_{s(i-1)} + b_i^{(1)}\xi_{fi} + b_i^{(2)}\xi_{si} + C_i^{(1)}\xi_{f(i+1)} + C_i^{(2)}\xi_{s(i+1)} = d_i^{(1)} \tag{A.1}$$

and

$$a_i^{(3)}\xi_{f(i-1)} + a_i^{(4)}\xi_{s(i-1)} + b_i^{(3)}\xi_{fi} + b_i^{(4)}\xi_{si} + C_i^{(3)}\xi_{f(i+1)} + C_i^{(4)}\xi_{s(i+1)} = d_i^{(2)} \tag{A.2}$$

for $1 \le i \le NC$

with $a_1^{(k)} = C_{NC}^{(k)} = 0$ for $1 \le k \le 4$.

Equations (A.1) and (A.2) can be conveniently written in compact bitridiagonal matrix form as Equation (4.11) for row by row solution and as Equation (4.18) for column by column solution.

The bitridiagonal algorithm (Von Rosenberg, 1969) is the only direct solution technique for solving systems of linear equations similar to Equations (A.1) and (A.2). The algorithm is a step by step triangular decomposition method that yields a recursion equation that substantially reduces computations and computer core storage.

The algorithm is as follows:

First Computes,

$$\beta_i^{(1)} = b_i^{(1)} - a_i^{(1)} \lambda_{i-1}^{(1)} - a_i^{(2)} \lambda_{i-1}^{(3)}$$

$$\beta_i^{(2)} = b_i^{(2)} - a_i^{(1)} \lambda_{i-1}^{(2)} - a_i^{(2)} \lambda_{i-1}^{(4)}$$

$$\beta_i^{(3)} = b_i^{(3)} - a_i^{(3)} \lambda_{i-1}^{(1)} - a_i^{(4)} \lambda_{i-1}^{(3)}$$

$$\beta_i^{(4)} = b_i^{(4)} - a_i^{(3)} \lambda_{i-1}^{(2)} - a_i^{(4)} \lambda_{i-1}^{(4)}$$

with $\beta_1^{(k)} = b_1^{(k)}$ for $1 \leq k \leq 4$

and

$$\sigma_i^{(1)} = d_i^{(1)} - a_i^{(1)} \gamma_{i-1}^{(1)} - a_i^{(2)} \gamma_{i-1}^{(2)}$$

$$\sigma_i^{(2)} = d_i^{(2)} - a_i^{(3)} \gamma_{i-1}^{(1)} - a_i^{(4)} \gamma_{i-1}^{(2)}$$

with $\sigma_1^{(1)} = d_1^{(1)}$ and $\sigma_1^{(2)} = d_1^{(2)}$

and

$$\mu_i = \beta_i^{(1)} \beta_i^{(4)} - \beta_i^{(2)} \beta_i^{(3)}$$

The $\beta_i^{(k)}$, $\sigma_i^{(k)}$ and μ_i are computed to aid in the computation of the following functions and need not to be stored after computation of

$$\lambda_i^{(1)} = (\beta_i^{(4)} c_i^{(1)} - \beta_i^{(2)} c_i^{(3)})/\mu_i$$

$$\lambda_i^{(2)} = (\beta_i^{(4)} C_i^{(2)} - \beta_i^{(2)} C_i^{(4)})/\mu_i$$

$$\lambda_i^{(3)} = (\beta_i^{(1)} C_i^{(3)} - \beta_i^{(3)} C_i^{(1)})/\mu_i$$

$$\lambda_i^{(4)} = (\beta_i^{(1)} C_i^{(4)} - \beta_i^{(3)} C_i^{(2)})/\mu_i$$

and

$$\gamma_i^{(1)} = (\beta_i^{(4)}\ \sigma_i^{(1)} - \beta_i^{(2)}\ \sigma_i^{(2)})/\mu_i$$

$$\gamma_i^{(2)} = (\beta_i^{(1)}\ \sigma_i^{(2)} - \beta_i^{(3)}\ \sigma_i^{(1)})/\mu_i$$

For back solution, the values of $\lambda_i^{(k)}$ and $\gamma_i^{(k)}$ are required, so these values must be stored. The back solution is

$$\xi_{fi} = \gamma_i^{(1)} - \lambda_i^{(1)}\ \xi_{f(i+1)} - \lambda_i^{(2)}\ \xi_{s(i+1)}$$

$$\xi_{si} = \gamma_i^{(2)} - \lambda_i^{(3)}\ \xi_{f(i+1)} - \lambda_i^{(4)}\ \xi_{s(i+1)}$$

for $(NC-1) \geq i \geq 1$

with

$$\xi_{fNC} = \gamma_{NC}^{(1)},\ \xi_{sNC} = \gamma_{NC}^{(2)}$$

APPENDIX B

LISTING OF COMPUTER PROGRAM

```
C
C
C     ****************************************************************
C     *                                                              *
C     *                                                              *
C     *        THIS PROGRAM SOLVES TWO VERTICALLY INTEGRATED         *
C     *                                                              *
C     *        NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS WHICH DESCRIBE *
C     *                                                              *
C     *        THE TRANSIENT POSITIONS OF FRESHWATER POTENTIAL,      *
C     *                                                              *
C     *        SALTWATER POTENTIAL AND FRESH-SALT WATER              *
C     *                                                              *
C     *        INTERFACE IN INLAND AQUIFER SYSTEM                    *
C     *                                                              *
C     *                                                              *
C     *        THE PROGRAM WAS DEVELOPED AND WRITTEN                 *
C     *                                                              *
C     *                            BY                                *
C     *                                                              *
C     *                 MOHAMMED MOZZAMMEL HOQUE                     *
C     *                                                              *
C     *                 WATER RESOURCES DIVISION                     *
C     *                                                              *
C     *                 SCHOOL OF CIVIL ENGINEERING                  *
C     *                                                              *
C     *                 OKLAHOMA STATE UNIVERSITY                    *
C     *                                                              *
C     *                       STILLWATER                             *
C     *                                                              *
C     *                        OKLAHOMA                              *
C     *                                                              *
C     *++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++*
C     *                                                              *
C     * ------------------MODEL VARIABLES AND PARAMETERS-------------*
C     *                                                              *
C     * HF(I,J)      INITIAL FRESHWATER HEAD AT POINT (I,J),(L),(FT) *
C     * HS(I,J)      INITIAL SALTWATER HEAD AT POINT (I,J),(L)       *
C     * HFT(I,J)     FRESHWATER HEAD AT THE BEGINNING OF TIME STEP AT *
C     *              POINT (I,J),(L)                                 *
C     * HST(I,J)     SALTWATER HEAD AT THE BEGINNING OF TIME STEP AT *
C     *              POINT (I,J),(L)                                 *
C     * HFF(I,J)     FRESHWATER HEAD AT THE PREVIOUS TIME STEP       *
C     *              AT POINT (I,J),(L)                              *
C     * HSS(I,J)     SALTWATER HEAD AT THE PREVIOUS TIME STEP AT     *
C     *              POINT (I,J),(L)                                 *
C     * HFA(I,J)     FRESHWATER HEAD AT POINT (I,J),(L)              *
C     * HSA(I,J)     SALTWATER HEAD AT POINT (I,J),(L)               *
C     * Z1(I,J)      INITIAL INTERFACE ELEVATION AT POINT (I,J),(L),(FT) *
C     * Z2(I,J)      INTERFACE ELEVATION AT THE END OF TIME STEP     *
C     *              AT POINT (I,J),(L)                              *
C     * ZC           CRITICAL RISE OF INTERFACE,(L)                  *
C     * Z3(I,J)      INTERFACE UPCONED AT POINT (I,J),(L)            *
C     * TH(I,J)      THICKNESS OF THE AQUIFER AT POINT (I,J),(L),(FT) *
C     * BF(I,J)      THICKNESS OF THE FRESHWATER REGION              *
C     *              AT POINT (I,J),(L)                              *
C     * BS(I,J)      THIKNESS OF THE SALTWATER REGION                *
C     *              AT POINT(I,J),(L)                               *
```

```
C    * ZO(I,J)        ELEVATION OF THE BOTTOM OF THE AQUIFER                  *
C    *                AT POINT (I,J),(L),(FT)                                 *
C    * TEMF(I,J)      VECTOR FOR TEMPORARY STORAGE OF FRESHWATER POTENTIAL*
C    *                AT POINT (I,J)                                          *
C    * TEMS(I,J)      VECTOR FOR TEMPORARY STORAGE OF SALTWATER POTENTIAL *
C    *                AT POINT (I,J)                                          *
C    * DELX(I)        GRID SPACING IN X-DIRECTION ,(L),(FT)                   *
C    * DELY(J)        GRID SPACING IN Y-DIRECTION,(L),(FT)                    *
C    * NC             NUMBER OF COLUMNS IN THE MODEL                          *
C    * NR             NUMBER OF ROWS IN THE MODEL                             *
C    * I              MODEL COLUMN NUMBER                                     *
C    * J              MODEL ROW NUMBER                                        *
C    * TIME           TOTAL TIME OF SIMULATION,(T)                            *
C    * DELT           INITIAL TIME STEP,(T),(DAYS)                            *
C    * THR            TIME OF SIMULATION IN HOUR                              *
C    * TDA            TIME OF SIMULATION IN DAY                               *
C    * TYR            TIME OF SIMULATION IN YEAR                              *
C    * CDLT           MULTIPLICATION FACTOR FOR INCREASING SUBSEQUENT         *
C    *                TIME STEP                                               *
C    * NUMT           TOTAL NUMBER OF TIME STEP IN SIMULATION                 *
C    * ITER           A COUNTER TO COUNT THE NUMBER OF ITERATION COMPLETED*
C    * KOUNT          A COUNTER TO COUNT THE NUMBER OF TIME STEP COMPLETED*
C    * MAXIT          MAXIMUM NUMBER OF ITERATIONS ALLOWED IN EVERY           *
C    *                TIME STEP                                               *
C    * TOL            CLOSURE CRITERION FOR CONVERGENCE,(L)                   *
C    * ERR            STEADY STATE ERROR CRITERION ,(L)                       *
C    * OMEG           LSOR   OVERRELAXATION FACTOR                            *
C    * LP             PRINTER UNIT NUMBER                                     *
C    * IN             READER UNIT NUMBER                                      *
C    * POR(I,J)       POROSITY OF THE AQUIFER AT POINT (I,J)
C    * ALPH           UNCONFINED/CONFINED INDICATOR                           *
C    *                ALPH=1; IF THE AQUIFER IS UNCONFINED                    *
C    * BETA           ISOTROPY/ANISOTROPY INDICATOR                           *
C    *                BETA=1;IF THE AQUIFER IS ANISOTROPIC                    *
C    * DENF           DENSITY OF FRESHWATER,(M/L**3)                          *
C    * DENS           DENSITY OF SALTWATER,(M/L**3)                           *
C    * SC(I,J)=       SPECIFIC STORAGE/STORAGE COEFFICIENT OF THE AQUIFER
C    *                AT POINT (I,J)                                          *
C    * KFX(I,J)       PERMEABILITY OF FRESHWATER REGION IN X-DIRECTIO         *
C    *                AT POINT (I,J),(L/T),(GPD/FT2)                          *
C    * KFY(I,J)       PERMEABILITY OF FRESHWATER REGION IN Y-DIRECTION        *
C    *                AT POINT (I,J),(L/T),(GPD/FT2)                          *
C    * KSX(I,J)       PERMEABILITY OF SALTWATER REGION IN X-DIRECTION         *
C    *                AT POINT (I,J),(L/T)                                    *
C    * KSY(I,J)       PERMEABILITY OF SALTWATER REGION IN Y-DIRECTION         *
C    *                AT POINT (I,J),(L/T)                                    *
C    * VIF            VISCOSITY OF FRESHWATER,(FT/L**2)
C    * VIS            VISCOSITY OF SALTWATER,(FT/L**2)
C    * TFR            HARMONIC MEAN OF KFX*BF/DELX AT (I+1/2,J),(L/T)         *
C    * TFC            HARMONIC MEAN OF KFY*BF/DELY AT (I,J+1/2),(L/T)         *
C    * TSR            HARMONIC MEAN OF KSX*BS/DELX AT (I+1/2,J),(L/T)         *
C    * TSC            HARMONIC MEAN OF KSY*BS/DELY AT (I,J+1/2),(L/T)         *
C    * QF(I,J)        FRESHWATER DISCHARGE/RECHARGE AT POINT (I,J),(L**3/T)
C    *                ,(GPD)                                                  *
C    * QS(I,J)        SALTWATER DISCHARGE/RECHARGE AT POINT (I,J),(L**3/T)*
C    *                ,(GPD)                                                  *
C    * NQ             NUMBER OF SOURCE/SINK                                   *
C    * K1             INDICATOR OF UNIFORMITY OF SPACING IN X-DIRECTION       *
```

```
C    *               K1=1,IF X-SPACING IS EQUAL                              *
C    * K2            INDICATOR OF UNIFORMITY OF SPACING IN Y-DIRECTION       *
C    *               K2=1,IF Y-SPACING IS EQUAL                              *
C    * K3            INDICATOR OF UNIFORMITY OF INITIAL FRESHWATER HEAD      *
C    *               K3=1,IF INITIAL FRESHWATER HEAD IS UNIFORM              *
C    * K4            INDICATOR OF UNIFORMITY OF INITIAL INTERFACE            *
C    *               K4=1,IF INITIAL INTERFACE IS UNIFORM                    *
C    * K5            INDICATOR OF UNIFORMITY OF PERMEABILITY IN              *
C    *               X-DIRECTION;K5=1,IF X-DIRECTION PERMEABILITY            *
C    *               IS UNIFORM                                              *
C    * K6            INDICATOR OF UNIFORMITY OF PERMEABILITY                 *
C    *               Y-DIRECTION; K6=1,IF Y-DIRECTION PERMEABILITY           *
C    *               IS UNIFORM                                              *
C    * K7            INDICATOR OF UNIFORMITY OF POROSITY                     *
C    *               K7=1,IF POROSITY IS UNIFORM                             *
C    * K8            INDICATOR OF UNIFORMITY OF AQUIFER THICKNESS            *
C    *               K8=1,IF THICKNESS IS UNIFORM                            *
C    * K9            INDICATOR OF UNIFORMITY OF STORAGE COEFFICIENT          *
C    *               K9=1,IF STORAGE COEFFICIENT IS UNIFORM                  *
C    * K10           INDICATOTOR OF UNIFORMITY OF AQUIFER BOTTOM ELEVATIN*
C    *               K10=1,IF BOTTOM ELEVATION IS UNIFORM                    *
C    * DI            INDICATOR FOR LSOR SOLUTION DIRECTION                   *
C    *               FOR SOLUTION IS IN THE Y-DIRECTION DI=1.,OTHERWISE      *
C    *               ANY OTHER VALUE                                         *
C    * F1            FACTOR FOR MULTIPLYING SPACING IN X-DIRECTION           *
C    * F2            FACTOR FOR MULTIPLYING SPACING IN Y-DIRCTION            *
C    * F3            FACTOR FOR MULTIPLYING INITIAL FRESHWATER HEAD          *
C    * F4            FACTOR FOR MULTIPLYING INITIAL INTERFACE                *
C    * F5            FACTOR FOR MULTIPLYING PERMEABILITY IN X-DIRECTION      *
C    * F6            FACTOR FOR MULTIPLYING PERMEABILITY IN Y-DIRECTION      *
C    * F7            FACTOR FOR MULTIPLYING POROSITY                         *
C    * F8            FACTOR FOR MULTIPLYING THICKNESS OF AQUIFER             *
C    * F9            FACTOR FOR MULTIPLYING STORAGE COEFFICIENT              *
C    * F10           FACTOR FOR MULTIPLYING BOTTOM ELEVATION                 *
C    *                                                                       *
C    *************************************************************************
C
C
C    * MAIN PROGRAM
C
C
      DOUBLE PRECISION DABS,TEMF,TEMS
C
      DIMENSION KFX(19,14),KFY(19,14),KSX(19,14),KSY(19,14),SC(19,14),
     1ZO(19,14),QF(19,14),QS(19,14),Z1(19,14),HS(19,14),HF(19,14),
     2BF(19,14),BS(19,14),TH(19,14),DELX(19),DELY(14),TFR(19,14),
     3TFC(19,14),TSR(19,14),TSC(19,14),HSA(19,14),HFA(19,14),HFF(19,14),
     4HSS(19,14),POR(19,14),TEMF(19,14),TEMS(19,14),LAM1(19),LAM2(19),
     5LAM3(19),LAM4(19),GAM1(19),GAM2(19),AAF(19),BBF(19),CCF(19),
     6DDF(19),AAS(19),BBS(19),CCS(19),DDS(19),CF1(19),CS1(19),CF2(19),
     7CS2(19),DF(19),DS(19),EEF(19),EES(19),HFT(19,14),HST(19,14),
     8Z2(19,14),Z3(19,14)
C
      REAL KFX,KFY,KSX,KSY,MMF,MMS,MEW,LAM1,LAM2,LAM3,LAM4
C
      DATA IN,LP/5,6/
      DATA NC,NR/19,14/
C
```

```
C
C     ----------TO READ AND WRITE INPUT DATA FOR SIMULATION------------
      CALL DATA(KFX,KFY,Z1,ZO,HF,TH,DELX,DELY,POR,SC,DELT,DENF,DENS,
     $ALPH,NUMT,MAXIT,OMEG,TOL,ERR,QF,QS,NQ,CDLT,VIS,VIF,NC,NR)
C
      DELTA  = DELT
C
C     -------------TO COMPUTE INITIAL SALTWATER POTENTIALS--------------
      SIGM = DENF/(DENS-DENF)
      DO 4 J = 1,NR
      DO 4 I=1,NC
    4 HS(I,J)=(Z1(I,J)+HF(I,J)*SIGM)/(1.+SIGM)
C
      WRITE(LP,5)
    5 FORMAT(1H1,50X,22HINITIAL SALTWATER HEAD)
      WRITE(LP,3)
    3 FORMAT(51X,22H----------------------)
      DO 6 J=1,NR
    6 WRITE(LP,7) J,(HS(I,J),I=1,NC)
    7 FORMAT(//10X,4HROW=,I2//(10X,8G15.7))
C
      DO 8 J=1,NR
      DO 8 I=1,NC
      Z2(I,J)=Z1(I,J)
      HFT(I,J)=HF(I,J)
    8 HST(I,J)=HS(I,J)
C
      DO 35 J=1,NR
      DO 35 I=1,NC
      KSX(I,J)=KFX(I,J)*(VIF*DENS/VIS*DENF)
   35 KSY(I,J)=KFY(I,J)*(VIF*DENS/VIS*DENF)
C
      DO 36 J=1,NR
      DO 36 I=1,NC
      QF(I,J)=QF(I,J)/7.48
      QS(I,J)=QS(I,J)/7.48
      KFX(I,J)=KFX(I,J)/7.48
      KFY(I,J)=KFY(I,J)/7.48
      KSX(I,J)=KSX(I,J)/7.48
   36 KSY(I,J)=KSY(I,J)/7.48
C
C     --------------TO START WITH TIME LOOP----------------------------
      TIME = 0.
      DO 90 KOUNT=1,NUMT
      DELT = DELTA
   10 TIME = TIME + DELT
C
      DO 20 J=1,NR
      DO 20 I = 1,NC
      HFF(I,J)=HFT(I,J)
   20 HSS(I,J)=HST(I,J)
C
      WRITE(LP,30) KOUNT,DELT
   30 FORMAT(1H1,9X,10HTIME STEP=,I2,5X,20HLENGTH OF TIME STEP=,G15.7)
C
C
C     -------------TO CALL SUBROUTINE SOLV------------------------------
      CALL SOLV(KFX,KFY,DELX,DELY,NC,NR,BF,BS,Z2,ZO,TH,HFA,HFF,TFR,TFC,
```

```
     1TSR,TSC,HSA,HSS,TEMF,TEMS,POR,ALPH,OMEG,MAXIT,TOL,QF,QS,SIGM,SC,
     2DELT,VIF,VIS,LAM1,LAM2,LAM3,LAM4,GAM1,GAM2,KSX,KSY,BBF,AAF,DDF,
     3CCF,BBS,AAS,DDS,CCS,CF1,CS1,CF2,CS2,DF,DS,EEF,EES)
C
C      -------------UPDATE THE SALTWATER AND FRESHWATER POTENTIALS-------
      NRR=NR-1
      NCC=NC-1
      DO 50 J=2,NRR
      DO 50 I=2,NCC
      HFT(I,J)=HFA(I,J)
   50 HST(I,J)=HSA(I,J)
C
C
C      -------------TO COMPUTE NEW INTERFACE POSITION--------------------
      DO 60 J=2,NRR
      DO 60 I=2,NCC
   60 Z2(I,J)=HST(I,J)*(1.+SIGM)-HFT(I,J)*SIGM
C
C      ---TO INITIALIZE SOME VARIABLES-----
      DO 75 J=1,NR
      DO 75 I=1,NC
   75 Z3(I,J)=0.
C
C
      DO 80 J=2,NRR
      DO 80 I=2,NCC
   80 Z3(I,J)=Z2(I,J)-Z1(I,J)
C      -------------TO CALL SUBROUTINE PRINT---------------------------
      CALL PRINT(HFT,HST,Z2,Z3,NC,NR,TIME)
C
      DO 85 J=1,NR
      DO 85 I=1,NC
      ZC=Z3(I,J)
      IF(ZC.GE.173.) GO  TO 2
   85 CONTINUE
C
      DELTA=DELT*CDLT
C
   90 CONTINUE
    2 WRITE(LP,1)
    1 FORMAT(1H1)
      STOP
C
      END
```

```
C     ------------------------------------------------------------
C
      SUBROUTINE DATA(KFX,KFY,Z1,ZO,HF,TH,DELX,DELY,POR,SC,DELT,DENF,
     $DENS,ALPH,NUMT,MAXIT,OMEG,TOL,ERR,QF,QS,NQ,CDLT,VIS,VIF,NC,NR)
C
C     ------------------------------------------------------------
C
      DIMENSION KFX(NC,NR),KFY(NC,NR),Z1(NC,NR),ZO(NC,NR),SC(NC,NR),
     $HF(NC,NR),TH(NC,NR),DELX(NC),DELY(NR),QF(NC,NR),QS(NC,NR),
     $POR(NC,NR)
C
      REAL KFX,KFY
C
      DATA IN,LP/5,6/
      READ(IN,10) NUMT,MAXIT,NQ
   10 FORMAT(3I3)
      WRITE(LP,20) NUMT,MAXIT,NQ
   20 FORMAT(1H1,9X,5HNUMT=,I3,2X,6HMAXIT=,I3,2X,6HPUMPS=,I3)
      WRITE(LP,40) NC,NR
   40 FORMAT(//10X,7HCOLUMN=,I3,5X,4HROW=,I3)
C     TO READ AND WRITE TIME PARAMETERS
      READ(IN,70) DELT,OMEG
   70 FORMAT(2F11.7)
      WRITE(LP,80) DELT,OMEG
   80 FORMAT(//10X,5HDELT=,G15.7,5X,5HOMEG=,G15.7)
      READ(IN,90)ALPH,BETA
   90 FORMAT(2F11.5)
      IF(ALPH.EQ.0.) WRITE(LP,100) ALPH
      IF(ALPH.EQ.1.) WRITE(LP,110) ALPH
  100 FORMAT(//10X,5HALPH=,G15.7,5X,16HCONFINED AQUIFER)
  110 FORMAT(//10X,5HALPH=,G15.7,5X,18HUNCONFINED AQUIFER)
C     TO READ AND WRITE CODE FOR DATA
      READ(IN,120) K1,K2 ,K3,K4 ,K5,K6,K7,K8,K9,K10
  120 FORMAT(10I2)
      WRITE(LP,121)
  121 FORMAT(//10X,29HUNIFORMITY/NONUNIFORMITY CODE)
      WRITE(LP,125) K1,K2,K3,K4,K5,K6,K7,K8,K9,K10
  125 FORMAT(//10X,10I2)
      IF(K1.EQ.1) WRITE(LP,140)
  140 FORMAT(//10X,32HSPACING IN X-DIRECTION ARE EQUAL)
      IF(K2.EQ.1) WRITE(LP,150)
  150 FORMAT(//10X,32HSPACING IN Y-DIRECTION ARE EQUAL)
      IF(K3.EQ.1) WRITE(LP,160)
  160 FORMAT(//10X,34HINITIAL FRESHWATER HEAD IS UNIFORM)
      IF(K4.EQ.1) WRITE(LP,170)
  170 FORMAT(//10X,28HINITIAL INTERFACE IS UNIFORM)
      IF(K5.EQ.1) WRITE(LP,180)
  180 FORMAT(//10X,38HPERMEABILITY IN X-DIRECTION IS UNIFORM)
      IF(K6.EQ.1) WRITE(LP,190)
  190 FORMAT(//10X,38HPERMEABILITY IN Y-DIRECTION IS UNIFORM)
      IF(K7.EQ.1) WRITE(LP,200)
  200 FORMAT(//10X,19HPOROSITY IS UNIFORM)
      IF(K8.EQ.1) WRITE(LP,210)
  210 FORMAT(//10X,28HAQUIFER THICKNESS IS UNIFORM)
      IF(K9.EQ.1) WRITE(LP,220)
  220 FORMAT(//10X,47HSTORAGE COEFFICIENT/SPECIFIC STORAGE IS UNIFORM)
      IF(K10.EQ.1) WRITE(LP,230)
  230 FORMAT(//10X,35HAQUIFER BOTTOM ELEVATION IS UNIFORM)
```

```
C     TO READ AND WRITE THE MULTIPLICATION FACTOR
      READ(IN,240) F1,F2,F3,F4,F5,F6,F7,F8,F9,F10
  240 FORMAT(5F11.5)
      IF(F1.EQ.O.) F1 = 1.
      IF(F2.EQ.O.) F2 = 1.
      IF(F3.EQ.O.) F3 = 1.
      IF(F4.EQ.O.) F4 = 1.
      IF(F5.EQ.O.) F5 = 1.
      IF(F6.EQ.O.) F6 = 1.
      IF(F7.EQ.O.) F7 = 1.
      IF(F8.EQ.O.) F8 = 1.
      IF(F9.EQ.O.) F9 = 1.
      IF(F10.EQ.O.) F10 = 1.
      WRITE(LP,250) F1,F2,F3,F4,F5,F6,F7,F8,F9,F10
  250 FORMAT(//10X,22HMULTIPLICATION FACTORS//(10X,5G10.5))
C     TO READ AND WRITE SPACINGS IN X-DIRECTION
      IF(K1.EQ.1) GO TO 270
      READ(IN,260)(DELX(I),I=1,NC)
  260 FORMAT(5F11.5)
      GO TO 300
  270 READ(IN,280) DELX(1)
  280 FORMAT(F11.5)
      DO 290 I = 1,NC
  290 DELX(I) = DELX(1)
  300 DO 310 I = 1,NC
  310 DELX(I) = DELX(I)*F1
      WRITE(LP,312)
  312 FORMAT(1H1,45X,12HGRID SPACING)
      WRITE(LP,314)
  314 FORMAT(46X,12H------------)
      WRITE(LP,320)(DELX(I),I = 1,NC)
  320 FORMAT(//10X,11HX-DIRECTION//(10X,8G12.7))
C     TO READ AND WRITE SPACINGS IN Y-DIRECTION
      IF(K2.EQ.1) GO TO 340
      READ(IN,330)(DELY(J),J = 1,NR)
  330 FORMAT(5F11.5)
      GO TO 370
  340 READ(IN,350) DELY(1)
  350 FORMAT(F11.5)
      DO 360 J = 1,NR
  360 DELY(J) = DELY(1)
  370 DO 380 J = 1,NR
  380 DELY(J) = DELY(J)*F2
      WRITE(LP,390)(DELY(J), J = 1,NR)
  390 FORMAT(//10X,11HY-DIRECTION//(10X,8G12.7))
C     TO READ AND WRITE INITIAL FRESH WATER HEADS
      WRITE(LP,400)
  400 FORMAT(1H1,50X,24HINITIAL FRESHWATER  HEAD)
      WRITE(LP,405)
  405 FORMAT(51X,24H------------------------)
      CALL ARRY(HF,K3,NC,NR,F3)
C     TO READ AND WRITE INITIAL INTERFACE ELEVATION
      WRITE(LP,410)
  410 FORMAT(1H1,50X,27HINITIAL INTERFACE ELEVATION)
      WRITE(LP,415)
  415 FORMAT (51X,27H---------------------------)
      CALL ARRY(Z1,K4,NC,NR,F4)
C     TO READ AND WRITE PERMEABILITY IN X-DIRECTION
```

```
      WRITE(LP,420)
  420 FORMAT (1H1,50X,27HPERMEABILITY IN X-DIRECTION)
      WRITE(LP,425)
  425 FORMAT(51X,27H---------------------------)
      CALL ARRY(KFX,K5,NC,NR,F5)
C     TO READ AND WRITE PERMEABILITY IN Y-DIRECTION
      IF(BETA.EQ.1.) GO TO 429
      WRITE(LP,424)BETA
  424 FORMAT(1H1,10X,5HBETA=,G15.7,5X,17HISOTROPIC AQUIFER)
      DO 426 J=1,NR
      DO 426 I=1,NC
  426 KFY(I,J)=KFX(I,J)
      WRITE(LP,438)
  438 FORMAT(//51X,27HPERMEABILITY IN Y-DIRECTION)
      WRITE(LP,439)
  439 FORMAT(51X,27H---------------------------)
      DO 427 J=1,NR
  427 WRITE(LP,428) J,(KFY(I,J),I=1,NC)
  428 FORMAT(//10X,4HROW=,I2//(10X,8G15.7))
      GO TO 434
  429 WRITE(LP,430)
  430 FORMAT(1H1,50X,27HPERMEABILITY IN Y-DIRECTION)
      WRITE(LP,432)
  432 FORMAT(51X,27H---------------------------)
      CALL ARRY(KFY,K6,NC,NR,F6)
C     TO READ AND WRITE POROSITY OF AQUIFER
  434 WRITE(LP,435)
  435 FORMAT(1H1,50X,19HPOROSITY OF AQUIFER)
      WRITE(LP,437)
  437 FORMAT(51X,19H-------------------)
      CALL ARRY(POR,K7,NC,NR,F7)
      IF(ALPH.EQ.1.)GO TO 442
C     TO READ AND WRITE STORAGE COEFFICIENT/SPECIFIC STORAGE
      WRITE(LP,440)
  440 FORMAT (1H1,50X,19HSTORAGE COEFFICIENT)
      WRITE(LP,441)
  441 FORMAT(51X,19H-------------------)
      GO TO 448
  442 WRITE(LP,445)
  445 FORMAT(1H1,50X,16HSPECIFIC STORAGE)
      WRITE(LP,446)
  446 FORMAT(51X,16H----------------)
  448 CALL ARRY(SC,K9,NC,NR,F9)
C     TO READ AND WRITE AQUIFER BOTTOM ELEVATION
      WRITE(LP,450)
  450 FORMAT(1H1,50X,24HAQUIFER BOTTOM ELEVATION)
      WRITE(LP,551)
  551 FORMAT(51X,24H------------------------)
      CALL ARRY(Z0,K10,NC,NR,F10)
C     TO READ AND WRITE THICKNESS OF AQUIFER
      WRITE(LP,455)
  455 FORMAT (1H1,50X,20HTHICKNESS OF AQUIFER)
      WRITE(LP,456)
  456 FORMAT (51X,20H--------------------)
      CALL ARRY(TH,K8,NC,NR,F8)
C     TO READ AND WRITE THE DENSITIES
      READ(IN,460) DENF,DENS
  460 FORMAT(2F11.5)
```

```
      WRITE(LP,470) DENF,DENS
  470 FORMAT(1H1,9X,5HDENF=,G15.7,2X,5HDENS=,G15.7)
      READ(IN,480) TOL,CDLT
  480 FORMAT(2F11.5)
      WRITE(LP,490) TOL,CDLT
  490 FORMAT(//10X,4HTOL=,G15.7,2X,5HCDLT=,G15.7)
      READ(IN,484) VIF,VIS
  484 FORMAT(2F10.6)
      WRITE(LP,486) VIF,VIS
  486 FORMAT(//10X,4HVIF=,G15.7,2X,4HVIS=,G15.7)
C     TO READ AND WRITE SOURCE/SINK
      DO 500 I=1,NC
      DO 500 J=1,NR
      QS(I,J)=0.
  500 QF(I,J)=0.
      IF(NQ.EQ.0.) GO TO 540
      DO 510 K=1,NQ
  510 READ(IN,520) I,J,QF(I,J)
  520 FORMAT(2I2,F10.5)
      WRITE(LP,521)
  521 FORMAT (//50X,11HSOURCE/SINK)
      WRITE(LP,522)
  522 FORMAT(50X,11H-----------)
      DO 525 J=1,NR
  525 WRITE(LP,530) J,(QF(I,J),I=1,NC)
  530 FORMAT(//10X,4HROW=,I2//(10X,8G15.7))
  540 CONTINUE
      RETURN
      END

C
C     ------------------------------------------------------------------
C
      SUBROUTINE ARRY(ARR,NCOD,NC,NR,FAC)
C
C     ------------------------------------------------------------------
C
      DIMENSION ARR(NC,NR)
      DATA IN,LP/5,6/
      IF(NCOD.EQ.1) GO TO 40
      DO 10 J = 1,NR
   10 READ(IN,20)(ARR(I,J),I=1,NC)
   20 FORMAT(7F10.5)
      GO TO 60
   40 READ(IN,50) ARR(1,1)
   50 FORMAT(F11.5)
      DO 55 J=1,NR
      DO 55 I=1,NC
   55 ARR(I,J)=ARR(1,1)
   60 DO 70 J = 1,NR
      DO 70 I = 1,NC
   70 ARR(I,J) = ARR(I,J)*FAC
      DO 80 J = 1,NR
   80 WRITE(LP,90) J,(ARR(I,J),I = 1,NC)
   90 FORMAT(//10X,4HROW=,I2//(10X,8G15.7))
      RETURN
      END
```

```
C
C     ------------------------------------------------------------------
C
      SUBROUTINE SOLV(KFX,KFY,DELX,DELY,NC,NR,BF,BS,Z2,ZO,TH,HFA,HFF,
     1TFR,TFC,TSR,TSC,HSA,HSS,TEMF,TEMS,POR,ALPH,OMEG,MAXIT,TOL,QF,QS,
     2SIGM,SC,DELT,VIF,VIS,LAM1,LAM2,LAM3,LAM4,GAM1,GAM2,KSX,KSY,BBF,
     3AAF,DDF,CCF,BBS,AAS,DDS,CCS,CF1,CS1,CF2,CS2,DF,DS,EEF,EES)
C
C     ------------------------------------------------------------------
C
      DOUBLE PRECISION DABS,TEMF,TEMS
C
      DIMENSION KFX(NC,NR),KFY(NC,NR),DELX(NC),DELY(NR),BF(NC,NR),
     $Z2(NC,NR),ZO(NC,NR),TH(NC,NR),HFA(NC,NR),HFF(NC,NR),SC(NC,NR),
     2TFR(NC,NR),TFC(NC,NR),TEMF(NC),TEMS(NC),BS(NC,NR),POR(NC,NR),
     3QF(NC,NR),QS(NC,NR),KSX(NC,NR),KSY(NC,NR),LAM1(NC),LAM2(NC),
     4LAM3(NC),LAM4(NC),GAM1(NC),GAM2(NC),HSA(NC,NR),HSS(NC,NR),
     $TSR(NC,NR),TSC(NC,NR),BBF(NC),AAF(NC),CCF(NC),DDF(NC),BBS(NC),
     $AAS(NC),DDS(NC),CCS(NC),CF1(NC),CS1(NC),CF2(NC),CS2(NC),EEF(NC),
     $EES(NC),DF(NC),DS(NC)
C
      REAL KFX,KFY,KSX,KSY,MMF,MMS,MEW,LAM1,LAM2,LAM3,LAM4
C
      DATA IN,LP/5,6/
      DATA DI/1./
C
      ITER = 0
      DO 5 J=1,NR
      DO 5 I=1,NC
      HFA(I,J)=HFF(I,J)
    5 HSA(I,J)=HSS(I,J)
      IF(DI.NE.1.) WRITE(LP,32)
   32 FORMAT(//10X,30HSOLUTION IS IN THE X-DIRECTION)
      IF(DI.EQ.1.) WRITE(LP,86)
   86 FORMAT(//10X,30HSOLUTION IS IN THE Y-DIRECTION)
C     -------------TO START WITH ITERATION LOOP-------------------------
    6 ITER=ITER+1
      ERR = 0.
C
      DO 8 J=1,NR
      DO 8 I=1,NC
    8 BS(I,J)=Z2(I,J)
      DO 10 J=1,NR
      DO 10 I=1,NC
      BF(I,J)=HFF(I,J)-BS(I,J)
      IF(ALPH.NE.1.) BF(I,J)=TH(I,J)-BS(I,J)
   10 CONTINUE
C     ----------TO CALCULATE THE HARMONIC MEAN ---------
      NRR=NR-1
      NCC=NC-1
      DO 30 J=1,NRR
      DO 30 I=1,NCC
      TFR(I,J)=(2.*KFX(I+1,J)*BF(I+1,J)*KFX(I,J)*BF(I,J))/(KFX(I,J)*BF(I
     1,J)*DELX(I+1)+KFX(I+1,J)*BF(I+1,J)*DELX(I))
      TFC(I,J)=(2.*KFY(I,J+1)*BF(I,J+1)*KFY(I,J)*BF(I,J))/(KFY(I,J)*BF(I
     1,J)*DELY(J+1)+KFY(I,J+1)*BF(I,J+1)*DELY(J))
      TSR(I,J)=(2.*KSX(I+1,J)*BS(I+1,J)*KSX(I,J)*BS(I,J))/(KSX(I,J)*BS(I
     1,J)*DELX(I+1)+KSX(I+1,J)*BS(I+1,J)*DELX(I))
```

```
      TSC(I,J)=(2.*KSY(I,J+1)*BS(I,J+1)*KSY(I,J)*BS(I,J))/(KSY(I,J)*BS(I
     1,J)*DELY(J+1)+KSY(I,J+1)*BS(I,J+1)*DELY(J))
   30 CONTINUE
      IF(DI.EQ.1.) GO TO 85
C
C     ----------TO COMPUTE THE ELEMENTS OF COEFFICIENT MATRIX-----------
C     --------------AND THE COLUMN VECTORS-----------------------------
      DO 80 J=2,NRR
      DO 60 I=2,NCC
      BBF(I)=TFR(I,J)/DELX(I)
      AAF(I)=TFR(I-1,J)/DELX(I)
      DDF(I)=TFC(I,J)/DELY(J)
      CCF(I)=TFC(I,J-1)/DELY(J)
C
      AAS(I)=TSR(I-1,J)/DELX(I)
      BBS(I)=TSR(I,J)/DELX(I)
      DDS(I)=TSC(I,J)/DELY(J)
      CCS(I)=TSC(I,J-1)/DELY(J)
C
      CS1(I)=POR(I,J)*(1.+SIGM)/DELT
      CF2(I)=POR(I,J)*SIGM/DELT
      IF(ALPH.NE.1.) GO TO 40
      CF1(I)=(POR(I,J)*SIGM+SC(I,J)*BF(I,J)+ALPH*POR(I,J))/DELT
      CS2(I)=(POR(I,J)*(1.+SIGM)+SC(I,J)*BS(I,J))/DELT
      GO TO 50
   40 CONTINUE
      CF1(I)=(POR(I,J)*SIGM+SC(I,J)*BF(I,J)/TH(I,J)+ALPH*POR(I,J))/DELT
      CS2(I)=(POR(I,J)*(1.+SIGM)+SC(I,J)*BS(I,J)/TH(I,J))/DELT
   50 CONTINUE
C
      EEF(I)=-(AAF(I)+BBF(I)+CCF(I)+DDF(I)+CF1(I))
      EES(I)=-(AAS(I)+BBS(I)+CCS(I)+DDS(I)+CS2(I))
C
      MMF=QF(I,J)/(DELX(I)*DELY(J))-CF1(I)*HFF(I,J)+CS1(I)*HSS(I,J)
      MMS=QS(I,J)/(DELX(I)*DELY(J))-CS2(I)*HSS(I,J)+CF2(I)*HFF(I,J)
C
      DF(I)=MMF-AAF(I)*HFA(I-1,J)-EEF(I)*HFA(I,J)-CS1(I)*HSA(I,J)-BBF(I)
     $ *HFA(I+1,J)-CCF(I)*HFA(I,J-1)-DDF(I)*HFA(I,J+1)
      DS(I)=MMS-AAS(I)*HSA(I-1,J)-EES(I)*HSA(I,J)-CF2(I)*HFA(I,J)-BBS(I)
     $ *HSA(I+1,J)-CCS(I)*HSA(I,J-1)-DDS(I)*HSA(I,J+1)
   60 CONTINUE
C
C     ----------TO COMPUTE INTERMEDIATE VECTORS FOR SOLUTION ALGORITHM--
C
      BET1=EEF(2)
      BET2=CS1(2)
      BET3=CF2(2)
      BET4=EES(2)
C
      SIGM1=DF(2)
      SIGM2=DS(2)
C
      MEW=BET1*BET4-BET2*BET3
C
      LAM1(2)=BET4*BBF(2)/MEW
      LAM2(2)=-BET2*BBS(2)/MEW
      LAM3(2)=-BET3*BBF(2)/MEW
      LAM4(2)=BET1*BBS(2)/MEW
```

```
C
      GAM1(2)=(BET4*SIGM1-BET2*SIGM2)/MEW
      GAM2(2)=(BET1*SIGM2-BET3*SIGM1)/MEW
C
      DO 65 K=3,NCC
      BET1=EEF(K)-AAF(K)*LAM1(K-1)
      BET2=CS1(K)-AAF(K)*LAM2(K-1)
      BET3=CF2(K)-AAS(K)*LAM3(K-1)
      BET4=EES(K)-AAS(K)*LAM4(K-1)
C
      MEW=BET1*BET4-BET2*BET3
C
      LAM1(K)=BET4*BBF(K)/MEW
      LAM2(K)=-BET2*BBS(K)/MEW
      LAM3(K)=-BET3*BBF(K)/MEW
      LAM4(K)=BET1*BBS(K)/MEW
C
      SIGM1=DF(K)-AAF(K)*GAM1(K-1)
      SIGM2=DS(K)-AAS(K)*GAM2(K-1)
C
      GAM1(K)=(BET4*SIGM1-BET2*SIGM2)/MEW
      GAM2(K)=(BET1*SIGM2-BET3*SIGM1)/MEW
   65 CONTINUE
C     ------BACK SUBSTITUTION FOR CALCULATING SOLUTION VECTORS----------
      TEMF(NCC)=GAM1(NCC)
      TEMS(NCC)=GAM2(NCC)
C
      NO3=NC-2
      DO 70 KNO4=2,NO3
      NO4=NC-KNO4
      TEMF(NO4)=GAM1(NO4)-LAM1(NO4)*TEMF(NO4+1)-LAM2(NO4)*TEMS(NO4+1)
      TEMS(NO4)=GAM2(NO4)-LAM3(NO4)*TEMF(NO4+1)-LAM4(NO4)*TEMS(NO4+1)
   70 CONTINUE
C
C     -------INTERPOLATION OF SALTWATER AND FRESHWATER POTENTIALS-------
      DO 75 I=2,NCC
      HFA(I,J)=HFA(I,J)+OMEG*TEMF(I)
      HSA(I,J) = HSA(I,J) + OMEG*TEMS(I)
C
C     --------------TO TEST FOR CONVERGENCE-----------------------------
C
      ECHKF=DABS(TEMF(I))
      ECHKS=DABS(TEMS(I))
      IF(ECHKF.GT.ERR) ERR = ECHKF
      IF(ECHKS.GT.ERR) ERR=ECHKS
   75 CONTINUE
   80 CONTINUE
      GO TO 90
C
   85 CONTINUE
C
      DO 280 I=2,NCC
      DO 260 J=2,NRR
C
      BBF(J)=TFR(I,J)/DELX(I)
      AAF(J)=TFR(I-1,J)/DELX(I)
      DDF(J)=TFC(I,J)/DELY(J)
      CCF(J)=TFC(I,J-1)/DELY(J)
```

```
C
      AAS(J)=TSR(I-1,J)/DELX(I)
      BBS(J)=TSR(I,J)/DELX(I)
      DDS(J)=TSC(I,J)/DELY(J)
      CCS(J)=TSC(I,J-1)/DELY(J)
C
      CS1(J)=POR(I,J)*(1.+SIGM)/DELT
      CF2(J)=POR(I,J)*SIGM/DELT
      IF(ALPH.NE.1.) GO TO 240
      CF1(J)=(POR(I,J)*SIGM+SC(I,J)*BF(I,J)+ALPH*POR(I,J))/DELT
      CS2(J)=(POR(I,J)*(1.+SIGM)+SC(I,J)*BS(I,J))/DELT
C
      GO TO 250
  240 CONTINUE
C
      CF1(J)=(POR(I,J)*SIGM+SC(I,J)*BF(I,J)/TH(I,J)+ALPH*POR(I,J))/DELT
      CS2(J)=(POR(I,J)*(1.+SIGM)+SC(I,J)*BS(I,J)/TH(I,J))/DELT
  250 CONTINUE
      EEF(J)=-(AAF(J)+BBF(J)+CCF(J)+DDF(J)+CF1(J))
      EES(J)=-(AAS(J)+BBS(J)+CCS(J)+DDS(J)+CS2(J))
C
      MMF=QF(I,J)/(DELX(I)*DELY(J))-CF1(J)*HFF(I,J)+CS1(J)*HSS(I,J)
      MMS=QS(I,J)/(DELX(I)*DELY(J))-CS2(J)*HSS(I,J)+CF2(J)*HFF(I,J)
C
C
      DF(J)= MMF-AAF(J)*HFA(I-1,J)-BBF(J)*HFA(I+1,J)-CCF(J)*HFA(I,J-1)
     $        -EEF(J)*HFA(I,J)-CS1(J)*HSA(I,J)-DDF(J)*HFA(I,J+1)
C
      DS(J)=MMS-AAS(J)*HSA(I-1,J)-BBS(J)*HSA(I+1,J)-CCS(J)*HSA(I,J-1)
     $        -EES(J)*HSA(I,J)-CF2(J)*HFA(I,J)-DDS(J)*HSA(I,J+1)
C
 260  CONTINUE
C
C
C
      BET1=EEF(2)
      BET2=CS1(2)
      BET3=CF2(2)
      BET4=EES(2)
      SIGM1=DF(2)
      SIGM2=DS(2)
C
      MEW=BET1*BET4-BET2*BET3
C
      LAM1(2)=BET4*DDF(2)/MEW
      LAM2(2)=-BET2*DDS(2)/MEW
      LAM3(2)=-BET3*DDF(2)/MEW
      LAM4(2)=BET1*DDS(2)/MEW
C
      GAM1(2)=(BET4*SIGM1-BET2*SIGM2)/MEW
      GAM2(2)=(BET1*SIGM2-BET3*SIGM1)/MEW
C
      DO 265 K=3,NRR
      BET1=EEF(K)-CCF(K)*LAM1(K-1)
      BET2=CS1(K)-CCF(K)*LAM2(K-1)
      BET3=CF2(K)-CCS(K)*LAM3(K-1)
      BET4=EES(K)-CCS(K)*LAM4(K-1)
C
```

```
      MEW=BET1*BET4-BET2*BET3
C
      LAM1(K)=BET4*DDF(K)/MEW
      LAM2(K)=-BET2*DDS(K)/MEW
      LAM3(K)=-BET3*DDF(K)/MEW
      LAM4(K)=BET1*DDS(K)/MEW
C
      SIGM1 =DF(K)-CCF(K)*GAM1(K-1)
      SIGM2 =DS(K)-CCS(K)*GAM2(K-1)
C
      GAM1(K)=(BET4*SIGM1-BET2*SIGM2)/MEW
      GAM2(K)=(BET1*SIGM2-BET3*SIGM1)/MEW
 265  CONTINUE
C
      TEMF(NRR)=GAM1(NRR)
      TEMS(NRR)=GAM2(NRR)
C
      NO3=NR-2
      DO 270 KNO4=2,NO3
      NO4=NR-KNO4
      TEMF(NO4)=GAM1(NO4)-LAM1(NO4)*TEMF(NO4+1)-LAM2(NO4)*TEMS(NO4+1)
      TEMS(NO4)=GAM2(NO4)-LAM3(NO4)*TEMF(NO4+1)-LAM4(NO4)*TEMS(NO4+1)
 270   CONTINUE
C
      DO 275 J=2,NRR
      HFA(I,J)=HFA(I,J)+OMEG*TEMF(J)
      HSA(I,J)=HSA(I,J)+OMEG*TEMS(J)
C
C     TEST FOR CONVERGENCE
C
      ECHKF=DABS(TEMF(J))
      ECHKS=DABS(TEMS(J))
      IF(ECHKF.GT.ERR) ERR=ECHKF
      IF(ECHKS.GT.ERR) ERR=ECHKS
  275 CONTINUE
  280 CONTINUE
C
   90 IF(ITER.LE.MAXIT) GO TO 115
      WRITE(LP,110) ITER
  110 FORMAT(//10X,I2,5X,39HITER EXCEEDED LIMIT AND EXEC TERMINATED)
      STOP
  115 IF(ERR.GT.TOL) GO TO 6
      WRITE(LP,120) ITER,ERR
  120 FORMAT(//10X,29HNUMBER OF ITERATION REQUIRED=,I2,5X,6HERROR=,
     $G15.7)
C
      RETURN
      END
```

```
C
C     ------------------------------------------------------------------
C
      SUBROUTINE PRINT(HFT,HST,Z2,Z3,NC,NR,TIME)
C
C     ------------------------------------------------------------------
C
      DIMENSION HFT(NC,NR),HST(NC,NR),Z2(NC,NR),Z3(NC,NR)
C
      DATA IN,LP/5,6/
C
      THR=24.*TIME
      TDA=TIME
      TYR = TDA/365.
      WRITE(LP,10) THR,TDA,TYR
   10 FORMAT(//10X,13HTIME IN HOUR=,G15.7,5X,13HTIME IN DAYS=,G15.7,5X,
     114HTIME IN YEARS=,G15.7)
C
      WRITE(LP,15)
   15 FORMAT(//50X,18HFRESHWATER HEAD,FT)
      WRITE(LP,16)
   16 FORMAT(50X,18H------------------)
      DO 30 J = 1,NR
   30 WRITE(LP,40)J,(HFT(I,J),I=1,NC)
   40 FORMAT(//10X,4HROW=,I2//(10X,8G15.7))
C
      WRITE(LP,44)
   44 FORMAT(1H1,50X,18HSALTWATER  HEAD,FT)
      WRITE(LP,45)
   45 FORMAT(51X,18H------------------)
      DO 50 J = 1,NR
   50 WRITE(LP,60)J,(HST(I,J),I=1,NC)
   60 FORMAT(//10X,4HROW=,I2//(10X,8G15.7))
C
      WRITE(LP,64)
   64 FORMAT(1H1,50X,22HINTERFACE ELEVATION,FT)
      WRITE(LP,66)
   66 FORMAT(51X,22H----------------------)
      DO 70 J = 1,NR
   70 WRITE(LP,80)J,(Z2(I,J),I=1,NC)
   80 FORMAT(//10X,4HROW=,I2//(10X,8G15.7))
C
      WRITE(LP,85)
   85 FORMAT(1H1,50X,20HINTERFACE UPCONED,FT)
      WRITE(LP,90)
   90 FORMAT(51X,20H--------------------)
      DO 110 J=1,NR
  110 WRITE(LP,120)J,(Z3(I,J),I=1,NC)
  120 FORMAT(//10X,4HROW=,I2//(10X,8G15.7))
      RETURN
      END
```

APPENDIX C

SEQUENCE AND FORMAT OF INPUT DATA

TABLE II

SEQUENCE AND FORMAT OF MODEL INPUT DATA

Parameters	Format
NUMT, MAXIT, NQ	3I3
DELT, OMEG	2F11.7
ALPHA, BETA	2F11.5
K1, K2, K3, K4, K5, K6, K7, K8, K9, K10	10I2
F1, F2, F3, F4, F5, F6, F7, F8, F9, F10	5F11.5
DELX(I)	5F11.5/F11.5
DELY(J)	5F11.5/F11.5
HF(I,J)	7F10.5/F11.5
Z1(I,J)	7F10.5/F11.5
KFX(I,J)	7F10.5/F11.5
KFY(I,J)	7F10.5/F11.5
POR(I,J)	7F10.5/F11.5
SC(I,J)	7F10.5/F11.5
ZO(I,J)	7F10.5/F11.5
TH(I,J)	7F10.5/F11.5
DENF, DENS	2F11.5
TOL, CDLT	2F11.5
VIF, VIS	2F10.6
I, J, QF(I,J)	2I2,F10.5

APPENDIX D

INPUT CARD DECK

NUMT= 7 MAXIT= 30 PUMPS= 9

COLUMN= 19 ROW= 14

DELT= 150.0000 OMEG= 1.200000

ALPH= 0.0000000 CONFINED AQUIFER

UNIFORMITY/NONUNIFORMITY CODE

1 1 0 0 0 0 1 0 1 1

SPACING IN X-DIRECTION ARE EQUAL

SPACING IN Y-DIRECTION ARE EQUAL

POROSITY IS UNIFORM

STORAGE COEFFICIENT/SPECIFIC STORAGE IS UNIFORM

AQUIFER BOTTOM ELEVATION IS UNIFORM

MULTIPLICATION FACTORS

1.0000	1.0000	1.0000	1.0000	1.0000
1.0000	1.0000	1.0000	1.0000	1.0000

GRID SPACING

X-DIRECTION

960.0000	960.0000	960.0000	960.0000	960.0000	960.0000	960.0000	960.0000
960.0000	960.0000	960.0000	960.0000	960.0000	960.0000	960.0000	960.0000
960.0000	960.0000	960.0000					

Y-DIRECTION

960.0000	960.0000	960.0000	960.0000	960.0000	960.0000	960.0000	960.0000
960.0000	960.0000	960.0000	960.0000	960.0000	960.0000		

INITIAL FRESHWATER HEAD

ROW= 1

950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000
950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000
950.0000	955.0000	955.0000					

ROW= 2

950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000
950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	955.0000
955.0000	955.0000	957.0000					

ROW= 3

950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000
950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	955.0000
955.0000	957.0000	960.0000					

ROW= 4

950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000
950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	957.0000
960.0000	960.0000	960.0000					

ROW= 5

950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000
950.0000	950.0000	950.0000	950.0000	950.0000	947.0000	955.0000	960.0000
960.0000	960.0000	960.0000					

ROW= 6

950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000
950.0000	950.0000	947.0000	945.0000	945.0000	950.0000	960.0000	960.0000
960.0000	960.0000	960.0000					

ROW= 7

950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000
947.0000	947.0000	945.0000	945.0000	945.0000	950.0000	960.0000	960.0000
960.0000	960.0000	960.0000					

ROW= 8

950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	948.0000	947.0000
945.0000	945.0000	945.0000	945.0000	950.0000	955.0000	960.0000	960.0000
960.0000	960.0000	960.0000					

ROW= 9

950.0000	950.0000	950.0000	950.0000	950.00C0	950.0000	947.0000	945.0000
944.0000	944.0000	945.0000	947.0000	955.0000	957.0000	960.0000	960.0000
960.0000	960.0000	960.0000					

ROW=10

950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	948.0000	947.0000
945.0000	945.0000	947.0000	950.0000	957.0000	957.0000	960.0000	960.0000
960.0000	960.0000	960.0000					

ROW=11

950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	948.0000	950.0000
947.0000	947.0000	950.0000	955.0000	957.0000	960.0000	960.0000	960.0000
960.0000	960.0000	960.0000					

ROW=12

950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	952.0000
950.0000	955.0000	955.0000	957.0000	960.0000	960.0000	960.0000	960.0000
960.0000	960.0000	960.0000					

ROW=13

950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	955.0000	955.0000
955.0000	955.0000	960.0000	960.0000	960.0000	960.0000	960.0000	960.0000
960.0000	960.0000	960.0000					

ROW=14

950.0000	950.0000	950.0000	950.0000	950.0000	955.0000	955.0000	955.0000
955.0000	960.0000	960.0000	960.0000	960.0000	960.0000	960.0000	960.0000
960.0000	960.0000	960.0000					

INITIAL INTERFACE ELEVATION

ROW= 1

370.0000	370.0000	375.0000	375.0000	375.0000	375.0000	375.0000	380.0000
380.0000	380.0000	380.0000	380.0000	380.0000	380.0000	380.0000	380.0000
385.0000	390.0000	400.0000					

ROW= 2

367.0000	370.0000	370.0000	375.0000	375.0000	375.0000	375.0000	380.0000
380.0000	380.0000	380.0000	380.0000	380.0000	380.0000	380.0000	385.0000
395.0000	400.0000	400.0000					

ROW= 3

365.0000	368.0000	370.0000	370.0000	375.0000	375.0000	375.0000	380.0000
380.0000	380.0000	380.0000	380.0000	380.0000	390.0000	400.0000	400.0000
400.0000	400.0000	420.0000					

ROW= 4

360.0000	367.0000	370.0000	370.0000	370.0000	375.0000	378.0000	380.0000
380.0000	380.0000	390.0000	400.0000	400.0000	390.0000	380.0000	400.0000
420.0000	420.0000	420.0000					

ROW= 5

360.0000	365.0000	370.0000	370.0000	370.0000	370.0000	375.0000	380.0000
380.0000	380.0000	390.0000	400.0000	400.0000	390.0000	380.0000	400.0000
420.0000	420.0000	420.0000					

ROW= 6

360.0000	360.0000	365.0000	370.0000	370.0000	370.0000	380.0000	380.0000
380.0000	400.0000	400.0000	400.0000	380.0000	370.0000	380.0000	400.0000
420.0000	420.0000	420.0000					

ROW= 7

360.0000	360.0000	360.0000	370.0000	370.0000	370.0000	380.0000	380.0000
400.0000	400.0000	400.0000	380.0000	370.0000	375.0000	390.0000	400.0000
420.0000	420.0000	420.0000					

ROW= 8

360.0000	360.0000	360.0000	365.0000	370.0000	375.0000	380.0000	400.0000
400.0000	400.0000	380.0000	360.0000	370.0000	380.0000	390.0000	400.0000
420.0000	420.0000	420.0000					

ROW= 9

360.0000	360.0000	360.0000	360.0000	370.0000	380.0000	390.0000	400.0000
400.0000	360.0000	360.0000	360.0000	380.0000	380.0000	400.0000	410.0000
420.0000	420.0000	420.0000					

ROW=10

360.0000	360.0000	360.0000	360.0000	370.0000	380.0000	390.0000	400.0000
400.0000	340.0000	350.0000	360.0000	380.0000	380.0000	400.0000	420.0000
420.0000	420.0000	420.0000					

ROW=11

360.0000	360.0000	360.0000	360.0000	358.0000	360.0000	370.0000	378.0000
370.0000	350.0000	345.0000	355.0000	370.0000	380.0000	400.0000	420.0000
420.0000	420.0000	420.0000					

ROW=12

360.0000	360.0000	360.0000	350.0000	350.0000	350.0000	360.0000	338.0000
335.0000	350.0000	360.0000	370.0000	380.0000	390.0000	410.0000	420.0000
420.0000	420.0000	420.0000					

ROW=13

360.0000	355.0000	345.0000	330.0000	330.0000	330.0000	325.0000	330.0000
345.0000	350.0000	360.0000	370.0000	380.0000	400.0000	410.0000	420.0000
420.0000	420.0000	420.0000					

ROW=14

350.0000	340.0000	330.0000	310.0000	300.0000	310.0000	330.0000	340.0000
345.0000	355.0000	365.0000	375.0000	390.0000	410.0000	420.0000	420.0000
420.0000	420.0000	420.0000					

PERMEABILITY IN X-DIRECTION

ROW= 1

16.50000	16.50000	16.50000	16.50000	16.50000	16.50000	16.00000	16.00000
15.50000	15.50000	15.50000	15.00000	15.00000	15.00000	15.00000	15.00000
15.00000	15.00000	15.00000					

ROW= 2

16.50000	16.50000	16.50000	17.00000	16.50000	16.50000	16.50000	16.00000
16.00000	15.50000	15.50000	15.50000	15.00000	15.00000	15.00000	15.00000
15.00000	15.00000	16.50000					

ROW= 3

17.00000	17.00000	17.00000	17.00000	17.00000	17.00000	16.50000	16.50000
16.50000	16.00000	16.00000	15.50000	15.50000	15.50000	15.50000	16.00000
17.00000	19.00000	19.00000					

ROW= 4

17.00000	17.00000	17.00000	17.50000	17.00000	17.00000	17.00000	17.00000
16.50000	16.00000	16.00000	16.00000	15.50000	16.00000	16.00000	17.00000
19.00000	20.00000	20.00000					

ROW= 5

17.00000	17.00000	17.00000	17.50000	17.00000	17.00000	17.00000	17.00000
17.00000	16.50000	16.50000	16.00000	16.00000	16.00000	17.00000	19.00000
20.00000	20.00000	19.00000					

ROW= 6

17.00000	17.50000	17.50000	18.00000	18.00000	17.00000	17.00000	17.00000
17.00000	17.00000	17.00000	16.00000	16.00000	18.00000	19.00000	19.00000
19.00000	18.50000	20.00000					

ROW= 7

18.00000	18.00000	18.00000	19.00000	19.00000	17.50000	17.00000	18.00000
17.00000	17.00000	17.00000	16.00000	17.00000	16.00000	17.00000	17.00000
17.00000	17.00000	17.00000					

ROW= 8

18.00000	19.00000	19.00000	20.00000	20.00000	18.00000	18.00000	17.50000
17.50000	17.00000	17.00000	16.00000	16.00000	15.00000	15.00000	15.00000
15.50000	16.00000	17.50000					

ROW= 9

19.00000	20.00000	19.00000	21.00000	21.00000	19.00000	19.00000	18.50000
18.00000	17.00000	17.00000	16.00000	15.50000	15.00000	15.00000	15.00000
15.00000	15.00000	16.00000					

ROW=10

20.00000	20.00000	20.00000	21.00000	21.00000	20.00000	19.50000	18.00000
17.00000	16.50000	16.00000	15.50000	15.00000	15.00000	15.00000	15.00000
15.00000	15.00000	15.00000					

ROW=11

20.00000	20.00000	20.00000	20.50000	20.00000	20.00000	18.00000	17.00000
17.50000	16.00000	15.50000	15.50000	15.00000	15.00000	15.00000	15.00000
15.00000	15.00000	15.00000					

ROW=12

19.00000	19.00000	20.00000	20.00000	18.50000	18.50000	17.00000	16.50000
16.00000	15.50000	15.50000	15.00000	15.00000	15.00000	15.00000	15.00000
15.00000	15.00000	15.00000					

ROW=13

18.00000	18.00000	18.00000	18.00000	18.00000	17.00000	16.50000	16.50000
16.00000	15.50000	15.50000	15.00000	15.00000	15.00000	15.00000	15.00000
15.00000	15.00000	15.00000					

ROW=14

18.00000	18.00000	17.50000	17.50000	17.00000	17.00000	16.50000	16.00000
15.50000	15.50000	15.50000	15.00000	15.00000	15.00000	15.00000	15.00000
15.00000	15.00000	15.00000					

BETA= 0.0000000 ISOTROPIC AQUIFER

PERMEABILITY IN Y-DIRECTION

ROW= 1

16.50000	16.50000	16.50000	16.50000	16.50000	16.50000	16.00000	16.00000
15.50000	15.50000	15.50000	15.00000	15.00000	15.00000	15.00000	15.00000
15.00000	15.00000	15.00000					

ROW= 2

16.50000	16.50000	16.50000	17.00000	16.50000	16.50000	16.50000	16.00000
16.00000	15.50000	15.50000	15.50000	15.00000	15.00000	15.00000	15.00000
15.00000	15.00000	16.50000					

ROW= 3

17.00000	17.00000	17.00000	17.00000	17.00000	17.00000	16.50000	16.50000
16.50000	16.00000	16.00000	15.50000	15.50000	15.50000	15.50000	16.00000
17.00000	19.00000	19.00000					

ROW= 4

17.00000	17.00000	17.00000	17.50000	17.00000	17.00000	17.00000	17.00000
16.50000	16.00000	16.00000	16.00000	15.50000	16.00000	16.00000	17.00000
19.00000	20.00000	20.00000					

ROW= 5

17.00000	17.00000	17.00000	17.50000	17.00000	17.00000	17.00000	17.00000
17.00000	16.50000	16.50000	16.00000	16.00000	16.00000	17.00000	19.00000
20.00000	20.00000	19.00000					

ROW= 6

17.00000	17.50000	17.50000	18.00000	18.00000	17.00000	17.00000	17.00000
17.00000	17.00000	17.00000	16.00000	16.00000	18.00000	19.00000	19.00000
19.00000	18.50000	20.00000					

ROW= 7

18.00000	18.00000	18.00000	19.00000	19.00000	17.50000	17.00000	18.00000
17.00000	17.00000	17.00000	16.00000	17.00000	16.00000	17.00000	17.00000
17.00000	17.00000	17.00000					

ROW= 8

18.00000	19.00000	19.00000	20.00000	20.00000	18.00000	18.00000	17.50000
17.50000	17.00000	17.00000	16.00000	16.00000	15.00000	15.00000	15.00000

15.50000	16.00000	17.50000

ROW= 9

19.00000	20.00000	19.00000	21.00000	21.00000	19.00000	19.00000	18.50000
18.00000	17.00000	17.00000	16.00000	15.50000	15.00000	15.00000	15.00000
15.00000	15.00000	16.00000					

ROW=10

20.00000	20.00000	20.00000	21.00000	21.00000	20.00000	19.50000	18.00000
17.00000	16.50000	16.00000	15.50000	15.00000	15.00000	15.00000	15.00000
15.00000	15.00000	15.00000					

ROW=11

20.00000	20.00000	20.00000	20.50000	20.00000	20.00000	18.00000	17.00000
17.50000	16.00000	15.50000	15.50000	15.00000	15.00000	15.00000	15.00000
15.00000	15.00000	15.00000					

ROW=12

19.00000	19.00000	20.00000	20.00000	18.50000	18.50000	17.00000	16.50000
16.00000	15.50000	15.50000	15.00000	15.00000	15.00000	15.00000	15.00000
15.00000	15.00000	15.00000					

ROW=13

18.00000	18.00000	18.00000	18.00000	18.00000	17.00000	16.50000	16.50000
16.00000	15.50000	15.50000	15.00000	15.00000	15.00000	15.00000	15.00000
15.00000	15.00000	15.00000					

ROW=14

18.00000	18.00000	17.50000	17.50000	17.00000	17.00000	16.50000	16.00000
15.50000	15.50000	15.50000	15.00000	15.00000	15.00000	15.00000	15.00000
15.00000	15.00000	15.00000					

POROSITY OF AQUIFER

ROW= 1

0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000					

ROW= 2

0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000					

ROW= 3

0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000					

ROW= 4

0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000					

ROW= 5

0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000					

ROW= 6

0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000					

ROW= 7

0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000					

ROW= 8

0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000					

ROW= 9

0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000					

ROW=10

0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000					

ROW=11

0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000					

ROW=12

0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000					

ROW=13

0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000					

ROW=14

0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000	0.3000000
0.3000000	0.3000000	0.3000000					

STORAGE COEFFICIENT

ROW= 1

0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03
0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03
0.5000001E-03 0.5000001E-03 0.5000001E-03

ROW= 2

0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03
0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03
0.5000001E-03 0.5000001E-03 0.5000001E-03

ROW= 3

0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03
0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03
0.5000001E-03 0.5000001E-03 0.5000001E-03

ROW= 4

0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03
0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03
0.5000001E-03 0.5000001E-03 0.5000001E-03

ROW= 5

0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03
0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03
0.5000001E-03 0.5000001E-03 0.5000001E-03

ROW= 6

0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03
0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03
0.5000001E-03 0.5000001E-03 0.5000001E-03

ROW= 7

0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03
0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03
0.5000001E-03 0.5000001E-03 0.5000001E-03

ROW= 8

0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03
0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03 0.5000001E-03
0.5000001E-03 0.5000001E-03 0.5000001E-03

ROW= 9

```
0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03
0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03
0.5000001E-03  0.5000001E-03  0.5000001E-03
```

ROW=10

```
0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03
0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03
0.5000001E-03  0.5000001E-03  0.5000001E-03
```

ROW=11

```
0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03
0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03
0.5000001E-03  0.5000001E-03  0.5000001E-03
```

ROW=12

```
0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03
0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03
0.5000001E-03  0.5000001E-03  0.5000001E-03
```

ROW=13

```
0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03
0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03
0.5000001E-03  0.5000001E-03  0.5000001E-03
```

ROW=14

```
0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03
0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03  0.5000001E-03
0.5000001E-03  0.5000001E-03  0.5000001E-03
```

AQUIFER BOTTOM ELEVATION

ROW= 1

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW= 2

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW= 3

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW= 4

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW= 5

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW= 6

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW= 7

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW= 8

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW= 9

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW=10

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW=11

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW=12

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW=13

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW=14

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

THICKNESS OF AQUIFER

ROW= 1

890.0000	890.0000	892.0000	895.0000	895.0000	900.0000	900.0000	902.0000
905.0000	908.0000	910.0000	915.0000	918.0000	920.0000	920.0000	920.0000
920.0000	920.0000	920.0000					

ROW= 2

885.0000	885.0000	888.0000	890.0000	892.0000	895.0000	895.0000	900.0000
900.0000	905.0000	905.0000	908.0000	910.0000	915.0000	915.0000	920.0000
920.0000	920.0000	920.0000					

ROW= 3

880.0000	880.0000	884.0000	885.0000	885.0000	888.0000	890.0000	890.0000
895.0000	895.0000	900.0000	900.0000	905.0000	905.0000	910.0000	915.0000
920.0000	920.0000	920.0000					

ROW= 4

875.0000	875.0000	875.0000	880.0000	880.0000	880.0000	885.0000	885.0000
885.0000	890.0000	895.0000	895.0000	898.0000	900.0000	910.0000	915.0000
920.0000	920.0000	920.0000					

ROW= 5

870.0000	870.0000	875.0000	875.0000	875.0000	875.0000	875.0000	880.0000
880.0000	885.0000	885.0000	890.0000	892.0000	900.0000	910.0000	915.0000
920.0000	920.0000	920.0000					

ROW= 6

865.0000	865.0000	868.0000	868.0000	870.0000	870.0000	872.0000	875.0000
875.0000	880.0000	880.0000	885.0000	895.0000	905.0000	910.0000	915.0000
915.0000	915.0000	915.0000					

ROW= 7

862.0000	862.0000	862.0000	865.0000	865.0000	865.0000	865.0000	870.0000
870.0000	875.0000	878.0000	890.0000	900.0000	905.0000	905.0000	910.0000
910.0000	910.0000	910.0000					

ROW= 8

858.0000	858.0000	858.0000	858.0000	858.0000	858.0000	860.0000	865.0000
861.0000	870.0000	890.0000	892.0000	900.0000	902.0000	902.0000	905.0000
905.0000	900.0000	900.0000					

ROW= 9

855.0000	855.0000	856.0000	852.0000	845.0000	852.0000	855.0000	858.0000
860.0000	875.0000	885.0000	890.0000	895.0000	895.0000	898.0000	900.0000
900.0000	890.0000	890.0000					

ROW=10

852.0000	850.0000	850.0000	848.0000	845.0000	845.0000	845.0000	850.0000
860.0000	875.0000	880.0000	880.0000	880.0000	885.0000	890.0000	885.0000
890.0000	880.0000	880.0000					

ROW=11

848.0000	845.0000	842.0000	840.0000	835.0000	835.0000	840.0000	845.0000
860.0000	865.0000	865.0000	865.0000	865.0000	865.0000	870.0000	870.0000
870.0000	870.0000	870.0000					

ROW=12

842.0000	840.0000	835.0000	830.0000	828.0000	830.0000	835.0000	845.0000
845.0000	847.0000	850.0000	850.0000	850.0000	850.0000	855.0000	855.0000
855.0000	855.0000	855.0000					

ROW=13

835.0000	835.0000	830.0000	825.0000	825.0000	825.0000	830.0000	835.0000
835.0000	835.0000	835.0000	838.0000	838.0000	840.0000	840.0000	840.0000
840.0000	840.0000	840.0000					

ROW=14

830.0000	825.0000	820.0000	820.0000	820.0000	820.0000	820.0000	820.0000
820.0000	820.0000	820.0000	825.0000	825.0000	825.0000	825.0000	827.0000
828.0000	830.0000	835.0000					

```
DENF=   1.000000      DENS=   1.025000

TOL=  0.1000000       CDLT=   2.000000

VIF=    1.000000      VIS=   1.000000

                              SOURCE/SINK
                              -----------

ROW= 1

  0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000
  0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000
  0.0000000    0.0000000    0.0000000

ROW= 2

  0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000
  0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000
  0.0000000    0.0000000    0.0000000

ROW= 3

  0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000
  0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000
   288000.0    0.0000000    0.0000000

ROW= 4

  0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000
  0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000
  0.0000000    0.0000000    0.0000000

ROW= 5

  0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000
  0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000
  0.0000000    0.0000000    0.0000000

ROW= 6

  0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000
  0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000     288000.0
  0.0000000    0.0000000    0.0000000

ROW= 7

  0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000
  0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000    0.0000000
  0.0000000    0.0000000    0.0000000
```

ROW= 8

0.0000000	0.0000000	0.0000000	0.0000000	288000.0	0.0000000	0.0000000	0.0000000
288000.0	0.0000000	288000.0	0.0000000	0.0000000	288000.0	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW= 9

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW=10

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW=11

0.0000000	0.0000000	0.0000000	0.0000000	288000.0	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW=12

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW=13

0.0000000	288000.0	0.0000000	0.0000000	288000.0	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW=14

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

INITIAL SALTWATER HEAD

ROW= 1

935.8538	935.8538	935.9756	935.9756	935.9756	935.9756	935.9756	936.0977
936.0977	936.0977	936.0977	936.0977	936.0977	936.0977	936.0977	936.0977
936.2195	941.2195	941.4634					

ROW= 2

935.7805	935.8538	935.8538	935.9756	935.9756	935.9756	935.9756	936.0977
936.0977	936.0977	936.0977	936.0977	936.0977	936.0977	936.0977	941.0977
941.3416	941.4634	943.4146					

ROW= 3

935.7317	935.8049	935.8538	935.8538	935.9756	935.9756	935.9756	936.0977
936.0977	936.0977	936.0977	936.0977	936.0977	936.3416	936.5854	941.4634
941.4634	943.4146	946.8293					

ROW= 4

935.6099	935.7805	935.8538	935.8538	935.8538	935.9756	936.0488	936.0977
936.0977	936.0977	936.3416	936.5854	936.5854	936.3416	936.0977	943.4146
946.8293	946.8293	946.8293					

ROW= 5

935.6099	935.7317	935.8538	935.8538	935.8538	935.8538	935.9756	936.0977
936.0977	936.0977	936.3416	936.5854	936.5854	933.4146	940.9756	946.3416
946.8293	946.8293	946.8293					

ROW= 6

935.6099	935.6099	935.7317	935.8538	935.8538	935.8538	936.0977	936.0977
936.0977	936.5854	933.6584	931.7073	931.2195	935.8538	945.8538	946.3416
946.8293	946.8293	946.8293					

ROW= 7

935.6099	935.6099	935.6099	935.8538	935.8538	935.8538	936.0977	936.0977
933.6584	933.6584	931.7073	931.2195	930.9756	935.9756	946.0977	946.3416
946.8293	946.8293	946.8293					

ROW= 8

935.6099	935.6099	935.6099	935.7317	935.8538	935.9756	934.1465	933.6584
931.7073	931.7073	931.2195	930.7317	935.8538	940.9756	946.0977	946.3416
946.8293	946.8293	946.8293					

ROW= 9

935.6099	935.6099	935.6099	935.6099	935.8538	936.0977	933.4146	931.7073
930.7317	929.7561	930.7317	932.6829	940.9756	942.9268	946.3416	946.5854
946.8293	946.8293	946.8293					

ROW=10

935.6099	935.6099	935.6099	935.6099	935.8538	936.0977	934.3904	933.6584
931.7073	930.2439	932.4390	935.6099	942.9268	942.9268	946.3416	946.8293
946.8293	946.8293	946.8293					

ROW=11

935.6099	935.6099	935.6099	935.6099	935.5610	935.6099	933.9026	936.0488
932.9268	932.4390	935.2439	940.3660	942.6829	945.8538	946.3416	946.8293
946.8293	946.8293	946.8293					

ROW=12

935.6099	935.6099	935.6099	935.3660	935.3660	935.3660	935.6099	937.0244
935.0000	940.2439	940.4878	942.6829	945.8538	946.0977	946.5854	946.8293
946.8293	946.8293	946.8293					

ROW=13

935.6099	935.4878	935.2439	934.8782	934.8782	934.8782	939.6343	939.7561
940.1221	940.2439	945.3660	945.6099	945.8538	946.3416	946.5854	946.8293
946.8293	946.8293	946.8293					

ROW=14

935.3660	935.1221	934.8782	934.3904	934.1465	939.2683	939.7561	940.0000
940.1221	945.2439	945.4878	945.7317	946.0977	946.5854	946.8293	946.8293
946.8293	946.8293	946.8293					

APPENDIX E

SAMPLE OUTPUT

TIME STEP= 1   LENGTH OF TIME STEP=   150.0000

SOLUTION IS IN THE X-DIRECTION

NUMBER OF ITERATION REQUIRED=21   ERROR= 0.9254092E-01

TIME IN HOUR=   3600.000   TIME IN DAYS=   150.0000   TIME IN YEARS= 0.4109589

FRESHWATER HEAD

ROW= 1

950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000
950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000
950.0000	955.0000	955.0000					

ROW= 2

950.0000	949.7161	949.4683	949.1841	948.9756	948.8081	948.6819	948.5471
948.4917	948.4871	948.5298	948.6025	948.6980	948.8079	948.9109	948.9927
949.5132	953.8135	957.0000					

ROW= 3

950.0000	949.4019	948.8340	948.3289	947.8459	947.5076	947.2454	946.9922
946.8835	946.8701	946.9578	947.1108	947.3259	947.4553	947.5217	947.3594
945.4531	953.6951	960.0000					

ROW= 4

950.0000	949.0325	948.1130	947.3037	946.6335	946.0652	945.6333	945.3013
945.1057	945.0740	945.1038	945.2754	945.6516	946.1960	946.8496	947.5642
949.4421	954.4941	960.0000					

ROW= 5

950.0000	948.6343	947.2817	946.0847	945.1047	944.4595	943.9119	943.4167
943.0815	943.0227	943.1165	943.4958	944.0510	944.7292	945.5410	946.4124
950.0342	954.8518	960.0000					

ROW= 6

950.0000	948.2244	946.3899	944.6094	943.1794	942.5632	941.9600	941.3079
940.6829	940.4458	940.6865	941.5481	942.5559	943.2903	943.8098	942.2036
949.4414	954.9751	960.0000					

ROW= 7

950.0000	947.7666	945.4209	942.8350	940.4644	940.4514	940.0903	939.1128
937.3416	937.7312	937.6868	939.7798	941.0217	941.4478	943.9067	946.2549
950.7458	955.4685	960.0000					

ROW= 8

950.0000	947.3789	944.5654	941.0789	935.3826	938.4661	938.8872	937.2083
932.0605	935.3245	932.8545	938.6443	939.9365	936.9565	944.0891	948.2471
952.1243	956.1489	960.0000					

ROW= 9

950.0000	947.2354	944.3918	941.4819	938.8169	939.1387	939.4099	938.9031
937.8142	939.0151	939.2805	941.4753	942.7710	943.7280	946.9800	950.2646
953.4902	956.7947	960.0000					

ROW=10

950.0000	947.1199	944.3171	941.5315	939.0684	939.8162	940.7173	941.2012
941.5593	942.9409	943.7175	945.0859	946.3394	947.8867	949.9744	952.2751
954.8606	957.4485	960.0000					

ROW=11

950.0000	946.9072	944.2573	941.1553	935.9832	940.5149	942.7207	944.0967
945.2397	946.5313	947.6960	948.8242	949.9607	951.2607	952.7097	954.3042
956.2080	958.1150	960.0000					

ROW=12

950.0000	946.1885	944.6382	942.8130	940.7046	943.4446	945.7273	947.6509
949.0151	950.3638	951.4539	952.3752	953.2974	954.2471	955.1592	956.2546
957.5037	958.7610	960.0000					

ROW=13

950.0000	943.0488	945.5940	944.7668	940.5967	947.1484	949.8945	951.2898
952.4009	954.5896	955.5562	956.1663	956.7039	957.1187	957.6311	958.1365
958.7646	959.3945	960.0000					

ROW=14

950.0000	950.0000	950.0000	950.0000	950.0000	955.0000	955.0000	955.0000
955.0000	960.0000	960.0000	960.0000	960.0000	960.0000	960.0000	960.0000
960.0000	960.0000	960.0000					

SALTWATER HEAD

ROW= 1

935.8538	935.8538	935.9756	935.9756	935.9756	935.9756	935.9756	936.0977
936.0977	936.0977	936.0977	936.0977	936.0977	936.0977	936.0977	936.0977
936.2195	941.2195	941.4634					

ROW= 2

935.7805	935.5764	935.3367	935.1777	934.9763	934.8127	934.6904	934.6794
934.6260	934.6213	934.6633	934.7346	934.8276	934.9358	935.0383	935.2593
935.9888	940.3040	943.4146					

ROW= 3

935.7317	935.2212	934.7158	934.2253	933.8726	933.5439	933.2893	933.1624
933.0571	933.0439	933.1311	933.2817	933.4929	933.8574	934.1606	934.0083
932.3684	940.2014	946.8293					

ROW= 4

935.6099	934.8357	934.0122	933.2234	932.5713	932.1360	931.7876	931.5129
931.3228	931.2930	931.5627	931.9709	932.3376	932.6301	933.0325	934.2102
936.5254	941.4543	946.8293					

ROW= 5

935.6099	934.3987	933.2004	932.0337	931.0779	930.4500	930.0371	929.6743
929.3477	929.2944	929.6270	930.2380	930.7773	931.1960	931.7527	933.0891
937.1042	941.8066	946.8293					

ROW= 6

935.6099	933.8787	932.2095	930.5938	929.1995	928.5996	928.2510	927.6174
927.0129	927.2568	927.4971	928.3340	928.8384	929.3157	930.0635	929.1978
936.5288	941.9272	946.8293					

ROW= 7

935.6099	933.4312	931.1445	928.8608	926.5522	926.5400	926.4280	925.4814
924.2290	924.6152	924.5686	926.1284	927.0981	927.6348	930.3958	932.9365
937.7991	942.4082	946.8293					

ROW= 8

935.6099	933.0525	930.3088	927.0291	921.8062	924.7246	925.2595	924.0999
919.3130	922.2603	919.5818	924.5391	926.0386	923.5815	930.5796	934.8801
939.1440	943.0725	946.8293					

ROW= 9

935.6099	932.7095	929.7700	926.8381	924.5532	924.7769	925.1545	924.8438
923.8574	924.0457	924.3794	926.4099	928.1218	929.3696	932.9236	936.5037
940.0334	943.4797	946.8293					

ROW=10

935.6099	932.6301	929.7415	926.9358	924.8289	925.4937	926.4937	927.1052
927.3340	927.5415	928.4070	929.9824	931.6641	933.3464	935.9199	938.7131
941.4211	944.1313	946.8293					

ROW=11

935.6099	932.4573	929.7410	926.7844	923.8401	926.0557	928.1418	929.5288
930.4363	931.2998	932.3669	933.7090	935.1907	936.7683	938.7153	940.7957
942.7920	944.8071	946.8293					

ROW=12

935.6099	931.9553	930.1536	928.1272	926.3496	928.5981	930.8777	932.3650
933.6516	935.2212	936.5117	937.6611	938.8328	940.0647	941.4436	942.8191
944.1428	945.4814	946.8293					

ROW=13

935.6099	931.0278	931.0415	929.8376	927.9070	932.0078	934.4910	935.9436
937.3105	939.5999	940.7769	941.6199	942.4207	943.2937	944.0798	944.8137
945.4832	946.1543	946.8293					

ROW=14

935.3660	935.1221	934.8782	934.3904	934.1465	939.2683	939.7561	940.0000
940.1221	945.2439	945.4878	945.7317	946.0977	946.5854	946.8293	946.8293
946.8293	946.8293	946.8293					

INTERFACE ELEVATION

ROW= 1

370.0000	370.0000	375.0000	375.0000	375.0000	375.0000	375.0000	380.0000
380.0000	380.0000	380.0000	380.0000	380.0000	380.0000	380.0000	380.0000
385.0000	390.0000	400.0000					

ROW= 2

367.0000	369.9844	370.0625	374.9141	374.9961	374.9883	375.0234	379.9648
379.9883	379.9844	379.9961	380.0078	380.0039	380.0469	380.1289	385.0938
395.0039	399.9141	400.0000					

ROW= 3

365.0000	367.9844	369.9805	370.0781	374.9297	374.9922	375.0391	379.9609
379.9922	379.9922	380.0547	380.1094	380.1641	389.9336	399.7109	399.9570
408.9727	400.4453	420.0000					

ROW= 4

360.0000	366.9570	369.9727	370.0000	370.0703	374.9570	377.9531	379.9727
379.9961	380.0430	389.9141	399.7852	399.7695	389.9844	380.3398	400.0430
419.8477	419.8516	420.0000					

ROW= 5

360.0000	364.9648	369.9375	369.9844	369.9922	370.0625	375.0391	379.9688
379.9844	380.1563	390.0391	399.9180	399.8203	389.8594	380.2109	400.1523
419.8984	419.9922	420.0000					

ROW= 6

360.0000	360.0430	364.9844	369.9609	369.9922	370.0469	379.8828	379.9922
380.2070	399.6914	399.9141	399.7617	380.1289	370.3242	380.2031	408.9531
420.0156	420.0039	420.0000					

ROW= 7

360.0000	360.0039	360.0820	369.8867	370.0625	370.0781	379.9258	380.2188
399.7227	399.9648	399.8320	380.0664	370.1445	375.1055	389.9492	400.1953
419.9180	419.9883	420.0000					

ROW= 8

360.0000	359.9844	360.0352	365.0273	378.7422	375.0547	380.1445	399.7578
409.4023	399.6836	388.6680	360.3203	370.1133	388.5742	390.1875	400.1914
419.9258	420.0078	420.0000					

ROW= 9

360.0000	359.9922	359.9922	360.0859	370.0547	379.9648	389.9336	399.9297
399.7813	360.2344	360.0273	360.1289	379.8516	380.2188	399.9688	410.0703
419.9648	420.0234	420.0000					

ROW=10

360.0000	359.9883	359.9961	360.0469	369.9844	379.8594	389.8438	399.7344
399.3242	340.5547	350.0195	360.0586	379.8516	380.1523	400.0703	419.8555
420.0234	420.0156	420.0000					

ROW=11

360.0000	359.9961	359.9883	359.9766	366.7617	360.1914	370.0313	377.6758
369.7500	350.0508	345.2383	355.2148	370.1289	380.1758	400.1211	419.9219
420.0078	420.0117	420.0000					

ROW=12

360.0000	360.0195	359.8281	349.9844	350.0313	349.9570	359.5469	338.1953
335.3477	350.0078	359.9883	369.9766	379.9922	390.1172	409.9375	419.9453
420.0078	420.0156	420.0000					

ROW=13

360.0000	363.5234	344.9766	330.0781	337.9219	329.8789	325.2344	330.2031
344.9492	350.1133	360.0742	370.0664	380.1563	399.9961	410.0859	419.9414
420.0117	420.0117	420.0000					

ROW=14

350.0000	340.0000	330.0000	310.0000	300.0000	310.0000	330.0000	340.0000
345.0000	355.0000	365.0000	375.0000	390.0000	410.0000	420.0000	420.0000
420.0000	420.0000	420.0000					

INTERFACE UPCONED

ROW= 1

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW= 2

0.0000000	-0.1562500E-01	0.6250000E-01	-0.8593750E-01	-0.3906250E-02	-0.1171875E-01	0.2343750E-01	-0.3515625E-01
-0.1171875E-01	-0.1562500E-01	-0.3906250E-02	0.7812500E-02	0.3906250E-02	0.4687500E-01	0.1289063	0.9375000E-01
0.3906250E-02	-0.8593750E-01	0.0000000					

ROW= 3

0.0000000	-0.1562500E-01	-0.1953125E-01	0.7812500E-01	-0.7031250E-01	-0.7812500E-02	0.3906250E-01	-0.3906250E-01
-0.7812500E-02	-0.7812500E-02	0.5468750E-01	0.1093750	0.1640625	-0.6640625E-01	-0.2890625	-0.4296875E-01
8.972656	0.4453125	0.0000000					

ROW= 4

0.0000000	-0.4296875E-01	-0.2734375E-01	0.0000000	0.7031250E-01	-0.4296875E-01	-0.4687500E-01	-0.2734375E-01
-0.3906250E-02	0.4296875E-01	-0.8593750E-01	-0.2148438	-0.2304688	-0.1562500E-01	0.3398438	0.4296875E-01
-0.1523438	-0.1484375	0.0000000					

ROW= 5

0.0000000	-0.3515625E-01	-0.6250000E-01	-0.1562500E-01	-0.7812500E-02	0.6250000E-01	0.3906250E-01	-0.3125000E-01
-0.1562500E-01	0.1562500	0.3906250E-01	-0.8203125E-01	-0.1796875	-0.1406250	0.2109375	0.1523438
-0.1015625	-0.7812500E-02	0.0000000					

ROW= 6

0.0000000	0.4296875E-01	-0.1562500E-01	-0.3906250E-01	-0.7812500E-02	0.4687500E-01	-0.1171875	-0.7812500E-02
0.2070313	-0.3085938	-0.8593750E-01	-0.2382813	0.1289063	0.3242188	0.2031250	8.953125
0.1562500E-01	0.3906250E-02	0.0000000					

ROW= 7

0.0000000	0.3906250E-02	0.8203125E-01	-0.1132813	0.6250000E-01	0.7812500E-01	-0.7421875E-01	0.2187500
-0.2773438	-0.3515625E-01	-0.1679688	0.6640625E-01	0.1445313	0.1054688	-0.5078125E-01	0.1953125
-0.8203125E-01	-0.1171875E-01	0.0000000					

ROW= 8

0.0000000	-0.1562500E-01	0.3515625E-01	0.2734375E-01	8.742188	0.5468750E-01	0.1445313	-0.2421875
9.402344	-0.3164063	8.667969	0.3203125	0.1132813	8.574219	0.1875000	0.1914063
-0.7421875E-01	0.7812500E-02	0.0000000					

ROW= 9

0.0000000	-0.7812500E-02	-0.7812500E-02	0.8593750E-01	0.5468750E-01	-0.3515625E-01	-0.6640625E-01	-0.7031250E-01
-0.2187500	0.2343750	0.2734375E-01	0.1289063	-0.1484375	0.2187500	-0.3125000E-01	0.7031250E-01
-0.3515625E-01	0.2343750E-01	0.0000000					

ROW=10

0.0000000	-0.1171875E-01	-0.3906250E-02	0.4687500E-01	-0.1562500E-01	-0.1406250	-0.1562500	-0.2656250
-0.6757813	0.5546875	0.1953125E-01	0.5859375E-01	-0.1484375	0.1523438	0.7031250E-01	-0.1445313
0.2343750E-01	0.1562500E-01	0.0000000					

ROW=11

0.0000000	-0.3906250E-02	-0.1171875E-01	-0.2343750E-01	8.761719	0.1914063	0.3125000E-01	-0.3242188
-0.2500000	0.5078125E-01	0.2382813	0.2148438	0.1289063	0.1757813	0.1210938	-0.7812500E-01
0.7812500E-02	0.1171875E-01	0.0000000					

ROW=12

0.0000000	0.1953125E-01	-0.1718750	-0.1562500E-01	0.3125000E-01	-0.4296875E-01	-0.4531250	0.1953125
0.3476563	0.7812500E-02	-0.1171875E-01	-0.2343750E-01	-0.7812500E-02	0.1171875	-0.6250000E-01	-0.5468750E-01
0.7812500E-02	0.1562500E-01	0.0000000					

ROW=13

0.0000000	8.523438	-0.2343750E-01	0.7812500E-01	7.921875	-0.1210938	0.2343750	0.2031250
-0.5078125E-01	0.1132813	0.7421875E-01	0.6640625E-01	0.1562500	-0.3906250E-02	0.8593750E-01	-0.5859375E-01
0.1171875E-01	0.1171875E-01	0.0000000					

ROW=14

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

TIME STEP= 5   LENGTH OF TIME STEP=   2400.000

SOLUTION IS IN THE X-DIRECTION

NUMBER OF ITERATION REQUIRED= 3   ERROR= 0.7449192E-01

TIME IN HOUR=   111600.0   TIME IN DAYS=   4650.000   TIME IN YEARS=   12.73973

FRESHWATER HEAD

ROW= 1

950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000
950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000
950.0000	955.0000	955.0000					

ROW= 2

950.0000	949.6450	949.3210	949.0059	948.7493	948.5452	948.3843	948.2449
948.1702	948.1631	948.2004	948.2834	948.3955	948.5164	948.6294	948.7322
949.2017	953.7144	957.0000					

ROW= 3

950.0000	949.2783	948.5991	947.9890	947.4534	947.0398	946.7136	946.4434
946.3040	946.2734	946.3425	946.5127	946.7468	946.9592	947.1184	946.8000
942.7344	953.3552	960.0000					

ROW= 4

950.0000	948.8765	947.8071	946.8594	946.0698	945.4368	944.9390	944.5532
944.3181	944.2529	944.3137	944.5464	944.9587	945.5095	946.1707	946.9968
948.9409	954.3076	960.0000					

ROW= 5

950.0000	948.4436	946.9280	945.5703	944.4573	943.6990	943.0771	942.5325
942.1475	942.0415	942.1616	942.5969	943.2114	943.9299	944.7395	945.6343
949.6042	954.6343	960.0000					

ROW= 6

950.0000	947.9905	945.9736	944.0347	942.4441	941.7163	941.0593	940.3154
939.5945	939.4395	939.6416	940.5647	941.5432	942.2810	942.7537	939.2122
948.8635	954.7332	960.0000					

ROW= 7

950.0000	947.5313	944.9678	942.2297	939.5706	939.5427	939.1394	938.0334
936.1738	936.5920	936.4712	938.6621	939.9272	940.3342	943.0242	945.3679
950.2722	955.2222	960.0000					

ROW= 8

950.0000	947.1545	944.1299	940.3457	932.3445	937.4253	937.8970	936.0903
928.1824	933.9038	929.3569	937.3521	938.7102	933.6072	943.1145	947.5149
951.6707	955.9089	960.0000					

ROW= 9

950.0000	947.0247	943.9983	940.8770	937.9216	938.3235	938.5562	937.9792
936.7397	937.8896	938.1096	940.4175	941.8120	942.6814	946.1958	949.6191
953.0452	956.5591	960.0000					

ROW=10

950.0000	946.9434	943.9729	941.0020	938.2776	939.1211	939.9993	940.4626
940.8340	941.9280	942.7759	944.1726	945.5093	947.0515	949.2634	951.7393
954.4531	957.2278	960.0000					

ROW=11

950.0000	946.7266	943.9509	940.6042	933.1045	939.7434	942.0859	943.4958
944.5872	945.7698	946.8926	948.0415	949.2319	950.5593	952.1052	953.8567
955.8577	957.9238	960.0000					

ROW=12

950.0000	945.9011	944.4102	942.4583	939.9854	942.9854	945.3472	947.1211
948.4331	949.8142	950.9102	951.8457	952.7810	953.7380	954.7458	955.9133
957.2410	958.6145	960.0000					

ROW=13

950.0000	940.6301	945.2832	944.4275	938.2817	946.8281	949.5886	950.9663
952.1177	954.2708	955.2415	955.8606	956.3870	956.8530	957.3809	957.9507
958.6155	959.3047	960.0000					

ROW=14

950.0000	950.0000	950.0000	950.0000	950.0000	955.0000	955.0000	955.0000
955.0000	960.0000	960.0000	960.0000	960.0000	960.0000	960.0000	960.0000
960.0000	960.0000	960.0000					

SALTWATER HEAD

ROW= 1

935.8538	935.8538	935.9756	935.9756	935.9756	935.9756	935.9756	936.0977
936.0977	936.0977	936.0977	936.0977	936.0977	936.0977	936.0977	936.0977
936.2195	941.2195	941.4634					

ROW= 2

935.7805	935.4944	935.2188	934.9788	934.7490	934.5547	934.4128	934.3667
934.3086	934.3030	934.3420	934.4248	934.5386	934.6775	934.8303	935.0872
936.0330	940.2361	943.4146					

ROW= 3

935.7317	935.0964	934.4834	933.9180	933.4636	933.0823	932.7878	932.6074
932.4868	932.4634	932.5547	932.7441	932.9978	933.3394	933.6729	933.8696
934.4036	940.3445	946.8293					

ROW= 4

935.6099	934.6631	933.6985	932.7927	932.0464	931.5042	931.0859	930.7683
930.5530	930.5212	930.7507	931.1465	931.5522	931.9575	932.5491	933.8145
936.3682	941.2881	946.8293					

ROW= 5

935.6099	934.1987	932.8213	931.5261	930.4521	929.7410	929.2373	928.7957
928.4478	928.4072	928.7095	929.3015	929.8630	930.3904	931.1787	932.7837
936.7383	941.6011	946.8293					

ROW= 6

935.6099	933.6782	931.8027	930.0188	928.5234	927.8140	927.3286	926.6650
926.0693	926.1641	926.4475	927.2712	927.9341	928.5840	929.6411	930.8132
936.2737	941.7166	946.8293					

ROW= 7

935.6099	933.2083	930.7473	928.2798	926.0303	925.7617	925.4963	924.5649
923.3069	923.5696	923.6274	925.1345	926.1970	926.9595	929.6797	932.5835
937.3704	942.1746	946.8293					

ROW= 8

935.6099	932.8384	929.9302	926.6648	923.2898	924.1528	924.4199	923.2593
920.5120	921.5310	920.8003	923.8120	925.3271	925.0425	930.0156	934.3359
938.6855	942.8433	946.8293					

ROW= 9

935.6099	932.7500	929.8335	926.9072	924.6272	924.8621	925.2368	924.9180
923.9185	924.0938	924.4158	926.4314	928.1240	929.3599	932.9065	936.4817
940.0120	943.4661	946.8293					

ROW=10

935.6099	932.6653	929.7993	927.0039	924.9165	925.5815	926.5786	927.1853
927.4023	927.5989	928.4526	930.0139	931.6790	933.3491	935.9133	938.7002
941.4060	944.1211	946.8293					

ROW=11

935.6099	932.4946	929.7957	926.8342	923.9097	926.1340	928.2190	929.6040
930.5046	931.3569	932.4143	933.7437	935.2114	936.7778	938.7163	940.7903
942.7830	944.8000	946.8293					

ROW=12

935.6099	932.0022	930.2034	928.1758	926.4155	928.6619	930.9397	932.4241
933.7056	935.2676	936.5505	937.6912	938.8521	940.0757	941.4480	942.8174
944.1377	945.4771	946.8293					

ROW=13

935.6099	931.0491	931.0715	929.8552	927.9380	932.0454	934.5269	935.9780
937.3416	939.6270	940.8003	941.6377	942.4329	943.3013	944.0835	944.8142
945.4817	946.1519	946.8293					

ROW=14

935.3660	935.1221	934.8782	934.3904	934.1465	939.2683	939.7561	940.0000
940.1221	945.2439	945.4878	945.7317	946.0977	946.5854	946.8293	946.8293
946.8293	946.8293	946.8293					

INTERFACE ELEVATION

ROW= 1

370.0000	370.0000	375.0000	375.0000	375.0000	375.0000	375.0000	380.0000
380.0000	380.0000	380.0000	380.0000	380.0000	380.0000	380.0000	380.0000
385.0000	390.0000	400.0000					

ROW= 2

367.0000	369.4609	371.1211	373.8867	374.7305	374.9297	375.5469	379.2305
379.8359	379.8906	380.0000	380.0703	380.2500	381.1133	382.8594	389.2773
409.2773	401.0938	400.0000					

ROW= 3

365.0000	367.8125	369.8438	371.0664	373.8672	374.7734	375.7461	379.1641
379.7930	380.0547	381.0313	381.9961	383.0273	388.5352	395.8438	416.6445
601.1641	419.9063	420.0000					

ROW= 4

360.0000	366.1172	369.3438	370.1211	371.1016	374.1914	376.9570	379.3633
379.9375	381.2461	388.2227	395.1406	395.2813	389.8672	387.6758	406.5117
433.4492	420.4961	420.0000					

ROW= 5

360.0000	364.3945	368.5469	369.7500	370.2383	371.4141	375.6328	379.3164
380.4492	383.0273	390.6133	397.4766	395.9180	388.8008	388.7383	418.7539
422.0938	420.2656	420.0000					

ROW= 6

360.0000	361.1797	364.9609	369.3750	371.6914	371.7109	378.0938	380.6445
385.0547	395.1406	398.6758	395.5234	383.5625	380.6953	405.1328	594.8516
432.6719	421.0469	420.0000					

ROW= 7

360.0000	360.2773	361.9219	370.2734	384.4102	374.5117	379.7656	385.8164
408.6211	402.6641	409.8672	384.0234	376.9805	391.9609	395.8945	421.1992
421.2891	420.2617	420.0000					

ROW= 8

360.0000	360.1836	361.9336	379.4219	561.0938	393.2461	385.3281	410.0078
613.6914	426.6133	578.5273	382.2031	389.9961	582.4492	406.0547	407.1680
419.2734	420.2109	420.0000					

ROW= 9

360.0000	360.0938	360.6328	365.2734	389.8086	382.9023	389.0820	399.4141
408.5547	370.2773	375.1602	366.1016	380.5039	396.8906	402.0273	411.8750
419.5547	420.2969	420.0000					

ROW=10

360.0000	360.0938	360.4766	364.2773	386.8711	380.3867	386.2617	392.8008
387.3242	352.0742	353.6406	362.3633	377.8477	385.1367	402.1719	417.6602
420.1328	420.2656	420.0000					

ROW=11

360.0000	361.6797	361.3359	373.9844	553.2578	378.5352	370.3672	370.8398
364.3906	352.4922	351.3359	360.3984	373.5313	385.1172	403.1094	418.3477
420.1602	420.1328	420.0000					

ROW=12

360.0000	374.1133	359.8828	354.8789	380.9141	353.1016	352.0898	342.1133
342.3828	351.4922	360.5664	370.2695	380.8945	393.1211	409.3477	419.0430
420.2070	420.1484	420.0000					

ROW=13

360.0000	546.9297	361.3672	346.2344	512.9102	339.1875	330.5781	335.0273
345.0156	352.7539	362.1836	371.9844	383.7617	400.9141	412.0313	419.3281
420.1797	420.1328	420.0000					

ROW=14

350.0000	340.0000	330.0000	310.0000	300.0000	310.0000	330.0000	340.0000
345.0000	355.0000	365.0000	375.0000	390.0000	410.0000	420.0000	420.0000
420.0000	420.0000	420.0000					

INTERFACE UPCONED

ROW= 1

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW= 2

0.0000000	-0.5390625	1.121094	-1.113281	-0.2695313	-0.7031250E-01	0.5468750	-0.7695313
-0.1640625	-0.1093750	0.0000000	0.7031250E-01	0.2500000	1.113281	2.859375	4.277344
14.27734	1.093750	0.0000000					

ROW= 3

0.0000000	-0.1875000	-0.1562500	1.066406	-1.132813	-0.2265625	0.7460938	-0.8359375
-0.2070313	0.5468750E-01	1.031250	1.996094	3.027344	-1.464844	-4.156250	16.64453
201.1641	19.90625	0.0000000					

ROW= 4

0.0000000	-0.8828125	-0.6562500	0.1210938	1.101563	-0.8085938	-1.042969	-0.6367188
-0.6250000E-01	1.246094	-1.777344	-4.859375	-4.718750	-0.1328125	7.675781	6.511719
13.44922	0.4960938	0.0000000					

ROW= 5

0.0000000	-0.6054688	-1.453125	-0.2500000	0.2382813	1.414063	0.6328125	-0.6835938
0.4492188	3.027344	0.6132813	-2.523438	-4.082031	-1.199219	8.738281	18.75391
2.093750	0.2656250	0.0000000					

ROW= 6

0.0000000	1.179688	-0.3906250E-01	-0.6250000	1.691406	1.710938	-1.906250	0.6445313
5.054688	-4.859375	-1.324219	-4.476563	3.562500	10.69531	25.13281	194.8516
12.67188	1.046875	0.0000000					

ROW= 7

0.0000000	0.2773438	1.921875	0.2734375	14.41016	4.511719	-0.2343750	5.816406
8.621094	2.664063	9.867188	4.023438	6.980469	16.96094	5.894531	21.19922
1.289063	0.2617188	0.0000000					

ROW= 8

0.0000000	0.1835938	1.933594	14.42188	191.0938	18.24609	5.328125	10.00781
213.6914	26.61328	198.5273	22.20313	19.99609	202.4492	16.05469	7.167969
-0.7265625	0.2109375	0.0000000					

ROW= 9

0.0000000	0.9375000E-01	0.6328125	5.273438	19.80859	2.902344	-0.9179688	-0.5859375
8.554688	10.27734	15.16016	6.101563	0.5039063	16.89063	2.027344	1.875000
-0.4453125	0.2968750	0.0000000					

ROW=10

0.0000000	0.9375000E-01	0.4765625	4.277344	16.87109	0.3867188	-3.738281	-7.199219
-12.67578	12.07422	3.640625	2.363281	-2.152344	5.136719	2.171875	-2.339844
0.1328125	0.2656250	0.0000000					

ROW=11

0.0000000	1.679688	1.335938	13.98438	195.2578	18.53516	0.3671875	-7.160156
-5.609375	2.492188	6.335938	5.398438	3.531250	5.117188	3.109375	-1.652344
0.1601563	0.1328125	0.0000000					

ROW=12

0.0000000	14.11328	-0.1171875	4.878906	30.91406	3.101563	-7.910156	4.113281
7.382813	1.492188	0.5664063	0.2695313	0.8945313	3.121094	-0.6523438	-0.9570313
0.2070313	0.1484375	0.0000000					

ROW=13

0.0000000	191.9297	16.36719	16.23438	182.9102	9.187500	5.578125	5.027344
0.1562500E-01	2.753906	2.183594	1.984375	3.761719	0.9140625	2.031250	-0.6718750
0.1796875	0.1328125	0.0000000					

ROW=14

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

TIME STEP= 1   LENGTH OF TIME STEP=   150.0000

SOLUTION IS IN THE Y-DIRECTION

NUMBER OF ITERATION REQUIRED=22   ERROR= 0.8881968E-01

TIME IN HOUR=   3600.000   TIME IN DAYS=   150.0000   TIME IN YEARS= 0.4109589

FRESHWATER HEAD

ROW= 1

950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000
950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000
950.0000	955.0000	955.0000					

ROW= 2

950.0000	949.7158	949.4656	949.1699	948.9541	948.7751	948.6311	948.4812
948.4094	948.3953	948.4326	948.4998	948.5991	948.7124	948.8247	948.9216
949.4617	953.7852	957.0000					

ROW= 3

950.0000	949.4067	948.8354	948.3228	947.8242	947.4644	947.1738	946.8977
946.7544	946.7251	946.7969	946.9458	947.1599	947.2966	947.3750	947.2368
945.3655	953.6460	960.0000					

ROW= 4

950.0000	949.0488	948.1328	947.3145	946.6250	946.0313	945.5681	945.2065
944.9751	944.9146	944.9224	945.0845	945.4519	945.9988	946.6648	947.4141
949.3350	954.4373	960.0000					

ROW= 5

950.0000	948.6636	947.3193	946.1221	945.1272	944.4563	943.8772	943.3457
942.9707	942.8779	942.9419	943.2993	943.8433	944.5222	945.3535	946.2542
949.9189	954.7910	960.0000					

ROW= 6

950.0000	948.2661	946.4553	944.6797	943.2427	942.6023	941.9658	941.2776
940.6079	940.3330	940.5376	941.3701	942.3669	943.0996	943.6328	942.0503
949.3276	954.9133	960.0000					

ROW= 7

950.0000	947.8235	945.5100	942.9358	940.5613	940.5347	940.1438	939.1294
937.3152	937.6628	937.5791	939.6436	940.8647	941.2795	943.7446	946.1187
950.6418	955.4102	960.0000					

ROW= 8

950.0000	947.4460	944.6777	941.2097	935.5125	938.5859	938.9819	937.2666
932.0803	935.3020	932.7915	938.5544	939.8162	936.8274	943.9570	948.1257
952.0320	956.0977	960.0000					

ROW= 9

950.0000	947.3040	944.5125	941.6265	938.9727	939.2837	939.5400	939.0010
937.8767	939.0398	939.2695	941.4312	942.6992	943.6387	946.8796	950.1724
953.4175	956.7554	960.0000					

ROW=10

950.0000	947.1897	944.4368	941.6780	939.2297	939.9744	940.8665	941.3279
941.6484	942.9990	943.7439	945.0813	946.3040	947.8303	949.9053	952.2075
954.8040	957.4114	960.0000					

ROW=11

950.0000	946.9714	944.3657	941.2915	936.1357	940.6680	942.8674	944.2234
945.3416	946.6094	947.7417	948.8479	949.9590	951.2429	952.6729	954.2651
956.1711	958.0884	960.0000					

ROW=12

950.0000	946.2319	944.7195	942.9216	940.8242	943.5686	945.8477	947.7581
949.1096	950.4392	951.5100	952.4082	953.3091	954.2437	955.1409	956.2307
957.4819	958.7476	960.0000					

ROW=13

950.0000	943.0718	945.6401	944.8306	940.6650	947.2202	949.9634	951.3491
952.4529	954.6311	955.5928	956.1946	956.7144	957.1270	957.6294	958.1284
958.7542	959.3872	960.0000					

ROW=14

950.0000	950.0000	950.0000	950.0000	950.0000	955.0000	955.0000	955.0000
955.0000	960.0000	960.0000	960.0000	960.0000	960.0000	960.0000	960.0000
960.0000	960.0000	960.0000					

SALTWATER HEAD

ROW= 1

935.8538	935.8538	935.9756	935.9756	935.9756	935.9756	935.9756	936.0977
936.0977	936.0977	936.0977	936.0977	936.0977	936.0977	936.0977	936.0977
936.2195	941.2195	941.4634					

ROW= 2

935.7805	935.5759	935.3340	935.1643	934.9556	934.7805	934.6409	934.6155
934.5459	934.5317	934.5686	934.6343	934.7312	934.8425	934.9541	935.1699
935.9382	940.2761	943.4146					

ROW= 3

935.7317	935.2261	934.7170	934.2192	933.8513	933.5017	933.2197	933.0703
932.9312	932.9026	932.9739	933.1206	933.33C6	933.7026	934.0171	933.8884
932.2827	940.1538	946.8293					

ROW= 4

935.6099	934.8516	934.0317	933.2339	932.5630	932.1028	931.7239	931.4209
931.1956	931.1377	931.3860	931.7847	932.1428	932.4377	932.8518	934.0635
936.4207	941.3984	946.8293					

ROW= 5

935.6099	934.4272	933.2371	932.0703	931.0999	930.4473	930.0034	929.6047
929.2395	929.1531	929.4568	930.0461	930.5745	930.9941	931.5696	932.9348
936.9917	941.7473	946.8293					

ROW= 6

935.6099	933.9194	932.2734	930.6626	929.2612	928.6379	928.2571	927.5879
926.9402	927.1467	927.3518	928.1602	928.6536	929.1294	929.8906	929.0[illegible]1
936.4180	941.8669	946.8293					

ROW= 7

935.6099	933.4866	931.2314	928.9590	926.6467	926.6216	926.4802	925.4976
924.2034	924.5486	924.4636	925.9956	926.9448	927.4709	930.2375	932.8040
937.6978	942.3513	946.8293					

ROW= 8

935.6099	933.1179	930.4182	927.1567	921.9326	924.8416	925.3518	924.1565
919.3325	922.2385	919.5205	924.4514	925.9209	923.4556	930.4507	934.7617
939.0540	943.0222	946.8293					

ROW= 9

935.6099	932.9795	930.2561	927.4429	925.0967	925.6418	926.1350	925.8528
924.7524	924.9224	925.1418	927.2532	928.9712	929.8970	933.5408	936.9995
940.4067	943.6643	946.8293					

ROW=10

935.6099	932.8682	930.1824	927.4924	925.3459	926.3137	927.4272	928.1182
928.4211	928.3057	929.2634	930.8130	932.4883	933.9849	936.4946	939.2236
941.7607	944.3042	946.8293					

ROW=11

935.6099	932.6550	930.1130	927.1133	922.2488	926.5105	928.8962	930.4055
931.3030	932.0598	933.0466	934.3691	935.8167	937.3149	939.1960	941.2324
943.0940	944.9646	946.8293					

ROW=12

935.6099	931.9343	930.4541	928.4602	926.4146	929.0908	931.5479	932.8909
934.1401	935.7947	937.0830	938.2029	939.3259	940.4849	941.8435	943.1511
944.3730	945.6077	946.8293					

ROW=13

935.6099	928.9363	930.9900	929.8372	925.9641	932.1636	934.7263	936.1992
937.6362	939.8867	941.0681	941.8992	942.6521	943.5391	944.2747	945.0027
945.6145	946.2317	946.8293					

ROW=14

935.3660	935.1221	934.8782	934.3904	934.1465	939.2683	939.7561	940.0000
940.1221	945.2439	945.4878	945.7317	946.0977	946.5854	946.8293	946.8293
946.8293	946.8293	946.8293					

INTERFACE ELEVATION

ROW= 1

370.0000	370.0000	375.0000	375.0000	375.0000	375.0000	375.0000	380.0000
380.0000	380.0000	380.0000	380.0000	380.0000	380.0000	380.0000	380.0000
385.0000	390.0000	400.0000					

ROW= 2

367.0000	369.9727	370.0625	374.9297	375.0039	374.9883	375.0234	379.9805
379.9961	379.9844	380.0000	380.0078	380.0039	380.0391	380.1211	385.0938
394.9922	399.9063	400.0000					

ROW= 3

365.0000	367.9922	369.9727	370.0703	374.9258	374.9844	375.0508	379.9648
379.9961	379.9961	380.0469	380.1055	380.1484	389.9336	399.6914	399.9414
408.9648	400.4570	420.0000					

ROW= 4

360.0000	366.9531	369.9805	370.0000	370.0742	374.9570	377.9453	379.9883
380.0078	380.0547	389.9219	399.7852	399.7695	389.9883	380.3242	400.0313
419.8398	419.8398	420.0000					

ROW= 5

360.0000	364.9648	369.9375	369.9922	369.9961	370.0742	375.0469	379.9570
379.9844	380.1484	390.0430	399.9102	399.8125	389.8594	380.2031	400.1523
419.8945	419.9883	420.0000					

ROW= 6

360.0000	360.0430	364.9883	369.9727	369.9961	370.0547	379.8984	379.9922
380.2266	399.6875	399.9141	399.7539	380.1094	370.3125	380.1953	408.9492
420.0234	420.0039	420.0000					

ROW= 7

360.0000	360.0000	360.0781	369.8789	370.0547	370.0898	379.9297	380.2148
399.7227	399.9688	399.8359	380.0703	370.1367	375.1172	389.9453	400.2109
419.9258	419.9883	420.0000					

ROW= 8

360.0000	359.9844	360.0273	365.0273	378.7305	375.0586	380.1406	399.7422
409.4102	399.6914	388.6719	360.3203	370.1016	388.5742	390.1875	400.1953
419.9258	419.9961	420.0000					

ROW= 9

360.0000	359.9922	359.9922	360.0898	370.0508	379.9609	389.9258	399.9180
399.7734	360.2148	360.0273	360.1250	379.8398	380.2188	399.9766	410.0781
419.9727	420.0156	420.0000					

ROW=10

360.0000	360.0000	359.9961	360.0625	369.9844	379.8789	389.8516	399.7227
399.3203	340.5625	350.0391	360.0703	379.8516	380.1602	400.0586	419.8633
420.0234	420.0078	420.0000					

ROW=11

360.0000	359.9883	359.9961	359.9727	366.7617	360.2031	370.0430	377.6797
369.7500	350.0703	345.2344	355.2109	370.1133	380.1875	400.1172	419.9141
420.0000	420.0078	420.0000					

ROW=12

360.0000	360.0234	359.8281	349.9961	350.0156	349.9727	359.5469	338.1953
335.3516	350.0039	359.9922	369.9844	379.9922	390.1250	409.9414	419.9609
420.0117	420.0039	420.0000					

ROW=13

360.0000	363.5078	344.9766	330.0938	337.9180	329.8867	325.2344	330.1953
344.9609	350.1016	360.0703	370.0742	380.1523	400.0156	410.0781	419.9648
420.0195	420.0039	420.0000					

ROW=14

350.0000	340.0000	330.0000	310.0000	300.0000	310.0000	330.0000	340.0000
345.0000	355.0000	365.0000	375.0000	390.0000	410.0000	420.0000	420.0000
420.0000	420.0000	420.0000					

INTERFACE UPCONED

ROW= 1

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW= 2

0.0000000	-0.2734375E-01	0.6250000E-01	-0.7031250E-01	0.3906250E-02	-0.1171875E-01	0.2343750E-01	-0.1953125E-01
-0.3906250E-02	-0.1562500E-01	0.0000000	0.7812500E-02	0.3906250E-02	0.3906250E-01	0.1210938	0.9375000E-01
-0.7812500E-02	-0.9375000E-01	0.0000000					

ROW= 3

0.0000000	-0.7812500E-02	-0.2734375E-01	0.7031250E-01	-0.7421875E-01	-0.1562500E-01	0.5078125E-01	-0.3515625E-01
-0.3906250E-02	-0.3906250E-02	0.4687500E-01	0.1054688	0.1484375	-0.6640625E-01	-0.3085938	-0.5859375E-01
8.964844	0.4570313	0.0000000					

ROW= 4

0.0000000	-0.4687500E-01	-0.1953125E-01	0.0000000	0.7421875E-01	-0.4296875E-01	-0.5468750E-01	-0.1171875E-01
0.7812500E-02	0.5468750E-01	-0.7812500E-01	-0.2148438	-0.2304688	-0.1171875E-01	0.3242188	0.3125000E-01
-0.1601563	-0.1601563	0.0000000					

ROW= 5

0.0000000	-0.3515625E-01	-0.6250000E-01	-0.7812500E-02	-0.3906250E-02	0.7421875E-01	0.4687500E-01	-0.4296875E-01
-0.1562500E-01	0.1484375	0.4296875E-01	-0.8984375E-01	-0.1875000	-0.1406250	0.2031250	0.1523438
-0.1054688	-0.1171875E-01	0.0000000					

ROW= 6

0.0000000	0.4296875E-01	-0.1171875E-01	-0.2734375E-01	-0.3906250E-02	0.5468750E-01	-0.1015625	-0.7812500E-02
0.2265625	-0.3125000	-0.8593750E-01	-0.2460938	0.1093750	0.3125000	0.1953125	8.949219
0.2343750E-01	0.3906250E-02	0.0000000					

ROW= 7

0.0000000	0.0000000	0.7812500E-01	-0.1210938	0.5468750E-01	0.8984375E-01	-0.7031250E-01	0.2148438
-0.2773438	-0.3125000E-01	-0.1640625	0.7031250E-01	0.1367188	0.1171875	-0.5468750E-01	0.2109375
-0.7421875E-01	-0.1171875E-01	0.0000000					

ROW= 8

0.0000000	-0.1562500E-01	0.2734375E-01	0.2734375E-01	8.730469	0.5859375E-01	0.1406250	-0.2578125
9.410156	-0.3085938	8.671875	0.3203125	0.1015625	8.574219	0.1875000	0.1953125
-0.7421875E-01	-0.3906250E-02	0.0000000					

ROW= 9

0.0000000	-0.7812500E-02	-0.7812500E-02	0.8984375E-01	0.5078125E-01	-0.3906250E-01	-0.7421875E-01	-0.8203125E-01
-0.2265625	0.2148438	0.2734375E-01	0.1250000	-0.1601563	0.2187500	-0.2343750E-01	0.7812500E-01
-0.2734375E-01	0.1562500E-01	0.0000000					

ROW=10

0.0000000	0.0000000	-0.3906250E-02	0.6250000E-01	-0.1562500E-01	-0.1210938	-0.1484375	-0.2773438
-0.6796875	0.5625000	0.3906250E-01	0.7031250E-01	-0.1484375	0.1601563	0.5859375E-01	-0.1367188
0.2343750E-01	0.7812500E-02	0.0000000					

ROW=11

0.0000000	-0.1171875E-01	-0.3906250E-02	-0.2734375E-01	8.761719	0.2031250	0.4296875E-01	-0.3203125
-0.2500000	0.7031250E-01	0.2343750	0.2109375	0.1132813	0.1875000	0.1171875	-0.8593750E-01
0.0000000	0.7812500E-02	0.0000000					

ROW=12

0.0000000	0.2343750E-01	-0.1718750	-0.3906250E-02	0.1562500E-01	-0.2734375E-01	-0.4531250	0.1953125
0.3515625	0.3906250E-02	-0.7812500E-02	-0.1562500E-01	-0.7812500E-02	0.1250000	-0.5859375E-01	-0.3906250E-01
0.1171875E-01	0.3906250E-02	0.0000000					

ROW=13

0.0000000	8.507813	-0.2343750E-01	0.9375000E-01	7.917969	-0.1132813	0.2343750	0.1953125
-0.3906250E-01	0.1015625	0.7031250E-01	0.7421875E-01	0.1523438	0.1562500E-01	0.7812500E-01	-0.3515625E-01
0.1953125E-01	0.3906250E-02	0.0000000					

ROW=14

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

TIME STEP= 5   LENGTH OF TIME STEP=   2400.000

SOLUTION IS IN THE Y-DIRECTION

NUMBER OF ITERATION REQUIRED= 3   ERROR= 0.9532338E-01

TIME IN HOUR=   111600.0   TIME IN DAYS=   4650.000   TIME IN YEARS=   12.73973

FRESHWATER HEAD

ROW= 1

950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000
950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000	950.0000
950.0000	955.0000	955.0000					

ROW= 2

950.0000	949.6477	949.3240	949.0063	948.7471	948.5396	948.3745	948.2307
948.1523	948.1418	948.1755	948.2551	948.3665	948.4910	948.6028	948.7085
949.1699	953.7026	957.0000					

ROW= 3

950.0000	949.2859	948.6096	947.9985	947.4580	947.0391	946.7061	946.4290
946.2822	946.2429	946.3054	946.4695	946.7014	946.9175	947.0620	946.7666
942.7000	953.3340	960.0000					

ROW= 4

950.0000	948.8901	947.8276	946.8809	946.0869	945.4485	944.9434	944.5491
944.3030	944.2253	944.2759	944.5000	944.9089	945.4604	946.1179	946.9590
948.9082	954.2866	960.0000					

ROW= 5

950.0000	948.4631	946.9578	945.6001	944.4814	943.7212	943.0923	942.5354
942.1328	942.0134	942.1218	942.5493	943.1599	943.8694	944.6816	945.5710
949.5649	954.6123	960.0000					

ROW= 6

950.0000	948.0178	946.0151	944.0837	942.5000	941.7634	941.0955	940.3425
939.6150	939.4380	939.6309	940.5352	941.5115	942.2339	942.7175	939.1631
948.8230	954.7107	960.0000					

ROW= 7

950.0000	947.5669	945.0188	942.2920	939.6223	939.6084	939.1951	938.0815
936.1919	936.6072	936.4648	938.6479	939.9067	940.2864	942.9849	945.3308
950.2390	955.2024	960.0000					

ROW= 8

950.0000	947.1973	944.1807	940.4260	932.4131	937.5042	937.9590	936.1719
928.2258	933.9497	929.3818	937.3477	938.7146	933.5852	943.0828	947.4836
951.6436	955.8923	960.0000					

ROW= 9

950.0000	947.0669	944.0605	940.9568	938.0149	938.4148	938.6384	938.0601
936.8096	937.9426	938.1538	940.4380	941.8198	942.6772	946.1787	949.5967
953.0234	956.5461	960.0000					

ROW=10

950.0000	946.9792	944.0288	941.0732	938.3440	939.2043	940.0840	940.5444
940.9082	941.9885	942.8242	944.2029	945.5256	947.0559	949.2573	951.7261
954.4375	957.2173	960.0000					

ROW=11

950.0000	946.7744	943.9990	940.6833	933.1819	939.8188	942.1636	943.5708
944.6563	945.8296	946.9419	948.0774	949.2542	950.5696	952.1062	953.8513
955.8474	957.9160	960.0000					

ROW=12

950.0000	945.9233	944.4497	942.5125	940.0347	943.0447	945.4092	947.1809
948.4883	949.8623	950.9507	951.8765	952.8010	953.7490	954.7507	955.9116
957.2356	958.6099	960.0000					

ROW=13

950.0000	940.6501	945.2991	944.4675	938.3152	946.8608	949.6245	951.0012
952.1504	954.2991	955.2664	955.8792	956.3997	956.8608	957.3853	957.9509
958.6135	959.3027	960.0000					

ROW=14

950.0000	950.0000	950.0000	950.0000	950.0000	955.0000	955.0000	955.0000
955.0000	960.0000	960.0000	960.0000	960.0000	960.0000	960.0000	960.0000
960.0000	960.0000	960.0000					

SALTWATER HEAD

ROW= 1

935.8538	935.8538	935.9756	935.9756	935.9756	935.9756	935.9756	936.0977
936.0977	936.0977	936.0977	936.0977	936.0977	936.0977	936.0977	936.0977
936.2195	941.2195	941.4634					

ROW= 2

935.7805	935.4961	935.2212	934.9792	934.7468	934.5486	934.4031	934.3535
934.2908	934.2810	934.3171	934.3975	934.5105	934.6528	934.8049	935.0645
936.0278	940.2300	943.4146					

ROW= 3

935.7317	935.1030	934.4932	933.9265	933.4680	933.0811	932.7805	932.5928
932.4648	932.4331	932.5173	932.7009	932.9517	933.2983	933.6262	933.8130
934.3713	940.3308	946.8293					

ROW= 4

935.6099	934.6760	933.7183	932.8130	932.0632	931.5151	931.0903	930.7646
930.5381	930.4949	930.7153	931.1013	931.5022	931.9082	932.4961	933.7620
936.3276	941.2673	946.8293					

ROW= 5

935.6099	934.2170	932.8501	931.5547	930.4756	929.7620	929.2517	928.7986
928.4336	928.3794	928.6714	929.2542	929.8125	930.3337	931.1233	932.7468
936.7034	941.5803	946.8293					

ROW= 6

935.6099	933.7051	931.8442	930.0630	928.5669	927.8572	927.3633	926.6882
926.0786	926.1570	926.4270	927.2393	927.8987	928.5383	929.5781	930.7686
936.2410	941.6965	946.8293					

ROW= 7

935.6099	933.2424	930.8005	928.3396	926.1045	925.8315	925.5540	924.6121
923.3523	923.5896	923.6460	925.1274	926.1772	926.9395	929.6392	932.5420
937.3381	942.1553	946.8293					

ROW= 8

935.6099	932.8794	929.9902	926.7197	923.3596	924.2351	924.4924	923.3127
920.5659	921.5591	920.8289	923.8225	925.3069	925.0239	929.9893	934.3052
938.6589	942.8264	946.8293					

ROW= 9

935.6099	932.7095	929.7700	926.8381	924.5532	924.7769	925.1545	924.8438
923.8574	924.0457	924.3794	926.4099	928.1218	929.3696	932.9236	936.5037
940.0334	943.4797	946.8293					

ROW=10

935.6099	932.6301	929.7415	926.9358	924.8289	925.4937	926.4937	927.1052
927.3340	927.5415	928.4070	929.9824	931.6641	933.3464	935.9199	938.7131
941.4211	944.1313	946.8293					

ROW=11

935.6099	932.4573	929.7410	926.7844	923.8401	926.0557	928.1418	929.5288
930.4363	931.2998	932.3669	933.7090	935.1907	936.7683	938.7153	940.7957
942.7920	944.8071	946.8293					

ROW=12

935.6099	931.9553	930.1536	928.1272	926.3496	928.5981	930.8777	932.3650
933.6516	935.2212	936.5117	937.6611	938.8328	940.0647	941.4436	942.8191
944.1428	945.4814	946.8293					

ROW=13

935.6099	931.0278	931.0415	929.8376	927.9070	932.0078	934.4910	935.9436
937.3105	939.5999	940.7769	941.6199	942.4207	943.2937	944.0798	944.8137
945.4832	946.1543	946.8293					

ROW=14

935.3660	935.1221	934.8782	934.3904	934.1465	939.2683	939.7561	940.0000
940.1221	945.2439	945.4878	945.7317	946.0977	946.5854	946.8293	946.8293
946.8293	946.8293	946.8293					

INTERFACE ELEVATION

ROW= 1

370.0000	370.0000	375.0000	375.0000	375.0000	375.0000	375.0000	380.0000
380.0000	380.0000	380.0000	380.0000	380.0000	380.0000	380.0000	380.0000
385.0000	390.0000	400.0000					

ROW= 2

367.0000	369.4258	371.1016	373.8867	374.7266	374.8984	375.5352	379.2578
379.8164	379.8398	379.9727	380.0820	380.2656	381.1211	382.8828	389.2930
410.3359	401.3125	400.0000					

ROW= 3

365.0000	367.7813	369.8281	371.0352	373.8594	374.7539	375.7539	379.1367
379.7617	380.0313	380.9844	381.9492	382.9531	388.5273	396.1875	415.6641
601.2227	420.1992	420.0000					

ROW= 4

360.0000	366.1016	369.3320	370.0898	371.1094	374.1719	376.9609	379.3789
379.9336	381.2656	388.2852	395.1445	395.2227	389.8125	387.6133	405.8750
433.0977	420.4883	420.0000					

ROW= 5

360.0000	364.3672	368.5352	369.7305	370.2344	371.3828	375.6211	379.3203
380.4570	383.0117	390.6445	397.4375	395.9102	388.9023	388.7813	419.7695
422.2305	420.2930	420.0000					

ROW= 6

360.0000	361.1875	365.0000	369.2227	371.2344	371.5977	378.0664	380.5078
384.6133	394.9102	398.2656	395.3984	383.3789	380.7070	403.9961	594.9844
432.9531	421.1211	420.0000					

ROW= 7

360.0000	360.2539	362.0586	370.2344	385.3828	374.7500	379.8984	385.8242
409.7578	402.8789	410.8828	384.3008	376.9922	393.0547	395.8008	420.9805
421.2969	420.2617	420.0000					

ROW= 8

360.0000	360.1563	362.3633	378.4609	561.2148	393.4688	385.8242	408.9375
614.1641	425.9297	578.7031	382.8086	388.9883	582.5664	406.2422	407.1602
419.2656	420.1836	420.0000					

ROW= 9

360.0000	360.0664	360.7422	364.9180	389.1094	382.7422	389.1641	399.2266
408.2656	370.1289	374.8867	366.1602	380.2813	396.6563	402.0117	411.8750
419.5469	420.2578	420.0000					

ROW=10

360.0000	360.0977	360.6094	364.2188	387.8086	380.6602	386.3555	392.8125
387.1563	352.0039	353.5781	362.4453	377.8008	385.0703	402.1445	417.6563
420.1367	420.2656	420.0000					

ROW=11

360.0000	361.2969	361.6523	372.8594	553.0156	378.7344	370.4258	370.9219
364.4336	352.4414	351.3047	360.3867	373.4961	385.0977	403.1133	418.3398
420.1992	420.1523	420.0000					

ROW=12

360.0000	375.1484	360.3398	354.7031	381.6445	353.3438	352.1523	342.1406
342.3867	351.4688	360.5352	370.2695	380.8828	393.1328	409.3320	419.0391
420.2109	420.1563	420.0000					

ROW=13

360.0000	547.0000	361.9609	345.3516	512.8438	339.4219	330.6094	335.0391
344.9805	352.7344	362.1484	371.9727	383.7539	400.9102	412.0039	419.3359
420.1992	420.1094	420.0000					

ROW=14

350.0000	340.0000	330.0000	310.0000	300.0000	310.0000	330.0000	340.0000
345.0000	355.0000	365.0000	375.0000	390.0000	410.0000	420.0000	420.0000
420.0000	420.0000	420.0000					

INTERFACE UPCONED

ROW= 1

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

ROW= 2

0.0000000	-0.5742188	1.101563	-1.113281	-0.2734375	-0.1015625	0.5351563	-0.7421875
-0.1835938	-0.1601563	-0.2734375E-01	0.8203125E-01	0.2656250	1.121094	2.882813	4.292969
15.33594	1.312500	0.0000000					

ROW= 3

0.0000000	-0.2187500	-0.1718750	1.035156	-1.140625	-0.2460938	0.7539063	-0.8632813
-0.2382813	0.3125000E-01	0.9843750	1.949219	2.953125	-1.472656	-3.812500	15.66406
201.2227	20.19922	0.0000000					

ROW= 4

0.0000000	-0.8984375	-0.6679688	0.8984375E-01	1.109375	-0.8281250	-1.039063	-0.6210938
-0.6640625E-01	1.265625	-1.714844	-4.855469	-4.777344	-0.1875000	7.613281	5.875000
13.09766	0.4882813	0.0000000					

ROW= 5

0.0000000	-0.6328125	-1.464844	-0.2695313	0.2343750	1.382813	0.6210938	-0.6796875
0.4570313	3.011719	0.6445313	-2.562500	-4.089844	-1.097656	8.781250	19.76953
2.230469	0.2929688	0.0000000					

ROW= 6

0.0000000	1.187500	0.0000000	-0.7773438	1.234375	1.597656	-1.933594	0.5078125
4.613281	-5.089844	-1.734375	-4.601563	3.378906	10.70703	23.99609	194.9844
12.95313	1.121094	0.0000000					

ROW= 7

0.0000000	0.2539063	2.058594	0.2343750	15.38281	4.750000	-0.1015625	5.824219
9.757813	2.878906	10.88281	4.300781	6.992188	18.05469	5.800781	20.98047
1.296875	0.2617188	0.0000000					

ROW= 8

0.0000000	0.1562500	2.363281	13.46094	191.2148	18.46875	5.824219	8.937500
214.1641	25.92969	198.7031	22.80859	18.98828	202.5664	16.24219	7.160156
-0.7343750	0.1835938	0.0000000					

ROW= 9

0.0000000	0.6640625E-01	0.7421875	4.917969	19.10938	2.742188	-0.8359375	-0.7734375
8.265625	10.12891	14.88672	6.160156	0.2812500	16.65625	2.011719	1.875000
-0.4531250	0.2578125	0.0000000					

ROW=10

0.0000000	0.9765625E-01	0.6093750	4.218750	17.80859	0.6601563	-3.644531	-7.187500
-12.84375	12.00391	3.578125	2.445313	-2.199219	5.070313	2.144531	-2.343750
0.1367188	0.2656250	0.0000000					

ROW=11

0.0000000	1.296875	1.652344	12.85938	195.0156	18.73438	0.4257813	-7.078125
-5.566406	2.441406	6.304688	5.386719	3.496094	5.097656	3.113281	-1.660156
0.1992188	0.1523438	0.0000000					

ROW=12

0.0000000	15.14844	0.3398438	4.703125	31.64453	3.343750	-7.847656	4.140625
7.386719	1.468750	0.5351563	0.2695313	0.8828125	3.132813	-0.6679688	-0.9609375
0.2109375	0.1562500	0.0000000					

ROW=13

0.0000000	192.0000	16.96094	15.35156	182.8438	9.421875	5.609375	5.039063
-0.1953125E-01	2.734375	2.148438	1.972656	3.753906	0.9101563	2.003906	-0.6640625
0.1992188	0.1093750	0.0000000					

ROW=14

0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000					

APPENDIX F

CONVERGENCE AND STABILITY

TABLE III

CONVERGENCE AND STABILITY OF NUMERICAL MODEL

Time Step	Length of Time Step (Days)	Total Time of Simulation (Days)	No of Iteration Required	
			LSOR Solution in x-Direction	LSOR Solution in y-Direction
1	150	150	21	22
2	300	450	4	4
3	600	1050	3	3
4	1200	2250	3	3
5	2400	4650	3	3

2

VITA

Md Mozzammel Hoque

Candidate for the Degree of

Doctor of Philosophy

Thesis: NUMERICAL SIMULATION OF SALTWATER UPCONING IN INLAND AQUIFERS

Major Field: Civil Engineering

Biographical:

Personal Data: Born in Badartani, Barisal District, East Pakistan, August 1, 1948, the son of Hafez Abdul Latif and Mrs. Saleha Latif.

Education: Graduated from Badartuni Bilateral High School, Barisal, East Pakistan, in March, 1964; earned the Bachelor of Science degree in Agricultural Engineering from the Bangladesh Agricultural University, Mymensingh, Bangladesh, in March, 1972; earned the Master of Science degree in Irrigation Engineering from the University of Southampton, Southampton, England, in April, 1979; completed the requirements for the Doctor of Philosophy degree in Civil Engineering at Oklahoma State University, Stillwater, Oklahoma, in May, 1983.

Professional Experience: Lecturer/Assistant Professor, Department of Irrigation and Water Management, Bangladesh Agricultural University, Mymensingh, August, 1972 to September, 1977; Commonwealth Scholar, October, 1977, to April, 1979; Graduate Research/Teaching Assistant, School of Civil Engineering, Oklahoma State University, September, 1979 to present.

Membership in Professional Societies: American Water Resources Association; American Water Well Association; Oklahoma Society of Professional Engineers; National Society of Professional Engineers.

Membership in Honorary Society: Chi Epsilon; Sigma Xi